ARITHMETIC

EUGENE HERZ
AND
MARY G. BRANTS

MANUAL

PARTS VII
AND VIII

ARITHMETIC

BY

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TEACHER'S MANUAL

FOR

PARTS VII AND VIII

ADVANCED LESSONS

THE JOHN C. WINSTON COMPANY
CHICAGO PHILADELPHIA TORONTO

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FOREWORD

Bearing in mind that a thorough knowledge of arithmetic is perhaps more frequently the cause of success in life than is any other single factor, one can hardly overestimate the importance of this subject to the future welfare of the child, nor can one fail to realize how great is the responsibility which rests on those whose duty it is to provide for his education in this branch.

No book or series of books can possibly illustrate every use to which numbers can be put, but if the principles underlying their use are properly taught, the child can reason for himself the proper application of his knowledge to any given problem. Furthermore, as he must know not merely how to solve a problem, but how to solve it in the quickest and simplest manner, he must know not merely the various processes, but their construction as well; he must be able to analyze to such an extent that when a problem is presented to him, he can distinguish the facts which are relevant from those which are irrelevant, he can separate the known from the unknown, he can arrange the known in logical order for his processes, and he can use the shortest processes possible. An attempt to give the pupil this ability is the motive for this work.

The vehicle used to obtain the result is a series of progressive lessons, which, with ample practice, take the pupil step by step through the construction of each process to be learned, thus giving him the opportunity of following the teacher's explanation, and of referring to past lessons at any time. In this way the pupil who is slower to grasp new ideas than the average can keep up with his class, and every pupil can at all times refresh his memory on any points which he may have forgotten or which may have escaped him in the classroom, and which have so often been lost to him forever.

The time-saving methods used by the most expert arithmeticians are introduced as part of the routine work; thus, these become a part of the child's general education without any special effort on his part.

It is not intended that the lessons or definitions are to be learned verbatim, any more than it is intended that the examples given are to be memorized; both are there for the purpose of showing the pupil the reason for, and the application of, the processes, and the exercises are there to give him practice and to test his knowledge of what he has learned.

The exercises are prepared in such manner that they form an automatic and continuous review of what has been learned, but further review work is given at regular intervals.

The series consists of Three Books and Teacher's Manuals, as follows:

Primary Lessons.....	Parts I and II. (Teacher's Manual only.)
Elementary Lessons...	Parts III and IV. (With Manual for the Teacher.)
Intermediate Lessons..	Parts V and VI. (With Manual for the Teacher.)
Advanced Lessons.....	Parts VII and VIII. (With Manual for the Teacher.)

The first two parts are so arranged in the Teacher's Manual that the lessons and exercises can be given largely as games, play work, number stories, in language work, etc., all used more or less incidentally, till the child is gradually prepared for work requiring an increasing degree of conscious effort.

The work contained in each of the eight parts is that which is usually taught in the corresponding grade, and it is recommended that this routine be followed. However, special provision has been made for such variations in the grading as are required in some localities, by means of a series of notes in the Teacher's Manuals which enable the teacher to follow either method with equal facility.

GENERAL NOTES FOR THE TEACHER

1. Read the foreword carefully.
2. Follow the detailed notes for each lesson.
3. Each lesson is to be thoroughly demonstrated, explained and discussed in the classroom before being used by the pupil for study and reference. The time required for each lesson depends on the ability of the class. Be thorough.
4. In demonstrating, use the objects of the pupil's environment for concrete material, and let him have first-hand experience.
5. Illustrate every essential point on the blackboard.
6. Do everything possible to make the recitations interesting and enjoyable.
7. Introduce the competitive spirit wherever possible.
8. Remember that proficiency in arithmetic can be analyzed as resulting principally from these three factors:
   (*a*) A thorough knowledge of the various processes and methods.
   (*b*) The ability to select the process or method most applicable to the given problem.
   (*c*) The elimination of all unnecessary work.
9. Make the pupil realize that a thorough knowledge of arithmetic will be of great value to him throughout his life.
10. Before beginning the year's work, make a careful survey of the topics to be covered, giving due consideration to the Notes Regarding Grading in the Teacher's Manuals; then plan your schedule so that you will not have to slight over some of the later work on account of lack of time.

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PART VII

AN INTELLIGENCE TEST

This set of test questions will aid you in grouping your pupils properly, in determining who are the weak ones and in checking the results of your efforts. Defective hearing and sight will also be disclosed thereby.

Make a tabulation of the number of problems attempted, the number correctly solved, and the percentage attained by each child, and keep it for future comparison. (Average the per cent attempted and the per cent solved of those attempted; viz.: 8 attempted out of 10 = 80%; 7 solved out of 8 attempted = 87½%; average 83¾%.)

Name	Beginning of Year			Middle of Year			End of Year		
	Attempted	Solved	%	Attempted	Solved	%	Attempted	Solved	%
Adams, Bessie	8	7	83¾						

Do not give the pupils the result of the test, nor show them directly wherein they failed, but help each group in its weak subjects without permitting them or the others to realize what you are doing.

At the end of the first half of the school year give the same test again and compare the results; repeat again at the end of the school year. If you desire, the test may be given more frequently.

How to give the test:

Write the first five problems (properly numbered) on the board during the pupils' absence and keep them covered until the proper moment designated hereafter.

When ready to begin the test, provide them with paper and read the second five problems to the pupils very slowly and let them make such memoranda thereof as they will. Give them the number of each problem as you read it.

Tell them they are to number their answers to correspond with the problem numbers.

Now remove the covering from the board and have them begin. (Those on the board come first.)

Allow 30 minutes for the actual working of the problems.

The Problems:

The first five to be written on the board:

1. A carload of coal consisted of 24 T. 20% of the coal was sold to A and 30% to B; what quantity would C and D each receive, if each got $\frac{1}{2}$ of what was left?
2. What will $12\frac{1}{2}$ A. of land cost, if $4\frac{1}{2}$ A. cost \$675.?
3. Find the interest on \$450. at 6% for 123 days.
4. A farm containing 10 A. 80 sq. rd. is 80 rd. long; find the perimeter.
5. If a quarter-section of land costs \$800., what should it be sold for to gain 40%?

The second five to be read to the pupils:

6. James Brown received \$363.18 when he sold a tract of land for his neighbor for \$12,106.; what rate of commission did he earn?
7. $(.0006 \div .002) + (.06 \times .0055) = ?$
8. Tom Jones owned $\frac{3}{5}$ of a boat worth \$6,000.; he sold $\frac{1}{2}$ of his share for \$4,800.; what did he gain?
9. If \$9.75 interest was earned on \$650. at 6%, how long was the money loaned?
10. What number divided by $84\frac{1}{2}$ equals $120\frac{1}{2}$?

The Answers:

1.	6 T.;	6.	3%;
2.	$1,875.;	7.	.30033;
3.	$9.23;	8.	$3,000.;
4.	202 rd.;	9.	90 da.;
5.	$1,120.;	10.	10,182¼.

Until further notice, the blackboard drill should now be varied from day to day, and should cover rapid addition and subtraction in all the forms taught in Parts IV, V, and VI, as well as the use of aliquot parts in multiplication and division. Give these four or five minutes a day if more time is not available. Time tests should be given frequently.

Competition is a wonderful incentive for good work and continued effort. Make use of it by giving "class honors" frequently and letting the children strive for them. As an example, you can give the honor of "Class Denominate Number Expert" to the boy or girl who has done the best work in denominate numbers after Lesson 6 is completed, and so on throughout the work.

DENOMINATE NUMBERS

Lesson 1

Let the child make a table each time as far as his work carries him. Drill in sorting-letters-style till he can identify the numbers of import in the work. Example 1, is in Linear Measure; after his talk on it, place 3, 12, 36 (one at a time) on the board and let him say: "3, the number of feet in 1 yard"; "12, the number of inches in 1 foot", etc.; later, just this: "36″= 1 yd." etc.

The drill must grow with the need of the problems until the full table is made by use and learned. Then call for memory work on the tables.

The distinction between a simple and a compound denominate number is here studied for the first time, and

this distinction must be made clear to the pupils. Compare with formation of compound words which he knows, as steamboat, railroad.

Let him create some compound denominate numbers that you may be enabled to see that he understands that the several parts thereof must come from the same table.

Make the child realize which is the basic unit in each example.

Reduction to smaller and to larger denominations should be presented co-relatedly as each of these processes is more easily understood by reason of the other. Explain carefully the method of proving by approximation, and use frequently. It will satisfy the children.

Exercise 2—Written.

Answers:

1.	17 ft.;	**5.**	49 pt.;	**9.**	98 mo.;
2.	212 in.;	**6.**	7,580 min.;	**10.**	50 qt.;
3.	7 pt.;	**7.**	70 oz.;	**11.**	90 in.;
4.	157 qt.;	**8.**	92 in.;	**12.**	14 qt.

Exercise 3—Written.

Answers:

1. 3 hr. 30 min.;
2. 4 da. 12 hr. 20 min.;
3. 21 yd. 6 in.;
4. 6 gal. 1 qt. 1 pt.;
5. 3 pk. 1 qt. 1 pt.;
6. 6 bu. 1 pk. 2 qt. (dry);
1 bbl. 19 gal. (liquid);
7. 3 hr. 15 min.;
8. 6 da. 7 min.;
9. 15 lb. 6 oz.;
10. 6 sq. yd. 8 sq. ft. 38 sq. in.

Lesson 2

This lesson covers a very important and frequently neglected phase of reduction, and offers the opportunity of introducing examples in every conceivable form.

In reducing to smaller denominations which are two or more denominations apart, as for instance, $\frac{1}{6}$ yd. to inches, let the pupils carry the work through each denomination in this manner: $\frac{1}{6}$ yd. = $\frac{1}{2}$ ft.; $\frac{1}{2}$ ft. = 6 inches; but demonstrate also that as there are 36 inches in a yard, $\frac{1}{6}$ yd. = 6 inches.

Exercise 6—Written.

Answers:

1. $4\frac{1}{2}$ ft.;
2. $3\frac{1}{8}$ pk.;
3. $3\frac{1}{16}$ pk.;
4. $4\frac{1}{2}$ hr.;
5. $4\frac{1}{4}$ da.;
6. $12\frac{1}{4}$ ft.;
7. $5\frac{2}{3}$ yd.;
8. $6\frac{5}{12}$ da.;
9. $8\frac{1}{4}$ ft.;
10. $3\frac{3}{4}$ bu.;
11. $\frac{1}{4}$ yd.;
12. $\frac{1}{16}$ pk.;
13. $5\frac{1}{8}$ gal.;
14. $\frac{7}{9}$ yd.;
15. $\frac{7}{8}$ gal.;
16. $\frac{41}{80}$ hr.;
17. $\frac{5}{6}$ sq. yd.;
18. 1 qt. 1 pt.;
19. 1 ft. 4 in.;
20. 9 doz.

Lessons 3, 4, 5, and 6

These lessons cover work similar in nature to that which the pupils have had in previous Parts, excepting that numbers of three or more denominations are now used. Drill on barrel and hogshead; also on square rods, acres and square miles as these denominations have not previously been used.

Exercise 7—Written. Call for Proofs.

Answers:

1. \$56.10;
2. \$3.00;
3. $789\frac{11}{20}$ cwt.;
4. $46\frac{3}{4}$ T.;

5. $37.38;
6. $50.88;
7. $27.84;
8. $17\frac{1}{3}$ yd.; $10.40;
9. $6\frac{1}{6}$ yd.;
10. $10\frac{7}{8}$ yd.;
11. $66.60;
12. $50.30.

Exercise 8—Written. Call for Proofs.

Answers:

1. 1 hhd. 1 bbl. $29\frac{1}{2}$ gal.;
2. 2 mi. 259 rd. $4\frac{1}{2}$ yd.;
3. 6 gal. 2 qt. 1 pt.;
4. 11 bu. 1 pk. 6 qt.;
5. 20 sq. yd. 6 sq. ft. 72 sq. in.; $74.60;
6. 3 gr. $7\frac{2}{3}$ doz.; $1.31;
7. 946 yd.;
8. 11 hr. 55 min.

Exercise 9—Written. Call for Proofs.

Answers:

1. 248 da. 23 hr.;
2. 41 hhd. 1 bbl.;
3. $245.50;
4. 1 mi. 316 rd. 2 yd.;
5. 87 rd.; $9.57;
6. 75 cu. yd. 18 cu. ft. 576 cu. in.

Exercise 11—Written. Call for Proofs.

Answers:

1. 43 da. 23 hr. 55 min.;
2. 12 mi. 160 rd. 3 yd.;
3. 40 cu. yd. 18 cu. ft. 869 cu. in.;
4. 2 sq. ft. 8 sq. in.;

5. 8 T. 10 cwt. 50 lb.;
6. 4 hhd. 1 bbl. 7 gal.;
7. 176 yd.;
8. Side = 4 yd. 2 ft. 6 in.;
   Area = $23\frac{13}{36}$ sq. yd.;
9. 1 T. 11 cwt. 70 lb.

Exercises 10 and 12 afford an opportunity of clinching the tables, for their application, and for the development of speed. Try to keep the lessons brisk. The slow ones will struggle to keep pace with the others. Work hard for quick identification of number to special application given.

Exercise 13—Written.

Answers:

1. 864 pieces;
2. 4 sq. rd.;
3. $\frac{5}{8}$ ft.;
4. 143 lengths;
5. 30 times;
6. $1.17;
7. .125;
8. $3,180.00;
9. $15.00 Profit;
   50% Profit;
10. 100 bu.;
    $1.00 per bu.;
11. $63.50 Total Cost;
    50¢ Saved;
12. Hay is more profitable;
    Difference $1,920.00;
13. 1,890,000 magazines;
14. 40 tools;
15. 1,125 lb.;
16. 280 packages;
17. 215 plants;
18. 32 jars;
19. 8 hr. 30 min.;
20. 40 sq. yd.

Give your "class honor" now for denominate number work.

Lesson 7

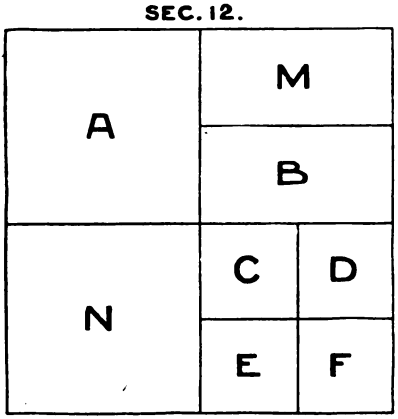
Not too much time should be devoted to this lesson; the principal points to be brought out are:

(*a*) That a township is a 6-mile square containing 36 sq. mi.;

(*b*) That a section is a 1-mile square containing 1 sq. mi. or 640 acres.

(*c*) That each section is divided into four quarters.

Let the children locate farms at sight, as:



Q. Locate M.

Ans. M = N. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Sec. 12.

Q. Locate E.

Ans. E = S.W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$ of Sec. 12.

Call for number of acres in each.

Call for miles of fencing required for each.

Call for rods of fencing required for each.

Exercise 15—Written.

Answers:

1. $\frac{1}{2}$ section;
2. $\frac{1}{4}$ section;
3. $\frac{1}{8}$ section;
   $\frac{1}{2}$ quarter-section;
4. $\frac{1}{16}$ section;
   $\frac{1}{4}$ quarter-section;
5. 1 mi. by $\frac{1}{2}$ mi.;
   320 rd. by 160 rd.;
6. $\frac{1}{2}$ mi. by $\frac{1}{2}$ mi.;
   160 rd. by 160 rd.;
7. $\frac{1}{2}$ mi. by $\frac{1}{4}$ mi.;
   160 rd. by 80 rd.;
8. $\frac{1}{4}$ mi. by $\frac{1}{4}$ mi.;
   80 rd. by 80 rd.;
10. 20 acres;
11. $7\frac{1}{2}$ acres;
12. 5 acres;
13. $2\frac{1}{2}$ acres;
14. $2\frac{1}{2}$ acres;
15. $2\frac{1}{2}$ acres;
16. 160 rods; 20 rods;
17. 1 mile;
18. 20 rods;
19. 320 rods;
20. 4 miles;
21. 80 rods;
22. 320 rods.

Lesson 8

This lesson not only familiarizes the pupils with the customs governing the purchase of paper, but offers invaluable work in the practical application of ratio and proportion.

Place a 22″ × 34″ oblong on the board and cut it into two 17″ × 22″ oblongs to show the ratio.

Exercise 17—Written.

Answers:

1. $7.20;
2. 88 lb.; Ratio = 2;
3. 144 lb.; Ratio = ½;
4. $1.62;
5. $222.60;
6. $12.00;
7. $400.00;*
8. $3.60;
9. $30.00;
10. $228.00.

Lesson 9

This lesson is of great benefit in three distinct ways:

1st. It affords abundant drill in multiplication and division.

2d. It teaches a subject which will be of considerable value to the child at all times.

3d. *And not least.* It teaches concentration and analysis, as the child must concentrate and analyze to ascertain the number of ems of different sizes of type to an inch.

This lesson can be made very interesting by having the children bring newspapers, magazines, and books to school, and have them measure the various sizes of type faces and pages for the number of points, picas, and ems. Let them measure their school books; they will enjoy it immensely.

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* In this example the size of the sheet has no bearing on the answer; (4 T. @ 5¢ per lb. = $400.00). Note how many of the pupils realize this from a reading of the example.

Exercise 19—Written.

Answers:

1. 468 points;
   39 picas;
2. $8\frac{1}{4}$ in.;
   $49\frac{1}{2}$ picas;
3. 1,602 points long;
   162 points wide;
   $133\frac{1}{2}$ picas long;
   $13\frac{1}{2}$ picas wide;
4. 81 ems in 1 line;
   108 ems long;
   8,748 ems per page;
5. 1,728 ems 12-point;
   6,912 ems 6-point;
6. 42 picas wide;
   54 picas long;
7. 1,512 ems;
8. 8 in. long;
9. 4 columns;
   1,512 ems per column;
   6,048 ems per page;
10. $8\frac{2}{3}$ in.;
    52 picas.

Lesson 10

Note Regarding Grading.

Where Lesson 10 has been taught in the fifth year, it must nevertheless now be given as review work.

The pupils should be familiar with the legal weights (for their state) of a bushel of the more common farm products. This lesson may be given in several installments to facilitate memorization. Bottles of grain of the different kinds should be used to help the children familiarize themselves with the several varieties; let them handle the grain and learn the weights at the same time. Test them occasionally.

The tables with which this subject closes are those used in certain professions requiring a technical education and need not be learned, but the pupils should be made familiar with their names and their uses.

Note Regarding Grading.

Where it is required that work on the Table of Circular Measure be given in the seventh year, Lesson 2 (including Exercises 4 and 5) of Part VIII should now be given.

Note Regarding Grading.

Where it is required that work in foreign money be given in the seventh year, Lesson 7 (including Exercise 14) of Part VIII should now be given.

Exercise 21—Oral Review.

More review work of a similar nature may be given, time permitting.

Exercise 22—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. June 6, 1919; \$255.78;
2. 3,880 bd. ft.;
3. \$600.00;
4. \$54.88;
5. $29\frac{7}{17}\%$;
6. $5\frac{1}{3}$ min. per report; 12 trips;
7. \$21.06;
8. 1 sq. yd. 7 sq. ft. 49 sq. in.;
9. 5,184 cu. in.;
10. 3 da. 4 hr. 45 min. 30 sec.:
11. 434,634;
12. 239,299;
13. 389,096;
14. 729,944;
15. 189,930;
16. 264,977;
17. 96,815;
18. 615,138;
19. 463,108;
20. 155,970;
21. 366,097;
22. 598,853;
23. 27,293;
24. 199,017;
25. 239,844;
26. 34,360;
27. 33,304;
28. 33,791;
29. 434;
30. 639;
31. 728;
32. 864;
33. 707;
34. 901;
35. 27,218;
36. 22,742;
37. 31,434;
38. 6,656;
39. 19,285;
40. 29,580;
41. 29,029;
42. 37,666.

LESSON 11

This interesting and useful lesson should be thoroughly mastered. It constitutes a thorough test of the pupil's knowledge of the proper application of multiplication and division. Call for equivalents often, as: 2 lamps for 8 hours, or 1 lamp for 16 hours, or 4 lamps for 4 hours, etc.

Exercise 24—Written.

Answers:

1. 20 men;
2. 30 men;
3. 50¢;
4. 42 hr.;
5. $3,937.50;
6. 7 hr.;
7. 40 lamps;
8. 60 hr.

Give your "class honor" again for denominate number work.

FRACTIONS

LESSON 12

The subject matter of this lesson is not new to the pupil; only the form is new, and this should be properly mastered. Use simple forms. Let children tell steps each time—parts first; then results.

Exercise 26—Written.

Answers:

1. $1\frac{1}{2}$;
2. $\frac{3}{14}$;
3. $1\frac{1}{10}$;
4. $\frac{2}{3}$;
5. $2\frac{13}{14}$;
6. $1\frac{1}{2}$;
7. $1\frac{1}{3}$;
8. $\frac{3}{5}$;
9. $\frac{2}{3}$;
10. $1\frac{1}{5}$;
11. $\frac{3}{4}$;
12. $\frac{1}{2}$;
13. $3\frac{1}{2}$;
14. $5\frac{1}{6}$;
15. $3\frac{1}{8}$;
16. $\frac{7}{8}$;
17. $2\frac{4}{5}$;
18. 3;
19. $1\frac{1}{7}$;
20. $2\frac{7}{10}$;
21. $62\frac{1}{2}$;
22. $600\frac{1}{4}$;
23. 280;
24. 35;
25. $\frac{1}{5}$;
26. $\frac{1}{10}$;
27. $\frac{1}{20}$;
28. $\frac{1}{3}$;
29. $\frac{1}{10}$;
30. $\frac{9}{16}$;
31. $\frac{13}{16}$;
32. $\frac{1}{2}$;
33. $\frac{20}{27}$;
34. $\frac{21}{26}$;
35. $\frac{4}{15}$;
36. $1\frac{1}{2}$;
37. 2;
38. $\frac{3}{8}$;
39. $\frac{2}{5}$;
40. $1\frac{1}{4}$.

MULTIPLICATION

Lesson 13

Cross Multiplication is so useful in every day life, regardless of the nature of one's occupation, that it should form part of the arithmetical equipment of every grammar school graduate.

Use small numbers of two orders at first to demonstrate, then gradually use larger numbers, but use no numbers of more than two orders at any time. Follow the text carefully step by step.

Coax him to keep his eye on the problem or numbers, to encourage concentration. Urge him to do these very promptly.

Exercise 28—Written.

Answers:

1. 682;	9. 1,092;	17. 1,958;	25. 1,230;
2. 483;	10. 1,462;	18. 1,968;	26. 1,659;
3. 672;	11. 1,518;	19. 2,584;	27. 1,302;
4. 882;	12. 864;	20. 3,528;	28. 2,883;
5. 528;	13. 1,376;	21. $15.70;	29. 1,455;
6. 768;	14. 868;	22. 1,296;	30. 4,096.
7. 1,008;	15. 851;	23. 462;	
8. 1,428;	16. 3,038;	24. 2,080;	

TIME AND WAGES

Lesson 14

Teach the pupils the necessity of recognizing aliquot parts of 48 hours or of any other *weekly hour basis.*

In finding a fraction of a week's wages, such as $\frac{43}{48}$ of $25. the importance of multiplying 43 × 25 and dividing by 48, rather than dividing 25 by 48 and multiplying by 43, is great, both for accuracy and for speed.

Exercise 30—Written.

Answers:

1. Adams, $16.88;
   Jones, $24.00;
2. Bailey, $3.50;
   Davis, $5.00;
3. $338.00;
4. $18.00;
5. $7.14;
6. 40 hr.;
7. 37½ hr.;
8. $36.00;
9. $20.00.

Lesson 15

The principle of transposition, applied here to the hours worked and the rate of wages when the rate of wages is an aliquot part of the weekly hour basis, should be thoroughly mastered, as other applications thereof appear later in connection with other processes.

Exercise 32—Written.

Answers:

1. $6.67;
2. $6.67;
3. $18.75;
4. $10.19;
5. $7.33;
6. $7.33;
7. $13.13;
8. $25.00;
9. $18.38;
10. $1.50.

Give your "class honor" now for work in time and wages.

Exercise 33—Oral Review.

More review work of a similar nature may be given, time permitting. Time them. Then coax them to "outrun" themselves. Let them try them over and over to win.

Exercise 34—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. Aug. 14, 1918; $398.53;
2. $10,000.;
3. $66\frac{2}{3}\%$;
4. $9.66;
5. 2,030 bd. ft.; $71.05;
6. $74\frac{3}{27}\%$;
7. 48 hr.; $2.37;
8. 69,722;
9. 394,199;
10. 648,357;
11. 165,391;
12. 169,745;
13. 202,393;
14. 212,196;
15. 119,188;
16. 146,888;
17. 99,892;
18. 188,889;
19. 193,989;
20. 79,099;
21. 429,969;
22. 207,899;
23. 96,365;
24. 396,584;
25. 187,532;
26. 104,812;
27. 460,362;
28. 476,672;
29. 246,304;
30. 617,148;
31. 750,984;
32. 4,464;
33. 27;
34. 72;
35. 36;
36. 28;
37. 27;
38. 47;
39. 34;
40. 27;
41. 28;
42. 34;
43. 26;
44. 97;
45. 34;
46. 24;
47. 36.

If you have not yet repeated the intelligence test given at the beginning of the year, now is a good time to do so.

MENSURATION

Lesson 16

This lesson must be thoroughly clear to the pupils, otherwise the following lessons cannot be properly assimilated. Call on a child to talk on various things connected with the circle (at board). Let another tell *some more.* Who is ahead? You will develop terms, recognition of relative parts and English.

Lesson 17

The ratio 3.1416 and its designation "pi" must be firmly impressed upon the pupil's mind. Train for use of $3\frac{1}{7}$ as ratio at times. This strengthens on decimals and common fractions.

Also spare no effort to help the pupil to properly familiarize himself with the equation form. Always get him to say

$C = \pi D$, then work it. $D = \frac{C}{\pi}$; $R = \frac{D}{2}$; $D = 2R$. Let him get the habit of making his own rules and to *work by them*, so he gets the value and joy out of them.

Exercise 37—Written.

Answers:

1. 2 in.;
2. 6 ft.; or, 2 yd.;
3. 12.5664 yd.;
4. $6\frac{1}{2}$ in.;
5. 62.832 ft.;
6. 4 in.;
7. 3.1416 yd.;
8. 50 yd.;
9. 2.87 yd.;
10. 10 yd. 2 ft. 11.8416 in.

Exercise 38—Written.

Answers:

1. 88 in.;
2. 28.64 + in.;
3. 13 in.;
4. 220 times;
5. 17.83 in.;
6. 7 times (approximately);
7. 20 laps;
8. 1 lap and 12 yd.

Lesson 18

The simplicity of converting a circle into a rectangle by first changing it to triangles can be so clearly demonstrated that this method should be taught in preference to the slightly shorter method of πR^2 which is not easily demonstrated and which involves the process of "squaring" which is more advantageously taught in Part VIII.

Remember, the child knows how to find the area of a rectangle and how to find the circumference of a circle; now, by merely using $\frac{1}{2}$ the circumference as the base of the rectangle and the radius as the altitude, the process is identical to that of finding the area of the rectangle.

Let the child cut circles as indicated and have him compare them to find that the curved base becomes more nearly straight as the number of parts becomes greater.

Exercise 40—Written.

Answers:

1. 50.2656 sq. ft.;
2. 78.54 sq. ft.;
3. 113.0976 sq. in.;
4. 7,854 sq. yd.;
5. .19635 sq. in.;
6. 78.54 sq. in.;
7. 176.715 sq. ft.;
8. D = 14 in.; Area = 154 sq. in.;
9. Area of square = 196 sq. in.;
   Difference in areas = 42 sq. in.;
   Ratio 11 to 14;
10. 12.5664 sq. ft.;
11. 3.1416 sq. ft.;
12. 9.4248 sq. ft.;
13. 50.2656 sq. yd.;
14. 25.1328 sq. in.;
15. 3.1416 sq. ft.;
16. 154 sq. in.;
17. $9\frac{5}{8}$ sq. in.;
18. $\frac{11}{14}$ sq. in.

Lesson 19

No difficulty of any kind will be experienced with this lesson. Let prisms be made. Teach how to add the lapels for construction purposes. Be sure to keep all units in square measure and multiply by abstract numbers only, to obtain square units in result.

Exercise 41—Written.

Answers:

1. 164 sq. in.;
2. 144 sq. ft.;
3. 96 sq. in.;
4. 200 sq. in.;
5. 112 sq. yd.;
6. 28 sq. in.;

7. 800 sq. in.;
8. 52 sq. ft.;
9. $10\frac{2}{3}$ sq. yd.;
10. $36\frac{1}{6}$ bd. ft.;
11. 152 sq. in.;
12. $433\frac{1}{3}$ sq. in.;
13. 150 sq. in.;
14. 128 sq. in.;
15. 152 sq. in.;
16. 1,336 sq. ft.

Lesson 20

Demonstrate that the length of the curved side when straightened is the same as the length of the circumference of the base, and that the width of the curved side is the same as the altitude of the cylinder. Let cylinders be made. If the children have any difficulty let them handle the paper cylinders they made and tell you just what it is they do not understand so that you may help them. They can show it when they cannot express it.

Exercise 42—Written.

Answers:

1. C = 4 in.;
2. 32 sq. in.;
3. 1.27+ in.;
4. 1.27+ sq. in.;
5. 34.55+ sq. in.;
6. 19.1− in. by 36 in.;
7. 687.58+ sq. in.;
8. Area = 37.7 sq. ft.; 40.7 sq. ft. metal;
9. 8.25 sq. in.;
10. Area = 188.5 sq. in.; 4 tubes.

Exercise 44—Written.

Answers:

1. 44 sq. in.;
2. 1,120 sq. in.;
3. $133\frac{4}{7}$ sq. ft.;
4. 880 sq. ft.;
5. 696 sq. in.;
6. 27 A's in B.

Lesson 21

This lesson can be made very interesting by having the pupils prove their answers by diagrams; or, better still, by

actually laying out and cutting paper to correspond with the arithmetical result which in every case must have been previously obtained. Good construction work may be secured here. Teach him to mark it off by division each time.

Exercise 46—Written.

Answers:

1. 15 cards;
   No waste;
2. 4 sheets;
   Remainder 2″ by 20″;
3. 9 pieces;
   24 sq. in. remainder;
4. 8 letter heads;
   No remainder;
5. 32 from a sheet;
   16,000 from a ream;
6. 16 circulars;
   No waste;
   1 ream;
7. 14″ by 21″;
   9 covers;
8. 32 pages;
9. 11 letter heads; (As shown in the example in the lesson);
10. 640 pieces.

Lesson 22

If possible, have the children make cylinders out of heavy paper.

Finding the volume of a cylinder is easily understood by comparison with the process of finding the volume of a prism. In both cases, the area of the base tells us how many cubic units there are in each layer, and the altitude tells us how many layers there are in the solid. Be sure that the children talk and understand "cubic units in each layer."

In its relation to Lesson 18 this lesson introduces but one new factor, that being altitude.

If possible, demonstrate Example 6 of Exercise 48 by the use of a tumbler of water. Ask the question: "Were the water to freeze, what shape would the piece of ice be?" Use tunnels, water pipes, lead pencils, lead in the pencil, etc.

Exercise 48—Written.

Answers:

1. 174 cu. in.;
2. 315 cu. yd.;
3. 138 cu. ft.;
4. 65.48+ gal.;
5. 4.91− gal.;
6. 226.20− cu. in.;
7. 6.28+ cu. ft.;
8. 1,256,640 cu. ft.;
9. 3,723.37+ cu. yd.;
10. 942.48 gal.

Exercise 49—Oral Review.

More review work of a similar nature may be given, time permitting, or call for this lesson again after a week or so. Don't make them *feel* the repeat. See if they are gaining speed. Give the slower ones most of the oral work for a short time.

Exercise 50—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. $4.56;
2. 69%;
3. 54 sq.yd. 4 sq.ft. 126 sq. in.;
4. 125.664 cu. ft.;
5. 1,256.64 sq. ft.;
6. 432 cu. in.; 54 cubes;
7. 408 sq. in.;
8. 3 lb. 3 oz.;
9. 129,501;
10. 96,213;
11. 36,113;
12. 42,750;
13. 72,403;
14. 439;
15. 274;
16. 806;
17. 340;
18. 319;
19. 509;
20. 10,416;
21. 33,867;
22. 45,136;
23. 840,213;
24. 1,582,880;
25. 4,006,976;
26. 31,889;
27. 764,875;
28. 420,156;
29. 499,996;
30. 169,354;
31. 448,199;
32. 417,016;
33. 84,268;
34. 86,108;
35. 208,056;
36. 257,602;
37. 368,982.

Give your "class honor" now for mensuration work.

PERCENTAGE

Lesson 23

Teach both methods of deducting successive discounts, and demonstrate that it is usually easier to deduct the discounts one after the other from the successive net amounts when a different set of discounts is to be used for each example, and that it is usually easier to find and use the percentage which represents the net amount when one set of discounts is to be used for several examples. Some cannot see the short cut until they have tried both ways.

Exercise 52—Written. Call for Proofs.

Answers:

1. $20.30 Discount; $119.70 Net;
2. $18.80 Discount; $61.20 Net;
3. $38.00 Discount; $162.00 Net;
4. $126.00 Discount; $324.00 Net;
5. $10.80;
6. $129.60;
7. 31.6%;
8. $2.70; $8.10; $9.72; $8.64; $3.24; $8.10;
9. $53.00;
10. $62.80.

Lesson 24

Demonstrate this lesson on the board very carefully, using all of the examples given in the text.

Exercise 54—Written. Call for Proofs.

Answers:

1. $150.00;
2. $118.75;
3. $12\frac{1}{2}\%$;
4. $250.00;
5. $182.40;
6. $160.00;
7. $1,440.00 Net; 72%;
8. $1,336.50 Net; 81%;
9. $213.75 Net; $85\frac{1}{2}\%$;
10. $650.00 Gross; 81%;
11. $5,000.00 Gross; 72%;
12. $1,000.00 Gross; $87\frac{3}{10}\%$;
13. $3,645.00 Net; $72\frac{9}{10}\%$.

Problem Project Suggestions

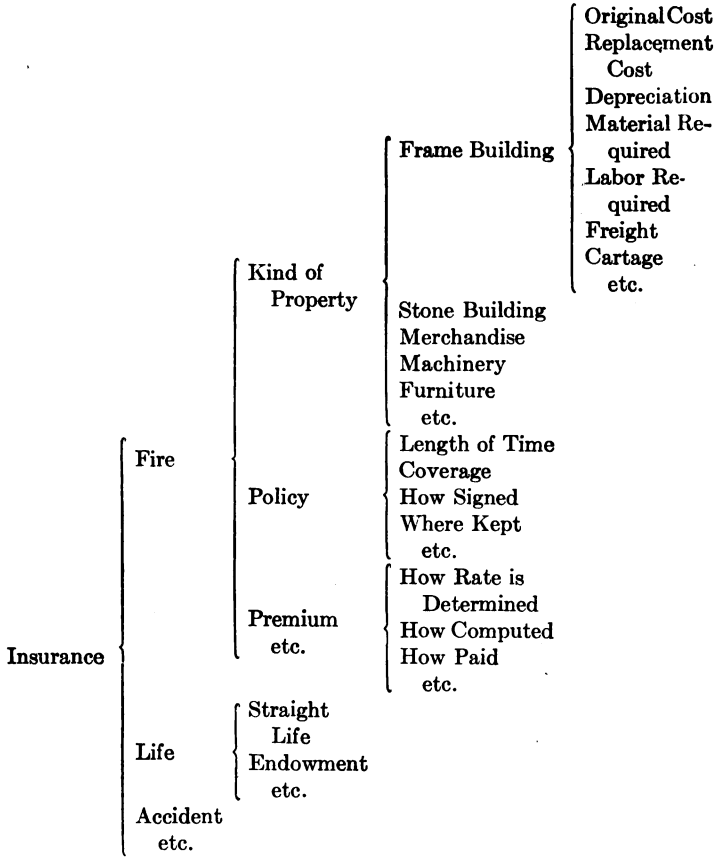
When planning to teach a given subject, motivate with such work as is found in the child's activities. Try to find something of such interest to him that the making or doing of that thing will be *quite real*—an experience that is *life itself* to him. It may be the planning and making of a sled; a garment; buying land; running a store, insurance office, or bank; forming a corporation; etc. Use the *idea* and let it include all the activities that usually swing around it, and these, of course, will lead into branching lines. Go steadily with the child, and the subject will carry him right into life every time. Drills will come in, and the child will be willing to study because the life project calls for it. In this way, interest will be a constant factor and progress is certain. Try it faithfully.

The diagram on Page 23 shows how a given unit of the child's experience may ramify and lead into various collateral subject lines which constitute life's broad experience.

In this diagram only one kind of insurance on one kind of property has been worked out in full. Each of the other kinds of property can be worked out in the same manner; then each of the other topics relating to fire insurance can be taken up in its turn. Next, we can take up the various kinds of life insurance; then accident insurance; and so on until every phase of the subject has been covered in detail.

A few moments in constructing a diagram of this kind for a problem project covering any given subject will place a wealth of material at your disposal which would otherwise escape your notice. Start with the given subject and think of its principal characteristics; then take up the subdivisions of each of these characteristics. See if there are any collateral subjects that should receive consideration. Sometimes the children, themselves, will suggest a new line of reasoning while the project is under discussion; if so, make a note of it.

Diagram of Insurance Problem Project.



From now on, the project method can be used frequently wherever possible.

This problem project will arouse a desire on the pupil's part to know all about insurance. Keep the work lively, but correct *at once* any misconception on the pupil's part regarding any subject.

The important things to be brought out are.

(*a*) Insurance is a promise by an insurance company to pay to the insured a specified sum of money in case of loss or injury of a specified nature.

(*b*) The written contract which is given to the insured is the "policy." The amount promised in the policy is called the "face" of the policy. The amount charged for taking the risk is the "premium."

(*c*) There are many different kinds of insurance, but those most commonly used are: Life, Accident, Fire, Marine, Employers' Liability, and Fidelity.

(*d*) The rate is usually stated at a certain price per $100. of insurance, but frequently a rate per cent is stated. The rate depends entirely upon the hazards connected with the risk.

Exercise 57—Written.

Answers:

1. $22.50;
2. $86.40;
3. $26,037.50;
4. $4,100.00 premium;
   $10,000.00 paid to wife;
5. $20,000.00.

Lesson 26

This lesson presents no new elements, but the meaning of the several terms which are used should be clearly understood.

Watch closely to satisfy yourself that the children have the ability to find the amount of the sale or invoice. The problem project will give the pupils a thorough understanding of this subject.

Exercise 59—Written. Call for Proofs.

Answers:

1. $17.00 Com.; $178.50 Net Proceeds;
2. $1.20;
3. $400.00;
4. $64.97;
5. $6,275.00 Net Proceeds;
6. $4,000.00; 80 acres;
7. 30 bbl.;
8. $40.00;
9. 500 bbl.;
10. $6\frac{2}{3}\%$.

Lesson 27

This lesson introduces a new form of percentage and furnishes excellent practice.

Problem project work can be used to advantage here.

Carefully explain the economic need for taxation and its application. The children should read the large numbers freely; encourage talk on *big money*. They like it; it teaches the use of numbers all the while. The problem project will prove of great interest and benefit to the pupils.

Exercise 61—Written.

Answers:

1. $250.00;
2. $328.90;
3. $119.25;
4. $1,500.00;
5. $49,617.90;
6. $1\frac{1}{4}\%$;
7. $140.35;
8. $2,437,200.00;
9. $4,082.27;
10. $2\frac{1}{2}\%$;
11. $107.00;
12. $442.00;
13. $272.00.

Lesson 28

Spend as much time as is necessary to clearly demonstrate the illustrated example. Use only as many problems as the class needs. Avoid drudgery; it is better to return to the subject in a day or two if necessary.

Exercise 63—Written.

Answers:

1. Aug. 6, 1918; $155.00;
2. Mar. 5, 1924; $49.25;
3. $1,076.00.

Note Regarding Grading.

Where Lessons 29 to 31 have been taught in the sixth year, they must nevertheless now be given in their entirety as review work, with such saving in time as is made possible by the previous teaching.

LESSON 29

Follow the method of analysis shown in the text; *i. e.*:

Total Int. ÷ Int. on $1. = No. of dollars in Principal.
Total Int. ÷ Int. at 1% = No. of % in Rate.
Total Int. ÷ Int. for 1 yr. = No. of yr. in Time.

This one exception is to be emphasized:

Total Amt. ÷ Amt. of $1. = No. of dollars in Principal.

Make frequent use of the formulæ for the various processes.

Exercise 65—Written. Call for Proofs.

Answers:

1. $400.00;
2. $540.00;
3. $870.00;
4. $1,250.00;
5. $2,400.00;
6. $400.00;
7. $3,400.00;
8. $418.80.

LESSONS 30 AND 31

These lessons will be quickly understood and mastered.

Exercise 66—Written. Call for Proofs.

Answers:

1. 3 mo.;
2. $1\frac{1}{2}$ yr.;
3. 9 mo.;
4. $9\frac{1}{3}$ mo.;

5. $3\frac{1}{3}$ yr.;
6. 3 yr. 2 mo.;
7. 45 da.;
8. 60 da.;
9. 1 yr. 6 mo.;
10. 10 mo.

Exercise 68—Written. Call for Proofs.

Answers:

1. 4%;
2. $5\frac{1}{2}$%;
3. 6%;
4. 8%;
5. 4%;
6. $4\frac{1}{2}$%;
7. 5%;
8. 3%;
9. 6%;
10. $4\frac{1}{2}$%;
11. 6%;
12. 1 yr. 4 mo.;
13. $612.00;
14. 25 yr.

Lesson 32

This lesson covers a most valuable application of transposition, and shows how some difficult examples in interest can be converted into problems so simple that they can easily be solved without the use of paper. The whole process is merely a device for using the numerators of an example in cancellation in the most convenient order. Spare no effort to teach this lesson properly. You will have another chance to see if the child "relates" numbers. Work hard to save him if he does not relate by this time or he will soon be lost.

Exercise 70—Written.

Answers:

1. $1.85;
2. $1.04;
3. $2.82;
4. $1.30;
5. $1.07;
6. $2.47;
7. $0.03;
8. $0.64;
9. $2.64;
10. $0.77.

Lesson 33

No difficulty will be experienced in connection with this lesson as the pupils are now thoroughly familiar with

interest in its various forms and applications. Explain the meaning of "semi-annually" and "bi-annually" and help them to remember the meaning of the prefix "bi" by referring to the fact that a *bi*cycle has *two* wheels.

Exercise 72—Written.

Answers:

1. $70.92;
2. $49.45;
3. $22.67;
4. $11.17;
5. $76.13.

Note Regarding Grading.

Where it is required that bank discount be taught in the seventh year, Lesson 28 (including Exercises 59 and 60) of Part VIII should now be given.

Give your "class honor" now for percentage work.

ACCOUNTS

LESSON 34

This lesson has three distinct purposes:

(*a*) To familiarize the pupil with banking practice;

(*b*) To train the pupil in the keeping of accounts;

(*c*) To furnish additional work in compound interest.

Therefore, in teaching this lesson see that each of these elements receives proper consideration. Stimulate the class through their own banking experience if possible. Excellent opportunity is here furnished for problem project work.

Demand careful and neat work where ruling is required.

Exercise 74—Written.

Answers:

1. Balance $175.00;
2. Interest 63¢;
3. Balance $900.00;
4. Interest $16.00.

Lesson 35

This lesson covers additional features in connection with banking practice and the keeping of accounts, and introduces a new element; *i. e.*, the reconcilement of a bank account. Cover each of these three phases of the lesson.

A check book should be at hand and the children should see checks filled out or should fill them out. Use problem project work here.

Use the word "reconcilement" often and urge the child to use it also; explain that "reconcilement" means "an adjustment of a difference."

Exercise 76—Written.

Answers:

1. Balance $817.27;
2. Balance $1,183.89;
3.

Reconcilement Sept. 30.

Balance as per Check Book.....		$817.27
Checks Outstanding:		
#17,477	$321.62	
78.....	45.00	
		366.62
Balance as per Bank Statement.....		$1,183.89

4. Balance $18,697.07;
5. Balance $20,978.23;
6.

Reconcilement Dec. 31.

Balance as per Check Book.....		$18,697.07
Checks Outstanding:		
#11,419	$1,347.21	
21.....	711.48	
22.....	222.47	
		2,281.16
Balance as per Bank Statement.....		$20,978.23

Give your "class honor" now for accounting work. Call him the "Class Expert Accountant."

Note Regarding Grading.

Where it is required that powers and roots be taught in the seventh year, Lessons 8 to 12 (including Exercises 18 to 26) of Part VIII should now be given.

Note Regarding Grading.

Where it is required that equations be taught in the seventh year, Lessons 13 to 15 (including Exercises 28 to 31) of Part VIII should now be given.

Note Regarding Grading.

Where it is required that the right triangle be taught in the seventh year, Lesson 16 (including Exercises 32 and 33) of Part VIII should now be given.

Note Regarding Grading.

Where it is required that pyramids, cones, and spheres be taught in the seventh year, Lessons 21 to 24 (including Exercises 42 to 50) of Part VIII should now be given.

Exercise 77—Oral Review.

More review work of a similar nature may be given, time permitting.

Exercise 78—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. $45.33;	5. 10%;	9. 58,649;
2. 565.488 cu. in.;	6. 60 da.;	10. 62,295;
3. 1,884.96 sq. in.;	7. 6%;	11. 77,710;
4. $144.18;	8. $111.70;	12. 63,420;

13. 188,641;
14. 106,820;
15. 329,940;
16. 460,845;
17. 118,496;
18. 434,028;
19. 197,613;
20. 430;
21. 604;
22. 741;
23. 832;
24. 947;
25. 787;
26. 64,126;
27. 145,439;
28. 366,725;
29. 421,884;
30. 313,093;
31. 477,277;
32. 382,763;
33. 612,866;
34. 120,880;
35. 109,278;
36. 773,992;
37. 56,104.

General Review

All work for general review purposes should be selected from, or based on, the various exercises which constitute the year's work in the text book.

Give your review work in three parts; viz.:

(*a*) Oral work on principles, etc.
(*b*) Oral problems.
(*c*) Written problems.

Now repeat the intelligence test given at the beginning of the year, and give each pupil's record for Parts V to VII to the teacher who will have that pupil next year.

Examination (If Desired)

That the examinations (if desired) may be a thorough test of the pupil's knowledge of the work covered, and that the teacher may know what next to undertake, the questions should be selected from the various exercises which constitute the year's work in the text book but the teacher may introduce other examples of a similar nature. Examinations should be held in oral as well as in written arithmetic, and should test the pupil's knowledge of both theory and application.

TEACHER'S MANUAL

ADVANCED LESSONS

PART VIII

AN INTELLIGENCE TEST

This set of test questions will aid you in grouping your pupils properly, in determining who are the weak ones and in checking the results of your efforts. Defective hearing and sight will also be disclosed thereby.

Make a tabulation of the number of problems attempted, the number correctly solved, and the percentage attained by each child, and keep it for future comparison. (Average the per cent attempted and the per cent solved of those attempted; viz.: 8 attempted out of 10 = 80%; 7 solved out of 8 attempted = 87½%; average 83¾%.)

Name	Beginning of Year			Middle of Year			End of Year		
	Attempted	Solved	%	Attempted	Solved	%	Attempted	Solved	%
Adams, Bessie	8	7	83¾						

Do not give the pupils the result of the test, nor show them directly wherein they failed, but help each group in its weak subjects without permitting them or the others to realize what you are doing.

At the end of the first half of the school year give the same test again and compare the results; repeat again at the end of the school year. If you desire, the test may be given more frequently.

How to give the test:

Write the first five problems (properly numbered) on the board during the pupils' absence and keep them covered until the proper moment designated hereafter.

When ready to begin the test, provide them with paper and read the second five problems to the pupils very slowly and let them make such memoranda thereof as they will. Give them the number of each problem as you read it.

Tell them they are to number their answers to correspond with the problem numbers.

Now remove the covering from the board and have them begin. (Those on the board come first.)

Allow 30 minutes for the actual working of the problems.

The Problems:

The first five to be written on the board:

1. If $3\frac{1}{2}$ yd. of tapestry cost \$42., what will $\frac{3}{4}$ yd. cost at the same rate?
2. I sent \$520. to an agent to invest in land and to pay his 4% commission; how many dollars were invested in land?
3. Find the altitude of a triangle which has a base of 20.6 ft. and an area of 82.4 sq. ft.
4. What is the ratio of the length of the lines of a 2-inch square to those of a 3-inch square? What is the ratio of the areas?
5. Mr. Miller sold a tract of land for \$15,000. and gained \$3,000.; what per cent of the cost did he gain?

The second five to be read to the pupils:

6. Find the volume of a rectangular pyramid 8 in. high and 3 in. on each side of the base.
7. Write a rule or give a formula telling how to find the area of a circle when the radius is given.

8. Make three drawings of an acre tract of land, showing different possible dimensions.
9. Find the interest on an $850. note for 96 days at 5%.
10. A field which is four times as long as it is wide, has a perimeter of 480 rd.; what are its dimensions?

The Answers:

1. $9.;
2. $500.;
3. 8 ft.;
4. 2 : 3;
   4 : 9;
5. 25%;
6. 24 cu. in.;
7. $\frac{C \times R}{2}$ or πR^2;
8. 80 rd. by 2 rd.;
   40 rd. by 4 rd.;
   20 rd. by 8 rd.; etc.;
9. $11.33;
10. Length 192 rd.;
    Width 48 rd.

The daily blackboard drill should include work in rapid addition, subtraction, multiplication and division in all of the forms taught in Parts IV, V, VI, and VII. Let the children read numbers often besides writing them. Encourage only one trial in these drills to get accuracy and the best effort of the child. If he is very slow wait for him. Time him frequently and show him he is gaining. See whether the slow child is using the "ten" combinations or not.

Competition is a wonderful incentive for good work and continued effort. Make use of it by giving "class honors" frequently and letting the children strive for them. As an example, you can give the honor of "Class Time Expert" to the boy or girl who has done the best work in Standard Time after Lesson 4 is completed, and so on throughout the work.

NOTATION AND NUMERATION

Lesson 1

As we rarely use the periods which are of higher rank than "trillions," their names and values need not be memorized but they should be gone over as a matter of general interest.

Use recent bond issues, expenses of the World War, etc., to show reason for learning billions and trillions.

DENOMINATE NUMBERS

Lesson 2

Note Regarding Grading.

Where Lesson 2 has been previously taught, it must nevertheless now be given as review work.

The Table of Circular Measure is so simple that it will be very quickly learned. The point to be emphasized in this lesson is that there are 360° in every circumference *regardless of its size.* To show this plainly, let 4 children draw 4 circles of different sizes on the board. Demand careful drawing. Remember, this work is elementary geometry.

Exercise 5—Written.

Answers:

1. 90°; 5,400′; 324,000″;
2. 9° 55′ 45″;
3. 80 miles;
4. 1.8 inches;
5. 8,115″.

Lesson 3

These are the basic points to be dwelt on in this lesson:

(*a*) The location of the equator, and the fact that it is a complete circumference dividing the earth's surface into the northern and southern hemispheres.

(*b*) The location of the Prime Meridian, and the fact that it, together with the 180th Meridian, forms a complete circumference dividing the earth's surface into the eastern and western hemispheres.

(*c*) That the meridians of longitude are arcs running north and south but that they indicate distance east and west from the Prime Meridian.

Harry's Imaginary Trips and Troubles will keep up an interest on account of the real travel situations that arise.

The Table of Equivalents will be more readily understood and retained if a blackboard demonstration is made, showing by computation how each of the equivalents is determined. Be sure they know, 15° 15′ 15″ of longitude corresponds to 1 hr. 1 min. 1 sec. of time. From this they can reason that since 15° 15′ 15″ corresponds to 1 hr. 1 min. 1 sec., the number of hours corresponding to any number of degrees is determined by finding the number of times that 15° is contained in the given number of degrees; also that dividing any number of minutes of longitude by 15′ determines the corresponding number of minutes of time; also that dividing any number of seconds of longitude by 15″ determines the corresponding number of seconds of time.

Work this illustration on the board, first as three separate examples, then as a regular division example in compound denominate numbers:

Example: How much time corresponds to 32° 5′ 12″?

$$15^\circ \overline{)\,32^\circ}\quad 2 \text{ and } 2^\circ \text{ rem.}$$

$2^\circ = 120'$; $120' + 5' = 125'$;

$$15' \overline{)\,125'}\quad 8 \text{ and } 5' \text{ rem.}$$

$5' = 300''$; $300'' + 12'' = 312''$;

$$15'' \overline{)\,312''}\quad 20\tfrac{4}{5}$$

$$15\ (^\circ\ '\ '')\ \overline{)\,32^\circ\ \ 5'\ \ 12''}\quad \begin{matrix} 2 & 8 & 20\frac{4}{5} \end{matrix}$$

$$\begin{matrix} 30 \\ \overline{2} = 120 \\ \overline{125} \\ 120 \\ \overline{5} = 300 \\ \overline{312} \\ 300 \\ \overline{12} \end{matrix}$$

Therefore, 32° 5′ 12″ corresponds to 2 hr. 8 min. $20\frac{4}{5}$ sec.

To find the number of °, ′, or ″ corresponding to any number of hr., min., or sec., we would naturally reverse the

process and multiply 15°, 15′, or 15″ by the number of hr., min., or sec., because 15 of each of these units of longitude corresponds to 1 of each unit of time.

Example: How many ° ′ ″ corresponds to 3 hr. 12 min. $6\frac{4}{5}$ sec.?

$$\begin{array}{r} 15'' \\ \times\ 6\frac{4}{5} \\ \hline 102''; \end{array} \quad 102'' = 1'\ 42'';$$

$$\begin{array}{r} 3 \quad 12 \quad 6\frac{4}{5} \\ \times \qquad\qquad 15\ (^\circ\ '\ '') \\ \hline 45^\circ \quad 180' \quad 102'' \\ 48^\circ \quad 1' \quad 42'' \end{array}$$

$$\begin{array}{r} 15' \\ \times\ 12 \\ \hline 180'; \end{array} \quad 180' + 1' = 181'; \quad 181' = 3^\circ\ 1';$$

$$\begin{array}{r} 15^\circ \\ \times\ 3 \\ \hline 45^\circ; \end{array} \quad 45^\circ + 3^\circ = 48^\circ;$$

Therefore, 3 hr. 12 min. $6\frac{4}{5}$ sec. corresponds to 48° 1′ 42″.

Use a globe and an imaginary sun to show why the time as shown by the clock is later the farther east one goes. Let the child who does not understand turn the globe himself. Problem projects can be used to good advantage here.

Exercise 8—Written.

Answers:

1. 40 min. 35 sec.;
2. 87° 36′ 30″;
3. 6:51:45 A. M.;
4. Harry from the East;
   Father from the West;
5. Harry + 75° 3′ 15″;
   Father + 120° 3′ 15″;
6. (*a*) 94° 13′ 15″; 6 hr. 16 min. 53 sec.;
   (*b*) 74° 57′ 5″; 4 hr. 59 min. $48\frac{1}{3}$ sec.;
   (*c*) 14° 2′ 12″; 56 min. $8\frac{4}{5}$ sec.;

(*d*) 92° 58′ 0″; 6 hr. 11 min. 52 sec.;
(*e*) 20° 30′ 12″; 1 hr. 22 min. $\frac{4}{5}$ sec.;

7. 40° 32′ 45″; 2 hr. 42 min. 11 sec.;
8. 10:19:19 A. M.;
9. 52° 30′; East;
10. 7:50:18 P. M. Wednesday.

LESSON 4

This lesson is of considerable importance to those pupils who may later enter the commercial field; it offers no difficulties and will be quickly understood. Problem projects can be used here.

Mr. Brown leaves Denver and wakes up in Chicago; he finds his time all wrong; what change must he make? What about the clocks in Chicago? etc. Talk real people and their troubles.

Give your "class honor" now for time work.

LESSON 5

This lesson has a very broadening effect on the child's general education quite separate and apart from its value as an arithmetical step. Problem projects can be used here.

If the class does not grasp the need of the International Date Line quickly, use the following for additional exposition.

Were it possible for a traveler starting, we will say, from Washington, D. C., at noon on Monday to travel westward as rapidly as the earth rotates eastward, he would make the entire journey around the earth in 24 hours, during all of which time it would be noon to him as he would remain in exactly the same position in relation to the sun. He would complete the journey at Washington 24 hours after starting, which would be noon on Tuesday, but not having passed through a night, he might well be puzzled to know when Monday ended and Tuesday began.

Exercise 11—Written.

Answers:

1. 1 hr. 2 min. 6 sec. difference in time;
   15° 31′ 30″ difference in longitude;
2. 2 hr. 9 min. 10 sec. difference in time;
   32° 17′ 30″ difference in longitude;
3. 4 hr. 4 min. 30 sec. difference in time;
   61° 7′ 30″ difference in longitude;
4. 8 hr. 57 min. 15 sec. difference in time;
   134° 18′ 45″ difference in longitude;
5. 5 hr. 4 min. 8 sec. difference in time;
   76° 2′ 0″ difference in longitude;
6. 0 hr. 16 min. $56\frac{8}{15}$ sec.;
7. 61° 32′ 30″;
8. 11 hr. 30 min. 42 sec.

Lesson 6

(Lessons 6 and 7 may be left till the last, or made optional.)

Note Regarding Grading.

Where Lesson 6 has been taught in the sixth year, it should nevertheless be given in its entirety as review work,with such saving in time as is made possible by the previous teaching.

In this lesson adhere strictly to the text. When the metric units and their equivalents in Linear, Avoirdupois, Dry and Liquid Measures are thoroughly understood, no difficulty will be experienced with the construction of the metric system tables, but any attempt to construct the tables prematurely must result in endless confusion and ultimate failure. Let children try to question each other in game fashion—sorting their terms quickly, etc.

All effort is now to be limited to the teaching of the values of the Greek and Latin prefixes: Myria, Kilo, Hecto, Deka, deci, centi, and milli.

Now the pupil is to be taught how to construct the metric system tables by combining the prefixes just learned with the units previously learned. It is desired that the pupil should be able to *construct* the tables—not memorize them.

Exercise 13—Written.

Answers:

1. 1,790.036 m.;
2. 642.17 g.;
3. 10,527.75 l.;
4. $2,947.77;
5. $32.11;
6. 48,960 g.;
7. 1,713.6 oz.; 107.1 lb.

LESSON 7

Note Regarding Grading.

Where Lesson 7 has been previously taught, it should nevertheless be given as review work.

An easy method of memorizing the values of the foreign coins in United States money is as follows:

(*a*) An English penny (plural "pence") is worth a trifle over $0.02;

(*b*) 12 English pence (1 shilling) are worth $0.02 × 12 = $0.24 + 3 mills or $0.243;

(*c*) 20 English shillings (1 pound) are worth $0.243 × 20 = $4.86+;

(*d*) 1 French franc (100 centimes) is worth 5¢ less than a shilling, or $0.193;

(*e*) The Belgian franc, The Italian lira, The Spanish peseta, The Swiss franc — Each is worth $0.193 the same as the French franc.

The values of the other monetary units need not be learned, as these are given for reference purposes only.

Problem projects can be used—let different parts of the class represent different countries.

Exercise 14—Written.

Answers:

1. $243.33;	9. £6.;	17. $55.97;
2. $23.36;	10. £50. 7s. 3d.;	18. 75 lira;
3. $9.85;	11. £86. 3d.;	19. 430 peseta;
4. $1.42;	12. £500.;	20. 60 fr.;
5. $1,951.26;	13. $33.78;	21. 500 fr.;
6. $9.65;	14. $57.90;	22. 90 fr.
7. $0.15;	15. $15.44;	
8. $13.18;	16. $86.85;	

Exercise 15—Written.

Answers:

1. 9:40 A. M.;	6. $14.60;
2. $26.77;	7. $1,737.00;
3. 5 hr. forward;	8. 46 fr. 6 c.;
4. $2.55;	9. $144.75;
5. $669.14;	10. $386.00;
$1,362.62;	11. Set it back 7 hr.
$2,433.25;	
$4,465.01;	

Give your "class honor" now for foreign money and metric system work.

Exercise 16—Oral Review.

Train the mind to run in advance of speech pretty briskly here; let the child add aloud while getting the total. He should see combinations of units to shorten the work. Give him more of these problems if necessary.

Exercise 17—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. 416,000,000,008,016;
   38,164,009,012;
   58,000,006,000,006;
2. (*a*) 10° 6′ 50″;
   (*b*) 309,630″;
   (*c*) 5,440$\frac{1}{3}$′;
3. 1 hr. 1 min. 1 sec.;
4. 161° 21′ 15″;
   10 hr. 45 min. 25 sec.;
5. 2:02 A. M.;
6. 7:59:58 P. M.;
7. 1,419.8576;
8. 3,345,790$\frac{2}{5}$;
9. $278.18;
10. 6%;
11. 96,896;
12. 482,975;
13. 64,888;
14. 333,637;
15. 206,296;
16. 442,670;
17. 96,838;
18. 312,732;
19. 702,566;
20. 224,449;
21. 494,298;
22. 206,351;
23. 137,111;
24. 766,044;
25. 373,576;
26. 538,747;
27. 59,772;
28. 444,225;
29. 398,176;
30. 551,002;
31. 34;
32. 41;
33. 52;
34. 53;
35. 37;
36. 64;
37. 35,260;
38. 39,030;
39. 29,047;
40. 33,000;
41. 28,508;
42. 34,201.

POWERS AND ROOTS

Note Regarding Grading.

Where Lessons 8 to 12 have been taught in the seventh year, they must nevertheless now be given in their entirety as review work with such saving in time as is made possible by the previous teaching.

Lesson 8

The pupils must understand that every number has many powers, the square and the cube being respectively the powers

of the second and third degrees. Train the eye to see rapidly exponents, indexes, roots and powers as the quick call from the teacher may demand. The terms are quickly learned in this way.

Exercise 19—Written.

Answers:

1.	225.;	**15.**	$12\frac{1}{4}$;	**29.**	$\frac{27}{343}$;
2.	$18\frac{7}{9}$;	**16.**	$72\frac{1}{4}$;	**30.**	.015625;
3.	1,225.;	**17.**	$11\frac{1}{9}$;	**31.**	h^3;
4.	15,625.;	**18.**	d^2;	**32.**	$\frac{8}{27}$;
5.	160,000.;	**19.**	x^2;	**33.**	$20\frac{51}{64}$;
6.	2,809.;	**20.**	a^2;	**34.**	$3\frac{3}{8}$;
7.	6,561.;	**21.**	32,768.;	**35.**	c^3;
8.	5,625.;	**22.**	91,125.;	**36.**	m^3;
9.	676.;	**23.**	884.736;	**37.**	15.625;
10.	.6889;	**24.**	1,000,000.;	**38.**	.0144;
11.	$\frac{1}{4}$;	**25.**	9,261.;	**39.**	1.3924;
12.	$\frac{9}{16}$;	**26.**	$15\frac{5}{8}$;	**40.**	$\frac{64}{125}$.
13.	$6\frac{1}{4}$;	**27.**	262,144.;		
14.	$\frac{121}{144}$;	**28.**	110,592.;		

Lesson 9

Demonstrate the meaning of square root and cube root on the blackboard by squaring and cubing simple numbers and pointing out the roots and the powers.

Lessons 10 and 11

Have lists of perfect squares on the board and let children tell at sight how many figures there are in the roots. Give plenty of time to pointing off.

As is shown in the text, the process of extracting the square root of a number is the exact reverse of the three steps used in squaring a number by cross multiplication, with which

process the pupils are entirely familiar; this is by far the simplest and most readily comprehended explanation of this difficult operation. Show the processes of squaring by cross multiplication and of extracting the square root side by side and step for step on the blackboard, for form and relation. Use drawings for a clearer understanding and for illustration. Show that squaring the root proves the work.

Exercise 22—Written. Call for Proofs.

Answers:

1. 13;	**6.** 41;	**11.** 35;
2. 12;	**7.** 70;	**12.** 67;
3. 23;	**8.** 18;	**13.** 56;
4. 14;	**9.** 32;	**14.** 98.
5. 25;	**10.** 83;	

Exercise 24—Written. Call for Proofs.

Answers:

1. 15.;	**5.** 819.;	**9.** 31.622+;
2. 35.;	**6.** 1.18;	**10.** 6.708+.
3. 125.;	**7.** 3.952+;	
4. 53.;	**8.** 1.414+;	

Exercise 25—Written. Call for Proofs.

Answers:

1. $4\frac{1}{3}$;	**6.** 17 in.;
2. $\frac{9}{10}$;	**7.** 6 yd.;
3. 1.224+;	**8.** 22 yd.;
4. $2\frac{3}{4}$;	**9.** 1,008 boards;
5. .881+;	**10.** 17,576 cu. in.

LESSON 12

This method is fast becoming popular. Be sure the child chooses very carefully his approximate divisor even into

hundredths, so he may have an accurate answer through hundredths. He may be glad to use this method when he handles small powers or products. Some children do.

Exercise 26—Written. Call for Proofs.

Answers:

1. 83;	6. 56;	11. 1.73+;
2. 67;	7. 98;	12. 1.41+;
3. 41;	8. 23;	13. 2.24−;
4. 32;	9. 2.5;	14. 3.16+;
5. 35;	10. 3.5;	15. 3.46+.

Exercise 27—Written. Call for Proofs.

Answers:

Time Carefully

1.	264,667;	13.	10,350;
2.	184,178;	14.	960,498;
3.	84,943;	15.	62,452;
4.	209,352;	16.	1,380 and 5 rem.;
5.	272,313;	17.	252;
6.	11,737;	18.	5,161;
7.	35,376;	19.	722 and 16 rem.;
8.	33,558;	20.	909 and 5 rem.;
9.	6,625;	21.	819;
10.	17,928;	22.	35;
11.	60,073;	23.	1.18;
12.	6,200,064;		

Give your "class honor" now for work in powers and roots.

EQUATIONS

Note Regarding Grading.

Where Lessons 13 to 15 have been taught in the seventh year, they must nevertheless now be given in their entirety as review work with such saving in time as is made possible by the previous teaching.

This work is to be gradually developed as outlined in the text. Care must be taken to see that each step is mastered before the next one is attempted.

Drill most carefully on transposition and the accompanying changing of signs.

If necessary, turn back to these lessons in a short time and review them.

Have the children use the equation from now on wherever it will save time or simplify the work.

Exercise 31—Written. Call for Proofs.

Answers:

1. 50 yd.;
2. 13 oz.;
3. $5\frac{5}{11}$ min. after 1 o'clock;
4. 32 ft.;
5. 18;
6. 10 and 12;
7. 8 ft.;
8. 12 ft. by 15 ft.;
9. 100 ft. by 125 ft.;
10. 12 ft. high;
    10 ft. wide;
    15 ft. long.

If you have not yet repeated the intelligence test given at the beginning of the year, now is a good time to do so.

MENSURATION

LESSON 16

Note Regarding Grading.

Where Lesson 16 has been taught in the seventh year, it must nevertheless now be given as review work with such saving in time as is made possible by the previous teaching.

Teach this lesson in three parts:

(*a*) That the square on the hypotenuse is equal to the sum of the squares on the two legs.

(*b*) That since the square on the hypotenuse is equal to the sum of the squares on the two legs, there-

fore, the square on the hypotenuse minus the square on either leg equals the square on the other leg.

(*c*) That when we know the square on any side of a triangle, we find the length of that side by extracting the square root.

Exercise 32—Oral.

Let the children run the oral work—you will readily see where any weakness lies—then come in and help.

Exercise 33—Written.

Answers:

1. 25 ft.;	**9.** 28.28+ ft.;
2. 76 ft.;	**10.** 176.77+ ft.;
3. 18 in.;	**11.** 23.58+ yd.;
4. 10 ft.;	**12.** 53.15+ ft.;
6. 7.07+ in.;	**13.** 57.21+ ft.;
7. 240 ft.;	**14.** 729. cu. in.
8. 127.27+ ft.;	

Lesson 17

It is of vital importance that the pupil should understand that every equilateral triangle and every isosceles triangle can be divided into two right triangles. Teach them to diagram all of their work—in some cases insist on drawings carefully made to scale with ruler—in some cases insist on pencil sketches made without ruler and possibly only roughly approximating scale, but drawn and marked to show the proper locations and lengths of the known dimensions to facilitate the determining of the unknown. Do this, using simple numbers or measurements until the principle is learned; many easy ones will fix the rule. Compare, for children apothem in polygon with altitude in triangle.

Exercise 34—Oral.

Let the children run it.

Exercise 35—Written.

Answers:

1. 5 ft.;	**6.** 21.65 in.;
2. 24 ft.;	**7.** 173.2+ sq. in.;
3. 12 sq. ft.;	**8.** 41.5+ sq. in.;
4. 432 sq. ft.;	**9.** 93.528 sq. in.;
5. 8.66+ in.;	**10.** 201.24 sq. in.

.866+ in. for each 1 in. of side;

LESSON 18

Make certain that the pupils understand that similar triangles may occupy different positions without affecting their similarity. This is clearly demonstrated by reference to Figure 7 which is composed of the triangles shown in Figures 4 and 5, and that these are similar is proven by Figure 6.

A practical demonstration of this method of making measurements by laying out triangles in a near-by field is of great value and interest to the pupils and should be undertaken if conditions permit. Problem projects are plentiful here; teach how to clear of fractions by reducing to L. C. D. Make use here of algebraic equations.

Exercise 37—Written.

Answers:

1. 55 ft.;	**4.** 885 ft.;
2. 64 ft.;	**5.** 30 in.;
3. 156 ft. 3 in.;	**6.** 22 ft. 6 in.

LESSON 19

Protractor must be made or bought.

Lesson 20

In this lesson we show, for the first time, that the area of a circle equals πR^2. Spare no effort to have this clearly understood. Let him substitute what he knows and he will see that $\frac{CR}{2} = \pi R^2$.

Finding the length of an arc or the area of the sector of a circle is merely the combining of several processes which the pupil has previously learned, and should offer no difficulty. Call attention to the fact that an arc is measured along the curved line and that this cannot be done with a ruler.

Select pupils across the room, say those in seat 3 in each row, to do the questioning. Tomorrow give those in seat 4 a chance, and so on.

Use the equation form wherever possible.

Exercise 41—Written.

Answers:

1.	1.5708 yd.;	**10.**	8 ft.;
2.	2.0944 ft.;	**11.**	2 ft.;
3.	$6\frac{1}{2}$ in.;	**12.**	25 ft.;
4.	4 in.;	**13.**	40°;
5.	62.832 ft.;	**14.**	180°;
6.	12.5664 sq. ft.;	**15.**	628.32 yd.;
7.	58.905 sq. ft.;	**16.**	20 yd.;
8.	981.75 sq. yd.;	**17.**	31.416 yd.
9.	19.635 sq. in.;		

Note Regarding Grading.

Where Lessons 21 to 24 have been taught in the seventh year, they must nevertheless now be given in their entirety as review work with such saving in time as is made possible by the previous teaching.

Do not proceed until you are certain that the pupils understand clearly what is meant by the slant height, the altitude, the lateral area, and the entire area of a pyramid.

Show that the slant height, the altitude and a line joining these two at the base form a right triangle of which the slant height is the hypotenuse. Tell them to play it is glass, then see the lines.

Also show that the slant height, one edge of the pyramid, and a line joining these two at the base form a right triangle of which the edge of the pyramid is the hypotenuse.

Now let the children take prisms and pyramids and *talk* before the class. Let them tell all they know about these solids.

The child must be shown that finding the area of a lateral face of a pyramid is nothing more than finding the area of a triangle and this he already understands.

Exercise 42—Oral and Written.

Work hard to get imaging all the time. Make them see it in the air.

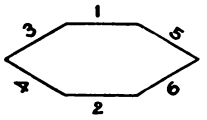
Demonstrate that the volume of a pyramid is $\frac{1}{3}$ as great as the volume of a prism having the same altitude and base, and the child will have no difficulty in understanding that:

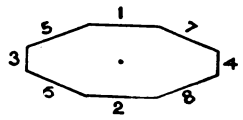
Area of base $\times$ Altitude $\div$ 3 = Volume.

To make this clear, have one of the pupils or all make a prism and a pyramid of same dimensions, both with open ends; have them bring sawdust, salt, or sand. Let the child fill the prism using the pyramid as a measure. He will see that it takes three measures full; then he knows that a pyramid holds just $\frac{1}{3}$ as much as a prism of the same measurements; if he gets the whole thing (prism) first, he will have no difficulty with the pyramid.

Be ready to show the children how to draw a prism, a pyramid, an octagon, etc., rapidly; it will save time and

energy both for you and for the pupils; draw according to the numbers suggested:





Lines drawn from the apex directly to the center of the base and to the vertices of the figure desired, form a pyramid. Use the equation form wherever possible.

Exercise 43—Construction Work.

Insist on each child making one of each. Use salt, sand, sawdust; anything to compare volumes.

Exercise 44—Written.

Answers:

1. Volume, 1,500 cu. ft.;
   Perimeter, 60 ft.;
   Slant Height, 21.36 ft.;
2. 384 sq. in.;
3. Volume, 216.5+ cu. in.;
   Slant Height, 15.61+ in.;
4. 927.6 sq. ft.;
5. 5,760 pieces;
6. 120 cu. in.;
7. 25 ft.;
8. 25.29+ in.;
9. 48 cu. in.;
10. Volume, 93,391,360 cu. ft.;
    Slant Height, 613.45+ ft.;
    Lateral Area, 937,351+ sq. ft.

Lesson 22

This lesson should be taught in the same manner as Lesson 21 on pyramids, with the necessary omission of those

remarks which do not apply in the case of cones. Use the equation form wherever possible.

Exercise 46—Written.

Answers:

1. Area of base, 314.16 sq. ft.;
   Volume, 1,570.8 cu. ft.;
   Circumference, 62.832 ft.;
   Slant height, 18.02+ ft.;
2. 301.59+ sq. in.;
3. 35.49+ in.;
4. Volume, 197.92+ cu. in.;
   Slant height, 21.21+ in.;
5. 41.888 cu. in.;
6. 200 cu. in.;
7. 10 ft.;
8. 5 ft.;
9. 301.59+ cu. in.;
10. $7\frac{1}{2}$ in.

Lesson 23

Finding the lateral area of a frustum is not a difficult process (average perimeter × slant height). Give child drawings of a trapezoid

then

then

so he will see that average perimeter × slant height = lateral area.

Exercise 48—Written.

Answers:

1. Area of side, 490.09—sq. in.;
   Area of bottom, 78.54 sq. in.;
   Total area, 568.63—sq. in.;
2. 136 sq. in.;
3. 301.59+ sq. in.;
4. 490.09— sq. in.;
5. 4 in.;
6. 150 sq. in.;
7. 20 in.;
8. 11 ft.;
9. 10 in.;
10. 36 in.

Follow the text carefully; illustrate and demonstrate wherever possible; avoid permitting the pupils to memorize a lot of formulæ which, to them, are meaningless—instead, show how each formula is obtained by reasoning, so that the pupil can, when necessary, reason along the same lines and construct his own formulæ. The children love to make the demonstrations given in the text; encourage it, for in no other way do they grasp the rules so quickly. Ask for letter formulæ all the time now.

Exercise 50—Written.

Answers:

1. Area great circle, 314.16 sq. in.;
   Area curved surface, 1,256.64 sq. in.;
2. 615.75+ sq. ft.;
3. 19.635 sq. in.;
4. 201,062,400 sq. mi.;
5. $109.08;
6. Area great circle, 3,141,600 sq. mi.;
   Radius, 1,000 mi.;
   Diameter, 2,000 mi.;
7. $33\frac{1}{2}$ cu. in.;
8. 113.1− cu. in.;
9. 16.96+ in.;
10. Diameter, 7.07+ in.;
    Volume, 185.04+ cu. in.

Give your "class honor" now for mensuration work.

Exercise 51—Oral Review.

Use letters and symbols very freely. Hold a contest on abbreviations and signs. See Part VIII, page 169.

Let the addition work be oral and watch the various grouping methods carefully. Let some add downward,

others upward. Stop them on the spot to correct them—then try again. Give a few minutes of this work every day now without fail.

More review work of a similar nature may be given, time permitting.

Exercise 52—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. 48.7;	**15.** 278,388;	**29.** 34,164;
2. 61,162,984.;	**16.** 234,367;	**30.** 32,923;
3. $\frac{17}{19}$;	**17.** 316,932;	**31.** 343,744;
4. 25 ft.;	**18.** 348,208;	**32.** 391,329;
5. 6.92+ in.;	**19.** 682;	**33.** 382,046;
6. 12 in.;	**20.** 559;	**34.** 537,884;
7. 60°;	**21.** 783;	**35.** 302,632;
8. 5 in.;	**22.** 889;	**36.** 243,895;
9. 35.49+ in.;	**23.** 707;	**37.** 124,408;
10. 13. in.;	**24.** 785;	**38.** 24,829;
11. 84,330;	**25.** 35,430;	**39.** 481,378;
12. 232,596;	**26.** 30,884;	**40.** 123,009;
13. 804,172;	**27.** 33,855;	**41.** 111,009;
14. 136,053;	**28.** 30,966;	**42.** 103,019.

GRAPHIC CHARTS AND METERS

LESSON 25

This lesson will prove very interesting and instructive to the pupils. Encourage them in constructing graphs whenever the opportunity presents itself. Work for neatness and for comprehension of the advantages of each of the several ways of graphing. Let them tell how various things might be done.

Problem projects can be used to advantage here.

Exercise 54—Written.

Answers:

1. Ratio 1 to 86;
2. Ratio 37 to 1;
7. New England..... 25°
   Middle Atlantic..... 76°
   East North Central..... 72°
   West North Central..... 47°
   South Atlantic..... 47°
   East South Central..... 32°
   West South Central..... 36°
   Mountain..... 11°
   Pacific..... 14°
   United States..... 360°
8. Dry Goods..... 20% 72°
   Furniture..... 50% 180°
   Carpets..... 17% 61°
   Curtains..... 5% 18°
   Miscellaneous..... 8% 29°
   Total Sales..... 100% 360°

Lesson 26

The ability to read meters and compute gas and electric bills correctly may save the pupils many dollars in later years. Teach both subjects thoroughly. If you can get a discarded electric meter from the local power company it will help a lot. Use problem projects. Have different "gas men" chosen from class; keep active readings going on; call on different pupils right along; if they miss, call on them again soon after till they learn how. In going to the "Gas Office," let some complain about their bills; let others be officials and work it out for them in detail if need be; they love to do that.

Explain carefully the meaning of "kilowatt hour" and its abbreviation, "k. w. hr."

Exercise 56—Written.

Answers:

1. $2.40;
2. $2.16;
3. Gross $4.68; Net $4.38;
4. Gross $15.24; Net $14.94;
5. Gross $7.80; Net $6.60;
6. Gross $6.36; Net $5.34;
7. $1.65;
8. Gross $1.62; Net $1.44.

Give your "class honor" now for chart and meter work.

PERCENTAGE

LESSON 27

The advantages of the method shown over the tedious process of listing each partial payment are self-evident. This method is so simple that the interest on almost any installment account can be computed without the use of paper and pencil. This is vital to every child. The world does not know how much is lost through ignorance regarding this subject. Use problem projects. Let them run an installment furniture house. Let some go there to buy furniture, then have them make their monthly payments, and after computing the interest have them settle their accounts.

Exercise 58—Written.

Answers:

1. $4.55;
2. $637.50;
3. $21.38;
4. $1.15;
5. $60.92;
6. $44.00;
7. $1,243.75;
8. $5.28;
9. $11,550.00;
10. $5.60.

Note: Do not forget the oral drill on rapid addition.

Lesson 28

Note Regarding Grading.

Where Lesson 28 has been taught in the seventh year, it must nevertheless now be given as review work with such saving in time as is made possible by the previous teaching.

When a note bearing interest is discounted, the two interest calculations must be kept entirely separate. First find what the note and interest will amount to at maturity, then discount this total amount for the required time. Explain one problem very carefully; leave the work; have children who wish to learn ask questions about the work, or point out what they do not understand. Encourage questions, then go ahead. Use problem projects. Let them have a bank window in room and go there to have notes discounted, some without interest others with interest. It will "liven" it up.

Exercise 60—Written.

Answers:

1. \$394.00;
2. \$347.08;
3. \$1,499.40;
4. \$282.07;
5. \$818.69;
6. \$400.00; 90 days;
7. \$240.00; 5%
8. \$510.20;
9. \$350.00;
10. \$360.00;
11. \$900.00.

Lesson 29

It will be noted that mortgages and bonds are treated in one lesson and that attention is called to the fact that stocks are never to be confused with bonds. Lay particular stress on this distinction which, unfortunately, is known by far too few people. Mortgages and bonds are closely related as a bond is really a part of a mortgage.

Use problem projects. Let different children represent the various parties to a mortgage and carry out a transaction in detail, even to the recording of the "satisfaction."

Exercise 62—Written.

Answers:

1. $2,375.00;
2. $2,286.00;
3. $1,320.00;
4. $4,350.00;
5. (*a*) Dec. 1, 1925, $40,000.00;
   Dec. 1, 1926, $60,000.00;
   Dec. 1, 1927, $100,000.00;
   (*b*) June 1, 1921, $6,000.00;
   Dec. 1, 1925, $6,000.00;
   June 1, 1926, $4,800.00;
   Dec. 1, 1926, $4,800.00;
   Dec. 1, 1927, $3,000.00;
   (*c*) $75,600.00;
6. $7.50.

Lesson 30

Here, again, pains must be taken to emphasize the distinction between (*a*) bonds and stocks, (*b*) bondholders and stockholders, (*c*) interest and dividends.

Organize a corporation in your school room and have it represent a successful enterprise for a time, then an unsuccessful one, that the pupils may better visualize the fate of an investor in "wild cat" securities. The problem project will give the children a most thorough comprehension of this subject.

Exercise 64—Written.

Answers:

1. $7\frac{1}{2}\%$;
2. 5%;
3. $23,000.00;
4. $1,000,000.00;
5. $24,000.00;
6. $38,500.00;
7. 20%;
8. $2\frac{1}{2}\%$;
9. $500.00;
10. $121,000.00;
    55 shares.

Note: Remember the addition drill—urge for more and more speed.

After disposing of the preliminary work covered by this lesson, the actual computation of yield must be very carefully undertaken. Every man and woman should be able to compute the percentage of income on a contemplated investment without depending upon the statements of salesmen and others who, themselves, seldom are correctly informed. Lay stress upon the advantages to be gained by those who know how to invest their savings properly.

Example #1 is illustrative of a case where stock is bought at a discount and held as a permanent investment; in such cases it must always be assumed that the market value of the stock remains unchanged, for no cognizance can be taken of any change in the market value until such time as the stock is actually disposed of; therefore, the percentage of income must be computed on the basis of the cost price.

Examples #2 and #3 show bonds bought at a discount and at a premium respectively. As every bond is worth its par value at maturity, the discount or premium must be distributed over the life of the bond. Thus, the value of the bond changes proportionately from year to year as the date of maturity approaches; therefore, the percentage of income must be computed on the basis of the *average value;* that is, the value which is *half way* between the *purchase price* and the *par value.* Go slowly but surely here. Talk it over with the children and let them ask for help freely.

Examples #4 and #5 show stocks which were sold for more and less respectively than what was paid for them. In cases of this kind, the gain or loss is distributed over the period during which the stock was owned, and the percentage of income must be computed on the basis of the *average value;* that is, the value which is *half way* between the *purchase price* and the *selling price.* Continue the project (Exercise 65); let each child buy some stock at par and market value. Then watch the yield.

Exercise 67—Written.

Answers:

1. \$4,432.50;
2. 40 shares;
3. $10\frac{5}{7}\%$ on 1 share;
   $10\frac{5}{7}\%$ on 15 shares;
4. $66\frac{13}{24}$;
5. 90 shares;
   $133\frac{5}{24}$;
   \$12,000.00;
6. $6\frac{18}{100}\%$;
7. $4\frac{76}{100}\%$;
8. $5\frac{47}{100}\%$;
9. The stock is $\frac{1}{6}$ of 1% more profitable;
10. $6\frac{33}{100}\%$;
11. $8\frac{16}{100}\%$.

Lesson 32

If the child can be made to realize the value, to him, of a thorough knowledge of insurance matters, this lesson will be much more thoroughly and quickly mastered than if it is approached from a purely arithmetical standpoint.

Each class of insurance should be discussed separately and the examples relating thereto should then be worked.

Run over the first part rapidly. When you take up the Insurance Office Project, let children vie with each other to find out things; asking pointed questions—watching papers; *i. e.*, getting right papers, correct names, etc. See that your officials are *well posted.* Give them policies to study and give them reference books so they may be a fountain head of knowledge.

Exercise 70—Written.

Answers:

1. \$29.01;
2. (*a*) \$4,925.00 premium;
   (*b*) \$5,000.00 collected;
   (*c*) Age 45;
3. (*a*) \$7,650.00 premium;
   (*b*) \$10,000.00 paid to beneficiary;
   (*c*) Int. on \$10,000.00 for 10 years = \$6,000.;

4. $180.00;
5. (*a*) Manufacturer gained $1,500.00;
   (*b*) Bonding company lost $1,500.00;
6. (*a*) Employer gained $25,625.00;
   (*b*) Insurance company lost $25,625.00;
   (*c*) Accident cost employer $15,860.00;
   (*d*) Accident cost insurance company $30,000.00.

Give your "class honor" now for percentage work.

Exercise 71—Oral Review.

Give plenty of drill on addition—watch the slow ones most—make them see the groups. They cannot take their own time about it if the work is oral—let them compete to see who finds the most groups and who adds most rapidly.

More review work of a similar nature may be given, time permitting.

Exercise 72—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. 10° 14′ 32″;
2. (*a*) 2,304;
   (*b*) 262,144;
3. (*a*) 819;
   (*b*) $4\frac{1}{3}$;
4. 28.28+ ft.;
5. $2\frac{3}{4}$ ft.;
6. 29.45+ sq. in.;
7. Volume, 3,840 cu. ft.;
   Slant height, 23.32+ ft.;
8. $76.00;
9. $676.00;
10. $5\frac{71}{100}\%$;
11. 28;
12. 53;
13. 72;
14. 526;
15. 718;
16. 643;
17. 133,425;
18. 291,788;
19. 392,290;
20. 147,467;
21. 774,917;
22. 408,165;
23. 265,202;

24. 90,263;
25. 381,344;
26. 316,989;
27. 322,075;
28. 178,415;
29. 333,717;
30. 478,928;
31. 345,400;
32. 251,228;
33. 255,645;
34. 151,632;
35. 363,552;
36. 360,372;
37. 41,630;
38. 32,560;
39. 33,115;
40. 27,910;
41. 42,705;
42. 27,733.

PARTNERSHIP

LESSON 33

Let several children impersonate the partners in some of the examples to see if they agree on the division of their profit, etc. Be sure they notice if investments are equal or unequal. Lead the child to be on the alert for the wording that tells the story.

Problem projects are plentiful here.

Exercise 73—Oral.

Divide room into groups—A, B or A, B, C. Keep each one asking questions and watching his profits. Interest will be red hot.

Exercise 74—Written.

Answers:

1. Johnson, $5,307.00;
   Brown, $3,538.00;
2. Alexander, $1,246.80;
   Wilson, $1,090.95;
   Hendricks, $779.25;
3. (Encourage accurate long division)
   Black, $4,749.03;
   White, $3,885.57;
4. A's profit, $4,510.00;
   B's profit, $3,690.00;
5. A 39%; B 36%;
   C 25%;
6. $48,000.00;
7. X withdrew $400.00;
   Y invested $400.00.

Exercise 75—Written.

Answers:

1. A withdrew $804.00;
2. B invested $804.00;
3. C invested $4,020.00;
4. $4,000.00;
5. $2,000.00;
6. $666.67;
7. $3,373.33;
8. $2,353.33;
9. $2,273.34;
10. No balances.

Give your "class honor" now for partnership work.

Exercise 76—Oral Review.

More review work of a similar nature may be given, time permitting.

Exercise 77—Written Review.

More review work of a similar nature may be given, time permitting.

Answers:

1. $16.61;
2. 314.16 sq. in.;
3. 12.72+ in.;
4. 4 in.;
5. 5 P. M.;
6. $492.50;
7. 10.39+ in.;
8. $8.55;
9. 68.1;
10. 40.06 yd.;
11. $2,334,800,000.00;
12. (*a*) $38\frac{9}{10}\%$ approximately;
    (*b*) $33\frac{1}{3}\%$;
    (*c*) 37+%;
13. 49,032;
14. 15,066;
15. 17,204;
16. 21,312;
17. 43,772;
18. 28,224;
19. 81,732;
20. 54,612;
21. 341,999;
22. 234,522;
23. 99,944;
24. 484,471;
25. 928;
26. 589,052;
27. 411,825;
28. 8,976;
29. 437;
30. 43,924;
31. 60,898;
32. 105,839;

33. 297;	**36.** 913;	**39.** 437,433;	**42.** 71,904;
34. 412;	**37.** 725;	**40.** 64,715;	**43.** 155,237;
35. 527;	**38.** 248;	**41.** 440,187;	**44.** 209,355.

General Review

All work for general review purposes should be selected from, or based on, the various exercises which constitute the year's work in the text book.

Give your review work in three parts; viz.:

(*a*) Oral work on principles, etc.

(*b*) Oral problems.

(*c*) Written problems.

Now repeat the intelligence test given at the beginning of the year, and give each pupil his record for the tests given in Parts V to VIII.

Special Intelligence Test in Arithmetic

The following 10 problems are taken from a series of 22 problems prepared as an Intelligence Test in Arithmetic for College Freshmen and High School Seniors by Dr. L. L. Thurstone, Division of Psychology, Carnegie Institute of Technology, Pittsburgh, Pa., to whom we hereby acknowledge our indebtedness for their use.

The time allowed for the solving of the entire series of 22 problems is 30 minutes; the pro rata time which should be allowed for the solving of the following 10 problems is about 14 minutes, but considering the fact that your pupils are much younger, it will be interesting to note how many of the pupils can solve the 10 problems in 30 minutes. Each problem is within the scope of the child's education.

If facilities are at hand, papers containing the problems should be prepared in advance and given to the pupils at the proper time; otherwise the problems should be written on the board during the pupils' absence and kept covered until the proper moment.

The Problems:

1. A firm builds a warehouse four stories high. The interior dimensions of the building are 50 by 200 ft. How many square feet of floor space have they provided for themselves?
2. If concrete curbing and gutter on a street cost 65 cents per running foot, what is the cost of the curbing and gutter on both sides of a city block 500 ft. long?

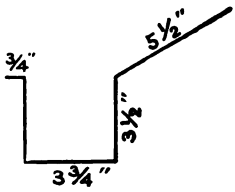


Figure 1

3. See Figure 1. This figure is a cross section of an iron gutter to be made by a tinsmith. It is made by bending a piece of material of the proper width. How wide must the material be?
4. If a steel rail weighs 60 lb. per yard, how many tons of rail will be required to lay ½ mile of single-track railroad?
5. Calculate the weight of a steel plate one foot square, ½″ thick with a rectangular hole which measures 3″ × 8″. Steel weighs 0.3 pound per cubic inch.
6. A contractor offers to lay an asphalt pavement at $3.60 per square yard. The street is 50 ft. wide. How much must the lot owners on both sides of the street be assessed for each foot in the width of their lots?
7. A tank of water is being drained at the rate of two cubic feet of water per second and supplied at the rate of one cubic foot per second. After two minutes there are one hundred cubic feet of water in the tank. How much water was in the tank before the pipes were opened?
8. What must be the length of a bolt under the head to go through $9\frac{3}{32}$″ thickness of plank and allow $1\frac{7}{8}$″ outside for taking a nut?

9. What will be the expense for a cement sidewalk on the two sides of a corner lot 50 ft. by 100 ft. if the walk is 5 ft. wide and costs 10 cents per square foot?
10. What is the area of the surface of a boiler plate 3′ 9″ by 1′ 6″? (Give your answer in square feet.)

The Answers:

1. 50,000 sq. ft. (This answer includes 10,000 sq. ft. in basement, and makes no allowance for stairways, etc.)
2. $650.00;
3. 17 in.;
4. 52.8 tons of rail;
5. 18 lb.;
6. $10.00;
7. 220 cu. ft.;
8. $10\frac{31}{32}$ in.;
9. $77.50;
10. $5\frac{5}{8}$ sq. ft.

Examinations (If Desired)

That the examinations (if desired) may be a thorough test of the pupil's knowledge of the work covered, the questions should be selected from the various exercises which constitute the year's work in the text book, but the teacher may introduce other examples of a similar nature. Examinations should be held in oral as well as in written arithmetic, and should test the pupil's knowledge of both theory and application.

A Suggestion

Show the weak ones why and where they failed; see if you cannot encourage them to do a little reviewing in their spare time. Perhaps they did not care before now but are just beginning to realize their plight—a word of encouragement may mean everything to their future welfare—try anyway, it can do no harm.

Teacher's Memoranda