FIRST LESSONS

IN

GEOMETRY:

WITH PRACTICAL APPLICATIONS

IN

MENSURATION,

AND

ARTIFICERS' WORK AND MECHANICS.

BY CHARLES DAVIES,

AUTHOR OF A FULL COURSE OF MATHEMATICS.

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DAVIES' COURSE OF MATHEMATICS.

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DAVIES' SHADOWS AND LINEAR PERSPECTIVE.

DAVIES' DIFFERENTIAL AND INTEGRAL CALCULUS—With numerous applications.

PREFACE.

The design of the present work is to afford an elementary text-book of a practical character, adapted to the wants of a community, where every day new demands arise for the applications of science to the useful arts. There is little to be done, in such an undertaking, except to collect, arrange, and simplify, and to adapt the work, in all its parts, to the precise place which it is intended to fill.

The introduction into our schools, within the last few years, of the subjects of Natural Philosophy, Astronomy, Mineralogy, and Chemistry, has given rise to a higher grade of elementary studies; and the extended applications of the mechanic arts call for additional information among practical men.

To understand the most elementary treatise on Natural Philosophy, or the simplest work on the Mechanic Arts, some knowledge of the principles of Geometry is indispensable; and yet, those in whose hands such works are generally placed, feel that they have hardly time to go through with a full course of exact demonstration.

The system of Geometry is a connected chain of rigorous logic. Every attempt to compress the reasoning, by abridging it at the expense of accuracy, has been uniformly and strongly condemned.

It is the object of the present work to present all the important truths of Geometry in such a way as to render them accessible to the general reader, without departing from the exactness of the geometrical methods. This, it was thought, could be done only by omitting the demonstrations altogether, and relying for the impression of each particular truth on the accuracy of the enunciation and the illustrations of the diagram. In this way, it is believed that all the properties of the geometrical figures may be learned in a few weeks; and after these properties are carried out in their practical applications, the mind receives a conviction of their truth little short of what is afforded by rigorous demonstration.

The work is divided into five parts. Part I. Explains the properties of the geometrical figures. It is, indeed, a complete course of Geometry, (if the term is admissible,) with the difference only that the demonstrations are omitted.

Part II., entitled "Practical Geometry," explains the construction of the Geometrical figures, the construction of scales, and the various uses to which they are applied.

Part III. contains the application of the principles of Geometry to the mensuration of surfaces and solids. A separate rule is given for each case, and the whole is illustrated by numerous and appropriate examples.

Part IV. is the application of the preceding parts to Artificers' Work. It contains full explanations of all the scales and measures used by mechanics—the construction of these scales—the uses to which they are applied —and specific rules for the calculations and computations which are necessary in practical operations.

Part V. explains the nature and properties of matter, the laws of motion and equilibrium, and the principles of all the simple machines.

From the above explanations, it will be seen that the work is entirely practical in its objects and character. Many of the examples have been selected from a small work somewhat similar in its object, recently published in Dublin, by the Commissioners of National Education. Some examples have also been taken from Bonnycastle's Mensuration, and the Library of Useful Knowledge was freely consulted in the preparation of Part V.

The author has indulged the hope that the Arithmetic, First Lessons in Algebra, and First Lessons in Geometry, will form a practical course of mathematical instruction adapted to the wants of Academies and the higher grade of schools.

Hartford, July, 1839.

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GEOMETRY.

PART I.

SECTION I.

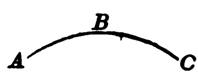
DEFINITIONS AND REMARKS.

1. A *Line* is length, without breadth or thickness.

2. The *Extremities of a Line* are called points; and any place between the extremities, is also called a point.

3. A *Straight Line*, is the shortest distance from one point to another. Thus, *AB* is a straight line, and the shortest distance from *A* to *B*.

4. A *Curve Line*, is one which changes its direction at every point. Thus, *ABC* is a curve line.



The word *Line*, when used by itself means a straight line; and the word *Curve*, means a curve line.

QUEST.—1. What is a line? 2. What are the extremities of a line called? What is any place between the extremities called? 3. What is a straight line? Make it on the black board. 4. What is a curve line? Make one. When the word *line* is used by itself, what does it mean?

5. A *Surface* is that which has length and breadth, without height or thickness.

6. A *Plane Surface* is that which lies even throughout its whole extent, and with which a straight line, laid in any direction, will exactly coincide.

7. A *Curved Surface* has length and breadth without thickness, and like a curve line is constantly changing its direction.

8. A *Solid* or *Body* is that which has length, breadth, and thickness. Length, breadth, and thickness, are called *Dimensions*. Hence, a solid has three dimensions, a surface two, and a line one. A point has no dimensions, but position only.

9. *Geometry* treats of lines, surfaces, and solids.

10. A *Demonstration* is a course of reasoning which establishes a truth.

QUEST.—5. What is a surface? Has a surface thickness? 6. What is a plane surface? If you lay a straight line on a plane surface will it touch it in its whole length? Is the surface of a looking-glass a plane surface? Is the surface of a sheet of paper a plane? 7. What is a curved surface? Is the surface of a ball plane or curved? Is the surface of the earth plane or curved? Will a straight line coincide with a curved surface? 8. What is a solid? What are length, breadth, and thickness called? How many dimensions has a solid? What are they? How many dimensions has a surface? What are they? How many dimensions has a line? What is it? Has a point dimensions? What has it? 9. What does Geometry treat of? 10. What is a demonstration?

11. A *Theorem* is something to be proved by demonstration.

12. A *Problem* is something proposed to be done.

13. A *Proposition* is something proposed either to be done or demonstrated—and may be either a problem or a theorem.

14. A *Corollary* is an obvious consequence, deduced from something that has gone before.

15. An *Hypothesis* is a supposition on which a system of reasoning may be founded.

16. A *Scholium* is a remark on one or more preceding propositions.

17. An *Axiom* is a self evident truth.

AXIOMS.

1. Things which are equal to the same thing are equal to each other.

2. If equals be added to equals the wholes will be equal.

3. If equals be taken from equals the remainders will be equal.

QUEST.—11. What is a theorem? 12. What is a problem? 13. What is a proposition? May it be a problem? May it be a theorem? 14. What is a corollary? 15. What is an hypothesis? 16. What is a scholium? 17. What is an axiom? What is the first axiom? What the second? What is the third axiom?

4. Things which are double of the same thing are equal to each other.

5. Things which are halves of the same thing are equal to each other.

6. A whole is greater than any of its parts.

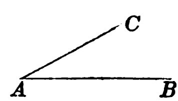
7. A whole is equal to the sum of all its parts.

8. Things which being applied to each other, coincide throughout their whole extent, are equal.

SECTION II.

OF ANGLES.

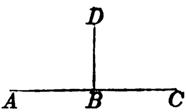
1. An *Angle* is the opening or inclination of two lines which meet each other in a point. Thus the lines *AC*, *AB*, form an angle at the point *A*. The lines *AC*, *AB*, are called the ***sides*** of the angle; and the point *A*, at which they meet, is called the ***vertex*** of the angle. An angle is generally read by placing the letter at



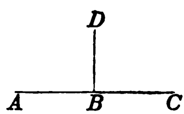
QUEST.—What is the fourth axiom? What the fifth? What the sixth? What the seventh? What the eighth? 1. What is an angle? What are the lines called which form the angle? What is the point of intersection called? How is the angle generally read? How else may it be read?

the vertex in the middle. Thus, we say the angle *CAB*. We may, however, say simply, the angle *A*.

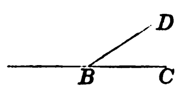
2. One line is perpendicular to another, when it inclines no more to the one side than to the other. The angles on each side are then equal to each other. Thus, if the line *DB* is perpendicular to *AC*, the angle *DBA* is equal to *DBC*.



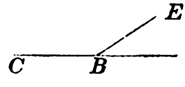
3. When two lines are perpendicular to each other, the angles which they form are called right angles. Thus, *DBA* and *DBC* are right angles. Hence, all right angles are equal to each other.



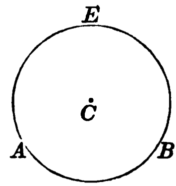
4. An acute angle is less than a right angle. Thus, *DBC* is an acute angle.



5. An obtuse angle is greater than a right angle. Thus, *EBC* is an obtuse angle.

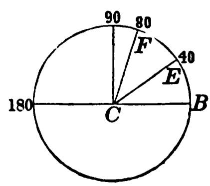


QUEST.—2. When is one line perpendicular to another? Are the angles on each side then equal? 3. When two lines are perpendicular to each other, what are the angles on each side called? Are all right angles equal to each other? Make two right angles. Point out the equal angles. 4. What is an acute angle? Make one. 5. What is an obtuse angle? Make one.



6. The circumference of a circle is a curve line, all the points of which are equally distant from a certain point within, called the centre. Thus, if all the points of the curve *AEB* are equally distant from the centre *C*, this curve will be the circumference of a circle.

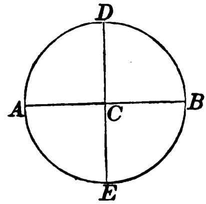
7. The circumference of a circle is used for the measurement of angles. For this purpose it is divided into 360 equal parts, called degrees, each degree is divided into 60 equal parts called minutes, and each minute into 60 equal parts called seconds. The degrees, minutes, and seconds, are marked thus, $^\circ$, $'$, $''$; and $9^\circ\ 18'\ 10''$, are read, 9 degrees, 18 minutes, and 10 seconds.



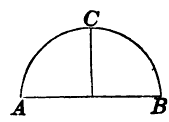
8. Suppose the circumference of a circle to be divided into 360 equal parts, beginning at the point *B*. If, through the point of division marked 40, we draw *CE*, then, the angle *ECB* will be equal to 40 degrees. If

QUEST.—6. What is the circumference of a circle? Make one, and point out the centre. 7. For what is the circumference of a circle used? Into how many parts is it supposed to be divided? What is each part called? Into how many parts is each degree divided? Each minute into how many parts? Show how the degrees, minutes, and seconds are marked. 8. Explain how an angle is measured on the circumference of a circle. Draw a line which shall make with another line an angle of 30°.—Then a line which shall make an angle of 60°.

we draw CF through the point of division marked 80, it will make with CB an angle equal to 80 degrees.



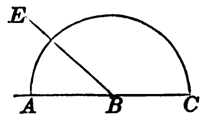
9. If two lines AB, DE, are perpendicular to each other, the four angles BCD, DCA, ACE, and ECB will be equal. These two lines will divide the circumference of the circle into the four equal parts BD, DA, AE, and EB, and each part will measure one of the right angles. But one quarter of 360 degrees, is 90 degrees. Hence, one right angle contains 90 degrees, two right angles 180 degrees, three right angles 270 degrees, and four right angles 360 degrees.



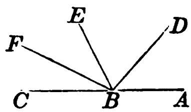
10. One quarter of the circumference is called a *quadrant*, and contains 90 degrees. One half of the circumference is called a *semi-circumference*, and contains 180 degrees. Thus, AC is a quadrant, and ACB is a semi-circumference.

QUEST.—9. If two lines are perpendicular to each other, how many right angles will be formed? Into how many equal parts will these lines divide the circumference? How many degrees does one right angle contain? How many degrees in two right angles? In three right angles? In four right angles? 10. What is one quarter of the circumference called? How many degrees does it contain? What is half the circumference called? How many degrees does it contain?

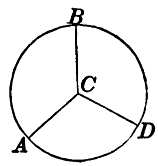
11. If a straight line *EB* meets another straight line *AC*, the sum of the angles *ABE* and *EBC*, will be equal to two right angles, since these two angles are measured by half the circumference.



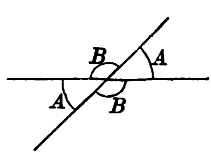
And if there be several angles *CBF*, *FBE*, *EBD*, *DBA*, formed on the same side of a line, their sum for a like reason, will be equal to two right angles.



12. The sum of all the angles *ACB*, *BCD*, *DCA*, which can be formed about any point as *C*, is equal to four right angles, or 360 degrees, since they are measured by the entire circumference.



13. If two lines intersect each other, the opposite angles *A* and *A* are called *vertical angles*. These angles are equal to each other, and so also, are the opposite angles *B* and *B*.

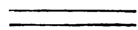


QUEST.—11. If one straight line meets another, what is the sum of the two angles on the same side equal to? What is the sum of several angles formed on the same side of a straight line equal to? 12. What is the sum of all the angles which can be formed about the same point equal to? 13. If two straight lines intersect each other, what are the opposite angles called? Are these angles equal or unequal?

SECTION III.

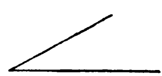
OF PARALLEL, OBLIQUE, AND PERPENDICULAR LINES.

1. Two straight lines are said to be parallel when they are at the same distance from each other at every point. Parallel lines will never meet each other.



2. Two curves are said to be parallel or *concentric*, when they are at the same distance from each other. Parallel curves will not meet each other.

3. Oblique lines are those which approach each other, and meet if sufficiently prolonged.

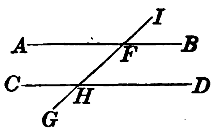


4. Lines which are parallel to the horizon, or to the water level, are called horizontal lines. Thus, the eaves of a house are horizontal.

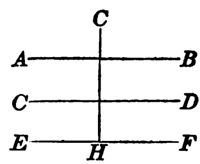
QUEST.—1. When are two straight lines said to be parallel? Will parallel lines meet each other? 2. When are two curves said to be parallel? Will parallel curves meet each other? 3. What are oblique lines? Make two oblique lines. 4. What are horizontal lines? Are the eaves of a house horizontal?

5. Lines which are perpendicular to the horizon, or to the water level, are called vertical lines. Thus, trees and plants grow vertically, or in lines perpendicular to the horizon.

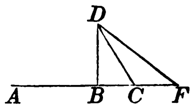
6. If two parallel lines *CD*, *AB*, are cut by a third line *IG*, the angles *IHD* and *AFG*, are called *alternate* angles. These angles are equal to each other. The angle *IHD* is also equal to the angle *IFB*, and to the opposite angle *CHG*.



7. If a line be perpendicular to one of several parallel lines, it will be perpendicular to all the others. Thus, if *AB*, *CD* and *EF*, be parallel, the line *CH* drawn perpendicular to *AB*, will also be perpendicular to *CD* and *EF*.



QUEST.—5. What are vertical lines? Are the corners of a house vertical? Point out the lines of a window which are horizontal. Point out those which are vertical. Point out the lines of a room which are horizontal: also those which are vertical. 6. If two parallel lines are cut by a third line, point out the alternate angles. Are these angles equal or unequal? Name all the angles which are equal to each other. 7. If several lines are parallel, and a line be drawn perpendicular to one of them, will it be perpendicular to all the others?



8. From the same point *D*, only one line *DB*, can be drawn, which shall be perpendicular to *AB*.

If oblique lines be drawn, as *DC*, *DF*, then :—

1st.—The perpendicular *DB*, will be shorter than any of the oblique lines.

2nd.—The oblique lines which are nearest the perpendicular, will be less than those which are more remote.

SECTION IV.

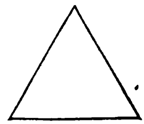
OF PLANE FIGURES.

1. A plane figure is a portion of a plane, terminated on all sides by lines, either straight or curved.

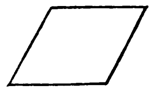
If the lines are straight, the space they enclose is called a *rectilineal* figure, or polygon. The lines themselves, taken together, are called the *perimeter* of the polygon. Hence, the perimeter of a polygon is the sum of all its sides.

Quest.—8. From the same point, how many perpendiculars can be drawn to a line? If oblique lines be drawn, which will be the least? Which the greatest? 1. What is a plane figure? If the lines are straight, what is the plane figure called? What is the sum of the lines called?

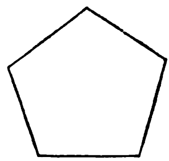
2. A polygon of three sides, is called a triangle.



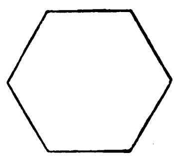
3. A polygon of four sides, is called a quadrilateral.



4. A polygon of five sides, is called a pentagon.



5 A polygon of six sides, is called a hexagon.



6. A polygon of seven sides, is called a heptagon.

7. A polygon of eight sides, is called an octagon.

8. A polygon of nine sides, is called a nonagon.

QUEST.—2. What is a polygon of three sides called? 3. What is a polygon of four sides called? 4. What is a polygon of five sides called? 5. What is a polygon of six sides called? 6. What is a polygon of seven sides called? 7. A polygon of eight sides? 8. A polygon of nine sides?

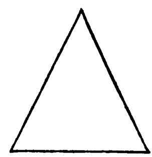
9. A polygon of ten sides, is called a decagon.

10. A polygon of twelve sides is called a dodecagon.

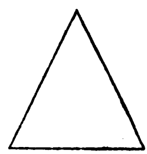
11. Three straight lines, are the smallest number which can enclose a space.

12. There are several kinds of triangles.

First.—An equilateral triangle, which has its three sides all equal.

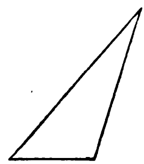


Second.—An isosceles triangle, which has two of its sides equal.

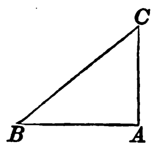


QUEST.—9. What is a polygon of ten sides called? 10 Of twelve sides? Make each of the polygons on the board. 11. What is the smallest number of straight lines which can enclose a space? 12. When is a triangle equilateral? When is a triangle isosceles?

Third.—A scalene triangle, which has its three sides all unequal.



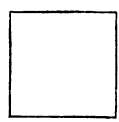
Fourth.—A right angled triangle, which has one right angle. In the right angled triangle *BAC*, the side *BC* opposite the right angle, is called the hypothenuse.



13. The *base* of a triangle is the side on which it stands. Thus, *BA* is the base of the right angled triangle *BAC*. The line drawn from the opposite angle perpendicular to the base, is called the altitude. Thus, *AC* is the altitude.

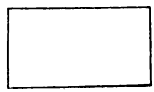
14. There are several kinds of quadrilaterals.

First.—The *square*, which has all its sides equal, and all its angles right angles.

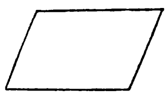


QUEST.—What is a scalene triangle? What is a right angled triangle? What is the side opposite the right angle called? 13. What is the base of a triangle? What is the altitude of a triangle? Make on the board the different kinds of triangles. 14. What is a square?

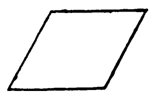
Second.—The *rectangle*, which has its angles right angles, and its opposite sides equal and parallel.



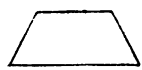
Third.—The *parallelogram*, which has its opposite sides equal and parallel, but its angles not right angles.



Fourth.—The *rhombus*, which has all its sides equal, and the opposite sides parallel, without having its angles right angles.

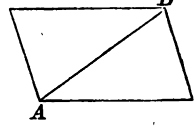


Fifth.—The *trapezoid*, which has only two of its sides parallel.



15. The base of a figure is the side on which it stands, and the altitude is a line drawn from the top, perpendicular to the base.

16. A diagonal, is a line joining the vertices of two angles not adjacent. Thus, *AB* is a diagonal.



QUEST.—What is a rectangle? What is a parallelogram? What is a rhombus? What is a trapezoid? 15. What is the base of a figure? 16. What is a diagonal?

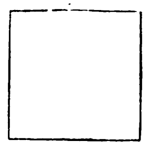
SECTION V.

OF THE UNIT OF LENGTH, AND THE MEASURE OF SURFACES.

1. If the length of a line be computed in feet, one foot is the unit of the line, and is called the *linear unit.*

If the length of a line be computed in yards, one yard is the linear unit. If it be computed in rods, one rod is the linear unit; and if it be computed in chains, one chain is the linear unit.

2. If we describe a square on the unit of length, such square is called the unit of surface. Thus, if the linear unit be 1 foot, one square foot will be the unit of surface.

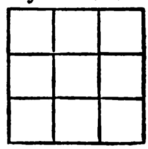


QUEST.—1. If the length of a line be computed in feet, what is the unit of the line? What is this unit called? If the length of a line be computed in yards, what is the linear unit? If it be computed in rods, what is the linear unit? 2. If a square be described on the unit of length, what is it called? If the unit of length be one foot, what is the unit of surface?

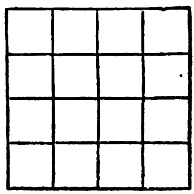
Of Surfaces.

3. If the linear unit is 1 yard, one square yard will be the unit of surface; and this square yard contains 9 square feet.

1 yard = 3 feet.



1 chain = 4 rods.



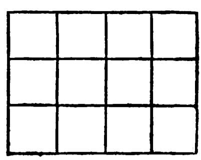
4. If the linear unit is 1 chain, the unit of surface will be 1 square chain, which will contain 16 square rods.

5. Lands are generally estimated in acres, roods, and perches or square rods.

1 Acre = 4 roods = 160 Perches or square rods.

1 Rood = 40 perches = $\frac{1}{4}$ of an Acre.

6. If we have a rectangle whose base is 4 feet, and altitude 3 feet, it is evident that it will contain 12 square feet. These 12 square feet are the *measure of the surface* of the rectangle.



QUEST.—3. If the unit of length be one yard, what is the unit of surface? 4. If the linear unit be one chain, what is the unit of surface? 5. In what are lands generally estimated? How many roods in an acre? How many perches in an acre? 6. What is the content of a rectangle whose base is 4 feet, and altitude 3 feet? What are these twelve square feet called?

It is plain that the number of squares in any rectangle, will be expressed by the units of its base, multiplied by the units in its altitude. This product is called the measure of the rectangle.

In geometry, we often say, the rectangle of two lines, by which we mean, the rectangle of which those lines are the two adjacent sides.

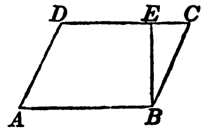
7. The *Area* of a figure, is the measure of its surface. It should be remembered that, the unit of the number which expresses the area, is a square of which the linear unit is the side.

8. The area of a rectangle is equal to the product of its base by its altitude. If the base of a rectangle is 30 yards, and the altitude 5 yards, the area will be 150 square yards.

9. The area of a square is equal to the product of its two equal sides ; that is, to the square of one of its sides.

QUEST.—What is the number of squares in any rectangle equal to? When you speak of the rectangle of two lines, what do you mean? 7. What is the area of a figure? What is the unit of the number which expresses the area? 8. What is the area of a rectangle equal to? If the base of a rectangle is 10, and altitude 4 feet, what is its area? If the base be 12 yards, and altitude 5 yards, what is its area? If the base be 9 rods, and altitude 8 rods, what is its area? 9. What is the area of a square equal to? If the side of a square is 3 feet, what is its area? If the side be 9 yards, what is its area? If the side be 8 rods, what is its area?

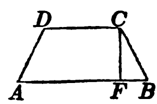
10. The altitude of a parallelogram is the perpendicular distance between two of its parallel sides. Thus, EB is the altitude of the parallelogram $ABCD$.



11. The area of a parallelogram is equal to its base multiplied by its altitude. Thus, the area of the parallelogram $ABCD$, is equal to $AB \times BE$.

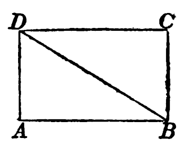
If the base is 20, and altitude 15 feet, the area will be 300 square feet.

12. The area of a trapezoid is equal to half the sum of its parallel sides multiplied by the perpendicular distance between them. Thus,

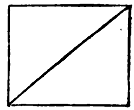


$$\text{area } ABCD = \tfrac{1}{2}(AB+CD) \times CF.$$

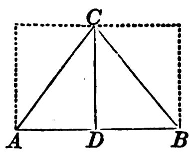
13. The diagonal DB divides the rectangle $ABCD$ into two equal triangles. Hence, a triangle is half a rectangle, having the same base and altitude.



QUEST.—10. What is the altitude of a parallelogram? 11. What is the area of a parallelogram equal to? If the base of a parallelogram be 10 feet, and the altitude 5 feet, what is the area? 12. To what is the area of a trapezoid equal? If the two parallel sides of a trapezoid are 8 and 10, and the altitude 7, what is the area? 13. How does the diagonal of a rectangle divide it? If a rectangle and a triangle have the same base and altitude, how do they compare with each other?



14. A triangle is also half a parallelogram, having the same base and altitude.



15. The area of a triangle is equal to half the product of the base by the altitude; for, the base multiplied by the altitude gives a rectangle, which is double the triangle. Thus, the area of the triangle *ABC*, is equal to half the product of $AB \times CD$.

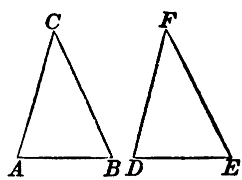
If the base of a triangle is 12, and the altitude 8 yards, the area will be 48 square yards.

QUEST.—14. If a triangle and a parallelogram have the same base and altitude, how do they compare with each other? 15. What is the area of a triangle equal to? If the base of a triangle is 20 feet, and its altitude 5 feet, what is its area? If the base is 40 yards, and the altitude 6 yards, what is the area? If the base is 16 rods, and altitude 10 rods, what is the area?

SECTION VI.

PROPERTIES OF THE TRIANGLE.

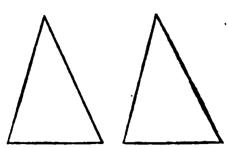
1. If two triangles have two sides, and an included angle of the one, equal to two sides and the included angle of the other, each to each, the remaining parts will also be equal.



That is, if we have the two triangles, *ABC* and *DEF*, having

$AC=DF$, $CB=FE$, and angle $C=F$, then will
Angle $A=D$, angle $B=E$, and $AB=DE$.

2. If two triangles have two angles, and the included side of the one equal to two angles and the included side of the other, the remaining parts will also be equal.

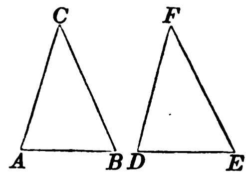


QUEST.—1. Name the parts of one triangle, which being equal to the corresponding parts of another, will cause the remaining parts of the triangles also to be equal. 2. If two triangles have a side, and the adjacent angles in each equal, will the remaining parts also be equal?

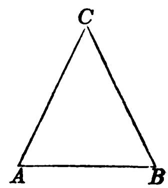
That is, if we have two triangles ABC and DEF, having

Angle $A=D$, angle $B=E$ and $AB=DE$, then will

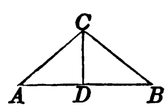
$AC=DF$, $CB=FE$ and angle $C=F$.



3. The angles opposite the equal sides of an isosceles triangle are equal. Thus, if ABC be an isosceles triangle, the angle $A=B$.



4. A line drawn from the vertical angle, perpendicular to the base of an isosceles triangle, will divide the base into two equal parts. Thus, if CD is perpendicular to AB, then $AD=DB$.

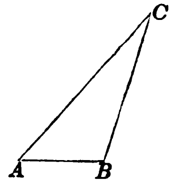


The perpendicular CD will also divide the vertical angle into two equal parts.

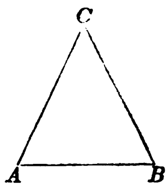
QUEST.—3. What is an isosceles triangle (See § IV, Art. 12.)? Are the angles opposite the equal sides equal? 4. If a line be drawn from the vertical angle of an isosceles triangle, perpendicular to the base, how will it divide the base? How will it divide the vertical angle?

Properties of the Triangle.

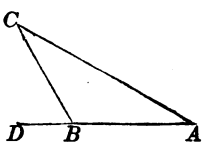
5. The greater side of every triangle is opposite the greater angle, and the greater angle opposite the greater side. Thus, if B is the greater angle, AC will be the greater side.



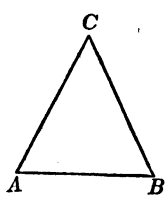
6. In the equilateral triangle, the three angles are likewise equal. Thus, if $AC=AB=BC$, then angle $A=C=B$.



7. If one side of a triangle, as AB, be produced out, the outward angle CBD, is equal to the sum of the inward angles A and C.



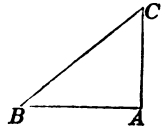
8. The sum of the three angles of every triangle is equal to two right angles or 180 degrees. That is, $A+B+C=180$ degrees.



QUEST.—5. If you know the greater side of a triangle, do you know the greater angle? Why? If you know the greater angle, do you know the greater side? Why? 6. Are the angles of an equilateral triangle equal to each other? 7. If one side of a triangle be produced out, what will the outward angle be equal to? 8. What is the sum of the three angles of any triangle equal to?

Properties of the Triangle.

9. In every right angled triangle, the sum of the two acute angles is equal to 90 degrees. Thus,



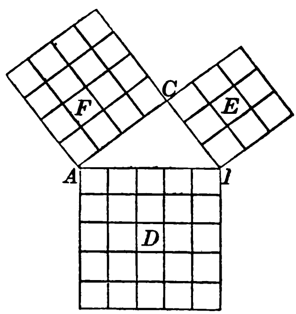
$$B+C=90 \text{ degrees};$$

This is evident, since

$$A+B+C=180 \text{ degrees},$$

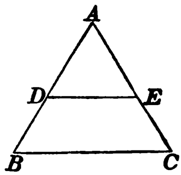
and $A=90$ degrees.

10. In every right angled triangle, the square described on the hypothenuse, is equal to the sum of the squares described on the other two sides.



Thus, if ABC be a right angled triangle, right angled at C, then will the square D described on AB be equal to the sum of the squares E, and F described on the sides CB and AC. This is called the carpenter's theorem.

QUEST.—9. In a right angled triangle, what is the sum of the two acute angles equal to? 10. In a right angled triangle, what is the square on the hypothenuse equal to? What theorem is this called?

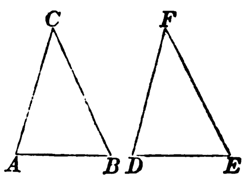


11. If a line be drawn parallel to the base of a triangle, it will cut the two other sides proportionally. Thus, if DE be drawn parallel to the base BC, we shall have

$$AB : AC : : AD : AE;$$

that is, the parts AD and AE will have to each other the same ratio as the sides AB and AC.

12. Similar triangles are those which have all the angles of the one, equal to the corresponding angles of the other, each to each. Thus, if the two triangles ABC and DEF have the angle $A=D$, $B=E$, and $F=C$, they will be similar.

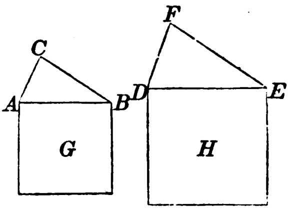


The sides which lie opposite equal angles are called *homologous* sides. Thus, AB and DE are homologous sides; also, AC and DF, and likewise CB and FE.

The homologous sides of similar triangles are proportional Thus,

$$AB : AC : : DE : DF.$$

QUEST.—11. If a line be drawn parallel to the base of a triangle, how will it divide the two other sides? 12. What are similar triangles? What are the sides called which lie opposite equal angles? Are these sides proportional?



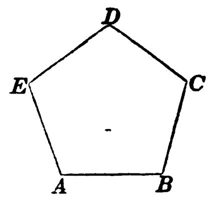
13. The areas of similar triangles are to each other as the squares described on their homologous sides.

The similar triangles *ABC*, and *DEF*, are to each other, as the squares *G* and *H*, described on the homologous sides *AB* and *DE*.

Thus, *ABC* : *DEF* : : square *G* : to square *H*.

SECTION VII.

PROPERTIES OF POLYGONS.



1. A regular polygon is one which has all its sides equal to each other, each to each, and all its angles equal to each other, each to each.

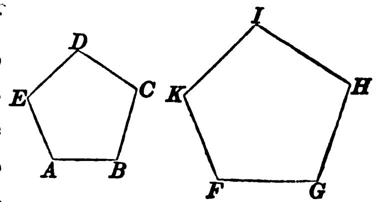
Thus, if the polygon *ABCDE* be regular, we have

$AB = BC = CD = DE = EA$: also

angle $A = B = C = D = E$.

QUEST.—13. How are the areas of similar triangles to each other, 1. What is a regular polygon ?

2. Similar polygons are those which have the angles of the one equal to the angles of the other, each to each, and the sides about the equal angles proportional. Hence, similar polygons are alike in shape, but may differ in size.



The sides which are like situated in two similar polygons, are called ***homologous sides***, and these sides are proportional to each other.

Thus, if $ABCDE$, and $FGHIK$ are two similar polygons: then

Angle $A=F$, $B=G$, $C=H$, $D=I$, and $E=K$.

Also $AB : FG :: BC : GH$

and $AB : FG :: CD : IH$

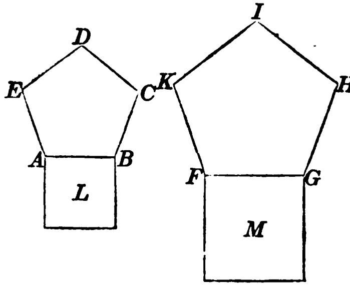
also $AB : FG :: DE : IK$

and $AB : FG :: EA : KF$.

3. Similar polygons are to each other as the squares

QUEST.—2. What are similar polygons? Are similar polygons of like shape? May they vary in size? What are the sides called which are like situated? Are the homologous sides proportional? Make two similar hexagons—mark the equal angles and the homologous sides. Also, write down the proportional sides. 3. How are similar polygons to each other? If 3 and 4 represent the homologous sides of two similar polygons, what proportion will those polygons bear to each other?

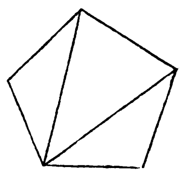
described on their homologous sides.



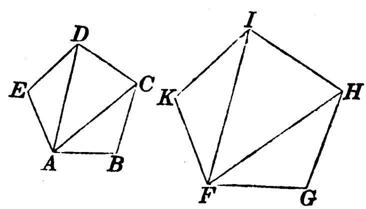
Thus, the two similar polygons *ABCDE*, *FGHIK*, are to each other as the squares described on the homologous sides *AB* and *FG*: that is

ABCDE : *FGHIK* : : square *L* : square *M*.

4. Any polygon may be divided by diagonals, into as many triangles less two, as the polygon has sides. Thus, if the polygon has five sides, there will be three triangles; if it has six sides, there will be four; if seven sides, five; if eight sides, six; &c.



5. Two similar polygons may be divided by diagonals, into the same number of similar triangles, which will be like placed.

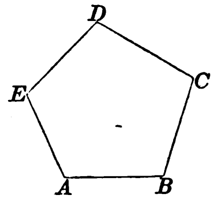


Thus, if we have two similar polygons,

QUEST.—4. Into how many triangles may every polygon be divided by diagonals? 5. If two similar polygons be divided by diagonals alike drawn in each, will the triangles thus formed be similar? Will they be like placed?

ABCDE, and *FGHIK*, the first will give the triangles *ABC*, *ACD*, and *ADE*; and the second, the similar triangles *FHG*, *FHI*, and *FIK*.

6. The sum of all the inward angles of any polygon is equal to twice as many right angles, wanting four, as the figure has sides. Thus, if the polygon has five sides, we have



$A+B+C+D+E=10$ right angles -4 right angles $=6$ right angles.

7. If the polygon is a quadrilateral, then the sum of the angles will be equal to four right angles.

8. When the polygon is regular, its angles will be equal to each other (Art. 1). If, then, the sum of the inward angles be divided by the number of angles, the quotient will be the value of one of the angles. We shall find the value in degrees, by simply placing 90° for the right angle.

Thus, for the sum of all the angles of an equilateral triangle, we have (Art. 6).

QUEST.—6. What is the sum of all the inward angles of any polygon equal to? 7. What is the sum of the inward angles of a quadrilateral equal to? 8. If a polygon is regular, are its angles equal or unequal? When the polygon is regular, if the sum of the angles be divided by the number, what will the quotient be? What is the value of either of the angles of an equilateral triangle?

$$6 \times 90^\circ - 4 \times 90^\circ = 540^\circ - 360^\circ = 180^\circ$$

and for each angle

$$180^\circ \div 3 = 60^\circ :$$

Hence, each angle of an equilateral triangle, is equal to 60 degrees.

9. For the sum of all the angles of a square, we have

$$8 \times 90^\circ - 4 \times 90^\circ = 720^\circ - 360^\circ = 360^\circ$$

and for each of the angles

$$360^\circ \div 4 = 90^\circ.$$

10. For the sum of all the angles of a regular pentagon, we have

$$10 \times 90^\circ - 4 \times 90^\circ = 900^\circ - 360^\circ = 540^\circ :$$

and for each angle

$$540^\circ \div 5 = 108^\circ.$$

11. For the sum of all the angles of a regular hexagon, we have

$$12 \times 90^\circ - 4 \times 90^\circ = 1080^\circ - 360^\circ = 720^\circ$$

and for each angle

$$720^\circ \div 6 = 120^\circ.$$

12. For the sum of the angles of a regular heptagon, we have

$$14 \times 90^\circ - 4 \times 90^\circ = 1260^\circ - 360^\circ = 900^\circ :$$

QUEST.—9. How do you find either of the angles of a square? What is either angle equal to? 10. How do you find the angle of a regular pentagon? What is it equal to? 11. How do you find the angle of a regular hexagon? What is it equal to? 12. What is the value of an angle of a regular heptagon?

and for one of the angles

$$900° \div 7 = 128° \ 34' +.$$

13. For the sum of the angles of a regular octagon, we have

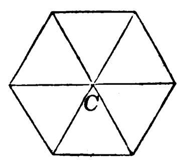
$$16 \times 90° - 4 \times 90° = 1440° - 360° = 1080° :$$

and for each angle

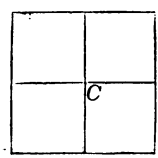
$$1080° \div 8 = 135°.$$

14. It may be well to remark that there are only three kinds of regular figures, which can be arranged around any point, as *C*, so as exactly to fill up all the space. These are,

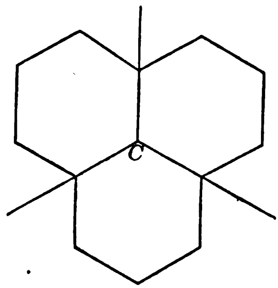
First.—Six equilateral triangles, in which each angle about *C* is equal to 60°, and their sum to $60° \times 6 = 360°$.



Second.—Four squares, in which each angle is equal to 90°, and their sum to $90° \times 4 = 360°$.

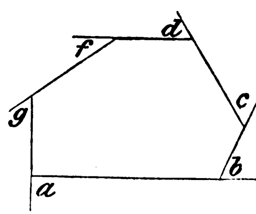


QUEST.—13. What is the value of an angle of a regular octagon? 14. How many figures are there which will exactly fill the angular space about a point? How many equilateral triangles will do it? Why? How many squares? Why? How many hexagons? Why?



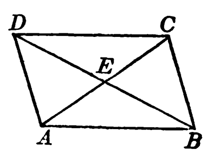
Third.—Three hexagons, in which each angle is equal to 120°, and the sum of the three to

$$120° \times 3 = 360°.$$



15. If all the sides of any polygon be produced out, the sum of al. the outward angles will be equal to four right angles. Thus,

$$a+b+c+d+f+g=4\times 90°=360°.$$

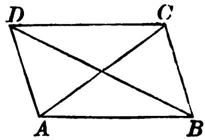


16. The diagonals of a rectangle, and also those of a parallelogram bisect each other. Thus we have

$$AE=EC, \text{ and } BE=ED.$$

QUEST.—15. If all the sides of a polygon be produced out, what will the sum of the outward angles be equal to? 16. How do the diagonals of a parallelogram divide each other?

17. The sum of the squares of the diagonals of a rectangle, and also of a parallelogram, is equal to the sum of the squares of the four sides. Thus

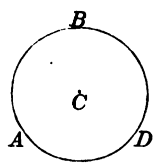


$$\overline{AC}^2+\overline{BD}^2=\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{DA}^2.$$

SECTION VIII.

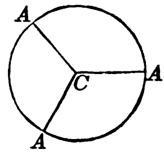
OF THE CIRCLE.

1. A circle is a plane figure, bounded by a curve line, all the points of which are equally distant from a certain point called the centre. The curve line is called the circumference. Thus, the space enclosed by the curve *ABD* is called a circle: the curve *ABD* is the circumference, and the point *C* the centre.



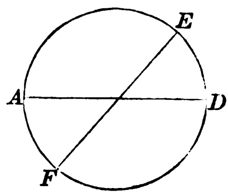
QUEST.—17. What is the sum of the squares of the diagonals of a parallelogram equal to? 1. What is a circle? What is the curve line called? Make the circumference of a circle on the black board. Point out the circle. Point out the centre. Also the circumference.

2. Any line, as *CA*, drawn from the centre *C* to the circumference, is called a *radius*, and two or more such lines, are called radii.

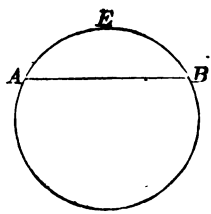


All the radii of a circle are equal to each other.

3. The *diameter* of a circle is any line, as *AD* or *EF*, passing through the centre and terminating in the circumference. Every diameter of a circle divides it into two equal parts, called semi-circles, or half circles.



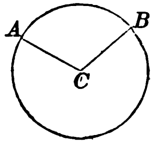
4. An *arc* of a circle, is any part of the circumference. Thus, *AEB* is an arc.



QUEST.—2. What is a radius? Make one. What are radii? Are all radii equal? 3. What is a diameter of a circle? How does every diameter divide the circle? What are these parts called? 4. What is an arc of a circle?

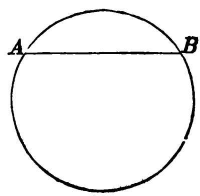
Of the circle.

5. A *sector* of a circle, is any part of a circle bounded by two radii and the arc included between them. Thus, *ACB* is a sector.



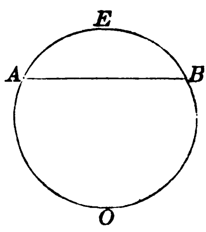
6. A *chord* of a circle, is a line drawn within a circle, and terminating in the circumference, but not passing through the centre. Thus, *AB* is a chord.

A chord divides the circle into two unequal parts.



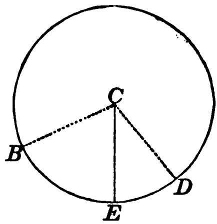
7. A *segment* of a circle, is a part cut off by a chord. Thus, *AEB* is a segment.

The part *AOB*, is also a segment, although the term is generally applied to the part which is less than a semi-circle.

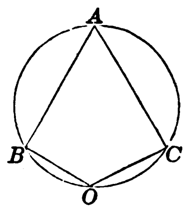


QUEST.—5. What is a sector of a circle? 6. What is a chord of a circle? Does a chord divide the circle into equal or unequal parts? 7. What is the segment of a circle? Are the two parts into which a chord divides a circle, equal or unequal? To which is the term segment applied?

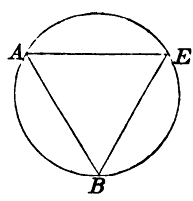
8. *An angle at the centre*, is one whose vertex is at the centre of the circle. Thus, *BCE*, or *ECD*, is an angle at the centre.



9. *An angle at the circumference*, is one whose angular point is in the circumference. Thus, *BAC* or *BOC* is an angle at the circumference.

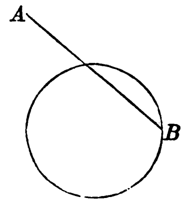


10. *An angle in a segment*, is formed by two lines drawn from any point of the segment to the two extremities of the arc. Thus, *ABE* is an angle in a segment.

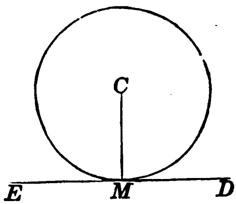


QUEST.—8. What is an angle at the centre? 9. What is an angle at the circumference? 10. What is an angle in a segment? Make a figure and point out an angle at the circumference. Also an angle at the centre. Also an angle in a segment.

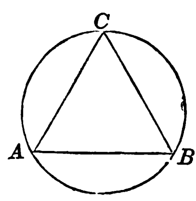
11. A *secant* line, is one which meets the circumference in two points, and lies partly within and partly without. Thus, *AB* is a secant line.



12. A *tangent* is a line which has but one point in common with the circumference. Thus, *EMD* is a tangent. The point *M* at which the tangent touches the circumference is called the *point of contact.* The tangent line is perpendicular to the radius passing through the point of contact. Thus, *CM* is perpendicular to *EMD.*

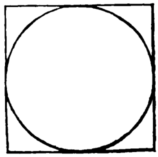


13. A figure is said to be inscribed in a circle when all the angular points of the figure are in the circumference. The circle is then said to circumscribe the figure. Thus, the triangle *ABC* is *inscribed* in the circle, and the circle *circumscribes* the triangle.

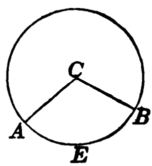


QUEST.—11. What is a secant line? 12. What is a tangent? What is the common point called? What position has the tangent line with the radius passing through the point of contact? 13. When is a figure said to be inscribed in a circle? What is then said with respect to the circle?

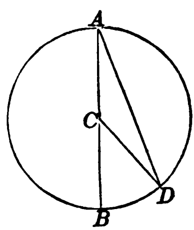
14. A figure is said to be circumscribed about a circle, when all the sides of the figure touch the circumference. The circle is then said to be *inscribed* in the figure.



15. An angle at the centre of a circle is measured by the arc contained by the sides of the angle. This arc is said to subtend the angle. Thus, the angle *ACB* is measured by the degrees in the arc *AEB*, and is subtended by the arc *AEB*.



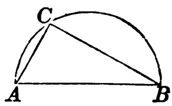
16. An angle at the circumference of a circle, is measured by half the arc which subtends it. Thus, the angle *BAD* is measured by half the one *BD*. Hence, it follows, that when an angle at the centre and an angle at the circum-



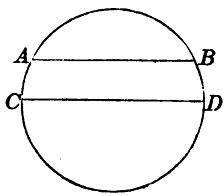
QUEST.—14. When is a figure said to circumscribe a circle? What is then said of the circle? 15. What measures an angle at the centre of a circle? What is said of this arc? 16. By what is an angle at the circumference of a circle measured? If an angle at the centre and an angle at the circumference both stand on the same arc, how will they compare with each other?

ference stand on the same arc, *BD*, the angle at the centre will be double the angle at the circumference.

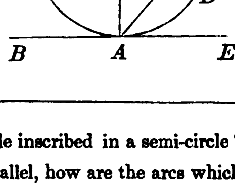
17. An angle inscribed in a semi-circle, is a right angle. Thus, if *AB* be the diameter of a circle, then will the angle *ACB* be equal to 90 degrees. This angle is measured by one half the semi-circumference, that is, by one half of 180°, or by 90°.



18. Two parallel chords intercept equal arcs. That is, if the chords *AB* and *CD* are parallel, the arcs *AC* and *DB*, which they intercept, will be equal.

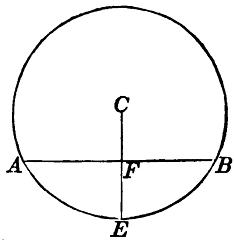


19. The angle formed by a tangent and a chord passing through the point of contact, is measured by half the arc of the chord. Thus, the angle *CAE*, is measured by half the arc *ADC*; and the angle *CAB*, by half the arc *CFMA*.



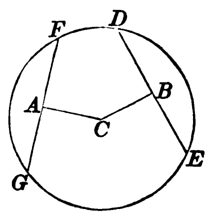
QUEST.—17. What is the value of an angle inscribed in a semi-circle? Why? 18. If two chords of a circle are parallel, how are the arcs which they intercept or include? 19. How is the angle measured which is formed by a tangent and a chord? If the arc of the chord be 60 degrees, what will the angle formed by the tangent and chord be equal to?

20. If from the centre of a circle a line be drawn perpendicular to a chord, it will bisect the chord, and also the arc of the chord. Thus, *CFE* drawn from the centre *C*, perpendicular to *AB*, bisects *AB* at *F*, and also makes $AE = EB$.

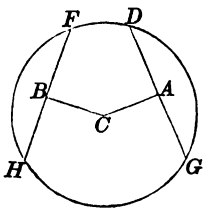


21. The distance from the centre of a circle to a chord, is measured on a perpendicular to the chord.

22. In the same, or in equal circles, chords which are equally distant from the centre, are equal. Thus, if $CA = CB$, then will the chord $FG =$ chord *DE*.



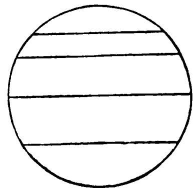
23. If the chord *DG* is equal to the chord *FH*, they will be equally distant from the centre: that is, *CB* will be equal to *CA*.



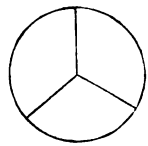
QUEST.—20. If from the centre of a circle a line be drawn perpendicular to a chord, how will it divide the chord? How will it divide the arc of the chord? 21. How is the distance from the centre of a circle, to a chord measured? 22. If chords are equally distant from the centre of a circle, will they be equal or unequal? 23. If two chords are equal to each other, will they be equally or unequally distant from the centre?

Of the Circle.

24. If several lines be drawn within a circle, the greatest is the diameter, and those nearest the centre are greater than those more remote.

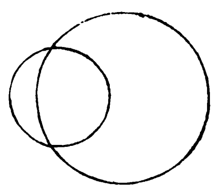


25. There is no point except the centre, from which three equal lines can be drawn to the circumference.



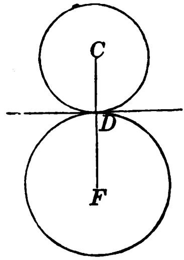
26. A straight line cannot cut the circumference of a circle in more than two points.

27. Two circumferences cannot cut each other in more than two points.

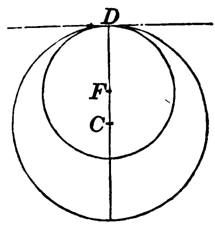


QUEST.—24. If several lines are drawn within a circle, which is the greatest? Which is the least? 25. What point is that within a circle from which three or more equal lines may be drawn to the circumference? Is there any other such point? 26. In how many points can a straight line cut the circumference of a circle? 27. In how many points can the circumferences of two circles intersect each other?

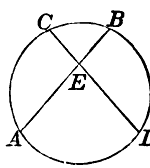
28. If two circles touch each other externally, their centres and the point of contact are in the same straight line. Thus, the centres C and F, and the point of contact D are in the same straight line CDF.



29. If two circles touch each other internally, their centres and point of contact are in the same straight line. Thus, the centres F and C, and the point of contact D, are in the same straight line.



30. If two chords intersect each other the product, or rectangle of the parts of the one, is equal to the rectangle of the parts of the other. Thus, the two chords AB and CD, which intersect each other at E, give



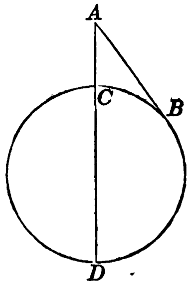
$$AE \times EB = CE \times ED.$$

QUEST.—28. If two circles touch each other externally, and the centres be joined by a straight line, will this line also contain the point of contact? 29. If two circles touch each other internally, and the centres be joined, will the line contain the point of contact? 30. If two chords intersect each other, how will the rectangle of the parts of the one compare with the rectangle of the parts of the other?

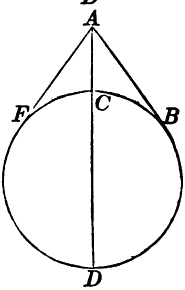
Of the Circle.

31. If from a point without a circle, a tangent AB, be drawn and also a secant ACD, then will the square of the tangent be equal to the rectangle of the parts of the secant. That is,

$$\overline{AB}^2 = AD \times AC.$$

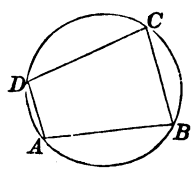


32. If from any point, as A, without the circumference of a circle, two tangents AB, AF be drawn, these tangents will be equal to each other, and the line ACD passing through the centre of the circle, will bisect the angle included between them.



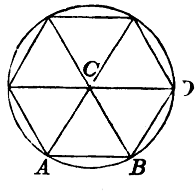
33. If a quadrilateral be inscribed in a circle, the sum of either two of its opposite angles is equal to two right angles. Thus, in the quadrilateral $ABCD$, we have

$$A + C = 180°, \text{ and } B + D = 180°$$

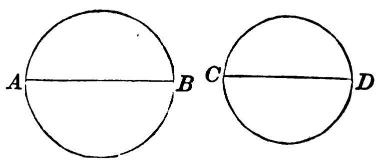


QUEST.—31. If from a point without a circle, you draw a tangent, and also a secant line, what will the square of the tangent be equal to? 32. If two tangent lines be drawn, how will they compare with each other? If from the same point, a line be drawn through the centre of the circle, how will it divide the angle included between the tangents? 33. If a quadrilateral be inscribed in a circle, what is the sum of either two of the opposite angles equal to?

34. If the radius of a circle be applied six times as a chord, it will reach exactly round the circumference. Thus, if you take the radius CA and lay it off from A to B, then from B to D, &c., you will find that after laying it off six times, you will exactly reach to the point A. Hence, the side of a regular hexagon (§ IV. Art. 5), is equal to the radius of the circumscribing circle.



35. The circumferences of circles are proportional to their diameters. If we represent the diameter AB by D, and the circumference of the circle by C, and the diameter CD by d, and the circumference by c; we shall have

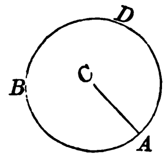


$$D : d :: C : c.$$

36. The circumference of a circle is a little more than three times as great as the diameter. If the diameter is 1, the circumference will be 3,1416.

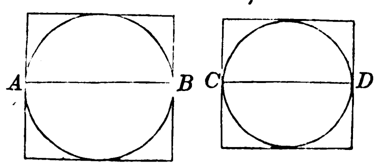
QUEST.—34. If the radius of a circle be applied as a chord, how many radii will go round the circumference? To what then is the side of a regular hexagon equal? 35. How are the circumferences of circles to each other? 36. How many times is the circumference of a circle greater than its diameter? If the diameter is 1 what is the circumference?

37. The area of a circle is equal to the product of half the radius, into the circumference. Thus, the area of the circle, whose centre is C, is equal to half the radius CA, multiplied by the circumference: that is,

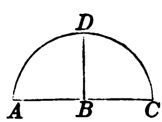


$$\text{area} = \tfrac{1}{2}\, CA \times \text{circumference } ABD.$$

38. The areas of circles are to each other as the squares described on their diameters: that is, the areas of two circles are to each other as the squares described on the diametres AB and CD.



39. If from any point of the circumference of a circle, a line be drawn perpendicular to a diameter, the square of the perpendicular will be equal to the rectangle of the segments of the diameter. That is, if DB be perpendicular to AC, we have



$$\overline{DB}^2 = AB \times BC.$$

QUEST.—37. What is the area of a circle equal to? If the diameter is 1, and circumference 3,1416, what is the area? 38. How are the areas of two circles to each other? If the diameters are 2 and 4, in what proportion are the areas? 39. What is the square of a perpendicular to a diameter equal to?

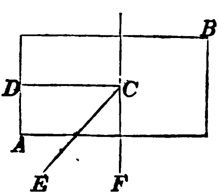
SECTION IX.

OF PLANES AND THEIR ANGLES.

1. A plane has been defined (§ I. Art. 6), to be a surface which lies even throughout its whole extent, and with which a straight line, laid in any direction, will exactly coincide.

2. The common intersection of two planes is the line in which they cut each other. This line is always a straight line.

3. Two planes which intersect each other form an angle. This angle is measured by two lines, one in each plane, and both perpendicular to the common intersection at the same point.



Let AB be a plane coinciding with the plane of the paper, and ECF a plane intersecting it in the line FC. Now, if from any point of the common intersection as C, we draw CD in the plane

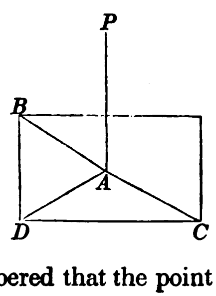
QUEST.—1. What is a plane? If a straight line be laid upon a plane, will it exactly coincide with the plane? 2. What is the line of intersection of two planes? Is this line straight or curved? 3. May two planes form an angle? How is this angle measured?

AB, and *CE* in the plane *ECF*, and both perpendicular to *CF* at *C*, then will the angle *DCE* measure the inclination between the two planes.

It should be remembered that the line *EC* is directly over the line *CD*.

4. If the angle *ECD* is a right angle, the planes will be perpendicular to each other.

5. A line is said to be perpendicular to a plane, when it inclines no more to one side than to the other. The point at which the perpendicular meets the plane is called the *foot* of the perpendicular.



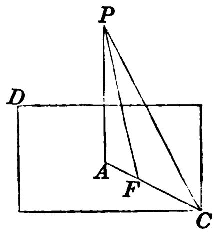
6. When a line is perpendicular to a plane, it will be perpendicular to every line of the plane which it meets. Thus, if *AP* is perpendicular to the plane *BC* at the point *A*, it will be perpendicular to the lines *AD*, *AB* and *AC*, drawn in the plane. It should be remembered that the point *P* is directly over the point *A*.

QUEST.—4. If the angle formed by the measuring lines is a right angle, what angle will the planes make with each other? 5. When is a line said to be perpendicular to a plane? What is the point called at which the perpendicular meets the plane? 6. If a line is perpendicular to a plane, will it be perpendicular to other lines of the plane which it meets?

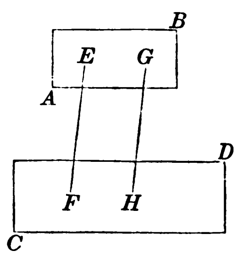
Of Planes.

7. If several lines are parallel to each other, and one of them is perpendicular to a plane, all the others will likewise be perpendicular to the plane.

8. A perpendicular is the shortest distance from a point to a plane. If oblique lines be drawn to the plane, those nearest the perpendicular will be less than those more remote. Thus, if *PA* be perpendicular to the plane *DC*, it will be the shortest line from *P* to the plane; and the line *PF* near *PA*, will be less than *PC*, which is more remote.



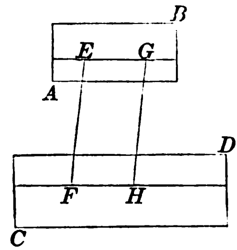
9. Two planes are said to be parallel when they are at the same perpendicular distance from each other. Parallel planes will never meet each other, how far soever they may be produced. If the two planes *AB* and *CD* are paral-



QUEST.—7. If several lines are parallel to each other, and one of them is perpendicular to a plane, what position will the other lines also have with the plane? 8. What is the shortest line which can be drawn from a point to a plane? If a perpendicular and several oblique lines be drawn, which of the oblique lines will be the least? 9. When are two planes said to be parallel? Will parallel planes meet each other if produced? If two or more lines be drawn perpendicular to two parallel planes, will they be equal or unequal?

lel, the lines *EF* and *GH* drawn perpendicular to them will be equal, and parallel.

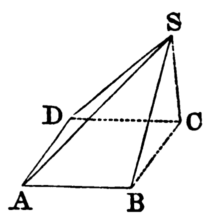
10. If two parallel planes are cut by a third plane, the lines of intersection will be parallel. Thus, if the two parallel planes *AB* and *CD* be cut by the plane *FHGE*, the lines of intersection *FH* and *EG* will be parallel.



11. A *Solid Angle* is the angular space included between several planes which meet at a point.

Thus, the solid angle *S*, is formed by the union of the planes *ASB*, *BSC*, *CSD*, and *DSA*.

Three planes, at least, are requisite to form a solid angle.



QUEST.—10. If two parallel planes are cut by a third plane, what position will the lines of intersection have with each other? 11. What is a solid angle? What is the least number of planes which can form a solid angle?

SECTION X.

OF SOLIDS BOUNDED BY PLANES.

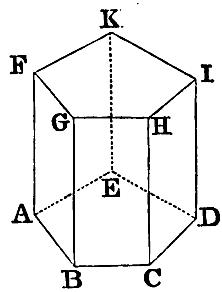
1. A solid or body was defined (§ I. Art. 8), to be that which has length, breadth, and thickness.

2. Every solid bounded by planes is called a *polyedron.*

3. The planes which bound a polyedron are called *faces*. The straight lines in which the faces intersect each other, are called the *edges* of the polyedron ; and the points at which the edges intersect, are called the *vertices* of the angles, or vertices of the polyedron.

QUEST.—1. What is a solid ? 2. What is a polyedron ? 3. What are the planes called which bound a polyedron ? What are the lines called in which the faces intersect each other ? What are the points called in which the edges intersect each other ?

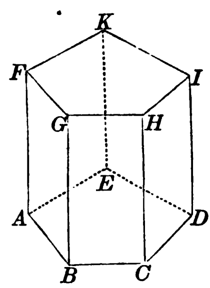
4. A prism is a solid, whose ends are equal polygons, and whose side faces are parallelograms.



Thus, the prism whose lower base is the pentagon *ABCDE*, terminates in an equal and parallel pentagon *FGHIK*, which is called the *upper base.* The side faces of the prism are the parallelograms *DH*, *DK*, *EF*, *AG* and *BH*. These are called the *convex* or *lateral* surface of the prism.

5. The altitude of a prism is the distance between its upper and lower bases: that is, it is a line drawn from a point of the upper base, perpendicular to the lower base.

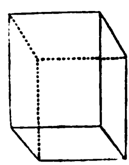
6. A right prism is one in which the edges *AF*, *BG*, *EK*, *HC*, and *DI* are perpendicular to the bases. In the right prism, either of the perpendicular edges is equal to the altitude. In the oblique prism the altitude is less than the edge.



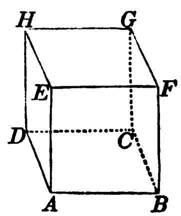
QUEST.—4. What is a prism? What is the lower base of a prism? What is the upper base? What are the side faces of the prism, taken together, called? 5. What is the altitude of a prism? 6. What is a right prism? In a right prism what is one of the perpendicular edges equal to? In the oblique prism is the edge greater or less than the altitude?

7. A prism whose base is a triangle, is called a *triangular* prism: if the base is a quadrangle it is called a quadrangular prism: if a pentagon, a pentagonal prism: if a hexagon it is called a hexagonal prism, &c.

8. A prism whose base is a parallelogram, and all of whose faces are also parallelograms, is called a parallelopipedon. If all the faces are rectangles, it is called a rectangular parallelopipedon. If all the faces are squares it is called a *cube*. The cube is bounded by six equal faces at right angles to each other.

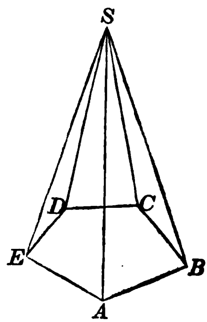


9. The opposite faces of a parallelopipedon are equal to each other. Thus, the parallelogram *BD* is equal to the opposite parallelogram *FH*, the parallelogram *BE* to *CH*, and *BG* to *AH*.



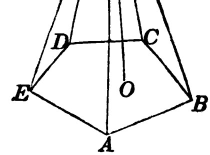
QUEST.—7. What is a triangular prism? What is a quadrangular prism? What is a pentagonal prism? 8. What is a parallelopipedon? When all the faces are rectangles, what is it called? If all the faces are squares, what is it then called? How is a cube bounded? 9. Are the opposite faces of a parallelopipedon equal or unequal? Point out the equal faces?

Of Solids bounded by Planes.



10. A pyramid is a solid, formed by several triangles united at the same point *S*, and terminating in the different sides of a polygon *ABCDE*.

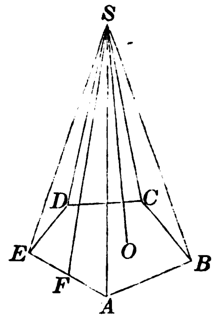
The polygon *ABCDE*, is called the base of the pyramid; the point *S*, is called the *vertex*, and the triangles *ASB*, *BSC*, *CSD*, *DSE*, and *ESA*, form its *lateral*, or *convex* surface.



11. The *altitude* of a pyramid, is the perpendicular let fall from the vertex, upon the plane of the base. Thus, *SO* is the altitude of the pyramid *S—ABCDE*.

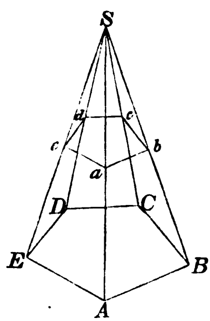
QUEST.—10. What is a pyramid? What is the base of a pyramid? Point it out. What is the vertex of a pyramid? What are the triangles called, which bound the pyramid? 11. What is the altitude of a pyramid? Point out the altitude.

12. The *slant height* of a regular pyramid, is a line drawn from the vertex, perpendicular to one of the sides of the polygon which forms its base. Thus, *SF* is the slant height of the pyramid *S—ABCDE*.



13. When the base of the pyramid is a regular polygon, and the perpendicular *SO*, passes through the middle point of the base, the pyramid is called a regular pyramid, and the line *SO*, is called the *axis*.

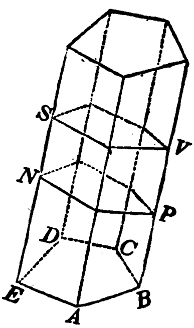
14. If from the pyramid *S—ABCDE*, the pyramid *S—abcde* be cut off by a plane parallel to the base, the remaining solid, below the plane, is called the *frustum* of a pyramid.



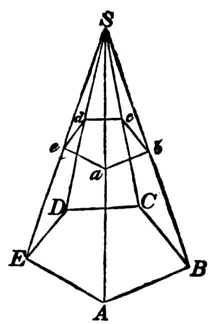
The altitude of a frustum is the perpendicular distance between the upper and lower planes.

QUEST.—12. What is the slant height of a regular pyramid? Point out the slant height. 13. When is a pyramid regular? In a regular pyramid, what is the axis? 14. What is the frustum of a pyramid? What is the altitude of a frustum?

15. A pyramid whose base is a triangle, is called a *triangular pyramid:* if the base is a quadrangle, it is called a quadrangular pyramid,—if a pentagon, it is called a pentagonal pyramid—if the base is a hexagon, it is called a hexagonal pyramid, &c.



16. In every prism, the sections formed by parallel planes, are equal polygons. Thus, if the prism, whose base is the polygon *ABCDE*, be cut by the planes *NP* and *SV*, parallel to the base, the polygons *NP*, and *SV*, will each be equal to the base.



17. If a pyramid be cut by a plane parallel to the base, the section will be similar to the base. Thus, if the pyramid *S—ABCDE*, be cut by the plane *abcde* parallel to the base, the polygon *abcde* will be a figure similar to the base.

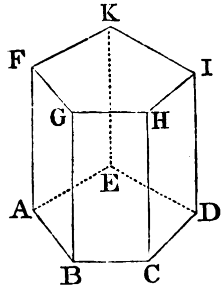
QUEST.—15. What is a triangular pyramid? What is a quadrangular pyramid? What is a pentagonal pyramid? What a hexagonal pyramid? 16. If a prism be cut by a plane, parallel to the base, how will the section compare with the base? 17. If a pyramid be cut by a plane parallel to the base, will the section be similar to the base?

Of Solids bounded by Planes.

18. The convex surface of a right prism, is equal to the perimeter of its base, multiplied by the altitude. Thus,

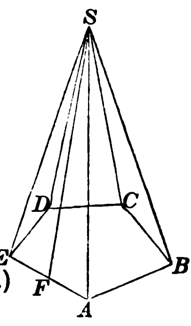
$BG\,(AB+BC+CD+DE+EA)$

is equal to the convex surface.



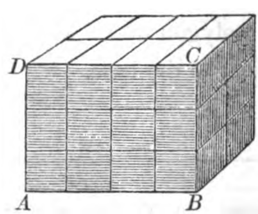
19. The convex surface of a regular pyramid, is equal to the perimeter of the base, multiplied by half the slant height. Thus, the convex surface of the pyramid S—$ABCDE$, is equal to

$\frac{1}{2}SF\,(AB+BC+CD+DE+EA.)$



20. The *content* of a solid is the number of cubes which it contains.

In order to find the content of a solid, suppose $ABCD$ to be the base of a parallelopipedon. Let us



QUEST.—18. What is the convex surface of a right prism equal to? 19. What is the convex surface of a regular pyramid equal to? 20. What is the content of a solid?

suppose $AB=4$ feet, and $BC=3$ feet. Then the number of square feet in the base will be equal to $3\times4=12$ square feet. Therefore, 12 equal cubes of one foot each, may be placed by the side of each other on the base. If the parallelopipedon be 1 foot in height, it will contain 12 cubic feet: were it 2 feet in height, it would contain two tiers of cubes, or 24 cubic feet: were it 3 feet in height, it would contain three tiers of cubes, or 36 cubic feet. Therefore, ***the solid content of a parallelopipedon is equal to the product of its length, breadth, and height.***

21. It may be remarked here, that there are three kinds of quantity in geometry,—viz. Lines, Surfaces, and Solids. Each of these has its own unit. The unit of a line, which we have called the linear unit (§ V. Art. 1), is a line of a known length, as a foot, a yard, a rod, &c.

QUEST.—If one side of the base of a parallelopipedon be 4 and the other 3 feet, how many square feet will it contain? How many cubes of 1 foot each, may be placed on the base? If the parallelopipedon be 1 foot in height, how many cubes will it contain? If it be 2 feet in height, how many would it contain? If it were 3 feet in height, how many would it contain? What then, is the solid content of a parallelopipedon equal to? 21. How many kinds of quantity are there in geometry? What are they? What is the unit of measure of a line? If a line be 30 yards long, what is the unit of measure?

The unit of surface, is a square, whose sides are the unit of length.

The unit of solidity is a cube whose edges are the unit of length.

For example, if the bounding lines of a surface be estimated in yards, the content will be square yards; and if the bounding lines of a solid be yards, its surface will be estimated in square yards, and its solid content in cubic yards.

22. The solid content of any prism, is equal to the area of the base, multiplied by the altitude.

23. The solid content of a pyramid, is equal to the area of the base, multiplied by one third of the altitude; or, equal to one third of the product of the base, multiplied by the altitude.

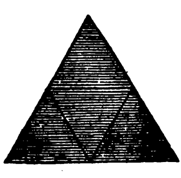
QUEST.—What is the unit of measure for surfaces? What is the unit of measure for solids? If the linear unit be 1 yard, what will be the unit of surface? What the unit of solidity? 22. What is the solid content of a prism equal to? 23. What is the solid content of a pyramid equal to?

SECTION XI.

OF THE FIVE REGULAR SOLIDS.

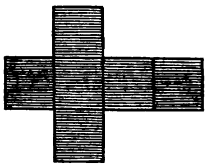
A *Regular solid*, is one whose faces are all equal polygons, and whose solid angles are equal. There are five such solids.

1. The *Tetraedron*, or equilateral pyramid, is a solid bounded by four equal triangles.

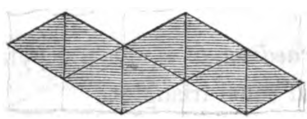


QUEST.—What is a regular solid? How many regular solids are there?
1. What is the regular Tetraedron or equilateral pyramid?

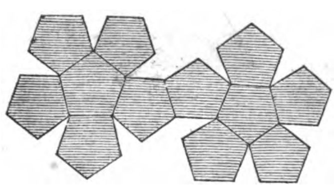
2. The *Hexaedron* or *cube*, is a solid, bounded by six equal squares.



3. The *Octaedron*, is a solid, bounded by eight equal triangles.

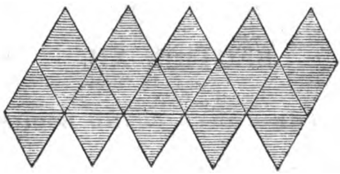


4. The *Dodecaedron*, is a solid, bounded by twelve equal pentagons.



QUEST.—2. What is the regular Hexaedron, or cube? 3. What is an Octaedron? 4. What is a Dodecaedron?

5. The *Icosaedron*, is a solid, bounded by twenty equal triangles.



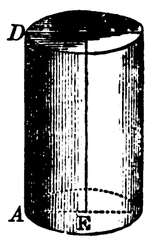
6. The regular solids may easily be made of paste-board.

Draw the figures of the regular solids accurately on pasteboard, and then cut through the bounding lines: this will give figures of pasteboard similar to the diagrams. Then, cut the other lines half through the pasteboard, after which, turn up the parts, and glue them together, and you will form the bodies which have been described.

QUEST.—5. What is the Icosaedron? 6. Describe the manner of making the regular solids with pasteboard.

SECTION XII.

OF THE THREE ROUND BODIES.



1. *A Cylinder* is a solid, described by the revolution of a rectangle *AEFD*, about a fixed side *EF*.

As the rectangle *AEFD*, turns around the side *EF*, like a door upon its hinges, the lines *AE* and *FD* describe circles, and the line *AD*, describes the *convex surface* of the cylinder.

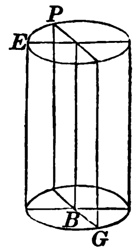
The circle described by the line *AE*, is called the *lower base* of the cylinder, and the circle described by *DF*, is called the *upper base.*

The immoveable line *EF*, is called the *axis* of the cylinder.

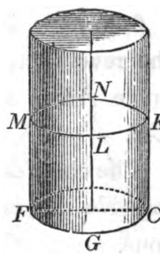
A cylinder, therefore, is a round body with circular ends.

QUEST.—1. How is a cylinder described? Point out the line which describes the convex surface of the cylinder. Point out the line which describes the lower base of the cylinder. Also the one which describes the upper base. What is the immoveable line called? What is a cylinder?

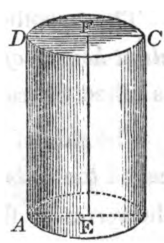
Of the Round Bodies.



2. If a plane be passed through the axis of a cylinder, it will intersect it in a rectangle *PG*, which is double the revolving rectangle *EB*.



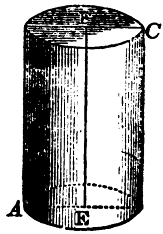
3. If a cylinder be cut by a plane parallel to the base, the section will be a circle equal to the base. Thus, *MLKN*, is a circle equal to the base *FGC*.



4. The convex surface of a cylinder is equal to the circumference of the base, multiplied by the altitude. Thus, the convex surface of the cylinder *AC* is equal to

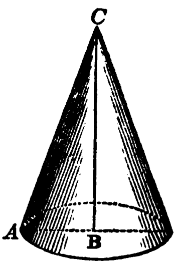
circumference of base $\times$ *AD*.

QUEST.—2. If a plane be passed through the axis of a cylinder, in what figure will it intersect the cylinder? How does this rectangle compare with the revolving rectangle? 3. If a cylinder be cut by a plane, parallel to the base, what will the section be? 4. What is the convex surface of a cylinder equal to?



5. The solidity of a cylinder is equal to the area of the base, multiplied by the altitude. Thus, the solidity of the cylinder *AC*, is equal to

area of base × *FE*.



6. A cone, is a solid, described by the revolution of a right angled triangle *ABC*, about one of its sides *CB*.

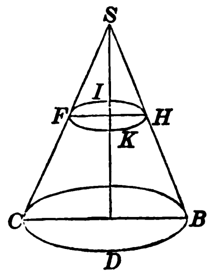
The circle described by the revolving side *AB*, is called the *base* of the cone.

The hypothenuse *AC*, is called the *slant height of the cone*, and the surface described by it, is called the *convex surface* of the cone.

The side of the triangle *CB*, which remains fixed, is called the *axis* or *altitude* of the cone, and the point *C*, the *vertex* of the cone.

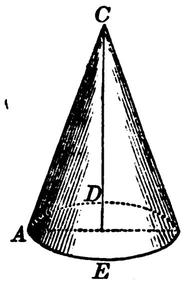
QUEST.—5. What is the solidity of a cylinder equal to? 6. How is a cone described? Point out the line which describes the base of the cone. What is the hypothenuse of the revolving triangle called? What does it describe? What is the side of the triangle which remains fixed called? What is the vertex of the triangle called?

7. If a cone be cut by a plane parallel to the base, the section will be a circle. Thus, the section *FKHI* is a circle. If from the cone *S—CDB*, the cone *S—FKH* be taken away, the remaining part is called the ***frustum*** of a cone.



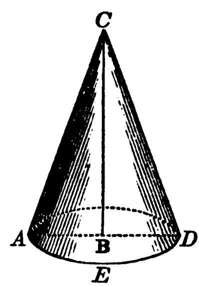
8. The convex surface of a cone is equal to the circumference of the base multiplied by half the slant height. Thus, the convex surface of the cone *C—AED* is equal to

circumference $AED \times \frac{1}{2}CA$.



9. The solidity of a cone is equal to the area of the base multiplied by one-third of the altitude. Thus, the solidity of the cone *C—AED* is equal to

base $AED \times \frac{1}{3}CB$.

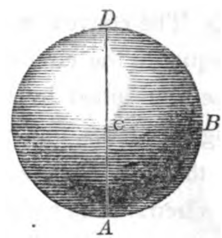


QUEST.—7. If a cone be cut by a plane parallel to the base, what will the section be? If the upper part be taken away what is the lower part called? 8. What is the convex surface of a cone equal to? 9. What is the solidity of a cone equal to?

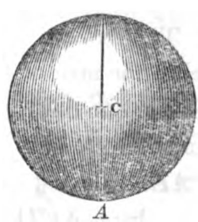
10. Since the solidity of a cylinder is equal to the base multiplied by the altitude (Art. 5), and that of a cone to the base multiplied by one third of the altitude, it follows that if a cylinder and cone have equal bases and altitudes, the *cone will be one-third of the cylinder.*

11. *A Sphere* is a solid terminated by a curved surface all the points of which are equally distant from a certain point called the centre.

12. The sphere may be described by revolving a semicircle *ABD* about the diameter *AD*. The plane will describe the solid sphere, and the semi-circumference *ABD* will describe the surface.

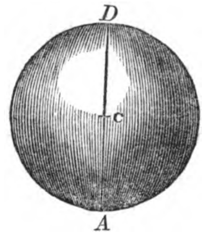


13. *The radius* of a sphere is a line drawn from the centre to any point of the circumference. Thus, *CA* is a radius.



QUEST.—10. If a cylinder and cone have the same base and altitude, which will be the greater? How much will the cylinder exceed the cone? 11. What is a sphere? 12. How may a sphere be described? What will the plane describe? What will the semicircumference describe? 13. What is the radius of a sphere?

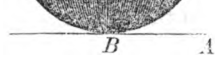
14. The diameter of a sphere is a line passing through the centre, and terminated by the circumference. Thus, *AD* is a diameter.



15. All diameters of a sphere are equal to each other; and each is double a radius.

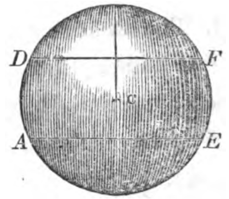
16. The axis of a sphere is any line about which it revolves; and the points at which the axis meets the surface, are called the *poles*.

17. A *Plane* is *tangent* to a sphere when it has but one point in common with it. Thus, *AB* is a tangent plane.

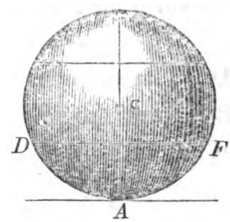


QUEST.—14. What is the diameter of a sphere? 15. Are all diameters equal? How does a diameter compare with a radius? 16. What is the axis of a sphere? What are the poles of a sphere? 17. When is a plane said to be tangent to a sphere?

18. A *zone* is a portion of the surface of a sphere, included between two parallel planes which form its bases. Thus, the part of the surface included between the planes *AE* and *DF* is a zone. The bases of this zone are two circles whose diameters are *AE* and *DF*



19. One of the planes which bound a zone may become tangent to the sphere, in which case the zone will have but one base. Thus, if one plane be tangent to the sphere at *A*, and another plane cut it in the circle *DF*, the zone included between them will have but one base.



20. A *spherical segment* is a portion of the solid sphere included between two parallel planes. These parallel planes are its bases. If one of the planes is tangent to the sphere, the segment will have but one base.

QUEST.—18. What is a zone? What are the bases of a zone? 19. If one of the planes which bound the zone becomes tangent to the sphere, how many bases will the zone have? 20. What is a spherical segment? What are the parallel planes called? If one of the parallel planes becomes tangent to the sphere, how many bases will the segment have?

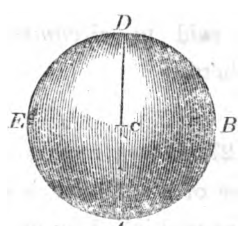
21. *The altitude* of a zone or segment, is the distance between the parallel planes which form its bases.

22. Every plane passing through a sphere intersects the solid sphere in a circle, and the surface of a sphere in the circumference of a circle.

23. If the intersecting plane passes through the centre of the sphere, the circle is called a *great circle.* If it does not pass through the centre, the circle of section is called a *small circle.*

24. The surface of a sphere is equal to the product of its diameter by the circumference of a great circle. Thus, the surface of the sphere whose centre is *C*, is equal to

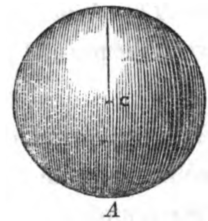
circumference $ABDE \times AD$.



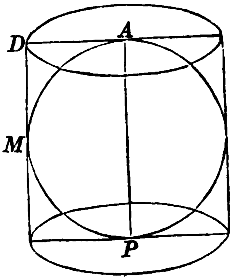
QUEST.—21. What is the altitude of a zone or segment? 22. If a plane be passed through a sphere, in what will it intersect the solid sphere? In what will it intersect the surface? 23. If the intersecting plane passes through the centre of the sphere, what is the circle called? If it does not pass through the centre of the sphere, what is the circle called? 24. What is the surface of a sphere equal to?

25. The solidity of a sphere is equal to its surface multiplied by one-third of the radius. Thus, the sphere whose centre is C, is equal to

$$\text{surface} \times \tfrac{1}{3}CA.$$



26. If the semicircle PMA, and the rectangle PD, be both revolved around the diameter PA, the semicircle will describe a sphere, and the rectangle a cylinder. This cylinder is said to *circumscribe* the sphere.



27. It was remarked, in Art. 4, that the convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude; and in Art. 24, that the surface of a sphere is equal to its diameter into the circumference of a great circle: hence, *the surface of a*

QUEST.—25. What is the solidity of a sphere equal to? 26. If a semicircle and a rectangle revolve about the diameter of the semicircle, what will the semicircle describe? What will the rectangle describe? 27. How does the surface of a sphere compare with the convex surface of the circumscribing cylinder?

sphere is equal to the convex surface of the circumscribing cylinder.

28. The solidity of a sphere is equal to two-thirds of the solidity of the circumscribing cylinder.

29. The cylinder, the cone, and the sphere are called the three round bodies.

QUEST.—28. What portion of the cylinder is the inscribed sphere? 29. What are the three round bodies?

PART II.

SECTION I.

PRACTICAL GEOMETRY.

1. Practical Geometry explains the methods of constructing or describing the Geometrical figures.

For these constructions, certain instruments are necessary. These we shall now describe.

DIVIDERS, OR COMPASSES.

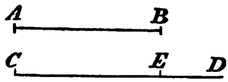
2. The dividers is the most simple and useful of the instruments used for describing figures. It consists of two legs, *ba* and *be*, which may be easily turned around a joint at *b*.

QUEST.—1. What is Practical Geometry? 2. What are the dividers used for? Of how many parts are they composed? How are these parts moved?

PROBLEM I.

On any line as CD, to lay off a distance equal to AB.

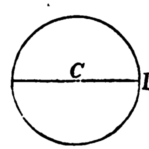
3. Take up the dividers with the thumb and second finger, and place the fore finger on the joint at *b*. Then, set one foot of the dividers at *A*, and extend the legs with the thumb and fingers, until the other foot reaches to *B*. Then raising the dividers, place one foot at *C*, and mark with the other the distance *CE*, this will evidently be equal to *AB*.



PROBLEM II.

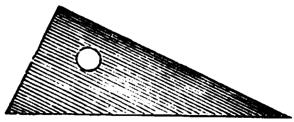
To describe from a given centre the circumference of a circle having a given radius.

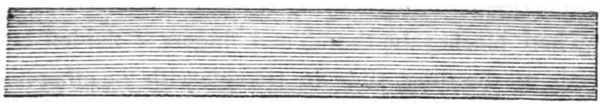
4. Let *C* be the given centre, and *CB* the given radius. Place one foot of the dividers at *C*, and extend the other leg until it shall reach to *B*. Then turn the dividers around the leg at *C*, and the other leg will describe the required circumference.



QUEST.—3. Explain the manner of laying off a given distance on a given line? 4. Explain the manner of describing from a fixed centre, the circumference of a circle that shall have a given radius?

RULER AND TRIANGLE.





5. A Ruler of a convenient size, is about twenty inches in length, two inches wide, and one-fifth of an inch in thickness. It should be made of a hard material, perfectly straight and smooth.

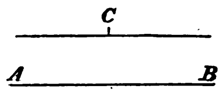
The hypothenuse of the right angled triangle, which is used in connexion with it, should be about ten inches in length, and it is most convenient to have one of the sides considerably longer than the other. We can resolve with the ruler and triangle the two following problems.

PROBLEM III.

To draw through a given point a line which shall be parallel to a given line.

6. Let *C* be the given point, and *AB* the given line.

QUEST.—5. What are the convenient dimensions of a ruler to be used in drawing? What should be the dimensions of a triangle? 6. Explain the manner of drawing with the ruler and triangle, a line which shall pass through a given point and be parallel to a given line?



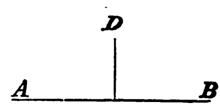
Place the hypothenuse of the triangle against the edge of the ruler, and then place the ruler and triangle on the paper, so that one of the sides of the triangle shall coincide exactly with *AB*—the triangle being below the line *AB*.

Then placing the thumb and fingers of the left hand firmly on the ruler, slide the triangle with the other hand along the ruler until the side which coincided with *AB* reaches the point *C*. Leaving the thumb of the left hand on the ruler, extend the fingers upon the triangle and hold it firmly, and with the right hand mark with a pen or pencil a line through *C*: this line will be parallel to *AB*.

PROBLEM IV.

To draw through a given point a line which shall be perpendicular to a given line.

7. Let *AB* be the given line, and *D* the given point.



Place, as before, the hypothenuse of the triangle against the edge of the ruler. Then place the ruler and triangle so that one of the sides of the triangle shall coincide exactly with the line *AB*. Then

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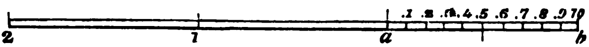
QUEST.—7. Explain the method of drawing a line through a given point that shall be perpendicular to a given line?

slide the triangle along the ruler until the other side reaches the point *D*. Draw through *D* a straight line, and it will be perpendicular to *AB*.

SCALE OF EQUAL PARTS.



8. A scale of equal parts is formed by dividing a line of a given length into equal portions.

If, for example, the line *ab* of a given length, say one inch, be divided into any number of equal parts, as 10, the scale thus formed is called a scale of ten parts to the inch. The line *ab*, which is divided, is called the *unit of the scale*. This unit is laid off several times on the left of the divided line, and the points marked 1, 2, 3, &c. The unit of scales of equal parts, is, in general, either an inch or an exact part of an inch. If, for example, the unit of the scale *ab*, were one inch, the scale would be one of ten parts to the inch; if it were half an inch, the scale would be one of ten parts to half an inch, or of 20 parts to the inch.

QUEST.—8. What is a scale of equal parts? What is the *unit* of the scale? Is the unit generally laid off at the left or right hand of the smaller divisions? What is generally taken for the unit of scales of equal parts? Explain the manner of taking from the scale of equal parts the distance two inches and six-tenths?

Let it be required to take from the scale a line equal to two inches and six-tenths.

Place one foot of the dividers at 2 on the left, and extend the other to ,6, which marks the sixth of the small divisions: the dividers will then embrace the required distance.

PROBLEM V.

To lay down, on paper, a line of a given length, so that any number of its parts shall correspond to the unit of the scale.

9. Suppose that the given line were 75 feet in length, and it were required to draw it on paper, on a scale of 25 feet to the inch.

The length of the line 75 feet, being divided by 25, will give 3, the number of inches which will represent the line on paper.

Therefore, draw the indefinite line *AB*, on which lay

off a distance *AC* equal to 3 inches: *AC* will then represent the given line of 75 feet drawn to the required scale.

QUEST.—9. Explain the manner of laying off on paper a line of a given length, so that any number of its parts shall correspond to the unit of the scale? When the length of the line to be laid down is known, give the rule for finding the length to be taken from the scale? When the length of a line is given on the paper, how will you find the true length of the line?

REMARK I. The last problem explains the manner of laying down a line upon paper, in such a manner that a given number of parts shall correspond to the unit of the scale, whether that unit be an inch or any part of an inch.

When the length of the line to be laid down is given, and it has been determined how many parts of it are to be represented on the paper by a distance equal to the unit of the scale, we find the length which is to be taken from the scale by the following

RULE.

Divide the length of the line by the number of parts which is to be represented by the unit of the scale: the quotient will show the number of parts which is to be taken from the scale.

EXAMPLES.

1. If a line of 640 feet in length is to be laid down on paper, on a scale of 40 feet to the inch; what length must be taken from the scale?

40)640(16 inches.

2. If a line of 357 feet is to be laid down on a scale of 68 feet to the unit of the scale, (which we will suppose half an inch), how many parts are to be taken?

Ans. { 5,25, parts, or 2,625 inches.

REMARK II. When the length of a line is given on the paper, and it is required to find the true length of the line which it represents, take the line in the dividers and apply it to the scale, and note the number of units, and parts of an unit to which it is equal. Then, multiply this number by the number of parts which the unit of the scale represents, and the product will be the length of the line.

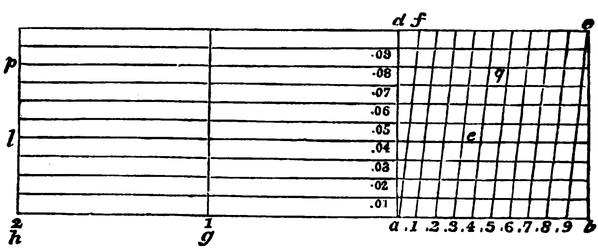
1. Suppose the length of a line drawn on the paper, to be 3,55 inches, the scale being 40 feet to the inch: then,

$$3{,}55 \times 40 = 142 \text{ feet, the length of the line.}$$

2. If the length of a line on the paper is 6,25 inches, and the scale be one of 30 feet to the inch, what is the true length of the line.

Ans. 187,5 feet.

DIAGONAL SCALE OF EQUAL PARTS.



10. This scale is thus constructed. Take *ab* for the unit of the scale, which may be one inch, $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{3}{4}$ of an

QUEST.—10. Explain the construction of the scale of equal parts?

inch, in length. On *ab* describe the square *abcd*. Divide the sides *ab* and *dc* each into ten equal parts. Draw *af* and the other nine parallels as in the figure.

Produce *ba* to the left, and lay off the unit of the scale any convenient number of times, and mark the points 1, 2, 3, &c. Then, divide the line *ad* into ten equal parts, and through the points of division draw parallels to *ab* as in the figure.

Now, the small divisions of the line *ab* are each one tenth (,1) of *ab*; they are therefore ,1 of *ad*, or ,1 of *ag* or *gh*.

If we consider the triangle *adf*, the base *df* is one tenth of *ad* the unit of the scale. Since the distance from *a* to the first horizontal line above *ab*, is one-tenth of the distance *ad*, it follows that the distance measured on that line between *ad* and *af* is one-tenth of *df*: but since one-tenth of a tenth is a hundredth, it follows that this distance is one hundredth (,01) of the unit of the scale. A like distance measured on the second line will be two hundredths (,02) of the unit of the scale; on the third, ,03; on the fourth, ,04, &c.

If it were required to take, in the dividers, the unit of the scale and any number of tenths, place one foot of the dividers at 1, and extend the other to that figure between *a* and *b* which designates the tenths. If two or more units are required, the dividers must be placed on a point of division farther to the left.

When units, tenths, and hundredths, are required, place one foot of the dividers where the vertical line through the point which designates the units, intersects the line which designates the hundredths: then, extend the dividers to that line between *ad* and *bc* which designates the tenths: the distance so determined will be the one required.

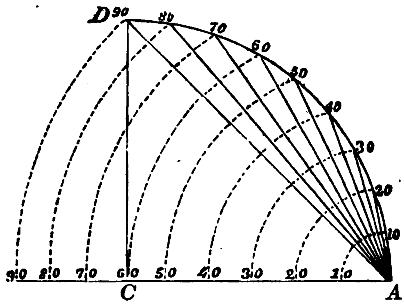
For example, to take off the distance 2,34, we place one foot of the dividers at *l*, and extend the other to *e*: and to take off the distance 2,58, we place one foot of the dividers at *p* and extend the other to *q*.

Remark I.—If a line is so long that the whole of it cannot be taken from the scale, it must be divided, and the parts of it taken from the scale in succession.

Remark II.—If a line be given upon the paper, its length can be found by taking it in the dividers and applying it to the scale.

Quest.—Show how to take off 1,35, also 2,47, also 1,78. If the line is so long that the whole of it cannot be taken at once, what do you do? If the line be given on paper, what do you do?

SCALE OF CHORDS.



11. If, with any radius, as AC, we describe the quadrant AD, and then divide it into 90 equal parts, each part is called a degree.

Through A, and each point of division, let a chord be drawn, and let the lengths of these chords be accurately laid off on a scale: such a scale is called a *scale of chords.* In the figure, the chords are drawn for every ten degrees.

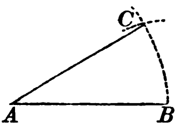
The scale of chords being once constructed, the radius of the circle from which the chords were obtained, is known; for, the chord marked 60 is always equal to the radius of the circle. A scale of chords is generally laid down on the scales which belong to cases of mathematical instruments, and is marked CHO.

QUEST.—11. Explain the construction of the scale of chords. What chord is equal to the radius of the circle?

PROBLEM VI.

To lay off, at a given point of a line, with the scale of chords, an angle equal to a given angle.

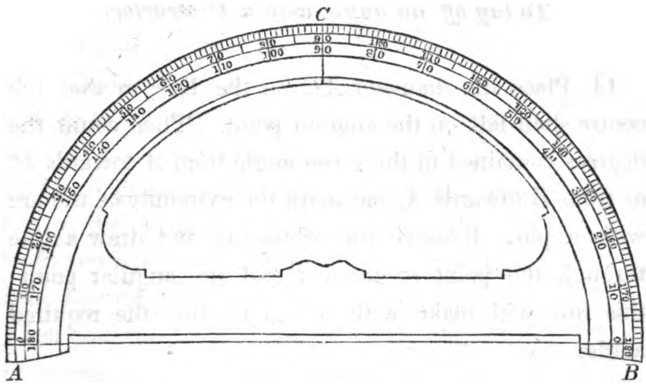
12. Let AB be the line, and A the given point.



Take from the scale the chord of 60 degrees, and with this radius and the point A as a centre, describe the arc BC. Then take from the scale the chord of the given angle, say 30 degrees, and with this line as a radius, and B as a centre, describe an arc cutting BC in C. Through A and C draw the line AC, and BAC will be the required angle.

QUEST.—12. Explain the manner of laying off an angle with the scale of chords.

SEMICIRCULAR PROTRACTOR.



13. This instrument is used to lay down, or protract angles. It may also be used to measure angles included between lines already drawn upon paper.

It consists of a brass semi-circle *ACB* divided to half degrees. The degrees are numbered from 0 to 180, both ways; that is, from *A* to *B*, and from *B* to *A*. The divisions, in the figure, are only made to degrees. There is a small notch at the middle of the diameter *AB*, which indicates the centre of the protractor.

QUEST.—13. Describe the circular protractor.

PROBLEM VII.

To lay off an angle with a Protractor.

14. Place the diameter AB on the line, so that the centre shall fall on the angular point. Then count the degrees contained in the given angle from A towards B, or from B towards A, and mark the extremity of the arc with a pin. Remove the protractor, and draw a line through the point so marked and the angular point: this line will make with the given line the required angle.

GUNTERS' SCALE.

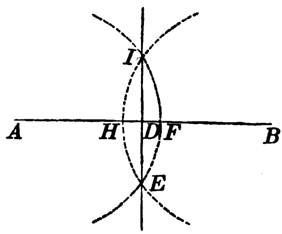
15. This is a scale of two feet in length, on the faces of which, a variety of scales are marked. The face on which the divisions of inches are made, contains, however, all the scales necessary for laying down lines and angles. These are, the scale of equal parts, the diagonal scale of equal parts, and the scale of chords, all of which have been described.

QUEST.—14. Explain the manner of laying off an angle with the circular protractor. 15. What is Gunter's scale?

PROBLEM VIII.

To bisect a given straight line: that is, to divide it into two equal parts.

16. Let AB be the given line. With A as a centre and a radius greater than half of AB, describe an arc IFE. Then remove the foot of the dividers from A to B, and with the same radius describe the arc EHI.

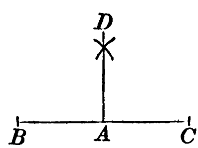


Then join the points I and E by the line IE: the point D where it intersects AB, will be the middle of the line AB.

PROBLEM IX.

At a given point in a given straight line, to erect a perpendicular to the line.

17. Let A be the given point, and BC the given line. From A lay off any two distances AB and AC equal to each other. Then, from the points B and C, as centres, with a radius greater than

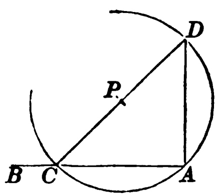


QUEST.—16. Describe the manner of bisecting an angle with the dividers. 17. Explain the manner of drawing a perpendicular to a given line at a given point.

BA, describe two arcs intersecting each other in *D*: draw *AD*, and it will be the perpendicular required.

SECOND METHOD.

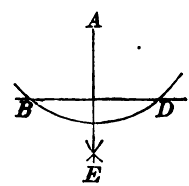
When the point A is near the end of the line.



18. Place one foot of the dividers at any point, as *P*, and extend the other leg to *A*. Then with *P* as a centre and radius from *P* to *A* describe the circumference of a circle. Through *C*, where the circumference cuts *BA* and the centre *P*, draw the line *CPD*. Then draw *AD*, and it will be perpendicular to *CA*, since *CAD* is an angle in a semi-circle.

PROBLEM X.

From a given point without a straight line, to let fall a perpendicular on the line.



19. Let *A* be the given point and *BD* the given line.

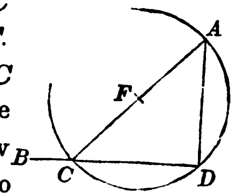
From the point *A* as a centre, with a radius sufficiently great, describe an arc cutting the line *BD* in the two points *B* and *D*: then mark the point *E*, equally distant from the points *B* and *D*, and draw *AE*: and *AE* will be the perpendicular required.

QUEST.—18. Explain the second method. 19. Explain the method of drawing a perpendicular to a line from a point without.

SECOND METHOD.

When the given point A, is nearly opposite one end of the given line.

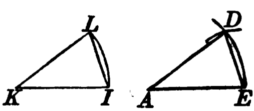
20. Draw AC to any point, as C of the line BD. Bisect AC at F. Then with F as a centre and FC or FA as a radius, describe the semi-circle CDA. Then draw DA, and it will be perpendicular to BD at D.



PROBLEM XI.

At a point, in a given line, to make an angle equal to a given angle.

21. Let A be the given point, AE the given line, and IKL the given angle.

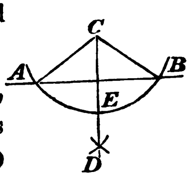


From the vertex K, as a centre, with any radius, describe the arc IL, terminating in the two sides of the angle. From the point A as a centre, with a distance AE equal to KI, describe the arc ED; then take the chord LI, with which, from the point E as a centre, describe an arc cutting the indefinite arc DE, in D; draw AD, and the angle EAD will be equal to the given angle K.

QUEST.—20. Give the second method. 21. Explain the manner of making an angle equal to a given angle.

PROBLEM XII.

To divide a given angle, or a given arc, into two equal parts.

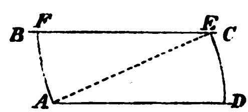


22. Let C be the given angle, and AEB the arc which measures it.

From the points A and B as centres, describe with the same radius two arcs cutting each other in D: through D and the centre C draw CD: the angle ACE will be equal to the angle ECB, and the arc AE to the arc EB.

PROBLEM XIII.

Through a given point to draw a parallel to a given line.



23. Let A be the given point, and BC the given line.

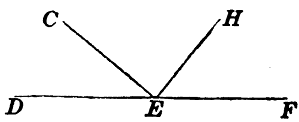
From A as a centre, with a radius greater than the shortest distance from A to BC, describe the indefinite arc ED: from the point E as a centre, with the same radius, describe the arc AF; make $ED=AF$, and draw AD: then will AD be the parallel required.

QUEST.—22. Show how to divide a given angle or a given arc into two equal parts. 23. Explain the manner of drawing through a given point, a line that shall be parallel to a given line.

PROBLEM XIV.

Two angles of a triangle being given, to find a third.

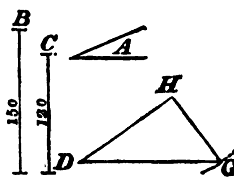
24. Draw the indefinite line *DEF*. At the point *E*, make the angle *DEC* equal to one of the given angles, and then the angle *CEH* equal to the other: the remaining angle *HEF* will be the third angle required.



PROBLEM XV.

Having given two sides and the included angle of a triangle, to describe the triangle.

25. Let the line $B=150$ feet, and $C=120$ feet, be the given sides; and $A=30$ degrees, the given angle: to describe the triangle on a scale of 200 feet to the inch.



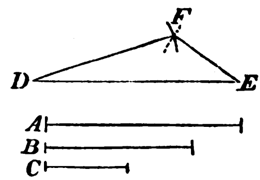
Draw the indefinite line *DG*, and at the point *D*, make the angle *GDH* equal to 30 degrees; then lay off *DG* equal to three quarters of an inch, and it will represent the side $B=150$ feet: make *DH* equal to six-tenths of an inch, and it will represent $C=120$ feet: then draw *GH*, and *DGH* will be the required triangle.

QUEST.—24. When two angles of a triangle are given, explain the manner of finding the third. 25. Explain the manner of describing a triangle when two sides and the included angle are known.

PROBLEM XVI.

The three sides of a triangle being given, to describe the triangle.

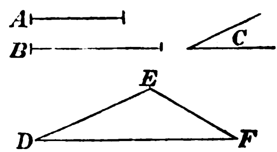
26. Let *A*, *B* and *C*, be the sides. Draw *DE* equal to the side *A*. From the point *D* as a centre, with a radius equal to the second side *B*, describe an arc: from *E* as a centre, with a radius equal to the third side *C*, describe another arc intersecting the former in *F*; draw *DF* and *EF*, and *DEF* will be the triangle required.



PROBLEM XVII.

Having given two sides of a triangle and an angle opposite one of them, to describe the triangle.

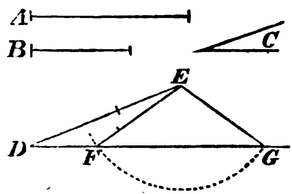
27. Let *A* and *B* be the given sides, and *C* the given angle which we will suppose is opposite the side *B*. Draw the indefinite line *DF* and make the angle *FDE* equal to the angle *C*: take $DE = A$, and from the point *E* as a centre, with a radius



QUEST.—26. Explain the manner of describing a triangle when the three sides are known. 27. Explain the manner of constructing a triangle when two sides and an angle opposite one is known, the known angle being acute.

equal to the other given side *B*, describe an arc cutting *DF* in *F*; draw *EF*: then will *DEF* be the required triangle.

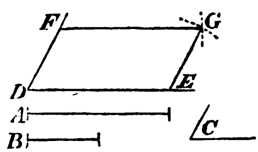
If the angle *C* is acute, and the side *B* less than *A*, then the arc described from the centre *E* with the radius $EF=B$ will cut the side *DF* in two points, *F* and *G*, lying on the same side of *D*: hence there will be two triangles, *DEF*, and *DEG*, either of which will satisfy all the conditions of the problem.



PROBLEM XVIII.

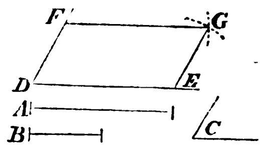
The adjacent sides of a parallelogram, with the angle which they contain, being given, to describe the parallelogram.

28. Let *A* and *B* be the given sides, and *C* the given angle.



Draw the line $DE=A$; at the point *D*, make the angle $EDF=C$; take $DF=B$: describe two arcs, the one from *F* as a centre, with a

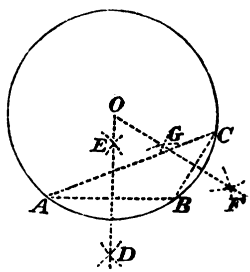
QUEST.—Explain the construction when the side opposite the given angle is the least. 28. Explain the manner of constructing a parallelogram when two adjacent sides and the included angle are known.



radius $FG = DE$, the other from E, as a centre, with a radius $EG = DF$; through the point G, where these arcs intersect each other, draw FG, EG; then $DEGF$ will be the parallelogram required.

PROBLEM XIX.

To describe the circumference of a circle which shall pass through three given points.



29. Let A, B and C be the three given points.

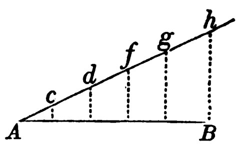
Join these points by straight lines AB, BC, CA. Then bisect any two of these straight lines by the perpendiculars OF, OD, as in Problem VIII, and the point O, where these perpendiculars intersect each other, will be the centre of the circle.

Place one foot of the dividers at this centre and extend the other to A, B, or C, and then with this radius, let the circumference be described.

QUEST.—29. Explain the manner of describing the circumference of a circle which shall pass through three given points.

PROBLEM XX.

To divide a given line AB, into any number of equal parts.

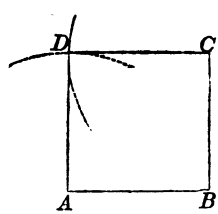


30. Let AB be the given line to be divided. Let it be required, if you please, to divide it into five equal parts.

Throughout A, one extremity of the line, draw Ah, making an angle with AB. Then lay off on Ah, five equal parts, Ac, cd, df, fg, gh, after which join h and B. Through the points of division c, d, f, and g, draw lines parallel to hB, and they will divide AB into the required number of equal parts.

PROBLEM XXI.

To describe a square on a given line.



31. Let AB be the given line. At the point B, draw BC perpendicular AB, by Problem IX, and then make it equal to AB.

Then, with A as a centre, and radius equal to AB, describe an arc; and with C as

QUEST.—30. Explain the manner of dividing a line into any number of equal parts. 31. Explain the manner of describing a square on a given line.

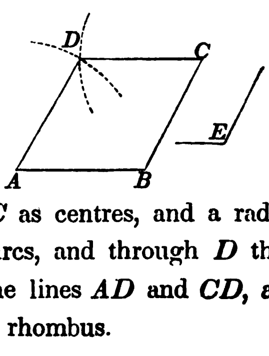
a centre, and the same distance AB, describe another arc, and through D, their point of intersection, draw AD and CD; then will $ABCD$ be the required square.

PROBLEM XXII.

To construct a rhombus, having given the length of one of the equal sides and one of the angles.

32. Let AB be equal to the given side, and E, the given angle.

At B, lay off an angle ABC, equal to E, by Problem XI, and make BC equal to AB. Then with A and C as centres, and a radius equal to AB, describe two arcs, and through D their point of intersection, draw the lines AD and CD, and $ABCD$ will be the required rhombus.



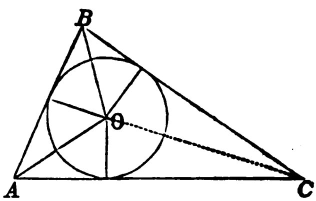
QUEST.—32. Show how to construct a rhombus, having given the length of one of the equal sides and one of the angles.

PROBLEM XXIII.

To inscribe a circle in a given triangle.

33. Let ABC be the given triangle.

Bisect either two of the angles, as A and C, by the lines AO and CO, and the point of intersection O will be the centre of the inscribed circle. Then, through the point of intersection O, draw a line perpendicular to either side, and it will be the radius.



PROBLEM XXIV.

In a given circle to inscribe.

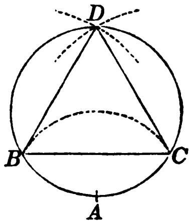
First.—An equilateral triangle.

Second.—A regular hexagon.

Third.—A regular dodecagon.

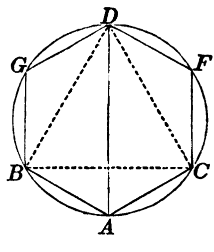
QUEST.—33. Show how to inscribe a circle in a given triangle.

For the equilateral triangle.



34. With any point *A*, as a centre, and radius equal to the radius of the circle, describe an arc cutting the circumference in *B* and *C*. Then bisect the arc *BDC* by Problem XII, after which, draw *BC*, *BD* and *CD*, and *BDC* will be an equilateral triangle.

For the hexagon.



35. Describe the equilateral triangle as before. Then bisect the arc *CD* in *F*, and the arc *BD* at *G*, and draw *AC*, *CF*, *FD*, *DG*, *GB*, and *BA*, and *ACFDGB* will be the hexagon required. Or the hexagon may be inscribed by applying the radius six times around the circumference.

For the dodecaedron.

36. Bisect the arcs which subtend the chords of the

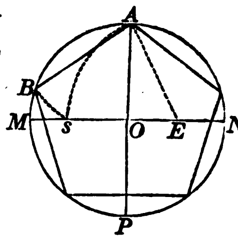
Quest.—34. Explain the manner of inscribing an equilateral triangle in a given circle. 35. Explain the manner of inscribing a regular hexagon in a given circle. 36. Explain the manner of inscribing a regular dodecagon in a circle.

hexagon, and through the points of bisection draw chords, and there will be formed a regular dodecaedron.

PROBLEM XXV.

To inscribe in a circle a regular pentagon.

37. Draw the diameters *AP* and *MN* at right angles to each other, and bisect the radius *ON* at *E*. From *E* as a centre, and *EA* as a radius, describe the arc *As*; and from the point *A*, as a centre and radius *As*, describe the arc *sB*.



Join the points *A* and *B*, and the line *AB* being applied five times around the circle, will form the required pentagon.

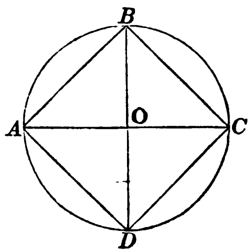
38. For the decagon, bisect the arcs which subtend the sides of the pentagon, and join the points of bisection; and the lines so drawn will form the regular decagon.

QUEST.—37. Explain the manner of inscribing a regular pentagon in a given circle. 38. Explain the manner of inscribing a dodecagon.

PROBLEM XXVI.

To inscribe a square in a given circle.

39. Let *ABCD* be the given circle. Draw two diameters *DB* and *AC* at right angles to each other, and through the points *A*, *B*, *C* and *D*, draw the lines *AB*, *BC*, *CD* and *DA*: then *ABCD* will be an inscribed square.

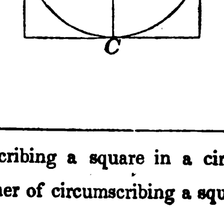


40. By bisecting the arcs *AB*, *BC*, *CD*, and *DA*, and joining the points of bisection, we can form an octagon; and by bisecting the arcs which subtend the sides of the octagon, we can inscribe a polygon of sixteen sides.

PROBLEM XXVII.

To circumscribe a square about a circle.

41. Draw two diameters *AB* and *CD* at right angles to each other; and through their extremities *A*, *B*, *C* and *D*, draw lines respectively parallel to the diameters *CD* and *AB*: a square will thus be formed circumscribing the circle.

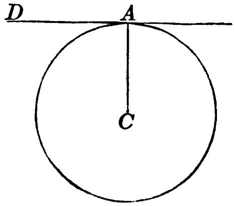


QUEST.—39. Explain the manner of inscribing a square in a circle. 40. Also, an octagon. 41. Explain the manner of circumscribing a square about a circle.

PROBLEM XXVIII.

To draw a line which shall be tangent to the circumference of a circle at a given point.

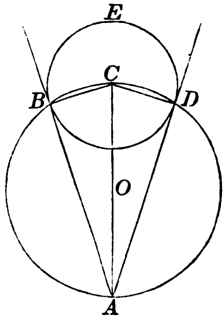
42. Let *A* be the given point. Through *A* draw the radius *AC*, and then draw *DA* perpendicular to the radius at the extremity *A*. The line *DA* will be tangent to the circumference at the point *A*.



PROBLEM XXIX.

Through a given point without a circle to draw a line, which shall be tangent to the circumference.

43. Let *A* be the given point without the given circle *BED*. Join the centre *C* and the given point *A*, and bisect the line *CA* at *O*.



With *O* as a centre, and *OA* as a radius, describe the circumference *ABCD*. Through *B* and *D* draw the lines *AB* and *AD*, and they will be tangent to the circle *BED* at the points *B* and *D*.

QUEST.—42. Explain the manner of drawing a tangent line to a circle at a given point of the circumference. 42. Explain the manner of drawing a tangent line to a circle through a given point without.

PART III.

SECTION I.

MENSURATION OF SURFACES.

1. The area of any figure, has already been defined to be the measure of its surface. (Part I. § V. Art. 7). This measure is merely the number of squares which the figure is equal to.

A square whose side is one inch, one foot, or one yard, &c., is called the *measuring unit;* and the area or content of a figure is expressed by the number of such squares which the figure contains.

2. In the questions involving decimals, the decimals are generally carried to four places, and then taken to the nearest figure. That is, if the fifth decimal figure is 5, or greater than 5, the fourth figure is increased by one.

QUEST.—1. What is the area of a figure? What is the measure? What is a square whose side is 1 foot, 1 yard, &c. called? How is the area or content of a figure expressed? 2. In questions involving decimals, to how many places are the figures generally carried? What is meant by taking the nearest figure?

3. Surveyors, in measuring land, generally use a chain called Gunters' chain. This chain is four rods, or 66 feet in length, and is divided into 100 links.

4. An *acre* is a surface equal in extent to 10 square chains; that is, equal to a rectangle of which one side is ten chains, and the other side one chain.

One-quarter of an acre, is called a ***rood.***

Since the chain is 4 rods in length, 1 square chain contains 16 square rods; and therefore, an acre, which is 10 square chains, contains 160 square rods, and a rood contains 40 square rods. The square rods are called perches.

5. Land is generally computed in acres, roods, and perches, which are respectively designated by the letters *A. R. P.*

When the linear dimensions of a survey are chains or links, the area will be expressed in square chains or square links, and it is necessary to form a rule for reducing this area to acres, roods, and perches. For this purpose, let us form the following

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QUEST.—3. What chain is generally used by land surveyors? What is the length of this chain? How is it divided? 4. What is an acre of land? What is a quarter of an acre called? What are square rods called? 5. In what is land generally computed? How is each denomination designated?

TABLE.

1 square chain = 10000 square links.
1 acre = 10 square chains = 100000 square links.
1 acre = 4 roods = 160 perches.
1 square mile = 6400 square chains = 640 acres.

6. Now, when the linear dimensions are links, the area will be expressed in square links, and may be reduced to acres by dividing by 100000, the number of square links in an acre: that is, by pointing off five decimal places from the right hand.

If the decimal part be then multiplied by 4, and five places of decimals pointed off from the right hand, the figures to the left will express the roods.

If the decimal part of this result be now multiplied by 40, and five places for decimals pointed off, as before, the figures to the left will express the perches.

If one of the dimensions be in links, and the other in chains, the chains may be reduced to links by annexing two ciphers: or, the multiplication may be made without annexing the ciphers, and the product reduced to

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How many square links in a square chain? How many square chains in an acre? How many acres in one square mile? 6. If the linear dimensions are links, in what will the area be expressed? How will this be reduced to acres? How can the decimal part be then reduced to roods? How then to perches? If one dimension be links, and the other chains, how may the chains be brought to links?

acres and decimals of an acre, by pointing off three decimal places at the right hand.

When both the dimensions are in chains, the product is reduced to acres by dividing by 10, or pointing off one decimal place.

From which we conclude: that,

1st. *If links be multiplied by links, the product is reduced to acres by pointing off five decimal places from the right hand.*

2d. *If chains be multiplied by links, the product is reduced to acres by pointing off three decimal places from the right hand.*

3d. *If chains be multiplied by chains, the product is reduced to acres by pointing off one decimal place from the right hand.*

7. Since there are 16,5 feet in a rod, a square rod is equal to

$$16{,}5 \times 16{,}5 = 272{,}25 \text{ square feet.}$$

If the last number be multiplied by 160, we shall have

$$272{,}25 \times 160 = 43560 = \text{the square feet in an acre.}$$

QUEST.—If both the dimensions are chains, how may the product be reduced to acres? Give, then, the three general results. 7. How many square feet in a square rod?

Since there are 9 square feet in a square yard, if the last number be divided by 9, we obtain

4840 = the number of square yards in an acre.

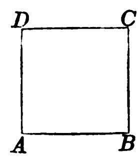
PROBLEM I.

8. To find the area of a Square, a Rectangle, a Rhombus, or a Parallelogram.

RULE

Multiply the base by the perpendicular height and the product will be the area.

EXAMPLES.



1. Required the area of the square *ABCD* each of whose sides is 36 feet.

We multiply two sides of the square together, and the product is the area in square feet.

Operation.

$36 \times 36 = 1296$ *sq. ft.*

2. How many acres, roods, and perches, in a square whose side is 35,25 chains?

Ans. 124 *A.* 1 *R.* 1 *P.*

QUEST.—8. Give the rule for finding the area of a square, a rectangle, rhombus or parallelogram.

3. What is the area of a square whose side is 8 feet 4 inches? (See Arithmetic, § 171).

Ans. 69 *ft.* 5′ 4″.

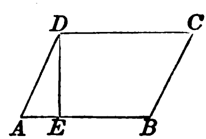
4. What is the content of a square field whose side is 46 rods?

Ans. 13 *A.* 0 *R.* 36 *P.*

5. What is the area of a square whose side is 4769 yards?

Ans. 22743361 *sq. yds.*

6. What is the area of the parallelogram *ABCD*, of which the base *AB* is 64 feet, and altitude *DE*, 36 feet?



We multiply the base 64, by the perpendicular height 36, and the product is the required area.

Operation.

$64 \times 36 = 2304$ *sq. ft.*

2. What is the area of a parallelogram whose base is 12,25 yards, and altitude 8,5?

Ans. 104,125 *sq. yds.*

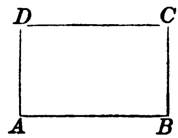
7. What is the area of a parallelogram whose base is 8,75 chains, and altitude 6 chains.

Ans. 5 *A.* 1 *R.* 0 *P.*

8. What is the area of a parallelogram whose base is 7 feet 9 inches, and altitude 3 feet 6 inches?

Ans. 27 *sq. ft.* 1′ 6″.

9. To find the area of a rectangle *ABCD*, of which the base $AB=45$ yards, and the altitude $AD=15$ yards.



Here we simply multiply the base by the altitude, and the product is the area.

Operation.

$45\times15=675$ *sq. yds.*

10. What is the area of a rectangle whose base is 14 feet 6 inches, and breadth 4 feet 9 inches?

Ans. 68 *sq. ft.* 10′ 6″.

11. Find the area of a rectangular board whose length is 112 feet, and breadth 9 inches.

Ans. 84 *sq. ft.*

12. Required the area of a rhombus whose base is 10,51, and breadth 4,28 chains.

Ans. 4 *A.* 1 *R.* 39,7 *P*+.

13. Required the area of a rectangle whose base is 12 feet 6 inches, and altitude 9 feet 3 inches.

Ans. 115 *sq. ft.* 7′ 6″.

PROBLEM II.

9. To find the area of a triangle, when the base and altitude are known.

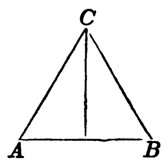
RULE.

1st. *Multiply the base by the altitude, and half the product will be the area.*

Or, 2d. *Multiply the base by half the altitude and the product will be the area.*

EXAMPLES.

1. Required the area of the triangle *ABC*, whose base *AB* is 10.75 feet and altitude 7,25 feet.



We first multiply the base by the altitude, and then divide the product by 2.

Operation.

$10,75 \times 7,25 = 77,9375$
and
$77,9375 \div 2 = 38,96875$
$=$ area.

2. What is the area of a triangle whose base is 18 feet 4 inches, and altitude 11 feet 10 inches?

Ans. 108 *sq. ft.* 5′ 8″.

QUEST.—9. Give the rule for finding the area of a triangle when the base and altitude are known?

3. What is the area of a triangle whose base is 12,25 chains, and altitude 8,5 chains?

Ans. 5 *A.* 0 *R.* 33 *P.*

4. What is the area of a triangle whose base is 20 feet, and altitude 10,25 feet.

Ans. 102,5 *sq. ft.*

5. Find the area of a triangle whose base is 625 and altitude 520 feet.

Ans. 162500 *sq. ft.*

6. Find the number of square yards in a triangle whose base is 40 and altitude 30 feet.

Ans. $66\frac{2}{3}$ *sq. yds.*

7. What is the area of a triangle whose base is 72,7 yards, and altitude 36,5 yards?

Ans. 1326,775 *sq. yds.*

PROBLEM III.

10. To find the area of a triangle when the three sides are known.

RULE.

1*st.* *Add the three sides together and take half their sum.*

QUEST.—10. Give the rule for finding the area of a triangle when the three sides are known.

2nd. From this half sum take each side separately.

3rd. Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.

EXAMPLES.

1. Find the area of a triangle whose sides are 20, 30 and 40 rods.

20	45	45	45
30	20	30	40
40	25 1st rem.	15 2d rem.	5 3rd rem.
2)90			
45 half sum.			

Then, to obtain the product, we have

$$45 \times 25 \times 15 \times 5 = 84375;$$

from which we find

$$\text{area} = \sqrt{84375} = 290{,}4737 \text{ perches.}$$

2. How many square yards of plastering are there in a triangle, whose sides are 30, 40 and 50 feet.

Ans. $66\frac{2}{3}$.

3. The sides of a triangular field are 49 chains, 50,25 chains, and 25,69 chains: what is its area?

Ans. 61 *A.* 1 *R.* 39,68 *P.*

4. What is the area of an isosceles triangle, whose base is 20, and each of the equal sides 15?

Ans. 111,803.

5. How many acres are there in a triangle whose three sides are 380, 420 and 765 yards.

Ans. 9*A.* 0 *R.* 38 *P.*

6. How many square yards in a triangle whose sides are 13, 14, and 15 feet.

Ans. $9\frac{1}{3}$.

7. What is the area of an equilateral triangle whose side is 25 feet?

Ans. 270,6329 *sq. ft.*

8. What is the area of a triangle whose sides are 24, 36, and 48 yards?

Ans. 418,282 *sq. yds.*

PROBLEM IV.

11. To find the hypothenuse of a right angled triangle when the base and perpendicular are known.

RULE.

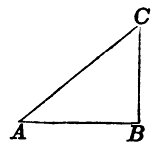
1*st. Square each of the sides separately.*

2*nd. Add the squares together.*

3*rd. Extract the square root of the sum, which will be the hypothenuse of the triangle.*

QUEST.—11. How do you find the hypothenuse of a triangle, when the base and perpendicular are known?

EXAMPLES.



1. In the right angled triangle ABC, we have,

$$AB=30 \text{ feet}, \ BC=40 \text{ feet},$$

to find AC,

We first square each side, and then take the sum, of which we extract the square root, which gives

$$AC=\sqrt{2500}=50 \text{ feet}.$$

Operation.

$$\overline{30}^2= \ 900$$
$$\overline{40}^2=1600$$
$$\text{sum}=2500$$

2. The wall of a building, on the brink of a river is 120 feet high, and the breadth of the river 70 yards: what is the length of a line which would reach from the top of the wall to the opposite edge of the river.

Ans. 241,86 feet.

3. The side roofs of a house of which the eaves are of the same height, form a right angle with each other at the top. Now, the length of the rafters on one side is 10 feet, and on the other 14 feet: what is the breadth of the house?

Ans. 17,204 feet.

4. What would be the width of the house, in the last example, if the rafters on each side were 10 feet?

Ans. 14,142 feet.

5. What would be the width if the rafters on each side were 14 feet?

Ans. 19,7989 feet.

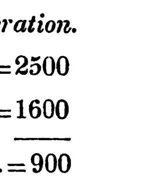
PROBLEM V.

12. When the hypothenuse and one side of a right angled triangle are known to find the other side.

RULE.

Square the hypothenuse and also the other given side, and take their difference: extract the square root of this difference, and the result will be the required side.

EXAMPLES.



1. In the right angled triangle *ABC*, there are given

$AC=50$ ft., and $AB=40$ ft.

required the side *BC*.

We first square the hypothenuse and the other side, after which we take the difference, and then extract the square root, which gives

Operation.

$\overline{50}^2=2500$

$\overline{40}^2=1600$

Diff. $=900$

$$BC=\sqrt{900}=30 \text{ feet.}$$

2. The height of a precipice on the brink of a river

QUEST.—12. How do you find one side of a right angled triangle, when the hypothenuse and the other are known?

is 103 feet, and a line of 320 feet in length will just reach from the top of it to the opposite bank: required the breadth of the river.

Ans. 302,9703 feet.

3. The hypothenuse of a triangle is 53 yards, and the perpendicular 45 yards: what is the base?

Ans. 28 yards.

4. A ladder 60 feet in length, will reach to a window 40 feet from the ground on one side of the street, and by turning it over to the other side, it will reach a window 50 feet from the ground: required the breadth of the street.

Ans. 77,8875 feet.

PROBLEM VI.

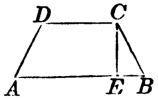
13. To find the area of a trapezoid.

RULE.

Multiply the sum of the parallel sides by the perpendicular distance between them, and then divide the product by two:—the quotient will be the area.

EXAMPLES.

1. Required the area of the trapezoid *ABCD*, having given



$AB=321,51$ ft., $DC=214,24$ ft., and $CE=171,16$ ft.

QUEST.—13. How do you find the area of a trapezoid?

We first find the sum of the sides, and then multiply it by the perpendicular height, after which, we divide the product by 2, for the area.

Operation.

$321{,}51+214{,}24=535{,}75=$ sum of parallel sides.

Then,

$535{,}75\times171{,}16=91698{,}97$

and, $\frac{91698{,}97}{2}=45849{,}485$ $=$the area.

2. What is the area of a trapezoid, the parallel sides of which, are 12,41 and 8,22 chains, and the perpendicular distance between them 5,15 chains?

Ans. 5 *A.* 1 *R.* 9,956 *P.*

3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches?

Ans. 230 *sq. ft.* 5′ 7″.

4. Required the area of a trapezoid whose parallel sides are 20,5 and 12,25, and the perpendicular distance between them 10,75 yards.

Ans. 176,03125 *sq. yds.*

5. What is the area of a trapezoid whose parallel sides are 7,50 chains, and 12,25 chains, and the perpendicular height 15,40 chains?

Ans. 15 *A.* 0 *R.* 33,2 *P.*

6. What is the content when the parallel sides are 20 and 32 chains, and the perpendicular distance between them 26 chains?

Ans. 67 *A.* 2 *R.* 16 *P.*

PROBLEM VII.

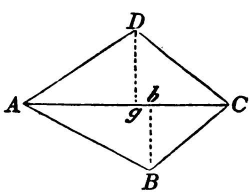
14. To find the area of a quadrilateral.

RULE.

Measure the four sides of the quadrilateral, and also one of the diagonals: the quadrilateral will thus be divided into two triangles, in both of which all the sides will be known. Then, find the areas of the triangles separately, and their sum will be the area of the quadrilateral.

EXAMPLES.

1. Suppose that we have measured the sides and diagonal AC, of the quadrilateral $ABCD$, and found



$AB=40{,}05$ ch, $CD=29{,}87$ ch,

$BC=26{,}27$ ch, $AD=37{,}07$ ch,

and $AC=55$ ch:

required the area of the quadrilateral.

Ans. 101 *A.* 1 *R.* 15 *P.*

QUEST.—14. How do you find the area of a quadrilateral?

REMARK.—Instead of measuring the four sides of the quadrilateral, we may let fall the perpendiculars Bb, Dg, on the diagonal AC. The area of the triangles may then be determined by measuring these perpendiculars and the diagonal AC. The perpendiculars are $Dg=$ 18,95 ch, and $Bb=17,92$ ch.

2. Required the area of a quadrilateral whose diagonal is 80,5 and two perpendiculars 24,5 and 30,1 feet?

Ans. 2197,65 *sq. ft.*

3. What is the area of a quadrilateral whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches?

Ans. 6347 *sq. ft.* 3'.

4. How many square yards of paving in a quadrilateral whose diagonal is 65 feet, and the two perpendiculars 28 and $33\frac{1}{2}$ feet?

Ans. $222\frac{1}{12}$ *sq. yds.*

5. Required the area of a quadrilateral whose diagonal is 42 feet, and the two perpendiculars 18 and 16 feet.

Ans. 714 *sq. ft.*

6. What is the area of a quadrilateral in which the diagonal is 320,75 chains, and the two perpendiculars 69,73 chains, and 130,27 chains?

Ans. 3207 *A.* 2 *R.*

PROBLEM VIII.

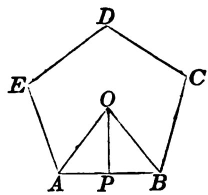
15. To find the area of a regular polygon.

RULE.

Multiply half the perimeter of the figure by the perpendicular let fall from the centre on one of the sides, and the product will be the area.

EXAMPLES.

1. Required the area of the regular pentagon *ABCDE*, each of whose sides *AB*, *BC*, &c., is 25 feet and the perpendicular *OP*, 17,2 feet.



We first multiply one side by the number of sides and divide the product by 2:—this gives half the perimeter which we multiply by the perpendicular for the area.

Operation.

$\frac{25\times5}{2}=62{,}5=$ half the perimeter. Then,

$62{,}5\times17{,}2=1075$ *sq. ft.* = the area.

2. The side of a regular pentagon is 20 yards and the perpendicular from the centre on one of the sides 13,76382: required the area.

Ans. 688,191 *sq. yds.*

QUEST.—15. How do you find the area of a regular polygon?

3. The side of a regular hexagon is 14, and the perpendicular from the centre on one of the sides 12,1243556: required the area.

Ans. 509,2229352 *sq. ft.*

4. Required the area of a regular hexagon whose side is 14,6, and perpendicular from the centre 12,64 feet.

Ans. 553,632 *sq. ft.*

5. Required the area of a heptagon whose side is 19,38, and perpendicular 20 feet.

Ans. 1356,6 *sq. ft.*

6. Required the area of an octagon whose side is 9,941 yards and perpendicular 12 yards.

Ans. 477,168 *sq. yds.*

16. The following table shows the areas of the ten regular polygons when the side of each is equal to 1: it also shows the length of the radius of the inscribed circle.

Number of sides.	Names.	Areas.	Radius of inscribed circle.
3	Triangle,	0,4330127	0,2886751
4	Square,	1,0000000	0,5000000
5	Pentagon,	1,7204774	0,6881910
6	Hexagon,	2,5980762	0,8660254
7	Heptagon,	3,6339124	1,0382617
8	Octagon,	4,8284271	1,2071068
9	Nonagon,	6,1818242	1,3737387
10	Decagon,	7,6942088	1,5388418
11	Undecagon,	9,3656404	1,2028437
12	Dodecagon,	11,1961524	1,8660254

QUEST.—16. How are similar polygons to each other? How do you find the area of a regular polygon from the table?

Now, since the areas of similar polygons are to each other as the squares described on their homologous side (see Part I. § VII. Art. 3), we have

1^2 : tabular area : : any side squared : area.

Hence, to find the area of a regular polygon, we have the following

RULE.

1st. *Square the side of the polygon.*

2d. *Multiply the square so found, by the tabular area set opposite the polygon of the same number of sides, and the product will be the required area.*

EXAMPLES.

1. What is the area of a regular hexagon whose side is 20?

$\overline{20}^2 = 400$ and tabular area $= 2{,}5980762$.

Hence,

$2{,}5980762 \times 400 = 1039{,}23048 =$ the area.

2. What is the area of a pentagon whose side is 25?

Ans. 1075,298375.

3. What is the area of a heptagon whose side is 30?

Ans. 3270,52116.

4. What is the area of an octagon whose side is 10 feet?

Ans. 482,84271 *sq. ft.*

5. The side of a nonagon is 50: what is its area?

Ans. 15454,5605.

6. The side of an undecagon is 20: what is its area?
Ans. 3746,25616.

7. The side of a dodecagon is 40: what is its area?
Ans. 17913,84384.

PROBLEM IX.

17. To find the area of a long and irregular figure, bounded on one side by a straight line.

RULE.

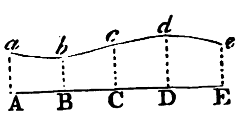
1st. *Divide the right line or base into any number of equal parts, and measure the breadth of the figure at the points of division, and also at the extremities of the base.*

2nd. *Add together the intermediate breadths, and half the sum of the extreme ones.*

3rd. *Multiply this sum by the base line, and divide the product by the number of equal parts of the base.*

EXAMPLES.

1. The breadths of an irregular figure, at five equidistant places, *A*, *B*, *C*, *D* and *E*, being 8,20 ch, 7,40 ch, 9,20 ch, 10,20 ch. and



QUEST.—17. How do you find the area of a long and irregular piece of ground?

8,60 chains, and the whole length 40 chains; required the area.

8,20		35,20	
8,60		40	
2)16,80		4)1408,00	
8,40	mean of the extremes.	352,00	square ch.
7,40			
9,20			
10,20			
35,20	sum		

Ans. 35 *A.* 32 *P.*

2. The length of an irregular piece of land being 21 ch, and the breadths, at six equidistant points, being 4,35 ch, 5,15 ch, 3,55 ch, 4,12 ch, 5,02 ch, and 6,10 chains: required the area.

Ans. 9 *A.* 2 *R.* 30 *P.*

3. The length of an irregular figure is 84 yards, and the breadths at six equidistant places are 17,4; 20,6; 14,2; 16,5; 20,1, and 24,4: what is the area?

Ans. 1550,64 *sq. yds.*

4. The length of an irregular field is 39 rods, and its breadths at five equidistant places, are 4,8; 5,2; 4,1; 7,3, and 7,2 rods: what is its area?

Ans. 220,35 *sq. rods.*

5. The length of an irregular field is 50 yards, and its breadths at seven equidistant points are 5,5; 6,2; 7,3; 6; 7,5; 7; and 8,8 yards: what is its area?

Ans. 342,916 *sq. yards.*

6. The length of an irregular figure being 37,6, and the breadths at nine equidistant places, 0; 4,4; 6,5; 7,6; 5,4; 8; 5,2; 6,5; and 6,1: what is the area?

Ans. 219,255.

PROBLEM X.

18. To find the circumference of a circle when the diameter is known.

RULE.

Multiply the diameter by 3,1416, *and the product will be the circumference.*

EXAMPLES.

1. What is the circumference of a circle whose diameter is 17?

We simply multiply the number 3,1416 by the diameter, and the product is the circumference.	*Operation.* $3,1416 \times 17 = 53,4072$ which is the circumference.

2. What is the circumference of a circle whose diameter is 40 feet?

Ans. 125,664 *feet.*

QUEST.—18. How do you find the circumference of a circle when the diameter is known?

3. What is the circumference of a circle whose diameter is 12 feet?

Ans. 37,6992 *feet.*

4. What is the circumference of a circle whose diameter is 22 yards?

Ans. 69,1152 *yards.*

5. What is the circumference of the earth—the mean diameter being about 7921 miles?

Ans. 24884,6136 *miles.*

PROBLEM XI.

19. To find the diameter of a circle when the circumference is known.

RULE.

Divide the circumference by the number 3,1416, *and the quotient will be the diameter.*

EXAMPLES.

1. The circumference of a circle is 69,1152 yards: what is the diameter?

We simply divide the circumference by 3,1416, and the quotient 22 is the diameter sought.

Operation.

```
3,1416)691152(22
        62832
        -----
         62832
         62832
         -----
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QUEST.—19. How do you find the diameter of a circle when the circumference is known?

2. What is the diameter of a circle whose circumference is 11652,1944 feet?

Ans. 3709. *ft.*

3. What is the diameter of a circle whose circumference is 6850?

Ans. 2180,4176.

4. What is the diameter of a circle whose circumference is 50?

Ans. 15,915.

5. If the circumference of a circle is 25000,8528; what is the diameter?

Ans. 7958.

PROBLEM XII.

20. To find the length of a circular arc, when the number of degrees which it contains, and the radius of the circle are known.

RULE.

Multiply the number of degrees by the decimal ,01745, *and the product arising by the radius of the circle.*

EXAMPLES.

1. What is the length of an arc of 30 degrees, in a circle whose radius is 9 feet.

QUEST.—20. How do you find the length of an arc when you know the number of degrees and the radius of the circle?

We merely multiply the given decimal by the number of degrees, and by the radius.	*Operation.* $,01745 \times 30 \times 9 = 4,7115$, which is the length of the required arc.

21. REMARK.—When the arc contains degrees and minutes, reduce the minutes to the decimals of a degree, which is done by dividing them by 60.

2. What is the length of an arc containing 12° 10′ or $12\frac{1}{6}$°, the diameter of the circle being 20 yards?

Ans. 2,1231.

3. What is the length of an arc of 10° 15′ or $10\frac{1}{4}$°, in a circle whose diameter is 68.

Ans. 6,0813.

PROBLEM XIII.

22. To find the length of the arc of a circle when the chord and radius are given.

RULE.

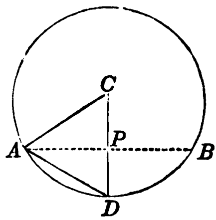
1st. *Find the chord of half the arc.*

2d. *From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by three, and the quotient will be the length of the arc, nearly.*

QUEST.—21. If the arc contains minutes, what do you do? 22. How do you find the length of the arc of a circle when diameter and chord of the arc are known?

EXAMPLES.

1. The chord $AB=30$ feet, and the radius $AC=20$ feet: what is the length of the arc ADB.



First, draw CD perpendicular to the chord AB: it will bisect the chord at P, and the arc of the chord at D. Then $AP=15$ feet. Hence.

$$\overline{AC}^2-\overline{AP}^2=\overline{CP}^2 \text{ : that is}$$

$$400-225=175 \quad \text{and} \quad \sqrt{175}=13{,}228=CP.$$

Then, $\quad CD-CP=20-13{,}228=6{,}772=DP.$

Again, $\quad AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{225+45{,}859984}$:

hence, $\quad AD=16{,}4578=$ chord of the half arc.

Then, $\quad \dfrac{16{,}4578\times 8-30}{3}=33{,}8874=$ arc ADB.

2. What is the length of an arc the chord of which is 24 feet and the radius of the circle 20 feet?

Ans. 25,7309 *ft.*

3. The chord of an arc is 16 and the diameter of the circle 20: what is the length of the arc?

Ans. 18,5178.

4. The chord of an arc is 50, and the chord of half the arc is 27: what is the length of the arc?

Ans. $55\frac{1}{3}$.

PROBLEM XIV.

23. To find the area of a circle when the diameter and circumference are both known.

RULE.

Multiply the circumference by half the radius and the product will be the area.

EXAMPLES.

1. What is the area of a circle whose diameter is 10, and circumference 31,416 ?

If the diameter be 10, the radius is 5, and half the radius $2\frac{1}{2}$: hence the circumference multiplied by $2\frac{1}{2}$ gives the area.	*Operation.* $31,416 \times 2\frac{1}{2} = 78,54$, which is the area.

2. Find the area of a circle whose diameter is 7, and circumference 21,9912 yards.

Ans. 38,4846 *yds.*

3. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet, and circumference 10,9956.

Ans. 1,069016.

4. What is the area of a circle whose diameter is 100, and circumference 314,16.

Ans. 7854.

QUEST.—23. How do you find the area of a circle when the diameter and circumference are both known ?

5. What is the area of a circle whose diameter is 1, and circumference 3,1416.

Ans. 0,7854.

6. What is the area of a circle whose diameter is 40, and circumference 131,9472?

Ans. 1319,472.

PROBLEM XV.

24. To find the area of a circle when the diameter only is known.

RULE.

Square the diameter, and then multiply by the decimal, ,7854.

EXAMPLES.

1. What is the area of a circle whose diameter is 5?

We square the diameter which gives us 25, and we then multiply this number and the decimal ,7854 together.	*Operation.* ,7854 $\overline{5}^2 =$ 25 39270 15708 area = 19,6350

QUEST.—24. How do you find the area of a circle when the diameter only is known?

2. What is the area of a circle whose diameter is 7?

Ans. 38,4846.

3. What is the area of a circle whose diameter is 4,5?

Ans. 15,90435.

4. What is the number of square yards in a circle whose diameter is $1\frac{1}{6}$ yards?

Ans. 1,069016.

5. What is the area of a circle whose diameter is 8,75 feet?

Ans. 60,1322 *sq. ft.*

PROBLEM XVI.

25. To find the area of a circle when the circumference only is known.

RULE.

Multiply the square of the circumference by the decimal ,07958, *and the product will be the area very nearly.*

EXAMPLES.

1. What is the area of a circle whose circumference is 3,1416?

QUEST.—25. How do you find the area of a circle when the circumference only is known?

We first square the circumference, and then multiply by the decimal ,07958.

Operation.

$$\overline{3,1416}^2 = 9,86965056$$
$$,07958$$
$$\text{area} = ,7854+$$

2. What is the area of a circle whose circumference is 91?

Ans. 659,00198.

3. Suppose a wheel turns twice in tracking $16\frac{1}{2}$ feet, and that it turns just 200 times in going round a circular bowling green: what is the area in acres, roods and perches?

Ans. 4 *A.* 3 *R.* 35,8 *P.*

4. How many square feet are there in a circle whose circumference is 10,9956 yards?

Ans. 86,5933.

5. How many perches are there in a circle whose circumference is 7 miles?

Ans. 399300,608.

PROBLEM XVII.

26. Having given a circle, to find a square which shall have an equal area.

QUEST.—26. Having given the diameter of a circle, how will you find the side of an equivalent square? Having given the circumference of a circle, how will you find the side of an equivalent square?

RULE.

1st. *The diameter* $\times$,8862 = *side of an equivalent square.*

2d. *The circumference* $\times$,2821 = *side of an equivalent square.*

EXAMPLES.

1. The diameter of a circle is 100: what is the side of a square of an equal area?

Ans. 88,62.

2. The diameter of a circular fish pond is 20 feet, what would be the side of a square fish pond of an equal area?

Ans. 17,724 *ft.*

3. A man has a circular meadow of which the diameter is 875 yards, and wishes to exchange it for a square one of equal size: what must be the side of the square?

Ans. 775,425.

4. The circumference of a circle is 200: what is the side of a square of an equal area?

Ans. 56,42.

5. The circumference of a round fish pond is 400 yards: what is the side of a square fish pond of equal area?

Ans. 112,84.

6. The circumference of a circular bowling green is 412 yards: what is the side of a square one of equal area?

Ans. 116,2252 *yds.*

PROBLEM XVIII.

27. Having given the diameter or circumference of a circle, to find the side of the inscribed square.

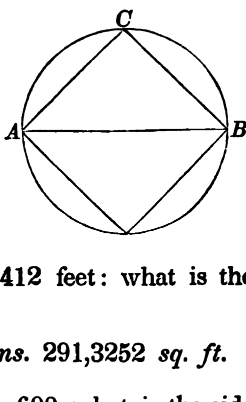
RULE.

1st. *The diameter* $\times$,7071 = *side of the inscribed square.*

2d. *The circumference* $\times$,2251 = *side of the inscribed square.*

EXAMPLES.

1. The diameter *AB* of a circle is 400: what is the value of *AC*, the side of the inscribed square?



Here,

,7071 $\times$ 400 = 282,8400 = *AC*.

2. The diameter of a circle is 412 feet: what is the side of the inscribed square.

Ans. 291,3252 *sq. ft.*

3. If the diameter of a circle be 600, what is the side of the inscribed square?

Ans. 424,26.

QUEST.—27. Having given the diameter of a circle, how will you find the side of an inscribed square? Having given the circumference of a circle, how will you find the side of an inscribed square?

4. The circumference of a circle is 312 feet: what is the side of the inscribed square?

Ans. 70,2312 *ft.*

5. The circumference of a circle is 819 yards: what is the side of the inscribed square?

Ans. 184,3569 *yds.*

6. The circumference of a circle is 715: what is the side of the inscribed square?

Ans. 160,9465.

PROBLEM XIX.

28. To find the area of a circular sector.

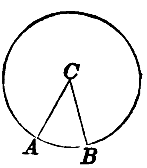
RULE.

1st. *Find the length of the arc by Problem* XII.

2d. *Multiply the arc by one half the radius, and the product will be the area.*

EXAMPLES.

1. What is the area of the circular sector *ACB*, the arc *AB* containing 18°, and the radius *CA* being equal to 3 feet.



First, $,01745 \times 18 \times 3 = ,94230 =$ length *AB*.

Then, $,94230 \times 1\frac{1}{2} = 1,41345 = $ area.

QUEST.—28. How do you find the area of a circular sector?

2. What is the area of a sector of a circle in which the radius is 20 and the arc one of 22 degrees?

Ans. 76,7800.

3. Required the area of a sector whose radius is 25 and the arc of 147° 29′.

Ans. 804,2448.

4. Required the area of a semicircle in which the radius is 13.

Ans. 265,4143.

5. What is the area of a circular sector when the length of the arc is 650 feet and the radius 325?

Ans. 105625 *sq. ft.*

PROBLEM XX.

29. To find the area of a segment of a circle.

RULE.

1st. *Find the area of the sector having the same arc with the segment by the last Problem.*

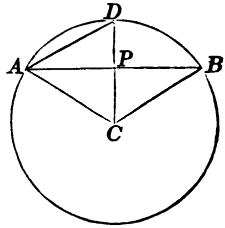
2d. *Find the area of the triangle formed by the chord of the segment and the two radii through its extremities.*

3d. *If the segment is greater than the semicircle, add the two areas together; but if it is less, subtract them and the result in either case will be the area required.*

QUEST.—29. How do you find the area of the segment of a circle?

EXAMPLES.

1. What is the area of the segment ADB, the chord $AB=24$ feet, and $CA=20$ feet.



First, $CP=\sqrt{\overline{CA}^2-\overline{AP}^2}$

$=\sqrt{400-144}=16$

Then,

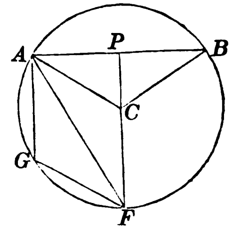
$PD=CD-CP=20-16=4$

And, $AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{144+16}=12{,}64911$:

then, arc $ADB=\dfrac{12{,}64911\times8-24}{3}=25{,}7309.$

arc	$ADB=25{,}7309$		$AP=12$
half radius	$=\quad 10$		$CP=16$
area sector	$ADBC=257{,}309$	area	$CAB=192$
area	$CAB=192$		
	$65{,}309=$ area of segment ADB.		

2. Find the area of the segment AFB, knowing the following lines, viz: $AB=20{,}5$; $FP=17{,}17$; $AF=20$; $FG=11{,}5$ and $CA=11{,}64$.



Arc $AGF=\dfrac{FG\times8-AF}{3}=\dfrac{11{,}5\times8-20}{3}=24$:

sector $AGFBC=24\times11{,}64=279{,}36$:

but $CP=FP-AC=17{,}17-11{,}64=5{,}53$:

Then, area $ACB=\dfrac{AB\times CP}{2}=\dfrac{20{,}5\times5{,}53}{2}=56{,}6825.$

Then, area of sector $AFBC = 279{,}36$
do. of triangle $ABC = \underline{\ 56{,}6825}$
gives area of segment $AFB = \underline{336{,}0425}$

3. What is the area of a segment, the radius of the circle being 10, and the chord of the arc 12 yards?

Ans. 16,324 *sq.yds.*

4. Required the area of the segment of a circle whose chord is 16, and the diameter of the circle 20?

Ans. 44,5903.

5. What is the area of a segment whose arc is a quadrant—the diameter of the circle being 18?

Ans. 63,6174.

6. The diameter of a circle is 100, and the chord of the segment 60: what is the area of the segment?

Ans. 408, nearly.

PROBLEM XXI.

30. To find the area of a circular ring: that is, the area included between the circumferences of two circles, having a common centre.

QUEST.—30. How do you find the area of a circular ring?

RULE.

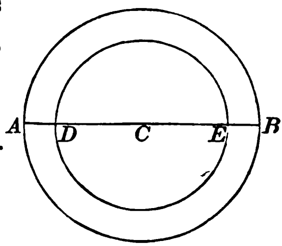
1*st*. *Square the diameter of each ring, and subtract the square of the less from that of the greater.*

2*nd*. *Multiply the difference of the squares by the decimal ,7854, and the product will be the area.*

EXAMPLES.

1. In the concentric circles having the common centre *C*, we have

$AB = 10$ yards, and $DE = 6$ yards: what is the area of the space included between them?



$$\overline{AB}^2 = \overline{10}^2 = 100$$
$$\overline{DE}^2 = 6^2 = 36$$
$$\text{Difference} = \overline{64}$$

Then, $64 \times ,7854 = 50,2656 = \text{area}$.

2. What is the area of the ring when the diameters of the circles are 20 and 10?

Ans. 235,62.

3. If the diameters are 20 and 15, what will be the area included between the circumferences?

Ans. 137,445.

4. If the diameters are 16 and 10, what will be the area included between the circumferences?

Ans. 122,5224.

5. Two diameters are 21,75 and 9,5; required the area of the circular ring.

Ans. 300,6609.

6. If the two diameters are 4 and 6, what is the area of the ring.

Ans. 15,708.

PROBLEM XXII.

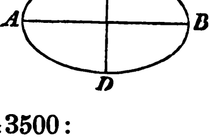
31. To find the area of an ellipse.

RULE.

Multiply the two axes together, and their product by the decimal ,7854, *and the result will be the required area.*

EXAMPLES.

1. Required the area of an ellipse, whose transverse axis $AB=70$ feet, and the conjugate axis $DE=50$ feet.



$$AB \times DE = 70 \times 50 = 3500:$$

Then, $,7854 \times 3500 = 2748,9 =$ area.

2. Required the area of an ellipse whose axes are 24 and 18.

Ans. 339,2928.

QUEST.—31. How do you find the arc of an ellipse?

3. What is the area of an ellipse whose axes are 35 and 25?

Ans. 687,225.

4. What is the area of an ellipse whose axes are 80 and 60?

Ans. 3769,92.

5. What is the area of an ellipse whose axes are 50 and 45?

Ans. 1767,15.

SECTION II.

OF THE MENSURATION OF SOLIDS.

1. The mensuration of solids is divided into two parts.

1st.—The mensuration of the surfaces of solids: and

2dly.—The mensuration of their solidities.

We have already seen that the unit of measure for plane surfaces, is a square whose side is the unit of length. (See Part I. § V).

QUEST.—1. Into how many parts is the mensuration of solids divided? What is the unit of measure for plane surfaces?

2. A curve line which is expressed by numbers is also referred to an unit of length, and its numerical value is the number of times which the line contains the unit.

If then, we suppose the linear unit to be reduced to a straight line, and a square constructed on this line, this square will be the unit of measure for curved surfaces.

3. The unit of solidity is a cube, whose edge is the unit in which the linear dimensions of the solid are expressed; and the face of this cube is the superficial unit in which the surface of the solid is estimated. (See Part I. § X. Art. 20).

4. The following is a table of solid measure.

1 cubic foot	= 1728 cubic inches.
1 cubic yard	= 27 cubic feet.
1 cubic rod	= $4492\frac{1}{8}$ cubic feet.
1 ale gallon	= 282 cubic inches.
1 wine gallon	= 231 cubic inches.
1 bushel	= 2150,42 cubic inches.

QUEST.—2. How is a curve line expressed by numbers? If we suppose the linear unit to be reduced to a straight line, and a square constructed on this line, what is this square? 3. What is the unit of solidity? 4. Repeat the table of solid measure?

OF POLYEDRONS, OR SOLIDS BOUNDED BY PLANES.

PROBLEM I.

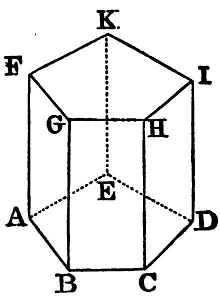
5. To find the surface of a right prism.

RULE.

1. *Multiply the perimeter of the base by the altitude and the product will be the convex surface: and to this add the area of the bases when the entire surface is required.*

EXAMPLES.

1. Find the entire surface of the regular prism whose base is the regular polygon $ABCDE$ and altitude AF—when each side of the base is 20 feet and the altitude AF, 50 feet.



$AB+BC+CD+DE+EA=100$; and $AF=50$: then $(AB+BC+CD+DE+EA)\ AF=$ convex surface becomes $100\times 50=5000$ square feet which is the convex surface. For the area of the end, we have

QUEST.—5. How do you find the surface of a right prism?

$\overline{AB}^2 \times$ tabular number = area $ABCDE$, (see page 136), that is, $\overline{20}^2 \times$ tabular number, or $400 \times 1{,}720477 =$ 688,1908 = the area $ABCDE$.

Then, convex surface =	5000	square feet.
lower base	688,1908	do.
upper base	688,1908	do.
entire surface	6376,3816	

2. What is the surface of a cube, the length of each side being 20 feet?

Ans. 2400 *sq. ft.*

3. Find the entire surface of a triangular prism, whose base is an equilateral triangle, having each of its sides equal to 18 inches, and altitude 20 feet.

Ans. 91,949 *sq. ft.*

4. What is the convex surface of a regular octagonal prism, the side of whose base is 15 and altitude 12 feet?

Ans. 1440 *sq. ft.*

5. What must be paid for lining a rectangular cistern with lead at 2*d* a pound, the thickness of the lead being such as to require 7*lb.* for each square foot of surface: the inner dimensions of the cistern being as follows: viz. the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches?

Ans. £ 2 3*s.* $10\frac{5}{9}$*d.*

PROBLEM II.

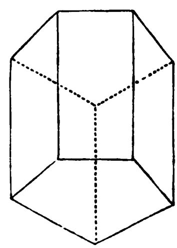
6. To find the solidity of a prism.

RULE.

Multiply the area of the base by the perpendicular height, and the product will be the area.

EXAMPLES.

1. What is the solidity of a regular pentagonal prism whose altitude is 20, and each side of the base 15 feet.



To find the area of the base we have by Problem VIII of § I

$\overline{15}^2=225$: and $225\times1{,}7204774=387{,}107415=$ the area of the base : hence,

$387{,}107415\times20=7742{,}1483=$solidity.

2. What is the solid content of a cube whose side is 24 inches ?

Ans. 13824 *solid inches.*

3. How many cubic feet in a block of marble, of

QUEST.—6. How do you find the solidity of a prism ?

which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches?

Ans. $21\frac{1}{9}$ *solid feet.*

4. How many gallons of water, ale measure, will a cistern contain whose dimensions are the same as in the last example?

Ans. $129\frac{17}{47}$.

5. Required the solidity of a triangular prism whose altitude is 10 feet, and the three sides of its triangular base, 3, 4, and 5 feet?

Ans. 60 *solid feet.*

6. What is the solidity of a square prism whose height is $5\frac{1}{2}$ feet, and each side of the base $1\frac{1}{3}$ foot?

Ans. $9\frac{7}{9}$ *solid feet.*

7. What is the solidity of a prism, whose base is an equilateral triangle, each side of which is 4 feet, the height of the prism being 10 feet?

Ans. 69,282 *solid feet.*

8. What is the number of cubic or solid feet in a regular pentagonal prism of which the altitude is 15 feet and each side of the base 3,75 feet?

Ans. 362,913.

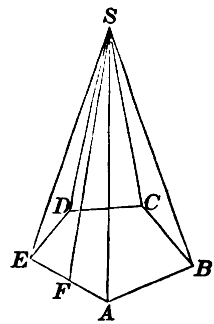
PROBLEM III.

7. To find the surface of a regular pyramid.

RULE.

Multiply the perimeter of the base by half the slant height, and the product will be the convex surface: to this add the area of the base, if the entire surface is required.

1. In the regular pentagonal pyramid $S-ABCDE$, the slant height SF is equal to 45, and each side of the base is 15 feet: required the convex surface, and also the entire surface.



$15 \times 5 = 75 =$ perimeter of the base

$75 \times 22\frac{1}{2} = 1687,5$ square feet $=$ area of convex surface.

And $\overline{15}^2 = 225$, then $225 \times 1,7204774 = 387,107415 =$ the area of the base.

Hence, convex surface $= 1687,5$
area of the base $= 387,107415$
entire surface $= 2074,607415$ square feet.

2. What is the convex surface of a regular triangular

QUEST.—7. How do you find the surface of a regular pyramid?

pyramid the slant height being 20 feet and each side of the base 3 feet.

Ans. 90 *sq. ft.*

3. What is the entire surface of a regular pyramid whose slant height is 15 feet, and the base a regular pentagon, of which each side is 25 feet.

Ans. 2012,798 *sq. ft.*

PROBLEM IV.

8. To find the convex surface of the frustum of a regular pyramid.

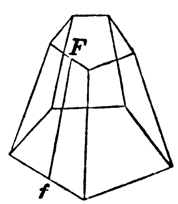
RULE.

Multiply half the sum of the perimeters of the two bases by the slant height of the frustum, and the product will be the convex surface.

EXAMPLES.

1. In the frustum of the regular pentagonal pyramid each side of the lower base is 30 and each side of the upper base is 20 feet, and the slant height fF is equal to 15 feet. What is the convex surface of the frustum?

Ans. 1875 *sq. ft.*



QUEST.—8. How do you find the convex surface of the frustum of a regular pyramid?

2. How many square feet are there in the convex surface of the frustum of a square pyramid, whose slant height is 10 feet, each side of the lower base 3 feet 4 inches, and each side of the upper base 2 feet 2 inches?

Ans. 110.

3. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 55 feet, each side of the lower base 8 feet, and each side of the upper base 4 feet?

Ans. 2310 *sq. ft.*

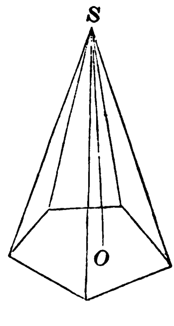
PROBLEM V.

9. To find the solidity of a pyramid.

RULE.

Multiply the area of the base by the altitude and divide the product by three—the quotient will be the solidity.

EXAMPLES.



1. What is the solidity of a pyramid the area of whose base is 215 square feet and the altitude $SO=45$ feet?

First, $215\times45=9675$:

then $9675\div3=3225$

which is the solidity expressed in solid feet.

QUEST.—9. How do you find the solidity of a pyramid?

2. Required the solidity of a square pyramid, each side of its base being 30 and its altitude 25.

Ans. 7500 *solid feet.*

3. How many solid yards are there in a triangular pyramid whose altitude is 90 feet, and each side of its base 3 yards?

Ans. 38,97117.

4. How many solid feet in a triangular pyramid the altitude of which is 14 feet 6 inches, and the three sides of its base 5, 6 and 7 feet?

Ans. 71,0352.

5. What is the solidity of a regular pentagonal pyramid, its altitude being 12 feet, and each side of its base 2 feet. *Ans.* 27,5276 *solid feet.*

6. How many solid feet in a regular hexagonal pyramid, whose altitude is 6,4 feet, and each side of the base 6 inches. *Ans.* 1,38564.

7. How many solid feet are contained in a hexagonal pyramid the height of which is 45 feet, and each side of the base 10 feet.

Ans. 3897,1143.

8. The spire of a church is an octagonal pyramid, each side of the base being 5 feet 10 inches, and its perpendicular height 45 feet. Within is a cavity, or hollow part, each side of the base of which is 4 feet 11 inches, and its perpendicular height 41 feet: how many yards of stone does the spire contain?

Ans. 32,197353.

PROBLEM VI.

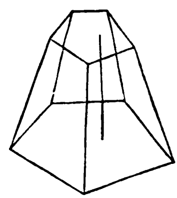
10. To find the solidity of the frustum of a pyramid.

RULE.

Add together the areas of the two bases of the frustum and a geometrical mean proportional between them; and then multiply the sum by the altitude and take one-third of the product for the solidity.

EXAMPLES.

1. What is the solidity of the frustum of a pentagonal pyramid the area of the lower base being 16 and of the upper base 9 square feet, the altitude being 7 feet.



First, $16 \times 9 = 144$: then $\sqrt{144} = 12$ the mean.

Then, area of lower base $= 16$

" upper base $= 9$

mean of bases $= 12$

37

height 7

$3)\ 259$

solidity $= 86\frac{1}{3}$ solid feet.

QUEST.—10. How do you find the solidity of the frustum of a pyramid?

2. What is the number of solid feet in a piece of timber whose bases are squares, each side of the lower base being 15 inches, and each side of the upper base being 6 inches;—the length being 24 feet?

Ans. 19,4776.

3. Required the solidity of a regular pentagonal frustum, whose altitude is 5 feet, each side of the lower base 18 inches, and each side of the upper base 6 inches.

Ans. 9,31925 *solid feet.*

4. What is the content of a regular hexagonal frustum, whose height is 6 feet, the side of the greater end 18 inches, and of the less end 12 inches?

Ans. 24,681724 *cubic feet.*

5. How many cubic feet in a square piece of timber, the areas of the two ends being 504 and 372 inches, and its length 31½ feet?

Ans. 95,447.

6. What is the solidity of a squared piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the smaller 12 inches?

Ans. 28,5 *cubic feet.*

7. What is the solidity of the frustum of a regular hexagonal pyramid, the side of the greater end being 3 feet, that of the less 2 feet, and the height 12 feet?

Ans. 197,453776 *solid feet.*

SECTION III.

OF THE MEASURES OF THE THREE ROUND BODIES.

1. To find the surface of a cylinder.

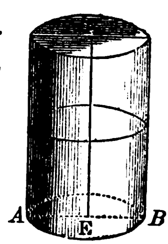
PROBLEM I.

RULE.

Multiply the circumference of the base by the altitude, and the product will be the convex surface; and to this, add the areas of the two bases, when the entire surface is required.

EXAMPLES.

1. What is the entire surface of the cylinder in which AB, the diameter of the base is 12 feet, and the altitude EF 30 feet.



First, to find the circumference of the base (See § I., Problem X.): we have $3{,}1416 \times 12 = 37{,}6992 =$ circumference of the base.

QUEST.—1. How do you find the surface of a cylinder?

Then, $37,6992 \times 30 = 1130,9760 =$ convex surface.

Also, $\overline{12}^2 = 144$: and $144 \times ,7854 = 113,0976 =$ area of the base.

Then,	convex surface	$=1130,9760$
	lower base	113,0976
	upper do.	113,0976
	Entire area	$=1357,1712$

2. What is the convex surface of a cylinder, the diameter of whose base is 20, and altitude 50 feet?

Ans. 3141,6 *sq. feet.*

3. Required the entire surface of a cylinder, whose altitude is 20 feet, and the diameter of the base 2 feet?

Ans. 131,9472 *feet.*

4. What is the convex surface of a cylinder, the diameter of whose base is 30 inches, and altitude 5 feet?

Ans. 5654,88 *sq. inches.*

5. Required the convex surface of a cylinder, whose altitude is 14 feet, and the circumference of the base 8 feet 4 inches?

Ans. 116,6666, &c., *sq. ft.*

PROBLEM II.

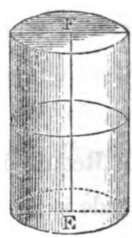
2. To find the solidity of a cylinder.

RULE.

Multiply the area of the base by the altitude, and the product will be the area.

EXAMPLES.

1. What is the solidity of a cylinder, the diameter of whose base is 40 feet, and altitude EF, 25 feet?



First, to find the area of the base, we have (by Problem XV. § 1.),

$\overline{40}^2=1600$, then $1600\times,7854=$ $1256,64=$area of the base. Then, $1256,64\times25=31416$ solid feet, which is the solidity.

2. What is the solidity of a cylinder, the diameter of whose base is 30 feet, and altitude 50 feet?

Ans. 35343 *cubic feet.*

3. What is the solidity of a cylinder whose height i 5 feet, and the diameter of the end 2 feet?

Ans. 15,708 *solid feet.*

QUEST.—2. How do you find the solidity of a cylinder?

4. What is the solidity of a cylinder whose height is 20 feet, and the circumference of the base 20 feet?

Ans. 636,64 *cubic feet.*

5. The circumference of the base of a cylinder is 20 feet, and the altitude 19,318 feet: what is the solidity?

Ans. 614,93 *cubic feet.*

6. What is the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet?

Ans. 2120,58 *cubic feet.*

7. Required the solidity of a cylinder whose altitude is 20 feet, and the circumference of whose base is 5 feet 6 inches.

Ans. 48,1459 *cubic feet.*

8. What is the solidity of a cylinder, the circumference of whose base is 38 feet, and altitude 25 feet?

Ans. 2872,838 *cubic feet.*

9. What is the solidity of a cylinder the circumference of whose base is 40 feet, and altitude 30 feet?

Ans. 3819,84 *solid feet.*

10. The diameter of the base of a cylinder is 84 yards, and the altitude 21 feet: how many solid or cubic yards does it contain?

Ans. 38792,4768.

PROBLEM III.

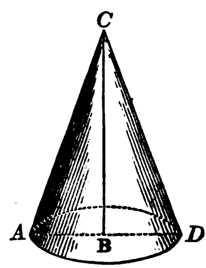
3. To find the surface of a cone.

RULE.

Multiply the circumference of the base by the slant height, and divide the product by 2; the quotient will be the convex surface, to which add the area of the base when the entire surface is required.

EXAMPLES.

1. What is the convex surface of the cone whose vertex is C,—the diameter AD of its base being $8\frac{1}{2}$ feet, and the side CA, 50 feet.



First, $3{,}1416 \times 8\frac{1}{2} = 26{,}7036 =$ circum. of base.

Then, $\frac{26{,}7036 \times 50}{2} = 667{,}59 =$ convex surface.

2. Required the entire surface of a cone whose side is 36, and the diameter of its base 18 feet.

Ans. 1272,348 *sq. ft.*

QUEST.—3. How do you find the surface of a cone? What is the difference between the entire surface and the convex surface?

3. The diameter of the base is 3 feet, and the slant height 15 feet: what is the convex surface of the cone?

Ans. 70,686 *sq. ft.*

4. The diameter of the base of a cone is 4,5 feet, and the slant height 20 feet: what is the entire surface?

Ans. 157,27635 *sq. ft.*

5. The circumference of the base of a cone is 10,75, and the slant height is 18,25: what is the entire surface?

Ans. 107,29021 *sq. ft.*

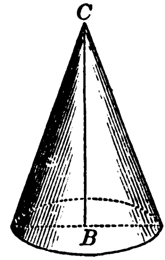
PROBLEM IV.

4. To find the solidity of a cone.

RULE.

Multiply the area of the base by the altitude, and divide the product by 3: *the quotient will be the solidity.*

EXAMPLES.



1. What is the solidity of a cone, the area of whose base is 380 square feet, and altitude *CB*, 48 feet?

QUEST.—4. How do you find the solidity of a cone?

We simply multiply the area of the base by the altitude, and then divide the product by 3.

Operation.

$$\begin{array}{r} 380 \\ 48 \\ \hline 3040 \\ 1520 \\ \hline 3)18240 \\ \hline \text{area}=6080 \end{array}$$

2. Required the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet.

Ans. 706,86 *cubic feet.*

3. Required the solidity of a cone whose altitude is $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22,5609 *cubic feet.*

4. What is the solidity of a cone, the diameter of whose base is 18 inches, and altitude 15 feet?

Ans. 8,83575 *cubic feet.*

5. The circumference of the base of a cone is 40 feet, and the altitude 50 feet: what is the solidity?

Ans. 2122,1333 *solid feet.*

PROBLEM V.

5. To find the surface of the frustum of a cone.

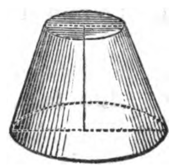
QUEST.—5. How do you find the convex surface of the frustum of a cone? How do you find the entire surface?

RULE.

Add together the circumferences of the two bases, and multiply the sum by half the slant height of the frustum; the product will be the convex surface, to which add the areas of the bases, when the entire surface is required.

EXAMPLES.

1. What is the convex surface of the frustum of a cone, of which the slant height is $12\frac{1}{2}$ feet, and the circumferences of the base 8,4 and 6 feet.



We merely take the sum of the circumferences of the bases, and multiply by half the slant height, or side.

Operation.

$$\begin{array}{rr} & 8{,}4 \\ & 6 \\ \hline & 14{,}4 \\ \text{half side} & 6{,}25 \\ \hline \text{area} = & 90 \ \textit{sq. ft.} \end{array}$$

2. What is the entire surface of the frustum of a cone, the side being 16 feet, and the radii of the bases 2 and 3 feet?

Ans. 292,1688 *sq. ft.*

3. What is the convex surface of the frustum of a cone, the circumference of the greater base being 30 feet, and of the less 10 feet; the slant height being 20 feet?

Ans. 400 *sq. ft.*

4. Required the entire surface of the frustum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet.

Ans. 439,824 *sq. ft.*

PROBLEM VI.

6. To find the solidity of the frustum of a cone.

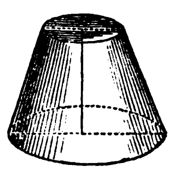
RULE.

1st. *Add together the areas of the two ends and a geometrical mean between them.*

2d. *Multiply this sum by one-third of the altitude and the product will be the solidity.*

EXAMPLES.

1. How many cubic feet in the frustum of a cone whose altitude is 26 feet, and the diameters of the bases 22 and 18 feet?



First, $\overline{22}^2 \times ,7854 = 380,134 =$ area of lower base:

QUEST.—6. How do you find the solidity of the frustum of a cone?

and $18^2 \times ,7854 = 254,47 =$ area of upper base.

Then, $\sqrt{380,134 \times 254,47} = 311,018 =$ mean.

Then, $(380,134 + 254,47 + 311,018) \times \frac{26}{3} = 8195,39$

which is the solidity.

2. How many cubic feet in a piece of round timber the diameter of the greater end being 18 inches, and that of the less 9 inches, and the length 14,25 feet?

Ans. 14,68943.

3. What is the solidity of the frustum of a cone, the altitude being 18, the diameter of the lower base 8 and that of the upper base 4?

Ans. 527,7888.

4. What is the solidity of the frustum of a cone, the altitude being 25, the circumference of the lower base 20, and that of the upper base 10?

Ans. 464,216.

5. If a cask, which is composed of two equal conic frustums joined together at their larger bases, have its bung diameter 28 inches, the head diameter 20 inches, and the length 40 inches, how many gallons of wine will it contain, there being 231 cubic inches in a gallon?

Ans. 79,0613.

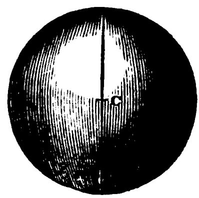
PROBLEM VII.

7. To find the surface of a sphere.

RULE.

Multiply the circumference of a great circle by the diameter, and the product will be the surface.

EXAMPLES.



1. What is the surface of the sphere whose centre is *C*, the diameter being 7 feet?

Ans. 153,9384 *sq. ft.*

2. What is the surface of a sphere whose diameter is 24?

Ans. 1809,5616.

3. Required the surface of a sphere whose diameter is 7921 miles.

Ans. 197111024 *sq. miles.*

4. What is the surface of a sphere the circumference of whose great circle is 78,54?

Ans. 1963,5.

QUEST.—7. How do you find the surface of a sphere?

5. What is the surface of a sphere whose diameter is $1\frac{1}{8}$ feet?

Ans. 5,58506 *sq. ft.*

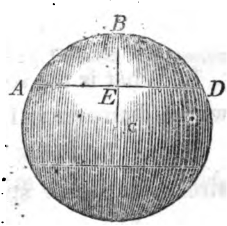
PROBLEM VIII.

8. To find the convex surface of a spherical zone.

RULE.

Multiply the height of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface.

EXAMPLES.



1. What is the convex surface of the zone, *ABD*, the height *BE* being 9 inches, and the diameter of the sphere 42 inches?

First, $42 \times 3,1416 = 131,9472 =$ circumference.
height $= 9$
surface $= 1187,5248$ square inches.

2. The diameter of a sphere is $12\frac{1}{2}$ feet: what will be the surface of a zone whose altitude is 2 feet?

Ans. 78,54 *sq. ft.*

QUEST.—8. How do you find the convex surface of a spherical zone?

3. The diameter of a sphere is 21 inches: what is the surface of a zone whose height is $4\frac{1}{2}$ inches?

Ans. 296,8812 *sq. in.*

4. The diameter of a sphere is 25 feet, and the height of the zone 4 feet: what is the surface of the zone?

Ans. 314,16 *sq. ft.*

PROBLEM IX.

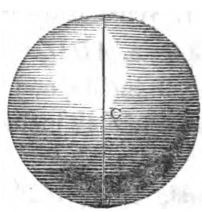
9. To find the solidity of a sphere.

RULE I.

Multiply the surface by one-third of the radius and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12 feet?



First, $3,1416 \times 12 = 37,6992 =$ circumference of sphere,

$$
\begin{array}{lr}
\text{diameter} & = \quad 12 \\
\hline
\text{surface} & = 452,3904 \\
\text{one-third radius} = & 2 \\
\hline
\text{solidity} & = 904,7808
\end{array}
\text{ cubic feet.}
$$

2. The diameter of a sphere is 7957,8: what is its so lidity?

Ans. 263863122758,4778.

QUEST.—9. How do you find the solidity of a sphere by the first rule?

3. The diameter of a sphere is 24 yards: what is its solid content?

Ans. 7238,2464 *cubic yds.*

4. The diameter of a sphere is 8: what is its solidity?

Ans. 268,0832.

RULE II.

10. *Cube the diameter and multiply the number thus found, by the decimal ,5236 and the product will be the solidity.*

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 20?

Ans. 4188,8.

2. What is the solidity of a sphere whose diameter is 6?

Ans. 113,0976.

3. What is the solidity of a sphere whose diameter is 10?

Ans. 523,6.

QUEST.—10. How do you find the solidity of a sphere by the second rule?

PROBLEM X.

11. To find the solidity of a spherical segment with one base.

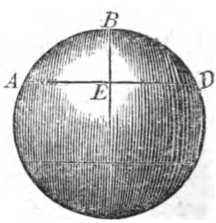
RULE.

1st. *To three times the square of the radius of the base add the square of the height.*

2d. *Multiply this sum by the height, and the product by the decimal* ,5236: *the result will be the solidity of the segment.*

EXAMPLES.

1. What is the solidity of the segment *ABD*, the height *BE* being 4 feet, and the diameter *AD* of the base being 14 feet?



First,

$(7^2 \times 3 + 4^2) = 147 + 16 = 163$:

Then, $163 \times 4 \times ,5236 = 341,3872$ solid feet, which is the solidity of the segment.

2. What is the solidity of the segment of a sphere, whose height is 4, and the radius of its base 8?

Ans. 435,6352.

QUEST.—11. How do you find the solidity of a spherical segment?

3. What is the solidity of a spherical segment, the diameter of its base being 17,23368, and its height 4,5?

Ans. 572,5566.

4. What is the solidity of a spherical segment, the diameter of the sphere being 8, and the height of the segment 2 feet?

Ans. 41,888 *cubic feet.*

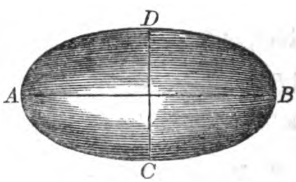
5. What is the solidity of a segment, when the diameter of the sphere is 20, and the altitude of the segment 9 feet?

Ans. 1781,2872 *cubic feet.*

OF THE SPHEROID.

12. A spheroid is a solid, described by the revolution of an ellipse about either of its axes.

13. If an ellipse *ACBD*, be revolved about the transverse or longer axis *AB*, the solid described, is called a *prolate spheroid:* and if it be revolved about the shorter axis *CD*, the solid described is called an *oblate spheroid.*



QUEST.—12. What is a spheroid? 13. What if an ellipse be revolved about the transverse axis, what is the solid which it describes called? If it be revolved about the conjugate axis, what is the solid called?

The earth is an oblate spheroid—the axis about which it revolves being about 34 miles shorter than the diameter perpendicular to it.

PROBLEM XI.

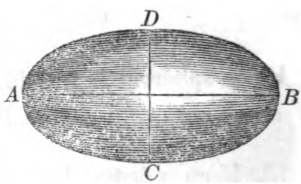
14. To find the solidity of an ellipsoid.

RULE.

Multiply the fixed axis by the square of the revolving axis, and the product by the decimal ,5236—the result will be the required solidity.

EXAMPLES.

1. In the prolate spheroid *ACBD*, the transverse axis $AB=90$, and the revolving axis $CD=70$ feet: what is the solidity?



Here, $AB=90$ feet: $\overline{CD}^2=\overline{70}^2=4900$: hence $AB\times\overline{CD}^2\times,5236=90\times4900\times,5236=230907,6$ cubic feet, which is the solidity.

2. What is the solidity of a prolate spheriod, whose fixed axis is 100 and revolving axis 6 feet?

Ans. 1884,96.

QUEST.—Is the earth an oblate or a prolate syheroid? What is the difference between the two diameters? 14. Give the rule for finding the solidity of an ellipsoid?

3. What is the solidity of an oblate spheroid, whose fixed axis is 60, and revolving axis 100?

Ans. 314160.

4. What is the solidity of a prolate spheroid, whose axes are 40 and 50?

Ans. 41888.

5. What is the solidity of an oblate spheroid, whose axes are 20 and 10?

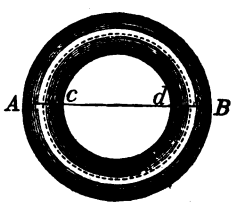
Ans. 2094,4.

6. What is the solidity of a prolate spheroid, whose axes are 55 and 33?

Ans. 31361,022.

OF CYLINDRICAL RINGS.

15. A cylindrical ring is formed by bending a cylinder until the two ends meet each other. Thus, if a cylinder be bent round until the axis takes the position *mon*, a solid will be formed, which is called a cylindrical ring.



The line *AB* is called the outer, and *cd* the inner diameter.

PROBLEM XII.

16. To find the convex surface of a cylindrical ring.

QUEST.—15. How is a cylindrical ring formed? 16. How do you find the convex surface of a cylindrical ring?

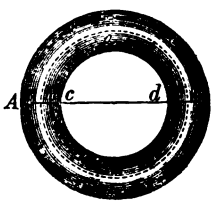
RULE.

1st. *To the thickness of the ring add the inner diameter.*

2nd. *Multiply this sum by the thickness, and the product by* 9,8696—*the result will be the area.*

EXAMPLES.

1. The thickness Ac of a cylindrical ring is 3 inches, and the inner diameter cd, is 12 inches: what is the convex surface?



$Ac+cd=3+12=15$: then

$15\times 3\times 9{,}8696=444{,}132$ square inches = the surface.

2. The thickness of a cylindrical ring is 4 inches, and the inner diameter 18 inches: what is the convex surface.

Ans. 868,52 *sq. in.*

3. The thickness of a cylindrical ring is 2 inches, and the inner diameter 18 inches: what is the convex surface?

Ans. 394,784 *sq. in.*

PROBLEM XIII.

17. To find the solidity of a cylindrical ring.

QUEST.—17. How do find the solidity of a cylindrical ring?

Mensuration of Cylindrical Rings.

RULE.

1st. *To the thickness of the ring add the inner diameter.*

2nd. *Multiply this sum by the square of half the thickness, and the product by* 9,8696—*the result will be the required solidity.*

EXAMPLES.

1. What is the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches? $8+3=11$: then, $11\times(\frac{3}{2})^2\times9{,}8696=244{,}2726$, which expresses the solidity in cubic inches.

2. The inner diameter of a cylindrical ring is 18 inches, and the thickness 4 inches: what is the solidity of the ring?

Ans. 868,5248 *cubic inches.*

3. Required the solidity of a cylindrical ring whose thickness is 2 inches, and inner diameter 12 inches?

Ans. 138,1744 *cubic inches.*

4. What is the solidity of a cylindrical ring, whose thickness is 4 inches, and inner diameter 16 inches?

Ans. 789,568 *cubic inches.*

PART IV.

SECTION I.

OF MEASURES.

1. The Carpenters' Rule, sometimes called the sliding rule, is used for the measurement of timber, and artificers' work. By it the dimensions are taken, and by means of certain scales, the superficial and solid contents may be computed.

2. The rule consists of two equal pieces of box wood, each one foot long, and connected together by a folding joint.

3. One face of the rule is divided into inches, half inches, quarter inches, eights of inches and sixteenths of inches. When the rule is opened, the inches are numbered from 1 to 23—the last number 24, at the end, being omitted.

Quest.—1. What is the carpenters' rule used for? 2. Describe the rule? 3. How is the rule divided on one face? When the rule is opened, how are the inches numbered? How long is the rule?

4. The edge of the rule is divided decimally: that is, each foot is divided into ten equal parts, and each of those again into ten parts, so that the divisions on the edge of the scale are hundredths of a foot. The hundredths are numbered on each arm of the scale from the right to the left.

By means of the decimal divisions it is easy to convert inches into the decimal of a foot.

Thus, if we have 6 inches, we find its corresponding decimal on the edge of the rule to be 50 hundredths of a foot, or ,50. Also 9 inches correspond to ,75; 8 inches to ,67 nearly, and 3 inches to ,25.

5. The multiplication of numbers is more easily made when the numbers are expressed decimally than when expressed in feet and inches.

Let us take an example. A board is 12 feet 6 inches long, and 2 feet 3 inches wide: how many square feet does it contain?

We see from the edge of the rule, that 6 inches correspond to ,50, and 3 inches to ,25. Hence, we have

---

QUEST.—4. How is the edge of the rule divided? Explain the manner of converting inches into the decimals of a foot? 5. Explain the manner of multiplying feet and inches, by means of decimals?

By cross multiplication.	By decimals.
12 6′	12,50
2 3′	2,25
25	6250
3 1′ 6″	2500
28 1′ 6″ content.	2500
	28,1250 content.

6. Besides the scale of feet and inches, already referred to, there are, on the same side, two small scales, marked M and E; the first is numbered from 1 to 36, and the second from 1 to 26. The object of these scales is to change a square into what is called in carpentry *an eight square*, or regular octagon.

Having formed the square which is to be changed to the octagon, find the middle of each side, and then the divisions of the scale marked M, show the distances to be laid off on each side of the centre points, to give the angles of the octagon.

For example, if the side of the square is 6 inches, the distance to be laid off, is found by extending the dividers from 1 to 6. If the side of the square is 12 inches, the distance to be taken reaches from 1 to 12; and so on for any distance from 1 to 36.

7. The scale marked E, is for the same object, only the distances are laid off from the angular points of the square, instead of from the centre.

QUEST.—6. What is the object of the scales marked M and E? Explain the use of the scale marked M. 7. Explain the use of the scale marked E.

Thus, if we have a square whose side is 9 inches, and wish to change it into an octagon, take from the scale E the distance from 1 to 9, and mark it off from each angle of the square, on the sides : then join the points, and the figure so formed will be a regular octagon.

If the side of the square is 18 inches, the distance to be taken reaches from 1 to 18, and so for any distance between 1 and 26—the numbers on the scale pointing out the distances to be laid off when the side of the square is expressed in inches.

8. Turning the rule directly over there will be seen on one arm several scales of equal parts, which are similar to those described in Part II, § I.

Fitting into the other arm, is a small brass slide, of the same length as the rule. On the face of the slide are two ranges of divisions, which are precisely alike. The upper, is designated by the letter B, and is to be used with the scale on the rule directly above, which is designated by A; the lower divisions on the slide designated by the letter C, are to be used with the scale mark GIRT LINE, and also designated by the letter D. The scales B and C on the slide, are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, and 1, from the left hand towards the right. From the middle point

QUEST.—8. On the opposite face of the rule, what scales are found? By what letters are the scales on the slide designated? By what letters are the scales adjacent designated?

1, the numbers go on 12, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Now, the values which the parts of this scale may represent will depend on the value given to the unit at the left hand. If the unit at the left be called 1, then the 1 at the centre point will represent 10, and the 2 at the right 20, the 3 at the right 30, and the ten 100, and similarly for the intermediate divisions.

If the left hand unit be called 10, then the 1 at the centre point will represent 100, the 2, two hundred, the 3, three hundred, and so on for the divisions to the right.

We shall now explain the use of the sliding Rule by examples.

PROBLEM I.

9. To multiply two numbers together.

RULE.

1*st. Mark a number on the scale A to represent the multiplier.*

2*nd. Then shove the slide until* 1 *on B stands opposite the multiplier on A.*

3*rd. Then pass along on B until you find a number to represent the multiplicand—the number opposite on A will represent the product.*

QUEST.—9. Explain the manner of multiplying two numbers together by the sliding rule.

EXAMPLES.

1. Multiply 24 by 14.

Move the slide until 1 on *B* is opposite the 2nd long mark at the right of 12, which is the division corresponding to 14. Then pass along *B* to the fourth of the larger lines on the right of 2: this line marks the division on the scale *A*, which shows the product. Now we must remark that the unit on the product line is always ten times greater than the unit 1 at the left of the slide; and since in the example this unit was 10, it follows that the 3 on *A*, will stand for 300, and each of the smaller divisions for 10; hence the product as shown by the scale is nearly 340, and by judging by the eye, we write it 336.

2. What is the product of 36 by 22.

Move the slide till 1 on *B* stands at 22 on *A*: then pass along on *B* to the 6th line between 3 and 4: the figures on *A* will then stand for hundreds, and the product will be pointed out a little to the right of the 9th line, between 7 and 8; or it will be 792.

3. A board is 16 feet 9 inches long, and 15 inches, or 1 foot and 3 inches wide, how many square feet does it contain?

First, 16 feet 9 inches = 16,75 feet;

and 15 inches = 1,25 feet:

Place 1 on *B* at the line corresponding to 16, between

12 and 2 on *A*, and then move over three-fourths of the distance to the next long line to the right. Then looking along on *A*, one quarter of the distance between 1 and 2, we find the area of the board to be 21 feet, which is correct, very nearly.

4. The length of a board is 15 feet 8 inches, and the breadth 1 foot 6 inches: what is the superficial content?

15 feet 8 inches = 15,7 nearly

1 foot 6 inches = 1,5 feet.

Then, place 1 on *B* at 15,7 on *A*, and 1 and a half on *B* will mark 23 and a half feet on *A*, which is the area very nearly.

10. Below the slide, and on the same side with the scales already described, is a row of divisions marked GIRT LINE, and numbered from 4 to 40. This line is also designated on the scale by the letter *D*. The object of this girt line, which is to be used in conjunction with the sliding scale, is to find the solid content of timber.

11. The *quarter girt*, as it is called in the language of mechanics, is one quarter the circumference of a stick of timber at its middle point. The quarter girt, in squared timber, is found by taking a mean between the breadth and thickness.

QUEST.—10. Explain the manner in which the girt line is numbered. 11. What is the quarter girt? How do you find the quarter girt?

Thus, if the breadth at the middle point is 4 feet 6 inches, and the thickness 3 feet 4 inches, we have

4	6	breadth
3	4	depth
2)7	10	
3	11	quarter girt.

and hence the quarter girt is 3 feet 11 inches.

12. If a stick of timber tapers regularly from one end to the other, the breadth and depth at the middle point may be found by taking the mean of the breadth and depth at the ends.

Thus, if the breadths at the ends are 1 foot 6 inches, and 1 foot 3 inches, the mean breadth will be 1 foot $4\frac{1}{2}$ inches. And, if the depths at the ends are 1 foot 3 inches, and 1 foot, the mean depth or thickness will be 1 foot $1\frac{1}{2}$ inches; and the quarter girt will be 1 foot 3 inches.

PROBLEM II.

13. To find the solid content of a stick of timber by the scale, when the length and quarter girt are known.

QUEST.—12. When a stick of timber tapers regularly, how do you find the quarter girt? 13. Explain the manner of finding the content of a stick of timber by the sliding rule.

RULE.

1*st.* *Reduce the length of the timber to feet and decimals of a foot, and the quarter girt to inches.*

2*nd.* *Note on scale C the number which expresses the length, and move the slide until this number falls at* 12 *on the girt line.*

3*rd.* *Pass along on the girt line till you find the number which expresses the quarter girt in inches, and the division which it marks on C will show the content of the timber in cubic feet.*

EXAMPLES.

1. A piece of square timber is 3 feet 9 inches broad, 2 feet 7 inches thick, and 20 feet long: how many solid feet does it contain?

ft.	*in.*	
3	9	
2	7	
2)6	4	
3	2	quarter girt = 38 inches.

Now, move the slide until 20 on *C* falls at 12 on the girt line. If we take 1 on *C* at the left for 10, 2 will represent 20, which is placed opposite 12 on *D*. Then passing along the girt line to division 38, we find the content on *C* to be a little over 200, say $200\frac{1}{2}$.

2. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and less ends are 1 foot 6

inches, and 1 foot 3 inches; and the thickness at the greater and less end, 1 foot 3 inches and 1 foot: what is the solid content?

Here, the mean breadth is 1 foot $4\frac{1}{2}$ inches, the mean thickness 1 foot $1\frac{1}{2}$ inches, and the quarter girt 1 foot 3 inches, or 15 inches.

Therefore, place 18,5 on *C*, at 12 on *D*, and pass along the girt line to 15—the number on *C*, which is a little more than 28 and a half, will express the solid content.

TABLE FOR BOARD MEASURE.

14. Besides the carpenter's rule with a slide, which we have just described, there is another folding rule without a slide, and on the face of which is a table to show the content of a board from 1 to 20 feet in length, and from 6 to 20 inches in width.

The upper line of the table shows the length of the board in feet, and the column at the left shows the width of the board in inches, from 6 to 20. For convenience, however, the table is often divided into two parts which are placed by the side of each other.

EXAMPLES.

1. If your board is 6 inches wide, and 14 feet long, cast your eye along the top line till you come to 14—di-

Quest.—14. Explain the table for finding the content of boards.

rectly under you will find 7, which shows that the board contains 7 square feet.

2. If your board is 10 inches wide, and 16 feet long, cast your eye along the top line till you come to 16; then pass along down till you come to the line of 10—the number thus found is 13-4, which shows that the board contains 13 and 4 twelfths square feet.

The right hand side of the table begins at 13 inches on the left hand column.

3. What is the content of a board which is 13 feet long, and 19 inches wide?

Look along the upper line to 13: then descend to the line 19, where you will find the number 20-7, which shows that the board contains 20 and 7 twelfths square feet.

4. If your board is 17 inches wide and 14 feet long, you will look under 14 till you come on to the line 17, where you will find the number 19-10; which shows that the board contains 19 and 10 twelfths square feet.

5. If you have a board 24 feet long, and 20 inches wide, first take the area for 20 feet in length, and then for 4 feet. Thus,

for 20 feet by 20 inches,	33	4
for 4 feet by 20 inches,	6	8
their sum gives	40	0 square feet.

Note.—Add as above for any different lengths or widths.

If your stuff is $1\frac{1}{2}$ inches thick, add half to it.

If 2 inches thick, you must double it.

The table on the four-fold Rule is not divided.

BOARD MEASURE.

15. This is a measure two feet in length, of an octagonal form, that is, having eight faces.

On the line running round the measure, at the centre, we find the faces of the measure marked, in succession, by the figures 8, 9, 10, 11, 12, 13, 14, and 15; and we shall designate each face by the figure which thus marks it. We will likewise observe, that figures corresponding to these, are also sometimes placed at one end of the measure.

Now, these figures at the centre of the measure correspond to the length of the board to be measured. Thus, if the board were 13 feet in length—place the thumb on the line 13 at the centre, and then apply the measure across the board, and the number on the face 13, which the width of the board marks, will express the

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Quest.—15. How long is the board measure? How many faces has it? How are they distinguished from each other? What do the figures at the centre correspond to? How would you measure a board 13 feet long? How would you measure a board 14 feet long? How would you measure a board 18 feet long.

number of square feet in the board. Thus, if the width of the board extended from 1 to 15, the board would contain 15 square feet.

If the board to be measured was 14 feet long, its content would be measured on face 14. If the board were 18 feet long, measure its width on face 8, and also on face 10, and take the sum for the true content of the board.

The Rules described above, are made by Jones & Co., of Hartford, Conn.

SECTION II.

OF TIMBER MEASURE.

1. The methods of measuring both the superficial content of boards and the solid content of timber, by rules and scales, have already been given. We shall now give the more accurate methods by means of figures.

PROBLEM I.

2. To find the area of a board or plank.

RULE.

Multiply the length by the breadth, and the product will be the content required.

QUEST.—1. What methods of measuring timber have already been explained? 2. Give the rule for finding the area of a board or plank.

NOTE.—3. If the board is tapering, add the breadths of the two ends together, and take half the sum for a mean breadth, and multiply the result by the length.

NOTE 4.—The examples may either be done by cross multiplication, or the inches may be reduced to the decimals of a foot, and the numbers then multiplied together.

EXAMPLES.

1. What is the area of a board whose length is 8 feet 6 inches, and breadth 1 foot 3 inches?

By cross multiplication.

ft.	*in.*		
8	6		
1	3		
8	6′		
2	1	6″	
10	7′	6″	content.

By Decimals.

ft.	*in.*	
8	6	=8, 5 *feet.*
1	3′	=1,25′
Product		=10,625 *sq. ft.*

2. What is the content of a board 12 feet 6 inches long, and 2 feet 3 inches broad?

Ans. 28 *feet* 1′ 6″ or 28,125 *sq. ft.*

3. How many square feet in a board whose breadth at one end is 15 inches, at the other 17 inches—the length of the board being 6 feet?

Ans. 8.

QUEST.—3. If the board is tapering, how is it found? 4. In how many ways may the examples be done?

4. How many square feet in a plank, whose length is 20 feet, and mean breadth 3 feet 3 inches?

Ans. 65.

5. What is the value of a plank whose breadth at one end is 2 feet, and at the other 4 feet—the length of the plank being 12 feet, and the value per square foot 10 cents?

Ans. $3,60.

PROBLEM II.

5. Having given one dimension of a plank or board, to find the other dimension such that the plank shall contain a given area.

RULE.

Divide the given area by the given dimension, and the quotient will be the other dimension.

EXAMPLES.

1. The length of a board is 16 feet, what must be its width that it may contain 12 square feet?

16 feet $=$ 192 inches

12 square feet $= 144 \times 12 = 1728$ square inches.

Then, $1728 \div 192 = 9$ inches, the width of the board.

QUEST.—5. If one dimension of a plank be given, explain the manner of finding the other, so that the plank shall contain a given area?

2. If a board is 6 inches broad, what length must be cut from it to make a square foot?

Ans. 2 *feet.*

3. If a board is 8 inches wide, what length of it will make 4 square feet?

Ans. 6 *feet.*

4. A board is 5 feet 3 inches long, what width will make 7 square feet?

Ans. 1 *foot* 4 *inches.*

5. What is the content of a board whose length is 5 feet 7 inches, and breadth 1 foot 10 inches?

Ans. 10 *feet* 2′ 10″.

PROBLEM III.

6. To find the solid content of squared or four-sided timber, which does not taper.

RULE.

Multiply the breadth by the depth, and then multiply the product by the length: the result will be the solid content.

EXAMPLES.

1. A squared piece of timber is 15 inches broad, 15 inches deep, and 18 feet long: how many solid feet does it contain?

Ans. 28,125.

QUEST.—6. How do you find the content of squared timber which does not taper?

2. What is the solid content of a piece of timber, whose breadth is 16 inches, depth 12 inches, and length 12 feet?

Ans. 16 *ft.*

3. The length of a piece of timber is 24,5 feet; its ends are equal squares, whose sides are each 1,04 feet: what is the solidity?

Ans. 26,4992 *solid feet.*

PROBLEM IV.

7. To find the solidity of a squared piece of timber which tapers regularly.

RULE.

1st. *Add together the breadths at the two ends and also the depths.*

2nd. *Multiply these sums together, and to the result add the products of the depth and breadth at each end.*

3d. *Multiply the last result by the length, and take one-sixth of the product, which will be the solidity.*

EXAMPLES.

1. How many cubic feet in a piece of timber whose ends are rectangles, the length and breadth of the larger

QUEST.—7. How do you find the solidity of a squared piece of timber when it tapers regularly?

Of Timber Measure.

being 14 inches and 12 inches; and of the smaller, 6 and 4 inches: the length of the piece being $30\frac{1}{2}$ feet.

$$\begin{array}{ccc} 14 & 12 & 16\times 20=320 \\ \underline{6} & \underline{4} & 14\times 12=168 \\ 20 & 16 & 6\times 4=\underline{24} \\ & & 512 \text{ square inches.} \end{array}$$

But, 512 square inches $=\frac{32}{9}$ square feet.
Then, $\frac{32}{9}\times 30\frac{1}{2}\times\frac{1}{6}=18\frac{2}{27}$ solid feet.

2. How many solid inches in a mahogany log, the depth and breadth at one end being $81\frac{1}{2}$ inches and 55 inches, and of the other 41 and $29\frac{1}{2}$ inches; the length of the log being $47\frac{1}{4}$ inches.

Ans. 126340,59375.

2. How many cubic feet in a stick of timber whose larger end is 25 feet by 20, the smaller 15 feet by 10, and the length 12 feet?

Ans. 3700.

3. What is the number of cubic feet in a stick of hewn timber, whose ends are 30 inches by 27 and 24 inches by 18—the length being 24 feet?

Ans. 102.

4. The length of a piece of timber is 20,38 feet, and the ends are unequal squares: the side of the greater is $19\frac{1}{8}$ inches, and of the less $9\frac{7}{8}$ inches: what is the solid content?

Ans. 30,763 *cubic feet.*

5. The length of a piece of timber is 27,36 feet: at the greater end, the breadth is 1,78 feet, and the thickness 1,23 feet; and at the less end, the breadth is 1,04 feet and the thickness 0,91 feet: what is its solidity?

Ans. 41,8179 *cubic feet.*

8. NOTE.—If the timber does not taper regularly, measure parts of the stick, the same as if it had a regular taper, and take the sum of the parts for the entire solidity.

PROBLEM V.

9. Knowing the area of the end of a square piece of timber which does not taper, it is required to find the length which must be cut off in order to obtain a given solidity.

RULE.

1st. *Reduce the given solidity to cubic inches.*

2d. *Divide the number of solid inches by the area of the end expressed in inches, and the quotient will be the length in inches.*

EXAMPLES.

1. A piece of timber is 10 inches square, how much must be cut off to make a solid foot?

$10 \times 10 = 100$ square inches.

Then, $1728 \div 100 = 17,28$ inches.

QUEST.—8. How do you find the content when it tapers irregularly? 9. Knowing the area of the end, how will you find the length to be cut off so as to give a solid foot?

2. A piece of timber is 20 inches broad, and 10 inches deep: how much in length will make a solid foot?

Ans. $8\frac{16}{25}$ *inches.*

3. A piece of timber is 9 inches broad and 6 inches deep: how much in length will make 3 solid feet?

Ans. 8 *feet.*

PROBLEM VI.

10. To find the solidity of round or unsquared timber.

RULE.

1st. *Take the girt or circumference, and then divide it by* 5.

2d. *Multiply the square of one-fifth of the girt by twice the length, and the product will be the solidity very nearly.*

EXAMPLES.

1. A piece of round timber is $9\frac{3}{4}$ feet in length, and the girt is 13 feet: what is its solidity?

First, $13 \div 5 = 2,6$ the fifth of the girt.

Also, $\overline{2,6}^2 = 6,76$; and $9,75 \times 2 = 19,50$

Again, $6,76 \times 19,5 = 131,82$ cubic feet, which is the required solidity.

QUEST.—10. How do you find the solidity of round, or unsquared timber?

2. The length of a tree is 24 feet, and the girt throughout 8 feet: what is the content?

Ans. 122,88 *cubic feet.*

3. Required the content of a piece of timber, its length being 9 feet 6 inches, and girt 14 feet?

Ans. 148,96 *cubic feet.*

11. NOTE—If the timber tapers, or the girt be different at different points, then, gird the timber at as many points as may be necessary and divide the sum of the girts by their number for the mean girt—of which, take one-fifth, and proceed as before.

4. If a tree, girt 14 feet at the thicker end and 2 feet at the smaller end, be 24 feet in length, how many solid feet will it contain?

Ans. 122,88.

5. A tree girts at five different places as follows: in the first, 9,43 feet; in the second 7,92 feet; in the third 6,15 feet; in the fourth 4,74 feet; and in the fifth 3,16 feet: now, if the length of the tree be 17,25 feet, what is its solidity?

Ans. 54,42499 *cubic feet.*

QUEST.—11. If the timber tapers, how will you find the solidity?

SECTION III.

BRICKLAYERS' WORK.

1. ARTIFICERS' WORK in general, is computed by three different measures: viz—

1st. The linear measure, or as it is called by mechanics, running measure.

2d. Superficial or square measure, in which the computation is made by the square foot, square yard, or by the square containing 100 square feet, or yards.

3d. By the cubic or solid measure, when it is estimated by the cubic foot, or the cubic yard. The work, however, is often estimated in square measure, and the materials for construction in cubic measure.

2. The dimensions of a brick generally bear the following proportions to each other: viz.

Length = twice the width, and

Width = twice the thickness, and

hence, the length is equal to four times the thickness.

3. The common length of a brick is 8 inches, in which

QUEST.—1. By what measures is artificer's work computed? 2. What proportion do the dimensions of a brick bear to each other? 3. What is the common length of a brick—its breadth—thickness? How many cubic inches in a brick of 8 inches long? How many such brick make a cubic foot?

case the width is 4 inches, and the thickness 2 inches. A brick of this size contains

$8 \times 4 \times 2 = 64$ cubic inches; and since a cubic foot contains 1728 cubic inches, we have

$1728 \div 64 = 27$ the number of bricks in a cubic foot.

4. If the brick is 9 inches long, then the width is $4\frac{1}{2}$ inches, and the thickness $2\frac{1}{4}$; and then each brick will contain

$9 \times 4\frac{1}{2} \times 2\frac{1}{4} = 91\frac{1}{8} =$ cubic inches in each brick; and

$1728 \div 91\frac{1}{8} = 19$ nearly, the number of bricks in a cubic foot. In the examples which follow, we shall suppose the brick to be 8 inches long.

PROBLEM I.

5. To find the number of bricks required to build a wall of given dimensions.

RULE.

1st. *Find the content of the wall in cubic feet.*

2d. *Multiply the number of cubic feet by the number of bricks in a cubic foot, and the result will be the number of bricks required.*

EXAMPLES.

1. How many bricks, of 8 inches in length, will be

QUEST.—4. How many cubic inches in a brick 9 inches long? How many such brick in a cubic foot? 5. How do you find the number of bricks necessary to build a wall of given dimensions?

required to build a wall 30 feet long, a brick and a half thick, and 15 feet in height?

Ans. 12150.

2. How many bricks, of the usual size, will be required to build a wall 50 feet long, 2 bricks thick, and 36 feet in height?

Ans. 64800.

5. Note.—In these examples no allowance has been made for the mortar. The thickness of mortar between the courses is nearly a quarter of an inch, so that four courses will give nearly 8 inches in height. The mortar, therefore, adds nearly one-eighth to the height, but as one-eighth is rather too large an allowance, we need not consider the mortar which goes to increase the length of the wall.

3. How many brick would be required in the first and second examples if we make the proper allowance for mortar?

Ans. { 1st. 10631¼
2d. 56700.

4. Bricklayers generally estimate their work at so much per thousand bricks. To find the value of things estimated by the thousand, see Arithmetic, page 192.

What is the cost of a wall 60 feet long, 20 feet high,

Quest.—5. How much space does the mortar between the courses occupy? What allowance then must be made for mortar in estimating the amount of bricks?

and two and a half bricks thick, at $7,50 per thousand, which price we suppose to include the cost of the mortar ?

If we suppose the mortar to occupy a space equal to one-eighth the height of the wall, we must find the quantity of bricks under the supposition that the wall was $17\frac{1}{2}$ feet in height.

Ans. $354,37$\frac{1}{2}$.

NOTE.—In estimating the bricks for a house, allowance must be made for the windows and doors.

OF CISTERNS.

6. It frequently occurs that cisterns are to be constructed which shall hold given quantities of water, and it is an useful practical problem to calculate their exact dimensions.

7. It was remarked in arithmetic, page 102, that a hogshead contains 63 gallons, and that a gallon contains 231 cubic inches. Hence, $231 \times 63 = 14553$, the number of cubic inches in a hogshead.

8. If, therefore, it is required to find the number of hogsheads which a cistern of given dimensions will contain, we have the following

QUEST.—7. What is the number of cubic inches in a hogshead? How are they found? 8. Give the rule for finding the content in hogsheads of a cistern?

RULE.

1st. *Find the solid content of the cistern in cubic inches.*

2d. *Divide the content so found by* 14553, *and the quotient will be the number of hogsheads.*

EXAMPLES.

1. The diameter of a cistern is 6 feet 6 inches, and height 10 feet: how many hogsheads does it contain?

The dimensions reduced to inches are 78 and 120. To find the solid content, see page 165. Then, the content in cubic inches, which is 573404,832, gives

$$573404,832 \div 14553 = 39,40 \text{ hogsheads nearly.}$$

9. If the height of a cistern be given, and it is required to find the diameter, so that the cistern shall contain a given number of hogsheads, we have the following

RULE.

1st. *Reduce the height of the cistern to inches, and the content to cubic inches.*

2d. *Multiply the height by the decimal* ,7854.

3d. *Divide the content by the last result, and extract the square root of the quotient, which will be the diameter of the cistern in inches.*

QUEST.—9. If the height of a cistern is known, how will you find the diameter, so that the cistern shall hold a given quantity?

EXAMPLES.

1. The height of a cistern is 10 feet, what must be its diameter that it may contain 40 hogsheads?

Ans. 78,6 *inches, nearly.*

10. If the diameter of a cistern be given, and it is required to find the height so that the cistern shall contain a given number of hogsheads, we have the following

RULE.

1st. *Reduce the content to cubic inches.*

2d. *Reduce the diameter to inches, and then multiply its square by the decimal* ,7854.

3d. *Divide the content by the last result and the quotient will be the height in inches.*

EXAMPLES.

1. The diameter of a cistern is 8 feet: what must be its height that it may contain 150 hogsheads.

Ans. 25 *ft.* 1 *in. nearly.*

QUEST.—10. If the diameter be known, how will you find the height, so that the cistern shall contain a given quantity?

SECTION IV.

MASONS' WORK.

1. To masonry belong all sorts of stone work. The measure made use of is either superficial or solid.

2. Walls, columns, blocks of stone or marble, are measured by the cubic foot; and pavements, slabs, chimney pieces, etc. are measured by the square or superficial foot. Cubic or solid measure is always used for the materials, and the square measure is sometimes used for the workmanship.

EXAMPLES.

1. Required the solid content of a wall 53 feet 6 inches long, 12 feet 3 inches high, and 2 feet thick?

Ans. 1310$\frac{3}{4}$ *feet.*

2. What is the solid content of a wall the length of which is 24 feet 3 inches, height 10 feet 9 inches, and thickness 2 feet?

Ans. 521,375 *feet.*

3. In a chimney piece we find the following dimensions :—

Length of the mantel and slab,	4	feet,	2	inches.
Breadth of both together,	3	"	2	"
Length of each jamb,	4	"	4	"
Breadth of both,	1	"	9	"

Required the superficial content.

Ans. 21 *feet,* 10′.

SECTION V.

CARPENTERS' AND JOINERS' WORK.

1. Carpenter's and joiner's work is that of flooring, roofing, &c., and is generally measured by the square of 100 square feet.

2. In carpentry, a roof is said to have a *true pitch* when the length of the rafters is three-fourths the breadth of the building. The rafters then, are nearly at right angles. It is therefore customary to take once and a half times the area of the flat of the building, for the area of the roof.

EXAMPLES.

1. How many squares, of 100 square feet each, in a floor 48 feet 6 inches long, and 24 feet 3 inches broad?

Ans. 11 and $76\frac{1}{8}$ *sq. ft.*

2. A floor is 36 feet 3 inches long, and 16 feet 6 inches broad: how many squares does it contain?

Ans. 5 and $98\frac{1}{8}$ *sq. ft.*

3. How many squares are there in a partition 91 feet 9 inches long, and 11 feet 3 inches high?

Ans. 10 and 32 *sq. ft.*

4. If a house measure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and

the roof be of the true pitch, what will the roofing cost at $1,40 per square?

Ans. $33,733.

SECTION VI.

SLATERS' AND TILERS' WORK.

1. In this work, the content of the roof is found by multiplying the length of the ridge by the girt from eaves to eaves. Allowances, however, must be made for the double rows of slate at the bottom.

EXAMPLES.

1. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches; what is its content?

Ans. 1566,9375 *sq. ft.*

2. What will the tiling of a barn cost, at $3,40 per square of 100 feet, the length being 43 feet 10 inches, and breadth 27 feet 5 inches, on the flat, the eave board projecting 16 inches on each side, and the roof being of the true pitch?

Ans. $65,26.

SECTION VII.

PLASTERERS' WORK.

1. Plasterers' work is of two kinds, viz.: ceiling, which is plastering on laths; and rendering, which is plastering on walls. These are measured separately.

2. The contents are estimated either by the square foot, the square yard, or by the square of 100 feet.

Inriched mouldings, &c., are rated by the running or lineal measure.

In estimating plastering, deductions are made for chimneys, doors, windows, &c.

EXAMPLES.

1. How many square yards are contained in a ceiling 43 feet 3 inches long, and 25 feet 6 inches broad?

Ans. $122\frac{1}{2}$ *nearly.*

2. What is the cost of ceiling a room 21 feet 8 inches, by 14 feet 10 inches, at 18 cents per square yard?

Ans. \$6,42$\frac{1}{9}$.

3. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height to the under side of the cornice 9 feet 3 inches. The cornice girts $8\frac{1}{2}$ inches, and projects 5 inches from the wall on the upper part next

the ceiling, deducting only for one door 7 feet by 4, what will be the amount of the plastering?

Ans. { 53 *yds.* 5 *ft.* 3′ 6″ *of rendering.*
18 *yds.* 5 *ft.* 6′ 4″ *of ceiling.*
37 *feet* 10′ 9″ *of cornice.* }

NOTE.—The area of the cornice is found by taking the length of the middle line, which for the length of the house is 14 feet, and for the breadth 12 feet 9 inches, and the multiplying each of these numbers by the girt, which is $8\frac{1}{2}$ inches.

SECTION VIII.

PAINTERS WORK.

1. Painters' work is computed in square yards. Every part is measured where the colour lies, and the measuring line is carried into all the mouldings and cornices.

Windows are generally done at so much a piece. It is usual to allow double measure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting in a room which is 65 feet 6 inches in perimeter, and 12 feet 4 inches in height?

Ans. $89\frac{11}{54}$ *sq. yds.*

2. The length of a room is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches: how many yards of painting are in it, deducting a fire place of 4 feet by 4 feet 4 inches, and two windows, each 6 feet by 3 feet 2 inches?

Ans. $73\frac{2}{27}$ *sq. yds.*

SECTION IX.

PAVERS' WORK.

1. Pavers' work is done by the square yard, and the content is found by multiplying the length and breadth together.

EXAMPLES.

1. What is the cost of paving a side walk, the length of which is 35 feet 4 inches, and breadth 8 feet 3 inches, at 54 cents per square yard?

Ans. 17,48 9.

2. What will be the cost of paving a rectangular court yard, whose length is 63 feet, and breadth 45 feet, at 2*s.* 6*d.* per square yard; there being, however, a walk running lengthwise 5 feet 3 inches broad, which is to be flagged with stone costing 3 shillings per square yard?

Ans. £40 5*s.* $10\frac{1}{2}$*d.*

SECTION X.

PLUMBERS' WORK.

1. Plumbers' work is rated at so much a pound, or else by the hundred weight. Sheet lead, used for gutters, &c., weighs from 6 to 12 *lbs.* per square foot. Leaden pipes vary in weight according to the diameter of their bore and thickness.

The following table shows the weight of a square foot of sheet lead, according to its thickness; and the common weight of a yard of leaden pipe according to the diameter of the bore.

Thickness of lead.	Pounds to a square foot.	Bore of leaden pipes.	Pounds per yard.
Inch. $\frac{1}{10}$	5,899	$0\frac{3}{4}$	10
$\frac{1}{9}$	6,554	1	12
$\frac{1}{8}$	7,373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8,427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9,831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11,797	2	24

EXAMPLES.

1. What weight of lead of $\frac{1}{10}$ of an inch in thickness, will cover a flat 15 feet 6 inches long, and 10 feet 3 inches broad, estimating the weight at 6 *lbs.* per square foot?

Ans. 8 *cwt.* 2 *qr.* $1\frac{1}{4}$ *lb.*

2. What will be the cost of 130 yards of leaden pipe of an inch and a half bore, at 8 cents per pound, supposing each yard to weigh 18 *lbs.*

Ans. $187,20.

PART V.

INTRODUCTION TO MECHANICS.

SECTION I.

OF MATTER AND BODIES.

1. MATTER is a general name for every thing which has substance, and is always capable of being increased or diminished. Whatever we can touch, taste, smell or see, is matter.

2. A BODY is any portion of matter.

3. SPACE is mere extension, in which all bodies are situated. Thus, when a body has a certain place, it is said to occupy that portion of space which it fills. Space has three dimensions, length, breadth and thickness.

QUEST.—1. What is matter? 2. What is a body? 3. What is space? How many dimensions has space?

4. There are certain characteristics which belong to all bodies. These are called the *essential properties* of bodies. They are, *Impenetrability*, *Extension*, *Figure*, *Divisibility*, *Inertia*, and *Attraction*.

5. IMPENETRABILITY is the property, in virtue of which, a body must fill a certain space, and which no other body can occupy at the same time. Thus, if you fill a vessel full of water, and then plunge in your hand, or a stick, some of the water will be forced over the top of the vessel. Your hand or the stick removes the water, and does not occupy the space until after the water is displaced.

6. EXTENSION.—Since a body occupies space, it must, like any portion of space, have the dimensions of length, breadth and thickness. These are called the *dimensions of extension*, and vary in different bodies. The length, breadth, and depth of a house are very different from those of an inkstand.

Length and breadth are generally measured in a horizontal direction. Height and depth are the same dimension: height is measured upward, and depth downward.

QUEST.—4. What are the properties common to all bodies? 5. What is impenetrability? Can two bodies occupy the same space at the same time? 6. What are the dimensions of extension? Do these vary in different bodies? How are length and breadth generally measured? What is the difference between height and depth?

Thus, we say a mountain is 400 feet high, and a river 50 feet deep.

7. FIGURE is merely the limit of extension. Figure is also called *form* or *shape*.

If all the parts of a body are arranged in the same way, about a line or a centre, the body is said to be *regular* or *symmetrical*; and when the parts are not so arranged, the body is said to be irregular. Nature has given regular forms to nearly all her productions.

8. DIVISIBILITY denotes the susceptibility of matter to be continually divided. That is, a portion of matter may be divided, and each part again divided, and each of the parts divided again, and so on, continually, without ever arriving at a portion which will be absolutely nothing.

Suppose, for instance, you take a portion of matter, say one pound or one ounce, and divide it into two equal parts, and then divide each part again into two equal parts, and so on continually. Now, all the parts will continually grow smaller and smaller, but no one of them will ever become equal to nothing, since the half of a thing must always have some value.

QUEST.—7. What is figure? When is a body said to be regular or symmetrical? When irregular? 8. What is divisibility? May matter be divided without limit?

9. INERTIA is the resistance which matter makes to a change of state. Bodies are not only incapable of changing their actual state, whether it be that of motion or rest, but they seem endowed with the power of resisting such a change. This property is called *inertia.*

If a body is at rest it will remain so, unless something be applied from without to move it; and if it be moving, it will continue to move, unless something stops it.

10. ATTRACTION OF COHESION.—It has been remarked in Art. 8, that matter is divisible into small parts or portions. The smallest parts into which we can suppose a body divided, are called *particles* or *atoms.* These particles adhere to each other, and form masses or bodies. The force which unites them is called the *attraction of cohesion.* Without this power, solid bodies would crumble to pieces and fall to atoms.

The attraction of cohesion exist also in liquids. It is the attraction of cohesion which holds a drop of water in suspension at the end of the finger, and causes it to take a spherical form.

The attraction of cohesion is stronger in some sub-

QUEST.—9. What is inertia? Can bodies change their actual state? Do they offer resistance to such change? If a body is at rest will it remain so? If in motion will it continue to move? 10. What are particles or atoms? What is the force called which unites them together? Does this attraction exist in liquids? Is it greater in some bodies than in others? In which is it the greatest, the tough or the weak bodies?

stances than in others. Those in which it is the weakest are easily broken, or the attraction is easily overcome; while those in which it is greater, are proportionably stronger.

11. Attraction of Gravitation.—The attraction of cohesion unites the particles of matter, and these by their aggregation form masses or bodies. The *attraction of gravitation* is the force by which masses of matter tend to come together. The attraction of cohesion acts only between particles of matter which are very near each other, while the attraction of gravitation acts between bodies widely separated.

The attraction between two bodies is mutual; that is, each body attracts the other just as much, and no more, than it is attracted by it. But if the bodies are left free, the smaller will move towards the larger: for, as they are urged together by *equal forces*, the smaller will obey the force faster than the larger. Thus, the earth being larger than any body near its surface, forces all bodies towards it, and they immediately fall unless the attraction of gravitation is counteracted.

It should, however, be borne in mind that every body

Quest.—11. What is the attraction of gravitation? What is the difference between the attraction of gravitation and the attraction of cohesion? Is the attraction of gravitation between two bodies mutual? What do you understand by its being mutual? Why does a small body fall to the earth? Does the body attract the earth just as much as the earth does the body?

attracts the earth just as strongly as the earth attracts the body; and the body moves towards the earth, only because the earth is larger, and therefore not as rapidly moved by their mutual attraction.

12. Weight.—The force which is necessary to overcome the attraction of gravitation is called *weight.* Thus, if we have two bodies, and one has twice as much tendency to descend towards the earth as the other, it will require just twice as much force to support it, and hence we say that it is twice as heavy.

SECTION II.

LAWS OF MOTION, AND CENTRE OF GRAVITY.

1. Motion is a change of place. Thus, a body is said to be in motion when it is continually changing its place.

It has been observed in Art. 9, that bodies are indifferent to rest or motion. Hence, a body cannot put itself in motion, or stop itself after it has began to move.

That which puts a body in motion, or which changes

Quest.—12. What is weight? If one body has twice as much matter in it as another, how much heavier will it be? 1. What is motion? Can a body put itself in motion? Can it stop itself when in motion?

its motion after it has begun to move, is called *force* or *power*. Thus, the stroke of the hammer is the force which drives the nail, the effort of the horse the force which moves the carriage, and the attraction of gravitation the force which draws bodies to the earth.

2. Velocity.—The rate at which a body moves, or the rapidity of its motion, is estimated by the space which it passes over in a given portion of time, and this rate is called its *velocity*. Thus, if in one minute of time a body passes over 200 feet, its velocity is said to be 200 feet per minute; and if another body, in the same time, passes over 400 feet, its velocity is said to be 400 feet per minute, or double that of the first.

3. When a body moves over equal distances in equal times, its velocity is said to be *uniform*. Thus, if a body move at the rate of 30 feet a second, it has a uniform velocity, for it always passes over an equal space in an equal time.

4. Bodies which receive uniform accelerations of velocity, that is, equal accelerations in equal times, are said to have motions *uniformly accelerated*. Thus, if a

Quest.—What is force or power? What draws a body to the earth? 2. What is velocity? If a body passes over 20 feet in a second, what is its velocity? 3. When is the velocity of a body said to be uniform? 4. What is an uniformly accelerated motion? If a body fall toward the earth, in what direction will it descend?

body fall freely towards the earth, by the attraction of gravitation, it will descend in a line perpendicular to its surface. In the first second it will fall through 16 feet; in the second second, having the velocity already acquired and being still acted on by the force of gravity, it will descend through 32 feet; in the third second, it will descend through 48 feet; in the fourth second, through 64 feet, and so on, adding to its velocity in every additional second. This is a motion uniformly accelerated, for the velocity is equally increased in each second of time.

5. Momentum is the force with which a body in motion would strike against another body. If a body of a given weight, say 10 pounds, were moving at the rate of 30 feet per second, and another body of the same weight were to move twice as fast, the last would have double the momentum of the first. Hence, when the bodies are equal the momentum will depend on the velocity. But if two unequal bodies move with the same velocity, their momentum will depend upon their weight. Hence, the momentum of a body will depend on its weight and ve-

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Quest.—How far will it fall in the first second of time? How far in the second second? How far in the third second, etc. What kind of a motion will the body have? 5. What is momentum? If two equal bodies move with different velocities, what will their momentums be proportional to? If the velocities are equal and the bodies unequal, what will the momentum depend on? Generally what is the momentum of a body equal to?

locity: that is, it will be equal to the weight multiplied by the velocity.

If the weight of a body be represented by 5 and its velocity by 6, its momentum will be $5 \times 6 = 30$.

If the weight of a body be represented by 8, and its velocity by 2, its momentum will be represented by $16 \times 2 = 32$.

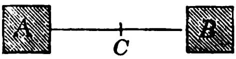
6. ACTION and REACTION.—When a body in motion strikes against another body, it meets with resistance. The force of the moving body is called *action*, and the resistance offered by the body struck is called *reaction;* and it is a general principle, that action and reaction are equal. Thus, if you strike a nail with a hammer, the action of the hammer against the nail, is just equal to the reaction of the nail against the hammer. Also, if a body fall to the earth, by the attraction of gravitation, the action of the body when it strikes the earth, is just equal to the reaction of the earth against the body.

7. CENTRE OF GRAVITY.—The centre of gravity is that point of a body about which all the parts will exactly balance each other. Hence, if the centre of gravity be supported, the body will not fall, for all the parts will balance each other about the centre of gravity.

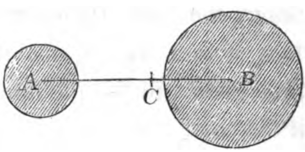
The centre of gravity of a body is not changed by

QUEST.—6. What is action? What is reaction? Are action and reaction equal? 7. What is the centre of gravity of a body? If the centre of gravity is supported will the body fall? Is the centre of gravity changed by altering the position of the body?

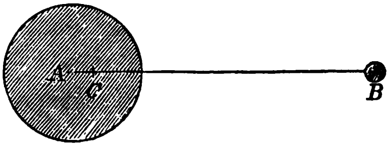
changing its position. Thus, if a body be suspended by a cord, attached at its centre of gravity, it will remain balanced, in every position of the body.



8. If we have two equal bodies *A* and *B*, connected together by a bar *AB*, the centre of gravity will be at *C*, the middle point of *AB*, and about this point the bodies will exactly balance each other.



9. If we have two unequal bodies, *A* and *B*, the centre of gravity *C*, will be nearer the larger body than the smaller, and just as much nearer as the larger body exceeds the smaller. Thus, if *B* is three times greater than *A*, then *BC* will be one third of *AC*.



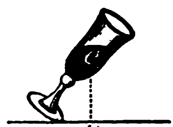
10. If one of the bodies is very large in comparison with the other, the centre of

QUEST.—8. If two equal bodies be connected by a bar, at what point of the bar will the centre of gravity be found? 9. If the bodies are unequal, where will the centre of gravity be found? How much nearer will it be to the larger body? 10. May one body be so much larger than another as to bring the common centre of gravity within the larger body?

gravity may fall within the larger body. Thus the centre of gravity of the bodies *A* and *B*, falls at *C*.

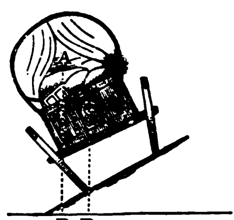
11. The vertical line drawn through the centre of gravity, is called the *line of direction* of the centre of gravity.

If the line of direction of the centre of gravity falls within the base on which the body stands, the body will be supported; but if the line falls without the base, the body will fall.



Thus, if in a wine glass, the centre of gravity be at *C*, the glass will fall the moment the line *CD* falls without the base.

12. Let us suppose a cart on inclined ground, to be loaded with stone, so that the centre of gravity of the mass shall fall at *C*. In this position the line of direction *CD*, falls within the base, and the cart will stand.



But if the cart be loaded with hay, so as to bring the centre of gravity at *A*, the line of direction *AB*, will fall without the base, and the cart will be upset.

QUEST.—11. What is the line of direction of the centre of gravity? When will a body be supported, and when will it fall? 12. If the centre of gravity is near the base, is the body less, or more likely to fall than when it is further from the base? Give the illustration.

SECTION III.

OF THE MECHANICAL POWERS.

1. There are six simple machines, which are called *Mechanical powers.* They are, the *Lever,* the *Pulley,* the *Wheel* and *Axle,* the *Inclined Plane,* the *Wedge,* and the *Screw.*

2. To understand the power of a machine, four things must be considered.

1st. The power or force which acts. This consists in the effort of men or horses, of weights, springs, steam, &c.

2d. The resistance which is to be overcome by the power. This generally is a weight to be moved.

3d. We are to consider the centre of motion, or *fulcrum,* which means a prop. The prop or fulcrum is the point about which all the parts of the machine move.

4th. We are to consider the respective velocities of the power and resistance.

3. A machine is said to be in equilibrium when the

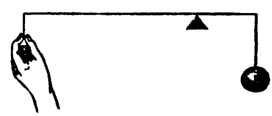
QUEST.—1. How many simple machines are there? What are they called? 2. What things must be considered in order to understand the power of a machine? 3. When is a machine said to be in equilibrium?

resistance exactly balances the power, in which case all the parts of the machine are at rest.

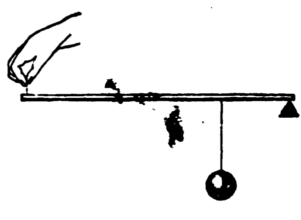
We shall first examine the lever.

4. The *Lever*, is a straight bar of wood or metal, which moves around a fixed point, called the fulcrum. There are three kinds of levers.

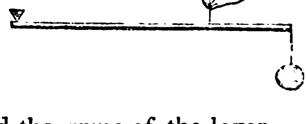
1st. When the fulcrum is between the weight and the power.



2d. When the weight is between the power and the fulcrum.



3d. When the power is between the fulcrum and the weight.



The parts of the lever from the fulcrum to the weight and power, are called the *arms* of the lever.

QUEST.—4. What is a lever? How many kinds of levers are there? Describe the first kind. Where is the weight placed in the second kind? Where is the power placed, in the third kind?

5. An equilibrium is produced in all the levers when the weight multiplied by its distance from the fulcrum, is equal to the product of the power multiplied by its distance from the fulcrum. That is,

The weight is to the power, as the distance from the power to the fulcrum, to distance from the weight to the fulcrum.

EXAMPLES.

1. In a lever of the first kind the fulcrum is placed at the middle point: what power will be necessary to balance a weight of 40 pounds?

2. In a lever of the second kind, the weight is placed at the middle point: what power will be necessary to sustain a weight of 50 lbs.

3. In a lever of the third kind, the power is placed at the middle point: what power will be necessary to sustain a weight of 25 lbs.

4. A lever of the first kind is 8 feet long, and a weight of 60 lbs. is at a distance of 2 feet from the fulcrum: what power will be necessary to balance it?

Ans. 20 *lbs.*

5. In a lever of the first kind, that is 6 feet long, a

QUEST.—5. When is an equilibrium produced in all the levers? What is then the proportion between the weight and power?

weight of 200 *lbs.* is placed at 1 foot from the fulcrum: what power will balance it?

Ans. 40 *lbs.*

6. In a lever of the first kind, like the common steelyard, the distance from the weight to the fulcrum is one inch: at what distance from the fulcrum must the poise of 1 lb. be placed, to balance a weight of 1 lb? A weight of $1\frac{1}{2}$ lbs? Of 2 lbs? Of 4 lbs?

7. In a lever of the third kind, the distance from the fulcrum to the power is 5 feet, and from the fulcrum to the weight 8 feet: what power is necessary to sustain a weight of 40 lbs?

Ans. 64 *lbs.*

8. In a lever of the third kind, the distance from the fulcrum to the weight is 12 feet, and to the power 8 feet: what power will be necessary to sustain a weight of 100 lbs?

Ans. 150 *lbs.*

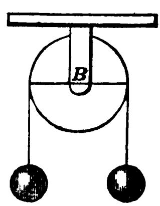
6. Remarks.—In determining the equilibrium of the lever, we have not considered its weight. In levers of the first kind, the weight of the lever generally adds to

Quest.—6. Has the weight been considered in determining the equilibrium of the levers? In a lever of the first kind, will the weight increase or diminish the power? How will it be in the two other kinds?

the powers, but in the second and third kinds, the weight goes to diminish the effect of the power.

In the previous examples, we have stated the circumstances under which the power will exactly sustain the weight. In order that the power may overcome the resistance, it must of course be somewhat increased. The lever is a very important mechanical power, being much used, and entering indeed, into all the other machines.

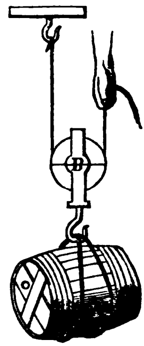
OF THE PULLEY.



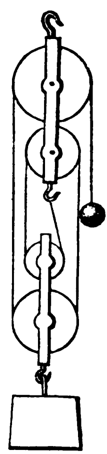
1. The pulley is a wheel, having a groove cut in its circumference, for the purpose of receiving a cord which passes over it. When motion is imparted to the cord, the pully turns around its axis, which is generally supported by being attached to a beam above.

2. Pulleys are divided into two kinds, fixed pulleys and moveable pulleys. When the pulley is fixed, it does not increase the power which is applied to raise the weight, but merely changes the direction in which it acts.

QUEST.—1. What is a pulley? 2. How many kinds of pulleys are there? Does a fixed pulley give any increase of power?



3. A moveable pulley gives a mechanical advantage. Thus, in the moveable pulley, the hand which sustains the cask does not actually support but one half the weight of it—the other half is supported by the hook to which the other end of the cord is attached.



4. If we have several moveable pulleys the advantage gained is still greater, and a very heavy weight may be raised by a small power. A longer time, however, will be required, than with the single pulley. It is indeed a general principle in machines, that *what is gained in power, is lost in time,* and this is true for all machines. There is also an actual loss of power, viz. the resistance of the machine to motion, arising from the rubbing of the parts against each other, which is called the *friction* of the machine. This varies in the different machines, but

QUEST.—3. Does a moveable pulley give any mechanical advantage? In a single moveable pulley how much less is the power than the weight? 4. Will an advantage be gained by several moveable pulleys? State the general principle in machines. What does the actual loss of power arise from? What is this rubbing called? Does this vary in different machines?

must always be allowed for, in calculating the power necessary to do a given work. It would be wrong, however, to suppose that the loss was equivalent to the gain, and that no advantage is derived from the mechanical powers. We are unable to augment our strength, but by the aid of science, we so divide the resistance, that by a continued exertion of power we accomplish that which it would be impossible to effect by a single effort.

If in attaining this result, we sacrifice time, we cannot but see that it is most advantageously exchanged for power.

5. It is plain, that in the moveable pulley, all the parts of the cord will be equally stretched, and hence, each cord running from pulley to pulley, will bear an equal part of the weight; consequently the *power will always be equal to the weight, divided by the number of cords which reach from pulley to pulley.*

EXAMPLES.

1. In a single immoveable pulley, what power will support a weight of 60 lbs ?

2. In a single moveable pulley, what power will support a weight of 80 lbs ?

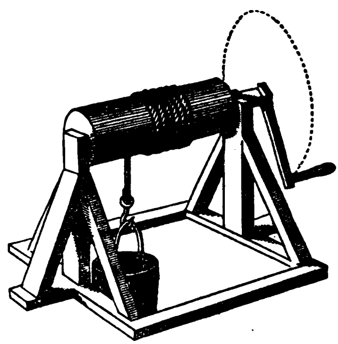
QUEST.—5. In the moveable pulley, what proportion exists between the cord and the weight?

In two moveable pulleys with 5 cords, (see last fig.,) what power will support a weight of 100 *lbs*?

Ans. 20 *lbs.*

WHEEL AND AXLE.

1. This machine is composed of a wheel or crank—firmly attached to a cylindrical axle. The axle is supported at its ends by two pivots, which are of less diameter than the axle around which the rope is coiled, and which turn freely about the points of support. In order to balance the weight, we must have



The power to the weight, as the radius of the axle, to the length of the crank, or radius of the wheel.

EXAMPLES.

1. What must be the length of a crank or radius of a wheel, in order that a power of 40 lbs. may balance

QUEST.—1. Of what is the machine called the wheel and axle composed? How is the axle supported? Give the proportion between the power and the weight?

a weight of 600 *lbs.* suspended from an axle of 6 inches radius?

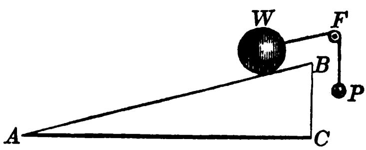
Ans. $7\frac{1}{2}$ *feet.*

2. What must be the diameter of an axle that a power of 100 lbs. applied at the circumference of a wheel of 6 feet diameter may balance 400 *lbs*?

Ans. $1\frac{1}{2}$ *feet.*

INCLINED PLANE.

1. The inclined plane is nothing more than a slope or declivity, which is used for the purpose of raising weights. It is not difficult to see that a weight can be forced up an inclined plane, more easily than it can be raised in a vertical line. But in this, as in the other machines, the advantage is obtained by a partial loss of power.



Thus, if a weight *W*, be supported on the inclined plane *ABC*, by a cord passing over a pulley at *F*, and the cord from the pulley to the weight be parallel to the length of the plane *AB*, the power *P*, will balance the weight *W*, when

$$P : W :: \text{height } BC : \text{length } AB.$$

QUEST.—1. What is an inclined plane? What proportion exists between the power and weight when they are in equilibrium?

It is evident that the power ought to be less than the weight, since a part of the weight is supported by the plane.

EXAMPLES.

1. The length of a plane is 30 feet, and its height 6 feet: what power will be necessary to balance a weight of 200 lbs?

Ans. 40 *lbs.*

2. The height of a plane is 10 feet, and the length 20 feet: what weight will a power of 50 lbs. support?

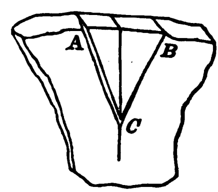
Ans. 100 *lbs.*

3. The height of a plane is 15 feet, and length 45 feet: what power will sustain a weight of 180 lbs?

Ans. 60 *lbs.*

THE WEDGE.

1. The wedge is composed of two inclined planes, united together along their bases, and forming a solid *ACB*. It is used to cleave masses of wood or stone. The resistance which it overcomes is the attraction of cohesion of the body which it is employed to separate. The wedge acts principally by being struck



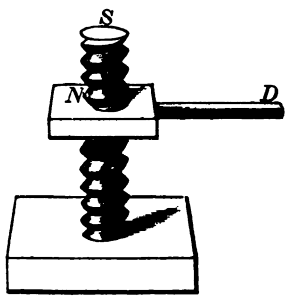
QUEST.—1. What is the wedge? What is it used for? What resistance is it used to overcome?

with a hammer, or mallet on its head, and very little effect can be produced by it, by mere pressure.

All cutting instruments are constructed on the principle of the inclined plane or wedge. Such as have but one sloping edge, like the chisel, may be referred to the inclined plane, and such as have two, like the axe and the knife, to that of the wedge.

THE SCREW.

1. The screw is composed of two parts—the screw *S*, and the nut *N*.



The screw *S*, is a cylinder with a spiral projection winding around it, called the *thread*. The nut *N* is perforated to admit the screw, and within it is a groove into which the thread of the screw fits closely.

The handle *D*, which projects from the nut, is a lever which works the nut upon the screw. The power of the screw depends on the distance between the threads. The closer the threads of the screw, the greater will be

QUEST.—1. Of how many parts is the screw composed? Describe the screw? What is the thread? What the nut? What is the handle used for? To what uses, is the screw applied?

the power, but then the number of revolutions made by the handle *D*, will also be proportionally increased—so that we return to the general principle—what is gained in power is lost in time. The power of the screw may also be increased by lengthening the lever attached to the nut.

The screw is used for compression, and to raise heavy weights. It is used in cider and wine presses, in coining, and for a variety of other purposes.

General Remarks.—All machines are composed of one or more of the six machines which we have described. We should remember, that friction diminishes very considerably the power of machines.

There are no surfaces in nature which are perfectly smooth. Polished metals, although they appear smooth, are yet far from being so. If, therefore, the surfaces of two bodies come into contact, the projections of the one will fall into the hollow parts of the other, and occasion more or less resistance to motion. In proportion as the surfaces of bodies are polished, the friction is diminished, but it is always very considerable, and it is computed that it generally destroys one-third the power of the machine.

Oil, or grease, is generally used to lessen the friction. It fills up the cavities of the rubbing surfaces, and thus makes them slide more easily over each other.

SECTION IV.

OF SPECIFIC GRAVITY.

1. The specific gravity of a body is the relation which the weight of a given magnitude of that body bears to the weight of an equal magnitude of a body of another kind.

2. If two bodies are of the same bulk, the one which weighs the most is said to be specifically heavier than the other. On the contrary, one body is said to be specifically lighter than another when a certain bulk or volume of it, weighs less than an equal bulk of that other.

Thus, if we have two equal spheres, each one foot in diameter, the one of lead and the other of wood, the leaden one will be found to be heavier than the wooden one; and hence, its specific gravity is greater. On the contrary, the wooden sphere being lighter than the leaden one, its specific gravity is less.

3. The greater specific gravity of a body indicates a greater quantity of matter in a given bulk, and consequently the matter must be more compact, or the particles nearer together. This closeness of the particles is called *density*. Hence, if two bodies are of equal bulk or volume, their weights or specific gravities will be proportional to their densities.

4. If two bodies are of the same specific gravity, or density, their weights will be proportional to their bulks.

5. A body specifically heavier than a fluid will sink on being immersed in it. It will, however, descend less rapidly through the fluid than through the air, and less power will be required to sustain the body in the fluid than out of it. Indeed, it will lose as much of its weight as is equal to the weight of a quantity of fluid of the same bulk. If a body is of the same specific gravity with the fluid, it loses all its weight, and requires no force but the fluid to sustain it. If it be lighter, it will be but partially immersed, and a part of the body will remain above the surface of the fluid.

6. We may conclude, from what has been said in the last article,

1st. That when a heavy body is weighed in a fluid, its weight will express the difference between its true weight and that of an equal bulk of the fluid.

2d. If the body have the same specific gravity with the fluid, its weight will be nothing.

3d. If the body be lighter than the fluid, it will require a force equal to the difference between its own weight, and that of an equal bulk of the fluid, to keep it entirely immersed, that is, to overcome its tendency to rise.

7. In comparing the weights of bodies, it is necessary to take some one as a standard, with which to compare all others. Rain water is generally taken as this standard.

A cubic foot of rain water is found, by repeated experiments, to weigh $62\frac{1}{2}$ pounds, avoirdupois, or 1000 ounces. Now, since a cubic foot contains 1728 cubic inches, it follows that 1 cubic inch weighs ,03616898148 of a pound. Therefore, if the specific gravity of any body be multiplied by ,03616898148, the product will be the weight of a cubic inch of that body in pounds avoirdupois. And if this weight be then multiplied by 175, and the product divided by 144, the quotient will be the weight of a cubic inch in pounds Troy—since 144 *lbs.* Avoirdupois is just equal to 175 *lbs.* Troy.

8. Since the specific gravities of bodies are as the weights of equal bulks, the specific gravity of a body will be to the specific gravity of a fluid in which it is immersed, as the true weight of the body, to the weight lost in weighing it in the fluid. Hence, the specific gravities of different fluids are to each other as the weights lost by the same solid immersed in them.

PROBLEM I.

To find the specific gravity of a body, when the body is heavier than water.

RULE.

1st. *Weigh the body first in air and then in rain water, and take the difference of the weights, which is the weight lost.*

2d. *Then say, as the weight lost is to the true weight,* *so is the specific gravity of the water to the specific gravity of the body.*

EXAMPLES.

1. A piece of platina weighs 70,5588 lbs. in the air, and in water only 66,9404 lbs.: what is its specific gravity, that of water being taken at 1000?

First, 70,5588 – 66,9404 = 3,6184 lost in water.

Then, 3,6184 : 70,5588 : : 1000 : 19500, which is the specific gravity, or weight of a cubic foot of platina.

2. A piece of stone weighs 10 *lbs.* in air, but in water only $6\frac{3}{4}$ *lbs*: what is its specific gravity?

Ans. 3077.

PROBLEM II.

To find the specific gravity of a body when it is lighter than water.

RULE.

1st. *Attach another body to it of such specific gravity that both may sink in the water, as a compound mass.*

2d. *Weigh the heavier body and the compound mass separately, both in water and in open air, and find how much each loses by being weighed in water.*

3d. *Then say, as the difference of these losses is to the weight of the lighter body in the air, so is the specific gravity of water to the specific gravity of the lighter body.*

EXAMPLES.

1. A piece of elm weighs 15 *lbs.* in open air. A piece of copper which weighs 18 *lbs.* in air and 16 in water is attached to it, and the compound weighs 6 *lbs.* in water: what is the specific gravity of the elm?

Copper.	Compound.
18 in air.	33 in air.
16 in water.	6 in water.
2 loss.	27 loss.

Then, 27—2=25=difference of losses.

Then, as 25 : 15 : : 1000 : 600, which is the specific gravity of the elm.

2. A piece of cork weighs 20 *lbs.* in air, and a piece of granite weighs 120 *lbs.* in air, and 80 *lbs.* in water. When the granite is attached to the cork the compound mass weighs $16\frac{3}{4}$ *lbs.* in water: what is the specific gravity of the cork?

Ans. 240.

PROBLEM III.

To find the specific gravity of fluids.

RULE.

1st. *Weigh any body whose specific gravity is known, both in the open air, and in the fluid, and take the difference, which is the loss of weight.*

2d. *Then say, as the true weight is to the loss of weight, so is the specific gravity of the solid, to the specific gravity of the fluid.*

EXAMPLES.

1. A piece of iron weighs 298,1 ounces in the air, and 259,1 ounces in a fluid, the specific gravity of the iron is 7645: what is the specific gravity of the fluid?

First, 298,1 − 259,1 = 39 loss of weight:

Then, 298,1 : 39 : : 7645 : 1000, which is the specific gravity of the fluid: hence the fluid is water.

2. A piece of lignumvitæ weighs $42\frac{3}{4}$ ounces in a fluid, and $166\frac{5}{8}$ ounces out of it; what is the specific gravity of the fluid—that of lignumvitæ being 1333?

Ans. 991, which shows the fluid to be liquid turpentine or Burgundy wine.

NOTE.—In a similar manner the specific gravities of all liquids may be found from the following table.

TABLE OF SPECIFIC GRAVITIES.

	Spec. grav.		wt. cub. in.		Spec. grav.		wt. cub. in.
Platina	19500	-	11·285	Mercury	13568	-	7·872
Do. hammered	20336	-	11·777	Pure cast gold	19258	-	11·145
Cast zinc	7190	-	4·161	Amber	1078	wt. cub. ft.	
Cast iron	7207	-	4·165	Brick	2000	-	125·00
Cast tin	7291	-	4·219	Sulphur	2033	-	127·06
Bar iron	7788	-	4·507	Cast nickel	7807	-	4513
Hard steel	7816	-	4·523	Cast cobalt	7811	-	4520
Cast brass	8395	-	4·856	Paving stones	2416	-	151·00
Cast copper	8788	-	5·085	Common stone	2520	-	157·50
Pure cast silver	10474	-	6·061	Flint and spar	2594	-	162·12
Cast lead	11352	-	6·569	Green glass	2642		

Of Specfiic Gravities.

	Spec. grav.	wt. cub. ft. lbs.
White glass	2892	
Pebble	2664	166·50
Slate	2672	167·00
Pearl	2684	
Alabaster	2730	
Marble	2742	171·38
Chalk	2784	174·00
Limestone	3179	193·68
Wax	897	
Tallow	945	
Camphor	989	
Bees' wax	965	
Honey	1456	
Bone of an ox	1659	
Ivory	1822	
Air at the earth's surface	14	
Liquid turpentine	991	
Olive oil	915	
Burgundy wine	991	
Distilled water	1000	
Sea water	1028	
Milk	1030	
Beer	1034	

	Spec. grav.	wt. cub. ft.
Cork	240	15·00
Poplar	383	23·94
Larch	544	34·00
Elm and West India fir	556	34·75
Mahogany	560	35·00
Cedar	596	37·25
Pitch pine	660	41·25
Pear tree	661	41·31
Walnut	671	41·94
Elder tree	695	43·44
Beech	696	43·50
Cherry tree	715	44·68
Maple and Riga fir	750	46·87
Ash & Dantzic oak	760	47·50
Apple tree	793	49·56
Alder	800	50·00
Oak, Canadian	872	54·50
Box, French	912	57·00
Logwood	913	57·06
Oak, English	970	51·87
Oak, 60 years old	1170	73·12
Ebony	1331	83·18
Lignumvitæ	1333	83·31

REMARK.—In the table of specific gravities, the cubic foot of water, which weighs 1000 ounces, is taken as the standard, and the figures in the column of specific gravity shows the weight of a cubic foot in ounces, or how many times each substance is heavier or lighter than water. The other column shows the weight in ounces of a cubic inch, or the weight in pounds of a cubic foot.

PROBLEM IV.

The specific gravity and weight of a body being given, to find its solidity.

RULE.

As the tabular specific gravity of the body is to its weight in ounces avoirdupois, so is 1 *cubic foot, to the content in cubic feet.*

EXAMPLES.

1. What is the solid content of a block of marble, that weighs 10 tons, its specific gravity being 2742.

First, 10 tons = 358400 ounces.

Then, 2742 : 358400 : : 1 : $130\frac{970}{1371}$, which is the content in cubic feet?

NOTE.—If the answer is to be found in cubic inches, multiply the ounces by 1728.

2. How many cubic inches in an irregular block of marble, which weighs 112 pounds, allowing its specific gravity to be 2520?

3. How many cubic inches of gunpowder are there in 1 pound weight, its specific gravity being 1745?

Ans. $15\frac{3}{4}$, *nearly.*

4. How many cubic feet are there in a ton weight, of dry oak, its specific gravity being 925.

Ans. $38\frac{138}{185}$.

THE END.