THE APPLETON SCHOOL BOOKS

ARITHMETIC

BY

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BOOK II

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PREFACE

1. **The dominating thought** in the preparation of this series has been that not only is it one of the chief functions of the teaching of arithmetic to lead the child to think, but also that this thinking should usually relate to concrete materials adapted to his comprehension and interest. The controlling method is therefore inductive rather than deductive; concrete rather than abstract.

2. **The framework** around which the subject is built consists of the essential processes of arithmetic, which the child must learn to perform with accuracy, speed and intelligence. The authors have followed what they believe to be the most natural path to the accomplishment of these ends, *viz.*, that leading *from the concrete to the abstract.*

3. To be **really concrete** it is not sufficient to use named units instead of unnamed ones. If 379 + 486 is abstract, 379 bu. + 486 bu. is no less so. Data for problems have therefore often been sought in the child's own activities—weighing, measuring, observing, drawing, building, inquiring, reading.

4. While **motor activity** is emphasized, the manual work is not pushed to an extreme. No special material is required. Optional work is occasionally suggested, making use of simple materials and instruments easily procured or improvised.

5. **Exercises for abstract drill** have been included at suitable points. The teacher can readily dictate similar exercises if more are needed to secure the requisite practice.

6. **Related problems.** Usually the problems of each page are grouped about a *central thought*, so as to secure a certain unity in the subject-matter. The reviews are, of course, miscellaneous.

7. **Form study** thoroughly graded is made an integral part of the work.

8. **The commercial applications** receive due attention without being given exaggerated importance. Reduced *photographic fac-similes* of a number of business forms are given.

9. **Much information** is incidentally included in the problems.

10. **The problems are natural.** Factitious and unreal problems have been excluded. (E. g., "How many seconds in 8,372 mo.? How many oz. Troy in a long ton avoirdupois? Divide $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{9}{16}$ of $\frac{14}{25}$ by $\frac{5}{8}$ of $\frac{3}{14}$ of $\frac{8}{15}$ of $\frac{35}{36}$.") The child's environment offers a wealth of material for problems relative to real conditions, more than ample for the development and mastery of all needed processes of arithmetic, and for the attainment of all the disciplinary benefits of its study.

11. **Obsolete** processes or types of problems have been omitted. The numerical needs of modern life are so many and so varied that no time can be spared for the antiquated or the artificial.

12. **Small numbers** are generally used. Every operation of arithmetic, however complex, consists of a succession of operations with very small numbers. If these are first thoroughly mastered, their repetition in longer problems offers no difficulty. Large numbers are not prematurely forced on the child, but, when prepared for them, he is led to them gradually and naturally in problems relating to actual conditions.

13. **Much oral work** precedes the written work in each topic.

14. **Frequent reviews** and occasional **general reviews** are given.

15. The **order of topics has** been determined to meet the needs of the child-mind. Its craving for variety and change is healthy and normal and must be heeded, but it can not be properly satisfied by a disjointed and fragmentary treatment. Nor, on the other hand, may the child-mind be forced to follow the practice of highly trained adult-minds in the complete and final treatment of each topic at one time. In due course, however, each leading subject is made the center of instruction, and its principles are summarized and emphasized.

16. The need to **teach through the eye** has been constantly borne in mind. The illustrations are, however, never simply pictorial, but always *illustrative* and usually integral parts of the text.

THE AUTHORS.

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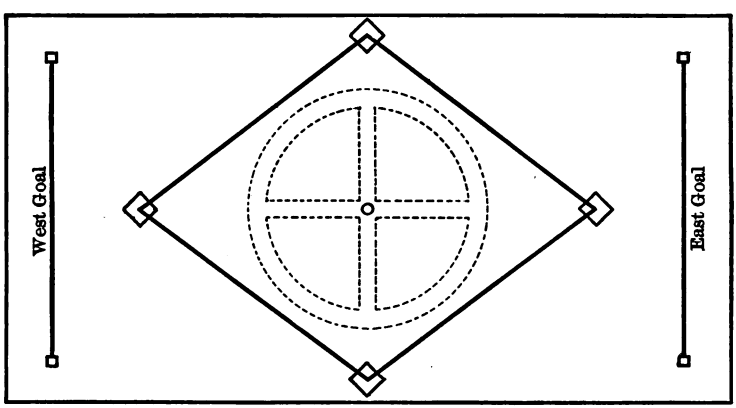
BOOK II

REVIEW AND PREPARATION

1. If an inch in a drawing represents 40 ft., how many feet does $\frac{1}{4}$ of an inch represent? $\frac{1}{2}$ of an inch? 3 inches? $3\frac{1}{2}$ inches?

2. In such a drawing what length represents 20 feet? 10 ft.? 30 ft.? 60 ft.? 75 ft.?



In the drawing of the playground 1 inch represents 20 ft.

3. Find the length of each side. What is the distance around the playground?

4. What is the length of each goal? What is the distance between the goals? How far are they from the ends of the playground? Measure other distances.

5. What is the area of the playground?

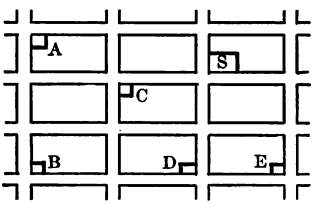
1. If your school has a playground estimate or measure its length and breadth. Select a convenient scale and make a drawing of the playground. This is called **drawing to scale.**

2. Estimate or measure the length and breadth of the school-room. Make to scale a drawing of the room.

3. Find the size and position of the desks and locate them in your drawing; do the same for windows and doors.

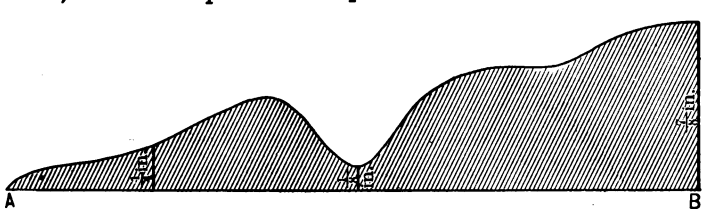
4. In your village or city what is the length of a block? How many blocks are there to the mile?

5. If there are 8 blocks to the mile and 1 inch in a drawing represents a block, how many inches represent $\frac{1}{2}$ of a mile?



6. Find the distances of the houses of several pupils from the school, counting parts of blocks as whole blocks, and using scale 8 long blocks or 12 short blocks to the mile.

7. The hill represented in the picture has been cut to a level, A B, for a railway. If one inch in the picture represents 80 ft., find the depths at the places indicated.



8. What is the length of the line A B, letting 1 in. represent 80 ft.?

9. Make a picture of a railway cut differing from the one shown in the figure. Select a scale and measure its depths at various places.

1. A certain post-office has 4 mail-carriers and each has a route with 1,000 addresses. If the delivery averages 1 piece to every fifth person, how many pieces does each carrier distribute at one delivery? How many pieces do the 4 carriers distribute?

2. There are two deliveries a day; how many pieces of incoming mail are handled for delivery by the post-office per day? If there is the same amount of outgoing mail, what is the total number of pieces handled?

3. If the postage averages 2 cents per piece, how many dollars' worth of stamps are canceled in a day?

4. 15 money-orders were sold for which the post-office received $2.25; also 480 stamped envelopes at 26¢ a dozen; 300 single postal cards and 200 with return card attached. How much did the office receive from these sources?

5. The four carriers are paid at the rate of $2 a day, the postmaster at the rate of $5, and 2 clerks at the rate of $1.75 each; what does the labor in this post-office cost the government per month of thirty days?

6. There are 120 lock-boxes in a post-office, $\frac{4}{5}$ of which are rented at 25¢ a month each; what is the income from these boxes in a year?

7. Make and solve 3 problems, using the data of the table:

Salaries per Year.		Receipts.	
Postmaster	$2,000	400 boxes	$2 each
Assistant	1,500	Stamps	5,000
2 clerks	800 each	Money-orders	500
4 carriers	725 each	Cards	400

8. Make problems about the postal service in your city.

1. The usual weight of a bale of hops is 200 lb.; how many bales are there in 1 ton? In a car-load of 10 tons?

2. 2,000 roots are required to plant an acre of hops; how many are required to plant 25 acres? What is the cost of planting 25 acres at $1 per 1,000 plants?

3. The wire trellis to support the vines costs $75 an acre; what is the cost for 25 acres?

4. Picking the hops costs at the rate of 1¢ a pound and $3\frac{1}{2}$ lb. of green hops make 1 lb. of dry hops; what is the cost of picking enough to make 1 ton of dry hops?

5. One cord of willow wood costing $5 will dry 1,000 lb. of hops; according to Exercise 1, what is the cost of enough wood to dry 100 bales?

6. The following are the ingredients of a fertilizer for vineyards: phosphate, ground bone, and potash in equal amounts. If an acre requires 900 lb. of fertilizer, how many pounds of each ingredient are used on an acre?

7. How many tons of such fertilizer are needed for a 5-acre vineyard?

8. A fertilizer for growing strawberries consists of 1 part nitrate of soda, 2 parts phosphate, 2 parts ground bone, and 2 parts of potash; how many pounds of each ingredient are there in 700 lb. of fertilizer?

9. $1\frac{3}{4}$ tons of fertilizer will fertilize 5 acres of strawberries; how many pounds are used per acre?

10. If a cow gives 450 lb. of milk in a month and each 100 lb. contains 4 lb. of fat, how much fat is there in the month's product?

11. According to Exercise 10, how many pounds of milk would a dairy of 20 cows produce in a month? If the skimmed milk is $\frac{4}{5}$ of this amount, what is the weight of the month's product of cream?

1. Find the cost of the kit of tools given in this list:

KIT OF COMMON TOOLS.

Kinds.	Cost.	Kinds.	Cost.
One 20″ rip-saw	$1.60	One T-bevel-gage	$0.25
" back-saw	1.00	" pair compasses	0.20
" claw-hammer	0.40	" oilstone	0.50
" mallet	0.25	" oil-can	0.15
" steel square	0.80	" screw-driver	0.20
" 6-inch try-square	0.25	" bench-brush	0.30
" marking-gage	0.25		

2. Find the cost of the kit of tools given in this list:

KIT OF INDIVIDUAL TOOLS.

Kinds.	Cost.	Kinds.	Cost.
One cross-cut saw	$0.80	Two turning-chisels	$0.45
" jack-plane	0.60	One parting tool	0.40
" smoothing-plane	0.50	" round-nose tool	0.40
Four chisels	0.90	" pair calipers	0.25
Three gouges	0.70	" two-foot rule	0.15
Two turning-gouges	0.55	" oilstone slip	0.15

3. A manual-training room is to be furnished with benches for 12 boys; if each bench costs $3, what will the 12 cost?

4. Each bench is to be supplied with a kit of individual tools as given in Exercise 2; how much will the 12 kits cost?

5. The room is to be supplied with 4 kits of common tools; how much will they cost?

6. Each pupil supplies himself with a 15-cent apron and 25 cents' worth of material; how much do these things cost for the whole class?

7. If each pupil uses 10 ft. of lumber in a month, how much is used by the whole class in the same time?

8. Make and solve other problems about supplies for manual training work.

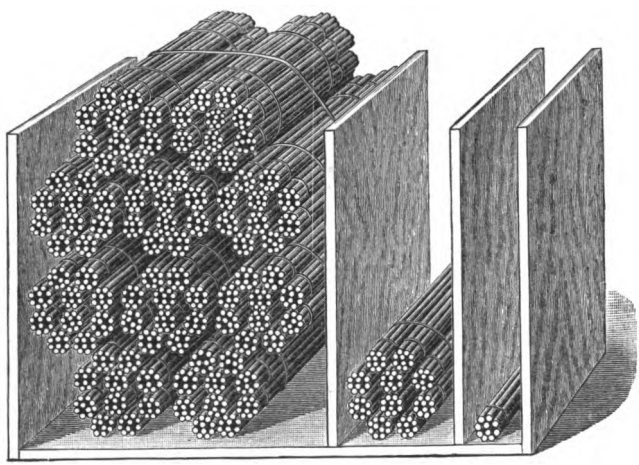
NOTATION AND NUMERATION

Ten-thousands

Oral.

The needs of business and science constantly call for numbers larger than those that can be expressed by thousands. In Book I the names and symbols for all integers to 10,000 have been given.

The method of naming or reading numbers is called **numeration** and that of writing, **notation.**



1. How many units make 1 ten?

2. How many tens make 1 hundred?

3. How many hundreds make 1 thousand?

4. What method of grouping is used in forming numbers to thousands?

Similarly, 10 thousands are grouped into 1 **ten-thousand.** The number of ten-thousands is written at the left, next to thousands' place.

5. Read: 20,000; 25,000; 35,000; 97,000.

10 ten-thousands are grouped into 1 hundred-thousand, and the number of hundred-thousands is written at the left, next to ten-thousands' place.

1. In 762,804 what figure stands in units' place? In tens' place? In hundreds' place? In thousands' place? In ten-thousands' place? In hundred-thousands' place?

2. Write 325,689 and above each figure write the name of its place.

3. What does the **position** of each figure in a number tell? What does its value tell?

For convenience, the figures are separated by commas into sets of three each as far as possible, beginning at the right. These sets are called **periods.**

In reading, the word "thousands" is omitted in the case of ten-thousands and of hundred-thousands. The whole period is read as if it were units' period and then the word "thousand" is added. Thus, in the number 762,804, the second period is read: "Seven hundred sixty-two thousand."

This is similar to what is done in speaking of a number of people, or houses, or dollars. We say, "Seven hundred sixty-two dollars," not "Seven hundred dollars, sixty dollars, two dollars."

Read:

4. 100,000. 5. 125,000. 6. 125,568.
7. 634,305. 8. 723,325. 9. 690,437.

10. Add the numbers in each of the above columns.

11. Read: In 1900 the mines of the world produced:

80,643 tons of tin. 787,841 tons of lead.
446,373 tons of zinc. 543,735 tons of copper.

12. The capacity of the car-ferryboat Père Marquette is 30 cars; if the weight per car is 42 tons, how many hundred pounds does the boat carry when loaded?

13. Write twenty-five thousand, five hundred fifteen.

1. Read the numbers in the table. The table shows the number of dollars earned by various railways during the third week of December for three consecutive years.

Gross Earnings.

Gross for the third week of December:

	1903.	1902.	1901.
Kana. & Mich.	25,451	25,520	15,643
Père Marq.....	199,934	206,784	152,870
Texas & Pac...	292,936	261,336	316,119
Tol. & O. Ct..	62,312	55,305	40,451
Wheel. & L. E.	66,107	67,605	57,692

2. Bring similar items cut or copied from newspapers and read the numbers in class.

Write in figures:

3. Five hundred five thousand, seventy-five.

4. Five hundred thousand, seven hundred twenty-five.

5. Five hundred seventy-five thousand, seven hundred five.

6. Mount Everest is 29,002 ft. high and Pike's Peak is 14,147 ft. high. Find the difference in their heights.

7. At the mines 2,240 lb. of coal is regarded as a ton; how many pounds are there in a car-load of coal weighing 40 tons at the mines?

8. The State of Colorado has the form of a rectangle. It is 380 mi. long and 273 mi. wide; how many square miles are there in its area?

9. $\frac{2}{5}$ of the area of Colorado is covered by the Rocky Mountains; how many square miles is this?

10. The State of Wyoming has the form of a rectangle 254 mi. long and 249 mi. wide; how many square miles are there in its area?

11. What is the difference between the areas of Colorado and Wyoming?

12. There are 640 acres in a square mile; how many square miles are there in 128,000 acres?

1. What is meant by Roman notation?

2. Why is it so called? It is little used now. Name some of its uses.

The numerals with which we write numbers are called Arabic or Hindu numerals, because the Hindus invented them and the Arabs introduced them into Europe.

3. Write the Roman symbols as usually seen on clock faces.

4. Write in Roman numerals the numbers from 10 to 50.

5. For what numbers do the following letters stand: I, V, X, L?

The symbol for 100 is **C**, for 500 is **D**, and for 1,000 is **M**.

Thus, CCXL means in modern notation 240;
DCCIX means in modern notation 709;
MDCCC means in modern notation 1,800.

6. What number does V represent? IV is how many less than V?

7. What number does X represent? IX is how many less than X?

8. What number does L represent? XL is how many less than L?

9. Similarly CD is 100 less than D. Read CD; CDXIX.

10. CM is how many less than M? Read CMIV; MCMIV.

11. Write in Roman notation 1904; 1900; 1899; 1892.

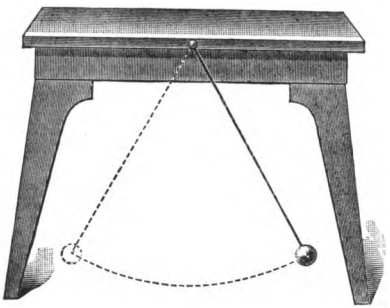
12. Write in Roman notation the number of the year in which Columbus discovered America.

13. Write in Roman notation the number of the present year.

14. If convenient, bring some book which has chapter numbers in Roman numerals and read them.

15. Make and solve other problems like Exercise 12.

1. Fasten a small piece of lead or clay to a string and suspend from a hook or nail, as shown in the picture. Adjust it so that the string may move freely. Such an instrument is called a **pendulum.** Make the string 2 ft. long, set the ball swinging and count the number of swings in 1 minute.



2. Calculate the number of swings the pendulum would make if it continued to swing at the same rate for 1 hour; for 24 hours.

3. Make a similar observation with a pendulum $2\frac{1}{2}$ ft. long. Calculate the number of swings for 1 hour; for 1 day.

4. Make a similar observation with a pendulum 3 ft. long; 4 ft. long. Calculate the number of swings of each for 1 hour; for 1 day.

5. Dean saw the steam when a mill-whistle blew and observed that the sound reached him 5 seconds after he saw the steam. The velocity of sound is 1,090 ft. per second. How far was Dean from the mill?

6. Clara saw the smoke of a gun and heard the report 4 seconds afterward; how far was Clara from the gun?

7. Casper was 11,990 ft. distant from a quarry when he heard the report of a blast; how many seconds had elapsed since the explosion?

8. A boy standing opposite to a cliff heard the echo of his voice 6 seconds after having shouted; how far was he from the cliff?

9. If convenient, make some observations in sound and solve two problems similar to Exercise 5.

Oral.

1. A garden in the form of a rectangle 30 yd. long and 40 yd. wide is to be represented on the blackboard. Name a convenient length to represent 1 yd.

2. What is the distance around the garden mentioned in Exercise 1? What is the distance around the drawing?

3. How may lengths of 30 ft., 40 ft.; of 30 yd., 40 yd.; of 30 mi., 40 mi. be represented for the purpose of comparison?

4. What is the cost of 100 stamped envelopes at 26¢ a dozen?

5. If a postman has in his sack 200 pieces of mail averaging 2 oz. in weight, what is the weight of the mail?

6. What is the cost of 25 two-cent stamps, 2 dozen stamped envelopes, and 50 postal cards?

7. In 100 lb. of milk there are 4 lb. of fat; what part of the milk is fat?

8. A cow produces 400 lb. of milk in a month; according to Exercise 7, how many pounds of fat does she produce in this time? In 3 months at the same rate?

9. What is the cost of picking 5 tons of hops at 1¢ per pound?

10. How many bales of hops weigh 25 tons? What did it cost to dry these hops at $1 per bale?

11. A common kit of manual-training tools costs about $6; how much does it cost to supply 25 boys with these tools?

12. Read: 2,005; 20,005; 200,005. Name the places in each number beginning with units.

13. Instead of writing, state the figures beginning at ten-thousands for seventy-five thousand, three hundred five.

14. In what way are figures grouped into periods in large numbers? Why are they so grouped? Name the periods in 376,542; in 300,000; in 2,300.

DECIMALS

Tenths

Oral.

1. In \$2.10 what figure shows the number of whole dollars? What figure shows the part of a dollar?

2. What name is given to the dot which separates the number of whole dollars from the part of a dollar?

3. How many tenths of a dollar are expressed by \$.20 or by \$.2?

4. Read as dollars and tenths of a dollar:
\$1.5; \$2.5; \$8.9; \$125.5; \$12.6; \$.1; \$.4.

Similarly, a number of feet and tenths of a foot, or of pounds and tenths of a pound, or of anything else may be expressed by using the decimal point.

In reading a whole number and a number of tenths the decimal point is read "and." Thus, 1.2 ft. is read "One *and* two tenths feet."

Read:

5. 20.1 dollars; 20.1 feet; 20.1 pounds.

6. 10.3 dollars; 10.3 quarts; 10.3 yards.

7. .8 of a dollar; .8 of a foot; .8 of a yard.

8. 6.8; 8.8; 5.8; 105.6; 55.5; .9.

The place at the right next to units' place is called tenths' place. The decimal point separates these two places.

Numbers expressed by use of the decimal point are said to be **written decimally.**

Write decimally:

9. Eleven and five-tenths.

10. Seven and nine-tenths.

11. Eighteen and one-tenth.

12. Sixty and four-tenths.

13. Express in cents: \$.1; \$.3; \$.8; \$.5; \$.9; \$2.4; \$3.5; \$11.1; \$14.7.

1. If 40 yards of cloth are divided into ten equal parts, how many yards are there in one part? How many are there in seven of these parts?

2. Find $\frac{1}{10}$ of 40 yards. Find $\frac{3}{10}$ of 40 yards.

3. Find $\frac{1}{10}$ of 30 pounds; of 50 yd.; of 1,200 ft.; of 6,520 oz.

4. Find $\frac{9}{10}$ of \$20; of \$100; of 120 tons; of 970 days.

5. Write decimally: $\frac{3}{10}$; $\frac{1}{10}$; $\frac{5}{10}$; $\frac{9}{10}$; $\frac{7}{10}$; $\frac{6}{10}$.

6. Find .1 of 120 in.; of 70 weeks; of 400 mi.; of 20 qt.

7. Find .7 of 90 days; of 120 minutes; of 140 ft.; of 70 tons.

8. How many tenths of a yard in two whole yards? In 3 yards? In 5 yd.? In 1 yd. and 5-tenths of a yard?

9. How many whole pounds are there in 30 tenths pounds? In 50 tenths pounds? In 20 tenths pounds?

10. Express in units and tenths: 17 tenths; 23 tenths; 81 tenths; 395 tenths.

11. Express decimally: 19 tenths; 29 tenths; 7 tenths; 175 tenths; 204 tenths.

12. Express in tenths: 2; 3; 5; 2.1; 2.3; 2.5; 23.4.

13. How many tenths of a dollar are ten times one-tenth of a dollar? How many dollars?

14. How many tenths of a dollar are ten times two-tenths of a dollar? How many dollars?

15. 10 times .1 = ? 10 times .2 = ?

16. 10 times 35 tenths = ? 10 times 3.5 = ?

17. 10 times 405 tenths = ? 10 times 40.5 = ?

18. How many places is the decimal point moved to the right in multiplying by 10?

19. Multiply by 10 by moving the decimal point:
.3; .7; .9; 1.1; 1.9; 20.1; 29.8; 196.7; 2,879.3.

1. How many are 3 times 5 tops? How many are 3 times 5 tenths? Express 15 tenths decimally.

2. How many are 7 times 6 hours? How many are 7 times 6 tenths? Express the result decimally.

Express the results decimally:

3. 5 times 9 tenths.
4. 5 times .9.
5. 8 times 4 tenths.
6. 8 times .4.
7. 5 times 8 tenths.
8. 5 times .8.

Multiply:

9.	10.	11.	12.	13.
.7	.9	.8	.5	.6
9	7	6	4	4

Express the results decimally:

14. 3 times 12 tenths.
15. 3 times 1.2.
16. 2 times 18 tenths.
17. 2 times 1.8.
18. 4 times 202 tenths.
19. 4 times 20.2.

Multiply:

20.	21.	22.	23.	24.
2.9	3.9	10.6	104.9	200.1
5	9	7	6	8

25. The length of about 39.4 in., called a **meter**, is used as a unit in the system known as the Metric or International System of weights and measures. It is the standard system of nearly all the leading civilized countries except the British Empire and the United States. It is permitted by law in the latter countries. How many inches are there in 3 meters? In 5 meters? In 10 meters? In 9 meters? In 100 meters?

26. A square is 2.4 ft. on a side. Find its perimeter.

27. A triangle is 13.7 in. on each side. Find its perimeter.

28. An ox killed and dressed is about .8 as heavy as when alive. An ox which weighs 880 lb. when alive, weighs how many pounds dressed?

1. A man bought 1 bu. each, of oats at $.40 a bushel, of wheat at $.90 a bushel, of beans at $1.20 a bushel, and of grass-seed at $3.30 a bushel; how much did he pay for all?

2. A farmer sowed 4 pieces of land and used 5.3 bu., 2.5 bu., 10.1 bu., and .8 bu. of seed respectively; how much did he sow in all?

5.3 bu.
2.5
10.1
.8
bu.

3. When wheat is bought at 60¢ per bushel and sold at 70¢, what is the gain per bushel?

4. An acre of land produced 28.5 bu. of wheat. When sown with oats it produced 32.3 bu. How many more bushels of oats than wheat did it produce?

32.3 bu.
28.5
bu.

5. Timothy seed averages about 48 lb. per bushel. The usual amount sown per acre is 12 quarts. At this rate, how many pounds are required for a 20-acre field?

6. At 16 qt. per acre, how many bushels are needed to sow 10 acres?

7. The following mixtures of seeds are sown for hay:

MIXTURE No. 1.

Seed.	Pounds per acre.
Oat grass	20
Red clover	10
Orchard grass	10

MIXTURE No. 2.

Seed.	Pounds per acre.
Timothy	16
Redtop	16
Red clover	8

Make and solve 5 problems about these mixtures.

8. Make and solve 5 problems about the following:

MIXTURE No. 1.

Seed.	Pounds per acre.
Kentucky blue grass	8
White clover	4
Perennial rye	9
Redtop	8

MIXTURE No. 2.

Seed.	Pounds per acre.
Red fescue	20
Redtop	10
Kentucky blue grass	8
White clover	2

Oral.

1. What is the cost of 10 dictionaries at $9.85 each? How is the result found?

2. When 100 cans of tomatoes cost $8.50, what is the price per can? How is the result found?

3. When 10 pumps cost $28.40, what is the price of each? How is the result found?

4. At the rate of .45 mi. per minute, how far does a train travel in 100 minutes? How is the result found?

5. How is a number expressed decimally multiplied by 10? By 100? How is such a number divided by 10? By 100?

6. How many bushels of apples can a man pick in 10 days at the rate of 36 bu. a day? What part does he pick in 5 days? How many bushels?

7. Following the plan of Exercise 6, find how many bushels of apples a man can pick in 5 days at the rate of 42 bushels a day.

8. 5 is how many halves? To multiply by 5 is to multiply by how many halves? What is the easiest way to multiply by 5?

Multiply by 5:

9. 17.	10. 23.	11. 86.	12. 95.
13. 66.	14. 117.	15. 206.	16. 819.
17. 5.6.	18. 9.16.	19. 8.9.	20. $7\frac{1}{2}$.

Find the cost of 5 articles of each kind:

21. Incubators, $26.	22. Brooders, $9.50.
23. Hay-carrier, $2.85.	24. Hay-fork, $ .95.
25. Feed-grinders, $10.80.	26. Windmill, $16.50.

27. What is the weight of 5 car-loads of coal, each weighing 40.2 tons?

FRACTIONS

Terms

1. Draw a line of convenient length; divide it as nearly as you can into 3 equal parts.

2. What fraction expresses the relation of one of these parts to the whole line? What fraction expresses the relation of 2 of these parts to the whole line? 3 of them?

3. When a quantity is divided into 3 equal parts what are the parts called? In the fractions $\frac{1}{3}$ and $\frac{2}{3}$ which number shows that the fractions express thirds?

4. Draw a square and divide it into 4 equal parts.

5. What fraction expresses the relation of one of these parts to the whole square? What fraction expresses the relation of 2 of these parts to the whole square? 3 of them? 4 of them?

6. When a quantity is divided into 4 equal parts, what are the parts called? Which number in the fractions $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ shows that the fractions express fourths?

The number that names the kind of parts expressed by the fraction is called the **denominator** of the fraction.

7. In the above fractions which number tells how many of these parts are expressed by the fractions?

The number that tells how many parts are expressed by a fraction is called the **numerator.**

The numerator and denominator are called the **terms** of the fraction.

8. In $\frac{7}{8}$ what is the numerator? The denominator?

9. Answer the same questions for $\frac{5}{16}$, $\frac{8}{9}$, $\frac{6}{8}$, $\frac{7}{10}$, $\frac{20}{10}$.

10. What takes the place of the denominator in .3? In .20? In .7? In .5? In .9?

1. Which is greater, 7 yd. or 5 yd.?

2. Which is greater, $\frac{7}{8}$ or $\frac{5}{8}$? $\frac{6}{7}$ or $\frac{4}{7}$? $\frac{3}{8}$ or $\frac{5}{8}$?

3. If two unequal fractions have equal denominators, which fraction is the greater?

4. Which fraction is greater, $\frac{1}{4}$ or $\frac{1}{5}$? $\frac{1}{6}$ or $\frac{1}{8}$? Why?

5. Which is less, $\frac{1}{7}$ or $\frac{1}{8}$? $\frac{2}{9}$ or $\frac{2}{10}$? Why?

6. If two unequal fractions have equal numerators, which fraction is the greater?

Arrange the numbers in each set according to size, placing the least at the top:

7.	8.	9.	10.
$\frac{8}{3}$	$\frac{10}{4}$	$\frac{7}{3}$	$\frac{9}{4}$
$\frac{8}{12}$	$\frac{10}{2}$	$\frac{7}{5}$	$\frac{9}{5}$
$\frac{8}{9}$	$\frac{10}{10}$	$\frac{7}{2}$	$\frac{9}{16}$
$\frac{8}{8}$	$\frac{10}{13}$	$\frac{7}{9}$	$\frac{9}{20}$
$\frac{8}{7}$	$\frac{10}{11}$	.7	.9

11. The areas of Denmark, Venezuela, England, and Germany are respectively $\frac{10}{15}$, $\frac{10}{3}$, $\frac{10}{20}$, $\frac{10}{6}$ of that of the British Isles. Arrange the names of these countries in order according to their areas.

12. Of all the cotton goods produced in the United States in 1900, about $\frac{1}{3}$ was made in Massachusetts, $\frac{1}{11}$ in South Carolina, $\frac{1}{14}$ in Pennsylvania, $\frac{1}{19}$ in Georgia, $\frac{1}{13}$ in Rhode Island, $\frac{1}{20}$ in Connecticut, $\frac{1}{12}$ in North Carolina, and $\frac{1}{15}$ in New Hampshire. Arrange the names of these states in order according to the amount of cotton goods produced.

13. Of all the boots and shoes made in the United States in 1900, $\frac{9}{100}$ were made in New Hampshire, $\frac{9}{20}$ in Massachusetts, and $\frac{9}{99}$ in New York. Arrange the names of these states in order according to the quantity of boots and shoes manufactured.

Oral.

1. How many years are there from 1800 to 1900? From 1900 to 2000?

2. 100 years are called a **century.** How many years are there in $\frac{1}{10}$ of a century? In $\frac{5}{10}$? In .8?

3. How many years are there from 1890 to 1900? From 1900 to 1910?

4. 10 years are called a **decade.** A decade is what part of a century? 2 decades are what part of a century?

5. What time does one division of the diagram represent? Draw a line to the same scale to represent 7 centuries.

6. How many years are there in .5 of a decade? In $\frac{4}{10}$ of a decade? In .3? In .9? In .7?

7. How many days are there in .1 of a 30-day month? In $\frac{3}{10}$ of it? In .2? In .5? In $\frac{7}{10}$? In .9?

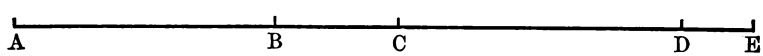
8. How many minutes are there in $\frac{3}{10}$ of an hour? In .7 of an hour? In $\frac{8}{10}$? In .4?

9. How many minutes are there in 1.2 hours? In 2.1 hours? In 5.6 hours? In 10.5 hours?

Written.

10. Add 3.5 hr., 7.1 hr., 5 hr., 10.9 hr., $\frac{6}{10}$ hr.

11. A train in traveling between the cities A and B required $1\frac{4}{5}$ hr., between B and C $\frac{3}{5}$ hr., between C and D $2\frac{2}{10}$ hr., and between D and E $\frac{2}{5}$ hr. Express these periods of time decimally and find how long, exclusive of time for stops, it took the train to travel from A to E.



12. How many hours and tenths of an hour are there from 6 A. M. to 7:12 A. M.? From 7:12 A. M. to 9 A. M.? From 9 A. M. to 11:30 P. M.?

Oral.

1. Which swings the faster, a long or a short pendulum?

2. A large clock has a pendulum that swings once every second. How many times does it swing in 1 minute? In 1 hour? In 10 hours?

3. The sound of a whistle was heard 3 seconds after the steam was seen; taking the velocity of sound as 1,090 ft. per second, how far away was the whistle?

4. In writing numbers decimally, what is used to separate the whole number from the decimal part?

5. How many tenths are expressed by .2? .3? .9? 1.9? 2.9? 3.0? 20.1?

6. How does multiplying a decimal by 10 change the position of the decimal point?

7. State where the decimal point will be when each of the following is multiplied by 10: 2.8; 3.7; .5; 20.2; 62.0; 207.2; 2.00.

8. How does dividing a number by 10 change the position of the decimal point?

9. State where the decimal point will be when each of the following is divided by 10: 2.7; 2.0; 2; 16.5.

Multiply:

10.	.6	11.	.8	12.	.9	13.	7.1
	7		9		9		3
14.	2.1	15.	4.2	16.	80.1	17.	200.1
	6		5		3		7

Add:

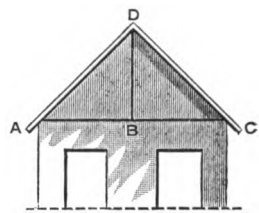
18.	10.1	19.	7.0	20.	1.9	21.	1.9
	6.3		2.1		4.5		.2
	4.5		.7		.3		9.1

FORM STUDY

Perpendiculars

1. The angles of a square are called **right angles.** What kind of angles do the adjacent sides of a rectangle form with one another?

2. Two straight lines which are sides of a right angle are said to be **perpendicular** to each other. Designate the lines in the picture that are perpendicular to each other.



3. What kind of an angle is formed by the top and one side of a door?

4. The adjacent edges of a book are perpendicular to each other. Point out in the school-room lines that are perpendicular to each other.

5. Right angles may be constructed by using an object which has one of its corners a right angle. Construct a right angle with the aid of your book-cover. In drawing, a *triangle* or *square* is used, as shown in the picture. Carpenters also use the square.

6. Draw a line 4 in. long. At each end and on the same side of this line draw a line $\frac{3}{4}$ as long and perpendicular to it. Connect the end points of the perpendiculars. What is the area of the figure?

1. If 1 inch in a drawing represents 100 mi., how many miles does $\frac{1}{4}$ of an inch represent? $\frac{3}{4}$ in.? $\frac{3}{10}$ in.? .7 in.? .9 in.?

2. How long is a line that represents 200 miles? 150 miles? 325 miles? 70 miles? 140 miles?

3. Draw a figure to represent a rectangle 250 mi. long and 125 mi. wide.

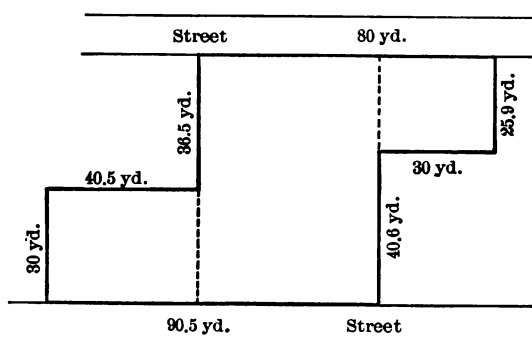
4. If a rectangle is 80 mi. wide, how long must it be to have an area equal to that of Florida, 54,000 sq. mi.?

5. Draw a rectangle representing the area of Florida, using 1 in. to represent 100 mi.

6. A mile contains 5,280 ft.; how many yards are there in a mile?

7. Central Park in New York city is a rectangle about $2\frac{1}{2}$ mi. long and $\frac{1}{2}$ mi. wide. Draw a plan of it, using 1 in. for 440 yd.

8. Find the perimeter of the whole plot in the figure.



9. What kind of angles do the sides of the above figure form with one another? State the lengths of two perpendicular sides; of another pair of perpendicular sides.

10. What are the lengths of the dotted lines in the figure? Find the area of the whole plot.

Written.

1. Mr. Roe, who bought the plot shown on page 22, fenced it at 10¢ per running foot; what did the fence cost?

2. He covered the two small pieces with sod at 7.3¢ per square yard; what did that cost him?

3. He built a cement walk 6 ft. wide along both street fronts at $1.10 per square yard; what did this cost?

4. Mr. Roe sold the piece at the left for $4 per front foot, the middle piece for $5.50 per front foot, and the other piece at $6.90; how much did he receive for the whole plot?

5. How many pupils can stand at a blackboard 14 ft. long, if each is allowed 28 inches?

6. How long is a parade consisting of 35 carriages, allowing 7.1 yd. for the length of a team and wagon, and 20 ft. for the distance between each pair?

7. How many feet are there in one yard? In $\frac{2}{3}$ of a yard? In $\frac{1}{3}$ of a yard? In $8\frac{2}{3}$ yd.? In $69\frac{1}{3}$ yd.?

8. A boy found a fence to be 32 times as long as a stick. He afterward measured the stick and found it to be 21 in. long. How long is the fence?

9. When William was 10 years old he was 1 yd. 10 in. tall. When he was 12 years old he was 1 yd. 1 ft. 6 in. tall. How much had he grown in the two years?

PLAN. 1 yd. 1 ft. 6 in. = 1 yd. 0 ft. 18 in.
1 10 1 10

10. What is the area of a rectangle of base 3.26 ft. and altitude 5 ft.?

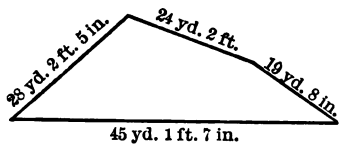
Written.

1. A three-cornered lot has its sides, 112 yd. 2 ft. 4 in., 109 yd. 2 in., and 127 yd. 1 ft. 11 in. long. What is the length of the fence around it?

112 yd.	2 ft.	4 in.
109		2
127	1	11

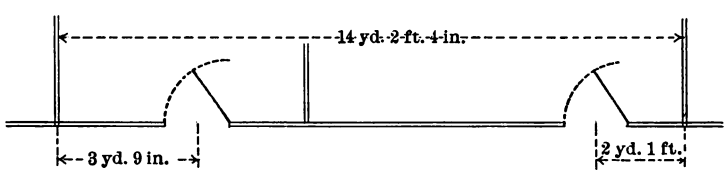
SUGGESTION.—The sum of a column may need reduction. Thus, in the first, 11 in. + 2 in. + 4 in. = 17 in. = 1 ft. 5 in.

2. A ditch 75 yd. long is to be dug. The first day 30 yd. 2 ft. 6 in. are dug, the second day 31 yd. 9 in. What length remains to be dug?



3. Thomas measured the fence around a lot and found the sides to have the lengths marked in the figure; what was the length of the fence?

4. Find the distance from the middle of one door to that of the other, as shown in the floor plan.



If convenient, make and solve a similar problem, using measurements found by yourself.

5. A boy walked 260 yd. 1 ft. 8 in. north, then 179 yd. 2 ft. 10 in. south; how far did he walk in all? How far was he from his starting-point?

6. Henry and James began to whitewash a fence 100 yd. long, beginning at the ends and working toward the middle. After a time Henry had whitewashed 24 yd. 2 ft. 6 in., and James 19 yd. 1 ft. 8 in. How much remained to be done?

DECIMALS

Hundredths

Oral.

1. Read: \$3.05; \$1.25; \$.65; \$.08; \$4.90; \$10.06.

2. How many cents are there in a dollar? What part of a dollar is one cent?

3. In \$3.05 what figure shows the number of whole dollars? What figures show the part of a dollar?

4. What separates the number of whole dollars from the part of a dollar?

5. How many hundredths of a dollar are expressed by \$.08? By \$.04? By \$.20? By \$.25? By \$.90? By \$.75?

Similarly, a number of feet and hundredths of a foot, or of pounds and hundredths of a pound, or of anything else may be expressed by using the decimal point.

Read:

6. 2.05 dollars; 2.05 feet; 2.05 ounces.

7. 7.25 dollars; 7.25 pounds; 7.25 yards.

8. .99 of a dollar; .99 of a quart; .99 of a year.

9. 3.25; 8.90; 7.08; .63; 73.40; 17,289.12.

10. What is the first place at the right of the decimal point called?

The second place is called **hundredths' place.** Hence, in writing a number of hundredths, there always must be a tenths' place.

11. How many places must be used to express completely a number of hundredths?

Written. Express decimally; then as whole numbers and fractions:

12. 12 and 39 hundredths.

13. 5 and 5 hundredths.

14. 32 hundredths.

15. 1 and 1 hundredth.

Oral.

1. What part of a dollar is one cent?

2. $\frac{1}{2}$ of a dollar = ___ cents. $\frac{50}{100}$ of a dollar = ___ cents.

3. Compare $\frac{1}{2}$ of a dollar with $\frac{50}{100}$ of a dollar.

4. How many hundredths are there in any quantity? How many hundredths in $\frac{1}{2}$ of it?

5. How many pounds are there in $\frac{1}{2}$ of a ton? In $\frac{50}{100}$ of it? In .50 of it?

6. How many feet are there in $\frac{1}{2}$ of a mile? In $\frac{50}{100}$ of it? In .50 of it?

7. What is $\frac{1}{4}$ of 100? How many hundredths are there in $\frac{1}{4}$ of 100 hundredths? How many hundredths of a dollar in $\frac{1}{4}$ of a dollar?

8. Compare $\frac{1}{4}$ of a dollar with $\frac{25}{100}$ of a dollar.

9. How many pounds are there in $\frac{1}{4}$ of a ton? In 25 hundredths of a ton? In .25 of a ton?

10. How many yards are there in $\frac{1}{4}$ of a mile? In $\frac{25}{100}$ of a mile? In .25 of a mile?

11. How many cents are there in $\frac{1}{4}$ of a dollar? In $\frac{3}{4}$ of a dollar? How many hundredths of a dollar are there in $\frac{3}{4}$ of a dollar?

12. How many hours are there in $\frac{3}{4}$ of a day? How many hours in .75 of a day?

Written.

13. Express decimally: $\frac{1}{4}$; $\frac{1}{2}$; $\frac{2}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{4}{5}$.

14. Express decimally: $1\frac{1}{4}$ hr.; $2\frac{3}{4}$ da.; $125\frac{3}{4}$ mi.

15. What part of a gallon is a quart? How many hundredths of a gallon?

16. How many hundredths of a bushel are three pecks?

17. An engineer's chain is 100 ft. long; how many feet are there in $\frac{1}{2}$ of it? In .50 of it? In $\frac{3}{4}$ of it?

Oral.

1. How many pounds are there in $\frac{1}{4}$ of a ton? If $\frac{1}{4}$ of a ton of hay is divided into 100 equal parts, how many pounds are there in 1 part? How many are there in 8 of these parts?

2. Find $\frac{1}{100}$ of 500 lb. Find $\frac{3}{100}$ of 500 lb. Find .09 of 500 lb.

3. Find $\frac{1}{100}$ of 1,000 yd.; of 2,000 lb.; of 200 ft.; of 800 gal.

4. Find $\frac{6}{100}$ of \$100; of 400 mi.; of 500 ft.; of 300 yr.

5. Write decimally: $\frac{3}{100}$; $\frac{1}{100}$; $\frac{15}{100}$; $\frac{25}{100}$; $\frac{99}{100}$; $\frac{70}{100}$.

6. Find .01 of 800 hr.; of 1,800 lb.; of 27,000 lb.; of \$25,000.

7. Find .08 of 200 hr.; of 1,200 lb.; of 3,800 ft.

8. How many hundredths of a yard are there in 2 whole yards? In 5 yd.? In 1 yd. and 50 hundredths of a yard?

9. How many whole hours are there in 300 hundredths hours? In 500 hundredths hours? In 2,000 hundredths hours?

Express in integers and hundredths, using fractional forms:

10. 117 hundredths.
11. 223 hundredths.
12. 996 hundredths.
13. 1,090 hundredths.
14. 2,703 hundredths.
15. 2,240 hundredths.

Express decimally:

16. 163 hundredths.
17. 5,280 hundredths.
18. 144 hundredths.
19. 63,300 hundredths.
20. 1,728 hundredths.
21. 4,990 hundredths.

22. Express in hundredths: 1.20; 4.50; 7.25; 10.05; 9.75; 20.09; 16.3; 24; 12.5.

23. 100×625 hundredths $= ?$ $100 \times 6.25 = ?$ $100 \times 62.5 = ?$

24. How many places is the decimal point moved to the right in multiplying by 100?

Oral.

1. How may the result of multiplying a decimal number by 10 be found easily? The result of multiplying by 100?

2. $10 \times .3 = ?$ $.1 \text{ of } 3 = ?$

3. $10 \times 3.2 = ?$ $.1 \text{ of } 32 = ?$

4. $100 \times .05 = ?$ $.01 \text{ of } 5 = ?$

5. $100 \times 3.25 = ?$ $.01 \text{ of } 325 = ?$

6. How many decimal places is the decimal point moved to the left in dividing by 10? In dividing by 100?

7. Divide by 10:

3.5; 35; .5; 20; 20.8;
9.5; 9,675; 27.5; 2,005; 200.5.

8. Divide by 100:

350; 3,500; 35; 625; 6,250; 27; 15; 1,025.

9. The following are prices of various kinds of tea sold in 10-lb. lots:

Common Japan.....	$.22	Extra Hyson.....	$.29
Choice Japan.....	.27	Fancy Oolong.....	.37
Green Japan.....	.23	Ceylon.....	.25

What is the cost of 10 lb. of each kind? What is the cost of 100 lb. of each kind?

Written.

10. What is the cost of 30 lb. of Ceylon tea? Of choice Japan? What is the cost of 300 lb. of green Japan?

11. When 40 lb. of tea cost $15.60, what is the cost of one pound? Of 10 pounds?

12. The following are prices of fish-nets per running yard according to width and quality:

A. $.07	B. $.11	C. $.09
D. .14	E. .15	F. .13

What is the cost of 500 yd. of net of grade A? Make and solve 5 other problems.

1. How many feet are 3 times 5 feet? How many hundredths are 3 times 5 hundredths? Express decimally 15 hundredths.

2. How many days are 3 times 25 days? How many hundredths are 3 times 25 hundredths? Express decimally.

3. $3 \times$ 8 hundredths $=$?

4. $5 \times$ 12 hundredths $=$?

5. $6 \times$ 11 hundredths $=$?

6. $6 \times$ 20 hundredths $=$?

Multiply:

7.	8.	9.	10.	11.
.07	.18	.60	.03	.75
9	3	5	12	8

Express decimally the result:

12. 3 times 108 hundredths.

13. 3 times 1.08.

14. 2 times 205 hundredths.

15. 2 times 2.05.

16. 15 times 375 hundredths.

17. 15 times 3.75.

Multiply:

18.	19.	20.	21.
1.05	20.06	12.65	19.99
5	10	18	35

22. What does it cost to travel 100 miles on a railroad at \$.03 a mile? To travel 300 miles? 500 miles? 1,000 miles?

23. What does it cost to ride from New York City to Buffalo, a distance of 440 miles, at the rate of \$.02 a mile?

24. Archie's cyclometer stood at 147.64 miles when he started on a bicycle ride. On his return it stood at 161.87 miles. How far did he ride?

161.87 miles.
147.64
miles.

25. Three tanks for distilling water produce 1.20 gal., .75 gal., and 2.08 gal. respectively per hour. How many gallons do they distil together in one hour? In 10 hours? In 100 hours? How many casks of 32 gallons each is this?

1.20 gal.
.75
2.08
gal.

Hundredths occur in problems relating to dollars and cents. Making and footing bills, or accounts, is one of the chief uses of decimals, requiring the addition and multiplication of hundredths.

Oral.

1. What is the cost of 2 lb. of tea at $.35 a pound?
2. What is the cost of 3 lb. of coffee at $.40 a pound?
3. What is the cost of 5 lb. of lard at $.08 a pound?

State the cost of:

4. 4 yd. lace, $.15.
5. $2\frac{1}{2}$ yd. silk, $1.00.
6. 8 sticks of braid, $.06.
7. $\frac{1}{2}$ doz. doilies, $.12 each.
8. 8 yd. woolen, $.75.
9. 10 yd. calico, $.06$\frac{1}{2}$.

Written.

10. Copy the items of the following bill; find the cost of each article and the total of the bill:

John Wanamaker

Broadway, 4th Ave. 8th & 9th and 10th Sts.

Purchases for New York,

Mr. Charles Sampson,
512 West 115th Street,
New York City,

Date.	Due upon account previously rendered.			Daily Total.	Credits.
	2½ Yd. Waist Goods	.50			
	3 " " "	.50			
	7 " Lawn	.20			
	12 Handkerchiefs	.12 1/2			
	1 Blanket	3.50			
	1 Comfort	2.50			
	6 Towels	.25			
	2 Sheets	.75			
	4 "	.60			
	4 Cases	.12 1/2			

Find the amount of each of the following bills:

1. A customer bought of a baker and confectioner:

Candy	$ .50	Cheese	$ .18	Bread	$ .20
Cookies	.10	Wafers	.08	Cake	.30
Pies	.50	Nuts	.25	Fruit	.35

2. A family bought from a grocer:

Jan. 1.	Figs	$ .18	Jan. 9.	Eggs	$ .35	Jan. 20.	Gelatine	$ .15
	Dates	.08		Soap	.25		Bananas	.10
	Coffee	.35		Clothes line	.30		Saltines	.15
	Peas	.30		Corn	.30		Salmon	.25
	Oranges	.25	Jan. 15.	Butter	.90	Jan. 26.	Crackers	.10
	Bananas	.10		Coffee	.30		Butter	.90
	Nuts	.15		Celery	.10		Oil	.13
	Butter	.90		Potatoes	.20		Coffee	.35
	Celery	.15		Oil	.13		Tea	.60

3. A lady purchased at a dry-goods store:

3	bolts of lace at	$1.00	4	sheets at	$ .70
18	cotton bats "	.11	2	sheets "	.58
9½	yd. outing flannel "	.12	18	yd. muslin "	.07

4. A restaurant keeper purchased at the market:

2	lb. beefsteak at	$ .16	2½	lb. veal at	$ .15
3	" chicken "	.18	3	" boiled ham "	.13
4	" kettle roast "	.08	3½	" beef stew "	.10
2	qt. oysters "	.35	2½	" sausage "	.12

5. Bought of Baker and Taylor: 25 copies Shakespeare (school ed.), $1.25; 70 copies Scott's "Lady of the Lake," $.75; 1 gross (12 doz.) copy books, $.15 each; 12 class registers, $.25.

6. Bought of Macy and Co.: Aug. 1, 25 yd. Am. prints, $.07; 24 yd. Janesville sheeting, $.80; 12 yd. damask toweling, $.15; Aug. 15, 5 cotton spreads, $3.25; 15 yd. pillow tubing, $.25.

7. Using your own name as purchaser, write bills for Exercises 5 and 6.

8. How are bills usually receipted?

Written.

1. If for every 100 miles of railway 2 general officers are employed, how many are employed for 500 miles? If each receives $10.97 a day, how much do they all receive in one day?

2. If there are 18 office clerks for every 100 miles of road and 34,776 clerks in the United States, how many hundred miles of railroad in the country? How many miles?

3. If there are 24 firemen per 100 miles, how many are there per 1,000 miles? If each receives $2.16 a day, what is the amount of their wages?

Employees.	Total Number.	Number per 100 Miles.	Daily Wages.
General Officers	4,780	2	$10.97
General Office Clerks	34,778	18	2.19
Station Agents	32,294	17	1.77
Enginemen	45,292	23	3.78
Firemen	47,166	24	2.16
Conductors	32,092	16	3.17

4. The first column in the above table shows the number of railway employees in the United States in 1901 in the classes named. Find the total number in these classes.

5. In 1901 there were 16 conductors per 100 miles of road; what did they earn in 30 days at $3.17 a day?

6. According to Exercise 5, what would a company whose line extended 1,000 miles pay its conductors in one month?

7. If 100 miles of railroad require 2 general officers at $10.97 a day and 23 enginemen at $3.75 a day, how much more does it cost for enginemen than for officers?

8. Make and solve 3 other problems using the facts of the table.

Oral.

1. Name some figure all of whose angles are right angles.

2. Name a figure one of whose angles is a right angle.

3. How may a right angle be drawn? Name another way.

4. From a point in a line draw a perpendicular; how many right angles are formed?

5. Explain how a carpenter uses a square to mark a line across a board, perpendicular to one edge.

6. If one inch on a map represents 50 mi., how many miles do 10 in. represent? $2\frac{1}{2}$ in.? $12\frac{1}{2}$ in.?

7. What is meant by "drawing to scale"? Explain how a drawing of a county 20 mi. square could be made on the blackboard.

8. How many feet are there in a mile? How many yards?

9. What is the area of a rectangle 40 yd. long and 30 yd. wide?

10. How is the area of any rectangle found? If the length is given as 40 ft. and the width 10 yd., how is the area found?

11. From a rod of iron 4 ft. and 6 in. long a piece 2 ft. and 8 in. is cut. How long is the part that is left?

Written. Add:

12.	13.	14.
17 ft. 8 in.	13 ft. 11 in.	72 yd. 1 ft.
9 9	72 7	66 2

15.	16.
17 yd. 1 ft. 7 in.	6 gal. 3 qt. 1 pt.
28 2 9	9 2 0

Oral.

1. What is the name of the first decimal place at the right next to the decimal point? Of the second place?

2. How many hundredths of a dollar are expressed by: \$.05? \$.03? \$.39? \$.99? \$1.25? \$.50?

Read:

3. 3.35 dollars. 4. 6.75 ft. 5. 2.49 lb.

6. 18.75 bu. 7. 6.05 yd. 8. 12.86 mi.

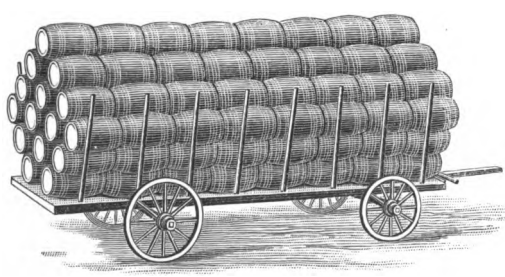
9. How many pounds are there in a ton? In $\frac{1}{2}$ ton? In $\frac{3}{4}$ ton? In $\frac{1}{5}$ ton? In $\frac{4}{5}$ ton? In .1 ton? In .3 ton?

Multiply:

10.	11.	12.	13.
.09	.11	.25	.25
9	9	4	8

Written.

14. Express decimally: 75 hundredths; 43 hundredths; 143 hundredths; 1,440 hundredths; 9 hundredths.



15. Express as common fractions: .25; .75; .50; .40.

16. How many barrels are there on this wagon? Each barrel is $2\frac{4}{5}$ ft. long; how many feet long is the load? Each barrel weighs 6.5 pounds; what is the weight of the load?

Multiply:

17.	18.	19.	20.
4.36	3.09	17.25	18.6
11	12	15	25
21.	**22.**	**23.**	**24.**
60.06	12.09	26.15	20.08
42	45	75	95

Oral.

1. What is the cost of 5 yards of cloth at $22\frac{1}{2}¢$ a yard?

2. What is a short way of multiplying $22\frac{1}{2}$ by 5?

Multiply by 5:

3. 18. 4. 64 ft. 5. 104 mi. 6. 44 hr.

7. 7.2. 8. 14.8. 9. \$7.60. 10. 12.96.

11. What is meant by the numerator of a fraction? By the denominator? By the terms?

12. Illustrate your answer to each question of Exercise 11.

13. Which is the greater: $\frac{6}{7}$ or $\frac{5}{7}$? $\frac{5}{9}$ or $\frac{7}{9}$?

14. Which is the greater of two unequal fractions having the same number for denominators?

15. Which is the greater: $\frac{7}{5}$ or $\frac{7}{2}$? $\frac{9}{7}$ or $\frac{9}{5}$? $\frac{5}{9}$ or $\frac{5}{13}$?

16. Which is the greater of two unequal fractions having the same number for numerators?

17. How many years are there in a century? How many are there in a decade?

18. How many decades are there in .7 of a century? In $\frac{7}{10}$ of a century?

19. How many decades are there in $\frac{2}{10}$ of a century? In $\frac{1}{5}$ of a century?

Written. Express decimally and add:

20.	21.	22.	23.
$11\frac{1}{2}$	$27\frac{3}{5}$	$12\frac{3}{10}$	$99\frac{9}{10}$
$10\frac{2}{5}$	$13\frac{9}{10}$	$10\frac{3}{5}$	$86\frac{3}{10}$
$17\frac{4}{5}$	$18\frac{1}{2}$	$99\frac{1}{2}$	$40\frac{4}{5}$
$16\frac{7}{10}$	$5\frac{4}{5}$	$66\frac{7}{10}$	$20\frac{1}{2}$

Express decimally and subtract:

24.	25.	26.	27.
$88\frac{1}{2}$	$76\frac{9}{10}$	$105\frac{3}{5}$	$93\frac{7}{10}$
$29\frac{3}{5}$	$47\frac{4}{5}$	$83\frac{1}{2}$	$49\frac{1}{2}$

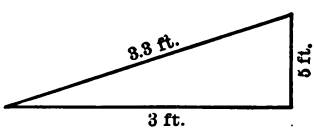
1. Measure in feet and inches the heights of various pupils in the class. Who is the tallest? Who is the shortest?

2. According to Exercise 1, how much does the height of each pupil measured differ from the height of the tallest? From the height of the shortest?

3. Draw lines on the board to represent the heights of several pupils, using $\frac{1}{2}$ in. for each inch of height.

4. Draw lines on paper to represent the heights of several pupils, using $\frac{1}{8}$ in. for each inch in height.

5. What is the perimeter of the right-angled triangle shown in the figure?



6. The amounts of sugar produced in 1902 by Louisiana, Porto Rico, British West Indies, and Brazil were respectively $\frac{3}{8}$, $\frac{3}{80}$, $\frac{3}{24}$, and $\frac{3}{12}$ of the amount produced by Cuba. Arrange the names of these countries in order according to the amount of sugar produced.

7. The weights of a cubic inch of copper, yellow pine, and ivory are respectively $\frac{8}{18}$, $\frac{7}{18}$, $\frac{1}{18}$ times that of a cubic inch of anthracite coal. Arrange the names of the substances in order of their weight.

8. A train travels $72\frac{1}{2}$ mi. between A and B, $24\frac{3}{10}$ mi. between B and C, $90\frac{2}{5}$ mi. between C and D, and $16\frac{4}{5}$ mi. between D and E. The stations are located in order from A to E. How many miles is it from A to E?

9. A lady bought $4\frac{1}{8}$ yd. of green ribbon, 17 yd. of blue ribbon, and $16\frac{2}{3}$ yd. of white ribbon, all at 65¢ per yard; how many yards did she buy? What was the total cost?

Express decimally:

10. 324 hundredths.

11. 149 hundredths.

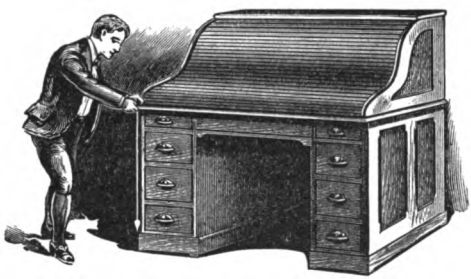
12. 203 hundredths.

13. 1,024 hundredths.

FORM STUDY

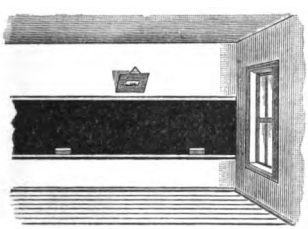
Lines and Surfaces

A line drawn in the direction in which a body falls, if dropped, is called a **vertical** line.



1. Suspend your pencil or some other small object by a string. The string assumes a vertical position.

2. Tie a weight to a string and test several lines supposed to be vertical. What line is being tested in the picture above?



A line perpendicular to a vertical line is called a **horizontal** line. The top of the blackboard in the picture represents a horizontal line.

3. Point out the horizontal lines in the picture.

A surface in which all straight lines are horizontal is called a **horizontal** or **level** surface.

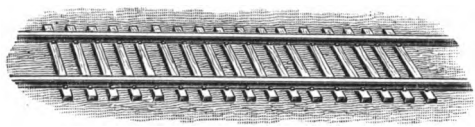
4. Point out various surfaces which appear to be level.

Carpenters use a spirit-level to determine whether a surface is level.

5. If you can procure a spirit-level, test various surfaces.

1. How does the distance between the vertical sides of a window-pane at the top compare with that at the bottom?

2. Do the two lines of rails of a railroad-track ever meet? How does their distance apart at any place compare with that at any other place?



3. Point out other pairs of lines that are the same distance apart everywhere. This distance is always taken on lines perpendicular to the lines considered.

Two lines that are everywhere the same distance apart are said to be **parallel.**

4. Point out the pairs of parallel lines in Figure 1; in Figure 2.

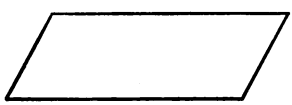


Figure 1

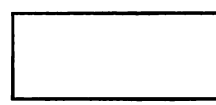


Figure 2

Figure 3

A four-sided figure whose opposite sides are parallel is called a **parallelogram.**

5. What kind of a figure is Figure 2? Figure 3? How many pairs of parallel lines are there in Figure 4?

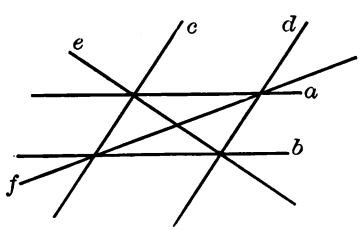
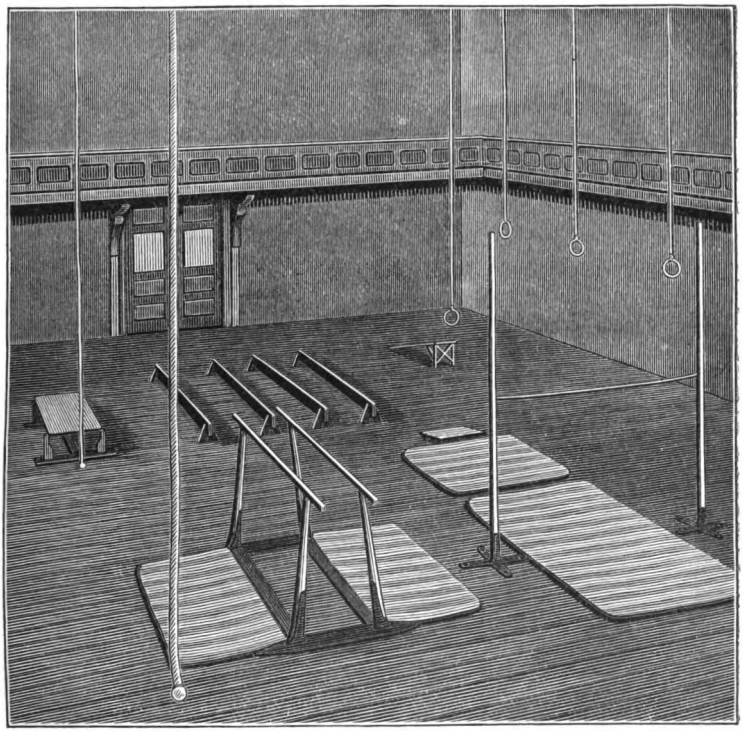


Figure 4

6. Name several objects that are parallelograms. Is a rectangle a parallelogram? Are all parallelograms rectangles? Is a square a rectangle?

7. Draw a parallelogram in which the angles are not right angles, and whose adjacent sides are 2.5 in. and $6\frac{2}{3}$ in.



1. Name 5 horizontal lines.

2. Name any level surfaces.

3. Name 5 vertical lines in the picture.

4. Name 5 sets of parallel lines in the picture.

5. Name any lines perpendicular to other lines.

6. Draw a horizontal line; a vertical line; two lines perpendicular to each other.

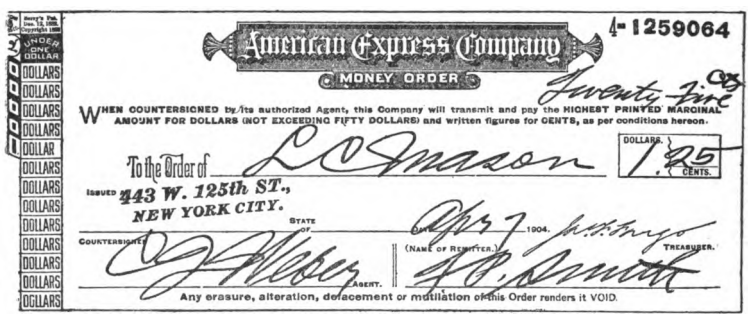
7. Place a pasteboard triangle so that one side rests on a straight line. Move the triangle from place to place along the line and draw parallels.

BUSINESS APPLICATIONS

Money-Orders

Money may be paid by means of postal money-orders, which are obtainable at post-offices, or by means of express money-orders obtainable at the offices of the express companies.

The following order was purchased by F. P. Smith in New York City to pay L. C. Mason in Detroit, Mich., $1.25.



FEES CHARGED FOR POSTAL MONEY-ORDERS

For payment in the United States (which includes Hawaii and Porto Rico), in Canada, in Cuba, and in the Philippines.

For	orders	for	sums	not	exceeding		$2.50	3	cents
"	"	over	$2.50	and not	"		5.00	5	"
"	"	"	5.00	"	"	"	10.00	8	"
"	"	"	10.00	"	"	"	20.00	10	"
"	"	"	20.00	"	"	"	30.00	12	"
"	"	"	30.00	"	"	"	40.00	15	"
"	"	"	40.00	"	"	"	50.00	18	"
"	"	"	50.00	"	"	"	60.00	20	"
"	"	"	60.00	"	"	"	75.00	25	"
"	"	"	75.00	"	"	"	100.00	30	"

The maximum amount for which a single money-order may be issued is $100. For a larger sum additional orders must be sent.

The rates for express money-orders are practically the same.

Written.

1. What is the fee for a money-order for \$1.75? For \$2.50? For \$2.55? For \$4.20? For \$6? For \$18?

2. Carl sent for a camera which cost \$15; a tennis-racket, \$3.75; and a rifle, \$25. What was the fee for the money-order? The total cost?

3. A lady ordered a rug for \$25, 20 yards of carpet at \$1.32 a yard, and a couch cover for \$8.75; what did she pay for all? What was the total cost of the money-order?

4. A musician ordered a mandolin for \$8, a guitar for \$7.50, a violin for \$55.75, and a banjo for \$12. What was the total cost of the money-order?

5. Mr. Mason bought of Ward and Company 1 box of soap at \$4; 1 barrel of flour, \$3.50; and 50 lb. of dried meat at 9¢ a pound. What was the cost of a money-order to pay for these goods?

6. Mrs. Martin bought of Thomas Smith 3 yd. of lace at \$.65 a yard, 1 dozen handkerchiefs for \$4.75, 4 yards of silk at \$1.15 a yard, and a table-cloth for \$8; what was the cost of the money-order to pay for these goods?

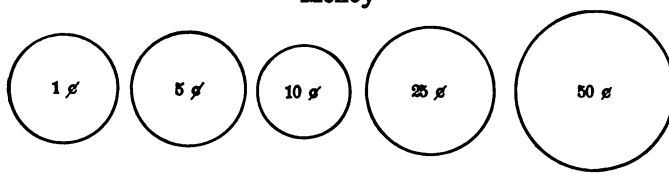
7. Mr. Carr bought of Milton Woodward & Co. a harness for \$25, a saddle for \$21.85, a horse-blanket for \$3.75, and a set of sleigh-bells for \$2.75; what did he pay for all? What was the total cost of the money-order?

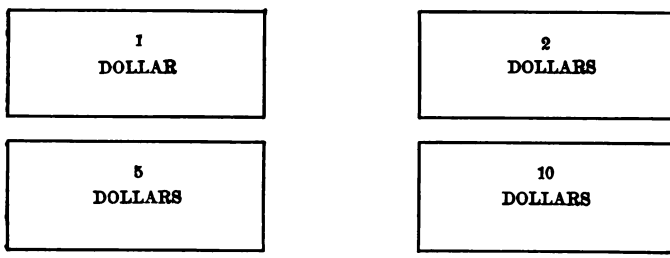
8. A publisher ordered a postal scale, price \$2.40, and weight when packed 20 oz. He purchased a money-order to pay for the scale and postage at the rate of 1¢ an oz. How much did he pay the postmaster?

9. Make a list of articles and their prices from newspapers and magazines. Find the cost of money-orders to pay for such goods.

Prepare and mark paper forms similar to these :

Money





Articles for Sale

Hat 75 ¢ *a*	Skates $ 1.25 *b*	Reader 30 ¢ *c*	Candy 35 ¢ *d*
1 Ton Coal $ 6.75 *e*	1 Sk. Flour $ 1.20 *f*	1 Bu Potatoes 60 ¢ *g*	1 Lb. Sugar 55 ¢ *h*
1 Loaf Bread 5 ¢ *i*	1 Cake 40 ¢ *j*	1 Doz. Oranges 30 ¢ *k*	Wafers 10 ¢ *l*

As many sets may be made as are needed for the work ; other tickets representing articles for sale may be devised.

1. Some pupils may act as salesmen and others as customers. Let each customer take at least 3 tickets and pay for them in money, offering the correct amount.

2. Let each customer take 3 tickets and pay with the smallest single piece of money whose value exceeds his purchase. Require the salesman to make the correct change.

Oral.

1. Carl purchased the three articles in the first column on page 42, or those shown by tickets *a*, *e*, *i*; how much did they cost?

2. Susie purchased those in the second column, or *b*, *f*, *j*; how much did she pay for them?

3. Myron purchased *d*, *h*, *l*; how much did they cost?

Find the cost of:

4. *a*, *b*, *c*, *d*.	5. *e*, *f*, *g*, *h*.	6. *i*, *j*, *k*, *l*.
7. *a*, *b*, *c*, *f*.	8. *e*, *f*, *i*, *j*.	9. *g*, *h*, *k*, *l*.
10. *c*, *d*, *g*, *h*.	11. *b*, *c*, *f*, *g*.	12. *f*, *g*, *j*, *h*.

Written.

13. Write the names of the articles and prices indicated in Exercises 4 and 5; find their total cost.

Find the total cost after writing the names and prices of the articles as indicated in Exercises:

14. 4 and 6.	15. 5 and 6.	16. 7 and 8.
17. 8 and 9.	18. 7 and 9.	19. 10 and 11.
20. 10 and 12.	21. 11 and 12.	22. 4, 7 and 10.
23. 5, 8 and 11.	24. 6, 9 and 12.	25. 4, 5 and 6.
26. 7, 8 and 9.	27. 10, 11 and 12.	28. 6, 8 and 10.

29. Write a bill with your own name as purchaser, and another pupil's name as dealer, for the items indicated in Exercise 14. (See page 30 for a correct form of a bill.)

30. Similarly write a bill for the items indicated in Exercise 26.

31. Make other combinations like those of Exercises 14 to 28, and write bills.

32. If convenient, bring bills to class and compute the amount of each.

Written.

A subscription agency offers the following sets of magazines at the prices quoted:

ALL OFFERS ARE FOR ONE FULL YEAR	
McClure's and Leslie's Popular Monthly	$1.80
McClure's and Scribner's Magazine	3.50
McClure's and Harper's Magazine	3.65
McClure's and Century Magazine	4.50
McClure's and Leslie's Popular Monthly and Munsey's	2.65
McClure's and Leslie's Popular Monthly and Cosmopolitan	2.65
McClure's and Munsey's and Cosmopolitan	2.65
McClure's and Leslie's Popular Monthly and Ainslee's	2.25
McClure's and Munsey's and Ainslee's	2.25
McClure's and Ainslee's and Pearson's	2.10
McClure's and Cosmopolitan	1.80
McClure's and Munsey's	1.85
McClure's and Ainslee's	1.45
McClure's and Ledger Monthly	1.30
McClure's and Youth's Companion	2.55

1. If the regular subscription price of McClure's and Leslie's magazines is $1 each per year, how much is saved in a year by accepting the offer of the agency?

2. If the regular price of Scribner's is $3, how much is saved by accepting the second offer of the agency?

3. Answer the same question for the third, fourth, and fifth offers, taking the regular price of Harper's as $4, of the Century as $4, and of Munsey's as $1.

4. Make and solve 10 other problems, taking the regular price of the Cosmopolitan as $1, of Ainslee's as $1.50, of Pearson's as $1, of the Ledger monthly as $1, and of the Youth's Companion as $1.75.

5. What is gained per dollar over the regular price by accepting the agency's offer on the first four sets?

6. If the agency reduces the present offer $\frac{1}{10}$ on all orders of $10 or over, what would be the total gain over the regular price in purchasing the first four sets?

7. What is the gain per dollar of the regular price in the fifth of the above offers? In the eleventh?

8. If convenient, bring similar advertisements and make and solve problems about them.

Oral.

1. How many eighths are there in 2 wholes? In 7? In 9? In 10? In 25? In 100?

2. Express in eighths: 3; 6; 7; 8; 25; 40; 50.

3. How many sevenths are there in 10? In 3? In 11? In 7? In 12? In 9? In 1?

4. 6 = — thirds. 10 = — fourths. 8 = — halves. 8 = — sevenths.

5. $15 = \frac{x}{6}$, $(x = ?)$; $12 = \frac{x}{100}$, $(x = ?)$; $8 = \frac{x}{9}$, $(x = ?)$.

6. $12 = \frac{a}{10}$, $(a = ?)$; $8 = \frac{a}{7}$, $(a = ?)$; $5 = \frac{a}{10}$, $(a = ?)$.

7. $7 = \frac{y}{5}$, $(y = ?)$; $6 = \frac{y}{5}$, $(y = ?)$; $5 = \frac{y}{6}$, $(y = ?)$.

8. A gardener in putting $\frac{1}{12}$ pk. of ground bone about each of his rose-bushes used 8 pk.; how many rose-bushes had he?

9. For one dinner of a certain army 4,000 lb. of beef were required; allowing $\frac{1}{5}$ of a pound for each man, how many men were there in the army?

10. A certain hotel provided 20 bu. of potatoes for one day's dinner on the basis of $\frac{1}{25}$ bu. per guest; how many guests were there in the hotel?

11. How many thirds of a yard are there in a yard? How many whole yards in 12 thirds of a yard?

12. How many whole pounds are forty tenths of a pound?

13. If $\frac{5}{8}$ of a pound of butter costs 15¢, what is the price of $\frac{1}{8}$ of a pound? Of a whole pound?

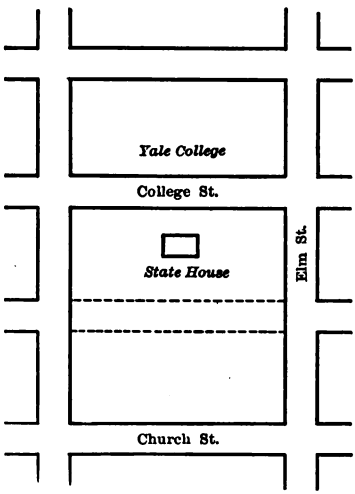
14. If $\frac{3}{10}$ of wheat bread is water, how much water is there in 100 one-pound loaves?

15. If .6 by weight of round steak is water, how much water is there in 10 lb. of steak?

1. What is meant by drawing to a scale?

2. If $\frac{1}{2}$ in. in a drawing represents 30 miles, how many miles do $3\frac{1}{2}$ in. represent?

3. In the picture of the New Haven Public Square, $\frac{1}{8}$ in. represents 100 ft. Find the length of a side of the square.



4. Find its area in square yards.

5. Measure Yale College grounds in the same way and find their area.

6. How many fence posts 3 yards apart must a railroad company furnish for a fence 2 miles long?

7. What is the cost of 25 street-car tickets at \$.04 each? At \$.05 each? What is the difference in cost?

8. Mr. Perkins sent a money-order in payment for a set of books which cost \$9.25; what did the money-order cost him?

9. If 100 lb. of sugar cost \$6, what is the cost of 1 lb.? Of 20 lb.? Of 50 lb.?

10. Express decimally, then as whole numbers and fractions:

125 and 1 hundredth. 36 and 3 hundredths.
16 and 5 hundredths. 28 and 39 hundredths.
17 and 25 hundredths. 25 and fifteen hundredths.

11. Make problems for the following:

100 yd. of silk cost \$140. $\frac{1}{2}$ of a ton of coal costs \$3.50.
100 yd. of ribbon cost \$25. $\frac{3}{4}$ of a pound of tea costs \$.60.

Written.

1. If .04 of any quantity of milk is cream, how many quarts of cream are there in 250 qt. of milk? In 360 qt.? In 482 qt.? Express decimally the results.

2. The three sides of a triangle are each $\frac{73}{100}$ ft. Express decimally the lengths of the sides and find the perimeter of the figure.

3. Mr. Adams owned a lot with a frontage of 107.25 ft. He bought two other lots adjacent to it, having frontages of $20\frac{2}{3}$ yd. and 60 ft. respectively. How many feet frontage did his three lots have?

4. A lot cost \$175. The house on it cost 12.35 times as much. Find the cost of the house.

5. John had 20 marbles and William had .7 as many; how many had William?

6. $12 \times 12 \times 12 = ?$ How many cubic inches are there in a cubic foot?

7. A cubic inch of mercury weighs about .49 lb. Find the weight of a cubic foot of mercury.

8. In 1900 about 14,200 persons were engaged in the glove industry in the United States. Of this number .31 were men, .67 women, and .02 were children. Find the number of each. Test your work by adding the results and comparing with the total number as given.

9. Express in inches: 1 yd.; 4 yd.; .07 yd.; 5 yd.

10. Express in quarts:

1 gal.;	.01 gal.;	.9 gal.;	2.5 gal.
.5 pk.;	.25 pk.;	.75 bu.;	.25 bu.
.50 bu.;	2.75 bu.;	5.5 bu.;	1.25 bu.

11. Add 17 yd. 2 ft. 8 in.; 12 yd. 1 ft. 9 in.; 15 yd.

12. The Place of Concord in Paris is 217 meters by 357 meters. Find its area in square meters.

FRACTIONS AND FORM STUDY

Addition and Subtraction

1. How many are 3 pages and 4 pages? 3 fifths and 4 fifths?

2. What is the sum of 3 qt. and 8 qt.? Of 3 fifths and 8 fifths?

3. Add .3 and .5; $\frac{2}{10}$ and $\frac{5}{10}$; .04 and .05; $\frac{9}{100}$ and $\frac{4}{100}$.

4. $\frac{3}{7}+\frac{3}{7}=?$ 5. $\frac{2}{3}+\frac{5}{3}=?$ 6. $\frac{1}{6}+\frac{4}{6}=?$

7. $\frac{11}{14}+\frac{2}{14}=?$ 8. $\frac{3}{10}+\frac{4}{10}=?$ 9. $\frac{1}{6}+\frac{3}{6}+\frac{7}{6}=?$

Add:

10. $\frac{11}{4}$, $\frac{15}{4}$, $\frac{3}{4}$

11. $\frac{1}{9}$, $\frac{9}{9}$, $\frac{10}{9}$

12. $\frac{3}{11}$, $\frac{2}{11}$, $\frac{4}{11}$

13. $\frac{5}{12}$, $\frac{11}{12}$, $\frac{7}{12}$

14. $\frac{3}{10}$, $\frac{9}{10}$, $\frac{7}{10}$

15. A man spent $\frac{2}{9}$ of his money for shoes and $\frac{5}{9}$ for books; what part of his money did he spend for both?

16. Two adjacent sides of a parallelogram are $\frac{4}{7}$ of a ft. and $\frac{2}{7}$ of a ft.; what is its perimeter?

17. A man set out on a journey; he walked $\frac{1}{10}$ of the way, rode $\frac{3}{10}$ on the electric car, and $\frac{5}{10}$ on the steam railway; what part of his journey had he then accomplished? How may these fractions be expressed more easily in writing?

18. How many are twelve books less four books?

19. How many are twelve sevenths less four sevenths?

20. How many are 12 hundredths less 4 hundredths?

Subtract:

21. $\frac{13}{8}$, $\frac{6}{8}$

22. $\frac{12}{15}$, $\frac{7}{15}$

23. $\frac{16}{18}$, $\frac{11}{18}$

24. $\frac{13}{10}$, $\frac{4}{10}$

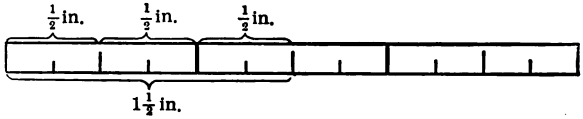
25. $\frac{26}{100}$, $\frac{6}{100}$

26. How may the fractions of Exercises 24 and 25 be expressed more easily in writing?

1. How many halves of an inch are there in 1 inch?

2. How many halves of an inch are there in $1\frac{1}{2}$ inches? In 3 in.?

3. $1\frac{1}{2}$ in. is what part of 3 in.?



4. How many fourths are there in 3 in.? $\frac{3}{4}$ of an inch is what part of 3 in.? Point this out in the figure.

$\frac{3}{4}$ *of an inch* $= \frac{1}{4}$ of 3 inches, or 3 *inches divided by 4.*

Show by diagram that:

5. $\frac{3}{5}$ of $1 = \frac{1}{5}$ of 3, or $3 \div 5$. 6. $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2, or $2 \div 3$.

7. $\frac{5}{6} = 5 \div 6$. 8. $\frac{4}{5} = 4 \div 5$. 9. $\frac{8}{3} = 8 \div 3$.

A fraction means the quotient of the numerator divided by the denominator.

Thus, $\frac{14}{7}$ means 14 divided into 7 equal parts, or 2.

Similarly, $\frac{1}{2}$ means 1 divided into 2 equal parts, or $1 \div 2$.

10. $\frac{2}{3} = \frac{1}{3}$ of —, or $2 \div$ —.

11. $\frac{2}{3}$ of 1 bu. $= \frac{1}{3}$ of — bu., or 2 bu. $\div$ —.

12. $\frac{5}{6}$ of a gal. $= \frac{1}{6}$ of — gal., or 5 gal. $\div$ —.

13. John worked 4 half days and Henry worked $\frac{1}{2}$ of 4 days; how many whole days did each work?

Perform the following divisions:

14. $18 \div 7$. 15. $140 \div 3$. 16. $17 \div 6$.

17. $2{,}141 \div 9$. 18. $21 \div 4$. 19. $3{,}043 \div 7$.

20. $73 \div 12$. 21. $3{,}140 \div 17$. 22. $5{,}246 \div 125$.

A number that is expressed as a whole number and a fraction is called a **mixed number.**

Express as mixed numbers:

23. $\frac{14}{3}$. 24. $\frac{11}{2}$. 25. $\frac{5}{3}$. 26. $\frac{6}{5}$.

27. $\frac{12}{9}$. 28. $\frac{23}{4}$. 29. $\frac{81}{10}$. 30. $\frac{125}{100}$.

Oral or written.

1. A lady bought $\frac{1}{4}$ yd. of black silk and $\frac{17}{4}$ yd. of red silk; how many yards of silk did she buy? Express the result as a mixed number.

2. A garden is $\frac{85}{4}$ ft. wide and $\frac{127}{4}$ ft. long; how long is the fence around it? Express as a whole number.

Add and express the results as mixed numbers:

3. $\frac{8}{5}$	4. $\frac{5}{8}$	5. $\frac{7}{10}$	6. $\frac{5}{13}$
$\frac{14}{5}$	$\frac{6}{8}$	$\frac{4}{10}$	$\frac{9}{13}$
7. $\frac{3}{4}$	8. $\frac{7}{9}$	9. $\frac{12}{11}$	10. $\frac{13}{15}$
$\frac{6}{4}$	$\frac{3}{9}$	$\frac{16}{11}$	$\frac{28}{15}$

Fractions that can be expressed as integers or as mixed numbers are called **improper fractions**; others are called **proper fractions.**

This does not mean that one kind is more properly a fraction than the other, but simply states the names which are in common use to designate these two kinds of fractions.

11. In a proper fraction, which is larger, the numerator or the denominator? In an improper fraction?

Point out the improper fractions:

12. $\frac{3}{5}$.	13. $\frac{11}{3}$.	14. $\frac{12}{4}$.	15. $\frac{16}{5}$.
16. $\frac{4}{9}$.	17. $\frac{9}{4}$.	18. $\frac{12}{17}$.	19. $\frac{9}{20}$.
20. $\frac{21}{21}$.	21. $\frac{24}{17}$.	22. $\frac{16}{23}$.	23. $\frac{36}{9}$.
24. $\frac{42}{14}$.	25. $\frac{25}{23}$.	26. $\frac{23}{24}$.	27. $\frac{8}{17}$.

28. Reduce the improper fractions of the list above to mixed or whole numbers.

Express as improper fractions:

29. $2\frac{3}{5}$.	30. $1\frac{1}{4}$.	31. $3\frac{1}{3}$.	32. $1\frac{4}{5}$.
33. $4\frac{1}{7}$.	34. $9\frac{2}{3}$.	35. $6\frac{4}{9}$.	36. $2\frac{2}{3}$.
37. $27\frac{2}{3}$.	38. $41\frac{4}{5}$.	39. $12\frac{1}{2}$.	40. $8\frac{1}{2}$.

Oral or written.

1. How wide must a piece of cloth be to make a ruffle 1 in. wide after having folded $\frac{1}{8}$ in. along each edge?

2. How many such pieces can be cut from a piece of cloth 5 in. wide?

3. If the edges of a piece of cloth 10 in. square are turned in $\frac{3}{4}$ in., what is the length of a side of the square thus formed?

4. How large must a square piece of cloth be cut so that it shall be 1 foot square after turning in $\frac{7}{8}$ in. for a hem?

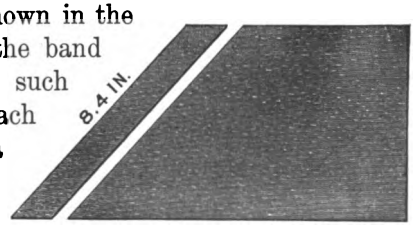
5. The cloth for making a book-bag is 24 in. long and 12 in. wide; how many book-bags can be cut from 6 yd. of duck 1 yd. wide? What will the cloth cost at 20¢ a yard?

6. Allowing 2¢ per bag for needles, thread, and tape, what is the cost of the materials for the book-bags?

7. Circular pieces of cloth 21 in. across are used to make cooking caps; how many can be cut from 5 yd. of cloth 22 in. wide?

8. A girl received 18 cents for making a shirt-waist on which she worked 3 hours. How much did she earn in an hour? If she makes shirt-waists at this price for a factory, how much does she earn in a 10-hour day? In a week?

9. Florence folded a piece of cloth 6 in. wide and cut a bias band 1 in. wide as shown in the picture. How long was the band which she made from 6 such strips, allowing $\frac{1}{2}$ in. for each seam? What part of a yard of cloth was cut into strips?



10. How many pieces of tape $6\frac{1}{2}$ in. long can be cut from a roll containing 5 yd. 5 in.?

1. What kind of angles does the dotted line in the figure form with the sides which it meets? How is this line related to the sides?

2. Cut out of paper a parallelogram.

3. Cut the parallelogram into two parts by a line perpendicular to the base as along the dotted line in the figure.

The perpendicular distance between either pair of parallel sides of a parallelogram is called an **altitude** of the figure.

4. Fit the two parts together so as to form a rectangle.

5. How would the area of the rectangle be found?

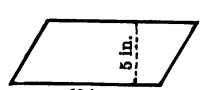
6. Compare the area of the rectangle with that of the figure. Compare their altitudes. Compare their bases.

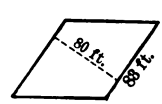
7. How may the area of the parallelogram be found?

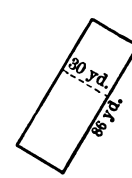
8. Cut out another parallelogram with base 4 in. and altitude 2 in.; what is the area of this parallelogram in square inches?

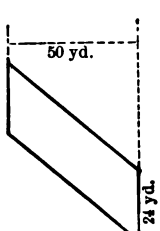
9. The product of what two numbers equals the number of square inches in the area of the parallelogram?

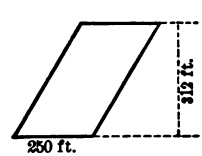
10. Find the area of each of the following parallelograms:

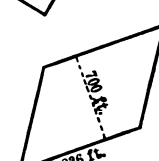




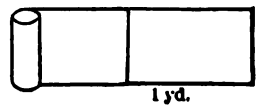




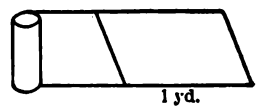




1. Compare the amount of velvet in the rectangular piece with that in the bias piece, as shown in the picture.



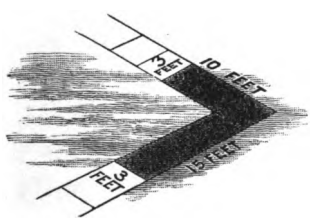
2. How many rectangular pieces of cardboard 9 in. by 12 in. can be cut from a piece a yard square?



Find the areas of parallelograms with the following bases and altitudes:

	3.	4.	5.	6.
Bases:	45 yd.	2.75 ft.	46 in.	315 ft.
Altitudes:	1.75 yd.	15 ft.	11.5 in.	.75 ft.

7. Find the area of the shaded portion of the walk shown in the picture.



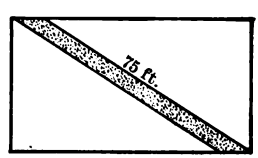
8. How many panes of glass are required for a factory with 145 windows, if each window contains 18 panes? If each pane is 12.5 in. by 9 in., how many square feet of glass are used?

9. How many acres in a farm in the shape of a rectangle which is 250 rods long and 125 rods wide? (160 sq. rd. = 1 acre.)

10. What is the altitude of a parallelogram that is 675 in. long and contains 135 sq. in.?

11. What is the cost of laying a cement walk $75\frac{1}{2}$ ft. long and 6 ft. wide at $\$0.16\frac{1}{2}$ a square foot?

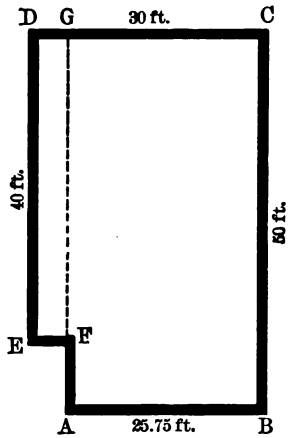
12. A man builds a gravel walk 5 ft. wide across a plat of ground, as indicated in the figure; how many square feet are there in its surface?



13. How many feet long is a park which contains 836,000 sq. yd. and is 1,425 ft. wide?

Written.

1. The figure represents the ground-plan of a house. Point out the parallelograms in the figure. What kind of angles have they? What other name have such parallelograms?



2. How many feet are there in the line A G? In the line G F? In the line A F? In the line E F? In the perimeter of the plan?

3. How many square feet are there in the area of the part A B C G? In the part E F G D? In the whole ground-plan? How many boards 10 feet long and 1 foot wide does it take to cover the floor?

4. If the house is 20 feet high from the foundation to the roof, what is the area of the side that stands on B C? Of the side that stands on D C? On D E? On E F? On F A? On A B?

5. How many square feet of boards are needed to cover the side B C? C D? D E? E F? A F? A B? To cover the six sides?

6. The roof takes 15 thousand shingles; what do they cost at $3.50 per thousand?

7. The cellar contains 10,800 cubic feet of earth; what will it cost to excavate the cellar at a cost of $.60 per cubic yard?

8. If the house costs at the rate of $3 for every square foot of the ground-plan, what does the whole house cost?

9. The house stands in the middle and lengthwise of a lot 50 ft. wide and 120.5 ft. long. How many feet of lot are there on each side of the house? At the end?

Oral. Supply the blanks; the tables of p. 244 may be consulted:

1. 12 in. = — ft. — sq. in. = 1 sq. ft.
2. — ft. = 1 rd. — rd. = 1 mi.
3. 3 ft. = — yd. — sq. ft. = 1 sq. yd.
4. — things = 1 doz. — doz. = 1 gross.
5. — sq. rd. = 1 acre. — acres = 1 sq. mi.
6. — yd. = 1 mi. — ft. = 1 mi.
7. How many feet in 1 yd. 2 ft.? In 5 yd. 1 ft.?
8. How many quarts in 2 gal. 2 qt.? In 5 gal. 3 qt.?
9. How many quarts in 2 pk. 3 qt.? In 3 pk. 5 qt.?
10. How many yards and feet besides in 25 ft.? In 17 ft.? In 35 ft.? In 23 ft.?
11. How many quarts and pints besides in 17 qt.? In 25 qt.? In 11 qt.?
12. How many bushels and quarts besides in 33 qt.? In 70 qt.? In 130 qt.?

Written. Add:

13. 9 ft. 7 in.
    3 5

14. 3 gal. 3 qt. 1 pt.
    5 2 1

15. 8 ft. 6.25 in.
    5 10.75

16. 25 lb. 3 oz.
    9 14

Subtract:

17. 13 ft. 0 in.
    3 5

18. 9 gal. 2 qt. 0 pt.
    5 2 1

19. 14 ft. 5 in.
    5 10.75

20. 35 lb. 1 oz.
    9 14

Multiply:

21. 13 lb. 5 oz.
    8

22. 1 da. 3 hr. 40 min.
    10

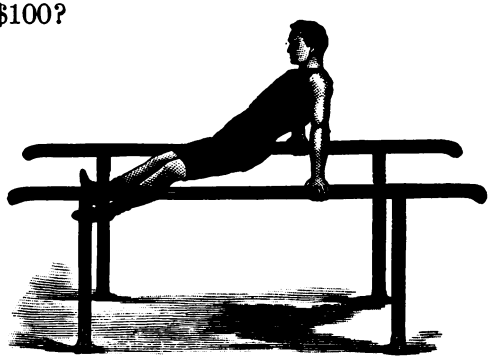
Written.

1. What will be the cost of a cement walk 125 ft. long and 5 ft. wide at $.15 per square foot?

2. How many tons of hay are needed to feed 25 horses for one year, allowing 10,000 lb. for each horse?

3. An agent bought for his employer 8 cottages at $1,200 each. He charged $24 per cottage. What was his rate of commission per $100?

4. What kind of straight lines are shown in the picture?



5. Of a 15,000 edition of a magazine, the number taken by the news-stands is $\frac{1}{2}$ of that mailed to subscribers; how many were there in each of these two lots?

6. When the wind is blowing 45 miles an hour, how long will it take a storm to cross New York State, 325 miles?

7. When the pressure of the wind is 10.12 lb. per square foot, what is the total pressure on a chimney whose surface is 10 ft. by 2 ft. 4 in.?

8. A certain village fire department has a hook-and-ladder truck which cost $350, a hose cart which cost $180, two horses worth $150 each, and other equipment worth $125; what is the total value of the outfit?

9. If the cost of the department building and of the alarm system is $8,000, how much money is invested in the whole property?

10. If 6 men are employed, 3 for the day and 3 for the night, each at a salary of $40 per month, what is the annual cost of the paid service?

1. What is meant by parallel lines? Illustrate.

2. What is a parallelogram? Illustrate your answer.

3. Are all rectangles parallelograms? Are all parallelograms rectangles? Draw figures to illustrate your answers.

4. What is a vertical line? A horizontal line? A level surface? Name something to illustrate each.

5. What is meant by an altitude of a parallelogram? How many altitudes has a parallelogram? Illustrate.

6. How is the area of a parallelogram found? What is the area of a parallelogram whose base is 4 ft. and whose altitude is 1.5 ft.?

7. What is the sum of $\frac{6}{11}$, $\frac{3}{11}$, $\frac{5}{11}$, $\frac{8}{11}$, $\frac{9}{11}$?

8. How are fractions having like denominators added? How are they subtracted?

9. What is meant by a proper fraction? By an improper fraction? Illustrate your answers.

10. How is any decimal multiplied by 10? By 100?

Express as a whole number and a proper fraction:

11. $\frac{9}{8}$. 12. $\frac{21}{6}$. 13. $\frac{42}{11}$. 14. $\frac{25}{4}$.

15. 1.25. 16. 1.6. 17. 2.50. 18. 19.75.

Written. Express as an improper fraction:

19. $3\frac{1}{4}$. 20. $7\frac{8}{3}$. 21. $6\frac{4}{5}$. 22. $10\frac{3}{4}$.

23. $11\frac{1}{7}$. 24. $20\frac{2}{3}$. 25. $40\frac{1}{5}$. 26. $12\frac{1}{2}$.

27. 6.3. 28. 12.5. 29. 9.6. 30. 19.75.

31. Determine the numbers to fill the blanks in the table:

Cows.	Product.	Amount per Cow.	Total.	Price.	Value.
110	Butter	130 lb.		$0.18	
10	Cheese	300 lb.		0.09	
55	Milk	380 gal.		0.08	

32. Make and solve 3 problems about this table.

Oral.

1. At $\$\frac{1}{8}$ each, what is the cost of 24 cans of cocoa?

2. A man walks $\frac{1}{20}$ mi. per minute; how far does he walk in 30 min.?

3. A boy runs $\frac{1}{15}$ mi. per minute; how far does he run in 15 min.?

4. In 1900, $\frac{249}{250}$ of all the collars and cuffs made in the United States were made in the State of New York; what part of the whole number was made in all the rest of the country?

5. In June $\frac{1}{10}$ of all the days were rainy, $\frac{3}{10}$ were cloudy, and the rest were clear; what part of the whole number of days was clear?

6. Find the perimeter of a parallelogram in inches whose adjacent sides are 3.75 ft. and $3\frac{1}{2}$ ft.

7. Find the perimeter of a door whose height is 6 ft. 6 in. and whose breadth is $3\frac{1}{2}$ ft.

8. If $\frac{7}{8}$ of a yard of cloth costs 35¢, what is the cost of $\frac{1}{8}$ yd.? Of 1 yd.?

9. What is the cost of 25 qt. of milk at 4¢ a quart? At 5¢ a quart? At 6¢ a quart?

10. Multiply by 100: 3.15; 4.05; 2.1; 210; 7.6.

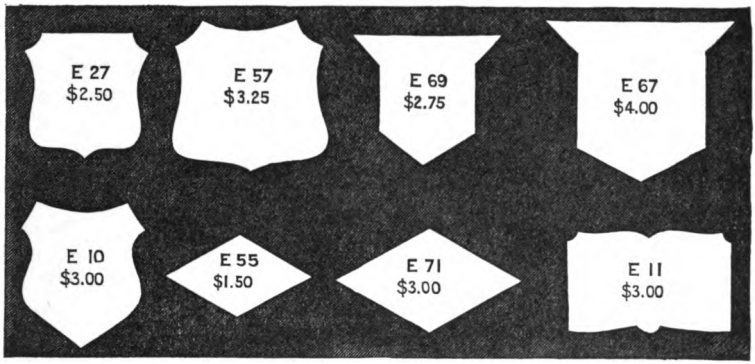
11. At $1.25 per $100, what is the cost of insuring goods valued at $200? At $500? At $1,000?

12. The ground-plan of a house is in the form of a rectangle 40 ft. long and 30 ft. wide. The cost of building it was $1.50 per square foot of the ground-plan. What was its whole cost?

13. A coat has a row of 8 buttons set $2\frac{1}{2}$ in. apart; how long is the row?

14. An ordinary gas-burner consumes 7 ft. of gas per hour; how many feet does it consume in 10 hr.? In a week when lighted 5 hr. per day?

1. The picture shows various forms of gummed labels with their prices per 10,000.



Find the cost of 100,000 of each kind of label.

2. The labels are packed in boxes of 1,000 each. How many boxes are used for 500,000 labels?

3. Which label costs the more, E 55 or E 69? How much more per 100,000?

4. Make and solve 3 other problems about the labels.

5. While taking a railway trip Paul noticed that there were marked on each freight car its weight when empty and the load it could carry. A number of these were:

	Weight.	Capacity.		Weight.	Capacity.
1st.	26,400 lb.	40,000 lb.	5th.	36,400 lb.	60,000 lb.
2nd.	32,000 "	50,000 "	6th.	31,000 "	60,000 "
3rd.	30,800 "	50,000 "	7th.	29,750 "	50,000 "
4th.	33,600 "	60,000 "	8th.	32,000 "	60,000 "

Express in tons or tons and pounds each of these weights.

6. Express in tons or tons and pounds the weight of each car when fully loaded.

DIVISIBILITY AND FRACTIONS

Divisibility

Oral.

1. Name the multiples of 2 from 2 to 30. If each of these is divided by 2, what is the remainder? These numbers are said to be **divisible** by 2.

2. Name the multiples of 3 from 3 to 30. If each of these are divided by 3, what is the remainder? These numbers are said to be **divisible** by 3.

An integer is said to be **divisible** by another if the quotient is an integer and the remainder zero.

3. Integers divisible by 2 are called **even** numbers, and all other integers are called **odd** numbers. Name the even numbers from 2 to 10; from 20 to 30; from 50 to 60.

4. In what numerals do the even numbers end? In what numerals do the odd numbers end?

5. Name the numbers divisible by 5 from 5 to 20; from 50 to 100.

6. In what digits do numbers divisible by 5 end?

7. Which of the following numbers are even, and which are odd: 7, 15, 12, 29, 32, 35, 16, 21, 44, 95?

8. Tell at sight which of the above numbers are divisible by 5.

9. Two numbers that are divisible by the same number are said to have a **common divisor.** What common divisor have all even numbers? All multiples of 3? Of 5? Of 12?

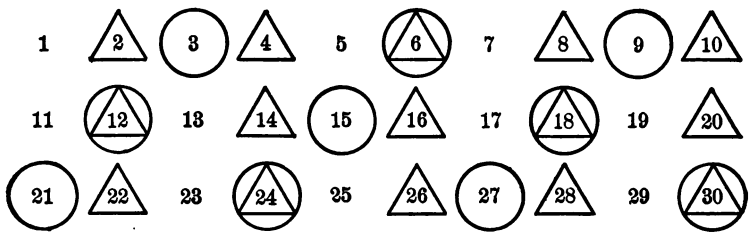
Find a common divisor of each pair of numbers:

10. 18, 27. 11. 24, 36. 12. 16, 40. 13. 15, 35.

14. Name the fractions of the following list in which the numerator and denominator have a common divisor:

$\frac{2}{4}$; $\frac{3}{5}$; $\frac{4}{6}$; $\frac{5}{8}$; $\frac{3}{8}$; $\frac{6}{8}$; $\frac{5}{10}$; $\frac{4}{100}$; $\frac{25}{100}$; $\frac{17}{25}$; $\frac{9}{10}$.

1. Find the numbers in the table divisible by 2. Read those divisible by 3.



2. By what form are all the numbers divisible by 2 enclosed?

3. By what form are all the numbers divisible by 3 enclosed?

4. How are all the numbers divisible by both 2 and 3 enclosed?

5. Name a common divisor of all the numbers in the table; of the numbers enclosed by a triangle; by a circle.

6. Name a common divisor of the numbers enclosed by both a triangle and a circle.

7. Write on the board in ten rows the numbers from 1 to 100. Cross in white all the even numbers; in red all the numbers divisible by 3; in yellow all the numbers divisible by 5.

8. What kind of numbers are those not crossed? Are there any odd numbers crossed?

9. What digits occupy units' place in all numbers divisible by 5?

10. With what colors are the numbers divisible by 2 and 3 crossed? Read these numbers. Read the multiples of 6 from the table.

11. Read the numbers not divisible by 2, 3, and 5.

Oral.

1. 3 books and 2 books are 5 —.

2. 3 pencils and 4 pencils are 7 —.

3. 3 books and 4 pencils are 7 —.

Only numbers of the *same kind* can be added or subtracted. 3 books (things) and 4 pencils (things) are 7 —.

4. 3 gallons (12 quarts) and 2 quarts are 14 —.

5. 3 fourths and 1 half (2 fourths) are 5 —.

6. How much cloth is there in $\frac{1}{8}$ of a yard and $\frac{1}{2}$ of a yard?

PLAN. 1. $\frac{1}{2}$ yd. = — eighths of a yard.
2. $\frac{1}{8}$ yd. and $\frac{4}{8}$ yd. = — yd.

7. What part of a day is $\frac{1}{2}$ da. and $\frac{1}{3}$ da.?

PLAN. 1. $\frac{1}{2}$ da. = — sixths of a day.
2. $\frac{1}{3}$ da. = — sixths of a day.
3. $\frac{3}{6}$ da. + $\frac{2}{6}$ da. = — da.

8. How many days are there in the sum of $\frac{1}{2}$ da. and $\frac{2}{3}$ da.?

9. What part of an hour is $\frac{1}{3}$ of an hour and $\frac{1}{4}$ of an hour?

PLAN. 1. $\frac{1}{3}$ hr. = — twelfths of an hr.
2. $\frac{1}{4}$ hr. = — twelfths of an hr.
3. $\frac{4}{12}$ hr. + $\frac{3}{12}$ hr. = — hr.

10. Clarence had $\frac{1}{2}$ of a dollar and spent $\frac{1}{4}$ of a dollar; what part of a dollar had he left?

PLAN. 1. $\$\frac{1}{2}$ = — fourths of a dollar.
2. $\$\frac{2}{4} - \$\frac{1}{4} = \$$—.

11. Susie solved $\frac{1}{3}$ of a page of problems one day and $\frac{1}{2}$ of a page the next; on which day did she solve the more? What part of a page more?

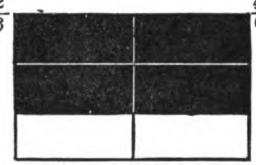
Only fractions of the *same kind*, that is, those with *like denominators*, can be added or subtracted.

12. $\frac{1}{2} + \frac{3}{8} = —$. 13. $\frac{3}{4} + \frac{5}{8} = —$. 14. $\frac{2}{3} + \frac{5}{6} = —$.

15. $\frac{5}{8} - \frac{1}{2} = —$. 16. $\frac{7}{8} - \frac{3}{4} = —$. 17. $\frac{3}{4} - \frac{2}{3} = —$.

1. Show what part of the figure represents $\frac{2}{3}$ of it.

2. Into how many equal parts does the middle line divide each third? Into how many equal parts is the figure divided by all the cross-lines? How many sixths are there in $\frac{2}{3}$?



3. Draw a figure like the one above and divide each part into halves by a line parallel to the middle one. Find how many twelfths there are in $\frac{2}{3}$.

4. Draw a circle and by lines from the center divide its area into fifths as nearly as you can; then into tenths. Find how many tenths there are in $\frac{2}{5}$; in $\frac{3}{5}$; in $\frac{4}{5}$.

5. If the number of equal parts into which a unit is divided is made twice as great, how is each part changed in size? If the number is made 3 times as great? 4 times? 5 times?

6. How many sixths of a day must one work to work $\frac{1}{3}$ of a day? How many sixths of a day must one work to work $\frac{1}{2}$ of a day?

7. If some bags of grain contain only $\frac{1}{2}$ as much as others, how many of the smaller must be taken to contain as much grain as 1 of the larger? If some are $\frac{1}{4}$ as large as others?

8. Which term of a fraction shows the number of parts into which the unit is divided? Which shows how many are taken?

9. How many sixteenths must be taken to equal $\frac{7}{8}$? Why? How many tenths must be taken to equal $\frac{3}{5}$? Why?

10. What effect on the value of the fraction has multiplying or dividing both terms by 2?

11. Show as in Exercise 9 that $\frac{3}{9} = \frac{9}{27}$. What effect has multiplying or dividing both terms of a fraction by 3? By any other number?

Show with and without diagrams the relations between:

1. $\frac{2}{4}$ and $\frac{1}{2}$. 2. $\frac{6}{8}$ and $\frac{3}{4}$. 3. $\frac{9}{12}$ and $\frac{3}{4}$.

4. $\frac{6}{9}$ and $\frac{2}{3}$. 5. $\frac{4}{12}$ and $\frac{1}{3}$. 6. $\frac{10}{12}$ and $\frac{5}{6}$.

When the terms of a fraction have been divided by a common divisor to form a fraction of smaller terms, the given fraction is said to be **simplified** or **reduced to lower terms.**

When all the common divisors have been removed the fraction is said to be simplified as far as possible, or **reduced to its lowest terms.**

Thus: $\frac{10}{12} = \frac{2 \times 5}{2 \times 6} = \frac{5}{6}$ is reduced to its lowest terms.

Also: $\frac{45}{60} = \frac{5 \times 9}{5 \times 12} = \frac{9}{12} = \frac{3 \times 3}{3 \times 4} = \frac{3}{4}$ is reduced to its lowest terms.

Reduce to lowest terms:

7. $\frac{9}{12}$. 8. $\frac{4}{16}$. 9. $\frac{10}{15}$. 10. $\frac{18}{21}$. 11. $\frac{25}{26}$.

12. $\frac{28}{21}$. 13. $\frac{36}{45}$. 14. $\frac{64}{50}$. 15. $\frac{10}{100}$. 16. $\frac{50}{100}$.

17. $\frac{75}{100}$. 18. $\frac{80}{100}$. 19. $\frac{20}{100}$. 20. $\frac{25}{100}$. 21. $\frac{2500}{100}$.

Write these decimals in the form of common fractions and reduce to lowest terms:

22. .05. 23. .05. 24. .50. 25. .55. 26. .75.

27. .90. 28. .15. 29. .25. 30. 6.25. 31. 1.05.

32. 1.5. 33. 2.5. 34. 1.25. 35. 10.25. 36. 26.25.

37. Add $\frac{3}{8}$ and $\frac{1}{8}$ and reduce the result to its lowest terms.

38. Add $\frac{1}{12}$ and $\frac{5}{12}$ and reduce the result to its lowest terms.

39. Add $\frac{2}{15}$, $\frac{5}{15}$, $\frac{1}{15}$, $\frac{4}{15}$, and reduce the result to its lowest terms.

40. The five countries of the world having the most miles of railway are the United States, Russia, Germany, France, and Great Britain. Russia has $\frac{6}{30}$ as many miles as the United States, Germany has $\frac{5}{30}$, France has $\frac{4}{30}$, and Great Britain has $\frac{3}{30}$; what part of the mileage of the United States do these together equal?

Oral.

1. How much is 3 times 2 pounds?
2. How much is 3 times 2 tenths? 3 times $\frac{2}{10}$?
3. How much is 3 times 2 fifths? 3 times $\frac{2}{5}$?
4. How much is 4 times 3 dollars? 4 times $\frac{3}{13}$?
5. How may a fraction be multiplied by an integer?

Multiply by 6:

6. $\frac{2}{5}$. 7. $\frac{3}{7}$. 8. $\frac{4}{11}$. 9. $\frac{11}{3}$. 10. $\frac{5}{17}$.

11. $5 \times \frac{4}{6} = ?$ 12. $4 \times \frac{3}{7} = ?$ 13. $5 \times \frac{2}{9} = ?$
14. $13 \times \frac{4}{9} = ?$ 15. $12 \times \frac{3}{5} = ?$ 16. $7 \times \frac{1}{45} = ?$
17. $8 \times \frac{3}{7} = ?$ 18. $2 \times \frac{3}{13} = ?$ 19. $25 \times \frac{4}{9} = ?$
20. $7 \times \frac{3}{7} = ?$ 21. $12 \times \frac{11}{12} = ?$ 22. $6 \times \frac{11}{6} = ?$

Written.

23. About $\frac{1}{75}$ of all the sewing-machines made in the United States in 1900 were made in Pennsylvania, 11 times as many in Massachusetts, 16 times as many in New Jersey, and 19 times as many in Illinois; what part of the whole number made were made in each State named? In all of them?

24. About $\frac{1}{5}$ of air is oxygen and the rest nitrogen; how many cubic feet of each are there in a room 12 ft. wide, 16 ft. long, and 10 ft. high?

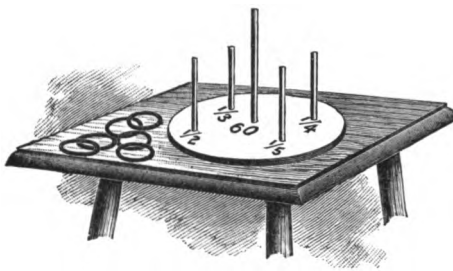
25. A man breathes about 20 times in a minute; how many times does he breathe in an hour? In $\frac{1}{2}$ hr.? In $\frac{1}{4}$ hr.? In $\frac{3}{4}$ hr.? In $\frac{1}{5}$ hr.? In $\frac{3}{5}$ hr.? In $\frac{4}{5}$ hr.?

26. At each breath he takes about 30 cu. in. of air; how many thousand cubic inches does he breathe in 1 hr.?

27. According to Exercises 24–26, how many cubic feet of oxygen does he breathe in an hour? How many cubic feet of nitrogen?

28. How many cubic inches of each gas are breathed in $\frac{1}{4}$ of an hour? In $\frac{3}{4}$? In $\frac{1}{5}$? In $\frac{4}{5}$?

1. The picture shows a board in which 5 pins are set, the center one being the highest. The player stands at a distance and throws 5 rings, one at a time. Each ring that drops on the center pin counts 60. If a ring drops on the pin marked $\frac{1}{3}$, it counts off $\frac{1}{3}$ of 60. If one drops on the pin marked $\frac{1}{2}$, it counts off $\frac{1}{2}$ of 60, and so on. Only those that drop on pins count. Charlotte threw 2 rings on 60, one on $\frac{1}{3}$, one on $\frac{1}{5}$, and the other missed entirely; what was her count?



2. The following numbers show how the rings fell in a contest between George and Myron:

George	$\frac{1}{5}$,	60,	$\frac{1}{4}$,	60,	$\frac{1}{5}$.
Myron	60,	0,	$\frac{1}{5}$,	$\frac{1}{3}$,	$\frac{1}{4}$.

Who won the game, and by how many points?

3. The number at the center and the fractions may be changed as desired. In Exercise 2, if the number at the center had been 120, what would have been the count?

4. Ralph made a board similar to the one shown in the picture, but put 1 at the center, and $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ at the other pins. The rule was to add all the counts made with 5 rings; the player won who had the greatest number. In 5 throws, the first ring missed, the second dropped on 1, the third on $\frac{1}{2}$, the fourth on $\frac{1}{8}$, and the fifth on $\frac{1}{2}$; what was the count?

5. Who won the following game played as described in Exercise 4?

Louise	$\frac{1}{2}$,	0,	$\frac{1}{8}$,	$\frac{3}{4}$,	0.
Susie	0,	1,	$\frac{1}{2}$,	$\frac{1}{2}$,	0.

6. Make and solve 5 problems for these games.

DECIMALS AND FORM STUDY

Thousandths

1. How many blocks like Figure 1 does Figure 2 contain? How many blocks like Figure 2 does Figure 3 contain?

Figure 1

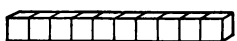


Figure 2

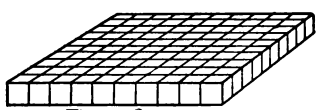


Figure 3

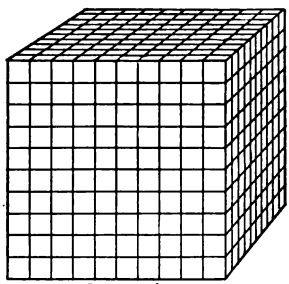


Figure 4

How many blocks like Figure 3 does Figure 4 contain? Like Figure 2? Like Figure 1?

2. If Figure 4 represents a cubic foot, what part of a cubic foot does Figure 3 represent? Figure 2? Figure 1?

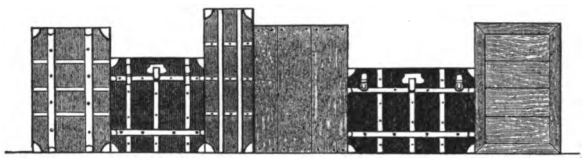
3. How many thousandths of a cubic foot are there in a cubic foot? In $\frac{1}{10}$ of a cubic foot? In $\frac{1}{100}$ of a cubic foot? In $\frac{3}{100}$ of a cubic foot? In $\frac{70}{100}$ of a cubic foot?

4. In a pile of 1,000 bricks one brick is what part of the whole pile? $\frac{50}{100}$ of this pile contains how many thousandths of it? $\frac{1}{2}$ of this pile contains how many thousandths?

5. How many thousandths in $\frac{1}{10}$ of a unit? In .3 of a unit? In $\frac{1}{100}$ of a unit? In .07 of a unit? In .37 of a unit?

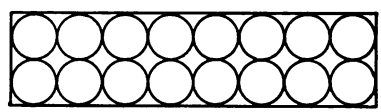
A number of thousandths is expressed by 3 decimal places, the third place at the right of the decimal point being called **thousandths' place.** Thus, 3 thousandths is expressed decimally by .003. 15 thousandths is expressed decimally by .015; and 625 thousandths by .625.

1. A baggageman puts these trunks and boxes into a space of 20 ft. Their widths are 2 ft. 3 in., 2 ft. 8 in., 1 ft. 6 in., 2 ft. 9 in., 3 ft. 8 in., 2 ft. 6 in.; how much space is left?



2. Would boxes having the widths 3 ft., 4 ft. 3 in., $1\frac{1}{2}$ ft., and 3.75 ft. make a row on a truck 12 ft. long?

3. The figure is a diagram of the bottom of a wagon-box, showing the number of barrels it carries at a load. If each barrel is 22 in. wide at its greatest width, what is the length and breadth of the wagon-box?



4. Flat steel bars 1 inch wide and $\frac{1}{2}$ of an inch thick weigh 1.706 lb. per foot of length; what is the weight of 10 ft. of this steel? Of 100 ft.? Of 1,000 ft.?

5. The following table gives the weights of several sizes of steel bars:

Inches Thick.	1	2	3	4
Inches Wide.	Pounds.	Pounds.	Pounds.	Pounds.
2	1.706	3.412	5.118	6.824
3	2.560	5.120	7.680	10.240
4	3.413	6.826	10.239	13.652
5	4.266	8.532	12.798	17.064

What is the weight of 10 ft. of each kind in the first column?

6. A bar 5 in. wide and 1 in. thick weighs how much more per ft. than a bar 2 in. wide and 2 in. thick?

7. Make and solve 3 other problems from the above table.

1. What is the shape of a wagon wheel? How do the spokes compare in length? How do the lines O A, O C, O B in Figure 2 compare?

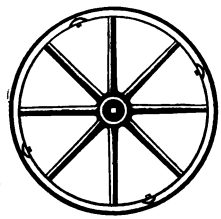


Figure 1.

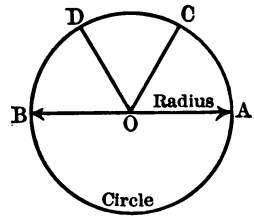


Figure 2.

2. Tie a string to your pencil and pin down one point as shown in Figure 3. By moving the pencil around the pin, keeping the pencil vertical and the string taut, a closed line is drawn. Draw several such lines. A string and crayon may be used at the blackboard, the string being held by one finger of the left hand.

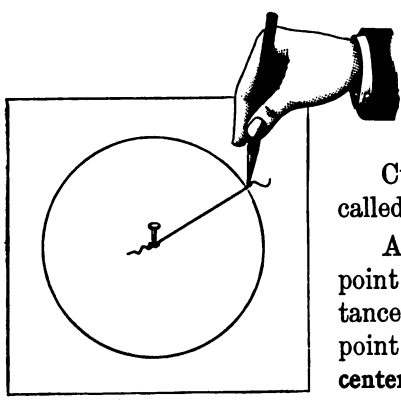


Figure 3.

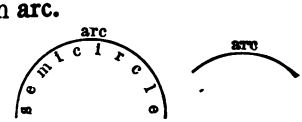
Curves like those drawn are called **circles.**

A **circle** is a plane curve every point of which is at the same distance from a fixed point. This point is within and is called the **center.**

3. What part of a wagon wheel is at the center? What point is the center of the circle in Figure 2?

Any portion of a circle is called an **arc.**

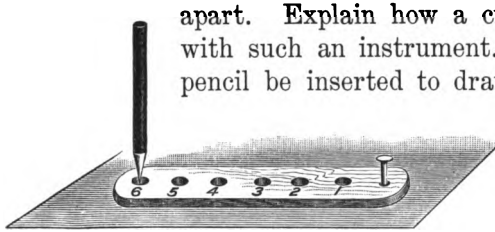
The straight line joining the center to any point of the circle is called a **radius** (plural, **radii**).



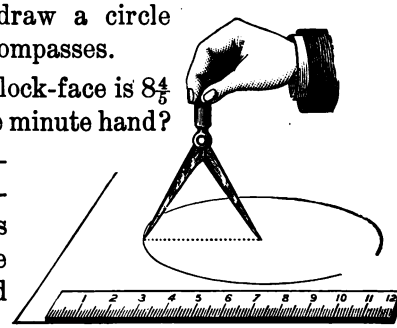
4. Name lines that represent radii in Figures 1, 2.

A straight line drawn through the center and terminated by the circle is called a **diameter,** as A B in Figure 2.

1. The picture shows a piece of wood with holes 1 in. apart. Explain how a circle may be drawn with such an instrument. Where would the pencil be inserted to draw a circle of 5 in. radius? One of 3 in. radius?



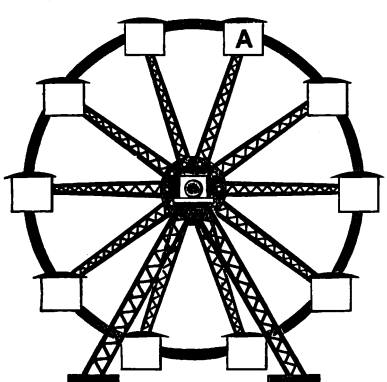
2. Arcs and circles can be drawn better by use of an instrument called *compasses*. Explain from the picture how to draw a circle of 4-inch radius with the compasses.



3. If the diameter of a clock-face is $8\frac{4}{5}$ in., what is the length of the minute hand?

4. The diameter of a bicycle wheel is 26 in. Allowing $1\frac{5}{8}$ in. for the thickness of the tire and 1 in. for the diameter of the hub, find the length of a spoke.

5. The radius of the Ferris wheel at the World's Fair, 1893, was about 125 ft.; how high from the ground was Albert when his car, A, reached the top?



6. The diameter of the earth is about 8,000 mi. and that of the sun about 850,000 mi. For rough comparison the diameter of the sun may be taken as 100 times that of the earth. If a circle of diameter $\frac{1}{4}$ in. represents the earth, how may the sun be represented?

1. What is the name of the first decimal place at the right of the decimal point? Of the second? Of the third?

Write decimally:

2. Five thousandths.
3. 175 thousandths.
4. 100 thousandths.
5. Twenty-five thousandths.
6. 100 and 75 thousandths.
7. 200 and 5 thousandths.

8. A yard of copper wire weighs 3.125 oz.; what is the weight of 10 yd. of the wire? Of 100 yd.? Of 1,000 yd.?

9. How is a decimal multiplied by 10? By 100?

Add:

10.	11.	12.	13.
1.706	3.412	5.118	6.824
2.560	5.120	7.180	10.240
3.413	6.826	10.239	13.652
4.266	8.532	12.789	17.064

Find the product of:

14. 17.06 × 2.56

15. 68.26 × 853.2

16. 13.652 × 1706

Find the quotient of:

17. $225 \div .25$.
18. $1875 \div 7.5$.
19. $1331 \div 1.1$.
20. $146.41 \div .121$.
21. $13003 \div 1.47$.
22. $25641 \div 3.33$.

23. What is the altitude of a triangle? How many altitudes has a triangle? Draw a triangle and all of its altitudes.

24. What is an odd number? An even number? In what digits do all even numbers end? All odd numbers?

25. When is one number said to be divisible by another?

26. What is a common divisor of several numbers? Illustrate your answer.

1. How can you tell without dividing whether or not a number is divisible by 5?

2. Which of the following numbers are divisible by 5: 63? 50? 875? 360? 417? 405?

3. What kind of a figure is a circle? What is a diameter? A radius? The length of a diameter is how many times that of the radius?

4. What is an arc? Illustrate your answer.

5. A bicycle wheel is 26 in. in diameter; what is the length of the radius?

6. A section of the earth through the center would be approximately a circle. The diameter of the earth is about 8,000 mi.; what is the distance from the surface to the center?

7. How must fractions be expressed before they can be added or subtracted?

Add:

8. $\frac{1}{2} + \frac{5}{8}$. 9. $\frac{2}{3} + \frac{5}{6}$. 10. $\frac{2}{3} + \frac{2}{5}$.

11. $\frac{1}{4} + \frac{3}{8}$. 12. $\frac{3}{4} + \frac{5}{8}$. 13. $\frac{1}{2} + \frac{1}{8}$.

14. $\frac{2}{5} + \frac{3}{10}$. 15. $\frac{1}{5} + \frac{7}{10}$. 16. $\frac{4}{5} + \frac{9}{10}$.

Subtract:

17. $\frac{7}{10} - \frac{3}{5}$. 18. $\frac{5}{8} - \frac{1}{2}$. 19. $\frac{7}{8} - \frac{3}{4}$.

20. $\frac{9}{11} - \frac{4}{5}$. 21. $\frac{7}{8} - \frac{1}{2}$. 22. $\frac{5}{6} - \frac{2}{3}$.

23. What effect has multiplying both terms of a fraction by the same number? What effect has dividing both terms by the same number?

24. What is meant by "reducing a fraction to lower terms"? "To its lowest terms"? Illustrate your answers.

25. When is a fraction said to be simplified? Illustrate.

Reduce to lowest terms:

26. $\frac{9}{27}$. 27. $\frac{13}{52}$. 28. $\frac{17}{51}$. 29. $\frac{45}{60}$.

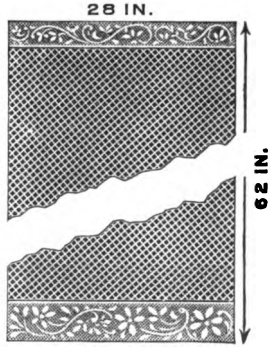
30. $\frac{14}{42}$. 31. $\frac{22}{55}$. 32. $\frac{12}{100}$. 33. $\frac{18}{60}$.

1. The table shows the average cost per year of various necessities for one person:

Year.	Breadstuffs.	Meats.	Clothing.
1896	$10.504	$7.058	$13.602
1897	10.587	7.529	13.808
1898	12.783	7.694	14.663
1899	13.483	7.988	15.021
1900	14.898	8.906	16.324
1901	14.904	9.430	15.098
1902	20.534	11.628	15.533
1903	17.473	9.269	17.136

Find the change in cost of each item from year to year.

2. The figure shows the length and width of a curtain. Find the number of yards of cloth in 6 pairs of these curtains. What is the cost of a dozen pairs at $1.75 a pair?



3. What is the perimeter of a parallelogram whose adjacent sides are 2.6 in. and 9.7 in.?

4. How many years are there in $\frac{5}{6}$ of a year and $\frac{3}{4}$ of a year? In $\frac{5}{6}$ of a year and 9 mo.?

5. Lucy had $\frac{3}{4}$ of a dollar and spent $\frac{1}{2}$ of a dollar; what part of a dollar had she left?

6. Express the following numbers decimally:

$\frac{3}{10}$; $\frac{5}{100}$; $\frac{25}{100}$; $\frac{65}{10}$; $\frac{12}{100}$; $\frac{12}{10}$; $\frac{60}{10}$; $\frac{117}{100}$; $\frac{40}{10}$; $\frac{700}{100}$.

Add and simplify the results:

7. $\frac{12}{7}$, $\frac{3}{7}$, $\frac{8}{7}$

8. $\frac{3}{5}$, $\frac{7}{5}$, $\frac{1}{5}$

9. $\frac{2}{9}$, $\frac{1}{9}$, $\frac{7}{9}$

10. $\frac{4}{12}$, $\frac{7}{12}$, $\frac{1}{12}$

DECIMALS AND MEASUREMENT

Pointing Off in Multiplication

1. The base of a rectangle is 4.5 in. and its altitude is 2.3 in.; how is its area found?

Observe the relation between these three multiplications:

45 tenths	or 4.5	The result of multiplying	4.5
23	23	4.5 by 23 is how many times	2.3
135	135	as great as that of multiply-	135
90	90	ing it by 2.3?	90
1035 tenths	103.5	$\frac{1}{10}$ of 103.5 = ?	10.35

The numbers multiplied to form a product are called the **factors** of the product.

Factors may not be integers, but divisors must be.

2. Name the factors in the above multiplication.

3. How does the number of decimal places in the product compare with the total number in the factors?

4. Find the area of a parallelogram of base 21.36 ft. and altitude 3.8 ft.

Observe the relation between these three multiplications:

2136 hundredths	or 21.36	The result of mul-	21.36
38	38	tiplying 21.36 by 38	3.8
17088	17088	is how many times as	17088
6408	6408	great as that of mul-	6408
81168 hundredths	811.68	tiplying it by 3.8?	81.168
		$\frac{1}{10}$ of 811.68 = ?	

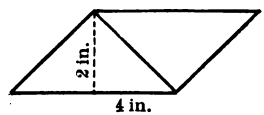
The *number of decimal places* in the *product* equals the *total number* in the *factors.*

5. Multiply 25.36 by 98.5; 174.3 by 4.58.

1. Draw a parallelogram. Draw a line connecting a pair of opposite corners, or vertices. This line is called a **diagonal.**

2. Draw rectangles and other parallelograms and their diagonals. How many diagonals has every parallelogram?

3. Draw a parallelogram like the figure, and of the size indicated. Draw the altitude and diagonal. Cut out the triangles into which the diagonal divides the parallelogram and show that they are equal.



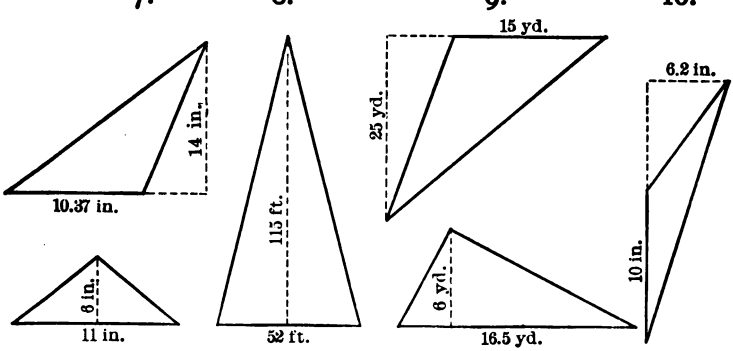
4. What is the area of the parallelogram? How is it found? Each triangle is what part of the parallelogram?

5. How does the base of the triangle compare with that of the parallelogram? How do the altitudes compare?

6. The number of square units of area in a triangle equals one-half of the product of what numbers?

Find the areas of the following triangles:

7. 8. 9. 10.



11. How is the area of a triangle found? Find the area of a triangle of base 16.05 in. and altitude 9.9 in.

PLAN. 1. $9.9 \times 16.05 = ?$ 2. $\frac{1}{2}$ of 158.895 sq. in. $=$ —sq. in.

12. Find the area when the base is 6.25 in. and the altitude 4 in.

Written.

A square foot of lumber 1 in. or less in thickness is called a **board foot.** In lumber more than 1 in. thick, the number of board feet depends upon the thickness.

Thus, a board 8 in. wide and 3 ft. long and $\frac{1}{2}$ in. thick contains 2 board feet, and a board 8 in. wide, 3 ft. long, and $1\frac{1}{2}$ in. thick contains 3 board feet.

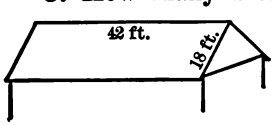
How many feet of lumber are there in:

1. 3 boards each 16 ft. long, 10 in. wide, and 1 in. thick?
2. 12 planks each 12 ft. long, 14 in. wide, and 2 in. thick?
3. 150 boards each 14 ft. long, 16 in. wide, and $\frac{3}{4}$ in. thick?
4. 25 joists each 16 ft. long, 4 in. wide, and 4 in. thick?

Find the cost of the following at $22 per M (1,000):

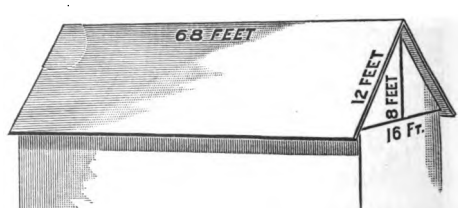
	Feet Long.	Inches Wide.	Inches Thick.
5. 28 boards.....	16	$10\frac{1}{2}$	$\frac{1}{2}$
6. 52 boards.....	14	14	1
7. 65 scantlings...	16	4	2

8. How many shingles will be needed to cover a roof 42 ft. along the ridge and 18 ft. down the slope, allowing 800 shingles per 100 square feet?



In roof and wall measurements 100 square ft. are often called a **square** (the equivalent of a 10-ft. square).

9. Shingles are sold in bundles, 4 bundles to the thousand and a fraction of a bundle is not sold. Using the dimensions indicated in the figure, find the cost of shingles for the roof at $2.45 per M and for the two gable ends at $3.80 per M.



Written.

1. A windmill pumps 2 gallons of water a minute; how many gallons does it pump in 1 hr.? In 10 hr.?

2. When the wind blows 30 miles per hour a good windmill furnishes sufficient power to grind 25 bu. of grain per hour; at this rate, how many bushels would it grind in 10 hours? In 50 hours? In 100 hours?

3. A Wisconsin farmer found it possible to grind 50 bu. of corn per week for 28 weeks of the year; how many bushels did he grind? How many pounds did he grind in a week? (A bushel of corn weighs 56 lb.)

4. If the mill mentioned in Exercise 3 ran on the average 2 days per week, and the attendant received $1.40 per day, what was the cost per hundredweight of grain? What was the cost per pound?

5. It has been shown that when a gas-engine is used 4.822 bu. of corn are ground per hour for every horse power; how many pounds is this? Express the result to the nearest pound.

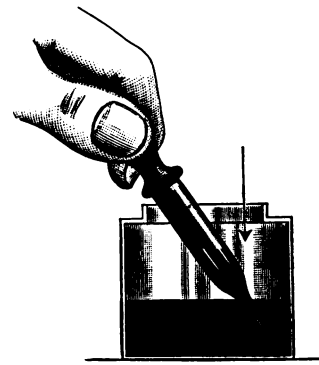
6. According to Exercise 5, how many bushels would a 5 horse-power engine grind in 10 hours? How many pounds?

7. When the gas required for the engine in grinding one hundredweight costs 1.3 cents, what will gas enough to grind one ton cost? $\frac{1}{2}$ ton? $\frac{3}{4}$ ton? $\frac{1}{5}$ ton? $\frac{2}{5}$ ton? $\frac{4}{5}$ ton? 1,500 lb.? 1,200 lb.? 800 lb.? 1,800 lb.?

8. If it takes 10 ft. of gas to grind 100 pounds of grain, what will it cost to grind a ton at $1.30 per 1,000 ft.?

9. A farmer can grind sufficient feed for a herd of 30 cows with a 12-foot windmill. Such a windmill and grinding equipment cost $225. If the grinding of his feed by others cost $50 per year, in how many years would the outfit pay for itself?

The picture shows a pipette or fountain-pen filler being filled with ink. When the fingers relax and allow the rubber part to expand, the pressure of the air on the surface of the ink forces it to rise up into the pipette. Although the atmosphere is relatively light, it exerts a heavy pressure because of its great height above the earth's surface.



1. A cubic foot of air weighs about 1.3 oz.; what is the weight of 100 cu. ft.? 1,000 cu. ft.? 5,000 cu. ft.?

2. The flue of a chimney is 1 ft. square and 50 ft. high; how many cubic feet in it?

3. According to Exercise 1, what is the weight of the air in the flue? What pressure would this volume of air produce on the bottom of the chimney?

4. If the flue was 1,000 ft. high, how many pounds would the air press on the base?

5. If the flue was 1 mile high, how many pounds would the air press on the base?

6. Regard the thickness of the atmosphere to be $5\frac{1}{2}$ mi.; how many cubic feet rest on every square foot of the earth's surface?

7. If each cubic foot weighs 1.3 oz., how many pounds does the air press on each square foot of surface? On each square inch?

8. If your book-cover is 7 in. long and 5 in. wide, what is the air-pressure on its surface at 15 lb. per square inch?

9. The air presses equally on all sides of an object. An electric-light bulb has a surface of 20 sq. in.; what is the air-pressure upon the outside of the bulb?

1. How is the area of a triangle found? Illustrate your answer.

Find the area of each triangle:

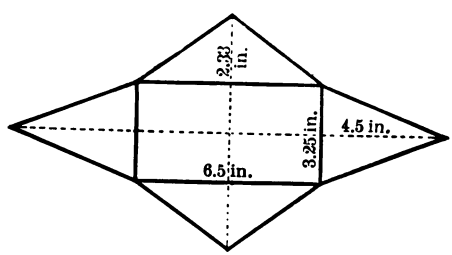
	2.	3.	4.	5.
Base.....	3 in.	9 ft.	$2\frac{1}{2}$ yd.	6 ft.
Altitude....	4 in.	3 ft.	1 yd.	$3\frac{1}{2}$ ft.

6. What is a "board foot"? How is lumber more than 1 in. thick measured?

7. How many board feet are there in a plank 12 in. wide, 12 ft. long, and $2\frac{1}{2}$ in. thick?

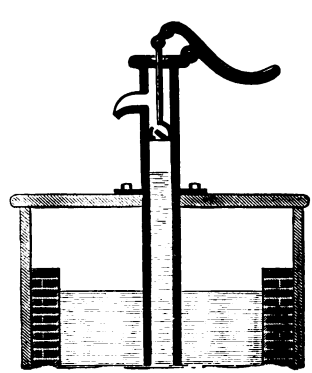
8. How many bunches of shingles of 250 each are there in 1,000 shingles?

9. In the measurement of a roof for tinning what is a "square"? What is the cost of tinning a roof 30 ft. by 40 ft. at $4 per square?



10. Find the area of a figure like the picture, and having the measurements as shown, by finding the area of each part separately and then adding.

11. In the ordinary cistern pump the air is drawn out of the pipe; what forces the water up the pipe?



12. In every inch of pipe there is 1 cu. in. of space. 1 cu. in. of water weighs $\frac{1}{2}$ oz. What is the weight of the water in 2 in. of pipe? 32 in.? 480 in.? The air forces the water up the pipe with a pressure of 15 lb. How high will the water rise, if the pipe is long enough?

DECIMALS AND FRACTIONS

Division

Oral and written.

1. The area of a rectangle is 25.5 sq. in. and the altitude is 10 in.; what is the length of the base? How is the area of a rectangle found?

SUGGESTION.—Since 10 × base = 25.5, the base = $\frac{1}{10}$ of 25.5. How is $\frac{1}{10}$ of a number expressed decimally found?

2. The area of a rectangle is 3.5 sq. ft. and the base is 10 ft.; what is the altitude?

SUGGESTION.—Since 10 × altitude = 3.5, the altitude = — of 3.5. How is $\frac{1}{10}$ of a number expressed decimally found?

3. The area of a parallelogram is 327.5 sq. in. and the base is 100 in.; find the altitude. Use A for altitude.

SUGGESTION.—Since 100 × A = 327.5, A = $\frac{1}{100}$ of 327.5. How is $\frac{1}{100}$ of a number expressed decimally found?

4. The area of a rectangle is 3.6 sq. ft. and the altitude is .6 ft.; what is the length of the base? Use B for base.

SUGGESTION.—Since .6 × B = 3.6, B = 3.6 ÷ —.
Why does 3.6 ÷ .6 = 36 ÷ 6? $6\overline{)36}$

5. The area of a parallelogram is 4.50 sq. yd. and the base is 2.25 yd.; what is the altitude?

SUGGESTION.—Since 2.25 × A = 4.50, A = 4.50 ÷ —. $225\overline{)450}$
Why does 4.5 ÷ 2.25 = 450 ÷ 225?

In dividing by a number containing a decimal it is convenient to *multiply both divisor and dividend by 10, 100, . . ., so as to make the divisor a whole number.* Why does this not alter the result?

Place the first figure of the quotient over the last figure of the dividend used to produce it.

1. The area of a rectangle is 4.472 sq. ft. and its altitude is .26 ft.; how may the length of the base be found?

2. In the division at the right 26 is taken instead of .26 and 447.2 instead of 4.472; why does $447.2 \div 26 = 4.472 \div .26$?

3. How is the position of the decimal point in the quotient related to that of the decimal point in the dividend?

$$\begin{array}{r} 17.2 \\ 26\overline{)447.2} \\ \underline{26} \\ 187 \\ \underline{182} \\ 5.2 \\ 5.2 \end{array}$$

In dividing decimals, *if the divisor be made a whole number and the first figure of the quotient be placed over the last figure of the part of the dividend used*, the *decimal point* of the *quotient* will stand *over the decimal point of the dividend.*

4. How is the work of division tested?

Divide and test:

5. $3.75 \div 5$.
6. $3.06 \div 6$.
7. $3.06 \div .06$.
8. $.625 \div .25$.
9. $4.494 \div .70$.
10. $81.54 \div 27$.
11. $42.63 \div .213$.
12. $423.4 \div 7.3$.
13. $6.636 \div .79$.
14. $69.76 \div .032$.
15. $885.8 \div 1.11$.
16. $97.336 \div .47$.
17. $121.67 \div 2.3$.
18. $80.48 \div 1.64$.
19. $160.05 \div .05$.

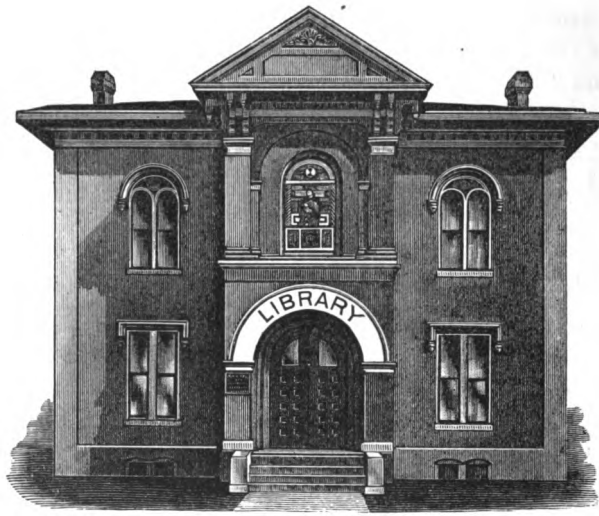
20. At 2.4¢ per mile, the fare from New York to Chicago would be $21.60; find the distance.

21. At an average speed of 25.6 mi. per hour, how long will it take a train to go from Chicago to Denver (1,028 mi.)?

22. A rod contains 16.5 ft.; how many rods are there in a mile (5,280 ft.)? In 2,541 ft.?

23. The pendulum of a clock made 120 swings in a minute; how many seconds did it take the pendulum to make 1 swing?

24. A cubic inch of mercury weighs about .5 lb.; how many inches high must a tube whose base is 1 square inch be in order to hold 100 lb. of mercury?



1. Find the width of the library shown in the picture to the nearest eighth of an inch.

2. What is its actual width, if $\frac{1}{16}$ in. on the picture represents 1 ft. on the object?

3. Find by means of this scale the height of the library; the width of a window; the height of the windows in each story; the height of the arch over the door; the height and width of the doors; the height and breadth of the art window over the door. Make other problems.

4. What is the height of the semicircular arch over the door? Its width? Its diameter? Its inside and outside radii?

5. If the radius of an automobile wheel is 16.5 in. and that of a carriage wheel is 2.25 ft., find the difference in the height of the two wheels.

6. The front wheel of a wagon has a radius of $18\frac{3}{4}$ in., and the rear wheel has a radius of 23.8 in.; how many inches is the rear wheel higher than the front wheel?

1. What is $\frac{1}{3}$ of 6 lb.? 6 lb. $\div 3 =$ —. What is $\frac{1}{3}$ of 6 tenths? $.6 \div 3 =$ —.

2. What is $\frac{1}{3}$ of 6 fifths? 6 fifths $\div 3 =$ —. $\frac{6}{5} \div 3 =$ —.

3. What is $\frac{1}{3}$ of 6 elevenths? $\frac{1}{3}$ of $\frac{6}{11} =$ —. $\frac{6}{11} \div 3 =$ —.

4. Draw a circle and draw radii to divide it into 8 equal parts as nearly as you can. Shade six of these parts.

5. In Exercise 4, how many eighths of the circle are there in $\frac{1}{2}$ of the shaded part? $\frac{1}{2}$ of $\frac{6}{8} =$ — eighths. $\frac{6}{8} \div 2 =$ —.

Find by diagram:

6. $\frac{1}{3}$ of $\frac{3}{4}$. 7. $\frac{1}{2}$ of $\frac{4}{8}$. 8. $\frac{1}{3}$ of $\frac{6}{9}$.

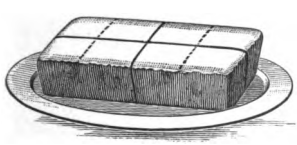
9. $\frac{1}{4}$ of $\frac{8}{12}$. 10. $\frac{1}{4}$ of $\frac{8}{15}$. 11. $\frac{1}{5}$ of $\frac{10}{12}$.

Answer without diagram:

12. $\frac{1}{4}$ of $\frac{8}{12}$. 13. $\frac{1}{5}$ of $\frac{10}{17}$. 14. $\frac{1}{6}$ of $\frac{18}{20}$.

15. A train runs $\frac{7}{10}$ of the distance from New York to Buffalo in 7 hours; what part of the distance does it run in 1 hour? In how many hours does it cover the whole distance?

16. Mr. Perkins spends $\frac{9}{11}$ of his annual salary and saves the rest. $\frac{1}{3}$ of the sum spent is paid out for food. What part of his entire salary is spent for food?



17. If each fourth of a cake is divided into two equal parts, how many parts are there in all? Each is what part of the whole cake?

18. Draw a rectangle like the top of the cake in Exercise 17. Shade $\frac{1}{4}$ of it. Show that $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. $\frac{1}{4} \div 2 =$ —.

Find without diagram:

19. $\frac{1}{2}$ of $\frac{1}{3}$. 20. $\frac{1}{3}$ of $\frac{1}{3}$. 21. $\frac{1}{5}$ of $\frac{1}{2}$. 22. $\frac{1}{3} \div 4$.

23. $\frac{1}{5} \div 3$. 24. $\frac{1}{3} \div 5$. 25. $\frac{1}{2} \div 8$. 26. $\frac{1}{12} \div 5$.

1. What is $\frac{1}{2}$ of $\frac{1}{4}$? Point this out in the diagram. How many of these parts are there in $\frac{1}{2}$ of $\frac{3}{4}$? Point this out in the diagram.

Find by diagram:

2. $\frac{1}{2}$ of $\frac{3}{5}$.
3. $\frac{1}{2}$ of $\frac{5}{6}$.
4. $\frac{1}{3}$ of $\frac{2}{5}$.
5. $\frac{1}{4}$ of $\frac{3}{5}$.
6. $\frac{1}{4}$ of $\frac{2}{3}$.
7. $\frac{1}{3}$ of $\frac{3}{5}$.
8. What is $\frac{1}{2}$ of $\frac{1}{6}$? What is $\frac{1}{2}$ of 5 times $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{5}{6}$ = —.
9. What is $\frac{1}{5}$ of $\frac{1}{4}$? What is $\frac{1}{5}$ of 3 times $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{3}{4}$ = —.

Find without diagram:

10. $\frac{1}{3}$ of $\frac{4}{7}$.
11. $\frac{1}{2}$ of $\frac{3}{8}$.
12. $\frac{1}{3}$ of $\frac{7}{9}$.
13. $\frac{5}{6} \div 4$.
14. $\frac{4}{7} \div 3$.
15. $\frac{5}{8} \div 4$.

Oral.

16. William and Frank raised chickens; William owns $\frac{3}{5}$ of the stock; he sells $\frac{1}{4}$ of his share to James; what share of the whole does James own?

17. Frank sells $\frac{1}{3}$ of his share to Henry; what part of the whole does Henry own?

18. A cubic inch of anthracite coal weighs about $\frac{2}{5}$ as much as a cubic inch of water. A cubic inch of poplar weighs about $\frac{1}{5}$ as much as a cubic inch of anthracite coal. Compare the weights of poplar and water.

19. A rug is $\frac{7}{4}$ yd. long and $\frac{1}{2}$ as wide. How wide is it?

20. A rectangle has a base $\frac{3}{5}$ of a yard long and an altitude $\frac{1}{4}$ as long; how long is the altitude?

21. The perimeter of a triangle is $\frac{7}{8}$ ft. and its altitude is $\frac{1}{4}$ as long; how long is its altitude?

22. What is the area of a parallelogram whose base is 2 feet and whose altitude is $\frac{1}{3}$ of a foot?

23. How many square feet are there in the area of a triangle of base $\frac{6}{7}$ of a foot and altitude $\frac{1}{6}$ of a foot?

Domestic Postage: To all parts of the United States, including Hawaii, Porto Rico, and the Philippine Islands; also to Canada, Mexico, and Cuba.

FIRST-CLASS MATTER: Letters or sealed matter 2 cents an ounce or fraction thereof. Postal cards 1 cent each; with paid reply card 2 cents each.

SECOND-CLASS MATTER: Newspapers and other periodical publications, when sent by publishers or news-agents, 1 cent per pound or fraction thereof; when sent by others, 1 cent for each four ounces or fractional part thereof.

1. What is the postage on a letter weighing $\frac{3}{5}$ oz.? $2\frac{1}{2}$ oz.? $4\frac{1}{4}$ oz.? $1\frac{3}{4}$ oz.? $1\frac{3}{5}$ oz.? 3 oz.? $6\frac{1}{2}$ oz.?

2. A publisher sends 15,000 copies of his paper by mail. Each paper with its wrapper weighs $4\frac{1}{2}$ oz. The papers are weighed at the post-office in bulk and he pays for the total weight at 1¢ per pound. What is his postage bill?

3. One of the subscribers remails his copy to a friend; how much postage must he pay?

4. Anna found that one of the monthly magazines weighed $13\frac{1}{4}$ oz. If 125,000 copies of it were sent by mail, what was the postage bill of the publisher for that month?

5. What does it cost a subscriber to remail a copy?

6. What is the greatest weight a newspaper can have that requires 3¢ postage when remailed?

7. How many ounces or less does a letter weigh on which the postage is 6¢?

8. A manuscript weighs 1 lb. $1\frac{1}{4}$ oz. What is the postage on it at letter rate?

9. What is the cost of a gross of postal cards with paid reply cards? What is the cost of 500?

10. What is the postage on 10,000 copies of 4-oz. newspapers when sent by the publishers?

THIRD-CLASS MATTER: Books, circulars, pamphlets, proof sheets, or other printed matter, 1 cent for each two ounces or fractional part thereof, sent to a single address.

FOURTH-CLASS MATTER: Merchandise and merchandise samples, 1 cent for each ounce or fraction thereof sent to a single address.

REGISTERED MATTER: Eight cents in addition to the regular postage.

SPECIAL DELIVERY: Ten cents in addition to the regular postage of first-class matter.

1. Sarah sent a book weighing $28\frac{1}{4}$ oz. by mail, as a Christmas present, and had it registered; how much postage did she pay?

2. Fred ordered by mail a toy sail-boat weighing 22 oz. when packed for shipment. The price was 50¢, and the purchaser was to pay the cost of sending; how much did it cost Fred, including postage on his letter ordering the boat?

Find the cost of each of the following articles, if ordered by mail, the purchaser paying the postage:

	ARTICLE.	PRICE.	SHIPPING WEIGHT.
3.	Lady's pocketbook	$0.25	4 oz.
4.	Man's wallet.....	1.10	6 oz.
5.	Man's alligator belt....	0.98	8 oz.
6.	Bicycle lamp.....	1.15	16 oz.

7. If the postage on a book is 11¢, what do you know about the weight of the book?

8. A mail-order house advertises opera-glasses at $3.75; postage 50¢ extra; what is the greatest weight the glasses can have when packed in a box weighing $\frac{1}{4}$ lb.?

9. If scales are available estimate and record the postage that would be required to send various books and papers by mail; then test your estimates by weighing the objects and referring to the rates.

1. What is meant by reducing fractions to their lowest terms? Illustrate your answer.

Reduce to lowest terms:

2. $\frac{40}{60}$. 3. $\frac{15}{75}$. 4. $\frac{39}{65}$. 5. $\frac{72}{144}$. 6. $\frac{12}{20}$.

7. $\frac{17}{85}$. 8. $\frac{25}{95}$. 9. $\frac{16}{84}$. 10. $\frac{36}{96}$. 11. $\frac{22}{121}$.

12. What kind of fractions can be added or subtracted? Illustrate your answer.

Add:

13. $\frac{1}{3} + \frac{5}{6}$. 14. $\frac{2}{3} + \frac{7}{9}$. 15. $\frac{1}{2} + \frac{3}{4}$.

16. $\frac{1}{4} + \frac{3}{8}$. 17. $\frac{3}{4} + \frac{5}{8}$. 18. $\frac{3}{5} + \frac{7}{10}$.

Subtract:

19. $\frac{9}{10} - \frac{2}{5}$. 20. $\frac{11}{20} - \frac{2}{5}$. 21. $\frac{11}{8} - \frac{3}{4}$.

22. $\frac{12}{9} - \frac{2}{3}$. 23. $\frac{7}{8} - \frac{3}{4}$. 24. $\frac{7}{12} - \frac{1}{6}$.

25. Before dividing one decimal by another, what is done to both divisor and dividend? Where is the quotient placed? Where does the decimal point of the quotient always stand?

Divide:

26. $9.696 \div 1.6$. 27. $22.50 \div 1.5$. 28. $14.4 \div .06$.

29. $1.728 \div .12$. 30. $30.10 \div .5$. 31. $7075 \div 2.5$.

32. How is any fraction multiplied by $\frac{1}{2}$? By $\frac{1}{3}$? By $\frac{1}{5}$?

33. Multiply by $\frac{1}{2}$: $\frac{3}{4}$; $\frac{3}{5}$; $\frac{2}{3}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{3}{8}$.

34. Multiply by $\frac{1}{3}$: $\frac{3}{5}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{4}{5}$; $\frac{7}{8}$.

35. Multiply by $\frac{1}{5}$: $\frac{2}{3}$; $\frac{3}{4}$; $\frac{7}{8}$; $\frac{4}{5}$; $\frac{3}{5}$; $\frac{1}{3}$; $\frac{4}{9}$.

36. What is the postage on 4 oz. of merchandise? On 7 oz.? On 8 oz.? On 16 oz.? On $\frac{1}{2}$ lb.?

37. The area of Turkey is $\frac{29}{100}$ of that of Venezuela, that of Russia (in Europe) is $\frac{54}{100}$, that of the United States is $\frac{97}{100}$, Austria-Hungary $\frac{68}{100}$, Mexico $\frac{20}{100}$, and France $\frac{5}{100}$; which country has the largest area? The smallest? Arrange the names of the countries in order according to their areas.

Oral.

1. What process does a fraction indicate? Illustrate your answer.

2. What is a mixed number? Illustrate your answer.

Express as mixed numbers:

3. $\frac{13}{7}$. 4. $\frac{101}{11}$. 5. $\frac{29}{4}$. 6. $\frac{35}{8}$.

7. $\frac{40}{8}$. 8. $\frac{65}{8}$. 9. $\frac{17}{2}$. 10. $\frac{25}{6}$.

11. How is a fraction divided by a whole number? Illustrate your answer.

Perform the divisions:

12. $\frac{1}{3} \div 3$. 13. $\frac{1}{5} \div 2$. 14. $\frac{2}{5} \div 3$.

15. $\frac{3}{4} \div 3$. 16. $\frac{5}{8} \div 2$. 17. $\frac{3}{8} \div 5$.

Written.

18. A windmill grinds 5.2 bu. of corn per hour for 9 hr. and 45 min.; how many bushels does it grind in this time?

19. When the air-pressure is 15 lb. per square inch, how many pounds pressure are there on a surface containing 8.75 sq. in.?

20. At 15 lb. per square inch, what is the air-pressure on the total surface of a book 4 in. by 6 in. by 1 in.?

21. How many cards 3 in. by 6 in. can be cut from 50 sheets of cardboard 18 in. by 24 in. in size?

22. In taking an account of stock, a merchant found that he had 4 pieces of silk costing 98¢ per yard. The lengths of the pieces were: 7 yd. 1 ft. 10 in.; 12 yd.; 6 yd. 8 in.; 3 yd. 2 ft.; find the total number of yards and the cost of the whole.

Write decimally:

23. Four hundred eight thousandths.

24. Four hundred and eight thousandths.

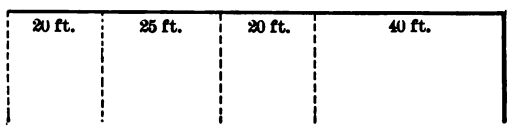
25. 9,352 thousandths. 26. 82 and 321 thousandths.

27. $\frac{1}{100}$; $\frac{5}{1000}$; $\frac{200}{1000}$; $\frac{25}{1000}$; $\frac{15}{1000}$; $\frac{986}{1000}$; $\frac{100}{1000}$.

1. Find the cost of $\frac{1}{2}$ gross of photographic paper (1 gross = 12 dozen) at 94¢ per gross; $\frac{5}{6}$ of a gross of pencils at \$1.50 per gross; $\frac{3}{4}$ doz. inkstands at \$.35 each.

2. At 3¢ per foot what is the cost of sufficient picture-molding to go around a room 14 ft. by 17 ft.?

3. A man bought a 25-foot lot at \$15 per foot. Later he bought two adjacent 20-foot lots at \$24 per foot. Still later he bought a corner lot adjacent to one of these at \$41 per foot. If he had bought all of them at a uniform price per foot and at the same total cost, how much would he have paid per foot?



4. The lots of Exercise 3 were 125 ft. deep; what is the cost of a fence surrounding the whole plot at 60¢ per running foot?

5. Find the cost of sodding a margin 15 ft. wide inside the fence along the front and side of the lot at 4¢ per square foot.

6. What will be the cost of a flagstone walk 5 ft. wide outside of the fence along the two street frontages, if the stone and labor cost 50¢ per square foot?

7. What will the owner make by selling the entire piece at \$48 per foot of the main frontage?

8. In 1900 the three States producing the largest quantities of manufactured ice were Pennsylvania, 735,000 tons; New York, 455,000 tons; Missouri, 285,000 tons. Draw on the board straight lines representing these numbers, letting 1 inch represent 25,000 tons.

9. A line $\frac{5}{6}$ ft. long would represent similarly the product of Illinois; express this amount in tons.

POWERS AND ROOTS

Powers

1. What is the area of a square 2 inches on a side?

2. What is the area of a square 3 feet on a side? What is such a square called?

3. What is the area of a square flower-bed 4 ft. on a side?

4. What are the computations for finding the areas of the above squares?

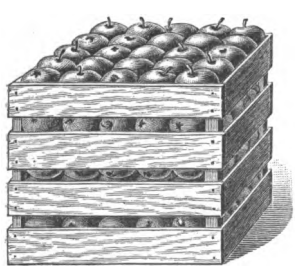
5. How do the factors in each product compare?

$2 \times 2 = 4, \qquad 3 \times 3 = 9, \qquad 4 \times 4 = 16.$

The product of *two equal factors* is called the **square** of either factor, or the **second power** of it.

Thus, in Exercise 5 above, 4 is the square (or second power) of 2, 9 is the square (or second power) of 3, and 16 is the square (or second power) of 4.

6. 25 is the square of what number? 36 is the second power of what number? 49 is the second power of what number? 81 is the second power of what number? 64 is the square of what number? 100 is the square of —.



7. State orally the squares of all integers from 1 to 10 inclusive.

8. Apples are packed in crates as shown in the picture; how many apples are there in a layer? How many apples are there in a box of 5 layers? Of 6 layers? When apples are $\frac{1}{2}¢$ each, what is the cost of a box of each size?

9. Write a table of the squares of all integers from 1 to 10 inclusive.

Oral.

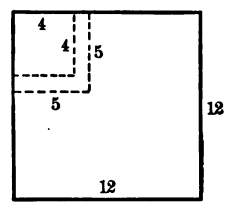
1. The area of a square rug is 25 sq. ft.; what is the length of a side?

2. How many squares are there in the side of a square checker-board containing 64 squares?

3. The area of a square register is 81 sq. in.; what is the length of one side?

4. The area of a square plot is 16 sq. yd.; how many yards are there in the length of one side?

5. How many 4-inch squares can be cut from 1 sq. ft. of cardboard? How many 3-inch squares?



One of the *two equal factors* of a number is called its **square root.** Since $2 \times 2 = 4$, 2 is the square root of 4; since $3 \times 3 = 9$, 3 is the square root of 9. Similarly 5 is the square root of 25.

6. What is the square root of 36? Of 81? Of 64? Of 49? Of 100? Of 144? Of 400? Of 2,500?

Written.

7. If an 8-inch square be cut from a square foot of cardboard, and as many 3-inch squares as possible from what is left, how many 3-inch squares are made? How many square inches of cardboard are wasted?

8. Draw on the blackboard a square 20 in. on a side. Rule it into square inches. With heavy lines mark off a square containing 121 sq. in. and as many squares as possible containing 49 sq. in. each. How many square inches of space are not used?

9. Test the work of Exercise 8 by adding the areas of all the parts and comparing with the area of the whole square.

10. What is the difference between the square roots of 100 and 121?

ANALYSIS OF PROBLEMS

Directions

PROBLEM: To find the altitude of a rectangle whose base is 8 in. and whose area is 152 sq. in.

1. *Read* carefully, noting that the base and the area of a rectangle are given, and that the altitude is required.

2. *Plan:* Recall that the area of a rectangle is the product of the numbers expressing its length and breadth. Hence 152 is 8 times the required number and must be divided by 8.

3. *Computation.* $8\overline{)152}$ 19 4. *Test.* $8 \times 19 = 152$.

In the solution of every problem there are four main steps:

a. **Read the problem;** note carefully what is given and what is required.

b. **Plan the work;** determine how to find what is required from what is given.

c. **Make the computations** as planned.

d. **Test the result.**

1. Read and determine what is given and what is required: A train runs from New York to Cleveland, 624 mi., in 16 hr.; find the speed in miles per hour.

2. Plan the work to be done in solving Exercise 1.

3. Make the computation. $16\overline{)624}$ 4. What is the test?

5. Read: The area of a triangle is 176 sq. ft.; its altitude is 11 ft.; what is its base?

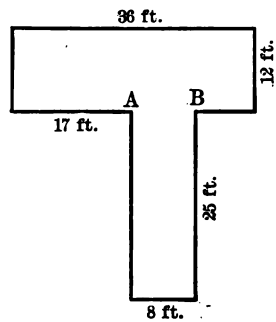
6. Plan the solution.

SUGGESTION.—When the calculation contains more than one process, it is better to indicate all of them in the plan. Thus, in this problem: 1. $2 \times 176 =$ the altitude times the base. 2. $352 =$ 11 times the base. The base $= 352 \div 11 = —$.

Plan the solution; do not make the computation:

1. How many feet per minute is a train moving when traveling 42 mi. per hour?

2. A train left Cincinnati at 8:15 A. M. and arrived at St. Paul, 702 miles distant, at 2:57 A. M. the next day; find its average speed in miles per hour.



3. The base of a rectangle is 16 in., its area is 3 sq. ft.; find the altitude in inches.

4. Show how to find the perimeter of the figure.

5. Draw a figure like this. Draw a line from A to B; show how to find the area of the figure.

6. A man earns \$75 a month. He spends \$10 per month for room rent; \$3.50 per week for board, \$125 per year for clothing and other expenses; in how many years will he save \$1,000 at this rate?

7. 40 ft. of wire weighed 1 lb.; what was the weight of 31 mi. of this wire?

8. A, B, and C own a store; A owns $\frac{4}{7}$ of it and B owns $\frac{1}{2}$ as much as A; what part does C own?

9. The base of a rectangle is 17 in.; its perimeter is 48; what is its altitude?

10. A 100-acre farm contains 4 lots. Three of the lots contain $75\frac{1}{2}$ acres, $20\frac{3}{4}$ acres, and $7\frac{5}{8}$ acres respectively; how many acres does the fourth lot contain?

NOTE.—Hereafter the planning of a problem should be made a distinct and important feature of its solution. In very simple problems, the plan and the work need not be separated, but any uncertainty as to *how* a problem is to be worked shows clearly that a separate, correct plan is necessary. NEVER WORK AT RANDOM.

1. $\frac{2}{3}$ of the distance from Detroit to Chicago is 190 mi.; what is the whole distance?

PLAN. 1. $\frac{1}{3}$ of the distance $=$ $\frac{1}{2}$ of 190 mi.
2. $\frac{3}{3}$, or the whole distance, $= 3 \times 95$ mi. $=$ — mi.

2. A dressmaker paid \$1.40 for $\frac{7}{8}$ of a yard of velvet; what was the price per yard?

PLAN. 1. $\frac{1}{8}$ yd. cost $\frac{1}{7}$ of \$1.40 $=$ \$—.
2. $\frac{8}{8}$ yd., or 1 yd. cost $8 \times$ \$.20 $=$ \$—.

Plan and solve:

3. When 13 ounces of cinnamon cost \$.65, what is the cost of cinnamon per pound?

4. In 1900, $\frac{5}{39}$ of the population of the United States was 10 millions. Find the population.

5. \$24 is $\frac{3}{5}$ of the cost of a wagon; what is its cost? What is $\frac{8}{15}$ of its cost?

6. $\frac{7}{8}$ of a ton of coal costs \$7.35; what is the cost of a ton? Of $\frac{3}{4}$ of a ton?

7. $\frac{3}{7}$ of the number of passengers on a street-car is 12; how many are there in the car? If $\frac{3}{4}$ of the whole number are women, how many are women?

8. $\frac{5}{17}$ of the population of Chicago in 1900 was 500,000; what was the population?

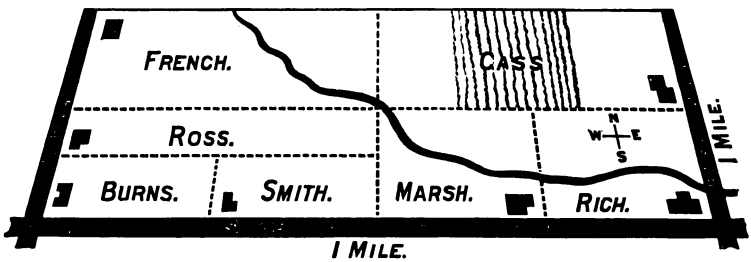
9. $\frac{3}{20}$ of the population of Pittsburg in 1900 was 48,000; what was the population? What was $\frac{5}{8}$ of it?

10. $\frac{2}{9}$ of the total newspaper product in the United States in 1900 was 60,000 tons; what was the total product?

11. At an entertainment $\frac{7}{11}$ of the audience sat on the main floor and 360 in the gallery; how many people were there in the audience?

12. A society of 300 men march in a Fourth of July procession. They march 4 abreast in ranks 6 ft. apart; how long is the procession which they form?

1. How many miles are there in the distance around the tract of land as shown in the picture? How many rods are there in this distance (1 mi. = 320 rods)?



2. The land is divided into fourths, eighths, and sixteenths. What part of the whole is each farm? How many rods are there in the length of the Cass farm? Its breadth?

3. How many rods of line-fence are there between the Cass farm and the French farm? Between the Cass farm and the Rich farm?

4. Make and answer 5 other questions about the fences.

5. What part of the square mile does the Cass farm occupy? The Rich farm? The Ross farm? The Smith farm?

A square mile of land contains 640 acres and is called by surveyors a **section.**

6. How many acres are there in the French farm? In the Smith farm? In the Burns farm?

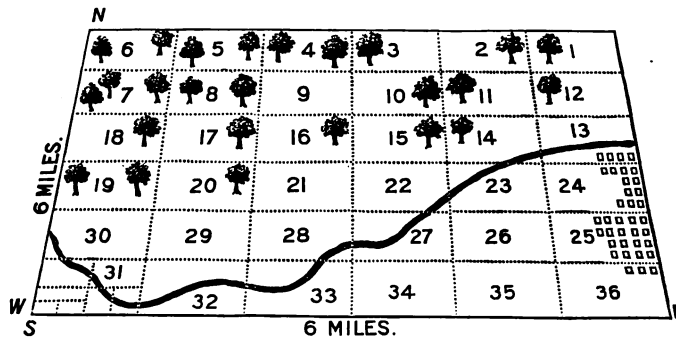
7. Make and solve 3 similar problems.

8. How many acres are there in a quarter-section? Which of the above farms occupy a quarter-section?

9. From the dimensions in rods of the Smith farm find its area in square rods. How many acres does it contain? How many square rods are there in an acre?

10. What farms occupy the east half (E $\frac{1}{2}$) of the section? Which the north half (N $\frac{1}{2}$)? Which the S W $\frac{1}{4}$?

In surveying the newer States, the government surveyors divided the land as nearly as possible into **townships** 6 miles square. The townships are slightly narrower at the top than at the bottom on account of the curvature of the earth. Each



township is divided into 36 parts called sections and these are recorded by number as shown in the figure. The township is regarded as square for all computations.

1. How many square miles are there in a township?

2. In every township the government set apart section number 16 for school purposes; how many acres is this? In a locality where such land sold for $25 an acre, what was the gift worth?

3. Section 31 is the one shown on p. 95; what part of the area of a township is this section? How many acres are there in a township?

4. How many acres are there in a row of sections in a township? In a column?

5. What will be the cost of a line-fence between section 9 and section 10 at $1.75 a rod?

6. If half of a township is timber land worth $10 an acre and the rest is farming land worth $25 an acre, what is the value of the township?

7. Make and solve 3 other problems about land.

Oral.

1. What is the second power of 5? Of 6? Of 7? Of 10?

2. What other name has the second power?

3. What is meant by the square root of a number?

4. What is the square root of 64? Of 49? Of 121?

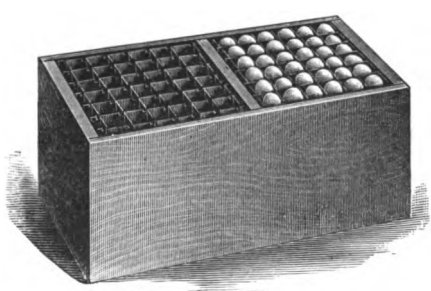
5. The area of a square lot is 144 sq. yd. Its side is —.

Written. Plan before solving:

6. A case of canned corn contains 3 square layers of 4 cans on a side. The corn costs $2.88 a case; at what price per case must it be sold to make a profit of $.03 per can?

7. If marshmallows are packed in a box 5 on a side, how many are there in 1 layer? In 2 layers? If a box of 2 layers costs 10¢, how many marshmallows does 1 cent buy?

8. Eggs are packed in crates in layers as shown in the picture; how many eggs are there in a layer in one end of the box? How many eggs are there in a crate of 5 layers? What is such a crate of eggs worth at 13¢ a dozen?



9. What are the four steps in the solution of a problem? Illustrate by the following: "How many square yards of paving are there on a rectangular court 20 yd. by 35 yd.?"

10. What is the size of a township? What is meant by a section of land?

11. How many acres are there in a section of land?

12. How many square rods are there in an acre? How many acres are there in a square mile?

1. Find the cost of a roll, 150 running feet, of wire netting at 38¢ per 100 ft.

2. A man wishes to fence a rectangular lot 60 ft. by 110 ft. with netting at 40¢ per 100 ft. If broken rolls are not sold, how much must he pay for the netting?

3. Find the cost of 12 yards of broadcloth at $2.30 per yard and 18 yards of lining silk at 59¢ per yard.

4. Find the cost of 27 fruit-jars at 84¢ per dozen, 18 tumblers at 72¢ per dozen, 9 goblets at $1.80 per dozen, and 40 cups at $1.50 per dozen.

5. James received by mail a package of stationery weighing 4 lb. $9\frac{1}{2}$ oz.; what was the amount of postage on it?

6. Mr. Perkins sent a money-order in payment for a set of drawing instruments, price $4.85 and weight $1\frac{3}{4}$ lb. The money order included the cost of postage; what was the total cost of the order?

7. Charles ordered a set of books. The price was $3.25, which he paid with a money-order; what was the total cost of the order? He sent the books by mail as a Christmas present to a friend; if the books weighed 2.75 lb., what did it cost to send them?

8. $\frac{3}{4}$ of the production of book paper in 1900 was 100,000 tons; what was the total product?

9. $\frac{19}{27}$ of the total product of writing-paper in 1900 was 100,000 tons; what was the total product?

10. When the area of a rectangle and the length of its base are given, how is the altitude found?

11. When the area of a square is given, what expresses the length of a side? Apply to "The area of a square is 64 sq. in."

12. When the area of a triangle and the length of its altitude are given, how is the base found?

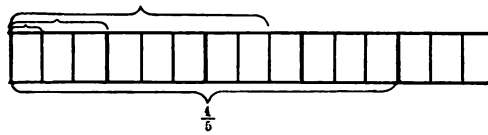
FRACTIONS

Multiplication

1. What is $\frac{1}{3}$ of 6 yards? What are $\frac{2}{3}$ of 6 yards?

2. $\frac{1}{3}$ of $\frac{1}{5}$ = —.
$\frac{1}{3}$ of $\frac{4}{5}$ = —.

3. Since $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{3 \times 5}$ what is $\frac{2}{3}$ of $\frac{4}{5}$?



4. In the diagram find $\frac{1}{3}$ of $\frac{1}{5}$; also $\frac{1}{3}$ of $\frac{4}{5}$; also $\frac{2}{3}$ of $\frac{4}{5}$.

Find by diagram:

5. $\frac{3}{5}$ of $\frac{2}{3}$. 6. $\frac{2}{3}$ of $\frac{2}{3}$. 7. $\frac{3}{4}$ of $\frac{3}{4}$.

8. $\frac{2}{5}$ of $\frac{2}{3}$. 9. $\frac{3}{4}$ of $\frac{1}{2}$. 10. $\frac{2}{5}$ of $\frac{3}{4}$.

Find without diagram:

11. $\frac{3}{4}$ of $\frac{5}{8}$. 12. $\frac{2}{3}$ of $\frac{2}{3}$. 13. $\frac{5}{6}$ of $\frac{3}{4}$.

14. $\frac{4}{5}$ of $\frac{2}{5}$. 15. $\frac{4}{5}$ of $\frac{4}{5}$. 16. $\frac{5}{6}$ of $\frac{5}{6}$.

The results found in Exercises 2–16 are called the **products of the fractions.**

The product of any two fractions, as $\frac{2}{3}$ and $\frac{3}{4}$, is found from the products of the numerators and denominators, as in $\frac{2 \times 3}{3 \times 4}$.

The common factors of the terms are then removed, as in the work shown.

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{4}}} = \frac{1}{2}$$

It is convenient to draw a line through the numbers in both numerator and denominator which are divided by the same number, as above. The quotients are the numbers left for multiplication. This process is called **canceling.**

17. Mr. Smith owned $\frac{2}{3}$ of a quarry and sold $\frac{4}{5}$ of his share to Mr. White; what part of the quarry did Mr. White buy?

18. If the quarry was worth $60,000, what was Mr. White's share worth?

1. What is the cost of $\frac{1}{2}$ of a pound of butter at 20¢ a pound? What is the cost of $2\frac{1}{2}$ lb.?

For $\frac{1}{2}$ lb. the cost is $\frac{1}{2}$ *of* 20¢.

For $2\frac{1}{2}$ lb., it is $2\frac{1}{2}$ *times* 20¢.

We find $2\frac{1}{2}$ *times* 20 by adding 2 *times* 20 and $\frac{1}{2}$ *of* 20.

The words *times* and *of* have been used here with the same meaning.

When the multiplier is a proper fraction the symbol × has the same meaning as "of." Thus, $\frac{2}{5} \times 30$ means $\frac{2}{5}$ of 30.

2. What is the meaning of $\frac{3}{4} \times 16$? $\frac{7}{8} \times 36$? $\frac{2}{3} \times \frac{3}{7}$?

3. Draw a line on the blackboard $\frac{2}{3}$ of a foot long. Draw another $2\frac{1}{2}$ times as long.

$2\frac{1}{2} \times \frac{2}{3}$ ft. $= 2 \times \frac{2}{3}$ ft. and — of $\frac{2}{3}$ ft.

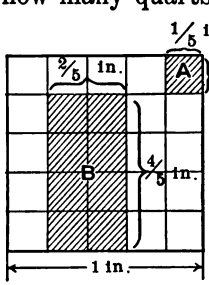
4. Draw a line 8 inches long and divide it into half-inch spaces. Show that $\frac{1}{2}$ of 8 in. = 8 times $\frac{1}{2}$ in.

5. $6 \times \frac{2}{3} =$ —. $\frac{2}{3}$ of 6 = —.

6. $\frac{1}{2}$ of $\frac{2}{3}$ = —. $\frac{2}{3}$ of $\frac{1}{2}$ = —.

The factors in a product are interchangeable in the case of fractions just as in the case of integers.

7. A gasoline stove uses $\frac{4}{5}$ of a quart of gasoline per hour; how many quarts does it use in $\frac{5}{6}$ of an hour?



PLAN. $\frac{5}{6}$ of $\frac{4}{5}$ qt. $= \frac{5 \times 4}{6 \times 5}$ qt. = — qt.

8. What is the width of the shaded square A? What is its height? Its area?

$\frac{1}{5} \times \frac{1}{5} = ?$

9. What is the width of the rectangle B? Its height? Its area?

$\frac{2}{5} \times \frac{4}{5} = ?$

10. Draw a diagram like this and shade it to show that:

$\frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$; $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$; $\frac{4}{5} \times \frac{2}{5} = \frac{8}{25}$.

Written.

1. Reduce to lowest terms: $\frac{4}{6}$; $\frac{3}{12}$; $\frac{15}{20}$; $\frac{24}{36}$.

2. A street is $\frac{9}{10}$ of a mile long; $\frac{2}{3}$ of it is paved; what part of a mile is paved? Express the product and cancel the common factors.

3. Reduce to lowest terms: $\frac{2\times2}{2\times3}$; $\frac{3\times1}{3\times4}$; $\frac{5\times3}{5\times4}$.

4. A cook has $\frac{5}{6}$ of a sack of flour and uses $\frac{3}{5}$ of it for a baking; what part of a sack does she use? Express the product and cancel the common factors.

5. A garden-bed is $1\frac{5}{16}$ yd., or $\frac{21}{16}$ yd. long and $\frac{12}{35}$ as wide; how many yards wide is it?

SUGGESTION.—$\frac{12}{35}$ of $\frac{21}{16}$ in. may be expressed by $\frac{3\times4\times3\times7}{5\times7\times4\times4}$ in. What is the result?

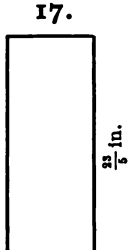
6. What is $\frac{4}{5}$ of $\frac{11}{12}$ mi.? Cancel.

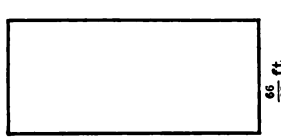
7. The length of the Amazon is $\frac{33}{28}$ times that of the Mississippi, and the length of the Yangtse (China) is $\frac{9}{11}$ of that of the Amazon. The Yangtse = — times the Mississippi?

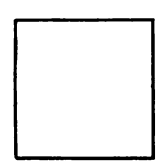
Find by canceling:

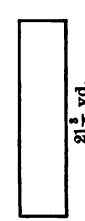
8. $\frac{9}{16}$ of $\frac{10}{21}$.
9. $\frac{8}{33}$ of $\frac{9}{4}$.
10. $\frac{12}{35}$ of $\frac{14}{27}$.
11. $\frac{10}{27}$ of $\frac{15}{7}$.
12. $\frac{28}{5}$ of $\frac{17}{4}$.
13. $\frac{7}{6}$ of $\frac{3}{5}$.
14. $\frac{18\times21}{7\times9}$.
15. $\frac{3\times4\times12}{10\times48}$.
16. $\frac{63\times72\times10}{10\times32}$.

Find the area of:

17. 

18. 

19. 

20. 

Written.

1. Indicate the cost of one drum at \$5.10 per dozen. Indicate the cost of 20 drums.

After having indicated the processes, as $\frac{20 \times \$5.10}{12}$, the value should be found by canceling.

2. Indicate the cost of 1 bracelet at \$11.65 per dozen. Indicate the cost of 16 bracelets. Find the value by canceling.

As above, indicate and find the cost of:

3. 675 dolls at \$26.40 per gross.
4. 15 doz. candles at \$1.75 per gross.
5. 20 doz. steel pens at 66¢ per gross.
6. 168 scarf pins at \$9.75 per hundred.
7. 18 handkerchiefs at \$2.70 per dozen.
8. 600 drawing pads at \$17.40 per gross.
9. 32 nickel watches at \$14.70 per dozen.
10. 21 bottles of ammonia at 88¢ per dozen.
11. 78 toy steam engines at \$65.56 per hundred.
12. 1 chest (88 lb.) Japan tea at $42\frac{1}{4}$¢ per pound.
13. 1 sack (76 lb.) Java coffee at $22\frac{3}{8}$¢ per pound.
14. 1 sack (125 lb.) Rio coffee at $15\frac{7}{8}$¢ per pound.
15. 86 neckties at \$28.08 per gross. (1 gross = 12 dozen.)
16. In New York on the longest day daylight lasts about 17 hours and on the shortest day 10 hours; what fraction of the entire day is daylight in each case? What part is darkness?
17. On paper ruled in squares draw a rectangle 3 squares wide and 8 squares long. Shade a portion of it black to represent the darkness of the longest day.
18. Draw another rectangle of the same size and shade a portion of it black to represent the amount of darkness on the shortest day.

Parcels weighing several pounds are usually sent by express. The charge varies with the distance and is stated at a rate per 100 pounds.

When the rate per 100 pounds is $2.50 the charges for smaller weights are:

Weight.....	1 lb.	2 lb.	3 lb.	4 lb.	5 lb.	7 lb.	10 lb.
Charge.....	$0.25	$0.30	$0.45	$0.50	$0.55	$0.65	$0.75

Weight.....	15 lb.	20 lb.	25 lb.	30 lb.	35 lb.	40 lb.	50 lb.
Charge.....	$0.80	$0.90	$1.00	$1.10	$1.20	$1.25	$1.25

Parcels weighing 50 lb. and over are charged at the rate of $2\frac{1}{2}¢$ per pound. Weights between those named in the table are charged for at the rate for the next higher weight.

1. The express rate from Chicago to New York or to Boston is $2.50; what does it cost to send a 12-lb. parcel by express from Chicago to New York? A 57-lb. parcel? A 65-lb. parcel? A 98-lb. parcel?

2. What is the express charge from Chicago to Boston for parcels weighing $4\frac{1}{2}$ lb.? 16 lb.? 32 lb.? 74 lb.? 110 lb.?

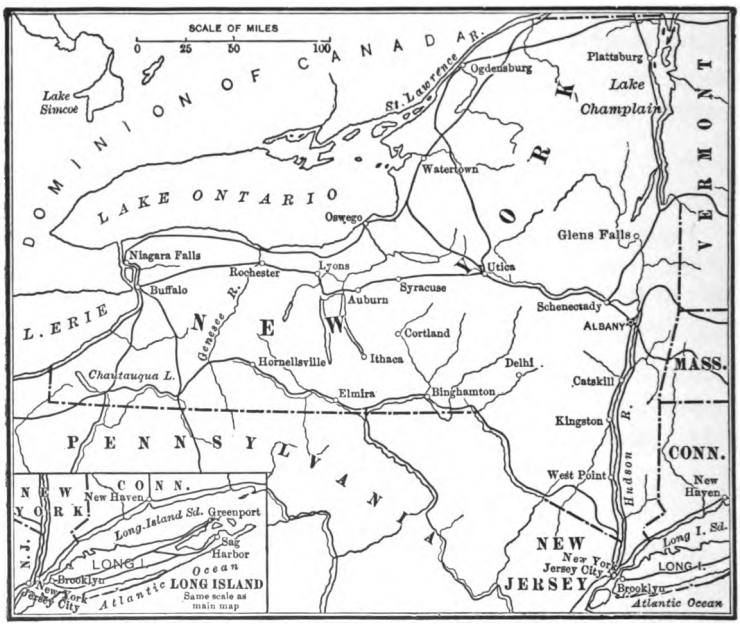
3. Which is cheaper and how much: To send a 2-lb. parcel of merchandise from Chicago to Boston by mail or by express? (See page 86.)

4. A teacher in Chicago ordered prepaid from Boston a set of books weighing 65 lb.; what was the express charge paid by the publishing house? The books cost $45.50; what did the money-order cost the teacher?

5. Miles, who lives in Boston, received for Christmas by express from Chicago a sled weighing 7 lb.; what express charge did the sender pay?

6. A person in Boston ordered a sled weighing $5\frac{1}{2}$ lb. sent from Chicago. The price was $2.60; he paid the expressage and sent the money by postal money-order; what was the whole cost?

1. According to the scale of miles given on the map, to how many miles does 1 inch on the map correspond? $\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $\frac{1}{8}$ in.? $\frac{1}{10}$ in.?



Find the distance within 10 miles:

2. From Buffalo to Albany.

3. From Elmira to Buffalo.

4. From Syracuse to Rochester.

5. From Syracuse to Binghamton.

6. From Albany to New York City.

7. From Binghamton to New York.

8. How many miles apart are two cities that are 1.4 in. apart on this map? If possible, find two such cities.

9. Make and solve 3 other problems about the map above.

Oral.

1. What is the cost of 100 lb. of butter at 16¢ a pound? What is the cost of $\frac{1}{4}$ as much, or of 25 lb.?

2. If, in order to light a schoolroom sufficiently, the window surface should be $\frac{1}{5}$ of the floor space, how much window surface is needed in a room whose floor contains 600 sq. ft.?

3. According to Exercise 2, how much window surface does a school building require that contains 25 rooms averaging 600 sq. ft. of floor space?

4. 25 units are how many fourths? To multiply a number by 25 is to multiply it by how many fourths? What is the easiest way to multiply it by 25?

Multiply by 25:

5. 16.	6. 7.5.	7. 45 lb.	8. $16.80.
9. 25 yd.	10. $17.40.	11. 29.2.	12. 179.60.
13. 144.	14. 62 lb.	15. .008.	16. 32 da.

Find the cost of 25 articles of each kind:

17. Trunks, $5.50.
18. Chairs, $3.75.
19. Tables, $18.75.
20. Coats, $16.60.
21. Wagons, $62.25.
22. Sofas, $25.25.
23. Dictionaries, $14.50.
24. Typewriters, $60.

25. When 25 books cost $11.50, what is the price of each?

26. If a sheep consumes on the average, besides pasturage, 400 lb. of hay in a year, how many tons of hay are needed to feed a flock of 25 sheep for one year?

27. If a cow consumes on an average, besides pasturage, 4.5 tons of hay in a year, how many tons are required to feed a herd of 25 cows for one year?

28. If the average yield of hay is 1.5 tons per acre, how many acres are needed to produce the feed in each case mentioned in Exercises 26, 27?

Oral or written.

Bulky goods are usually shipped by freight. The charge is by weight and articles are grouped in different classes, according to bulk and character, with different rates.

What is the freight charge on shipments of the following weights from Chicago to Boston at 82¢ per 100 lb.:

1. 125 lb.? 2. 248 lb.? 3. 850 lb.? 4. 1,240 lb.?

5. Between Boston and Chicago, and frequently between other places, shipments weighing less than 100 lb. are charged as 100 lb., irrespective of weight. This being the case, would it be cheaper to send a 6-lb. parcel by express or by freight (see p. 103 for express rates)? A 10-lb. parcel? A 30-lb. parcel?

6. In Exercise 5 no account has been taken of the fact that express charge includes calling for the parcel and delivering it and that the freight charge includes neither. If 25¢ is paid for cartage at each end of the route when a parcel is shipped by freight, what is the largest weight that can be sent more cheaply by express than by freight?

7. Mr. Henderson of Boston orders a 25-lb. lot of paper from Chicago. It is sold f.o.b. in Chicago (delivered *free on board* in Chicago). The cartage in Boston will cost Mr. Henderson 30¢. Is it cheaper to have the paper sent by express or freight and how much?

8. When the freight rate is 95¢ per 100 lb. and the cartage at each end is 40¢ what is the total cost of transportation of a box of books weighing 350 lb.?

9. When the freight rate is 60¢ per 100 lb. and shipments weighing less than 100 lb. are charged as 100 lb., how much is saved by making one shipment of three 90-lb. packages instead of a separate shipment for each?

10. If convenient, ascertain the freight rates from your city to several others and make problems about them.

Oral.

1. Write the multiples of $12\frac{1}{2}¢$ from $12\frac{1}{2}¢$ to \$1.00.

2. What part of $25¢$ is $12\frac{1}{2}¢$? What part of \$1.00 is $25¢$? What part of \$1.00 is $12\frac{1}{2}¢$?

3. How many times $12\frac{1}{2}¢$ are $37\frac{1}{2}¢$? What part of a dollar is $37\frac{1}{2}¢$?

4. How many times $12\frac{1}{2}¢$ are $62\frac{1}{2}¢$? What part of a dollar is $62\frac{1}{2}¢$?

5. What part of a dollar is $87\frac{1}{2}¢$? $75¢$? $50¢$? $25¢$?

6. At $12\frac{1}{2}¢$ each, how many cakes of soap can be bought for \$.25? For $37\frac{1}{2}¢$? For $62\frac{1}{2}¢$? For $87\frac{1}{2}¢$? For $75¢$?

7. At $12\frac{1}{2}¢$ per yard, how many yards of lace can be bought for $37\frac{1}{2}¢$? For \$1.00? For $87\frac{1}{2}¢$? For \$1.25? For \$1.50?

8. At $12\frac{1}{2}¢$ per yard, what is the cost of 3 yards of ribbon? Of 5 yd.? 7 yd.? 8 yd.? 11 yd.?

SUGGESTION.—The cost of 11 yards is the cost of 8 yd. + 3 yd.

9. As in Exercise **8**, what is the cost of 26 yd.?

SUGGESTION.—The cost of 26 yards is the cost of 3 × 8 yd. + 2 yd.

10. Similarly, find the cost of 35 yd. of ribbon; 44 yd.; 80 yd.; 97 yd.

11. Write the multiples of $33\frac{1}{3}¢$ from $33\frac{1}{3}¢$ to \$1.00.

12. What part of a dollar is $33\frac{1}{3}¢$? How many times $33\frac{1}{3}¢$ are $66\frac{2}{3}¢$? $66\frac{2}{3}¢$ is what part of \$1.00?

13. How many times $33\frac{1}{3}¢$ is \$1.00? $\$1.33\frac{1}{3}$? $\$1.66\frac{2}{3}$? \$2.00? $\$2.66\frac{2}{3}$? $\$2.33\frac{1}{3}$? \$5.00?

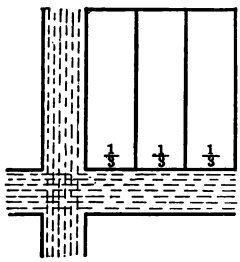
14. What is the price of a dollar article when marked down $\frac{1}{3}$? How much does one pay for such an article?

15. At $33\frac{1}{3}¢$ per pint, how many pints of olive oil can be bought for $67¢$? For \$1.33? For \$2.00?

16. At $33\frac{1}{3}¢$ per pound, how many pounds of coffee can be bought for \$1.00? For \$1.67?

Oral.

1. A real estate owner divided a plot so that the frontage of each part was $\frac{1}{3}$ of the frontage of the whole; into how many lots did he divide the plot?



2. How many thirds are there in one whole thing?

3. A housekeeper divided a roll of carpet so that the length of each strip was $\frac{1}{5}$ of that of the piece; into how many strips did she cut the piece?

4. How many fifths are there in one whole thing?

5. How does the diagram show that $1 \div \frac{1}{3} = 3$?

[redacted]

Written.

Show by diagram that:

6. $1 \div \frac{1}{2} = 2$. 7. $1 \div \frac{1}{4} = 4$. 8. $1 \div \frac{1}{5} = 5$.

9. $1 \div \frac{1}{6} = 6$. 10. $1 \div \frac{1}{7} = —$. 11. $1 \div \frac{1}{8} = —$.

12. Show from the diagram that $1 \div \frac{2}{4}$ is $\frac{1}{2}$ as many as $1 \div \frac{1}{4}$.

[redacted]

Show by diagram that:

13. $1 \div \frac{2}{6}$ is $\frac{1}{2}$ of $1 \div \frac{1}{6}$. 14. $1 \div \frac{2}{8}$ is $\frac{1}{2}$ of $1 \div \frac{1}{8}$.

15. $1 \div \frac{2}{10}$ is $\frac{1}{2}$ of $1 \div \frac{1}{10}$. 16. $1 \div \frac{3}{6}$ is $\frac{1}{3}$ of $1 \div \frac{1}{6}$.

17. $1 \div \frac{4}{8}$ is $\frac{1}{4}$ of $1 \div \frac{1}{8}$. 18. $1 \div \frac{5}{10}$ is $\frac{1}{5}$ of $1 \div \frac{1}{10}$.

Oral. State the quotients:

19. $1 \div \frac{2}{6}$. 20. $1 \div \frac{3}{6}$. 21. $1 \div \frac{2}{8}$. 22. $1 \div \frac{4}{8}$.

23. $1 \div \frac{2}{5}$. 24. $1 \div \frac{3}{4}$. 25. $1 \div \frac{2}{3}$. 26. $1 \div \frac{3}{5}$.

27. How may the result of dividing 1 by any fraction be written?

1. Which is the greater, the quotient of $1 \div \frac{1}{3}$ or $2 \div \frac{1}{3}$? How many times as great? Show this from the diagram.

Show by diagram that:

2. $3 \div \frac{1}{4}$ is 3 times as great as $1 \div \frac{1}{4}$.
3. $3 \div \frac{1}{5}$ is 3 times as great as $1 \div \frac{1}{5}$.
4. $4 \div \frac{1}{3}$ is 4 times as great as $1 \div \frac{1}{3}$.
5. $6 \div \frac{1}{4}$ is 6 times as great as $1 \div \frac{1}{4}$.

6. $1 \div \frac{2}{3} = —$. $3 \div \frac{2}{3} = —$. $2 \div \frac{2}{3} = —$.
7. $1 \div \frac{4}{5} = —$. $6 \div \frac{4}{5} = —$. $3 \div \frac{4}{5} = —$.
8. $1 \div \frac{2}{5} = —$. $4 \div \frac{2}{5} = —$. $2 \div \frac{2}{5} = —$.
9. $1 \div \frac{3}{5} = —$. $8 \div \frac{3}{5} = —$. $4 \div \frac{3}{5} = —$.

10. In Exercise 6, $3 \div \frac{2}{3} = 3 \times \frac{3}{2}$; in Exercise 7, $6 \div \frac{4}{5} = 6 \times \frac{5}{4}$. Write the corresponding relations for Exercises 8 and 9.

In a similar way find the quotients:

11. $8 \div \frac{5}{4}$. 12. $9 \div \frac{7}{12}$. 13. $18 \div \frac{6}{7}$. 14. $11 \div \frac{2}{3}$.
15. $9 \div \frac{5}{11}$. 16. $5 \div \frac{6}{7}$. 17. $7 \div \frac{9}{16}$. 18. $8 \div \frac{3}{5}$.
19. $3 \div \frac{4}{5}$. 20. $6 \div \frac{2}{7}$. 21. $7 \div \frac{4}{5}$. 22. $7 \div \frac{8}{9}$.

Simplify the results by canceling:

23. If one garment requires $\frac{3}{4}$ yd. of cloth, how many such garments can be cut from 12 yd.?

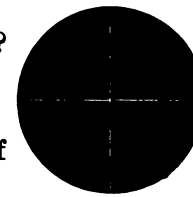
24. A real estate owner divided 6 acres of land into lots of $\frac{3}{8}$ of an acre each; how many lots did he have?

25. A bootblack used $\frac{2}{15}$ of a box of blacking to each pair of shoes; how many pairs did he black with 6 boxes?

26. A cook used $\frac{2}{3}$ lb. of sugar for each cake; how many cakes could she make with 10 lb. of sugar?

27. A baker used $\frac{3}{5}$ lb. of flour to a loaf of bread; how many loaves could he make from a barrel of flour? (1 barrel of flour weighs 196 lb.)

1. Find from the diagram the quotient of $1 \div \frac{1}{4}$; also of $\frac{1}{2} \div \frac{1}{4}$.



2. Which is the greater; $1 \div \frac{1}{4}$ or $\frac{1}{2} \div \frac{1}{4}$? How many times as great?

Show by diagram that:

3. The quotient of $\frac{1}{3} \div \frac{1}{6}$ is $\frac{1}{3}$ of that of $1 \div \frac{1}{6}$.

4. The quotient of $\frac{1}{2} \div \frac{1}{8}$ is $\frac{1}{2}$ of that of $1 \div \frac{1}{8}$.

5. The quotient of $\frac{1}{4} \div \frac{1}{8}$ is $\frac{1}{4}$ of that of $1 \div \frac{1}{8}$.

6. Find from the diagram the quotient of $3 \div \frac{1}{2}$; also that of $\frac{3}{2} \div \frac{1}{2}$.

7. Which is the greater, $3 \div \frac{1}{2}$ or $\frac{3}{2} \div \frac{1}{2}$? How many times as great?

Show by diagram that:

8. $\frac{3}{2} \div \frac{1}{4}$ is $\frac{1}{2}$ of $3 \div \frac{1}{4}$.

9. $\frac{3}{4} \div \frac{1}{8}$ is $\frac{1}{4}$ of $3 \div \frac{1}{8}$.

Find without diagram:

10. $\frac{3}{2} \div \frac{1}{4}$.

11. $\frac{3}{4} \div \frac{1}{8}$.

12. $\frac{3}{4} \div \frac{1}{2}$.

13. $\frac{2}{3} \div \frac{1}{2}$.

14. $\frac{5}{6} \div \frac{1}{5}$.

15. $\frac{7}{8} \div \frac{1}{5}$.

16. $2 \div \frac{3}{5} = 2 \times$ —. $\frac{2}{7} \div \frac{3}{5} =$ — of $2 \times \frac{3}{5}$.

17. $3 \div \frac{2}{5} = 3 \times$ —. $\frac{3}{4} \div \frac{5}{2} =$ — of $3 \times \frac{5}{2}$.

18. $5 \div \frac{2}{3} = 5 \times$ —. $\frac{5}{6} \div \frac{3}{2} =$ — of $5 \times \frac{3}{2}$.

19. $7 \div \frac{4}{5} = 7 \times$ —. $\frac{7}{9} \div \frac{5}{4} =$ — of $7 \times \frac{5}{4}$.

20. In Exercise 16, $\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times$ —; in 17, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times$ —. Write the corresponding relations for Exercises 18 and 19.

21. In dividing one fraction by another, by what may the dividend be multiplied?

In this way find the quotients; simplify the results by canceling:

22. $\frac{2}{3} \div \frac{3}{4}$.

23. $\frac{5}{6} \div \frac{7}{8}$.

24. $\frac{5}{8} \div \frac{2}{3}$.

25. $\frac{6}{11} \div \frac{4}{5}$.

26. $\frac{4}{5} \div \frac{7}{9}$.

27. $\frac{3}{8} \div 1\frac{1}{12}$.

GENERAL REVIEW

Summary

1. If two fractions have the same number for denominators and unequal numerators, which fraction is the larger?

2. If two fractions have the same number for numerators and unequal denominators, which fraction is the larger?

3. If the numerator of a fraction is multiplied by a certain number, how is the value of the fraction changed?

4. What is the effect on the value of a fraction, if both numerator and denominator are multiplied or divided by the same number?

5. The value of a fraction may be divided by a number by dividing the numerator by that number. Explain why this is true.

6. The value of a fraction may be divided by a number by multiplying the denominator by that number. Explain why this is true.

7. How is an integer multiplied by 10? By 100?

8. How is a decimal multiplied by 10? By 100?

9. How is a decimal divided by 10? By 100?

10. How is any number multiplied by 5 in the shortest way?

11. Write a mixed number. Explain how to change it to an improper fraction.

12. What is meant by canceling? Illustrate by an example.

13. How is the work of division of fractions tested?

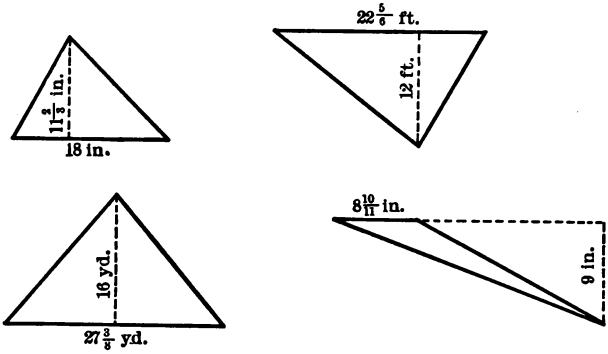
14. What is $\frac{1}{8}$ of 100? What is $\frac{1}{3}$ of 100? $\frac{1}{5}$ of 100?

15. Name some important parts of 100. What part of 100 is $87\frac{1}{2}$?

16. Illustrate what is meant by reducing a fraction to its lowest terms.

Written.

1. Find the area of each of the following triangles:



Make out bills for:

2. 6,000 lb. of small egg coal at \$7 a ton; 4,000 lb. of large egg coal at \$6.75 per ton; 2,000 lb. cannel at \$7.25 per ton; carrying in, 25¢ per ton.

3. 7 lb. mixed nuts at 15¢ per pound; 2 doz. oranges at 18¢ a dozen; 2 gal. of vinegar at 4¢ per quart; 1 gal. of maple sirup at 24¢ per quart.

Multiply:

4.	5.	6.	7.
6.25	8.09	1.25	99.2
30.9	6.3	17	.86
8.	**9.**	**10.**	**11.**
24.5	46.78	19.6	1.276
9.8	9.7	4.5	28

Divide. No quotient need be expressed beyond 3 decimal places:

12.	13.	14.	15.
$1.9\overline{)16.32}$	$27\overline{)178.5}$	$.39\overline{).7839}$	$2.4\overline{)9.648}$
16.	**17.**	**18.**	**19.**
$4.5\overline{)90.46}$	$1.26\overline{)3.672}$	$84\overline{)700.2}$	$53\overline{)9.007}$

1. How many pickets $2\frac{1}{2}$ in. wide, set $4\frac{1}{4}$ in. apart, will be required to make a fence 90 ft. long? What is the cost of the pickets at $1\frac{3}{8}¢$ apiece?

2. Taking $1\frac{1}{4}$ cu. ft. = 1 bu., how many bushels are there in a bin 8 ft. long, $4\frac{1}{2}$ ft. wide, 3 ft. 4 in. deep?

3. If the pressure of the atmosphere is 14.7 lb. per square inch, what is the pressure on the top of a table $1\frac{1}{4}$ yd. long and $\frac{2}{3}$ yd. wide?

4. A man bought a lot for \$6,000, the price being $62\frac{1}{2}¢$ per square foot; how many square feet did it contain?

5. The frontage was 60 ft.; how deep was the lot? What was the price per front foot?

6. If a glass of soda-water costs a druggist $2\frac{5}{8}¢$, how much does he make in selling 380 glasses at 5¢ each?

7. A border 3 feet wide is painted on a floor 24 ft. long, 21 ft. wide; what are the dimensions of the unpainted part? Of the border? What part of the whole floor is painted?

8. A gas burner consumed 550 cu. ft. of gas in 80 hours at a cost of 66 cents, what is the cost of 1 hour's light? What is the cost for 4 burners used 30 nights, 4 hours each night?

9. If 500 sheep are fed 10,500 pounds of hay in 7 days, how much hay are 500 sheep fed in one day? How much is 1 sheep fed in a day? How much are 800 sheep fed in 16 days?

10. A contractor had agreed to finish a building costing \$300,000 by a certain date, and, if not finished, he was to pay a fine of a certain amount for each day. The building was finished 12 days after time and his fine was \$240. What was his fine per day?

11. According to Exercise 10 what was the daily fine per thousand dollars? At the same rate, what would have been the daily fine on a building worth \$75,000?

1. A publisher's bill for sending magazines by mail was $263; what was the weight of the magazines?

2. What is the cost of a money order for $5.01? $9.99? $15.25? $2.75? $65.23? $89.99? $77.55?

3. Simplify:

$\frac{8}{10}$; $\frac{4}{12}$; $\frac{25}{20}$; $\frac{6}{18}$; $\frac{16}{28}$; $\frac{2}{34}$; $\frac{15}{45}$; $\frac{7}{56}$.

4. What is the cost of 3 dozen shirt-waists at 98¢ each?

5. What is the cost of 5 lb. of coffee at $.39 a pound?

6. If the freight rate is 75¢ per 100 pounds, what is the cost of sending a box of goods weighing 225 lb.?

7. A rectangular lot 540 ft. by 390 ft. is surrounded by a cement walk 9 ft. wide; find the area of the walk.

8. The cement of the walk mentioned in Exercise 7 rests upon a bed of crushed stone 6 in. deep; find the cost of the crushed stone at $1.44 per cubic yard.

9. Outside of the above walk a curbing is placed; find the cost of the curbing at $.40 per running foot.

10. If the area of a rectangle is 104 sq. in. and its width is 9 in., how long is it?

11. A man owns a tract of land with street frontage of 654 feet. He divides it into 12 lots of equal frontage; what is the frontage of each lot?

12. The distance around a square lot is 167 ft.; how long is one side of the lot?

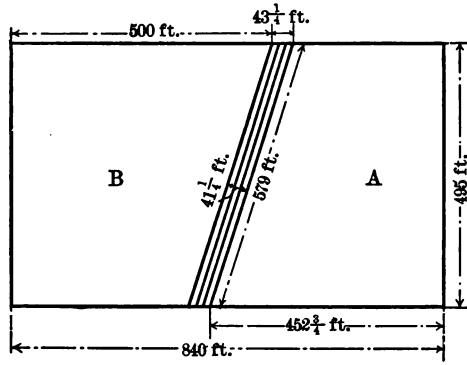
13. A boy in mowing a lawn 20 ft. by 30 ft. with a mower that cuts a swath 15 in. wide goes back and forth the short way of the lawn; how many feet does he travel in mowing the whole lawn?

Find:

14. $\frac{1}{2}$ of $\frac{1}{5}$.
15. $\frac{2}{7}$ of $\frac{1}{4}$.
16. $\frac{3}{4}$ of $\frac{1}{2}$.
17. $\frac{1}{6}$ of $\frac{2}{7}$.
18. $\frac{7}{8}$ of 64.
19. $\frac{5}{8}$ of 56.
20. $\frac{4}{5}$ of 45.
21. $\frac{7}{8}$ of 64.
22. $\frac{4}{9}$ of 72.

1. A piece of land is cut by a railroad as indicated in the figure; what is the area belonging to the railroad company? The area of the whole tract? The area of the parts A and B together?

2. If the railroad company fences its own land, how many yards of fence must it build on this tract? How many additional yards of fence are needed to enclose tract A? Tract B?

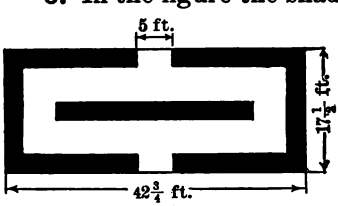


3. A rug 9 × 12 ft. covers $\frac{2}{5}$ of the floor of a room 12½ ft. wide; how long is the room?

4. The distance from New York to San Francisco by the proposed Panama route is 4,700 mi. If a boat travels 20 mi. per hour, how many days will it take for the voyage?

5. From New York to Hongkong by the proposed Panama route is about 13,900 mi.; in how many days would this voyage be made by a ship steaming 24 mi. per hour?

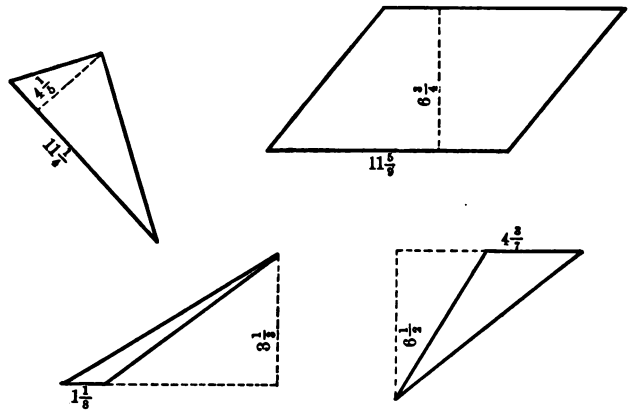
6. In the figure the shaded portions represent flower beds 2½ ft. wide. The unshaded part represents a gravel walk 5 ft. wide. How many running feet of wire netting will be required to separate the flower beds from the walk?



7. How many square feet are there in a board fence 5⅔ ft. high enclosing the garden, except at the entrances marked?

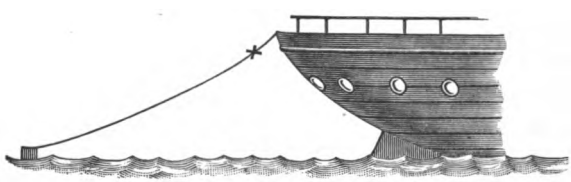
8. Find the area of the flower beds; of the walks. Verify by adding. What should this sum be?

1. Find the area of each of the following figures:



2. A steamer requires 6 days to go from San Francisco to Sitka, about 1,300 miles; what is the speed per hour in miles?

3. To measure the speed of a steamer, a float (log) sufficiently large to remain stationary was thrown overboard; the rope attached to the log had knots 44 ft. apart. As the ship moved on, a sailor, counting the number of knots drawn out, counted 14 knots in 30 seconds. How many feet was the ship going per minute? Per hour? How many miles per hour?



This is one of the ways in which the speed of a ship is measured. The nautical mile is about 6,087 ft. and the knots are placed at such distances apart that the number counted in 30 seconds is the same as the number of nautical miles the ship is sailing per hour. Hence the expression, "sailing 12 knots an hour."

4. Make and solve three problems about the ship's log.

Millions

1. How many thousands make 1 ten-thousand?

2. How many ten-thousands make 1 hundred-thousand?

Similarly, 10 hundred-thousands are grouped to make 1 **million**; 10 million are grouped into 1 **ten-million**, and 10 ten-millions are grouped into 1 **hundred-million.** The three places, millions', ten-millions', and hundred-millions', belong to **millions' period.** The following table shows the names of the first nine places or orders and the first three periods for integers. The number used for illustration is six hundred seventy-five million, three hundred twenty-one thousand, five hundred thirty-six.

	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units (Ones)
	6	7	5	3	2	1	5	3	6
Periods		Millions 3d			Thousands 2d			Units 1st	

The period at the left in a number may not be complete. That is, it may contain only one or two figures, as in 6,000 or in 65,000,233. How many must the other periods contain?

Each period beginning at the left is read as if it were a number by itself, the name of the period being added except in the case of units' period when it is understood.

Read:

3. 4,000,000. 4. 4,100,000. 5. 7,326,000.

6. 9,001,430. 7. 8,600,020. 8. 8,625,365.

1. A merchant gained $1,325.75 the first year, $2,195.50 the second year, and lost $989.85 the third year; how many dollars did he gain in the three years?

2. A ranchman paid $565 for sheep, $1,250 for cattle, and $863 for horses; how much had he left out of $3,000?

3. A landowner received $25,673 from the sale of lands, $2,565 for rent, and expended $565 for repairs; how much had he left?

4. A man started in business with $2,765.25 in cash and a stock of goods worth $5,850.25. At the end of the first year he has $3,650.50 in cash and a stock of goods worth $4,985. Has he gained or lost, and how much?

5. A man bought a horse and carriage for $300.50. He sold the horse for $125 at a loss of $50.50. What was the cost of the carriage?

6. The coinage of gold at the United States mints in 1880 was $62,308,279, in 1900 it was $99,272,942.50; how much more was coined in 1900 than in 1880?

7. Find the total coinage for each year from the following table:

Year.	Gold.	Silver.	Other Metals.
1880	$62,308,279.00	$27,141,673.75	$391,395.95
1885	27,773,012.50	28,962,176.20	191,622.04
1890	20,467,182.50	39,202,908.20	384,792.14
1895	59,616,357.50	5,698,010.25	882,430.56
1900	99,272,942.50	36,295,321.45	2,031,137.39

8. Read each result in Exercise 7. How many periods are there in each result? Name them.

Write in numerals:

9. Eight million.

10. Eighty-five million.

11. Four million two hundred seventy-five thousand.

1. Read the numbers in the statement of the New York banks for the last week of December, 1903:

The week's averages of the New York banks compare as follows with the same date one and two years ago:

	1903	1902	1901
Loans.....	$892,262,800	$875,321,500	$857,960,200
Deposits.....	865,918,700	865,953,600	910,869,300
Circulation...	45,304,400	45,656,400	31,856,100
Specie.....	161,151,800	153,744,300	161,618,200
Legal tender..	67,902,000	69,293,300	71,990,600
Total res...	$229,054,300	$223,037,600	$235,608,800
Res. required.	216,479,675	216,488,400	227,717,450
Surplus....	$12,574,625	$6,549,200	$7,891,350

Bring similar items cut or copied from newspapers or magazines and read the numbers in class.

2. Read: The numbers of people speaking various languages in 1801 and in 1890 were:

People.	1801.	1890.
English.....	20,520,000	111,100,000
French.....	31,450,000	51,200,000
German.....	30,320,000	75,200,000
Italian.....	15,070,000	33,400,000
Spanish.....	26,190,000	42,800,000
Portuguese.....	7,480,000	13,000,000
Russian.....	30,770,000	75,000,000

3. From Exercise 2 find the total number of people speaking the seven languages in 1801; in 1890.

4. Find how many more people spoke each language in 1890 than in 1801.

5. Which language was spoken by the most people in 1801? How many more people used it than used each of the other languages?

6. Answer the questions of Exercise 5 for the year 1890.

State.	Area in Sq. Mi.	Population.	State.	Area in Sq. Mi.	Population.
Alabama.....	52,250	1,828,697	Nevada.....	110,700	42,335
Alaska.....	590,884	63,592	New H'pshire	9,305	411,588
Arizona.....	113,020	122,931	New Jersey..	7,815	1,883,669
Arkansas.....	53,850	1,311,564	New Mexico.	122,580	195,310
California....	158,360	1,485,053	New York...	49,170	7,268,894
Colorado.....	103,925	539,700	N. Carolina..	52,250	1,893,810
Connecticut..	4,990	908,420	N. Dakota...	70,795	319,146
Delaware.....	2,050	184,735	Ohio.....	41,060	4,157,545
Dist. of Col...	70	278,718	Oklahoma Ty	39,030	398,331
Florida.....	58,680	528,542	Oregon.....	96,030	413,536
Georgia.....	59,475	2,216,331	Pennsylvania	45,215	6,302,115
Hawaii.....	6,449	154,001	Rhode Island	1,250	428,556
Idaho.....	84,800	161,772	S. Carolina..	30,570	1,340,316
Illinois.....	56,650	4,821,550	S. Dakota...	77,650	401,570
Indiana.....	36,350	2,516,462	Tennessee...	42,050	2,020,616
Indian Ty....	31,400	392,060	Texas.....	265,780	3,048,710
Iowa.....	56,025	2,231,853	Utah.....	84,970	276,749
Kansas.....	82,080	1,470,495	Vermont....	9,565	343,641
Kentucky....	40,400	2,147,174	Virginia.....	42,450	1,854,184
Louisiana....	48,720	1,381,625	Washington..	69,180	518,103
Maine.....	33,040	694,466	W. Virginia..	24,780	958,800
Maryland....	12,210	1,188,044	Wisconsin...	56,040	2,069,042
Massachusetts	8,315	2,805,346	Wyoming....	97,890	92,531
Michigan....	58,915	2,420,982	Possessions.		
Minnesota....	83,365	1,751,394	Guam (La-		
Mississippi....	46,810	1,551,270	drones)....	150	9,000
Missouri.....	69,415	3,106,665	Philippine Is.	140,000	8,000,000
Montana.....	146,080	243,329	Porto Rico ..	3,600	953,243
Nebraska....	77,510	1,066,300	Samoan Isl'ds	79	5,800

1. The total population, excluding that of possessions, is 76,303,387; what is the total including them?

Find the total area and population of the states bordering on:

2. The Atlantic Ocean.
3. The Pacific Ocean.
4. The Gulf of Mexico.
5. The Mississippi River.
6. The Great Lakes.
7. Your own State.

RATIO AND AVERAGE

Ratio

Oral.

1. What part of 9 qt. are 2 qt.? 5 qt.? 3 qt.? 7 qt.?

2. What part of 10 bu. are 2 bu.? 4 bu.? 5 bu.? 8 bu.?

3. What fraction of 7 days are 2 days? 3 days? 5 days? 6 days?

4. What fraction of 1 square foot is 1 sq. in.? 12 sq. in.? 36 sq. in.? 72 sq. in.?

5. What fraction of a day is 1 hr.? 1 hr. 45 min.? 6 hr.? 6 hr. 15 min.?

6. What part of $\frac{12}{17}$ is $\frac{4}{17}$? What part of $\frac{12}{25}$ is $\frac{4}{25}$?

7. What fraction of \$1 is $16\frac{2}{3}¢$? $25¢$? $33\frac{1}{3}¢$? $37\frac{1}{2}¢$? $62\frac{1}{2}¢$?

8. What part of 12 is 4? What is the quotient of 4 divided by 12?

The quotient of two numbers is often called their **ratio.**

The questions

What part of 12 is 4?
What fraction of 12 is 4?
What is the quotient of $4 \div 12$?
What is the ratio of 4 to 12?

} are all answered by the fraction $\frac{1}{3}$.

9. State the ratio of 16 oz. to 20 oz.; of 10 min. to 15 min.

What is the ratio of the gain to the cost?

	10.	11.	12.	13.	14.
Cost.....	\$8	$\$\frac{3}{4}$	\$.50	\$1.75	\$26.00
Gain.....	2	$\frac{1}{4}$	$.12\frac{1}{2}$	.25	5.20

1. An apartment-house contains 16 apartments. The total monthly rental is $410. If all the apartments were rented at the same rate, what would be the price of each? This is called the *average* rent for the apartments.

2. On three consecutive days a traveler spent $7.85, $6.15, and $6; how much did he spend in all? Had he spent an equal amount each day, how much must it have been to make the same total expenditure? This is called his *average* expenditure for the 3 days.

The ratio of the sum of several numbers to their number is called their **average**. $\frac{6+8+13}{3} = ?$ Thus, the average of 6, 8, and 13 is found as shown.

3. Write the ages to the nearest year of 4 pupils in your grade. Find the average age of these pupils.

4. If you have brothers or sisters, write their ages together with your own and find the average age of all.

The following table shows the attendance of 5 children for 7 weeks:

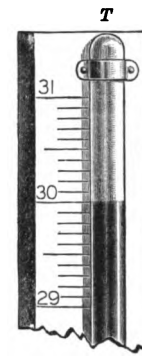
Name.	1st Week.	2d.	3d.	4th.	5th.	6th.	7th.
Mary...	5 days.	4 da.	5 da.	5 da.	5 da.	3 da.	4 da.
Helen..	5 "	5 "	4 "	3 "	5 "	5 "	4 "
Leslie..	3 "	5 "	4 "	4 "	5 "	5 "	5 "
Charles.	5 "	5 "	5 "	5 "	5 "	5 "	5 "
James..	5 "	4 "	5 "	3 "	5 "	4 "	5 "

5. According to the Table, find the average weekly attendance of each pupil.

6. For each week, find the average daily attendance of all the pupils.

7. How many whole blocks do you pass in coming to school? Count and note the number of houses in each of these blocks on the side usually taken by you. What is the average number of houses per block?

1. The barometer is an instrument for measuring air-pressure. The pressure on the surface, *S*, shown in the picture, forces the mercury up the tube, *T*. The scale in the picture shows how high the mercury stands in the tube. How many inches high does it stand?



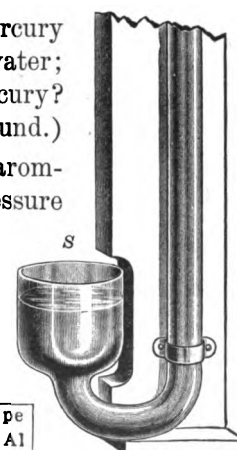
2. If the cross-section of the tube is 1 sq. in., the volume of the mercury in 1 inch of the tube is 1 cu. in.; how many cubic inches of mercury are there in the tube?

3. 1 cu. in. of water weighs .58 oz.; what is the weight of 30 cu. in.?

4. The weight of any quantity of mercury is 13.6 times that of an equal volume of water; what is the weight of 30 cu. in. of mercury? (Find the result correct to the nearest pound.)

5. What causes the mercury in the barometer to rise and fall? What is the air-pressure per square inch?

When the mercury rises gradually it is a general indication of fair weather. When it falls rapidly it indicates rain or snow.



6. Read the weather report. How do the barometer readings indicate snow or a storm? What is the average of these readings?

THE WEATHER.

THERMOMETER IN NEW YORK.

January 2, 1903.			January 2, 1904.		
8 A.M.	12 M.	2:30 P.M.	8 A.M.	10 A.M.	12 M.
33	39	42	19	18	18

BAROMETER IN NEW YORK.

8 A.M....30.03 ; 10 A.M.....29.99 | 12 M.....29.92

LOCAL FORECAST.

UNITED STATES WEATHER BUREAU, NEW YORK, January 2.—For New York and vicinity: Snow to-night. Sunday generally fair and colder; brisk to high northeast to north winds, shifting to northwest Sunday.

Plan the solution and make the computation:

1. If 8 hours of labor are understood to constitute a day's work and a workman receives $1.50 a day on this basis, how much does he receive for 36 hours' work? How much does he receive per hour? For 3 hours' work?

NOTE.—Fractional parts of a cent should be omitted from the results.

2. On the basis of Exercise 1, how many dollars does a man earn in a week who works $8\frac{1}{2}$ hours on Monday, 6 hours on Tuesday, 8 hours on Wednesday, 7 hours on Thursday, 9 hours on Friday, and 4 hours on Saturday?

3. The following table shows the number of hours worked by A, B, C, D in one week at a factory; also their wages per day on the basis of an 8-hour day:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	WAGES.	TOTAL.
A	8	$7\frac{1}{4}$	$7\frac{1}{2}$	6	9	8	$1.25	
B	$7\frac{1}{2}$	$8\frac{1}{2}$	8	$8\frac{3}{4}$	6	8	1.50	
C	8	$6\frac{1}{2}$	0	8	0	6	2.00	
D	6	$5\frac{1}{2}$	$6\frac{3}{4}$	9	$8\frac{1}{2}$	8	1.75	

On the basis of an 8-hour day, how many days did each man work during the week?

4. What is the average number of hours per day for each man in Exercise 3?

5. What is the average wage according to Exercise 4 for the 6 days?

6. What are the total wages of each for the week?

7. What is the total pay-roll for the four men?

8. How many hours did C's week lack of being six 8-hour days?

9. A man earned $840 a year and spent $780; what part of his earnings did he spend? What is the ratio of his expenditures to his income?

The following table shows the number of birds and the number of insects destroyed by them in a period of 5 days:

Black-billed Cuckoo.		Yellow-billed Cuckoo.
Number of birds.....	46	109
Destroyed		Destroyed
Caterpillars.....	906	1,865
Beetles.....	44	92
Grasshoppers.....	96	242
Spiders	15	86
Others.....	130	112

1. According to the above table, what was the average number of insects destroyed by the black-billed cuckoos? By the yellow-billed cuckoos? Give the results to the nearest whole number.

2. Find the total number of birds and the total number of insects. What was the average number of insects destroyed per bird?

3. What part of the whole number of insects eaten were caterpillars?

4. If 5 days are taken as the time in which these insects were eaten, what is the average number destroyed per day?

5. A Swanson hawk destroys on the average 100 grasshoppers per day; how many grasshoppers do 300 of these birds destroy in a month of 30 days?

6. Taking the average weight of each insect as 15.4 grains, what is the weight of the insects destroyed in a month? (7,000 grains = 1 lb.)

7. It is estimated that a grasshopper devours daily a quantity of food equal to his own weight; if the grasshoppers had not been killed, how many tons of produce would the farmer have lost in one month?

1. In 1895 Pennsylvania provided a bounty of 50¢ each on hawks, owls, and certain other birds and animals. In 1 year $60,000 were paid in bounties. How many of the animals were destroyed?

2. Most of the animals destroyed were hawks and owls. Granted that 5,000 chickens are killed annually in Pennsylvania by hawks and owls, and that they are worth 25¢ each, what is the annual loss from the chickens killed?

3. An owl or hawk is supposed to destroy on an average 1,000 mice annually. Granted that mice destroy on an average at least 2¢ worth of produce annually, how much does one owl or hawk save the farmers annually by destroying mice?

4. According to Exercises 1 and 3, taking all the animals as hawks and owls, how much would the birds, if allowed to live, have saved the farmers of Pennsylvania?

5. According to Exercises 3 and 4, how many dollars would the birds have saved the farmers in excess of what they destroyed in poultry?

6. How many dollars did the State throw away in bounties and in the loss of agricultural products in one year?

7. How many dollars did the State spend for every dollar saved under the Act of 1885?

8. Many birds eat seeds, which, if not destroyed, grow into weeds. Suppose that in your State (for area in square miles, see p. 120) there are 25 seed-eating birds per square mile and that each bird eats $\frac{1}{8}$ oz. of seed per day, how many pounds are destroyed per day? How many tons? At this rate how many tons are destroyed from June 1st to Oct. 31st, inclusive?

9. According to the last questions of Exercise 8, how many car-loads of 60,000 lb. of these seeds are destroyed in your State in a season?

Oral.

1. What are the names of the first four places in a whole number?

2. What is the name of the fifth place? The sixth?

3. How many periods are there in a number of 6 places?

4. What is the name of the seventh place in a whole number? What is the name of the third period?

Write:

5. Three million, two hundred seventy thousand, sixty-five.

6. Twenty-five million, ninety thousand, one hundred twenty.

7. Eighty-seven million, eighty-seven thousand, forty-nine.

8. Twenty million, forty thousand, one hundred forty.

9. What is a ratio? Give several examples.

10. What other questions mean the same as: "What is the ratio of 9 to 27?"

11. A man invested \$500 and received an income of \$25. What is the ratio of the income to the investment?

12. Illustrate what is meant by average. Name two kinds of problems in which it has been used.

13. Four cars together carry 240,676 lb. of granite; what is the average number of pounds which one car carries?

14. In 1884 the export from the United States of iron and steel and articles manufactured from them amounted to \$21,900,000; in 1900 the value of the same exports was \$121,900,000. What was the difference between the values of the exports for the two years given?

15. The price of wheat was: Monday, $\$.67\frac{1}{2}$; Tuesday, $\$.68\frac{3}{4}$; Wednesday, $\$.66\frac{1}{8}$; Thursday, $\$.67\frac{5}{8}$; Friday, $\$.67\frac{3}{8}$; Saturday, $\$.66\frac{7}{8}$. Find the average price for the week.

PERCENTAGE

Rate Per Hundred

1. Find the cost of 600 coats at \$492 per hundred.

\$492
6

2. Find the cost of 350 chairs at \$112 per hundred.

\$112
$3\frac{1}{2}$

3. A boy is permitted to fish in a pond on condition that he gives the owner 30 per hundred of the fish caught. He caught 120 fish. How many did he give the owner?

SUGGESTION: 30 of the hundred equal parts of the fish caught $= \frac{30}{100}$ of 120 = ?

4. Fractions with the denominator 100 are extensively used in practical and business arithmetic. $\frac{1}{2} = \frac{?}{100}$. $\frac{3}{2} = \frac{?}{100}$.

5. $1 = \frac{?}{100}$. $\frac{1}{5} = \frac{?}{100}$. $\frac{3}{5} = \frac{?}{100}$. $\frac{4}{5} = \frac{?}{100}$.

Express as hundredths:

6. $\frac{1}{4}$. 7. $\frac{3}{4}$. 8. $\frac{7}{4}$. 9. $\frac{5}{2}$.

10. $\frac{11}{5}$. 11. $\frac{6}{5}$. 12. $\frac{3}{5}$. 13. $\frac{1}{20}$.

14. $\frac{17}{20}$. 15. $\frac{7}{20}$. 16. $\frac{12}{20}$. 17. $\frac{1}{5}$.

18. $\frac{1}{10}$. 19. $\frac{2}{5}$. 20. $\frac{3}{20}$. 21. $\frac{9}{20}$.

22. 1 = — hundredths. $\frac{1}{3} = \frac{33\frac{1}{3}}{100}$. Why? $\frac{2}{3} = \frac{?}{100}$.

23. 1 = — hundredths. $\frac{1}{24} = \frac{4\frac{1}{6}}{100}$. Why? $\frac{11}{24} = \frac{?}{100}$.

Express in hundredths and fractions of a hundredth:

24. $\frac{1}{6}$. 25. $\frac{5}{6}$. 26. $\frac{4}{3}$. 27. $\frac{1}{8}$.

28. $\frac{6}{8}$. 29. $\frac{7}{8}$. 30. $\frac{11}{8}$. 31. $\frac{1}{12}$.

32. Write as an ordinary fraction 5 hundredths. Reduce it to its lowest terms.

33. Write as an ordinary fraction $12\frac{1}{2}$ hundredths. Then cancel $12\frac{1}{2}$ from both terms.

1. Of 600 pupils in a school 2 per hundred were absent; how many were absent?

2. A contractor bought 12,000 bricks; 28 per hundred were defective; how many defective bricks were there in the lot?

3. An army consists of 6,500 men; of these 11 per hundred are officers, the rest privates; how many officers are there?

4. Two thousand persons wished to attend a political meeting, but the hall would accommodate only 47 per hundred of them; find the capacity of the hall.

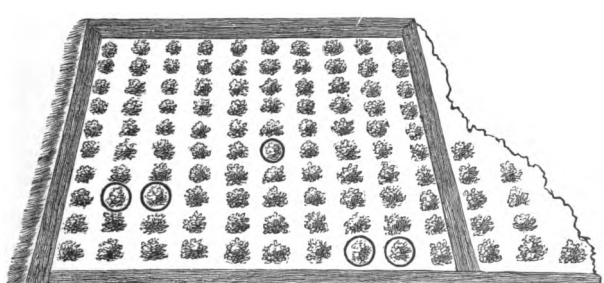
5. A defeated candidate received 35 per hundred of the 8,620 votes cast; how many did he receive?

In expressions like the above, **per cent** is commonly used instead of *per hundred.* Per cent is derived from the Latin *per centum* meaning *per hundred.*

Thus, 6 per cent (6%) means 6 per hundred. The symbol % is read *per cent.*

6. Read Exercises 1–5, using per cent whenever per hundred occurs. This does not change the meaning.

7. How many strawberry-plants are there in a square outlined in the picture? Those marked with circles are dead.



If there are as many dead in each hundred plants, how many per cent of the plants in the field are dead? How many per cent are alive?

1. Helen colored some water with cochineal. With a measuring-glass she took 80 parts of the red water and 20 parts of clear water and poured them into a glass. How many per cent of the solution were taken from the clear water?

2. Wesley colored some water with bluing and some with cochineal. He measured 40 parts of red water and 60 parts of blue water and mixed them together. What per cent of the solution was taken from the red water? From the blue?

3. If convenient, make a solution using 30% of red water and 70% of clear water; one using 75% of red water and 25% of blue; also other solutions.

4. Draw a square and divide it into 10 equal rectangles. Divide each rectangle into 10 equal squares. How many small squares are there in the figure?

5. Shade 1% of the squares in the figure of Exercise 4. Shade 5% of them; 10%; 25%; 50%; 75%; 80%.

6. Draw a circle and shade 25% of it; 50% of it; 75%.

7. Draw a rectangle whose breadth is 20% of its length; one whose breadth is 75% of its length.

The percentage of salt in the bodies of water named is:

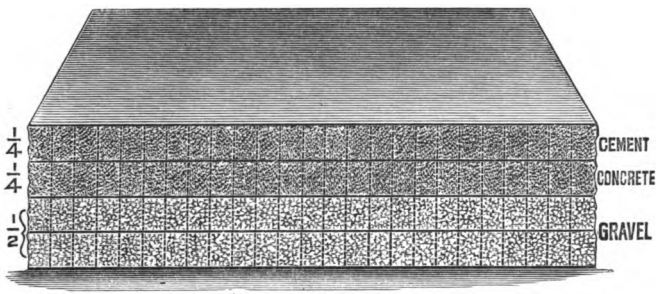
Caspian Sea	1.2	Atlantic Ocean	3.7
Black Sea	1.2	Mediterranean Sea	3.9
Great Salt Lake	20.0	Dead Sea	6.5

8. In the bodies of water named, how many pounds of salt are there in each 100 pounds of water?

9. Find the amount of salt in 1,700 pounds of water from each.

10. Make and solve 5 other problems using the facts of the table.

The picture shows a section of 1 yard of cement walk. In building such a walk a space is excavated the width of the proposed walk and half filled with coarse gravel. Then, $\frac{1}{2}$ of the rest is filled with concrete (cement and fine gravel), and the rest with cement and sand. The dotted lines in the figure are used simply to show the number of hundredths in each part.



1. How many rows of rectangles are there in the section? How many rectangles are there in each row? How many rectangles are there in the whole section? How many are there in $\frac{1}{2}$ of it? In $\frac{1}{4}$ of it?

2. What part of the material used in building a cement walk is coarse gravel? How many parts in 100 parts are coarse gravel? How many per cent?

3. What part of the material used in building such a walk is concrete? How many parts in 100 parts are concrete? How many per cent?

4. What part of the material used is cement and sand? How many parts per hundred? How many per cent?

5. 20% of anything are how many hundredths of it? What part of it? What percentage of anything is $\frac{1}{5}$ of it? $\frac{2}{5}$ of it? $\frac{3}{5}$ of it? $\frac{4}{5}$ of it?

How many are:

6. 20% of 200? 7. 10% of 1,000? 8. 75% of 1,600?

9. 25% of 500? 10. 50% of 2,000? 11. 60% of 1,200?

1. From the dimensions given in the figures, find the area of the space covered by the poultry-house.

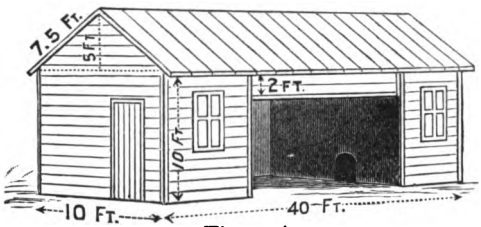


Figure 1.

2. From the figures, find the number of square feet of lumber required for the walls; the roof.

3. How many yards of wire netting are required to cover the opening to the shed? What is its cost at 15¢ per square yard?

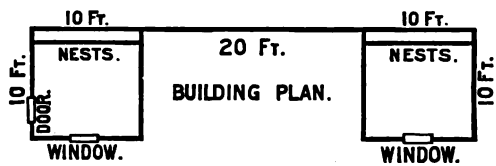


Figure 2.

4. If a flock of 100 hens average 192 eggs each in a year, how many dozen do they lay?

5. What is the value of these eggs at 22¢ a dozen?

6. If the average net profit of the hens is $1, what does it cost per hen to keep the flock?

7. What percentage of the cost is the annual net profit?

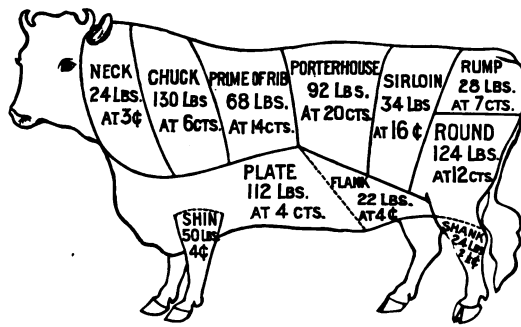
8. What is the value of a dozen broilers having an average weight of $1\frac{3}{4}$ lb. at 18¢ a pound?

The standard weights of different kinds of fowls are:

Kinds.	Weight.	Kinds.	Weight.
Plymouth Rocks...	$8\frac{1}{2}$ lb.	Cochins.....	10 lb.
Wyandottes.....	$7\frac{1}{2}$ "	Langshans.....	$8\frac{1}{2}$ "
Light Brahmas....	$10\frac{3}{4}$ "	Minorcas.....	$7\frac{1}{4}$ "
Dark Brahmas.....	$9\frac{3}{4}$ "	Spanish.....	$7\frac{1}{4}$ "

9. What is the value of 5 average Plymouth Rocks at 8¢ a pound? Make and solve 5 other problems.

1. The diagram shows the different kinds of meats as obtained by a standard method of cutting beef. The figures, representing the number of pounds and the cost of each kind, are approximate.



Find the total weight of beef in one animal as shown in the diagram.

2. Find the value of each kind and the total value of one animal.

3. Which kind contains the greatest number of pounds? The least? Which has the greatest value per pound? The greatest total value?

4. A Chicago wholesale house cuts beef so that the rib contains 9% of the whole animal; how many pounds of rib are cut from an animal weighing 1,075 lb.?

5. By this method 16% of loin is cut; how many pounds of loin are cut from an animal weighing 1,075 lb.? From one weighing 980 lb.?

6. 24% of a beef weighing 1,050 lb. is tallow and suet; how many pounds is this?

7. The following table gives the parts into which dressed mutton is cut for market and the average price of each kind. Find the number of pounds in an animal and the total value.

Leg.....	22.5 lb., 10¢.	Rib.....	14.5 lb., 9¢.
Loin.....	17.5 lb., 9¢.	Chuck.....	19.8 lb., $1\frac{1}{4}$¢.

8. Answer questions about mutton similar to those in Exercise 3.

1. The larva of the silkworm passes through 5 stages of about 5, 4, 5, 6, and 10 da.; what is the lifetime of a larva?

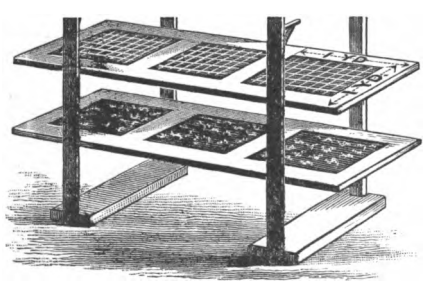
2. When a larva is full grown it spins a cocoon of silk around its body, making about 65 oscillations of its head per minute for a period of approximately 72 hours; about how many movements does it make in weaving the cocoon?

3. The silkworm moth lays about 500 eggs, of which 70% are deposited on the first day, 25% on the second, and the rest on the third; how many is this for each day?

4. After the larva stage, the worm is a chrysalis for about 20 days; it then emerges as a moth. During the next three days it lays its eggs, and lives about 12 days thereafter. Using the result of Exercise 1, find the approximate lifetime of a silkworm.

5. If the larvæ hatched from 1 ounce of eggs during the first stage require 11 lb. of mulberry-leaves, during the second stage 30 lb., during the third 120 lb., during the fourth 300 lb., and during the fifth, 1,650, how many pounds are required to mature the larvæ from 1 ounce of eggs?

6. The larvæ from 1 oz. of eggs should cover at birth 1 sq. yd. of breeding-bed. This space should be doubled twice during the first stage, twice during the second, and once during each of the third, fourth, and fifth stages; how many square yards do they occupy at last?



7. The figure represents some feeding beds or trays. According to the measurements given in the picture and the data of Exercise 6, how many trays are needed for the larvæ in the fifth stage from 1 oz. of eggs?

1. If 8 inch cubes are placed in a column, one above another, how high is the column?

2. If they are placed on a base 2 inches square, as shown in Figure 2, how high is the column?

3. If the 8 cubes are placed on a rectangular base 1 in. by 2 in., how high is the column?

4. If a dish of the size of Figure 2 is filled with water and the water is poured into a dish of the size of Figure 1, to what depth will the latter be filled?

5. If the tall dish with a base of 1 sq. in. contained 10 cu. in. and a low one had a base of 10 sq. in., to what depth would the contents of the former fill the latter?

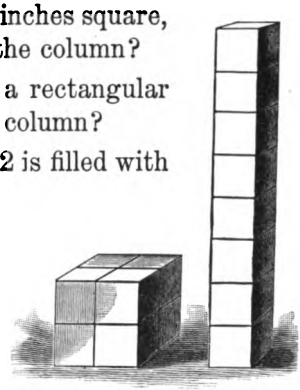


Figure 2. Figure 1.

6. The picture shows a standard rain-gage. The top is a funnel that receives the water falling on 10 sq. in. and collects it into a small tube whose base is 1 sq. in. When the water that would cover 10 sq. in. 1 in. deep is collected into a tube with a base of 1 sq. in., how high does the water stand?

7. If the water stands 5 in. in the tube to what depth would it stand, if spread over 10 sq. in.?

8. When the water stands 1 in. in the tube, to what depth would it stand if spread over 10 sq. in.?

9. What is the rainfall, or precipitation per square inch, when the water stands 2 in. in the tube?

10. Make and solve 5 other problems about the rain-gage.

Oral.

1. What is the meaning of "per cent"? Illustrate your answer by several examples.

2. How many per cent of a number is .75 of it? .25 of it? .12½ of it? .40 of it? .89 of it? .99 of it?

3. A farmer planted some corn of which 80 kernels per hundred came up; how many per cent came up? How many per cent decayed?

4. What is the symbol for "per cent"? Read: 75% of \$80 = \$60; 6% of \$100 = \$6.

Written.

5. On paper ruled in squares draw a square 10 in. units on a side. Shade 1% of its area. Shade a square whose area is 25% of that of the given square.

6. Draw on the board a square 10 in. on a side. What is its area? How many square inches are there in 25% of it? In 75% of it? In 40%? In 53%?

7. Draw a rectangle 10 in. long whose area shall equal 10% of that of the square of Exercise 5; draw other rectangles each 10 in. long to equal 20%; 30%; 50%; 70% of the given square.

8. Draw a rectangle 1 inch wide whose area is 5% of that of the square of Exercise 5; draw others of the same width with areas 7%; 3%; 15%; 29% of the area of the square. How long would such a rectangle be if it equals 100% of the square?

9. To test flower-seeds, 200 seeds were planted; 40 plants sprouted; how many per cent of the seeds were good?

10. If the seeds mentioned in Exercise 9 were bought at 25¢ per ounce, what was the price of good seeds per ounce?

11. Fifty pounds of corn contain 35 lb. of starch; what per cent of corn is starch?

1. Read:

The receipts of the Western Union Telegraph Company during the years 1880, 1890, and 1903 were respectively: $12,782,894; $22,387,029; and $29,167,687. Their expenses for the same years were: $6,948,957; $15,074,304; and $20,953,215.

2. Write in figures:

Five million, two hundred eighty thousand.

Seven million three hundred seventy-one thousand, six hundred fifty.

3. Write the number of persons in your family and that in 3 other families that you know. What is the average number of persons per family?

4. Henry's age is $12\frac{3}{4}$ years, John's $10\frac{1}{2}$ years, George's $11\frac{7}{12}$ years. Find their average age.

5. Mary's weight is $96\frac{1}{2}$ lb.; James's, $82\frac{3}{4}$ lb.; Helen's, $102\frac{5}{6}$ lb.; Louise's, $88\frac{11}{16}$ lb. Find their average weight.

6. The attendance at a certain school was: Monday, 621; Tuesday, 630; Wednesday, 638; Thursday, 627; Friday, 634. Find the average daily attendance.

7. During a certain period ending with 1898 England had 19 sovereigns and the average length of their reigns was $24\frac{3}{19}$ years; when did the period begin?

8. In 1901 the imports of Mexico from the United States were 44% of the exports to the United States; how many dollars' worth of imports did Mexico receive per $100 worth of exports to the United States? For every $500? For every $50? For every $25?

9. The foreign commerce of the United States has increased 11% since 1890; that of Africa, 49%; that of the whole world, 13%. What do these statements mean?

10. If 75% of a certain ore is copper, how many pounds of copper are there in 100 lb. of ore? In 4,000 lb. of ore?

FRACTIONS

Mixed Numbers

Oral.

1. $2\frac{1}{2} =$ — halves.
2. $5\frac{1}{3} =$ — thirds.
3. $5\frac{3}{4} =$ — fourths.
4. $2\frac{4}{5} =$ — fifths.
5. Add $3\frac{1}{2}$ and $5\frac{1}{2}$.
6. Add $6\frac{2}{3}$ and $17\frac{1}{3}$.
7. Add $4\frac{1}{2}$ and $7\frac{1}{4}$.
8. Add $12\frac{5}{6}$ and $12\frac{1}{3}$
9. Subtract $4\frac{1}{2}$ from $7\frac{1}{4}$.
10. Subtract $9\frac{2}{5}$ from $17\frac{3}{5}$.
11. Multiply $4\frac{1}{2}$ by 2; by 3; by 4; by 5; by 6; by 10.
12. A woman cut garments $\frac{1}{3}$ of a 10-hour day and sewed the remainder of the day; how many hours did she sew?

Written.

13. The length of a rectangular picture-frame is $3\frac{5}{9}$ in. and the width $4\frac{1}{2}$ in; what is the area of the opening?

SUGGESTION.—Reduce the numbers to improper fractions, indicate the product, and simplify by canceling.

14. A blotter is $3\frac{3}{4}$ in. wide and $9\frac{1}{5}$ in. long; how many square inches does it cover?
15. How many garments requiring $\frac{4}{5}$ of a yard of cloth can be cut from $4\frac{4}{5}$ yd.?

SUGGESTION.—Reduce the mixed number to an improper fraction, indicate the process and cancel as usual.

16. How many packages of $3\frac{3}{8}$ of a pound each can be made from $6\frac{3}{4}$ lb. of pepper?
17. How is the work of division tested in the case of whole numbers? In fractions the product of the divisor and quotient is always the dividend.

Divide and test:

18. $1\frac{5}{8} \div \frac{7}{4}$.
19. $2\frac{1}{2} \div \frac{5}{7}$.
20. $6\frac{3}{5} \div 3\frac{2}{3}$.
21. $2\frac{6}{7} \div \frac{5}{12}$.
22. $\frac{3}{5} \div 4\frac{2}{7}$.
23. $16\frac{5}{6} \div 3\frac{1}{8}$.
24. $5\frac{1}{2} \div 2\frac{1}{4}$.
25. $7\frac{9}{16} \div 2\frac{1}{4}$.
26. $11\frac{4}{9} \div 3\frac{3}{8}$.

1. Write the multiples of 3 from 3 to 39; of 4 from 4 to 40.

2. What numbers are common to both sets? What are the common multiples of 3 and 4 from 1 to 39? Which is the smallest of these multiples?

3. What are the common multiples of 4 and 5 from 1 to 50? Which is the smallest of these numbers?

4. Some boys formed a military company and found that they could march 3, 4, or 5 abreast in full ranks; what number is a multiple of 3, 4, and 5? What is the least number that is a multiple of 3, 4, and 5? What is the least number of boys that could have been in the company?

The smallest common multiple of two or more numbers is called their **least common multiple** (l. c. m.).

5. What is the l. c. m. of 4 and 6? Of 5 and 6? Of 3 and 5? Name other common multiples for each set.

Find by inspection the l. c. m. of:

6. 6, 9. 7. 4, 8. 8. 3, 4, 6. 9. 4, 8, 10.

10. 4, 6. 11. 8, 10. 12. 3, 5, 10. 13. 5, 6, 15.

14. Find by factoring the l. c. m. of 12, 21, 39. Each factor must occur as often in the l. c. m. as in any of the given numbers. Hence, the l. c. m. equals $2 \times 2 \times 3 \times 7 \times 13 = 1{,}092$.

$12 = 2 \times 2 \times 3$.
$21 = 3 \times 7$.
$39 = 3 \times 13$.

Find by factoring the l. c. m. of:

15. 36, 42. 16. 18, 28. 17. 12, 18, 27.

18. 27, 24. 19. 14, 28. 20. 25, 60, 10.

21. 14, 22. 22. 22, 32. 23. 32, 48, 72.

24. A florist when decorating a hall found that he had just enough roses to enable him to put either 12, 15, or 18 roses into each vase and have no roses left over; how many roses did he have?

1. In 1900, $\frac{3}{35}$ of all the farms in New Jersey contained less than 10 acres; $\frac{3}{5}$ contained from 10 to 99 acres.

$\frac{3}{5} = \frac{21}{35}$. How is this found? $\frac{3}{35} + \frac{21}{35} = \frac{?}{35}$. What part of the whole number of farms contained less than 100 acres?

2. $1 = \frac{?}{35}$. $\frac{35}{35} - \frac{24}{35} = ?$ What part of the whole number of farms in New Jersey contained 100 acres or more?

3. On $\frac{1}{5}$ of the total number of farms the principal source of income was vegetables; on $\frac{2}{7}$, live stock; on $\frac{6}{35}$, dairy produce; what part of the whole number had none of these as the principal source of income?

4. Before adding or subtracting the fractions in the above exercises, how were the fractions changed?

When fractions are to be added or subtracted they should be changed to fractions having the same denominator. This denominator is called their **common denominator.**

Change the fractions of each row to equal fractions having the common denominator as indicated:

Fractions.	Common Denominator.	Fractions.	Common Denominator.
5. $\frac{5}{16}$, $\frac{2}{32}$.	64.	6. $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{2}$.	16.
7. $\frac{1}{3}$, $\frac{4}{7}$, $\frac{5}{2}$.	42.	8. $\frac{5}{9}$, $\frac{6}{3}$, $\frac{11}{6}$.	18.
9. $\frac{12}{7}$, $\frac{2}{2}$, $\frac{1}{4}$.	28.	10. $\frac{3}{5}$, $\frac{9}{10}$, $\frac{12}{4}$.	40.

11. In Exercises 5–10 select those in which the new denominator is the least number that could be used thus. What is the least common multiple of the given denominators in each exercise?

The least common multiple of the given denominators is called the **least common denominator** (l. c. d.) of the fractions.

Find the least common denominators of:

12. $\frac{2}{3}$, $\frac{4}{5}$.	13. $\frac{5}{8}$, $\frac{3}{4}$.	14. $\frac{4}{7}$, $\frac{3}{5}$.	15. $\frac{5}{6}$, $\frac{8}{9}$.
16. $\frac{2}{9}$, $\frac{7}{8}$.	17. $\frac{3}{10}$, $\frac{7}{8}$.	18. $\frac{7}{9}$, $\frac{1}{6}$.	19. $\frac{5}{7}$, $\frac{2}{3}$.

Change to equal fractions with l. c. d.:

1. $\frac{3}{5}$, $\frac{5}{8}$. 2. $\frac{8}{5}$, $\frac{3}{2}$. 3. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{6}$.

4. $\frac{3}{2}$, $\frac{2}{5}$, $\frac{7}{10}$. 5. $\frac{2}{7}$, $\frac{1}{4}$, $\frac{5}{2}$. 6. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{15}$.

Change to fractions equal to the given fractions with l. c. d. and add:

7.	8.	9.	10.	11.
$\frac{3}{4}$	$\frac{8}{9}$	$\frac{3}{5}$	$\frac{7}{12}$	$\frac{5}{6}$
$\frac{5}{8}$	$\frac{5}{6}$	$\frac{11}{10}$	$\frac{11}{8}$	$\frac{3}{8}$

12. Add $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{8}$. It is convenient in adding several fractions to arrange the work thus:

	24THS.
$\frac{2}{3}$	16
$\frac{1}{4}$	6
$\frac{5}{8}$	15
$\frac{37}{24}$	37

13. A man sold $\frac{2}{3}$ of an acre at one time, $\frac{1}{2}$ of an acre at another, and $\frac{3}{4}$ at another; how many acres did he sell altogether?

14. If a farmer laid $\frac{1}{8}$ of a mile of drain-tile one year, $\frac{3}{4}$ of a mile the second year, and $\frac{5}{8}$ of a mile the third year, how many miles did he lay in three years?

15. If from $\frac{7}{8}$ of a bolt of silk $\frac{3}{5}$ of the whole bolt are sold, what part of the whole bolt remains?

Reduce to least common denominator and add:

16.	17.	18.	19.
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{3}{4}$
$\frac{2}{5}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{13}{10}$
$\frac{7}{10}$	$\frac{5}{6}$	$\frac{11}{18}$	$\frac{4}{5}$

Reduce to least common denominator and subtract:

20.	21.	22.	23.
$\frac{17}{3}$	$\frac{11}{3}$	$\frac{16}{25}$	$\frac{11}{8}$
$\frac{11}{12}$	$\frac{15}{7}$	$\frac{3}{5}$	$\frac{5}{4}$

24. From a bolt of cloth a merchant sold $\frac{1}{8}$ at one time, $\frac{1}{3}$ at another, and $\frac{1}{7}$ at another; how much did he sell in all?

25. A rectangle is $\frac{1}{6}$ of a foot on one side and $\frac{3}{5}$ on another; what is the length of its perimeter?

1. A train leaves Chicago for St. Louis at 1:30 P. M. and makes the run in $8\frac{3}{4}$ hours; at what time does it arrive in St. Louis? What time of day is $10\frac{1}{4}$ hr. past noon?

hr.
$1\frac{1}{2}$
$8\frac{3}{4}$
9 hr. + $1\frac{1}{4}$ hr. = — hr.

2. A chest of tea contains $69\frac{5}{16}$ lb. and $12\frac{1}{4}$ lb. are sold; how many pounds are left?

lb.
$62\frac{5}{16}$
$12\frac{1}{4}$
50 lb. + $\frac{1}{16}$ lb. = — lb.

In adding or subtracting mixed numbers it is usually more convenient to treat the fractions and whole numbers separately than to change the numbers to improper fractions.

3. A man walked $11\frac{1}{4}$ miles on Monday and $6\frac{7}{14}$ miles on Tuesday; how many miles did he walk altogether?

4. William is $11\frac{3}{4}$ years old and Thomas is $9\frac{5}{12}$ years old; William is how much older than Thomas?

5. A gold dollar contains $23\frac{11}{20}$ grains of gold and $2\frac{5}{10}\frac{8}{0}$ grains of alloy; what is the weight of the gold dollar?

6. The half-eagle or 5-dollar gold piece contains $99\frac{2}{2}\frac{2}{5}$ more grains of gold than the gold dollar; how much gold does the half-eagle contain?

Add:

7.	8.	9.	10.	11.
$3\frac{2}{3}$	$6\frac{1}{5}$	$4\frac{5}{12}$	$2\frac{1}{8}$	$4\frac{1}{3}$
$5\frac{3}{4}$	$7\frac{2}{3}$	$2\frac{1}{2}$	$5\frac{2}{3}$	$3\frac{1}{4}$
12.	**13.**	**14.**	**15.**	**16.**
$3\frac{4}{5}$	$1\frac{1}{4}$	$7\frac{3}{8}$	$2\frac{3}{8}$	$6\frac{5}{9}$
$2\frac{5}{10}$	$1\frac{11}{15}$	$4\frac{1}{12}$	$6\frac{13}{16}$	$2\frac{5}{12}$

Subtract.

17.	18.	19.	20.	21.
$50\frac{3}{4}$	$63\frac{7}{12}$	$84\frac{17}{20}$	$61\frac{8}{13}$	$7\frac{7}{8}$
$17\frac{5}{12}$	$21\frac{3}{10}$	$49\frac{9}{16}$	$27\frac{4}{15}$	$2\frac{1}{2}$

1. From a jar containing $18\frac{1}{2}$ lb. of butter $7\frac{3}{4}$ lb. are sold; how many pounds remain in the jar? $18\frac{1}{2}$ lb. $= 17\frac{6}{4}$ lb. Why?

$18\frac{1}{2} = 17\frac{6}{4}$
$7\frac{3}{4} \quad 7\frac{3}{4}$

2. A park contains $63\frac{1}{8}$ acres of which $13\frac{3}{16}$ acres are given to lakes and lagoons; how many acres are dry land? $63\frac{1}{8}$ acres $= 62\frac{18}{16}$ acres. Why?

$63\frac{1}{8} = 62\frac{18}{16}$
$13\frac{3}{16} \quad 13\frac{3}{16}$

3. Mary bought $12\frac{1}{2}$ yards of cloth and used $7\frac{5}{8}$ yards for a dress; how many yards had she left?

4. A man traveled $125\frac{6}{13}$ miles one day, $250\frac{3}{4}$ miles the next, and $113\frac{1}{2}$ miles the next; how many miles did he travel in the three days?

5. Charles walks $1\frac{1}{2}$ miles to school and George walks $\frac{3}{4}$ of a mile; who walks the farther, and how much?

6. From a jar containing 20 lb. of butter, a grocer sold $3\frac{3}{4}$ lb. to one customer and $6\frac{5}{8}$ to another; how many pounds are left in the jar?

7. From 20 yards of cloth a merchant sold $10\frac{3}{4}$ yards, $5\frac{1}{2}$ yards, and $2\frac{3}{8}$ yards; how many yards had he left?

Subtract:

8.	9.	10.	11.	12.
$16\frac{2}{5}$	$9\frac{2}{3}$	$2\frac{3}{4}$	$11\frac{3}{7}$	$105\frac{3}{8}$
$7\frac{9}{10}$	$5\frac{11}{12}$	$1\frac{5}{6}$	$9\frac{1}{2}$	$89\frac{11}{20}$
13.	**14.**	**15.**	**16.**	**17.**
$12\frac{1}{2}$	$16\frac{1}{3}$	$\frac{11}{4}$	$\frac{27}{8}$	$25\frac{4}{7}$
$\frac{3}{3}$	$\frac{17}{5}$	$1\frac{1}{6}$	$2\frac{7}{12}$	$\frac{25}{14}$
18.	**19.**	**20.**	**21.**	**22.**
$6\frac{2}{3}$	$3\frac{4}{9}$	$17\frac{2}{3}$	$6\frac{1}{2}$	$32\frac{1}{11}$
$3\frac{2}{5}$	$1\frac{1}{6}$	$\frac{17}{4}$	$\frac{7}{5}$	$\frac{16}{3}$
23.	**24.**	**25.**	**26.**	**27.**
$8\frac{1}{7}$	$6\frac{7}{8}$	$18\frac{1}{5}$	$87\frac{5}{6}$	$12\frac{1}{9}$
$4\frac{3}{4}$	$2\frac{9}{10}$	$8\frac{1}{17}$	$25\frac{3}{25}$	$7\frac{2}{3}$

1. Small triangular pieces of cloth used to shape garments are called gussets. They are of various triangular shapes. How many gussets like Figure 1 can be cut from a 4-in. square?

2. State the size of the smallest piece of cloth that will cut 4 gussets like Figure 2.

3. What is the smallest piece of cloth that will make 6 gussets like Figure 3?

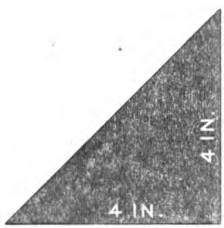


Figure 1.

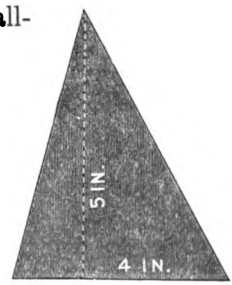


Figure 2.

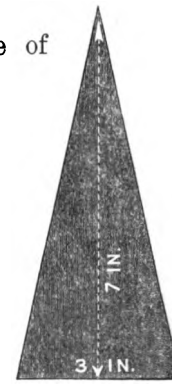
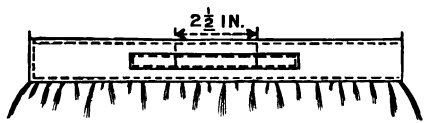


Figure 3.

4. Show by drawings the shape of the pieces of cloth mentioned in Exercises 2 and 3, and how the gussets should be cut.

5. A breadth of a skirt is $23\frac{1}{4}$ in. wide after a seam of $\frac{1}{4}$ in. has been made along each edge; how wide was the cloth?

6. A tape loop sewed to a band has a free length of $2\frac{1}{2}$ in. as shown in the picture. If $\frac{1}{4}$ in. is turned under at each end, and the tape is stitched down $\frac{1}{2}$ in. from each end, how long was the piece of tape?



7. Pillow-cases are made from pillow-tubing; one end of the pillow-case is seamed and the other hemmed. Allowing $\frac{1}{2}$ in. for the seam and 3 in. $+ \frac{1}{8}$ in. for the hem, how long must the tubing be cut to make a finished pillow-case $32\frac{3}{4}$ in. long?

1. There are $20\frac{5}{8}$ lb. of butter in a jar which weighs $4\frac{4}{5}$ lb.; what is the combined weight of the butter and the jar?

2. If from a 5-gallon jug of sirup $\frac{1}{3}$ is poured out at one time and $\frac{1}{5}$ of the remainder at another, how many gallons remain in the jug?

3. A man bought at different times $20\frac{2}{3}$ acres of land, $50\frac{3}{4}$ acres, and $17\frac{6}{8}$ acres; how many acres did he buy?

4. A boy walked $\frac{1}{2}$ of a mile in one day, $\frac{3}{4}$ of a mile the next day, and $1\frac{1}{2}$ miles the next day; how far did he walk in the three days?

5. A man spent $\frac{1}{6}$ of his salary for rent, $\frac{1}{4}$ for food, $\frac{1}{10}$ for clothes, $\frac{1}{5}$ for miscellaneous items and saved the rest; what part did he save?

6. A boy took a bicycle trip into the country to visit his uncle. He was gone $6\frac{3}{4}$ hours, $2\frac{5}{6}$ of which he spent with his uncle, and $2\frac{1}{12}$ hours in riding back; how many hours did he take to ride out?

7. Mr. Smith's farm contains $102\frac{5}{8}$ acres; Mr. Brown's farm contains $10\frac{1}{4}$ acres more than Mr. Smith's and $26\frac{5}{16}$ less than Mr. Green's farm; how many acres are there in Mr. Brown's farm? In Mr. Green's? In all three together?

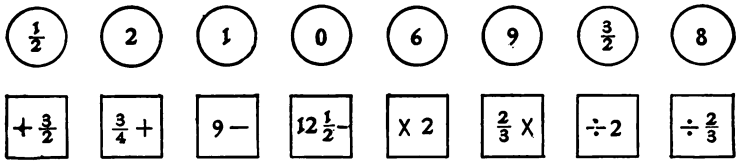
Fill out the blanks and test the work:

	8.	9.	10.	11.	12.
Addend:	( )	$5\frac{3}{5}$	$8\frac{1}{2}$	$11\frac{1}{4}$	( )
Addend:	$3\frac{1}{2}$	$3\frac{1}{6}$	$2\frac{5}{12}$	$6\frac{1}{8}$	$10\frac{1}{3}$
Sum:	$8\frac{1}{2}$	( )	( )	( )	$15\frac{5}{8}$

	13.	14.	15.	16.	17.
Minuend:	$12\frac{2}{3}$	$21\frac{1}{2}$	$4\frac{5}{12}$	14	$3\frac{7}{15}$
Subtrahend:	( )	$12\frac{1}{6}$	$2\frac{11}{18}$	$6\frac{3}{7}$	( )
Remainder:	$6\frac{1}{2}$	( )	( )	( )	$1\frac{2}{3}$

18. $5\frac{5}{12}$ hours after noon are how many hours before midnight?

A game is played with 8 round cards and 8 square cards marked as indicated.



The round cards are placed on the table face up. The square cards are shuffled and placed on the table face down. The first player draws a square card, looks at it and then takes any round card he wishes. He places the two cards side by side so as to indicate an operation, and scores the result of the operation. Then the second player draws a square card, takes any round card remaining, places them side by side similarly, and scores the result of the operation. They continue thus alternately until all the square cards are drawn. The player wins who has the larger total score.

1. In a game, John drew first $\frac{3}{4}+$. He took the card 9, because he gets the largest score with it. What was his score?

2. Then Henry drew $\frac{2}{3}\times$ and took the card 8. What was his score?

3. Then John drew $12\frac{1}{2}-$ and took 0 scoring ——.

4. At the end of the game they had the cards placed thus:

JOHN.		HENRY.	
$\frac{3}{4}+$	9	$\frac{2}{3}\times$	8
$12\frac{1}{2}-$	0	6	$+\frac{3}{2}$
2	$\div\frac{2}{3}$	$\frac{3}{2}$	X 2
9 −	$\frac{1}{2}$	1	÷ 2

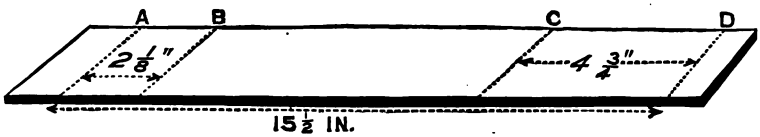
What was John's total score? Henry's? Which wins?

5. If convenient, cut out round and square pieces of card-board, mark them as indicated above, play games and give scores.

NOTE.—The game may be varied by playing for the smaller total score; also by first dealing out all the square cards, four to each player. Four players can also play. If a ninth card of each sort is added (for example +3, 4), three players can play.

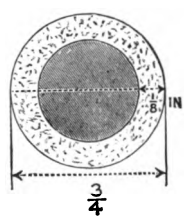
Oral.

1. Four half-inch holes are bored at equal intervals in a towel-rack. The distance from each hole to the next one is 4 in., and the outer ones are $1\frac{1}{2}$ in. from the ends of the rack; what is the length of the rack?

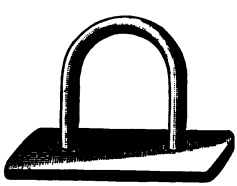


2. Distances in inches are laid off on an iron band as shown in the figure; find the distance from B to C.

3. Flat rings, as shown in the picture, are used for packing valves. The outer diameter of the ring and its width are shown. What is the diameter of the circle to be cut from the center?

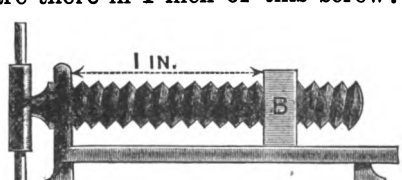


4. The picture shows an iron staple set in a piece of iron $\frac{1}{8}$ in. thick. The length of the staple from one end to the other above the plate is $3\frac{7}{8}$ in. Before it was riveted it projected through the plate $\frac{1}{8}$ in. at each end. How long must the piece of rod be cut to make one staple?



5. How many such staples can be cut from a rod 27 in. long? From one 54 in. long?

6. How many threads are there in 1 inch of this screw? This number is called the **pitch** of the screw. When the screw is turned around once, how far is the block, B, moved? How many turns must the screw make to move B $\frac{1}{2}$ in.? $\frac{3}{4}$ in.? $\frac{1}{4}$ in.? $1\frac{1}{4}$ in.? How far do 4 turns move B?



Oral.

1. Do 12 dozen eggs at 19¢ a dozen cost more or less than 12×20¢? How much more or less? How many are 12×20? What is the cost of 12 dozen eggs at 19¢ a dozen?

2. Do 9 lb. of butter at 28¢ a pound cost more or less than 9×30¢? How much more or less? How many are 9×30? What is the cost of 9 lb. of butter at 28¢ a pound?

3. What is the cost of 1 dozen pairs of lace curtains at 98¢ a pair?

4. What is the cost of 15 yd. of muslin at 11¢ a yard? Of 24 yd. of gingham at 9¢ a yard?

5. What is the cost of 5 lb. of spring chicken at 18¢ a pound? Of 13 lb. of pork at 11¢ a pound?

6. What is the cost of 101 boxes of soap at $2.75 a box?

7. What is the cost of 3 shirt-waists at $.99 each? Of 4 pairs of gloves at 98¢ a pair?

8. What is the cost of 5 lb. of coffee at 29¢ a pound? Of 9 lb. at 19¢ a pound?

9. How much do 11 yards of carpet cost at 79¢ per yard?

10. What is the area of a square 21 in. on a side?

11. What is the area of a triangle of base 12 in. and altitude 11 in.?

12. How many miles does a train run in 18 hr. at the rate of 31 mi. per hour?

13. How many miles does an automobile travel in 11 hr. at the rate of 21 mi. per hour?

14. What is the weight of a load of apples containing 22 barrels, if each barrel of apples weighs 130 lb.?

15. How many square rods are there in a field 17 rd. long and 9 rd. wide?

1. A rectangle is 12 in. long and 6.5 in. wide; is its area greater or less than 72 sq. in.? Why? Is its area greater or less than 84 sq. in.? Why? Find the exact area in the usual way.

2. A rectangle is 5.5 ft. wide and 11.3 ft. long; is its area greater or less than 55 sq. ft.? Why? Is its area greater or less than 72 sq. ft.? Why? Find the exact area in the usual way.

3. What is the cost of 15 lb. of sugar at $5\frac{1}{2}¢$ a pound? Is the cost more or less than 75¢? Why? Is it more or less than 90¢? Why? Find the exact cost in the usual way.

4. A train runs from New York to Montreal, 383 miles, in 12 hours; estimate the rate of speed per hour. Is the rate more or less than 30 mi. per hour? Why? Is the rate more or less than 40 mi. per hour? Why? Find the exact rate.

Estimate the results before solving:

5. A rectangular box is 8 ft. by 4 ft. by $5\frac{1}{2}$ ft. Find its volume.

6. What is the area of a triangle of base 6.5 in. and altitude 14.5 in.?

7. What is the area of a rectangle of base $5\frac{1}{2}$ yd. and altitude $3\frac{1}{2}$ yd.?

8. When bottles of ink cost $15.12 per gross, what is the cost per bottle?

9. When bottles of library paste cost $5.04 per gross, what is the cost per bottle?

10. A crate of eggs contains 6 layers of 6 dozen each; how many eggs are there in the crate?

11. What is the cost of 2 16-quart crates of blueberries at 8 cents a quart?

12. How many hours does it take a train to travel 440 miles at the rate of 35 miles an hour?

Oral.

1. State a short way of finding the cost of 11 yd. of cloth at 29¢ a yard.

2. State a short way of finding the cost of a gross of paste-bottles at 9¢ each.

3. State a short way of finding the area of a triangle 22 in. long and 19 in. wide.

4. What is the cost of one dozen pairs of lace curtains at $1.99 a pair?

5. What is meant by estimating the result before solving a problem? Illustrate.

6. Estimate the area of a rectangle whose length is $17\frac{1}{2}$ ft. and whose width is $5\frac{3}{4}$ ft.

7. What is a mixed number? How is a mixed number related to an improper fraction? Illustrate.

8. How is one fraction divided by another? Illustrate by $\frac{2}{3} \div \frac{4}{5}$.

9. What is meant by a multiple of a number? What is the least common multiple of several numbers?

10. Name the least common multiple of 4 and 14; of 12 and 24; of 4 and 18; of 6 and 27.

11. When the least common multiple can not be found by inspection, what method may be used?

12. What is meant by a common denominator? What is the least common denominator of several fractions?

13. How are least common multiple and least common denominator associated?

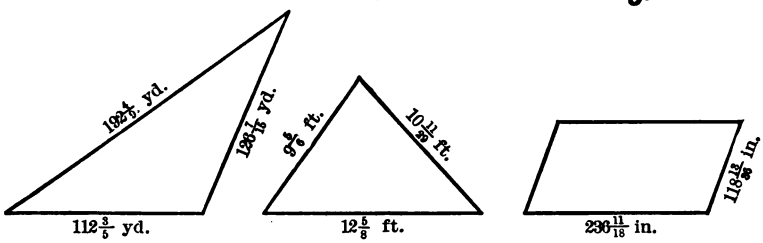
14. What change should be made in fractions with different denominators before adding or subtracting? Illustrate.

15. A man worked $\frac{1}{2}$ da. on Monday, $\frac{2}{3}$ da. on Tuesday, and $\frac{3}{4}$ da. on Wednesday; how many days did he work in all?

16. How may subtraction of fractions be tested?

Find the length of fence around each of the following fields:

1. 2. 3.



4. A rectangular garden is $21\frac{5}{8}$ ft. wide and $67\frac{7}{12}$ ft. long; how long a fence will be required to enclose it?

5. A bicycle rider goes $\frac{1}{2}$ mile in the first minute of a race, $\frac{3}{8}$ of a mile in the second minute, and $\frac{7}{10}$ of a mile in the third; how far does he ride altogether?

6. Mr. Smith burned $\frac{1}{4}$ of a ton of coal in his furnace in October; $\frac{7}{8}$ of a ton in November; $1\frac{3}{16}$ tons in December; $2\frac{1}{5}$ tons in January; $1\frac{7}{40}$ tons in February; $1\frac{3}{4}$ tons in March; $1\frac{1}{16}$ tons in April; and $\frac{3}{10}$ of a ton in May; how many tons of coal did he use altogether?

7. A farmer planted $63\frac{3}{4}$ acres of his farm in wheat; $16\frac{7}{8}$ acres in oats; $46\frac{5}{8}$ acres in corn; and $6\frac{1}{2}$ acres in rye; how many acres did he plant?

8. Add at sight by giving the sum after the 2d, 3d, 4th addend, and so on to the end: $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$.

9. Add as in Exercise 8 by repeating $3\frac{1}{3}$ until the sum is 30.

Add by repeating each of the following until the sum equals or exceeds 100:

10. $3\frac{1}{3}$. 11. $12\frac{1}{2}$. 12. $6\frac{1}{4}$. 13. $33\frac{1}{3}$ 14. $16\frac{2}{3}$.

15. How many feet of iron rod are needed to make 2 dozen wire staples, requiring $2\frac{5}{8}$ in. each?

1. How many flagstones 2 ft. 8 in. by 5 ft. are required to lay a walk 5 ft. wide and 56 ft. long?

2. If it requires $3\frac{1}{2}$ yd. of silk to make a waist, how many waists can be made from a piece of silk 35 yd. long?

3. On July 4, 1904, at New York the sun rose at 4:27 A. M. and set at 7:40 P. M.; how long was it from sunrise to sunset?

4. What part of the day (24 hours) did the sun shine?

5. Draw a rectangle 12 in. long and divide it into two parts, one representing the time from sunrise to sunset for to-day, the other from sunset to sunrise.

6. Mark on squared paper a square 10 little squares each way and shade 20% of it black; how many squares remain unshaded? Of these shade 20% red. How many remain unshaded? What percentage of the whole is unshaded?

7. Mark on squared paper a rectangle 5 squares wide and 20 squares long. Shade 40% of it black; shade 25% of the remainder red; what percentage of the whole remains unshaded? What fraction? What is the ratio of the entire shaded part to the white? Of the black to the red? Of the red to the white? Of the white to the black?

8. A field yielded 30 bu. of oats one season, 40 bu. the next, and 56 bu. the third; what was the average yield for the three seasons?

9. The area of Austria-Hungary is 240,942 sq. mi. and of Belgium 11,373 sq. mi.; the area of the former is about how many times that of the latter?

10. The area of Great Britain is 120,000 sq. mi. and that of the United States 3,000,000 sq. mi. The area of the latter is how many times that of the former?

11. The area of Germany is about 200,000 sq. mi. and the population in 1900 was 56,000,000; how many persons were there in Germany per square mile at that time?

APPROXIMATION

Cost

1. What is the cost of 3 yards of muslin at $12\frac{1}{2}¢$ a yard? How much does a customer who makes this purchase pay for the cloth?

2. What is the cost of $8\frac{2}{3}$ yards of ribbon at $10¢$ a yard? How much does a customer who makes this purchase pay for the ribbon?

3. How much does a customer pay for $8\frac{1}{4}$ lb. of rice at $9¢$ a pound?

4. If the computed cost of an article is $\$.37\frac{1}{2}$, what is the amount usually paid for it?

5. One cent is what part of a dollar? To how many decimal places need the value of anything be expressed to show its value correct to the nearest cent, or within one-half of a cent?

In determining results of this kind *all decimal places beyond the third are useless and should not be calculated.*

6. On a certain railroad, the distance from New York to Chicago is 1,024 miles and the fare is $17. How much is this per mile correct to the nearest tenth of a cent?

7. The cost of operating a railway may be classified thus:

Cost of maintenance of way	.2733
Cost of maintenance of equipment	.1551
Conducting transportation	.5284
General expenses	.0432

In the year ending June 30, 1900, the operating expenses of the Missouri Pacific railway were ten million, five hundred thousand dollars; how would the sum be distributed under the above heads? Find the results correct to the nearest $1,000.

Plan the solution, but do not solve:

1. The cyclometer of a bicycle reads at the close of each day as follows: Monday, 841.62 mi.; Tuesday, 873.84 mi.; Wednesday, 910.05 mi.; Thursday, 919.42 mi.; find the average daily ride.

2. On Monday morning a cistern contained 627.5 gal. of water. Tuesday it rained and 127.46 gal. ran into the cistern; Thursday it rained and 472.01 gal. ran into the cistern; the following Monday morning there were 710.91 gal. of water in the cistern; find the average daily consumption of water.

3. In 1900, 2,895,661 dozen pairs of leather gloves and mittens were made in the United States; how many pairs was this per inhabitant, the population being 76,303,387?

4. The number of men's gloves and mittens was .783 of the total number; the number of women's gloves, .112; of boys', .085; and girls', .02; find the number of each.

5. The distance from Cincinnati to St. Louis is 341 mi.; how long will it take a train to make the run, if it goes at the rate of 28.3 mi. per hour the first three hours, 26 mi. the next 2.7 hours, and 30.7 mi. the remainder of the time?

6. By density of population is meant the average number of inhabitants per square mile. For Massachusetts the density is 348.9. The land area is 8,040 sq. mi.; find the population of Massachusetts.

7. The population of Mississippi is 1,551,220; its density of population is 33.5; find its land area.

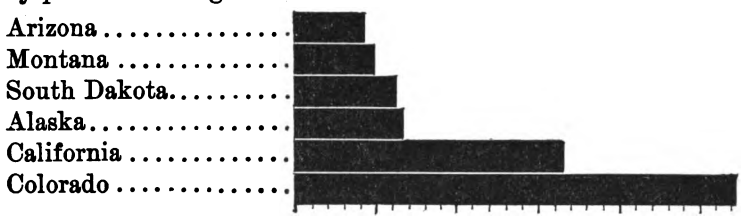
8. The population of Wisconsin is 2,069,042; its land area is 54,450; find its density of population.

9. Two steamers left New York for Liverpool at 10 A. M., Saturday. The average speed of the first was 386.4 mi. per day; of the second, 495.35 mi.; how far apart were they by 10 P. M., Wednesday?

1. The following numbers express approximately in millions of dollars the value of the gold produced by the leading sources in 1901:

Alaska.....	$6\frac{3}{4}$.	California.....	$16\frac{3}{8}$.	Montana.....	4.75.
Arizona	4.	Colorado.....	$27\frac{1}{2}$.	South Dakota...	6.5.

These values are more easily compared if they are represented by parallel rectangles. Thus:



Draw the same set of rectangles on the blackboard, using 1 inch for each division of the scale.

The following tables show the States or Territories of the United States which lead in the production of various commercial articles, and the approximate amounts produced recently:

2. *Silver.*	*Million dollars.*
Nevada.....	1.5
California	4
Utah.....	12
Montana.....	18
Colorado	26

3. *Copper.*	*100,000 tons.*
Utah.....	0.2
California.....	0.3
Arizona	0.5
Michigan.....	0.7
Montana.....	2.5

4. *Cotton.*	*Million bales.*
Arkansas	$\frac{7}{8}$
Alabama.....	1
Mississippi.....	$1\frac{1}{8}$
Georgia.....	$1\frac{1}{4}$
Texas	3

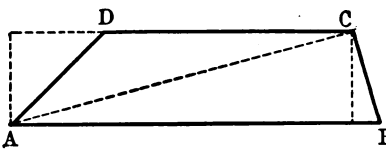
5. *Sugar.*	*100,000 tons.*
Porto Rico.....	0.8
Philippine Is.....	0.9
Hawaiian Is.....	3
Louisiana.....	3
Georgia	0.3

Draw a set of rectangles to represent the numbers in each of the above tables.

FORM STUDY AND MEASUREMENT

Trapezoids

1. Which are the parallel sides in the figure? What is the altitude of the triangle A B C? Of triangle A D C?



Why are they equal? A four-sided figure having one and only one pair of opposite sides parallel is called a **trapezoid.**

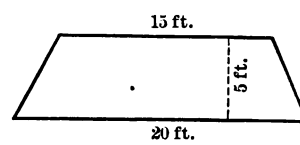
2. How is the area of triangle A B C found? Of triangle A D C?

3. Compare the area of the trapezoid with the sum of the areas of the triangles.

4. $\frac{1}{2}$ h times the lower base $+$ $\frac{1}{2}$ h times the upper base is the same as $\frac{1}{2}$ h times the lower base $+$ the upper base; how may the area of the trapezoid be found?

5. Find the area of the trapezoid in the figure.

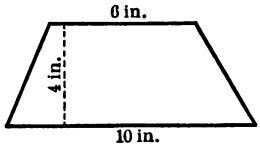
The number of square units of area in a trapezoid is one-half the product of the number of units in the altitude and in the sum of the lengths of the bases.



6. Let s represent the sum of the two parallel sides of a trapezoid and a its altitude; what stands for its area?

Find the areas of the following trapezoids:

7.



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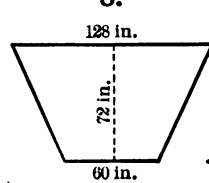

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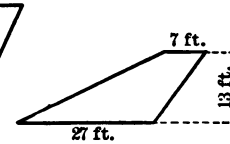
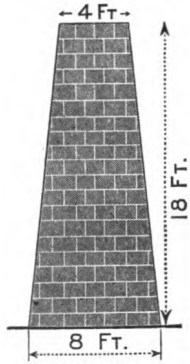


1. A field is in the form of a trapezoid. The parallel sides are 116 yd. and 84 yd. and the distance between them is 50 yd.; how much will it cost to plow it and sow it with wheat at 6¢ per 10 square yards?



2. What will the stone for the wall in the figure cost at 97¢ per square foot of the face?

3. The board supporting the end of a wagon seat is in the form of a trapezoid; how many square inches are there in such a board 14 in. wide at the top, 20 in. wide at the bottom, and 10 in. high?

4. The sail of a ship is 40 ft. wide at the bottom, 30 ft. at the top, and 30 ft. high; how many square yards are there in it?

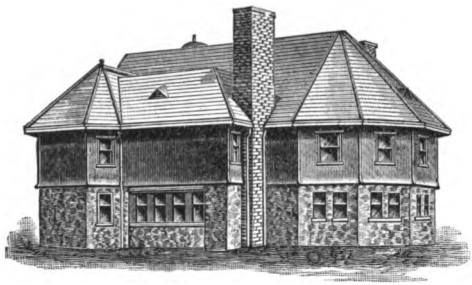
5. A railway is surveyed across a lot 400 yd. square. The strip of land is 10 yd. wide, and has one side along the diagonal; its parallel sides are 566 yd. and 546 yd. in length. At 32¢ per square foot, what is the cost of the land taken? What is the area of each part left?

Find the unknown parts in the following:

	Lower Base.	Upper Base.	Altitude.	Area.
6.	23	17	12	——
7.	72	28	—	1,000
8.	—	25	12	480
9.	32	—	9	270

10. What is the area of a trapezoid of bases 8 in. and 6 in. and of altitude 4 in.?

1. How many triangles are visible in the roof of the building shown in the picture?



2. How many rectangular forms are visible in the second story of the building, not counting windows? How many forms apparently square are visible?

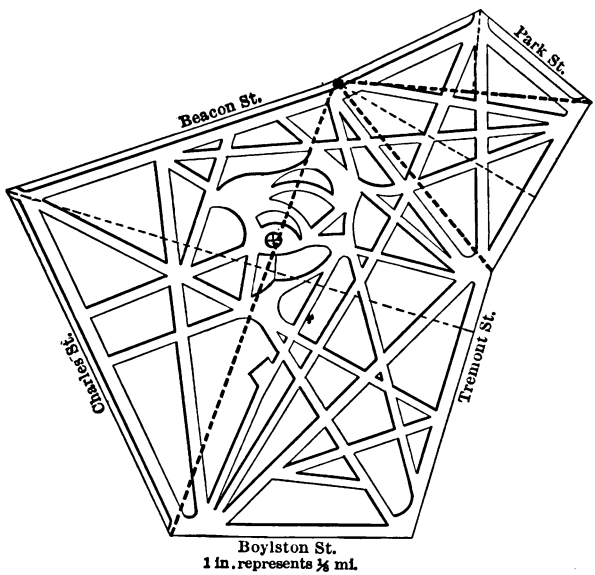
3. Each of the triangles on the end at the right has an altitude of 20 ft. and a base of 8 ft.; find the area of this part of the roof. Find the areas of the squares in the second story just beneath these triangles.

4. Each side of the shade in the picture is 3 in. wide at the top, 6 in. wide at the bottom, and 9 in. high; how many square inches of material are there in one side? In the whole lamp-shade?

5. The glass faces of a street-lamp are trapezoids. The top is a 6-in. square, the base is an 8-in. square, and the middle frame is a 12-in. square. The altitude of each upper trapezoid is 9 in., and of each lower one 17 in. How many square inches of glass are there in the lamp?

6. The four sides of a cake tin are trapezoids. The dimensions of the bottom are 4 in. by 8 in. and of the top are 6 in. by 10 in. The depth down each side is 3 inches. How many square inches of metal are there in the whole tin?

1. The picture represents Boston Common. By use of the scale indicated, find the length of each side and the perimeter of the Common.



2. It may easily be seen to consist of 1 trapezoid and 3 triangles. Measure the necessary lines by use of the scale and compute the area of each part.

3. Find the total area of the Common.

4. The circle containing a cross indicates the position of the Army and Navy Monument; how far is this monument from Beacon Street? From Boylston Street?

5. Central Park in New York city is $\frac{1}{2}$ mi. wide and $2\frac{1}{2}$ mi. long; how many acres does it contain?

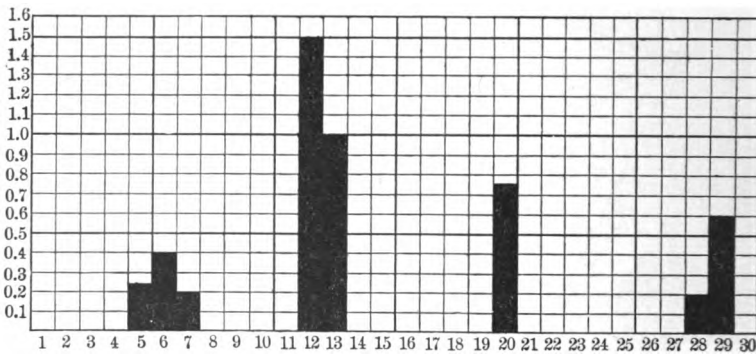
6. What is the ratio of the area of Central Park to that of Boston Common?

7. Name some of the forms shown in the map.

1. The following are the records by days of the rainfall at a certain place during September :

Day.	5th.	6th.	7th.	12th.	13th.	20th.	28th.
Inches.	.25	.4	.2	1.5	1	.75	.2

What was the rainfall for the month?



2. The adjacent lines in the figure are $\frac{1}{10}$ in. apart; how high is each of the dark columns? How do the heights compare with the records of rainfall in Exercise 1? What numbers in the figure show on what days occurred the various amounts of rain?

3. Find from the diagram the rainfall on the 29th.

4. Robert set out a 2-quart pail and measured the rainfall for April. The records were:

Day.	5th.	6th.	10th.	11th.	12th.	24th.	30th.
Inches.	.8	.3	.2	1	.5	.6	.2

Represent the rainfall graphically as in Exercise 2.

5. What is the difference between the greatest and least rainfall for the month?

1. From New York to Boston is 217 miles; how long does it take a train traveling 42 mi. per hour to go from New York to Boston? Express the result to .01 of an hour.

2. A train goes 12 mi. in 16.4 min.; what is the speed per minute to the nearest .01 of a mile?

Find the result to the nearest hundredth:

3. $29.5 \div 7.5$. 4. 48.6×2.7. 5. $856.432 \div 1.6$.

6. 11.485×2.8. 7. $72.80 \div 3{,}465$. 8. $14.2 \div 7.25$.

9. What fraction of 1 square yard is 12 sq. in.? 144 sq. in.? 156 sq. in.? 288 sq. in.?

10. How many window-curtains $\frac{2}{3}$ yd. long can be cut from a piece of goods 30 yd. long?

Find the quotients:

11. $8 \div \frac{4}{5}$. 12. $21 \div \frac{7}{8}$. 13. $7 \div \frac{11}{12}$.

14. $7 \div \frac{1}{5}$. 15. $16 \div \frac{3}{7}$. 16. $5 \div \frac{15}{17}$.

17. $8 \div \frac{2}{3}$. 18. $14 \div \frac{2}{9}$. 19. $4 \div \frac{9}{10}$.

20. How many are 20% of 300? Of 500? Of 1,700?

21. How many are 60% of 400? Of 4,000? Of 6,400?

22. How many are 5% of 700? Of 100? Of 900?

23. How many are 100% of 100? Of 900? Of 70? Of 3?

24. How many are 150% of 100? Of 200? Of 400?

25. How many are 200% of 100? Of 200? Of 80? Of 5?

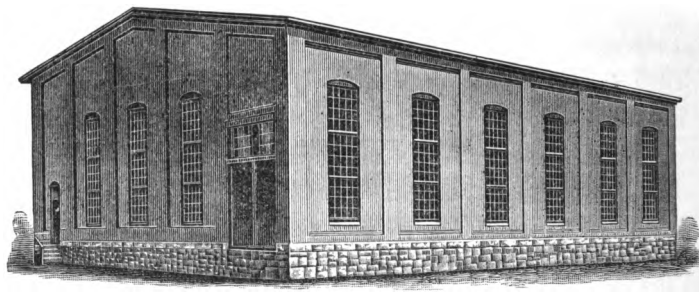
26. The table shows the average annual rainfall of various places to September, 1902.

Chicago.....	34.8	Sacramento...	20.9	Pensacola.....	57.1
Boston.....	45.0	Denver.....	14.5	Mobile.....	62.2
St. Louis.....	41.1	New York City.	44.8	Yuma.....	3.0

Which of these places had the greatest rainfall? The least? What is the difference in inches between these extremes? Find how much the rainfall of each place exceeds the smallest in the list. Find how much the rainfall of each place falls below the greatest in the list.

1. How much would one pay for 5 yd. of lawn at $33\frac{1}{3}¢$ a yard?

2. The picture shows a power-house 60 ft. long and 50 ft. wide; what area does it cover?



3. How many windows are shown? There are twice as many in the whole building. Each window is 3 ft. wide and 10 ft. high; find the total window surface. Add to this 96 sq. ft. for door surface.

4. The side walls are 16 ft. high and the front wall 19 ft. at the highest point; find the total surface of the four walls of the building.

5. Deduct from the total surface of the walls the surface of the windows and doors; what is the surface of the brick-work?

6. The average thickness of the wall is 8 in. In such a wall the number of bricks required to 1 sq. yd. of surface is 135. How many thousand bricks did it take to build the walls of the power-house?

7. Find the ratio of two cubes of edges 3 in. and 5 in.

8. Construct a barometer line from these readings:

Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
$27\frac{1}{2}$ in.	28 in.	30 in.	29 in.	$31\frac{1}{2}$ in.	$29\frac{1}{2}$ in.	31 in.

POWERS AND ROOTS AND MEASUREMENT

Powers and Roots

1. What is the volume of a cube 2 in. on a side?

2. What is the volume of a cube 3 ft. on a side? What is this unit for measuring volume called?

3. What is the volume of a cubical block of granite 4 ft. on a side?

The computations for finding the volumes of the above cubes are

$2 \times 2 \times 2 = 8, \quad 3 \times 3 \times 3 = 27, \quad 4 \times 4 \times 4 = 64.$

The product of three equal factors is called the **cube** or the **third power** of any of the factors.

Thus, in the above exercises, 8 is the cube or third power of 2; 27 is the cube or third power of 3; and 64 is the cube or third power of 4.

4. Find the product of 5, 5, and 5. 125 is the cube of what number?

5. Find the product of 6, 6, and 6; of 7, 7, and 7; of 8, 8, and 8. What is the cube of 6? Of 7? Of 8?

6. How many cubic yards of masonry are there in a cubical pier 15 ft. on a side?

7. What is the length of an edge of a cube containing 8 cu. in.? Of one containing 27 cu. in.? 1,728 cu. in.? Each result is seen to be one of the equal factors in the products

$2 \times 2 \times 2 = 8, \quad 3 \times 3 \times 3 = 27, \quad 12 \times 12 \times 12 = 1{,}728.$

One of the *three equal factors* of a number is called its **cube root.**

Thus, in the above equations 2 is the cube root of 8, 3 is the cube root of 27, and 12 is the cube root of 1,728.

8. How many inches in the side of a 1-foot cube?

9. What is the cube root of 8? Of 27? Of 64? Of 125?

1. What kinds of angles do the adjacent edges of Figure 1 form? Figure 1 is a **rectangular prism.**

2. The base of a rectangular prism is 3 ft. by 4 ft., and the altitude of the prism 5 ft. What is the area of the base? The area of the sides, or the lateral area of the prism? The volume of the prism?

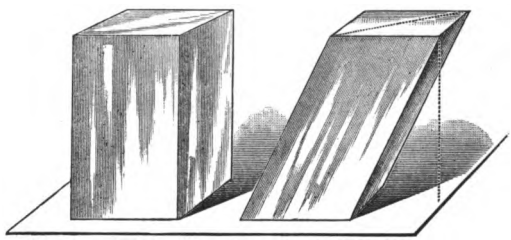


Figure 1. Figure 2.

3. A prism whose lateral edges are oblique to the base is called an **oblique prism.** Figure 1 represents a right prism; what does Figure 2 represent? The altitude of an oblique prism is the perpendicular distance between the bases. Compare the altitudes of Figures 1 and 2.

4. What kind of faces has each of these prisms? A prism whose faces are parallelograms is called a **parallelepiped.**

5. Wrap a piece of paper around a right parallelepiped. Leave one end open and trim even with the solid. Remove the solid. Form a hollow parallelepiped in the same way from an oblique solid, having a base and an altitude equal to those of the right prism. Fill one form with sand and pour the contents into the other. How do their volumes compare?

6. A *right* parallelepiped and an *oblique* parallelepiped *having equal bases and altitudes* are —.

7. How is the volume of a rectangular parallelepiped found? The volume of any parallelepiped is the product of the base and altitude.

8. Show how to find the volume of an oblique parallelepiped whose base contains 50 sq. in. and whose altitude is 15 inches.

1. What kind of bases has Figure 2 in the picture?

A prism with triangular bases is called a **triangular prism.**

2. Richard modeled from clay two triangular prisms so that their altitudes and their bases were equal. He weighed the solids and found that their weights were equal. The two triangular prisms placed together formed a parallelepiped.

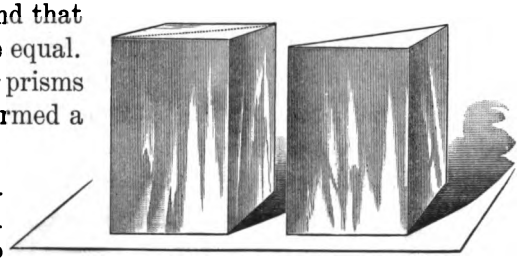


Figure 1. Figure 2.

How is the volume of a parallelepiped computed? According to Richard's experiment, how would you compute the volume of a triangular prism?

3. If convenient, make from cardboard hollow forms like Figures 1 and 2 and compare their volumes by filling one with sand and pouring its contents into the other prism. Explain how the volume of a triangular prism may be found.

4. Any prism may be divided into triangular prisms by diagonal planes. Count the triangular prisms in Figure 3. How is the volume of each found? What is the sum of their volumes? How does the sum of their bases compare with the base of the given figure? How do their altitudes compare?

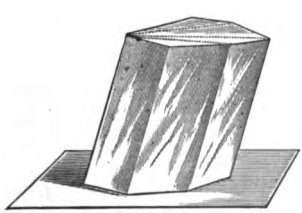


Figure 3.

5. How may the volume of any prism be found?

6. Find the volume of a prism whose base is 15 sq. ft. and whose altitude is 50 ft.

1. What is the volume of a square prism whose altitude is 10 in. and the side of whose base is 8 in.?

2. A triangular stone pier has a base of 25 sq. yd. and a height of 35 ft.; what is the number of cubic feet in its volume?

3. What is the cost of the pier mentioned in Exercise 2 at the rate of $8.50 per cubic yard?

4. If the area of the bottom of an excavation in the form of a prism is 9 sq. yd., and the depth is 2 yd., how many cubic yards are taken from the excavation?

5. What will be the cost of the excavation in Exercise 4 at the rate of 25 cents per cubic yard? What will it cost to draw away the earth at 50 cents per cubic yard? How many wagon-loads of $\frac{3}{4}$ cu. yd. each will there be?

6. What will be the cost of plastering a 10-room house, the rooms averaging 4 yd. by 5 yd. and 3 yd. high, at 15¢ per square yard?

On account of the extra work around windows and doors, masons seldom make any allowance for these areas in estimating the cost of the work.

7. Air is composed of oxygen and nitrogen. In any quantity of pure air about 20% is oxygen and 79% is nitrogen; how many cubic feet of each are there in 2,700 cubic feet of air?

8. How many cubic feet of air will your schoolroom hold? How many cubic feet of oxygen are there in the room when the air is pure?

9. If the average amount of air inhaled by a pupil at one breath is 25 cubic inches, how many times must he breathe to inhale 1 cu. ft.? If he breathes 20 times a minute, how long will it take him to inhale a cubic foot?

10. How long will it take 30 pupils to breathe once all the air in a schoolroom 25 ft. by 30 ft. by 12 ft.?

1. William weighed a wooden box, whose inside measures were 4 in. by 4 in. by 9 in. and found that it weighed 14 oz. He filled it with soil and found that it weighed 8 lb. 6 oz.; what was the weight of the soil in the box?

2. How many cubic inches did the box contain? What part of a cubic foot is this?

3. What would a cubic foot of the soil weigh? What would a wagon-load weigh, the wagon-box measuring inside 6 ft. by $3\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft.?

4. According to Exercise 3, how many wagon-loads would be required to make a 6-in. layer of soil over a lot 40 ft. by 120 ft.?

5. A loamy soil is found to weigh 96 lb. per cubic foot and to contain $\frac{1}{2}$ of 1% of lime; how much lime is there in the soil which covers an acre to a depth of 1 foot?

6. If this soil contains .1 of 1% of phosphoric acid, how much phosphoric acid is there in the whole amount?

7. If a soil weighs 84 lb. per cubic foot and contains $\frac{1}{4}$ of 1% potash, how much potash is there in 10 acres of such soil $1\frac{1}{2}$ ft. deep?

8. A box 6 in. by 8 in. by 12 in. weighs 2 lb. 4 oz. Filled with soil it weighs 32 lb. 12 oz.; what is the weight of the soil used?

9. What part of a cubic foot does the box contain? What is the weight of the soil per cubic foot?

10. According to Exercise 9, what is the weight in tons of a layer of the soil 2 ft. deep over an acre?

11. By passing the soil used in Exercise 8 through a fine sieve it was found that 22 lb. 14 oz. of sand passed through; what part of the soil was sand? What percentage?

12. How many tons of sand in an acre of such soil 1 foot deep? $1\frac{1}{2}$ feet deep?

1. The following table shows the amounts of various foods consumed and the milk and butter produced from two cows at the Illinois agricultural station in one year:

Food and Products.	Rose.	Nora.
Food consumed:	*Pounds.*	*Pounds.*
Clover hay	3,873.10	3,547.50
Corn silage and green corn	8,107.50	8,082.00
Rape	2,482.00	2,482.00
Cowpeas	750.00	750.00
Corn meal	1,692.70	1,653.50
Wheat bran	1,256.50	1,158.00
Ground oats	670.10	567.60
Gluten meal	516.50	573.00
Old process linseed-meal	848.00	785.30
Milk produced	11,329.00	7,759.40
Butter fat produced	564.82	298.64

Find the total number of pounds of food consumed by each animal in the year.

2. Find the difference in the amounts of milk produced; in the amounts of butter produced.

3. If the milk weighed 2.3 lb. per quart, how many quarts did each cow produce in a year?

4. What is the total value of the milk produced by each cow at 5¢ a quart? Of the butter at 16¢ a pound?

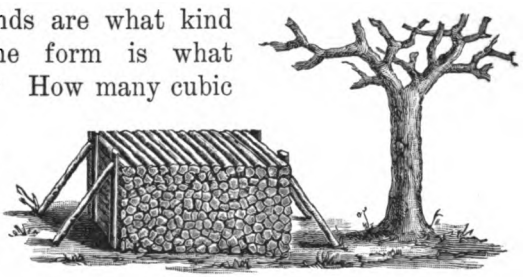
5. If $\frac{3}{4}$ lb. of butter is lost per 100 lb. of milk by setting in pans instead of using a separator, how many pounds of butter are lost by setting 800 lb. in pans? What is the money loss, if butter is worth 20¢ a pound?

6. If the loss is $\frac{1}{5}$ of the total butter product, how many pounds of butter does 1 ton of milk yield?

7. A good cow averages 5,000 lb. of milk a year, how many pounds of butter are lost by setting the milk in pans?

Wood for fuel is usually cut into 4-foot lengths. A pile of this wood 8 ft. long and 4 ft. high is called a **cord.**

1. The sides of a cord of 4-foot wood are what kind of figures? The ends are what kind of figures? The form is what kind of a prism? How many cubic feet are there in a cord of wood?



2. How many cords of wood are there in a pile 32 ft. long and 4 ft. high? In a pile 32 ft. long and 2 ft. high?

3. How many cords of wood are there in a yard containing 12 rows 32 feet long and 4 feet high?

4. How many cords of wood are there in a pile 64 ft. long and 10 ft. high?

4-foot wood is usually cut at the wood-yard into 16-inch lengths for use in stoves. This is called **stove wood.**

5. How many stove lengths can be cut from 1 4-foot length?

A pile of wood 8 ft. long, 4 ft. high, and one stove length thick is called a cord of stove wood. Wood is usually sold at the wood-yard in this form.

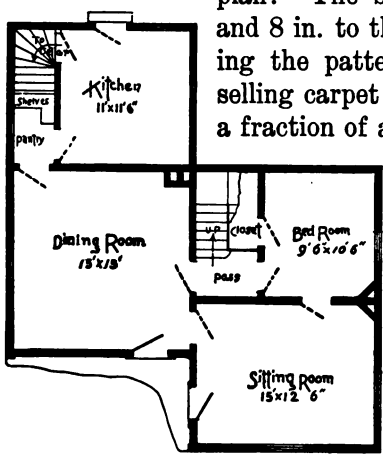
Thus, if a dealer says maple wood is $2.50 a cord, he refers to stove wood.

6. How many cords of 16-inch stove wood can be cut from a cord of 4-foot wood?

7. How many cords of 4-foot wood are required to cut 75 cords of stove wood? 96 cords? 69 cords? 81 cords? 243 cords?

8. A dealer buys 4-foot wood at $4 a cord, and after cutting it into stove wood, sells it for $2.50 a cord; how much does he gain per cord on the 4-foot wood?

1. How many yards of carpeting $\frac{3}{4}$ yd. wide will be required to carpet the sitting-room as shown in the floor plan? The strips are to run lengthwise, and 8 in. to the strip is allowed for matching the pattern. It is not customary in selling carpet to cut strips lengthwise, but a fraction of a yard in length is sold.



2. Find the number of yards of carpet required to carpet the above room if the strips run the other way.

3. How many yards of carpet 1 yard wide, that can be matched without waste, must be bought to carpet the bedroom if the strips run in the direction requiring the fewer yards?

4. A rug 9 ft. by 12 ft. is placed centrally in the dining-room, and filling at $33\frac{1}{3}¢$ per yard is used to cover the rest of the floor. If the rug costs \$18, what is the cost of covering the dining-room floor?

Wall-paper is usually sold in rolls of 16 yd. each, the paper being 18 in. wide. Fractions of a roll are not sold. The number of strips that can be cut from a roll depends upon matching the pattern. In the bedroom of the above house the ceiling paper is to run the short way, 5 strips being cut from a roll. In the other rooms the paper is cut so as to use 3 sixteen-foot strips to the roll.

5. Find the cost of the paper for the ceilings of the bedroom, dining-room, and sitting-room at 40¢ a roll.

6. Taking the height of the rooms to be 9 ft., 5 strips can be cut from a roll. Allowing $\frac{1}{10}$ of the measured surface for doors, windows, baseboards, and frieze, find the cost of the paper for the three rooms, at 45¢ a roll.

PERCENTAGE

Percentage of Numbers

1. What part of 7 is 7? How many hundredths of 7 is 7? What percentage of 7 is 7?

2. How many per cent of itself is any number?

3. 12 is 6% of what number?

PLAN. 1. 1% of the number is —.
2. 100% of the number is —.

4. 16 is 4% of what number?

5. 10% of ? = 15.

6. 25% of N = 7; find N.

7. 20% of ? = 12.

8. 8% of N = 24; find N.

9. Fill the blanks in the table:

STATE.	POPULATION.	PERSONS OF SCHOOL AGE. 5–20 YEARS.	PER CENT OF TOTAL.
Illinois.....	4,800,000	1,600,000	—
Indiana.....	———	800,000	32
Kansas.....	1,500,000	———	33⅓
Tennessee...	2,000,000	———	40
Texas.....	———	1,200,000	40

10. 25% of a number is what fractional part of it? Answer the same question for 50% of it; 75%; 10%; 60%; 40%; 80%; 33⅓%; 66⅔%; 35%; 85%; 15%; 12½%; 37½%; 62½%; 87½%; 16⅔%; 83⅓%; 84%; 6%; 5%; 4%.

11. ⅓ of a number is what percentage of it? Answer the same question for ½ of it; ¼; ⅕; ⅙; ⅛; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{20}$; $\frac{1}{25}$; $\frac{1}{50}$; ⅔; ¾; ⅖; ⅚; ⅜; ⅘; $\frac{7}{10}$; $\frac{5}{12}$; $\frac{4}{25}$; $\frac{11}{30}$.

12. Which of the following percentages would be found more easily, if expressed as common fractions: 25% of 1,000; 40% of 1,000; 46½% of 1,000; 12½% of 888; 81% of 400?

1. 10 lb. of apples were placed in an oven to dry. After drying they weighed 1 lb. 12 oz.; how many pounds of water evaporated from the apples? What part of the weight was lost in drying?

2. Potatoes contain 78% of water; how many pounds of water are there in a bushel of potatoes? (1 bu. of potatoes weighs 60 lb.)

3. Flour contains $12\frac{1}{2}\%$ of water; how much water is there in a barrel of flour? (1 bbl. of flour weighs 196 lb.)

4. Draw a line 8 in. long to represent the weight of a barrel of flour. Make one part heavier to represent the part that is water.

5. 8 oz. of dry corn were placed in water at a temperature of 60° and allowed to remain 18 hr. At the end of that time it weighed $10\frac{1}{2}$ oz.; what part of the soaked corn was water? Make a drawing to illustrate the result.

6. Assuming that a bushel of dry corn weighs 56 lb. and that 1 lb. of water equals 1 pt., how much water would a bushel of dry corn absorb under the conditions stated in Exercise 5?

7. Walter took a 3-lb. sample of soil from a forest and dried it. Afterward it weighed 2 lb. 4 oz.; what part of the soil was water? What percentage? 1 cu. ft. of the soil weighed 72 lb.; how many pounds of water did it contain?

8. Suppose the roots of forest-trees to extend to an average depth of 5 ft. According to Exercise 7, how much water would there be about the roots of a grove 4 rd. long and 20 rd. wide? Express the result in pounds, also in cubic feet. (1 cu. ft. of water weighs $62\frac{1}{2}$ lb.)

9. A grown person should drink about 2 quarts of water a day. A ship with 1,600 persons aboard (passengers and crew) carries a supply of fresh water for a voyage of 78 days; how many cu. ft. of tank space are needed? (1 pt. weighs 1 lb.)

1. Fresh beef furnishes on the average 15.5% of protein, the chief tissue-building element of all foods; how many ounces of protein are there in 300 oz. of fresh beef?

2. Green corn furnishes on the average 3.1% by weight of protein; how many ounces of protein are there in 200 oz. of green corn?

3. 13.1% of eggs are protein. How many ounces of protein are there in 200 ounces of eggs?

The following table shows the average relative amounts of waste, water, protein, fat, and heat units derived from various staple foods:

Animal Food.	Waste Per Cent.	Water Per Cent.	Protein Per Cent.	Fat Per Cent.	Heat Units Per Pound.
Beef.....	16.9	52.6	15.5	15.0	910
Veal.....	21.6	52.0	15.4	11.0	745
Mutton....	10.3	39.0	13.8	36.9	1,770
Pork, fresh	12.6	48.0	13.5	25.9	1,320
Chickens...	42.1	43.7	12.8	1.4	305
Fish (cod)..	30.2	58.5	11.1	0.2	220
Eggs	12.1	65.5	13.1	9.3	635

Vegetable Food.	Carbohydrates Per Cent.	Water Per Cent.	Protein Per Cent.	Fat Per Cent.	Heat Units Per Pound.
Flour.....	75.6	12.0	11.4	1.0	1,635
Beans, dried	63.1	12.6	22.5	1.8	1,520
White bread	54.2	35.3	9.2	1.3	1,200
Corn, green	20.4	75.4	3.1	1.1	440
Rice.....	79.4	12.3	8.0	0.3	1,620

4. According to the above table, how many units of heat are supplied by the fuel in 1 lb. of fresh beef? How many units are there in 5 lb.? Make and solve 10 other problems.

1. Wheat sold at \$0.80 in October and at \$0.85 in December; for how much more did a bushel of wheat sell in December? What per cent more?

2. A real-estate agent bought a house for \$2,500 and sold it for \$3,000; what per cent did he gain?

PLAN. 1. \$3,000 − \$2,500 = \$ —. 2. \$500 = — of \$2,500.
3. \$500 is —% of \$2,500. 4. The agent gained —%.

3. Mr. Williams sold a house for \$200 more than it cost. The gain was 5% of the cost; find the cost.

4. A bicycle, sold at a loss of 10%, brought \$45; the cost = \$—.

5. In 1900 the population of New York City was 3,400,000. In 1890 it was 2,500,000; what was the gain per cent in 10 years?

Find the percentage of gain or loss in each case:

	6.	7.	8.	9.	10.
Cost.....	\$16	\$25	\$40	\$1.20	\$2.00
Selling Price...	\$20	\$15	\$48	\$1.60	\$1.80

Find the cost:

	11.	12.	13.	14.
Selling Price.....	\$2	\$6	\$130	\$1,500
Gain or Loss.....	25%, gain.	20%, loss.	30%, gain.	10%, loss.

15. If wheat produces 82% of its weight in flour, how much flour will be obtained from 42 bags of wheat, each weighing 125 lb.?

16. If 18% by weight of wheat is taken out as bran, etc., in grinding it for flour, how many bushels of wheat are required to make a barrel of flour? (1 bu. of wheat weighs 60 lb.; 1 bbl. of flour weighs 196 lb.)

17. A tailor made $6\frac{1}{2}$ dozen coats with cloth at \$1.95 per yard. Three yards are required for each coat, and the making cost \$19.80 per dozen. He sold the coats at \$8.25 each; what was his percentage of gain?

1. A house, sold at a gain of 20%, brought \$2,100. Find its cost.

PLAN. 1. Let c = cost. Then $\frac{1}{5}c$ = gain.
2. $c + \frac{1}{5}c = —$. $\frac{1}{5}c = \$—$.
3. $c = —$ of \$2,100 = \$—.

2. 25 yards of cloth cost \$4; what is the selling price of 15 yd. at a profit of 25%?

PLAN. 1. \$4.00 ÷ 25 = \$—, the cost of 1 yd.
2. 125% of \$0.16 = \$—, the selling price of 1 yd.
3. 15 × \$— = \$—, the selling price of 15 yd.

3. A suit of clothes was sold for \$22 at a gain of 10%. Find the cost.

PLAN. 1. $c + .10\,c$, or $1.10\,c$ = \$22.
2. Therefore, c = \$—.

4. A dealer bought a car-load of coal, 38,000 lb. He sold it at \$7.50 per ton, and thus gained \$28.50 on the car-load. Find the rate of gain per cent.

5. In 1900, there were in the United States about 200 million apple-trees, a gain of $66\frac{2}{3}\%$ since 1890. Find the number in 1890.

6. In 1899, there were produced in the United States about $9\frac{1}{2}$ million bales of cotton. (A bale contains about 500 lb.) This was a gain of $26\frac{2}{3}\%$ since 1889. Find the production in 1889.

7. A dealer sold two typewriters, costing the same price. On one he gained twice as much as on the other. His total gain was \$45. What was the gain on each?

8. A dealer sold two automobiles of equal cost so as to gain 5 times as many dollars on one as he lost on the other. His total gain was \$200. What was the gain, or loss, on each?

9. A dealer sold a piano at a gain of 30%. The purchaser, after having used it a year, sold it for \$195 at a loss of 40% of its cost to him. Find the cost to the dealer.

1. Horace sifted soil with a coarse sieve, thus retaining gravel in the sieve; he sifted the residue through a finer sieve, retaining sand in the sieve. He weighed soil, gravel, and sand in 6 experiments and found the weights as follows:

Sample.	1st.	2d.	3d.	4th.	5th.	6th.
Soil taken..	24 oz.	36 oz.	72 oz.	84 oz.	90 oz.	120 oz.
Gravel.....	6 "	10 "	27 "	20 "	15 "	36 "
Sand.....	10 "	16 "	32 "	36 "	25 "	53 "

What percentage of each soil is gravel? Sand? Gravel and sand together?

2. Henry sifted 2 lb. of soil through a coarse sieve. The gravel remaining in the sieve weighed 5.4 oz. What percentage of the soil was gravel?

3. John weighed out 10 oz. of green oak leaves; he dried them thoroughly and weighed them again. This time they weighed 5.1 oz. The loss of weight is due to the evaporation of water from the leaves while drying. What percentage of the green leaves was water?

4. According to Exercise 3, how many pounds of water are there in a ton of green oak leaves? How many cubic feet? ($62\frac{1}{2}$ lb. water = 1 cu. ft.)

5. When 10 oz. of green sycamore leaves were dried, the residue weighed $4\frac{1}{2}$ oz.; what percentage of the green leaves was water? How many pounds of water in a ton of these leaves? How many cubic feet?

6. Water evaporates from the leaves of plants, which accounts largely for the amount of water required by them. Chester found by growing a sunflower in an earthen vase that a full-grown sunflower used 1 lb. 4 oz. of water in 24 hours. Taking this as the rate of evaporation, how much water is evaporated from 100 sunflowers in a day?

1. At a test station, a bushel (60 lb.) of clover-seed, costing $5.25 per bushel, was tested and found to contain 80% of good seed; how many pounds of good seed did it contain?

2. How many bushels of this seed must be bought to obtain a bushel of good seed? According to the price given, what is the cost of a bushel of good seed?

3. In another test, a bushel costing $5.00, was found to contain 75% of good seed. Find the cost of a bushel of good seed at this rate.

4. In a third test, a bushel costing $1.10 was found to contain 10% of good seed; which was the cheaper, the seed at $5 per bushel or that at $1.10?

5. A bushel of standard grass-seed weighs 14 lb. and 3 bushels contain 25.8 lb. of good seed; how many pounds of good seed are there per bushel? What percentage?

6. Determine the numbers to fill the blanks in the table:

Red Clover Sample No.	Market Price per bu. (60 lb.)	Good Seed per cent.	Good Seed lbs. per bu.	Price Paid per bu. for Good Seed.
1.....	$5.50	93	——	$5.88
2.....	5.25	—	45.72	6.90
3.....	5.00	92	55.2	——
4.....	——	93	55.8	5.10
5.....	4.75	—	48	5.94
6.....	4.00	87.3	——	4.59
7.....	3.50	46.2	27.72	——

7. Which of the above samples is the most economical seed to buy and which is the least?

8. If a standard bushel of clover seed contains 48 lb. of good seed, which of the samples of Exercise 6 are above standard? Below?

1. An ice-wagon has a bed 10 ft. 6 in. by 3 ft. 9 in., inside measure, and the ice is packed 4 ft. 2 in. high; how many cubic feet of ice are there?

2. A cubic foot of water weighs $62\frac{1}{2}$ lb., and ice is .92 as heavy as water; what is the weight of the ice mentioned in Exercise 1?

3. How many bricks, 8 in. by 4 in. by 2 in., are there in a pile $12\frac{2}{3}$ ft. $\times$ $8\frac{1}{3}$ ft. $\times$ $5\frac{5}{6}$ ft.?

4. What is the value of the bricks just mentioned at $17.50 per M?

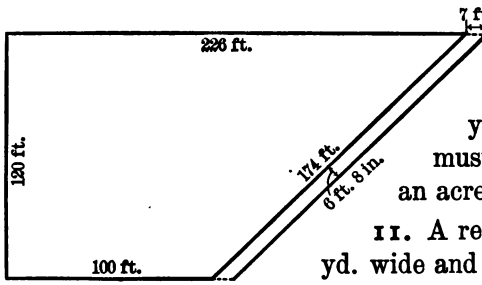
5. A street 40 ft. wide and $\frac{3}{4}$ mi. long is to be covered with crushed stone to a depth of 3 in. Find the cost of the stone at $1.45 per cubic yard.

6. How many wagon-loads will the quantity of stone mentioned in Exercise 5 make, if the inside measurements of the wagon are $4\frac{1}{2}$ ft. by $9\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft.?

7. Find the cost of varnishing a floor 84 ft. long and 42 ft. wide at 3.4¢ per square yard.

8. What is the number of tiles $\frac{3}{4}$ of an inch wide that can be laid in a row 1 foot long?

9. Find the area of the land in the figure and the area of the sidewalk.



10. If a rectangular field is 110 yd. long, how wide must it be to contain an acre?

11. A rectangular lot is $16\frac{2}{3}$ yd. wide and $41\frac{2}{3}$ yd. long. Find the length of the fence around it.

12. How many yards of carpet $\frac{3}{4}$ yd. wide are needed to cover a room 15 ft. by 18 ft.?

1. A merchant bought a gross of hats for $216 and sold them at $2 each; how many per cent did he gain?

2. What is meant by a cord of 4-foot wood? How many cubic feet are there in 4 cords?

3. How many cords of 16-inch stove wood can be cut from 3 cords of 4-foot wood?

4. In a sample of clover-seed weighing 60 lb. per bushel, 3,000 weed seeds were found per pound; how many weed seeds were there in the bushel?

5. How many weed seeds does a farmer sow per acre, if he uses 15 lb. of this grass-seed per acre?

6. A 60-ride railway ticket between Baltimore and Washington costs $15.45; how much is this per ride?

7. The regular fare between Baltimore and Washington is $1.20 each way; at least how many round trips must a man make per month to gain by purchasing a 60-ride ticket?

8. In a railroad journey James noticed that some freight-cars have their measurements in feet and inches stamped on them, thus:

Length...	33 — 9	50 — 0	44 — 0	33 — 10	34 — 0
Width....	6 — 2	9 — 4	8 — 6	8 — 1	8 — 6
Height...	7 — 0	8 — 10	8 — 7	7 — 1	8 — 0

How many cubic feet in the contents of each car? Plan the work, cancel when possible, and find the results within 1 cu. ft., but not exceeding the actual capacity of the cars.

9. Various commercial fertilizers are sold. They are intended to supply nitrogen, phosphoric acid, and potash to the soil. If nitrogen is valued at 16¢ a pound, phosphoric acid at $4\frac{1}{2}$¢ per pound, and potash at 5¢ a pound, what is the value of 100 lb. of a fertilizer of which $2\frac{1}{2}$% is nitrogen, 10% available phosphoric acid, and $1\frac{1}{2}$% potash?

1. What is the cube of each number from 1 to 10?

2. What is meant by the cube root of a number? What is the cube root of 64? Of 125? Of 1,728?

3. A piece of woolen cloth lost 18% of its length in shrinking. The present length is 41 yards. If the cloth cost $60 before shrinking, what was the price per yard?

4. Beef loses about 15% of its weight by boiling and 20% by roasting; how much will a piece of beef, weighing 10 lb. when raw, weigh after having been roasted? Boiled?

5. A straight glass jar, 12 in. high, was filled $\frac{2}{3}$ full of milk. The next day, the cream on top was $\frac{1}{2}$ in. thick. What percentage of the milk poured in was cream?

6. The water surface of Kentucky is 1% of the land surface; the total area is 40,000 sq. mi. Find the area of the land surface of Kentucky; the water surface.

7. The land surface of New Hampshire is about 9,000 sq. mi. The water surface is $3\frac{1}{3}$% of the land surface. Find the total area.

8. By passing some soil through sieves, Thomas found that 18% of his sample was coarse gravel, 37% gravelly sand, and 11 oz. fine earth; what was the weight of the sample?

9. When wheat is ground, $77\frac{1}{2}$% by weight of the product is fine flour; how many bushels of wheat are required to produce 1,000 barrels of fine flour?

10. The distance from Chicago to Minneapolis is 70% of that from Boston to Toronto. The sum of the two distances is 1,020 miles. Find each.

11. A maker of picture-frames is asked the cost of a frame 24 in. by 36 in., outside measure. The molding selected costs him 16¢ per running foot, the glass 40¢, the back and nails 15¢, labor 25¢. He wishes to set the price so as to gain $12\frac{1}{2}$%. What price should he set?

BUSINESS APPLICATIONS

Commission

1. A real-estate agent receives for his services 5% of the rents collected. If he collects $25 rent each month, how much does he receive in a year?

2. An agent sold a car-load of fruit for $142. He retained 6% for his services. How much did he remit to the shipper?

3. The percentage received by an agent for handling goods is called his **commission.** What is the commission in Exercise 2?

4. A real-estate agent sold a 200-acre farm at $75 an acre. If he charged $2\frac{1}{2}\%$, what was his commission?

5. A commercial traveler receives 7% commission on the orders he takes. In 6 months he sells goods to the amount of $9,800. How much does he earn?

6. An auctioneer received $15 for selling $500 worth of goods; what was his rate of commission?

7. A broker bought 10,000 bu. of wheat for a customer at a commission of $\frac{1}{8}$ of a cent per bushel; what was the amount of his commission?

8. A merchant gave bills aggregating $300 to a collector who succeeded in collecting $240, but reported the rest uncollectable. If the collector charged 8% of the amount collected, how much of the $300 did the merchant lose? What was the percentage of loss?

9. Mr. Jones is an agent for an automobile company that pays him a commission of 10% on his sales. In one month he sold one automobile at $760, and another at $925. How much did he earn? His expenses were $96. How much did he make above expenses?

1. A real-estate agent received $125 commission for selling a house at $5,000; how many per cent did he receive?

2. A commission merchant remitted $142.50 to the owner of the goods and retained $7.50 commission; for how much did he sell the goods? What was his rate of commission?

3. An agent sold 6 cottages at $1,000 each for $2\frac{1}{2}\%$ commission; what was his commission per cottage?

4. A real-estate agent sold a house and lot for $3,200. His commission was $2\frac{1}{2}\%$. How much did the owner receive?

5. Mr. Baker, a cattle broker, paid $1,100 for sheep to stock his ranch. This includes the broker's commission of 10%. What did the sheep cost the broker?

PLAN. 1. Let C = the cost. 2. C + 10% of C = $1,100.
3. 1.10 C = $—. 4. Therefore C = $—.

6. A commercial traveler, who receives a commission of 5% on his sales, earned $28 in ten days. Find the amount of the sales.

PLAN. 1. Let S = the amount of his sales.
2. .05 × S = $—. 3. Therefore S = $—.

7. A real-estate agent collects rents amounting to $400 and remits $384 to the owner; what is his rate of commission?

8. A man sold his house and lot through an agent, who charged a commission of 2% and remitted $9,310. What was the selling price?

PLAN. 1. Let S = selling price.
2. S − .02 × S = $9,310. 3. Therefore S = $—.

9. A merchant gave bills to the amount of $300 to an agency for collection. The agency charged 2% for collecting $\frac{1}{2}$ of the amount, but 5% for collecting the other half, on account of the trouble in collecting. How many dollars did the merchant receive?

1. A merchant offered a reduction of 5% on purchases at one time amounting to at least $50. A woman bought $65.20 worth of goods. What reduction was made her?

When for any reason a reduction is made from a fixed price, such a reduction is called a **discount.**

At the close of a season damaged goods were offered at the following discounts. Find the reduced price:

	2.	3.	4.	5.
Marked price.....	$1.15	$39	$15	$42
Discount.....	20%	33⅓%	10%	16⅔%

6. A car-load of fruit valued at $160 was slightly damaged in transit. The buyer agreed to take it at a discount of 25%. What did he pay? If the fruit cost the shipper $135, what was his loss?

The price after the discount has been taken off is called the **net price.** Determine the numbers to fill the blanks:

	Marked Price.	Discount.	Net Price.
7.	$4.00	6 %	———
8.	$1.75	20 %	———
9.	$12.00	—	$11.40
10.	$225.00	—	$202.50
11.	———	10 %	$8.10
12.	———	15 %	$17.00
13.	$8.40	8⅓%	———
14.	$4.80	12½%	———
15.	$855.00	4½%	———
16.	———	2 %	$10.29
17.	$1,025.00	—	$1,000.00
18.	———	12½%	$87.50
19.	$56.20	5 %	———

1. When goods are sold on 60 days credit, allowing 2% discount if paid for within 10 days, how much is gained by paying a bill of $480 within 10 days?

Many articles of merchandise have fixed prices which are published in catalogues and price-lists. This price is known as the **list price.**

State orally the numbers to fill the blanks:

	List Price.	Discount.	Net Price.
2. Shakespeare (Temple Edition)..	$26.00	20%	——
3. Roosevelt's Rough Riders	$2.00	15%	——
4. Hawthorne's Works, 8vo. cloth.	$10.00	25%	——
5. Poe's Works, 6 vol., cloth.	$7.50	20%	——
6. Cooper's Leatherstocking Tales.	——	25%	$3.75
7. David Harum.	——	30%	$1.05
8. Dickens's Works, 12 vol., cloth.	$6.00	——	$4.50
9. Kipling's Works, 15 vol.	$15.00	——	$10.00

When two or more discounts are quoted, the first discount is taken from the list price, the second discount from the price after the first discount has been taken off, and so on.

Thus, if the list price is $9 and the discounts are 20%, 10%, and 5%, what is the net price?

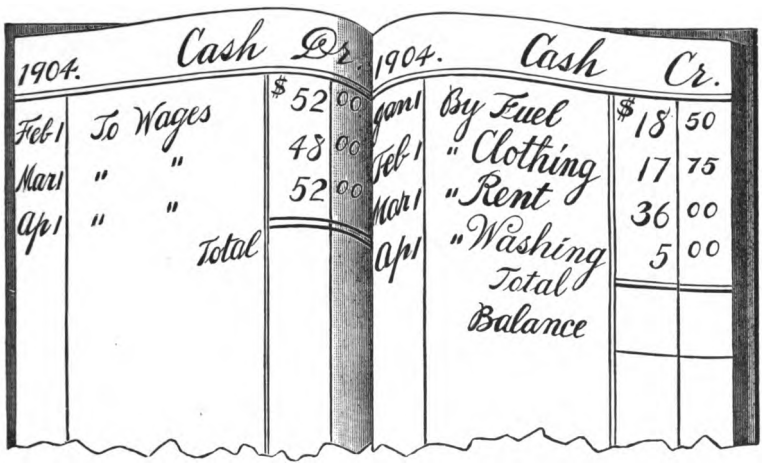
Plan. 1. $9.00 − $1.80 = $—. 2. $7.20 − $.72 = $—.
3. $6.48 − $.32 = $—, net price.

10. A bill of goods amounted to $163 (list-price). It was bought at 10%, 10%, and 5%, off, and the goods were sold at 10% more than the list price. Find the percentage of profit.

11. To what single discounts are the following discounts equivalent: 25% and 10%? $\frac{1}{3}$ and 25%? 40% and 10% and 5%?

12. Compare the net prices of a carriage listed at $330, sold at the following discounts: (*a*) 30%, 20%, and 10%; (*b*) 20%, 10%, and 30%; (*c*) 10%, 30%, and 20%; (*d*) 10%, 20%, and 30%. What is the similarity in these cases?

It is desirable to keep an account of the money received and paid out. This is called a **cash account.**



Dr. stands for debtor and *Cr.* for creditor. *Cash* may be thought of as a debtor to the person keeping the account for all money received and creditor for all money paid out.

1. On which side of the account are entered the amounts of money, or cash, received? On which side are entered the amounts of money, or cash, paid out?

2. In the above account, on what items was money received from January 1 to April 1?

3. What is the total of the debtor side? Credit side?

4. How much should be entered opposite the word **balance** to make the footing of the credit side also $152?

Finding how much must be added to one side of an account to make the footings of the two sides equal is called *balancing* the account.

The double lines under the footings show that the account has been balanced to date, and the amount on hand, if such there be, is the same as the balance on the credit side.

Make out and balance:

1. Mr. Minor had on hand, March 1, $5. Earned to March 20, $25. March 31, sold a watch for $15. March 10, purchased a pair of shoes for $4. March 30, paid for board $10.

2. In debt, May 1, $25.25. Received, May 2, $50. Bought, May 5, a suit of clothes for $14.50. Paid board, May 20, $9. Incidentals to May 30, $5.75. Received May 31, month's wages, $62.

3. Balance on hand, July 1, $335. Received from rents, July 3, $75. Paid for board, July 15, $35. Paid for laundry, July 18, $3.35. Paid for incidentals, July 20, $28.87.

4. On hand, May 1, $25. Received for wages, May 15, $5. Paid for board, May 16, $7.50. Paid note, May 20, $25. Received for wages, May 31, $40.

5. On hand, July 1, $365. Paid for labor, July 3, $12.45. Paid for groceries, July 5, $18.35. Received interest, July 8, $175. Paid for dry-goods, July 15, $67.75. Paid taxes, July 23, $95.75.

6. On hand, Apr. 1, $25. Paid rent, Apr. 2, $15.50. Paid labor, Apr. 15, $17.75. Paid for merchandise, Apr. 25, $500.66. Received by sale of merchandise to Apr. 28, $600.25.

7. Balance on hand, Jan. 1, $500. Received from rents, March 1, $50; July 1, $75; Dec. 1, $125. Received from interest, Nov. 1, $205. Paid grocery bills, Apr. 1, $45.75; July 1, $60; Dec. 5, $75. Paid dry-goods bill, Sept. 1, $100.75. Paid for incidentals to Dec. 31, $125.35.

8. Paid for groceries, Oct. 1, $35.55. Paid for gas, Oct. 3, $6.25. Paid for coal, Oct. 15, $50. Paid for milk, Oct. 20, $3.25. Paid for dry-goods, Oct. 25, $35. Received from interest, Oct. 28, $100. Received salary for Oct., Oct. 31, $225.

Oral.

1. What part of a dollar is $12\frac{1}{2}¢$? What is the cost of a dozen collars at $12\frac{1}{2}¢$ each?

2. What part of \$100 is $\$12\frac{1}{2}$? What is the cost of 9 harnesses at $\$12\frac{1}{2}$ each?

3. What part of \$10 is $\$2\frac{1}{2}$? What is the cost of 84 hats at $\$2\frac{1}{2}$ each?

4. What part of \$100 is $\$33\frac{1}{3}$? What is the cost of 6 suites of furniture at $\$33\frac{1}{3}$ a suite?

5. What part of \$100 is $\$16\frac{2}{3}$? What is the cost of 18 tables at $\$16\frac{2}{3}$ each?

6. A real-estate agent bought some property for \$2,400 and sold it so as to gain $33\frac{1}{3}\%$; for how much did he sell it?

7. 25 = — fourths. To divide a number by 25 is to divide it by how many fourths? How is this done?

Written. Divide by 25:

8. 175. 9. \$3.75. 10. 2,550. 11. 425 mi.

12. \$46.50. 13. 20,125 lb. 14. 3,030 ft. 15. 50,775.

16. When 25 bunches of lath cost \$8.75, what is the price per bunch?

17. When 25 bunches of shingles cost \$90, what is the cost per bunch?

18. What is the cost of 42 dozen pencils at $33\frac{1}{3}¢$ per dozen?

19. $33\frac{1}{3}$ = — thirds. To divide by $33\frac{1}{3}$ is to divide it by how many thirds? How is this done?

20. When $33\frac{1}{3}$ yd. of cloth cost \$30, what is the price per yard?

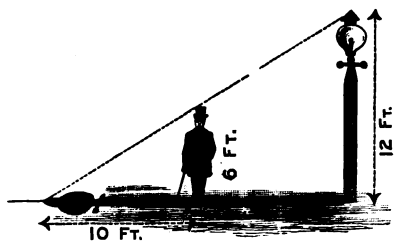
21. To divide by 15 is to divide by 3 and also by —. Divide 90 by 15.

22. Give other examples of division in which it is easier to divide by the factors of the divisor.

RATIO AND MEASUREMENT

Heights by Shadows

1. How do the shadows of two objects of equal height compare at the same time of day?

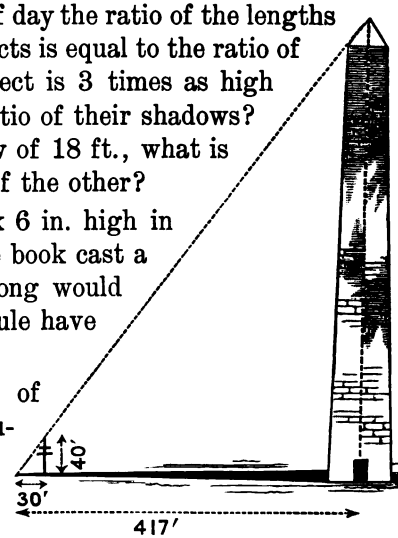


2. The picture shows that a man 6 ft. tall casts a shadow 10 ft. long at a certain time of day; how long is the shadow cast by the lamp-post at the same time?

3. At any given time of day the ratio of the lengths of the shadows of two objects is equal to the ratio of their heights. If one object is 3 times as high as another, what is the ratio of their shadows? If the first casts a shadow of 18 ft., what is the length of the shadow of the other?

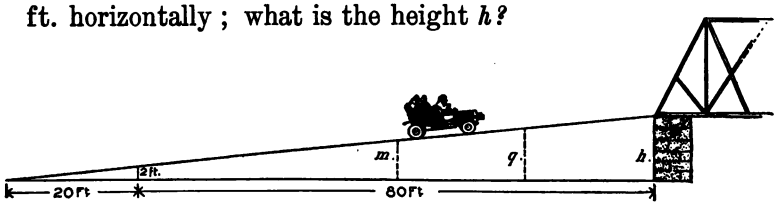
4. Helen placed a book 6 in. high in the sun and found that the book cast a shadow 4 in. long; how long would the shadow of a 12-inch rule have been?

5. At a certain time of day the Washington monument casts a shadow of 417 ft. and a telegraph pole 40 ft. high casts a shadow of 30 ft.; what is the height of the monument?



6. The obelisk in Central Park, New York city, casts a shadow of 100 ft. when a man 6 ft. tall casts a shadow of 8.5 ft. Find the height of the obelisk.

1. The approach to this bridge rises 2 ft. for every 20 ft. horizontally; what is the height h?



m and q in the picture are 20 ft. apart; also q and h.

2. What is the height m? What is the height q?

3. If the height h were 5 ft. only, and if the approach were 100 ft. long horizontally, what would be the rise of the approach for each 10 ft.?

The rise in a road is called its grade, and it is often indicated by stating the rise for some horizontal distance; e. g., the *grade* of the bridge approach is *2 ft. in 20 ft.*

The grade per cent means the number of feet the grade rises vertically per 100 ft., on the level. The grades of roads are seldom more than 3% or 4%.

4. What is the grade in Exercise 3?

5. What is the grade per cent in Exercise 3?

6. What is meant by a 12% grade? By a 5% grade? By a .5% grade? By a $\frac{2}{3}$% grade?

7. How many feet does a 1% grade rise in a mile? A 10% grade? A 12% grade?

8. A horse that can draw 1,000 lb. on a level can draw:

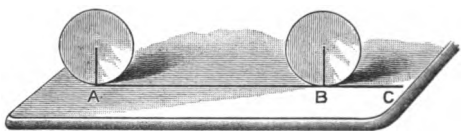
900 lb.	on a	1%	grade,	810 lb.	on a	2%	grade
750 "	"	2.3%	"	720 "	"	2.5%	"
640 "	"	$3\frac{1}{2}$%	"	600 "	"	4%	"

Find the number of feet rise per hundred of horizontal length in each case.

9. According to Exercise 8, on what grade can a horse draw 75% as much as on the level?

1. Draw a circle of 1 in. radius on heavy cardboard. Cut out the circle and mark some point on it.

2. Draw a straight line 10 in. long. A C in the picture represents this line. Letter the left end A; place the circle upright with the mark at A and roll the circle along the line A C until the mark again touches the line. Mark this point B. What line has the same length as the length of the circle? Measure carefully the line A B within .1 in.



3. What is the length of the diameter of the circle? Find the ratio of the length A B to the length of the diameter.

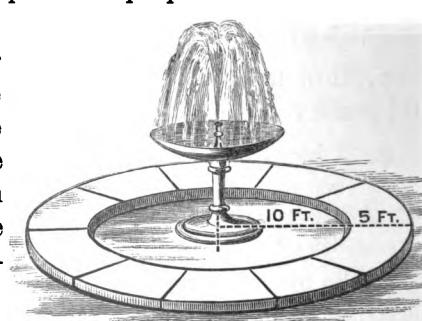
4. Repeat the experiment with a circle of radius 5 in.

5. Measure the diameter of some round bodies, as ink-bottles or cylinders. Also measure the circumference by a string and find the ratio of the circumference to the diameter in each case.

6. How near do the results in all these cases come to 3.14?

This ratio can not be expressed exactly, but 3.14, or $\frac{22}{7}$, is sufficient for all practical purposes. It is denoted by π (pī).

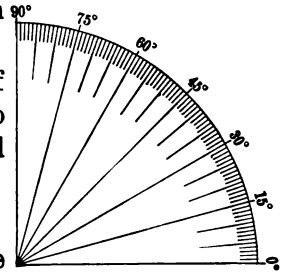
7. By what number may the length of the diameter of a circle be multiplied to determine very nearly the length of the circle? Find the length of a circle of diameter 5 in.



8. What is the diameter of the fountain as shown in the picture. What is the perimeter of the inside of the walk? The outside?

1. What is an angle? The vertex of an angle? A right angle? A perpendicular?

2. How many right angles will exactly fill the space in a plane about a point? Tear off the four corners of a rectangular sheet of paper and arrange them so as to show this.



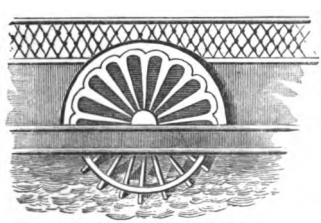
In order to describe the size of angles a right angle is supposed to be divided into 90 equal angles, called degrees.

90 degrees (90°) = 1 right angle.

3. How many degrees are there in two right angles? In three right angles? In four?

4. A pie is cut into 4 equal pieces; how many degrees are there in the angle between the sides of each piece? How many degrees are there in the angle of each piece, if it is cut into 6 equal pieces? Into 8 equal pieces?

5. A Detroit and Buffalo steamer has a paddle-wheel 28 feet in diameter. If the buckets are hung 4 feet apart, how many are there on the wheel? What is the angle between the lines from the center to a pair of adjacent buckets?



6. A wheel has 8 spokes; what is the angle between each spoke and the next?

7. A ship is sailing north; through how many degrees must it turn to change its course to northeast?

8. Through how many degrees does the minute-hand turn in an hour? In $\frac{1}{2}$ hr.? In $\frac{1}{4}$ hr.? In 10 min.? In 5 min.?

9. What is the angle between the hour-hand and the minute-hand at 2 o'clock? At 5 o'clock?

Much writing may be saved by the use of short forms, or abbreviations.

Thus lb. for pound, mi. for mile, $\times$ for times, $+$ for plus, B for the base of a figure, A for the altitude, and many others that have been used frequently.

1. If A stands for the number of square units in the area of a rectangle, L for the number of units of its length, and B for the number in its breadth, then the relation between these three numbers may be stated in abbreviated form thus: $A = L\,B$. State the same relation in full.

It is customary not to use $\times$ for times between letters, or between a number and a letter. When no sign is used multiplication is understood.

2. If A stands for the area of a parallelogram, L for the length of its base, and H for its height, what does $A = L\,H$ mean?

3. If A stands for the area of a triangle, B for its base, and H for its height, what does $A = \frac{1}{2}\,B\,H$ mean?

Determine the numbers to fill the blanks, concerning rectangles:

	4.	5.	6.	7.	8.	9.
A =	36	600	840	?	64	?
L =	9	?	150	8	?	1,273
B =	?	20	?	$5\frac{1}{2}$	$5\frac{1}{3}$	629

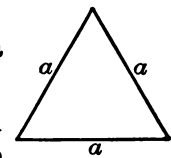
Concerning parallelograms:

	10.	11.	12.	13.	14.	15.
A =	?	30	100	?	50	10
B =	5	10	?	63	?	2.5
H =	6	?	20	$5\frac{1}{3}$	$7\frac{1}{2}$	?

Concerning triangles:

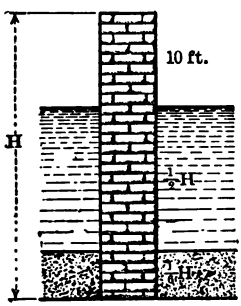
	16.	17.	18.	19.	20.	21.
A =	?	25	49	?	12	50
B =	6	?	7	$1\frac{1}{2}$	$\frac{1}{2}$	$12\frac{1}{2}$
H =	15	6	?	7	?	?

1. If the length of each side of a triangle is represented by a, what represents the length of all the sides?



2. If s represents the length of a side of a square, what represents its perimeter?

3. If b represents the number of units in the breadth of a rectangle and if the rectangle is 4 times as long as wide, what represents the length of the rectangle? Its perimeter?



4. The foundation (the part in the earth) of a bridge pier is $\frac{1}{6}$ of the total height of the pier; the part in the water is $\frac{1}{2}$ of the total height, and the top of the pier is 10 ft. above the water; what is the total height?

PLAN. 1. $\frac{1}{2}$ H + $\frac{1}{6}$ H + — ft. = H.

2. H = — ft.

Test by putting the value found for H in Step 1.

5. One-half of the distance between St. Louis and New Orleans is 280 miles more than $\frac{1}{10}$ of the distance; what is the distance between these places?

PLAN. 1. Let D = the distance in miles.

2. $\frac{1}{2}$ D = $\frac{1}{10}$ D + —. D = —.

6. Boston and Cleveland are 682 mi. apart. A place on a line between them is d miles from Boston; how far is it from Cleveland?

7. If the area of New Hampshire were increased by 9,005 sq. mi., it would be doubled; what is the area of New Hampshire?

8. A man bought an equal number of 2-cent and 1-cent postage-stamps. He paid for all $1.47; how many of each kind did he buy?

Oral.

1. State some familiar instance in which letters are used to represent numbers. Illustrate.

2. What is the meaning of $a\ b$? What is the value of $a\ b$, when $a = 2$, $b = 5$?

3. What is the meaning of $p\ v$, when $p = 4$, $v = 9$? Of $a + 5\ m$, when $a = 7$, $m = 3$?

4. When a stands for the altitude of a figure, b for the base, state what stands for the area of: a parallelogram; a rectangle; a triangle.

5. If b and b' represent the bases of a trapezoid and h the altitude, what represents the area?

6. A half of one-seventh of the number of days in a certain month is 2; what month is it?

7. Mary is $\frac{1}{2}$ as old as Clara and the sum of their ages is 24 yr.; how old is each?

8. Who is a commission merchant? What is meant by the rate of commission? Illustrate.

9. A commission merchant sold a car-load of potatoes for \$225; what was his commission at the rate of 10%?

Oral and written.

10. If the car mentioned in Exercise 9 contained 300 bu. of potatoes, what did the customer receive per bushel? To how many decimal places need one express the result?

11. What is a discount? How may there be several discounts for the same list price? Illustrate.

12. To what single discount is 20% and 10% equivalent?

13. What is the meaning of list price? Of net price? Illustrate.

14. What is the meaning of ratio? How may the height of various objects be found by measuring shadows? Illustrate the method.

Oral and written.

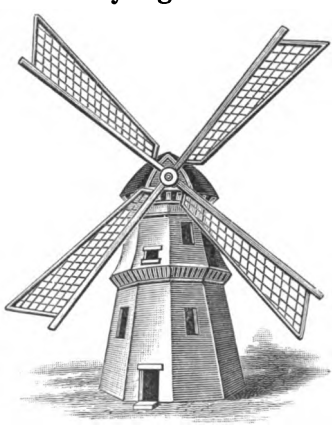
1. Name one way by which the length of a circle may be measured. Name another way.

2. What is the ratio of the length of any circle to that of its diameter?

3. What is the length of a circle whose diameter is 8 in.? Of a circle whose diameter is 4 ft.? 3 yd.? 7 in.? 1.2 ft.?

4. How many degrees are there in the angle of a half-open knife? If convenient, open a knife so as to show an angle of 45°; 90°; 135°; 180°.

5. A belt wheel has 16 spokes; how many degrees are there between each pair of spokes?



6. Windmills in Holland are like the picture; how many degrees are there in the angle between the arms of each pair of sails?

7. How many degrees are traversed by the hour-hand of a clock from 10 o'clock A. M. to 6 P. M. the next day?

8. What is the length of a circle whose radius is 15 in. long?

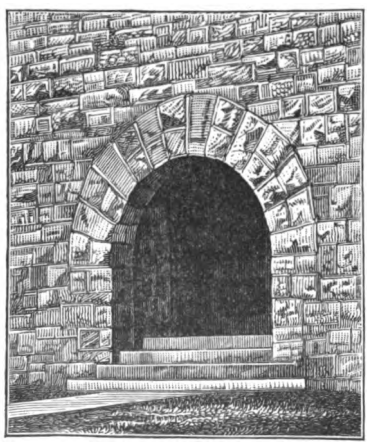
9. Standard wheels for the front axles of farm-wagons are 44 in. high; those for rear wheels are 55 in. high. How many inches longer is the perimeter of a rear wheel than that of a front wheel?

10. The height of a medium front wheel is 36 in.; how many inches more are there in the perimeter of a standard wheel than in that of a medium wheel?

11. A horse that can draw 1,000 lb. on the level can draw 640 lb. on a $3\frac{1}{3}\%$ grade; by how many per cent is the load decreased?

1. A boy stands so as to face the east and turns to the left through 90°, what point does he face? If he turns through 180°? Through 270°? If he turns to the right through 90°? Through 180°?

2. Through how many degrees does the hour-hand of a watch turn between 12 o'clock and 4 o'clock? Between 2 o'clock and 7 o'clock?



3. Measure the height of the archway in the picture, in which 1 inch represents 10 ft.

4. Measure the distance from the second stone step to the beginning of the half-circle.

5. From these two heights find the radius of the arch; the diameter; the distance around the outside of the semi-circle.

6. If a man receives 75¢ a 4-foot cord for cutting wood and 50¢ a 4-foot cord for sawing it into stove wood, what is the cost of cutting 125 cords of stove wood?

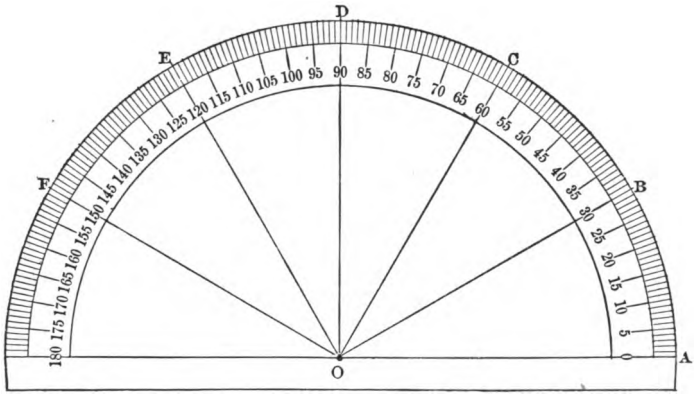
7. A commission merchant sold 150 bushels of peaches at $2.25 per bushel. His rate of commission was 4½%; what was his commission?

8. What is meant by a 2½% grade? A 6% grade? A 7½% grade?

9. At a special sale a firm offered goods at the following discounts; find the reduced price:

Marked price.....	$3.75	$14	$59	$87	$99
Discount.....	10%	16⅔%	20%	3%	5¼%

The figure shows an instrument used to measure angles. It is called a **protractor.** The semicircular part is divided into 180 equal parts, each part representing one degree.



1. How many degrees are there in the angle A O D? How many in the angle A O C? In the angle A O B? In A O E?

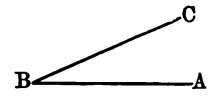
2. Draw angles that look like each of the following with sides much longer. Measure each, if a protractor is available.

3. With the protractor construct an angle of 90°. Of 45°. Of 30°. Of 60°. Of 135°. Of 160°. Of 180°.

4. Draw angles of various sizes without using the protractor. Estimate and record the number of degrees in each. Measure each with the protractor and test your estimate.

5. Draw a triangle at random and measure its angles. Find their sum. Repeat with four other triangles.

An angle lettered on its sides and at its vertex as shown is read "Angle A B C."



1. Read each angle in Figure 1 that is less than 90°. Such an angle is called an **acute angle.**

2. Read each angle in Figure 2 that is greater than 90° and less than 180°. Such an angle is called an **obtuse angle.**

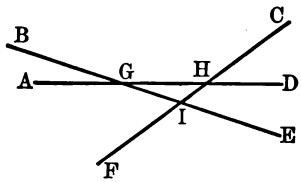


Figure 1.

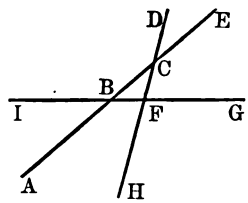


Figure 2.

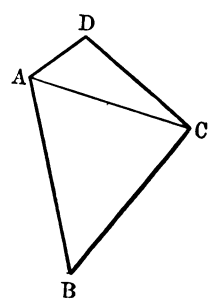
3. What is the sum of the angles of a square?

4. Draw a parallelogram. Measure each angle. What is the sum of the four angles?

5. Treat a trapezoid similarly.

6. Make a figure with at least 2 acute angles, 3 obtuse angles, and 1 right angle.

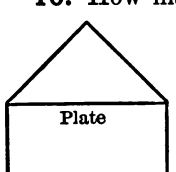
7. Draw an irregular figure with 4 sides. Measure the angles and find their sum.



8. Compare the results of Exercises 4, 5, and 7.

9. Measure the angles of the triangles in the picture.

10. How many degrees are there in $\frac{1}{2}$ of a right angle? In $\frac{1}{6}$ of a right angle? In $\frac{1}{3}$ of a right angle? In $\frac{2}{3}$? In $\frac{1}{4}$? In $\frac{1}{8}$? In $\frac{5}{6}$?



A roof with such a slant is said to have *half-pitch.*

11. In a half-pitch roof, what angle do the sides make at the ridge? What angle do they make with the plate?

1. Draw an arc of a circle. Draw radii from its end points to the center.

The arc is said to *subtend* the angle between the radii.

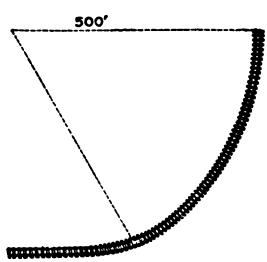
2. Draw a circle and two diameters at right angles. Into how many parts is the circle divided? What angle does each subtend?

The size of an arc is often expressed in degrees, an arc of one degree being an arc which subtends an angle of 1°.

3. By use of a protractor and a circle of radius 3 in., draw arcs of 15°; 25°; 90°; 110°; 160°; 180°.

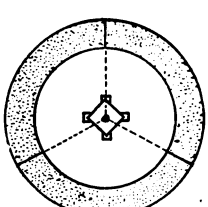
4. Draw arcs of 190°; 210°; 250°; 270°; 300°; 360°. What part of an entire circle is each arc you have drawn?

5. What is the length of a circle of radius 5 feet? What is the length in feet of a 180-degree arc of this circle? Of a 30-degree arc? Of a 270-degree arc?



6. The railway curve shown in the picture is a 60-degree arc of a circle. The rails are standard gage (4 ft. 8½ in. apart); find the length of each rail in this arc.

7. A fountain is surrounded by 3 equal flower-beds as shown in the picture; how many degrees are there in the whole angle at the center of the fountain?



8. What part of 360° is the angle belonging to one flower-bed? How many degrees are there in each of these angles?

9. How many degrees are there in the arc belonging to each flower-bed?

Oral.

1. John had a certain number of marbles. Represent the number by n. What represents the number, if he buys 2 more?

2. What represents the number if he gives away 4?

3. James has twice as many as John had at first; how many has James?

4. Frank has 2 more than twice this number; how many has Frank?

5. Tom has one less than 3 times n marbles; how many has Tom?

6. When John had n and James 2 n, how many did both have?

7. John gave $\frac{1}{2}$ of his to Tom. How many had Tom then? How many had John left?

8. If the number which John had was 12, how many had Frank? James? Tom?

Written. State the following in figures or letters, using signs for operations:

9. One hundred is forty more than sixty.

10. The length (l) of a box is twice its breadth (b).

11. The number of quarts (q) in a certain number (g) of gallons.

12. A man is y years old. His father is twice as old.

13. The sum of the ages of father and son in Exercise 12 is 60 years.

14. A statue is 3 times as high as its pedestal, the height of the pedestal being h.

15. The number denoted by x is to be subtracted from that denoted by y.

16. A pair of gloves costs c cents; what would the cost be, if the price were raised 5 cents?

1. If there are 8 books on one shelf and 12 books on another, how many books are there on both shelves? If there are a books on one shelf and b books on another, how many books are there on both shelves? Write this sum.

2. What is the value of $a+b$ when $a=6$ and $b=14$? When $a=8$ and $b=20$? When $a=16$ and $b=14$?

3. A merchant sold goods which cost \$6 for \$8; how many dollars did he gain? If goods costing \$$b$ were sold at a profit for \$$a$, how many dollars were gained? Write an expression indicating this profit.

4. What is the value of $a-b$ when $a=25$ and $b=10$? When $a=15$ and $b=7$? When $a=30$ and $b=14$?

5. If a barrel of apples costs \$3, what will 3 barrels cost? 4 barrels? 12 barrels? n barrels?

6. If there are 4 rows of apple-trees in an orchard and 8 trees in each row, how many trees are there in the orchard? How many trees are there in an orchard with b rows of 8 trees each? How many trees are there in an orchard with b rows of a trees each? How is the product of a and b written?

7. How many tons of coal at \$5 a ton can be bought for \$25? How many tons at b dollars a ton can be bought for a dollars?

The operation a divided by b is written $a \div b$ or $\frac{a}{b}$.

8. What is the value of $\frac{a}{b}$ when $a=30$ and $b=6$? When $a=42$ and $b=7$? When $a=56$ and $b=7$?

Find the value of each expression when $a=2$, $b=1$, $c=3$, $d=4$:

9. $a+b$.
10. $2a+3b$.
11. $a-b$.
12. $2a-b$.
13. ab.
14. $\frac{2}{3}cd$.
15. $\frac{a}{b}$.
16. $\frac{a+c}{d}$.
17. $\frac{d-b}{c}$.

1. A contractor, wishing to dig a cistern, finds it most convenient to make it 5 ft. square; how deep must it be dug to contain 750 gal.? (Use 1 cu. ft. $= 7\frac{1}{2}$ gal.)

PLAN. 1. $750 \div 7\frac{1}{2} = —$, the number of cu. ft. required.
2. Let d stand for the depth.
3. $5 \times 5 \times d = —$, the number of cu. ft.
4. Therefore $d = —$, the number of ft. of depth required.

2. The metal types used in printing are made of 4 parts by weight of lead to 1 part of antimony; how much lead is there in 86 lb. of type? Antimony?

PLAN. 1. Let a stand for the number of pounds of antimony in the type.
2. The number of pounds of lead is 4 times —.
3. $a + 4a = —$. Why?
4. Therefore $a = —$. 86 lb. $- a = —$.

3. A rectangular park is twice as long as it is wide and the distance around it (perimeter) is 18 miles; find its length and breadth in miles.

PLAN. 1. Let x stand for the number of miles in the width of the park.
2. The number of miles in the length is $— x$.
3. The perimeter is $2x + 4x = — x = —$.
4. Therefore, the length $= —$, and the breadth $= —$.

4. The perimeter of a rectangular farm is $1\frac{1}{2}$ miles. The length is $\frac{1}{4}$ of a mile more than the breadth; find its dimensions.

5. A man left $\frac{3}{4}$ of his estate to his children and divided the remainder equally among 4 charitable institutions, each receiving \$6,000; what was the value of his estate?

6. A newsboy delivers 75 papers per day. He delivers twice as many in the morning as in the evening; how many does he deliver in the morning?

7. A man owned $\frac{9}{14}$ of a store; he sold $\frac{2}{3}$ of his share for \$3,000; find the value of the store.

1. How many degrees are there in the sum of the angles of a triangle?

2. What is the sum of the acute angles of a right-angled triangle?

3. What is an acute angle? An obtuse angle? How many obtuse angles can a triangle have?

4. What is a cash account? What are the names of the two sides of the account?

5. On which side of the account are items of money expended recorded? Of money received? How is cash thought of, so that this may be done?

6. What is meant by balancing an account?

7. Make out a cash account for the following:

Mr. Roe received $35 interest Jan. 1, 1904, for some money lent; $105, January 15, for the sale of some produce; and $18, January 24, from the rent of a house. He paid out $15 for clothing, Jan. 5, $10 for provisions, Jan. 13, and $50 for a wagon, Jan. 30.

8. If n represents a number, what does $5\,n$ represent?

9. Write n, $2\,n$, $3\,n$, ... to $10\,n$, and write the values of these numbers when $n = 2, 3, 4, 5, \ldots . 10$. Of what are the numbers of the first set multiples? Of the second set? Of the third? Of the eighth?

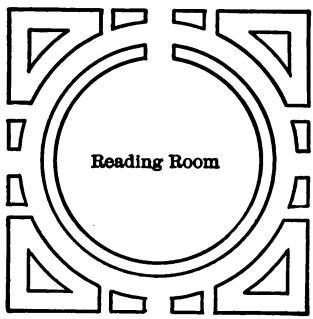
10. If n represents Henry's age, what represents his age 5 years ago? What represents his age 5 years from now?

11. If Henry is x years of age and George is y years of age, what represents the sum of their ages? $\frac{1}{2}$ of the sum of their ages?

12. If the adjacent sides of a rectangle are x and y, what represents the perimeter of the rectangle?

13. Belle had a apples and Susie half as many. How many had Susie?

1. Make out and balance the following account: Mr. Smith had on hand, Mar. 1, $225; received from rents, Mar. 15, $47; paid for board, Mar. 18, $25; received salary, Mar. 31, $155.



2. The figure shows the plan of a library; the diameter of the reading-room is 100 ft.; find the distance around it.

3. How many degrees are there in a right angle? In $\frac{1}{3}$ of a right angle? How many degrees are there in $\frac{1}{2}$ of $\frac{1}{5}$ of a right angle?

4. $2n$ may stand for any even number. Why? Write the next even number larger than $2n$. Write the next even number smaller than $2n$.

Add:

5.	6.	7.
75.728	10.276	2.767
80.213	4.727	1.891
47.612	5.879	2.426
45.276	4.343	1.200

8. If on an average 75% of the eggs placed in an incubator hatch and 90% of the chickens transferred to the brooder live to become marketable fowls, how many fowls are grown from 80 doz. eggs?

9. Diminish 100 by 10% of itself; the result by $33\frac{1}{3}$ of itself; this result by $33\frac{1}{3}\%$ of itself; what is the final result? What percentage of 100 is it?

10. A merchant began with $5,000 capital, and at the end of each year added his profit to his capital. His profit each year is shown by the following percentage of the capital invested: first year, 20%; second year, 25%; third year, 10%; what was his capital for the fourth year?

INTEREST AND TAXES

Difference between Dates

1. How many days are there from January 1 to February 25, including Jan. 1, but excluding Feb. 25?

2. How many days are there from April 1 to May 20, including April 1, but excluding May 20?

3. The Spanish-American War began April 21, 1898, and ended April 11, 1899. How long did it last?

PLAN. 1. April 11 is — days earlier than April 21.
2. The difference between the dates is — days less than 1 year, or — mo. and — da.

4. How long after the beginning of the American Revolution, April 19, 1775, did the French Revolution, May 5, 1789, take place?

5. How long after the Battle of Hastings, Oct. 14, 1066, did the Battle of Waterloo, June 18, 1815, take place?

PLAN. 1. From Oct. 14, 1066, to Oct. 14, 1814, was — years.
2. From Oct. 14, 1814, to June 14, 1815, was — mo.
3. From June 14 to June 18 was — days.
4. Therefore, from Oct. 14, 1066, to June 18, 1815, was — yr., — mo., — da.

6. Victoria was born May 24, 1819, and was crowned queen, June 28, 1836; how old was she at this time?

7. Napoleon was born Aug. 15, 1769, and died May 5, 1821; what was his age at the time of his death?

8. John Milton was born Dec. 9, 1608, and died Nov. 8, 1674; what was his age at the time of his death?

9. Longfellow was born June 18, 1819, and Lowell was born Feb. 22, 1819; what was the difference in their ages?

10. Queen Elizabeth was born Sept. 7, 1533, and died March 24, 1603; how old was she at the time of her death?

Oral.

1. Mr. Howard borrowed \$500 of Mr. Tripp for one year. At the end of the year he returned the \$500 and paid Mr. Tripp \$6 per \$100 for the use of the money; how much did he pay for the use of the \$500?

The payment for the use of money is called **interest.** The rate of payment may be stated by naming the sum to be paid for the use of \$100 for one year.

2. At the rate of \$6 per \$100 for one year, what is the interest for one year on \$200? On \$500? On \$800? On \$1,000? On \$2,000? On \$3,200?

3. At the rate of \$5 per \$100 for one year, what is the interest for three years on \$300? On \$600? On \$800? On \$1,000? On \$4,000? On \$6,800?

Instead of saying "interest at the rate of \$6 per \$100 per year," it is customary to say, "interest at 6% per annum," or, more briefly, "interest at 6%," per year or per annum being understood.

4. What is the interest on \$100 for 1 year at 4%? On \$600? On \$3,200? On \$1.00 (or 100 cents)?

5. What is the interest on \$1 for 1 year at 5%? At $3\frac{1}{2}$%? At 6%?

6. If the rate of interest is 6%, what is the interest on \$100 for 1 year? For 2 years? For 3 years? For 12 years?

7. At 6% what is the interest on \$1.00 for 1 year? For 2 years? For 4 years? For 7 years? For 18 years?

8. At 3% what is the interest on \$100 for 1 year? For 5 years? For $\frac{1}{2}$ year? For $2\frac{1}{2}$ years? For $\frac{3}{4}$ year?

9. If a savings-bank pays interest at 2% semi-annually, what is the interest on a deposit of \$100 for $\frac{1}{2}$ year? Add the interest to the deposit and find the interest at the same rate for the next half year.

The sum of money bearing interest is usually called the **principal.** Find the interest on the following:

	1.	2.	3.	4.
Principal.....	$200	$250	$483	$1,500
Rate.....	4%	5%	6%	3%
Time.....	1 yr.	2 yr.	1 yr.	5 yr.

5. Mr. Todd bought a house for $1,800. He had $1,100 cash on hand and borrowed the rest from a friend to whom he agreed to pay interest at 5% per annum; how much interest did he pay for each year?

6. At the end of the first year Mr. Todd had saved $185. He paid the interest due and paid the rest of his money to diminish the indebtedness; how much did he still owe? How much interest did he owe at the end of the second year?

7. What is the difference in the annual income from $8,600 when invested at 5% and at $4\frac{1}{2}$% interest?

8. What is the interest on $10 for one year at 6%? For $\frac{1}{2}$ of a year? For $\frac{1}{3}$ of a year? For $\frac{1}{12}$ of a year? For 1 mo.? For 4 mo.? For 6 mo.? For 9 mo.? For 11 mo.?

Find the interest on the following at 6%:

	9.	10.	11.	12.
Principal...	$200	$250	$600	$880
Time.....	1 yr. 4 mo.	2 yr. 3 mo.	4 yr. 8 mo.	1 yr. 10 mo.

13. If $100 has been on interest at 6% per annum long enough to earn $12, how many years has it been lent?

14. How long must $200 be on interest at 5% to earn $20? $50? $35? $15?

15. Which is the greater, the interest on $100 for 3 yr. at 6%, or that on $100 for 4 yr. at 5%?

16. A bank pays 4% interest on money deposited, and receives 6% interest on money lent; how much does the bank make on $10,000 in 5 yr. in this way?

17. What is the interest at 4% on $500 for 5 yr.?

It is customary to deposit money, except a small amount for incidentals, in a bank. The bank will pay the money, or any part of it, to any one who presents a written order from the depositor. Such an order is called a **check.**

2902 BROADWAY.

No. 126 New York June 20 1904

The Corn Exchange Bank
UNIVERSITY BRANCH.

PAID JUN. 22. 1904

Pay to the order of James Carver

Thirty-five & Dollars

$ 35 00 Albert C. Arnold.

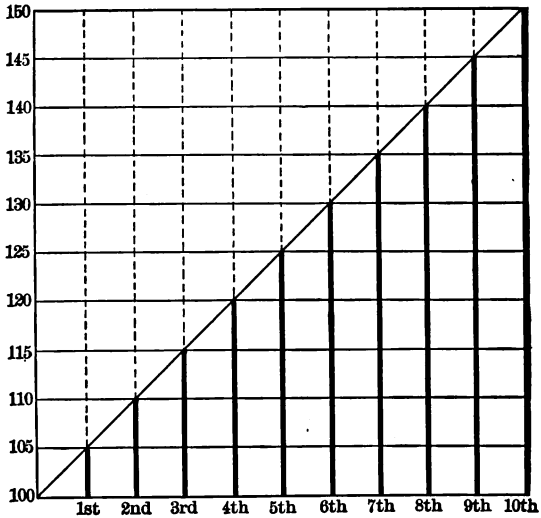
1. Read the check. Who made it? On what bank is this order drawn? To whom did the bank pay the money?

Blank checks are furnished by the banks in book form. The one issuing the check, called the **maker** of the check, should keep a record of the essential facts about it in the portion of the book made for this purpose. The deposits may also be entered as shown.

No. 126 Date June 20 1904	AMOUNT		Date 1904	Deposits:		$365	30
Pay to J. Carver	$35	00	June 8	Cash		125	00
For Clothing							
No. Date 190							
Pay to							
For							
No. Date 190							
Pay to					Total Deposits		
For					Checks Drawn		
Total					Balance		

2. If Mr. Arnold had $490.30 in the bank before the check was paid, how much had he afterward? Find the balance in the account above. Make other problems.

1. What is the interest on \$100 at 5% for 1 year? What is the interest plus the principal?



The principal plus the interest for any given time is called the **amount** for that time.

2. In the figure, each space between the lines running lengthwise represents \$5 and the space on these lines represents 1 year. How many dollars does the first heavy line represent? The second? These lines represent the interest on \$100 at 5% for the time indicated.

3. What interest does the fourth line represent? The tenth line?

4. The figures at the left express the amounts of \$100 for various years at 5%. What is the amount of \$100 for 2 years at 5%? How does the figure show this? What is the amount for 5 years?

Read from the figure the interest on \$100 at 5% for:

	5.	6.	7.	8.	9.
Years....	1	2	3	4	5

Read from the figure the number of years required to earn:

	10.	11.	12.	13.	14.
Interest..	\$5	\$10	\$20	\$15	\$40

15. Make a similar table for the rate 4%.

Oral.

1. 6 months are what part of a year? 4 months? 3 months? 8 months? 9 months? 1 month?

2. Express 2 months as a fractional part of a year. Also 4 mo., 6 mo., 8 mo., 10 mo.

3. $\frac{1}{4}$ yr. $+ \frac{1}{6}$ yr. $=$ — yr. $=$ — mo.

4. $\frac{1}{3}$ yr. $+ \frac{1}{4}$ yr. $=$ — yr. $=$ — mo.

5. 1 yr. $- \frac{1}{12}$ yr. $=$ — yr. $=$ — mo.

6. In how many months would the interest on a sum of money equal that for $\frac{1}{4}$ yr. $+ \frac{1}{6}$ yr.? That for $\frac{1}{3}$ yr. $+ \frac{1}{4}$ yr.? That for 1 yr. $- \frac{1}{12}$ yr.?

7. What is the interest on \$300 for 1 year at 4%? What is the interest for $\frac{1}{3}$ yr., or 4 mo.? For $\frac{1}{6}$ yr., or 2 mo.? For $\frac{1}{4}$ yr. $+ \frac{1}{6}$ yr., or 5 mo.?

8. As in Exercise 7, find the interest on \$300 at 4% for 7 mo.; for 11 mo.; for 2 yr. 7 mo.; for 3 yr. 11 mo.

What is the interest on \$800 at 3% for:

9. 1 yr.? 10. 2 mo.? 11. 3 mo.? 12. 4 mo.?

13. 5 mo.? 14. 6 mo.? 15. 7 mo.? 16. 8 mo.?

17. 9 mo.? 18. 10 mo.? 19. 11 mo.? 20. 12 mo.?

Written. Find the interest on:

21. \$675 for 11 mo. at 4%.

22. \$500 for 3 yr. 9 mo. at 3%.

23. \$500 for 5 yr. 3 mo. at 6%.

24. \$200 for 2 yr. 3 mo. at $3\frac{1}{2}$%.

25. \$385 for 5 yr. 6 mo. at $4\frac{1}{2}$%.

26. \$1,000 for 5 yr. 4 mo. at 5%.

27. \$1,500 for 3 yr. 3 mo. at 4%.

28. \$1,000 for two yr. 6 mo. at 6%.

29. \$325 from January 1, 1901, to Apr. 1, 1904, at 3%.

30. Make and solve other problems about interest.

Cities, towns, counties, and States require their citizens to pay a sum of money each year for the support of schools, charitable institutions, public improvements, and many other common interests. Such a charge is called a **tax.**

1. A man's property is valued at $4,000. If his school tax is $\frac{1}{4}$ of 1% of this amount, how many dollars is he taxed for this purpose?

Property is seldom taxed on its full value. An officer, called an **assessor,** estimates the value of all taxable property in a certain district for the purpose of taxation. The value that he sets is called the **assessed value.**

2. If a man's property valued at $10,000 is assessed at $\frac{3}{4}$ of its full value, on how many dollars does he pay taxes?

One tenth of a cent is sometimes called a **mill** and the tax per dollar is usually expressed in mills.

Thus, a tax of $\frac{1}{4}$ of 1% is the same as $2\frac{1}{2}$ mills per dollar.

3. If the tax in Exercise 2 is $2\frac{1}{2}$ mills on each dollar assessed, what are the man's taxes? What percentage of the assessed value are his taxes?

4. The board of a school district wishes to build a new schoolhouse costing $1,500. If the assessed valuation of the property of the district is $125,000, how much must be collected on every dollar?

5. According to Exercise 4, what tax will a man pay who owns property assessed at $10,000?

6. What is the real value of the man's property mentioned in Exercise 5, if it is assessed at $\frac{2}{3}$ of its value?

7. If the man who collects the tax receives 1% of the tax for his work, how much does the collector in Exercise 4 receive?

8. What is the collector's fee at 1% for collecting a tax of $.002 per $1 on an assessment of $250,000?

LEASE—SHORT. 10 6-02

It is Hereby Agreed, BETWEEN Lewis Grant part y of the first part, and Charles Whitcomb part y of the second part, as follows: The said part y of the first part, in consideration of the rents and covenants herein specified, do es hereby **Let and Lease** to the said part y of the second part, the following described premises, situated and being in the village of Wayne County of Wayne and State of Michigan, to wit: house and lot No. 122 for the term of five (5) years from and after the first day of April 1904 on the terms and conditions hereinafter mentioned, to be occupied for residence.

Provided, That in case any rent shall be due and unpaid, or if default shall be made in any of the covenants herein contained, then it shall be lawful for the said part y of the first part or his certain attorney, heirs, representatives and assigns, to re-enter into, re-possess the said premises, and the said part y of the second part, and each and every other occupant, to remove and put out.

And the said part y of the second part do es hereby hire the said premises for the term of five (5) years as above mentioned, and do es covenant and promise to pay to the said part y of the first part, or his representatives and assigns, for rent of said premises for said term the sum of one thousand (1000) Dollars, the rent to be paid in equal annual installments.

Witness our hands and seals this tenth day of March 1904.

Lewis Grant

Charles Whitcomb

Money paid for the use of houses, land, furniture, etc., is called **rent**. The object used is said to be rented. A contract is usually signed, stating the time for which the property is rented, the amount of rental and the mode of payment, the responsibility of the renter for proper care of the property, etc. Such a contract is called a **lease**.

1. According to the lease shown on page 212, to whom did Lewis Grant rent his property?

2. What was the total rental, and how paid, as stated at the end of the lease?

3. How much rent did Mr. Whitcomb pay annually?

4. B leased a house at $750 per annum, payable monthly in advance; how much was his rent per month?

5. A lady rented a piano at $4.50 per month. She paid $4.00 for cartage and had the piano 6 months. What was the total cost?

6. If the rent of a house is $250 per year, how much is it per month? How much is the rent for 7 months? For 19 months?

7. A man arranged to rent a bicycle by the hour and to pay his bill monthly. The first month he used the wheel 162 hours and paid $15.12. What was the rate per hour? The second month he rode 117 hours, and the third month 78 hours. What was the amount of his bill for each month?

8. A merchant rented a block containing 2 stores and 4 suites of rooms for $750 per year. He subrented one store for $200 a year and each suite of rooms for $8 a month. How much rent did the other store cost him per month?

9. An apartment-house contains 6 suites of rooms that rent for $25 a week each; 10 suites that rent for $50 a month each; and 5 that rent for $8 a month. What is the annual rental for the building?

10. Make up the items necessary to fill out a blank lease for a certain piece of property.

Written.

Insurance companies have been formed to afford partial protection (insurance) against monetary loss by fire, storms, shipwreck, theft, injury by accidents, death, and so on.

The written or printed agreement which a company makes with a customer is called a **policy.** The amount paid the company is the **premium.** This is usually stated at a rate per $100 for a period of 1 or 3 years.

1. A man had his house insured for $2,700 at $1.10 per $100; what is the premium?

2. What is the annual premium on a brick house valued at $6,500, the rate being $.45 per $100 for 1 year?

Find the premiums for insuring dwelling property against loss by fire for the following amounts at the specified rates:

3. House, $1,550, contents for $500, at $1.05 per $100.

4. House, $3,400, contents for $1,000, at $.95 per $100.

5. House, $2,500, contents for $800, at $.90 per $100.

6. House, $4,500, contents for $1,500, at $1.10 per $100.

7. House, $7,000, contents for $3,000, at $1.20 per $100.

8. What is the annual premium for insuring a business block against loss by fire for $9,000 at $1.50 per $100, its first-floor contents for $5,000 at $1.15 per $100, and the other contents for $3,500 at $1.50 per $100?

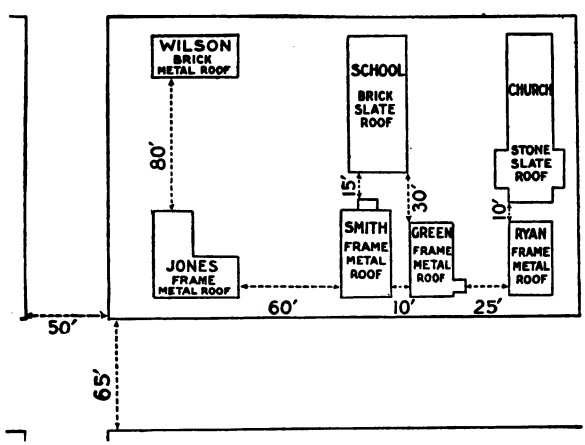
9. Find the premium for insuring a building valued at $55,500 against damage by cyclones at $1 per $1,000.

10. If convenient, bring insurance policies and find the premium on each. They may be obtained at insurance agencies, since all policies are valueless beyond the period for which they are drawn.

11. Find out about the rates of insurance in your locality, also the approximate value of some property. Find the premium that would be paid on the insurance of this property.

Insurance rates are fixed by an association representing the various companies (Underwriters' Association) and vary with time and place.

SINGLE DWELLINGS AND CONTENTS:	RATE PER $100 INSURED.
Brick	$0.30
Brick veneered	0.40
Frame, detached, not less than 50 ft. on all sides,	0.50
" " " " " 25 " " " "	0.60
" " " " " 20 " " " "	0.70
" all others	0.75
Churches and contents, brick or stone	0.75
" " " frame	1.00
SCHOOLHOUSES:	
Brick or stone, with metal or slate roof	0.60
Brick or stone, with shingle roof	0.75
Frame	1.00



What premiums must be paid for the following insurances:

1. The schoolhouse, $10,000 for 5 years?
2. Mr. Ryan, house, $2,500, furniture, $800 for 2 years?
3. Make and solve 3 other problems about above houses.

The National Government does not tax land or personal property, but it taxes certain manufactured products. For example, alcoholic liquors and tobacco. Such charges are called **internal-revenue** taxes. A charge is also made on many articles brought into the country from foreign countries. Taxes levied on imported articles are called **customs** or **duties.**

1. If the tax on a gallon of wine of a certain brand is 30 cents, what is the tax on 1,700 gallons?

2. When the tax on a box of cigars of a certain brand is 20 cents, what is the tax on 25,000 boxes?

THE DUTY ON:

Carpets is	40%	Skein silk is	35%
Clothing is	50%	Silk fabric is	60%
Porcelain is	55%	Cotton garments is	25%
Flannels is	35%	Fur articles is	35%
Leather goods is	35%	Linen goods is	45%
Velvet is	15%	Oriental rugs is	40%

3. According to the table, what is the duty on 150 yards of carpet at 75¢ a yard?

4. What is the duty on 35 yards of silk at 95¢ a yard?

5. What is the duty on an Oriental rug which cost $250? On one which cost $475?

6. Make and solve 5 other problems from the table.

7. In 1902 the United States Government received from:

LIQUORS.	TOBACCO.	STAMPS.	MISCELLANEOUS.
$193,126,913	$51,937,425	$13,442,792	$13,360,130

How much did it receive from all these sources?

8. From what source did it receive the most? The least? The amount received from liquors was how much more than that from tobacco? Than that from stamps?

9. What percentage of the total was miscellaneous?

Make and solve problems:

1. Emperor William II of Germany was born January 27, 1859.

2. William Ewart Gladstone was born Dec. 29, 1809.

3. The Battle of Bunker Hill was fought June 17, 1775.

4. Cornwallis surrendered at Yorktown, Oct. 19, 1781.

5. Cervera's fleet was destroyed at Santiago, July 3, 1898.

6. What is interest? What does "interest at 6%" mean? What is the principal? The amount?

7. What is a bank check? Who is the maker of a check? Who may make checks to be paid by a bank?

8. How is a record kept of the amount of checks drawn and the date of issue?

9. What are taxes? By whom are taxes levied? What is the name of the officer who estimates the value of property to be taxed? Of the officer who collects taxes?

10. How is a small tax-rate usually expressed? What is meant by a tax-rate of $2\frac{1}{2}$ mills?

11. What is the amount of a farmer's tax at 2 mills per dollar, whose farm, worth \$20,000, is assessed at $\frac{3}{4}$ of its value?

12. How does the United States Government levy taxes? Name some property that it does not tax. What are government taxes called?

13. What is the duty on 17 cases of leather goods valued at \$5.00 per case when the duty is 35%?

Find the interest on:

14. \$675 for 11 mo. at 4%.

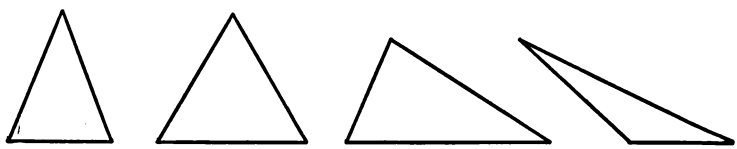
15. \$500 for 3 yr. 9 mo. at 3%.

16. \$200 for 2 yr. 3 mo. at $3\frac{1}{2}$%.

MEASUREMENT

Problems—Angles

1. Draw triangles of the same shape as the figures and having sides four times as long.

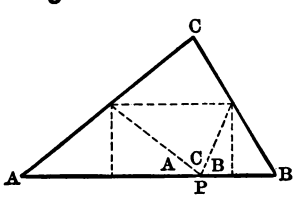


2. With a protractor measure the angles in each triangle and record the results thus: Figure 1, 70°, 70°, 40°.

3. Add the number of degrees in the angles of each triangle. What is the difference between the greatest and smallest of these sums?

4. Draw at random several other triangles of various shapes. Measure the angles as in Exercise 2. Find the sum of the angles in each triangle. Which sum is the greatest? The least? What is this difference?

5. Cut out the triangles as drawn for Exercise 1. Fold a vertex of each to the opposite base. Then fold the other vertices of the triangle as shown in the figure.



6. The angles A, B, C of the triangle are all at one point as P in the figure; what kind of an angle is B P A? What is the sum of the angles A, B, C?

7. By how many degrees do the results of Exercise 3 differ from those of Exercise 6?

8. Draw a 4-sided figure (quadrilateral) and measure the four angles of the figure; find their sum.

1. What is the volume of a prism whose base is 10 sq. yd. and whose altitude is 2 yd.?

2. If b stands for the base of a prism and a for its altitude, what product stands for its volume?

3. If v stands for the volume of a prism and a for its altitude, what quotient stands for b? Given v and b, what quotient stands for a?

Find the unknown numbers:

	4.	5.	6.	7.
$a =$	3 in.	———	5 in.	———
$b =$	16 sq. in.	12 sq. in.	———	25 sq. ft.
$v =$	———	196 cu. in.	125 cu. in.	725 cu. ft.

8. If r stands for the radius of a circle and π for the ratio of the length of the circle to the diameter, what stands for the length of the circle?

9. π is approximately $\frac{22}{7}$. Using this value, find the length of a circle of radius 6 in. correct to .1 in.

10. If c stands for the length of the circle, what quotient stands for r?

Find the unknown numbers:

	11.	12.	13.	14.
$c =$	66 ft.	——	———	154 in.
$r =$	——	7 yd.	630 in.	———

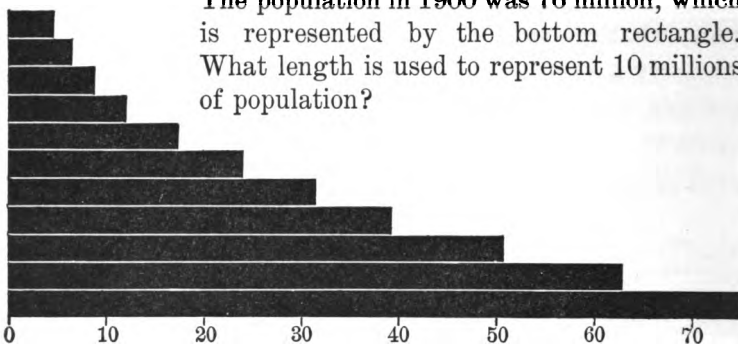
15. If b stands for the base and a for the altitude of a parallelogram, what product stands for its area?

16. If A stands for the area of a parallelogram and a for its altitude, what stands for its base?

17. Given $A = 126$ sq. in. and $a = 3$ in., what is the length of b?

18. If b stands for the base of a triangle and a for its altitude, what stands for its area?

1. In the figure the rectangles represent the population of the United States for each decade from 1800 to 1900. The population in 1900 was 76 million, which is represented by the bottom rectangle. What length is used to represent 10 millions of population?



2. Measure the rectangles from 1830 to 1890 and determine the population for each period within one million.

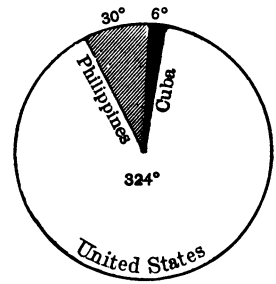
3. What percentage is each population, after the first, of the one preceding it? Take only the number of millions and compute the results correct to 1%.

4. Determine the numbers to fill the blanks:

Population.	1900.	1890.	Increase per cent.
St. Louis.....	580,000	450,000	....
San Francisco.....	340,000	300,000	....
Milwaukee.....	290,000	.....	45%
Memphis.....	.....	60,000	16⅔%
Los Angeles.....	.....	50,000	100%
Columbus.....	130,000	90,000	....
Atlanta.....	90,000	70,000	....

5. Arrange the cities in order of their population and represent the population graphically, similar to the figure at the top of the page, putting the names of the cities in a column at the left.

1. The area of the circle in the figure is divided into parts representing approximately the population of the United States, of Cuba, and the Philippine Islands. The population of Cuba is represented by the black part of the circle; what is the angle of the part representing the population of the United States? Of the Philippines?



2. 4°.25 corresponds to 1 million of population; what is the number of millions in the population of the Philippines? In that of the United States? In that of Cuba?

3. Taken to the nearest million the area of the Russian Empire is 8 million square miles, of which 2 million square miles are in Europe and the rest in Asia; how many square miles are there in Asiatic Russia?

4. Draw a circle of 2-inch radius to represent the area of the Russian Empire. Divide it by radii into two parts to represent the areas of the two parts of the Russian Empire.

5. The area of the Chinese Empire is approximately $4\frac{1}{2}$ million square miles, of which China proper contains $1\frac{1}{8}$ million sq. mi. and the dependencies contain the rest; what is the area of the dependencies?

6. Draw a circle of 2-inch radius and divide it to represent the parts of the Chinese Empire.

7. The total area of the French Republic is 738,681 sq. mi., of which France contains 204,092 sq. mi. and Algeria 198,950 sq. mi.; what is the area of its other possessions?

8. Draw a circle of 2-inch radius to represent the area of the French Republic. Divide it by radii into 3 parts to represent approximately the areas of France, Algeria, and the other possessions, taking the above areas as 750,000 square miles, 200,000 square miles, and 200,000 square miles.

The table shows the average height and weight of Boston school-children.

Age.	Boys.		Girls.	
	Height in Inches.	Weight in Pounds.	Height in Inches.	Weight in Pounds.
9	52.00	60.1	47.58	57.07
10	53.51	70.6	49.37	62.35
11	54.90	75.3	51.34	68.84
12	56.78	85.9	53.42	78.31
13	59.60	94.4	55.88	88.65
14	61.51	99.9	58.16	98.43
15	64.20	116.0	59.94	106.08
16	65.83	125.8	61.10	112.03
17	67.44	135.2	61.92	115.53
18	67.44	138.2	61.95	115.16

1. From the measurements for boys find the increase in height for each successive year; the increase in weight.

2. From the measurements for girls find the increase in height for each successive year; the increase in weight.

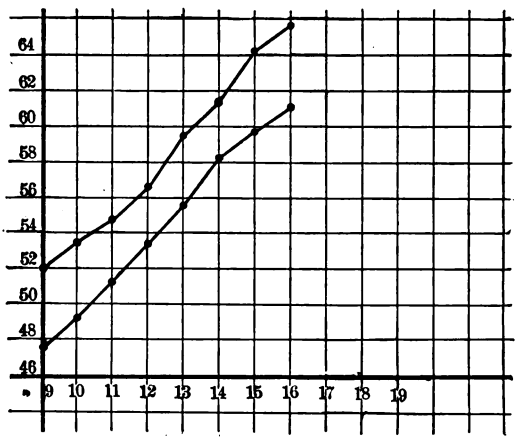
3. What is the total increase in the height of boys during the period from 9 to 12 years of age? From 12 to 15? From 15 to 18? During what period is the increase greatest?

4. What is the total increase in the height of girls during the period from 9 to 12 years of age? From 12 to 15? From 15 to 18? During what period is the increase greatest?

5. What is the total increase in the weight of boys during the period from 9 to 12 years of age? From 12 to 15? From 15 to 18? During what period is the increase greatest?

6. What is the total increase in the weight of girls during the period from 9 to 12 years of age? From 12 to 15? From 15 to 18? During what period is the increase greatest?

The upper line in the figure represents the rate of growth of a boy from 9 years to 16 years of age according



to the table; the lower line represents the rate of growth of a girl.

1. Explain how the points are located on the vertical lines so as to determine the curve.

2. In a similar way make curves on squared paper to represent the rate of increase in weight.

3. At what age does a girl attain her full height? Does a boy attain his earlier or later than this?

4. How do the lines show that a boy's height is always greater than a girl's?

5. Make similar comparisons in regard to weight.

6. During what period of years does the weight change most rapidly? The height?

7. In what period of years is the difference between weights and between heights of boys and girls most marked?

8. If convenient, measure the heights of pupils and represent the results on squared paper.

Various foods furnish different amounts of nutriment, so that a food costing little may be of high value and *vice versa*. For example, 1 lb. of round beefsteak contains $5\frac{1}{2}$ oz. of nutriment, and a pound of stew beef 12 oz. The steak will cost about 14¢ a pound and the stew beef 5¢.

1. A pound of sirloin of beef costing 20¢ a pound furnishes 2.5 oz. of protein or nourishment, and 1 lb. of fresh codfish costing 10¢ furnishes 1.8 oz.; which is the cheaper food, sirloin or codfish?

The table shows the amounts of various foods that 10¢ will buy and the respective amounts of nourishment furnished:

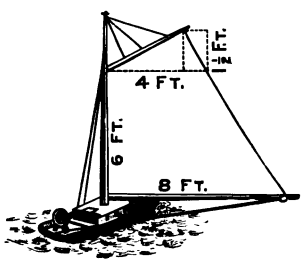
Food.	Price.	Amounts for 10 Cents.			
		Food.	Protein.	Fat.	Units of Energy.
Beef, sirloin....	20¢ a lb.	0.50 lb.	0.08 lb.	0.08 lb.	515
Beef, round....	16	0.63	0.11	0.08	560
Beef, shoulder..	12	0.83	0.13	0.08	595
Beef, dried.....	25	0.40	0.10	0.03	315
Mutton, leg....	20	0.50	0.07	0.07	445
Pork, salt.....	12	0.83	0.02	0.68	2,950
Codfish, salt....	7	1.43	0.22	0.01	220
Eggs.....	24¢ a doz.	0.63	0.07	0.06	385
Milk.....	6¢ a qt.	3.33	0.11	0.13	1,030
Cheese.....	16¢ a lb.	0.63	0.16	0.20	1,185
Corn meal.....	$2\frac{1}{2}$	4.00	0.31	0.07	6,540
Wheat bread...	$7\frac{1}{2}$¢ a loaf	1.33	0.13	0.02	2,235
Oatmeal	4¢ a lb.	2.50	0.34	0.16	4,500
Rice.....	8	1.25	0.08	....	2,025
Beans, white...	5	2.00	0.35	0.03	3,040
Potatoes	60¢ a bu.	10.00	0.15	0.01	2,950

2. Which article of food furnishes the most protein for the same cost? The most fat? The greatest number of energy or heat units? Make 10 other problems.

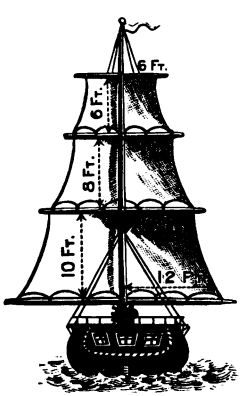
1. Claude let his toy balloon ascend by a string. A light breeze carried the balloon along at the rate of 2 mi. per hour. He followed the balloon for 40 minutes; how many miles did he walk?

2. A strong wind blowing 20 mi. per hour blew Mark's hat across a ravine. The hat was carried along a minute before it landed; how far was it carried in miles? In yards? In feet?

3. Charlotte sailed her boat in a breeze blowing 7 mi. per hour. The boat moved only $\frac{2}{5}$ as fast as the wind; how far did she sail in an hour? In 45 min.?



4. From the dimensions given in the picture, find the area of the sail of the boat. When a breeze of 7 miles per hour blows directly against a surface it exerts a pressure of .23 lb. per square foot. Find the total pressure on the sail of the boat.



5. Find the total area of the sails on this ship. When a strong wind exerts a pressure of 1.62 lb. per square foot, what is the total pressure on these sails?

6. A strong wind blows at the rate of 18 mi. per hour. If a schooner sailing in such a wind goes $\frac{2}{3}$ as fast as the wind, how many miles does it sail in $2\frac{1}{2}$ hours?

7. How long does it take a violent gale blowing at the rate of 58 mi. an hour to cross Lake Michigan where it is 65 miles wide?

8. From Ludington to Milwaukee across Lake Michigan is 91 miles; how long would it take a gale blowing 45 mi. per hour to travel from Ludington to Milwaukee?

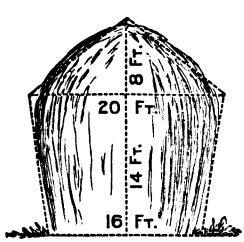
1. Find from the following table the pressure exerted on 2 sq. ft. of surface by each of the winds named.

VELOCITY AND PRESSURE OF WINDS.

KINDS.	AVERAGE VELOCITY. MILES PER HOUR.	AVERAGE PRESSURE. POUNDS PER SQ. FT.
Gentle breeze	7 or less	0.23 or less
Fresh breeze.....	11	0.64
Strong wind	18 or more	1.62 or more
High wind.....	27	3.64
Gale.....	36	6.48
Strong gale	45	10.12
Violent gale.....	58	17.12
Hurricane	76	29.26

2. Find how many pounds pressure each wind would exert in blowing directly against the side of a house 50 ft. long and 18 ft. high.

3. A railroad engine is running directly against a high wind. If the surface exposed to the wind is 55 sq. ft., how many pounds pressure from the wind does the engine overcome?

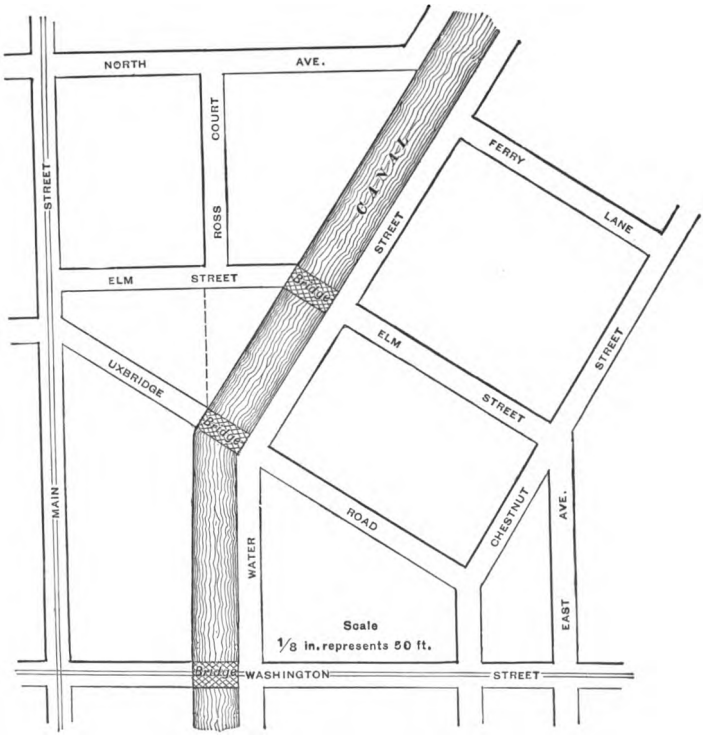


4. The surface enclosed by dotted lines in the picture represents the area of the side of a haystack; how many square feet are there in this surface? How many pounds pressure would each kind of wind given in the table exert on the haystack in blowing directly against this surface?

5. A strong wind carries a balloon along for $5\frac{1}{2}$ hr. at 27 mi. per hr.; how many miles is the balloon carried?

6. A hurricane blowing 76 miles per hour tore a shingle from a house, carried it for $\frac{3}{4}$ of an hour and dropped it at a distant village; how far from the house was the village?

Find from the scale all measurements necessary.



Make and solve problems using the following data:

1. The street-paving cost $1.50 per sq. yd.

2. The construction of the canal cost $.50 per sq. yd.

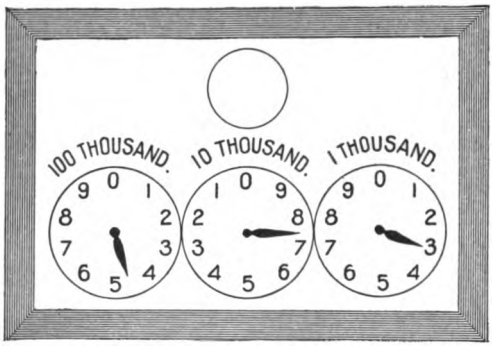
3. The equipment for lighting the streets cost $2,000 per mile.

4. The construction of sewers on every street cost at the rate of $1.00 per running foot.

5. It cost $3.50 per foot to construct the street-car line.

6. Land in blocks that border on the canal is worth 25¢ per sq. ft.; land elsewhere is worth 50¢ per sq. ft.

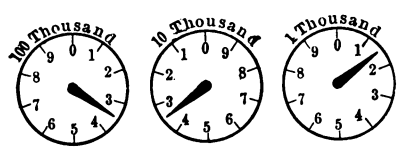
The picture shows the three dials of a gas-meter. The figures on the dial at the right denote hundreds. When the hand of this dial has made a revolution the hand on the center dial points to 1; that is, it indicates 1 thousand feet. The figures on the dial at the left denote ten-thousands, and the hand moves one space as often as the center hand makes one revolution.



In reading the center and left-hand dials, if the hand lies between two digits the smaller is taken as the reading, except between 9 and 0, where 9 is taken. On the right-hand dial it is customary to take the nearest digit.

Thus, the meter in the picture reads 47,300. Note that the hand on the center dial turns in the opposite direction from those on the end dials.

1. The figure shows the state of the meter in Mr. Roe's house at each of 2 readings. Read the meter for each date.



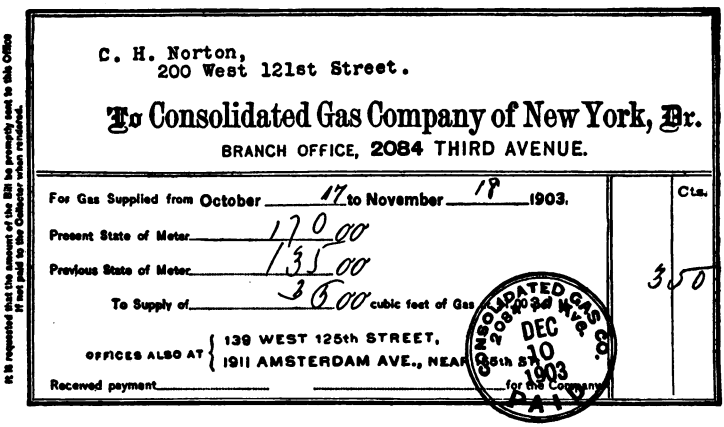
JANUARY 5, 1904.

FEBRUARY 6, 1904.

2. How much gas was consumed from Jan. 5, 1904, to Feb. 6, 1904? What was the cost of the gas at $1 per thousand feet?

3. At this price what is the cost of 9,800 ft.? Of 3,700 ft.? Of 600 ft.?

The following is the ordinary form of a gas-bill:



1. Make a bill for Exercise 2 on page 55, similar to the one shown above.

2. The burners in a house consumed the following number of feet of gas in each month named:

Jan.	Feb.	Mar.	Apr.	May	June
2,800	3,000	2,800	2,400	1,600	1,800

How many thousand feet were consumed in the six months? What was the cost at $1 per 1,000 feet?

3. If an ordinary gas-burner consumes 7 ft. of gas per hour and a Welsbach burner consumes 4 ft. per hour, how much would two burners, one of each kind, consume in 10 hours?

4. When gas costs $1 per M (per 1,000), how much is saved per hour by the use of a Welsbach burner? Per day of 6 hours? Per month of 30 days at the same rate?

5. If convenient, read the gas-meter of your school building at various times and make out bills for the gas consumed, using the rate actually charged.

GENERAL REVIEW

Problems

Add and test:

1.	2.	3.	4.
$375.65	$478.26	$268.42	$827.99
163.42	232.47	127.80	777.85
987.63	128.35	988.75	676.90
473.25	456.26	878.29	788.75
662.32	634.12	144.35	425.50
834.88	478.56	800.20	100.83
247.92	328.83	723.00	676.97
726.87	736.32	110.75	900.08
145.23	278.32	273.89	276.07
274.77	127.26	139.76	878.92
986.38	648.57	276.91	144.27

Find the products of:

5. $284.8 \times 27.$
6. $76.7 \times .23.$
7. $499.7 \times .03.$
8. $23.4 \times 7.09.$
9. $983.2 \times 99.6.$
10. $88.3 \times 9.06.$
11. $497.8 \times 1.04.$
12. $17.8 \times .87.$

Multiply:

13.	14.	15.
$1729.85	$2799.87	$1728.97
29	79.8	34.6

16.	17.	18.
$72658	$7267.43	$2897.8
.125	8.7	27.5

Divide to the nearest thousandth and test:

19. $\$237.66 \div 17.$
20. $\$6{,}934.25 \div .01.$
21. $\$70.616 \div .009.$
22. $\$3{,}692.73 \div .27.$

Add:

1. $\frac{4}{5} + \frac{7}{8} + \frac{2}{3}$.
2. $\frac{1}{10} + \frac{2}{9} + \frac{5}{8}$.
3. $\frac{3}{4} + \frac{5}{6} + \frac{1}{2}$.
4. $\frac{1}{2} + \frac{1}{4} + \frac{3}{5}$.
5. $\frac{1}{12} + \frac{4}{9} + \frac{3}{8}$.
6. $\frac{1}{15} + \frac{5}{18} + \frac{3}{4}$.
7. $\frac{9}{10} + \frac{2}{3} + \frac{1}{4}$.
8. $\frac{5}{6} + \frac{4}{5} + \frac{1}{18}$.
9. $\frac{1}{2} + \frac{2}{7} + \frac{3}{4}$.
10. $\frac{4}{9} + \frac{2}{7} + \frac{5}{6}$.
11. $\frac{4}{9} + \frac{3}{5} + \frac{1}{2}$.
12. $\frac{6}{7} + \frac{1}{5} + \frac{4}{21}$.
13. $\frac{2}{9} + \frac{1}{8} + \frac{2}{5}$.
14. $\frac{7}{9} + \frac{1}{6} + \frac{7}{15}$.

Subtract:

15. $\frac{5}{8} - \frac{1}{3}$.
16. $\frac{11}{12} - \frac{1}{4}$.
17. $\frac{14}{17} - \frac{1}{11}$.
18. $\frac{4}{9} - \frac{1}{4}$.
19. $\frac{10}{17} - \frac{2}{7}$.
20. $\frac{13}{19} - \frac{1}{12}$.
21. $\frac{6}{7} - \frac{1}{2}$.
22. $\frac{24}{25} - \frac{2}{3}$.
23. $\frac{15}{21} - \frac{3}{24}$.
24. $\frac{5}{6} - \frac{1}{3}$.
25. $\frac{18}{19} - \frac{3}{4}$.
26. $\frac{17}{35} - \frac{3}{11}$.
27. $\frac{3}{8} - \frac{1}{9}$.
28. $\frac{13}{18} - \frac{2}{9}$.
29. $\frac{11}{18} - \frac{6}{13}$.
30. $\frac{4}{7} - \frac{2}{9}$.
31. $\frac{12}{13} - \frac{4}{5}$.
32. $\frac{13}{15} - \frac{2}{19}$.

Multiply:

33. $\frac{6}{7} \times \frac{2}{3}$.
34. $\frac{4}{11} \times \frac{2}{3}$.
35. $230\frac{4}{9} \times 12$.
36. $\frac{5}{6} \times \frac{1}{7}$.
37. $\frac{9}{10} \times \frac{3}{5}$.
38. $179\frac{3}{4} \times 14$.
39. $\frac{2}{3} \times \frac{5}{6}$.
40. $\frac{5}{13} \times \frac{4}{7}$.
41. $276\frac{1}{8} \times 23$.
42. $\frac{8}{9} \times \frac{1}{5}$.
43. $\frac{9}{22} \times \frac{1}{8}$.
44. $199\frac{1}{7} \times 35$.
45. $\frac{7}{6} \times \frac{5}{8}$.
46. $\frac{6}{17} \times \frac{2}{7}$.
47. $267\frac{4}{9} \times 28$.
48. $\frac{3}{8} \times \frac{1}{7}$.
49. $\frac{11}{12} \times \frac{5}{9}$.
50. $100\frac{2}{3} \times 98$.
51. $\frac{4}{9} \times \frac{2}{7}$.
52. $\frac{13}{17} \times \frac{3}{8}$.
53. $340\frac{1}{5} \times 79$.

Divide:

54. $\frac{8}{9} \div 4$.
55. $\frac{8}{17} \div \frac{1}{3}$.
56. $\frac{9}{13} \div \frac{4}{3}$.
57. $\frac{1}{8} \div 3$.
58. $\frac{5}{12} \div \frac{1}{9}$.
59. $\frac{24}{25} \div \frac{2}{7}$.
60. $\frac{4}{7} \div 6$.
61. $\frac{3}{14} \div \frac{1}{6}$.
62. $\frac{13}{17} \div \frac{8}{9}$.
63. $\frac{3}{4} \div 5$.
64. $\frac{9}{22} \div \frac{2}{5}$.
65. $\frac{11}{13} \div \frac{4}{7}$.
66. $\frac{2}{5} \div 7$.
67. $\frac{11}{13} \div \frac{5}{7}$.
68. $\frac{12}{17} \div \frac{5}{6}$.
69. $\frac{5}{6} \div 6$.
70. $\frac{10}{11} \div \frac{4}{9}$.
71. $\frac{18}{29} \div \frac{2}{3}$.

1. 20% of potatoes is water; how many pounds of water are there in 100 lb. of potatoes? In 500 lb.? In 250 lb.? In 625 lb.?

2. 1% of flour is fat; how many pounds of fat are there in 200 lb. of flour? In 150 lb.? In 500 lb.? In 625 lb.?

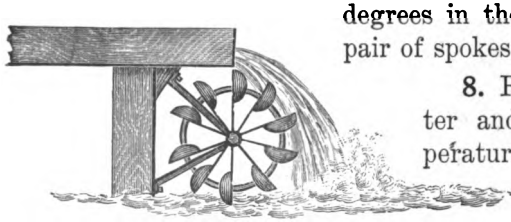
3. 37% of mutton is fat; how many pounds of fat are there in 500 lb. of mutton?

4. The distance from New York to San Francisco is about 3,300 mi. By how many inches is this distance represented on a map whose scale is 1 in. for every 100 mi.?

5. If a barometer reads 30 in. on 3 days in a week, 29.5 in. on 2 other days, and 30.1 in. on 2 others, what is the average height for the week?

6. If the thermometer registers 70° one day, 75° the next, and 84° the next, what is the average temperature for the three days?

7. How many buckets has the water-wheel? How many degrees in the angle between a pair of spokes?



8. Read the thermometer and record the temperature at the same hour each morning for a week. Find the average temperature for the week. By how many degrees does this differ from the highest reading? From the lowest? Represent by a broken line.

9. A workman received $12 per week of 6 working days. His wages were raised to $14.40 per week; how much was the increase per day? How many per cent?

10. A mailing clerk was to address circulars according to a certain list. After having addressed 3,200, $\frac{3}{7}$ of the names on the list had not been used; how many names were there?

1. Write in Roman notation:
19; 26; 35; 39; 43; 56; 49; 100; 97; 63; 45; 90; 75.

2. A house costing \$24,000 rents for \$1,200. The taxes and expenses are \$300 per year. What is the annual percentage of income on the investment?

3. How many years, months, and days were there from the assassination of President Lincoln, April 14, 1865, to that of President McKinley, Sept. 7, 1901?

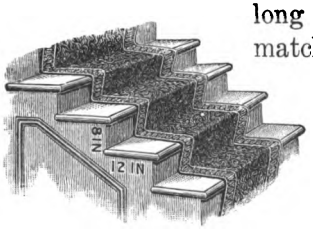
4. The silkworm larvæ from 1 ounce of eggs produce about 130 lb. of cocoons under ordinary conditions. With special care the product may be increased by $33\frac{1}{3}\%$. How many pounds would this equal?

5. How many acres are there in a farm $1\frac{1}{2}$ miles long and $\frac{3}{4}$ of a mile wide?

Find the interest at 4% on:

6. \$1,000 for 2 yr. 6 mo.
7. \$285 for 4 yr. 4 mo.
8. \$500 for 5 yr. 3 mo.
9. \$675.50 for 3 yr. 11 mo.

10. How many yards of carpet must be bought to cover the floor of a room $3\frac{3}{4}$ yd. by 5 yd., the strips running the long way, and $1\frac{1}{2}$ yd. being allowed for matching? (Width of carpet $\frac{3}{4}$ yd.)



11. There are 14 steps to the stairs shown in the picture; what did the carpet cost at 60¢ a yard?

12. The "East Room" at the White House is 82 ft. × 40 ft. Find the cost of carpeting it with carpet 27 in. wide at \$3 a yard.

13. What is the cost of renovating a rug $8\frac{1}{4}$ ft. × $10\frac{1}{2}$ ft. at 10¢ per square yard?

14. What is the height of a prism whose volume is 945 cu. ft. and whose base contains 9 sq. yd.?

1. Suppose that in your State 70% of the area is under cultivation and that there are 4 insect-eating birds per acre. If each bird eats 12 injurious insects per day, how many insects are killed weekly by the birds in your State?

2. Before the Spanish-American War Spanish exports to Cuba amounted to 136 million dollars, to Porto Rico 44 million dollars, to the Philippines 49 million dollars; in 1901 they fell to 66 million, 13 million, and 27 million respectively to these places; what was the total reduction of Spain's exports? What was the reduction in each case?

3. If the wages of 10,000 men are cut 10 cents a day, how many dollars will the company save in wages in a year of 300 working-days?

4. In Minnesota and in the Dakotas 15 million acres are given to raising wheat. If, by using a new variety of seed, the present yield of 27 bushels per acre is increased 2 bushels an acre, what will be the annual crop in these 3 States? At 75¢ per bushel, what will be the increase in the value of the wheat-crop by the use of the new seed?

5. The Canadian power canal at Sault de Ste. Marie furnishes 20,000 horse-power. The canal on the Michigan side furnishes $2\frac{1}{2}$ times as much. How many horse-power does the latter furnish?

6. In a Fall River cotton-mill 58 hours are considered a week's work. The average wage paid is \$7.50 a week, which is increased $\frac{1}{10}$ by the profit-sharing plan; what is the complete average wage per day? Per hour?

7. Of the 800 employees in a soap factory 325 are boys; the average wage for men is \$10 a week, and for boys \$5.50; what is the amount of the weekly pay-roll?

8. In a profit-sharing concern each employee earning \$500 a year receives a dividend equal to $\frac{1}{8}$ of his wages; how much does he receive from the company in a year?

1. Square 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9; in what different digits do these squares end? In what digits may the squares of integers end?

2. Cube 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9; in what different digits do these cubes end? In what digits may the cubes of integers end?

3. What is the air-pressure on 1 sq. ft. of surface at 15 lb. per square inch? Using the weight of a cubic foot of water as 62.5 lb., find the total pressure per square foot at a depth of 10 ft. under water? 100 ft.?

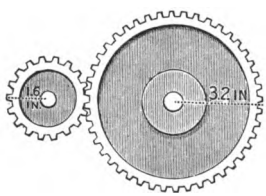


Figure 1.

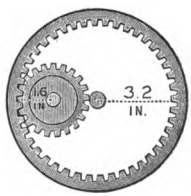


Figure 2.

4. Find the distance between the centers of the cog-wheels in Figure 1; in Figure 2.

5. When the larger wheel revolves once, how many times does the smaller wheel revolve?

6. Two men engage in business, one furnishes $40,000 and the other $60,000; what is the ratio of their capitals? They gain $6,000 in one year, which is shared in the ratio of their capital; how many dollars does each receive?

7. The average daily paid circulation of a newspaper is 95,000; if the receipts for one day are $1,900, what is the price per copy?

8. How many feet of inch boards will be necessary to build a sign-board 40 ft. long and 12 ft. high? What will the lumber cost at $6 per M?

9. If a fan having 15 rays opens out so that the outer rays form a straight line, how many degrees are there between any two adjacent rays?

1. In a house using 6 Welsbach burners and 6 common burners, the gas costs 3 cents an evening for each common burner and $1\frac{3}{4}$ cents for each Welsbach; how much less does the gas for the Welsbach burners cost during a month?

2. If pure air contains 20.04% of oxygen, how many cubic inches of oxygen are there in 1 cu. ft. of air? If 70.02% of pure air is nitrogen, how many cubic inches of nitrogen are there in 1 cu. ft. of air? If the remainder is carbonic-acid gas, what percentage of air is carbonic-acid gas?

3. If sound travels 1,000 ft. per second when the temperature is 32° Fahr., how many seconds after firing will the report of a cannon be heard at a point 4 miles distant?

4. Sound travels 4 times as fast in water as in air; according to the rate given in Exercise 3, how many seconds will it take sound to travel a mile in water?

5. A railroad-train crosses a bridge $\frac{1}{2}$ a mile long at a speed of 15 mi. per hour; in how many minutes does the engine pass over the bridge?

6. How many bricks, $4\frac{1}{4}$ in. by 9 in., are required to pave a passageway $4\frac{1}{2}$ ft. wide and 120 ft. long? What is the cost of the bricks at $6.75 per thousand?

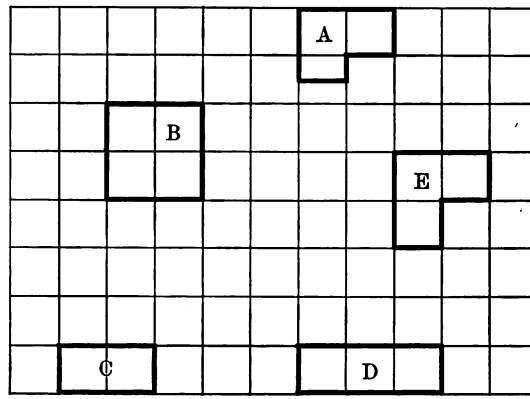
7. How many feet are there in a mile? How many inches? If an inch on a map represents 100 miles, 3 miles are represented by what part of an inch?

8. If the area of the island of Cuba is 46,000 sq. mi. and that of New York State 49,000 sq. mi., find the ratio of the latter to the former.

9. Make problems from the following data:

Name.	Date of Birth.	Name.	Date of Birth.
James Madison	Mar. 16, 1751.	Geo. Washington	Feb. 22, 1732.
J. G. Whittier	Dec. 17, 1807.	H. W. Longfellow	Feb. 27, 1807.

1. In making applications to their companies, insurance agents use squared paper to show the distances between neighboring buildings. If the side of each square represents 10 ft., how far apart, according to the figure, are the buildings, E and B?



2. How far apart are C and D? C and B? E and D?

3. What is the greatest length and greatest breadth of each building?

4. What is the freight charge at 55¢ per 100 lb. on 12 stoves, each weighing 450 lb.?

5. If $\frac{3}{8}$ of the number of glass bottles made in the United States in 1900 were 30 million dozen, how many million dozen bottles were made?

6. What was the value of the lumber produced in Kentucky in 1900, if 12 million dollars was $\frac{6}{7}$ of the value?

7. If the school tax is $\frac{1}{3}$ of 1%, what is the school tax on property worth $25,000?

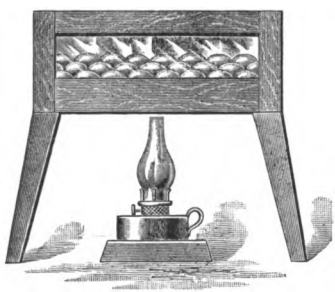
8. What is the duty at 60% on 425 yd. of silk at $2.15 per yard?

9. If celery-plants are planted 6 in. apart in rows 5 ft. apart, how many plants are there in a field 4 rd. by 40 rd.?

10. How many yards of lining $1\frac{1}{4}$ yd. wide will be required to line 25 yd. of cloth $\frac{3}{4}$ yd. wide?

1. It has been estimated that during some storms the waves on Lake Michigan strike the shore with a force equal to 6,000 lb. per square foot and average 20 ft. in height; with what force does such a wave strike a wall 100 ft. long and 30 ft. high?

2. This incubator holds 5 dozen eggs. If 80% of them hatch, how many chickens will there be?



3. In a certain total eclipse the sun was completely covered at 2 o'clock 47 min. 23 sec. and reappeared at 2 o'clock 50 min. 48 sec.; what was the length of the total eclipse?

4. At $1.56 per cubic yard, what is the cost of the stone for a wall $1\frac{1}{2}$ ft. thick, built within a cellar 30 ft. by 40 ft. and 7 ft. deep?

5. The edge of a cube is $4\frac{1}{2}$ in.; how many cubic inches does it contain?

6. How many cubic feet are there in a rectangular solid whose edges are $1\frac{1}{5}$ ft., $2\frac{3}{4}$ ft., and $3\frac{1}{3}$ ft.?

7. The dimensions of a box are $14\frac{2}{3}$ in., $12\frac{3}{4}$ in., and $3\frac{7}{11}$ in.; how many cubic inches does it contain?

8. A tank is $28\frac{4}{5}$ in. long., $6\frac{5}{16}$ in. wide, and $8\frac{4}{7}$ in. deep; how many cubic inches does it contain? How many gallons?

9. Find how many cubic yards of masonry there are in a foundation wall in the form of a rectangle whose inside dimensions are 30 ft. by 50 ft. and 9 ft. high, and whose thickness is 18 in. Draw a ground-plan of the wall.

10. If slates cost $12.40 per M and if 4 slates are required per square foot, what is the cost of slates for a roof 54 ft. along the ridge and 20 ft. down the slope?

1. Express in hundredths and thousandths:
84 thousandths. Thus: 8 hundredths, 4 thousandths.
27 thousandths; 43 thousandths; 91 thousandths.

Write decimally:

2. 32 and 28 thousandths.
3. 5 and 139 thousandths.
4. 19 thousandths.
5. 1 and 1 thousandth.

6. From the center of a wagon hub to the outside of the tire is 2.25 ft.; what is the height of the wheel in feet and inches?

7. When the wind blows 15 mi. per hour the power of a windmill is sufficient to grind about $46\frac{1}{2}$ bu. per hour; how many bushels would it grind in 87 days of 10 hr. each?

8. What is the difference between the square roots of 81 and 144?

9. Three brothers owned an apartment building jointly, all having equal shares. One sold $\frac{2}{3}$ of his share to his cousin; what fraction of the building did the cousin own?

10. They sold the building for $18,000; how much did each receive?

11. Find the area of a rectangle in square feet, the length of whose base is $\frac{2}{3}$ ft. and whose altitude is $\frac{3}{4}$ ft.

12. What is the ratio of the population of Tennessee, 2 millions, to that of the United States, 76 millions? Of Texas, 3 millions, to that of the United States?

13. Name a fraction in which the terms have a common divisor.

14. If $\frac{1}{9}$ of wheat flour is water, how much water is there in a barrel of flour (196 lb.)?

15. On Oct. 1 a gas meter read 34,800. On Nov. 1 it read 39,400; how many cubic feet of gas passed through the meter in the interval? What did it cost at $1.20 per M?

1. A screw of pitch 9 is being driven into a board; how many turns are necessary to drive the screw $1\frac{1}{3}$ in. into the board?

2. How many complete tens are there in 35? In 96? In 84? How many complete ten-thousands in 35,000? In 96,000? In 84,325?

3. What is the weight of 10 baskets of coal each weighing 84.7 lb.?

4. A house cost \$2,175. The lot on which it stands cost .2 as much. How many dollars did the lot cost?

5. A farmer sowed 14 qt. of timothy and 14 qt. of clover seed per acre. The yield was 5 tons of hay per acre. How many bushels of seed did he sow on 10 acres? How many tons of hay did he cut on 10 acres?

6. Draw a line 20 in. long on the board. Draw perpendiculars at its end points and on the same side of it .7 as long as the first line. Connect the ends of the perpendiculars. What is the area of the figure formed?

7. From a log 35 ft. long a length of 17 ft. 8 in. was cut; how long was the piece that was left?

8. There are 100 small cartridges in a box; how many are there in $\frac{25}{100}$ of a box? In $\frac{1}{4}$ of a box? In $\frac{75}{100}$ of a box? In $\frac{3}{4}$ of a box?

9. Point out vertical lines about the schoolroom.

10. Point out horizontal lines about the schoolroom.

11. A rectangular desk is 5 ft. 8 in. long and 28 in. wide; how many feet of molding are required to go once around the desk?

12. A letter-carrier walks $2\frac{3}{8}$ mi. on his first trip, $2\frac{9}{8}$ mi. on the second, and $2\frac{7}{8}$ mi. on the third; how far does he walk altogether? Express the result as a mixed number.

13. At \$1.25 per M, what is the cost of 5,000 ft. of gas?

1. How many acres are there in a rectangular piece of land 1 mile wide and 15 miles long?

2. Find the cost of $4\frac{1}{2}$ lb. of steak at 18¢ per pound; 3 doz. cookies at 12¢ a dozen; $2\frac{3}{8}$ lb. of butter at 24¢ per pound.

3. The expenses of conducting the Library of Congress in 1902 were: $384,185 for cost of service, $91,300 for books, $45,000 for furniture, $95,000 for printing and binding. What were the total expenses?

4. If the top hoop of a barrel has a radius of 8.75 in., what is the greatest width of the top of the barrel?

5. Find the area of each triangle into which a diagonal divides a parallelogram of base 7.5 ft. and altitude 3.75 ft.

6. How is the area of a right-angled triangle found from its perpendicular sides?

7. What is a diagonal of a parallelogram? How many diagonals has such a figure?

8. What is the cost of 500 ft. of joists at $22 per M?

9. If x denotes the distance from Boston to a certain town on the line between Boston and Cleveland (682 mi. apart), what denotes the distance from Cleveland?

10. A bill for nails amounted to $276. The price of the nails was 2.3¢ per pound; how many pounds of nails were bought? How many tons?

11. At the rate of 32 mi. per hour, how long does it take a train to travel from New York to New Orleans, 1,344 miles?

Fill out the blanks:

	12.	13.	14.	15.	16.	17.
Dividend	14.4	21.6	25.6	5.12	5.12	——
Divisor	.6	3.6	——	1.28	12.8	3.2
Quotient	——	——	6.4	——	——	1.4

1. At 1.9¢ per mile, the fare from New York to Chicago by a certain route would be $17.35; find the distance.

2. How many acres are there in a square mile? How many acres are there in a section? In a half-section? In a quarter-section?

3. How many rods of fence are necessary to enclose a quarter-section?

4. How many feet of 6-inch boards are required to make a fence 4 boards high around a quarter-section?

5. At how many pounds is a 90-lb. package rated when sent by freight?

6. What is the cost of a postal money-order for $15? For $1? For $10?

7. How many steps $\frac{5}{6}$ of a yard long does a man take in walking 5 mi.? (1 mi. = 1,760 yd.)

8. A man wishes to plant a hedge about his lot which is 100 ft. by 200 ft. The hedge is set in 1 foot from the boundary of the lot. His plants cost $5 a hundred and are set out 3 per foot. He pays a man 2 cents apiece for setting out the plants. What does it cost him to plant the hedge?

9. Draw a rectangle of length 10 in. and width 5 in. Shade 60% of it; 95% of it.

Subtract:

10.	11.	12.	13.	14.
$7\frac{1}{12}$	$21\frac{3}{5}$	$5\frac{1}{5}$	$14\frac{7}{9}$	$41\frac{5}{18}$
$\frac{7}{18}$	$18\frac{1}{10}$	$3\frac{1}{6}$	$3\frac{3}{4}$	$12\frac{1}{24}$

15. From a rectangular piece of cloth 8 in. wide and $7\frac{1}{2}$ in. long are cut without waste 2 rectangular pieces 2 in. wide and $7\frac{1}{2}$ in. long, one rectangular piece $2\frac{1}{2}$ in. wide and 4 in. long, one right-angled triangle with base 4 in. and altitude 4 in., and one trapezoid with bases 1 in. and 5 in. and altitude — in. What is the area of the trapezoid?

1. What is meant by a power of a number? Illustrate the second power.

2. Find the area of a trapezoid of bases 8 yd. and 6 yd. and of altitude 5 ft. In what unit do you express these lengths before making the computation?

3. The readings of the thermometer for 3 days were: 68°, $74\frac{1}{2}°$, $72\frac{1}{2}°$. Find the average temperature for the 3 days.

4. The total liabilities of all the houses failing in 1902 were \$45,376,472, $\frac{1}{8}$ of which belonged to the grocers; how many dollars were their liabilities?

5. What is the average rainfall for San Diego for the six months given in the table?

Place.	Dec.	Jan.	Feb.	Mar.	Apr.	May.
San Diego, Cal.....	2.34	1.96	2.36	1.47	0.87	0.36
San Francisco.....	5.11	4.98	3.72	3.28	2.14	0.68
Sitka, Alaska.....	10.11	9.75	10.51	10.02	6.24	4.94
Salt Lake City.....	1.59	1.52	1.38	1.92	2.36	1.78
New York.....	3.33	3.89	3.76	4.10	3.40	3.05

6. Make 5 problems about the table above.

7. When 100 yd. of net cost \$14, what is the cost of 25 yd.? What is the cost of 25 yd. at \$7 per 100? What is the cost of 75 yd. at \$16 per 100?

8. If buttonholes $\frac{3}{4}$ in. long are set $\frac{1}{4}$ in. from one edge of a pleat and $\frac{3}{8}$ of an inch from the other, how wide is the pleat?

9. A man bought 125 acres of land at \$75 an acre. He sold $\frac{3}{5}$ at \$80 an acre and $\frac{2}{5}$ at \$85; how much did he gain?

10. A train goes from New York to Tacoma, Washington (3,169 mi.), in 93.5 hours; find the average speed per hour correct to the nearest mile.

DENOMINATE NUMBERS

Tables

Linear

12 inches (in.)	= 1 foot (ft.)
3 feet (ft.)	= 1 yard (1 yd.)
16.5 feet	= 1 rod (rd.)
320 rods (rd.)	= 1 mile (mi.)
1760 yards (yd.)	= 1 mile
5280 feet	= 1 mile
39.37 inches	= 1 meter

Time

60 minutes (min.)	= 1 hour (hr.)
24 hours (hr.)	= 1 day (da.)
7 days (da.)	= 1 week (wk.)
365 days or	
12 months (mo.)	= 1 year (yr.)
10 years	= 1 decade
10 decades	= 1 century

Liquid

2 pints (pt.)	= 1 quart (qt.)
4 quarts (qt.)	= 1 gallon (gal.)
231 cu. in.	= 1 gal.

Counting

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross

Dry

2 pints (pt.)	= 1 quart (qt.)
8 quarts (qt.)	= 1 peck (pk.)
4 pecks (pk.)	= 1 bushel (bu.)
32 quarts	= 1 bu.
2150.4 cu. in.	= 1 bu.

Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds (lb.)	= 1 hundredweigh
2000 pounds	= 1 ton (t.)
2240 pounds	= 1 gross ton
7000 grains	= 1 pound

Value

10 cents (¢)	= 1 dime
10 dimes	= 1 dollar
100 cents	= 1 dollar ($)

Angle

90°	= 1 right angle (rt. ∠)
180°	= 1 straight angle (st. ∠)
360°	= 1 circumference

Square

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet (sq. ft.)	= 1 square yard (sq. yd.)
30¼ sq. yd.	= 1 square rod (sq. rd.)
160 sq. rd.	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
36 sq. mi.	= a township

Cubic

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.)	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord

Troy Weight.

20 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweight	= 1 ounce
12 ounces	= 1 pound (5,760 gr.)