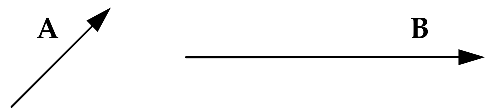
1. The Basics

a) Given vectors **A** and **B** as shown, draw the following:



- **A** + **B**

- **A** – **B**

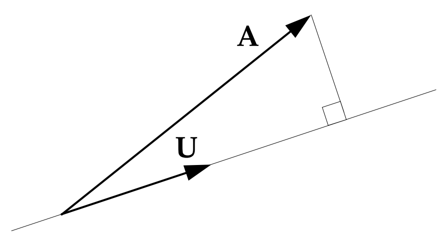
- –½ **A**

b) Write two equations for calculating the dot product **A** · **B**, where **A** = [A_x A_y A_z] and **B** = [B_x B_y B_z].

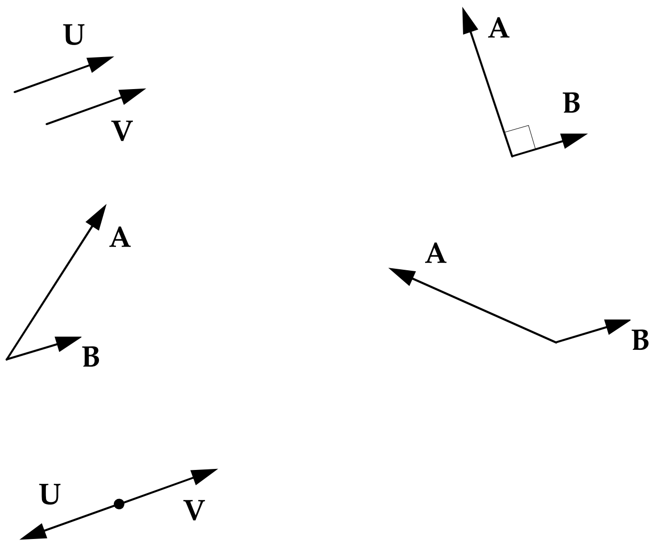
- **A** · **B** =

- **A** · **B** =

c) Draw **A** · **U** on the diagram, given that |**U**| = 1.

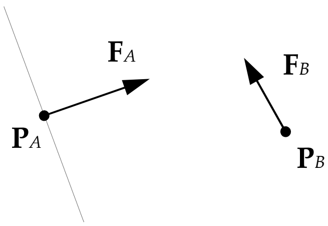


d) For each pair of vectors **A** and **B**, or **U** and **V**, write an inequality indicating the sign of the dot product… or if possible, write the exact value of the dot product. Note that |**U**| = |**V**| = 1, while |**A**| ≠ 1 and |**B**|≠ 1.



2. Can you see me?

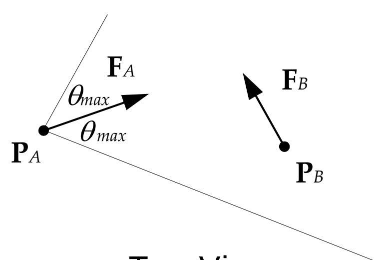
Two characters are standing on a roughly horizontal planar surface. The position of character A is $\mathbf{P}_A$ and its forward-facing unit vector is $\mathbf{F}_A$. Likewise the position and forward vector of character B are $\mathbf{P}_B$ and $\mathbf{F}_B$ respectively.



Top View

a) Use the sign of a dot product to determine whether character B is in front of or behind character A.

b) Assume both characters have a vision cone extending θ_{max} radians to either side of their **F** vectors. Write an expression (using a dot product) indicating whether or not character A can "see" character B.
BONUS: How can we avoid finding the inverse cosine, $\cos^{-1}(\theta_{max})$?



Top View

3. Ray Versus Sphere

In many games, collision geometry is represented using spheres. Ray casts are a common way to query the collision world (e.g. line of sight queries, bullet traces, leg IK ground checks, etc.)

You are given a **sphere** defined by a center point $\mathbf{C}$ and a radius r. You are also given a **ray** defined by the parametric equation $\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{U}$, where $\mathbf{P}_0$ is the start point of the ray, $\mathbf{U}$ is a unit vector lying in the direction of the ray, and t is an arbitrary real-valued parameter in the range $[0, +\infty)$. Determine whether or not the ray intersects the sphere.

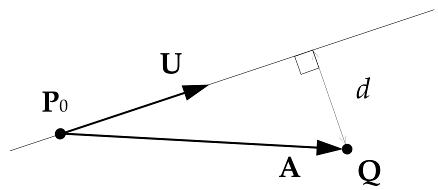
4. Wind Tunnel

The designers want to implement a shaft of wind that will affect any character or object that enters its cylindrical boundary.

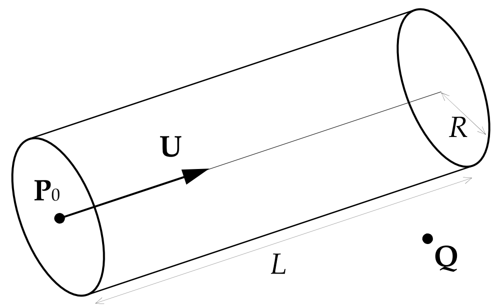
a) You are given an arbitrary point **Q** in 3D space, and an infinite line represented by the locus of points **P**(t) defined as follows:

$$\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{U},$$

where $\mathbf{P}_0$ is a fixed point on the line, and **U** is a unit vector defining the line's direction. Find the perpendicular distance d from **Q** to the line.



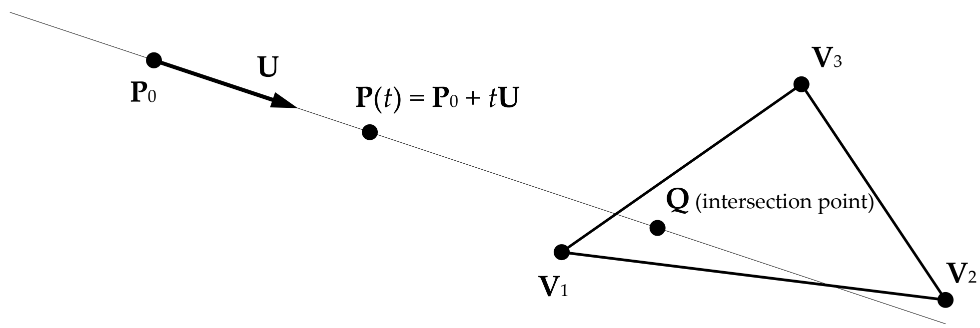
b) The cylindrical wind tunnel can be defined by adding a radius r and length L to the infinite line from part (a). Assuming the position of our object or character is **Q**, write an expression that can be used to determine whether it will be affected by the wind or not.



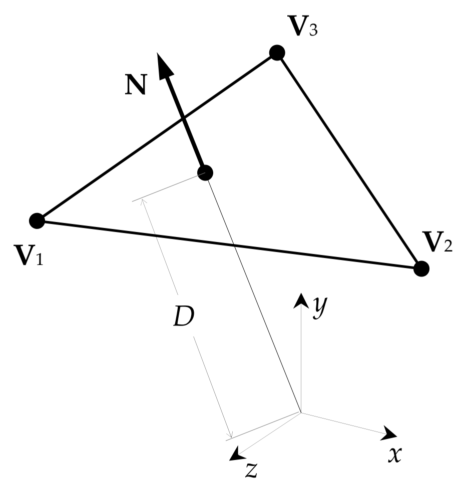
5. Ray Verus Triangle

Ray casts against triangles are also very common in games.

You are given a **triangle** defined by the three vertices $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$, and an **infinite line** defined by the parametric equation $\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{U}$, where $\mathbf{P}_0$ is any fixed point on the line, $\mathbf{U}$ is a unit vector lying in the direction of the line, and t is an arbitrary real-valued parameter. Determine if the **line intersects the triangle**, by following the three steps outlined below.



a) Find the equation of the plane, in the form $(\mathbf{N} \cdot \mathbf{P}) + D = 0$. (*i.e.* find $\mathbf{N}$ and D, given $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$.) Note that this is the same as writing $Ax + By + Cz + D = 0$, where $\mathbf{N} = (A, B, C)$ is the normal of the plane, D is the signed perpendicular distance from the plane to the origin, and $\mathbf{P} = (x, y, z)$ represents any arbitrary point on the plane.



b) Find the point **Q** where the line $\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{U}$ intersects the plane $(\mathbf{N} \cdot \mathbf{P}) + D = 0$. How can you find the point **Q**, given your t?

c) We now know the intersection point **Q**, and that it lies on the plane. Determine whether it lies *inside* or *outside* the triangle. (The intersection only "counts" if **Q** is inside.)