ARITHMETIC

FOR

HIGH SCHOOLS:

CONTAINING

THE ELEMENTARY AND THE HIGHER PRINCIPLES AND APPLICATIONS OF THE SCIENCE.

A NEW AND IMPROVED EDITION, WITH AN APPENDIX ON STANDARD MEASURES AND WEIGHTS, &C.

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PREFACE.

THE first ten Chapters of this work, with an Appendix containing Geometrical Definitions and Practical Mensuration, constitute the author's revised Elementary and Practical Arithmetic. With the additional Chapters, the work will be found to contain a very complete arrangement of principles and exercises, adapted alike to elementary and to higher education in this Science.

The concluding Chapter, on mathematical Probabilities and their applications to Life Annuities and Life Insurance, has not been given on account of its practical utility to the business man, though it is manifestly not without such utility; but because its expositions must be matters of interest to the liberal scholar, in a merely theoretical point of view. These subjects have probably not been treated, in a demonstrative manner, in any other educational work known in this country.

In the author's elementary work, which, as abovementioned, consists of the first ten Chapters of this, with an Appendix, the Teacher will find all that is essential to a preparatory or a business education in Arithmetic. When the pupil is to be put on a more extensive course, let him be transferred from that work to the place at which he has arrived in this, and thus be saved the loss of time and the perplexity which must ensue from studying two *different treatises* on the same subject, whether they be by the same, or by different authors. In this way the pupil's course will be constantly *progressive.* The author has esteemed it a matter of great importance to adapt his two Arithmetics, as well as his two Algebras, to this uninterrupted progression of studies.

A KEY containing all the Miscellaneous Exercises in this work, and the Miscellaneous Exercises in Mensuration in the elementary one, with their *solutions*, has been published, to save time to the Teacher, and to facilitate his necessary labors, whatever may be his competency as an Arithmetician.

In the first editions of his Arithmetics, the author adventured some rather violent changes in the common *arrangement* and *nomenclature* of this science. So far as he has learned, these innovations have been approved by all who have used or examined his books. He is himself fully confirmed in his convictions of their propriety; and retains them in the revised editions, as improvements in the logic of the science, and as thereby facilitating its acquisition.

TRANSYLVANIA UNIVERSITY,
March 7th, 1856.

ANALYSIS OF CONTENTS.

☞ This ANALYSIS is designed to be used in oral examinations, in review. The Teacher will name the *topic* as presented in this table; the Learner will respond according to his knowledge of the subject.

For example; the Teacher will say, "Arithmetic;" the Learner will respond, "Arithmetic is the *science of numbers*; when practically applied, it becomes the *art of calculation*."

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ARITHMETIC.

CHAPTER I.

PRELIMINARY DEFINITIONS.—NUMERATION.—NOTATION.

(1.) Arithmetic is the *science of numbers:* when practically applied, it becomes the *art of calculation.*

Science is knowledge reduced to a *system,* so as to be conveniently taught, and readily applied: *Art* is knowledge applied to practical purposes.

The *Rules* of art are founded on the *principles* of science.

Unity and Number.—Quantity.

(2.) A *Unit* is any thing regarded simply as *one: Number* consists, properly, in a *repetition of units,* as *two,* three, &c.; but a unit is also called the number *one.*

Can you recite the *names* of numbers, from *one* to a *hundred?*

(3.) *Quantity* is any thing which admits of being *measured,* so as to be expressed in *units* of that quantity.

Thus a *line* is a quantity, since a line may be measured, so as to be expressed in *inches,* or feet, &c.; as when we say that a line is *ten inches* long.

Numbers are quantities; for every number is necessarily measured by a *unit.*

Is *time* a quantity? Is *industry* a quantity? Is *weight* a quantity? Is *hope* a quantity? Is *distance* a quantity? Is *virtue* a quantity? Are *length, breadth,* and *height* quantities?

Abstract and Concrete Numbers.

(4.) An *abstract* number is a number without any *kind of units* expressed; as the numbers *one*, *five*, *ten*, a *hundred*.

(5.) A *concrete* number is a number of *some kind of units* expressed; as the numbers *one book*, *five men*, a *hundred* dollars.

Is *twenty* an abstract or a concrete number? Is *nine* pounds an abstract or concrete number? Is *two hundred* miles an abstract or a concrete number? Is *one thousand* an abstract or a concrete number?

Give two other examples of *abstract*, and two of *concrete* numbers.

Similar and Dissimilar Numbers.

(6.) *Similar* concrete numbers are such as express the *same kind of units;* as *three* dollars and *five* dollars.

(7.) *Dissimilar* concrete numbers are such as express *different kinds of units;* as *two* dollars and *five* miles.

Are *four* inches and *seven* inches similar or dissimilar concrete numbers? Are *nine* pounds and *twelve* yards similar or dissimilar? Are *one* cent and *ten* dollars similar or dissimilar? *Twenty* men and *five hundred* men?

Give another example of *similar* concrete numbers; and another of *dissimilar* concrete numbers.

The groundwork of a thorough knowledge of Arithmetic must be laid in the principles of *Numeration* and *Notation;* for on these principles depend the four fundamental operations in Arithmetic—*Addition*, *Subtraction*, *Multiplication* and *Division*.

NUMERATION.

(8.) Numeration is the method of *naming* numbers by *units*, *tens*, *hundreds*, &c.

Thus the name *Eleven* denotes *ten* and *one*;
Twelve " ten and two;
Thirteen " ten and three;
Fourteen " ten and four; &c.
Twenty " *two tens;*
Thirty " three tens; &c.
Twenty one " two tens and one;
Twenty two " two tens and two; &c.,
A *Hundred* is *ten* tens;
A *Thousand* is ten hundred;
A *Million* is a thousand *thousands*,
A *Billion* is a thousand *millions;*
A *Trillion* is a thousand *billions;*

and so on, through *quadrillions*, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, &c.

What two numbers are implied in the name *fifteen?* In the name *sixteen?* In the name *seventeen?* In *eighteen?* In *nineteen?*

What is implied in the name *forty?* In *fifty?* Twenty-three? Thirty-one? Forty-five? Fifty-four? Sixty? Sixty-seven? Seventy?

A Quadrillion is how many? A Quintillion? A Sextillion? &c.

Different Orders of Units.

(9.) The naming of numbers by *units*, *tens*, *hundreds*, &c., introduces different *orders of units* in Numeration.

The numbers, *two*, *three*, *four*, &c., are repetitions of the simple unit *one*, which is a *unit* of the *first order*.

Twenty, thirty, forty, &c., are respectively two *tens*, three *tens*, four *tens*, &c.; and in these repetitions of *ten*, *ten* is regarded as a *unit* of the *second order*.

In repetitions of a *hundred*, as two *hundred*, three *hundred*, &c., one *hundred* is a *unit* of the *third order*.

In like manner, one *thousand* is made a *unit* of the *fourth order;* and so on.

How many *units* of the *first* and *second orders*, respectively, are contained in the number *thirteen?* How many in the number *twenty-five?* In the number *thirty-four?* In the number *forty-nine?* In the number *seventy-seven?*

How many *units* of *distinct orders* make up the number *five hundred* and *twenty-one?* How many *units* of *distinct orders* make up the number *nine hundred* and *fifty-two?*

Scale of Numeration.

(10.) *Ten* of any *lower order* of units make *one* of the next higher order; or *one* of a higher order makes *ten* of the next lower order.

Thus *ten units* (of the first order) make *one* ten.

How many *tens* make *one* hundred? How many *hundreds* make *one* thousand? How many *hundreds* make *two* thousand? How many hundreds make *five* thousand?

One million is how many hundred thousand? *One billion* is how many hundred millions? *One trillion* is how many hundred billions?

Numeration Table.

(11.) The *ascending orders* of units are given in the following Table (to be recited from *right to left*).

Billions, &c.,	*Hund. of mill.,*	*Tens of mill.,*	*Millions,*	*Hund. of thous.,*	*Tens of thous.,*	*Thousands,*	*Hundreds,*	*Tens,*	*Units,*

Recite the *orders of units*, ascending, from simple Units to Decillions. Recite them, descending, beginning with Hundreds.—Beginning with Thousands. Beginning with Tens of thousands. Beginning with Hundreds of thousands. Beginning with Millions. Beginning with Billions.

What are the *relative values* of these different orders of units? (10.)

NOTATION.

(12.) NOTATION is the method of *denoting* numbers by numeral *figures.*

These figures, sometimes called the *digits* of numbers, are

1 *one*, 2 *two*, 3 *three*, 4 *four*, 5 *five*, 6 *six*, 7 *seven*, 8 *eight*, 9 *nine;* and 0 *zero* or *cipher*, which has *no value.*

0 is used to occupy *vacant places* in Notation.

The Higher Orders of Units denoted.

(13.) The ascending *orders of units* are denoted by figures in a row, from right to left; the first on the right denotes *units*, the second *tens*, the third *hundreds;* and so on, according to the Numeration Table (11).

Thus, in 123, the 1 is one *hundred*, the 2 is two *tens*, or *twenty*, the 3 is three *units;* and the whole number denoted is *one hundred and twenty-three.*

What would be the value of 4 in the *first place* on the right? In the *second* place? In the *third* place? In the *fourth* place?

Local Values of Figures.

(14.) The value of a figure is increased *ten-fold* for each place it is removed from *units* towards the left, in a row of figures: this constitutes its *local value.*

In 25, 2 has the local value 2 *tens*, which is *ten times* the simple 2; in 125, 1 has the local value 1 *hundred*, which is *ten* times *ten*, the value that 1 would have in the second place.

The figure 3, in the *second place* from the right, would denote how many times the simple 3? In the third place from the right? In the fourth?

RULE I.

(15.) *To numerate or read a row of Figures.*

Call the successive figures *units*, *tens*, *hundreds*, &c., from *right to left* (11); and then read them, according to their respective values, from *left to right*.

EXAMPLE.

To read the figures 70304521.

Calling the figures, one after another, *units*, *tens*, *hundreds*, &c., from right to left, we find the last figure 7 to be *tens of millions;* then, reading from left to right, we say,

Seventy millions, three hundred and four thousand, five hundred and twenty-one.

EXERCISES.

Read the following rows of figures

1 - -	100	11 - -	100000	21 - -	50000000
2 - -	210	12 - -	201834	22 - -	63281314
3 - -	309	13 - -	350710	23 - -	70380078
4 - -	1000	14 - -	403008	24 - -	89034060
5 - -	3820	15 - -	500500	25 - -	10382000
6 - -	4075	16 - -	1000000	26 - -	100000000
7 - -	5003	17 - -	2070860	27 - -	202202202
8 - -	61234	18 - -	3803907	28 - -	360000000
9 - -	80709	19 - -	4006009	29 - -	731137731
10 - -	99036	20 - -	8100001	30 - -	901901901

RULE II.

(16.) *To write in Figures any given Number.*

Set proper figures, from *left to right*, to denote the descending *orders of units*, from the highest in the given number down to simple units—supplying each vacant place with a 0

EXAMPLE.

To write in figures the number

Three millions, twenty-five thousand, and thirty.

The descending orders of units in this number, are

3 *millions*, 2 *tens of thous.*, 5 *thousands*, and 3 *tens*;

hence we write it thus:

3 0 2 5 0 3 0,

in which the vacant places of *hundreds of thousands*, *hundreds*, and *units*, are filled with 0s.

EXERCISES.

Write in figures each of the following numbers:

1. One hundred.
2. Two hundred and one.
3. Three hundred and ten.
4. Four hundred and five.
5. Five hundred and fifteen.
6. Six hundred and twenty.
7. Seven hund. and thirty-four.
8. Eight hundred and eleven.
9. Nine hun. and ninety-nine.
10. One thousand.
11. Two thousand and nine.
12. Five thousand and ten.
13. Seven thous. one hundred.
14. Three thousand and five.
15. Eight thous. and nineteen.
16. Nine thous. and eleven.
17. Ten thousand.
18. Twelve thousand and ten.
19. Twenty thous. and nine.
20. Four thousand and ninety.
21. Eleven thous. and eleven.
22. Thirty thous. and sixteen.
23. Forty-one thous. & twelve.
24. Fifty thous. nine hundred.
25. Seventy-eight thous. & one.
26. Ninety thous. twenty-five.

27. One hundred thousand and one hundred.
28. Two hundred and thirty thous. and three hundred.
29. Five hundred and one thous. two hund. and three.
30. Seven hund. and thirteen thous. four hund. and fifty.
31. Nine hund. and ninety-nine thous. and seventy-five.
32. Eight hund. and fifty-one thous. one hund. and twenty-one.
33. Seven hundred and eleven thousand five hundred and nineteen.
34. One million, two hundred and fifty-four thousand.
35. Two millions, forty thous., six hundred and twenty.
36. Fifty millions, one hundred thousand, seven hundred.
37. Sixty-one millions, four hundred and ten thousand.
38. Two hun. and five millions, four hundred and ninety-one
39. Four hundred and ten millions, six hundred and one thousand.
40. Nine hundred millions, one thousand, eight hundred and sixty-nine.

French and English Numeration.

(17.) In the *French* system of Numeration, which prevails in continental Europe and America, a *thousand millions* make *one billion*, a *thousand billions* make *one trillion*, &c.

In the *English* system, which was formerly used in this country, a *million millions* make *one billion*, a *million billions* make *one trillion*, &c.

This system, it is said, is now becoming obsolete in England. We sometimes find the higher numbers named in accordance with it, in scientific books, and it is necessary to notice the difference between the two systems.

In the French system, the number

3 840 930 670 820

is 3 trillions, 840 *billions*, 930 *millions*, 670 *thousand*, 820 ;

In the English system, the same number would be called,

3 *billions*, 840930 *millions*, 670820.

CHAPTER II.

ADDITION.—SUBTRACTION.—MULTIPLICATION.—DIVISION.

ADDITION.

(18.) ADDITION consists in uniting two or more numbers in *one sum*. Thus 5 and 4 *added together* make 9; or the *sum* of 5 and 4 is 9.

What is the *sum* of 3 and 2? What is the sum of 3 and 2 and 4? What is the sum of 6 and 4? What is the sum of 7 and 5 and 3?

The *sum* found may be regarded as a *whole*, of which the given numbers are the *parts*.

What is the *sum* of 4 and 6 and 8? Then what is the *whole?* and what are its *parts?* What is the sum of 10 and 6 and 4? Then what is the whole? and what are its parts?

Commit to memory the *elementary sums* of numbers; thus 1 and 1 are 2, 1 and 2 are 3, &c.; 2 and 1 are 3, 2 and 2 are 4, &c., as given, from *left to right*, in the following

Addition Table.

1 *and*	1 *are* 2	2 *are* 3	3 *are* 4	4 *are* 5	5 *are* 6	6 *are* 7	7 *are* 8	8 *are* 9	9 *are* 10
2 *and*	1 *are* 3	2 *are* 4	3 *are* 5	4 *are* 6	5 *are* 7	6 *are* 8	7 *are* 9	8 *are* 10	9 *are* 11
3 *and*	1 *are* 4	2 *are* 5	3 *are* 6	4 *are* 7	5 *are* 8	6 *are* 9	7 *are* 10	8 *are* 11	9 *are* 12
4 *and*	1 *are* 5	2 *are* 6	3 *are* 7	4 *are* 8	5 *are* 9	6 *are* 10	7 *are* 11	8 *are* 12	9 *are* 13
5 *and*	1 *are* 6	2 *are* 7	3 *are* 8	4 *are* 9	5 *are* 10	6 *are* 11	7 *are* 12	8 *are* 13	9 *are* 14
6 *and*	1 *are* 7	2 *are* 8	3 *are* 9	4 *are* 10	5 *are* 11	6 *are* 12	7 *are* 13	8 *are* 14	9 *are* 15
7 *and*	1 *are* 8	2 *are* 9	3 *are* 10	4 *are* 11	5 *are* 12	6 *are* 13	7 *are* 14	8 *are* 15	9 *are* 16
8 *and*	1 *are* 9	2 *are* 10	3 *are* 11	4 *are* 12	5 *are* 13	6 *are* 14	7 *are* 15	8 *are* 16	9 *are* 17
9 *and*	1 *are* 10	2 *are* 11	3 *arc* 12	4 *are* 13	5 *are* 14	6 *are* 15	7 *are* 16	8 *are* 17	9 *are* 18
10 *and*	1 *are* 11	2 *are* 12	3 *are* 13	4 *are* 14	5 *are* 15	6 *are* 16	7 *are* 17	8 *are* 18	9 *are* 19
11 *and*	1 *are* 12	2 *are* 13	3 *are* 14	4 *are* 15	5 *are* 16	6 *are* 17	7 *are* 18	8 *are* 19	9 *are* 20
12 *and*	1 *are* 13	2 *are* 14	3 *are* 15	4 *are* 16	5 *are* 17	6 *are* 18	7 *are* 19	8 *are* 20	9 *are* 21

The *sign* +, called *plus*, placed between numbers, signifies that the numbers are to be *added together;* thus 5 + 4, 5 *plus* 4, signifies 5 and 4 added together.

How many are 4 + 3 + 2?	How many are 5 + 4 + 3?
How many are 6 + 5 + 4?	How many are 7 + 6 + 5?
How many are 8 + 7 + 6?	How many are 9 + 8 + 7?

The *sign* =, denotes *equal to;* thus 10 + 5 + 4 = 19, denotes that the sum of 10 and 5 and 4 is *equal to* 19.

For large numbers, employ

RULE III.

(19.) *To add two or more Numbers together.*

1. Set the numbers one under another, with *units* under *units*, tens under tens, &c.

2. Proceeding from *right to left*, add up each column of figures, and under each set its amount, if *less than* 10.

3. If the amount of a column be 10 *or more*, set down its *right hand figure*, and add the left figure or figures to the next column; but set down the whole amount of the last column.

EXAMPLE.

What is the amount of 930 dollars, 6754 dollars, and 8621 dollars, when united in *one sum?*

```
          930
         6754
         8621
        -----
Answer. 16305
```

Having set *units* under *units*, tens under tens, and hundreds under hundreds, we say 1 and 4 are 5, and set 5 under the *units column*.

Then, 2 and 5 and 3 are 10; this is 10 *tens*, equal to 1 *hundred;* we therefore set 0 under the *tens* column, and add 1 to the next column.

Then 1 and 6 and 7 and 9 are 23; this is 23 *hundreds*, equal to 2 *thousand* and 3 *hundred;* we therefore set 3 under the *hundreds* column, and add 2 to the next column, which makes 16 for that column.

The *left-hand* figure in the amount of any column, is the number of *tens* in that amount; and these *tens* are *units* of the next order on the left (10); hence the left-hand figure must be added to the next column.

The Operation Proved.

(20.) Addition is proved by *adding the columns downwards.* This changes the order in which the figures are taken, and will be likely to show whether any error has been committed.

EXERCISES.

1. John has 95 chestnuts, Thomas has 180, and Charles 270; what number have they all together?

Ans. 545 chestnuts.

2. A farmer being asked how many sheep he had, replied: "in one field I have 410, in another, 500, in another 602;" how many had he? *Ans.* 1512 sheep.

3. A merchant bought cloth for 375 dollars, linen for 83 dollars, silk for 234 dollars, and calico for 75 dollars. What sum did he expend for the whole? *Ans.* 767 dollars.

4. A gentleman bought a carriage for 350 dollars, a pair of horses for 240 dollars, and a set of harness for 100 dollars; what did the whole amount to? *Ans.* 690 dollars.

5. Going out to collect money, I received from one person 13 dollars, from another 124 dollars, from another 89 dollars, and from another 20 dollars. What was the whole sum collected? *Ans.* 246 dollars.

6. An agriculturist raised on one field 685 bushels of grain; on another 97 bushels, on another 330, and on another 1000 bushels. How many bushels did he raise altogether?

Ans. 2112 bushels.

7. Allowing a person's estate to be estimated as follows, viz: real estate 9000 dollars, personal property 1375 dollars, cash 300, and recoverable debts 875 dollars; what would be the value of his estate? *Ans.* 11550 dollars.

8. Admitting I bought of one person 500 bushels of wheat, of another 934 bushels, of another 83 bushels, and of another 125 bushels; how many bushels did I buy in all?

Ans. 1642 bushels.

9. Three farmers deposite flour in the same warehouse; the first, 43 barrels; the second, 150 barrels; and the third, 89 barrels. What quantity do they all deposit?

Ans. 282 barrels.

10. A merchant bought 4 pieces of cloth; the first for 225 dollars, the second for 310 dollars, the third for 279 dollars, and the fourth for 95 dollars. What did the whole cost him? *Ans.* 909 dollars.

11. If a merchant buy a stock of goods for 5000 dollars, for what sum must he sell the goods to gain 475 dollars?

Ans. 5475 dollars.

12. Bought a barrel of sugar for 15 dollars, a barrel of molasses for 13 dollars, and a sack of coffee for 20 dollars. For what sum must the whole be sold to gain 10 dollars?

Ans. 58 dollars.

13. A person on a journey travels, the first week 255 miles; the second, 240 miles; the third and fourth, each 200 miles. How far did he travel in the four weeks?

Ans. 895 miles.

14. Four persons engage in speculation; A gains 75 dollars, B 100 dollars, C and D each 235 dollars; what sum was gained by them all? *Ans.* 645 dollars.

15. A draper sold four bales of linen; the first and second contained each 480 yards, the third and fourth each 542 yards. How many yards did he sell? *Ans.* 2044 yards.

16. Bought of A 325 bushels of wheat; of B, 280 bushels; of C as much as from A; and of D as much as from B; what quantity of wheat did I buy from them all?

Ans. 1210 bushels.

17. A gentleman bought three plantations at 3750 dollars each, and sold them again at such prices as gained 1000 dollars on the whole; for what sum did he sell the three plantations?

Ans. 12250 dollars.

18. Bought at one time 375 barrels of flour, for 1875 dollars; and at another 400 barrels, for 2000 dollars; how many barrels were bought in all, and for what sum of money?

Ans. 775 barrels, for 3875 dollars.

19. A lends to B 2500 dollars, to C 3000 dollars, and has 5325 dollars left; what sum had A at first?

Ans. 10825 dollars.

20. A speculator bought stock at one time for 325 dollars, and at another time for 705 dollars. In selling the whole he made a profit of 175 dollars; for what sum did he sell?

Ans. 1205 dollars.

21. Three persons form a partnership in trade. A puts in 4250 dollars, B 2000 dollars, and C as much as A and B together; what is their whole stock in trade?

Ans. 12500 dollars.

22. A gentleman is 15 years older than his wife, and she is 20 years older than their eldest son, who is 29 years of age. Find the gentleman's age, and the age of his wife.

Ans. His age is 64 years; hers 49.

23. A merchant bought cloth for 375 dollars, and silk for 95 dollars. In selling, he gained 50 dollars on the cloth, and 45 dollars on the silk; for what sum did he sell the whole?

Ans. 565 dollars.

24. The produce of two farms was as follows, viz: of the first, 785 bushels of wheat, and 250 of rye; of the second, 1000 bushels of wheat, and 113 of rye. What was the entire produce of the farms?

Bushels of *wheat* must be united in one sum, and bushels of *rye* in another; for *dissimilar quantities* (7) cannot be added together. *Ans.* 1785 bushels of wheat, and 363 of rye.

25. A grocer paid 300 dollars for sugar, 174 dollars for coffee, 85 dollars for rice, and 56 dollars for tobacco. He sold the sugar at a profit of 25 dollars, and the other articles at cost; what did he receive for the whole?

Ans. 640 dollars.

26. A merchant bought 4 bales of cotton; the first and second contained 470 yards each, the third and fourth 532 yards each. What was the number of yards purchased?

Ans. 2004 yards.

27. Bought live-stock as follows, viz: of A 13 cows, 16 oxen, and 120 sheep; of B 24 cows, 30 oxen, and 153 sheep; and of C 100 cows, and 425 sheep. What was the amount of stock purchased? *Ans.* 137 cows; 46 oxen; 698 sheep.

28. A father bequeathed to his only daughter 2500 dollars, and to each of his two sons 500 dollars more than to his daughter. What was the amount of the several bequests?

Ans. 8500 dollars.

29. Bought a quantity of cloth for 386 dollars, of cotton for 200 dollars, and of silk for 150 dollars. The cloth was sold at a profit of 73 dollars, the cotton at a profit of 35 dollars, and the silk at cost; what sum was received for the whole? *Ans.* 844 dollars.

30. A farmer has in store at one place 500 bushels of wheat, 325 of oats, and 50 of corn; and at another, 475 bushels of wheat, 75 of oats, and 83 of corn. What amount of produce has the farmer in store?

Ans. 975 bushels of wheat; 400 of oats; 133 of corn

SUBTRACTION.

(21.) SUBTRACTION consists in taking a *less number* from a greater, to find their *difference.*

The less number is called the *subtrahend,* and the greater the *minuend;* the difference is the *remainder* of the greater number.

Thus 4 from 9 leaves 5; then 4 is the *subtrahend*, 9 the *minuend,* and 5 the *difference,* or *remainder.*

What is the *difference* between 5 and 8? Between 6 and 10? Between 9 and 15? Between 8 and 17? Between 10 and 19?

What *remains* when 8 is subtracted from 12? When 9 is subtracted from 13? When 10 is subtracted from 17? When 11 is subtracted from 20?

Addition and Subtraction.

(22.) Addition and Subtraction are the *reverse of each other:* in Addition, the *parts* are given, to find the *sum* or *whole;* in Subtraction, the *sum* or whole and one of its parts are given, to find the *other part.*

The *sum* being 10, and one of the parts 7, what is the *other part?* The sum being 13, and one part 6, what is the other part? The sum being 19, and one part 10, what is the other part?

The *sign* —, called *minus,* placed between two numbers, signifies that the one before which it stands, is to be *subtracted from the other.*

Thus 9 — 4, 9 *minus* 4, signifies that 4 must be subtracted from 9.

How many is 8 — 3? 13 — 7? 15 — 4? 18 — 11? 20 — 10?
How many is 10 — 3? 14 — 9? 17 — 8? 21 — 10? 25 — 15?
How many is 30 — 20? 50 — 30? 70 — 20? 90 — 60? 100 — 70?

Constant Difference.

(23.) The *difference* between two numbers evidently *remains the same*, when those numbers are *equally increased* or *diminished*.

What is the difference between 4 and 7? Between 4 + 1 and 7 + 1?
What is the difference between 5 and 9? Between 5 + 7 and 9 + 7?
What is the difference between 3 and 8? Between 3 + 10 and 8 + 10?

For large numbers we have

RULE IV.

(24.) *To subtract a less Number from a greater.*

1. Set the less number under the greater, with *units* under *units*, tens under tens, &c.

2. Proceeding from *right to left*, take each lower figure from the one above it, and underneath set the remainder.

3. If the lower figure *exceed the upper*, add 10 to the upper figure; from the *sum* subtract the lower figure, and then add 1 to the *next lower figure* before subtracting it.

EXAMPLE.

What number will *remain* when 80657 is subtracted from 2451039?

$$\begin{array}{r} 2451039 \\ 80657 \\ \hline 2370382 \end{array}$$

Having set the less number under the greater, with *units* under *units*, tens, under tens, &c., we say, 7 from 9 leaves 2, and set the 2 underneath.

The 5 being greater than the 3 above it, we add 10 to 3, and say, 5 from 13 leaves 8; then, adding 1 to the 6, we say 7 from 10 leaves 3; adding 1 to 0, we say, 1 from 1 leaves 0; 8 from 15 leaves 7.

There being no figure under the 4, the 1 to be added there makes 1 for that place; 1 from 4 leaves 3; nothing from 2 leaves 2.

The 10 added to any *upper figure* is always equal to the 1 added to the *next lower figure* (10); so that these additions do not affect the *difference between the two given numbers* (23).

The Operation Proved.

(25.) Subtraction is proved by *adding the difference to the less number:* the sum must be equal to the greater number.

EXERCISES.

1. William had 325 apples, but gave James 148 of them; how many apples had William left? *Ans.* 177 apples.

2. A person who undertook a journey of 735 miles, has traveled 93 miles of the distance; how far has he yet to travel? *Ans.* 642 miles.

3. From a farm which contained 2350 acres, 1234 acres were sold; how many acres remained of the original farm? *Ans.* 1116 acres.

4. A young man received from his father 5325 dollars, of which he paid 2500 dollars for a house; how many dollars had he remaining? *Ans.* 2825 dollars.

5. A merchant deposited 5800 dollars in bank, but afterwards made a draft upon it for 3270 dollars; how much remained in bank? *Ans.* 2530 dollars.

6. A farmer who had 4000 bushels of wheat in his granary, took out 2100 to be sent to market; how many bushels remained in the granary? *Ans.* 1900 bushels.

7. A vintner bought 4036 gallons of wine, and afterwards sold to the amount of 2373 gallons how many gallons had he remaining? *Ans.* 1663 gallons

8. Suppose I borrow of my neighbor 1000 dollars, and three months afterwards return him 385 dollars; what balance would still be owing? *Ans.* 615 dollars.

9. A drover bought cattle for 1495 dollars, and sold the same at a loss of 270 dollars; for what sum did he sell the cattle? *Ans.* 1225 dollars.

10. A gentleman sold a farm for 6700 dollars, which was 530 dollars more than he paid for it; what did he pay for the farm? *Ans.* 6170 dollars.

11. A weaver made 30 pieces of cotton, containing 1200 yards; of which he has sold 17 pieces, containing 875 yards. How many pieces, and how many yards remain?

Ans. 13 pieces; and 325 yards.

12. Bought of A 385 barrels of flour, and 2805 bushels of corn; of which I sold to B 109 barrels of flour, and 936 bushels of corn. What quantity of each remains unsold?

Ans. 276 barrels; 1869 bushels.

13. A salter bought 35850 pounds of beef, and 150000 pounds of pork. Having exported 20500 pounds of the beef, and 75900 of the pork, what quantity of each has he still on hand? *Ans.* 15350 pounds of beef; 74100 of pork.

14. A farmer raised 1200 bushels of wheat, and 213 of oats. He sold to A 835 bushels of wheat, and 179 of oats, and the remainder of the crop to B. What amount of produce did he sell to B? *Ans.* 365 bushels of wheat; and 34 of oats.

15. A grocer bought coffee for 420 dollars, and sugar for 545 dollars. He sold the coffee for 500 dollars, and the sugar for 603 dollars; what did he gain on each?

Ans. 80 dollars; and 58 dollars.

16. A merchant bought 375 yards of cloth, for 1645 dollars; of which he has sold 103 yards for 685 dollars. What quantity of the cloth remains on hand, and for what sum must it be sold, to lose nothing? *Ans.* 272 yards; 960 dollars.

17. A manufacturer sold 2 bales of cotton, which together contained 2000 yards. Allowing the first bale to have contained 985 yards, how many yards were in the second bale?
Ans. 1015 yards.

18. A gentleman who owned a tract of land containing 15735 acres, sold from it, at different times, to the amount of 6141 acres. How many acres had he then remaining?
Ans. 9594 acres.

19. A testator bequeathed to his son and daughter 25479 dollars, of which the son had 18875 dollars. What was the daughter's portion? *Ans.* 6604 dollars.

20. Four persons contribute towards the founding of a literary institution; A gives 2500 dollars, and B 3300; C gives 375 dollars less than A, and D 283 less than B. What are the sums contributed by C and D?
Ans. 2125 dollars; 3017 dollars.

21. Having 4800 dollars on hand, I wish to borrow as much as will enable me to purchase a farm at 5390 dollars; what sum must I borrow? *Ans.* 590 dollars

22. Sold a lot of hams for 275 dollars, which was at a profit of 43 dollars; and a lot of cheese for 305 dollars, which was at a profit of 39 dollars. What did each kind cost me?
Ans. 232 dollars; and 266 dollars.

23. A merchant bought silk for 3710 dollars, and linen for 1759 dollars. On account of damage received, the silk was sold at a loss of 123 dollars; and the linen at a loss of 370 dollars; for what sum was each article sold?
Ans. 3587, and 1389 dollars.

24. A person who had a journey of 1000 miles to perform, traveled the first week 240 miles, and the second 237 miles. How many miles then remained to be traveled? *Ans.* 523

25. A gentleman who had 3000 dollars on hand, bought land for 1835 dollars, and stock for 370 dollars. How many dollars had he then remaining? *Ans.* 795 dollars

26. Out of 4379 dollars which a person collected, he paid 734 dollars to A, 360 dollars to B, and 839 dollars to C. What sum had he then remaining? *Ans.* 2446 dollars.

27. An agriculturist raised 2376 bushels of wheat, and 930 bushels of rye. Having sold 1000 bushels of wheat, and 437 of rye, what quantity of each has he remaining?

Ans. 1376 bushels; and 493 bushels.

28. A testator bequeathed 10000 dollars so that each of his two sons should receive 3500 dollars, and his daughter the remainder. What was the daughter's portion?

Ans. 3000 dollars.

29. A's estate is worth 50000 dollars, B's is worth 3785 dollars less than A's, C's is worth 2500 dollars less than B's, and D's 1324 dollars less than C's. What is the value of D's estate? *Ans.* 42391 dollars.

30. A person who had a journey of 1500 miles to make, went the first day 165 miles, the second 170 miles, the third 183 miles, and the fourth 182 miles. How many miles then remained to be traveled? *Ans.* 800 miles.

MULTIPLICATION.

(26.) MULTIPLICATION consists in finding the *product* of a number, or of a part of a number, when *taken any given number of times.*

Thus 3 times 5 is 15; that is, 5 multiplied by 3 *produces* 15.

The number to be multiplied is called the *multiplicand,* and the *multiplying* number the *multiplier;* the two together are called the *factors* of their product.

What is the *product* of 3 times 2? Which number is the *multiplicand*? The *multiplier*? What are the *factors*?

The addition of the *same number to itself,* repeatedly, is a *multiplication* of that number; thus 4+4+4 is three times 4.

How many is 6 + 6, or *twice* 6? 6 + 6 + 6, or 3 times 6?
How many is 7 + 7, or twice 7? 7 + 7 + 7, or 3 times 7?

Commit to memory the *elementary products,* once 1 is 1, once 2 is 2, &c.; twice 1 is 2, twice 2 is 4, &c., as given from *left to right,* in the following

Multiplication Table.

Once	1	2	3	4	5	6	7	8	9	10	11	12
Twice	*is* 2	*is* 4	*is* 6	*is* 8	*is* 10	*is* 12	*is* 14	*is* 16	*is* 18	*is* 20	*is* 22	*is* 24
3 *times*	*is* 3	*is* 6	*is* 9	*is* 12	*is* 15	*is* 18	*is* 21	*is* 24	*is* 27	*is* 30	*is* 33	*is* 36
4 *times*	*is* 4	*is* 8	*is* 12	*is* 16	*is* 20	*is* 24	*is* 28	*is* 32	*is* 36	*is* 40	*is* 44	*is* 48
5 *times*	*is* 5	*is* 10	*is* 15	*is* 20	*is* 25	*is* 30	*is* 35	*is* 40	*is* 45	*is* 50	*is* 55	*is* 60
6 *times*	*is* 6	*is* 12	*is* 18	*is* 24	*is* 30	*is* 36	*is* 42	*is* 48	*is* 54	*is* 60	*is* 66	*is* 72
7 *times*	*is* 7	*is* 14	*is* 21	*is* 28	*is* 35	*is* 42	*is* 49	*is* 56	*is* 63	*is* 70	*is* 77	*is* 84
8 *times*	*is* 8	*is* 16	*is* 24	*is* 32	*is* 40	*is* 48	*is* 56	*is* 64	*is* 72	*is* 80	*is* 88	*is* 96
9 *times*	*is* 9	*is* 18	*is* 27	*is* 36	*is* 45	*is* 54	*is* 63	*is* 72	*is* 81	*is* 90	*is* 99	*is* 108
10 *times*	*is* 10	*is* 20	*is* 30	*is* 40	*is* 50	*is* 60	*is* 70	*is* 80	*is* 90	*is* 100	*is* 110	*is* 120
11 *times*	*is* 11	*is* 22	*is* 33	*is* 44	*is* 55	*is* 66	*is* 77	*is* 88	*is* 99	*is* 110	*is* 121	*is* 132
12 *times*	*is* 12	*is* 24	*is* 36	*is* 48	*is* 60	*is* 72	*is* 84	*is* 96	*is* 108	*is* 120	*is* 132	*is* 144

The *sign* ×, called *into*, placed between two numbers, signifies that the two numbers are to be *multiplied together*; thus 9 × 4, 9 *into* 4, signifies 9 multiplied by 4.

How many is 6 × 7?	5 × 9?	8 × 3?	4 × 11?	8 × 9?
How many is 9 × 9?	7 × 8?	8 × 6?	12 × 4?	6 × 9?
How many is 3 × 7?	5 × 8?	9 × 3?	11 × 9?	12 × 11?

Constant Product.

(27.) The Product of two numbers remains *the same*, when the multiplicand and multiplier are taken the *one for the other.*

Thus 25 *times* 7 is equal to 7 *times* 25

For 25 times 7 must be 7 times as many as 25 times 1, which is 25; that is, 25 times 7 must be equal to 7 times 25.

Prove that 14 times 9 is equal to 9 times 14.
Prove that 31 times 11 is equal to 11 times 31.

A *concrete number* (5) cannot be taken *concretely* as a *multiplier*; for, as a multiplier, a number can denote only *repetitions* of the multiplicand.

For example, 3 *hats* at 5 *dollars* each would cost 3 *times* 5 dollars, which is 15 dollars: we multiply 5 dollars by 3, not by 3 *hats.*

RULE V.

(28.) *To multiply by a Number not exceeding* 12, or 12 *with* 0s *annexed.*

1. Multiply each figure of the multiplicand, from *right* to *left*, and under each set its product, when *less than* 10.

2. When the product is 10 *or more*, set down its *right hand figure*, and add the left figure or figures to the next product; but set down the whole of the last product.

3. *Ciphers in the right of the factors*, are omitted in multiplying; but as many 0s must be placed in the *right of the product.*

EXAMPLE.

What will 30 acres of land amount to at 543 dollars per acre?

$$\begin{array}{r} 543 \\ 30 \\ \hline 16290 \end{array}$$

Ans. 16290 dollars.

The land will amount to 30 *times the price per acre.*

After placing a 0 in the right of the product, we say, 3 times 3 is 9; 3 times 4 is 12; 3 times 5 is 15, and the 1 carried from the 12 makes 16.

The *left hand figure* of any product is so many *units* of the next order on the left (10), and must therefore be added to the next product.

Multiplying by 30 must produce 10 *times* as much as multiplying by 3. This *tenfold* increase is assigned to the 1629 by the 0 annexed, which removes each of its figures one place farther towards the left (14).

When the Multiplier is 10, 100, or 1000, &c.

(**29.**) A number is multiplied by 10, 100, or 1000, &c., by annexing to that number as many 0s as there are in the right of the *multiplier.*

Thus 100 times 29 is 2900; the 29 being increased in value *tenfold* for each 0 annexed (14).

EXERCISES.

1. Mary bought two books, at 31 cents apiece. How many cents did she pay for both of them?

She paid 31 *cents* + 31 cents, or *twice* 31 cents.

Ans. 62 cents.

2. A farmer sold 3 horses at 125 dollars each. What sum did he receive for them? *Ans.* 375 dollars.

3. What is the value of 20 shares of road stock, at 95 dollars for each share? *Ans.* 1900 dollars.

4. What would be the weight of 30 bales of cotton, allowing 450 pounds to each bale? *Ans.* 13500 pounds.

5. If a steamboat can run 305 miles in one day, how many miles could it run in 4 days? *Ans.* 1220 miles.

6. How many pounds of flour are in 10 barrels of flour, there being 196 pounds in each barrel? *Ans.* 1960 pounds.

7. There being 1760 yards in one mile, what number of yards is there in 5 miles? *Ans.* 8800 yards.

8. A farmer sold 100 acres of land at 43 dollars per acre; what did the whole amount to? *Ans.* 4300 dollars.

9. It requires 660 feet to make one furlong. How many feet make 1 mile, which is 8 furlongs? *Ans.* 5280 feet.

10. A merchant bought 60 sacks of coffee, at 13 dollars a sack. What did the whole amount to? *Ans.* 780 dollars.

11. A butcher bought 7 oxen, at an average of 32 dollars a head. What did he pay for the whole of them?

Ans. 224 dollars.

12. A planter bought 8 mules, at an average of 125 dollars a head. What did he pay for the whole of them?

Ans. 1000 dollars.

13. A manufacturer made 9 coils of rope, each of which contained 139 yards. What was the whole number of yards?

Ans. 1251 yards.

14. An agriculturist bought 110 acres of land, at 175 dollars per acre. What did the whole amount to? *Ans.* 19250 dolls.

15. It requires 4840 square yards to make an acre. What is the number of square yards in 120 acres?

Ans. 580800 square yards.

16. If 10 masons can build a wall in 34 days, in what time ought one mason to build the same wall?

It would take 1 mason 10 *times as long* as it would 10 masons. *Ans.* 340 days.

17. If 9 men could dig a certain ditch in 19 days, in what time ought one man to dig the same ditch? *Ans.* 171 days.

18. How long ought one man to subsist on a stock of provisions which would suffice 7 men for 29 days?

Ans. 203 days.

19. If 20 pieces of artillery demolish a fortress in 48 hours, in what time ought one piece to demolish the fortress?

Ans. 960 hours.

20. If 115 bushels of oats will feed one horse for 12 months, what quantity of oats would feed 60 horses the same time? *Ans.* 6900 bushels.

21. Allowing 120 clerks to accomplish a certain amount of writing in 53 days, in what time ought one clerk to accomplish an equal amount of writing? *Ans.* 6360 days.

22. Allowing a ship to sail at the rate of 170 miles per day, what distance would she sail in 30 days?

Ans. 5100 miles.

23. Allowing an acre of ground to produce 105 bushels of corn, what would be the produce of 100 acres?

Ans. 10500 bushels.

24. A merchant bought 125 yards of cloth, at 4 dollars a yard. What did the whole amount to?

The cloth amounted to 125 *times* 4 *dollars;* but 4 *times* 125 is the same number as 125 *times* 4 (27).

Ans. 500 dollars.

25. A planter sold 325 bales of cotton, at 40 dollars per bale. What did the whole amount to?

Ans. 13000 dollars.

26. If a steamship run at the rate of 309 miles per day, how far will she run in 20 days? *Ans.* 6180 miles.

27. If 301 head of cattle be sold at 50 dollars a head, what will be the proceeds of the sale?

Ans 15050 dollars.

28. If 1406 acres of land be sold at 80 dollars per acre, what will be the proceeds of the sale?

Ans. 112480 dollars.

29. Allowing the standard weight of a bushel of wheat to be 60 pounds, what should be the weight of 2090 bushels?

Ans. 125400 pounds.

RULE VI.

(30.) *To multiply by any Number exceeding* 12, *and containing two or more significant figures.*

1. Multiply by each *significant figure*, separately, of the multiplier.

2. Set the different rows of product *one under another*, with the first figure of each under the *multiplying figure;* and in that order *add them together.*

3. *Ciphers in the right of the factors*, are omitted in multiplying; but as many 0s must be placed in the *right of the product.*

EXAMPLE.

To find the Product of 305 times 6794.

```
   6794
    305
  -----
  33970
 20382
-------
2072170
```

We say 5 times 4 is 20, and set the first figure 0 under the *multiplying figure* 5; 5 times 9 is 45, and 2, carried from the 20, makes 47, &c.

Then, 3 times 4 is 12; we set the first figure 2 of this product under the *multiplying figure* 3; &c. The two rows of product figures are added together, in the order in which they are placed.

The first product figure is set under the multiplying figure, to *increase the product* in the same degree in which the *multiplying figure is increased* in value, by distance from the *units place* (14).

The Operation Proved.

(31.) Multiplication may be proved by multiplying the multiplicand by the *multiplier less one*, and adding the multiplicand to the product thus obtained. The sum must be equal to the product found with the *entire multiplier*.

EXERCISES.

30. There are 24 *hours* in one day; then how many hours make *a year* of 365 days?

The number of hours in *a year*, is 365 *times* 24 *hours*; but 24 *times* 365 will produce the same number (27), and the multiplication will be shortened by taking the *less number for the multiplier*. *Ans.* 8760 hours.

31. A hogshead of wine or brandy contains 63 gallons. How many gallons would there be in 250 hogsheads?

Ans. 15750 gallons.

32. What sum should be paid for a plantation containing 765 acres, at 43 dollars per acre? *Ans.* 32895 dollars.

33. If a man can walk 35 miles in a day, how far could he walk, at that rate, in a year, or 365 days?

Ans. 12775 miles.

34. A merchant sold 475 barrels of flour, at the rate of 13 dollars a barrel. What did the whole amount to?

Ans. 6175 dollars.

35. A manufacturer exported 234 bales of cotton cloth, each containing 2400 yards. What was the number of yards exported? *Ans.* 561600 yards.

36. A farmer had in wheat 205 acres, which produced 27 bushels per acre. What was the whole number of bushels produced? *Ans.* 5535 bushels.

37. A speculator bought 150 mules, which he sold at a profit of 29 dollars a head. What amount of profit did he make? *Ans.* 4350 dollars.

38. In a certain orchard there are 43 rows of trees, and 57 trees in each row. What is the number of trees in the orchard? *Ans.* 2451 trees.

39. If a physician make, on an average, 14 dollars a day, what will be the amount of his earnings in a year, or 365 days? *Ans.* 5110 dollars.

40. A gentleman sold a tract of land, containing 2307 acres, at 123 dollars an acre. What did the whole amount to? *Ans.* 283761 dollars.

41. What would be the population of a State containing 201 counties, allowing 18036 inhabitants, on an average, to each county? *Ans.* 3625236 inhabitants.

42. The circumference of the Earth is about 25000 miles, and the distance to the Sun is 3800 times the Earth's circumference. How many miles then is it to the sun?

Ans. 95000000 miles.

43. The Earth revolves on its axis once in 24 hours, and moves 68000 miles an hour in its orbit around the Sun. How far then are we carried along the Earth's orbit during one revolution of the Earth on its axis? *Ans.* 1632000 miles.

Successive Multiplications.

(32.) Multiplying by *one number*, and the product thence arising by *another*, and so on, is equivalent to multiplying by the *product of the several multipliers;* and it is immaterial in what order the multipliers are taken.

Thus $100 \times 5 \times 4$, or $100 \times 4 \times 5, = 100 \times 20$;

$239 \times 7 \times 6$, or $239 \times 6 \times 7, = 239 \times 42$.

44. A field containing 27 acres was sold at 245 dollars per acre; what did it amount to?

Instead of multiplying 245 by 27, we may, according to the preceding observation, multiply it by 9, and the *product thence arising* by 3, since $9 \times 3 = 27$.

Ans. 6615 dollars.

45. A manufacturer exported 45 bales of cotton cloth, containing 316 yards to the bale; what was the whole number of yards? *Ans.* 14220 yards.

46. A person on a journey traveled 54 days, at the rate of 36 miles per day; how far did he go in that time?

Ans. 1944 miles.

47. If the making of a railroad cost 23050 dollars a mile, what will 63 miles of the road amount to?

Ans. 1452150 dollars.

48. Allowing a garrison of soldiers to consume 735 pounds of meat per day, what quantity would they consume in 72 days? *Ans.* 52920 pounds.

49. Allowing 84 gallons of rum to fill one puncheon, how many gallons will be required to fill 123 puncheons?

Ans. 10332 gallons.

50. Allowing a pipe to throw into a certain reservoir 560 gallons of water per hour, how many gallons will thus be poured into it in 96 hours? *Ans.* 53760 gallons.

51. A crop of cotton was put up in 132 bales, weighing 456 pounds each; what did the entire crop weigh?

Ans. 60192 pounds.

52. Two towns which are 144 miles apart, are to be connected by a railroad which will cost 33460 dollars a mile. What will be the entire cost of the road?

Ans. 4818240 dollars.

DIVISION.

(33.) Division consists in finding how many times one number *contains another*, or *what part* one number is of another.

The number to be divided is called the *dividend*, the dividing number the *divisor*, and the number or part found the *quotient*.

If we divide 15 by 5, the *quotient* will be 3, because 5 is *contained in* 15, 3 *times*.

What is the quotient of 6 divided by 3? Of 12 divided by 4?
What is the quotient of 20 divided by 4? Of 36 divided by 9?
What is the quotient of 56 divided by 8? Of 63 divided by 7?

One-half is one of the *two equal parts* of any quantity; *two-thirds* are two of the *three equal parts* of any quantity, and so on.

What is meant by *one third*? One *fourth*? Three *fourths*?
What is meant by *one fifth*? Two *fifths*? Four *fifths*?
What is meant by *one sixth*? Three *sixths*? Five *sixths*?

A less Number divided by a greater.

(34.) The Quotient of a *less number* divided by a greater is *the part* that the less is of the greater; and is denoted by the less over the greater, with a line between them.

1 divided by 2 is $\frac{1}{2}$, *one-half*, because 1 is *one-half* of 2; 2 divided by 3 is $\frac{2}{3}$, *two-thirds*, because 2 is two-thirds of 3.

How much is 1 divided by 3? and why? How much is 1 divided by 4? and why? How much is 3 divided by 4? and why? How much is 1 divided by 5? and why? How much is 2 divided by 7? and why?

The *reciprocal* of a number is *a unit divided* by that number; thus the reciprocal of 2 is $\frac{1}{2}$, and the reciprocal of 3 is $\frac{1}{3}$.

What is the reciprocal of 4? Of 5? Of 6? Of 7?

Relation of the Quotient to the Dividend.

(35.) The Quotient is always that part of the dividend which is denoted by the *reciprocal of the divisor.*

Thus the quotient of 15 divided by 5 is 3, and 3 is $\frac{1}{5}$ of 15.

The quotient of 2 divided by 3, is $\frac{2}{3}$ (34), and *two-thirds* of any quantity is $\frac{1}{3}$ of 2 *such quantities.*

How do you find $\frac{1}{2}$ of any given number? How would you find $\frac{1}{3}$ of any given number? How would you find $\frac{1}{5}$ of any given number?

$\frac{3}{4}$ of 1 is what part of 3? $\frac{2}{3}$ of 1 is what part of 2?
$\frac{5}{6}$ of 1 is what part of 5? $\frac{4}{7}$ of 1 is what part of 4?
$\frac{3}{8}$ of 1 is what part of 3? $\frac{7}{9}$ of 1 is what part of 7?

Remainder in Division.

(36.) A *remainder*, in Division, is an *overplus of the dividend* above the repetitions of the divisor contained in it; and may be divided separately, to *complete the quotient* (34).

Thus 5 is contained in 17, 3 *times* with 2 *over;* this 2 divided by 5 gives $\frac{2}{5}$, which, annexed to 3, makes the *complete quotient*, $3\frac{2}{5}$, three and *two-fifths.*

What is the quotient of 17 divided by 2? Of 20 divided by 3?
What is the quotient of 35 divided by 4? Of 47 divided by 5?
What is the quotient of 50 divided by 6? Of 65 divided by 7?

Multiplication and Division.

(37.) Multiplication and Division are the *reverse of each other:* in Multiplication two *factors* are given to find their *product;* in Division a product and *one of its factors* are given to find the *other factor.*

The product being 60, and one factor 5, what is the other factor?
The product being 72, and one factor 8, what is the other factor?
The product being 85, and one factor 9, what is the other factor?

The *sign* ÷, called *by*, placed between two numbers, signifies that the first number is to be *divided by the second;* thus 36 ÷ 9, 36 *by* 9, signifies that 36 is to be divided by 9.

What is the quotient of 56 ÷ 8? Of 63 ÷ 9? Of 77 ÷ 11?
What is the quotient of 64 ÷ 8? Of 84 ÷ 7? Of 100 ÷ 12?

Division is also denoted by the dividend over the divisor, with a line between them.

Thus $\frac{36}{9}$ denotes 36 divided by 9.

Constant Quotient.

(38.) The quotient evidently remains *the same*, when the dividend and divisor are *both multiplied* or both *divided* by the same number.

Thus 3 is contained in 15 just as often as 4 *times* 3 is contained in 4 *times* 15.

RULE VII.

(39.) *To divide by a Number not exceeding* 12, *or* 12 *with* 0's *annexed.*

1. Take figures enough in the *left of the dividend* to contain the divisor, and set down the number of times the divisor goes therein, noticing the *overplus*, if any.

2. Take the next figure of the dividend, with the preceding *overplus*, if any, prefixed, and set the number of times the divisor goes therein on *the right* of the first quotient; if the divisor *will not go therein*, set down 0, and include the next figure in dividing; and so on.

3. *Ciphers in the right of the divisor* are omitted in dividing; but as many figures must be omitted in the right of the dividend, and *annexed to the remainder:* if there be no other remainder, these figures will form the remainder.

4. *Under the remainder*, if any, set the given divisor, to *complete the quotient.*

EXAMPLE I.

To find how many times 9 is contained in 23472.

$$\begin{array}{r} 9)\underline{23472} \\ 2608 \end{array}$$

We say, 9 in 23, *twice*, and 5 *over;* prefixing this 5 to the 4, we say, 9 in 54, 6 times; 9 in 7, 0 time; 9 in 72, 8 times.

The *overplus* of any particular place in the dividend, is so many *tens* in the next place on the right (10); and is made *tens* to the next figure by *prefixing it to that figure.*

EXAMPLE II.

To find how many times 120 is contained in 13127.

$$\begin{array}{r} 12|0)\underline{1312|7} \\ 109\frac{47}{120} \end{array}$$

In dividing, we omit the 0 in the right of the divisor, and the 7 in the right of the dividend.—12 in 13 goes *once,* and 1 *over;* prefixing this 1 to the next figure, we say, 12 in 11, 0 time; including the next figure, we say, 12 in 112, 9 times, and 4 *over;* to this 4 we annex the 7 omitted, and get the *remainder* 47, under which we set the *whole given divisor* 120.

The *quotient thus found* is evidently the same as if we had used the entire divisor 120, and taken one figure more, each time, of the dividend.

When the Divisor is 10, 100, *or* 1000, &c.

(40.) A number is divided by 10, 100, or 1000, &c., by cutting off from the *right of the dividend* as many figures as there are 0's in the divisor.

The other figures of the dividend will be the *quotient,* and those cut off, the *remainder.*

Thus 37560 ÷ 100 gives the *quotient* 375, and the *rem.* 60.

The Operation Proved.

(41.) Division may be proved by multiplying the *divisor* and *quotient* together, and adding the *remainder*, if any, to the product; the result must be equal to the *dividend*

EXERCISES.

1. How many barrels of apples, at 2 dollars a barrel, may be bought for 150 dollars?

The number of barrels that may be bought is the *number of times that* 2 *is contained in* 150. *Ans.* 75 barrels.

2. How many yards of broadcloth, at 3 dollars a yard, may be purchased for 387 dollars? *Ans.* 129 yards.

3. How many cords of wood, at the rate of 4 dollars per cord, may be purchased for 621 dollars? *Ans.* $155\frac{1}{4}$ cords.

4. How many superfine beaver hats, at 5 dollars apiece, may be purchased for 3700 dollars? *Ans.* 740 hats.

5. How many dozen of shoes, at the rate of 6 dollars per dozen, may be purchased for 775 dollars? *Ans.* $129\frac{1}{6}$ dozen.

6. There being 7 days in a week, it is required to find how many weeks there are in 728 days. *Ans.* 104 weeks.

7. If one box will hold 80 pair of shoes, how many of such boxes will be required to hold 1840 pair? *Ans.* 23 boxes.

8. At the rate of 90 dollars per acre, how many acres of land could be bought for 5237 dollars? *Ans.* $58\frac{17}{90}$ acres.

9. At the rate of 10 dollars per barrel, how many barrels of flour could be purchased for 1890 dollars?

Ans. 189 barrels.

10. At 11 dollars each, per month, how many laborers could be hired a month for 2530 dollars? *Ans.* 230 laborers.

11. If 2 acres of ground sell for 365 dollars, what is the price per acre?

If 2 acres bring 365 dollars, 1 *acre brings* $\frac{1}{2}$ *of* 365 dollars.

Ans. $182\frac{1}{2}$ dollars.

12. If a ship sail 485 miles in 3 days, what will be her daily rate of sailing? $\frac{1}{3}$ *of* 485 *miles*. *Ans.* 161$\frac{2}{3}$ miles.

13. A farmer has 4 plantations which are of the same size, and together contain 3787 acres. How many acres are in each? *Ans.* 946$\frac{3}{4}$ acres.

14. A speculator sold 50 mules for the sum of 7350 dollars. What did he receive, on an average, for each? *Ans.* 147 dollars.

15. A person on a journey traveled the distance of 1705 miles in 6 weeks. At what rate did he travel per week? *Ans.* 284$\frac{1}{6}$ miles.

16. A planter put 28350 pounds of cotton in 70 bales. What was the average number of pounds in each bale? *Ans.* 405 pounds.

17. A butcher sold 8 beeves, which together weighed 61205 pounds. What was the average weight of each? *Ans.* 7650$\frac{5}{8}$ pounds.

18. A farmer raised 65700 bushels of corn, on 900 acres of ground. What was the number of bushels per acre? *Ans.* 73 bushels.

19. If the making of 100 miles of road cost, in the aggregate, 273100 dollars, what was the average cost per mile? *Ans.* 2731 dollars.

20. A grazier sold 110 head of fat cattle, for the sum of 5830 dollars. At what rate per head were the cattle sold? *Ans.* 53 dollars.

21. A salter packed 24360 pounds of pork in 120 barrels If the barrels contained equal quantities, how many pounds were in each? *Ans.* 203 pounds.

22. A plantation containing 1200 acres was sold for 176400 dollars. At what price per acre was the plantation sold? *Ans.* 147 dollars.

RULE VIII.

(42.) *To divide by any Number exceeding* 12, *and containing two or more significant figures.*

1. Take figures enough in *the left of the dividend* to contain the divisor, and set down the number of times the divisor goes therein.

2. Multiply the divisor by the quotient figure, and subtract the product from those figures of the dividend which *were taken in dividing.*

3. Set down the next figure of the dividend, on the *right of the remainder,* if any; divide into the number thus obtained, and set the quotient figure on the *right of the previous one;* if the divisor *will not go in the number,* set a 0 in the quotient, and bring down the next figure of the dividend.

4. Multiply the divisor by the last quotient figure; subtract the product from the *number last divided;* and so on, as before, until the operation is completed.

EXAMPLE.

To find how many times 690 is contained in 210490.

$$
\begin{array}{l}
690)210490(305\tfrac{40}{690}. \\
\quad\;\; 207 \\
\quad\;\; \overline{\;\;\;349} \\
\quad\;\;\;\;\; 345 \\
\quad\;\;\;\;\; \overline{\;\;\;\;40}
\end{array}
$$

Omitting the 0 in the right of the divisor, and one figure in the right of the dividend, as under the former Rule, we say,

69 in 210, 3 *times*, and set the 3 on the right; we multiply 69 by 3, and subtract the product 207 from 210.

On the right of the *remainder* 3, we set the next figure 4 of the dividend, and say, 69 in 34, 0 time; bringing down the next figure 9 of the dividend, we say, 69 in 349, 5 times; we multiply 69 by 5, and subtract the product from 349.

To the remainder 4 we annex the 0 omitted in the dividend, and find the true remainder 40, under which we set the whole given divisor 690, to complete the quotient (36).

This Rule differs from Rule VII. only in requiring the *products* and *remainders* to be written down. By Rule VII., as the divisor is a small number, the *multiplications* and *subtractions* are carried on *mentally*: both Rules depend on the same principles.

Proof by Addition.

(43.) The operation by the last Rule may be proved, most readily, by adding up the *remainder*, if any, and the several *products* of the divisor and quotient figures, *in the order in which they stand*: the sum must be equal to the *dividend*.

For the preceding example the proof may be presented thus:

	207	
	345	
	40	
The Sum	210490	is equal to the dividend.

EXERCISES.

24. At the rate of 21 miles per day, how many days would a person be employed in walking 1323 miles?

The number of days required is the *number of times that* 21 *is contained in* 1323. *Ans.* 63 days.

25. A gentleman sold a lot containing 31 acres, for 3875 dollars. What was the price per acre?

The *price per acre* was $\frac{1}{31}$ of 3875 dollars,

Ans. 125 dollars.

26. A speculator bought a lot of cattle for 5494 dollars, and found that he had paid at the rate of 41 dollars a head; how many did he purchase? *Ans.* 134 head.

27. A person on a journey accomplished 2028 miles in 52 days. At what rate did he travel per day? *Ans.* 39 miles.

28. A hogshead of wine or brandy contains 63 gallons; how many hogsheads then will be required to hold 2835 gallons? *Ans.* 45 hogsheads.

29. A ship on a cruise was estimated to have sailed 1690 miles in 13 days. At what rate was that per day?

Ans. 130 miles.

30. A merchant sold a quantity of broadcloth for 3220 dollars, and the price per yard was 14 dollars. How many yards did he sell? *Ans.* 230 yards.

31. A speculator sold 15 pair of carriage horses, for 5475 dollars. What did he get, on an average, per pair?

Ans. 365 dollars.

32. A field containing 16 acres produced 2193 bushels of corn. What was the average product per acre?

Ans. $137\frac{1}{16}$ bushels.

33. An agriculturist bought a tract of land for 15170 dollars, at 74 dollars per acre. What was the number of acres?

Ans. 205 acres.

34. Allowing a ship to sail at the rate of 170 miles per day, in how many days would she make a voyage of 6630 miles? *Ans.* 39 days.

35. Allowing 38057 pounds of cotton to have been packed in 84 bales, what was the average quantity in each bale?

Ans. $453\frac{5}{84}$ pounds.

36. Allowing a road which is 180 miles in length, to have cost 369000 dollars, what was the average cost per mile?

Ans. 2050 dollars.

37. Allowing 950 horses to have been sold for 148200 dollars, what was the average sum received for each?

Ans. 156 dollars.

38. It takes 196 pounds of flour to make one barrel of flour; how many barrels then are there in 21364 pounds?
Ans. 109 barrels.

39. A tract of land containing 975 acres, was sold for the sum of 256425 dollars; at what rate was it sold per acre?
Ans. 263 dollars.

40. If one man can perform a piece of work in 347 hours, in what time ought 15 men to perform the same work?

15 men could do the work in $\frac{1}{15}$ *of the time in which one man could do it.* *Ans.* $23\frac{2}{15}$ hours.

41. In how many days ought 25 men to complete an excavation which would require one man 475 days?
Ans. 19 days.

42. A grazier has 340 head of cattle, whose aggregate value is 9860 dollars; what is their value per head?
Ans. 29 dollars.

43. How long might 16 men subsist on a stock of provisions which would suffice one man 3251 days?
Ans. $203\frac{3}{16}$ days.

44. A tract of land containing 105750 acres, is to be divided into 235 equal parts; how many acres will there be in each part? *Ans.* 450 acres.

45. It requires 1760 yards to make a mile; how many miles then are there in 65120 yards? *Ans.* 37 miles.

46. How long ought 18 horses to be fed on a quantity of oats which is sufficient for one horse 1499 days?
Ans. $83\frac{5}{18}$ days.

47. The sum of 175605 dollars is to be divided equally among 345 men; what will the portion of each man be?
Ans. 509 dollars.

48. If a field containing 39 acres produce 2189 bushels of wheat, what will be the number of bushels per acre?
Ans. $56\frac{5}{39}$ bushels.

49. It requires 144 square inches to make one square foot; how many square feet are there in 46800 square inches?

Ans. 325 square feet.

50. A cistern which holds 7250 gallons, is to be filled with water by a pipe which pours into it 290 gallons per hour; in what time will the cistern be filled?

Ans. 25 hours.

51. Allowing a steamship to run 375 miles per day, in what time would she make a voyage of 7875 miles?

Ans. 21 days.

52. It requires 1728 cubic inches to make one cubic foot; how many cubic feet then will 525312 cubic inches make?

Ans. 304 cubic feet.

53. A reservoir containing 106950 gallons of water, is to be emptied by a pump which discharges 465 gallons per hour. In what time will the reservoir be emptied?

Ans. 230 hours.

54. If the population of a State containing 135 counties, be 3180600, what is the average number of inhabitants to each county? *Ans.* 23560.

55. If the velocity of light is 192500 miles per second, and the distance from the Sun to the Earth is 95000000 miles; how long is light in coming from the Sun to the earth?

Ans. $493\frac{97500}{192500}$ seconds.

Successive Divisions.

(44.) Dividing by *one number*, and the quotient thus obtained by *another*, and so on, is equivalent to dividing by the *product of the several divisors.*

The last *remainder* × the last divisor *but one*, + the preceding remainder, × the next preceding divisor, + the next preceding remainder, and so on, will be the true *remainder* of the dividend.

For example, dividing 836 by 3, and the quotient thus obtained by 7, is equivalent to dividing the first number by the product 21 of 3 and 7.

```
3)836
7)278  2 over.
   39  5 over.
```

The quotient is 39, and the *remainder* is $5 \times 3 + 2 = 17$.

The remainder 2 is *units* of the given dividend ; the remainder 5 is *units* of the dividend 278, or *first quotient ;* and since the *first quotient* × the first divisor 3, produces *units of the given dividend*, it follows that the remainder 5 × the divisor 3, produces *units of the given dividend*, to which the 2 must be added, for the entire *remainder* of the dividend.

56. Allowing a railroad car to run at the rate of 45 miles an hour, in what time would it run 6120 miles?

Ans. 136 hours.

57. A hogshead of ale or beer contains 54 gallons ; how many hogsheads then might be filled with 11070 gallons ?

Ans. 205 hogsheads.

58. If 81 men take equal shares of 13846 dollars, how many dollars will fall to the share of each man ?

Ans. $170\frac{76}{81}$ dollars.

59. A field which contains 96 acres, produced 14304 bushels of corn ; what was the average yield per acre ?

Ans. 149 bushels.

60. A canal which is 132 miles in length, cost 268637 dollars ; what did this canal cost per mile, one mile with another ? *Ans.* $2035\frac{17}{132}$ dollars.

Cancellation in Division.

(45.) *Equal factors* may be canceled from a dividend and its divisor, without *altering the quotient;* for this is equivalent to dividing the dividend and divisor by the same number (38.)

For example, suppose that 132 is to be divided by 48.

$$132 = 12 \times 11;$$
$$\text{and} \quad 48 = 12 \times 4.$$

The division and cancellation may be denoted by placing the dividend over the divisor, and drawing lines across the *canceled equal factors;* thus

$$\frac{132}{48} = \frac{\cancel{12} \times 11}{\cancel{12} \times 4} = 2\tfrac{3}{4}.$$

Cancellation may always be thus employed to *simplify* Division, when the dividend and divisor contain *equal factors.*

To give other examples ;—

$$\frac{81}{36} = \frac{\cancel{9} \times 9}{\cancel{9} \times 4} = 2\tfrac{1}{4}; \qquad \frac{99}{22} = \frac{\cancel{11} \times 9}{\cancel{11} \times 2} = 4\tfrac{1}{2}.$$

Use of the Parenthesis, or Vinculum.

(46.) A *parenthesis* ( ) enclosing a numerical expression, or a *vinculum* ——— drawn over it, connects the value of that expression with the *sign* which immediately precedes or follows it.

Thus $20 - (4+5)$, the *sum* of 4 and 5 subtracted from 20.

$20 - (4 \times 5) \times 6$, or $20 - 6 \times (4+5)$, denotes 6 *times* the sum of 4 and 5 subtracted from 20.

$\overline{20 - (4+5)} \times 6$, or $6 \times \overline{20 - (4+5)}$ denotes 6 times the *difference* between 20 and the sum of 4 and 5.

The use of these, as well as of the other signs which have been explained, will be fully seen hereafter.

MISCELLANEOUS EXERCISES.

ON NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.—ABBREVIATED MULTIPLICATION AND DIVISION.

1. Find the sum of 19 thousand and thirteen, 105 thousand and ten, 3 millions 94 thousand and sixty, and 45 millions two hundred and five. *Ans.* 48218288.

2. Find the sum of 135 thousand four hundred, 500 millions thirty-five thousand, 350 millions and forty, and 4 billions 24 millions 30 thousand. *Ans.* 4874200440.

3. Find the difference between 9 trillions 31 millions 360 thousand five hundred and three, and 10 billions 5 millions 273 thousand 8 hundred and four. *Ans.* 8990026086699.

4. Find the difference between 360 billions 204 millions. 34 thousand three hundred, and 2 trillions 375 millions 183 thousand 7 hundred and sixteen. *Ans.* 1640171149416.

5. Find the product of 350 thousand and nineteen, multiplied by the sum of 5 thousand four hundred, and 100 thousand two hundred and ten. *Ans.* 36965506590.

6. Find the product of 4 millions 3 thousand seven hundred, multiplied by the sum of 300 and 21 thousand nine hundred and seventy-seven. *Ans.* 89190424900.

7. Find the quotient of 95 trillions 200 millions, divided by the difference between 275 millions 100 thousand and 75 millions 100 thousand. *Ans.* 475001.

8. Find the Quotient of 495 millions 66 thousand 570, divided by the Difference between 150 thousand 100 and 55 thousand four hundred and 41. *Ans.* 5230.

9. The Sum of two numbers being 346790, and the less number 39035, what is the greater number? (22)
Ans. 307755

10. The Sum of two numbers being 5706803, and the greater number 3983000, what is the less number?
Ans. 1723803.

11. The Product of two numbers being 64605700, and the multiplier 740, what is the multiplicand? (37)

Ans. 87305.

12. The Product of two numbers being 72958584, and the multiplicand 8076, what is the multiplier?

Ans. 9034.

13. The Sum of two numbers is 12384, and the Difference is 3600; what are the two numbers?

Since the Difference of two numbers, added to the less number, makes the *greater number*, it is plain that

The Sum of two numbers + their *difference, is twice the greater number;* and the Sum of two numbers − their *difference, is twice the less number.*

The two numbers required above are therefore

$\frac{1}{2}$ of (12384+3600), and $\frac{1}{2}$ of (12384−3600).

Ans. 7992 and 4392.

14. The Sum of two numbers is 3864, and their Difference is 2400; what are the two numbers? *Ans.* 3132 and 732.

15. The Sum of two numbers is 45831, and their Difference is 3500; what are the two numbers?

Ans. 24665½ and 21165½.

16. A and B together have 9875 dollars, and A has 1260 dollars more than B; what sum has each of them?

Ans. A has 5567½, and B 4307½ dollars.

17. A borrowed of B 8794 dollars, of which he paid to B at one time 2340 dollars, and at another time 1375 dollars; what balance remains to be paid?

The balance that remains is the Difference between the sum borrowed and the aggregate of the two payments made, which may be denoted thus:

$$8794 - (2340 + 1375).$$

Ans. 5079 dollars.

18. Put in store at one time 500 pounds of hemp; at another time 3800 pounds; and at another 2005 pounds. Having withdrawn 3473 pounds, what quantity remains in store?

Ans. 2832 pounds

19. Three persons propose to purchase a manufactory, valued at 25850 dollars; A agrees to pay 5000 dollars, B twice as much as A. and C the remainder. What sum will C have to pay? *Ans.* 10850 dollars.

20. A planter sold cotton amounting to 3460 dollars, and out of these proceeds purchased groceries for 150 dollars, and other provisions for 375 dollars; how much of the first sum had he remaining? *Ans.* 2935 dollars.

21. A cabinet-maker sold furniture to the amount of 4000 dollars, and received in payment, at different times, 200 dollars, 475 dollars, and 904 dollars. How much of the debt remains to be paid? *Ans.* 2421 dollars.

22. A bought of B 875 acres of land for 23400 dollars. For 500 acres of the tract he paid 11379 dollars; how many acres were in the remainder of the tract? and for what sum was it purchased? *Ans.* 375 acres; and 12021 dollars.

23. A merchant exchanges a stock of goods worth 6725 dollars, and a house worth 3120 dollars, for a tract of land valued at 6900 dollars,—the farmer paying the balance in money. What sum must the merchant receive?

Ans. 3945 dollars.

24. A gentleman purchased at one time 7 acres, and at another time 12 acres of land, at 234 dollars an acre; what did the whole amount to?

The whole amounted to 234 dollars multiplied by the *sum* of 7 and 12; that is, 234 dollars × 19.

$$\begin{array}{r} 234 \\ 2106 \\ \hline \end{array}$$

Ans. 4446 dollars.

When the Multiplier is 13, 14, or 15, &c., the operation may be *abbreviated* thus:—multiply by the *units* figure, 9 in this example, and set the first product figure, 6, one place *to the right of the units* in the multiplicand; and, in this order, *add the product to the multiplicand.*

In like manner we may multiply by 1 *with 0's and a significant figure* annexed, as 103, 1004, &c.; observing to set the first product figure *one more place* to the right of the multiplicand than there are intervening 0's in the multiplier.

25. A merchant sold a piece of cloth containing 45 yards, another piece containing 57 yards, and another containing 63 yards, at 14 dollars a yard. What did the whole amount to?

Ans. 2310 dollars.

26. Farmer A had in wheat 205 acres, which produced 27 bushels per acre; and farmer B had 320 acres, which produced 19 bushels per acre. What quantity of wheat was raised by them both? *Ans.* 11615 bushels.

27. A speculator bought 150 head of cattle, and 47 mules. He made a profit of 13 dollars a head on the former, and 17 on the latter; what was gained by the speculation?

Ans. 2749 dollars.

28. A has 340 acres of land worth 18 dollars an acre, and B has 239 acres worth 22 dollars an acre. How many acres have the two together? and what is the value of the whole? *Ans.* 579 acres; and 11378 dollars.

29. One manufacturer exported 234 bales of cloth, each containing 103 yards; another exported 209 bales, each containing 107 yards. Which of the two exported the greater quantity? and by how many yards?

Ans. The first, by 1739 yards.

30. Two persons start from the same place, at the same time, and travel in contrary directions. One proceeds at the

rate of 39 miles per day, and the other at the rate of 44 miles per day; how many miles will they be apart at the end of 21 days?

The number of miles they will be apart, is the *sum* of 39 miles and 44 miles multiplied by 21; that is, 83 miles × 21.

$$\begin{array}{r} 83 \\ 166 \\ \hline \end{array}$$

Ans. 1743 miles.

When the Multiplier is 21, 31, or 41, &c., the operation may be *abbreviated* thus:—multiply by the *tens* figure, 2 in this example, and set the first product figure, 6, under the *tens* figure of the multiplicand, and thus *add the product to the multiplicand.*

In like manner we may multiply by any *significant figure with* 0'*s and a unit* annexed, as 201, 3001, &c.; observing to set the first product figure *one more place* to the left of the units in the multiplicand than there are intervening 0's in the multiplier.

31. A bought of B, 475 acres of land at 31 dollars per acre, 236 acres of which he sold to C at 18 dollars, and the remainder to D at 41 dollars per acre. What did A gain or lose by these transactions? *Ans.* Lost 678 dollars.

32. An importer bought 124 bales of linen; each bale contained 24 pieces, and each piece 21 yards. What was the whole number of pieces? and the whole number of yards?
Ans. 2976 pieces; 62496 yards.

33. A farmer has three tracts of land. The first and second contain 280 acres each, and the third contains twice as much as both the other two; how many acres does the farmer own? and what would the whole amount to at 201 dollars per acre? *Ans.* 1680 acres; 337680 dollars.

34. A gave B 3 horses, and 31 head of cattle, for 51 bar-

rels of flour, and 301 barrels of corn. A sold the flour at 6 dollars, and the corn at 3 dollars, a barrel; while B sold his horses at 120 dollars, and his cattle at 17 dollars, each. Which of the two gained by the trade? and how much?

Ans. A gained 322 dollars.

35. A has 339 acres of land, and B has 240 acres. Allowing each tract to be worth 54 dollars per acre, how much does the whole value of A's land exceed that of B's?

The value of A's land exceeds the value of B's by the product of 54 dollars multiplied by the *excess of* 339 *above* 240; that is, by 54 dollars × 99.

$$\begin{array}{r} 5400 \\ 54 \\ \hline 5346 \end{array}$$

Ans. 5346 dollars.

When the Multiplier is any number of 9's, the operation may be *abbreviated* thus:—annex as many 0's to the multiplicand as there 9's in the multiplier, and from the result *subtract the multiplicand.*

In this example, the annexing of 00 to the 54 produces 100 times 54 (29); then 54 subtracted leaves 99 times 54.

36. What would be the *sum of the products* obtained by multiplying 375 dollars by 9, five hundred and thirty seven dollars by 99, and six hundred and forty one dollars by 999?

Ans. 696897 dollars

37. An ironmonger bought 99 tons of iron at 39 dollars per ton, 19 tons at 43 dollars per ton, and 20 tons at 40 dollars per ton. What would he gain or lose by selling the whole at 41 dollars per ton?

Ans. He would gain 180 dollars.

38. A merchant bought 290 yards of cloth at 9 dollars per yard. Having sold 137 yards of it, at 14 dollars per yard, and 81 yards, at 15 dollars per yard; what would he make on the whole by selling the remainder at 16 dollars per yard? *Ans.* 1675 dollars.

39. A planter sold 139 bales of cotton, at an average of 41 dollars per bale. Out of the proceeds he bought 29 mules, at 100 dollars each, and 4 pair of oxen, at 81 dollars a pair; what sum had he left from the sale of his cotton?

Ans. 2475 dollars.

40. A sends to market 209 tons of coal; B sends as much as A, wanting 10 tons; and C sends as much as A and B together. What is each man's proceeds of sale, at 13 dollars per ton? *Ans.* A's 2717 dollars; B's 2587; C's 5304.

41. A farmer bought land from A at 60 dollars an acre, and the *same quantity* from B at 85 dollars an acre. The whole amounted to 53215 dollars; how many acres did he buy from each?

To buy *one acre* from each, required 60 dollars + 85 dollars, that is, 145 dollars; then the *number of acres* bought from each, is the number of times that 145 *is contained in* 53215.

```
145) 53215 (367 acres.
      971
      1015
      0000
```

The operation of dividing, by several figures, may be *abbreviated*, by subtracting the products of the divisor and quotient figures *mentally*. In thus subtracting, we add as many *tens* as may be necessary to the upper figure, and then add as many *units* to the next product. Thus,

We say 145 in 532, 3 times; 3 times 5 is 15; 15 from 22 leaves 7, which we set down; 3 times 4 is 12, and 2 make 14; 14 from 23 leaves 9; 3 times 1 is 3, and 2 make 5; 5 from 5 leaves nothing.

Bringing down the 1 from the dividend, we say 145 in 971, 6 times; 6 times 5 is 30; 30 from 31 leaves 1; 6 times

4 is 24, and 3 make 27; 27 from 27 leaves 0; 6 times 1 is 6, and 2 make 8; 8 from 9 leaves 1.

Bringing down the 5 from the dividend, we proceed in the same manner as before, and find *no remainder*.

42. A planter has 2280 dollars to lay out for mules and oxen; and wishes to purchase the same number of each. If he pay 65 dollars a head for mules, and 30 for oxen, how many of each can he buy? *Ans.* 24 of each.

43. A merchant sold cloth for 423 dollars, cotton for 125 dollars, and silk for 300 dollars; and invested the whole proceeds in sugar at 12 dollars per barrel. How many barrels of sugar did he purchase? *Ans.* $70\frac{8}{12}$ barrels.

44. How many yards of cloth at 7 dollars a yard, at 8 dollars a yard, and at 9 dollars a yard—the quantity of each kind to be the same—could be purchased for 1800 dollars?

Ans. 75 yards of each kind.

45. A person who undertook a journey of 1000 miles, traveled the first 7 days at the rate of 35 miles per day. How long will he be in accomplishing the remaining distance, at the rate of 33 miles per day? *Ans.* $22\frac{29}{33}$ days.

46. In how many days could 10 men accomplish the same amount of work that 13 men could perform in 349 days?

One man would do the work in 13 *times* 349 *days*, and 10 men would do it in $\frac{1}{10}$ of the time in which *one man* would do it. *Ans.* $453\frac{7}{10}$ days.

47. How long should 12 teams be employed in doing an amount of hauling which 23 teams could accomplish in 65 days? *Ans.* $124\frac{7}{12}$ days.

48. A company of 100 men have provisions sufficient for 4 months. If 33 men leave the company, how long will the same provisions suffice the remainder? *Ans.* $5\frac{65}{67}$ months.

49. If 27 barrels of flour be worth 135 dollars, what are 350 barrels worth, at the same price per barrel?

Ans. 1750 dollars.

50. If 39 acres of ground produce 2184 bushels of corn, how many bushels will 280 acres produce at the same rate? *Ans.* 15680 bushels.

51. A merchant bought 250 yards of cloth for 1750 dollars, and sold 133 yards of it at the same price at which he bought it. What did the cloth sold amount to? *Ans.* 931 dollars.

52. A gentleman having on hand 6975 dollars, bought 5 pair of oxen, at 45 dollars a pair, and paid the remainder of his money for 230 acres of land. What was the price of the land per acre? *Ans.* $29\frac{80}{230}$ dollars.

53. A cistern which holds 10000 gallons, is to be filled with water by 3 pipes discharging into it. The first pipe discharges 200 gallons per hour, the second and third each 150 gallons per hour; in what time will the three pipes, running together, fill the cistern? *Ans.* 20 hours.

54. A carpenter can earn 45 dollars a month, but his necessary expenditures are at the rate of 24 dollars a month. He wishes to purchase a certain lot of ground, which contains 19 acres, and is held at 35 dollars per acre; in what time may he save enough to make the purchase? *Ans.* $31\frac{14}{21}$ months.

55. A plantation containing 1200 acres was exchanged for another containing 1000 acres, and worth 96 dollars an acre. At what price per acre was the first plantation rated? *Ans.* 80 dollars.

56. A gentleman having on hand 5730 dollars, purchased 23 shares of bank stock, at 109 dollars a share, and divided the remainder of his money equally between three benevolent institutions. What sum did each institution receive? *Ans.* $1074\frac{1}{3}$ dollars.

57. An army of 5000 men has provisions for 6 months. If 2500 more be added to this army, how long will the same

stock of provisions supply the whole number, without any change of rations? *Ans.* 4 months.

58. A farmer sold wheat for 900 dollars, corn for 274 dollars, and other produce for 329 dollars. Out of these proceeds he bought three pair of oxen, at 55 a pair, and paid the remainder for 65 acres of land; what did the land cost him per acre? *Ans.* $20\frac{38}{65}$ dollars.

59. A bought a piece of ground for 1080 dollars, which was at the rate of 27 dollars per acre; and B purchased 3 tracts, each containing 125 acres, for 18750 dollars. What quantity did A purchase? And what did B pay per acre?

Ans. 40 acres; and 50 dollars.

60. A cistern whose capacity is 15000 gallons, is supplied with water by two pipes, each discharging 325 gallons per hour; but, by leakage, the cistern loses, during the time of filling, at the rate of 100 gallons per hour. In what time would the two pipes fill the cistern? *Ans.* $27\frac{150}{550}$ hours.

CHAPTER III.

PRIME NUMBERS.—COMMON MEASURE.—COMMON MULTIPLE.

PRIME NUMBERS.

(47.) A PRIME NUMBER is one which *is not the product* of two numbers, each greater than a *unit.*

A *composite number* is one which is the product of two numbers, each greater than a *unit.*

Thus 5 is a *prime* number; and 6 is a *composite* number, since 6 is the product of the factors 2 and 3.

Is 2 a *prime*, or a *composite* number? Is 3 a prime, or a composite number? 4? 6? 13? 15? 18? 25?

Name all the *prime* numbers from 1 to 49.

An *even* number is one which can be divided by 2, without a remainder; and an *odd* number is one which cannot be divided by 2, without a remainder.

Name all the *even* numbers to 50.—The *odd* numbers to 55.

Are all *prime* numbers *odd*, or *even* numbers? Are *composite* numbers *odd*, or *even* numbers?

Decomposition of Numbers.

(48.) *Decomposing a composite* number consists in resolving it into its *prime factors*, that is, into factors each of which shall be a *prime number.*

Thus 6 is *decomposed* when it is resolved into 3×2;

8 is decomposed when it is resolved into $2 \times 2 \times 2$.

What are the *prime factors* of 12? What are the prime factors of 18? What are the prime factors of 50? What are prime factors of 75? What are the prime factors of 63? What are the prime factors of 96?

The following are some of the *Prime Numbers* :—

1	31	79	137	193	257	317	389	457	523	601	661
2	37	83	139	197	263	331	397	461	541	607	673
3	41	89	149	199	269	337	401	463	547	613	677
5	43	97	151	211	271	347	409	467	557	617	683
7	47	101	157	223	277	349	419	479	563	619	691
11	53	103	163	227	281	353	421	487	569	631	701
13	59	107	167	229	283	359	431	491	571	641	709
17	61	109	173	233	293	367	433	499	577	643	719
19	67	113	179	239	307	373	439	503	587	647	727
23	71	127	181	241	311	379	443	509	593	653	733
29	73	131	191	251	313	383	449	521	599	659	739

RULE IX.

(49.) *To resolve a Composite Number into its Prime factors.*

1. Divide the given number by any *prime number*, greater than *unity*, that will divide it without a remainder.

2. Divide the quotient in like manner; and so on, until the quotient becomes a *prime number*. The several *divisors* and the last *quotient* will be the prime factors of the given number.

EXAMPLE.

To resolve 210 into its *prime factors.*

```
2)210
 3)105
   5)35
      7
```

The prime divisor 2 resolves 210 into 2×105 (37); the divisor 3 resolves 105 into 3×35; the divisor 5 resolves 35 into 5×7; hence 210 is resolved into $2 \times 3 \times 5 \times 7$.

EXERCISES.

1. Resolve 735 into its *prime factors*.

Ans. 5, 7, 3, and 7

2. Resolve 330 into its prime factors.

Ans. 2, 3, 5, and 11.

3. Resolve 510 into its prime factors.

Ans. 2, 3, 5, and 17.

4. Resolve 390 into its prime factors.

Ans. 5, 2, 3, and 13.

5. Resolve 550 into its prime factors.

Ans. 5, 2, 5, and 11.

6. Resolve 930 into its prime factors.

Ans. 2, 3, 5, and 31.

7. Resolve 1330 into its prime factors.

Ans. 2, 5, 7, and 19.

8. Resolve 1610 into its prime factors.

Ans. 2, 5, 7, and 23.

9. Resolve 4350 into its prime factors.

Ans. 2, 3, 5, 5, and 29.

10. Resolve 6020 into its prime factors.

Ans. 2, 2, 5, 7, and 43.

COMMON MEASURE.

(50.) One number is called *a measure* of another, if it is contained in the other a number of times, without *a remainder;* and

A *Common Measure* of two or more numbers, is any number that is contained in *each of them* a number of times, without *a remainder.*

Thus 3 is a *common measure* of 12 and 15.

Name a *common measure* of 18 and 27. Name a common measure of 32 and 48. Of 12, 18, and 30. Of 36, 54, and 72.

Greatest Common Measure.

(51.) The *Greatest Common Measure* of two or more numbers, is the greatest number that is contained in *each of them* a number of times, without a remainder.

Thus 9 is the greatest common measure of 18 and 27.

What is the *greatest common measure* of 16 and 24? What is the greatest common measure of 30 and 40? What is the greatest common measure of 8, 12, and 32? Of 20, 60, and 80?

When two numbers have no common measure but *unity*, they are said to be *prime to each other;* thus 16 and 21 are prime to each other.

A common measure is sometimes, though not so properly, called a *common divisor;* and the greatest common measure, the greatest common *divisor*.

RULE X.

(52.) *To find the Common Measures of two or more numbers.*

1. Resolve each number into its *prime factors;* and select those factors which are *common to all the numbers.*

2. Any *one*, or the product of any *two or more*, of these common factors, will be *a common measure*—and the product of all these common factors will be the *greatest* com mon measure, of the given numbers.

EXAMPLE.

To find the *common measures* of

390, 930, and 4350.

By resolving each number into its prime factors, we find

$$390 = 2 \times 3 \times 5 \times 13;$$
$$930 = 2 \times 3 \times 5 \times 31;$$
$$4350 = 2 \times 3 \times 5 \times 5 \times 29;$$

The factors which are *common to the three given numbers*, that is, found in each of those numbers, are 2, 3, and 5.

Then each of these common factors is *a common measure* of the given numbers; also $2 \times 3 = 6$, $2 \times 5 = 10$, and $3 \times 5 = 15$ are *common measures;* and $2 \times 3 \times 5 = 30$ is the *greatest common measure.*

EXERCISES.

1. Find the *greatest common measure* of 252, 180, and 288. *Ans.* 36.

2. Find the greatest common measure of 120, 144, and 168. *Ans.* 24.

3. Find the greatest common measure of 240, 336, and 432. *Ans.* 48.

4. Find the greatest common measure of 392, 504, and 560. *Ans.* 56.

5. Find the greatest common measure of 504, 567, and 630. *Ans.* 63.

6. Find the greatest common measure of 336, 588, and 756. *Ans.* 84.

7. Find the greatest common measure of 288, 480, and 672. *Ans.* 96.

8. Find the greatest common measure of 460, 1035, and 1150. *Ans.* 115.

9. Find the greatest common measure of 620, 1116, and 1488. *Ans.* 124.

10. Find the *several common measures* of 42, 210, and 126. *Ans.* 2, 3, 7, 6, 14, 21, and 42

(53.) *Another Method of finding the Greatest Common Measure of two or more numbers.*

1. Divide the *least number* into each of the others.

2. Take the *divisor* and *remainders* for a new set of numbers, with which proceed as before; and so on, until there is no remainder. The *last divisor* will be the greatest common measure of the given numbers.

Thus, to find the greatest common measure of 390, 930, and 4350.

```
390)930(2               390)4350(11
    150 remainder.            60 remainder.

60)150(2                60)390(6
    30 remainder.           30 remainder.

             30)60(2
                no remainder.
```

The *last divisor*, 30, is the greatest common measure required.

When two or more remainders *are equal*, only one of them must be used in the next division.

For finding the Greatest Common Measure, this method is preferable to that by Rule X.—Its correctness depends on the principle, that

The Greatest Common Measure of two or more numbers, is the same as that of the *least* of those numbers and the *remainders*, if any, after dividing the least number into each of the others.

Take, for example, the numbers 12 and 28, or

$$12, \text{ and } 12 \times 2+4.$$

It is plain that *any measure* of 12, will also measure 12×2; and, measuring 12×2, if it measure $12 \times 2 + 4$, it must also measure 4.

Hence there can be no *common* measure of 12 and $12 \times 2 + 4$ which is not a *common* measure of 12 and 4; the *greatest* common measure, therefore, of 12 and 4 is the greatest common measure of 12 and 28.

But 4 is the *remainder* after dividing 12 into 28; hence the principle above stated is true for *two numbers;* and in like manner it may be proved for any three or more numbers.

To show the application of this principle to the preceding method.—The last divisor is the greatest *measure of itself;* hence it is the greatest common measure of the preceding *divisor* and *remainders*, and therefore of the next preceding *divisor* and *remainders*, and so on; it is therefore the greatest common measure of the *given numbers.*

11. Find the Greatest Common measure of 324 and 480. *Ans.* 12.

12. Find the greatest common measure of 972 and 1260. *Ans.* 36.

13. Find the greatest common measure of 744 and 1680. *Ans.* 24.

14. Find the greatest common measure of 636 and 1080. *Ans.* 12.

15. Find the greatest common measure of 375 and 1100. *Ans.* 25.

16. Find the greatest common measure of 120 and 1440. *Ans.* 120.

17. Find the greatest common measure of 780 and 1560. *Ans.* 780.

18. Find the greatest common measure of 720, 1008, and 1152 *Ans.* 144.

COMMON MULTIPLE.

(54.) One number is called a *multiple* of another, if it contains the other a number of times, without *a remainder;* and

A *Common Multiple* of two or more numbers, is any number that contains *each of them* a number of times, without a remainder.

Thus 30 is a *common multiple* of 10 and 6.

Name a *common multiple* of 5 and 8. Name a common multiple of 5, and 11. Of 3, 4, and 8. Of 4, 3, and 6. Of 2, 5, and 10.

Least Common Multiple.

(55.) The *Least Common Multiple* of two or more numbers, is the smallest number that contains *each of them* a number of times, without a remainder.

Thus 15 is the least common multiple of 3 and 5.

What is the *least common multiple* of 4 and 5? What is the least common multiple of 3 and 6? Of 2, 3, and 5? Of 3, 4, and 6?

When one number is a *measure* of another, the latter is a *multiple* of the former. Thus 6 is a measure of 30, and 30 is a multiple of 6.

RULE XI.

(56.) *To find the Least Common Multiple of two or more numbers.*

1. Set the numbers in a line from left to right, and divide *two or more* of them by any *prime number*, greater than *unity*, that will divide them without a remainder.

2. Set the quotients and the *undivided* numbers in a line

below, and divide them, as before; and so on, until no two numbers in the lowest line can be so divided.

3. Multiply together the *divisors* and numbers in the *lowest line*, for the least common multiple of the given numbers.

☞ If *no two* of the given numbers can be divided as above, the product of all the given numbers will be their least common multiple.

EXAMPLE.

To find the *least common multiple* of 6, 12, and 15.

2)6	12	15
3)3	6	15
1	2	5

$2 \times 3 \times 1 \times 2 \times 5 = 60$, the least common multiple of the given numbers.

The divisors 2 and 3 resolve the given numbers into their *prime* factors; and by using the same divisor for *two or more of the numbers*, we obtain the *smallest collection* of prime factors out of which can be produced each of the given numbers; that is, we thus obtain the factors of the least *common multiple.*

☞ Any number of *common multiples* of given numbers may evidently be found, by multiplying their least common multiple by 2, 3, 4, &c.

EXERCISES.

1. Find the *least common multiple* of 4, 7, 9, and 21.
Ans. 252

2. Find the least common multiple of 3, 9, 12, and 15.
Ans. 180

3. Find the least common multiple of 4, 6, 8, and 10.
Ans. 120.

4. Find the least common multiple of 6, 4, 12, and 20.
Ans. 60.

5. Find the least common multiple of 8, 7, 10, and 14.
Ans. 280.

6. Find the least common multiple of 5, 6, 10, and 24.
Ans. 120.

7. Find the least common multiple of 5, 10, 13, and 24.
Ans. 1560.

8. Find the least common multiple of 2, 7, 13, and 15.
Ans. 2730.

9. Find the least common multiple of 6, 7, 2, and 17.
Ans. 714.

10. Find the least common multiple of 11, 4, 5, and 19.
Ans. 4180.

11. Find the common multiples of 6, 14, 20, 8, 12, and 24.
Ans. 840, 1680, 2520, &c.

MISCELLANEOUS EXERCISES

ON PRIME FACTORS, COMMON MEASURES, AND COMMON MULTIPLES.

1. An agriculturist has 2145 bushels of wheat, with which he wishes to fill a number of sacks of equal capacity. What are the several smallest capacities, either of which would meet the requisition ?

Each of the required capacities must be a *prime factor* or divisor of the given number of bushels.

Ans. 1, 3, 5, 11, or 13 bushels.

2. A farmer has 66 bushels of corn, and 90 of wheat, which he wishes to put into sacks of equal size, and without

mixing the two kinds of grain. How many bushels must each sack contain?

Each sack must evidently contain a *common measure* of 66 and 90 bushels. *Ans.* 2, 3, or 6 bushels.

3. How many acres of land would admit of being divided into a number of farms containing 150, 200, or 250 acres, each?

The required number of acres must evidently be a *common multiple* of the given numbers.

Ans. 3000 acres, or 6000 acres, &c.

4. An upholsterer has 125 yards of carpeting of one kind, 175 of another, and 225 of another. He wishes to divide the whole into pieces of equal length, and the longest that can be obtained; what must be the length of each piece?

Ans. 25 yards.

5. A certain school consists of 132 junior, and 99 senior students. How might each of these two classes be divided, so that the whole school should be distributed into equal sections? *Ans.* Into sections of 3, 11, or 33.

6. What is the smallest sum of money for which a person could purchase, either a number of mules at 32 dollars a head, or a number of cows at 14 dollars a head,—the same sum to be employed in either purchase? *Ans.* 224 dollars.

7. A can build 7 rods of fencing in a day, B can build 9 rods, and C 12 rods, in a day. What amount of fencing would afford a number of full days' work for any one of the three? *Ans.* 252, 504, or 756 rods, &c.

8. A gentleman has a piece of ground, the sides of which measure 225 feet, 297 feet, and 369 feet. He wishes to enclose it with a fence having panels of uniform length; what is the longest panel that can be used for that purpose?

Ans. 9 feet.

9. What is the smallest sum for which I could purchase a

number of plows at 14 dollars each, or a number of carts at 30 dollars each, or a number of wagons at 90 dollars each,—allowing the whole sum to be expended in either purchase?

Ans. 630 dollars.

10. If one team can haul to market 10 barrels of flour, another 12 barrels, and another 15 barrels; what number of barrels would make a number of full loads for any of the three teams? *Ans.* 60, or 120 barrels, &c.

11. Having 140 acres of land at one place, and 252 at another, I wish to divide the whole into fields which shall be of equal size, and the largest that will meet such requisition. What must be the number of acres in each field?

Ans. 28 acres.

12. A wine merchant has 111 gallons of Madeira, 185 gallons of Port, and 259 gallons of Malaga, with which he wishes to fill a number of casks, and without mixing the different kinds of wine. What must be the contents of each cask? *Ans.* 37 gallons.

13. Three regiments of soldiers, containing, respectively, 1538 men, 2307 men, and 3845 men, are to be formed, separately, into battalions, the largest that will admit the same number of men in each. What must be the number in each battalion? *Ans.* 769 men.

14. A and B purchased horses at the same rate per head; A's horses amounted to 623 dollars, and B's to 1068 dollars; what was the number purchased by each?

Ans. 7 by A; 12 by B.

15. How many bushels would fill a number of barrels, each containing 3 bushels, or a number of sacks, each containing 4 bushels, or a number of casks, each containing 14 bushels,—the quantity to be the same in each case?

Ans. 84, or 168 bushels, &c.

16. What is the smallest sum for which I could purchase

a number of mules, at 75 dollars a head, or a number of horses, at 125 dollars a head? and what number of each could I purchase for that sum?

Ans. 375 dollars; 5 mules, or 3 horses.

17. A has 413 dollars, B 531 dollars, and C 590 dollars; and they agree to purchase oxen at the same price per head, provided each man can thus invest all his money. How many oxen can each man purchase?

Ans. A, 7; B, 9; and C, 10.

18. For what sum could I hire workmen, for one month, at 15 dollars, 21 dollars, or 24 dollars each, allowing the whole sum to be thus expended?

Ans. 840, or 1680 dollars, &c.

19. Three hundred and eighty-five Irishmen, 455 Frenchmen, and 700 Germans are to be ferried over a river. What are the largest equal companies into which they may all be divided, so that those of the same nation shall go over together? *Ans.* Companies of 35.

20. A, B, C, and D start together, and travel the same way around an island which is 600 miles in circuit; A goes 20 miles per day, B 30, C 25, and D 40. How long must their journeyings continue, in order that they may all come together again? *Ans.* 120 days.

CHAPTER IV.

FRACTIONS.

(57.) A FRACTION is one or more of the *equal parts* into which any quantity may be supposed to be divided.

One half is one of the *two equal parts* of any quantity; *two thirds* are two of the *three equal parts* of any quantity; and so on.

What is meant by *one third?*	One *fourth?*	Three *fourths?*
What is meant by *one fifth?*	Two *fifths?*	Four *fifths?*
What is meant by *one sixth?*	Three *sixths?*	Five *sixths?*

Any quantity consists of how many *halves* of that quantity? Of how many *thirds?* Of how many *fourths?* Of how many *tenths?* Of how many *hundredths?*

Numerator and Denominator.

(58.) The *numerator* of a Fraction is the *number of equal parts* in the fraction: the *denominator* shows the number of such parts in the *whole quantity* divided.

Thus in $\frac{3}{4}$, *three fourths*, 3 is the *numerator*, and 4 the *denominator.*

In the fraction $\frac{2}{3}$, which number is the *numerator?* and what does it show? Which the denominator? and what does it show? In $\frac{5}{8}$? In $\frac{4}{9}$?

The numerator and denominator are called the *terms* of the fraction

Fractions express Division.

(59.) Every fraction is equal to its numerator *divided by its denominator.*

Thus the fraction $\frac{4}{9}$ is the quotient of 4 divided by 9, since it expresses the *part that* 4 *is of* 9, (34). Hence also

Every fraction is equal to that *part of its numerator* which is denoted by the *reciprocal of its denominator* (35).

$\frac{4}{9}$ is equal to $\frac{1}{9}$ of 4; *four ninths* of any quantity is $\frac{1}{9}$ of 4 such quantities.

$\frac{5}{9}$ of 1 is what part of 5? $\frac{7}{10}$ of 1 is what part of 7?
$\frac{3}{11}$ of 1 is what part of 3? $\frac{5}{12}$ of 1 is what part of 5?

Proper and Improper Fractions.

(60.) A *proper fraction* is one whose numerator is *less than its denominator;* and whose value is therefore less than *a unit* or *whole one.*

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, &c. are proper fractions.

(61.) An *improper fraction* is one whose numerator is *equal to*, or *greater* than, its denominator. Its value in *units* is found by *dividing its numerator* by its denominator.

Thus $\frac{8}{3}$ is an *improper fraction;* and its value in *units* is $2\frac{2}{3}$ (59).

What is the value in *units* of $\frac{5}{3}$? What is the value of $\frac{8}{4}$?
What is the value in *units* of $\frac{17}{5}$? What is the value of $\frac{24}{6}$?
What is the value in *units* of $\frac{25}{7}$? What is the value of $\frac{42}{8}$?

Integral and Mixed Numbers.

(62.) An *integral* number, or simply an *integer*, is a number in which there is *no fraction;* as 1, 3, 5, &c.

A *mixed number* consists of an *integer* and a *fraction:* as $5\frac{1}{4}$.

(63.) An *integer* is made an *improper fraction* by taking any number for a *denominator*, and multiplying the integer by that number for a *numerator.*

Thus the *integer* 3 is equal to $\frac{3}{1}$, $\frac{6}{2}$, $\frac{9}{3}$, or $\frac{12}{4}$, &c.

The *integer* 4 is equal to how many *halves?*—how many *thirds?*
The integer 5 is equal to how many *thirds?*—how many *sixths?*

(64.) A *mixed number* is made an *improper fraction*, by multiplying its integer by the *denominator* annexed, and *adding the numerator* to the product, for a numerator to be placed over said denominator.

Thus $3\frac{2}{5}$ is equal to $\frac{17}{5}$; the numerator 17 being $5 \times 3 + 2$.

$3\frac{1}{4}$ is equal to how many 4ths? $4\frac{3}{7}$ is equal to how many 7ths?
$5\frac{3}{10}$ is equal to how many 10ths? $6\frac{5}{12}$ is equal to how many 12ths?
$7\frac{5}{9}$ is equal to how many 9ths? $9\frac{7}{10}$ is equal to how many 10ths?

Constant value of à Fraction.

(65.) The value of a Fraction remains *the same*, when its numerator and denominator are both multiplied, or both divided, by the *same number*.

Thus $\frac{3}{4}$ is equal to $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.

For if any quantity were divided into 4 *fourths*, each *one* of these fourths, divided into *two equal parts*, would make 2 *eighths* of the quantity; then 3 *fourths* would make 6 *eighths*.

$\frac{3}{5}$ is equal to how many 10ths?—how do you prove it?
$\frac{2}{6}$ is equal to how many 18ths?—how do you prove it?

The preceding principle also follows from regarding a Fraction as the quotient of its numerator divided by its denominator (59) (38).

REDUCTION OF FRACTIONS.

(66.) The *reduction* of a quantity, in general, consists in changing its *expression*, without *altering its value.*

Thus the mixed number $3\frac{2}{5}$ may be *reduced to the improper fraction* $\frac{17}{5}$. (64).

A fraction is *reduced to its lowest term* when its numerator and denominator are made the *smallest* that will express the value of the given fraction.

Thus $\frac{27}{36}$ reduced to its lowest terms is $\frac{3}{4}$,—found by dividing 27 and 36 both by 9, which is their *greatest common measure* (65).

RULE XII.

(67.) *To reduce a Fraction to its Lowest Terms.*

1. Divide both terms of the fraction by their *greatest common measure;* the quotients will be the lowest terms of the given fraction. Or

2. Divide both terms by *any common measure,* and the quotients by any common measure, and so on, until the quotients become *prime to each other.*

EXAMPLE.

To reduce $\frac{90}{120}$ to its lowest terms.

$$\frac{90}{120} = \frac{3}{4}.$$

The greatest common measure of 90 and 120 is 30 (53), by which we divide both terms of the given fraction. This does not *alter the value* of the fraction (65).

Or, we might divide, successively, by 10 and 3 (44).

EXERCISES.

1. Reduce $\frac{25}{75}$ and $\frac{216}{360}$ to their lowest terms.
Ans. $\frac{1}{3}$ and $\frac{3}{5}$.

2. Reduce $\frac{30}{90}$ and $\frac{125}{150}$ to their lowest terms.
Ans. $\frac{1}{3}$ and $\frac{5}{6}$.

3. Reduce $\frac{45}{50}$ and $\frac{279}{403}$ to their lowest terms.
Ans. $\frac{9}{10}$ and $\frac{9}{13}$.

4. Reduce $\frac{55}{75}$ and $\frac{99}{225}$ to their lowest terms.
Ans. $\frac{11}{15}$ and $\frac{11}{25}$.

5. Reduce $\frac{39}{84}$ and $\frac{180}{468}$ to their lowest terms.
Ans. $\frac{13}{28}$ and $\frac{5}{13}$.

6. Reduce $\frac{284}{300}$ and $\frac{600}{845}$ to their lowest terms.
Ans. $\frac{71}{75}$ and $\frac{120}{169}$.

7 Reduce $\frac{345}{800}$ and $\frac{436}{940}$ to their lowest terms.
Ans. $\frac{69}{160}$ and $\frac{109}{235}$.

Common Denominator.

(68.) Two or more fractions are said to have a *common denominator*, when they have the *same number* for a denominator.

Thus $\frac{2}{7}$, $\frac{4}{7}$, and $\frac{6}{7}$ have a common denominator.

Two or more fractions are reduced to their *least common denominator*, when their common denominator is made the *smallest* by which the value of each fraction can be expressed.

The fractions $\frac{3}{4}$ and $\frac{5}{6}$ are reduced to their *least common denominator*, when these fractions are made $\frac{9}{12}$ and $\frac{10}{12}$, respectively. This is done by multiplying *both terms* of the first by 3, and both terms of the second by 2 (65).

The least common denominator, 12, is the *least common multiple* of the given denominators 4 and 6.

RULE XIII.

(69.) *To reduce two or more Fractions to a Common Denominator.*

1. Multiply each numerator by all the denominators except its own, for the *new numerators;* and multiply all the denominators together, for a *common denominator.*

2. If the *least common denominator be required,*—take the *least common multiple* of the given denominators, for the common denominator. Divide this multiple by each given *denominator,* and multiply the quotient by the corresponding numerator, for the *new numerators.*

EXAMPLES.

1. To reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator.

For the *new numerators,* we have

$$2 \times 6 \times 8 = 96; \quad 5 \times 3 \times 8 = 120; \quad 7 \times 6 \times 3 = 126,$$

and for the *common denominator,* $3 \times 6 \times 8 = 144$.

The given fractions thus become

$\frac{96}{144}$, $\frac{120}{144}$, and $\frac{126}{144}$, respectively.

2. To reduce the same fractions, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$ to their *least common denominator.*

The least *common multiple* of the given denominators is 24 (56); then 24 is the *common denominator* required; and the *new numerators* are

$$(24 \div 3) \times 2 = 16; \; (24 \div 6) \times 5 = 20; \; (24 \div 8) \times 7 = 21$$

The given fractions are thus reduced to

$\frac{16}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$, respectively.

This method will not always find the *least* common denominator, unless each of the given fractions is in its *lowest terms.*

Thus, in the preceding Example, if we take $\frac{14}{16}$ instead of its equal $\frac{7}{8}$, the least common multiple will be 48, which, as has been seen, is not the *least* common denominator by which the equivalent of each fraction can be expressed.

The *values of the fractions are not altered in* reducing them to a common denominator, because both terms of each fraction are, in the operation, multiplied by the same numbers, (65). This will be evident upon inspecting the preceding Examples.

EXERCISES.

1. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{8}$ to a common denominator.
Ans. $\frac{80}{120}$, $\frac{96}{120}$, and $\frac{45}{120}$.
2. Reduce $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{8}{13}$ to a common denominator.
Ans. $\frac{351}{585}$, $\frac{260}{585}$, and $\frac{360}{585}$.
3. Reduce $\frac{5}{8}$, $\frac{6}{7}$, and $\frac{3}{12}$ to a common denominator.
Ans. $\frac{420}{672}$, $\frac{576}{672}$, and $\frac{168}{672}$.
4. Reduce $\frac{5}{11}$, $\frac{9}{7}$, and $\frac{3}{2}$ to a common denominator.
Ans. $\frac{70}{154}$, $\frac{198}{154}$, and $\frac{231}{154}$.
5. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to the *least* common denominator.
Ans. $\frac{18}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.
6. Reduce $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{7}{12}$ to the least common denominator.
Ans. $\frac{24}{36}$, $\frac{16}{36}$, and $\frac{21}{36}$.
7. Reduce $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{9}{10}$ to the least common denominator.
Ans. $\frac{24}{30}$, $\frac{25}{30}$, and $\frac{27}{30}$.
8. Reduce $\frac{4}{16}$, $\frac{5}{20}$, and $\frac{21}{45}$ to the least common denominator.
Ans. $\frac{15}{60}$, $\frac{15}{60}$, and $\frac{28}{60}$

ADDITION OF FRACTIONS.

(70.) The Sum of two or more Fractions is found by means of a *common denominator.*

For example, to find the Sum of $\frac{2}{3}$ and $\frac{3}{4}$.

These fractions are equal, respectively, to $\frac{8}{12}$ and $\frac{9}{12}$; the sum of 8 *twelfths* and 9 *twelfths* is $\frac{17}{12}$, equal to $1\frac{5}{12}$; then

$$\frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}.$$

What is the Sum of $\frac{1}{2}$ and $\frac{1}{3}$? What is the Sum of $\frac{1}{2}$ and $\frac{3}{4}$?
What is the Sum of $\frac{3}{4}$ and $\frac{5}{6}$? What is the Sum of $\frac{2}{3}$ and $\frac{4}{9}$?
What is the Sum of $\frac{3}{5}$ and $\frac{5}{8}$? What is the Sum of $\frac{3}{4}$ and $\frac{9}{16}$?

RULE XIV.

(71.) *For the Addition of Fractions.*

1. If the fractions have not a *common denominator,* reduce them to a common denominator.

2. Add all the numerators together, and place the Sum, as a numerator, over their common denominator.

3. *Mixed numbers* may be added under the form of *improper fractions* (64); or their *fractional* and *integral* parts may be added separately.

EXAMPLE.

To add together the mixed numbers $7\frac{3}{4}$, $8\frac{5}{6}$, $15\frac{2}{3}$.

$$\begin{array}{rcl} 7 & \quad & \frac{3}{4} = \frac{9}{12} \\ 8 & & \frac{5}{6} = \frac{10}{12} \\ 15 & & \frac{2}{3} = \frac{8}{12} \\ \hline 32\frac{1}{4} & & \frac{27}{12} = 2\frac{1}{4}. \end{array}$$

Reducing the *fractional* parts of the given numbers to their least common denominator (69), we find them equal to

$\frac{9}{12}$, $\frac{10}{12}$, and $\frac{8}{12}$, respectively.

These fractions added together make $\frac{27}{12}$, which is $2\frac{3}{12}$ (61), or $2\frac{1}{4}$; and this sum added to the sum, 30, of the *integral* parts of the given numbers, makes the *entire sum* $32\frac{1}{4}$.

Instead of the preceding method, we might reduce the given mixed numbers to the *improper fractions*

$\frac{31}{4}$, $\frac{53}{6}$, $\frac{47}{3}$, (64),

and then reduce these improper fractions to a *common denominator*, &c.; but this would not be so convenient a method.

☞ In all subsequent Exercises, *improper fractions* in the Answers are to be reduced to *integral* or mixed *numbers* (61); and proper fractions to their *lowest terms* (67).

EXERCISES.

1. What sum should be paid for a vest at $4\frac{3}{4}$ dollars, and a hat at $5\frac{7}{8}$ dollars? *Ans.* $10\frac{5}{8}$ dollars.

2. What sum should be paid for a cord of wood at $3\frac{1}{2}$ dollars, a barrel of flour at $5\frac{3}{4}$ dollars, and a shote at $2\frac{1}{3}$ dollars? *Ans.* $11\frac{7}{12}$ dollars.

3. Bought a quantity of corn for $15\frac{7}{8}$ dollars, a ton of hay for 13 dollars, and a lot of pork for $19\frac{3}{4}$ dollars. What did the whole amount to? *Ans.* $48\frac{5}{8}$ dollars.

4. Sold wheat for 275 dollars, oats for $37\frac{3}{8}$ dollars, and rye for $27\frac{4}{5}$ dollars. What did the whole amount to? *Ans.* $339\frac{7}{40}$ dollars

5. A manufacturer sold four pieces of cloth. The first piece contained $39\frac{3}{4}$ yards, the second $41\frac{5}{8}$ yards, the other two each $93\frac{1}{4}$ yards; how many yards did he sell?

Ans. $267\frac{19}{24}$ yards.

6. A farmer paid three laborers for a month's work as follows: to the first, $15\frac{1}{2}$ bushels of corn; to the second, $19\frac{1}{4}$ bushels; to the third $23\frac{7}{8}$ bushels. How much corn did he pay them all? *Ans.* $58\frac{5}{8}$ bushels.

7. A person on a journey traveled the first day 31 miles; the second and third each $29\frac{3}{7}$ miles; the fourth and fifth each $27\frac{5}{8}$ miles. How far did he go in the five days?

Ans. $145\frac{3}{28}$ miles.

8. A stage coach ran for two hours at the rate of $8\frac{7}{10}$ miles per hour, and for two hours more at the rate of $7\frac{1}{3}$ miles per hour; how far was that in the whole time?

Ans. $31\frac{28}{45}$ miles.

9. Bought of a grocer a sack of coffee, for $13\frac{5}{8}$ dollars; a barrel of sugar for $18\frac{2}{5}$ dollars, and a keg of rice for $5\frac{1}{16}$ dollars. What sum should be paid for the whole?

Ans. $37\frac{7}{80}$ dollars.

10. Sold to A, 25 barrels of apples, for $56\frac{1}{4}$ dollars; to B, $30\frac{1}{2}$ barrels, for 75 dollars; and to C, $10\frac{3}{4}$ barrels, for $21\frac{7}{8}$ dollars. Required the quantity sold, and the sum received.

Ans. $66\frac{1}{4}$ barrels; $153\frac{1}{8}$ dollars.

11. Laid out for goods, at one time, $\frac{4}{7}$ of a dollar; at another time, $3\frac{2}{5}$ dollars; at another, $21\frac{1}{3}$ dollars; and at another, $9\frac{3}{4}$ dollars. What was the whole sum disbursed?

Ans. $35\frac{83}{420}$ dollars.

12. Bought in market a pound of butter for $18\frac{3}{4}$ cents, a dozen eggs for $12\frac{1}{2}$ cents, a quarter of veal for $56\frac{1}{4}$ cents, and a quart of peas for $6\frac{1}{4}$ cents. What did the whole amount to?

Ans. $93\frac{3}{4}$ cents.

13. A merchant sold to one person, 4 yards of cloth for 24 dollars; to another, $9\frac{3}{8}$ yards for $43\frac{3}{16}$ dollars; and to another, $13\frac{1}{8}$ yards for $40\frac{1}{5}$ dollars. Required the quantity of cloth sold, and the sum received.

Ans. $26\frac{1}{2}$ yards; $107\frac{31}{80}$ dollars.

14. On a journey I traveled the first day $41\frac{1}{2}$ miles, the second $40\frac{3}{4}$ miles, the third and fourth each 45 miles, the fifth and sixth each $39\frac{1}{5}$ miles. What distance did I accomplish in the six days? *Ans.* $250\frac{13}{20}$ miles.

15. Bought of a farmer a quarter of beef for $8\frac{1}{2}$ dollars, a cord of wood for $2\frac{3}{4}$ dollars, a ton of hay for 13 dollars, a quantity of corn for $18\frac{1}{4}$ dollars, and a lot of bacon for $15\frac{7}{8}$ dollars. What did the whole amount to?

Ans. $58\frac{3}{8}$ dollars.

16. If I purchase from one person $5\frac{1}{4}$ tons of coal, from another $13\frac{2}{5}$ tons, from another $23\frac{1}{3}$ tons. and from two others each 17 tons; what will be the whole quantity purchased?

Ans. $75\frac{59}{60}$ tons.

17. An upholsterer sold a lot of chairs for $37\frac{1}{2}$ dollars, a set of window blinds for $18\frac{7}{8}$ dollars, a bureau for 35 dollars, and three mattresses for $16\frac{1}{4}$ dollars each. What did all these articles amount to? *Ans.* $140\frac{1}{8}$ dollars.

18. A farmer bought at one time $97\frac{1}{4}$ acres of land, for 1000 dollars; at another, $127\frac{2}{5}$ acres, for $1375\frac{1}{2}$ dollars; at another, $500\frac{3}{8}$ acres, for 6831 dollars; and at another $333\frac{1}{3}$ acres, for $4013\frac{3}{16}$ dollars. What was the whole quantity of land that he purchased? and the sum that he paid for it?

Ans. $1058\frac{43}{120}$ acres; $13219\frac{11}{16}$ dollars.

SUBTRACTION OF FRACTIONS.

(72.) The Difference between two Fractions is found by means of a *common denominator*.

For example, to find the Difference between $\frac{2}{5}$ and $\frac{4}{7}$.

These fractions are equal, respectively, to $\frac{14}{35}$ and $\frac{20}{35}$; and 14 *thirty fifths* taken from 20 *thirty fifths* leaves $\frac{6}{35}$; then

$$\frac{4}{7} - \frac{2}{5} = \frac{6}{35}.$$

What is the difference between $\frac{2}{3}$ and $\frac{3}{4}$? Between $\frac{2}{5}$ and $\frac{2}{3}$?
What is the difference between $\frac{5}{6}$ and $\frac{7}{10}$? Between $\frac{4}{5}$ and $\frac{8}{15}$?
What is the difference between $\frac{5}{7}$ and $\frac{11}{14}$? Between $\frac{9}{10}$ and $\frac{9}{20}$?

A proper fraction is subtracted from an *integer*, by first subtracting the fraction from a *unit*, and then subtracting a unit from the integer.

Thus to subtract $\frac{3}{5}$ from 7, we say, $\frac{3}{5}$ from 1 or $\frac{5}{5}$ leaves $\frac{2}{5}$, and 1 from 7 leaves 6 ; then $7 - \frac{3}{5} = 6\frac{2}{5}$.

RULE XV.

(73.) *For the Subtraction of Fractions.*

1. If the fractions have not a *common denominator*, reduce them to a common denominator.

2. Subtract the less numerator from the greater, and place the remainder, as a numerator, over their common denominator.

3. *A mixed number* may be taken in subtraction under the form of an *improper fraction* (64) ; or its *fractional* and *integral* parts may be taken separately.

EXAMPLES.

1. To subtract $25\frac{3}{4}$ from $439\frac{2}{3}$.

$$\begin{array}{rl} 439 & \frac{2}{3} = \frac{8}{12} \\ 25 & \frac{3}{4} = \frac{9}{12} \\ \hline 413\frac{11}{12} & \end{array}$$

Reducing the *fractional* parts of the given mixed numbers to a common denominator, we find them equal to

$\frac{8}{12}$ and $\frac{9}{12}$, respectively,

As the $\frac{9}{12}$, in the subtrahend, cannot be subtracted from the $\frac{8}{12}$, in the minuend, we add a *unit*, that is, 12 *twelfths*, and say 9 *twelfths* from 20 *twelfths* leaves $\frac{11}{12}$. We then add 1 to the 5, and say 6 from 9, &c.

2. To subtract $1843\frac{2}{7}$ from 2745

$$\begin{array}{r} 2745(\frac{7}{7}) \\ 1843\ \frac{2}{7} \\ \hline 901\ \frac{5}{7} \end{array}$$

Here we annex, mentally, $\frac{7}{7}$, equal to a *unit*, to the upper number, and say 2 *sevenths* from 7 *sevenths* leaves $\frac{5}{7}$. Then 1 to 3 makes 4; 4 from 5, &c.

In the preceding Examples, the subtractions might have been performed, after reducing the given numbers to *improper fractions*, (63), (64), and these fractions to a common denominator; but the methods which have been pursued are preferable.

EXERCISES.

1. From a barrel of wine which contained $31\frac{1}{2}$ gallons, $17\frac{1}{4}$ gallons were drawn. What quantity remained in the barrel? *Ans.* $14\frac{1}{4}$ gallons

2. If flour be purchased at $4\frac{1}{16}$ dollars a barrel, and sold at $5\frac{3}{8}$ dollars a barrel, what will be the gain per barrel?
Ans. $1\frac{5}{16}$ dollars.

3. A person who had to make a journey of $500\frac{1}{4}$ miles, has traveled $275\frac{3}{8}$ miles on his way. How far has he yet to go?
Ans. $224\frac{7}{8}$ miles.

4. A farmer having 1000 acres of land, sells to one of his neighbors $479\frac{7}{16}$ acres. How many acres will he have remaining? *Ans.* $520\frac{9}{16}$ acres.

5. A manufacturer who had on hand $700\frac{5}{8}$ yards of cloth, has sold 534 yards of it. What quantity remains on hand?
Ans. $166\frac{5}{8}$ yards.

6. A merchant bought a quantity of provisions for $38\frac{7}{8}$ dollars, and immediately sold the same for 48 dollars. What did he gain by the sale? *Ans.* $9\frac{1}{8}$ dollars.

7. Bought coal at different times to the amount of $376\frac{3}{5}$ bushels. Having consumed 183 bushels of the same, what quantity of the coal is still on hand? *Ans.* $193\frac{3}{5}$ bushels.

8. A bought of B 75 yards of cloth; of which he has sold $18\frac{3}{4}$ yards to C, and $20\frac{5}{8}$ yards to D; how much of the cloth remains unsold? *Ans.* $35\frac{5}{8}$ yards.

9. A gentleman having 3000 dollars to divide among his three sons, gives $753\frac{1}{2}$ dollars to the first, 1284 dollars to the second, and the remainder to the third. What sum does the third receive? *Ans.* $962\frac{1}{2}$ dollars.

10. If I should collect from A 200 dollars, from B and C each $175\frac{1}{2}$ dollars, and then pay to D $56\frac{7}{8}$ dollars; how many dollars would I have remaining? *Ans.* $494\frac{1}{8}$ dollars.

11. A merchant bought two pieces of cotton, each containing $34\frac{3}{8}$ yards, from which he has sold $18\frac{3}{4}$ yards. How many yards has he left? *Ans.* 50 yards.

12. Bought 40 cords of wood for $81\frac{3}{4}$ dollars. Having sold 20 cords of the same for $45\frac{1}{4}$ dollars, what would I gain by selling the remainder for $43\frac{3}{4}$ dollars? *Ans.* $7\frac{1}{4}$ dollars.

13. From a tract of land containing 1300 acres a farmer sold, at different times, to the amount of 934$\frac{2}{9}$ acres. What quantity then remained in the tract?

Ans. 365$\frac{7}{9}$ acres.

14. Out of the sum of 2345$\frac{7}{8}$ dollars, which a person had collected, he paid 350 dollars for house rent, and 187 dollars for other expenses; what sum had he remaining?

Ans. 1808$\frac{7}{8}$ dollars.

15. If two lots of ground, one of which was purchased for 1354$\frac{1}{2}$ dollars, and the other for 800 dollars, should both be sold for 2579$\frac{5}{8}$ dollars; what would be gained or lost by the sale?

Ans. 425$\frac{1}{8}$ dollars gained.

16. A manufacturer put into one bale 200 yards of cloth, and into another 187$\frac{3}{8}$ yards. How many yards must be put into a third bale, so that the three together shall contain 500 yards?

Ans. 112$\frac{5}{8}$ yards.

17. A farmer who had raised 1000$\frac{3}{4}$ bushels of wheat, sold 320$\frac{1}{2}$ bushels of his crop to A, 200$\frac{3}{8}$ bushels of it to B, and the remainder of it to C. How many bushels did C purchase?

Ans. 479$\frac{7}{8}$ bushels.

18. If a quantity of cloth be purchased for 321$\frac{1}{4}$ dollars, a quantity of silk for 137 dollars, and a quantity of linen for 93$\frac{7}{8}$ dollars; what will be the gain or loss if the whole be sold for 600 dollars?

Ans. Gain 47$\frac{7}{8}$ dollars.

19. Bought 350 acres of land for 4327$\frac{1}{4}$ dollars. Having sold 137$\frac{5}{8}$ acres for 1387$\frac{1}{8}$ dollars, I desire to know how many acres remain, and what will be the gain or loss on the whole if the remainder be sold for 2300 dollars.

Ans. 212$\frac{3}{8}$ acres; 640$\frac{1}{8}$ dollars loss.

MULTIPLICATION OF FRACTIONS.

(74.) The Product from multiplying by a Fraction is that *part of the multiplicand* which is denoted by the multiplier.

The product of 20 multiplied by $\frac{3}{4}$, for example, is 15, which is obtained by taking 3 *fourths* of 20, or 3 *times* $\frac{1}{4}$ of 20, (26).

The product, 15, is less than the multiplicand, 20; because the multiplier $\frac{3}{4}$ is less than *a unit.*

What is the product of 12 multiplied by $\frac{1}{2}$? Of 16 × $\frac{3}{4}$?
What is the product of 15 multiplied by $\frac{2}{3}$? Of 24 × $\frac{1}{6}$?
What is the product of 36 multiplied by $\frac{7}{9}$? Of 60 × $\frac{5}{12}$?

Compound Fractions

(75.) A *compound* fraction is a fraction of a fraction; and is therefore equal to the *product of the two fractions.*

Thus $\frac{2}{3}$ of $\frac{5}{6}$ is a compound fraction; and is equal to $\frac{5}{6} \times \frac{2}{3}$ (74).

RULE XVI.

(76.) *For the Multiplication of Fractions.*

1. Multiply the *numerators together* for a numerator, and the denominators together for a denominator.

2. An *integer* and *a fraction* are multiplied together, by *multiplying the numerator*, or dividing the denominator, by the integer.

3. A *mixed number* may be taken in multiplication under the form of an *improper fraction* (64); or its *fractional* and *integral* parts may be taken separately.

4*

EXAMPLES.

1. To multiply $\frac{5}{7} \times \frac{3}{4}$; that is, to find $\frac{3}{4}$ of $\frac{5}{7}$.

$$\frac{5}{7} \times \frac{3}{4} = \frac{15}{28}.$$

One 4th of 1 *seventh* of any quantity is evidently *one* 28*th* of the quantity ; then $\frac{1}{4}$ of 5 *sevenths* is *five* 28*ths ;* and $\frac{3}{4}$ of 5 *sevenths* is 3 times *five* 28ths, which is *fifteen* 28ths.

The product is therefore obtained by multiplying the numerators together, and the denominators together.

2. To multiply $\frac{5}{24}$ by 6 ; that is, to find 6 times *five* 24*ths.*

$$\frac{5}{24} \times 6 = \frac{30}{24} = 1\frac{6}{24} = 1\frac{1}{4}.$$

Or, *dividing the denominator* by the integer 6 we obtain

$\frac{5}{4} = 1\frac{1}{4}$, as before.

Dividing the denominator *multiplies the value of the fraction*, because the denominator is thus diminished.

3. To multiply $5\frac{2}{3}$ by $2\frac{1}{2}$; that is, to find *twice* $5\frac{2}{3}$, together with $\frac{1}{2}$ of $5\frac{2}{3}$.

$$\begin{array}{lr} 5\frac{2}{3} \times 2 = & 11\frac{1}{3} \\ \frac{1}{2} \text{ of } 5\frac{2}{3} = & 2\frac{5}{6} \\ \hline & 14\frac{1}{6} \end{array}$$

We first say, *twice* 2 thirds is 4 *thirds*, equal to $1\frac{1}{3}$; set down $\frac{1}{3}$; twice 5 is 10, and 1 makes 11.

Then, $\frac{1}{2}$ of 5 is 2, with 1 over ; this 1 is $\frac{3}{3}$, and added to the $\frac{2}{3}$, makes 5 *thirds ;* $\frac{1}{2}$ of $\frac{5}{3}$ is $\frac{5}{6}$ (75).

The partial products $11\frac{1}{3}$ and $2\frac{5}{6}$ are added together, for the *entire product.*

This last Example may also be performed by reducing the given numbers to *improper fractions* (64) ; then multiplying the numerators together, and the denominators, and reducing the product to *units* (61).

EXERCISES.

1. What should be paid for $\frac{3}{4}$ of a yard of linen, at the rate of $\frac{7}{8}$ of a dollar per yard?

If a yard of the linen costs $\frac{7}{8}$ of a dollar, $\frac{3}{4}$ of a yard will cost $\frac{3}{4}$ of $\frac{7}{8}$ of a dollar; or $\frac{7}{8}$ of a dollar $\times$ $\frac{3}{4}$ (75).

Ans. $\frac{21}{32}$ of a dollar.

2. What should be paid for $\frac{2}{3}$ of a barrel of apples, if the whole barrel be worth $\frac{15}{16}$ of a dollar? *Ans.* $\frac{5}{8}$ of a dollar.

3. A person owning $\frac{7}{12}$ of a tract of land, sells $\frac{3}{5}$ of his share to A; what part of the whole tract does A purchase?

Ans. $\frac{7}{20}$ of the whole.

4. What should be paid for $\frac{7}{8}$ of a pound of tea, at the rate of $\frac{4}{5}$ of a dollar per pound? *Ans.* $\frac{7}{10}$ of a dollar.

5. A merchant owning $\frac{1}{4}$ of a ship, sells $\frac{2}{7}$ of his share to A, and the rest of it to B. What part of the ship does B purchase? *Ans* $\frac{5}{28}$ of the ship.

6. Bought $\frac{3}{5}$ of an acre of ground, at the rate of 18 dollars per acre; required the sum to be paid for it.

Ans. $10\frac{4}{5}$ dollars.

7. Sold 25 bushels of clover seed, at $7\frac{1}{4}$ dollars per bushel; what did it amount to? *Ans.* $181\frac{1}{4}$ dollars.

8. In how many days ought one man to accomplish a work equivalent to what 12 men performed in $7\frac{1}{4}$ days?

Ans. 87 days.

9. What would be the profit on 75 barrels of flour, purchased at $3\frac{3}{4}$ dollars a barrel, and sold at $4\frac{1}{2}$ dollars a barrel?

Ans. $56\frac{1}{4}$ dollars.

10. What would be the value of 3 pieces of cloth, each containing $25\frac{1}{4}$ yards, at $6\frac{3}{4}$ dollars per yard?

Ans. $511\frac{5}{16}$ dollars.

11. In how many days ought one man to accomplish an undertaking which 17 men could perform in $13\frac{3}{8}$ days?

Ans. $227\frac{3}{8}$ days.

12. Find the distance a person would travel in $6\frac{1}{2}$ days, allowing him to proceed for 3 days, at the rate of $30\frac{1}{3}$ miles per day, and the rest of the time at the rate of 27 miles per day. *Ans.* $185\frac{1}{2}$ miles.

13. Find the entire cost of $\frac{3}{4}$ of a pound of pepper at $\frac{1}{4}$ of a dollar a pound, $\frac{3}{4}$ of a hundredweight of flour at $2\frac{1}{2}$ dollars a hundred, and $2\frac{1}{8}$ yards of cloth at 7 dollars a yard? *Ans.* $16\frac{15}{16}$ dollars.

14. What would be the profit or loss on $35\frac{7}{8}$ yards of silk, purchased at $\frac{3}{4}$ of a dollar per yard, if $16\frac{1}{4}$ yards of it be sold at $1\frac{1}{4}$ dollars a yard, and the remainder at $\frac{5}{8}$ of a dollar a yard? *Ans.* Profit $5\frac{43}{64}$ dollars.

15. What should be paid for $\frac{5}{8}$ of a yard of cloth, at the rate of 8 dollars per yard?

A fraction is multiplied by its *own denominator* by merely *canceling the denominator.* In this example, the product of $8 \times \frac{5}{8}$ is at once known to be 5. *Ans.* 5 dollars.

16. What should be paid for $\frac{3}{4}$ of $\frac{4}{5}$ of a yard of muslin, at the rate of $\frac{5}{16}$ of a dollar per yard?

The value to be found is evidently $\frac{3}{4}$ *of* $\frac{4}{5}$ *of* $\frac{5}{16}$ *of a dollar.*

$$=\frac{3 \times 4 \times 5}{4 \times 5 \times 16} \ (75) \ =\frac{3 \times \not{4} \times \not{5}}{\not{4} \times \not{5} \times 16} \ (45) \ =\frac{3}{16} \text{ of a dollar.}$$

In multiplying two or more fractions together, *equal factors* may thus always be *canceled in the resulting numerator and denominator;* for this divides the numerator and deno- inator by the same number (65).

17. What should be paid for $\frac{1}{2}$ of $\frac{2}{3}$ of a pound of tea, at the rate of $\frac{7}{8}$ of a dollar per pound? *Ans.* $\frac{7}{24}$ of a dollar.

18. What should be paid for $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a yard of silk, at the rate of $\frac{15}{16}$ of a dollar per yard? *Ans.* $\frac{3}{8}$ of a dollar

DIVISION OF FRACTIONS.

(77.) The Quotient from dividing by a Fraction, as well as by an integer, is the number of times that the *dividend contains the divisor*, or the *part that the dividend is* of the divisor.

Thus $\frac{6}{7}$ divided by $\frac{2}{7}$ gives the *quotient* 3, because 3 times $\frac{2}{7}$ is $\frac{6}{7}$; and $\frac{2}{7}$ divided by $\frac{5}{7}$ gives the *quotient* $\frac{2}{5}$, because $\frac{2}{7}$ is 2 *fifths* of $\frac{5}{7}$.

What is the quotient of $\frac{8}{9}$ divided by $\frac{2}{9}$? Of $\frac{1}{3}$ divided by $\frac{2}{3}$?
What is the quotient of $\frac{12}{13}$ divided by $\frac{4}{13}$? Of $\frac{3}{10}$ divided by $\frac{7}{10}$?
What is the quotient of $\frac{30}{48}$ divided by $\frac{6}{48}$? Of $\frac{9}{20}$ divided by $\frac{18}{20}$?

The *reciprocal* of a Fraction is the fraction *inverted;* and is equal to *a unit* divided by the fraction.

Thus the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; equal to 1 or $\frac{3}{3}$ divided by $\frac{2}{3}$.

What is the reciprocal of $\frac{4}{5}$? Of $\frac{3}{7}$? Of $\frac{5}{9}$?

The reciprocal of a *mixed number* is that of its equivalent *improper fraction;* thus the reciprocal of $5\frac{2}{3}$, or $\frac{17}{3}$, is $\frac{3}{17}$.

What is the reciprocal of $2\frac{1}{2}$? Of $7\frac{3}{4}$? Of $10\frac{1}{4}$?

Mixed Fractions.

(78.) A *mixed fraction* is one which contains a fraction in *one or both of its terms.* It is reduced to a *simple fraction* by dividing its numerator by its denominator.

Thus $\frac{2}{3\frac{1}{2}}$, numerator 2, denominator $3\frac{1}{2}$, is a mixed fraction.

By reducing the numerator to *halves* (63), and the denominator to *halves*, and dividing, we have

$$2 \div 3\tfrac{1}{2} = \tfrac{4}{2} \div \tfrac{7}{2} = \tfrac{4}{7},$$ which is a *simple fraction.*

RULE XVII.

(79.) *For the Division of Fractions.*

1. Divide the numerator of the dividend by the numerator of the divisor, and the denominator by the denominator; or *multiply the dividend by the reciprocal of the divisor.*

2. A *fraction* is divided by *an integer*, by *dividing the numerator*, or multiplying the denominator, by the integer.

3. A *mixed number* may be taken in division under the form of an *improper fraction* (64); or its *integral* and *fractional* parts may be divided separately.

EXAMPLES.

1. To divide $\frac{9}{35}$ by $\frac{2}{3}$.

As we cannot divide numerator by numerator, and denominator by denominator, without remainders; we multiply the dividend by the *reciprocal of the divisor;* thus

$$\tfrac{9}{35} \div \tfrac{2}{3} = \tfrac{9}{35} \times \tfrac{3}{2} = \tfrac{27}{70}.$$

The correctness of this method may be shown thus: the dividend is equal to

$$\frac{9 \times 2 \times 3}{35 \times 2 \times 3} \quad (65).$$

Dividing the numerator of this dividend by the numerator of the divisor, and the denominator by the denominator, we find the *quotient* $\frac{9 \times 3}{35 \times 2}$ or $\frac{9}{35} \times \frac{3}{2}$.

2. To divide $163\frac{2}{3}$ by 5.

$$\begin{array}{r} 5)\underline{163\frac{2}{3}} \\ 32\frac{11}{15}. \end{array}$$

We say, 5 in 16, 3 times and 1 over; 5 in 13, *twice* and 3 over; this 3 and the $\frac{2}{3}$ make $3\frac{2}{3}$, equal to $\frac{11}{3}$; then $\frac{1}{5}$ of $\frac{11}{3}$ is $\frac{11}{15}$ (75).

The division in this case might also be performed, by reducing the dividend to an *improper fraction* (64), and multiplying this fraction by the *reciprocal* $\frac{1}{5}$.

From principles which have been established it follows, that, in all cases,

(80.) Dividing a given number is equivalent to multiplying it by the *reciprocal of the divisor*

EXERCISES.

1. How many yards of calico, at $\frac{1}{4}$ of a dollar per yard, may be purchased for $\frac{7}{8}$ of a dollar?

The number of yards is the number of times that $\frac{1}{4}$ *is contained in* $\frac{7}{8}$. *Ans.* $3\frac{1}{2}$ yards.

2. How many weeks would a family be in consuming $19\frac{4}{5}$ barrels of flour, at the rate of $\frac{3}{4}$ of a barrel per week? *Ans.* $26\frac{2}{5}$ weeks.

3. How many yards of silk could be purchased for $12\frac{1}{2}$ dollars, at the rate of $\frac{7}{8}$ of a dollar per yard? *Ans.* $14\frac{2}{7}$ yards.

4. How many hours would a person be in walking $175\frac{4}{5}$ miles, at the rate of 3 miles per hour? *Ans.* $58\frac{3}{5}$ hours.

5. A merchant laid out for broadcloth 5727 dollars, paying $5\frac{3}{4}$ dollars per yard. How many yards did he purchase? *Ans.* 996 yards.

6. A farmer purchased a farm for $4379\frac{1}{16}$ dollars, paying $16\frac{7}{8}$ dollars per acre. How many acres did he purchase? *Ans.* $259\frac{1}{2}$ acres.

7. How many barrels of wine are there in 2753 gallons, allowing $31\frac{1}{2}$ gallons to make one barrel? *Ans.* $87\frac{25}{63}$ barrels.

8. What quantity of salt may be purchased for $\frac{1}{4}$ of a dollar, at $\frac{5}{8}$ of a dollar per bushel?

$\frac{1}{4}$ of a dollar will buy the *same part* of a bushel that $\frac{1}{4}$ of a dollar is of $\frac{5}{8}$ of a dollar, which will be found by dividing $\frac{1}{4}$ by $\frac{5}{8}$ (77). *Ans.* $\frac{2}{5}$ of a bushel.

9. What quantity of iron may be purchased for $22\frac{1}{2}$ dollars, at the rate of 45 dollars per ton? *Ans.* $\frac{1}{2}$ of a ton.

10. What quantity of land may be purchased for $15\frac{3}{4}$ dollars, at the rate of 30 dollars per acre? *Ans.* $\frac{21}{40}$ of an acre.

11. If a person could accomplish a certain work in $25\frac{1}{2}$ days, what part of it could he perform in $3\frac{1}{2}$ days? *Ans.* $\frac{7}{51}$ of the work.

12. A laborer agreed to work 30 days for a certain sum of money; having worked but $17\frac{1}{2}$ days, what part of the stipulated sum ought he to receive? *Ans.* $\frac{7}{12}$ of it.

13. A mason having undertaken to build a wall of specified dimensions in $62\frac{1}{2}$ days, what part of the wall ought he to accomplish in $6\frac{1}{4}$ days? *Ans.* $\frac{1}{10}$ of it.

14. What is the price of cloth per yard, when $\frac{2}{3}$ of a yard costs 5 dollars?

If 2 *thirds* of a yard costs 5 dollars, 1 *third* of a yard costs $\frac{1}{2}$ of 5 dollars, and 3 *thirds* of a yard, which is a whole yard, costs $\frac{3}{2}$ of 5 dollars, which is 5 dollars $\times$ $\frac{3}{2}$ (74), or 5 dollars $\div$ $\frac{2}{3}$ (80). *Ans.* $7\frac{1}{2}$ dollars.

15. What is the price of hay per ton, when $\frac{3}{4}$ of a ton costs 10 dollars? *Ans.* $13\frac{1}{3}$ dollars.

16. What should be paid for an acre of ground, when $\frac{5}{8}$ of an acre is sold for $21\frac{1}{4}$ dollars? *Ans.* 34 dollars.

17. What should be paid for a ton of coal, when $\frac{4}{5}$ of a ton is purchased for $18\frac{3}{4}$ dollars? *Ans.* $23\frac{7}{16}$ dollars.

18. If $\frac{7}{10}$ of an acre of ground produce $19\frac{1}{2}$ bushels of wheat, what is the produce per acre? *Ans.* $27\frac{6}{7}$ bushels.

19. A railroad car ran 37 miles in $\frac{7}{12}$ of an hour; at what rate was that per hour? *Ans.* $63\frac{3}{7}$ miles.

20. Allowing a person to walk $3\frac{1}{4}$ miles in $\frac{5}{6}$ of an hour, at what rate does he walk per hour? *Ans.* $3\frac{9}{10}$ miles.

21. A company of laborers have finished $\frac{7}{10}$ of a certain work in $43\frac{1}{2}$ days. In what time will the whole work be accomplished at that rate? *Ans.* $62\frac{1}{7}$ days.

22. What is the price of flour per barrel, when $5\frac{1}{2}$ barrels cost $49\frac{1}{2}$ dollars?

The *price per barrel* will be found by dividing $49\frac{1}{2}$ dollars by $5\frac{1}{2}$, because *that price* $\times$ $5\frac{1}{2}$ must produce $49\frac{1}{2}$ dollars; the product $49\frac{1}{2}$ and one of its factors being given, to find the other factor (37). *Ans.* 9 dollars.

23. What is the price of silk per yard, when $3\frac{1}{2}$ yards are purchased for $4\frac{3}{8}$ dollars? *Ans.* $1\frac{1}{4}$ dollars.

24. If a man travels $27\frac{3}{5}$ miles in $\frac{3}{4}$ of a day, at what rate does he travel per day? *Ans.* $36\frac{4}{5}$ miles.

25. What is the price of land per acre, when 13 acres are disposed of for $71\frac{1}{2}$ dollars? *Ans.* $5\frac{1}{2}$ dollars.

26. What is the price of coal per ton, when $\frac{3}{5}$ of a ton costs $9\frac{3}{4}$ dollars? and what would $10\frac{1}{4}$ tons amount to?

Ans. $16\frac{1}{4}$ dollars; and $166\frac{9}{16}$ dollars.

27. If 25 cords of wood sell for $68\frac{3}{4}$ dollars, what is the price per cord? and what should be paid for $\frac{3}{4}$ of a cord?

Ans. $2\frac{3}{4}$ dollars; and $2\frac{1}{16}$ dollars.

28. If a man walk $62\frac{1}{3}$ miles in $18\frac{1}{2}$ hours, at what rate will he walk per hour? and how many miles would he go in 20 hours at the same rate?

Ans. $3\frac{41}{111}$ miles; and $67\frac{43}{111}$ miles.

29. If a railroad car run 230 miles in $10\frac{1}{4}$ hours, what will be its rate per hour? and how far would it run in 23 hours at the same rate?

Ans. $22\frac{18}{41}$ miles; and $516\frac{4}{41}$ miles.

MISCELLANEOUS EXERCISES

ON FRACTIONS.—ABBREVIATED MULTIPLICATION AND DIVISION.

1. Find the Sum, in *units*, of $\frac{13}{4}$, $\frac{23}{4}$, and $\frac{37}{4}$.
Ans. $18\frac{1}{4}$.

2. Find the Sum, in units, of $\frac{17}{6}$, $\frac{35}{6}$, and $\frac{53}{6}$.
Ans. $17\frac{3}{6}$.

3. Find the Sum, in units, of $\frac{31}{10}$, $\frac{84}{10}$, and $\frac{73}{10}$.
Ans. $18\frac{8}{10}$.

4. Find the Difference, in *fifths*, between 75 and $31\frac{2}{5}$.
Ans. $\frac{218}{5}$.

5. Find the Difference, in sevenths, between 83 and $54\frac{3}{7}$.
Ans. $\frac{200}{7}$.

6. Find the Difference, in tenths, between 130 and $95\frac{7}{10}$.
Ans. $\frac{343}{10}$.

7. Reduce $5\frac{2}{4}$, $9\frac{3}{6}$, and $10\frac{8}{12}$ to *improper* fractions in their *lowest terms*. *Ans.* $\frac{11}{2}$, $\frac{19}{2}$, and $\frac{32}{3}$.

8. Reduce $9\frac{5}{10}$, $10\frac{4}{8}$, and $13\frac{3}{12}$ to improper fractions in their lowest terms. *Ans.* $\frac{19}{2}$, $\frac{21}{2}$, and $\frac{53}{4}$.

9. Reduce $7\frac{2}{8}$, $12\frac{2}{10}$, and $20\frac{3}{15}$ to improper fractions in their lowest terms. *Ans.* $\frac{29}{4}$, $\frac{61}{5}$, and $\frac{101}{5}$.

10. Reduce $6\frac{1}{2}$, $10\frac{3}{4}$, and $12\frac{2}{3}$ to improper fractions having the *least common denominator*. *Ans.* $\frac{78}{12}$, $\frac{129}{12}$, and $\frac{152}{12}$.

11. Reduce $8\frac{3}{4}$, $12\frac{5}{8}$, and $20\frac{3}{16}$ to improper fractions having the least common denominator. *Ans.* $\frac{140}{16}$, $\frac{202}{16}$, and $\frac{323}{16}$.

12. Reduce $9\frac{2}{5}$, $10\frac{7}{10}$, and $35\frac{1}{15}$ to improper fractions having the least common denominator.
Ans. $\frac{282}{30}$, $\frac{321}{30}$, and $\frac{1052}{30}$.

13. One person expends 5 dollars for coal, at 7 dollars per ton; and another, 6 dollars, at 9 dollars per ton. Which of them obtains the greater quantity of coal?
Ans. The first, by $\frac{1}{21}$ of a ton.

14. A paid 20 dollars for iron, at 37 dollars per ton; B 25 dollars, at 42 dollars per ton; and C 30 dollars, at 50 dollars per ton. Which of them bought the largest? and which the smallest quantity?

Ans. C the largest, and A the smallest quantity.

15. Bought at one time $147\frac{2}{3}$ bushels of coal, and at another time $320\frac{1}{5}$ bushels. Having consumed $156\frac{1}{4}$ bushels, I desire to know what quantity of the coal purchased is still on hand. *Ans.* $311\frac{37}{60}$ bushels.

16. From the sum of 1500 dollars which I deposited in bank, having drawn, at different times, 200 dollars, $137\frac{3}{4}$ dollars, $313\frac{1}{2}$ dollars, and $79\frac{3}{8}$ dollars; what sum have I yet in bank? *Ans.* $769\frac{3}{8}$ dollars.

17. Bought a quantity of iron for 95 dollars, and of coal for $81\frac{7}{8}$ dollars. The iron was sold for $115\frac{3}{4}$ dollars, and the coal for 100 dollars; what profit was made on both commodities? *Ans.* $38\frac{7}{8}$ dollars.

18. A merchant bought one piece of cloth containing $53\frac{1}{2}$ yards, another containing $39\frac{3}{8}$ yards, and another containing 40 yards. Having sold 13 yards, from the first piece, $24\frac{3}{4}$ from the second, and $19\frac{1}{8}$ from the third; he wishes to know the whole number of yards he has remaining.

Ans. 76 yards.

19. A speculator bought 1000 acres of land for $1587\frac{3}{5}$ dollars, and 500 acres for $737\frac{7}{8}$ dollars. Having sold $945\frac{1}{2}$ acres for 2000 dollars, he wishes to know how much land he has remaining, and for what sum he shall sell the remainder, so as to lose nothing on the whole.

Ans. $554\frac{1}{2}$ acres; $325\frac{19}{40}$ dollars.

20. Going out to collect money, I received from A $37\frac{1}{2}$ dollars, from B 20 dollars more than from A, from C $5\frac{3}{4}$ dollars more than from B, and from D as much as from the other three together. What was the whole sum collected?

Ans. $316\frac{1}{2}$ dollars.

21. A contributed towards a charitable purpose $23\frac{1}{2}$ dollars, B contributed twice as much as A, C as much as B, D as much as A and B together, and E as much as all the rest. What was the whole contribution? *Ans.* 376 dollars.

22. Find the value of the expression

$$\left(5\tfrac{1}{2} + \tfrac{2}{5} + \tfrac{3}{4} - \tfrac{1}{2} \text{ of } \tfrac{3}{4}\right) \times 4. \qquad \textit{Ans. } 25\tfrac{1}{10}$$

23. Find the value of the expression

$$\left(10\tfrac{2}{5} - 5\tfrac{3}{4} + 2\tfrac{1}{3} - \tfrac{2}{3} \text{ of } \tfrac{5}{6}\right) \times \tfrac{4}{5}. \qquad \textit{Ans } 5\tfrac{32}{225}$$

24. Find the value of the expression

$$\left(19\tfrac{4}{5} + \tfrac{7}{10} - 15 + \tfrac{1}{3} \text{ of } 2\tfrac{1}{4}\right) \times \frac{3}{5\frac{1}{2}}. \qquad \textit{Ans. } 3\tfrac{9}{22}.$$

25. Find the value of the expression

$$\left(254 + \tfrac{2}{7} + \tfrac{3}{8} - \tfrac{4}{5} \text{ of } 15\right) \div \frac{2\frac{1}{2}}{5}. \qquad \textit{Ans. } 485\tfrac{9}{28}.$$

26. Find the value of the expression

$$\left(\frac{5}{3\frac{1}{2}} + \tfrac{1}{2} \text{ of } \tfrac{2}{3} \text{ of } \tfrac{2}{5} \text{ of } 20\right) \div \frac{3\frac{1}{4}}{6\frac{1}{2}}. \qquad \textit{Ans. } 8\tfrac{4}{21}.$$

27. What will $16\frac{2}{3}$ acres of land amount to at 125 dollars per acre?

$$125 \text{ dollars} \times 16\tfrac{2}{3}.$$

When the Multiplier is *one half*, or *one third*, or *one fourth*, &c., of any number of *tens*, *hundreds*, or *thousands*, &c.; the operation may be *abbreviated* by taking those tens, hundreds, or thousands for the multiplier, and then taking the *same part of the product* thus obtained that the given multiplier is of the assumed one.

Thus, in the present example, the multiplier

$$16\frac{2}{3} \text{ is } \frac{1}{6} \text{ of } 100, \text{ since } 6 \text{ times } 16\frac{2}{3} \text{ is } 100;$$

the required product is therefore

$$125 \times \frac{100}{6} = \frac{12500}{6} = 2083\frac{1}{3}.$$

Ans. $2083\frac{1}{3}$ dollars.

28. What should be paid for $2\frac{1}{2}$ tons of coal at 13 dollars a ton, $12\frac{1}{2}$ tons of hay at 27 dollars a ton, and $33\frac{1}{3}$ cords of wood at 7 dollars a cord? *Ans.* $603\frac{1}{3}$ dollars.

29. What should be paid for 125 sheep at $3\frac{1}{3}$ dollars a head, 34 cows at $16\frac{2}{3}$ dollars a head, and 19 mules at $133\frac{1}{3}$ dollars a head? *Ans.* $3516\frac{2}{3}$ dollars.

30. Having on hand $19\frac{4}{5}$ tons of iron, if I sell $10\frac{1}{4}$ tons of it at 45 dollars a ton, and the remainder at 43 dollars a ton, what will the whole amount to? *Ans.* $871\frac{9}{10}$ dollars.

31. Paid 70 dollars for wood, at 2 dollars per cord; and afterwards sold $\frac{1}{4}$ of the quantity, at $3\frac{1}{4}$ dollars per cord. What did the wood sold amount to? *Ans.* $28\frac{7}{16}$ dollars.

32. If 3 masons can build a wall in $13\frac{1}{2}$ days, in what time ought one mason to build another wall of the same height and thickness, but $2\frac{1}{4}$ times as long?

Ans. $91\frac{1}{8}$ days.

33. A speculator bought 189 acres of land at 10 dollars an acre, and $250\frac{1}{4}$ acres at 13 dollars an acre. He sold $\frac{2}{3}$ of the first tract at $18\frac{1}{2}$ dollars an acre, and $\frac{2}{5}$ of the second at 19 dollars an acre; what would he gain on the whole by selling the remainder of both tracts at 20 dollars an acre?

Ans. $3352\frac{13}{20}$ dollars.

34. How many tons of hay, at $16\frac{2}{3}$ dollars per ton, may be purchased for 246 dollars?

$$246 \text{ dollars} \div 16\frac{2}{3} \text{ dollars}.$$

When the Divisor is *one half*, or *one third*, or *one fourth*, &c., of any number of *tens*, *hundreds*, or *thousands*, &c.; the operation may be *abbreviated* by taking such equivalent improper fraction for the divisor.

Thus, in the present example, the divisor $16\frac{2}{3}$ being equal to $\frac{100}{6}$, we have

$$246 \div \frac{100}{6} = 246 \times \frac{6}{100} = \frac{1476}{100} = 14\frac{19}{25}.$$

Ans. $14\frac{19}{25}$ tons.

35. How many barrels of pork, at $12\frac{1}{2}$ dollars a barrel, may be purchased for 347 dollars? *Ans.* $27\frac{19}{25}$ barrels.

36. How many acres of ground, at 25 dollars an acre, may be bought for 1340 dollars? and how many at 75 dollars an acre may be bought for 3480 dollars?

The divisors are equal to $\frac{100}{4}$ and $\frac{300}{4}$, respectively.

Ans. $53\frac{3}{5}$ acres; and $46\frac{2}{5}$ acres.

37. A sold to B 30 acres of land for $622\frac{1}{2}$ dollars; and B sold to C $12\frac{3}{4}$ acres of the same land, at the same price per acre. What did C pay for the land he bought?

Ans. $264\frac{9}{16}$ dollars.

38. In how many days ought 14 men to accomplish a piece of work which 5 men could do in $33\frac{1}{2}$ days?

Ans. $11\frac{27}{28}$ days.

39. Paid out $59\frac{15}{16}$ dollars for silk at $\frac{7}{8}$ of a dollar per yard, and sold $\frac{1}{2}$ of the quantity purchased, at a profit of $\frac{3}{8}$ of a dollar per yard. What did the part sold amount to?

Ans. $42\frac{13}{16}$ dollars.

40. If $3\frac{1}{4}$ hundredweight of hemp sell for $16\frac{1}{4}$ dollars, what will $10\frac{2}{3}$ hundredweight bring at the same rate?

Ans. $53\frac{1}{3}$ dollars.

41. A bought of B $13\frac{1}{2}$ tons of hay, at 9 dollars per ton, and of C $16\frac{2}{3}$ tons at $10\frac{1}{2}$ dollars per ton. He then sold to

D 9 tons at 12 dollars, and the rest of his purchase to E at 13 dollars per ton; what did he gain on the hay?

Ans. $83\frac{1}{2}$ dollars.

42. If 3 men can plough $15\frac{1}{2}$ acres of ground in 4 days, how many acres ought 5 men to plough in $7\frac{2}{5}$ days?

Ans. $47\frac{19}{24}$ acres.

43. If a railroad car can run 35 miles in $3\frac{1}{2}$ hours, and 41 miles in 4 hours, and $62\frac{1}{2}$ miles in 5 hours; what will be its average rate per hour? *Ans.* $11\frac{2}{25}$ miles.

44. A bought of B $783\frac{1}{2}$ bushels of wheat for $\$587\frac{5}{8}$ dollars, and sold 500 bushels of it to C for $562\frac{1}{2}$ dollars. How much did he gain or lose per bushel on what he sold?

Ans. Gained $\frac{3}{8}$ of a dollar.

45. Bought 175 cords of wood for $437\frac{1}{2}$ dollars, and sold 93 cords of it at a profit of half a dollar per cord. At what rate must the remainder be sold, to gain $97\frac{3}{4}$ dollars on the whole? *Ans.* $3\frac{1}{8}$ dollars per cord.

46. A person who has a journey of 570 miles to perform, proceeds for 9 days at the rate of $33\frac{1}{3}$ miles per day. How much must his daily rate be increased or diminished, to complete the journey in 9 more days?

Ans. Diminished $3\frac{1}{3}$ miles.

47. A person bought 19 barrels of apples, at $2\frac{1}{4}$ dollars per barrel. Having sold $12\frac{1}{2}$ barrels of them at $2\frac{1}{2}$ dollars a barrel, at what price per barrel must he sell the remainder, to gain $5\frac{3}{4}$ dollars on the whole? *Ans.* $2\frac{17}{26}$ dollars.

48. A purchased of B 40 yards of cloth for 260 dollars. He then sold to C $\frac{3}{5}$ of his purchase at a profit of $\frac{3}{8}$ of a dollar per yard, and the remainder to D, at a loss of $\frac{1}{8}$ of a dollar per yard; what did A gain or lose by these several transactions? *Ans.* Gained 7 dollars.

CHAPTER V.

DECIMAL FRACTIONS.—DECIMAL OR FEDERAL MONEY.

DECIMAL FRACTIONS.

(81.) A Decimal Fraction is a number of *tenths, hundredths*, or *thousandths*, &c., denoted by one or more figures on the *right of units*, and after a point (.) which distinguishes them from *integers* (62).

Thus .3, three *tenths;* .35, thirty-five *hundredths.*

The 1st figure after the decimal point denotes *tenths*, the 2d *hundredths*, the 3d *thousandths*, and so on; but they may all together be expressed in the denomination of the *right hand figure.*

Thus .35 denotes 3 *tenths* and 5 *hundredths*, or 35 *hundredths.*

What does .1, decimal point, 1, denote? What does .12 denote? What does .234 denote? .3546? .05? .006? .067? .0009?

The simple term *decimal* is often used to designate a decimal Fraction.

A *Vulgar Fraction* is one which is denoted by a numerator and denominator; as $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{23}{100}$.

Scale of Decimals.

(82.) In Decimals, as in integers, *ten* of any lower order make *one* of the next higher order; or *one* of a higher order makes *ten* of the next lower order.

Thus 10 *thousandths* make 1 *hundredth;* 10 *hundredths* make 1 *tenth.*

2 *units* are how many *tenths?* 3 *tenths* are how many *hundredths?* 5 hundredths are how many *thousandths?* 40 thousandths are how many hundredths?

From the preceding it follows, that

(83.) Each 0 between the (.) and the first significant decimal figure, *diminishes* the decimal to *one tenth* of its value without the 0

Thus .03 is *one tenth* of .3.

How may the figure 1 be made to denote 1 *tenth?* How may it be made to denote 1 *hundredth?* 1 thousandth?

How may the figure 5 be made to denote 5 *hundredths?* 5 thousandths?

0's annexed to decimals do not alter the values of the decimals; thus

$$.1 = .10 = .100 = .1000, \text{ \&c.}$$

Mixed Decimals.

(84.) A *mixed* Decimal is a decimal fraction with a vulgar fraction annexed to denote a part of 1 *tenth*, or 1 *hundredth*, &c.

.5½ denotes 5½ *tenths*, that is, 5 *tenths* and ½ of 1 *tenth*.

.25¾ denotes 25¾ *hundredths*, or 25 *hundredths* and ¾ of 1 *hundredth*.

What does .3⅓ denote? .04⅕? .123½? .0005¼?

Notation of Decimals.

RULE XVIII.

(85.) *To denote, decimally, a Number of tenths, hundredths, or thousandths, &c.*

Prefix the (.) to the number, with 0's interposed, if necessary, to put the last figure in the *given denomination*, when the successive figures are called *tenths, hundredths, thousandths*, ten-thousandths, &c., from the (.) towards the right.

EXAMPLES.

1. To denote, decimally, 54 *ten-thousandths.*

.0054.

The (.) and 00 must be prefixed to 54, to put the 4 in the donomination of *ten-thousandths*, when the successive figures are called *tenths*, *hundredths*, &c., from the (.) towards the right.

2. To denote, decimally, 125 and 7 *hundredths.*

125.07.

The (.) and 0 prefixed to 7 make the 7 denote 7 *hundredths*, to be placed on the right of the integral number 125.

EXERCISES.

Write in *decimal figures* each of the following Fractions.

1. Fifteen *hundredths.*
2. Nineteen *thousandths.*
3. Six *ten-thousandths.*
4. Twenty-four *thousandths.*
5. Five *Hund. thousandths.*
6. Thirty-nine *millionths.*
7. One hundred *thousandths.*
8. Ten *ten-millionths.*
9. Forty-nine *hundredths.*
10. Seventeen *ten-thousandths*
11. Fifty-two *thousandths.*
12. Eight *hund. thousandths.*

Write in integers and *decimals* each of the following Mixed Numbers—observing that, in the verbal expression, the comma (,) separates the fraction from the integer.

13. Four thousand and nine, and five *thousandths.*
14. Fifty-four thousand, three hundred and two *thousandths.*
15. Six hundred and twenty, and twelve *hundredths.*
16. Nine hundred and one, and five hundred and one *millionths.*

17. One million, and four thousand three hundred and ten *hundred-thousandths*.

18. Twenty thousand and seventeen, and nineteen *ten-thousandths*.

19. Forty-seven thousand, and two hundred and twenty-one *thousandths*.

20. Five millions two hundred and one thousand, and three *tenths*.

21. Seven hundred millions, and three hundred and nine *thousandths*.

The principles of the preceding Rule will also enable the pupil to *read any decimal* fraction.—To prevent ambiguity in the enunciation of Mixed Numbers it will sometimes be expedient to insert the word *decimal* before the fraction ; thus

300.005, three hundred and *decimal* 5 *thousandths*.
500.0002, five hundred and *decimal* 2 *ten-thousandths*.

RULE XIX.

(86.) *To reduce a Decimal to a Vulgar Fraction.*

1. Remove the (.), and under the given number of *tenths*, or *hundredths*, or *thousandths*, &c., set the proper denominator 10, or 100, or 1000, &c.

2. The Vulgar Fraction thus formed may often be reduced to *lower terms*.

EXAMPLE.

To reduce .125 to a vulgar fraction.

$$.125 = \tfrac{125}{1000} = \tfrac{1}{8}.$$

The given decimal being 125 *thousandths*, we take 1000 for a *denominator*, and reduce the $\tfrac{125}{1000}$ to its lowest terms $\tfrac{1}{8}$ (67).

EXERCISES.

1. Reduce .5 to a *vulgar fraction* in its lowest terms. *Ans.* $\frac{1}{2}$.
2. Reduce .25 to a vulgar fraction in its lowest terms. *Ans.* $\frac{1}{4}$.
3. Reduce .75 to a vulgar fraction in its lowest terms. *Ans.* $\frac{3}{4}$.
4. Reduce .375 to a vulgar fraction in its lowest terms. *Ans.* $\frac{3}{8}$.
5. Reduce .625 to a vulgar fraction in its lowest terms. *Ans.* $\frac{5}{8}$.
6. Reduce .875 to a vulgar fraction in its lowest terms. *Ans.* $\frac{7}{8}$.
7. Reduce .1875 to a vulgar fraction in its lowest terms. *Ans.* $\frac{3}{16}$.
8. Reduce .0625 to a vulgar fraction in its lowest terms. *Ans.* $\frac{1}{16}$.
9. Reduce .5625 to a vulgar fraction in its lowest terms. *Ans.* $\frac{9}{16}$.
10. Reduce .9375 to a vulgar fraction in its lowest terms. *Ans.* $\frac{15}{16}$.
11. Reduce .0075 to a vulgar fraction in its lowest terms. *Ans.* $\frac{3}{400}$.

RULE XX.

(87.) *To reduce a Vulgar Fraction to a Decimal.*

1. Divide the denominator into the numerator with as many 0's *annexed to the latter* as may be necessary to find an accurate quotient.

2. *Point off in the right of the quotient* as many decimal figures as there were 0's annexed to the numerator; *prefixing 0's to the quotient* when necessary to make up the number.

EXAMPLE.

To reduce $\frac{3}{125}$ to a decimal fraction.

125)3000(24 ; then $\frac{3}{125} = .024$.

Annexing three 0's to the numerator, and dividing 3000 by the denominator, we find the quotient 24, to which one 0 must be prefixed, to make up *three decimal figures*, for the 000 annexed to the numerator.

The fraction is equal to its numerator 3 divided by its denominator 125 (59); each 0 annexed to the numerator *multiplies* the fraction by 10 (76 . . . 2); but the making of a decimal figure in the quotient *divides* the quotient by 10, since each quotient figure is thus made *one-tenth* of its former value; the foregoing operation has therefore the effect of multiplying and dividing the Fraction by the same number, which *does not alter its value.*

EXERCISES.

1. Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to decimals.
   Ans. .5 and .25.
2. Reduce $\frac{3}{4}$ and $\frac{3}{8}$ to decimals.
   Ans. .75 and .375.
3. Reduce $\frac{5}{8}$ and $\frac{7}{8}$ to decimals.
   Ans. .625 and .875.
4. Reduce $\frac{1}{16}$ and $\frac{1}{20}$ to decimals.
   Ans. .0625 and .05.
5. Reduce $\frac{3}{40}$ and $\frac{2}{25}$ to decimals.
   Ans. .075 and .08.
6. Reduce $\frac{5}{16}$ and $\frac{3}{75}$ to decimals.
   Ans. .3125 and .04.
7. Reduce $\frac{9}{16}$ and $\frac{3}{50}$ to decimals.
   Ans. .5625 and .06.

(88.) A *mixed decimal* may be reduced to a simple one, by substituting for the vulgar fraction annexed, the figures of its *equivalent decimal.*

Thus $.6\frac{1}{25}$ is equal to .604, since $\frac{1}{25} = .04$.

8. Reduce $.25\frac{1}{2}$ to a simple decimal. *Ans.* .255.
9. Reduce $.31\frac{1}{4}$ to a simple decimal. *Ans.* .3125.
10. Reduce $.18\frac{3}{4}$ to a simple decimal. *Ans.* .1875.
11. Reduce $.23\frac{1}{8}$ to a simple decimal. *Ans.* .23125.
12. Reduce $.90\frac{1}{16}$ to a simple decimal. *Ans.* .900625.

Repeating Decimals.

(89.) A *Repeating Decimal,* also called a *repetend,* is a decimal in which the same figure or figures recur in immediate and continual succession.

In reducing $\frac{1}{3}$ to a decimal, we obtain the *repetend* .3333, &c., in which there is the repeating figure 3.

In reducing $\frac{2}{11}$ to a decimal, we obtain the *repetend* .181818, &c., in which there are the repeating figures 18.

A *mixed repetend* is a decimal in which other figures *precede a repetend* or repeating decimal. Thus in reducing $\frac{5}{12}$ to a decimal, we obtain the *mixed repetend* .416666, &c., in which the *precedent figures* are 41.

A *repetend is indicated* by placing a point over the repeating figure, or over the first and last repeating figures, when there are more than one.

Thus $.\dot{3}$; $.\dot{1}\dot{8}$; $.\dot{4}1\dot{6}$ denote repetends.

$.275\dot{3}$ denotes a *mixed* repetend, in which there are the precedent figures 275 and the repeating figure 3.

Repeating Decimals Reduced to Vulgar Fractions.

(90.) A *repeating decimal* is always equal to a vulgar fraction whose numerator is the *repeating figure or figures,* and denominator as many 9's as there are repeating figures.

For, by reducing $\frac{1}{9}$ to a decimal, we obtain the repetend $.\dot{1}$;

then $.\dot{1} = \frac{1}{9}$; and consequently, $.\dot{2} = \frac{2}{9}$; $.\dot{3} = \frac{3}{9}$, &c.

By reducing $\frac{1}{99}$ to a decimal, we obtain the repetend $.0\dot{1}$; then $.0\dot{1} = \frac{1}{99}$; and consequently, $.0\dot{2} = \frac{2}{99}$; $.0\dot{3} = \frac{3}{99}$, &c.

The same method of illustration will apply to any repetend consisting of three, four, or five, &c., repeating figures.

The value of a *mixed repetend* may be expressed by a *mixed decimal;* and this decimal may be reduced to a vulgar fraction.

Thus in $.41\dot{6}$ the repeating figure 6 annexed to .41, is equivalent to $\frac{6}{9}$ or $\frac{2}{3}$ annexed (90); then

$$.41\dot{6} = .41\tfrac{2}{3} = \frac{41\frac{2}{3}}{100} = \frac{125}{3} \div 100 = \frac{125}{300} = \frac{5}{12}.$$

EXERCISES.

1. Reduce $.\dot{5}$ to an equivalent vulgar fraction. *Ans.* $\frac{5}{9}$.
2. Reduce $.\dot{1}\dot{5}$ to an equivalent vulgar fraction. *Ans.* $\frac{5}{33}$.
3. Reduce $.\dot{5}1\dot{3}$ to an equivalent vulgar fraction. *Ans.* $\frac{19}{37}$.
4. Reduce $.\dot{5}0\dot{3}$ to an equivalent vulgar fraction. *Ans.* $\frac{503}{999}$.
5. Reduce $.\dot{3}03\dot{6}$ to an equivalent vulgar fraction. *Ans.* $\frac{92}{303}$.
6. Reduce $.24\dot{1}\dot{2}$ to an equivalent vulgar fraction. *Ans.* $\frac{199}{825}$.

Approximate Decimals.

(91.) An *approximate Decimal* is one which expresses a *near*, but not the *exact* value of a vulgar fraction, or other quantity.

In reducing $\frac{1}{3}$, for example, to a decimal, we find the successively *approximating* decimals .3, .33, .333, .3333, &c.

The sign + is commonly affixed to an approximate decimal; thus

$\frac{1}{3}$ = .333 +, 333 thousandths, *nearly*.

Instead of the sign +, we shall employ a *comma'*, after the manner of an *apostrophe*, to denote an approximate decimal; thus

$\frac{1}{3}$ = .333', 333 thousandths, *nearly*.

FEDERAL MONEY.

(92.) Federal Money, or money of the United States, is expressed in units according to the *decimal scale* of numeration, that is, the system of numbering by *tens*.

The units of Federal Money are

Mills, Cents, Dimes, Dollars, and Eagles.

10 mills,	*m.*	make	1 cent,	*ct.*
10 cents		"	1 dime,	*d.*
10 dimes, or 100 *cts.*,		"	1 dollar,	$.
10 dollars		"	1 Eagle,	*E.*

The federal character $ is prefixed to dollars; thus $5 is 5 dollars.

How many *mills* make 1 dollar?

The denominations of Federal Money most commonly used in accounts, are *dollars* and *cents*,—Eagles being expressed in *dollars*, dimes in *cents*, and smaller values in fractions of a cent.

Thus we say, 45 *dollars* 37½ *cents*, instead of 4 *E.* 5 *dol.* 3 *d.* 7 *cts.* 5 *m.*

Notation of Federal Money.

Cents being expressed in numbers less than 100, and *mills* in numbers less than 10, we have

RULE XXI.

(93.) *For the Decimal Expression of Federal Money.*

1. Regard dollars as *integers*, and make cents and mills *decimals* of a dollar, by prefixing to them the decimal *point* (.); but

2. Interpose a 0 next the point when the number of cents is *less than* 10,—and 00 next the point when only a fraction of a cent or mills are given.

EXAMPLES.

5 *dollars* and 12 *cents* is expressed by $5.12;
7 *dollars* and 6¼ *cents* is expressed by $7.06¼;
9 *dollars* and 8 *mills* is expressed by $9.008.

It will be useful to observe here, conversely, that,

(94.) In a Decimal *of a dollar*, the first figure after the (.) denotes *dimes;* the second denotes *cents;* the first two together denote *cents;* the third, *mills;* the fourth, *tenths* of a mill, &c.

Thus $.5 is 5 *dimes*, equal to 50 *cents;*
$.435 is 43 *cents* and 5 *mills;*
$.0625 is 6 *cents* 2 *mills* and 5 *tenths* of a mill.

By observing the preceding Rule for the expression of Federal Money, this subject is brought under the same Rules of Addition, Subtraction, &c., as *Decimal Fractions.*

ADDITION OF DECIMALS.

RULE XXII.

(95.) *For the Addition of Decimals.*

1. Set *tenths* under *tenths*, hundredths under hundredths, &c., and add up the several columns as in *integers*.

2. In the right of the *sum* make as many *decimal figures* as will be equal to the greatest number of decimal figures in any one of the given numbers.

EXAMPLE.

To find the Sum of .25 + 84.346 + .73 + 275.937.

```
    .25
  84.346
    .73
 275.937
 -------
 361.263
```

Having set *tenths* under *tenths*, hundredths under hundredths, &c.—this order also causing *units* to fall under *units*, tens under tens, &c., when mixed numbers are to be added—we add up the columns as in integers (82).

In the right of the sum found we make *three decimal figures*, .263, this being the greatest number of decimal figures in any one of the given numbers.

EXERCISES.

1. Bought at one time 215.5 acres of land, at another 23.56 acres, and at another 32.32 acres; what was the whole quantity purchased? *Ans.* 271.38 acres,

2. A person on a journey traveled the first day 35 miles, the second 29.3 miles, the third 40.15 miles, and the fourth 39 miles. How far was that in the four days?

Ans. 143.45 miles.

3. A merchant bought one piece of cloth containg 30 yards, another containing 36.25 yards, and two others each containing 28.5 yards. What was the whole number of yards purchased? *Ans.* 123.25 yards.

4. Sold to one person 25.4 bushels of coal, to another 56 bushels, to another 75.25 bushels, to another 80.125 bushels, and two others each 95.2 bushels. What was the whole quantity sold? *Ans.* 427.175 bushels.

5. Find the sum of 100 *dollars* 72½ *cents*, 25 *dollars* 6¼ *cents*, and 119 *dollars* 48¾ *cents.*

$$\begin{array}{r} \$100.72\tfrac{1}{2} \\ 25.06\tfrac{1}{4} \\ 119.48\tfrac{3}{4} \\ \hline 245.27\tfrac{1}{2}. \end{array}$$

Ans. 245 *dols.* 27½ *cts*

The *dollars* are set down as *integers*, and the *cents* as *decimals* (93); then the ¾, ¼, and ½ make $\frac{6}{4} = 1\frac{1}{2}$ (71); set down ½, and add 1 to 8.

In the sum we point off the 27 for *cents*, or *hundredths* of a dollar.

6. Find the sum to be paid for a hat at 5 dollars 87½ cents, a vest at 3 dollars 18¾ cents, and a pair of shoes at 2 dollars 62½ cents. *Ans.* $11.68¾.

7. Find the sum to be paid for a quarter of beef at 7 dollars, a barrel of flour at 4 dollars 56¼ cents, a lot of groceries at 13 dollars 37½ cents, and a lot of butter at 2 dollars 6¼ cents. *Ans.* $27.00.

8. Find the sum to be paid for a quire of paper at 25 cents, a bottle of ink at 12½ cents, a dozen of books at 1 dollar 18¾ cents, and a bunch of quills at 37½ cents. *Ans.* $1.93¾.

9. Sold a barrel of sugar for 15 dollars, a sack of coffee for 13 dollars 5 cents, a keg of rice for 5 dollars 43¼ cents,

and a box of candles for 9 dollars 8 cents. What did the whole amount to? *Ans.* \$42.56¼.

10. Find the sum that should be paid for a set of chairs at 18 dollars, a pair of tables at 35 dollars 50 cents, a looking-glass at 5 dollars 18¾ cents, and a bedstead at 9 dollars 31¼ cents. *Ans.* \$68.00.

11. A merchant's bill was as follows: for 3½ yards of cloth, 21 dollars; for 3 pair of stockings, 1 dollar 87½ cents; for a dozen skeins of silk, 75 cents; required the amount of the bill. *Ans.* \$23.62½.

12. A farmer sold produce as follows, namely; wheat for 300 dollars, corn for 97 dollars 93¾ cents, hay for 56 dollars 12½ cents, and oats for 18 dollars 6¼ cents. Required his amount of sales. *Ans.* \$472.12.

SUBTRACTION OF DECIMALS.

RULE XXIII.

(96.) *For the Subtraction of Decimals.*

1. Set the less value under the greater, with *tenths* under *tenths*, hundredths under hundredths, &c., and subtract as in *integers.*

2. In the right of the *remainder* make as many *decimal figures* as will be equal to the greatest number of decimal figures in either of the given numbers.

3. When the minuend has *no decimal figures*, or not so many as the subtrahend, conceive the deficient places to be occupied by decimal 0's.

EXAMPLE.

To find the Difference between 525 and 9.87534.

```
525.00000
  9.87534
---------
515.12466
```

The places for *decimals* over the .87534 must be regarded as occupied by 0's ; then 4 from 10 leaves 6, &c.

EXERCISES.

1. Having on hand 125.5 tons of coal, if I sell 13.75 tons, how many tons will I have remaining ? *Ans.* 111.75 tons.

2. Having purchased 575.75 yards of cotton, if I sell 350.125 yards, how many yards will I have remaining ?
Ans. 225.625 yards.

3. A person bought 1000 acres of land, of which he has sold 462.375 acres ; how many acres has he remaining ?
Ans. 537.625 acres.

4. A merchant bought 3860.5 bushels of salt, of which he has sold, at different times, to the amount of 783 bushels ; what quantity of salt has he still on hand ?
Ans. 3077.5 bushels.

5. A manufacturer made 5790.75 yards of cloth, of which he has sent off, to various places, to the amount of 3764.5 yards ; how many yards of the cloth has he still on hand ?
Ans. 2026.25 yards.

6. A person collected 325 dollars, and out of that sum paid bills amounting to 93 dollars $6\frac{1}{4}$ cents ; how much had he remaining ?

$$\begin{array}{r}\$325.00 \\ 93.06\frac{1}{4} \\ \hline 231.93\frac{3}{4}\end{array}$$

Ans. 231 *dols.* $93\frac{3}{4}$ *cts.*

The dollars are set down as *integers*, and the cents as *decimals* (93) ; we then subtract the $\frac{1}{4}$ from a whole one, or $\frac{4}{4}$, and add 1 to 6.

In the difference we point off the 93 for *cents* or *hundredths* of a dollar.

7. If a lot of goods were purchased for 579 dollars, and sold for 650 dollars 87½ cents, what sum would be gained?

Ans. $71.87½.

8. What would be made on a quantity of lumber, bought for 225 dollars 18¾ cents, and sold for 300 dollars 50 cents?

Ans. $75.31¼.

9. Required the loss on a lot of flour, purchased for 372 dollars 12½ cents, and sold for 321 dollars 56¼ cents.

Ans. $50.56¼.

10. A merchant bought a piece of cloth for 120 dollars, and a piece of silk for 85 dollars 68¾ cents. He sold both pieces for 316 dollars 56¼ cents; what profit did he make?

Ans. $110.87½.

11. A manufacturer purchased a quantity of raw cotton for 400 dollars, which he made into cloth at an expense of 132 dollars 6¼ cents. What profit will he make by selling the cloth for 700 dollars? *Ans.* $167.93¾.

12. A grazier bought cattle for 160 dollars, and sheep for 50 dollars 50 cents. He sold the cattle for 225 dollars 37½ cents, and the sheep for 83 dollars 93¾ cents; what did he gain by these transactions? *Ans.* $92.81¼.

13. A speculator purchased wheat for 344 dollars, and bacon for 88 dollars 18¾ cents. He sold his wheat for 300 dollars 75 cents, and his bacon for 100 dollars 12½ cents; what did he gain or lose by the speculation?

Ans. Lost $31.31¼.

14. Having deposited in bank 1000 dollars, and having drawn out 74 dollars 50 cents, 390 dollars 87½ cents, and 213 dollars 68¾ cents; what sum have I still in bank?

Ans. $320.93¾.

15. Bought a house and lot in a city for 3000 dollars, and paid for improvements on the same 316 dollars 93¾ cents. If the property be sold for 4500 dollars, what amount of profit will be realized? *Ans.* $1183.06¼.

MULTIPLICATION OF DECIMALS.

RULE XXIV.

(97.) *For the Multiplication of Decimals.*

1. Multiply as in *integers;* and in the product make as many *decimal figures* as there are decimal figures in *both the factors.*

2. *Prefix* 0'*s to the product* when necessary to make up the required number of decimal figures.

EXAMPLE.

To multiply .19 by .5; that is, to find $\frac{5}{10}$ of $\frac{19}{100}$ (74).

$$\begin{array}{r} .19 \\ .5 \\ \hline .095 \end{array}$$

Multiplying as in integers, we find the product 95; to which we prefix a 0 and the (.), to make *three decimal figures* for the three in the multiplicand and multiplier.

If we multiply the two quantities together under the form of *vulgar fractions*, we shall have

$$\frac{19}{100} \times \frac{5}{10} = \frac{95}{1000} = .095.$$

By a like method it may be shown that, in every case of decimal multiplication, the number of decimal figures belonging to the product is the same as the number in *both the factors.*

(98.) *When the Multiplier is* 10, *or* 100, *or* 1000, &c.; the Product will be found by removing the (.) as many places *to the right* in the multiplicand as there are 0's in the multiplier—0's being annexed to the multiplicand when necessary.

Thus .235 × 100 produces 23.5; 3.5 × 1000 produces 3500.

EXERCISES.

1. An agriculturist had in wheat 75 acres, which produced 24.5 bushels per acre. What was the entire produce?
Ans. 1837.5 bushels.

2. A person accomplished a journey in 12.5 days, by traveling at the rate of 40.8 miles per day. What was the length of the journey? *Ans.* 510 miles.

3. A merchant bought 13 pieces of cloth, each containing 35.75 yards. How many yards did he purchase?
Ans. 464.75 yards.

4. A farmer sold a tract of land containing 375.3 acres, at 100 dollars per acre. What did the whole amount to?
Ans. $37530.

5. A railroad car has been running for 9.75 hours, at the rate of 39.1 miles per hour. How many miles has it run?
Ans. 381.225 miles.

6. A merchant who owned .125 of the cargo of a ship, sold .3 of his share to his brother. What part of the entire cargo did he sell? *Ans.* .0375 of it.

7. A field containing 18.125 acres produced 8.5 barrels of corn per acre. What was the entire produce of the field?
Ans. 154.0625 barrels.

8. A road which is 37.8 miles in length was made at an expense of 1000 dollars per mile. What was the entire cost of the road? *Ans.* $37800.

9. A speculator purchased an interest of .23 in a manufacturing establishment, and then sold .12 of his purchase What part of the entire establishment did he sell?
Ans. .0276 of it.

10. Find the sum that should be paid for 7 yards of cloth, at 5 dollars 6¼ cents per yard.

$$\begin{array}{r} \$5.06\frac{1}{4} \\ 7 \\ \hline 35.43\frac{3}{4}. \end{array}$$

Ans. 35 dollars 43¾ cents.

We set down the $6\frac{1}{4}$ cents as a *decimal* of a dollar (93). In multiplying, we say 7 times $\frac{1}{4}$ is $\frac{7}{4}$, equal to $1\frac{3}{4}$; 7 times 6 is 42, and 1 makes 43.

In the product we make *two decimal places* for the two in the multiplicand.

11. What should be paid for 9 hundredweight of tobacco, at 10 dollars $37\frac{1}{2}$ cents per hundredweight?

Ans. \$93.37$\frac{1}{2}$.

12. What should be paid for 8 yards of cloth, at 9 dollars $56\frac{1}{4}$ cents a yard, and 12 yards of linen at $87\frac{1}{2}$ cents a yard?

Ans. 87.00.

13. What should be paid for 9 head of cattle at 13 dollars $18\frac{3}{4}$ cents a head, and 7 mules at 80 dollars 50 cents a head?

Ans. 682.18$\frac{3}{4}$.

14. What should be paid for 5 bushels of wheat at 1 dollar $6\frac{1}{4}$ cents a bushel, and 30 bushels of corn at 50 cents a bushel?

Ans. \$20.31$\frac{1}{4}$.

15. Find the sum that should be paid for 17 barrels of flour, at 7 dollars $93\frac{3}{4}$ cents a barrel?

The multiplication will be facilitated by reducing the *mixed decimal* .93$\frac{3}{4}$ to the *simple* decimal .9375 (88); we shall then have \$7.9375 × 17.

Ans. \$134.9375.

16. Find the sum that should be paid for 23 cords of wood at 4 dollars $37\frac{1}{2}$ cents per cord, and 131 bushels of coal at $12\frac{1}{2}$ cents a bushel.

Ans. \$117.00.

17. A merchant bought 37 yards of cloth at 3 dollars $6\frac{1}{4}$ cents per yard, and sold the same at 5 dollars $18\frac{3}{4}$ cents per yard. What amount of profit did he make on the cloth?

Ans. 78.625.

18. A farmer purchased 234.5 acres of land at 43 dollars per acre, which he sold again at 57 dollars $56\frac{1}{4}$ cents per acre. What amount of profit did he make on the land?

Ans. \$3414.906'.

DIVISION OF DECIMALS.

RULE XXV.

(99.) *For the Division of Decimals.*

1. Divide as in *integers;* and in the quotient make a many *decimal figures* as there are decimal figures in the dividend *more than in the divisor.*

2. *Prefix* 0's *to the quotient* when necessary to make up the required number of decimal figures.

3. When the divisor has *more decimal figures* than the dividend, or is greater than the dividend (regarding both as *integers*), annex *decimal* 0's to the dividend, to supply the deficiency.

4. Annex 0's to the *remainder*, if any, and continue the division to any required exactness—counting these 0's as *decimal figures of the dividend.*

EXAMPLES.

1. To divide .375 by 12.5.

12.5).375(.03.

Dividing as in integers, we find the quotient 3, to which a 0 and the (.) must be prefixed, to make *two decimal figures* for the two which the dividend has more than the divisor.

2. To divide 45 by 35.7.

```
35.7)45.0(1.26'
     357
     ---
      930
      714
      ----
      2160
```

The divisor having one decimal figure, while the dividend 45 has none, we annex a *decimal* 0 to the dividend. The division is continued by annexing 0 to the remainder 93, and also to the next remainder.

The 0's annexed make *three decimal* figures in the dividend ; and there being *one* in the divisor, there must be *two* in the quotient.

The Quotient must have just as many decimal figures as there are decimal figures in the dividend *more than in the divisor*, because the divisor and quotient multiplied together must produce the dividend (97 . . . 1).

Thus in the first example, $12.5 \times .03 = .375$.

The 0 *is prefixed to the quotient*, because this is necessary to make the product of the divisor and quotient equal to the dividend. We also see that the number of decimal figures in the dividend cannot be taken *less than the number in the divisor.*

(100.) *When the Divisor is* 10, *or* 100, *or* 1000, &c., the Quotient will be found by removing the (.) as many places *to the left* in the dividend as there are 0's in the divisor—0's being prefixed to the dividend when necessary.

Thus $23.5 \div 100$ gives .235 ; and $3.5 \div 1000$ gives .0035.

EXERCISES.

1. How many hours would a person be in going 22.36 miles, at the rate of 4.3 miles per hour? *Ans.* 5.2 hours.

2. A farmer raised 1837.5 bushels of oats on 75 acres; what was the average produce per acre?

Ans. 24.5 bushels.

3. A merchant has 490.75 yards of linen in 13 pieces; what is the average length of the pieces?

Ans. 37.75 yards.

4. A traveler performed a journey of 610 miles in 12.5 days; at what rate did he travel per day?

Ans. 40.8 miles.

5. A tract of land containing 375.3 acres, was sold for 37530 dollars; what was the price per acre? *Ans.* \$100.

6. At the rate of 8.5 barrels per acre a field of corn produced 154.0625 barrels; how many acres did the field contain? *Ans.* 18.125 acres.

7. A steamboat has run 381.225 miles in 39.1 hours; what was her average rate per hour? *Ans.* 9.75 miles.

8. A road which cost 5783.5 dollars, was made at the average cost of 1000 dollars per mile. How long is the road?

Ans. 5.7835 miles.

9. A barrel of wine contains 31.5 gallons; how many barrels then would be required to contain 4410 gallons?

Ans. 140 barrels.

10. Allowing a piece of ground to produce at the rate of 25.75 bushels of wheat per acre, how many acres would produce 1000 bushels? *Ans.* 38.834 acres.

11. What quantity of wine, at 1 dollar 37½ cents per gallon, may be bought for 25 dollars and 50 cents?

By using *decimals*, we have \$25.50 ÷ \$1.375, (93), (88).

Ans. 18.545' gallons.

12. What quantity of coal, at 18 dollars 75 cents per ton, may be purchased for 13 dollars?

13 dollars will buy the same *part of a ton* that 13 dollars is of 18 dollars 75 cents; that is \$13 ÷ \$18.75.

Ans. .693' of a ton.

13. How many hundredweight of flour, at 2 dollars 18¾ cents per hundredweight, may be bought for 25 dollars?

Ans. 11.428 hund'w't.

14. What quantity of land, at the rate of 25 dollars per cre, may be purchased for 9 dollars 62¼ cents?

Ans. .384' of an acre.

15. How many bushels of clover seed, at 5 dollars $18\frac{3}{4}$ cents per bushel, may be purchased for 30 dollars ?

Ans. 5.783' bushels.

16. How many yards of cloth, at 4 dollars and 50 cents per yard, may be purchased for 19 dollars and 75 cents ?

Ans. 4.388' yards.

17. How many barrels of corn, at 3 dollars and 85 cents per barrel, may be purchased for 100 dollars ?

Ans. 25.974' barrels.

18. If $3\frac{1}{2}$ cords of wood sell for 12 dollars and 75 cents, what is the price per cord ?

The *price per cord* will be found by dividing \$12.75 by $3\frac{1}{2}$, because *that price* $\times$ $3\frac{1}{2}$ must produce \$12.75 ; this product and one of its factors being given, to find the other factor (37).

The $\frac{1}{2}$ may be used decimally (87) ; we have then

\$12.75 $\div$ 3.5 *Ans.* 3.642'.

19. If $5\frac{1}{4}$ yards of broadcloth cost 21 dollars 25 cents, what should be paid for a yard of the same cloth ?

Ans. \$4.047'.

20. If $\frac{5}{8}$ of a lot of ground be worth 73 dollars $87\frac{1}{2}$ cents, what is the whole of the lot worth at that rate ?

Ans. \$118.20.

21. If $4\frac{1}{2}$ cords of wood cost 9 dollars, what is the price per cord ? and what would $7\frac{3}{4}$ cords amount to at the same rate ? *Ans.* \$2 ; and \$15.50.

22. What is the price of wheat per bushel, when $25\frac{1}{8}$ bushels sell for 37 dollars $68\frac{3}{4}$ cents ? and what should be paid for 40 bushels at the same rate ?

Ans. \$1.50 ; and \$60.

23. What is the price of butter per pound when $13\frac{1}{2}$ pounds sell for 1 dollar 62 cents ? and what should be paid for 145 pounds of butter at the same price ?

Ans. 12 cents ; and \$17.40.

24. Bought 50 bushels of salt for 31 dollars 25 cents, and sold it at a profit of 25 cents per bushel. At what price per bushel was it sold?

$31.25 ÷ 50 gives the price at which it was bought per bushel. *Ans.* $.875.

25. Bought 32½ barrels of corn for 81 dollars 25 cents, and sold it at a profit of 61¼ cents per barrel. At what price per barrel was it sold? and what was the whole profit made? *Ans.* $3.1125; $19.906'.

26. What should be paid for 10 tons of hay, when .7 of a ton sells for 13 dollars 12½ cents?

$13.125 ÷ .7 will give the *price per ton*, because that price × .7 must produce the value of .7 of a ton.

Ans. $187.50.

27. What should be paid for 15½ yards of silk when ⅝ of a yard costs 1 dollar 12½ cents? *Ans.* $27.90.

28. Allowing ¾ of a yard of cloth to cost 5 dollars 43¾ cents, what should be paid for 13⅞ yards at the same rate?

Ans. $100.59375.

29. A person having 300 dollars on hand would disburse it for equal quantities of sugar and coffee. What quantity of each can he purchase, if the sugar be at 9 cents, and the coffee at 15 cents per pound?

9 + 15 = 24; then 24 cents will buy *one pound* of each.

Ans. 1250 pounds of each.

30. If wheat be at 1 dollar, rye at 50 cents, and corn at 37½ cents per bushel; how many bushels of each may be purchased for 500 dollars? *Ans.* 266.666 bushels of each.

MISCELLANEOUS EXERCISES

ON DECIMAL FRACTIONS, AND FEDERAL MONEY.—ABBREVIATED MULTIPLICATION AND DIVISION.

1. What is the Sum of 230 and 3 *tenths*, 29 and 13 *hundredths*, 173 and 5 *hundredths*, 75 *thousandths*, and 1350 and 3 *ten-thousandths?* *Ans.* 1782.5553.

2. What is the Sum of 625 *thousandths*, 162 and 5 *hundredths*, 346 and 9 *tenths*, 375 *thousandths*, and 8374 and 15 *ten-thousandths?* *Ans.* 8883.9515.

3. What is the Product of 375 and 125 *thousandths* multiplied by the difference between 75 *hundredths* and 31 *thousandths?* *Ans.* 269.714875.

4. What is the Product of 25 *millionths* multiplied by the Sum of 300 and 5 *tenths*, 17 *thousandths*, and 10 and $6\frac{1}{4}$ *tenths?* (88) *Ans.* .00777855.

5. What is the Quotient of 803 and 154 *hundred-thousandths* divided by the Difference between 25 *hundred-thousandths* and 24564 and 625 *ten-thousandths?*

Ans. .0326'.

6. What is the Quotient of 7340 and 16 *hundred-thousandths* divided by the Difference between 6357 and 8 *tenths* and $3\frac{1}{4}$ *hundredths?* (88) *Ans.* 1.154.

7. Find the value of $(.25 + .125 + 2.5 - .05 - .005) \times .04$ in a vulgar fraction in its lowest terms?

Ans. $\frac{141}{1250}$.

8. Find the value of $(100 + .1 + .34 + .09 + 3.2 - 60) \div 50$ in a vulgar fraction in its lowest terms.

Ans. $\frac{4373}{5000}$.

9. Find the value of $(\frac{3}{4} + 5\frac{1}{2} + 10 + .3\frac{1}{4} + 2\frac{1}{10}) - (7 + \frac{2}{5} + 5\frac{1}{3})$ in an integer and *decimal thousandths.*

Ans. **$5.941\frac{2}{3}$.**

10. Find the amount of a merchant's bill for $3\frac{1}{2}$ yards of cloth at 7 dollars $68\frac{3}{4}$ cents per yard, $12\frac{1}{4}$ yards of silk at 1 dollar $31\frac{1}{4}$ cents per yard, and 16 skeins of silk thread at $6\frac{1}{4}$ cents a skein.

In these Exercises reduce Vulgar Fractions to *Decimals*.

Ans. $43.984'.

11. Bought 20 barrels of flour for 102 dollars 50 cents, and sold the same at a profit of $87\frac{1}{2}$ cents a barrel. At what price per barrel was it sold? and what was the entire profit made? *Ans*. $6; and $17.50.

12. A gives B $117\frac{1}{2}$ yards of silk, at $93\frac{3}{4}$ cents per yard, for 20 yards of cloth at 4 dollars $37\frac{1}{2}$ cents per yard, and enough of calico at $12\frac{1}{2}$ cents per yard to pay the balance. How much calico must A receive? *Ans*. 181.25 yards.

13. A merchant bought 25 yards of cloth at 4 dollars $87\frac{1}{2}$ cents per yard, and sold it at an entire profit of 50 dollars $68\frac{3}{4}$ cents. At what price per yard was the cloth sold?

Ans. 6.9025.

14. What is the price of sugar per hundredweight when $\frac{3}{4}$ of a hundredweight costs 6 dollars $37\frac{1}{2}$ cents? and what should be paid for $5\frac{1}{2}$ hundredweight of sugar at the same rate? *Ans*. $8.5; and $46.75.

15. How much rice at $4\frac{1}{2}$ cents a pound would be an equivalent for 100 pounds of coffee at $12\frac{1}{2}$ cents a pound, 300 pounds of sugar at 7 cents a pound, and $25\frac{1}{4}$ pounds of tea at $1 a pound? *Ans*. 1305.555' pounds.

16. A person wishes to purchase a quantity of coffee and as much rice. The coffee is at $13\frac{1}{2}$ cents, and the rice at 5 cents, per pound; what quantity of each can he purchase for $15\frac{1}{2}$ dollars? *Ans*. 83.783' pounds.

17. A farmer bought a plantation containing 400 acres, at $20\frac{1}{2}$ dollars per acre, and sold $\frac{1}{2}$ of it at a profit, on that half, of 213 dollars $12\frac{1}{2}$ cents. At what price per acre was the land sold? *Ans*. $21.565'.

18. A gives B 37 hundredweight of hemp, at 4 dollars 33$\frac{1}{3}$ per hundredweight; for 1000 pounds of bacon at 6$\frac{1}{4}$ cents per pound, 20 dollars in cash, and as much sugar at 7$\frac{1}{4}$ cents per pound as will pay the balance. How much sugar must A receive? *Ans.* 1073.563' pounds.

19. A miller sold 75 barrels of flour at 4 dollars 87$\frac{1}{2}$ cents a barrel, and with the proceeds intends to purchase, in equal quantities, wheat at 75 cents a bushel, rye at 31$\frac{1}{4}$ cents a bushel, and corn at 37$\frac{1}{2}$ cents a bushel. What quantity of each can he purchase? *Ans.* 254.347' bushels.

20. A merchant bought 100 yards of cloth at 3 dollars 93$\frac{3}{4}$ cents per yard, and 82$\frac{1}{2}$ yards at 4 dollars 12$\frac{1}{2}$ cents per yard. At what average price per yard should he sell the whole, to realize a profit which shall be equal to $\frac{1}{4}$ of the cost? *Ans.* $5.027'.

21. A speculator bought 210 barrels of flour at 4 dollars a barrel, 965$\frac{1}{2}$ bushels of oats at 33$\frac{1}{3}$ cents a bushel, and 50 barrels of pork at 5 dollars 6$\frac{1}{4}$ cents a barrel. He sold the flour and pork at an advance of 2$\frac{1}{2}$ dollars a barrel, and the oats at a loss of 3 cents a bushel; what was the result of the speculation? *Ans.* He gained $621.03'.

22. A bought of B 122$\frac{1}{2}$ bushels of wheat, and of C 75$\frac{1}{4}$ bushels, at 93$\frac{3}{4}$ cents per bushel. He made 60 bushels into flour, and sold the flour at a profit of 12$\frac{1}{2}$ dollars; if he sell the remainder of the wheat at 81$\frac{1}{4}$ cents per bushel, what will be his entire profit or loss? *Ans.* Loss $4.718'.

23. A speculator bought 50 barrels of flour at 4 dollars 6$\frac{1}{4}$ cents per barrel, and 75 bushels of wheat at 68$\frac{3}{4}$ cents per bushel. Having sold 20 barrels of the flour at 5 dollars a barrel, and the whole of the wheat at 75 cents a bushel, at what price per barrel must the remainder of the flour be sold to make his profit 100 dollars on the whole?

Ans. $6.614' per barrel.

Abbreviated Multiplication of Decimals.

By the Rule for the Multiplication of Decimals (97), the product will sometimes contain more decimal figures than it is necessary to find. In such cases the multiplication may be *abbreviated* as follows:

To multiply 37.14586 by 92.83, for only *three decimal figures in the product.*

```
 37.14586
    38.29
  -------
  3343127
    74292
    29716
     1114
 --------
 3448.249
```

We set the *units figure,* 2, of the multiplier under that decimal place of the multiplicand which is the *last to be retained in the product.* The remaining figures of the multiplier are set in *reversed order*—the 92 being reversed on the right of the (.), and the .83 on the left.

In multiplying we begin with that figure of the multiplicand which stands directly over the *multiplying figure;* thus in multiplying by the 9 we begin with the 8; in multiplying by the 2 we begin with the 5, &c.; but

To secure an *average correctness* in the first figures of the several products, we must add to each the *nearest number of tens* that would arise from multiplying the rejected right hand figure of the multiplicand; thus we say 9 time 8 is 72, and add 5, which would be carried from multiplying the 6.

When the rejected right hand figure would give a product *midway between two numbers of tens,* as 15, 25, 35, &c., we take the *greater* number as the nearest value, because such product would generally be increased from multiplying the other rejected figures.

. The several products thus formed will all begin with the *same order of decimals* as the last decimal figure in the required product. The .0008 multiplied by the 9, which is 9 *tens*, or 90, produces 72 *thousandths* (97); the .005 × 2 produces 10 *thousandths;* and so on; and *thousandths* is the lowest order required in the product. The first figures of the several products must therefore be set *one under another*, to be added together.

The last decimal figure in the product found as above, will often be the same that would be found in that place by the common rule, and will very seldom be wrong by more than 1. By that Rule the entire product, in the present case, would be

3448.2501838.

When the multiplicand has not as many decimal figures as are required in the product, 0's must be annexed to supply the deficiency.

24. Find the Product of 73.1285 × 4.1316 to *two decimal figures.* *Ans.* 302.12.

25. Find the Product of 130.375 × .47348 to *three decimal figures.* *Ans.* 61.728.

26. Find the Product of 570.794 × 1.7383 to *four decimal figures.* *Ans.* 992.2111

Abbreviated Division of Decimals.

When there are several figures in the Divisor, and it is necessary to find only a few decimal figures in the Quotient, the division may be *abbreviated* as follows.

To divide 2508.92806 by 92.4135 to *two decimal figures* in the Quotient.

```
92.41,35(2508.92806(27.14
         18483
         -----
          6606
          6469
          ----
           137
            92
           ---
            45
            37
```

We first consider how many figures the quotient will contain. There will be *four figures* in the quotient—two integral figures from dividing 92 into 2508, and the two required decimals.

We first divide by as many figures, 9241, in the left of the divisor as there are to be *figures in the quotient;* and instead of affixing the next figure of the dividend to the remainder, for a new dividend, we *reject another figure from the divisor*, and divide 924 into 6606; and so on.

In multiplying the divisors by the quotient figures, it is necessary to add the *tens* that would arise from multiplying the rejected figure on the right, as in abbreviated multiplication of decimals.

When the Divisor has not as many figures as are required in the quotient, the division must proceed according to the common Rule, until the figures in the divisor are one more than those *remaining to be found in the quotient.*

27. Find the Quotient of 2857.35 ÷ 743.672 to *two decimal figures.* *Ans.* 3.84.

28. Find the Quotient of 738.973 ÷ 205.864 to *three decimal figures.* *Ans.* 3.589.

29. Find the Quotient of 1384.47 : 237.416 to *four decimal figures.* *Ans.* 5.8314

CHAPTER VI.

WEIGHTS AND MEASURES.—CURRENCIES.—MONOMIALS AND POLYNOMIALS.—DUODECIMALS.—ALIQUOT PARTS.

Different Orders of Measuring Units.

(101.) A Quantity (3) is sometimes expressed in two or more *different orders* of measuring units.

Thus 5 *dollars* 25 *cents* is a quantity, or sum of money, expressed in *two different orders* of measuring units.

In the expression 3 *pounds* 4 *ounces* are how many different orders of measuring units? In 4 *days* 7 *hours* 20 *minutes* are how many different orders of measuring units?

In Federal Money, as has been seen (92), the measuring units rise from lower to higher orders by a *tenfold increase*, as in abstract numbers.

In other kinds of quantity, the relative values of the measuring units are to be learned from the following Tables:

(102.) *Troy Weight*

Is used in weighing *jewels*, *gold*, *silver*, *liquors*, and, generally, the most valuable commodities.

24	grains (*gr.*) make	1	pennyweight,	*dwt.*
20	pennyweights .	1	ounce,	*oz.*
12	ounces . . .	1	pound,	*lb.*

One *gr.* is what part of a *dwt.*? 1 *dwt.* is what part of an *oz.* One *oz.* is what part of a *lb.*?

(103.) *Avoirdupois Weight*

Is used in weighing *groceries*, all the coarser *metals*, and, generally, all coarse and cheap commodities.

16	drams (*dr.*)	make	1 ounce,	*oz.*
16	ounces		1 pound,	*lb.*
28	pounds		1 quarter,	*qr.*
4	quarters		1 hundredweight,	*cwt.*
20	hundredweight . .		1 ton,	*T.*

This Table is according to former usage, which is still the usage of Great Britain, and of the Custom Houses of the United States.

By the statutes of several of our States,

25	pounds	make	1 quarter,
4	quarters	.	1 hundredweight.

This makes a *ton* consist of 2000 pounds, while by the first Table a *T.* is 2240 *lb.*

But even in these States the first Table is often followed in weighing the coarsest commodities, such as *plaster*, *coal*, *iron*, *hemp*, &c.

One pound in Avoirdupois Weight is equal to 1*lb.* 2*oz.* 11*dwt.* 16*gr.* in Troy Weight.

One *dr.* is what part of an *oz.*? 1 *lb.* is what part of a *qr.*?

(104.) *Apothecaries' Weight*

Is used in compounding *medicines*, which, however, are bought and sold by Avoirdupois Weight.

20	grains (*gr.*)	make	1 scruple,	℈.
3	scruples		1 dram,	ʒ.
8	drams		1 ounce,	℥.
12	ounces		1 pound,	℔.

The *pound*, *ounce*, and *grain* in Apothecaries' Weight, are the same as in Troy Weight.

One *gr.* is what part of a scruple? One scruple is what part of a dram?

(105.) *Dry Measure*

Is used in measuring *grain*, *fruit*, *salt*, and, in general, all such commodities as are estimated in the *heap* or aggregate.

2 pints (*pt.*)	make	1 quart,	*qt.*
8 quarts	. . .	1 peck,	*pk.*
4 pecks	. . .	1 bushel,	*bu.*

The English *quarter*, in Dry Measure, is 8 bushels, and the *chaldron* is a coal measure of 36 bushels; but coal is usually sold by *weight*.

One *pt.* is what part of a *qt.*? 1 *qt.* is what part of a *pk.*?

(106.) *Beer Measure*

Is used in measuring *beer* and other *malt liquors*, and, in some places, *milk* and *water*.

2 pints, (*pt.*)	make	1 quart,	*qt.*
4 quarts	. . .	1 gallon,	*gal.*
36 gallons	. . .	1 barrel,	*bar.*
54 gallons	. . .	1 hogshead,	*hhd.*

The English *firkin* is 9 gallons; also 2 *firkins* make 1 *kilderkin*.

One *qt.* is what part of a *gal.*? 1 *gal.* is what part of a *bar.*?

(107.) *Wine Measure*

Is used in measuring *wine*, and, in general, all *liquids* excepting such as fall under Beer Measure.

4	gills (*gi.*)	make	1 pint,	*pt.*
2	pints		1 quart,	*qt.*
4	quarts . . .		1 gallon,	*gal.*
31½	gallons . . .		1 barrel,	*bar.*
63	gallons . . .		1 hogshead,	*hhd.*
2	hogsheads . .		1 pipe,	*pi.*
2	pipes . . .		1 tun,	*tun.*

Also 42 gallons make 1 *tierce*, and 84 *gal.* make one puncheon. The *gallon* in Wine Measure is .81' of a *gal.* in Beer Measure.

One *gi.* is what part of a *pt.*? 1 *gal.* is what part of a *bar.*?

(108.) *Linear Measure*.

Is used in measuring ***lines;*** that is, ***length, distance, height,*** &c.

12	inches (*in.*)	make	1 foot,	*ft.*
3	feet		1 yard,	*yd.*
5½	yards		1 rod or pole,	*rd.*
40	rods		1 furlong,	*fur.*
8	furlongs or 1760 *yd.*		1 mile,	*mi.*

Also 3 miles make 1 *league*, L.—used to express distances at sea.

The term *barley-corn* was formerly used for *one third* of an *inch:* the term *line* is sometimes used for *one twelfth* of an *inch.*

A *hand* is 4 inches—used in measuring the height of horses; a *fathom* is 6 feet—used in measuring the depth of water.

One *in.* is what part of a *ft.*? 1 *ft.* is what part of a *yd.*?

(109.) *Cloth Measure*

Is used in measuring *cloth*, *silk*, *lace*, &c. ; being a species of Linear Measure.

2¼	inches (*in.*)	make	1 nail,	*na.*
4	nails	. . . .	1 quarter,	*qr.*
4	quarters	. . .	1 yard,	*yd.*

Also 3 quarters make 1 Flemish Ell; 4 *qr.* 1¼ *in.*, 1 Scotch Ell; 5 *qr.*, 1 English Ell; and 6 *qr.* 1 French Ell.

The *yard* in Cloth Measure is the same as in Linear Measure.

One *in.* is what part of a *na.*? 1 *na.* is what part of a *qr*?

(110.) *Square Measure*

Is used in measuring *surfaces*, that is, any extension in *length* and *breadth*, without regard to thickness.

A *square inch* is an inch *long* and an inch *wide*; a *square foot* is a foot long and a foot wide; and so on.

Square measure is found by *multiplying together length and breadth*; thus 2 *in.* long and 1 *in.* wide makes 2 *square inches*; 2 *in.* long and 2 *in.* wide makes 4 *square inches*; and so on. Hence

144	square inches (*sq. in.*)	make	1 square foot,	*sq. ft*
9	square feet	. . . . . .	1 square yard,	*sq. yd*
30¼	square yards	. . . . . .	1 perch or *sq.* rod,	*P.*
40	perches	. . . .	1 rood,	*R.*
4	roods	. . . .	1 acre,	*A.*

Also 640 acres make 1 *square mile*, or Section of land; and 6 *miles square*, which is 36 square miles, make a Township.

An *inch square* is an inch long and an inch wide, being the same as a *square inch;* but 2 *inches square* is 2 *in.* long and 2 *in.* wide, which makes 4 *square inches;* 3 *inches* square is 3 *in.* long and 3 *in.* wide, which makes 9 *sq. in.*, &c.

4 inches square is how many square inches? 5 feet square is how many square feet? 10 miles square is how many square miles?

Which is the greater, *half a square foot* or a *square half foot?*

(111.) *Cubic or Solid Measure*

Is used in measuring *solids,* that is, any extension in *length, breadth,* and *thickness.*

A *cubic inch* is an inch *long*, an inch *wide*, and an inch *thick;* a *cubic foot* is a foot long, a foot wide, and a foot thick; and so on.

Cubic or solid measure is found by *multiplying together length, breadth, and thickness.* Thus 3 *in.* long, 2 *in.* wide, and 2 *in.* thick would make 12 *cubic or solid inches.* Hence

1728	cubic inches (*cu. in.*) make	1 cubic foot,	*cu. ft.*
27	cubic feet	1 cubic yard,	*cu. yd.*

Also 128 cubic feet make 1 *cord.* A cord of wood is usually put up 8 *ft.* long, 4 *ft.* wide, and 4 *ft.* high. One foot in length of such a pile is called a *cord foot;* and contains 16 cubic feet.

50 cubic feet of timber are allowed to weigh *a ton.* Of round timber such a quantity is allowed for *a ton* as, when hewn, will make 40 cubic feet.

231 *cu. in.* is the capacity of a *gallon* in Wine Measure;
282 *cu in.* is the capacity of a gallon in Beer Measure.

The British Imperial gallon contains 277.274 *cu. in.*; and the Imperial bushel, being 8 Imperial gallons, contains 2218.192 *cu. in.*

The standard bushel in the United States is the same as the British Winchester bushel, and contains 2150.4 cubic inches.

(112.) *Circular Measure*

Is used in expressing any part of the *circumference* of a circle, *latitude*, *longitude*, and the motions of the heavenly bodies.

60 seconds (″) make	1 minute,	′
60 minutes . . .	1 degree,	°
360 degrees, the *circumference of any circle.*		

A *degree* has no determinate linear extent; being the 360th part of the circumference on which it is taken, it is greater or less as that circumference is greater or less.

A degree on the *Equator of the Earth* is about 69½ miles. One *minute* on the circumference of the Earth is called a *geographical* or *nautical mile*; the mile of Linear Measure being denominated a *statute mile.*

(113.) *Measure of Time.*

Time is measured in *days* by the revolution of the Earth around its *axis*, and in *years* by the revolution of the Earth around the Sun.

60 seconds (*sec.*) make	1 minute,	*min.*
60 minutes	1 hour,	*hr.*
24 hours	1 day,	*da.*
7 days	1 week,	*wk.*
365 days	1 common year,	*yr.*
366 days, every 4th year, called *leap year.*		
100 years	1 century.	

A year consists of 12 *months*—January, February, March, April, May, June, July, August, September, October, November, and December.

The number of days in each month is as follows:

> Thirty days has September,
> April, June, and November;
> February has twenty-eight alone,
> And all the rest have thirty-one;
> But leap year comes one year in four,
> When February has one day more.

The true period of the Earth's revolution around the Sun is 365 *da.* 5 *hr.* 48 *min.* 51.6 *sec.* This constitutes the *astronomical year*, and contains an excess of nearly 6 hours above the common *civil year* of 365 days.

To adjust the civil reckoning to astronomical time, *one day* is added to February every fourth year, which makes the *leap year* of 366 days. But one day is more than the aforementioned excess amounts to in 4 years; and to obviate the error which would thence result, the following rule is adopted:

When the number of the year is divisible by 4, *without a remainder, it is made* LEAP YEAR; *but a centurial year, as* 1800, 1900, *&c., is not made leap year unless it is divisible by* 400, *without a remainder.*

(114.) *English Money*

Is the national currency of the kingdom of Great Britain.

4 farthings (*qr.*)	make	1 penny,	*d.*
12 pence	. . . .	1 shilling,	*s.*
20 shillings	. . . .	1 pound,	£.

Also 5 shillings make 1 *crown*, and 21 shillings 1 *guinea*.

English money is also called *Sterling* money, which is a term expressive of *purity*.

The *Pound Sterling* is represented by a gold coin called a *Sovereign*, which is valued by law in the United States at \$4.84; but its value as compared with our gold coin is \$4.866.

When the Sovereign is valued at \$4.84 we shall find that

One *shilling* sterling is equal to $24\frac{1}{5}$ *cents*.

(115.) *Values of a Shilling in Different States.*

At the adoption of Federal Money, by Act of Congress, in 1786, the paper money, in the English denominations, of the different States, had depreciated in *different degrees*—which caused the *pound*, *shilling*, &c., to have different *Federal* values in different States.

These different values of the *Shilling* are as follows:

1*s.* = $12\frac{1}{2}$ *cents*, or 8*s.* = \$1, in New York, Ohio, and North Carolina.

1*s.* = $13\frac{1}{3}$ *cents*, or $7\frac{1}{2}$*s.* = \$1, in New Jersey, Pennsylvania, Delaware, and Maryland.

1*s.* = $16\frac{2}{3}$ *cents*, or 6*s.* = \$1, in New England, Virginia, Kentucky, and Tennessee.

1*s.* = $21\frac{3}{7}$ *cents*, or $4\frac{2}{3}$*s.* = \$1, in South Carolina and Georgia.

In some of the new States the Shilling is valued according to the New York, in others according to the New England currency, and others adhere exclusively to Federal Money.

MONOMIALS AND POLYNOMIALS.

(116.) A Monomial Quantity, or simply a *Monomial*, is a quantity expressed by a *single name* of measuring units.

5 *pounds* is a *monomial;* 10 *yards* is a monomial.

A Polynomial Quantity, or simply a *Polynomial*, is a quantity expressed by *two or more names* of measuring units.

5 *pound* 3 *ounces* is a *polynomial;* 10*yd.* 3*qr.* 2*na.* is a polynomial.

A Polynomial is composed of two or more *monomials*, which may thence be called the *terms* of the polynomial.

How many *terms* has the Polynomial 3*hhd.* 9*gal.* 2*qt.* 3*pt.*?

Monomial quantities have usually been called *denominate numbers;* and Polynomials, *compound numbers*, or compound quantities.

REDUCTION.

(117.) The *Reduction* of a quantity, in general, consists in changing its *expression*, without altering its *value;* and this is done when the quantity is expressed in a *lower*, or a *higher* order of measuring *units*.

Thus 5 *dollars* reduced to *cents*, makes 500 cents; 234 cents, reduced to dollars, makes 2 dollars 34 cents.

The Reduction of *Monomials and Polynomials* presents a variety of cases, which all come within the applications of the following general Rule.

RULE XXVI.

(118.) *To Reduce a Monomial to a Different Order of Units, in the same kind of measure.*

1. To reduce a Monomial to a *lower order of units.*—Multiply the given monomial by the number of *the lower units* in *one* of the same order with the monomial; the product will be in the lower order of units.

2. To reduce a Monomial to a *higher order of units.*—Divide the given monomial by the number of *its own order* in *one* of the higher units; the quotient will be in the higher order of units, and the remainder, if any, will be in the *same order as the given monomial.*

EXAMPLE I.

To reduce 5£. 14*s.* 9*d.* to *pence.*

```
 5£. 14s. 9d.
 20
114s.
 12
1377d.                Ans. 1377 pence.
```

We first reduce the monomial 5£. to *shillings,* which are of a *lower order* of units. Since 20 shillings make 1£. (114), 5£. is 5 × 20*s.*, that is, 100*s.*; adding the 14*s.*, we have 114*s.*

We then reduce these 114*s.* to *pence,* which are of a *lower order* of units. As 12 pence make 1*s.* (114), 114*s.* is 114 × 12*d.*, that is, 1368*d.*, to which we add the 9*d.*, making 1377 pence.

The reduction of the given Polynomial thus consists in the successive reductions of *monomials,* according to the *first part of the* RULE.

EXERCISES.

These Exercises require a familiar acquaintance with the preceding TABLES. In all questions involving Avoirdupois Weight (103), 28 *pounds* are taken for a *qr.* of a *cwt.*

1. Reduce 4*lb.* 7*oz.* 13*dwt.* to pennyweights. *Ans.* 1113*dwt.*

2. Reduce 7*lb.* 10*dwt.* 2*gr.* to grains. *Ans.* 40562*gr.*
3. Reduce 3*T.* 2*cwt.* 3*qr.* to quarters. *Ans.* 251*qr.*
4. Reduce 9*cwt.* 1*qr.* 13*oz.* to ounces. *Ans.* 16589*oz.*
5. Reduce 14℥. 2ʒ. 12*gr.* to grains. *Ans.* 6852*gr.*
6. Reduce 8℔. 1ʒ. 15*gr.* to grains. *Ans.* 46155*gr.*
7. Reduce 15*bu.* 2*pk.* 7*qt.* to quarts. *Ans.* 503*qt.*
8. Reduce 9*bu.* 5*qt.* 1*pt.* to pints. *Ans.* 587*pt.*

9. Reduce 3*pi.* 1*hhd.* 40*gal.* to gallons. *Ans.* 481*gal.*
10. Reduce 4 *tuns* 5*hhd.* 3*qt.* to quarts. *Ans.* 5295*qt.*
11. Reduce 13*m.* 7*fur.* 25*r.* to rods. *Ans.* 4465*r.*
12. Reduce 30*m.* 16*fur.* 15*p.* to poles. *Ans.* 10255*p*
13. Reduce 20*yd.* 3*qr.* 2*na.* to nails. *Ans.* 334*na*
14. Reduce 31*yd.* 3*na.* 2*in.* to inches. *Ans.* 1124$\frac{3}{4}$*in*
15. Reduce 14*A.* 1*R.* 20*P.* to perches. *Ans.* 2300*P.*

16. Reduce 10*cu. yd.* 17*cu. ft.* to *cu. ft.* *Ans.* 287*cu. ft.*

17. Reduce 4*cu. yd.* 100*cu. in.* to *cu. in.* *Ans.* 186724*cu. in.*

18. Reduce 20*wk.* 5*da.* 3*hr.* 5*min.* to *min.* *Ans.* 208985*min*

19. Reduce 1*yr.* 100*da.* 20*hr.* 5*min.* to *min.* *Ans.* 670805*min.*

20. Reduce 9*A.* 13*P.* 4*sq. yd.* to *sq. yd.* *Ans.* 43957$\frac{1}{4}$*sq. yd.*

EXAMPLE II.

To reduce 3561 *farthings* to a Polynomial in £. *s.* &c.

```
4)3561qr.
 12)890d. 1qr.
  20)74s. 2d.
     3£. 14s. 2d. 1qr.
```

We first reduce the 3561 *qr.* to *pence*, which are of a *higher order* of units. Since 4 farthings make 1*d.*, 3561 *qr.* ÷ 4 gives 890 *pence*, with the remainder 1*qr.*

We reduce the 890*d.* to *shillings*, a higher order of units. Since 12 pence make 1*s.*, 890*d.* ÷ 12 gives 74 *shillings*, with the remainder 2*d.*

We next reduce the 74*s.* to *pounds*. Since 20*s.* make 1£, 74*s* ÷ 20 gives 3 *pounds*, with the remainder 14*s.*

The *last quotient* and the several *remainders* make the Polynomial

3£. 14*s.* 2*d.* 1*qr.*

The required Polynomial is obtained by the successive reductions of *monomials*, according to the *second part of the preceding* RULE.

21. Reduce 874 *pennyweights* to a polynomial in *lb.*, *oz.*, &c. *Ans.* 3*lb.* 7*oz.* 14*dwt.*

22. Reduce 785 *pounds* to a polynomial in *cwt.*, *qr.*, &c. *Ans.* 7*cwt.* 0*qr.* 1*lb.*

23. Reduce 730 *quarts* to a polynomial in *bu* , *pk.*, &c. *Ans.* 22*bu.* 3*pk.* 2*qt.*

24. Reduce 890 *gallons* to a polynomial in *tuns*, *pi.*, &c. *Ans.* 3 *tuns*, 1*pi.* 8*gal.*

25. Reduce 500 *inches* to a polynomial in *yd.*, *ft.*, &c. *Ans.* 13*yd.* 2*ft.* 8*in.*

26. Reduce 375 *nails* to a polynomial in *yd.*, *qr.*, &c. *Ans.* 23*yd.* 1*qr.* 3*na.*

27. Reduce 4750 *sq. in.* to a polynomial in *sq. yd.*, &c.
Ans. 3*sq. yd.* 5*ft.* 142*in.*.

28. Reduce 3795 *perches* to a polynomial in *A.*, *R.*, &c.
Ans. 23*A.* 2*R.* 35*P.*

29. Reduce 9374 *cu. in.* to a polynomial in *cu. ft.*, &c.
Ans. 5*cu. ft.* 734*in.*

30. Reduce 4034 *seconds* to a polynomial in *deg.*, *min.*, &c. *Ans.* 1*deg.* 7*min.* 14*sec.*

31. Reduce 3875 *seconds* to a polynomial in *hr.*, *min.*, &c. *Ans.* 1*hr.* 4*min.* 35*sec.*

32. Reduce 4375 *minutes* to a polynomial in *da.*, *hr.*, &c.
Ans. 3*da.* 0*hr.* 55*min.*

33. Reduce 3470 hours to a polynomial in *wk.*, *da.*, &c.
Ans. 20*wk.* 4*da.* 14*hr.*

EXAMPLE III.

To reduce the fraction $\frac{2}{7}$£ to *integers* in *shillings*, *pence*, &c.

The given monomial is to be reduced to *lower units;* we therefore apply the *first part of the preceding* RULE.

$$\tfrac{2}{7}£ \text{ is } \tfrac{2}{7} \text{ of } 20s., \text{ that is, } \tfrac{2}{7} \times 20s. = \tfrac{40}{7}s. = 5\tfrac{5}{7}s. \quad (61).$$

Reserving the *integer* 5*s.*, and reducing $\frac{5}{7}s.$ to *pence*, we say

$$\tfrac{5}{7}s. \text{ is } \tfrac{5}{7} \text{ of } 12d., \text{ that is, } \tfrac{5}{7} \times 12d. = \tfrac{60}{7}d. = 8\tfrac{4}{7}d.$$

Reserving the *integer* 8*d.*, and reducing $\frac{4}{7}$ to *farthings*, we say

$$\tfrac{4}{7}d. \text{ is } \tfrac{4}{7} \text{ of } 4qr., \text{ that is, } \tfrac{4}{7} \times 4qr. = \tfrac{16}{7}qr. = 2\tfrac{2}{7}qr.$$

The several *integers* reserved and the *last result* must now be arranged in a Polynomial; thus

$$5s.\ 8d.\ 2\tfrac{2}{7}qr.$$

In like manner a *decimal monomial* may be reduced to *integers*; thus

To reduce .23£ to *integers* in *s.*, *d.*, &c.

$$.23£ \text{ is } .23 \times 20s. = 4.60s.$$

Reserving the integer 4*s.*, we say

$$.60s. \text{ is } .60 \times 12d. = 7.20d.$$

Reserving the integer 7*d.*, we say

$$.20d. \text{ is } .20 \times 4qr. = 0.80qr.$$

Thus we find the Polynomial

$$4s.\ 7d.\ 0.80qr.$$

In reducing a monomial vulgar Fraction, as above, the products will usually be in the form of *improper fractions*, which must be reduced to *units*, (61).

34. Reduce $\frac{4}{9}$*lb.* to integers in *oz. dwt.*, &c.
Ans. 5*oz.* 6*dwt.* 16*gr.*

35. Reduce .17*lb.* to integers in *oz.*, *dwt.*, &c.
Ans. 2*oz.* 0*dwt.* 19.2*gr.*

36. Reduce $\frac{2}{3}$*qr.* to integers in *lb. oz.*, &c.
Ans. 18*lb.* 10*oz.* 10$\frac{2}{3}$*dr.*

37. Reduce .19*T.* to integers in *cwt.*, *qr.*, &c.
Ans. 3*cwt.* 3*qr.* 5.6*lb.*

38. Reduce $\frac{7}{12}$*pk.* to integers in *qt.*, *pt.*, &c.
Ans. 4*qt.* 1*pt.* 1$\frac{1}{3}$*gi.*

39. Reduce .31*bu.* to integers in *pk.*, *qt.*, &c.
Ans. 1*pk.* 1*qt.* 1.84*pt.*

40. Reduce $\frac{3}{5}$*pi.* to integers in *hhd.*, *gal.*, &c.
Ans. 1*hhd.* 12*gal.* 2$\frac{2}{5}$*qt.*

41. Reduce .6 *tun* to integers in *pi.*, *hhd.*, &c.
Ans. 1*pi.* 25*gal.* 1.6*pt.*

42. Reduce $\frac{387}{880}$*m.* to integers in *fur.*, *r.*, &c.
Ans. 3*fur.* 20*r.* 4*yd.*

43. Reduce .985*yd.* to integers in *qr.*, *na.*, &c.
Ans. 3*qr.* 3*na.* 1.71*in.*

44. Reduce $\frac{2}{3}$*A.* to integers in *R.*, *P.*, &c.
Ans. 2*R.* 26*P.* $20\frac{1}{6}$*yd.*

45. Reduce .83*A.* to integers in *R.*, *P.*, &c.
Ans. 3*R.*, 12*P.*, 24.2*yd.*

46. Reduce $\frac{7}{12}$ *cu. yd.* to integers in *cu. ft.*, &c.
Ans. 15*cu. ft.*, 1296*cu. in.*

47. Reduce .3 *cu. yd.* to integers in *cu. ft.*, &c.
Ans. 8*cu. ft.*, 172.8*cu. in.*

48. Reduce $\frac{5}{14}$ *degrees* to integers in *min.*, &c.
Ans. 21*min.* $25\frac{5}{7}$*sec.*

49. Reduce .37 *deg.* to integers in *min.*, &c.
Ans. 22*min.* 12*sec.*

50. Reduce $\frac{7}{10}$ *wk.* to integers in *da.*, *hr.*, &c.
Ans. 4*da.* 21*hr.* 36*min.*

EXAMPLE IV.

To reduce 10*s.* 6*d.* 2*qr.* to a Fraction of a *pound.*

We commence with the *lowest term* of the given polynomial, and reduce, successively, to *higher units*, by the *second part of the preceding* RULE.

Since 4 farthings make 1*d.*, 2*qr.* is $\frac{2}{4}$*d.* = $\frac{1}{2}$*d.* To this we add the 6*d.*, and divide by 12 to reduce to *shillings;* thus

$$6\tfrac{1}{2}d. \div 12 \text{ gives } \tfrac{13}{24}s.$$

To this result we add the 10*s*., and divide by 20 to reduce to *pounds;* thus

$$10\tfrac{13}{24}s. \div 20 \text{ gives } \tfrac{253}{480}£,$$

which is equal to the given polynomial.

The same reductions may be performed decimally; thus

$$2qr. \div 4 = .5d.; \quad 6.5d. \div 12 = .541's.;$$
$$10.541's. \div 20 = .527'£.$$

(119.) *Another Method of Reducing a Polynomial to a Fraction of a Higher Unit.*

1. Reduce the given polynomial to its *lowest named units*, for a numerator; and reduce the *higher unit* to the same name, for a *denominator.*

2. The Vulgar Fraction thus formed may, when requisite, be reduced to a *decimal.*

Thus, to reduce 10*s.* 6*d.* 2*qr.* to the fraction of a £.

$$10s.\ 6d.\ 2qr. = 506qr.; \quad \text{and } 1£ = 960qr.$$

The required Fraction will therefore be $\tfrac{506}{960}£ = \tfrac{253}{480}£$.

51. Reduce 8*oz.* 15*dwt.* 18*gr.* to a fraction of a *lb.*

Ans. $\tfrac{703}{960}$*lb.*

52. Reduce 10*oz.* 13*dwt.* 20*gr.* to a decimal of a *lb.*

Ans. .890'*lb.*

53. Reduce 2*qr.* 14*lb.* 12*oz.* to a fraction of a *cwt.*

Ans. $\tfrac{283}{448}$*cwt.*

54. Reduce 9*cwt.* 1*qr.* 10*lb.* to a decimal of a *T.*

Ans. .466'*T.*

55. Reduce 4*yd.* 2*ft.* 9*in.* to a fraction of a *rod.*

Ans. $\tfrac{59}{66}$*rod.*

56. Reduce 6*fur.* 30*p.* 4*yd.* to a decimal of a *m.*
Ans. 846'*m.*

57. Reduce 2*qr.* 3*na.* 2*in.* to a fraction of a *yd.*
Ans. $\frac{107}{144}$*yd.*

58. Reduce 1*qr.* 2*na.* $1\frac{1}{2}$*in.* to a decimal of a *yd.*
Ans. .416'*yd.*

59. Reduce 8*sq. ft.* 100*sq. in.* to a decimal of a *sq. yd.*
Ans. .966'*yd.*

60. Reduce 3*R.* 20*P.* 9*sq. yd.* to a decimal of an *A.*
Ans. .876'*A.*

61. Reduce 3*hr.* 4*min.* 20*sec.* to a decimal of a *day.*
Ans. .128'*da.*

A Polynomial may be reduced to the denomination of either of *its terms,* by reducing its other terms to that denomination, and *adding together its several parts.*

To reduce 7£ 10*s.* 8*d.* 2*qr.* to *shillings.*

7£ = 140*s.*; and 8*d.* 2*qr.* = .708'*s.*

We have then 140*s.*+10*s.*+.708'*s.*=150.708'*s.*

62. Reduce 25*bu.* 3*pk.* 3*qt.* 1*pt.* to *pk.*
Ans. 103.437'*pk.*

63. Reduce 2*T.* 15*cwt.* 3*qr.* 18*lb.* to *cwt.*
Ans. 55.910'*cwt.*

64. Reduce 4*m.* 5*fur.* 30*r.* 3*yd.* to *rods.*
Ans. 1510.545*r.*

65. Reduce 5 *tuns* 3*hhd.* 20*gal.* 1*qt.* to *gal.*
Ans. 1469.25*gal.*

66. Reduce 3*A.* 2*R.* 19*P.* 5*sq. yd.* to *P.*
Ans. 579.165*P.*

67. Reduce 10*T.* 15*cwt.* 1*qr.* 25*lb.* to *cwt.*
Ans. 215.473*cwt.*

68. Reduce 12*A.* 3*R.* 21*P.* 25*sq. yd.* to *A.*
Ans. 12.886'*A.*

Reduction of Currencies.

To reduce 75£. 15*s*. 9*d*. Sterling to Federal Money, according to the legal value of the pound sterling in the United States.

By reducing the 15*s*. 9*d*. to a *decimal* of a £, we shall have

75.7875£, which, multiplied by $4.84, gives

$366.8115.

It is evident that by dividing any sum in Federal Money by $4.84, we should obtain *pounds sterling*.

69. Reduce 5*s*. 6*d*. in New York to Federal Money

Express the whole sum in shillings and *decimal* of a *s*., and multiply by the number of *cents* in one shilling (115). In this multiplication, use *cents* in a *decimal* of a $, (93).

Ans. $0.687'.

70. Reduce 16*s*. 9*d*. in Georgia tc Federal Money.

Ans. $3.589'.

71. Reduce 14*s*. 8*d*. in Pennsylvania to Federal Money.

Ans. $1.955'.

72. Reduce 2*s*. 3*d*. in New England to Federal Money.

Ans. $0.375.

73. Reduce 100£. 15*s*. 10*d*. sterling to Federal Money, according to the legal value of the £ sterling in the United States. *Ans*. $487.828'.

74. Reduce $1000 to Sterling or English Money, according to the legal value of the £ sterling in the United States.

Ans. 206.611'£ = 206£. 12*s*. 2*d*. 2.56*qr*.

ADDITION, SUBTRACTION, &c., OF MONOMIALS.

(120.) *Dissimilar* Monomials in the *same kind of measure*, may be reduced to similar monomials, that is, monomials of the *same order of units;* and then be added together, or subtracted the one from the other.

To find the sum of $\frac{3}{4}$£ and $\frac{2}{3}s.$ in *pence.*

By reducing, we find $\frac{3}{4}$£$=180d.$, and $\frac{2}{3}s.=8d.$, (118..1);

$$\text{then } 180d.+8d.=188d.$$

In *Multiplying a monomial*, the multiplier can be regarded only as denoting *repetitions* of the multiplicand, or *a part* of the multiplicand (74); and the general Rules of multiplication are applicable.

(121.) In *Dividing a monomial*, when the quotient is to be regarded as the number of times the dividend *contains the divisor*, or the part the dividend is of the divisor*:* these two terms must be taken in the *same order of units.*

Thus, to find how many times $15s.$ is contained in 3£.

By reduction we find 3£$=60s.$*;* then $60\div15$ gives 4 *times.*

EXERCISES.

1. Find the Sum of $\frac{1}{4}T.$, $2cwt.$, and $\frac{1}{2}qr.$ in *lb.*

   Ans. 798*lb.*

2. Find the Sum of $10bu.$, $3\frac{1}{2}pk.$, and $2qt.$ in *bu.*

   Ans. $10\frac{15}{16}bu.$

3. Find the Sum of $\frac{1}{4}m.$ $\frac{3}{4}fur.$, and $20r.$ in *m.*

   Ans. $\frac{13}{32}m.$

4. Find the Sum of $3.7bu.$, $3pk.$, and $4qt.$ in *bu.*

   Ans. $4.575bu.$

5. Find the Sum of $.4lb.$, $3oz.$, and $.5dwt.$ in *dwt.*

   Ans. $156.5dwt.$

6. Find the Sum of .2*A.*, 3.1*R.*, and 4*P.* in *sq. yd.*
Ans. 4840*sq. yd.*

7. Find the Difference between 3*hhd.* and 4½*gal.*, in *qt.*
Ans. 738*qt.*

8. Find the Difference between .3*T.* and 7.3*cwt.* in *cwt.*
Ans. 1.3*cwt.*

9. Find the Difference between .45*mi.* and .3*fur.* in *m.*
Ans. .4125*m.*

10. Find the Difference between 3*A.* and 30½*P.* in *P.*
Ans. 449½*P.*

11. Find the Difference between .75*A.* and 1*R.* in *sq. yd.*
Ans. 2420*sq. yd.*

12. How many yards of silk, at 5 shillings per yard, can be purchased for 9½£? (121). *Ans.* 38*yd.*

13. How many pounds of butter, at 9 pence per pound, can be purchased for 21¼ shillings? *Ans.* 28⅓*lb.*

14. A person having a lot of ground which contained 1¼ acres, sold 39*P.* of it to his neighbor. What part of the lot did he sell? *Ans.* .195 of it.

15. A laborer who had 25 rods of ditching to execute, has accomplished 51½ yards of it. What part of the whole work has he accomplished? *Ans.* .374' of it.

16. An agriculturist bought, at one time, 2*T.* of plaster, and at another 15½*cwt.* How many acres of meadow can he sow with the whole, at the rate of 100*lb.* per acre?
Ans. 62$\frac{4}{25}$*A.*

17. A wine merchant has 3 *tuns* and 1 *pipe* of wine, which he wishes to put into barrels of 31½ *gal.* each. How many barrels will be requisite? *Ans.* 28*bbl.*

18. Bought, at different times, in adjoining parcels, 3*A.* 3½*R.*, and 20*P.* of ground. I wish to divide the whole into lots of 40*P.* each; how many of such lots will there be?
Ans. 16 lots.

ADDITION OF POLYNOMIALS.

Two or more Polynomials in the *same kind of measure*, might be reduced to similar monomials, and then added together. They may also be added under the polynomial form —which operation is usually called *Compound Addition*.

RULE XXVII.

(122.) *For Polynomial or Compound Addition.*

1. Set the polynomials with *similar terms one under another*, in separate columns.

2. Proceeding from right to left, add up each column of similar terms, and under each set its amount, *if less than the next higher unit.*

3. *If not less than such unit*, divide the amount by that number of its own name which makes the next higher unit; set the *remainder*, if any, under the column, and add the *quotient* to the next column of similar terms.

EXAMPLE.

	£.	s.	d.
To add together	13 ,	17 ,	2 ,
	49 ,	18 ,	4 ,
and	84 ,	9 ,	5 .
	148 ,	4 ,	11 .

Having set *pounds* under *pounds*, shillings under shillings, &c., we add up the column of *d.*, and set down the amount 11*d.*, which is less than 1*s.*

Adding up the column of *s.*, we find the amount to be 44*s.*, which we divide by 20, since 20*s.* make 1£ ; the *remainder* 4*s.* is set under that column, and the *quotient* 2£ is added to the next column, (118 . . . 2).

EXERCISES.

1. Find the Sum of 125£. 13*s.* 5*d.*, 19£. 4*s.* 10*d.* 2*qr.*, and 12£. 16*s.* 8*d.* 3*qr.* *Ans.* 157£. 15*s.* 0*d.* 1*qr.*

2. Find the Sum of 23*lb.* 8*oz.* 16*dwt.*, 36*lb.* 5*oz.* 8*dwt.* 16*gr.*, and 300*lb.* 2*oz.* 9*dwt.* 13*gr.*

Ans. 360*lb.* 4*oz.* 14*dwt.* 5*gr.*

3. Find the Sum of 3*T.* 9*cwt.* 2*qr.* 16*lb.*, 10*T.* 15*cwt.* 1*qr.*, and 54*T.* 7*cwt.* 3*qr.* 20*lb.*

Ans. 68*T.* 12*cwt.* 3*qr.* 8*lb.*

4. Find the Sum of 13*bu.* 2*pk.* 7*qt.* 1*pt.*, 150*bu.* 1*pk.* 5*qt.*, and 200*bu.* 3*pk.* 5*qt.* 1*pt.* *Ans.* 365*bu.* 0*pk.* 2*qt.*

5. Find the Sum of 3*hhd.* 20*gal.* 3*qt.*, 29*hhd.* 13*gal.* 2*qt.*, and 200*hhd.* 12*gal.* 1*qt.* *Ans.* 232*hhd.* 46*gal.* 2*qt.*

6. Find the Sum of 4*m.* 5*fur.* 20*p.*, 29*m.* 3*fur.* 16*p.* 4*yd.*, and 34*m.* 7*fur.* 13*p.* 1*yd.* *Ans.* 69*m.* 0*fur.* 9*p.* 5*yd.*

7. Find the Sum of 15*yd.* 3*qr.* 1*na.*, 75*yd.* 3*qr.* 3*na.* 1*in.*, and 100*yd.* 1*qr.* 2*na.* 1*in.* *Ans.* 192*yd.* 0*qr.* 2*na.* 2*in.*

8. Find the Sum of 24*A.* 3*R.* 20*P.*, 100*A.* 2*R.* 16*P.*, 4*sq. yd.*, and 95*A.* 1*R.* 29*P.* 20*sq. yd.*

Ans. 220*A.* 3*R.* 25*P.* 24*sq. yd.*

9. Find the sum of 200*A.* 1*R.* 24*P.* 20*sq. yd.*, 50*A.* 2*R.*, and 500*A.* 3*R.* 19*P.* 16*sq. yd.*

Ans. 751*A.* 3*R.* 4*P.* 5¾*sq. yd.*

10. A farmer raised from one field 150*bu.* 3*pk.* of wheat, from another 75*bu.* 1*pk.* 7*qt.*, and from another 200*bu.* 5*qt.* What was the whole quantity of wheat?

Ans. 426*bu.* 1*pk.* 4*qt.*

11. A merchant has in one piece 34*yd.* 3*qr.* of cloth, in another 21*yd.* 2*qr.*, and in two others each 19*yd.* 3½*qr.* How many yards has he in the four pieces? *Ans.* 96 yards.

12. An agriculturist sold at one time 3*T.* 19*cwt.* 2*qr.* of hemp, at another 5*T.* 13*cwt.*, and at another 2*T.* 16*cwt.* 3*qr.* 20*lb.* What amount of hemp did he sell?

Ans. 12*T.* 9*cwt.* 1*qr.* 20*lb.*

SUBTRACTION OF POLYNOMIALS.

Two Polynomials in the *same kind of measure* might be reduced to similar monomials, and then subtracted the one from the other. But the subtraction may be performed on the polynomials—forming what is usually called *Compound Subtraction.*

RULE XXVIII.

(123.) *For Polynomial or Compound Subtraction.*

1. Set the less polynomial under the greater, with *similar terms one under the other.*

2. Proceeding from right to left, subtract each lower term from the one above it, and underneath set the remainder.

3. If the lower term *exceed the upper*, add to the upper term that number of its own name which makes the next higher unit; from the sum subtract the lower term, and add 1 to the next lower term, before subtracting it.

EXAMPLE.

To subtract 85£. 13*s.* 7*d.* from 100£. 10*s.*

£.	s.	d.
100	10	0
85	13	7
14	16	5

We set *pounds* under *pounds*, and shillings under shillings, &c.; having supplied the place of *pence* in the upper line with 0.

As we cannot take 7*d.* from 0*d.*, we add 12*d.*, which makes 1*s.*, to the upper term. and say 7*d.* from 12*d.* leaves

5*d.* Then 1*s.* to 13*s.* makes 14*s.*; and since this exceeds the 10*s.*, we add 20*s.*, equal to 1£, and say 14*s.* from 30*s.* leaves 16*s.*; then 1 to 5 makes 6, and 6 from 10 leaves 4, &c.

EXERCISES.

1. Find the Difference between 60£. 17*s.* and 35£. 13*s.* 6*d.*
Ans. 25£. 3*s.* 6*d.*

2. Find the Difference between 200*lb.* 9*oz.* 1*dwt.* and 180*lb.* 10*oz.* *Ans.* 19*lb.* 11*oz.* 1*dwt.*

3. Find the Difference between 150 *T.* 13*cwt.* and 75 *T.* 3*cwt.* 1*qr.* *Ans.* 75 *T.* 9*cwt.* 3*qr.*

4. Find the Difference between 100*bu.* 2*pk.* and 21*bu.* 1*pk.* 1*qt.* *Ans.* 79*bu.* 0*pk.* 7*qt.*

5. Find the Difference between 21 *tuns* 2*hhd.* 3*gal.* and 3 *tuns* 13*gal.* *Ans.* 18 *tuns* 1*hhd.* 53*gal.*

6. Find the Difference between 150*yd.* 3*qr.* 2*na.* and 2*qr.* 3*na.* *Ans.* 150*yd.* 0*qr.* 3*na.*

7. Find the Difference between 123*A.* 2*R.* and 30*A.* 3*R.* 13*P.* *Ans.* 92*A.* 2*R.* 27*P.*

8. A jeweler purchased 34*lb.* 9*oz.* 13*dwt.* of silver ware, of which he has sold 19*lb.* 4*oz.* 18*gr.* What quantity has he remaining? *Ans.* 15*lb.* 5*oz.* 12*dwt.* 6*gr.*

9. An agriculturist raised 30 *T.* 13*cwt.* 1*qr.* of hemp, of which he has sent to market 21 *T.* 15*cwt.* 21*lb.* What quantity of hemp has he still on hand? *Ans.* 8 *T.* 18*cwt.* 0*qr.* 7*lb.*

10. A farmer raised 500*bu.* 3*pk.* 7*qt.* of wheat. Having sold 300*bu.* 2*pk.* 5*qt.* of this crop, what quantity of wheat has he still unsold? *Ans.* 200*bu.* 1*pk.* 2*qt.*

11. A speculator bought a tract of land containing 960*A.* 2*R.* 26*P.* Having sold from the tract to the amount of 509*A.* 3*R.*, how much of it remains unsold?
Ans. 450*A.* 3*R.* 26*P.*

Interval of Time between two given Dates.

(124.) *In subtracting a prior from a later date,*—Add to the days elapsed in the month of the later date (when requisite), as many as make the month of the *prior date ;* and allow 12 months to a year.

How long was it from March 20th, 1823, to April 10th, 1848?

y.	*m.*	*da.*
1848	4	10
1823	3	20
25	0	21

March being the 3*d*, and *April* the 4*th* month in the year, we designate them by these numbers, respectively.

Since March has 31 days, 11 days of it remained after the 20th. Adding these 11 days to the 10 days of April, we have 21 days.

But $31-20+10=10+31-20$; hence the 21 days (and the surplus days in every case) will be found as above directed (124).

12. Find the interval of time between May 16th, 1834, and September 4th, 1848. *Ans.* 14*y.* 3*m.* 19*da.*

13. A person was born on the 3d of April, 1807; required his age on the 15th of December, 1854.

Ans. 47*y.* 8*m.* 12*da.*

14. How long was it from the discovery of America, October 21st, 1492, to the founding of Jamestown, May 23d, 1607? *Ans.* 114*y.* 7*m.* 2*da.*

15. How long was the founding of Jamestown prior to the birth of Washington, February 22d, 1732? and what was Washington's age at his decease, December 14, 1799?

Ans. 124*y.* 8*m.* 30*da.*; and 67*y.* 9*m.* 21*da.*

MULTIPLICATION OF POLYNOMIALS.

Any Polynomial quantity might be reduced to a monomial, and then multiplied. When the multiplication is performed on the polynomial, it forms what is usually called *Compound Multiplication.*

RULE XXIX.

(125.) *For Polynomial or Compound Multiplication.*

1. Proceeding from right to left, multiply each term, separately, and under each set its product, *if less than the next higher unit.*

2. *If not less than such unit,* divide the product by that number of its own name which makes the next higher unit; set the *remainder,* if any, under the term, and add the *quotient* to the product of the next term.

EXAMPLE.

	£.	s.	d.	
To multiply	25	16	3	by 3.
			3	
	77	8	9	

Proceeding from right to left, we say 3 times 3*d.* is 9*d.*; 3 times 16*s.* is 48*s.*, which we divide by 20, since 20*s.* make 1£.; the remainder 8*s.* is set under that term, and the quotient 2£. is added to the next product, (118 . . . 2).

This Rule depends on the same principles as the Rule for polynomial Addition.

EXERCISES.

1. What should be paid for 4 yards of broadcloth, at 1£. 3*s.* 8*d.* per yard? *Ans.* 4£. 14*s.* 8*d.*

2. Required the aggregate weight of 5 silver goblets, each weighing 1*lb.* 9*oz.* 13*dwt.* *Ans.* 9*lb.* 5*dwt.*

3. Bought 6 loads of hay, whose average weight was 19*cwt.* 3*qr.* 23*lb.* What was their entire weight?

Ans. 119*cwt.* 2*qr.* 26*lb.*

4. An apothecary sold 7 bottles of quinine, each weighing 12*oz.* 13*dr.* What was the weight of the whole?

Ans. 5*lb.* 9*oz.* 11*dr.*

5. A brewer sold to each one of 9 men 18*gal.* 3*qt.* 1*pt.* of beer. What quantity did he sell in all?

Ans. 169*gal.* 3*qt.* 1*pt.*

6. A vintner bought of 10 persons each 3*hhd.* 24*gal.* 2*qt.* of wine. How much did he buy from them all?

Ans. 33*hhd.* 56*gal.*

7. If a man travel at the rate of 33*m.* 7*fur.* 30*r.* per day, how far will he travel in 11 days? *Ans.* 373*m.* 5*fur.* 10*r.*

8. A merchant sold 19 pieces of linen, each piece containing 16*yd.* 3*qr.* 2*na.* How many yards did he sell?

Ans. 320*yd.* 2*qr.* 2*na.*

9. A farmer has 13 fields whose average contents are 24*A.* 3*R.* 10*P.* How much land do all the fields contain?

Ans. 322*A.* 2*R.* 10*P.*

10. If a steamboat run at the rate of 12*m.* 3*fur.* 19*r.* per hour, what distance will it run in 14 hours?

Ans. 174*m.* 26*r.*

11. A teamster hauled 16 loads of coal, averaging 2*T.* 17*cwt.* 1*qr.* each. What was their entire weight?

Ans. 45*T.* 16*cwt.*

12. A manufacturer made 17 pieces of cloth, measuring 39*yd.* 3*qr.* each. How much cloth was there in all?

Ans. 675*yd.* 3*qr.*

13. An agriculturist had 15 acres of ground in hemp, and found his crop to be at the rate of 17*cwt.* 3*qr.* 10*lb.* per acre. What was the entire crop? *Ans.* 13*T.* 7*cwt.* 2*qr.* 10*lb.*

14. A brewer filled 3 hogsheads with beer, out of which he has sold to the amount of 75*gal.* 3*qt.* 1*pt.* What quantity remains of the 3 hogsheads?

Ans. 1*hhd.* 32*gal.* 0*qt.* 1*pt.*

15. A merchant bought 7 pieces of silk, containing 47*yd.* 2*qr.* each. Having sold to one lady 11*yd.*, and to three others each 10*yd.* 3*qr.*, how many yards of the silk remain on hand? *Ans.* 289*yd.* 1*qr.*

DIVISION OF POLYNOMIALS.

Any Polynomial quantity might be reduced to a monomial, and then divided. When the division is performed on the polynomial, it forms what is usually called *Compound Division*

RULE XXX.

(126.) *For Polynomial or Compound Division.*

1. Proceeding from *left to right*, divide each term of the polynomial, for the corresponding term of the quotient.

2. *When a remainder occurs*, reduce it to the next *lower units;* add the term in the same order of units, if any, and divide the result for the quotient term in that order of units.

EXAMPLE.

To divide 285£. 17*s.* 5*d.* by 3; that is, to find $\frac{1}{3}$ of this polynomial.

£.	*s.*	*d.*	
3)285	17	5	*qr.*
95	5	9	$2\frac{2}{3}$

Proceeding from left to right, we find 3 in 285, 95 times; 3 in 17, 5 times, with 2*s.* over; reducing the 2*s.* to *pence*, and adding the 5*d*, we have 29*d.*; then 3 in 29, 9 times, with 2*d.* over; reducing this 2*d.* to *qr.*, we have 8*qr.*; then 3 in 8 gives $2\frac{2}{3}$*qr.*

EXERCISES.

1. If 4 yards of cloth sell for 9£, 17*s.* 8*d.*, what is the price per yard? *Ans.* 2£. 9*s.* 5*d.*

2. If 5 silver candlesticks weigh 10*lb.* 7*oz.* 18*dwt.*, what is the average weight of each? *Ans.* 2*lb.* 1*oz.* $11\frac{3}{5}$*dwt.*

3. If 6 barrels of pork weigh 12*cwt.* 2*qr.* 23*lb.*, what is the average weight of each? *Ans.* 2*cwt.* 0*qr.* $13\frac{1}{6}$*lb.*

4. If 7 acres of ground produce 150*bu.* 2*pk.* 1*qt.* of wheat, what is the produce per acre? *Ans.* 21*bu.* 2*pk.* $\frac{1}{7}$*qt.*

5. If 8 casks together contain 250*gal.* 3*qt.* 1*pt.* of spirits, what are the average contents of each?

Ans. 31*gal.* 1*qt.* $3\frac{1}{2}$*gi.*

6. If a person travel 300*m.* 2*fur.* 25*p.* in 9 days, at what rate will he travel per day? *Ans.* 33*m.* 2*fur.* $38\frac{1}{3}$*p.*

7. A merchant has 10 pieces of cloth, of equal length, and together containing 575*yd.* 2*qr.* 3*na.* What is the length of each piece? *Ans.* 57*yd.* 2*qr.* $1\frac{1}{10}$*na.*

8. A farmer having a tract of land containing 486*A.* 2*R.* 30*P.*, wishes to divide it into 12 fields of equal size. What quantity will be in each? *Ans.* 40*A.* 2*R.* $9\frac{1}{6}$*P.*

9. A cellar measuring 1570 *cu. yd.* 18 *cu. ft.* was excavated by a laborer in 30 days. At what rate did he dig per day? *Ans.* 52*cu. yd.* $9\frac{3}{5}$*cu. ft.*

(127.) *When the Divisor is a Polynomial.*

Reduce the Divisor and Dividend both to *monomials* of the same order of units, and divide as in abstract numbers.

10. How many yards of silk at 7*s.* 6*d.* per yard, may be purchased for 3£. 14*s.* 10*d.*?

7*s.* 6*d.* = 90*d.*, and 3£. 14*s.* 10*d.* = 898*d.* (118 . . .1)

then 898 ÷ 90 gives 9.977' yards.

11. How many hundredweight of iron, at 19*s*. 8*d*. per *cwt*., may be bought for 20£. 15*s*.? *Ans*. 21.101'*cwt*.

12. How many acres of ground can be sown with 75*bu*. 1*pk*. of wheat, allowing 1*bu*. 3*pk*. to an acre? *Ans*. 43 acres.

13. In what time will a ship perform a voyage of 1000*L*. 2*m*., if she sail at the rate of 60*L*. 1*m*. per day?

Ans.16.585 days.

14. How many spoons weighing 3*oz*. 6*dwt*. each can be made out of 6*lb*. 7*oz*. 4*pwt*. of silver? *Ans*. 24 spoons.

15. A planter has 113*T*. 9*cwt*. 2*qr*. of sugar, which he wishes to put into hogsheads containing 12*cwt*. 2*qr*. each. How many hogsheads will be requisite?

Ans. 181.56 hogsheads.

DUODECIMALS.

(128.) Duodecimals are a kind of polynomials which result from conceiving a *linear*, *square*, or *cubic foot* to be divided into 12 *equal parts*, each of these parts again into 12 equal parts; and so on.

12*ths* of a linear, square, or cubic *foot* are called *primes*;

12*ths* of a prime are called *seconds*;
12*ths* of a second are called *thirds*, &c.

Primes, seconds, thirds, &c., are denoted by one, two, three, &c., *accents*, which are called the *indices* of the terms; thus

$3'\ \ 4''\ \ 5'''$, 3 *primes*, 4 *seconds*, 5 *thirds*.

Linear, Square, and Cubic inches expressed in Duodecimals.

(129.) In *linear* measure, *primes* are linear inches; in *square* measure, *seconds* are square inches; in *cubic* measure, *thirds* are cubic inches.

Thus 1′ is $\frac{1}{12}$ of a *ft.*, which is 1*in.* in linear measure;
1″ is $\frac{1}{12}$ of $\frac{1}{12}$, or $\frac{1}{144}$, of a *ft.*, which is 1 *sq. in.* in square measure.
1‴ is $\frac{1}{12}$ of $\frac{1}{12}$, of $\frac{1}{12}$, or $\frac{1}{1728}$, of a *ft.*, which is 1 *cu. in.*, in cubic measure.

In 3′, *square* measure, how many *sq.* inches? In 5′? In 8′?
In 2″, cubic measure, how many *cu.* inches? In 4″? In 6″?

Square and Cubic measure—how found.

(130.) Square measure, or measure of *surface*, is found by multiplying together *length* and *breadth*, in the same order of units.

Thus 4 *in.* long and 3 *in.* wide make 12 *square inches.*

Cubic measure, or measure of *solidity*, is found by multiplying together *length*, *breadth*, and *thickness*, in the same order of units.

4 *in.* long, 3 *in.* wide, and 2 *in.* thick, makes 24 *cu. inches.*

Product of two Duodecimal Terms.

(131.) The product of any two terms in Duodecimals has for its *index* the sum of the indices of the two terms;—*feet* being understood to have *no index.*

Thus if we take 3*ft.* in length and 2′ in breadth, we have

$$3\,ft. \times 2' = 3ft. \times \tfrac{2}{12}ft. = \tfrac{6}{12}\,sq.\,ft.\ (130) = 6'\,sq.\,ft.$$

And if we take 3′ in length and 2″ in breadth, we have

$$\tfrac{3}{12}ft. \times \tfrac{2}{144}ft. = \tfrac{6}{1728}\,sq.\,ft. = 6'''\,sq.\,ft.$$

In these examples, the products 6′ and 6‴ have their indices (′ and ‴) equal, respectively, to the *sums of the indices* of the two terms multiplied together.

Reduction, Addition, &c., are performed on Duodecimals in the same manner as on other polynomials. We have here however, a peculiar case in Multiplication, and also one in Division.

RULE XXXI.

(132.) *To Multiply one Duodecimal Polynomial by another.*

1. Proceeding from *right to left*, multiply each term of the multiplicand by each term of the multiplier; mark each product term with the proper *index* (131), and set similar terms one under another.

2. When any product below *feet* is 12 or more, divide it by 12; set down the remainder, if any, and add the quotient to the next product.

3. Add up the similar product terms, as in polynomial Addition (122), for the entire product.

EXAMPLE.

To find the number of square feet in a plank 16*ft.* 8*in.* long, and 2 *ft.* 5*in.* wide, (130).

16*f.*	8′	
2*f.*	5′	
6	11′	4″
33	4′	
40 *s.f.*	3′	4″

Marking inches or *primes* with the index ′, 8′ × 5′ gives 40″; the product 40 having an index ″ equal to the sum of the indices of the two terms 8′ and 5′ (131); 40″ ÷ 12 gives 3′ 4″ (118 . . . 2); we set 4″ on the right, and then say 16*ft.* × 5′ gives 80′; adding the 3′, and dividing by 12, we obtain 6*sq. ft.* 11′

Next, $8' \times 2ft.$ gives $16'$, equal to $1sq.\,ft.\ 4'$; setting the $4'$ under $11'$, and adding 1 to 16×2, we find $33\,sq.\,ft.$ The two polynomial products are then added together, for the entire product,

$$40sq.\,ft. \quad 3' \quad 4''$$

To find the number of *sq. in.* in the $3'\ 4''$, we have

$$3' \times 12 + 4'' = 40'' \text{ or } \textit{square inches } (129).$$

Without employing Duodecimals, we have

$$16ft.\ 8in. = 16\tfrac{8}{12}ft.;\ 2ft.\ 5in. = 2\tfrac{5}{12}ft.$$

$$\text{and } 16\tfrac{8}{12} \times 2\tfrac{5}{12} = 40\tfrac{5}{18}\ sq.\,ft.$$

If the given length and breadth were reduced to *inches*, and then multiplied together, we should find the product in *square inches*, which would be reduced to *square feet* by dividing it by 144.

The measure of a surface, as expressed in square feet, square inches, &c., is called its *area*.

EXERCISES.

1. How many square feet are there in a pavement which is 30*ft.* 10*in.* long, and 7*ft.* 5*in.* wide?

Ans. 228 *sq. ft.* 8′ 2″ = 228 *sq. ft.* 98*sq. in.*

2. How many square feet of plank will make a close fence 80*ft.* 8*in.* long, and 6*ft.* 4*in.* high?

Ans. 510*sq. ft.* 10′ 8″=510*sq. ft.* 128*sq. in.*

3. How many square feet, and also how many square yards, are in a ceiling 18*ft.* 5*in.* long, and 12*ft.* 10*in.* wide?

Ans. 236 *sq. ft.* 50*in.* = 26*sq. yd.* 2*ft.* 50*in.*

4. How many square yards of plastering would be required for one side of a wall which is 50*ft.* 6*in.* in length, and 20*ft.* 4*in.* in height? *Ans.* 114*sq. yd.* 120*sq. in.*

5. Find the number of *cubic feet* in a piece of timber which is 9*ft.* 10*in.* long, 3*ft.* 4*in.* wide, and 2*ft.* 6*in.* thick.

Multiplying the *length* by the *breadth*, we get the product

32*sq. ft.* 9′ 4″.

Multiplying this product by the *thickness*, we get the *solidity*

81 *cu. ft.* 11′ 4″, (130).

To find the number of *cubic inches* in the 11′ 4″, we must reduce these terms to *thirds;* thus $11' \times 12 + 4'' = 136''$, and $136'' \times 12 = 1632'''$ or *cubic inches* (129).

Without employing Duodecimals, the dimensions might be taken in *feet* and *fractions* of a foot, and thus multiplied together.

If the dimensions were reduced to *inches*, and then multiplied together, we should find the solidity in *cubic inches*, which would be reduced to *cubic feet* by dividing it by 1728.

6. How many cubic feet are there in a hewn log which is 22*ft.* 8*in.* long, 1*ft.* 10*in.* wide, and 1*ft.* 2*in.* thick?

Ans. 48 *cu. ft.* 5′ 9″ 4‴=48*cu. ft.* 832*cu. in.*

7. How many cubic feet are there in a piece of scantling which is 15*ft.* long, 1*ft.* 2*in.* wide, and 8 inches thick?

Ans. 11 *cu. ft.* 8′=11*cu. ft.* 1152*cu. in.*

8. How many cubic feet were dug from a cellar which measures 42*ft.* 10*in.* long, 12*ft.* 6*in.* wide, and 8 feet deep? How many cubic yards?

Ans. 4283*cu. ft* 4′= 158*cu. yd.* 17*cu. ft.* 4′.

(133.) *To Divide one Duodecimal Polynomial by another.*

The method of doing this is shown in the following

EXAMPLE.

To find the *breadth* of a surface whose *area* is 40*sq. ft.* 3′ 4″ and *length* 16*ft.* 8*in.*

16*f.* 8′)40*f.*	3′	4″	(2*ft.* 5′
33*f.*	4′		
6*f.*	11′	4″	
6*f.*	11′	4″	
0	0	0	

We divide the left hand term, 16*ft.*, of the divisor into the left hand term, 40*ft.*, of the dividend, and obtain the quotient term 2*ft.* We multiply the *entire divisor* by the 2*ft.*, and subtract the product 33*f.* 4′ from the corresponding part of the dividend.

To the remainder, 6*f.* 11′, we subjoin the next term, 4″ of the dividend; we now divide 16*ft.* into 6*f.* 11′ which is 83′, and obtain the quotient term 5′; we multiply the entire divisor by the 5′, and find that the operation is completed.

The proper *index* for any quotient term may always be known, by considering that the left hand term of the divisor × the quotient term must produce a term with the same index as that into which the division is made; thus 16*ft.* divided into 83′ gives 5′, because 16*ft.* × 5′ produces 80′ (131).

Without employing Duodecimals, the given polynomials might be taken in *feet* and *fractions* of a foot.

9. The area of a surface is 228 *sq. ft.* 8′ 2″, and its length is 30*ft.* 10*in.*; what is its breadth? *Ans.* 7*ft.* 5*in.*

10. The area of a surface is 510 *sq. ft.* 10′ 8″, and its breadth is 6*ft.* 4*in.*, what is its length?

Ans. 80*ft.* 8*in.*

ALIQUOT PARTS.

(134.) An ALIQUOT PART of a quantity is an exact *half*, *third*, or *fourth*, and so on, of the quantity.

Thus 10*s*. is an *aliquot part* of 1£, being $\frac{1}{2}$ of 1£.

What aliquot part is 6 *gr*. of 1 *dwt.*?	5 *dwt*. of 1 *oz.*?	Of 2 *oz.*?
What aliquot part is 7 *lb*. of 1 *qr.*?	2 *qr*. of 1 *cwt.*?	Of 3 *cwt*.
What aliquot part is 2 *qt*. of 1 *gal.*?	1 *qt*. of 1 *pk.*?	Of 3 *pk.*?
What aliquot part is 8 *r*. of 1 *fur.*?	2 *fur*. of 1*m.*?	Of 4 *m.*?
What aliquot part is 10*P*. of 1 *R.*?	2 *R*. of 1 *A.*?	Of 5 *A.*?

It is often convenient to regard the lower orders of units in a *polynomial multiplier* as aliquot parts of one or more of the higher units.

EXAMPLE.

To find the value of 2*lb*. 5*oz*. 12*dwt*. of silver ware, at \$45.12½ per *lb*.

			\$45.125			
			2			
The value of	2*lb*.	is	90.250 ;	*twice* the value of 1*lb*.		
" "	of 4*oz*.	is	15.0416,	$\frac{1}{3}$ of	"	of 1*lb*.
" "	of 1*oz*.	is	3.7604,	$\frac{1}{4}$ of	"	of 4*oz*.
" "	of 10*dwt*.	is	1.8802,	$\frac{1}{2}$ of	"	of 1*oz*.
" "	of 2*dwt*.	is	.3760,	$\frac{1}{5}$ of	"	of 10*dwt*.
			\$111.3082,	*value of the whole.*		

For the *aliquot parts* we say, 4*oz*. is $\frac{1}{3}$ of a *lb*., 1*oz*. is $\frac{1}{4}$ of 4*oz*., 10 *dwt*. is $\frac{1}{2}$ of an *oz*., 2*dwt*. is $\frac{1}{5}$ of 10 *dwt*.—The values of these several parts, added to the value of 2*lb*., make up the value of the whole quantity.

For convenience, the aliquot parts should be so apportioned, when practicable, that none of the divisors shall exceed 12.

No general Rule can be given for calculating by *aliquot parts*; but practice will soon render the method easy.

EXERCISES.

1. Find the value of *3lb. 4oz. 17dwt.* of jewelry, at \$50.50 per *lb.*

Aliquot parts may be taken for the *oz.* and *dwt.* thus; *4oz.* is $\frac{1}{3}$ of a *lb.*, *10dwt.* is $\frac{1}{8}$ of *4oz.*, *5dwt.* is $\frac{1}{2}$ of *10dwt.*, and *2dwt.* is $\frac{1}{5}$ of *10dwt.* *Ans.* \$171.909'.

2. Find the sum that should be paid for *13cwt. 2qr. 14lb.* of soap, at \$3,62$\frac{1}{2}$ per *cwt.* *Ans.* 49.39'.

3. Find the sum that should be paid for *3 T. 10cwt. 3qr.* of iron, at \$30.37$\frac{1}{2}$ per ton. *Ans.* \$107.45'.

4. A farmer sold *125bu. 3pk. 1qt.* of wheat, at \$0.87$\frac{1}{2}$ per bushel. What did the whole amount to?

Ans. \$110.058.

5. A merchant sold *10yd. 3qr. 2na.* of silk, at \$1.50 per yard. What did the whole amount to? *Ans.* \$16.3125.

6. A townsman bought a lot of ground containing *3A. 2R. 25P.*, at \$75 per acre. What did he pay for the lot?

Ans. \$274.218'.

7. An agriculturist sold *19cwt. 3qr. 21lb.* of hemp, at \$7.50 per hundredweight. What did the hemp amount to?

Ans. \$149.53'.

8. A farmer bought *12bu. 2pk. 5qt. 1pt.* of clover seed, at \$10 per bushel. What did the whole amount to?

Ans. \$126.718'.

9. Allowing an acre of ground to produce 30 bushels of wheat, what would be the produce of a field containing *20A. 1R. 24P.*? *Ans.* 612 bushels.

10. A merchant sold *15yd. 3qr. 3na.* of silk, at 1.37$\frac{1}{2}$ per yard, and *5yd. 2qr.* of lace, at \$2.50 per yard. What did the whole amount to? *Ans.* \$35.661'.

11. Find what would be the expense of putting up 230*r.* $2\frac{3}{4}$*yd.* of fencing, at the rate of $0.75 per rod.

Ans. $172.875.

12. Allowing an acre of meadow ground to produce 2*T.* 16*cwt.* 2*qr.* of hay, what would be the produce of a meadow containing 12*A.* 2*R.* 10*P.*? *Ans.* 35*T.* 9*cwt.* $3\frac{1}{8}$*qr.*

MISCELLANEOUS EXERCISES

ON MONOMIALS AND POLYNOMIALS, CURRENCIES, DUODECIMALS, AND ALIQUOT PARTS.

1. What will 2*lb.* 8*oz.* 13*dwt.* of silver ware amount to at the rate of $$0.31\frac{1}{4}$ per *dwt.*? *Ans.* $204.0625.

2. What will 2*bu.* 3*pk.* 3*qt.* of strawberries amount to at the rate of $$0.12\frac{1}{2}$ per *quart*? *Ans.* $11.375.

3. What will 2*hhd.* 40*gal.* 3*qt.* of beer amount to, if retailed at $0.03 per *pint*? *Ans.* $35.70.

4. What will 3*bar.* 16*gal.* 3*qt.* of brandy amount to, if retailed at $$0.06\frac{1}{4}$ per *gill*? *Ans.* $222.50.

5. Find the expense of putting up 1*m.* 3*fur.* 20*r.* of fencing, at the rate of $0.75 per *rod.* *Ans.* $345.00.

6. At the rate of 3 gills per day, what quantity of brandy will a toper drink in a year of 365 days?

Ans. 34*gal.* 1*pt.* 3*gi.*

7. At the rate of 5 pints per day, what quantity of milk will a family consume in a year of 365 days?

Ans. 228*gal.* 0*qt.* 1*pt.*

8. At the rate of 200*lb.* per acre, what quantity of plaster will be required to sow a field containing 95 acres?

Ans. 8*T.* 9*cwt.* 2*qr.* 16*lb.*

9. A farmer sold corn to the amount of $100, which he laid out for wheat at 75 cents per bushel. How much wheat did he purchase? *Ans.* 133*bu.* 1*pk.* 2*qt.* $1\frac{1}{3}$*pt.*

10. A merchant invested the profits of five years' business, amounting to 7349.31\frac{1}{4}$, in land at 24.12\frac{1}{2}$ per acre. How much land did he purchase? *Ans.* 304*A.* 2*R.* 21.44*P.*

11. What will 4*cwt.* 3*qr.* 19*lb.* of hemp amount to, at 6.87\frac{1}{2}$ per hundredweight?

The 3*qr.* 19*lb.* may be reduced to a decimal of a *cwt.* (118 . . . 2), and the price per *cwt.* be then multiplied by the whole quantity in *cwt.*; or the calculation may be made by means of *aliquot* parts. *Ans.* $33.818'.

12. Required the sum that should be paid for 13*yd.* 1*qr.* 3*na.* of lace, at 0.93\frac{3}{4}$ per yard. *Ans.* $12.597'.

13. Required the sum that should be paid for 10 *T.* 13*cwt.* 2*qr.* 23*lb.* of coal, at 5.37\frac{1}{2}$ per ton. *Ans.* $57.431'.

14. An iron-monger bought iron at $45 per ton, and sold 13*cwt.* 2*qr.* 15*lb.* of the same at 62\frac{1}{2}$ per ton. What profit was made on the quantity sold? *Ans.* $11.928'.

15. A merchant bought in New York 135 *yd.* 3*qr.* of linen, at 2*s.* 3*d.* per yard, and 74*yd.* 2$\frac{1}{2}$*qr.* of silk at 6*s.* 9*d.* per yard. Required the whole amount in Federal Money. *Ans.* $101.144'.

16. A farmer sold in Philadelphia 400*bu.* 3*pk.* of wheat, at 6*s.* 11*d.* per bushel, and 175$\frac{1}{4}$ *bu.* of oats, at 3 shillings per bushel. Required the whole amount in Federal Money. *Ans.* $439.644'.

17. A lot of broadcloth imported from Liverpool amounted to 125£. 16*s.* 10*d.* sterling. Required the amount in Federal Money, according to the legal value of the pound sterling in the United States. *Ans.* $609.07'.

18. A farmer sold 3 *T.* 16*cwt.* 3*qr.* 21*lb.* of hemp, at $5 per *cwt.*, and invested the proceeds in land at 37\frac{1}{2}$ per acre. What quantity of land did the farmer purchase? *Ans.* 10*A.* 1*R.* 1.28*P*

19. A laborer dug 130*r.* 4*yd.* $2\frac{1}{2}$*ft.* of ditching at \$$2\frac{1}{2}$ per rod, for which he is to take \$100 in cash, and wheat at $87\frac{1}{2}$ cents per bushel. To what quantity of wheat will he be entitled? *Ans.* 259*bu.* 2*pk.* 4*qt.* 1.6'*pt.*

20. A grocer bought, at different times, $3\frac{1}{5}$*cwt.*, $2\frac{1}{4}$*qr.* and 49*lb.* of soap, of which he has sold 2*cwt.* 1*qr.* Find the quantity remaining in *lb.* *Ans.* $218\frac{2}{5}$*lb.*

21. A merchant had 3 pieces of cloth containing 29*yd.* 3*qr.* each, of which he has sold, to different persons, $5\frac{1}{8}$*yd.*, $3\frac{1}{2}$*qr.*, and 10*yd.* $1\frac{1}{2}$*qr.* Find the remainder in yards.

Ans. $72\frac{7}{8}$*yd.*

22. A townsman who had a lot of ground containing $5\frac{1}{4}$*A.*, sold to each of two persons $3\frac{1}{2}$*R.*, at the rate of \$100 per acre. What is the remainder of it worth at the same rate?

Ans. \$350.

23. A miller bought at one time 200*bu.* 3*pk.* of wheat, at another 313*bu.* 1*pk.*, and at another 194*bu.* Having made 405*bu.* 1*pk.* of these purchases into flour, how much wheat has he still on hand? *Ans.* 302*bu.* 3*pk.*

24. A merchant had 5 pieces of cotton, containing 33*yd.* 3*qr.* each, which he sold in equal portions to ten customers. What quantity was bought by each customer?

Ans. 16*yd.* $3\frac{1}{2}$*qr.*

25. An agriculturist raised 500*bu.* 3*pk.* 4*qt.* of oats from one field, and he found the produce to be at the rate of 29*bu.* 1*pk.* 1*qt.* per acre. How many acres did the field contain?

Ans. 17*A.* 16.8*P.*

26. A bought a tract of land containing 570*A.* 3*R.*; of which he sold to one person 90*A.* 1*R.* 20*P.*, and three times that quantity to another, at \$$27\frac{1}{2}$ per acre. What is $\frac{2}{3}$ of the remainder worth at the same rate? *Ans.* \$3813.

Direct and Inverse Ratio.

(137.) The *direct ratio* of the *first* of two quantities to the *second*, is the quotient of the first divided by the second; (135);

The *inverse ratio* of the first quantity to the second, is the direct ratio of the second to the first; thus the inverse ratio of 7 to 5 is $\frac{5}{7}$.

What is the *inverse ratio* of 9 to 4? Of 8 to 15? Of 5 to 30?
What is the inverse ratio of 20 to 7? Of 4 to 19? Of 8 to 24?

The term *ratio*, when used alone, always means *direct* ratio.

Comparison of Fractions.

(138.) Two Fractions having a *common denominator* are to each other as their *numerators;* and two fractions having a common *numerator* are to each other *inversely as their denominators.*

Thus the ratio of $\frac{2}{5}$ to $\frac{3}{5}$ is $\frac{2}{5} \div \frac{3}{5} = \frac{2}{3}$ (135), which is the ratio of the numerator 2 to the numerator 3.

And the ratio of $\frac{5}{8}$ to $\frac{5}{9}$ is $\frac{5}{8} \div \frac{5}{9} = \frac{9}{8}$ (135), which is the *inverse ratio* of the denominator 8 to the denominator 9 (137).

What is the ratio of $\frac{3}{7}$ to $\frac{4}{7}$? Of $\frac{7}{16}$ to $\frac{15}{16}$? Of $\frac{20}{27}$ to $\frac{13}{27}$?
What is the ratio of $\frac{3}{7}$ to $\frac{3}{8}$? Of $\frac{9}{14}$ to $\frac{9}{25}$? Of $\frac{20}{30}$ to $\frac{20}{37}$?

Variation.

(139.) One quantity *varies directly* as another when both *increase* or *decrease* together in the same ratio.

Thus the *value* of a given commodity varies directly as the *quantity*, since the value will be *doubled*, or *trebled*, &c., when the quantity is doubled, or trebled, &c.

(140.) One quantity *varies inversely* as another when one of them *increases* in the same ratio in which the other *decreases.*

Thus the *time* in which a laborer will earn a given sum, varies inversely as his *rate of wages*, since one of these quantities will *increase* in the same ratio in which the other is *diminished.*

Will the *value* and the *quantity* of a piece of cloth vary *directly* or *inversely* with each other?—The *time* and the *number of men* required for a given work?—The *number of men* and the *amount of provisions* that will suffice them for a given time?—The *weight* of an article and the *distance* it may be carried for a given sum of money?—The *length* and the *breadth* of a garden containing a given area?—The *weight* of the five-cent loaf of bread and the *price of flour*?

PROPORTION.

(141.) Proportion consists in an *equality of ratios.*

Four quantities are in proportion when the ratio of the first to the second is equal to the ratio of the third to the fourth. Thus the numbers 6, 3, 8, 4 are in *proportion*, since $\frac{6}{3}=\frac{8}{4}$.

The *first* and *third* terms are the *antecedents*, the second and fourth the *consequents;* the *first* and *fourth* are the two *extremes*, the second and third the two *means.*

The fourth term is called a *fourth proportional* to the other three taken in order; thus 4 is a fourth proportional to 6, 3, and 8.

What is the *fourth proportional* to 10, 2, and 15, that is, the number to which 15 has the same ratio that 10 has to 2? What is the fourth proportional to 16, 4, and 20? To 4, 8, and 10? To 5, 20, and 12? To 2, 1, and 10? To $\frac{1}{2}$, 1, and $2\frac{1}{2}$?

Direct and Inverse Proportion.

(142.) A *direct proportion* consists in an equality between two direct ratios (141).

An *inverse proportion* consists in an equality between a *direct* and an *inverse* ratio.

The numbers 6, 3, 8, 4 are in *direct* proportion; the same numbers in the order 6, 3, 4, 8 are in *inverse* proportion, since the direct ratio of 6 to 3 is equal to the inverse ratio of 4 to 8 (137).

What is the *inverse fourth proportional* to 12, 6, and 8, that is, the number to which 8 has the inverse ratio of 12 to 6? What is the inverse fourth proportional to 8, 2, and 3? To 3, 9, and 15? To 24, 3, and 4? To 4, 20, and 30?

The term *proportion*, used alone, always means *direct* proportion.

A Proportion is denoted by a *double colon* (: :), or the sign $=$, between the two ratios of the proportion; thus

$6 : 3 :: 8 : 4$, 6 is to 3 as 8 is to 4

or $6 : 3 = 8 : 4$, the ratio of 6 to 3 equals the ratio of 8 to 4.

To denote an *inverse* Proportion we shall employ the sign $\neq$ between the two ratios of such proportion; thus

$6 : 3 \neq 4 : 8$, 6 is to 3 *inversely* as 4 is to 8.

Inverse Converted into Direct Proportion.

(143.) An *inverse* is converted into a *direct* Proportion by interchanging either antecedent and its consequent, that is, by taking the antecedent and its consequent the *one for the other.*

Thus from the *inverse* proportion $6 : 3 \neq 4 : 8$,
we get the *direct* proportion $3 : 6 :: 4 : 8$, or $6 : 3 :: 8 : 4$.

Product of the Extremes = that of the Means.

(144.) In every direct proportion the product of the *first* and *fourth* terms is equal to the product of the *second* and *third.*

In the proportion $3 : 6 :: 4 : 8$, we have the *equal ratios*

$$\tfrac{3}{6} \text{ and } \tfrac{4}{8}.$$

By reducing these equal fractions to a *common denominator*, we get the equal numerators 3×8 and 4×6 (138); which shows that the product of the two extremes in a proportion will always be equal to that of the two means.

On this principle depends

RULE XXXII.

(145.) *To find a Fourth Proportional to three given Terms.*

1. Multiply the *second* and *third* together, and divide the product by the first term; the quotient will be the *fourth term.*

2. The first and second terms must be taken in the *same order of units* (136).

3. When the third term is a *polynomial*, it will often be most convenient to reduce it to a *monomial.*

4. The *fourth* term will be found in the *same order of units* as the third term.

5. An *inverse fourth proportional* may be found by interchanging the first and second terms, and then proceeding as above (143).

EXAMPLE.

To find a fourth proportional to

3yd., *5yd. 2qr.*, and 2£. 10*s.*

Reducing the first and third terms both to *quarters*, and the third to *shillings*, we have 12*qr.*, 22*qr.* and 50*s.*

Multiplying the second and third together, and dividing the product by the first, we find $91\frac{8}{12}$*s.*, which is 4£. 11*s.* 8*d.* (118 ... 2).

The fourth term is equal to the product of the second and third *divided by the first*, because the first × the fourth = the second × the third (144).

EXERCISES.

1. Find a fourth proportional to

4yd., *7yd. 2qr.* and $26. *Ans.* $48.75.

2. Find a fourth proportional to

3oz., *4lb. 15dwt.* and $4. *Ans.* $65.

3. Find a fourth proportional to

$15, $2.25, and 3*A.* 10*P.* *Ans.* 1.8375*R.*

4. Find an inverse fourth proportional to

5 *men*, 12 *men*, and 18 *days*. *Ans.* $7\frac{1}{2}$ days.

5. Find an inverse fourth proportional to

9 *days*, $4\frac{1}{2}$ *days*, and 5*bu.* 1*pk.* *Ans.* 10*bu.* 2*pk.*

In practical questions in Proportion, there will always be given a term of *supposition*, a similar term of *demand*, and a third term which is *directly* or *inversely* to a required fourth term as the term of supposition is to that of demand. But the following Rule will always arrange the terms in *direct* Proportion.

RULE XXXIII.

(146.) *For Solving Questions in Proportion.*

1. Take for the *third term* that which is of the same kind as the required fourth term, or *answer* to the question.

2. If, from the nature of the question, the answer will be greater than the third term, take the *greater* of the two remaining terms for the *second term*—otherwise, take the *less*; the term still remaining will be the *first term.*

3. Find the *fourth proportional* for the answer, (145).

EXAMPLE.

If 6 *men* can perform a certain work in 30 *days*, in what time ought 13 *men* to perform the same work?

We take the 30 *days* for the *third term,* because *time* is required in the answer; and since 13 *men* would require *less time* than 6 *men,* the answer will be less than 30 days; we must therefore take 6 *men,* the less of the two remaining terms, for the *second term,* and 13 men for the *first term.*

13 *men* : 6 *men* : : 30 *da.* : the *time required.*

The fourth proportional, or Answer to the question, is

$$30 \textit{ days} \times 6 \div 13 = 13\tfrac{11}{13} \textit{ days}.$$

In this question, 6 *men* is the term of *supposition,* 13 *men* is the similar term of *demand,* and the time required is *inversely* as the number of men; that is,

6 *men* : 13 *men* $\neq$ 30 *days* : the *time required.*

By interchanging the first and second terms, this becomes a *direct proportion,* as above, (143).

By the ANALYSIS of a question is meant its solution, on elementary principles, without the direction of special Rules.

The preceding question is *analyzed* as follows:

One man would require 6 times as long as 6 *men* to do the work; 30 *days* $\times$ 6; 13 *men* would require $\frac{1}{13}$ as long as 1 man; 30 *days* $\times 6 \div 13 = 13\frac{11}{13}$ *days*.

EXERCISES.

1. If 9 acres of land sell for \$230.62½, what should 5 acres bring at the same rate? *Ans.* \$128.125.

2. If 1.5 tons be hauled 40 miles for a given sum, how far ought 3 tons to be hauled for the same sum? *Ans.* 20 miles.

3. How much cloth may be bought for \$73.75, when 4.25 yards of the same kind cost \$12.75? *Ans.* 24.583' yards.

4. If 7 masons can build a house in 28 days, in what time ought 17 masons to build the house? *Ans.* $11\frac{9}{17}$ days.

5. If 5 yards of silk cost \$6.25, what should be paid for 12*yd.* 3*qr.* of silk, at the same rate? *Ans.* \$15.937'.

6. Allowing 4 horses to consume 13*bu.* 3*pk.* of oats in a week, how much would 9 horses require for a week? *Ans.* 30.937'*bu.*

7. If the transportation of 10*cwt.*, 100 miles, cost \$25, what should be paid for the conveyance of 33*cwt.* 2*qr.* the same distance? *Ans.* \$83.75.

8. If 7 men can do a certain work in $\frac{3}{8}$ of a day, in what time ought 9 men to do the same work? *Ans.* $\frac{7}{24}$ of a day.

9. If a person, by traveling 10 hours a day, perform a journey in 31 days; in how many days ought he to perform the same journey, if he travel 13 hours a day? *Ans.* $23\frac{11}{13}$ days.

10. If 10 head of cattle require $20A.\ 2R.$ of pasture ground, for a summer, how many acres ought 25 head to have, for the same time? *Ans.* $51A.\ 1R.$

11. A cistern is filled with water, by 2 pipes, in $3hr.\ 25m.$ In what time would it be filled by 5 pipes of like size? *Ans.* $1hr.\ 22min.$

12. A sum of money having been equally divided among 19 men, each man received $\$3\frac{1}{4}$. If the number of men had been 30, what would have been the share of each? *Ans.* \$2.058'.

13. Allowing $15A.\ 30P.$ to produce $403bu.\ 2pk.$ of wheat, how many bushels would be raised from a field containing 40 acres, at the same rate? *Ans.* $1062\frac{58}{81}bu.$

14. If 25 sacks, each measuring $4bu.$, will contain a given quantity of corn; how many sacks, each measuring $3\frac{1}{2}bu.$, will contain the same quantity? *Ans.* $28\frac{4}{7}$ sacks.

15. A post, standing in a stream, has $\frac{1}{5}$ of its length in the earth, $\frac{2}{3}$ in the water, and 5 feet above the water. What is the length of the post?

ANALYSIS. $\frac{1}{5}+\frac{2}{3}=\frac{13}{15}$; and $1-\frac{13}{15}=\frac{2}{15}$.

The post has therefore $\frac{2}{15}$ of its length above the surface of the water; $\frac{2}{15}$ of its length is then 5 feet; $\frac{1}{15}$ of it is $\frac{1}{2}$ of 5 feet, and the whole length is

$$\frac{15}{2} \text{ of } 5 \text{ feet} = \frac{75}{2} = 37\tfrac{1}{2} \text{ feet.}$$

The *proportion* in the question is, the part, $\frac{2}{15}$, above the water, is to a *unit*, as the length, *5ft.*, of the part above the water, is to the *entire length*. *Ans.* $37\frac{1}{2}$ feet.

16. A farmer sold $\frac{1}{3}$ of his land to A, $\frac{1}{4}$ of it to B, and the remainder, which was 100 acres, to C. How much land did the farmer own? *Ans.* 240 acres.

17. In a certain school, $\frac{1}{4}$ of the pupils study Arithmetic, $\frac{3}{8}$ of them study Languages, and the remaining 36 are employed on various other subjects. Required the number in the school. *Ans.* 96 pupils.

18. A person failing in business owes $5000, and is able to pay but $2000. How much can he pay per dollar to his creditors? *Ans.* $0.40.

19. A traveler having gone 375.5 miles on his journey, finds that $\frac{3}{8}$ of it remains to be traveled. What was the length of his journey? *Ans.* 600.8 miles.

20. A gentleman who owned $\frac{2}{3}$ of a manufactory, sold $\frac{1}{4}$ of his share for $3000. What was the estimated value of the whole establishment? *Ans.* $18000.

21. How many miles must a person walk in $5\frac{1}{2}$ days, to accomplish a journey of 500.5 miles, at the same rate, in 15 days? *Ans.* 183.516' miles.

22. A bankrupt owes $5349.75, and has property amounting to $2300. In an equitable distribution of his property, how much will a creditor receive whose claim is $400?
Ans. $171.970'.

23. A borrowed of B $500, which he kept $3\frac{1}{2}$ years. On a subsequent occasion A lends B $375; how long ought B to keep this latter sum in return for the accommodation he had afforded A? *Ans.* $4\frac{2}{3}$ years.

24. Allowing a man to do a certain work in 3 days, and a boy to do it in 5 days, in what time ought the two together to do the work?

ANALYSIS. The man could do $\frac{1}{3}$, and the boy $\frac{1}{5}$, of the work, in 1 day; then both together could do $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$ of the work in 1 day; hence they could do $\frac{1}{15}$ in $\frac{1}{8}$ of a day, and the *entire work* in $\frac{15}{8}$ of a day. *Ans.* $1\frac{7}{8}$ days.

25. A can dig a ditch in 5 days, B in 6 days, and C in 8 days. In what time could the three together dig the ditch? *Ans.* $2\frac{2}{59}$ days.

26. If 4*T.* 13*cwt.* of iron be conveyed 50 miles for $30 how far should 9*T.* 5*cwt.* 3*qr.* be conveyed for the same sum? *Ans.* 25.033' miles.

27. Two masons together build a wall in 10 days. One of them could have built the wall himself in 15 days; in what time could the other have done it? *Ans.* 30 days.

28. A merchant bought three pieces of cloth, each containing 25*yd.* 2*qr.*, for $500; and sold 50*yd.* of it at cost. What did the 50 yards amount to? *Ans.* $326.79'.

29. If $\frac{3}{4}$ of $\frac{4}{5}$ of an acre of land sell for $$18.18\frac{3}{4}$, what would a lot containing 7*A.* 2*R.* 13*P.* bring at that rate? *Ans.* $229.80'.

30. How many yards of linen which is $\frac{1}{2}$ *yd.* wide will be equivalent to 30 *yd.* of another kind which is $\frac{3}{4}$ of a yard wide? *Ans.* 45 yards.

31. How many yards of carpeting which is $\frac{3}{4}$ of a yard wide, will be required to cover a floor that measures 25 feet in length, and 20 feet in breadth? *Ans.* $74\frac{2}{27}$ yards.

32. A farmer has a field 100 poles in length, and 45.25 poles in width. He wishes to lay off another field to contain the same quantity of ground, and be 80 poles in length; what must be its breadth? *Ans.* 56.5625 poles.

33. The governor of a besieged place has provision for 54 days, at the rate of $1\frac{1}{2}$*lb.* of bread to each man per day, but is desirous to prolong the siege to 80 days, in expectation of succor; in that case what must the ration of bread be? *Ans.* $1\frac{1}{80}$*lb.* per day

PARTITIVE PROPORTION.

(147.) PARTITIVE PROPORTION is Proportion applied to dividing a given quantity into *two or more parts* which shall have a given *ratio*, one to another.

The terms of the given ratio, or ratios, may be called the *proportional terms.*

For example, to divide $150 into *three parts* which shall be to one another in the proportion of 2, 3, and 5; that is, the first part to the second as 2 to 3, and the second to the third as 3 to 5.

In this example, 2, 3, and 5 are the *proportional terms.*

This division of Arithmetic is commonly called PARTNERSHIP or FELLOWSHIP.

RULE XXXIV.

(148.) *To Divide a given Quantity into two or more parts which shall have a given Ratio, one to another.*

1. Add together all the given proportional terms. Then,
2. The *sum* of those terms *will be to any one* of the terms, *as* the quantity to be divided, *is to the part* corresponding to that term.

EXAMPLE.

To divide $150 between A, B, and C in the proportion of 2, 3, and 5.

The sum of the given proportional terms is $2+3+5=10$;

then, 10 : 2 : : $150 : A's part, $30,
10 : 3 : : $150 : B's part, $45,
10 : 5 : : $150 : C's part, $75, (145).

ANALYSIS. Suppose to the whole sum \$150 to be divided into 2+3+5, or 10 *equal parts;* then it is evident that A must have *two*, B *three*, and C *five* of those parts; that is,

A's part is $\frac{2}{10}$ of \$150, or \$150 $\times \frac{1}{5}$;
B's part is $\frac{3}{10}$ of \$150, or \$150 $\times \frac{3}{10}$;
C's part is $\frac{5}{10}$ of \$150, or \$150 $\times \frac{1}{2}$.

The results obtained by the Analysis will be the same as those obtained by the Rule.

EXERCISES.

1. Divide \$240 between three persons in such a manner that their shares shall be as the numbers 5, 4, and 3.
Ans. \$100; \$80; and \$60.

2. A gentleman divided \$10000 between his son and daughter in the proportion of 3 to 2. What were the respective shares? *Ans.* \$6000, and \$4000.

3. A testator bequeathed \$15000 to his widow, daughter, and son in the proportion of 3, 5, and 7. What were the respective shares? *Ans.* \$3000; \$5000; and \$7000.

4. A merchant employed three clerks at the annual salaries of \$300, \$400, and \$500. At the end of the year, having become bankrupt, he has but \$650 to be divided proportionally among them. What will be the portion of each?

The *proportional terms* are 300, 400, and 500; or, without altering the ratios, 3, 4, and 5, since $\frac{300}{400}=\frac{3}{4}$, and $\frac{400}{500}=\frac{4}{5}$.
Ans. \$162.5; \$216.66'; and \$270.83'.

5. An insolvent debtor owes to A \$250, to B \$100, and to C \$300. He is able to pay \$420; what should each of the three creditors receive?
Ans. \$161.538'; \$64.615'; and \$193.846'.

6. It is required to divide the number 180 into three parts which shall be to one another as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

Reducing the proportional terms to a common *denominator*, they become $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$; and these are to one another as their numerators 6, 8, and 9 (138); hence these numerators may be taken for the *proportional terms*.

Ans. $46\frac{22}{23}$, $62\frac{14}{23}$, and $70\frac{10}{23}$.

7. A person proposed to divide \$1000 between his two sons in the proportion of $\frac{1}{3}$ to $\frac{1}{2}$, provided either of them could ascertain the amount offered to him. What would be their respective shares? *Ans.* \$400, and \$600.

8. The sum of \$500 is to be divided between A, B, and C in the proportion of $\frac{2}{3}$, $1\frac{1}{4}$, and $2\frac{1}{2}$. What will be the share of each? *Ans.* \$75.471'; \$141.509'; \$283.

9. Two persons form a partnership in trade, with a capital of \$3000, of which the first contributed \$1800, and the second the remainder. They gain \$900; what is each one's share of this sum? *Ans.* \$540, and \$360.

10. The sum of \$1000 is to be divided among four persons in the proportion of 1, $1\frac{1}{2}$, 2, and $2\frac{1}{2}$. What are the several shares? *Ans.* \$142.857, \$214.285, \$285.714, \$357.142.

11. A, B, and C in partnership gained \$360. What is each partner's share of gain, allowing that $\frac{1}{8}$ of the capital employed belonged to A, $\frac{1}{4}$ of it to B, and the remainder to C? *Ans.* \$45, \$90, and \$225.

12. Three persons freighted a ship with 340 tuns of wine, of which the first had 110 tuns, the second 97 tuns, and the third the remainder. In a storm the seamen were obliged to throw overboard 85 tuns; what must each person sustain of the loss? *Ans.* $27\frac{1}{2}$ tuns, $24\frac{1}{4}$, and $33\frac{1}{4}$ tuns.

(149.) ***When a Partitive Proportion has respect to Different Periods of Time.***

1. Multiply each term operating through a period of time *by its time;* take the products for the *proportional terms*, and then proceed according to Rule XXXIV.

2. The different periods of time must be taken in the *same order of units.*

EXAMPLE.

10. A and B trade together; A furnished $200 for 7 months, and B $300 for 9 months. They gained $100; what were their respective shares of gain?

200 × 7 = 1400; $200 for 7*m.* is equivalent to $1400 for 1*m.*;
300 × 9 = 2700; $300 for 9*m.* is equivalent to $2700 for 1*m.*

Having made the time the same, that is, 1 month, in both cases, we take the products 1400 and 2700, or 14 and 27, for the *proportional terms*, without regard to time.

Ans. $34.146' and $65.853'.

13. Three persons rent a pasture for $20. A puts in 20 sheep for 4 months, B 36 sheep for 3 months, and C 45 sheep for 2 months; how much of the rent should accordingly be paid by each? *Ans.* $5.755'; $7.769; and $6.474'.

14. A, B, and C, contracted to make a road for $5000. A furnished 30 laborers for 45 days, B 42 laborers for 34 days, and C 50 laborers for 30 days; what are their respective shares of the $5000?

Ans. $1577.84'; $1669'; and $1753.15'.

15. A, B, and C, in partnership, have made $400. What are their respective shares of profit, supposing A's capital in the business to have been $500 for 10 months, B's $900 for 1 year and 3 months, and C's $600 for 2 years?

Ans. $60.79'; $164.133'; $175.075'.

MEDIAL PROPORTION.

(150.) MEDIAL PROPORTION is Proportion applied to adjusting the quantities of two or more ingredients, at *different rates* of value, for a compound of a given *mean rate* of value.

For example, to find in what proportion, rye at 37 *cents* per bushel, and oats at 25 *cents* per bushel, must be taken to form a mixture of the two which shall be worth 30 *cents* a bushel.

This division of Arithmetic is commonly called ALLIGATION.

RULE XXXV.

(151.) *To find the Proportion of two or more Ingredients, at different Rates of value, for a Compound of a given Mean Rate of value.*

1. For *two ingredients*—take the quantities *inversely* as the *differences* between their respective rates of value and the given *mean rate.*

2. For *three or more ingredients*—find the proportions for one rate which is *less*, and another which is *greater*, than the given mean rate, as above; then for one of these two rates and another, or for two others, in like manner, and so on, until all the different rates are included; and add together all the proportional terms *found for the same rate.*

EXAMPLE.

To find in what proportion rye at 37 *cents* a bushel, and oats at 25 *cents* a bushel, must be taken for a mixture which shall be worth 30 *cents* a bushel.

The differences between the rates of the two ingredients and the *mean rate* 30 cents, are,

for the *rye* $37-30=7$, and for the *oats* $30-25=5$.

The quantity of *rye* must be to that of *oats inversely* as 7 is to 5; that is, the quantity of *rye* must be to that of *oats* as 5 is to 7 (143). In other words, since $5 + 7 = 12$, 5 *twelfths* of the mixture must be *rye*, and 7 *twelfths* of it *oats*.

ANALYSIS. On 1 bushel of *rye* there is an *excess* of 7 *cents*, and on 1 bushel of *oats* a *deficiency* of 5 *cents*, in relation to the *mean rate* 30 cents.

Then $\frac{1}{7}$*bu.* of rye is in excess 1 cent, and $\frac{1}{5}$*bu.* of oats is deficient 1 cent, in relation to the mean rate. This equal *excess* and *deficiency* counterbalance each other; hence $\frac{1}{7}$*bu.* of rye and $\frac{1}{5}$*bu.* of oats, mixed together, will be at the *mean rate.*

$\frac{1}{7} : \frac{1}{5} :: 5 : 7$ (138); hence the mixture must be in the proportion of 5*bu.* of rye to 7*bu.* of oats, as found by the Rule,—which shows the correctness of the first part of the Rule.

EXERCISES.

1. In what proportion must corn at 40 *cents* a bushel, and oats at 25 *cents* a bushel, be taken, to form a mixture which shall be worth 33 *cents* a bushel?

Ans. 8*bu.* of corn to 7*bu.* of oats.

2. In what proportion must one kind of tea, at 75 cents a *lb.*, and another, at 90 cents a *lb.*, be taken for a mixture which shall be worth 83 cents a pound?

Ans. 7*lb.* of the first to 8*lb.* of the second.

3. In what proportion must one kind of coffee, at 9 cents a *lb.*, and another, at 13 cents a *lb.*, be taken to form a mixture which shall be worth $12\frac{1}{2}$ cents a pound?

Ans. $\frac{1}{2}$*lb.* of the first to $3\frac{1}{2}$*lb.* of the second.

4. In what proportion must one kind of wine, at 90 cents a *gal.*, and another, at 75 cents a *gal.*, be taken for a mixture which shall be worth $87\frac{1}{2}$ cents a gallon?

Ans. $12\frac{1}{2}$*gal.* of the first to $2\frac{1}{2}$*gal.* of the second.

5. A farmer wishes to purchase two different qualities of land, at \$20 and \$35 per acre, in such proportion that the average rate shall be $\$27\frac{1}{2}$ per acre. In what proportion must the two kinds be purchased?

Ans. $7\frac{1}{2}$ acres, or equal quantities, of each.

EXAMPLE

Of three Ingredients at different Rates.

6. In what proportion must rye at 37 *cents*, oats at 23 *cents*, and corn at 32 *cents* a bushel, be taken for a compound which shall be worth 31 *cents* a bushel?

A mixture of the *rye* and *oats*, at the *mean rate* 31 cents, would require

8 bushels of *rye* to 6 bushels of *oats;*

and a mixture of the *oats* and *corn*, at the mean rate 31 cents, would require

1 bushel of *oats* to 8 bushels of *corn* (151 . . . 1).

These two mixtures, mixed *together*, will evidently be at the given *mean rate*, and will contain 8*bu.* of rye, 6 + 1, or 7*bu.* of oats, and 8*bu.* of corn.

The proportion of *oats* in the mixture of the three ingredients is found by adding together the proportional terms, 6 and 1, found for the *oats* in the mixture of the same ingredients taken *two and two.*

$8 + 7 + 8 = 23$; so that $\frac{8}{23}$ of the mixture must be rye, $\frac{7}{23}$ of it oats, and $\frac{8}{23}$ of it corn, whatever be the quantity of the mixture.

The following arrangement of the several rates will facili tate the operation:

$$\text{mean rate } 31\,;\quad \left.\begin{matrix}37\\23\\32\end{matrix}\right\rangle \qquad \begin{matrix} 8bu. \text{ of } rye\,; \\ 6+1=7bu. \text{ of } oats\,; \\ 8bu. \text{ of } corn. \end{matrix}$$

The rates of the several *ingredients* are set one under another, with the *mean rate* 31 cents on the left; each rate which is *less* than the mean rate 31 is linked with one which is *greater*, and each one which is *greater* is linked with one which is *less*.

The difference between the rate of *each ingredient* and the *mean rate* is set on the right, opposite to the rate, or rates, *with which the former is linked.*

These differences show the proportions for the rates *against which they stand.* When there are two or more differences, as the 6 and 1, against the same rate, they must be added together.

This arrangement is nothing more than a practical expedient for applying the second part of the preceding Rule.

7. A merchant wishes to mix three kinds of tea, at 90 cents, $1, and $1.50 per *lb.*, so that the mixture shall be worth $1.25 per *lb.* In what proportion must the different kinds be taken?

Ans. 25*lb.* at 90*cts.* to 25*lb.* at $1, and 60*lb.* at $1.50.

8. A grocer mixed brandy at 30 cents per *gal.*, and wine at $1 a *gal.*, with water, and found the compound to be worth 50 cents per gallon. In what proportion were the several ingredients taken, the water being rated at 0?

Ans. 50*gal.* of brandy to 70 of wine and 50 of water.

9. A farmer has one tract of land worth $15 an acre, another worth $22 an acre, and another worth $25 an acre. In what proportion must he sell from the several tracts, that the average price received shall be $20 an acre?

Ans. 7 acres at $15 to 5 at $22 and 5 at $25.

(152.) *Different Proportions of the same Ingredients*

May be taken, whenever there are two or more rates *greater*, and two or more *less*, than the *mean rate*. This is shown in the following example:

10. Four different kinds of sugar, at 5 *cts.*, 8 *cts.* 13 *cts.*, and 14 *cts.*, a *lb.*, are to be formed into a mixture which shall be worth 10 cents a *lb.* What proportions of the different kinds must be taken?

10;	5	3*lb.* at 5 cts.
	8	4*lb.* at 8 cts.
	13	5*lb.* at 13 cts.
	14	2*lb.* at 14 cts.

10;	5	3*lb.* at 5 cts.
	8	3+4=7*lb.* at 8 cts.
	13	5+2×7*lb.* at 13 cts.
	14	2*lb.* at 14 cts.

We take any one rate which is *less* than the mean rate 10, and any one which is *greater*, and adjust the proportions for those two rates. We proceed in like manner with either of these two rates and another, or two others, until all are included; and add together the proportional terms found for the same rate.

Different results will be obtained, according to the different ways of *coupling the ingredients.*

Find other Answers to the preceding question.

Ans. 7*lb.* at 5 cents, 3*lb.* at 8 cents, 7*lb.* at 13 cents, 5*lb.* at 14 cents.

4*lb.* at 5 cents, 7*lb.* at 8 cents, 2*lb.* at 13 cents, 7*lb.* at 14 cents;

7*lb.* at 5 cents, 4*lb.* at 8 cents, 5*lb.* at 13 cents, 7*lb.* at 14 cents.

COMPOUND RATIO.

(153.) A COMPOUND RATIO is the ratio of the *product of two or more antecedents* to the product of their consequents.

Thus the *compound ratio* of 3 and 4 to 5 and 7 is the ratio of the product 3×4 to the product $5\times7,=\frac{12}{35}$.

COMPOUND PROPORTION.

(154.) A COMPOUND PROPORTION consists in an equality between a *compound* and a *simple* ratio. Thus

$$\left.\begin{matrix}2:3\\6:8\end{matrix}\right\} :: 5 : 10 \text{ is a Compound Proportion;}$$

in which the *compound* ratio $2\times6:3\times8$ is equal to the *simple* ratio 5 : 10.

Compound Proportion is applicable to the solution of questions which would require two or more simple proportions.

RULE XXXVI.

(155.) *For Solving Questions in Compound Proportion.*

1. Take for the *third term* that which is of the same kind as the answer to the question.
2. Take the remaining terms in *couples of the same kind*, and place each couple as required for questions in simple proportion (146 . . . 2).
3. Multiply the first terms together for a *divisor*, and the second and third together for a *dividend;* the quotient will be the answer required.
4. Each antecedent and its consequent must be taken in the *same order of units*, and the third term reduced, when necessary, as in finding a fourth proportional.

EXAMPLE.

If a footman can go 150 miles in 5 days, by walking 12 hours each day, in how many days may he go 275 miles, by walking 10 hours each day?

$$\left.\begin{matrix} 150m. : 275m. \\ 10h. \quad 12h. \end{matrix}\right\} :: 5\ days : time\ required.$$

We take 5 *days* for the third term, because the answer will be the number of *days* in which he would go 275*m.*

It would require a *greater* number of days to go 275*m.* than it would to go 150*m.*; the greater of these two terms must therefore be taken for the *second term* (146 . . . 2).

When he walks 10*h.* a day he will require a *greater* number of days than when he walks 12*h.* a day; hence the greater of these two terms must be taken for the *second term.* The operation is

$$(275\times12\times5)\div(150\times10)=16500\div1500=11 \text{ days.}$$

ANALYSIS. 1 *mile* would be traveled in $\frac{1}{150}$ of the 5 *days*,

$$\frac{5\,da.}{150};$$

275*m.* would be traveled in 275 times as many days,

$$\frac{5\times275}{150};$$

if he walked but 1*h.* a day, the number of days would be

$$\frac{5\times275\times12}{150};$$

if he walked 10*h.* a day the number of days would be

$$\frac{5\times275\times12}{150\times10}.$$

From the dividend and divisor thus obtained, we may *cancel,* successively, the factors 5, 6, 5, 2, 5; this will reduce the expression to 11, the *required number of days.*

EXERCISES.

1. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks? *Ans.* 896 pounds.

2. If 5 oxen require an acre of grass for 9 days, how many acres will 20 oxen require for $30\frac{1}{2}$ days? *Ans.* $13\frac{5}{9}$ acres.

3. If a man travel 100 miles in 3 days of 13 hours each, how far might he travel in 33 days of $14\frac{1}{4}$ hours each? *Ans.* $1205\frac{10}{13}$ miles.

4. If the conveyance of 20 *cwt.*, 40 miles, cost \$$15.87\frac{1}{2}$, what should be charged for the conveyance of 50 *cwt.* 3*qr.*, 100 miles? *Ans.* \$100.70'.

5. If 2 yards of cloth, which is $1\frac{1}{2}$*yd.* wide, cost \$10.25, what should be paid for 13 yards, of like quality, which is $1\frac{3}{4}$ *yd.* wide? *Ans.* \$77.72'.

6. If 6000*lb.* of bread will supply a garrison of 100 men, for 2 months, how long will 12000*lb.* supply three such garrisons? *Ans.* $1\frac{1}{3}$ months.

7. Allowing 4 men to mow 19*A.* 3*R.* 27*P.* of meadow, in 5 days, how long ought 7 men to be employed in mowing 45 acres? *Ans.* 6.454' days.

8. If $11\frac{1}{2}$*oz.* of bread costs $6\frac{1}{4}$ cents, when flour is at \$5 a barrel, how much bread should be bought for 75 cents when flour is at \$6 a barrel? *Ans.* 7*lb.* 3*oz.*

9. Allowing the transportation of 15 *cwt.*, 100 miles, to amount to \$45.50, how far ought 37*cwt.* 1*qr.* 20*lb.* to be carried for \$100? *Ans.* 88.079' miles.

10. If 17 head of cattle consume 5*A.* 2*R.* 10*P.* of pasture, in 30 days, how many acres would be consumed by 40 head, in 50 days? *Ans.* 21*A.* 3*R.* $10\frac{19}{51}$*P.*

11. If 25 men can dig a ditch 80*ft.* long, 4*ft.* wide, and 3*ft.* deep, in 2 days, in what time ought 30 men to dig one 300*ft.* long, 5*ft.* wide, and 4*ft.* deep? *Ans.* $10\frac{5}{12}$ days.

CONJOINED PROPORTION.

(156.) A CONJOINED PROPORTION is a kind of compound Proportion in which the ratio of one of the antecedents to its consequent is made to depend on *equivalences* among the terms of the proportion.

Its nature and use will be seen under

RULE XXXVII.

(157.) *For Solving Questions in Conjoined Proportion.*

1. Set *equivalent terms* on the left and right of the sign =, and so that terms of the same kind shall be on *opposite sides*, in the different expressions; also set the *odd term* on the side which is opposite the other term of the same kind.

2. Multiply together all the terms on the same side with the odd term, for a *dividend*, and all the others for a *divisor*; the quotient will be the required term, or answer to the question.

EXAMPLE.

If 3*qr.* of cloth be worth 4*gal.* of wine, and 2*gal.* of wine be worth 5*lb.* of tea, how many *qr.* of cloth will be equal in value to 12*lb.* of tea?

The equivalent terms are 3*qr.* and 4*gal.*; 2*gal.* and 5*lb.*; the *odd term* is 12*lb.*, for which an *equivalent is to be found.*

The arrangement will be

$$3\ qr. = 4\ gal.$$
$$2gal. = 5\ lb.$$
$$12\ lb. = \text{how many } qr. \text{ of cloth?}$$

the odd term, 12*lb.*, being on the side which is opposite to the 5*lb.*

The operation is $(3\times2\times12)\div(4\times5)=72\div20=3\frac{3}{5}qr.$

The terms being *equivalent*, that is, equal *in value*, on opposite sides; if we had the equivalent of the 12*lb.*, the product of the values on one side would be equivalent to the product of those on the other.

Hence the product on the side on which the number of terms is complete $\div$ the incomplete product on the other side, gives the term which is *wanting on this latter side.*

ANALYSIS. From the equivalence of the terms we have

$$1lb. = \tfrac{1}{5} \text{ of } 2\ gal.;$$
$$= \tfrac{1}{5} \text{ of } \tfrac{2}{4} \text{ of } 4\ gal.;$$
$$= \tfrac{1}{5} \text{ of } \tfrac{2}{4} \text{ of } 3\ qr.;$$

hence $12lb. = \tfrac{12}{5}$ of $\tfrac{2}{4}$ of $3\ qr. = 3\tfrac{3}{5}qr.$, as before.

The preceding question might be solved by two simple proportions, in which regard would be had to the equivalence of the terms, or by a compound proportion; but the method by the Rule which has been given is the most convenient.

EXERCISES.

1. If 7*bu.* of wheat be worth as much as 3 cords of wood, and 9 cords of wood as much as 2 tons of hay; how many bushels of wheat should be exchanged for 5 tons of hay?
Ans. $52\tfrac{1}{2}$ bushels.

2. If 3 barrels of corn be given for 7*bu.* of wheat, and 4*bu.* of wheat for 13 of rye, and 15 of rye for 20 of oats; how many bushels of oats would be an equivalent for 10 barrels of corn? *Ans.* $101\tfrac{1}{9}$ bushels.

3. If A can do as much work in 5 days as B can do in 8 days, and B as much in 4 days as C can do in 11 days; in how many days could A do the same that C could do in 20 days? *Ans.* $4\tfrac{6}{11}$ days.

4. If $10\frac{1}{2}$ yards of silk cost \$15.75, and \$6 will purchase 1*yd.* of broadcloth, and $4\frac{1}{4}$*yd.* of the cloth be bartered for 25*yd.* of Irish linen; how many yards of the silk would be an equivalent for 40 yards of the linen? *Ans.* $27\frac{1}{5}$ yards.

5. Allowing that in a certain factory 6 girls do as much work in a day as 4 boys, and 8 boys as much as 6 men; how many men would be required to do as much work as 20 girls? *Ans.* 10 men.

6. Supposing A to earn as much money in 4 months as B earns in 6*m.*, and B as much in 5*m.* as C in 7*m.*, and C as much in 10*m.* as D in 3*m.*; in what time could D earn the same that A could earn in 12*m.*? *Ans.* $7\frac{1}{2}\frac{4}{5}$ months.

7. If 12*lb.* in the United States be equal to 10*lb.* at Amsterdam, and 100*lb.* at Amsterdam be equal to 120*lb.* at Paris; how many pounds at Paris are equal to 150*lb.* in the United States? *Ans.* 150 pounds.

MISCELLANEOUS EXERCISES

ON SIMPLE PROPORTION, PARTITIVE, MEDIAL, COMPOUND, AND CONJOINED PROPORTION.

1. If a person can earn $\$62.87\frac{1}{2}$ in a month, by working $9\frac{1}{4}$ hours per day, what ought he to earn in a month by working $11\frac{1}{2}$*h.* per day? *Ans.* \$78.168'.

2. A company of emigrants has a supply of bread for 25 days, at an allowance of $1\frac{1}{4}$*lb.* per day. How long would the supply last them, at an allowance of 12*oz.* per day? *Ans.* $41\frac{2}{3}$ days.

3. A lot of ground which is 40*p.* 2*yd.* long, and 27*p.* wide, is equivalent to another which is $53\frac{1}{2}$*p.* long. What is the breadth of the latter? *Ans.* 20.37' poles.

4. If a given sum of money will supply a number ______ with oats, for 3 months, when oats is at $\$\frac{1}{4}$ per *bu.*, how long will the same sum supply them with the same article, at $0.37½ per bushel? *Ans.* 2 months.

5. How many yards of carpeting which is $\frac{7}{8}$ of a yard wide, will be sufficient for a room 18*ft.* 9*in.* long, and 16½ feet wide? *Ans.* $39\frac{2}{7}$ yards.

6. If A could do a piece of work in 10 days, B in 12 days, and C in 15 days, in how many days could the three together do the work? *Ans.* 4 days.

7. A young man squandered $\frac{1}{5}$ of his fortune in one year; $\frac{2}{3}$ of the remainder went in the next six months, when he had $3000 left. What was the amount of his fortune?

Ans. $11250.

8. A and B together can mow a meadow in 5 days, and B could do it himself in 8 days. In what time could A mow the meadow? *Ans.* $13\frac{1}{3}$ days.

9. A farmer sold $\frac{2}{5}$ of his whole amount of land, at $25 per acre, and received for it $10000. What amount of land did the farmer own? *Ans.* 1000 acres.

10. A cistern receives water through 2 pipes, one of which would fill it in 8 hours, and the other in 5 hours; but by leakage the cistern loses at the rate of $\frac{1}{10}$ of its whole capacity per hour. In what time will the 2 pipes running together fill the cistern? *Ans.* $4\frac{4}{9}$ hours.

11. The sum of $1000 is to be divided among four persons in the proportion of 1, 1½, 2, and 2½; what are the several shares? *Ans.* $142.857', $214.285', $285.714', $357.142'.

12. Four persons in partnership gain $1800. One third of the capital employed belonged to the first, $\frac{1}{4}$ of it to the second, and the remainder equally to the other two; what amount of gain should be assigned to each?

Ans. To the 1st $600, to the 2d $450, to the 3d and 4th each $375.

13. Divide \$1700 among 4 persons, so that A's share shall be to B's as 1 to 2, B's to C's as $\frac{1}{3}$ to 1, and C's to D's as 3 to 4.

We must employ but *one proportional term* for each share:

$$\tfrac{1}{3} : 1 :: 2 : 6, \quad \text{and } 3 : 4 :: 6 : 8, \ (145);$$

the several shares are therefore in the proportion of 1, 2, 6, and 8. *Ans.* \$100, \$200, \$600, \$800.

14. Divide \$70 between A, B, and C in such a manner that A's share shall be to B's as 2 to 3, and B's to C's as 4 to 5. *Ans.* \$16, \$24, and \$30.

15. A farmer divided 500 acres of land between his three sons, giving to the first $1\frac{1}{2}$ times as much as to the second, and to the second $1\frac{1}{4}$ times as much as to the third. What were the shares? *Ans.* $227\frac{9}{33}$, $151\frac{17}{33}$, and $121\frac{7}{33}$ acres.

16. Three persons in a joint speculation lose \$800. A's portion of the capital employed was $\frac{3}{4}$ of B's, and B's was $\frac{2}{3}$ of C's; what amount of the loss should be assigned to each? *Ans.* \$184.615', \$246.153', \$369.23'.

17. Two persons trade in partnership. A contributes at first \$1000, and 6 months afterwards \$500 more; B contributes at first \$2000, but 4 months afterwards withdraws \$600. In 12 months the profits amount to \$800; what is each one's share of the same?

A employed \$1000 for 6*m.*—equivalent to \$6000 for 1*m.*,
and \$1500 for 6*m.*—equivalent to \$9000 for 1*m.*;

then A's capital was equivalent to \$15000 for 1*m.*, (149).

In a similar manner find the equivalent for B's capital. *Ans.* \$350.877', and \$449.122'.

18. A, B, and C form a partnership for 12 months. A and B at once advanced \$2500 each, as their portion of the capital stock. At the end of 3 months C advances \$3000, and B

withdraws \$1000. The profits amount to \$1500; what is each partner's share of the same?

Ans. \$576.923', \$403.846', \$519.23'.

19. A gentleman bequeathed the sum of \$5000 to his widow, son, and daughter, in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. The widow dying soon after, the whole sum was divided in due proportion between the two children; how much did each receive? *Ans.* \$2777$\frac{7}{9}$, and \$2222$\frac{2}{9}$.

20. How many pounds of tea at 85*cts.* a pound, and at 90*cts.* a pound, must be mixed with 5*lb.* at \$1 a pound, that the mixture may be worth 94*cts.* a pound?

The proportions of the *three ingredients* for a mixture at the mean rate 94 cents, will be found to be,

6*lb.* at 85*c.* to 6*lb.* at 90*c.* and 13*lb.* at \$1, (151);

then 13*lb.*, the term found for the rate whose *quantity is given,* : 5*lb.*, that given quantity, : : 6*lb.*, the term found for any other rate, : the quantity required at that rate.

Ans. 2$\frac{4}{13}$*lb.* of each.

21. How many ounces of gold which is 23 *carats* fine, and how many 20 carats fine must be compounded with 8*oz.* which is 18 carats fine, that the compound may be 22 carats fine?

A *carat* is a weight of 4 grains; but the term is also used as above in expressing the degree of purity or fineness of gold. Thus when pure gold is combined with some baser metal, called *alloy;* if 20 parts in every 24 of the compound be pure gold, the alloyed gold is said to be 20 *carats* fine; and so for other numbers.

Ans. 48*oz.* of the first, and 8*oz.* of the second.

22. How many gallons of brandy at 25*cts.* per gallon, and how much water must be mixed with 5 gallons of brandy at 40*cts.* per gallon, that the adulterated compound may rate at 30 cents per gallon? *Ans.* 1$\frac{3}{7}$*gal.* of each.

23. How many gallons of vinegar at 20 cents a *gal.*, and at 50 cents a *gal.* should be mixed with 4 *gal.* at 25 cents a *gal.* and 2 *gal.* at 16 cents a *gal.*, that the whole may be worth 28 cents a gallon?

The 4 *gal.* and the 2 *gal.* together amount to 132 cents; hence these 6 *gal.* average 22 cents per gallon. By substituting 6 gallons at 22 cents for the 4 *gal.* and 2 *gal.* at their respective rates, the question becomes of the same kind as the 20th.

Ans. 6*gal.*, and $3\frac{9}{11}$*gal.*

24. A farmer wishes to mix 10*bu.* of corn at 35 cents per *bu.*, and 8*bu.* of rye at 40 cents per *bu.* with such a quantity of oats at 25 cents per bushel that the whole may be worth \$0.33⅓ per bushel. What must be the quantity of oats?

Ans. $8\frac{2}{5}$ bushels.

25. A vintner wishes to mix wines at \$0.75 and \$1.25 per gallon in such proportion and quantities as to produce 100 *gal.* worth \$0.87½ per gallon. What quantities of the two kinds must be taken?

The proportion of the two kinds for a mixture at the mean rate 87½ cents, is,

37½*gal.* and 12½*gal.* (151);

then $37\frac{1}{2} + 12\frac{1}{2} : 37\frac{1}{2} :: 100$: the quantity of the first kind;

and $37\frac{1}{2} + 12\frac{1}{2} : 12\frac{1}{2} :: 100$: the quantity of the 2d kind.

Ans. 75*gal.* and 25*gal.*

26. How many pounds of each of three different kinds of coffee, rating at 12 cents, 13 cents, and 15½ cents per pound, must be taken for a mixture of 100*lb.* which shall rate at 14 cents per *lb.*? *Ans.* 25*lb.*, 25*lb.* and 50*lb.*

27. Allowing a mechanic to earn \$62.87½ in a month, by working 9¼ hours per day, what ought he to earn in 3 months by working 11½ hours per day? *Ans.* \$234.50'.

28. If $50bu.$ of wheat be exchanged for $80\frac{1}{2}bu.$ of rye, and $3bu.$ of rye for $4\frac{1}{2}bu.$ of corn, and $10bu.$ of corn for $12bu.$ $3pk.$ of oats, and $3\frac{1}{3}bu.$ of oats be worth \$1 ; what is the value of $100bu.$ of wheat ? *Ans.* \$92.373'.

29. Three pipes of equal size will fill a cistern with water in $13h.$ $40m.$ In how many hours would 5 such pipes fill a cistern whose capacity is $2\frac{1}{2}$ times that of the first one ?

Ans. $20\frac{1}{2}$ hours.

30. A contractor engaged to pave 15 miles of road in 12 months, and for that purpose employed 100 men. Seven months have now elapsed, and but 6 miles of the road have been completed ; how many more men must be employed, to finish the work in the time prescribed ? *Ans.* 110 men.

31. If $10lb.$ at London be equivalent to $9lb.$ at Amsterdam, and $45lb.$ at Amsterdam to $49lb.$ at Bruges, and $98lb.$ at Bruges to $116lb.$ at Dantzic ; how many pounds at Dantzic are equivalent to 112 pounds at London ?

Ans. 129.92 pounds.

32. If $1\frac{1}{5}$ ells of Hamburg make 1 ell in Holland, and 7 ells in Holland make 4 in France, and 7 in France make 5 yards in England ; how many yards in England are equivalent to 588 ells at Hamburg ? *Ans.* 200 yards.

CHAPTER VIII.

PERCENTAGE, AND ITS APPLICATIONS.

PERCENTAGE.

(158.) PERCENTAGE is an allowance at a certain rate for *every hundred.*

Per centum, or its contraction *per cent.*, is *Latin,* and signifies *by the hundred.*

One per cent. on any number is *one* for every *hundred; two per cent.* is *two* for every *hundred; three per cent.* is *three* for every *hundred,* &c.

How much is 1 *per cent.* on $200? On $300? On $400?
How much is 2 per cent. on $200? On $300? On $350?
How much is 3 per cent. on $300? On $350? On $500?

Ratio of Percentage.

(159.) The *ratio* of Percentage is the ratio of the *rate per cent.* to 100, and is therefore equal to the *rate per unit.*

Thus the *ratio* of percentage for 4 *per cent.* is $\frac{4}{100}$, or .04, which is plainly the rate for each *unit* of the quantity on which the percentage would be computed.

What is the *ratio* of Percentage for 1 *per cent.*? For 3 per cent? For 5 per cent.? For 7 per cent.? For 9 per cent.? For 10 per cent.? For 12 per cent.?

(160.) The ratio of Percentage is usually expressed *decimally;* and the proper decimal may always be found by dividing the *rate per cent.* by 100.

Thus the ratio for $2\frac{1}{2}$ per cent. is $2.5 \div 100 = .025$;

the ratio for $\frac{1}{2}$ per cent. is $.5 \div 100 = .005$, (100) What is the *ratio* of Percentage for $1\frac{1}{8}$ per cent.? For $1\frac{1}{4}$ per cent.? For $\frac{1}{4}$ per cent.? For $2\frac{3}{4}$ per cent.? For $\frac{3}{4}$ per cent.? For $3\frac{1}{4}$ per cent.?

Basis of Percentage.

(161.) The number or quantity on which Percentage is computed, at any given rate, may be called the *Basis* of percentage.

Thus when we say 2 per cent. on $300, the *basis* of percentage is $300.

If the Rate per cent. is 2, and the amount of Percentage $4, what is the *basis* of percentage? If the Rate per cent. is 3, and the amount of Percentage $9, what is the *basis* of percentage? If the rate per cent. is 4, and the amount of Percentage $20, what is the *basis* of percentage?

Percentage, in its simpler forms, is applied to Profit and Loss in trade, Taxes, Duties, Insurance, Commission, Stocks, &c.

(162.) Taxes

Are charges imposed by Law on *property*, and frequently on *persons*, for public purposes.

A *poll* or *capitation* tax is a tax on the person, without regard to property.

A tax on property is sometimes *specific*, that is, a specified sum on certain articles; but it is most commonly *ad valorem*, or a specified *per centum* on the value.

(*Ad valorem* is Latin, and means *according to value*).

(163.) Duties

Are charges, either *specific* or *ad valorem*, imposed by Law on imported goods, for purposes of *revenue*. They are paid in the Custom Houses at the *ports of entry*.

The *invoice* is a written statement of the articles and their cost.

Tare, *draft, leakage*, &c., are allowances made for the box, cask, &c., containing the articles, or for waste, &c., before the duty is computed.

Net weight is the weight of the goods after all allowances are made.

(164.) INSURANCE

Is an obligation on a Company or an Individual, to pay for any *loss of property* by fire, storms, or other casualty.

The price or *premium* paid for insurance is usually a specified percentum on the amount insured.

The writing which binds the Company, or *insurer*, to the person insured, is called a *policy*.

(165.) COMMISSION

Is a compensation to an Agent, Factor, Correspondent, or Commission Merchant, for buying or selling for another ; and is usually a certain percentum on the amount of purchase or sale.

Brokerage is a commission charged by Brokers, or dealers in *money, stocks*, &c., on the amount of exchange, purchase, or sale which they effect for another.

(166.) STOCK OR CAPITAL

Is money or other property employed in any way to produce a profit ; as in manufactures, banking, &c. *Bonds* of the Government are also called Government Stock.

The stock of a Company, who are called the *stockholders*, is divided into *shares*, usually of $100 each.

The *par value* of a share of stock is what it originally cost. Stock is *above par*, or at an advance, when it sells for more than it cost ; and *below par*, or at a discount, when it sells for less than it cost.

The rise or fall in stock is expressed by a percentum on its *par value*.

RULE XXXVIII.

(167.) *To find Percentage on a given Number.*

Multiply the given number, or *basis* of percentage, by the *ratio* of percentage; the product will be the amount of percentage.

EXAMPLE.

A merchant bought a quantity of cloth for \$50.75, and sold the same at a profit of $33\frac{1}{3}$ *per cent.* What amount of profit did he make?

The *ratio* of percentage is $.33\frac{1}{3}$, which is the profit *per dollar* (159);

then $\$50.75 \times .33\frac{1}{3} = \$16.91\frac{2}{3}$, the *entire profit.*

When the rate *per cent.* is an aliquot part of 100, the amount of Percentage will be found, most readily, by taking the same part of the *basis* of percentage.

Thus $33\frac{1}{3}$ being $\frac{1}{3}$ of 100, the Percentage in the present example is

$\frac{1}{3}$ of \$50.75.

PROPORTION. $100 : 50.75 : : 33\frac{1}{3}$: Percentage or Profit required.

From the preceding rule it follows, that

(168.) The amount of Percentage divided by the *ratio* of percentage, gives the *basis* of percentage, or the number on which the percentage was computed.

EXERCISES.

1. A grocer bought a hogshead of sugar for \$55.75, and sold it at a profit of $12\frac{1}{2}$ per cent. What amount of profit did he make? *Ans.* \$6.968'

2. What would be the annual premium of insurance on a manufactory, valued at $20,000, at 1½ per cent. ?

Ans. $300.

3. A merchant bought silk for $160, which, on account of damage received, he sold at a loss of 5½ per cent. What was the entire loss? *Ans.* $8.80.

4. What would be the cost of insurance on a store house, valued at $5000, and a stock of goods amounting to $7500.50, at 2½ per cent. ? *Ans.* $312.51'.

5. The annual insurance on a paper mill, at 2 per cent., amounts to $165.50. What is the value insured? (168).

Ans. $8275.

6. What would be the amount of duty to be paid on an invoice of broadcloth amounting to $5465.75, at 30 per cent. ?

Ans. $1639.725.

7. A flour dealer bought 130 barrels of flour, at $4.12½ per barrel, and sold it at a profit of 10 per cent. What was his entire profit? *Ans.* $53.625.

8. An invoice of Irish linens paid a duty of $330. What was the amount invoiced, allowing the rate of duty to have been 33 per cent. ? *Ans.* $1000.

9. By selling a lot of iron at an entire profit of $22.50, I made 9 per cent., on the cost of it. What was the cost of the iron? *Ans.* $250.

10. A manufacturer sold cotton cloth at a profit of 20 per cent. on the cost of making it, which was $0.12½ per yard. At what price was the cotton sold?

The price at which the cotton was sold was 12½ + 12½ × .20; which is equivalent to the *basis* of percentage × (a *unit* + the *ratio*);

thus $.12½ × 1.20 *Ans.* $0.15.

11. A farmer bought land at \$44.75 per acre. At what price must he sell the land to make a profit of 25 per cent.? *Ans.* \$55.937'.

12. A merchant purchased a quantity of cloth, at \$6.30 per yard. At what price must he sell the cloth to gain $33\frac{1}{3}$ per cent? *Ans.* \$8.40.

13. The stock of an Insurance Company is 5 per cent. *below par.* What is the value of \$10000 of the stock at that rate?

The value of the stock is $10000 - 10000 \times .05$;

which is equivalent to the *basis* of percentage $\times$ (a *unit*— the *ratio*); thus $\$10000 \times .95$. *Ans.* \$9500.

RULE XXXIX.

(169.) *To find what Percentum one given number is of another.*

Divide the number which is made the *percentage* by that which is made the *basis* of percentage. The quotient will be the *ratio* of percentage, and when $\times$ 100 will give the required *rate per cent.*

EXAMPLE.

On an investment of \$82750 a person gained $\$1379.16\frac{2}{3}$. What was the gain *per cent.*?

The first number is the *basis*, and the second is to be made the *percentage.*

$\$1379.1666' \div 82750 = .01\dot{6}\dot{6}$, the gain on \$1;

$\$.0166 \times 100 = \$1.\dot{6}\dot{6}$, the gain on \$100, or gain *per cent.*

The repeating decimal $.\dot{6}\dot{6}$ is equal to $\frac{2}{3}$ (90); hence the gain *per cent.* is $1\frac{2}{3}$.

PROPORTION. $82750 : 100 :: 1379.16\frac{2}{3}$: gain *per cent.*

14. A person paid a tax of \$52.88$\frac{1}{4}$ on property valued at \$3525.50; at what rate per cent. was the tax assessed?

Ans. 1$\frac{1}{2}$ per cent.

15. The property of a village amounts to \$100000, and is to be taxed to the amount of \$2250, for public improvements. At what percentum must the tax be laid?

Ans. 2$\frac{1}{4}$ per cent.

16. If silk were purchased at \$1.50 per yard, and sold at \$2 per yard, what would be the gain per cent.?

The gain on \$1.50 would be \$2−1.50=\$0.50; and we have to find what percentum this is on the cost of the silk.

Ans. 33$\frac{1}{3}$ per cent.

17. If a lot of books were purchased at \$2.50 per dozen, and sold at \$3.75 per dozen, what would be the gain per cent.? *Ans.* 50 per cent.

18. A merchant bought hats at \$36 per dozen, and sold them at \$4.62 apiece. What was his gain or loss per cent.?

Ans. 54 per cent. gain.

19. A shop-keeper bought shoes at \$18.75 per dozen, and sold them at \$1.37$\frac{1}{2}$ a pair. What was his percentum of profit or loss? *Ans.* 12 per cent. loss.

20. The landed property of a State is valued at \$38400000; at what rate per cent. must it be taxed to produce an annual revenue of \$96000? *Ans.* $\frac{1}{4}$ per cent.

21. A tax of \$1133.33$\frac{1}{3}$ was raised from property amounting to \$340000. At what percentum was the tax levied?

Ans. $\frac{1}{3}$ per cent.

22. A merchant bought flour at \$5.75 per barrel, and sold it at \$7.18$\frac{3}{4}$ per barrel. What did he gain per cent. on the flour? *Ans.* 25 per cent.

23. A gentleman purchased land at \$37.50 per acre, and sold the same at \$42.18$\frac{3}{4}$ per acre. What was the percentum of profit made? *Ans.* 12$\frac{1}{2}$ per cent.

RULE XL.

(170.) *To find a number to which Percentage added at a given Rate, will make a given Sum.*

Divide the given sum by 1 *plus* the *ratio* of percentage; the quotient will be the required number, or *basis* of percentage.

EXAMPLE.

An agent is intrusted with $500 to purchase goods, out of which sum he is to retain a commission of 2 per cent. on the *amount of purchase.* What amount of purchase can he make?

The *ratio* of percentage is .02, which is the agent's commission on *each dollar* to be laid out in the purchase of the goods (159); then

$500 ÷ 1.02 = $490.196', the *amount of purchase.*

Of every $1.02 in the agent's hands, he can lay out $1 and retain a commission of 2 *cents*, which is 2 per cent. on $1; hence the whole number of dollars to be laid out is the number of times that 1.02 is contained in 500.

PROPORTION. 100 + 2 : 100 : : 500 : the *required number.*

24. A drover sold a lot of cattle for $900, which was at a profit of 20 per cent. on what he paid for them. What did he pay for the cattle? *Ans.* $750.

25. A grocer sells sugar at $0.12½ per *lb.*, and in so doing makes a profit of 25 per cent. on the cost. What did the sugar cost per *lb.*? *Ans.* $0.10.

26. An agent receives a remittance of $1200 to purchase cloth, and is to retain 1¼ per cent. on the purchase. What amount of purchase can he make? *Ans.* $1185.18'.

27. A manufacturer sold a lot of shoes for $400, which was $33\frac{1}{3}$ per cent. advance on the cost of making them. What was the cost of the shoes? *Ans.* $300.

28. A merchant having sold a lot of silks for $1012.95, finds that his profit is at the rate of 50 per cent. What was the cost of the silks? *Ans.* $675.30.

RULE XLI.

(171.) *To find a Number from which Percentage subtracted at a given Rate, will leave a given Remainder.*

Divide the given remainder by 1 *minus* the *ratio* of percentage; the quotient will be the required number, or *basis* of per centage.

EXAMPLE.

When Bank Stock sells at 5 per cent. *discount*, what amount of stock can be purchased for $475?

It must be understood that the "amount of stock" is estimated according to its *par value*, that is, its original cost, and not according to what it brings at any given time.

The *ratio* of percentage is .05, which is the *discount* on $1 of stock (159); then

$$\$475 \div .95 = \$500, \text{ the } \textit{amount of stock}.$$

Since 95 *cents* will buy $1 of stock, it is plain that the whole number of dollars of stock that may be purchased, is the number of times that $0.95 is contained in $475.

PROPORTION. 100 − 5 : 100 : : 475 : the required *amount of stock*.

29. A merchant sold a lot of damaged flour at \$3.75 per barrel, which was at a loss of 12½ per cent. on the cost of it. What did the flour cost per barrel? *Ans.* \$4.285.

30. When rail road stock sells at a discount of 7½ per cent., what amount of stock can be purchased for \$2775? *Ans.* \$3000.

31. A gentleman sold his household furniture for \$3242.74½, which was at a loss of 10 per cent. on the cost of it. What was the cost of the furniture? *Ans.* \$3603.05.

32. What amount of stock in the capital of a manufacturing company, at a discount of 3½ per cent, may be purchased for \$1930? *Ans.* \$2000.

33. A grocer sold a quantity of damaged coffee for \$701.25, and thereby sustained a loss of 15 per cent. What was the cost of the coffee? *Ans.* \$825.

34. A merchant obtains on a stock of goods amounting to \$5000 an insurance, at ½ per cent., which includes the value of the goods and the premium of insurance. For what amount is the policy taken?

The amount insured diminished ½ per cent. leaves \$5000. *Ans.* \$5025.125'.

35. By selling a piece of damaged silk at \$1.25 per yard, a merchant sustains a loss of 16⅔ per cent. At what price was the silk purchased? and what was the amount of loss on 20 yards of it? *Ans.* \$1.50; and \$5.

26. A manufactory, valued at \$2500, is insured, at 1¼ per cent., for such a sum, that, in case of its being destroyed by fire, the proprietors may claim, at the Insurance office, the value of the property, together with the premium paid for insurance. What is the amount insured?

Ans. \$2531.64'.

MISCELLANEOUS EXERCISES

ON THE APPLICATIONS OF PERCENTAGE.

1. A steamboat is valued at $35000, and its proprietors obtain an insurance on $\frac{3}{4}$ of its value, at $3\frac{1}{4}$ per cent. What is the premium to be paid? *Ans.* 853.12\frac{1}{2}$.

2. An upholsterer has his warehouse, valued at $8000, insured for $\frac{4}{5}$ of its value, at $1\frac{1}{2}$ per cent., per annum; and $10000 worth of furniture insured at $1\frac{3}{4}$ per cent., per annum. What does he pay for insurance annually? *Ans.* $271.

3. What would be the amount of duty, at 9 per cent., on 3*cwt.* 3*qr.* 20*lb.* of steel, which was purchased at 21.12\frac{1}{2}$ per *cwt.*? *Ans.* $7.46'.

4. The property of a town amounting to $5000000 is to be taxed to the amount of $100000, for the purpose of constructing a railroad. At what rate per cent. must the tax be assessed? *Ans.* 2 per cent.

5. Bought a quantity of wheat for 700.62\frac{1}{2}$; paid for transportation and other charges on it 43.06\frac{1}{4}$; and then sold it at a profit of 15 per cent. What was the amount of profit made? *Ans.* $111.553.

6. A sells for B 50 bales of cotton, averaging 450*lb.*, at $7\frac{1}{2}$ cents per *lb.*, and expends the proceeds for provisions, after retaining a commission of $1\frac{3}{4}$ per cent. on the sale, and $\frac{1}{2}$ per cent. on the purchase. What sum is expended for provisions? *Ans.* $1649.71'.

7. What would be the percentum of profit or loss on a quantity of spirits, if purchased at 0.43\frac{3}{4}$ a gallon, and sold at $12\frac{1}{2}$ cents a pint? *Ans.* $128.57' per cent. profit.

8. A merchant sold a lot of cloth at $6.50 per yard, and thus realized a gain of 25 per cent. If the cloth had been sold at $5 per yard, what would have been his percentum of gain or loss? *Ans.* $3\frac{11}{13}$ per cent. loss

9. An agent receives $3275 to invest in merchandise, at a commission of 1½ per cent. on the amount of purchase that can be made after this percentum is deducted. What will be the amount of his commission? *Ans.* $48.4'.

10. The tax on a certain landed estate amounts to $734.25. What is the estimated value of the estate, the tax being levied at ¾ per cent.? *Ans.* $97900.

11. What amount of stock in the capital of a canal company, at a discount of 3½ per cent., could be purchased for $3860? and what amount at an advance of 4 per cent., could be purchased for $6240? *Ans.* $4000, and $6000.

12. A merchant bought 37*yd.* 3*qr.* of cloth, at $4.87½ per yard, and 49*yd.* 2½*qr.* of silk, at 93¾ cents per yard. For what sum must the whole be sold to make a profit of 33⅓ per cent.? *Ans.* $307.40.

13. A commission merchant is to sell 12000*lb.* of cotton, and invest the proceeds in sugar—retaining 1¾ per cent. on the sale, and the same on the purchase. Cotton selling at 7 cents, and sugar at 5 cents per *lb.*, what quantity of sugar can the merchant buy? *Ans.* 16222.11'*lb.*

14. On a stock of leather which was sold at 18¾ per cent. profit, a merchant clears $237.50. What was the cost of the leather? *Ans.* $1266.66'.

15. A church which cost $20000 is insured, at ¼ per cent., for such a sum, that, in case of its being destroyed by fire, the Insurance Company shall be liable for the cost of the building and the premium of insurance. What is the sum insured? *Ans.* $20050.125'.

16. By selling a piece of damaged cloth at $2.50 per yard, a merchant sustained a loss of 16⅔ per cent. What did he lose on 20 yards? *Ans.* $10.00.

17. A commission merchant sold a lot of cloth for $200, at a profit to the owner of 20 per cent. If it had been sold for $175, what would have been the percentum of profit or loss, the commission in each case being 2 per cent.?

Ans. 5 per cent. profit.

18. A broker purchased for another $3000 of Bank Stock, at an advance of 3 per cent., and charged $\frac{3}{4}$ per cent. on the sum disbursed. What did the broker pay for the stock? and what is his commission? *Ans.* $3090, and $23.17'.

19. A shipment of goods from New York to Mobile, amounting to $5362.36, is insured, at $2\frac{1}{8}$ per cent., for such a sum, that, in case of a total loss at sea, the Insurance Company shall be liable for $\frac{3}{4}$ of the value of the goods, together with the premium of insurance. For what amount was the policy taken? *Ans.* $4109.08'.

20. A broker in Lexington receives $5000 in Southern funds, with instructions to purchase Stock in the Northern Bank of Kentucky. These funds are at a discount of $2\frac{1}{2}$ per cent., while the Bank Stock is at an advance of 10 per cent.; and the broker charges a commission of $\frac{3}{4}$ per cent. on the stock purchased. What amount of Stock can he purchase?

Ans. $4398.82'.

CHAPTER IX.

INTEREST.—EQUATION OF PAYMENTS, &C.

INTEREST.

(172.) INTEREST is the price or premium paid for the *use of money;* and is estimated at a certain percentum, *annually,* on the sum for which it is paid.

Thus the Interest of $300 for one year, at 6 per cent., is $18; for 2 years it is $36; for 3 years it is $54, and so on.

The *Principal* is the sum for which Interest is paid; the *Amount* is the sum of *principal* and *interest.*

What is the Interest of $100 for one year, at 6 per cent.? What is the Interest of $200 for 3 years, at 6 per cent.? For 4 years?

What is the Amount of $100 for one year, at 7 per cent.? What is the Amount of $300 for 2 years, at 8 per cent.? For 3 years?

Legal Interest.

(173.) The *legal rate* of Interest is the rate prescribed by the *law* of the State in which the debt is contracted; and is the rate always understood when none is named.

On debts in favor of the United States, interest is computed at 6 per cent.

In the individual States, the legal rate is 6 per cent,—excepting that it is 5 per cent. in Louisiana—7 per cent. in New York, South Carolina, Michigan, Wisconsin, and Iowa—8 per cent. in Georgia, Alabama, Mississippi, Florida, and Texas—and 10 per cent. in California.

Conventional Interest.

(174.) *Conventional* Interest is the rate of interest agreed upon between the *debtor* and *creditor*, as sanctioned by Law in some States; and is seldom allowed to exceed 12 per cent. per annum.

Usury is any rate of interest above that which the Law will sanction; and, in most States, is prohibited under various penalties; such as a forfeiture of double the usury, and sometimes a forfeiture of the debt.

Simple and Compound Interest.

(175.) *Simple* interest is that which is computed on a given *Principal* only; *Compound* interest is interest on both Principal and the *simple interest* when the latter remains *unpaid*, after it is understood to be *due*.

Compound Interest is not sanctioned by Law, on money lent, or debts contracted in ordinary commercial transactions.

The term *interest*, used alone, always means *simple* interest.

RULE XLII.

(176.) *To calculate Interest on any given Principal.*

1. Multiply the Principal by the *ratio* of percentage (159); the product will be the interest for *one year*, and may be multiplied to find the *interest for any number of years.*

2. Interest for *months* and *days* may be found by taking proper parts of one year's interest—in which case it is customary to consider a year as 12 *months* of 30 *days* each.

EXAMPLE.

To find the Interest of \$220.12½ for 4*y*. 7*m*. 10*da*., at 6 per cent.

The *ratio* of percentage is .06, the interest of \$1 for *one year* (159).

\$220.125		
.06		
13.20750,	interest for *one year;*	
4		
52.83000	"	for 4 years;
6.603'	"	for 6*m*. = ½*y*
1.100'	"	for 1*m*.
.366'	"	for 10*da*. = ⅓*m*.
\$60.899'	"	for 4*y*. 7*m*. 10*da*.

The interest for 7*m*. 10*da*. is found by means of *aliquot parts* (134).—The several parts of the interest added together make the *whole interest* required.

By Compound Proportion :—

$$\left.\begin{array}{l}\$100 : \$220.125 \\ 1y. : 4y.\ 7m.\ 10da.\end{array}\right\} :: \$6 : \text{Int. required } (155)$$

In calculating Interest for *months* and *days*, less than a year, strict accuracy would require that the number of days in those months be taken, and 365 days allowed to a year.

Thus, Interest for the months of *May* and *June*, which together contain 61 days, would be $\frac{61}{365}$ of *one year's interest.*

The method by the preceding Rule is commonly used, for convenience, though, in particular cases, it gives slightly too much, or too little, interest, for the time as reckoned from one *given date* to another.

EXERCISES.

1. A borrowed of B $500 for 2 years 5 months and 12 days. What did the interest amount to, at 6 per cent.? *Ans.* $73.50.

2. Borrowed of my neighbor $175.75, at 6 per cent. interest. What amount will I owe him, if the money be kept 3*y.* 11*m.*? *Ans.* $217.05.

3. C loaned to D $750.50, for 1*y.* 8*m.* 20*da.* What did the interest amount to at 7 per cent.? *Ans.* $90.476'.

4. Loaned to a friend $436.75, at 5 per cent. interest. What sum will discharge the debt at the end of 1*y.* 2*m.* 15*da.*? *Ans.* $463.136'.

5. An account with a merchant of $75.87½ bore interest, at 6 per cent., for 2*y.* 4*m.* 10*da.* What amount was then due? *Ans.* $86.623'.

6. A farmer hired a laborer for $125. Having deferred the payment for 3*y.* 1*m.* 25*da.*, what amount should the farmer now pay, allowing interest at 6 per cent.? *Ans.* $148.645'.

7. Bought a plantation for $4500, on a credit of 2*y.* 6*m.*, by paying interest at 8 per cent. What will be the amount at the expiration of that time? *Ans.* $5400.

8. A debt of $500.50 has been due, with interest at 7 per cent., for 3*y.* 8*m.* 20*da.* What sum will discharge the debt? *Ans.* $630.90'.

9. Judgment was issued against the defendant in a suit for $325.37½, with interest at 7 per cent., for 2*y.* 10*m.* 25*da.* What was the amount of the judgment? *Ans.* $391.48'.

10. An agent sold property to the amount of $345.30, after taking out his commission, and having kept the money for 8*m.* 21*da.*, now pays it, with interest at 8 per cent. What is the amount? *Ans.* $365.32'.

11. What is the interest on $1000 from February 20th, 1850, to November 15th, 1853, at 5 per cent. ?

The *interval of time* between these dates would be found by the method explained in Polynomial Subtraction (124); but in calculations like the present it is customary to consider *every month as containing* 30 *days*.

y.	*m.*	*d.*
1853	11	15
1850	2	20
3	8	25

February being the 2d, and *November* the 11th month, we denote them by these numbers respectively; then subtracting polynomially, we find the interval of time to be 3*y.* 8*m.* 25*da.*

Ans. $186.80'.

12. What is the interest on $350.90 from January 10th, 1850, to June 1st, 1852, at 6 per cent. ?

Ans. $50.354'.

13. What is the interest on $425.30 from March 4th, 1851, to May 19th, 1853, at 7 per cent. ?

Ans. $65.744'.

14. What is the interest on $504.12½ from September 12th, 1850, to February 3d, 1853, at 8 per cent. ?

Ans. $96.455'.

15. What is the interest on $634.25 from August 13th, 1850, to July 10th, 1854, at 10 per cent. ?

Ans. $247.88'.

16. What is the interest on $730.37½ from April 10th, 1852, to October 5th, 1855, at 12 per cent. ?

Ans. $305.53'.

(177.) *Concise Method of computing Interest for Months and Days—allowing* 12 *months of* 30 *days each to a year.*

1. *For* 6 *per cent.*—Multiply the Principal by half the number of *months*, and divide the product by 100 ; or multiply by $\frac{1}{6}$ of the number of *days*, and divide by 1000.

2. *For any other rate*—find the interest for 6 per cent., as above, and increase or diminish it proportionably for the other rate.

EXAMPLES.

1. To find the interest of \$50.10 for 8*m*. at 6 per cent.

$$\$\frac{50.10\times 4}{100}=\$2.0040.$$

The interest of \$1 for *one year* is $\frac{6}{100}$; for 8*m*. it is therefore

$$\tfrac{8}{12} \text{ of } \tfrac{6}{100}=\tfrac{6\times 8}{12\times 100}=\tfrac{1}{2} \text{ of } \tfrac{8}{100}=\tfrac{4}{100},$$

the numerator 4 being $\frac{1}{2}$ the number of *months ;* and the given Principal $\times \frac{4}{100}$, the *interest of* \$1, gives the interest of that principal.

2. To find the interest of \$500.10 for 18 *days*, at 6 per cent.

$$\$\frac{500.10\times 3}{1000}=\$1.50030.$$

The interest of \$1 for 1*y*. is $\frac{6}{100}$; for 18*da*. it is therefore

$$\tfrac{18}{360} \text{ of } \tfrac{6}{100}=\tfrac{6\times 18}{360\times 100}=\tfrac{6\times 18}{36\times 1000}=\tfrac{1}{6} \text{ of } \tfrac{18}{1000}=\tfrac{3}{1000},$$

the numerator 3 being $\frac{1}{6}$ of the number of *days ;* and the given Principal $\times \frac{3}{1000}$, the *interest of* \$1, gives the interest of that principal.

Having found the *Interest at* 6 *per cent.*, $\frac{5}{6}$ of that would be the interest at 5 *per cent. ;* $\frac{7}{6}$ of it would be the interest at 7 *per cent. ;* and so on.

When the time is in *months* and *days*, find the interest for each, separately, and add the two results together, for the whole interest.

When $\frac{1}{6}$ of the given number of days cannot be exactly expressed in *integers* and *decimals*, as $\frac{1}{6}$ of 20 days; it will be best to multiply the Principal by the whole number of days, and then divide the product by 6000.

The remarks respecting the accuracy of the result which accompany the Example to Rule XLII., are also applicable to the present method.

17. Find the interest on $230 for 11*m*. 15*da*., at 6 per cent. *Ans.* $13.225.

18. Find the interest on $1234.75 for 120*da*., at 6 per cent. *Ans.* $24.695.

19. Find the interest on $4360.12½ for 54*da*., at 5 per cent. *Ans.* 32.70'.

20. Find the interest on $1385.50 for 23*da*., at 7 per cent. *Ans.* $6.196'.

21. Find the interest on $2360.25 for 7*m*., at 8 per cent. *Ans.* $110.145.

22. Find the interest on $3879.06¼ for 9*da*., at 7 per cent. *Ans.* $6.788'.

23. Find the interest on $9000.87½ for 17*m*., at 7 per cent. *Ans.* $892.586'.

24. Find the interest on $2300.25 for 137*da*., at 5 per cent. *Ans.* $43.768'.

25. Find the interest on $8730.62½ for 5*m*. 23*da*., at 6 per cent., and also at 8 per cent.

Ans. $251.73'; $335.64'.

Partial Payments on Notes or Bonds.

(178.) No one method has been universally approved for computing the Balance due on Notes, &c., bearing interest, on which *partial payments* have been made.

The following has been adopted by the Supreme Court of the United States, and by most of the individual States.

1. Whenever a payment, or the *aggregate of payments* made, will cancel the *interest due,* add the interest to the Principal, and from the amount subtract the payment, or aggregate of payments up to that time.

2. The remainder is to be regarded as a *new Principal,* dating from the time of the last payment; and payments on this principal are to be subtracted in the same manner as before; and so on.

EXAMPLE.

$4000. *Washington, Jan. 1st,* 1850.

On demand I promise to pay James Wealthy, Four thousand dollars, with interest, for value received.

John Ready.

On this Note the following payments are *endorsed:*

July 1st, 1851, received $300;
April 11th, 1853, received $700.

What balance is due on the Note on the 20th of August, 1855?

The legal rate of Interest at Washington, D. C., is 6 per cent. (173). This is therefore the rate of interest which must be allowed on the Note, the debt being supposed to have been contracted in *Washington;* and the calculation for the Balance is as follows:

Principal, January 1st, 1850, ...		$4000.00
Interest on it to July 1st, 1851, . .	$360	
The payment $300 will not cancel this interest.		
Interest on the $4000 to April 11th, 1853,		786.66
Amount then due,		$4786.66
Subtract the payments made, $300+700, .		1000.00
Principal, April 11th, 1853, . . .		$3786.66
Interest on it to August 20th, 1855, . .		535.81
Amount or balance then due,		$4322.47

The intervals of time between the dates will be found as in Ex. 11th, page 215.

Interest for *months* and *days* may be computed by the general Rule (176), or, sometimes more concisely, by the other method (177).

$950. *Philadelphia, June 26th*, 1853.

26. On demand I promise to pay Timothy Friend, Nine hundred and fifty dollars, with interest, for value received.

Jacob Faithful.

Endorsements.—March 20th, 1854, received $430.
May 15th, 1854, received $234.75.

What balance was due, June 1st, 1855? *Ans.* $353.01'.

$3000. *New York, May 1st*, 1852.

27. On demand I promise to pay to John Prosperous, Three thousand dollars, with interest, for value received.

William Needy.

Endorsements.—November 1st, 1852, received $1000,
October 10th, 1853, received $93.75.
December 20th, 1854, received $300.50.

What balance was due, January 1st, 1855?

Ans. $2029.83'.

$500. *New Orleans, May 1st*, 1850.

28. Twelve months after date I promise to pay to Caleb Strong, Five hundred dollars, with interest, for value received.

James Mindful.

Endorsements.—May 1st, 1851, received $200.
June 10th, 1852, received $100.50.
July 15th, 1853, received $50.75.

What balance was due, September 20th, 1854 ?

Ans. $217.16'

Partial Payments on Accounts.

(**179.**) The following method is frequently adopted for computing the Balance due on an account, bearing interest on which *partial payments* have been made.

1. Compute the interest on the account from the time it *became due*, and the interest on *each payment* from the time it was made, to the *time of settlement*.

2. Subtract the sum of all the payments and their interest from the sum of the account and its interest; the remainder will be the *balance due*. But

The same result may also be obtained, and frequently with greater convenience, as follows:

1. Multiply the Principal due at first, and the remainders after the payments are successively subtracted, each by the *number of days* it was separately at interest.

2. Divide the sum of the products by 6000. The quotient will be the interest, at 6 *per cent.* (from which may be found the interest at *any other rate*), to be added to the last remainder, for the *balance due*.

EXAMPLE.

A merchant's account amounting to \$230 was due January 1st, 1854; on which there was paid, March 20th, 1854, \$80, and June 30th, 1854, \$100. What was the balance due, October 15th, 1854, allowing interest at 6 per cent.?

From Jan. 1st to March 20th, is 78*da.*; \$230 × 78 = 17940

From March 20th to June 30th, 102*da.*; $\frac{80}{150}$ × 102 = 15300

From June 30th to Oct. 15th, 107*da.*; $\frac{100}{50}$ × 107 = 5350

The sum of these products is 38590;

6000)38590(6.43;

then \$50 + \$66.81 = \$56.43 was the *required balance.*

Only one of the days of the two given dates must be included in the interval of time for which interest is computed; and it will be most convenient to find such intervals by means of the following Table, which shows

(180.) *The Number of Days between any two Dates, to the extent of one Year.*

From any day of	To the corresponding day of the following											
	Jan.	Feb.	Mar.	Ap.	May.	June	July	Aug	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March,	306	337	365	31	61	92	122	153	184	214	245	275
April,	275	306	334	365	30	61	91	122	153	183	214	244
May,	245	276	304	335	365	31	61	92	123	153	184	214
June,	214	245	273	304	334	365	30	61	92	122	153	183
July,	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Each of the numbers in the preceding Table is the number of days from any day of the month standing opposite on the left, to the corresponding day of the month which stands above.

The use of the table will be seen from the following Example.—To find the number of days from June 20th to March 13th.

Opposite to June and under March, we find 273, which is the number of days from June 20th to March 20th; and since March 13th is 7 days short of March 20th, the required number of days is $273-7=266$.

In Leap years, the numbers in the preceding Table will be 1 *more*, after the 28th of February, because, in such years, February has 29 days.

29. An account for $350 was due, January 1st, 1854; on which $120 was paid, June 10th, and $100, September 25th, 1854. What balance was due on the 20th of December, 1854, allowing interest at 6 per cent.?

Ans. $145.29'.

30. An account for $485 became due, July 1st, 1854, on which was paid, October 1st, 1854, $100, December 15th, 1854, $180, and May 26th, 1855, $90. What balance is due, August 1st, 1855, allowing interest at 6 per cent.?

Ans. $134.06'.

31. With the same items as in the preceding question, what balance would be due, if the computation were made according to the method adopted for partial payments on Notes or Bonds? (178). *Ans.* $134.37'.

Why should this last *Answer* be greater than the preceding one?

(181.) *Rate per cent., or Time of Interest,—how found*

1. The *Rate per cent.* is found by dividing the given Interest by the interest of the given Principal, at 1 *per cent.*, for the given time.

2. The *Time*, in *years*, months, or days, is found by dividing the given Interest by the interest of the given Principal, at the given rate, for 1 *year*, month, or day, respectively.

32. At what rate per cent. would $250 amount to $287.50 in 2*y*. 6*m*.?

The given interest is $287.50 − 250 = $37.50;

and the interest of $250, at 1 *per cent*,, for 2*y*. 6*m*., is $6.25 (176); then the required rate per cent. is $37.50 ÷ 6.25.

Ans. 6 per cent.

33. In what time will $300 amount to $373.50, at 7 per cent. interest?

The given interest is $373.50 − 300 = $73.50;

and the interest of $300, at 7 per cent. for 1 *year*, is $21;

then the required number of years is 73.50 ÷ 21.

Ans. 3.5 years.

34. At what rate per cent. must $1000 be put at interest, to amount to $1120 in 1 year and 6 months?

Ans. 8 per cent.

35. In what time will $475.37½ amount to $532.42, if the rate of interest be 6 per cent.? *Ans.* 2 years.

36. In what time will $100, or any other principal, double itself, if put on intérest at 6 per cent.? *Ans.* 16⅔ years.

37. In what time will $1000, or any other principal, double itself, if the rate of interest be 7 per cent.?

Ans. 14$\frac{2}{7}$ years.

Present Worth of a Future. Debt.

(182.) The PRESENT WORTH of a debt *not due*, and not bearing interest, is that *principal* which, put at interest, would *amount to the debt* by the time the debt becomes due.

The present worth subtracted from the future debt leaves the *discount*, which is the deduction that should be made for the *present payment* of the debt.

RULE XLIII.

(183.) *To find the Principal, or Present Worth of a future debt, when the Amount, Time, and Rate of interest are given.*

Divide the given Amount by the amount of $1, at the given rate of interest, for the given time ; the quotient will be the *principal* required.

38. A debt of $500 will be due in 3 years, without interest. What is the *present worth* of the debt, allowing the rate of interest to be 6 per cent. ?

The amount of $1, at 6 per cent., for $3y.$, is $1.18 ; then $500 ÷ 1.18 gives the *present worth*, that is, the *principal* which would amount to the debt by the time the debt becomes due. *Ans.* $423.72.

39. What principal would amount to $650 in 2 years if the rate of interest be 5 per cent. ? *Ans.* $590.909.

40. What is the present worth of $750 due in 2 years 3 months, 20 days, allowing the rate of interest to be 7 per cent. ? *Ans.* $645.827'.

41. What *discount* should be allowed for the present payment of a debt of $1000, due in 1 year and 6 months, when the interest is 8 per cent. ? *Ans.* $107.143

42. Sold property amounting to $3000, on a credit of 12 months, without interest. What should be discounted for present payment, allowing the rate of interest to be 8 per cent.?

Ans. $222.223.

43. A note for $1200 has 1*y.* 10*m.* 15*da.* to run, without interest. What sum in hand would be an equivalent for the note, supposing the rate of interest to be 6 per cent.?

Ans. $1078.65'.

44. A merchant buys goods amounting to $4200, on a credit of 6 months, but purposes to pay immediately, for the proper discount. What sum in ready money will discharge the debt, interest being at 8 per cent.? *Ans.* $4038.46'.

Proceeds of a Note in Bank.

(184.) A Note on which money is obtained from a Bank has usually from two to four months in which to *mature*, that is, become due; and it is customary to extend the time specified in the Note 3 or 4 days, called *days of grace*, before payment is required.

The Bank deducts the interest from the *face of the note*, or principal for which the Note is written—*days of grace* being included in the calculation; this is called *discounting* the Note. The remainder is the sum paid for the Note, and is called the *avails* or *proceeds* of the Note.

For example, suppose that a Note for $500, payable in 60 days, is discounted in a Bank, at 6 per cent. interest.

Including 3 days of grace, the time on the Note will be 63 days. The interest of $500 for 63 days is $5.25, which is the Bank *discount*. This subtracted from $500, leaves $494.75, the *proceeds* of the Note.

The borrower therefore receives $494.75, and at the expiration of 63 days must pay the $500.

RULE XLIV.

(185.) *To find the Principal from which the Interest, or Bank Discount, deducted will leave a Given Sum.*

Divide the given sum by $1 *minus* the interest of $1 for the given time—days of grace included; the quotient will be the principal required.

In finding the interest of $1, in the application of this Rule, the decimal should contain at least *four figures*, if it does not terminate with two or three.

Three days of grace are allowed in the United States and Great Britain.

45. What must be the principal of a Note, payable in 60 days, that, when discounted in Bank, at 6 per cent. interest, the proceeds shall be $500?

The interest of $1 for 63 days is $0.0105 (177); $1—$0.0105=$0.9895.

Then $500 ÷ .9895 gives the principal required.

Ans. $505.30'.

46. What would be the proceeds of a Note for $1000, due in 90 days, if discounted in Bank, at 6 per cent. interest?

Ans. $984.50.

47. A Note for $300, payable in 4 months, or 120 days, was discounted in Bank, at 8 per cent. Required the sum received for it. *Ans.* $291.80.

48. A merchant wishes to borrow in Bank $2500, for 90 days. For what principal must his Note be drawn, rating the interest at 6 per cent.? *Ans.* $2539.36'.

49. A person wishes to pay a debt of $375.25 by having a Note discounted in Bank, for 60 days. For what sum must the Note be made, allowing the rate of interest to be 6 per cent.? *Ans.* $379.23'.

EQUATION OF PAYMENTS.

(186.) The EQUATION OF PAYMENTS consists in reducing two or more different times at which payments of money are to be made, without interest, to one equitable *mean time* for the payment of the whole.

The different cases which the subject presents will be exemplified in connection with the appropriate Rules.

RULE XLV.

(187) *To find the proper Credit for the Sum of two or more Payments due at different times, without interest.*

Multiply each payment *not due* by its own period of credit, and divide the sum of the products by the sum of *all the payments.* The quotient will be the time required.

The multipliers must all be taken in the *same order of units.*

EXAMPLE.

A owes B $600, of which $300 is to be paid in hand, $200 in 6 months, and the remaining $100 in 18 months. If the whole were reduced to *one payment*, in what time ought that payment to be made

$$\begin{aligned} \$200\times 6 &= 1200 \\ 100\times 18 &= 1800 \\ 600)\overline{3000} & \text{ (5 months.} \end{aligned}$$

The credit on the *sum of the payments* should be such that the *discount* (182) on that sum, would equal the *sum of the discounts* on the separate payments.

The preceding Rule substitutes *interest* for *discount,* and is therefore inaccurate, since the discount, being the interest on the *present worth* of a debt, is less than the interest on the debt itself. For convenience, the Rule is adopted in business.

In the preceding Example,

The interest on \$200 for 6*m.*=the interest on \$1 for 1200*m.*,
interest on \$100 for 18*m.*=the interest on \$1 for 1800*m.*

The whole interest involved is therefore equal to the interest on \$1 for 1200+1800=3000 *months;* and the same interest would accrue on the \$600 in 3000 ÷ 600=5 *months.* Hence 5 months is the credit to be allowed on the \$600.

EXERCISES.

1. A is indebted to B \$900; of which \$200 will be due in 6 months, \$300 more in 9 months, and the remainder in 12 months. What would be the proper time for the payment of the whole at once? *Ans.* $9\frac{2}{3}$ months.

2. A merchant bought goods amounting to \$5000; of which he was to pay \$3000 in hand, and the remainder in 6 months. It is since agreed that the whole shall be paid at one time; what is the proper credit to be allowed?

Ans. $2\frac{2}{5}$ months.

3. A sum of money was to be paid as follows; viz., $\frac{1}{4}$ of it in 2 years, $\frac{1}{2}$ of it in 3 years, and the remainder in 4 years and 6 months. The debtor proposing to pay the whole at one time, what is the proper credit to be allowed?

The payments to be multiplied in this case will be represented by $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$. The *divisor* to be employed will be the sum of these fractions, that is, a *unit.*

Ans. $3\frac{1}{8}$ years.

4. A plantation is to be paid for in three equal instalments, in 6, 9, and 12 months from the day on which it was sold. If

the three payments be converted into one, how long a credit should be allowed on it?

The three instalments may each be represented by $\frac{1}{3}$; or we may find the *mean time* for payment from assuming any one number, as $1, to represent each instalment, in which case the *divisor* to be employed will be 3. *Ans.* 9 months.

5. A engaged to pay to B $200 on the first day of January, $300 on the 15th of April, and $400 on the 20th of August. They now agree to make but one payment of the whole, and wish to know the mean time for that payment.

We may compute the mean time of payment from any *assumed date*, as January the 1st, when the $200 becomes due.

The $300 had a credit of 104 days, and the $400 a credit of 231 days, *after January the* 1st.

$$\$300 \times 104 = 31200$$
$$400 \times 231 = 92400$$
$$900)\overline{123600}\ (137\tfrac{1}{3} \text{ days.}$$

We find that the mean time is $137\frac{1}{3}$ days after the 1st of January. 137 is the nearest number of *whole days.*

To find the date, we may use the Table already given (180). Among the numbers opposite to January, the nearest to 137 is 120, which brings the time from January 1st to May 1st; then 137 will make it 17 days later, that is, May 18th.
Ans. The 18th of May.

6. A is indebted to B $400, of which $100 will be due on the 1st of July, $150 on the 3d of September, and the remainder on the 16th of December—without interest. On what day might the whole be paid, consistently with justice?
Ans. The 26th of September.

7. On the 5th of September, 1854, a merchant bought goods amounting to $8000 ; of which $4000 was to be paid in 4 months, $2000 in 6 months, and the remainder in 8 months. It was afterwards arranged that but one payment should be made of the whole ; what was the proper day of payment?

* In questions like this, the time is reckoned by the *Calendar*, without reference to the number of days in the particular months. Thus the first payment above, was due on the 5th of January.

Ans. Feb. 17th, 1855.

RULE XLVI.

(188.) *When Partial Payments are made on a Debt not due, and not bearing interest, to find the proper Extension of Credit on the Balance of the Debt.*

Multiply each payment by the number of days it is made before the Debt is due, and divide the *sum of the products* by the Balance remaining unpaid ; the quotient will be the number of days the *credit on the balance should be extended.*

8. A house was sold on the first of January, for $1000, to be paid in 6 months, without interest. Of this debt $300 was paid on the 1st of April, and $500 on the 15th of May ; on what day is the remaining $200 equitably due?

The debt $1000 is due on the 1st of July. The $300 is paid 91 days, and the $500 is paid 47 days, *before the 1st of July* (180).

$$\$300 \times 91 = 27300$$
$$500 \times 47 = 23500$$
$$200) \quad 50800 \; (254 \text{ days.}$$

The remaining $200 is justly due 254 days *after the 1st of July.* By means of the Table already referred to, the time for the payment of the $200 is found to be the *12th of the following March.*

The interest on the $300 for the 91 days, and the $500 for the 47 days, they are paid *before due,* is equal to the interest of $50800 for 1 *day*, and is balanced by the interest of the $200 for the 254 days this last payment is *deferred.*

9. A merchant bought goods to the amount of $700, on a credit of 6 months. At the expiration of 3 months he paid $300, and one month afterwards $200 more; what *extension of credit* ought he to have on the balance of the debt?

Ans. 6½ months.

10. The sum of $900 is to be paid in 12 months, without interest. If $300 be. paid on this debt at the end of 5 months, $200 more at the end of 7 months, and 100 more at the end of 9 months; how much beyond the 12 months might the payment of the balance justly be deferred?

Ans. 11⅓ months.

11. On the 10th of April, 1854, a person gave his Note for $600, at 9 months, without interest. On the 20th of June following he paid $200 of the debt, and on the 25th of August following, $300. When will the balance be due, if the proper extension be made on account of these payments?

Ans. April 12, 1857.

12. On the 25th of August, 1855, a merchant bought goods to the amount of $5000, on a credit of 6 months. On the first of November following he paid $400 of this debt, and on the 10th of December $300 more; when will the balance of the debt be equitably due?

Ans. March 13, 1856.

EQUATION OF ACCOUNTS.

(189.) The *Equation of an Account* containing charges or *debits* at different dates, consists in finding the *mean date* at which the *whole debit* may be charged, or the date at which it becomes *due* when time is allowed on its separate items.

Different cases will be presented according as the same or different periods of credit are allowed on the separate items of the account.

RULE XLVII.

(190.) *To find the mean Date at which an Account may be charged—or becomes due, with the same Time on its separate Items.*

1. Multiply each debit, after the *first*, by its time from the *first date*, and divide the sum of the products by the sum of all the debits; the quotient will be the time after the first date for the *mean date* of the Account.

2. The *time of credit*, if any, must be reckoned from the *mean date*, for the date at which the whole debit becomes *due*.

EXAMPLE.

An account with a merchant consists of the following items.

January 1,	Merchandise	. .	$30;
March 5,	"	. .	$25;
April 10,	"	. .	$40;
May 20,	"	. .	$50.

When is this Account *due*, with a Credit of 6 months on each item?

$$\begin{array}{l} \$30 \\ \$25 \times 63 = 1575 \\ \$40 \times 99 = 3960 \\ \$50 \times 139 = 6950 \\ \hline 145 \;)\; 12485 \;(86 \text{ days.} \end{array}$$

From Jan. 1st to March 5th is 63 days, by which we multiply the $25; from Jan. 1st to April 10th is 99 days, by which we multiply the $40, &c.

We find the *mean date* of the Account to be 86 days *after Jan. 1st*, which is the 28th of March (180).

The whole Account for $145 may therefore be regarded as transacted on the 28th of March; and, allowing 6 months Credit, it becomes *due* on the 28th of September. If paid before this latter date, the Account would be fairly entitled to *discount;* if not paid till afterwards, it would be chargeable with *interest.*

This method of *equating* an Account depends on the same principles as Rule XLV.

13. Find the mean date of the following Account, that is, the date at which the whole Account might be charged:

June 10th,	Merchandise . .	$20;
Aug. 5th,	" . .	$35;
Oct. 25th,	" . .	$50.

Ans. September 2d.

14. The following Account is entitled to 3 months credit; from what date would the whole Account be chargeable with interest?

May 1st,	Merchandise . .	$60;
July 4th,	" . .	$75;
Sept. 15th,	" . .	$100;

Ans. October 19th.

15. The following Account was made on 4 months credit; from what date would the whole Account be chargeable with interest?

Jan. 20, Merchandise	. .	$50;
March 1st,	" . .	$75;
April 10th,	" . .	$100;
June 30,	" . .	$120.

Ans. August 18th.

RULE XLVIII.

(191.) *To find the mean Date at which an Account becomes due, with different Periods of Credit on its separate Items.*

1. Find the date at which each debit falls due.

2. Multiply each debit, after the one which is *first due*, by the period of its falling due *after the first is due;* and divide the sum of the products by the sum of all the debits.

3. The quotient will be the period of the Account's falling due *after the first debit is due.*

EXAMPLE.

A merchant has the following Account against one of his customers:

Jan. 1, Merchandise	. .	$50, on 6 months credit;
Feb. 10, "	. .	$60, on 4 months credit;
Mar. 20, "	. .	$70, on 8 months credit.

On what day may the whole Account be considered as *due?*

The $50 is due July 1;
$60 is due June 10;
$70 is due Nov. 20,

The \$60 is *first due;* the \$50 is due 21 days afterwards, and the \$70, 163 days afterwards (180). The operation therefore is

$$\begin{array}{l} \$60 \\ \ 50 \times \ 21 = \ 1050 \\ \ \underline{70 \times 163 = 11410} \\ 180) \qquad 12460(69 \text{ days.} \end{array}$$

We find that the Account is due 69 days after the 10th of June, which is *Aug.* 18*th.* If not then paid, it would be entitled to interest from that date.

16. Find on what day the following Account is *due:*

April 5, Merchandise	. .	\$20, at 3 months ;
June 15, "	. .	\$40, at 4 months ;
Aug. 25, "	. .	\$60, at 2 months.

Ans. October 3d.

17. Find on what day the following Account is *due:*

May 20,	Merchandise,	\$25, at 4 months ;
July 10,	"	\$100, at 3 months ;
Sept. 5,	"	\$120, at 2 months.

Ans. October 21st.

18. Find on what day the following account is *due* :

January 1st,	Merchandise,	\$ 50, at 3 months ;
March 20th,	"	\$ 75, at 4 months ;
June 30th,	"	\$ 84, at 4 months ;
August 15th,	"	\$120, at 5 months.

Ans. October 3d.

Equation of Accounts Current.

(**192.**) An *Account Current* is a statement of mercantile transactions in which a person is *Debtor* for merchandise sold, or money paid, to him, and *Creditor* for merchandise purchased, or money received, from him.

The *Equation* of such an Account consists in finding the date at which the *Balance* of the Account becomes *due*, or subject to *interest.*

RULE XLIX.

(**193.**) *To find the Date at which the Balance of an Account Current becomes due, or subject to Interest.*

1. Find the mean date at which the Debtor side, and also the Creditor side, of the Account becomes subject to interest (190 or 191).

2. Multiply the *smaller side* of the Account by the time between the two dates thus found, and divide the product by the *balance*, or difference between the *Dr.* and *Cr.* sides of the Account.

3. The quotient will be the time the Balance falls due *before the earlier date*, when the Balance is on the side of that date; but *after the later date* when the Balance is on the side of that date.

EXAMPLES.

I. A has an Account with B, in which the debtor and creditor sides, when *equated* separately (190 or 191), are found to be as follows:

1855.	*Dr.*	1855.	*Cr.*
June 1, . .	$700	June 16, . .	$500.

When does the Balance, $200, become *due*, or subject to *interest?*

$500, the smaller side of the Account ;
15 days between the mean dates.
200)7500 (37 days.

The Balance, $200, is on the side of the *earlier date*, June 1 ; then the $200 becomes subject to *interest* 37 days *before* June 1, which is April 25th.

The reason of this method may be shown as follows : The operation shows that the interest of $200 for 37 days is equal to the interest of $500 for 15 days ; hence the interest of $700 for 37 days is equal to the interest of $500 for 37+15, 52 days. If therefore the Account had been settled on the 25th of April, the debtor and creditor sides would have been subject to the *same discount*, which, in the Equation of Accounts, is considered equal to *interest ;* and the Balance, $200, would then have been *due in cash*.

II. Suppose the Balance to be on the side of the *later date ;* thus

1855.	*Dr.*	1855.	*Cr.*
June 1, . .	$500	June 16, . .	$700

The interest of $500 for 37+15 days being equal to the interest of $700 for 37 days, as before; if the Account be settled 37 days *after June* 16, that is, July 23d, its two sides will then be subject to the *same interest*, and the Balance $200 will be *due in cash*.

In this second example, the balance, $200, is thus found to be due on the 23*d of July*. If paid before this date, it would be entitled to a *discount* (182) ; if not paid till afterwards, it would be chargeable with *interest*.

19. The Debtor and Creditor sides of an Account, after having been equated separately, (190 or 191), are found to be as follows:

1855.	*Dr.*	1855.	*Cr.*
May 1, . .	$300	June 20, . .	$250

When does the Balance, $50, become due, or subject to interest?

Ans. August 24th, 1854.

20. What sum would pay the Balance of the account in the preceding Example, on the 1st of November, 1855, allowing interest at 6 per cent.?

The sum to be paid is what the $50 will amount to from August 24th to November 1st.

Ans. $53.56'.

21. The Debtor and Creditor sides of an Account, after having been equated separately, (190 or 191), are found to be as follows:

1855.	*Dr.*	1855.	*Cr.*
Aug. 20, . .	$500	Nov. 10, . .	$800

When does the Balance, $300, become due, or subject to interest?

Ans. March 27th, 1856.

22. What sum would pay the Balance of the account in the preceding Example, on the 1st of January, 1855, if the rate of interest be 6 per cent.?

The sum that should be paid is the *present worth* (182) of the $300 from January 1st to March 27th.

Ans. $295.82'.

MISCELLANEOUS EXERCISES

ON INTEREST, THE EQUATION OF PAYMENTS, &C.

1. Bought a plantation for $6000, of which one fourth is to be paid in hand, and the remainder in 2*y*. 6*m*., with interest at 8 per cent. What will be the Amount of the remainder, at the expiration of the credit? *Ans.* $5400.

2. A person buys a house for $3600, to be paid in three equal instalments, in one, two, and three years, with interest at 6 per cent. What will be the entire Amount to be paid? *Ans.* $4032.

3. A speculator borrowed $5000, which he immediately invested in land. Six months afterwards he sold the land for $7500, on a credit of 12 months, with interest. Money being at 6 per cent., what is the speculator's profit at the end of the 12 months, at which time he pays the $5000. *Ans.* $2500.

4. $2485.75 *New Orleans, June 3d,* 1852.

On or before the 1st of January next, I promise to pay to Enos Goodfellow, Two thousand four hundred and eighty-five dollars and seventy-five cents, for value received.

Timothy Trustworthy.

Endorsements.—March 4th, 1853, received $100.
May 20th, 1853, received $340.50.

What balance was due on the 25th of December, 1854? *Ans.* $2337.87'.

5. A planter consigned to a commission merchant 45000*lb.* of cotton, which the latter sold at 9½ cents per *lb.*, and charged ½ per cent. The proceeds due the planter were retained for 1*y*. 10*m.*; what amount was then due him, allowing interest at 7 per cent.? *Ans.* $4799.50'

6. A farmer bought of a merchant goods amounting to $175.12½, on a credit of 12 months; but paid the debt in 2 months and 10 days. What sum should have been discounted from the debt, allowing the rate of interest to be 8 per cent.?

Ans. $10.59'.

7. A loaned B $320 for 2*y.* 9*m.* 25*da.*, at the end of which time the Amount was found to be $383.155'. At what rate per cent. was the interest computed? *Ans.* 7 per cent.

8. $2500.18¾. *Philadelphia, June 1st,* 1851.

On the 1st day of January, 1852, I promise to pay to William Kind, the sum of Two thousand five hundred dollars, 18¾ cents, with interest; for value received.

Simon Thankful.

Endorsements.—January 1st, 1852, received $1000.
October 10th, 1852, received $35.25.
August 16th, 1853, received $200.

The balance of the Note was not paid until the 1st of January, 1854; what amount was then due?

Ans. $1541.15'.

9. A debt of $375.37½, in the District of Columbia, remained unpaid until it had legally amounted to $500. How long was it on interest? *Ans.* 5.533' years.

10. A gentleman in New York proposes to loan a sum of money which, at lawful interest, shall amount to $5000 in 2 years and 6 months. What must be the amount of the loan?

Ans. $4255.319'

11. An account of $200.50 was due, January 1st, 1854. On this debt there was paid, March 10th, $80, and June 20th, $75.25; what balance was due on the 1st of October, 1854, with interest at 6 *p. c.?* (179). *Ans.* $50.34'.

12. A merchant bought a stock of goods amounting to $7381.25 on a credit of 6 months, but paid the debt in 3*m.*

21*da*. from the time of purchase. What discount should have been allowed him, interest being at 6 per cent.?

Ans. $83.919'.

13. A debt of $3000.75 will be due in 2*y*. 7*m*. 18*da*. without interest. What sum in hand would be an equivalent for the debt, allowing the rate of interest to be 7 per cent.?

Ans. $2533.77'.

14. An account for $300 was due July 1st, 1854. On this debt there was paid, August 20th, $60, October 15th, $70, and December 1st, $80. What balance was due on the 1st of January, 1855,—according to each of the two methods which have been given for applying *partial payments*, with interest at 6 per cent.? *Ans*. $96.48 and $96.53.

15. A merchant bought 43*cwt*. 3*qr*. of sugar, at $5.25 per *cwt*., which he immediately sold at $7 per *cwt*., on a credit of 90 days, and then had the purchaser's Note for the amount discounted in Bank, at 6 per cent. What profit did the merchant make? *Ans*. $71.81'.

16. Wishing to raise the sum of $3760.50, I design, for this purpose, to put a Note in Bank, for 120 days. For what principal must the Note be drawn, interest being at 8 per cent.? *Ans*. $3866.04'.

17. A owes B 5000; of which $1200 is to be paid in 9 months, $3000 in 1 year and 3 months, and the remainder in 2 years. In what time might the whole debt be paid, without injustice to either? *Ans*. 15 months.

18. A person owes a debt of $1500, towards the payment of which he is able to raise but $900. He offers this sum, to be applied in part to a payment on the debt, and in part to paying the interest, at 8 per cent., in advance, on his Note, at 12 months, for the remainder. His creditor agreeing, for what principal must the Note be drawn? *Ans*. $652.17'.

19. A rice plantation was to be paid for as follows: $\frac{1}{4}$ of the purchase money in hand, $\frac{1}{3}$ of it in 12 months, and the remainder in 1 year and 9 months. The parties have since agreed that the whole shall be paid at one time; when should the payment be made? *Ans.* In $12\frac{3}{4}$ months.

20. A person owes a Note of $800 in Bank, and has $500, with which he proposes to pay a part of the Note, and the discount, at 6 per cent., on a Note, at 120 days, to be given for the remainder. How much is he able to pay on the Note? and what will be the discount on the remainder?

Ans. $493.73', and $6.27'.

21. On the 5th of September, 1854, a merchant bought goods to the amount of $5000, on a credit of 6 months. On the 20th of November, he paid $2000 on this debt, and on the 1st of December $1500; to what date might the payment of the balance be equitably deferred?

Ans. October 25th.

22. A merchant has a customer charged with $48.50 on the 1st of January, $80.25 on the 10th of March, and $100 on the 20th of May, and 6 months credit is allowed on the account. What amout is due on the 1st of December, if interest be charged at 7 per cent., after the account is due? (190). *Ans.* $231.59'.

23. A held a Note against B for $473.50, due the 3d of April; and B. held one against A for $500.62½, due the 10th of June—no interest accruing in either case, until the Note is due. Settlement was had on the 5th of May; what was then the balance between the two, allowing the rate of interest to be 6 per cent.? *Ans.* A owed B $21.61'.

24. A merchant bought goods, on the 1st of August, to the amount of $1200, on 3 months credit, on the 10th of September to the amount of $2500, on 4 months' credit, and

on the 20th of October to the amount of $2923, on 6 months' credit. What is the amount of the account on the 1st of January, allowing the rate of interest to be 6 per cent.? (191). *Ans.* $6583.49'.

25. What would $200 amount to in 3 years, at 6 per cent. *compound interest*, if the interest be payable annually? (175). *Ans.* $238.203'.

26. What would $1000 amount to in 2 years, at 8 per cent., *compound interest*, if the interest were payable *semi-annually?* *Ans.* $1169.858'.

27. The Debtor side of an Account Current amounts to $325.50, and is due on the 1st of May; the Creditor side amounts to $200, and is due on the 20th of of June. What balance is due on the account on the 1st of August, if interest be at 5 per cent.? *Ans.* $128.48'.

28. A speculator buys 320 mules at an average of $62½ each, and to pay for them has a Note discounted in Bank, for 120 days, at 6 per cent. In 30 days he sells his mules at an average of $87½, and puts the proceeds at interest, at 10 per cent., until his Note is due in Bank; what does he gain by these transactions? *Ans.* $8304.75'.

29. The Debtor side of an Account Current amounts to $500, and is due on the 10th of August; the Creditor side amounts to $700.50, and is due on the 1st of November. What balance is due on the account on the 20th of September, if interest be at 8 per cent.? *Ans.* $190.22'.

30. A merchant took a farmer's Note for $325.50, due, without interest, on the 1st of June, 1852; and some time afterwards the farmer got possession of a Note on the merchant for $500, due, without interest, on the 20th of January, 1854. Settlement was had on the 15th of August, 1853; how stood the matter of debt between them, interest being at 7 per cent.? *Ans.* $132.468' due the farmer.

CHAPTER X.

POWERS AND ROOTS OF NUMBERS.—INVOLUTION AND EVOLUTION.

POWERS AND ROOTS.

(194.) The FIRST POWER of a number is the number itself, thus the first power of 5 is 5.

The *second power*, or *square*, of a number, is the product of the number *multiplied into itself;* thus the second power or square of 5 is 5 × 5, that is, 25.

The *third power*, or *cube*, of a number, is the product of the number multiplied into its second power or square; thus the third power, or cube, of 5 is 5×5×5, that is 125.

What is meant by the *fourth power* of a number? What is meant by the *fifth power* of a number? The *sixth power* of a number?

What is the *square* of 2? What is the *cube* of 3? What is the square of 4? What is the cube of 6? What is the fourth power of 3?

(195.) The *second root*, or square root, of a number, is that number whose *square* is equal to the given number; thus the square root of 9 is 3.

The *third root*, or cube root, of a number is that number whose cube is equal to the given number; thus the cube root of 8 is 2.

What is meant by the *fourth root* of a number? What is meant by the *fifth root* of a number? What is meant by the *sixth root* of a number?

What is the square root of 16? What is the cube root of 27? What is the square root of 36? What is the cube root of 1000?

Any *power* or *root* whatever of *unity* is unity, since any number of 1's multiplied together produce 1.

Thus 1×1=1; 1×1×1=1; 1×1×1×1=1; and so on.

Powers and Roots of Fractions, &c.

(196.) A *power*, or a *root*, of a Fraction is obtained by finding the power, or the root, of its *numerator* and *denominator*, separately.

Thus the square of $\frac{2}{3}$ is $\frac{2}{3} \times \frac{2}{3}$, that is, $\frac{4}{9}$; and the *square root* of $\frac{4}{9}$ is therefore $\frac{2}{3}$.

What is the square of $\frac{3}{4}$? What is the cube $\frac{2}{3}$? What is the square of $\frac{5}{7}$? What is the cube $\frac{2}{5}$? What is the cube of $\frac{5}{6}$?

What is the square *root of* $\frac{9}{16}$? What is the cube root of $\frac{8}{27}$? What is the square root of $\frac{25}{100}$? What is the cube root of $\frac{125}{1000}$?

(197.) A power, or a root, of a Mixed Number may be obtained by finding the power, or the root, of its equivalent *improper fraction.*

Thus the square *root* of $5\frac{4}{9}$ is equal to the square root of $\frac{49}{9}$, which is $\frac{7}{3}$, equal to $2\frac{1}{3}$.

What is the square *root* of $2\frac{1}{4}$? What is the cube root of $3\frac{3}{8}$? What is the square root of $11\frac{1}{9}$? What is the cube root of $2\frac{10}{27}$?

Perfect and Imperfect Powers.

(198.) A *perfect power*, of any order, is a number which has an *exact root* of the corresponding order,—otherwise, the number is called an *imperfect power.*

A *square number* is any number, integral or fractional, which has an exact square root; and a *cube* number is one which has an exact cube root.

Name all the *square numbers*, in succession, from unity to the square of 12. Name several *cube numbers*, beginning with unity. Name three fractions which are perfect *squares*. Name three which are perfect *cubes*.

An imperfect power is also called a *Surd;* and its root is called an *irrational* number, because its *ratio* to unity cannot be exactly determined.

Exponents of Powers and Roots.

(199.) An *exponent* is an integer annexed to a number to denote a *power*, or a fraction annexed to denote a *root* of that number.

Thus 5^2 denotes the *second power*, or square, of 5; and 5^3 denotes the *third power*, or cube, of 5.

So $5^{\frac{1}{2}}$ denotes the square *root* of 5; and $5^{\frac{1}{3}}$ the cube *root* of 5.

In these expressions, 2, 3, $\frac{1}{2}$, and $\frac{1}{3}$ are *exponents*.

A root is also sometimes denoted by the *radical sign*, $\sqrt{}$, with a number over it, called an *index*; thus $\sqrt{9}$ denotes the square root, and $\sqrt[3]{9}$ the cube root, of 9.

INVOLUTION.

(200.) Involution consists in raising a given number to any required *power*. This may always be effected by multiplying the number into itself until it becomes a factor as many times as there are *units in the exponent of the power*.

Thus $9^2 = 9 \times 9 = 81$; and $9^3 = 9 \times 9 \times 9 = 729$, (194).

A *higher power* of a given number may also be obtained by multiplying together two or more lower powers (of the same number), the sum of whose *exponents* is equal to the exponent of the required power.

Thus $9^2 \times 9^2 = 9 \times 9 \times 9 \times 9$, the *fourth power* of 9; and $9^2 \times 9^3$ will produce the *fifth power* of 9.

The preceding statements will afford sufficient direction in any required case of Involution, without formal Rules.

EVOLUTION.

(201.) Evolution consists in extracting any required *root* of a given number, regarded as the corresponding power of the root to be found.

The only roots for which arithmetical Rules are usually given are the square root and cube root. The general method of extracting roots belongs to a more advanced stage of a mathematical course.

Extraction of the Square Root.

(202.) Extracting the square root consists in finding a number *whose square is equal to a given number.* The Rule for doing this depends on the following principles:

1. If a Number be separated into *periods of two figures* each, from right to left,—these periods will correspond, respectively, to the *units*, *tens*, *hundreds*, &c., in the square *root* of the number.

For since the square of 10 is 100, the square of the *tens figure* in the root leaves *two vacant places* in the right of the given number; these two places must therefore correspond to the *units* in the root.

And since the square of 100 is 10000, the square of the *hundreds* in the root leaves *four vacant places* in the right of the given number; and the first two corresponding to the *units*, as shown above, the next two must correspond to the *tens* in the root; and so on.

2. If a Number be divided into any two parts, the *square* of the number will be equal to the *square of the first part* + *twice the first* × *the second* + *the square of the second part*.

Thus the square of 16 is equal to the square of (10+6).

$$\begin{array}{r} 10+6 \\ 10+6 \\ \hline 60+36 \\ 100+60 \\ \hline 100+120+36. \end{array}$$

In squaring 10+6, we first multiply it by 6, and obtain 60+36; we then multiply it by 10, and obtain 100+60. By adding the two products together we obtain the entire square

$$100+120+36.$$

This square is composed of $10^2=100$, *twice* $10\times6=120$, and $6^2=36$.

RULE L.

(203.) *To extract the Square Root of a given Number.*

1. Separate the number into *periods of two figures each*, from right to left:—the left hand period will sometimes have but one figure.

2. Take the greatest integral square *root* of the left hand period, for the first figure of the root required; subtract the *square* of this figure from said period, and to the remainder affix the next period, for a dividend.

3. Divide the dividend, exclusive of its right hand figure, by *twice the root* already found; and annex the quotient figure to both the root and the divisor.

4. Multiply the divisor thus increased, by the quotient figure; subtract the product from the dividend; to the remainder affix the next period; divide by twice the root now found; and so on, till the operation is completed.

EXAMPLES.

1. To extract the Square Root of 1369.

```
   13'69(37
    9
   ------
 67)469
    469
```

The greatest integral square *root* of the left hand period, 13, is 3, which is the first figure of the required root. The square of 3 subtracted from 13 leaves the remainder 4; and the next period affixed to this makes the dividend 469.

Excluding the right hand figure, 9, we divide 46 by 6, which is *twice* the *root* 3, and annex the quotient figure 7 to both the root and the divisor. Multiplying the divisor 67 by the 7, completes the operation.

The first figure 3 in the root is 3 *tens* (202.... 1), and its square is 900, which leaves the remainder 469.

The given number 1369 is equal to the square of the 3 *tens* + *twice* 3 tens × the *units* in the root + the square of those *units* (202.... 2); and since the square of 3 *tens* has been subtracted, the remainder

469 = *twice* 3 tens × the *units* + the square of the *units*.

The composition of this remainder shows how the *divisor must be formed*, so that, divided into the remainder, it will give the *units* of the root; namely, by doubling the root, 3 *tens*, already found.

The same reasoning will apply to a number containing *three or more periods*, by always regarding the *figures found* in the root as the *first part* of the root, and those *to be found* as its second part.

2. To extract the Square root of 368449.

```
  36'84'49(607
  36
120)  8449
      8449
```

The square root of the left hand period, 36, is 6, the square of which subtracted leaves no remainder in that period.

We take the next period, 84, to find the next figure in the root. Doubling the root 6, for a divisor, and excluding the 4, we say 12 in 8, 0 time.

Including the next period, 49, and doubling the root, 60, now found, for a divisor, we say 120 in 844, 7 times, and annex the 7 to the root and the divisor—which produces the same result as merely annexing 0 to the root 6 and divisor 12, and dividing 120 in 844, &c.

Square Root of Decimals.

(204.) In extracting the Square Root of a Decimal Fraction, the *periods* must be taken from the decimal point *towards the right;* and a 0 must be annexed, if necessary, to complete the last period.

The last period must be complete, because, by the principles of decimal multiplication, the square of a Decimal Fraction must contain *twice as many* decimal figures as are in the root.

The number of decimal figures to be made in the root, is therefore the same as the number of *decimal periods.*

When an exact root cannot be found, decimal periods of 00 each may be annexed, and the root continued in *decimals* to any required exactness.

EXERCISES.

1. Find the square root of 784, and of 11236.

Ans. 28, and 106.

2. Find the square root of 2025, and of 38809.

Ans. 45, and 197.

3. Find the square root of 7396, and of 75076.

Ans. 86, and 274.

4. Find the square root of 22801, and of 473344.

Ans. 151, and 688.

5. Find the square root of 36100, and of 904401.

Ans. 190, and 951.

6. Find the square root of $346\frac{1}{8}$.

3'46.12'50.

First reduce $\frac{1}{8}$ to the equivalent decimal .125. Point off the integral part of the number into periods of two figures from the decimal point *towards the left*, and the decimal into periods *towards the right;* also annex a 0 to complete the last period (204).

Ans. 18.604'.

7. Find the square root of .582169. *Ans.* .763.

8. Find the square root of .3478312. *Ans.* .5897'.

9. Find the square root of .0073474. *Ans.* .0857'.

10. Find the square root of 737.8742. *Ans.* 27.16'.

11. Find the square root of 43.73731. *Ans.* 6.613'.

12. Find the square root of 90374376. *Ans.* 9506.543'.

13. Find the square root of 23473783. *Ans.* 4844.975'.

Application of Square Root.

(205.) The *square root* of any number of square inches or square feet, &c., is evidently the number of *linear inches* or feet, &c., in each of the four sides of the square which contains that number of square inches, or square feet, &c.

Thus the square root of 144 is 12, the number of linear inches in each side of a square foot; the square root of 9 is 3, the number of linear feet in each side of a square yard, &c. (110).

14. How long must the side of a square field be which shall contain just 10 acres of ground? *Ans.* 40 poles.

15. How long must the side of a square lot be which shall contain just one acre of ground? *Ans.* 12.649' poles.

16. A merchant bought a bale of cloth containing just as many pieces as there were yards in each piece. The number of yards was 1089; what was the number of pieces?
Ans. 33 pieces.

17. What must be the sides of two squares, one of which shall contain 2 square miles, and the other 3 square miles of land? *Ans.* 452.548'*p.*, and 554.256'.

18. A company of men on a journey expended $6084; each man expending as many dollars as there were men in the company. What was the number of men?
Ans. 78 men.

19. A regiment consisting of 5476 men is to be formed into a solid square. What will be the number of men in each side of the square? *Ans.* 74 men.

20. What would be the expense of enclosing 15 acres 2 roods 18 perches of ground, in the form of a square, at the rate of $2.12½ per rod for the fencing? *Ans.* $424.82'.

EXTRACTION OF THE CUBE ROOT.

(206.) Extracting the cube root consists in finding a number *whose cube is equal to a given number.* The Rule for this depends on the following principles :

1. If a Number be separated into *periods of three figures each*, from right to left,—these periods will correspond, respectively, to the *units*, *tens*, *hundreds*, &c., in the cube *root* of the number.

For since the cube of 10 is 1000, the cube of the *tens figure* in the root leaves *three vacant places* in the right of the given number ; these three places must therefore correspond to the *units* in the root.

And since the cube of 100 is 1000000, the cube of the *hundreds* in the root leaves *six vacant places* in the right of the number ; and the first three corresponding to the *units*, as shown above, the next three must correspond to the *tens* in the root ; and so on.

2. If a Number be divided into any two parts, the *cube* of the number will be equal to the *cube of the first part* + 3 *times the square of the first* × *the second* + *three times the first* × *the square of the second* + *the cube of the second part.*.

Thus the cube of 16 is equal to the cube of (10+6).

$$\begin{array}{r} 100+120+36\ \ (202\ldots.2) \\ 10+6 \\ \hline 600+720+216 \\ 1000+1200+360 \\ \hline 1000+1800+1080+216 \end{array}$$

This cube is composed of $10^3=1000$, 3 times $10^2\times6=1800$, 3 times $10\times6^2=1080$, and $6^3=216$.

RULE LI.

(207.) *To extract the Cube Root of a given Number.*

1. Separate the number into *periods of three figures each*, from right to left—the left hand period will sometimes have but one or two figures.

2. Take the greatest integral cube *root* of the left hand period, for the first figure of the root required; subtract the *cube* of this figure from said period, and to the remainder affix the next period, for a dividend.

3. Take 3 times the *square of the root* already found, for an incomplete divisor; divide it into the dividend, exclusive of its two right hand figures, and annex the quotient to the root.

4. *Complete the divisor*, by annexing to it 00, and adding the product which arises from annexing the last figure in the root to 3 times the *other part of the root*, and multiplying the result by the last figure.

5. Multiply the completed divisor by the last figure in the root; subtract the product from the dividend; and to the remainder affix the next period, for a new dividend.

6. *Find the next incomplete divisor* by adding to the last complete divisor the *product which completed it*, and the square of the *last figure* in the root; divide, and complete the divisor, as before; and so on.

In applying this Rule it will be convenient to have the following

Table of Roots and Cubes.

Roots,	1,	. . 2,	. . 3,	. . 4,	. . 5,	. . 6,	. . 7,	. . 8,	. . 9.
Cubes,	1,	. . 8,	. 27,	. 64,	125,	216,	343,	512,	729.

EXAMPLE.

To extract the Cube Root of 95443993.

	95'443'993(457
	64
48	31443
4800+625=5425	27125
5425+625+ 25=6075	4318993
607500+9499= 616999	4318993

The greatest integral cube root of the left hand period, 95, is 4, the *cube* of which subtracted leaves the remainder 31; and the next period affixed to this gives the

dividend 31443.

Three times the square of 4, the root already found, is 48, which we divide into 314. The quotient would appear to be 6; but the divisor being as yet *too small*, we take 5 for the quotient.

To complete the divisor, we annex 00 to it, and add 625; the 625 being obtained by annexing the 5 in the root to 3 times the 4, and multiplying the result, 125, by 5.

Multiplying the *completed divisor*, 5425, by 5, subtracting, and affixing the next period to the remainder, we find the

dividend 4318993.

To find the next incomplete divisor, 6075, we add to the last complete divisor the product, 625, which completed it, and the square, 25, of the 5 in the root. Dividing 6075 into 43189, the quotient is 7.

To complete this divisor, we annex 00 to it, and add 9499, which is obtained by annexing the 7 to 3 times the 45, and multiplying the result, 1357, by 7. The divisor now found, multiplied by 7, produces the last dividend, and thus the operation is completed.

The preceding Rule for extracting the cube root depends on the principles which have already been stated (206): the application of these principles to the demonstration of the Rule is shown in the author's Algebra.

Cube Root of Decimals.

(208.) In extracting the Cube Root of a Decimal Fraction, the periods must be taken from the decimal point *towards the right;* and a 0 or 00 must be annexed, if necessary, to complete the last period.

The last period must be complete, because, by the principles of decimal multiplication, the cube of a Decimal Fraction must contain 3 *times as many* decimal figures as are in the root.

The number of decimal figures to be made in the root, is therefore the same as the number of *decimal periods.*

When an exact root cannot be found, decimal periods of 000 each may be annexed, and the root continued in *decimals* to any required exactness.

EXERCISES.

1. Find the cube root of 103823. *Ans.* 47.
2. Find the cube root of 262144. *Ans.* 64.
3. Find the cube root of 2406104. *Ans.* 134.
4. Find the cube root of 22906304. *Ans.* 284.
5. Find the cube root of 20796875. *Ans.* 275.
6. Find the cube root of 28372625. *Ans.* 305.
7. Find the cube root of 131872229. *Ans.* 509.
8. Find the cube root of 9873¼.

9'873'.250

First reduce ¼ to the equivalent decimal .25. Point off the integral periods from the decimal point *towards the left,* and the decimal periods *towards the right;* also annex a 0 to complete the last period (208). *Ans.* 21.4 [illegible]

9. Find the cube root of .389017. *Ans.* .73.
10. Find the cube root of 734.673. *Ans.* 9.023'.
11. Find the cube root of 7386. *Ans.* 19.474'
12. Find the cube root of 479.2735. *Ans.* 7.825.
13. Find the cube root of .202262003. *Ans.* 587.

Application of Cube Root.

(209.) The *cube root* of any number of cubic inches, or cubic feet, &c., is evidently the number of *linear inches*, or feet, &c., in each *edge* of the cube which contains that number of cubic inches, or cubic feet, &c.

Thus the cube root of 1728 is 12, the number of linear inches in each edge of a cubic foot; the cube root of 27 is 3, the number of linear feet in each edge of a cubic yard, (111).

14. What must be the length of each edge of a cubical block of marble which shall contain 1000 cubic feet?

Ans. 10 feet.

15. What must be the length, breadth, or depth of a cubical box, that its capacity may be 2000 cubic feet?

Ans. 12.598' feet.

16. What must be the depth of a cubical cistern, that its capacity may be 5000 gallons of water? (106).

Ans. 9.344' feet.

17. What must be the dimensions of a cubical granary which shall contain 3000 bushels of wheat? *Ans.* 15.513 feet

MISCELLANEOUS EXERCISES

ON THE APPLICATION OF SQUARE ROOT AND CUBE ROOT.

1. A city whose corporate area is in the form of a circle, contains 3.1416 square miles, and is 6.2832 miles in circumference. Had the same amount of area been incorporated in the form of a *square*, what would have been the compass of the city? *Ans.* 7.088' miles.

2. What must be the dimensions of a field to contain 10 acres of ground, and have its length equal to twice its breadth?

One half of the given area is the area of a *square* whose side is equal to the breadth of the field.

Ans. 28.284' *p.* wide, and 56.568' *p.* long.

3. A warehouse whose base shall occupy 10000 square feet, is intended to have its breadth only one third of its length. What must be the length and breadth?

Ans. 173.205 feet, and 57.735 feet.

4. A farmer intending to enclose 50 acres of land, wishes to know what difference in the amount of fencing there would be between having the enclosure in the form of a square, and having its length equal to twice its breadth?

Ans. 21.702' poles.

5. Allowing the solidity of a cube to be 1331 cubic inches, what is the number of square inches contained in its surface?

Recollect that the surface of the cube consists of *six equal squares.* *Ans.* 726 square inches.

6. A cubical cistern is to be constructed which shall contain 300 barrels of water. How many square yards will there be in the bottom and the four sides of this cistern?

Ans. 81.05' square yards

7. A farmer wishes to construct a crib which shall contain 2000 bushels, and have its breadth and height each equal to one half of its length. What must be the length of the crib? *Ans.* 21.512' feet.

8. How many square feet will there be in the bottom and the four sides of a cubical reservoir which shall contain 10000 gallons of water? *Ans.* 693.017' square feet.

9. The capacity of the reservoir being the same as in the preceding question, how many square feet would be contained in the bottom, the two sides, and the two ends, allowing its length to be double each of its other dimensions?

Ans. 698.482' square feet.

10. What must be the depth of a cubical cellar whose capacity shall be equal to that of another which is 30 feet long, 20 feet wide, and $10\frac{1}{2}$ feet deep? *Ans.* 18.469' feet.

11. A person enclosed a garden 16*r.* 2*yd.* in length, and 10*r.* in breadth, at $3.75 per rod for paling; and his neighbor enclosed the same area, at the same rate, but chose to have his in the form of a square. What did the latter gain or lose by such determination. *Ans.* Gained $5.842'.

12. What would be the difference in the number of square feet in the four sides of a wine vat of 10 barrels' capacity, whether it be of a cubical form, or of a length equal to double each of its other dimensions?

Ans. 2.674 square feet.

13. A certain reservoir for water is 150 feet in length, 100 feet in breadth, and 20 feet in depth, and is lined at the bottom and sides with plank which cost $1.50 per 100 square feet. Had the reservoir been in the form of a *cube*, with the same capacity, what would have been gained or lost in the cost of the plank for lining it? *Ans.* $38.89' gained.

Abbreviated Evolution.

In extracting Roots, when the successive divisors have become the same in their first two or three *left hand figures;* and the figures remaining to be found in the root, are less in number than those in the last *complete divisor;* the operation may then be continued as in Abbreviated Division of Decimals.

Thus, to extract the square root of 2 to *six decimal figures.*

```
         2(1.414214'
           1
  24)     100
           96
  281)     400
           281
  2824)  11900
         11296
  282)     604
           565
  28)       39
            28
  2)        11
```

We proceed according to the common Rule (203) in finding the first four figures, 1.414, in the root.

The divisors 281 and 2824 are the same in their two left hand figures; and the figures remaining to be found in the root are less in number than in the divisor 2824.

The operation is continued by rejecting the right hand figure of 2824, and of the succeeding divisors, and using their other figures and the *remainders* in the same manner as in Abbreviated Division of Decimals.

The correctness of this method will be evident from considering that the value of each succeeding figure in the Root, depends on the first two or three figures in the left of the corresponding divisor.

In the preceding example, the root carried to the same extent by the common Rule, is

1.414213'.

There will always be some uncertainty in regard to the last figure, found by the abbreviated method; but this is of little importance when the root contains several decimal figures.

A similar method of abbreviation may be adopted for the *Cube Root.*

CHAPTER XI.

PROGRESSIONS, AND THEIR APPLICATION TO COMPOUND INTEREST AND ANNUITIES.

ARITHMETICAL PROGRESSION.

(210.) An ARITHMETICAL PROGRESSION is a series of quantities which continually increase or decrease by a *common difference.*

Thus 1, 3, 5, 7, 9, &c., is an Arithmetical Progression which increases by the continual addition of the *common difference* 2.

State the Progression commencing with 2, and increasing by the common difference 3. State the Progression commencing with 20, and decreasing by the common difference 4.

The *first* and *last terms* of a Progression are called the two *extremes*, and all the intermediate terms the *means*.

The principles of Arithmetical Progression are contained in the following propositions.

The Last Term.

(211.) The *last term* of an *increasing* Arithmetical Progression, is equal to the first term + the product of the common difference × the number of terms *less one;* and in a *decreasing* Progression it is equal to the first term — the same product.

This may be seen in the Progressions,

$$4,\quad 4+3,\quad 4+2\times3,\quad 4+3\times3,\quad 4+4\times3;$$
$$16,\quad 16-3,\quad 16-2\times3,\quad 16-3\times3,\quad 16-4\times3;$$

in which the first terms are 4 and 16, the common difference is 3, and the *number of terms* 5.

From the preceding it follows that

(212.) The *common difference* of the terms in an Arithmetical Progression, is equal to the difference between the *two extremes* ÷ the number of terms *less one.*

The Sum of the two Extremes.

(213.) The *sum of the two extremes* in an Arithmetical Progression, is equal to the sum of any two terms *equidistant from them,* or to twice the middle term when the number of terms is *odd.*

This may be seen in the Progression

$$4, \quad 4+3, \quad 4+2\times3, \quad 4+3\times3, \quad 4+4\times3,$$

in which the sum of the two extremes, 4 and $4+4\times3$, is equal to the sum of $4+3$ and $4+3\times3$, which are equidistant from the extreme terms, or to twice the middle term $4+2\times3$.

From which it follows that

(214.) An *arithmetical mean* between two given terms, is equal to *half the sum* of those terms.

For the sum of the two given terms, regarded as the two extremes of an Arithmetical Progression, is equal to twice the middle or mean term.

The Sum of all the Terms.

(215.) The *sum of all the terms* in an Arithmetical Progression, is equal to half the sum of the two extremes × the number of terms.

To prove this we add the several terms of an Arithmetical Progression to those of the same Progression *reversed.* Thus

4,	7,	10,	13,	16;
16,	13,	10,	7,	4
20,	20,	20,	20,	20.

The sum 20+20+20, &c., of the *two series* is equal to the sum, 20, of the two extremes, 4 and 16, in either series × the number of terms; hence the sum of either series is equal to half the sum of the two extremes × the number of terms.

The preceding principles are to be applied to the following

EXERCISES.

1. The first term of an increasing Arithmetical Progression is 3, the common difference of the terms is 5, and the number of terms 100; what is the *last term?* and the sum of all the terms?

The last term is equal to $3+5\times99$ (211); and the sum of all the terms is equal to $(3+3+5\times99)\div2\times100$ (215).

Ans. 498; and 25050.

2. The first term of a decreasing Arithmetical Progression is 4680, the common difference of the term is 3, and the number of terms 120; what is the last term? and the sum of all the terms? *Ans.* 4323; 540180.

3. What is the *common difference* of the terms of an Arithmetical Progression whose first term is 10, last term 150, and number of terms 21? (212). *Ans.* 7.

4. What is the *third term* in an Arithmetical Progression whose second term is 26, and fourth term 100? (214).

Ans. 63.

5. If the first term of an Arithmetical Progression be 15, the last term 80, and the number of terms 5; what are the three intermediate terms?

Find the common difference of the terms (212), and from that and either of the given terms find the required terms.

Ans. $31\frac{1}{4}$, $47\frac{1}{2}$, and $63\frac{3}{4}$.

6. If the first term of an Arithmetical Progression be 36, the last term 150, and the number of terms 4 ; what are the intermediate terms? *Ans.* 74, and 112.

7. What is the sum of 1000 terms of an increasing Arithmetical Progression in which the first term is $\frac{1}{2}$, and the common difference of the terms also $\frac{1}{2}$? (211 & 215).

Ans. 250250.

GEOMETRICAL PROGRESSION.

(216.) A *geometrical progression* is a series of quantities in which each succeeding term has the *same ratio* to the term which immediately precedes it.

Thus 1, 2, 4, 8, 16, &c., is a Geometrical Progression in which each succeeding term is *double* the one which immediately precedes it; and the *ratio of the progression* is therefore 2.

And 27, 9, 3, 1, $\frac{1}{3}$, &c., is a Geometrical Progression in which each succeeding term is *one third* of the one which immediately precedes it; that is, the *ratio of the progression* is $\frac{1}{3}$.

State the Progression whose first term is 2, and ratio 3. State the Progression whose first term is 1, and ratio $\frac{1}{2}$. State the Progression whose first term is $\frac{1}{2}$, and ratio 4.

The first and last terms of a Progression are called the two *extremes*, and all the intermediate terms the means.

The principles of Geometrical Progression are contained in the following propositions.

The Last Term.

(217.) The *last term* of a Geometrical Progression is equal to the first term × that *power of the ratio* which is expressed by the number of terms *less one.*

Thus if the first term be 1, and the *ratio* of the Progression 3, the series will be

$$1,\ 1\times3,\ 1\times3\times3,\ 1\times3\times3\times3,\ 1\times3\times3\times3\times3,\ \&c.$$
$$\text{or } 1,\ 1\times3,\quad 1\times3^2,\quad 1\times3^3,\quad 1\times3^4,\ \&c.;$$

in which it is plain that the last term, as 1×3^4, will always be equal to the first term, 1, multiplied by that power of the ratio, 3, which is expressed by the number of terms *less one.*

From this it follows that

(218.) The *last term* of a Geometrical Progression ÷ the first term, gives that *power of the ratio* which is expressed by the number of terms *less one.*

Product of the two Extremes.

(219.) The *product of the two extremes* in a Geometrical Progression, is equal to the product of any two terms *equidistant from them*, or to the *square* of the middle term when the number of terms is *odd.*

Thus in the Progression

$$2,\quad 2\times3,\quad 2\times3^2,\quad 2\times3^3,\quad 2\times3^4,$$

the product $2\times2\times3^4$ of the first and last terms is equal to that of the *second* and *fourth* terms which are equidistant from them, or to the *square* of the middle term 2×3^2.

From the preceding it also follows that

(220.) A *geometrical mean* between two given terms, is equal to the *square root* of the product of those terms.

For the product of the two given terms, regarded as the extremes of a Geometrical Progression, is equal to the square of the middle or mean term.

The Sum of all the Terms.

(221.) The *sum of all the terms* in a Geometrical Progression, is equal to the difference between the *first term* and the *product* of the last term × the *ratio*, ÷ the difference between the *ratio* and a *unit*.

For take the Progression

$$4, \quad 4\times 3, \quad 4\times 3^2, \quad 4\times 3^3, \quad 4\times 3^4, \text{ the } \textit{ratio} \text{ being } 3.$$

Multiplying each term by the *ratio*, we obtain the series

$$4\times 3, \quad 4\times 3^2, \quad 4\times 3^3, \quad 4\times 3^4, \quad 4\times 3^5,$$

the sum of which is 3 *times* the sum of the first series.

If the first series be subtracted from the second, the remainder will be

$4\times 3^5-4$, which must be *twice the given series.*

But this remainder is the difference between the first term, 4, and the product of the last term, 4×3^4, multiplied by the ratio 3; while 2 is the difference between this ratio and a *unit.* The sum of the series will therefore be found according to the proposition stated.

(222.) If the number of terms in a *decreasing* Geometrical Progression were *infinite,* that is, increased *without limit,* their sum would be equal to the *first term* ÷ the difference between the *ratio* and a *unit.*

Thus if the series 9, 3, $\frac{1}{1}$, $\frac{1}{3}$, and so on, in which the ratio is $\frac{1}{3}$, were continued to an *infinite number of* terms; the last term would be *diminished without limit*, that is, it would be 0; and by the preceding proposition, the sum of all the terms would be

$$(9-0\times\tfrac{1}{3})\div(1-\tfrac{1}{3})=9\div\tfrac{2}{3}=13\tfrac{1}{2}.$$

On this principle we may also compute the value of a *Repeating Decimal* (89). For example, the repetend .4444, &c., is equal to

$$\tfrac{4}{10}+\tfrac{4}{100}+\tfrac{4}{1000}+\tfrac{4}{10000}, \text{ and so on without limit.}$$

Now this is a *decreasing* Geometrical Progression in which the ratio is $\frac{1}{10}$; the sum of all the terms is therefore

$$\tfrac{4}{10}\div\tfrac{9}{10}=\tfrac{4}{9}=\tfrac{2}{3};$$

which is the same value of the Repetend that would be found by the method before given (90).

The preceding propositions are to be applied to the following

EXERCISES.

1. The first term of a Geometrical Progression is 3, the *ratio* of the progression is 5, and the number of terms 9; what is the *last term?* and the *sum* of all the terms?

The last term is equal to 3×5^8 (217); and the sum of all the terms is equal to $(3\times5^8\times5-3)\div(5-1)$ (221).

Ans. 1171875; and 1464843.

2. The first term of a Geometrical Progression is 100, the ratio of the progression is $\frac{1}{3}$, and the number of terms 7; what is the last term? and the sum of all the terms?

Ans. $\frac{100}{729}$; and $149\frac{679}{729}$.

3. What is the ratio of a Geometrical Progression whose first term is 234, fourth term 1872, and number of terms 4?

In the preceding question we may readily find the *third power* of the ratio (218), and thence the ratio itself by evolution. *Ans.* 2.

4. What is the *second term* in a Geometrical Progression whose first term is 5, and third term 1125 ? (220).
Ans. 75.

5. If the first and fourth terms of a Geometrical Progression be 34 and 918 respectively ; what are the two intermediate terms ?

Find the *ratio* of the progression (218), and from that and either of the given terms find the required terms.
Ans. 102 and 306.

6. If the first term of a Geometrical Progression be 15, the last term 960, and the number of terms 4 ; what are the intermediate terms ? *Ans.* 60 and 240.

7. What is the sum of an *infinite* number of terms of a Geometrical Progression whose first term is 1000, and ratio $\frac{1}{2}$?
Ans. 2000.

8. How far would a person travel in 6 days, allowing him to go 40 miles the first day, and to diminish his rate in such a manner that each succeeding day's journey shall be $\frac{3}{4}$ of the one immediately preceding ? *Ans.* $131\frac{67}{128}$ miles.

9. If a body should move 2000 feet the first second, half that distance the next second, half the latter distance the next second, and so on, forever, what is the utmost distance it could go ? *Ans.* 4000.

10. If 11 yards of cloth were sold at 1 cent for the first yard, 3 for the second, 9 for the third, and so on, what would be the price of the last yard ? and what would the whole amount to ? *Ans.* $590.49 ; and $885.73.

COMPOUND INTEREST (175)

AS CONNECTED WITH GEOMETRICAL PROGRESSION.

(**223.**) In Compound Interest, the *Principal* is the *first term* of a geometrical progression; the amount of \$1 for one year is the *ratio* of the progression; the number of years +1 is the number of *terms;* and the *Amount* of principal and interest is the *last term.*

At 6 per cent., \$1 amounts to \$1.06 in *one year.*

In Compound Interest, the Amount for each year is the *Principal for the next year;* and it is plain that the Amount for any year, at 6 per cent., is equal to the Principal $\times$ 1.06.

Then \$1 amounts

to \$1.06 in 1 *year,*

to \$1.06 $\times$ 1.06 in 2 *years,*

to \$1.06 $\times$ 1.06 $\times$ 1.06 in 3 *years,*

to \$1.06 $\times$ 1.06 $\times$ 1.06 $\times$ 1.06 in 4 *years;* and so on.

Denoting the preceding *powers* of 1.06 by the proper *exponents,* the Principal \$1 and its Amounts for 1, 2, 3, and 4 *years* gives the series

$$\$1,\quad \$1.06,\quad \$1.06^2,\quad \$1.06^3,\quad \$1.06^4,$$

which is a *geometrical progression* corresponding with the proposition (223).

For the Principal \$5 the progression for 4 years, at 6 per cent., would be

$$\$5,\quad \$5\times 1.06,\quad \$5\times 1.06^2,\quad \$5\times 1.06^3,\quad \$5\times 1.06^4.$$

The preceding principles give the following Rule:

RULE LXII.

(224.) *For the Computation of Compound Interest.*

1. Raise the amount of $1, for one year, to that *power* which is denoted by the number of years the Principal is at interest.

2. Multiply said power by the Principal ; the product will be the *Amount* of Principal and compound Interest.

The application of this Rule, for 5, 6, or 7 per cent., will be greatly facilitated by the following

(225.) *Table containing the Powers, to the 50 order, of the Amount of $1, for one year, at 5, 6, or 7 per cent.*

	5 p. cent.	6 p. cent.	7 p. cent		5 p. cent.	6 p. cent.	7 p. cent.
1	1.050000	1 060000	1.070000	26	3.555673	4.549383	5 807351
2	1.102500	1.123600	1.144900	27	3.733456	4.822346	6.213866
3	1.157625	1.191016	1.225043	28	3.920129	5.111687	6.648836
4	1.215506	1.262477	1.310796	29	4.116136	5.418388	7.114255
5	1.276282	1.338226	1.402552	30	4.321942	5.743491	7.612253
6	1.340096	1.418519	1.500730	31	4.538039	6.088101	8.145110
7	1.407100	1.503630	1.605781	32	4.764941	6.453387	8.715268
8	1.477455	1.593848	1.718186	33	5.003188	6.840590	9.325337
9	1.551328	1.689479	1 838459	34	5.253348	7.251025	9.978110
10	1.628895	1.790848	1.967151	35	5.516015	7.686087	10.676578
11	1.710339	1.898299	2.104852	36	5 791816	8.147252	11.423939
12	1.795856	2.012196	2.252192	37	6.081407	8.636087	12.223614
13	1.885649	2.132928	2.409845	38	6 385477	9.154252	13.079277
14	1.979932	2.260904	2.578534	39	6.704751	9.703507	13.994827
15	2.078928	2.396558	2.759031	40	7.039989	10.285718	14 974465
16	2.182875	2.540352	2.952164	41	7.391988	10.902861	16.022677
17	2.292018	2.692773	3.158815	42	7.761587	11.557033	17.144265
18	2.406619	2 854339	3.379931	43	8.149667	12.250455	18.344363
19	2 526950	3.025599	3.616526	44	8.557150	12.985482	19.628469
20	2.653298	3.207135	3.869683	45	8.985008	13.764611	21.002461
21	2.785963	3.399564	4.140561	46	9.434258	14.590487	22.472634
22	2.925261	3 603537	4.430400	47	9.905971	15.465917	24.045718
23	3.071524	3.819750	4.740528	48	10.401267	16.393872	25.728918
24	3.225100	4.048935	5.072365	49	10.921333	17.377504	27.529943
25	3.386355	4.291871	5.427431	50	11.467400	18.420154	29.457039

The *order* of the Power is to be found in the 1*st*, or the 5*th* column of this Table; the Power itself will then be the corresponding number under the given rate per cent.

Thus opposite to 1, and under 5 *per cent.*, is 1.05, which is the 1*st* power of the Amount, \$1.05, of \$1, for one year, at 5 per cent.; opposite to 2, and under 5 per cent., is 1.1025, which is the 2*nd* power of the Amount of \$1, for one year, at 5 per cent.; and so on.

By the preceding Rule, each of these Powers is the *Amount of* \$1, *at Compound Interest*, for the corresponding number of years.

Thus the Amount of \$1, at compound interest, for 10 years, at 5 per cent., is seen to be \$1.628895.

EXAMPLE.

To find what \$1000 would amount to in 20 years, at 6 per cent., compound interest.

Opposite to 20 in the 1st column of the Table, and under 6 per cent., we find 3.207135, which is the 20th power of 1.06, or the Amount of \$1, at Compound Interest, for 20 years, at 6 per cent. Then

\$3.207135 × 1000 = \$3207.135, the Am't of \$1000 for 20*y.*;
and \$1000 subtracted from this Amount leaves
the *compound interest.*

(226.) *To find the Principal from a given Amount at Compound Interest.*

Divide the given Amount by the amount of \$1, at the given rate per cent., for the given number of years.

This is merely the converse of the preceding Rule.

EXERCISES.

1. What would $500 amount to in 5 years, allowing compound interest, at 6 per cent.? *Ans.* $669.113.

2. What would $325.50 amount to in 12 years, allowing compound interest, at 5 per cent.? *Ans.* 584.551.

3. What would be the compound interest of $2000 for 10 years, at 7 per cent.?

Subtract the given Principal from the Amount, found by the RULE; the remainder will be the *compound interest.*

Ans. $1934.302.

4. What would be the compound interest of $5630.75 for 20 years, at 5 per cent.? *Ans.* $9309.307.

5. What would be the compound interest of $7325.12½ for 50 years, at 6 per cent.? *Ans.* $127604.805.

6. What *principal* would amount to $1000 in 5 years, at 6 per cent., compound interest? (226). *Ans.* $747.25.

7. What principal would amount to $10000, in 10 years, at 7 per cent., compound interest? *Ans.* $5083.49.

8. A debt of $600 will be due in 3 years, without interest. What is the *present worth* of the debt, allowing money to be worth 6 per cent., at compound interest?

The present worth is the *principal* which would amount to $600, in 3 years, at 6 per cent., compound interest.

Ans. $503.77.

9. What is the present worth of $1200, due in 4 years, without interest, on the supposition that money can be loaned at 6 per cent., compound interest? *Ans.* $950.51.

10. A owes B $3250 to be paid in 5 years, without interest. What sum in hand would be an equivalent for the debt, if the present worth could be put at interest, at 7 per cent., and the interest compounded annually?

Ans. $2317.20.

ANNUITIES.

(227.) An Annuity is properly a sum of money which is payable *annually;* but the term is also applied to a sum which is to be paid semi-annually, quarterly, or at any regular intervals.

Pensions, salaries, rents, &c., are of the nature of annuities.

A *Perpetuity* is a *perpetual* annuity; that is, an annuity which is unlimited in duration, and is thus said to continue *forever.* Of this kind may be the interest which a Government pays on borrowed money.

An Annuity *forborne,* or *in arrears,* is one on which the periodical payments have remained unpaid, after becoming due.

An Annuity *in reversion* is one on which the periodical payments are to commence at a specified *future time,* or on the occurrence of a specified future event.

Amount of Annuities—how found.

(228.) The Amount of an Annuity, at *simple interest,* is equal to the *sum of the terms* of an Arithmetical Progression whose first term is the *annuity,* common difference the interest on the annuity for *one year,* and number of terms the number of *years* for which the annuity is taken.

For example, to find what the rent of a house, at $100 a year, will amount to in 4 years, allowing interest at 6 per cent.

At the end of the 4th year, there will be the rent, $100, for that year; the rent for the 3d year, with *one year's interest;* the rent for the 2d year, with 2 years' interest; and the rent for the 1st year, with 3 years' interest.

In 4 years the rent will therefore amount to the sum of the series

$100, $106, $112, $118,

which is an Arithmetical Progression in accordance with the preceding proposition.

(229.) The Amount of an Annuity, at *compound interest*, is equal to the *sum of the terms* of a Geometrical Progression whose first term is the *annuity*, ratio the amount of $1 for *one year*, and number of terms the number of *years* for which the annuity is taken.

If we take the example under the preceding proposition, and compute the Amount of the Annuity for 4 years at *compound interest*, the result will be the sum of the series

$\$100, \quad \$100 \times 1.06, \quad \$100 \times 1.06^2, \quad \$100 \times 1.06^3,$

which is a Geometrical Progression agreeing with the proposition last stated.

The principles which have been established give

RULE LXIII.

(230.) *To find the Amount of an Annuity.*

1. At *Simple Interest*,—multiply the interest of the Annuity for *one year* by the number of years *less one;* add twice the annuity to the product, and multiply the sum by half the number of years.

2. At *Compound Interest*,—multiply the Annuity by that *power* of the amount of $1 for *one year* which is expressed by the number of years, subtract the annuity from the product, and divide the remainder by the interest of $1 for one year.

EXAMPLE.

An annuity of $40 a year remained unpaid till the end of 5 years; what amount was then due, allowing interest at 5 per cent.?

1. At *Simple Interest.* The interest of $40 for one year, at 5 per cent., is $2.00; this multiplied by the number of years *less one*, gives $2.00 × 4 = $8.

Adding twice the annuity to this product, and multiplying by half the number of years, we find the required amount to be

$$\$(8+80)\times 2\tfrac{1}{2}=\$220.$$

2. At *Compound Interest.* Multiplying $40 by the 5th power of the amount, 1.05, of $1 for one year, (225), we have

$$\$40\times 1.276282=\$51.05128.$$

Subtracting the annuity from this product, and dividing the remainder by the interest, .05, of $1 for one year, we find the required amount, in this case, to be

$$\$11.05128\div .05=\$221.025'.$$

A little reflection will show the dependence of the first part of the RULE on propositions (228 and 215), and of the second part on (229 and 221).

EXERCISES.

1. A laborer's wages were $125 a year, and remained unpaid until the end of 3 years; what amount was then due him, allowing simple interest at 7 per cent.? *Ans.* $401.25.

2. The rent of a house, which is $250 a year, has remained unpaid for 10 years; what amount is now due, allowing simple interest at 10 per cent.? *Ans.* $3625.

3. An annuity of $300 per annum has been forborne for 25 years. What is the present amount of this annuity, allowing compound interest at 6 per cent.? *Ans.* $16459.35'.

4. What is the amount due on a pension of \$500 per annum, which has remained unpaid for 13 years, allowing compound interest at 5 per cent.? *Ans.* \$8856.49'.

5. What difference is there between the amount at *simple*, and at *compound* interest, at 6 per cent., of an annuity of \$375 per annum, on which the payments have been suspended for 20 years? *Ans.* \$2019.59'.

RULE LXIV.

(231.) *To find the Present Worth of an Annuity.*

Find the Amount of the Annuity for the given time; and then find the Principal which would produce that Amount, in the given time, for the *present worth* of the annuity.

EXAMPLE.

A person is entitled to an annuity of \$40 a year, for 5 years; what is the *present worth* of this annuity, that is, its value in ready money, allowing compound interest at 5 per cent.?

In the Example under the former Rule (230), the amount of an annuity of \$40 a year, for 5 years, at 5 per cent., compound interest, was found to be

\$221.025'.

The Principal which would produce this amount in 5 years, at 5 per cent., compound interest, that is, the present worth of the annuity, is

\$221.025 ÷ 1.276282 = \$173.179'. (226).

By the same Rule may be found the present worth of an Annuity at *simple interest;* observing to work by the rules for simple interest (230 .... 1, and 183).

(**232.**) The Present Worth of a Perpetuity, or *perpetual* annuity, is that *principal* whose annual interest is equal to the annual payment on the Perpetuity.

Thus the present worth of \$100 payable annually *forever*, allowing interest at 5 per cent., is

$$\$100 \div .05 = \$2000. \quad (168).$$

(**233.**) The Present Worth of an Annuity *in reversion*, is that *principal* which, by the time the reversion expires, will amount to what will then be the present worth of the Annuity.

To find the present worth of an annuity of \$40 a year, which is to continue 5 years, but not to commence till the expiration of 3 years from the present time, allowing compound interest at 5 per cent.

In the Example under the Rule, the present worth of an annuity of \$40 a year, for 5 years, at 5 per cent., compound interest, is found to be \$173.179.

This will be the present worth of the annuity at the *expiration of the* 3 *years* during which the annuity is *in reversion*. The *principal* which would amount to this sum in 3 years, at 5 per cent., compound interest, is

$$\$173.179 \div 1.157625 = \$149.59. \quad (226).$$

which is therefore the present worth of \$40 a year, for 5 years, after a reversion of 3 years.

EXERCISES.

6. What is the present worth of an annuity of \$200 a year, for 12 years, allowing simple interest, at 6 per cent.?

Ans. \$1855.813'.

7. The annual rent of an estate is \$500. What would be the present worth of the rents for 7 years, allowing compound interest at 6 per cent.? *Ans.* \$2791.18'.

8. What is the present worth of an annuity of $500 a year, which is to continue forever, allowing the rate of interest to be 5 per cent.? *Ans.* 10000.

9. A perpetuity of $300 per annum is in reversion for 20 years. What is its value in ready money, allowing compound interest at 5 per cent.? *Ans.* $2261.33'.

10. An annuity of $1000 per annum is to continue for 15 years, but is not to commence till the expiration of 10 years from the present time. What sum in hand would be an equivalent for this annuity, allowing compound interest at 6 per cent.? *Ans.* $5423.25'.

MISCELLANEOUS EXERCISES

IN PROGRESSIONS, COMPOUND INTEREST, AND ANNUITIES.

1. What sum would be accumulated in one year by laying up 1 *cent* the 1st day, 2 the 2d, 3 the 3d, and so on; allowing 365 days to the year? *Ans.* $667.95.

2. A man setting out on a journey, travels 12 miles the first day, 16 the 2d, 20 the 3d, and so on, till at last he traveled 64 miles in one day; how many days did he travel? *Ans.* 14 days.

3. Allowing a person to commence trading on a capital of $1000, and to increase it by $\frac{1}{4}$ of itself each year for 10 years; what would then be the amount of his capital? *Ans.* $7450.580'.

4. Four persons on comparing their ages find that the first is as much younger than the second as the second is younger, and the fourth older, than the third. The ages of the first and third are $21\frac{3}{8}$ and $56\frac{1}{4}$ years respectively; how old is the fourth? *Ans.* $73\frac{13}{24}$ years.

5. A lady who was married on a New-year's day, received from her father $1 towards her fortune, and the sum was

tripled on the 1st day of each month to the end of the year. What was the amount of her fortune? *Ans.* $265720

6. If a body were to move 60 miles the first hour, 40 miles the second, 26⅔ miles the third, and so on *forever;* what is the utmost distance it would reach?

Ans. 180 miles.

7. If $500 were put at interest, at 6 per cent., and the interest collected annually, and put out at the same rate, and so on with all the interest annually due; what would be the amount in 20 years? *Ans.* $1603.56.

8. A gentleman, on the birth of his first son, wishes to put at interest, at 7 per cent., a sum of money which, by adding the interest annually due to the principal, shall amount to $5000 by the time his son is 21 years of age. What sum must be put at interest? *Ans.* $1207.56.

9. The rents of an estate, at $500 per annum, have remained unpaid for 7 years. What amount is now due, allowing *simple* interest, and what, allowing *compound* interest, at 6 per cent.? *Ans.* $4130; and $4196.91'.

10. A person at the age of 22 put $100 at interest, at 6 per cent., and $100 each year afterwards, until he was 40 years old. He also collected the interest annually, and converted the same into *principal;* what amount was, by these means, accumulated? *Ans.* 3090.56'.

11. A disabled officer has a pension of $300 per annum, and wishes to convert this income, for 5 years to come, into an equivalent in ready money. Required the sum that should be paid him, allowing compound interest at 6 per cent.?

Ans. $1263.70'.

12. What is the difference in present value between a term of 15 years in an estate of $800 per annum, and the possession of the same estate forever, after the expiration of the 15 years, allowing money to be worth 5 per cent. at compound interest? *Ans.* $607.45'.

CHAPTER XII.

PERMUTATIONS AND COMBINATIONS.

PERMUTATIONS.

(234.) PERMUTATIONS are the different *orders of succession* in which a given number of things may be taken—either the *whole number together*, or the whole number taken *two* and *two*, or *three* and *three*, &c.

Thus the different Permutations of the letters *a*, *b*, and *c*, when *all are taken together*, are

abc, *acb*, *bac*, *cab*, *bca*, *cba*.

And the different Permutations of the same letters when taken *two* and *two*, are *ab*, *ba*, *ac*, *ca*, *bc*, *cb*.

Number of Permutations.

(235.) The number of *permutations* that may be formed of a given number of *different* things, is equal to the given number × the given number *minus one*, × the given number *minus two*, × the given number *minus three*, and so on, until the number of *factors multiplied together* is equal to the number of things taken in each permutation.

Suppose that the *four* letters, *a*, *b*, *c*, *d*, are to be subjected to *permutations*.

If we write one of the letters, as *a*, before each of the other 3 letters, we shall have 3 *permutations* of the 4 letters, taken *two* and *two*, in which *a* stands first; in like manner there may be formed 3 *permutations* of the 4 letters, taken *two* and *two*, in which *b* stands first; and so for each of the four letters.

Hence there may be

4 × 3 *permutations* of the 4 letters, taken *two* and *two*.

By taking *b*, *c*, and *d*, and proceeding after the same manner as before, we should have

3×2 *permutations* of these 3 letters, taken *two* and *two* ;

By writing *a* before each of these permutations there would be formed 3×2 permutations of the 4 letters, taken *three* and *three*, in which *a* stands first; in like manner there may be formed 3×2 *permutations* of the 4 letters, taken *three* and *three*, in which *b* stands first; and so for each of the four letters.

Hence there may be

$4\times 3\times 2$ *permutations* of the 4 letters, taken *three* and *three*.

The results thus obtained are in accordance with the proposition which has been stated (235), and similar illustration may be employed in all like cases.

(236.) The number of *different permutations* that may be formed of a given number of things—all taken together—when *some of those things are alike*, is equal to the number that could be formed if the things were *all different* (235), divided by the number that could be formed of as many of them as *are alike*.

When there are *two or more sets of like things*, the divisor to be used will be the *product* of the numbers of permutations that could be formed of *each set*, supposing the things to be different (235).

For example, to find the number of *different permutations* that may be formed of *aaabc*, taken all together.

If the letters were all different from each other, the number of permutations that could be formed would be

$$5\times 4\times 3\times 2\times 1=120 \quad (235).$$

In that case the three letters corresponding to the three *a*'s, would admit of $3\times 2\times 1=6$ permutations, or different orders of succession; whereas *aaa* admits of only *one order of succession.*

The required number of permutations is therefore

$$\frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1}=\frac{120}{6}=20.$$

To find the number of *different permutations* that may be formed of *aababcd*, taken all together.

If the letters were all different from each other, the number of permutations that could be formed, would be

$$7\times 6\times 5\times 4\times 3\times 2\times 1=5040 \quad (235).$$

In that case the number of permutations of the three letters corresponding to the three *a*'s would be $3\times 2\times 1$; and of those corresponding to the two *b*'s would be 2×1; whereas the three *a*'s, and the two *b*'s admit each of only *one order* of succession.

The required number of permutations is therefore

$$\frac{5040}{3\times 2\times 1\times 2\times 1}=420$$

EXERCISES.

1. In how many different ways may a class containing 10 pupils be arranged? (235). *Ans.* 3628800.

2. How long would it require for 5 persons to seat themselves in a different order each day at dinner?
Ans. 120 days.

3. In how many different ways might the names of the 12 months be placed, one after another? *Ans.* 479001600.

4. How many different numbers may be expressed by the 10 digits, allowing 5 figures to each number? (235).

Ans. 30240.

5. How many different successions of 5 men could be formed out of a company consisting of 15 men?

Ans. 360360.

6. A farmer wishes to select a team of 6 horses out of a drove containing 17 horses. How many different choices for the team would it be possible to make? *Ans.* 12376.

7. In how many different orders of succession may the letters in the word *Aaron* be placed? (236). *Ans.* 60.

8. How many variations might occur in the succession of the figures in the number 32233? *Ans.* 10.

9. In how many different orders of succession may the letters in the word *Virginia* be arranged? *Ans.* 6720.

10. How many variations might occur in the succession of the letters in the word *Constitution?* *Ans.* 9979200.

COMBINATIONS.

(237.) COMBINATIONS are the different *collections* which may be formed out of a given number of things, by taking the same number in each collection—without regard to the *order of succession.*

Thus the different Combinations that may be formed out of the letters *a*, *b*, and *c*, when taken *two at a time*, are

ab, *bc*, *ac*.

ab and *ba* are not different *combinations*, but different *permutations*, of the letters *a* and *b*.

In Permutations we have regard to the *order of succession*, and may therefore have two permutations of *two things*. In

Combinations we do not consider the order of succession; so that the combination of two or more things is the same, in whatever order they are taken.

Number of Combinations.

(238.) The number of *different combinations* that may be formed out of a given number of things, is equal to

the number of *permutations* that may be formed in the like case, divided by $1\times2\times3$, and so on;

until the number of factors composing the *divisor* is equal to the number of things in each combination.

Suppose we wish to find the number of *combinations* that may be formed out of 4 *letters*, by taking 3 *letters* in each combination.

The number of *permutations* that may be formed of 4 letters, when taken *three* and *three*, is

$$4\times3\times2=24, \quad (235).$$

Now there can be but *one combination* of 3 letters taken *all together*, while there may be $3\times2\times1$ *permutations* of those letters, which is 6 permutations for each combination.

The required number of combinations is therefore

$$24\div(1\times2\times3)=4.$$

(239.) The number of *different combinations* that may be formed by taking *one* from each of two or more *sets of different things*, will be found by multiplying together the number of things in the *different sets* respectively.

For example, to find how many *different collections* of 3 men might be chosen out of 3 companies containing 4, 5, and 6 men, respectively, by taking *one from each*.

Each of the 4 men in the first company may be combined, separately, with every one of the 5 men in the second company—which gives

$4 \times 5 = 20$ combinations of 2 men in the first *two companies.*

Again; each of these 20 combinations of 2 men may be combined, separately, with every one of the 6 men in the third company—which gives

$$4 \times 5 \times 6 = 120 \text{ combinations of 3 men,}$$

with one man from each of the 3 companies in each combination.

EXERCISES.

1. How many different combinations of 2 kinds of metal could be formed of 5 different metals? *Ans.* 10.

2. How many different combinations of 4 letters may be formed out of the first 12 letters of the Alphabet? *Ans.* 495.

3. How many different collections of 3 persons could be taken out of a company of 13 persons? *Ans.* 286.

4. How many different companies of 4 persons may be taken out of 4 companies containing 5, 7, 8, and 9 persons, respectively? (239). *Ans.* 2520.

5. How many variations might occur in forming a class of 5 pupils, by taking one from each of 5 other classes consisting of 6, 9, 13, 10, and 12 pupils, respectively? *Ans.* 84240.

MISCELLANEOUS EXERCISES

ON THE GENERAL PRINCIPLES OF ARITHMETIC.

1. A person bought $\frac{4}{5}$ of a piece of ground for $\$73\frac{1}{2}$, and sold $\frac{2}{3}$ of his purchase for what it cost him. What part of the whole piece did he sell? and for what sum?

Ans. $\frac{8}{15}$; and $49.

2. An upholsterer bought a quantity of carpeting for $150, at $\$1\frac{1}{4}$ per yard, and sold $\frac{3}{4}$ of it at a profit of $\frac{1}{3}$ of a dollar per yard. What amount of profit did he make on the quantity sold? *Ans.* $30.

3. A bought of B 34 yards, and of C 46 yards of cloth, at $\$5\frac{1}{2}$ per yard. Having sold $\frac{1}{4}$ of these purchases to D, at a profit of $\$1\frac{1}{4}$ per yard; at what rate must the remainder be sold, that his profit may be $150 on the whole?

Ans. $\$7\frac{7}{12}$ per yard.

4. A merchant sold some remnants of cloth, containing $3\frac{1}{2}$ yards, $2\frac{1}{8}$ yards, $3\frac{1}{4}$ quarters, and $1\frac{1}{2}$ quarters, at the rate of $3 per yard. What did the whole amount to?

Ans. $20.436'.

5. A person who had a journey of 735*m.* 5*fur.* to perform, went 13 days at the rate of 40*m.* 3*fur.* 20*p.*, and 2 days at the rate of $39\frac{1}{2}$*m* miles per day. What distance then remained to be traveled? *Ans.* 130*m.* 7*fur.* 20*p.*

6. A grocer exchanged 29*gal.* 3*qt.* 1*pt.* of brandy, at $43\frac{3}{4}$ cents per gallon, for rye at $31\frac{1}{4}$ cents per bushel. What quantity of rye did he thus obtain?

Ans. 41*bu.* 3*pk.* 2*qt.* .8*pt.*

7. A bought of B 42*T.* 13*cwt.* 2*qr.* of iron, of which he sold 18*T.* 14*cwt.* to C, and the remainder to D. What part of the whole quantity did he sell to each?

Ans. .438', and .561'.

8. What should be paid for plastering one side of a wall which is 30*ft.* 10*in.* long, and 8*ft.* 9*in.* high, at $18\frac{3}{4}$ cents per square yard? *Ans.* $5.62'.

9. What would be the cost of excavating a cellar which is 36 feet long, 25*ft.* 8*in.* wide, and 8*ft.* 10*in.* deep, at the rate of 1.06\frac{1}{4}$ per cubic yard? *Ans.* $321.189'.

10. A reservoir for water is 10*ft.* 3*in.* in length, 8 feet in breadth, and 4*ft.* 11*in.* in depth. How many barrels of water will it contain? *Ans.* 68*bar.* 22*gal.* 1*qt.* 1.7'*pt.*

11. A farmer wishes to construct a crib which shall contain 1000 bushels. What must be the length of the crib, allowing its height to be 7 feet, and its breadth 9*ft.* 6*in.*? *Ans.* 18.713 feet.

12. A southern merchant purchased in New York, 95*yd.* 3*qr.* of calico at 1*s.* 6*d.* per yard, $39\frac{1}{2}$ yards of cloth at 20*s.* per yard, and 45*yd.* $2\frac{1}{2}$*qr.* of silk at 8*s.* 6*d.* per yard. What was the amount of his bill in Federal money? *Ans.* $165.179'.

13. A company of emigrants has a supply of bread for 25 days, at an allowance of $1\frac{1}{4}$*lb.* per day. How long would the supply last them at an allowance of $\frac{3}{4}$ of a pound per day? *Ans.* $41\frac{2}{3}$ days.

14. If $20 will supply a family with flour, at 5\frac{1}{2}$ per barrel, for $3\frac{1}{3}$ months; how long will the same sum supply them with flour at 4\frac{3}{4}$ per barrel? *Ans.* $3\frac{1}{5}\frac{9}{7}$ months.

15. If A could build a wall in 3 days, B in 5 days, and C in 6 days; in what time could the three togther build the wall? *Ans.* $1\frac{3}{7}$ days.

16. If the penny-loaf weighs 12 ounces when wheat is sold at 3*s.* 4*d.* per bushel; what ought to be the weight of a loaf worth 9*d.* when wheat is sold at 10*s.* per bushel? *Ans.* 36 ounces.

17. A, B, and C hire a pasture for $24. A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months; what share of the rent should be paid by each?

Ans. $9.60; $3.60; and $10.80.

18. The sum of $2000 is to be divided among three persons in such a manner that the first shall have $\frac{2}{3}$ as much as the second, and the second $\frac{4}{5}$ as much as the third. What are the shares? *Ans.* $457\frac{1}{7}$; $685\frac{5}{7}$; and $857\frac{1}{7}$.

19. In a joint speculation A furnished $5000, B $4000, and C $3500. At the end of 6 months, A withdrew $1500, B $500, and C increased his stock by $\frac{1}{2}$ of its original amount. At the end of 12 months a dissolution occurred, when their profits had amounted to 3765.12\frac{1}{2}$; what were the respective shares of profit?

Ans. $1293.07'; $1140.94'; and $1331.10'.

20. How many ounces of gold which is 15 carats fine must be mixed with 3*oz.* 18 carats fine, and 5*oz.* 23 carats fine, that the compound may be 20 carats fine? *Ans.* $1\frac{4}{5}$*oz.*

21. If 18 men build a wall 40 feet long, 3 feet thick, and 16 feet high, in 12 days; how many men will build a wall 360 feet long, 8 feet thick, and 10 feet high, in 60 days?

Ans. 54 men.

22. A vintner has wine at 3*s.* per gallon, and would mix it with water, so as to obtain 144 gallons which shall be worth 2*s.* 6*d.* per gallon. How much wine, and how much water must be taken? *Ans.* 120*gal.*, and 24*gal.*

23. Suppose 9*lb.* of pure gold immersed in a vessel full of water to expel 3*lb.* of water, 9*lb.* of pure silver to expel 6*lb.* of water, and 9*lb.* of a composition of gold and silver to expel 4*lb.* of water; what are the respective quantities of gold and silver in the composition?

Ans. 6*lb.* of pure gold; and 3*lb.* of pure silver.

24. If London remit £1000 sterling to Spain, by way of Holland, at 35*s*. Flemish per £ sterling; thence to France, at 58*d*. Flemish per crown; thence to Venice, at 100 crowns per 60 ducats; and thence to Spain, at 360 maravedis per ducat; how many piastres, of 272 maravedis each, will the £1000 sterling amount to in Spain? *Ans.* 5750½ piastres.

25. A merchant sold goods which cost him \$250, at a profit of 25 per cent., on a credit of 6 months, and afterwards deducted 5 per cent., for immediate payment. What percentum of profit did he make? *Ans.* 18¾ per cent.

26. A merchant sold goods which cost him \$300, at a profit of 60 per cent., on credit, and then expended 40 per cent. of the debt in securing its payment. What was his percentum of profit or loss? *Ans.* 4 per cent. *loss.*

27. Suppose a merchant to sell at a profit of 33⅓ per cent., and that his annual expenses are \$2000. What must his annual sales amount to in order to his saving \$3000 per annum? *Ans.* \$15000.

28. The taxable polls in a State number 540853, and are assessed at \$1. The landed property of the State is worth \$76800000; at what percentum must the land be taxed that the revenue from both sources may amount to \$694453? *Ans.* ⅕ per cent.

29. A merchant imported 90 bags of coffee, which weighed in the gross 150*lb*. each, and were invoiced at 6½ cents per pound. An allowance of 4 per cent. was made for *waste*, and the *duty* paid was 20 per cent.; what was the amount of duty? *Ans.* \$168.48.

30. What amount of stock in the capital of an Insurance Company, at a discount of 3½ per cent., could be purchased for \$3860? and what amount, at an advance of 4 per cent., could be purchased for \$6240? *Ans.* \$4000; and \$6000.

31. By selling a lot of carpeting at \$1.25 per yard, an upholsterer realized a profit of 20 per cent. If the same had been sold at 87½ cents per yard, what would have been his percentum of profit or loss? *Ans.* 15.946' per cent. *loss.*

32. A Railroad Company sells its bonds, bearing interest at 6 per cent., at a discount of 25 per cent. What would be the annual produce of \$20000 invested in such stock? and what rate of interest would be made on the investment?

Ans. \$1600; and 8 per cent.

33. The stock of a certain Bank is at an advance of 10 per cent., and the annual dividends or profits on this stock are at the rate of 9 per cent. What rate of interest would be realized on money invested in such stock? and what sum must be thus invested to produce an *annual income* of \$3000?

Ans. 8.181' per cent.; and \$36666⅔.

34. \$1500.12½ *Cincinnati, Jan. 1st,* 1854.

Three months after date I promise to pay to John Smith & Co. One thousand five hundred dollars 12½c., for value received, *Simon True.*

On this Note there are the following endorsements:

April 1st, 1854, received \$500.00.
July 10, 1854, received \$450.50.
May 25, 1855, received \$325.12½.

What balance is due on the 18th of August, 1855?

Ans. \$274.44.

35. A merchant's account for \$300, in Louisville, was due on the 1st of July, 1854; on which there was paid, August 20th, \$120, and October 5th, \$75.50. What balance was due on the 1st of January, 1855—according to each of the two methods which have been given for deducting *partial payments?* *Ans.* \$109.85: and \$109.91.

36. The sum of $1000 was loaned, at conventional interest, for 3 years 7*m.* 15*da.*, when it was found to have amounted to $1326.25. What was the rate of interest?

Ans. 9 per cent.

37. In what time will $1000, or any other principal, double itself, if put at interest, at 10 per cent.? *Ans.* 10 years.

38. A person sold a tract of land for $15375.75, to be paid in three equal instalments, in 1 year 6*m.*, 2 years, and 3*y.* 6*m.*, without interest. What sum in hand ought to pay for the land, if the rate of interest be 8 per cent.?

Ans. $12998.52.

39. A person owes a Note in Bank of $2500, towards the payment of which he can raise but $1900. This sum he proposes to apply towards paying the Note, and to paying the discount on a new Note, at 60 days, for the remainder of the debt. For what sum must the new Note be drawn, if interest be at 6 per cent.? *Ans.* $606.36.

40. A gentleman wishes to divide $1000 between his two sons, whose ages are 15 and 17 years, respectively, in such proportion that their shares, put on interest at 8 per cent., shall amount to the same sum against the youths attain the age of 21. What are the two shares?

Ans. $471.425; and $528.574.

41. A plantation was sold on the conditions that $\frac{1}{4}$ of the purchase money should be paid on the 1st of March, $\frac{1}{3}$ of it on the 4th of June, and the remainder on the 20th of September. What would be the equitable mean time for the payment of the whole? *Ans.* The 25th of June.

42. A merchant bought goods on the 20th of August, to the amount of $3200, on a credit of 6 months. Of this debt he paid $1000 on the 1st of October, and $750 on the 1st of December; on what day ought the remainder in justice to be paid? *Ans.* The 10th of July.

43. The following account was paid on the 30th of December; what amount was then due, allowing interest at 6 per cent. ?

January 5th,	Merchandise,	$30.75,	on 6 months' credit;
March 9th,	"	$90.20,	on 4 months' credit;
June 20th,	"	$89.37,	on 3 months' credit.

Ans. $215.29.

44. A father bequeathed an estate amounting to $20000 to his three sons, at the ages of 10, 12, and 14, in such proportion that the different shares, at 6 per cent. interest, should be of the same amount to each at the age of 21. What were the several shares ?

Ans. $6159.82'; $6639.39'; and $7200.77'.

45. What balance was due on the following account, on the 30th of November, allowing a credit of 4 months on each item, and interest at 7 per cent. ?

1854.	*Dr.*	1854.	*Cr.*
Jan. 1st, . . .	$25.00	March 10th, . .	$10.00
Feb. 10th, . . .	$73.20	May 15th, . . .	$30.50
May 6th, . . .	$85.50	July 20th, . . .	$54.30

Ans. $92.875.

46. What must be the side of a square which shall be equal in area to a surface which is 320 yards long, and 75 yards wide ? *Ans.* 154.919' yards.

47. A field containing 15 acres is to be laid out in such a manner that its length shall be equal to three times its breadth. What must be the dimensions of the field ?

Ans. 28.284' poles, and 84.852'*p.*

48. What would be the difference in area between two fields of the same compass or perimeter—one of them to be in the form of a square—the other to be 75 rods in length, and 30*r.* 3*yd.* in breadth ? *Ans.* 3*A.* 13.92*P.*

49. What must be the dimensions of a granary which shall contain 2000 bushels of wheat—its length to be equal to twice its breadth, and its breadth equal to twice its height?

Ans. 27.1'*ft.*, 13.55'*ft.*, and 6.77'*ft.*

50. How many square feet are contained in the surface of a cubical rock of granite whose solidity is 1331 cubic feet?

Ans. 726 square feet.

51. A gentleman has an oblong garden containing 5*A.* 3*R.* of ground, and wishes to make another which shall contain the same area, in the form of a square. What must be the length of each side of the square? *Ans.* 30.331' poles.

52. A brewer has a cistern which contains 6 barrels of beer, and whose length and height are each equal to twice its breadth. What are the dimensions of the cistern?

Ans. Length and height 4.13'*ft.*; breadth 2.065'*ft.*

53. What would be the expense of plastering the bottom and walls of a cubical reservoir which shall contain 100 barrels of water, at $0.37½ per square yard?

Ans. 14.612'.

54. Placing 100 eggs in a straight line, at a yard's distance one from another, and the first a yard from a basket; how far must a person travel to bring the eggs, one by one, to the basket? *Ans.* 10100 yards.

55. A person received $300 at 12 payments, each successive payment increasing by $4; what was the first? and what the last payment? *Ans.* $3, and $47.

56. A gentleman bequeathed to the eldest of his four sons $4000, and to the youngest $9000, while his second and third were to have the geometrical and arithmetical *means*, respectively, between the portions of the other two. What did the second and third receive? *Ans.* $6000, and $6500.

57. Allowing a sum of money to increase at the rate of $500 the first year, $400 the second, $320 the third, and

so on at the same rate forever; what would be the utmost amount of the increase? *Ans.* $2500.

58. What must be paid for 32 yards of cloth, at the rate of 1 farthing for the first yard, 3*qr.* for the second, 9*qr.* for the third, and so on in triple ratio to the last?

Ans. 965114681693£. 13*s.* 4*d.*

59. What will $5 amount to in 50 years, at 5 per cent. Compound Interest? *Ans.* 57.337.

60. A is indebted to B $4750, to be paid in two equal instalments, in 3 and 6 years, without interest. What sum in hand would be an equivalent for the debt, on the supposition that money will produce 6 per cent. at compound interest? *Ans.* $3668.377'.

61. A pension of $300 per annum has remained unpaid for 25 years. What amount is due on this pension at simple interest, and also at compound interest, at 6 per cent.?

Ans. $12900; and $16459.35.

62. A person's dividend from his stock in Bank is $530 a year. What is the present value of this income for 5 years to come, computing by simple, and also by compound, interest, at 7 per cent.? *Ans.* $2237.77; and $2173.10.

63. What annuity, to continue 20 years, can be purchased for $10000, allowing compound interest at 5 per cent.?

Ans. $802.42.

64. What annual income ought to be realized, for 25 years, from the present investment of $20000, computing by comp. interest, at 6 per cent.? *Ans.* $1564.94'.

65. For what sum might the Government of a country undertake to pay an annuity of $1000 a year, forever, on the supposition that money may always be invested at 6 per cent.? *Ans.* 16666\frac{2}{3}$.

66. For what sum might an annuity of $400 a year, for 10 years, to commence in 5 years, be purchased, allowing compound interest at 6 per cent.? *Ans.* $2199.95'.

67. In how many different ways may the names of the 12 months in the year be arranged one after another?

Ans. 479001600.

68. In how many different ways might the seven prismatic colors, *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*, have been arranged in the solar spectrum? *Ans.* 5040.

69. In an exhibition of a Public School, 5 speakers are to be taken from a class of 15 students. How many different selections of the five might be made? and in how many different ways might the five chosen succeed one another in the delivery of their speeches? *Ans.* 3003, and 120.

70. A die is a small cube whose six faces are marked with the numbers from 1 to 6 inclusive. How many different combinations of 5 numbers might be exhibited on their superior faces in throwing *five dice* together? *Ans.* 6 combinations.

71. Out of a Company consisting of 100 soldiers six are to be taken for a particular service. How many different selections of the six might be made? and in how many different ways might the six chosen be disposed with regard to the *order of succession*? *Ans.* 1192052400; and 720.

72. In how many different orders of succession may the letters in the word *America* be arranged? *Ans.* 2520.

73. How many different classes of 5 students might be formed out of 5 other classes containing 9, 11, 13, 17, and 20 students, respectively? *Ans.* 437580.

74. In how many different orders of succession may the figures in the number 5367395 be arranged? *Ans.* 1260.

75. A person who enjoyed a perpetuity of $1000 per annum, provided in his will that, after his decease, it should descend to his only son for 10 years, to his only daughter for the next 20 years, and to a benevolent Institution forever afterwards. What was the value of each bequest at the time of his decease, allowing compound interest at 6 per cent.?

Ans. $7360.08'; $6404.74'; and $2901.83'.

CHAPTER XIII.

EXCHANGE,—FOREIGN COINS AND CURRENCIES.

EXCHANGE.

(240.) EXCHANGE, in Commerce, is a transaction by which a debt is paid to a person at a distance, without the transmission of money.

This is effected by means of a

Draft or Bill of Exchange.

(241.) A Draft or Bill of Exchange is a written order from one person, called the *drawer*, to another called the *drawee*, for the payment of a certain sum, at a specified time, to a third person called the *payee*, or *to his order*.

The Payee becomes an *endorser* of the Bill by writing his name across the back of it; and the bill is thus made payable to the *bearer*, that is, to any holder of the bill.

If the Payee endorse that payment shall be made to the order of any particular person, that person is the *endorsee*, and may himself endorse the Bill in the same manner; and the bill may receive any number of successive endorsers.

The *Drawee* becomes the *accepter* of the Bill, and binds himself to pay as directed, by writing his name on the bill, under the word "Accepted," usually across the face of the bill.

Example in Exchange.

Suppose that John Smith of Louisville is indebted to James Brown of New York, and Thomas Jones of New York to William Nelson of Louisville; and that payments are to be made to the amount of $500.

John Smith pays this sum to William Nelson for a *draft* or bill on Thomas Jones, which reads as follows:

$500. LOUISVILLE, *January* 1*st*, 1856.

Thirty days after date, pay to the order of John Smith, Five hundred dollars, and charge the same to the account of

Yours, &c.

To THOMAS JONES, WILLIAM NELSON.
Merchant, New York.

John Smith writes on the back of this bill, "Pay to the order of James Brown," subscribes his name, and sends the bill to James Brown. The latter presents it to Thomas Jones, who, agreeing to pay as directed, writes "Accepted," and subscribes his name on the bill.

This bill, either with or without the acceptance of Thomas Jones, might pass, by endorsement, through different hands, answering the purposes of *money*, like a Bank bill, until, becoming due, it is presented to the drawee for payment.

A Bill of Exchange is said to be *negotiable* when, as in the preceding Example, it may be passed from one Payee to another, and thus become a medium of Commerce, in the same manner as money.

Domestic and Foreign Bills.

(242.) A Domestic or Inland Bill of Exchange is one which is payable in the same Country or State in which it is drawn. Such bills are usually called *drafts*—or *checks*, when made on a Bank in which the drawer has funds deposited.

A Foreign Bill of Exchange is one which is payable in a different Country or State from the one in which it is drawn.

When a Bill of Exchange is drawn on a distant Country, it is usual to make three bills of the *same import*, called respectively the *first*, *second*, and *third* of Exchange, and collectively a *Set of Exchange*. To provide against miscarriage, these are sent by different conveyances, and when either of them is accepted or paid, the others are *void*.

The following will serve as an Example of a Bill drawn in New York on London.

£1000. New York, *March 4th*, 1856.

Sixty days after sight of this first of Exchange, (second and third of the same date and tenor unpaid), pay to the order of George Greedy, Esq., One thousand pounds sterling, with or without farther advice.

Smith, Jones & Co.

To Messrs. Rothschild & Co.,
Brokers, London.

Laws and Customs respecting Bills of Exchange.

(243.) 1. The *Drawer* and all the *endorsers* of a Bill of Exchange are liable for its amount to the *holder* or owner of the Bill, if the *drawee* fail to pay it at maturity. But to bind them to this liability they must receive due notice in writing. This notice, given by an officer styled a *Notary*, is called a *protest*; and will be for *non-acceptance*, or else for *non-payment* of the Bill.

A protest for *non-acceptance* binds the drawer for the immediate payment of the Bill, even though it should not have reached its maturity.

2. When the time in which a Bill will mature is given in *months*, Calendar months are always to be understood, without regard to the number of days.

Thus a Bill dated on the 28th of January, payable *one month* after date, will mature on the 28th of February; if dated on the 29th, 30th, or 31st of January, it would mature on the last day of February.

Days of Grace are, in this country and Great Britain, *three days* allowed for payment, beyond the time for which the Bill is drawn.

When the last day of grace is Sunday, or a public *holiday*, as the 4th of July, the Bill must be paid on the preceding day.

Promissory Notes may be made *negotiable*, and passed from one owner to another, in the same manner, and subject to the same Laws and Customs as Bills of Exchange.

Par of Exchange.

(244.) The *Par of Exchange* between two countries is the value of a given amount of the currency of the one, as expressed in the currency of the other.

The true or *intrinsic* par of Exchange depends on the amount of *pure gold* or *silver* in the coins compared.

For example, To find the value, in Federal Money, of the English *Sovereign*, or Pound Sterling.

The United States Eagle is a gold coin, worth $10, weighing 258 *grains*, $\frac{9}{10}$ of which is *pure gold*. Then the Eagle contains

$$258 \times \frac{9}{10} = 232.2 \text{ gr. of pure gold.}$$

The English Sovereign is a gold coin whose *full weight* is 5 *dwt*. 3.274 *gr*., = 123.274 *gr*., $\frac{11}{12}$ of which is pure gold. Then the Sovereign contains

$$123.274 \times \frac{11}{12} = 113.0011' \text{ gr. of pure gold.}$$

For the Federal value of the *Sovereign* we have therefore the proportion, 232.2 : 113.0011 : : \$10 : \$4.866'.

The weight of the Sovereign, as given above, is deduced from the English standard, which requires 1869 sovereigns to 40 pounds troy of standard metal, that is, metal which is *eleven twelfths* pure gold. But in England, the sovereign weighing but 5 *dwt.* $2\frac{3}{4}$ *gr.* is a *legal tender* in the payment of debts.

Taken at this weight, the sovereign will be found to be worth \$4.84, which is its value, as fixed by Law, in the Custom Houses of the United States.

For the Par of Exchange between the United States and other Countries, the reader is referred to (249); and for a list of Foreign Coins which have been made current, by Law, in the U, S., to (250).

Course of Exchange.

(245.) The *Course of Exchange* between two countries, is the variable price paid in the one for Bills of exchange *payable in the other.*

Exchange between two countries will be *at par* when the *debts* and *credits* between them balance each other—*above par* in the country which owes a *balance of debt* to the other —and *below par* in the country to which a balance is due from the other.

Thus when New York owes London just as much as London owes New York, the debtors in the two places will *exchange liabilities* with one another. A New York creditor draws a Bill on his London debtor, payable to a New York debtor, which the latter endorses to his London creditor; and by transactions of this kind the indebtedness of each place to the other is discharged by *exchanges at par.*

If New York owe a balance of debt to London, there will be competition in New York for Bills on London—each debtor striving to avoid the expense of transmitting gold or silver, and Bills on London will sell *above par.* At the same time, there being a greater amount to be drawn for by London on New York, than will be needed in the way of exchange, there will be competition in London in the sale of Bills on New York, and this will cause them to fall *below par.*

The *premium* or advance that can be obtained on Bills of Exchange, will never exceed the expense (including insurance) attendant on the transmission of gold or silver to the place on which the Bills are drawn. For, rather than pay a higher premium, the debtor will transmit the precious metals.

Exchange between the U. S. and England.

(246.) To understand the present Course of Exchange between the United States and England, reference must be made to a change which has been produced in the relative value of the Pound Sterling by an alteration in the standard U. S. Eagle.

By the standard first adopted the Eagle was to weigh 270 *gr.*, $\frac{11}{12}$ of it to be pure gold, and $\frac{1}{12}$ alloy; and its value was \$10.

By Act of Congress, taking effect August 1st, 1834, the present standard was adopted. By this the weight of this coin was diminished 12 *gr.*, and its proportion of alloy increased from $\frac{1}{12}$ to $\frac{1}{10}$, while its value still remained \$10.

The relative value of gold, in the U. S., was thus *enhanced;* the *old Eagle* came to be worth \$10.62½; and the English Pound Sterling, from being worth but \$4.44⅘, in Federal Money, was enhanced, intrinsically, to \$4.866', or at least, to

$4.84', which is about 9 *per cent.* above the *old par* of $4.44\frac{4}{9}$.

Now it has been found most convenient to retain the *old value* of the Pound as the basis of Exchange with England, and to express its present exchangeable value by a *premium on the former value.*

Hence when it is said that Bills of Exchange on England are at a *premium of* 9 *per cent.*, it must be understood that the English Pound is reckoned at $4.44\frac{4}{9}$; and that the exchange is really *about at par.*

ARBITRATION OF EXCHANGE.

(247.) The *Arbitration of Exchange* consists in computing Exchange between two countries through the medium of exchanges between these two and one or more other countries.

This will be illustrated by the following

EXAMPLE.

A merchant in New York has to pay £100 in London, and Bills on London are at a premium of 10 per cent. The exchange with Paris is at the rate of 5.4 *francs* to the Dollar, and with Hamburg 35 cents per *marc* banco. The exchange between Paris and London is 25.8 *francs* per £, and between Hamburg and London 13.5 *marcs* banco per £. It is required to determine what sum, in Federal Money, will pay the £100,

1st. By a Direct Exchange with *London;*

2d. By an Arbitrated Exchange through *Paris*,

3d. By an Arbitrated Exchange through *Hamburg.*

1. *With London.*—£1 in Exchange is $4.444'.

After adding $0.444 for the premium of 10 per cent.,

£1 = $4.888 ; then £100 = $488.8.

2. *Through Paris.*—By arranging the terms in a *Conjoined Proportion* (157)

we have \$1 = 5.4 *francs.*
25.8 *fr.* = £1,
£100 = *how many* \$?

Ans. (1 × 25.8 × 100) ÷ (5.4 × 1) = \$477.77.

3. *Through Hamburg.*—Again arranging the terms in a Conjoined Proportion,

we have \$0.35 = 1 *marc.*
13.5*m.* = £1,
£100 = *how many* \$?

Ans. (.35 × 13.5 × 100) ÷ (1 × 1) = \$472.5.

We thus find that a direct exchange with London will require \$488.8; an exchange through Paris \$477.77; and through Hamburg, \$472.5.

By the same method an Exchange may be arbitrated between two countries through *two or more* intervening countries; and thus the most advantageous medium through which to make a foreign payment may be ascertained.

These transactions, it will be understood, are to be effected by Bills of Exchange between the different places; and require that the person so arbitrating exchanges should have, in the intermediate places, Correspondents or Agents to assist him.

Relative Values of Gold and Silver.

(248.) The relative values of Gold and Silver in any country are ascertained from the proportional amount of pure gold and silver in its principal coins—taking these coins of the *value, weight* and *purity* prescribed by the laws of that country.

Thus in the United States the *gold* Eagle, of \$10, is required to weigh 258 grains, $\frac{9}{10}$ of it to be pure gold, and $\frac{1}{10}$ alloy.

The *silver* Dollar is required to weigh $412\frac{1}{2}$ grains, $\frac{9}{10}$ of it to be pure silver, and $\frac{1}{10}$ alloy.

The Eagle then contains $258 \times .9 = 232.2$ *gr.* of pure gold ;
The Dollar contains $412.5 \times .9 = 371.25$ *gr.* of pure silver ;
232.2 *gr.* of gold are equal in value to 3712.5 *gr.* of silver.

$$3712.5 \div 232.2 = 15.988'.$$

The value of gold in the United States is therefore 15.988' *times* that of the same weight of silver ; in other words their relative values are as 15.988 to 1.

The relative values of Gold and Silver are not the same in all countries.

In the U. S. these values are as 15.988' to 1 ;
in England, as 14.28 to 1 ;
in France, as 15.50 to 1 ;
in Spain, as 16.00 to 1 ;
in China, as 14.25 to 1.

These differences in the relative values of Gold and Silver in different countries, will cause the one or the other of these metals to be employed in the payment of foreign debts—when the circumstances of trade require the transmission of money—according as the one or the other will be *increased in value* in the country to which it is sent.

Thus in England, France, or China, silver is, relatively to gold, more valuable than in the United States. Silver, rather than gold, will therefore be sent from the U. S. to those countries.

The relative valuation of these metals is sometimes changed in the same country. This occurred in the U. S., in the year 1834, as has been formerly shown (246).

(249.) *Foreign Coins and Moneys of Account.*

The *Coin*, or *Specie*, of a country consists of pieces of metal, chiefly of *gold* and *silver*, of fixed value, and *stamped* by public authority, to be used as *money*.

Moneys of Account are those denominations of money in which accounts are kept—being those in which sales are effected; they are generally, but not always, represented by corresponding *coins*.

Great Britain.

4 farthings make one penny;
12 pence 1 shilling (*silver*);
20 shillings 1 pound sterling (*gold*); = \$4.84 to \$4.86.

In British North America— £1 = \$4.00.

In the British W. Indies, the £ varies in different islands, and is always less than the £ sterling.

France.

100 centimes make 1 franc (*silver*); = \$0.186.

Holland and Belgium.

100 centimes make 1 florin or guilder (*silver*); = \$.040.
In 1832 the coinage of Belgium was conformed to that of France.

Denmark.

12 pfenings make 1 skilling;
16 skillings 1 marc;
6 marcs 1 rix-dollar (*silver*); = \$0.52½.

Portugal.

400 rees, make 1 cruzado;
1000 rees 1 milree or crown (*silver*); = \$1.12.

Norway.

120 skillings make 1 rix-dollar specie (*silver*); = \$1.05.

Sweden.

12 rundstycks make 1 skilling;
48 skillings 1 rix-dollar specie (*silver*); = \$1.06.

Russia.

100 copecks make 1 rouble (*silver*); = \$0.75.

Prussia.

12 pfenings make 1 grosch (*silver*);
30 groschen 1 thaler or dollar (*silver*); = \$0.69.

Austria.

60 kreutzers make 1 florin (*silver*); = \$0.48⅖.

Spain.

2 maravedis make 1 quinto;
16 quintos 1 rial of old plate;
10⅝ rials of old plate 1 dollar (*silver*); = \$1.00.

Sicily.

20 grani make 1 taro;
30 tari 1 oncia (*gold*); = \$2.48¼.

Papal States.
10 bajocchi make 1 paoli;
10 paoli 1 scudo or crown (*silver*);
= $1.

Naples.
10 grani make 1 carlino;
10 carlini 1 ducat (*silver*);
= $0.79.

Turkey and Egypt.
3 aspers make 1 para;
40 paras 1 piastre (*silver*);
= in Turkey, $0.03 to $0.05.
in Egypt, $0.048.

Greece.
100 lepta make 1 drachme (*silver*);
= $0.166.

Mexico.
8 rials make 1 dollar (*silver*);
= $1.00.

Brazil.
1000 rees make 1 milree;
1200 rees (*silver*) = $0.994.

In the other S. American States, 8 rials make 1 dollar, sometimes more, sometimes less than, $1.00.

(250.) *Foreign Coins made current, and Moneys of Account determined, in the United States, by Acts of Congress.*

A Foreign Coin is made *current* when, by Law, it is made receivable, at a fixed value, in the payment of debts. This supposes the Coin to be of standard *weight* and *purity*. But other Foreign Coins will also circulate, at values corresponding to their weight and purity.

Pound Sterling of G. Britain	$4.84
Pound of British N. America	4.00
Franc of France & Belgium	0.186
Livre Tournois of France...	0.185
Florin of Austria..........	0.485
Milree of Portugal.........	$1.12
Milree of Azores..........	0.83⅓
Marc Banco of Hamburg..	0.35
Livre of Lombardo........	0.16
Livre of Leghorn..........	0.16
do. Tuscany..........	0.16
do. Sardinia...........	0.186
Ducat of Naples..........	0.80
Ounce of Sicily...........	2.40

Rix Dollar of Bremen....	$0.78¾
Florin of Netherlands......	0.40
Do. S. States of Germany..	0.40
Guilder of Netherlands.....	0.40
Real Vellon of Spain.......	0.05
Real Plato of Spain........	0.10
Thaler or Rix Dollar of Prussia & N. States of Germ'y	0.69
Specie Dollar of Denmark..	1.05
do. Sweden and Norway.	1.06
Tael of China.............	1.48
Rupee of British India....	0.445
Pagoda of India..........	1.84
Silver Rouble of Russia...	0.75

CHAPTER XIV.

MATHEMATICAL PROBABILITIES, AND THEIR APPLICATION TO LIFE ANNUITIES AND LIFE INSURANCE.

(251.) The THEORY OF PROBABILITIES has respect to events which may be regarded as *equally contingent;* and, in the sense here intended,

A *contingent event* is one of a number of events, some only of which will certainly occur, while no reason can be perceived why any one of them should occur *rather than any other;* as when *one person* is to be taken, by lot, from a company consisting of *five persons*

Measure of Probability.

(252.) The Probability of a contingent event is measured, and expressed, by the *ratio of the number of chances* favorable to that event to the whole number of chances favorable and unfavorable to the same event.

Suppose that *one* person only is to be taken *by lot* from among *five persons,* represented by

A, B, C, D, and E.

The Probability that the lot will fall on any particular one of the five, as A, is expressed by $\frac{1}{5}$, since he has *one* chance in *five.* In like manner the Probability that the lot will fall on any other one, as B, is $\frac{1}{5}$, since each person has one chance in five.

Opposite Probabilities.

(253.) The Probability of the occurrence of a contingent event, and the probability of its *non-occurrence,* are opposite probabilities, the sum of the measures of which is *unity.*

Suppose, as before, that one person is to be taken by lot from among five persons,

A, B, C, D, and E.

The Probability that the lot will fall on A is $\frac{1}{5}$, because he has one chance in five; the probability that the lot *will not fall on* A is $\frac{4}{5}$, because there are *four* chances in *five* against its falling on A; and the sum of $\frac{1}{5}$ and $\frac{4}{5}$ is *unity*.

(254.) The Probability of the *non-occurrence* of a contingent event, is the same thing as the *improbability* of that event; and is measured by a *unit* minus the *probability* of the same event.

Thus, in the preceding example, the Probability of the lot's falling on A is $\frac{1}{5}$; the *improbability* of its falling on A is $\frac{4}{5}$; and $\frac{4}{5} = 1 - \frac{1}{5}$.

It follows from the preceding principles that, in the Theory of Probabilities, a *unit* is the measure of *certainty*.

For it is *certain* that a contingent event will either happen or not happen, and the *opposite probabilities* thus existing are together measured by *unity* (253).

Compound Probabilities.

(255.) The Probability of one, *indifferently*, of two or more designated contingent events, is measured by the *sum of the separate probabilities* of the same events.

EXAMPLE.

If one person is to be taken by lot from among five persons, A, B, C, D, and E; the Probability that the lot will fall on *one of the three*, A, B, and C, is

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5},$$

because these three together have *three chances in five*.

(256.) The Probability of the *concurrence of two or more* contingent events, is measured by the *product of the separate probabilities* of the same events.

EXAMPLE.

Suppose that two tickets are to be drawn, *successively*, from among 9 tickets, of which 4 are *prizes*, and 5 are *blanks*.

The Probability that a *prize* will be obtained at the first drawing is $\frac{4}{9}$ (252); and as 3 prizes would then remain among 8 tickets—supposing a prize to have been already drawn—the Probability that a prize will be obtained at the second drawing is $\frac{3}{8}$; then the Probability that *two prizes* will be obtained at the two drawings is

$$\frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}.$$

Each of the 9 tickets in the first drawing might be combined with each of the 8 in the second, which gives $9 \times 8 = 72$ *chances* or different combinations of two tickets. Each of the 4 *prizes* in the first might combine with each of the 3 prizes which would remain for the second drawing—supposing a prize to have been drawn first,—which gives $4 \times 3 = 12$ *chances* favorable to the drawing of *two prizes* (252).

(257.) The Probability of the first of two contingent events, or, if the *first fail*, of the second event, is measured by the probability of the first *plus* the product of the probability of the second $\times$ the *improbability of the first*.

EXAMPLE.

If one person is to be taken by lot from among *five* persons, A, B, C, &c., and another from among *six* other persons, F, G, H, &c.; then the probability that the lot will fall on A, in the first case, or *if not on A*, that it will fall on F, in the second case, is

$$\tfrac{1}{5} + \tfrac{1}{6} \times \tfrac{4}{5} = \tfrac{1}{5} + \tfrac{4}{30} = \tfrac{1}{3}.$$

The $\frac{1}{5}$ in this compound expression, is the probability that A will be taken out of the first company (252); $\frac{1}{6} \times \frac{4}{5}$ is the probability that F will be taken out of the second *but not A out of the first* (256); and the compound Probability in question is the sum of these separate probabilities (255).

Absolute and Relative Values of Contingent Payments of Money.

(258.) The *absolute* value of a sum which is payable only on the occurrence of a contingent event, is found by multiplying that sum by the *probability of the event.*

EXAMPLE.

Suppose that *one* ticket is to be drawn from among 10 tickets, *two* of which are *prizes* of \$100 each, and the rest are *blanks.*

The Probability that the ticket to be drawn will be a prize, is $\frac{2}{10}$ or $\frac{1}{5}$, since there are two chances in 10 in favor of a prize (252).

The *certainty* of drawing a prize would be worth \$100; *one-fifth* of this certainty, that is, a probability amounting to *one-fifth* that a prize will be drawn, is therefore worth

$$\$100 \times \tfrac{1}{5} = \$20.$$

Thus the absolute or abstract value of a ticket in this lottery, is equal to the amount of a prize, \$100, multiplied into the *probability of a ticket's drawing a prize.*

Contingent events, in the long run, *conform to the laws of probability;* so that in an indefinitely great number of such events, the actual *occurrences* will be in very near accordance with the abstract probabilities.

In the lottery of the preceding Example, the value of a ticket has been shown to be \$20. If a ticket at this price draw a *prize,* the gain will be \$80; if it draw a blank, the loss will be \$20; and the result of *a single trial* must be \$80 gained, or \$20 lost.

The probability of gaining \$80 is $\frac{1}{5}$, while the probability of losing \$20 is $\frac{4}{5}$; and the contingent gain and loss are theoretically *equivalent;* thus

$$80 \times \tfrac{1}{5} = 20 \times \tfrac{4}{5} \ (258).$$

In any number of trials, less than *five,* in such a lottery as this, the gains and losses *could not be equal:* in *five trials* there might be drawn *one prize* and *four blanks,* when the gain would be equal to the losses; and an equivalence between gains and losses would be *actually approximated* in proportion as the number of trials is increased.

The *Relative Value* of a Sum which is payable on the occurrence of a contingent event, depends on the *fortune of its expectant.*

A sum which is of little importance to a wealthy man, may be of great importance to one of smaller fortune; and cannot therefore be prudently *risked* by the latter on the same *contingency* as it might be by the former.

A person of large fortune might repeat the risks of lottery dealing so often as to bring his losses, with almost entire certainty, within any given amount; while one of limited means, who could not continue the chances in case of losses, would incur the risk of being ruined.

LIFE ANNUITIES.

(259.) A LIFE ANNUITY is a sum of money to be paid *annually* during the life of a person, called the *Annuitant*, but the payment is to cease at his death.

A Life Annuity, under the name of *pension*, is sometimes bestowed on a person on account of past services to his country; and is sometimes purchased from a Company by the present payment of an equitable sum of money.

A *temporary* Life Annuity is one which is limited to a given number of years, and is liable to fail at any time by the decease of the Annuitant.

Present Value of a Life Annuity.

(260.) The Present Value of a Life Annuity is estimated according to the *probabilities* of the annuitant's living *one*, *two*, *three*, &c., *years*, or through the period to which the Annuity may be limited.

The Probabilities that a person at any given age, will live one, two, three, &c., years, are obtained from observations on the *usual rate of mortality*.

The following TABLE commences with 10,000 persons *at birth*, and shows the number who *die*, and the number who survive, for each year, until all are dead. It was formed from the registers of births and deaths in the city of Carlisle (England), between the years 1779 and 1787. Its accuracy has been confirmed by observations at various other places, and by the experience of the oldest Life Annuity and Insurance Companies.

TABLE

Of Mortality based upon observations at Carlisle, showing the rate of extinction of 10,000 *lives.*

AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.	AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.	AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.
0	10000	1539	35	5362	55	70	2401	124
1	8461	682	36	5307	56	71	2277	134
2	7779	505	37	5251	57	72	2143	146
3	7274	276	38	5194	58	73	1997	156
4	6998	201	39	5136	62	74	1841	166
5	6797	121	40	5075	66	75	1675	160
6	6676	82	41	5009	69	76	1515	156
7	6594	58	42	4940	71	77	1359	146
8	6536	43	43	4869	71	78	1213	132
9	6493	33	44	4798	71	79	1081	128
10	6460	29	45	4727	70	80	953	116
11	6431	31	46	4657	69	81	837	112
12	6400	32	47	4588	67	82	725	102
13	6368	33	48	4521	63	83	623	94
14	6335	35	49	4458	61	84	529	84
15	6300	39	50	4397	59	85	445	78
16	6261	42	51	4338	62	86	367	71
17	6219	43	52	4276	65	87	296	64
18	6176	43	53	4211	68	88	232	51
19	6133	43	54	4143	70	89	181	39
20	6090	43	55	4073	73	90	142	37
21	6047	42	56	4000	76	91	105	30
22	6005	42	57	3924	82	92	75	21
23	5963	42	58	3842	93	93	54	14
24	5921	42	59	3749	106	94	40	10
25	5879	43	60	3643	122	95	30	7
26	5836	43	61	3521	126	96	23	5
27	5793	45	62	3395	127	97	18	4
28	5748	50	63	3268	125	98	14	3
29	5698	56	64	3143	125	99	11	2
30	5642	57	65	3018	124	100	9	2
31	5585	57	66	2894	123	101	7	2
32	5528	56	67	2771	123	102	5	2
33	5472	55	68	2648	123	103	3	2
34	5417	55	69	2525	124	104	1	1

(261.) The Probability, according to the preceding Table, that a person at any *given age*, will attain any designated *higher age*, is the ratio of the number who attain the *higher* to the number who attain the *given* age.

For example, to find what is the probability that a person who is 30 years old will attain the age of 60.

From the Table we find of 5642 *survivors* at the age of 30, only 3643 attaining the age of 60 ; the probability in question is therefore the ratio

$$\frac{3643}{5642}, \ (252).$$

EXAMPLE

In computing the Present Value of Life Annuities.

To find the *present value* of an Annuity of \$1 on the life of a person aged 100, on the supposition that money is worth 5 per cent. at *compound interest.*

If the Annuitant live *one year*, \$1 will be paid at the end of the year.

The *present value* of \$1 payable in 1 year is

$$\$1 \div 1.05 = \$0.95238 \ (226).$$

The *probability* that the Annuitant will live *one year*, is $\frac{7}{9}$ (261) ; the present value of the first payment on the Annuity, subject to the *contingency of its failure* from the decease of the Annuitant, is therefore

$$\$0.95238 \times \frac{7}{9} = \$0.74074' \ (258).$$

If the Annuitant live *two years*, \$1 will also be paid at the end of the 2 years.

The present value of \$1 payable in 2 years, is

$$\$1 \div 1.1025 = \$0.90702.$$

The probability that he will live *two years* is $\frac{5}{9}$ (261); the present value of the second payment on the Annuity, subject to the *contingency of its failure*, is therefore

$$\$0.90702 \times \tfrac{5}{9} = \$0.50390'.$$

If the Annuitant live *three years*, $1 will also be paid at the end of the 3 years.

The present value of $1 payable in 3 years, is

$$\$1 \div 1.15762 = \$0.86383.$$

The probability that he will live *three years* is $\frac{3}{9}$ (261); the present value of the third payment on the Annuity, subject to the contingency of its failure, is therefore

$$\$0.86383 \times \tfrac{3}{9} \doteq \$0.28794'.$$

If the Annuitant live *four years*, $1 will also be paid at the end of the 4 years.

The present value of $1 payable in 4 years, is

$$\$1 \div 1.2155 = \$0.82270'.$$

The probability that he will live four years is $\frac{1}{9}$ (261); the present value of the fourth payment on the Annuity, subject to the contingency of its failure, is therefore

$$\$0.82270 \times \tfrac{1}{9} = \$0.09141.$$

There would be no more payments on this Annuity, according to our Table of Mortality; hence the entire present value of all the payments, according to the probabilities that these payments would be realized, is

$$\$0.74074' + .5039 + .28794 + .09141 = \$1.62399.$$

The *Present Value* of an Annuity of $1 on any given life, *multiplied by any other Annuity*, gives the present value of that annuity on the same life.

(262.) The Present Value of an Annuity of $1 on any given life, is equal to (the present value of an annuity of $1 on a life *one year older* + 1) × the present value of $1 payable in *one year*, × the *probability* of the given life's continuing *one year*.

To find the Present Value of an Annuity of $1 on a life at the age of 99, its value on a life at 100 having already been found to be $1.62399.

According to the preceding proposition, the required value is ($.162399 + 1) × · 95238 × $\frac{9}{11}$ = 2.04466'.

To show the correctness of this method, we remark,

First. $1.62399 multiplied by · 95238, the present value of $1 payable in *one year*, produces the *present value* of the value $1.62399 of the Annuity *after the age of* 100.

But the value thus obtained depends on the Annuitant's living *one year*, the probability of which is $\frac{9}{11}$ (261) ; hence, if we multiply again by $\frac{9}{11}$, we find the *present value of the Annuity estimated from the age of* 100 *onwards* (258).

Secondly. $1 × · 95238 is the present value of $1 payable in *one year ;* and this multiplied by the probability $\frac{9}{11}$ of the Annuitant's living one year, gives the *present value of the Annuity from the age of* 99 *to that of* 100.

The *sum of the present values* of the Annuity for the two periods into which its duration is divided, as above, is the entire Present Value of the Annuity ; and this sum results from the operation above indicated.

From the Present Value of an Annuity on a life at 99, may be found, in like manner, its present value on a life at 98 ; and so on. In this way may be computed the Table on the following page.

TABLE

Showing the Present Value of an Annuity of $1, at 4 or 5 per cent., compound interest—on a Single Life, according to the Carlisle Table of Mortality.

AGE.	4 P. C.	5 P. C.	AGE.	4 P. C.	5 P. C.	AGE.	4 P. C.	5 P. C.
1	16.556	13.995	34	16.219	14.260	67	7.700	7.227
2	17.728	14.983	35	16.041	14.127	68	7.380	6.941
3	18.717	15.824	36	15.856	13.987	69	7.049	6.643
4	19.233	16.271	37	15.666	13.843	70	6.709	6.336
5	19.592	16.590	38	15.471	13.695	71	6.358	6.015
6	19.747	16.735	39	15.272	13.542	72	6.026	5.711
7	19.790	16.790	40	15.074	13.390	73	5.725	5.435
8	19.766	16.786	41	14.883	13.245	74	5.458	5.190
9	19.693	16.742	42	14.694	13.101	75	5.239	4.989
10	19.585	16.669	43	14.505	12.957	76	5.024	4.792
11	19.460	16.581	44	14.308	12.806	77	4.825	4.609
12	19.336	16.494	45	14.104	12.648	78	4.622	4.422
13	19.210	16.406	46	13.889	12.480	79	4.394	4.210
14	19.082	16.316	47	13.662	12.301	80	4.183	4.015
15	18.956	16.227	48	13.419	12.107	81	3.953	3.799
16	18.837	16.144	49	13.153	11.892	82	3.746	3.606
17	18.723	16.066	50	12.869	11.660	83	3.534	3.406
18	18.608	15.987	51	12.566	11.410	84	3.329	3.211
19	18.488	15.904	52	12.258	11.154	85	3.115	3.009
20	18.363	15.817	53	11.945	10.892	86	2.928	2.830
21	18.233	15.726	54	11.627	10.624	87	2.776	2.685
22	18.095	15.628	55	11.300	10.347	88	2.683	2.597
23	17.951	15.525	56	10.966	10.063	89	2.577	2.495
24	17.801	15.417	57	10.625	9.771	90	2.416	2.339
25	17.645	15.303	58	10.286	9.478	91	2.398	2.321
26	17.486	15.187	59	9.963	9.199	92	2.492	2.412
27	17.320	15.065	60	9.663	8.940	93	2.600	2.518
28	17.154	14.942	61	9.398	8.712	94	2.650	2.569
29	16.997	14.827	62	9.137	8.487	95	2.674	2.596
30	16.852	14.723	63	8.872	8.258	96	2.628	2.555
31	16.705	14.617	64	8.593	8.016	97	2.492	2.428
32	16.552	14.506	65	8.307	7.765	98	2.332	2.278
33	16.390	14.387	66	8.010	7.503	99	2.087	2.045

Annuities on Joint Lives.

(263.) The Present Value of an Annuity which depends on the continuance of *both* of two lives, is estimated according to the *probabilities* that both lives will continue *one*, *two*, *three*, &c., *years*; and in like manner for an Annuity which depends on *three* or more lives.

EXAMPLE.

To find the Present Value of an Annuity of \$1, which depends on the continuance of two lives, one at the age of 90, and the other at the age of 95—at 5 per cent., compound interest.

If *both* lives continue *one year*, \$1 will be paid at the end of the year. The present value of \$1 payable in one year, is

$$\$1 \div 1.05 = \$0.95238 \ (226).$$

The probability that the *first* life will continue one year, is $\frac{105}{142}$, and the probability that the *second* life will continue one year, is $\frac{23}{30}$ (261); then the probability that *both* lives will continue one year, is

$$\tfrac{105}{142} \times \tfrac{23}{30} \ (256).$$

The present value, therefore, of the first payment on the Annuity, subject to the *contingency of its failure* from the decease of one or both of the lives, is

$$\$0.95238 \times \tfrac{105}{142} \times \tfrac{23}{30} = \$0.5399'.$$

In the same way we might compute the present values of the 2d, 3d, &c., payments, to the number of *nine*, which brings the older life to the *limit of mortality*, according to our Table, when the Annuity would cease.

The sum of all these present values would be the entire Present Value of the Annuity.

Survivorship, or an Annuity on the Survivor of two or more Lives.

(264.) The Present Value of an Annuity which depends on the continuance of *either* of two lives, is estimated according to the *improbabilities* that both lives will *fail* before the end of *one, two, three, &c., years;* and in like manner for an Annuity which depends on a survivorship among *three* or more lives.

EXAMPLE.

To find the Present Value of an Annuity of \$1, which is to continue so long as either of two lives, one at the age of 90, the other at the age of 95, shall survive,—allowing compound interest at 5 per cent.

If *either* of the two lives continue *one year*, \$1 will be paid at the end of the year.

The present value of \$1 payable in one year, is

$$\$1 \div 1.05 = \$0.95238' \ (226).$$

The probability that the first life will *not continue* to the end of one year, is $1 - \frac{105}{142}$, and the probability that the second will *not continue* to the end of one year is $1 - \frac{23}{30}$ (254); then the probability that *both* will not continue to the end of one year, is

$$(1 - \tfrac{105}{142}) \times (1 - \tfrac{23}{30}), \cdots (256)$$

Consequently the probability that both lives will *not fail* before the end of one year, which is the same as the probability that the first payment on the Annuity will be *realized*, is

$$1 - \overline{(1 - \tfrac{105}{142}) \times (1 - \tfrac{23}{30})} \cdots \cdot (254).$$

This measure of probability multiplied into \$0.95238 will give the present value of the first payment on the Annuity, subject to the contingency of its failure.

In like manner may be computed the present values of the 2d, 3d, &c., payments, to the number of *nine*—when, according to our Table of Mortality, the older life must be dropped; and the Annuity must thenceforth be considered with reference to the probabilities of the younger life's continuing to the same *limit of mortality*.

LIFE INSURANCE.

(265). Life Insurance is an obligation assumed, usually by an incorporated company, to pay on the *decease* of the person on whose life the insurance is effected, a certain sum to the one for whose benefit it is effected.

The Insurance may embrace the *whole life*, or be limited to a given number of years. In the latter case the obligation of the Company will not exist unless the life insured shall fail *within the given number of years.*

The Premium for Life Insurance is usually in *annual* payments, the first of which is paid *in advance;* and its proportional amount is computed with reference to,

The *probabilities* that the life insured will survive *one, two, three*, &c., years; and, the *rate* of Interest at which the Premiums paid may be invested.

Rate of Interest in Life Insurance.

(266.) The Rate of Interest at which money may be invested for the periods of time embraced in the transactions of Life Insurance, cannot be certainly determined.

The *higher* is the assumed Rate of Interest, the *lower* will the computed Premiums be, and conversely. If the assumed rate be not realized in the future experience of the Company, the means will be wanting of paying the *sums insured.*

In England, where the legal rate of interest is 5 per cent., it is considered unsafe to transact life insurance at more than 3 or 4 per cent. interest; in the United States, where money produces a higher profit, the rate assumed might probably be 5 *per cent.*

Present Value of a Sum Insured.

(267.) The Present Value of $1 *insured on a given life*, will be found by subtracting the Present Value of an Annuity of $1 on the same life from the Present Value of a *perpetuity* of $1, and dividing the remainder by 1 + the *present value of the perpetuity.*

EXAMPLE.

To find the Present Value of $1 payable at the end of the year in which a person now 50 years old may die, allowing interest at 5 per cent.

The present value of an Annuity of $1 on the given life, according to the Table on page 318, is $11.66; and the Present Value of a Perpetuity, or perpetual annuity, of $1, is

$$1 \div \cdot 05 = \$20 \ (232).$$

Then the present value of $1 to be paid at the end of the year in which the given life may fail, is

$$\frac{\$20 - 11.66}{1 + 20} = \frac{\$8.34}{21} = \$0.397143.$$

To show the correctness of this method, we remark,

First, The present value of the Annuity subtracted from the present value of the Perpetuity, leaves the value $8.34 of a *perpetuity of* $1, commencing with the payment of $1 at the end of the year in which the *given life may fail.*

Secondly. The divisor 1 + 20 is the present value of a *perpetuity* of \$1, commencing with the *present payment of* \$1.

The ratio of the \$8.34 to the \$21 must be the same as the ratio of \$1, payable at the end of the year in which the given life may fail, to \$1.

Hence the *quotient* found, which is the value of this ratio, must be the present value of \$1 payable at the end of the year in which the given life may fail.

The Present Value of \$100 payable as in the Example, would be .397143 × 100 = \$39.7143 ; of 1000, \$397.143 ; and so on.

Premiums in Life Insurance.

(268.) The Annual Premium which, irrespective of *expense* or *profit* to the Insurance Company, should be given for a sum to be paid on the *failing of a given* life, is equal to the *present value* of that sum, divided by a *unit* + the present value of an Annuity of \$1 on the same life.

EXAMPLE.

To find the annual Premium which, irrespective of expense or profit to the Insurance Company, should be given for \$1000 to be paid at the decease of a person now 50 years old, allowing interest at 5 per cent.

The present value of \$1000 to be paid at the end of the year in which the given life may fail, as shown under the former Example, is

\$397.143.

The Premiums *after the first* being payable in *one, two, three,* &c., years, their present value is the same as the present value of an equal Annuity on the life of the person.

The present value of an Annuity of $1 on the life of a person, at the age of 50, according to the Table on page 318, is 11.66; and the present value of an Annuity equal to the annual Premium, is therefore

$11.66 × the Premium.

By adding the *first* Premium to the present value of the succeeding ones, we shall have the present value of *all the premiums* which must be equal to the present value of the *sum insured.*

Hence the *premium* + the *premium* × 11.66 = $397.143,

or, the *premium* × (1 + 11.66) = 397.143;

which gives the *premium* = $397.143 ÷ 12.66 = $31.369.

Insurance on Joint Lives.

(269.) In a *joint insurance* on two lives, the obligation of the Company is, to pay the sum insured as soon as either of the two lives fails.

The Present Value of the sum insured in this case, would be found from an Annuity on the joint continuance of the two lives (263), in the same manner as for an insurance on a single life (267); and from this Present Value the Annual Premium may be computed as for a single life.

Temporary Life Insurance.

(270.) In a *temporary* Life Insurance, the obligation of the Company is, to pay the sum insured on the failing of the given life, *provided it shall fail within the period* comprehended by the insurance.

The Present Value of a *temporary* Life Insurance may be found from that of an insurance on the whole life—as in the following

EXAMPLE.

To find the Present Value of $1000 to be paid on the decease of a person 50 years old, provided he die within 5 years.

The present value of $1000 insured on the whole of the given life, heretofore found, is $397.143, (267).

The present value of $1000 insured on a life 55 years old, found by the same method, is $459.666.

$1000 insured on the whole of the given life would therefore be worth $459.666 at the end of 5 years; and the *present value* of this is

$$\$459.666 \div 1.27628 = \$360.161 \ (226).$$

The *probability* that the given life will survive 5 years, is $\frac{4073}{4397}$ (261); and therefore the present value of $1000 insured on the whole of the given life, *if that life survive 5 years*, is

$$\$360.161 \times \tfrac{4073}{4397} = \$333.621.$$

Then the Present Value of $1000 to be paid on the decease of the given life, *if that life fail within 5 years*, must be

$$\$397.143 - \$333.621 = \$63.522.$$

(271.) The Annual Premium which, irrespective of *expense* or *profit* to the Company, should be paid for a *temporary* Life Insurance, may be found by dividing the Present Value of the sum insured by a *unit* + the present value of an Annuity of $1 on the given life *for one year less than the period of insurance.*

Thus, to recur to the preceding Example, observe that the first Premium must be paid immediately, and that the four remaining premiums are equal to an Annuity of the same amount, on the same life, for 4 years.

The value of an Annuity of \$1 on the given life 4 years hence, that is, at the age of 54, according to the Table on page 318, at 5 per cent, would be

$$\$10.624.$$

The present value of this sum payable in 4 years, is

$$\$10.624 \div 1.2155 = \$8.74.$$

The probability that the given life will survive 4 years, is $\frac{4143}{4397}$ (261); and hence the present value of the Annuity, subject to the contingency of its continuing 4 years, is

$$\$8.74 \times \frac{4143}{4397} = \$8.235.$$

The present value of an Annuity of \$1 on the whole of the given life, is \$11.66; consequently the value of the *temporary* annuity on the given life for 4 years, is

$$\$11.66 - \$8.235 = \$3.425.$$

The Annual Premium is then the Present Value of the sum insured, as found in the preceding Example, divided by $1 + 3.425$; that is,

$$\$63.522 \div 4.425 = \$14.355.$$

Different allowances for *expenses* and *profits* on Capital employed, will be made by different Insurance Companies; and hence will arise differences in their *rates* or *premiums* for Insurance, with the same rate of interest assumed.

Mutual Insurance.

(272.) In Mutual Life Insurance, the *funds* of the Company consist wholly or chiefly of the Premiums paid; and any *surplus* which remains after the obligations and safety of the Company are provided for, are distributed, or credited, from time to time, among the *insured*, or *policy holders*.

By this method no injustice will result to the insured from *excessive premiums*.

TABLE

Of Premiums for Life Insurance charged by the "MUTUAL LIFE INSURANCE COMPANY OF NEW YORK."

ANNUAL PAYMENT FOR THE INSURANCE OF $1000.

AGE.	FOR LIFE.	FIVE YEARS.	AGE.	FOR LIFE.	FIVE YEARS.	AGE.	FOR LIFE.	FIVE YEARS.
14	$14.71	$7.60	28	$21.70	$11.31	42	$34.05	$17.26
15	15.11	7.82	29	22.35	11.64	43	35.30	17.89
16	15.52	8.05	30	23.02	11.98	44	36.63	18.58
17	15.94	8.28	31	23.73	12.33	45	38.04	19.36
18	16.38	8.52	32	24.47	12.69	46	39.53	20.22
19	16.83	8.77	33	25.23	13.07	47	41.11	21.17
20	17.30	9.02	34	26.03	13.46	48	42.78	22.20
21	17.78	9.28	35	26.87	13.86	49	44.55	23.30
22	18.28	9.55	36	27.75	14.28	50	46.42	24.48
23	18.80	9.82	37	28.67	14.71	51	48.39	25.74
24	19.34	10.10	38	29.64	15.17	52	50.49	27.09
25	19.89	10.38	39	30.66	15.65	53	52.71	28.56
26	20.47	10.68	40	31.73	16.15	54	55.07	30.16
27	21.07	10.99	41	32.86	16.69	55	57.58	31.90

The preceding Table exhibits the sums which are to be annually paid to the Company, during the whole of the life on which Insurance is taken, for $1000 to be paid whenever that life shall fail; or during *five years*, for $1000 to be paid provided that life shall fail *within the five years*.

Thus the Annual Premium, *for life*, for $1000 insured, on a life at 50 years, is $46.42; and, for *five years*, it is $24.48.

By computations heretofore made, allowing Interest at 5 per cent., and allowing nothing for *expenses* or *profits* to the Company, the first of these two premiums would be $31.369; and the second would be $14.355.

The expenses of the Company are such as rents of offices, compensation to officers and agents, &c.; and unless good provision be made for these expenses, as well as for meeting their obligations to the insured, the Company must become insolvent.

As an interesting subject of reflection, though not necessary in the calculations of Life Insurance, we give here the theory of what is called the

Expectation of Life.

(273.) By the *Expectation of Life* is meant the *average* number of years remaining to a person at a given age, according to an ascertained *rate of mortality;* and is equal to the *sum* of the probabilities that the person will live over *one*, *two*, *three*, &c., years, increased by $\frac{1}{2}$.

EXAMPLE.

To compute the Expectation of Life of a person at the age of 100.

According to the *Carlisle Table of Mortality*, of 9 persons at the age of 100, only 7 live through the 1st succeeding year.

With respect to the 1st year, therefore, the 9 persons have an expectation, in the aggregate, of 7 years of life, which gives to each $\frac{7}{9}$ of a year; and $\frac{7}{9}$ is the probability that any one of the *nine* will live *one year*, (261).

Again, of the same 9 persons, only 5 live through the 2d succeeding year. Hence with respect to the 2d year, the 9 persons have an expectation of 5 years of life, which gives to each an expectation of $\frac{5}{9}$ of a year; and $\frac{5}{9}$ is the probability that any one of the *nine* will live *two years*.

In like manner it will be found that each of the *nine* persons has an expectation of $\frac{3}{9}$ for the 3d, and of $\frac{1}{9}$ for the 4th year.

In the 5th year the last of the *nine* persons dies; and if the probabilities were only that the several lives would fail at the *terminations* of the successive years; we should have for the *Expectation of Life* of a person at the age of 100, the sum of the *partial expectations* found above, viz.:

$$\frac{7}{9}+\frac{5}{9}+\frac{3}{9}+\frac{1}{9}.$$

But any one of these lives may fail *in the course of* any one of the five years; and the probability that it will fail in the *course of some one* of them, if it do not at the *termination* of a year, amounts to a certainty. And since the probabilities of the life's failing are about the same for every time in the year; we can only balance these probabilities by supposing it to fail at the *middle* of the year. This supposition adds $\frac{1}{2}$ of a year to the Expectation found above.

We have then for the whole Expectation of Life at the age of 100,

$$\frac{7}{9}+\frac{5}{9}+\frac{3}{9}+\frac{1}{9}+\frac{1}{2}=\frac{41}{18}=2.277' \text{ years.}$$

In this way has been computed the following

TABLE.

Showing the Expectation of Life, according to the Carlisle Table of Mortality.

Age.	Expectation in years and 100ths.	Age.	Expectation in years and 100ths.	Age.	Expectation in years and 100ths.	Age.	Expectation in years and 100ths.	Age.	Expectation in years and 100ths.
Birth	38.72	21	40.75	42	26.34	63	12.81	84	4.39
1	44.68	22	40.04	43	25.71	64	12.30	85	4.12
2	47.55	23	39.31	44	25.09	65	11.79	86	3.90
3	49.82	24	38.59	45	24.46	66	11.27	87	3.71
4	50.76	25	37.86	46	23.82	67	10.75	88	3.59
5	51.25	26	37.14	47	23.17	68	10.23	89	3.47
6	51.17	27	36.41	48	22.51	69	9.70	90	3.28
7	50.80	28	35.69	49	21.81	70	9.18	91	3.26
8	50.24	29	35.00	50	21.11	71	8.65	92	3.37
9	49.57	30	34.34	51	20.39	72	8.16	93	3.48
10	48.82	31	33.68	52	19.68	73	7.72	94	3.53
11	48.04	32	33·03	53	18.97	74	7.33	95	3.53
12	47.27	33	32.36	54	18.28	75	7.01	96	3.46
13	46.51	34	31.68	55	17.58	76	6.69	97	3.28
14	45.75	35	31.00	56	16.89	77	6.40	98	3.07
15	45.00	36	30.32	57	16.21	78	6.12	99	2.77
16	44.27	37	29.64	58	15.55	79	5.80	100	2.28
17	43.57	38	28.96	59	14.92	80	5.51	101	1.79
18	42.87	39	28.28	60	14.34	81	5.21	102	1.30
19	42.17	40	27.61	61	13.82	82	4.93	103	0.83
20	41.46	41	26.97	62	13.31	83	4.65	104	0.50

This Table shows that at *birth* the *expectation of life* is 38.72 years; at the age of *one year* it is 44.68 years; and so on, being greatest at *seven* years. At the age of 92 the expectation would seem to increase, and to decrease again at 96; and to be the same at the ages of 90 and 97. These last particulars are quite anomalous.

APPENDIX.

STANDARD MEASURES AND WEIGHTS.—FRENCH MEASURES.

The Measures and Weights of a country are regulated by *Law*; Government provides the *standards*, and requires that the measures and weights employed in trade shall conform to them.

The English standards have, with little variation, been adopted in the United States; and the entire system of measures and weights in these countries is, by Law, referred, ultimately, to the standard *yard*.

The English *yard* is said to have been at first adopted from the length of the arm of King Henry VII.

The old English or Saxon *pound* was derived from the weight of *grains* of wheat; 32 grains taken from the middle of the ear, and well dried, made a *pennyweight*; 20 pennyweights made an *ounce*, and 12 ounces a *pound*. This pound was afterwards altered to the *Troy* pound, which was $\frac{1}{16}$ heavier than the Saxon pound; and the Troy penny-weight was divided into 24 grains—one of these grains thus becoming heavier than a grain of wheat.

The Troy *ounce* was divided by Apothecaries into *drams*, drams into *scruples*, and scruples into *grains* in compounding their drugs.

The *Avoirdupois* pound was introduced for weighing butcher's meat in the market; but gradually came to be used for all coarse commodities.

The standards of these measures and weights was definitely fixed, by Act of Parliament, in the year 1824. By that act the Imperial or standard Measures and weights of England are the following :

1. The Imperial *yard* (referred now to an invariable standard in nature) is defined to be, in length, to a pendulum vibrating *seconds*, at the level of the sea, in the latitude of London, as 36 to 39.1393, or 1 to 1.087'.

2. The Imperial *Troy Pound* consists of 12 ounces, each ounce of 20 pennyweights, each pennyweight of 24 grains; and the weight of *one grain* is defined to be such that a *cubic inch* of water, at the temperature of 62° Fahrenheit, when the barometer is at 30 inches, shall weigh 252.458 grains.

3. The Imperial *Avoirdupois Pound* is the weight of 7000 grains.

4. The Imperial *Gallon* contains 10 pounds, avoirdupois, of distilled water, at the temperature of 62° Fahrenheit, when the barometer is at 30 inches; and is equivalent to 277.274 cubic inches.

5. The Imperial *Bushel* contains 8 Imperial Gallons, or 2218.192 cubic inches.

The Constitution of the United States empowers Congress "To coin money, regulate the value thereof, and of foreign coin, and fix the *standard of weights and measures.*" This power has been exercised, as to weights and measures, only so far as to establish (in the year 1836) standards for the Custom Houses, and to cause copies of the same to be delivered to the Governor of each State, with a view to uniformity throughout the United States.

These standards are,

1. The *yard* of 3 feet, or 36 inches, from a brass scale of

82 inches, made by Troughton, an English artist; and is the same as the standard of England.

2. The Troy and Avoirdupois *Pounds*, the same as those of England.

3. The *Gallon*, containing 231 cubic inches, the same as the old wine gallon of England.

4. The *Bushel*, containing 2150.4 cubic inches, the same as the English *Winchester* Bushel. The bushel, in the form of a cylinder, is 18½ inches in diameter, and 8 inches deep.

The old Wine and Beer Measures are not recognized among the legal standards of England, nor Beer Measure by the laws of the United States. They continue in use, however, to some extent, it is said, in the former country, and the latter in this country, without the regulation of law.

Prior to the proceedings of Congress having reference to a uniformity of Weights and Measures in this country, the individual States were under the necessity of establishing a system, each for itself; and these State systems, in some instances, are still in force, with slight variations from the national system, and from each other.

Thus in the State of New York, which has been followed by Ohio, the standard *yard* (said to be the same that was in use in New York at the Declaration of American Independence) is defined to be, in length, to a pendulum vibrating seconds, in a vacuum, in Columbia College, as 1 to 1.086'; the standard of weight is the Avoirdupois Pound, such that a cubic foot of distilled water, at the maximum density, in a vacuum, weighs 62½ pounds; the standard *liquid* Gallon contains 8 pounds, and the *dry* Gallon 10 pounds, of distilled water, at the maximum density, at the level of the sea, under the mean pressure of the atmosphere.

The national standards, it is said, have been furnished to all the States; but they have not been adopted and introduced by all; and thus a perfect uniformity in Weights and Measures does not prevail in the United States.

This want of exact uniformity, however, is not so great an inconvenience as are the different scales of *units* in the Tables of Weights and Measures (102), (103), &c. These Tables would seem to have been the result of caprice or accident. By not having been conformed to the *decimal scale* of numeration, as was done in the case of Federal Money, they greatly complicate many of the practical applications of Arithmetic.

The inconveniences just mentioned, as belonging to the English and American metrical systems, are obviated in that of the French, in which the decimal scale has been adopted.

FRENCH MEASURES AND WEIGHTS.

The standard of measure in the French system is the distance, on a meridian, from the *equator* to the *pole* of the Earth. This distance having been determined (by methods which cannot here be explained), the *ten-millionth* part of it is assumed for the *standard unit* in linear measure, and is denominated the *Metre.*

The *Metre* is equal to 39.371' English or American *inches.*

The *Metre* is divided into 10 *decimetres;* the Decimetre into 10 *centimetres;* the Centimetre into 10 *millimetres.*

10 metres	make	1 decametre;
10 decametres	"	1 hectometre;
10 hectrometres	"	1 kilometre;
10 kilometres	"	1 myriametre.

The unit of *square* measure is the *Are*, which is a square *decametre*. Its subdivisions, decreasing in a decimal ratio, are *deciares*, *centiares*, and *milliares*.

10 ares	make	1	decare;
10 decares	"	1	hectare;
10 hectares	"	1	kilare;
10 kilares	"	1	myriare.

The unit of *solid* measure is the *Stere*, which is a cubic *metre*. Its subdivisions, decreasing in a decimal ratio, are *decisteres* and *centisteres*.

10 steres make 1 decastere.

The unit of measures of *capacity* is the *Litre*, which is a cubic *decimeter*. The litre is divided into 10 *decilitres*, beyond which there appear to be no subdivisions.

10 litres	make	1	decalitre;
10 decalitres	"	1	hectolitre;
10 hectolitres	"	1	kilolitre.

The unit of *weight* is the *Gramme*, which is the weight of a cubic *centimeter* of water, at its maximum density, and is about 15.434 *grains*, troy weight. Its subdivisions, decreasing in a decimal ratio, are *decigrammes*, *centigrammes*, and *milligrammes*.

10 grammes	make	1	decagramme;
10 decagrammes	"	1	hectogramme;
10 hectogrammes	"	1	kilogramme;
10 kilogrammes	"	1	myriagramme.

In the preceding Tables, the *divisions* and *subdivisions* of the standard unit are denoted by means of the *Latin* prefixes, *deci*, *centi*, *milli*; thus decimetre $\frac{1}{10}$ of a metre; centimetre, $\frac{1}{100}$ of a metre; millimetre, $\frac{1}{1000}$ of a metre. The *multiples* of the standard unit are expressed by means of the *Greek* prefixes, *deca*, *hecto*, *kilo*, *myria*; thus decametre, 10 metres; hectometre, 100 metres, &c.

This system of Measures and Weights is universally admitted to be the best that has ever been devised; and is used, to some extent, particularly for scientific purposes, by other nations than the French, with whom it originated.

"No system of metrology hitherto invented can be compared with this of the French in a scientific point of view; nevertheless the decimal subdivisions have been found unsuited to the purposes of retail traffic, to which, in fact, only a binary system, or the division of the unit into halves and quarters, seems applicable. Accordingly, it has been found necessary to permit a modified system for each purpose; so that there are, in fact, at present in France three different systems of measures; the ancient, which was never wholly abandoned; the decimal system; and a binary system, or *système usuel*, having the decimal standards for its basis, with binary divisions, to which the names of the ancient weights and measures are given, the word *usuel* being annexed to prevent confusion."

Brande's Encyclopedia.