ADVANCED ARITHMETIC

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THE

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PREFACE

The work in Arithmetic in the upper elementary grades must accomplish two aims. It must retain or secure accuracy and reasonable rapidity in the mechanical manipulation of figures; and it must develop, so far as the maturity of the pupil and the time limits of the school program permit, the power to utilize the arithmetical processes presented in the solution of problems of the types that one may reasonably expect to find in ordinary life. These aims have been kept constantly in mind.

To eliminate the useless, the obsolete, and the unnecessarily technical and complex, and thus to secure sufficient opportunity for emphasis of the essential, is one of the commendable tendencies of the time. In the effort to accomplish this, and yet to retain what should be found in a standard arithmetic, which necessarily is frequently used for reference, topics are presented in the simplest manner consistent with clearness and accuracy.

The authors realize that the importance of certain topics varies in different localities and in different schools; and they suggest that where, for any reason, some topic is not deemed fundamentally necessary to secure the essential results desired, it may be treated as useful and interesting information, and need not be mastered in the thorough manner required in the case of other topics. It is believed, however, that no topics have been included that have not a legitimate place, or that do not deserve about the relative emphasis which is given in this series.

Where time problems are given as tests of readiness and quickness of work, as on pages 9, 12, 18 and 22, additional practice should be given to pupils who require more than average time for their solutions. Teachers are urged to devise tests revealing the relative ability of the pupils, and to supplement the drill for the slower ones in such a way as to prevent unreasonable divergences in power at the end of the year's work. The time tests are inserted in order to call the attention of the teacher to this need for a careful observation of the powers and progress of the *individual pupil.*

The extended treatment accorded to the Metric System (which is in universal use throughout the world except among the English-speaking peoples) is deemed advisable at this time, because of our rapidly-growing intercourse with the American lands to the southward, and because of the claims which the science, as well as the commerce, of the great world makes upon it. Its nomenclature has been carried to the field of electricity and other sources of power, and may be expected to enter further fields of universal interest. There is no good reason why either the teaching or the study of this marvelously simple and interesting system should be dreaded or slighted as it has been in the past. The man or woman of to-day cannot afford to be ignorant of it, and youth is the proper time for its mastery.

For expert advice in the proportioning of the work, and its adaptation to specific grades, and for sedulous care in the arrangement of its parts, in supervising its make-up, acknowledgment is made to Mr. James C. Thomas, whose long and fruitful experience in school-book publishing qualifies him to be of the greatest service in all the innumerable details.

Credit is due to Mr. Charles L. Spain, Assistant Superintendent of the Detroit City Schools, for the valuable material supplied in the "Exercises for Practice" given on pages 251-266.

In the confident hope that we have combined, in their proper proportions, exercises to develop accuracy and facility in working with numbers, and topics of genuine value, with illustrative problems of a really practical nature, insuring the development of judgment on the part of the pupil, we present this advanced book of the series to the consideration of all interested in the teaching of the subject.

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September, 1914

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PART I

REVIEW OF DEFINITIONS AND PRINCIPLES

Numeration is reading numbers written in figures.

Notation is writing numbers in figures.

Arabic figures consist of nine *digits* (taking their name from the fingers of the human hand) and the cipher, which is used to fill vacant places in written numbers. There is no figure to represent the tenth digit, or unit. When this is reached the circuit of units is complete, and this circuit is called a *ten*, and written in the next *order*, or place of figures.

A full period consists of three orders, numbered from the right. The first order is *units;* the second, *tens;* the third, *hundreds*.

The periods that are in common use, counting from the right, are the *first* period, *thousands'* period, and *millions'* period.

The next periods which follow these in order, counting from the right, are *millions'* period, *billions'* period, and *trillions'* period.

In theory there is no limit to the number of periods that may be written. Names have been applied to twelve or more, but those beyond billions' period are rarely employed. While periods are counted from the right, they are read from the left; the name of the period being pronounced after the period is read.

In writing numbers, or in reading them, we begin with the highest order of the highest period, which is the first place of figures at the left.

In finding new numbers by addition, subtraction, or multiplication, we begin with the lowest order of the lowest period, which is the first place of figures to the right.

In finding new numbers by division, we begin with the highest order of the highest period.

In reading the figures of a period, we name the hundreds (saying "hundred"), but we substitute other words for the tens and units named together, and we generally omit the word *units* when the units are stated separately.

In the Roman notation, letters are employed in an arbitrary way to represent numbers, and there are no places of figures. A horizontal mark placed over a letter thus used multiplies its numerical value by one thousand.

Numbers to be added are called *addends*. The result of the addition of two numbers is called their *sum*. Thus, 2, 3, and 4 are addends producing 9, which is their sum.

A number that is to be diminished, or made less, is called a *minuend*.

A number that is to be taken from another is called a *subtrahend*.

The result obtained by subtracting the subtrahend from the minuend is called their *difference*.

Thus if 8 be the minuend, and 3 be the subtrahend, the difference between them will be 5.

A number to be multiplied is called a *multiplicand*.

A number by which another is multiplied is called a *multiplier*.

The result of the multiplication of one number by another is called the *product*.

Thus if 12 be the multiplicand, and 6 be the multiplier, the product will be 72.

Except as a matter of convenience, it does not matter which of the numbers is deemed the multiplier and which the multiplicand; for in reality the two numbers are multiplied together.

Thus 12 multiplied by 8 is the same as 8 multiplied by 12.

The multiplicand and the multiplier are called *factors* of the product.

Thus 8 and 12 *are factors of* 96.

A number to be divided is called a *dividend*.

A number by which the dividend is to be divided is called a *divisor*.

The number which shows how many times the divisor is contained in the dividend is called the *quotient.*

Thus if 12 be divided by 4, the quotient will be 3.

Division is the separation of a product into its fac tors.

The dividend is equal to the product of the divisor and the quotient. These are factors of it. The dividend divided by the quotient is equal to the divisor.

Point off and read:

1. 1289764523
2. 786543210798
3. 1000000000000
4. 707070707070707
5. 87962398042163
6. 2020202020202
7. 9060705040306
8. 1564329876521
9. 2968701218260
10. 99110011990022
11. 302036280798687
12. 270006230007160
13. 872965439178602
14. 289763492860435
15. 202020202000
16. 8714030021107
17. 84075629100001
18. 390060807003
19. 20080007000
20. 10101010101010

Write:

1. Ten thousand ten.
2. One hundred fifty thousand fifteen.
3. Two million two hundred thousand.
4. Sixteen million sixteen thousand sixteen.
5. Twenty eight million four hundred fifteen thousand one hundred.
6. Two billion one million two hundred thousand seventy-six.
7. Four trillion four hundred billion two million.

8. Sixteen billion sixty million fifteen thousand.
9. Twenty-four billion sixty million sixty thousand.
10. Seventy million seventeen thousand six.
11. Twenty-nine billion six million twenty thousand.
12. Fifty million one hundred fifty thousand thirteen.
13. Forty billion one hundred seventy million.
14. Eight hundred thousand nine hundred sixty four.
15. Twenty four million twenty six thousand two hundred.

ADVANCED LESSONS IN NUMERATION AND NOTATION

The numeration and notation of our time include certain substitutes for *ten*, *hundred*, *thousand*, *ten thousand*, and for certain fractional quantities. These substitutes are syllables to be united with certain words to avoid dealing unnecessarily with large numbers.

FOUR NEW WORD ELEMENTS

The syllables *deca*, *hecto*, *kilo*, and *myria* are here to be learned in their most general application. Later on in this book they will be applied specifically to the Metric System of weights and measures; and in statements of electrical quantity, current, etc., the pupil will encounter them occasionally through life. They are presented here in their broadest sense, and in the widest variety of connections. The pupil is now to master these syllables simply as to their significance, however applied, and need not concern himself beyond this with unnecessary particulars.

Ten = deca

The Ten Commandments are called the *Decalogue*. A period of ten years is called a *decade*. An animal having ten feet is called a *decapod*. Ten meters (a measure of length) are called a *decameter*.

Hundred = hecto or hecaton

The old word *hecatomb* is applied, in newspapers and magazines, to a terrible disaster in which many lives are lost by fire. In ancient days it meant the sacrifice of a hundred oxen at once, on the altars of some heathen god. A *tome* is a volume; and a *hecatontome* formerly meant a library, or set, of one hundred books. The *hectograph* is supposed to make a hundred copies of a letter. A hundred meters are called a *hectometer*.*

Thousand = kilo

The *kilowatt* is used by machinists and scientists as a measure of power equal to a thousand watts. The kilovolt is used by electricians as a measure of electrical force equal to a thousand volts. A thousand meters is called a *kilometer*.

Ten-thousand = myria

We often speak of a myriad of leaves, a myriad of people, etc., using the word indefinitely, to mean a vast number. But the true *myriad* of old times was exactly ten thousand. Ten thousand meters are called a *myriameter*.

The old word *gram* meant a mark, a letter, or something written; and we find it in *telegram*, *monogram*, *cryptogram*, etc. In our time it means also a certain small measure of weight. This is its meaning in arithmetic.

What is the meaning of *decagram?* of *hectogram?* of *kilogram?* of *myriagram?*

---

* In such words as *hectometer*, *decagram*, etc., the letter *k* is often substituted for the *c*, although the latter has the preference among lexicographers.

The *liter* (pronounced lēter) is a measure of capacity. What is the meaning of *decaliter?* of *hectoliter?* of *kiloliter?* of *myrialiter?*

The *ampere* is used by electricians as a measure of electrical *current*. What is the meaning of *kiloampere?*

The *coulomb* (pronounced kōō-lŏm) is used by electricians as a measure of electrical *quantity*. What is the meaning of *myriacoulomb?*

1. Write in grams the amount of one decagram; of two decagrams; of five decagrams; of eight decagrams.

2. Write in liters the amount of four decaliters; of six decaliters; of nine decaliters; of seven decaliters.

3. Write in meters the amount of two decameters; of seven decameters; of eight decameters.

4. Write in years the length of one decade; of four decades; of five decades; of six decades.

5. Write in grams the amount of two hectograms; of six hectograms; of four hectograms; of seven hectograms.

6. Write in liters the amount of three hectoliters; of five hectoliters; of two hectoliters.

7. Write in meters the amount of seven hectometers; of four and one-half hectometers; of three hectometers.

8. Write the number of books in ten hecatontomes; in fifty hecatontomes; in twenty-five hecatontomes.

9. Write the number of oxen sacrificed in an ancient hecatomb; in five hecatombs; in forty hecatombs.

10. Write the number of feet of six decapods; of four decapods; of eight decapods.

11. Write the number of grams in four kilograms; in nine kilograms; in three kilograms.

12. Write the number of liters in six kiloliters; in eight kiloliters; in seven kiloliters.

13. Write the number of meters in two kilometers; in six kilometers; in four kilometers.

14. Write the number of watts in five kilowatts; in nine kilowatts; in three kilowatts; in four kilowatts.

15. Write the number of volts in a kilovolt; in five kilovolts; in nine kilovolts.

The compound words containing *deca*, *kilo*, etc., may be called *deca* compounds, *kilo* compounds, etc.

Read in *deca* compounds the following:

1.	90 meters	120 meters	fifteen meters
2.	70 liters	14 liters	ninety liters
3.	80 grams	65 grams	thirty grams

Read in *hecto* compounds the following:

4.	450 meters	600 meters	900 meters
5.	850 grams	200 grams	700 grams
6.	500 liters	675 liters	280 liters

Read in *kilo* compounds the following:

7.	5,000 grams	6,000 grams	9,500 grams
8.	3,000 liters	4,500 liters	8,000 liters
9.	7,000 meters	6,800 meters	4,000 meters
10.	8,000 watts	4,000 watts	9,000 watts
11.	7,000 volts	6,000 volts	8,000 volts
12.	6,000 amperes	8,500 amperes	4,000 amperes

Read in *myria* compounds the following:

13.	100,000 grams	50,000 grams	20,000 grams
14.	10,000 meters	90,000 meters	60,000 meters
15.	10,000 liters	40,000 liters	70,000 liters
16.	90,000 coulombs	80,000 coulombs	25,000 coulombs

ADDITION

Time Problems

Note the time required for performing the work of addition in these ten examples, without copying them. Write only the results on the tablet.

1.	2.	3.	4.	5.
2,486	3,972	7,916	2,023	3,974
5,997	8,889	8,479	9,986	8,237

6.	7.	8.	9.	10.
4,473	5,129	20,302	50,405	27,812
9,787	8,683	19,198	29,686	39,489

1. The National House of Representatives has a membership of 435 (based on the census of 1910), and the Senate has a membership of 96. How many members are there in all?

2. Pittsburgh is 468 miles east of Chicago, and New York (city) is 445 east of Pittsburgh. How far apart, by this railway connection, are New York and Chicago?

3. The area of the United States, exclusive of detached possessions, is stated at 3,624,122 square miles. Including 119,184 of detached possessions, what is the total area?

4. The area of the six New England States is as follows: Maine, 33,040 square miles; New Hampshire, 9,341; Vermont, 9,504; Massachusetts, 8,266; Rhode Island, 1,248; Connecticut, 4,965. What is the total area of the New England States?

In the addition of long columns it is well to write the number to be carried on the completion of each column, to avoid an unnecessary repetition of the work in case this number shall slip from the mind through any momentary interruption. The number to be carried may be written temporarily in a small figure under the recorded figure, as in the first example below.

Add in like manner the columns in the succeeding examples.

1.	2.	3.	4.	5.
2,496	2,340	1,982	3,927	4,455
1,983	5,864	4,651	8,220	6,872
7,218	3,217	3,928	4,065	9,987
4,520	8,629	5,416	3,928	2,436
6,924	4,837	7,718	4,639	1,002
8,765	5,426	2,003	7,856	20,005
4,312	3,798	5,694	9,742	45,170
9,736	4,125	23,178	6,319	2,856
10,841	7,864	4,056	5,005	3,792
5,298	3,917	2,092	6,654	4,891
1,121	20,813	1,188	2,390	7,726
1,083	1,024	2,140	2,945	3,555
2,904	12,793	4,695	30,298	4,930
1,326	2,684	2,781	28,487	1,798
68,527				
5 7 6 5				

The small memorandum figures, temporarily penciled lightly below the recorded figures, may be subsequently erased.

Another way is to write the complete sum on the completion of each column, for a partial sum. The sum of units' column, then the sum of tens' column, then the sum of hundreds' column, etc., being each recorded separately in full, the various partial sums may then be added to make the sum total.

This plan is followed in the first example below. In like manner add the columns of the succeeding examples.

1.	2.	3.	4.	5.
2,106	8,843	1,298	5,436	8,436
3,285	2,796	2,046	20,139	2,879
9,728	5,487	7,987	7,745	9,498
5,941	4,863	5,693	4,897	8,196
2,005	2,211	8,489	2,854	2,022
8,706	1,976	7,886	3,959	23,023
3,491	5,425	5,985	8,788	38,604
7,825	3,397	7,798	9,654	9,719
24,683	6,669	4,976	8,977	8,840
9,231	27,842	18,554	8,888	9,678
2,007	5,006	1,088	9,765	8,925
10,203	7,985	21,899	14,898	4,099
9,846	4,391	48,327	9,978	89,700
2,857	8,256	8,654	5,431	4,658
3,406	12,897	9,187	35,288	9,766
5,620	2,403	4,488	2,876	2,765
63,197	5,062	9,763	5,944	4,932
38,489	1,087	2,099	5,030	6,004
99,057	20,099	8,908	8,895	7,586

Partial sums for example 1:

```
      93
     69
    7 9
   84
  21
 -------
 302,683
```

For a test of the correctness of work done in adding columns, the work may be repeated in reverse order. Beginning at the top of the column, and adding downward, we find that the figures fall in different combinations; and for this reason a mistake previously made is not likely to be repeated.

Time Problems

Note the time required for performing the work of subtraction in these ten examples without copying them. Write only the results on the tablet.

1.	2.	3.	4.	5.
2,164	3,986	4,000	2,859	10,100
1,907	2,497	3,179	1,964	2,786

6.	7.	8.	9.	10.
10,724	29,847	32,809	40,400	21,610
9,847	18,858	14,987	3,976	19,143

1. The area of Texas is 265,896 square miles. How much larger in area is Texas than Rhode Island, which has an area of 1,248 square miles?

2. The population of the United States in 1910, exclusive of the detached possessions, was 92,228,531. Including the detached possessions, it was 101,102,677. What was the population of the detached possessions?

3. If, as computed, the water area of the earth is approximately 144,500,000 square miles, and the total surface of the earth is approximately 196,907,000 square miles, how much more water surface than land is there?

4. The ancients believed one-seventh of the earth's surface to be water. This would be approximately 20,642,857 square miles. How great was their error as to the amount?

5. The distance from New York to San Francisco by sea, by way of Cape Horn, is reckoned at 13,000 miles. By way of the Panama Canal it is reckoned at 5,278 miles. How much the shorter is the Panama route?

SUBSTITUTIONS IN SUBSTRACTION

Addition is so intimately related to subtraction, that frequently it is substituted for the latter in making change and in supplying balances that are wanting. It is likewise used to prove the accuracy of work performed in subtraction.

$5.00	$1.73
1.73	.02
$3.27	.25
	1.00
	2.00
	$5.00

Here is illustrated the modern method of making change for a purchase, as compared with the employment of subtraction for the purpose: If you sell goods to the amount of $1.73, and receive a five dollar bill from the purchaser, you may take a pencil and perform the subtraction, as indicated above, and may then take from the cash drawer bills and coins to the amount of $3.27, to return as change. By the modern plan the work of subtraction is obviated, no pencil is used, and the work is all done mentally as the pieces of money are taken from the drawer.

Starting with the amount of the sale, $1.73, you take out 2 cents to make up the incomplete "quarter," and say, mentally, "$1.75." You then take out another "quarter," to complete the dollar, and say, mentally, "$2.00;" you then take out a dollar, and say, mentally, "$3.00;" you then take out, perhaps, a two-dollar bill, and say, mentally, "$5.00." In all this you do not count, at all, the money that you are paying out in change. You simply count from the subtrahend up to the minuend. Since the money has to be counted up in this way in any event, you save time by making this suffice as a substitute for subtraction.

The money is handed to the customer in precisely the way in which it has been taken from the cash register, except that the words which before were said mentally are now spoken. All this, if correctly and satisfactorily done, may save the customer the trouble of re-counting the money.

1. What steps would you take in giving out from a cash register change for a dollar bill, the amount of the purchase being 12 cents?

2. How would you proceed in making change for a ten-dollar bill, the purchase amounting to $3.48?

3. What pieces of money and in what order, would you give in change for a two-dollar bill, the purchase amounting to $1.06?

4. What coins and bills would you be likely to give, in order, in making change for two five-dollar bills the purchase amounting to $5.28?

5. What pieces of money would you be likely to give, in order, in change for $1, the purchase amounting to 8 cents?

6. How would you change a ten-dollar bill and a five-dollar bill, the purchase amounting to $11.27?

7. How would you make change for three five-dollar bills, the purchase amounting to $12.39?

8. Taking out $7.33, what change would you give back, in order, for $10?

9. In cashing a postal money order for $11.27, if you should give 65 cents in trade, what pieces of money would you be likely to give in change?

10. A subscription for a present to cost $95 amounts to only $91.35. If a gentleman makes up the balance needed, taking it from his cash drawer, how does he probably count it out?

Since the subtrahend and the difference together equal the minuend, in subtraction, any one of the three can be easily supplied if the others are given. Supply the subtrahends:

1.	2,837		1,956
2.	3,500		3,080
3.	5,962		3,000
4.	75,160		42,839
5.	19,763		11,274
6.	20,000		18,276
7.	401,286		297,497
8.	954,362		100,000
9.	400,000		286,934
10.	79,080		16,111
11.	240,000		20,700
12.	546,280		697,149
13.	717,400		200,099
14.	100,000		66,752
15.	327,861		100,000

SUPPLYING THE BALANCE

Supply the missing balance in the following additions, beginning at the top of the column and adding downward. Prove the work by subsequently adding the columns upwards.

1.	5.	9.
25	360	20,817
186	1,007	19,463
29	4,268	21,204
78	5,176	19,146
396	**2,497**	**90,275**

2.	6.	10.
124	296	171,740
258	384	20,000
396	5,102	13,130
512	6,001	2,019
439	4,040	303,160
2,456	**18,296**	**700,000**

3.	7.	11.
1,085	71,500	999,000
203	20,003	82,164
6,017	19,264	741,178
4,080	78,571	936,875
2,108	93,210	200,200
19,154	**434,180**	**5,357,000**

4.	8.	12.
2,839	248	1,219,086
4,726	597	2,319,492
5,131	6,030	7,706,548
2,907	4,298	8,540,692
1,600	10,027	9,100,280
22,085	**35,216**	**38,249,673**

MULTIPLICATION

The multiplication table, presumed to have been thoroughly learned progressively in the work of the preceding grade, is here presented as a whole, for rapid tests of the pupil's command of it. If at this point any part of this table is doubtful in the mind of any pupil, he should detect this fact here through self-examination, and make good the deficiency by perfecting his mastery of it before proceeding farther.

Multiplication Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Time Problems

Note the time required for performing the work of multiplication in these ten examples without copying them. Write only the results on the tablet.

1.	2.	3.	4.	5.
297	384	960	872	749
12	15	9	14	8

6.	7.	8.	9.	10.
2,084	9,678	2,563	4,976	2,180
15	11	13	15	14

SUBSTITUTIONS IN MULTIPLICATION

325	4)32,500
25	8,125
1625	
650	
8,125	

Here we have the number 325 multiplied by 25; another method of securing the same result is to annex two ciphers to the 325, thus multiplying it by one hundred, and then to divide the product by 4.

1. What is the difference in the number of figures employed in the two operations?

2. Which operation do you prefer, and why?

3. If instead of multiplying by 50 we annex two ciphers to the number given, by what must we divide the product? Why?

4. If instead of multiplying by $12\frac{1}{2}$ we annex two ciphers to the number given, by what must we divide the product? Why?

5. If instead of multiplying by $16\frac{2}{3}$ we annex two ciphers to the number given, by what must we divide the product? Why?

6. If instead of multiplying by 20 we annex two ciphers to the number given, by what must we divide the product? Why?

7. If instead of multiplying by $33\frac{1}{3}$ we annex two ciphers to the number given, by what must we divide the product? Why?

8. If instead of multiplying by $14\frac{2}{7}$ we annex two ciphers to the number given, by what must we divide the product? Why?

9. If instead of multiplying by $11\frac{1}{9}$ we annex two ciphers to the number given, by what must we divide the product? Why?

1. Multiply 492 by 25, and the product by 50, in the easiest way.

2. Multiply 39 by $33\frac{1}{3}$, and the product by 10, in the easiest way.

3. Multiply 960 by $12\frac{1}{2}$, and the product by $33\frac{1}{3}$, in the easiest way.

4. Multiply 360 by $16\frac{2}{3}$, and the product by 25, in the easiest way.

5. Multiply 847 by $14\frac{2}{7}$, and the product by 10, in the easiest way.

6. Multiply 8739 by $11\frac{1}{9}$, and the product by 50, in the easiest way.

7. Multiply 9424 by $12\frac{1}{2}$, and the product by 25, in the easiest way.

8. Multiply 555 by 20, and the product by 10, in the easiest way.

24	2	24
2	4	8
48	8	192
4		
192		

Here we have the number 24 multiplied *continuously* by 2 and by 4. The product of 2 and 4 is 8. Multiplying the original number by 8, we find the same result as when we multiplied it successively by 2 and by 4.

> *When a number is multiplied continuously by the factors of a multiplier, it is multiplied by that multiplier.*

24	24	48	2	24
2	4	96	4	6
48	96	144	6	144

Here we have the number 24 multiplied *separately* by 2 and by 4; and the sum of the two products is 144. The sum of 2 and 4 is six. Multiplying the original number by this sum, we find the same result as when we added the two products.

> *When a number is multiplied separately by the addends of a sum, the products together are equal to the original number multiplied by the sum of the addends.*

1. Prove that 36 multiplied continuously by 3 and by 4 equals 36 multiplied by 12.

2. Prove that the product of 36 and 3 added to the product of 36 and 4 equals the product of 36 and 7.

3. Prove that 125 multiplied continuously by 4 and 12 equals the product of 125 and 48.

4. Prove that the product of 95 and 7 added to the product of 95 and 2 equals the product of 95 and 9.

5. Prove that 1486 multiplied continuously by 9 and by 3 equals the product of 1486 and 27.

6. Prove that the product of 1486 and 9 added to the product of 1486 and 3 equals the product of 1486 and 12.

7. Show that $(2458\times4)\times9=2458\times36$.

8. Show that $(2458\times4)+(2458\times9)=2458\times36$.

9. Show that $(32980\times25)\times5=32980\times125$.

10. Show that $(32980\times25)+(32980\times5)=32980\times30$.

22,486 ×9 =224,860 − 22,486

Subtraction is sometimes employed as a substitute for multiplication. Thus if a number is to be multiplied by 9, we may, instead, annex to it a cipher, thus multiplying it by 10. From this product we may subtract the original number, and the result will be the same as if the multiplication had been performed in the regular way.

Multiply by Substitution

1. Multiply 1,786 by 9.
2. Multiply 2,758 by 99.
3. Multiply 3,280 by 9.
4. Multiply 26 by 99.
5. Multiply 4,280 by 9.
6. Multiply 85,000 by 9.
7. Multiply 45 by 99.

Time Problems

Note the time required for performing the work of division in these ten examples, without copying them.

1.	2.	3.	4.	5.
5)2,180	9)3,717	4)6,092	8)9,288	7)5,096
6.	**7.**	**8.**	**9.**	**10.**
3)2,634	13)4,420	14)4,200	15)9,615	12)7,452

If 4 barrels of apples cost $10, what will 12 barrels cost at the same price?

```
4)10.00        $2.50
  $2.50           12
              ------
              $30.00
```

As a matter of reasoning, the first step is to find the cost of 1 barrel. This is found by dividing the cost of four barrels by the number of barrels. The cost of one barrel is found to be $2.50. The next step is to find the cost of 12 barrels, which will be 12 times $2.50, or $30.00.

Shorter methods for problems of this kind are found in cancellation and in proportion, to be studied later on. But for the purpose of making the reasoning clear, the following problems are to be solved like the one given above.

1. If six acres of land cost $7,200, what will be the cost, at the same price, of 40 acres?

2. A certain water tank will hold 5,250 gallons. If it receives 450 gallons an hour by one pipe, and discharges 325 gallons in the same time, how much will it contain at the end of ten hours from the beginning of the flow of water?

3. If a man earns $80 in 20 days, how much will he earn at the same rate in 75 days?

4. If he spends $2.25 each day, how much will he have saved in 75 days?

5. Two trains, one going at the rate of 40 miles an hour and the other at the rate of 47 miles an hour, are speeding toward each other from opposite ends of a railway line. How much nearer together will they be at the end of three hours?

6. When they go in opposite directions at the stated rates of speed, how much farther are they apart at the end of three hours?

$$\begin{array}{r} 18 \\ 9 \\ \hline 162 \end{array} \qquad\qquad \begin{array}{r} 9\overline{)162} \\ 18 \end{array} \qquad\qquad \begin{array}{r} 18\overline{)162} \\ 9 \end{array}$$

Multiplication is so related to division that each may be proved by the other. Here the product of 18 and 9 is shown to be 162. If 162 be divided by 9, the quotient will be 18. If the same dividend be divided by 18, the quotient will be 9.

The dividend is equal to the product of the divisor and the quotient.

1. Prove that $49 \times 15 = 735$.
2. Prove that $448 \div 14 = 32$.
3. Show that $204 \div 12 = 17$.
4. Show that $21 \times 52 = 1{,}092$.
5. Prove the division of 7320 by 18.
6. Prove the multiplication of 98 by 74.

7. Prove that $29 \times 18 = 522$.
8. Show that $3,741 \div 43 = 87$.
9. Show that $298 \times 156 = 46,488$.

$$4\overline{)128} \quad 2\overline{)32} \quad 16 \qquad 4 \times 2 = 8 \qquad 8\overline{)128} \quad 16$$

Here the number 128 is divided continuously by 4 and 2, and the quotient is 16. 4 and 2 are factors of 8. The number 128 is divided by their product, which is 8, and the quotient is seen to be the same as before.

> *To divide a number continuously by the factors of a divisor is to divide it by that divisor.*

1. Prove that $(144 \div 4) \div 3 = 144 \div 12$.
2. Prove that $(625 \div 5) \div 5 = 625 \div 25$.
3. Show that $(1,728 \div 4) \div 6 = 1,728 \div 24$.
4. Show that $(4,096 \div 32) \div 2 = 4,096 \div 64$.
5. Divide 7,480 continuously by 2 and 5. What is the shortest way to reach the same result?
6. Divide 400 continuously by 5 and 4. What is the shortest way to reach the same result?
7. Prove that 150,000 divided continuously by 5 and 8 is equal to the same dividend divided by 40.
8. Prove that 2,048 divided continuously by 2, 2, and 2 is equal to the same dividend divided by 8.
9. Show that $(6,561 \div 9)$ divided by 9 is equal to $6,561 \div 81$.
10. Show that $(7,203 \div 3) \div 7 = 7,203 \div 21$.

RATIO AND PROPORTION

2 : 3

In this way we express the relation which one number bears to another. The word *ratio* is used to indicate this relation. The colon is used to separate the terms of the ratio. The first number is called the *antecedent*, and the second is called the *consequent*.

The ratio of 2 to 3 is the same as the ratio of 4 to 6; for in each case the antecedent is two-thirds of the consequent. Two equal ratios are separated by a double colon, thus: **2 : 3 :: 4 : 6.**
This is read "2 is to 3 as 4 is to 6"

2 is two-thirds of 3, just as 4 is two-thirds of 6. In other words, the ratio of 2 to 3 is equal to the ratio of 4 to 6.

In like manner,

3 : 6 :: 5 : 10

3 is one-half of 6, and 5 is one-half of 10. Therefore, the ratio of 3 to 6 is equal to the ratio of 5 to 10.

An equality of ratios is called a *proportion*.

Read the proportions here indicated:

1.	7 : 8 :: 14 : 16	6.	4 : 5 :: 8 : 10
2.	3 : 6 :: 5 : 10	7.	49 : 50 :: 98 : 100
3.	10 : 11 :: 20 : 22	8.	25 : 75 :: 3 : 9
4.	15 : 20 :: 60 : 80	9.	2 : 4 :: 18 : 36
5.	50 : 75 :: 100 : 150	10.	9 : 10 :: 36 : 40

11. What are the antecedents in the 1st of these proportions? in the 2nd? in the 3rd?

12. What are the consequents in the 4th of these proportions? in the 5th? in the 6th?

13. Read the antecedents and the consequents in the 7th, and 8th of these proportions.

14. Read the antecedents and the consequents in the 9th, and 10th of these proportions.

$$4 : 5 :: 8 : 10$$

Of this proportion the first and last terms are called the *extremes*. The second and third terms are called the *means*.

Which numbers are the extremes?

Which numbers are the means?

If we multiply together the extremes, what will be their product?

If we multiply together the means, what will be their product?

> *In any proportion the product of the means will equal the product of the extremes.*

Find the products of the means and of the extremes in these proportions:

1. 2 : 7 :: 4 : 14
2. 3 : 5 :: 6 : 10
3. 2 : 9 :: 4 : 18
4. 4 : 6 :: 8 : 12
5. 5 : 7 :: 10 : 14
6. 3 : 2 :: 6 : 4
7. 2 : 6 :: 3 : 9
8. 30 : 25 :: 6 : 5
9. 50 : 40 :: 5 : 4
10. 90 : 100 :: 9 : 10

Ratio may be expressed in the form of a fraction. Thus the ratio 2 : 3 may be expressed by $\frac{2}{3}$; and the ratio 4 : 6 by $\frac{4}{6}$. The proportion 2 : 3 :: 4 : 6 may be written $\frac{2}{3} = \frac{4}{6}$.

Since the two extremes and the two means are both factors of the same product, it follows that if one of the terms of a proportion is wanting, it can be easily supplied by means of the other three.

Thus if the product of the means is 56, and one of the extremes is 14, we know that the other extreme is $56 \div 14$, or $\frac{56}{14}$, or 4. If the product of the extremes is 24, and one of the means is 8, we know that the other mean is $24 \div 8$, or $\frac{24}{8}$, or 3.

Either mean is equal to the product of the extremes divided by the other mean. Either extreme is equal to the product of the means divided by the other extreme.

1. 3 is to 7 as 6 is to what number?
2. 4 is to 5 as 8 is to what number?
3. 6 is to 8 as what number is to 16?
4. 5 is to 12 as what number is to 24?
5. 2 is to what number as 4 is to 6?
6. 7 is to what number as 10 is to 70?
7. What number is to 6 as 4 is to 8?
8. What number is to 9 as 36 is to 4?

The principle of proportion is employed constantly in business transactions.

If two apples cost 6 cents, what will 4 apples cost at the same rate?

One way of solving this problem is to divide 6 by 2, in order to find the cost of 1 apple; then to multiply the cost of one apple, which is found to be 3 cents, by 4. The product will be 12 cents, the cost of four apples.

Making use of the principle of proportion, we may say, "2 apples are to 6 cents as 4 apples are to 12 cents."

The proportion may be expressed in the following ways:

We may say,—

$$2 : 6 :: 4 : 12 \qquad \frac{2}{6} = \frac{4}{12}$$

Or we may say "2 apples are to 4 apples as 6 cents are to 12 cents," and the proportion may be expressed in the following ways:

$$2 : 4 :: 6 : 12 \qquad \frac{2}{4} = \frac{6}{12}$$

What are the means and what are the extremes in the following proportions expressed as fractions?

1. $\frac{2}{3} = \frac{4}{6}$
2. $\frac{2}{9} = \frac{4}{18}$
3. $\frac{1}{7} = \frac{3}{21}$
4. $\frac{5}{9} = \frac{20}{36}$
5. $\frac{8}{10} = \frac{4}{5}$
6. $\frac{2}{9} = \frac{10}{45}$
7. $\frac{3}{7} = \frac{9}{21}$
8. $\frac{4}{9} = \frac{36}{81}$
9. $\frac{2}{5} = \frac{8}{20}$
10. $\frac{7}{12} = \frac{14}{24}$

Supply the missing terms in these proportions:

1. 2 : 8 :: 4 : ?
2. 3 : 6 :: ? : 10
3. 4 : ? :: 7 : 14
4. ? : 9 :: 15 : 45
5. 24 : 48 :: 7 : ?
6. $\frac{2}{3} = \frac{8}{?}$
7. $\frac{3}{4} = \frac{?}{24}$
8. $\frac{12}{?} = \frac{16}{48}$
9. $\frac{?}{10} = \frac{16}{40}$
10. $\frac{7}{9} = \frac{21}{?}$

Cancellation may be very largely employed in solving problems in proportion, thus shortening the work.

$$2 : 5 :: 4 : ? \qquad \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2}} = 10$$

Here the two means and but one of the extremes are given. By cancellation, the means are multiplied together and divided by the one extreme to find the other extreme.

Solve by cancellation:

$$3 : 9 :: ? : 36 \qquad \frac{3 \times \overset{4}{\cancel{36}}}{\cancel{9}} = 12$$

Here the two extremes and but one of the means are given. By cancellation the extremes are multiplied together and divided by one of the means, to find the other extreme.

By cancellation supply the missing terms in these proportions:

1. 3 : 7 :: 9 : ?
2. 4 : 28 :: 2 : ?
3. 1 : 12 :: 12 : ?
4. 20 : 30 :: 16 : ?
5. $\frac{3}{7} = \frac{?}{21}$
6. $\frac{4}{?} = \frac{24}{54}$
7. $\frac{9}{?} = \frac{36}{48}$
8. $\frac{?}{9} = \frac{16}{36}$
9. $\frac{?}{36} = \frac{24}{72}$
10. $\frac{19}{20} = \frac{57}{?}$

11. If 3 horses cost $174, what will 5 horses cost, at the same rate?

12. If 6 men can clear 2 acres of timber in a certain time, 3 men can clear how many acres in the same time?

13. 20 recitations of 40 minutes each occupy how many hours?

14. If 20 front feet of a city lot can be bought for $4,000, for how much can 45 feet be bought at the same rate?

15. If 3 jars contain 15 pounds of butter, how much will 5 such jars contain?

16. A real estate man trades 40 front feet of city ground, at $240 a front foot, for a Western farm of 80 acres? How much is the farm worth per acre?

17. How many jugs of 3 gallons each can be filled from 2 barrels ($31\frac{1}{2}$ gallons each) of linseed oil?

18. How many garden beds 4 ft. x 30 ft. can be made in a tract 40 ft. x 90 ft., exclusive of paths?

19. How many dozen garments, each requiring 1.75 yards can be made from 35 pieces of cloth of 12 yards each?

20. A farmer exchanges 30 bushels of apples at $1.50 per bushel for coal at $7.50 per ton. How many tons does he receive?

21. How much butter, at 35 cts. a pound, will pay for 70 yds. of muslin at 15 cts. per yard?

22. In a vacation of 2 weeks, working 3 days in the week, Henry gathered 21 bushels of hickory nuts which he sold at 5 cts. a quart. What did he earn for each of these working days?

23. A man works at 90 cts. an hour, 8 hours per day, for 20 weeks, of $5\frac{1}{2}$ working days each. What are his wages in all? His wages will pay the expenses of 2 persons at $11 per week for how many weeks?

24. If one machine turns out one product in a minute, how many such machines will it require to turn out one hundred such products in the same time? in one *hundred* minutes?

25. 12 great-gross of buttons at 2 cts. apiece, are exchanged for 8 dozen gown patterns; what is the cost of each pattern?

26. 9 horses each eating 6 ears of corn 3 times each day for 100 days will consume as much corn as 3 horses eating 9 ears of corn 2 times a day will consume in how many days?

27. 27 men working 8 hours a day for 3 days on a sewer will do as much work as 8 men working how many hours per day for 27 days?

28. To cover with leather the black squares of 8 checkerboards of 64 squares each (red and black) at 10 cts. for 4 squares will cost how much?

29. Divide 10 times the number of days in an ordinary year by the exact number of weeks in seven years.

30. Ten coils of rope, each 45 feet in length, will make how many pairs of halters, each $7\frac{1}{2}$ feet long?

31. 8 students' lamps worth $3.60 each are exchanged for 2 pairs of vases of equal value; what are the vases worth apiece?

32. 14 automobiles, worth on an average $900 each, are exchanged for 2 droves of horses numbering 35 in each drove; what was the average value of the horses?

33. The ratio of the votes cast to the population, in a certain commonwealth was as 2 : 14. If the votes were 140,000, what was the population?

34. If the ratio of the illiterates to the entire adult population of a certain community is 3 : 54, and the adult population is 36,000, what is the number of illiterates?

35. The ratio of the bulk of a pound of water to that of a pound of ice is 11 : 12. 44 cubic inches of water will make how many cubic inches of ice?

36. The ratio of the length of a building to its height is 4 : 3, and its length is 120 feet. How tall is the building?

37. If the ratio of pine trees to maples in a forest is 17 : 9, and there are 340 pine trees, how many maples are there?

38. If in an epidemic 3 persons in 20 of a population are stricken, and there are 348 patients, what is the population?

39. If 2 property holders in 13 in a county are delinquent tax payers, and there are 260 delinquent, what is the total number of property holders?

40. The ratio of the wheat raised to the area of a certain farm is 57 bushels to 3 acres, and the wheat fields produce 17,100 bushels; what is the number of acres?

41. If 3 persons in 19 are subject to military service in a certain country and the population is 114,000, how many persons are subject to such service?

42. 96 sacks of potatoes containing 2.5 bushels per sack are exchanged for 5 dozen barrels of flour at $3 the barrel. What are the potatoes worth the bushel?

43. A rancher trades 3 dozen cows worth $30 each, for 5 span of horses; what are the horses worth apiece?

2 : 8 :: 8 : 32

Here one number is repeated to serve for both means of the proportion. Such a number is called a *mean proportional* between the extremes.

1. 3 : 9 :: 9 : ?	6. 4 : 8 :: 8 : ?
2. ? : 7 :: 7 : 147	7. 5 : 25 :: 25 : ?
3. 4 : 12 :: 12 : ?	8. 4 : 6 :: 6 : ?
4. 3 : 9 :: 9 : ?	9. 2 : 13 :: 13 : ?
5. 5 : 15 :: 15 : ?	10. 8 : 63 :: 63 : ?

The use of the mean proportional in the part of arithmetic which treats of roots of numbers, especially as applied to measurements, is important. This belongs to more advanced study.

IDENTICAL RATIOS

If $1\frac{1}{2}$ bushels of apples cost $1\frac{1}{2}$ dollars, what will 18 bushels cost?

$$1\frac{1}{2} : 1\frac{1}{2} :: 18 : 18$$

Here it is not necessary to go through the process of multiplication and division, since it is plain that, in any proportion,

> *If the terms of the first ratio are alike, the terms of the second ratio will be alike.*

With a little ingenuity, the terms of a ratio in concrete problems may often be made alike when they do not so appear at first. Thus $2\frac{2}{3}$ yds. may be counted 8 ft.; $1.50 may be counted 15 dimes; $75,000 may be counted 75 thousand dollars; 27 may be counted $2\frac{1}{4}$ dozen; $2\frac{1}{3}$ dozen may be counted 28; etc.

1. If $1\frac{3}{8}$ days' work will be sufficient to clear of weeds $1\frac{3}{8}$ acres of garden, how many such days' work will be required to clear $8\frac{1}{2}$ such acres?

2. If $1.75 (14 "bits"*) will buy 14 collar buttons, how many will $2.75 (22 "bits") buy?

3. If $87\frac{1}{2}$ cts. will buy 70 buttons, what number will $10.50 buy? (7 "bits" : 7 tens of buttons :: 84 "bits" : 84 tens of buttons.)

4. If 55 yards of cloth cost $16.50, what will 33 yards cost? (165 ft. : 165 dimes :: 99 feet : ? dimes.)

5. If the rent of a house for 2 years and 4 months is $1,400, what is it for 1 year? (28 months : 28 fifty-dollar bills :: 12 months : ? fifty-dollar bills.)

6. If 96 casters cost $12.00, what will 64 casters cost? (Casters for 24 chairs : 24 half-dollars :: casters for 16 chairs : ? half dollars.)

7. A man dying at 3 score and 10 years of age left $70,000 as a missionary offering proportioned to his years. At the same rate, what would he have left to this cause had he lived to the age of 87 years and nine months? (State so as to show similar terms in ratios.)

*There is no coin to represent the value of one-eighth of a dollar, though this value is very commonly used in calculations. Unfortunately, it has no universally-accepted name. "Shilling" has other uses. "Bits" (in the plural number) is used in a large part of this country to indicate multiples of eighths of a dollar; thus, "two bits" signifies a "quarter," "four bits" a half dollar, "six bits" seventy-five cents, etc. The ready calculator of prices thinks of $2.75 as eleven quarters, or twenty-two bits; of $1.50 as six quarters, or twelve bits; of $1.87½ as seven and one-half "quarters," or fifteen bits, etc.

8. If the cost of maintaining a pet animal for 2 weeks is \$1.40, what will be the cost of maintaining it for 100 days at the same rate? (14 days : 14 dimes :: 100 days : ? dimes.)

9. At $87\frac{1}{2}$ cts. a day, for $3\frac{1}{2}$ hours each day, what will be the cost of the services of an assistant for 100 hours? (7 half-hours : 7 "bits" :: 200 half-hours : ?"bits")

10. At 75 cts. for 60 picture cards, what number will \$1.25 buy? (3 "quarters" : 3 score :: 5 "quarters" : ? score.)

11. A certain quarter section of land cost \$16,000. At that rate, what was the value of $7\frac{1}{2}$ acres?

12. If 35 cts. will buy 42 picture cards, what number will \$3.90 buy? ($3\frac{1}{2}$ dimes : $3\frac{1}{2}$ doz. cards :: 39 dimes : ? cards.)

13. If 3 dozen miners dig 36 tons of coal in a certain time, how much coal can 74 men dig in the same time?

14. If $1\frac{1}{2}$ dozen loaves of bread are worth \$1.80 ($1\frac{1}{2}$ dozen dimes), 27 loaves are worth how much?

15. If $2\frac{1}{4}$ dozen plums cost 27 ($2\frac{1}{4}$ dozen) cts., what will 13 plums cost at the same rate?

16. If in $3\frac{1}{2}$ weeks of working days a boy earns \$21, in how many days can he earn \$37?

17. If 9 yards of cloth cost \$2.70, what would $\frac{2}{3}$ of a yard cost?

18. Three base ball teams at a hotel paid a total hotel bill of \$270; at that rate, what would have been the bill of $1\frac{4}{9}$ teams?

19. If \$6.25 will buy 25 candy boxes, what number will \$10.00 buy?

$$(5 + 6 + 7) \div 3 = 6$$

An average is a mean, or medium, in numbers. Thus if three children are respectively 5, 6, and 7 years of age, their average age will be 6 years. The sum of 5, 6, and 7 years is 18 years; and the number of the years divided by the number of the persons indicates the average age of the persons.

It is to be remembered that an average is artificial rather than real, and sometimes there is nothing to correspond to it. Thus if I have 20 boards that are 3 feet in length, and 20 more that are 5 feet in length, the average length will be exactly 4 feet, though there be not a single board to correspond to it. A community may have a few inhabitants owning immense wealth, and a great number of others wretchedly poor; and in that case the average wealth of the inhabitants may make a showing of general prosperity, though there be not among them a single person of moderate wealth.

An average of the averages will not answer for the average of the whole. One total must be divided by another total in order that the true average may appear.

1. Of three wheat fields of 40 acres each, one yields 18 bushels to the acre, another 24 bushels, and the other 15 bushels; what is the average yield per acre?

2. In a certain department store 4 buyers receive an average salary of $125 per month; 2 assistant buyers receive an average salary of $75 per month; 20 salesmen receive an average salary of $35 per month; and 4 bundle wrappers receive an average salary of $25 per month. What is the average salary per month received by these employees?

3. Of five hogs that were slaughtered by a farmer, one weighed "dressed" 328 pounds, another 416 pounds,

another 519 pounds, another 432 pounds, and the last 316 pounds. What was the average weight of the carcasses?

4. If the limit of the most profitable fattening of swine is 300 pounds, what was the average excess weight of these five swine?

5. Between certain stations on a railway line, three trains maintain a speed of 40 miles the hour, two of 28 miles and one of 49 miles; what is the average speed of the trains?

6. Longevity is the length of a human life. Of six members of a certain family, one reached the age of 14 years, another of 28 years, another of 47 years, two of 74 years each, and one of 87 years. What was the average longevity of the family?

7. If in a day's business in a retail store the profit on $100 sales is $25, the profit on $50 sales is $20, the profit on $85.00 sales is $27.00, and the profit on $35 sales is $12, what is the average profit on the total amount of sales? What is the average per cent of profit on the sales?

8. The expenses of a certain business house for six months of the year are, respectively, $2,700, $2,300, $2,400, $2,250, $2,025 and $1,800; what is the average monthly expense for this period?

MAJORITIES AND PLURALITIES

A majority is the greater of two numbers, or more than half of a number regarded as a total. Where officers are elected by voters, the one who receives a majority of the total votes cast is chosen. Where there are more than two candidates for the same office, it may happen that no one receives a majority of the whole number of votes. In that case he who receives the highest number of votes is said to have a plurality, or excess, over the votes received by any of his competitors individually.

For some purposes of legislation, and in some matters of parliamentary usage a two-thirds vote of legislative chambers or of conventions or societies is required.

1. What is the smallest number that will be a majority of 368 votes?—of 369 votes?

2. What is the smallest number that will constitute a two-thirds majority of a total vote of 128? of 129? of 127?

3. Of three candidates for Congressman in a certain congressional district, one received 12,462 votes, another 9,139; the third 7,462. What was the total vote cast? What was the smallest number that would have constituted a majority of it?

4. What did the successful candidate lack of a majority of the votes cast?

5. What was his plurality over the second candidate?—over the third candidate?

6. When President Andrew Johnson was impeached in 1868, and tried before the U. S. Senate, that chamber contained 2 members from each of 27 States. A two-thirds majority is necessary to conviction and removal from office in impeachment trials. What number of votes would have been necessary to convict and remove President Johnson?

7. Thirty-five votes were cast for conviction and removal of the President, and nineteen were cast in his favor. How many necessary votes were lacking for conviction and removal?

In the Presidential election of 1860 the *popular* votes were divided among the four candidates as follows:

For Abraham Lincoln,	1,866,452
For Stephen A. Douglas,	1,375,157
For John C. Breckenridge,	847,953
For John Bell,	590,631

8. What was the aggregate popular vote?

9. What was the smallest number of votes that would have constituted a popular majority?

10. What was Lincoln's plurality over Douglas?—over Breckenridge?—over Bell?

In the Presidential election of 1860 the *electoral* votes were divided among the candidates as follows:

For Abraham Lincoln,	180
For John C. Breckenridge,	72
For John Bell,	39
For Stephen A. Douglas,	12

11. What was the aggregate electoral vote?

12. What was the smallest number of electoral votes that would have constituted a majority?

13. What was Lincoln's electoral majority?

14. Territory may be annexed to the United State through a treaty, by a two-thirds majority of the Senate alone, or through a joint resolution of Congress, with a mere majority of both houses. What was the election problem presented in 1844 to those who desired the annexation of Texas?

15. What is the advantage of having an odd number of votes in a social committee or legislative chamber?

PERCENTAGE

Review of Principles.

A fraction having for its denominator 100 is usually spoken of as a *per cent.* Strictly speaking, however, *per cent* is a phrase meaning *in the hundred,* and it applies to whole numbers. Thus instead of saying or writing $\frac{1}{100}$ or .01, we may say "*one per cent,*" or *one in the hundred.* Here *one* is a *whole number,* considered in its relation to *one hundred.* The sign per cent is %.

A fractional per cent is a fraction of one per cent. Thus .5%, or $\frac{1}{2}$%, is .5 of one per cent. .01% is one hundredth of one per cent. A per cent may be expressed in figures in three different ways. Thus one-tenth may be written 10%, or .1, or $\frac{1}{10}$.

Express the following in *per cent:*

1. .2	6. .12$\frac{1}{2}$	11. $\frac{3}{4}$
2. .35	7. .84	12. 12$\frac{9}{10}$
3. .01	8. .25	13. 6$\frac{1}{4}$
4. .25	9. .07	14. .18
5. .16$\frac{2}{3}$	10. $\frac{1}{2}$	15. $\frac{1}{5}$

Express the following in decimal fractions:

1. 25%	6. 75%	11. .01%	16. 125%
2. 2%	7. 37$\frac{1}{2}$%	12. .06%	17. 1000%
3. 16$\frac{1}{2}$%	8. 90%	13. $\frac{1}{2}$%	18. $\frac{116}{100}$%
4. 8%	9. 5%	14. $\frac{3}{4}$%	19. $\frac{210}{100}$%
5. 10%	10. 80%	15. .001%	20. $\frac{1}{20}$%

Per cents changed to common fractions should be reduced to the lowest terms; thus—

$$50\% = \frac{50}{100} = \frac{1}{2}$$

Express the following in common fractions:

1. 25%
2. .25%
3. 20%
4. .20%
5. $16\frac{2}{3}\%$
6. $.16\frac{2}{3}\%$
7. $14\frac{2}{7}\%$
8. $.14\frac{2}{7}\%$
9. $12\frac{1}{2}\%$
10. $.12\frac{1}{2}\%$
11. $11\frac{1}{9}\%$
12. $.11\frac{1}{9}\%$
13. 10%
14. .1%
15. $9\frac{1}{11}\%$
16. $.9\frac{1}{11}\%$
17. $12\frac{1}{2}\%$
18. $.12\frac{1}{2}\%$
19. $14\frac{2}{7}\%$
20. $.14\frac{2}{7}\%$
21. $16\frac{2}{3}\%$
22. $.16\frac{2}{3}\%$
23. $33\frac{1}{3}\%$
24. $.33\frac{1}{3}\%$
25. 50%
26. .50%
27. 625%
28. 62.5%
29. 6.25%
30. .625%
31. $\frac{1}{4}\%$
32. $1\frac{1}{4}\%$
33. $\frac{1}{2}\%$
34. $2\frac{1}{2}\%$
35. $\frac{1}{3}\%$
36. $3\frac{1}{3}\%$
37. $\frac{1}{4}\%$
38. $4\frac{1}{4}\%$
39. $.5\frac{3}{4}\%$
40. $\frac{1}{5}\%$
41. $3\frac{2}{5}\%$
42. $4\frac{4}{5}\%$
43. $\frac{1}{6}\%$
44. $2\frac{5}{6}\%$
45. $\frac{1}{7}\%$
46. $7\frac{1}{7}\%$
47. $\frac{1}{8}\%$
48. 375%
49. 37.5%
50. 3.75%

> *To find a given per cent of a number, multiply the number by the per cent expressed as a decimal fraction.*

6% of $156 is $156 × .06, or $9.36.

1. What is 7% of $960?
2. How much is 19% of 123?
3. Find 15% of 15.
4. What is 2% of 2?
5. Find 4% of 384.
6. How much is 10% of 102?

To find what per cent one number is of another the former expressed decimally is divided by the latter.

Thus 2 is 25% of 8.

$$8\overline{)2.00}$$
$$.25$$

1. 4 is what part of 12?
2. $150 is what % of $300?
3. 40 sheep is what % of 75 sheep.
4. 60 bushels is what % of 58 bushels?
5. 24 soldiers is what % of 960 soldiers?
6. 18 trees is what % of 54 trees?

To find the number of which a given quantity is a stated per cent, the given quantity, called the percentage, is divided by the stated per cent expressed decimally.

Thus if $80 is 2%, the whole amount will be $4,000. $80 ÷ .02 = $4,000.

1. 4 is 5% of what number?
2. 8 is 16% of what number?
3. 17 is 5% of what number?
4. 24 is $16\frac{2}{3}$% of what number?
5. 18 is 40% of what number?
6. 72 is $8\frac{1}{4}$% of what number?

PRACTICE PROBLEMS

1. What is 8% of $50?
2. $40 is what % of $50?
3. $40 is 8% of what sum?
4. What is 15% of $960?
5. $65 is what % of $35?

6. \$75 is 25% of what sum?
7. What is 75% of \$820?
8. \$16.20 is what % of \$64.80?
9. \$360 is 6% of what sum?
10. What is 15% of \$2,500?
11. \$900 is what % of \$4,500?
12. \$825 is 15% of what sum?
13. What is 65% of \$128.40?
14. \$72 is what % of \$96?
15. \$38 is 6% of what sum?
16. What is 94% of 94?
17. \$32 is what % of \$800?
18. \$78 is 12% of what sum?
19. What is 47% of \$47?
20. \$63 is what % of \$72?
21. \$128 is 87% of what sum?
22. What is 75% of $\frac{4}{5}$?
23. 80 is what % of 320?
24. 300 is 60% of what number?
25. What is 72% of 656?
26. 16 is 75% of what number?
27. 18 is what % of 300?
28. What is 12% of 27,500?
29. 90 is 25% of what number?
30. 75 is what % of 90?

As a matter of convenience, and for ready use in calculations, it is well to have at instant command the equivalents of a number of simple fractions stated in per cents. Some that are included in the following table are presented chiefly for illustration and for reference.

1. $1\% = \frac{1}{100}$	5. $5\% = \frac{1}{20}$	9. $.7\frac{9}{13}\% = \frac{1}{13}$
2. $2\% = \frac{1}{50}$	6. $.6\frac{1}{4}\% = \frac{1}{16}$	10. $.8\frac{1}{3}\% = \frac{1}{12}$
3. $3\frac{1}{3}\% = \frac{1}{30}$	7. $.6\frac{2}{3}\% = \frac{1}{15}$	11. $.9\frac{1}{11}\% = \frac{1}{11}$
4. $4\% = \frac{1}{25}$	8. $.7\frac{1}{7}\% = \frac{1}{14}$	12. $10\% = \frac{1}{10}$

13. $11\frac{1}{9}\% = \frac{1}{9}$	19. $33\frac{1}{3}\% = \frac{1}{3}$	25. $80\ \% = \frac{4}{5}$
14. $12\frac{1}{2}\% = \frac{1}{8}$	20. $37\frac{1}{2}\% = \frac{3}{8}$	26. $83\frac{1}{3}\% = \frac{5}{6}$
15. $14\frac{2}{7}\% = \frac{1}{7}$	21. $50\ \% = \frac{1}{2}$	27. $28\frac{4}{7}\% = \frac{2}{7}$
16. $16\frac{2}{3}\% = \frac{1}{6}$	22. $62\frac{1}{2}\% = \frac{5}{8}$	28. $42\frac{6}{7}\% = \frac{3}{7}$
17. $20\ \% = \frac{1}{5}$	23. $66\frac{2}{3}\% = \frac{2}{3}$	29. $71\frac{3}{7}\% = \frac{5}{7}$
18. $25\ \% = \frac{1}{4}$	24. $75\ \% = \frac{3}{4}$	30. $87\frac{1}{2}\% = \frac{7}{8}$

In examples like the following, it is convenient to make use of the common fraction equivalent to the per cents stated.

1. What % is 20% of $83\frac{1}{3}$%?
2. What % is 10% of 10%?
3. What % is $33\frac{1}{3}$% of 18%?
4. What % is $33\frac{1}{3}$% of 15%?
5. What % is 20% of $83\frac{1}{3}$%?
6. What % is $14\frac{2}{7}$% of 77%?
7. What % is $9\frac{1}{11}$% of 22%?
8. What % is 20% of $83\frac{1}{3}$%?
9. What % is $14\frac{2}{7}$% of $87\frac{1}{2}$%?
10. What % is $33\frac{1}{3}$% of $37\frac{1}{2}$%?
11. What % is $33\frac{1}{3}$% of 75%?
12. What % is 20% of $62\frac{1}{2}$%?
13. What % is $66\frac{2}{3}$% of 75%?
14. What % is $33\frac{1}{3}$% of $37\frac{1}{2}$%?
15. What % is $33\frac{1}{3}$% of $16\frac{2}{3}$%?

Express in a common fraction, and also in a decimal fraction, each of these per cents:

1. 2%	7. 9%	13. 14%
2. 4%	8. 8%	14. 17%
3. 3%	9. 10%	15. 19%
4. 5%	10. 11%	16. 18%
5. 7%	11. 15%	17. 16%
6. 6%	12. 13%	18. 12%

19. 20%	23. 25%	27. 22%
20. 21%	24. 26%	28. 24%
21. 1%	25. 30%	29. 23%
22. 27%	26. 29%	30. 28%

$$1 + 1\% = 101\% \text{ of } 1$$
$$1 - 1\% = 99\% \text{ of } 1$$

Where a number is increased or diminished by a per cent of itself, the original number may be considered as 100% and the changed per cent will then apply to the original number. Thus if 8 be increased by 25% of itself, the enlarged number will be 125% of 8, which is 10. To find the original number when the enlarged number is given, the latter is divided by the enlarged per cent expressed as a decimal fraction. Thus $10 \div 1.25 = 8$. This is because $8 \times 1.25 = 10$.

Likewise if 8 be diminished by 25%, the diminished number will be 75% of 8, which is 6. To find the original number when the diminished number is given, the latter is divided by the diminished per cent expressed as a decimal fraction. Why? Thus $6 \div .75 = 8$.

PRACTICE PROBLEMS

1. A clerk receiving $100 the month has his salary increased 15%. What is it then?

2. A man paying a rental of $600 per year finds that this rental is to be increased 12%. What will it be then?

3. If I add to my bank deposit $120, which is 60% of what I had already on deposit, what was the latter sum?

4. If I take from the bank $37.50, which is 5% of my deposit, how much of the latter do I have in the bank?

5. If a man sells a house for \$2,760, which is 15% more than he paid for it, what was the original purchase price?

6. If a railway line which has been extended 18% of its original length is now 554.6 miles long what was its original length?

7. If a stock of goods is sold for \$200,000 at a loss of $2\frac{1}{2}$%, what was its purchase price?

8. The present enrollment of a school, 486 pupils, is 20% more than that of last year. What was last year's enrollment?

9. The circulation of a certain newspaper, now 39,875, has increased 10% over that of last year. What was last year's circulation?

10. By his opposition to a certain measure a Congressman lost 13 per cent of his supporters at the previous election, receiving for re-election only 2,398 votes How many votes did he receive before?

11. After losing 18% of his investment in a commercial enterprise, a man retains of it \$6,192.60. What was the amount of his investment?

12. A boy who weighs 72 pounds has gained $12\frac{1}{2}$ per cent since his last birthday. What was his weight then?

13. In an examination of 415 students in orthoepy, 332 improperly accented the word *construe* on the last syllable. The others pronounced the word correctly. What per cent of the students were correct in their pronunciation of the word?

14. Of 100 young men who started in business for themselves, 84% failed and abandoned the enterprise. Of the remainder 50% failed but compromised with

their creditors, and were thus enabled to continue the business to the time of their death, but their estates did not make good their deficits. Of the remainder, 50% failed but compromised with their creditors, and being thus enabled to go on with their business, paid off their indebtedness in full and prospered. The remainder succeeded from the beginning, were never insolvent nor delinquent in the payment of their debts, and maintained their success through life. What per cent of the whole number of men did the last class constitute?

15. If in 1913 there were in the United States 12,000,000 farmers, owning farms to the total value of $40,000,000,000, as estimated, what was the average value of the farmers' landed possessions? If the value of the farm crops for the year was $9,500,000,000, what per cent was this on the total value of the farms, as estimated?

EXPENSE AND INVESTMENT

The necessary cost of conducting a business is the expense, which must be deducted from the income to find the profit. The costs of shop rent, the salaries of workmen, the insurance and taxes on merchandise, etc., are expenses to be considered. It is important, however, to note the difference between an expense and an investment. The constant repairing of an old machine, which at best can never improve in value, is an expense. The discarding of the old machine is a loss. But the purchase of a new improved machine which will turn out more and better work and will speedily pay for itself, immediately adds value to the equipment of the shop, and is to be considered an investment. The repairing of a railway track is an expense to the company that owns it. An extension of the track to a point where new business can be secured is an investment.

In business a distinction is made between expense and investment. The outlay necessary to carry on a business, apart from the capital used in establishing it, is *expense*. An outlay that will "pay for itself", by increasing the amount and value of the business or by reducing the expense, is *investment*.

1. A certain railway company expends $75,000 in repairing side tracks, and $10,000 in replacing condemned cars. Within the same period it expends $160,000 in extending its line, to secure the hauling of wheat from a local market in such amount as will pay present interest on this outlay, eventually pay off the principal, and meanwhile yield a profit over the expense of operating the extension. Which of these items are to be classed as expense, and which as investment?

2. A certain factory has an old form of machine which does one-fourth less work than a new and improved machine would do with the same outlay for power and management. If this old machine is destroyed by an accident, and the new machine, is bought at double the present price of the old, what part of the outlay for it is to be regarded as investment?

3. A certain man pays $1200 a year for the rent of his store. By expending $200 in making alterations of the building, he can sublet a portion of it without loss to his trade, and the outlay will be replaced in a year by the sub-rent. By doing this he can avoid the expense of a night watchman at $5 a month. What part of the $200 expended is expense, and what part is investment?

The *gross profit* in any business transaction is the amount of gain from it without regard to any attendant expense or to any loss resulting from it. Where there are attendant losses or expenses or both, they take away from the gross profit; and they may be sufficient to destroy it, and turn the profit into a loss. *Net profit* is the profit from which any losses and expenses have been deducted, and from which no further deduction is to be made. Only gross profit and loss are generally calculated for individual transactions in business. For the results of a continuous business for a certain period, the expenses and losses are deducted, and the net profit found.

In a statement of profit, gross profit is understood unless net profit is named.

The profit or loss on any form of property that is sold is calculated on the purchase price, and stated as a per cent of it. Thus if an article which cost \$12, be sold for \$15, there is a profit of one-fourth or 25%; whereas if it be sold for \$9.60, there is a loss of one-fifth, or 20%.

Some of the following problems may be solved without written work:

1. At what price is a building which cost \$5,000 sold at a profit of $12\frac{1}{2}$%? at a loss of 4%?

2. At what price is a building lot which cost \$2,000 sold at a profit of 6%? at a loss of 3%?

3. At what price must an overcoat that cost \$16 be sold to make a profit of 30%? to cause a loss of 2%?

4. At what price must a pair of shoes which cost \$3.40 be sold to make a profit of 25%? to cause a loss of 5%?

5. At what price must I sell glassware which cost me \$1,200 to cover a breakage of $1\frac{2}{3}$% value and make a profit of 15%?

6. An abandoned beach hotel which cost $80,000 is sold for $48,000 at what per cent of loss? The purchaser re-opens it for a new class of patronage, and sells it to a company for the original cost. What was his per cent of profit?

7. What per cent of profit must be made on the sale of goods costing $50,000 to cover an expense of $7,500 and a net gain of the same amount?

8. Of a shipment of fruit, $6\frac{1}{4}\%$ was condemned as spoiled. At what percent must the remainder be sold to gain 12% on the whole?

9. The sales in a store were $960 for one day, which would have meant a profit of 20% but for the unfortunate acceptance of a counterfeit 20-dollar bill which could not be traced to the payer. What was the net per cent of gain?

10. What is a gain of $11\frac{1}{9}\%$ on $1,800? What is a loss of 15%?

11. If $1,640 worth of food products are advanced 10% n price, what will be their aggregate price?

PRACTICE PROBLEMS

1. If goods valued at $9,600 be reduced 15% in price, what will be their aggregate price?

2. If overcoats marked at $16.00 each be advanced 15%, what will be their price? If they be marked down 15%, what will it be?

3. A house and lot was purchased for $8,000. The house was moved off and sold for $2,000 and the cost of moving. At what price must the lot be sold, to realize a total gain of 25% on the investment?

4. The sales of a certain store were $72,000 for the year, and the profit made was $8,000. What was the per cent of profit?

5. A merchant sold goods for which he paid $30,000 at an average of 30% higher price, but lost 5% from the failure of certain debtors. What was the amount of his profit?

6. A man bought an orchard for $4,000. He sold for $600 the fruit of one season, and sold the orchard for $4,400 in the fall. What was his per cent of profit?

7. A man bought 4 suburban lots at $700 each. On 2 he made a gain of 35%, and on the other two he lost 10% when he sold them. What was his net gain on the entire purchase and sale?

8. A factory employing 40 equally paid operators of machines, reduces its force by 25%, and increases by 25% the wages of those that remain. Does it now pay more or less in wages than before?

9. A mill is sold for $856,000, at an advance of $14\frac{2}{7}$% on its cost price. What did it cost?

10. At what price must a horse costing $125 be sold to gain for its owner 20%?

11. A teamster paid $100 each for 2 horses and 1 wagon. At what price must he sell the outfit to repay him for $20 spent on harness and secure for him a gain of 10%?

12. With an expenditure of $25 a week a man gains $37.50 each week. What is his per cent f gain on the money expended? What will be the amount of receipts of expenditures and gain for 52 weeks?

13. To increase his capital from $30,000 to $45,000, what per cent of it must be added?

14. A man has $14,000 invested in a lumber business, and $26,000 in an artificial stone enterprise. In the former he loses 8% of his investment. What amount must he gain on the other to equal 10% on the whole?

15. A "cut-off" line of track will enable a railway to save 13 miles of the 104 miles of distance between two cities. What will be the per cent of the distance saved?

16. Warlike rumors caused a sudden fall in the market price of a foreign corporation's stock, involving a loss in value of 8%. What is the present loss to the owner of $40,000 worth of the stock?

17. If 10% of $5,000 worth of clothing becomes shelf-worn, and loses 25% of its value, what is the amount of the loss?

18. A mine produces lead at a profit of $35\frac{1}{2}$% on the investment. A new use for the by-products causes an additional $2\frac{1}{2}$% of income calculated on the same investment. What will be the net gain on the investment if 20% of the returns from the by-products be required to meet the additional expense incurred because of them?

19. If a piece of meat which cost 18 cts. per pound loses one-third of its weight in the cooking, at what price per pound must it be sold, when cooked, to make a profit of 20%?

20. A man bought a stock of potatoes at 90 cts. per bushel, lost 10% of the amount by frost, and sold the remainder securing a profit of 20% on the original purchase price. What did he charge per bushel for them.

INSURANCE

Insurance is a contract whereby, for an agreed consideration called a *premium* one party undertakes to indemnify or guarantee another against loss from a specified contingency or peril, called a *risk*.

The contract is set forth in a document called a *policy*. Sometimes a temporary memorandum called a binder is issued pending the preparation of the policy.

Fire Insurance insures against loss from injury to property, or destruction of it, by fire.

Marine Insurance insures against loss by injury to, or disappearance of, ships, cargoes, or freight, by perils of the sea.

Casualty Insurance, of various kinds, insures against loss resulting from accidental injury to property, such as live stock, plate glass, etc.

Fidelity Insurance insures against loss arising from the default or dishonesty of public officers, or of clerks or agents in the employ of the insured.

Life Insurance insures a person against loss through the death of another.

Accident Insurance insures a person against disability caused by an accidental injury to him; and in case of his death from the injury received, it insures an indemnity to a specified beneficiary.

In addition to these there are some other forms of insurance.

To be equitably insured, one must have an *Insurable Interest* in what is covered by the insurance. This is to prevent an insurance from having the nature of a betting or gambling contract. The insured must have such an interest as would suffer legal damage in the event of the realization of the peril insured against, in the case of fire and marine insurance. In life insurance, ties of blood and affection are sufficient in this country.

Tradesmen in monarchies are sometimes held to have an insurable interest in the life of the reigning sovereign, since in case of his death a period of mourning might ruin the sale of fashionable apparel in certain lines, and thus cause the dealer who has stocked up in them a material loss of trade.

PROPERTY INSURANCE

1. If I deem my house worth $5,000, and desire to insure it for one year for three-fifths of its value, what must I pay for the policy when it costs $7.50 for each $1,000 of insurance taken?

2. A farmhouse and barn and other buildings pertaining to them, valued at $15,000, are insured for one year for two-thirds of their value, at 75 cents for each hundred dollars of insurance. What is the cost of the policy?

3. If the furniture in my cottage is worth $1,000, and I wish to insure it for 60% of its value having to pay $1.66⅔ for each hundred dollars of insurance, what will the policy cost me?

4. A church pays, in all, $180 yearly, for insurance to the amount of $10,000, in each of three separate companies, the rate of insurance being the same in each company. What is the rate of insurance?

5. If my house, insured for $4,000, is destroyed and I am unable to prove that it was really worth over $2,000, I can recover from the insurance company no more than the latter amount. If when insured against fire for $5,000 it was destroyed, and I can prove its value only to the amount of 60% of this sum, what amount of compensation do I receive?

6. To insure my house for three years at a time, I have to pay no more than the insurance would cost by the year for two years. My insurance for a three-year period is $25.00. What would be the cost for a period of a single year?

7. By insuring a shop for five years at a time, the owner pays only as much as the cost would be for three separate periods of one year each. If he pays for $150

for the five-year period, what would the insurance cost him for a period of one year? For a period of three years? For five separate periods of one year each?

8. Insurance on furniture is strictly construed in accordance with the exact terms of the policy. If the furniture is removed at all without express permission from the building in which it is insured, the insurance fails. If I have furniture to the amount of $1,200 insured for three-quarters of its value, and during a day of housecleaning half of it, equal to half the value of the whole, is temporarily placed in an out building on the premises which is not mentioned in the policy, and the house and outbuilding both take fire and all the furniture is destroyed, what amount can I recover on my loss of furniture?

9. A rug worth $150 is hung out on a clothes line in the yard to air, and is ruined by sparks from a passing fire engine. If it formed a part of the household furniture that is insured for $1,000, can the owner recover its value from the insurance company? What could be recovered if it had been thus destroyed while hung on a line on a porch of the residence?

10. A dozen rare books in a library are insured against fire for $1,200 as highly prized incunabula (books printed not later than the year 1500). They are destroyed by fire, and it is then proved that they were spurious incunabula recently manufactured, and of no greater possible value than $3 apiece. What is the limit of indemnity that the company should pay for them?

11. Of a stock of goods that cost $100,000 and was insured against fire for three-fifths of the cost at $15.50 for each $1,000 of insurance, one-half the value is destroyed. What is the net amount of loss sustained by the insurance company?

12. A pair of supposed silver foxes, valued at $1,800 apiece, are insured for 90% of their value. They die in captivity, and it is proved by the insurance company that they were not of pure blood, and that their value was not more than $600 apiece. What is the most that can be recovered from their insurance?

13. A horse insured for $300 is choked to death, being ignorantly tied with a running noose of rope for a halter about its neck. Is the owner entitled to the insurance? If by a compromise 40% of the policy is paid, what sum does the owner receive?

14. A county officer having to give security for $60,000 for the faithful performance of his official duties, a fidelity insurance company protects the county against loss through a default by him, at a cost of .4% annually. What does this surety cost the officer in a term of four years?

15. A city officer having to give security for $50,000, two fidelity insurance companies furnish the security, each to the amount of $25,000, at .3½% annually. Judgment to the amount of $5,000 is secured against the officer for misconduct in office before the end of his term, and he is found to be $2,500 in arrears in his account with the city. What is the net loss to each of the Fidelity companies?

LIFE INSURANCE

The age nearest the birthday of the person insured is the age considered.

1. At the age of 21 a man insured his life for $2,000 on what is called the Ordinary plan, for an annual payment of $37.52. What does he pay in fifteen years?

2. Had he deferred the insurance for five years, it would have cost him $42.10 for each year. How

much more would this have been than the amount which the insurance actually did cost him for the same period of ten years?

3. By a plan of Limited Payments, a life may be insured for a stated period covering ten payments, fifteen payments, or twenty payments. What is the total cost of a Fifteen-payment life insurance policy for $1,000 taken by a man at the age of thirty at $40.25 for each annual payment?

4. Endowment life policies call for a limited number of payments, and are for certain periods. At the end of the period, if the insured person is living and has made all the payments according to agreement, the amount of the insurance is paid to him. If a man insures his life on the Endowment plan for $1,000 at the age of 21, paying $30.36 annually for a thirty-year period, and lives beyond the time, what does he pay in all for the insurance?

ACCIDENT INSURANCE

1. A man has paid $18 for 4 years for an accident insurance policy. Owing to an accident, he is disabled for a period of 6 weeks, during which time he receives $25 a week. What has been the net financial value of the insurance to him? What has been the net loss to the insurance company?

2. By the accidental discharge of a weapon, a man is slightly wounded, and is disabled from work for three weeks. His loss of salary is $75 for the period. The insurance company pays him $20 a week for the time lost. The insurance policy cost him $12. What is his net financial loss from the accident? What would have been his loss but for his insurance?

3. A man has paid $15 a year for 6 years for accident insurance. He loses $2\frac{1}{2}$ weeks' time from being disabled by a falling sign on the street, and receives from the insurance company $62.50. He has suffered no loss of wages. What is the net gain to the insurance company from his insurance?

TAXES

Taxes are levied under State and National laws, for the support of the local, State, and National governments, and for the carrying out of their undertakings.

Real estate consists of lands and buildings, the land including forests, mines, quarries, etc.

Personal property consists of movable possessions, such as clothing and jewelry, household furnishings, domestic animals and farm products, the merchandise and productions of stores and shops, machines and engines, vehicles, mortgages and notes, credits, bonds, and stocks, money, etc.

Direct taxes are levied on real estate and personal property by State authority. States may also provide for levying indirect taxes, such as inheritance taxes, incomes, licenses, corporation taxes, etc. They may also authorize the levying of a poll tax on all voters, regardless of possessions or income. The poll tax is usually small in amount, and is the same for rich and for poor. "Poll" means *head;* and it was customary in the olden time to state a certain number of men by giving the number of their heads, as we do to this day in the matter of cattle.

The revenues of the National government are derived chiefly from duties levied on imports under the tariff law; from the Internal Revenue, which is levied on a few classes of products, such as fermented and distilled liquors, tobacco and its manufactured forms, and oleomargarine; and from taxes on corporations and (recently) on incomes.

1. What is the rate of taxation in your county for county purposes? In your State for State purposes?

2. If property is taxed at the rate of $1.25 on the $1,000, what is the per cent of taxation? How many mills is that on the dollar?

3. In a certain village the assessment valuation of the property is, in round numbers, $300,000. The amount of tax necessary to be raised is $5,000. What will be the rate of taxation for village purposes?

4. In a certain township (town) a tax of $20,000 is to be raised. If there are 500 citizens to pay a poll tax of $1, how much of the tax must be laid on property?

5. Personal property taxes are assessed where the owner of the property resides, wherever the personal property may be, and whether he owns real estate there or does not. Therefore a community may profit largely from the personal taxes of residents possessed of great wealth in this form. If the real estate of a village is assessed at $600,000, and the personal property at $4,000,000, what amount will be realized for village purposes by a tax of 2 mills on the dollar?

6. If a man is assessed at $64,000, what does every mill added to the rate of taxation cost him in the increase of his tax?

7. On an assessment valuation of $5,000,000 what rate of taxation must be imposed to raise the amount of $750,000?

8. The rate of taxation for State purposes in a certain State is 1 mill on the dollar. The assessment valuation being only one-fifth of the real value, what is the actual rate of assessment?

9. In a village containing property to the assessment value of $200,000, the rate of taxation is 3%. If a poll tax of $2 can be collected from 800 citizens, how much can the assessed rate of taxation be reduced?

10. A certain farm is valued at $30,000 and the personal property upon it at $10,000. What amount of tax will the owner pay for one year if the property is assessed at $\frac{2}{3}$ its value, and the state and local taxation equals $2\frac{1}{2}$%.

11. If the rate of taxation is .2%, what tax must be paid on 120 acres of coppice valued at $12 the acre, and assessed at $\frac{3}{4}$ its value?

12. A renter in a city, having household furniture to the value of $1,200, and owning bonds to the value of $5,000, is assessed for what amount, if the property is assessed at one-third of its value, and rate is $2\frac{1}{4}$%?

13. A farm adjoining a city is valued at $15,000, and is assessed at $\frac{3}{4}$ its value, the rate of taxation being 1%. If $\frac{1}{3}$ of the farm is annexed to the city, in which the rate of taxation is $2\frac{1}{2}$%, what will be the amount of the taxes levied upon the whole property? upon the part annexed to the city?

14. If the assessed value of the property in a certain county is $18,596,482, and the local taxes levied upon it for State and local purposes is $650,876.87, what is the total rate of taxation?

15. In Chicago, property has been assessed at one-fifth of its actual value, as a matter of convenience in calculating the amount of taxes, the rate being adjusted accordingly. In 1908 the total value of all private property in Chicago was stated at $2,385,951,995. What was its valuation for taxation?

16. If the amount necessary to be raised in a certain corporation is assessed at $42,000, and there are 300 polls, the poll tax being $1.50, what must be the amount of the tax laid on the property?

17. If the assessment valuation of a certain village is $1,800,000, and there is to be raised for a special purpose, $48,000, what will be the rate of taxation required for this purpose?

18. What will this add to the tax of a resident who owns property assessed at $6,000?

19. If a tax of 4 mills is assessed on the property of a State for school purposes, how much will this be to the owner of property assessed at $9,000?

20. The National Income Tax law went into effect March 1, 1913, taxing personal incomes equaling or exceeding $3,000 per year. Since the tax is levied for the Calendar year, what amount of income from March 1st to December 31st, was necessary to subject the recipient to this tax in that year?

21. In 1908 the receipts of the National government from Internal Revenue amounted to $251,711,-126.70. The expense of collecting this sum was $4,650,049.89. What per cent of the receipts was paid for the collection of the Internal Revenue?

22. In the State of New York in 1907 direct taxes (city, village, county, and town) were levied to the amount of $180,942,341.27, and indirect taxes were levied to the amount of $32,339,707.49. No direct taxes were levied for State purposes, because of the great burden of local taxes, which are direct. What per cent of the total taxes levied did the indirect taxes constitute?

What principal at 6%, will produce annually an income of $420?

$$\$420 \div .06 = \$7{,}000$$

Here the interest on the entire principal is divided by the interest on one dollar of it. The quotient is the number of dollars in the principal.

The entire interest divided by the interest on one dollar of it will equal the principal.

1. What principal at 4% will earn $326.40 in 1 year?
2. What principal at 5% will earn $32.00 in 1 year 9 months, 10 days?
3. What principal will earn $900 in 1½ years at 6%?
4. What principal will earn $2,606.80 at 6% in 30 days?
5. What principal at 9% will earn $427.50 in 2 years 4½ months?
6. What principal will earn $70.10 at 8% in 5 years, 10 months, and 3 days?
7. What principal, at 3%, will earn $21 in 1 year?
8. What principal, at 5%, will earn $26.50 in 1 year, 5 months, 20 days?
9. What principal at 3½% will earn $639.24 in 2 years?
10. What principal will earn $98.75 at 5% in 2 years?

The principal and the interest added together constitute the **amount.**

Find the principal of which the amount is $530 at 6% for 1 year.

$1.00 + $.06 = $1.06   530 ÷ 1.06 = $500

The amount of $1 for the time and at the rate of interest stated is $1.06.

> *The whole amount divided by the amount of one dollar will equal the number of dollars of the principal.*

1. What principal, at 5% for 5 years, will produce the amount of $500?
2. What principal, at 6% for 3 years, will produce the amount of $572.30?
3. What principal, at 7% for 4 years, will produce the amount of $11,636.16?
4. What principal, at 9% for $3\frac{1}{2}$ years, will produce the amount of $1,643.75?
5. What principal, at 8% for 4 years, 9 months, will produce the amount of $2,036.88?
6. What principal, at 10% for $6\frac{1}{2}$ years, will produce the amount of $16,498.35?
7. What principal, at 3% for 3 years, 4 months, will produce the amount of $313.17?
8. What principal, at 4% for 7 years, 6 months, will produce the amount of $705.90?
9. What principal, at $6\frac{1}{4}$% for 8 years, will produce the amount of $45,750?
10. What principal, at $5\frac{1}{2}$% for 6 years, will produce the amount of $1,045.80?

Considering the interest year as made up of 360 days, the interest for each day at 5% is $\frac{1}{360}$ of 5 cts, or $\frac{5}{360}$ or $\frac{1}{72}$ of a cent, for each dollar of the principal. The principal, multiplied by the number of days, must therefore be then multiplied by $\frac{1}{72}$ or (what is the same thing) divided by 72, to find the total interest in cents.

A similar explanation will account for all the following rules, which have the appearance of being purely arbitrary:

For Finding Interest Expressed in Cents by the Number of Days

For the interest at 5%, multiply the principal by the number of days, and divide by 72.

For the interest at 6%, multiply the principal by the number of days, and divide by 60.

For the interest at 7%*, multiply the principal by the number of days, and divide by 52.

For the interest at 8%, multiply the principal by the number of days, and divide by 45.

For the interest at 9%, multiply the principal by the number of days, and divide by 40.

For the interest at 10%, multiply the principal by the number of days, and divide by 36.

For the interest at 12%, multiply the principal by the number of days, and divide by 30.

For the interest at 15%, multiply the principal by the number of days, and divide by 24.

For the interest at 20%, multiply the principal by the number of days, and divide by 18.

By this method find the interest.

1. On $456 for 6 months at 5%.
2. On $520 for 9 months at 6%.

*In many States the laws prohibit a higher rate of interest than 6%. A rate of interest higher than the limit allowed by law is called *usury*.

3. On $1,000 for 195 days at 6%.
4. On $2,530 for 7 months at 9%.
5. On $246 for 18 months at 6%.
6. On $4,856 for 100 days at 5%.
7. On $9,200 for 10 months at 7%.
8. On $8,800 for 7 months at 8%.
9. On $9.650 for $2\frac{1}{2}$ yr. at 9%.
10. On $2,290 for $4\frac{1}{2}$ yr. at 10%.
11. On $1,972 from May 1, to Aug. 1, 1915, at 5%.
12. On $8,460 from Jan. 1 to June 1, 1912, at 6%.
13. On $1,246 from Mar. 1 to Sept. 1, 1912, at 6%.

EXACT INTEREST

The National Government and some of the States take no notice of the "interest year" of 360 days, and of a month rated at thirty days regardless of fact. They deal with the exact number of days actually involved, and for each such day the interest allowed is $\frac{1}{365}$ of the interest for a full year. Trust companies also are apt to make use of exact interest.

What is the exact interest on $1,000 from June 30, to Dec. 31, 1912, at 6%?

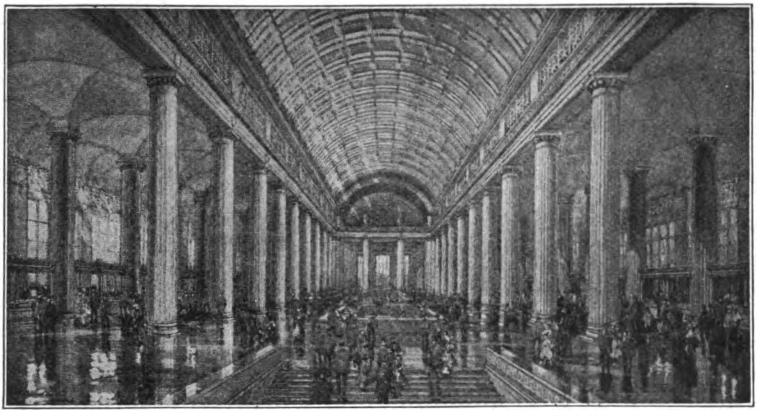
The days are $31+31+30+31+30+31=184$.

The interest for 1 year is $\$1,000 \times .06 = \$60$. $\frac{184}{365}$ of $60 $= \$\frac{11040}{365} = \$30.53$.

1. In the problem given, the exact interest differs by how much from the interest as commonly reckoned?

2. What is the exact interest on $1,500 from Jan. 1, to July 1, 1913, at 6%? By how much does it differ from the interest, as commonly reckoned?

3. A certain trust company pays exact interest at 5% on $9,476 from Apr. 3 to Nov. 9, 1912. What interest does it pay, and how does this differ from the interest as commonly reckoned?



Interior of a Metropolitan Bank

A bank is an institution which receives on deposit, lends, and exchanges money.

In this country, banks are incorporated either by the National government or by the State government.

National banks are authorized by the National government and are inspected by National officers.

State banks are authorized by the State government and are inspected by State officers.

A very large part of the banking capital of the country is invested in State banks.

Only National government banks issue bank notes.

Savings banks, established under State laws, receive deposits of money, generally in small amounts, from many individuals, and pay but a low rate of interest for it. They lend the money to borrowers at a higher rate of interest. The National government does not charter or inspect savings banks, but it conducts a Postal Savings Service in connection with the Post Office.

Owing to the laxity of the laws in some of the States, individuals or partnerships sometimes call their offices "banks", although they possess no bank charters and are not subject to any official inspection.

Trust companies, organized under State laws, not only do a banking business, but also settle estates, take care of the property of minors, and perform other services in the nature of trust.

In order to provide for the establishment of Federal Reserve Banks, to furnish an elastic currency, to afford means of rediscounting commercial paper, and to establish a more effective supervision of banking in the United States, the government of the United States inaugurated in 1914 a system of Federal Reserve banks.*

Federal Reserve banks are to be found only in the twelve specified Federal Reserve cities, viz.: New York, Chicago, Philadelphia, Boston, St. Louis, Cleveland, San Francisco, Minneapolis, Kansas City, Atlanta, Richmond, and Dallas. No Federal Reserve bank is permitted to begin business with a capital of less than $4,000,000. Only a member bank of its district is permitted to own at any one time more than $25,000 of the stock (par value) of any one of these banks.

1. If a capitalist owns the maximum amount permitted in a Federal Reserve bank in each of the cities named, what is the par value of his investment?

2. What was the minimum capital required for the beginning of business by eight Federal Reserve banks?

3. The shares of stock of a Federal Reserve bank are of the par value of $100 each. How many shares are

* A Federal Reserve bank is essentially "a banker's bank." It sustains with its members much the same relation that ordinary banks sustain with their depositors.

required to be taken, to amount to the minimum sum required for beginning the business of the bank?

4. Three State banks and two business houses each purchase the maximum permitted amount of stock in a Federal Reserve bank. What is the amount of their stock in it?

5. If a certain Federal Reserve bank has a capital of $20,000,000, and yields a dividend of 6% annually on the capital stock, what will be the amount of the dividends distributed in one year?

FARMERS' NATIONAL BANK

DEPOSITED TO THE CREDIT OF

____

FORT DEARBORN, ILL. __________ 191_

(CHECKS ARE RECEIVED FOR COLLECTION)

DRAFTS		
CHECKS		
CURRENCY		
TOTAL		

All kinds of banks receive money on deposit for safe keeping. Some banks make a small charge for their care of money deposited in small amounts, and some pay a low rate of interest for large deposits left with them on checking account. Generally, however, the depositor neither receives interest upon, nor pays for the care of, money deposited on checking account.

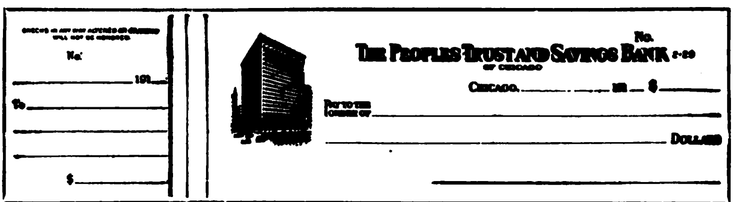
With the money to be deposited, at any time, the depositor hands to the receiving teller of the bank a deposit slip filled out with his name, the date and the items of amount deposited, and the nature of the deposit, whether it consists of bills, coin, or checks on banks.

A blank deposit slip is shown on the preceding page.

1. Write a fictitious deposit slip for a deposit of John Doe.*

2. Write a deposit slip for a deposit of Richard Roe in an imaginary local bank.

Check books contain blanks which can be easily and rapidly filled out. They also contain stubs from which the checks may be torn off, and which are prepared to contain a memorandum of each check drawn; so that when all the blank checks have been used, the stubs will contain a record of all the moneys withdrawn from the bank by means of them.



When the depositor wishes to withdraw from the bank any of the money deposited to his credit, he fills out a check, which is an order for the amount to be withdrawn. The check may be made payable to himself or to some other person to whom he wishes to make a payment. The person who presents the check to the paying teller, to be "cashed", must indorse it. This is done by writing his name on the back of the check.

* "John Doe" and "Richard Roe" have been for centuries legal designations for supposititious or unknown personages.

If the depositor makes a check payable to himself and indorses it, any one may present it for payment. If he makes it payable to some particular person, that person (called the *payee*) must indorse it, whether he transfers it to any one else or presents it for payment at the bank.

Checks should be presented promptly for payment. When checks are received by a bank for deposit they are credited as cash, for they are immediately collected from the banks on which they are drawn.

1. If you receive a check made payable to yourself and you lose it before you have indorsed it, can the finder cash it at the bank without forging your name?

2. If you receive a check made payable to yourself and you indorse it and then lose it, can the finder cash it at the bank?

3. If you receive a check made payable to "bearer", can any one cash it at the bank without your indorsement of it? Is it best to make checks in this way?

A check made to be paid to any one in an official capacity cannot be received by him in a personal capacity. Thus if you make a check payable to "John Doe, Treas.", and John Doe is the treasurer of a lumber company, he cannot claim it for a personal payment to him, although he may allege that you owe him the amount of it on a personal debt. It must be credited to the lumber company.

1. If you receive a check made payable to yourself, and if, instead of presenting it personally at the bank, you send it to another person with the mere indorsement of your name (which is called an indorsement in blank), and it is lost in transit, can any finder of it cash it?

2. If in sending it you indorse it with an order to

pay that person (giving his name), can any other person who finds it cash it?

A check so indorsed is said to be "indorsed in full."

3. Write a check payable to Richard Roe, sign it Richard Roe, and indorse it in blank with the name Richard Roe.

4. Write and indorse in the same manner a check payable to John Doe.

5. Write a check payable to Richard Roe. Sign it Richard Roe, and indorse it in blank with the name Richard Roe.

6. On a check made payable to John Doe, and signed by John Doe, write an indorsement in full, authorizing payment to Richard Roe.

7. When you indorse a check which proves to be worthless, you become fully liable for the amount of it unless you indorse it in such a way as to disclaim such liability. This can be done by writing over your indorsement the words "without recourse".

8. Exchange checks with another pupil. Indorse one of his checks in blank.

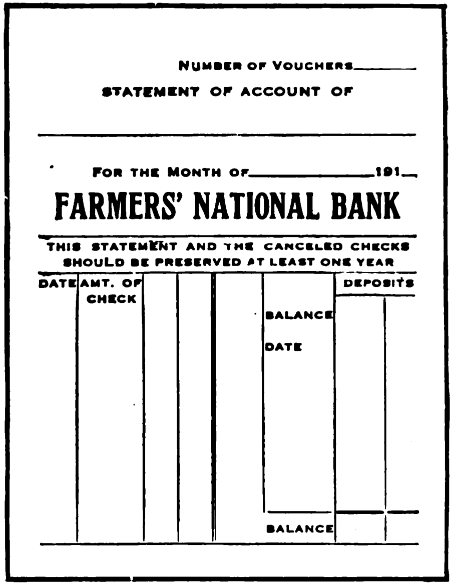
9. Indorse his other check in full.

10. Write a check for a fictitious amount to be paid to John Doe, and sign the name Richard Roe.

11. Write a check for a fictitious amount to be paid to Richard Roe, and sign the name John Doe.

The use of checks renders it easy to pay bills by mail, and in various ways it lessens the risk of loss in the transmission of money. At stated periods, usually at the close of each month, the paid checks are returned by banks to the persons who issued them; and they thus serve as receipts, since they show that the moneys have been paid.

With the returned checks is sent a statement of account.


NUMBER OF VOUCHERS______

STATEMENT OF ACCOUNT OF

FOR THE MONTH OF__________191__

FARMERS' NATIONAL BANK

THIS STATEMENT AND THE CANCELED CHECKS SHOULD BE PRESERVED AT LEAST ONE YEAR

DATE | AMT. OF CHECK | BALANCE | DATE | DEPOSITS | BALANCE

A check issued by a bank against another bank is called a draft, or bill of exchange. It may be purchased by individuals for their own use in making payments at a distance where the bank has credit. A small charge is made by the bank for the use of its paper in this way. Where both banks are in the same country, the word *draft* is more commonly used. Where they are in different nations, *bill of exchange* is the term generally employed.

A New York house owes a Chicago house. Another Chicago house owes another New York house a like amount. Individuals in each city adjust the matter at their respective banks. Only a relatively small amount of cash is transferred from city to city.

But for this method of adjusting accounts between business houses in different cities, vast amounts of money would be constantly transported in all directions, and very largely returned to their starting place.

Banks lend money generally in small amounts and for short periods of time, such as thirty days or sixty days, more rarely ninety days. In such transactions the word month is avoided, because it is so indefinite in meaning; and the period of the loan is stated in days. While a day is really $\frac{1}{365}$ of a year, bankers reckon it as $\frac{1}{360}$ of a year, or $\frac{1}{30}$ of a month, unless there is an understanding otherwise. A day is therefore $\frac{1}{60}$ of a sixty-day period. Where money is loaned at 6%, which is the usual rate of interest, the interest on any principal for sixty days is .01. The interest for six days is .001 of the principal, or one mill in the dollar.

The bankers' method of reckoning interest on the basis of the sixty-day period is so simple that results can be reached mentally with little expenditure of effort or of time. Bankers treat the 6% as a decimal fraction, and follow this memorandum:

At 6% *the interest for* 60 *days is* .01 *of any principal.*
At 6% *the interest for* 6 *days is* .001 *of any principal.*

Since the interest for the sixty day period is one-hundredth of the principal, it is found by merely moving the decimal point two places to the left.

In dealing with smaller periods of time, we are to regard these as fractions of the sixty-day period. Thus 30 days will be one-half of it; 20 days, one-third of it; 15 days, one-fourth of it; 12 days, one-fifth of it; 10 days, one-sixth of it; etc.

To find the interest for 9 days, the interest for 10 days may be found, and one-tenth subtracted from it. If the period is 14 days, the interest for 15 days may be found, and one-fifteenth subtracted from it; or the interest for 12 days may be found, and one-sixth added to it.

Solve these problems by inspection, the interest being 6%:

1. What is the interest on $620 for 60 days?
2. What is the interest on $7,450 for 60 days?
3. What is the interest on $927 for 60 days?
4. What is the interest on $8,296 for 60 days?
5. What is the interest on $2,850 for 60 days?
6. Find the interest on $2,680 for 30 days.
7. Find the interest on $1,842 for 30 days.
8. Find the interest on $864 for 30 days.
9. Find the interest on $296 for 30 days.
10. Find the interest on $744 for 30 days.
11. Determine the interest on $144 for 10 days.
12. Determine the interest on $2,970 for 10 days.
13. Determine the interest on $1,296 for 10 days.
14. Determine the interest on $1,080 for 10 days.
15. Determine the interest on $588 for 10 days.
16. What is the interest on $4,230 for 20 days?
17. What is the interest on $297 for 20 days?
18. What is the interest on $1,512 for 20 days?
19. What is the interest on $4,230 for 20 days?
20. What is the interest on $3,096 for 20 days?
21. State the interest on $5,360 for 15 days.
22. State the interest on $1,728 for 15 days.
23. State the interest on $144 for 15 days.

24. State the interest on $5,992 for 15 days.
25. State the interest on $7,560 for 15 days.
26. Find the interest on $3,945 for 12 days.
27. Find the interest on $1,620 for 12 days.
28. Find the interest on $865 for 12 days.
29. Find the interest on $7,290 for 12 days.
30. Find the interest on $9,775 for 12 days.

THE SIXTY-DAY METHOD

Find the interest on $480 at 6% for 78 days.

For 60 days,	$4.80
For 15 days,	1.20
For 3 days,	.24
For 78 days,	$6.24

Here we have found the interest for the sixty-day period, then for a quarter of that period, and finally for one-twentieth of it. The complete amount of interest is found by adding these partial amounts.

In the following problems, partial statements of interest may be written, and added together without further writing.

1. Find the interest on $2,480 for 75 days.
2. Find the interest on $3,087 for 80 days.
3. Find the interest on $4,812 for 40 days.
4. Find the interest on $960 for 50 days.
5. Find the interest on $2,970 for 62 days.
6. What is the interest on $5,940 for 73 days?
7. What is the interest on $39,510 for 23 days?
8. What is the interest on $6,000 for 2 days?

9. What is the interest on $812 for 21 days?
10. What is the interest on $1,545 for 100 days?
11. State the interest on $640 for 75 days.
12. State the interest on $18,510 for 4 days.
13. State the interest on $9,260 for 14 days.
14. State the interest on $8,988 for 9 days.

BANK DISCOUNT

In lending money on notes for short periods, banks require the interest to be paid in advance. That is, they deduct it from the face of the note, and the borrower receives the remainder.

The money which a borrower actually receives is called the *proceeds* of the note. The interest deducted in advance is called *Bank Discount.* The note given to the bank to secure the loan does not promise to pay interest, since this is paid at the time when the note is received. Only the face of such a note is to be paid.

$500.00 Ft. Dearborn, Ill., 1914.

Sixty days *after date* I *promise to pay to the* Farmers' National Bank, Ft. Dearborn, Ill., Five Hundred and 00/100 Dollars, *for value received.*

John Doe.

Sometimes a note bearing interest is given to a bank to secure a short-period loan. In that case the interest on the note to its maturity is computed and added to the principal, to find the whole amount that is to be paid when due. Upon this *amount* the bank discount is calculated, and from it the bank discount is deducted, the borrower receiving the proceeds.

1. What is the bank discount on the note on page 75?

2. What are the proceeds of it?

3. Draw a note for a fictitious sum at 30 days.

4. What would be the bank discount?—the proceeds?

5. Draw a note for another fictitious sum at 60 days.

6. What would be the bank discount?—the proceeds?

7. Find the bank discount of $450 for 60 days at 6%. Find the proceeds.

8. Find the bank discount of $180 for 30 days at 6%. Find the proceeds.

9. Find the bank discount of $5,000 for 30 days at 5%. Find the proceeds.

10. Find the bank discount of $10,000 for 60 days at 4%. Find the proceeds.

11. What is the bank discount of $4,000 for 60 days at 6%? What are the proceeds? What would be the interest on the proceeds for the period of the loan? Find the difference between this true interest and the bank discount.

12. What is the bank discount of $6,000 for 90 days, at 6%? Find the true interest on the proceeds for the period of the loan. Find the difference between this true interest and the bank discount.

13. The nominal amount of $10,000 is loaned by a bank separately for 12 periods of 30 days each, and for one period of 6 days, all within a single calendar year (leap year), at 6%. What does the

bank receive in interest, in all? How does this differ from the true interest on the proceeds?

14. What is the bank discount of $1,800 for 60 days at 6%? What are the proceeds? What would be the interest on the proceeds for the period of the loan?

15. If you give a bank your note for $2,360 for 90 days at 6%, what will be the proceeds?

16. A note for $3,600 for 30 days is discounted at a bank. What are the proceeds? They are what per cent of the face of the note?

17. What is the bank discount of $546 for 60 days, at 6%, and what are the proceeds? They are what per cent of the face of the note?

18. If I wish to borrow $250 from a bank for 60 days at 6%, for what amount must the note be drawn?

19. In order that the proceeds of a note discounted at a bank for 60 days at 6% may be $160, what must be the face of the note?

20. In order that the proceeds of a note discounted at a bank for 30 days at 6% may be $850, what must be the face of the note?

21. In order to secure the sum of $248 as proceeds of a note at bank discount, the interest being 6% and the period 60 days, for what sum must the note be given?

NEGOTIABLE PAPER

A very large part of the business of to-day is conducted upon borrowed capital. Men of large means often possess neither the training nor the inclination to engage in the business of trade, manufacturing, construction, etc., and prefer to lend their capital to those who, lacking in money, possess the qualifications for success in business.

While the borrowing of money is still spoken of as "an accommodation", it rests upon business principles. The borrower gives his note, which is a promise to pay a specified sum at a specified time, with interest at a specified rate. It must be stated in the note that it is given "for value received". In borrowing money for the purchase of houses and lands, a note is often given for a long period. In some forms of business, loans are desired for very short periods at certain seasons, as for "moving the crops," paying taxes, completing purchases, etc.

While the term "capitalist" is applied to persons possessing large amounts of money which they lend at interest, there are many men of moderate wealth who possess *some* surplus money which they lend and who to that extent are also capitalists. Notes are given for any amounts of money, and at varying rates of interest. Any interest above the amount permitted by law is called *usury;* and for the taking of usury legal penalties are provided in many States. The laws relating to usury are not uniform among the States. What is permitted to be charged in some States is prohibited in others.

Six per cent is the highest interest charge allowed in most of the States. In some States it is more than this, and in some States less; and the law in any State is subject to change.

The payment of a note may be secured by a mortgage on property, or by the indorsement of persons who vouch for the payment by becoming "securities" for it. The securities must indorse the note at the time it is given.

Notes are usually made out upon printed blanks, and are of various forms, some providing penalties for a failure to make payment when it is due, and waiving certain legal rights of the borrower.

Here is a note of *simple form:*

$1,500.00 Detroit, Mich., June 15, 1914.

Sixty Days *after date* I *promise to pay to*

John Doe *or order* One Thousand Five Hundred Dollars,

with interest at 6%, *for value received.*

Richard Roe.

A note is *negotiable paper* when it is so drawn that it may be legally transferred by the holder of it to another party. If it is made payable to a specified person "or bearer", or simply to "the bearer", it is negotiable without any indorsement by the person transferring it. If it is not so drawn as to be payable to "bearer" or "to the order of" the payee named, it is not negotiable, but is payable only to the payee named.

When a note is transferred to another party, the indorsement made upon it by the person so disposing of it may be either an *indorsement in full* or an indorsement *without recourse.* Thus if, in selling to John Doe a note that has been drawn payable to me, I write on the back of it "Pay to the order of John Doe," and sign my name, I indorse the note *in full,* and become liable for the payment of it in case the maker of the note fails to pay it. If I simply write on the back of the note the words "without recourse," and sign my name, I cease to be liable for the payment of the note in case the maker of it does not pay it.

A negotiable note may be indorsed *"in blank"* so as to be negotiable or transferable thereafter to any number of persons in succession without any further indorsement. The holder of a note indorses it "in blank" by simply writing his name upon the back of it.

A note that is negotiable will become non-negotiable by a "restrictive indorsement".* Thus if a note is drawn payable to me "or bearer", and I sell it to John Doe, I can make a "restrictive indorsement" by writing on the back of it "Pay to John Doe", and by signing my name.

Even if a note does not specify that it is to draw interest, the holder is entitled to draw interest upon it from the date of maturity (in case it is not paid at that time) until it is paid.

PARTIAL PAYMENTS

When a note is given for a long period, the interest is usually to be paid periodically, and not to be deferred until the principal becomes due. A part of the principal may likewise be paid from time to time. Such a payment is called a partial payment. The amount paid should be stated on the back of the note, together with the date on which it is received.

The subject of Partial Payments in Arithmetics, is one in which there has been much confusion, owing to the diverse laws of different States. *Simple Interest* contemplates only the payment of interest on the principal. Interest paid on interest is called *compound interest.* In Partial Payments compound interest is to be avoided, even for a short period.

If a partial payment that is made is more than the interest due at the time, the surplus is deducted from

* This *"restrictive indorsement"* must not be confounded with *"qualified indorsement"*; the latter is indorsement *"without recourse."*

the principal, and the reduced principal is used for the next calculation of interest due. But if the partial payment made is less than the interest due at the time, and the remainder of the interest then due is added to the principal, there will be compound interest on such remainder. This might be avoided by keeping two separate accounts, one of the interest and one of the

$400.00 Denver, Colo., July 1, 1914.

Three years *after date* I *promise to pay to*

the order of John Doe Four Hundred Dollars,

with interest at 6%, *for value received.*

Richard Roe.

Received on this note:
Jan. 1, 1911, $100. John Doe.
July 1, 1911, $50. John Doe.
Jan. 1, 1912, $75. John Doe.
July 1, 1912, $50. John Doe.
Jan. 1, 1913, $100. John Doe.

principal. Sometimes, by mutual agreement between the maker and the holder of the note, where there are numerous partial payments, each of the two parties has allowed interest to the other on moneys received from him from the time of the drawing of the note to the time of final settlement, as if each were borrowing from the other. To some persons this seems perfectly just. To others it seems unreasonable to allow interest on money that was paid to reduce indebtedness.

The tendency is towards unity of practice in partial payments, since nearly all of the States have adopted the rule sustained by the Supreme Court of the Nation in causes that have come before it. This is known as the United States Rule, and is in substance as follows:

> *When a partial payment or the sum of two or more partial payments is equal to, or more than, the interest due, it is to be subtracted from the amount (principal +interest due) at the time; and the remainder is to be considered a new principal, from that time to the next payment.*

A note is said to *mature* when it becomes due, and that time is called the *date of maturity.* In former times it was the general custom to extend the time of payment for three days beyond the stated period, to provide against accidental delays in making payment. The added days were called "days of grace." With the improvement of means of communication and transportation wrought by the telegraph and the telephone, the railway and the automobile, the need for such indulgence has generally disappeared; and in most of the States the "days of grace" have been abolished by law.

If the note does not state the place at which it is to be paid, the demand for payment must be presented to the maker at his place of business or at his residence.

Some forms of notes do not name any date of maturity, but are payable at any time "on demand". Of this class are bank notes. If a note is not paid at maturity when presented for payment, it is said to be "dishonored"; and any persons who may have indorsed it as security for its payment become liable for the payment of it, provided they are *immediately* notified of the fact by means of a "*protest*". This is a sworn statement made before a notary public (or other officer empowered to administer an oath) that the note has been presented for payment, but that payment has not been made. It should be presented to the indorsers within the day following the date of maturity.

A note that does not show that it is given for a consideration (that is, "for value received") is not binding in law.

1. Find the amount of this note (p. 81) due on settlement at maturity.

2. Find the amount due at maturity Jan. 15, 1913, on a note drawn at Indianapolis, Ind., Jan. 15, 1911, for $1,000 with interest at 6%, having the following credits indorsed upon it:

July 15, 1911, $ 80. July 15, 1913, $29.
Jan. 15, 1912, $107.

3. What was due at maturity, Aug. 10, 1913, on a note drawn at San Francisco, Cal., Feb. 10, 1911, for $650 with interest at 6%, having the following partial payments indorsed upon it.

Aug. 10, 1911, $109.50. Aug. 10, 1912, $219.50.
Feb. 10, 1911, $116.50. Feb. 10, 1914, $10.00.

4. What amount was due at maturity Apr. 30, 1910, on a note drawn at St. Louis, Mo., July 15, 1908, for $800, with interest at 5%, having these credits indorsed upon it?

Oct. 15, 1908, $210. Jan. 15, 1910, $5.
Apr. 15, 1909, $215.

5. What is the amount that was due at maturity, Sept. 10, 1910, on a note drawn in St. Paul, Minn., July 20, 1907, for $1,200 at 6%, the note having indorsed upon it the following partial payments:

June 30, 1908	$50	Jan. 30, 1909	$15
Sept. 15, 1908	25	Sept. 30, 1909	25

6. Find what was due March 15, 1909, on a note drawn at Chattanooga, Tenn., Oct. 31, 1906, for $4,500 at 6% the note having indorsed upon it the following partial payments

Mar. 15, 1907	$ 40	May 15, 1907	$ 500
Nov. 15 1907	100	Jan. 1, 1908	1,400

7. Find the amount that was due Oct. 12, 1913, on a note for $1,000, drawn at Santa Fe, N. M., March 12, 1911, at 6%, the partial payments indorsed upon it being these:

Mar. 12, 1912	$125	Jan. 12, 1913,	$250
Oct. 27, 1912	125		

DRAFTS

To collect a debt from a debtor in another town or city, the creditor may "draw" on him for the amount due. This is done by sending to a bank in the debtor's home town or city an order to pay the amount. This order is called a *draft.*

If the debtor is ordered to pay the draft "at sight", the paper is called a *sight draft.* If the order calls for payment at a stated later time, it is called a *time draft.*

Generally the draft is sent to a bank with which the debtor does business. The draft may be made payable to the creditor himself, and sent to the bank for collection, or it may be made payable to the bank itself. The order is addressed directly to the debtor drawn upon, the address being written generally in the lower left corner of the paper.

$150.00 Milwaukee, Wis., July 1, 1914.

At sight pay to the order of Self

One Hundred and Fifty Dollars,

for value received, and charge the same to the account of

To RICHARD ROE, JOHN DOE.

Chicago, Ill.

Instead of making a draft payable to himself or to the bank which collects it, the drawer of it may make it payable to a third party. If it is a time draft, the bank receiving it immediately ascertains from the party addressed if it is satisfactory and will be paid at the time stated; and in that case the party addressed writes across the *face* of it the word "accepted", and adds his signature below.

$240.00 Harrisburg, Pa., Sept. 4, 1914.

Thirty days *from sight, pay to the order of* Fulano & Zutano*

Two Hundred and Forty Dollars ($240 and 00/100), *for value received. and charge to the account of*

To RICHARD ROE, JOHN DOE.

Philadelphia, Pa.

1. Is this a sight draft or a time draft?
2. By whom is it drawn? On whom is it drawn?

* These names, like "John Doe" and "Richard Roe" with us, are used hypothetically in the nineteen nations of Spanish-speaking peoples.

3. Rewrite this draft in such a way as to make it a sight draft.

$240.00 — Harrisburg, Pa., Sept. 4, 1914.

Thirty days *from sight, pay to the order of* Fulano & Zutano

Two Hundred and Forty Dollars ($240 and 00/100), *for value received, and charge to the account of*

To RICHARD ROE, — JOHN DOE.

Philadelphia, Pa.

4. Write an acceptance of it by Richard Roe.

5. Why is not this acceptance not said to be "indorsed"? What does "indorsed" mean?

A check drawn by one bank upon another is called a "bank draft". It is used largely to avoid the needless transmission of actual money from one city to another. (P71.)

The cashier signs for the bank making the draft, and the name of his institution appears at the top of the paper, as on a letter head, while the name of the bank drawn upon appears below.

SMELTERS' NATIONAL BANK,

$1,000.00 — Vanadium, Colo., Jan. 5, 1914.

Pay to the order of John Doe

One Thousand Dollars

in current funds.

To FARMERS' NATIONAL BANK, — RICHARD ROE,

Ft. Dearborn, Ill. — Cashier.

1. On what bank is this draft made?
2. What bank makes the draft?
3. Who signs this draft, and in what capacity?
4. Where is the name of the party addressed?

COMPOUND INTEREST

TABLE SHOWING AMOUNT OF $1.00 AT COMPOUND INTEREST, EXTENDED TO FIVE DECIMALS FOR EACH OF TWENTY PERIODS FROM 1 TO 7 PER CENT.

YEAR	1 PER CENT	2 PER CENT	3 PER CENT	4 PER CENT	5 PER CENT	6 PER CENT	7 PER CENT
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000	1.07000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360	1.14490
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102	1.22504
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248	1.31080
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823	1.40255
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852	1.50073
7	1.07213	1.14869	1.22987	1.31593	1.40710	1.50363	1.60578
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385	1.71819
9	1.09368	1.19509	1.30477	1.42331	1.55133	1.68948	1.83846
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085	1.96715
11	1.11567	1.24337	1.38423	1.53945	1.71034	1.89830	2.10485
12	1.12682	1.26824	1.42576	1.60103	1.79586	2.01220	2.25219
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293	2.40985
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090	2.57853
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656	2.75903
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035	2.95216
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277	3.15882
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434	3.37993
19	1.20811	1.45681	1.75351	2.10685	2.52695	3.02560	3.61653
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714	3.86968

Compound interest is interest compounded, when due, with the principal. If, when due, the interest is not paid, but is added to, or compounded with, the principal, to form a new principal for the next interest period, and this is maintained through the full period of the loan, the compound interest will be found by subtracting the original principal from the amount at the end of such period.

Interest is compounded by savings banks.

The same result can be secured by means of simple interest if this is collected promptly when due, and reinvested or loaned without loss of time. On savings accounts at savings banks, interest is usually compounded twice a year, the interest being payable every six months.

The calculation of compound interest for any considerable length of time involves so many steps that it is generally avoided by the use of Compound Interest Tables. The table given on the preceding page shows the compound amount of one dollar for twenty annual interest periods, or for twenty years, at 1%, 2%, 3%, 4%, 5%, 6% and 7%. Since the amount for one dollar for a short period is so small, the interest is calculated with great exactness. The amount of any principal for the full period desired is found by multiplying the amount for one dollar by the whole number of dollars in the given principal, which may be large; hence the necessity for a very close approach to exactness in the amount of one dollar, causing the decimal to be carried out to a number of places very unusual elsewhere.

The table is made out for annual periods. Where the payments are semiannual, half the rate per cent for the annual period may be taken, and the result will be equal to the full rate per cent for the semiannual period. Likewise one-fourth of the rate per cent for the annual period may be taken as the equivalent of the full rate per cent for a "quarterly period" (of three months).

By finding the compound amount of a certain principal at the end of ten years, and using this for the principal at compound interest for ten years more, the compound amount of the original principal for twenty years may be found.

The compound interest on any principal for a stated period may be found by subtracting the original principal from the compound *amount.*

1. Find by the table the compound amount of $1,000 at 6% for ten years, payable annually.

2. Find the compound interest of the same.

3. How much greater is this than the simple interest would be?

4. Find by the table the compound amount of $1,000 at 6% for ten years, payable semiannually?

5. How much greater is this than the compound amount of the same principal at the same rate per cent, the interest being paid semiannually.

6. There is placed to the credit of a boy, on the day of his birth, the amount of $1,000, to draw compound interest at 4%, payable semiannually until he shall attain his majority. What amount is due him on his twenty-first birthday?

7. Find the compound amount of $2,500 for two years and three months at 6% payable annually.

8. What is the compound interest on $500 for three and one-half years at 6% payable semiannually?

9. What would be the compound amount of $1,000 for four years at 8% payable quarterly?

10. Find the compound amount of $1,200 for 6 years at 4%, payable quarterly.

11. What is the nearest full year at which the original principal will double itself at compound interest at 6%, payable annually?

12. What is the nearest full year at which the original principal will treble itself at compound interest at 6% payable annually?

The prosperity of the small farmers, and the great number of them, in certain European lands, has been due largely to the Credit Foncier (*Crā-dē' fôns-yā'*), a government system of small loans on land, to run a long time at a very low rate of interest.

1. If by means of a Credit Foncier 6,000 farmers secure each a loan of $800 for 10 years at 4%, what will be the total interest paid for one year?—for the ten years?

2. If it is a fact, as alleged by statisticians, that the farmers of the United States in 1913 owed $6,000,000,000, on which they paid interest (including commissions and renewal charges) to the amount of $510,000,000, what was the rate of interest paid on the total indebtedness?

3. If by a system of rural credits similar to the Credit Foncier the loans to the farmers of the United States had been at 4%, what would have been the aggregate amount of interest received from them, there being no commissions or renewal charges? What would have been the saving to the farmers? What would have been the saving at 6%?

4. If a man is enabled by a Credit Foncier loan of $800 to purchase 20 acres of ground with necessary buildings, tools, and appliances for fruit and vegetable gardening, the interest being at 4% for 8 years; and if in that time he has saved over interest and expenses an average annual amount of $200, and the land has trebled in value, what is his gain for the period covered?

5. If in order to secure a loan of $500 for 5 years in successive periods not longer than two years each, one must pay 6% per annum and $25 commission to an agent at the outset, and $10 to the agent for each renewal, what per cent does the use of the $500 cost the borrower for the first period?—for the second period?—for the third period?

MEASUREMENTS

Review of Principles

All bodies possess the three dimensions: length, breadth, and thickness.

In measuring length, the other two dimensions are not considered. Whatever instrument of measurement be used, even if it be the finest thread, it must have *some* breadth and thickness; but since these are left out of account, a mathematical line is considered as having only the one dimension, length.

A surface has two dimensions: length and breadth. The number of units of the area of a surface is equal to the number of units of its length multiplied by the number of the units of its breadth.

The number of units of an area divided by the number of units of its length will be the number of units of its breadth.*

The number of units of an area divided by the number of units of its breadth will be the number of units of its length.

A solid, or volume, has three dimensions: length, breadth, and thickness. The number of units of its contents is equal to the product of three factors, the numbers expressing the units of its length, breadth, and thickness.†

* It is customary to say, "The area equals the length multiplied by the breadth," although a multiplier must always be in reality an abstract number.

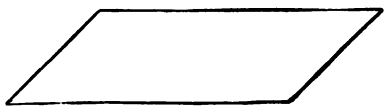
† What is here called a "solid" is a measured portion of space, which may be in reality either filled or hollow and vacant.

The number of units contained in a solid, or volume, is equal to the number of units of any one of its dimensions multiplied by the product of the units of its other two dimensions.*

The number of units contained in a solid, or volume, divided by the number of units of any one of its dimensions is equal to the product of the units of its other two dimensions.†

The number of units contained in a solid, or volume, divided by the product of the units of any two of its dimensions, will equal the number of units of its third dimension.‡

Any figure of four sides is a *quadrilateral.*



A Parallelogram

A quadrilateral whose opposite sides are parallel is called a *parallelogram.*

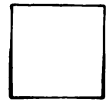
The area of a parallelogram is equal to the length of its base multiplied by its altitude, or breadth, which is not the same as its slant height.

* It is customary to say that the cubic contents are ascertained by multiplying the length by the product of the breadth and thickness; or by multiplying the breadth by the product of the length and thickness; or by multiplying the thickness by the product of the length and breadth.

† As ordinarily expressed, the product of the length and breadth may be found by dividing the cubic contents by the thickness; the product of the length and thickness may be found by dividing the cubic contents by the breadth; the product of the breadth and thickness may be found by dividing the cubic contents by the length.

‡ Or, as commonly expressed, the length is equal to the cubic contents divided by the product of the breadth and thickness; the breadth is equal to the cubic contents divided by the product of the length and thickness; the thickness is equal to the cubic contents divided by the product of the length and breadth.

A *rectangle* is a figure having four straight lines and four right angles. The opposite sides are equal.

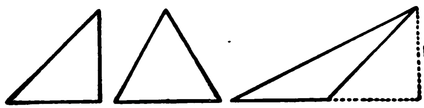


A Square

A *square* is a rectangle of which all the sides are equal.

The area of a rectangle is equal to the number of units of its length multiplied by the number of units of its breadth.

A *triangle* is a figure having three sides and three angles.

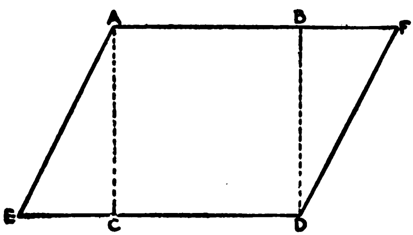


Triangles

A right-angled triangle is a triangle of which one of the angles is a right angle (of lines perpendicular to each other).

The area of a triangle is equal to one-half the product of its base by its altitude, or height.

A *rhomboid* is a parallelogram, of which the adjacent sides are not equal, and the angles are not right-angles.



A Rhomboid

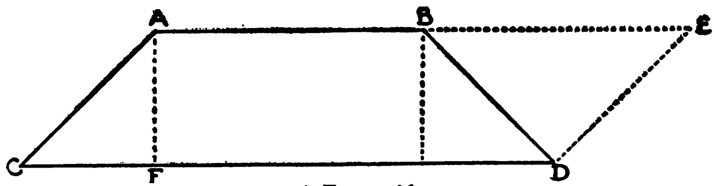
It is seen that this rhomboid may be divided into one rectangle and two equal triangles. The area of the rhomboid is equal to the length of its base line multiplied by the breadth of the figure, measured perpendicularly.

1. If the line *CD* (or *AB*) equals 12 inches, the line *EC* equals 4 inches, and the distance *AC* equals 10 inches, what is the area of the figure?—of the rectangle *ABCD?*—of each of the triangles?

2. If the line *CD* (or *AB*) equals 1 ft., *AC* equals 9 in., and *EC* equals 4 inches, what is the area of the figure?—of the rectangle *ABCD?*—of each of the triangles?

3. If the line *CD* (or the line *AB*) equals 14 in., *AC* equals 7 in., and *EC* equals 3 inches, what is the area of the figure?—of the rectangle *ABCD?*—of each of the triangles?

A *trapezoid* is a figure bounded by straight lines, of which only two are parallel.



A Trapezoid

The trapezoid may be made a rhomboid by extending the upper line to the length of the base line, and drawing the dotted line *ED* parallel with *AC*. The area of the rhomboid may then be found, and the area of the triangle added may be subtracted from it.

1. If, in the trapezoid above, the line *AB* is 7 in., and the line *CD* is 12 in., and the perpendicular distance between the lines is 4 in., what will be the area of the added triangle?—of the rhomboid?

2. If, in the trapezoid above, the line *AB* is four in., and the line *CD* is 9 in., and the perpendicular distance between the lines 3 in., what is the area of the rhomboid, which includes the added triangle? What is the area of the triangle?—of the trapezoid?

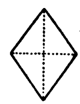
Another way of finding the area of the trapezoid is to find the *average* length of the base and upper lines and to multiply this by their perpendicular distance apart.*

Thus, if the line *ab* is 6 in., and the line *cd* is 10 in., the average length of the two lines is 8 in. This is found by adding the lines together and dividing their sum by 2. And if the perpendicular distance between the lines is 4 inches, the area of the figure will be 8×4 square inches, or 32 square inches.

1. Find the area of a trapezoid of which the upper and base lines have an average length of 15 in., and are separated by a perpendicular distance of 5 in.

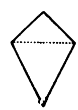
2. Find the area of a trapezoid of which the upper line is 10 ft., the base line 18 ft., and the perpendicular distance between them is $1\frac{1}{2}$ ft.

A *rhombus* is a parallelogram of which the sides are equal and the angles are not right angles. It is frequently called a diamond. Since it is equivalent to two similar and equal triangles, arranged base to base, its area is equal to half the product of their length by their breadth.



Rhombus or Diamo .

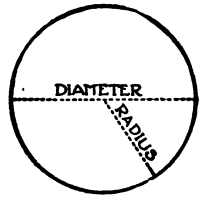
A *trapezium*† is a quadrilateral of which no two sides are parallel. It may be considered as made up of two triangles, and its area is equal to half the product of their length by their breadth.



Trapezium

* This form of expression is not strictly accurate, but is used here (and will be used hereafter) as a matter of convenience and of custom. Length cannot be multiplied by breadth. See note, page 142, Intermediate Arithmetic.

† A trapezium may be very irregular in form, having no two sides equal.



A circle is a figure bounded by a curved line, called the *circumference*, all parts of which are equally distant from the center of the figure.

A line from the center to the circumference is called a *radius*. The plural of this word is *radii*.

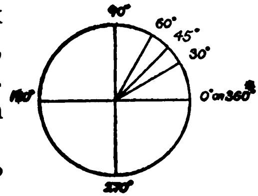
A line drawn through a circle, passing through its center, is called a diameter. The diameter is equal to two radii.

Every circle may be divided into three hundred sixty parts, called degrees.

The sign for *degree* is a tiny circle, above the line (°). The length of a degree depends upon the size of the circle.

1. How many degrees are there in one-half of a circle?
2. How many degrees are there in one-fourth of a circle?
3. How many degrees are there in one-eighth of a circle?
4. If the diameter of a circle is eight inches, what is the radius?
5. How many degrees are there in one-half of a circle?
6. How many degrees are there in one-fourth of a circle?
7. How many degrees are there in one-eighth of a circle?
8. The outer circle of the face of a clock is divided into sixty parts, for the minute hand to pass over. How many degrees are there in each of these parts?

9. The inner circle of the clock face is divided into twelve spaces, for the hour hand to pass over. How many degrees are there in each space?



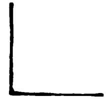
10. 90° is what part of a circle?

11. 180° is what part of a circle?

12. 270° is what part of a circle?

Angles

When two lines meet, they form an *angle*.

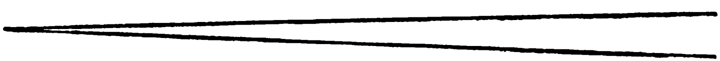


An angle is the difference in direction of two lines.

An angle of 90° is called a right-angle.

The size of an angle does not depend on the length of the lines.

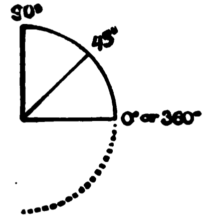
Here is a very small angle with long lines.



Here is a very large angle with short lines: 

It is easy to measure an angle. Take the point where the lines meet for the center of a circle, and draw a circumference with a radius of any convenient length; then note how many degrees of the circumference lie between the points where the lines of the angle cross it. A circle can be drawn most conveniently with compasses, or dividers, the arms of which can be opened to form a circle of the radius desired.

1. A perpendicular line meets a horizontal line in an angle of how many degrees?



2. When it is three o'clock, how many degrees apart are the hands of the clock?

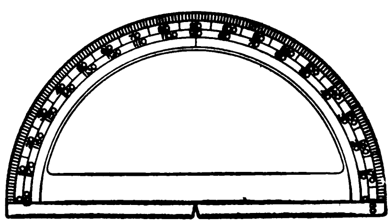
3. A line half way between horizontal and perpendicular is raised how many degrees?

4. If in writing you slant the letters at 82°, how many degrees do they lack of being perpendicular?

5. Where a perpendicular line meets a horizontal line, as where the wall of a room meets the floor, the angle, called a right angle, represents what part of a circle?

6. An angle greater than a right angle is called an *obtuse* angle. *Obtuse* means *blunt*, or *dull*. Is an angle of 85° an obtuse angle? Is an angle of 96°? Is an angle of 75°? Is an angle of 150°?

7. An angle that is less than a right angle is called an *acute* angle. *Acute* means *sharp*. Is an angle of 88° an acute angle? Is one of 100°? Is one of 2°? Is one of 94°? Is one of 45°?



The instrument called the protractor enables us to find the measure of any angle in degrees, and to construct an angle of any desired measure.

For use in practical astronomy and in surveying, engineering, etc., protractors are sometimes made on a large scale, so as to indicate not only the 360 degrees in every circle, but also sixty minutes in each degree, and sixty seconds in every minute. The minutes and seconds here are not measures of time, but of arc; they are fractions of the circumference.

1. A quarter of a circle, or ninety degrees, is called a *quadrant*. How many quadrants are there in a circle? —a half circle? How many minutes are there in a quadrant? How many seconds?

2. The upper quadrant to the right is called the *first* quadrant; the upper one to the left is called the *second* quadrant; the lower quadrant to the left is called the *third* quadrant; the lower one to the right is called the *fourth* quadrant. Draw a circle divided into quadrants, and number these.

3. A sixth of a circle is called a sextant. This name is given also to an instrument for taking observations at sea. How many degrees are there in a sextant? How many minutes? How many seconds? Draw a sextant indicating every five degrees.

4. The Equator of the heavens, which is directly above the Equator of the earth, is divided into twelve parts, called Signs. How many degrees are there in a Sign? How many minutes? How many seconds? Draw a Sign, indicating every five degrees of it.

5. How many Signs are there in a quadrant of the Equator of the heavens?—in a sextant of the Equator of the heavens?

6. The axis of the earth lacks $23\frac{1}{2}°$ of being perpendicular to the earth's orbit, or path through the heavens. What angle does it make with the orbit?

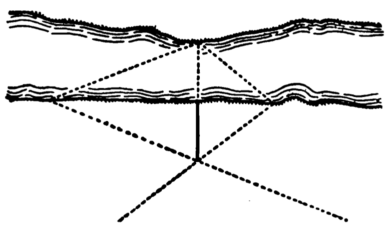
7. All the interior angles of a square, taken together, are equal to how many degrees? This number is how much more than the sum of two sextants?—than the sum of a sextant and a quadrant?

8. Since an angle is the divergence of two straight lines which meet at a point, what will be the result if the lines diverge until they form, together, a single straight line? While there appears to be no angle at all, the divergence of the lines is how many degrees?

9. Two lines, therefore, meeting at an angle of 180°, constitute what?

10. How many degrees are there in an angle that measures one-sixth of a circle? How many times this number of degrees are there in two right angles?—in six right angles?

11. A boy desiring to measure the width of an impassable river flowing through level land drew on the edge of the bank, parallel with the stream, a base line, at each end of which he drove a stake to which he attached the end of a ball of twine. At one end of the base line, with an instrument, he sighted a small spot in a rock on the opposite bank, and noted the angle made by the line of sight with the base line. He then turned his instrument to an equal angle on the other side of the base line, and had an assistant carry the twine a long way along the line of sight. At the other end of the base line he sighted the same spot in the rock, then turned his instrument to an equal angle on the other side of the base line, and had his assistant carry the second ball of twine along the line of sight. From the point where the strings crossed, he measured straight to the base line, and announced the measure as being that of the width of the stream. Was this correct?



Illustrate his plan by a diagram.

A triangle, or trigon,* is a figure having three sides and three angles.

> *All the angles of a triangle added together are always equal to one hundred eighty degrees, or two right angles.*

1. The triangles may be of any variety of form. If we know the measure of one of the angles of a triangle, we can find the sum of the other two by subtracting the known one from 180°. If we know the sum of two of the angles of a triangle, how can we find size of the third?

2. If in a right angled triangle one of the angles measures 30°, what will be the size of the other acute angle?

3. If a triangle has its sides all equal, its angles will all be equal. What will be the measure of each angle?

4. If one angle of a triangle be 130°, and the other two angles are equal to each other, what is the size of each of these?

5. If the sum of two angles of a triangle is 160°, what is the size of the third angle?

6. If one angle of a triangle measures 40°, what is the sum of the other two?

7. If the sum of two angles of a triangle equals 160°, what is the size of the third angle?

8. Can a triangle contain more than one obtuse angle? Can it contain more than one right angle?

*The branch of higher mathematics which deals with this figure is called *trigonometry*.

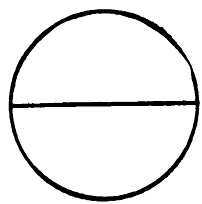
9. If you double the length of each line of a triangle, will the angles remain the same? If you reduce by one-half the lines of a triangle, will the angles remain the same?

10. Draw a triangle with two sides equal, the third side being much longer or shorter than these. How many of the angles of the triangle will be equal to each other?

11. The angles of four triangles, added together, would equal the angles of how many squares, added together?

We learn from geometry that the circumference of a circle is 3,14159+ times the diameter. The decimal may be carried very far. It has been carried to several hundred figures, without coming to an end. For most scientific purposes the number used is 3.1416. For ordinary practical purposes the number $3\frac{1}{7}$ is used.

To find the circumference of a circle, multiply the diameter by 3.1416 *or (for many purposes) by* $3\frac{1}{7}$.



1. If the diameter of a circle is one inch, what is the circumference?

2. If the diameter is seven inches, what is the circumference?

3. If the circumference is forty-four inches, what is the diameter?

4. Which number is the greater, $3\frac{1}{7}$ or 3.1416. What is the difference between them?

5. If the Equator were a perfect circle of 25,000 miles, what would be its diameter in whole miles?

6. The diameter of a circular park is 300 feet. What is its circumference, calculated to two decimal places? What is the length of the radius?

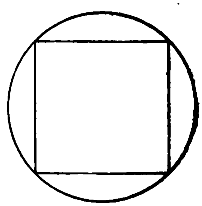
7. If this park contains six radiating paths from the center to the circumference, what is the total length of these paths?

8. A circular park has a diameter of 360 feet. What is its circumference? What is the length, in feet, of one degree of this circle?

9. The circumference of the earth at the Equator is 24,859.76 miles. What is the length of a degree in miles, on the Equator?

10. A degree of a circle of the earth passing east and west through Chicago measures 51.483 miles. What is the length of the entire circumference of the circle?

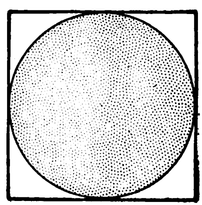
A square drawn within a circle, and having its diagonals equal to the diameter of the circle, is said to be *inscribed*.



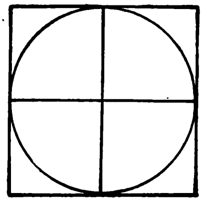
A square drawn about a circle, and having its side equal to the diameter of the circle, is said to be *escribed*.

The escribed square has double the area of the inscribed square.

A circle inscribed within a square occupies more than three-fourths of the area of the square. A close approximation to its area is .7854 of the escribed square. This decimal fraction should not be forgotten.



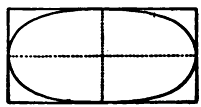
The area of a circle may be found by multiplying the area of the escribed square by .7854.



The area of the escribed square is of course equal to the square of the diameter.

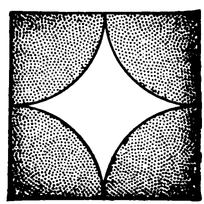
The square of the radius is one-fourth of the square of the diameter. Hence to find the area of the circle from the square of the radius, the decimal must be *four times* as large as .7854.

The area of a circle may be found by multiplying the square of its radius by 3.1416, *which is four times* .7854.



An ellipse may be called a circle that is flattened more or less, having diameters of different lengths.

A simple way of finding the area of an ellipse is to multiply the escribed rectangle by .7854, as in the case of the circle. The escribed rectangle is, of course, the product of the long and short diameters.



1. Here the four quarters of an inscribed circle are differently arranged in the escribed square. What is the area of the central figure bounded by four curved lines?

2. A circular park with a diameter of 300 feet has a surrounding driveway, 100 feet wide. What is the area included within the outer line of the driveway? Draw diagram.

3. If a square be inscribed within this driveway, what will be its area? Draw the square.

4. Draw an inscribed circle. Its area will equal half the area of the circle escribed.

5. What is the area of each of the four corner spaces lying between the inscribed circle and the square?

6. A cow is tethered to a stake in a grass field by a rope 100 feet long, attached to one of its fore feet. What is the area included within the sweep of the rope around the stake?

7. If the rope be attached to one of the hind feet of the cow, giving the animal a reach of five feet more from the stake, how much additional area will she have to graze over?

8. A circular lake three miles in diameter is drained until it is only two miles in diameter. What area has been reclaimed by the receding of the water? Draw diagram.

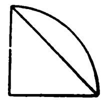
9. By irrigation from a central artesian well, with radiating ditches extending half a mile in each direction, a circular area of arid land has been reclaimed. How many acres does it contain? Draw diagram.

10. If the radiating ditches be extended to twice their length, what will be the gain in irrigated area? Draw diagram.

11. Bottles two inches in diameter are packed in a box one foot square, inside measure. How much of the area of the bottom of the box is covered by the bottles? Would this be the same if there were but one bottle, and if it were one foot in diameter? Draw diagram.

12. A certain revolving search light illuminates the land to a distance of five miles. What area is included in the circle of its illumination?

13. A wire one-twelfth of an inch in diameter is wound around a pipe one inch in diameter (outside measure) for a distance of a yard. What is the length of the winding wire?



14. If the diameter of the circle be one inch, what will be the size of the quadrant?

15. The straight line joining the ends of an arc is called its chord. What is the area of the triangle included between the radii and the chord of the quadrant?

16. What is the area of the space included between the curved line and the chord?

17. By means of a lever ten feet long, to the outer end of which a horse is attached, the animal, walking in a circle, turns a perpendicular windlass. If the lever be made five feet longer, how much larger will be the circle around which the horse walks? The circumference of a circle being 3.1416 times its diameter, how much longer will be the path of the horse around the windlass?

18. If the minute hand of a clock be twelve inches long, over what area of the clock face will it pass in twenty minutes?

19. If the hour hand be nine inches long, over what area of the clock face will it pass in the same time?

20. A square inscribed in a circle contains one hundred square feet. What is the area of one-fourth of the escribed square.

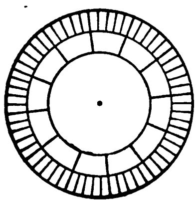
21. A square having a side of ten feet has an inscribed circle. In this is an inscribed square, and within the latter is an inscribed circle, its area being one-half that of the escribed circle. Find the areas of the two circles.

22. Draw a circle of any area. Then by means of an inscribed square, draw a circle having half the area of the first.

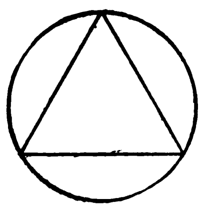
23. Draw a circle of any size, then, by means of an escribed square, draw a circle having double the area of the first.

24. A farmer builds a circular barn having a diameter of 40 feet. Its circular wall will have 3.1416 times the length of the diameter. What will be the length of it? Suppose this length of wall were used to inclose a square. What would be the length of one of the sides. What would then be the area of the barn?

25. What is the area of the circular barn? What is the gain in area from having the barn circular in form?



Here is a triangle of equal sides, inscribed in a circle.

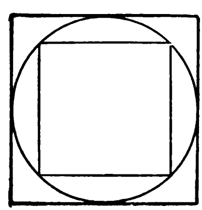


To draw a regular triangle in a circle, we must divide the circle into 360 parts or degrees. The unfinished clock face has 6 in each minute space.

1. Into how many parts does the triangle divide the circumference?

2. How many degrees apart are its angles?

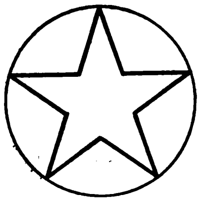
3. With the protractor, construct a triangle of equal sides.



Here is a square inscribed in a circle.

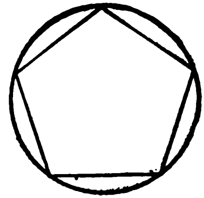
4. Into how many parts does it divide the circumference?

5. How many degrees apart are its angles?



6. With the protractor, construct a square.

Here are a five-pointed star and a pentagon inscribed in a circle.



A regular pentagon is a figure having five equal sides, and a regular hexagon is a figure having six equal sides.

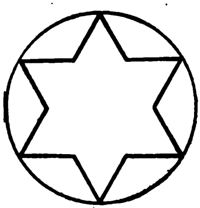
7. Into how many parts does each divide the circumference?

8. How many degrees apart are their angles?

9. With the protractor, construct a pentagon.

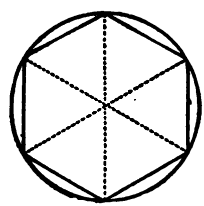
Here is a hexagon inscribed in a circle.

Each side of a regular hexagon is equal to the radius.



10. How many degrees apart are the angles of the hexagon?

11. Into how many parts does it divide the circumference?



12. With the protractor, construct a regular hexagon.

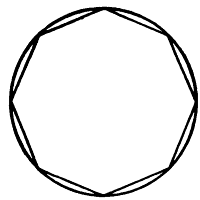
13. Into how many parts does it divide the circumference?

14. If a six-pointed star be drawn in the circle, how many degrees apart must the points be?

15. If a regular hexagon be drawn in a circle, its points will be those of a six-pointed star; and the spaces between

its points will be equal to half the diameter. The distance around a hexameter-shaped garden bed is how many times the distance across it from point to point?

16. Can square blocks of equal size be placed together so as to leave no spaces between them? Can circles? Can triangles of equal size? Can hexagons?

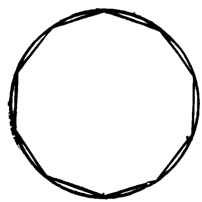


An octagon is a figure having eight equal sides.

Here is an octagon inscribed in a circle.

17. How many degrees apart are its angles?

18. With the protractor, construct an octagon.



Here is a decagon inscribed in a circle.

19. Into how many parts does it divide the circumference?

20. How many degrees apart are its angles?

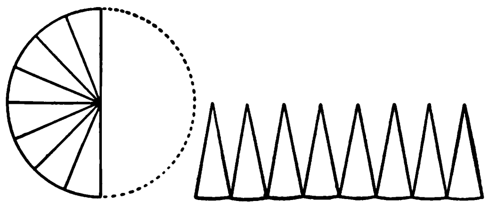
Any inscribed figure having more than four sides is called a polygon.

How many polygons have we studied, and what are they?

It will be seen that an inscribed square takes up more of the circle than an inscribed triangle; an inscribed pentagon takes up more than an inscribed square; an inscribed hexagon takes up more than an inscribed pentagon, etc.

All the inscribed figures are made up of triangles, if their angles are joined by lines to the center of the circle; and the areas of the triangles can be easily measured. But no one can tell with absolute exactness the area of a circle. This is called "squaring the circle." It has never been accomplished fully; but we can come very near to ascertaining the exact area by considering the circle as made up of a great number of triangles.

> *The greater the number of sides the inscribed regular polygon may have, the nearer it will approach to the circumference of the circle, and the greater will be the area that it will contain.*



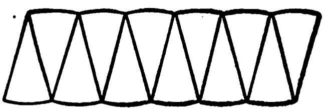
The circle divided into triangles

Where a circle is divided by a great number of radii, it is made up of many triangles, having narrow bases, slightly curved. If the bases of these triangles were not curved at all, it would be easy to find the area of each. We could multiply its base by its altitude, and take one-half of the product. If we form so many triangles that the bases are exceedingly narrow, the curvature of each will be so slight that each base will seem like a straight line. Hence, we imagine the circle to be divided into so many triangles that we cannot see the difference between their tiny bases and straight lines.* The bases of all of them, taken together, equal the circumference. The height of each is the radius. Hence the area of all of them, taken

*These imaginary triangles are imagined to be *infinite* in number—that is, without any limit as to number.

together, will be found by multiplying the circumference by the radius and taking half of the product. This is near enough for all practical purposes.

The triangles may be arranged in this way to form a nearly perfect rectangle. Its length will be equal to half the circumference, and its breadth will be equal to the radius. An old rule for finding the area of a circle was, "Multiply half the circumference by half the diameter."



The same triangles arranged in a row having half the length of the circumference, and having a breadth equal to the radius.

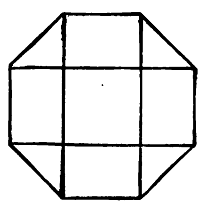
The broken line bounding a polygon is called its *perimeter*.

It is an important fact that the more sides a polygon has, the more space will be inclosed with the same length of perimeter.

> *The circle contains more space than can be otherwise inclosed with its length of circumference.*

1. A farmer built a square barn whose perimeter was eighty feet. What was its area.

2. Another farmer built a barn in the shape of an octagon whose perimeter was eighty feet. What was its area?



Dividing the interior by cross lines through the angles, we can find by measuring that it contains:

1. A central square 10 ft. x 10 ft.
2. Four rectangles, each 10 ft. x 7.07 ft.

3. Four triangles whose base and altitude are each 7.07 ft.

Find the sum of all these areas.

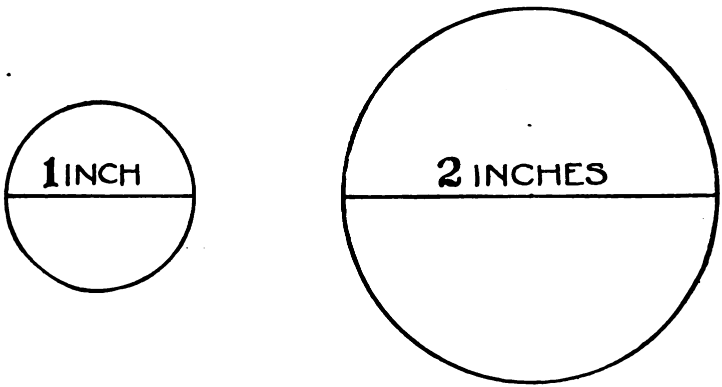
3. A third farmer built a circular barn whose circumference was eighty feet. What was its diameter? What was its area?

4. How much more area did the octagonal barn contain than the square barn?

5. How much more area did the circular barn contain than the square barn?

6. What is the area of a shed 30 ft. x 10 ft.? What is its perimeter?

7. How much space would be gained by building the same wall in a circular form?



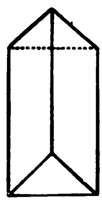
*Circles are in proportion to each other as the squares of their diameters.**

*This is in accordance with a general rule, which is not limited to circles, but may be applied to similar plane figures of whatever form. The areas of plane figures are to each other as the squares of their corresponding lines.

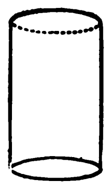
A circle having a diameter of two inches contains *four times* the area of a circle having a diameter of one inch, though its diameter is only twice as large.

A prism is a solid, or volume, having the upper and lower bases equal and parallel to each other, and having the lateral (side) faces equal to each other. The upper and lower bases may be triangular, square, hexagonal, etc, When the lateral faces are very narrow and numerous. the prism approaches the figure of the cylinder.

A cylinder is a regular solid, or volume, having parallel and equal circles for its upper and lower bases, and bounded laterally by a surface uniformly curved.



Triangular prism



Cylinder

The surface of a prism is equal in area to the sum of its bases and its lateral faces. The surface of a cylinder is equal to the area of its bases plus the area of its lateral surface.

1. What is the area of the surface of a prism of four equal sides whose height is 4 feet, and whose bases are 6 inches square?

2. What is the area of the surface of a cylinder whose diameter is 8 inches, and whose height is 6 feet?

3. What is the area of the surface of a prism whose base is a right-angled triangle of 3 inches, 4 inches, and five inches, and whose altitude is 1 foot?

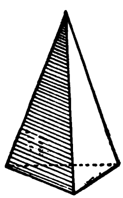
4. What is the area of the *convex* (outward-curved) surface of a cylinder whose diameter is 6 inches and whose height is $1\frac{1}{2}$ feet?

> *The cubic contents of a prism or of a cylinder are equal to the area of one of its bases multiplied by its altitude, or height.*

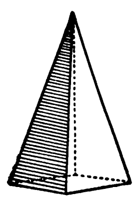
1. What are the cubic contents of a cylinder whose bases have each an area of 28.27 square inches, and whose altitude is 16 inches?

2. What are the cubic contents of a prism whose bases have each an area of $9\frac{1}{2}$ square inches, and whose altitude is 8 inches?

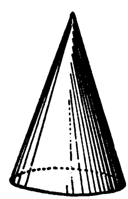
A pyramid is a solid, or volume, having for its base a triangle, rectangle, or other polygon, and having for its lateral faces triangles meeting at the vertex, or highest point.



Triangular Pyramid



Quadrangular Pyramid



Cone

In a *regular* pyramid the triangles are equal.

Where the lateral faces of a pyramid are numerous and narrow. it approaches the figure of a cone.

A cone is a solid or volume having for its base a circle, and bounded by a surface curved and tapering uniformly to an apex.

The area of the lateral surface of a pyramid is equal to the sum of the areas of the triangles of which it is composed. The altitude of these triangles is not the altitude of the pyramid, but its *slant* height. Since the area of each of these triangles is equal to half the product of its base and its altitude, and since the perimeter of the base of the pyramid is equal to the sum of the bases of these triangles, it follows that

> *The area of the lateral surface of a regular pyramid is equal to the perimeter of the base multiplied by one-half the slant height; or it is equal to the sum of the areas of the lateral triangles.*

1. What is the lateral area of a regular quadrangular pyramid whose perimeter is 16 inches, and whose slant height is nine inches?

2. What is the area of a regular pentagonal pyramid, the perimeter of which equals 6 ft. 3 in., and the slant height of which is 10 inches?

3. The length of the perimeter of a certain pyramid is 24 inches; the slant height, 1 foot. What is the lateral area?

4. The slant height of a certain pyramid is 15 ft., the base of each of its six sides is 10 ft. What is the lateral area?

> *The area of the convex surface of a cone is equal to one-half the product of the circumference of its base by its slant height.*

1. The diameter of a certain cone is 8 inches, and its slant height is 1 ft. What is the area of its convex surface?

2. The slant height of a certain cone is 10 inches, and the circumference of its base is 3 ft. What is its convex surface?

3. The diameter of a certain cone is 2 ft. Its slant height is three times its diameter. What is the area of its convex surface?

4. The slant height of a certain cone is 18 inches. The diameter of the base is one-third of the slant height. What is the area of the convex surface?

It can be shown that a cup having the shape of a cube can be filled exactly three times by a cup in the shape of an inverted pyramid having base and altitude the same as those of the cube-shaped cup. Likewise it can be shown that a cup having the shape of a cylinder can be filled exactly three times by a cup in the shape of an inverted cone having the diameter and altitude the same as those of the cylinder-shaped cup. Therefore it follows that

The cubic contents of a pyramid or of a cone are equal to one-third the product of its base and its altitude.

In geometry the ratio of the volume of a pyramid to the volume of a cube having the same base and altitude, and of the volume of a cone to the volume of a cylinder having the same base and altitude, is demonstrated by *reasoning*, without any experiment.*

* A tinner can easily make a cube-shaped cup of tin, and also a cone-shaped cup having the diameter and altitude equal to those of the cube-shaped cup; and the demonstration may be made by the use of water.

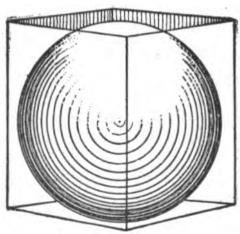
1. What are the cubic contents of a cone having a diameter of 7 inches and a height of 9 inches?

2. Find the cubic contents of a pyramid having a base containing 64 square feet, and an altitude of 6 feet.

3. What would be the contents, in cubic miles, of a cone having a diameter of 2 miles and an altitude of 3 miles?

4. The Pyramid of Khufu, in Egypt, has a square base, measuring 750 feet on each side, and its height was originally 482 feet. Find in cubic yards its contents as originally completed, according to these figures.

A globe, or sphere, is a solid, or volume, bounded by an evenly curved surface, every part of which is equally distant from the center.



Every cube contains within it a sphere having the same diameter that the cube has. By removing the parts of the cube not included in the sphere, we may bring the sphere itself to light. The material to be removed lies chiefly at the corners of the cube, and along the edges of the cube. Only at the central points of the six faces of the cube is there no removal to be made. When a cube is thus converted into a sphere, it can be shown by weighing the removed portion that this is somewhat less than half of the cube. The sphere will weigh somewhat more than half the weight of the cube. Very exact weighing would show the sphere to equal .5236 of the inclosing cube. The removed portion would prove to be .4764 of the cube. Hence,

The cubic contents of a globe, or sphere, may be found by taking .5236 of the inclosing cube.

1. What are the cubic contents of a globe having a diameter of 8 inches?

2. If the diameter of the earth were exactly 8,000 miles, and if the earth were a perfect sphere, what would it contain in cubic miles?

3. A turner makes a wooden globe from a cubic foot of wood, with no waste of diameter. What part of the wood is wasted in making the globe?

4. What is the volume of a sphere 1 foot in diameter?

5. What is the volume of a globe 3 feet in diameter?

6. A globe 3 ft. in diameter is how many times the size of a globe 1 ft. in diameter?

A *great circle* of a globe or sphere is the largest circle that can be drawn around it. The center of a great circle is also the center of the globe itself.

The area of a globe, or sphere, is equal to four times its great circle.

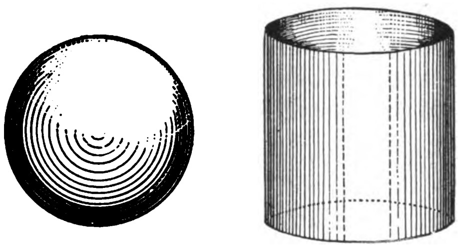
1. The diameter of a globe is 3 ft. What is the area?

2. The diameter of a globe is 10 ft. What is the area?

3. The circumference of a globe is 31.416 ft. What is the area?

4. The circumference of a globe is 78.54. What is the area?

The surface of the globe is equal to the convex surface of a cylinder having its diameter and altitude each equal to the diameter of the globe.



Equal Convex Areas

The convex surface of a cylinder is equal to the product of its circumference and its altitude. It follows, therefore, that

The surface of a globe, or sphere, is equal to the product of its great circle (circumference) and its diameter.

The surface of a sphere is equal to the convex surface of what kind of a cylinder?

2. The convex surface of a cylinder whose diameter equals its altitude is equal to the surface of what kind of a sphere?

3. The surface of a globe is equal to how many times its great circle?

4. The convex surface of a cylinder whose diameter equals its altitude is equal to how many times the circle of its base?

5. The convex surface of a fruit can whose diameter equals its height has how many times the area of one of the circular ends?

6. When a round apple is cut into halves, the curved surface of each half equals how many times the flat surface?

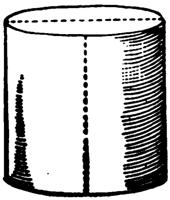
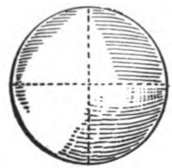
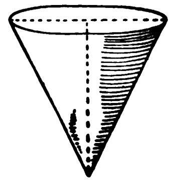
7. If a round apple is cut into quarters, how will the curved surface of each quarter compare in area with the two flat surfaces?

For the purpose of ascertaining its area, the circle (see p. 110) was conceived to be divided into an infinite number of triangles, whose vertices all met at one point, the center of the circle, and whose bases were infinitely narrow.

In like manner, a globe, or sphere, may be conceived to be made up of an infinite number of pyramids, whose vertices all meet at one point, the center of the globe, and whose bases, infinitely small, form the surface of the globe (see p. 119.) In that case, the radius of the globe is the altitude of every one of the pyramids; and since the cubic contents of each pyramid would be equal to one-third of the product of its altitude by its base, the cubic contents of all the pyramids taken together would be equal to one-third of the product of this common altitude by the sum of all their bases (the surface of the sphere).

If follows that

The cubic contents of a globe, or sphere, are equal to one-third of the product of its radius by the square contents of its surface.

Here we see a cylinder, a globe, and a cone (inverted), all of the same height and diameter. The cylinder will contain exactly as much as the contents of both the globe and the cone.

The volume of the cylinder equals the area of its circle multiplied by its height. The volume of the globe is two-thirds of this amount, for it equals the area of its great circle, multiplied by two-thirds of its height, or diameter. The volume of the cone is one-third the volume of the cylinder, and one-half the volume of the globe; for it equals the area of its great circle multiplied by one-third its height.

If the cylinder is in the form of a hollow cup, and if the globe and the cone are likewise hollow and are filled with water, the globe and the cone may be emptied into the cylinder, and their contents will exactly fill it.

1. A certain carboy, a globe in shape, contains 12 gal. of muriatic acid. If its contents be poured into a cylindrical tank having the same inside diameter and height, how many gallons more will the latter hold? What part of the contents of a carboy will be contained in an inverted hollow cone having the same inside diameter and height?

2. A globe of marble rests upon a marble cylinder of the same diameter. The height of the cylinder is four

times its diameter. The weight of the globe is what part of the weight of the entire column?

3. A marble cylinder, the height of which is five times its diameter, is capped by a marble cone having the same diameter. The height of the cone is equal to its diameter. The weight of the cone is what part of the weight of the entire column?

4. One-half of a globe is fitted to the base of a cone of the same material having the same diameter and having a height equal to its diameter. If the globe originally weighed twelve pounds, what will this combination weigh?

APPLICATIONS OF LONG MEASURE

The inch is not the lowest denomination in linear measure. Tape lines and rulers generally indicate the half, the quarter, the eighth, and the sixteenth of one inch. The quarter inch may be divided into three spaces called lines.*

A foot is one-third of a yard, or one-sixth of a fathom, or two thirty-thirds of a rod.

A yard is one-half of a fathom, or two-elevenths of a rod.

A rod is one-three-hundred-twentieth of a mile.

The mile is not the highest denomination in linear measure. The distance of three miles is called a *league.*

The Statute mile used on land in the United States is equal to 5280 feet, or 1760 yards, or 320 rods.

The Geographical, or nautical, mile used at sea in the Coast Survey is considerably longer, containing 6,080.27 feet.

* In button measure one-fortieth of an inch is called a line.

The Surveyors' Chain (ch.) of 100 links (li), measures 66 feet, or 22 yards, or 4 rods, or one-eightieth of a mile. So important is the measuring of land, that all persons should understand chain measure and the relative values of the denominations named.

1. How many lines are there in $2\frac{3}{4}$ inches?

2. A building lot having a frontage of 22 feet on a business street is valued at $1760. How much is that per front foot?†

3. Two headlands at the entrance of a bay are $23\frac{1}{2}$ geographical miles apart. What is the distance in statute miles?

4. Through inexactness in the survey, one side of a certain section of land is only $78\frac{1}{2}$ chains long. What is its length in feet? What does it lack of a mile?

5. A certain railway has, within ten "line miles", or miles of line, 28 "track miles", or miles of track. How much track has it for each "line mile"?

6. A certain rectangular field is 15 chains long and 10 chains wide What length of barbed wire will surround it four times?

7. What is the length of a link in the Surveyors' Chain?—of ten links?—of twenty links?—of thirty links?

8. A lot having a frontage of 50 feet and a depth of 150 feet will require what length of wall to inclose it?

9. If the sounding line of a ship shows a depth of 1,785 fathoms, how deep is the sea at the place of the sounding?

10. On a globe 10 feet in diameter the highest mountain in the world would be proportionately represented

† "Front foot" means foot of frontage on the street.

by an elevation of 1 line. What part of the earth's diameter is the height of that mountain?

APPLICATIONS OF SQUARE MEASURE

The denominations of square measure have a very wide range, from the square inch to the Congressional township, which, being six miles square, contains thirty-six sections, or square miles.

Because of the curvature of the earth, which would naturally cause the north lines of Congressional townships to be shorter than the south lines, and for other reasons, these divisions are not perfectly formed. Likewise the sections are not, in practice, *exact* squares. The acre, from ancient days, was not intended to be square, but of oblong shape, convenient for plowing. A tract of 10 acres, or of 40, or of 160, or of 640 may be an exact square; but to lay off a single acre in a square would involve a decimal of an inch.

A square chain is 16 square rods; 10 square chains make one acre.

These values should be firmly fixed in the memory.

It is in the measurement of small areas that the greatest exactness is possible. The standard measures of the inch, the foot, and the yard are preserved at the capital of the nation for comparison, these denominations having been, in their origin, purely arbitrary.

The square yard is equal to 9 square feet, and is $\frac{4}{121}$ of a square rod. The square rod is equal to $30\frac{1}{4}$ square yards, and is $\frac{1}{160}$ of an acre.

1. What would be the number of acres in a perfectly formed Congressional township?

2. What would be the number of square yards in a perfectly measured section of land?

3. What is the number of square feet in an acre? What is the number of square yards?

4. What is the area of the floor space in your schoolroom?

5. What is the entire area of wall space in your schoolroom, including openings for windows and doors?

6. What must be the dimensions of a rug for a room 18 feet in length and 14 feet in breadth, a yard of uncovered floor space being left on each side of the rug? Make a diagram.

7. What is the area of $7\frac{1}{2}$ linear yards of linoleum which is 6 feet broad?

8. Make a diagram of a floor fifteen feet long and twelve feet broad completely covered with a carpet having a border 9 inches in width. How many yards of border must be procured for such a floor? How many linear yards of carpet 27 inches in breadth? (Solve this problem in the easiest way.)

9. How many square yards are there in an acre? How many square feet?

10. A square field of 40 acres has a roadway 6 feet in breadth entirely around it, inside the fence. How many acres are left for cultivation?

11. In a certain State the public roads have generally a uniform width of 80 feet. If their breadth were reduced one-half, and the land thus given up were cultivated under a system of model farming and gardening, how many acres would be rendered available for such purposes along roadways amounting to 1,000 miles, *exclusive* of the interruptions of cross roads, streams, etc.?

12. In a library 16 feet long and 12 feet broad, the floor has a border of marquetry 18 inches wide, and

the library table rests upon a rug of 9 feet long and 5 feet broad. How much of the floor inside the marquetry is uncovered? Make a diagram of the floor.

13. A parlor 22 feet long and 16 feet broad is covered with a carpet except a space 2 feet broad around the outer edge. The carpet has a border 18 inches broad. How many yards of border must there be? How many linear yards of carpet 27 inches broad must have been procured for the floor, provided there was no waste in matching the figures? Make a diagram of the floor.

14. The walls of a dining room extend 6 feet above the wainscoting, and the room is 12 feet square. The ceiling border and the side border are each 18 inches broad. Allowing 36 square feet for the interruptions of doors, and making no allowance for any waste in matching the patterns of the wall paper, how many square feet of wall paper will be required? How many linear yards of bordering for ceiling and side walls?

15. Usually wall paper is 18 inches broad, a single roll of it containing 8 yards, and a double roll 16 yards. How many rolls would be required for the dining room?

16. How much wall paper will be required to paper the two sides of a hall 23 feet long and 10 feet in height, allowing for four doors each $4\frac{1}{2}$ feet broad and 7 feet in height, there being a border 18 inches broad?

17. What will be the cost of kalsomining the ceilings of two rooms 14 feet square, and of one room 18 feet long and 12 feet broad, and another room 10 feet square, at $1\frac{1}{4}$c per square foot?

18. What is the area of the plastering in your schoolroom?

19. What will be the cost of painting the ceiling, walls, doors, and woodwork of a room 40 feet long, 25 feet

broad, and 12 feet in height, at 15 cents per square foot, no allowance being made for the windows?

20. What area of plastering will be required for the two sides, one end, and the ceiling of a hall 18 feet long and 9 feet broad, 10 feet in height, allowing for a base board 1 foot high, and for five doors each $4\frac{1}{2}$ feet wide and $6\frac{1}{2}$ feet high?

21. The roof of a house has an area of 1,200 square feet for each of two slopes. Allowing 10 shingles for each square foot, how many bundles of shingles are required for the roofing, there being 250 shingles in a bundle?

22. A square building has a roof of four slopes, each containing 700 square feet. How many bundles of shingles are required for the roof?

23. If shingles are 16 inches long, and each projects one fourth of its length beyond the shingle above it, how many inches of each will be exposed to the weather, and how many thicknesses of shingle will cover each part of the roof from the lowest layer to the boards laid along the crest? Make diagram of profile at the edge of the roof, showing the arrangement of the layers, and making the first layer double

24. If one roofer lays 1,000 shingles in half a working day, how long will it take two men to shingle a roof having an area of 2,400 square feet?

25. A "board foot" is an area of one square foot.* If there were no allowance for the tongue of a board of flooring which fits into the groove of the next board, or for waste in cutting and fitting, how many board feet of flooring would be required for your schoolroom?

* In stating the board feet, no account is made of a quarter of an inch of excess or deficit in width. This dimension is given to the nearest half inch of a board of whatever thickness not exceeding one inch.

For a parlor 16 feet long and 13 feet broad? For a chamber 14 feet square?

26. How many board feet are there in 60 boards, each board being 16 feet long, eight inches broad and $\frac{3}{4}$ of an inch thick?

27. How many board feet would be required for the wainscoting five feet high of a room 14 feet square, if allowance of one-sixth be made for interruptions of doors, and windows, for the matching of boards, or for waste in cutting, and if no account be taken of the molding above or of the base board below, the boards used being 1 inch thick?

28. How many board feet would be required for the floor of an assembly hall 100 feet long and 50 feet broad, leaving out of account the matching and the waste in cutting, the flooring being 1 inch thick?

29. If a man can lay 100 square feet of this floor in half a day, how long will five men require to lay the floor?

30. A store room 80 feet long and 20 feet broad has a ceiling of stamped steel with a curved or beveled border cutting off 14 inches from the edge all around. What area is covered by the stamped steel? What is the length of the border used?

APPLICATIONS OF CUBIC MEASURE

The denominations of cubic measure in practical use are but few, since they increase so rapidly in magnitude. Only the cubic inch, the cubic foot, and the cubic yard, are used in practice, for there is little occasion even to make mention of cubic rods or of cubic miles.

Where wood was commonly used as fuel it was universally measured by the cord, as it is in a limited way at the present time; but for other purposes it is and has been measured by the cubic foot.

A cord of wood is a pile 8 feet long and 4 feet broad, the sticks being 4 feet in length.

The number of cubic inches in a cubic foot (1728) and of cubic feet in the cubic yard (9) should be fixed in the mind.

1. How many cubic feet are there in a cord of wood? in 6 cords? in 10 cords?

2. A bushel contains 2150.42 cubic inches. How many cubic inches are there in 10 bushels? How many cubic feet?

3. If in building a bin you allow 5 cubic feet for 4 bushels of grain, do you allow too much, or too little?

4. In that case, what is the amount of the error?

5. A gallon contains 231 cubic inches. How many gallons are there in a cubic foot?

6. Counting 400 cubic feet to the ton, how much hay is contained in a stack 60 feet long and 20 feet wide, built up to a flat top 12 feet above the ground?

7. A schoolroom 30 feet long and 23 feet wide, with a ceiling 12 feet high, is deemed sufficiently large with proper ventilation, for use by a class of forty-two pupils. How many cubic feet of space does this calculation allow for each pupil?

8. A cubic foot of air weighs, on an average, $1\frac{1}{2}$ ounces. What is the weight of the air contained in such a schoolroom?

9. A cellar is 100 feet long and 25 feet wide, and is excavated to a depth of nine feet below the level of the ground. Allowing a cubic yard to a wagon load, how many loads of earth were carried away in excavating it?

10. The weight of a cubic foot of pure water at ordinary temperatures is 62.5 pounds. Other substances

except gases are compared with water in weight, and the figures which state their weight in terms of water are said to express their *specific gravity*. Thus a cubic foot of hardwood which weighs three-fourths, or seventy-five hundreds, as much as water is said to have a specific gravity of .75. What is the weight of a cubic foot of such hardwood?

11. Lead has a specific gravity of 11.3. What is the weight of a cubic foot of lead?

12. If a block of stone has a specific gravity of 2.5, what will be the weight of a cubic foot of it?

13. It is not easy to ascertain by direct measurement the cubic contents of a body of irregular form. The bulk of such a body can often be found, however, by immersing it in a tank of regular form and noting how much water it displaces. If into a tank with a bottom 6 inches square (inside measurement) and with straight sides, nuggets of gold are dropped until the water rises one-sixteenth of an inch, what is the amount of their bulk?

14. The specific gravity of gold in its pure state is 19.3. What would be the weight of the nuggets if they were without alloy?

15. What would be the weight of a cubic foot of pure gold?

16. The specific gravity of pure platinum is 22. What is the weight of a cubic inch of platinum?

17. The specific gravity of silver, when pure, is about 10.5. What is the weight of a cubic inch of pure silver? What would be the weight of a cubic foot of it?

18. The specific gravity of zinc is 7. How much greater is the weight of a cubic inch of silver than that of a cubic inch of zinc?

PART II

REVIEW OF PRINCIPLES

With most pupils the work of the eighth grade offers the last chance for the study of arithmetic in school. In view of the importance of this work as a preparation for the business to be transacted in a lifetime, the class should *make the most of the opportunity.*

A brief review of principles, in the form of problems, will show each pupil for himself whether he has a *ready, working* knowledge of these principles or whether he is lacking in this respect. It will also enable the teacher to "take account of stock" before entering upon the new work of the year.

The problems may be used as time tests; for it is not so much the amount of knowledge that may be recalled to mind as it is the *ready mastery of processes* that is here to be shown.

I. Foundation Work

1. Write in Arabic figures the equivalent of MDCCCLXV.

2. Write in Roman numerals the equivalent of 119; of 1914; of 1776; of 449; of 1066.

3. If the Romans had had places of figures, as we have, what would have been the value of VII? Of IV?

4. Write a number of four periods, all full. Read it.

5. What is the meaning of the word elements *deca*, *hecto*, *kilo*, and *myria*?

6. What is the meaning of the word elements *deci*, *centi*, and *milli*?

7. Write in columns seven numbers, each containing four places, making light and small memoranda of numbers to be "carried."

8. Write seven other numbers in columns, and add, writing as a partial result the sum of each column separately; then add the partial results.

9. Write a minuend of seven places of figures, and a subtrahend of six places. Perform the subtraction, and prove the correctness of the work.

10. 5,192, 2,643, 7,182, 1,121, and what other number, added together, will equal 24,860? Supply the balance without subtracting.

11. Recite the "elevens" of the multiplication table.

12. Recite the "twelves" of the multiplication table.

13. Recite the "thirteens"*of the multiplication table.

*See Note, page 17, Intermediate Arithmetic.

14. Recite the "fourteens" of the multiplication table.

15. Recite the "fifteens" of the multiplication table.

16. Multiply 428,804 by 25 in the easiest way.

17. Multiply 84,246 by $16\frac{2}{3}$ in the easiest way.

18. Multiply 91,638 by $33\frac{1}{3}$ in the easiest way.

19. Muitiply 56,6314 by $14\frac{2}{7}$, in the easiest way.

20. Multiply 886,456 by $12\frac{1}{2}$ in the easiest way.

21. Multiply 86 by $37\frac{1}{2}$, using the iraction first.

22. Multiply 15 by 22 $\frac{3}{5}$ using the fraction last.

23. Multiply 259,614 by 999 in the easiest way.

24. How can you tell in advance if a number is exactly divisible by 3?

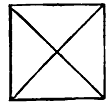
25. How can you tell in advance if a number is exactly divisible by 5?

II. Factoring, H. C. F., and L. C. M.

1. What is the highest common factor of 82, 96, and 112?
2. What is the highest common factor of 125 and 325?
3. What is their lowest common multiple?
4. What is the lowest common multiple of 24, 15, and 81?
5. Find the highest common factor of 96, 84, 24, and 32.
6. What is the lowest common multiple of 18, 14, and 2?
7. What is the lowest common multiple of 9, 18, 27 and 36?
8. What is their highest common factor?

III. Practical Measurements

1. What is meant by the altitude of a triangle?
2. How is the area of the triangle ascertained?
3. What is the area of a triangle of which the base is 4 in., and the altitude is 6 in.?



4. Draw a figure two inches square. What is the amount of its square contents?

5. From this figure construct a square figure having twice the amount of area.*

6. From the first square construct a square figure having one-half the amount of area.

7. Construct a square measuring three inches each way.

8. Construct a square of double the area.

9. Construct a square of half the area of the first one.

10. One picture having twice the diameter of another has how many times its area?

11. Draw a plat of a Congressional township, and number the sections.

12. What is meant by a school section?

13. What is the difference between a Congressional township and a civil township?

14. Draw a plat of an acre in three different forms, marking the length in rods, of each boundary line.

IV. Fractions

1. Write a common fraction; a mixed number.
2. Write an improper fraction; a complex fraction.
3. Divide $\frac{2}{3}$ by 8. Divide 8 by $\frac{2}{3}$.
4. Divide $\frac{2}{3}$ by $\frac{3}{4}$.
5. Add $\frac{4}{9}$ and $\frac{4}{15}$.
6. Subtract $\frac{4}{15}$ from $\frac{4}{9}$.
7. Multiply $\frac{4}{15}$ by $\frac{4}{9}$.
8. Find $\frac{4}{9}$ of $\frac{4}{15}$.

*See page 147, Intermediate Arithmetic.

9. Explain the division of a whole number by a fraction, giving an example.

10. Explain the division of a fraction by a whole number, giving an example.

11. Explain the division of a fraction by a fraction, giving an example.

12. Group these numbers in two ways by the use of parentheses, and give the result for each grouping:

$$\frac{1}{2}+\frac{2}{3}\times 3=?$$

13. Write as a complex fraction $\frac{2}{3}\div\frac{3}{4}$.

14. Write as a complex fraction $2\div\frac{2}{3}$.

15. Write as a complex fraction $\frac{2}{3}\div 2$.

V. Decimal Fractions and Cancellation

1. In the multiplication of decimals, what must be the number of decimal places in the product?

2. In the division of decimals, what must be the number of decimal places in the quotient?

3. Change $\frac{3}{15}$ to a decimal fraction.

4. Change .75 to a common fraction.

5. What is the value of $\frac{.8}{.2}$?

6. What is the product of .1 used five times as a factor?

7. Change $6\frac{1}{4}$ to a decimal fraction.

8. Change .125 to a common fraction.

9. What is the product of 1000 and .0001?

10. What is the quotient of $500\div .005$?

11. Solve by cancellation:

$$(15\times 50\times 45\times 10)\div(5\times 5\times 25).$$

12. Solve by cancellation:

$$(72\times 28\times 48\times 84)\div(48\times 24\times 14).$$

13. Solve by cancellation:

$(63 \times 25 \times 42 \times 24) \div (6 \times 7 \times 3)$.

14. Solve by cancellation:

$(14 \times 28 \times 21) \div (7 \times 3)$.

15. Solve by cancellation:

$(9 \times 21 \times 36) \div (3 \times 2 \times 9)$.

16. Solve by cancellation:

$(9 \times 3) \times (14 \times 7 \times 28) \div (3 \times 7 \times 6)$.

17. Solve by cancellation:

$(2 \times 4 \times 2 \times 4 \times 2) \div (4 \times 2 \times 4 \times 2)$.

VI. Per Cents

1. What per cent of a number is $\frac{1}{8}$ of it? —is $\frac{1}{12}$ of it? —is $\frac{1}{18}$ of it? —is $\frac{2}{25}$ of it?

2. What is 20% of $83\frac{1}{3}$%?

3. What is the difference between 1% and .01%?

4. Find 40% of 250.

5. Find 47% of 9.

6. 24 is what per cent of 36?

7. 9 is 4% of what number?

8. If a merchant gains 10% and then loses 10%, is his capital the same as it was at first?

9. If a farmer has added 25% to the area of his farm, and then sold 25% of his land, has it gained or lost anything in area?

10. What is $66\frac{2}{3}$% of 15%?

11. What is 25% of 20%?

12. What is $12\frac{1}{2}\%$ of 80%?

13. What is the difference between 5% and .05%?

14. 6 is 6% of what number?

15. 4 is 2% of what number?

16. 8 is what per cent of 10?

17. 25 is what per cent of 30?

18. Of a company of one hundred soldiers 5% were in the hospital, 2% absent on furlough, and 7% absent on special service, at the time of roll call. What per cent were not at roll call?

19. In a certain cash drawer, 30% of the cash is in one dollar bills, 18% in two dollar bills, 20% in five dollar bills, 20% in ten dollar bills, and the remainder in fractional currency. What per cent of the whole is in the fractional currency?

20. 86% of the income of a certain railway company for a certain year was devoted to expenses and investments. Expenses constituted 46% of the whole income. What per cent was devoted to investments? What per cent of the whole income was left for dividends for the stockholders.

21. If 80% of the cargo of a wrecked ship is saved, and 40% of this has depreciated 50% in value, what is the per cent of loss of the cargo?

VII. Interest, Discounts, and Commissions

1. What is the interest on $450 for 2 years, 4 months, and 3 days at 6%?

2. What is the interest on $1914 for 4 years, 8 months, and 18 days at 6%?

3. What is the interest on $180 for 16 months and 18 days, at 4%?

If you borrow $400 for 60 days at bank discount, at 6%, what amount will you actually receive?

5. To receive actually $100 borrowed at bank discount for 60 days at 6%, for what sum must you draw the note?

6. By the "twelve per cent method," write the interest on $1 for 6 years, 10 months, and six days at 12%; at 6%. Write the interest on $1,500 at 6% for the same time.

7. If the trade discounts on a bill for $360.00 are 20%, 5% and 3%, what is the net price?

Does it make any difference in what order these discounts are stated?

8. A commission merchant sells wheat to the amount of $15,000, on a commission of 2%. What sum does he receive for his services?

9. If by selling fruit on a commission of 5% a boy earns $10, what is the amount of his sales?

10. A collector remits to his customer $218 after deducting a commission of 5%. What was the amount of his collections?

11. A broker sells 100 bales of cotton, averaging 490 pounds to the bale, at 16 cents the pound, what is the amount of his commission at $2\frac{1}{2}$%?

12. If the net price of an article is $15, and the trade discount is 20%, what is the list price of it?

13. If the list price of an article is $25, and the trade discount is 20%, what is the net price?

14. If a piece of goods is listed at $18, and you buy it at a net price of $15, what is the trade discount?

15. Does it make any difference whether discounts are 15%, 10%, and 5%, or whether they are 5%, 10% and 15%?

16. If a certain kind of stock is bought at 58% of its face value and goes upward to 92%, how many points does it rise?

17. What % does it increase in value?

18. If stock bought at $87\frac{1}{2}$% of its face value declines $12\frac{1}{2}$ points, at what is it rated?

19. What % has it lost in value?

20. If a certain stock declines 25% below par, and then advances 10 points, at what % is it quoted?

21. Illustrate the difference between per cents and points in stating an advance or decline in value of stocks.

GENERAL PROBLEMS

1. One-eighth of the employees of a mill were discharged. One-quarter of their number were re-employed. What part of the original number were still unemployed?

2. A man owns one-eighth of the stock in a factory. He buys another eighth, then three more eighths, and then sells a quarter of the whole stock. What part of the stock has he left then?

3. A certain man devotes one-tenth of his income to charity, two-thirds of this amount being turned over to a benevolent association, and one-third being dispensed personally by himself. What part of his income does he personally dispense in charity?

4. A teacher having one-sixth of the year for vacation devotes one-fourth of that period to travel, and the remainder to special studies. What part of the year does she devote to these special studies?

5. Six-tenths of a certain road was of burnt clay construction, and one-third only of this amount was built by voluntary service contributed by users of the highway. What part of the road was not so built?

6. A man owns one-third of the cargo of a ship that is lost at sea. He receives from an insurance company compensation for two-thirds of his loss. His net loss is what part of the value of the whole cargo?

7. Two-thirds of a man's property consists of railway bonds. He gives to his daughters one-fourth of his entire property, all in bonds. What part of the whole estate does he retain in bonds?

8. Two-thirds of a church debt being unpaid, a member contributed one-quarter of the whole amount to its extinguishment. What part then remained unpaid?

9. How many degrees are there between the hands of a clock at three o'clock?

10. How many degrees does the visible arch of the sky measure, on a plain?

11. How many decades are there in 17 centuries?

12. What months have thirty days, each?

13. What years, in the next twenty, will be leap years?

14. A dozen boxes, containing each forty cans of vegetables worth 15 cents the can are exchanged for forty pairs of ducks. What is the worth of each duck?

15. Twenty wagons, each holding a ton of merchandise and making five trips daily, are replaced by ten auto vans, each carrying two tons. How many trips must they make to carry the same freight?

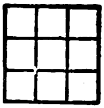
16. If one brushful of paint is sufficient for painting the flat side of a half of a globe*, how many brushfuls will be required to paint the entire surfaces of six similar globes?

17. If pared round apples are divided into quarters for cooking, how much is their absorbing surface increased by the quartering?

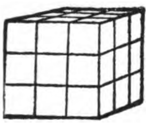
*See Elementary Arithmetic, page 184.

POWERS AND ROOTS

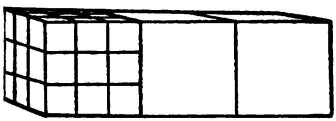
Conventional* representation of the number 3, as a number of the first power.



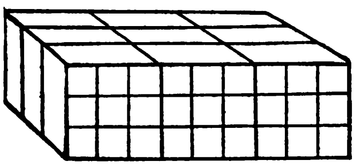
Conventional representation of the number 9, as the second power of 3.



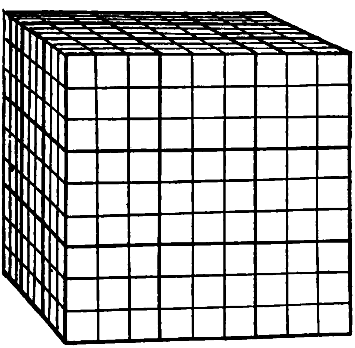
Conventional representation of the number 27, as the third power of 3.



Equally logical representation of the number 81, as the fourth power of 3.



Equally logical representation of the number 243, as the fifth power of 3.



Equally logical representation of the number 729, as the sixth power of 3.

*That which is done universally by common consent is said to be conventional. The tacit agreement by which we use the line, the square, and the cube, respectively, to represent the first, second, and third powers of numbers, and by which we speak of an area as "equal to the product of its length and its breadth," is called the "Cartesian Convention," in honor of the French philosopher Descartes (*day-cart'*).

INVOLUTION

3

A number that is not multiplied by itself is said to be of the *first power.*

9 is what power of 3?

$$3 \times 3 = 9$$

What number is here repeated to form the two factors of a composite number? What is the composite number?

> *A number that is repeated to form the two factors of a composite number is said to be raised to the second power.*

The process of ascertaining the *powers* of numbers is called, in arithmetic, *involution.*

$$3 \times 3 \times 3 = 27$$

What number is here repeated to form the three factors of a composite number? What is the composite number?

27 is what power of 3?

> *When a number is repeated to form the three factors of a composite number, it is said to be raised to the third power.*

As a convenient way of illustrating a number of the first power, a line may be used. Thus a line three inches long may be used to indicate the number 3.

As a convenient way of illustrating a number of the second power, a square figure may be drawn. Thus a square three inches long and three inches broad will contain nine square inches.

This illustration is so very common that people often speak of the second power of a number as the "*square*" of that number. Thus it is said that the square of 2 is 4; the square of 3 is 9; the square of 8 is 64; etc. This is a mere matter of convenience and custom. If a boy has a fishline twenty-five feet long, its length in feet is *the second power of five*, and there is nothing square about it; still, the custom is so general that we should keep in mind what is generally meant when people speak of the "square" of a number.

As a convenient way of illustrating a number of the third power, a cubic figure may be drawn. Thus a cube three inches long, three inches broad, and three inches thick will contain twenty-seven cubic inches.

The second powers of numbers are much used in the measurement of areas of land, of floors and tables, etc.

The third powers of numbers are much used in the measurement of solid bodies, such as timber, ground or rock to be excavated, artificial stone or other material to be manufactured, etc.

A number used four times as a factor might likewise be represented by a *line made up of small cubes;* as $3 \times 3 \times 3 \times 3$ by three cubes of 3, arranged in a row.

A number used five times as a factor might be represented by a *square made up of small cubes;* as $3 \times 3 \times 3 \times 3 \times 3$ by nine cubes of 3, arranged in the form of a square.

A number used six times as a factor might be represented by a large *cube made up of small cubes;* as 27 cubes of 3, arranged to form a large cube, which would be a cube of 9.

4^2

The power to which a number is to be raised is indicated by a small raised figure following it. This small figure is called the *exponent*. Thus 4^2 indicates that 4 is to be raised to the second power. $4^2 = 16$.

For the first power of a number, no exponent is written.

1. Raise 6 to the third power.
2. What is the value of 10^2?
3. What is the fourth power of 2?
4. State the value of 4^2.
5. Raise 10 to the third power.
6. What is the value of 25^2?
7. Find the value of 7^3.
8. Raise 12 to the third power.
9. What is the second power of 11?
10. Find the value of 6^4.
11. Raise 90 to the second power.
12. What is the second power of 120?
13. What is the value of 120^3?
14. What is the sixth power of 2?
15. Find the value of 15^3.
16. What is the third power of 75?
17. Raise 256 to the second power.
18. What is the value of 100^3?
19. State the second power of 14.
20. What is the fourth power of 9?

$$3^2 \times 3^3 = 3^5$$

This may be shown by writing out the factors of each number, thus $(3 \times 3) \times (3 \times 3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3$.

To multiply a power of a number by another power of the same number, add the exponents of the two powers.

1. Find the value of $2^2 \times 2^3$.
2. Multiply 4^2 by 4^3.
3. How much is 3×3^2?
4. Find the value of $3^2 \times 3^3$.
5. How much is $2^3 \times 2^3$?
6. What is the value of $5^2 \times 5^2$?
7. Multiply 2^2 by 2^3.
8. Find the equivalent of $x^2 \times x^2 \times x^2$.
9. How much is $5^2 \times 5^3$?
10. Multiply 3^3 by 3^3.

$$3^5 \div 3^2 = 3^3$$

This can be demonstrated by expressing the numbers in their factors, thus—

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$, or 243.

$3^2 = 3 \times 3$, or 9.

$243 \div 9 = 27$, or 3^3.

To divide a power of a number by another power of the same number, subtract the exponent of the divisor from the exponent of the dividend.

1. Find the value of $8^6 \div 8^3$.
2. What number is equal to $4^7 \div 4^4$?
3. Find the value of $(3^2 \times 3^4) \div 3^3$.

4. What number equals $(2^2)^2 \div 2^3$?

5. State the equivalent of $(3^3 \times 3^3) \div 3^4$.

6. State the equivalent of—

$$(3 \times 3 \times 3 \times 3 \times 3) \div (3 \times 3 \times 3).$$

7. What is the value of $7^7 \div 7^4$?

8. What is the value of $10^8 \div 10^6$?

9. What number is equal to $20^6 \div 20^3$?

10. What is the equivalent of $12^5 \div 12^2$?

A number that is used twice as a factor is called the *square root* or second root, of the product. The square root, or second root, of a number is indicated by placing in front of it a sign called the radical sign, which was originally the script *r*. It is made thus: $\sqrt{\ }$.

$$\sqrt{36}$$

Here it is indicated that the second root of 36 is to be found, or used in a calculation. The second root of this quantity (called a radical quantity) is 6.

A number that is used three times as a factor is called the *cube* root, or third root, of the product. It is indicated by a small figure 3 written with the radical sign. Thus $\sqrt[3]{64}$ indicates that the third root of 64 is to be found, or used in calculation. The third root of this radical quantity is 4.

The second root of the second root is called the *fourth root.*

The fourth root of 16 is 2, and is indicated thus: $\sqrt[4]{16}$. The second root of the third root, or the third root of the second root (for they are the same) is the *fifth root.* The fifth root of 32 is 2, and is indicated thus: $\sqrt[5]{32}$. In like manner, the second root of the fourth root is the sixth root.

EVOLUTION

The process of ascertaining a root of a number is called, in arithmetic, *evolution.* This word is used with a very different meaning in other sciences.

A number that can be represented by a *perfect square* is a number the square root of which is a whole number, and hence can be determined exactly. Thus 9 is a number that can be represented by a perfect square, while 8 is not.

A number that can be represented by a *perfect cube* is one whose cube root, or third root, is a whole number, and hence can be determined exactly. Thus 8 is a number that can be represented by a perfect cube, while 9 is not.

Here are the fifteen perfect squares contained in the multiplication tables that have been studied, *and the root of each.*

Squares	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Roots..	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

1. Write the squares, or second powers, of all the numbers from 1 to 10.

2. Write the cubes, or third powers, of all the numbers from 1 to 10.

3. What are the values of these radical quantities: $\sqrt{49}$, $\sqrt{36}$, $\sqrt{81}$, $\sqrt{25}$ and $\sqrt{64}$?

4. State the values of these radical quantities: $\sqrt[3]{27}$, $\sqrt[3]{125}$, $\sqrt[3]{8}$, $\sqrt[3]{64}$.

5. What are the second roots of 100; 400; 2500; 900; 1600?

6. Read these radical quantities: $\sqrt{625}$; $\sqrt[3]{27}$; $\sqrt{4900}$; $\sqrt{121}$; $\sqrt[3]{216}$.

To find the second root of a fraction, find the second root of both of its terms.

Thus the second root of $\frac{4}{9}$ is $\frac{2}{3}$.

1. Find the square root of $2\frac{1}{4}$.
2. Find the square root of $6\frac{1}{4}$.
3. Find the square root of $2\frac{7}{9}$.
4. Find the square root of $1\frac{9}{16}$.
5. Find the square root of $1\frac{11}{25}$.
6. Find the value of $\sqrt{12\frac{1}{4}}$.
7. Find the value of $\sqrt{5\frac{4}{9}}$.
8. Find the value of $\sqrt{3\frac{1}{16}}$.
9. Find the value of $\sqrt{1\frac{24}{25}}$.
10. What is the value of $\sqrt{1\frac{13}{36}}$.
11. What is the value of $\sqrt{7\frac{1}{9}}$.
12. What is the value of $\sqrt{2\frac{14}{25}}$.
13. What is the value of $\sqrt{1\frac{15}{49}}$.
14. What is the value of $\sqrt{3\frac{1}{16}}$.
15. What is the value of $\sqrt{3\frac{6}{25}}$.
16. Find the value of $\sqrt{1\frac{32}{49}}$.
17. Find the value of $\sqrt{1\frac{17}{64}}$.
18. Find the value of $\sqrt{11\frac{1}{9}}$.
19. Find the value of $\sqrt{2\frac{2}{49}}$.
20. Find the value of $\sqrt{1\frac{19}{81}}$.
21. What is the square root of $12\frac{13}{9}$?
22. What is the square root of $4\frac{21}{25}$?
23. What is the square root of $3\frac{13}{36}$?
24. What is the square root of $2\frac{23}{49}$?

25. What is the square root of $1\frac{57}{64}$?
26. Find the square root of. $1\frac{40}{81}$.
27. Find the square root of $1\frac{21}{100}$.
28. Find the square root of $2\frac{46}{49}$.
29. Find the square root of $1\frac{63}{81}$.
30. Find the square root of $1\frac{23}{121}$.
31. Find the value of $\sqrt{1\frac{25}{144}}$.
32. Find the value of $\sqrt{1\frac{52}{144}}$.
33. Find the value of $\sqrt{1\frac{81}{144}}$.
34. Find the value of $1\frac{81}{144}$.

$$\frac{2^2}{3^2}=\frac{2\times2}{3\times3}=\frac{4}{9} \qquad \sqrt{\frac{4}{9}}=\frac{\sqrt{4}}{\sqrt{9}}=\frac{2}{3}$$

$$\sqrt{\frac{16}{10}}=\frac{4}{\sqrt{10}}$$

Caution is to be observed in the matter of decimal mixed numbers. For example, 1.6 is the product of 4 and .4; and neither of these can be its root. If we write the number thus, $\frac{16}{10}$, we clearly see that the roots of both terms of the fraction must be found.

The second root of any number of one or two places of figures that can be represented by a perfect square will be a single figure; the second root of any number having three or four places of figures will consist of two figures. The second root of any number having five or six places of figures will consist of three figures, etc.

To determine how many places of figures there are in the second root of any number, divide the number into root periods of 2 figures each, beginning at the right. Indicate the root periods by dots placed above the figures.

Thus the number 8,836 is seen to contain two root periods of two figures each, and its square root will consist of two figures. Sometimes the last root period is not a full one, as 30,025.

1. How many figures will there be in the square root of 16,384? Read the root periods separately.

2. How many figures will there be in the square root of 54,596? Read the root periods separately.

3. How many figures will there be in the square root of 39,112,516? Read the root periods separately.

4. Divide into root periods the number 788,544.

5. Divide into root periods the number 11,669,056.

6. Divide into root periods the number 30,250,000.

If the number includes a decimal fraction, the periods of the decimal part must be marked from the decimal point to the right, thus: 1728.864.

Here is a table *of the square roots, or second roots, of numbers from 1 to 100. By using the high denominations, larger numbers made up of smaller units may be brought within this limit. Where a number that can be represented by a perfect

* This introduction of Tables of Square Roots and Cube Roots—the first in a school Arithmetic—will save to pupils much needless labor in detail work. The use of these tables here is similar to the use of logarithmic tables in higher mathematics, and is equally consistent.

(See page 155 for the corresponding Table of Cube Roots.)

SQUARE ROOTS OF NUMBERS

From 1 to 100, Carried to Three Places of Decimals.

Number	Square Root	Number	Square Root	Number	Square Root.
1	1	34	5.831	67	8.185
2	1.414	35	5.916	68	8.246
3	1.732	36	6	69	8.307
4	2	37	6.083	70	8.367
5	2.236	38	6.164	71	8.426
6	2.449	39	6.245	72	8.485
7	2.646	40	6.325	73	8.544
8	2.828	41	6.403	74	8.602
9	3	42	6.481	75	8.660
10	3.162	43	6.557	76	8.718
11	3.317	44	6.633	77	8.775
12	3.464	45	6.708	78	8.832
13	3.606	46	6.782	79	8.888
14	3.742	47	6.856	80	8.944
15	3.873	48	6.928	81	9
16	4	49	7	82	9.055
17	4.123	50	7.071	83	9.110
18	4.243	51	7.141	84	9.165
19	4.359	52	7.211	85	9.220
20	4.472	53	7.280	86	9.274
21	4.583	54	7.348	87	9.327
22	4.690	55	7.416	88	9.381
23	4.796	56	7.483	89	9.434
24	4.899	57	7.550	90	9.487
25	5	58	7.616	91	9.539
26	5.099	59	7.681	92	9.592
27	5.196	60	7.746	93	9.644
28	5.291	61	7.810	94	9.695
29	5.385	62	7.874	95	9.747
30	5.477	63	7.937	96	9.798
31	5.568	64	8	97	9.849
32	5.657	65	8.062	98	9.899
33	5.745	66	8.124	99	9.950
				100	10

square ends in two ciphers, its second root will be the second root of the digits with one cipher added. Thus the second root of 900 is 30. Where a number that cannot be represented by a perfect square ends in two ciphers, its second root will be the second root of the digits, with the decimal point moved one place to the right (correct as far as carried).

Thus the second root of 800, so carried, is 28.28.

Square roots are much employed in construction work It is often necessary to find the length of the hypotenuse of a right-angled triangle, when the lengths of the other two sides are known; also to find the length of a square whose contents are known.

> *The square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.*

APPLICATIONS OF SQUARE ROOTS

1. What is the second root of 97?

2. Find the second root of 5600.

3. One side of a right-angled triangle is 4 yards, and another is 7 yards. What is the hypotenuse?

4. What will be the dimensions of a square containing 80 square rods?

5. 74 square inches of gold leaf will make how large a square?

6. 60 square yards of painting will cover how large a square?

7. Two Congressional townships of land are equal to a square of what dimensions?

8. A certain pot of paint is sufficient to cover 40 square yards; another, to cover 32 square yards. How large a square would they together cover?

9. 86 square yards of roofing would cover how large a square roof?

10. 72 square yards of flooring would make how large a square floor?

11. A rope nine yards long is stretched from the top of a building to a stake driven into the ground 12 feet from the base of the wall. How tall is the building?

12. A ladder 27 feet long leans against a wall from which its base is separated by a distance of 18 feet. How high does it strike the wall?

13. A right-angled triangular lot is four rods long, and its slanting side is five rods long. How wide is the lot?

14. A right-angled triangular lot is 21 feet long and 15 feet broad at its broadest. How long is its slanting side?

15. If Phillipsville station is 4 leagues due east of Wapping Station, and the shortest line between Phillipsville Station and Rallytown is 8 leagues, Rallytown lying due north of Wapping Station, what is the distance from Phillipsville Station to Rallytown?

16. What is the longest line that can be drawn through a lot measuring 8 rods by 4 rods?

17. At one corner of a level rectangular field 8 rods long and 6 rods broad is a tower 165 feet high. How long a wire will be required to reach from the top of the tower to the ground at the corner diagonally opposite?

18. Tannersville is 24 miles due south from Smithville. What is the shortest line from a point 7 miles east from Tannersville to a point 2 miles west from Smithville? Make diagram. (Use leagues.)

19. A straight wagon road, starting 5 miles due east of a certain station on a directly north-and-south railway, meets and crosses it just 6 miles due north of the station. How long is the wagon road to the crossing?

20. A certain town is planned with a circular boulevard, two miles in inner diameter, to surround it. What is the size of the largest square that can be laid out inside the inner line of the boulevard? Illustrate with diagram.

21. In a certain dwelling there is a square dining room, the diagonal of which is equal to one side of the library. How much larger is the library than the dining room? Illustrate with diagram.

The third root, or cube root, of a number which can be represented by a perfect cube contains as many places of figures as the number itself contains periods of three figures each; so that *the root period of such a number corresponds to the period used in numeration.*

Here is a table containing the third roots of numbers from 1 to 100. Where a number that can be represented by a perfect cube ends in three ciphers, its third root will be the root of the digits with one cipher added. Thus the third root of 8000 is 20. Where a number which cannot be represented by a perfect cube ends in three ciphers, its (incomplete) third root will be the third root of its digits with the decimal point moved one place to the right. Thus the third root of 9000 is 20.80.

APPLICATIONS OF CUBE ROOTS

1. If a box 8 feet long, 3 feet broad, and 2 feet deep (inside measure) is filled with sand, what would be the measure of a cube-shaped box that the sand would fill?

2. If I have 84 cubic inches of candy, what will be the size of a cubical box to contain it?

3. 90 loads of earth (each a cubic yard) have been removed in excavating a cellar. How large a cube would they make?

4. An excavation 2 rods square and $8\frac{1}{4}$ feet deep is equal in capacity to how large a cube?

CUBE ROOTS OF NUMBERS

From 1 to 100, Carried to Three Places of Decimals

Number	Cube Root	Number	Cube Root	Number	Cube Root
1	1	34	3.240	67	4.062
2	1.260	35	3.271	68	4.082
3	1.442	36	3.302	69	4.102
4	1.587	37	3.332	70	4.121
5	1.710	38	3.362	71	4.141
6	1.817	39	3.391	72	4.160
7	1.913	40	3.420	73	4.179
8	2	41	3.448	74	4.198
9	2.080	42	3.476	75	4.217
10	2.154	43	3.503	76	4.236
11	2.224	44	3.530	77	4.254
12	2.289	45	3.557	78	4.273
13	2.351	46	3.583	79	4.291
14	2.410	47	3.609	80	4.309
15	2.466	48	3.634	81	4.327
16	2.520	49	3.659	82	4.344
17	2.571	50	3.684	83	4.362
18	2.621	51	3.708	84	4.380
19	2.668	52	3.733	85	4.397
20	2.714	53	3.756	86	4.414
21	2.759	54	3.780	87	4.431
22	2.802	55	3.803	88	4.448
23	2.844	56	3.826	89	4.465
24	2.884	57	3.849	90	4.481
25	2.924	58	3.871	91	4.498
26	2.962	59	3.893	92	4.514
27	3	60	3.915	93	4.531
28	3.037	61	3.936	94	4.547
29	3.072	62	3.958	95	4.563
30	3.107	63	3.979	96	4.579
31	3.141	64	4	97	4.595
32	3.175	65	4.021	98	4.610
33	3.208	66	4.041	99	4.626
				100	4.642

5. A room 5 yards long, 4 yards wide, and 3 yards high is equal in capacity to how large a cube?

6. A common brick contains 64 cubic inches. How large a cube would it make?

7. A bin is 5 yards long, 3 yards wide, and 2 yards deep. What is the size of a cube having the same capacity?

8. A certain product is put up into 180 boxes, each 4 feet long, 2 feet broad, and 2 feet deep. If the same material is put into 360 boxes of cubical shape, what will be the measure of each box?

9. A certain column of solid bronze contains 17 cubic feet. How large a cube can be made of it?

10. A mass of steel 6 feet long, 5 feet broad, and 3 feet thick would form a cube of what dimensions?

ROOTS OF FRACTIONS

$$\sqrt{2\tfrac{1}{2}} = \sqrt{\tfrac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} = (2.236 \div 1.414) = 1.582$$

1. What is the second root of $\frac{3}{4}$?
2. What is the equivalent of $\sqrt{2\frac{4}{5}}$?
3. Find the second root of $3\frac{4}{9}$.
4. What is the value of $\sqrt{\frac{7}{10}}$?
5. What is the third root of $\frac{2}{3}$?
6. Find the third root of $\frac{4}{15}$.
7. Find the second root of $5\frac{4}{9}$.
8. What is the second root of $18\frac{4}{5}$?
9. What is the third root of $25\frac{1}{2}$?
10. Write the equivalent of $\sqrt{14\frac{3}{4}}$.
11. Add $\sqrt{3}$ and $\sqrt[3]{3}$.
12. Subtract $\sqrt[3]{34}$ from $\sqrt{34}$.

GENERAL PROBLEMS IN EVOLUTION

1. If I have materials for $8\frac{1}{2}$ square yards of roof. How large a square can I cover with them?

2. If I have plaster for $10\frac{3}{4}$ square yards of ceiling. How large a square ceiling can I cover with it?

3. There are $15\frac{3}{4}$ square yards in a certain sail. How large a square sail would the cloth have made?

4. A rug is 4 yards long and $2\frac{1}{2}$ yards wide. How large a square rug could have been made of the same materials?

5. A foot path 4 feet wide and 60 feet long occupies ground that would make a square of what size?

6. What is twice the second root of 78?

7. What is three times the third root of 48?

8. Subtract the third root of 90 from the second root of 80.

9. A certain cube contains 7 cubic inches. What is the area of all its faces?

10. One face of a certain cube contains 6 square feet. What are the contents of the cube?

11. A certain cube-shaped box contains (inside measure) 12 cubic feet. What is the inside area of one of its sides?

12. A farmer, finding that some hogs had rooted up and destroyed a garden 60 feet long and 12 feet broad, asked his son to figure out the "square root" of the damage done. How large a square would the garden space have made?

13. Two masses of solid masonry, one measuring 2 feet in each dimension, and the other measuring three feet in each dimension, contain material for a single cube-shaped mass of what size?

14. The ceilings of five rooms each 12 feet square are kalsomined. The same material, and work would have sufficed to kalsomine a single square ceiling of what dimensions?

15. Three flower beds, each 4 feet square, on a lawn, occupy as much room as one square flower bed of what dimensions?

16. One dozen cubic blocks measuring, each, 2 inches in their dimensions contain as much as one cube-shaped block of what dimensions?

17. 3456 alphabet blocks 1 inch in each dimension contain enough wood to make a single cube-shaped block of what dimensions?

18. What will be the total area of all the surfaces of the small blocks?—of the large block?

THE EXTRACTION OF SQUARE ROOTS*

Suppose that I have 625 apple trees that I wish to set out in a square orchard, but do not know the square root of that number. Knowing that the square of 20 is 400, that the square of 30 is 900, and that the area of the orchard lies between these numbers, I conclude that its square root will be more than 20 and less than 30. I begin with the square of 20, and set out 400 trees in 20 rows, 20 in each row. I find that I now have 225 trees left. To preserve the shape of the square, I must add trees on two sides of it; that is to say, additions must be made to 40 feet in total length. 40 is contained 5 times in 200, with a remainder of 25. So there must be five rows added on each of the two sides. In the corner of the two additions there will be a small square space for 5 rows of 5 trees each, which will exactly use up the remainder of the trees. The planted area will thus be a complete square of 25 feet.

The square of 25 is seen to be equal to the square of 20, plus twice the product of 20 and 5, plus the square of 5.

*This is to explain how the roots in the table were found, and is to be utilized for finding any roots beyond the limits of the table.

A similar statement may be made of the squares of all numbers consisting of tens and units; that is, of all numbers of two places of figures.

> *The square of any number consisting of tens and units is equal to the square of the tens, plus twice the product of the tens and the units, plus the square of the units.*

Find the second root of 625.

Dividing 625 into root periods, we find that there are two of these. Hence the second root will consist of two places of figures. We first find the largest square in the root period at the left. It is 4, and its root is 2. We write 2 as the first figure of the root at the right. We write the square of 2, which is 4, under the figure in the root period at the left, and add two ciphers, (since 4 in hundreds' place is really 400), then subtract this square from the original number. We now double the root found, for a trial divisor of the remainder exclusive of the last figure.

$$\begin{array}{r|l} & \dot{6}2\dot{5}\;|\underline{2} \\ & \underline{400} \\ 4 & 225 \end{array}$$

4 is contained 5 times in 22. We therefore write 5 for the second figure of that root, and annex it to the divisor, which now becomes 45. Multiplying the trial divisor by the new figure in the root, and subtracting as in long division, we find that there is no remainder. Hence 625 proves to be a "perfect square," and 25 is its root.

$$\begin{array}{r|l} & \dot{6}2\dot{5}\;|\underline{25} \\ & \underline{400} \\ 45 & 225 \\ & 225 \end{array}$$

Where the given number contains more than two periods, the next period is brought down after the first is disposed of. The root, as far is found, is doubled for a trial divisor of the new dividend exclusive of the last figure, and a new root figure is found as before, and is annexed to the trial divisor; and so on until all the periods of the original number are exhausted. If at any time the new dividend will not contain double the amount of the root found, we place a cipher in the place of the root figure, and bring down another period.

In this way, find the second root

1. Of 256	9. Of 961	17. Of 1849
2. Of 361	10. Of 3969	18. Of 6889
3. Of 484	11. Of 3225	19. Of 9216
4. Of 576	12. Of 1089	20. Of 1089
5. Of 625	13. Of 961	21. Of 7569
6. Of 676	14. Of 7569	22. Of 7921
7. Of 784	15. Of 20736	23. Of 9801
8. Of 841	16. Of 7921	24. Of 12664

PRACTICE PROBLEMS.

1. To test a "square corner" of a fence or a building, and see if it is accurately constructed, and is really an angle of 90°, I measure off 6 feet from the corner on one side, and 8 feet on the other, and then measure across from one terminal point to the other. If the corner is accurately made, what will be the length of the diagonal line?

2. Since an acre contains 10 square chains, what will be the approximate length of one side of a square acre?

3. If I construct a right-angled triangle of ground, one of its sides being 9 rods long and the other 4 rods long, describing a square, on its hypotenuse, what area will the square contain?

4. A road passes along two sides of a rectangular field 20 rods long and 10 rods wide. How much shorter is the distance diagonally through the field than along the two sides of it by way of the road?

5. A pole 33 feet tall, planted 15 feet from the wall of a taller building, leans until it touches the building. How high up is the wall touched?

6. Two sides of a right angled triangle are 84 inches and 36 inches long. What is the length of the hypotenuse of the triangle?

STOCKS AND BONDS

STOCKS

To a very great extent the business of the country is now conducted by corporate companies, called *corporations*. All who own any of the stock of a corporation are members of it, and have votes in it in proportion to the number of shares of stock which they possess.

A corporation is regarded in law as an *artificial person*, created by law, for specified purposes and having specified powers. However great the number of its members, the corporation is regarded as a unit, or as a single person, in its dealings with individuals and with the public.

When any of its members die, or retire from membership by disposing of their stock, others take their place; and the life of the corporation is continuous.

The members of a corporation may be widely scattered over the country, leaving the management of its affairs to a Board of Directors whom they elect; such stockholders are subject only to a limited liability in case of the corporation's failure to meet its obligations. The profits of the corporation are usually divided periodically (annually, semi-annually or quarterly) among the holders of its stock, and are called dividends.

The certificate of stock issued to the members of a corporation states the number of shares held, the face value of each, and how the stock may be transferred.

Stock certificates vary somewhat in statement, but the Common Stock Certificates are usually in the general form here illustrated.

The market value of stocks may be very different from their face value. When they are the same they are said to be *at par*, or equal. The values of most securities of this kind are apt to fluctuate.

Large corporations sometimes issue stock of two kinds or classes, *common* and *preferred* stock. The preferred stock guarantees to the holders a certain amount of the earnings, before any payment is made to the holders of the common stock.

If after satisfying the guaranties of the preferred stock there is nothing left for further payment, the holders of the common stock receive nothing. It may happen, however, that the holders of the common stock will receive the larger returns, if the profits of the corporation are large; for usually no limit is placed upon the dividends they may receive after the dividends have been paid on the preferred stock.

Stocks are valued according to the dividends which they pay, other considerations being equal. Thus 100 shares yielding a return of 5% may be worth no more than 50 shares yielding 10%.

PRACTICE PROBLEMS

1. If a gas and electric company, incorporated, pays quarterly dividends, two of them being 3% and two of them 2%, what income is derived annually from 10 shares, of $100 each?

2. What will be the cost of 100 shares of stock of a certain railway (par value $100) if you buy them at 125%, and pay $\frac{1}{8}$% brokerage?

3. If the common stock of a great steel manufacturing corporation is sold at 72%, (par value $100 per share) what will be the cost of 300 shares of it, including the brokerage at $\frac{1}{8}$%?

4. If the preferred stock (par value $100 per share) of a certain coal mining corporation sells at 115%, whatwill be the cost of 10 shares of it, including the brokerage of $\frac{1}{8}$%?

5. What profit is made by purchasing 100 shares ($100 each) of stock of a certain railway at 97% and selling them at 107%, paying $\frac{1}{8}$% brokerage for each transaction?

6. How much stock of a certain street car company, incorporated, must be purchased to insure an income of $600 from it if the stock pays semi-annual dividends of 4%?

The following problems will illustrate the possible risks in stock ownership, and also the possible gains by coöperative management.

1. A business corporation with a capital stock of $60,000 is formed to purchase a certain canal abandoned for freight purposes, and to utilize it for irrigation and other purposes. Its gross earnings are $7,000 per year. Three men, by securing a bare majority of the shares of stock, elect themselves Directors and, controlling the Board, vote themselves salaries and contracts for unnecessary supervision and repairs, to the total amount of $5,500 yearly, and the taxes of the corporation are 2%. What is left to be divided among the other stock holders?

2. What is the net income from their shares?

3. What per cent does the stock yield them?

4. If with a dividend of 6% the stock was valued at par, what is now its proportional market value?

5. What aggregate loss have the three controlling Directors sustained in the decline of their stock in market value?

6. What gain have they secured in their income from the possession of a controlling amount of stock?

7. A small railway company and a small steamboat company, each capitalized at $200,000, by rivalry and mismanagement, have reduced the net profits to 1.5%. Neither corporation is permitted by its charter to purchase the stock of the other. A "holding company" incorporated for the stated purpose of buying stocks, secures a controlling amount of the stock of both of the other corporations, and insures their coöperative management, causing them to yield a net profit of 7.5%. What is now the market value of the stock if, with net earnings at 6% the railway and steam barge companies' stocks are valued at par?

BONDS

While corporate companies usually provide the necessary capital for the conduct of their business, through the sale of *Stock Certificates*, they may also provide for additional capital by the sale of *Bonds*, which are generally secured by the tangible property of the corporation, and the bonds become the first lien on the business.

In case of default in the payment when due, either of the accrued interest or of the principal, the holders of the bonds are in most instances legally empowered to sell the property of the corporation issuing the bonds, and re-imburse themselves from the proceeds.

Bonds issued by the Federal Government, by a State, or by a county, must be previously authorized by legislation, or by a direct vote of the people, who become the guarantors.

It will be seen that the essential difference between stocks and bonds is that, in the former the stockholders are the *real owners*, and in the latter the bondholders become the *creditors* of the corporation issuing the bonds.

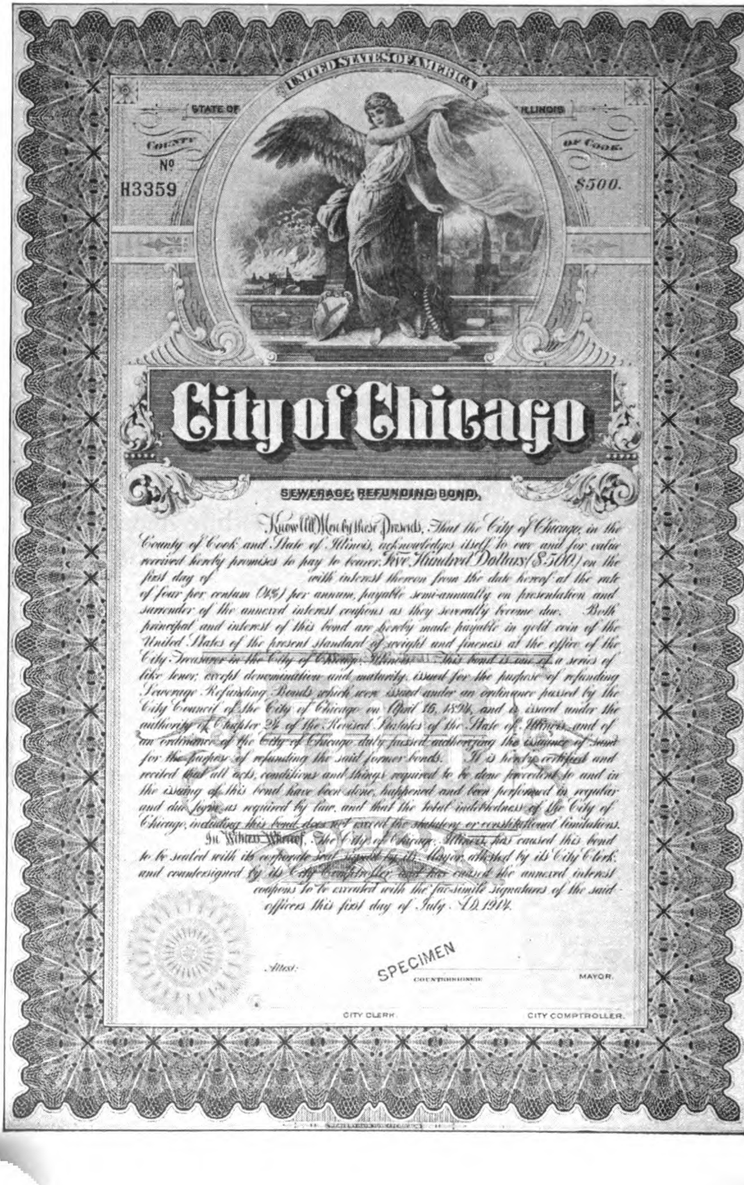
Bonds bear a fixed rate of interest, while the stock proceeds are governed by the earning power of the business.

In addition to the signature of the proper executive officers, all stocks and bonds must have affixed to them the seal of the Government, the State, the county, or the business corporation issuing them.

Facsimile of the Seal of the State of New York, reduced to about ⅔ of the usable size.

Facsimile of the great Seal of the United States, reduced to about ½ of the usable size.

Facsimile of the Seal used by a business corporation, reduced to about ⅔ of the usable size.


UNITED STATES OF AMERICA

STATE OF ILLINOIS

COUNTY OF COOK.

Nº H3359

$500.

City of Chicago

SEWERAGE REFUNDING BOND.

Know All Men by these Presents, That the City of Chicago, in the County of Cook and State of Illinois, acknowledges itself to owe and for value received hereby promises to pay to bearer Five Hundred Dollars ($500) on the first day of with interest thereon from the date hereof at the rate of four per centum (4%) per annum, payable semi-annually on presentation and surrender of the annexed interest coupons as they severally become due. Both principal and interest of this bond are hereby made payable in gold coin of the United States of the present standard of weight and fineness at the office of the City Treasurer in the City of Chicago, Illinois. This bond is one of a series of like tenor, except denomination and maturity, issued for the purpose of refunding Sewerage Refunding Bonds which were issued under an ordinance passed by the City Council of the City of Chicago on April 16, 1894, and is issued under the authority of Chapter 24 of the Revised Statutes of the State of Illinois and of an ordinance of the City of Chicago duly passed authorizing the issuance of said for the purpose of refunding the said former bonds. It is hereby certified and recited that all acts, conditions and things required to be done precedent to and in the issuing of this bond have been done, happened and been performed in regular and due form as required by law, and that the total indebtedness of the City of Chicago, including this bond, does not exceed the statutory or constitutional limitations.

In Witness Whereof, The City of Chicago, Illinois, has caused this bond to be sealed with its corporate seal, signed by its Mayor, attested by its City Clerk, and countersigned by its City Comptroller, and has caused the annexed interest coupons to be executed with the fac-simile signatures of the said officers this first day of July, A.D. 1914.

Attest: SPECIMEN

COUNTERSIGNED MAYOR.

CITY CLERK. CITY COMPTROLLER.

Bonds, like stock certificates, necessarily vary more or less in statement. The illustration on the preceding page shows the general form of a corporation bond and the illustrations on this page the general form of the detachable coupons.

Detachable coupons, or small, dated certificates, are generally issued with and as a part of each bond. These are to be separated from the bonds and presented for payment on the dates named upon them.*

These coupons are usually transferable, being deemed equivalent to cash, and are collectible by any person holding them.

Bonds without coupons are called *registered bonds,* and their transfer from one holder to another must be recorded upon the books of the corporation issuing them.

PRACTICE PROBLEMS

1. A village issues corporation bonds to the amount of $40,000 to build a schoolhouse. The bonds bear 5½% interest. What is the amount of interest paid to the bondholders in one year?

2. Allowing .12½% for brokerage, and buying 2% below par, 4 years before maturity, what is the profit on a corporation bond bearing 6%, at the time of its maturity?

3. An American business corporation operating a rubber plantation in Mexico issues bonds for $60,000, payable in 15 years. If they are sold at $93 for $100 of par value, what is realized from them when 80% of the issue is "taken," or sold?

*The coupons attached to a bond are numbered from right to left, or from the bottom up. In this way they may be detached in the order of number and date. For the bond here illustrated thirty-nine coupons are required—one for each semi-annual interest payment from January 1915 to January 1934—covering a period of 20 years.

4. In order to secure an income of $1,200 annually, how many bonds of the denomination of $100, bearing 6% interest, must I buy?

5. A bank buys 50 U. S. bonds of the denomination of $1,000, paying for them a premium of $3\frac{1}{4}$%, and a brokerage fee of $\frac{1}{8}$%. What do the bonds cost the bank?

6. How much must be paid for the Gas and Electric Company bonds of a certain city, at a premium of 3%, the brokerage being .25%?

7. If I paid $3,200 for bonds of the face value of $4,000 and receive 5% interest on the face of the bonds, what do I receive in a year, and what per cent do I receive on my investment?

8. If ten 6% Bridge bonds of a certain city, are bought at $90.00 for $100, and immediately exchanged for eleven 4% bonds, both kinds to run for ten years, what is my gain or loss through the exchange for the period?

9. Which is the better investment: a 5% bond bought at $92 for $100 face value, or 6% bond bought at par?

10. If the 6% bonds of a certain railway company are sold at 9% below par, and are paid off at the end of ten years, what per cent of profit is made by investing in them?

11. What must be paid for 5% bonds in order to receive an income of 6% on my investment?

12. If a man receives $480 for the coupons of his bonds, bearing 6%, what amount of the bonds does he hold?

13. To secure an income of 8%, how much below par must I buy bonds bearing 6%?

14. If you invest $5,000 in 4% bonds at par, and receive three semi-annual payments of interest and then sell the

bonds at a premium of 6%, what is the amount of your gain, and what per cent is it for eighteen months on your investment?

15. If a commercial corporation issues bonds to the amount of $60,000 to run eight years at 7%, and after paying interest for five years fails, and after a year of delay, pays only 78% of face of the bonds, what does the holder of each bond receive in all?

16. What would he have received on a 6% bond for the full period?

PARTNERSHIPS

A *partnership* is a union of two or more persons, with unlimited liability, for the conducting of some form of business or professional work. A mercantile partnership is called a firm, and its members are known as *partners*.

The capital stock of a firm is the amount invested in its business. Its assets are all the property it owns, including amounts owed to it. Its liabilities are the debt it owes. Equal partners are partners who own the business and property of the firm in equal shares. Having contributed equally to the capital of the firm, they are entitled to share its profits equally.

The capital invested by a partner may vary at different times; and in reckoning the share of the profits to which each is entitled, both the *amount invested* and the *time of continuance* of the investment must be considered. Thus if one partner invests $5,000 for 2 years, his claim is equal to that of a partner who invests $10,000 for 1 year, unless by special agreement some other arrangement is made.

The profits of a partnership are divided among the partners in the ratio of the amounts, by periods, of the capital supplied by them.

1. John Doe and Richard Roe conduct a partnership business for 18 months. John Doe invests $800 for 12 months and $1,000 for the remaining 6 months. Richard Roe invests $1,000 for 6 months, and $800 for the remaining 12 months. Their profits are $2,000. What is the share of each in the profits?

2. A, B, and C conduct a partnership business for 2 years, A and B investing $2,000 each, and C, $3,000. Their total profits for the time are $15,000. What is the share of each for each of the years?

3. A invests $3,200 for 6 months, and $4,300 for the remainder of the year, in a partnership with B, who invests $3,800 for the full year. The profits of the business are $4,500. What is the share of each?

4. Fours boys conduct a refreshment stand in a village on the Fourth of July. A invests $2.50, B, $5.00; C, $4.50 and D, $8.00. The profits are $42.80. What is the share of each?

5. Three young women conduct a millinery establishment in partnership for 3 years, A invests $250 for one year, $200 for another year, and $150 for the third year. B invests $80 for one year, $100 for another year, and $150 for the third year. C invests $150 for the entire time. Their profits are $6,900 for the 3 years. What is the share of each?

6. A, B, and C invest $10,000 each in a store, in partnership. A withdraws $5,000 at the end of one year, B the same amount at the end of two years, and C the same amount at the end of three years. If the net profits for four years equal $40,000, what is the share of each partner in them?

7. If settlement is made at the close of each year, and the partnership extended for the year to come, and in the last year there is a net loss of $10,000 what is each partner's share of the loss?

Exchange, in arithmetic, relates to the making of payments by means of drafts. Where the exchange takes place between different cities in one country, it is called *domestic* exchange. Where the exchange is effected between cities in different countries, it is called *foreign* exchange.

Drafts

Where the obligations of the business houses (including banks) of one city to those of another city are pretty evenly balanced by obligations of the latter to the former, there will be no occasion for the transmission of cash from one city to the other for the settlement of the obligations; the obligations of the business houses of the one city will largely cancel those of the other, and this is effected by the use of drafts.

Where, however, there is much more money due in one city *from* the business houses of another city than is due from these *to* them, there must be a shipment of money to "settle the balance." Because of the desire to avoid this as far as possible, bank drafts drawn on the city in which the surplus of credits is held command a slight premium; while drafts drawn on the city in which the surplus of debits are held depreciate slightly in value, and are sold at a discount.

Thus, if the business houses of St. Louis owe to those of New York a large surplus over the indebtedness of the latter to them, drafts drawn on New York will command a premium* in St. Louis, and drafts on St. Louis will be procurable at something less than their face value in New York.

According to the condition of the money market, drafts may be "at par" or they may "appreciate" or "decline."

*Some amount over the face value.

Where exchange is rated above or below par, the premium or discount is calculated on the face value of a draft.

1. Find out the cost of a sight draft for $500 where there is in the money market a premium of $\frac{3}{4}$%.

2. Where exchange is at a discount of $\frac{3}{4}$%, what will be the cost of a sight draft for $500?

3. What is the value of a sight draft for $1,200, where exchange is at a premium of $\frac{1}{2}$%?

4. What will be the cost of a sight draft for $625, where exchange is at a discount of $\frac{1}{2}$%?

Time Drafts

In the case of time drafts, the element of time has to be taken into account. The premium, or discount, is calculated on the face of the draft. The bank discount for the time specified is deducted from the face of the draft; and the premium or discount is added to or subtracted from the remainder.

1. A sixty-day sight draft for $300 must be bought where exchange is at premium of $\frac{1}{2}$%. What is the bank discount? What is the difference between the bank discount and the face of the draft? What is the premium on the face of the draft? What is the cost of the draft?

2. What can be obtained for a draft for $400 payable in 90 days from sight, and discounted at the time of its acceptance? What was its cost at a discount of $\frac{3}{4}$%?

3. What must I pay for a 60-day sight draft for $600, the premium being $\frac{1}{4}$%?

4. What will be the cost of a draft for $1,000 payable 60 days after sight, exchange being at a premium of $\frac{1}{4}$%?

5. What will be the cost of a draft for $600 payable 30 days after sight, the exchange being $\frac{1}{2}$% discount?

Drafts drawn on banks of foreign countries are called *bills of exchange.*

In stating the terms on which bills of exchange may be purchased, it is customary not to state the rate of premium or discount, but to add to the standard unit of money, in the country to which the draft is to be sent, the actual premium on it, or to subtract the actual discount from it. Thus if $4.88 is paid in New York for each pound of exchange drawn on London or Liverpool, the exchange in New York is quoted, "London, $4.88."

In quoting the exchange drawn on Paris, the amount stated is the money in francs that can be bought in New York for $1.00 of American money. Thus if $1.00 will buy 5.15$\frac{3}{4}$ francs in exchange drawn on Paris, the quotation is "Paris, 5.15$\frac{3}{4}$."

Exchange drawn on Berlin is so stated as to indicated the cost in American money of 4 marks. Thus if for every 4 marks drawn 95$\frac{1}{4}$ cents must be paid, the quotation is "Berlin, 95$\frac{1}{4}$."

1. What is the cost in Boston of a bill of exchange for $6,000 drawn on Liverpool, when the exchange is quoted "Liverpool, $4.8775?"

2. What is the cost in New York of a bill of exchange for $4,000 drawn on Hamburgh, when the exchange is quoted "Hamburgh, 95$\frac{1}{4}$?"

3. What is the cost in Charleston of a bill for $2,400 drawn on Bordeaux, when the exchange is quoted "Bordeaux, 15$\frac{3}{4}$?"

4. What is the cost, in New Orleans, of a bill of exchange for $6,000 drawn on Marseilles, when the exchange is quoted "Marseilles, 15$\frac{1}{2}$?"

ISSUED IN $10. $20. $50. AND $100.

Tourist companies, express companies, and sometimes banks, issue travelers' checks, in convenient amounts of $200, $100, and sometimes $20 and $10, or other sums, for travelers to carry with them on tours to foreign lands. These are bound together in the form of a folded check book, and are detachable one at a time. The purchaser of a book of such checks must sign each one of them in the office at which he procures it, and afterwards, when he cashes it, at one of the foreign agencies of the company or bank issuing it, or at one of their agencies in his own country.

1. If I buy travelers' checks to the amount of $486.65, and pay $\frac{1}{2}\%$ of the aggregate face of them for the accommodation, what do the checks cost me? How much do I receive for them in London?*

2. If, instead of cashing all the checks in London, I cash those for half the amount in Paris,† what do I receive for them? If in Rome what? If in Madrid what?

3. If I cash all the checks in Berlin* and Munich, what do I receive for them?

4. If I cash half the amount of the checks in Moscow and half of them in Amsterdam, what do I receive for them?‡

*For foreign equivalent, see page 175 †See page 178. ‡See page 180.

MONEY OF OTHER LANDS

So extensive are the trade and travel among the nations of to-day, that some acquaintance with the values of the principal denominations of money used in foreign lands is important in the study of arithmetic.

The British Empire, including many lands scattered all over the globe, has not adopted for general use the Decimal System of money, which we find so convenient, though the latter is in use in the Dominion of Canada, which is a part of the Empire; but there is coined in England a piece of money which is a close approximation to our dollar (the double florin, worth 97.4 cents).

It is to be noted that the basis of our coinage, the dollar, is about midway in value between the high British basis, the pound sterling, and the low French basis, the franc; for it is but little more than one-fifth of the former, and but little more than five times the latter.

The exact value of the British pound sterling in our money is *four dollars, eighty-six cents, six and one-half mills* ($4.8665). The sign of the pound sterling is £, which is the script *L*, in quaint form, the initial of the Latin word *libra*, which in ancient times meant a Roman pound weight. When the name was changed from *libra* to the French word *livre*, and thence to the English word *pound*, the old initial, as used to indicate money, was not changed. One-twentieth of the ancient pound was called by the name *solidus*, which has become changed to *shilling*, and now designates one-twentieth of the pound sterling, the initial (s.) remaining unchanged. One-twelfth of the shilling was called the *denarius*. This word, applied now to a bronze coin, has been changed to *penny;* but the old initial (d.) to designate it is still retained. The farthing, a bronze coin, is equal to one-fourth of a penny, or to one-half of a cent of our money.

Besides these coins, the British make use of the *crown* (equal to two shillings). "Pounds," "shillings," and "pence," however, are the names commonly in use, whatever coins may be meant. The pound is really money of *account;* for the coin corresponding to it is always called a *sovereign* when considered as a *coin.* The guinea, likewise a gold coin, is worth 1 shilling more, for it is equal to 21 shillings. It is now but little used. While in Canada the coins are based on the Decimal System, like our own, there is much use made of English money in reckoning, particularly in Eastern Provinces.

English Money

4 farthings (far.) make 1 penny (d.).

12 pence make 1 shilling (s.).

20 shillings make 1 pound sterling (£).

1. How many pence are there in 4s. 6d.?

2. How many shillings are there in £3 4s.?

3. What is the value in U. S. money of £100?

4. What is the value in English money of $486.65?

5. One dollar is what decimal fraction of a pound sterling?

6. State in pounds sterling the value of $100.

7. If you pay 2s. 6d. for a book, and sell it for 3s., what is the % of your profit?

8. If you buy a souvenir in England for 10s. 4d. and damage it to the amount of one-fourth of its value, what is it then worth?

9. At 2s. the pound, what is the cost of a quarter of a pound of tea?

10. How many penny rolls can be bought for 3s. 6d.?

11. What will be the exchange value, in U. S. money, of a set of the English coins that have been named?

12. If you exchange $15 for English money, without charge for services, what will you receive?

13. If you exchange £6 3s. 8d. for U. S. money, with· charge for services, what will you receive?

14. If the cash drawer in a British shop is found to contain 15 sovereigns, 40 crowns, 18 florins, 60 shillings, 90 pennies, 50 half-pennies, and 20 farthings; what is the sum of its contents?

15. What is the value of £15 18s. 9d. in U. S. money?

16. What is the value of $6.75 in English money?

17. As a matter of convenience in the rapid reckoning of small accounts, what approximate values would you assign to the pound, the shilling and the penny?

18. If in the course of a day you expended £4 7s. 9d., what would be your rapid calculation of the approximate amount?

19. How much would such an approximate amount differ from the exact amount?

Other Monetary Systems

For convenience in computation, the other monetary systems of the modern world are generally *decimal*, *centesimal*, or *millesimal*.

In a *decimal* system, the denominations increase in a tenfold ratio. Our cents, dimes, dollars, and eagles* are an illustration of this system. The decimal system is in use, likewise, in Canada.

In a *centesimal* system, the denominations increase in a hundred-fold ratio. In most nations of the Old World this system is used.

In a *millesimal* system, the denominations increase in a thousand-fold ratio. This system is in use in Brazil and Portugal.

*Our eagles are commonly known as ten-dollar gold pieces.

While there is much variety in the basic units of value among the different nations, there are several countries of Europe in which the value of the basic units is exactly the same. Among these countries are France, Italy, Spain, Roumania, Servia, and Bulgaria.

In France the basic unit is called the *franc,* and is equal to one hundred *centimes* (*sahn-teem*)*. The franc is commonly reckoned as being equivalent to one-fifth of our dollar, or twenty cents; its exact value being $0.193. The *centime* is deemed equal to one-fifth of our cent. In Italy the basic unit is called the *lira* (lee′rah), and is equal to one hundred *centesimi* (*cen-tes′ee-mee,* plural of *centesimo*).

In Spain the basic unit is called the *peseta* (*pay-say-tah*), and is equal to one hundred *centimos* (*sen′tee-moes*).

In Roumania the basic unit is called the *ley* (lay), and is equal to one hundred *bani* (*bah′nee,* plural of *ban*).

In Bulgaria the basic unit is called the *lev,* and is equal to one hundred *stotinki,* (plural of *stotinka*).

In Servia the basic unit is called the *dinar* (*dee′nar*), and is equal to one hundred *paras* (*pah′ rahs*).

The franc, the lira, the peseta, the ley, the dinar, and the lev are all exactly equal; each being reckoned at about twenty cents in our money.

Likewise the centime, centesimo, centimo, ban, para, and stotinka are all exactly equal, each being reckoned as worth one-fifth of our cent. It follows that the franc, the lira, the peseta, the ley, and the lev may all be written in one column, and added together, since they are only different in names for the same value. Likewise the

*There are some coins of a value between the centesimal denominations. Thus in France the sou (soo) is equal to 5 centimes, or to one of our cents.

centime, the centesimo, the centimo, the ban, and the stotinka may all be written in one column, and added together.

Since all these systems are centesimal, they are treated as our dollars and cents, the decimal point separating the francs, pesetas, lire,* leys, or levs from the centimes, centesimi, centimos, bani, or stotinki. Thus two francs and seventy-five centimes are written 2.75; three lira and fifty centesimi are written 3.50; four pesetas and seventy-five centimos are written 4.75.

Problems relating to Monetary Units of equal value.

Reduce to U. S. money 2 francs, 45 centimes.

```
  $0.193
    2.45
 -------
     965
    772
   386
 -------
$0.47285
```

The value of a franc is $0.193. 2 francs, 45 centimes are to be regarded as 2.45 francs. Discarding the fraction of a cent, we find the value to be 47 cents.

Problems relating to Monetary Units of different values.

1. What is the sum of 4 francs, 5 centimes; 6 francs, 15 centimes; 25 francs, 10 centimes; and 15 francs, 75 centimes?

2. What is the sum of 16 lire, 20 centesimi; 20 lire, 5 centesimi; 4 lire, 65 centesimi; and 10 lire, 10 centesimi?

3. Add 14 pesetas, 10 centimos; 16 pesetas, 18 centimos; 25 pesetas, 14 centimos; and 25 pesetas, 58 centimos.

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*Pronounced lee′ray; plural of lira.

4. What is the exact value, in our money, of 24 lire, 15 centesimi?

5. What is the exact value, in our money, of 40 francs, 24 centimes?

6. What is the exact value, in our money, of 60 pesetas, 25 centimos?

7. What is the exact value, in our money, of 28 leva (plural of lev), 26 stotinki?

8. What is the approximate value, in our money, of 15 francs? Of 50 lire? Of 75 pesetas? Of 42 leva? Of 75 centimos? Of 60 centimes? Of 50 centesimi?

In Holland the guilder is the monetary unit, and is almost exactly equal to two-fifths of our dollar ($0.402), having a little more than double the value of the franc, lira, etc. It is equal to one hundred Dutch cents, or gulden, each of which is about double the value of the centime, centimo, etc.

In Germany the *mark* is equal to $0.238, or a little less than our quarter of a dollar, and is the equivalent of one hundred pfennigs, the pfennig being worth about one-quarter of our cent.

In Russia the ruble is worth a little more than our half-dollar ($0.515), and is the equivalent of one hundred kopeks, the kopek being about equal to half of our cent.

In Japan the yen is almost exactly equal to our half-dollar, and is the equivalent of one hundred sen; the sen being equal to half of our cent.

In Austria-Hungary the *krone* (kro-ny) is worth less than the German mark, and one cent more than the French franc, its exact value in our money being $0.203. It is usually counted as equal to twenty of our cents. It is the equivalent of one hundred *heller*, and a heller is reckoned as worth one-fifth of our cent, or $0.002.

1. Add 4 guilders, 6 (Dutch) cents; 15 guilders, 12 cents; 26 guilders, 80 cents; and 4 guilders, 50 cents. State in terms of our money the exact equivalent of the sum.

2. What is the sum of 20 marks, 15 pfennig; 42 marks, 25 pfennig; 68 marks, 40 pfennig; and 12 marks, 20 pfennig? What does the sum equal, exactly, in our money?

3. Add 20 rubles, 15 kopeks; 28 rubles, 65 kopeks; 29 rubles, 11 kopeks; and 15 rubles, 26 kopeks. State in terms of our money the exact equivalent of the sum.

4. Add 60 yen, 30 sen; 45 yen, 15 sen; 26 yen, 45 sen; and 14 yen, 20 sen. What is their exact equivalent in our money?

5. Find the sum of 25 marks, 20 pfennigs; 14 marks, 35 pfennigs; 16 marks, 42 pfennigs; and 28 marks, 13 pfennigs. What is the exact value of the sum in our money?

6. If a relative traveling in Europe buys for you at St. Petersburg a souvenir for 2 rubles, at Berlin another for 2 marks, at Amsterdam another for 3 guilders and fifty cents, at Vienna another for 2 krone and 50 heller, what is the approximate sum of the purchases in U. S. money?

7. If at Paris he buys a souvenir for 2 francs 50 centimes, at Rome another for 4 lira 10 centesimi, and at Barcelona another for 3 pesetas 50 centimos, what is the approximate sum of the purchases in U. S. money?

8. What is, in U. S. money, the approximate value of 4 marks, 7 kronen (plural of krone), 6 guilders, and 6 rubles?

9. What is the approximate value, in our money, of 8 guilders, 25 cents; 2 marks, 50 pfennigs; 8 kronen, 50 heller; 4 rubles, 20 kopeks?

10. What, in U. S. money, is the approximate value of 20 francs, 30 lira, and 16 pesetas?

11. What, in U. S. money, is the value of 4 francs, 10 centimes; 3 lira, 50 centimes; and 10 pesetas, 20 centimos?

In Portugal the milreis (mil′-race), which is worth $1.08 in American money, is conceived to be made up of 1000 reis (race). The rei (ray) is not coined, but is used in accounting. It is reckoned as worth one-tenth of a cent, or $0.001.

In Brazil the milreis is worth $0.546, and the rei is reckoned as worth one-eighteenth of a cent. The milreis is written with the dollar mark before the three ciphers, thus: 1$000.

1. What is the value of 180$000, Brazilian, in U. S. money?

2. What is the value of $5.46 in milreis, Brazilian?

3. What is the value of $109 in milreis, Portuguese?

4. How many reis are there in 48$000?

5. 70000 reis are how many milreis?

6. Change $100 to milreis, Brazilian.

7. Change $100 to milreis, Portuguese.

8. If at Rio Janeiro you purchase a souvenir worth, in our money, $5.46, what will be the amount of your bill in Brazilian money? Write the amount.

9. If at a restaurant in Bahia you are charged 1500 reis for a lunch, what does it equal in American money.

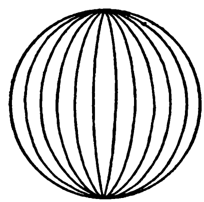
10. If a friend writes you from Lisbon that he paid 3000 reis for an automobile ride, what do you understand to be the cost of the ride in American money?

LONGITUDE AND LATITUDE

Short distances on land are measured .by means of the surveyor's chain, which is sixty-six feet long, and has one hundred links.

Distances on the water, and long distances by land, are measured by observing the sun and other heavenly bodies, which seem to pass over the heavens and entirely around the world in twenty-four hours.

Distance east and west, measured in this way, is called *longitude.* This old word meant *length;* and the ancient peoples who lived on the shores of the "long-east-and-west" Mediterranean Sea supposed that the *length* of the world was east and west. They did not know that the world is round, and they gave us the word *longitude.* It is customary to measure longitude from some great observatory, where the heavenly bodies are observed through the best instruments. Since the one at Greenwich (pronounced grin'nij), near London, England, is the best known in the world, longitude is generally reckoned from that one.



To mark the distances east and west, imaginary lines are drawn north and south, from the North Pole to the South Pole. These imaginary lines are called *meridians.* In geography, then, a meridian is an imaginary north-and-south line extending from one Pole to the other. Every place on the earth is on some meridian. 180 *meridians are marked on maps.** Since in geography each meridian ends at the Poles, each is a half-circle, or semicircle. While all the meridians

*These are called Principal Meridians.

start together at the Poles, they grow farther and farther apart until they reach the midway line between the Poles, which is called the Equator. Here they are farthest apart.

1. Since the sun appears to pass over all the three hundred and sixty degrees of the earth's circumference in twenty-four hours, how long a time is taken for it to pass, seemingly, over one degree?

2. If the sun is seen on the meridian of your schoolhouse at noon, when will it be seen on the meridian of a place two degrees west of the schoolhouse?

3. What time is it at that place when it is noon at the schoolhouse? What time is it then at a place two degrees east of the schoolhouse?

4. Since West Longitude is reckoned half around the world west from Greenwich, and East Longitude is reckoned half around the world east from Greenwich, how many degrees of each are there?

5. If it is ten o'clock in the morning at one place, and at the same time it is eleven o'clock in the morning at another place, how far apart are the meridians of the two places?

6. If a man sets his watch at twelve o'clock when the sun appears on the meridian of his home, and without changing his watch (which is supposed to keep good time) travels westward until he finds that the sun appears on his meridian at ten o'clock by the watch, how many degrees westward has he traveled?

7. If a man starts at sunrise and travels three degrees west before sunset, will he have the same period of daylight that he would have had if he had remained at home? Will his day be longer or shorter? How much, and why?

8. If he starts at sunrise and travels three degrees east before sunset, will his day be longer or shorter? How much and why?

The length of a degree of longitude, measured in miles, is constantly changing as you pass northward or southward. At the Equator it is longest, for there the meridians are farthest apart. Near the Poles the length is very small; and at the Poles it is nothing at all, for there the meridians all come together in a point.

9. At the Equator a degree of longitude is 69.65 miles. As far from the Equator as Toledo, Ohio, it is only 53.43 miles. What is the difference in miles?

The Equator, which crosses every meridian at right angles, is the line from which distance is measured in degrees north and south. All circles around the globe to the north or to the south of the Equator are smaller, and they grow less toward the Poles. They mark distance north or south from the Equator. Distance thus measured is called *latitude.* This old word meant *breadth*; and the ancients believed that the breadth of the world lay north and south, as its length lay east and west. The Equator is the largest circle that can be drawn upon the globe. It is about 25,000 miles in length. Two opposite meridians meeting at the Poles, and taken together, would form a circle as large as the Equator if the world were exactly round; but since it is slightly flattened at the Poles, the two meridians would measure somewhat less than the equator.

The imaginary circles to indicate latitude, or distance north or south from the Equator, are called *Parallels of Latitude.* Unlike the Meridians, they never meet. All degrees of latitude would measure exactly the same number of miles, but for the slight flattening of the earth at the Poles. As it is, they vary but little and are longest near the Poles. A degree of latitude at the Equator is 68.704 miles. At the Poles it is 69.407 miles.

1. What part of the Earth's circumference is the distance from the Equator to one of the Poles?

2. How many degrees of North Latitude can there be?

3. How many degrees of South Latitude can there be?

4. Were the ancients wholly wrong in considering the earth *longer* east and west than north and south?

5. What is the greatest difference in miles, in the length of the degrees of latitude?

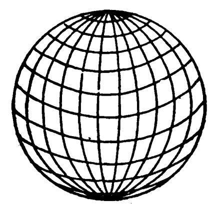
6. How many degrees of East Longitude are there?

7. How many degrees of West Longitude are there?

8. How many degrees of latitude are there altogether?

9. How many degrees of longitude are there altogether?

If the exact latitude and longitude of a place are known, the place can be found by observation of the heavenly bodies, and without any other measurements.



Every place has its parallel of latitude, as well as its meridian; but *only a limited number of the parallels are drawn upon the map.*

All places lying on the same meridian always have really the same time, whatever their latitude, because they are equally far west or east. No two places lying on the same parallel of latitude can ever have really the same time, whatever their longitude, because one is farther east or west than the other, and the sun appears to pass westward.

1. A degree of longitude in latitude 42° N. is 51.483 miles in length. That is about the latitude of Providence, R. I., and of Chicago, Ill. What is the distance around the world in that latitude?

2. A degree of longitude at the Equator is 69.172 miles in length. How great is the distance around the world at the Equator?

3. A degree of longitude in Latitude 30° N. (New Orleans) is 59.956 miles in length. What is the distance around the world in that latitude?

4. A degree of longitude in Latitude 40° N. (Indianapolis) is 53.063 miles. What is the distance around the world in that latitude?

5. Degrees of latitude are of nearly the same length wherever measured. The first degree of latitude north or south from the Equator is 68.704 miles in length. The last degree, reaching the Pole, is 69.407 miles in length. What is their difference in length?

6. If you were at the Equator, the Pole Star would be seen on the horizon. Its altitude (height) would be 0°. If you were at the North Pole, the star would seem to be right over-head. How many degrees high would it be?

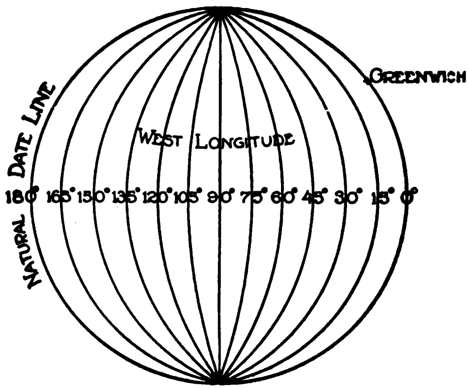
7. Your latitude is the same as the altitude of the Pole Star. How many degrees is it?* How many degrees high is the star, where you live?

8. For every degree of the earth's surface that you travel north, the Pole Star rises one degree. How high would it be to you if you should travel six degrees north from the place where you are?

9. For every degree of the earth's surface that you travel south, the Pole Star sinks one degree. How high would it be to you if you should travel six degrees south?

*Supply this information by referring to a map.

LONGITUDE AND TIME



Since the earth rotates on its axis from west to east, a north-and-south line, called a meridian, on its surface seems to pass directly beneath the sun at one time in the 24 hours of the day. That time is called "high noon," or "noon by the sun." To the inhabitants of the earth it seems that the sun is moving while the earth remains at rest. As the meridian moves on to the eastward, the sun seems to sink in the western sky. Then it disappears from sight until the meridian has so far completed its circuit that the sun appears again, this time on the eastern horizon, whence it seems to rise in the sky until another "high noon" is reached. The meridian, meanwhile, with the turning of the globe, has described a complete circle of 360 degrees in 24 hours, which is 15 degrees for each hour. It has passed through 15 *minutes of arc* (circular measure) in each minute of time; likewise, through 15 *seconds* of *arc* in each second of time.

If the stars, as well as the sun, could be seen in the day time, and a certain star were seen exactly north or south of the center of the sun's disk at high noon, the meridian would return to its place directly beneath the star after one complete rotation of the earth, but would *keep on turning four minutes of time longer* before it would be directly beneath the sun again. This is because the sun does not keep its place among the stars, but seems to move through them in a regular course lasting through a year. Thus we say that the sun is at one time in a certain *Sign*, called Aries, then in another Sign, called Taurus, etc.; the Signs being convenient divisions of a great belt of stars called the Zodiac, extending around the whole heaven. Really, it is the revolution of the earth around the sun that causes the seeming movement of the sun through all this belt of stars. Sometimes the sun seems to move more rapidly, and sometimes more slowly; for the earth's revolution about it is not uniform. Our clocks are made to keep *average* time, so as to make even periods of 24 hours each. Thus the earth makes more than a complete rotation between two high noons. A day by the sun is longer than a day by the stars; but the rotation of the earth on its *axis* is *regular*, as is proved by observing the fixed stars.

The sun passes over 15° of longitude in 1 hour of time. The sun passes over 15′ of longitude in one minute of time. The sun passes over 15″ of longitude in 1 second of time.*

A north-and-south line imagined to run through any point or place on the earth's surface, and to extend to

*Apparently.

the Poles, is called the *meridian* of the point or place. All points north or south of that point are on the same meridian with it. No point east or west of it can be on the same meridian with it.

The meridian indicates the *longitude*, or distance in degrees east or west from a certain meridian, called the *Prime* Meridian. As a matter of convenience, the meridian of Greenwich (grin'nij), London, England, is made the Prime Meridian. Since longitude east and west is measured from it, the Prime Meridian is said to be longitude 0° (Lon. 0°).

There are 180 degrees of longitude measured eastward from that of Greenwich (Lon. E.), and 180 degrees measured westward from it (Lon. W.). Lon. 180° may be either E. or W., since it is reached by measuring half-way around the earth in both directions.

The meridians, which mark even hours of time, are 15° apart; and the sun requires $\frac{1}{15}$ of an hour, or 4 minutes of time, to pass over the distance from one degree of longitude to the next.

1. How many degrees apart are the meridians Lon. 3° E. and Lon. 2° W.?

2. What is the difference in time by the sun (called "local time") between these two meridians? At which meridian is the time later?

3. When it is noon at Lon. 16°E., what time is it at Lon. 80° 30′ W.?

4. When it is 1 p. m. at 0°, what time is it at Lon. 120°E.?

5. If I set my watch at Greenwich and carry it west with me to Lon. 90°W., without resetting it, how much too slow will it be?

6. If I journey eastward from Greenwich, without resetting my watch until I find that it is 2½ hours slow, what longitude have I reached?

7. Rome is in Lon. 12° 27′ 14″ E., and Philadelphia is in Lon. 75° 9′ 5″ W. What is the difference in time between these cities?

8. Chicago is in Lon. 87° 34′ 8″ W., and Boston is 71° 3′ 30″ W. When it is noon at Chicago, what time is it at Boston?

9. Paris is in Lon. 2° 20′ 30″ E. When it is noon at Paris, what time is it at Chicago?

10. What is the meridian thirty degrees east of Lon. 100° W.

11. What is the meridian of one hundred degrees east of Lon. 30° W.?

12. San Francisco is in Lon. 122° 25′ 42″ W. Washington, D. C. is in Lon. 77° 03′ 06″ West. What is the difference in time between these cities?

13. Honolulu, H. I., is in Lon. 157° 52′ W. New York is in Lon. 53° 58′ 24″ W. What is the difference in time between them?

14. Berlin is in Lon. 13° 23′ 44″ E. How much "slower" is its time than the time at Greenwich?

15. In some old books and on some maps, the meridian of Washington is taken as the Prime Meridian, or meridian from which longitude is to be reckoned east and west.

Washington is in Lon. 77° 3′ 56. 7″ W. from Greenwich. The meridian of Berlin, which is sometimes used in Europe as a Prime Meridian, is 13° 23′ 44″ E. from Greenwich.

How far apart are the meridians of Berlin and of Washington?

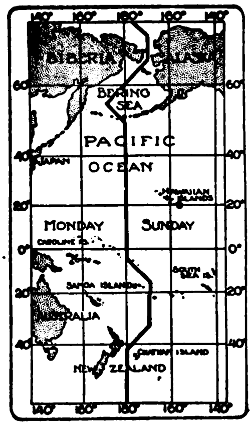
16. San Francisco is in Lon. 122° 25′ 42″ W. from Greenwich. What is its Lon. W. from Washington?

17. Philadelphia is in Lon. 75° 9′ 54″ W. from Greenwich. What is its longitude from Washington?

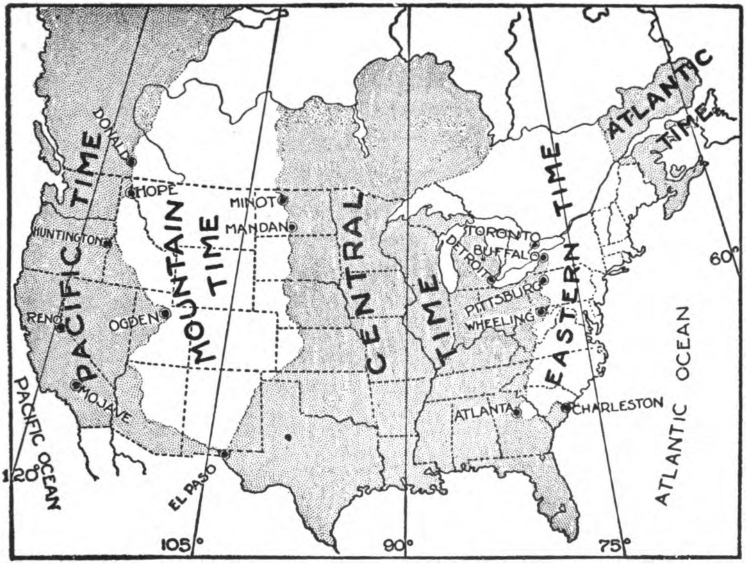
18. Find from a map through what states in this country the one-hundredth meridian passes.

19. When it is noon at Greenwich, what time is it at Lon. 180°?*

20. When it is noon at Lon. 180°, what time is it at Greenwich?



*It is easy to name the hour, but not so easy to name the day. If it is Wednesday noon at Greenwich, it will seem to be the midnight of Tuesday to a man who has traveled westward from Greenwich through the 180°. It will seem to be the midnight of Wednesday to a man who has traveled from Greenwich eastward through the 180°. To the man who travels westward each day is longer than he would find it if he remained at one place; for he is in one longitude when his day begins, and in another when it closes. Consequently, when he has gone half round the world, he has gained half a day's time over the local time. Likewise, a man traveling eastward finds that each of his days is shorter than he would have found it had he remained at one place. Hence when he has gone half round the world, he has lost half a day. When the westward-bound man and the eastward-bound man meet at Lon. 180°, they find that the half-day gained by the one and the half-day lost by the other cause them to be a day apart in their reckoning. One says that it is Tuesday, while the other says that it is Wednesday. It happens that there is practically no land at Lon. 180°, since the meridian runs through the Pacific. This is the natural "date line" of the world; but where the line runs between islands desiring to have the same day, the date line has been varied by international agreement so as to swerve to the east of Lon. 180° in the southern part of the Pacific, and to the west of it in the northern part of the same ocean, while in the central part it follows the meridian. When it is Sunday east of this line it is Monday to the west of it. If a ship going westward reaches the Date Line on Sunday morning, the time is called Monday morning, one whole day being dropped. When a ship going eastward reached the Date Line at 6 a. m. Monday morning, the time is called 6 a. m. Sunday morning and the passengers thus have another Sunday in immediate succession.



STANDARD TIME

All places on the same degree of longitude have the same local time, however far apart they may be north and south; but by far the greater amount of travel in the world is in an easterly or westerly direction, and to one traveling east or west the local time changes constantly. It is especially important that railways shall have an unvarying standard of time for long distances. Hence a system of Standard Time has been adopted for this great country, by which its area has been divided into four great time sections, known as the Divisions of Eastern Time, Central Time, Mountain Time, and Pacific Time. At all points in any one of these Divisions, the time is made artificially the same. When it is noon in the Eastern Division, it is 11 o'clock in the Central Division, 10 o'clock in the Mountain Division, and 9 o'clock in the Pacific Division. Thus the Divisions are, successively, one hour apart.

When travelers going east or west arrive at the boundary line of one of these divisions, they set their watches ahead or back, to correspond with the time in the next division. The Southern Pacific Railway makes no use of Mountain Time, but passes directly through from Pacific Time to Central Time.

It will be noted that the Maritime Provinces of the Dominion of Canada make use of what is called Atlantic time, which is the time of the meridian of Lon. 60° W. This time is not employed in the United States.

1. When it is 12:15 a. m. at Chicago, what time is it in New York (Standard Time)?

2. When it is 4:32 p. m. at San Francisco, what time is it in Chicago?

3. When it is 9:45 at New York, what time is it at Denver?

4. How much does the Atlantic time of the Dominion of Canada differ from Greenwich time?

5. Detroit is separated from Canada by a narrow strait, or river. Under the artificial arrangement of Standard Time, what is the difference in the time between the two banks of the stream?

6. Buffalo, being at the line of division between Eastern Time and Central Time, makes use of both. How far apart are clocks and watches found to be in a city so situated? Mention some other cities on the dividing lines of Standard Time Divisions?

7. If El Paso should make use of Mountain Time it would have the time of what meridian? Would this be near the local time of the place?

8. How far apart are Pacific Time and Atlantic Time?

THE METRIC SYSTEM

In the time of the French Revolution, over a century ago, there was a disposition to change many things, among them the calendar and the systems of weights and measures. The proposed new calendar failed; but the new scheme for weighing and measuring was so scientific in plan and so convenient in its use, that it has grown in favor with the world, until now it is used around the globe, except among the English-speaking peoples. Even in the United States and in the British Empire it is in use in a limited way, being employed very generally in scientific laboratories and books, and is recognized by law.

It has all the advantages of the decimal system, which we find so convenient in American money, since we can multiply or divide its denominate numbers by merely moving a decimal point.

The need for a more general knowledge of this system in this country is growing from day to day, in view of our increasing trade with the nations which use it exclusively. Without mastering it we cannot readily understand the trade catalogues of their business houses or the bills sent us for articles purchased; nor can we make them readily understand our own price lists and bills of goods sold to them without writing these in terms of the metric system. No ambitious pupil of the present day can afford to slight the metric system in his study of arithmetic.

The basis of the metric system is a ten-millionth of the quadrant (or a forty-millionth of the circumference) of the earth, as calculated when the system was devised. This basis is a measure called the *meter*, a little more than one and one-tenth yards in length. It has been found that there was a slight error in the computation of the earth's quadrant, but there has been no disposition to change the standard unit on that account. The auxiliary syllables used to designate ten, one hundred, one thousand, and ten thousand units of the metric system have been previously presented.

1. How many units are signified by the auxiliary *deca?*

2. What is meant by the auxiliary *hecto?*

3. What is the meaning of the auxiliary *kilo?*

4. What is indicated by the auxiliary *myria?*

5. How do we form a word to express ten units of a certain kind? Illustrate.

6. How do we form a word to express one hundred units of a certain kind? Illustrate.

7. How do we form a word to express a thousand units of a certain kind? Illustrate.

8. How do we form a word to express ten thousand units of a certain kind? Illustrate.

For more extended exercise in *myria*, *kilo*, *hecto*, and *deca*, presented on pages 5 and 6, the dictations which follow are given. The learner did not there concern himself with the denominations further than to read and write the numbers that express them. The purpose was to master *myria*, *kilo*, *hecto*, and *deca*, as belonging to numeration and notation.

Review Problems

To aid in a clearer understanding of this increasingly important study,* pupils should keep constantly before them the following summary of facts:

The **meter** *is the unit measure of* **length.**
The **square meter** *is the unit measure of* **surface.**
The **cubic meter** *is the unit measure of* **volume.**
The **gram** *is the unit measure of* **weight.**
The **liter** *is the unit measure of* **capacity.**

1. 4,200 liters are how many kiloliters and how many hectoliters?
2. 340 grams are how many hectograms and how many decagrams?
3. 20,000 amperes are how many kiloamperes?
4. 532 meters are how many hectometers, how many decameters, and how many meters?
5. 25 grams are how many decagrams and how many grams?
6. 96 liters are how many decaliters and how many liters?
7. 25,000 coulombs are how many myriacoulombs?
8. 4,000 meters are how many kilometers?
9. 220 grams are how many hectograms?

1. 5,020 liters are how many kiloliters and how many decaliters?
2. 220 meters are how many hectometers and how many decameters?
3. Write in kiloamperes 5,000 amperes.
4. Write in decameters 20 meters.
5. Write in kilowatts 8,000 watts.

*The Panama Canal, and the growth of international trade resulting therefrom, add greatly to the importance of a mastery of the Metric System; for all the nations of the Western Hemisphere to the south of us employ this system in their trade.

In the following problems use both common and decimal fractions:

1.	Write in hectometers	125	meters.
2.	Write in myriacoulombs	32,500	coulombs.
3.	Write in kilowatts	8,250	watts.
4.	Write in kiloamperes	150,000	amperes.
5.	Write in hectograms	425	grams.

6. Write in decameters $22\frac{1}{2}$ meters.

The auxiliaries learned deal only with whole numbers.

THREE NEW WORD AUXILIARIES

For fractional quantities, three other substitutions of syllables are used in like manner.

The substitutes are *deci*, *centi*, and *milli*. They are to be mastered now in their broadest application, the pupil not concerning himself here in any special way with the things to which they are applied, but becoming familiar with the significance of the substitute syllables themselves.

Deci means *one-tenth*, and is applied to denominations of the Metric System. We speak of an army or company as being *decimated* when it loses one-tenth of its number. A chemical is said to be *decinormal* when it has but one-tenth of its normal strength. A *deciare* is one-tenth of an are, which is a measure of area. A *decistere* is one-tenth of a *stere*, which is a measure of cubic contents. A *decimeter* is one-tenth of a meter; etc.

Centi means *one-hundredth*, and is used in the Metric system. A centigrade thermometer is one on which every "grade", or degree, is one-hundredth of the difference in heat between the freezing point of water and the boiling point. A *centime* is the hundredth part of a franc, in French money. Our own *cent* is the hundredth part of a dollar. A *cent* enary year is the hundredth year after an event that is commemorated. A chemical is said to be *centinormal* when it has but one-hundredth of its normal strength. A *centi* are is one-hundredth of an are. A *centi* stere is one-hundredth of a stere. A *centi* liter is one-hundredth of a liter; etc.

Milli means *one-thousandth*, and is used with denominations of the Metric System. Our *mill* (which is not coined, but is used in financial reckonings) is one-thousandth of a dollar. A milliampere, in electricity, is a thousandth part of an ampere. A *milligram* is one-thousandth of the weight of a gram. A millimeter is one-thousandth of the length of a meter. A *millistere* is one-thousandth of the bulk of a stere. A *milliare* is one-thousandth of the area of an are.

Write in compounds of *deci*, *centi*, or *milli* the following:

1. One-thousandth of a gram.
2. Three-tenths of a stere.
3. Seven-hundredths of an are.
4. Five-tenths of a meter.
5. Four-thousandths of a liter.
6. Three-thousandths of a meter.
7. .002 liter.
8. .1 gram.
9. .02 meter.
10. .007 are.

125 millimeters = 1.25 decimeters, or 12.5 centimeters.

This may be read "one hundred twenty-five-thousandths of a meter," or "one and one-fourth decimeters," or "twelve and one-half centimeters," or "one decimeter, two centimeters, five millimeters," or "one decimeter, two and one-half centimeters," or "one hundred twenty-five millimeters."

In like manner, read in different ways the following:

1.	.352 meter	6.	.839 coulomb	11.	.187 stere
2.	.174 are	7.	.796 watt	12.	1.1 watts
3.	.217 stere	8.	.453 meter	13.	12.55 amperes
4.	.518 liter	9.	1.25 liters	14.	.203 are
5.	.745 gram	10.	2.348 grams	15.	2.25 steres

13275428

It will be seen that this number may be read in all of the following ways:

1.3275428	myriameters	13,275.428	meters
13.275428	kilometers	132,754.28	decimeters
132.75428	hectometers	1,327,542.8	centimeters
1,327.5428	decameters	13,275,428	millimeters

METRIC LONG MEASURE

Multiplication and Division Tables

10 millimeters make 1 centimeter.

10 centimeters make 1 decimeter.

10 decimeters make 1 *meter*.

10 meters make 1 decameter.

10 decameters make 1 hectometer.

10 hectometers make 1 kilometer.

10 kilometers make 1 myriameter.

In 1 myriameter there are 10 kilometers.

In 1 kilometer there are 10 hectometers.

In 1 hectometer there are 10 decameters.

In 1 decameter there are 10 *meters*.

In 1 meter there are 10 decimeters.

In 1 decimeter there are 10 centimeters.

In 1 centimeter there are 10 millimeters.

A *Meter* is equal to 39.37 inches. For ordinary purposes it is assumed to be *one and one-tenth yards*, this being a little above its true value, which is 1.093 yards.

A *Decameter* is *nearly eleven yards*, being 10.93 yards, or 393.7 inches.

A *Hectometer* is 328 feet 1 inch, and is commonly rated at 328 feet, which is a convenient length for a block in a village or city.

A *Kilometer* is .62137 miles, and is commonly counted as *six-tenths of a mile.* This measure is so constantly used in travel in many lands, and in books and newspapers, that it is very important to know.

A *Myriameter* is usually counted as *six and a quarter miles*, though it falls a trifle short of that distance, being 6.21 miles.

A *Decimeter* is *nearly four inches*, its equivalent being 3.927 inches.

A *Centimeter* is more than a third of an inch, being .393 of an inch.

A *Millimeter* is nearly *one-twenty-fifth of an inch*, being .039 of an inch. It is used chiefly in laboratory and studio work requiring minute measurements.

1. Of these measures, what three are you likely to use most, or to have most need to fix in mind?

2. At $1.80 a meter, what is the value of 6 decameters of silk?

3. How many miles are there in 621 kilometers?

4. A yard is equal to 0.9144 meters. A mile contains 1760 yards. How many meters does it contain? How many decameters? How many hectometers?

5. A rod contains 5½ yards. How many meters does it contain? How many decimeters?

6. 1 foot equals 30.48 centimeters. What part of a meter is it?

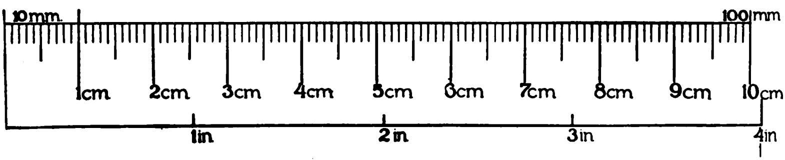
7. Sound travels in dry air at the rate of 1087 feet in a second. How many meters does it travel? How many decameters?

8. 1 inch is equal to 2.54 centimeters. What part of a decimeter is it?

9. A meter of wire can be cut into how many inch lengths, and with what waste?

10. At $3.00 per meter, what does it cost to lay a pavement 327.9 feet long?

Practice With the Decimeter



A DECIMETER RULER WITH CENTIMETERS AND MILLIMETERS MARKED

1. Copy this ruler on thin cardboard for a measure, and find the length of this book in decimeters—in centimeters—in millimeters. Find its thickness between the covers.

2. How many meters of cord will be needed to tie 6 parcels, each requiring 25 centimeters?

3. If the squares on a checkerboard measure each 4 centimeters, what will be the length, in decimeters, of a row?

4. At 40 cents a meter, what will be the cost of 2.5 decimeters of cloth?

5. Of books each measuring 25 millimeters in thickness, how many will fill a shelf 4 decimeters in length?

6. If a shrub grows 3 centimeters a month, how many decimeters taller will it be in 10 months?

7. If a meter of gold lace is worth $4, what is the worth of 125 millimeters of it?

8. If a certain strip of fur costs 80 cents per decimeter, what is the cost of 250 centimeters of it?

9. If a strip of cloth 20 meters in length shrinks one meter, what will be the shrinkage of each decimeter?

10. If 3 meters of cloth is cut into 6 equal lengths, how long is each strip?

11. How many 2 decimeter rulers, placed end to end, will measure 3.6 meters?

12. A book is 4 centimeters thick between the covers. It contains 400 pages, how many leaves are there in 1 millimeter of thickness?

13. An ivory comb two decimeters in length contains 46 teeth, the spaces between points of the teeth being equal in width. What is the width of each space?

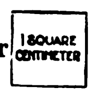
14. A picture is copied on scale reduced to .7 of its diameter. A line in the original picture measures five centimeters. How many millimeters does it measure in the copy?

15. What is the cost of 150 decimeters of cloth at $1.25 per meter?

METRIC SQUARE MEASURE

Multiplication and Division Tables

100 square millimeters make 1 square centimeter (sq. cm.).



100 square centimeters make 1 square decimeter (sq. dm.).

100 square decimeters make 1 *Square Meter* (sq. m.).

100 square meters make 1 square decameter (sq. Dm.).

100 square decameters make 1 square hectometer (sq. Hm.).

100 square hectometers make 1 square kilometer (sq. Km)

In 1 square kilometer there are 100 square hectometers.

In 1 square hectometer there are 100 square decameters.

In 1 square decameter there are 100 square meters.

In 1 square meter there are 100 square decimeters.

In 1 square decimeter there are 100 square centimeters.

In 1 square centimeter there are 100 square millimeters.

In square measure the Metric denominations increase and decrease in a hundred-fold ratio. Of the abbreviations used to designate them, those above the meter contain each one capital letter.

A *Square Meter* is less than one and one-fourth square yards. It is called a *centare* (sahn-tair) when applied to land measure.

A *Square Decameter*, containing 100 square meters, or centares, is a measure of land known as the *are* (*air*). It contains 119.6 square yards, and is a convenient measure for small garden plots and lawns, and for sites of buildings. It is equal to .024 of an acre.

A *Square Hectometer* applied to land measurement, is called a *hectare*. This contains 100 ares, and is nearly equal to *two and one-half acres*, being 2.471 acres, or .386 of a section. Farms and estates in many lands are surveyed and described in hectares. In countries where intensive cultivation of the land prevails, there are many hundreds of peasant farms not larger than a hectare.

A *Square Kilometer*,* containing ten thousand ares, applied to land measurement, is called a *myriare*. It is equal to one hundred hectares, or 247.1 acres, or .386 of a section. The areas of many foreign countries and their principal civic divisions are stated in miliares.

A centiare is, of course, a hundreth of an are.

1. How many entire hectares would be contained in a Congressional township?

2. How many entire ares would be contained in an acre?

3. How many centiares are there in a hectare?

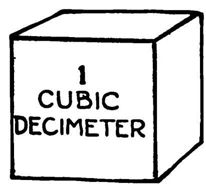
4. How many centiares are there in a square kilometer?

5. A French farmer divided equally among his 5 sons an estate of 2 hectares. How many ares did each son receive?

METRIC CUBIC MEASURE

The *Cubic Meter* is the unit of capacity for the measurement of large volumes. The *Cubic Decimeter* is likewise very important, as it is the basis of capacity for dry and liquid measures.

While the denominations in square measure increase and decrease in a hundred fold ratio, those in cubic measure vary regularly in a ratio of one thousand.



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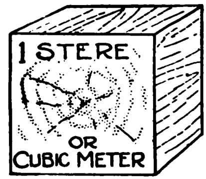
Multiplication and Division Tables

1000 cubic millimeters make 1 cubic centimeter (ccm.)

1000 cubic centimeters make 1 cubic decimeter (cdm.).

*Between the hectare and the myriare is the *kiliare*, of one thousand ares. It is not in the form of a square, and is little used. Note the use *are*, *area*, and *acre*, all appearing on this page.



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1000 cubic decimeters make 1 cubic *meter* (c.m.) or *stere* (stair).

1000 cubic meters make 1 cubic decameter (cDm.).

1000 cubic decameters make 1 cubic hectometer (cHm.).

1000 cubic hectometers make 1 cubic kilometer (cKm.).

In 1 cubic kilometer there are 1000 cubic hectometers (cHm.).

In 1 cubic hectometer there are 1000 cubic decameters (cDm.).

In 1 cubic decameter there are 1000 *cubic meters* (cm.).

In 1 cubic meter there are 1000 cubic decimeters (cdm.).

In 1 cubic decimeter there are 1000 cubic centimeters (ccm.).

In 1 cubic centimeter there are 100 cubic millimeters (cmm.).

1. A *stere* is equal to about 1.3 cubic yards (1.308). The stere is the cubic meter applied to the measurement of wood. It need not be in cubic form. Hence the next denomination for wood measure need not be in cubic form; and we have the *decastere*, consisting of 10 cubic meters, as well as the *hectostere*, consisting of 100 cubic meters, before we come to the *kilostere* of 1000 cubic meters, which may be conceived as in cubic form.

1. In a stere are how many cubic centimeters of wood?

2. Draw a diagram of a decastere of wood, marking the dimensions on the lines in metric long measure.

3. In a kilostere are how many decasteres?

4. How many hectosteres would be required to form a cube?

5. What is the difference between a cubic hectometer and a hectostere?

6. What is the difference between a cubic decameter and a decastere?

7. What is the difference between a cubic kilometer and a kilostere?

8. How many cubic centimeters are there in a decastere?

9. A shipment of guanacaste wood from Central America forms a pile two meters high, 1 meter broad, and 30 meters long. How many decasteres does it contain?

10. How many cubic hectometers of earth are removed to make an excavation 12 meters long, 8 meters broad and 5 meters deep?

11. What will be the cost of teaming in the removal of 2500 cubic decimeters of gravel at $1 per cubic meter?

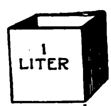
12. Reduce 2 cKm. to cDm.

13. Add 6 cm., 90 cDm. and 4 cHm.

14. Subtract 30 cdm. from 4 cDm.

15. Subtract 17 ccm. from 2 cHm.

METRIC DRY AND LIQUID MEASURE



(Reduced.)

The basic unit of dry and liquid measure in the Metric System is the *liter*, represented by a vessel in cubic form of which the inside measure is 1 decimeter. It is usually reckoned as equal to *one and one-twentieth quarts* (liquid measure), being 1.05 6 quarts. It is equal to .908 dry-measure quarts.

The decaliter contains more than *two and three-fifths* gallons, and more than *one and one-fourth pecks*.

The denominations in Metric dry and liquid measure increase and decrease in a ten-fold ratio. The abbreviations are as follows: for milliliters, ml.; for centiliters, cl.; for deciliters, dl.; for liters, l.; for decaliters, Dl.; for hectoliters, Hl.; for kiloliters, Kl.

1. Prepare a multiplication table for this measure.

2. Prepare a division table for this measure.

3. Approximately, how many decaliters are there in thirteen gallons?

4. Approximately, how many bushels are there in 8 decaliters?

5. At 8 cents the liter, what is the cost of 3 kiloliters?

6. Reduce 8 decaliters to deciliters.

7. A centiliter is what part of a hektoliter?

8. Reduce 1,298, 765 milliliters, successively, to each of the denominations above them.

9. Reduce 1 kiloliter, successively, to each of the denominations below it.

10. A liter of water weighs 2.2046 pounds. What is the weight of a myrialiter of water?

11. An aquarium 2 meters long, 1½ meters broad, and 1 meter deep contains how many liters?

12. A bin in a certain granary is 4 meters long, 3 meters broad and 175 centimeters deep. How many kiloliters does it contain?

METRIC WEIGHT

The basic unit of weight in the Metric System is the *gram*, which, like the liter, is founded ultimately upon the meter. The gram is the weight of a cubic centimeter of distilled water at maximum density (15.432 grains).

In practice, since the gram is so small, the *kilogram* is taken as a basic unit, it being a convenient measure for this purpose. The kilogram is about equal to *two and one-fifth* pounds, being very slightly in excess of this weight (2.2046 lbs.). So common is its use that its name is shortened into *kilo*. Next below this is the *hectogram*, which is less than *a quarter of a pound*, Avoirdupois (3.527 ounces). One hundred kilos (10 myriagrams) or 220.46 lbs., is called a *quintal*. Ten quintals constitute the *Metric ton**, which is a little less than the long ton of Avoirdupois weight, being 2204.6 pounds.

(Reduced.)

The metric denominations of weight, when written in full, increase and decrease in a ten-fold ratio. Their abbreviations are as follows: for milligram, mg.; for centigram, cg.; for decigram, dg.; for gram, g.; for decagram, Dg.; for hectogram, Hg.; for kilogram, Kg.

Multiplication and Division Tables of the More Familiar Commercial Denominations

10 hectograms make 1 kilo.
100 kilos make 1 quintal.
10 quintals make one metric ton.

In 1 metric ton there are 10 quintals.
In 1 quintal there are 100 kilos.
In 1 kilo there are 10 hectograms.

1. Construct multiplication and division tables in full, giving the scientific equivalents of the commercial terms *kilo*, *quintal*, etc., and omitting these terms.

2. What is the weight of a liter of distilled water?

*This is called also the *tonneau*, (tun-o) or *millier*, (me-yay).

3. If a kilo of cheese costs $1.20, what is the cost per hectogram?

4. Of baled hay weighing 25 kilos per bale, 4 tonneaus will include how many kilos?

5. Approximately, what is the weight in pounds of a crate of fruit which weighs 10 kilos?

6. A gram is a little more than fifteen *grains*, Avoirdupois or Troy (15.432), and is therefore a little more than a quarter of a dram, Troy. Approximately how many ounces, Troy, are there in a hectogram? How many pounds?

7. 22046 tons are equal to how many tonneaus?

8. How many centigrams are there in one liter of alcohol? The weight is .763 the weight of water.*

9. If the specific gravity of linseed oil is .935, what is the weight of a decaliter of that oil?

10. A milligram is equal to .0154 grains. What is the weight (metric) of 2 ounces of gold (Troy).

11. The specific gravity of lead is 11.4. What is the weight of a liter of melted lead?

12. Reduce, successively, a kiloliter to each of the denominations below.

13. Reduce 6,354,792 milliliters, successively, to each denomination above.

14. Approximately how many ounces, Troy, are there in 2 kilos?

15. The old silver penny weighed 24 grains. How many such pennies would be required to make up a kilo?

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*The ratio of the weight of any substance to that of water is called the *specific gravity* of the substance. Thus the specific gravity of absolute alcohol is .763.

MISCELLANEOUS WORK

DIRECT AND INVERSE PROPORTION*

Where the element of time is considered in connection with that of labor, or investment of money, etc., the two can be reduced to a single unit. Thus $500 invested for 10 months is the equivalent of $5000 for 1 month. That is to say, $(500\times10)=(5000\times1)$; and 500 : 1 :: 5000 : 10. 4 men working 6 days are the equivalent of 24 men working 1 day; 25 men working 8 hours a day are the equivalent of 200 men working 1 hour a day, or of 20 men working 10 hours a day.

1. If of two partners A invests $10,000 for two years and then $2000 for 2 years, how much must B invest continuously in order that their investments may be equal in the four years?

2. If 20 men working 10 hours a day could perform a certain piece of work in 12 days, how many men working 8 hours a day will be required to accomplish it in the same time?

3. If 6 pumps fill 6 equal tanks in 6 hours, how many such pumps would be required to fill 90 such tanks in 1 hour?

4. If 12 cows eat 504 bushels of grain in 4 weeks, how many cows would eat 252 bushels in 2 weeks?

Proportion is either *direct* or *inverse*.

It is *direct* when the antecedent and consequent of the first ratio correspond directly with those of the second ratio.

Thus if 2 boxes of a certain size will hold 12 cans each of a certain size, 8 such boxes will hold 48 of the cans; and the proportion will be expressed thus:

2:12::8:48

The larger the number of boxes, the larger the number of cans they will hold in all.

*Many educators believe, this topic to be unnecessary. It is presented here as a statement of fact, and may be omitted if so desired. Problems involving inverse proportion are generally solved by analysis, the value of one unit being found and then that of the required unit.

Proportion is *inverse* when the antecedent and consequent of the first ratio correspond indirectly with those of the second ratio. Thus if 2 six-can boxes will contain 12 cans, 48 cans will require 6 eight-can boxes.

Here the greater the number of cans in a box, the smaller will be the number of the boxes required.

An inverse proportion may be made direct by rearranging the terms. Thus we may say that 12 cans are in proportion to 2 six-can boxes as 48 cans are to 6 eight-can boxes; and the proportion may be expressed directly thus:

$$12 : 2\times 6 :: 48 : 6\times 8$$

The amounts of merchandise that can be purchased for a certain sum, when the prices vary, present problems in inverse proportion: Thus a dollar will buy 4 pounds of coffee at a price of 25 cents, or 3 pounds at a price of $33\frac{1}{3}$ cents. The larger the price, the smaller the amount that the dollar will purchase. Here the price of the dearer coffee is to the amount of the cheaper coffee as the price of the cheaper coffee is to the amount of the dearer coffee; and we write the proportion thus:

$33\frac{1}{3}$ cents. : 4 pounds :: 25 cents : 3 pounds.

1. The greater the amount of weeds in a field, the less the value of the crops. Is this proportion direct or inverse?

2. The more learned a criminal is, the more dangerous is he to society. Is the proportion direct or inverse?

3. In the old proverb "The more haste the less speed," is the proportion direct or inverse?

4. If a certain sum of money will buy 4 pounds of macaroni at 15 cents the pound, how much will it buy of macaroni at 12 cents the pound?

5. If an electric light bulb will last $33\frac{1}{3}$ days (and nights) burning continuously, how many days (and nights) will it last, burning 8 hours in every 24?

6. If the fast train, running at the rate of 40 miles an hour, makes the trip from Millerville to Arno in $2\frac{1}{2}$ hours, how long a time will be required by the slow train which runs at 25 miles an hour?

If A owes B $100 payable at the end of one year, and settles the account at once, some reduction should be made for the advance payment. In determining the sum to be paid *now* we must consider that in making the earlier payment A loses the interest which the money would have earned for the year; and consequently the amount he should pay must be such a sum as, put out at interest, would at the end of the year amount to $100.*

The difference between the amount of the debt at the time of maturity and the present worth of the debt is sometimes called its *true discount.* However, this term is little used in the business world at the present time. *Present worth* is here to be calculated on a basis of 6% unless some other rate is stated for it.

1. What is the present worth of a note for $500 due one year hence without interest? In other words, what principal with 6% interest would amount to $500 in one year? (See p. 41; also p. 44). The problem is to find a number such that 106% of it will equal 500; or a principal such that 106% of it will equal $500.

2. A farmer offers to sell his farm for $8000, of which one-half must be paid down, and one-half after one year, without interest. What is the present worth of the unpaid half?

3. What is the present worth of $1500 due without interest in 3 years, 6 months, and 10 days?

4. I hold a note for $350 due without interest in 90 days. What is its present worth?

5. A youth who will become of age in 4 years will then inherit $10,500. What is the present worth of that sum?

Sometimes the debt at maturity will consist of both principal and accrued interest. The present worth must be a principal which at a certain per cent would equal the *total amount* of the debt.

6. Thus if a note to run four years was given just three years ago, and is for $200 at 6% the interest to this date having been paid, its present worth is a principal which, at 6%, would equal $212 in one year? What is it?

7. What is the present worth of a note for $4,000 without nterest, given for goods purchased on a credit of 90 days?

8. What ready money will pay off a debt of $450 due six months hence at 6% interest?

9. Find the present worth of $980 due in six months with accrued simple interest for a full period of four years, at 6%.

*In actual business practice *exact* present worth is seldom computed. The creditor will ordinarily state some amount for which he will close the account.

The following problems based on actual experience will afford suggestions to pupils, and stimulate their business perception.

1. Two college students working on a farm in summer discovered a bed of molding sand, and secured an option on the apparently worthless tract at a low figure. They brought about the incorporation of a company capitalized at $10,000 to supply the sand to foundries, and they received ten shares of $100 each for their option. If the stock paid an average annual dividend of 35% for eight years, and was then sold at par, what was the total amount received by each of the boys?

2. Rival owners of an abrupt bluff and of a flat valley adjacent to it, in a suburb, could do practically nothing with either piece of property. A stock company was formed to own both tracts, and transformed them into a beautiful slope. If the properties separately held were worth $4,000 apiece and in the hands of the corporation, $100,000 for the whole what was the profit gained by means of the incorporation?

3. A business corporation capitalized at $50,000 was formed to develop and market a bed of lithographic stone, discovered on a farm. The stone proved to be usuable only in surface quantities of one square inch or less, and was worthless for lithographing, but could be sold for other purposes, as was permitted in the charter; the corporation paid an annual dividend of 2%. The stock was valued at 25% of its face value. In the same proportion, what would it have been worth if the company, as contemplated, had paid a dividend of 35%?

4. A corporate company owning and operating a mine of brown lead ore paid an annual dividend of 12%. Vanadium was found to be one of the products. If it added 150% to the dividend, what was the value of the stock, estimated on the basis of par value for a dividend of 6%?

5. In a certain town there were three academies—one having a *classical* course, another a *business* course, and the third a *manual training* course—and all conducted as private enterprises. A stock company was incorporated to purchase them all and to conduct them as departments of a unified institution. If the real estate and good will of these academies was worth, in all, $45,000, and if the new company was capitalized at $60,000 and its stock commanded a steady preminm of 6%, what was the gain in value from this unifying of the enterprises?

6. In a certain factory where the product required the successive use of three different machines, and each workman used all three kinds, a new manager had a different set of workmen for each kind of machine, observing the scientific principle of "division of labor" (where each workman managed only one machine, continuously). There was such a gain in efficiency that the output was increased 12% with no increase of expense. If the annual profits of the factory were $20,000 before, what were they now?

7. A bright young workman suggested a different arrangement of the raw material, the machines, and the finished product in the long factory building, so that the raw material would be at one end, the finished product at the other, and the machines in regular, successive cross lines between them, the material passing always forward, and with the least carrying from place to place. By this plan, there was a gain of 8% in efficiency. What was the amount of gain to the factory?

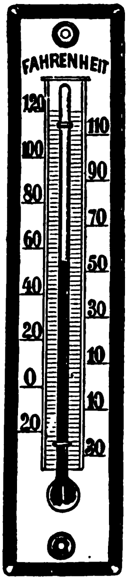
8. By feeding his cows, this year, food that cost 10%, less than the food cost last year, a dairyman has increased the value of his milk product 20%. What is the gain in efficiency of the scientific food over that of the unscientific food of last year?

9. An ignorant gardener killed six toads in his garden. If the ravages of insects cost him $33\frac{1}{3}$% of the potential value of his crop, and if each toad would have diminished these ravages by 2%, and if his products brought him $600, what was his loss from the killing of each toad?

10. The sleep of plants stops their growth for the time, and plants cannot sleep when it is light. If by means of a forcing house certain plants are deprived of 40% of their natural sleep, and if the natural sleep would average one-third of the twenty-four hours, by what per cent is the growth of these plants accelerated?

11. A company invests $12,000 in "stump lands," from which the pine timber has been removed, and $3,000 in machinery to uproot and pulverize the stumps, for the extraction of turpentine. The annual profit is 15% on the investment, for four years; at the end of which time the land has doubled in value, and the machinery is sold for half the cost price. What is the real per cent of annual profit?

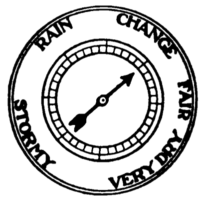
WEATHER FORECAST



A Standard Thermometer

For City and vicinity—Partly cloudy Wednesday and Thursday, probably local thundershowers, continued warm Wednesday; not so warm Thursday; fresh southerly winds Wednesday, becoming variable Thursday.
For Central territory—Partly cloudy Wednesday and Thursday, probably local thundershowers; not so warm Thursday in the northern portion; moderate, southwest winds.
Sunrise, 4:19; sunset, 7:14; moonset, 10:17 p. m.

AN ANEROID BAROMETER*



TEMPERATURE
During 24 hours

Maximum, 2 p. m.............92
Minimum, 5 a. m.............75

3 a. m.....77	11 a. m.....85	7 p. m.....88
4 a. m.....76	Noon......88	8 p. m.....85
5 a. m.....75	1 p. m.....90	9 p. m.....84
6 a. m.....75	2 p. m.....92	10 p. m...80.5
7 a. m.....77	3 p. m.....92	11 p. m......80
8 a. m.....80	4 p. m.....92	Midnight...79
9 a. m.....84	5 p. m.....92	1 a. m.....79
10 a. m.....84	6 p. m.....91	2 a. m.....78

Mean temperature, 83.5; normal for the day, 60
Excess since Jan. 1, 363.
Precipitation for 24 hours to 7 p. m., 0. Deficiency since Jan. 1, 2.17 inches.
Wind, S. W.; max., 24 miles an hour, at 8:35 a. m.
Relative humidity, 7 a m., 67%; 7 p. m., 52%.
Barometer, sea level, 7. a. m., 30.02; 7 p. m., 30.01.

A Daily Paper's Weather Record.


A Standard Barometer

Class Room Drill

Write a definition of **local** and **variable.**
Write a definition of **maximum** and **minimum.**
Write a definition of **mean** and **normal.**
What is meant by **humidity?**
What is meant by **precipitation?**
What is meant by **deficiency?**
Write a description of the **thermometer.**
Write a description of the **barometer.**
What is meant by **"sea-level"?**
How many **points of the compass** can you name?

*Some ingenious pupil will be found in almost every school, who can construct from this illustration a *card dial* with moveable pointer, to show the "Weather Forecast" for *school room use*. A daily paper supplying the required information is now accessible to most schools.

The Fahrenheit* thermometer is the standard measure of temperature in the United States. The Centigrade thermometer is used with the Metric system.

1. Light travels at the rate of 186,300 miles† per second. Sound travels at the rate of 1,087 feet the second. During a thunder storm Harold Smith counted three seconds between a flash of lightning and the report it produced. What was the approximate distance, *in feet*, to the storm clouds?

2. If there had been 20 seconds between the flash and the report, what would have been the approximate distance *in miles?*

3. A gallon contains 231 cubic inches. An acre of ground contains 43,560 square feet. What would be the weight, *in tons*, of a rainfall of one inch in depth over a quarter-section of land, estimating the weight of each gallon of water at $8\frac{1}{3}$ pounds?

4. The Wapsie River has an average width of 175 ft. for $2\frac{1}{2}$ miles. A rise of 2 ft. 10 in. resulted from a heavy rainfall. What amount of water, *in cubic feet*, was required to raise its level over this area to the extra height indicated?

5. A farmer had, in process of curing, five acres of new mown hay. Counting on fair weather and failing to profit by his daily paper's "Weather Forecast," his crop was damaged to the extent of 35% during an unexpected rainstorm. How much did he lose, on 11 tons of hay, counting the full market value at $11.40 the ton?

6. In a certain fruit belt the temperature dropped unexpectedly, over night, from 51 degrees to 35 degrees above zero. The peach crop that year in a certain locality had yielded 3,768 baskets. In the season following, under normal conditions, the yield was 7,348 baskets. What was this increased product worth at 55c the basket?

7. The precipitation in a certain township in one season was 26.8 inches. The wheat yield that year in the township aggregated 137.540 bushels. In the succeeding year the precipitation during the same period was 19.7 inches, and the wheat yield aggregated 68,430 bushels. What was the difference in the value of the crops at 95c the bushel?

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*Pupils should be encouraged to consult the dictionary freely regarding the pronunciation, meaning, and use of all unfamiliar words.

†The flight of light may here be considered as *instantaneous*.

The majority of problems actually arising in life are not solved at once with exactness, but only *approximately*. One is often interested only in approximating the amount of the answer, and in determining whether this is *more* or *less* than a given amount. The habit of inspecting a problem and roughly estimating the answer (in advance) is of value in preventing the danger of being satisfied with an absurdly incorrect result.

1. A man owes the following amounts, \$173.57, \$54.55, \$46.10 and \$198.28. Does he owe more or less than \$500?

2. I can save from my wages \$3.50 per day. Working 26 days per month, about how long will it take me to save \$1,000?

3. A fruit ranch yields 2,600 boxes of peaches which sell at 48 cents the box. Will the receipts exceed \$1,300?

4. 4,000 boxes of apples bring \$1.14 the box. Will the proceeds exceed or be less than \$5,000?

5. I have borrowed \$1,385 for one year at 8%. Shall I pay more or less than \$100 interest?

6. A man purchased a house for \$6,100, and sold it later for \$6,800. Did he gain more or less than 10%?

7. Give the approximate cost of 16 dozen eggs at 24 cents.

8. Give the interest for one year on \$990 at 9%.

9. Give the income from \$22,240 at 6%; 7%; 11%.

10. What is 9% of 6,500,000? Is it 58,500 or 585,000?

11. 50×49? $1,100 \div 50.4$? $18,527 + 1,460$? $527 + 110 + 92$?

12. 6 521, 865 divided by 276 about 2000 or about 20,000?

13. The distance covered by an automobile in 21 hours when averaging 18½ miles the hour.

14. The number of printed pages required to print a manuscript of 17,500 words, the page averaging 400 words.

15. I own stocks costing \$11,500, and sell them for \$13,100. What is my percentage of gain?

16. The assessed valuation of a school district is \$21,945,865. The expenses of conducting the schools are \$22,000. What is the necessary rate of taxation for school purposes?

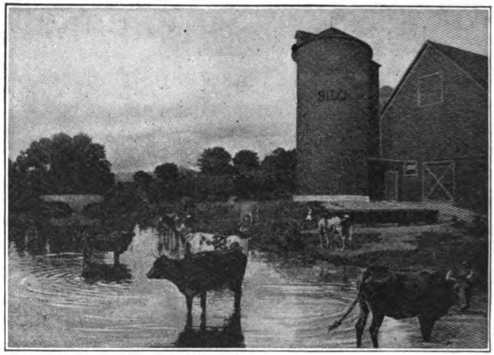
**Approximation work* has had consideration in the earlier lessons of this series, but only so far as seemed consistent with the needs of the younger pupils, with whom *exactness* should be the prevailing idea. Before the work of this grade is ended, an extended application of the principle may advantageously be made, with numerous original problems, which the pupils themselves may invent.

Supplement

The succeeding pages supply material much of which does not necessarily belong to the usual school course in arithmetic, but *it is such as should be readily accessible to pupils in the advanced grades.* This work will afford invaluable drill, and should be utilized to as large an extent in the regular class assignments as time and circumstance will permit. Vocational instruction is now required in many schools throughout the country. The following subjects are considered:

NOTE.—The text of this supplement will prove a fruitful source for material in the preparation of *original problems* (similar to those given under the several headings); and *much of this kind of work all pupils should be encouraged to do.*

Where the problem permits, the pupil should inspect it and find approximate answer before undertaking the exact solution.



The silo, which is a chief factor in modern dairying, supplies to cows in the colder period of the year the fresh green food of the summer time, much as the canneries supply to us our fruits, vegetables, and berries for winter use.

To secure the pressure necessary for the best preservation of silage, the silo should be of a height equal at least to twice its diameter. The circular form allows the greatest area in proportion to the wall space, and not only is economical as to capacity, but *reduces the friction* of the wall in the settling of the silage.

1. If a silo has an inside area of 19.63 square yards, and its height, exclusive of roof, is 2½ times its inside diameter, what is the height of its wall?

2. A certain silo is 14 feet in diameter, and is filled with silage to a height of 25 feet. If the silage weighs 34 pounds to the cubic foot, what is the amount of it in tons?

3. A field devoted to the raising of Indian corn for use as silage, and producing 54 tons, will supply 16 cows for how long a period, allowing to each cow 35 pounds each day?

4. If 37 pounds of silage be fed each day to each cow on a certain farm, how many cows will 74,000 pounds of silage from corn and vegetables supply for 200 days?

5. Standard milk is 87% water, 4% fat, .7% ash, 3.3% protein, and the remainder is made up of carbohydrates. What per cent of standard milk is carbohydrates?

6. Cream is 74% water, 2.5% protein, 4.5 carbohydrates, .5% ash, and the remainder is fat. What % of cream is fat?

7. A certain Shorthorn cow of record yielded 18,075 pounds of milk in twelve months. How much was this in gallons for each day, counting a quart as 1.75 pounds.

8. From the milk yielded in one year by this cow, 735 pounds of butter was made. At 30c the pound, what was the worth of this butter product?

9. A certain Jersey cow of record yielded 17,557 pounds of milk in one year, and from the milk 1,175 pounds of butter was made. What per cent of the milk did the butter fat constitute?

10. A certain Guernsey cow of record yielded in one year 910 pounds of butter fat, in 17,285 pounds of milk. What was the per cent of butter fat?

11. A certain Holstein cow of record yielded an average of 14,134 pounds of milk a year for a period of five years. What was the total milk product in that period?

12. If one cow yields 17 quarts of milk each day, the milk containing butter fat to the amount of 3%, and another cow yields 12 quarts of milk each day, the milk containing butter fat to the amount of 4.5%, which cow is the more profitable for butter making?

13. An Indiana farmer shipped 448 gallons of milk in one week to a Chicago dealer, who sold it for $72.80, charging the dairyman $1.68 for his services. What did the dairyman receive for each can? At 8 cents the quart, what did the consumers pay for the milk?

1. In 1913 the Dominion of Canada imported from the United States 13,160,000 dozen eggs, paying a duty of 3 cents on each dozen. What was the total duty on these eggs?

2. Before the art of poultry management reached its recent development, the egg product of a single hen was estimated at 70 eggs in a year. What was the annual egg product of 40 hens? At 25 cents the dozen, what were the eggs worth?

3. A record of 303 eggs in one year as the product of a single hen has been made. What would be the value of this product at 35 cents the dozen for the eggs?

4. If 14 Brahma hens produce an average of 12½ dozen eggs each per year, the eggs weighing each one-seventh of a pound, what will be the weight of the eggs produced by them in a year?

5. If 14 Leghorn hens produce an average of 175 eggs each in a year, the eggs averaging 10 to the pound, what will be the aggregate weight of the eggs produced?

6. If 14 Black Minorca hens lay an average of 15 dozen eggs each in a year, the eggs averaging ⅛ of a pound each in weight, what will be the aggregate weight of the eggs laid by them in the year?

7. Four Pekin ducks produced 100 eggs each in one season, and four others produced 150 eggs each. What was the total number of eggs?

8. The standard weight of Plymouth Rock hens is 7½ pounds; of other Plymouth Rock fowls, 9½ pounds. What is the weight of 10 hens and 40 fowls of standard weight?

9. The standard weight of Orpington hens is 8 pounds, and of other Orpington fowls 10 pounds. What is the weight of 10 hens and 40 other fowls, all Orpingtons, of standard weight?

10. The average standard weight of Bronze turkeys is 26 pounds. What will be the weight of 40 Bronze turkeys of the average standard weight?

11. The standard weight of Wyandotte hens is 6½ pounds; of other Wyandotte fowls, 8½ pounds. What is the weight of 10 hens and 40 other fowls, all Wyandottes of standard weight?

12. The limit of the most profitable turkey fattening is said to be 30 pounds. According to this, if 12 Bronze turkeys are fattened to a weight of 36 pounds each, what amount of turkey flesh has been produced at a lower profit?

13. If 10 turkey hens produce an average of 24 eggs each, within a year, and each egg produces a poult which grows to a weight of 20 pounds, what is the total weight of the turkey product from the year's laying?

14. The standard weight of the Pekin drake is 8 pounds, and of the Pekin duck is 7 pounds. What is the standard weight of 16 of each?

15. The standard weight of the Rouen drake is 9 pounds, and of the Rouen duck is 8 pounds. What is the standard weight of 16 of each?

16. The standard weight of the Aylesbury drake is 9 pounds, and of the Aylesbury duck is 8 pounds. What is the standard weight of 10 of each?

BEE CULTURE

1. A certain bee keeper with 40 hives derives from each an average of 28 pounds of comb honey in a year. What is the total amount?

2. If the bee keeper's labor of a half-hour a day is worth $2.75 each week, and there is a monthly expense of 45 cts., what is the total expense for the year?

3. A standard honeycomb frame for bee hives is 17⅝ inches long by 9⅛ inches broad. What is it area?

4. A worker bee emerges from its cell 21 days after the laying of the egg, and it begins field work 2 weeks later. How long before the "honey flow" period of field work begins must the eggs of bees be laid?

5. In a winter a colony of bees uses 30 pounds of honey If the bee keeper prepares a substitute made of granulated sugar, at 5 cents a pound, what will be the saving, if the honey is worth 25 cents a pound?

6. Cases of unseparatored honey average not less than 23 pounds per section. At this rate, how many pounds of honey are there in a case of 24 sections?

7. In a certain apiary of 94 colonies of bees, 10,600 pounds of honey is produced in one year. The product is sold for $1,060. The expense, apart from the work of the bee keeper, is $100. What is the average production of honey by the colony? What is paid per pound for the honey?

8. A certain colony of bees consisted of one queen bee, 300 drones, and ten times as many workers. When the drones were all destroyed, and 30% of the workers were lost, how many bees did the colony include?

9. Granulated honey is made into 1-pound bricks, and wrapped in paraffin paper. What is the value of 110 such bricks at 25 cents each?

FOOD VALUES

The four groups of true nutrients of the human body are the *protein* (pro′te-in), or nitrogenous compounds, the *fats*, the *carbohydrates*, and the *mineral matter*. The protein compounds are the only ones that both build up the tissues and furnish energy. The carbohydrates (car′bo-hy′drates) and the fats are not building materials, but *fuel* for the body. The mineral matter required by the body is very small in amount, and is supplied through a great variety of foods, generally in excess amounts. The care in preparing a scientific diet is to secure a proper balancing of the protein compounds, the fats, and the carbohydrates.

The average man engaged in moderately active work requires about one-fifth of a pound of protein daily, to keep up his tissues, if he has enough of fats and carbohydrates to supply heat and energy. If the protein is not properly balanced, or supplemented, with these, he requires more of it.

The *nutritive ratio* is the ratio of protein to carbohydrates in the food required by a human being or by an animal. The nutritive ratio for a man at active work in the open air is found to be 1:6.9. For a growing youth it is found to be 1:5.2.

This comparison is made with carbohydrates only, as a matter of convenience. Fat will replace $2\frac{1}{4}$ times its weight in carbohydrates.

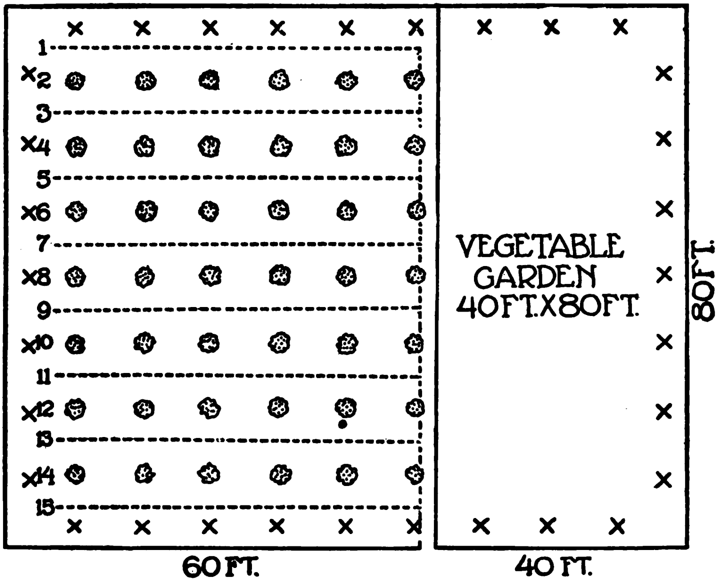
Meats, eggs, and cheese are rich in protein and in fat. Vegetables are rich in carbohydrates. Bread contains both protein and carbohydrates, but should be taken with cheese or meat, to supply enough protein. Corn bread contains more fat and less protein than wheat bread.

1. If cereals and their products supply 62% of the carbohydrates, and vegetables and fruits together 16% of the carbohydrates, what per cent of the total carbohydrates do both of these classes supply?

2. If meat and poultry supply 16% of the total food material in the average American home, and dairy products 18%, cereals and their products 31%, vegetables and fruits, together, 25%, how much of the total food materials do these items constitute?

3. If meat and poultry supply 30% of the protein, the dairy products 10% of it, cereals and their products 43% of it, and vegetables and fruits together 9% of it, how much of the protein is supplied by these four kinds of food?

4. If meat and poultry supply 59% of the fat, dairy products 26% of the fat, cereals and their products 9% of the fat, and fruits and vegetables together 2% of the fat, what per cent of the total fat do these four kinds of food supply?



In a certain fruit garden 80 feet long and 60 feet broad, the fruit bearing trees and plants are as follows:*

1. Thirty-two grape vines, 10 feet apart, around the entire area.
2. Three rows of dwarf pear trees, 6 in each row. (Rows 2, 10 and 14).
3. One row of peach trees, 6 in number. (Row 4).
4. One row of cherry trees, 6 in number. (Row 8).
5. One row of dwarf apple trees, 6 in number. (Row 6).
6. One row of plum trees, 6 in number. (Row 12).
7. One row of blackberry bushes, 20 in number. (Row 1).
8. Two rows of blackcap bushes (black raspberries) 20 in each row. (Rows 3 and 5).

*From a Bulletin issued by the Department of Agriculture, Washington, D. C.

10. Three rows of strawberry plants, 100 in each row. (Rows 11, 13, 15).

1. What part of an acre does this garden constitute?

2. How many trees, bushes, vines and plants does it contain?

3. How many different kinds of fruit are represented in this garden?

4. How far apart are the trees?

5. How far apart are the blackberry and raspberry bushes?

6. How far apart are the strawberry plants?

7. What part of the fruit garden is occupied by the trees?

A vegetable garden adjoining, 80 feet long and 40 feet broad, contains the following plants:

1. One row: ½ row rhubarb, ½ row asparagus (the row occupying 4 feet).

2. One row, salsify (oyster plant, 1½ feet).

3. One row parsnips (1½ feet).

4. Two rows beets (3 feet).

5. One row eggplant; plants set 18 inches apart; two dozen (the row occupying 3 feet).

6. Two rows tomatoes; plants set 2 feet apart; 2 dozen (the two rows occupying 6 feet).

7. One row summer squash, 12 hills, 3 feet apart (the row occupying 3 feet).

8. Two rows cucumber, 24 hills, 3 feet apart (the two rows occupying 1 foot).

9. Two rows early cabbage, 4 dozen plants, set 18 inches apart (the two rows occupying 4 feet).

10. Two rows late cabbage, 4 dozen plants, set 18 inches apart (the two rows occupying 4 feet).

11. One row early celery, 6 dozen plants, set 6 inches apart (the row occupying 2 feet).

12. Eight rows peas, plant in double rows, 4 inches apart; follow by 6 rows late celery, 36 dozen plants (the eight rows occupying 16 feet).

13. Two rows lima beans, 4 dozen hills, 18 inches apart (the two rows occupying 4 feet).

14. Six rows bunch beans; in succession, seeds sown in drills, seeds placed about 6 inches apart in the row; follow by late cabbage, turnips or spinach (the six rows occupying 12 feet).

15. Two rows radishes, 4 sowings, planted in double rows 6 inches apart (the two rows occupying 3 feet).

16. Two rows lettuce, two sorts, adapted for early and late use (the two rows occupying 3 feet).

17. One row parsley, and peppergrass (the row occupying 1½ feet).

The space occupied by the last three plants is given over to winter squashes by planting these before other crops are off the ground.

1. How many different varieties of plants are raised in this garden?

2. How many rows of plants does the garden contain?

3. What fraction of an acre does the garden contain?

4. How far apart are the rows, and what is the length of each?

LUMBER

The "rule" of a lumberman is a flat measuring stick with a transverse flat head of metal, and with figures marked upon it at different lengths, to indicate the contents of logs according to length and diameter.

To estimate the amount of available timber in a saw log, deduct two inches from each side, as an allowance for slabs with bark, and then multiply the square of one quarter of the remaining diameter in inches by the length of the log in feet.

The *Scribner Rule* is the oldest in use, but is more favorable to the purchaser of logs of large diameter, and has given place generally to the *Doyle Rule*, which is more favorable to the purchaser of logs of small diameter. This is constructed by deducting 4 inches from the small diameter of the log as an allowance for slab, squaring one quarter of the remainder, and multiplying the result by the length of the log in feet.*

Thus if the log be 16 feet long and 26 inches in diameter, the removal of the slabs will leave a diameter of 22 inches. A square of one quarter of this remaining diameter is $\frac{486}{16}$, or $\frac{121}{4}$, which, multiplied by the number of feet in the length of the log, gives the product of $\frac{1936}{4}$, or 484, which is the number of feet of lumber contained in the log.

1. How many feet would you estimate to be contained in a log 16 feet long and 20 inches in diameter.

2. What is your estimate of the timber contained in five logs each 12 feet long, and each having a diameter of 34 inches?

3. A log 16 feet long and 28 inches in diameter contains how many feet of lumber?

4. What are the actual contents of the log in cubic feet? What is the difference in amount between this measure and the lumber measure?

5. A carpenter constructed for a book store the following described platforms, for holding books:

16 platforms 4 ft., 2 in. x 3 ft., 4 in. x 7 in.
12 platforms 5 ft., 6 in. x 3 ft., 2 in. x 7 in.
8 platforms 6 ft., 3 in. x 2 ft., 10 in. x 7 in.

How much inch lumber was required for all of these platforms? Draw a diagram of a platform of the largest size indicated.

*See Bulletin No. 36 of the Forestry Service in the U. S. Department of Agriculture.

COAL

1. If a coal mine yields 600 long tons of coal per month, what amount does it yield in a year?

2. If the coal cars on railways carry 23 tons each, on an average, how many carloads will the mine supply in a month?

3. If the coal produced by the mine in one month is sold by the ordinary ton, how many tons will there be of it?

4. If the price at the mine is $1.50 per long ton, what is its total value there?

5. If its price at retail is $7.25 per ton, what is its total retail value?

6. A coal bin 20 feet long and 10 feet wide, filled to a height of 5 feet, will contain how many tons of coal, $1\frac{1}{3}$ cubic yards being allowed for each ton?

7. A coal cellar 18 feet long and 20 feet wide must be filled to what height to contain 12 tons of coal?

8. West Virginia mined, in 1911, coal to the amount of 59,831,580 tons. What is four per cent of this amount?

OIL

1. A barrel of coal oil is larger than the ordinary barrel, and usually contains 42 gallons. How many lamps, holding each one pint, could be filled from one of these barrels?

2. An oil car in the form of a tank for railway transportation will hold 250 barrels of oil. What is its capacity, stated in gallons? How many ordinary barrels of liquid would the tank hold?

3. How many barrels of oil can be stored in a cylinder-shaped tank 15 feet in diameter and 15 feet in height?

4. Of a certain quantity of crude oil, as it comes from the oil well, 25% is refined into gasoline, 20% into kerosene, and 28% is usable as fuel oil, the remainder being waste. What is the per cent of waste?

5. If oil is worth 90 cents a barrel at wholesale, what is the value of the oil stored in a tank having a diameter of 30 feet and an altitude of 20 feet?

6. At 87 cents a barrel, what is the value of an oil-car load of oil?

7. What must be the flow of oil per minute at the oil well to produce 50 barrels in 24 hours?

8. The total output of oil in the United States during a certain year was valued at $134,044,752. What is 10 per cent of this amount? What is 6 per cent? What is 8 per cent?

MECHANICS

1. If the balance wheel, or flywheel, of a certain engine is rotating at the rate of 140 rotations a minute, and is 3 feet in diameter, how far does a spot or point on its outer circumference move in that time?

2. The governors of a certain engine are 18 inches apart. If they are separated 2 inches farther, how much larger an inside circle do they describe?

3. A band saw having a length of 10 feet revolves at the rate of 143 revolutions in 2 minutes. How far does the point of one of its teeth travel in that time?

4. A circular saw 2 feet in diameter makes 200 rotations in a minute. How far does the point of one of its teeth travel in that time?

5. The cylinder of a pump is 2 feet long, inside measure, exclusive of the thickness of the piston. What must be the area of its bases, inside, in order that it may draw in or throw out a barrel of water in three strokes?

6. A wheel of 200 cogs turns one of 30 cogs. When the first has made 30 rotations, how many has the second one made?

7. If the steam in an engine boiler has a pressure of 160 pounds to the square inch, what is the pressure it exerts on a piston 1 foot in diameter?

8. The steam chest, or cylinder of a certain engine, is 18 inches in length, with a diameter of 8 inches. If it is filled with steam twice every second, how many cubic feet of steam does it use in a minute?

9. A windmill having a cylinder 2½ inches in diameter is sufficient to supply water for a certain purpose. If double the amount of water is required, what must be the diameter of a new cylinder to replace the one in use?

ELECTRICITY

Electricity is a force, the nature of which is a matter of theory. In addition to its use as a means for conveying intelligence, electricity at the present time supplies light, heat, or power, or all three of these, the current for all flowing sometimes through a single wire. Electricity in large quantities is supplied generally from power houses in which it is generated from dynamos. For local uses a small dynamo may be operated; and for certain household purposes (as for door bells) a battery* is often employed. The strength, current and quantity of electricity can be measured by its effects.

The *electromotive force* (expressed by E. M. F.) is the working or driving force of an electric current. The strength of a current, other things being equal, is directly proportional to the E. M. F. The unit of E. M. F. is the *volt*. An instrument called the *voltmeter* indicates the *voltage*, or number of volts supplied by a dynamo or by a battery.

Electric currents move easily through some substances, which are therefore said to be *good conductors*. Copper and aluminum are among the best conductors. Some substances offer such resistance to an electric current that they are called *nonconductors*, and may be used as insulators to confine a current. Even the best of conductors offer some resistance

*A battery is a jar or an assembly of jars in which a current of electricity is generated by the action of a weak acid on metal bars immersed in it.

to an electric current. The unit of resistance is called the *ohm*. An instrument called the *ohmmeter* indicates directly the electrical resistance of a substance. Ohm's law, which lies at the foundation of practical electrical science, is this:

The current (in amperes) flowing through any substance is equal to the E. M. F. (in volts) in one second divided by the resistance of the substances (in ohms).

An instrument called the *ammeter* indicates the current, or flow of electricity.

The volts ÷ the ohms = the amperes.
The volts ÷ the amperes = the ohms.
The amperes × the ohms = the volts.

1. How many amperes will be sent through a wire which has a resistance of 4 ohms by a current having an E. M. F. of 40 volts?

2. How strong will be the current if the resistance is 5 ohms, and the E. M. F. is 60 volts?

3. If the E. M. F. gives 120 volts, and the current is 20 amperes, what is the resistance?

4. The filament in an electric light bulb of 16-candle power, when heated, has a resistance of about 150 ohms A copper wire $\frac{1}{8}$ of an inch in diameter and 1,500 feet long has a resistance of about 1 ohm. What must be the length of a similar wire to equal in resistance the carbon loop, or filament, of an electric light bulb of 16-candle power, the resistance being directly proportional to the length of the wire?

5. A dynamo generating an E. M. F. of 240 volts is sending a current of 30 amperes through a wire. What is the resistance?

6. A wire having a resistance of 9 ohms is conducting a current of 30 amperes. What is the voltage, or E. M. F.?

7. To drive the required current through the carbon of a 16-candle electric light bulb, the usual voltage employed is

about 100. If the total resistance is 160 ohms, what is the current? What is the quantity of electricity consumed in one minute, with this current?

8. About 10 amperes are usually employed to maintain an electric arc light. If the voltage is 2,000*, what is the resistance? What is the quantity of electricity used in a minute?—in an hour? How many myriacolombs are used in an hour?

9. The *watt*† is a unit of power. It is the amount of power conveyed by a current of one ampere through a resistance of one ohm. One horsepower is equal to 746 watts. One kilowatt is equal to how many horsepower? One myriawatt to how many horsepower?

10. To send a message by means of a telegraph wire on land, about $\frac{1}{500}$ of an ampere is necessary. If the resistance is 50 ohms, what must be the voltage?

11. What will be the resistance in 300 miles of ordinary iron telegraph wire if a mile of it has a resistance of 13 ohms?

12. Since the resistance of a wire varies *inversely* as the square of the diameter of its section‡, what will be the resistance of a wire ¼ of an inch in diameter if the resistance is 1 ohm for a similar wire ⅛ of an inch in diameter?

ARTIFICIAL STONE

Much use is made of artificial stone in the construction of pavements, columns, walls of buildings, etc. This is known as *concrete*. It is composed of cement mixed with sand, crushed rock, or gravel. Often it is molded into blocks, which are laid in walls like natural stone. To a growing extent it is run into large molds to form, when hardened, entire walls without division into blocks. When entire columns are molded, they

*Of current. †The volt, watt, coulomb, ohm, and ampere are named for illustrious scientists. ‡*Section* means *cutting;* and the area of the section of a wire or other body is the area of the end where the same is cut in two transversely. The section, or cutting, may be imaginary, since it is not necessary to do any actual cutting in order to determine the diameter.

are usually fortified within with spiral hoops or bands of steel, to resist a tendency to lateral expansion from pressure above.

There is much variety in the proportions of the ingredients, in view of the different purposes for which the product is intended. It will be readily seen that a cubic foot each of cement, sand, and crushed stone or gravel will not make three cubic feet of the mixture, for the fine grains of the cement and of the sand will be largely taken up in filling the little spaces among the coarser fragments of the stone or gravel, without adding to the bulk of the latter. Thus it is found necessary to allow about 1.6 cubic feet of materials for the production of 1 cubic foot of concrete.

A barrel of cement contains 4 cubic feet. Cement is often supplied in sacks, each containing one-fourth of a barrel. The other ingredients are measured in cubic yards and feet.

1. In a simple concrete formed of equal volumes of cement and sand, the cement required for 1 cubic yard is 4.8 barrels. What fraction of a cubic yard does it constitute?

2. What remaining fraction of the cubic yard must be made up of the sand used?

3. Since equal bulks of sand and cement are used to make this composition, find what is the difference between the aggregate amount of the two materials and the amount of the product. How do you explain the difference?

4. A kind of concrete known as the "one-two-three" mixture, and used for pavements, makes use of one volume of cement, two of sand, and three of crushed stone. For 1 cubic yard of this concrete 6.96 sacks of cement are used. What will be the total bulk of all the ingredients for 1 cubic yard of this concrete? How much does this exceed the bulk of the finished concrete?

5. Another kind of concrete, used in building walls, is called the "one-three-five" mixture. It contains one volume

of cement, three of sand, and five of crushed stone. For 1 cubic yard of this concrete 4.64 sacks of cement are used. What is the total bulk of the materials used to make 1 cubic yard of this concrete?

6. How much of these materials is used in filling the "voids," or interstices of the rock and sand?

GOOD ROADS PROBLEMS

1. The maximum grade of ascent or descent for important roads has been fixed, generally, at 5%, or 5 feet of rise or fall in 100 feet of length. For a rise of 528 feet, what would be the length of road, at this minimum grade?

2. Gutter grades, at the sides of roadways, should have a minimum fall of 6 inches in 100 feet, to the culverts. If the culverts are 600 feet apart, how much will the gutters slope to meet them, at this minimum grade?

3. For a road 15 feet or less in width, the middle line, or crown, should be 5¼ inches higher than the sides. For a greater width of road, the crown should be raised ½ inch for each foot of distance from the boundary. What should be the height of the crown of a road 30 feet wide?—36 feet wide?

4. If to build a sand-clay roadway 12 feet wide, with an average depth of 6 inches of clay, requires a cubic yard of sand-clay for every 4½ feet of road length, how many cubic yards of clay will be required for every mile of the roadway?

5. In building a burnt-clay road in the South for 300 feet, as a test, the following expenses were incurred:

30½ cords of wood, at $1.30	$39.65
20 loads of bark, chips, etc.	6.00
Labor, at $1.25 per day, and teams at $3.00 per day	38.30

What was the total cost per foot of length, and what, at this rate, would be the cost per mile?

6. If in such a road the clay has been burnt to a depth of 12 inches, and when rolled and compacted the burnt surface is but 8 inches deep, what has been the per cent of the shrinkage?

7. If a ton of broken rock for a macadam road will answer for 3.13 square yards of surface, the depth of the broken rock being 5 inches after rolling, how many tons of this material will be required for 62,600 square yards of macadam road?

8. If a cubic yard of broken stone for a macadam road weighs $1\frac{1}{3}$ tons, what will be the weight of such material required for a road 1,200 yards long and 6 yards wide?

TRANSPORTATION

1. If the time required by a railway train to pass from San Francisco to New York is 103 hours, and the distance by rail is 3,250 miles, at what average rate of speed must the train travel?

2. Half a century ago a steamer crossed the Atlantic in 9 days. If the distance covered was 3,100 miles, what was the average rate of speed of the vessel per day?—per hour?

3. If a knot at sea equals 1.15 miles per hour, what will be the speed of a vessel "making" 20 knots? How far will such a vessel go in a day?

4. The distance from New York to Denver is 1,930 miles. How many hours would be required for a train to cover this distance at the rate of 40 miles an hour?

5. One train leaves a certain station, going eastward at the rate of 30 miles an hour, and another leaves the same station going westward at the rate of 40 miles an hour. If these rates are maintained, how far apart will the trains be at the end of 7 hours?

6. New York and Albany are 142 miles apart. If a train leaves New York going northward at the same time when a

train leaves Albany going southward and the trains travel at the average rate of 39 miles an hour, how far apart will they be in 40 minutes?

7. If a steamer makes the trip from Bombay to London, via Suez, (6,332 miles) at the average rate of 24 miles per hour, what time is required for the trip?

8. Last year a farmer hauled over bad roads 3040 bushels of wheat to a railway station, being able to carry only 38 bushels in each trip, and to make only four trips in a day. This year, with a good macadam road built, he can haul 60.8 bushels on each trip, and can make five trips in a day. What is his saving in time through the improvement of the road?

9. Valuing the time of the team and driver at $5.50 per day, what was the saving in money value?

TRAVEL

In any country where the centesimal system of money denominations is in use, written statements of amounts of foreign moneys look very natural to us, for they are written as we write dollars and cents. Thus 5 francs and 15 centimes are written 5.15. Many travelers make it a rule in such cases to *think* of the denominations as dollars and cents, until the summing up of the items of a bill is effected and the total is found, after which they convert the sum into approximate or exact equivalents in our money. For a rough estimate, sufficient for most purposes in traveling, where small sums are spent for personal expenses, a *franc* or *lira* or *drachma* or *peseta* is called 20 cents; a *ruble*, a *yen* or a *milreis*, a half-dollar; a *mark* or a *krone*, a "quarter"; etc.

In the following problems the money unit of the country visited and its centesimal parts are left to be stated by thepupil:

1. If, when visiting **Marseilles** your expense for a luncheon and for table service is 2.05,for carriage hire 10.15, and for

a souvenir 1.20, how will you compute the total? What will be its approximate equivalent in American money? What its exact equivalent?

2. If in **Rome** the charge for your hotel room is 6.10, for meals and table service 9.10, for street car and carriage expense 20.15, for a guide 10.20, and for souvenirs 14.50, what will be the total? What will be its approximate equivalent in American money?

3. If in **Copenhagen** you pay 1.90 for a luncheon, and 7.50 for a drive about the city, what will be the cost of both, and what its approximate equivalent in American money?

4. If a student friend writing from **Corinth** tells you that his expenses for board and lodging are 8.50 a day, and that his other expenses amount to 2.75 a day, how would you state the total amount in its U. S. equivalent?

5. If in **Rotterdam** one of your traveling friends states that he paid 1.50 to a hack driver to take him to the park where he could see the statue of Erasmus, and 3.10 for souvenirs, together with 1.10 each for breakfast, lunch, and dinner, what was the amount of these expenses, roughly stated in U. S. money? What was the exact amount?

6. If you visit **Panama** and find that your hotel expenses are 4.00 per day, and your incidentals, .75 more, what is the aggregate for the day, in U. S. money?

7. If a friend in **Rio de Janiero** writes to you that he has invested 5,000 milreis in *mate*, or Paraguayan tea, with a view to finding a new market for it, what do you understand to be, in U. S. money, the amount of his investment?

8. If in **Moscow** an American tourist pays for a week's board and lodgings 16.20, for souvenirs 4.20, and for carriage and guides 60.00; what is the approximate expenditure for these, stated in U. S. money? What is the exact amount?

Means For Measuring Dry Substances: *

Pint

Quart

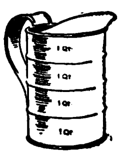
Peck

For Measuring Liquids:

Gill

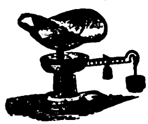
Pint

Quart



Gallon

For Weighing:



Scales

TABLES OF DENOMINATE NUMBERS

Dry Measure

2	pints	=1	quart (qt.)
8	quarts	=1	peck (pk.)
4	pecks	=1	bushel (bu.)

Liquid Measure

4	gills	=1	pint (pt.)
2	pints	=1	quart (qt.)
4	quarts	=1	gallon (gal.)
31½	gallons	=1	barrel (bbl.)
63	gallons	=1	hogshead (hhd.)

Avoirdupois Weight

16	ounces (oz.)	=1	pound (℔.)
100	pounds	=1	hundredweight (cwt.)
2000	pounds	=1	ton (T.)

One pound Avoirdupois = 7000 grains.

Troy Weight

24	grains (gr.)	=1	pennyweight (pwt.)
20	pennyweights	=1	ounce (oz.)
12	ounces	=1	pound (lb.)

One Pound Troy = 5760 grains.

Apothecaries' Weight

60	grains (gr.)	=1	dram (dr. or ʒ)
8	drams	=1	ounce (oz. or ℥)
12	ounces	=1	pound (lb. or ℔)

One Pound Apothecaries' weight = 5760 grains

Apothecaries' Liquid Measure

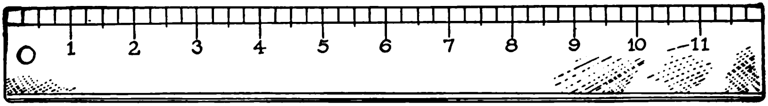
60	minims (m.)	=1	fluid dram (ʒ)
8	fluid drams	=1	fluid ounce (℥)
16	fluid ounces	=1	pint (fl. oz. or O)
8	pints	=1	gallon (cong.)

Measure of Time

60	seconds (sec.)	=1	minute (min.)
60	minutes	=1	hour (hr.)
24	hours	=1	day (da.)
7	days	=1	week (wk.)
365	days	=1	common year (yr.)
366	days	=1	leap year

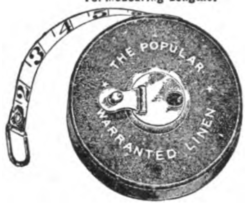
*All schools are not fully equipped with the various measures and instruments required in the study of Denominate Numbers. For this reason suitable illustrations accompany these tables.

TABLES OF DENOMINATE NUMBERS

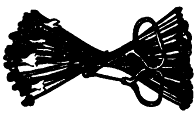


Foot Rule, reduced to one third of its true length.

For Measuring Lengths:

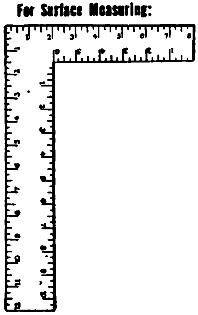


Tape Measure



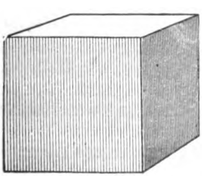
Surveyor's Chain*

For Surface Measuring:



Carpenter's Square

Illustrating Solids:



Cube

Linear Measure

12	inches (in.)	= 1	foot	(ft.)
3	feet	= 1	yard	(yd.)
5½	yards, or 16½ feet	= 1	rod	(rd.)
40	rods	= 1	furlong	(fur.)
320	rods, or 5280 feet	= 1	mile	(mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

Surveyors' Linear Measure

7.92	inches (in.)	= 1	link	(l.)
25	links	= 1	rod	(rd.)
100	links	= 1	chain	(ch.)
80	chains	= 1	mile	(mi.)

Square Measure

144	square in. (sq. in.)	= 1	square foot	(sq.ft.)
9	square feet (sq.ft.)	= 1	square yard	(sq.yd.)
30¼	sq. yd., or 272¼ sq.ft	= 1	square rod	(sq.rd.)
160	square rods	= 1	acre	(A.)
640	acres	= 1	square mile	(sq.mi.)

1 A. = 160 sq.rd. = 4840 sq.yd. = 43,560 sq.ft.

Surveyors' Square Measure

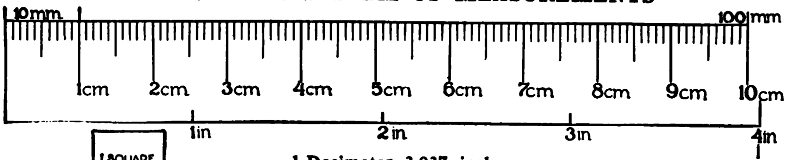
16	square rods	= 1	square chain	(sq.ch.)
10	square chains	= 1	acre	(A.)
640	acres	= 1	square mile	(sq.mi.)
1	square mile	= 1	section	(sec.)
36	sections	= 1	Cong. township	(T.)

Cubic Measure

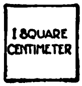
1728	cubic inches (cu.in)	= 1	cubic foot	(cu.ft.)
27	cubic feet	= 1	cubic yard	(cu.yd.)
128	cubic feet	= 1	cord	(cd.)
16	cubic feet	= 1	cord foot	(cd.ft.)
8	cord feet	= 1	cord	(cd.)

*Instead of the standard (Gunter's) chain, some surveyors use a steel tape 50 feet long, divided into foot lengths, each of these being marked off into tenths.

THE METRIC SYSTEM OF MEASUREMENTS



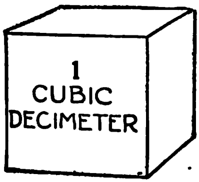
1 Decimeter 3.937 inches



0.155 sq. inches

Tables of Metric Measures of Length

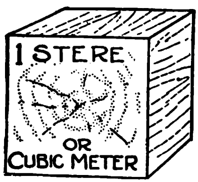
10 *millimeters* (mm)	=1	*centimeter* (cm)
10 centimeters	=1	decimeter (dm)
10 decimeters	=1	*meter* (m)
10 meters	=1	decameter (Dm)
10 decameters	=1	hectometer (Hm)
10 hectometers	=1	kilometer (Km)
10 kilometers	=1	myriameter (Mm)



3.5315 cu. ft.

Table of Metric Measure of Surfaces

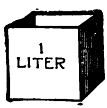
100 *sq. millimeters* (smm)	=1	*sq. centimeter* (scm)
100 sq. centimeters	=1	sq. decimeter (sdm)
100 sq. decimeters	=	{1 *sq. meter* (sm) {1 centare (ca)
100 centares	=1	are (a)
100 ares	=1	hectare (Ha)



35.3 cu. ft.

Table of Metric Measures of Volume

1000 *cu. millimeters* (cmm)	=1	*cu. centimeter* (ccm)
1000 cu. centimeters	=1	*cu. decimeter* (cdm)
1000 cu. decimeters	=1	*cu. meter* (cm), or stere (s)



0.908 qt. (dry)
1.0567 qt. (liq.)

Tables of Metric Measures of Capicaty

10 milliliters (ml)	=1	centiliter (cl)
10 centiliters	=1	deciliter (dl)
10 deciliters	=1	*liter* (l)
10 liters	=1	decaliter (Dl)
10 decaliters	=1	hectoliter (Hl)
10 hectoliters	=1	kiloliter (Kl)

15.432 grains

2.2046 lb. (avoir)

Table of Metric Measures of Weight

10 *milligrams* (mg)	=1	*centigram* (cg)
10 centigrams	=1	decigram (dg)
10 decigrams	=1	*gram* (g)
10 grams	=1	decagram (Dg)
10 decagrams	=1	hectogram (Hg)
10 hectograms	=1	*kilogram* (Kg)
10 kilograms	=1	myriagram (Mg)
10 myriagrams	=1	quintal (Q)
10 quintals	=1	*Metric Ton* (MT)

The Meter, 39.37 inches, is the foundation unit of the Metric System

METRIC EQUIVALENTS

Length

Myriameter	10,000 meters	6.2137 miles.
Kilometer	1,000 meters	.62137 mile.
Hectometer	100 meters	328 feet 1 inch.
Decameter	10 meters	393.7 inches.
Meter	1 meter	39.37 inches.
Decimeter	0.1 meter	3.937 inches.
Centimeter	0.01 meter	.3937 inch.
Millimeter	0.001 meter	.0394 inch.

Surface

Hectare	10,000 square meters	2.471 acres.
Are	100 square meters	119.6 square yards.
Centare	1 square meter	1,550 square inches.

Capacity

Kiloliter, or Stere	1,000 liters	1 cu. meter	1,308 cu. yds.	
Hectoliter	100 liters	0.1 cu. meter	2.837 bu.;	26.417 gal.
Decaliter	10 liters	10 cu. decim.	1.135 pk.;	2.6417 gal.
Liter	1 liter	1 cu. decim.	.908 quart:	1.0567 qts.
Deciliter	0.1 liter	0.1 cu. decim.	6.1022 cu. in.;	0.845 gill
Centiliter	0.01 liter	10 cu. centim.	.6102 cu. in.:	0.338 fl. oz.
Milliliter	0.001 liter	1 cu. centim.	.061 cu.in.:	0.27 fl. dram

Weight

Metric Ton, Millier, or Tonneau	1,000,000 grams	1 cubic meter	2204.6 lbs.
Quintal	100,000 grams	1 hectoliter	220.46 lbs.
Myriagram	10.000 grams	1 dekaliter	22.046 lbs.
Kilogram, or Kilo	1,000 grams	1 liter	2.2046 lbs.
Hectogram	100 grams	1 deciliter	3.5274 oz.
Decagram	10 grams	10 cubic centimeters	.3527 oz.
Gram	1 gram	1 cubic centimeter	15.432 gr.
Decigram	0.1 gram	.1 cubic centimeter	1.5432 gr.
Centigram	0.01 gram	10 cubic millimeters	.1543 gr.
Miligram	0.001 gram	1 cubic millimeter	.0154 gr.

Helpful information not included in the Regular Tables

A cubic foot of air weighs approximately 1¼ ounces.

A cubic foot of water weighs approximately 62½ lbs.

A gallon of water weighs approximately 8⅓ lbs.

A fathom, used in measuring the depth of water, is 6 ft.

A furlong is equal to ⅛ of a mile.

A league measures 3 nautical miles.

A knot (nautical mile) is equal to 6080.27 ft., approximately 1 1-15 common miles.

A hand, used in measuring the height of horses, is 4 in.

A pace in walking is 2½ feet. For stepping distance it is 3 3-10 feet, or 1-5 of a rod.

14 lbs. equal 1 stone.

160 stone equal 1 long ton.

1 long ton ,used in U. S. custom houses, in mines, and sometimes elsewhere, is equivalent to 2240 lbs.

A cubic yard of earth is usually considered a load.

24¾ cubic feet of masonry is called a perch.

Masons and brick-layers regard an inclosing wall, measured around the outside, as one straight wall; thus practically the corners are measured twice.

The grains are the same in Avoirdupois, Troy, and Apothecaries weight. The Avoirdupois pound is greater than the Troy or Apothecaries pound, but the Avoirdupois ounce is less than the Troy or Apothecaries ounce.

APPROXIMATE COMPOSITION OF AMERICAN FOOD PRODUCTS

	Protein	Fat	Carbo-hydrates	Ash (mineral)	Water	Refuse
White bread	9.2	1.3	53.1	1.1	35.3	....
Graham bread	8.9	1.8	52.1	1.5	35.7	....
Soda crackers	9.8	9.1	73.1	2.1	5.9	....
Corn meal	9.2	1.9	75.4	1.0	12.5	....
Oat meal	16.7	7.3	66.2	2.1	7.7	....
Buckwheat	6.4	1.2	77.9	.9	13.6	....
Butter	1.0	85.0	....	3.0	11.0	....
Cheese	26.8	35.3	3.3	3.8	30.8	....
Milk	3.3	4.0	5.0	0.7	87.0	....
Buttermilk	3.0	.5	4.8	0.7	91.0	....
Cream	2.5	18.5	4.5	0.5	74.0	....
Beef	16.7	16.1	....	0.8	51.7	16.4
Veal	16.5	7.8	....	0.7	58.2	17.5
Pork	14.5	23.2	....	0.8	50.3	14.3
Mutton	13.7	25.5	....	0.7	44.0	16.6
Sausage	16.9	27.5	1.1	3.1	50.7	3.3
Soups	3.2	2.1	4.3	1.3	89.0	....
Chicken	13.7	6.8	....	0.7	45.4	33.7
Turkey	16.1	18.4	....	0.8	42.4	22.7
Fish	14.9	3.0	.4	0.9	52.9	35.5
Oysters	6.0	1.3	3.3	1.1	88.3	....
Potatoes	1.8	0.1	14.7	0.8	62.6	20.0
Sweet potatoes	1.4	0.6	21.9	0.9	55.2	20.0
Beans	7.1	0.7	22.0	1.7	68.5	....
Peas	7.0	0.5	16.9	1.0	74.6	....
Beets	1.3	0.1	7.7	0.9	70.0	20.0
Cabbage	1.4	0.2	4.8	0.9	77.7	15.0
Onions	1.4	0.3	8.9	0.5	78.9	10.0
Turnips	0.9	0.1	5.7	0.6	62.7	30.0
Parsnips	1.3	0.4	10.8	1.1	66.4	20.0
Squash	0.7	0.2	4.5	0.4	44.2	50.0
Tomatoes	0.9	0.4	3.9	0.5	94.3	....
Cucumbers	0.7	0.2	2.6	0.4	81.1	15.0
Lettuce	1.0	0.2	2.5	0.8	80.5	15.0
Rhubarb	0.4	0.4	2.2	0.4	56.6	40.0
Rice	8.0	0.3	79.0	0.4	12.3	....
Tapioca	0.4	0.1	88.0	0.1	11.4	....
Sugar	....	....	100.0	....	....	....
Molasses	....	....	70.0	....	....	....
Honey	....	....	81.0	....	....	....
Candy	....	....	96.0	....	....	....
Apples	0.3	0.3	10.8	0.3	63.3	25.0
Bananas	0.8	0.4	14.3	0.6	48.9	35.0
Grapes	1.0	1.2	14.4	0.4	58.0	25.0
Lemons	0.7	0.5	5.9	0.3	62.5	30.0
Oranges	0.6	0.1	8.5	0.4	63.4	27.0
Pears	0.5	0.4	12.7	0.4	76.0	10.0
Strawberries	0.9	0.6	7.0	0.6	85.9	5.0
Watermelon	0.2	0.1	2.7	0.1	37.5	59.4
Muskmelon	0.3	....	4.6	0.3	44.8	50.0
Dates—dried	1.9	2.5	70.6	1.2	13.8	10.0
Figs—dried	4.3	0.3	74.2	2.4	18.8	....
Raisins—dried	2.3	3.0	68.5	3.1	13.1	10.0
Almonds	11.5	30.2	9.5	1.1	2.7	45.0
Brazilnuts	8.6	33.7	3.5	2.0	2.6	49.6
Butternuts	3.8	8.3	0.5	0.4	.6	86.4
Chestnuts	6.6	4.9	45.9	1.4	21.1	15.0
Cocoanuts	4.6	41.6	22.9	1.1	5.4	25.0
Hickory nuts	5.8	25.5	4.3	0.8	1.4	62.2
Filberts	7.5	31.3	6.2	1.1	1.8	52.4
Pecans	5.2	33.3	6.2	0.7	1.4	53.2
Peanuts	19.5	29.1	18.5	1.5	6.9	24.5
English walnuts	6.9	26.6	6.8	.6	1.0	58.1

LEGAL EQUIVALENTS OF A BUSHEL,* IN POUNDS, FOR VARIOUS PRODUCTS.

STATES	Wheat	Rye	Buckwheat	Shelled† Corn	Oats	Barley	Apples	Potatoes	Sweet Potatoes	Beans	Peas	Beets	Onions	Flaxseed
Alabama	60	56		56	32	47			55	60	60			
Arizona	60	56			32	45				55				
Arkansas	60	56	52	56	32	48	50	60	50	60	60		57	56
California	60	54	40		32	50								
Colorado	60	56	52		32	48		60		60			57	
Connecticut	60	56	48		32	48	48	60	54	60	60	60	52	55
Delaware	60													
Florida	60	56		56	32	48	48		60	60			56	
Georgia	60	56	52	56	32	47			55	60	60		57	56
Idaho	60	56	42		36	48	45	60						56
Illinois	60	56	52	56	32	48			50	60			57	56
Indiana	60	56	50	56	32	48		60	55	60			48	
Iowa	60	56	52	56	32	48	48	60	46	60			57	56
Kansas	60	56	50		32	48	48	60	50	60			57	56
Kentucky	60	56	56	56	32	47		60	55	60	60		57	56
Louisiana	60	56												
Maine	60	50	48		32	48	44	60		60	60	60	52	
Maryland					26			56						
Massachusetts	60	56	48	50	32	48	48	60	54	60	60		52	55
Michigan	60	56	48	56	32	48	48		56	60	60		54	56
Minnesota	60	56	50	56	32	48	50		55	60	60	50	52	
Mississippi	60	56	48	56	32	48			60	60	60		57	56
Missouri	60	56	52	56	32	48	48		56	60	60		57	56
Montana	60	56	52	56	32	48	45	60		60	60	50	57	56
Nebraska	60	56	52	56	32	48			50	60	60		57	56
Nevada														
N. Hampshire	60	56			32			60		62	60			
New Jersey	60	56	50		30	48	50		54	60	60		57	55
New Mexico														
New York	60	56	48		32	48	48		54	60	60		57	55
North Carolina	60	56	50		32	48					60			55
North Dakota	60	56	42	56	32	48	50		46	60	60	60	52	56
Ohio	60	56	50	56	32	48	50		50	60	60	56	55	56
Oklahoma	60	56	42	56	32	48			46	60	60	60	52	56
Oregon	60	56	42		32	46	45	60						
Pennsylvania	60	56	48		32	47		56					50	
Rhode Island	60	56	48	56	32	48	48		54	60	60	50	50	56
South Carolina														
South Dakota	60	56	42	56	32	48			46	60	60	60	52	56
Tennessee	60	56	50	56	32	48	50		50	60	60	50	56	56
Texas	60	56	42	56	32	48	45		55	60			57	56
Utah														
Vermont	60	56	48		32	48	46	60		62	60	60	52	
Virginia	60	56	52	56	30	48			56	60	60		57	56
Washington	60	56	42		32	48	45	60						56
West Virginia	60	56	52		32	48		60		60				56
Wisconsin	60	56	50		32	48	50		54	60	60	50	57	56
Wyoming														
U. S. Customs	60	56	42		32	48	60	60			60			56
Dist. of Col.								60						

* The **standard** bushel used for grain measure contains 2150.42 cu. in., approximately 1¼ cu. ft. It is in cylinder form having a diameter of 18½ in. and a depth of 8 in. The **heaped** bushel is employed in measuring coarse commodities and is equal to 2747.07 cu. in., approximately 1⅝ cu. ft.

† Two bushels of corn in the ear are considered equivalent to one bushel of **shelled** corn.

FOREIGN MONEY VALUES *

Country	Money Unit	Equivalent In Lower Denominations	Value of Unit in U.S. Money
Abyssinia	Levant Dollar	23 kharafs	$ .45 1
Argentina	Peso	100 centavos	.96 5
Austria-Hungary	Krone	100 heller	.20 3
Belgium	Franc	100 centimes	.19 3
Bolivia	Boliviano	100 centavos	.38 9
Brazil	Milreis	1000 reis	.54 6
Bulgaria	Lev	100 stotink	.19 3
Canada	Dollar	100 cents	1.00
Chile	Peso	100 centavos	.36 5
China	Tael	100 fen, or cadareen	.72 9
Colombia	Dollar	100 centavos	1.00
Costa Rica	Colon	100 centavos	.46 5
Cuba	Dollar	100 cents	1.00
Denmark	Krone	100 ore	.26 8
Dominican Rep.	Dollar	100 cents	1.00
Ecuador	Sucre	100 centavos	.48 7
Egypt	Pound	100 piasters	4.94 3
France	Franc	100 centimes	.19 3
Germany	Mark	100 pfennigs	.23 8
Great Britain	Pound	240 pence	4.86 65
Greece	Drachma	100 lepta	.19 3
Guatemala	Peso	100 centavos	.43 4
Haiti	Gourde	100 centimes	.96 5
Holland	Guilder	100 cents	.40 2
Honduras	Peso	100 centavos	.43 4
India	Rupee	192 pi	.32 4
Italy	Lira	100 centesimi	.19 3
Japan	Yen	100 sen	.49 8
Mexico	Peso	100 centavos	.49 8
Morocco	Dollar	20 dirhem	.40
Nicaragua	Cordoba	100 centavos	1.00
Norway	Krone	100 ore	.26 8
Panama	Balboa	100 cents	1.00
Paraguay	Peso	100 centesimos	.43 4
Persia	Toman	200 chahis	1.70 4
Peru	Libra	100 centavos	4.86 65
Portugal	Milreis	1000 reis	1.08
Roumania	Lev	100 bani	.19 3
Russia	Ruble	100 kopecks	.51 5
Salvador	Peso	100 centavos	.43 4
Servia	Dinar	100 paras	.19 3
Siam	Tical	4 salungs	.37
Spain	Peseta	100 centimos	.19 3
Sweden	Krone	100 ore	.26 8
Switzerland	Franc	100 centimes, or rappes	.19 3
Turkey	Lira, or Pound	100 piasters	4.39 6
Uruguay	Peso	100 centesimos	1.03 4
Venezula	Bolivar	100 centimos	.19 3

*These are **denominations**, not necessarily coins. For the pronunciation of any of these names about which you are in doubt, consult a dictionary.

States	Admission or Ratification of U. S. Constitution		Area in Sq. Miles	Population, 1910
Alabama	Dec. 14,	1819	51,998	2,138,093
Arizona	Feb. 14,	1912	113,956	204,354
Arkansas	June 15,	1836	53,335	1,574,449
California	Sept. 9,	1850	158,297	2,377,549
Colorado	Aug. 1,	1876	103,948	799,024
Connecticut	Jan. 9,	1788	4,965	1,114,756
Delaware	Dec. 7,	1787	2,370	202,322
Florida	Mar. 3,	1845	58,666	752,619
Georgia	Jan. 2,	1788	59,265	2,609,121
Idaho	July 3,	1890	84,313	325,594
Illinois	Dec. 3,	1818	56,665	5,638,591
Indiana	Dec. 11,	1816	36,354	2,700,876
Iowa	Dec. 28,	1846	56,147	2,224,771
Kansas	Jan. 29,	1861	82,158	1,690,949
Kentucky	June 1,	1792	40,598	2,289,905
Louisiana	April 30,	1812	48,506	1,656,388
Maine	Mar. 15,	1820	33,040	742,371
Maryland	April 28,	1788	12,327	1,295,346
Massachusetts	Feb. 7,	1788	8,266	3,366,416
Michigan	Jan. 26,	1837	57,980	2,810,173
Minnesota	May 11,	1858	84,682	2,075,708
Mississippi	Dec. 10,	1817	46,865	1,797,114
Missouri	Aug. 10,	1821	69,420	3,293,335
Montana	Nov. 8,	1889	146,572	376,053
Nebraska	Mar. 1,	1867	77,520	1,192,214
Nevada	Oct. 31,	1864	110,690	81,875
New Hampshire	June 21,	1788	9,341	430,572
New Jersey	Dec. 18,	1787	8,224	2,537,167
New Mexico	Jan. 6,	1912	122,634	327,301
New York	July 26,	1788	49,204	9,113,614
North Carolina	Nov. 21,	1789	52,426	2,206,287
North Dakota	Nov. 2,	1889	70,837	577,056
Ohio	Feb. 19,	1803	41,040	4,767,121
Oklahoma	Nov. 16,	1907	70,057	1,657,155
Oregon	Feb. 14,	1859	96,699	672,765
Pennsylvania	Dec. 12,	1787	45,126	7,665,111
Rhode Island	May 29,	1790	1,248	542,610
South Carolina	May 23,	1788	30,989	1,515,400
South Dakota	Nov. 2,	1889	77,615	583,888
Tennessee	June 1,	1796	42,022	2,184,789
Texas	Dec. 29,	1845	265,896	3,896,542
Utah	Jan. 4,	1896	84,990	373,351
Vermont	Mar. 4,	1791	9,564	355,956
Virginia	June 25,	1788	42,627	2,061,612
Washington	Nov. 11,	1889	69,127	1,141,990
West Virginia	June 20,	1863	24,170	1,221,119
Wisconsin	May 29,	1848	56,066	2,333,860
Wyoming	July 10,	1890	97,914	145,965
Territories	**Organized**			
Alaska	July 27,	1868	590,884	64,356
Dis. of Columbia	Mch. 30,	1791	70	331,069
Hawaii	April 30,	1900	6,449	191,909
Colonies	**Acquired**		3,624,122	92,228,531
Porto Rico	Dec. 10,	1898	3,435	1,118,012
Philippine Islands	Dec. 10,	1898	115,026	7,635,000
Tutuila etc.	Dec. 2,	1899	77	5,000
Guam,	Dec. 10,	1898	210	12,000
Panama Canal Zone	Feb. 26,	1904	436	50,000
In	U. S. Service abroad			55,608
	Total		3,743,306	101,104,151

The *vara* (vah'rah) is a measure of land in Latin America, having different values in Mexico, Cuba, Brazil, Colombia, Argentina, the Central American States, etc. In all these countries except Brazil it is somewhat less than a yard in length.

In Texas and in California the vara is used, it being an inheritance from the early settlers of these States, who were Spaniards and Mexicans. In Texas the vara is $33\frac{1}{3}$ inches. In California it is 33 inches; the square vara is .84 of a square yard, and 5761.9 square varas equal 1 acre.

The *labor* (lah-bore'), likewise is a denomination in the land measure of Latin America. It is a square measuring on each side one thousand varas. In Texas it is the equivalent of $177\frac{1}{7}$ (177.136) acres. It contains one million square varas.

A *legua* (lay'goo-ah), or *Spanish square league,* contains 25 labores (lah-bor'es, plural of *labor*) and differs in value in various countries of Latin America, owing to the difference in the values of the labores. In Texas it is the equivalent of 4,428.4 acres.

Our rapidly increasing commercial relations with Latin America, often involving amounts of real estate, render some acquaintance with these land measures desirable as a matter of general information.

Spanish Land Measures Used in Texas:

1 vara = $33\frac{1}{3}$ inches	1 mile = 1900.8 varas
36 varas = 100 feet	1,000,000 square varas = 1 labor
108 varas = 100 yards	25 labors = 1 legua
1900.8 varas = 1 mile	1 acre = 5645.376 square varas
1 inch = .03 varas	1 labor = 177.136 acres
1 yard = 1.08 varas	1 legua = 4428.4 acres

PERPETUAL CALENDAR

COVERING 212 YEARS, VIZ. 1750—1961.

DAY OF THE MONTH					Jan. Oct.	Apr. July *Jan.* *	Sept. Dec.	June	Feb. Mar. Nov.	Aug. *Feb.* *	May.	DAY OF WEEK
1	8	15	22	29	a	b	c	d	e	f	g	Mon.
2	9	16	23	30	g	a	b	c	d	e	f	Tues.
3	10	17	24	31	f	g	a	b	c	d	e	Wed.
4	11	18	25		e	f	g	a	b	c	d	Thurs.
5	12	19	26		d	e	f	g	a	b	c	Fri.
6	13	20	27		c	d	e	f	g	a	b	Sat.
7	14	21	28		b	c	d	e	f	g	a	Sun.

To find the day of the week corresponding to any date, find the small letter directly under the month and opposite the day of the month; the same small letter also appears in the vertical column that contains the number of the year; and if the line in which it stands is followed out to the right, the day of the week is found. Thus, the small letter under March and opposite 18 is *b*; *b* appears again directly over 1904, and at its right is the word *Friday*. March 18 fell on Friday in 1904, and also in 1898, 1892, etc. The calendar has other uses, as for finding the months which begin on Sunday in a particular year, etc.

Jan. Oct.	Apr. July *Jan.* *	Sept. Dec.	June	Feb. Mar. Nov.	Aug. *Feb.* *	May.
1753	1754	1755	1750	1751	1757	1752
1759	1765	1760	1761	1756	1763	1758
1764	1771	1766	1767	1762	1768	1769
1770	1776	1777	1772	1773	1774	1775
1781	1782	1783	1778	1779	1785	1780
1787	1793	1788	1789	1784	1791	1786
1792	1799	1794	1795	1790	1796	1797
1798	1805	1800	1801	1802	1803	1809
1804	1811	1806	1807	1813	1808	1815
1810	1816	1817	1812	1819	1814	1820
1821	1822	1823	1818	1824	1825	1826
1827	1833	1828	1829	1830	1831	1837
1832	1839	1834	1835	1841	1836	1843
1838	1844	1845	1840	1847	1842	1848
1849	1850	1851	1846	1852	1853	1854
1855	1861	1856	1857	1858	1859	1865
1860	1867	1862	1863	1869	1864	1871
1866	1872	1873	1868	1875	1870	1876
1877	1878	1879	1874	1880	1881	1882
1883	1889	1884	1885	1886	1887	1893
1888	1895	1890	1891	1897	1892	1899
1894	1901	1902	1896	1909	1898	1905
1900	1907	1913	1903	1915	1904	1911
1906	1912	1919	1908	1920	1910	1916
1917	1918	1924	1914	1926	1921	1922
1923	1929	1930	1925	1937	1927	1933
1928	1935	1941	1931	1943	1932	1939
1934	1940	1947	1936	1948	1938	1944
1945	1946	1952	1942	1954	1949	1950
1951	1957	1958	1953		1955	1961
1956			1959		1960	

*For dates occurring in Jan. or Feb. of leap year, use *italic* names of months, above.

PROBLEMS ON THE PERPETUAL CALENDAR

1. The Declaration of Independence was adopted July 4, 1776. On what day of the week was it adopted?

2. A certain man was born in the first term of President Grover Cleveland. His birthday is November 28, and it is remembered that he was born on Sunday, but he is not certain as to the exact year. In what year was he born?

3. A political pamphlet of historic value printed while James K. Polk was President is much prized by its owner, who is a bibliophile. It is dated Wednesday, January 10, but the number of the year is torn off. What was the year of its publication?

4. While a certain will, dated 1902, is in the hands of the court, another will is discovered, bearing date of Friday, Oct. 26. The number of the year is either 1901 or 1906, but the experts in penmanship cannot determine which it is. Is the new-found will the older of the two or the later? In which of the years was it written?

5. It is remembered that a certain member of a family was born on Friday, the "unlucky day." His birthday is July 19, and his birth was in the time when Benjamin Harrison was President, but the year is not certain. In what year was he born?

6. Since a man attains his majority on the day before his birthday, on what date will a person born March 6, 1914, attain his majority? What will be the day of the week?

7. If the Tuesday nearest that date shall be an election day for local purposes, on what date will it fall? Will he then be a voter?

8. Once in 28 years the calendar for the year is repeated exactly, the same dates falling on the same week days. In what year will the calendar of 1914 be repeated first? When will it be repeated a second time?

9. President Abraham Lincoln's life was closed April 15, 1865. On what day of the week did he pass away?

10. If Captain Herschel, who prepared this calendar, completed the work on June 2, 1750, on what day of the week was it completed?

EXERCISES FOR PRACTICE

in

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

Arranged By

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SUGGESTIONS TO TEACHERS

The following Exercises for Practice have been arranged to assist pupils to develop accuracy and speed in the fundamental operations. Pupils should practice until they can **correctly** work every example in each exercise within the time limit prescribed for each grade in this table.*

Grade	4	5	6	7	8
Number of minutes required to work each exercise	3¼	2¾	2¼	2	1¾

Directions for Conducting Practice Exercises

1. A copy of this book should be in the hands of each pupil, and he should open the book to the exercise to be used.

2. Each pupil is to have a blank sheet of paper, which he places directly under the first row of examples to be worked. All answers are to be written on this paper. To avoid crowding the answers, the pupil should draw the paper slightly to the left after he writes each answer. Answers to examples in the second and succeeding rows are to be placed under the answers for the first row. No pupil is to write answers in the book. Exercises XV, XXXV, XLI and XLIV must be copied on paper before the practice exercise is begun.

3. Every exercise is to be accurately timed by means of a watch or clock. Start and stop pupils promptly.

4. As soon as the time has expired, the teacher will quickly read the answers to the examples. As she reads, each pupil checks the examples which he has worked incorrectly. As soon as the errors have been checked, the papers are to be collected by the teacher.

5. On the first day these exercises are given, all pupils are to begin with Exercise I. Pupils who are perfect in Exercise I will, on the following day, take Exercise II; while those who fail on Exercise I must repeat this exercise until they are perfect in it. As the time required for all exercises is uniform, no two pupils need necessarily work on the same exercise during any one class period. Each pupil may thus progress at his own pace.

*This time schedule may not in every case be reached by a class so early as the teacher hopes and expects: but it should be kept in view as a standard *to be attained by continued practice.*

I. Addition

6	9	7	6	2	3	5	1	4	9	5	5
7	1	2	3	1	7	9	0	7	4	4	6
7	7	3	3	8	7	2	4	9	2	9	2

5	9	5	7	2	1	6	1	6	3	3	8
7	6	0	7	8	4	4	9	5	2	6	6
5	9	7	8	7	6	0	9	2	6	4	8

8	9	5	1	1	4	9	4	9	2	4	4
4	5	1	5	8	3	3	1	7	7	0	5
3	3	0	7	2	4	1	0	6	3	5	2

II. Subtraction

17	56	98	77	19	34	76	58	49	46	49	65
6	4	7	5	7	4	5	8	3	4	5	3

56	75	97	88	49	83	66	99	89	67	28	35
3	4	6	5	7	2	6	1	5	5	4	3

23	76	83	72	56	67	43	59	66	55	69
2	2	1	2	5	7	3	9	5	5	4

III. Addition

4	7	4	1	2	2	5	7	5	3
0	2	4	9	1	4	0	1	4	9
1	3	3	3	8	5	1	3	0	5
3	2	5	0	2	1	1	2	1	8
6	5	8	3	2	6	7	8	1	8

9	2	6	6	7	8	1	3	1
1	6	1	0	9	8	7	5	1
9	5	4	4	8	6	2	8	7
0	4	8	9	6	7	2	3	1
1	6	6	9	5	5	9	3	8

IV. Addition

55	31	47	24	14	24	32	70
32	48	51	62	15	33	36	18

17	43	20	20	26	81	42	22
72	21	13	50	73	13	27	16

48	33	52	75	32	17	12	26
31	33	41	21	23	62	26	51

V. Multiplication

25	46	58	94	87	53	85	67	26	49
2	3	7	5	6	7	4	8	9	5

24	19	28	91	87	39	83	16	54	57
4	2	5	3	7	4	9	7	6	3

VI. Addition

32	24	31	25	70	42	50	36
16	13	22	33	17	31	30	20
31	62	11	10	11	15	19	32

20	42	25	52	17	10	23	50
19	13	20	15	51	55	61	16
30	22	42	12	20	32	14	12

VII. Subtraction

68	57	79	32	61	89	75	78	39	48	59	35
27	34	25	21	20	55	60	18	22	36	54	15

44	52	76	53	67	31	44	89	74	42	80	33
31	41	74	33	20	11	22	67	53	32	50	21

VIII. Division

8)48	7)56	9)81	5)45	7)84	4)36	7)63	6)54
5)55	8)56	9)63	7)35	6)60	9)90	4)36	9)45
4)20	7)42	7)70	9)36	8)40	5)35	6)36	6)48
9)99	8)64	9)54	9)27	3)24	6)42	6)66	8)80
7)77	6)72	5)50	3)21	7)28	4)40	5)40	5)25
4)24	3)27	8)96	4)48	3)33	3)30	6)24	8)32
5)15	6)18	7)28	3)36	4)44	2)22	4)16	

IX. Addition

35	63	48	37	67	38	59	62	67	42
17	27	13	15	26	32	25	18	26	29

49	17	36	27	35	64	34	39	55	24
42	55	15	54	25	26	48	22	18	28

X. Addition

5	7	6	9	9	7	1
3	5	1	4	5	7	1
5	4	3	8	7	0	1
1	7	4	5	9	4	0
5	6	9	6	6	9	7
7	8	6	9	1	3	8
7	3	4	1	7	7	9

4	6	9	6	6	4
4	4	6	4	7	5
1	2	4	7	9	1
1	9	8	8	7	8
6	7	2	8	7	6
6	5	5	5	5	7
7	2	7	9	4	8

XI. Subtraction

56	39	51	45	75	31	92	99
48	28	43	32	54	17	67	75

76	92	29	68	57	44	87	46
25	58	16	49	16	36	68	23

21	74	46	88	62	53	77	55
19	35	12	69	22	34	24	51

XII. Addition

2	6	5	4	1	3	2	5
3	4	4	1	2	7	5	1
2	4	7	5	3	2	3	9
1	1	3	7	9	7	5	7
4	4	3	3	7	8	1	6
1	4	2	2	8	4	4	7
2	2	5	6	5	2	2	5
5	6	3	4	2	6	3	3
3	2	2	4	3	3	6	1

XIII. Multiplication

158	572	327	689	215	379
4	7	9	3	8	5

427	526	917	669	537	329
3	5	4	2	8	8

XIV. Addition

75	77	85	96	56	89	66	47
38	35	39	34	79	46	54	95

37	46	26	35	32	58	96	34
85	68	75	77	99	47	45	77

XV. Division

$23\overline{)598}$ $17\overline{)306}$ $72\overline{)864}$

$38\overline{)456}$ $35\overline{)735}$ $27\overline{)729}$

XVI. Addition

58	15	18	22	16	12	52
19	13	47	29	11	25	29
20	32	20	20	66	37	14

28	50	40	26	25	13	14
22	29	37	17	34	22	28
27	17	14	36	14	35	42

XVII. Subtraction

63	41	74	63	81	70	84
27	25	36	19	17	29	45

30	43	78	55	42	63	57
22	39	49	37	17	44	48

48	41	64	47	40	22	53
39	33	56	38	39	15	44

XVIII. Addition

22	69	35	61	75	37
29	41	36	54	37	54
70	50	57	37	22	71

67	27	53	37	90	31
43	22	75	25	79	57
51	95	27	87	88	68

XIX. Multiplication

48	57	72	85	92	83	50	76
30	40	50	20	60	70	80	40

84	35	48	53	71	38	93
90	60	40	80	90	20	30

XX. Addition

38	67	32	21	36	30	74	72
64	57	54	43	29	63	44	78
34	43	76	97	60	75	61	51
78	86	89	66	83	27	20	65
95	95	54	10	40	81	57	36

XXI. Division

2)430	5)895	3)171	2)642	5)325
6)474	9)756	4)372	2)198	8)472
3)522	9)468	7)259	3)147	6)888

XXII. Addition

111 222	635 340	326 362	784 215	869 120
362 425	441 236	150 228	655 132	252 525
631 225	725 262	795 203	438 351	651 325
521 420	573 315	435 323	726 252	522 175

XXIII. Subtraction

745 48	473 51	763 86	394 62	223 58
624 55	250 30	415 97	432 21	848 59
421 38	555 35	317 58	590 60	500 73

XXIV. Addition

127	423	377	265	188	177	509
346	369	208	127	506	718	306

223	351	746	462	119	227	148
547	329	136	228	239	136	226

XXV. Multiplication

427	342	872	951
12	27	36	27

XXVI. Addition

368	556	274	775	456	515
457	256	169	137	359	387

448	374	556	271	664	457
452	459	368	579	236	353

XXVII. Subtraction

678	465	546	388	394	189
354	123	124	235	181	145

857	311	385	618	621	785
640	200	352	303	611	463

157	141	383	468	124
133	120	202	357	114

XXVIII. Addition

34	73	29	19	22	25
23	29	28	77	43	72
26	93	35	81	46	84
11	58	15	52	24	20
53	54	93	51	70	12
17	13	20	30	83	64
56	16	47	48	21	27

XXIX. Division

20)600	30)900	40)120	50)550
70)490	60)300	80)240	20)800
40)280	70)630	60)420	80)400
30)270	80)480	80)640	50)250
20)140	90)810	70)350	20)120
50)700	70)280	50)300	60)660
80)800	90)630		50)750

XXX. Subtraction

579	856	634	755	384
445	587	312	557	272
227	883	359	472	786
105	595	244	186	555
449	200	326	791	327
228	185	222	693	207

XXXI. Addition

34	42	85	69	86
17	88	24	83	35
91	89	55	14	43
63	28	67	22	33
49	29	95	47	56
30	44	70	98	67
62	20	85	73	35
74	88	60	25	20
83	92	37	40	45

XXXII. Addition

764	843	597	634	488
886	568	488	397	536

927	848	368	456	543
384	153	645	844	567

XXXIII. Multiplication

452	147
121	362

572
251

XXXIV. Addition

288	167	472	144	263	321
326	221	149	186	175	490
281	482	201	214	199	129

161	255	221
185	245	178
127	105	102

XXXV. Division

89)8722 84)7644 37)2442

21)2625 84)6804

XXXVI. Addition

645	524	249	626
256	190	462	257
192	309	318	199

488	676	507	496
262	166	345	605
413	234	159	619

XXXVII. Subtraction

753	228	374	783	831
386	169	185	299	564

823	912	973	211	243
337	864	699	186	178

433	741	256	555
277	666	167	478

XXXVIII. Addition

971	456	998	878	246	751
783	326	875	717	202	344
111	411	502	616	115	561
888	600	751	135	494	228
438	812	209	802	787	794

XXXIX. Multiplication

2344	3271
689	422

688
75

XL. Addition

422	492	555	862
567	835	122	545
895	976	263	535
361	524	962	262
227	327	165	213
426	831	447	991
772	991	887	104

XLI. Division

72)16992

33)792

21)378

XLII. Addition

457	717	617	
913	308	297	
124	890	434	
718	383	456	
323	262	913	
444	999	363	422
353	424	802	560
232	282	791	789
666	415	345	671

XLIII. Subtraction

865341	723597	583121	895752
46233	83701	76243	59863

400271	687094	284463
38105	54321	18989

XLIV. Division

999)22977

31)868

39)936

XLV. Subtraction

357912352	975432186	545123615
87527286	78523321	80828268

934105526	773642
65687810	55566