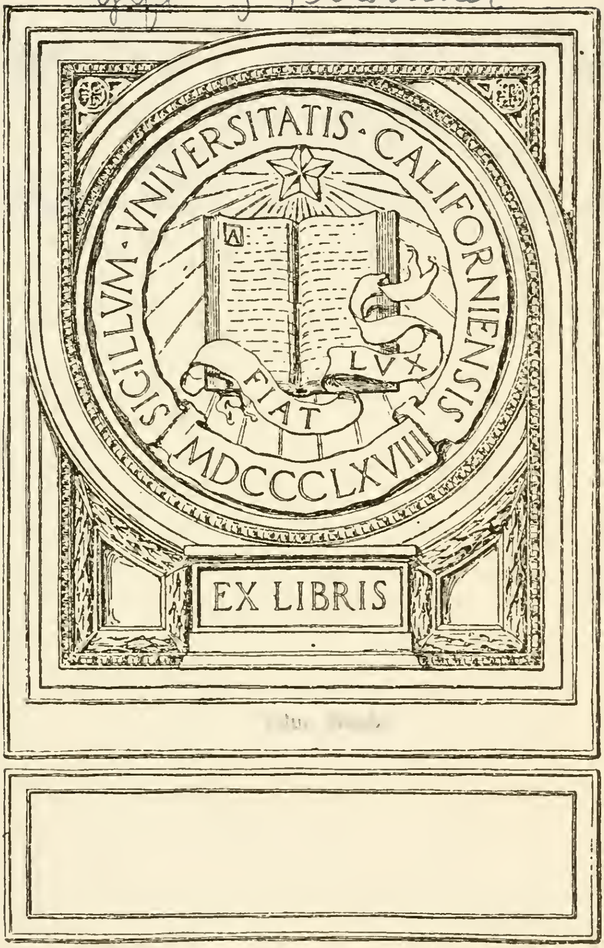
ARITHMETIC

EUGENE HERZ
AND
MARY G. BRANTS

PARTS VII
AND VIII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ARITHMETIC

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PARTS VII AND VIII

ADVANCED LESSONS

THE JOHN C. WINSTON COMPANY

CHICAGO PHILADELPHIA TORONTO

FOREWORD

Bearing in mind that a thorough knowledge of arithmetic is perhaps more frequently the cause of success in life than is any other single factor, one can hardly overestimate the importance of this subject to the future welfare of the child, nor can one fail to realize how great is the responsibility which rests on those whose duty it is to provide for his education in this branch.

No book or series of books can possibly illustrate every use to which numbers can be put, but if the principles underlying their use are properly taught, the child can reason for himself the proper application of his knowledge to any given problem. Furthermore, as he must know not merely how to solve a problem, but how to solve it in the quickest and simplest manner, he must know not merely the various processes, but their construction as well; he must be able to analyze to such an extent that when a problem is presented to him, he can distinguish the facts which are relevant from those which are irrelevant, he can separate the known from the unknown, he can arrange the known in logical order for his processes, and he can use the shortest processes possible. An attempt to give the pupil this ability is the motive for this work.

The vehicle used to obtain the result is a series of progressive lessons, which, with ample practice, take the pupil step by step through the construction of each process to be learned, thus giving him the opportunity of following the teacher's explanation, and of referring to past lessons at any time. In this way the pupil who is slower to grasp new ideas than the average can keep up with his class, and every pupil can at all times refresh his memory on any points which he may have forgotten or which may have escaped him in the classroom, and which have so often been lost to him forever.

The time-saving methods used by the most expert arithmeticians are introduced as part of the routine work; thus, these become a part of the child's general education without any special effort on his part.

It is not intended that the lessons or definitions are to be learned verbatim, any more than it is intended that the examples given are to be memorized; both are there for the purpose of showing the pupil the reason for, and the application of, the processes, and the exercises are there to give him practice and to test his knowledge of what he has learned.

The exercises are prepared in such manner that they form an automatic and continuous review of what has been learned, but further review work is given at regular intervals.

The series consists of Three Books and Teacher's Manuals, as follows:

Primary Lessons.......	Parts I and II. (Teacher's Manual only.)
Elementary Lessons...	Parts III and IV. (With Manual for the Teacher.)
Intermediate Lessons...	Parts V and VI. (With Manual for the Teacher.)
Advanced Lessons.....	Parts VII and VIII. (With Manual for the Teacher.)

The first two parts are so arranged in the Teacher's Manual that the lessons and exercises can be given largely as games, play work, number stories, in language work, etc., all used more or less incidentally, till the child is gradually prepared for work requiring an increasing degree of conscious effort.

The work contained in each of the eight parts is that which is usually taught in the corresponding grade, and it is recommended that this routine be followed. However, special provision has been made for such variations in the grading as are required in some localities, by means of a series of notes in the Teacher's Manuals which enable the teacher to follow either method with equal facility.

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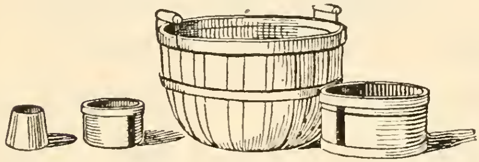
Tables of Weights and Measures

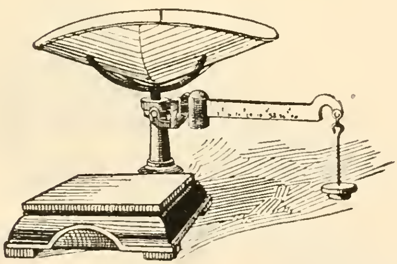
(For Ready Reference)

Dry Measure



2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

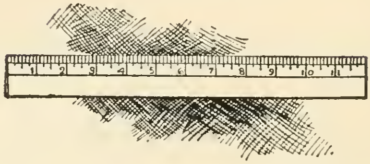
Avoirdupois Weight



16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T.)
2,000 pounds	= 1 short ton
2,240 pounds	= 1 long ton (used at mines and U. S. Custom House)

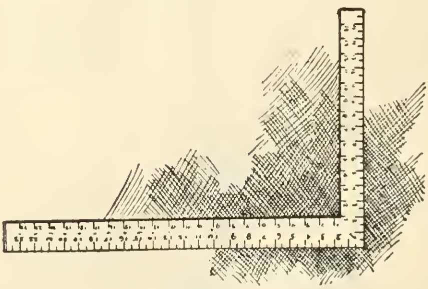
TABLES OF WEIGHTS AND MEASURES

Linear Measure



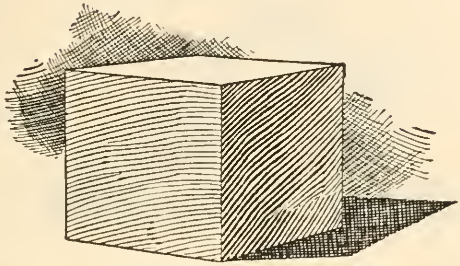
12	inches (in.)	= 1 foot (ft.)
3	feet	= 1 yard (yd.)
$5\frac{1}{2}$	yards	= 1 rod (rd.)
320	rods	= 1 mile (mi.)
1,760	yards	= 1 mile
5,280	feet	= 1 mile
6	feet	= 1 fathom (used in measuring the depth of water)

Square Measure



144	square inches (sq. in.) . .	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)
640	acres	= 1 section (sec.)
100	square feet	= 1 square (sq.)

Cubic Measure.



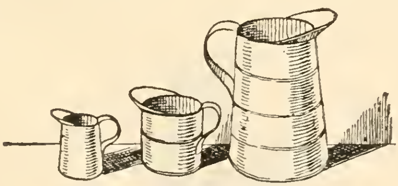
1,728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)
128 cubic feet	=	1 cord (cd.)

1 gallon contains 231 cubic inches.

1 bushel contains 2,150.42 cubic inches or $1\frac{1}{4}$ cu. ft. (nearly).

1 cubic foot of water contains $7\frac{1}{2}$ gallons and weighs $62\frac{1}{2}$ pounds.

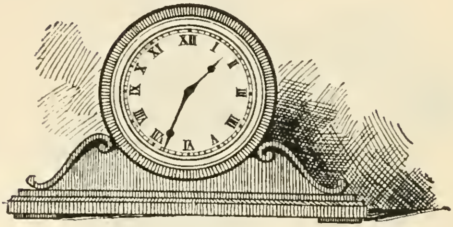
Liquid Measure



4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)
$31\frac{1}{2}$ gallons	=	1 barrel (bbl.)
2 barrels	=	1 hogshead (hhd.)

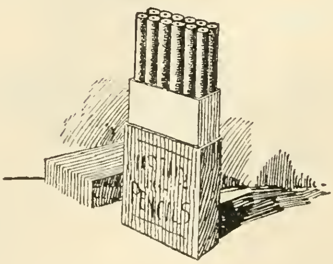
TABLES OF WEIGHTS AND MEASURES

Time Measure



60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
28, 29, 30, 31 days	= 1 month (mo.)
12 months	= 1 year (yr.)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

Table Used in Counting Merchandise



12 things	= 1 dozen (doz.)
12 dozen	= 1 gross (gr.)
12 gross	= 1 great gross (gt. gr.)

ARITHMETIC

PART VII

ADVANCED LESSONS

ADVANCED LESSONS

PART VII

DENOMINATE NUMBERS

Lesson 1

Reduction

A "denominate number" is a number which is used with the name of a measure, as 7 yd.; 5 hours; $2.; 40 min.

Look at the door. Name a denominate number. Name three more.

A denominate number of only one denomination, as 8 yd., is a "simple denominate number."

Look at the window pane. Name a simple denominate number.

A denominate number of more than one denomination, as 2 yd. 5 ft., is a "compound denominate number."

Look at the clock and name such a number. Find another in the room.

Changing the denomination of a number without changing its value is called "reduction."

6 ft. = ? in.; 2 yd. = ? ft.; 24 in. = ? ft.; 36 in. = ? yd.; 1 ft. = ? part of a yd.;

Exercise 1—Oral.

Reduce:

1. 4 yd. to ft.;	**3.** 7 ft. to in.;
2. $2\frac{1}{2}$ lb. to oz.;	**4.** 2 hr. to min.;

5. 3 mo. to da.;
6. 72 in. to yd.;
7. 9 yd. to ft.;
8. 48 oz. to lb.;
9. 120 sec. to min.;
10. 72 in. to ft.;
11. 6 in. to ft.;
12. 8 oz. to lb.;
13. 3 pk. to bu.;
14. 7 pt. to qt.;
15. 80 min. to hr.;
16. 10 hr. to min.

EXAMPLE: Reduce 4 yd. 2 ft. 6 in. to inches.

3 ft. × 4 = 12 ft.;
12 ft. + 2 ft. = 14 ft.;
12 in. × 14 = 168 in.;
168 in. + 6 in. = 174 in., Ans.

Steps

(1) Reduce yards to feet and add feet, if any.
(2) Reduce feet to inches and add inches, if any.

Prove examples like this by approximation:
4 yd. 2 ft. 6 in. = almost 5 yd.; 5 yd. = (36″ × 5) 180 in.
The answer is 6 in. less, or 174 in.

To reduce a compound denominate number to a smaller denomination, begin with the largest denomination and reduce it to the next smaller denomination, then to the next smaller, and so on.

EXAMPLE: Reduce 6 da. 7 min. to minutes.

24 hr. × 6 = 144 hr.;
60 min. × 144 = 8,640 min;
8,640 min. + 7 min. = 8,647 min., Ans.

Steps

(1) Reduce da. to hr. and add hr., if any.
(2) Reduce hr. to min. and add min., if any.

When a denomination is skipped reduce one step at a time as before, using the skipped one in its proper turn.

Exercise 2—Written.

Reduce:

1. 5 yd. 2 ft. to ft.;
2. 5 yd. 2 ft. 8 in. to in.;
3. 3 qt. 1 pt. to pt.;
4. 4 bu. 3 pk. 5 qt. to qt.;
5. 3 pk. 1 pt. to pt.;
6. 5 da. 6 hr. 20 min. to min.;
7. 4 lb. 6 oz. to oz.;
8. 7 ft. 8 in. to in.;
9. 8 yr. 2 mo. to mo.;
10. 1 bu. 2 pk. 2 qt. to qt.;
11. 2 yd. 1 ft. 6 in. to in.;
12. 3 gal. 2 qt. to qt.

EXAMPLE: Reduce 174 in. to larger denominations.

12) 174 (no. of in.)
3) 14 (no. of ft.) + 6 in.
4 (no. of yd.) + 2 ft.
4 yd. 2 ft. 6 in., Ans.

Steps

(1) Reduce in. to ft. saving remainder, if any.
(2) Reduce ft. to yd. saving remainder, if any.
(3) Write last quotient plus all remainders.

To reduce a simple denominate number to larger denominations, reduce to the next larger denomination, then to the next larger, and so on, saving all the remainders. The last quotient plus all the remainders is the answer.

Exercise 3—Written.

Reduce:

1. 12,600 sec. to hr. and min.;
2. 6,500 min. to largest denominations;
3. 762 in. to largest denominations;
4. 51 pt. (liquid) to largest denominations;
5. 51 pt. (dry) to largest denominations;
6. 202 qt. to largest denominations;

7. 195 min. to largest denominations;
8. 8,647 min. to largest denominations;
9. 246 oz. to largest denominations;
10. 8,966 sq. in. to largest denominations.

Lesson 2

Reduction of Fractional Denominate Numbers

> **EXAMPLE:** Reduce $\frac{1}{6}$ yd. to inches.
> $\frac{1}{6}$ yd. = $\frac{1}{6}$ of 3 ft., which is $\frac{3}{6}$ or $\frac{1}{2}$ ft.;
> $\frac{1}{2}$ ft. = $\frac{1}{2}$ of 12 in., or 6 in., Ans.
> Also: $\frac{1}{6}$ yd. = $\frac{1}{6}$ of 36 in., or 6 in., Ans.

Reduce the fraction of the given denomination to the next smaller denomination; then to the next smaller again, until the required denomination is reached.

Exercise 4—Oral.

1. $\frac{1}{3}$ da. = ? hr.;
2. $\frac{1}{3}$ hr. = ? min.;
3. $\frac{3}{4}$ bu. = ? pk.;
4. $\frac{1}{2}$ pk. = ? qt.;
5. $\frac{1}{4}$ gal. = ? pt.;
6. $\frac{1}{8}$ bu. = ? qt.;
7. $\frac{3}{8}$ lb. = ? oz.;
8. $\frac{7}{8}$ da. = ? hr.
9. Can days be changed to hours?
10. Can $\frac{1}{2}$ da. be changed to hours?
11. Can bushels be changed to pecks?
12. Can $\frac{1}{4}$ bu. be changed to pecks?
13. Can 3 pk. be changed to a bu.?
14. How many pk. make a bu.?
15. Will 3 pk. make a part of a bu.?
16. What part of a bu. will 3 pk. make?

When any denominate number or part of a denominate number does not make a whole unit of the next larger denomination, find what part it does make and use that fraction.

Exercise 5—Oral.

1. 6 in. = ? part of 1 ft.;
2. 8 oz. = ? part of 1 lb.;
3. 2 ft. = ? part of 1 yd.;
4. 6 doz. = ? part of 1 gr.;
5. 12 hr. = ? part of 1 da.;
6. Change 15 da. to mo.;
7. Change 5 da. to mo.;
8. Change 3 pt. to gal.;
9. Change 3 pk. to bu.;
10. Change 10 oz. to lb.
11. Can you identify these as quickly as a mailman does his letters? Try.

8 (Say rapidly "8 qt. = 1 pk.")

4;	9;	5,280;
24;	2;	366;
$5\frac{1}{2}$;	3;	1,760;
160;	$30\frac{1}{4}$;	128;
100;	1,728;	16;
640;	60;	144;
27;	320;	7;
2;	$31\frac{1}{2}$;	2,240;
365;	63;	12;
30;	2,000;	20.

Exercise 6—Written.

Reduce:

1. 4 ft. 6 in. to feet;
2. 3 pk. 1 qt. to pecks;
3. 3 pk. 1 pt. to pecks;
4. 4 hr. 30 min. to hours;

5. 4 da. 6 hr. to days;
6. 12 ft. 3 in. to feet;
7. 5 yd. 2 ft. to yards;
8. 6 da. 10 hr. to days;
9. 8 ft. 3 in. to feet;
10. 3 bu. 3 pk. to bushels;
11. 9 in. to yards;
12. 1 pt. to pecks;
13. 5 gal. 1 pt. to gallons;
14. 2 ft. 4 in. to yards;
15. 3 qt. 1 pt. to gallons;
16. 30 min. 45 sec. to hours;
17. 7 sq. ft. 72 sq. in. to square yards;
18. $\frac{3}{16}$ pk. to smaller denominations;
19. $\frac{4}{9}$ yd. to smaller denominations;
20. $\frac{1}{16}$ gt. gr. to smaller denominations.

Lesson 3

Addition of Denominate Numbers

EXAMPLE:

wk.	da.	hr.
4	6	8
5	3	12
–	4	7
15	–	4
26	0	7

The sum of the hour column is 31 hr. which is reduced to 1 da. 7 hr.; the 7 hr. are written in the hour column and the 1 da. is carried to the day column; the sum of the day column (including the 1 da. carried from the hour column) is 14 da. which is reduced to 2 wk. 0 da.; the 0 da. are written in the day column and the 2 wk. are carried to the week column; the sum of the week column (including the 2 wk. carried from the day column) is 26 wk., which is written in the week column.

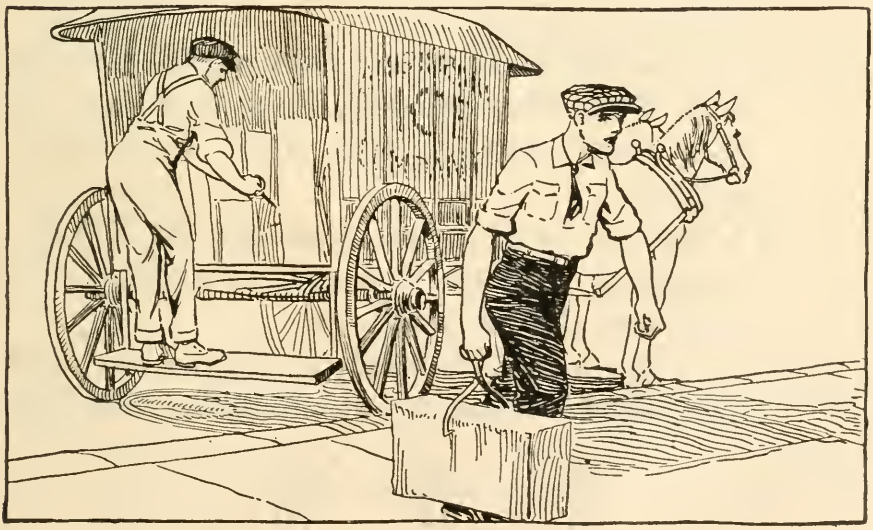
In adding denominate numbers, a separate column must be used for each denomination to be added, as we must always bear in mind that unlike numbers, quantities or things cannot be added.

After finding the sum of the addends of the smallest denomination, the sum must be reduced so that all units of a larger denomination which are contained in it may be carried and added to the addends of the larger denomination.

Exercise 7—Written.

Add and prove:

The Iceman's Problems



1. To fill a large refrigerator, 4 loads of ice were necessary; how much did it cost to fill this refrigerator if ice is worth 30¢ per 100 lb. and the four loads weighed as follows:

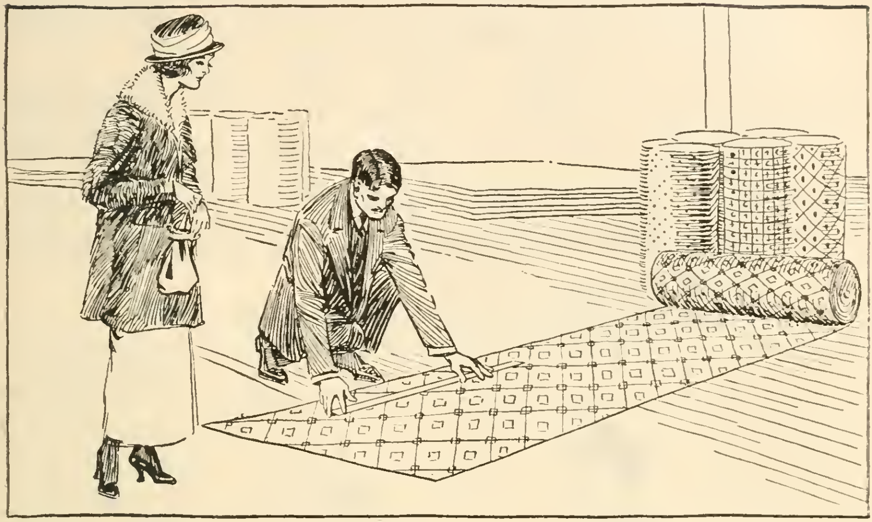
T.	cwt.	lb.
2	6	25
2	4	50
2	8	75
1	3	30
1	4	20

Can the iceman save time in finding his total delivery by adding groups? Try it. First add all the lb., then all the cwt., then the T. Now express the sum in cwt. because the price is 30¢ per cwt. Now make out the bill.

2. If he delivered 3 cwt. 50 lb.; 2 cwt. 25 lb.; and 6 cwt. 25 lb.; what is the value of the whole delivery at 25¢ per cwt.?
3. If he received 20 T. on one delivery, 17 T. 6 cwt. 50 lb. on another delivery, and 2 T. 3 cwt. 5 lb. on a third delivery, how many cwt. did he receive?
4. If he packed 14 T. 5 cwt. in one car, 20 T. 6 cwt. in another, and 12 T. 4 cwt. in another, how many tons did he pack?
5. If he sold ice to restaurants at 25¢ per cwt., how much did he collect for the following deliveries:

T.	cwt.	lb.
1	18	70
1	14	80
1	12	40
2	1	50
—	2	10

The Merchant's Problems



6. What is the cost of the oil cloth needed to cover 4 halls, if a square yard costs $1.44 and the area of the halls is as follows:

sq. yd.	sq. ft.
6	8
10	8
12	7
4	7

7. A wholesale stationer received 3 orders for pencils; if he sold the pencils for 1¢ each and the orders were for the following quantities, how much did he receive for all the pencils?

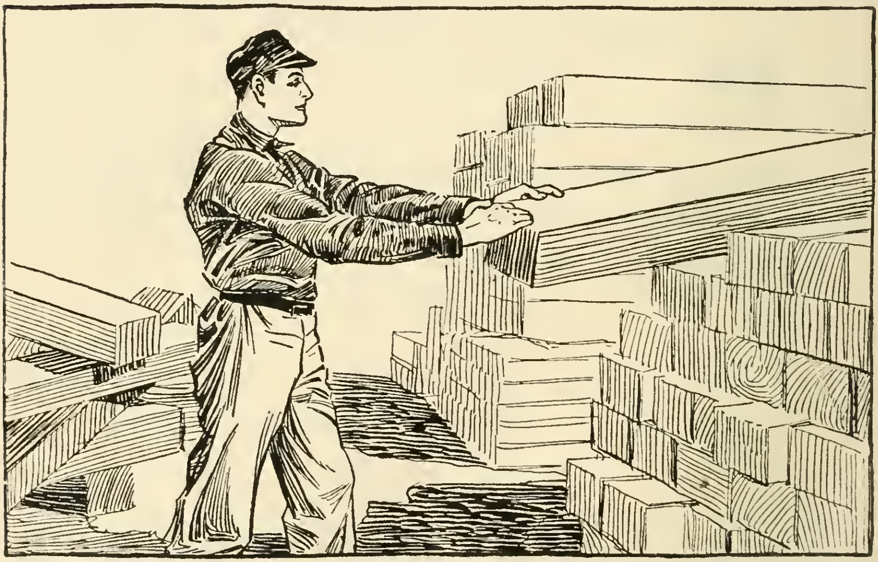
gr.	doz.
4	6
8	2
6	8

8. If one bolt of ribbon has 6 yd. 2 ft. left on it, another bolt has 8 yd. 1 ft., another bolt has 2 yd. 1 ft., how many yards of ribbon are there? What is the total value at 60¢ a yard?

9. If $1\frac{1}{3}$ yd. of ribbon is added to a bolt containing 4 yd. 2 ft. 6 in., how many yards are there in all?

10. If one strip of carpet measures 5 yd. 2 ft. 7 in. and another measures 4 yd. 2 ft. 9 in., how many yards are there in all?

The Laborer's Problems



11. A laborer working for 40¢ an hour would receive how much money for doing these 3 pieces of work if 8 hours is considered a day's work:

da.	hr.	min.
4	6	45
8	4	30
7	3	15

12. A laborer working at 40¢ an hour, worked as follows during three weeks; how much did he earn if 9 hours is considered a day's work:

da.	hr.	min.
4	6	30
5	7	15
3	4	—

Lesson 4

Subtraction of Denominate Numbers

EXAMPLE:

yd.	ft.	in.
4	1	18
4	2	6
1	1	10
3	0	8

As 10 in. cannot be subtracted from 6 in., 1 of the 2 ft. must be changed to 12 in. and added to the 6 in., making 18 in.; 18 in. − 10 in. = 8 in.; 1 ft. − 1 ft. = 0 ft.; 4 yd. − 1 yd. = 3 yd.

As in addition, a separate column must be used for each denomination to be subtracted.

When the subtrahend of any denomination is larger than the minuend of that denomination, 1 unit of the next larger denomination of the minuend must be changed into units of the smaller denomination and added to the other units of that denomination before subtracting.

Exercise 8—Written.

Subtract and prove:

1.

hhd.	bbl.	gal.
3	1	18
1	1	20

2.

mi.	rd.	yd.
5	240	4
2	300	5

3.

gal.	qt.	pt.
12	2	—
5	3	1

4.

bu.	pk.	qt.
18	1	4
6	3	6

5. If a piece of cloth containing 4 sq. yd. 2 sq. ft. 72 sq. in. is removed from a bolt containing 25 sq. yd., how much material remains on the bolt? What is it worth at $3.60 per sq. yd.?

6. From a box containing 1 gt. gr. of buttons, 8 gr. $4\frac{1}{3}$ doz. are sold; how many buttons remain? What are they worth at 36¢ per gross?

7. A number of men were digging a trench a mile long; how many yards had they still to dig when 148 rd. had been completed?

8. A train required 20 hr. 45 min. to run from Chicago to New York; how long had it still to travel at 9.30 P. M. if it left Chicago at 12.40 P. M.

LESSON 5

Multiplication of Denominate Numbers

EXAMPLE: 5 yd. 2 ft. 7 in. × 6 = ?

yd.	ft.	in.
5	2	7
		6
35	0	6

The product of 7 in. × 6 is 42 in.; 42 in. is reduced to 3 ft. 6 in.; write 6 in. in the product and carry 3 ft.; the product of 2 ft. × 6 is 12 ft.; 12 ft. + 3 ft. (carried) = 15 ft.; 15 ft. is reduced to 5 yd. 0 ft.; write 0 ft. in the product and carry 5 yd.; the product of 5 yd. × 6 is 30 yd.; 30 yd. + 5 yd. (carried) = 35 yd.; write 35 yd. in the product.

The product of the smallest denomination must be reduced so that all units of a larger denomination which may be contained in it can be carried and added to the product found by multiplying the larger denomination.

Exercise 9—Written.

Multiply and prove:

1.

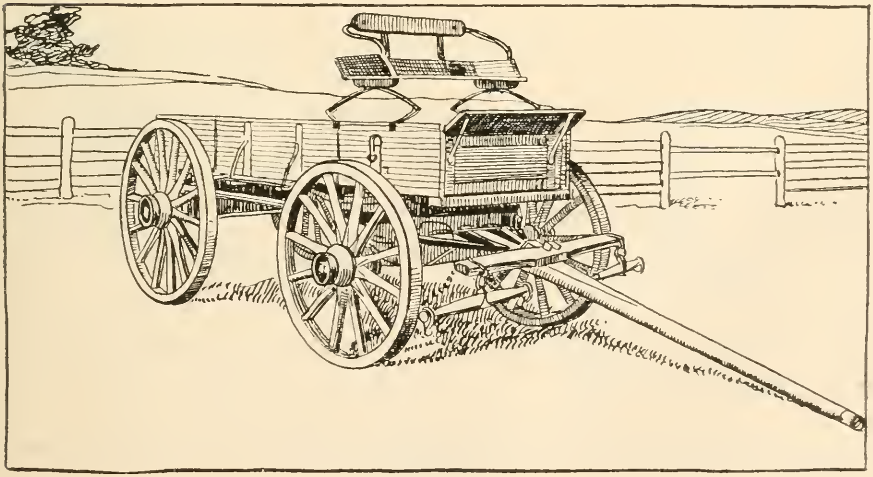
da.	hr.	min.
12	10	45
		20

2.

hhd.	bbl.	gal.
4	1	7
		9

3. What is the cost of laying carpet in four rooms at $3.00 per square yard, if the area of each room is 20 sq. yd. 4 sq. ft. 18 sq. in.? (Suggestion: Find total area first.)

4. If the circumference of a wagon wheel is 3 yd. 1 ft. 6 in., how far has the wagon traveled when the wheel has turned 1,000 times?



5. What is the total length of 9 pieces of rope, if each piece measures 9 rd. 3 yd. 2 ft.? What is the rope worth @ 2¢ per yard?
6. Find the volume of 40 blocks of granite, the volume of each block being 1 cu. yd. 24 cu. ft. 144 cu. in.

Lesson 6

Division of Denominate Numbers

EXAMPLE: (Short Division) 5 hr. 40 min. 40 sec. ÷ 2 = ?

```
    hr.  min.  sec.
     2    50    20
   ________________
2) 5    40    40
```

5 hr. ÷ 2 = 2 hr. and 1 hr. remaining to be changed to minutes which are added to the other 40 min. of the dividend; 60 + 40 = 100 min.; 100 min. ÷ 2 = 50 min. and no minutes remaining to be changed to sec.; 40 sec. ÷ 2 = 20 sec.

EXAMPLE: (Long Division) 500 A. 83 sq. rd. 8 sq. yd. ÷ 45 = ?

```
        A.  sq. rd.  sq. yd.
        11     19      19
     ________________________
45)   500     83       8
       45
       --
        50
        45
        --
Rem.     5 = 800
             ---
             883
              45
             ---
             433
             405
             ---
     Rem.     28 = 847
                   ---
                   855
                    45
                   ---
                   405
                   405
                   ---
```

500 A. ÷ 45 = 11 A. and 5 A. remaining to be changed into 800 sq. rd. which are added to the 83 sq. rd., making 883 sq. rd.; 883 sq. rd. ÷ 45 = 19 sq. rd. and 28 sq. rd. remaining to be changed into 847 sq. yd. which are added to the 8 sq. yd., making 855 sq. yd.; 855 sq. yd. ÷ 45 = 19 sq. yd.

In dividing to find one of the equal parts of a compound denominate number, we divide the largest denomination first, and any remainder from this denomination is changed into units of the next smaller denomination and these are added to the other units of that denomination before dividing it. The remainder from the smallest denomination is written in the form of a fraction as in ordinary division.

EXAMPLE: How many ribbon bows can be made from 10 yd. 2 ft. 6 in. of ribbon, if 2 ft. 6 in. are used for each bow?

10 yd. 2 ft. 6 in. = 390 in.;
2 ft. 6 in. = 30 in.;
390 in. ÷ 30 in. = 13 (number of bows), Ans.

In dividing to find how many times one compound denominate number is contained in another compound denominate number, we first reduce both compound denominate numbers to simple denominate numbers of the same denomination, then we divide in the usual way.

Exercise 10—Oral.

A test for you. Study to answer these promptly and in good English.

1. In adding and subtracting denominate numbers, why do we use a separate column for each denomination?
2. In adding denominate numbers, what is done with the sum of the first column before finding the sum of the next column?

3. Explain just what you would do to subtract 13 cwt. from 1 T. 2 cwt.
4. Tell how to multiply 4 yd. 2 ft. 6 in. by 10.
5. Tell how to reduce 2,400 ft. to rods.
6. Tell how to divide 12 lb. 8 oz. by 10.
7. How do we change inches to feet? Feet to yards? Yards to rods? Rods to miles?
8. How do we change square inches to square feet? Square feet to square yards? Square yards to square rods? Square rods to acres? Acres to square miles?
9. How do we change cubic yards to cubic feet? Cubic feet to cubic inches?
10. How do we change hogsheads to barrels? Barrels to gallons? Gallons to quarts? Quarts to pints? Pints to gills?
11. How would you find how many times 1 hr. 20 min. is contained in 5 hr. 20 min.?

Exercise 11—Written.

Solve and prove:

1. 263 da. 23 hr. 30 min. ÷ 6 = ?
2. 150 mi. 6 rd. 3 yd. ÷ 12 = ?
3. 406 cu. yd. 23 cu. ft. 50 cu. in. ÷ 10 = ?
4. 10 sq. yd. 8 sq. ft. 96 sq. in. ÷ 48 = ?
5. 341 T. ÷ 40 = ?
6. 41 hhd. 1 bbl. ÷ 9 = ?
7. A trench 1 mile long is divided into 10 equal sections; how many yards long is each section?
8. The perimeter or distance around a square is 19 yd. 1 ft.; what is the length of one of the sides? What is the area in sq. yd.?

9. What is the average weight of each of the following 5 machines:
   The first weighs 1 T. 4 cwt. 78 lb.;
   The second weighs 9 cwt. 50 lb.;
   The third weighs 2 T.;
   The fourth weighs 2 T. 3 cwt.;
   The fifth weighs 2 T. 1 cwt. 22 lb.

Exercise 12—Oral.

1. What table of measures is used for measuring distance? Height? Length? Be ready to write it on the board quickly.
2. What table of measures is used for weighing all common articles such as coal, meat, sugar, etc.? Say this table.
3. What table of measures is used for measuring liquids such as water, milk, oil, etc.? Be ready to say the table.
4. What table of measures is used for measuring the quantity of vegetables and grains such as oats, wheat, potatoes, etc.? Be ready to write it.
5. What table of measures is used for counting articles such as buttons, eggs, etc.? Write all the numbers and let your classmates identify them. Be ready.
6. What table of measures is used for measuring time? Say this table.
7. Name the table used for measuring the area of surfaces. Do you remember this table?
8. What table of measures is used for measuring the volume of solids? Be ready to write it.

9. How many and what dimensions has a line? A surface? A solid?
10. What is the size of a board foot? What is the volume of a board foot?
11. Draw a board foot; a cubic foot; a square foot; a foot.
12. Can you identify each quickly?

Exercise 13—Written.

1. How many pieces of tile 6″ × 6″ will be needed to cover a floor 12′ × 18′? Can you use cancellation here? Try $12 \times 18 \div \frac{1}{2} \div \frac{1}{2}$.
2. In a certain orchard there are 400 trees planted in 20 rows of 20 trees each; if the area of this orchard is 10 A., how much area is allowed for each tree?
3. If $3\frac{1}{3}$ yd. of brass tubing weigh 1 lb., how many feet of tubing weigh 1 oz.?
4. How many 36 ft. lengths of wire are needed to build a wire fence 6 wires high around a field 16 rd. × 10 rd.?
5. How many times can a $2\frac{1}{2}$ gal. pail be filled from a tank containing 600 pints?
6. What is the value of 4 gal. 3 qt. 1 pt. of gasolene @ 24¢ per gallon?
7. What decimal fraction of a great gross is equal to 1 gr. 6 doz.?
8. At $5.00 per dozen, what is the cost of 4 gt. gr. 5 gr. of books?
9. A grocer bought 50 bu. potatoes @ 60¢ per bu., and sold $\frac{1}{2}$ of them @ 20¢ per pk. and the other

half @ 25¢ per pk.; what was the grocer's profit? What was the per cent of profit?

10. A grocer invests $100.00 in apples which he sells at 28% profit; if he sells the apples at 32¢ per pk., how many bushels did he buy? What did he pay for each bushel?

11. A family using 12 T. of coal during the winter bought 17,000 lb. @ $5.00 per T. and the balance @ $6.00 per T.; what was the total cost of the coal? How much could have been saved by buying the entire supply at $5.25 per T.?

12. If an acre planted in hay yields $1\frac{1}{2}$ T. when hay is worth $12.00 per T., while an acre planted in oats yields 30 bu. when oats is worth 50¢ per bu., is it more profitable to raise hay or oats? If you had a square mile of land what could you gain by raising the more profitable crop?

13. If a printing press can print 7,500 completed magazines in 1 hr. of continuous printing, how many magazines can it print in 6 wk. of 48 hr. each, allowing 1 hr. out of each 8 hr. day for necessary delays on account of oiling, changing paper, etc.?

14. If it takes a man 1 hr. 10 min. to make a certain tool, how many such tools could he make in 46 hr. 40 min. of work? Compare carefully.

15. A cubic foot of water weighs 62.5 lb.; what is the weight of the water which would completely fill a tank 6 ft. long, 2 ft. wide, and 18 in. deep?

16. How many 1 lb. 8 oz. packages of sugar can be filled from a barrel containing 4 cwt. 20 lb.?

17. In a bed of pansies, each plant is given 36 sq. in. of ground space; how many plants can be planted in 5 sq. yd. 8 sq. ft. 108 sq. in.?

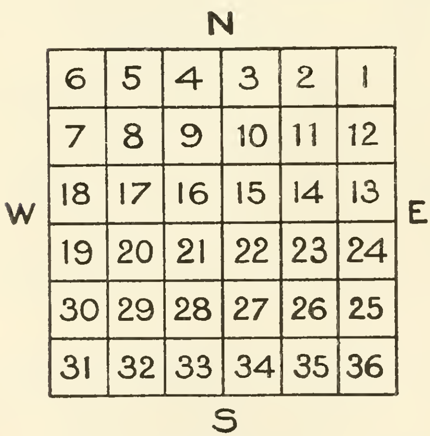
18. How many jars of jam each containing 2 qt. 1 pt. can be filled from a pail containing 20 gal.?

19. A carpenter laying flooring requires 4 hr. 15 min. to lay 6 sq. yd. 6 sq. ft.; how long will it take him to lay the floor in a room 4 yd. long and 3 yd. 1 ft. wide?

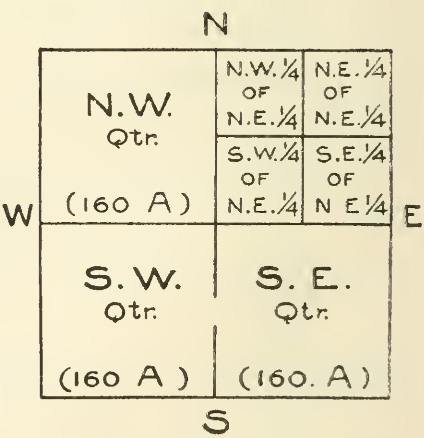
20. A carpenter laying flooring requires 4 hr. 15 min. to lay 6 sq. yd. 6 sq. ft.; how much flooring can he lay in 25 hr. 30 min.?

Lesson 7

Measuring Land



(Figure 1)
1 Township.
36 Sections.
36 Square Miles.



(Figure 2)
1 Section.
1 Square Mile.
640 Acres.

Figure 2 shows a section divided into 4 quarters, the N. E. quarter of the section being divided into 4 quarters of 40 A. each.

The greater part of the land in the United States is divided into townships.

A township is a piece of land 6 miles square; that is, it is 6 miles long and 6 miles wide; hence, a township contains 36 square miles or "sections," a "section" being 1 square mile. These 36 sections are numbered from 1 to 36 as shown in the illustration. Count them.

Each section is divided into four quarters and each of these four quarters is divided into four quarters; hence, a section will have a North East (N. E.) quarter, a North West (N. W.) quarter, a South East (S. E.) quarter, and a South West (S. W.) quarter, and each of these quarters will have a N. E., N. W., S. E., and S. W. quarter. N. E. $\frac{1}{4}$; S. E. $\frac{1}{4}$; N. W. $\frac{1}{4}$; S. W. $\frac{1}{4}$.

As a square mile or section contains 640 acres, each quarter-section contains 160 acres,, and each quarter of a quarter-section contains 40 acres.

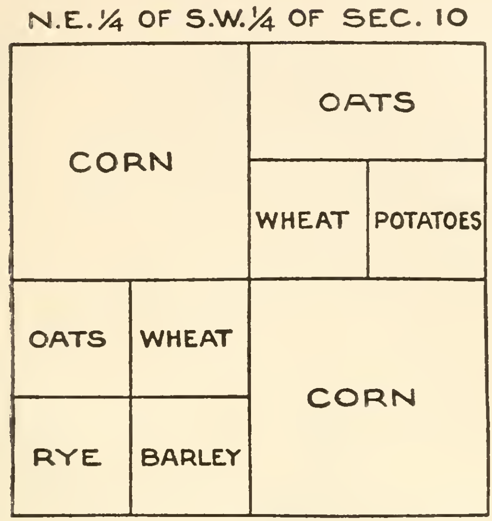
Exercise 14—Oral and Written.

1. Draw the plan of a township.
2. How long and how wide is a township?
3. How many square miles are there in a township?
4. What is each square mile in a township called?
5. Say how the sections in a township are numbered.
6. Number the sections on your plan.
7. Draw the plan of a section.
8. What is the area of a section in square miles? In acres?
9. Show the N. E. $\frac{1}{4}$; N. W. $\frac{1}{4}$; S. E. $\frac{1}{4}$; S. W. $\frac{1}{4}$.
10. How many acres are there in each of these quarters?
11. Show a 160-acre farm on your plan. Locate it as a part of the section.
12. Show the East $\frac{1}{2}$ of N. E. $\frac{1}{4}$.

13. Show a 40-acre farm on your plan. How do you write its location?
14. How many acres are there in a quarter of a quarter-section?
15. Show a 320-acre farm on your plan. Read its location.
16. How many acres are there in the N. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$? (See Figure 2, Page 20.)
17. How many acres are there in the S. E. $\frac{1}{4}$ of N. E. $\frac{1}{4}$? (See Figure 2, Page 20.)

Exercise 15—Written.

1. A 320-A. farm is what part of a section?
2. A 160-A. farm is what part of a section?
3. An 80-A. farm is what part of a section? What part of a quarter-section?
4. A 40-A. farm is what part of a section? What part of a quarter-section?
5. What are the dimensions of a half-section in miles? In rods?
6. What are the dimensions of a quarter-section in miles? In rods?
7. What are the dimensions of half of a quarter-section in miles? In rods?
8. What are the dimensions of quarter of a quarter-section in miles? In rods?
9. Draw a plan of a section and shade the South half of the N. E. quarter with horizontal lines.
10. If the drawing shows a quarter of a quarter-section, how many acres are planted in corn?
11. How many acres are planted in oats?



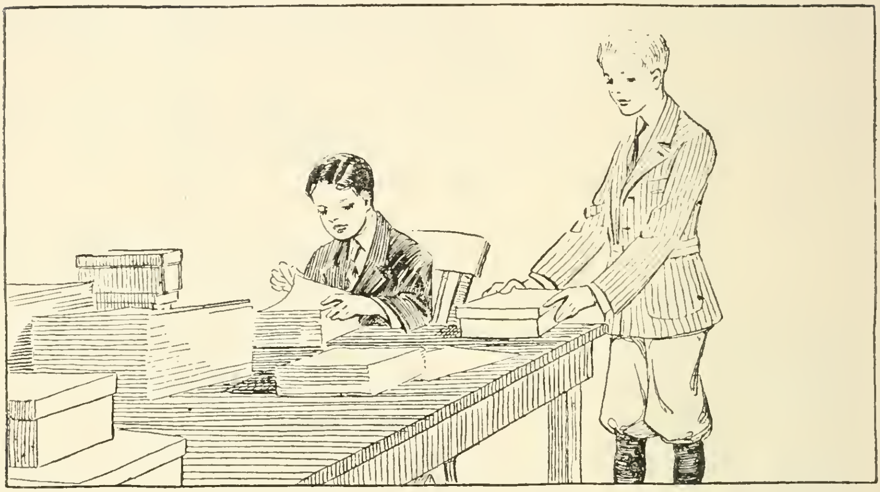
A Quarter of a Quarter-Section

12. How many acres are planted in wheat?
13. How many acres are planted in potatoes?
14. How many acres are planted in rye?
15. How many acres are planted in barley?
16. How many rods of fence are required for one of the corn fields? How many rods for the fence between wheat and potatoes?
17. How many miles of fence are required to go around the N. E. $\frac{1}{4}$ of S. W. $\frac{1}{4}$?
18. How many rods of fence are required on the south side of the rye field?
19. How many rods of fence are required on the north side of the entire Section 10?
20. What is the perimeter of the entire Section 10 in miles?
21. What is the perimeter of the rye field?
22. What is the total perimeter of the two corn fields? (Answer in rods.)

Lesson 8

Paper Measure

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)
500 sheets = 1 printer's ream.



EXAMPLE: Find the cost of a ream of 34″ × 44″ bond paper @ 10¢ per lb., if a ream of 17″ × 22″ = 16#.

34″ × 44″ = 1,496 sq. in.
17″ × 22″ = 374 sq. in.

1,496 sq. in. is a larger area than 374 sq. in.; therefore, the ratio of the weights must be 1,496:374 = $\frac{1496}{374}$ or 4;

1 ream of 34″ × 44″ paper therefore weighs 16# × 4, or 64#, and @ 10¢ per lb. costs $6.40, Ans.

Writing paper and fine stationery is sold by the quire consisting of 24 sheets, or by the ream consisting of 480 sheets.

Paper used for printing purposes is sold by the pound, on the basis of a certain weight for a ream of 500 sheets;

thus, if a ream of paper of a certain size weighs 16#, and the price of the paper is 10¢ per pound, the ream would cost 10¢ × 16, or $1.60; 250 sheets or $\frac{1}{2}$ ream would weigh $\frac{1}{2}$ of 16#, or 8# and would cost 80¢; 3 reams would weigh 16# × 3, or 48# and would cost $4.80.

The basis of weight naturally varies with the size of the sheet; hence, if we know the basis of weight for a ream of paper of any size, we can easily find the weight of a ream of any other size by comparing the two areas.

EXAMPLE: Find the cost of 3,000 sheets of 22″ × 34″ paper @ 8¢ per lb. on the basis of a ream of 17″ × 22″ = 20#.

34″ paper will weigh more than 17″ paper; therefore, the ratio is 34:17 = $\frac{34}{17}$ or 2.

1 ream of 22″ × 34″ paper therefore weighs 20# × 2, or 40#;
3,000 sheets = 6 rm.; 40# × 6 = 240#;
240# @ 8¢ = $19.20, Ans.

Here the dimension 22″ is common to both sizes of paper; therefore, we use the ratio of the two unlike dimensions only, instead of the ratio of the two areas.

When the two sizes of paper have one dimension the same, this dimension can cause no change in weight or money; therefore, find the ratio of the unlike dimensions only, as this ratio is the same as the ratio of the two areas.

Exercise 16—Oral.

1. Say the table of Paper Measure.
2. If fine writing paper costs 20¢ per quire, what is the cost of 1 ream?

3. If linen writing paper cost 1¢ per sheet, what is the cost of 1 ream?
4. How many sheets are there in a printer's ream?
5. If a ream of a certain kind of paper weighs 20# and the price is 8¢ per lb., what is the cost of the entire ream?
6. If a ream of a certain kind of paper weighs 30#, what is the weight of 7 reams of the same paper?
7. If a ream of a certain size of paper weighs 40#, what is the weight of a ream of the same kind of paper when the sheets are double the size?
8. If you know the weight of a ream of a certain kind of paper of a certain size, how can you find the weight of a ream of the same paper of any other size? Is there such a problem in Exercise 17?
9. When one dimension is common to two sizes of paper, how do you find the ratio quickly? Can you find such a problem in Exercise 17?
10. What kind of paper is sold 480 sheets to the ream? What kind is sold 500 sheets to the ream?

Exercise 17—Oral and Written.

A. Tell how to do these examples. Point out the comparison in each example.

1. Find the cost of 750 sheets of bond paper @ 12¢ per lb. on the basis of 1 rm. 22″ × 34″ = 40#.
2. Find the weight of 1 ream of 36″ × 48″ ledger paper on the basis of 1 rm. 24″ × 36″ = 44#. What difference do you notice in the size of the sheets?

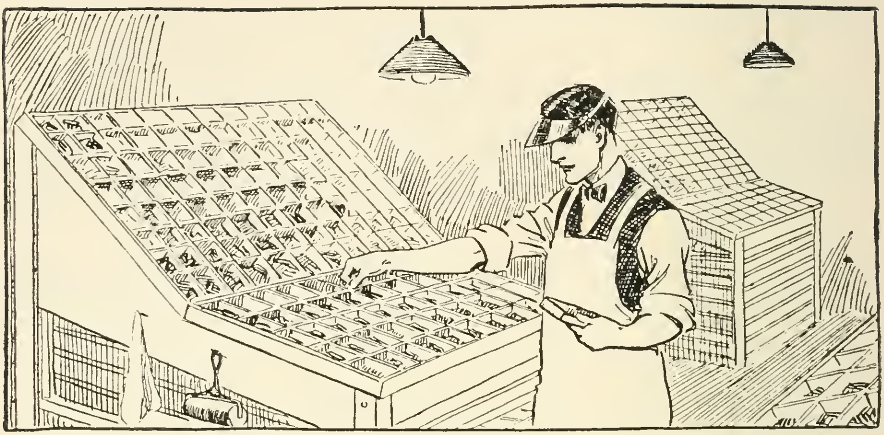
3. Find the weight of 8 rm. 17″ × 22″ flat paper on the basis of 1 rm. 22″ × 34″ = 36#. Compare the weights of 1 ream of each size.
4. Find the cost of 1 rm. 18″ × 24″ linen-finish paper @ 9¢ per lb. on the basis of 1 rm. 36″ × 48″ = 72#.
5. Find the cost of 20 rm. 42″ × 42″ super-coated paper @ 7¢ per lb. on the basis of 1 rm. 42″ × 56″ = 212#.
6. Find the cost of 6,000 sheets of 17″ × 22″ machine-finish paper @ $6\frac{1}{4}$¢ per lb. on the basis of 1 rm. 22″ × 34″ = 32#.
7. Find the cost of 4 tons of 10″ × 30″ manila wrappers @ 5¢ per lb. on the basis of 1 rm. 30″ × 40″ = 36#.
8. Find the cost of 4,000 white manila 10″ × 30″ wrappers @ $5.00 per cwt. on the basis of 1 rm. 30″ × 40″ = 36#.
9. Find the cost of 10 rm. 35″ × 40″ brown wrappers @ $120.00 per T. on the basis of 24″ × 35″ = 30# to the ream.
10. Find the cost of 10,000 sheets of 42″ × 56″ machine-finish paper @ $5.70 per cwt. on the basis of 1 rm. 21″ × 28″ = 50#.

B. Now work all of the examples.

Lesson 9

Printers' Type Measure

12 points = 1 pica
6 picas (72 points) = 1 inch



In reading a newspaper, magazine, or book (this arithmetic, for example) have you noticed the perfect uniformity of all the characters in each style of type used, and the uniformity of the width of the columns and pages? If you haven't noticed these things, it is on account of the *very* uniformity which exists, for, if even a single letter in a line were $\frac{1}{100}$ of an inch larger or smaller than the others, you would quickly notice that something was wrong.

To obtain this uniformity, printers require units of measurement much smaller than the inch or even 16th's or 32d's of an inch; so they have divided the inch into 72d's (approximately) and they call $\frac{1}{72}$ of an inch a "point," and 12 of these points are called a "pica," so there are 6 picas to an inch.

The height of type is usually measured by points, thus: as there are 72 points to an inch, 6-point type is $\frac{1}{12}$ of an inch high; 8-point type is $\frac{1}{9}$ of an inch high; 12-point type is $\frac{1}{6}$ of an inch high; etc.

The length and width of the surface covered by the printing on a page is usually measured by picas, thus:

as there are 6 picas to an inch, a page of type 4 inches wide is 24 picas wide; a page of type $3\frac{1}{2}$ inches wide is 21 picas wide; a page of type $10\frac{1}{2}$ inches long is 63 picas long; etc.

The area of the surface covered by the printing on a page is usually measured by a unit called an "em." This unit is a square whose face is as many points high and wide as there are points in the size of type being used on a particular piece of printing. Thus, if 6-point type is being used, an "em" is 6 points high and 6 points wide; therefore, there would be 12 six-point "ems" ($72 \div 6$) in a line 1 inch long, and 12 six-point "ems" in a column 1 inch long and 1 "em" wide; in other words, there are 144 six-point "ems" to the square inch. Eight-point type runs 9 "ems" to the inch ($72 \div 8$) or 81 "ems" to the square inch; etc.

This paragraph is set in 12-point type, 21 picas wide; this gives us 6 lines to the inch, because there are 6 times 12 points in 72 points. Since a 12-point em is 12 points square, there are 6 ems to the inch, or 36 ems to the square inch; therefore, this paragraph is 21 ems wide and 7 ems deep and contains 147 ems.

This paragraph is set in 8-point type, 16 picas wide; this gives us 9 lines to the inch, because there are 9 times 8 points in 72 points. Since an 8-point em is 8 points square, there are 9 ems to the inch, or 81 ems to the square inch; therefore, this paragraph is 24 ems wide and 7 ems deep, and contains 168 ems.

This paragraph is set in 6-point type, 12 picas wide; this gives us 12 lines to the inch, because there are 12 times 6 points in 72 points. Since a 6-point em is 6 points square, there are 12 ems to the inch, or 144 to the square inch; therefore, this paragraph is 24 ems wide and 7 ems deep, and contains 168 ems.

Exercise 18—Oral.

1. Why do printers require units of measurement smaller than the inch?
2. Into how many parts do they divide an inch for the unit which is called a point? How many points are there in 1 inch?
3. Into how many parts do they divide an inch for the unit which is called a pica? How many picas are there in 1 inch?
4. Points in 1 inch = ? Picas in 1 inch = ? How many points are there in 1 pica?
5. What is the height of 6-point type in points? In parts of an inch?
6. What is the height of 8-point type in points? In parts of an inch?
7. What is the height of 12-point type in points? In parts of an inch?
8. What is the height of 24-point type in points? In parts of an inch?
9. What is the height of 10-point type in points? In parts of an inch?
10. How many 6-point lines are there to an inch in the length of a column? How many 8-point lines? 12-point lines? 24-point lines? 36-point lines?
11. How many picas are there in 1 inch? In 4 inches? In 8 inches?
12. Does the size of the type or the length of a line have anything to do with the number of points to an inch? Has it anything to do with the number of picas to an inch?

13. How long is a printed page, if it is 48 picas long? If it is 72 picas long? How wide is it if it is 60 picas wide?
14. How many points high is a 6-point em? How many points wide? How many 6-point ems are there in 1 inch of a line of printing? How many 6-point ems are there in 1 inch of a column of printing? How many 6-point ems are there in 1 square inch of printing?
15. How many points high and wide is an 8-point em? How many 8-point ems are there in 1 inch horizontally? How many vertically? How many are there in 1 square inch?
16. Give the dimensions of a 12-point em. Tell all you can about it.
17. How many ems of 6-point type are there in a line 5 inches long? How many in a line 8 inches long? 12 inches long?
18. How many ems long is a page of 8-point type, if the column measures 48 picas in length? 60 picas? 72 picas?
19. How many ems of 8-point type are there in a line 6 inches long? How many ems of 24-point type?
20. How many 8-point ems are there in a square inch? How many 12-point ems? How many 6-point ems?

Exercise 19—Oral and Written.

A. Tell just how you will work each of these examples:

1. A line of type is $6\frac{1}{2}$ in. long; how many points long is it? How many picas long is it?

2. A line of type is 594 points long; what is its length in inches? In picas?
3. The Daily News has a column $22\frac{1}{4}''$ long and $2\frac{1}{4}''$ wide; how many points long and wide is the column? How many picas long and wide?
4. The type-page of a magazine is $9''$ wide and $12''$ long; if this magazine is set in 8-point type, how many ems are there in one line? How many ems long is a column? How many ems are there on a page?
5. The type-page of a book is 36 picas wide and 48 picas long; how many ems of 12-point type does it contain? How many ems of 6-point type?
6. How many picas wide and long is the page of a book, if it is 504 points wide and 648 points long?
7. How many more ems are there on a $4'' \times 6''$ page when the type is set in 6-point than when it is set in 8-point?
8. A certain page of 12-point type contains 1,728 ems; if the page is 36 picas wide, how many inches long is it?
9. A certain magazine has columns $2\frac{1}{4}''$ wide and $10\frac{1}{2}''$ long; if the type-page is $9''$ wide and $10\frac{1}{2}''$ long, how many columns are there on a page? How many ems of 9-point type are there in a column? In a page?
10. What would be the length (in inches) of a line set in 8-point type, that it might contain as many ems as a line $6\frac{1}{2}''$ long set in 6-point type? How many picas wide would it be?

B. Now work all the examples.

Lesson 10

(For Reference)

Legal Weights of a Bushel

(In Pounds)

	Wheat	Rye	Oats	Barley	Shelled Corn	Potatoes	Onions	Beans	Peas	Apples
United States......	60	56	32	48	..	60	..	..	60	..
Alabama..........	60	56	32	47	56	60	..	60	60	..
Arizona...........	60	56	32	45	54	..	..	60	..	..
Arkansas..........	60	56	32	48	56	60	57	60	60	50
California.........	60	54	32	50	52	..	..	..	..	..
Colorado..........	60	56	32	48	56	60	57	60	..	..
Connecticut.......	60	56	32	48	56	60	52	60	60	48
Delaware..........	60	..	..	..	56	..	..	..	..	..
Dist. of Columbia ..	..	..	32	..	56	60	..	..	..	..
Florida...........	60	56	32	48	56	60	56	60	..	48
Georgia...........	60	56	32	47	56	60	57	60	60	..
Idaho.............	60	56	36	48	56	60	..	..	..	45
Illinois............	60	56	32	48	56	60	57	60	60	50
Indiana...........	60	56	32	48	56	60	48	60	..	..
Iowa.............	60	56	32	48	56	60	57	60	..	48
Kansas............	60	56	32	48	56	60	57	60	..	48
Kentucky.........	60	56	32	47	56	60	57	60	60	..
Louisiana.........	60	56	32	48	56	..	..	..	..	..
Maine............	60	50	32	48	56	60	52	60	60	44
Maryland.........	..	..	26	..	..	56	..	..	..	..
Massachusetts.....	60	56	32	48	56	60	52	60	60	48
Michigan..........	60	56	32	48	56	60	54	60	60	48
Minnesota.........	60	56	32	48	56	60	52	60	60	50
Mississippi........	60	56	32	48	56	60	57	60	60	..
Missouri..........	60	56	32	48	56	60	57	60	60	48
Montana..........	60	56	32	48	56	60	57	60	60	45
Nebraska.........	60	56	32	48	56	60	57	60	60	..
New Hampshire....	60	56	32	..	56	60	..	62	60	..
New Jersey........	60	56	30	48	56	60	57	60	60	50
New York.........	60	56	32	48	56	60	57	60	60	48
North Carolina.....	60	56	32	48	56	..	..	..	60	..
North Dakota.....	60	56	32	48	56	60	52	60	60	..
Ohio..............	60	56	32	48	56	60	55	60	60	50
Oklahoma.........	60	56	32	48	56	60	52	60	60	..
Oregon............	60	56	32	46	56	60	..	..	..	45
Pennsylvania......	60	56	32	47	56	56	50	..	..	..
Rhode Island......	60	56	32	48	56	60	50	60	60	48
South Carolina.....	..	..	..	..	..	..	..	..	..	..
South Dakota......	60	56	32	48	56	60	52	60	60	..
Tennessee.........	60	56	32	48	56	60	56	60	60	50
Texas............	60	56	32	48	56	60	57	60	..	45
Vermont..........	60	56	32	48	56	60	52	62	60	46
Virginia..........	60	56	30	48	56	56	57	60	60	..
Washington........	60	56	32	48	56	60	..	..	..	45
West Virginia......	60	56	32	48	56	60	..	60	..	..
Wisconsin.........	60	56	32	48	56	60	57	60	60	50

The table here given shows the number of pounds of various products considered by law as constituting a bushel. It is not necessary for you to learn this entire table, but you should learn the figures which appear opposite your state.

Exercise 20—Oral.

In this state, what is the legal weight of:

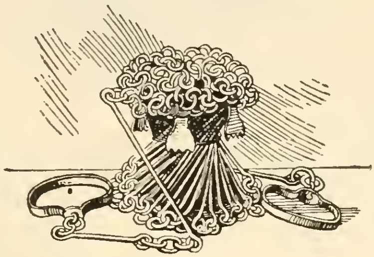
1. A bushel of wheat? Of rye? Of oats?
2. A bushel of barley? Of shelled corn? Of potatoes?
3. A bushel of onions? Of beans? Of peas? Of apples?
4. 10 bushels of wheat? Of rye? Of onions?
5. 1 pk. of potatoes? Of beans? Of peas?
6. 1 qt. of oats?
7. 3 pk. of corn?
8. 6 bu. of beans?
9. $\frac{1}{2}$ bu. of barley?
10. $\frac{3}{4}$ bu. of corn?
11. 1 pk. of oats?
12. 3 pk. of potatoes?
13. 1 qt. of oats?
14. .75 bu. of rye?
15. 1.5 bu. of barley?
16. .25 bu. of peas?
17. .5 bu. of beans?
18. Which is the lightest of these products?
19. Which is the heaviest of these products?
20. Pick out all the things weighing 60 lb. to the bushel.
21. Can you explain why there is so great a difference in weight between oats and wheat?

Tables of Measures Used in Certain Professions

(For Information and Reference)

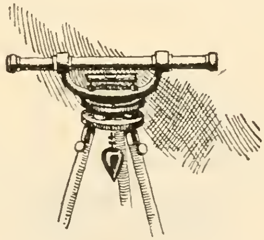
Used by Surveyors and Civil Engineers for Measuring Land

Surveyors' Linear Measure



7.92	inches (in.)	= 1 link (l.)
25	links	= 1 rod (rd.)
4	rods	= 1 chain (ch.)
80	chains	= 1 mile (mi.)

Surveyors' Square Measure

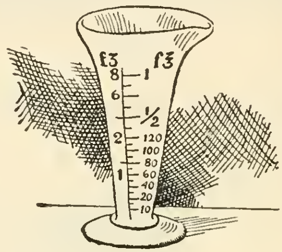


16 square rods (sq. rd.)	= 1 square chain (sq. ch.)
10 square chains	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 square mile	= 1 section (sec.)
36 sections	= 1 township (T.)

(NOTE: Point out the differences between this table and the Table of Common Square Measure.)

Used by Druggists, Chemists, and Physicians for Measuring Chemicals

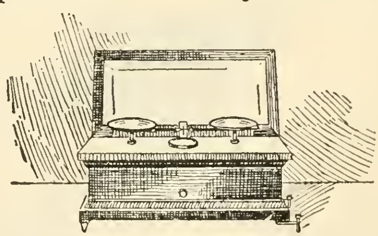
Apothecaries' Liquid Measure



60 minims (m.)	= 1 fluid dram (ʒ)
8 fluid drams	= 1 fluid ounce (℥)
16 fluid ounces	= 1 pint (*o*)
8 pints	= 1 gallon (cong.)

(Notice that 1 pint is $\frac{1}{8}$ gallon just as in Common Liquid Measure, but that each pint is divided into many smaller parts here, while in Common Liquid Measure $\frac{1}{4}$ pint or gill is the smallest unit used.)

Apothecaries' Dry Measure

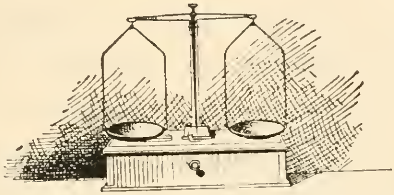


20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (lb.)

(Note: Does this table more closely resemble Common Dry Measure or Avoirdupois Weight?)

Used by Goldsmiths, Silversmiths, and Jewelers for Measuring Precious Metals and Stones

Troy Weight



24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Exercise 21—Oral Review.

1. William Smith sold merchandise amounting to $250.00 to Frank Jones; who is the debtor and who is the creditor in this transaction?
2. Find the interest on $200.00 for 120 days at 6%. At 3%. At $4\frac{1}{2}$%.
3. Add:

(*a*)	(*b*)	(*c*)
4,634	3,872	2,798
397	597	398

4. Subtract:

(*a*)	(*b*)	(*c*)
5,632	2,108	4,531
296	799	195

5. Multiply:

(*a*)	(*b*)	(*c*)
$24 \times 16\frac{2}{3}$	$72 \times 66\frac{2}{3}$	48×87.5

6. Divide:

(a)	(b)	(c)
$15 \div 33\frac{1}{3}$	$800 \div 25$	$12 \div 12.5$

(Practice till you can do #3, 4, 5, and 6 correctly in from three to five minutes.)

7. Reduce 4 min. 20 sec. to minutes. To seconds.

8. How many inches are there in $\frac{1}{3}$ yard? In $\frac{2}{3}$ yd.?

9. How many square miles are there in a township? How many sections are there in a township?

10. Say the table of Paper Measure.

Answer these rapidly:

11. 5% of $400. = ?

12. 3% of $600. = ?

13. 4% of $500. = ?

14. 8% of $200. = ?

15. $33\frac{1}{3}$% of $300. = ?

16. $12\frac{1}{2}$% of $80. = ?

17. 25% of $600. = ?

18. 75% of $800. = ?

Exercise 22—Written Review.

1. On March 8, 1919, a boot and shoe retailer bought the following merchandise:

48 pr. Calf Shoes......	@ $2.75
36 pr. Kid Shoes......	@ 3.00
12 pr. Rubbers........	@ 1.00

To pay for this merchandise he gave a 90-day note bearing 6% interest; what was the date of maturity of the note? What amount was paid in full settlement of the note?

2. How many board feet of lumber are there in the following shipment:

50 pieces	$12' \times 8'' \times 2''$	= ?
60 pieces	$10' \times 12'' \times 3''$	= ?
80 pieces	$8' \times 6'' \times 4''$	= ?
Total	....	= ?

3. A salesman received $48.00 commission for selling a certain bill of goods; his commission was figured at 8%. Make a good statement about $48.00 so you can begin to work; under it write 1% of the sales. Then 100% or total sales.

4. A wholesale drug dealer allowed a trade discount of 30% and a cash discount of 2% 10 days on all goods sold to retail druggists; how much would a retail druggist have to pay for a bill of goods which at list prices amounted to $80.00?

5. What per cent of 8.5 is 2.5?

6. A messenger boy who delivers reports has 40 business houses on his route; if it takes him 3 hr. 40 min. to cover the complete route, what is the average time required to deliver each report? How many complete trips can he make in 44 working hours?

7. What is the cost of 4M sheets of $28'' \times 42''$ book paper at $5.85 per cwt. on the basis of 1 rm. $42'' \times 56'' = 90$#?

8. Find $16\frac{2}{3}$% of 10 sq. yd. 8 sq. ft. 6 sq. in.

9. How many cubic inches are there in $\frac{1}{9}$ cu. yd.?

10. Reduce 276,330 sec. to days, hours, minutes, and seconds.

Subtract, but do not copy:

(Time for these 12 examples is less than $4\frac{1}{2}$ minutes.)

11.	12.	13.	14.	15.	16.
874,362	487,398	687,481	768,042	488,398	763,904
439,728	248,099	298,385	38,098	298,468	498,927

17.	18.	19.	20.	21.	22.
249,763	863,901	848,729	511,872	863,784	948,728
152,948	248,763	385,621	355,902	497,687	349,875

Add, but do not copy:

(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

23.	24.	25.	26.	27.	28.
4,875	48,712	38,641	7,741	2,349	1,593
2,387	986	864	8,431	7,257	5,945
5,963	3,639	41,386	9,876	7,812	9,198
8,846	53,872	64,138	2,346	9,136	7,758
5,222	91,808	94,815	5,966	6,750	9,297

Copy and divide:

(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

29. $10,416 \div 24$;

30. $33,867 \div 53$;

31. $45,136 \div 62$;

32. $61,344 \div 71$;

33. $44,541 \div 63$;

34. $73,882 \div 82$.

Copy and multiply:

Time for these 8 examples is less than $4\frac{1}{2}$ minutes.)

35. 439×62;

36. 274×83;

37. 806×39;

38. 128×52;

39. 551×35;

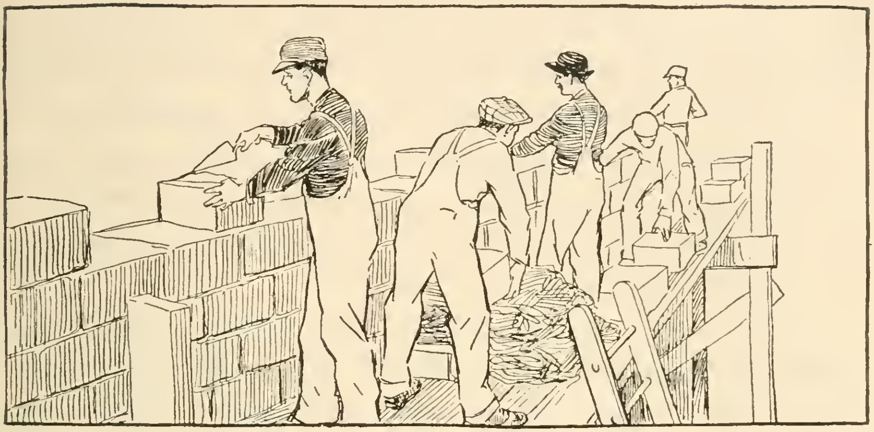
40. 340×87;

41. 319×91;

42. 509×74.

Lesson 11

Special Working Units



The average man engaged in any occupation can, in one hour, do a certain amount of work; in two hours he can do twice that amount of work, or as much work as two men can do in one hour; thus, it naturally follows that when a piece of work can be done, for instance, by 10 men in 6 hours, the total time consumed by the 10 men in doing this work is the same as 60 hours of one man's work; in other words, the work requires 60 "man-hours," a "man-hour" being 1 man's effort for 1 hour of time.

The number of "man-hours" required for any piece of work is the product of the number of men engaged on the work multiplied by the number of hours consumed in doing the work.

When the number of "man-hours" necessary for a piece of work is known, we find either the number of men to be engaged or the number of hours to be consumed, by dividing the number of "man-hours" by the known factor; thus, a piece of work which can be completed in 24 man-hours, can be done by 1 man in 24 hours, by 2 men in 12 hours, by 3 men in 8 hours, by 4 men in 6 hours, by 6 men in 4 hours, by 8 men in 3 hours, by 12 men in 2 hours, or by 24 men in 1 hour.

Another "special working unit" in common use in railroading is the "ton-mile"; this is the equivalent of hauling 1 ton a distance of 1 mile. The number of tons hauled multiplied by the number of miles they are hauled equals the "ton-miles."

There are many other "special working units," such as the "kilowatt-hour," the "foot-pound," the "light-year," the "acre-inch," etc., all based on the same general principle of using the product of two units of measure.

Exercise 23—Oral.

1. When a certain piece of work can be completed by 2 men in 1 hour, how long would it take 1 man to do the work? How many man-hours are needed to do this work?
2. How many man-hours are needed to do a certain job, if 10 men can do it in 10 hours?
3. To build a certain wall, 5 men worked 8 hours each and 5 other men worked 12 hours each; how many man-hours were needed to build this wall?
4. To plow a certain field, 48 man-hours of work were required; how many men should be engaged to finish this work in 8 hours? In 6 hours? In 4 hours?

5. To dig a certain trench, 36 man-hours of work were required; how quickly can 3 men dig this trench? 4 men? 6 men? 9 men? 12 men? 18 men? 24 men?
6. A freight engine had to haul 400 tons of freight 2 miles to place it on a side-track; how many ton-miles were involved in this operation?
7. If the cost of moving machinery by freight is 10¢ per ton-mile, what is the cost of moving 10 tons 10 miles?
8. The electrical energy required to keep 1 electric lamp burning 50 hours is equal to that required to keep 10 lamps burning for how many hours?
9. At 1¢ per lamp per hour, what is the cost of burning 6 lamps 5 hours?
10. Could the burning of 1 lamp for 1 hour be called a lamp-hour?
11. Tell in good English what is meant by a man-hour.
12. Tell in good English what is meant by a ton-mile.

Exercise 24— Written.

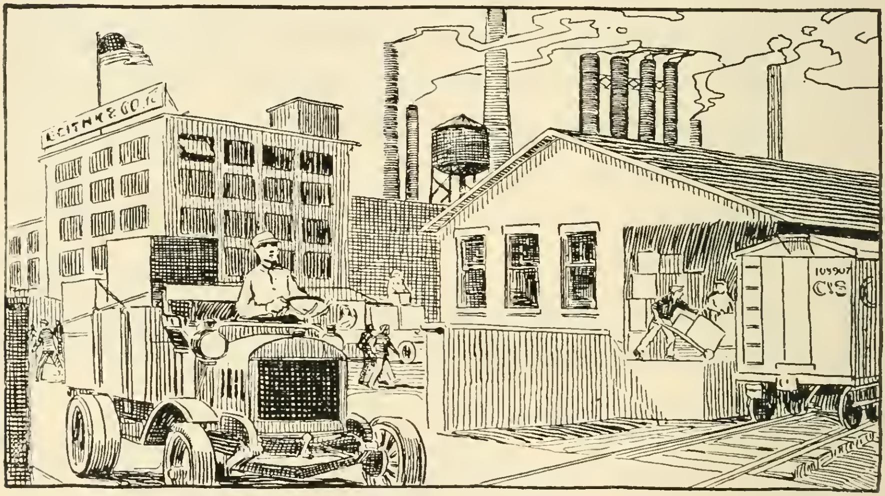
Be this manufacturer for to-day.

1. You estimated that it would require 1,500 man-hours to execute a contract for steel rails and you had only 10 working days in which to do the work; if your men worked $7\frac{1}{2}$ hours per day, how many men were needed?
2. A contractor who works for you has a weekly pay roll amounting to $720.00; if his men work 40 hours per week and the rate is 60¢ per man-hour, how many men has he?

3. In one department of your business 35 men are paid a total of $2,520. for working 3 weeks of 48 hours each; what is the rate of wages per man-hour?

4. The weekly pay roll in another department amounts to $1,108.80; if the rate per man-hour is 55¢ and there are 48 men employed, how many hours per week do they work?

5. If 1,050 tons of your freight are hauled 250 miles at a cost of $1\frac{1}{2}$¢ per ton-mile, what is the total cost?



6. A contractor agreed to build an addition to your factory in 16 weeks and estimated that the work would require 76,800 man-hours; being unable to furnish more than 50 men with the necessary tools and machinery, he decided to work two shifts of 50 men each—a day shift and a night shift. If the day shift worked 9 hours per day 6 days a week, how many hours per day for 6 days a week did the night shift work?

7. In one of the offices electricity costs .8 cents per lamp per hour; how many lamps can be burned 30 hours for $9.60?

8. How long could 50 lamps be burned for $22.50, if the cost per lamp per hour were .75 cents?

FRACTIONS

Lesson 12

Fractions as You Want to Know Them Now

Exercise 25—Oral.

Answer at sight:

1. $\frac{1}{2}$ of $\frac{1}{2}$ = ?
2. $\frac{4}{5}$ of $\frac{5}{7}$ = ?
3. $\frac{1}{3}$ of $\frac{3}{7}$ = ?
4. $\frac{1}{2}$ of $\frac{3}{5}$ = ?
5. $\frac{7}{8}$ of $\frac{8}{9}$ = ?
6. $\frac{1}{5}$ of $\frac{15}{8}$ = ?
7. $\frac{1}{3}$ of $\frac{8}{9}$ = ?
8. $\frac{7}{8}$ of $\frac{16}{7}$ = ?
9. $\frac{1}{7}$ of $\frac{49}{50}$ = ?
10. $\frac{2}{3}$ of $\frac{4}{5}$ = ?
11. $\frac{7}{10}$ of $\frac{30}{35}$ = ?
12. $\frac{1}{9}$ of $\frac{27}{5}$ = ?
13. 2 plus $\frac{1}{2}$ of $\frac{1}{2}$ = ?
14. 1 + $\frac{1}{3}$ of $\frac{3}{4}$ = ?
15. 1 minus $\frac{1}{2}$ of $\frac{1}{2}$ = ?
16. 5 − $\frac{1}{3}$ of $\frac{9}{10}$ = ?
17. $\frac{1}{2}$ of $\frac{1}{2}$ added to $\frac{3}{4}$ of 12 = ?
18. $\frac{2}{3}$ of $\frac{3}{4}$ less $\frac{1}{2}$ of $\frac{1}{2}$ = ?
19. $\frac{1}{2}$ of $1\frac{1}{2}$ plus $\frac{1}{2}$ of $2\frac{1}{2}$ = ?
20. $\frac{3}{4}$ of $\frac{1}{2}$ + $5\frac{1}{2}$ + $6\frac{1}{2}$ = ?

21. What did you add to 2 in example #13? 2 + $\frac{1}{2}$ of $\frac{1}{2}$ = ? Read it; it will tell you just what to do.

22. When two or more steps occur, what is done first? Example: $\frac{1}{2}$ of $\frac{1}{2}$ added to $\frac{3}{4}$ of 12 = ?

Exercise 26—Written.

> **EXAMPLE:** If 6 yd. of goods cost $1.50, what will one yd. cost?
> $1½. ÷ 6 = $\frac{1}{6}$ of $1½., or $¼., Ans.

1. $4\frac{1}{2}$ ÷ 3 or ? of ? = ?
2. $1\frac{1}{2}$ ÷ 7 or ? of ? = ?
3. $2\frac{1}{5}$ ÷ 2 or ? of ? = ?
4. $7\frac{1}{3}$ ÷ 11 or ? of ? = ?
5. $20\frac{1}{2}$ ÷ 7 or ? of ? = ?
6. $10\frac{1}{2}$ ÷ 7 = ?
7. $6\frac{2}{3}$ ÷ 5 = ?
8. $7\frac{1}{5}$ ÷ 12 = ?
9. $9\frac{1}{3}$ ÷ 14 = ?
10. $10\frac{4}{5}$ ÷ 9 = ?

Another way to see the same step:

EXAMPLE: Dividend / Divisor $\frac{1\frac{1}{2}}{6} = 1\frac{1}{2} \div 6$, or $\frac{1}{4}$, Ans.

EXAMPLE: $\frac{4\frac{1}{2}}{5} = \frac{9}{2} \div 5$, or $\frac{9}{2} \times \frac{1}{5}$, or $\frac{9}{10}$, Ans.

11. $\frac{1\frac{1}{2}}{2} = ?$

12. $\frac{3\frac{1}{2}}{7} = ?$

13. $\frac{10\frac{1}{2}}{3} = ?$

14. $\frac{10\frac{1}{3}}{2} = ?$

15. $\frac{12\frac{1}{2}}{4} = ?$

16. $\frac{10\frac{1}{2}}{12} = ?$

EXAMPLE: If $1\frac{1}{2}$ yd. cost \$6.00 what will one yd. cost?

Dividend / Divisor $\frac{\$6.}{1\frac{1}{2}} = \$6. \div 1\frac{1}{2}$, or $\$6. \times \frac{2}{3}$, or \$4., Ans.

17. $\frac{7}{2\frac{1}{2}} = ?$

18. $\frac{10}{3\frac{1}{3}} = ?$

19. $\frac{12}{10\frac{1}{2}} = ?$

20. $\frac{9}{3\frac{1}{3}} = ?$

21. $60 + \frac{12\frac{1}{2}}{5} = ?$

22. $600 + \frac{12\frac{1}{2}}{50} = ?$

23. $\frac{880}{\frac{22}{7}} = ?$

24. $\frac{110}{\frac{22}{7}} = ?$

EXAMPLE: $\frac{1\frac{3}{4}}{6} = ?$

$\frac{1\frac{3}{4}}{6} = \frac{1.75}{6.00}$ or $\frac{175}{600}$ or $\frac{7}{24}$, Ans.

Very often fractions of this kind can be cleared most easily by changing both terms to decimals of a like number of places and reducing to lowest terms.

25. $\frac{1\frac{1}{2}}{7\frac{1}{2}} = ?$

26. $\frac{3\frac{4}{5}}{38} = ?$

27. $\frac{2\frac{1}{2}}{50} = ?$

28. $\frac{8\frac{3}{4}}{26\frac{1}{4}} = ?$

29. $\frac{1\frac{1}{8}}{11\frac{1}{4}} = ?$

30. $\frac{4\frac{1}{2}}{8} = ?$

EXAMPLE:

$\frac{1\frac{1}{2}}{6} = \frac{1\frac{1}{2} \times 2}{6 \times 2}$ or $\frac{3}{12}$ or $\frac{1}{4}$, Ans. (We use 2 as the number to multiply by, because 2 is the denominator of the fraction $\frac{1}{2}$.)

Since the quotient in an example in division is not changed by multiplying the dividend and the divisor by the same number, we can multiply the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

EXAMPLE:

$\frac{1\frac{2}{3}}{7\frac{1}{2}} = \frac{1\frac{2}{3} \times 6}{7\frac{1}{2} \times 6}$ or $\frac{10}{45}$ or $\frac{2}{9}$, Ans. (We use 6 as the number to multiply by, because 6 is the least common denominator of $\frac{2}{3}$ and $\frac{1}{2}$.)

When either term of a fraction contains a fraction, as $\frac{1\frac{1}{2}}{6}$, $\frac{4\frac{2}{3}}{3\frac{1}{2}}$, etc., the whole is called a "complex fraction." When both terms of a complex fraction contain fractions, clear by multiplying both terms by that number which would be the least common denominator of the two fractions.

31. $\frac{3\frac{1}{4}}{4} = ?$

32. $\frac{2\frac{2}{3}}{5\frac{1}{3}} = ?$

33. $\frac{3\frac{1}{3}}{4\frac{1}{2}} = ?$

34. $\frac{5\frac{1}{4}}{6\frac{1}{2}} = ?$

35. $\frac{1\frac{1}{5}}{4\frac{1}{2}} = ?$

36. $\frac{2\frac{1}{2}}{1\frac{2}{3}} = ?$

37. $\frac{3\frac{1}{3}}{1\frac{2}{3}} = ?$

38. $\frac{1\frac{1}{4}}{3\frac{1}{3}} = ?$

39. $\frac{2\frac{1}{5}}{5\frac{1}{2}} = ?$

40. $\frac{3\frac{1}{8}}{2\frac{1}{2}} = ?$

MULTIPLICATION

Lesson 13

Cross Multiplication

"Cross Multiplication" is a method of multiplication in which it is unnecessary to write any partial products, as the final product is obtained at once.

```
 23
 21
 23
46
483
```

In multiplying numbers of two orders as 23 × 21, it will be noticed that the units' figure of the product is obtained by multiplying the units' figures of the multiplier and of the multiplicand; thus, 3 × 1 = 3. The tens' figure of the product is the sum of the tens' figure of the multiplicand multiplied by the units' figure of the multiplier, plus the units' figure of the multiplicand multiplied by the tens' figure of the multiplier; thus, (2 × 1) + (3 × 2) = 8. The hundreds' figure of the product is obtained by multiplying the tens' figures of the multiplicand and of the multiplier; thus, 2 × 2 = 4.

```
 23
 21
483
```

Now, if we had added 2 and 6 in tens' place without writing them, we would have had 483 for the answer just as before, but we would have written no partial products; thus:

3 × 1 = 3; write 3;
2 × 1 = 2; 3 × 2 = 6; 2 + 6 = 8; write 8;
2 × 2 = 4; write 4.

Remember, we multiply in this order:

First: Units × Units.

```
23
21
 3
```

Second: (Units × Tens) plus (Tens × Units).

$$\begin{array}{r} 23 \\ 21 \\ \hline 83 \end{array}$$

Third: Tens × Tens.

$$\begin{array}{r} 23 \\ 21 \\ \hline 483 \end{array}$$

EXAMPLE: 34 × 26 = 884.

$$\begin{array}{r} 34 \\ 26 \\ \hline 884 \end{array}$$

First: 4 × 6 = 24; write 4 and carry 2;
Second: 3 × 6 = 18; 4 × 2 = 8; 18 + 8 + 2 (carried) = 28; write 8 and carry 2;
Third: 3 × 2 = 6; 6 + 2 (carried) = 8; write 8.

EXAMPLE: 47 × 35 = 1,645.

$$\begin{array}{r} 47 \\ 35 \\ \hline 1{,}645 \end{array}$$

First: 7 × 5 = 35; write 5 and carry 3;
Second: 4 × 5 = 20; 7 × 3 = 21; 20 + 21 + 3 (carried) = 44; write 4 and carry 4;
Third: 4 × 3 = 12; 12 + 4 (carried) = 16; write 16.

If there is a carrying figure from any product or sum, carry to the next place as usual.

Exercise 27—Oral.

$$\begin{array}{r} 14 \\ 21 \\ \hline 14 \\ 28 \\ \hline 294 \end{array}$$

1. In the multiplication here shown, how is the units' figure of the product obtained?
2. Show how the tens' figure of the product is obtained? Point it out.
3. Tell us where the hundreds' figure of the product comes from.
4. Begin the multiplication. No writing of partial products.

$$\begin{array}{r} 43 \\ \times 21 \\ \hline \end{array}$$

5. Show where you get the tens' figure of the product.
6. Show where you get the hundreds' figure of the product.
7. What is done with the carrying figures in cross multiplication?

Exercise 28—Written.

Multiply the following without writing partial products:

1. 31 × 22	2. 23 × 21	3. 32 × 21	4. 42 × 21
5. 24 × 22	6. 32 × 24	7. 42 × 24	8. 42 × 34
9. 42 × 26	10. 43 × 34	11. 46 × 33	12. 32 × 27

Do not copy the following to multiply; write nothing but the answers:

13. 43 × 32;
14. 28 × 31;
15. 37 × 23;
16. 98 × 31;
17. 89 × 22;
18. 48 × 41;
19. 76 × 34;
20. 84 × 42.

21. Find the cost of the following merchandise, without writing partial products:

45# Rice	@	14¢ per lb	$?.??
24 Cans Peaches	@	17¢ per can	?.??
14# Cocoa	@	38¢ per lb	?.??
Total			$?.??

22. 72 × 18;
23. 21 × 22;
24. 65 × 32;
25. 82 × 15;
26. 79 × 21;
27. 93 × 14;
28. 93 × 31;
29. 97 × 15;
30. 64 × 64.

(Practice this till you can do the last nine in 5 minutes or less.)

TIME AND WAGES

Lesson 14

How Wages Are Figured

Most tradesmen are paid by the hour (or fraction of an hour) for their labor, based on a certain sum, as \$25.00, \$30.00, etc., for a week's work consisting of a certain number of hours, as 42, 45, 48, etc.; thus, if a man is working on a 48-hr. basis at \$25.00 per week, he receives $\frac{1}{48}$ of \$25.00, or $52\frac{1}{12}$¢ for every hour he works, and his wages for 46 hr. would be $\frac{46}{48}$ of \$25.00, or \$23.96.

EXAMPLE: 12 hr. @ \$24.00 per week on a 48-hr. basis = ?
$\frac{12}{48} = \frac{1}{4}$; $\frac{1}{4}$ of \$24.00 = \$6.00, Ans.

Always use aliquot parts when possible, as:

8 hr. = $\frac{8}{48}$ or $\frac{1}{6}$ of a 48-hour week;
12 hr. = $\frac{12}{48}$ or $\frac{1}{4}$ of a 48-hour week; etc.

EXAMPLE: 9 hr. @ \$30.00 per week on a 48-hr. basis = ?

8 hr.	= $\frac{1}{6}$ wk.; $\frac{1}{6}$ of \$30.00	= \$5.00;
1 hr.	= $\frac{1}{8}$ of 8 hr.; $\frac{1}{8}$ of \$5.	= 0.63;
9 hr.		= \$5.63, Ans.

When the number of hours worked is an aliquot part of a week plus or minus a fraction of such aliquot part, find the amount corresponding to the aliquot part and add or subtract the amount corresponding to the fraction of the aliquot part.

EXAMPLE: 43 hr. @ \$25.00 per week on a 48-hr. basis = ?

$\frac{43}{48}$ of \$25.00 = \$$\frac{1075}{48}$;

$$48\overline{)\$1075.00}\quad \$22.40$$

Ans., \$22.40

When the use of aliquot parts is impossible, find the proper fractional part of the rate per week, but always multiply by the numerator before you divide by the denominator because you will usually have to multiply a difficult fraction if you first divide by the denominator to find the rate per hour and then multiply by the numerator.

EXAMPLE: $42\frac{3}{4}$ hr. @ \$27.00 per week on a 48-hr. basis = ?

$$\frac{42\frac{3}{4}}{48} = \frac{42.75}{48.00};\ \frac{4275}{\cancel{4800}_{16}} \times \frac{\cancel{2700}^{9}}{1} = \frac{38475}{16} \text{ or } \$24.05, \text{ Ans.}$$

$$\frac{42\frac{3}{4}}{48} = \frac{42\frac{3}{4} \times 4}{48 \times 4} \text{ or } \frac{171}{192};\ \frac{\cancel{171}^{57}}{\cancel{192}_{64}} \times \frac{2700}{1} = \frac{153900}{64} \text{ or } \$24.05, \text{ Ans.}$$

When fractions of an hour are given as $\frac{42\frac{3}{4}}{48}$, clear by multiplying both terms of the fraction by 4 and continue as before, or clear by using decimals.

Examine this and state which is the shortest way.

Exercise 29—Oral.

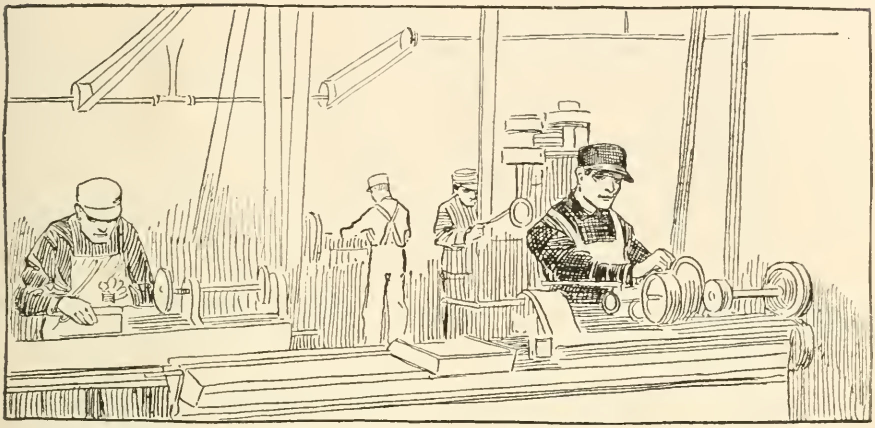
1. If a carpenter is paid on the basis of 45 hr. per week, what fraction of a week's wages does he receive for 1 hour's work?

2. A machinist is paid on the basis of 48 hr. per week what fraction of a week's wages does he receive for 23 hours' work?
3. An electrician is paid on the basis of 42 hours per week; what fraction of a week's wages does he receive for $39\frac{3}{4}$ hours' work?
4. At the rate of $32.00 for 48 hours' work, how would you find the amount of wages to be paid to a bricklayer for working 8 hours?
5. A teamster who is paid on the basis of $24.00 for 52 hours' work per week, worked $47\frac{1}{2}$ hours; how would you find the amount due him?
6. When aliquot parts cannot be used, why is it easier to multiply by the numerator first and then divide by the denominator, than it is to divide first to find the rate per hour and then multiply to find the amount for a certain number of hours?
7. What aliquot part of 48 hours is 6 hours? 8 hours? 12 hours? 16 hours? 24 hours?
8. What aliquot part of 42 hours is 6 hours? 7 hours? 14 hours? 21 hours?
9. What aliquot part of 45 hours is 9 hours? 15 hours? 18 hours? 27 hours? 30 hours? 36 hours?
10. When the number of hours worked is an aliquot part of the number of hours per week, what is the easiest way of finding the amount of wages to be paid?
11. How can you find the wages for 9 hours on the basis of 48 hours per week, using aliquot parts? How for 7 hours?

12. How can you find the wages for 10 hours on the basis of 48 hours per week? For 13 hours?

Exercise 30—Written.

Mr. Rice's Problems Concerning His Men's Wages.



1. In Department "A" the men are paid on the basis of 48 hr. per week; how much money would be paid to each of the following men:

   Adams worked 45 hr. @ $18.00 per week.
   Jones worked 48 hr. @ $24.00 per week.

2. In Department "B" the men are paid on a 48-hr. basis; how much money did he pay to each of the following workers:

   Bailey worked 8 hr. @ $21.00 per week.
   Davis worked 10 hr. @ $24.00 per week.

3. In Department "C" where the men are paid on a 45-hr. basis, what would be the total amount of the following pay roll:

2 men worked 45 hr. each @ $30.00 per week.
4 men worked 45 hr. each @ $18.00 per week.
3 men worked 42 hr. each @ $15.00 per week.
6 men worked 30 hr. each @ $21.00 per week.
5 men worked 30 hr. each @ $24.00 per week.

4. S. Smith works in one of the departments where the working hours are from 8.00 to 12.00 o'clock A. M. and from 12.30 to 5.00 o'clock P. M. from Monday to Friday inclusive, and from 8.00 A. M. to 1.30 P. M. on Saturday (without a stop for lunch); how much would Smith receive for working 36 hr. @ $24.00 per week?

5. J. Brown went to work at $8.00 per week in the shop where the working hours are from 8.30 to 12.00 A. M. and from 1.00 to 5.00 P. M. from Monday to Friday inclusive, and from 8.30 A. M. to 1.00 P. M. on Saturday; how much would he receive for working full time from Monday morning to Friday night?

6. At the rate of $24.00 for 48 hours' work, how many hours must James Fitzgerald work to receive $20.00?

7. At the rate of $33.00 for 45 hours' work, how many hours must Samuel Jones work to receive $27.50?

8. If Tom Daly is paid $15.00 for working 20 hours, how much would he receive for working a full week consisting of 48 hours?

9. At what rate of wages for a week of 48 hours must Wm. Beach work to receive $7.50 for 18 hours' work?

Lesson 15

Transposition in Figuring Wages

EXAMPLE: $7\frac{1}{4}$ hr. @ \$12.00 per week on a 48-hr. basis = ?
$7\frac{1}{4}$ hr. @ \$12.00 = 12 hr. @ \$7.25;
$\frac{12}{48} = \frac{1}{4}$; $\frac{1}{4}$ of \$7.25 = \$1.81, Ans.

We can "transpose" or change the order of the numerators in an example in cancellation without affecting the result, as is very readily proven. By using this simple device in figuring wages, much time and work can be saved.

Is 2×4 the same as 4×2? Is $10 \times 12\frac{1}{2}$ the same as $12\frac{1}{2} \times 10$?

Supposing we wanted to find the amount of wages to be paid for $37\frac{1}{4}$ hours' work at \$16.00 per week on a basis of 48 hours per week; if we transpose the hours worked and the rate per week, the example would read: 16 hours @ \$37.25 per week on a 48-hour basis, and could be worked very easily by aliquot parts as follows: $\frac{16}{48} = \frac{1}{3}$ week; $\frac{1}{3}$ of \$37.25 = \$12.42, which is the answer. We chose $\frac{16}{48}$ because 16 is an aliquot part of 48. Watch for the relation of one number to another.

$\frac{16}{48}$ of \$37.25 gives the same answer as $\frac{37\frac{1}{4}}{48}$ of \$16.00.

Therefore, when the rate per week is an aliquot part of the weekly hour basis, call the hours "dollars" and the dollars "hours" and work as usual. In this way, wages for almost any number of hours at \$6.00, \$8.00, \$12.00, etc., per week on a 48-hour basis can be figured by the use of aliquot parts without using paper.

Exercise 31—Oral.

1. What is meant by transposition?
2. Transpose these numbers: 8, 12.
3. When the rate per week is $8.00 and the basis is 48 hours, how can we simplify the work of finding the wages for $37\frac{3}{4}$ hr.?
4. When the basis is 45 hr. per week, and the rate is $15.00, how can we find the wages for $41\frac{1}{2}$ hours quickly?
5. What amount of wages should be paid for 30 hours' work at $22.00 per week on a 44-hour basis?

Exercise 32—Oral and Written.

A. Tell how to work each of the following examples:

On the basis of 48 hours per week, what amount of wages should be paid for:

1. 16 hr. @ $20.00 per week?
2. 20 hr. @ $16.00 per week?
3. $37\frac{1}{2}$ hr. @ $24.00 per week?
4. $40\frac{3}{4}$ hr. @ $12.00 per week?

On the basis of 45 hours per week, what amount of wages should be paid for:

5. 15 hr. @ $22.00 per week?
6. 22 hr. @ $15.00 per week?
7. $26\frac{1}{4}$ hr. @ $22.50 per week?
8. 37.5 hr. @ $30.00 per week?

On the basis of 42 hours per week, what amount of wages should be paid for:

9. $36\frac{3}{4}$ hr. @ $21.00 per week?

10. $4\frac{1}{2}$ hr. @ $14.00 per week?

B. Now work all of them, but do as much of the work as possible mentally.

Exercise 33—Oral Review.

1. Find the interest on $300.00 for 30 days at 6%. For 60 days at 5%.

2. Add:

(*a*)	(*b*)	(*c*)
3,687	1,325	1,498
498	897	867

3. Subtract:

(*a*)	(*b*)	(*c*)
3,675	4,722	3,621
398	699	295

4. Multiply:

(*a*)	(*b*)	(*c*)
$72 \times 16\frac{2}{3}$	32×25	96×12.5

5. Divide:

(*a*)	(*b*)	(*c*)
$12 \div 33\frac{1}{3}$	$600 \div 25$	$400 \div 16\frac{2}{3}$

(Time yourself on #2, 3, 4, and 5 and state time used.)

6. How many square miles are there in a township? How many sections?

7. A certain wall was built by 8 men in 12 hours; how many man-hours of work were required? How many men would have been able to build this wall in 6 hours?

8. Multiply without writing partial products:

(*a*)	(*b*)	(*c*)
67	25	89
23	15	31

9. At the rate of $28.00 for 48 hours' work, how much should be paid to a conductor who worked 36 hours?
10. At the rate of $36.00 for 48 hours' work, how much should be paid to a motorman who worked 28 hours?
11. What is the interest on $75. for 60 days at 6%?

Exercise 34—Written Review.

1. A furniture dealer bought the following merchandise from a manufacturer on April 16, 1918:

   24 chairs @ $5.25
   12 tables @ 18.00
   6 lamps @ 8.33⅓

   To pay for this merchandise, he gave a note bearing 5% interest, due in 120 days. What was the date of maturity of the note? What amount was paid in full settlement?
2. A real estate broker received a commission of $600.00 for selling a piece of property; if his commission was figured at 6%, what was the amount of sale?
3. What per cent of 48.75 is 32.5?
4. What is the cost of 3,500 sheets of 21″ × 28″ book paper @ $6.00 per cwt. on the basis of 42″ × 56″ = 92# per ream?

5. How many board feet of lumber are there in the following shipment:

60 pieces 8′ × 6″ × 2″
75 pieces 12′ × 10″ × 1″
100 pieces 12′ × 4″ × 2″

How much is this lumber worth @ $35.00 per M.?

6. $\frac{2}{3}$ is what % of $\frac{9}{10}$?

7. The men employed by a certain contractor work on this schedule: 5 days per week, 8.00 to 12.00 A. M.; 12.30 to 5.15 P. M.; Saturday, 8.00 A. M. to 12.15 P. M.; how many hours per week must these men work to receive full time? How much would a man working for $24.00 per week lose by being absent from work one afternoon?

Add, but do not copy:

(Time for these 7 examples is less than $4\frac{1}{2}$ minutes.)

8.	9.	10.
29,841	248,940	486,312
476	37,297	449
97	8,767	86,498
480	542	73,812
9,520	71	419
497	487	98
48	97,282	586
776	741	138
27,987	72	45

11.	12.	13.	14.
48,982	92,872	92,349	97,431
27,911	28,947	21,572	87,287
88,498	47,926	88,472	27,478

Subtract, but do not copy:

(Time for these 12 examples is less than $4\frac{1}{2}$ minutes.)

15.	16.	17.	18.
768,175	943,275	429,381	676,275
648,987	796,387	329,489	487,386

19.	20.	21.	22.
978,947	272,983	913,462	487,896
784,958	193,884	483,493	279,997

23.	24.	25.	26.
289,177	892,947	285,102	484,801
192,812	496,363	97,570	379,989

Copy and multiply:

(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

27. 4,746 × 97; **30.** 9,796 × 63;
28. 9,728 × 49; **31.** 8,632 × 87;
29. 2,864 × 86; **32.** 124 × 36.

Copy and divide:

(Time for these 15 examples is less than $4\frac{1}{2}$ minutes.)

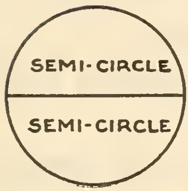
33. 1,269 ÷ 47; **38.** 4,465 ÷ 95; **43.** 1,222 ÷ 47;
34. 2,736 ÷ 38; **39.** 1,564 ÷ 46; **44.** 9,409 ÷ 97;
35. 864 ÷ 24; **40.** 729 ÷ 27; **45.** 918 ÷ 27;
36. 1,316 ÷ 47; **41.** 644 ÷ 23; **46.** 864 ÷ 36;
37. 918 ÷ 34; **42.** 884 ÷ 26; **47.** 1,620 ÷ 45.

MENSURATION

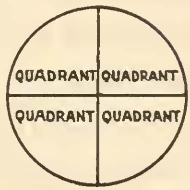
Lesson 16

The Circle

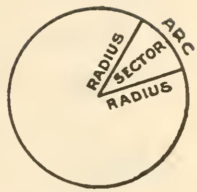
A circle.



A circle divided into two semi-circles by a diameter.



A circle divided into four quadrants by two diameters drawn at right angles.



A circle showing two radii, a sector, and an arc.

A "circle" is a plane figure bounded by one continuous curved line, which at all points is a uniform distance from a point in the center of the figure.

The distance around a circle is its "circumference" just as the distance around a square or oblong is its "perimeter."

A straight line drawn from one point in the circumference to another point in the circumference through the center is a "diameter." A diameter divides a circle into two halves, each of which is called a "semi-circle," "semi" meaning half.

Two diameters drawn at right angles to each other divide a circle into four quarters, each quarter being called a "quadrant."

A straight line drawn from the center to a point in the circumference is called a "radius," and several such lines are called "radii."

Any part of the circumference of a circle is called an "arc," and the figure bounded by two radii and an arc is called a "sector" of the circle.

Exercise 35—Oral.

Go to the board to show all of these:

1. Be ready to draw a circle.
2. Draw the diameter or "through line" of this circle and say how long it is.
3. Into how many parts is the circle now divided? What is each of these parts called?
4. Draw another diameter at right angles to the first diameter. How long is this second diameter? Into how many parts is the circle now divided? Name each part.
5. Rule two radii so that the ruler will show a distance of 1 inch between the points where the two radii touch the circumference. What is the length of each of these radii?
6. What is the part of the circumference between the two radii called? What is the remainder of the circumference called?
7. What is the figure bounded by the two radii and that part of the circumference which lies between the two radii, called?
8. What is a circle? What is half a circle called? What is a quarter of a circle called?
9. What is the curved boundary of a circle called?
10. What is a line drawn through the center, touching the circumference at two opposite points, called? Such a line divides the circle into what?

11. What is a line drawn from the center to the circumference of a circle called? Give one word for many of them.
12. What is any part of the circumference of a circle called?
13. What is the figure bounded by two radii and an arc called?
14. How many sides has it?

Lesson 17

The Ratio of the Circumference to the Diameter

Measure the circumference and diameter of a coin, a wheel, a plate, and a spool. Arrange them in order. Compare each C with its D.

(Coin) C : D = ?
(Wheel) C : D = ?
(Plate) C : D = ?
(Spool) C : D = ?

Write the ratio for each of these. Is the ratio about the same each time? What do we know is the relation of C to D?

The ratio shown by your comparison is a little over 3, or nearly 3.1416; therefore, for all ordinary purposes, 3.1416 or $3\frac{1}{7}$ will be used. This ratio is expressed by the Greek letter π which is called "pi" (pronounced pī).

By using the letter C to represent the circumference, D for diameter, R for radius, and π for pi or 3.1416, we can show that the circumference equals the diameter multiplied by pi in this manner:

$$C = D \times \pi.$$

Making a statement in this form is called an "equation." An equation is a statement showing the equality of two quantities by placing one before and one after an equality sign.

Exercise 36—Oral.

1. Express the ratio 3.1416 as a common fraction (approximately).
2. If you have D, what must you do to get C?
3. If you have C, how would you find D?
4. If the diameter of the driving wheel on a locomotive is 10 feet, tell how to find the circumference of the wheel. How many feet would this wheel travel on the track in making one revolution?
5. Express the relation of D to R.
6. How can we find the radius when the diameter is known?
7. How can we find the radius when the circumference is known?
8. How can we find the circumference when the radius is known?
9. If C stands for circumference, D for diameter, R for radius, and π for pi, read the following equations:

   (*a*) $C = D \times \pi$;
   (*b*) $D = C \div \pi$;
   (*c*) $D = 2R$;
   (*d*) $C = 2R \times \pi$;
   (*e*) $D = \frac{C}{\pi}$;
   (*f*) $R = \frac{D}{2}$;

   This is a very short way of telling your rules.
10. Using equations, make statements to show:

   (*a*) That the diameter equals twice the radius;

(*b*) That the diameter equals the circumference divided by pi;
(*c*) That the circumference equals the diameter multiplied by pi;
(*d*) That the circumference equals twice the radius times pi;
(*e*) That the radius equals one-half the diameter;
(*f*) That the radius equals one-half of the quotient of the circumference divided by pi;

Exercise 37—(*a*) Oral. Tell how to work each of the following examples:
(*b*) Written. Find the circumference, diameter, or radius as required:

1. D = 4 in.; R = ?
2. R = 3 ft.; D = ?
3. D = 4 yd.; C = ?
4. C = 20.4204 in.; D = ?
5. R = 10 ft.; C = ?
6. C = 25.1328 in.; R = ?
7. Find the circumference of a table which has a diameter of 1 yd.
8. Find the radius of a circular lake which has a diameter of 100 yd.
9. Find the diameter of a circle which has a circumference of 9 yd. Carry it to 2 decimal places.
10. Find the circumference of a circle which has a radius of 1 yd. 2 ft. 3 in.

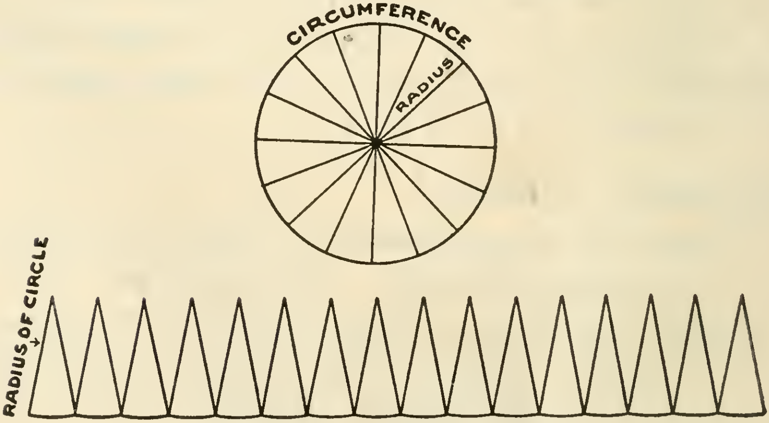
Exercise 38—Written.

1. Find the circumference of Tom's bicycle wheel if the diameter is 28″. (Use $3\frac{1}{7}$ for pi.)
2. The spokes in a certain wheel are 2 in. apart at the rim; what is the diameter of the wheel if there are 45 spokes? (Use 3.1416)

3. In the same wheel, what is the length of each spoke from the rim to the hub of the wheel, if the hub is 2.64 in. in diameter?
4. The circumference of a locomotive wheel is 24 ft.; how often must it revolve while the locomotive travels 1 mile?
5. The cogs on a cog wheel are $\frac{1}{2}$ in. apart; what radius must the wheel have if there are 224 cogs? (Use 3.1416)
6. The drum of a windlass has a diameter of 2 ft.; how many times must the drum revolve to move a block of granite 44 ft.? (Use $3\frac{1}{7}$.)
7. A circular running-track has a diameter of 84.033 ft.; how many laps must one run on this track to cover a mile? (Use 3.1416)
8. In running a 100-yard dash on this track, how many complete laps and how many yards in addition would one have to run?

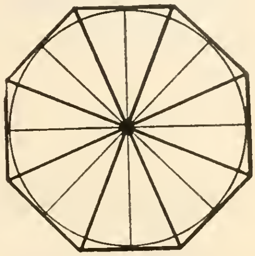
Lesson 18

Finding the Area of a Circle



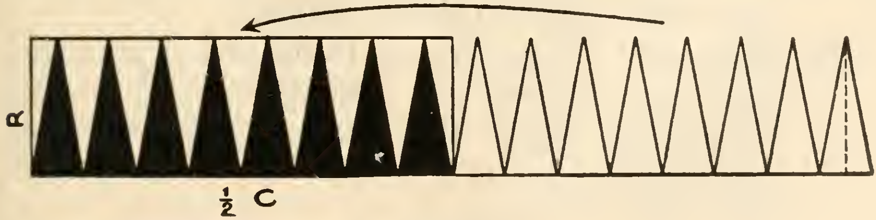
If you divide a circle into sectors and cut them apart as shown in the illustration, you will find that each sector would be a triangle, were it not for the fact that it has a slightly curved base instead of a straight base.

Cut out a circle having very few sectors; arrange them in a straight line. Cut out another circle of the same size having very many sectors; arrange them in a straight line. Compare the two groups of sectors.



Now study this figure very carefully. Can you see the radii of the circle? Can you see the altitude of one of the triangles? Is it the same? Now study the circumference of the circle and the perimeter of the eight-sided figure or octagon. You can see that there must be a little difference between the lengths of these, but supposing we used a sixteen-sided figure in place of an eight-sided figure then the perimeter would be much more nearly the same as the circumference of the circle, and if we used a figure with ever-so-many small sides there would be practically no difference whatever, and the area of all the little triangles would then be practically the same as the area of the circle.

As a matter of fact, that is exactly how we find the area of a circle, only we make a rectangle out of the little triangles by fitting them together as shown here



and we consider $\frac{1}{2}$ of the circumference of the circle as the base of this rectangle and the radius of the circle as the altitude of this rectangle which represents the area of the circle.

Since a circle with a diameter of 10 inches has a circumference of 31.416 in. and a radius of 5 in., therefore,

$$\frac{31.416 \text{ sq. in.} \times 5}{2} = 78.54 \text{ sq. in., area of circle.}$$

Your short rule or equation for finding the area of a circle therefore is:

$$\frac{C \times R}{2} = \text{Area; or } \frac{C \times D}{4} = \text{Area.}$$

Exercise 39—Oral.

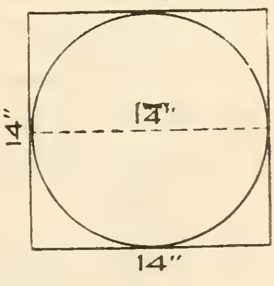
1. How do we find the area of a rectangle whose base is 3.1416 in. and whose altitude is $\frac{1}{2}$ in.?
2. If a circle has a circumference of 3.1416 in. and a diameter of 1 in., what is the length of its radius?
3. In finding the area of a circle, we think of the circumference of the circle as being what dimension of a rectangle?
4. In finding the area of a circle, we think of the radius of the circle as being what dimension of a rectangle?
5. How do we find the area of a circle if we know the radius and the circumference?
6. What must we get first or think first if we know the diameter and the circumference and want to find the area? R is what part of D? Then $\frac{C}{2} \times \frac{D}{2} = C \times \frac{D}{4}$.

7. How do we find the area of a circle if we know only the radius?
8. How do we find the area of a circle if we know only the diameter?
9. How do we find the area of a circle if we know only the circumference?

Exercise 40—Written.

Find the area of the following circles, using the simplest method in each case:

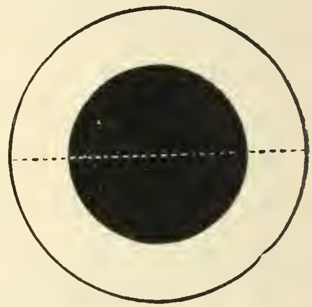
1. R = 4 ft.; C = 25.1328 ft.
2. D = 10 ft.; C = 31.416 ft.
3. R = 6 in.
4. C = 314.16 yd.
5. D = $\frac{1}{2}$ in.
6. A phonograph record has a diameter of 10 in.; what is its area?
7. A circular flower bed has a diameter of 15 ft.; what is its area?
8. What is the diameter of the largest circle which can be cut out of a 14-in. square? What is the area of this circle using $3\frac{1}{7}$ for π?
9. What is the area of the square? What is the difference between the area of the circle and the area of the square out of which it was cut? What is the ratio of the area of the circle to the area of the square? Make a new rule for finding area. If D is given, this would be a fine rule to use.



10. A bull's eye target has two sections; the inner section being white, and the outer section being black. If the diameter of the entire target is 4 ft., what is the area?

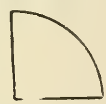
11. If the diameter of the white section of this target is 2 ft., what is its area?

12. How can you find the area of the black section? What is the area of the black section of the target?



13. A circular cement walk 2 yd. wide runs around a flower bed; if the diameter of the flower bed is 6 yd., what is the area of the cement walk?

14. A semi-circular paper pattern has a diameter of 8 in.; what is the area of this pattern?



15. A quadrant has a radius of 2 ft.; what is its area?

16. What is the area of the bottom of a round tin pail, the diameter being 14″? (Use $3\frac{1}{7}$ in cases of this kind.)

17. What is the area of the bottom of a drinking-cup, the radius being $1\frac{3}{4}$″? (Use $3\frac{1}{7}$ in cases of this kind.)

18. What is the area of a coin having a diameter of 1″? (Can you use $\frac{11}{14}$ of the square of the diameter in this case?)

Lesson 19

Finding the Area of the Surface of a Right (Rectangular) Prism

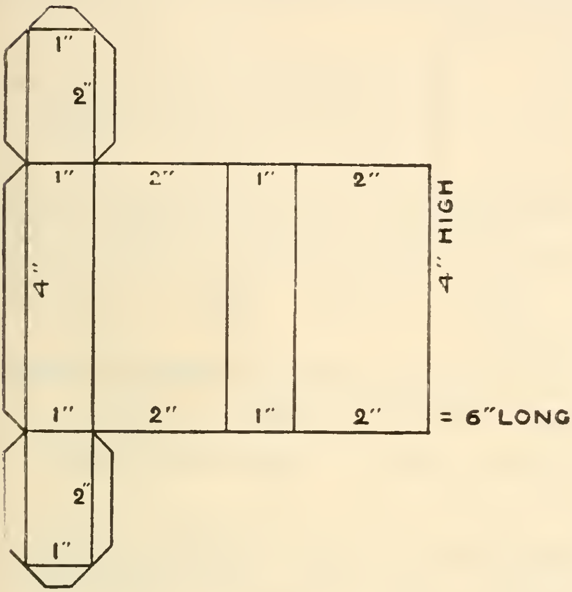


Figure I

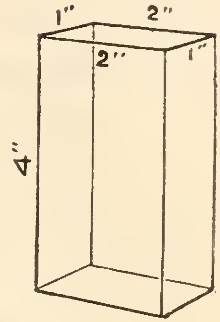


Figure II

Make a prism out of heavy paper.

Besides being able to find the volume of solids, we must also be able to find the area of their surfaces, for it is only in this way that we can tell how much lumber is required to construct a packing case, how much card-board is needed to make a shoe box, etc.

In the prism shown, the surface around is a rectangle 6″ long by 4″ high, containing 24 sq. in.

The two bases are rectangles 2″ long 1″ wide, each containing 2 sq. in. or a total of 4 sq. in.

The entire surface therefore contains 24 sq. in. + 4 sq. in., or 28 sq. in.

Exercise 41—Written.

Make drawings of the surfaces of the following rectangular prisms (to scale of $\frac{1}{4}''$ to 1″, or $\frac{1}{2}''$ to 1″) in the same way as is shown in Figure I, and find the area of the entire surface of each:

1. 3″ × 4″ × 10″;
2. 3′ × 6′ × 6′;
3. 4″ × 4″ × 4″;
4. 2″ × 2″ × 2′;
5. 2 yd. × 4 yd. × 8 yd.;
6. 1″ × 2″ × 4″.

7. How many square inches of cardboard are needed to make a box for an umbrella, the size of the box being 4″ wide, 4″ deep, 4′ high, if no allowances are made?

8. How many square feet of zinc are needed to line a case 2′ deep, 3′ wide, 4′ long?

9. How many square yards of paper are needed to line a case 4′ × 4′ × 4′?

10. How many board feet of 1″ lumber are needed to make a box with a cover, the outside dimensions being 20″ × 30″ × 40″?

11. Find the area of the entire surface of a prism 2″ × 6″ × 8″.

12. Find the area of the entire surface of a cube $8\frac{1}{2}''$ each way.

13. Find the area of the entire surface of a cube having 5″ edges.

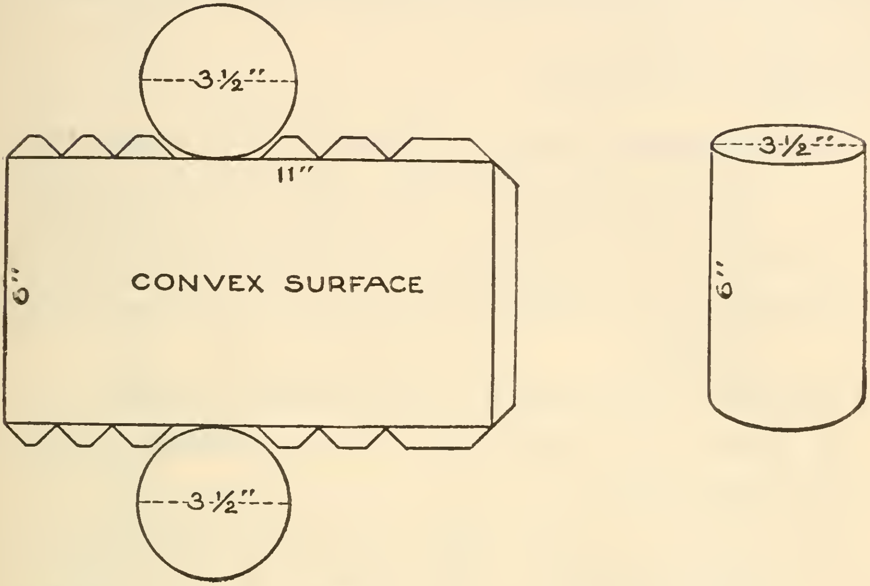
14. Find the area of the entire surface of a prism 4″ × 4″ × 6″.

15. Find the area of the entire surface of a cigar box 2″ × 6″ × 8″.

16. Find the area of the four walls, floor, and ceiling of a room 14′ × 22′ × 10′.

Lesson 20

Finding the Area of the Surface of a Cylinder



Make a cylinder out of heavy paper.

A "cylinder" is a solid bounded by a uniformly curved side and two parallel circular ends or bases of equal size. Pipes, tin cans, and round pencils usually have the shape of cylinders.

To find the area of the entire surface of a cylinder, we must find the sum of the areas of the curved side and the two circular bases.

You already know how to find the areas of the two circular bases; therefore, the only new point for you to learn is how to find the area of the curved side. Can you tell how? (Use the terms of the cylinder.)

In the drawing here shown, we find the circumference of the base to be 11″; therefore, the curved side is 11″ long and 6″ wide, its area being 11 sq. in. × 6 = 66 sq. in.; the area of each base is $\frac{1}{2}$ of 11 sq. in. × ($3\frac{1}{2} \div 2$) = $9\frac{5}{8}$ sq. in. or $19\frac{1}{4}$ sq. in. in both bases, or 66 sq. in. + $19\frac{1}{4}$ sq. in. = $85\frac{1}{4}$ sq. in. in the area of the entire surface.

Exercise 42—Written.

1. Cut a paper into an oblong 4″ × 8″, and form a hollow cylinder 8″ long; what is the circumference of this cylinder? Watch the bases carefully.
2. What is the area of the curved side of this cylinder?
3. What is the diameter of one of the bases of this cylinder?
4. What is the area of each of the two bases?
5. What is the total area of the curved side and the two bases?
6. A section of rain pipe is 6″ in diameter and 36″ long; what are the dimensions of a sheet of galvanized iron of the correct size to make this pipe, allowing $\frac{1}{4}$″ for the seam?
7. What is the area of this sheet of iron?
8. A cylindrical smoke-stack is 12′ high and 1′ in diameter; what is the area of its curved side? How many square feet of metal were needed to make this stack, if the metal overlaps 3″ at the seam?
9. What is the area of the curved side of a pencil 7″ long and $\frac{3}{8}$″ in diameter?
10. A mailing tube is 20″ long and 3″ in diameter; what is the area of its curved side? How many such tubes could be cut from a sheet of cardboard 20″ by 37.7″, allowing nothing for the seam?

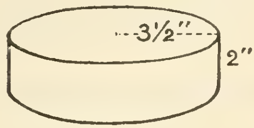
Exercise 43—Oral.

1. A rectangular prism has how many sides? How many bases? How many surfaces?
2. How do we find the area of the entire surface of a rectangular prism?
3. Name the dimensions of each of the surfaces of a prism $2'' \times 4'' \times 8''$.
4. How many sides has a cylinder? How many bases? How many surfaces?
5. What is the shape of the side of a cylinder when the curve is straightened?
6. What is the shape of each of the bases of a cylinder?
7. Take a cylinder and tell how we find the dimensions of the curved side.
8. How do we find the area of the curved side of a cylinder?
9. How do we find the area of each of the two bases of a cylinder?
10. Take a cylinder and tell the class all you can about it. Take 2 minutes.
11. Take a prism and tell the class all you can about it. Take 2 minutes.
12. A rectangular prism has how many edges?
13. The dimensions of a rectangular prism are $5'' \times 10'' \times 15''$; how many of its edges are 5″ long? How many of its edges are 10″ long? How many of its edges are 15″ long?
14. A cylinder has how many edges?
15. The diameter of a cylinder is 10″; what is the length of each of its edges?

Exercise 44—Written.

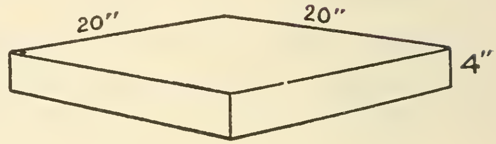
Problems on prisms and cylinders.

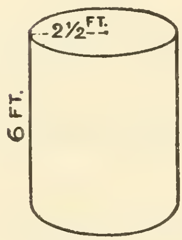
(Use Common Fractions.)

1. 

Convex surface = ?

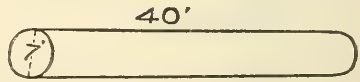
2. Entire surface = ?

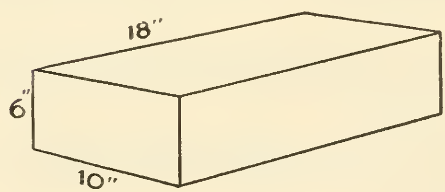


3. 

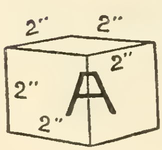
Entire surface = ?

4. Convex surface = ?

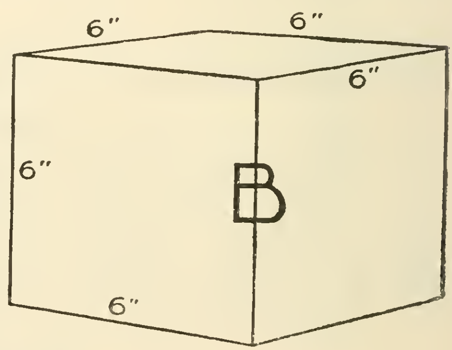


5. 

Entire surface = ?

6. 

How many like A can you find in B?

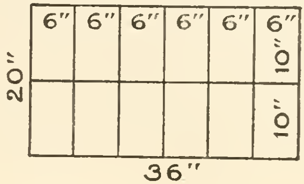


Lesson 21

Cutting Material to Avoid Waste

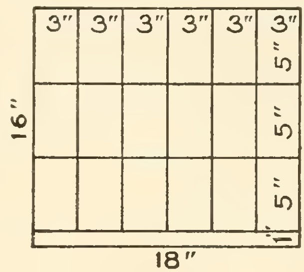
EXAMPLE: How many sheets 6″ by 10″ can be cut from a sheet of paper 20″ by 36″?

$\underset{6)\overline{36}}{6} \times \underset{10)\overline{20}}{2}$ = 12 sheets.



EXAMPLE: How many cards 3″ by 5″ can be cut from a sheet 16″ by 18″?

$\underset{5)\overline{16}}{3} \times \underset{3)\overline{18}}{6}$ = 18 cards.
(1″ by 18″ remainder)



Material such as paper, tin, cloth, etc., often comes in large sheets or pieces out of which smaller sheets or pieces of certain sizes are to be cut. We have paper cutting for our work; let us see if we are economical.

While it is always possible to cut a sheet having an area of 48 sq. in. into two sheets each having an area of 24 sq. in., it is not possible to obtain two sheets having dimensions of 6″ by 4″ unless the original sheet has dimensions of 12″ by 4″ or dimensions of 6″ by 8″; therefore, great savings are made by using material of the proper size.

To tell quickly how many rectangular sheets of a certain size can be cut from a rectangular sheet of another size, and also to determine how much waste there will be, we divide one of the dimensions of the large sheet by one of the dimensions of the small sheet to see how many it will make, then we divide the other dimension of the large sheet by the other dimension of the small sheet (*in each case using as the divisor, that dimension which will leave the smallest remainder*), and then we find the product of these two quotients.

EXAMPLE: How many letter heads $8\frac{1}{2}''$ by $11''$ can be cut from a sheet $30\frac{1}{2}''$ by $34''$?

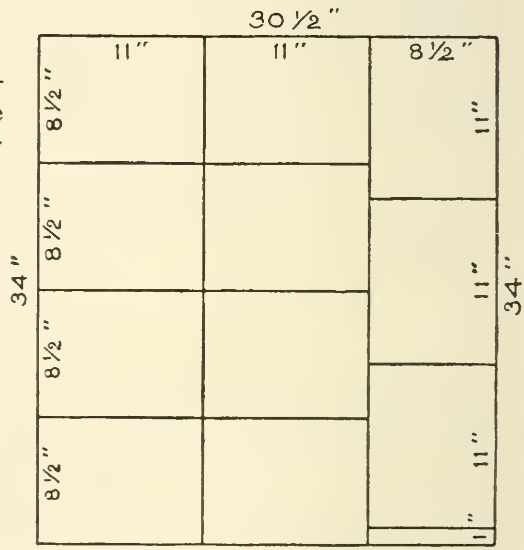
$$\begin{array}{ccc} 4 \times & 2 & = 8 \\ 8\frac{1}{2})\overline{34} & 11)\overline{30\frac{1}{2}} & \end{array}$$

($8\frac{1}{2}''$ by $34''$ rem.)

$$\begin{array}{ccc} 1 \times & 3 & = 3 \\ 8\frac{1}{2})\overline{8\frac{1}{2}} & 11)\overline{34} & \end{array}$$

($1''$ by $8\frac{1}{2}''$ rem.)

Total, $\overline{11}$ Letter Heads.



Sometimes it is most economical to cut the material so that there will be a strip remaining which will be wide enough to be used in the opposite direction.

Exercise 45—Oral.

1. What is the area of a piece of tin $3'' \times 8''$?
2. What is the area of a piece of tin $2'' \times 6''$?
3. The answer to Question 1 was what? The answer to Question 2 was what? How many times as large as No. 2 is No. 1?

4. Can we cut No. 1 into two pieces of the dimensions of No. 2? Why?
5. What dimensions should No. 1 have in order that we might cut it into two of No. 2?
6. What other dimensions might No. 1 have which would also enable us to cut it into two of No. 2?
7. Explain how you can find out how many 5″ × 8″ cards can be cut from a sheet 24″ × 25″ in size?
8. How can you tell if there will be any waste?
9. In each of the divisions, how can you tell which dimension of the small sheet should be used as the divisor?
10. How can we sometimes cut the material to better advantage than at first appears possible?

Exercise 46—Written.

1. How many cards 4″ × 6″ can be cut from a sheet 18″ × 20″? How much waste will there be? Show this by a diagram drawn to the scale of $\frac{1}{4}$.
2. How many sheets of tin 8″ × 10″ can be cut from a sheet 18″ × 20″? What are the dimensions of the remainder if there is one?
3. How many pieces of cloth 4″ square can be cut from a piece 12″ × 14″? What is the area of the remainder if there is one?
4. How many letter heads $8\frac{1}{2}$″ × 11″ can be cut from a sheet of paper 22″ × 34″? What is the area of the remainder if there is one?
5. How many note heads $5\frac{1}{2}$″ × $8\frac{1}{2}$″ can be cut from a sheet of paper 34″ × 44″? How many can be cut from a ream of such paper?

6. How many circulars $6'' \times 9\frac{1}{2}''$ can be printed at one time on a sheet of paper $24'' \times 38''$? How many square inches would be wasted? How many reams of $24'' \times 38''$ paper would be needed for 8,000 circulars?
7. The front and the back covers of a magazine are printed on one sheet of paper; if the finished magazine is $10\frac{1}{2}'' \times 14''$, what is the size of the complete cover as it comes off the press? How many complete covers can be printed from a sheet $42'' \times 63''$?
8. If both sides of a sheet of paper $24'' \times 38''$ are printed and made into a book $6'' \times 9\frac{1}{2}''$ in size, how many pages will the book have?
9. How many letter heads $8\frac{1}{2}'' \times 11''$ can be cut from a sheet $30\frac{1}{2}'' \times 34''$?
10. A pan of candy is $20'' \times 30''$; into how many pieces $\frac{3}{4}'' \times 1\frac{1}{4}''$ can it be cut?

Lesson 22

Finding the Volume of a Cylinder

A "cylinder," as you know, is a solid bounded by a uniformly curved side and two parallel circular ends or bases of equal size.

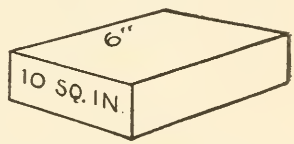
As in the case of the prism, we find the volume of a cylinder by finding how many cubic units there are in each layer of the cylinder, and multiplying this by the number of layers. Therefore, we find the area of one of the bases, make that one unit high so that we may know how many cubic units there are in each layer, and multiply by the altitude as that shows the number of layers, to find the volume.

EXAMPLE: Find the volume of a cylinder whose altitude is 10 in., the diameter of the base being 4 in.

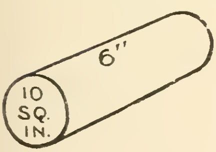
The area of the base = 12.5664 sq. in.; if 1 in. high, there are 12.5664 cu. in. in each layer; as the height is 10 in., there are 10 layers; therefore, the volume = 12.5664 cu. in. × 10, or 125.664 cu. in., Ans.

Exercise 47—Oral.

1. Volumes call for how many dimensions?
2. Volumes call for the use of what table?
3. The altitude of a prism is 6 in. and its base has an area of 10 sq. in.; how many cubic inches are there in each layer? How many layers are there? How many cubic inches are there in the prism? What is the volume of this prism?



4. The altitude of a cylinder is 6 in. and its base has an area of 10 sq. in.; how many cubic inches are there in each layer? How many layers are there? How many cubic inches are there in the cylinder? What is the volume of this cylinder?



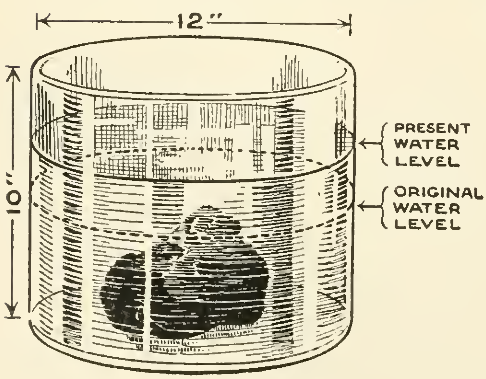
5. How many cubic inches are there in each layer of a cylinder if its base has an area of 5 sq. in.? How many layers are there if the altitude of the cylinder is 10 in.? What is the volume?

6. Name some prisms that you see or know about.
7. Name some cylinders that you see or know about.
8. How do we find the volume of a prism?
9. How do we find the volume of a cylinder?
10. Talk about a prism for one minute.
11. Talk about a cylinder for one minute.
12. How do you find the area of the base of a cylinder if you know the diameter?

Exercise 48—Written.

Find the volume of the following cylinders:

1. Area of base = 14.5 sq. in.; Altitude = 1 ft.
2. Area of base = 45 sq. yd.; Altitude = 21 ft.
3. Area of base = 23 sq. ft.; Altitude = 2 yd.
4. An oil tank is $1\frac{2}{3}$ ft. in diameter and 4 ft. high; how many gallons of oil will it hold if there are $7\frac{1}{2}$ gallons in each cubic foot?
5. A cylindrical fish tank or aquarium is 12″ in diameter and 10″ high; how many gallons of water does it contain when completely filled?



6. The tank here shown was half full of water when a stone was placed in the bottom of it, making the water rise 2″; what was the volume of the stone?

(NOTE: The volume of any insoluble solid object of any shape can be measured by the volume of water it displaces. In this case the stone has a volume equal to a cylinder 12″ in diameter and 2″ high, since it made a cylindrical body of water 12″ in diameter rise 2″.)

7. If a tank had a diameter of 4 ft., and placing a stone in it made the water rise 6 in., what would be the volume of the stone? Draw carefully.
8. How many cubic feet of gas must enter a gas tank 200 ft. in diameter, to make it rise from a height of 10 ft. to a height of 50 ft.? Draw carefully.
9. A silo is 40 ft. in diameter and 80 ft. high; what is its volume in cubic yards?
10. A cylindrical cistern is 10 ft. deep and 4 ft. in diameter; how many gallons of water will it hold?

Exercise 49—Oral Review.

1. How many sections are there in a township? How many square miles? What are the dimensions of a township?
2. If a bricklayer's wages are $30.00 for a week of 48 hours, how much will he receive for working 24 hours?
3. If a plasterer's wages are $24.00 for a week of 48 hours, how much will he receive for working 30 hours?
4. What is the interest on $400.00 for 90 days at 6%?
5. What is the ratio of the circumference of a circle to the diameter?
6. If the diameter of a circle is 10″, what is the circumference?
7. If the dimensions of a box are 3′ × 4′ × 5′, what are the dimensions of each of the 6 surfaces?

8. What is the perimeter of a 4″ square? What is the area?

9. Multiply:

(a)	(b)	(c)
$60 \times 12\frac{1}{2}$;	56×25;	$33 \times 33\frac{1}{3}$.

10. Divide:

(a)	(b)	(c)
$30 \div 33\frac{1}{3}$;	$800 \div 16\frac{2}{3}$;	$12 \div 25$.

(Time for #9 and #10 should be 3 minutes or less.)

Exercise 50—Written Review.

1. What is the interest on $438.00 from March 8, 1920, to May 22, 1920, at 5%?
2. When a case containing 3 reams of paper was opened, 465 sheets were found to be damaged; what percentage of the paper was undamaged?
3. Stating your answer in sq. yd., sq. ft., and sq. in., what is the area of a circle having a circumference of 942.48 in.?
4. What is the volume of a section of water main 4′ in diameter and 10′ long?
5. What is the area of the curved surface of 10 sections of this water main?
6. What is the volume of a box 4″ × 6″ × 18″? How many 2″ cubes will it hold?
7. What is the total area of all the surfaces of this box? (Answer in square inches.)
8. If this box is made of wood $\frac{1}{4}$″ thick, and a cubic foot of this wood weighs 54#, what is the weight of this box?

Add, but do not copy:
(Time for these 5 examples is less than $4\frac{1}{2}$ minutes.)

9.	10.	11.	12.	13.
83,062	2,763	1,845	1,972	4,638
912	38,745	4,618	219	7,319
8,746	428	7,594	35,211	8,242
29,428	36	8,730	25	3,625
35	46,812	3,948	740	1,745
7,318	7,429	9,378	4,583	9,721
				8,631
				4,728
				9,386
				5,581
				7,003
				1,784

Copy and divide:
(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

14. $27,218 \div 62$; **17.** $29,580 \div 87$;
15. $22,742 \div 83$; **18.** $29,029 \div 91$;
16. $31,434 \div 39$; **19.** $37,666 \div 74$.

Copy and multiply:
(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

20. 434×24; **23.** $6,831 \times 123$;
21. 639×53; **24.** $3,805 \times 416$;
22. 728×62; **25.** $7,312 \times 548$.

Subtract, but do not copy:
(Time for these 12 examples is less than $4\frac{1}{2}$ minutes.)

26.	27.	28.	29.
316,817	943,814	740,002	800,005
284,928	178,939	319,846	300,009

30.	31.	32.	33.
386,412	745,038	713,845	385,219
217,058	296,839	296,829	300,951

34.	35.	36.	37.
731,987	431,875	589,463	386,411
645,879	223,819	331,861	17,429

PERCENTAGE

Lesson 23

Successive Trade Discounts

In your previous work you have learned that "trade discount" is an amount allowed by wholesalers to retailers so that retailers can sell at list or advertised prices and still make a profit.

As some merchants allow more than one trade discount, we can have several discounts to be deducted one after the other; these are called "successive discounts."

In deducting successive discounts, each discount is figured on the net amount remaining after deducting the previous discount; therefore, a discount of 10% and 10% is not the same as a discount of 20%, because 100% less 10% = 90% and 90% less 10% of itself = 81%, while 100% less 20% = 80%.

	100%		100%
Less 10% of	10%	Less 20% of	20%
	90%		80%
Less 10% of	9%		
	81%		

Again, discounts of 10% and 5% do not equal 15%.

	100%		100%
Less 10% of	10%	Less 15% of	15%
	90%		85%
Less 5% of	$4\frac{1}{2}$%		
	$85\frac{1}{2}$%		

EXAMPLE:

$180.00 Less 20%, 10%, and 5% = ?

	$180.00
Less 20%	36.00
	144.00
Less 10%	14.40
	129.60
Less 5%	6.48
	$123.12

EXAMPLE:

$180.00 Less 5%, 10%, and 20% = ?

	$180.00
Less 5%	9.00
	171.00
Less 10%	17.10
	153.90
Less 20%	30.78
	$123.12

Successive discounts may be deducted in any order without affecting the final result; thus, a discount of 20%, 10%, and 5% is the same as a discount of 5%, 10%, and 20%, etc.

The difference between the original or "gross" price and the "net" price is the discount.

EXAMPLE: $180.00 Less 20%, 10%, and 5% = ?

	100%	
Less 20%	20%	
	80%	
Less 10%	8%	68.4% of $180.00 = $123.12;
	72%	
Less 5%	3.6%	
	68.4%	

From this it can be seen that taking 68.4% of any amount gives the same result as deducting 20%, 10%, and 5%.

Another method of figuring successive discounts, which saves considerable time when the same set of discounts is to be deducted from several amounts, is to find what per cent of 100% remains after the discounts are deducted and then find this percentage of the various list prices.

Exercise 51—Oral.

1. Is a trade discount of 20% and 10% the same as a trade discount of 30%?
2. Explain why.
3. Is a trade discount of 20% and 10% the same as a trade discount of 10% and 20%?
4. What is the general rule regarding the order in which trade discounts may be deducted?
5. If you are asked to find the net cost of an article listed at $150.00 the discount being 10% and 10%, state how you would proceed.
6. What is the gross price of an article?
7. What is the net price of an article?
8. What is the difference between the gross price and the net price of an article?
9. What is the easiest way of finding successive discounts when the same set of discounts is to be deducted from several amounts?
10. What per cent of 100% remains when a discount of 10% and 10% has been deducted?
11. If you were asked to deduct 10%, 20%, and 5% from six different amounts, how would you proceed to find the total discount?
12. When is it easier to find the per cent of the gross price which represents the net price? When is it easier to deduct the successive discounts one at a time?

Exercise 52—Written.

Find the trade discount and the net amount of each of the following bills and prove your work:

1. $140.00 less 10% and 5%;
2. $80.00 less 15% and 10%;
3. $200.00 less 10% and 10%;
4. $450.00 less 20% and 10%.
5. A wholesale grocer allows a discount of 20% and 10% from list prices on soap; what is the net price of a box containing 150 cakes of soap listed at 10¢ per cake?
6. A hat manufacturer allows a trade discount of 20% and 10% on his $3.00 hats; what is the net cost of 5 dozen of such hats?
7. What per cent of discount is equivalent to successive discounts of 20%, 10%, and 5%?
8. If a music publisher allows a discount of 40% and 10% from list prices, what would be the net amount he would charge for each of the following items:

10 Pieces of Sheet Music....	@ 50¢
20 Pieces of Sheet Music....	@ 75¢
12 Music Books............	@ $1.50
8 Music Collections........	@ $2.00
24 Songs..................	@ 25¢
12 Duets	@ $1.25

9. What is the discount on $125.00 at the rate of 20%, 20%, and 10%?
10. At the rate of 25%, 10%, and 10%, what is the discount on $160.00?

LESSON 24

Finding the Gross Amount When the Rates of Discount and the Net Amount Are Given

EXAMPLE: After deducting 10% and 10%, the net amount of an invoice is $324.00; what is the gross amount of the invoice?

$324.00 = 100% less 10% and 10%;	100%
therefore,	− 10% (10% of 100%)
$324.00 = 81% of the Gross Amount;	90%
$324.00 ÷ 81% = $400.00, Ans.	− 9% (10% of 90%)
	81% Net Rate Per Cent

As you have already learned, the gross amount always is 100%; therefore, when the net amount and the rates of discount are known, we must subtract the rates of discount successively from 100% so that we may know what per cent the net amount is of the gross amount, and then we must divide the net amount by this per cent to find 100% or the original bill.

Exercise 53—Oral.

Give the total discount rate per cent, also the net rate per cent of the following:

1. 10% and 10%;	7. 5% and 20%;
2. 20% and 10%;	8. 5% and 10%;
3. 10% and 20%;	9. 25% and 10%;
4. 10% and 5%;	10. 25% and 5%;
5. 25% and 20%;	11. 20% and 20%;
6. 20% and 5%;	12. 30% and 10%.

13. If the net amount of an invoice is $80.00 after deducting 20% and 20%, what is the first thing you would do if you were asked to find the gross amount?
14. Since a discount of 20% and 20% is the same as a discount of 36%, what per cent of the gross amount is $80.00 in Question 13?
15. Knowing that $80.00 is a certain per cent of the gross amount, how do we find the gross amount?
16. If 10% and 10% discount on a certain invoice is $19.00, how would you find the gross amount of the invoice?
17. If the discount on a certain invoice is $43.00, and the net amount of the invoice is $88.00, how would you find the gross amount?
18. In the example stated in Question 17, how would you find the rate of discount?
19. In Question 17, how would you find what per cent the net amount is of the gross amount?

Exercise 54—Written.

Solve and prove:

1. After deducting 20% and 10% discount, the net amount of an invoice is $108.00; what is the gross amount of the invoice?
2. After deducting 20% and 10%, the net amount of an invoice is $85.50; what is the gross amount of the invoice?
3. A clock manufacturer changed his trade discount from 20% to 10%; what per cent did he increase his net price?

4. If 25% and 10% discount on an invoice is $81.25, what is the gross amount of the invoice?
5. If 20% and 5% discount on an invoice is $57.60, what is the net amount of the invoice?
6. If 25% and 20% discount on an invoice is $64.00, what is the gross amount of the invoice?

Find the missing items in the following:

	Gross Amount	Net Amount	Rate of Discount	Rate Per Cent Net Amount is of Gross Amount
7.	$2,000.	?	20% and 10%	?
8.	$1,650.	?	10% and 10%	?
9.	$250.	?	10% and 5%	?
10.	?	$526.50	10% and 10%	?
11.	?	$3,600.	20% and 10%	?
12.	?	$873.	3% and 10%	?
13.	$5,000.	?	10% and 10% and 10%	?

Lesson 25

Insurance

Exercise 55—Oral.

In an Insurance Office.

Class is to select two girls and two boys to be the insurance men with offices in opposite sides of the room. Some of the pupils are to be ready to inquire about insurance for fire.

1. Questions are to be asked regarding value, rate, premium and amount of insurance possible:

Appointed insurance men in office must answer.

Choose some of these questions and get the appointed insurance men in office to answer you.

(*a*) For frame house;
(*b*) For brick house;
(*c*) For large apartment building;
(*d*) For a store;
(*e*) For frame house next to dry cleaning place;
(*f*) For doctor's office;
(*g*) For a dry cleaning place;
(*h*) For a brand-new house;

Get all the information you can.

Ask father, mother, teacher, and insurance men.

Class members may stand and call for a clearer understanding on any point.

Other pupils are to carry out this part of the work. Two or three pupils go as members of one family to take out insurance.

2. How much insurance can you get or do you want?
3. Who tells the value of the place?
4. Who decides how much you can get?
5. Is the rate always the same?
6. If it differs, why?
7. Go take out insurance to cover your building.
8. Go take out insurance to cover your furniture.
9. Go take out insurance to cover your personal property. (Tell what it is.)

Class members may stand and call for a clearer understanding on any point.

Other pupils are to carry out this part of the work. Two or three pupils go as members of one family to take out insurance

10. Ask for rates for 1 yr., 3 yr., 5 yr.
11. Choose.
12. Pay whom for the insurance? (The Insurance Company.)
13. What is that money called? (Premium.)
14. What is your contract or important paper called?
15. Where should you keep it? Why?
16. What is the "face" of the policy?

Exercise 56—Oral.

1. What is the premium on a \$2,000. insurance policy at $\frac{1}{2}\%$?
2. My barn is worth \$800. If I have an insurance policy on $\frac{3}{4}$ of its value at $\frac{1}{2}\%$, what must I pay for my insurance?
3. The Continental Insurance Company insures John Black's furniture against loss by fire to the extent of \$800. at 90¢ per yr. per \$100. Who is the insured? Who is the insurer? What is the written agreement called? How much premium did he pay? State the face of the policy. What kind of insurance is this?

   (Examine an insurance policy and be ready to talk about it.)
4. Can you name some other insurance?

5. What is Life Insurance? Accident Insurance? Marine Insurance? Employers' Liability Insurance?

Exercise 57—Written.

1. A stock of merchandise valued at $5,000. is insured at 45¢ per $100.; what is the premium on this policy?
2. A building valued at $8,000. is insured for 90% of its value at $1.20 per $100.; what is the premium?
3. A vessel worth $200,000. is insured for 80% of its value at $4\frac{1}{4}$%, and its cargo, worth $450,000., is insured for 90% of its value at $4\frac{3}{4}$%; what is the total premium on the two policies?
4. A man aged 30, insured his life in favor of his wife for $10,000., agreeing to pay an annual premium of $20.50 per $1,000. as long as he lived. He died at the age of 50. How much had he paid in premium during the 20 yr.? How much did his wife get from the insurance company?
5. If a premium of $80.00 was paid for insurance at $\frac{1}{2}$% to cover $\frac{4}{5}$ of the value of my house, what was the value of my house?

Lesson 26

Commission and Brokerage

You have already learned that "commission" is an amount of money which is paid by one person who is called the "principal" to another person who is called the "agent," for some service which the agent performs for the principal.

A "commission merchant" is one who sells produce or merchandise of a similar nature for his principal; however, while an ordinary salesman who sells on commission assumes no responsibility in regard to collecting for the goods he sells, a commission merchant is the loser if he fails to collect for the merchandise he sells.

A shipment of merchandise to be sold by a commission merchant is called a "consignment," and the amount of money which is to be paid to the principal after commissions and all other charges are deducted is called the "net proceeds."

A "broker" also sells on a percentage basis, but his business is usually confined to real estate, stocks, bonds, etc. The amount he receives for his work is called the "brokerage."

Exercise 58—Oral.

Commission and Brokerage Project

Select two girls to be real estate brokers, and two boys on the opposite side of the room to be commission merchants.

1. (The commission merchants are to impersonate Henderson & Co.; one of the pupils can impersonate John Smith, the shipper; others can impersonate the buyers; others can ask the questions shown; everyone may ask for information.)

   John Smith, who owns a fruit farm in Michigan, sent 100 crates of berries to Henderson &

Co., Commission Merchants, Chicago, Illinois. The berries were sold for $7.00 per crate, and a commission of 10% was charged. The freight on the shipment amounted to $5.00.

(*a*) In this transaction, who is the principal?
(*b*) Who is the agent?
(*c*) What did this consignment consist of?
(*d*) What amount was received for the berries?
(*e*) What amount of commission was charged?
(*f*) How much was paid to the principal as his *net proceeds* after deducting the commission and the freight charges?

2. (The real estate brokers are to impersonate Frederick Sharp; others can impersonate the seller, Henry Wise; others can impersonate the buyer; others can ask the questions shown; everyone may ask for information.)

A real estate broker named Frederick Sharp sold a farm belonging to Henry Wise for $10,000., receiving 5% as his brokerage.

(*a*) In this transaction, who is the principal?
(*b*) Who is the agent?
(*c*) Who is the broker?
(*d*) How much brokerage was paid? Who received it?
(*e*) What did the net proceeds amount to? Who received them?

Now some may ask these questions and others may answer; the real estate brokers must be ready to settle any point about brokerage; the commission merchants must be ready to settle any point about commission.

3. In commission and brokerage, who is the principal?
4. Who is the agent?
5. What is a commission merchant?
6. What is a real estate broker?
7. What is the amount called which a commission merchant receives for his services? Is it more or less than 100%?
8. What is the amount called which a principal receives? Is it more or less than 100%?
9. What is the amount called which a broker receives for his services? Is it more or less than 100%?
10. State whether commission or brokerage was paid on the following transactions, and give the amount so paid:
    (*a*) The sale of merchandise amounting to $500. at 6%;
    (*b*) The sale of fruit amounting to $300. at 12%;
    (*c*) The sale of produce amounting to $800. at 8%;
    (*d*) The sale of a house and lot for $8000. at 5%;
    (*e*) The sale of Liberty Bonds for $800. at $\frac{1}{8}$%.

Exercise 59—Written.

Solve and prove:

1. A commission merchant sold the following consignment of merchandise:

20 boxes Oranges..........	@ $2.00
24 boxes Lemons..........	@ 3.00
25 boxes Grape Fruit......	@ 2.80
12 bunches Bananas.......	@ 1.50

He received $8\frac{1}{2}$% commission; the freight, drayage, etc., amounted to $4.50; what amount of commission was paid? What did the net proceeds amount to?

2. What is the commission at 4% on 50 baskets of peaches sold at 60¢ per basket?

3. The commission on the sale of a certain lot of merchandise was paid at the rate of 8% and amounted to $32.00; what was the amount of the sale?

4. A consignment of 3,465 bu. of wheat was sold at $83\frac{1}{3}$¢ per bu.; what was the agent's commission at $2\frac{1}{4}$%?

5. A house and lot was sold for $6,575.00; the brokerage was 4%, and the attorney's fees, etc., amounted to $37.00; what were the net proceeds of this sale? If you were the broker, how would you send this money to the principal? Why?

6. If the brokerage on the sale of a certain farm was paid at the rate of $3\frac{1}{2}$% and amounted to $140.00, for what amount was the farm sold? How many acres were there if the price per acre was $50.00?

7. If 8% commission on the sale of a consignment of potatoes amounts to $11.40, and the potatoes were sold at $4.75 per bbl., how many barrels were there in the consignment?

8. The net proceeds from the sale of a consignment of eggs amounted to $38.40; if 4% commission was charged by the commission merchant, what amount was received for the eggs?

9. The net proceeds from the sale of a consignment

of apples amounted to \$665.00; if 5% commission was charged, and the apples were sold for \$1.40 per bbl., how many barrels were there in the consignment?

10. The net proceeds from the sale of a factory site amounted to \$7,560.00; if the brokerage amounted to \$540.00, what was the rate per cent of brokerage?

Lesson 27

Taxes

A "tax" is a sum of money which must be paid by the citizens to help defray the expenses of the national, state, county, and city governments, and for public schools, improvements, etc.

Taxes are of several kinds:

Real estate taxes on the assessed valuation of houses, lots, etc.

Personal property taxes on the assessed valuation of movable property.

Income taxes on salary and other income.

"Assessed valuation" means the estimated value of the property which is subject to the tax, or upon which tax is paid, as determined by the Assessor.

Taxes are charged on a percentage basis, but the percentage is sometimes stated as being a certain number of mills per dollar, or a certain number of cents per hundred dollars.

A certain portion of a person's income is usually not subject to income taxes; this is called an "exemption."

Exercise 60—Oral.

Tax Project

Select an Assessor; a Board of Review for the adjustment of complaints; a Tax Office Cashier.

Let the Assessor go around and assess the real and personal property of the property owners; the teacher will give the tax rates for the different townships (different parts of the class room); those who think they are assessed too high can go before the Board of Review for a hearing; others must pay their taxes at the Cashier's Office; some may ask the following questions and others may answer; all may ask for information; the Officials must be ready to settle disputes.

1. A man who owns a house and lot having an assessed valuation of $3,500. has to pay a tax at the rate of 2%.

(*a*) Name some of the purposes for which this tax may be used when collected.

(*b*) Is this a real estate tax, a personal property tax, or an income tax?

(*c*) What is meant by "assessed valuation of $3,500."?

2. A rich man has personal property of an assessed valuation of $100,000. and lives in a city where the tax rate is $1.75 per $100.

(*a*) What kind of a tax does this man pay?

(*b*) What rate per cent is this tax?

(*c*) Tell how to find the amount of this tax.

(*d*) What is the amount of this man's tax?

3. Why must citizens pay taxes?

4. What is a real estate tax?

5. What is a personal property tax?
6. What is an income tax?
7. Name three kinds of income that a man may receive.
8. What is meant by "assessed valuation"?
9. What is meant by "exemption"?
10. What is a Tax Assessor?
11. What is a Board of Review?

Exercise 61—Written.

1. If the tax rate is 2%, what is the tax on a house and lot having an assessed valuation of $12,500.?
2. If the tax rate is 13 mills on the dollar, what is the tax on a farm having an assessed valuation of $25,300.?
3. If the tax rate is $1.25 per $100., what is the tax on a piece of property having an assessed valuation of $9,540.?
4. If the assessed valuation placed on property is $33\frac{1}{3}$% of the actual value, and the tax rate is 5%, what would be the tax on an estate having an actual value of $90,000.?
5. If the assessed valuation of all the taxable property in a town is $1,530,000., and the following table shows the items that make up the tax rate, what is the total tax to be collected?

Per $100.

Schools......	$0.255	City.......	$2.16
Drainage....	.113	County....	.52
Parks.......	.085	State......	.11

6. What rate of taxation must be used to raise $10,942.50 in a town having a total assessed valuation of $875,400.?

7. In a certain town there is real and personal property assessed at $3,245,000.; if the rate of taxation is such that a total of $45,430. will be raised, how much must a man pay who has property assessed at $10,025.?

8. If a tax rate of 18 mills on the dollar yields $43,869.60, what is the assessed valuation of the taxable property?

9. A tax collector's commission is $2\frac{1}{2}\%$ on the amount of taxes collected. What will be his commission if the assessed value of the property is $6,875,400. and the rate of taxation is 2.37\frac{1}{2}$ per $100.00?

10. A certain community wishes to build a new town-hall to cost $48,500.; what rate of taxation must be used if the assessed valuation of the property is $2,000,000. and 3% of the amount collected must be paid to the collector?

11. How much income tax must Mr. Jones, a single man, pay if his income is $5,400. during a certain year and he is entitled to an exemption of $1,000., the tax rate being 2% on the first $2,500. of taxable income and 3% on the balance?

Mr. Tower's Tax Trouble

12. Mr. Tower has a married man's exemption under certain income tax laws of $2,000. plus $400. for each child under the age of 16 years. How much income tax must he pay if he has 3 young

children and his income is $14,350. during a certain year, the tax rate being as follows:

2% on the first $2,500. subject to tax;
3% on the next 2,500. subject to tax;
4% on the next 2,500. subject to tax;
5% on the next 2,500. subject to tax;
8% on the next 2,500. subject to tax;
10% on the next 2,500. subject to tax.

13. The following year, Mr. Tower's income amounted to $12,500. Allowing the same exemptions as before, how much tax must he pay, the tax rates being as follows:

2% on the first $2,000. subject to tax;
$2\frac{1}{2}$% on the next $2,000. subject to tax;
3% on the next $2,000. subject to tax;
$3\frac{1}{2}$% on the next $2,000. subject to tax;
4% on all over $8,000. subject to tax.

Lesson 28

Computing Interest When There Are Partial Payments

You already know that "interest" is money paid, or to be paid, for the use of money.

As we can charge interest on a sum for only such time as the sum remains unpaid, all partial payments on the principal must naturally affect the amount of interest; therefore, whenever a partial payment is made, the amount of the partial payment must be subtracted from the principal and the difference so found is the new principal to the date of the next partial payment.

Partial payments should be endorsed on the back of the note.

EXAMPLE: What is the amount due on Oct. 10, 1924, on the following note, payments having been made as shown by the endorsements on the reverse side:

The Face of the Note.

$450 00/100 Chicago, Ill. Oct. 10, 1923
Twelve months after date I promise to pay
to the order of Harry C. Black
Four Hundred & Fifty & 00/100 Dollars
At First National Bank
Value received with interest at 6 % per annum
No. 63 Due 10/10/24 Will B. Green

The Endorsements on the Reverse Side of the Note.

Received on Within Note
Feb. 10, 1924 $50.00
Harry C. Black
May 10, 1924 $50.00
Harry C. Black
Aug. 10, 1924 $100.00
Harry C. Black

Explanation:

Principal:

Principal 10/10/23	$450.00
Payment 2/10/24	50.00
Principal 2/10/24	400.00
Payment 5/10/24	50.00
Principal 5/10/24	350.00
Payment 8/10/24	100.00
Principal 8/10/24	250.00

Interest:

10/10/23 to 2/10/24 ($450. 4 mo. @ 6%)	$9.00	
2/10/24 to 5/10/24 ($400. 3 mo. @ 6%)	6.00	
5/10/24 to 8/10/24 ($350. 3 mo. @ 6%)	5.25	
8/10/24 to 10/10/24 ($250. 2 mo. @ 6%)	2.50	
Total Interest		$22.75
Amount due 10/10/24		$272.75, Ans.

Exercise 62—Oral.

1. What are the dates and the amounts of the payments on the note shown in the example?
2. Why is the interest from 10/10/23 to 2/10/24 figured on $450.?
3. What is the balance of the principal after deducting the payment of 2/10/24?
4. Why is the $50. payment of 5/10/24 deducted from the $400.00 principal?
5. Explain the entries made on 8/10/24 and state why each of these entries was made.
6. Explain each of the interest calculations, and show why each is figured on a different principal.
7. What is interest?
8. How do we figure the interest when there are partial payments?
9. If someone holds your note for $500.00 and you pay $100.00 on it, what kind of a receipt should you demand for the payment?
10. Why is the interest on any sum for 60 days at 6% equal to 1% of the sum?

What is the interest on each of these notes:

	Principal	Time	Rate
11.	$350.	60 days	6%
12.	$400.	30 days	6%
13.	$825.	120 days	6%
14.	$200.	90 days	6%
15.	$600.	60 days	5%
16.	$400.	30 days	3%
17.	$300.	60 days	4%
18.	$500.	90 days	6%

Exercise 63—Written.

What is the date of maturity and the amount payable at the date of maturity of each of these notes:

1. Principal, \$350.00; Date, Apr. 6, 1918; Time, 4 months; Int., 6%;
Partial payments: May 6, 1918, \$100.00;
July 6, 1918, \$100.00.

2. Principal, \$148.00; Date, Dec. 6, 1923; Time, 90 days; Int., 5%;
Partial payments: Jan. 6, 1924, \$50.00;
Feb. 6, 1924, \$50.00.

3.

\$ 3200 00/100 New York, Jan. 8, 1920
Ninety days after date I promise to pay
to the order of Frank Wilson
Thirty two Hundred & 00/100 Dollars
At City National Bank
Value received with interest at 5% per annum
No. 16 Due Apr. 7, 1920 Harry French

Payments Endorsed:
April 8, 1920, \$1,200.
June 8, 1920, \$1,000.

What amount was paid October 20, 1920, on which date the note was settled in full?

Lesson 29

Finding the Principal When the Time, Rate, and Interest Are Given

EXAMPLE: What principal will yield \$8.70 in 90 days at 6%?

Interest on \$1.00 for 90 days at 6% = $1\frac{1}{2}$¢;

\$8.70 ÷ \$0.015 = \$580.00, Ans.

Proof: Int. on \$580. for 90 days at 6% = \$8.70.

$$I \div (T \times R) = P$$

$$T \times R = \frac{90}{360} \times \frac{6}{100}, \text{ or } \$\frac{3}{200};\ I = \$8.70;\ \$8.70 \div \$\frac{3}{200} = \$8.70 \times \frac{200}{3}, \text{ or } \$580., \text{ Ans.}$$

EXAMPLE: The amount due at the maturity of a 30-day 6% note is \$452.25; what is the principal?

Interest on \$1.00 for 30 days at 6% is 5 mills; therefore, each \$1.00 amounts to \$1.005. If all the principal amounted to \$452.25 there were as many dollars as \$1.005 is contained times in \$452.25.

\$1.005)\$452.250 = 450 times; therefore, the principal is \$450., Ans.

$$A \div (T \times R + \$1.00) = P$$

$$T \times R = \frac{30}{360} \times \frac{6}{100}, \text{ or } \$\frac{1}{200};\ \$\frac{1}{200} + \$\frac{200}{200} = \$\frac{201}{200};$$

$$\$452.25 \div \frac{201}{200} = \$450., \text{ Ans.}$$

As you have learned, four elements—principal, rate per cent, time, and interest—are concerned in every interest example; therefore, if any three of these elements are given, the fourth can be found. Thus, in the work which you have been doing you have always known the principal, the rate per cent, and the time, and you have found the interest. $P \times R \times T = I$.

To find the principal which will yield a certain amount of interest at a certain rate per cent in a given time, we divide the given interest by the interest on \$1.00 for the given time at the given rate per cent, and as the interest on \$1.00 for the given time at the given rate equals $T \times R$, therefore, $I \div (T \times R) = P$.

Exercise 64—Oral.

1. The interest for 60 days at 6% is what per cent of the principal?
2. If we know the interest for 60 days at 6%, how can we find the principal? Show this by letters and signs.
3. If we know the interest for 90 days at 6%, how can we find the principal? Show this by letters and signs.
4. If we know the interest for 60 days at 5%, how can we find the principal? Show this by letters and signs.
5. If we know the interest for 120 days at 5%, how can we find the principal? Show this by letters and signs.
6. What per cent of the principal is the amount due at the maturity of a 60-day 6% note?
7. If the correct answer to Question 6 is 101%, and you know the amount due at maturity, how do you find the principal in such an example?
8. How do you find the principal of a 30-day 5% note, if the amount due at maturity is known? Show this by letters and signs.
9. What four elements are involved in every interest example?
10. How many of these elements must be known, and how many may be unknown in any example?

Exercise 65—Written.

Solve, and prove by finding the interest:

(Total interest ÷ interest on $1. = number of dollars in principal; I ÷ (T × R) = P).

What principal will yield:

1. $12.00 in 180 days at 6%?
2. $2.25 in 30 days at 5%?
3. $40.60 in 1 yr. 2 mo. at 4%?
4. $156.25 in 2 yr. 6 mo. at 5%?
5. $98.00 in 245 days at 6%?

Solve, and prove by finding the amount:

(Total amount ÷ amount of $1. = number of dollars in principal; A ÷ (T × R + $1.00) = P).

What principal will amount to:

6. $406.00 in 4 months at $4\frac{1}{2}$%?
7. $3,655.00 in 1 yr. 3 mo. at 6%?
8. $429.27 in 180 days at 5%?

Lesson 30

Finding the Time When the Principal, Rate, and Interest Are Given

EXAMPLE: In what time will $420.00 yield $1.75 at 5%?

The interest for 1 year at 5% on $420.00 = $21.00;

$1.75 ÷ $21.00 = $\frac{175}{2100}$, or $\frac{1}{12}$ year or 1 month, Ans.

$$\$1\tfrac{3}{4} \div \$21. = \frac{7}{4} \times \frac{1}{\not{21}_3}, \text{ or } \frac{1}{12} \text{ year or 1 month, Ans.}$$

I ÷ (P × R) = T

P × R = $420. × $\frac{5}{100}$, or $21.; I = $1.75; $1.75 ÷ $21. = $\frac{1}{12}$ year or 1 month, Ans.

To find the time in which a certain principal will yield a certain amount of interest at a certain rate per cent, we divide the total interest by 1 year's interest on the given principal at the given rate per cent, and as 1 year's interest on the given principal at the given rate per cent equals P × R, therefore, I ÷ (P × R) = T.

EXAMPLE: In what time will $500.00 amount to $525.00 at 6%?

Amount	=	$525.00	Interest on $500.00 for 1 yr. at 6% = $30.00;
Principal	=	500.00	
Total Interest	=	25.00	$25.00 ÷ $30.00 = $\frac{5}{6}$ year or 10 mo., Ans.

When the amount due at maturity is given, subtract the principal from the amount to find the total interest, then continue as before. A − P = I.

Exercise 66—Written.

Solve, and prove by finding the interest:

(Total interest ÷ interest for 1 year = number of years in time; I ÷ (P × R) = T).

In what time will:

1. $880.00 yield $10.45 at $4\frac{3}{4}$%?
2. $448.00 yield $36.96 at $5\frac{1}{2}$%?
3. $960.00 yield $45.60 at $6\frac{1}{3}$%?
4. $810.00 yield $31.50 at 5%?
5. $416.50 yield $83.30 at 6%?

Solve, and prove by finding the amount:

In what time will:

6. $72.00 amount to $86.82 at $6\frac{1}{2}$%?
7. $464.00 amount to $465.74 at 3%?

8. \$876.00 amount to \$883.30 at 5%?

9. \$84.00 amount to \$89.67 at $4\frac{1}{2}$%?

10. \$1,080.00 amount to \$1,143.00 at 7%?

Lesson 31

Finding the Rate Per Cent When the Principal, Time, and Interest Are Given

EXAMPLE: At what rate per cent will \$450.00 yield \$11.25 in 6 mo.?

The interest on \$450.00 at 1% for 6 months = \$2.25; therefore, the rate must be as great as \$11.25 ÷ \$2.25 = $\frac{1125}{225}$, or 5%, Ans.

$$I \div (P \times T) = R$$

$P \times T$ = \$450. × $\frac{1}{2}$, or \$225.; I = \$11.25; \$11.25 ÷ \$225. = .05 or 5%, Ans.

To find the rate per cent at which a certain principal will yield a certain interest in a certain time, we divide the total interest by the interest at 1% on the given principal for the given time, and as the interest at 1% on the given principal for the given time is equal to $\frac{1}{100}$ of (P × T), therefore, if we divide I by (P × T) the answer will be in hundredths instead of per cents.

EXAMPLE: At what rate per cent will \$816.00 amount to \$907.80 in $2\frac{1}{2}$ years?

Amount	= \$907.80	The interest on \$816.00 at 1% for $2\frac{1}{2}$
Principal	= 816.00	years equals \$20.40;
Total Interest	= 91.80	\$91.80 ÷ \$20.40 = $\frac{9180}{2040}$, or $4\frac{1}{2}$%, Ans.

When the amount due at maturity is given, subtract the principal from the amount to find the total interest, and continue as before. $A - P = I$.

Exercise 67—Oral.

1. If we know that 1 year's interest on a certain principal at a certain rate per cent is $12.00, in what time would the same principal yield $6.00 at the same rate per cent?
2. In what time would the same principal yield $24.00?
3. What process did you use to find your answers to Question 1 and Question 2?
4. In what time will $100.00 yield $1.00 at 4%?
5. How do we find the time in which a given principal will yield a certain interest at a certain rate per cent? Show this by letters and signs.
6. If we know that the interest at 1% on a certain principal for a certain time is $8.00, at what rate per cent would the same principal yield $40.00 in the same time?
7. What process did you use to find your answer to Question 6?
8. At what rate per cent will $200.00 yield $6.00 in 1 year?
9. How do we find the rate per cent at which a certain principal will yield a certain interest in a certain time? Show this by letters and signs.
10. When finding the time or the rate per cent, what must be done when the amount due at maturity is known, but the interest is unknown?
11. Looking at Exercise 68, state how you will prove your answers for Examples #1 to #5.
12. Looking at Exercise 68, state how you will prove your answers for Examples #6 to #10.

13. Looking at Exercise 68, state how you will prove your answer for Example #11.
14. Looking at Exercise 68, state how you will prove your answer for Example #12.
15. Looking at Exercise 68, state how you will prove your answer for Example #13. State how you will prove your answer for Example #14.

Exercise 68—Written.

Solve and prove:

(Total interest ÷ interest at 1% = number of % in rate; I ÷ (P × T) = R).

At what rate per cent will:

1. $675.00 yield $6.75 in 3 months?
2. $420.00 yield $7.70 in 120 days?
3. $1,260.00 yield $170.10 in 2 years 3 months?
4. $808.00 yield $8.08 in 45 days?
5. $546.00 yield $11.83 in $6\frac{1}{2}$ months?

At what rate per cent will:

6. $4,800.00 amount to $4,809.00 in 15 days?
7. $72.00 amount to $74.10 in 210 days?
8. $4,124.00 amount to $4,216.79 in 9 months?
9. $42.00 amount to $42.98 in 4 mo. 20 da.?
10. $98.80 amount to $106.21 in 1 yr. 8 mo.?

Are you sure this is true each time:

Total interest ÷ {
- interest on $1 for full time, at full rate = principal.
- interest for 1 yr. on full principal at full rate = time.
- interest at 1% on full principal for full time = rate.

11. At what rate will $650. yield $97.50 interest in 2 yr. 6 mo.?
12. In what time will $922.50 yield $49.20 interest at 4%?
13. How much money loaned for 3 yr. 8 mo. at 5% will yield $112.20 interest?
14. In what time will $5,000. yield $5,000. interest at 4%?

Lesson 32

Transposition in Figuring Interest

EXAMPLE: Find the interest on $90.00 for 42 days at 5%.

Int. on $90.00 for 42 days = Int. on $42.00 for 90 days;
Int. for 60 days at 6% = $0.42
Int. for 30 days at 6% = 0.21
Int. for 90 days at 6% = $0.63; Int. at 5% = $\frac{5}{6}$ of $0.63;
$\frac{5}{6}$ of $0.63 = $0.53, Ans.

You will remember that in figuring wages we can transpose the hours worked and the rate per week to shorten the work, when the rate per week is an aliquot part of the weekly hour basis; as, 37¼ hours work at $16.00 per week on a 48-hour basis is the same as 16 hours at $37.25 per week, the answer being $12.42 (⅓ of $37.25).

$$\frac{16}{48} \text{ of } \$37.25 \text{ is equal to } \frac{37\frac{1}{4}}{48} \text{ of } \$16.00.$$

In the same manner, we can save much work in figuring interest by transposing the principal and the time when the principal is an aliquot part of 360, since 360 days is the basis on which commercial interest is figured; thus, in figuring the interest on $60.00 for 197 days at 6%, we can call the dollars "days" and the days "dollars," and find the interest on $197.00 for 60 days at 6%, the answer being $1.97.

$\frac{60}{360} \times \frac{6}{100} \times \$197.$ is equal to $\frac{197}{360} \times \frac{6}{100} \times \$60.$

Exercise 69—Oral.

1. The interest on $30.00 for 78 days is the same as the interest on $78.00 for how many days?
2. The interest on $90.00 for 56 days is the same as the interest for 90 days on how many dollars?
3. How can you find the interest on $60.00 for 273 days most quickly?
4. What is the interest on $60.00 for 273 days at 6%?
5. When can work be saved by transposing the time and the principal in figuring interest?
6. What must be done when the rate is more or less than 6%?
7. How would you find the interest on $120.00 for 18 days at 5%? See if you can give the answer.
8. How would you find the interest on $45.00 for 160 days at 7%? See if you can give the answer.
9. How would you find the interest on $180.00 for 411 days at 6%?
10. How would you find the interest on $90.00 for 544 days at 4½%?

Exercise 70—Written.

Using transposition, find the interest on:

1. $75.00 for 148 days at 6%.
2. $15.00 for 312 days at 8%.

3. \$120.00 for 188 days at $4\frac{1}{2}\%$.
4. \$150.00 for 78 days at 4%.
5. \$80.00 for 96 days at 5%.
6. \$40.00 for 318 days at 7%.
7. \$180.00 for 1 day at 6%.
8. \$50.00 for 84 days at $5\frac{1}{2}\%$.
9. \$30.00 for 488 days at $6\frac{1}{2}\%$.
10. \$210.00 for 22 days at 6%.

Lesson 33

Compound Interest

EXAMPLE: Find the compound interest on \$500.00 for 3 years, at 6%, the interest being compounded annually.

\$500.00 at 6% = \$30.00 Interest for 1st year; Amount \$530.00;
\$530.00 at 6% = \$31.80 Interest for 2d year; Amount \$561.80;
\$561.80 at 6% = \$33.71 Interest for 3d year; Amount \$595.51;
Total..........\$95.51 Compound Interest for 3 years.

or:

Final Amount..........\$595.51
Original Principal....... 500.00
\$95.51 Compound Interest for 3 years.

When the interest for stated periods is added to the principal and the amount so found is used as the principal for the next interest period, the total interest so added to the several principals is called "compound interest."

Savings banks usually allow compound interest, adding the interest to the principal quarterly or semi-annually to form each new principal. Try to find out how the savings banks in your locality pay interest.

Bear in mind that:

6% annually = 3% semi-annually, or $1\frac{1}{2}$% quarterly.
5% annually = $2\frac{1}{2}$% semi-annually, or $1\frac{1}{4}$% quarterly.
4% annually = 2% semi-annually, or 1% quarterly.
3% annually = $1\frac{1}{2}$% semi-annually, or $\frac{3}{4}$% quarterly.

Exercise 71—Oral.

1. In figuring compound interest, how often does the principal change in a year if the interest is payable annually?
2. In figuring compound interest, how often does the principal change in a year if the interest is payable semi-annually?
3. In figuring compound interest, how often does the principal change in a year if the interest is payable quarterly?
4. What is the longest period of time that one principal can be used if the interest is payable quarterly?
5. What is the longest period of time that one principal can be used if the interest is payable semi-annually?
6. What is the longest period of time that one principal can be used if the interest is payable annually?
7. How do savings banks usually pay interest?
8. What is the difference between the final amount and the original principal called?
9. In figuring the compound interest on $375.00 for 18 months at 4% compounded semi-annually, how many separate interest calculations must

you make? How long a period of time will be covered by each interest calculation?

10. State how you would find the compound interest on $1,000.00 for 2 years at 3% payable quarterly.

Exercise 72—Written.

Find the compound interest on:

1. $450.00 for 3 years at 5% payable annually.
2. $600.00 for 2 years at 4% payable semi-annually.
3. $1,000.00 for 9 months at 3% payable quarterly.
4. $275.00 for 1 year at 4% compounded quarterly.
5. What will $75.00 amount to in 6 months at 3% compounded quarterly?

ACCOUNTS

Lesson 34

Savings Bank Accounts

When money is placed in a bank, it is said to be "deposited"; when it is taken out of a bank, it is said to be "withdrawn"; the amount remaining on deposit at any time is called the "balance."

When a "savings account" is opened, the bank gives the depositor a bank book in which an entry is made to show the amount deposited; thereafter when money is deposited or withdrawn, this bank book must be presented so that the amount deposited or withdrawn can be recorded and the new balance shown.

Savings banks usually pay interest at 3%, $3\frac{1}{2}$% or 4%, compounded quarterly or semi-annually, but they deduct amounts withdrawn during any interest period from the balance at the beginning of the period if possible, otherwise from the first deposits made; thus, an amount must be left on deposit to the end of the interest period to earn any interest.

All deposits made during the first few business days of any month draw interest from the first of that month; other deposits draw interest from the first of the following month, and no interest is paid on fractions of a dollar.

In the savings account here shown, the withdrawals, amounting to $185.00, would be applied against the deposits of March 3, $150.00, and March 31, $40.00, leaving only $5.00 on which interest would be paid from April 1st, and $43.00 on which interest would be paid from May 1st to July 1st which is the end of the semi-annual period, and this at 3% amounts to $0.24 which is entered as a deposit and added to the balance.

Dr. NATIONAL TRUST & SAVINGS BANK
CHICAGO, ILLINOIS

In account with:- **No.** 715,706

George Palmer **Cr.**

Date		Initial	Withdrawals		Deposits		Balance	
1920 Mar.	3	E. J.			150	00	150	00
	15	F. L.	85	00			65	00
	31	E. J.			40	00	105	00
Apr.	30	E. J.			43	50	148	50
May	4	F. L.	100	00			48	50
July	1	Interest				24	48	74

A Page from a Savings Bank Book.

Exercise 73—Oral.

1. On the bank book shown in the illustration, which entries refer to amounts put into the bank?
2. Which entries refer to amounts taken out of the bank?

3. What is meant by a "deposit"?
4. What is meant by a "withdrawal"?
5. What is meant by a "balance"?
6. At what rate per cent do savings banks usually pay interest? What kind of interest do they pay?
7. If an amount is withdrawn before the end of the interest period, is any interest paid thereon for the time it was on deposit?
8. When does an amount deposited during the first few days of a month start to draw interest? When does an amount deposited at any other time start to draw interest?
9. What name is given any sum of money put into a bank?
10. What name is given any sum of money taken out of a bank?
11. What name is given to the amount you have remaining in a bank at any time?
12. Why is a bank book given by the bank to the depositor when a savings account is opened?
13. Why must the bank book be presented every time money is deposited or withdrawn?
14. Explain what is meant when we say that a savings bank pays interest quarterly?
15. Explain what is meant when we say that a savings bank pays interest semi-annually?
16. Is any interest paid on fractions of a dollar?
17. How is the interest paid to the depositor?
18. If you have $100. on deposit in a savings bank which pays interest quarterly at the rate of

4% per annum, how much interest could you withdraw every three months? Would the withdrawing of this interest every three months reduce your $100. balance? Explain fully.

Exercise 74—Written.

1. Rule paper to represent a savings bank book, record the following transactions, and find the balance of the account:

   Deposit April 6, $150.00;
   Deposit May 14, $100.00;
   Withdrawal June 9, $75.00.

2. If the bank in Question 1 pays interest at 3% compounded semi-annually on Jan. 1st and July 1st, and all deposits made on or before the 5th of any month draw interest from the first of that month, what amount of interest will be credited on this account July 1st? Make the necessary entry on the account.

3. Rule paper, record the following transactions, and find the balance of the account:

   Deposits: July 3, $1,400.00; Sep. 6, $200.00;
   Withdrawals: Aug. 1, $300.00; Nov. 1, $400.00.

4. If the bank in Question 3 pays interest at 4% compounded semi-annually Jan. 1st and July 1st, and all deposits made on or before the 5th of any month draw interest from the first of that month, what amount of interest will be credited on this account Jan. 1st? Make the necessary entry on the account.

Lesson 35

Bank Accounts Which Are Subject to Check

Bank accounts on which checks can be issued, or "checking accounts," as they are very often called, are used by individuals, firms, and companies to conduct their finances.

Money deposited in these accounts is entered in the depositor's bank book, but draws no interest (excepting in special cases).

To withdraw money, the depositor is not required to present his bank book as in the case of savings accounts; instead, he issues checks for the amounts he wishes to withdraw and makes them payable to the parties to whom he wishes to pay the money.

These checks, after being cashed by the bank, are charged to the depositor's account and are returned to him with a statement of his account on the first day of each month.

No. 141	No. 141 Chicago, June 30, 1920
Date 6/30/20	
Balance 425.00	**FIRST NATIONAL BANK**
Deposit 6/28 40.00	OF CHICAGO, ILLINOIS
Total 465.00	
Check payable to Wm Collins	*Pay to the order of* William Collins $ 50 00/100
Amount 50.00	Fifty and 00/100 ——— *Dollars*
Balance 415.00	Thos. Hall

The "payee" of a check is the party to whom it is payable.

Reconcilement of a Bank Account

June 30, 1920

Balance as per Check Book,		$415.00
Checks Outstanding:		
# 140	$75.00	
# 141	50.00	125.00
Balance as per Bank Statement		$540.00

Since the bank does not know of the existence of any check until it is presented for payment, any checks not cashed on the last day of a month would naturally not be charged by the bank against the depositor's account; therefore, to make his account agree with the bank, the depositor must add all uncashed outstanding checks to his balance. Balancing in this manner is called a "reconcilement" of a bank account.

To ascertain what checks are outstanding uncashed at the end of the month, all cashed checks received from the bank must be sorted numerically and compared with the stubs.

Exercise 75—Oral.

1. In the check here illustrated, who is paying a sum of money? Who is receiving a sum of money? What sum of money is being paid?
2. What has the First National Bank to do with this transaction?
3. What will the First National Bank do with this check after paying it?
4. Who is the payee of this check?

5. Supposing you had a bank account and issued a check on April 30th for $100.00 payable to a party in another city and mailed it to him, would your bank be able to cash this check during April? When would they be likely to cash it?
6. In the case mentioned in Question 5, how would you be able to reconcile your bank account on April 30th?
7. What is meant by reconciling a bank account?
8. How can you ascertain which checks are outstanding unpaid at the end of the month?
9. Explain each of the entries on the stub shown in the illustration.
10. Explain each of the entries in the reconcilement shown in the illustration.

Exercise 76—Written.

1. Find the balance as of September 30th on the following bank account:

Balance Sep. 1		$1,412.75;
Deposit Sep. 4		473.86;
Checks Issued:		
#17,474	Sep. 4	87.13;
75	Sep. 8	146.87;
76	Sep. 15	468.72;
77	Sep. 29	321.62;
78	Sep. 30	45.00.

2. What balance would the bank show on the account given in Question 1 Sep. 30, if checks #17,477

and #17,478 were sent to distant cities and therefore were not cashed until early in October?

3. Prepare a reconcilement of the bank account given in Question 1 and Question 2 as of Sep. 30th.

4. Find the balance as of Dec. 31st of the following bank account:

Balance	Dec. 1	$18,745.36;
Deposits:		
	Dec. 5	1,486.32;
	Dec. 13	2,394.97;
	Dec. 20	1,663.13;
Checks Issued:		
#11,416	Dec. 3	1,741.32;
17	Dec. 3	431.87;
18	Dec. 19	296.98;
19	Dec. 28	1,347.21;
20	Dec. 28	841.38;
21	Dec. 30	711.48;
22	Dec. 30	222.47.

5. On Dec. 31st the bank returned the following checks with a monthly statement of the account given in Question 4:

#11,416;
17;
18;
20;

What balance appeared on this statement?

6. Construct a reconcilement of the bank account given in Question 4 and Question 5 as of Dec. 31st.

7. Using the form shown in the illustration, write a check dated New York, Oct. 3, 1918, drawn on the Citizens' National Bank of New York for $100.00, payable to Harry Knowles, signed by yourself.
8. Write a check dated today showing how you would pay your teacher $15.46 if you had money on deposit at the Continental Bank of Cincinnati, Ohio.
9. Write a check dated today showing how Wilson & Co. of Philadelphia, Pa., could pay you $18.75 if they had money on deposit at the Old Trust Company of Boston, Mass.
10. Write a check on the First Bank and Trust Company of San Francisco, Calif., in the sum of $1,000.00, with your teacher as the payee, using today's date.

Exercise 77—Oral Review.

1. What is the interest on $60.00 for 318 days at 6%?
2. What are the dimensions of a township? How many sections does a township contain?
3. What is the ratio of the circumference of a circle to the diameter?
4. If a teamster's wages are $24.00 for a week of 48 hours, how much will he receive for working $18\frac{1}{2}$ hours?
5. Multiply:

(*a*)	(*b*)	(*c*)
$64 \times 37\frac{1}{2}$;	$56 \times .75$;	$66 \times 33\frac{1}{3}$.

6. Divide:

(a)	(b)	(c)
$600 \div 16\frac{2}{3}$;	$11 \div 25$;	$60 \div 33\frac{1}{3}$.

7. What is the commission on the sale of produce amounting to $400.00 at $12\frac{1}{2}\%$?
8. What is the circumference of a circle if the diameter is 10 inches?
9. What is the perimeter of a 10-yard square? What is the area?
10. How many cubic inches are there in a prism 2 in. wide, 3 in. thick, and 12 in. high?
11. What is the volume of a 10-inch cube? What is the area of its entire surface?

Exercise 78—Written Review.

1. What is the interest on $346.00 from June 9, 1918, to Aug. 14, 1920, at 6%?
2. What is the volume of a section of stove pipe 6″ in diameter and 20 inches long?
3. What is the area of the curved surface of 5 sections of this stove pipe?
4. Find the net amount of an invoice of $178.00 subject to 10% and 10% discount.
5. The discounts on an invoice are equivalent to 27.1%; if two of the rates are 10% and 10%, what is the other rate?
6. In what time will $87.60 amount to $88.33 at 5% interest?
7. At what rate per cent will $126.00 amount to $143.01 in 2 yr. 3 mo.?

8. Find the compound interest on $2,750.00 for 1 yr. at 4% payable quarterly.

Add, but do not copy:

(Time for these 3 examples is less than $4\frac{1}{2}$ minutes.)

9.	10.	11.
1,974	4,612	7,387
4,132	3,734	6,219
7,385	5,875	7,385
3,198	6,298	5,873
5,988	7,129	9,821
3,085	3,975	6,185
7,142	7,309	5,318
1,251	7,111	3,071
2,038	5,873	6,936
6,712	4,109	3,174
1,788	1,291	2,682
9,174	3,712	8,638
4,782	1,267	5,021

Copy and multiply:

(Time for these 8 examples is less than $4\frac{1}{2}$ minutes.)

12. $1,812 \times 35$;
13. $4,387 \times 43$;
14. $3,815 \times 28$;
15. $6,345 \times 52$;
16. $7,315 \times 63$;
17. $1,288 \times 92$;
18. $5,167 \times 84$;
19. $7,319 \times 27$.

Copy and divide:

(Time for these 6 examples is less than $4\frac{1}{2}$ minutes.)

20. $14,620 \div 34$;
21. $19,328 \div 32$;
22. $39,273 \div 53$;
23. $35,776 \div 43$;
24. $56,820 \div 60$;
25. $33,054 \div 42$.

Subtract, but do not copy:

(Time for these 12 examples is less than $4\frac{1}{2}$ minutes.)

26.	27.	28.	29.
353,872	475,318	598,712	843,829
289,746	329,879	231,987	421,945

30.	31.	32.	33.
611,841	741,028	681,491	831,874
298,748	263,751	298,728	219,008

34.	35.	36.	37.
439,821	288,487	912,091	210,005
318,941	179,209	138,099	153,901

ADVANCED LESSONS

PART VIII

NOTATION AND NUMERATION

Lesson 1

The Higher Periods

Write eight; eight hundred; eight thousand; eight million. Which of these is the largest? Tell why.

Each of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 has two values; one of these values is absolute and is expressed by the form of the figure; the other value is relative and depends upon the position that the figure occupies in a number. Thus, the figure "8" always has an absolute value of "eight," but in the number "894" the figure "8," besides having an absolute value of "eight," also has a relative value of "eight hundred," since it occupies hundreds' place in this particular number.

Read evenly:

80,452;	995,800;	65,100,142;
912,816;	1,265,104;	99,999,999;
999,999;	9,999,999;	100,000,000;
1,000,000;	10,000,000;	999,999,999.

In the same manner as 1,000 follows 999 and 1,000,000 follows 999,999, so we can have any number of places and periods, but more than five periods are seldom necessary to express a numerical value, as few things are so vast in their scope that more periods are necessary.

The first five periods—with three of which you are already entirely familiar—and the names of the places they comprise are as follows:

The Places or Orders	Hundreds of Trillions	Tens of Trillions	Trillions	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
	8	4	6,	3	9	8,	7	4	6,	8	7	2,	3	6	1
The Periods	Trillions			Billions			Millions			Thousands			Units		

The names of the next seven periods in their order are: quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions.

Exercise 1—Oral.

1. In the number 764,384 what is the absolute value of each of the digits?
2. In the number 764,384 what is the relative value of each 4?
3. What determines the absolute value of a digit?
4. What determines the relative value of a digit?
5. What is the name of the first place of the 3d period (6,000,000)? What order is this when units' place is counted as the first? What places make up this full period?
6. What is the first place of the 4th period called? What places make up this period?
7. What is the first place of the 5th period called? What places make up this period?
8. What is the first place in any period called? The second place? The third place?

9. Name 5 periods; begin with units' period.
10. Beginning with units, name all the places to trillions' place.

Exercise 2—Oral.

Point off, and read the following statements smoothly:

1. Rays of heat from the sun travel 94,500,000 miles before they reach us in June; they travel 91,500,000 miles in January.
2. The distance across the earth's orbit is 186,000,000 miles the long way.
3. The area of the earth's surface is about 197,000,000 sq. mi.
4. The earth has about 1,700,000,000 people.
5. During the World War it was necessary for the United States to borrow money by issuing Liberty Bonds. The subscriptions for the Fourth Liberty Loan exceeded $6,000,000,000.00.
6. During the five years 1912–1916 inclusive, the average yearly production of the United States was:

   2,761,252,000 bu. corn.
   1,296,406,000 bu. oats.

7. The lumber cut in the United States during the year 1917 was 35,821,239,000 ft.
8. To June 29, 1918, the Senate of the United States passed war bills aggregating $21,500,000,000.00 for conducting the World War.
9. The national wealth of the United States including all the real and personal property of every

description was estimated to be $187,739,000,-000.00 in the year 1912.

10. Before the war the national wealth of France was approximately $63,000,000,000. and the national income about $7,500,000,000.

Exercise 3—Written.

Write:

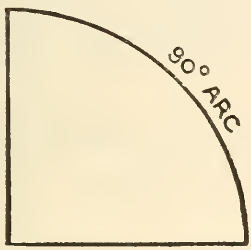
1. Eight hundred forty-six billion, three hundred seventy-two million, one hundred forty-five thousand, nine hundred.
2. Four hundred sixteen billion, three hundred twenty-five thousand, four hundred sixty-two.
3. Eight hundred seventy-four trillion, three hundred twenty-one billion, four hundred sixty-two million, one hundred.
4. Six hundred five trillion, four hundred forty-five million, eight hundred thirty-one.
5. Thirty-eight trillion, seven hundred forty-six billion, one hundred forty-five.
6. Three hundred eighty-six trillion, four hundred ninety-eight million, two hundred twenty thousand, one.
7. Three hundred seven trillion, thirty-seven billion, thirty million, seven hundred thousand, thirty.
8. Eight hundred twelve billion, eight hundred million, twelve thousand, eight hundred twelve.
9. Sixty trillion, six hundred billion, six million, six hundred thousand, sixty-six.
10. Three hundred forty-five trillion, nine hundred eleven thousand.

DENOMINATE NUMBERS

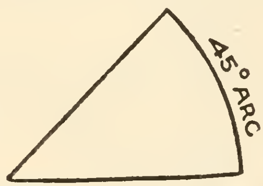
LESSON 2

Table of Circular Measure

60 seconds (″)	=	1 minute (′)
60 minutes...	=	1 degree (°)
90 degrees...	=	1 quadrant (quad.)
360 degrees...	=	1 circle (⊙)



A Quadrant
or
$\frac{1}{4}$ Circle



$\frac{1}{2}$ Quadrant
or
$\frac{1}{8}$ Circle

Since there are 360 degrees in every circumference, a degree of arc is always $\frac{1}{360}$ of a complete circumference, regardless of the size of the circle. As a circumference may be an inch, a foot, a yard, a mile, or 25,000 miles in length, one degree of arc will represent as many different distances as there are different circumferences, because the number of inches, yards, etc., in a degree varies with the size of the circle, but the number of degrees in every circumference always is 360 regardless of the size of the circle. Remember that a circumference 1 inch in length contains just as many degrees as one 25,000 miles in length.

For making the more accurate measurements in surveying and astronomy each degree is divided into 60 equal parts called minutes, and each minute is divided into 60 equal parts called seconds.

Exercise 4—Oral.

1. Draw a circle. How many degrees are there in your circle?
2. If each one in the class draws a circle, does each circle contain 360°?
3. Does the size of the circle make any difference?
4. How many degrees are there in $\frac{1}{4}$ of a circle? In $\frac{1}{2}$ of $\frac{1}{4}$?
5. One degree is what part of a circumference?
6. If the circumference is 25,000 miles, 1° of that is found how?
7. If the circumference is 9 inches, how do you find the length of 1°?

The "horizon" is the line where the sky and earth seem to meet.

8. How many degrees of arc are there in the entire horizon?
9. How many degrees of arc are there in the horizon between points exactly east and exactly west? How many degrees between points exactly north and exactly south?
10. Point with your arm toward the eastern horizon. What is the position of your arm while pointing?

The point directly overhead is called the "zenith."

11. How many degrees of arc are there between any point on the horizon and the zenith?

12. From horizon to horizon through the zenith is how many degrees? Does the direction of such an arc make any difference?

13. Point with your arm toward the zenith. What is the position of your arm while it is pointing toward the zenith?

14. How many degrees are there between the hands of the clock when it is 3 o'clock?

15. If a star is 30° from the zenith on an arc toward the southern horizon, how many degrees is it from the southern horizon? Point with your arm in the direction where such a star would be. What is the position of your arm while it is pointing toward this star?

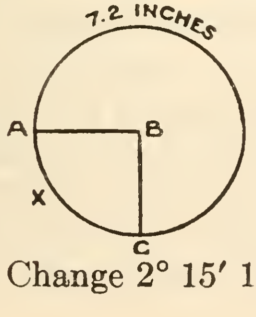
Exercise 5—Written.

1. How many degrees of arc are there in $\frac{1}{4}$ circumference? How many minutes of arc? How many seconds of arc?

2. Reduce 35,745″ to ° ′ ″.

3. 

Circumference = 28,800 miles; 1° = ?

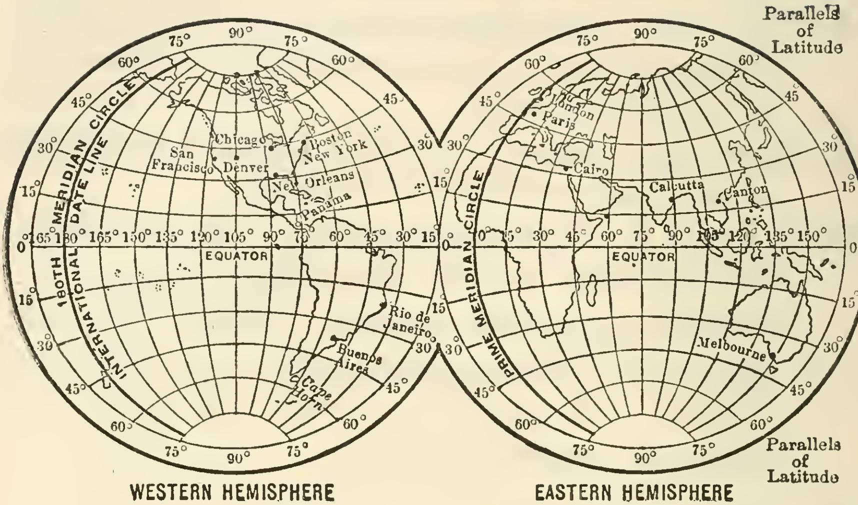
4. 

Find the length of x.

5. Change 2° 15′ 15″ to seconds.

Lesson 3

Longitude and Time



The World

Besides measuring distances on the earth's surface in units of miles, rods, etc., we can, since the earth is almost a sphere, measure these distances along arcs of imaginary circles, and express them in units of degrees, minutes, and seconds of arc, in the same manner as we measure other circles.

The equator, being a circumference of the earth, contains 360° and each of these degrees is 1° of longitude. The 0° point on the equator is on an arc (called the Prime Meridian) drawn from pole to pole through Greenwich, near London, England, where one of the world's greatest observatories is located, and all places are East or West, depending upon their location on other "meridians of longitude" east or west of this Prime Meridian. No point can be called more than

180° east or west of Greenwich, since, if it were 190° east it would be only 170° west and it would be so called.

There are other imaginary lines running parallel to the equator to show distances north and south of the equator, and these are called "parallels of latitude."

All places on the same meridian have the same longitude; all places on the same parallel have the same latitude.

Exercise 6—Oral.

Follow Harry's imaginary trips and troubles.

1. He is traveling on an imaginary circle equally distant from the poles; what imaginary circle is this?
2. He has a trip of how many degrees if he tries to reach either of the poles?
3. Longitude is distance east or west of the Prime Meridian. If he is where the Prime Meridian crosses the equator, write the longitude of his place.
4. If he travels the circumference of the equator how many degrees does he travel?
5. How many ° can he travel to the greatest east longitude? How many to the greatest west longitude?
6. Why is it wrong to say 260° East?
7. He visits several cities that have east longitude. Name 3.
8. Follow him to 3 cities in west longitude. Name them.

9. Follow him to the cities that have approximately the following locations, and tell what cities they are:
   (*a*) 90° West Longitude; 30° North Latitude.
   (*b*) 105° West Longitude; 40° North Latitude.
   (*c*) 30° East Longitude; 30° North Latitude.
   (*d*) 0° East Longitude; 50° North Latitude.
10. From the map tell his longitude if he visits Buenos Aires.

Exercise 7—Oral.

Harry studies the map and clock. Since the earth rotates on its axis from west to east once every 24 hr., he has some questions to answer. Can you help him?

1. (*a*) 24 hr. of time corresponds to 360° longitude;
   (*b*) 1 hr. of time corresponds to ? longitude; ($\frac{1}{24}$ of 360°);
   (*c*) 1 min. of time corresponds to ? longitude; ($\frac{1}{60}$ of 15°);
   (*d*) 1 sec. of time corresponds to ? longitude; ($\frac{1}{60}$ of 15′).
2. (*a*) 360° longitude corresponds to 24 hr. of time;
   (*b*) 1° longitude corresponds to ? min. of time; ($\frac{1}{360}$ of 24 hr.);
   (*c*) 1′ longitude corresponds to ? sec. of time; ($\frac{1}{60}$ of 4 min.);
   (*d*) 1″ longitude corresponds to ? sec. of time; ($\frac{1}{60}$ of 4 sec.).
3. (*a*) 15° longitude corresponds to how much time?
   (*b*) 15′ longitude corresponds to how much time?

(*c*) 15″ longitude corresponds to how much time?

(*d*) 15° 15′ 15″ longitude corresponds to how much time?

4. How many of each unit of longitude correspond to one of each unit of time? 1 hr., 1 min., 1 sec. = ?°, ?′, ?″.

5. If there is a difference in time between two places, what process will find the corresponding difference in longitude?

6. As New York is about 15° east of Chicago, which city sees the sun first? How many hours sooner?

7. How far are two cities apart if A is 30° East and B is 60° West? Did you add or subtract? Why? What is the difference in time?

8. If M is 90° E. and N is 105° E., what is the difference in longitude? Add or subtract? Why?

9. How many degrees of longitude correspond to 3 hr. of time? $2\frac{1}{2}$ hr.?

10. Two cities are 90° apart; what is the difference in time?

11. A has 80° 12′ W. longitude; B has 10° 3′ E. longitude; what is the difference in longitude?

12. If both places have E. longitude, how do you find the difference in longitude? If both have W. longitude? If one has E. longitude and the other W. longitude?

Exercise 8—Written.

Harry and his father are traveling.

1. Harry is in one city, his father is in another located 10° 8′ 45″ away; what is the difference in their time?

2. Another time Harry was in Chicago 5 hr. 50 min. 26 sec. west of Greenwich while his father was visiting in Greenwich; how many ° ′ ″ were between them?
3. Harry is in Washington 77° 3′ 45″ west of London; if it is 12 o'clock noon in London, what time has Harry?
4. Harry and his father met in New Orleans at 6 o'clock one evening; as their watches were still keeping the times of the places from which they came, Harry's watch showed the time to be 7 o'clock while his father's watch showed 4 o'clock; from which direction did Harry come? His father?
5. If the longitude of New Orleans is 90° 3′ 15″ West; from what longitude did Harry come? His father?
6. Drill: Find the difference in longitude and the difference in time:

   (*a*) 82° 6′ West; 12° 7′ 15″ East;
   (*b*) 100° 2′ 15″ East; 25° 5′ 10″ East;
   (*c*) 10° 1′ 6″ East; 4° 1′ 6″ West;
   (*d*) 105° 6′ 7″ West; 12° 8′ 7″ West;
   (*e*) 10° 15′ 6″ East; 10° 15′ 6″ West.

7. The longitudes of two ships at sea are respectively 30° 12′ 15″ W. and 10° 20′ 30″ E.; what is the difference in their longitude? What is the difference in their time?
8. When the meridian on which your school is located passes under the sun it is noon; what time is it then in a city located 25° 10′ 15″ farther west?

9. What difference is there in the longitudes of your school and a school located at a point where it is 3.30 P. M. when you have noon? Would such a school be east or west of you?

10. What is the time and day in San Francisco, California, 122° 25′ 30″ W. longitude when it is 4 A. M. of Thursday at Greenwich near London, England?

LESSON 4

Standard Time in the United States

The Four Time Zones—1920

In 1883 the railroads of the United States established a set schedule which divides the United States into four time zones or divisions, known as follows: Eastern Time, Central Time, Mountain Time, and Pacific Time. The heavy zigzag lines separate these divisions on the map.

The time used in each of these divisions is that of longitude 75° W., 90° W., 105° W. and 120° W. and standard or railroad time is the same throughout a whole division. The division lines touch important railroad terminals. All this was done to save confusion and danger of accidents resulting from differences in time.

Exercise 9—Oral.

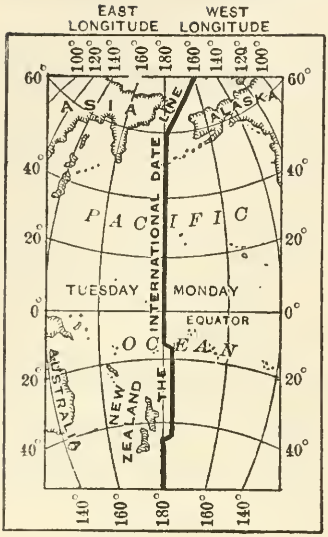
Harry has "time trouble" in the United States. Help him.

1. Harry traveled from coast to coast; he covered about how many degrees?
2. His watch said 5 A. M. when he left the 75th meridian; when should he change his watch, when he crossed the division line between the two time zones or when he reached the 90th meridian? Should he move it back or ahead? How much?
3. How much did he change his time to be correct if he went from the 75th meridian to the 120th? Did he set his watch ahead or back?
4. When it was 9.30 A. M. (Standard Time) in New York, what time did his father have if he was in Chicago?
5. When his father had 3.45 P. M. in Iowa, what time had Harry in Delaware?
6. When Harry's father had 12.00 midnight in Colorado, what time had Harry in Illinois?
7. Harry wanted to find out what good there was in having Standard Time; can you tell him?

8. He could name 5 noted cities that have Eastern Time; can you?
9. He could name 5 noted cities that have Mountain Time; can you?
10. He could name 2 noted cities that have Pacific Time; can you?
11. He met his father in Wyoming; Harry's watch said 2 P. M., his father's said noon; if his father had changed his watch to Standard Time, what change must Harry make? Which way had Harry traveled? How many degrees?
12. Harry met his father in Chicago; Harry had come from eastern Texas, his father from western Ohio; who was ahead by his watch if neither had changed time on the trip?
13. Harry knew on what meridian in the Eastern Time Zone sun time and Standard Time are alike; do you know?
14. He knew on what meridian in the Central Time Zone sun time and Standard Time are alike; do you know?
15. Could you tell him on what meridian in the Mountain Time Zone sun time and Standard Time are alike?
16. Could you tell him on what meridian in the Pacific Time Zone sun time and Standard Time are alike?
17. Harry said that there are more places in the United States that use Standard Time or railroad time than there are places that use sun time; do you think he was right? Why?

Lesson 5

The International Date Line



The International Date Line

The 180th meridian was selected by the nations of the world as the dividing line between dates and between the East and the West (called the International Date Line) because it marks the distance half way around the earth from Greenwich, and also because it is located almost entirely in the Pacific Ocean, which avoids the confusion that might result were a densely populated location selected. The actual date line varies some places from the 180th meridian to avoid populated land.

When a ship crosses the 180th meridian from east to west at noon on Monday, Monday noon instantly changes to Tuesday noon (or add 24 hr.) and should this ship at once turn about and recross from west to east, Tuesday noon would again change back to Monday noon (or subtract 24 hr.)

Exercise 10—Oral.

Harry goes to sea.

1. Harry's ship crosses the International Date Line from west to east at 11.59 P. M. Saturday night and continues traveling toward the east; how

long will it have been Saturday on board this ship when Sunday begins?

2. If Harry's ship crosses the International Date Line from east to west at 11.59 P. M. Saturday night and continues traveling toward the west, how long will it have been Sunday on board this ship when Monday begins?
3. Harry studies the location of the International Date Line and wonders why it is so irregular; can you tell him?
4. When Harry crossed the International Date Line, how far was he from the Prime Meridian? How many hours difference in time?
5. What can you tell Harry about time in different parts of the world?
6. Harry's ship crossed the 45th meridian (west longitude) at 10 A. M. Tuesday; what time was it then on each of the other meridians shown on the map at the beginning of Lesson 3 going around both ways to the 180th meridian?
7. When it is 1 A. M. on the Prime Meridian, what time is it on each of the other meridians shown on the map in Lesson 3?
8. When it is 3 P. M. on the 90th meridian (east longitude), what time is it on each of the other meridians shown on the map in Lesson 3?

Exercise 11—Written.

Drill: Find the difference in time; then the difference in longitude:

1. 1 hr. 2 min. 6 sec. P. M. and 12 noon.

2. 2 hr. 6 min. 10 sec. P. M. and 4 hr. 15 min. 20 sec. P. M.
3. 12 noon and 4 hr. 4 min. 30 sec. P. M.
4. 3 hr. 2 min. 45 sec. A. M. and 12 noon.
5. 1 A. M. and 6 hr. 4 min. 8 sec. A. M.
6. Change $4° 14' 8''$ to its equivalent in time units.
7. Reduce 4 hr. 6 min. 10 sec. to its equivalent in longitude.
8. Reduce $172° 40' 30''$ to its equivalent in time units.

Lesson 6

The Metric System

The "metric system" is a decimal system of weights and measures devised and adopted by the French in 1789. On account of its simplicity, it is coming more and more into general use. For scientific and laboratory purposes it is used almost universally.

Since the World War the metric system has become more vital to the United States than ever before; besides, every South American country now uses this method and for this reason finds it easier to trade with European countries.

In the metric system, 10 units of one measure are equal to 1 unit of the next larger measure in the same way as in our decimal system.

There are five units of measure used in the metric system.

meter (m.) unit of length.
(1 m. = 39.37 in.)
square meter (sq. m.) unit of area.
cubic meter (cu. m.) unit of volume.

gram (g.) unit of weight.
(1 g. = .035 oz.)
liter (l.) unit of capacity.
(1 l. = $\frac{10}{11}$ qt. Dry—for grains, etc.)
(1 l. = $1\frac{1}{19}$ qt. Liquid—for all liquids.)

Now, bearing in mind that the metric system is based on the decimal principle, anyone can construct all of the tables used in the metric system by learning these few simple Greek and Latin prefixes which designate the different decimal values:

Greek	Myria (M.) meaning........	10,000.
	Kilo (K.) meaning..........	1,000.
	Hecto (H.) meaning.........	100.
	Deka (D.) meaning.........	10.
	Use name of unit for........	1.
Latin	deci (d.) meaning...........	.1
	centi (c.) meaning..........	.01
	milli (m.) meaning..........	.001

This gives us the following comparative table:

Decimal Scale	10,000. (1,000.× 10)	1,000. (100.×10)	100. (10.× 10)	10. (1.× 10)	1. (.1 × 10)	.1 (.01 × 10)	.01 (.001 × 10)	.001 (—)
U.S.Money	—	—	—	Eagle	dollar	dime (d.)	cent. (c.)	mill (m.)
Length....	Myria (M.)	Kilo (K.)	Hecto (H.)	Deka (D.)	meter (m.)	deci (d.)	centi (c.)	milli (m.)
Weight ...	Myria (M.)	Kilo (K.)	Hecto (H.)	Deka (D.)	gram (g.)	deci (d.)	centi (c.)	milli (m.)
Capacity..	Myria (M.)	Kilo (K.)	Hecto (H.)	Deka (D.)	liter (l.)	deci (d.)	centi (c.)	milli (m.)

(Note the similarity between dime and deci, cent and centi, mill and milli and the abbreviations thereof. Also note that the abbreviations for the terms larger than 1 are in large letters, while those for the terms smaller than 1 are in small letters.)

The tables of the metric system are constructed by using the proper unit of measure with the various prefixes to show the proper decimal values, as follows:

Table of Length Measure

(1 meter = 39.37 inches)

10 millimeters (m. m.)......	= 1 centimeter (c. m.)
10 centimeters..............	= 1 decimeter (d. m.)
10 decimeters...............	= 1 meter (m.)
10 meters...................	= 1 Dekameter (D. m.)
10 Dekameters...............	= 1 Hectometer (H. m.)
10 Hectometers..............	= 1 Kilometer (K. m.)
10 Kilometers...............	= 1 Myriameter (M. m.)

Table of Weight Measure

(1 gram = .035 oz.)

Same as Table of Length Measure excepting that "gram" must be substituted for "meter."

Build your own Table of Weight Measure.

Table of Capacity Measure

$$\text{(1 liter} = \begin{cases} \frac{10}{11} \text{ qt. Dry)} \\ 1\frac{1}{19} \text{ qt. Liquid)} \end{cases}$$

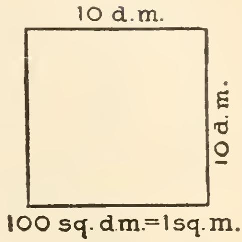
Same as Table of Length Measure excepting that "liter" must be substituted for "meter."

Build your own Table of Capacity Measure.

Table of Square Measure

Just as 1 square foot contains 144 square inches, (12 in. long and 12 in. wide), so 1 square centimeter contains 100 square millimeters (10. m. m. long and 10 m. m. wide); therefore, 100 units of one square measure are equal to 1 unit of the next larger measure.

The table is the same as the Table of Length Measure excepting that 100 must be substituted for 10, because each area is 10 units square, and the word "square" must be prefixed to each measure.



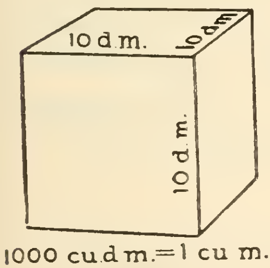
100 square millimeters (sq. m. m.) = 1 square centimeter (sq. c. m.).

Build your own Table of Square Measure.

Table of Cubic Measure

Just as 1 cubic foot contains 1,728 cubic inches, (12 in. long, 12 in. wide and 12 in. thick), so 1 cubic centimeter contains 1,000 cubic millimeters (10 m. m. long, 10 m. m. wide and 10 m. m. thick); therefore, 1,000 units of one cubic measure are equal to 1 unit of the next larger measure.

The table is the same as the Table of Length Measure excepting that 1,000 must be substituted for 10, and the word "cubic" must be prefixed to each measure.



1,000 cubic millimeters (cu. m. m.) = 1 cubic centimeter (cu. c. m.)

Build your own Table of Cubic Measure.

Exercise 12—Oral.

Children are to ask each other:

1. What prefix means 1,000? 100? 10,000? 10? .001? .1? .01?
2. What are the abbreviations for each of the prefixes in Question 1?

3. Which prefixes have large letters for their abbreviations? Which have small letters?
4. In the metric system Table of Length Measure, how many units of one measure are equal to 1 unit of the next larger measure? What is the unit of length called?
5. In the metric system Table of Weight Measure, how many units of one measure are equal to 1 unit of the next larger measure? What is the unit of weight called?
6. In the metric system Table of Capacity Measure, how many units of one measure are equal to 1 unit of the next larger measure? What is the unit of capacity called?
7. In the metric system Table of Square Measure, how many units of one measure are equal to 1 unit of the next larger measure? Why is this right? What is the unit of area called?
8. In the metric system Table of Cubic Measure, how many units of one measure are equal to 1 unit of the next larger measure? Why so many? What is the unit of volume called?

What is equal to:

9. 10 millimeters?
10. 10 centigrams?
11. 10 deciliters
12. 100 square meters?
13. 1,000 cubic Dekameters?
14. 10 Kilometers?
15. 10 Dekagrams?
16. 10 Hectoliters?
17. 10 milligrams?
18. 10 centimeters?
19. 1,000 m. m.?
20. 100 K. m.?
21. 100 d. g.?
22. 1,000 cu. d. m.?
23. 100 sq. D. m.?
24. 10 centiliters?

Exercise 13—Written.

Name the fundamental unit of each. Arrange in order and add:

1. 6 m. m., 3 c. m., 9 D. m., 7 H. m., 1 K. m.
2. 2 c. g., 20 d. g., 55 D. g., 9 D. g., 15 c. g.
3. 20 l., 15 d. l., 25 c. l., 9 K. l., 15 H. l., 6 l.
4. Find cost of No. 3 if each liter costs 28¢.
5. Find value of #2 if each gram costs $.05.
6. If a box of biscuit weighs 340 g. what will a gross of boxes weigh? (Answer in grams.)
7. Answer No. 6 in oz. In lb.

Lesson 7

Foreign Money

In order that we may have business dealings with people in foreign countries, it is necessary that we know the value of the different denominations of money used in such countries.

Table of English Money

12 pence (d.) . .	= 1 shilling (s.) . .	= $0.243+
20 shillings	= 1 pound (£) . . .	= 4.8665

Table of French Money

100 centimes (c.) = 1 franc (fr.) = $0.193

Money used in other European Countries

A Belgian franc
A Spanish peseta
An Italian lira
A Swiss franc
} = $0.193 each.

(Each of these has the same value as the French franc.)

You may be interested to know the value of the monetary unit used in some of the other principal countries of the world; these you need not learn, but you can use them for reference:

Country	Monetary Unit	U. S. Value
Canada	dollar	$1.00
Germany	mark	0.2382
Japan	yen	0.4985
Norway	crown	0.268
Sweden	crown	0.268
South American Countries:		
Argentine Republic	peso	0.9648
Bolivia	boliviano	0.3893
Brazil	milreis	0.5462
Chile	peso	0.365
Peru	libra	4.8665
Venezuela	bolivar	0.193

Exercise 14—Written.

Study first to decide which is the best way of doing these.

Find the value of the following in United States money:

Time yourself.

1. £50
2. £4 16s.
3. £2 6d.
4. 5s. 10d.
5. £400 19s. 2d.
6. 50fr.
7. 75c.
8. 68fr. 30c.

Find the value of the following in English money:

9. $29.20
10. $245.08
11. $418.58
12. $2,433.25

} Time yourself.

Find the value of the following in United States money:

13. 175 Belgian francs.
14. 300 Italian lira.
15. 80 Spanish peseta.
16. 450 Swiss francs.
17. 290 French francs.

} Time yourself.

Find the value of the following:

18. $14.47 in Italian lira.
19. $82.99 in Spanish peseta.
20. $11.58 in Swiss francs.
21. $96.50 in French francs.
22. $17.37 in Belgian francs.

} Time yourself.

Exercise 15—Written.

A Trip through Europe

John Walker's father is a buyer for Marshall & Co.'s large store and goes abroad every summer on business. Last summer he took John along. "Watch your time and money, John," was his father's advice.

1. They left Chicago, Ill., at 12.40 P. M., on one of the fast trains which reaches New York in 20 hours; what time of the day did this train arrive in New York?

2. They sailed July 8th from New York City on a British ship bound for Southampton. While on board ship, Mr. Walker paid £5 10s. for miscellaneous expenses; how much was this in United States money?
3. John, who had set his watch properly in New York City, made no change in his time while on the ocean, but his father set his watch several times on the way so that it agreed with London time on their arrival at Southampton. What was the difference in time as shown by these two watches? Did John have to set his watch forward or backward to correct it?
4. They immediately went by train from Southampton to London, to which point they had bought tickets in Chicago. Arriving in London, they took a taxicab to the hotel, paying the chauffeur 10s. 6d.; how much was this in United States money?
5. The next day Mr. Walker bought the following invoice of Nottingham lace:

   500 yards 2″ Lace @ 5s. 6d.
   800 yards 3″ Lace @ 7s.
   1,000 yards 4″ Lace @ 10s.

   How much did each item on this bill amount to in United States money? How much did the total bill amount to in United States money?
6. That evening they settled their hotel bill for two days at 15s. per day per person, and left for Brussels, Belgium; what was the amount of the hotel bill in United States money?

7. At Brussels, Mr. Walker bought the following Belgian lace:

6 Hectometers.. @ 5 fr. per m.
1 Kilometer .... @ 6 fr. per m.

What was the total amount of this bill in United States money?

8. When getting ready to leave Brussels for Paris, Mr. Walker arranged for transportation through the hotel clerk and found his entire bill amounted to 80 fr. Having no Belgian money, he handed the clerk £5 in English money; how many francs and centimes did he receive as change?

9. After making a number of purchases at Paris, Mr. Walker decided they would spend ten days sight-seeing in France, Switzerland, Spain and Italy before sailing for home. Their entire expenses for these ten days were as follows:

In France..........223 francs;
In Switzerland......200 francs;
In Spain...........152 peseta;
In Italy............175 lira;

How much was this in United States money?

10. They bought tickets from Italy direct to Chicago for 2,000 lira; how much was this in United States money?

11. In Rome, Italy, John had set his watch according to the time of longitude 15° E. and he did not change it until he reached Chicago; what had he to do to correct his watch to Standard Central Time?

Exercise 16—Oral Review.

Use your eyes quickly—keep looking ahead.

1. Read:

(*a*) 986,438,721,863,410;

(*b*) 876,439,000,000,001;

(*c*) 38,000,468,000,612.

2. Add 8,000,000; 9,000,000; 17,000,000; 20,000,000.

3. Add $6\frac{1}{2}$; $13\frac{1}{4}$; $10\frac{1}{2}$; $5\frac{1}{4}$.

4. Add .25; .75; .75; .75; .25;

5. Add $66\frac{2}{3}\%$; $66\frac{2}{3}\%$; $33\frac{1}{3}\%$; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$.

6. Give the sum of $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{1}{2}$.

7. Define longitude.

8. How much time corresponds to 360° of longitude? 90°? 15°? 1°? 1′? 1″?

9. Name the metric system unit used in measuring ribbons. In measuring the area of a lot. The amount of grain in a bin. The weight of a car.

10. What single word means 10,000 meters? 1,000 m.? 100 l.? 10 g.? .1 m.? .01 g.? .001 l.?

Exercise 17—Written Review.

1. Write:

(*a*) Four hundred sixteen trillion, eight thousand, sixteen.

(*b*) Thirty-eight billion, one hundred sixty-four million, nine thousand, twelve.

(*c*) Fifty-eight trillion, six million, six.

2. Reduce:

(*a*) 36,410″ to highest terms;

(*b*) 86° 30″ to seconds of arc;

(*c*) 90° 40′ 20″ to minutes of arc.

3. Two cities are 15° 15′ 15″ apart; what is the difference in their local time?

4. The longitudes of two cities are respectively: 95° 50′ 35″ E. and 65° 30′ 40″ W. What is the difference in their longitude? What is the difference in their time?

5. What time is it 45° 30′ east of you when it is 11 P. M. in your locality?

6. What time is it 75° 30″ west of you when it is 1 A. M. in your locality?

7. Add 684.32; 25.684; 695.404; 2.6568; .7923; 4.0005; 7.

8. From 12,000,000 take 8,654,209$\frac{7}{9}$.

9. Find the compound interest on \$3,000.00 for 1$\frac{1}{2}$ years at 6% payable semi-annually.

10. At what rate per cent will \$76.00 amount to \$77.52 in 120 days?

Subtract, but do not copy:

(Time for these 12 examples is less than 4 minutes.)

11.	**12.**	**13.**	**14.**
386,419	741,038	563,875	619,024
289,523	258,063	498,987	285,387

15.	**16.**	**17.**	**18.**
400,004	738,056	293,712	611,041
193,708	295,386	196,874	298,309

19.	**20.**	**21.**	**22.**
901,315	612,041	814,002	619,312
198,749	387,592	319,704	412,961

Copy and multiply:

(Time for these 8 examples is less than 4 minutes.)

23. 2,587 × 53;
24. 9,342 × 82;
25. 7,624 × 49;
26. 8,041 × 67;
27. 1,758 × 34;
28. 5,923 × 75;
29. 4,328 × 92;
30. 6,407 × 86.

Copy and divide:

(Time for these 6 examples is less than 4 minutes.)

31. 14,246 ÷ 419;
32. 33,087 ÷ 807;
33. 37,440 ÷ 720;
34. 36,146 ÷ 682;
35. 19,351 ÷ 523;
36. 20,736 ÷ 324.

Add, but do not copy:

(Time for these 6 examples is less than 4 minutes.)

37.	**38.**	**39.**	**40.**	**41.**	**42.**
7,341	6,194	1,234	1,787	2,631	4,879
8,629	9,387	5,874	8,641	1,649	3,092
6,295	8,495	9,021	3,953	9,345	2,877
3,129	4,729	3,849	7,865	5,076	8,985
4,728	6,028	6,128	2,013	6,032	6,139
5,138	4,197	2,941	8,741	3,775	8,229

POWERS AND ROOTS

Lesson 8

What Powers Are—Squaring and Cubing

The product obtained by using any number several times as a factor is a "power" of that number. Thus:

5 used twice as a factor gives us the 2d power of 5, or 25 (5×5).

5 used three times as a factor gives us the 3d power of 5, or 125 ($5 \times 5 \times 5$).

The 2d power of any number is called the "square" of that number; the 3d power is called the "cube" of that number.

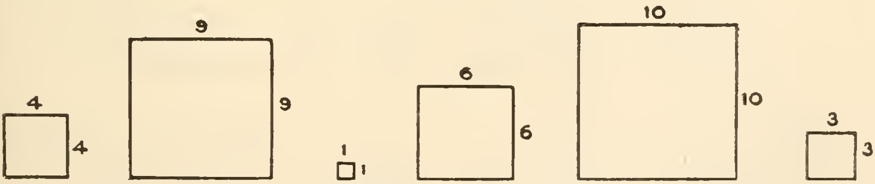
$5 \times 5 = 25$, also written 5 square, or 5^2.

$5 \times 5 \times 5 = 125$, also written 5 cube, or 5^3.

The small figure telling the *power* of the number is called the exponent. What exponents were used here?

Exercise 18—Oral.

1. Give the areas of the following:

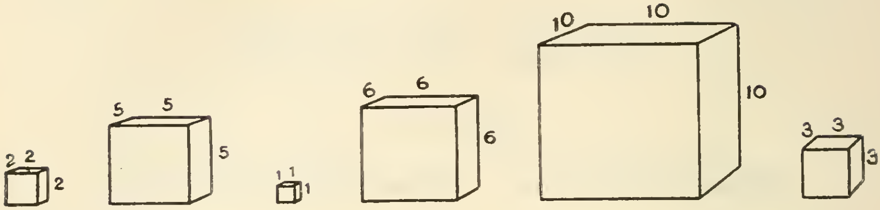


2. $4^2 = ?$; $9^2 = ?$; $1^2 = ?$; $6^2 = ?$; $10^2 = ?$; $3^2 = ?$.

3. Give the second power of 5; 4; 7; 8; 6; a; x.

4. Give the square of 12; c; 3; a; 4.

5. Give the volumes of the following:



6. $2^3 = ?$; $5^3 = ?$; $1^3 = ?$; $6^3 = ?$; $10^3 = ?$; $3^3 = ?$.

7. Give the third power of 3; 2; 6; 5; 4; 1.

8. Give the cube of 2; m; 4; 1; c.

9. Read the exponents here: 4^2; m^3; 4^2; 7^2; 8^3; 2^4.

10. Read and find the values of: 2^2; $(\frac{1}{2})^2$; $(\frac{1}{4})^2$; 12^2; $.2^2$;

11. By cross multiplication find the square of any number composed of 2 digits.

Example: $23^2 =$ 23 × 23	$3 \times 3 =$	9	1st step.
	$2(3 \times 2) =$	12	2d step.
529	$(2 \times 2) + 1 =$	5	3d step.

Exercise 19—Written.

Find the squares of:

1. 15;	**6.** 53;	**11.** $\frac{1}{2}$;	**16.** $8\frac{1}{2}$;
2. $4\frac{1}{3}$;	**7.** 81;	**12.** $\frac{3}{4}$;	**17.** $3\frac{1}{3}$;
3. 35;	**8.** 75;	**13.** $\frac{5}{2}$;	**18.** d;
4. 125;	**9.** 26;	**14.** $\frac{11}{12}$;	**19.** x;
5. 400;	**10.** .83;	**15.** $3\frac{1}{2}$;	**20.** a.

Find the cubes of:

21. 32;	**25.** 21;	**29.** $\frac{3}{7}$;	**33.** $2\frac{3}{4}$;
22. 45;	**26.** $2\frac{1}{2}$;	**30.** .25;	**34.** $1\frac{1}{2}$;
23. 9.6;	**27.** 64;	**31.** h;	**35.** c;
24. 100;	**28.** 48;	**32.** $\frac{2}{3}$;	**36.** m.

Find the values of:

37. 2.5^3; **38.** $.12^2$; **39.** 1.18^2; **40.** $(\frac{4}{5})^3$.

Lesson 9

What Roots Are

Any one of the equal factors which produce a number is called a "root" of that number; thus a number is the "root" of all of its "powers."

The root of a 2d power is called a "square root"; therefore, since $5 \times 5 = 25$, the square root of 25 is 5. 5 is *one* of the two equal factors that produce 25.

The root of a 3d power is called a "cube root"; therefore, since $5 \times 5 \times 5 = 125$, the cube root of 125 is 5. 5 is *one* of the three equal factors that produce 125.

To indicate square root, the symbol $\sqrt{}$ called a "radical sign" is used; thus, $\sqrt{25} = 5$. A small figure ^{2} called an "index" is sometimes written in the radical sign to indicate square root; thus, $\sqrt[2]{25} = 5$, but this is unnecessary as square root is the simplest form of root, and square root is understood unless some other index is used, as:

(*a*) $\sqrt{25}$ means square root of 25. Ans., 5.

(*b*) $\sqrt[3]{125}$ means cube root of 125. Ans., 5.

Fxercise 20—Oral.

1. What is the square root of 4? Of 49? Of 144? Of $\frac{1}{4}$?

2. What is the cube root of 8? Of 64? Of 27? Of $\frac{1}{8}$?

3. If the cube is 8, what is the root from which it came?

4. $\sqrt{36} = 6$; explain which is the power. Which is the root?
5. $\sqrt[3]{27} = 3$; explain which is the root. Which is the power?
6. Indicate that the square root of 169 is to be extracted. Indicate that the cube root of m^3 is to be extracted.
7. What name is given to the root sign?
8. Read, and find the values of:

$\sqrt{81} = ?$ $\sqrt[3]{27} = ?$ $\sqrt[2]{144} = ?$ $\sqrt[3]{8} = ?$

$\sqrt{.64} = ?$ $\sqrt[3]{1} = ?$

9. Read the index figures in Question 8.
10. Read the powers in Question 8.

Lesson 10

How to Extract the Square Root (Integers)

As you have already learned, the square of a number is the product obtained by using that number twice as a factor; hence, to find the square root when we know the square, we must reverse the process and find the number which was used twice as a factor to produce such square.

Root	Square	Root	Square	Root	Square
$1^2 =$	1;	$10^2 =$	100;	$100^2 =$	10,000;
$9^2 =$	81;	$99^2 =$	9,801;	$999^2 =$	998,001.

By looking over the table here given which shows the smallest and largest numbers containing one, two, and three digits, you will notice that when a number composed of one digit is squared, the square contains either one or two digits; when a number composed of two digits is squared, the square contains either three or four digits; and when a number composed of three digits is squared, the square contains either five or six digits; so if we allow 2 places for each digit in the root we will certainly have enough for even the highest; therefore, by beginning at the *units* always and separating a number into *periods of two digits each,* we know that the square root will contain as many digits as there are *periods* in the number.

Exercise 21—Oral.

For practice, tell at sight how many figures there will be in the square root of each of the following numbers:

1. 144;	**6.** 1,681;	**11.** 1,024;
2. 169;	**7.** 900;	**12.** 8,281;
3. 529;	**8.** 289;	**13.** 3,844;
4. 6,561;	**9.** 625;	**14.** 5,625;
5. 6,889;	**10.** 2,500;	**15.** 7,225.

Separate the following numbers into periods to show how many figures the square roots will contain:

16. 144;	**23.** 529;	**30.** 10,000;
17. 625;	**24.** 400;	**31.** 56,874;
18. 4,900;	**25.** 1,225;	**32.** 4,168;
19. 15,625;	**26.** 7,225;	**33.** 137;
20. 90,601;	**27.** 9,801;	**34.** 96;
21. 93,636;	**28.** 6,889;	**35.** 5;
22. 169;	**29.** 324;	**36.** 15,498.

In working an example by cross multiplication, we first multiply the units; second, we find the sum of the units multiplied by the tens plus the tens multiplied by the units; third, we multiply the tens; hence, when the multiplier and the multiplicand are exactly alike, as they always are in squaring a number, the first step is equivalent to *squaring the units*, $3^2 = 9$; the second step is equivalent to finding *the product of twice the tens multiplied by the units*, $2\ (1 \times 3) = 6$; and the third step is equivalent to *squaring the tens*, $1^2 = 1$.

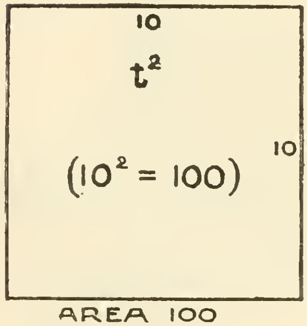
Finding the Square by Cross Multiplication

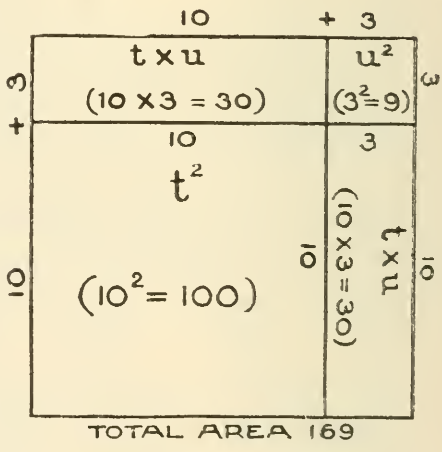
First Step
$$\begin{array}{r} 13 \\ \underline{13} \\ 9 \end{array}$$

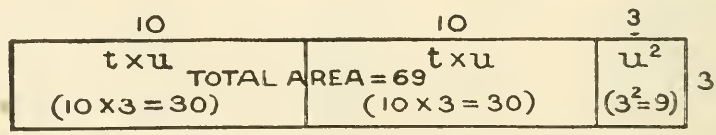
Second Step
$$\begin{array}{r} 13 \\ \underline{13} \\ 69 \end{array}$$

Third Step
$$\begin{array}{r} 13 \\ \underline{13} \\ 169 \end{array}$$

All the parts in the square of 13 = 169.





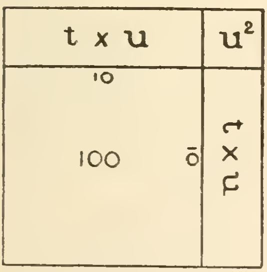


$(10 \times 2) + 3 = \text{length}$

$3 = \text{width}$

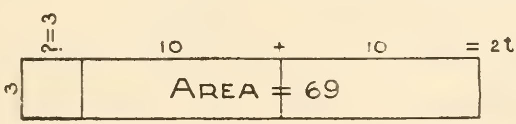
$23 \times 3 = \text{area } 69$

Reversing this process, we separate the square (169) into periods of two figures each, and starting with the reverse of the third step, we find the greatest square in the left hand period (1 hundred) from which we know the tens' figure of the root is 1; we then subtract the 1 hundred and bring down the next period (69) which resulted originally from the second and first steps of the multiplication, and, therefore, contains twice the tens multiplied by the units (cross multiplication) plus the square of the units; we therefore divide 69 by twice the tens (20) and find this quotient to be 3; therefore, 3 is the units' figure, but before we complete the second step, we add the units' figure (3) to 20 giving us 23, so that when we multiply by 3 we have also reversed the first step, since the 3 which we added to 20 is squared when 23 is multiplied by 3.



Cross Multiplication Reversed for Finding the Square Root

	1 3, Ans.
	1'69 (1 ten
t^2 or 1^2 =	1 00
10 × 2 = 20 (2 tens)	69 (3 units
3 (units)	(width)
23 × 3 =	69



Trial Divisor = 20

Real Divisor = 23 because 23 is the real length.

Exercise 22—Written.

Find the square root of each of the following and prove:

1. 169;	**5.** 625;	**9.** 1,024;
2. 144;	**6.** 1,681;	**10.** 6,889;
3. 529;	**7.** 4,900;	**11.** 1,225;
4. 196;	**8.** 324;	**12.** 4,489;

13. 3,136; **14.** 9,604.

Lesson 11

How to Extract the Square Root (Decimals and Fractions)

EXAMPLE: $\sqrt{2\frac{7}{9}} = ?$

$\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}}$, or $\frac{5}{3}$, or $1\frac{2}{3}$, Ans.

Proof: $(1\frac{2}{3})^2 = 2\frac{7}{9}$

Since both terms of $\frac{25}{9}$ are perfect squares, we can extract the square root of each term to find the corresponding term of the root.

In finding the square root of a fraction we must find the root of the numerator and of the denominator separately, and this we cannot do unless each of the two terms of the fraction is a perfect square.

When either the numerator or the denominator is not a perfect square, the fraction must be reduced to a decimal before finding the root, in which case the root will be in the form of a decimal.

Remember, we can extract the square root of a common fraction (as such) only when each of its two terms is a perfect square, otherwise the common fraction must be reduced to a decimal before we can extract the root.

EXAMPLE: $\sqrt{1{,}500.} = ?$

	3 8. 7 2+, Ans.	
	15 00.00'00	(3
$3^2 =$	9	
$30 \times 2 = 60$	6 00	(8
8		
$68 \times 8 =$	5 44	
$38.0 \times 2 = 76.0$	56.00	(7
.7		
$76.7 \times .7 =$	53.69	
$38.70 \times 2 = 77.40$	2.31 00	(2
.02		
$77.42 \times .02 =$	1.54 84	
	.76 16 Remainder.	

Proof: $38.72^2 = 1{,}499.2384$
Plus Remainder .7616
1,500.

We annex a period of two ciphers for every decimal place we wish to use in the root and proceed as before. Each new trial divisor is obtained by annexing a cipher to the root figures previously found (as if they were tens), and multiplying by 2.

In finding the square root of numbers containing decimals, the periods in the *integer are separated toward the left beginning with units,* and the periods in the *decimal are separated toward the right from units;* as, $\overset{\frown}{68}\overset{\frown}{23}.\overset{\frown}{45}\overset{\frown}{50}$; an even number of decimal places is always necessary so that the denominator of the decimal fraction may be a perfect square, otherwise the denominator in the root cannot be expressed decimally; as we cannot express the root of thousandths decimally, we annexed a cipher to .455 making it .4550, and the root of ten-thousandths is hundredths.

Exercise 23—Oral.

1. Separate the following numbers into periods of two figures each in the manner necessary for extracting the square root, and read the figures in each period.

(*a*) 468,736;
(*b*) 1,637,462;
(*c*) 8,746.3621;
(*d*) 96,386.472;

2. How many figures will there be in the square root of a number containing 1 or 2 figures? 3 or 4 figures? 5 or 6 figures?

3. How many decimal places are there in the square of .9? Of .19? Of .09? Of .123? How many of these squares contain an odd number of decimal places? How many of these squares contain an even number of decimal places?

4. Why does the square of a decimal always contain an even number of decimal places?

5. If you were asked to extract the square root of .463 (.463 contains an odd number of decimal places), how would you proceed so that you could point off properly in the root?

6. How do we find the square root of $\frac{25}{36}$? Of $\frac{49}{84}$?

7. Which method would you use to find the square root of $\frac{4}{9}$? Of $\frac{3}{4}$? Of $\frac{4}{6}$? Of $\frac{7}{8}$?

8. Without using paper, what do you think is the square root of 169; 290; 6,400; 1; 600.

Exercise 24—Written.

Find the square roots of the following and prove your answers:

1. 225.; **3.** 15,625.;
2. 1,225.; **4.** 2,809.;

5. 670,761.;	8. 2.;	(carry 3 places)	
6. 1.3924;	9. 1,000.;		
7. 15.625;	10. 45.;		

Exercise 25—Written.

Find the square roots of the following and prove your answers:

1. $18\frac{7}{9}$; 2. $\frac{81}{100}$; 3. $1\frac{1}{2}$; 4. $\frac{121}{16}$; 5. $\frac{7}{9}$.
6. Find the length of the side of a square whose area is 289 sq. in.
7. The floor of a square room contains 324 sq. ft.; how many yards long and wide is it?
8. What is the length of the perimeter of a square containing $30\frac{1}{4}$ sq. yd.?
9. How many boards 6″ wide will be needed to build a fence around a square lot containing 15,876 sq. ft.?
10. What is the volume (in cubic inches) of a cube if the area of one of its faces is 676 sq. in.?

Lesson 12

How to Extract the Square Root by Factoring

EXAMPLE: $\sqrt{6,561} = ?$

82. = other factor		82. (Quotient) } Unequal
80.) 6561.		80. (Divisor) } Factors
640	Proof by	2) 162.
161	Division	81. (Average Factor)
160	81.	(The quotient (81) being the
1	81.) 6,561.	same as the divisor (81), we
*Approximately	6 48	know that we have found the
this seems to	81	square root.)
be one factor.	81	

Find $\sqrt{6,561}$ by approximating one factor then dividing to find the other approximately. Of course, they will not be equal factors until you average them.

```
                EXAMPLE:  √2,401 = ?
      40             40 (Quotient  ⎫
 60)2401             60 (Divisor)  ⎭ Unequal Factors.
    2400          2)100
       1             50 (Average Factor)
Proof by Division
      48             48 (Quotient) ⎫
 50)2401             50 (Divisor)  ⎭ Unequal Factors.
    200            2)98
     401             49 (New Average Factor)
     400
       1
Proof by Division
      49
 49)2401          (Now the quotient (49) is the same as
    196             as the divisor (49), so we know that
     441            we have found the square root.)
     441
```

If your proof by division gives a quotient which is different than the divisor, you, of course, know that you have not yet found the square root. In that case use your divisor and your quotient as two new factors, the average of which will give you a closer approximation to the actual square root. You can then prove again by division and if necessary average again till your quotient is the same as your divisor.

In extracting the square root of a number which is not a perfect square, we can carry out the work by this method to any number of decimal places that may be desired.

Exercise 26—Written.

Find the square roots by approximating factors and prove:

1. 6,889;	**6.** 3,136;	**11.** 3;
2. 4,489;	**7.** 9,604;	**12.** 2;
3. 1,681;	**8.** 529;	**13.** 5;
4. 1,024;	**9.** 6.25;	**14.** 10;
5. 1,225;	**10.** 12.25;	**15.** 12.

You may like to use this method when your numbers are small.

Exercise 27—Written Review.

Add in five or six minutes; then prove:

1.	**2.**	**3.**	**4.**	**5.**
47,312	36,872	1,312	72,164	63,128
8,642	4,120	16,198	3,190	4,386
97,628	90,000	8,888	501	72,101
7,463	3,612	7,212	83,120	99,999
87,211	12,762	46,328	1,648	1,687
16,411	36,812	5,005	48,729	31,012

Subtract in four minutes; then prove:

6.	**7.**	**8.**	**9.**	**10.**
13,724	36,874	35,687	46,342	18,234
1,987	1,498	2,129	39,717	306

Multiply in four minutes; then prove:

11. 4,621 × 13;
13. 138 × 75;
12. 19,872 × 312;
14. 7,623 × 126;
15. 1,201 × 52.

Divide in four minutes; then prove:

16. 38,645 ÷ 28;

17. 13,608 ÷ 54;

18. 41,288 ÷ 8;

19. 13,012 ÷ 18;

20. 50,000 ÷ 55.

Work in four minutes; then prove:

21. $\sqrt{670,761}$

22. $\sqrt{1,225}$

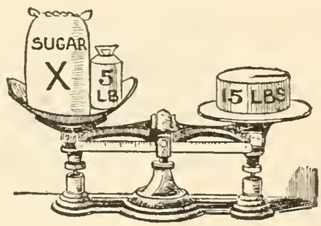
23. $\sqrt{1.3924}$

EQUATIONS

Lesson 13

Numbers and Quantities Represented by Letters

Very often it is convenient, and frequently it is necessary, to use a letter to represent an unknown number or quantity while solving a problem; in such cases, when we have found the value of the letter, we have, of course, found the value of the number or quantity which that particular letter represents.



As an example we will say that a grocer had a certain number of pounds of sugar on a scale and by placing a 5-pound weight with it, a 15-pound weight was required to balance the scale. Now, if we let x represent the unknown number of pounds of sugar we find that

$$x + 5 = 15.$$

Can you find the value of x? Try.

A statement in the form of this one "$x + 5 = 15$" is called an "equation," for, as you know, an equation is a statement showing the equality of two quantities by placing one before and one after the equality sign.

The quantity written before or to the left of the equality sign is called the "first member" of the equation, and the one written after or to the right of the equality sign is called the "second member" of the equation. We usually use the last three letters ($x\ y\ z$) of the alphabet to represent unknown quantities.

Finding the value of the letter which represents the unknown quantity is called "solving" the equation.

Exercise 28—Oral.

1. In the equation $y + 10 = 12$, is y a known or an unknown quantity? What is 10? What is 12?
2. Read this equation: $z - 5 = 2$. Is z a known or an unknown quantity? What is 5? What is 2?
3. Make an equation to show that 4 added to a certain quantity equals 12.
4. Make an equation to show that 8 subtracted from a certain quantity equals 2.
5. If $x + 2 = 5$, 5 is how much more than x? Does this equation express the same condition: $x = 5 - 2$? What is the value of x?
6. If you subtract 4 from both members of this equation: $y + 4 = 9$, how will the equation read? What is the value of y?
7. Solve this: $z + 3 = 6$.
8. Solve this: $a + 9 = 10$.
9. Solve this: $b + 7 = 11$.
10. Solve this: $c + 4 = 14$.
11. Make an equation to show that all the chairs in a room and 3 chairs more make a total of 9 chairs.
12. Make an equation to show that all the water in a tank excepting 10 gallons equals 90 gallons.
13. Make an equation to show that all the oats in a bin excepting 7 bushels equals 18 bushels.

Lesson 14

Solving Equations (Adding and Subtracting)

EXAMPLE:	EXAMPLE:
$x + 3 = 12$	$x + 3 = 6$
$-\quad 3 \quad 3$	$+\quad 3 \quad 3$
$x = 9$	$x + 6 = 9$

Since the two members of an equation are equal in value, it is easily understood that if we add the same number to both members, or if we subtract the same number from both members, the equality will not be disturbed.

EXAMPLE:	EXAMPLE:
$x + 3 = 12$	$x + 3 = 6$
same as $x = 12 - 3;$	same as $x = 6 - 3;$
because $x = 9$	because $x = 3$

The same result is obtained by transposing a quantity from one member to the other, at the same time changing the sign from + to − or from − to + as the case may be.

In solving equations, first transpose all the known quantities to the second member, changing signs from + to − or from − to + as is necessary.

In all equation work, remember that unlike numbers must be kept separate:

$3x + 2x = 5x;$ $\qquad 3x - 2x = x;$

$3x + 3 = 3x + 3;$ $\qquad 3x - 2 = 3x - 2;$

$3x + 2x + 5x = 10x;$ $\qquad 3x + 2x + 2y = 5x + 2y;$

$3x - x - 2y = 2x - 2y;$ $\qquad 4z + 5z = 9z.$

Exercise 29—Oral.

First say how you will transpose the known quantities to the second member, then solve:

1. $x + 4 = 8$;	**11.** $14 = c + 5$;
2. $y - 3 = 7$;	**12.** $18 = y - 2$;
3. $z + 8 = 12$;	**13.** $7 = x + 1$;
4. $a + 5 = 10$;	**14.** $11 = y - 5$;
5. $b - 4 = 3$;	**15.** $12 = z + 10$;
6. $5 + c = 11$;	**16.** $18 = 10 + b$;
7. $8 + x = 16$;	**17.** $14 = 17 - z$;
8. $12 + y = 15$;	**18.** $13 = 1 + x$;
9. $10 + z = 11$;	**19.** $11 = 3 + z$;
10. $8 - x = 6$;	**20.** $12 = 15 - y$.

21. What number increased by 8 equals 12?

22. What number decreased by 9 equals 6?

23. What number is 5 more than 10?

24. What number is 12 less than 20?

25. If $x + 20 = 35$, what is the value of x?

Solve:

26. $3x + 4x = ?$	**37.** $5x - x = 12$;
27. $6y - 4y = ?$	**38.** $2z + 4z + 5 = 17$;
28. $7z + 3 = ?$	**39.** $4y - y - 3 = 9$;
29. $4x - 2 = ?$	**40.** $3x = 10 + x$;
30. $5x + 2y = ?$	**41.** $8z = 20 - 2z$;
31. $2z + 4z + z = ?$	**42.** $5y + 2y = 24 - y$;
32. $a + 2a + 3a = ?$	**43.** $7z - 3z = 9 + z$;
33. $z + 4z - 2z = ?$	**44.** $9x + 5x = 24 + 2x$;
34. $y + 5y - y = ?$	**45.** $4y + 3y + 4 = 20 - y$;
35. $x + x + 3x = ?$	**46.** $3\mathrm{b} + 4\mathrm{b} = 24 - \mathrm{b}$;
36. $3y + 2y = 10$;	**47.** $5\mathrm{c} - 3\mathrm{c} = 10 + \mathrm{c}$.

Lesson 15

Solving Equations (Multiplying and Dividing)

EXAMPLE:	EXAMPLE:
$x = 3$ $2\overline{)2x = 6}$	$\frac{1}{3}x = 4$ $\times \quad 3$ $x = 12$

Since both members of an equation are equal in value, if we multiply both members by the same number, or divide both members by the same number, the equality will not be disturbed.

Since $x + x + x = 3x$, then $3 \times x$ must also equal $3x$; therefore, an expression such as $3x$ means $3 \times x$, but expressions of this kind are always written without the sign of multiplication. $4y$ means $4 \times y$; $8z$ means $8 \times z$; etc. $y \div 3$ may be written as $\frac{1}{3}y$, or as $\frac{y}{3}$; $z \div 4 = \frac{1}{4}z$, or $\frac{z}{4}$; etc.

EXAMPLE:

$$\frac{x}{3} = \frac{1}{2};\ \text{L. C. D. } (6); \frac{2x}{6} = \frac{3}{6};$$
$$2x = 3;\ x = 1\tfrac{1}{2}.$$

When there are denominators in the equation, both members must be reduced to the L. C. D. and then the denominators can be dropped without disturbing the equality. This step is called "clearing of fractions."

EXAMPLE:	EXAMPLE:
$2x = 6$ same as $x = 6 \div 2$; because $x = 3$	$\frac{1}{3}x = 4$ same as $x = 4 \times 3$; because $x = 12$

Here again, we can obtain the same result by transposing a quantity from one member to the other, at the same time changing the sign from $\div$ to $\times$ or from $\times$ to $\div$ as the case may be.

Exercise 30—Oral.

First say how you will solve each equation, then give the value of x, y, or z:

1. $4x = 12$;
2. $5y = 10$;
3. $2z = 20$;
4. $24 = 6x$;
5. $32 = 8z$;
6. $\frac{1}{3}y = 4$;
7. $\frac{1}{2}x = 5$;
8. $\frac{2}{3}z = 6$;
9. $12 = \frac{3}{4}x$;
10. $15 = 1\frac{1}{2}y$;
11. $\frac{1}{2} = \frac{x}{6}$;
12. $\frac{1}{4} = \frac{x}{12}$;
13. $\frac{2}{3} = \frac{x}{6}$;
14. $\frac{x}{4} = \frac{1}{8}$:
15. $\frac{x}{2} = \frac{2}{3}$;
16. $\frac{y}{3} = 5$;

Solve:

17. $3 \times y = ?$
18. $4 \times z = ?$
19. $12 \times a = ?$
20. $8 \times b = ?$
21. $b \times 8 = ?$
22. $c \times 3 = ?$
23. $x \times 5 = ?$
24. $100 \times c = ?$
25. $x \div 3 = ?$
26. $y \div 8 = ?$
27. $\frac{1}{3}y = \frac{?}{3}$;
28. $\frac{1}{4}x = ?$
29. $\frac{y}{6} = ? \div ?$
30. $\frac{1}{5}z = ? \div ?$
31. $a \div 6 = ?$
32. $\frac{b}{12} = ?$

Exercise 31—Oral and Written.

A. State the equation that you will use in solving each of these problems:

1. The distance between two trees is 5 times the distance between one of the trees and a shrub. If

the distance between the tree and the shrub is 10 yd., how far apart are the two trees?

(Let x represent the distance between the two trees; therefore, $\frac{1}{5}x = 10$ yd., and $x = 50$ yd., Ans. Proof: $\frac{1}{5}$ of 50 yd. = 10 yd.)

2. I have two books, one with a red cover and one with a blue cover. The red book weighs 5 oz. less than 6 times as much as the blue book and the weight of the blue book is 3 oz. What is the weight of the red book?

3. The minute hand on a watch moves 12 times as fast as the hour hand. The two hands point in exactly the same direction at 12 o'clock. What time will it be when they next point in exactly the same direction?

(Let x represent the distance the hour hand must move.)

4. The height of a certain oak tree is 8 times the height of John, and John is $\frac{1}{3}$ as tall as a young poplar tree. If the height of the poplar tree is 12 ft., what is the height of the oak tree?

5. One-half of a certain number is 3 more than one-third of the number. What is the number?

6. The sum of two numbers is 22, and one of the numbers is 2 less than the other; what are the two numbers?

7. The width of Park Avenue is 30 ft. from curbing to curbing. The combined width of both sidewalks is 7 ft. less than one-half the entire width of the street including the sidewalks. What is the width of each sidewalk?

8. The perimeter of a rug is 54 ft.; if the difference between the length and the width is 3 ft., what are the dimensions of the rug?
9. The perimeter of a building lot is 450 ft.; if the combined length of the two long sides is 50 ft. greater than the combined length of the two short sides, what are the dimensions of the lot?
10. The height of a room is 3 ft. less than the length but 2 ft. more than the width; if the combined length, width and height is 37 ft., what are the dimensions of the room?

B. Now solve the problems; also prove each answer.

MENSURATION

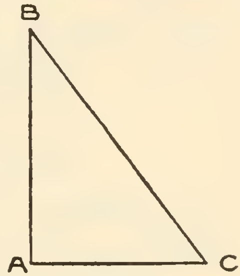
Lesson 16

Right Triangles

Read carefully.

A right triangle is a triangle having one right angle. The two other angles of a right triangle are acute angles.

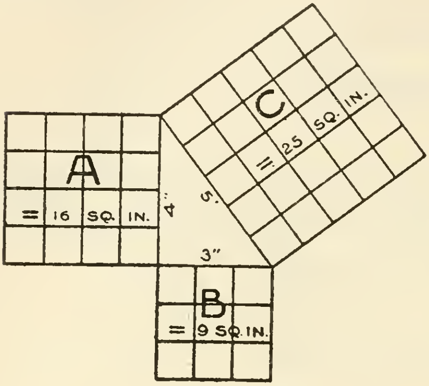
The two sides which form the right angle are called the "legs" of the triangle, and the remaining side is called the "hypotenuse"; thus, in the right triangle *ABC* shown in Figure 1, the sides *AB* and *AC* are the legs and *BC* is the hypotenuse.



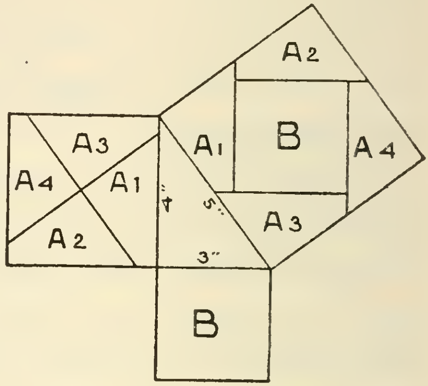
(Figure 1)

Over two thousand years ago, a Greek mathematician named Pythăgoras discovered the fact that when squares are drawn on all three sides of a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two other sides. This rule is known as the Pythagōrean theorem.

To prove that this is so, we need only to make three cardboard squares to correspond with the three sides of a right triangle as shown in Figure 2. Taking the square which is neither the largest nor the smallest, that is Square *A*, find the point where its diagonals cross, that being the exact center of the square. Now divide and cut this square into four equal parts by drawing two lines through the center of the square, the first line running parallel to the hypotenuse of the given triangle, and the second line crossing the first line at right angles, as shown in Figure 3. Now lay Square B and the four parts of Square A on Square C as indicated in Figure 3.



(Figure 2)



(Figure 3)

It naturally follows, that since the square on the hypotenuse ($C = 25$ sq. in.) is equal to the sum of the squares on the two other sides ($A = 16$ sq. in. $+ B = 9$ sq. in.) then $C - B = A$, and $C - A = B$; in other words, the square on the hypotenuse minus the square on either leg equals the square on the other leg.

Since the length of one side of a square is equal to the square root of the area, we can find the length of any side of a triangle when we know the area of the square on that side by extracting the square root of such area. Thus, in Figure 2, if we know that the area of A is 16 sq. in., we know that the length of that side is $\sqrt{16}$ in., or 4 in.; in like manner, the area of B is 9 sq. in., therefore, the length of that side is $\sqrt{9}$ in., or 3 in.; and the area of C is 25 sq. in., therefore, the hypotenuse is $\sqrt{25}$ in., or 5 in.

Count the blocks in the three squares shown in Figure 2 above? Is it true?

Make one of your own and use 6″ and 8″ as the legs to find the hypotenuse. Prove by blocking it as he did. Now prove by cutting it and fitting the parts.

The sign △ is often used to indicate a triangle; ⚠ for triangles.

Exercise 32—Oral.

The children of one row are to ask the following questions of the others:

1. Referring to the triangle shown in Figure 2, state the length of each of the two legs. Of the hypotenuse.
2. What kind of a triangle is this? Why?
3. What is the ratio of a square constructed on the hypotenuse of a right triangle to the sum of squares constructed on the two legs?
4. State the rule covering the theorem discovered by Pythagoras regarding squares constructed on the sides of right triangles.
5. Again referring to Figure 2, how can we find the area of Square *C* when we know the areas of *A* and *B*? How can we find *A* when we know *B* and *C*? How can we find *B* when we know *A* and *C*?
6. How can we find the square on either leg of a right triangle when the squares on the hypotenuse and on the other leg are known?
7. If we know the length of any side of a △, how do we find the square on that side?
8. If we know the square on any side of a △, how do we find the length of that side?

9. If the square on the hypotenuse is 64 sq. in., what is the length of the hypotenuse?
10. If the squares on the two legs of a right triangle are respectively 36 sq. in. and 64 sq. in., what is the area of the square on the hypotenuse? What is the length of the hypotenuse? What is the length of each of the two legs?

Exercise 33—Written.

1. The legs of a right triangle are respectively 15 ft. and 20 ft.; what is the length of the hypotenuse?
2. The hypotenuse of a right △ is 95 ft. and one of the legs is 57 ft.; what is the length of the other leg?
3. The hypotenuse of a right triangle is $22\frac{1}{2}$ in. and one of the legs is $13\frac{1}{2}$ in.; what is the length of the other leg?
4. The legs of a right △ are respectively 6 ft. and 8 ft. long; what is the length of the hypotenuse?

   (Make a chalk mark on the floor 8 feet from a wall, and make one on the wall 6 feet from the floor; prove, by using a string of the length corresponding to the hypotenuse of this triangle, that the wall is perpendicular.)
5. Using the principle outlined in Question 4, open a door so that it forms an exact right angle with the wall.
6. The legs of a compass are 5 in. long; what is the distance from the point of one leg to the point of the other leg when the compass is opened to a right angle?

7. A smoke-stack 40 ft. high is to be held rigid by 4 wires running from the top of the stack to points 30 ft. from the bottom of the stack; allowing 10 ft. for fastening each of these 4 wires, what is the total length of the wire required to anchor this stack?

8. A baseball diamond is 90 ft. square; how far is it from first base to third base?

9. What is the length of the diagonal of a 20-ft. square?

10. The area of a square lot is 15,625 sq. ft.; what is the length of its diagonal?

11. Find the longest line in a rectangle 20 yd. long and $12\frac{1}{2}$ yd. wide.

12. Find the diagonal of the ceiling of a room 40′ long and 35′ wide.

13. Find the length of the line from the upper corner of a room to the lower corner diagonally opposite; the room measurements are: length, 45 ft.; height, 15 ft.; width, 32 ft.

    (Suggestion: Get the diagonal of the ceiling first.)

14. Find the volume of a cube whose entire surface is 486 sq. in.

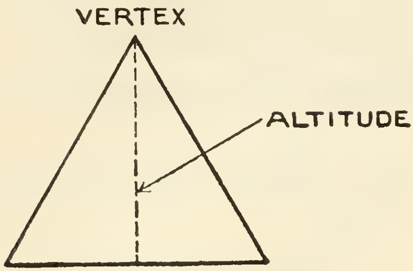
Lesson 17

Isosceles and Equilateral Triangles

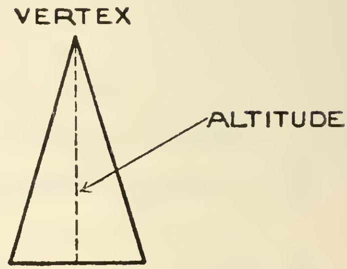
A triangle having three equal sides is an "equilateral" triangle.

A triangle having two equal sides is an "isosceles" triangle; therefore, every equilateral triangle is also isosceles.

A right triangle may be isosceles, but can it ever be equilateral? Why not?



Equilateral Triangle

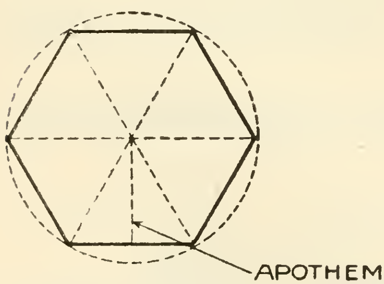


Isosceles Triangle

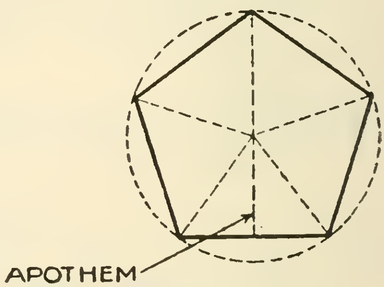
Read carefully so you can question well.

A straight line drawn from any vertex of an equilateral triangle to the middle of the opposite side is perpendicular to that side, and divides the side as well as the triangle into two equal parts.

A straight line drawn from the vertex where the two equal sides of an isosceles triangle meet to the middle of the base is perpendicular to the base, and divides the base as well as the triangle into two equal parts.



A Regular Hexagon



A Regular Pentagon

A regular hexagon is composed of six equilateral triangles.

A regular pentagon is composed of five isosceles triangles.

A perpendicular line drawn from the center of one of the sides to the center of a many-sided figure is called an "apothem"; it corresponds to the altitude of a triangle.

Exercise 34—Oral.

1. Draw an equilateral △ on the board.
2. Draw an isosceles △.
3. Draw a right triangle which is also an isosceles △.
4. Show that a right triangle can also be an isosceles △.
5. Draw an altitude in an isosceles △. Tell what it does to the base. To the △.
6. Has every triangle an altitude? Tell about it.
7. A regular hexagon has how many sides?
8. Of how many triangles is a regular hexagon formed? Of what kind of triangles is it formed?
9. A regular pentagon has how many sides? How many △s? What kind of △s?
10. Of how many triangles is a regular octagon formed?
11. What is an apothem of a hexagon?
12. What is the altitude of a triangle?
13. An equilateral triangle has how many equal sides? How many equal angles?
14. An isosceles triangle has how many equal sides? How many equal angles?

Exercise 35—Written.

1. The base of an isosceles triangle is 6 ft. and the altitude is 4 ft.; what is the length of each of the two equal sides?

2. The base of an isosceles triangle is 36 ft. and each of the two equal sides is 30 ft. in length; what is the altitude?
3. What is the area of the △ referred to in Question 1?
4. What is the area of the △ referred to in Question 2?
5. What is the altitude of an equilateral triangle having sides 10″ long? If you find the altitude is 8.66 + in. when the sides are 10 in. long, what would be the altitude for each 1″ of side?
6. Knowing that the altitude of an equilateral triangle equals .866 of the side, find the altitude of an equilateral triangle having 25-inch sides.
7. What is the area of an equilateral triangle having sides 20″ in length?
8. What is the area of a regular hexagon having sides 4″ long?
9. What is the area of the largest regular hexagon which can be constructed in a circle having a a 6″ radius?
10. A regular pentagon whose sides are 12″ long is constructed in a circle having a 9″ radius; what is the area of this pentagon?

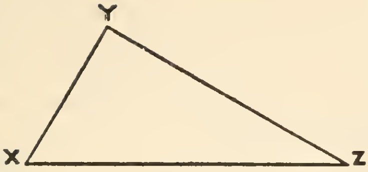
Lesson 18

Similar Triangles

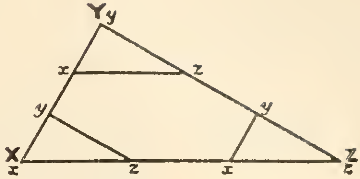
Triangles that have the same shape, though they be different in size, are called "similar triangles."

Naturally, the corresponding angles of similar triangles are equal, and the corresponding sides of similar triangles are proportional, otherwise the triangles could not have the same shape.

(Figure 1)

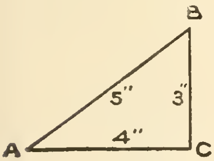


(Figure 2)

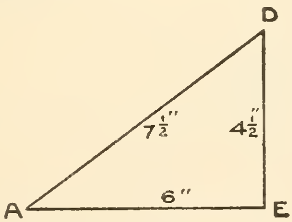


(Figure 3)

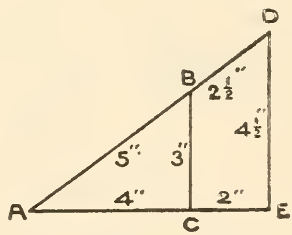
That Triangle xyz (Figure 1) is similar to Triangle XYZ (Figure 2) is proven by the fact that each of the angles of Triangle xyz is equal to the corresponding angle of Triangle XYZ as is shown by Figure 3 where Triangle xyz is placed on Triangle XYZ in three different positions. That these two triangles are similar is further proven by the fact that each of the sides of Triangle XYZ is three times as long as the corresponding side of Triangle xyz.



(Figure 4)



(Figure 5)



(Figure 6)

Test: See if the angles of Figure 4 coincide with the corresponding angles of Figure 5. Cut your own △ and fit them to coincide or, as we say, "superimpose" one on the other. Try placing your smaller triangle in three different positions as shown in Figure 3.

Knowing that corresponding sides of similar triangles are proportional, it is quite evident that if we know the length of any two sides of a given triangle and the length of one corresponding side of a similar triangle, the length of the other corresponding side may be found by proportion; thus:

EXAMPLE: Find the length of *AE*, if—
Triangle *ABC*, $AB = 5''$; $AC = 4''$;
Triangle *ADE*, $AD = 7\frac{1}{2}''$; $AE = ?$;
$7\frac{1}{2} : 5 = ? : 4$; or $\frac{7\frac{1}{2}}{5} = \frac{?}{4}$;
Using L. C. D. (20), $\frac{30}{20} = \frac{?}{4}$;
$\frac{30}{20} = \frac{6}{4}$; therefore (∴) $AE = 6''$, Ans.

In this example the length of *AE* is an unknown quantity which we are required to find. Let us call this unknown quantity x and see if we can work the example a little differently:

EXAMPLE: Find the length of x, if—
Triangle *ABC*, $AB = 5''$; $AC = 4''$;
Triangle *ADE*, $AD = 7\frac{1}{2}''$; $AE = x$;
$7\frac{1}{2} : 5 = x : 4$; or $\frac{7\frac{1}{2}}{5} = \frac{x}{4}$;
Reducing to L. C. D. (20) we have $\frac{7\frac{1}{2}}{5} = \frac{30}{20}$; $\frac{x}{4} = \frac{5x}{20}$,
If $\frac{30}{20} = \frac{5x}{20}$, then $30 = 5x$ or 5 times the unknown quantity;
If 30 is 5 times the unknown quantity, then the unknown quantity is $\frac{1}{5}$ of 30, or 6; ∴ $AE = 6''$, Ans.

Another example based on the same two triangles (Figures 4 and 5) follows. In this example we are required to find the length of *DE*, while in the previous example, we found the length of *AE*.

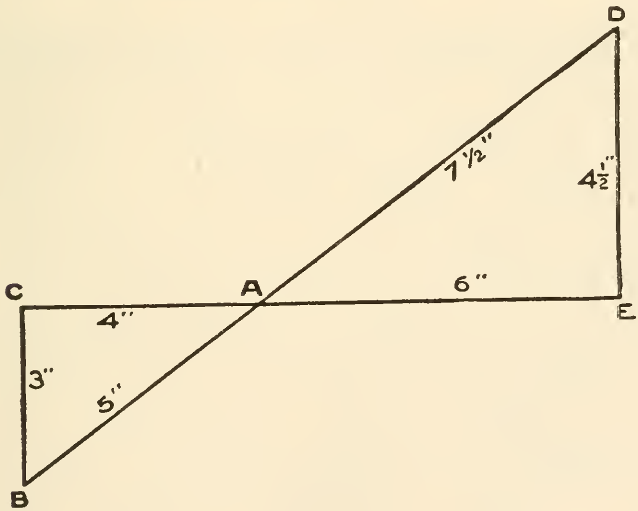
EXAMPLE: Find the length of x, if—
Triangle ABC, $AC = 4''$; $BC = 3''$;
Triangle ADE, $AE = 6''$; $DE = x$;

$6:4 = x:3$; or $\frac{6}{4} = \frac{x}{3}$; L. C. D. (12); $\frac{18}{12} = \frac{4x}{12}$;

After bringing to L. C. D. we can drop the denominator or "clear of fractions," therefore, $18 = 4x$;

If $18 = 4x$, $x = \frac{1}{4}$ of 18 or $4\frac{1}{2}$; $\therefore DE = 4\frac{1}{2}''$, Ans.

Triangles may occupy various positions and still be similar and, therefore, proportional. Thus, if Triangle *ABC* were inverted and attached to Triangle *ADE*, the figure would appear as follows:



(Figure 7)

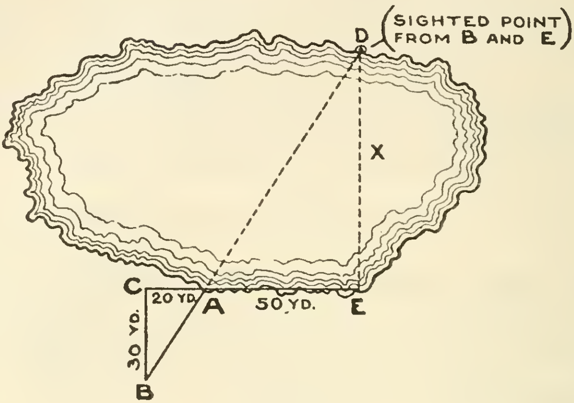
Are the same corresponding sides here?

Are the same corresponding angles here? Are they equal?

Does this position affect their similarity?

Are the figures similar?

Many practical measurements can be made by the use of triangles as is shown by the following:



(Figure 8)

Let us assume that we wish to measure the distance across a lake *DE*. We lay out the line *CE* with one end at *E*, and make it long enough so that we may construct similar triangles thereon; we then lay out the line *CB* perpendicular to *CE* and parallel to *DE*. By standing at *B* and looking at the point *D* which we recognize by some peculiarity of land or plant formation, we locate the point *A* where *BD* crosses *CE*.

We now measure and find *CA* to be 20 yd., *AE* to be 50 yd., and *CB* to be 30 yd.; this enables us to find the length of *DE* by the following example in proportion:

EXAMPLE: Find the length of x, if—
Triangle ABC, $AC = 20$ yd.; $BC = 30$ yd.;
Triangle ADE, $AE = 50$ yd.; $DE = x$;
Then $50:20 = x:30$; or $\frac{50}{20} = \frac{x}{30}$; L. C. D. (60); $\frac{150}{60} = \frac{2x}{60}$;
Clearing of fractions, $150 = 2x$;
Dividing by 2, $75 = x$; $\therefore$ $DE = 75$ yd., Ans.

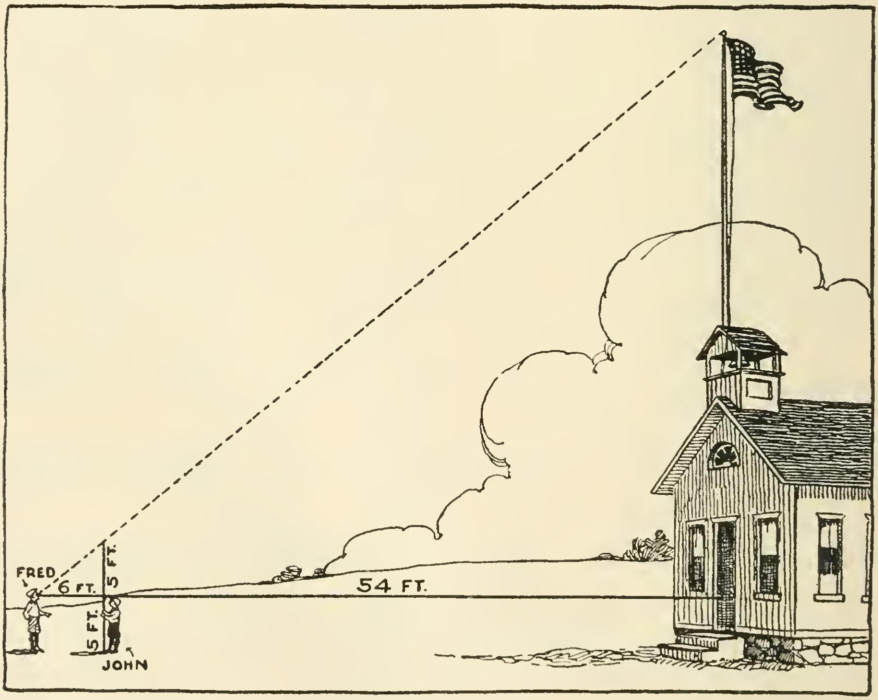
Exercise 36—Oral and Written.

Some boy may draw two similar triangles on the board. Members of 2 rows may do the questioning. Ask these questions and others:

1. Triangles of the same shape have what name?
2. Must similar triangles be of the same size?
3. What are corresponding sides?
4. If one side of a given triangle is exactly twice as long as the corresponding side of a similar triangle, how will the two other sides of the given triangle compare with the two other corresponding sides of the similar triangle?
5. Draw two similar triangles, one having sides $\frac{1}{4}$ longer than the corresponding sides of the other.
6. Draw two similar △, one of which has sides 75% longer than the corresponding sides of the other.
7. Must similar triangles occupy similar positions?
8. Draw two similar triangles, one of which has sides bearing the ratio of 1 to 2 to the corresponding sides of the other, and let one triangle occupy an inverted position in relation to the other.
9. In Figure 8, what would be the length of *DE* if *AE* were 40 yd.? 60 yd.? 80 yd.?
10. In Figure 8, what would be the length of *DE* if *CB* were 20 yd.? 40 yd.? 60 yd.?
11. If two of the angles of a triangle are equal to the two corresponding angles of another triangle, could the third corresponding angles be unequal? Draw several similar triangles and explain fully.
12. If $x = 10$ yd., what is the value of $2x$? Of $5x$? Of $10x$? Of $\frac{1}{2}x$? Of $\frac{1}{5}x$? Of $\frac{1}{10}x$?

13. If x represents an unknown quantity, what does $3x$ represent? $8x$? $\frac{1}{8}x$?
14. If $24 = 4x$, what is the value of x? Of $5x$? Of $\frac{1}{2}x$?
15. If $\frac{7}{10} = \frac{x}{20}$, what is the value of x?
16. If $\frac{7}{20} = \frac{x}{10}$, what is the value of x?
17. If $\frac{2}{3} = \frac{x}{2}$, what must be done before we can find the value of x? Reduce $\frac{2}{3} = \frac{x}{2}$ to L. C. D. Now find the value of x.
18. If $\frac{3}{4} = \frac{x}{3}$, what is the value of x?
19. Read and solve: $15 : 3 = x : 3$. $10 : 2 = x : 4$.
20. What is the meaning of proportional?

Exercise 37—Written.



1. Two boys had a friendly argument regarding the height of the top of the flag-staff shown in

the picture. Fred said he thought the height to be about 40 ft. from the ground, and John thought it to be about 70 ft., so they decided to ascertain its height by the use of similar triangles. John took a 10-ft. stick and stood 54 ft. from a point directly under the flag-staff; Fred had to stand 6 ft. farther from the staff to be able to sight the top of the staff over the top of the 10-ft. stick, and Fred's eyes were exactly 5 ft. from the ground; what was the height of the top of the flag-staff from the ground?

2. If Fred had to stand 5 ft. instead of 6 ft. farther from the staff than John to be able to sight the top of the staff over the 10-ft. pole, what would be the height of the top of the staff from the ground?

3. What is the height of a tree which casts a shadow 150 ft. long when a boy 4 ft. 2 in. tall casts a shadow 4 ft. long?

4. A certain mountain has its highest peak exactly over the center of the base, and the base is a quarter of a mile wide; how high is the highest peak if it can be seen over the top of a 25-ft. tree which stands 1,500 yards from the base of the mountain, when the observer whose eyes are 5 ft. from the ground stands 40 yards beyond the tree?

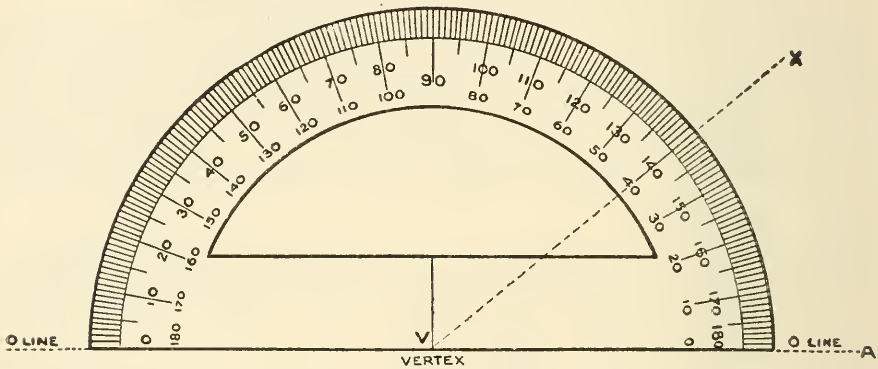
5. If a card $7\frac{1}{2}$ in. wide held 2 ft. from one of your eyes enables you to sight the width of a door which is 6 ft. beyond the card, how wide is the door? (Close one eye while trying this.)

6. If a card 4″ wide held 2′ 6″ from the eye completely hides the width of a picture 3′ wide from sight, how far from the eye is the picture?

Lesson 19

Table of Angular Measure

60 seconds (″)......	=	1 minute (′)
60 minutes.........	=	1 degree (°)
90 degrees.........	=	1 right angle (∟)



A PROTRACTOR

The instrument commonly used for measuring angles and arcs is called a "protractor" (see illustration). To measure an angle or arc place the vertex of the angle at the center of the protractor with one side of the angle running along the 0-line of the protractor; the reading where the other side of the angle falls is the number of degrees in the angle or arc. The angle $AVX = 40°$.

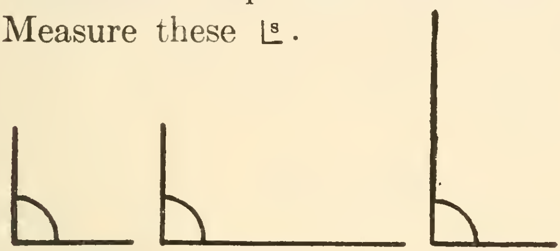
The sign for an angle is ∠; for several angles ∠s.

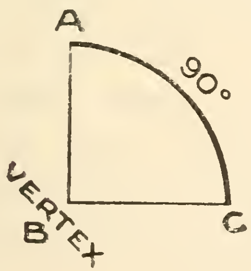
The sign for a right angle is ∟; for several right angles ∟s.

Exercise 38—Oral and Written.

(Make a protractor or buy one.)

1. Draw a circle. Locate C; D; R; show an arc; a quadrant; a sector.
2. How many ° are there in a circle? (Use protractor.)
3. How many ° are there in a right angle?
4. How many right angles are there in a circle?
5. Are the sides equal in these three ∠s? Prove.
6. Measure these ∠s.



7. Did the different lengths of sides affect the ∠s?
8. Draw a quadrant. Take $\frac{1}{2}$ of it with protractor.
9. How many ° are there in $\frac{1}{2}$ of $\frac{1}{4}$ of a circle?
10. $\frac{1}{2}$ of $\frac{1}{4}$ of a circle is what part of a circle?
11. 

What is the point called where the two sides of an ∠ meet? An angle is read with this point in the center: as ∠*ABC* or ∠*CBA*.
12. Measure these angles with your protractor:

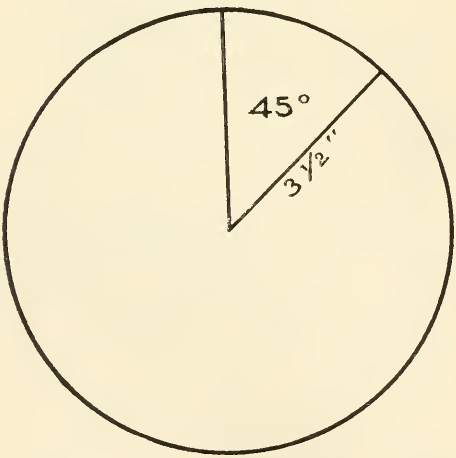
13. What is the sum of these two angles?
14. What is the difference between these two angles?

Exercise 39—Written.

1. Draw a triangle.
2. Letter the $\angle^s$ A, B, C.
3. Measure each carefully. Put your results on your drawing.
4. Find the sum of the 3 $\angle^s$.
5. Cut out the 3 $\angle^s$ carefully and place all vertices (plural of vertex) at one point on a straight line.
6. Does this prove your answer to Question 4?
7. This makes how many right angles?
8. When it makes 2 right angles call it a *straight angle.* (180°)

LESSON 20

Measuring the Length of Arcs and the Area of Sectors of Circles



(SCALE 1/4)

Read carefully.

You have learned that the circumference of a circle is equal to "pi" (π) times the diameter, and you also have learned that there are 360° in every circumference; hence, to find the length of any arc, we first find the length of the entire circumference of which the arc is a part, and then we find the required number of 360ths of such circumference corresponding to the number of degrees in the arc.

In the figure here shown, the circle has a $3\frac{1}{2}''$ radius or a $7''$ diameter, and the circumference, therefore, equals $7'' \times 3.1416 = 22''$ (nearly), and 45° is $\frac{45}{360}$ or $\frac{1}{8}$ of 360°; therefore, the length of this arc is $\frac{1}{8}$ of $22''$ $= 2\frac{3}{4}''$.

To find the area of a sector of a circle, we must first find the area of the entire circle and then take the required number of 360ths of such area corresponding to the number of degrees in the sector.

In the figure here shown, we find the circumference to be $22''$, and the area is found by multiplying the circumference by $\frac{1}{2}$ the radius; therefore, $22'' \times 1\frac{3}{4}$ $= 38\frac{1}{2}$ sq. in. area, and this sector contains 45° or $\frac{45}{360}$ of the entire area; $\frac{45}{360} = \frac{1}{8}$; $\frac{1}{8}$ of $38\frac{1}{2}$ sq. in. $= 4.8125$ sq. in., area of the sector.

Another way of finding the area of a circle, and one which is more generally used when the diameter or radius is known, is to multiply "pi" by the square of the radius; thus, $3.1416 \times (3\frac{1}{2})^2 = 38.5$ sq. in. (nearly). Can you show why this is true?

Exercise 40—Oral.

Pupils may ask questions of their classmates. Teacher will select the rows. Be ready to answer these questions in good English.

1. How do we find the circumference of a circle when we know the diameter?
2. How do we find the circumference of a circle when we know the radius?
3. How many degrees of arc are there in every circumference?

4. What fraction of an entire circumference is an arc of 180°? Of 90°? Of 45°?
5. Knowing the circumference of a circle, how can we find the length of any arc of that circumference?
6. How do we find the area of a circle? This sign ⊙ is often used to indicate a circle.
7. In what other way can we find the area of a ⊙?
8. Knowing the area of a ⊙, how can we find the area of any sector of that circle?
9. Explain why $\frac{C \times R}{2}$ = Area.
10. Study this out to see if you can tell why books say πR^2 = area of a circle. Watch the substitution:

$$\text{Area of } \odot = \frac{\not{C} \times R}{2}$$

$$\text{Area of } \odot = \frac{\pi \times \not{D} \times R}{2}$$

$$\text{Area of } \odot = \frac{\pi \times \not{2}R \times R}{\not{2}}$$

$$\text{Area of } \odot = \pi R^2.$$

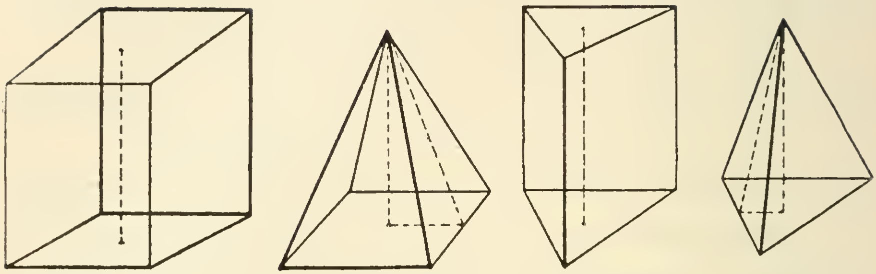
Exercise 41—Written.

1. A circle has a diameter of 1 yd.; what is the length of 180° of its circumference?
2. The radius of a ⊙ is 2 ft.; what is the length of 60° of its circumference?
3. An arc of 90° of the circumference of a certain ⊙ measures 5.1051 in. in length; what is the diameter of this ⊙?

4. An arc of 45° of the circumference of a certain ⊙ measures 3.1416 in. in length; what is the radius of this ⊙?
5. The diameter of a certain circle is 20 ft.; what is the length of 360° of its circumference?
6. A certain ⊙ has a radius of 4 ft.; what is the area of a sector of this ⊙ measuring 90°?
7. A certain circle has a diameter of 10 ft.; what is the area of a sector of this circle measuring 270°?
8. A ⊙ has a circumference of 314.16 yd.; what is the area of a sector of this ⊙ measuring 45°?
9. What is the area of $\frac{1}{4}$ pie if the radius of the pie is 5″?
10. The area of a semi-circular flower bed is 25.1328 sq. ft.; what is the length of its diameter?
11. The area of a certain quadrant is 3.1416 sq. ft.; what is the length of its radius?
12. The area of a sector measuring 72° of a ⊙ is 392.7 sq. ft.; what is the radius of this ⊙?
13. The area of a certain sector of a circle is 706.86 sq. in.; if the diameter of the circle is 90 in., how many degrees are there in this sector?
14. The area of a certain sector of a ⊙ is 508.9392 sq. yd.; the circumference of the ⊙ is 113.0976 yd.; how many degrees are there in this sector?
15. The area of a certain circle is 31,416 sq. yd.; what is the circumference of this circle?
16. A circular lake has an area of 1,256.64 sq. yd.; what is the radius of this lake?
17. What is the length of 90° of the circumference of the lake mentioned in Question 16?

Lesson 21

Pyramids



Rectangular Prism | Rectangular Pyramid | Triangular Prism | Triangular Pyramid

Read carefully.

A "pyramid" is a solid whose base is a triangle, square, or other polygon, and whose sides or lateral faces (corresponding in number to the number of sides in the base) are triangles meeting at a common point called the vertex.

If the base of a pyramid is a regular polygon (that is, a polygon whose sides are exactly equal, as an equilateral triangle, a square, pentagon, hexagon, octagon, etc.) and the lateral faces are all isosceles triangles, then the pyramid is a "regular pyramid." There are other pyramids, but our lesson is confined to regular pyramids.

The distance from the vertex to the center of any side of the base is called the "slant height." The "slant height" of a pyramid is really the altitude of one of its faces.

The distance from the vertex to the center of the base of the pyramid is the "altitude." In solids, we must look very carefully for the altitude.

The "lateral area" of a pyramid is the sum of the areas of its lateral faces. The "entire area" is the sum of the lateral area plus the area of the base.

Since each of the lateral faces of a regular pyramid is an isosceles triangle whose altitude is equal to the slant height of the pyramid, and whose base is equal to one side of the base of the pyramid, therefore:

$\frac{1}{2}$ of slant height × 1 side of base = area of 1 lateral face;

Area of 1 lateral face × number of lateral faces
or
Perimeter of base × $\frac{1}{2}$ of slant height
} = lateral area;

Lateral area + area of base = entire area.

Since the volume of a prism is found by multiplying the area of the base by the altitude, and since it is a determined fact that the volume of a pyramid is exactly $\frac{1}{3}$ as great as the volume of a prism having the same base and altitude, therefore:

Base × altitude ÷ 3 = volume of pyramid, or

$$\frac{B \times \text{Alt.}}{3} = \text{Volume.}$$

Exercise 42—Oral and Written.

Use the blackboard and scales. Make drawings rapidly or have them ready. Volunteer to draw them.

1. What is a polygon having three equal sides called? Four equal sides? Five? Six? Eight?

2. Draw the five polygons referred to in Question 1, and under each drawing write its correct name.

3. A prism has how many bases? A pyramid has how many bases?
4. A triangular prism has how many sides or lateral faces? What shape is each of these faces?
5. A triangular pyramid has how many sides or lateral faces? What shape is each of these faces?
6. A rectangular prism has how many sides or lateral faces? What shape is each of these faces?
7. A rectangular pyramid has how many sides or lateral faces? What shape is each of these faces?
8. If the base of a prism is an octagon, how many lateral faces has the prism? What is the shape of each of the faces? If the base is a hexagon, how many lateral faces has the prism? What shape is each of the faces? If the base is a pentagon, how many lateral faces has the prism? What shape is each of the lateral faces?
9. If the base of a pyramid is an octagon, how many lateral faces has the pyramid and what is the shape of each? If the base is a hexagon? If the base is a pentagon?
10. How do we find the area of a $\triangle$? How do we find the area of one of the lateral faces of a triangular pyramid? Of any other kind of a pyramid?
11. How do we find the lateral area of a pyramid when we know the area of 1 lateral face? How, when we know the perimeter of the base and the slant height of the pyramid?
12. How do we find the entire area of a pyramid?
13. Cut a prism out of a potato or make one out of clay; weigh it and note the weight.

14. Without changing the size of the base or the altitude of the prism you have just made, cut a pyramid out of the prism; weigh it and note the weight; also weigh the waste material which was cut away.
15. How does the weight of the pyramid compare with the weight of the prism?

Exercise 43—Class work (Construction).

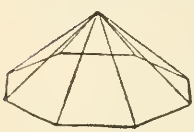
Everybody make these. Base 2″ × 2″, Altitude 4″.

1. Make a prism and a pyramid of heavy paper, using the same base and altitude for one that you use for the other; leave the end of the pyramid and one end of the prism open. Fill your prism full of sawdust by using the pyramid as a measure; how many times did you have to fill your pyramid? What is the volume of the prism as compared with the volume of the pyramid?
2. Say how to find the volume of a pyramid.
3. Take a prism in your hand and say four facts that you have learned about it. Can you give more facts?
4. Take a pyramid in your hand and see if you know all the interesting things about it that you *should* know. Point out the important things as you proceed.
5. How do we find the volume of any prism?
6. If the base of a prism is a 2″ square (we could write this 2″ □), and its altitude is 6″, what is its volume? If each cubic inch of the prism weighed 1 lb., what would be its weight?

7. If this prism was changed into a pyramid having the same altitude and base, what part of its volume would remain? What would the pyramid weigh?
8. What is the volume cf a pyramid whose base is a 3″ square, and whose altitude is 10″? How do we find the volume of any pyramid?
9. Draw, and state the shape of the figure made by the three lines representing the following three dimensions of a pyramid:
   (*a*) A line representing the altitude;
   (*b*) A line representing the slant height;
   (*c*) A line joining (*a*) and (*b*);
   In the figure you have drawn, what name is given to the line which represents the slant height?
10. What is the volume of a prism whose base is a 2″ square, and whose altitude is 12″?
11. What is the volume of a pyramid whose base is a 2″ square, and whose altitude is 12″?

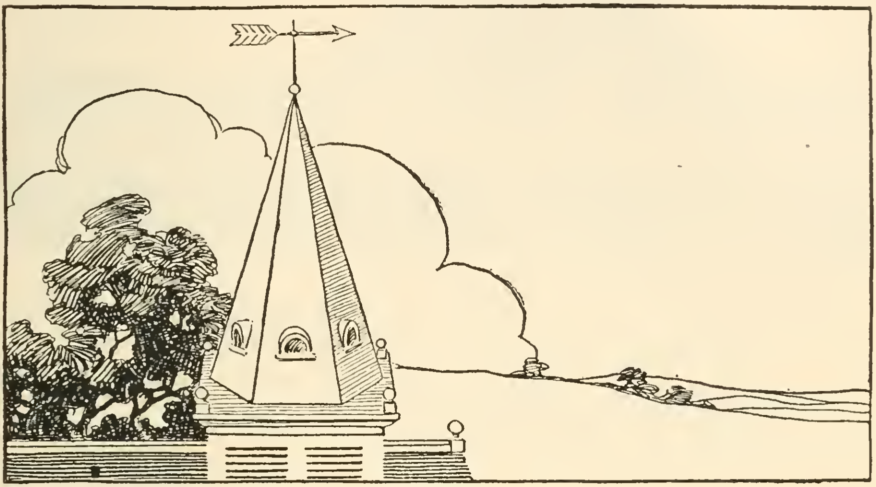
Exercise 44—Written.

1. A pyramid has a 15′ □ for its base, and its altitude is 20 feet; what is its volume? What is its perimeter? Its slant height?



2. The base of a pyramid has the shape of an octagon having 8-in. sides; its slant height is 12 in.; what is its lateral area?
3. The base of a pyramid is a △ having 10″ sides; its altitude is 15″; what is its volume? What is its slant height?

4. How many sq. ft. of slate are required to slate a steeple having the shape of a pyramid whose base is a 15′ square, and whose altitude is 30′?



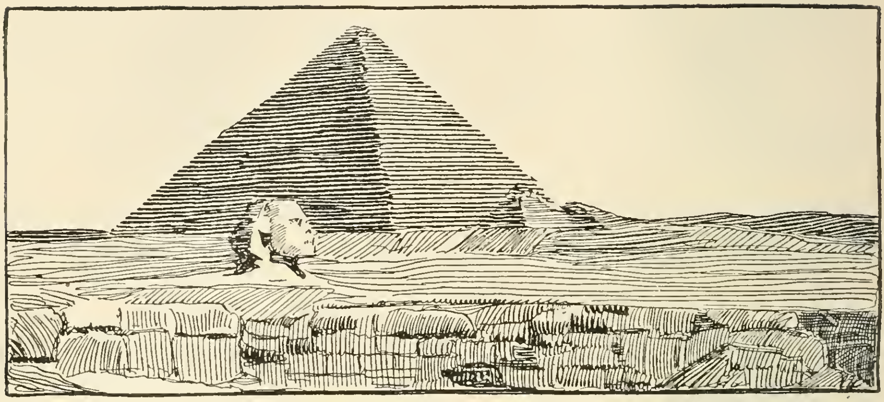
5. The base of a hexagonal church steeple has sides 12′ long; its slant height is 40′; how many pieces of slate 8″ square will be needed to slate this steeple, if a 6″ square of each piece of slate is exposed to the weather?

6. What is the volume of the largest pyramid which can be cut out of a prism whose base is a 6″ square, and whose altitude is 10″?

7. The volume of a pyramid having a square base is 6,250 cu. ft.; its altitude is 30 ft.; what is the length of each side of the base?

8. The volume of a pyramid having a square base is 2,048 cu. in.; its altitude is 24 in.; what is its slant height?

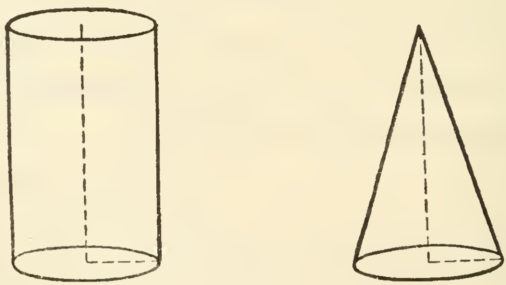
9. The slant height of a pyramid is 5″; its altitude is 4″; its base is a □; what is its volume?



10. The Great Pyramid of Cheops in Egypt is 764 ft. square at the base, and its altitude is 480 ft.; what is its volume? What is its slant height? What is its lateral area?

Lesson 22

Cones



Cylinder Cone

Read carefully.

A "cone" is a solid having a circular base, and one curved side which tapers uniformly from the base to a vertex directly over the center of the base.

The distance on a straight line from the vertex to any point on the circumference of the base is the slant height.

The distance from the vertex to the center of the base of the cone, is the altitude.

There are cones of other descriptions, but they are not included in our lesson.

The lateral area of a cone is the area of its curved surface or side. The entire area of a cone is the sum of its lateral area plus the area of its base.

Since the lateral surface of a cone may be regarded as being an isosceles triangle whose altitude is equal to the slant height of the cone and whose base is equal to the circumference of the circular base of the cone, therefore:

$\frac{1}{2}$ of slant height $\times$ circumference of base = lateral area;

Lateral area + area of base = entire area.

Since the volume of a cylinder is found by multiplying the area of the base by the altitude, and since it is a determined fact that the volume of a cone is exactly $\frac{1}{3}$ as great as the volume of a cylinder having the same base and altitude, therefore:

Area of base $\times$ altitude $\div$ 3 = volume of cone.

Exercise 45—Oral and Written.

Use scales, if possible.

1. Make a cone with the same base and altitude as a cylinder; fill the cylinder with sawdust by using the cone as a measure. Is the relation the same as between a prism and a pyramid of the same dimensions?
2. A cylinder has how many bases? A cone has how many bases?

3. A cylinder has how many sides? Of what shape?
4. A cone has how many sides? Of what shape?
5. What is the shape of the curved surface of a cylinder when it is straightened out?
6. What does the curved surface of a cone resemble when it is straightened out?
7. How do we find the area of a triangle?
8. How do we find the area of the curved surface of a cone?
9. How do we find the area of a circle most quickly when the circumference and diameter are known?
10. How do we find the area of a ⊙ most quickly when the radius is known? How when the diameter is known?
11. How do we find the entire area of a cone?
12. Cut a cylinder out of a potato or make one out of clay; weigh it and note the weight.
13. Without changing the size of the base or the altitude of the cylinder you have just made, cut a cone out of the cylinder; weigh it and note the weight; also weigh the waste material which was cut away.
14. How does the weight of the cone compare with the weight of the cylinder? How does the weight of the waste compare with the weight of the cylinder?
15. How do we find the volume of any cylinder?
16. If the area of the base of a cylinder is 4 sq. in., and its altitude is 6 in., what is its volume? If each cubic inch of this cylinder weighed 1 lb., what would be its weight?

17. If this cylinder was changed into a cone having the same altitude and base, what part of its volume would remain? What would it weigh?
18. What is the volume of a cone whose base has an area of 9 sq. in., and whose altitude is 10 in.?
19. How do we find the volume of any cone?
20. Draw, and state the shape of the figure made by the three lines representing the following three dimensions of a cone:
    (*a*) A line representing the altitude;
    (*b*) A line representing the slant height;
    (*c*) A line joining (*a*) and (*b*);
21. In the figure you have drawn, what name is given the line which represents the slant height?
22. Take a cylinder in your hand and tell the class all you know about it. Describe it fully and clearly so that those who do not know as well as you may learn.
23. Take a cone in your hand and tell all about it. Use good English so that all may understand.

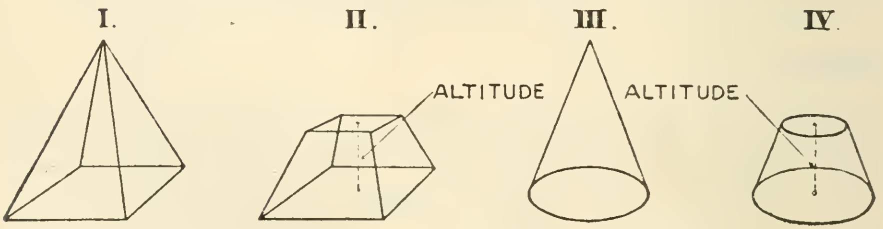
Exercise 46—Written.

1. A cone has a 20-ft. circle for its base and its altitude is 15 ft.; what is the area of its base? What is its volume? What is the circumference of its base? What is its slant height?
2. The base of a cone is a circle of 8″ radius; its slant height is 12″; what is the area of its lateral surface?
3. A cone has a ⊙ 12″ in diameter for its base and its slant height is 36″; what is its altitude?/

4. The area of the base of a cone is 28.2744 sq. in.; its altitude is 21 in.; what is its volume? What is its slant height?
5. What is the volume of the largest cone that can be cut out of a cylinder whose base is 4″ in diameter, and whose altitude is 10″?
6. What is the volume of the largest cone that can be cut out of a cylinder containing 600 cu. in.?
7. The volume of a certain cone is 785.4 cu. ft.; if its altitude is 30 ft., what is the diameter of its base?
8. The volume of a cone is 37.6992 cu. ft.; its altitude is 4 ft.; what is its slant height?
9. The slant height of a cone is 10″; its altitude is 8″; what is its volume?
10. The circumference of the base of a cone is 28.2744 in.; its altitude is 6″; what is its slant height?

Lesson 23

Frustums



Pyramid — Frustum of a Pyramid — Cone — Frustum of a Cone

Read carefully.

A "frustum" of a pyramid or of a cone is that part of a pyramid or of a cone which lies between the base and a plane parallel to the base. If you cut off the upper part of a pyramid or of a cone so that the cut is parallel to the base, the remaining part is a frustum.

The surface between the two bases is the lateral surface.

The area of the lateral surface is the lateral area.

The lateral area plus the areas of the two bases, is the entire area.

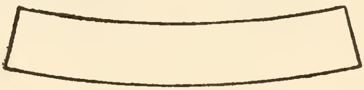
The distance between the centers of the two bases is the altitude.

How would you find the area of the frustum in Figure II?

Open it out on the board:

How would you find the area of the frustum in Figure IV?

Open it out on the board:

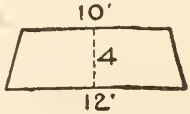


To find the lateral area of a frustum, find the average of the perimeters of the two bases by adding the two perimeters and dividing the sum by 2, then multiply this average perimeter by the slant height. Can you see how this simplifies your process?

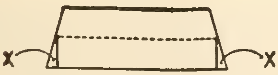
Exercise 47—Oral.

Some boy may draw these on the board for the class before class time.

1. Can you make this into a rectangle? What are the dimensions of the rectangle?



2. Can you make a rectangle of it by using the average line between the two bases? Where must the two pieces marked "X" be placed?



3. Mention some things you know which are formed like the frustum of a cone. Mention some which are formed like the frustum of a pyramid. (Think of any long walk you have taken.)
4. How many bases has the frustum of a pyramid? Of a cone?
5. The number of lateral faces of the frustum of a pyramid depends on what?
6. How many lateral faces has the frustum of a cone?
7. What constitutes the lateral surface of the frustum of a pyramid? Of a cone?
8. What constitutes the lateral area of the frustum of a pyramid? Of a cone?
9. What constitutes the entire area of the frustum of a pyraimd? Of a cone?
10. From what point to what point is the slant height of the frustum of a pyramid measured? Of a cone?
11. From what point to what point is the altitude of the frustum of a pyramid measured? Of a cone?
12. If one of the lateral faces of the frustum of a pyramid measures 4″ in width along the upper base, and 6″ in width along the lower base, at what point between the two bases would it measure 5″ in width? What would be the average width of this face? If this face were 5″ in width at all points between the two bases what would be its shape? How would its area be found? How do you find the area of one of the lateral faces of the frustum of a pyramid?

13. How do you find the average perimeter of the bases of any frustum? How do you find the area of the lateral surface of any frustum?

Exercise 48—Written.

Make large excellent drawings before you begin your calculations.

1. A round tin pail measures 16″ in diameter at the top, and 10″ in diameter at the bottom; its slant height is 12″; how much tin is there in the side of the pail? How much in the bottom? How much in the entire pail?
2. The dimensions of a photographer's tray are 10″ × 10″ at the top, and 8″ × 8″ at the bottom, and the slant height is 2″; what is the area of the sides and bottom of this tray?
3. The diameter of a flower pot is 10″ at the top, and 6″ at the bottom; its slant height is 12″; what is the area of its lateral surface?
4. A circular pan measures 20″ in diameter at the top and 8″ in diameter at the bottom; its slant height is 10″; what is the area of the material out of which it is made?
5. The area of the material in a pan is 208 sq. in.; it measures 10″ × 10″ at the top and 8″ × 8″ at the bottom; what is its slant height?
6. The frustum of a pyramid has for its lower base an equilateral △ whose sides measure 6″, and for its upper base an equilateral △ whose sides measure 4″; the slant height of the frustum is 10″; what is its lateral area?

7. The lateral surface of the frustum of a cone measures 565.488 sq. in.; the diameter of its upper base is 10″; its slant height is 12″; what is the diameter of its lower base?

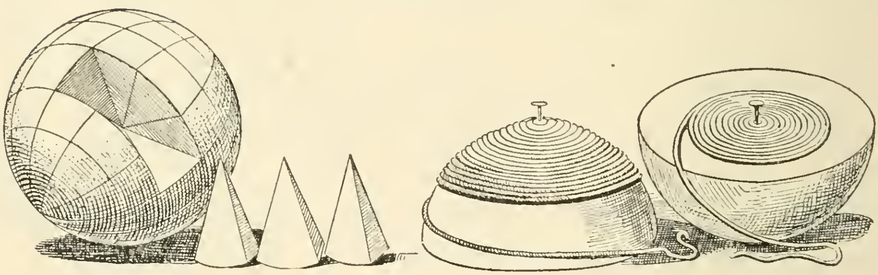
8. The lateral surface of the frustum of a cone measures 311.0184 sq. ft.; the radius of its upper base is 4 ft.; the radius of its lower base is 5 ft.; what is its slant height?

9. The upper base of the frustum of a pyramid is a 10″ □; its lower base is a 22″ □; its altitude is 8″; what is its slant height?

10. The upper base of the frustum of a cone is 6″ in diameter; the lower base is 60″ in diameter; its slant height is 45″; what is its altitude?

Lesson 24

Spheres



Read carefully.

A "sphere" is a round solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The circle made by cutting a sphere into two equal parts or hemispheres, by a plane passing through the center, is called a "great circle" of the sphere. Name a great circle of the earth.

The distance from surface to surface through the center is the diameter of the sphere, and is the same as the diameter of the great circle.

The distance from the center to any point on the surface is the radius of the sphere, and is the same as the radius of the great circle.

The circumference of a sphere is the same as the circumference of its great circle. The circumference is the longest curved line around the sphere.

By placing a tack in the center of the curved surface of one of the hemispheres, and another tack in the center of the flat surface of the other hemisphere, and winding cord carefully around each to cover the two surfaces, you will find that exactly twice as much cord is required to cover the curved surface as is required to cover the flat surface; hence, the area of the curved surface of the entire sphere equals four times the area of its great circle. The area of the great circle equals πR^2; therefore, the area of the curved surface equals $4\pi R^2$. You can also prove this by cutting several circles out of paper; make the circles the same diameter as the diameter of a wooden ball (you can cut an old croquet ball in half); tear one circle at a time into small pieces and paste to cover the ball. How many circles are needed?

If a sphere were cut into small pyramids as is shown in the illustration, each of the pyramids would have an altitude equal to the radius of the sphere, and the combined area of the bases of the small pyramids would be approximately the same as the area of the surface of the sphere. Since we find the volume of a pyramid by multiplying the area of its base by $\frac{1}{3}$ the altitude, we can find the volume of a sphere by multiplying the area of its curved surface by $\frac{1}{3}$ its radius. Therefore, since $4\pi R^2$ = area, $\frac{R}{3} \times 4\pi R^2$ or $\frac{4}{3}\pi R^3$, = volume.

Exercise 49—Oral.

Choose one of your classmates to answer. His row asks him any of these questions. After losing or answering successfully five of these, another row takes it up, etc.

1. Name several things you know which are spheres in shape.
2. When a sphere is cut into two equal parts by a plane passing through the center, what is each of the two parts called?
3. What name is given to the circular plane surface made by cutting a sphere into two equal parts?
4. How is the diameter of a sphere measured? How the radius? How the circumference?
5. With what measurement of the great circle of a sphere does the diameter of the sphere correspond? The radius? The circumference?
6. Does it require more cord to cover the curved surface or the flat surface of a hemisphere? How many times as much? How many times as much is required to cover the curved surfaces of both hemispheres as is required to cover the flat surface of one hemisphere?
7. What is the ratio of the area of the curved surface of a sphere to the area of the flat surface of one of its hemispheres?

8. How do we find the area of any circle? How do we find the area of the great circle of a sphere? How do we find the area of the curved surface of a sphere?
9. If a sphere is cut into small pyramids as is shown in the illustration, what dimension of the sphere would correspond to the altitude of each of the pyramids?
10. Which dimension of the sphere would correspond to the combined area of the bases of all of the small pyramids?
11. How do we find the volume of any pyramid?
12. If we know the radius of a sphere and the area of its curved surface, how can we find the volume?
13. Since πR^2 = area of any circle, state in the form of an equation the formula for finding the area of the curved surface of a sphere.
14. What change do you make in your equation to show volume?
15. Given the radius of a sphere, state the shortest way of finding its volume.
16. Take a ball and tell all the rules you know about it.

Exercise 50—Written.

1. The radius of a sphere is 10″; what is the area of its great circle? What is the area of its curved surface?
2. The diameter of a sphere is 14 ft.; what is the area of its curved surface?
3. What is the area of the cover of a baseball $2\frac{1}{2}''$ in diameter?

4. The diameter of the earth is approximately 8,000 miles; what is the area of the earth's surface?
5. The outside of the dome of an astronomical observatory is in the form of a hemisphere 50 ft. in diameter; at 25¢ per square yard, what would be the cost of painting the outside of this dome?
6. The area of the surface of the moon is about 12,566,400 sq. mi.; what is the area of its great circle? What is the length of its radius? What is the length of its diameter?
7. What is the volume of an orange whose area is 50.25 sq. in. and whose radius is 2″?
8. What is the volume of the largest sphere that can be turned from a 6″ cube of wood?
9. Circular discs of flat sheet metal are often pressed by machinery into the form of hemispheres. The area of the circular disc must equal the area of the finished hemisphere. What is the diameter of the disc required to make a hemisphere whose diameter is 12″?
10. What would be the diameter of the sphere which could be made by joining the two hemispheres pressed out of two brass discs each 10″ in diameter? What would be the volume of this sphere.

Exercise 51—Oral Review.

1. How many degrees of arc are there in a circle? In a semicircle? In a quadrant? From the equator to the North Pole? From the North Pole to the South Pole?

2. How many degrees are there in a right angle?
3. How much time corresponds to 180° of longitude? How much time corresponds to 90°? 15°? 1°? 1′? 1″?
4. What is the unit of length in the metric system? Of volume? Weight? Capacity? Area?
5. What single word means 100 m.? 10,000 l.? 1,000 g.? 10 g.? 1 m.? .1 l.? .01 m.? .001 g.?
6. How do we find the area of a triangle? Of a circle? Of a trapezoid?
7. How do we find the area of the lateral surface of a cylinder? Of a cone? Of a prism? Of a pyramid?
8. How do we find the volume of a cube? Of a prism? Cylinder? Pyramid? Cone?
9. How do we find the circumference of a circle?
10. How do we find the perimeter of the base of a pyramid? Of a prism? Cylinder? Cone?
11. How do we find the average perimeter of the bases of the frustum of a pyramid?
12. How do we find the average perimeter of the bases of the frustum of a cone?

Add the following:

13.	14.	15.	16.
46,314	39,876	35,212	56,598
38,726	67,968	64,911	67,873
34,912	46,436	17,878	43,289
62,848	34,999	43,219	21,196
36,944	89,722	38,777	55,439
56,497	32,464	58,649	58,935

Exercise 52—Written Review.

1. $\sqrt{2{,}371.69} = ?$
2. $394^3 = ?$
3. $\sqrt{\frac{289}{361}} = ?$
4. What is the hypotenuse of a right triangle whose legs measure respectively 15 ft. and 20 ft.?
5. An equilateral triangle has sides measuring 8 in.; what is its altitude?
6. An arc of 45° of the circumference of a certain circle measures 4.7124″ in length; what is the diameter of this circle?
7. The area of a certain sector of a circle is 52.36 sq. in.; if the diameter of this circle is 20 in., how many degrees are there in this sector?
8. The volume of a pyramid having a square base is 48 cu. in.; its altitude is 4″; what is its slant height?
9. The slant height of a cone is 36″; the radius of its base is 6″; what is its altitude?
10. The area of a sphere is 2,123.7216 sq. in.; what is its radius?

Copy and multiply:

(Time for these 8 examples is less than 4 minutes.)

11. 1,874 × 45;
12. 3,692 × 63;
13. 8,741 × 92;
14. 5,039 × 27;
15. 7,326 × 38;
16. 4,783 × 49;
17. 3,234 × 98;
18. 6,218 × 56.

Copy and divide:

(Time for these 6 examples is less than 4 minutes.)

19. 49,786 ÷ 73;
20. 15,652 ÷ 28;

21. 18,009 ÷ 23;
22. 22,225 ÷ 25;
23. 44,541 ÷ 63;
24. 29,045 ÷ 37.

Add, but do not copy:

(Time for these 6 examples is less than 4 minutes.)

25.	26.	27.	28.	29.	30.
5,419	8,732	9,328	7,514	3,928	4,609
7,429	1,872	4,687	3,975	2,448	6,987
3,921	8,728	6,215	4,988	7,397	5,025
9,627	4,187	8,199	2,875	5,622	1,987
2,639	4,879	1,628	5,319	7,875	5,188
6,395	2,486	3,798	6,295	6,894	9,127

Subtract, but do not copy:

(Time for these 12 examples is less than 4 minutes.)

31.	32.	33.	34.
863,472	749,802	621,804	831,841
519,728	358,473	239,758	293,957

35.	36.	37.	38.
612,041	541,387	716,284	300,412
309,409	297,492	591,876	275,583

39.	40.	41.	42.
975,319	716,304	824,508	612,387
493,941	593,295	713,499	509,368

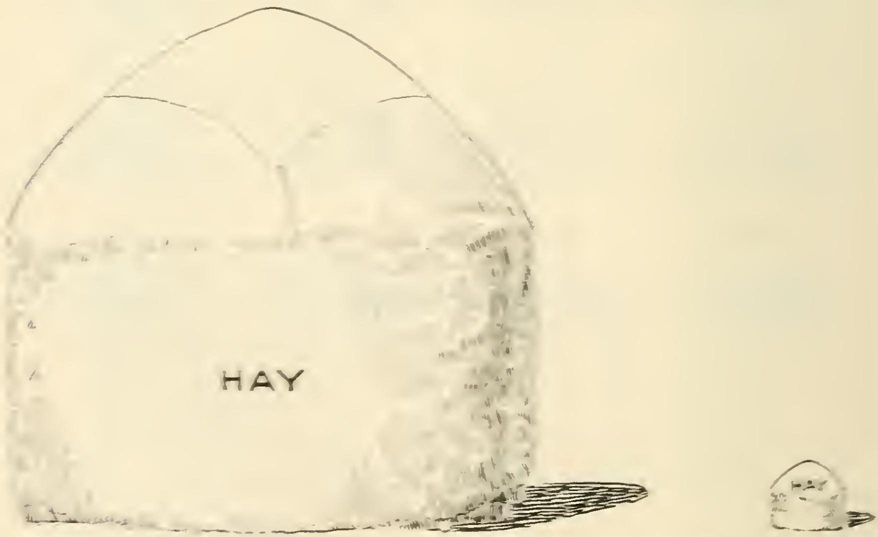
GRAPHIC CHARTS AND METERS

LESSON 25

Graphic Charts

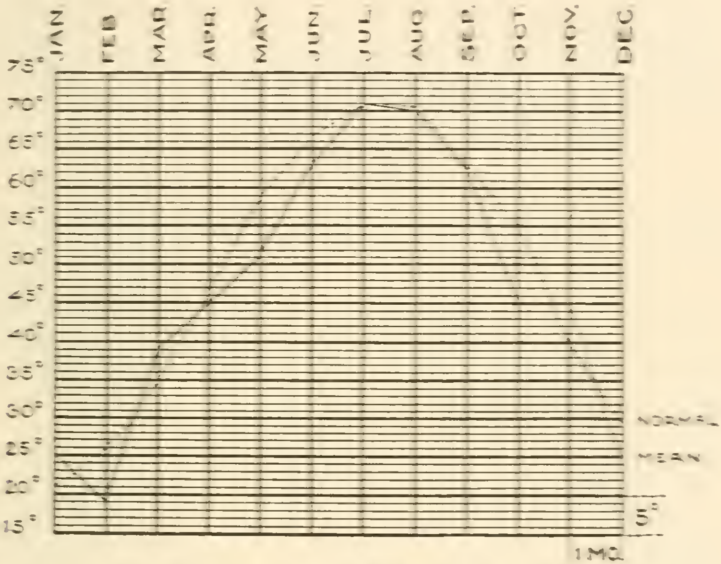
Graphic charts or "graphs" as they are frequently called, can be made in a great variety of ways and can be used for many purposes. In short, any drawing or chart which shows at a glance comparative statistics or other information, may be called a "graph."

To show the comparative production of a certain commodity for various periods of time, or for various sections of the country, "pictorial graphs" are often used (see Figure 1). This style of graph is most useful when the commodity can be faithfully portrayed.



New York = 5,554,000 T. Florida = 65,000 T.
Hay Production, 1914
(Figure 1)

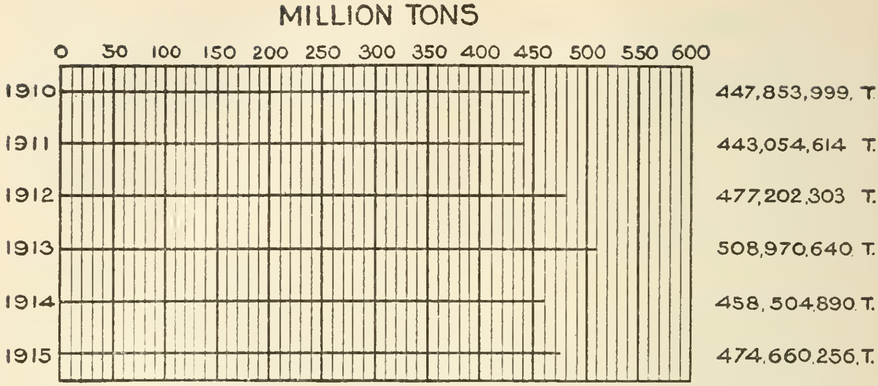
To record variations within definitely known limits, the "line graph" is most generally used (Figure 2). This style of graph is convenient for recording changes in prices, in production, in temperature, etc. The line by which the desired information is recorded is frequently called the "curve."



The Temperature in Chicago for a Year
(Figure 2)

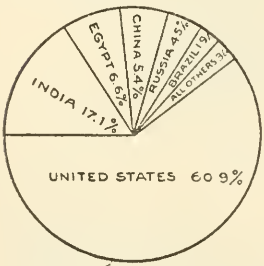
This line graph shows two separate kinds of information. The continuous line or "curve" shows the "mean" or *actual* average temperature for the twelve months of a certain year, while the dotted line or "curve" shows the "normal" or *usual* average temperature for the corresponding months of preceding years.

For almost every purpose involving comparisons, the "bar graph" is adaptable (Figure 3). The bar graph and the line graph are more frequently used than any other forms of graphs.

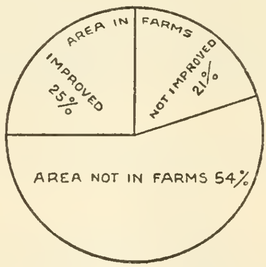


Coal Production of United States, 1910–1915.
(Figure 3)

To illustrate percentages, the "circle graph" is very useful (Figures 4, 5, and 6). In this style of graph, the complete circle represents 100%, and the relative sizes of the various sectors show the comparative percentages.

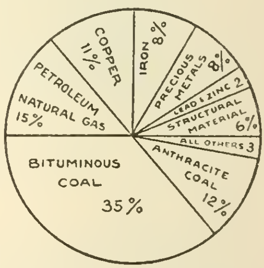


World's Cotton Production, 1913
(Figure 4)



Area of U. S. Land

(Figure 5)



Value of 1909 U. S. Mining Products
(Figure 6)

Exercise 53—Oral.

1. What information is shown by the graph in Figure 1? Read this graph.

2. What information does the continuous curve in Figure 2 show? What information is shown by the dotted line? Read this graph. When would you choose to use this one?

3. What is compared by the graph in Figure 3? From this graph, compare the coal production of 1912 with that of 1915.
4. Tell about the graph in Figure 4.
5. Explain Figure 5.
6. What information do you see in the graph in Figure 6? Read some relations found in this one.
7. Which style of graph would you prefer to use, if you wanted to show the apple production of Michigan in comparison with that of Illinois?

Exercise 54—Written.

1. What is the ratio of the hay production in Florida to the hay production in New York as found by a comparison of the areas of the two drawings in Figure 1?
2. Draw a pictorial graph showing that in 1914 Pennsylvania produced about 148,000,000 tons of coal, while Oklahoma produced about 4,000,-000 tons.
3. Draw a line graph showing that the price of wheat on October 1st of each year from 1910 to 1918 inclusive was as follows:

1910	$0.94	1914	$0.94
1911	0.88	1915	0.91
1912	0.83	1916	1.36
1913	0.78	1917	2.01
	1918	$2.06	

4. Draw a line graph showing (by a continuous line) the membership of a certain club on the first day of each month during the year 1918, and show also on the same graph (by a dotted line) the membership on the first day of the corresponding month in previous years:

Date	Membership, 1918	Membership Previous Years
Jan. 1st	410	390
Feb. 1st	430	420
Mar. 1st	435	425
Apr. 1st	440	430
May 1st	430	440
Jun. 1st	425	435
Jul. 1st	420	430
Aug. 1st	415	420
Sep. 1st	410	410
Oct. 1st	415	405
Nov. 1st	420	400
Dec. 1st	425	395

5. Draw a bar graph to show the population of the United States by millions for each census year from 1850 to 1910:

Census Year	Population
1850	23,000,000
1860	31,000,000
1870	39,000,000
1880	50,000,000
1890	63,000,000
1900	76,000,000
1910	92,000,000

6. Draw a bar graph showing the steel production of the United States by millions of tons for each year, 1906 to 1916:

Year	Tons of Steel
1906	23,000,000
1907	23,000,000
1908	14,000,000
1909	24,000,000
1910	26,000,000
1911	24,000,000
1912	31,000,000
1913	31,000,000
1914	24,000,000
1915	32,000,000
1916	43,000,000

7. Draw a circle graph showing the percentage of the population distributed in each of the following divisions:

(Use your protractor to divide the circle into sectors containing the correct number of degrees.)

Division	Percentage
New England	7.
Middle Atlantic	21.
East North Central	20.
West North Central	13.
South Atlantic	13.
East South Central	9.
West South Central	10.
Mountain	3.
Pacific	4.
Total	100.

8. Draw a circle graph showing the monthly sales of a large wholesale house to be divided as follows:

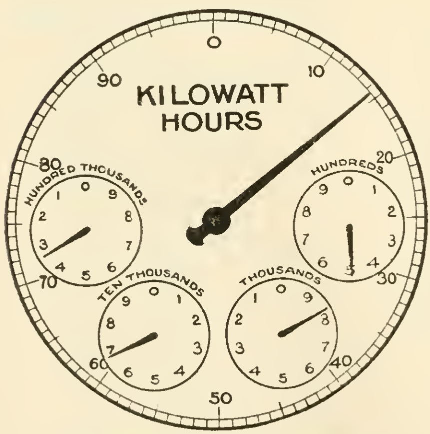
Commodity	Sales
Dry Goods	$415,000.
Furniture	1,037,500.
Carpets	352,750.
Curtains	103,750.
Miscellaneous	166,000.
Total	$2,075,000.

Lesson 26

Meters

Read very carefully.

The unit used for the measurement of electrical energy is the "kilowatt," and the special working unit in commercial use is that indicating the continuous use of 1 kilowatt of energy for 1 hour of time, and is called the "kilowatt hour" (k. w. hr.).

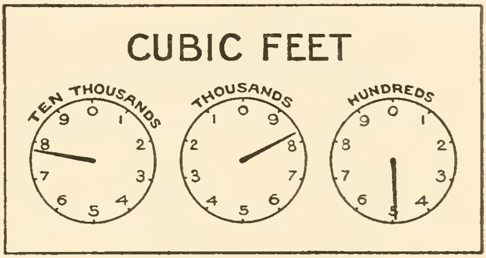


The Dial of an Electric Meter

Electricity is sold to the consumer on the basis of a certain rate per kilowatt hour, and a meter is used to record the number of kilowatt hours to be charged. The dial of the electric meter here illustrated shows more than 3 hundred-thousands, more than 6 ten-thousands, more than 8 thousands, more than 5 hundreds, 1 ten and 4 units of kilowatt hours; to be exact, this meter records 368,514 kilowatt hours.

To ascertain the consumption of electricity for any given period of time, the meter reading at the beginning of the period is subtracted from the meter reading at the end of the period, the difference between the two readings being the consumption for the period.

The consumption of gas is measured by cubic feet. The gas, after being manufactured, is stored in large tanks called "gas holders" from which it is forced under pressure into the mains. From the gas mains it passes through the meter (where it is measured) into the gas pipes on the consumer's premises.



The Dial of a Gas Meter

The dial of the gas meter here illustrated shows more than 7 ten-thousands, more than 8 thousands, and 5 hundreds of cubic feet, the exact meter reading being 78,500 cu. ft. Gas is charged to the consumer at a certain rate per 1,000 cu. ft. and the quantity consumed is determined by subtraction in the same manner as in the case of an electric meter.

Exercise 55—Oral.

Some boy may draw dials of both meters on the board. Choose one of your classmates to be the "gas man" or "electric man" who comes to read the meter.

Some girl may change the hands on the meters for him to read. Ask the following questions:

1. What is meant by a kilowatt hour?
2. Why do electric lighting companies install meters on the premises of their patrons?
3. What do such meters show?
4. How can we ascertain the quantity of electrical energy to be charged for a certain period?
5. An electric meter shows 4,319 kilowatt hours consumption; between what two figures is the hand on the thousands' dial? On the hundreds' dial? On the tens' and units' dial?
6. What unit is used for the measurement of gas?
7. How does a gas company measure the quantity of gas used by each of its patrons?
8. The rate charged for gas is based on what?
9. The reading on a gas meter on Nov. 6th was 7,000; on Dec. 6th it was 9,000; what information can you obtain from this data?
10. Read the two meters illustrated in Lesson 26.
11. Study your gas and electric meters at home; copy the position of the hands on a piece of paper and bring it to school; be ready to read the meters.

Exercise 56—Written.

Office and officials are to be in one part of the room. Go to the office to pay bills. Ask questions if you want information about your bills.

1. The reading of an electric meter on Jan. 10th was 1,624; on Feb. 10th it was 1,648; at 10¢

per k. w. hour, what was the amount of the bill rendered on Feb. 10th?

2. A discount of 1¢ per k. w. hr. is allowed on the bill in Question 1 for prompt payment on or before Feb. 20th; what was the net amount paid if this bill was settled Feb. 19th?

3. A certain lighting company charges 10¢ per k. w. hr. for the first 30 k. w. hr. and 6¢ per k. w. hr. for all in excess of 30 k. w. hr., and allows a discount of 1¢ per k. w. hr. on the 10¢ portion of the bill for prompt payment; what is the gross amount of the bill rendered July 15th if the meter reading was 3,978 on June 15th and 4,036 on July 15th? What was the net amount paid?

4. If the reading of the meter in Question 3 was 4,212 on July 15th, what was the gross amount of the bill for the month ending July 15th? What was the net amount paid in settlement of this bill?

5. An electric lighting company charges 11¢ per k. w. hr. for the first 40 k. w. hr., 7¢ for the second 40 k. w. hr., and 4¢ for all in excess of 80 k. w. hr.; this company allows a discount of $1\frac{1}{2}$¢ per k. w. hr. on the 11¢ and 7¢ portions of the bill; one of its meters read 2,611 on Sept. 20th, and 2,706 on Oct. 20th; what was the gross amount of the bill rendered Oct. 20th? What was the net amount paid in settlement of this bill?

6. If the reading of the meter in Question 5 was 2,638 on Sept. 20th, what was the gross amount of the bill for the month ending Oct. 20th? What was the net amount paid in settlement of the bill?

7. A gas company charges 37¢ for the first 350 cu. ft. consumed, and 88¢ per thousand for the excess over 350 cu. ft.; one of its meters read 9,500 Oct. 8th and 11,300 Nov. 8th; what was the amount of the bill rendered Nov. 8th?

8. If the rate in Question 7 were 90¢ per thousand cu. ft. with a discount of 10¢ per thousand cu. ft. for prompt payment, what would be the gross amount of the bill? What would be the net amount paid in settlement of the bill?

9. Draw the dials of an electric meter reading 8,643.

10. Draw the dials of a gas meter reading 82,700.

PERCENTAGE

Lesson 27

Interest on Installment Accounts

Read all of this very carefully.

You have already learned that since we charge interest on a sum for only such time as the sum remains unpaid, all partial payments on the principal must naturally affect the amount of interest, as the principal will change as many times as there are partial payments.

It is customary that when goods are purchased on the installment plan, that is, under an agreement to make a certain number of equal partial payments at certain equal intervals of time (usually one payment per month), interest is charged on the diminishing balance from month to month, and is paid each month when the partial payment is made on the principal.

While interest of this kind can be computed in the usual manner by making as many interest calculations as there are partial payments, much time and labor can be saved by using averages in problems of this kind.

Practice Exercise:

1 month = what part of a year?

6% per annum = ? % for 1 month?

At 6% per annum, what is the interest for 1 month on:

on:	\$100.	\$150.	\$75.	\$25.
	\$200.	\$240.	\$60.	\$20.
	\$300.	\$360.	\$40.	\$10.

EXAMPLE: A parlor suite was bought for $150.00 on terms of $30.00 cash and $10.00 per month with interest at 6% per annum; what was the interest on this account?

Total Sale..............$150.00
Cash Paid.............. 30.00
$120.00 = 12 Payments @ $10.00 each.
6% Int. for 1 mo. on $120.00 (largest interest payment) = $0.60
6% Int. for 1 mo. on 10.00 (smallest interest payment) = 0.05
$0.65

Average interest payment = ½ of $0.65, or $0.32½
12 Interest payments @ $0.32½ each = $3.90, Total Interest.

To use averages in figuring interest on installment accounts, we find the interest to be paid with the first installment (that being the largest interest payment), then we find the interest to be paid with the last installment (that being the smallest interest payment), then we add these two amounts of interest and divide the sum by 2 to find the average interest payment and multiply this average by the number of installments to be made; the answer so found is the interest on the entire account.

Always figure the interest at the rate of 6% per annum by using the "1% 60-day" method (½% per month), and when the rate is other than 6%, reduce your total interest to such other rate by the use of fractions, as: $\frac{5}{6}$ for 5%, $\frac{7}{6}$ for 7%, etc.

Exercise 57—Oral.

Do this work by Section A in the room (½ of class) buying from Section B (other ½ of class). Buy, ask questions, and both sections must balance bills.

Mrs. Wilson bought a piano for $300.00, paying $50.00 cash and agreeing to pay $10.00 per month with interest at 6% per annum.

1. What is an account of this kind called? Why?
2. What is the unpaid balance on the principal of this account during the first month? During the last month?
3. How many $10.00 payments must be made on this account? How many months will it take to pay this account?
4. What is the greatest amount on which interest will have to be paid for 1 month? How much interest will be paid on this sum?
5. What is the smallest amount on which interest will have to be paid for 1 month? How much interest will be paid on this sum?
6. What is the total of the largest and smallest interest payments on this account? What is the average interest payment?
7. How many partial payments are to be made? What is the total interest on this account?
8. What part of this account draws no interest?
9. How do we find the interest on any sum for 1 month at 6% per annum?
10. How do we find the largest interest payment to be made? How do we find the smallest interest payment to be made?
11. How do we find the interest on an installment account most quickly?
12. How do we proceed when the rate is 5%? How, when the rate is 7%?

Exercise 58—Written.

Act out the following: Row "A" buys of Row "B"; "C" of "D", etc.

1. A furniture dealer sold a parlor suite for $160.00, receiving $30.00 cash at the time of the sale, and an agreement calling for $10.00 per month with interest at 6%; what is the interest on this account?
2. Mr. Brown bought a house and lot for $5,500.00, paying $500.00 cash, and agreeing to pay $100.00 per month with 6% interest; what is the interest on this account?
3. Mrs. Jackson bought a grand piano for $500.00, paying $50.00 cash and agreeing to pay $25.00 per month with interest at 6% per annum; what is the interest on this account?
4. A watch was sold for $75.00 on terms of $25.00 cash and $5.00 per month with 5% interest; what was the amount of interest on this account?
5. A kitchen range was sold for $60.00 on terms of $15.00 cash and $7.50 per month with interest at 7%; what was the total amount paid for the range?
6. A building lot was sold for $900.00 on terms of $100.00 cash and $80.00 every second month with interest at 6% per annum; what was the interest on this account?
7. A man bought an automobile for $1,200.00 paying $200.00 cash and agreeing to pay $50.00 per month with 5% interest; what was the total cost of this automobile?

8. A phonograph costing $125.00 was sold on terms of $15.00 cash and $5.00 per month with 5% interest; what was the interest on this account?
9. A building was sold for $22,000.00 on terms of $2,000.00 cash and $1,000.00 per year with $5\frac{1}{2}$% interest; find the total interest on this sale.
10. What is the interest on the sale of a diamond ring for $135.00 if $15.00 is paid in cash at the time of the sale, and the balance is paid at the rate of $8.00 per month with 7% interest?

Lesson 28

Bank Discount

Read carefully.

When banks loan money to their depositors on promissory notes, or when they purchase notes signed by third parties from their depositors, they figure interest from the date they part with the money to the date of the maturity of the note, and deduct this interest from the amount of the note, giving the borrower the remainder.

The amount given the borrower is called the "proceeds" of the note.

The amount retained by the bank as interest is called "bank discount."

EXAMPLE: Find the proceeds of a note for $300.00 dated March 1st, 1920, due in 60 days, bearing no interest, discounted at 6% on March 1st, 1920.

Amount due at the maturity of the note = $300.
Bank discount for 60 days at 6%....... = 3.
Proceeds........................... = $297.

Since the bank collects the full amount of the note at maturity and gives the borrower only the proceeds, the difference between these two amounts is the amount of the bank discount.

EXAMPLE: Find the proceeds of a note for $250. dated April 6th, 1921, due July 5th, 1921, bearing 6% interest, discounted at 5% on May 6th, 1921.

Principal of note	=	$250.00
Interest Apr. 6 to July 5, 90 days at 6%	=	3.75
Amount due at maturity	=	$253.75 (Value of note)
Bank discount *May 6* to July 5, 60 days at 5%	=	2.11
Proceeds	=	$251.64

When discounting a note which bears interest, the interest due at the maturity of the note is added to the principal, and this entire amount is discounted; always discount the full value of the note.

Exercise 59—Oral.

Play going to a bank (some place in the room) to have your note discounted. Different children may take different notes; some with interest, others without. Ask these questions. Be sure of your money.

1. In the first example shown in this lesson, what amount is given by the bank to the borrower? What is this amount called?

2. What amount must the borrower pay to the bank when the note matures? What is this called?

3. Who gets the difference between the amount given by the bank to the borrower and the amount the borrower must repay to the bank? Why? What is this difference called?

4. In the second example, what amount will be collected by the bank when this note matures? Of what does this amount consist?
5. Why did the bank charge only $2.11 for discounting this note, instead of $3.75 which is the amount of interest to be collected on the note?
6. When a note for $200. bearing no interest is discounted for 60 days at 6%, what amount does the borrower receive? What amount will the bank collect when the note matures? Who gets the difference? Why?
7. In Question 6, what is the amount of the bank discount? What is the amount of the proceeds?
8. On what amount is bank discount computed when a note bearing interest is discounted?
9. For what length of time does the bank compute its bank discount?
10. A 90-day note for $100. bearing 6% interest is discounted by a bank at 5% 60 days before maturity; state exactly how you would find the proceeds of this note.

Exercise 60—Written.

Find the proceeds of each of the following notes:

	Date of Note	Principal	Time	Rate of Int.	Date Discounted	Disc. Rate
1.	Apr. 6, 1919	$400.	3 mo.	...	Apr. 6, 1919	6%
2.	May 8, 1920	$350.	120 da.	...	Jul. 7, 1920	5%
3.	Aug. 4, 1916	$1,500.	4 mo.	6%	Aug. 4, 1916	6%
4.	Dec. 8, 1923	$280.	90 da.	6%	Jan. 22, 1924	6%
5.	Oct. 19, 1921	$816.	60 da.	5%	Nov. 18, 1921	6%

6. The proceeds of a note are \$394.; the bank discount at 6% is \$6.; what is the principal of the note? For what length of time was the note discounted?

(Suggestion: Proceeds + Bank Discount = Principal;
\$394. + \$6. = \$400.;
\$6. is the interest at 6% on \$400. for how long?
Interest for 1 yr. = \$24.; $\frac{6}{24} = \frac{1}{4}$ year, or 3 mo.)

7. The proceeds of a note are \$239.00; the bank discount for 1 month is \$1.00; what is the principal of the note? What is the rate of discount?

8. I wish to borrow \$500.00 (proceeds) from a bank for 4 months at 6%; for what amount must I sign a note?

(Suggestion: 4 mo. interest at 6% = 2%; therefore, \$500.00 is 98% of the principal.
If \$500.00 = 98% of principal,
100% = $\frac{100}{98}$ of \$500.00, or
100% = \$500.00 ÷ .98)

9. A merchant went to his bank and arranged to borrow \$346.50 (proceeds) for 3 months at 4%; for what amount did he sign a note?

10. The proceeds of a note discounted for 30 days at 6% amounted to \$358.20; what was the principal of the note?

11. A 4-months' note bearing 6% interest was discounted 2 months before maturity at 6%, the proceeds being \$908.82; what was the principal of the note?

Lesson 29

Mortgages and Bonds

Read carefully.

When property is pledged as security for the payment of a debt, it is done by the signing of a document called a "mortgage." This mortgage is recorded at the office of the County Recorder so that any interested person may know that someone has a temporary claim against the property. When the debt is settled, the original mortgage and a form called a "satisfaction" are obtained from the person to whom the debt was paid, and this "satisfaction" is filed with the County Recorder, after which the property is again free from this claim, encumbrance, or lien.

The person who gives a mortgage is a "mortgager."

The person in whose favor a mortgage is given is a "mortgagee."

While a mortgage remains unpaid, the fire insurance policies are left in the hands of the mortgagee so that he may be protected in case of fire.

Mortgages are of two kinds: "chattel" and "real estate." A "chattel mortgage" pledges merchandise, machinery, or other personal property as security, while a "real estate mortgage" pledges houses, lots, or other real property as security.

The interest on a mortgage is payable at regular stated intervals, such as once every six months, once every year, etc., just as is the interest on a long time promissory note.

When a large sum of money is to be borrowed on a mortgage, it is frequently done by issuing many "bonds" for small amounts, the total of which is equal to the amount to be borrowed on the mortgage. In such cases the mortgage is made in favor of a "trustee" who holds it for the protection of the bondholders, as each of the bonds is really a part of the mortgage.

Most bonds have "coupons" attached, each coupon representing the interest on the bond for a quarterly or semi-annual period; each of these coupons is dated ahead to correspond with the date on which it falls due, and when that date arrives, it must be clipped off the bond and cashed through a bank just as if it were a check for so much interest.

Government bonds are similar to other bonds, but, of course, they need no mortgage to protect them, as they are an obligation of the government itself, and it is beyond question that they will be paid at maturity.

Never confuse "bonds" with "stocks," for they are as different as day and night. Stocks will be considered in the next lesson.

Exercise 61—Oral.

Act these out in the class room. Appoint a recorder. Watch the papers handled. Talk freely.

1. Frank Graham borrowed $1,000.00 on March 5, 1920, on his house and lot, from William Rich, for 3 years at 6% payable semi-annually.
   (*a*) What kind of a mortgage was Mr. Graham required to sign? Who was the mortgager? Who was the mortgagee?
   (*b*) What did Mr. Rich do with this mortgage so that any interested person might find out

that he had a claim or lien against the property? Who holds the fire insurance policies until this mortgage is paid?

(*c*) When was the first interest payment due on this mortgage? What amount of interest was due on that date? When was the second interest payment due? What amount? Name the dates on which each of the other interest payments were due.

(*d*) When does this mortgage fall due? What amount must be paid on that date?

(*e*) When this mortgage was paid in full, what two documents did the mortgager demand from the mortgagee? What did the mortgager do to make it known that this lien on the property had been satisfied?

2. When machinery is pledged to secure payment of a debt, what kind of a mortgage is given?

3. Henry Walker, a printer, borrowed $400.00 on his printing presses from the Merchants Loan Company by giving a mortgage dated Oct. 4, 1920, due in one year, bearing 7% interest payable quarterly.

(*a*) What kind of a mortgage was given in this transaction? Who was the mortgager? Who was the mortgagee?

(*b*) State what the Merchants Loan Company did to protect its interests when this mortgage was received. Who has possession of the fire insurance policies while this mortgage remains unpaid?

(*c*) State the dates on which the interest must be paid. State the amount of each interest payment.

(*d*) When does this mortgage fall due? What amount must be paid on that date?

(*e*) State what Henry Walker did to protect his interest after paying his mortgage.

4. The Empire Manufacturing Company, wishing to raise $500,000.00 to enlarge its manufacturing facilities, issues a series of one thousand $500.00 bonds dated July 1, 1919, protected by deposit with the First Trust Co., as Trustee, of a mortgage on all its buildings. The bonds bear 6% interest payable Jan. 1st and July 1st of each year and mature as follows:

$\frac{1}{5}$ or $100,000. July 1st, 1924.
$\frac{1}{5}$ or 100,000. July 1st, 1925.
$\frac{1}{5}$ or 100,000. July 1st, 1926.
$\frac{1}{5}$ or 100,000. July 1st, 1927.
$\frac{1}{5}$ or 100,000. July 1st, 1928.

(*a*) Who is the mortgager? Who is the mortgagee? What kind of a mortgage was given?

(*b*) What did the First Trust Co. do for the protection of the bondholders when this mortgage was received? What was done with the fire insurance policies?

(*c*) State how much interest must be paid on each $500.00 bond on Jan. 1st, 1920. How much on July 1st of every year?

(*d*) State how the bondholders will collect their interest on Jan. 1st and July 1st of each year.

(*e*) On what date would this mortgage be fully satisfied?

(*f*) What would the Empire Manufacturing Company do to protect its interests after fully satisfying this mortgage?

5. The interest on the Fourth Liberty Loan issued by the United States in 1918 to help finance the World War is payable semi-annually at $4\frac{1}{4}\%$; for what amount is each semi-annual interest coupon on a $100.00 bond?

6. How do you collect the interest on such a bond?

7. Is this issue of bonds protected by a mortgage? If not, what assurance has a bondholder that the bond will be paid at maturity?

Exercise 62—Written.

1. A mortgage for $9,500.00 dated Aug. 1st, 1918, bearing 5% interest payable semi-annually, matures in 5 years. What is the total amount of interest which would be paid on this mortgage?

2. A 3-year mortgage for $8,700.00 bearing 6% interest matures Oct. 1, 1923, on which date $4,700. and the interest to date is paid and the balance is renewed for another 3-year period. What is the total interest on this mortgage?

3. A house and lot worth $8,000.00 is mortgaged for 60% of its value for 5 years at $5\frac{1}{2}\%$; what is the total amount of interest on this mortgage?

4. A property owner, wishing to borrow the largest amount on which he can, without inconvenience, pay 6% interest, figures that he can spare $261.00

a year to meet the interest payments. What amount can he borrow?

5. A certain bond issue is dated Dec. 1, 1920, and consists of four hundred $500.00 bonds bearing 6% interest payable semi-annually on June 1st and Dec. 1st of each year. The bonds are in three series:

   Series "A" is for 20% of the total, and expires Dec. 1, 1925.

   Series "B" is for 30% of the total, and expires Dec. 1, 1926.

   Series "C" is for 50% of the total, and expires Dec. 1, 1927.

   (*a*) What sum must be paid on the principal on each of these dates: Dec. 1, 1925; Dec. 1, 1926; Dec. 1, 1927?

   (*b*) What amount of interest must be paid on each of these dates: June 1, 1921; Dec. 1, 1925; June 1, 1926; Dec. 1, 1926; Dec. 1, 1927?

   (*c*) What is the total interest on this issue of bonds?

6. The interest coupons on a $500.00 6% bond are payable Jan. 1st and July 1st of each year. If I bought this bond on April 1st how much accrued interest would I have to pay? How and when would I get this accrued interest back?

Lesson 30

Corporations and Their Capital Stock

Read all of this very carefully.

When a business is owned by one person or by a few individuals in partnership, the capital needed for its operation is naturally limited to the financial resources of the one person or few people who own it.

As most of the larger commercial enterprises of the present day operate on so great a scale that practically no one person and but very few small groups of persons could possibly command the vast sums of money which are needed for their operation, it was necessary to devise a method whereby many persons might invest their money in such enterprises without necessarily devoting their time to the management of the business. To meet this necessity, the modern "corporation" was evolved.

A "corporation" is an artificial body created and chartered by the law of the state to transact its specific business as if it were an individual. Can you give the name of a corporation doing business in the United States?

The capital of a corporation is limited to a specified amount by its charter, and this specified amount is divided into a certain number of shares, each of which has a like "par" or face value. Thus, if a corporation is capitalized for $100,000. and this is divided into 1,000 shares, then each share has a par value of $100.

The shares are represented by "stock certificates" which show the par value of each share, the number of shares owned, and the name of the owner; therefore, a stock certificate of a corporation is proof that the owner is a partner in the business to the extent that the ratio of the number of shares held by him bears to the total number of shares of the corporation.

The affairs of a corporation are managed by a "board of directors" who are elected by the stockholders. In voting for the directors, each share of stock usually entitles the owner to one vote for each director to be elected; thus it is readily seen that the owner of a few shares of stock has very little to say regarding the management of the business. The directors elect the "officers" who sign the official papers of the corporation, such as checks, notes, etc.

The profits of a corporation are distributed to the stockholders in the form of "dividends" which are paid at such time and in such amounts as the directors may decide; therefore, the income from shares of stock depends entirely upon the profits of the corporation and is not a debt which must be paid, as is the interest on bonds. Furthermore, while the bondholders are usually protected by a mortgage and are therefore the first to receive their money if the corporation, for any reason, goes out of business, the stockholders are the owners of the business and are, therefore, the last to receive their money—if there is any left after paying all the other claims, and those stockholders who did not pay the full face or par value for their stock are liable to be called on to pay the difference between what they have paid and the par value, and if the corporation conducts a bank, the stockholders are liable to pay double the par value of their stock so that the depositors may suffer no loss. Thus, we say again: Do not confuse bonds with stocks.

Exercise 63—Oral.

1. The authorized capital stock of a corporation is 5,000 shares of a par value of $100. each; what is the total capitalization of this corporation?

What is the par value of 20 shares of this stock? What is the par value of 100 shares?

2. What percentage of this business belongs to a man who owns 500 shares of stock? 2,500 shares? 4,000 shares? 5,000 shares?

3. If this corporation pays an annual dividend of 8%, what would be the total amount of such an annual dividend? How much would a person who owns 1 share receive? How much would the owner of 20 shares receive? 100 shares? 1,000 shares?

4. Who elects the directors of a corporation? Who elects the officers? Has the owner of a few shares of stock very much to say regarding the management of the business?

5. A corporation is authorized by its charter to manufacture machinery; can this corporation go into the banking business? Into the railroad business? Explain why you answer as you do.

6. A corporation, in accordance with its charter, has an authorized capitalization of $100,000. consisting of 1,000 shares of a par value of $100. each; can this corporation sell 1,500 shares? Explain why?

7. By what authority is a corporation permitted to organize and exist?

8. Is a stockholder a creditor of a corporation? Is a bondholder a creditor? Explain fully.

9. Are the dividends on shares of stock a debt which must be paid regardless of the profits of a corporation? How about the interest on bonds? Explain fully.

10. A corporation goes out of business and its property is sold for $175,000.00; the bondholders' claims amount to $50,000.00; ordinary creditors' claims amount to $50,000.00; stockholders' claims amount to $100,000.00; in what order are these claims paid? Who are the losers? Why?

11. Supposing the stockholders in Question 10 had bought their stock at $50.00 per share, for shares of a par value of $100.00, how much might the holder of each share be called on to pay in addition to the $50.00 he has paid? Explain fully.

12. Supposing the corporation in Question 10 were engaged in the banking business, and the stockholders have bought their shares for $50.00 per share for each $100.00 share, how much might the holder of each share be called on to pay in addition to the $50.00 he has paid? Explain fully.

Exercise 64—Written.

1. The net profits of a corporation for a certain year are $15,000.; the capital stock is $200,000.; what per cent of dividends may be declared by the directors?

2. What per cent of dividends may the directors declare if the net earnings are $4,375. and the capital stock is $87,500.?

3. The net profits of a corporation capitalized at $100,000. enabled the directors to declare a dividend of 8% and set aside the remainder amounting to $15,000. to be used as dividends in future years; what were the net profits?

4. The directors of a corporation declared a 5% dividend amounting to $50,000.; what was the amount of the capital stock of this corporation?

5. The capital stock of a corporation is $300,000. and the par value of each share is $100.; this company pays a semi-annual dividend of 4%; what must its annual net profits amount to?

6. What would be the annual income of a man who owns 5,500 shares of stock of a par value of $100. each, if the corporation pays $1\frac{3}{4}$% dividends quarterly?

7. The net profits of a corporation are sufficient to pay dividends of $2\frac{3}{4}$% quarterly on a capitalization of $150,000. after setting aside $10,000. for emergencies and allowing $3,500. to remain undistributed. What per cent of dividends might the directors have declared if they had chosen to do so?

8. What per cent of dividends might the directors of a corporation declare quarterly if the total annual earnings of the business are $75,000., the total annual expenses are $45,000., and the capital stock is $300,000.?

9. A stockholder received $770.00 as his 7% dividend on 22 shares of stock; what was the par value of this stock?

10. A stockholder received $3,300. as his 6% dividend on $45\frac{5}{11}$% of the stock of a corporation; what was the entire capital stock? If the par value of the shares is $1,000., how many shares did this stockholder own?

Exercise 65—Oral and Written.

Problem Project.

Organize a corporation—all the children must be stockholders. (Build your corporation on what you know.)

1. Study the purpose of your corporation.
   - For gain?
   - What can it do, and what can't it do?
2. Capital needed; how much?
3. Directors
   - Who elects them? How?
   - What are their duties?
4. Officers
   - Who elects them? How?
   - What are their duties?
5. Issue capital stock certificates.
   - (*a*) Preferred — Fixed rate of dividends, paid first out of profits.
   - (*b*) Common — Unlimited rate of dividends, paid after preferred dividends are paid.
6. Buy and sell capital stock.
   - (*a*) At par value.
   - (*b*) At market value.
   - (*c*) Brokerage.
   - (*d*) Income — Rate indicated. Calculate your dividends.

N. B.

Art Work: Hang posters to boom your project.

English: Write letters asking and answering questions about your project.

Lesson 31

Rate of Income (Yield) on Stocks and Bonds Bought at a Premium or Discount

Read carefully.

Many of the larger corporations have two kinds of capital stock, "preferred stock" and "common stock."

"Preferred stock" is stock on which dividends at a certain fixed rate per cent must be paid before any other dividends may be paid.

"Common stock" is stock on which unlimited dividends may be paid after preferred dividends have been provided for.

Thus, where the net earnings are only slightly in excess of the preferred dividends, the preferred stock is the more valuable because the preferred dividends must be paid in full before any dividends can be paid on the common stock; but where the net earnings are greatly in excess of the preferred dividends, the common stock is the more valuable because the dividends thereon might, under certain circumstances, be many times greater than on the preferred stock.

Bonds, of course, always specify the rate of interest that they bear.

The "par" value of stock is the value per share which is indicated on the certificate of stock.

The "market" value of stock is the value at which the shares can be sold in the market at a given time.

If the market value is greater than the par value, the stock is said to be "above par" or "at a premium."

If the market value is less than the par value, the stock is said to be "below par" or "at a discount."

All that has been said regarding the par value and the market value of stocks may also be said about bonds.

Bonds and stocks are bought and sold through brokers who receive a brokerage of about $\frac{1}{8}\%$ on the par value from the buyer and also from the seller of the bonds or stocks. Thus, when stock is bought at 98, a \$100. share costs the buyer \$98. + \$$\frac{1}{8}$ brokerage = \$98.12$\frac{1}{2}$, but the seller receives only \$98. − \$$\frac{1}{8}$ brokerage = \$97.87$\frac{1}{2}$; therefore, the broker receives 12$\frac{1}{2}$¢ from the buyer and 12$\frac{1}{2}$¢ from the seller, or 25¢ in all.

EXAMPLE #1: Find the yield on ten \$100. shares of 6% stock bought at 95.

Cost: 10 shares @ \$95. + \$$\frac{1}{8}$........................ = \$951.25
Dividend on \$1,000 @ 6%........................ = 60.00
\$60.00 annual dividends on an investment of \$951.25 = $\frac{60.00}{951.25}$ of 100%; or $\frac{6000}{95125}$ of 100%; or $6\frac{31}{100}\%$, Ans.

Dividends are always paid on the par value of the stock, but to find the rate per cent of income on money invested in stocks, we must consider the actual cost (including brokerage) and not the par value, and we must assume that the market value of the stock will remain unchanged because there is no way of determining the price for which the stock can be sold at some future time until that time actually arrives.

After the stock has been sold and we have determined our loss or gain on the transaction, such loss or gain must be taken into consideration as shown in Examples #4 and #5.

EXAMPLE #2: Find the yield on a 5-year 6% \$100. bond bought at $94\frac{7}{8}$.

Par value payable in 5 years......	=	\$100.00	Average investment \$97.50.
Cost $\$94\frac{7}{8} + \$\frac{1}{8}$..................	=	95.00	
Discount gained during life of bond	=	\$5.00	= \$1.00 per year

Annual interest to be received (6% on \$100.00)........ = \$6.00
Plus one year's portion of discount.................... = 1.00
Total annual earnings.............................. = \$7.00

\$7.00 annual earnings on an average investment of \$97.50 = $7\frac{18}{100}$%, Ans.

$$97.50\overline{)7.000000} = .0718, \text{ Rate.}$$

EXAMPLE #3: Find the yield on a 10-year 7% \$1,000. bond bought at $109\frac{7}{8}$.

Cost \$1,098.75 + \$1.25..........	=	\$1,100.00	Average investment \$1,050.
Par value payable in 10 years.....	=	1,000.00	
Premium lost during life of bond	=	\$100.00	= \$10. per yr.

Annual interest to be received (7% on \$1,000.)....... = \$70.00
Minus one year's portion of premium................ = 10.00
Net annual earnings.............................. = \$60.00

\$60.00 annual earnings on an average investment of \$1,050. = $5\frac{71}{100}$%, Ans.

Bonds are always due and payable at their par value on a definitely known date; therefore, any premium paid for the bond is, in fact, a sacrifice of a portion of the interest to be collected during the life of the bond, while any amount that may be saved by buying the bond at a discount is really an addition to the interest to be collected during the life of the bond. Such premium or discount must be distributed over the life of the bond and only one year's portion thereof considered when finding the rate of income, and we must figure the rate of income on the average value of the bond; that is, the value which is half way between the cost price and the par value.

EXAMPLE #4: Find the yield on a \$100.00 share of stock paying 6% annual dividends, bought at $93\frac{7}{8}$ and sold 3 years later at $97\frac{1}{8}$.

Sold for \$$97\frac{1}{8}$ − \$$\frac{1}{8}$............. = \$97.00 } Average investment
Cost \$$93\frac{7}{8}$ + \$$\frac{1}{8}$................ = 94.00 } \$95.50.
Increase in value during 3 years.. = \$3.00 = \$1.00 per yr.
Annual dividends (6% on \$100.00).................... = \$6.00
Plus one year's portion of increase in value............ = 1.00
Total annual earnings.............................. = \$7.00
\$7.00 annual earnings on an average investment of \$95.50 = $7\frac{33}{100}$%, Ans.

EXAMPLE #5: Find the yield on a \$100.00 share of stock paying 8% annual dividends, bought at $102\frac{7}{8}$ and sold 2 years later at $99\frac{1}{8}$.

Cost \$$102\frac{7}{8}$ + \$$\frac{1}{8}$.............. = \$103.00 } Average investment
Sold for \$$99\frac{1}{8}$ − \$$\frac{1}{8}$............ = 99.00 } \$101.00.
Decrease in value during 2 years = \$4.00 = \$2.00 per year.
Annual dividends (8% on \$100.00).................... = \$8.00
Minus one year's portion of decrease in value.......... = 2.00
Net annual earnings............................... = \$6.00
\$6.00 annual earnings on an average investment of \$101.00 = $5\frac{94}{100}$%. Ans.

When stock is actually sold for more or less than was paid for it, the gain or loss must be distributed over the period during which the stock was owned, and the yield must be based on the average value of the stock in the same manner as we figure the yield on bonds which were bought at a premium or discount.

Exercise 66—Oral.

Use the following in asking your chosen corporation officials for information. They must be sure to know.

1. What is the difference between preferred stock and common stock as you understand it?

2. If a corporation has both preferred and common stock outstanding and its net earnings are only slightly in excess of the amount required for the dividends on the preferred stock, which class of stock would be the more valuable? Why?
3. In Question 2, which would be the more valuable if the net earnings were greatly in excess of the preferred dividends? Why?
4. What is the par value of a share of stock or of a bond?
5. What is the market value of a share of stock or of a bond?
6. When is a bond sold at a premium? At a discount?
7. How do stock brokers charge their brokerage on sales of stocks or bonds?
8. How would you find the rate of income (yield) on a $100.00 share of 7% stock bought at $104\frac{7}{8}$?
9. If a $1,000, 5% bond, bought at $94\frac{7}{8}$, matures in 5 years, what amount will be collected at maturity? How much discount is earned on this bond? Is this discount earned in one year or during the life of the bond? What should be done with the discount so that the correct percentage of annual income may be ascertained? What is the average investment in this case?
10. How would you find the yield on the bond in Question 9?
11. If a $100.00, 7% bond, bought at $109\frac{7}{8}$, matures in 10 years, was the bond bought at a premium or at a discount? How much premium or discount? What is the average investment in this case?

12. How would you find the yield on the bond in Question 11?

13. The following list shows the sales (for a week) of some of the securities on the Chicago Stock Exchange:

Stocks

Company	Sales	High	Low	Close	Net Change from Previous Week
Am. Radiator..........	6	298	295	298	−2
do Pfd..............	2	116	116	116	−6
Am. Shipbuilding.....	967	119½	108½	119½	+1½
Quaker Oats...........	5	260	260	260	+5
do Pfd..............	11	99½	99	99	...
Sears Roebuck........	906	163	160	163	+3

Bonds

C. C. Rys.	5%	$1,000.	61	61	61	...
C. Rys.........	5%	$1,000.	58¼	58¼	58¼	−1
Chicago Tel. Co.,	5%	$1,000.	96½	96½	96½	...
Swift & Co......	5%	$7,500.	97	96¾	96¾	−¼

Name the items which closed above par. Name those which closed below par.

14. State a possible reason why Am. Radiator common stock should be quoted at 298 while the same company's preferred stock is quoted at only 116?

15. Which stock would be the better investment as regards yield, Am. Radiator at 298 or Quaker Oats at 260, if the dividend rates of the two stocks are alike?

Exercise 67—Written.

(All stocks in the following examples are to be considered as of $100.00 par value and brokerage is to be computed at $\frac{1}{8}$% unless otherwise stated.)

1. What is the cost of 45 shares of U. S. Iron Co. common stock at 98$\frac{3}{8}$?
2. I sold 60 shares of Am. Wool Co. common stock at 58$\frac{1}{8}$ and invested the proceeds in Am. Wool Co. preferred stock at 86$\frac{7}{8}$; how many shares of preferred stock did I receive?
3. A certain common stock pays 12% dividends annually; what would be the per cent of income on 1 share of this stock bought at 111$\frac{7}{8}$? What per cent on 15 shares?
4. A stock called "Consolidated Railway Corporation 7% Preferred" sells on a basis which nets the investor 10$\frac{1}{2}$%; what is this stock quoted on the stock exchange?
5. A packing company's 8% stock is selling on a basis which nets the investor 6%; how many shares of this stock must be bought to insure an annual income of $720.00? What is this stock quoted on the stock exchange? What amount must be invested?
6. What is the per cent of income or yield on a 6-year 5% $100.00 bond bought at 93$\frac{7}{8}$?
7. What is the per cent of yield on eight 10-year 6% $100.00 bonds bought at 109$\frac{7}{8}$?
8. What is the yield on a 50-year 5% $1,000. bond bought at 89$\frac{7}{8}$?
9. What is the difference in the rate of income

between these two investments, and which is the more profitable:

A share of 7% stock bought at $109\frac{7}{8}$, or

A 6-year 6% \$100.00 bond bought at $98\frac{7}{8}$?

10. A 20-year 6% \$500.00 bond bought at $95\frac{7}{8}$ yields what per cent of income?
11. Find the yield on a \$100.00 share of stock paying 7% annual dividends bought at $94\frac{7}{8}$ and sold 6 years later at $101\frac{1}{8}$.

Lesson 32

Insurance

Read carefully.

"Insurance" is a sum of money promised to be paid by an Insurance Company to the insured person or company in case of loss or injury of a certain kind.

The written contract given the insured is the "policy."

The amount of money promised in the policy is the "face" of the policy.

The amount charged for insurance is the "premium."

There are many kinds of insurance, but those most commonly used are:

Life Insurance, which is insurance against the loss of life.

Accident Insurance, which is insurance against disability on account of accident.

Fire Insurance, which is insurance against the loss or injury of property by fire.

Marine Insurance, which is insurance against the loss of property at sea.

Employers' Liability Insurance, which is insurance against loss by reason of claims for damages by workmen on account of injury they may sustain while employed.

Fidelity Insurance, which is insurance against loss on account of the dishonesty of persons occupying positions of trust.

Most insurance premiums are stated at a certain price per $100.00 of insurance, but frequently the premium is stated at a certain rate per cent. The rate of premium, of course, depends entirely upon the hazards connected with the risk; thus, fire insurance rates on a wooden building are usually much higher than on a stone building, etc.

Exercise 68—Oral.

Children must be ready to ask and answer these when class room becomes an Insurance Office; so be careful.

The National Insurance Company insures William Burns' furniture against loss by fire to the extent of $1,000. at 80¢ per year per $100.00:

1. Who is the insured? Who is the insurer?
2. What is this written agreement called?
3. What name is given to the amount paid to the insurance company for assuming this risk? How much is this amount on this policy?
4. What is the face of this policy?
5. What kind of insurance is this?
6. Examine an insurance policy and tell us what you can about it.
7. Mention and explain some other kinds of insurance.

8. What is life insurance? Accident insurance?
9. What is marine insurance? Employers' liability insurance?
10. What is fidelity insurance?
11. If the premium is stated at a certain price per $100.00 of insurance, how do we find the entire premium on a policy?
12. If the premium is stated at a certain rate per cent, how do we find the entire premium on a policy?
13. On what does the rate of premium chiefly depend?
14. Where should we keep insurance policies?
15. Mention several kinds of insurance that every owner of an automobile should have.
16. What kind of insurance should a landlord have?

Exercise 69—Oral and Written.

Problem Project

Insurance

Make your Classroom an Insurance Office

1. Different desks for different departments.
   - (*a*) Marine
   - (*b*) Fire
   - (*c*) Life
     1. Straight
     2. Endowment
   - (*d*) Employers' Liability
   - (*e*) Fidelity
   - (*f*) Accident

2. Posters in conspicuous places. (From Art Dept.)
   - "Pay Premiums Here"
   - "Fire Insurance"
   - "Accident Insurance, etc."

3. Get ready to question officials preparatory to taking out insurance. Seek to know all about it now.
   { This can be done partly by writing letters asking for information, and answering them; or by reading some received.

   (Correct English is essential.)

4. Take out insurance now; watch:
   { Rate and premium paid.
   Does policy cover correctly?
   Privileges granted?
   What is done with policy.

   (Thoroughness in Topic 3 will show now.)

5. Collecting on a policy
   (*a*) Adjuster
   (*b*) Signing of papers
   { After fire.
   After accident.
   After maturity.

Exercise 70—Written.

Play your part in any of these in your Insurance Office.

1. A farmer valued his barn at $900., his house at $2,400., his farm implements at $1,200. and his furniture at $750.; he insured all of these items at 85% of their value at $\frac{65}{100}\%$; what was the premium on this policy?
2. A certain life insurance policy stipulates that if the premium is paid for 20 years the face of the policy will be paid to the insured. A man aged 25 buys a $5,000. policy of this kind agreeing to pay $49.25 per $1,000. annually, and he lived to collect the amount of the policy:
   (*a*) How much did he pay in premiums?

(*b*) How much did he collect?

(*c*) How old was he when he collected?

3. A certain policy stipulates that after the premium has been paid for 20 years no further payments need be made, but that at the time of the death of the insured the face of the policy shall nevertheless be paid to the beneficiary. A man aged 35 bought a $10,000. policy on this plan at $38.25 annually per $1,000., and lived to the age of 73.

(*a*) How much was paid as premium on this policy?

(*b*) How much was paid to the beneficiary at the time of this man's death?

(*c*) Allowing 6% simple interest, how much less would have been the insurance company's profit on this policy had this man died at the age of 63?

4. A man paid an annual premium of $15.00 on an accident insurance policy which provides for the payment of $30.00 per week during disability caused by accident. If this man was disabled for 10 weeks after having paid the premium for 8 years, how much did this policy save him?

5. A manufacturer bonded his cashier for $5,000., paying therefor an annual premium of $5.00 per $1,000. After 20 years of faithful work, this cashier became addicted to the use of liquor and embezzled $2,000.

(*a*) Without considering interest, how much did this manufacturer gain or lose by having had this insurance?

(*b*) How much did the bonding company gain or lose?

6. A steel manufacturer bought an employers' liability insurance policy at an annual cost of $875. under the terms of which the insurance company's liability was limited to $10,000. on account of injury to any one person, or $30,000. on account of injuries caused by any one accident. After paying the annual premium for 5 years, a serious accident caused injuries to a number of workmen, who under the Workmen's Compensation Law were entitled to receive $45,860., of which amount not over $10,000. was due any one man.

(*a*) Without considering interest, how much did the manufacturer gain by having this insurance?

(*b*) How much did the insurance company lose?

(*c*) How much of the $45,860. did the employer have to pay?

(*d*) How much did the insurance company have to pay?

Exercise 71—Oral Review.

See if every child can ask and answer a question on this today. Somebody else must ask you a question.

1. How do we find the area of a circle? Of a triangle? Of a trapezoid? Of a parallelogram?

2. How do we find the volume of a cube? Of a prism? Of a pyramid? Of a cone? Of a cylinder? Of a sphere?

3. How do we find the area of the lateral surface of a cylinder? Of a prism? Of a pyramid? Of a cone? Of a frustum of a cone? Of a frustum of a pyramid? Of a sphere?
4. What single word means 1,000 meters? 100 liter? 10,000 grams? 10 meters? 1 liter? .1 gram? .01 meter? .001 liter?
5. What is the unit of volume in the metric system? Of weight? Of capacity? Of length? Of area?
6. How many degrees are there in a semi-circle? In a right angle?
7. What is the principal difference between a bond and a share of stock? Why are government bonds issued at a lower rate of interest than other bonds?
8. A corporation's net profits are several times as great as the dividends on its preferred stock; would you rather own 10 shares of the preferred stock or of the common stock of this corporation? Why?
9. Is a stockholder a creditor of a corporation? Is a bondholder? In case of the failure of a corporation would you rather be one of its bondholders or one of its stockholders? Why?
10. What is the difference in time between two points if they are separated by 90° of longitude? By 180°? By 1°? By 1′? By 1″?

Exercise 72—Written Review.

1. Reduce 36,872″ to degrees, minutes, and seconds.
2. (*a*) $48^2 = ?$ (*b*) $64^3 = ?$
3. (*a*) $\sqrt{670{,}761} = ?$ (*b*) $\sqrt{18\frac{7}{9}} = ?$

4. The legs of a right triangle are 20 ft. long; what is the length of the hypotenuse?
5. The radius of a circle is 3 ft. 6 in.; what is the length of 45° of its circumference?
6. The diameter of a circle is 10 in.; what is the area of a sector measuring 135° of this circle?
7. What is the volume of a pyramid which has a 24-ft. square for its base, and whose altitude is 20 ft.? What is its slant height?
8. A furniture dealer advertised as follows: "We furnish four rooms complete for $99.00 payable $4.00 cash and $5.00 per month." If he charged 6% interest on every sale of this kind, how much interest would he collect on 16 sales?
9. The proceeds of a note discounted for 90 days at 6% amounted to $665.86; what was the principal of the note?
10. What is the yield on a 15-year 5% $1,000. bond bought at $92\frac{3}{8}$, allowing $\frac{1}{8}$% for brokerage?

Copy and divide:

(Time for these 6 examples is less than 4 minutes.)

11. 11,648 ÷ 416;
12. 19,822 ÷ 374;
13. 37,368 ÷ 519;
14. 18,936 ÷ 36;
15. 33,746 ÷ 47;
16. 59,156 ÷ 92.

Subtract, but do not copy:

(Time for these 12 examples is less than 4 minutes.)

17.	18.	19.	20.
375,412	639,741	712,041	864,342
241,987	347,953	319,751	716,875

21.	22.	23.	24.
931,204	728,041	413,841	286,319
156,287	319,876	148,639	196,056

25.	26.	27.	28.
741,093	612,058	761,432	870,002
359,749	295,069	439,357	691,587

Copy and multiply:

(Time for these 8 examples is less than 4 minutes.)

29. 519 × 643; **33.** 437 × 585;
30. 809 × 592; **34.** 648 × 234;
31. 628 × 550; **35.** 541 × 672;
32. 347 × 724; **36.** 708 × 509.

Add, but do not copy:

(Time for these 6 examples is less than 4 minutes.)

37.	38.	39.	40.	41.	42.
5,328	3,639	6,759	5,189	8,643	1,898
7,639	8,723	3,423	1,939	9,075	3,990
9,348	8,529	8,979	2,579	7,638	3,853
8,985	3,987	7,598	9,536	4,928	6,492
7,982	4,893	4,381	3,870	5,632	3,658
2,348	2,789	1,975	4,797	6,789	7,842

PARTNERSHIP

Lesson 33

Division of Profits and Losses

(Let the class transact this example—one half of class representing A's interest, the other half B's.)

EXAMPLE: A and B are partners sharing profits or losses according to their several investments; A has $12,000. invested, and B has $18,000. invested; their profits during a certain year are $4,000.; how much of the profit should each partner receive?

A's investment	= $12,000., or	40%
B's investment	= 18,000., or	60%
Total investment	= $30,000., or	100%
A's share of profits	= 40% of $4,000., or	$1,600.
B's share of profits	= 60% of $4,000., or	2,400.
Total profits	= 100%	$4,000.

A partnership is the association of two or more persons for the purpose of carrying on an enterprise or business according to agreement.

The agreement which the partners enter into, and which stipulates what the relations between the partners shall be, how much each shall invest, how the profits or losses shall be divided, how long the agreement shall be in force, etc., is called the "partnership agreement."

A partnership dissolves upon the expiration of the partnership agreement, upon the death of one of the partners, or by mutual consent of the partners.

Profits or losses are sometimes divided in proportion to the amount invested by each of the several partners, and sometimes according to a fixed basis, as 50% each if there are two partners, $33\frac{1}{3}$% each if there are three partners, etc.

(Two rows can represent A, two rows B and two rows C. Watch your money.)

EXAMPLE: A owns $\frac{5}{8}$ of a business, and B owns $\frac{3}{8}$ of it; they decide to sell $\frac{1}{3}$ interest in the business to C; what part of the business will each partner own after C is admitted to partnership?

Before the Sale	The Sale	After the Sale
A's share = $\frac{5}{8}$	$\frac{1}{3}$ of A's share = $\frac{5}{24}$	A's share = $\frac{10}{24}$ ($\frac{5}{8} - \frac{5}{24}$)
B's share = $\frac{3}{8}$	$\frac{1}{3}$ of B's share = $\frac{3}{24}$	B's share = $\frac{6}{24}$ ($\frac{3}{8} - \frac{3}{24}$)
C's share = 0	Total C buys = $\frac{8}{24}$	C's share = $\frac{8}{24}$ ($\frac{5}{24} + \frac{3}{24}$)
Total... = $\frac{8}{8}$		Total... = $\frac{24}{24}$

(NOTE: The ratio of A's share to B's share is 5 to 3 both before and after making the sale.)

Exercise 73—Oral.

1. What is the association of two or more persons for the purpose of carrying on an enterprise or business called?
2. What do partners do on entering into business so that there may be no dispute regarding the amount of money each is to invest, how the profits shall be divided, etc.?
3. What name is given to the agreement between partners?
4. Name three ways in which a partnership dissolves?
5. What is meant when we say that profits or losses are to be divided in proportion to each partner's investment?

6. What is meant when we say that profits or losses are to be divided according to a fixed basis?
7. If it is agreed between three partners that A is to receive $\frac{1}{2}$ of the profits, B $\frac{1}{3}$ and C $\frac{1}{6}$, is this a case of dividing the profits according to a fixed basis or in proportion to the investments of the partners?
8. If A has $6,000. invested and B has $4,000. invested, and they divide the profits on the basis of 60% and 40%, is this a case of dividing the profits according to a fixed basis or in proportion to the investments of the partners?
9. Henry and Frank are partners dealing in apples; Henry has 3 apples and Frank has 6 apples; they decide to sell $\frac{1}{3}$ of their apples to William for 9¢; how much of this 9¢ should each boy receive?
10. In Question 9, how many apples will each boy have after the sale is made?

Exercise 74—Written.

1. Frederick Johnson and George Brown are partners sharing profits and losses on the basis of Johnson 60% and Brown 40%; for a certain year their gross earnings were $49,365. and their expenses were $40,520.; what was each partner's share of the net profit?
2. Robt. Alexander, Benj. Wilson, and Chas. Hendricks are partners sharing profits and losses on the basis of 40%, 35%, and 25% respectively; for a certain year their gross earnings were

$31,612.35 and their expenses were $34,729.35; what was each partner's share of the loss?

3. Henry Black and George White are partners sharing profits and losses in proportion to their investment; at the end of a certain year they find that they have made a profit of $8,634.60; what is each partner's share of this profit if Black's capital account showed he had $11,748.00 invested and White's capital account showed he had $9,612.00 invested? (Be very accurate.)

4. From the following information find each partner's share of the net profit or loss for the year, the arrangement between the partners being that $\frac{1}{2}$ of the profits are to be divided equally between them, the other $\frac{1}{2}$ being divided in proportion to their investments:

Gross Earnings...$36,400.;
Expenses........ 28,200.;
A's Capital.......18,000.;
B's Capital.......12,000.

5. A and B are partners in business; their capital accounts show that A has $6,552. invested and B has $6,048. invested; they decide to admit C as a partner with a 25% interest; what share of the business will each partner own after C is admitted?

6. Frank Powell has $32,000. invested in business when he decides to admit Walter Pearson as a partner; Pearson is to invest an amount which shall be 60% of their combined capital; how much did Pearson invest?

7. X and Y are partners in business; X's investment is \$4,480.; Y's investment is \$3,680.; they decide to bring their capital accounts to a 50% – 50% basis by investing or withdrawing as the case may be; what amount does each partner invest or withdraw?

Exercise 75—Written.

A Grocery Partnership

1. A and B are partners in the grocery business; A's investment is \$4,824.; B's investment is \$3,216.; they decide to equalize their capital accounts by investing or withdrawing as the case may be; what amount must A invest or withdraw?
2. What amount must B invest or withdraw?
3. They decide to admit C as a partner with $\frac{1}{3}$ interest, he to invest a sum similar to that of each of the two other partners; what amount must C invest?
4. At the end of the year they found that their gross earnings were \$36,000. and their expenses were \$24,000.; what was each partner's net profit?
5. At the end of the second year their capital accounts stood as follows:

   A \$4,040.; B \$3,020.; C \$2,940.;

   They decided to discontinue business at this time and after selling all of their merchandise and other property and settling all of their debts, they had exactly \$8,000. cash left; how much did they lose in selling out?

6. How much of this loss should be charged to each partner?
7. How much of the $8,000. cash belongs to A?
8. How much belongs to B?
9. How much belongs to C?
10. After paying the proper amount to each of the partners, what balances would appear on the partners' accounts?

Exercise 76—Oral Review.

1. Read: (*a*) 48,874,936,724; (*b*) 473,000,001,002.
2. What is the cost of 72 dozen eggs at $33\frac{1}{3}$¢ per dozen?
3. If $87\frac{1}{2}$ lb. of butter cost $28.00, what is the cost per lb?
4. Add, and read the answers at sight:

(*a*)	(*b*)	(*c*)
4,634	3,872	9,726
7,362	1,498	1,265

5. Subtract, and read the answers from left to right:

(*a*)	(*b*)	(*c*)
974	882	6,321
699	595	895

6. What is the interest on $60.00 at 6% for 293 days?
7. How much wages will a boy earn if he works 37 hours at $12.00 per week on a 48-hour basis?
8. Draw the following or hold the object in your hand before the class and make an interesting talk on each:

EXAMPLE: A Square.

This is a square because it is an area bounded by four straight equal sides and all its angles are right angles. One side may be the length or width. The sum of the four sides is the perimeter. Area is found by multiplying the number of square units in 1 row by the number of rows, or area = length × width. If area is given extract the square root to get one side. To find diagonal, use Pythagorian Theorem; $H^2 = A^2 + B^2$; $H = \sqrt{A^2 + B^2}$.

(*a*) A square;
(*b*) An oblong;
(*c*) A rectangle;
(*d*) A triangle;
(*e*) A trapezoid;
(*f*) A parallelogram;
(*g*) A circle;
(*h*) A cube;
(*i*) A prism;
(*j*) A cylinder;
(*k*) A cone;
(*l*) A pyramid;
(*m*) A sphere;
(*n*) A frustum.

9. Be ready to define the following:

(*a*) Simple Interest;
(*b*) Bank Discount;
(*c*) Trade Discount;
(*d*) Cash Discount;
(*e*) Premium and Discount;
(*f*) Insurance Premium;
(*g*) Commission;
(*h*) Brokerage;
(*i*) Taxes;
(*j*) Compound Interest.

10. Be ready to give a clear idea of the following topics:

(*a*) A bond;
(*b*) A share of stock;
(*c*) A corporation;
(*d*) A stockholder;
(*e*) A bondholder;
(*f*) Partnership;
(*g*) Dividends;
(*h*) Par value of stock;
(*i*) Market value of stock;
(*j*) Yield;
(*k*) Assets;
(*l*) Liabilities;

(*m*) Capital;
(*n*) Lien;
(*o*) Installments;
(*p*) Mortgage.

Exercise 77—Written Review.

1. What is the interest on $438.00 from Aug. 8, 1921, to May 8, 1922, at 5%?
2. What is the area of the largest circle which can be drawn in a square whose area is 400 sq. in.?
3. What is the hypotenuse of a right triangle if each of its legs is 9″ in length?
4. What is the altitude of an isosceles triangle whose base is 6″ and whose equal sides are 5″ long?
5. The difference in longitude between Chicago and New York is approximately 15°; when it is 4 P. M. in Chicago, what time is it in New York?
6. A $500. bond bears 6% interest payable Jan. 1st and July 1st of each year; how much cash would be required to buy this bond at 97 and accrued interest on April 1st? (Omit brokerage.)
7. Find the diagonal of a 6-inch cube.
8. A phonograph is sold for $200.00 on terms of $20.00 cash and $10.00 per month plus 6% interest; what is the interest on this installment account?
9. $\sqrt{4,637.61} = ?$
10. What is the length of one side of a square which has an area of 1,604.8036 sq. yd.?
11. Express the values of the following sales:

100 – $1,000. Liberty Bonds = ?
2,000 – $500. Liberty Bonds = ?
2,000 – $100. Liberty Bonds = ?

1,000 –	\$500. Liberty Bonds	= ?
60,000 –	\$50. Liberty Bonds	= ?
800,000 –	\$100. Liberty Bonds	= ?
50,000 –	\$5,000. Liberty Bonds	= ?
20,000,000 –	\$100. Liberty Bonds	= ?
Total value.		= ?

12. (*a*) This value is approximately what % of the total issue of \$6,000,000,000.?

(*b*) The last sale is what % of the total issue?

(*c*) If the total amount subscribed was \$8,246,-523,000. the issue was oversubscribed by what % approximately?

Copy and multiply:

(Time for these 8 examples is less than 4 minutes.)

13. 681 × 72;	17. 706 × 62;
14. 558 × 27;	18. 784 × 36;
15. 782 × 22;	19. 973 × 84;
16. 888 × 24;	20. 738 × 74.

Subtract, but do not copy:

(Time for these 12 examples is less than 4 minutes.)

21.	22.	23.	24.
539,285	621,741	397,412	867,412
197,286	387,219	297,468	382,941

25.	26.	27.	28.
100,874	631,025	612,318	298,741
99,946	41,973	200,493	289,765

29.	30.	31.	32.
300,412	538,763	658,729	284,302
299,975	494,839	597,831	178,463

Copy and divide:

(Time for these 6 examples is less than 4 minutes.)

33. 11,286 ÷ 38; **36.** 83,996 ÷ 92;
34. 30,076 ÷ 73; **37.** 60,900 ÷ 84;
35. 24,769 ÷ 47; **38.** 16,120 ÷ 65.

Add, but do not copy:

(Time for these 6 examples is less than 4 minutes.)

39.	40.	41.	42.	43.	44.
12,874	8,649	38,741	5,196	6,641	7,639
319	42,487	349,965	863	45,875	148,759
6,487	29	416	5,887	93,241	213
29	538	3,148	43,971	1,859	6,874
413,528	7,625	40,329	6,329	7,612	58
4,196	5,387	7,588	9,658	9	45,812

DEFINITIONS

(Parts I to VIII, Inclusive.)

Abstract Number......A number which is used without the name of an object.

Account..............A record showing all business transactions with any person.

Acute Angle...........An angle which is sharper (or less) than a right angle.

Addends..............The numbers which are to be added in an example in addition.

Addition..............Uniting two or more numbers or quantities into one number or quantity.
The numbers to be added are called "addends."
The answer is called the "sum" or "total."
The sign of addition is +, which is called "plus."

Agent................(In Commission.) The person who is engaged by the principal to perform some service.

Aliquot Part..........An equal part of a number.

Altitude..............Height.

Amount..............(In Interest.) The principal plus the interest.

Angle................The opening between two straight lines which meet in a point.

Angular Measure......The table of measures used for measuring angles.

Apex.................The highest point or summit of a plane or solid figure.

Apothecaries' Dry Measure............The table of measures used by druggists and physicians in measuring dry chemicals.

Apothecaries' Liquid Measure............The table of measures used by druggists and physicians in measuring liquid chemicals.

Apothem.............A perpendicular line drawn from the center of one of the sides to the center of a many-sided figure.

Arabic Numerals.......The numbers in common use, as 1, 2, 3, etc.

Arc...................Any part of a circumference less than the whole.

Area. . . . . .The space within the limits or boundaries of a surface.
Arithmetic. . . . . .The science of numbers.
Average. . . . . .A medium number which can be used in place of each of several unequal numbers.
Avoirdupois Weight. . . .The table of weights used for weighing all common articles, such as groceries, meats, hay, etc.
Balance. . . . . .The difference between the two sides of an account.
Bank Discount. . . . . . . .Interest deducted in advance by a bank from the amount of a note.
Base. . . . .(In Mensuration.) The side of a figure on which the figure appears to rest.
Base. . . . .(In Percentage.) The number or quantity on which the percentage is computed.
Bill. . . . . .See "Sales-slip."
Billions' Period. . . . . . . .The period of the 4th rank, counting units' period as the first.
Board Measure. . . . . . . .The unit of measure used for measuring boards is the "Board Foot," which equals $1' \times 1' \times 1''$.
Bond. . . . . .An evidence of indebtedness usually secured by a mortgage.
Brackets. . . . . .Marks used to enclose numbers which are to be treated as one number. ( ); [ ]; { }.
Broker. . . . . .One who sells stocks, bonds, real estate, etc., on a percentage basis.
Brokerage. . . . . .The compensation paid to a broker.
Buyer. . . . . .One who buys.
Cancellation. . . . . .Reducing the numerator of one fraction and the denominator of another to simplify multiplication.
Capital Stock. . . . . .The total amount for which a corporation can issue stock certificates as specified in its charter.
Carry. . . . . .To convert 10 of any order into 1 of the next higher order.
Cash Account. . . . . .An account with Cash.
Cash Book. . . . . .The book in which the Cash Account is kept.
Cash Discount. . . . . .An amount allowed for the payment of a bill on or before a certain date.
Change. . . . . .To convert 1 of any order into 10 of the next lower order.

DEFINITIONS

Check................An order drawn on a bank to pay a sum of money from funds on deposit.

Circle................A plane figure bounded by one continuous curved line which at all points is a uniform distance from a point in the center of the figure.

Circular Measure......The table of measures used for measuring circles and arcs.

Circumference.........The distance around a circle. The line which forms the boundary of a circle.

Commercial Interest...Interest computed on the basis of 360 days to the year.

Commission...........An amount of money paid by one person who is called the "principal" to another person who is called the "agent," for some service the agent performs for the principal.

Common Fraction.....A fraction of which both the numerator and the denominator are written.

Common Stock........Stock on which unlimited dividends may be paid after preferred dividends have been provided for.

Common Year ....A year containing 365 days.

Complement..........The difference between a number and 10, 100, 1,000, etc.

Complex Fraction......A fraction containing a fractional denominator or numerator.

Composite Number....Any number which is composed of two or more factors.

Compound Denominate Number............A denominate number which consists of two or more denominations.

Compound Fraction...A fraction of a fraction.

Compound Interest....When interest for stated periods is added to the principal and the amount so found is used as the principal for the next interest period, the total interest is called "Compound Interest."

Compound Proportion..A proportion consisting of more than two pairs of terms.

Concrete Number......A number which is used with the name of an object.

Cone.................A solid having a circular base and one curved side which tapers uniformly from the base to a vertex directly over the center of the base.

Consecutive Numbers..Numbers which follow one another without interruption of any kind; as 1, 2, 3, 4, 5, etc.

Consignment..........A shipment of merchandise to be sold for the account of the shipper.

Corporation...........An artificial body created and chartered by the law of the state to transact its specific business as if it were an individual.

Counting.............To name one by one to find the number of units in a group.

Credit................An entry which shows that the person or thing on whose account the entry appears, has parted with something of value.

Creditor..............One to whom money is owing.

Cube..................(In Mensuration.) A solid having six square sides or faces of equal size, joined so that every angle is a right angle.

Cube..................(In Powers and Roots.) The power of the third degree of any number.

Cube Root............The root of a power of the third degree.

Cubic Contents........The space occuped by a solid. (Volume.)

Cubic Measure........The table of measures used for measuring the cubic contents or volume of solids.

Cylinder..............A solid bounded by a uniformly curved side and two parallel circular ends or bases of equal size.

Date of Maturity......The date on which a note or other obligation matures or becomes due.

Debit.................An entry which shows that the person or thing on whose account the entry appears, has received something of value.

Debtor...............One who owes money.

Decillions' Period......The period of the 12th rank, counting units' period as the first.

Decimal..............Numbered by tens.

Decimal Fraction......(More commonly called "Decimal.") A fraction with a denominator of 10, 100, etc., the denominator being indicated by writing the numerator to the right of a decimal point.

DEFINITIONS

Decimal Point.........A point or period (.) used to indicate that the numbers written to the right of it form a decimal fraction.

Denominate Number. . .A number used with the name of a measure. (See also: "Simple Denominate Number" and "Compound Denominate Number.")

Denominator..........That term of a fraction which shows into how many equal parts a unit has been divided.

Diameter.............The distance across a circle through the center.

Difference............The answer found by subtraction.

Digit.................Any single figure, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Dimension............The extent of a line, area, or solid.

Disbursements.........Amounts of money paid out.

Discount..............A reduction from the regular price of an article for any reason. (See also: "Cash Discount," "Trade Discount," and "Bank Discount.").

Discount..............The difference between the par value and the market value of stocks and bonds when the par value is the greater.

Divided by (÷).......The sign of division.

Dividend.............The number to be divided in an example in division.

Dividend.............A distribution of profits by a corporation.

Division..............Finding how many times one number is contained in another number, and finding one of the equal parts of a number.

The number to be divided is called the "dividend."

The number which shows into how many parts the dividend is to be divided, or which shows the size of the parts when the number of parts is to be found, is called the "divisor." The divisor is the number you divide by.

The answer is called the "quotient."

The sign of division is ÷, called "divided by."

Divisor...............The number which shows into how many parts the dividend is to be divided, or which shows the size of the parts when the number of parts is to be found, in an example in division. The number you divide by.

Dry Measure.........The table of measures used in measuring grains, fruit, vegetables, etc. (See also: "Apothecaries' Dry Measure.")

Due Date.............See "Date of Maturity."

Elapsed Time.........The time between two dates.

Em..................The square of a type face.

Endorsement..........Any writing on the back of a note, check, etc.

Equals (=)...........The sign of equality.

Equation.............A statement showing the equality of two quantities, one of which is placed before and one after an equal sign.

Equator..............A line running completely around the earth midway between the North Pole and the South Pole. The line from which latitude is computed.

Equilateral Triangle....A triangle having three equal sides.

Equivalent............Having the same value.

Even Numbers........All numbers divisible by 2 without remainder; therefore, all numbers with 0, 2, 4, 6, and 8, in units' place.

Exact Interest.........Interest computed on the basis of 365 days (in Leap Year 366 days) to the year.

Exponent.............The number which indicates the degree of power.

Extremes.............The first and fourth terms of a proportion.

Factor................Any number which is contained in another number without a remainder.

Fraction..............A number which shows one or more of the equal parts of a unit. (See also: "Compound Fraction" and "Complex Fraction.")

Frustum..............The lower part of a pyramid or of a cone which remains after a plane has been cut through parallel to the base.

Grand Total..........The sum of several totals.

Graph or Graphic Chart.Any drawing or chart which renders possible the visualization of comparative statistics or other information.

Great Circle of a Sphere..A circle made by cutting a sphere into two equal parts by a plane passing through the center.

Greatest Common Divisor.............The largest number which will divide two or more numbers without remainder.

	(The Greatest Common Divisor of two or more numbers is also the Highest Common Factor of those numbers.)
Gross Price	The price of an article before deducting any discount. The "List Price."
Gross Profit	The amount of profit realized from the sale of an article without making any deduction for such expenses as rent, light, etc.
Group	Several persons or things.
Highest Common Factor	The largest factor which is common to two or more numbers. (The Highest Common Factor of any two or more numbers is also the Greatest Common Divisor of those numbers.)
Hexagon	A six-sided polygon.
Horizontal	Parallel to the surface of the earth.
Hundreds of Millions' Place	The place of the 9th order.
Hundreds of Thousands' Place	The place of the 6th order.
Hundreds' Place	The place of the 3d order.
Hundredths' Place	The 2d place to the right of the Decimal Point.
Hundred-Thousandths' Place	The fifth place to the right of the Decimal Point.
Hypotenuse	The side of a right triangle which joins the two legs.
Improper Fraction	A fraction which means 1 unit or more than 1 unit.
Index	The number which indicates the degree of root.
Insurance	A written contract whereby one party agrees to pay a certain sum to another party in case of loss or injury of a certain kind.
Integer	A whole number.
Interest	Money paid or to be paid for the use of money. (See also: "Compound Interest.")
International Date Line	A line passing, with few exceptions, along the 180th meridian, which is the dividing line between dates.
Inventory	Merchandise on hand.
Invoice	A bill showing the quantities, price, etc., of articles bought from a wholesaler or manufacturer.

Isosceles Triangle......A triangle having two equal sides.
Land Measure.........The table of measures used for measuring land.
Lateral Area..........The area of the lateral faces or sides of a solid.
Lateral Faces..........The sides of a solid.
Latitude..............Distance north or south of the equator.
Leap Year............A year containing 1 extra day, making 366 days in all.
Least Common Denominator........The smallest denominator common to two or more fractions.
Least Common Multiple............The smallest number which contains two or more other numbers without remainder.
Ledger................The book in which accounts are kept.
Legs of a Triangle.....The two sides of a right triangle which form the right angle.
Linear Measure........The table of measures used for measuring lines and distances. (See also: "Surveyors' Linear Measure.")
Liquid Measure........The table of measures used for measuring common liquids. (See also: "Apothecaries' Liquid Measure.")
List Price.............The price at which an article is listed in a catalogue or price-list.
Location..............One object's or number's position in relation to the other objects or numbers in a group.
Long Division.........The method of division in which partial dividends are written.
Longitude.............Distance east or west of the prime meridian.
Loss..................The difference between the cost and the selling price, when the cost is the greater.
Lowest Terms.........The simplest form in which a number or fraction can be written.
Maker................The person who "makes" or signs a promissory note.
Manufacturer.........One who makes or manufactures merchandise.
Market Value.........The value at which stocks or bonds can be sold at a particular time.
Means................The second and third terms of a proportion.
Mensuration..........Taking the measurements of anything.
Merchandise..........Any article which can be bought and sold.

Merchandise Counting Table..............The table used for counting merchandise of all kinds.

Metric System.........A decimal system of weights and measures devised by the French.

Millions' Period.......Millions', Tens of Millions', and Hundreds of Millions' Places.

Millions' Place........The place of the 7th order.

Millionths' Place.......The sixth place to the right of the Decimal Point.

Minuend..............The number from which we subtract in an example in subtraction.

Minus (−)............The sign of subtraction.

Mixed Decimal........A number which shows both an integer and a decimal fraction.

Mixed Number........A number which shows one or more units, plus one or more parts of a unit; therefore, a combination of a whole number and a fraction.

Mortgage.............A document pledging property as security for the payment of a debt. (See also: "Satisfaction of Mortgage.")

Mortgagee............The person in whose favor a mortgage is given.

Mortgager............The person who gives a mortgage.

Most Convenient Denominator........See "Least Common Denominator."

Multiple..............A number which contains another number more than once without remainder.

Multiplicand..........The number which is to be repeated in an example in multiplication.

Multiplication.........Finding a number or quantity by repeating another number or quantity a given number of times.

The number which is to be repeated is called the "multiplicand."

The number which shows how many times the multiplicand is to be repeated is called the "multiplier."

The answer is called the "product."

The sign of multiplication is ×, called "multiplied by" when the multiplier follows it, and "times" when the multiplier comes before it.

Multiplied by (×)..... The sign of multiplication.

Multiplier............. The number which shows how many times the multiplicand is to be repeated in an example in multiplication.

Net Price............. The price of an article after deducting all discounts.

Net Proceeds.......... The amount of money realized from a consignment after all commissions and other expenses are deducted.

Net Profit............ The final profit remaining after deducting all expenses of every nature.

Nonillions' Period...... The period of the 11th rank, counting units' period as the first.

Note.................. See: "Promissory Note."

Numerator............. That term of a fraction which shows how many parts of the unit are being spoken of.

Oblique............... A position which slants.

Oblique Angle......... An angle which is either acute or obtuse. (Any angle which is not a right angle.)

Oblong............... The plane figure formed by joining the ends of two straight lines of one length to two straight lines of another length, so that they form four right angles.

Obtuse Angle.......... An angle which is blunter (or greater) than a right angle.

Octagon.............. An eight-sided polygon.

Octillions' Period...... The period of the 10th rank, counting units' period as the first.

Odd Number.......... All numbers not divisible by 2 without remainder; therefore, all numbers with 1, 3, 5, 7, or 9, in units' place.

Order................ The place occupied by a number, as:
1st order is Units' Place.
2d order is Tens' Place.
9th order is Hundreds of Millions' Place.

Paper Measure........ The table of measures used in measuring paper.

Parallel.............. Running in the same direction with an equal distance between.

Parallelogram......... Any plane figure bounded by four straight lines, having two sets of parallel sides.

Partial Dividends. . . . . The several dividends necessary in finding the quotient in an example in long division.
Partial Products . . . . The several products which, when added, form the final product in an example in multiplication.
Partnership. . . . . The association of two or more persons for the purpose of carrying on an enterprise or business according to agreement.
Par Value. . . . . The value of stocks and bonds as indicated thereon.
Payee. . . . . The person in whose favor a promissory note, bill of exchange, or check is made.
Pentagon. . . . . A five-sided polygon.
Per Annum . . . . By the year.
Per Cent. . . . . By the hundred. Also: Hundredths.
Percentage. . . . . Calculation by hundredths. Also: The answer of an example in percentage.
Perimeter. . . . . The sum of the sides which form the boundary of a figure.
Period. . . . . Units', Tens', and Hundreds' Places when considered as a group.
Pi (Pronounced Pī). . . . A Greek letter representing the ratio of the circumference of a circle to the diameter (3.1416).
Pica . . . . $\frac{1}{6}$ inch. (Printers' Type Measure.)
Place. . . . . The order in which a number is written, as:
Units' Place is the 1st order.
Tens' Place is the 2d order.
Hundreds of Millions' Place is the 9th order.
Plus (+) . . . . The sign of Addition.
Point. . . . . $\frac{1}{72}$ inch. (Printers' Type Measure.)
Polygon . . . . A plane figure having more than four sides and angles.
Power . . . . The product obtained by using any number several times as a factor.
Preferred Stock. . . . . Stock on which dividends at a certain fixed rate must be paid before any other dividends may be paid.
Premium. . . . . The difference between the par value and the market value of stocks and bonds, when the market value is the greater.

Premium............The amount paid for insurance.
Prime Factor..........A factor which cannot itself be separated into other factors.
Prime Meridian........The meridian which runs from pole to pole through Greenwich (near London), England, from which longitude is computed.
Prime Number........Any number which is divisible only by 1 and by itself without remainder.
Principal..............(In Commission.) The person who engages an agent to perform some service.
Principal..............(In Interest.) The sum of money on which interest is paid. The Base.
Printers' Type Measure. The table of measures used in measuring type.
Prism................A solid having rectangular sides and two parallel ends or bases.
Proceeds..............The amount of a note remaining after deducting Bank Discount. (See also: "Net Proceeds.")
Product..............The answer found by multiplication.
Profit................The difference between the cost and the selling price, when the selling price is the greater.
Progressive Numbers...Numbers which follow one another in any regular order, as 2, 4, 6, 8, etc.; 5, 10, 15, 20, etc.
Promissory Note.......A promise (in writing) to pay a certain sum of money at a certain time.
Proper Fraction.......A fraction which means less than 1 unit.
Proportion............The comparison of equal ratios. (See also: "Simple Proportion" and "Compound Proportion.")
Protractor............An instrument used for measuring angles.
Pyramid..............A solid whose base is a triangle, square, or polygon and whose sides or lateral faces (corresponding in number to the number of sides in the base) are triangles meeting at a common point called the vertex.
Quadrant.............One fourth of a circle, the figure being bounded by an arc and two radii which form a right angle.
Quadrillions' Period....The period of the 6th rank, counting units' period as the first.
Quintillions' Period....The period of the 7th rank, counting units' period as the first.

DEFINITIONS

Quotient	The answer found by division.
Radius (plural "Radii")	A straight line drawn from the center of a circle to a point in the circumference.
Rank	See "Location."
Rate	A certain per cent of the base.
Ratio	The relation which one number or quantity bears to another number or quantity of the same kind.
Receipt	A paper showing payment or delivery.
Receipts	Amounts of money received.
Rectangle	Any plane figure which is bounded by four straight sides, and has four right angles.
Rectangular Prism	See "Right Prism."
Rectangular Solids	Cubes and Right Prisms.
Reduction	Changing the form of a number or fraction without changing its value.
Remainder	The answer found by subtraction. The part of a dividend which remains undivided in an example in division.
Retailer	One who sells merchandise in small quantities to the general public.
Right Angle	An angle formed by two straight lines meeting in a point in such a way that if both lines were lengthened to cross each other, all four angles so formed would be exactly alike.
Right Prism	A solid having four square or oblong sides and two parallel square or oblong ends.
Right Triangle	A triangle having one right angle.
Roman Numerals	The numbers which are often used on watches, clocks, etc., as I, V, X, L, C, D, M.
Root	Any one of the equal factors which produce a power.
Sales-check	See "Sales-slip."
Sales-slip	A paper showing the cost and quantity of merchandise purchased.
Satisfaction of Mortgage	A document showing that a mortgage has been settled or satisfied.
Scale	The relation of the size of the drawing of an object to the size of the object itself.

Sector................Any part of a circle less than the whole, the figure being bounded by an arc and two radii.
Seller................One who sells.
Semi-circle............One half of a circle, the figure being bounded by an arc and a diameter of the circle.
Septillions' Period......The period of the 9th rank, counting units' period as the first.
Sextillions' Period......The period of the 8th rank, counting units' period as the first.
Short Division.........The method of division in which no partial dividends are written.
Simple Denominate Number............A denominate number which consists of only one denomination.
Slant Height..........The shortest distance between the vertex and a point on the perimeter of the base of a pyramid or cone; or, the shortest distance between points on the perimeters of the upper and lower bases of a frustum.
Special Working Unit...A unit of measure consisting of the product of two units of measure.
Specific Gravity.......The ratio of the weight of any volume of any liquid or solid substance to the weight of an equal volume of distilled water; or, the ratio of the weight of any volume of any gas to the weight of an equal volume of air.
Sphere................A round solid bounded by a uniformly curved surface, every point of which is equally distant from a point within called the center.
Square................(In Mensuration.) The plane figure formed by joining the ends of four straight lines of equal length, so that they form four right angles.
Square................(In Powers and Roots.) The power of the second degree of any number.
Square Measure.......The table of measures used for measuring the area of surfaces. (See also: "Surveyors' Square Measure.")
Square Root..........The root of a power of the second degree.
Stock Certificate.......A certificate showing ownership of stock in a corporation. (See also: "Preferred Stock" and "Common Stock.")

Straight Line.........The shortest distance between two points.

Subtraction...........Taking one number or quantity from another number or quantity.

The number from which we subtract is called the "minuend."

The number which we subtract is called the "subtrahend."

The answer is called the "difference" or "remainder."

The sign of subtraction is −, called "minus."

Subtrahend...........The number which we subtract in an example in subtraction.

Successive Trade Discounts..........Several trade discounts to be deducted one after the other.

Sum...................The answer found by addition.

Surveyors' Linear Measure............The table of measures used by surveyors and civil engineers in measuring distances.

Surveyors' Square Measure............The table of measures used by surveyors and civil engineers in measuring area.

Taxes................Sums of money which must be paid by the citizens to help defray the expenses of the government.

Temperature..........The degree of warmth or coldness of an object.

Tens of Millions' Place. .The place of the 8th order.

Tens of Thousands' Place...............The place of the 5th order.

Tens' Place...........The place of the 2d order.

Tenths' Place.........The first place to the right of the Decimal Point.

Ten-Thousandths' Place...............The 4th place to the right of the Decimal Point.

Terms................The several parts of a fraction, or of a proportion.

Thermometer..........An instrument used for measuring temperature.

Thousands' Period.....Thousands', Tens of Thousands', and Hundreds of Thousands' Places.

Thousands' Place......The place of the 4th order.

Thousandths' Place....The third place to the right of the Decimal Point.

Time Measure........The table of measures used for measuring time.
Times (×)...........The sign of multiplication.
Total................See "Sum."
Trade Discount........An amount allowed by wholesalers to retailers so that retailers can sell at list prices and still make a profit. (See also: "Successive Trade Discounts.")
Transposition.........Changing the order of numbers or things.
Trapezoid.............A plane figure bounded by four straight sides, of which only one pair of sides are parallel.
Triangle..............A plane figure bounded by three straight sides joined to form three angles. (See also: "Equilateral Triangle," "Isosceles Triangle," and "Right Triangle.")
Triangular Prism......A solid having three square or oblong sides, and two parallel triangular ends or bases.
Trillions' Period.......The period of the 5th rank, counting units' period as the first.
Troy Measure.........The table of measures used by goldsmiths, silversmiths, and jewelers for weighing precious metals and stones.
Unit..................One person or thing.
Units' Period...........Units', Tens', and Hundreds' Places.
Units' Place...........The place of the 1st order.
Vertex (plural, "Vertices").........The point where two lines meet in an angle.
Vertical...............At right-angles to the surface of the earth.
Volume...............The space occupied by a solid. (Cubic Contents).
Whole Number........A number which shows one or more units or whole things. An integer.
Wholesaler............One who sells merchandise in large quantities, and usually deals only with other merchants.
Working Units........See "Special Working Units."
Yield.................The rate of income from an investment.

ABBREVIATIONS AND SIGNS

(Parts I to VIII, Inclusive.)

Account	Acct. or a/c.
Acre	A.
Altitude	Alt.
Amount	amt.
Angle	∠
Answer	Ans.
At	@
Barrel	bbl.
Base	B.
Board Foot	bd. ft.
Brackets	( ); [ ]; { }.
Bushel	bu.
Cent	ct. or ¢
Centi (.01)	c.
Centime	c.
Chain	ch.
Circle	⊙
Circumference	C.
Commission	com.
Cord	cd.
Credit	Cr.
Creditor	Cr.
Cubic Foot	cu. ft.
Cubic Inch	cu. in.
Cubic Meter	cu. m.
Cubic Yard	cu. yd.
Day	da.
Debit	Dr.
Debtor	Dr.
Deca (10)	D.
Deci (.1)	d.
Decimal Point	.
Degree	°
Diameter	D.

Dime	d.
Discount	disc.
Divided by	÷
Dollar	$
Dozen	doz.
Dram	ʒ
Equals	=
Fifty	L.
Five	V.
Five Hundred	D.
Foot	ft. or ′
Franc	fr.
Free on Board Cars	F. O. B.
Gallon (Liquid Measure)	gal.
Gallon (Apothecaries' Measure)	cong.
Gill	gi.
Grain	gr.
Gram	g.
Greatest Common Divisor	G. C. D.
Great Gross	gt. gr.
Gross	gr.
Hecto (100)	H.
Highest Common Factor	H. C. F.
Hogshead	hhd.
Hour	hr.
Hundred	C.
Hundredweight	cwt.
Inch	in. or ″
Interest	Int.
Kilo (1,000)	K.

Latitude. . . . . .lat.
Least Common
Denominator. . . . .L. C. D.
Least Common
Multiple. . . . . .L. C. M.
Link . . . . .l.
Liter . . . . .l.
Longitude. . . . . .long.
Merchandise . . . . . . .mdse.
Meter. . . . . .m.
Mile. . . . . .mi.
Mill. . . . . .m.
Mili (.001) . . . . .m.
Minim. . . . . .m.
Minus. . . . . –
Minute (Time) . . . . .min.
Minute (Angle and
Arc) . . . . ′
Month. . . . . .mo.
Multiplied by . . . . . . ×
Myria (10,000) . . . . .M.
Number. . . . . .No. or #
(# written before a number)
One. . . . . .I.
Ounce
(Avoirdupois). . . . .oz.
Ounce
(Apothecaries') . . . ℥
Peck. . . . . .pk.
Penny (plural,
"pence"). . . . . .d.
Pennyweight. . . . . . .pwt.
Per Cent. . . . .%
Percentage . . . . .P.
Pi. . . . . .π
Pint (Liquid or Dry).pt.
Pint (Apothecaries'). *O*
Plus. . . . .+
Pound (Avoirdupois).lb. or #
(# written after a number)

Pound (Sterling). . . .£
Power . . . . .², ³, etc.
Quadrant . . . . .quad.
Quart . . . . .qt.
Quire. . . . . .qr.
Radius (plural,
"Radii") . . . . .R.
Rate . . . . .R.
Ratio . . . .:
Ream . . . . .rm.
Remainder . . . . rem.
Right Angle. . . . . ∟
Rod . . . . .rd.
Root . . . .$\sqrt{}$ (called
"radical sign") or rt.
Scruple . . . .℈
Second (Time). . . . . .sec.
Second (Angle and
Arc) . . . . ″
Section . . . . .sec.
Shilling . . . . .s.
Square . . . . .sq. or □
Square Chain . . . . . .sq. ch.
Square Foot. . . . . . . .sq. ft.
Square Inch. . . . . . . .sq. in.
Square Meter . . . . . .sq. m.
Square Mile . . . . .sq. mi.
Square Rod . . . . .sq. rd.
Square Root . . . . . . .sq. rt.
Square Yard . . . . . . .sq. yd.
Ten . . . . .X.
Therefore . . . .∴
Thousand. . . . . .M.
Times . . . .×
Ton. . . . . .T.
Township. . . . . .T.
Triangle . . . .△
Week. . . . . .wk.
Yard . . . . .yd.
Year . . . . .yr.

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