ARITHMETIC

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PARTS V AND VI

INTERMEDIATE LESSONS

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FOREWORD

Bearing in mind that a thorough knowledge of arithmetic is perhaps more frequently the cause of success in life than is any other single factor, one can hardly fail to realize how great is the responsibility which rests on those whose duty it is to provide for the child's education in this branch.

No book or series of books can possibly illustrate every use to which numbers can be put, but if the principles underlying their use are properly taught, the child can reason for himself the proper application of his knowledge to any given problem. Furthermore, as he must know how to solve a problem in the quickest and simplest manner, he must know not merely the various processes, but their construction as well; he must be able to analyze to such an extent that when a problem is presented to him, he can distinguish the facts which are relevant from those which are irrelevant, he can separate the known from the unknown, he can arrange the known in logical order for his processes, and he can use the shortest processes possible. An attempt to give the pupil this ability is the motive for this work.

The vehicle used to obtain the result is a series of progressive lessons, which, with ample practice, take the pupil step by step through the construction of each process to be learned, thus giving him the opportunity of following the teacher's explanation, and of referring to past lessons at any time. In this way the pupil who is slower to grasp new ideas than the average can keep up with his class, and every pupil can at all times refresh his memory on any points which he may have forgotten or which may have escaped him in the classroom, and which have so often been lost to him forever.

The time-saving methods used by the most expert arithmeticians are introduced as part of the routine work; thus, the child learns these without any special effort.

It is not intended that the lessons or definitions are to be learned verbatim, any more than it is intended that the examples given are to be memorized; both are there for the purpose of showing the pupil the reason for, and the application of, the processes, and the exercises are there to give him practice and to test his knowledge of what he has learned.

The exercises form a continuous review of what has been learned, but further review work is given at regular intervals.

The series consists of Three Books and Teacher's Manuals, as follows:

Primary Lessons	Parts I and II. (Teacher's Manual only.)
Elementary Lessons	Parts III and IV. (With Manual for the Teacher.)
Intermediate Lessons	Parts V and VI. (With Manual for the Teacher.)
Advanced Lessons	Parts VII and VIII. (With Manual for the Teacher.)

The first two parts are so arranged in the Teacher's Manual that the lessons and exercises can be given largely as games, play work, number stories in language work, etc., all used more or less incidentally, till the child is gradually prepared for work requiring an increasing degree of conscious effort.

The work contained in each of the eight parts is that which is usually taught in the corresponding grade, and it is recommended that this routine be followed. However, special provision has been made for such variations in the grading as are required in some localities, by means of a series of notes in the Teacher's Manuals which enable the teacher to follow either method with equal facility.

The authors wish to express their deep appreciation to Mr. E. C. Hinkle and Mr. J. R. Clark, both of the Department of Mathematics, Chicago Normal College, for their critical examination of the manuscript and their valuable suggestions for its improvement.

CONTENTS

PART V

LESSON
NUMBER PAGE

FRACTIONS

CONTENTS

CONTENTS

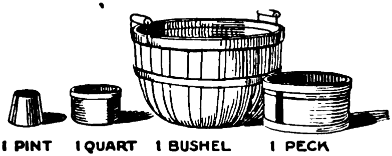
PART VI

CONTENTS

Tables of Weights and Measures

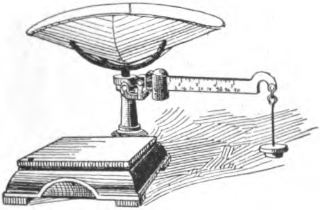
(For Ready Reference)

Dry Measure



2 pints (pt.)........................ = 1 quart (qt.)
8 quarts............................ = 1 peck (pk.)
4 pecks............................. = 1 bushel (bu.)

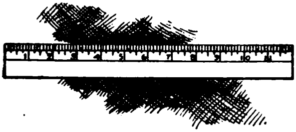
Avoirdupois Weight



16 ounces (oz.)......... = 1 pound (lb.)
100 pounds............. = 1 hundredweight (cwt.)
20 hundredweight...... = 1 ton (T.)
2,000 pounds............. = 1 short ton
2,240 pounds............. = 1 long ton (used at mines and U. S. Custom House)

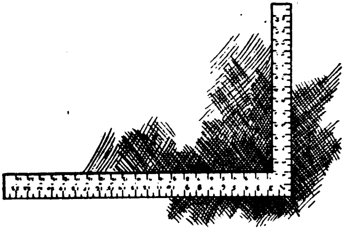
TABLES OF WEIGHTS AND MEASURES

Linear Measure



12 inches (in.)...... = 1 foot (ft.)
3 feet............ = 1 yard (yd.)
5½ yards........... = 1 rod (rd.)
320 rods............ = 1 mile (mi.)
1,760 yards........... = 1 mile
5,280 feet............ = 1 mile
6 feet............ = 1 fathom (used in measuring the depth of water)

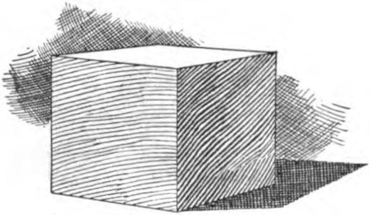
Square Measure



144 square inches (sq. in.).. = 1 square foot (sq. ft.)
9 square feet........... = 1 square yard (sq. yd.)
30¼ square yards.......... = 1 square rod (sq. rd.)
160 square rods........... = 1 acre (A.)
640 acres................. = 1 square mile (sq. mi.)
640 acres................. = 1 section (sec.)
100 square feet........... = 1 square (sq.)

TABLES OF WEIGHTS AND MEASURES

Cubic Measure.



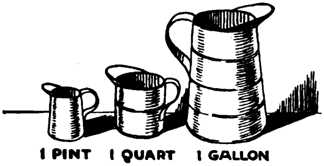
1,728 cubic inches (cu. in.)... = 1 cubic foot (cu. ft.)
27 cubic-feet............. = 1 cubic yard (cu. yd.)
128 cubic feet............ = 1 cord (cd.)

1 gallon contains 231 cubic inches.

1 bushel contains 2,150.42 cubic inches or 1¼ cu. ft. (nearly).

1 cubic foot of water contains 7½ gallons and weighs 62½ pounds.

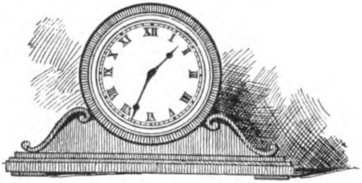
Liquid Measure



4 gills (gi.)................... = 1 pint (pt.)
2 pints...................... = 1 quart (qt.)
4 quarts..................... = 1 gallon (gal.)
31½ gallons.................... = 1 barrel (bbl.)
2 barrels..................... = 1 hogshead (hhd.)

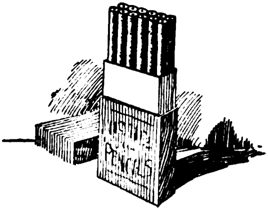
TABLES OF WEIGHTS AND MEASURES

Time Measure



60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
28, 29, 30, 31 days = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

Table Used in Counting Merchandise



12 things = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
12 gross = 1 great gross (gt. gr.)

ARITHMETIC

PART V

INTERMEDIATE LESSONS

INTERMEDIATE LESSONS

PART V

FRACTIONS

Lesson 1

How Fractions Are Formed

A "whole number" is a number which shows one or more units or whole things; as an apple; an orange; a pie.

A "fractional number" or "fraction" is a number which shows one or more of the equal parts of a unit; as $\frac{1}{2}$ of a pie; $\frac{1}{4}$ of an apple.

A "mixed number" is a number which shows one or more units plus one or more parts of a unit; therefore, a "mixed number" is a combination of a "whole number" and a "fraction"; as \$1$\frac{1}{2}$.

To form a fraction it is necessary that we know two things:

First: Into how many equal parts is the unit divided?

Second: How many of the equal parts are being spoken of?

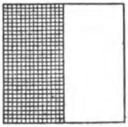
If the unit is divided into two equal parts, each part is called a "half," and if we are speaking of one of these halves, we say "one-half"; if we speak of both halves, we say "two-halves."

If the unit is divided into three equal parts, each part is called a "third," and we can speak of "one," "two," or "three-thirds."

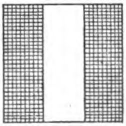
If the unit is divided into four equal parts, each part is called a "fourth" or a "quarter."

If the unit is divided into five equal parts, we have "fifths;" if into six parts, we have "sixths," and so on.

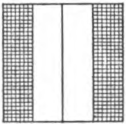
I.



II.



III.



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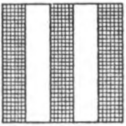

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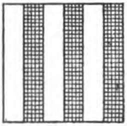


Exercise 1—Oral.

Each of these drawings shows a square.

1. Into how many parts is Square I divided? What is each of these parts called? How many "halves" are there in the whole square? What part of this square is shaded? How many halves make one?
2. Into how many parts is Square II divided? Name each part. How many "thirds" are there in the whole square? What part of this square is shaded? How many thirds make one?
3. Into how many parts is Square III divided? What is each of these parts called? How many "fourths" are there in any whole thing? What part of this square is shaded? How many fourths make one?
4. Into how many parts is Square IV divided? What is each of these parts called? How many "fifths" are there in any whole thing? What part of this square is shaded? How many fifths make one?

5. Into how many parts is Square V divided? What is each of these parts called? How many "sixths" are there in any whole thing? What part of this square is shaded? How many sixths make one?
6. Look at the unshaded part of Squares I and II showing one-half and one-third and say whether one-half is larger or smaller than one-third. Is a third of an apple as large as one-half an apple?
7. Is one-third larger or smaller than one-fourth?
8. Is one-sixth larger or smaller than one-fifth?
9. When an apple is divided into four parts, is each part larger or smaller than when the apple is divided into two parts?
10. Six is a larger number than three; why is not one-sixth of any whole thing larger than one-third of it?

LESSON 2

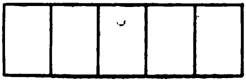
Writing Fractions

To write a fraction, we use two numbers which we write one above the other, with a short line between to separate them. These two numbers are called the two "terms" of the fraction.

The number which is written below the line tells us into how many equal parts the unit is divided, and shows whether we are speaking of "halves," "thirds," "fourths," etc. This term of the fraction is called the "denominator" because it is the name of the fraction, and "nominate" means "to name."

The number which is written above the line tells us how many parts are being spoken of, and shows whether we are speaking of one or two halves; one, two, or three thirds; one, two, three, or four fourths, etc. This term of the fraction is called the "numerator," because it shows the number of parts being spoken of, and "numerate" means "to number."

Exercise 2—Oral and Written.



1. Draw a whole oblong; cut it into 5ths. Show 3 fifths; show 4 fifths.
2. Fold a paper oblong into 4ths.
3. Read:

   $\frac{1}{2}$; $\frac{2}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{3}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$; $\frac{4}{4}$; $\frac{1}{5}$;
   $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{5}{5}$; $\frac{1}{6}$; $\frac{2}{6}$; $\frac{3}{6}$; $\frac{4}{6}$; $\frac{5}{6}$; $\frac{6}{6}$.
4. Read all the numerators; all the denominators.
5. Point out 5 figures that name the equal parts of the unit.
6. In writing $\frac{5}{8}$, what does the number which is written below the line show? What is this term of the fraction called?
7. In writing $\frac{5}{8}$, what does the number which is written above the line show? What is this term of the fraction called?
8. Name the two terms of $\frac{7}{8}$, and say what each shows and where each is written.
9. What is a whole number? What is a fractional number or fraction?
10. Draw a whole oblong and one-half more. How much have you in all? Write the number which

shows how much you have; call it a mixed number.

11. What kind of a number is each of the following: $6; \frac{2}{3}; 7; 2\frac{3}{4}; \frac{7}{8}; 5; \frac{10}{11}; 9; 6\frac{7}{8}$.
12. Say three whole numbers. Write six on the board.
13. Say three fractional numbers. Write three on the board.
14. Say three mixed numbers. Write five on the board.

Exercise 3—Written.

Write, using figures instead of words:

1. One-half.
2. One and one-third.
3. Two-fourths.
4. Three and three-fifths.
5. Four-sixths.
6. Six and five-sevenths.
7. Seven-eighths.
8. Nine and four-ninths.
9. Two-tenths.
10. Two and three-elevenths.
11. Six and four-twelfths.
12. Seven-thirteenths.
13. Eight and nine-fourteenths.
14. Two-fifteenths.
15. Ten and eleven-sixteenths.
16. Two-fifths.
17. Five and three-sixths.
18. Two-thirds.
19. Four-ninths.
20. Four and seven-sixteenths.

LESSON 3

Finding Fractional Parts of Whole Numbers

EXAMPLE:

$\frac{1}{4}$ of 12 = 3.
$\frac{3}{4}$ of 12 = 3 × 3 or 9.
Note: $\frac{3}{4}$ is three times as great as $\frac{1}{4}$.

EXAMPLE:

$\frac{1}{3}$ of 9 = 3.
$\frac{2}{3}$ of 9 = 2 × 3 or 6.
Note: $\frac{2}{3}$ is two times as great as $\frac{1}{3}$.

In your previous lessons, you have learned that when you divide a number by 4, you are finding one-fourth of the number; when you divide a number by 5, you are finding one-fifth of the number; when you divide a number by 10, you are finding one-tenth of the number, etc.; therefore, to find a fractional part of any whole number, you divide the whole number by the denominator of the fraction, and to find several fractional parts, you multiply the answer so found, by the numerator.

Exercise 4—Oral.

1. Find $\frac{1}{2}$ of 10; of 12; of 14; of 8; of 16.
2. Find $\frac{1}{3}$ of 12; $\frac{2}{3}$ of 12; $\frac{3}{3}$ of 12; $\frac{1}{3}$ of 6; $\frac{2}{3}$ of 6.
3. Find $\frac{3}{4}$ of 8; $\frac{2}{4}$ of 12; $\frac{1}{5}$ of 15; $\frac{3}{5}$ of 15; $\frac{4}{5}$ of 15.
4. How many pints are there in one quart? A pint is what part of a quart? What is a pint of milk worth if a quart is worth 12¢?
5. How many feet are there in a yard? A foot is what part of a yard? What is a foot of calico worth if a yard is worth 9¢? Two feet equals what part of a yard? What are two feet of calico worth at 9¢ a yard?

6. How many pecks are there in a bushel? What part of a bushel is one peck? Two pecks? Three pecks? If potatoes cost $1.00 a bushel, what is the cost of a peck? Of two pecks? Of three pecks?
7. At 32¢ a pound, what is the cost of $\frac{1}{4}$ pound of beef? Of $1\frac{1}{4}$ pounds?
8. At 40¢ a dozen, what is the cost of $\frac{3}{4}$ dozen of eggs? Of $1\frac{3}{4}$ dozen?
9. At 30¢ a dozen, what is the cost of $\frac{2}{3}$ dozen of oranges? Of $1\frac{2}{3}$ dozen?
10. At 20¢ a dozen, what is the cost of $1\frac{1}{2}$ dozen of bananas?
11. At 48¢ a pound, what is the cost of $\frac{1}{8}$ pound of coffee? Of $\frac{3}{8}$ pounds?
12. Tell how you find $\frac{3}{4}$ of 64; $\frac{7}{8}$ of 24; $\frac{3}{10}$ of 50.

Exercise 5—Oral and Written.

A. Tell how to work each of these examples:

1. $\frac{1}{4}$ of 24 = ?
2. $\frac{3}{4}$ of 24 = ?
3. $\frac{1}{3}$ of 96 = ?
4. $\frac{2}{3}$ of 96 = ?
5. $\frac{1}{8}$ of 88 = ?
6. $\frac{7}{8}$ of 88 = ?
7. $\frac{1}{8}$ of 160 = ?
8. $\frac{3}{8}$ of 160 = ?
9. $\frac{1}{10}$ of 30 = ?
10. $\frac{7}{10}$ of 30 = ?
11. $\frac{1}{11}$ of 55 = ?
12. $\frac{7}{11}$ of 55 = ?
13. $\frac{1}{4}$ of 872 = ?
14. $\frac{3}{4}$ of 640 = ?
15. $\frac{1}{3}$ of 963 = ?
16. $\frac{2}{3}$ of 765 = ?
17. $\frac{5}{8}$ of 936 = ?
18. $\frac{7}{9}$ of 819 = ?
19. $\frac{7}{10}$ of 900 = ?
20. $\frac{4}{11}$ of 550 = ?
21. $\frac{7}{12}$ of 888 = ?
22. $\frac{3}{16}$ of 480 = ?

B. Now work each of the examples.

LESSON 4

Finding Fractional Parts of Fractions

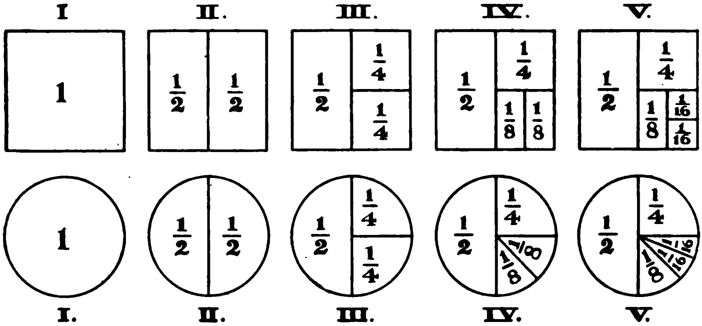


Illustration:

Fold a paper to show halves. Fold again to show half of each half.

What is $\frac{1}{2}$ of $\frac{1}{2}$? Write it on your paper.

Fold again so we can see $\frac{1}{2}$ of $\frac{1}{4}$.

How many parts does the paper show now? ($\frac{8}{8}$).

What is $\frac{1}{2}$ of $\frac{1}{4}$? Write it on your paper in the right place.

Fold again so we can see $\frac{1}{2}$ of $\frac{1}{8}$.

How many parts does the paper show now? ($\frac{16}{16}$).

What is $\frac{1}{2}$ of $\frac{1}{8}$? Write it properly on your paper.

You have learned that in any single thing, there are two halves, three thirds, four fourths, etc. Now if there are two halves in a single thing, and there are also four fourths in the same thing, then each of the halves must be as large as two of the fourths. Now find one-half of one of the halves and see that it is as large as one of the fourths, as is shown by Drawing III; therefore, $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$.

FRACTIONS

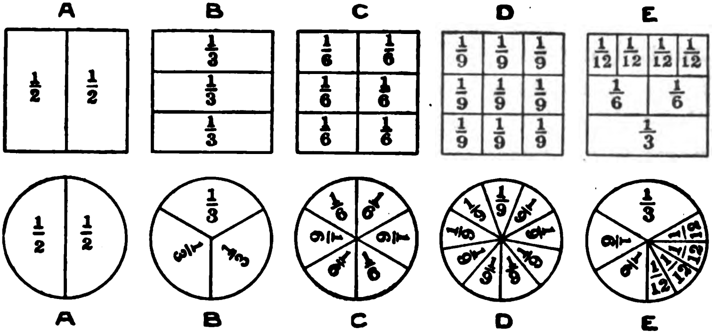
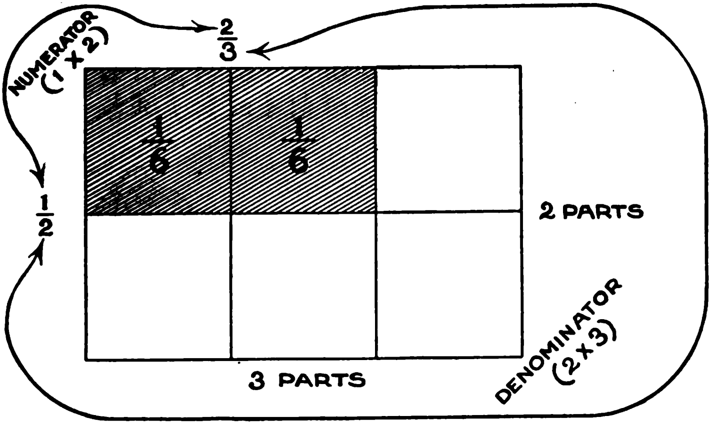


Illustration:

Fold a paper up and down to show thirds.
Fold so you have two parts on the left and right sides.
How many equal parts have you now?
Put your hand on $\frac{2}{3}$; show $\frac{1}{2}$ of $\frac{2}{3}$; name it $\frac{2}{6}$.
Draw it on the board and see if we can discover the number work for $\frac{2}{6}$. $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$.



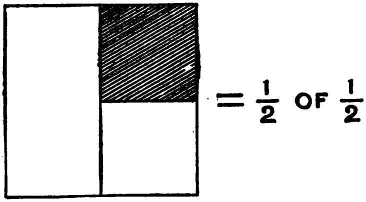
$\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$

The denominator or name of the answer is found by multiplying the two denominators. Here we first cut a unit into three thirds and then we cut each of the three *thirds* into two *halves* so there must be six equal parts in all, because 2 × 3 = 6.

The numerator or number of parts in the answer is found by multiplying the two numerators. Here we are using only *one* of the halves of only *two* of the thirds, so we are using only two of the six parts, because 1 × 2 = 2.

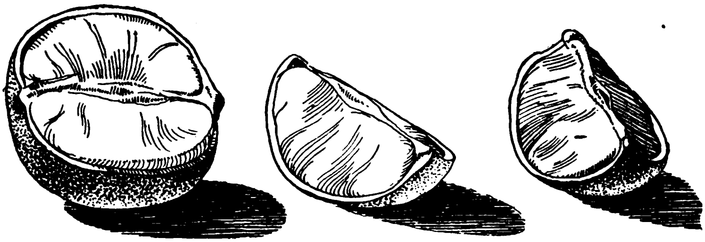
Exercise 6—Oral and Written.

(Be ready to draw these.)



1. Find: $\frac{1}{2}$ of $\frac{1}{2}$; $\frac{1}{2}$ of $\frac{1}{4}$; $\frac{1}{2}$ of $\frac{1}{8}$; $\frac{1}{4}$ of $\frac{1}{2}$; $\frac{1}{4}$ of $\frac{1}{4}$.

2. Find: $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{1}{4}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{1}{2}$; $\frac{3}{4}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{1}{4}$.



3. John had an orange which he cut into two equal parts. He kept one of these parts for himself;

he gave half of the other piece to Mary. What part of the orange did Mary get?

4. How many halves are there in 1? How many thirds are there in 1? How many sixths are there in 1?
5. In which of the drawings do you see $\frac{1}{2}$ of $\frac{1}{3}$? How much is $\frac{1}{2}$ of $\frac{1}{3}$? In which drawing do you see $\frac{1}{3}$ of $\frac{1}{2}$? How much is this? In the same way find $\frac{1}{3}$ of $\frac{1}{3}$; $\frac{2}{3}$ of $\frac{1}{3}$.
6. Which of the drawings shows how many sixths there are in $\frac{1}{2}$? How many sixths are there in $\frac{1}{2}$? In the same way find how many sixths there are in $\frac{1}{3}$. How many ninths are there in $\frac{1}{3}$? How many ninths are there in $\frac{2}{3}$?
7. In the same way find $\frac{2}{3}$ of $\frac{1}{2}$; $\frac{1}{2}$ of $\frac{3}{3}$; $\frac{1}{3}$ of $\frac{2}{2}$.
8. Draw $\frac{3}{3}$; shade $\frac{2}{3}$ of it; draw right and left lines to show $\frac{1}{3}$ of your thirds; shade $\frac{1}{3}$ of your $\frac{2}{3}$; shade another $\frac{1}{3}$ of your $\frac{2}{3}$; $\frac{2}{3}$ of $\frac{2}{3} = ?$
9. In the same way find $\frac{1}{2}$ of $\frac{1}{6}$; $\frac{1}{4}$ of $\frac{1}{3}$; $\frac{3}{4}$ of $\frac{1}{3}$.
10. A pie is cut into 6 equal parts; what is each part called? What are three of the parts called? What are five of the parts called?
11. A pie is cut into two equal parts and each of these parts is cut into three equal parts; what part of the whole pie is each of the small parts? Three of the small parts are what part of the whole pie? What are five of the parts?
12. How many twelfths are there in 1? How many thirds are there in 1? Into how many parts must you cut thirds to get 12ths? How many twelfths are there in $\frac{1}{3}$?

Exercise 7—Oral.

At sight work each of these examples:

1. $\frac{1}{2}$ of $\frac{2}{4} = ?$
2. $\frac{1}{2}$ of $\frac{2}{5} = ?$
3. $\frac{1}{3}$ of $\frac{3}{8} = ?$
4. $\frac{1}{4}$ of $\frac{4}{7} = ?$
5. $\frac{1}{5}$ of $\frac{5}{9} = ?$
6. $\frac{1}{6}$ of $\frac{6}{10} = ?$
7. $\frac{1}{3}$ of $\frac{3}{11} = ?$
8. $\frac{1}{2}$ of $\frac{2}{15} = ?$
9. $\frac{1}{3}$ of $\frac{3}{19} = ?$
10. $\frac{1}{4}$ of $\frac{4}{6} = ?$
11. $\frac{1}{5}$ of $\frac{5}{8} = ?$
12. $\frac{1}{10}$ of $\frac{10}{12} = ?$
13. $\frac{1}{2}$ of $\frac{2}{8} = ?$
14. $\frac{2}{2}$ of $\frac{2}{8} = ?$
15. $\frac{1}{3}$ of $\frac{3}{5} = ?$
16. $\frac{2}{3}$ of $\frac{3}{5} = ?$
17. $\frac{1}{4}$ of $\frac{4}{17} = ?$
18. $\frac{3}{4}$ of $\frac{4}{17} = ?$
19. $\frac{1}{5}$ of $\frac{5}{9} = ?$
20. $\frac{2}{5}$ of $\frac{5}{9} = ?$
21. $\frac{4}{5}$ of $\frac{5}{7} = ?$
22. $\frac{6}{8}$ of $\frac{8}{12} = ?$
23. $\frac{7}{9}$ of $\frac{9}{10} = ?$
24. $\frac{8}{12}$ of $\frac{12}{13} = ?$
25. $\frac{6}{10}$ of $\frac{10}{50} = ?$
26. $\frac{7}{10}$ of $\frac{20}{50} = ?$
27. $\frac{1}{2}$ of $\frac{8}{10} = ?$
28. $\frac{3}{2}$ of $\frac{8}{10} = ?$

Exercise 8—Oral.

A. Say how you would work each of these examples:

1. How many hours are there in a day? In $\frac{1}{2}$ of a day? In $\frac{2}{4}$ of a day? In $\frac{3}{6}$ of a day? In $\frac{4}{8}$ of a day?
2. How many things are there in $\frac{1}{3}$ of a dozen? How many in $\frac{2}{6}$ of a dozen? How many in $\frac{4}{12}$ of a dozen?
3. If Alice practices on the piano $\frac{1}{2}$ of an hour each day, how many half hours will she practice in 6 days? This is the same as how many hours?
4. How many ounces are there in $\frac{3}{8}$ of a pound? How many in $\frac{5}{8}$ of a pound?

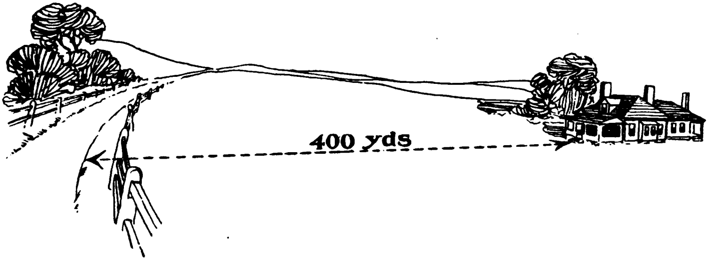
5. If $\frac{7}{8}$ of the pupils in a class of 40 pupils pass their examinations, how many pupils will be promoted? How many will not be promoted?

6. A shoe merchant added $\frac{1}{8}$ of the cost to the cost of his goods to make a little profit. At this rate, what would be the selling price of shoes which cost him $3.20 a pair? What would be the selling price of slippers which cost him 80¢ a pair if he added $\frac{3}{8}$ of the cost to the cost?

7. Find the cost of $\frac{11}{12}$ doz. oranges @ 60¢ a doz. Find the cost of $\frac{7}{12}$ doz. Of $\frac{1}{6}$ doz.

8. A bin which holds 400 lb. of flour is only $\frac{3}{4}$ full; how many pounds of flour does it contain? How many pounds would it contain if $\frac{1}{3}$ of the flour were removed? How many pounds if $\frac{1}{2}$ of the remainder were removed?

9. Frank is 54 inches tall; his brother is $\frac{5}{6}$ as tall; how tall is Frank's brother?



10. The distance between the road and a house is 400 yd.; how many yards are there in $\frac{3}{4}$ of this distance? In $\frac{7}{8}$ of the distance? In $\frac{5}{8}$ of the distance?

B. Now work all of these examples orally.

LESSON 5

Addition and Subtraction of Fractions with Like Denominators or Names

Just as we add 2 apples + 2 apples = 4 apples, or say 4 chairs − 2 chairs = 2 chairs, etc., so we add 2 quarters + 2 quarters = 4 quarters and say 4 eighths − 2 eighths = 2 eighths. The number of parts changes, but the name does not.

EXAMPLE:	**EXAMPLE:**	**EXAMPLE:**
$\frac{1}{\text{fourth}} \overset{\frown}{+} \frac{1}{\text{fourth}} \overset{\frown}{=} \frac{2}{\text{fourths}}$	$\frac{3}{8} \overset{\frown}{-} \frac{2}{8} \overset{\frown}{=} \frac{1}{8}$	$\frac{2}{16} \overset{\frown}{+} \frac{3}{16} \overset{\frown}{+} \frac{7}{16} \overset{\frown}{=} \frac{12}{16}$
What did you add? What did you call your 2?	What did you subtract? What did you call your 1?	What did you add? What did you call your 12?

Exercise 9—Oral.

1. $\frac{2}{4} + \frac{1}{4} = \frac{?}{4}$
2. $\frac{1}{12} + \frac{3}{12} + \frac{4}{12} = \frac{?}{12}$
3. $\frac{3}{4} - \frac{1}{4} = \frac{?}{4}$
4. $\frac{3}{9} + \frac{4}{9} - \frac{1}{9} = \frac{?}{9}$
5. $\frac{3}{6} + \frac{2}{6} = \frac{?}{6}$
6. $\frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{?}{8}$
7. $\frac{7}{8} - \frac{2}{8} = \frac{?}{8}$
8. $\frac{5}{6} - \frac{1}{6} - \frac{2}{6} = \frac{?}{6}$
9. $\frac{4}{8} + \frac{3}{8} = ?$
10. $\frac{9}{16} + \frac{1}{16} + \frac{5}{16} = ?$
11. $\frac{7}{12} - \frac{4}{12} = ?$
12. $\frac{7}{16} - \frac{2}{16} + \frac{9}{16} = ?$
13. $\frac{1}{4} + \frac{2}{4} + \frac{1}{4} = ?$
14. $\frac{2}{8} + \frac{1}{8} + \frac{3}{8} = ?$
15. $\frac{1}{10} + \frac{1}{10} + \frac{5}{10} = ?$
16. $\frac{1}{2} + \frac{3}{2} + \frac{1}{2} = ?$
17. $\frac{2}{2} + \frac{1}{2} + \frac{4}{2} = ?$
18. $\frac{1}{3} + \frac{2}{3} = ?$
19. $\frac{8}{4} + \frac{1}{4} = ?$
20. $\frac{6}{8} + \frac{2}{8} = ?$
21. $\frac{9}{10} + \frac{5}{10} = ?$
22. $\frac{1}{4} + \frac{3}{4} = ?$
23. $\frac{9}{12} + \frac{1}{12} + \frac{1}{12} = ?$
24. $\frac{7}{9} + \frac{1}{9} + \frac{1}{9} = ?$
25. $\frac{2}{5} + \frac{1}{5} + \frac{3}{5} = ?$
26. $\frac{8}{5} + \frac{6}{5} + \frac{2}{5} = ?$
27. $\frac{1}{3} + \frac{1}{3} + \frac{3}{3} = ?$
28. $\frac{1}{4} + \frac{2}{4} + \frac{1}{4} = ?$

29. How do we add fractions with like denominators?

30. How do we subtract fractions with like denominators?

To add a fraction to a whole number, we merely write the fraction after the whole number making a mixed number, thus:

EXAMPLE:	EXAMPLE:	EXAMPLE:
$3 + \frac{3}{4} = 3\frac{3}{4}$.	$4 + \frac{1}{4} + \frac{1}{4} = 4\frac{2}{4}$.	$2 + \frac{4}{8} = 2\frac{4}{8}$.

Exercise 10—Oral.

1. $3 + \frac{1}{2} = ?$
2. $8 + \frac{2}{7} = ?$
3. $10 + \frac{1}{2} = ?$
4. $2 + \frac{1}{2} + \frac{1}{2} = ?$
5. $4 + \frac{1}{3} + \frac{1}{3} = ?$
6. $10 + \frac{1}{5} + \frac{3}{5} = ?$
7. $2\frac{1}{2} + \frac{1}{2} = ?$
8. $6 + \frac{2}{3} = ?$
9. $\frac{1}{2} + 8 = ?$
10. $\frac{2}{3} + 2 = ?$
11. $\frac{1}{3} + 5 = ?$
12. $7 + \frac{2}{5} + \frac{2}{5} = ?$
13. $\frac{7}{8} + \frac{7}{8} = ?$
14. $\frac{4}{5} + \frac{1}{5} + 2 = ?$
15. $40 + \frac{5}{8} = ?$
16. $100 + \frac{1}{2} = ?$
17. $20 + \frac{4}{7} = ?$
18. $8 + \frac{9}{10} = ?$
19. $9 + \frac{4}{9} + \frac{4}{9} = ?$
20. $\frac{5}{5} + \frac{1}{5} = ?$
21. $3 + \frac{5}{5} + \frac{1}{5} = ?$
22. $5 + \frac{2}{6} + \frac{3}{6} = ?$

LESSON 6

Reduction of Whole and Mixed Numbers and Improper Fractions

When the numerator of a fraction is smaller than the denominator, the fraction is called a "proper fraction," as $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{12}$, etc. A proper fraction always means less than 1.

Since a unit can be cut into any number of equal parts, when we show all of the parts we are really showing a complete unit, only in another form; thus, $\frac{2}{2} = 1$; $\frac{3}{3} = 1$; $\frac{4}{4} = 1$; etc.

If we have one or more complete units plus one or more parts of a unit, this can be shown in the form of a mixed number, as $1\frac{1}{2}$, $1\frac{2}{3}$, $2\frac{1}{4}$, etc., or it can be shown in fractional form by adding the extra parts to the number of parts in the units, thus: $1\frac{1}{2} = \frac{3}{2}$; $1\frac{2}{3} = \frac{5}{3}$; $2\frac{1}{4} = \frac{9}{4}$.

When the numerator of a fraction is as large or larger than the denominator, the fraction is called an "improper fraction," as $\frac{3}{3}$, $\frac{7}{6}$, $\frac{16}{12}$, etc. An improper fraction always means 1 or more than 1.

Practice Exercise:

Select the proper fractions: Select the improper fractions:

$\frac{4}{5}$; $\frac{1}{4}$; $\frac{8}{3}$; $\frac{7}{2}$; $\frac{9}{6}$; $\frac{10}{2}$; $\frac{3}{2}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{9}{5}$; $\frac{8}{3}$; $\frac{5}{2}$; $\frac{7}{12}$; $\frac{11}{15}$; $\frac{3}{4}$; $\frac{1}{10}$.

To change the form of any number or fraction without changing its value is called "reduction."

When we desire to reduce a whole number to an improper fraction we multiply the number of parts of the desired denomination that there are in one unit, by the number of units to be reduced, because there are that many parts in each unit of the whole number. As an example, to show 5 in the form of eighths, we multiply 8 by 5, because there are 8 eighths in each of the 5 units; therefore, $5 = \frac{8}{8} \times 5$ or $\frac{40}{8}$; 5 also equals $\frac{30}{6}$, $\frac{20}{4}$, $\frac{10}{2}$, etc.

Practice Exercise:

Change 2 to fourths; 8 to halves; 5 to thirds; 10 to 20ths; 3 to 6ths; 10 to 30ths; 4 to 8ths. Change to 12ths: 1; 2; 3; 5; 10.

A mixed number can be reduced to an improper fraction by first reducing the whole number which it contains, and then adding the fraction which it contains; as an example, $2\frac{2}{3} = \frac{8}{3}$ because $2 = \frac{6}{3}$ and $\frac{6}{3} + \frac{2}{3} = \frac{8}{3}$.

Practice Exercise:

Change $2\frac{1}{2}$ to halves; $1\frac{1}{3}$ to thirds; $2\frac{2}{5}$ to fifths; $3\frac{1}{3}$ to thirds; $1\frac{1}{7}$ to sevenths; $2\frac{7}{10}$ to tenths.

To reduce an improper fraction to a whole number or mixed number, we reverse the work, and divide the numerator by the denominator; thus, $\frac{40}{8} = 5$ because each 8 eighths = 1, and $40 \div 8 = 5$. In the same way, $\frac{8}{3} = 2\frac{2}{3}$ because each 3 thirds = 1, and $8 \div 3 = 2\frac{2}{3}$.

Practice Exercise:

Change to whole or mixed numbers:

$\frac{10}{2}$; $\frac{8}{3}$; $\frac{9}{5}$; $\frac{16}{3}$; $\frac{20}{5}$; $\frac{27}{3}$; $\frac{40}{4}$; $\frac{44}{2}$; $\frac{25}{5}$; $\frac{27}{10}$; $\frac{16}{3}$; $\frac{60}{3}$; $\frac{84}{3}$; $\frac{11}{7}$; $\frac{61}{3}$; $\frac{12}{11}$; $\frac{18}{5}$.

In your previous work you have always used "and" to show a remainder in division, as $\begin{array}{r} \text{2 and 2 rem.} \\ 3\overline{)8\phantom{\text{and 2 rem.}}} \end{array}$, but from now on the remainder must be changed into a fraction of the divisor excepting when the wording of the example tells us that the remainder cannot be divided.

EXAMPLE: $8 \div 3 = ?$

$$\begin{array}{r} 2\frac{2}{3} \\ 3\overline{)8} \end{array}$$

Because:

$$\begin{array}{l} \frac{1}{3} \text{ of } 6 = 2 \\ \frac{1}{3} \text{ of } 2 = \ \frac{2}{3} \\ \hline \frac{1}{3} \text{ of } 8 = 2\frac{2}{3} \end{array}$$

EXAMPLE: $\$8. \div 3 = ?$

$$\begin{array}{r} \$2\frac{2}{3}. \\ 3\overline{)\$8.} \end{array}$$

Here we are finding $\frac{1}{3}$ of $8.

EXAMPLE: $\$8. \div \$3. = ?$

$$\begin{array}{r} 2\frac{2}{3} \text{ times} \\ \$3.\overline{)\$8.\qquad} \end{array}$$

Here we find that $3. are contained in $8., $2\frac{2}{3}$ times.

EXAMPLE: How many $3. hats can be bought for $8.?

$$\begin{array}{r} 2 \text{ and } \$2. \text{ rem.} \\ \$3.\overline{)\$8.\qquad\qquad} \end{array}$$

Here we find that we can buy 2 hats, but there is a remainder of $2. as we cannot buy $\frac{2}{3}$ of a hat.

Exercise 11—Oral.

1. What is the name given to a fraction whose numerator is not so great as its denominator? Does such a fraction mean more or less than 1? Give five such fractions to write on the board.
2. What is the name given to a fraction whose numerator is as great or greater than its denominator? Does such a fraction mean more or less than 1? Give five such fractions to write.
3. $\$\frac{1}{2}$ is worth how many $\$\frac{1}{4}$? Which is worth more, $\$\frac{1}{2}$ or $\$\frac{2}{4}$?
4. Changing the form of a number or fraction without changing its value is called what?
5. How many thirds are there in 1? In 5? In 10?
6. How is a whole number reduced to a fraction?

7. How many halves are there in 1? In $1\frac{1}{2}$? In $3\frac{1}{2}$?
8. How is a mixed number reduced to a fraction?
9. How many whole dollars have you if you have $\$\frac{3}{2}$? $\$\frac{6}{2}$? $\$\frac{7}{2}$?
10. How is an improper fraction reduced to a whole or mixed number?
11. From now on, how must the remainder in an example in division be shown in most cases?
12. What number is used as the numerator when the remainder in an example in division is changed into the form of a fraction? What number is used as the denominator?
13. What is a fraction?

Exercise 12—Oral.

1. In a certain restaurant, a fourth of a pie is served for 10¢; what will be the income from 8 pies?
2. In the same restaurant, one-half of a grape-fruit is served for 10¢; how much will be the income from 7 grape-fruit?
3. How many half-pound packages can be made from 9 pounds of tea? How many quarter-pound packages?
4. A grocer emptied 24 packages of pepper into a can; if each package contained $\frac{1}{8}$ pound, how many pounds were there in all?
5. What part of a gallon is a quart? What is the cost of 1 quart of milk @ 44¢ a gallon? Of 3 quarts?
6. What part of a peck is a quart? What is the cost of 1 quart of potatoes @ 40¢ a peck? Of 7 quarts?

ADDITION

LESSON 7

Grouping Numbers

When two digits having a sum of 10 are separated by some other digit, add the other digit first, then add the group just as if there were no other digit between.

EXAMPLE:

54
64
73
46
25
85
347

Units: *Seven; teen;* twenty-seven.
Tens: Seven; fourteen; twenty-four; thirty-four.

Exercise 13—Oral.

Add, grouping wherever possible:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
36	58	87	88	92	43	94	33	71	63
48	91	56	73	82	95	84	77	48	72
27	73	92	41	39	37	73	86	39	55
62	39	54	37	28	15	26	32	72	48
46	36	73	41	67	66	37	54	88	43
74	48	37	59	42	82	43	93	42	57

(Can you do these 10 examples in 6 minutes? Some can do them in 4 minutes; see if you can.)

MOST CONVENIENT DENOMINATOR

LESSON 8

Least Common Denominator

(Some Good Things to Know)

A number is divisible by 2 *if its last digit is an even number.*

A number is divisible by 4 *if its last two digits are divisible by 4.*

A number is divisible by 8 *if its last three digits are divisible by 8.*

A number is divisible by 3 *if the sum of its digits is divisible by 3.*

A number is divisible by 6 *if it is an even number and the sum of its digits is divisible by 3.*

A number is divisible by 9 *if the sum of its digits is 9, or if the sum of the digits in the sum is 9:* thus, 1,989 is divisible by 9 because $1+9+8+9=27$ and $2+7=9$.

A number is divisible by 5 *if its last digit is 5 or 0.*

A number is divisible by 10 *if its last digit is 0.*

Exercise 14—Oral.

1. How can you tell quickly whether or not a number is divisible by 2? Is 16? Is 25? Is 17?
2. How can you tell quickly whether or not a number is divisible by 3? Is 14? Is 29? Is 33? Is 45?
3. How can you tell quickly whether or not a number is divisible by 4? Is 48? Is 24? Is 260?

4. How can you tell quickly whether or not a number is divisible by 5? Is 10? Is 25? Is 91? Is 105?

5. How can you tell quickly whether or not a number is divisible by 6? Is 16? Is 162? Is 71? Is 135?

6. How can you tell quickly whether or not a number is divisible by 8? Is 116? Is 2,116? Is 143? Is 2,144?

7. How can you tell quickly whether or not a number is divisible by 9? Is 270? Is 117? Is 18,117? Is 14,210?

8. How can you tell quickly whether or not a number is divisible by 10? Is 500? Is 206? Is 355? Is 3,550?

9. Tell quickly which of the following numbers are divisible by 2; then tell which are divisible by 3; then by 4; by 5; by 6; by 8; by 9; by 10;
(*a*) 423; (*c*) 387; (*e*) 446; (*g*) 409;
(*b*) 250; (*d*) 231; (*f*) 344; (*h*) 743.

The numbers which are contained without remainder in any other number are called "factors" of that number, thus:

$$\left.\begin{array}{lll} 2 \times & 2 \times 2 \times 3 & = 24 \\ 4 \times & 2 \times 3 & = 24 \\ 8 \times & 3 & = 24 \\ 4 \times & 6 & = 24 \\ 2 \times & 12 & = 24 \end{array}\right\} \text{Therefore, 2, 3, 4, 6, 8, and 12 are all factors of 24.}$$

Any number which contains a factor evenly is a "multiple" of the factor; thus 8, 12, 16, 20, etc., are all multiples of 4.

The smallest number which contains several factors evenly is called the "least common multiple" of the several factors, and when the least common multiple (L. C. M.) is used as a denominator for several fractions, it is called the "least common denominator" or "most convenient denominator."

To find the least common denominator (L. C. D.) or most convenient denominator, think in the multiplication table of the largest factor and find the smallest number in that table which contains all of the factors evenly.

EXAMPLE: What number will hold 3, 4, and 6 evenly?

Here the largest factor is 6; therefore, think in the Multiplication Table of 6's; 12 is the smallest number in the Table of 6's which will hold 3, 4, and 6 without remainder; therefore, 12 is the L. C. D. for 3, 4, and 6.

Exercise 15—Oral.

Think in the multiplication table of the largest factor:

1. What is the smallest number in that table which will hold 2 and 3 evenly?
2. What is the smallest number in that table which will hold 3 and 4 evenly?
3. What is the smallest number in that table which will hold 4 and 8 evenly?
4. What is the smallest number in that table which will hold 3 and 2 evenly?
5. What is the smallest number in that table which will hold 8 and 4 evenly?
6. What is the smallest number in that table which will hold 2, 3 and 4 evenly?
7. What is the smallest number in that table which will hold 2, 3, 4 and 6 evenly?

8. What is the smallest number in that table which will hold 2, 3, 4, 6 and 12 evenly?
9. What is the smallest number in that table which will hold 3, 4 and 6 evenly?
10. What is the smallest number in that table which will hold 6 and 4 evenly?
11. What is the smallest number in that table which will hold 5, 2 and 10 evenly?
12. What is the smallest number in that table which will hold 6, 4, 2 and 12 evenly?
13. What is the smallest number in that table which will hold 3 and 8 evenly?
14. What is the smallest number in that table which will hold 3, 8, 12 and 4 evenly?
15. What is the smallest number in that table which will hold 3, 8, 12 and 24 evenly?
16. What is the smallest number in that table which will hold 3, 6 and 9 evenly?
17. What is the smallest number in that table which will hold 5 and 4 evenly?
18. What is the smallest number in that table which will hold 2, 5 and 4 evenly?
19. What is the smallest number in that table which will hold 2, 5, 4 and 10 evenly?
20. What is the smallest number in that table which will hold 8 and 5 evenly?

When a number contains no factors other than itself and 1, the number is said to be a "prime number."

The prime numbers to 25 are: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23. Do you think you can remember them?

MOST CONVENIENT DENOMINATOR

Exercise 16—Written.

1. If 3 is one of the factors of 12, what is the other factor?
2. If 12 is one of the factors of 72, what is the other factor?
3. What are two factors of 20? What are two other factors of 20?
4. Name 2 factors for 27.
5. Name 3 factors for 27.
6. Name 2 factors for 64.
7. Name 3 factors for 64.
8. Name 2 factors for 120.
9. Name 2 other factors for 120.
10. Name 3 factors for 120.

Exercise 17—Oral Review.

1. What is the cost of $4\frac{2}{3}$ doz. rolls @ 12¢ a dozen?
2. What part of a foot is 1 inch? What part of a yard is 1 inch?
3. How many eighths are there in 6? In 10? In 12?
4. Find $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{5}{6}$ of $\frac{6}{7}$; $\frac{3}{4}$ of $\frac{1}{2}$.
5. $\frac{3}{4} + \frac{1}{4} = ?$ $\frac{2}{7} + 1\frac{3}{7} = ?$ $5\frac{1}{9} + 3\frac{6}{9} = ?$
6. How can you tell quickly whether or not a number is divisible by 2? By 3? By 4? By 5? By 6? By 8? By 9? By 10?
7. What numbers not higher than 10 will divide without remainder:

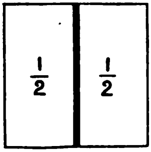
   (*a*) 320; (*b*) 468; (*c*) 597;
8. $7\frac{4}{7} - 2\frac{3}{7} = ?$ $6\frac{2}{3} - \frac{1}{3} = ?$ $8\frac{1}{3} - 7 = ?$
9. $\frac{42}{6} = ?$ $\frac{56}{7} = ?$ $\frac{25}{5} = ?$
10. $6\frac{2}{3} = \frac{?}{3}$; $9\frac{1}{6} = \frac{?}{6}$; $10\frac{1}{4} = \frac{?}{4}$.

FRACTIONS

Lesson 9

Reduction

NO. 1.



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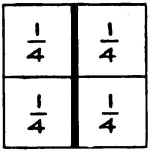

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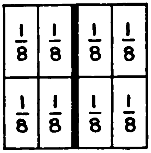

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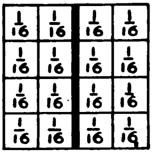


Looking at these four squares you will notice that each of them is divided into two halves by a heavy line, and that No. 2 is divided into $\frac{4}{4}$, No. 3 into $\frac{8}{8}$, and No. 4 into $\frac{16}{16}$. Now by comparing the squares, you will find that $\frac{1}{2}$ of No. 1 is the same size as $\frac{2}{4}$ of No. 2, or $\frac{4}{8}$ of No. 3, or $\frac{8}{16}$ of No. 4. From this you see that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ are all alike in actual value, but are merely different forms in which this value can be expressed.

Notice how the form of the fraction changes, but the value remains $\frac{1}{2}$ in the following:

Each $\frac{1}{2}$ cut in two. Each $\frac{1}{4}$ cut in two. Each $\frac{1}{8}$ cut in two.

$$\frac{1\,(\times 2)}{2\,(\times 2)} = \frac{2}{4}; \qquad \frac{2\,(\times 2)}{4\,(\times 2)} = \frac{4}{8}; \qquad \frac{4\,(\times 2)}{8\,(\times 2)} = \frac{8}{16};$$

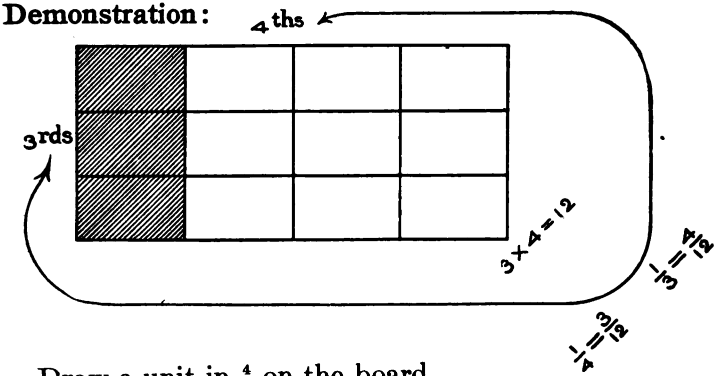
Two 16ths joined. Two 8ths joined. Two 4ths joined.

$$\frac{8\,(\div 2)}{16\,(\div 2)} = \frac{4}{8}; \qquad \frac{4\,(\div 2)}{8\,(\div 2)} = \frac{2}{4}; \qquad \frac{2\,(\div 2)}{4\,(\div 2)} = \frac{1}{2};$$

We learn from this that the value of a fraction is not changed when both terms are multiplied by the same number; neither is its value changed when both terms are divided by the same number.

Therefore, to reduce a fraction to higher or lower terms, it is only necessary to find how many times larger or smaller is the new denominator than the old, then raise or lower the numerator as many times. The new fraction must have the same value as the old one.

Demonstration:



Draw a unit in $\frac{4}{4}$ on the board.

Shade $\frac{1}{4}$ lightly. Into how many parts must the 4th be cut to make 12ths?

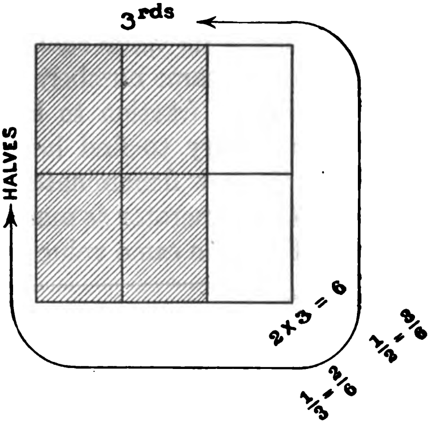
Draw lines to change the whole unit into 12ths.

$$\frac{1}{4} = \frac{?}{12}$$

EXAMPLE: $\frac{1}{4} = \frac{?}{12}$.

Worked: The new denominator (12) is 3 times as large as the old denominator (4); therefore, the new numerator must be 3 times as large as the old numerator (1), and $1 \times 3 = 3$; therefore, $\frac{1}{4} = \frac{3}{12}$.

Demonstration:



Draw a unit in $\frac{6}{6}$ on the board.
Shade $\frac{4}{6}$ lightly.
Put your hand on $\frac{1}{3}$ of the unit.
How many 6ths are there in $\frac{1}{3}$? In $\frac{2}{3}$?

$$\frac{4}{6} = \frac{?}{3}$$

EXAMPLE: $\frac{4}{6} = \frac{?}{3}$.

Worked: The new denominator (3) is $\frac{1}{2}$ as large as the old denominator (6); therefore, the new numerator must be $\frac{1}{2}$ as large as the old numerator (4), and $\frac{1}{2}$ of 4 = 2; therefore, $\frac{4}{6} = \frac{2}{3}$.

When a fraction is in its simplest form, it is said to be reduced to its "lowest terms." A fraction is in its lowest terms when its numerator and its denominator cannot be further divided by the same factor.

FRACTIONS

Exercise 18—Oral.

1. $\frac{1}{2} = \frac{?}{4}; \frac{?}{6}; \frac{?}{8}; \frac{?}{10}; \frac{?}{12}; \frac{?}{14}; \frac{?}{16}; \frac{?}{100}; \frac{?}{200}$.
2. $\frac{2}{3} = \frac{?}{6}; \frac{?}{9}; \frac{?}{12}; \frac{?}{15}; \frac{?}{18}; \frac{?}{30}; \frac{?}{300}$.
3. $\frac{3}{4} = \frac{?}{8}; \frac{?}{12}; \frac{?}{16}; \frac{?}{20}; \frac{?}{24}$.
4. $\frac{7}{8} = \frac{?}{16}; \frac{?}{24}; \frac{?}{32}; \frac{?}{40}$.
5. $\frac{12}{24} = \frac{?}{12}; \frac{?}{8}; \frac{?}{6}; \frac{?}{4}; \frac{?}{2}$.
6. $\frac{10}{16} = \frac{?}{8}; \frac{9}{15} = \frac{?}{5}$.
7. Reduce to lowest terms:

   $\frac{12}{16}; \frac{32}{36}; \frac{24}{32}; 4\frac{16}{20}; \frac{24}{16}; \frac{30}{40}; \frac{12}{36}; \frac{9}{90}$.
8. When is a fraction in its lowest terms?
9. How do we reduce a fraction to higher terms?
10. How do we reduce a fraction to lower terms?

Exercise 19—Written.

1. $14 = \frac{?}{8}$.
2. $\frac{11}{15} = \frac{?}{30}$.
3. $\frac{25}{45} = \frac{?}{9}$.
4. $10\frac{7}{8} = \frac{?}{8}$.
5. $\frac{186}{12} = ?$ units.
6. $3\frac{3}{4} = \frac{?}{4}; \frac{?}{8}$.
7. $8\frac{6}{8} = \frac{?}{8}$. How many 4ths?
8. $\frac{20}{3} = 6\frac{?}{3}$. How many 6ths?
9. $\frac{90}{12} = 7\frac{?}{12}$.
10. $\frac{16}{3} = ?$

Exercise 20—Written.

1. What is the cost of $\frac{3}{4}$ of a ton of coal when a ton costs $6.80?
2. How many pounds are there in $\frac{7}{8}$ of a ton (1 ton = 2,000 lb.)?

3. What part of a hundredweight is 75 pounds, expressed in a fraction of lowest terms?
4. What part of a day is 18 hours, expressed in a fraction of lowest terms?
5. What part of an hour is 40 minutes, expressed in a fraction of lowest terms?
6. What part of a square foot is 84 sq. in., expressed in a fraction of lowest terms?
7. At 24¢ for a half pound, what is the cost of 7 ounces of butter?
8. At 24¢ for a half dozen, what is the cost of 2 dozen of eggs?
9. What is the cost of a yard and a half of calico when half a yard costs 5¢? How many inches are there in a yard and a half?
10. 72 sq. in. is what part of 1 sq. ft.? 16 sq. in. is what part of 1 sq. ft.? 288 sq. in. = ? sq. ft.
11. What is the cost of 288 sq. in. of carpet when a square foot costs 40¢?

LESSON 10

Addition and Subtraction of Fractions with Unlike Denominators or Names

Just as it is impossible to add or subtract unlike things, so it is impossible to add or subtract unlike fractions, that is, fractions with unlike denominators; therefore, before unlike fractions can be added or subtracted, they must be made into like fractions, that is, fractions with like denominators.

To reduce two or more unlike fractions to like fractions, we find the most convenient denominator, and use this as the denominator of each of the new fractions, reducing each of the numerators of the unlike fractions as necessary.

This new denominator is called the "most convenient denominator" or "least common denominator" (L. C. D.) of the several fractions, because it is the smallest denominator to which all of the fractions can be reduced.

When the several fractions have been reduced to the L. C. D., we merely add or subtract the numerators to find the numerator of the answer, and the L. C. D. or name is also the denominator or name of the answer because the name does not change in adding or subtracting.

EXAMPLE: $\frac{4}{5} - \frac{2}{3} = ?$

Unlike Like

$\frac{4}{5} = \frac{12}{15}$

$\frac{2}{3} = \frac{10}{15}$ All 15ths.

$\frac{2}{15}$

Proof: $\frac{10}{15} + \frac{2}{15} = \frac{12}{15}$

EXAMPLE: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = ?$

First find the L. C. D. for 2, 4, and 8, which is 8; therefore, each of these fractions must be reduced to eighths before they can be added.

Unlike Like

$\frac{1}{2} = \frac{4}{8}$

$\frac{1}{4} = \frac{2}{8}$ All 8ths.

$\frac{1}{8} = \frac{1}{8}$

Total $\frac{7}{8}$

To prove, subtract the sum of all the addends but one, from the entire sum; the remainder will be the missing addend if the work is correct.

EXAMPLE:

$\frac{1}{2} = \frac{4}{8}$

$\frac{1}{4} = \frac{2}{8}$

$\frac{1}{8} = \frac{1}{8}$

$\frac{7}{8}$ Entire sum.

$\frac{6}{8}$ Sum of two addends.

Proof: $\frac{1}{8}$ Missing addend.

If the answer is an improper fraction it must be reduced to a whole or mixed number of lowest terms, and if it is a proper fraction which can be reduced, it must be reduced to lowest terms.

EXAMPLE: $\frac{2}{3} + \frac{1}{4} + \frac{1}{2} = ?$

First find the L. C. D. for 3, 4, and 2, which is 12; therefore, each of the fractions must be reduced to twelfths before they can be added.

Unlike	Like	
$\frac{2}{3} =$	$\frac{8}{12}$	
$\frac{1}{4} =$	$\frac{3}{12}$	All 12ths.
$\frac{1}{2} =$	$\frac{6}{12}$	
Total	$\frac{17}{12}$	$= 1\frac{5}{12}$

Exercise 21—Written.

Show each sum before and after reducing, and prove:

1. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = ?$
2. $\frac{1}{6} + \frac{2}{3} + \frac{4}{9} = ?$
3. $\frac{1}{2} + \frac{1}{4} + \frac{5}{8} = ?$
4. $\frac{7}{8} + \frac{2}{3} + \frac{3}{4} + \frac{1}{2} = ?$
5. $\frac{7}{10} + \frac{1}{2} + \frac{3}{5} = ?$
6. $\frac{1}{4} + \frac{1}{5} + \frac{9}{10} = ?$
7. $\frac{1}{8} + \frac{1}{4} + \frac{2}{3} + \frac{5}{24} = ?$
8. $\frac{7}{8} + \frac{3}{16} + \frac{1}{4} = ?$
9. $\frac{5}{8} + \frac{7}{16} + \frac{1}{2} = ?$
10. $\frac{1}{10} + \frac{3}{5} + \frac{1}{2} + \frac{1}{4} = ?$
11. $\frac{5}{6} + \frac{1}{3} + \frac{3}{9} = ?$
12. $\frac{3}{6} + \frac{1}{9} + \frac{1}{12} = ?$

Exercise 22—Written.

Subtract, showing each difference before and after reducing, and prove by adding the difference to the subtrahend:

1. $\frac{1}{8} - \frac{1}{16} = ?$
2. $\frac{2}{3} - \frac{3}{8} = ?$
3. $\frac{4}{5} - \frac{3}{10} = ?$
4. $\frac{5}{8} - \frac{5}{12} = ?$
5. $\frac{19}{16} - \frac{5}{8} = ?$
6. $\frac{4}{9} - \frac{3}{10} = ?$
7. $\frac{19}{20} - \frac{1}{5} = ?$
8. $\frac{17}{24} - \frac{5}{8} = ?$
9. $\frac{2}{3} - \frac{1}{4} = ?$
10. $\frac{7}{8} - \frac{2}{3} = ?$
11. $\frac{7}{10} - \frac{2}{5} = ?$
12. $\frac{12}{30} - \frac{3}{10} = ?$
13. $\frac{7}{8} - \frac{1}{24} = ?$
14. $\frac{6}{7} - \frac{15}{21} = ?$

LESSON 11

Addition and Subtraction of Fractions, Whole and Mixed Numbers

EXAMPLE: $46\frac{7}{8} + \frac{3}{4} = ?$

$$\begin{array}{rl} 46\frac{7}{8} = & 46\frac{7}{8} \\ \frac{3}{4} = & \underline{\frac{6}{8}} \\ & 46\frac{13}{8} = 47\frac{5}{8} \end{array}$$

EXAMPLE: $39\frac{5}{6} - 22\frac{7}{9} = ?$

$$\begin{array}{rl} 39\frac{5}{6} = & 39\frac{15}{18} \\ 22\frac{7}{9} = & \underline{22\frac{14}{18}} \\ & 17\frac{1}{18} \end{array}$$

When adding or subtracting fractions and mixed numbers, the fractions must be reduced to L. C. D. just as when adding or subtracting fractions and fractions.

EXAMPLE: $13 - 3\frac{1}{2} = ?$

$$\begin{array}{r} \overset{12\frac{2}{2}}{\cancel{13}} \\ \underline{3\frac{1}{2}} \\ 9\frac{1}{2} \end{array}$$

(1 unit of 13 is changed into $\frac{2}{2}$ making $12\frac{2}{2}$.)

EXAMPLE: $16 - \frac{11}{16} = ?$

$$\begin{array}{r} \overset{15\frac{16}{16}}{\cancel{16}} \\ \underline{\frac{11}{16}} \\ 15\frac{5}{16} \end{array}$$

(1 unit of 16 is changed into $\frac{16}{16}$ making $15\frac{16}{16}$.)

When subtracting fractions from whole numbers, 1 unit of the whole number must be changed into the form of a fraction, just as we change 1 ten into 10 units when units' place of the minuend contains a smaller number than units' place of the subtrahend in ordinary subtraction.

EXAMPLE: $12\frac{1}{4} - \frac{3}{4} = ?$

$$\begin{array}{r} \overset{11\frac{5}{4}}{\cancel{12\frac{1}{4}}} \\ \underline{\frac{3}{4}} \\ 11\frac{2}{4} = 11\frac{1}{2} \end{array}$$

(1 unit of 12 is changed into $\frac{4}{4}$ which are added to the $\frac{1}{4}$ making $11\frac{5}{4}$.)

EXAMPLE: $18\frac{5}{6} - 13\frac{11}{12} = ?$

$$\begin{array}{rr} 18\frac{5}{6} = & \overset{17\frac{22}{12}}{\cancel{18\frac{10}{12}}} \\ & \underline{13\frac{11}{12}} \\ & 4\frac{11}{12} \end{array}$$

(1 unit of 18 is changed into $\frac{12}{12}$ and these are added to the $\frac{10}{12}$, making $17\frac{22}{12}$.)

When subtracting fractions from mixed numbers, if the fraction which is part of the mixed number is smaller than the fraction to be subtracted, 1 unit of the mixed number must be reduced to a fraction and this must be added to the fraction which is already part of the mixed number; the subtraction can then be done.

Exercise 23—Oral.

1. If $\frac{3}{4}$ pound of salt is removed from a sack which contained $1\frac{1}{4}$ pounds, how much salt will remain in the sack?
2. If $\frac{7}{8}$ gallon of kerosene is removed from a can which contained $1\frac{3}{8}$ gallons, how much kerosene will remain in the can? (Reduce your answer to lowest terms.)

3. If a string $1\frac{1}{2}$ yd. long is fastened to the end of another string $\frac{5}{8}$ yd. long, what is the length of the string then?

4. If a pole $8\frac{1}{4}$ ft. long is broken into two pieces, and one of the pieces is $2\frac{3}{8}$ ft. long, how long is the other piece?

5. If it is $2\frac{3}{4}$ hr. after noon, how many hours is it to midnight?

6. If you received a box of candy for a present, and gave away $\frac{1}{4}$ of it and ate $\frac{1}{8}$ of it, what part was removed? What part remained?

7. If $\frac{1}{12}$ of the pupils in a class are marked "Fair" in their studies, $\frac{1}{6}$ are marked "Good" and the balance are marked "Excellent," what part of the class is marked "Excellent"? (Reduce your answer to lowest terms.)

8. $2 - \frac{1}{3} = ?$ $4 - \frac{1}{2} = ?$ $6 - \frac{1}{2} = ?$ $7 - \frac{3}{4} = ?$

9. Of the chairs in a room, $\frac{1}{4}$ are made of oak and the balance are made of birch; how many chairs are there in all if there are 6 birch chairs?

10. Of the pencils in a certain box, $\frac{3}{8}$ are long and the balance are short; how many pencils are there in this box if there are 15 short pencils?

11. $5 - \frac{1}{4} = ?$ $6 - \frac{2}{3} = ?$ $7 - \frac{1}{5} = ?$ $10 - \frac{3}{8} = ?$
$12 - \frac{1}{12} = ?$ $4 - \frac{7}{10} = ?$ $8 - \frac{7}{9} = ?$

Exercise 24—Written.

1. $38\frac{11}{16} + \frac{7}{12} = ?$

2. $27\frac{4}{7} + 41\frac{5}{14} = ?$

3. $32\frac{7}{11} - \frac{2}{9} = ?$

4. $60\frac{5}{16} - 18\frac{1}{8} = ?$

5. $86 - 42\frac{2}{9} = ?$

6. $100 - 66\frac{2}{3} = ?$

7. $97\frac{1}{3} - 52\frac{2}{3} = ?$

8. $39\frac{1}{6} - 18\frac{2}{3} = ?$

9. $79\frac{1}{10} - 13\frac{1}{5} = ?$

10. $44\frac{1}{7} - 16\frac{1}{2} = ?$

BRACKETS

Lesson 12

The Use of Brackets

EXAMPLE: 8 × (4 − 2) = ?

This means that 8 is to be multiplied by the difference between 4 and 2, hence we must first find that difference; 4 − 2 = 2; therefore, 8 ×(4 − 2) is the same as 8 × 2 = 16. (If there were no brackets in this example it would read: 8 × 4 − 2 = 30 which is an entirely different answer.)

When two or more numbers are to be treated in an example as though they were a single number, we place them in brackets. Such numbers must be worked first. Thus: (6 × 5) − 2 means 30 − 2.

EXAMPLE: (4 + 6) − (3 + 2) = ?

4 + 6 = 10; 3 + 2 = 5; 10 − 5 = 5, Ans.

EXAMPLE: 3 +(8 − 7) = ?

3 +(8 − 7) = 3 + 1; 3 + 1 = 4, Ans.

Exercise 25—Oral.

1. 6 + (4 ÷ 2) = ?
2. 5 × (6 ÷ 2) = ?
3. 9 ÷ (5 − 2) = ?
4. 8 − (2 × 3) = ?
5. (6 − 3) × (6 + 3) = ?
6. (8 + 2) ÷ (8 − 3) = ?

7. (4 × 2) − (3 × 2) = ?
8. (9 ÷ 3) + (8 ÷ 2) = ?
9. (4 + 2) − (3 − 1) = ?
10. (7 × 3) ÷ (1 + 1 + 1) = ?

Exercise 26—Written.

1. 46 × (18 ÷ 3) = ?
2. (28 × 3) − (17 × 4) = ?
3. (75 ÷ 25) × (46 + 4) ÷ 50 = ?
4. 12 × (9 ÷ 3) = ?
5. 20 + 8 − (3 × 2) = ?
6. (412 − 316) ÷ (24 × 2) = ?
7. [96 ÷ (12 × 4)] + [84 ÷ (6 × 7)] = ?
8. 140 − (32 − 4) = ?
9. 40 − 32 − 4 = ?
10. (40 + 32) ÷ (4 × 9) = ?

(Practice till you can do the next 6 examples in 4 minutes.)

11. $\begin{array}{r} 16{,}122 \\ -8{,}165 \\ \hline \end{array}$

12. $\begin{array}{r} 91{,}623 \\ -89{,}764 \\ \hline \end{array}$

13. $\begin{array}{r} 4{,}053 \\ -2{,}869 \\ \hline \end{array}$

14. $\begin{array}{r} 34{,}586 \\ -8{,}692 \\ \hline \end{array}$

15. $\begin{array}{r} 6{,}894 \\ -1{,}823 \\ \hline \end{array}$

16. $\begin{array}{r} 98{,}767 \\ -18{,}288 \\ \hline \end{array}$

MENSURATION

LESSON 13

Lines, Surfaces, and Solids Compared

"Mensuration" means "Taking the measurements of anything."

In your previous work you have learned that to measure "distance" we always measure a straight line between two objects or points; that a straight line has only "one dimension," namely—"length," and that "length" is measured by using "Linear Measure" which is a measure of "one dimension."

You have also learned that the "area" of a surface has "two dimensions," namely—"length and width," and that we measure the area of a surface by using "Square Measure," which is a measure of "two dimensions."

Now, supposing you were asked to measure a block of wood. You could measure the length of the block, you could measure the width of the block, and you could also measure the thickness of the block; so, you see, every "solid" object has "three dimensions"—"length, width (or breadth) and thickness (or depth)," and the space which is occupied by the solid is called its "volume" or "cubic contents."

To measure the "cubic contents" or "volume" of a solid we use "Cubic Measure" which is a measure of "three dimensions," because the solid has "three dimensions."

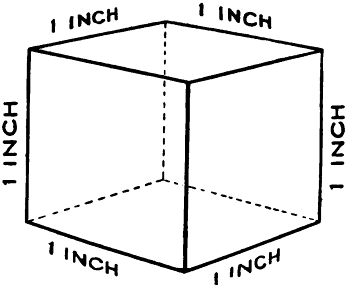
Exercise 27—Oral.

1. Place 2 dots on the board. Draw a straight line joining them. How many dimensions has a straight line? Name?
2. What table of measures do we use to measure short straight lines? Long straight lines? All straight lines?
3. What is meant by mensuration?
4. What is the space within the boundaries of a surface called? Show this at the front board. Look at the window; do you see area?
5. How many dimensions has a surface? Name them? Point them out on the door?
6. What table of measures do we use to measure area?
7. How many dimensions has a solid? Name them?
8. What name is given to the space occupied by a solid?
9. What table of measures is used to measure the volume of a solid?
10. Point out two things in this room that can be measured by Linear Measure. Point out two that can be measured by Square Measure. Point out two that can be measured by Cubic Measure.

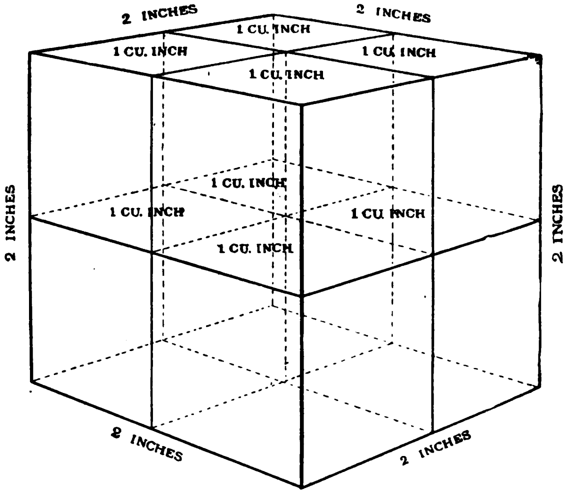
LESSON 14

The Cube and the Right Prism

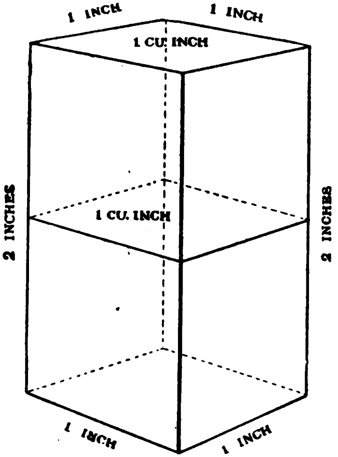
A "cube" is a solid having six square sides or "faces" of equal size, joined so that every angle is a right angle.



A One-inch Cube
Containing
1 Cubic Inch



A Two-inch Cube
Containing
8 Cubic Inches



A Right Prism
Containing
2 Cubic Inches

A "right prism" which means "right-angled prism" is a solid having four oblong sides and two square or oblong ends, joined so that every angle is a right angle. If you place one cube on top of another, you will have a right prism.

You will notice that the right prism in the picture contains 2 cubic inches because it is twice as high as the one-inch cube.

The large cube contains 8 cubic inches because it is not only 2 inches high, but also 2 inches deep and 2 inches wide. It therefore has 4 cubic inches in each layer, and there are two layers. Its volume is four times as great as that of the right prism and eight times as great as that of the small one-inch cube.

From this you see that when we multiply the number of cubic inches in one row along the length, by the number of rows in the width and by the number of layers in the thickness, we find the number of "cubic inches" in the volume; therefore,

1″ long, 1″ wide, 1″ thick = 1 cubic inch × 1 × 1 or 1 cubic inch.
2″ long, 1″ wide, 1″ thick = 2 cubic inches × 1 × 1 or 2 cubic inches.
2″ long, 2″ wide, 2″ thick = 2 cubic inches × 2 × 2 or 8 cubic inches.

When the measurements are in feet, the multiplication will give the volume in "cubic feet." Measurements in yards will give "cubic yards," and so on.

Because squares and oblongs are rectangles, cubes and right prisms are "rectangular solids."

Exercise 28—Oral.

1. What is the name of any solid which has six square sides or faces and all of its angles are right angles?
2. What is the name of any solid which has 4 oblong sides and 2 square or oblong ends, and all of its angles are right angles?
3. Why are cubes and right prisms called rectangular solids? What kind of a solid is a chalk box?
4. How many dimensions have rectangular solids? Name them.
5. How do we find the volume of rectangular solids?
6. What is the volume of a cube if the area of each of its sides is 1 sq. in.?

7. Read these dimensions: 1″ × 1″ × 12″. What does each of these dimensions mean?

8. In Question 7, what is the name of the solid? How many cubic inches does it contain?

9. Read these dimensions: 3″ × 3″ × 3″. What does each of these dimensions mean?

10. What is the name of the solid in Question 9? How many cubic inches are there in 1 row? How many rows are there? How many cubic inches in 1 layer? How many layers? How many cubic inches in all?

11. When the dimensions of a solid are given in inches, the volume is of what name? When the dimensions are given in feet? In yards?

12. Is an empty box measured in the same way as a solid? Why?

Exercise 29—Oral.

(A) What kind of a rectangular solid is each of the following?

(B) How many cubic units are there in 1 row along the length of each?

(C) How many rows are there in the width of each?

(D) How many cubic units are there in 1 layer of each?

(E) How many layers are there in each solid?

(F) What is the volume of each solid?

1. 1″ × 1″ × 1″;
2. 1″ × 2″ × 2″;
3. 2″ × 2″ × 2″;
4. 2″ × 2″ × 3″;
5. 1′ × 1′ × 2′;
6. 1′ × 1′ × 3′;
7. 1′ × 2′ × 3′;
8. 2′ × 3′ × 3′;

9. 3′ × 3′ × 3′;
10. 10′ × 10′ × 10′;
11. 4′ × 5′ × 10′;
12. 3 yd. × 4 yd. × 5 yd.;
13. 2 yd. × 5 yd. × 10 yd.;
14. 2 yd. × 6 yd. × 8 yd.

Lesson 15

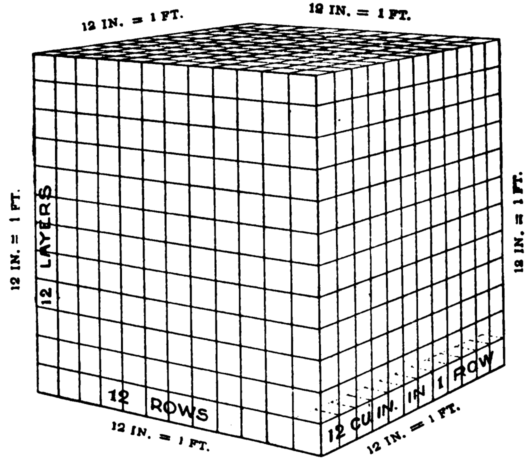
Cubic Measure

Cubic Measure is used for measuring the contents of bins, cars, elevators, cellars, and the volume of solids such as wood and stone.

1,728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord

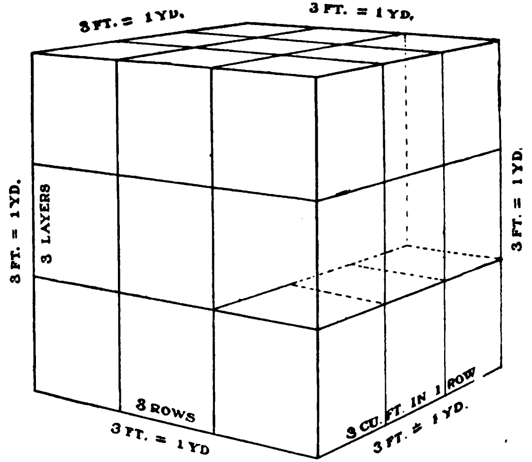
1,728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
128 cu. ft. = 1 cd.

Note: The cord is used for measuring cut wood and stone.



1,728 cu. in. = 1 cu. ft.

MENSURATION



27 cu. ft. = 1 cu. yd.

Exercise 30—Oral.

1. A cubic foot is a cube 1′ long, 1′ wide, 1′ thick; or 12″ long, 12″ wide, 12″ thick. Why are there 1,728 cu. in. in 1 cu. ft.? Can you see 1 row? How many rows are there? Can you see 1 layer? How many layers are there?
2. Why are there 27 cu. ft. in 1 cu. yd.?
3. How many cu. in. are there in 10 cu. ft.?
4. How many cu. ft. are there in 10 cu. yd.? How many in 1 cord of wood?
5. Say the table of Cubic Measure.
6. If it takes a man 1 hour to lay 1 cu. ft. of brick, how long will it take him to lay 1 cu. yd.?
7. If 1 cu. in. of metal weighs 1 ounce, how many ounces will 1 cu. ft. weigh?

8. If a special kind of glued lumber is worth $1.00 per cu. ft., what will be the cost of 1 cu. yd.?
9. Number in length × number in width = ?
10. Number in length × number in width × number in thickness = ?

Lesson 16

How to Find the Third Dimension when the Volume and Two Dimensions are Given

Since you multiplied the number of cubic units in 1 layer by the number of layers to find the number of cubic units in the volume, you will, of course, find the number of layers when you divide the number of cubic units in the volume by the number of cubic units in 1 layer.

EXAMPLE:
Volume = 24 cu. in.;
1 Layer = 6 cu. in.;
Therefore,
Number of Layers = 24 ÷ 6, or 4 Layers

As the "1 layer" may be along the bottom, the front, or the end of the solid, dividing the number in the volume by the number in 1 layer of any kind will tell you the number of layers of *that* kind there are in the solid; therefore,

$$\left.\begin{array}{c}\text{Number}\\ \text{in}\\ \text{volume}\end{array}\right\} \div \left\{\begin{array}{c}\text{Number in 1 layer}\\ \text{along the bottom}\\ \text{or}\\ \text{Number in length} \times\\ \text{number in width}\end{array}\right\} = \left\{\begin{array}{c}\text{Number of}\\ \text{layers in}\\ \text{thickness.}\end{array}\right.$$

$$\left.\begin{array}{c}\text{Number}\\ \text{in}\\ \text{volume}\end{array}\right\} \div \left\{\begin{array}{c}\text{Number in 1 layer}\\ \text{along the front}\\ \text{or}\\ \text{Number in length} \times\\ \text{number in thickness}\end{array}\right\} = \left\{\begin{array}{c}\text{Number of}\\ \text{layers in}\\ \text{width.}\end{array}\right.$$

$$\left.\begin{array}{c}\text{Number}\\ \text{in}\\ \text{volume}\end{array}\right\} \div \left\{\begin{array}{c}\text{Number in 1 layer}\\ \text{along the end}\\ \text{or}\\ \text{Number in width} \times\\ \text{number in thickness}\end{array}\right\} = \left\{\begin{array}{c}\text{Number of}\\ \text{layers in}\\ \text{length.}\end{array}\right.$$

EXAMPLE: What is the length of a solid which is 2″ wide, 2″ thick, and which has a volume of 12 cu in.?

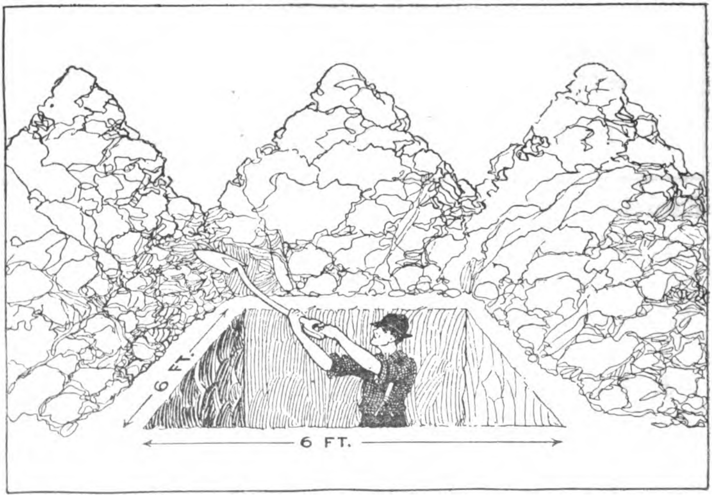
2 (Number in width)
× 2 (Number in thickness)
= 4 (Number in 1 layer along the end)
(Number 3 (Number of layers in length)
in 1 layer along the end) $4\overline{)12}$ (Number in volume)
Therefore, Length = 3″, Ans.
Proof: 3 × 2 × 2 = 12 Number of cu. in. in volume.

Exercise 31—Oral.

1. A box is 3″ long, 2″ wide, and contains 18 cu. in.; what is the depth of the box?
2. A trunk is 3′ wide, 3′ deep, and contains 45 cu. ft.; what is the length of the trunk?
3. A board is 1″ thick, 12″ long, and contains 144 cu. in.; what is the width of the board?
4. A solid is 3′ long, 3′ wide, and contains 27 cu. ft.; what is the thickness of this solid? How many cu. yd. does it contain? Name this solid?
5. A stick is 1″ wide, 1″ thick, and contains 39 cu. in.; how long is the stick?

6. A room is 10′ high, 20′ long, and contains 2,000 cu. ft.; how wide is the room?
7. What is the length of a brick 2″ thick, 4″ wide, containing 64 cu. in.? Name this solid.
8. What is the length of a board 1″ thick, 8″ wide, containing 96 cu. in.?
9. What is the depth of a box 1 yd. long, 1 yd. wide, containing 27 cu. ft.? Is this a right prism or a cube?
10. A block of glued lumber worth $1.00 per cu. ft. cost $3.00; how long is the block if it is 1′ wide and 1′ thick?

Exercise 32—Written.



1. How deep is a hole 6′ × 6′ if the ground removed in digging it measured 720 cu. ft.? Note: How many cu. ft. are there in 1 layer?

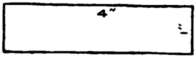
2. How many cu. in. are there in 8 cu. ft.?
3. How many cu. ft. are there in 27,648 cu. in.?
4. How many cu. ft. are there in a 4′ cube?
5. If a pillar 1′ × 1′ × 20′ is placed in a hole 1′ × 1′ × 10′, how many cu. ft. of the pillar will be inside of the hole? How many cu. ft. will be outside?
6. If a package 12″ × 12″ × 12″ is placed in a packing case 3′ × 3′ × 3′, how many cu. ft. of the packing case will remain unfilled? How many more packages of the same size could be placed in this case?
7. The packing case referred to in Question 6 contains how many cu. yd.? How many cu. ft.? Can you change that to cu. in.? How?
8. How many cu. ft. of brick are needed to build a wall 1′ thick, 18′ wide, and 36′ high? How many cu. yd. of brick are needed?
9. How many bricks 2″ × 4″ × 8″ are there in a cu. ft. of brick?
10. How many bricks will be needed to build a wall 1′ thick, 30′ long, and 54′ high, if there are 27 bricks in each cu. ft.? How many cu. yd. of brick will be needed?

LESSON 17

Drawing to Scale

When the drawing of an object is as large as the object itself, the drawing is said to be "actual size" or "full size," but when the drawing of an object is smaller or larger than the object itself, the relation of the size of the drawing to the size of the object is the "scale" on which the drawing is made; thus, if a square having sides 12″ long is represented by a drawing having sides 1″ long, the drawing is made on a scale of 1 to 12 or $\frac{1}{12}$ of the actual size. On this scale, 2 inches in the size of the drawing would represent 2 feet in the size of the object; 6 inches would represent 6 feet, and so on.

In marking dimensions on a drawing, the dimensions of the object and not the dimensions of the drawing are used; thus, an oblong 4″ × 1″ drawn to the scale of 1 to 4 or $\frac{1}{4}$ is marked in this manner:



(Scale $\frac{1}{4}$)

Exercise 33—Oral.

1. Could we draw a figure of the blackboard in actual size? Why not? Could we draw it to a scale?
2. What do you understand by the words "scale 1 to 6" when used in describing a drawing?
3. What do you understand by the words "scale $\frac{1}{2}$" when used in describing a drawing?
4. In a certain drawing, the scale is 1 to 2, or $\frac{1}{2}$; what does 1″ in such a drawing represent? 6″? 1′?
5. In a certain drawing, the scale is 1 to 4, or $\frac{1}{4}$; what does 2″ in such a drawing represent? 5″? 7″?

6. In a certain drawing, the scale is $\frac{1}{12}$; what does 1″ in such a drawing represent? 12″? 1′?
7. In marking the dimensions on a drawing, are the dimensions of the object or of the drawing used?
8. In a certain drawing, 1 inch represents 1 yard; on what scale is this drawing made?
9. In a certain drawing, $\frac{1}{4}$″ represents 1′; on what scale is this drawing made?
10. In a certain drawing, $\frac{1}{8}$″ represents 1′; on what scale is this drawing made?

Exercise 34—Written.

1. Make a drawing of an oblong 1″ × 2″ actual size, and mark the dimensions. Give the area.
2. Make a drawing of a 2″ × 2″ square on a scale of $\frac{1}{2}$, and mark the dimensions. Give the area.
3. Draw a rectangle 1′ × 3′ on a scale of $\frac{1}{12}$, and mark the dimensions. Give the area.
4. Draw a rectangle 2″ × 2″ on a scale of 4 to 1, and mark the dimensions. Give the area.
5. Draw a rectangle 12′ × 18′ on a scale of $\frac{1}{8}$″ = 1′; mark the dimensions, and write the scale under the drawing in the form of a fraction.
6. Draw a rectangle 6′ × 9′ on a scale of $\frac{1}{96}$; mark the dimensions, and write the perimeter of this rectangle under the drawing.

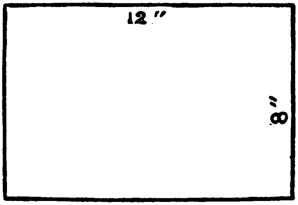
LESSON 18

Miscellaneous Rectangular Surfaces

You know that to find the area of a rectangle we find the product of the number in length multiplied by the number in width. Therefore, if a figure is not a complete square or oblong, we must separate it into such parts as will make complete squares or oblongs, and then find the sum of the areas of these parts; or we can find the area of the rectangle as it would be if it were complete and subtract the area of the missing part.

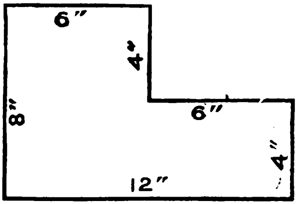
The addition method is the easier when the missing parts are many, and the subtraction method is the easier when the missing parts are few.

FIGURE "A"



(Scale $\frac{1}{8}$)

FIGURE "B"



(Scale $\frac{1}{8}$)

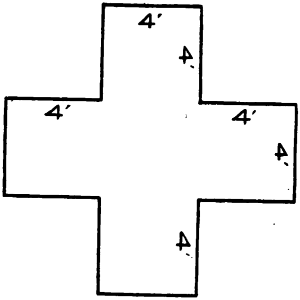
Exercise 35—Written.

1. What is the area of the rectangle shown in Figure "A"?
2. Measure it and state the scale on which it is drawn.
3. What is the perimeter of the rectangle it represents?
4. Separate Figure "B" into two rectangles by a dotted line. What is the area of the larger of the two rectangles?
5. What is the area of the smaller of the two rectangles into which Figure "B" has been divided?
6. What is the area of both of the rectangles into which Figure "B" has been divided?

7. What would be the area of Figure "B" if the one corner was not missing?
8. What is the area of the missing corner?
9. What is the area of the remaining part?
10. Is there any difference in the area of Figure "B" as found by addition in Question 6 and as found by subtraction in Question 9? What was the area by each of these methods?

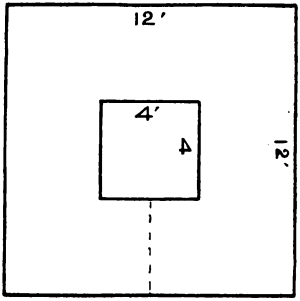
(To work your problem two ways is an excellent way to prove your work.)

FIGURE "C"



(Scale $\frac{1}{8}''= 1'$)

FIGURE "D"



(Scale $\frac{1}{8}''= 1'$)

11. What is the smallest number of rectangles into which Figure "C" can be divided?
12. How many 4′ × 4′ rectangles are there? What is their total area?
13. How many 4′ × 12′ rectangles are there? What is their total area?
14. What is the area of the entire cross represented by this drawing?
15. What is the distance around this cross, or the perimeter?

16. Figure "D" represents a lawn with a bed of flowers in the center. What is the area of the entire piece of ground represented by this drawing?
17. What is the area of the bed of flowers?
18. What is the area of the lawn?
19. What is the perimeter of the bed of flowers?
20. How many feet wide is the lawn at the dotted line?

Lesson 19

Lines, Angles, and Parallelograms

Where the earth and sky seem to meet is the "horizon." You have seen this line if you were ever on a large lake or on a large tract of flat land. Any line which is parallel to the horizon is a "horizontal" line; therefore, a straight line drawn parallel to the seeming flat surface of the earth is a horizontal line.

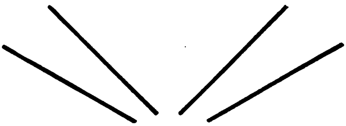
A straight line drawn at right angles to the seeming flat surface of the earth is a "vertical" line. When a ripe apple falls from a tree, it falls along a vertical line.

A straight line which slants is an "oblique" line.

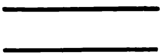
Lines are said to be "parallel" when they run in the same direction and have the same distance between them for their entire length. Parallel lines can never meet no matter how great a distance they may be extended.


A Horizontal Line


A Vertical Line

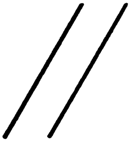

Several Oblique Lines

(When this book is lying flat on a table, whether it is closed or open, every one of the lines shown on Page 54 is in a horizontal position, but when you place the book in an upright position, the first of the lines remains horizontal, the second line becomes vertical, and the last four lines become oblique. Try this.)



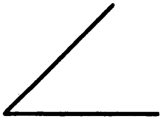
Parallel
Horizontal Lines

Parallel
Vertical Lines

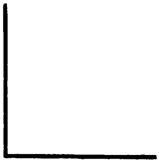


Parallel
Oblique Lines

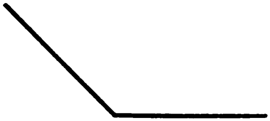
An angle which is sharper (or less) than a right angle is called an "acute angle" and an angle which is blunter (or greater) than a right angle is called an "obtuse angle." As acute and obtuse angles contain oblique lines, you see now why some people call them "oblique angles."



An Acute Angle



A Right Angle



An Obtuse Angle

You have already learned that squares and oblongs are called "rectangles" because they have four right angles and their opposite sides are parallel. Any plane figure which has its opposite sides parallel is called a "parallelogram;" therefore, every rectangle is a paral-

lelogram, but not every parallelogram is a rectangle because not every figure with parallel opposite sides has right angles.

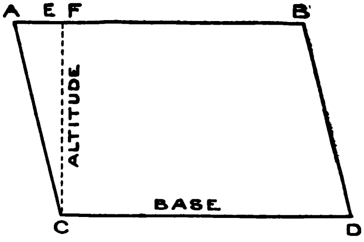


Figure 1

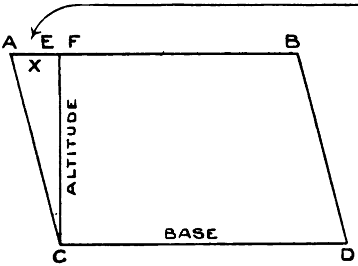


Figure 2

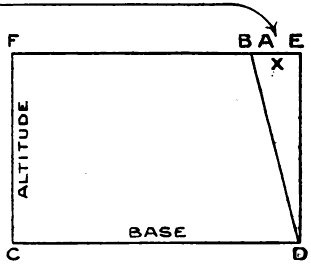


Figure 3

Figure 1 shows a parallelogram which is not a rectangle, because the angles in it are not right angles, two of them being acute angles and the other two being obtuse angles. However, as every parallelogram can be changed into a rectangle by drawing a line at right angles to the base and removing the triangular part so formed (X) from one end as shown in Figure 2 and adding it to the other end as shown in Figure 3, the area of a parallelogram will always be the same as the area of a rectangle having the same altitude and base. Can you point out the parallel sides?

The line drawn in Figure 2 shows the height or "altitude" of the parallelogram, and the side from which it was drawn at right angles is the "base." These are the two dimensions of a parallelogram.

Exercise 36—Oral.

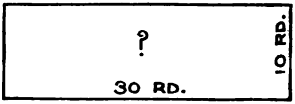
1. What is a horizontal line? Hold your pencil in a horizontal position. Find 4 horizontal lines in this room.
2. What is a vertical line? Hold your pencil in a vertical position. Find 4 vertical lines in this room.
3. What is an oblique line? Hold your arm in an oblique position. Show an oblique line in this room.
4. What are parallel lines? Find some in this room.
5. Is every rectangle a parallelogram? Why? What is the shape of the doors in this room? The windows? The blackboard?
6. Is every parallelogram a rectangle? Is Figure 1 a rectangle? Is Figure 2? Is Figure 3?
7. Is an acute angle greater or less than a right angle? Draw one.
8. Is an obtuse angle greater or less than a right angle? Find one in Figure 2.
9. What general name is given to acute and obtuse angles?
10. Draw the parallelogram shown in Figure 1 on a scale four times larger on a piece of paper. Draw the line showing the altitude as indicated in Figure 2. Cut off the triangle and place it in the position shown in Figure 3. Is there any

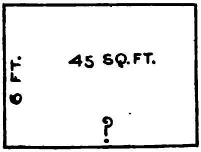
more or less paper in the rectangle you have made than there was in the original parallelogram? Is the area the same as it was originally?

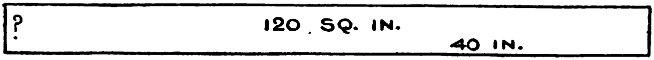
11. Show the altitude or height of the parallelogram. How is the altitude of a parallelogram found?
12. Show the base or edge on which it seems to rest. How is the base of a parallelogram found.
13. Now that you know that every parallelogram can be made into a rectangle, think this every time you find the area of a parallelogram. How can you find the area of a parallelogram?

What is the area of each of the following parallelograms:

14. Altitude 3″, base 6″;
15. Altitude 6″, base 9″;
16. Altitude 4′, base 11′;
17. Altitude 5′, base 10′;
18. Altitude 7′, base 8′.
19. How can a parallelogram which is not a rectangle be changed into one which is a rectangle?
20. Do these at sight. First tell how many units there are in 1 row:







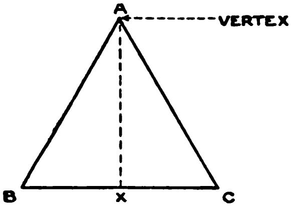
MENSURATION

LESSON 20

Triangles

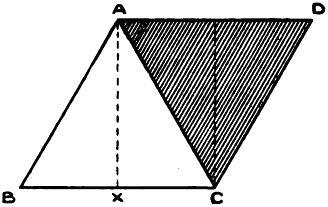
A "triangle" is a plane figure having three straight sides and three angles, the syllable "tri" meaning "three."

FIGURE "A"



A Triangle

FIGURE "B"



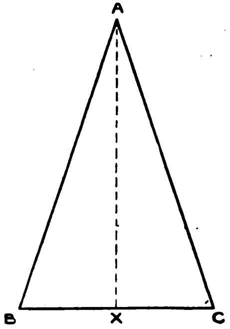
A Parallelogram
Made from Two Triangles of
Equal Size

Like a parallelogram, the triangle has two dimensions, one being the base, and the other altitude or height. The altitude is found by drawing a line at right angles to the base, from the highest point called the "vertex." Now, if you draw two triangles of equal size on a sheet of paper as shown in Figure "B," you will find that you have a parallelogram, and you will also find that the dimensions of each of these triangles are exactly the same as the dimensions of the parallelogram.

In other words, a parallelogram will contain exactly two triangles each having the same base and the same altitude as the parallelogram; therefore, the area of a triangle is exactly one-half the area of a parallelogram of the same dimensions.

Exercise 37—Oral.

1. Draw a triangle. How many sides has a triangle? How many angles? Which is the vertex?
2. What are the two dimensions of a triangle called?
3. How is the altitude of a triangle drawn?
4. A parallelogram of certain dimensions will contain how many triangles of the same dimensions?
5. The area of a triangle is what part of the area of a parallelogram of the same dimensions?
6. The area of a triangle is what part of the area of a rectangle of the same dimensions?
7. How is the area of a parallelogram found?
8. How is the area of a triangle found?
9. What is the altitude of the triangle in this drawing, if the scale is $\frac{1}{8}$? If $\frac{1}{8}$ in. = 1 ft.?

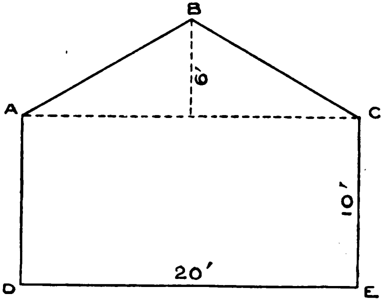


10. What is the base of this triangle?
11. What is the area of this triangle? Prove this by drawing or piecing.

What is the area of each of the following triangles:

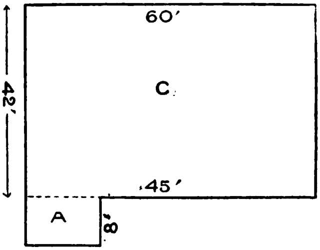
12. Altitude 6″, base 4″;

13. Altitude 8″, base 12″;
14. Altitude 10″, base 11″;
15. Altitude 5′, base 6′.



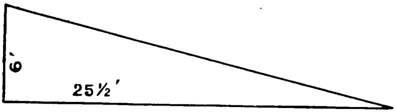
16. This drawing shows the wall of an attic room. What is the altitude of the triangular part of this wall?
17. What is the base of the triangular part of this wall?
18. What is the area of the triangular part of this wall?
19. What is the area of the rectangular part of this wall?
20. What is the total area of this wall?

Exercise 38—Written.

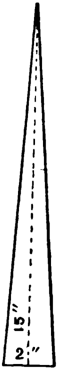


1. Find the area of "A."
2. Find the area of "C."

3. Find the area of "A" + "C."
4. Find the area of a square 22 ft. on the side.
5. Find the area of an oblong 12½ ft. long, and 8 ft. wide.
6. If a triangle is 10″ high and has a 6″ base, what is its area?
7. What is the area of a triangle whose base is 20′ and whose altitude is 16′?



8. Area = ?



9. Area = ?

Lathing, plastering, painting and kalsomining are usually figured by the square yard, and any fraction of a square yard in the net area is considered an entire square yard. As there are various customs regarding allowances for doors, windows, etc., written contracts must be made to avoid misunderstandings.

10. The kitchen of a bungalow is 10′ × 13′ and 9′ high, with 2 windows 3′ × 6′ and 2 doors 3′ × 7′.
    (*a*) How many sq. ft. are there in the walls and ceiling? In the windows and doors?
    (*b*) How many sq. ft. remain after deducting for the windows and doors? How many sq. yd.?
    (*c*) Find the cost of lathing @ 8¢ per sq. yd.
    (*d*) Find the cost of plastering @ 35¢ per sq. yd.
    (*e*) Find the cost of painting @ 30¢ per sq. yd.
    (*f*) Find the cost of kalsomining @ 12¢ per sq. yd.

ADDITION

LESSON 21

Grouping Numbers

Whenever the partial sum of a column can, by skipping a digit, be brought to even tens, as "twenty," "thirty," "forty," "fifty," "sixty," "seventy," "eighty," or "ninety," skip the digit but add it immediately afterward by merely saying it, and not repeating the tens; thus:

EXAMPLE:

48
27
98*
35
*64
89*
38
399

The digits marked * are added after the digits next below them.

Units: Fifteen; *twenty; eight;* thirty-two; *forty; nine.*

Tens: Eight; ten; nineteen; twenty-two; *thirty; six*; thirty-nine.

Exercise 39—Oral.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
46	38	97	89	65	97	97	88	86	87
28	97	46	77	79	88	36	97	49	43
93*	79	88	95	33	78	87	69	95	99
*46	55	77	84	86	85	48	85	88	78
29*	68	64	37	19	37	46	94	64	65
77	31	52	85	41	42	32	61	82	53

(Can you do 8 or 9 of these in 6 minutes?)

Exercise 40—Oral Review.

1. $\frac{1}{3} = \frac{?}{9}$; $\frac{3}{4} = \frac{?}{16}$; $\frac{2}{5} = \frac{?}{25}$.
2. Reduce to lowest terms: $\frac{16}{20}$; $\frac{48}{56}$; $3\frac{9}{27}$.
3. If a piece of string $7\frac{3}{4}$ yd. long is cut into two pieces, and one of the pieces is $3\frac{3}{8}$ yd. long, how long is the other piece?
4. $6 - (7 - 3) = ?$ $(5 + 3) \div (10 - 8) = ?$
5. How many and what dimensions has a straight line? A surface? A solid?
6. What kind of a rectangular solid is each of the following, and what is the volume of each:

   $2'' \times 3'' \times 3''$; $3' \times 3' \times 3'$;
   $2'' \times 2'' \times 2''$; 2 yd. × 3 yd. × 4 yd.;
   $2'' \times 2'' \times 6''$; 2 yd. × 5 yd. × 8 yd.

7. How do we find the area of a parallelogram?
8. How do we find the area of a triangle?
9. How do we find the area of a rectangle?
10. In a certain drawing the scale is 1 to 4; 6 inches in such a drawing represents what in the real object?

Exercise 41—Written Review.

1. $12\frac{1}{2} = \frac{?}{6}$.
2. $\frac{86}{12} = 7\frac{?}{6}$.
3. What part of a ton is 800 lb.? (Express your answer in a fraction of lowest terms.)
4. Find the sum of the following fractions, and reduce your answer to lowest terms: $\frac{3}{4} + \frac{5}{12} + \frac{7}{9} + \frac{1}{3} = ?$
5. $185 - (60 + 16 - 12) = ?$
6. What is the area of a parallelogram of the following dimensions: Altitude 13 ft., base 40 ft.

7. What is the area of a triangle of the following dimensions: Altitude 14 yd., base 16 yd.
8. Draw a rectangle 24′ × 36′ on the scale of $\frac{1}{8}''$ = 1′.
9. Draw a parallelogram with an altitude of 4′ and a base of 12′ on the scale of $\frac{1}{4}''$ = 1′.
10. Draw a triangle with an altitude of 8′ and a base of 4′ on the scale of $\frac{1}{2}''$ = 1′.

Add, without copying:

(Time for these 5 examples is less than 6 minutes.)

11.	**12.**	**13.**	**14.**	**15.**
3,946	6,889	5,284	5,780	7,486
8,826	2,487	3,474	9,431	9,828
7,322	8,298	9,886	7,737	7,137
4,416	7,374	3,749	4,896	9,296
6,547	4,747	1,942	2,753	7,436
9,149	1,689	2,637	4,126	1,837
2,587	9,999	1,849	1,218	2,786

Copy and divide:

(Time for these 4 examples is less than 6 minutes.)

16. 7,743 ÷ 87; **17.** 3,431 ÷ 47; **18.** 4,592 ÷ 56; **19.** 4,891 ÷ 67.

Subtract, without copying:

(Time for these 16 examples is less than 6 minutes.)

20.	**21.**	**22.**	**23.**
74,876	94,834	68,431	98,648
38,387	72,186	59,120	79,864

24.	**25.**	**26.**	**27.**
93,128	79,387	84,645	98,648
74,712	38,699	69,116	79,739

28.	**29.**	**30.**	**31.**
6,512	4,708	37,735	19,474
487	378	487	598
32.	**33.**	**34.**	**35.**
8,787	1,286	6,487	6,363
918	897	799	678

Copy and multiply:

(Time for these 10 examples is less than 6 minutes.)

36. 430 × 34;
37. 604 × 32;
38. 741 × 53;
39. 832 × 43;
40. 947 × 60;
41. 787 × 42;
42. 843 × 53;
43. 819 × 27;
44. 331 × 43;
45. 712 × 41.

46. Find the cost of lathing a room 12′ × 15′ and 10′ high @ 10¢ per sq. yd., allowing for 1 window 3′ × 6′ and 1 door 3′ × 7′.

47. If there are 100 laths in a bundle, and a bundle covers 5 sq. yd. of surface, how many bundles will be needed for the room in Question 46? How many laths? (Note: Any fraction of a bundle must be considered an entire bundle, as a fraction of a bundle cannot be bought.)

48. Find the cost of lathing and plastering a room 12′ × 16′ and 9′ high @ 40¢ per sq. yd., allowing for 2 windows 3′ × 6′ and 2 doors 3′ × 7′. How many bundles of laths will be needed?

49. Find the cost of lathing and plastering a room 15′ × 18′ and 10′ high @ 42¢ per sq. yd., allowing for 2 windows 4′ × 6′ and 2 doors 3′ × 7′.

FRACTIONS

LESSON 22

Multiplying Fractions and Whole Numbers

EXAMPLE:
3 times $\frac{1}{4} = \frac{3}{4}$.

EXAMPLE:
$\frac{3}{4}$ multiplied by $3 = \frac{9}{4}$ or $2\frac{1}{4}$.

EXAMPLE:
$\frac{3}{8} \times 7 = \frac{21}{8}$ or $2\frac{5}{8}$.
Note: Always reduce your answer to lowest terms.

If you were asked "How many cents are 3 times 1 cent?" it would not take you very long to give your answer "3 cents." Now stop and think what you really did to find this answer; you found the product of $1 \times 3 = 3$, and you called the answer cents because you multiplied cents. Now, supposing that instead of being asked to multiply *cents* you were asked to multiply *quarters*, then your answer would be 3 *quarters;* so you see, the name of the thing that the number stands for makes no difference because you always multiply the quantity; therefore, to multiply a fraction by a whole number, the *denominator* or *name* of the fraction does not change, you merely multiply the *numerator* or *number* which shows how many parts there are. 2 times $\frac{3}{5} = \frac{6}{5}$.

Sometimes the multiplication sign is used instead of the word "of;" thus, $\frac{1}{4} \times 12 = 3$ is exactly the same in effect as finding $\frac{1}{4}$ of $12 = 3$, which you learned some time ago; so you see, multiplying a fraction by a whole number gives the same result as finding that *fractional part* of the whole number.

EXAMPLE: $\frac{3}{4}$ of 12 = 9. Here it is easier to find $\frac{1}{4}$ of 12 = 3 and $3 \times 3 = 9$, because 4 is an equal part or factor of 12.

EXAMPLE: $\frac{3}{4} \times 7 = 5\frac{1}{4}$. Here it is easier to multiply $3 \times 7 = 21$ and $21 \div 4 = 5\frac{1}{4}$, because 4 is not an equal part or factor of 7.

When the denominator of the fraction is a factor of the whole number it is easier to find the fractional part of the whole number, but when the denominator of the fraction is not a factor of the whole number it is easier to multiply the fraction by the whole number.

EXAMPLE: What is the cost of $\frac{3}{4}$ yd. of silk @ $12. per yard?

$\frac{3}{4} \times$ $12. (or, for convenience, $\frac{3}{4}$ of $12.) = $9., Ans.

($12. is the multiplicand, because it is the number which is to be repeated.)

EXAMPLE: What is the cost of 7 yd. of cloth @ \$$\frac{3}{4}$. per yard?

7 × \$$\frac{3}{4}$. = \$$5\frac{1}{4}$., Ans.

(\$$\frac{3}{4}$. is the multiplicand because it is the number which is to be repeated.)

In all of your work you must remember that the multiplicand is the number which is to be repeated, and that the multiplier is the number which shows how many times the multiplicand is to be repeated; therefore, if the multiplicand has a name, as $5, 10 yards, $\frac{3}{4}$ bushel, etc., the product will have the same name.

Exercise 42—Oral.

1. In multiplying a fraction by a whole number does the numerator or the denominator change? Does it become larger?
2. Should the answer be reduced to lowest terms? To whole numbers?
3. When is it easier to find the fractional part of the whole number than to multiply the fraction by the whole number?
4. When is it easier to multiply the fraction by the whole number than to find the fractional part of the whole number?

Which is the easier way of finding the answer of each of the following examples; would you prefer $\frac{2}{3}$ of 9 or $\frac{2}{3} \times 9$? Tell why.

5. $\frac{2}{3}$ (of or $\times$) 9; $\frac{2}{3}$ (of or $\times$) 7; $\frac{4}{5}$ (of or $\times$) 15; $\frac{4}{5}$ (of or $\times$) 13.
6. $\frac{3}{4}$ (of or $\times$) 16; $\frac{5}{8}$ (of or $\times$) 24; $\frac{4}{7}$ (of or $\times$) 14; $\frac{5}{6}$ (of or $\times$) 18.
7. $\frac{3}{4}$ (of or $\times$) 5; $\frac{5}{8}$ (of or $\times$) 5; $\frac{4}{7}$ (of or $\times$) 5; $\frac{5}{6}$ (of or $\times$) 5.
8. $\frac{7}{9}$ (of or $\times$) 27; $\frac{5}{9}$ (of or $\times$) 8; $\frac{4}{9}$ (of or $\times$) 2; $\frac{2}{9}$ (of or $\times$) 2.
9. $\frac{2}{3}$ (of or $\times$) 18; $\frac{2}{3}$ (of or $\times$) 10; $\frac{2}{3}$ (of or $\times$) 3; $\frac{2}{3}$ (of or $\times$) 1.
10. $\frac{1}{10}$ (of or $\times$) 20; $\frac{3}{10}$ (of or $\times$) 40; $\frac{7}{10}$ (of or $\times$) 4; $\frac{9}{10}$ (of or $\times$) 7.

Exercise 43—Written.

1. $\frac{11}{100}$ of 8,000 = ?
2. $\frac{13}{64}$ of 1,280 = ?
3. $\frac{18}{23}$ of 46 = ?
4. $\frac{17}{32}$ of 1,472 = ?

5. $\frac{7}{50} \times 39 = ?$
6. $\frac{18}{31}$ of $465 = ?$
7. $\frac{8}{9}$ of $27,612 = ?$
8. $\frac{29}{30}$ of $240 = ?$
9. $\frac{11}{12}$ of $1,536 = ?$
10. $\frac{875}{1000} \times 6 = ?$

11. What is the cost of $\frac{7}{8}$ acres of land @ \$128. per acre?

12. What is the cost of 768 bu. of corn @ \$$\frac{5}{8}$. per bushel?

13. A gallon of water contains 231 cu. in.; how many cubic inches are there in $\frac{6}{7}$ gal.?

14. How long will it take an aeroplane flying at the rate of 1 mi. in $\frac{5}{6}$ min. to fly 96 mi.?

15. A brick is $\frac{2}{3}$ ft. in length; how long is a wall if there are 162 bricks placed end to end in each layer?

LESSON 23

Multiplying Fractions by Fractions

EXAMPLE:	**EXAMPLE:**	**EXAMPLE:**
$\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$ is the same as: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.	$\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$ is the same as: $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.	$\frac{5}{8} \times \frac{2}{5} = \frac{10}{40}$ or $\frac{1}{4}$ is the same as: $\frac{5}{8}$ of $\frac{2}{5} = \frac{10}{40}$ or $\frac{1}{4}$.

Multiplying a fraction by a fraction is exactly the same as finding a fractional part of a fraction, which you know how to do; we merely use the multiplication sign instead of the word "of."

Remember also, that the order in which the fractions are used has no effect on the result, as you will readily see by the following examples:

EXAMPLE:

$\frac{3}{5}$ of $\frac{2}{3} = \frac{6}{15}$ or $\frac{2}{5}$.
$\frac{2}{3}$ of $\frac{3}{5} = \frac{6}{15}$ or $\frac{2}{5}$.
$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$ or $\frac{2}{5}$.
$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$ or $\frac{2}{5}$.

EXAMPLE:

$\frac{2}{5}$ of $\frac{3}{4} = \frac{6}{20}$ or $\frac{3}{10}$.
$\frac{3}{4}$ of $\frac{2}{5} = \frac{6}{20}$ or $\frac{3}{10}$.
$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$ or $\frac{3}{10}$.
$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ or $\frac{3}{10}$.

Though we can use the numbers in any order for convenience, the multiplicand is always the number which is to be repeated, and if it has a name, the product will have the same name.

EXAMPLE: $\frac{3}{8}$ of $\frac{2}{3}$ yd. $= \frac{6}{24}$ yd. or $\frac{1}{4}$ yd., Ans.

Exercise 44—Oral.

1. In multiplying a fraction by a fraction, how do we find the numerator of the product?
2. In multiplying a fraction by a fraction, how do we find the denominator of the product?
3. Is there any difference between multiplying a fraction by a fraction and finding a fractional part of a fraction?
4. $\frac{2}{3}$ of 2 ft. = ? $\frac{2}{3}$ of 1 = ? $\frac{2}{3}$ pk. × $\frac{1}{2}$ = ?
5. $\frac{4}{5}$ of $\frac{3}{5}$ = ? $\frac{2}{5}$ of \$$\frac{2}{5}$. = ? $\frac{3}{5}$ × $\frac{1}{5}$ cwt. = ?
6. $\frac{8}{9}$ of $\frac{3}{4}$ lb. = ? $\frac{7}{9}$ of $\frac{5}{6}$ = ? $\frac{5}{9}$ × $\frac{2}{3}$ = ?
7. $\frac{4}{7}$ of $\frac{3}{8}$ = ? $\frac{2}{7}$ of $\frac{1}{8}$ bu. = ? $\frac{3}{7}$ × $\frac{7}{8}$ = ?
8. $\frac{9}{10}$ of \$$\frac{2}{3}$. = ? $\frac{7}{10}$ of $\frac{1}{2}$ = ? $\frac{3}{10}$ × $\frac{3}{4}$ gr. = ?

LESSON 24

Multiplying Whole Numbers and Mixed Numbers

To multiply a whole number by a mixed number or a mixed number by a whole number, we can reduce the mixed number to an improper fraction and multiply, as: $12 \times 1\frac{2}{3} = 12 \times \frac{5}{3}$ or 20; or we can write the example in the regular way, thus:

EXAMPLE: $12 \times 1\frac{2}{3} = ?$

$$\begin{array}{rl} 12 & \\ \underline{1\frac{2}{3}} & \\ 12 & (12 \times 1) \\ \underline{8} & \underline{(12 \times \frac{2}{3})} \\ 20 & (12 \times 1\frac{2}{3}) \end{array}$$

$(12 \times 1) + (12 \times \frac{2}{3}) = 20.$

When the mixed number is a number of several orders, the regular form of multiplication is usually much more rapid than the fractional method, and the number which will produce the fewest partial products, whether it be the multiplier or the multiplicand, is written below the other number in working the example.

EXAMPLE: What is the cost of 432½ lb. of black printing ink @ 18¢ per pound?

$$\begin{array}{rl} 432\frac{1}{2} & \\ \underline{.18} & \\ 3456 & (432 \times 8) \\ 432 & (432 \times 1) \\ \underline{9} & (\ \frac{1}{2} \times 18) \\ \$77.85 & \end{array}$$

Here 18¢ is the multiplicand, but time is saved by writing it below 432½, as we have fewer partial products by so doing.

EXAMPLE: What is the cost of 18½ gr. of pencils @ \$4.32 per gross?

$$\begin{array}{rl} \$4.32 & \\ \underline{18\frac{1}{2}} & \\ 3456 & (432 \times 8) \\ 432 & (432 \times 1) \\ \underline{216} & (432 \times \frac{1}{2}) \\ \$79.92 & \end{array}$$

Here \$4.32 is the multiplicand, and time is saved by writing it above 18½, as we have fewer partial products by so doing.

Exercise 45—Written.

1. How many hours are there in $12\frac{3}{4}$ days? (If there are 24 hours in 1 day, there must be $12\frac{3}{4}$ times 24 hours in $12\frac{3}{4}$ days.)
2. How many ties are there in $8\frac{2}{3}$ doz.?
3. How many inches are there in $18\frac{5}{6}$ yd.?
4. How many square inches are there in $14\frac{1}{3}$ sq. ft.?
5. How many cubic feet are there in $11\frac{2}{3}$ cu. yd.?
6. 246 sq. yd. of carpet @ \$$14\frac{2}{3}$. per sq. yd. = ?
7. What is the cost of $138\frac{3}{20}$ T. of coal @ \$6.20 per T.?
8. What is the cost of 24 gal. of paint @ $81\frac{3}{4}$¢ per gal.?
9. What is the profit on the sale of $42\frac{5}{36}$ yards of cloth which was bought at \$2.75 per yard, and sold for \$3.47 per yard?
10. How much was gained by buying 72 yards of lining when the price was $32\frac{3}{4}$¢ per yard, instead of buying it when the price was $48\frac{2}{3}$¢ per yard?

Lesson 25

Multiplying Fractions or Mixed Numbers by Mixed Numbers

When fractions or mixed numbers are to be multiplied by mixed numbers, it is usually easier to reduce the mixed numbers to improper fractions and multiply as we do when proper fractions are to be multiplied; thus:

EXAMPLE: $\frac{3}{4} \times 2\frac{3}{4} = ?$

$2\frac{3}{4} = \frac{11}{4}$;
$\frac{3}{4} \times \frac{11}{4} = \frac{33}{16}$;
$\frac{33}{16} = 2\frac{1}{16}$, Ans.

EXAMPLE: $2\frac{1}{3} \times 3\frac{7}{8} = ?$

$2\frac{1}{3} = \frac{7}{3}$; $3\frac{7}{8} = \frac{31}{8}$;
$\frac{7}{3} \times \frac{31}{8} = \frac{217}{24}$;
$\frac{217}{24} = 9\frac{1}{24}$, Ans.

When the mixed numbers are large, multiply all the parts, as:

EXAMPLE: What is the cost of 142½ acres of land @ $165½. an acre?

```
   $165½
    142½
   ─────
       ¼. = (½ of ½)
     82½  = (½ of 165)
     71   = (½ × 142)
    330 ⎫
   660  ⎬ = (165 × 142
  165   ⎭
 ───────
$23,583¾., Ans.
```

Exercise 46—Oral and Written.

A. Tell how to do these examples:

1. A wagon standing in front of a foundry was being loaded with iron castings of which a dozen weighed 8¾ lb.; what was the weight of 8 of such castings?

2. A merchant in placing boxes on a shelf found that each dozen boxes occupied 4⅔ cu. ft. of space; how many cu. ft. of space were needed for 1⅔ doz. boxes?

3. What is the cost of 8⅔ yd. of cotton @ 12½¢ yd.? (Note: When the answer of an example contains a fraction of a cent, drop the fraction if it is less than ½¢, but add another full cent in place of the fraction if it is ½¢ or more than ½¢.)

4. What is the cost of 14¼# of tea @ 66⅔¢ lb.?

5. What is the profit on 18½# of coffee bought @ 25¢ lb., and sold @ 33⅓¢ lb.?

6. What is the profit on $24\frac{5}{8}$ doz. oranges bought @ $31\frac{1}{4}$¢ doz., and sold @ $37\frac{1}{2}$¢ doz.?

7. $104\frac{1}{4}$ acres of land @ \$$137\frac{1}{2}$. an acre = ?

8. A merchant who was wrapping a package for express shipment found that he needed $3\frac{1}{2}$ sheets of wrapping paper; if each sheet of paper measured $12\frac{1}{2}$ sq. ft., how many sq. ft. of paper did he use?

9. In measuring milk, a baker used a measure which held $3\frac{1}{3}$ gal.; how many gallons of milk did he use if he filled the measure 3 times and used $\frac{1}{2}$ a measure-full besides?

10. $12\frac{1}{2} \times 8\frac{1}{2} = ?$

11. $6\frac{1}{8} \times 3\frac{1}{2} = ?$

12. $86\frac{3}{4} \times 48\frac{1}{2} = ?$

13. $119\frac{1}{3} \times 104\frac{1}{4} = ?$

B. Now work all of them.

(Time for last 4 = 5 min.)

LESSON 26

Division by a Fraction (The First Step)

We have used division when we found part of a number, but division, as you know, also means "Finding how many times one number is contained in another number."

In dividing by a fraction two steps are necessary: We first find how often the fraction is contained in 1 unit, and then we must multiply this result by the number of units or part of a unit.

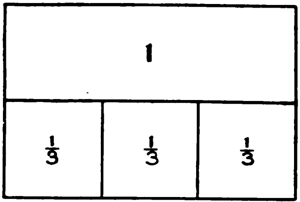
You know that there are $\frac{3}{3}$ in 1; now look for $\frac{2}{3}$ in 1. If you study illustrations A, B, and C carefully, you will have no difficulty in understanding that $\frac{2}{3}$ is contained in $\frac{3}{3}$ one and one-half times, just as 2 is contained in 3 one and one-half times, thus:

$$\frac{\textcircled{3}}{1} \div \frac{\textcircled{2}}{1} = \frac{3}{2} \text{ or } 1\frac{1}{2};\ \frac{\textcircled{3}}{3} \div \frac{\textcircled{2}}{3} = \frac{3}{2} \text{ or } 1\frac{1}{2}.$$

In both cases we are finding how many 2's there are in 3, then changing the remainder into a fraction:

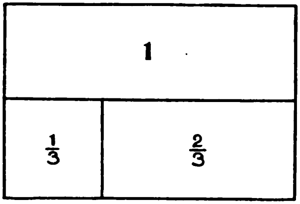
$$\begin{array}{r} 1\frac{1}{2} \\ 2\overline{)3\phantom{\frac{1}{2}}} \\ \underline{2}\phantom{\frac{1}{2}} \\ 1\phantom{\frac{1}{2}} \end{array}$$

A



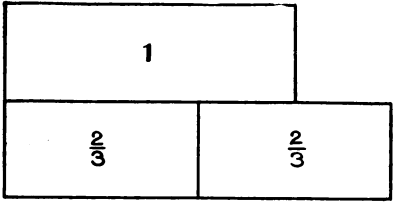
$\frac{3}{3} = 1$

B



$\frac{1}{3} = \frac{1}{2}$ OF $\frac{2}{3}$

C



$\frac{2}{3}$ IS CONTAINED IN $\frac{3}{3}$ $1\frac{1}{2}$ TIMES
OR $1 \div \frac{2}{3} = 1\frac{1}{2}$

From this you can see that to find the quotient of 1 divided by any fraction, we need only to invert the terms of the fraction; that is, interchange the numerator and the denominator; thus:

$$1 \div \frac{3}{4} = \frac{4}{3}. \qquad 1 \div \frac{2}{5} = \frac{5}{2}.$$

If the divisor is a mixed number, reduce it to an improper fraction before dividing, as:

$$1 \div 1\frac{1}{2} = 1 \div \frac{3}{2};\ 1 \div \frac{3}{2} = \frac{2}{3}.$$

Exercise 47—Oral.

1. What is the first thing we must find when we divide by a fraction? (1 ÷ fraction)
2. Prove on the blackboard by illustrations similar to A, B, and C that $1 \div \frac{3}{4} = \frac{4}{3}$ or $1\frac{1}{3}$. In the same way, prove that $1 \div \frac{2}{5} = \frac{5}{2}$ or $2\frac{1}{2}$.
3. Prove on the blackboard by arithmetic that $1 \div \frac{3}{4} = \frac{4}{3}$.
4. $1 \div \frac{1}{5} = ?$ $1 \div \frac{3}{5} = ?$ $1 \div \frac{4}{5} = ?$ $1 \div 5 = ?$ $1 \div 2 = ?$
5. $1 \div \frac{1}{6} = ?$ $1 \div \frac{5}{6} = ?$ $1 \div \frac{7}{6} = ?$ $1 \div 9 = ?$ $1 \div 11 = ?$
6. $1 \div \frac{2}{3} = ?$ $1 \div 3\frac{3}{4} = ?$ $1 \div 4\frac{1}{2} = ?$ $1 \div 4 = ?$ $1 \div 12 = ?$
7. $1 \div 3 = ?$ $1 \div 6 = ?$ $1 \div 8 = ?$ $1 \div 1\frac{1}{2} = ?$ $1 \div \frac{1}{4} = ?$
8. $1 \div \frac{2}{7} = ?$ $1 \div 7 = ?$ $1 \div 1\frac{2}{7} = ?$ $1 \div 1\frac{3}{4} = ?$ $1 \div \frac{9}{10} = ?$
9. $1 \div \frac{1}{10} = ?$ $1 \div 10 = ?$ $1 \div 1\frac{1}{10} = ?$ $1 \div 2\frac{1}{2} = ?$ $1 \div \frac{2}{3} = ?$
10. How can you tell quickly how many times any fraction is contained in 1 unit?

LESSON 27

Division by a Fraction (The Second Step)

After finding how many times a fraction is contained in 1 unit by inverting its terms, we merely multiply this result by any number to find how often the fraction is contained in that number; thus, to find the quotient of $12 \div \frac{3}{4}$, we first find the quotient of $1 \div \frac{3}{4} = \frac{4}{3}$, then we multiply this result by 12 because $\frac{3}{4}$ must be contained in 12 twelve times as often as it is contained in 1, and $\frac{4}{3} \times 12 = 16$.

Another example: $\frac{1}{2} \div \frac{2}{3}$; here we find that $1 \div \frac{2}{3} = \frac{3}{2}$, and in $\frac{1}{2}$ it would be contained only $\frac{1}{2}$ as often, and $\frac{1}{2}$ of $\frac{3}{2} = \frac{3}{4}$.

From this you see that to divide by a fraction, we invert the divisor and multiply.

EXAMPLE: $\frac{3}{4} \div \frac{1}{3} = ?$

$\frac{3}{4} \div \frac{1}{3} =$

$\frac{3}{4} \times \frac{3}{1} =$ (Read the divisor.)

$\frac{9}{4}$ or $2\frac{1}{4}$, Ans.

EXAMPLE: $\frac{5}{6} \div \frac{1}{6} = ?$

$\frac{5}{6} \div \frac{1}{6} =$

$\frac{5}{6} \times \frac{6}{1} =$ (Read the divisor.)

$\frac{30}{6}$ or 5, Ans.

If the dividend is a whole number, the process is just the same.

EXAMPLE: $8 \div \frac{3}{4} = ?$

$8 \div \frac{3}{4} =$

$8 \times \frac{4}{3} =$ (Read the divisor.)

$\frac{32}{3}$ or $10\frac{2}{3}$, Ans.

If the dividend is a mixed number, write it in the form of an improper fraction, thus:

FRACTIONS

EXAMPLE: $1\frac{2}{3} \div \frac{1}{4} = ?$

$1\frac{2}{3} \div \frac{1}{4} =$

$\frac{5}{3} \times \frac{4}{1} =$ (Read the divisor.)

$\frac{20}{3}$ or $6\frac{2}{3}$, Ans.

In all of your work remember that when a number is divided by exactly 1, it remains unchanged; when it is divided by more than 1, it is decreased; when it is divided by less than 1, it is increased; thus:

$4 \div 1 = 4$ because there are 4 one's in four.
$4 \div 2 = 2$ because there are 2 two's in four.
$4 \div \frac{1}{2} = 8$ because there are 8 halves in four.

Also remember that when a number is multiplied by exactly 1, it remains unchanged; when it is multiplied by more than 1, it is increased; when it is multiplied by less than 1, it is decreased; thus:

$4 \times 1 = 4$ because 4 one's are four.
$4 \times 2 = 8$ because 4 two's are eight.
$4 \times \frac{1}{2} = 2$ because 4 halves are two.

(Notice that multiplying by 2 gives the same answer as dividing by $\frac{1}{2}$, and multiplying by $\frac{1}{2}$ gives the same answer as dividing by 2.)

Exercise 48—Written on Board.

1. $\frac{2}{3} \div \frac{1}{2} = ?$ $\frac{3}{4} \div \frac{1}{2} = ?$ $\frac{5}{6} \div \frac{1}{2} = ?$
2. $\frac{2}{3} \div \frac{3}{4} = ?$ $\frac{3}{4} \div \frac{3}{4} = ?$ $\frac{5}{6} \div \frac{3}{4} = ?$
3. $\frac{1}{8} \div \frac{2}{3} = ?$ $\frac{3}{8} \div \frac{1}{6} = ?$ $\frac{5}{8} \div \frac{1}{4} = ?$
4. $12 \div \frac{2}{3} = ?$ $10 \div \frac{2}{5} = ?$ $5 \div \frac{1}{7} = ?$

5. $1\frac{2}{3} \div \frac{3}{5} = ?$ $1\frac{3}{8} \div \frac{1}{2} = ?$ $1\frac{1}{2} \div \frac{1}{3} = ?$
6. $\frac{2}{3} \div 3 = ?$ $\frac{3}{4} \div 6 = ?$ $\frac{5}{6} \div 7 = ?$
7. $\frac{3}{8} \div 1\frac{1}{3} = ?$ $8 \div 1\frac{3}{4} = ?$ $\frac{1}{5} \div 1\frac{1}{2} = ?$
8. $1\frac{1}{4} \div 1\frac{1}{4} = ?$ $3\frac{1}{2} \div 1\frac{2}{3} = ?$ $9 \div 2\frac{1}{4} = ?$
9. $5\frac{1}{4} \div \frac{1}{8} = ?$ $7\frac{1}{3} \div 8 = ?$ $8\frac{1}{2} \div \frac{1}{3} = ?$
10. $6\frac{2}{3} \div 4\frac{1}{2} = ?$ $9\frac{1}{9} \div 4 = ?$ $\frac{1}{8} \div 8 = ?$

(Time used for last ten = ? minutes.)

Exercise 49—Oral.

(Be ready to illustrate these at the board.)

1. After finding how many times a fraction is contained in 1, how do we find how many times it is contained in any number? How many times is $\frac{2}{3}$ contained in 1? How many times in 4?
2. How do we change a whole number into an improper fraction?
3. When a number is multiplied by 1, does it become larger or smaller?
4. When a number is multiplied by more than 1, does it become larger or smaller?
5. When a number is multiplied by less than 1, does it become larger or smaller?
6. When a number is divided by 1, does it become larger or smaller?
7. When a number is divided by more than 1, does it become larger or smaller?
8. When a number is divided by less than 1, does it become larger or smaller?
9. Dividing by 6 is the same as finding what part of a number?

Exercise 50—Written.

1. $\frac{11}{16} \div 4 = ?$
2. $\frac{9}{10} \div 16 = ?$
3. $20 \div \frac{5}{6} = ?$
4. $24 \div \frac{1}{6} = ?$
5. $\frac{8}{9} \div \frac{15}{16} = ?$
6. $\frac{3}{10} \div \frac{10}{11} = ?$
7. $\frac{2}{9} \div 8\frac{1}{3} = ?$
8. $\frac{4}{11} \div 12\frac{1}{2} = ?$
9. $14\frac{1}{2} \div \frac{2}{7} = ?$
10. $33\frac{1}{3} \div \frac{1}{10} = ?$
11. $9\frac{1}{2} \div 6 = ?$
12. $16\frac{2}{3} \div 8 = ?$
13. $14 \div 6\frac{2}{3} = ?$
14. $21 \div 4\frac{1}{5} = ?$
15. $8\frac{1}{3} \div 16\frac{2}{3} = ?$
16. $12\frac{1}{2} \div 10\frac{1}{4} = ?$

(Time for 5 is 10 min. or less.)

Exercise 51—Oral and Written.

A. Tell how to work the following examples:

1. How many pieces of tile each containing $\frac{1}{16}$ sq. ft. will be needed to cover a floor 12′ × 18′?
2. How many strips of wall paper each $1\frac{1}{2}$′ wide will be needed to paper a wall $10\frac{1}{2}$′ wide?
3. How many boards each $6\frac{1}{2}$″ wide must be placed side by side to cover a ditch $97\frac{1}{2}$″ long?
4. The length of a ballroom is $85\frac{1}{2}$ ft.; how many strips of wall paper $1\frac{1}{2}$ ft. wide will be needed to cover one of its $85\frac{1}{2}$ ft. walls?
5. How many $\frac{1}{2}$ pt. bottles of cream are there in $8\frac{2}{3}$ pt.?
6. How many $\frac{3}{4}$ lb. tins can be filled from an 8 lb. package of tea? What part of a pound of tea remains unused?
7. How many $1\frac{1}{2}$ lb. boxes can be filled from a 10 lb. pail of candy? How much candy remains unused?

8. How many $\frac{2}{3}$ gal. jars of molasses can be filled from a barrel containing $31\frac{1}{2}$ gal.? What part of a gallon remains unused?
9. How many pieces of ribbon $1\frac{1}{2}$ yd. long can be cut from a 10 yd. bolt? What will be the length of the remaining piece?
10. How many days will it take a man to walk 84 miles if he walks $18\frac{2}{3}$ miles each day?

B. Now work the examples.

LESSON 28

Cancellation

EXAMPLE: $\frac{2}{3}$ of $\frac{6}{7}$ = ? ($\frac{2}{3}$ of $\frac{6}{7}$ = $\frac{12}{21}$ or $\frac{4}{7}$, long way.)

$$\frac{2}{\underset{1}{\cancel{3}}} \text{ of } \frac{\overset{2}{\cancel{6}}}{7} = \frac{4}{7}.$$

Divide the denominator of $\frac{2}{3}$ and the numerator of $\frac{6}{7}$ by the common factor 3. The result obtained in this way is exactly the same as if the multiplication had been made without cancellation and the product ($\frac{12}{21}$) had then been reduced to $\frac{4}{7}$ by dividing both terms by the common factor 3.

You know that a fraction can be reduced to lower terms by dividing both of its terms by a common factor, and you know that its value is not in any way changed by this reduction.

In the same manner, before multiplying fractions we can reduce the numerator of one fraction and the denominator of another by dividing these terms by any factors which are found in both, just as though the terms were of the same fraction. Reducing the terms of fractions in this way is called "cancellation."

EXAMPLE: $\frac{7}{12}$ of $\frac{9}{10}$ = ?

$$\frac{7}{\underset{4}{\cancel{12}}} \text{ of } \frac{\overset{3}{\cancel{9}}}{10} = \frac{21}{40}.$$

Divide both the numerator of $\frac{9}{10}$ and the denominator of $\frac{7}{12}$ by the common factor 3, and multiply what is left to find the new numerator and denominator.

EXAMPLE: $\frac{7}{9}$ of $\frac{6}{7}$ of $\frac{2}{3}$ = ?

$$\frac{\overset{1}{\cancel{7}}}{9} \text{ of } \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}} \text{ of } \frac{2}{\underset{1}{\cancel{3}}} = \frac{4}{9}.$$

Divide by 7, then divide by 3.

EXAMPLE: $\frac{24}{25} \times \frac{5}{6} \times \frac{15}{16}$ = ?

$$\frac{\overset{3}{\cancel{24}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{25}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{15}}}{\underset{2}{\cancel{16}}} = \frac{3}{4}.$$

The factors 8, 5, 5, and 3 are here used.

Exercise 52—Written.

(Be very careful and accurate.)

1. $\frac{4}{5}$ of $\frac{1}{6}$ = ?
2. $\frac{3}{4}$ of $\frac{2}{9}$ = ?
3. $\frac{3}{5}$ of $\frac{10}{11}$ = ?
4. $\frac{12}{13}$ of $\frac{26}{27}$ = ?
5. $\frac{9}{10}$ of $\frac{5}{6}$ = ?
6. $\frac{1}{8} \times \frac{4}{5} \times \frac{10}{15}$ = ?
7. $\frac{3}{4} \times \frac{8}{9} \times \frac{1}{6}$ = ?
8. $\frac{3}{8} \times \frac{4}{5} \times \frac{10}{11}$ = ?
9. $\frac{4}{9} \times \frac{3}{8} \times \frac{6}{1}$ = ?
10. $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{10}$ = ?

ADDITION

LESSON 29

Grouping Numbers

Having learned to recognize groups of ten that are composed of two digits, you must now learn to recognize groups of ten that are composed of three digits; of these, there are only eight possible groups:

8	7	6	5	6	5	4	4
1	2	3	4	2	3	4	3
1	1	1	1	2	2	2	3
10	10	10	10	10	10	10	10

You will be greatly surprised by the speed with which you will be able to add long columns of numbers after having practiced this and the previous lessons on grouping numbers for a short time.

Exercise 53—Oral.

Add, grouping wherever possible:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
88	99	68	89	83	88	72	54	98	76
71	56	25	44	42	34	91	12	22	31
21	43	23	34	25	14	37	34	32	21
13	11	72	32	48	65	31	67	52	58
56	88	82	99	93	92	41	18	86	11
44	83	29	82	76	67	78	43	77	84
89	84	49	46	89	86	29	57	54	45

(Time, 8 examples in 6 minutes. Some can do 8 in 4 minutes.)

MEASURING TIME

LESSON 30

The Months and the Year

The earth makes one complete revolution on its axis every day of 24 hours, giving us daylight while we are turned toward the sun and darkness while we are turned away from the sun.

Besides revolving on its axis once in 24 hours, the earth revolves around the sun once in a little less than 365¼ days, giving us the four seasons, Spring, Summer, Autumn (or Fall), and Winter.

The earth, therefore, turns a little more than 365 times on its axis while making one journey around the sun.

Now, as we call 365 days a "common year," it can easily be seen that in four years the ¼ day left from each year will amount to an additional day, which is added to all years which are divisible by 4, giving us 366 days, which we call a "leap year;" but it does not take exactly 365¼ days for the earth to make a complete revolution around the sun—it takes enough less than 365¼ days to equal 3 days in every 400 years; therefore, in every 400 years, 3 of the years which would otherwise be leap years are common years. These 3 years are the years which end with two ciphers but are not divisible by 400.

Thus, every year which is divisible by 4 is a leap year excepting the even hundreds, and they are leap years only when divisible by 400.

In every year there are 12 months as follows:

1/ January	(Jan.)	= 31 days.	
2/ February	(Feb.)	= 28 days.	(In leap year, 29 days.)
3/ March	(Mar.)	= 31 days.	
4/ April	(Apr.)	= 30 days.	
5/ May	(May)	= 31 days.	
6/ June	(Jun.)	= 30 days.	
7/ July	(Jul.)	= 31 days.	
8/ August	(Aug.)	= 31 days.	
9/ September	(Sep.)	= 30 days.	
10/ October	(Oct.)	= 31 days.	
11/ November	(Nov.)	= 30 days.	
12/ December	(Dec.)	= 31 days.	
		365 days.	(In leap year, 366 days.)

Dates are written in three different ways:

1st. Written out in full: July 4th, 1776.
2d. With the month abbreviated: Jul. 4, 1776.
3d. With the number of the month: 7/4/1776.

In business, when the date referred to is within the present century, the first two figures of the century are omitted: August 4th, 1918, would be written: 8/4/18.

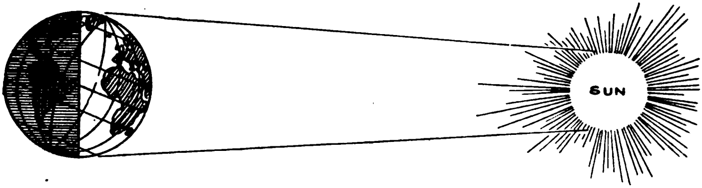
From the above list you will see that in common years the month of February has only 28 days, but in leap years it has 29 days, and is even then the only month which has less than 30 days. April, June, September, and November each have 30 days, and the remaining seven months each have 31 days.

MEASURING TIME

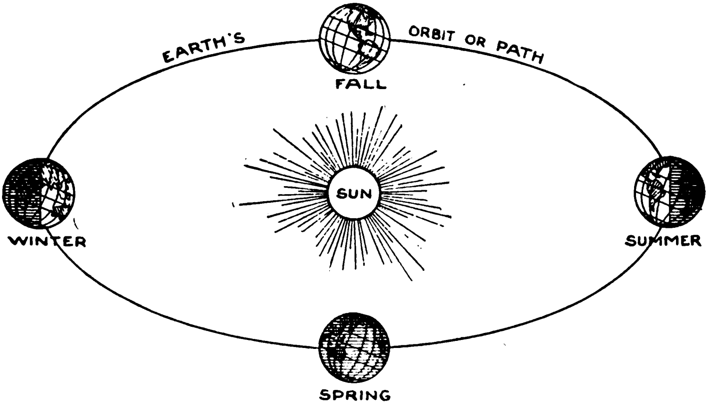
This little rhyme will help you to remember how many days there are in each of the months:

“Thirty days has September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.”

Exercise 54—Oral.



1. How long does it take the earth to make one complete turn on its axis? What does this give us?



2. How long does it take the earth to travel once around the sun? What does this give us?

3. How many days are there in a common year? How many in a leap year?
4. How can you tell whether or not any certain year is a leap year?
5. Which of these years are leap years:

1492; 1783; 1700; 1776; 1600; 1895; 1900; 1916; 1940; 2000; 2001; 2002; 2003; 2004.

6. Name the months of the year, and say how many days each month has.
7. Which month has 1 day more in leap years than in common years? How many days has that month in common years? How many in leap years?
8. Which are the months that have 30 days? How many of these are there?
9. Which are the months that have 31 days? How many of these are there?
10. Say the little rhyme which helps you to remember how many days there are in each of the months of the year.

LESSON 31

Measure of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
28, 29, 30, or 31 days = 1 month
12 months = 1 year
100 years = 1 century

MEASURING TIME

60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.
28, 29, 30, or 31 da. = 1 mo.
12 mo. = 1 yr.
100 yr. = 1 century

Note: 10 years is often called a "decade."

Though there are more days in some months than there are in others, in business a month is usually considered to be $\frac{1}{12}$ of a year, and a year is considered as consisting of 12 months of 30 days each, or a total of 360 days.

Exercise 55—Oral.

1. How many seconds are there in 8 minutes? In 10 minutes?
2. How many minutes are there in 6 hours?
3. How many hours are there in 2 days?
4. How many days are there in 7 weeks?
5. How many months are there in 12 years?
6. How many years are there in 7 centuries? How many years are there in 3 decades?
7. Say the table used in measuring time.
8. In business, a month is considered to be what part of a year?
9. In trade, a month is considered as consisting of how many days?
10. In commerce, a year is considered as consisting of how many days?

Exercise 56—Written.

1. On the basis of 360 days to the year, what part of a year is 270 days?
2. On the basis of 30 days to the month, how many months are there in 240 days?
3. How many hours are there in 6 weeks?
4. How many minutes are there in 4 days?
5. How many days are there in two common years and 1 leap year?
6. From Jan. 8th to Jan. 31st equals how many days?
7. From Jan. 8th to Feb. 20th equals how many days?
8. From Feb. 6th, 1916, to Mar. 8th, 1916, equals how many days?
9. From Apr. 10th to Jul. 4th equals how many days?
10. If a man earns $1,300.00 per annum (meaning by the year), how much is that for 4 months? How much for 11 months?

Exercise 57—Oral Review.

1. How can you tell quickly whether or not a number is divisible by 2, 3, 4, 5, 6, 8, 9, and 10?
2. What numbers not higher than 10 will divide:
   (*a*) 375,480; (*b*) 972,402; (*c*) 461,324;
   (*d*) 538,712; (*e*) 863,235; (*f*) 923,426.
3. The altitude of a parallelogram is 6′; the base is 12′; what is the area?
4. The altitude of a triangle is 6′; the base is 12′; what is the area?
5. How do we divide any number by a fraction?
6. $\frac{3}{8} \times \frac{1}{8} = ?$ $\frac{3}{8} \div \frac{1}{8} = ?$ $\frac{3}{8} + \frac{1}{8} = ?$ $\frac{3}{8} - \frac{1}{8} = ?$

7. Add:

(*a*)	(*b*)	(*c*)	(*d*)
394	644	593	194
982	472	559	983
198	734	378	848
437	459	254	675
839	682	589	369
343	789	868	238

(Practice till you can do these four examples in 5 minutes. Can you do them in less than 5 minutes?)

8. How many days are there in each of the months of the year?

9. Say the table used in measuring time.

10. What month has an extra day in leap year? How do you tell quickly whether any year is a common year or a leap year?

Exercise 58.—Written Review.

1. $\frac{5}{16} + \frac{7}{8} + \frac{3}{4} = ?$

2. $\frac{13}{24} \times \frac{8}{16} \times \frac{3}{26} = ?$

3. $\frac{14}{15} \div \frac{3}{5} = ?$

4. If a man is paid $5.00 per day and worked every day in the month of October (including Sundays) how much would he be paid?

5. From Feb. 8th, 1917, to May 31st, 1917, equals how many days?

6. $33\frac{1}{3} \div 18\frac{2}{3} = ?$

7. $66\frac{2}{3} \div 5\frac{5}{9} = ?$

8. $6\frac{1}{2} + 2\frac{1}{3} + 12\frac{2}{3} = ?$

9. $1{,}000 - 188\frac{2}{3} = ?$

10. $3{,}000\frac{1}{2} - 1{,}000\frac{3}{4} = ?$

ARITHMETIC

Copy and multiply:

(Time for these 2 examples is less than 6 minutes.)

11. 47,928 × 3,847;

12. 41,848 × 296;

Subtract, without copying:

(Time for these 11 examples is less than 6 minutes.)

13. 83,287,421 − 56,798,837;

14. 47,389,864 − 9,792,875;

15.	**16.**	**17.**	**18.**	**19.**
1,426	987	487	743	386
829	498	246	451	196

20.	**21.**	**22.**	**23.**
1,297	1,471	1,217	1,291
948	691	799	987

Add, without copying:

(Time for these 4 examples is less than 6 minutes.)

24. 8,978 + 2,747 + 4,728 + 9,628 + 7,170;

25. 6,626 + 2,747 + 1,928 + 9,947 + 2,120;

26. 4,876 + 3,921 + 4,687 + 3,842 + 8,468;

27. 4,898 + 5,487 + 3,968 + 9,421 + 4,861;

Copy and divide:

(Time for these 3 examples is less than 6 minutes.)

28. 17,873,352 ÷ 378;

29. 1,776 ÷ 48;

30. 3,604 ÷ 68.

DENOMINATE NUMBERS

LESSON 32

One-Step Reduction

A "denominate number" is a number which is used with the name of a measure, as 4 feet; 6 pints; 10 hours, etc.; therefore, every denominate number is a concrete number. However, not every concrete number is a denominate number, because a number is concrete when used with the name of any object, not necessarily with the name of a measure.

Very often it is necessary to change the denomination of a number without changing its value; this we call "reduction." Thus, 6 feet can be reduced to 72 inches (12 in. × 6 because 12 inches = 1 foot), or 6 feet can be reduced to 2 yards (6 ft. ÷ 3 ft. because 3 feet = 1 yard).

When we reduce to a smaller denomination we multiply the number which indicates how many units of the smaller denomination there are in 1 unit of the larger denomination by the number of units to be reduced; thus, in reducing 3 feet to inches, we multiply 12 in. by 3 because there are 12 inches in 1 foot and there are 3 ft. to be reduced. Remember—this is a multiplication step.

When we reduce to a larger denomination we divide by the number which indicates how many units of the smaller denomination there are in 1 unit of the larger denomination; thus, in reducing 6 feet to yards, we divide 6 ft. by 3 ft. because there are 3 feet in 1 yard, and the quotient tells us the number of times 3 ft. are found in 6 ft. Remember—this is a division step.

When a number is expressed in two denominations, as "8 weeks 6 days," and we desire to reduce it to the terms of the smaller denomination, we first find the number of days in 8 weeks, and then add the 6 days; thus, 8 wk. 6 da. = 62 days. To reduce this number to the terms of the larger denomination, we merely say $8\frac{6}{7}$ weeks, 6 days being $\frac{6}{7}$ of a week.

When a number is expressed in one denomination, as 62 days, and we desire to reduce it to two denominations, we divide 62 days by 7 days because there are 7 days in 1 week; thus, 62 da. ÷ 7 da. = $8\frac{6}{7}$, number of weeks, or, 8 wk. 6 da.

Exercise 59—Oral.

1. How many feet are there in 4 yd. 2 ft.? How many in 7 yd. 1 ft.?
2. How many inches are there in 6 ft. 6 in.? How many in 8 ft. 4 in.?
3. Reduce 8 ft. 3 in. to inches.
4. Reduce 4 qt. 1 pt. to pints.
5. Reduce 1 pt. 3 gi. to gills.
6. Reduce 3 gal. 1 qt. to quarts.
7. Reduce 4 pk. 3 qt. to quarts.
8. Reduce 6 bu. 2 pk. to pecks.
9. Reduce 3 wk. 4 da. to days.
10. Reduce 3 sq. yd. 2 sq. ft. to square feet.
11. How many gross are there in 19 dozen? How may dozen remain?

Reduce the following:

12. 30 sq. ft. to square yards and square feet.
13. 67 in. to feet and inches.
14. 14 ft. to yards and feet.
15. 43 da. to weeks and days.
16. 27 hr. to days and hours.
17. 18 oz. to pounds and ounces.
18. 7 gi. to pints and gills.
19. 19 qt. to gallons and quarts.
20. 23 pk. to bushels and pecks.
21. 23 pk. to bushels.
22. 21 qt. to pecks.
23. 24 oz. to pounds.
24. 30 in. to feet.
25. 40 sq. ft. to square yards.
26. 9 da. to weeks.
27. 90 sec. to minutes.
28. 18 doz. to gross.
29. 175 lb. to hundredweight.

Exercise 60—Written.

Reduce the following:

1. 14 da. 8 hr. to hours.
2. 45 min. 30 sec. to seconds.
3. 13 hr. 45 min. to minutes.
4. 2 yr. 214 da. to days.
5. 40 cwt. 50 lb. to pounds.
6. 8 T. 12 cwt. to hundredweight.
7. 4 mi. 220 rd. to rods.
8. 6 sq. ft. 72 sq. in. to square inches.
9. 12 hr. 17 min. to minutes.

10. 4 cu. ft. 864 cu. in. to cubic inches.
11. 1,000 rd. to miles and rods.
12. 865 sq. in. to square feet and square inches.
13. 1,512 sq. in. to square feet and square inches.
14. 438 da. to years and days.
15. 214 hr. to days and hours.
16. 1,002 min. to hours and minutes.
17. 468 sec. to minutes and seconds.
18. 71 cwt. to tons and hundredweight.
19. 4,450 lb. to hundredweight and pounds.
20. 430 oz. to pounds and ounces.
21. 430 oz. to pounds.
22. 448 da. to years.
23. 748 sq. in. to square feet.
24. 1,760 rd. to miles.
25. 1,013 cu. ft. to cubic yards.
26. How many square inches are there in a rectangle 4 ft. 3 in. × 6 ft. 8 in.? How many square feet and square inches? How many square feet?
27. What is the area of a square if one of the sides is 8 yd. 2 ft. long? Reduce your answer to square yards and square feet.
28. What is the area of a parallelogram with an altitude 4 ft. 6 in. and a base 7 ft. 3 in.? Reduce your answer to square feet and square inches.
29. What is the area of a triangle with an altitude 2 ft. 4 in. and a base 1 ft. 7 in.? Reduce your answer to square feet and square inches.
30. What is the area of a triangle with an altitude 2 yd. 2 ft. and a base 2 yd.? Reduce your answer to square yards and square feet.

DENOMINATE NUMBERS

LESSON 33

Addition of Denominate Numbers

EXAMPLE:

ft.	in.
3	6
4	9
8	
	14
17	5

The sum of the first column is 29 in. which is reduced to 2 ft. 5 in.; the 5 inches are written in the inch column and the 2 feet are carried to the feet column; the sum of the second column, including the 2 feet carried from the inch column, is 17 feet.

In adding denominate numbers, a separate column must be used for each denomination to be added. After finding the sum of the addends of the smallest denomination, the sum must be reduced so that all units of a larger denomination which are contained in it may be carried and added to the addends of the larger denomination.

EXAMPLE:

lb.	oz.
4	7
3	9
7	4
	8
15	12

The sum of the first column is 28 oz. which is reduced to 1 lb. 12 oz.; the 12 ounces are written in the ounce column and the 1 pound is carried to the pound column; the sum of the second column, including the 1 pound carried from the ounce column, is 15 pounds.

Exercise 61—Written.

Add and prove:

1.

da.	hr.
16	8
37	4
21	7
3	19

2.

lb.	oz.
14	7
19	8
21	3
16	8

3.

cwt.	lb.
9	50
17	75
23	46
18	72

4.

hr.	min.
4	37
8	42
6	19
28	30

5.

bu.	pk.
4	2
12	3
14	1
17	

6.

cu. yd.	cu. ft.
30	15
48	25
86	12
39	20

7. A butcher received four orders for beef; the first order was for 3 lb. 8 oz.; the second for 2 lb. 12 oz.; the third for 5 lb. 6 oz.; the fourth for 1 lb.; how much beef did he sell in all? How much money did he receive if he sold the beef for 32¢ per lb.?

8. A certain building lot is 125 ft. 8 in. long and 30 ft. 6 in. wide; what is the perimeter of this rectangle?

9. In a certain building, four of the halls were covered with tile; how many square yards of tiling were used if one of the halls contained 7 sq. yd. 7 sq. ft., another contained 12 sq. yd. 4 sq. ft., the third contained 14 sq. yd., and the fourth 10 sq. yd. 8 sq. ft.? What was the total cost of laying this tiling @ $5.40 per sq. yd.?

10. A train made four round trips between two cities in the following periods of time: 2 hr. 39 min.; 2 hr. 41 min.; 2 hr. 40 min.; 2 hr. 38 min.; how many hours did the train travel in all.

DENOMINATE NUMBERS

LESSON 34

Subtraction of Denominate Numbers

EXAMPLE:

ft.	in.
3	4
1	8
1	8

As 8 in. cannot be subtracted from 4 in., 1 of the 3 feet must be changed to 12 in. and added to the 4 in., making 16 in.; 16 in. − 8 in. = 8 in.; 2 ft. − 1 ft. = 1 ft.

WORKED:

ft.	in.
$\overset{2}{\cancel{3}}$	$\overset{16}{\cancel{4}}$
1	8
1	8

As in addition, a separate column must be used for each denomination to be subtracted.

When the subtrahend of any denomination is larger than the minuend of that denomination, 1 unit of the next larger denomination of the minuend must be changed into units of the smaller denomination and added to the other units of that denomination before subtracting.

Exercise 62—Written.

Subtract and prove by addition:

1.

lb.	oz.
118	6
40	10

2.

sq. ft.	sq. in.
8	17
3	18

3.

cu. ft.	cu. in.
26	400
12	900

4.

min.	sec.
40	18
13	15

5.

hr.	min.
10	45
4	50

6.

gr.	doz.
17	4
8	9

7. A merchant sold 8 yd. 2 ft. of muslin from a bolt which contained 25 yd.; how much muslin remained? What was it worth @ 9¢ per yd.?
8. A grocer sold 8 lb. 8 oz. of coffee from a bin containing 28 lb. 4 oz.; how much coffee remained? What was it worth @ 24¢ a pound?
9. If a man can walk a mile in 15 min. 45 sec., and he can run the same distance in 5 min. 55 sec., how much longer does it take him to walk than to run?
10. For the construction of a certain building, 36 T. 4 cwt. of steel were required; how much of this steel was still to be received after 19 T. 17 cwt. had arrived?

LESSON 35

Multiplication of Denominate Numbers

EXAMPLE: 8 ft. 3 in. × 6 = ?

ft.	in.
8	3
	6
49	6

The product of 3 in. × 6 is 18 in.; this is reduced to 1 ft. 6 in.; write 6 in. in the product and carry 1 ft.; the product of 8 ft. × 6 is 48 ft.; 48 ft. + 1 ft. carried is 49 ft.

The product of the smallest denomination must be reduced so that all units of a larger denomination which are contained in it may be carried and added to the product found by multiplying the larger denomination, just as we carry all tens from the units' product to tens' place in ordinary multiplication.

Exercise 63—Written.

Multiply:

1.	da.	hr.	2.	hr.	min.	3.	mi.	rd.
	7	4		13	45		4	160
		8			10			6

4.	sq. ft.	sq. in.	5.	cu. yd.	cu. ft.	6.	gt. gr.	gr.
	4	100		19	14		14	6
		4			6			7

7. What is the total area of 8 rugs each of which contains 13 sq. yd. 3 sq. ft.? What is the total value of these rugs @ $5.00 per sq. yd.?
8. If it takes 4 hr. 48 min. to do a certain piece of work, how long will it take to do 10 times as much work?
9. How many sq. ft. of glass will be needed for 12 windows if the upper sash of each window contains 8 sq. ft. 72 sq. in., and the lower sash of each window contains 14 sq. ft. 36 sq. in.? What will be the value of this glass @ 20¢ per sq. ft.?

LESSON 36

Division of Denominate Numbers

EXAMPLE (Short Division): 7 hr. 40 min. ÷ 2 = ?

hr.	min.
3	50
2)7	40

7 hr. ÷ 2 = 3 hr. and 1 hr. remaining to be changed into minutes which are added to the other 40 min. of the dividend; 60 + 40 = 100 min.; 100 min. ÷ 2 = 50 min.

In dividing a denominate number, we divide the largest denomination first, and any remainder from this denomination is changed into units of the next smaller denomination which are added to the other units of that denomination before dividing it. The remainder from the smallest denomination is written in the form of a fraction as in ordinary division.

EXAMPLE (Long Division): 53 sq. yd. 3 sq. ft. ÷ 40 = ?

	sq. yd.		sq. ft.	
	1		3	Quotient.
40)	53		3	
	40			
	13	=	117	
			120	
			120	

Here the remainder, 13 sq. yd., is changed into 117 sq. ft. which are added to the 3 sq. ft. making 120 sq. ft.; 120 sq. ft. ÷ 40 = 3 sq. ft.

Exercise 64—Written.

Divide and prove by multiplication:

1. 46 hr. 21 min. ÷ 3 = ?
2. 13 lb. 9 oz. ÷ 5 = ?
3. 43 gal. 3 qt. ÷ 7 = ?
4. 129 bu. 2 pk. ÷ 14 = ?
5. 1,146 lb. ÷ 24 = ?
6. 11 cu. yd. 7 cu. ft. ÷ 16 = ?
7. A roll of wrapping paper containing 53 sq. yd. 3 sq. ft. was entirely used in wrapping 40 packages; how much paper was used for each package?
8. What is the average weight of each of the following 5 machines: The first weighs 5 cwt. 43 lb.; the second weighs 3 cwt. 86 lb.; the third weighs 6 cwt.; the fourth weighs 4 cwt. 72 lb.; the fifth weighs 744 lb.?

9. If you cut a sheet of paper 1′ 5″ wide into 3 equal strips, how wide will each strip be?
10. The distance around a square is 14 yd. 2 ft.; what is the length of one side? What is the area of the square?
11. In working 4 examples you required:

    8 min. 15 sec. for the first,
    7 min. 45 sec. for the second,
    5 min. 10 sec. for the third,
    6 min. 15 sec. for the fourth.

    What was the total time required? What was the average time required for each example?

Exercise 65—Oral.

1. What is a denominate number?
2. Changing the form of a denominate number without changing the value is called what?
3. How do we reduce a denominate number to the next smaller denomination?
4. How do we reduce a denominate number to the next larger denomination?
5. In adding denominate numbers why must a separate column be used for each denomination?
6. In adding denominate numbers what is done with the sum of the smallest denomination before finding the sum of the next larger denomination?
7. In subtracting denominate numbers what is done when the subtrahend of any denomination is larger than the minuend of that denomination?
8. In multiplying denominate numbers what is done with the product of the smallest denomination

before finding the product of the next larger denomination?

9. In dividing denominate numbers what is done with the remainder found by dividing the largest denomination?
10. In dividing denominate numbers what is done with the remainder found by dividing the smallest denomination?

Reduce:

11. 15 in. to feet and inches.
12. 5 yd. to feet.
13. 17 da. to weeks and days.

Add:

14. 2 wk. 4 da. + 3 wk. 3 da.
15. 5 yd. 2 ft. + 4 yd. 2 ft.
16. 6 ft. 3 in. + 7 yd. 10 in.
17. 3 hr. 30 min. + 1 hr. 30 min.

Subtract:

18. 2 yd. − 1 ft.
19. 3 qt. − 1 pt.
20. 10 lb. − 8 oz.
21. 5 hr. − 50 min.

Multiply:

22. 3 ft. 2 in. × 6.
23. 5 yd. 1 ft. × 3.
24. 1 bu. 2 pk. × 4.
25. 5 lb. 8 oz. × 2.

Divide:

26. 3 yd. 1 ft. ÷ 2.
27. 8 lb. 4 oz. ÷ 4.
28. 1 hr. 30 min. ÷ 3.

AVERAGE

LESSON 37

Averaging by Groups

EXAMPLE: A contractor was paid $4,350.00 for building one house, and $5,400.00 for building another; what was the average amount he received for building each of the two houses?

$4,350.00 For 1 house.
$5,400.00 For 1 house.
$9,750.00 For 2 houses.

$9,750.00 ÷ 2 = $4,875.00, Average for each house.

You have already learned that the average of several unlike numbers is found by adding the numbers and dividing the sum by the number of addends.

EXAMPLE: A merchant sold 6 pairs of shoes @ $3.00 each, 4 pairs @ $3.25 each, and 2 pairs @ $4.00 each; what was the average selling price per pair?

($3.00 Group)	6 pairs	@ $3.00	=	$18.00
($3.25 Group)	4 pairs	@ 3.25	=	13.00
($4.00 Group)	2 pairs	@ 4.00	=	8.00
(Total Sales)	12 pairs			$39.00

$39.00 ÷ 12 = $3.25, Average for each pair of shoes.

When the same number appears several times among the numbers to be averaged, arrange the numbers in groups and use the product of each group to save time in finding the sum of all the numbers.

Exercise 66—Written.

1. A haberdasher sold 15 ties as follows: 4 @ 50¢, 3 @ 60¢, 8 @ 80¢; what was the average selling price per tie?
2. A tailor bought the following cloth:

    6 bolts containing 10 yd. 2 ft. each,
    4 bolts containing 8 yd. 1 ft. each,
    2 bolts containing 9 yd. 1 ft. each.

    What was the average quantity in each bolt? What was the total cost @ $3.00 per yard?
3. When Tom went to work he received $5.00 per week for the first 13 weeks, $6.00 per week for the second 13 weeks, and $7.00 per week for the next 26 weeks; what was his average weekly salary for the first year he worked?
4. A stationer sold 18 pencils @ 30¢ a dozen, 24 pencils @ 40¢ a dozen, and 18 pencils @ 50¢ a dozen; what was the average selling price per dozen pencils?
5. A butcher sold 3½ lb. meat @ 28¢ per lb., 2½ lb. @ 30¢ per lb., 4 lb. 8 oz. @ 32¢ per lb.; what was the total selling price? What was the average selling price per lb.?
6. A grocer sold 4½ doz. eggs @ 28¢ a doz., 3¼ doz. @ 32¢ a doz., and 6⅓ doz. @ 33¢ a doz.; what was the total selling price? What was the average selling price per doz.?
7. A farmer bought 100 A. of land @ $110.00 per A., and 100 A. @ $120.00 per A.; what was the average cost per A.?

AVERAGE

Exercise 67—Oral Review.

1. In what way is a square different from an oblong? Tell all you know about each.
2. In what way is a cube different from a right prism? Tell all you know about each.
3. How do you explain the fact that every rectangle is a parallelogram but not every parallelogram is a rectangle?
4. What would you measure by Liquid Measure?
5. Say the table of Dry Measure.
6. Tell something you buy for which you use Linear Measure.
7. Say the table of Square Measure.
8. What is Cubic Measure used for?
9. Give the table of Time Measure.
10. Write the table used in counting merchandise.

Exercise 68—Written Review.

1. $\frac{14}{24} \times \frac{8}{28} \times \frac{9}{16} = ?$
2. $\frac{38}{72} + \frac{19}{48} = ?$
3. $(96 \div 12) + (24 - 8) \times (\frac{1}{2} \text{ of } 6) = ?$
4. The dimensions 14′ and 20′ will produce what area if they represent:
    - (*a*) The length and width of an oblong?
    - (*b*) The length and width of a rectangle?
    - (*c*) The altitude and base of a parallelogram?
    - (*d*) The altitude and base of a triangle?
5. $45\frac{5}{11} \div 27\frac{3}{11} = ?$
6. Reduce:
    - (*a*) 49 da. 7 hr. to hours;
    - (*b*) 12 hr. 15 min. to minutes;

(*c*) 8 T. 3 cwt. to hundredweight;

(*d*) 5 cu. ft. 144 cu. in. to cubic inches.

7. 19 lb. 7 oz. + 43 lb. 8 oz. + 36 lb. 13 oz. + 10 lb. 6 oz. = ?

8. 44 sq. ft. 18 sq. in. – 16 sq. ft. 39 sq. in. = ?

9. 5 mi. 80 rd. × 8 = ?

10. 196 hr. 48 min. ÷ 36 = ?

11. A stove manufacturer sold 18 style A stoves @ $10.00 each, 32 style B stoves @ $15.00 each, and 40 style C stoves @ $19.50 each; what was the average selling price? What was the average profit per stove if the average cost of the stoves was $12.00?

Subtract, without copying:

(Time for these 19 examples is less than 6 minutes.)

12.	**13.**	**14.**	**15.**	**16.**
4,026	4,728	7,450	5,787	9,771
298	742	497	289	865

17.	**18.**	**19.**	**20.**	**21.**
9,686	2,717	4,727	1,664	6,581
598	896	729	797	384

22.	**23.**	**24.**	**25.**	**26.**
48,947	81,147	92,142	19,417	74,717
29,788	70,959	39,716	8,629	35,567

27.	**28.**	**29.**	**30.**
62,727	28,712	47,472	26,483
35,818	9,847	18,956	17,684

AVERAGE

Add, without copying:

(Time for these 10 examples is less than 6 minutes.)

31.	**32.**	**33.**	**34.**	**35.**
72,846	47,317	62,117	61,912	97,289
94,721	92,949	94,212	92,847	39,634
29,721	42,120	74,788	76,795	46,813

36.	**37.**	**38.**	**39.**	**40.**
47,863	86,889	24,874	88,947	48,288
47,912	96,387	86,487	99,327	49,329
27,121	48,499	98,383	19,474	99,479

Copy and multiply:

(Time for these 5 examples is less than 6 minutes.)

41. 34,787 × 4,799;
43. 4,796 × 84;
42. 26,943 × 3,787;
44. 8,673 × 409;
45. 608 × 21.

Copy and divide:

(Time for these 5 examples is less than 6 minutes.)

46. 6,768 ÷ 94;
48. 3,444 ÷ 82;
47. 2,666 ÷ 62;
49. 6,768 ÷ 94;
50. 7,221 ÷ 87.

Add, without copying:

(Time for these 5 examples is less than 4 minutes.)

51.	**52.**	**53.**	**54.**	**55.**
36,872	73,439	63,971	73,829	54,387
49,386	17,827	46,849	64,739	36,719
25,437	46,438	63,874	56,372	16,874
63,943	76,573	77,236	36,417	46,876

Exercise 69—Written.

The unit of measure used in roofing is the "square," this being an area of 100 sq. ft.

Shingles used in roofing are usually 16 in. or 18 in. long and 4 in. wide, but they are laid overlapping one another with only 4 in. or 4½ in. of the length of the shingle exposed to the weather. If 4 in. are exposed, 900 shingles will cover a square; if 4½ in. are exposed, 800 shingles will cover a square; however, including waste, 1,000 shingles are usually allowed for each square. As shingles come in bunches of 250 each, 4 bunches must be allowed for each square. A fraction of a bunch cannot be bought.

1. The roof of a bungalow has two slopes, each 50 ft. long and 15 ft. wide; answer the following:
   (*a*) What is the area of this entire roof?
   (*b*) How many squares are there in this roof?
   (*c*) How many bunches of shingles must be bought to cover this roof?
   (*d*) If shingles cost $2.60 per bunch, find the cost of the shingles needed for this roof.
   (*e*) The roofer was paid $6.00 per day and laid 2 squares per day; what did he earn?

2. Find the cost of shingling the two 45′ × 12′ slopes of a roof, using shingles costing $2.75 per bunch, nails, etc., costing $7.25, and the roofer receiving $6.25 per day for 5 days.

3. Find the number of pieces of slate needed to cover a roof having an area of 900 sq. ft. if the slate is 16″ long 9″ wide with 4″ of the length of each piece exposed to the weather.

ARITHMETIC

PART VI

INTERMEDIATE LESSONS

INTERMEDIATE LESSONS

PART VI

DECIMALS

LESSON 1

How Decimals Are Formed

In your previous work you have learned that the value of any digit depends upon the "place" in which it is written; for, as you know, when a digit is moved to the left from any place to the place of the next higher order, its value is multiplied by 10; and when it is moved to the right from any place to the place of the next lower order, its value is divided by 10; thus, reading toward the left, each of the digits "1" in the number 1,111,111 has a value 10 times as great as the digit in the place of the next lower order, and reading toward the right, each digit has a value $\frac{1}{10}$ as great as the digit in the place of the next higher order. It is for this reason that the Arabian system of notation which we use is called a "decimal" system of notation, because "decimal" means "numbered by tens."

Now, since the value of a digit written in units' place is $\frac{1}{10}$ as great as the value of the same digit written in tens' place, if, instead of stopping with units' place, we use another place to the right of units' place, the value of any digit written in this new place will be $\frac{1}{10}$ as great as if it were written in units' place, and will therefore represent so many "tenths"; and if we use still another place to the right of "tenths' place" the value of any digit written in this place will be $\frac{1}{10}$ as great as so many tenths, and will therefore be so many "hundredths," $\frac{1}{10}$ of $\frac{1}{10}$ being $\frac{1}{100}$. Therefore, any fraction with a denominator of 10 can be written as a "decimal fraction" or, to use a more common expression, as a "decimal," by merely writing the numerator of the fraction in tenths' place; and any fraction with a denominator of 100 can be written as a decimal by merely writing the numerator of the fraction in tenths' and hundredths' places.

In writing decimals, a period (.) which is called a "decimal point" is used to separate the decimal fraction from the whole number or "integer."

This is the same principle which you have used in all of your work with United States money, as "ten cents" is written $0.10 because it is one dime or $\frac{1}{10}$ of $1. and "one cent" is written $0.01 because it is $\frac{1}{100}$ of $1.

In reading decimals the word "and" is used where the decimal point appears; thus, 1.3 is read "one and three tenths," just as if it were written $1\frac{3}{10}$.

Exercise 1—Oral.

1. What part of one dollar is ten cents?
2. What part of one dollar is one cent?
3. How many tenths of dollars are there in each of these amounts?

$0.40; $0.60; $0.80; $0.30; $0.90;

4. How many hundredths of dollars are there in each of these amounts?

$0.05; $0.39; $0.46; $0.73; $0.98;

5. How many units, tenths and hundredths of dollars are there in each of these amounts?

$4.30; $5.73; $6.01; $7.58; $9.67;

6. How many units, tenths and hundredths are there in each of these numbers?

(*a*) .3; .6; .8; .9; .4;
(*b*) .36; .08; .72; .33; .07;
(*c*) 4.68; 3.5; 9.06; 8.75; 4.3;
(*d*) 5.06; 7.10; 6.11; .01; 1.75;

7. Why do we use a decimal point between units' place and tenths' place?

8. Moving any digit one place to the left to the place of the next higher order has what effect on its value?

9. Moving any digit one place to the right to the place of the next lower order has what effect on its value?

10. What are the names of the first seven places to the left of the decimal point?

11. What is the name of the first place to the right of the decimal point?

12. What is the name of the second place to the right of the decimal point?

13. How do we write any number of tenths in decimal form?

14. How do we write any number of hundredths in decimal form?

15. What name is given to a whole number?

Exercise 2—Written.

Write in decimal form:

1. $\frac{3}{10}$; $\frac{4}{10}$; $\frac{6}{10}$;
2. $\frac{8}{10}$; $\frac{7}{10}$; $\frac{5}{10}$;
3. $\frac{16}{100}$; $\frac{21}{100}$; $\frac{8}{100}$;
4. $\frac{92}{100}$; $\frac{48}{100}$; $\frac{73}{100}$;
5. $1\frac{5}{10}$; $8\frac{7}{10}$; $14\frac{9}{10}$;
6. $28\frac{3}{10}$; $46\frac{1}{10}$; $58\frac{6}{10}$;
7. $8\frac{8}{100}$; $48\frac{72}{100}$; $59\frac{18}{100}$;
8. $76\frac{49}{100}$; $98\frac{98}{100}$; $21\frac{4}{100}$;
9. 6; $\frac{6}{10}$; $\frac{6}{100}$;
10. $\frac{60}{100}$; $6\frac{6}{10}$; $6\frac{6}{100}$;
11. $1\frac{4}{10}$; $\frac{73}{10}$; $\frac{46}{10}$;
12. $1\frac{73}{100}$; $\frac{689}{100}$; $\frac{472}{100}$.

Write in the form of common fractions or mixed numbers:

13. .5; .7; .9;
14. .4; .8; .6;
15. .46; .78; .93;
16. .06; .11; .10;
17. 1.4; 4.8; 8.3;
18. 11.46; 46.31; 18.09;
19. 8.; .8; .08;
20. .88; 8.8; 8.80;

Lesson 2

The Value of Ciphers in Decimals

Since the value of every digit depends upon the place that it occupies, ciphers written to the right of the decimal point cannot possibly change the place or value of any digit written to the left of the decimal point, and can change the place and value of digits written to the right of the decimal point only if the ciphers are written between the decimal point and the digits; thus, 6.; 6.0; 6.00 are all alike in value, but 6.0; .6; .06 have entirely different values.

The third place to the right of the decimal point is "thousandths' place," because $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$, which is written .001

The fourth place is "ten-thousandths' place," because $\frac{1}{10}$ of $\frac{1}{1\,000}$ = $\frac{1}{10\,000}$, which is written .0001

The fifth place is "hundred-thousandths' place," because $\frac{1}{10}$ of $\frac{1}{10\,000}$ = $\frac{1}{100\,000}$, which is written .00001

The sixth place is "millionths' place," because $\frac{1}{10}$ of $\frac{1}{100\,000}$ = $\frac{1}{1\,000\,000}$, which is written .000001

It will help you greatly to read and write decimals if you remember that the number of decimal places in any decimal fraction is always the same as the number of ciphers that there would be in the denominator of that fraction were it written as a common fraction; thus:

$\frac{1}{10}$ = .1 (one cipher, one place) one tenth.
$\frac{1}{100}$ = .01 (two ciphers, two places) one hundredth.
$\frac{1}{1\,000}$ = .001 (three ciphers, three places) one thousandth.
$\frac{1}{10\,000}$ = .0001 (four ciphers, four places) one ten-thousandth.
$\frac{1}{100\,000}$ = .00001 (five ciphers, five places) one hundred-thousandth.
$\frac{1}{1\,000\,000}$ = .000001 (six ciphers, six places) one millionth.

Also remember that we read the entire decimal fraction first, this being the numerator of the fraction, then we supply the denominator; thus:

.465 is read "Four hundred sixty-five thousandths."
.0187 is read "One hundred eighty-seven ten-thousandths."
.00864 is read "Eight hundred sixth-four hundred-thousandths."

Exercise 3—Oral.

1. Can a cipher written to the right of the decimal point have any effect on the value of the integer which is written to the left of the decimal point? Why?
2. Can a cipher written to the right of the decimal point have any effect on the value of the decimal fraction if the cipher is not followed by any digit? Why?
3. What is the effect of writing a cipher between the decimal point and a decimal fraction?
4. What are the names of the first six places to the right of the decimal point?
5. If a common fraction has four ciphers in its denominator, how many decimal places will the corresponding decimal fraction have? What is the general rule regarding this?
6. When reading a decimal fraction do we read the entire decimal fraction before supplying the denominator? What is the general rule regarding this?

Read:

7. (*a*) 48.8; 97.46; 88.09;
(*b*) 146.875; 138.075; 149.009;
8. (*a*) 96.4763; 38.0796; 58.0018;
(*b*) 153.48763; 91.86087; 59.00009;
9. (*a*) 46.87; 39.0089; 53.000016;
(*b*) 96.3487; 143.00101; 48.100001;
10. (*a*) 306,070.4; 30,607.04; 3,060.704;
(*b*) 306.0704; 30.60704; 3.060704;

DECIMALS

Exercise 4—Oral and Written. .

Write in decimal form:

1. $\frac{6}{10}$; $\frac{8}{10}$;
2. $\frac{46}{100}$; $\frac{78}{100}$;
3. $\frac{586}{1\,000}$; $\frac{785}{1\,000}$;
4. $\frac{5\,687}{10\,000}$; $\frac{87}{10\,000}$;
5. $\frac{58\,726}{100\,000}$; $\frac{63}{1\,000}$;
6. $\frac{837\,462}{1\,000\,000}$; $\frac{19}{1\,000\,000}$;
7. $46\frac{38}{10\,000}$; $58\frac{7\,876}{10\,000}$;
8. $1{,}487\frac{3}{100}$; $14\frac{873}{1\,000}$;
9. $\frac{4\,687}{1\,000}$; $\frac{98\,763}{100}$;
10. $\frac{847}{10}$; $\frac{93\,687}{10\,000}$.

Read the following, then write them in the form of common fractions or mixed numbers:

11. .68; 9.87;
12. 86.346; 186.005;
13. .4687; 98.7632;
14. .37421; 19.00006;
15. .387609; 51.876001;
16. 86.387; 46.0019;
17. 408,709.; 40,870.9;
18. 4,087.09; 408.709;
19. 40.8709; 4.08709;
20. .408709; 0.408709;

Write in words:

21. .7;
22. 1.4;
23. 32.14;
24. 98.05;
25. 10.1;
26. 158.75;
27. 86.405;
28. 70.075;
29. 108.635;
30. 75.01;

Write in decimal form:

31. One and seven tenths.
32. Fifty-five and eighteen hundredths.
33. Seventy-nine and one hundredth.
34. One hundred one and one hundredth.
35. Eighty-eight and one hundred fourteen thousandths.
36. Seven hundredths.
37. Seventy-five hundredths.

38. Five hundred six and five hundredths.

39. One thousand, forty-five and eight tenths.

40. Fourteen thousand, three hundred and three hundredths.

LESSON 3

Addition of Decimals

EXAMPLE:

46.8
6.93
18.06
137.49
93.
302.28

Starting in the place of the lowest order (in this case hundredths' place) we add and carry just as usual, because each ten hundredths ($\frac{10}{100}$) equals one tenth ($\frac{1}{10}$) to be carried to tenths' column; and each ten tenths ($\frac{10}{10}$) equals one unit (1) to be carried to units' column.

As you know very well, units must always be added to units, tens to tens, etc., because unlike quantities cannot be added. Following this principle, tenths must always be added to tenths, hundredths to hundredths, etc.; therefore, in writing decimal numbers to be added, always arrange the columns so that the decimal points will come one under the other.

Exercise 5—Oral and Written.

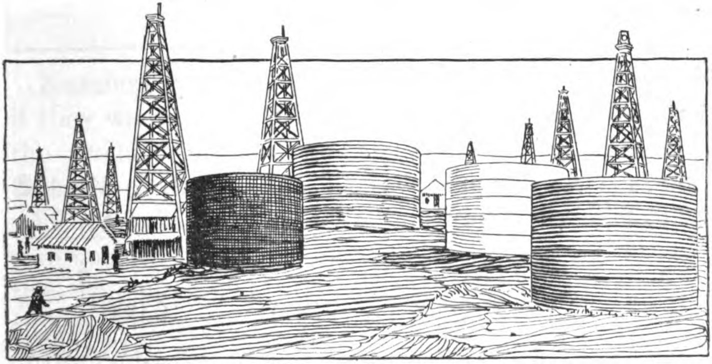
Read, then add and prove:

1.	2.	3.	4.
56.873	13.8627	96.06	147.03
875.42	147.0046	106.00008	28.634
938.763	318.6666	18.006	.13
46.0008	18.75	473.003	13.
114.0778	96.875	.0007	875.
18.37	43.006	13.01	8.75

5.	6.	7.	8.
36,872.	.10704	86.347	.14137
3,687.2	1.0704	294.13	1.4137
368.72	10.704	46.0002	14.137
36.872	107.04	.01347	14,137.
3.6872	1,070.4	100.	1,413.7
.36872	10,704.	.106	141.37

9. A certain farm had 17.46 A. planted in corn, 18.375 A. planted in oats, 39.065 A. planted in wheat, and 25.1 A. planted in rye; how many acres were planted?

10. A wholesaler sold five bolts of silk to a retailer; one bolt contained 14.625 yd., the second contained 16.75 yd., the third 18.5 yd., the fourth 15. yd., and the fifth 18.625 yd.; how many yards were there in all?



11. Four tanks contained the following quantities of oil: 130.387 gal., 145.206 gal., 137.428 gal., and 156.103 gal.; how many gallons were there in all?

12. What was the total weight of the following five car-loads of sand: 38.75 T., 41.375 T., 42.167 T., 39.625 T., and 40.667 T.?

Lesson 4

Subtraction of Decimals

EXAMPLE:

18.07
12.916
5.154

When there are more decimal places in the subtrahend then there are in the minuend, imagine that the minuend has ciphers in the vacant places and subtract as usual. When necessary we change 1 of any order into 10 of the next lower order in the minuend before subtracting.

In subtracting decimals, the decimal points must be written one under the other so that units can be subtracted from units, tenths from tenths, hundredths from hundredths, etc.

Exercise 6—Written.

Subtract and prove by addition:

1.	**2.**	**3.**	**4.**
56.874	792.473	43.872	316.402
32.613	308.219	21.936	187.12
5.	**6.**	**7.**	**8.**
521.87	418.72008	918.63	10,000.
218.7468	119.8635	427.3872	618.46375

9. In a certain tank there were 48.75 gal. of oil; if 19.375 gal. were sold out of this tank, how many gallons remained?

10. The area of one farm is 53.667 A.; the area of the adjoining farm is 118.75 A.; how much larger is the second farm than the first?

11. If a sheet of paper containing 9 sq. ft. is cut into two parts, and one of the parts contains 19.625 sq. in., what will be the area of the other part?

LESSON 5

Multiplication of Decimals

EXAMPLE: $5.72 \times 14 = ?$

$$\begin{array}{r} 5.72 \\ \underline{14} \\ 2288 \\ \underline{572} \\ 80.08 \end{array}$$

The product contains two decimal places because 5.72 is the same as $\frac{572}{100}$, and $\frac{572}{100} \times 14 = \frac{8008}{100}$ or 80.08;

(2 decimal places in the multiplicand + 0 decimal places in the multiplier = 2 decimal places in the product.)

Numbers containing decimals are multiplied just as if they were whole numbers or integers, excepting that the product will contain as many decimal places as the sum of the decimal places in the multiplicand and multiplier, for the same reason that in multiplying common fractions with denominators of 10, 100, etc., the denominator of the product contains as many ciphers as the sum of the ciphers in the denominators of the multiplicand and multiplier. Thus:

$3 \times .1 = .3$ because $3 \times \frac{1}{10} = \frac{3}{10}$;

$.3 \times .1 = .03$ because $\frac{3}{10} \times \frac{1}{10} = \frac{3}{100}$;

$.03 \times .1 = .003$ because $\frac{3}{100} \times \frac{1}{10} = \frac{3}{1000}$.

EXAMPLE: 13.41 × 8.7 = ?

$$\begin{array}{r} 13.41 \\ \underline{8.7} \\ 9387 \\ \underline{10728} \\ 116.667 \end{array}$$

The product contains three decimal places because 13.41 is the same as $\frac{1341}{100}$, and 8.7 is the same as $\frac{87}{10}$; $\frac{1341}{100} \times \frac{87}{10} = \frac{116667}{1000}$ or 116.667; (2 decimal places in the multiplicand + 1 decimal place in the multiplier = 3 decimal places in the product.)

EXAMPLE: .0047 × 2.7 = ?

$$\begin{array}{r} .0047 \\ \underline{2.7} \\ 329 \\ \underline{94} \\ .01269 \end{array}$$

Ciphers appearing between the decimal point and the other digits, like those in .0047, must never be omitted, otherwise the product cannot be properly pointed off. (4 decimal places in the multiplicand + 1 decimal place in the multiplier = 5 decimal places in the product, but as the product has only 4 places when the multiplication is completed, a cipher is written to the left of the product before the decimal point is written, making 5 decimal places.)

Exercise 7—Oral..

Multiply:

1. 4 × .2; 3 × .3; 9 × .1;
2. .4 × 3; 6 × .4; 18 × .1;
3. .3 × .2; .3 × .3; .8 × .1;
4. .5 × .5; .6 × .9; 1.2 × .8;
5. .11 × 6; .12 × 12; 1.1 × 1.1;

6. .08 × 8; 9 × .07; 12 × .06;
7. .06 × .9; .5 × .11; .12 × .7;
8. 1.2 × .04; .11 × 1.2; 1.2 × .12;
9. .008 × 7; .012 × 9; .011 × 11;
10. .6 × .08; .5 × 12; .8 × .5;

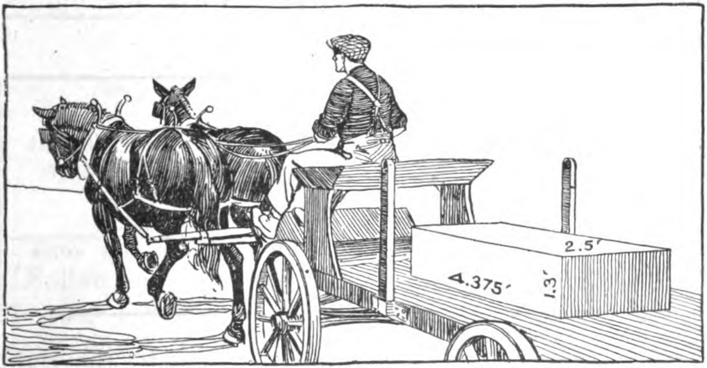
Exercise 8—Oral and Written.

Can you tell how you will point off in your product before you begin?

1.	**2.**	**3.**	**4.**
63.9	53.8	5.87	3.87
42	5.9	1.23	.462

5.	**6.**	**7.**	**8.**
.063	.042	.147	4.26
.105	.008	.021	11.3

9. What is the area of a building lot 125.8 ft. deep and 34.4 ft. wide?



10. What is the volume of a block of granite 1.3′ × 2.5′ × 4.375′?

11. What is the weight of 439 books, if each book weighs 1.43 lb.?
12. What is the value of 8.375 yd. silk @ $4.25 per yd.?

LESSON 6

Multiplication by Moving the Decimal Point

EXAMPLE:

486.32 × 10 = 4,863.2

PROOF:

$$\begin{array}{r} 486.32 \\ \underline{10} \\ 4{,}863.20 \end{array}$$

Note: Ciphers in the ending of a decimal have no value and should be dropped. In this example .20 is the same as .2 since $\frac{20}{100} = \frac{2}{10}$.

Since units' place is always the first place to the left of the decimal point, moving the decimal point one place to the right will naturally multiply any number by 10, because the place which was units' place then becomes tens' place, and the place which was tenths' place then becomes units' place.

EXAMPLE:

39.64 × 100 = 3,964.

PROOF:

$$\begin{array}{r} 39.64 \\ \underline{100} \\ 3{,}964.00 \end{array}$$

Compare your original number with the result:

39.64
3964.

What change has come about?

Moving the decimal point two places to the right multiplies any number by 100; three places multiplies by 1,000, etc.

EXAMPLE:

$$46.37 \times 1{,}000 = 46{,}370.$$

PROOF:

$$\begin{array}{r} 46.37 \\ 1000 \\ \hline 46{,}370.\cancel{0}\cancel{0} \end{array}$$

When there are not enough digits in the decimal to permit the decimal point to be moved, ciphers must be annexed.

EXAMPLE:

$$\begin{array}{r} 46.38 \times 50 = 463.8 \\ 5 \\ \hline 2{,}319.\cancel{0} \end{array}$$

PROOF:

$$\begin{array}{r} 46.38 \\ 50 \\ \hline 2{,}319.\cancel{0}\cancel{0} \end{array}$$

The principles which you have been using when multiplying whole numbers by multiples of 10, as 50 for example, are also used here. First move the decimal point one place to the right as you did to multiply by 10, then multiply by 5, because there are 5 tens in 50.

EXAMPLE:

$$\begin{array}{r} 398.7 \times 3{,}600 = 39870. \\ 36 \\ \hline 239220 \\ 119610 \\ \hline 1{,}435{,}320. \end{array}$$

PROOF:

$$\begin{array}{r} 398.7 \\ 3600 \\ \hline 2392200 \\ 11961 \\ \hline 1{,}435{,}320.\cancel{0} \end{array}$$

Following the same plan, to multiply by a multiple of 100, 1,000, etc., we move the decimal point as many places to the right as there are ciphers in the ending of the multiplier, then we complete the multiplication.

Exercise 9—Oral.

Multiply:

1. 4,687.9 × 10;
2. 972.46 × 10;
3. 15.687 × 10;
4. 86.37 × 100;
5. 5.963 × 100;
6. 934.7 × 100;
7. 8,749. × 100;
8. 368. × 1,000;
9. 49.3 × 1,000;
10. 8.65 × 1,000;
11. .478 × 1,000;
12. .059 × 1,000.

Exercise 10—Written.

Multiply and prove:

1.	**2.**	**3.**	**4.**
38.72	123.7	1.043	32.47
80	30	90	300
5.	**6.**	**7.**	**8.**
1.627	287.1	6321.	4.632
500	420	600	2000
9.	**10.**	**11.**	**12.**
13.71	.0123	.00463	137.2
3000	6100	5000	9000

(Time for all of these, 5 min.)

Exercise 11—Oral.

1. In adding and subtracting decimals, why must the decimal points be written one under the other?
2. In adding decimals what is done with the carrying figure from tenths' column?

3. When there are more decimal places in the subtrahend than there are in the minuend, how do we subtract?
4. Excepting the matter of pointing off the decimal places in the product, is there any difference between multiplying decimal numbers and integers or whole numbers?
5. How do we tell quickly how many decimal places there should be in the product in multiplying decimal numbers?
6. Why is the product of .1 × .1 pointed off .01?
7. How do we multiply a decimal number by 10? By 100? By 1,000?
8. Why is a number multiplied by 10 when the decimal point is moved one place to the right?
9. What should be done with ciphers in the ending of a decimal as .46300? Have these ciphers any value whatever?
10. What is done when there are not enough digits in the decimal to permit the decimal point to be moved two, three, four or any other number of places to the right as the case may be?
11. How do we multiply a decimal by 60? By 900? By 5,000?
12. How do we tell how many places to move the decimal point in multiplying? Do you move it to the right or to the left?

BUYING AND SELLING

LESSON 7

Bills and Invoices

A retail merchant or "retailer" is one who sells merchandise in small quantities to the public. Your grocer is a "retailer."

A wholesale merchant or "wholesaler" is one who sells merchandise in large quantities and usually deals only with other merchants. Your grocer buys of a "wholesaler."

A "manufacturer" is one who makes or manufactures merchandise.

You have already learned that when you buy merchandise in any large retail store (and in some small ones) you receive a sales-slip, sales-check, or bill showing the quantities and prices of the various articles you bought, the salesman's number, amount received, date, etc.; and when the goods are paid for, the sales-slip is receipted by the cashier before it is given to you.

Wholesalers and manufacturers follow a very similar method, but instead of calling their bills sales-slips, they call them "invoices," and as most wholesale sales are made on credit, the amount of the invoice is charged to the customer's account on the wholesaler's books.

In business, the sign # when written after a number means "pounds" (50#), but when it is written before a number it means "number" (#50); therefore, 50# means "50 pounds," but #50 means "number 50."

The Roman Numerals C meaning 100, and M meaning 1,000 are often used on invoices.

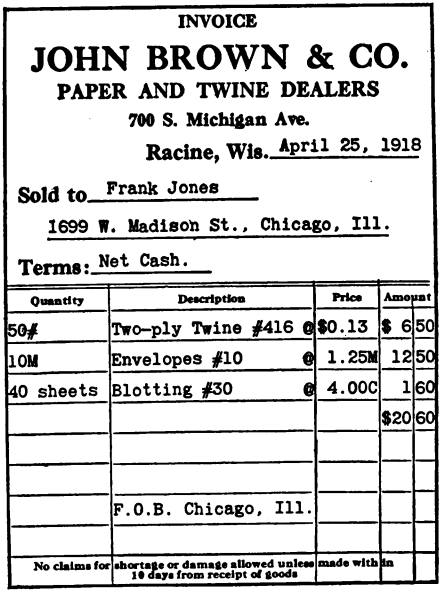
On the invoice here shown, you will see:

50# Two-ply Twine #416 @ $0.13
meaning 50 lb. Two-ply Twine
Number 416 @ 0.13

10 M Envelopes....#10 @ $1.25 M.
meaning 10,000 Envelopes Number 10 @ 1.25 per thousand.

40 sheets Blotting..#30 @ $4.00 C.
meaning 40 sheets Blotting Number 30 @ 4.00 per hundred.

The letters F. O. B. on an invoice mean "Free on board cars" at the city written after the letters; therefore, on this invoice F. O. B. Chicago, Ill., means that the shipper agrees to pay the freight to Chicago, Ill.



INVOICE

JOHN BROWN & CO.

PAPER AND TWINE DEALERS

700 S. Michigan Ave.

Racine, Wis. April 25, 1918

Sold to Frank Jones

1699 W. Madison St., Chicago, Ill.

Terms: Net Cash.

Quantity	Description	Price	Amount	
50#	Two-ply Twine #416 @	$0.13	$ 6	50
10M	Envelopes #10 @	1.25M	12	50
40 sheets	Blotting #30 @	4.00C	1	60
			$20	60
	F.O.B. Chicago, Ill.			
No claims for	shortage or damage allowed unless 10 days from receipt of goods	made with	in	

Exercise 12—Oral.

1. What name is given to a merchant who sells in small quantities to the public?

2. What name is given to a merchant who sells in large quantities and usually deals with other merchants only?
3. What name is given to one who makes or manufactures an article?
4. What name is usually given to the bills of wholesalers and manufacturers?
5. What does the sign # mean when written before a number?
6. What does the sign # mean when written after a number?
7. What does the letter C mean? What does the letter M mean?
8. On the invoice here shown, who is the buyer?
9. On the invoice here shown, who is the seller?
10. Read the line which shows the sale of Twine on this invoice. How much Twine was sold? What kind of Twine was sold? How much per pound is to be paid for the Twine? How much is that per 100 pounds?
11. Read the line which shows the sale of Envelopes on this invoice. How many Envelopes were sold? What kind of Envelopes were sold? How much per thousand is to be paid for the Envelopes?
12. Read the line which shows the sale of Blotting Paper on this invoice. How many sheets of Blotting were sold? What kind of Blotting was sold? How much per hundred sheets is to be paid for the Blotting? How much per sheet is to be paid for the Blotting?

13. What does F. O. B. mean when written or printed on an invoice? What does F. O. B. New York City, mean?

14. Read the items shown on each of the invoices in the next exercise.

Exercise 13—Written.

Find the total value of the merchandise on each of the following invoices:

1.

Quantity	Description	Price	Amount	
5C#	Butter. . . . . @	$0.30	??	??
20 doz.	Eggs. . . . . @	0.20	??	??
11 bbl.	Salt. . . . . @	2.00	??	??
3 bbl.	Flour. . . . . @	4.80	??	??
			??	??

2.

Quantity	Description	Price	Amount	
24	Chairs #14. . . . . @	$2.50	??	??
18	Chairs #20. . . . . @	5.00	??	??
12	8′ × 10′ Wilton Rugs. . @	25.00	??	??
			??	??

3.

Quantity	Description	Price	Amount	
6 doz.	Locks #42 . . . . per gross	$14.40	??	??
8 doz.	DoorChecks#121, per gross	432.00	??	??
9 doz.	Sets of Hinges #438, per gross	57.60	??	??
			??	??

DECIMALS

LESSON 8

Division of a Decimal by an Integer

EXAMPLE: $.8 \div 4 = ?$

$$\begin{array}{r} .2 \\ 4\overline{).8} \end{array}$$

The quotient contains one decimal place because $.8 = \frac{8}{10}$, and $\frac{1}{4}$ of $\frac{8}{10} = \frac{2}{10}$ or .2

Numbers containing decimals can be divided by integers just as if they, themselves, were integers excepting that the quotient will contain as many decimal places as there are decimal places in the dividend, since we are really dividing the numerator of a fraction; therefore, place your decimal point in the quotient exactly over the decimal point in the dividend. A part of tenths equals tenths; a part of thousandths equals thousandths, etc.

EXAMPLE: $1.269 \div 27 = ?$

$$\begin{array}{r} .047 \\ 27\overline{)1.269} \\ \underline{108} \\ 189 \\ \underline{189} \end{array}$$

The quotient contains three decimal places because $1.269 = \frac{1269}{1000}$, and $\frac{1}{27}$ of $\frac{1269}{1000} = \frac{47}{1000}$ or .047

Exercise 14—Oral.

1. $.9 \div 3 = ?$ $.8 \div 4 = ?$ $.6 \div 3 = ?$
2. $1.4 \div 7 = ?$ $2.4 \div 8 = ?$ $3.6 \div 6 = ?$
3. $.48 \div 12 = ?$ $.72 \div 6 = ?$ $.09 \div 3 = ?$

4. 1.44 ÷ 12 = ? 1.32 ÷ 11 = ? 1.08 ÷ 9 = ?
5. .072 ÷ 8 = ? .084 ÷ 12 = ? .096 ÷ 8 = ?

Exercise 15—Written.

Divide and prove by multiplication:

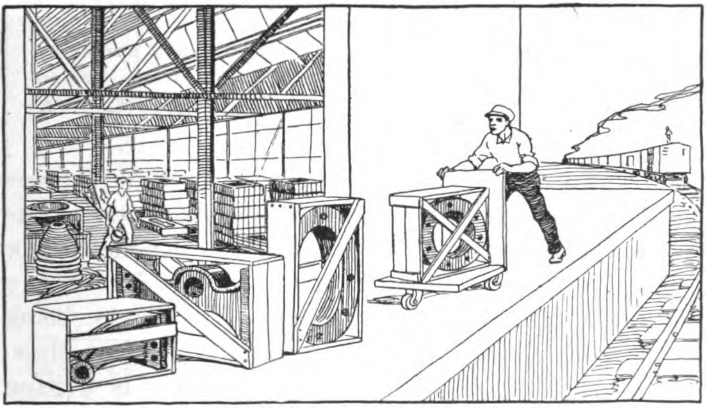
1. $42\overline{)2679.6}$ **2.** $58\overline{)312.04}$ **3.** $33\overline{)6.2337}$

4. 468.615 ÷ 105 = ? **5.** .00426 ÷ 213 = ?

6. If the area of a building lot is 3,816.8 sq. ft., and the width of the lot is 26 ft., what is the depth?

7. A train ran 46 mi. in 36.8 min.; what was the average time required for each mile?

8. A certain shipment consisting of 15 cases weighed 6,185.55#; what was the average weight of each of the cases?



9. The weight of four castings is as follows: 132.46#, 87.49#, 139.45#, 26.84#; what is the average weight of each casting?

10. The altitude of a parallelogram is 24 ft., and the area is 3,235.2 sq. ft.; what is the length of the base?

LESSON 9

Division of a Decimal by a Decimal

EXAMPLE: .9 ÷ .3 = ?

$$\begin{array}{r} 3 \\ .3\overline{).9} \end{array}$$

Tenths are contained in tenths a whole number of times, if at all; $\frac{9}{10} \div \frac{3}{10} = 3$.

EXAMPLE: 108.12 ÷ .12 = ?

$$\begin{array}{r} 901 \\ .12\overline{)108.12} \end{array}$$

Hundredths are contained in hundredths a whole number of times, if at all; $\frac{10812}{100} + \frac{12}{100} = 901$.

EXAMPLE: 4.68 ÷ .18 = ?

$$\begin{array}{r} 26 \\ .18\overline{)4.68} \\ 36 \\ \hline 108 \\ 108 \\ \hline \end{array}$$

Whether the example is worked by short division or by long division makes no difference; $\frac{468}{100} \div \frac{18}{100} = 26$.

As you know, division tells us how many times one number or quantity is contained in another number or quantity. Thus, by division we find that 5 cents are contained in 10 cents, two times. Five cents, being $\frac{5}{100}$ of a dollar, is written as a decimal fraction thus: \$0.05, and ten cents being $\frac{10}{100}$ of a dollar is written \$0.10; now just as 10 cents ÷ 5 cents = 2, so $\frac{10}{100} \div \frac{5}{100} = 2$, and

$$\begin{array}{r} 2 \\ .05\overline{).10} \end{array}$$

In the division of common fractions you learned that when the fractions have common denominators, the division can be completed by using only the numerators; thus, $\frac{4}{6} \div \frac{2}{6} = 2$ because when we divide one number by another number of the same kind, the answer always tells us the *number of times* that the one number is contained in the other.

Now, just as 2 *sixths* are contained in 4 *sixths* 2 *whole* times, so *tenths* are contained in *tenths* a *whole* number of times, if at all; *hundredths* are contained in *hundredths* a *whole* number of times, if at all; and so on. Therefore, when there are as many decimal places in the dividend as there are in the divisor, the decimals are really fractions with like denominators, and the quotient must be a *whole* number if the divisor is contained in the dividend at all.

$$\frac{1}{10} \div \frac{1}{10} = 1. \qquad \frac{1}{100} \div \frac{1}{100} = 1. \qquad \frac{1}{1\,000} \div \frac{1}{1\,000} = 1.$$

$$\begin{array}{r} 1| \\ .1\overline{).1}| \end{array} \qquad \begin{array}{r} 1| \\ .01\overline{).01}| \end{array} \qquad \begin{array}{r} 1| \\ .001\overline{).001}| \end{array}$$

$$\frac{9}{10} \div \frac{3}{10} = 3. \qquad \frac{24}{100} \div \frac{6}{100} = 4. \qquad \frac{72}{1\,000} \div \frac{8}{1\,000} = 9.$$

$$\begin{array}{r} 3| \\ .3\overline{).9}| \end{array} \qquad \begin{array}{r} 4| \\ .06\overline{).24}| \end{array} \qquad \begin{array}{r} 9| \\ .008\overline{).072}| \end{array}$$

EXAMPLE: $.9 \div .03 = ?$

$$\begin{array}{r} 30| \\ .03\overline{).90}| \end{array}$$

Here one cipher is annexed, changing $\frac{9}{10}$ into $\frac{90}{100}$; $\frac{90}{100} \div \frac{3}{100} = 30$.

EXAMPLE: $468. \div .0018 = ?$

$$\begin{array}{r} 260{,}000| \\ .0018\overline{)468.0000}| \\ \underline{36} \\ 108 \\ \underline{108} \end{array}$$

Here four ciphers are annexed so that the dividend may have as many decimal places as the divisor: $\frac{4\,680\,000}{10\,000} \div \frac{18}{10\,000} =$ 260,000.

When the divisor contains more decimal places than the dividend, annex ciphers to the dividend until it has as many decimal places as the divisor, which is the same as giving like denominators to common fractions.

Practice Exercise:

State where the decimal point must be placed in the quotient of each of the following:

$.5\overline{).5}$ $.4\overline{).8}$ $.2\overline{).6}$ $.8\overline{)1.6}$ $.9\overline{)8.1}$ $.7\overline{)2.1}$

$.16\overline{).32}$ $.18\overline{)3.60}$ $.35\overline{)7.00}$ $.8\overline{)16.}$ $.9\overline{)18.}$ $.4\overline{)1.6}$

$.08\overline{).16}$ $.03\overline{)1.2}$ $.07\overline{)21.}$ $.003\overline{).6}$ $.012\overline{).84}$ $.011\overline{)99.}$

EXAMPLE: 3. ÷ 5. = ?

$\frac{1}{5}$ of 3 = $\frac{3}{5}$ or $\frac{6}{10}$.

$$\begin{array}{r|l} & 6 \\ 5.\overline{)3} & \overline{0} \end{array}$$

Annexing the cipher to 3. changes it to $\frac{30}{10}$; $\frac{1}{5}$ of $\frac{30}{10}$ = $\frac{6}{10}$. A part of tenths equals tenths, as you know.

EXAMPLE: 9. ÷ 12. = ?

$$\begin{array}{r|l} & 75 \\ 12.\overline{)9} & \overline{00} \end{array}$$

Annexing two ciphers to 9. changes it to $\frac{900}{100}$; $\frac{1}{12}$ of $\frac{900}{100}$ = $\frac{75}{100}$.

Sometimes you will find it necessary to annex some ciphers to the right of the decimal point in the dividend before you can complete your division.

EXAMPLE: .48 ÷ .2 = ?

$$\begin{array}{r|l} 2 & 4 \\ .2\overline{).4} & \overline{8} \end{array}$$

As there are two decimal places in the dividend and only one decimal place in the divisor, the quotient has one decimal place; the decimal point is placed to show tenths in tenths a whole number of times.

When the dividend contains more decimal places than the divisor, each of the extra decimal places in the dividend gives us one decimal place in the quotient, because we still find that tenths are contained in tenths a whole number of times and the decimal point must be placed to show it.

EXAMPLE: 138.721 ÷ 12.46 = ?

```
             11|133+
12.46)138.72|100
      1246
       1412
       1246
        1661
        1246
         4150
         3738
          4120
          3738
           382
```

Here ciphers are annexed in the dividend so that the quotient can be figured to thousandths' place, and as there is then a remainder, the sign + is written in the quotient. Five decimal places in the dividend less two decimal places in the divisor leaves three decimal places in the quotient; the decimal point is placed to show hundredths in hundredths a whole number of times.

EXAMPLE: .12 ÷ 1.4 = ?

```
      |085+
1.4).1|200
     112
      80
      70
      10
```

Here ciphers are annexed because 14 is not contained in 12, and so that the quotient can be figured to thousandths' place. Four decimal places in the dividend less one decimal place in the divisor leaves three decimal places in the quotient; the decimal point is placed to show tenths in tenths a whole number of times.

Perhaps we see by this time that, as units are contained in units a certain number of units times if found at all in the number, so like decimals are found in like decimals a certain number of units times if any, after that the quotient runs into decimals.

	Units	Decimals
.06)	426.84	32

	Units	Decimals
.003)	4.500	

	Units	Decimals
126.8)	4.5	60000

Exercise 16—Oral.

Be ready to place the decimal point very promptly in the following:

1. $.06\overline{)1.26}$ $.7\overline{)42.}$ $.006\overline{).0012}$

2. $.006\overline{)18.0}$ $.05\overline{).2555}$ $.0005\overline{)5.}$

Divide the following at sight:

3. $.02\overline{).02}$ $.002\overline{)2.000}$ $.02\overline{)2.22}$

4. $.2\overline{)2.48}$ $.2\overline{).30}$ $.2\overline{)4.68}$

5. $.002\overline{).0002}$ $.004\overline{).0048}$ $2.\overline{).2}$

6. $.2\overline{)21.2}$ $2.\overline{)64.4}$ $2.\overline{).342}$

7. .9 ÷ .3; .8 ÷ .2; .6 ÷ .2;
8. 1.2 ÷ .4; 4.8 ÷ .8; 8.4 ÷ .7;
9. 3.6 ÷ 1.2; 14.4 ÷ 1.2; 13.2 ÷ 1.1;
10. 4.4 ÷ 11.; 4.4 ÷ 1.1; 4.4 ÷ .11;
11. 81. ÷ .9; 72. ÷ .8; 99. ÷ 1.1;
12. 6.4 ÷ .8; 64. ÷ .8; 6.4 ÷ 8.;
13. 63. ÷ 9.; 6.3 ÷ 9.; 6.3 ÷ .9;

Exercise 17—Written.

Divide and prove by multiplication:

1. 453.6 ÷ .9;
2. 49.32 ÷ 1.2;
3. 9.7236 ÷ .36;
4. 8,766.1 ÷ .049;
5. 574. ÷ .0014;
6. 247.62 ÷ 14.23;
7. 196.3 ÷ 1.73;
8. .13 ÷ 1.43;
9. .104 ÷ 28.7;

10. In the year 1910 the census which was taken of the population of the United States showed an average of 30.9 persons to each square mile of land; on this basis how large an area of land would be occupied by 41,097. persons?

11. According to the same census, there was 1 person to each 6.4 acres of land in the State of Illinois; on this basis how many persons would be found on 20 sq. mi. or 12,800 acres of land?

12. A train traveling at the average rate of 31.86 miles per hour would require how many hours to go from New York to San Francisco, a distance of 3,186 miles?

Lesson 10

Division by Moving the Decimal Point

EXAMPLE:

4,938. ÷ 10 = 493.8

PROOF:

$$\begin{array}{r} 493.8 \\ 10\overline{)4938.0} \end{array}$$

Just as any number can be multiplied by 10 by moving the decimal point one place to the right, so can any number be divided by 10 by moving the decimal point one place to the left, because the place which was tens' place then becomes units' place, and the place which was units' place then becomes tenths' place.

EXAMPLE:
592.8 ÷ 100 = 5.928

PROOF:

$$\begin{array}{r} 5.928 \\ 100\overline{)592.800} \end{array}$$

Moving the decimal point two places to the left divides any number by 100; three places divides by 1,000, etc.

EXAMPLE:
3.74 ÷ 100 = .0374

PROOF:

$$\begin{array}{r} .0374 \\ 100\overline{)3.7400} \end{array}$$

When there are not enough digits in the number to permit the decimal point to be moved, ciphers must be prefixed.

EXAMPLE: 86.49 ÷ 90 = ?

$$\begin{array}{r} .961 \\ 9\overline{)8.649} \end{array}$$

PROOF:

$$\begin{array}{r} .961 \\ 90\overline{)86.490} \\ \underline{810} \\ 549 \\ \underline{540} \\ 90 \\ \underline{90} \end{array}$$

To divide by a multiple of ten, as 90 for example, first move the decimal point one place to the left which divides by 10, then divide by 9 because there are 9 tens in 90.

```
EXAMPLE:  5,964.4 ÷ 3,100 = ?              PROOF:

              1.924                                1.924
          31)59.644                        3100)5964.400
             31                                 3100
             286                                28644
             279                                27900
               74                                 7440
               62                                 6200
               124                                12400
               124                                12400
```

Always move the decimal point as many places to the left as there are ciphers to be cancelled in the ending of the divisor.

Exercise 18—Oral.

Divide:

1. 3,648. ÷ 10;
2. 123.6 ÷ 10;
3. 23.47 ÷ 10;
4. .35 ÷ 10;
5. 8,269. ÷ 100;
6. 361.8 ÷ 100;
7. 4.5 ÷ 100;
8. 164. ÷ 1,000;
9. 18.1 ÷ 1,000;
10. 406. ÷ 100;
11. 103.02 ÷ 100;
12. 2.05 ÷ 10.

Exercise 19—Written.

Divide:

1. 3,682. ÷ 20;
2. 140.8 ÷ 40;
3. 1,305.09 ÷ 90;
4. .01554 ÷ 70;
5. 182.97 ÷ 300;
6. 13.71 ÷ 500;

7. 97,200. ÷ 900; 10. 4,628.7 ÷ 5,000;
8. .00639 ÷ 710; 11. 3.4821 ÷ 9,500;
9. 5,873. ÷ 4,000; 12. 18.619 ÷ 4,300.

Exercise 20—Oral.

1. In dividing a decimal by an integer how do we know where to place the decimal point in the quotient?
2. With the exception of pointing off the decimal places in the quotient, is there any difference between dividing decimals and dividing whole numbers or integers?
3. Why is the quotient of .12 ÷ 4 pointed off .03?
4. When the dividend and the divisor contain the same number of decimal places, how do we point off in the quotient?
5. When the divisor contains more decimal places than the dividend, what must be done?
6. When the divisor contains less decimal places than the dividend, how do we point off in the quotient?
7. How do we divide a number by 10? By 100? By 1,000?
8. Why is a number divided by 10 when the decimal point is moved one place to the left?
9. What is done when there are not enough digits in the number to permit the decimal point to be moved two, three, four, or five places to the left?
10. How do we divide by 80? By 700? Ly 9,000?
11. State which of the ciphers in these numbers have no value and should, therefore, not be written:

(*a*) 0,046.37; (*c*) 306.300;
(*b*) 0,406.090; (*d*) 009,000.040;

12. How do you tell how many places to move the decimal point in dividing by any multiple of 10, 100, etc.? Do you move it to the right or left?

Lesson 11

Changing Common Fractions to Decimals

EXAMPLE: Change $\frac{3}{8}$ to a Decimal.

$$\begin{array}{r} .375 \\ 8\overline{)3.000} \end{array}$$

$\frac{3}{8} = \frac{3}{8}$ of 1. or $\frac{1}{8}$ of 3.
Therefore, $\frac{3}{8} = .375$ or $.37\frac{1}{2}$

EXAMPLE: Change $\frac{17}{25}$ to a Decimal.

$$\begin{array}{r} .68 \\ 25\overline{)17.00} \\ \underline{150} \\ 200 \\ \underline{200} \end{array}$$

$\frac{17}{25} = \frac{17}{25}$ of 1. or $\frac{1}{25}$ of 17.
Therefore, $\frac{17}{25} = .68$

As you know very well, a common fraction such as $\frac{1}{4}$ when used without any explanation, means $\frac{1}{4}$ of an entire thing or of 1 unit; therefore, since 1 unit is written 1. in decimal form, $\frac{1}{4}$ of 1 unit is found by dividing 1. by 4; thus: $\begin{array}{r} .25 \\ 4\overline{)1.00} \end{array}$; therefore, $\frac{1}{4} = .25$ of 1. As $\frac{3}{4}$ must be 3 times as great as $\frac{1}{4}$, therefore, $\frac{3}{4} = .25 \times 3$ or .75 of 1.; also, as $\frac{3}{4}$ is $\frac{1}{4}$ of 3 as well as being $\frac{3}{4}$ of 1, we can change $\frac{3}{4}$ to a decimal by finding $\frac{1}{4}$ of 3; $\begin{array}{r} .75 \\ 4\overline{)3.00} \end{array}$; therefore, $\frac{3}{4} = .75$ of 1.

In the same way, any common fraction can be changed into a decimal by writing the numerator of the common fraction as an integer, and dividing it by the denominator.

EXAMPLE: Change $\frac{7}{24}$ to a Decimal.

$$\begin{array}{r} .29\frac{4}{24} \\ 24\overline{)7.00} \\ \underline{4\,8} \\ 2\,20 \\ \underline{2\,16} \\ 4 \end{array}$$

Therefore, $\frac{7}{24} = .29\frac{4}{24}$, $.29\frac{1}{6}$, or $.29+$

EXAMPLE: Change $\frac{17}{24}$ to a Decimal.

$$\begin{array}{r} .70\frac{20}{24} \\ 24\overline{)17.00} \\ \underline{16\,8} \\ 20 \end{array}$$

Therefore, $\frac{17}{24} = .70\frac{20}{24}$, $.70\frac{5}{6}$, or $.71-$

Usually, decimals are not carried further than hundredths' place, any remainder that there then may be can be shown as a common fraction in the quotient, or if the fraction is less than $\frac{1}{2}$ it can be dropped entirely and the sign (+) used in the quotient to show that it has been dropped; but if the fraction is more than $\frac{1}{2}$, then 1 should be added to the last figure in the quotient and the sign (−) used to indicate that the quotient is actually a little less than is written.

EXAMPLE: Change $\frac{5}{8}$ to a Decimal.

$$\begin{array}{r} .62\frac{1}{2} \\ 8\overline{)5.00} \end{array}$$

Therefore, $\frac{5}{8} = .62\frac{1}{2}$ (Do not drop the $\frac{1}{2}$.)

EXAMPLE: Change $\frac{10}{11}$ to a Decimal.

$$\begin{array}{r} .90\frac{10}{11} \\ 11\overline{)10.00} \end{array}$$

Therefore, $\frac{10}{11} = .90\frac{10}{11}$ (Do not drop the $\frac{10}{11}$.)

When changing common fractions with denominators not over 16, always show the fraction in the quotient—never drop it.

To change a mixed number to a mixed decimal, we merely change the fraction in the mixed number to a decimal fraction; thus: $14\frac{1}{2} = 14.5$; $28\frac{7}{24} = 28.29+$; $6\frac{5}{8} = 6.62\frac{1}{2}$ or 6.625:

Exercise 21—Written.

Change each of these common fractions or mixed numbers to a decimal fraction or mixed decimal:

1. $\frac{1}{2}$;
2. $\frac{1}{4}$;
3. $\frac{3}{4}$;
4. $1\frac{1}{5}$;
5. $\frac{4}{5}$;
6. $\frac{1}{6}$;
7. $1\frac{5}{6}$;
8. $\frac{12}{25}$;
9. $\frac{19}{72}$;
10. $\frac{18}{31}$;
11. $\frac{15}{26}$;
12. $\frac{13}{74}$.

LESSON 12

Changing Decimals to Common Fractions

EXAMPLE: Change .875 to a Common Fraction.

$$.875 = \frac{875}{1000} \text{ or } \frac{7}{8}$$

Since .25 means $\frac{25}{100}$, which can be reduced to $\frac{1}{4}$, any decimal can be changed to a common fraction by merely supplying the proper denominator and reducing to lowest terms.

EXAMPLE: Change $.33\frac{1}{3}$ to a Common Fraction.

$$\frac{100}{3} \div 100 = \frac{\cancel{100}}{3} \times \frac{1}{\cancel{100}} \text{ or } \frac{1}{3}$$

When the decimal ends with a common fraction, as $.33\frac{1}{3}$ which means $\frac{33\frac{1}{3}}{100}$, we reduce the numerator to an improper fraction, as $\frac{100}{3}$, and divide by the denominator.

EXAMPLE: Change 14.75 to a Mixed Number.
$14.75 = 14\frac{3}{4}$

EXAMPLE: Change $26.33\frac{1}{3}$ to a Mixed Number.
$26.33\frac{1}{3} = 26\frac{1}{3}$

To change a mixed decimal to a mixed number, we merely change the decimal fraction to a common fraction.

Exercise 22—Written.

Change each of these decimal fractions or mixed decimals to common fractions or mixed numbers:

1. .25; **3.** .4; **5.** $.12\frac{1}{2}$; **7.** .96; **9.** $1.33\frac{1}{3}$;
2. .75; **4.** .125; **6.** .45; **8.** $.66\frac{2}{3}$; **10.** 14.72;

Exercise 23—Oral.

1. How do we change a common fraction to a decimal?
2. What is the most accurate way of showing the complete quotient when changing a common fraction to a decimal?
3. If the fraction is not shown in the quotient, what must be done?
4. When should the fraction in the quotient never be dropped?

5. How do we change a mixed number to a mixed decimal?

6. How do we change a decimal such as .75 to a common fraction?

7. How do we change a decimal such as $.16\frac{2}{3}$ to a common fraction?

8. How do we change a mixed decimal to a mixed number?

9. State whether the following numbers are whole numbers, mixed numbers, common fractions, decimals, or mixed decimals:

(*a*) 14.75; (*c*) $18\frac{2}{3}$: (*e*) 463;
(*b*) .36; (*d*) $\frac{7}{12}$; (*f*) $14.66\frac{2}{3}$;

10. Reduce to common fractions:

(*a*) .25; (*c*) .50; (*e*) .6;
(*b*) .75; (*d*) .2; (*f*) .8;

Reduce to decimals:

(*g*) $\frac{3}{10}$; (*i*) $\frac{1}{4}$; (*k*) $\frac{3}{4}$;
(*h*) $\frac{9}{10}$; (*j*) $\frac{1}{2}$; (*l*) $\frac{4}{5}$;

Exercise 24—Oral Review:

1. (*a*) $.46 + 3.04 = ?$ (*b*) $5.32 + 4.68 = ?$
(*c*) $2.09 + .18 = ?$

2. (*a*) $3.97 - .16 = ?$ (*b*) $5. - .9 = ?$
(*c*) $14.71 - 1.06 = ?$

3. (*a*) $4. \times .6 = ?$ (*b*) $.8 \times .3 = ?$
(*c*) $.04 \times 9. = ?$

4. (*a*) $1.8 \div .3 = ?$ (*b*) $.24 \div 4. = ?$
(*c*) $48. \div .8 = ?$

5. (*a*) 1.6 × 10 = ? (*b*) .34 × 10 = ?
(*c*) .48 × 100 = ?

6. (*a*) 3 ÷ 10 = ? (*b*) 5.6 ÷ 10 = ?
(*c*) 6.5 ÷ 100 = ?

7. Change to decimals:
(*a*) $\frac{3}{4}$; (*b*) $\frac{1}{8}$; (*c*) $\frac{9}{10}$;

8. Change to common fractions:
(*a*) .8; (*b*) .25; (*c*) .5;

9. What is the weight of 6 erasers if each eraser weighs 1.2 oz.?

10. What is the weight of 1 ruler if 6 rulers weigh 12.6 oz.?

11. Say the table of Square Measure.

12. Say the table of Cubic Measure.

13. Say the table of Liquid Measure.

14. Say the table of Dry Measure.

Make the following subtractions at sight:

15.	39	46	97	458	863	936
	25	35	26	237	502	314
16.	60	90	70	57	64	91
	18	63	46	48	29	43
17.	158	167	182	428	586	678
	38	59	47	132	195	583
18.	472	668	534	136	346	498
	108	109	289	48	195	299

(Time yourself. Practice a while and after that time yourself again. Did you gain?)

DECIMALS

Exercise 25—Written Review.

(Time yourself for these four. How many minutes were needed? Report to the teacher.)

1. 36.8 + 468.07 + 19.235 + 146.002 + 18.75 + .19 = ?
2. 138.72 − 14.876 = ?
3. 159.003 × .126 = ?
4. 186.346 ÷ .18 = ?
5. A certain farm contains 96.875 A.; 35.125 A. are planted in corn, and the remainder is planted in wheat; how many acres are planted in wheat?
6. If a block of granite contains 12.22 cu. ft., how many cubic feet will 37.5 such blocks contain?
7. An aeroplane traveling 72.6 miles per hour travels how far in 1 minute?
8. A steamboat traveling 34.4 miles per hour, travels how far in .9 hours?
9. How many minutes are there in .04 hours?
10. How many inches are there in 4.6 feet?

Add, without copying:

(Time for these 7 examples is less than 5 minutes.)

11.	**12.**	**13.**
24,672	584,725	62
347	36,423	932
602	7,390	37,638
9,276	504	461
532	377	328,264
63	52,314	719
458	637	925
38,515	51	19,117

14.	**15.**	**16.**	**17.**
378,541	12,478	78,309	48,129
1,000	10,000	20	400
79,429	62,148	80,716	75,423
92,631	11,873	3,000	45,228
100	1,000	37,489	50

Subtract, without copying:

(Time for these 16 examples is less than 5 minutes.)

18.	**19.**	**20.**	**21.**
48,732	68,210	37,412	38,219
20,612	34,681	18,647	19,408

22.	**23.**	**24.**	**25.**
46,831	78,416	74,839	56,028
19,298	38,219	29,484	47,109

26.	**27.**	**28.**	**29.**
36,812	38,612	86,387	67,418
21,919	19,284	41,919	18,293

30.	**31.**	**32.**	**33.**
84,913	78,428	86,201	73,191
61,493	39,395	14,818	58,208

Copy and multiply:

(Time for these 5 examples is less than 5 minutes.)

34. 4,328 × 84; **36.** 5,986 × 72;

35. 3,986 × 93; **37.** 8,328 × 47;

38. 3,946 × 74.

Copy and divide:
(Time for these 15 examples is less than 5 minutes.)

39. 1,792 ÷ 64;
40. 414 ÷ 18;
41. 912 ÷ 57;
42. 442 ÷ 26;
43. 945 ÷ 63;
44. 855 ÷ 45;
45. 861 ÷ 41;
46. 962 ÷ 37;
47. 1,537 ÷ 53;
48. 3,770 ÷ 65;
49. 1,925 ÷ 77;
50. 528 ÷ 24;
51. 945 ÷ 45;
52. 1,833 ÷ 47;
53. 2,325 ÷ 31.

54. If a roll of wall paper contains a strip 1.5 ft. wide and 24 ft. long, how many sq. ft. are there in a roll? How many sq. yd. are there in a roll?

55. Find the number of square feet in the area of the four walls of a room 20 ft. square and 13.5 ft. high. Find the number of square yards.

56. Figuring 4 sq. yd. to the roll and making no allowance for openings, how many rolls of wall paper are needed to cover the walls of the room in Q. 55? What is this worth @ 45¢ per roll?

57. Find the cost of papering a room 18′ square and 13.5′ high, if the wall paper costs 40¢ per roll (1.5′ × 24′), the border costs $2\frac{1}{2}$¢ per foot, and the ceiling paper costs 25¢ per roll. (Make no allowance for openings.)

58. Find the cost of papering the walls and painting the ceiling of a room 12′ × 13.5′ and 9′ high, the paper costing 35¢ per roll, and the painting costing 20¢ per square yard. (Make no allowance for openings, and remember that a fraction of a roll cannot be bought.)

RATIO

LESSON 13

Finding the Ratio

"Ratio" is the relation which one number or quantity bears to another number or quantity of the same kind. Ratio tells us how many, or what part, one number is of another, and this you have been doing for a long time in your division work. So, when we compare 6 with 12 we say the ratio of 6 to 12 is $\frac{6}{12}$ or $\frac{1}{2}$ because 6 is $\frac{1}{2}$ of 12. The ratio of 5 to 30 is $\frac{5}{30}$ or $\frac{1}{6}$ because 5 is $\frac{1}{6}$ of 30. John has $5.00 and Mary has $10.00; how does John's money compare with Mary's? 5 to $10 = \frac{1}{2}$. Mary's compared with John's? 10 to $5 = \frac{10}{5}$ or 2. Mary's is twice as much as John's. Remember, only like numbers can be compared.

EXAMPLE: $8:4 = ?$

$8 \div 4 = 2$, Ans. The ratio of 8 to 4 is 2 because there are 2 four's in 8.

EXAMPLE: $4:8 = ?$

$4 \div 8 = \frac{1}{2}$, Ans. The ratio of 4 to 8 is $\frac{1}{2}$ because 4 is $\frac{1}{2}$ of 8.

When you find the ratio of one number to another number, you are doing nothing more than dividing the first number by the second, and the ratio is your quotient; therefore, when you see the sign of ratio (:) think of it as if it were the sign of division.

Exercise 26—Oral.

In Ruth's flower garden there are 24 flowers; 6 of them are tulips, 8 are violets, 2 are lilies, 4 are hyacinths, 3 are daffodils, and 1 is a narcissus.

1. What part of her flowers are tulips?
2. What part of her flowers are violets?
3. The lilies are what part of all the flowers?
4. The hyacinths are what part of all the flowers?
5. The daffodils are what part of all the flowers?
6. What part of the whole flower bed is the narcissus?
7. Compare the lilies with the hyacinths.
8. Compare the lilies with the tulips.
9. How do the daffodils compare with the tulips?
10. How do the hyacinths compare with the violets?
11. Compare the violets with the hyacinths.
12. Compare the tulips with the lilies.

13. 4 is what part of 8?
14. 6 is what part of 12?
15. 8 is what part of 24?
16. 2 is what part of 6?
17. 10 is what part of 15?
18. 12 is what part of 36?
19. 12 is what part of 144?
20. 8 is what part of 48?

Compare only like numbers.

21. 2 lb. is what part of 5 lb. ?
22. 6 oz. is what part of 30 oz.?
23. 2 oz. is what part of 1 lb. or 16 oz.?
24. 5 ft. is what part of 20 ft.?
25. 8 yd. is what part of 32 yd.?
26. Compare 32 yd. with 8 yd.
27. Compare 20 ft. with 5 ft.
28. Compare 60 oz. with 12 oz.
29. Compare $2,000. with $8,000.
30. Compare 400 bu. with 100 bu.

Exercise 27—Oral.

Compare these numbers. Tell the relation of the first to the second:

1. 8 to 4;
2. 12 to 3;
3. 20 to 4;
4. 8 to 16;
5. 12 to 14;
6. 50 to 100;
7. 7 to 15; ($7 : 15 = \frac{7}{15}$ A short way to write it.)
8. 9 to 3; ($9 : 3 = \frac{9}{3}$ A short way to write it.)
9. 3 to 9; ($3 : 9 = \frac{3}{9}$ A short way to write it.)
10. What is ratio?
11. What process is used to find ratio?

Exercise 28—Written.

Find the ratio of the following, reducing to lowest terms:

1. 100 : 300;

2. 250 : 500;

3. 33 : 66;

4. 18 : 20;

5. 2,000 : 6,000;

6. 11,000 : 55,000;

7. 222,000 : 888,000;

8. $\frac{3}{7} : \frac{9}{7} = \frac{3}{9}$ or $\frac{1}{3}$;

9. $\frac{7}{24} : \frac{21}{24} = ?$

10. $\frac{6}{8} : \frac{9}{24} = ?$

11. 40 : 20;

12. 75 : 15;

13. 5. : 2.5;

14. 8. : 3.;

15. $\frac{3}{4} : \frac{3}{8}$;

16. $\frac{2}{3} : \frac{3}{4}$;

17. $\frac{3}{5} : \frac{1}{3}$;

18. 2,000 : 500;

19. 750 : 25;

20. 600 : 2.

LESSON 14

Finding One Term of a Ratio When the Ratio and the Other Term Are Given

EXAMPLE: The ratio of 6 to ? = 2; or 6 ÷ ? = 2.

As 6 is 2 times as great as the missing number, the missing number must equal 6 ÷ 2, which is 3.

Proof: The ratio of 6 to 3 = 2.

EXAMPLE: The ratio of 6 to ? = $\frac{1}{2}$; or 6 ÷ ? = $\frac{1}{2}$.

As 6 is $\frac{1}{2}$ as great as the missing number, the missing number must equal 6 ÷ $\frac{1}{2}$; 6 ÷ $\frac{1}{2}$ = 6 × 2, or 12.

Proof: The ratio of 6 to 12 = $\frac{1}{2}$.

EXAMPLE: The ratio of ? to 8 = 4; or ? ÷ 8 = 4.

As the missing number ÷ 8 = 4, 4 × 8 must be the missing number, and 4 × 8 = 32. **Proof:** The ratio of 32 to 8 = 4.

EXAMPLE: The ratio of ? to 8 = $\frac{1}{4}$; or ? ÷ 8 = $\frac{1}{4}$.

As the missing number ÷ 8 = $\frac{1}{4}$, 8 × $\frac{1}{4}$ must be the missing number, and 8 × $\frac{1}{4}$ = 2. **Proof:** The ratio of 2 to 8 = $\frac{1}{4}$.

As the ratio of one number to another number is the quotient found by dividing the first number by the second, so, when the ratio and one of the numbers are given, we can find the missing number just as we do in division when the quotient and one of the numbers are given:

$$\begin{array}{lclcl}
\text{1st term} & \div & \text{2d term} & = & \text{Ratio} \\
\text{Dividend} & \div & \text{Divisor} & = & \text{Quotient} \\
6 & : & ? & = & 3 \\
6 & \div & ? & = & 3 \\
6 & \div & \underline{2} & = & 3 \text{ because } 6 \div 3 = \underline{2}
\end{array}$$

$$\begin{array}{lclcl}
\text{1st term} & \div & \text{2d term} & = & \text{Ratio} \\
\text{Dividend} & \div & \text{Divisor} & = & \text{Quotient} \\
? & : & 2 & = & 3 \\
? & \div & 2 & = & 3 \\
\underline{6} & \div & 2 & = & 3 \text{ because } 2 \times 3 = \underline{6}
\end{array}$$

Exercise 29—Oral.

What is the missing number in each of the following:

1. 12 to ? = 4; (12 ÷ ? = 4)
2. 12 to ? = $\frac{1}{5}$; (12 ÷ ? = $\frac{1}{5}$)
3. 16 to ? = 8; (Tell process.)
4. 6 to ? = $\frac{1}{8}$; (Tell process.)
5. ? to 12 = 6; (Tell process.)
6. ? to 12 = $\frac{1}{6}$; (Tell process.)
7. ? to 11 = 10; (Tell process.)
8. ? to 10 = $\frac{1}{10}$; (Tell process.)

When fractions have like denominators, use only the numerators to find the comparison or ratio.

When fractions have unlike denominators, reduce to like denominators before comparing, or divide the first fraction by the second and reduce your answer to lowest terms.

Exercise 30—Oral and Written.

A. First tell how to compare the following:

1. $\frac{2}{3}$ with $\frac{5}{3}$;
2. $\frac{4}{5}$ with $\frac{12}{5}$;
3. $\frac{9}{10}$ with $\frac{3}{10}$;
4. $\frac{8}{7}$ with $\frac{4}{7}$;
5. $\frac{7}{12}$ with $\frac{14}{12}$;
6. $\frac{7}{12}$ with $\frac{9}{12}$;
7. $\frac{5}{8}$ with $\frac{2}{8}$;
8. $\frac{10}{11}$ with $\frac{7}{11}$;
9. $\frac{100}{9}$ with $\frac{20}{9}$;
10. $\frac{20}{50}$ with $\frac{4}{50}$;
11. $\frac{10}{50}$ with $\frac{1}{25}$;
12. $\frac{1}{2}$ with $\frac{1}{4}$;
13. $\frac{7}{8}$ with $\frac{1}{2}$;
14. $\frac{3}{4}$ with $\frac{5}{12}$;
15. $\frac{1}{3}$ with $\frac{1}{6}$;
16. $\frac{3}{2}$ with $\frac{5}{3}$;
17. $\frac{8}{9}$ with $\frac{1}{6}$;
18. $\frac{3}{5}$ with $\frac{1}{4}$;
19. $1\frac{1}{2}$ with 3;
20. $2\frac{1}{2}$ with $3\frac{1}{2}$;
21. $2\frac{1}{5}$ with 22;
22. $3\frac{1}{3}$ with $4\frac{2}{3}$;
23. $10\frac{1}{2}$ with $20\frac{1}{2}$;
24. $12\frac{1}{2}$ with 25.

B. Now work all of them.

Exercise 31—Written.

1. If 6 pencils cost 12 cents, what will 12 pencils cost? Compare what you want to buy with what was bought. Will they cost more or less money? If they will cost more, the larger number leads in the ratio, if they will cost less the smaller number leads. The ratio then is? They will cost how many times 12 cents?

2. If 18 yd. of ribbon cost 90¢, what will 9 yd. cost?

3. If $2\frac{1}{2}$ yd. of gingham cost 75¢, what will 5 yd. cost?

4. If 20 lb. of sugar cost $1.40, what will 80 lb. cost?

5. Find the cost of 3 yd. of goods if 12 yd. cost $1.20.

6. Find the cost of $\frac{1}{4}$ of a yd. of broadcloth if $\frac{1}{2}$ yd. costs $2.00.

7. If I paid a man $10.00 for 10 hr. of work, how much must I pay for 50 hr. of work?

8. If 30 yd. of string flies 1 kite how many kites can be flown with 90 yd. of string?

Exercise 32—Oral and Written.

A. Tell how to do these examples:

1. If 144 eggs cost $3.36, what will 36 eggs cost?
2. If a man is paid $8.32 for 16 hr. of work, what must be paid him for 48 hr.?
3. If 2 yd. of oil-cloth cost $1.50, how much will 2 ft. cost?
   (Reduce to like denominations first.)
4. If $\frac{2}{3}$ yd. of goods cost $2.40, what will $5\frac{1}{3}$ yd. cost?
5. If 500 sheets of paper can be bought for $10.50, how many sheets of the same kind of paper can be bought for $31.50?
6. Find the cost of 15 T. of coal if $2\frac{1}{2}$ T. cost $30.00.
7. If 225# of paper cost $15.75, what will 150# cost?
8. Find the cost of $32\frac{1}{2}$ lb. of coffee, if $6\frac{1}{2}$ lb. cost $2.60.

B. Now work all of them.

Exercise 33—Written.

1. A son who inherited $\frac{9}{20}$ of an estate received $9,000.00; what was his brother's share if he received $\frac{3}{20}$ of the estate?
2. A man earned $3.50 in $\frac{7}{8}$ of a day; what did he earn in $2\frac{5}{8}$ da.?
3. If you can walk 220 yd. in 3 min., how long will it take to walk 1 mi.? (Compare only like numbers.)
4. What is the value of $\frac{14}{24}$ of a farm, if $\frac{7}{24}$ is worth $14,000?

5. A man working 6 hr. a day plows a certain field in 6 da.; if he worked 12 hr. a day how long would it take him? (Ask yourself the question: "Will it take more days or less days?")

6. If 8 men can build a certain wall in 15 da., how long will it take 24 men to do the same work?

7. If 12 men can plow a certain field in 6 da., how many men must work to plow it in 4 da.? More or less men?

8. If I can rent a store for 7 mo. for $1,400., how long can I rent it for $5,600.?

9. 56 = $\frac{7}{8}$ of ? (Since the missing number must equal $\frac{8}{8}$, the ratio is $\frac{8}{8}$ to $\frac{7}{8}$, or $\frac{8}{7}$; $\frac{8}{7}$ of 56 = 64. Notice that the original fraction ($\frac{7}{8}$) inverted ($\frac{8}{7}$) is the ratio.)

10. 56 = $\frac{4}{7}$ of ?
11. 70 = $\frac{5}{6}$ of ?
12. 84 = $\frac{12}{13}$ of ?
13. 34 = $\frac{2}{3}$ of ?
14. 42 = $\frac{3}{4}$ of ?
15. 10 = $\frac{2}{5}$ of ?

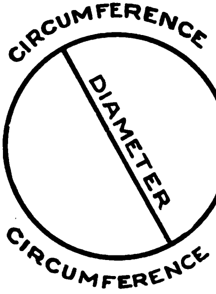
Lesson 15

Ratio of the Circumference of a Circle to the Diameter

A "circle" is a plane figure bounded by one continuous curved line which at all points is a uniform distance from a point in the center of the figure.

The distance around a circle is the "circumference."

The distance across a circle through the center is the "diameter."



The distance from the center of a circle to any point in the circumference is the "radius." Measure with a string or tape-measure the diameter and the circumference of any three circles, such as plates, wheels, ink stoppers, coins, saucers, etc. Write your results in corresponding columns.

When you have done this, see what ratio C has to D in each one. Do all the ratios seem to be about the same? Now you have found the relation of C to its D is a little more than 3. The ratio is nearly 3$\frac{1}{7}$ or decimally nearly 3.1416

C		D	Ratio
Plate ——	to	——	?
Saucer—	to	——	?
Wheel —	to	——	?

EXAMPLE: What is the circumference (C) of a pipe whose diameter (D) is 2"? C : D = 3$\frac{1}{7}$ or $\frac{22}{7}$.

$C = 3\frac{1}{7} \times D;$

$C = 3\frac{1}{7} \times 2'';$

$C = \frac{22}{7} \times \frac{2}{1}''$, or $\frac{44}{7}''$, or $6\frac{2}{7}''$, Ans.

EXAMPLE: What is the D of a hose which has a C of 44 in.?

$C = \frac{22}{7} \times D;$

$D = C \div \frac{22}{7};$

$D = 44'' \times \frac{7}{22}$, or 14", Ans.

Exercise 34—Oral.

Tell just what to do. (Use the ratio 3$\frac{1}{7}$ or $\frac{22}{7}$.)

1. If C is 144", how do you find D?
2. If D is 6 ft., how do you find C?
3. If R is 4$\frac{1}{2}$ ft., how do you find D?
4. If D is 40 ft., how do you find C?

5. If C is 24 ft., how do you find D?
6. If R is 8″, how do you find C?
7. If C is 200 rd., how do you find D?
8. If D is 2 rd., how do you find C?
9. If D is 14½ ft., how do you find R?
10. If R is 6½″, how do you find D?

Exercise 35—Written.

(Use the ratio 3.1416 this time.)

1. If the circumference of a cart-wheel is 4 yd., what is its diameter? (Carry out through 2 decimal places in the quotient.)
2. If the R of a wheel on Tom's bicycle is 1⅓ ft., what is the circumference of the wheel?
3. Find the D of a plate, whose C is 12.5664 in.
4. Find the R of a saucer whose C is 28.2744″.
5. What is the C of a boiler whose D is 6 ft.?
6. Find the D of a jumping-rope whose C is 1.5708 in.
7. Write in letters and signs the rule for finding C when D is given. C = ?
8. Write in letters and signs the rule for finding D when C is given. D = ?
9. Write in letters and signs the rule for finding R when D is given. R = ?
10. Write in letters and signs the rule for finding D when R is given. D = ?

PERCENTAGE

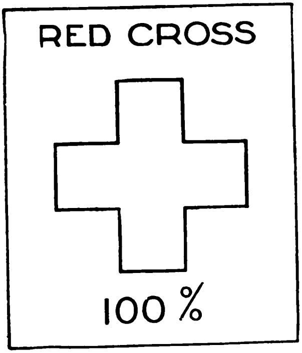
Lesson 16

What Percentage Is

In writing common fractions, we always write the denominator of the fraction under a short line, as $\frac{1}{100}$.

In writing decimal fractions, we always indicate the denominator by the number of decimal places to the right of the decimal point, as .01, which means $\frac{1}{100}$.

Now, as hundredths are used so often and in so many different ways, the term "per cent" which means "by the hundred" is used very often for "hundredths" instead of the fractional or decimal form; thus, 1 per cent of anything (written 1%) means .01 or $\frac{1}{100}$ of that thing. Remember that the whole of anything is always 100% or $\frac{100}{100}$ of that thing, and that 1% is always $\frac{1}{100}$ of the whole.



Exercise 36—Oral.

1. What did this sign mean when it was put in your windows during the Great War? Who gave it? Why the 100%? Who could not have one of these?
2. What is meant when we say a person is 100% American?
3. What is meant by saying soap is 99% pure?

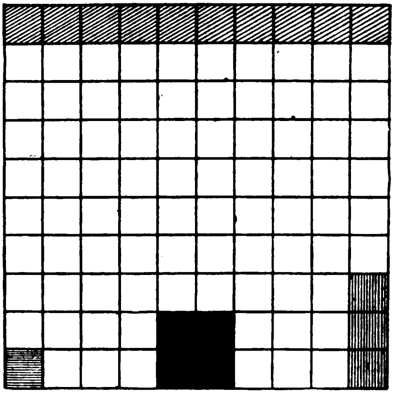
4. What is meant when we say, "The boy's grade was 85%"?
5. If you have 100 cents and spend 5 cents, how many hundredths of your money do you spend? What per cent of your money do you spend? What per cent of $1.00 is 5¢?
6. If you have 100 crayons and give away 10 of them, what per cent do you give away? What per cent do you have left?
7. If you have 100 rabbits and sell 50 of them, what per cent do you sell? What per cent remains?
8. If you have 100 cents and spend 6% of them, how many cents do you spend? How much is 6% of $1.00?
9. How many books are 50% of 100 books? Of 50 books? Of 20 books?
10. If there are 100 children at a picnic and 60% are girls, how many girls are there? How many boys are there? What per cent are boys?
11. What is 10% of $1.00? Of 50¢? Of 10¢?
12. What is 25% of $1.00? Of $2.00? Of $4.00?

Exercise 37—Oral.

Answer and point out each of the following:

1. Into how many equal parts is the square in the drawing on Page 54 divided?
2. What fraction of the whole square is each part?
3. What per cent of the whole square is each part?
4. What per cent of the whole square is shaded by horizontal lines?
5. What per cent of the whole square is shaded by vertical lines?

6. What per cent of the whole square is shaded by oblique lines?



The Whole of Anything = 100% of That Thing

7. What per cent of the square has solid shading?
8. What per cent of the whole square has shading of any kind?
9. What per cent of the square has no shading?
10. What part of a sheet of paper is 100% of it?
11. What part of a sheet of paper is 1% of it?
12. Read: $\frac{5}{100}$; $\frac{9}{100}$; $\frac{6}{100}$; $\frac{7}{100}$; $\frac{2}{100}$; .04; .07; .08; .25;
13. Read: 13; 8%; 14; 25%; 15; 90%; 37; 49%.
14. Read: $\frac{3}{100}$; .03; 3%; .08; $\frac{14}{100}$; 28%; 45%; $\frac{86}{100}$.

Exercise 38—Written.

Write each of the following in the form of per cent:

1. $\frac{1}{100}$;
2. $\frac{6}{100}$;
3. $\frac{25}{100}$;
4. $\frac{50}{100}$;
5. $\frac{100}{100}$;
6. .04;
7. .10;
8. .75;
9. .90;
10. 1.00;

Write in the form of common fractions:

11. 3%; **13.** 40%;
12. 12%; **14.** 60%;
15. 100%.

Write in the form of decimal fractions:

16. 5%; **18.** 30%;
17. 15%; **19.** 90%;
20. 100%.

21. Draw a square similar to the one in this lesson, and shade 50% of it with horizontal lines, 25% with vertical lines, and 10% with oblique lines; then write on the unshaded part the percentage that is not shaded.

LESSON 17

Finding Percentages

Since "per cent" means "hundredths," we find any percentage of any number by finding so many hundredths of it, using common fractions when the fraction can be reduced to very low terms, as 50% of 60 = $\frac{50}{100}$ or $\frac{1}{2}$ of 60, which is 30; otherwise using decimal fractions, as 7% of 125 =

$$\begin{array}{r}125\\ .07\\ \hline 8.75\end{array}$$

Remember this and always do it the shortest way.

The number on which the percentage is to be found is called the "base." The base (B) is always 100%.

The number which tells us how many per cent or hundredths of the base are to be taken is called the "rate" (R).

The number which represents a certain per cent of the base is called the "percentage" (P).

EXAMPLE: 8% of 25 = ?

25 base (B)
× .08 rate (R)
= 2.00 percentage (P)

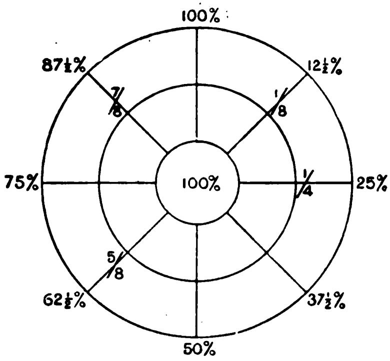
$$B \times R = P.$$

To save time in finding certain percentages, learn this table of aliquot parts of 100 and multiples thereof, an aliquot part of a number being an equal part of that number; thus, 25 is an aliquot part of 100 because it is $\frac{1}{4}$ of 100.

25 = $\frac{1}{4}$ of 100; therefore, 25% of a unit = $\frac{1}{4}$ of a unit.
75 = $\frac{3}{4}$ of 100; therefore, 75% of a unit = $\frac{3}{4}$ of a unit.
$33\frac{1}{3}$ = $\frac{1}{3}$ of 100; therefore, $33\frac{1}{3}$% of a unit = $\frac{1}{3}$ of a unit.
$66\frac{2}{3}$ = $\frac{2}{3}$ of 100; therefore, $66\frac{2}{3}$% of a unit = $\frac{2}{3}$ of a unit.
$16\frac{2}{3}$ = $\frac{1}{6}$ of 100; therefore, $16\frac{2}{3}$% of a unit = $\frac{1}{6}$ of a unit.
$83\frac{1}{3}$ = $\frac{5}{6}$ of 100; therefore, $83\frac{1}{3}$% of a unit = $\frac{5}{6}$ of a unit.
$12\frac{1}{2}$ = $\frac{1}{8}$ of 100; therefore, $12\frac{1}{2}$% of a unit = $\frac{1}{8}$ of a unit.
$37\frac{1}{2}$ = $\frac{3}{8}$ of 100; therefore, $37\frac{1}{2}$% of a unit = $\frac{3}{8}$ of a unit.
$62\frac{1}{2}$ = $\frac{5}{8}$ of 100; therefore, $62\frac{1}{2}$% of a unit = $\frac{5}{8}$ of a unit.
$87\frac{1}{2}$ = $\frac{7}{8}$ of 100; therefore, $87\frac{1}{2}$% of a unit = $\frac{7}{8}$ of a unit.
10% = $\frac{1}{10}$; 20% = $\frac{2}{10}$ or $\frac{1}{5}$; 30% = $\frac{3}{10}$; 40% = $\frac{4}{10}$ or $\frac{2}{5}$;
50% = $\frac{5}{10}$ or $\frac{1}{2}$;
60% = $\frac{6}{10}$ or $\frac{3}{5}$; 70% = $\frac{7}{10}$; 80% = $\frac{8}{10}$ or $\frac{4}{5}$; 90% = $\frac{9}{10}$.

Practice Exercise:

75 = $\frac{3}{4}$ of 100 or $\frac{1}{4}$ of? $66\frac{2}{3}$ = $\frac{2}{3}$ of 100 or $\frac{1}{3}$ of?
$87\frac{1}{2}$ = $\frac{7}{8}$ of 100 or $\frac{1}{8}$ of? $83\frac{1}{3}$ = $\frac{5}{6}$ of 100 or $\frac{1}{6}$ of?



Decimal Equivalents

Can you relate all of these with the center and do it quickly? Try 25%.

Put in as many more related spokes in this % wheel as you can, but you must know them well. Can you travel on the little circle and tell all the parts in common fractions?

EXAMPLE: Find $\frac{7}{8}$ of 1,640.

$$\begin{aligned} \tfrac{8}{8} &= 1{,}640 \\ -\tfrac{1}{8} &= \underline{205} \\ \tfrac{7}{8} &= 1{,}435 \end{aligned}$$

Whenever we multiply by a fraction which is only one part less than a unit, as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, etc., it is easier to find one part and subtract it from the number than to find one part and multiply it by the numerator. You will want to use this method very soon.

Try $\frac{4}{5}$ of 2,550; Try $\frac{2}{3}$ of 162;
Try $\frac{5}{6}$ of 1,284; Try $\frac{7}{8}$ of 8,480;
Try $\frac{3}{4}$ of 845; Try $\frac{9}{10}$ of 5,650.

EXAMPLE: Find 6% of $400.

100% = $400.;
1% = $4.;
6% = $24.; ($4. × 6)

$400. = 100% or all of it.
×.06
$24.00 = 6% of all of it.

To save time in multiplying by hundredths to find percentages, we can point off two places to find 1%, and multiply by the rate per cent. There are times when you will want to use this method.

Exercise 39—Written.

1. Find 5% of 300;
2. Find 4% of 250;
3. Find 1% of 680;
4. Find 2% of 3,000;
5. Find 6% of 1,200;
6. Find 8% of 900;
7. Find 3% of 1,500;
8. Find 1% of 300;
9. Find ½% of 300;
10. Find 7% of 910;
11. Find 12% of 1,000;
12. Find 6½% of 400;
13. Find 2½% of 200;
14. Find ¼% of 4,000.

Exercise 40—Oral.

1. What aliquot part of 100 is 25? What is the fractional equivalent of 25%? Of 75%?
2. What is the fractional equivalent of 33⅓%? Of 66⅔%?
3. What is the fractional equivalent of 16⅔%? Of 83⅓%?
4. What is the fractional equivalent of 12½%? Of 37½%?
5. What is the fractional equivalent of 62½%? Of 87½%?
6. What is the fractional equivalent of 10%? Of 20%?

7. What is the fractional equivalent of 30%? Of 40%?
8. What is the fractional equivalent of 50%? Of 60%?
9. What is the fractional equivalent of 80%? Of 90%?
10. What is the fractional equivalent of 70%? Of 100%?

Exercise 41—Oral.

1. What is 25% of 100? (25% = $\frac{1}{4}$; $\frac{1}{4}$ of 100 = 25.) Of 48? Of 40? Of 36?
2. What is $33\frac{1}{3}$% of 24? Of 60? Of 75? Of 90?
3. What is $16\frac{2}{3}$% of 12? Of 36? Of 60? Of 120?
4. What is $12\frac{1}{2}$% of 16? Of 40? Of 64? Of 80?
5. What is 10% of 40? Of 90? Of 900? Of 9,000?
6. What is 50% of 10? Of 60? Of 600? Of 6,000?
7. What is 20% of 40? Of 50? Of 500? Of 5,000?
8. What is 75% of 16? Of 40? Of 400? Of 4,000?
9. What is $66\frac{2}{3}$% of 24? Of 30? Of 300? Of 3,000?
10. What is $83\frac{1}{3}$% of 12? Of 60? Of 600? Of 6,000?
11. What is $37\frac{1}{2}$% of 48? Of 800? Of 88? Of 8,000?
12. What is $87\frac{1}{2}$% of 8? Of 80? Of 48? Of 8,000?
13. What is 30% of 50? Of 70? Of 90? Of 1,000?
14. What is 40% of 30? 60% of 70? 70% of 40?
15. What is 80% of 20? 90% of 40? 100% of 60?

Exercise 42—Oral.

What per cent is the equivalent of:

1. $\frac{1}{4}$; $\frac{3}{4}$;
2. $\frac{1}{3}$; $\frac{2}{3}$;
3. $\frac{1}{6}$; $\frac{2}{6}$; $\frac{3}{6}$; $\frac{4}{6}$; $\frac{5}{6}$;
4. $\frac{1}{8}$; $\frac{2}{8}$; $\frac{3}{8}$; $\frac{4}{8}$; $\frac{5}{8}$; $\frac{6}{8}$; $\frac{7}{8}$;
5. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$;

6. $\frac{1}{10}$; $\frac{2}{10}$; $\frac{3}{10}$; $\frac{4}{10}$; $\frac{5}{10}$;
7. $\frac{6}{10}$; $\frac{7}{10}$; $\frac{8}{10}$; $\frac{9}{10}$; $\frac{10}{10}$;
8. What is 1% of 100? 2% of 100? 8% of 100?
9. What is 4% of 100? Of 200? Of 300?
10. What is 6% of $500.00? Of $800.00? Of $1,000.00?
11. How do we find any per cent of any number?
12. What is the easiest way of finding the percentage when the rate is an aliquot part of 100, or of a multiple thereof? How do we multiply most quickly by a fraction which is only 1 part less than a unit?
13. What is the easiest way of finding the percentage when the rate is not an aliquot part of 100, or of a multiple thereof? How do we point off to find 1% of a number? How do we find 6% of a number quickly?
14. B = $5.00; R = 10%; P = ?
15. B = 300; R = 4%; P = ?
16. What does B stand for? R? P?
17. Write in letters and signs on the board how to find P when we know B and R.

Exercise 43—Written.

Solve and prove:

1. Find $33\frac{1}{3}\%$ of $416.34;
2. Find $87\frac{1}{2}\%$ of $536.40;
3. Find 75% of $832.36; Name B; R; P.
4. A man invested $37\frac{1}{2}\%$ of his savings in business; how much did he invest if his entire savings were $364.32? Name B; R; P.

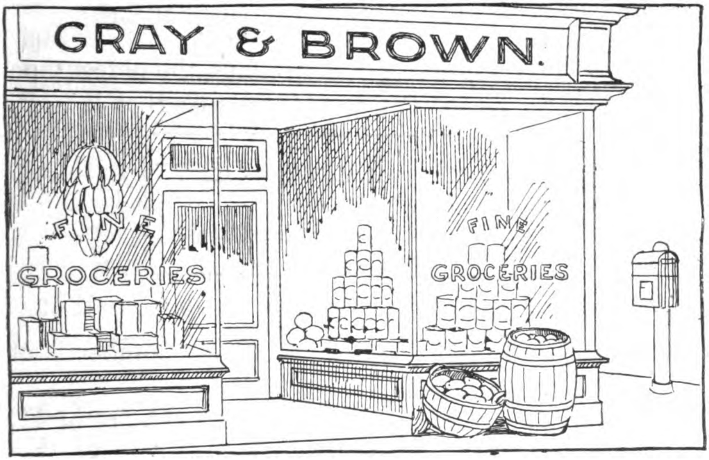
5. A man lost 66⅔% of his savings because he placed his money in a small private bank, instead of placing it in a large State Bank; his entire savings were $612.33; how much did he lose? How much had he left? Name B; R; P.

6. Find 8% of $614.50; Name B; R; P.

7. Find 19% of $36.40; Name B; R; P.

8. Find 56% of $83.75; Name B; R; P.

9. If a real estate dealer sold a house and lot for $4,500. and received 11% of the amount in cash, how much cash did he receive? Name B; R; P.



10. Gray and Brown started in the grocery business with a capital of $1,400. of which Gray invested 60% and Brown invested the balance; how much money did Gray and Brown each invest? What per cent of the entire capital was invested by Brown?

LESSON 18

Finding What Per Cent One Number Is of Another Number

EXAMPLE: 12 is what % of 48?

$12:48 = \frac{12}{48}$ or $\frac{1}{4}$;

$\frac{1}{4}$ of 100% = 25%, Ans.

To find what per cent one number is of another number, we do two things:

1st. Find what fractional part the one number is of the other by ratio.

2d. Find that fractional part of 100%.

This gives the same result as dividing the percentage by the base; therefore, $P \div B = R$ because $B \times R = P$.

EXAMPLE: 9 is what % of 11?

$9:11 = \frac{9}{11}$;

$\frac{9}{11}$ of 100% = $\frac{900}{11}$% or $81\frac{9}{11}$%, Ans.

Exercise 44—Oral.

1. How do we find what per cent one number is of another number?
2. 2 is what per cent of 4? (2 is what part of 4?; $2 = \frac{1}{2}$ of 4; $\frac{1}{2}$ of 100% = 50%.)
3. 2 is what per cent of 3? In this example is 2 the B, R, or P? What is 3?
4. 3 is what per cent of 9? In this example what is the B?
5. 5 is what per cent of 20? In this example what is the R?

6. What per cent of 8 is 1? In this example what is the P?
7. What per cent of 6 is 1? In this example is 6 the B, P, or R?
8. What per cent of 8 is 8? In this example which 8 is the B?
9. What per cent of 12 is 24? In this example is 24 the B, P, or R?
10. What per cent of 24 is 12? In this example is 24 the B, P, or R?
11. What per cent of 1,000 is 100?
12. Write in letters and signs on the board how to find R when B and P are given.

Exercise 45—Written.

Solve and prove:

1. If a merchant sells $50. worth of goods a day, what per cent of a day's sales has he made when he has sold $10. worth? ($10. is what part of $50.? $\frac{1}{5}$ of 100% = ?)
2. If a retailer's average daily sales are $75., what per cent over the average are his sales on a day when he sells $100. worth of merchandise?
3. 24 = P; 72 = B; R = ?
4. 18 is what per cent of 81?
5. 56 = B; 28 = P; R = ?
6. 35 is what per cent of 105?
7. B = 35; P = 105; R = ?
8. If a case of eggs containing 2 gross is damaged in shipment and 72 eggs are broken, what per cent of the eggs are broken?

9. If a merchant, through carelessness, leaves a bolt of silk containing 25 yd. in his show-window where the sun can shine on 6 ft. of the silk and these 6 feet are spoiled, what per cent of the entire bolt is spoiled? What per cent of the entire bolt remains unspoiled?
10. If 24 sq. in. are cut from a sheet of paper containing a square foot, what per cent was cut off? What % remains?

Lesson 19

Finding the Whole or 100% When Part of It Is Given

EXAMPLE: 6 = 10% of what number?
10% = $\frac{10}{100}$ or $\frac{1}{10}$; therefore, 6 = $\frac{1}{10}$;
If 6 = $\frac{1}{10}$, $\frac{10}{10}$ = 10 × 6, or 60, Ans.

To find 100% when part of 100% is given, first see what fraction the per cent that is given is of 100%, then find the entire quantity by ratio. This gives the same result as dividing the percentage by the rate; therefore, P ÷ R = B because B × R = P.

EXAMPLE: 48 = 75% of what number?
75% = $\frac{75}{100}$ or $\frac{3}{4}$; therefore, 48 = $\frac{3}{4}$;
If 48 = $\frac{3}{4}$, $\frac{4}{4}$ = $\frac{4}{3}$ of 48, or 64, Ans.

EXAMPLE: 14 = 7% of what number?
7% = $\frac{7}{100}$; therefore, 14 = $\frac{7}{100}$;
If 14 = $\frac{7}{100}$, $\frac{100}{100}$ = $\frac{100}{7}$ of 14, or 200, Ans.

PERCENTAGE

Exercise 46—Oral.

1. If $5. is $\frac{1}{2}$ of my money, how much have I?
2. If $2. is $\frac{2}{3}$ of my money, how much have I?
3. If $6. is 20% of my money, how much have I? In this example is $6. the B, P, or R?
4. If $10. is 25% of my money, how much have I? In this example what is the B?
5. If $10. is 50% of my money, how much have I? In this example what is the R?
6. If $80. is $16\frac{2}{3}$% of my money, how much have I? In this example what is the P?
7. P = $12.; R = $33\frac{1}{3}$%; B = ?
8. P = $60.; R = 20%; B = ?
9. R = $12\frac{1}{2}$%; P = $40.; B = ?
10. If $4. is 40% of my money, how much have I?
11. If $20. is 20% of my money, how much have I?
12. If $8. is 1% of my money, how much have I?
13. Write in letters and signs on the board how to find B when R and P are given.

Exercise 47—Written.

Solve and prove:

1. A grocer's sales for one day were $75.00, this being 10% of his total sales for the week; what were his total sales?
2. When John was 14 years old, his age was $33\frac{1}{3}$% of his father's age; what was his father's age?
3. After removing $66\frac{2}{3}$% of the water in a tub, the tub contains 20 gallons; how many gallons were there in the tub at first? (20 gal. = ? % of all the water.)

4. After removing 50% of the contents of a barrel, it still contains 375#; how many pounds were there at first?
5. After selling 90% of the grain in a bin, it still contains 60 bu.; how many bushels were there at first?
6. A primary class of 40 pupils contains $12\frac{1}{2}$% of all the children in that school. How many children are there in all?
7. 35 graduates were only 50% of a class. How many pupils were there in the class?
8. If John told you that 10% of his sales at the grocery store were $3.50, would you know how much he sold in all?

Exercise 48—Oral Review.

1. (*a*) $\frac{3}{4} \div \frac{1}{8} = ?$ (*b*) $\frac{4}{5} \times \frac{1}{4} = ?$ (*c*) $\frac{3}{8} + \frac{3}{4} = ?$
2. (*a*) $\frac{5}{6} = \frac{?}{24}$; (*b*) $\frac{47}{9} = ?$ (*c*) $1\frac{3}{5} = \frac{?}{15}$.
3. (*a*) $4.6 + .6 = ?$ (*b*) $3.97 - .14 = ?$ (*c*) $.04 \times .9 = ?$
4. Change to decimals:

   (*a*) $\frac{1}{6}$; (*b*) $\frac{3}{4}$; (*c*) $\frac{2}{3}$.
5. Change to common fractions:

   (*a*) .7; (*b*) .25; (*c*) $.12\frac{1}{2}$;
6. Make a problem using the table of Liquid Measure.
7. Make a problem using the table of Dry Measure.
8. Make a problem using the table of Linear Measure.
9. Make a problem using the table of Square Measure.
10. Make a problem using the table of Cubic Measure.
11. What is the ratio of 4 to 9?
12. What is the ratio of 7 to 3?

13. B = 20; R = 50%; P = ?
14. B = 40; P = 10; R = ?
15. R = 25%; P = 5; B = ?

Exercise 49—Written Review.

Solve and prove:

1. What is the ratio of 360 to 45?
2. What is the ratio of 54 to 81 expressed in the form of a common fraction?
3. What is the ratio of 156 to 624 expressed in the form of a decimal?
4. The ratio of 540 to ? = 5?
5. The ratio of ? to 18 = $\frac{4}{9}$?
6. $\frac{36}{70} \times \frac{8}{9} = ?$
7. Reduce:
- (*a*) 36 da. 5 hr. to hours.
- (*b*) 8 sq. ft. 14 sq. in. to square inches.
- (*c*) 5 sq. yd. 7 sq. ft. to square feet.
- (*d*) 15 T. 3 cwt. to hundredweight.

8. 5 lb. 6 oz. + 12 lb. 8 oz. + 14 lb. 13 oz. + 9 lb. 11 oz. = ?
9. 45 cu. ft. 12 cu. in. − 16 cu. ft. 40 cu. in. = ?
10. 5 mi. 240 rd. × 12 = ?
11. 138 hr. 24 min. ÷ 48 = ?
12. 75 ÷ (48 − 23) × 16 = ?

Copy and multiply:
(Time for these 8 examples is less than 5 minutes.)

13. 260 × 36;
14. 850 × 16;
15. 690 × 72;
16. 590 × 28;
17. 780 × 25;
18. 907 × 25;
19. 603 × 80;
20. 793 × 35.

ARITHMETIC

Copy and divide:

(Time for these 4 examples is less than 5 minutes.)

21. 3,915 ÷ 45;

22. 2,394 ÷ 38;

23. 1,817 ÷ 79;

24. 8,004 ÷ 92.

Subtract, but do not copy:

(Time for these 12 examples is less than 5 minutes.)

25.	**26.**	**27.**	**28.**
874,352	594,721	624,319	754,963
293,713	195,236	417,821	122,791

29.	**30.**	**31.**	**32.**
379,482	854,239	954,628	491,782
283,473	379,518	579,719	156,937

33.	**34.**	**35.**	**36.**
794,526	671,456	598,741	879,425
354,623	293,782	235,924	547,358

Add, but do not copy:

(Time for these 6 examples is less than 8 minutes.)

37.	**38.**	**39.**	**40.**	**41.**	**42.**
3,245	1,572	2,378	2,587	7,586	8,891
2,357	8,379	4,456	3,798	3,279	1,643
3,845	2,215	5,827	1,057	2,682	1,725
5,256	5,319	4,130	4,842	6,691	3,062
8,176	3,560	6,279	3,963	4,837	5,021
4,563	2,348	3,956	2,187	8,299	9,846
1,376	1,847	1,931	2,873	1,017	4,319
2,624	1,153	2,069	2,127	1,983	1,681

PERCENTAGE

LESSON 20

Profit and Loss

EXAMPLE: An article which was bought for $5.00 was sold at a profit of 10%; what was the profit and what was the selling price?

10% of $5.00 = $0.50, Profit.
$5.00 Cost + $0.50 Profit = $5.50, Selling Price.

"Profit" is the difference between the cost and the selling price, when the selling price is the greater.

"Loss" is the difference between the cost and the selling price, when the cost is the greater.

Profit and loss examples are figured in exactly the same way as percentage examples, the cost being used as the base or 100%; therefore:

B × R = P (P or L)
Cost Price × Rate % = Profit or Loss.
Cost Price + Profit = Selling Price.
Cost Price − Loss = Selling Price.

EXAMPLE: An article which cost $12.00 was sold for $14.40; what was the profit?

Selling Price	$14.40
Cost	12.00
Profit	2.40

Exercise 50—Oral.

1. What is profit?
2. What is loss?
3. If we know the cost and selling price of an article, how do we find the amount of profit or loss?

4. If we know the cost and the rate per cent of profit or loss, how do we find the amount of profit or loss?
5. In figuring profit or loss, do we use the cost or the selling price as 100%?
6. Cost + profit = ?
7. Cost − loss = ?
8. Is the cost the B, the R, or the P (P or L)?
9. State in letters and signs how to find P (P or L) when B and R are given.

Exercise 51—Written.

Find the profit or loss and the selling price on each of these items:

1. Cost $12.00; Profit 15%.
2. Cost $50.00; Loss 3%.
3. Cost $96.00; Profit 12½%.
4. Cost $84.00; Loss 8⅓%.

Find the profit or loss on each of these items, the cost and selling price being:

5. Cost $50.00; Selling Price $60.00.
6. Cost $75.00; Selling Price $70.50.
7. Cost $36.00; Selling Price $48.00.
8. Cost $48.00; Selling Price $36.00.

Solve and prove:

9. A desk which cost $15.00 is sold at a profit of 20%; what is the profit? What is the selling price?
10. A grocer bought a tub of butter containing 50 lb. for $12.50; if he sold this butter at a profit of 40%, how much per pound did he receive for it?

11. I sold a horse which cost me $125.00 at a loss of 20%; how much did I receive for the horse?

LESSON 21

Discount

EXAMPLE: On account of being shopworn, a $300.00 piano is sold at a discount of 20%; at what price was the piano sold?

20% of $300.00 = $60.00, Discount.
$300.00 − $60.00 = $240.00, Selling Price.

A "discount" is a reduction from the regular price of an article for any reason. An article sold at a discount is said to be "marked down" from the regular price. The regular price is the base or 100%.

All discounts are figured by percentage.

B × R = P (Disc.)

EXAMPLE: A $30.00 couch was marked down to $25.00; what was the discount? This discount was on what amount of money?

$30.00 − $25.00 = $5.00, Discount.
$5.00 Discount on $30.00.

Exercise 52—Oral.

1. What is discount?
2. How are discounts figured?
3. What is the discount on $20.00 at 5%? What is the B in this example?
4. What is the discount on $75.00 at 33⅓%? What is the R in this example?

5. If discount at the rate of 16⅔% equals $10., $10. is what part of the original price? Find the original price.
6. If discount at the rate of 12½% equals $12., $12. is what part of the original price? Find the original price.
7. If a $75.00 article is marked down 10%, for how much will it be sold? What is the P (Disc.) in this example?
8. A discount of $12.00 on an article marked $72.00 equals what part of the marked price? What per cent is that?
9. A discount of $8. is allowed on a $40. table; what per cent of discount is that?
10. State in letters and signs how to find P (Disc.) when B and R are given.
11. State in letters and signs how to find R when P (Disc.) and B are given.

Exercise 53—Written.

Solve and prove:

At a closing-out sale, all goods were marked down 15%; what was the marked-down price of articles which were originally marked:

1. $80.00; 2. $30.00; 3. $67.00.

4. If the reduction in the price of a bicycle on account of slight usage is 20%, and the amount of the reduction is $6.00, what was the original marked price?
5. If a $30.00 suit of clothes is marked down to $25.00, what is the discount? What amount

was the discount on? What part is the discount? What per cent is the discount?

6. If $2.00 straw hats are sold at the end of the Summer for $1.50, what is the discount? On what amount? What part is the discount? What per cent is the discount?

7. If $25.00 Winter coats are sold in the Spring for $23.00, what is the discount? On what amount? What per cent is the discount?

LESSON 22

Cash Discount

EXAMPLE: A bill of goods amounting to $100.00 was sold subject to a discount of 2% if paid in 10 days; what was the amount of the discount? What amount was paid to settle the bill?

2% of $100.00 = $2.00, Amount of discount.
$100.00 − $2.00 = $98.00, Amount paid to settle the bill.

A "cash discount" is an amount allowed off a bill for payment on or before a certain date. The amount of the bill is the base or 100%.

Cash discounts are almost always figured at a certain rate per cent agreed upon when the goods are purchased. Thus, if goods are purchased subject to a 2% discount if paid within 10 days, the invoice or bill is marked "2% 10 days" or "2/10" for short. If the longest time which can be taken for the payment of the bill is 30 days subject to a 2% discount if paid in 10 days, the bill is marked "2% 10 days, Net 30 days," or "2/10 – N/30" for short.

EXAMPLE: The discount on a bill of $80.00 amounted to $4.00; what was the rate?

$$\frac{\text{Discount } \$4.00}{\text{On } \$80.00} = \frac{1}{20}; \quad \frac{1}{20} \text{ of } 100\% = 5\%, \text{ Ans.}$$

Proof: 5% of $80.00 = $4.00.

Exercise 54—Oral.

1. What is 2% discount on a $50. bill? In this example what is the B? R? P (Disc.)?
2. What is the discount on a $200. bill at 10%? Is $200. the B, R, or P (Disc.)?
3. Find the discount on $500. at 6%. What must I pay to settle the bill?
4. Give Mr. Jones 3% discount on a $600. bill. What must he pay?
5. How much cash discount would you deduct in paying a bill amounting to $100.00 in 10 days, if the terms were "3% 10 days, Net 30 days"? How much would you pay?

6. How much cash discount would you deduct in paying a bill amounting to $100.00 in 10 days, if the terms were "5/10, N/30"? How much would you pay?
7. How much cash discount would you deduct in paying a bill amounting to $100.00 in 30 days, if the terms were "3/10, 2/30, N/60"? How much would you pay?
8. A gas bill for $5.00 states: "10% may be deducted if paid on or before February 16th;" how much discount would you deduct in paying this bill? How much would you pay?
9. What rate of discount is $2.00 on $100.00? What part of the amount was discount? Is $2.00 the B, R, or P (Disc.)?
10. What rate of discount is $10.00 on $200.00? What part of the amount was discount?
11. If the discount on a certain bill at 3% amounts to $3.00, what is the amount of the bill? Is $3.00 the B, R, or P (Disc.)?
12. If the discount on a certain bill at 4% amounts to $2.00, what is the amount of the bill?
13. What is cash discount?
14. On what basis is cash discount figured?
15. What do you understand by "3% 10 days, Net 30 days" on an invoice?

Exercise 55—Written.

Solve and prove:

Find the discount and the net amount to be paid on each of these bills:

1.	$43.50 @ 6%;	**3.**	$98.50 @ 2%;
2.	$87.12 @ 8%;	**4.**	$137.25 @ 4%.

Find the amount of each of these invoices, the discount and the rate per cent being as follows:

5. Discount $8.00; Rate 4%.
6. Discount $30.00; Rate 6%.
7. Discount $4.75; Rate 5%.
8. Discount $2.79; Rate 3%.

Find the rate per cent, the amount of the invoice and the discount being:

9. Invoice $75.00; Discount $4.50.
10. Invoice $125.00; Discount $2.50.

LESSON 23

Trade Discount

A "trade discount" is a discount allowed by wholesale merchants to retailers, so that the retailer can sell the article at a certain price and still make a profit. Thus, an article which is advertised to retail for $1.00 might be sold by the wholesaler to the retailer at a trade discount of 20% or 25%.

The price on which the trade discount is figured is called the "list price." The list price is the base or 100% for figuring trade discount.

In addition to allowing a "trade discount" from "list price," wholesalers almost always allow a "cash discount" for prompt payment, but as the "trade discount" does not depend on payment at a certain time, it is usually deducted by the wholesaler on his bill, while the "cash discount" must be deducted by the retailer before he pays the bill. In cases of this kind, the amount remaining after deducting the trade discount is the base or 100% for figuring the cash discount.

Exercise 56—Oral.

1. If the list price is $1.00 and the trade discount is 20%, what does the retailer pay for the article? What is the B?
2. If the list price is 50¢ and the trade discount is 10%, what does the retailer pay for the article? What is the R?
3. If the list price is 60¢ and the retailer pays 40¢, what is the per cent of trade discount? What is the P (Disc.)?
4. If the list price is $1.00 and the retailer pays 75¢, what is the rate per cent of trade discount? What is the B?
5. If the trade discount at the rate of 25% amounts to $1.00, what is the list price? What is the P?
6. If the trade discount at the rate of 33⅓% amounts to 50¢, what is the list price? What is the R?
7. If the list price is $2.00, the trade discount is 50%, and there is a cash discount of 2%, what is the net amount paid by the retailer?
8. If the list price is $1.25, the trade discount is 20%, and there is a cash discount of 3%, what is the net amount paid by the retailer?
9. What is a trade discount?
10. What is the price called on which the trade discount is figured?

11. How are trade discounts figured?
12. Can there be a trade discount and a cash discount on the same invoice? If so, would the trade discount be lost if the invoice were not paid in time to take advantage of the cash discount?

Exercise 57—Written.

Solve and prove:

Find the trade discount and the amount the retailer would be charged on each of these purchases:

1. List Price $412.00; Trade Discount 20%.
2. List Price $213.42; Trade Discount 33⅓%.
3. List Price $472.00; Trade Discount 25%.

Find the net amount to be paid by the retailer on each of these purchases:

4. List price $120.00, trade discount 16⅔%. What is the price charged? Now allow him a cash discount of 2% and find the net amount.
5. List price $75.00, trade discount 20%. What is the price charged? Now allow him a cash discount of 2½% and find the net amount.
6. List price $88.00, trade discount 12½%. What is the price charged? Now allow him a cash discount of 3% and find the net amount.

LESSON 24

Commission

"Commission" is an amount of money which is paid by one person who is called the "principal" to another person who is called the "agent" for some service which the agent performs for the principal.

Commission is almost always figured by percentage; therefore, commission is a certain % of the sale or investment.

The amount of the sale or investment is the base or 100%.

$$B \times R = P \text{ (Com.)}.$$

Salesmen are often paid a commission on the amount of their sales, instead of being paid a salary by the week or by the month.

Exercise 58—Oral.

1. A salesman is paid a commission of 10% for selling a bill of goods amounting to $175.00; what does his commission amount to? What is the B? What is the R? What is the P (Com.)?
2. A salesman's commission on a sale of $100.00 amounted to $15.00; what rate of commission did he receive? After paying the salesman, what per cent remained for the merchant?
3. A salesman whose commission was figured at 10%, received $32.50 on a certain sale; what was the amount of the sale?
4. Say in your own words what "commission" is.
5. Is the principal the person who pays or who receives commission?
6. Who is the agent?
7. How is commission figured?
8. Show by letters and signs how to figure commission.

Exercise 59—Written.

Solve and prove:

Find the amount of commission paid by a wholesale house to one of its salesmen on each of these sales:

1. Amount of Sale $325.00; Rate of Commission 8%.
2. Amount of Sale $387.50; Rate of Commission 10%.
3. Amount of Sale $275.00; Rate of Commission 12%.
4. Amount of Sale $180.00; Rate of Commission 15%.

Find the amount of each of these sales:

5. Amount of commission $75.00, rate 10%. $75.00 is what part of the sale?
6. Amount of commission $90.00, rate 15%. $90.00 is what part of the sale?
7. Amount of commission $96.00, rate 12%. $96.00 is what part of the sale?

What is the rate of commission in the following:

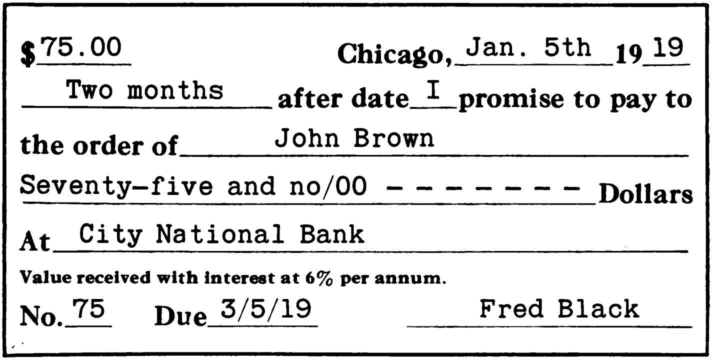
8. Amount of sale $125.00, commission $11.25. What part of the sale is the commission?
9. Amount of sale $345.00, commission $37.95. What part of the sale is the commission?
10. Amount of sale $360.00, commission $45.00. What part of the sale is the commission?

LESSON 25

Promissory Notes

When money is loaned, a written memorandum is usually given by the borrower to the person making the loan to show when and where the money is to be repaid; such a memorandum is called a "promissory note" because in it the borrower promises to repay the money.

A PROMISSORY NOTE



$75.00 Chicago, Jan. 5th 19 19

Two months after date I promise to pay to

the order of John Brown

Seventy-five and no/00 - - - - - - - - Dollars

At City National Bank

Value received with interest at 6% per annum.

No. 75 Due 3/5/19 Fred Black

The person who signs the note is called the "maker."

The person to whom the money is to be paid is called the "payee."

The date on which the note is made is called the "date" of the note.

The date on which the note is payable is called the "due date" or "date of maturity."

Exercise 60—Oral.

1. In the note shown in this lesson, who is the maker?
2. In this note, who is the payee?
3. If this note was given for borrowed money, who borrowed the money? Who loaned the money?
4. What amount was borrowed?
5. For what length of time was the money borrowed?
6. At what rate of interest was the money borrowed?
7. What is the date of the note?

8. What is the date of maturity of the note?
9. Is a "promissory note" a good name for this note? Why?

Lesson 26

Interest

EXAMPLE: Find the interest on $100.00 at 3% for 1 year.
3% of $100.00 = $3.00, which is the interest for 1 year.

EXAMPLE: Find the interest on $100.00 at 3% for ½ year.
3% of $100.00 = $3.00 for 1 year; ½ of $3.00 = $1.50, which is the interest for ½ year.

EXAMPLE: Find the interest on $50.00 at 4% for 3 months (¼ year).
4% of $50.00 = $2.00 for 1 year; ¼ of $2.00 = $0.50, which is the interest for 3 months.

"Interest" is money paid, or to be paid, for the use of money. In figuring interest, we again make use of percentage as we did in figuring discount and commission, but we have to take into consideration one new factor, and that is "time," for interest is always paid at a certain rate per cent for a certain length of time, and the time is always figured on the basis of one year. Thus, when we say that a savings bank pays 3% interest, we mean that it pays $3.00 for the use of $100.00 *for 1 year*, "for 1 year" being understood. We could say $0.03 on every dollar left in the bank for 1 year.

The term "per annum" which means "by the year" is also very often used to show that the rate of interest is for 1 year; thus, "6% per annum" means that interest is to be paid by the year at 6%. From this you see that to figure interest, three things have to be known, viz.:

First. The sum of money on which interest is to be paid, this being called the "principal." This is the base or 100%.

In the promissory note what is the principal?

Second. The rate per cent at which the interest is to be paid, this being called the "rate."

In the note what is the rate %?

Third. The length of time for which the interest is to be paid, this being called the "time."

For what length of time was the note to run?

The principal plus the interest is called the "amount."

First find the interest for 1 year, then find the interest for the required number of years or parts of a year.

Exercise 61—Written.

Solve and prove:

1. What is the interest on $100.00 for 1 year at 5%? On $200.00? On $300.00?
2. What is the interest on $300.00 for 1 year at 4%? At 5%? At 6%? At 7%?
3. What is the interest on $200.00 for 1 year at 6%? For $\frac{1}{2}$ yr.? For $\frac{1}{4}$ yr.? For $\frac{1}{3}$ yr.?
4. What is the interest on $100.00 for 1 year at 6%? For 6 mo.? For 4 mo.? For 3 mo.?

What is the interest at 6% on:

5. $300.00 for 2 yr.?
6. $250.00 for 4 yr.?

7. $275.00 for 2½ yr.?
8. $360.00 for 7 mo.?

What is the interest at 5% on:

9. $35.00 for 5 yr.?
10. $90.00 for 1 yr. 6 mo.?
11. $126.00 for 8 mo.?
12. $175.00 for 2 yr.?

What is the interest at 4% on:

13. $87.50 for 18 mo.?
14. $90.00 for 2 yr. 7 mo.?
15. $250.00 for 4 mo.?
16. $75.00 for 1 yr. 5 mo.?

What is the interest at 3% on:

17. $200.00 for ⅔ yr.?
18. $150.00 for 1⅓ yr.?
19. $75.00 for 8 mo.?
20. $225.00 for 1 yr. 4 mo.?

Exercise 62—Oral.

1. What is interest?
2. What element or factor must we consider when figuring interest that we did not have to consider in figuring commission?
3. What is meant by "Interest at 6% per annum"?
4. What name is given the sum of money on which interest is to be paid?
5. What three things must always be known before interest can be figured?
6. How do we figure the interest on any sum for 1 year at 5%?

7. How do we figure the interest on any sum for several years at 4%?
8. How do we figure the interest on any sum for a fraction of a year at 7%?
9. What part of a year is 1 mo.? 2 mo.? 3 mo.? 4 mo.? 5 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 10 mo.? 11 mo.? 12 mo.?

Exercise 63—Oral and Written.

Solve and prove on paper:

1. I borrowed $250.00 Jan. 6th of last year and paid it back Jan 6th of this year; if I paid interest at 6%, how much interest did I pay? What was the total amount I paid to settle my whole debt?

2. If I borrow $75.00 from a bank to-day at 6%, and pay my debt in 6 mo., what total amount will I have to pay to settle my debt?

3. What is the interest on $350.00 at 7%, for 6 mo., from March 8th to Sept. 8th?
4. What is the interest on $85.00 at 6%, for 4 mo., from July 6th to Nov. 6th?
5. What is the interest on $125.00 at 4%, for 9 mo., from May 8th to Feb. 8th?
6. What is the interest on $300.00 at 5%, for 3 mo., from Oct. 24th to Jan. 24th?
7. If I borrow $175.00 for 4 mo. at 6%, how much will I have to pay to settle my debt in full?
8. If I borrow $84.00 for 3 mo. at 4%, how much will I have to pay to settle my debt in full?
9. If I borrow $125.00 for 2 mo. at 6%, how much will I have to pay to settle my debt in full?
10. A merchant bought an invoice of merchandise amounting to $500.00 on terms of 3% 10 days, Net 30 days; in order to take the cash discount on the invoice he had to borrow $500.00, for 2 mo. at 6%, from a bank. Answer the following: (*a*) What was the amount of the discount on the invoice? (*b*) What was the amount of interest he had to pay the bank? (*c*) What did he save by borrowing the money to pay the invoice in 10 days?

Solve and prove orally:

11. What is the interest on $100. at 6%, for 1 year? For 6 months? For 1 month?
12. What is the interest on $300. at 4%, for 1 year? For 1 month? For 10 months?
13. What is the interest on $200. at 5%, for 1 year? For 6 months? For 3 months?

PERCENTAGE

LESSON 27

Elapsed Time

EXAMPLE: From Sept. 3d to Sept. 29th equals how many days?
29 (second date) − 3 (first date) = 26 days, Ans.

EXAMPLE: From Mar. 5th to June 12th equals how many days?
From Mar. 5th to Mar. 31 = 26 days (31 − 5)
April = 30 days
May = 31 days
to June 12 = 12 days (To June 12th only)
Total 99 days, Ans.

To find the number of days between two dates, first find how many days there are after the first date to the end of that month by subtracting the date from the days in that month; then add these days to the days in each of the months between the two dates, up to, and including, the second date.

EXAMPLE: Find the elapsed time from Apr. 6th, 1918, to Apr. 25th, 1919.
From Apr. 6, 1918, to Apr. 6, 1919 = 1 year
From Apr. 6, 1919, to Apr. 25, 1919 = 19 days
Ans., 1 year 19 days

When the length of time is more than a year, figure years and days.

EXAMPLE: What date is 45 days after June 8th?
From June 8th to June 30th = 22 days
From July 1st to July 23 = 23 days
Total 45 days
(Leaving 23 of the 45 days to be used in July.)
Therefore, July 23d is 45 days after June 8th.

To find what date is a given number of days after another date, start with the first date and use as many of the given days as are necessary to the end of that month, then use the other days for other months, until the days are all used.

EXAMPLE: What date is 120 days after Oct. 9th, 1917?

Oct. 9 to Oct. 31 = 22 days (Leaving 98 days)
Nov. = 30 days (Leaving 68 days)
Dec. = 31 days (Leaving 37 days)
Jan. = 31 days (Leaving 6 days)
Feb. 1 to Feb. 6 = 6 days (Leaving 0 days)
Total 120 days

Therefore, Feb. 6th, 1918, is 120 days after Oct. 9th, 1917.

Do not confuse 30 days with 1 month, or 60 days with 2 months, etc.; watch the words because *60 days* from any date means exactly *60 days,* while *2 months* means *2 months* which may contain 59, 60, 61, or 62 days.

Exercise 64—Oral.

What is the number of days, or years and days, from:

1. April 18th to April 27th?
2. June 12th to June 18th?
3. Feb. 7th to Feb. 29th?
4. Aug. 1st to Aug. 31st?
5. Nov. 4, 1920, to Nov. 8, 1924?
6. Mar. 13, 1874, to Mar. 26, 1887?
7. Dec. 11, 1941, to Dec. 11, 1946?
8. Jan. 18, 1928, to Jan. 28, 1931?
9. Feb. 12, 1920, to Feb. 29, 1924?
10. July 4, 1931, to July 24, 1932?

Exercise 65—Written.

Find the elapsed time from:

1. Mar. 6th, to May 14th.
2. Oct. 8th, to Jan. 6th.
3. Feb. 4, 1916, to Apr. 7, 1916.
4. May 8, 1922, to Oct. 14, 1922.
5. Apr. 7, 1930, to June 6, 1931.
6. Nov. 4, 1922, to Mar. 3, 1924.

Find the date which is:

7. 60 days after Feb. 8, 1912.
8. 180 days after Oct. 4, 1921.
9. 1 year 90 days after Dec. 8, 1922.
10. 3 years 2 months after Apr. 4, 1930.

LESSON 28

Commercial Interest

"Commercial Interest" is interest for the exact number of days between two dates, figured on the basis of 360 days to the year. Thus, 1 day's commercial interest equals $\frac{1}{360}$ of a year's interest, 17 days' commercial interest equals $\frac{17}{360}$ of a year's interest, and so on. The denominator of your fraction is always 360.

When interest is to be paid, commercial interest is always used unless otherwise stated.

When the interest from one date to another date is required, always find the exact number of days—not months, or months and days—but when the interest for a certain number of months is required, always think of so many 12ths of a year.

When the number of days between two dates is 360, 361, 362, 363, 364, 365, or (in leap year) 366, figure the interest for 1 year.

In figuring commercial interest you can work principal × rate % × the number of days as so many 360ths of one year, or you can use the "1% 60-day method" which is also known as the "6% method" of computing interest. The latter is much shorter, therefore, you should always use it, rather than the other method.

The 1% 60-Day Method, or The 6% Method.

EXAMPLE: Find the interest on $225.13 for 60 days at 6%.

$2.25 = 60 days' interest at 6%.

(Found by moving the decimal point two places to the left, the interest for 60 days at 6% being 1% of the principal.)

Since 60 days equals $\frac{60}{360}$ or $\frac{1}{6}$ of a year, and since $\frac{1}{6}$ of 6% equals 1%, the interest on any sum for 60 days at 6% per annum equals 1% of that sum; therefore, move the decimal point two places to the left to find the interest for 60 days at 6%.

EXAMPLE: Find the interest on $48.00 for 70 days at 6%.

$0.48 = 60 days' interest at 6%; $\frac{7}{\cancel{6}} \times \overset{8}{\cancel{48}}¢ = 56¢$, interest for 70 days.

(70 days = $\frac{70}{60}$ or $\frac{7}{6}$ of 60 days; $\frac{7}{6}$ is $\frac{1}{6}$ more than 48¢.)

To find the interest for any number of days, take so many 60ths of the 1%; thus, 30 days' interest = $\frac{30}{60}$ or $\frac{1}{2}$ of 60 days' interest; 90 days' interest = $\frac{90}{60}$ or $\frac{3}{2}$ of 60 days' interest; and so on.

EXAMPLE: Find the interest on $32.00 for 75 days at 5%.
$0.32 = 60 days' interest at 6%;
75 days = $\frac{75}{60}$ or $\frac{5}{4}$ of 60 days; therefore, the interest for 75 days at 6% = $\dfrac{5}{\cancel{4}} \times \overset{8}{\cancel{32}}\text{¢}$, or 40¢;
5% = $\frac{5}{6}$ of 6%; therefore, the interest for 75 days at 5% = $\dfrac{5}{\underset{3}{\cancel{6}}} \times \overset{20}{\cancel{40}}\text{¢}$, or $\frac{100}{3}$¢; $\frac{100}{3}$¢ = $33\frac{1}{3}$¢ or 0.33\frac{1}{3}$.

To find the interest for any rate other than 6%, find so many 6ths of your answer; thus, 5% interest = $\frac{5}{6}$ of 6% interest; 4% interest = $\frac{4}{6}$ or $\frac{2}{3}$ of 6% interest; and so on.

Exercise 66—Oral.

1. What part of a year's interest is 30 days' commercial interest? 60 days'?
2. What kind of interest is used when no particular kind is agreed upon or mentioned?
3. When the interest from March 8th to May 8th is to be figured, must you use 61 days, or can you use 2 months and figure for $\frac{1}{6}$ year?
4. When the number of days between two dates is more than 360 but not more than a complete year, for what length of time do you figure the interest?
5. What is the shortest method of figuring commercial interest?
6. Why is the interest on any sum for 60 days at 6% equal to 1% of the sum?

7. What is the shortest way of finding the interest on any sum for 60 days at 6%?
8. After finding the interest for 60 days, how do you find the interest for any number of days at 6%? 80 da.? 90 da.? 120 da.?
9. After finding the interest for any number of days at 6%, how do you find the interest at any other rate per cent? 3%? 4%? 5%?
10. What is the interest on $10.00 for 60 days at 6%?
11. What is the interest on $50.00 for 30 days at 6%?
12. What is the interest on $12.00 for 60 days at 5%?
13. What is the interest on $8.00 for 90 days at 6%?
14. What is the interest on $100.00 for 30 days at 6%?
15. What is the interest on $150.00 for 60 days at 3%?
16. What is the interest on $60.00 for 20 days at 6%?
17. What is the interest on $30.00 for 10 days at 6%?
18. What is the interest on $40.00 for 45 days at 6%?

Exercise 67—Written.

Solve:

1. A business man borrowed $375.00 from a banker for 90 days at 6%; how much interest did he have to pay?
2. If you loaned someone $75.00 for 45 days at 5%, how much interest would you receive?
3. If you borrowed $175.00 from a bank for 72 days at 7%, how much interest would you have to pay?

Find the amount of interest plus principal on:

4. $500.00 from Aug. 5, 1920, to Aug. 3, 1921, at 5½%.

5. $1,000.00 from Apr. 8, 1925, to June 7, 1927, at 6%.
6. $314.50 from Mar. 3, 1910, to May 3, 1910, at 6%.
7. $375.00 from Feb. 6, 1912, to June 6, 1912, at 6%.

Exercise 68—Written.

(If you can work the first six correctly within a time stated by your teacher, you need not work the last four.)

Solve:

1. A note for $125.00 dated March 5, 1925, matures in 60 days and bears 6% interest; what date does the note mature? What amount must be paid to settle the note?
2. A note for $460.00 dated Jan. 6, 1920, matures in 90 days and bears 5% interest; what date does the note mature? What amount must be paid to settle the note?

3. A note for $86.00 dated May 6, 1921, matures in 3 months and bears 7% interest; what date does the note mature? What amount must be paid to settle the note?
4. A 30-day note for $75.86, bearing 5% interest, matures March 5, 1924; what is the date of the note? What amount must be paid to settle it?
5. A note for $575.00, bearing 6% interest, is dated Apr. 6, 1923, and matures July 8, 1923; what is the time on this note? What is the amount due at maturity?
6. A note for $45.00, bearing 7% interest, is dated Aug. 8, 1923, and matures March 5, 1924; what is the time on this note? What is the amount due at maturity?
7. A note for $1,200.00, bearing 6% interest, is dated Mar. 8, 1929, and matures July 6, 1929; what is the time on this note? What is the amount due at maturity?
8. A note for $250.00 dated Aug. 4, 1920, matures in 4 months and bears 6% interest; what date does the note mature? What amount must be paid to settle the note?
9. A note for $75.00, bearing 7% interest, is dated Jan. 2, 1921, and matures Feb. 1, 1921; what is the time on this note? What amount must be paid to settle the note?
10. A 60-day note for $500.00, bearing 6% interest, falls due Sept. 30, 1920; what is the date of this note? What amount must be paid to settle this note at maturity?

ADDITION AND SUBTRACTION

Lesson 29

The Use of Complements

(Another Good Thing To Know)

The "complement" of any number is the difference between that number and 10, 100, 1,000, etc.; therefore, the complement added to any number completes a ten, a hundred, a thousand, etc.

1 is the complement of 9, because 9 + 1 = 10.
1 is also the complement of 99, because 99 + 1 = 100.
1 is also the complement of 999, because 999 + 1 = 1,000.
2 is the complement of 8, 98, 998, etc.

EXAMPLE:

8,647
+ 98
8,745

Add 100 and subtract 2, which is the same as adding 98, but much faster. Whenever possible work from left to right instead of from right to left.

EXAMPLE:

4,688
+ 699
5,387

Add 700 and subtract 1, which is the same as adding 699.

Whenever any number to be added to another number is almost an even ten, hundred, thousand, etc., add the even ten, hundred, or thousand, and subtract the complement of the number.

EXAMPLE:

7,439 − 197 7,242	Subtract 200 and add 3, which is the same as subtracting 197.

Whenever any number to be subtracted from another number, is almost an even ten, hundred, thousand, etc., subtract the even ten, hundred, or thousand, and add the complement of the number.

Exercise 69—Oral.

1. Can you save time when adding 95? How?
2. Can you save time when subtracting 95? How?
3. How can you save time when adding numbers like 699? 98? 899? 795? 96?
4. How can you save time when subtracting numbers like 99? 199? 395? 97? 298? 998?

Add:

5.	(*a*)	(*b*)	(*c*)
	8,639	4,728	3,986
	694	893	598
6.	(*a*)	(*b*)	(*c*)
	7,387	3,987	1,468
	94	398	795

(Time for these six is 3 minutes.)

Subtract:

7.	(*a*)	(*b*)	(*c*)
	3,987	4,631	7,286
	498	97	994

8.	(*a*)	(*b*)	(*c*)
	1,921	4,386	3,872
	297	1,997	791
9.	(*a*)	(*b*)	(*c*)
	2,634	6,925	1,342
	399	996	495

(Time for these nine is 5 minutes.)

LESSON 30

Addition of Horizontally Arranged Addends

Very often, in your work in adding, you will find the addends arranged horizontally instead of vertically. With a little practice, you will be able to add horizontally just as rapidly as you can add vertically. Write numbers in columns very carefully.

EXAMPLE:

POPULATION OF NEW ENGLAND STATES

State	Population (Urban)	Population (Rural)	Population (Total)
Maine......	381,443	360,928	742,371
N. H.......	255,099	175,473	430,572
Vt.........	168,943	187,013	355,956
Mass.......	3,125,367	241,049	3,366,416
R. I........	524,654	17,956	542,610
Conn.......	999,839	114.917	1,114,756
Total......	5,455,345	1,097,336	6,552,681

In examples of this kind, the sum of the totals horizontally and vertically should balance, and this sum of the totals is called the "grand total" (6,552,681).

Exercise 70—Written.

1. Find the total area of each of these States, the total land area and water area of all of the States, and the grand total area of all of the States:

LAND AND WATER AREA OF MIDDLE ATLANTIC STATES

State	Land Area (sq. mi.)	Water Area (sq. mi.)	Total Area (sq. mi.)
N. Y.	47,654	1,550	
N. J	7,514	710	
Pa.	44,832	294	
Total.			

2. A silk dealer has 6 salesmen, and the following is a statement of the silk and satin sales of each; find the total sales of each of the salesmen, the silk sales and the satin sales of all of the salesmen, and the grand total sales:

SALES FOR OCTOBER

Salesmen	Silk Sales		Satin Sales		Total Sales	
Smith.	$436	75	$293	65	$	
Jones.	386	47	346	28		
Brown.	193	62	450	60		
Johnson.	438	72	238	67		
Miller.	629	18	240	00		
Wilson.	346	28	286	38		
Total.	$		$		$	

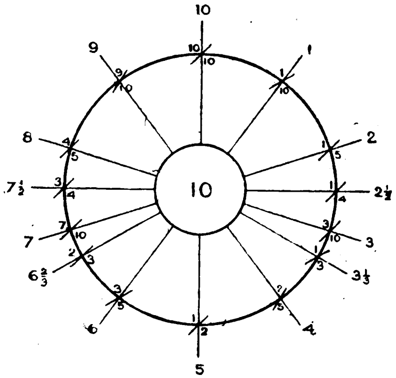
MULTIPLICATION AND DIVISION

LESSON 31

The Use of Aliquot Parts in Multiplication

(For Those Who Would Like To Speed)

An aliquot part of a number, as you know, is an equal part of a number; 50 is $\frac{1}{2}$ of 100; 75 is $\frac{3}{4}$ of 100; 25 is $\frac{1}{4}$ of 100.



ALIQUOT PARTS OF 10

Exercise 71—Oral.

1. What part of 100 is 25? 50? 75?
2. What part of 100 is $33\frac{1}{3}$? $66\frac{2}{3}$?
3. What part of 100 is $16\frac{2}{3}$? $83\frac{1}{3}$?
4. What part of 100 is $12\frac{1}{2}$? $37\frac{1}{2}$? $62\frac{1}{2}$? $87\frac{1}{2}$?
5. What part of 100 is 10? 20? 30? 40?
6. What part of 100 is 60? 70? 80? 90?

7. What part of 10 is $2\frac{1}{2}$? $3\frac{1}{3}$? 5? $6\frac{2}{3}$? $7\frac{1}{2}$?

8. 75 is $\frac{3}{4}$ of 100 or $\frac{1}{4}$ of ? $66\frac{2}{3}$ is $\frac{2}{3}$ of 100 or $\frac{1}{3}$ of ?

9. $83\frac{1}{3}$ is $\frac{5}{6}$ of 100 or $\frac{1}{6}$ of ? $37\frac{1}{2}$ is $\frac{1}{8}$ of ?

10. $62\frac{1}{2}$ is ? of 100 or $\frac{1}{8}$ of ? $87\frac{1}{2}$ is ? of 100 or $\frac{1}{8}$ of ?

11. 30 is ? of 100 or $\frac{1}{10}$ of ? 80 is ? of 100 or $\frac{1}{5}$ of ?

12. 40 is ? of 100 or $\frac{1}{5}$ of ? 60 is ? of 100 or $\frac{1}{5}$ of ?

13. 70 is ? of 100 or $\frac{1}{10}$ of ? 90 is ? of 100 or $\frac{1}{10}$ of ?

14. Why is the answer found by multiplying any number by 100 and taking $\frac{1}{4}$ of the product, the same as the answer found by multiplying that number by 25?

15. Why is the answer found by multiplying any number by 200 and taking $\frac{1}{3}$ of the product, the same as the answer found by multiplying that number by $66\frac{2}{3}$?

16. Why is the answer found by multiplying any number by 700 and taking $\frac{1}{8}$ of the product, the same as the answer found by multiplying that number by $87\frac{1}{2}$?

17. Why is the answer found by multiplying any number by 100 and adding $\frac{1}{4}$ of the product, the same as the answer found by multiplying that number by 125?

18. Why is the answer found by multiplying any number by 300 and adding $\frac{1}{10}$ of the product, the same as the answer found by multiplying that number by 330?

19. Why is the answer found by multiplying any number by 400 and subtracting $\frac{1}{10}$ of the product, the same as the answer found by multiplying that number by 360?

EXAMPLE: $864 \times 33\frac{1}{3} = ?$

($33\frac{1}{3} = \frac{1}{3}$ of 100)

86,400 (864×100)

28,800 ($\frac{1}{3}$ of 86,400)

Answer, 28,800.

To multiply by an aliquot part of 100, annex two ciphers (which multiplies by 100) and find the same fraction of this product that the aliquot part is of 100.

EXAMPLE: $712 \times 75 = ?$

($75 = \frac{3}{4}$ of 100 or $\frac{1}{4}$ of 300)

213,600 (712×300)

53,400 ($\frac{1}{4}$ of 213,600)

Answer, 53,400.

To multiply by an aliquot part of several hundred, multiply by the several hundred and find the proper fractional part of the product.

EXAMPLE: $746 \times 150 = ?$

($150 = 100 + \frac{1}{2}$ of 100)

74,600 (746×100)

37,300 ($\frac{1}{2}$ of 74,600)

111,900, Ans.

EXAMPLE: $484 \times 1{,}500 = ?$

($1{,}500 = 1{,}000 + \frac{1}{2}$ of 1,000)

484,000 ($484 \times 1{,}000$)

242,000 ($\frac{1}{2}$ of 484,000)

726,000, Ans.

When multiplying by a number made up of even tens, hundreds, etc., plus or minus an aliquot part thereof, as 150, 220, etc., multiply by the even tens or hundreds in the shortest manner, and add or subtract the proper aliquot part of this product: $150 = 100 + \frac{1}{2}$ of 100; $1{,}500 = 1{,}000 + \frac{1}{2}$ of 1,000.

Exercise 72—Oral and Written.

A. First tell just what to do, as: "multiply by 100 and divide by 3."

1. $18 \times 33\frac{1}{3} = ?$
2. $36 \times 25 = ?$
3. $48 \times 16\frac{2}{3} = ?$
4. $17 \times 99 = ?$
5. $16 \times 75 = ?$
6. $15 \times 66\frac{2}{3} = ?$
7. $16 \times 37\frac{1}{2} = ?$
8. $24 \times 110 = ?$
9. $687 \times 66\frac{2}{3} = ?$
10. $1{,}287 \times 75 = ?$
11. $1{,}763 \times 83\frac{1}{3} = ?$
12. $8{,}640 \times 25 = ?$
13. $4{,}387 \times 33\frac{1}{3} = ?$
14. $1{,}746 \times 660 = ?$
15. $7{,}348 \times 707 = ?$
16. $6{,}834 \times 125 = ?$
17. $5{,}248 \times 270 = ?$
18. $1{,}274 \times 550 = ?$

B. Now work all of them, and prove.

(Time for last ten should be from 5 to 10 minutes. See what your time is.)

LESSON 32

The Use of Aliquot Parts in Division

EXAMPLE: $732 \div 33\frac{1}{3} = ?$

$(33\frac{1}{3} = \frac{1}{3} \text{ of } 100)$

$$\begin{array}{l} 7.32 \ (732 \div 100) \\ \underline{\times\ 3} \\ 21.96\text{, Ans.} \end{array}$$

To divide by an aliquot part of 100, point off two decimal places (which divides by 100) and multiply by the number which shows how many times the aliquot part is contained in 100; thus, to divide by 25, we divide by 100 and multiply by 4, since 25 is contained

4 times as often as 100 in any number, there being four 25's in each 100.

EXAMPLE: $46.72 \div 66\frac{2}{3} = ?$

$(66\frac{2}{3} = \frac{2}{3}$ of 100 or $\frac{1}{3}$ of 200)

$$\begin{array}{l} .2336 \ (46.72 \div 200) \\ \underline{\times \quad 3} \\ .7008, \text{ Ans.} \end{array}$$

To divide by an aliquot part of several hundred, divide by the several hundred and multiply by the number which shows how many times that aliquot part is contained in the several hundred.

Exercise 73—Oral.

1. $\frac{7}{8}$ of 100 is the same as $\frac{1}{8}$ of how many hundred?
2. $\frac{5}{6}$ of 100 is the same as $\frac{1}{6}$ of ?
3. $\frac{2}{3}$ of 100 is the same as $\frac{1}{3}$ of ?
4. $\frac{3}{4}$ of 100 is the same as $\frac{1}{4}$ of ?
5. $\frac{5}{8}$ of 100 is the same as $\frac{1}{8}$ of ?
6. $\frac{3}{8}$ of 100 equals $\frac{?}{8}$ of 300?
7. $\frac{5}{6}$ of 100 equals $\frac{?}{6}$ of 500?
8. $\frac{2}{3}$ of 100 equals $\frac{?}{3}$ of 200?

Find the quotient of:

9. $675 \div 10 = ?$
10. $120 \div 33\frac{1}{3} = ?$
11. $60 \div 16\frac{2}{3} = ?$
12. $112 \div 20 = ?$
13. $22 \div 25 = ?$
14. $24 \div 50 = ?$
15. $90 \div 12\frac{1}{2} = ?$
16. $100 \div 8\frac{1}{3} = ?$
17. $33 \div 3\frac{1}{3} = ?$
18. $21 \div 2\frac{1}{2} = ?$
19. $900 \div 33\frac{1}{3} = ?$
20. $500 \div 16\frac{2}{3} = ?$
21. $600 \div 12\frac{1}{2} = ?$
22. $300 \div 8\frac{1}{3} = ?$

Exercise 74—Written.

Divide and prove:

1. $1{,}428 \div 87\frac{1}{2} = ?$
2. $8{,}635 \div 62\frac{1}{2} = ?$
3. $6{,}081 \div 37\frac{1}{2} = ?$
4. $47.28 \div 66\frac{2}{3} = ?$
5. $186.3 \div 75 = ?$
6. $16.25 \div 83\frac{1}{3} = ?$
7. $283.5 \div 87.5 = ?$
8. $68.7 \div 37.5 = ?$
9. $72.5 \div 62.5 = ?$
10. $14.25 \div 83.33\frac{1}{3} = ?$

Exercise 75—Oral Review.

1. (*a*) $\frac{6}{7} \div \frac{1}{2} = ?$ (*b*) $\frac{3}{5} \times \frac{8}{5} = ?$ (*c*) $\frac{3}{4} - \frac{3}{8} = ?$
2. A set of books which cost $50.00 was sold at a loss of 20%; what amount was received for the books?
3. A discount of $11.00 was allowed on a desk which was marked $44.00; what part of the marked price was allowed? How much was the discount? How much was paid for the desk?
4. The trade discount on an article amounts to $10.00; if the rate of discount is 20% what is the list price?
5. What is the interest on $50.00 for 60 days at 6%? How much for 30 days? For 90 days?
6. Add:

	(*a*)	(*b*)	(*c*)
	4,638	5,938	1,765
	795	497	998

7. Subtract:

	(*a*)	(*b*)	(*c*)
	3,374	1,941	4,673
	595	397	94

(Time for #6 and #7 is 2 minutes.)

8. Multiply: (*a*) 36×25 (*b*) $21 \times 33\frac{1}{3}$ (*c*) $16 \times 16\frac{2}{3}$

9. Divide: (*a*) $30 \div 16\frac{2}{3}$ (*b*) $12 \div 12.5$ (*c*) $33 \div 33\frac{1}{3}$

(Time for #8 and #9 is 5 minutes.)

10. (*a*) Say the table of Square Measure.
(*b*) Say the table of Cubic Measure.
(*c*) Say the table of Common Linear Measure.
(*d*) Say the table of Dry Measure.

Exercise 76—Written Review.

1. What is the ratio of 6 ft. 8 in. to 1 ft. 8 in.?
2. What is the ratio of 3 yd. 1 ft. to 33 yd. 1 ft.?
3. What per cent of 1 lb. 4 oz. is 5 oz.?
4. What is 15% of 1 sq. ft. 16 sq. in.?
5. After allowing a discount of 20% from the marked price, a merchant sold a picture for $4.80; what was the marked price?
6. A note for $525.00, bearing 6% interest is dated Jan. 8, 1920, and matures in 80 days; what is the date of maturity of the note? What amount must be paid to settle it?
7. $864.78 \div .426 = ?$
8. What per cent is 22.38 of 7.46?

Copy and divide:

(Time for these 6 examples is less than 5 minutes.)

9. $6{,}348 \div 23$;
10. $84{,}456 \div 92$;
11. $9{,}880 \div 38$;
12. $78{,}601 \div 83$;
13. $60{,}225 \div 73$;
14. $31{,}067 \div 47$.

Copy and multiply:

(Time for these 8 examples is less than 5 minutes.)

15. 271 × 31;
16. 936 × 13;
17. 670 × 76;
18. 560 × 29;
19. 779 × 24;
20. 897 × 23;
21. 609 × 79;
22. 797 × 36.

Add, but do not copy:

(Time for these 6 examples is less than 5 minutes.)

23.	**24.**	**25.**	**26.**	**27.**	**28.**
4,387	2,256	4,582	9,176	8,856	4,560
7,842	3,572	7,356	4,793	7,941	8,107
5,621	8,919	3,176	1,925	1,050	2,057
8,743	5,198	8,990	8,801	1,351	3,160
1,938	6,542	5,421	7,056	5,680	7,109
3,156	1,934	2,756	3,460	6,843	5,823

Subtract, but do not copy:

(Time for these 12 examples is less than 5 minutes.)

29.	**30.**	**31.**	**32.**
435,783	958,761	518,927	885,441
185,611	375,892	250,510	704,956

33.	**34.**	**35.**	**36.**
618,756	315,846	778,592	258,471
419,732	105,796	431,791	139,683

37.	**38.**	**39.**	**40.**
854,318	587,962	329,546	905,781
396,209	299,463	191,352	667,503

ACCOUNTS

LESSON 33

How Accounts Are Kept

So that a merchant may at all times know who owes him money, and to whom he owes money, he keeps a set of books containing an "account" with each such person or firm.

The merchant also keeps various other accounts from which he can tell whether the business is being conducted at a profit or at a loss, and he keeps a "Cash Account" which shows him how much money is received, how much money is paid out, and how much money he should have on hand.

All accounts excepting the Cash Account are kept in a book which is called a "Ledger."

The Cash Account is kept in a separate book which is called a "Cash Book," and this will be described later.

Every account in the Ledger has two money columns in which to record transactions, and these two columns are separated by a line running down the center of the page.

The left-hand column of an account is used for recording entries which show that the person or thing on whose account the entries appear, has received something of value; thus, all "charges" and "debts" against another person are entered on the left-hand side of the account, which is called the "debit" (dr.) side of the account (acct. or a/c).

The right-hand column of an account is used for recording entries which show that the person or thing on whose account the entries appear, has parted with something of value; thus, all amounts paid by another person are entered on the right-hand side of the account, which is called the "credit" (cr.) side of the account.

The general rule is to debit a person with everything of value he receives from you, and to credit him with everything of value you receive from him. Whoever owes money is a "debtor" (dr.); whoever has money owing him is a "creditor" (cr.).

(If Jones owes Brown $100.00, Jones is Brown's debtor, and Brown is Jones's creditor.)

To find how much money a debtor owes, or how much money is due a creditor, find the sum of all the debits on the account, then find the sum of all the credits, and subtract one sum from the other; if the *debit sum is the greater*, the difference or balance is the amount that is due from a *debtor;* if the *credit sum is the greater*, the difference or balance is the amount that is owing to a *creditor.*

After finding the balance, enter it on the smaller side of the account, which will make the two sums equal, and after entering the totals and ruling as shown in the illustration, bring down the balance to the opposite side of the account; thus, a debit balance is first entered on the credit side of the account to make the two sides equal and is then brought down to the debit side of the account which again makes the debit side larger than the credit side; this is called "balancing" an account.

JOHN SMITH.

Dr.								Cr.	
1918					1918				
Oct.	7	Mdse.	36	--	Nov.	1	Cash	100	--
	9	"	14	75		10	Note	50	--
	14	"	48	70		15	Cash	25	--
	28	"	31	25	Dec.	1	Balance	50	35
Nov.	1	"	25	--					
	4	"	19	50					
	8	"	38	75					
	15	"	11	40					
			225	35				225	35
Dec.	1	Balance	50	35					

Exercise 77—Oral.

1. Why does a merchant keep accounts? What does "acct." or "a/c" stand for?
2. What is the name given the book in which accounts are kept?
3. How many sides or money columns has an account?
4. What is the left-hand side called? What class of items are recorded there?
5. What is the right-hand side called? What class of items are recorded there?
6. What is the general rule for debiting and crediting?
7. What is the name which is given to one who owes money? To the one who has money owing him?
8. How do we find the balance on an account?
9. When the sum of the debits on an account is greater than the sum of the credits, what does that show? What is such a balance called?

10. When the sum of the credits on an account is greater than the sum of the debits, what does that show? What is such a balance called?
11. How do you balance an account when the debit side is the greater? What does "dr." mean?
12. How do you balance an account when the credit side is the greater? What does "cr." mean?

Play that you are in the grocery business, and you have on your books an account in the name of John Smith, such as is illustrated:

13. Call off all the debit items on this account.
14. Call off all the credit items on this account.
15. Which side of the account was the greater before entering the balance? How much greater?
16. Which side of the account is the greater after entering the balance and bringing it down?
17. Has the balance been affected in any way by balancing this account?
18. Does this account show that John Smith owes you money, or that you owe John Smith money?
19. Is John Smith your debtor or your creditor? Why?
20. Are you John Smith's debtor or creditor? Why?

Exercise 78—Written.

Play that you are in the wholesale toy business and that you buy and sell the following merchandise; rule paper, write up the accounts, and balance them:

1. On Jan. 6th you sell Fred Barnum mdse. amounting to $75.00; Jan. 15th, mdse. $87.50; Jan. 18th he pays you cash $150.00; Feb. 6th he buys mdse. $75.00; Feb. 15th he pays $50.00.

2. On Jan. 15th you sell to your teacher mdse. amounting to $45.00; On Jan. 25th, mdse. $55.00; On Feb. 1st your teacher returns mdse. amounting to $25.00; On Feb. 1st your teacher pays cash for the balance of the account.
3. On Mar. 1st you buy from Nelson Bros. mdse. amounting to $87.50; On Apr. 5th, mdse. $90.00; On Apr. 10th you pay $50.00 in cash and $75.00 in the form of a 60-day note; Apr. 20th you buy mdse. amounting to $35.00 and pay cash $25.00.
4. Fred Fuller is your debtor on Jan. 1st in the sum of $180.00 and his account is settled in full on Feb. 1st by the payment of cash.
5. George Green is your creditor on April 15th in the sum of $75.00, and his account is settled in full on May 1st by the payment of cash.

Balance each of the following accounts:

6.		7.		8.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
$93.00	$560.00	$76.00	$48.00	$640.00	$500.00
114.00		148.00	79.00		875.00
87.60		39.00	50.00		375.00
96.40		74.00	100.00		460.00
72.00					

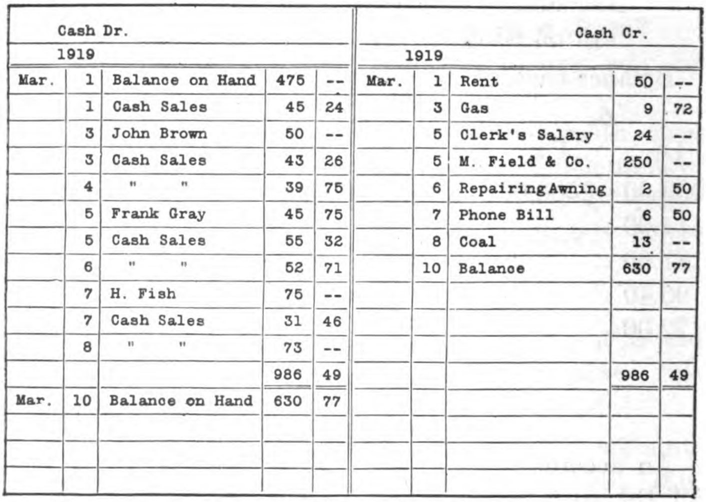
LESSON 34

Cash Account

An account which shows all the cash received and all the cash paid out by anyone is called a "Cash Account". If you think of the cash drawer as receiving what you place in it, and as parting with what you spend out of it, you will easily understand that Cash Account is debited with all cash received (called "receipts"), and is credited with all cash paid out (called "disbursements").

The Cash Account is kept in a book which is called a "Cash Book," of which all the left-hand or debit pages correspond with the debit side of a ledger account, and all the right-hand or credit pages correspond with the credit side of a ledger account.

A Cash Account will always show a debit balance (if it shows any balance whatever) since it is impossible to pay out more money than has been received. A Cash Account is balanced in the same manner as other accounts are balanced, as is shown in the illustration.



Cash Dr.					Cash Cr.				
1919					1919				
Mar.	1	Balance on Hand	475	--	Mar.	1	Rent	50	--
	1	Cash Sales	45	24		3	Gas	9	72
	3	John Brown	50	--		5	Clerk's Salary	24	--
	3	Cash Sales	43	26		5	M. Field & Co.	250	--
	4	" "	39	75		6	Repairing Awning	2	50
	5	Frank Gray	45	75		7	Phone Bill	6	50
	5	Cash Sales	55	32		8	Coal	13	--
	6	" "	52	71		10	Balance	630	77
	7	H. Fish	75	--					
	7	Cash Sales	31	46					
	8	" "	73	--					
			986	49				986	49
Mar.	10	Balance on Hand	630	77					

Cash Book
(Showing a left-hand page and a right-hand page.)

Exercise 79—Oral.

1. What is an account called which shows all the cash received and all the cash paid out by anyone?
2. What is the book called which contains the Cash Account?
3. Which side of the Cash Book is used for recording cash received?
4. Which side of the Cash Book is used for recording cash paid out?
5. Why does a Cash Account always show a debit balance if it shows any balance whatever?
6. How is a Cash Account balanced?
7. In the Cash Account shown in the illustration, which of the items represent cash receipts?
8. In this illustration, which of the items represent cash payments or disbursements?
9. What was the amount of cash on hand March 1st?
10. What was the amount of cash on hand March 10th?

Exercise 80—Written.

Rule paper to represent a Cash Book, make the following entries, and balance the account:

1. Receipts:

Apr. 1, Cash Sales, $43.00;
Paid by J. Smith on account, $50.00;
Apr. 3, Cash Sales, $25.00;
Paid by F. Jones on account, $25.00;
Apr. 6, Cash Sales, $17.00;
Paid by T. Burke, on account, $45.00.

Disbursements:

Apr. 4, Gas, $13.75;
Rent, $50.00;
Paid Murdock & Co. on account, $50.00;
Apr. 5, Salary, $30.00;
Apr. 6, Coal, $12.00;
Paid Johnson & Co. on account, $40.00.

2. June 1, Balance on Hand, $125.00;
1, Cash Sales, $30.00;
1, Paid for light, $7.50;
2, Cash Sales, $60.00;
3, Cash Sales, $45.00;
4, Paid Frank Murphy & Co. on account, $100.00;
4, Received from F. Gray in full, $20.00;

3. Supply dates and names suitable for the following entries and write up the Cash Book:

Receipts:	Disbursements:
$40.00	$35.00
50.00	25.00
45.00	18.00
35.00	16.00
85.00	41.00

4. Supply dates and names suitable for the following entries and write up the Cash Book:

Balance on hand, Monday morning:	$65.00
Received during the week:	14.00
	60.00
	25.00
	30.00
	40.00

Paid during the week:	$75.00
	21.00
	13.00
	62.00
	18.00

5. Tom kept an account of his cash receipts and expenditures during his vacation, and balanced his account each month. The following are the items he entered. Rule paper to represent a Cash Book, make the entries properly, and show how much money Tom had August 1st and September 1st:

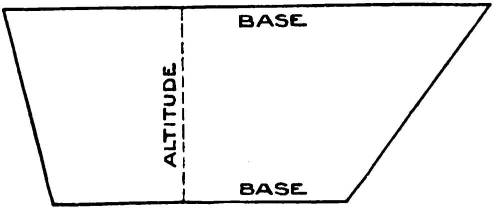
July 1.	Rec'd for delivering groceries	$0.50
4.	Paid for ticket for ball game	0.25
7.	Rec'd for cleaning store windows	0.75
8.	Paid for present for mother	1.00
15.	Rec'd for delivering papers (July 1 to 15)	2.50
16.	Paid for tennis slippers	1.50
22.	Rec'd for delivering groceries	0.50
22.	Paid for dictionary	1.25
31.	Rec'd for delivering papers (July 16 to 31)	2.50
Aug. 3.	Rec'd for running errands	0.25
4.	Paid for fishing tackle	0.50
5.	Rec'd for delivering groceries	0.75
7.	Paid for roller skates	2.00
15.	Rec'd for delivering papers (Aug. 1 to 15)	2.50
17.	Paid for bell for bicycle	0.60
18.	Rec'd for cleaning store windows	0.75
25.	Paid for school books	2.25
31.	Rec'd for delivering papers (Aug. 16 to 31)	2.50
31.	Paid for school supplies	1.00

MENSURATION

Lesson 35

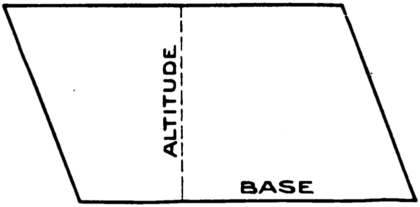
Trapezoids, Parallelograms, Rectangles, and Triangles Compared

A "trapezoid" is a plane figure having four straight sides, of which only one pair of sides are parallel; these parallel sides are called the two "bases" of the trapezoid and the distance between them is the "altitude."



A Trapezoid

A trapezoid differs from a "parallelogram," because a parallelogram has two pairs of parallel sides.

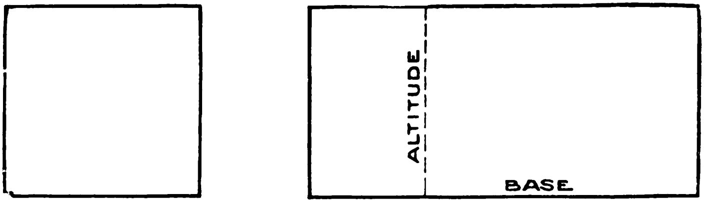


A Parallelogram

A "rectangle" is a parallelogram which has four right angles, and may be either square or oblong.

MENSURATION

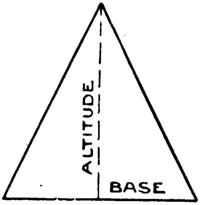
Rectangles



A Square

An Oblong

A "triangle" is a plane figure having three straight sides and three angles.



A Triangle

Exercise 81—Oral.

1. How many sides has a trapezoid? How many of the sides are parallel?
2. What are the parallel sides of a trapezoid called?
3. What is the distance between the two parallel sides called?
4. How many sides has a parallelogram? How many of the sides are parallel?
5. In what way does a trapezoid differ from a parallelogram?
6. How many sides has a rectangle? How many of the sides are parallel?
7. What is necessary in a four-sided figure to make it a rectangle?

8. What are squares—parallelograms, rectangles, or trapezoids? What are oblongs?
9. How many sides and angles has a triangle?
10. How is the altitude of a triangle measured?

Exercise 82—Written.

Draw:

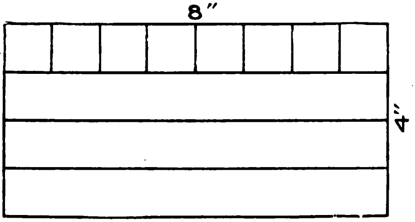
1. A trapezoid with an altitude of $\frac{3}{4}''$, one $2''$ base, and one $2\frac{3}{4}''$ base.
2. A trapezoid with an altitude of $1\frac{1}{2}''$, and bases of your own chosing.
3. A parallelogram (not a rectangle) with an altitude of $1''$, and a base of $2''$.
4. A parallelogram (not a rectangle) with an altitude of $\frac{1}{2}''$, and a base of your own chosing.
5. A rectangle with an altitude of $1\frac{1}{4}''$ and a $3''$ base.
6. A rectangle which is not an oblong.
7. A triangle with an altitude of $2''$ and a base of $\frac{3}{4}''$.
8. A triangle with an altitude of $1\frac{1}{2}''$ and a base of your own chosing.

LESSON 36

Finding the Area of Trapezoids, Parallelograms, Rectangles, and Triangles

As you have already learned, a rectangle has two dimensions, length and width; therefore, its area is measured by Square Measure, which is a measure of two dimensions. Thus, after we know what unit of measure we wish to use, we find how many of these units there are in one row, and multiply this by the number of rows in the rectangle.

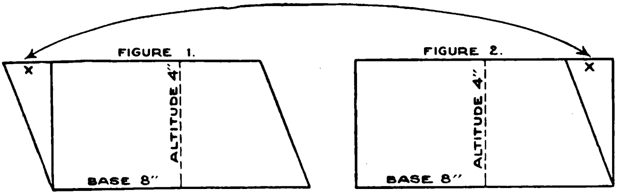
MENSURATION



(Scale $\frac{1}{4}$)

This drawing represents a rectangle containing 32 sq. in., because there are 4 rows, each of which contains 8 sq. in.; thus, the number in the length multiplied by the number in the width, or the number in the altitude multiplied by the number in the base, equals the area.

Since any parallelogram can be changed into a rectangle by drawing a line at right angles to the base and removing the triangular part so formed (x) from one end as shown in Figure 1, and adding it to the other end as shown in Figure 2, the area of a parallelogram will always be the same as the area of a rectangle having the same altitude and base; therefore, the number in the altitude multiplied by the number in the base equals the area of a parallelogram.



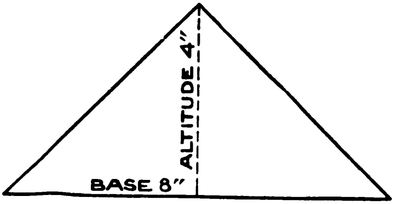
(Scale $\frac{1}{6}$)

(Scale $\frac{1}{6}$)

EXAMPLE: Altitude 4″ × Base 8″ = Area 32 sq. in.

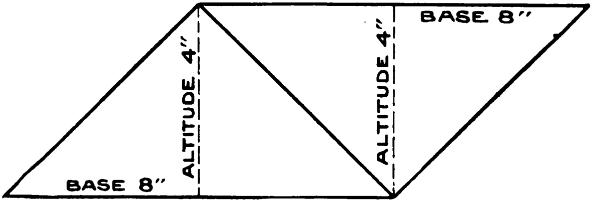
Since two triangles of equal size will make a parallelogram having the same altitude and base as one of the triangles, the area of a triangle is exactly one half the area of a parallelogram of the same dimensions; therefore, the product of the number in the altitude and the number in the base, divided by 2 equals the area of a triangle.

FIGURE A



(Scale $\frac{1}{4}$)
A Triangle

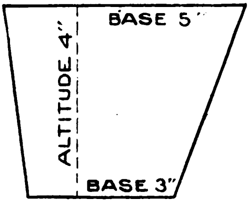
FIGURE B



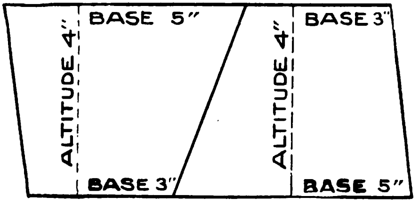
(Scale $\frac{1}{4}$)
A Parallelogram
(Showing Two Triangles of Equal Size)

EXAMPLE: Altitude 4″ × Base 8″ = 32 sq. in.;
32 sq. in. ÷ 2 = Area 16 sq. in.

Since two trapezoids of equal size will make a parallelogram having the same altitude as the trapezoids, but having a base equal in length to the two parallel sides or bases of the trapezoid, $\frac{1}{2}$ the sum of the number in the bases multiplied by the number in the altitude equals the area of a trapezoid. One-half the sum of the number in the bases is called the "average base."



(Scale $\frac{1}{4}$)
A Trapezoid

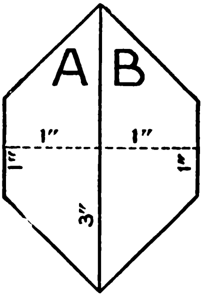


(Scale $\frac{1}{4}$)
A Parallelogram
(Showing Two Trapezoids of Equal Size)

EXAMPLE: Upper Base = 5″
Lower Base = 3″
Total 8″
Average Base 4″ × Altitude 4″ = Area 16 sq. in.

The area of figures of various shapes can be found by dividing such figures into parts having the shape of parallelograms, triangles, or trapezoids, and combining the areas of these parts.

One figure may be most easily measured by dividing it into two trapezoids; another may make a trapezoid and a triangle; still another may make a triangle and a rectangle; and so on.



(Scale ½)

A Figure Which Can Be Divided Into Two Trapezoids.

EXAMPLE: Area of Part A : (3″+ 1″ ÷ 2) × 1″ = 2 sq. in.
Area of Part B : (3″+ 1″ ÷ 2) × 1″ = 2 sq. in.
Total Area = 4 sq. in.

Exercise 83—Oral and Written.

1. Draw a trapezoid having an altitude of 2½″, one base of 4″ and one base of 2″, and cut this trapezoid and another just like it out of two sheets of paper at one time, and paste them together so that they form a parallelogram. (Use a strip of paper or a sticker to fasten them, so that the parallelogram may be the full size of the two trapezoids.)
2. What is the altitude of this parallelogram? How does this altitude compare with the altitude of either of the two trapezoids?
3. What is the base of this parallelogram? How does this base compare with the sum of the two bases of either of the trapezoids?

4. What is the area of this parallelogram?
5. What is the area of each of the trapezoids?
6. What is the area of a trapezoid having an altitude of 4″, one base of 3″, and one base of 5″?
7. Fold the parallelogram which you made of paper diagonally from upper left-hand corner to lower right-hand corner.
8. What is the area of each of the triangles you have now formed, if the area of the parallelogram is 15 sq. in.?
9. What is the area of a triangle having an altitude of 2″ and a base of 7½″?
10. Unfold the paper and draw a line at right-angles to the base of the parallelogram so that it will form a triangle at the one end of the parallelogram; cut this triangle off and paste it to the other end of the parallelogram to form a rectangle. (Use another strip of paper or a sticker.)
11. What is the area of this rectangle? How does this area compare with the area of the parallelogram out of which the rectangle was formed?
12. How does this area compare with the area of either of the two original trapezoids? How does it compare with the area of either of the triangles into which the parallelogram was folded?

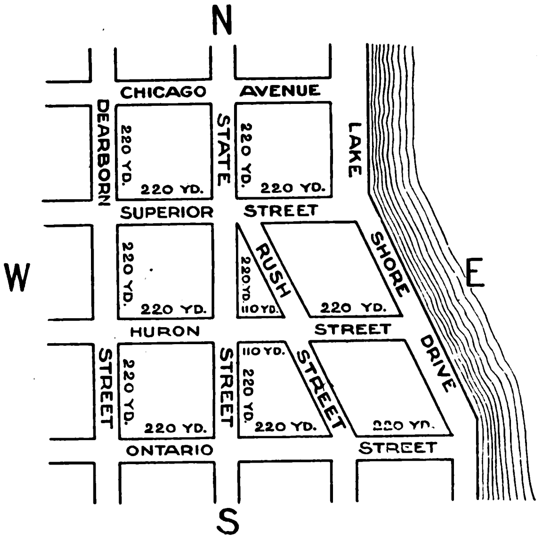
Exercise 84—Oral.

1. How do we find the area of a square?
2. How do we find the area of an oblong?
3. How do we find the area of a rectangle?
4. How do we find the area of a parallelogram?

5. How do we find the area of a triangle?
6. How do we find the area of a trapezoid?
7. How do we find the area of figures having odd shapes?
8. How do we change a parallelogram which is not a rectangle into one which is a rectangle?
9. What figure can be made out of two trapezoids of equal size?
10. What figure can be made out of two triangles of equal size?

Exercise 85—Oral and Written.

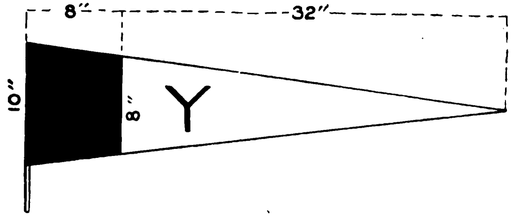
1. How many of the blocks in the map on Page 125 are rectangles?
2. How many of the blocks in this map are parallelograms, but not rectangles?
3. How many of the blocks in this map are triangles?
4. How many of the blocks in this map are trapezoids?
5. What is the area of the block bounded by Chicago Ave., State St., Superior St., and Dearborn St.?
6. What is the area of the block bounded by Superior St., Rush St., Lake Shore Drive, and Huron St.? What is the shape of this block?
7. What is the area of the block bounded by Huron St., Rush St., Lake Shore Drive, and Ontario St.?
8. What is the area of the block bounded by Rush St., Huron St., and State St.?
9. What is the area of the block bounded by Huron St., Rush St., Ontario St., and State St.?
10. What is the area of the block bounded by Dearborn St., Huron St., State St., and Ontario St.?



(Scale: ½ inch = ⅛ mile)

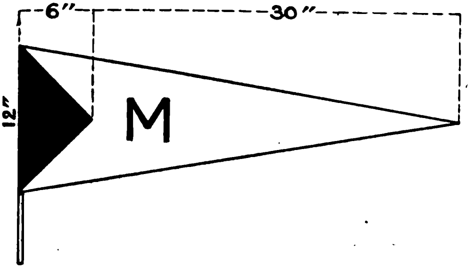
Exercise 86—Written.

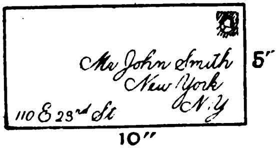
1. The dark part of this pennant is made of red felt



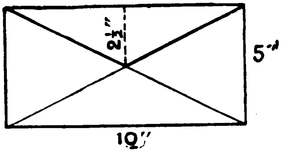
and the light part is made of white felt; how many square inches of red felt are there in this pennant? How many square inches of white felt are there in it?

2. The dark part of this pennant is made of green felt and the light part is made of white felt; how many square feet of felt are needed in all? How many square feet of each kind are needed?

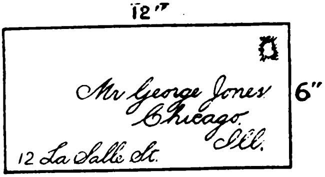




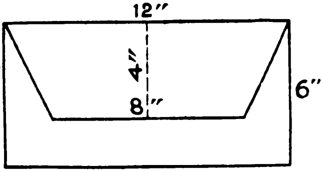
3. A paper envelope is 10" long and 5" wide; how many square inches of paper are there in the front of the envelope?



4. How many square inches of paper are there in the gummed flap of this envelope?

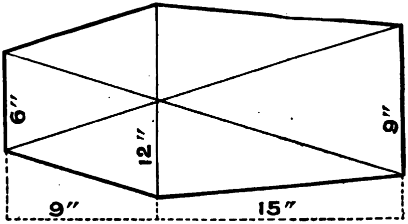


5. A paper envelope is 12" long and 6" wide; how many square inches of paper are there in the front of this envelope?



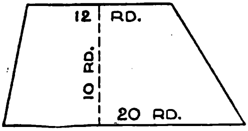
6. How many square inches of paper are there in the gummed flap of this envelope?

7. How many square inches of paper are needed to cover this kite? (Add 30 square inches for lapels after finding your answer.)

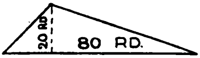


8. Find the area in acres of each of these tracts of land. (160 square rods = 1 acre.)

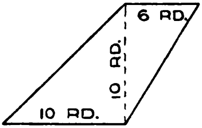
A



B

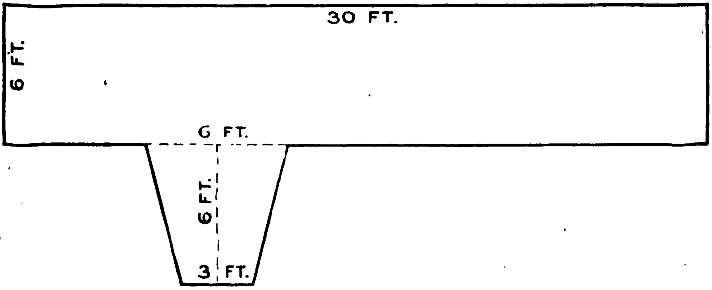


C



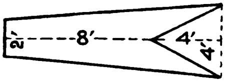
9. If corn is worth 60¢ per bushel, and yields 60 bu. to the acre, while wheat is worth $1.20 per bushel and yields 25 bu. to the acre, is it more profitable to plant corn or wheat on the farm shown in Figure (*b*)? How much more profitable?

10. What is the cost of laying this cement walk at $2.50 per square yard?

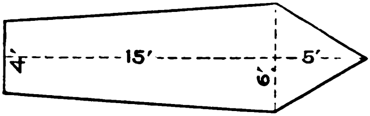


11. Find the area of each of the following figures:

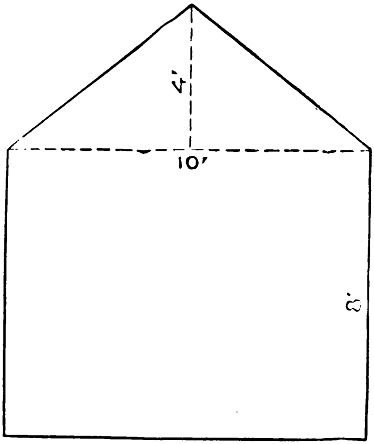
A



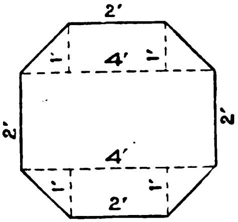
B



12. Find the area of each of the following figures:



A

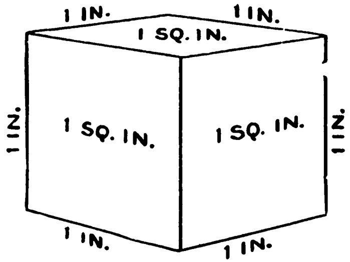


B

LESSON 37

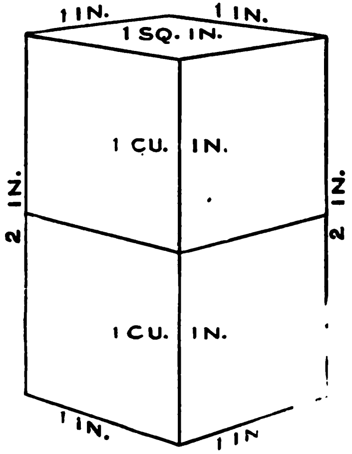
Cubes, Right Prisms, and Triangular Prisms Compared

As you have already learned, a "cube" is a solid having six square sides or "faces" of equal size, joined so that every angle is a right angle. A cube is a rectangular solid, just as a square is a rectangle.



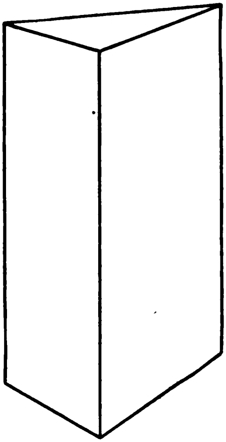
A One-inch Cube Containing
1 Cubic Inch

A "right prism" which means "right-angled prism" is a solid having four oblong sides and two square or oblong ends or bases, joined so that every angle is a right angle. A right prism is also a rectangular solid, just as an oblong is a rectangle.



A Right Prism Containing
2 Cubic Inches

A "triangular prism" is a solid having three square or oblong sides and two triangular ends or bases.



A Triangular Prism

Exercise 87—Oral.

1. How many sides or faces has a cube?
2. How many right angles are formed by the joining of the faces of a cube?
3. What is the shape of each of the faces of a cube?
4. Can any face of a cube be larger or smaller than any other face of the same cube?
5. How many sides has a right prism? What shape are they?
6. How many ends or bases has a right prism? What may be their shape?
7. How many right angles are formed by the joining of the sides and bases of a right prism?
8. How many sides has a triangular prism and what shape are they?

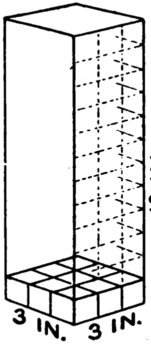
9. How many ends has a triangular prism and what shape are they? What name is given to the ends of any prism?
10. How many angles are formed by the joining of the sides and bases of a triangular prism?

LESSON 38

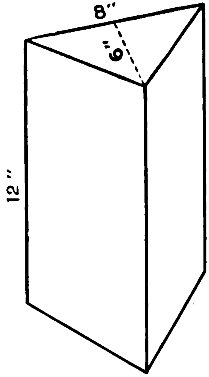
Finding the Cubic Contents or Volume of Cubes, Right Prisms, and Triangular Prisms

As you have already learned, a rectangular solid has three dimensions—length, width, and thickness; therefore, its volume is measured by Cubic Measure, which is a measure of three dimensions; thus, after we know what unit of measure we desire to use, we find how many of these units there are in each layer, and multiply this by the number of layers.

This drawing represents a right prism containing 108 cubic inches, because there are 12 layers, each of which contains 9 cubic inches; thus, the number of cubic units which will cover one end or base multiplied by the number of layers in the altitude equals the cubic contents or volume.

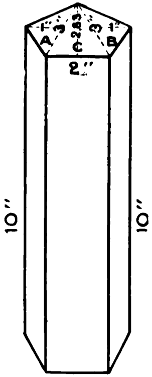


Since the volume of a triangular prism bears the same ratio to the volume of a right prism of the same altitude, as the area of one of its bases does to the area of one of the bases of the right prism, the volume of a triangular prism is equal to the volume of one layer at the base multiplied by the number of layers in the altitude.



EXAMPLE: One of the triangular ends has an altitude of 6″ and a base of 8″; therefore, the area of the end is 24 sq. in. and 1 layer contains 24 cu. in; the number of layers in the altitude of the prism is 12; 24 cu. in. in 1 layer × 12 (number of layers) = 288 cu. in., volume.

As we can find the area of figures of various shapes by dividing such figures into parts having the shape of parallelograms, triangles, or trapezoids, and combining the areas of these several parts, so we can find the volume of prisms of various shapes by dividing them into prisms having parallelograms, triangles, or trapezoids for bases, combining the areas of such bases to find the number of cubic units in 1 layer, and multiplying the number of units in 1 layer by the number of layers in the altitude.



EXAMPLE:

		sq. in.
Area (A)	3 sq. in. × 1 ÷ 2 =	1.5
Area (B)	3 sq. in. × 1 ÷ 2 =	1.5
Area (C)	2 sq. in. × 2.83 ÷ 2 =	2.83
Area of Entire Base	=	5.83

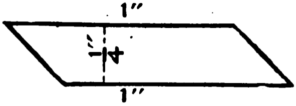
Area of Base = 5.83 sq. in.; therefore, 1 layer contains 5.83 cu. in.; 5.83 cu. in. × 10 (number of layers) = 58.3 cu. in., volume.

MENSURATION

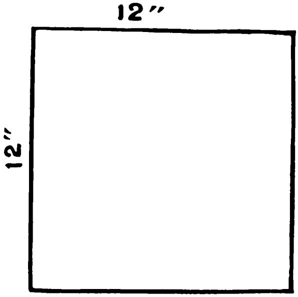
Exercise 88—Oral.

1. How do we find the volume of a cube?
2. How do we find the volume of a right prism?
3. How do we find the volume of any rectangular prism?
4. How do we find the volume of a prism having bases the shape of parallelograms?
5. How do we find the volume of a prism having bases the shape of triangles?
6. How do we find the volume of a prism having bases the shape of trapezoids?
7. How do we find the volume of prisms having bases of various shapes?
8. How many dimensions has a straight line? Name?
9. How many dimensions has a surface? Name them.
10. How many dimensions has a solid? Name them.

Exercise 89—Written.

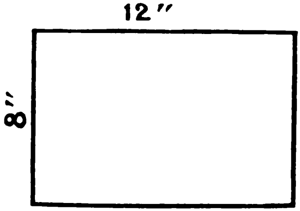


1. The base of a ruler is a parallelogram as shown in the drawing; the altitude is 12″; what is the volume? What would be the volume if the length of the ruler were 3 feet?

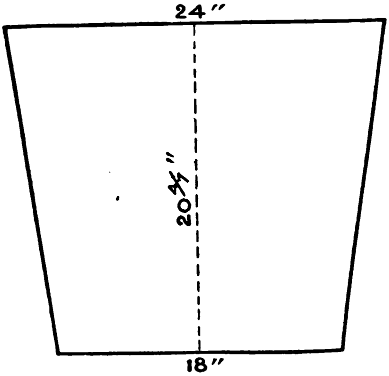


2. A pillar has a square base as shown in the drawing; the altitude is 15′; what is the volume?

3. A flight of stone steps has 20 steps, each of which has an oblong end as shown in the drawing, and is 15′ wide; how many cubic feet of stone does this flight contain?

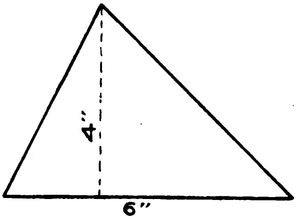


4. A watering trough is 8′ long, 24″ wide at the top, 18″ wide at the bottom, and 20$\frac{4}{7}$″ deep; how many cubic feet of water will it hold?

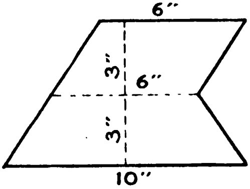


5. A retaining wall built of concrete is 8′ wide at the top, 10′ wide at the bottom, 12′ high, and 400′ long; how many cubic yards of concrete does it contain?

6. A triangular prism is 20″ long, and has a base as shown in the drawing; what is the volume?



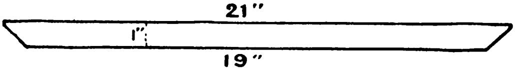
7. A block of granite is 18″ long, and its end is shaped as shown in the drawing; what is the volume in cubic inches?



8. A joist in a building is 6′ long, 8″ thick, and 10″ wide; what is its volume in cubic feet?

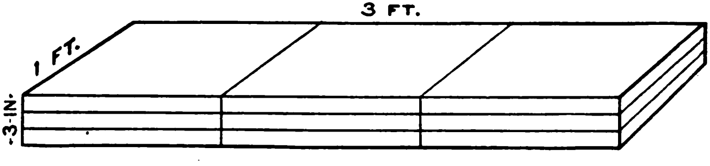
9. A wooden support is 2″ × 4″ × 10′; what is its volume?

10. A table has a beveled top 30″ long, 1″ thick, of which the upper surface is 21″ wide, and the lower surface is 19″ wide; what is the volume of this beveled top?



LESSON 39

Board Measure



9 Board Feet

The unit of measurement used for measuring lumber is the "board foot," which is the equivalent of a piece of lumber 1 ft. long, 1 ft. wide, and 1 in. thick; thus, a board 3 ft. long, 1 ft. wide, and 3 in. thick, would make 9 board feet.

To find the number of board feet in a piece of lumber, find the number of square feet in the area of its largest surface, and multiply this by the number of inches in the thickness.

Boards which are less than 1 in. thick, are usually figured as though they were exactly 1 in. thick.

Lumber in large quantities is usually sold at a certain price per "1,000 board feet."

Exercise 90—Oral.

1. How many board feet can be made from a cubic foot? Which is the larger?
2. If I gave you a cubic foot, what would you do to change it to board feet?
3. What are the dimensions of a board foot?
4. How do we find the number of board feet in any piece of lumber?

5. How many board feet are there in a board 1′ wide, 10′ long, and 1″ thick?
6. How many board feet are there in a board 2′ wide, 10′ long, and 1″ thick?
7. How many board feet are there in a board 2′ × 10′ × 2″?
8. How are boards usually measured when they are less than 1″ thick?
9. How many board feet are there in a board 2′ × 6′ × $\frac{3}{4}$″?
10. How is lumber sold when large quantities are dealt in?
11. What is the cost of 4M feet of lumber @ $40.00 per M?
12. In Question 11, what is meant by "4M feet"? What kind of feet are meant? What is meant by "@ $40.00 per M"?

Exercise 91—Written.

1. How many board feet are there in a board 16′ long, 2′ wide, and 2″ thick?
2. How many board feet are there in a board 12′ long, 6″ wide, and 3″ thick?
3. How many board feet are there in a board 14′ long, 18″ wide, and 5″ thick?
4. Find the cost of 6M feet of hardwood flooring @ $75.00 per M.
5. If lumber costs $40.00 per M feet, how much is that per board foot?

Find the cost of the following lumber:

6. 40 joists 18′ × 8″ × 4″ @ $30.00 per M.

7. 25 posts 12′ × 6″ × 4″ @ $35.00 per M.
8. 30 planks 12′ × 10″ × 3″ @ $32.50 per M.
9. 50 beams 16′ × 9″ × 8″ @ $27.50 per M.
10. 20 boards 8′ × 20″ × 3″ @ $43.75 per M.

Exercise 92—Written.

When boards have tongues and grooves, part of the width of the board is wasted. There is also a waste in fitting the lumber exactly to the space for which it is intended. To provide for this waste, carpenters usually add 25% or ¼ to the number of board feet when ordering lumber.

1. What will the lumber cost to cover a floor 15′ × 18′ at $40.00 per M, adding 25% for waste?
2. What will the lumber cost for a barn floor 20′ × 25′ if 2″ planks are used, and the cost per M is $30.00, adding 25% for waste?
3. A certain room is 18′ × 20′; the lumber used for the flooring of this room came in 12′ lengths, 3″ wide; how many pieces of lumber were needed allowing 25% for waste? What did the lumber cost @ $40.00 per M?
4. To make a stand for a statue, the following lumber is needed:

2 pieces 12″ × 6″ × 1″
4 pieces 12″ × 4″ × 1″
2 pieces 12″ × 12″ × 1″
2 pieces 8″ × 2″ × 1″

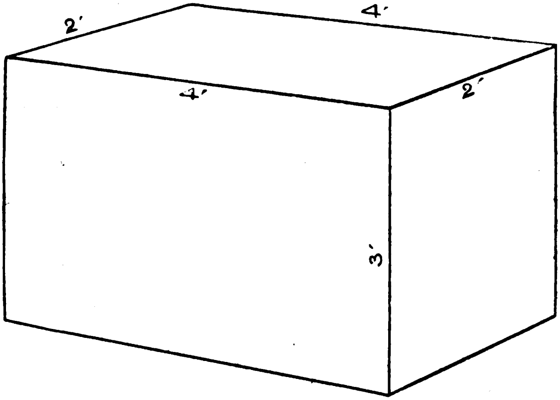
What is the total cost of this lumber at $90.C0 per M?

5. To make a tool-chest, Tom used the following lumber:

12 pieces 24″ × 6″ × ½″
2 pieces 18″ × 6″ × ½″
2 pieces 18″ × 12″ × ½″

How many square feet of ½″ lumber were used? How many board feet were used? What did the lumber cost at $20.00 per M?

6. How many square feet of 1″ lumber are needed to make the packing case (including the cover) shown in the drawing? How many board feet were used?



7. What is the volume of this packing case?

8. If the lumber used to make this case came in boards 12′ long, 4″ wide, and 1″ thick, how many boards were needed?

9. What is the cost of the lumber in this case @ $27.50 per M?

LESSON 40

Drawing to Scale

As you have already learned, when the drawing of an object is made larger or smaller than the actual size of the object, the relation of the size of the drawing to the size of the object is the "scale" on which the drawing is made; thus, if a square having sides 12″ long is represented by a drawing having sides 1″ long, the drawing is made on a scale of 1 to 12, or $\frac{1}{12}$ of the actual size. On this scale, 2 inches in the size of the drawing would represent 2 feet in the size of the object; 6 inches would represent 6 feet, and so on.

In the same manner, maps and plans are drawn to a scale, but naturally, 1″ in the size of a map might represent a great many miles of land; therefore, the scale might be 1″ = 100 miles, or 1″ = 300 miles, and so on.

This map of Ohio is drawn on a scale of 1″ = 115 miles.

Exercise 93—Written.

1. Measure the width of Ohio on this map; on the scale of 1″ = 115 miles, approximately how many miles wide is Ohio?
2. Approximately how many miles distant is Cincinnati from Cleveland?
3. Approximately how many miles distant is Cincinnati from Toledo?
4. Approximately how many miles distant is Toledo from Cleveland?
5. Springfield is on a straight line between Cleveland and Cincinnati; it is about 65 miles from Cincinnati, and about 160 miles from Cleveland; how many inches from Cleveland and how many inches from Cincinnati should a dot be placed on this map to represent the location of Springfield?
6. On a map drawn on a scale of 180 miles to the inch, the State of Wyoming appears as a trapezoid having an altitude of $1\frac{1}{2}$″, one base of 2″, and one base of $2\frac{1}{16}$″; what is the approximate area of Wyoming in square miles?
7. On this same map, Nevada appears as a rectangle $1\frac{3}{4}$″ × $1\frac{3}{16}$″ and a triangle having a base of $1\frac{3}{4}$″ and an altitude of $1\frac{1}{2}$″; what is the approximate area of Nevada in square miles?
8. On a map drawn on a scale of 240 miles to the inch, New York is $3\frac{1}{16}$″ from Chicago; What distance does this represent in miles?
9. From New York to Boston is about 190 miles; on a map drawn on a scale of 80 miles to the

inch, how many inches should there be between New York and Boston?

10. Tennessee is nearly a trapezoïd; find the State of Tennessee on any map, and figure its area.

Exercise 94—Oral Review.

1. Fred James buys merchandise amounting to $100.00 from William Bright; is James the creditor or the debtor in this transaction?
2. A piece of cloth 4 yards long is cut into 24 equal pieces; how long is each piece?
3. A board 8 feet long is cut into parts 8 inches long; how many parts are there?
4. A piano which was marked $250.00 was sold at a discount of 30%; for how much money was this piano sold?
5. If a 60-day note bearing 6% interest was given for the piano in Question 4, how much interest would there be on the note at maturity?
6. What is the interest on $400.00 for 30 days at 6%? For 90 days? For 120 days?
7. Add:

(*a*)	(*b*)	(*c*)
3,298	1,497	3,149
645	279	198

8. Subtract:

(*a*)	(*b*)	(*c*)
3,841	2,104	1,306
298	395	591

9. Multiply:

(*a*) $18 \times 33\frac{1}{3}$;

(*b*) $96 \times 12\frac{1}{2}$;

(*c*) $48 \times 87\frac{1}{2}$;

10. Divide:

(*a*) $11 \div 12.5$;

(*b*) $400 \div 25$;

(*c*) $90 \div 33\frac{1}{3}$;

(Time for #7, 8, 9, and 10, should be 4 minutes.)

Exercise 95—Written Review.

A Factory Fire.

1. During a fire in a clothing factory, 45% of the merchandise was destroyed by flames and 23% was damaged by water; if the stock was worth $4,635.00, what amount of damage was caused by the flames? By the water? What was the value of the undamaged stock?

2. At this fire, a bolt of cloth containing $53\frac{1}{3}$ sq. yd. was so badly damaged by water that only 10% of the cloth was usable; how many square feet of cloth were usable from this bolt?

3. Another bolt of cloth containing $48\frac{1}{2}$ sq. yd. worth $1.60 per sq. yd. was so badly damaged that it had to be sold at a discount of 75%; how much was received for this bolt of cloth?

4. A bolt of serge lining containing 20 sq. yd. 6 sq. ft. was so badly damaged that only 3 sq. yd. 4 sq. ft. were usable; what per cent of this bolt was not usable?

5. Out of a box containing $3\frac{1}{3}$ gr. of buttons, only 10% of the buttons were not broken; what per cent of the buttons were broken? How much was received for the unbroken buttons if they were sold at 75¢ per gr.?

6. One lot of cloth which originally cost $480.00 was sold at a discount of 80% and was paid for by a 30-day note bearing 5% interest; what was the amount paid at the maturity of the note?

7. To repair part of the damage done to the building, the following labor was required:

4 carpenters for 96 hours each, @ 75¢ per hour.
3 decorators for 36 hours each, @ $1.10 per hour.
2 plumbers for 48 hours each, @ 90¢ per hour.
How much wages did these tradesmen receive?

8. Buy the following material used by the carpenters:

125 boards 10′ × 12″ × 1″ @ $30.00 per M.
450 boards 8′ × 6″ × 2″ @ $40.00 per M.
48 posts 12′ × 2″ × 4″ @ $35.00 per M.

240 feet of quarter-round moulding @ $12.50 per M.

What is the total amount of this bill for lumber?

9. The flooring for the office came in boards 3″ wide and 12′ long; if the size of the office was 40′ × 60′, and 25% was added for waste, how many boards were needed?

Exercise 96—Written Review.

Subtract, but do not copy:

(Time for these 12 examples is less than 5 minutes.)

1.	2.	3.	4.
954,682	298,746	564,921	725,831
495,621	125,959	284,139	378,450

5.	6.	7.	8.
859,623	619,461	419,356	945,621
593,769	357,862	197,823	258,652

9.	10.	11.	12.
558,921	510,781	854,672	248,761
175,840	354,890	198,473	151,842

Add, but do not copy:

(Time for these 6 examples is less than 5 minutes.)

13.	14.	15.	16.	17.	18.
2,348	3,548	7,851	9,156	1,059	8,356
4,489	7,256	1,030	4,657	3,658	7,176
7,982	9,158	5,679	8,259	5,750	8,054
3,456	1,025	7,756	2,135	1,003	3,260
4,182	5,941	8,172	4,600	9,137	6,751
1,592	8,001	3,543	7,908	6,683	1,909

ARITHMETIC

Copy and divide:
(Time for these 6 examples is less than 5 minutes.)

19. 5,397 ÷ 21;
20. 87,397 ÷ 97;
21. 9,860 ÷ 34;
22. 78,003 ÷ 81;
23. 60,264 ÷ 72;
24. 32,997 ÷ 51.

Copy and multiply:
(Time for these 8 examples is less than 5 minutes.)

25. 289 × 29;
26. 945 × 12;
27. 682 × 73;
28. 559 × 28;
29. 783 × 23;
30. 889 × 25;
31. 707 × 63;
32. 785 × 37.

Subtract, but do not copy:
(Time for these 4 examples is less than 2 minutes.)

33.	**34.**	**35.**	**36.**
647,223	327,671	981,222	463,829
381,047	193,378	729,197	182,837

Add, but do not copy:
(Time for these 6 examples is less than 12 minutes.)

37.	**38.**	**39.**	**40.**	**41.**	**42.**
2,477	6,789	2,212	2,161	8,637	2,812
9,648	137	6,565	4,940	1,486	4,787
3,833	1,554	4,281	8,174	2,727	3,646
1,080	877	3,250	9,388	1,480	5,102
432	2,637	6,676	7,172	1,942	1,934
570	1,649	1,842	4,626	7,170	4,836
4,396	2,235	2,420	1,212	8,646	2,948
8,374	7,700	1,753	2,464	9,438	7,360
5,097	7,480	2,531	6,321	817	6,166
2,322	2,117	1,838	1,617	2,137	3,740

DEFINITIONS

(Parts I to VI, Inclusive)

Abstract Number	A number which is used without the name of an object.
Account	A record showing all business transactions with any person.
Acute Angle	An angle which is sharper (or less) than a right angle.
Addends	The numbers which are to be added in an example in addition.
Addition	Uniting two or more numbers or quantities into one number or quantity. The numbers to be added are called "addends." The answer is called the "sum" or "total." The sign of addition (+) is called "plus."
Agent	(In Commission.) The person who is engaged by the principal to perform some service.
Aliquot Part	An equal part of a number.
Altitude	Height.
Amount	(In Interest.) The principal plus the interest.
Angle	The opening between two straight lines which meet in a point.
Arabic Numerals	The numbers in common use, as 1, 2, 3, etc.
Area	The space within the limits or boundaries of a surface.
Arithmetic	The science of numbers.
Average	A medium number which can be used in place of each of several unequal numbers.
Avoirdupois Weight	The table of weights used for weighing all common articles, such as groceries, meats, hay, etc.
Balance	The difference between the two sides of an account.
Base	(In Mensuration.) The side of a figure on which the figure appears to rest. (In Percentage.) The number or quantity on which the percentage is computed.

ARITHMETIC

Bill	See "Sales-slip."
Board Measure	The unit of measure used for measuring boards is the "Board Foot," which equals $1' \times 1' \times 1''$.
Brackets	Marks used to enclose numbers which are to be treated as one number. ( ) ; [ ] ; { }.
Buyer	One who buys.
Cancellation	Reducing the numerator of one fraction and the denominator of another to simplify multiplication.
Carry	To convert 10 of any order into 1 of the next higher order.
Cash Account	An account with Cash.
Cash Book	The book in which the Cash Account is kept.
Cash Discount	An amount allowed for the payment of a bill on or before a certain date.
Change	To convert 1 of any order into 10 of the next lower order.
Circle	A plane figure bounded by one continuous curved line which at all points is a uniform distance from a point in the center of the figure.
Circumference	The distance around a circle. The line which forms the boundary of a circle.
Commercial Interest	Interest computed on the basis of 360 days to the year.
Commission	An amount of money paid by one person who is called the "principal" to another person who is called the "agent," for some service the agent performs for the principal.
Common Fraction	A fraction of which both the numerator and the denominator are written.
Common Year	A year containing 365 days.
Complement	The difference between a number and 10, 100, 1,000, etc.
Composite Number	Any number which is composed of two or more factors.
Concrete Number	A number which is used with the name of an object.
Counting	To name one by one to find the number of units in a group.
Credit	An entry which shows that the person or thing

	on whose account the entry appears, has parted with something of value.
Creditor	One to whom money is owing.
Cube	A solid having six square sides or faces of equal size, joined so that every angle is a right angle.
Cubic Contents	The space occupied by a solid. (Volume.)
Cubic Measure	The table of measures used for measuring the cubic contents or volume of solids.
Date of Maturity	The date on which a note or other obligation matures or becomes due.
Debit	An entry which shows that the person or thing on whose account the entry appears, has received something of value.
Debtor	One who owes money.
Decimal	Numbered by tens.
Decimal Fraction	(More commonly called "Decimal.") A fraction with a denominator of 10, 100, etc., the denominator being indicated by writing the numerator to the right of a decimal point.
Decimal Point	A point or period (.) used to indicate that the numbers written to the right of it form a decimal fraction.
Denominate Number	A number used with the name of a measure.
Denominator	That term of a fraction which shows into how many equal parts a unit has been divided.
Diameter	The distance across a circle through the center.
Difference	The answer found by subtraction.
Digit	Any single figure, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
Dimension	The extent of a line, area, or solid.
Disbursements	Amounts of money paid out.
Discount	A reduction from the regular price of an article for any reason. (See also: "Cash Discount" and "Trade Discount.")
Divided by (÷)	The sign of division.
Dividend	The number to be divided in an example in division.
Division	Finding how many times one number is contained in another number, and finding one of the equal parts of a number. The number to be divided is the "dividend."

	The number which shows into how many parts the dividend is to be divided, or which shows the size of the parts when the number of parts is to be found, is called the "divisor." The divisor is the number you divide by. The answer is called the "quotient." The sign of division (÷) is called "divided by."
Divisor	The number which shows into how many parts the dividend is to be divided, or which shows the size of the parts when the number of parts is to be found, in an example in division. The number you divide by.
Dry Measure	The table of measures used in measuring grains, fruit, vegetables, etc.
Due Date	See "Date of Maturity."
Elapsed Time	The time between two dates.
Equals (=)	The sign of equality.
Equivalent	Having the same value.
Even Numbers	All numbers divisible by 2 without remainder; therefore, all numbers with 0, 2, 4, 6, or 8 in units' place.
Exact Interest	Interest computed on the basis of 365 days (in Leap Year 366 days) to the year.
Extremes	The first and fourth terms of a proportion.
Factor	Any number which is contained in another number without a remainder.
Fraction	A number which shows one or more of the equal parts of a unit.
Grand Total	The sum of several totals.
Greatest Common Divisor	The largest number which will divide two or more numbers without remainder. (The Greatest Common Divisor of two or more numbers is also the Highest Common Factor of those numbers.)
Group	Several persons or things.
Highest Common Factor	The largest factor which is common to two or more numbers. (The Highest Common Factor of any two or more numbers is also the Greatest Common Divisor of those numbers.)

DEFINITIONS

Horizontal	Parallel to the surface of the earth.
Hundreds of Millions' Place	The place of the 9th order.
Hundreds of Thousands' Place	The place of the 6th order.
Hundreds' Place	The place of the 3d order.
Hundredths' Place	The second place to the right of the Decimal Point.
Hundred-Thousandths' Place	The fifth place to the right of the Decimal Point.
Improper Fraction	A fraction which means 1 unit or more than 1 unit.
Integer	A whole number.
Interest	Money paid or to be paid for the use of money.
Invoice	A bill showing the quantities, price, etc., of articles bought from a wholesaler or manufacturer.
Leap Year	A year containing 1 extra day, making 366 days in all.
Least Common Denominator	The smallest denominator common to two or more fractions.
Least Common Multiple	The smallest number which contains two or more other numbers without remainder.
Ledger	The book in which accounts are kept.
Linear Measure	The table of measures used for measuring lines and distances.
Liquid Measure	The table of measures used for measuring common liquids.
List Price	The price at which an article is listed in a catalogue or price-list.
Location	One object's or number's position in relation to the other objects or numbers in a group.
Long Division	The method of division in which partial dividends are written.
Loss	The difference between the cost and the selling price, when the cost is the greater.
Lowest Terms	The simplest form in which a number or fraction can be written.
Maker	The person who "makes" or signs a promissory note.

ARITHMETIC

Manufacturer	One who makes or manufactures merchandise.
Means	The second and third terms of a proportion.
Mensuration	Taking the measurements of anything.
Merchandise	Any articles which can be bought and sold.
Merchandise Counting Table	The table used for counting merchandise of all kinds.
Millions' Period	Millions', Tens of Millions', and Hundreds of Millions' places.
Millions' Place	The place of the 7th order.
Millionths' Place	The sixth place to the right of the Decimal Point.
Minuend	The number from which we subtract in an example in subtraction.
Minus (−)	The sign of subtraction.
Mixed Decimal	A number which shows both an integer and a decimal fraction.
Mixed Number	A number which shows one or more units, plus one or more parts of a unit; therefore, a combination of a whole number and a fraction.
Most Convenient Denominator	See "Least Common Denominator."
Multiple	A number which contains another number more than once without remainder.
Multiplicand	The number which is to be repeated in an example in multiplication.
Multiplication	Finding a number or quantity by repeating another number or quantity a given number of times. The number which is to be repeated is called the "multiplicand." The number which shows how many times the multiplicand is to be repeated, is called the "multiplier." The answer is called the "product." The sign of multiplication (×) is called "multiplied by" when the multiplier follows it, and "times" when the multiplier comes before it.
Multiplied by (×)	The sign of multiplication.
Multiplier	The number which shows how many times the multiplicand is to be repeated in an example in multiplication.

DEFINITIONS

Note	See "Promissory Note."
Numerator	That term of a fraction which shows how many parts of the unit are being spoken of.
Oblique	A position which slants.
Oblique Angle	An angle which is either acute or obtuse. (Any angle which is not a right angle.)
Oblong	The plane figure formed by joining the ends of two straight lines of one length to two straight lines of another length, so that they form four right angles.
Obtuse Angle	An angle which is blunter (or greater) than a right angle.
Odd Number	All numbers not divisible by 2 without remainder; therefore, all numbers with 1, 3, 5, 7, or 9 in units' place.
Order	The place occupied by a number, as: 1st order is Units' Place. 2d order is Tens' Place. 9th order is Hundreds of Millions' Place.
Parallel	Running in the same direction with an equal distance between.
Parallelogram	Any plane figure bounded by four straight lines, having two sets of parallel sides.
Partial Dividends	The several dividends necessary in finding the quotient in an example in long division.
Partial Products	The several products which, when added, form the final product in an example in multiplication.
Payee	The person in whose favor a promissory note, bill of exchange, or check is made.
Per Annum	By the year.
Per Cent	By the hundred. Also: Hundredths.
Percentage	Calculation by hundredths. Also: The answer of an example in percentage.
Period	Units', Tens,' and Hundreds' Places when considered as a group.
Place	The order in which a number is written, as: Units' Place is the 1st order. Tens' Place is the 2d order. Hundreds of Millions' Place is the 9th order.
Plus (+)	The sign of addition.

ARITHMETIC

Prime Factor	A factor which cannot itself be separated into other factors.
Prime Number	Any number which is divisible only by 1 and by itself without remainder.
Principal	(In Commission.) The person who engages an agent to perform some service. (In Interest.) The sum on which interest is paid. The Base.
Prism	A solid having rectangular sides and two parallel ends or bases.
Product	The answer found by multiplication.
Profit	The difference between the cost and the selling price, when the selling price is the greater.
Promissory Note	A promise (in writing) to pay a certain sum of money at a certain time.
Proper Fraction	A fraction which means less than 1 unit.
Proportion	The comparison of equal ratios.
Quotient	The answer found by division.
Radius (plural, Radii)	A straight line drawn from the center of a circle to a point in the circumference.
Rank	See "Location."
Rate	A certain per cent of the base.
Ratio	The relation which one number or quantity bears to another number or quantity of the same kind.
Receipt	A paper showing payment or delivery.
Receipts	Amounts of money received.
Rectangle	Any plane figure which is bounded by four straight lines, and has four right angles.
Rectangular Prism	See "Right Prism."
Rectangular Solids	Cubes and Right Prisms.
Reduction	Changing the form of a number or fraction without changing its value.
Remainder	The answer found by subtraction. The part of a dividend which remains undivided in an example in division.
Retailer	One who sells merchandise in small quantities to the general public.
Right Angle	An angle formed by two straight lines meeting in a point in such a way that if both lines were lengthened to cross each other, all four angles so formed would be exactly alike.

DEFINITIONS

Right Prism	A solid having four square or oblong sides and two parallel square or oblong ends.
Roman Numerals	The numbers which are often used on watches, clocks, etc., as I, V, X, L, C, D, M.
Sales-check	See "Sales-slip."
Sales-slip	A paper showing the cost and quantity of merchandise purchased.
Scale	The relation of the size of the drawing of an object to the size of the object itself.
Seller	One who sells.
Short Division	The method of division in which no partial dividends are written.
Square	The plane figure formed by joining the ends of four straight lines of equal length, so that they form four right angles.
Square Measure	The table of measures used for measuring the area of surfaces.
Straight Line	The shortest distance between two points.
Subtraction	Taking one number or quantity from another number or quantity. The number from which we subtract is called the "minuend." The number which we subtract is called the "subtrahend." The answer is called the "difference" or "remainder." The sign of subtraction (−) is called "minus."
Subtrahend	The number which we subtract in an example in subtraction.
Sum	The answer found by addition.
Temperature	The degree of warmth or coldness of an object.
Tens of Millions' Place	The place of the 8th order.
Tens of Thousands' Place	The place of the 5th order.
Tens' Place	The place of the 2d order.
Tenths' Place	The first place to the right of the Decimal Point.
Ten-Thousandths' Place	The fourth place to the right of the Decimal Point.
Terms	The several parts of a fraction, or of a proportion.
Thermometer	An instrument used for measuring temperature.

Thousands' Period	Thousands', Tens of Thousands', and Hundreds of Thousands' Places.
Thousands' Place	The place of the 4th order.
Thousandths' Place	The third place to the right of the Decimal Point.
Time Measure	The table of measures used for measuring time.
Times (×)	The sign of multiplication.
Total	See "Sum."
Trade Discount	An amount allowed by wholesalers to retailers so that retailers can sell at list prices and still make a profit.
Trapezoid	A plane figure bounded by four straight sides, of which only one pair of sides are parallel.
Triangle	A plane figure bounded by three straight sides joined to form three angles.
Triangular Prism	A solid having three square or oblong sides and two parallel triangular ends or bases.
Unit	One person or thing.
Units' Period	Units', Tens', and Hundreds' Places.
Units' Place	The place of the 1st order.
Vertex (plural, Vertices)	The point where two lines meet in an angle.
Vertical	At right-angles to the surface of the earth.
Volume	The space occupied by a solid. (Cubic Contents.)
Whole Number	A number which shows one or more units or whole things. An Integer.
Wholesaler	One who sells merchandise in large quantities, and usually deals only with other merchants.

ABBREVIATIONS AND SIGNS

(Parts I to VI, Inclusive)

Account	Acct. or a/c.
Acre	A.
Amount	amt.
Answer	Ans.
At	@
Barrel	bbl.
Base	B.
Board Foot	bd. ft.
Brackets	( ); [ ]; { }
Bushel	bu.
Cent	ct. or ¢
Commission	com.
Cord	cd.
Credit	Cr.
Creditor	Cr.
Cubic Foot	cu. ft.
Cubic Inch	cu. in.
Cubic Yard	cu. yd.
Day	da.
Debit	Dr.
Debtor	Dr.
Decimal Point	.
Degree	°
Dime	d.
Discount	disc.
Divided by	÷
Dollar	$
Dozen	doz.
Equals	=
Fifty	L.
Five	V.
Five Hundred	D.
Foot	ft. or ′
Free on Board Cars	F. O. B.
Gallon	gal.
Gill	gi.
Greatest Common Divisor	G. C. D.
Great Gross	gt. gr.
Gross	gr.
Highest Common Factor	H. C. F.
Hogshead	hhd.
Hour	hr.
Hundred	C.
Hundredweight	cwt.
Inch	in. or ″
Interest	Int.
Least Common Denominator	L. C. D.
Least Common Multiple	L. C. M.
Merchandise	mdse.
Mile	mi.
Mill	m.
Minus	−
Minute	min.
Month	mo.
Multiplied by	×
Number	No. or # (# written before a number)
One	I.
Ounce	oz.
Peck	pk.
Per Cent	%
Percentage	P.
Pint	pt.
Plus	+
Pound	lb. or # (# written after a number)

ARITHMETIC

Quart	qt.
Rate	R.
Ratio	:
Remainder	rem.
Rod	rd.
Second	sec.
Square Foot	sq. ft.
Square Inch	sq. in.
Square Mile	sq. mi.
Square Rod	sq. rd.
Square Yard	sq. yd.
Ten	X.
Thousand	M.
Times	×
Ton	T.
Week	wk.
Yard	yd.
Year	yr.

INDEX

Part V

INDEX

INDEX

Part VI

INDEX