Session 1845–6.

Presented to

Mr. Alexander Beaton,

a Student of the

Natural Philosophy Class

in Marischal College,

for an Essay on

the Steam Engine,

by David Gray.

Professor of Nat. Philosophy

THE THEORY

OF

THE STEAM ENGINE;

SHOWING

THE INACCURACY OF THE METHODS IN USE FOR CALCULATING THE EFFECTS OR THE PROPORTIONS OF STEAM-ENGINES,

AND SUPPLYING A SERIES OF

PRACTICAL FORMULÆ

TO DETERMINE

THE VELOCITY OF ANY ENGINE WITH A GIVEN LOAD, THE LOAD FOR A STATED VELOCITY, THE EVAPORATION FOR DESIRED EFFECTS, THE HORSE-POWER, THE USEFUL EFFECT FOR A GIVEN CONSUMPTION OF WATER OR FUEL, THE LOAD, EXPANSION, AND COUNTERWEIGHT FIT FOR THE PRODUCTION OF THE MAXIMUM USEFUL EFFECT, ETC.

WITH

AN APPENDIX,

CONTAINING CONCISE RULES FOR PERSONS NOT FAMILIAR WITH ALGEBRAIC SIGNS, AND INTENDED TO RENDER THE USE OF THE FORMULÆ CONTAINED IN THE WORK PERFECTLY CLEAR AND EASY.

BY COMTE DE PAMBOUR,

FORMERLY A STUDENT OF THE ECOLE POLYTECHNIQUE, LATE OF THE ROYAL ARTILLERY, ON THE STAFF IN THE FRENCH SERVICE, OF THE ROYAL ORDER OF THE LEGION D'HONNEUR, ETC.

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1839.

G. WOODFALL, ANGEL COURT, SKINNER STREET, LONDON.

INTRODUCTION.

In the *Treatise on Locomotive Engines*, the first French edition of which appeared in the beginning of 1835, we developed a new theory of the steam-engine. We then made the application of it merely to locomotive engines, because the subject constrained us so to do; but we announced at the same time, that this theory was no less necessary to calculate with accuracy, either the effects or the proportions of other steam-engines of every description.

Immediately after the publication of that Treatise, the theory we speak of was adopted in several works. An illustrious member of the *Institut*, of whom science has lately had to deplore the loss, M. Navier, who had just published in the *Annales des Ponts-et-Chaussées* of 1835, a paper on the use of locomotive engines, immediately resumed his work, and presented in 1836, in the same scientific publication, another paper, in which he acknowledged the accuracy of the theory we had just introduced, and abandoned, in consequence, the ordinary process of calculation to substitute ours in its place.

The English professor Whewell, whose name is so well known in science, inserted also the principles of this theory in the third edition of his *Treatise on Mechanics ;* many engineers made it the basis of their calculations for the use of locomotive engines upon divers railways; and finally, at the moment when the present work appears, we find that M. Wood, in the third edition of his *Treatise on Railroads*, London, 1838, has just adopted it likewise, (pages 555–557,) without mentioning there from what source he has it, but afterwards repairing that omission in a slip added at the beginning of his work.

However, we had as yet developed our theory only in its application to locomotives, and it had been considered by those who had admitted it after us, only as a theory specially applicable to those engines. It remained, then, first to demonstrate it in a general manner, and afterwards to apply it to the different kinds of steam-engines in use. But as the action of steam is extremely simple in locomotives, since those engines are rotative, non-condensing and unexpansive, much remained to be done to establish formulæ suitable to all the other systems of steam-engines. Such was the object of a series of papers which we presented to the Academy of Sciences of the *Institut*, in the beginning of 1837, and to which that illustrious body was pleased to grant its approbation, recognizing that the priority relative to the theory in question belonged to us.

It is these papers which we now publish, after having given them the development of which they appeared to be susceptible. We shall therefore begin the work by proofs of the inaccuracy of the theory employed before us; and this part of the work will be purposely established upon practical facts so simple, and explained in terms so exempt from any scientific erudition, that it cannot fail to be in the reach of all practitioners, and even of persons the least acquainted with these questions.

Afterwards, the general properties of steam, or the laws which regulate its action, will be developed. To the laws and formulæ already known on this subject, some will be added which will be found very useful for calculating the effects of steam. For instance, the successive formulæ by means of which the temperature of the steam generated under different pressures was calculated, did not present in their results a perfectly continued series: a new formula substituted for that of Tredgold, will now allow a complete and regular table to be offered on this subject. Another formula will equally be found in the work, which permits an immediate determination of the volume or density of the steam, by knowing only the pressure at which it is generated, and without recurring to the temperature. Finally, a new and very important law will be explained, according to which the steam, during the whole of its action in the engines, remains at the maximum density for its temperature; and this law will make it easy to take account, in the various calculations which may occur upon steam, of the changes of temperature it undergoes during its action; a problem which had not been solved as yet.

These preliminaries once established, the general theory of the steam engine will be explained, and its application will be shown to every particular system of engines. High-pressure engines, locomotives, Watt's and Cornish double-acting engines, Woolf's, Evans's, Watt's and Cornish single-acting engines, and finally atmospheric engines, will be successively treated in special chapters; and for each of them practical formulæ will be given, to solve the divers problems which may offer themselves, either in the management or in the building of those engines.

In this part of the work will be recognized the advantages resulting from the new theory, as many important questions which had not been solved by the ordinary calculation, nay, some, the existence of which could not even be admitted in that theory, will now be solved in the most easy manner. Among those questions we shall quote: the means of determining the friction proper to the engine itself; the calculation of the velocity of the piston for a given load, which remained impossible in the ordinary theory; the determination of the load to be adopted for an engine, in order to make it produce its maximum of useful effect, a problem inadmissible in the ordinary theory, since, by that theory, the engines are always supposed to work with the load determined by the pressure in the boiler, that is, with their maximum load; the research of the expansion and counterweight corresponding to the maximum of useful effect in single-acting and atmospheric engines, a question which as yet was a desideratum in science. And the importance of these problems will be recognized by merely announcing them, since their immediate result is to make an engine work in the most advantageous manner, an enquiry which Watt and Robison acknowledged not to have been as yet established upon principles *.

Among the practical results of the new theory, it will be remarked that it explains quite satisfactorily a question much debated at this moment among English engineers: namely, the apparently surprising effects of the Cornish engines. The fact is that the effects of those engines have only been put in question, because the calculation applied to them could not agree with the practical facts deduced from observation. But when they are calculated by the theory now presented, the results are found to agree perfectly with actual facts. It will be seen in the chapter relating to that kind of engines, that in their mean working condition, they may be expected to produce, per imperial bushel or 84 lbs. of Newcastle coals, a useful effect of 76,000,000 of pounds raised one foot; and it is known that the average of ten of those engines has given lately an authentic result of 70,000,000 of pounds raised one foot, for the above consumption of coals. This coincidence is, therefore, a fresh

* Watt on the Steam-Engine, p. 81.

confirmation of the new theory, and similar facts will be found in the work, relating to Watt's and atmospheric engines.

The incorrect manner in which the effects or the proportions of steam-engines have been hitherto calculated, appears to us to have been the principal cause of those continual disappointments experienced in the trying of steam-engines, and of the lawsuits which have often followed between the purchasers and makers. Engine builders thought they could assign beforehand the effects which would be produced by an engine when completed; but experience soon demonstrated that they could not be assured of accomplishing a determined object, except the engine were precisely modelled after another already tried; and that, in all other cases, they were in absolute uncertainty as to the real effects. If then, apprehensive of furnishing an engine too weak, they designedly gave it an excess of power, the consequence was that power was expended in mere loss, and that the engine, therefore, did not work with all the economy that had been expected from it; if, on the contrary, the engine was found too feeble, then it was incapable of performing its intended task, and it became necessary to give up the use of it; or else, the persons charged with the management of the engine, knowing its insufficiency, sought to compensate the want of force by increasing the pressure in the boiler, that is to say, by loading, or even by making fast the safety-valves, and dreadful explosions have often been the result. Thus, on board of steam-boats too weak to make way against the currents on which they were to navigate, the lives of many travellers have been compromised.

On the other hand, the analytical calculations by which we replace the old methods, are reduced to forms so simple, that they cannot offer the least embarrassment in the applications, nor have we hesitated to risk some repetitions in order to render the use of them still more clear and convenient. We therefore think that these researches may equally suit both engineers and practitioners, and we sincerely hope that they may be found to render some service.

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THEORY

OF THE

STEAM-ENGINE.

CHAPTER I.

PROOFS OF THE INACCURACY OF THE ORDINARY MODE OF CALCULATION.

Sect. I. *Mode of calculation now in use, to calculate the effects of a steam-engine.*

The object of this work is to demonstrate that the calculation of the effects or the proportions of steam-engines, either as it is practically used or as it is found indicated by authors who have treated this subject, is completely erroneous, and to develop a new theory which leads to accurate results. Our first chapter, then, will necessarily be devoted to proving the inaccuracy of the ordinary methods of calculation. From that we shall pass successively to the development of the theory proposed, and its application to the different systems of steam-engines in use.

The effect produced by a machine consists of two elements: the resistance set in motion, and the velocity communicated to that resistance. Hence results, that the calculations which first occur in the application of machines refer to the two following problems:

1st. The machine being supposed constructed, and the velocity of its motion given, to determine what resistance it can move.

2d. The machine being supposed constructed, and the resistance it has to move being known, to determine the velocity it can communicate to that resistance.

A third problem then presents itself as a natural consequence of the two preceding, viz.:

3d. The resistance being known as well as the velocity to be communicated to it, to determine the dimensions that ought to be adopted in the construction of the machine, in order to produce the effect proposed. In steam-engines, this third problem reduces itself to determining the size of the boiler, or, in other words, the evaporation it should be capable of, in order to obtain the effect proposed.

These three problems are the basis of all calculations on steam-engines. They may take divers modifications and give rise to several questions, which hereafter we shall notice, but whose solution will entirely depend on that of the three fundamental problems just mentioned. Thus, for instance, to find the useful effect of an engine of which the number of strokes of the piston is counted, that is, whose velocity is known, amounts to determining the resistance it can move at that velocity; since that resistance being once known, it suffices to multiply it by the given velocity, to have the useful effect required. The horse-power of an engine, and its effect for a given weight of fuel, being nothing more than the useful effect of the engine referred to particular units; that is, to the power of the horse considered as the unit of force produced, or to the consumption of a certain quantity of fuel considered as the unit of force applied; it is plain that these questions merge into that of the useful effect. This point, however, will be readily recognized when we come to treat specially of these questions.

Thus all enquiries relative to steam-engines reduce themselves finally to the three just announced: to find the load, to find the velocity, to find the evaporation.

The only mode of calculation hitherto in use to estimate either the effort of which a steam-engine is capable, or the useful effect it can produce, is first to perform the calculation under the supposition that the steam acts in the cylinder with the same elastic force as in the boiler, and without regarding the friction of the engine; then to reduce the result in a certain proportion indicated by a constant coefficient. This method, which we shall name the method of *coefficients*, was resorted to, because there being no means of knowing, *à priori*, the pressure of the steam in the cylinder, it was naturally enough at first concluded to be equal to that of the boiler. But as the result thus obtained, and which was called the *theoretic* result, was invariably found much higher than the *practical* results compared with it, a reduction was found to be necessary.

The necessity of this reduction was attributed to two causes: 1st, to the friction of the engine having been neglected in the calculation; and 2nd, to no account having been taken of losses resulting from the five following circumstances: the contraction of the passages through which the steam has to pass, the changes in the direction of the conducting pipes, the friction of steam in the steam-pipes, the waste of steam, and its partial condensation. And as these causes seemed likely to act similarly, not only in the same engine, as long as the passages of the steam were not varied, but in all engines of the same system, it was natural to suppose that they would produce, on the definitive result of the calculation, a reduction proportioned to its total value.

It was in consequence judged, that the real result might be attained by reducing the theoretic result in a certain constant proportion. It had been observed, also, that the ratio of the theoretical and practical effects was not the same in the different systems of steam-engines; this consequently led to the admitting of a different coefficient for each system.

This mode was the most natural, it was even the only one possible, so long as means were wanting to determine beforehand what would be the real pressure of the steam in the cylinder under given conditions. And notwithstanding we have undertaken to demonstrate the errors which result from the application of that calculation, and to substitute another in its place, we are far from wishing to depreciate the works in which that calculation is developed. It was felt, no doubt, that that method, from the very circumstance of its consisting in the use of a coefficient to represent in total, various effects which had never been submitted to direct admeasurement, could be but an approximation, a mere provisional method. It was used as we use a bad instrument, till we can get a better. Many of the works wherein it is explained, acknowledge at the same time that the theory of the steam-engine is as yet unknown or imperfectly studied. Moreover, these works do not all treat the subject in a manner perfectly similar, and therefore the observations we are about to make, cannot be addressed equally to all. We wish it, then, to be clearly understood, that in comparing the two calculations together, when it shall be needful so to do, our end is to establish the accuracy of the method we propose, and not to attack the writings of others.

To return to the method in use, this was the proceeding. The force applied to the piston was computed, in supposing the pressure of the steam in the cylinder equal to that of the steam in the boiler: that is to say, the area of the piston was multiplied by the pressure of the steam in the boiler, which gave the force exerted by the engine; this result was then multiplied by the velocity of the piston, and thus was obtained the *theoretic* effect of the engine. But the result of this calculation having been compared with that of some experiments made on engines of the same kind, the ratio between the two results had furnished a fractional coefficient, which was regarded as the constant ratio between the theoretical and practical effects of all engines of the same system; therefore, in multiplying the number expressing the theoretic effect by this fractional coefficient, a definitive product was obtained, which was the *practical* effect that could be expected from the engine.

Supposing, for instance, an engine without condensation, and expressing the area of the cylinder by a, the pressure of the steam per unit of surface in the boiler by π, and the velocity of the piston by v; $a\pi$ was the force applied by the engine, and $a\pi v$ the theoretic effect it ought to produce. As then some experience, instead of giving an effect equal to $a\pi v$, had given but a certain fraction of it, which we will express by k, the coefficient k was admitted as representing the constant ratio between the theoretical and practical effects. So that the useful effect of a non-condensing engine was represented by

$$ka\pi v;$$

or the theoretic effort of the engine being expressed oy $a\pi$, its useful effort, or the resistance the piston could move, was represented by

$$aR = ka\pi$$

R expressing that resistance supposed to be divided per unit of the surface of the piston.

The coefficients indicated by Tredgold, in order to pass from the pressure in the boiler to the part of it which is applied to produce the useful effect, are the following:

Non-condensing unexpansive engines	·60
Non-condensing expansive engines	·60
Non-condensing expansive engines, with two cylinders	·47
Watt's single-acting engines	·60
Single-acting expansive engines	·60
Watt's double-acting steam-engine	·63
Double-acting engine, with two cylinders . .	·48
Atmospheric engine (coefficient to be applied to the atmospheric pressure, and including a deduction for the pressure of the uncondensed steam on the opposite side of the piston) . .	·52
Atmospheric engine, with a separate condenser (coefficient to be applied like the preceding one)	·54

It must be understood that these coefficients, excepting those for the atmospheric engines, are to be applied to the *total* pressure of the steam in the boiler; that is, before any deduction is made, either for the pressure of the uncondensed steam or for the atmospheric pressure, on the opposite side of the piston. If, on the contrary, before applying the coefficient, the pressure in the boiler had been diminished by the pressure subsisting on the other side of the piston, in order to deduce first what is called the *effective* pressure of the steam, then other coefficients, smaller than those indicated here, ought to be used. For instance, if we suppose a high pressure steam-engine working at the *total* pressure of 65 lbs. per square inch, and that we apply to that pressure the coefficient ·60, we shall have for the useful part of the total pressure

$$65 \times \cdot 60 = 39 \text{ lbs.};$$

and if we deduct from this force, the atmospheric pressure which acts on the other side of the piston, the practical effort applied by the engine will be

$$65 \times \cdot 60 - 15 = 24 \text{ lbs.}$$

But, as the *effective* pressure of the steam in the boiler is

$$65 - 15 = 50 \text{ lbs.},$$

it is clear that, if we had had to pass from the *effective* pressure of the steam to the practical effort applied by the engine, a coefficient of ·50 ought then to have been used, instead of the coefficient ·60 applied to the *total* pressure in the boiler.

It is to be remarked also, that, as the theoretic effect of an engine is known only after deduction of the pressure on the other side of the piston, it follows that if the theoretic effect of the engine had been definitively calculated by this mode, and that we were to pass from it to the practical effect, it is the coefficient ·50 which ought then to be employed. Therefore, this last coefficient indicates in reality the reduction operated upon the *theoretic effects*, to conclude from them the *practical effects* of the engine; but the calculation comes to the same either way, provided a suitable coefficient be used.

Such was the solution of the first of the three problems above mentioned. The second, which consists in determining the velocity, had not been the object of any enquiry, by the mode of reasoning we have just exposed.

The third problem, or the evaporation of water necessary to produce a given effect, had been solved in a manner similar to the first. The rule consisted in calculating the volume described by the piston, and in supposing that volume to have been filled with steam at the same pressure as in the boiler, and then applying to it a constant coefficient. That determined in the preceding problem was usually employed, but it was applied as a divisor, with a view to augment the evaporation in proportion to the losses represented by that coefficient.

Thus, retaining the foregoing notations, and expressing by m the volume of steam formed at the pressure of the boiler referred to the volume of water that produced it, we perceive that the volume described by the piston during the unit of time, was av. From the signification of the letter m this volume of steam represented a volume of water expressed by

$$\frac{av}{m};$$

but as it was deemed subject to a loss represented by the fraction or coefficient k, the volume of water really necessary to supply the expenditure $\frac{av}{m}$, became

$$S = \frac{av}{km}.$$

Such was the calculation in use; it solved, as we have seen, only two of the three fundamental problems. We shall, at a future moment, return to what regards the velocity of the piston under a given load.

Besides what has been said, the received ideas relative to the pressure of the steam in the cylinder, consisted in deeming that, the pressure in the boiler being given and fixed, it were possible at pleasure to vary the pressure in the cylinder and to produce there any desired pressure, provided it were inferior to that of the boiler, by contracting more or less the orifice of the steam-passages; and it was thought that, when this orifice was entirely open, with the area usually given to it in fixed engines, to wit, $\frac{1}{25}$ of the area of the cylinder, the pressure of steam in the cylinder could differ but in an inconsiderable quantity from the pressure in the boiler.

However, as the *indicator* of Watt, applied to the cylinder of several engines, had demonstrated a certain diminution of pressure therein, when compared with the pressure in the boiler, the authors who took this fact into account, without perceiving its real cause, still attributed it to the circumstances already explained, and it became merely one of the elements in the explication of their definitive coefficient. Thus, in all cases, the pressure in the cylinder was considered as being equal or proportional to that in the boiler, and therefore constant, so long as no change took place in the pressure of the boiler; but in no wise as being regulated by the resistance, or as variable with the resistance, independently of all pressures in the boiler, which we shall demonstrate that it in reality is.

SECT. II. *Objections against that mode of calculation.*

The objections which first present themselves against that mode of calculation are the following:

1st. The coefficient adopted by many to represent the ratio of the theoretic effects to the practical, in high-pressure engines, was ·33; which was explained by saying that the remainder, or ·66 of the total force developed, was absorbed by the frictions and losses. Not that these frictions and losses had been measured and found such; but merely that the calculation, which might be inexact in its very principle, wanted so much of coinciding with experience.

To obtain conviction of the impossibility of justifying such an assertion as to the value of the frictions and losses, it suffices to peruse the explanation of it attempted by Tredgold, who follows this method.

He indicates that a deduction of 4 tenths should be made on the *total* pressure of the steam, (including the atmospheric pressure,) which amounts to making a reduction of ·5 on the ordinary *effective* pressure of those engines, or to using a coefficient of ·5 applied to the theoretic effect of the engine. He thus explains this loss in the effect produced *.

Force necessary to bring the steam into the cylinder .	·007
Force necessary to drive the steam into the atmosphere	·007
Loss from cooling in the cylinder and in the pipes . .	·016
Friction of the piston, losses, and waste	·200
Force necessary for the opening of the valves and the friction of the different parts of the engine . . .	·062
Loss in consequence of the steam being intercepted before the end of the stroke	·100
	·392

Reflecting that the numbers here given express fractions of the *total gross* power of the engine, we shall immediately be convinced of the impossibility of admitting such estimates. If, for instance, the engine had a *useful* effect of 100 horse-power, which, from the coefficient, supposes a gross effect of 200, it would require the power of 12 horses to move the machinery, of 40 to draw the piston, &c. The exaggeration is self-evident.

Besides, in applying this estimate of the frictions to a locomotive engine, which is also a high-pressure engine, and supposing it to work at 60 lbs. effective pressure, or at 75 lbs. *total* pressure per square inch, we perceive that, were the cy-

* Tredgold, Treatise on Steam-engine, Article 367.

linders 12 inches in diameter, or their surface 226 square inches, the force here reckoned as representing the friction of the piston would be $226 \times 75 \times \cdot 20 = 3390$ lbs. Now, our own experiments on the friction of the mechanical organs of the locomotive engine the *Atlas*, which has those dimensions and which works at that pressure, demonstrate that the force requisite to move, not only the pistons, but all the other mechanical organs, including wastes, if it be true that such exist in an engine in good order, is but 48 lbs. applied to the wheel, or $48 \times 5 \cdot 9 = 283$ lbs. applied to the piston *.

It is impossible, therefore, to admit estimates so exaggerated as these; and what will it be, when it becomes necessary to explain a loss, not merely of half, but of two thirds of the effect produced, as required by the coefficient ·33, adopted by many in practice, particularly for locomotive engines?

2d. This deduction, moreover, of $\frac{2}{3}$, considerable as it is, in very many cases does not suffice to harmonize the practical effects with the effects called theoretical.

In Wood's Treatise on Railroads, second edition, pp. 277—284, appears the calculation of five steam-engines, not locomotive but stationary, two working at low pressure and three at high pressure, in which the real effects are to the theoretic, in the pro-

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* Treatise on Locomotive Engines, Second Edition.

portions represented by the following coefficients:

·26—·29—·31—·27 and ·30.

Here then are examples, wherein it would be necessary to explain a loss of three fourths of the total power of the engine.

But, with respect to locomotives, the difficulty becomes still greater, for it often occurs, when the load of the engine is light, that it would be necessary to apply a coefficient less even than ·25; and yet in these engines the steam passages have an area of $\frac{1}{10}$ instead of $\frac{1}{25}$ of the area of the cylinder; they are immerged in the steam of the boiler itself, which precludes all possibility of waste; the cylinders are in contact with the flame issuing from the fire-box, which entirely prevents condensation. Thus there remains only the friction to explain the enormous loss sustained, of more than ·75 of the total power; and this friction measured in our own experiments, as will be seen farther on, never rises above ·18 of what is termed the theoretic effect of the engine.

3d. It has just been said that, in locomotive engines, the coefficient would in certain cases sink below ·25. But there are other cases on the contrary, wherein, for the same engine, it would rise to ·80, examples of which may be seen in our *Treatise on Locomotives,* in all the cases when the engine drew a heavy load. Thus all the loss hitherto so laboriously explained disappears on a sudden.

4th. The measure of the theoretic effect of the engine results from three elements, to wit: the surface of the piston, the pressure of the steam, and the velocity of the motion. The causes which are said to explain the reduction to which this theoretic effect is liable, are: first, the friction of the engine, then the contraction of the passages, their changes of direction, the friction of the steam, its waste and its condensation. Now of the last five causes, the condensation is the only one that can diminish the *pressure* of the steam during its passage, and that condensation is almost entirely obviated by the precautions used in practice: all the remaining causes of reduction act merely on the velocity. If then these causes produce definitively a reduction in the theoretic effect, it can only be by reason of their action on the velocity.

But, to calculate, by this method, the theoretic effect of an engine, the area of the piston is multiplied first by the pressure of steam in the boiler, which gives the theoretic effort; then this result is multiplied, not by the *theoretic* velocity of the engine which is unknown, but by its *observed* or practical velocity. All reduction then applicable to the theoretic velocity is, by the proceeding itself, already made in the calculation, and cannot be introduced into it anew.

Consequently, if notwithstanding the use of the practical velocity in the calculation, it be still necessary to retrench $\frac{2}{3}$ or $\frac{3}{4}$ from the result obtained, that loss of $\frac{3}{4}$ of the total effect must be due wholly to the friction, which is evidently impossible.

It is clear, then, that these inexplicable differences between the theory and the facts, can arise only from an error in the theory itself; that the results obtained from it are to be considered at most but as approximations, and not as being proper to determine, in an exact and analytical manner, either the effects or the proportions of steam-engines.

Sect. III. *Formulæ proposed by divers authors to determine the velocity of the piston under a given load; and proofs of their inaccuracy.*

We have said that, in the above theory, the velocity of the piston under a given load, had not been made the object of a special research. Some essays had, however, been made to estimate this, but in a different manner.

1st. Tredgold, in his *Treatise on Railways*, (p. 83,) proposes a formula, without, however, establishing it in any way on reasoning or on fact. This formula is as follows:

$$V = 240\sqrt{l\,\frac{P}{W}}.$$

V is the velocity of the piston in feet per minute; l the stroke of the piston; P the effective pressure of the steam in the boiler; and W the resistance of the load. This formula, he says, will give the velocity of the piston. But as no mention is made therein, either of the diameter of the cylinder, or of the quantity of steam furnished by the boiler per minute, it clearly cannot give the velocity required;

for if it could, the velocity of an engine would be the same with a cylinder of 1 foot diameter as with a cylinder of 4 feet, though the latter expends 16 times as much steam as the former. The heating surface, or evaporation of the boiler, would be equally indifferent. An engine would not move faster with a boiler evaporating a cubic foot of water per minute, than with another that should evaporate but $\frac{1}{4}$ or $\frac{1}{20}$. Hence this formula is unfounded.

2d. Wood, in his *Treatise on Railways*, (2nd edition, p. 351,) proposes, also without discussing it, the following formula:

$$V = 4\sqrt{l\,\frac{P}{W}}.*$$

V is the velocity of the piston in feet per minute, l the stroke of the piston, W the resistance of the load, and P the surplus of pressure in the boiler above what is necessary to balance the resistance W. This formula, containing no term to represent either the diameter of the cylinder or the evaporating force of the boiler, is, like the preceding, demonstrated inexact *à priori*.

We know of no other attempt made to attain the solution of this problem, and these are altogether unsatisfactory.

* In the third edition of his work, which is just published, 1838, Mr. Wood abandons this formula to adopt ours, as also our theory in general, developed in the Treatise on Locomotive Engines, 1835.

Consequently, of the three fundamental problems which we have presented, two have received inaccurate solutions by means of coefficients, and the third remains unsolved.

SECT. IV. *View of the theory proposed.*

We have so far demonstrated that there exists no analytical formula, nor any exact means for calculating the effects of steam-engines, and, consequently, for determining the proportions proper to be used in their construction, to obtain desired effects. A great number of engines are constructed and intended to fulfil required conditions; but the truth is that, unless they have been designed after others already executed, their precise effects are not known till they are submitted to trial after construction, that is, when it is too late to apply a remedy. In a machine, the most powerful of all known, and which is tending to become almost universal, errors cannot be without importance. Not only such errors have been frequently prejudicial to vast manufacturing enterprises, and the occasion of difficulties between the builders and the purchasers of engines, but they have also compromised the lives of travellers; for when a steam-vessel has been found incapable of accomplishing its destined task, the only remedy to the evil that has occurred to the engineer, has been that of overloading, or even of making fast, the safety valve, and frightful explosions have often been the result. No doubt then can be entertained as to the usefulness of new researches on the subject.

After having exposed the state of science in what concerns the theory of the steam-engine, it remains to shew on what principles is grounded the theory we are about to present.

We shall first explain this theory in all its simplicity, supposing the steam to preserve the same temperature throughout its action in the engine, limiting ourselves to rotative engines without expansion, and taking our basis on the consideration merely of the uniform motion which the engine necessarily attains after a very short lapse of time. A great number of theoretical and practical proofs will then corroborate the accuracy of our reasoning; and, finally, in the subsequent chapters, we shall resume the theory in all its generality, so as to render it applicable to all systems of steam-engines, taking into account the circumstances neglected in our first exposition.

It is well known that in every machine, the effort of the mover first being superior to the resistance, a slow motion is produced, which accelerates gradually till the machine attains a certain velocity which it does not surpass, the mover being incapable of sustaining a greater velocity with the mass it has to move; the machine having once attained this point, which requires but a very little time, the velocity continues the same, and the motion remains uniform during the rest of the work. It is but from this moment, viz., the commencement of uniform motion, that the effects of machines begin to be calculated, and the few minutes during which the velocity regulates itself, or the transitory effects from the velocity null to uniform velocity, are always neglected.

Now in every machine which has attained a uniform motion, the power is strictly in equilibrio with the resistance; for were it greater or less, there would be acceleration or retardation of motion, which is not the case. In a steam-engine, the force applied by the mover is no other than the pressure of the steam *against the piston* or *in the cylinder.* This pressure then, in the cylinder, is strictly equal to the resistance opposed by the load against the piston.

Consequently, the steam in passing from the boiler into the cylinder changes its pressure, assuming that which represents the resistance to the piston. This principle, of itself, explains all the theory of the steam-engine, and in a manner lays its play open.

It becomes, in fact, easy to render an account of what passes in a steam-engine set in motion. The steam confined in the boiler, at a certain degree of pressure, as soon as the regulator or distributing-cock is open, rushes into the steam-pipes, and from thence into the cylinders. Arriving in the cylinder, whose area is much greater than that of the pipes, the steam dilates at first, losing proportionally a part of its elastic force; but as the piston is as yet immoveable, and as the steam continues to flow in rapidly, the balance of pressure is soon established between the two vessels; and the piston, urged by all the force of the steam, begins slowly to move. The fly-wheel of the engine, its entire machinery, and the resistance opposed to it, begin then to acquire a small velocity, which accelerates by insensible degrees; and, if at the end of the stroke of the piston, the coming vapour were suddenly withheld, the piston would not stop instantaneously on that account; it would itself be impelled for some time by the effect of the velocity previously communicated to the mass. The result of this is, that at the following stroke, the steam finds the piston already slowly receding, at the moment it impresses thereon a new quantity of motion; which again passes on to the fly-wheel, and to the total mass, where it continues to accumulate. Receiving thus, at every stroke, a new impulse, the piston accelerates its motion by degrees, and, at length, acquires all the velocity the motive power is capable of communicating to it.

During all this time the steam continues to be generated in the boiler with the same rapidity, and to flow into the cylinder; but as the piston acquires a quicker motion and developes a greater volume before the steam, the latter dilates, assuming a lower pressure, till at length, the piston having assumed all the velocity that the steam can impress upon it, with the load that it supports, the pressure of the steam in the cylinder becomes equal to the resistance of the piston, and the motion remains in a state of uniformity, as has been said above.

Thus, from what precedes, we have the pressure which the steam really exercises against the piston; so that if P′ represent that pressure per unit of surface, and if R represent the resistance of the load against the piston, divided in like manner per unit of surface, the condition of the uniformity of motion will furnish the first equation of analogy

$$P' = R.$$

This equation establishes the intensity of the effort exerted by the power. Were the case merely one of equilibrium, this determination would suffice; but in a case of motion, not only the intensity of the force is to be considered, but also the velocity with which it is applied. Now, in the case before us, it is evidently the velocity of production of steam in the boiler, which indicates the velocity with which the above force is renewed or applied. To this latter element then of calculation we must recur, in order to obtain a second relation among the data of the problem, comprising the velocity of the motion.

This relation will be furnished by the consideration, that there is necessarily an equality between the quantity of steam produced and the quantity expended, a proposition which is self-evident. If, then, we continue to express by S the volume of water evaporated in the boiler per unit of time, and effectively transmitted to the cylinder, and by m the ratio of the volume of the steam formed under the pressure P of the boiler, to the volume of water that has produced it, it is clear that

mS

will be the volume of steam formed per unit of time, and under the pressure P, in the boiler. This steam passes into the cylinder and there assumes the pressure P′; but if it be supposed that the steam in this movement preserves its temperature, which, as to the engines under consideration, will make but little change in the results, the steam, in passing from the pressure P to the pressure P′, will increase its volume in the inverse ratio of the pressures. Thus, transmitted to the cylinder, the volume mS of steam, supplied at each unit of time by the boiler, will become

$$m\mathrm{S}\frac{\mathrm{P}}{\mathrm{P}'}.$$

On the other hand, v being the velocity of the piston and a the area of the cylinder, av will be the volume of steam expended by the cylinder per unit of time. Therefore, on account of the equality necessarily existing between the production and the expenditure of steam, we shall have the relation

$$av=m\mathrm{S}\frac{\mathrm{P}}{\mathrm{P}'},$$

which is the second relation sought.

Consequently, by eliminating P′ from these two equations, we have as a definitive analytical relation among the various data of the problem,

$$v=\frac{m\mathrm{S}}{a}\cdot\frac{\mathrm{P}}{\mathrm{R}}.$$

This relation, which is very simple, suffices for the solution of all questions relative to the determining of the effects or the proportions of steam-engines. As we shall develop its terms hereafter, on taking it up in a more general manner, we will leave it for the present under this form, which will render the discussion of it more easy and more clear.

We have, then, the velocity which the piston of an engine will assume under a given resistance R. If, on the contrary, the velocity of the motion be supposed known, and it be required to calculate what resistance the engine may move at that velocity, it will suffice to solve the same equation with reference to R, and we shall have

$$R = \frac{mSP}{av}.$$

Finally, supposing the velocity and the load to be given, and that it be required to know the evaporation proper for the boiler, that the given load may be set in motion at the desired velocity, the value of S must be drawn from the same relation, and it will be

$$S = \frac{avR}{mP}.$$

We limit our deductions here, because, as has been already observed, these three problems are the basis of all problems that can be proposed on steam engines, and that they are sufficient, moreover, to enable us to establish our theory and to compare it with the mode of calculation now in use. But on resuming the same questions with more detail in the following parts of the work, we shall give to the equations their full development, and treat of all the other accessory determinations which occur in problems relative to steam engines.

From what has been stated, it plainly appears that we ground all our theory on these two incontestable facts; 1st, that the engine having attained uniform motion, there is necessarily equilibrium between the power and the resistance, that is, between the pressure of the steam *in the cylinder*, and the resistance against the piston; which furnishes the first relation

$$P' = R.$$

And 2dly, that there is also a necessary equality between the production of the steam and its expenditure, which furnishes the second relation

$$v = \frac{mS}{a} \cdot \frac{P}{P'}.$$

And these two equations suffice for the solution of all the problems.

SECT. V.—*New proofs of the accuracy of the theory proposed, and of the inaccuracy of the ordinary theory.*

As we shall draw from the examination of locomotive engines, the greater part of the considerations we are now about to offer on the two theories, we will first observe, with respect to those engines, that we look upon them as being incontestably more proper than all others to make known the true theory of the motion and action of steam. The reasons of this preference are, 1st, that those engines are of a remarkable simplicity; 2d, that the determination of the resistance which they have to move is easy, and susceptible of great exactitude, since it consists merely in weighing the train they have to draw; whereas to estimate the resistance opposed to stationary engines often requires calculations both various and uncertain; 3d, that the friction of locomotive engines is known from our own experiments, and with a degree of precision that seems to be trustworthy, since that friction has been determined by several methods which have served to verify each other; 4th, that it is easy to observe a locomotive engine under a hundred circumstances different from each other, by varying at pleasure the load and the velocity, which may be done in very wide limits; whereas in stationary engines, it happens most frequently that the resistance to be moved is incapable of variation, whence results that the steam is never seen to act but in one manner, and thus the study of those engines reduces itself nearly to that of one particular case.

To return to the theory we have exposed, it visibly rests chiefly on this, that though the steam be formed in the boiler at a certain pressure P, yet in passing into the cylinder it assumes a pressure R, strictly determined by the resistance against the piston, whatever else may be the pressure in the boiler; so that, according to the intensity of that resistance, the pressure in the cylinder, far from being always equal to that of the boiler, or from differing always from it in any constant ratio, as is believed, may at times be fully equal to it, and at other times considerably different. Thus, when in the ordinary theory, the calculation is performed under the supposition that the steam acts in the cylinder at the pressure of the boiler, an error often very considerable, and independent of all the real losses to which the engine is liable, is introduced into the calculation; since a force is considered as applied, which is two or three times greater than the real one. No wonder then it became necessary to use a coefficient $\frac{1}{3}$ or $\frac{1}{4}$, which makes the supposed losses of the engine appear enormous, whereas the real error is in the very basis of the calculation itself.

We have already proved this mode of action of the steam in the cylinder, from the consideration of uniform motion; but in examining what takes place in the engine, we shall presently find many other proofs.

1st. The steam being generated at a certain degree of pressure in the boiler, passes into the steam-pipe, and from thence into the cylinder; there it dilates at first, because the area of the cylinder is ten or twenty times that of the pipe, and it would quickly rise to the same degree of pressure as in the boiler, were the piston immoveable. But as the piston, on the contrary, opposes only a certain resistance determined by the load imposed on the engine, 40 lbs. for instance, per square inch, it will give way as soon as the elastic force of the steam in the cylinder shall have attained that point. A piston sustaining a resistance of 40 lbs. per square inch is nothing more than a valve loaded with 40 lbs. per square inch. Were the communication perfectly free between the boiler and the cylinder, without tube or contraction, so that the two vessels should form but one, the piston would become a real valve to the boiler; and that valve yielding before the safety valve, which is loaded perhaps at 50 lbs. per square inch, the steam could never rise in the boiler above the pressure of 40 lbs. per square inch. Since the communication between those two vessels is not wholly free, the piston is not à valve to the boiler, but it still continues to be one to the cylinder. Wherefore the pressure in the cylinder can never exceed the resistance of the piston.

2dly. Another consideration will readily prove to us again that the pressure of the steam in the cylinder must necessarily be regulated, not by the pressure in the boiler, but by that of the resistance. In fact, were it actually true that the steam be expended in the cylinder, either at the pressure of the boiler, or at any other pressure that were in any fixed ratio whatever to that of the boiler, then, since the quantity of steam raised per minute in the boiler would be expended by the cylinder at one and the same pressure in all cases, and would consequently fill the cylinder a fixed number of times in a minute, it would follow that the engine, so long as it should work with the same pressure in the boiler and the same apertures or steam passages, would assume the same velocity with all loads. Now, we see that the very contrary takes place; for, the lighter the load, the greater becomes the velocity of the engine.

The effect produced is explained easily, in considering what really passes in the engine. If it be supposed that the evaporation, producing, for instance, 200 cubic feet of steam per minute at the pressure of the boiler, be sufficient to fill the cylinder 200 times, when the piston is loaded with the resistance R; as soon as that resistance R shall be replaced by a resistance $\frac{1}{2}$R, the same mass of steam assuming in the cylinder a pressure of only half of what it was before, will furnish 400 cylinders full of steam per minute at the new pressure. It is then clear that the resistance $\frac{1}{2}$R will be set in motion with a velocity double that of the resistance R; which does in fact accord with observation, if, in estimating that resistance, account be taken of all the partial resistances and frictions really opposed to the motion of the engine.

3dly. Applying the same reasoning inversely, we see that, were the pressure of the cylinder in a fixed ratio with that of the boiler, or were it constant so long as that of the boiler remained the same, then in calculating the effort of which the engine is capable, this would always be found the same, whatever might be the velocity of the piston. Thus, at any velocity whatever the engine would always be capable of drawing the same load. Now, this result again is contrary to experience;

and the reason of it is that the greater the velocity of the piston, the lower the pressure in the cylinder; whence results, that the load the engine is capable of moving diminishes in the same proportion.

4thly. Another proof no less evident is easily adduced. Were it true that the steam be expended by the cylinder at a pressure equal to that of the boiler, or in any fixed ratio to it, indicated by any coefficient whatever, since any one locomotive engine requires the same number of turns of the wheel, or the same number of strokes of the piston to traverse the same distance, it would follow that so long as these engines work at the same pressure, they ought in all cases to consume the same quantity of water for the same distance. Now, the quantity of water evaporated, far from being constant, decreases, on the contrary, with the load, as may be seen in the experiments we have published on this subject. The *Atlas* engine, for instance, evaporated 132 cubic feet of water in drawing 195·5 tons, and 95 cubic feet only in drawing 127·6 tons. Since the same number of cylinders-full of steam was expended in each case, the steam of the first must have been of a density different from that of the second; and here again it is manifest that, notwithstanding the equality of the pressure in the boiler, and of the opening of the regulator in the two cases, the density of the expended steam followed the intensity of the resistance, that is to say, the pressure of steam in the cylinder was regulated by the resistance.

5thly. From the same cause, since the consumption of fuel must be in proportion to the evaporation effected, it would follow too, were the ordinary theory exact, that the quantity of fuel consumed by a given engine would always be the same for the same distance, whatever might be the load. Now, we find again, by experiment, that the quantity of fuel, on the contrary, diminishes with the load, conformably to the explanation we have given of the effects of the steam in steam-engines.

6thly. It is clear, moreover, that if the pressure in the cylinder were, as it is thought, constant for a given pressure in the boiler, then after an engine has been found capable of drawing a certain load with a certain pressure, and of communicating to it a uniform motion, it would follow that the same engine could never draw a less load with the same pressure in the boiler, without communicating to it a velocity indefinitely accelerated; since the power having been found equal to the resistance in the first case, would necessarily be superior to the resistance in the second. Now, experience proves that in the second case the motion is quicker, but no less uniform than in the first; and the reason is, that though the steam be generated in the boiler at a pressure more or less elevated, which matters little, yet in passing into the cylinder it always assumes the pressure of the resistance; whence results that the power is no more superior to the resistance in the second case than in the first, and that the motion ought therefore to remain uniform.

7thly and lastly. On looking over our experiments on locomotives, the same engine will be seen sometimes drawing a very light load with a high pressure in the boiler, and sometimes, on the contrary, a very heavy load with a low pressure. It is then impossible to admit, as the ordinary theory would have us, that there is any fixed ratio whatever between the two pressures. This effect, moreover, is most easy to explain; for it depends simply on this, that in both cases the pressure in the boiler was superior to the resistance against the piston, and no more was needful in order that the steam, generated at that pressure, or at any other fulfilling merely that condition, might, on passing into the cylinder, assume the pressure of the resistance.

It is moreover to be observed, that these effects cannot take place in a locomotive steam-engine, without equally occurring in a stationary one; for the steam acts in the same manner in the cylinders of both, and it is unimportant whether during the action of that steam, the engine moves or remains at rest, or whether its own weight do or do not form a part of the load imposed on the piston.

All these proofs then establish clearly that the pressure of the steam in the cylinder, is strictly regulated by the resistance on the piston and by nothing else; and that all methods, like that of the coefficients, founded on the principle of its being in any fixed ratio whatever with the pressure in the boiler, are necessarily inaccurate.

It is, however, essential to observe, that we wish to establish by these reasonings, that, since the pressure in the cylinder is fixed *à priori*, it cannot depend on the pressure of the boiler; but we believe, on the contrary, as will be seen, Sect. VII., that the pressure in the cylinder being once regulated by the resistance on the piston, that of the boiler afterwards depends on it, in proportion to the size of the passages, the volume of steam produced, and the weight of the safety-valves. It would only be for want of making this needful distinction, that we could be thought to admit an entire independence between the two pressures.

Sect. VI.—*Comparison of the two theories in their application to particular examples.*

The foregoing already establishes sufficiently, in principle, the accuracy of the theory which we propose, and the inaccuracy of that which has hitherto been made use of. It may, however, be thought by some that the inaccuracy alleged against the latter is of trifling importance, and that in practical examples it gives results very near the truth at least, if not quite correct. We are now, therefore, about to submit that method as well as our own to the scrutiny of practice. When in action together, the difference of the results to which they lead will be apparent, and it will be recognized which of the two is more in harmony with the facts; and finally, a clear idea will be formed of the causes from whence the errors of the ordinary theory derive.

The coefficient of correction for high pressure steam-engines without expansion and without condensation, not being fixed to the same amount by the authors who have treated on these subjects, suppose it be attempted to determine it from the two following facts of which we were eye-witnesses.

1. The locomotive engine *Leeds*, which has two cylinders of 11 inches diameter; stroke of the piston, 16 inches; wheels, 5 feet; weight, 7·07 tons; drew a load of 88·34 tons, ascending a plane inclined $\frac{1}{1300}$, at the velocity of 20·34 miles per hour; the effective pressure in the boiler being 54 lbs. per square inch, or the total pressure 68·71 lbs. per square inch.

2. The same day the same engine drew a load of 38·52 tons, descending a plane inclined $\frac{1}{1094}$, at the velocity of 29·09 miles per hour; the pressure in the boiler being precisely the same as in the preceding experiment, and the regulator opened to the same degree. These experiments are given, pages 233, 234, of our *Treatise on Locomotives* (1st edition).

Reckoning, on the one part, the *theoretical* effort applied on the piston according to the ordinary calculation; and on the other part, the effect really produced, viz., the resistance of the load together with that of the air against the train, we find, referring the area of the pistons and the pressure to the square foot:

1st Case.	Theoretical effort applied to the piston, according to the ordinary calculation,	lbs.
	1·32 × (68·71 × 144)	13060
	Real effect	8846
	Coefficient of correction	·68
2d Case.	Theoretical effort, the same as above . .	13060
	Real effect	6473
	Coefficient of correction*	·50

The mean coefficient between the two is ·59. We thus find three different coefficients. Let the first be chosen, an error will be made in the second case; let the second be preferred, and an error will be made in the first case. Let the third be ad-

* The detailed calculation of the effects produced is this:

		lbs.
1st Case.	Resistance of the 88·34 tons, at 7 lbs. per ton (*v.* 2d edition of *Treatise on Locomotives*) .	618
	Gravity of 95·41 tons (train and engine) on a plane rising $\frac{1}{1300}$	164
	Resistance of the air against the train, at the velocity of the motion	134
	Resistance overcome at the velocity of the wheel	916

And as that resistance is here measured at the velocity of the wheel, it produced against the piston, a force augmented in the inverse ratio of the velocities of the piston and of the wheel, that is to say a resistance of 916 × 5·9 5404

Add for the pressure of the atmosphere against the piston, the engine being a high pressure one, 1·32 × (14·71 × 144)	2796
And for the pressure caused by the blast-pipe 1·32 × (3·4 × 144)	646
Total resistance against the piston, exclusive of friction,	8846

 2d Case

mitted, and the error will only be divided between the two cases. In any way error is inevitable, and that, of itself, suffices to prove that any method, like the ordinary theory, which consists in employing a constant coefficient, is necessarily erroneous, whatever be the coefficient adopted, and to whatever system of engine the application be made; for it is evident that the same fact would occur in every species of steam-engine. It might only be less striking, if the velocities in the two instances were less different, and that is what has hitherto prevented the error of that method from being perceived; for all the engines of the same system being imitated from each other, and working nearly at the same velocity, from a factitious limit that had been imposed on the velocity of the piston, the same coefficient of correction appeared tolerably to suit them all.

In stationary engines, moreover, it was impossible, for want of precise determinations of the

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2d Case.	Resistance of the 38·52 tons	270	
	Gravity on the plane descending $\frac{1}{1094}$	— 93	
		177	
	Resistance of the air	282	
	Resistance against the wheel . . .	459	
	Or against the piston, 459 × 5·9		2708
	Add for the pressure of the atmosphere . .		2796
	And for the pressure caused by the blast-pipe 1·32 × (5·1 × 144)		969
Total resistance against the piston, exclusive of friction,			6473

frictions, to separate that part of the result which is really attributable to them, from that part which constitutes a veritable error. But here we easily obtain the conviction that neither of these coefficients of correction represent, as we are told, the frictions, losses, and divers resistances of the engine; for direct experiments on the engine under consideration, which are given in our *Treatise on Locomotives*, enable us to estimate separately all those frictions and resistances. Now from these experiments, the friction of that engine, when isolated, is equivalent to a force of 82 lbs. applied to the wheel, and that friction afterwards augments by 1 lb. for every ton of load added to the engine.

Besides, the losses to which the engine is liable either from condensation or from waste of steam (during its passage from the boiler to the cylinder) are nothing, or at least inconsiderable.

It becomes easy, then, with these elements, to estimate the amount of the frictions really experienced by the engine. Now in calculating them separately thus, we find:

1st Case. Friction 1257 lbs., or ·10 of the theoretical result.
2d Case. Friction 873 lbs., or ·07 of the theoretical result *.

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* We have in fact:

		lbs.
1st Case.	Friction of the engine without load .	82
	Additional friction for a resistance equivalent to $\frac{916}{7} = 131$ tons . . .	131
		213
	Which produces against the piston $213 \times 5{\cdot}9$.	1257

2d Case.

Thus it appears that in these two cases the frictions omitted in the calculation amounted really to but 10 and 7 hundredths of the theoretic result; and if $\frac{1}{20}$ or ·05 of loss be added for the filling up of the vacant spaces in the cylinder, which we have not been able to estimate in lbs., they will amount to ·15 and ·12; whereas the coefficients of correction raise them to ·32 on the one part, and to ·50 on the other; that is, from two to four times what they really are. Deducting, then, from these coefficients, the true value of the frictions and losses, it will be found that the theoretical error which this method brings into the calculation, under the denomination of friction, is 17 per cent. of the *total force of the engine* in one case, and 38 per cent. in the other.

It will now be observed that, from the preceding determinations, viz., of the resistances first and then of the frictions, we have for each of the two cases which occupy our attention, the sum of the total effects really produced by the engine, that is:

		lbs.
1st Case.	Resistances	8846
	Frictions	1257
		10103

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		lbs.	
2d Case.	Friction of the engine without load .	82	
	Additional friction for a resistance equivalent to $\frac{459}{7} = 66$ tons . . .	66	
		148	
	Which produces against the piston $148 \times 5{\cdot}9$.		873

		lbs.
2d Case.	Resistances	6473
	Frictions	873
		7346

We are now, therefore, enabled to compare these effects produced, with the results, either of the ordinary calculation, or of that which we propose to substitute for it.

1st. In applying, first, the ordinary calculation with the mean coefficient ·59 determined above, and comparing its result with the real effect, we find:

		lbs.
1st Case.	Effort applied on the piston, from the ordinary calculation, $1{\cdot}32 \times (68{\cdot}71 \times 144) \times {\cdot}59$ . .	7705
	Effect produced, including all frictions and resistances	10103
	Error, over and above the frictions and resistances,	2398
2d Case.	Effort applied on the piston, from the ordinary theory, the same as above	7705
	Effect produced, including, &c.	7346
	Error, besides frictions and resistances	359
	Mean error of the two cases	1378

It is plain, then, that there would be risk of a very great error in attempting to calculate the effects of this engine with the coefficient ·59; but it is plain, moreover, that in applying any other coefficient *whatever*, the error would only be transferred from the one case to the other, without ever disappearing; and thus, in fact, the coefficient ·59 has almost rendered null the error of the second case above, by transferring it to the first.

To apply now our formula relative to the same problem, viz.,

$$aR = \frac{mSP}{v},$$

nothing more is requisite than to replace the letters m, S, P, and v by their respective values, taking care only to refer all the measures to the same unit.

Thus, P is the total pressure of the steam in the boiler, viz., 68·71 lbs. per square inch, or 68·71 × 144 lbs. per square foot.

m is the ratio of the volume of the steam, at the total pressure of 68·71 lbs. per square inch, to the volume of the same weight of water; and, according to tables which will be given in the following chapter, $m = 411$.

S is the volume of water evaporated per minute, and converted to use in the cylinders. Now, during the journey, of which the first of these experiments was a part, the engine evaporated 60·52 cubic feet of water per hour, (*Treatise on Locomotives*, 1st edition, page 175); which, after a deduction of $\frac{1}{5}$ for the loss of steam by the safety valve, measured, as explained in section VII., and of $\frac{1}{20}$ on the rest for the filling up of the vacant spaces in the cylinder, leaves an *effective* evaporation of ·77 cubic feet of water per minute. We here then have $S = {\cdot}77$.

Finally, v is the velocity of the piston; and as the engine moved at a velocity of 20·34 miles per hour in the first case, and of 29·09 miles per hour in the second, which correspond respectively to 298 and 434 feet per minute for the piston, we shall have successively $v = 298$ and $v = 434$.

Hence the formula gives:

	lbs.
1st Case. Effort exerted by the engine at the given velocity, from our calculation, $\frac{411 \times \cdot 77 \times (68 \cdot 71 \times 144)}{298}$	10507
Effect produced, including the frictions and resistances, as above	10103
Difference	404
2d Case. Effort exerted by the engine at the given velocity, $\frac{411 \times \cdot 77 \times (68 \cdot 71 \times 144)}{434}$	7215
Effect produced, including, &c.	7346
Difference	131
Mean difference of the two cases	267

We attain then the effect really produced, within a difference of only 267 lbs., a difference which is less than can generally be expected in experiments of this kind, wherein all depends on the management of the fire; whereas the preceding theory gives a *mean* and inevitable error of 1378 lbs., which is $\frac{1}{7}$ of the real effect of the first case, and $\frac{1}{5}$ of the real effect of the second.

2d. To continue the same comparison of the two theories, suppose it were required to calculate what quantity of water the boiler should evaporate per minute to produce either the first or the second effect. The mode of calculation followed by the ordinary theory consists, as we have said, in supposing, first, that the volume described by the piston has been filled with steam at the same pressure as in the boiler, and then in applying to it a coefficient of reduction for the losses.

Now, in the first case, the volume described by the piston at the given velocity, is

$$av = 1{\cdot}32 \times 298 = 393 \text{ cubic feet.}$$

If this volume had been filled with steam at the pressure of the boiler, it would have required an evaporation of water of

$$\frac{393}{411} = {\cdot}96 \text{ cubic foot of water.}$$

But the real evaporation was no more than ·77. Therefore the theoretic evaporation of the first case requires a coefficient of

$$\frac{{\cdot}77}{{\cdot}96} = {\cdot}81.$$

In the second case, the evaporation similarly computed, supposing the steam to have acted in the cylinder at the pressure of the boiler, is

$$\frac{1{\cdot}32 \times 434}{411} = 1{\cdot}39 \text{ cubic foot of water.}$$

So, for this case the necessary coefficient is ·55. In this problem, therefore, as in the preceding, no constant coefficient *whatever* can be satisfactory.

If, however, the calculation be performed with the mean coefficient ·68, there results:

1st Case. Evaporation per minute calculated by the ordinary theory, with the coefficient ·68, $\frac{1{\cdot}32 \times 298}{411} \times {\cdot}68$	·65
Real evaporation	·77
Error	·12

2d Case. Evaporation per minute, calculated by the ordinary theory, with the coefficient ·68,

$\frac{1{\cdot}32 \times 434}{411} \times {\cdot}68$ . . . . ·95

Real evaporation . . . . ·77

Error . . . . ·18

The mean error committed is then $\frac{1}{5}$ of the evaporation; and, for the very reason that it is a medium, it may, in extreme cases, become twice as much.

This is the error committed, when a coefficient is sought *expressly* for the evaporation. But when, instead of that, the coefficient ·59, determined in the preceding problem from the comparison of the theoretic and practical effects, is used as a divisor, as by many authors it is, far greater errors are induced; for we find:

1st Case. Evaporation per minute, calculated by the ordinary theory, with the coefficient ·59 as a divisor, $\frac{298 \times 1{\cdot}32}{411 \times {\cdot}59}$ . . . . 1·62

Real evaporation . . . . ·77

Error . . . . ·85

2d Case. Evaporation per minute $\frac{434 \times 1{\cdot}32}{411 \times {\cdot}59}$ . . . . . 2·36

Real evaporation . . . . ·77

Error . . . . 1·59

In our method, on the contrary, the evaporation necessary to put in motion the resistance aR at the velocity v, is given by the formula

$$S = \frac{aR \times v}{mP}.$$

Which gives:

1st Case. Evaporation given by our calculation,

$$\frac{10103 \times 298}{411 \times (68{\cdot}71 \times 144)} \ldots \ldots \cdot 74$$

Real evaporation . . . . ·77

Difference . . . . ·03

2d Case. Evaporation given by our calculation,

$$\frac{7346 \times 434}{411 \times (68{\cdot}71 \times 144)} \ldots \ldots \cdot 78$$

Real evaporation . . . . ·77

Difference . . . . ·01

3dly and finally, for the case wherein the velocity of the piston is sought, supposing the resistance given, no method like the ordinary one could do otherwise than lead to error, but on this head comparison is unnecessary, since the problem has never yet been solved.

We shall merely, therefore, show the verification of our theory. The formula relative to this problem is:

$$v = \frac{mSP}{aR}.$$

And we find:

1st Case. Velocity of the piston, in feet per minute, calculated by our theory,

$$\frac{411 \times {\cdot}77 \times (68{\cdot}71 \times 144)}{10103} \ldots \ldots 310$$

Real velocity 298

Difference 12

2d Case. Velocity by our calculation,

$$\frac{411 \times {\cdot}77 \times (68{\cdot}71 \times 144)}{7346} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 426$$

Real velocity 434

Difference 8

It consequently appears that in each of the three problems in question, the theory we propose leads to the true result; whereas the ordinary theory, besides that it leaves the third problem without solution, may, in the two others, lead to very serious errors.

Before abandoning this comparison we will recall attention to an effect, in the calculation of the ordinary theory, of which we have already spoken, but which is here found demonstrated by the facts. It is, that that calculation gives the same force applied by the engine in both the cases considered, notwithstanding their difference of velocity; and such will always be the result, since the calculation consists merely in multiplying the area of the piston by the pressure in the boiler, and reducing the product in a constant proportion. Thus the ordinary theory maintains in principle, that the engine may always draw the same load at all imaginable velocities. Again we see that, in the same computation, viz., that of the load or of the force applied, the evaporation of the engine is not mentioned; which implies that the engine would always draw the same load at all velocities, and whatever might be the evaporation of the boiler; which is impossible.

Lastly, we shall remark that, in the calculation made by the ordinary theory, in order to find the evaporation of the engine, no mention whatever is made of the resistance the engine is supposed to draw; so that the evaporation necessary to draw a given resistance, is independent of that resistance; another result equally impossible.

To these omissions, then, which we regard as errors of principle, and to other causes already noted, are to be attributed the deviations observable in the results of the ordinary theory in the examples proposed.

Sect. VII.—*Of the area of the steam-passages.*

There yet remains one point which needs examination, and that is the area of the steam-passages, or the size of the opening of the regulator.

The ordinary theory recognizes in this opening a very important effect on the engine, since it affirms that by increasing or diminishing it, any desired pressure may be produced in the cylinder. Yet no means are afforded us of taking account of this opening in the calculation; unless obliged, as we are already, to have a coefficient for the useful effects and for every species of engine, and another for the evaporation, modified also for every system of engines, and again a different coefficient for all velocities, we be required to have a new one also for every opening of the regulator. But these coefficients are not given, and notwithstanding that the action of the engine is considered to change with the opening of the regulator, yet the calculation is always the same, and made with the same coefficient, whatever that opening may be.

Now, when a stationary engine is at work, its regulator is in constant motion by the effect of the *governor*, and, as it were, unperceived by the engineer. The calculation then of the ordinary theory will be continually at fault; it will be inexact in all cases and at all moments wherein the regulator shall happen to have an opening different from that for which the coefficient employed shall have been determined.

In the theory which we propose, on the contrary, account is taken of the opening of the regulator, or at least of the effects it produces, though its direct measure does not appear ostensibly in the equations. To set this fact in a perfectly clear light, we will first of all establish what are the real effects of the regulator.

We will first prove that the degree of opening of the regulator can have no influence on the pressure in the cylinder, but that its reaction, on the contrary, is upon the pressure *in the boiler;* we will then shew that, whatever be the contraction of the regulator, the formulæ will keep account of it, and will continue to give the true effects produced; and finally, we will examine, under each circumstance, what changes do take place in those effects, by reason of the contracting of the orifice of the regulator.

1. It is supposed, in the ordinary theory, that the pressure of steam in the boiler being given and fixed, the contracting more or less of the aperture of the regulator may be made to produce at pleasure a certain pressure in the cylinder. But we have proved that the pressure in the cylinder is, on the contrary, always strictly determined, *à priori*, by the resistance on the piston; the greater or less opening, then, of the regulator can effect no change in it. Besides, how could the contracting of the passage change the *pressure* of the steam which issues through it? It may, we agree, change the *quantity*, because the smallness of the opening will prevent more than a certain portion from passing in a given time, but it certainly never can change its *pressure*. It will, in fact, always happen, that as soon as the steam, on passing into the cylinder, shall attain there the pressure of the resistance, the piston will recede and not allow the steam to assume a greater pressure. And if it be supposed that by enlarging the passage, the steam be made to flow in 10 times, 20 times, 30 times quicker, the piston will recede 10 times, 20 times, 30 times quicker also, since its motion is the result of the arrival of the steam; but never will the pressure of the steam exceed the resistance of the piston, since the piston being a valve to the cylinder, that would be supposing a boiler in which the pressure of the steam were greater than that of the valve.

The regulator, then, can make no change in the pressure in the cylinder, but this is what happens.

The quantity of steam of a given density, which flows through a determined orifice, being in proportion to the area of that orifice, it follows that when the opening of the regulator is contracted, the *quantity* of steam, at the pressure of the boiler, which passes into the cylinder, is thereby diminished; nevertheless the same quantity is still generated in the boiler. The steam which has ceased to find an issue towards the cylinder, will then accumulate in the boiler, and will there rise to a greater and greater density and elastic force, till at length it finds an issue somewhere; till, for instance, having attained the pressure necessary to raise the safety valves, it escape into the atmosphere. Then a balance will be established, according to which the surplus of steam generated above what can reach the cylinder, will find a constant issue by the safety valves; and the rest will pass through the orifice of the regulator and go into the cylinders to produce the motion of the piston. From this moment all will persevere in the same state, and the pressure in the boiler will continue as high as it must be, to keep the safety-valve open and give egress to the steam, as quickly as it is produced.

Hence it is plain that the contracting more or less of the regulator can have no action on the pressure in the cylinder, but that it has a very direct action on the pressure in the boiler.

2. We have just said that, according as the aperture of the regulator is contracted, the pressure of

the steam will rise in the boiler and its density increase at the same time; and so long as the steam shall not find an opening to escape entirely as fast as it is produced, this increase of density and elastic force will continue; for we suppose that the same mass of steam per minute is still generated in the boiler, and that the fire is maintained in the same state. Now the steam is retained in the boiler by two obstacles: the orifice of the regulator which opposes its passage on account of the density, and the safety-valve which opposes its passage on account of the pressure. Two cases then may now occur, according to which of the two obstacles shall give way first: either the steam becoming more and more dense, will in the end so reduce its volume, as to issue entirely by the orifice of the regulator, notwithstanding the contraction of the latter; or else the safety-valve, opposing less resistance to the elastic force than the narrowed orifice opposes to the density, the steam will escape by the safety-valve.

In the first of these two cases, then, the engine will be thus regulated: in the cylinder, as invariably, the pressure of the resistance; and in the boiler the pressure necessary for the corresponding density of the steam, to admit of its issuing entirely by the aperture afforded by the regulator.

And in the second case, the engine, on the contrary, will be thus regulated; in the cylinder still the pressure of the resistance, and in the boiler that of the safety-valve.

We must now consider separately each of these cases. Let us suppose that the safety-valve being set at a very high pressure, and the orifice of the regulator, on the contrary, being *but moderately* contracted, the steam accumulating in the boiler, has acquired the density which allows its issue by the orifice, before it has acquired the pressure which procures its escape by the safety-valve. Then it will happen that the *total* quantity of steam produced will pass into the cylinder, that it will there assume the pressure of the resistance, dilating itself in proportion; and by dividing the volume of the steam thus dilated by the area of the cylinder, we shall always have the velocity of efflux by the cylinder, which is nothing else but the velocity of the piston. Thus all will go on as before in the engine, and consequently the effects produced will always be given by the same formulæ, P being made of course to express the new pressure produced in the boiler, and S the new vaporisation, if that vaporisation has changed in consequence of the change of pressure.

Let us now suppose that the safety-valve is set at a low pressure, and that the regulator, on the contrary, is considerably contracted; so that the steam raises the valve before it acquires the density that would permit it to issue entirely by the regulator. The valve will then be raised, and a part of the steam which continues to be generated in the boiler, will be lost in the atmosphere; and necessarily the effects of the engine will be by so much diminished. But let it be observed, that, with respect to that part of the steam which is not lost, that is the part which finds an issue towards the cylinder, it may always be truly said, that it will there assume the pressure of the resistance, and act in the same manner as the total mass of the steam did before.

The only difference will be, then, that the effects produced, instead of being due to the *totality* of the steam, will now be due to a portion only of that steam.

Thus, provided our formulæ take account of this difference, they will thereby take into account the whole change that has taken place. Now this is precisely what they do, for we have said that the quantity S, in those formulæ, represents the *effective* vaporisation of the engine, that, in fact, which is really transmitted to the cylinders; or, in other words, the *total* vaporisation, minus that which is lost by the safety-valve. It will, then, suffice to substitute for S the real value proper to the case, and the formulæ will continue to represent what passes in the engine.

As the *total* quantity of water evaporated in a given time is measured directly in the feeding apparatus, all that remains to be sought is the means of estimating that which is lost by the safety-valve, in order to subtract it from the former. This valuation is easily made, by noting how much the valve is raised at the moment of the loss, which the length of the valve-levers, and the graduated scale with which they are furnished, render very easy to do; afterwards the regulator must be completely closed, so as to force the whole of the steam produced to escape by the valve, and note taken again of the degree of elevation which this will cause to the valve. Then the proportion of the first elevation to the second will give the ratio of the steam lost to the whole steam produced. This is the means we have employed for locomotives. Should this valuation not appear sufficiently precise, the waste steam may be condensed in a separate vessel, and the quantity of water measured. It will always, then, be easy to know the *effective* vaporisation of the engine, and consequently, by introducing it into the formulæ, we shall continue to have the true effects produced.

In the two preceding cases we have supposed that the boiler continues, after the contracting of the regulator, to produce the same quantity of steam. A third case, however, may occur, namely, that wherein the engineer shall lower the *damper*, the moment he sees the valve blow, and reduce his fire so as to stop the blowing of the valve. Then the mass of steam produced per minute will diminish; but since it is clear that the quantity which is produced, however small it may be, will always act in the engine in the same manner, it follows that, provided we substitute this new evaporation in the formulæ, we shall have also the new corresponding effects. Thus, for this third case as for the other two, the formulæ will always satisfy the exigencies, as soon as the substitutions proper to the supposed case shall have been made.

3. It will now be proper to examine what changes the effects of the engine will undergo in the three preceding suppositions. We have seen that the proposed formulæ will always give those effects, on the proper substitutions being made in them. Let us then examine the results of those substitutions.

In the first case, to wit, the fire continued at the same degree of intensity, and the orifice narrowed, though not sufficiently to make the valve blow, the pressure P in the boiler becomes greater. But in the exposed formulæ, the pressure P figures only as multiplied by m, which is the *relative* volume of the steam. This volume being inversely as the density, and the density itself varying very nearly in the direct ratio of the pressure, it follows that, unless a very great change of pressure take place, the product m P will remain constant. If it be supposed, as is generally admitted, that the evaporation of a given boiler is the same under different pressures of the steam, the quantity S will not vary either. In this case, then, the formulæ will give the same results; and consequently the engine will produce the same effects, after the contracting of the regulator as before that contraction was made.

In the second case, to wit, contraction of the regulator, attended with blowing of the safety-valve, there is still increase of pressure in the boiler, which, as we have just seen, produces no change in the effects. But moreover there is a certain loss by the safety-valves, and that loss diminishes by so much the *effective* vaporization S. There will then be a diminution of effect precisely proportional to the quantity of steam lost by the valve, which we have given the means of measuring.

Finally, in the third case, to wit, contraction of the regulator, accompanied by a reduction of the intensity of the fire, the blowing of the safety-valve will be suppressed only by producing a smaller mass of steam in the boiler. But since this mass of steam, generated and transmitted to the cylinders, is less than before the contraction of the regulator, it follows that the effect produced by the engine, or the result given by the formulæ, will be reduced just so much. Thus this third case is similar to the second, and will similarly be attended with a reduction of effect.

The first of the three cases which we have just presented, takes place without the smallest attention being paid to it, whenever the orifice of the regulator is but slightly diminished.

The second occurs almost continually in locomotive engines, because these having to overcome very variable resistances, according to the inclinations of the road they traverse, it is necessary to maintain an intense fire, and to keep the engine always ready to develope on an emergency an increase of power.

The third is that which happens generally in stationary engines, when the regulator is pretty much contracted, because the regulator, in those engines, being never reduced but when the work of the engine requires less force, the engineman takes advantage of that circumstance to diminish the intensity of the fire, and to produce no more evaporation than what is strictly necessary.

These three cases may then occur in the different engines, but the exposed formulæ will always adapt themselves to them.

SECT. VIII. *Of the differences which exist between the theory proposed and the ordinary theory.*

In terminating the general exposition of our manner of viewing the action of steam in steam-engines, we will resume in a few words the differences existing between the method we propose and that which has been in use hitherto.

1. The ordinary theory passes from the theoretic effects to the practical by means of a constant coefficient.

Ours rejects entirely the use of that coefficient, which we regard as resulting from a fundamental error in the calculation of what are termed the theoretic effects.

2. The ordinary theory acknowledges not knowing the pressure in the cylinder; it seeks to deduce it from that of the boiler.

Our theory determines, *à priori*, the pressure in the cylinder, as being, not equal nor proportional to that of the boiler, but equal to that of the resistance on the piston.

3. The ordinary theory determines the load which an engine is capable of drawing, without taking the velocity into the calculation. That is to say, it maintains that the engine will always draw the same load at any velocity that can be imagined.

Our theory brings the velocity into the calculation in such sort, that the greater the velocity the less will be the load the engine can draw.

4. The ordinary theory calculates the evaporation of the engine for a resistance and a velocity given, exclusively of any consideration of the resistance; that is to say, it maintains again that the evaporation necessary to effect the motion shall be independent of the resistance to be moved.

Ours, on the contrary, introduces the load and the velocity into the calculation.

5. The ordinary theory has no means of calculating the velocity that an engine will assume with a given resistance.

Ours gives this calculation with the same simplicity as the preceding.

6. The ordinary theory regards the regulator as determining the pressure in the cylinder. And yet in that calculation it takes no account of the variations of the regulator.

Ours regards the regulator as fixing the pressure in the boiler and not in the cylinder. It introduces the effects of the regulator into the formulæ.

7. The ordinary theory is but an approximation more or less exact.

Ours, on the contrary, which will be seen still more developed, is a method completely analytic in all its parts.

Nothing then can be more distinct than these two methods; and as, not only since the year 1835, when we first laid down these principles in our *Treatise on Locomotive Engines*, but even as late as December, 1837, the authors who have treated these questions, whether in their writings or in their public lectures, have employed the method of coefficients, we think that the recapitulation we have just made sufficiently establishes that their conception of these questions is altogether different from our own.

We do not then deem it necessary to insist any more on this subject, and shall now pass on to the complete development of the formulæ, of which we have as yet given but a general outline.

CHAPTER II.

OF THE LAWS WHICH REGULATE THE MECHANICAL ACTION OF THE STEAM.

SECT. I. *Relation between the temperature and the pressure of the steam in contact with the liquid.*

BEFORE entering upon considerations which have for their basis the effects of the steam, it may be necessary to lay down in a few words, some of the laws according to which the mechanical action of the steam is determined or modified.

In the calculation of steam engines it is requisite to consider four things in the steam.

Its *pressure*, which is also called tension or elastic force, and which is the pressure it exercises on every unit of the surface of the vessel that contains it.

Its *temperature*, which is the number of degrees marked by a thermometer immerged in it.

Its *density*, which is the weight of a unit of its volume.

And its *relative volume*, which is the volume of a given weight of steam compared to the volume of the same weight of water, or, in other words, to the volume of the water that has served to produce it. We deem it necessary to add here the word *relative*, in order to avoid the confusion which would otherwise arise continually between the absolute volume filled by the steam, which may depend on the capacity of the vessel that contains it, and the relative volume which is the inverse of the density. Thus, for instance, steam generated under the pressure of the atmosphere may fill a vessel of any size, but its relative volume will always be 1700 times that of water.

When the volumes occupied by the same weight of two different steams are compared together, it is evidently a comparison of what we call the relative volumes of those two steams. For, the two steams compared having the same weight, correspond to the same volume of water evaporated. But the relative volume of the steam is the quotient of the absolute volume of the steam by the corresponding volume of water. Therefore, it follows that the ratio of the relative volumes of the two steams is the same as the ratio of their absolute volumes; and this proposition must be kept in mind for what will follow hereafter.

The steam may be considered at the moment of its generation in the boiler, when still in contact with the liquid from which it emanates, or else as being separated from that liquid.

When the steam, after having been formed in a boiler, remains in contact with the generating water, it is observed that the same temperature corresponds invariably to the same *pressure*, and *vice versâ.* It is impossible then to increase its temperature, without its pressure and density increasing spontaneously at the same time. In this state the steam is therefore at its *maximum density and pressure for its temperature*, and then a constant connexion visibly exists between the temperature and the pressure.

If on the contrary the steam be separated from the water that generated it, and that the temperature be then augmented, the state of maximum density will cease, since there will be no more water to furnish the surplus of steam, or increase of density, corresponding to the increase of temperature. That invariable connexion above mentioned, between the temperature and the pressure, will then no longer exist, and, by accessory means, the one may at pleasure be augmented or diminished, without any necessity of a concomitant variation taking place in the other, as it happens in the case of the maximum density.

It is necessary then to distinguish between these two states of the steam.

One of the most important laws on the properties of steam, is that which serves to determine the elastic force of the steam in contact with the liquid, when the temperature under which it is generated is known; or, reciprocally, to determine that temperature when the elastic force is known. Not only this enquiry is of a direct utility, but we shall see in the sequel, that it serves equally to determine the density or the relative volume of the steam formed under a given pressure, a point of knowledge indispensable in the calculation of steam-engines.

Experiments on this subject had long been taken in hand, and they were very numerous for steam formed under pressures less than that of the atmosphere; but for high temperatures, the experiments extended but to pressures of four or five atmospheres. Some few only went as far as eight, and that without completing the scale in the interval. The extreme difficulty of researches of this kind, if made with proper attention, the heavy expenses they occasion, and the danger attending them, had prevented the experiments from being carried farther. But to the Academy of Sciences of the Institute of France we are indebted for a complete table on this subject. The academy confided the conduct of these delicate experiments to two distinguished scientific men, Messrs. Arago and Dulong, who evinced in them every nicety that a perfect knowledge of the laws of natural philosophy could suggest, to avoid the ordinary causes of error. Never were researches of this kind conducted on so vast a scale, nor with more accuracy. The pressure of the steam was measured by effective columns of mercury contained in tubes of crystal glass, which together extended to the height of 87 feet English. The instruments were constructed by the most skilful makers, and no expense was spared *. Therefore the greatest degree of confidence is to be attached to their results.

These beautiful experiments furnish a series of observations, from the pressure of 1 atmosphere to that of 24. To form however a table extending beyond this limit, Messrs. Dulong and Arago have sought to deduce from their observations a formula which might represent temperatures for still higher pressures without any noticeable error. They have in fact attained that end, by means of a formula which we shall presently report, and whose accord with experience is such, for all that part of the scale above four atmospheres, as to give room to think that, on being applied to pressures up to 50 atmospheres, the error in temperature would not in any case exceed 1 degree of the centigrade thermometer or 1·8 degree of Fahrenheit. They were enabled then, as well from the result of their observations as by means of that formula, to compose a table of temperatures of steam up to 50 atmospheres of pressure, with the certainty of committing no error worthy of note.

Though the formula of Messrs. Arago and Dulong may be applied to pressures comprised between 1 and 4 atmospheres, with an approximation that would suffice for most of the exigencies in the

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* Vide Exposé des recherches faites par ordre de l'Académie des Sciences, pour determiner les forces élastiques de la vapeur d'eau à de hautes températures. *Mémoires de l'Académie des Sciences*, Tome X.; *Annales de Chimie et de Physique*, Tome XLIII. 1830.

arts, they did not indicate the use of it for that interval, because in that part of the scale, other formulæ already known accord more exactly with the results of observation, and ought, in consequence, to be preferred. Among those formulæ, that originally proposed by Tredgold, and afterwards modified by his translator, Mr. Mellet, gave the most exact results; and no inconvenience arises from the use of it, when it is required merely to compose a table by intervals of half-atmospheres. But as, for the more commodious use of the formulæ which we have to propose in this work, we shall want to establish a table by intervals of pounds per square inch; we deem it better to employ a formula which we shall give with the others presently, and which, approaching as near as that of Tredgold to the results of direct observation, in the points furnished by experiment, has moreover the advantage of coinciding exactly at 4 or 4½ atmospheres with the formula of Messrs. Dulong and Arago, which is to form the continuation of it*.

* In fact, comparing, in French measure, the two formulæ with the observation, we find the following results, as it will be easy to verify hereafter.

Elastic force in atmospheres.	Observed temperature.	Temperature given by Tredgold's form. modified by Mellet.	Temperature given by the proposed formula.	Temperature given by the form. of Messrs. Arago and Dulong.
1	100·	99·96	100	„
2·14	123·7	123.54	123·34	„
2·8705	133·3	133·54	133·17	„
4·	„	145·43	144·88	„
4·5735	149·7	150·39	149·79	149·77

These formulæ, as well as other similar ones, have the inconvenience of suiting only a limited part of the scale of temperatures. That of Tredgold modified, as well as that which we propose to substitute for it, represent very closely the observations for the interval between 1 and 4 atmospheres; but below that point they are incorrect, and above it they are inferior in point of accuracy to that of Messrs. Dulong and Arago.

The latter accords remarkably with the facts, from 4 atmospheres to 24. In this interval its greatest difference with observation is ·4 degree of the centigrade thermometer or ·7 of Fahrenheit, and nearly all the other differences are only ·1 degree centigrade or ·18 Fahrenheit; but, as we have already said, it begins to deviate from the observation below 4 atmospheres.

Finally, among the formulæ proposed by different authors on the same subject, that of Southern is very suitable to steam formed under pressures inferior to that of one atmosphere; it deviates then from the truth only in very low pressures, as appears from the experiments of that engineer. But for pressures superior to 1 atmosphere it ceases to have the same accuracy: from 1 to 4 atmospheres

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It appears that the formula which we propose differs from the observed temperatures nearly as much as that of Tredgold modified; but as the difference from the observation is on the *minus* side instead of the *plus*, there results a coincidence at 4½ atmospheres with that of Messrs. Arago and Dulong.

it gives more error than that of Tredgold modified, and above 4 atmospheres the error rises rapidly to 1 and 1·5 degree of the centigrade thermometer, or 1·8 and 2·6 degrees of Fahrenheit; so that the formula of Messrs. Arago and Dulong, which is, besides, of more easy calculation, becomes then far preferable to it.

No one then of these formulæ suits the whole series of the scale of temperatures, and to hold exclusively to any one of them would be knowingly to introduce errors into the tables. As moreover, the true *theoretic* law which connects the pressures with the temperatures is unknown, and that these formulæ are formulæ of interpolation, established solely from their coincidence with the facts, and used merely to fill up the intervals of the experiments, according to what is wanted for the regular division of the tables, the only means of making use of them is to apply each respectively to that portion of the series which it suits. Then, from the comparison of their results with experience, one may rest assured that the error on the temperature will in no point exceed seven-tenths of a degree of Fahrenheit, or four-tenths of a degree of the centigrade thermometer. This was the means employed before us, and we shall adopt it in the formation of the tables we are about to present.

The formulæ, which will serve to compose these tables, are then the following, which we report here, not in their original terms but transformed, for greater convenience, into the measures usual in practice; that is, expressing the pressure p in pounds per square inch or in kilograms per square centimetre, and the temperature t, in degrees of Fahrenheit's or of the centigrade thermometer, reckoned in the ordinary manner.

Southern's formula, suitable to pressures less than that of the atmosphere (French measures):

$$p = \cdot 0034542 + \left(\frac{46 \cdot 278 + t}{145 \cdot 360}\right)^{5.13},$$

$$t = 145 \cdot 360 \sqrt[5.13]{p - \cdot 0034542} - 46 \cdot 278.$$

Tredgold's formula modified by Mr. Mellet, suitable to pressures of 1 to 4 atmospheres (French measures):

$$p = \left(\frac{75 + t}{174}\right)^6,$$

$$t = 174 \sqrt[6]{p} - 75.$$

Formula suitable, like the preceding, to pressures from 1 to 4 atmospheres (French measures):

$$p = \left(\frac{72 \cdot 67 + t}{171 \cdot 72}\right)^6,$$

$$t = 171 \cdot 72 \sqrt[6]{p} - 72 \cdot 67.$$

Formula of Messrs. Dulong and Arago, suitable to pressures from 4 to 50 atmospheres (French measures):

$$p = (\cdot 28658 + \cdot 0072003\, t)^5,$$

$$t = 138 \cdot 883 \sqrt[5]{p} - 39 \cdot 802.$$

Southern's formula, suitable to pressures less than that of the atmosphere (English measures):

$$p = \cdot04948 + \left(\frac{51\cdot3 + t}{155\cdot7256}\right)^{5.13},$$

$$t = 155\cdot7256 \sqrt[5.13]{p - \cdot04948} - 51\cdot3.$$

Tredgold's formula modified by Mr. Mellet, suitable to pressures from 1 to 4 atmospheres (English measures):

$$p = \left(\frac{103 + t}{201\cdot18}\right)^{6},$$

$$t = 201\cdot18 \sqrt[6]{p} - 103.$$

Formula suitable, like the preceding, to pressures from 1 to 4 atmospheres (English measures):

$$p = \left(\frac{98\cdot806 + t}{198\cdot562}\right)^{6},$$

$$t = 198\cdot562 \sqrt[6]{p} - 98\cdot806.$$

Formula of Messrs. Dulong and Arago, suitable to pressures from 4 to 50 atmospheres (English measures):

$$p = (\cdot26793 + \cdot0067585\, t)^5,$$

$$t = 147\cdot961 \sqrt[5]{p} - 39\cdot644.$$

Besides the formulæ which we have just reported, there exists another proposed by Mr. Biot, which, compared by that illustrious natural philosopher to the above-mentioned experiments on high pressures, to those of Taylor on pressures approaching nearer to 100 degrees centigrade, and to a numerous series of manuscript observations made by Mr. Gay-Lussac, from 100° to − 20 degrees centigrade, reproduces the results observed, with very slight accidental deviations, such as the experiments themselves are liable to. This formula, which has consequently the advantage over the preceding, of being applicable to all points of the scale, is the following:—

$$\log p = a - a_1 b_1^{20+t} - a_2 b_2^{20+t}.$$

Log p is the tabulary logarithm of the pressure expressed in millimetres of mercury at 0° centigrade; t is the centesimal temperature counted on the air thermometer, and the quantities a, a_1, a_2, b_1, b_2, are constant quantities which have the following values:

$$a = 5{\cdot}96131330259,$$
$$\log a_1 = \bar{1}{\cdot}82340688193,$$
$$\log b_1 = -{\cdot}01309734295,$$
$$\log a_2 = {\cdot}74110951837,$$
$$\log b_2 = -{\cdot}00212510583.$$

This formula cannot fail to be extremely useful in many delicate researches on the effects of steam; but to establish, by its means, a table of the form we require, the pressure ought first to be deduced from it for each degree of the air thermometer; then these degrees ought to be afterwards changed into degrees of the mercury thermometer; and as this would not give the temperatures corresponding to given pressures, by regular intervals, a subsequent interpolation would be still necessary to make the table in the proper disposition. These long operations induced us to give the preference to the previously cited formulæ, for the construction of the tables which we shall shortly present.

Sect. II. *Relation between the relative volumes and the pressures, at equal temperature, or between the relative volumes and the temperatures, at equal pressure, in the steam separated from the liquid.*

We have said that when the steam is in contact with the generating liquid, its pressure is necessarily connected with its temperature; and as the density of an elastic fluid depends only on its temperature and its pressure, it follows that the density is then always constant for a given temperature or pressure. But when the steam is separated from the liquid, that connexion between the temperature and the pressure no longer exists. The temperature of the steam may then be varied without changing its pressure, or reciprocally; and according as the one or the other of these two elements is made to vary, the density of the steam undergoes changes which have been an object of investigation among natural philosophers.

One very remarkable law in the effects of gas and steam is that which was discovered by Mariotte or Boyle, and has since been confirmed, as far as to pressures of 27 atmospheres, by Messrs. Arago and Dulong. It consists in this, that if the volume of a given weight of gas or of steam be made to vary without changing its temperature, the elastic force of the gas will vary in the inverse ratio of the volume it is made to occupy. That is to say, if v and v' express the volumes occupied by the same weight of steam, and p and p' the pressures which maintain the steam compressed under those respective volumes, the temperature, moreover, being the same in both cases, the following analogy will exist:

$$\frac{p}{p'}=\frac{v'}{v}.$$

And therefore, μ and μ' being the *relative* volumes of the steam at the pressures p and p', we shall have

$$\frac{p}{p'}=\frac{\mu'}{\mu}.$$

According to this law, if a given weight of an elastic fluid be compressed to half its primitive volume, without changing its temperature, the elastic force of that fluid will become double. But it is plain that this effect cannot take place in the steam in contact with the liquid, because it supposes that during the change of pressure the temperature remains constant, whereas we have seen that in such state the pressure always accompanies the temperature, and *vice versâ*.

Another property equally important in the appreciation of the effects of steam has been discovered by a celebrated chemist of our times, Mr. Gay-Lussac. It consists in this, that if the temperature of a given weight of an elastic fluid be made to vary, its tension being maintained at the same degree, it will receive augmentations of volume exactly proportional to the augmentations of temperature; and for each degree of the centigrade thermometer, the increase of volume will be ·00364 of the volume which the same weight of fluid occupies at the temperature zero. If the temperatures are taken from Fahrenheit's thermometer, each augmentation of 1 degree in the temperature will produce an increase of ·00202 of the volume occupied by the fluid at the temperature of 32°.

If then we call V the volume of the given weight of the elastic fluid, under any pressure, and at the temperature of 32 degrees of Fahrenheit, the volume it will occupy under the same pressure, and at the temperature t of Fahrenheit will be

$$v = V + V \times \cdot 00202\,(t - 32).$$

It follows that, between the volumes v and v' occupied by the same weight of steam, at the same pressure and under the respective temperatures t and t', there will be the following analogy:

$$\frac{v}{v'} = \frac{1 + \cdot 00202\ (t - 32)}{1 + \cdot 00202\ (t' - 32)},$$

which will also be true, when we replace the ratio of the two absolute volumes v and v', by the ratio of the *relative* volumes μ and μ' of the steam.

This law, supposing that the temperature of the steam changes, without the pressure undergoing any change, obviously cannot apply to the effects produced in steam in contact with the liquid, since in those the pressure changes necessarily and spontaneously with the temperature.

SECT. III. *Relation between the relative volumes, the pressures, and the temperatures, in the steam in contact or not in contact with the liquid.*

As it has just been observed, neither Boyle's law nor that of Gay-Lussac can apply alone to changes which take place in the steam remaining in contact with the liquid. But it is clear that from the two, a third relation may be deduced, whereby to determine the variations of volume which take place in the steam, by virtue of a simultaneous change in the temperature and in the pressure; and this relation may then comprehend the case of the steam in contact with the liquid, since it will suffice to introduce into the formulæ the pressures and temperatures which, in this state of the steam, correspond to each other.

Suppose then it be required to know the volume occupied by a given weight of steam, which passes from the pressure p' and temperature t', to the pressure p and temperature t. It may be supposed that the steam passes first from the pressure p' to the pressure p without changing its temperature, which, from Boyle's law, will give between the relative volumes of the steam, the analogy

$$\mu'' = \mu' \frac{p'}{p};$$

then supposing this steam to pass from the temperature t' to the temperature t, without changing its pressure, the relative volume of the steam, from the law of Gay-Lussac, will become

$$\mu = \mu'' \frac{1 + \cdot 00202\ (t - 32)}{1 + \cdot 00202\ (t' - 32)} =$$

$$= \mu' \frac{p'}{p} \cdot \frac{1 + \cdot 00202\ (t - 32)}{1 + \cdot 00202\ (t' - 32)}.$$

This formula will then express the law according to which the relative volume of the steam changes, by virtue of a given combination of pressure and temperature. Consequently, substituting in this equation for p and t, p' and t', the pressures and temperatures only which correspond to each other in the steam in contact with the liquid, we shall have the analogous changes which take place in the relative volume of the steam, when it is not separated from the water which generated it.

On the other hand, it is known by experience, that under the atmospheric pressure, or 14·706 lbs. per square inch, and at the temperature of 212° of Fahrenheit's thermometer, the relative volume of the steam in contact with the liquid is 1700 times that of the water which has produced it. Hence it is easy to conclude the relative volume of the steam at any given pressure p and at the corresponding temperature t. It suffices, in fact, to insert the above values for p', t', and μ', in the general equation obtained above, and the result will be

$$\mu = 1700 \times \frac{14{\cdot}706}{p} \times \frac{1 + {\cdot}00202\ (t-32)}{1 + {\cdot}00202 \times 180} =$$

$$= 18329 \frac{1 + {\cdot}00202\ (t-32)}{p}.$$

Thus we may, by means of this formula, calculate the relative volume of the steam generated under a given pressure, as soon as we know the temperature answering to that pressure in steam at the maximum of density for its temperature.

It is what we have done in the construction of the following table. The second column has been formed by calculating the temperature of the steam at the maximum density, from the formulæ which we have given in the first section of this chapter. Then using this series of temperatures in the formula which precedes, we have concluded the third column, or the relative volumes of the steam in contact with the liquid, under all the pressures comprised between 1 and 8 atmospheres. This table will, in consequence, dispense from all calculation with regard either to the research of the temperatures, or to that of the relative volumes of the steam; and its extent will suffice for all applications that occur in the working of steam engines.

When we speak of steam *generated* under a given pressure, we understand the steam considered at the moment of its generation, and consequently still in contact with the liquid. We have explained elsewhere that the volume of the steam, compared to that of the water which has produced it, is precisely what we call the *relative* volume of the steam.

TABLE of the volume of the steam generated under different pressures, compared to the volume of the water that has produced it.

Total pressure, in English pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.	Total pressure, in English pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.
1	102·9	20954	37	263·7	727
2	126·1	10907	38	265·3	710
3	141·0	7455	39	266·9	693
4	152·3	5695	40	268·4	677
5	161·4	4624	41	269·9	662
6	169·2	3901	42	271·4	647
7	176·0	3380	43	272·9	634
8	182·0	2985	44	274·3	620
9	187·4	2676	45	275·7	608
10	192·4	2427	46	277·1	596
11	197·0	2222	47	278·4	584
12	201·3	2050	48	279·7	573
13	205·3	1903	49	281·0	562
14	209·0	1777	50	282·3	552
15	213·0	1669	51	283·6	542
16	216·4	1572	52	284·8	532
17	219·6	1487	53	286·0	523
18	222·6	1410	54	287·2	514
19	225·6	1342	55	288·4	506
20	228·3	1280	56	289·6	498
21	231·0	1224	57	290·7	490
22	233·6	1172	58	291·9	482
23	236·1	1125	59	293·0	474
24	238·4	1082	60	294·1	467
25	240·7	1042	61	294·9	460
26	243·0	1005	62	295·9	453
27	245·1	971	63	297·0	447
28	247·2	939	64	298·1	440
29	249·2	909	65	299·1	434
30	251·2	882	66	300·1	428
31	253·1	855	67	301·2	422
32	255·0	831	68	302·2	417
33	256·8	808	69	303·2	411
34	258·6	786	70	304·2	406
35	260·3	765	71	305·1	401
36	262·0	746	72	306·1	396

Total pressure, in English pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.	Total pressure, in English pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.
73	307·1	391	92	323·5	317
74	308·0	386	93	324·3	313
75	308·9	381	94	325·0	310
76	309·9	377	95	325·8	307
77	310·8	372	96	326·6	305
78	311·7	368	97	327·3	302
79	312·6	364	98	328·1	299
80	313·5	359	99	328·8	296
81	314·3	355	100	329·6	293
82	315·2	351	105	333·2	281
83	316·1	348	120	343·3	249
84	316·9	344	135	352·4	224
85	317·8	340	150	360·8	203
86	318·6	337	165	368·5	187
87	319·4	333	180	375·6	173
88	320·3	330	195	382·3	161
89	321·1	326	210	388·6	150
90	321·9	323	225	394·6	141
91	322·7	320	240	400·2	133

Sect. IV. *Direct relation between the relative volumes and the pressures, in the steam in contact with the liquid.*

It has just been seen, from the formulæ given in the preceding section, that the density and the relative volume of the steam, whether separated from the liquid or not, are known in terms of the simultaneous pressure and temperature. It is likewise known that in the steam in contact with the liquid, the temperature depends immediately on the pressure. It should therefore be possible to find a relation proper to determine directly the relative volume of the steam in contact with the liquid, or, in other words, of the steam at the maximum density and pressure for its temperature, by means of the sole knowledge of the pressure under which it is formed.

The equation which gives the relative volume of the steam in any state whatever, in terms of its pressure and temperature, has been given above. We have also shewn the formulæ which serve to find the temperature in terms of the pressure, in steam in contact with the liquid. Eliminating then the temperature from the equation of the volumes and that of the temperatures, we shall obtain definitively the relation sought, or the relative volume of the steam at the maximum density, in terms of the pressure only.

But here starts the difficulty. First, Mr. Biot's formula not being soluble with reference to the temperature, does not admit the necessary elimination. In the next place, the assemblage of the three formulæ reported above, which are made to succeed each other, suit very well in the formation of tables of correspondence between the pressures and the temperatures, when that is the end proposed. Likewise, in an enquiry relative to the expansion of the steam in an engine, when it is known precisely within what limits of pressure that expansion will take place, it may immediately be discerned which of the three formulæ is applicable to the case to be considered, and then *t* may be eliminated between that formula and the equation of volumes. But if the question regards, for instance, the case wherein the steam generated in the boiler under a pressure of 8 or 10 atmospheres, might, according to the circumstances of the motion, expand during its action in the engine, either to a pressure less than 1 atmosphere, or to a pressure between 1 and 4 atmospheres, or in fine to a pressure superior to 4 atmospheres; then we shall not know which of the three formulæ to use in the elimination, and it will be impossible to arrive at a general equation representing the effect of the engine in all cases.

Besides, were we even to adopt any one of those equations, the radicals they contain would render the calculation so complicated as to make it unfit for practical applications.

The equations of temperature hitherto known cannot then solve the question that presents itself, that is to say, satisfy the wants of the calculation of steam-engines in this respect; and, consequently, the only means left is to seek, in a direct manner, an approximate relation, proper to give immediately the relative volume of the steam at the maximum density in terms of the pressure alone.

With this view Mr. Navier had proposed the expression:

$$\mu = \frac{1000}{\cdot 09 + \cdot 0000484\, p},$$

in which μ is the *relative* volume, or the ratio of the volume of the steam to that occupied by the same weight of water, and p the pressure expressed in kilograms per square metre. But this formula, though exact enough in high pressures, deviates considerably from experience in pressures below that of the atmosphere, which, however, come under consideration in condensing engines. Moreover, for non-condensing engines, it is possible to find one much more exact, as will presently be seen. We deem it then proper to propose, on this subject, the following formulæ:

Formula for *condensing engines* of various systems;

$$\mu = \frac{10000}{\cdot 4227 + \cdot 00258\, p.}$$

Formula for *non-condensing engines*;

$$\mu = \frac{10000}{1\cdot 421 + \cdot 0023\, p.}$$

In these formulæ, the pressure p is expressed in pounds per square foot.

The former of the two suits equally to pressures superior or inferior to that of the atmosphere, at least within the limits that it may occur to consider in applying them to steam-engines.

We know that the greatest pressure used in the boiler never surpasses 8 atmospheres, or 120 lbs. per square inch; and on the other hand, that it can, in no case, be required to calculate the effects of steam acting as a moving force in an engine, at a pressure inferior to 8 or 10 lbs. per square inch, or about $\frac{2}{3}$ of an atmosphere. In a condensing engine, for instance, the steam, after the communication with the condenser has been opened, never descends into the cylinder at a pressure less than 4 lbs. per square inch; the friction of the engine, besides, may be estimated at 1 lb. per square inch; and it is impossible to suppose a load which shall not, of itself and with the additional friction it occasions in the engine, produce a resistance against the piston of at least 3 lbs. per square inch. Thus the resistance to be overcome by force of the steam, cannot in any case be less than 8 lbs. per square inch; consequently the steam cannot descend into the cylinder at a pressure less than 8 lbs. per square inch. A formula which gives the exact volumes down to that pressure, is then all that can be necessary for the calculations that may occur, and we shall presently see that the proposed formula fulfils that condition.

The first of the formulæ might also, without any noticeable error, be applied to non-condensing engines. Since, however, in these the steam can hardly be spent at a total pressure less than two atmospheres, by reason of the atmospheric pressure, the friction of the engine and the resistance of the load, it is needless to require of the formula exact volumes for pressures less than 2 atmospheres. In this case, then, the second formula will be found to have a greater degree of accuracy, and we shall in consequence prefer it.

It will be remarked that, besides the necessity of these formulæ in the general calculation of the effect of steam-engines, they have the advantage moreover, for other purposes in the arts, of dispensing entirely with tables of temperature, and of supplying the place of tables of the volume of the steam, when these are not at hand.

Finally, to give a precise idea of the approximation given by the two formulæ just mentioned, we here subjoin a table of the values they furnish for the principal points of the scale of pressures.

Relative Volume of the Steam generated under different pressures, calculated by the proposed formulæ.

TOTAL PRESSURE of the Steam, in pounds per square inch.	VOLUME of the Steam, calculated by the ordinary formulæ.	VOLUME calculated by the proposed formula for condensing engines.	VOLUME calculated by the proposed formula for non-condensing engines.
5	4624	4386	„
6	3901	3771	„
7	3380	3307	„
8	2985	2946	„
9	2676	2655	„
10	2427	2417	„
11	2222	2218	„
12	2050	2049	„
13	1903	1904	„
14	1777	1778	„
15	1669	1668	„
20	1280	1273	1243
25	1042	1030	1031
30	882	864	881
35	765	745	768
40	677	654	682
45	608	583	613
50	552	526	556
55	506	479	509
60	467	440	470
65	434	407	436
70	406	378	406
75	381	354	381
80	359	332	358
85	340	312	338
90	323	295	320
105	281	254	276
120	249	222	243
135	224	198	217
150	203	178	196

SECT. V. *Of the constituent heat of the steam in contact with the liquid.*

There is yet an enquiry, relative to the properties of steam, which has long fixed the attention of natural philosophers: it is that of the quantity of heat, necessary to constitute the steam in the state of an elastic fluid under various degrees of elasticity.

It is well known that when water is evaporated under the atmospheric pressure, in vain new quantities of heat may be added by means of the furnace, neither the temperature of the water, nor that of the steam ever rise above 100° of the centigrade thermometer, or 212° of Fahrenheit. All the heat then which is incessantly added to the liquid must pass into the steam, but must subsist there in a certain state which is called *latent*, because the heat, though really transmitted by the fire, remains nevertheless without any effect upon the thermometer, nor does it afterwards become perceptible till the moment of disengaging itself, on the steam being condensed.

This latent heat evidently serves to maintain the molecules of water in the degree of separation suitable to their new state of elastic fluid; and it is then absorbed by the steam, in a manner similar to that which is absorbed by the water, on passing from the solid state, or state of ice, to the liquid. But it is important to know the quantity of the latent heat, in order to appreciate with accuracy the modifications the steam may undergo.

Some essays made by Watt had already elicited, that the steam, at the moment of its generation, or in contact with the liquid, contains the same quantity of total heat, at whatever degree of tension, or, in other words, at whatever degree of density it may be formed. The experiments of Messrs. Sharpe and Clement have since confirmed this result. From them is deduced, that the quantity of latent heat contained in the steam in contact with the liquid, is less and less, in proportion as the temperature is higher; so that the total heat, or the sum of the latent heat plus the heat indicated by the thermometer, form in all cases a constant quantity represented by 650° of the centigrade thermometer, or, 1170° of Fahrenheit's.

Southern, on the contrary, has concluded from some experiments on the pressure and temperature of steam, that it is the latent heat which is constant; and that, to have the total quantity of heat actually contained in steam formed at a given temperature, that temperature must be augmented by a constant number, representing the latent heat absorbed by the steam in its change of state.

Some authors have deemed this opinion more rational, but the observations we are about to report seem to us to set the former beyond all doubt.

It is known, that when an elastic fluid dilates itself into a larger space, the dilatation is invariably attended with a diminution of temperature. If, then, the former of the two laws is exact, it follows that the steam, once formed at a certain pressure, may be separated from the liquid, and provided it lose no portion of its primitive caloric, by any external agent, it may dilate into greater and greater space, passing at the same time to lower and lower temperatures, without ceasing on that account, to remain at the maximum density for its actual temperature. In effect, since we suppose that the steam has in reality lost no portion of its total heat, the consequence is that it always contains precisely as much as suffices to constitute it in the state of maximum density, as well at the new temperature as at the former.

If, on the contrary, Southern's law be exact, when the steam, once separated from the liquid, will diminish in density as it dilates into a larger space, it will not remain at the maximum density for the new temperature. To admit indeed that it would do so, would be to verify Watt's law, since the new steam would be at the maximum density, although containing precisely the same quantity of total heat as the old. But since we admit, on the contrary, that the primitive steam contained more heat than was necessary to constitute the new at the maximum density, it follows that the surplus heat, now liberated, will diffuse itself in the new steam; and as this is separated from the liquid, the increase of heat cannot have the effect of increasing the density of the steam, but will be altogether sensible in the temperature. Thus the result will be, a steam at a certain density, indicated by the spaces into which it is dilated, and at a temperature higher than what is suitable to that density, in steams at the maximum of density for their temperature.

Now, in a numerous series of experiments of which we shall speak hereafter, we have found that in an engine whose steam-pipes were completely protected against all external refrigeration, the steam was generated at a very high pressure in the boiler; and, after having terminated its action in the engine, escaped into the atmosphere at pressures very low and very varied; and that in every case, the steam issued forth precisely in the state of steam at the maximum of density for its temperature. Southern's law then is inadmissible, unless any one choose to suppose that in these varied changes of pressure, the steam lose, by contact with the very same external surfaces, always precisely and strictly just that quantity of heat, sometimes very considerable, at other times very small, by which its temperature should have increased. Consequently we regard the law of Watt as the only one supported by the facts.

The total quantity of heat contained in the steam in contact with the liquid, and under any pressure whatever, is then a constant quantity; and according as the sensible heat increases, the latent heat diminishes in an equal quantity.

On the other hand, according to the same law, if we conceive water to be enclosed in a vessel capable of sufficient resistance, and submitted to temperatures of greater and greater intensity; the latent heat of the steam thence arising, will be less and less as the sensible heat or temperature shall become greater; and as soon as the steam shall be generated at a temperature equal to 650° centigrade or 1170 degrees of Fahrenheit, it will cease to absorb heat in a latent state, and will no longer receive any portion of it, but which will be sensible on the thermometer. We must then conclude that at this point the steam will have a density equal to that of water; since in passing from one state to another, it requires no farther increase of caloric, as would be necessary if any farther increase of severance were to take place between the molecules. Thus the water, though still contained in the vessel, will all have passed into the state of steam. From this moment then, new quantities of heat may be applied to the vessel; but instead of acting on a liquid, it will now act only on an elastic fluid, and therefore all the increase of heat which is added, will, as in all gases, become sensible on the thermometer.

This observation explains the difficulty, which would otherwise present itself: viz., that beyond 650 degrees centigrade or 1170 of Fahrenheit, the preceding law could not subsist without the latent heat becoming a negative quantity, which had been the cause of this law being rejected by some authors.

SECT. VI.—*Of the conservation of the maximum density of the steam for its temperature, during its action in the engine.*

When an engine is at work, the steam is generated in the boiler at a certain pressure; from thence it passes into the cylinder, assuming a different pressure, and then, if it be an expansive engine, the steam after its separation from that of the boiler, continues to dilate itself more and more in the cylinder, till the end of the stroke of the piston. It is commonly supposed that, during all the changes of pressure which the steam may undergo, its temperature remains the same, and the consequent conclusion is that, during the action of the steam in the engine, its density or relative volume follows the law of Boyle or Mariotte; that is to say, the relative volume varies in the inverse ratio of the pressure. This supposition simplifies indeed the formulæ considerably, but we shall presently see that it is contrary to experience; and therefore it becomes necessary to seek what is the true law, according to which the steam changes temperature in the engine, at the same time that its pressure changes. And as calculations relative to the effects of steam, depend essentially on the volume it occupies, we must seek also what changes that volume undergoes, by reason of the variations of temperature and pressure, which take place in the steam during its action.

We shall then substitute for the relation precedently indicated, another more real, and, what is essentially necessary to calculate the effects of steam with accuracy, deduced from the facts themselves.

We have just said that the calculations relative to steam-engines suppose the steam to preserve invariably its original temperature, which allows the application of Boyle's or Mariotte's law to all the changes of density or of pressure it may undergo. However, as it is known that elastic fluids never dilate without cooling in some degree, this supposition obviously could not be realized, but on condition that the steam have time to recover from the bodies with which it is in contact, supposed to be sufficiently heated, the quantity of caloric necessary to restore its temperature, after expansion, to the same degree at which it was before. Now, the rapidity of the motion of the steam in the cylinders and the pipes will not suffer the admission of such an hypothesis.

To obtain satisfaction on this head, in a numerous series of experiments which will be found related in the second edition of our Treatise on Locomotives, we adapted to the boiler of a locomotive engine a thermometer and an air-gauge or manometer; we applied also two similar instruments to the pipe through which the steam, after having terminated its action in the engine, escaped into the atmosphere; and observed their simultaneous indications. The steam was generated in the boiler at a total pressure varying from 40 lbs. to 65 lbs. per square inch, and escaped into the atmosphere at a pressure varying, according to different circumstances, from 20 lbs. to 15 lbs. per square inch. Had the steam preserved its temperature during its action in the engine, it would have issued forth with the pressure, for instance, of 15 lbs. per square inch, but with the temperature proper to the pressure at which it had been formed, that is, 65 lbs. per square inch. Now, nothing like this took place: during some hundreds of experiments wherein we observed and registered these effects, we found invariably that the steam escaped precisely with the temperature suitable to its actual pressure; so that the thermometer, graduated to indicate the pressure in the steam of maximum density, gave identically the same degree of pressure as the air-manometer, and accorded equally with a siphon-manometer, which we had superadded to the apparatus at the point of the outlet of the steam. The steam then was generated in the boiler at a very high pressure and quitted the engine at a very low one; but, on its leaving the engine, as well as at the moment of its production, the steam was at the same temperature that it would have had, if immediately formed at the pressure which it had at the moment of the observation.

Consequently, we are to conclude from these experiments, that during its whole action in the engine, the steam remains in the state of steam at the maximum pressure or density for its temperature. Hence it results that, when the pressure of the steam changes in the engine, its temperature changes spontaneously at the same time, and *vice versâ;* so that they always preserve the mutual relation which connects the pressures and temperatures in the steam in contact with the generating liquid.

Now, we have shown in the fourth section of this chapter, that, with regard to steam in contact with the liquid, the *relative* volume, that is the ratio between the volume of the steam and the volume of an equal weight of water, may be expressed in terms of the pressure by the following very simple formula,

$$\mu = \frac{1}{n + qp}, \quad . \; . \; . \; . \; (a)$$

in which μ is the relative volume of the steam, p the pressure expressed in pounds per square foot, and the constant quantities n and q, have, according to the engines considered, the numerical values already indicated, viz.:

Condensing engines:

$n = \cdot00004227$ . . . . $q = \cdot000000258$;

Non-condensing engines:

$n = \cdot0001421$ . . . . $q = \cdot00000023$.

This relation, then, will be applicable to all the states of the steam during its action in the engine.

Now, according to equation (a), if we suppose that a certain volume of water represented by S, be transformed into steam at the pressure p, and that we call M the *absolute* volume of steam which will be produced by it, we shall have,

$$\mu = \frac{M}{S} = \frac{1}{n + qp}.$$

If afterwards the same volume of water be transformed into steam at the pressure p, and that we call M′ the *absolute* volume of the resulting steam, we shall have also,

$$\frac{M'}{S} = \frac{1}{n + qp'}.$$

Consequently, between the *absolute* volumes of steam which correspond to the same weight of water, we shall have the definitive relation,

$$\frac{M}{M'} = \frac{\frac{n}{q} + p'}{\frac{n}{q} + p}; \quad . \ . \ . \ . \ (b)$$

that is to say: the volumes of the steam will be, not in the inverse ratio of the pressures, as was supposed in admitting Boyle's or Mariotte's law, but in the inverse ratio of the pressures augmented by a constant quantity.

The last equation gives also

$$p = \frac{M'}{M}\left(\frac{n}{q} + p'\right) - \frac{n}{q}. \quad . \ . \ . \ (c)$$

And the two equations (b) and (c) will serve to determine, either M, or p, according to the one of these two quantities, which will be unknown.

These relations then must be substituted for that of Boyle or Mariotte, which is not applicable to the operation of steam in the steam-engine.

As, in all calculations relative to the effects of steam-engines, the volume occupied by a given weight of steam forms the important element of calculation, it is very obvious that the use of the above principle; that is, of the *conservation of the maximum density of the steam for its temperature*, during its action in the engine, and the formula by which we have represented it, will tend to the avoiding of many considerable errors in the results.

If we consider, for instance, an engine in which the steam generated at the pressure of 8 atmospheres, or 120 lbs. per square inch, shall expand to 10 lbs. per square inch; then in the usual mode of calculation, it will be supposed that the steam, during its expansion, will preserve its temperature, and that its volume will vary in the inverse ratio of the pressures. The volume of the steam at the pressure of 120 lbs. per square inch is 249 times that of the water which produced it. If its temperature remained unchanged during its action in the engine, its volume after the expansion would become

$$249 \times \frac{120}{10} = 2988.$$

The supposition, then, amounts to admitting that under the pressure of 10 lbs. per square inch, the volume of the steam would be 2988 times that of the water. Now, from accurate tables, this volume is 2427. An error, then, is induced of $\frac{1}{5}$ on the real volume of the steam, that is to say, on the effect of the engine; and this error will be almost entirely avoided by the use of our formula, since it gives in this case 2417, instead of 2427, that is to say, it differs inconsiderably from the true volume of the steam.

Let us, however, add, that in slight differences of pressure, such as take place in some engines, the error may become scarcely noticeable.

CHAPTER III.

GENERAL THEORY OF THE STEAM-ENGINE.

ARTICLE I.

OF THE EFFECT OF STEAM-ENGINES, IN THE CASE OF A GIVEN EXPANSION, WITH ANY VELOCITY OR LOAD WHATEVER.

SECT. I. *Of the different problems which present themselves in the calculation of steam-engines.*

AFTER having exposed succinctly, in the first chapter of this work, the manner in which we conceive the mode of action of the steam in steam-engines, we now proceed to the full development of the theory of which, as yet, we have given but a very imperfect sketch, and to the solution of the different problems that may occur in the working or in the construction of steam-engines.

We distinguish three cases in the working of a steam-engine; that in which it works at a given expansion of the steam, and with any load or velocity *whatever;* that in which it works at a given expansion, and with the load or velocity proper to produce its *maximum useful effect with that ex-*

pansion; and lastly, that in which the expansion having been previously regulated for the most favourable working of the steam in that engine, it is loaded, moreover, with the most advantageous load for that expansion; which consequently produces the *absolute maximum useful effect* for that engine.

We have already observed, that the three fundamental problems of the calculation of steam-engines consist in finding successively the velocity, the load, and the evaporation of the engine. After the solution of these three problems, that which first presents itself as a corollary to them, consists in determining the useful effect of the engine, which determination itself may be expressed under eight different forms, viz. by the number of pounds raised one foot by the engine in a minute; by the force of the engine in horse power; by the effect of 1 lb. of coal; by the effect of one cubic foot of water evaporated; by the number of pounds of coal or of cubic feet of evaporated water, necessary to produce one horse power; and finally, by the number of horses represented by each pound of fuel consumed or by each cubic foot of water evaporated. We have, then, to give successively the means of solving these different questions.

For the sake, however, of greater precision, the following are the problems we purpose to solve in a general manner, for each of the three cases above noticed, and for the different kinds of engines.

1. Given the load of an engine, in other respects

fully known, to determine what velocity the engine will assume with that load.

2. Knowing, on the contrary, the velocity at which it is intended to work the engine, to determine what load it can set in motion at that velocity.

3. Given the load to be moved by the engine and the velocity at which it is to move, to determine what evaporation the engine must be capable of, and consequently the dimensions requisite for the boiler, in order to produce the desired effects.

4. The evaporation, the pressure, and the dimensions of an engine being known, to calculate the useful effect it will produce in a given time, at a determined velocity or with a determined load.

To determine, from the same data, the horse power of the engine.

Having the same data, and moreover the consumption of fuel per hour, to find successively:—

The useful effect the engine will produce per pound of fuel.

The useful effect the engine will produce per cubic foot of water evaporated.

The weight of fuel that will produce one horse power.

The volume of evaporated water that will produce one horse power.

The horse power which will be produced by the consumption of one pound of fuel.

The horse power which will be produced by one cubic foot of water evaporated.

These various problems will be solved in the three cases mentioned above. Consequently in the two latter, the question will be to calculate the velocity, the load, and the effects, corresponding to the *relative* or *absolute maximum useful effect of the engine.*

In the ordinary theory of steam-engines, the solution of three questions only had ever been attempted; namely, to determine the load, the evaporation, and the useful effect (under different forms); and we have seen that their solution was defective. As for determining the velocity of the engine for a given load, no solution had ever been proposed; and the very nature of the theory employed did not permit of distinguishing in the engine, the existence of the three cases which do in reality occur. It is possible, then, that the questions we have just presented may at first appear rather obscure, expressed as they are in general terms, and inferring relations under which it is not usual to consider steam-engines; but they will be explained as we proceed, and their indispensable necessity will be felt, to calculate with accuracy either the proportions or the effects of steam engines of every kind.

SECT. II. *Of the velocity of the piston under a given load.*

In the 6th section of the preceding chapter, we have demonstrated that during all its action in the engine, the steam constantly remains at the state of maximum density for its temperature; and we have shewn that, accordingly, when the steam passes, in the engine, from a certain volume M′ to another volume M equally known, and that its pressure varies in consequence, and passes from the known pressure p' to another unknown pressure p, the pressure p may be determined by the following equation:

$$p = \frac{M'}{M}\left(\frac{n}{q} + p'\right) - \frac{n}{q} \quad \ldots\ldots \quad (c)$$

This preliminary relation once established, in order to embrace immediately the most complete mode of action of the steam, we will suppose an engine working with expansion and condensation, and with any pressure whatever in the boiler. Then, to pass afterwards to unexpansive engines, or to those without condensation, it will suffice to make the proper suppressions and substitutions in the general equations.

From what is already known of the proposed theory, the relation we seek between the various data of the problem, will be deduced from two general conditions: the former expressing that the engine has attained an uniform motion, and consequently, that the quantity of work applied by the power is equal to the quantity of action developed by the resistance; the second, that there is necessarily, equality between the mass of steam expended by the cylinder, and the mass of steam generated in the boiler.

Let P be the total pressure of the steam in the boiler, and P′ the pressure the same steam will have on arriving in the cylinder, a pressure which will always be less than P, except in a particular case, which we shall treat of shortly. The steam then will enter the cylinder at the pressure P′, and will continue to flow in with that pressure and to produce a corresponding effect, till the communication between the boiler and the cylinder is intercepted. The arrival of any new steam into the cylinder will then be stopped, but that which is already there will begin to dilate during the rest of the stroke of the piston, producing by its expansion a certain quantity of work, which will go to augment that already produced during the period of the admission of the steam.

P being, as has been said, the pressure of the steam in the boiler, and P′ the pressure it will assume on reaching the cylinder before the expansion, let π be the pressure of that steam at any point of the expansion. At the same time let l be the total length of the stroke of the piston, l' the portion traversed at the moment when the expansion begins, and λ that which corresponds to the point where the steam has acquired the pressure π. Lastly, let a be the area of the piston, and c the clearance of the cylinder, that is to say, the vacant space which exists at each end of the cylinder, beyond the portion traversed by the piston, and which necessarily fills with steam at every stroke; this space, including the adjoining passages, being represented by an equivalent length of the cylinder.

If the piston be taken at the moment when the portion of the stroke traversed is λ, and the pressure π, it will appear that if the piston traverse, moreover, an elementary space $d\lambda$, the elementary work produced in that motion will be $\pi\, a\, d\lambda$. But at the same time, the volume $a(l'+c)$, occupied by the steam before the expansion, will have become $a(\lambda+c)$. Hence, from the equation (c), indicated above, there will exist between the two corresponding pressures P′ and π, the analogy

$$\pi = \left(\frac{n}{q} + \mathrm{P}'\right)\frac{l'+c}{\lambda+c} - \frac{n}{q}.$$

Multiplying the two members of this equation by $a\, d\lambda$, we shall deduce

$$\pi\, a\, d\lambda = a(l'+c)\left(\frac{n}{q} + \mathrm{P}'\right)\frac{d\lambda}{\lambda+c} - \frac{n}{q} a\, d\lambda.$$

This expression will give then the quantity of elementary work produced by the expansion, while the piston traverses the space $d\lambda$; and if the integral be taken between the limits l' and l, we shall have the total effect produced by the expansion of the steam, from the moment of its being intercepted to the end of the stroke: viz.

$$a(l'+c)\left(\frac{n}{q} + \mathrm{P}'\right)\log\frac{l+c}{l'+c} - \frac{n}{q}\, a(l-l'),$$

an expression in which the logarithm is a hyperbolic one.

This quantity expressing the work performed in that portion of the stroke during which there was expansion, if we add to it the effect $P'al'$, produced during the anterior part l' of the stroke, or before the beginning of the expansion, we shall have for the total work developed by the steam during the whole stroke of the piston,

$$a\,(l'+c)\left(\frac{n}{q}+P'\right)\left\{\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right\}-\frac{n}{q}\,al.$$

But the engine being supposed to have attained uniform motion, the work developed by the mover must be equal to that developed by the resistance. Representing by R the total pressure exerted on the unit of surface of the piston by virtue of that resistance, or rather by virtue of the divers resistances which take place in the engine, the work it will have developed in one stroke, will have for its expression,

$$a\,R\,l.$$

We must therefore have the analogy

$$a\,(l'+c)\left(\frac{n}{q}+P'\right)\left\{\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right\}-\frac{n}{q}\,al=Ral\;.\;.\;.\;.\;(A),$$

which is the first general relation between the different data of the problem.

This equation expressing that the work developed by the power, is entirely found in the effect produced, it will be remarked that, for the analogy to take place, it is not necessary that the motion of the engine be strictly uniform. It may be composed of equal oscillations, beginning from zero of velocity, and returning to zero again; provided the successive oscillations be made in equal times, and that the changes of velocity take place by insensible degrees, so as to suffer no loss of *vis viva.*

It must be observed also, that, if in this expression we make $l'=l$, which amounts to supposing that the engine works without expansion, the equation reduces itself to $P'=R$; that is to say, the pressure of the steam in the cylinder will, in this case, be equal to the pressure of the resistance against the piston, as we have already demonstrated directly for unexpansive engines, of which we spoke in the first chapter.

We have just obtained the first general relation between the data and the incognita of the problem. Let us now seek a second analogy resulting from the equality between the production and the expenditure of the steam. If S be made to express the volume of water evaporated by the boiler in a unit of time, and transmitted to the cylinder, this volume on reaching the cylinder, transformed into steam at the pressure P′, will there become, from the relation already given (a),

$$\frac{S}{n+q\,P'}.$$

This will then be the volume of steam, at the pressure P′, supplied by the boiler in a unit of time, in one minute for instance. On the other hand, $a(l'+c)$ being the volume of the steam expended at each stroke of the piston, if there be K strokes per minute, the expense per minute will be

$$Ka\,(l'+c).$$

But expressing by v the velocity of the piston per minute, we shall have also $v = \mathrm{K}\, l$; which gives $\mathrm{K} = \frac{v}{l}$. Whence the above expenditure will be

$$\frac{v\, a\,(l' + c)}{l}.$$

Since, then, there is an equality between the production and the expenditure of the steam, we shall have the equation

$$\frac{\mathrm{S}}{n + q\,\mathrm{P}'} = v\, a \frac{l' + c}{l}, \quad . \quad . \quad . \quad (\mathrm{B})$$

which is the second general relation between the data and the incognita of the problem.

Consequently, on eliminating P′ from the two equations (A) and (B), we shall have as the final relation sought,

$$v = \frac{\mathrm{S}}{a} \cdot \frac{1}{n + q\mathrm{R}} \left\{ \frac{l'}{l' + c} + \log \frac{l + c}{l' + c} \right\} \quad . \quad . \quad . \quad (1)$$

In this equation the logarithm $\log \frac{l + c}{l' + c}$ is a hyperbolic logarithm. As it is known that these logarithms are deduced from those of the tables, by multiplying the latter by the constant number 2·302585, or approximatively by 2·303, the term $\log \frac{l + c}{l' + c}$ might, for practical purposes, be replaced by $2{\cdot}303 \log \frac{l + c}{l' + c}$, in which log. would then express an ordinary logarithm. But as tables of hyperbolic logarithms are found in several works, and as besides, we shall give in the sequel, a table which will dispense from all research on this head, we will not here make any change in the formulæ.

This equation is less simple than that which would be obtained in the same enquiry, by supposing the steam to preserve its temperature through the whole of its action in the engine; but that supposition, though producing often but slight differences in the definitive results of the calculations, is not really exact, since it is incontestable that the steam changes its pressure during the expansion, and that the experiments quoted above prove that it changes temperature in a manner exactly correspondent. The last formula which we have presented, has then the advantage of taking this important circumstance into account, and consequently of being more accurate in the applications. Besides, if in equation (1) the effect of the change of temperature be annulled, the formula becomes the same that we have presented in the first chapter, supposing the preservation of the temperature of the steam.

In effect, we have seen, from equation (a), that after the steam has assumed in the engine the pressure R, the *absolute* volume of that steam, which corresponds to the volume of water S, is given by the relation

$$\frac{S}{n+qR}.$$

On the contrary, when the steam is supposed to preserve its temperature, the volume varies in the inverse ratio of the pressure. If, then, we call

m the relative volume of the steam generated at the pressure P of the boiler, a relative volume which can be known by the tables already given, it is clear that the absolute volume of the steam correspondent to the volume S of water, will first be, under the pressure P, expressed by

$$m\,S;$$

and that, in passing afterwards to the pressure R, this volume will change in the inverse ratio of the pressures, that is to say, will become

$$m\,S\frac{P}{R}.$$

Therefore, to pass from one law to the other, we must write

$$\frac{S}{n+qR}=m\,S\frac{P}{R};$$

or, what comes to the same, we must, in the formulæ already obtained, make

$$n=o, \text{ and } \frac{1}{q}=m\,P.$$

Then the equation which gives the velocity, becomes

$$v=\frac{m\,P\,S}{a\,R}\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right);$$

which, for the case of unexpansive engines, or for $l'=l$, reduces itself to the following:

$$v=\frac{m\,P\,S}{a\,R}\cdot\frac{l}{l+c}.$$

And this is precisely the equation we made use of in the first chapter, if only we neglect in it the clearance of the cylinder c.

The quantity R contained in equation (1), is the total resisting pressure which takes place on the unit of surface of the piston in the motion. But this resisting pressure is evidently composed of three parts, namely, the resistance arising from the motion of the load, which we will call r; that arising from the friction proper to the engine, which we will express by $(f+\delta r)$, calling f the friction of the engine unloaded, and δ the augmentation of that friction per unit of the load r; and finally the pressure which may subsist on the face of the piston opposed to the arrival of the steam, which we will represent by p; the latter quantity p expressing the atmospheric pressure, when the engine is without condensation, or only the pressure of condensation in the cylinder, when the engine is a condensing one. The quantities r, f, p and δ, are besides, as well as R, referred to the unit of surface of the piston.

In the calculations relative to locomotive engines we shall introduce three terms more: the first to express the resistance of the air against the train in motion, a force which, increasing in the ratio of the square of the velocity, could not be neglected without error; the second to represent the resistance offered by the engine itself in the transport of its own weight on the rails; and the third, to take account of the force expended by the engine in animating its fire, according to the method in use in those engines. But as these divers circumstances do not in general occur in stationary engines, we will omit them at present, it being easy to reproduce them in the particular cases, as it may become necessary.

From what has just been said, the resistance R may be replaced by

$$R=(1+\delta)\,r+p+f.$$

We shall then substitute this value in that of v, and at the same time make

$$\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}=k;$$

an expression, which in the case of $l'=l$, that is to say, for unexpansive engines, reduces itself simply to the ratio

$$\frac{l}{l+c}.$$

Then, the value of v will become

$$v=\frac{S}{a}\cdot\frac{k}{n+qR},$$

or

$$v=\frac{S}{a}\cdot\frac{k}{n+q\{(1+\delta)\,r+p+f\}}\;\cdot\;\cdot\;\cdot\;\cdot\;(1)$$

It will be remarked that the quantity

$$\frac{S}{n+qR}$$

is nothing else but the *absolute* volume of the steam correspondent to S, in contact with the liquid at the pressure R. Therefore, to have the velocity v, we must calculate the volume of the steam which corresponds to the volume of water S, supposed immediately transformed into steam at a pressure equal to the resistance R, afterwards divide that volume by the area a of the piston, and lastly, multiply the quotient by the quantity k, of which we have a little before given the developed expression.

The formula (1) contains the general relation between all the data of the problem, and will serve us to solve successively the different questions we have proposed elucidating. It will, however, be observed that the homogeneity of the formula requires that the dimensions of the engine a, l and l' be expressed in the same unit as the volume of water evaporated S, and that the pressures per unit of surface P, r, and p, be also referred to the same unit as S. We mention this circumstance because these various quantities are usually referred to different units, according to what may be, in practice, the most convenient manner of expressing each.

Besides, from the mode of our reasoning itself, it is to be understood that the quantity S, in the equation, is the *effective* evaporation of the engine; that is, it represents the volume of water which really enters the cylinder in the state of steam, and there acts upon the piston. If then, from any mode of construction of the engine, it should occur that a portion of the steam generated in the boiler, escape without acting on the piston, that portion is not to be considered as included in the quantity S, and ought, therefore, to be deducted before all calculation.

The formula just obtained will give the velocity of the piston for any load r, when the dimensions and different data of the engine contained in the equation are known. This formula is general, and applies to every kind of rotative steam engine. If the engine be expansive, it will suffice to replace l' by the length of the stroke traversed when the steam begins to be intercepted; if the engine be unexpansive, it will suffice to make $l' = l$. If there be condensation, p must be replaced by the pressure of condensation; and if the engine be not a condensing one, p is to be replaced by the atmospheric pressure. However, before making these deductions relative to the different systems of engines, we shall continue to seek the general formulæ for all the problems we have undertaken to solve.

Let it only be observed, that the velocity of the piston in a given engine, is totally independent of the pressure at which the steam is formed in the boiler, and that, on the contrary, it depends essentially on the evaporation S of the boiler per unit of time, and on the total resistance $[(1+\delta)r+p+f]$ opposed to the motion of the piston.

Sect. III. *Of the load of the engine, for a given velocity.*

The analogy we have just obtained will show reciprocally the resistance a known engine can set in motion at a determined velocity. In effect, it suffices to draw from it the value of r; or rather, as

r is only the resistance per unit of surface of the piston, it will be preferable to have the whole resistance, by taking immediately the value of $a \times r$, that is,

$$ar = \frac{Sk}{(1+\delta)qv} - \frac{a}{1+\delta}\left(\frac{n}{q} + p + f\right) \quad . \; . \; . \; . \; (2).$$

From the form of this expression, it would appear at a first glance, that on making $v=0$, that is, on supposing the velocity null, the result would be an infinite load; but, on examining the formula more attentively, we soon perceive that the result would by no means be such.

In effect, if $v=0$, it follows also that $S=0$; for S is the quantity of steam which effectively traverses the cylinders in a unit of time; and no quantity of steam whatever can traverse the cylinders without moving the piston, and consequently creating some velocity in the engine. If, then, the velocity be supposed equal to zero, we must necessarily have at the same time $S=0$. But, making at once $v=0$ and $S=0$, we find

$$ar = \frac{0}{0},$$

and not $ar = \infty$, as it at first appeared.

Thus, in this case, the formula reduces itself to the indeterminate form; but it is to be observed, that the present formulæ give the effects of the engine, only after the uniform motion has taken place. Now we shall presently see that, for a given evaporation S, the uniform velocity can never be less than

$$v' = \frac{mS}{a} \cdot \frac{l}{l' + c},$$

since it is that which corresponds to the passage of the steam into the cylinders, at its state of greatest density, and that at any other density, that steam would form a larger volume, and consequently could not traverse the cylinders in the same time, without producing a greater velocity. All supposition of less velocity than this, is then inadmissible in this problem, as being incompatible with that state of uniformity of motion, for which alone the effects of machines are calculated.

SECT. IV. *Of the evaporation of the boiler, to produce wanted effects.*

To find the evaporation of which an engine ought to be capable, in order to set in motion a certain resistance r at a known velocity v, the value of S must be drawn from the same equation,

$$S = av \frac{n + q \{(1 + \delta) r + p + f\}}{k} \quad . \; . \; (3)$$

This equation gives the quantity of water the engine ought to be capable of evaporating and transmitting to the cylinder per minute. It will then be easy, according to the mode of construction intended to be used for the boiler, and the practical data proper to estimate the quantity of water evaporated by such form of boiler, to know what extent of heating surface should be given to the boiler of the engine, in order to obtain the proposed effects.

As the quantity S represents here the effective evaporation disposable by the engine, it is understood that, if the usual construction of the engines under consideration give rise to a certain loss of steam, either by safety-valves or otherwise, account of this must be taken, with as close an approximation as possible, by first adding that loss to the quantity S deduced from the preceding equation, then by estimating the heating surface suitable to the production of the useful steam augmented by the lost steam.

SECT. V. *Of the different expressions of the useful effect of the engines.*

1. The useful effect produced by the engine in the unit of time at the velocity v, is evidently $a r v$, since the velocity v is at the same time the space traversed by the piston in a unit of time. Consequently, by multiplying both members of equation (2) by v, we shall have the useful effect:

$$\text{u. E.} = a r v = \frac{Sk}{(1+\delta)q} - \frac{a v}{1+\delta}\left(\frac{n}{q} + p + f\right) \ldots (4)$$

This may be expressed in terms of the load, by multiplying the two members of equation (1) by $a r$. We have then for the useful effect the engine may produce with a given load,

$$\text{u. E.} = arv = \frac{S r k}{n + q\,\{(1+\delta)r + p + f\}} \ldots\ldots (4 \text{ bis})$$

It will be remarked, that in a given engine this useful effect does not depend on the pressure at which the steam is generated in the boiler, since the quantity P does not appear in the above equations; but that it depends essentially on the evaporation S effected by the boiler in a unit of time.

2. If it be required to know the horse-power which represents the effect of the engine, when working at the velocity v, or when loaded with the resistance r, it suffices to observe that what is called one-horse power represents an effect of 33,000 lbs. raised one foot per minute. All consists then in referring the useful effect produced by the engine in the unit of time, to the new measure just chosen, that is, to the power of one horse; and consequently it will suffice to divide the expression already obtained in equation (4) by 33,000.

Thus the horse-power of the engine, at the velocity v, or with the resistance r, will be

$$\text{u. HP.} = \frac{\text{u. E.}}{33000} \quad \ldots . (5)$$

We will here observe, that what is designated by *horse-power* would, with much more propriety, be termed *horse-effect*, since it is an effect and not a force. It should then be said, that an engine is of so many horse-effect, instead of saying that it is of so many horse-power.

3. In the two preceding questions, we have expressed the power of the engine from the total effect it is capable of developing, without regard to

its consumption of fuel or water. We are now about to express the same, either from the effect it produces per unit of fuel or of water expended; or from its consumption while performing a given work.

The useful effect obtained in equation (4), is that which is produced by the volume S of water transformed into steam; and as that volume of water S is evaporated in a unit of time, the result is, as has been said, the useful effect produced by the engine in a unit of time. But if it be supposed that during the unit of time, there be consumed N pounds of fuel, the useful effect produced by each pound of fuel will plainly be the N^{th} part of the above effect.

Hence the effect arising from the consumption of 1 lb. of fuel will be

$$\text{u. E. 1 lb. co.} = \frac{\text{u. E.}}{\text{N}} \dots\dots (6)$$

To apply this formula, it suffices to know the quantity of fuel consumed in the furnace per minute, that is to say, while the evaporation S is taking place. This datum may be determined by a direct experiment on the boiler itself, or by analogy with other boilers similarly disposed. And the datum once obtained, may be used for every other case, and for every supposition of velocity of the engine.

4. We have seen above, that the effect indicated by u. E. is that which is due to the volume of water S transformed into steam. If, then, it be

required to know the useful effect arising from each cubic foot of water, or from each unit of the volume S, it will obviously suffice to divide the total effect u. E. by the number of units in S. Thus, for the useful effect due to the evaporation of one cubic foot of water in the engine, we have

$$\text{u. E. 1 ft. wa.} = \frac{\text{u. E.}}{\text{S}} \quad . \; . \; . \; . \quad (7)$$

5. We have obtained above the useful effect produced by one pound of fuel. It consequently becomes easy to know the number of pounds of fuel which represent any given useful effect, as, for instance, one horse-power. A simple proportion is, in fact, enough, and we have for the quantity, in weight, of fuel requisite to produce one horse-power,

$$\text{Q. co. for 1 hp.} = \frac{33000 \text{ N}}{\text{u. E.}} \quad . \; . \; . \; . \quad (8)$$

6. By a simple proportion will also be found the quantity of water that must be evaporated, in order to produce one horse-power, viz.:

$$\text{Q. wa. for 1 hp.} = \frac{33000 \text{ S}}{\text{u. E.}} \quad . \; . \; . \; . \quad (9)$$

7. It may yet be required to know what horse-power will be produced by a pound of coal; which will evidently be

$$\text{u. HP. for 1 lb. co.} = \frac{\text{u. E.}}{33000 \text{N}} \quad . \; . \; . \; . \quad (10)$$

8. Finally, the horse-power produced by the

evaporation of 1 cubic foot of water will likewise be

$$\text{u. HP. for 1 ft. wa.} = \frac{\text{u. E.}}{33000\,\text{S}} \quad . \quad . \quad . \quad . \quad (11)$$

Substituting, then, in these several equations, for u. E. its value determined by the formula, (4,) we immediately deduce the numerical solution of the proposed problems.

Sect. VI. *Table for the numerical solution of the formulæ (rotative engines).*

As the formulæ we have just obtained, and those which are about to follow, contain hyperbolic logarithms, the use of which is inconvenient, we here subjoin a table which gives, without calculation, the principal elements of the equations, and will greatly simplify the matter.

In this table we have supposed the clearance of the cylinder $c = \cdot 05\,l$, as is the case in rotative steam-engines, of which we are now treating. In single-acting engines the clearance of the cylinder, including the adjoining passages, amounts to ·1 of the stroke, because the motion of the piston not being limited by a crank, it is more liable to strike the bottom of the cylinder.

We have not inserted in the table a column to represent the fraction

$$\frac{l' + c}{l}$$

because it is evident that $\frac{l'}{l}$ being known by the first column, the fraction

$$\frac{l'+c}{l}$$

will be equal to the former augmented by $\frac{c}{l}$, that is, by ·05.

Table for the numerical solution of the formulæ (rotative engines).

Portion of stroke performed before the expansion, or value of the fraction $\frac{l'}{l}$	Corresponding value of the fraction $\frac{l}{l'+c}$	Corresponding value of k, or of the expression $\frac{l'}{l'+c}+\log.\frac{l+c}{l'+c}$	Portion of stroke performed before the expansion, or value of the fraction $\frac{l'}{l}$	Corresponding value of the fraction $\frac{l}{l'+c}$	Corresponding value of k, or of the expression $\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}$
·10	6·667	2·613	·51	1·786	1·539
·11	6·250	2·569	·52	1·754	1·523
·12	5·882	2·526	·53	1·724	1·507
·13	5·556	2·485	·54	1·695	1·491
·14	5·263	2·446	·55	1·667	1·476
·15	5·000	2·408	·56	1·639	1·461
·16	4·762	2·371	·57	1·613	1·445
·17	4·546	2·336	·58	1·587	1·431
·18	4·348	2·301	·59	1·563	1·417
·19	4·167	2·268	·60	1·539	1·402
·20	4·000	2·235	·61	1·515	1·388
·21	3·846	2·203	·62	1·493	1·374
·22	3·704	2·173	·63	1·471	1·361
·23	3·571	2·142	·64	1·449	1·347
·24	3·448	2·114	·65	1·429	1·334
·25	3·333	2·085	·66	1·409	1·321
·26	3·226	2·059	·67	1·389	1·308
·27	3·125	2·032	·68	1·370	1·295
·28	3·030	2·006	·69	1·351	1·282
·29	2·941	1·980	·70	1·333	1·269
·30	2·857	1·955	·71	1·316	1·257
·31	2·778	1·931	·72	1·299	1·240
·32	2·703	1·908	·73	1·282	1·233
·33	2·632	1·884	·74	1·266	1·221
·34	2·564	1·862	·75	1·250	1·210
·35	2·500	1·840	·76	1·235	1·197
·36	2·439	1·818	·77	1·220	1·186
·37	2·381	1·797	·78	1·205	1·175
·38	2·326	1·776	·79	1·191	1·164
·39	2·273	1·755	·80	1·177	1·152
·40	2·222	1·736	·81	1·163	1·141
·41	2·174	1·716	·82	1·149	1·131
·42	2·128	1·697	·83	1·136	1·119
·43	2·083	1·678	·84	1·123	1·109
·44	2·041	1·660	·85	1·111	1·099
·45	2·000	1·642	·86	1·099	1·088
·46	1·961	1·624	·87	1·087	1·078
·47	1·923	1·606	·88	1·075	1·067
·48	1·887	1·589	·89	1·064	1·057
·49	1·852	1·572	·90	1·053	1·047
·50	1·818	1·555			

We limit ourselves to the preceding problems, because they are those which are most commonly wanted; but it is obvious that, by means of the same general analogies, any one of the quantities which appear in the problem may be determined, in case that quantity should be unknown, and that it were desired to determine it according to a given condition. Thus, for instance, might be determined the area of the piston, or the pressure in the boiler, or the pressure of condensation, &c., corresponding to given effects of the engine, as we have done for locomotive engines, in a preceding work (Treatise on Locomotives). But as these questions rarely occur, and as they offer no difficulty, we deem it sufficient to indicate here the manner of obtaining their solution.

ARTICLE II.

OF THE MAXIMUM OF USEFUL EFFECT WITH A GIVEN EXPANSION.

Sect. I. *Of the velocity of maximum useful effect.*

The preceding problems have been solved in the most general way, that is to say, supposing the engine to set in motion any load whatever at any velocity whatever, with the single condition that the load and velocity be compatible with the power of the engine. In constructing an engine for a determinate object, or to move a certain load with a given velocity, it must not be planned in such manner as to require the greatest effort of which it is capable, to perform that task which is to be its regular work; for in that case, it would have no power in reserve, to meet whatever emergencies may occur in the service. On the other hand, since the maximum effort of the engine with a given expansion, corresponds, as we shall presently see, to its maximum useful effect, it follows that we are not to expect regularly from the engine its maximum of useful effect, nor can the engine be constructed with such pre-intention. It is necessary however, when an engine is constructed, or to be constructed, to know what is the velocity at which it will produce its maximum useful effect, and what this maximum useful effect will be; for it is evidently that knowledge which must decide the regular working load of the engine, and mark the possible limits of its effects in case of emergency.

What is that velocity or that load, most advantageous for the work, and what are the divers effects which will then be produced by the engine? This is what now remains to determine, first, in supposing the expansion of the engine fixed *à priori*, then in making that expansion itself to vary, in order to obtain a further increase of effect.

To know the velocity corresponding to the greatest useful effect, it suffices to examine the expression of the useful effect produced by the engine under any velocity whatever, namely, (equa. 4):

$$\text{u. E.} = \frac{S k}{(1+\delta) q} - \frac{a v}{1+\delta}\left(\frac{n}{q} + p + f\right).$$

It is observable here, at the first glance, that since the velocity enters only into the negative terms, the less that velocity is, *for a given expansion*, the greater will be the useful effect of the engine. On the other hand, referring to the expression of the velocity of the engine under a given load, before having substituted for P′ its numerical value, viz. (equa. B):

$$v = \frac{S}{a(n + q P')} \cdot \frac{l}{l' + c},$$

we perceive that the velocity is the smallest possible, without loss of steam, when P′ is greatest; and as P′, which is the pressure of the steam in the cylinder, can in no case exceed P, which is the pressure in the boiler, the condition of the minimum velocity, or of the maximum useful effect, will be given by the equation P′ = P, or

$$v' = \frac{S}{a(n + q P)} \cdot \frac{l}{l' + c} \quad . \ . \ . \ . \ (12)$$

Expressing by *m* the volume of the steam under the pressure P, referred to the volume of the same weight of water, this formula may, from equation (*a*) take the form

$$v' = \frac{m S}{a} \cdot \frac{l}{l' + c} \quad . \ . \ . \ . \ (12 \text{ bis})$$

In this manner the calculation of the term $(n + q\,\mathrm{P})$ is avoided, since the quantity m is given by the tables of Chapter II., and may thence be taken with greater accuracy than from its approximative value

$$m = \frac{1}{n + q\,\mathrm{P}}.$$

This observation will equally apply to all the following formulæ, wherein the quantities n and q recur united under the form $(n + q\,\mathrm{P})$.

It is to be remarked, with respect to the preceding formula, that, mathematically speaking, the pressure P' can never be quite equal to P. In fact, since there exist pipes between the boiler and the cylinder, through which the steam must pass, and that the passages of those pipes form an obstacle to the free motion of the steam, there must necessarily be, on the side of the boiler, a small surplus of pressure equivalent to the resistance of the obstacle in question; otherwise the motion of the steam could not take place. This surplus of pressure, then, on the side of the boiler, prevents P' from becoming mathematically equal to P, and thus the real velocity will always be rather greater than v'. The difference between P' and P (we mean the difference merely arising from the obstacle just mentioned) will be by so much the less, as the area of the passages is larger and their way more direct; but as, with the dimensions of ordinary use in steam-engines, that difference is very trifling, we shall not notice it here. Seeking it, in fact, by known formulæ for the flowing of gases, we find that it is hardly appreciable by the instruments used for measuring the pressure in the boiler; consequently, to introduce them into the calculation would only complicate the formulæ, without rendering them more exact.

To return to the enquiry before us, the maximum useful effect will be given by the condition $P' = P$, or

$$v' = \frac{S}{a(n + qP)} \cdot \frac{l}{l' + c}.$$

This is, then, the velocity at which the engine must work, in order to obtain the greatest effect possible; and the equation $P' = P$ shows reciprocally, that, when that velocity takes place, the steam enters the cylinder at full pressure, that is, nearly at the same pressure which it had when in the boiler.

It is necessary to remark, that this velocity of maximum useful effect, or of full pressure in the cylinder, will not be the same in all engines, but that on the contrary, *cæteris paribus*, it will vary in a direct ratio with the evaporation S of the boiler, and in inverse ratio with the area of the cylinder. It may then be found, in one engine, the half or the double of what it would be in another; which shews how erroneous is the belief that, because the velocity of the piston of stationary engines does not in general exceed a certain velocity of 150 to 300 feet per minute, the steam of the boiler necessarily reaches the cylinder without changing its pressure. If it be supposed that, in a certain engine, the maximum of effect be produced at the velocity of 200 feet per minute, it will be easy to plan another, in which the same effect shall take place at a very different velocity. To such end, it would evidently suffice to retain the same boiler, and to diminish or augment the diameter of the cylinder; or else, on the contrary, to preserve the dimensions of the cylinder, and to change those of the boiler. Nay more; it will suffice to let the fire go down, which will diminish the evaporation, and the velocity of full pressure will vary at the same time. It cannot, then, be decided *à priori*, as it is in the theory usually applied to steam-engines, that the velocity of 150 to 250 feet per minute is, for all engines, the velocity of full pressure.

The fact is, that there are no other means of knowing the velocity of maximum useful effect or of full pressure for an engine, than by calculating it directly for that engine, which is the object of the formula we have given above. That formula, moreover, is of remarkable simplicity, and requires no other experimental knowledge than that of the evaporation of which the boiler is capable. As to that evaporation, it may be determined by a special experiment, or deduced from the heating surface of the boiler and the quality of the fuel, taking as a basis some special experiment on the subject, made, not on the boiler of the engine itself, but on some other boiler of similar construction. Thus the velocity of the maximum useful effect of an engine may always be calculated *à priori.*

SECT. II. *Of the load of maximum useful effect.*

To know what useful resistance an engine is capable of setting in motion at the above velocity of maximum useful effect, it suffices to introduce for v, in the general expression of the resistance, (equation 2,) the value we have just found; and we obtain for the corresponding load r',

$$ar' = \frac{a}{1+\delta} \cdot \frac{l'+c}{l} k\left(\frac{n}{q} + \mathrm{P}\right) - \frac{a}{1+\delta}\left(\frac{n}{q} + p + f\right) \quad (13)$$

Examining the same equation (2), which gives the resistance in the general case, we perceive that resistance to be the greater as the velocity of the motion is less. The resistance which we have just obtained, as corresponding to the velocity of maximum useful effect, or to the minimum velocity of the engine without loss of steam, is then, at the same time, the greatest resistance of which the engine is capable with the given expansion. Thus, the greatest useful effect will be obtained by making the engine work at its smallest velocity and with its maximum load, which moreover is evident *à priori.*

This load suitable to the production of the greatest useful effect, may also be found directly. It suffices for that, to consider the expression of the useful effect in terms of the load, that is, to examine the equation (4 bis). There we see that the maximum of effect, for a given expansion, corresponds to the maximum of the fraction

$$\frac{\mathrm{S}rk}{n+q\left[(1+\delta)\,r+p+f\right]},$$

in which r alone is variable; and whose maximum will evidently be given by the maximum of the resistance r. We are therefore led to the same solution as above.

SECT. III. *Mode of determining the friction of unloaded engines, and the additional friction per unit of the load; derived from the preceding enquiry.*

The analogy which we have obtained above, may serve to determine the friction of the engine working without load, and its additional friction per unit of the load. It will, in fact, be remarked that, since there is a maximum load corresponding to every pressure in the boiler, every load whatever may be made a maximum load for the engine, by lowering sufficiently the pressure. Suppose, then, that no load be imposed on the engine, but that the pressure in the boiler be lowered, by means of the safety-valve, till the engine can do no more than just overcome its own friction and keep itself in motion. With this reduced pressure, the friction of the engine alone becomes a maximum load for it. Let P″ be that pressure, determined by experiment; the preceding equation, then, will be true in changing P for P″ and making $r=0$. Therefore we shall have

$$f=\left(\frac{n}{q}+P''\right)\frac{l'+c}{l}k-\frac{n}{q}-p;$$

and this equation will determine the quantity f, or the friction of the engine without load.

To obtain afterwards the quantity δ, resort will be had to a similar means. Without making any change in the ordinary pressure of the engine, the load must be augmented more and more, till it appear that the engine would stop if any further resistance were imposed on it. Then we shall have obtained the maximum load corresponding to the pressure of the boiler. If, therefore, the load thus determined by experiment be denoted by r'', P being still the pressure of the boiler, equation (13) will give

$$1+\delta=\frac{1}{r''}\cdot\frac{l'+c}{l}\left(\frac{n}{q}+P\right)k-\frac{1}{r''}\left(\frac{n}{q}+p+f\right);$$

which determines the quantity δ, or the additional friction accruing to the engine per unit of the load r imposed on it.

If the load of the engine be not capable of augmentation, instead of increasing the load as has just been said, the pressure in the boiler must be lowered so as to bring it in equilibrio with the ordinary load of the engine; and the value of P thus found, must be introduced into the equation, as in the case in which the engine was made to work without a load.

The quantities f and δ being thus determined, the friction of the engine, with any given load r, will be

$$F = f + \delta r.$$

These are the means we used to determine the friction of the locomotive engines, either isolated or followed by their trains, and we propose them equally with respect to steam-engines of every kind.

It is to be remarked that if, having omitted previously to certify that the engine is working with its maximum load, we should happen erroneously to take a case of general velocity for a case of minimum velocity, and pretend to deduce therefrom the friction of the engine, the value thus found would necessarily be too great, and by so much the more exaggerated, as the load with which the engine is working may be more remote from the maximum. In effect, the effort of which the engine is capable diminishing as its velocity increases, it is plain that, in the above calculation, there would be introduced for r'' a quantity smaller than it ought to be; whence results that the quantity δ, and consequently the definitive friction of the engine, would appear too great. This explains how the ordinary theory, in comparing its theoretical results to those of experiment, is led to coefficients of reduction, which make the friction of the engine appear much more considerable than it really is.

SECT. IV. *Of the evaporation of the engine.*

The evaporation necessary to an engine, in order to exert a certain maximum effort r' at the minimum velocity v', will be given by equation (3), on substituting in it r' and v' instead of the general values r and v; or more simply, it may be derived immediately from equation (12), which amounts to the same, and gives

$$S=(n+q\,P)\,a\,v'.\frac{l'+c}{l} \quad \ldots\ldots \quad (14)$$

It will be observed that on substituting r' and v' for r and v in equation (3), we should have

$$S=a\,v'\frac{n+q\{(1+\delta)r'+p+f\}}{k};$$

but on introducing for r' its value given in equation (13), we fall into the preceding formula (14).

SECT. V. *Of the maximum useful effect of the engine.*

The maximum of useful effect the engine can produce in a unit of time, with a given expansion, will be known by the general formula (4), on introducing into it the velocity proper to produce that effect; or it may be deduced from the product of the two equations (12), (13), viz.

$$\text{max. u.E.}=ar'v'=\frac{S}{(1+\delta)\,q}\left\{k-\frac{l}{l'+c}\cdot\frac{n+q(p+f)}{n+q\,P}\right\} \quad \ldots\ldots \quad (15)$$

This maximum useful effect, it will be observed, in nowise depends either on the area of the cylinder, or on the velocity, that is the number of strokes of the piston per minute. If we suppose an engine working without expansion, which amounts to making $l' = l$ and reduces the expression k to the value $\frac{l}{l+c}$, we find even that the maximum useful effect does not depend on the length of stroke of the piston. In effect the quantity $\frac{l}{l+c}$, which still subsists in the equation, expresses merely a ratio, that of the volume traversed by the piston to the whole volume of the cylinder. On the other hand, the quantities f and δ are constant in a given engine, and vary but little in engines of the same system; the quantities n and q are constant coefficients, as has been seen; in fine, the pressure of condensation p depends on the mode of condensation used, and especially on the quantity and temperature of the water applied to produce the condensation; consequently it is a constant quantity under given conditions. We may then say, that the maximum useful effect of an engine depends essentially on but two things: the evaporating power S of the boiler, and the pressure P at which the steam is generated. This result, indeed, must appear evident *à priori*, for they are the only real causes of power. As to the dimensions of the cylinder, and of the stroke, they are no more than means of transmitting that power under one form or another, but are utterly unable to create it, or fundamentally to change it; and as to the velocity of the piston, that cannot in any way influence the maximum useful effect, since, for a given production of steam, that velocity is susceptible of all values according to the diameter given to the cylinder.

Here we see what error we commit, when we pretend to calculate the useful effect of engines, from the diameter of the cylinder, taking no account of the evaporation produced, which not only enters not into the calculation, but forms no part even of the observations.

The horse-power of the engine when exerting its greatest effort, or when it produces its greatest useful effect, will be as before,

$$\text{m. u. HP.} = \frac{\text{max. u. E.}}{33000} \quad \ldots\ldots (16)$$

The other modifications already given of the expression of the useful effect, will also be as follows:—

(17) $\text{m. u. E. 1 lb. co.} = \frac{\text{max. u. E.}}{N}$ .... Maximum useful effect of 1 lb. of fuel.

(18) $\text{m. u. E. 1 ft. wa.} = \frac{\text{max. u. E.}}{S}$ .... Maximum useful effect of 1 cubic foot of water evaporated.

(19) $\text{mi. Q. co. for 1 hp.} = \frac{33000\ N}{\text{max. u. E.}}$ .... Minimum quantity of fuel consumed per horse-power.

(20) mi. Q. wa. for 1 hp. $=\dfrac{33000\ S}{\text{max. u.E.}}$ .... Minimum quantity of water evaporated per horse-power.

(21) m.u. HP. 1 lb. co. $=\dfrac{\text{max. u. E.}}{33000\ N}$ .... Maximum horse-power produced by 1 lb. of fuel.

(22) m. u. HP. 1 ft. wa. $=\dfrac{\text{max. u. E.}}{33000\ S}$ .... Maximum horse-power produced by 1 cubic foot of water evaporated.

ARTICLE III.

OF THE ABSOLUTE MAXIMUM OF USEFUL EFFECT.

The preceding enquiries contain all that is necessary for engines in which the expansion is fixed *à priori*. They suffice also for unexpansive engines, because these are naturally included in the case of engines with a fixed expansion; and we have only to make in the formula $l'=l$, which gives at the same time $k=\dfrac{l}{l+c}$. But there still remains a problem to occupy our attention, for engines in which the expansion is susceptible of variation.

It has been shewn that, for a given expansion, the most advantageous mode of working the engine is to give it its maximum load, which we have made known above, and which may be calculated *à priori* by equation (13). Hence may be known what load to prefer for every expansion. But the question is now to determine, among the divers degrees of expansion that may be given to the engine, each accompanied by its maximum corresponding load, which of them will produce the greatest definitive useful effect.

To this end, recourse must be had to equation (15), which gives the maximum useful effect of the engine with the expansion l'; viz., in putting for k its value,

$$\text{max. u. E.} = \frac{S}{(1+\delta)q}\left\{\frac{l'}{l'+c} + \log\frac{l+c}{l'+c} - \frac{l}{l'+c}\cdot\frac{n+q(p+(f))}{n+q\,P}\right\};$$

and we must seek, among the values that may be given to l', that which will render this useful effect a maximum. Now, making equal to zero the differential coefficient of this equation, taken with reference to l', we find for the condition of the maximum sought

$$\frac{l'}{l} = \frac{\frac{n}{q} + p + f}{\frac{n}{q} + P} \quad \ldots . (34)$$

This equation may be written as follows:

$$\frac{l'}{l} = \frac{\dfrac{1}{n+q\,P}}{\dfrac{1}{n+q\,(p+f}};$$

and, under this form, it is easy to perceive that the second part is the ratio of the relative volumes of the steam generated under the respective pressures P and $(p+f)$.

Therefore, to find the value of the ratio $\frac{l'}{l}$, which corresponds to the production of the absolute maximum of useful effect, the relative volume of the steam generated under the pressure P of the boiler must be sought first, either by formula (a), or by the table given in Chapter II.; then, afterwards, the relative volume of the steam supposed immediately generated under a pressure indicated by the sum $(p+f)$, must be sought in the same manner; and the first volume divided by the second will give the wanted value of $\frac{l'}{l}$, that is the ratio which ought to exist between the portion of the stroke traversed before the beginning of the expansion, and the total stroke of the piston.

We also see that, in supposing $n=0$, this relation reduces itself to

$$\frac{l'}{l}=\frac{p+f}{\mathrm{P}}.$$

If, then, in the calculation of the effects of the engine, we neglected the change of temperature of the steam, the value of $\frac{l'}{l}$, proper to the production of the absolute maximum of useful effect, would be equal to the ratio between the two quanties $(p+f)$ and P. We must consequently consider this last equation as an approximation to the preceding one.

Introducing the value of $\frac{l'}{l}$ given by equation (34), in the formulæ of Article II., we shall have all the determinations relative to the maximum useful effect produceable by the engine with that expansion; and since that expansion is the most favourable for the engine, it follows that those determinations will correspond to the *absolute* maximum of useful effect of which that engine is capable.

The formulæ in which the value of $\frac{l'}{l}$ thus found is to be substituted, are those already given, and in the calculation of which, account is taken of the change of temperature of the steam. Since, however, the supposition of the preservation of the temperature of the steam, that is to say, the hypothesis of $n = 0$, $q = \frac{1}{m\text{P}}$, in which m represents the relative volume of the steam at the pressure P, greatly simplifies the equations, and gives an approximation near enough for a great number of cases in which the expansion is not carried very far, we present here the corresponding results of all the formulæ. They make known very nearly to the truth, the absolute maximum of useful effect that it is possible to obtain from a known engine, taking at the same time the most advantageous expansion and the most advantageous load.

(23) ...... $v'' = \frac{m\text{S}}{a} \cdot \frac{l\text{P}}{l(p+f) + \text{P}c}$ .... Velocity of the *absolute* maximum of useful effect.

(24) . . $ar'' = \frac{a}{1+\delta} \cdot \frac{l(p+f)+Pc}{l} \cdot \log \frac{(l+c)P}{l(p+f)+Pc}$ · Load of the piston, corresponding to the absolute maximum of useful effect.

(25) .... $S = \frac{av'}{m} \cdot \frac{l(p+f)+Pc}{lP}$ .... Effective evaporation per unit of time.

(26) ab. max. u. E. $= \frac{mSP}{1+\delta} \cdot \log \frac{(l+c)P}{l(p+f)+Pc}$ *Absolute* maximum of useful effect.

(27) ab. m. u. HP. $= \frac{\text{ab. max. u. E.}}{33000}$ .... Absolute maximum of useful horse-power.

(28) ab. m. u. E. 1 lb. co. $= \frac{\text{ab. max. u. E.}}{N}$ ... Absolute maximum of useful effect arising from 1 lb. of fuel.

(29) ab. m. u. E. 1 ft. wa. $= \frac{\text{ab. max. u. E.}}{S}$ ... Absolute maximum of useful effect due to the evaporation of 1 cubic foot of water.

(30) mi. Q. co. for 1 hp. $= \frac{33000\ N}{\text{ab. max. u. E.}}$ ...... Minimum quantity of fuel per horse-power.

(31) mi. Q. wa. for 1 hp. $= \frac{33000\ S}{\text{ab. max. u. E.}}$ ...... Minimum quantity of water evaporated per horse-power.

(32) ab. m. HP. 1 lb. co. $= \frac{\text{ab. max. u. E.}}{33000\ \text{N}}$ ...... Absolute maximum of horse-power produced by 1 lb. of fuel.

(33) ab. m. HP. 1 ft. wa. $= \frac{\text{ab. max. u. E.}}{33000\ \text{S}}$ ...... Absolute maximum of horse-power produced by 1 cubic foot of water evaporated.

(34 bis) .... $\frac{l'}{l} = \frac{p+f}{\text{P}}$ .... Expansion which produces approximatively the *absolute* maximum of useful effect.

The only remark we have to make on the subject of these formulæ is, that the load proper to the production of the *absolute* maximum of useful effect, in expansive engines, is not the maximum load of which the engine is capable. If, in effect, we refer to equation (13), which represents the maximum load with a given expansion, that is, developing in it the quantity k,

$$ar' = \frac{a}{1+\delta}\left(\frac{n}{q}+\text{P}\right)\left(\frac{l'}{l}+\frac{l'+c}{l}\log\frac{l+c}{l'+c}\right) - \frac{a}{1+\delta}\left(\frac{n}{q}+p+f\right);$$

and seek what value of l' will make it a maximum, we shall find that condition to be expressed by $l' = l$, and not by

$$\frac{l'}{l} = \frac{\frac{n}{q}+p+f}{\frac{n}{q}+\text{P}}.$$

Consequently, if the engine be required to move the greatest load of which it is capable, it must work without expansion; but that load is not the load which produces the *absolute* maximum of useful effect. This will be given by the solution of equation (34) introduced into the formula (13), or approximatively by the formula (24) as has just been seen above.

CHAPTER IV.

OF HIGH-PRESSURE ENGINES.

ARTICLE I.

THEORY OF HIGH-PRESSURE ENGINES, AND OF UNEXPANSIVE ENGINES IN GENERAL.

SECT. I. *Of the effects of the engine with any given load or velocity.*

In the preceding chapter we have given, in a general manner, the theory of the effects of steam acting in the cylinder of an engine, as well while it is coming directly from the boiler, as during its expansion in the cylinder, after its separation from the boiler. This theory, and the formulæ we have deduced from it, form the basis of all the calculations that the particular applications of steam in steam-engines can require; and nothing now remains but to show the modifications of it according to each system of engines, or according to each mode of applying the motive power of steam.

Such is the object we have now in view in the present and following chapters. We divide steam-engines into three kinds, and each kind into several classes, according to the differences which exist in the various systems of engines.

In the first kind we place *rotative unexpansive* steam-engines, comprising as subdivisions, high-pressure stationary engines, locomotives, and Watt's rotative or double-acting steam-engines.

In the second kind we place *rotative expansive* engines, which form likewise three classes: condensing engines, with expansion in a single cylinder, or Cornish engines; condensing engines, with expansion in two cylinders, or Woolf's and Edwards's engines; and, lastly, expansive engines without condensation, or Evans's engines.

In the third kind we place *single-acting* engines, which again form three classes, namely: Watt's single-acting engines, Cornish single-acting engines, and atmospheric engines.

We shall, in the present chapter, treat specially of high-pressure engines, and shall first give the theory of those engines, under a form so general as to embrace at the same time all unexpansive engines.

We have already said, in developing the general theory of the action of steam, that the formulæ suitable to the calculation of unexpansive engines, may be deduced from the general formulæ, by supposing in them the expansion null, that is to say, by making $l' = l$. Moreover, as in those engines, the expansion is capable of no variation, since there is no expansion, the third case considered in the general theory, cannot occur. Thus, there will be but two circumstances to notice in their work, namely: the case in which they work with *any given load whatever*, and that in which they work with their *maximum load, or load of greatest useful effect.*

We might, then, immediately conclude, from the general formulæ already developed, the particular formulæ suitable to the engines of which we are treating. But, as in the general theory which we have developed, there occurs more complication than is necessary for the case of unexpansive engines, and as we have been obliged to make some use of the differential and integral calculus in it, the knowledge of which is not familiar to the greater number of readers, we deem it better to leave aside the general formulæ already found, and to seek in a direct manner, those which are proper to the action of the steam, considered in unexpansive engines only.

We have already said, in the first chapter, that since the effects of steam-engines are never calculated till after the moment when they have attained uniform motion, it follows that there is necessarily equilibrium between the power and the resistance; that is, between the force applied by the mover, and the force resulting from the friction of the engine, the resistance of the load, and all the other divers resistances which oppose the motion of the engine. Consequently, if we call P the pressure of the steam per unit of surface in the boiler, P′ the unknown pressure that steam will have in the cylinder, and R the *total* resistance opposed to the motion of the piston, we shall have as a first equation of analogy

$$P' = R \quad . \quad . \quad . \quad . \quad (A)$$

We have said, besides, that there is necessarily equality also between the expenditure and the production of steam. Now, if we express by S the volume of water evaporated per unit of time in the boiler, this volume of water will first, in the boiler, be converted into steam at the pressure P; then in the cylinder it will change into steam at the pressure P′. But we have seen in Chapter II., that during the change of pressure of the steam, which, from our experiments, is always found attended with a corresponding change of temperature, the steam always remains at the maximum density for its temperature. Again, in the steam of a maximum density for its temperature, the relative volume of the steam, that is, the ratio of the volume occupied by the steam to the volume of the water that produced it, may be expressed by the very simple equation (a)

$$\mu = \frac{1}{n + qp},$$

in which μ is the relative volume sought, p the pressure, and n and q two constant numbers, of which we have given the values. The volume of steam at the pressure P′, transmitted per minute to the cylinder of the engine, will then be μ S, or

$$\frac{S}{n + qP'}.$$

On the other hand, if by v be expressed the velocity of the piston, and by a the area of the cylinder, $v\,a$ will be the volume described by the piston in a unit of time. It is however to be remarked that the steam admitted in the cylinder, fills not only the space actually traversed by the piston, but also the clearance of the cylinder and the adjoining passages, wherein a portion of it is lost at every stroke of the piston. Calling l the stroke of the piston, and c the clearance of the cylinder expressed by an equivalent portion of the useful length of the cylinder, then $a\,l$ will be the volume described by the piston at each stroke, but the real volume expended in the same time will be $a\ (l + c)$. The volume of steam expended at every stroke, will be then, to the volume described by the piston, in the ratio

$$\frac{l+c}{l};$$

and consequently there will be the same ratio between the volumes expended and the volumes described by the piston, in the unit of time.

The volume described by the piston in the unit of time is, as has been said, $a\,v$; the volume of steam, then, expended in the unit of time will be

$$av\,\frac{l+c}{l}.$$

And thus the equality between the production of the steam and its expenditure, will give for a second general analogy

$$\frac{S}{n+q\mathrm{P}'}=av\,\frac{l+c}{l}\quad .\ .\ .\ .\ \text{(B)}$$

Eliminating therefore P′ from the two equations (A) and (B), we obtain

$$v = \frac{l}{l+c} \cdot \frac{S}{a} \cdot \frac{1}{n+qR}.$$

On the other hand, expressing by r the resistance of the load, by f the friction of the engine when unloaded, by δ the increase accruing to that friction, per unit of the resistance r imposed on the engine, lastly, by p the pressure against the opposite face of the piston, proceeding from the atmosphere in non-condensing engines or from imperfect condensation in the cylinder in condensing ones, these four forces being referred besides to the unit of surface of the piston; we shall plainly have, for the *total* resistance R, the value

$$R = (1+\delta)r + f + p.$$

And, therefore, the velocity v of the engine with a known load, in terms of all the data of the problem, will be

$$v = \frac{l}{l+c} \cdot \frac{S}{a} \cdot \frac{1}{n+q[(1+\delta)r+p+f]} \quad . \; . \; . \; (1)$$

We derive reciprocally, for the value of the load ar, in terms of the velocity,

$$ar = \frac{l}{l+c} \cdot \frac{S}{(1+\delta)qv} - \frac{a}{1+\delta}\left(\frac{n}{q}+p+f\right) \; . \; . \; (2)$$

Similarly the evaporation capable of producing the velocity v, with the load ar, will be

$$S = \frac{l+c}{l} \cdot av\{n+q[(1+\delta)r+p+f]\} \; . \; . \; (3)$$

Finally, knowing, by means of these equations, the load and the velocity of the engine, their product will give, without further calculation, the useful effect of the engine, viz.

$$\text{u.E.} = a r v ; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

and successively, as in Section 5, Article I. of the preceding chapter, we shall have the horse-power and the other modes in which the useful effect of the engine may be represented.

SECT. II.—*Of the maximum useful effect of the engine.*

The preceding calculation refers to the general case in which the load or the velocity are given *à priori*, and without any particular condition; but if it be required to know what is the velocity or load suitable to the production of the maximum useful effect of the engine, we must examine the value of the useful effect $a r v$, which, from equation (2) is

$$\text{u.E.} = \frac{l}{l+c} \cdot \frac{\text{S}}{(1+\delta) q} - \frac{a v}{1+\delta} \left(\frac{n}{q} + p + f \right).$$

Now, this expression containing the velocity only in the negative terms, will evidently attain its maximum when the velocity shall be the smallest possible; and on the other hand, referring to equation (B), we perceive that the velocity will be a minimum, when the pressure P′ shall, on the contrary, have attained its highest value, that is, when it shall be equal to the pressure P, in the boiler. The velocity then of maximum useful effect will be furnished by equation (B), on making $P' = P$; or that velocity will be

$$v' = \frac{l}{l+c} \cdot \frac{S}{a} \cdot \frac{1}{n + q\,P} \quad . \; . \; . \quad (5)$$

Consequently, substituting this value in equation (2), we have for the load of maximum useful effect,

$$ar' = \frac{a}{1 + \delta}(P - p - f); \quad . \; . \; . \quad (6)$$

and, substituting it in equation (3), we have for the evaporation of the engine, in terms of the velocity of maximum useful effect,

$$S = \frac{l + c}{l} av' (n + q\,P) \quad . \; . \; . \quad (7)$$

Finally, the product $a\,r'\,v'$ will give the measure of the maximum useful effect of the engine, and it may be expressed, as above, under the different forms indicated in Section 5, Article 1, Chapter III.

Thus, therefore, we have all the formulæ necessary to the solution of the different problems, which may occur in the calculation of these engines.

ARTICLE II.

PRACTICAL FORMULÆ FOR THE CALCULATION OF HIGH-PRESSURE ENGINES; AND EXAMPLE OF THEIR APPLICATION.

In high-pressure unexpansive engines, the steam is generated in the boiler at a very high pressure.

It passes thence into the cylinder, where it acts successively above and below the piston, to impress on it an alternate motion, which being communicated to the great beam, is transmitted to a crank and changed into a continued rotatory motion applied to the shaft of the engine; and this afterwards determines the particular action of all the parts necessary to the operation of the engine. The steam then penetrates into the cylinder alternately by each extremity; but after it has driven the piston to the end of the stroke, it finds an aperture by which it escapes into the atmosphere, without being condensed. Consequently, instead of having, on that face of the piston opposed to the action of the steam, a vacuum more or less perfect resulting from the condensation, that is to say, a pressure null or nearly so, there always subsists, on the contrary, the atmospheric pressure, which immediately succeeds to that previously exerted by the steam. In these engines, then, the atmospheric pressure is to be counted one of the forces which oppose the motion of the piston; and, therefore, the quantity represented by p, should in this case have a value equal to 14·71 lbs. per square inch.

To be able to apply the formulæ which we have just developed, it is necessary to know the two quantities f and δ; that is, the friction of the engine when unloaded, and its additional friction, per unit of the load r. Special and circumstantial experiments would be requisite for this purpose, but till more precise determinations on the subject be obtained, we may take an evaluation of those two quantities from our own experiments on locomotives, which are also high-pressure unexpansive engines. In these the friction of the engine when unloaded, deduction being made of the force necessary to move its own weight along the rails, amounts to about 1 lb. per square inch of the surface of the piston, and the additional friction caused by any additional resistance is ·14 of that resistance. We shall take then

$$f = 1 \times 144 \text{ lbs. and } \delta = \cdot 14 *.$$

Finally, in these as well as in all rotative engines, where the motion of the piston is checked and

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* A locomotive engine with two cylinders, 11 inches in diameter, and with wheels uncoupled, has, when unloaded, a mean friction of 101 lbs.; and deducting 64 lbs. for the transport of its own weight, and the additional friction which that draught causes in the engine, there remain 37 lbs. for the resistance occasioned by the machinery. This resistance being measured at the velocity of the wheel, produces against the piston, a force increasing in the inverse proportion of the respective velocities; that is to say, a force of

$$37 \times 5 \cdot 9 = 218 \text{ lbs.}$$

which amounts to 1·15 lb. per square inch of the surface of the pistons.

Again, a load of 1 ton causes on a railway a resistance of 7 lbs., and occasions in the engine a surplus friction of 1 lb., which is an additional friction equal to $\frac{1}{7}$th of the resistance imposed on the engine. This evaluation may be found too high, with regard to a stationary engine, in which there is no friction of carrying wheels; but we let it remain, because in the absence of positive determinations, it is better to avoid the risk of rating the passive resistances too low.

regulated by a crank, the space left between the end of the stroke of the piston and the bottom of the cylinder, amounts, including the adjoining passages, to but $\frac{1}{20}$ of the useful length of the cylinder. Therefore we have

$$\frac{l+c}{l}=\frac{21}{20}=1{\cdot}05.$$

Introducing these values into the formulæ, except those of f and δ, which are but approximative; and replacing the constant quantities n and q by the values suitable to them in non-condensing engines, that is, in English measure, and when the pressure is expressed per square foot,

$$n={\cdot}0001421,$$

$$q={\cdot}00000023,$$

we obtain, for the numerical formulæ suitable to the calculation of the engines under consideration:

Practical formulæ for high-pressure engines.

GENERAL CASE.

$$v=\frac{S}{a}\cdot\frac{10000}{6{\cdot}6075+{\cdot}002415\,[(1+\delta)\,r+f]}.$$ Velocity of the piston, in feet per minute.

$$ar=4140750\,\frac{S}{(1+\delta)v}-\frac{a}{1+\delta}\,(2736+f)$$ Useful load of the piston, in pounds.

$$S=\frac{av}{10000}\{6{\cdot}6075+{\cdot}002415\,[(1+\delta)\,r+f]\}.$$ Effective evaporation, in cubic feet of water per minute.

u. E.	$= arv$	Useful effect, in pounds raised 1 foot per minute.
u. HP.	$= \frac{\text{u. E.}}{33000}$	Useful force, in horse-power.
u. E. 1 lb. co.	$= \frac{\text{u. E.}}{N}$	Useful effect of 1 lb. of coal, in pounds raised 1 foot.
u. E. 1 ft. wa.	$= \frac{\text{u. E.}}{S}$	Useful effect of 1 cubic foot of water, in pounds raised 1 foot.
Q. co. for 1 hp.	$= \frac{33000\, N}{\text{u. E.}}$	Quantity of coal, in pounds, which produces 1 horse-power.
Q. wa. for 1 hp.	$\frac{33000\, S}{\text{u. E.}}$	Quantity of water, in cubic feet, which produces one horse-power.
u. HP. 1 lb. co.	$= \frac{\text{u. E.}}{33000\, N}$	Horse - power produced by 1 lb. of coal.
u. HP. 1 ft. wa.	$= \frac{\text{u. E.}}{33000\, S}$	Horse - power produced by 1 cubic foot of water evaporated.

CASE OF MAXIMUM USEFUL EFFECT.

$v' = \frac{S}{a} \cdot \frac{10000}{1{\cdot}492 + {\cdot}002415\,P}$ .... Velocity of the piston, in feet per minute.

$ar' = \frac{a}{1+\delta}(P - f - 2118)$ .... Useful load of the piston, in pounds.

$S = \frac{av'}{10000}(1{\cdot}492 + {\cdot}002415\,P)$ .... Effective evaporation, in cubic feet of water per minute.

max. u. E. $= ar'v'$ .... Useful effect, in pounds raised to 1 foot per minute.

We do not add to the formulæ of the case of maximum useful effect, the different expressions of the useful effect, in horse-power, in weight of fuel, &c., because the formulæ which give them are the same as those of the general case.

To shew an application of these formulæ, let us suppose it be required to determine the effects to be expected from an engine of this system already constructed, and of which the dimensions and other data of the calculation are known, viz.:

Cylinder, 17 inches diameter; or $a = 1{\cdot}57$ sq. feet.

Stroke of the piston, 16 inches; or $l = 1{\cdot}33$ feet.

Effective evaporation of which the boiler is capable, ·67 cubic foot of water per minute; or $S = {\cdot}67$ cubic foot.

Consumption of coke in the same time, 8 lbs.; or $N = 8$ lbs.

Total pressure in the boiler, 65 lbs. per square inch; or $P = 65 \times 144$ lbs. per square foot.

Performing the calculation with these data, we obtain the following results, as the effects which this engine is capable of producing, at the velocity of maximum useful effect, and also at the respective velocities of 250 and 300 feet per minute:

			Maximum of useful effect.
v	=300	250	176
ar	=4146	5769	9777
$\frac{r}{144}$	=18·34	25·52	43·25
S	=·67	·67	·67
u. E.	=1243800	1442250	1724580
u. HP	=38	44	52
u. E. 1 lb. co.	=155475	180280	271390
u. E. 1 ft. wa.	=1856450	2152640	2574000
Q. co. for 1 hp.	=·212	·183	·153
Q. wa. for 1 hp.	=·018	·015	·013
u. HP. 1 lb. co.	=4·71	5·46	6·53
u. HP. 1 ft. wa.	=56	65	78

Such will be the effects produced. With respect, however, to those presented in the Table, as resulting from the combustion of one pound of fuel, it is to be observed that they refer to the use of coke; that is to say, it is from experiments made with that kind of fuel that we have concluded the quantity employed in the evaporation of ·67 cubic foot of water per minute. From Smeaton's experiments, it would appear that coke produces but $\frac{5}{6}$ of the effect produced by the same weight of coal. Therefore, it would follow, that if the fire, in these experiments, had been fed with coal instead of coke, the fuel necessary for the above evaporation, would have been but 6·66 lbs.; and the effect due to the consumption of 1 lb. of fuel, in the engine in question and in the case of maximum useful effect, would have become,

$$u.\ E.\ 1\ lb.\ co. = 258950.$$

However, as the experiments of Smeaton on that head have been regarded as not being conclusive, we do not lay a stress upon the result, but make the observation, only to call attention upon the difference that may arise from the use of the two sorts of fuel.

CHAPTER V.

LOCOMOTIVE ENGINES.

ARTICLE I.

THEORY OF LOCOMOTIVE ENGINES.

THE locomotive engines in use on railways are constructed on the same principles as the preceding, with regard to the application of the steam as a motive force. The steam is generated at a very high pressure in the boiler. It then passes into the cylinders, where it is admitted without interruption during the whole stroke of the piston; and finally it escapes into the atmosphere without having been condensed. These engines, therefore, act without expansion or condensation.

The steam having entered the cylinder, acts successively on either side of the piston, and thus communicates to it a rectilineal alternate motion, which, by means of a crank, changes to a rotatory motion applied to the wheels that support the engine; and the effect of this rotation is to carry forward the engine itself, followed by all its train.

As these engines are no more than a particular application of the preceding, the formulæ proper to calculate their effects will be similar to those just given. There, however, occur in them some accessory circumstances which must be introduced; and this renders it necessary to treat of them apart.

These circumstances are: 1st, that the engine is obliged to draw its own weight, which by so much diminishes its *useful* effect; 2d, that the waste steam being driven into the chimney through the orifice of the blast-pipe, to cause an artificial current of air intended to excite the fire, and thus to compensate for the smallness of the boiler, it results that a certain force is expended by the engine in driving this steam with the necessary velocity; 3d, that the train led by the engine, having in its motion to contend against the resistance of the air, and this force increasing as the square of the velocity, there results a variable resistance to be added to those already considered; 4th, and lastly, that some of these engines are liable to a considerable loss of steam by the safety-valves, and till this defect is entirely corrected, it is necessary to notice it.

In order to take account of these different circumstances, we will express by ρ the force requisite to move the engine itself; and by $p'v$ the pressure on the opposite face of the piston, resulting from the action of the blast-pipe, a pressure which, from our own experiments on this subject, we will suppose to be proportional to the velocity of the motion; we will express by $g\,v^2$ the resistance of the air against the train, and will introduce these three forces into the calculation, supposing them to be referred to the unit of surface of the piston and to its velocity. With regard to the loss of steam by the valves, it will be taken account of, as may be required, in the manner we have already indicated, and then subtracted from the total evaporation, in order to conclude from it the effective evaporation, or the true value of S.

This being premised, the resistance of the load against the piston, which we have expressed by r, will now become

$$r+\rho+g\,v^2$$

and the pressure p, on the opposite face of the piston, will become

$$p+p'v.$$

Consequently, the formulæ proper for the calculation of these engines, in the general case and in that of maximum useful effect, will be—

GENERAL CASE.

$$v=\frac{l}{l+c}\cdot\frac{\text{S}}{a}\cdot\frac{1}{n+q\,[(1+\delta)\,(r+\rho+g\,v^2)+p+p'\,v+f]}$$

$$a\,r=\frac{l}{l+c}\cdot\frac{\text{S}}{q\,(1+\delta)\,v}-\frac{a}{1+\delta}\left(\frac{n}{q}+p+p'\,v+f\right)-a\,(\rho+g\,v^2)$$

$$\text{S}=\frac{l+c}{l}\cdot av\,\{n+g\,[(1+\delta)\,(r+\rho+g\,v^2)+p+p'v+f]\}.$$

$$\text{u.E.}=a\,r\,v.$$

CASE OF MAXIMUM USEFUL EFFECT.

$$v' = \frac{l}{l+c} \cdot \frac{S}{a(n+qP)}.$$

$$a\,r' = \frac{a}{1+\delta}(P-p-p'v'-f) - a(\rho+gv'^2)$$

$$S = \frac{l+c}{l} \cdot a\,v'(n+qP).$$

$$\text{max.u.E.} = a\,r'\,v'.$$

The other different expressions of the useful effect will be expressed by the formulæ developed in Sect. V. Article I. Chapter III., which are suitable to all engines.

The velocity v, in these equations, is that of the piston. It is practically more convenient to use the velocity of the engine itself, as we have done in our Treatise on Locomotives; but to retain the same form in the application of our formulæ to all steam-engines, we shall make no change here in that respect. Howbeit, as the piston performs two strokes while the wheel makes one revolution, the velocity of the engine is easily deduced from that of the piston, by multiplying the latter by the ratio

$$\frac{\pi D}{2l} \quad \text{or} \quad 1{\cdot}5708\,\frac{D}{l},$$

in which π is the ratio of the circumference to the diameter, D the diameter of the wheel, and l the stroke of the piston.

In the application of these formulæ, it will be remarked that the formula which gives the velocity in the general case, contains still in the denominator one term $p'v$ and another $g v^2$; but to avoid the solution of an equation of the third degree, an evaluation may be made of the resulting velocity, and thus approximate values of each of those terms calculated, which being introduced into the equation as constant quantities, a value of v may be derived. With a little experience of these engines, the first trial thus made, leads to a value of v sufficiently near in many cases. If, however, it should appear from the result, that too great an error has been made in the original evaluation of v, for this first trial to be depended on, the result of it must be used as an approximation for a second, and the result of this for a third, if necessary. The difficulty, however, occurs only in the equation of the velocity, and not in any of the other formulæ.

ARTICLE II.

PRACTICAL FORMULÆ FOR THE CALCULATION OF LOCOMOTIVE ENGINES, AND EXAMPLE OF THEIR APPLICATION.

Now, to form the numerical equations proper for the calculation of these engines, the constant quantities in the algebraic formulæ just given, must be replaced by the values which experience has assigned to them.

The friction of a locomotive engine, with uncoupled wheels, is, as we have said, about 1 lb. per square inch of the surface of the piston; but in the engines with coupled wheels, that friction is rather greater, and is equivalent to 1·25 lb. per square inch of the piston. In order therefore to include those engines, we will adopt the latter determination, and take

$$f = 1{\cdot}25 \times 144 \text{ lbs.}$$

It has already been said, that the surplus accruing to that friction, when the engine draws a given resistance, amounts to $\frac{1}{7}$th of that resistance; we have then

$$\delta = {\cdot}14.$$

The pressure arising from the blast-pipe varies not only with the velocity of the engine but also with the evaporation of the boiler per minute, and with the size of the orifice of the blast-pipe, as will be seen by the experiments developed in the TREATISE ON LOCOMOTIVES; but to simplify the formulæ, we will here refer that effect to the mean evaporation of the engines and to the size of orifice commonly adopted. With these mean dimensions and data, it is found that, when the velocity is 10 miles per hour for the engine, or 150 feet per minute for the piston, the pressure arising from the blast-pipe is 1·75 lb. per square inch of the surface of the piston, and that it varies in the direct ratio of the velocity of the motion; whence is deduced

$$p' v = 1{\cdot}75 \times 144, \text{ when } v = 150;$$

and, consequently,

$$p' = \frac{1{\cdot}75}{150} \times 144 = {\cdot}0117 \times 144 \text{ lbs.}$$

The resistance of the air against a train of mean surface is 33 lbs. at the velocity of 10 miles per hour for the engine. This resistance in transmitting itself to the piston, increases in the inverse ratio of the respective velocities of the piston and of the engine, that is to say, of the velocities of the piston and of the circumference of the wheel. Consequently, if D represent the diameter of the wheel, that force becomes

$$33 \times \frac{3{\cdot}1416\ D}{2l};$$

and supposing it to be divided per unit of the surface of the piston, it produces a resistance expressed by

$$\frac{1}{a} \times 33 \times \frac{3{\cdot}1416\ D}{2l}.$$

Admitting then the mean proportions most in use in these engines, that is, taking the diameter of the wheel at 5 feet, the stroke of the piston at 16 inches or 1·33 foot, the diameter of the piston at 12 inches or 1 foot, the above resistance, divided per square foot of the surface of the piston, amounts to

$$\cdot 8615 \times 144.$$

And as we have said that it is the intensity of that force, when measured at the velocity of 150 feet per minute for the piston, and that it increases as the square of the velocity, we have

$$g\, v^2 = \cdot 8615 \times 144, \text{ when } v = 150,$$

which gives

$$g = \cdot 00003829 \times 144 \text{ lbs.}$$

Finally, as locomotive engines are high pressure engines, without condensation, the values of n and q which are suitable to them, are

$$n = \cdot 0001421$$
$$q = \cdot 00000023.$$

Introducing these values then in the algebraic formulæ above, they become as follows:

Practical formulæ for locomotive engines.

GENERAL CASE.

$$v = \frac{S}{a} \cdot \frac{10000}{7\cdot042 + \cdot00275\,(r+\varrho) + \cdot0040688\,v + \cdot00001518\,v^2}$$ Velocity of the piston, in feet per minute.

$$ar = 3632300\,\frac{S}{v} - a[2558 + \varrho + 1\cdot4779v + \cdot0055v^2]$$ Useful load of the piston, in pounds.

$$S = \frac{av}{10000} \cdot [7\cdot042 + \cdot00275\,(r+\varrho) + \cdot0040688\,v + \cdot00001518\,v^2]$$ Effective evaporation, in cubic feet of water per minute.

u. E............ $= arv$.................................Useful effect, in pounds raised 1 foot per minute.

u. HP..........$= \frac{\text{u. E.}}{33000}$ ....Useful force, in horse-power.

u. E. 1 lb. co. $= \frac{\text{u. E.}}{\text{N}}$ ....Useful effect of 1 lb. of coke, in pounds raised 1 foot.

u. E. 1 ft. wa. ... $= \frac{\text{u. E.}}{\text{S}}$ ....Useful effect of 1 cubic foot of water evaporated, in pounds raised 1 foot.

Q. co. for 1 hp... $= \frac{33000\ \text{N}}{\text{u. E.}}$ ....Quantity of coke, in pounds, which produces 1 horse-power.

Q. wa. for 1 hp. $= \frac{33000\ \text{S}}{\text{u. E.}}$ ....Quantity of water in cubic feet, which produces 1 horse-power.

u. HP. 1 lb. co... $= \frac{\text{u. E.}}{33000\ \text{N}}$ ....Horse - power produced per pound of coke.

u. HP. 1 ft. wa. $= \frac{\text{u. E.}}{33000\ \text{S}}$ ....Horse - power produced per cubic foot of water evaporated.

CASE OF THE MAXIMUM USEFUL EFFECT.

$v' = \frac{\text{S}}{a} \cdot \frac{10000}{1{\cdot}492 + {\cdot}002415\ \text{P}}$ ....Velocity of the piston, in feet per minute.

$ar' = a({\cdot}8772\,\text{P} - 2016 - \rho - 1{\cdot}4779\,v' - {\cdot}0055\,v'^2)$ Useful load of the piston, in pounds.

$S = \frac{av'}{10000}(1{\cdot}492 + {\cdot}002415\ P)$ ....Effective evaporation, in cubic feet of water per minute.

max. u. E....... $= ar'v'$ ....Useful effect, in pounds raised 1 foot per minute.

We omit, in the case of maximum useful effect, the horse-power and the other modes of expressing the effect of the engine, because the formulæ proper to those determinations are the same as those of the general case, which dispenses us from repeating them here.

To give now an example of the application of these formulæ, we will suppose a locomotive engine similar to the *Atlas*, which has the following dimensions and capabilities:

Two cylinders of 12 inches diameter; or $a = 1{\cdot}57$.

Stroke of the piston 16 inches; or $l = 1{\cdot}33$.

Clearance of the cylinder $\frac{1}{20}$ of the stroke; or $c = {\cdot}05\ l$.

Wheels 5 feet in diameter, coupled.

Total pressure in the boiler, 65 lbs. per square inch; or $P = 65 \times 144$ lbs. per square foot.

Effective evaporation, ·67 cubic feet of water per minute; or $S = {\cdot}67$.

Consumption of fuel in the same time 8 lbs.; or $N = 8$.

Resistance caused by the transport of the engine itself, $\frac{1}{320}$ of its own weight, or 80 lbs.; which makes against the piston a resistance of $80 \times 5.9 = 472$ lbs. or 2·09 lbs. per square inch of its surface. Thus $\rho = 2{\cdot}09 \times 144$ lbs.

Performing, then, the calculation with these data, we find the following results, at the velocity of maximum useful effect, and at the velocities of 250 and 300 feet per minute for the piston.

				Maximum useful effect.
v	=	300	250	176
ar	=	2151	4124	8576
$\frac{r}{144}$	=	9·52	18·25	37·95
S	=	·67	·67	·67
u. E.	=	645280	1031010	1512690
u. HP.	=	20	31	46
u. E. 1 lb. co.	=	80663	128880	189090
u. E. 1 ft. wa.	=	963150	1538850	2257800
Q. co. for 1 hp	=	·409	·256	·175
Q. wa. for 1 hp	=	·034	·021	·015
u. HP. 1 lb. co	=	2·44	3·91	5·73
u. HP. 1 ft. wa	=	29	47	68

Were it desired to refer these results to the measures usual on railways, that is, to count the velocity in miles per hour for the engine, and the load in tons drawn at the same velocity, then, from what has been said, it would suffice to multiply the above velocities by the factor

$$\frac{5{\cdot}9 \times 60}{5280};$$

because, in the ordinary proportions, the velocity of the engine is 5·9 times that of the piston, a mile contains 5280 feet, and an hour 60 minutes. The result of this first multiplication would convert the velocity of the piston in feet per minute, to that of the engine in miles per hour.

To convert afterwards the loads in pounds on the piston, which have just been calculated above, into loads of the engine in tons, the former must be multiplied by the factor

$$\frac{1}{5{\cdot}9 \times 7},$$

on account of the proportion of the velocities, and because the draught of 1 ton requires a force of 7 lbs.

Thus the three cases reported above would then be expressed as follows:

Velocities 20·11...16·76...11·83 miles per hour.

Loads...... 52 ...... 100 ..... 207 tons.

The other results would not be changed.

We will, however, observe, that these effects will be those of the engine, for the data introduced above; but by forcing the fire, in order to increase the evaporation S, or by augmenting the pressure P of the steam in the boiler, the engine may be made to produce greater effects; and, on the contrary, the effects will be less if the evaporation of the boiler is less than ·67 square foot per minute, or if a part of the steam is lost by the safety valves.

We have designedly chosen a locomotive and a stationary steam-engine of the same dimensions, in order to render it easy to judge of the disadvantages the locomotive engines lie under, by reason of the circumstances we have just mentioned, viz. :—the necessity of transporting their own weight, the resistance of the air against the train in great velocities, and the force consumed in blowing the fire in a furnace of small dimensions.

CHAPTER VI.

WATT'S ROTATIVE OR DOUBLE-ACTING ENGINES.

SECT. I. *Practical formulæ suitable to the calculation of these engines; and Example of their application.*

IN Watt's rotative or double-acting steam-engines, condensation is used, but not the expansion of the steam. The steam is generated in the boiler at a pressure which exceeds that of the atmosphere by about 1·5 to 3 lbs. per square inch. It then passes into the cylinder, flowing in without interruption during the whole stroke of the piston; but when the piston has terminated its descending stroke, a communication opens between the top of the cylinder and the condenser. The steam which filled the cylinder passes immediately into the condenser, leaving on the upper surface of the piston only a very slight pressure, owing to imperfect condensation. At this moment the communication from the boiler to the cylinder is changed. The steam, instead of continuing to enter above the piston, now comes in below it, driving the piston back to the top of the cylinder, and then is condensed again, while a new quantity of steam is admitted above the piston, and so on. The alternate motion thus communicated to the piston is transmitted to a crank, and therefore changed to one of rotation, imparted to the divers pieces of machinery used for various purposes in the arts.

These engines being unexpansive, the formulæ suitable to them are the same as those of stationary high-pressure engines, which have been treated of in Chapter IV.; only the quantity P, which expresses the pressure in the boiler, will represent a much lower pressure, and the quantity p, instead of representing the atmospheric pressure, will now stand for the pressure which still subsists in the cylinder, after imperfect condensation of the steam.

We shall not, then, repeat those formulæ here in their algebraic shape, but transform them at once into the corresponding numerical formulæ; to which end we will first seek the constant quantities that figure in them, in order to make the proper substitutions.

The quantity p represents the pressure of condensation under the piston; but it is necessary here, to make an observation relative to that pressure. In good engines, well provided with injection water, at a degree of temperature not exceeding 50° of Fahrenheit, the pressure in the condenser, taken by means of a manometer, is usually reduced to 1·5 lb. per square inch; but that is clearly not the true value of p, or the pressure of condensation in the steam cylinder itself, and under the piston. In effect, since the condensation of the steam takes place only by degrees, as it passes from the cylinder to the condenser, and as the velocity of its transit depends only on the difference of pressure between the uncondensed steam remaining in the cylinder and the imperfect vacuum in the condenser, it follows that an equilibrium of pressure between the emptying cylinder and the condenser can only be established *gradually*. The mean pressure of condensation, then, under the piston, must be superior to that of the condenser. Direct experiments on this subject, made with Watt's *indicator*, prove that in ordinary velocities, and with the usual dimensions of the passages, the mean pressure under the piston is ordinarily 2·5 lbs. higher than that of the condenser. The latter, then, being 1·5 lb., we perceive that the value of p will generally be

$$p = 4 \times 144 \text{ lbs.};$$

it ought, however, to be measured specially for each occasion.

As to the friction of these engines, some notions tolerably precise have been acquired by experience. It is a datum admitted among practical men, and founded on numerous trials made on Watt's engines, that their friction when working under a moderate load, varies from 2·5 lbs. per square inch of the piston, in the smaller and less elaborate engines; to 1·5 lb. in those of larger dimensions and better made; and this includes the friction of the different parts of the engine and the force necessary to work the feeding and discharging pumps, &c.

By a moderate load in these engines is understood a load of about 8 lbs. per square inch of the piston; and our own experiments on locomotives give room to think that the *additional* friction created in the engine by reason of that load, will be $\frac{1}{7}$th of the load, or 1 lb. per square inch. The above datum goes then to establish that Watt's engines, working without load, have a friction of 1·5 lb. to ·5 lb. per square inch, according to their dimensions: namely, 1·5 lb. for the smaller ones, or those termed 10 horse power, having a cylinder 17·5 inches in diameter; and ·5 lb. for those termed 100 horse power, or having a cylinder 48·5 inches in diameter; which makes 1 lb. for engines of mean power. This result agreeing with that which we have deduced from our researches on locomotives, the data precedently indicated may be admitted for engines of mean dimensions of this system; but it is to be recollected that these data can only serve in default of special and circumstantial experiments on the subject.

In an engine of this system of mean dimensions, we may then take

$$f = 1 \times 144, \quad \delta = \cdot 14.$$

The engine, too, working by condensation, we have,

$$n = \cdot 00004227$$
$$q = \cdot 000000258.$$

Finally, the clearance of the cylinder in these engines is ordinarily $\frac{1}{20}$th of the stroke, which gives

$$c = \cdot 05\, l.$$

Substituting in the algebraic formulæ, those among these values which are constant, we have the following numerical formulæ :—

Practical formulæ for Watt's rotative or double-acting engines.

GENERAL CASE.

$$v = \frac{S}{a} \frac{10000}{\cdot 4438 + \cdot 00271\,[(1+\delta)\,r + p + f]} \ldots$$ Velocity of the piston, in feet per minute.

$$ar = 3691400\, \frac{S}{(1+\delta)\,v} - \frac{a}{1+\delta}(164 + p + f)$$ Useful load of the piston, in pounds.

$$S = \frac{av}{10000}\{\cdot 4438 + \cdot 00271[(1+\delta)r + p + f]\}$$ Effective evaporation, in cubic feet of water per minute.

$$\text{u. E.} = arv \ldots\ldots$$ Useful effect, in pounds raised 1 foot per minute.

$$\text{u. HP.} \ldots\ldots = \frac{\text{u. E.}}{33000} \ldots\ldots$$ Useful force, in horse-power.

$$\text{u. E. 1 lb. co.} = \frac{\text{u. E.}}{\text{N}} \ldots\ldots$$ Useful effect of 1 lb. of coal, in pounds raised 1 foot.

$u. E. 1 ft. wa. = \frac{u. E.}{S}$	....	Useful effect of 1 cubic foot of water, in pounds raised 1 foot.
$Q. co. for 1 hp. = \frac{33000 N}{u. E.}$	....	Quantity of coal in lbs., which produces 1 horse power.
$Q. wa. for 1 hp. = \frac{33000 S}{u. E.}$	....	Quantity of water in cubic feet, which produces 1 horse-power.
$u. HP. 1 lb. co. = \frac{u. E.}{33000 N}$	....	Horse - power produced per pound of coal.
$u. HP. 1. ft. wa. = \frac{u. E.}{33000 S}$	....	Horse - power produced by 1 cubic foot of water evaporated.

CASE OF THE MAXIMUM USEFUL EFFECT.

$v' = \frac{S}{a} \cdot \frac{10000}{\cdot 4438 + \cdot 00271 P}$	....	Velocity of the piston, in feet per minute.
$ar' = \frac{a}{1+\delta}(P - p - f)$	....	Useful load of the piston, in pounds.
$S = \frac{av'}{10000}(\cdot 4438 + \cdot 00271 P)$	....	Effective evaporation, in cubic feet of water per minute.

max. u. E. $= a r' v'$ .... Maximum useful effect, in pounds raised 1 foot per minute.

As an application of these formulæ, we will perform the calculation on an engine constructed by Watt at the Albion Mills, and put to trial soon after it was built by Watt himself. The following were the dimensions of the engine:

Diameter of the cylinder, 34 inches; or $a =$ 6·287 sq. feet.

Stroke of the piston, 8 feet; or $l = 8$ feet.

Clearance of the cylinder, $\frac{1}{20}$th of the stroke; or $c = {\cdot}05\, l$.

Pressure in the boiler, 16·5 lbs. per square inch; or $P = 16{\cdot}5 \times 144$ lbs. per square foot.

Effective evaporation, ·927 cubic feet of water per minute; or $S = {\cdot}927$.

Consumption of coal in the same time, 6·71 lbs.; or $N = 6{\cdot}71$ lbs.

The engine had been constructed to work at the velocity of 256 feet per minute, which was considered as its proper velocity; but when put to trial by Watt, accompanied by Sir J. Rennie, it assumed, in performing its regular work, which was esteemed 50 horse-power, the velocity of 286 feet per minute for the piston, consuming per minute the quantity of water and coal which we have just stated.

On seeking the effects which this engine ought to produce, at its velocity of maximum useful effect, and then at those of 256 and 286 feet per minute, they are found to be:

				Maximum useful effect.
v	=	286	256	214
ar	=	5621	6850	9133
$\frac{r}{144}$	=	6·21	7·57	10·09
S	=	·927	·927	·927
u. E.	=	1607610	1753600	1957180
u. HP.	=	49	53	59
u. E. 1 lb. co.	=	239585	261340	291680
u. E. 1 ft. wa.	=	1734200	1891700	2111300
Q. co. for 1 hp	=	·138	·126	·113
Q. wa. for 1 hp	=	·019	·017	·016
u. HP. 1 lb. co	=	7·26	7·92	8·84
u. HP. 1 ft. wa	=	53	57	64

Such are the effects that might have been expected from this engine; and we see in consequence that, performing a work estimated at 50 horses, it was to be expected to assume the velocity which in fact it did; namely, 286 feet per minute.

SECT. II. *Considerations on the application of the ordinary mode of calculation to Watt's steam-engines.*

THE above calculations and the formulæ which precede them, testify our free admission that Watt's engines may work at a full pressure of steam in the cylinder, that is to say, at a pressure in the cylinder sensibly equal to that of the boiler, since the case of maximum effect here mentioned is nothing else but that. But we have proved that this effect cannot be produced at all velocities, as the calculation commonly applied to steam-engines would have it. It can be produced only at one velocity, that of the maximum of effect, and this will vary in every engine, so that it becomes necessary to calculate it separately for each.

Some persons think that stationary engines, and those of Watt in particular, working always at a very moderate velocity, and having, as they allege, very large passages for the circulation of the steam, must necessarily work at full pressure in the cylinder; and they conclude that, however true the theory we have explained may be, it is nevertheless unnecessary for calculating the effects of these engines, because they are found to realize the suppositions of the ordinary calculation. But, besides that the steam-passages are much narrower in Watt's engines than in the locomotives, since they are but $\frac{1}{25}$th instead of $\frac{1}{10}$th of the area of the cylinder, it is easy to obtain conviction, that Watt's engines, as well as those we have already treated, may work at pressures very much reduced in the cylinder, which fact sets all the suppositions and calculations of the ordinary theory at fault.

Not to return to the general demonstration of this fact, which results from principles we have already explained, the proof of it will be found in three authentic circumstances.

1st. Watt's *indicator* applied to the cylinder of several of his engines, while working at their usual velocity and in their regular state, made known that the pressure in the cylinder was several pounds per square inch below that of the boiler. Whatever reasons, then, be adduced to the contrary, there is not an equality between the two pressures.

2d. Experience has demonstrated to Watt, that in his engines, every cubic inch of water evaporated produced a cubic foot of steam of an elasticity sufficient to effect the motion. This practical datum he has registered in his notes to the article on steam-engines in Robinson's *Encyclopædia*. From this observation the steam is expended in the cylinder at a volume equal to 1728 times that of water. But the steam is formed in the boiler at the pressure of 16·5 lbs. per square inch, which gives but 1530 times the volume of the water. Therefore, the volume of the steam, in passing into the cylinder, becomes 1728 instead of 1530; that is to say, the steam undergoes a considerable diminution of pressure, and at the same time a corresponding augmentation of volume.

3d. It is known that Watt's engines, when put to the proof, will produce about half as much more than their nominal power, without any change being made in the engine. Hence, in their daily work, they do not work at full pressure; for if they did, it would be impossible to make them execute more.

Thus we see that these engines work habitually at a reduced pressure, as well as those of which we have treated hitherto. However, we do not give this observation as new in itself. A diminution of pressure in the cylinder, and even an augmentation of the volume of the steam, had already been recognized in these engines; but the observation of such a change, instead of leading to the theory of the steam-engine, such as we have developed it, was nothing but a material fact used to explain the *coefficient*, which we cannot admit.

As to that coefficient itself, it would be wrong to deem that, notwithstanding its inexactitude in principle, it might yet, without error, be used in practice to calculate Watt's steam-engines, for the reason alleged, that these engines, undergoing but slight variations of velocity, the application of a *constant* coefficient would suit them in all cases. This is certainly an error; for the smallest velocity of the piston in those engines being 150 feet per minute, and the greatest 300 feet per minute, it will be remarked that these are also the velocities of the piston in a locomotive engine, moving at from 10 to 20 miles an hour, as may be seen in the example of the ATLAS given in Chapter V. of this work. Consequently, there must always be, between the loads corresponding to these extreme velocities, or in other words, between the useful effects produced by the stationary engines, differences analogous to those which are observed for similar velocities in locomotives. This, besides, is made completely manifest by the calculation which we have given above.

If in fact we turn back to the useful effects obtained for the respective velocities of 286, 256, and

214 feet English per minute, and seek to represent them by a coefficient applied to what is called the *theoretic* effect, we shall have the following results:

Velocity.			Coefficient.
286	Theoretic effect	3236500	·50
	Real	1607610	
256	Theoretic effect	2897000	·61
	Real	1753600	
214	Theoretic effect	2013300	·81
	Real	1957180	

Thus, for these engines, as well as for those precedently treated of, no constant coefficient whatever can supply the place of the analytical calculation which we offer in its stead. And considering the great variation that takes place in that coefficient, from the very ordinary differences of the velocities 286, 256, and 214 feet per minute, for which the coefficient assumes the three values given above, we cannot but be convinced of the fallacy of the mode of calculation by coefficients; for however little the velocity of the engine to which the calculation is applied, differ from the velocity at which the coefficient has been originally determined, which is altogether unknown, no reliance whatever is to be placed on the effect indicated by that coefficient.

We thus see that Watt's engines make no exception to the general rule in this respect; and it may further be observed, that the experiment of Watt reported above, would give rise, in the ordinary theory, to precisely the same contradictions and inaccuracies as the experiments which we have discussed in Chapter I. on locomotives.

In effect, in this experiment the engine evaporating ·927 cubic foot of water per minute, and exercising a force of 50 horses, assumed a velocity of 286 feet per minute.

We find, then, that since the engine had a useful effect of only 50 horse-power, and that its *theoretic* force calculated according to that method, from the area of the cylinder, the effective pressure in the boiler and the velocity of the piston, was

$$\frac{6{\cdot}287 \times (16{\cdot}5 - 4) \times 144 \times 286}{33000} = 98 \text{ horse-power,}$$

it followed that, to pass from the theoretic effects to the practical required the coefficient of ·51. Consequently following the reasonings of that theory, we are led to these conclusions:—

1st. The observed velocity having been 286 feet per minute, the evaporation, calculated from the quantity of water which, being reduced to steam at the pressure of the boiler, might occupy the volume described by the piston, and afterwards divided, as indicated by several authors, by the coefficient, to account for the losses, would have been

$$\frac{\frac{1}{1530} \times 6{\cdot}287 \times 286}{{\cdot}51} = 2{\cdot}305 \text{ cubic feet of water per}$$

minute, instead of ·927.

2d. The engine having evaporated only ·927 cubic foot of water per minute, the velocity of the piston, calculated from the volume of steam formed at the pressure of the boiler, and afterwards reduced by the coefficient, not as indicated, since this problem has never been solved, but as one must naturally conclude from the signification attributed to that coefficient, could be no other than

$\frac{1530 \times {\cdot}927}{6{\cdot}287} \times {\cdot}51 = 115$ feet per minute, instead of 286.

3d. The coefficient found by the comparison of the theoretical to the practical effects being ·51, the frictions, losses and divers resistances of the engine amounted to ·49 of the effective power; whereas these frictions, losses and resistances consisting merely in the friction of the engine and the clearance of the cylinder, cannot be estimated at a higher rate than the following:—

Total friction, (additional friction included,) 2 lbs. per sq. inch, or as a fraction of the effective pressure, $\frac{2}{12}$	·17
Clearance of the cylinder, $\frac{1}{20}$th of the effective stroke, or	·05
	·22

Hence we see that Watt's experiment, as well as those made on locomotives, would be altogether inexplicable by that theory, and it would be the same with any experiment wherein should be noted the quantity of water evaporated.

Before quitting this example we must recall to notice that some authors employ *constant* coefficients also, but without using the same coefficient to determine the evaporation as to determine the useful effect. This mode of calculating originated in those authors having recognized by experience that the steam assumed in the cylinder a pressure and density less than in the boiler. But as they have not been able to fix *à priori* what was that pressure in the cylinder, and as they always seek to deduce it from that of the boiler, instead of concluding it immediately and in principle, as we do, from the resistance against the piston, the observed diminution of pressure could not be defined as to its limits, and it remained merely a practical fact of which they availed themselves to explain their *coefficient.* The change they make in the coefficient makes them avoid the first and second of the contradictions we have just signalized, but the third as well as all the objections which we have advanced elsewhere against the use of any constant coefficient whatever, remain in full force. That is to say, in this method, the power of the engine is always calculated independently of the evaporating force of the boiler, and the evaporation independently of the resistance to be moved; the effort of which the engine is capable is always found the same at all velocities; no account can be taken of the degree of opening of the regulator, except by introducing for that purpose a new series of coefficients, and the same for all the changes of velocity. In a word, it is the method which we have applied, in the first chapter, to the experiments on the locomotive engine the *Leeds*, and consequently it is directly liable to all the objections we have there made.

To return to the results of our formulæ, it is understood that the effects of steam-engines as we have shewn them, are producible only in as much as the various conditions of the calculation are fulfilled. But, as it often occurs that several important circumstances suffer change without any notice being taken of their alteration, we must here add: 1st, that as all effect produced depends directly and absolutely on the evaporation effected, it is utterly impossible to know the power or to compute the effects of a given steam-engine, without having previously measured or estimated its evaporation; 2d, that the pressure in the boiler, as well as that in the condenser, ought, for every case, to be observed by the manometer, because a change of pressure in the boiler is a circumstance occurring continually, and it is plain that if the engine works at a higher pressure, some of the effects produced will change in consequence; 3d, with respect especially to the effects attributable to the consumption of a given weight of fuel, we must observe that nothing can be more uncertain. Smeaton's experiments demonstrate that the different qualities of coal in use in England may produce effects varying in divers proportions between the numbers 86 and 133. He found, moreover, that refuse coal, such as is often used in some mines, compared to the same quality of coal in pieces of the size of an egg, produced effects in the proportion of 80 to 100; and finally that if the fire is ill managed, loaded in layers too thick and seldom stirred, the effects produced may be but $\frac{5}{6}$ths of what they would be with a clear well-stoked fire. Thus we see that all these causes combined may produce a difference of effect, between the least favourable and the most advantageous circumstances, in the proportions of

$$\frac{80}{100} \times \frac{5}{6} \times \frac{86}{133} = \frac{43}{100};$$

that is, the difference between the two may exceed the half.

It is clear, besides, that the effect due to a given weight of fuel will depend also on the more or less judicious construction of the boiler, as well as on the incrustations that may form upon it during the work. For these various reasons, then, it is impossible to found a mathematical comparison of the effects of steam-engines on the quantity of fuel reported to have been consumed by them in producing a given effect. This datum must evidently be accompanied by many others in order to be decisive.

In Watt's experiment, for instance, which has just been mentioned, the consumption of fuel was in the ratio of 7·24 lbs. of coal per cubic foot of water evaporated; whereas, from the medium observation of Watt himself, that evaporation is not generally produced at a consumption of less than 8·4 lbs. of the best coal. If, then, in the instance in question, the ordinary data had not been exceeded, the evaporation of ·927 cubic foot of water per minute, would have required a consumption of 7·79 lbs. of coal; and thus the effect arising from 1 lb. of fuel, would have been, in the case of maximum useful effect, only

$$\text{u. E. 1 lb. co.} = \frac{\text{max. u. E.}}{7{\cdot}79} = \frac{1957180}{7{\cdot}79} = 251240 \text{ lbs.};$$

a result which, as we see, is much inferior to that deduced from the experiment itself.

CHAPTER VII.

CORNISH DOUBLE-ACTING STEAM-ENGINES.

Practical formulæ for calculating those engines.

In the county of Cornwall both double-acting and single-acting engines are in use. The latter are a modification of Watt's single-acting engines. We shall not notice these at present, but direct our attention wholly to the former.

The Cornish double-acting engines, are the rotative or double-acting engines of Watt, with condensation, working at a higher pressure in the boiler, that is, at a total pressure of about 3·5 atmospheres, or more, and constructed to admit the use of the expansion of the steam, a force of which Watt availed himself only in his single-acting engines. These engines having but one cylinder, which serves at once for the direct admission of the steam and for its expansion, are no other than those we have treated of in Chapter III., in developing the general formulæ of the action of the steam on the piston, by direct admission, expansion and condensation. We shall not, then, here reproduce those formulæ under their algebraic form, but shall replace them by the corresponding numerical formulæ, to which end it will be necessary, first, to settle the value of the constant quantities which figure in them.

The friction of Watt's steam-engines, as has been said above, may be estimated at 1·5 lb. per square inch of the surface of the piston, when the piston is of the smallest dimensions in use in those engines, that is, about 17½ inches in diameter, and at ·5 lb. per square inch, when, on the contrary, the diameter of the piston is as much as about 48½ inches. It is for this reason that we have considered 1 lb. per square inch as representing, nearly enough for practical purposes, the friction of Watt's engines of mean dimensions. The Cornish engines being, as we have said, merely a modification of those of Watt, their friction may be estimated by the same rule, at least till special experiments be made on the subject. But as, in these engines, the expansion requires a cylinder of much greater capacity, it follows that a diameter of 48 inches, instead of being an extreme size for the piston, is, on the contrary, no more than a medium dimension. Consequently the friction proper to the mean dimensions of these engines, may be valued at ·5 lb. per square inch of the surface of the piston; which gives per square foot

$$f = \cdot 5 \times 144 \text{ lbs.}$$

In this valuation, however, we comprehend only the friction proper to the engine itself, and not that of any connecting rods, and other pieces of machinery more or less complex, which may serve to transmit the motion to points often very distant from the engine. The friction, or the resistance of these different parts, when they exist, ought to be valued separately, according to the circumstances of their construction, and then deducted from the result which we obtain for the useful effect; this representing, in our calculation, only the quantity of power as disposable on the very shaft of the engine. This observation, however, refers more particularly to pumping or single-acting engines.

Furthermore, we may, as in Watt's steam-engines, estimate also the additional friction caused by a given resistance, at $\frac{1}{7}$th of that resistance, or make

$$\delta = \cdot 14:$$

and admit that the pressure subsisting in the steam cylinder, after communicating with the condenser, will be about 4 lbs. per square inch, that is to say, we shall have

$$p = 4 \times 144 \text{ lbs.}$$

Thus, from what precedes, we might in the algebraic equations, replace the term $(p + f)$ by its mean value

$$4 \cdot 5 \times 144.$$

But as our value of the friction is only an estimation; as it is to be taken differently, according to the dimensions of the cylinder; and as, moreover, the pressure of condensation, p, varies also according to the construction of the engine, and likewise according to the quantity and temperature of the water used for condensation, we prefer to let the term $(p+f)$ remain in the equations, leaving its precise value to be given suitably to each case.

As to the coefficients of the relative volume of the steam, since the engine is a condensing one, they will have the values already given:—viz.

$$n = \cdot 00004227$$
$$q = \cdot 000000258.$$

Finally, the motion of the piston being regulated by a crank, the clearance of the cylinder is still, at a medium, $\frac{1}{20}$th of the stroke, as in all rotative engines, which gives

$$c = \cdot 05\, l.$$

Admitting, then, these valuations, and making the proper substitutions in the algebraic formulæ already developed, we shall have the following results, in which the value of k is

$$k = \frac{l'}{l'+c} + \log \frac{l+c}{l'+c}.$$

Practical formulæ for Cornish double-acting engines.

CASE OF AN INDEFINITE LOAD OR VELOCITY, WITH A GIVEN EXPANSION.

$$v = \frac{S}{a} \cdot \frac{10000\,k}{\cdot 4227 + \cdot 00258\,(1+\delta)\,r + \cdot 00258\,(p+f)}$$ Velocity of the piston, in feet per minute.

$$ar = 3875970\,\frac{k\,S}{(1+\delta)\,v} - \frac{a}{1+\delta}\,(164+p+f)$$ ... Useful load of the piston, in pounds.

$$S = \frac{a\,v}{10000\,k}\,[\cdot 4227 + \cdot 00258\,(1+\delta)\,r + \cdot 00258\,(p+f)]$$ Effective evaporation, in cubic feet of water per minute.

u. E. .... $= a\,r\,v$ .... Useful effect, in lbs. raised 1 foot per minute.

u. HP................ $= \frac{\text{u. E.}}{33000}$ .... Useful force, in horse-power.

u. E. 1 lb. co. ...... $= \frac{\text{u. E.}}{N}$ .... Useful effect of 1 lb. of coal, in lbs. raised 1 foot.

u. E. 1 ft. wa. ...... $= \frac{\text{u. E.}}{S}$ .... Useful effect of one cubic foot of water, in lbs. raised 1 foot.

$$\text{Q. co. for 1 hp....} = \frac{33000\,\text{N}}{\text{u. E.}}$$ .... Quantity of coal, in lbs., producing 1 horse-power.

$$\text{Q. wa. for 1 hp....} = \frac{33000\,\text{S}}{\text{u. E.}}$$ .... Quantity of water, in cubic feet, producing 1 horse-power.

$$\text{u. HP. for 1 lb. co.} = \frac{\text{u. E.}}{33000\,\text{N}}$$ .... Useful horse-power produced by 1 lb. of coal.

$$\text{u. HP. for 1 ft. wa.} = \frac{\text{u. E.}}{33000\,\text{S}}$$ .... Useful horse-power produced by 1 foot of water evaporated.

CASE OF MAXIMUM USEFUL EFFECT, WITH A GIVEN EXPANSION.

$$v' = \frac{l}{l'+c} \cdot \frac{\text{S}}{a} \cdot \frac{10000}{\cdot 4227 + \cdot 00258\,\text{P}}$$ .... Velocity of the piston, in feet per minute.

$$ar' = \frac{a}{1+\delta} \cdot \frac{l'+c}{l} k(164+p) - \frac{a}{1+\delta}(164+p+f)$$ Useful load of the piston, in pounds.

$$\text{S} = \frac{l'+c}{l} \cdot \frac{a\,v'}{10000}(\cdot 4227 + \cdot 00258\,\text{P})$$ .... Effective evaporation, in cubic feet of water per minute.

max. u. E. $= a r' v'$ .... Useful effect, in lbs. raised 1 foot per minute.

CASE OF ABSOLUTE MAXIMUM USEFUL EFFECT.

$\frac{l'}{l} = \frac{164+p+f}{164+P}$ .... Expansion which produces the absolute maximum of useful effect.

To make use of these formulæ, the first thing to be done is to determine the quantity k. But as the expansion at which the engine works is known, or rather that point of the stroke at which the expansion begins, or the ratio

$$\frac{l'}{l},$$

the corresponding value of k will be immediately concluded, by means of the table given in Chapter III. Article I., as well as that of the fraction

$$\frac{l}{l'+c}.$$

If the table above mentioned is not at hand, it will be necessary to seek directly the value of the term

$$\frac{l}{l'+c}.$$

Then, to obtain the term

$$\log. \frac{l+c}{l'+c},$$

which is an hyperbolic logarithm, the common logarithm must be taken of

$$\frac{l+c}{l'+c};$$

and this being multiplied by

$$2{\cdot}302585$$

will give the hyperbolic logarithm sought. So the value of k will be known as before, and then the various formulæ will offer no other difficulty, since they contain no terms beyond the first degree.

We have said that the Cornish engines generally work at the total pressure in the boiler of 50 lbs. per square inch. It has been seen also that their mean friction may be estimated at ·5 lb. per square inch of the surface of the piston, and the pressure of condensation under the piston at 4 lbs. per square inch. Now, the table of Chapter III. shows that the relative volume of the steam, under the pressure of 4·5 lbs. per square inch, is very nearly 5160 times that of water; and that under the pressure of 50 lbs. per square inch, the relative volume is expressed by 552. We see then, bearing in mind at the same time, that the formula of the expansion of absolute maximum useful effect expresses the ratio of the relative volumes of the steam, under the respective pressures P and $(p+f)$, that in the Cornish engines the absolute maximum of useful effect will be given in general by the equation

$$\frac{l'}{l} = \frac{552}{5160} = \cdot 11.$$

It is, however, to be remarked that, if the engine should condense less perfectly, or its friction be somewhat greater than we have supposed, the preceding fraction will tend rather to increase. On the other hand, if the pressure in the boiler be above 50 lbs. per square inch, the fraction will tend to diminish. Thus it may happen in some of these engines, that the maximum of useful effect shall be produced by cutting the steam when the piston has performed ·11 or ·12 of the stroke; and in others, when it has performed ·10 of it, or even less.

The expansion which produces the absolute maximum of useful effect having been determined by the last formula given above, substituting the value of l' thus found, in the formulæ of the case of maximum effect with a given expansion, we obtain all the determinations relative to the maximum useful effect which that expansion can produce; and since that expansion is the most favourable for the engine, these determinations will be found to be precisely those of the absolute maximum useful effect of which the engine is capable.

As an example of the application of these formulæ, we will suppose that the Watt engine, whose dimensions and capacities have been given in Article III. of the preceding chapter, is adjusted to generate the steam at the total pressure of 50 lbs. per square inch in the boiler, and to use the expansion according to the Cornish system; and we will then calculate its effects. This supposition will furnish a natural comparison of the two systems, and will enable us to recognise the considerable advantages to be derived from the use of the expansion of the steam.

We will suppose then that the same boiler abovementioned be retained, but that the cylinder be changed for another of greater capacity, in which the admission of the steam shall be intercepted when the piston has performed only one quarter of the stroke. Therefore we will suppose the following dimensions:

Diameter of the cylinder, 48 inches; or surface of the piston, $a = 12{\cdot}566$ square feet.

Stroke of the piston, 10 feet; or $l = 10$.

Space traversed by the piston before the expansion begins, $\frac{1}{4}$ of the stroke; or $l' = {\cdot}25\, l$.

From these dimensions, we may estimate the friction of the engine, when unloaded, at ·5 lb. per square inch of the surface of the piston; and if, moreover, we suppose the condensation to be operated to the same degree as in the Watt original engine, we shall have for f and p the values given above, that is:

$$f = {\cdot}5 \times 144 \text{ lbs.}, \quad p = 4 \times 144 \text{ lbs.}$$

We have seen, besides, that it requires the same quantity of total heat, and therefore nearly the same expenditure of fuel, to produce the same weight of steam under different degrees of tension. Whence it is to be concluded, that at the pressure of 50 lbs. per square inch, at which we purpose to work the engine, the boiler will retain the same evaporation of water per minute as before, consuming nearly the same quantity of fuel in the same time. We will admit then for the engine, in this respect, the same data already reported, and shall have

Total pressure in the boiler, 50 lbs. per square inch; or $P = 50 \times 144$ lbs. per square foot.

Effective evaporation, ·927 cubic foot of water per minute; or $S = \cdot 927$.

Consumption of fuel in the same time, 6·71 lbs.; or $N = 6{\cdot}71$.

Applying the formulæ to this engine, we shall find, for the effects produced at the velocity of maximum useful effect and at the respective velocities of 200 and 250 feet per minute, the following results:

Effects of the engine with the given expansion.

				Maximum of useful effect.
$\frac{l'}{l}$	=	·25	·25	·25
v	=	250	200	136
ar	=	17337	23909	39465
$\frac{r}{144}$	=	9·58	13·21	21·81
S	=	·927	·927	·927
u. E.	=	4334210	4781800	5356860
u. HP.	=	131·34	144·90	162·33
u. E. 1 lb. co.	=	645940	712640	798350
u. E. 1 ft. wa.	=	4675520	5158350	5778700

Q. co. for 1 hp......	=	·051	·046	·041
Q. wa. for 1 hp	=	·007	·006	·0057
u. HP. for 1 lb. co.	=	19·57	21·60	24·19
u. HP. for 1 ft. wa.	=	141·7	156·3	175·1

Maxima useful effects of the engine, with various expansions.

		Absolute max. of useful effect.		
$\frac{l'}{l}$	=	·50	·25	·11
v'	=	74	136	255
ar'	=	57247	39465	22865
$\frac{r'}{144}$	=	31·64	21·81	12·64
S	=	·927	·927	·927
max. u. E.	=	4238480	5356860	5819300
u. HP.	=	128·44	162·33	176·34
u. E. 1 lb. co.	=	631670	798350	867260
u. E. 1 ft. wa.	=	4572250	5778700	6277560
Q. co. for 1 hp......	=	·052	·041	·038
Q. wa. for 1 hp.	=	·007	·0057	·005
u. HP. for 1 lb. co.	=	19·14	24·19	26·28
u. HP. for 1 ft. wa.	=	138·6	175·1	190·2

These two tables shew what diversity of effect may be produced by the same engine, according to the degree of expansion at which it works and the load imposed on it. In calculating the maxima useful effects for different rates of expansion, they are found continually to increase, on diminishing the portion l' of the stroke described before the expansion, till we have $l' = ·11\ l$; and beyond that point, they are on the contrary found to diminish; which demonstrates by the facts, that the absolute maximum useful effect of the engine is really attained. We find in effect:

$\frac{l'}{l}$ = ·10.........max. u. E. = 5806300

·11..........................5819300 ab. max.

·12..........................5817820

From the results we have just obtained, it is observable, that by using a sufficient degree of pressure in the boiler of a Cornish engine, and by carrying the principle of the expansion far enough, a useful effect may be obtained nearly treble of that which would be produced in the Watt engine, by the same quantity of water evaporated. It is without reason, then, that the effects of the Cornish engines have been considered so extraordinary; and that they have sometimes even been reputed incredible.

The effect, however, of every engine must obviously depend: on the pressure at which the steam is generated in the boiler; on the improvements introduced into the construction of the boiler and the furnace, whence will result that the same quantity of fuel more judiciously applied, will evaporate a greater quantity of water; on the quality of the fuel employed; on the condensation more or less complete in the condenser; on the care taken to clear the boiler of the incrustations which form on it and obstruct its evaporating power; on the more perfect workmanship in the execution of the engine, which may diminish its friction; and, in fine, on the nature and number of the connecting rods, which, in transmitting the action of the engine to distant points, absorb in their motion a portion, more or less considerable, of the useful effect really produced by the engine.

For this reason, if it be required to compute with great accuracy, what would be, in certain circumstances, the effects of an engine already constructed, it will not suffice to take an approximate valuation of the friction, nor of the pressure of condensation in the cylinder. But the friction of the engine must be determined by the method developed in Sect. III. of Article II., Chapter III. The pressure subsisting in the steam cylinder, after communication with the condenser, must also be measured by Watt's *indicator;* and the two quantities f and p, thus determined, must be introduced into the formulæ, with the other data of the problem equally deduced from immediate admeasurement or observation.

CHAPTER VIII.

WOOLF'S OR EDWARDS' STEAM-ENGINES.

ARTICLE I.

THEORY OF WOOLF'S STEAM-ENGINE.

Woolf's or Edwards' engines are of the same kind as the preceding: that is to say, the expansion of the steam is used also, but instead of that expansion being effected in a single cylinder, it is performed in two unequal ones, which the steam traverses, successively dilating itself more and more.

The steam coming directly from the boiler is first admitted into the upper part of the small cylinder, and there acts at full pressure during a portion of the stroke. Then the communication with the boiler is intercepted, and the steam already received into the small cylinder expands till it has driven the piston to the bottom of its stroke. At this moment, a communication opens between the top of the small cylinder and the bottom of the large one. The steam which filled the upper part of the small cylinder passes then into the larger, and coming under the large piston and expanding, makes it perform a stroke upwards. But at the moment the communication opens from the small cylinder to the large one, the passage from the boiler to the small cylinder opens also, to let in a new supply of steam, which now penetrates under the smaller piston, so as to make it perform an ascending stroke.

The two pistons then rise together: the smaller, by the direct action of the steam from the boiler at first, and then by that steam expanded; the larger, by the expansion of the steam which has produced the preceding motion in the small cylinder. Thus, the two pistons are brought simultaneously to the top of their cylinders, and the same effect is renewed. Finally, after having terminated its action in the greater cylinder, the steam is received into a separate vessel and there condensed.

The two piston rods articulate on the beam of the engine, and both tend to communicate to it a motion of oscillation, which is afterwards converted, by means of a crank, into a circular motion; but as these articulations may be placed at different distances from the centre, the stroke of the larger piston may be made longer than that of the smaller, and thus the large cylinder may offer a capacity by so much the more considerable for the expansion of the steam.

It is plain that the mode of action of the steam in these engines is precisely the same as in the preceding; but as the difference in the surface of the two pistons, on which the steam acts during the expansion, must give rise to some modifications in the formulæ, we will briefly introduce those circumstances into the general theory.

Let P be the pressure of the steam in the boiler, and P′ the pressure that steam assumes on entering the small cylinder, before the expansion. Let A and *a* be the surfaces of the two pistons, L and *l* their respective strokes, and C and *c* the clearances of the cylinders. Let *l*′, in fine, be the portion of the stroke performed by the small piston before the expansion.

If we take the engine after it has attained uniform motion, the quantity of labour applied by the power will be equal to the quantity of action developed in the same time by the resistance. Now the effort applied by the power consists of the pressure exerted against the small piston, by the steam coming from the boiler, and of the pressure exerted against the large piston, by the steam coming from the small cylinder. The resistance, on the contrary, consists of the pressure produced against the small piston, by the reaction of the steam which serves as a motive force to the larger; of the pressure subsisting behind the large piston, after imperfect condensation of the steam which effected the preceding stroke; and, finally, of the load and of the friction of the engine.

We will then estimate successively the quantity of action developed by each of these five forces, during one oscillation of the beam, and will afterwards form the equation of the dynamical equilibrium of the engine.

1st. Referring to the calculation which has already been made in Chapter III., Article I., Sect. II., of the action of the expansion of the steam, it will be found that the quantity of labour developed by the direct entrance of the steam into the small cylinder, and by its expansion there, has for its expression

$$a\,(l'+c)\left(\frac{n}{q}+\mathrm{P}'\right)\left[\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right]-\frac{n}{q}\,al.$$

2d. To obtain the quantity of motive labour developed also in the large cylinder during one oscillation of the engine, it must be observed that it is the same steam which, after having first occupied the length l' of the small cylinder, with the pressure P', is now diffused partly under the small piston and partly above the large one, with a pressure corresponding to the space it fills.

If, then, we consider the point of the oscillation of the engine, at which the small piston has performed a portion λ of its stroke, and at which the pressure of the expanded steam, above the large piston or below the small one, is become π; it is plain that we shall have, between the two pressures π and P' and the spaces respectively occupied by the steam, the general relation (*c*), which we have originally written under the form

$$p=\frac{\mathrm{M}'}{\mathrm{M}}\left(\frac{n}{q}+p'\right)-\frac{n}{q};$$

and which expresses that, during its change of volume, the steam remains at the maximum density for its temperature.

But, since the strokes L and l of the two pistons are performed in the same time, it follows that when the smaller one shall have performed the length λ of its stroke, the larger one will have performed of its stroke

$$\frac{L}{l}\lambda.$$

Consequently, the space occupied by the expanded steam, as well above the large piston as below the small one, will be

$$A\left(\frac{L}{l}\lambda+C\right)+a(l-\lambda+c)=\frac{AL-al}{l}\lambda+a(l+c)+AC.$$

We will for a moment write it under the form

$$B\lambda+Q,$$

B representing the coefficient of λ, and Q the constant quantity.

This expression gives, then, the value of the present volume of the steam, under the pressure π. Now the same steam under the pressure P′, occupied the volume $a(l'+c)$. We have then, from the general analogy indicated a little above,

$$\pi=\frac{a(l'+c)}{B\lambda+Q}\left(\frac{n}{q}+P'\right)-\frac{n}{q}.$$

Consequently, proceeding as before, that is to say, multiplying both members of the equation by $\frac{AL}{l}d\lambda$, and then taking the integral between the limits $\frac{L}{l}\lambda=O$ and $\frac{L}{l}\lambda=L$, or $\lambda=O$ and $\lambda=l$, we shall have the value of the total labour produced by the expansion of the steam on the large piston, from the beginning of the stroke to the end, viz:

$$a(l'+c)\left(\frac{n}{q}+P'\right)\frac{AL}{Bl}\log\frac{Bl+Q}{Q}-\frac{n}{q}AL.$$

3d. To obtain the expression of the quantity of action developed by the resistance of the steam under the small piston, it is to be observed that that steam being at the pressure π, like the steam in the large cylinder, it will suffice to multiply the value of π obtained above, by $a\,d\lambda$, and to take the integral between the limits $\lambda=O$ and $\lambda=l$. That operation will give us the quantity of action developed by the resistance of the steam against the motion of the small piston, from the beginning of the stroke to the end. The result will obviously be the same as that just obtained, except that $\frac{AL}{l}$ will be replaced by a It will then be

$$a(l'+c)\left(\frac{n}{q}+P'\right)\frac{a}{B}\log\frac{Bl+Q}{Q}-\frac{n}{q}\,a\,l.$$

4th. Expressing still by p the pressure subsisting in the cylinder which communicates with the condenser, as this cylinder is here the great one, the quantity of resisting action developed by the force p, during one stroke, will be

$$p\,AL.$$

5th. If, in fine, we represent by R the resistance set in motion by the engine, measured not per unit of surface, but in absolute magnitude, and by h

the space which that resistance traverses at each stroke of the piston, the quantity of action produced by that force during one stroke will evidently be

$$R\,h.$$

Again, supposing the friction of the engine to be represented by two forces: one f exerted upon each unit of the surface of the small piston, and the other F exerted upon each unit of the surface of the large piston, the labour produced by these two forces together, during one stroke, will be

$$f\,a\,l + \mathrm{F\,A\,L}.$$

Besides, if we call δ the surplus friction of the engine, accruing from each unit of the resistance r, and measured at the same velocity as that resistance, the quantity of action produced by the latter force during one stroke, will be

$$\delta\,R\,h.$$

Thus, the quantity of action produced by these three resistances together, during one oscillation of the engine, will be

$$(1 + \delta)\,R\,h + f\,a\,l + \mathrm{F\,A\,L}.$$

We have, then, all the elements of the labour developed by the power and by the resistance. Consequently, forming the equation of their dynamical equilibrium, and transposing into the first member, the term which expresses the quantity of action developed by the third of the forces estimated, that is to say, by the reaction of the steam against the small piston, we shall have:

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}'\right)\left(\frac{l}{l'+c}+\log\frac{l+c}{l'+c}+\frac{\mathrm{AL}-al}{\mathrm{B}l}\log\frac{\mathrm{B}l+\mathrm{Q}}{\mathrm{Q}}\right)-\frac{n}{q}\mathrm{AL}$$
$$=(1+\delta)\,\mathrm{R}h+fal+\mathrm{FAL}+p\mathrm{AL}.$$

Finally, replacing the quantities B and Q by their values, we obtain the relation

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}'\right)\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}+\log\frac{\mathrm{A(L+C)}+ac}{a(l+c)+\mathrm{AC}}\right)-\frac{n}{q}\mathrm{AL}$$
$$=(1+\delta)\,\mathrm{R}h+fal+\mathrm{FAL}+p\mathrm{AL}\ldots\ldots\ldots\ldots\ldots(\mathrm{A})$$

This equation, then, is the first general relation that we shall establish between the data and the incognita of the problem. The second will, as in the preceding calculations, be deduced from the consideration of the equality between the production and the expenditure of the steam.

S being the volume of water evaporated per unit of time in the boiler,

$$\frac{\mathrm{S}}{n+q\,\mathrm{P}'}$$

will be the volume of the resulting steam, measured at the pressure P′, that is, at the pressure of the small cylinder, before the expansion. Again, v being the velocity of the small piston,

$$\frac{v}{l}\,a(l'+c)$$

will be the expenditure of steam per unit of time, measured at the moment of its passage into the small cylinder, before expansion, that is, at the same pressure P′. Thus the second relation will be as before,

$$\frac{S}{n+q\,P'}=\frac{v}{l}\,a\,(l'+c)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(B)$$

Consequently, eliminating P′ from these two equations and making, in order to simplify the formulæ,

$$\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}+\log\frac{A\,(L+C)+ac}{a\,(l+c)+AC}=k',$$

we shall obtain for the required value of v,

$$v=\frac{l}{L}\cdot\frac{S}{A}\cdot\frac{k'}{n+q\,\frac{1}{AL}\,[(1+\delta)\,R\,h+f\,a\,l+F\,A\,L+p\,A\,L]}.$$

This velocity will be that of the small piston; and from this velocity we conclude that of the large piston which will be $\frac{L}{l}\,v$, and that of the point of application of the resistance R, which will evidently be $\frac{h}{l}\,v$, or

$$V=\frac{h}{L}\cdot\frac{S}{A}\cdot\frac{k'}{n+q\,\frac{1}{AL}\,[(1+\delta)\,R\,h+f\,a\,l+F\,A\,L+p\,A\,L]};$$

since the resistance R traverses the distance h, and the large piston the distance L, in the same time that the small piston performs its stroke l.

It is to be observed that the value of V, whence all the other formulæ are afterwards deduced, is precisely similar to that which we have obtained in general, in Chapter III., with the exception only that the quantity $\frac{h}{L}\,k'$ replaces k, and that the factor

$$\frac{1}{\mathrm{A\,L}}\left[(1+\delta)\,\mathrm{R}h+fal+\mathrm{F\,A\,L}+p\,\mathrm{A\,L}\right]$$

replaces the similar factor

$$[(1+\delta)\,r+p+f].$$

It will be easy then to derive the formulæ of the divers effects of the engine, from the general formulæ given in Chapter III. We shall, moreover, remark that if, in the two expressions just obtained for the values of k' and V, we were to make $\mathrm{A}=a$, $\mathrm{L}=l=h$, and $\mathrm{C}=c$, these values would be reduced precisely to those of Chapter III.; and this in effect must be so, since the supposition of $\mathrm{A}=a$ and $\mathrm{L}=l$ amounts to reducing the two cylinders to one.

Performing, then, the calculations as before, we obtain the following formulæ:

Case of any load or velocity, with a given expansion.

$$\mathrm{V}=\frac{h}{\mathrm{L}}\cdot\frac{\mathrm{S}}{\mathrm{A}}\cdot\frac{k'}{n+q\dfrac{1}{\mathrm{AL}}[(1+\delta)\,\mathrm{R}h+fal+\mathrm{FAL}+p\mathrm{AL}]}.$$

$$\mathrm{R}=\frac{k'\mathrm{S}}{q(1+\delta)\mathrm{V}}-\frac{n}{q}\cdot\frac{\mathrm{AL}}{(1+\delta)\,h}-\frac{fal+\mathrm{FAL}+p\,\mathrm{AL}}{(1+\delta)\,h}.$$

$$\mathrm{S}=\frac{\mathrm{L}}{h}\cdot\frac{\mathrm{AV}}{k'}\left\{n+q\frac{1}{\mathrm{AL}}[(1+\delta)\,\mathrm{R}h+fal+\mathrm{FAL}+p\,\mathrm{AL}]\right\}$$

$$\text{u. E.}=\mathrm{RV}.$$

Case of maximum useful effect, with a given expansion.

$$V' = \frac{h}{l} \cdot \frac{l}{l'+c} \cdot \frac{S}{a} \cdot \frac{1}{n+qP}.$$

$$R' = \frac{l'+c}{l} \cdot \frac{al}{(1+\delta)h} k' \left(\frac{n}{q} + P\right) - \frac{n}{q} \cdot \frac{AL}{(1+\delta)h} - \frac{fal + FAL + pAL}{(1+\delta)h}.$$

$$S = \frac{l}{h} \cdot \frac{l'+c}{l} \cdot aV'(n+qP).$$

max. u. E. $= R'V'$.

Case of absolute maximum useful effect.

$$\frac{l'}{l} = \frac{AL}{al} \cdot \frac{\frac{n}{q} + \frac{1}{AL}(fal + FAL + pAL)}{\frac{n}{q} + P}.$$

This latter formula makes known, as we have explained elsewhere, the expansion which produces the absolute maximum of useful effect; and by introducing the value thus found into the formulæ which give the maximum effect for a given expansion, we have the maximum effect which that expansion can produce, and consequently the *absolute* maximum effect of the engine.

ARTICLE II.

PRACTICAL FORMULÆ FOR CALCULATING WOOLF'S ENGINES.

The formulæ which we have obtained above, offer no difficulty in their application. To resolve them numerically, recourse may be had to the table given in Article I. Chapter III., which furnishes, without calculation, the value of the terms

$$\frac{l}{l'+c} \text{ and } \frac{l'}{l'+c} + \log \frac{l+c}{l'+c}.$$

Then, having calculated the expression

$$\frac{A(L+C)+ac}{a(l+c)+AC},$$

its hyperbolic logarithm must be taken directly in tables of that system; or not having such tables, its ordinary logarithm will be taken in a table of common logarithms, and then multiplied by the number

$$2{\cdot}302585.$$

The product will be the hyperbolic logarithm required. In case the table of Chapter III. should not be at hand, the hyperbolic logarithm of the quantity

$$\frac{l+c}{l'+c}$$

would be found in the same way. The value, therefore, of the expression which we have represented by k' will be easily obtained, and thus the solution of the formulæ will offer no difficulty, for the quantities contained in them are connected in the first degree only.

To transform the equations which we have just obtained into accurate numerical formulæ, it would be necessary to know the precise value of the constant quantities, for which purpose special experiments would be requisite. However, till such experiments are made, we will calculate these formulæ numerically, contenting ourselves with such approximate value of the constant quantities, as may be deduced from the analogy of those engines with those already treated of.

In Watt's engines, which have but one cylinder, the friction of the engine, without load, amounts to 1·5 lb. per square inch of the surface of the piston, for a cylinder of small dimensions, and to ·5 lb. per square inch of the piston, for a cylinder of large dimensions, as has been explained above. In Woolf's engines there are two cylinders instead of one, and it will be observed that each of them requires nearly the same machinery and produces nearly the same friction as if it were alone in the engine. The safest way then of estimating the friction of these engines, is to attribute to each of the pistons the friction suitable to its dimensions in the Watt engine.

We will take, moreover, at the same rate as in the other engines, the additional friction per unit of the load, that is, we will make

$$\delta = \cdot 14.$$

The pressure of condensation in the cylinder communicating with the condenser, will also be, in good engines,

$$p = 4 \times 144 \text{ lbs.}$$

The clearance of the cylinder will be $\frac{1}{20}$ of the stroke, which gives

$$c = \cdot 05\, l, \text{ and } C = \cdot 05\ L$$

And in fine, since the engine is a condensing one, we must take

$$n = \cdot 00004227,$$
$$q = \cdot 000000258.$$

Admitting then these approximate valuations, and bearing in mind that the value of k' is

$$k' = \frac{l'}{l'+c} + \log \frac{l+c}{l'+c} + \log \frac{A(L+C)+ac}{a(l+c)+AC},$$

we obtain the following formulæ.

PRACTICAL FORMULÆ FOR WOOLF'S ENGINES.

Case of any load or velocity, with a given expansion.

$$V = \frac{h}{L} \cdot \frac{S}{A} \cdot \frac{10000\,k'}{\cdot 4227 + \cdot 00258 \frac{1}{AL}\left[(1+\delta)\,R\,h + fal + FAL + pAL\right]} \;\ldots$$

. . . . Velocity of the load, in feet per minute.

$$R = 3875970 \frac{k'S}{(1+\delta)V} - \frac{1}{(1+\delta)h}(164\,AL + fal + FAL + pAL) \;\ldots$$

. . . . Useful load of the engine, in lbs.

$$S = \frac{L}{h} \cdot \frac{AV}{10000\,k'}\left[\cdot 4227 + \cdot 00258(1+\delta)\frac{Rh}{AL} + \cdot 00258\,\frac{1}{AL}(fal + FAL + pAL)\right]$$

. . . . Effective evaporation, in cubic feet of water per minute.

u. E.	$= R V$	Useful effect, in lbs. raised one foot per minute.
u. HP...............	$= \frac{\text{u. E.}}{33000}$	Useful horse-power.
u. E. 1 lb. co. ...	$= \frac{\text{u. E.}}{N}$	Useful effect of 1 lb. of coal, in lbs. raised 1 foot.
u. E. 1 ft. wa. ...	$= \frac{\text{u. E.}}{S}$	Useful effect of 1 cubic foot of water, in lbs. raised 1 foot.
Q. co. for 1 hp....	$= \frac{33000\,N}{\text{u. E.}}$	Quantity of coal, in lbs., which produces one horse-power.
Q. wa. for 1 hp....	$= \frac{33000\,S}{\text{u. E.}}$	Quantity of water, in cubic feet, which produces one horse-power.
u. HP. for 1 lb. co.	$= \frac{\text{u. E.}}{33000\,N}$	Horse-power produced by 1 lb. of coal.
u. HP. for 1 ft. wa.	$= \frac{\text{u. E.}}{33000\,S}$	Horse-power produced by 1 cubic foot of water evaporated.

Case of maximum useful effect, with a given expansion.

$V' = \frac{h}{l} \cdot \frac{l}{l'+c} \cdot \frac{S}{a} \cdot \frac{10000}{\cdot 4227 + \cdot 00258\,P}$ .... Velocity of the load, in feet per minute.

$R' = \frac{l'+c}{l} \cdot \frac{al}{(1+\delta)h} k'(164 + P) - \frac{1}{(1+\delta)h}(164\,AL + fal + FAL + pAL)$ .... Useful load of the engine, in lbs.

$S = \frac{l}{h} \cdot \frac{l'+c}{l} \cdot \frac{aV'}{10000}(\cdot 4227 + \cdot 00258\,P)$ .... Effective evaporation, in cubic feet of water per minute.

max. u. E. $= R'V'$ .... Useful effect, in lbs. raised 1 foot per minute.

Case of absolute maximum of useful effect.

$\frac{l'}{l} = \frac{AL}{al} \cdot \frac{164 + \frac{1}{AL}(fal + FAL + pAL)}{164 + P}$ ...... Expansion which produces the absolute maximum of useful effect.

The expansion of the absolute maximum of useful effect being determined by this formula, then introducing it into the formulæ of the case of maximum effect for a given expansion, we have the maximum of useful effect producible by that expansion, and consequently the absolute maximum useful effect of the engine.

CHAPTER IX.

EVANS'S ENGINES.

PRACTICAL FORMULÆ FOR CALCULATING THESE ENGINES.

In Evans's engines, the steam generated at the pressure of about 8 atmospheres, or 120 lbs. per square inch, is admitted into the cylinder during about a third part of the stroke of the piston; the communication with the boiler is then intercepted, and the piston continues its motion by means of the expansion of the steam; after which the steam generally escapes into the atmosphere without any use being made of condensation.

The steam, therefore, in these engines acts by expansion, as in the Cornish engines; and thus the formulæ proper to calculate them, are the same as those which have been given in Chapter VII. The only difference is that the quantity P will represent a higher pressure of formation of the steam, and that the quantity p, instead of expressing the pressure due to the imperfect condensation of the steam, will express the atmospheric pressure.

We shall not give these formulæ under their algebraic form, since it would be a mere repetition of those of Chapter III.; but we will present them under their numerical form, taking the value of the constant quantities by approximation, from analogy with the Watt engines and the locomotives, of which we have given the determinations above.

To this end we will notice that the high pressure at which the steam is employed in this system, admits of a cylinder of very small capacity. The result is, that in a middle-sized engine of Evans, the diameter of the cylinder is scarcely half what it is in the smallest of Watt's engines that it has occurred to us to mention, and in which the friction is about 1·5 lb. per square inch. Supposing then the total friction the same in both, it is visible that considering it divided per square inch of the surface of the piston, it must, in Evans's engine, exert four times the resistance that it would in Watt's engine; and consequently in Evans's engines of mean force, the friction of the engine unloaded should be estimated at $1{\cdot}5 \times 4 = 6$ lbs. per square inch, which gives

$$f = 6 \times 144 \text{ lbs.}$$

Moreover, as before, we may admit the following determinations:

Additional friction of the engine, per unit of the resistance imposed on it, $\frac{1}{7}$th of that resistance; or $\delta = {\cdot}14$.

Atmospheric pressure, per square foot, $p = 14{\cdot}71 \times 144$ lbs.

Clearance of the cylinder, $c = {\cdot}05\,l$.

Finally, the coefficients of the relative volume of the steam will be, as in non-condensing engines

$$n = {\cdot}0001421,$$
$$q = {\cdot}00000023.$$

Admitting then these data, and bearing in mind that the value of k is

$$k = \frac{l'}{l'+c} + \log \frac{l+c}{l'+c},$$

we obtain the following equations:

PRACTICAL FORMULÆ FOR EVANS'S ENGINES.

Case of any load or velocity, with a given expansion.

$$v = \frac{S}{a} \cdot \frac{10000\,k}{6{\cdot}29 + {\cdot}0023\,(1+\delta)\,r + {\cdot}0023\,f} \ldots\ldots$$ Velocity of the piston, in feet per minute.

$$ar = 4347830\,\frac{k\,S}{(1+\delta)\,v} - \frac{a}{1+\delta}\,(2736 + f) \ldots\ldots$$ Useful load of the piston, in lbs.

$$S = \frac{a\,v}{10000\,k}\left[6{\cdot}29 + {\cdot}0023\,(1+\delta)\,r + {\cdot}0023\,f\right]$$ Effective evaporation, in cubic feet of water per minute.

$u. E. = arv$	Useful effect, in lbs. raised 1 foot per minute.
$u. HP. = \frac{u. E.}{33000}$	Useful horse-power.
$u. E. 1 lb. co. = \frac{u. E.}{N}$	Useful effect of 1 pound of coal, in lbs. raised 1 foot.
$u. E. 1 ft. wa. = \frac{u. E.}{S.}$	Useful effect of 1 cubic foot of water, in lbs. raised 1 foot.
$Q. co. for 1 hp. = \frac{33000 N}{u. E.}$	Quantity of coal, in lbs., producing 1 horse-power.
$Q. wa. for 1 hp. = \frac{33000 S}{u. E.}$	Quantity of water, in cubic feet, producing 1 horse-power.
$u. HP. for 1 lb. co. = \frac{u. E.}{33000 N}$	Horse-power produced by 1 pound of coal.
$u. HP. for 1 ft. wa. = \frac{u. E.}{33000 S}$	Horse power produced by 1 cubic foot of water evaporated.

Case of maximum useful effect with a given expansion.

$v' = \frac{l}{l'+c} \cdot \frac{S}{a} \cdot \frac{10000}{1{\cdot}421 + {\cdot}0023 P}$ .... Velocity of the piston, in feet per minute.

$a r' = \frac{a}{1+\delta} \cdot \frac{l'+c}{l} k\,(618+P) - \frac{a}{1+\delta}(2736+f)$ Useful load of the piston, in lbs.

$S = \frac{l'+c}{l} \cdot \frac{a v'}{10000}\,(1{\cdot}421 + {\cdot}0023\ P)$ .... Effective evaporation, in cubic feet of water per minute.

max. u. E. $= a\,r'\,v'$ .... Useful effect, in lbs. raised 1 foot per minute.

Case of absolute maximum useful effect.

$\frac{l'}{l} = \frac{2736+f}{618+P}$ .... Expansion, which produces the absolute maximum of useful effect.

This latter relation having made known the expansion which produces the absolute maximum of useful effect, if that expansion be introduced into the formulæ of maximum effect for a given expansion, all the determinations relative to the absolute maximum useful effect of the engine will be derived.

As, in this system, the steam is usually employed at the pressure of 120 lbs. per square inch, we see from the last formula, and adopting also the valuation just made of the friction, that the expansion of absolute maximum useful effect will generally be given by the relation

$$\frac{l'}{l} = \frac{3600}{17898} = \cdot 20 ;$$

that is to say, the greatest possible effects are to be obtained by intercepting the action of the steam at about a fifth part of the stroke.

CHAPTER X.

WATT'S SINGLE-ACTING ENGINES.

ARTICLE I.

THEORY OF WATT'S SINGLE-ACTING ENGINES.

SECT. I. *Of the regulation of the engine.*

IN Watt's single-acting engines, the steam is applied only on the upper side of the piston, and the action of the engine, to raise the water in the pumps, or to produce the useful effect, is exercised only during the descending stroke of the piston; and thence these engines are termed single-acting.

The communication being first opened between the boiler and the upper part of the cylinder on the one hand, and between the lower part of the cylinder and the condenser on the other, the steam passes into the cylinder, and presses with all its force above the piston, whereas below it, the steam which has effected the preceding stroke is condensed. The piston then begins its descending stroke. When it has performed a certain portion of the stroke, the communication from the boiler to the cylinder is intercepted. The piston, however, still continues its motion, by the effect of the expansion of the steam already admitted in the cylinder. But on its attaining nearly the end of the stroke, a valve, called the equilibrium valve, opens and establishes a free communication between the upper and lower portions of the cylinder. Then the steam, diffusing itself on both sides of the piston, puts the latter in a state of equilibrium, that is, presses equally on both its faces. Consequently there ensues a total cessation in the action of the motive power, and as the resistance opposed by the load remains the same, the piston, whose motion has already slackened during the decreasing action of the expansion, is quickly brought to a state of rest.

The piston then has reached the bottom of the cylinder, and as the equilibrium valve still continues open, the piston remains also in equilibrio in the steam. On the other hand, during the stroke just performed, a counterweight, suspended at the opposite end of the beam, has been raised at the same time with the load. It now, therefore, tends to descend again by its own weight, and consequently to raise the piston in the cylinder. The piston, too, no longer bears any load to oppose this motion, since the action of the pump to raise water, or of the load of the engine, is exercised only when the piston tends to descend, whereas at this moment it tends to move upwards. Nothing remains, then, but the friction of the engine, to oppose a resistance to the effort of the counterweight; but as the weight given to this is sufficient to overcome the friction of the engine, it follows that the piston is raised and brought back to the top of the cylinder.

Thus all has returned to the same state as at first, and the action of the steam recommences, to make the piston perform a new descending stroke.

Comparing these engines with those which we have hitherto treated of, they are found to differ in three points:—1st. The counterweight acts alternately as a resistance and as a motive force. 2d. During its action as a motive force, it effects the motion by means of its fall. 3d. There is neither continuity nor uniformity in the motion of the resistance. It will be proper, then, to examine whether these circumstances ought to change the bases whereon we have established the theory of the other engines.

1st. With respect to the alternate action of the counterweight as a resistance and as a motive force, no change in the mode of reasoning need result from that; for the counterweight acts precisely in the manner of an ordinary fly-wheel, such as are met with in the engines of which we have hitherto treated. In fact, the motion of the single-acting engine is composed of two distinct parts: 1st, the descending stroke, during which the water is raised in the pumps, that is to say, during which the useful effect is produced; and 2dly, the ascending stroke, effected by the counterweight, and unattended with any useful effect, that stroke serving merely to replace matters as they were originally, and to enable the steam to renew its action. Now, during the first period of this motion, a certain quantity of work is communicated to the counterweight, by raising it to a certain height; and during the second period, the counterweight does no more than restore this quantity of work to the engine, by its descent from the same height to which it was raised. Its office then is limited to receiving a certain quantity of action, in order to restore it at a proper time, during the suspension of the effort of the motive force; and consequently its action is of the same nature as that of an ordinary fly-wheel.

2dly. During the ascending stroke of the piston, the weight of the counterweight becomes the motive power, and the velocity produced in that stroke depends incontestably on the superiority of the counter-weight over the resistance then opposed by the engine. If then, we were to make a distinction between the velocity of the descending and of the ascending stroke of the piston, we ought to calculate the latter from the circumstances of the fall of the counterweight. But it will be remarked that a *complete* oscillation of the engine consists of an ascending and a descending stroke of the piston. It matters little that one of these be rapid and the other slow; a new oscillation will never take place till the time requisite for both shall have expired. If the respective velocities of the two strokes are equal, the motion of the piston will be regular throughout the oscillation; if they are unequal, that motion on the contrary will be accelerated at one point and retarded at another. But in either case, the true velocity of the engine, that which we want to know in order to calculate the useful effects, will always be the *mean* velocity of the two strokes.

In fact, as soon as we know the number of complete oscillations made by the engine per minute, we necessarily know its useful effect; since at every complete oscillation, we know that the load advances the length of one stroke.

Thus we see, first, that the velocity wanted, in order to calculate the useful effect, is neither the velocity of the rising stroke of the piston, nor that of the descending, but the mean velocity of the two strokes taken together. Now, this is necessarily regulated, *à priori*, by the production of steam in the boiler, and may be calculated directly from that datum. For, if the boiler produce per minute, and transmit to the cylinder, a certain quantity of steam, whose volume be known, it is plain that, since that volume of steam issues by the cylinder in one minute, dividing it by the content of the cylinder, we have the number of complete oscillations it will cause in the engine. This, therefore, will be the mean velocity of the piston, however rapid the motion of the piston may have been at one point of its oscillation, and however slow, on the contrary, its motion may have been at another point of the same oscillation.

As to the greater or less velocity of the piston during its ascending stroke, it will in no wise alter the mean velocity of the engine, as long as the production of steam be supposed the same in the boiler. If, in fact, the rising stroke of the piston be performed in a longer time, since the issue of the steam by the cylinder takes place only during the descending strokes, it will happen that the successive interruptions thus made in the efflux of the steam, will be so much the longer. But, since the production of steam per minute in the boiler is a fixed and determinate quantity, and that its issue by the cylinder will now be found suspended during a longer time, the steam during that interval will accumulate in the boiler. At the beginning then of the descending stroke, there will be a greater quantity ready to be applied to the effecting of the motion; and it will consequently supply a velocity in that stroke by so much the greater. The velocity, therefore, of the descending stroke will augment as that of the rising stroke diminishes. If, on the contrary, the rising stroke is performed more rapidly, the steam will accumulate in a smaller quantity during the interval of the rising strokes, and will in consequence afford a smaller supply of steam, that is to say, a less velocity of the piston, during the descending stroke. Thus, in all cases, the velocity of the descending stroke will vary in the inverse manner to that of the rising stroke, and the mean or definitive velocity will always be regulated by the number of cylinders-full of steam supplied by the boiler per minute. One case only can occur wherein this rule would fail, namely, that in which the motion of the ascending stroke were so slow that the time left for the issue of the steam during the descending stroke, should be insufficient for the total efflux of the steam generated per minute in the boiler, and that a part of the steam should escape by the safety-valves. Then the mean velocity could no longer be calculated, as above, on the total efflux of the steam by the cylinder, since that total efflux would not take place. But as, in single-acting engines, care is always taken to set a counterweight capable of executing the ineffective stroke, with a velocity nearly equal to that which it is purposed to give to the effective stroke, this excessive slowness of one of the two strokes never occurs in practice, nor is it necessary to consider such supposition.

Hence we see that the use of the counterweight is no hindrance to the velocity of the engine being always calculated from the production of steam in the boiler.

3dly. These engines have no fly-wheel, nor is the motion of the resistance either continued or uniform. But the preservation of the engine requires that, at the end of each alternate motion, the piston be brought to rest without shock and by insensible degrees. No loss therefore of *vis viva* is occasioned; and consequently, in these engines, as well as in the double-acting ones, there is always equality between the quantity of work applied by the power and that which is performed by the resistance. It is necessary, however, before we proceed, to explain how such a regulation of the engine may be attained in practice, both in the descending and in the ascending stroke of the piston.

For this purpose, care is taken, during the descending stroke, to admit no more steam into the cylinder, than what is just requisite to drive the piston by the expansion of that steam, to the point where the stroke should end; so that it may there stop of itself, without either falling short or going too far. This effect is easily obtained in a few trials, on setting the engine to work. A small quantity of steam is first admitted with caution into the cylinder, and this is augmented by degrees, till the piston is found to attain precisely the point proposed as the end of the stroke. Should the piston overpass that point, and strike against the springs which protect the bottom of the cylinder, it is a proof that the quantity of steam admitted is too great, and it must be reduced in consequence till a proper adjustment be obtained.

During the ascending stroke, on the contrary, as the piston rises in the cylinder only because the equilibrium valve allows the steam then in the upper part of the cylinder, to pass into the lower part and make room for the piston, this equilibrium valve is closed somewhat before the piston has reached the point where it is intended to stop. The piston then continues its motion for some time by virtue of its acquired velocity and by the effect of the counterweight; but the steam situated in the lower part of the cylinder dilates as the progress of the piston opens a greater space for it, and the steam which is intercepted in the upper part, is on the contrary gradually compressed, and consequently acquires more and more elastic force. Thus the difference between the two pressures increasing continually, and creating a greater and greater resistance against the motion of the piston, at last brings it gently to rest. In practice, this adjustment is also obtained by trials, in closing the valve a little sooner or a little later, according as the piston is seen to come too near or not near enough the top of the cylinder. Moreover, in this way of adjusting the equilibrium valve, it will be observed that the steam compressed in the upper part of the cylinder retains all the work developed by the piston, while coming to rest, and that in this state of compression, it immediately contributes to produce the new descending stroke of the piston; whence results that no action is lost, and that the expenditure of steam consists only in that portion which has been intercepted below the piston.

This previous regulation of the engine must necessarily change every time that the load varies, but when it is once effected, that is to say, as soon as the engine has attained its regular working state, the piston, whether rising or falling, is always brought to rest by insensible degrees, and without loss of *vis viva*. Thus, in the single-acting engines, as well as in those of the divers systems of which we have already treated, there is always equality between the quantity of work developed by the power and that which is performed by the resistance; and consequently we may apply to these engines, the same mode of reasoning that we have hitherto made use of. It is, however, to be observed, that a distinction must be made between the descending and the rising stroke of the piston, in as much as the equality of the work produced by the power and the resistance, takes place in each separately. This double condition will furnish two equations proper to calculate the expansion of the steam in the descending stroke, and the adjustment of the equilibrium valve in the ascending one. Afterwards, to determine the velocity of the engine, there will remain the condition that the expenditure of the steam be equal to its production. We shall first establish these three fundamental equations, and then seek to deduce from them the formulæ suitable to the solution of the divers questions that may occur in these engines.

Sect. II. *Of the effects of the engine in the case of a given counterweight, with an indefinite load or velocity.*

We distinguish three cases in the working of single-acting engines: that in which they work with a given counterweight, and with the load or the velocity *indefinite;* that in which they work with a given counterweight, and the load or the velocity which produces the *maximum of useful effect with that counterweight;* and finally, that in which the counterweight having been first regulated according to its most advantageous weight for the engine, the load imposed is also the most advantageous for that counterweight, which consequently produces the *absolute maximum of useful effect* that it is possible to obtain from the engine.

In the rotative or double-acting engines, which we have treated of farther back, it was seen that at the moment of the starting of the engine, the steam penetrated into the cylinder with a pressure equal to that of the boiler; but that, by the effect of the continuity of motion and the intervention of a fly-wheel, the impulse given to the piston during one stroke, was continued in the stroke following. The velocity of the piston then accelerated by degrees from the accession of new quantities of motion, and the pressure of the steam in the cylinder lowered at the same time, till at last the piston attained the greatest velocity capable of being impressed on it by the motive power, with the load imposed on the engine. Then the motion ceased to accelerate. It remained in the state of uniformity, or rather in that state which produced uniformity in the motion of the resistance; and that condition regulated the pressure which the steam definitively acquired in the cylinder, during all the duration of uniform motion. The pressure then, in the cylinder, might be found, in certain cases, much inferior to that of the steam in the boiler.

But this circumstance does not occur in the engines we are now considering, because the motion in them not being continued or brought to continuity by the use of a regular fly-wheel, the impulse given to the piston in one stroke, cannot transmit itself to the following stroke. The piston then is placed perpetually under such conditions as in rotative engines, it is placed only at the moment of departure; and consequently, the cause no longer existing of the diminution of pressure of the steam in the cylinder, the only difference of pressure occasioned between the two vessels is that resulting from the force requisite to traverse the narrow passages which separate the cylinder from the boiler. But as, with the dimensions usually adopted in steam-engines, the diminution of pressure attributable to this cause alone is found to be unimportant, we shall omit it here, as we have done in calculating the maximum load of rotative engines. On this head, however, it will readily be seen that, if it be deemed necessary to take account of this difference, it will suffice to attribute to the force which will appear in our calculation, a value not precisely equal to the pressure in the boiler, but somewhat inferior, according to its intensity given by the formulæ on the flowing of gas.

This being premised, and first considering the descending stroke of the piston, the quantity of work applied by the steam, as well during its direct action as during its expansion, must be found *in toto* in the work performed by the resistance. Now, adopting all the notations hitherto employed, that is, indicating by P the pressure of the steam in the boiler, by a the area of the cylinder, by l the stroke of the piston, and by l' the portion of that stroke performed before the expansion begins, and referring besides to the calculation which has been developed in Sect. II., Article I., Chapter III., we shall have for the work performed by the direct action and by the expansion of the steam in the cylinder, the following expression:

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}\right)\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right)-\frac{n}{q}\,al.$$

On the other hand, the resistance then consists of the load r measured per unit of the surface of the piston; the friction $(f'+\delta r)$ of the engine loaded with the resistance r, terming f' the friction of the engine unloaded, and δ the increase of that friction per unit of the load; the counterweight which we will express by Π, supposing it divided per unit of the surface of the piston; and finally, the pressure p subsisting under the piston, by reason of the imperfect condensation of the steam. The work performed during the stroke by these different resistances united, will therefore be

$$[(1+\delta)\,r+p+f'+\Pi]\,a\,l.$$

Consequently, we have the analogy

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}\right)\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right)-\frac{n}{q}\,al=[(1+\delta)\,r+p+f'+\Pi]\,al;$$

and making, for simplification,

$$k'=\frac{l'+c}{l}\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right),$$

the expression obtained above will become

$$\left(\frac{n}{q} + \mathrm{P}\right) k' = \frac{n}{q} + (1+\delta)\, r + p + f' + \Pi;$$

or

$$k' = \frac{\frac{n}{q} + (1+\delta)\, r + p + f' + \Pi}{\frac{n}{q} + \mathrm{P}} \quad \text{.........} \quad (A)$$

This will then be the first of the relations sought between the data and incognita of the problem.

To obtain a similar relation for the ascending stroke, we must express that the same equality exists between the quantities of action developed by the power and by the resistance.

Now the power is here the counterweight; and the resistance consists of the opposition exerted by the steam after the closing of the equilibrium valve, and of the friction of the engine. By friction of the engine, is here to be understood the sum of all the resistances overcome by the engine in effecting its motion; that is to say, not merely the friction of the various moving parts of the machinery, but also, when the engine is used to raise water, the resistance resulting from the pump piston penetrating into the water to be exhausted, a resistance which does not occur in the ascending stroke. The friction of the unloaded engine in this stroke, is not then exactly the same as the friction of the engine supposed unloaded in the descending stroke, and consequently we will express it by f'' instead of f'.

This premised, the quantity of work developed by the counterweight during the stroke, is

$$\Pi \, a \, l,$$

and that developed by the friction is

$$f'' \, a \, l.$$

As to the work developed by the resistance of the steam, it requires a short calculation, analogous to that which has been performed for the expansion.

During the ascending motion of the piston, and before the closing of the equilibrium valve, the pressure of the steam above the piston is necessarily rather higher than that of the steam below, as we have just seen, with respect to the pressure of the boiler compared with that of the cylinder. As, however, the difference between those two pressures may be left out without error, we will here suppose that, till the moment of the closing of the equilibrium valve, the piston is in equilibrio in the steam, that is, pressed equally on both its faces.

Now, during all the time that the equilibrium valve remains open, the two parts of the cylinder are filled with the steam which has undergone expansion in the preceding downward stroke. This steam, at the moment when it entered the cylinder, was at the pressure P, and filled the length $(l' + c)$ of the cylinder. It now finds itself distended through all the capacity of the cylinder, including the two vacant spaces not traversed by the piston. Therefore, from the relation (c), between the volumes and the pressures of the same weight of steam, during its action in the engine, demonstrated in Chapter III., Article I., Sect. II., the pressure of the steam, after being dilated in the two parts of the cylinder, has become

$$\pi = \left(\frac{n}{q} + P\right)\frac{l' + c}{l + 2c} - \frac{n}{q}.$$

This premised, let l'' be the length already traversed by the piston in its rising stroke, at the moment when the equilibrium valve closes. From this point, the piston still continuing its motion, by virtue of its acquired velocity and of the effort of the counterweight, the steam confined *above*, and which can no longer escape, acquires by compression still more and more elastic force; while that which subsists *below* assumes, on the contrary, a pressure still less and less.

Suppose, then, the piston arrived at a distance λ, from the beginning of its stroke, and at that point let π' be the pressure of the steam beneath the piston, and π'' that of the steam above it. Let the piston then traverse a further elementary space $d\lambda$, the corresponding labour, produced by the resistance of the steam, will be

$$(\pi'' - \pi')\, a\, d\lambda.$$

But, as the pressures π'' and π', which take place in the two parts of the cylinder, result from the spaces respectively occupied by the steam, whether above or below the piston, and that at the origin of this compression and dilatation of the steam, both the portions of the cylinder were filled with steam at the pressure π, we shall again have, between the pressures and the volumes respectively occupied by the steam, the relation

$$\pi'' = \left(\frac{n}{q} + \pi\right)\frac{l - l'' + c}{l - \lambda + c} - \frac{n}{q},$$

and

$$\pi' = \left(\frac{n}{q} + \pi\right)\frac{l'' + c}{\lambda + c} - \frac{n}{q}.$$

Whence the elementary labour above, will be

$$(\pi'' - \pi')\, a\, d\lambda = a\left(\frac{n}{q} + \pi\right)\left[(l - l'' + c)\frac{d\lambda}{l - \lambda + c} - (l'' + c)\frac{d\lambda}{\lambda + c}\right].$$

Or, substituting for π its value given above, this expression will become

$$(\pi'' - \pi')\, a\, d\lambda = a\frac{l' + c}{l + 2c}\left(\frac{n}{q} + \mathrm{P}\right)\left[(l - l'' + c)\frac{d\lambda}{l - \lambda + c} - (l'' + c)\frac{d\lambda}{\lambda + c}\right].$$

Now, as this effect of compression on the one hand, and dilatation on the other, takes place from the length l'' of the cylinder to the end of the stroke, taking the integral of that expression between the limits l'' and l, we have the quantity of total labour developed by the steam in its resisting action against the piston, namely :

$$a\frac{l' + c}{l + 2c}\left(\frac{n}{q} + \mathrm{P}\right)\left[(l - l'' + c)\log\frac{l - l'' + c}{c} - (l'' + c)\log\frac{l + c}{l'' + c}\right].$$

But if, in order to abridge, we make

$$k'' = \frac{l - l'' + c}{l}\log\frac{l - l'' + c}{c} - \frac{l' + c}{l}\log\frac{l + c}{l' + c},$$

the expression just obtained may be written under the form

$$k'' a l\left(\frac{n}{q} + \mathrm{P}\right)\frac{l' + c}{l + 2c}.$$

Consequently, referring to what has been said above, of the labour performed during the same stroke by the counterweight and the friction of the engine, the equality between the action developed by the power and that developed by the resistance, during the rising stroke of the piston, will be found to produce the equation

$$k'' a l \left(\frac{n}{q} + \mathrm{P}\right) \frac{l' + c}{l + 2c} + f'' a l = \Pi a l,$$

or

$$k'' = \frac{l + 2c}{l' + c} \cdot \frac{\Pi - f''}{\frac{n}{q} + \mathrm{P}}; \quad . \quad . \quad . \quad (\mathrm{B})$$

which is the second relation sought between the data and the incognita of the problem.

Finally, to obtain the third relation, expressing the equality between the expenditure and the production of the steam, it must be observed that at each stroke of the piston, there is condensed and consequently expended, only the steam which has passed under the piston, during the rising stroke. As to that which is intercepted above the piston, it is driven back towards the boiler and serves to produce the following stroke.

Now, the volume of the steam to be condensed at each stroke of the piston, taken at the moment of its separation, is

$$a(l'' + c);$$

and its pressure is then

$$\pi = \left(\frac{n}{q} + \mathrm{P}\right) \frac{l' + c}{l + 2c} - \frac{n}{q}.$$

If M be the number of strokes of the piston per minute, the volume of steam expended per minute will then be

$$M\, a\, (l'' + c).$$

But V being the mean velocity of the piston, or the space it traverses, rising and descending, in a minute, we have $V = 2\, M\, l$; or if, conformably to the usage very properly adopted with these engines, we reckon only the space v traversed by the piston in producing the useful effect, a space which is the half of that traversed in the two strokes, we shall have

$$v = M\, l, \quad \text{or } M = \frac{v}{l}.$$

Whence the above volume, expended by the cylinder, may be expressed by

$$a\, v\, \frac{l'' + c}{l}.$$

On the other hand, if we represent by S the volume of water evaporated per minute in the boiler, the volume of steam that will result from it, under the pressure P, at which it is generated, will be, according to what has been seen (equ. a)

$$\frac{S}{n + q\, P}.$$

This will be the volume occupied by the steam under the pressure P; but in passing to the pressure π, at which we measure the steam expended, that volume will become

$$\frac{S}{n + q\, P} \cdot \frac{n + q\, P}{n + q\, \pi} = \frac{S}{n + q\, \pi}.$$

Therefore, since the expenditure of the cylinder is equal to the production of the boiler, we shall have

$$a\,v\frac{l''+c}{l}=\frac{S}{n+q\,\pi};$$

or, substituting for π its value, and resolving the equation with reference to v,

$$v=\frac{l}{l''+c}\cdot\frac{l+2c}{l'+c}\cdot\frac{S}{a}\cdot\frac{1}{n+q\,P}\;\ldots\ldots\;(1)$$

which is the third relation sought.

Consequently adding to this, the two other relations already obtained, and resolving these equations successively with reference to the quantities r and S, we obtain definitively the following formulæ:

$$k'=\frac{\frac{n}{q}+(1+\delta)\,r+p+f'+\Pi}{\frac{n}{q}+P}\;\ldots\ldots\;(A)$$

$$k''=\frac{l+2c}{l'+c}\cdot\frac{\Pi-f''}{\frac{n}{q}+P}\;\ldots\ldots\;(B)$$

$$v=\frac{l}{l''+c}\cdot\frac{l+2c}{l'+c}\cdot\frac{S}{a}\cdot\frac{1}{n+q\,P}\;\ldots\ldots\;(1)$$

$$ar=\frac{a}{1+\delta}\,k'\left(\frac{n}{q}+P\right)-\frac{a}{1+\delta}\left(\frac{n}{q}+p+f+\Pi\right)(2)$$

$$S=\frac{l''+c}{l}\cdot\frac{l'+c}{l+2c}\,av(n+qP)\;\ldots\ldots\;(3)$$

$$\text{u. E.}=arv\;\ldots\ldots\;(4)$$

In these equations, k' and k'' represent the expressions indicated a little above, viz.:

$$k' = \frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log \frac{l+c}{l'+c}\right),$$

$$k'' = \frac{l-l''+c}{l} \log \frac{l-l''+c}{c} - \frac{l''+c}{l} \log \frac{l+c}{l''+c}.$$

SECT. III. *Of the velocity of the engine with a given load.*

The equations (A), (B), (1), (2), (3), and (4) resolve all the problems relative to the calculation of Watt's single-acting engines. As, however, the nature of the expressions represented by k' and k'' in the equations (A) and (B), do not admit of eliminating between these equations and equation (1); and that we cannot, therefore, attain a direct expression of the velocity in terms of the load, nor of the load in terms of the velocity, as we were enabled to do in the formulæ of the rotative engines, it will be proper to dwell a moment on the mode of calculation to be adopted, in seeking the solution of the various problems that may occur.

Suppose it were required to determine at what velocity an engine, fully known in other respects, would set in motion a given load r.

We shall first introduce the given value of r in equation (A),

$$k' = \frac{\frac{n}{q} + (1+\delta)r + p + f' + \Pi}{\frac{n}{q} + P}.$$

Thus will be obtained the value of k'. Then in the first of the two tables which we shall give presently, will be found immediately and without calculation the corresponding value of $\frac{l'}{l}$.

Introducing the value of $\frac{l'}{l}$ into the equation (B),

$$k'' = \frac{l+2c}{l'+c} \cdot \frac{\Pi - f''}{\frac{n}{q} + \mathrm{P}},$$

the result will give the value of k'', and consequently, seeking that value in the second of the two tables mentioned, we shall there find the corresponding value of $\frac{l''}{l}$.

Finally, the two values of $\frac{l'}{l}$ and $\frac{l''}{l}$ thus found must be substituted in the equation (1)

$$v = \frac{l}{l''+c} \cdot \frac{l+2c}{l'+c} \cdot \frac{\mathrm{S}}{a} \cdot \frac{1}{n+q\mathrm{P}};$$

which operation will make known the velocity sought.

Sect. IV. *Of the load of the engine with a given velocity.*

Suppose the velocity of the engine be given, and that it be required to find what load it can set in motion at that velocity.

It must be observed that drawing from equation (B), the value of $\frac{l+2c}{l'+c}$; and substituting it in equation (1), the latter becomes

$$v = \frac{1}{q} \cdot \frac{l}{l'' + c} \cdot \frac{S}{a} \cdot \frac{k''}{\Pi - f''};$$

or

$$\frac{l}{l'' + c} k'' = \frac{\Pi - f''}{S} q a v.$$

This is then the analogy proper to give the value of k''. Substituting for v the velocity given, we shall have the number which represents the expression

$$\frac{l}{l'' + c} k'';$$

and seeking this number in the Table No. II., we shall immediately find the corresponding value of $\frac{l''}{l}$ and that of k''.

The value of $\frac{l''}{l}$ being known, that of $\frac{l'}{l}$ is next to be calculated. To this end, the equation (B) furnishes the analogy,

$$\frac{l' + c}{l} = \frac{l + 2c}{l} \cdot \frac{\Pi - f''}{\left(\frac{n}{q} + P\right) k''}.$$

The value of k'' being known by the preceding research, substituting it then in the equation just given, we derive from it $\frac{l'}{l}$, and afterwards k' by means of the Table No. I. Consequently, introducing the value of k' into equation (2)

$$ar = \frac{a}{1+\delta} k' \left(\frac{n}{q} + P\right) - \frac{a}{1+\delta} \left(\frac{n}{q} + p + f' + \Pi\right),$$

we obtain definitively the load $a\,r$ corresponding to the velocity v.

Sect. V. *Of the evaporation necessary to produce desired effects.*

Suppose it be required to find what ought to be the evaporation of the engine, to set in motion a given load at a given velocity.

The values of $\frac{l'}{l}$ and $\frac{l''}{l}$ must be calculated by means of the two formulæ (A) and (B), and substituting these, with the given velocity v, in equation (3),

$$S=\frac{l''+c}{l}\cdot\frac{l'+c}{l+2\,c}\;a\,v\,(n+q\,P),$$

we shall thence conclude the evaporation sought.

Sect. VI. *Of the useful effects of the engine.*

Let it be required to find what useful effect the engine may produce with a given load.

We shall seek first the velocity corresponding to that load, by the proceeding indicated above; then multiplying that velocity by the given load, we shall obtain the corresponding useful effect, viz.:

$$u.\,E. = arv.$$

Were the velocity given instead of the load, we should seek the load corresponding to that velocity, and the product $a\,r\,v$ of that load by the given velocity would be the useful effect required.

All the divers expressions of the useful effect are to be derived from the knowledge of the product $a\,r\,v$, by the formulæ already given in Sect. V., Article I., Chapter III.

SECT. VII. *Of the load and the velocity which correspond to a given expansion.*

Finally, as in these engines the expansion is essentially connected with the load, in such sort that the one being known the other follows necessarily, it may happen that the expansion at which the engine is to work, be determined beforehand, and that it be required to find what load it can set in motion with that expansion, and what velocity it will assume with that load.

In this case the quantity $\frac{l'}{l}$ is given *à priori*, and consequently we have also the corresponding value of k', which is found simply by inspection in Table No. I. Therefore the equation (2), viz.

$$a\,r = \frac{a}{1+\delta}\,k'\left(\frac{n}{q} + \mathrm{P}\right) - \frac{a}{1+\delta}\left(\frac{n}{q} + p + f + \Pi\right)$$

will immediately make known the load r; and this being found, the corresponding velocity will be deduced, as in the first problem, which we have treated above, Sect. III.

Thus it appears that the various questions which may occur in the calculation of single-acting engines, are to be resolved less directly, but yet in a manner nearly as simple as those which concern the rotative, or double-acting engines.

SECT. VIII. *Determination of the friction of the unloaded engine, and of its additional friction per unit of the load.*

The making use of the formulæ which we have just presented, suppose to be known, or at least that means are had of estimating the friction f' of the engine unloaded in the descending stroke, the surplus δ accruing to that friction per unit of the load imposed on the engine, and finally the friction f'' of the engine in the ascending stroke. It remains then to shew the means of determining these three quantities.

In order to do this, it will be observed that equation (A) is general, or that we have always the analogy

$$\frac{n}{q}+(1+\delta)\,r+p+f+\Pi=\left(\frac{n}{q}+\mathrm{P}\right)k'.$$

This analogy will consequently subsist when the load of the engine shall be null, that is when $r=0$. So that, in this case, we have

$$\frac{n}{q}+p+f'+\Pi=\left(\frac{n}{q}+\mathrm{P}\right)k',$$

which gives

$$f'=\left(\frac{n}{q}+\mathrm{P}\right)k'-\left(\frac{n}{q}+p+\Pi\right).$$

This equation then will determine the quantity f', as soon as the quantities P, p, Π and k' become known.

Now, it is easy to know these quantities by direct experiment. Suppose the engine put in motion without any load, and that by admitting cautiously a very small quantity only of steam into the cylinder at each stroke of the piston, we have succeeded in discovering, by repeated trials, the portion $\frac{l'}{l}$ of the stroke of the piston, at which the steam flowing into the cylinder should be intercepted, in order that the piston may be driven just to the end of its stroke by the expansion of the steam. We have thus, by experiment, the quantity $\frac{l'}{l}$; and consequently recurring to the Table No. I., or to the developed expression of k', viz.

$$k' = \frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right),$$

it will be easy to find the value of k', corresponding to the observed value of $\frac{l'}{l}$.

Again, the counterweight Π is known, and by means of a manometer and Watt's *indicator*, may be measured the pressures P and p, subsisting in the boiler, and in the steam cylinder after imperfect condensation. All the elements then of the calculation will be had, and consequently, substituting them in the preceding equation

$$f' = \left(\frac{n}{q} + \mathrm{P}\right) k' - \left(\frac{n}{q} + p + \Pi\right),$$

we easily deduce the quantity f', or the friction of the engine in the descending stroke.

To find the quantity δ, it will suffice to set the engine to work with a known load, and to regulate by trials, as is usually done in practice, the portion of the stroke which the piston must be allowed to perform, before intercepting the flow of the steam into the cylinder. Then by experiment we have the quantity $\frac{l'}{l}$ suitable to the load r. Examining afterwards the counterweight and measuring the pressure in the boiler, as well as that which subsists in the cylinder after imperfect condensation of the steam, we shall likewise have by direct observation the quantities P, p and Π.

Now we have always the relation (A), namely

$$\frac{n}{q} + (1+\delta)\, r + p + f' + \Pi = \left(\frac{n}{q} + \mathrm{P}\right) k',$$

which gives

$$(1+\delta) = \frac{\left(\frac{n}{q} + \mathrm{P}\right) k' - \left(\frac{n}{q} + p + f' + \Pi\right)}{r}.$$

It will be easy then to deduce δ; for $\frac{l'}{l}$ being known, as has been said above, the quantity k'

may be immediately concluded from the table already indicated, and as the friction f' is equally known by the preceding research, we shall have all the elements of the value of δ.

Finally, to obtain the friction f'' of the rising stroke, the equation (B) gives

$$f'' = \Pi - \frac{l' + c}{l + 2c} k'' \left(\frac{n}{q} + P\right).$$

The engine then being supposed regulated by trials with any load, the two quantities $\frac{l'}{l}$ and $\frac{l''}{l}$ and the pressure P must be measured simultaneously on the engine. Then, recurring to the Table No. II., or to the developed expression of k'' viz.:

$$k'' = \frac{l - l'' + c}{l} \log \frac{l - l'' + c}{c} - \frac{l'' + c}{l} \log \frac{l + c}{l'' + c},$$

we shall know the value of k'' which corresponds to the observed value of $\frac{l''}{l}$. We have, moreover, by direct observation, that of $\frac{l'}{l}$; introducing therefore these values in the equation just given, we conclude from it the value of f''.

It is however to be remarked, with respect to the determination of f'', that in the engines employed for raising water, this friction is not strictly a constant quantity. It includes, in fact, the resistance opposed by the water to the pistons of the pumps, and this resistance varies as the square of the velocity of the motion. But as the velocity of the piston varies but little in these engines, it may suffice to determine the value of f'' for the ordinary velocity, and this value may afterwards be considered as a mean value applicable to all cases.

From what has been said then, the friction of the engine in each stroke, and the additional friction per unit of the load imposed on the engine in the descending stroke, may always be determined. After having measured these frictions in several engines, a mean friction may be deduced, which will serve to estimate the friction of engines, before their construction.

Sect. IX. *Tables for the numerical solution of the formulæ, for single-acting engines.*

The formulæ (A) and (B), which we have given in the preceding section, not admitting of a direct solution, and the use of them occurring nevertheless in all the problems relative to the effects of the engines, we here subjoin two tables which give their solution without calculation.

The first of these tables refers to the descending stroke of the piston, the second to the rising stroke. The correspondence between the numbers of the different columns will immediately show, either the value of k' when $\frac{l'}{l}$ is given, or on the contrary the value of $\frac{l'}{l}$ when k' is known. The same will be found with reference to $\frac{l''}{l}$ and k'' in the second table.

When the ratio $\frac{l'}{l}$ is determined, that of

$$\frac{l'+c}{l}$$

will be immediately concluded; since the latter ratio is nothing more than

$$\frac{l'}{l}+\frac{c}{l},$$

and that in every engine the clearance c of the cylinder is known.

When the equations contain the term

$$\frac{l+2c}{l'+c},$$

its value will be concluded from the ratio between the two following,

$$\frac{l+2c}{l} \text{ and } \frac{l'+c}{l},$$

which are both known from the values of $\frac{l'}{l}$ and $\frac{c}{l}$.

Thus no difficulty can, in any case, occur in the numerical solution of the formulæ.

In the tables we are about to offer, we have supposed $c = \cdot 1 l$; that is to say, we have taken the clearance of the cylinder, including the adjoining steam passages, as equal to one tenth of the useful stroke of the piston. This proportion is in effect adopted in single-acting engines, in order to avoid the accidents which might happen if the piston, carrying its stroke rather too far, should strike against the bottom of the cylinder.

We must here recall to mind that the quantities l' and l'' are the distances traversed by the piston, at the moment when the communication of the steam is intercepted; and, consequently, that these distances are supposed to be measured, not from the bottom of the cylinder, but from that point whence the piston effectively departs.

(No. 1.)—*TABLE for the numerical solution of the formulæ.* (*Single-acting engines.*)

Portion of the descending stroke performed before expansion, or value of the fraction $\frac{l'}{l}$.	Corresponding value of k', or of the expression $\frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right)$.	Portion of the descending stroke performed before expansion, or value of the fraction $\frac{l'}{l}$.	Corresponding value of k', or of the expression $\frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right)$.
·01	·263	·49	·857
·02	·286	·50	·863
·03	·308	·51	·869
·04	·329	·52	·875
·05	·349	·53	·881
·06	·368	·54	·887
·07	·387	·55	·892
·08	·406	·56	·897
·09	·424	·57	·902
·10	·441	·58	·907
·11	·458	·59	·912
·12	·474	·60	·917
·13	·490	·61	·921
·14	·505	·62	·925
·15	·520	·63	·929
·16	·535	·64	·933
·17	·549	·65	·937
·18	·563	·66	·941
·19	·577	·67	·945
·20	·590	·68	·949
·21	·603	·69	·952
·22	·615	·70	·955
·23	·627	·71	·958
·24	·639	·72	·961
·25	·651	·73	·964
·26	·662	·74	·967
·27	·673	·75	·970
·28	·684	·76	·973
·29	·694	·77	·975
·30	·704	·78	·977
·31	·714	·79	·979
·32	·724	·80	·981
·33	·734	·81	·983
·34	·743	·82	·985
·35	·752	·83	·987
·36	·761	·84	·989
·37	·770	·85	·990
·38	·778	·86	·991
·39	·786	·87	·992
·40	·794	·88	·993
·41	·802	·89	·994
·42	·810	·90	·995
·43	·817	·91	·996
·44	·824	·92	·997
·45	·831	·93	·998
·46	·838	·94	·999
·47	·845	·95	·999
·48	·851		

(No. II.)—*TABLE for the numerical solution of the formulæ.* (*Single-acting engines.*)

Portion of the ascending stroke performed before the closing of the equilibrium valve, or value of the fraction $\frac{l''}{l}$.	Corresponding value of k'', or of the expression $\frac{l-l''+c}{l}\log\frac{l-l''+c}{c}-\frac{l''+c}{l}\log\frac{l+c}{l''+c}$.	Corresponding value of the expression $\frac{l}{l''+c}k''$.
·50	·711	1·186
·51	·687	1·127
·52	·664	1·072
·53	·641	1·017
·54	·618	·966
·55	·595	·916
·56	·573	·869
·57	·551	·823
·58	·530	·780
·59	·509	·738
·60	·488	·698
·61	·468	·659
·62	·448	·622
·63	·428	·586
·64	·409	·552
·65	·390	·519
·66	·371	·488
·67	·352	·458
·68	·334	·429
·69	·317	·401
·70	·300	·375
·71	·283	·349
·72	·266	·325
·73	·250	·301
·74	·234	·279
·75	·219	·258
·76	·205	·238
·77	·190	·218
·78	·176	·200
·79	·162	·182
·80	·149	·165
·81	·136	·149
·82	·123	·134
·83	·112	·120
·84	·101	·107
·85	·089	·094
·86	·079	·083
·87	·069	·072
·88	·060	·061
·89	·052	·052
·90	·044	·044
·91	·036	·036
·92	·029	·029
·93	·023	·023
·94	·017	·017
·95	·012	·012

SECT. X. *Of the maximum useful effect with a given counterweight, and of the absolute maximum useful effect of the engine.*

The effects of engines with a given counterweight, and with *indefinite* load or velocity, have just been considered. But if different loads be imposed on the engine without any change being made in the counterweight, a certain velocity will be generated for each load, and, consequently, a corresponding useful effect. The useful effects thus produced will necessarily differ. It will then be necessary to seek, which, among these divers loads, will be most advantageous for the engine with the fixed counterweight, or which will produce the maximum of useful effect with that counterweight.

To this end, the direct method would be first to form the expression of the useful effect produced with an indefinite load, in terms of that load; then to seek, among all the possible values of the load, that which would make the useful effect a maximum. This is the mode we have followed in the similar research for rotative engines. Here the velocity of the engine with an indefinite load r has for its expression

$$v=\frac{l}{l''+c}\cdot\frac{l+2c}{l'+c}\cdot\frac{S}{a}\cdot\frac{1}{n+qP};$$

and, consequently, the corresponding useful effect is

$$arv=\frac{l}{l''+c}\cdot\frac{l+2c}{l'+c}\cdot\frac{Sr}{n+qP}.$$

To arrive at the expression of this useful effect in direct terms of the load, it would be necessary to substitute the values of l' and l'' for those quantities, in the second member of the equation. These values are given by the two equations (A) and (B), namely:

$$k' = \frac{\frac{n}{q} + (1+\delta)r + p + f' + \Pi}{\frac{n}{q} + \mathrm{P}},$$

$$k'' = \frac{l+2c}{l'+c} \cdot \frac{\Pi - f''}{\frac{n}{q} + \mathrm{P}},$$

in which k' and k'' are logarithmic functions of l' and l'', which we have expressed above. But these two functions not being of a nature to furnish a direct solution for l' and l'', it follows that those quantities cannot, in the expression arv, be replaced by their value in terms of r, and consequently, that this mode cannot lead to the desired result.

The only method then to follow, is to proceed by successive trials and approximations. A supposition must be made on the load r, or on the expansion $\frac{l'}{l}$ of the steam, which amounts to the same, since, in these engines, those two quantities are essentially connected with each other. Then must be sought, by the means indicated above, the corresponding velocity of the engine, and on forming the product of the supposed load by the velocity found, we shall have the corresponding useful effect. This proceeding must be renewed for a second load, then for a third, and so on; and observing always to vary the load on that side on which the useful effect is seen to increase, we shall finally attain that load which produces the maximum useful effect sought.

By the foregoing may be known the most advantageous load for the engine, *with a given counterweight*, which is the first problem we had proposed to ourselves in this section. But now it is plain that on varying the counterweight itself, there will exist for each of its values, a load the most advantageous and a corresponding maximum effect. It remains then to seek, among these different maxima effects produced by divers values of the counterweight, that which is the most considerable; in order to discover, among all the values that may be given to the counterweight, that which is the most advantageous for the engine. If, in effect, the solution of this question be attained, it is plain that by first giving to the counterweight the value thus found, and then laying on the engine the most advantageous load for that counterweight, we shall obtain the *absolute maximum* useful effect whereof the engine is capable.

To obtain the required solution, the analytic mode would again be similar to that which we have followed with respect to the double-acting engines, and which we have this instant recalled to mind; but this direct method is again inapplicable here, for the reasons already stated. Consequently, it is only by making successive suppositions on the value of the counterweight, then seeking by trials the corresponding maximum of useful effect, and finally, comparing with each other the divers maxima useful effects, that it will be possible to attain the numerical solution of the question proposed.

This proceeding, at the first glance, appears rather long; but on observing that the equations employed are very simple, and that in the successive essays the same numbers continually recur, it will soon be recognized that the calculation offers in reality but little difficulty; a difficulty, moreover, which occurs but once for all, and is nothing in comparison of the importance of the question, namely, to find the means of making an engine work ever afterwards with the greatest possible advantage.

ARTICLE II.

PRACTICAL FORMULÆ FOR WATT'S SINGLE-ACTING ENGINES, AND EXAMPLE OF THEIR APPLICATION.

To form the numerical equations suitable to the calculation of these engines, it must be observed that, since they are condensing engines, the value of the coefficients n and q of the relative volume of the steam, will be

$$n = \cdot 00004227$$
$$q = \cdot 000000258.$$

As to the pressure P of the steam in the boiler, it is generally from 16·5 to 18 lbs. per square inch, as in Watt's double-acting engines. The pressure p subsisting, not in the condenser, but in the steam cylinder itself, after imperfect condensation of the steam, is again about 4 lbs. per square inch, when the engine is well supplied with injection water.

Referring the pressures to the square foot, we have then generally

$$P = 16{\cdot}5 \times 144 \text{ lbs.}, \quad p = 4 \times 144 \text{ lbs.}$$

Finally, to have all the elements of the calculation, it would be necessary to know the precise values of the frictions f', f'' and δ, in order to substitute them in the algebraic equations already obtained. We want special experiments on this head; but to furnish at least an example of calculation which may show the process, if not the precise results, we will make, from analogy with Watt's rotative engines, an approximate valuation of those frictions.

These two kinds of engines differing but little from each other, we will take the friction f'' of the unloaded engine in the rising stroke, at the same value as in Watt's double-acting engines; that is, at ·5 lb. per square inch of the piston, for a cylinder of about 48 inches diameter. As this dimension is nearly the mean dimension of Watt's single-acting engines, because the expansion of the steam requires a capacious cylinder, we will consider the above valuation as suitable to middle-sized engines of this system.

We will estimate at the same value the friction f' of the unloaded engine, in the descending stroke; and, in fine, with respect to the additional friction δ, accruing to the engine per unit of the load imposed on it, we will admit, till an especial determination be obtained, the datum deduced from the observation of locomotive engines. Consequently we take

$$f' = f'' = \cdot 5 \times 144, \quad \delta = \cdot 14.$$

The numerical formulæ will be obtained by substituting the values of these constant quantities in the algebraic equations. But as the pressure in the boiler varies in different engines, as well as the pressure p of condensation, and as the valuations we have just given of the frictions are but temporary, we will introduce no other values into the numerical equations than those of n and q. The following will then be the formulæ obtained:

Practical formulæ for Watt's single-acting engines.

$$k' = \frac{164 + (1+\delta)\, r + p + f' + \Pi}{164 + P}$$ ....Regulation of the descending stroke of the piston.

$$k'' = \frac{l + 2c}{l' + c} \cdot \frac{\Pi - f''}{164 + P}$$ ....Regulation of the ascending stroke of the piston.

$$a r = \frac{a}{1+\delta} k' (164 + P) - \frac{a}{1+\delta} (164 + p + f' + \Pi)$$ Useful load of the piston, in pounds.

$v = \frac{l}{l'' + c} \cdot \frac{l + 2c}{l' + c} \cdot \frac{S}{a} \cdot \frac{10000}{\cdot 4227 + \cdot 00258\,P}$. Velocity of the piston, in feet per minute.

$S = \frac{l'' + c}{l} \cdot \frac{l' + c}{l + 2c} \cdot \frac{av}{10000} (\cdot 4227 + \cdot 00258\,P)$ Effective evaporation, in cubic feet of water per minute.

u. E. $= arv$ .... Useful effect, in pounds raised 1 foot per minute.

u. H P. $= \frac{\text{u. E.}}{33000}$ .... Useful horse power.

u. E. 1 lb. co. $= \frac{\text{u. E.}}{\text{N}}$ .... Useful effect of 1 lb. of coal, in pounds raised 1 foot.

u. E. 1 ft. wa. $= \frac{\text{u. E.}}{\text{S}}$ .... Useful effect due to the evaporation of 1 cubic foot of water, in lbs. raised 1 foot.

Q. co. for 1 hp. $= \frac{33000\,\text{N}}{\text{u. E.}}$ .... Quantity of coal, in pounds, which produces 1 horse-power.

Q. wa. for 1 hp. $= \frac{33000\,\text{S}}{\text{u. E.}}$ .... Quantity of water, in cubic feet, which produces 1 horse-power.

u. HP. for 1 lb. co.... $= \frac{\text{u. E.}}{33000 \text{ N}}$ ....Horse - power produced per pound of coal.

u. HP. for 1 ft. wa. $= \frac{\text{u. E.}}{33000 \text{ S}}$ ....Horse - power produced per cubic foot of water evaporated.

Now, to show an application of the preceding formulæ, we will suppose an engine of this system offering the following data:

Diameter of the cylinder, 48 inches; or surface of the piston, $a = 12{\cdot}566$ square feet.

Stroke of the piston, $l = 8$ feet.

Clearance of the cylinder, $\frac{1}{10}$ of the useful stroke of the piston; or $c = {\cdot}1\, l$.

Pressure in the boiler, 16·5 lbs. per square inch; or $\text{P} = 16{\cdot}5 \times 144$ lbs. per square foot.

Pressure of condensation in the steam-cylinder, 4 lbs. per square inch; or $p = 4 \times 144$ lbs. per square foot.

Effective evaporation, $\text{S} = {\cdot}506$ cubic foot of water per minute.

Consumption of coal in the same time, $\text{N} = 4{\cdot}25$ lbs.

Counterweight, 1·25 lb. per square inch of the surface of the piston; or $\Pi = 1{\cdot}25 \times 144$ lbs. per square foot of the surface of the piston.

With these divers data, it is proposed to determine the effects which the engine may produce. Leaving then at first the counterweight invariable, but making the engine work with different loads, or, which amounts to the same, with different expansions of the steam; varying afterwards the counterweight, and proceeding in the calculation, according to the mode indicated in Article I. of the present chapter, we obtain the following results:

Effects of the engine, with the counterweight given.

$$\frac{\Pi}{144} = 1{\cdot}25 \text{ lb.}$$

				Maximum useful effect.
$\frac{l'}{l}$	=	·66	·625	·50
$\frac{r}{144}$	=	8·52	8·30	7·33
ar	=	15411	15009	13251
v	=	100	105	130
S	=	·506	·506	·506
u. E.	=	1543050	1580270	1728580
u. HP.	=	47	48	52
u. E. 1 lb. co.	=	363070	371830	406730
u. E. 1 ft. wa.	=	3049510	3123060	3416180
Q. co. for 1 hp	=	·091	·089	·081
Q. wa. for 1 hp	=	·0108	·0106	·0097
u. HP. for 1 lb. co.	=	11·00	11·27	12·33
u. HP. for 1 ft. wa.	=	92·41	94·64	103·52

Maxima effects of the engine, with different counterweights.

			Absolute maximum of useful effect.	
$\frac{\text{II}}{144}$	=	1·00	1·25	1·50
$\frac{l'}{l}$	=	·50	·50	·50
$\frac{r}{144}$	=	7·54	7·33	7·10
ar	=	13640	13251	12849
v	=	125	130	133
S	=	·506	·506	·506
u. E.	=	1708840	1728580	1705030
u. HP.	=	51·8	52·4	51·7
u. E. 1 lb. co.	=	402080	406730	401180
u. E. 1 ft. wa.	=	3377150	3416180	3369610
Q. co. for 1 hp	=	·082	·081	·082
Q. wa. for 1 hp	=	·0098	·0097	·0098
u. HP. for 1 lb. co.	=	12·18	12·33	12·16
u. HP. for 1 ft. wa.	=	102·34	103·52	102·11

The first of these tables shows that with the constant counterweight of 1·25 lb. per square inch of the piston, *the maximum of useful effect* is produced by making the engine work with the expansion indicated by $\frac{l'}{l} = \cdot 50$, or with the corresponding load, which is here found to be 7·33 lbs. per square inch of the surface of the piston. An expansion, indicated by a number less than ·50, would, indeed, produce still greater useful effects; but as, with the pressure at which these engines work, the motion becomes too irregular, when the steam is intercepted before the half of the stroke, we deem it proper to stop at this practical limit.

The second table shows moreover, that among the divers counterweights that may be used to work the engine, that of 1·25 lb. per square inch of the surface of the piston, is the most advantageous; and that employing this counterweight with the expansion corresponding to $\frac{l'}{l} = \cdot 50$, or, in other words, with the load of 7·33 lbs. per square inch of the piston, the engine will produce the *absolute maximum* of useful effect that can possibly be expected from it.

In performing the calculation, it is also to be remarked, that for each different counterweight that may be supposed for the engine, it is always the same expansion, corresponding to $\frac{l'}{l} = \cdot 50$, which is the most advantageous. This observation will greatly simplify the research of the absolute maximum of useful effect; for it will obviously suffice, first to make different essays on the expansion with the given counterweight, which will make known the expansion of greatest useful effect; and then to apply that expansion to divers hypotheses on the counterweight, which will lead very promptly to the counterweight of the absolute maximum of useful effect sought. Nevertheless, as this remark is established only by the fact, and not by a general reasoning, it will be proper to verify as to the question, whether effectively the expansion suitable to the maximum useful effect for the first counterweight chosen, is equally the expansion of maximum useful effect for the counterweight determined by the calculation.

For the above engine, it is easy to be convinced that on changing either the expansion $\frac{l'}{l} = \cdot 50$, or the counterweight 1·25 lb. per square inch, there would be a diminution of useful effect; for on performing the calculation we find the following corresponding results:

$$\frac{\Pi}{144} = 1\cdot 25 \ldots\ldots \frac{l'}{l} = \cdot 50 \ldots\ldots\ldots\ldots\ldots\ldots \text{u. E.} = 1728580$$

$$\cdot 625 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1580270$$

$$\cdot 66 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1543050$$

$$\frac{l'}{l} = \cdot 50 \ldots\ldots \frac{\Pi}{144} = 1 \ldots\ldots\ldots\ldots\ldots\ldots \text{u. E.} = 1708840$$

$$1\cdot 25 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1728580$$

$$1\cdot 50 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1705030.$$

We do not suppose an expansion represented by a number less than $\frac{l'}{l} = \cdot 50$ for the reasons given above; but were it chosen to step out of this limit fixed by practical experience, the calculation must be continued by trying other values of the expansion and of the counterweight, as we shall do in the following chapter with respect to the Cornish engines.

The engine we have just calculated is the pattern single-acting engine, constructed by Watt at his factory in Soho. Its counterweight was 1·25 lb.

per square inch of the surface of the piston, and when the steam was intercepted after the piston had traversed 5 feet of the stroke, that is, in the case of $\frac{l'}{l} = \frac{5}{8} = \cdot 625$, the engine assumed a velocity of 96 feet per minute. This is, then, a verification of the preceding theory; for the smallest diminution in the quality of the fuel, that is to say, in the intensity of the fire, and, consequently, in the evaporation of the engine, may produce the slight difference which here appears between the velocity of 105 feet per minute, resulting from our calculation, and that of 96 feet per minute, the result of direct observation.

CHAPTER XI.

CORNISH SINGLE-ACTING ENGINES.

SECT. I. *Practical formulæ for calculating these engines; and example of their application.*

THE Cornish single-acting engines are nothing more than a modification of Watt's single-acting engines. All the difference consists in the steam being employed at a total pressure of 50 or 55 lbs., and sometimes at 75 or 80 lbs., instead of 16 or 18 lbs. per square inch; and in the expansion of the steam being carried much farther, since the steam is often intercepted, when the piston has performed but one-tenth of the stroke.

As this modification in no way affects the principle itself of the application of the steam as a motive power, but concerns merely the limits of two quantities which figure in the formulæ, it is plain that the theory we have developed with regard to Watt's single-acting engines, will equally apply to the Cornish engines, and that we shall only have to substitute for the pressure in the boiler and the expansion of the steam, that is, for the quantities P and $\frac{l'}{l}$, numbers different from those which relate to Watt's engines.

The Cornish engines, therefore, will be calculated by the following numerical formulæ:

Practical formulæ for the Cornish single-acting engines.

$$k' = \frac{164 + (1+\delta)\,r + p + f' + \Pi}{164 + \mathrm{P}}$$ .... Regulation of the descending stroke of the piston.

$$k'' = \frac{l+2c}{l'+c} \cdot \frac{\Pi - f''}{164+\mathrm{P}}$$ .... Regulation of the ascending stroke of the piston.

$$ar = \frac{a}{1+\delta}\,k'(164+\mathrm{P}) - \frac{a}{1+\delta}(164+p+f'+\Pi)$$ Load of the piston, in pounds.

$$v = \frac{l}{l''+c} \cdot \frac{l+2c}{l'+c} \cdot \frac{\mathrm{S}}{a} \cdot \frac{10000}{\cdot 4227 + \cdot 00258\,\mathrm{P}}$$ Velocity of the piston, in feet per minute.

$$\mathrm{S} = \frac{l''+c}{l} \cdot \frac{l'+c}{l+2c} \cdot \frac{av}{10000}(\cdot 4227 + \cdot 00258\,\mathrm{P})$$ Effective evaporation, in cubic feet of water per minute.

$$\text{u. E.} = arv$$ .... Useful effect, in pounds raised 1 foot per minute.

$$\text{u. HP.} = \frac{\text{u. E.}}{33000}$$ .... Useful horse-power.

$$\text{u. E. 1 lb. co.} = \frac{\text{u. E.}}{\text{N}}$$ .... Useful effect of 1 lb. of coal, in pounds raised 1 foot.

$$\text{u. E. 1 ft. wa.} = \frac{\text{u. E.}}{\text{S}}$$ .... Useful effect of 1 cubic foot of water, in pounds raised 1 foot.

$$\text{Q. co. for 1 hp.} = \frac{33000\ \text{N}}{\text{u. E.}}$$ .... Quantity of coal, in lbs., which produces 1 horse power.

$$\text{Q. wa. for 1 hp.} = \frac{33000\ \text{S}}{\text{u. E.}}$$ .... Quantity of water, in cubic feet, which produces 1 horse-power.

$$\text{u. HP. for 1 lb. co.} = \frac{\text{u. E.}}{33000\ \text{N}}$$ .... Horse - power produced per pound of coal.

$$\text{u. HP. for 1. ft. wa.} = \frac{\text{u. E.}}{33000\ \text{S}}$$ .... Horse - power produced per cubic foot of evaporated water.

For the mode of solution and for the perfect understanding of these formulæ, we refer to what has been said in the preceding chapter, on Watt's single-acting engines.

As an example of the calculation, we will suppose the single-acting engine of Watt, the effects of which were calculated in the last chapter, to be modified according to the Cornish system; that is to say, the steam to be at a total pressure of 50 lbs. per square inch; the cylinder to be replaced by another of greater diameter; and in fine, the steam to be intercepted in the cylinder, when a small portion only of the stroke is performed. We shall then take for the different data of the problem:

Diameter of the cylinder 80 inches; or surface of the piston, $a = 34{\cdot}910$ square feet.

Stroke of the piston, $l = 10$ feet.

Clearance of the cylinder, $\frac{1}{10}$ of the useful stroke of the piston, or $c = {\cdot}1\ l$.

Total pressure in the boiler, 50 lbs. per square inch, or $P = 50 \times 144$ lbs. per square foot.

Pressure of condensation in the steam cylinder, 4 lbs. per square inch, or $p = 4 \times 144$ lbs. per square foot.

Friction of the unloaded engine in each stroke, on account of the size of the cylinder, ·25 lb. per square inch of the surface of the piston; or $f' = f'' = {\cdot}25 \times 144$.

Additional friction of the engine, per unit of the resistance imposed on the piston, $\frac{1}{7}$ of that resistance; or $\delta = {\cdot}14$.

Effective evaporation, $S = {\cdot}506$ cubic foot of water per minute.

Consumption of coal in the same time, $N = 4{\cdot}25$ lbs.

Substituting then these values in the formulæ, and supposing that, after having adopted a counterweight, several degrees of expansion be successively tried, or in other words, several loads for the engine, we shall first obtain the results contained in the first of the following tables; then varying the counterweight itself, and essaying again for each value of the counterweight, different degrees of expansion, we shall obtain the results of the second table.

Effects of the engine with the counterweight given,
$\Pi = 1{\cdot}50 \times 144$ lbs.

			Maximum of useful effect.
$\frac{l'}{l}$	=·25	·12	·10
$\frac{r}{144}$	=23·16	15·22	13·74
$a\,r$	=116426	76523	69053
v	=28·72	47·85	53·28
S	=·506	·506	·506
u. E.	=3344260	3661850	3679170
u. HP	=101	111	111·5
u. E. 1 lb. co.	=786880	861610	865690
u. E. 1 ft. wa.	=6609220	7236860	7271100
Q. co. for 1 hp.	=·042	·038	·038
Q. wa. for 1 hp.	=·00499	·00456	·00454
u. HP. for 1 lb. co.	=23·85	26·11	26·23
u. HP. for 1 ft. wa.	=200	219	220

Maxima effects of the engine with different counterweights.

		Absolute maximum of useful effect.	
$\frac{\Pi}{144}$	=1·50	5·00	6·00
$\frac{l'}{l}$	=·10	·11	·11
$\frac{r}{144}$	=13·74	11·42	10·54
$a\,r$	=69053	57428	52993
v	=53·28	67·28	72·65
S	=·506	·506	·506
u. E.	=3679170	3863940	3850000
u. HP.	=111·5	117·1	116·7
u. E. 1 lb. co.	=865690	909160	905880
u. E. 1 ft. wa.	=7271100	7636250	7608700
Q. co. for 1 hp	=·038	·036	·036
Q. wa. for 1 hp	=·00454	·00432	·00434
u. HP. for 1 lb. co	=26·23	27·55	27·45
u. HP. for 1 ft. wa	=220	231	230·6

The first of these tables shows that, with the counterweight 1·50 lbs. per square inch of the surface of the piston, the most advantageous way of working the engine is with the expansion corresponding to $\frac{l'}{l} = \cdot 10$, or, in other words, with the load of 13·74 lbs. per square inch of the piston. Any other expansion, greater or less than that indicated by $\frac{l'}{l} = \cdot 10$, tends to diminish the useful effect; for the calculation gives the following results:

$$\frac{l'}{l} = \cdot 11 \ldots\ldots\ldots\ldots \text{u. E} = 3672490$$

$$\cdot 10 \ldots\ldots\ldots\ldots 3679170$$

$$\cdot 09 \ldots\ldots\ldots\ldots 3675790$$

Thus, this first research establishes the expansion or the load which produces the *maximum of useful effect with the counterweight given.*

Applying, moreover, the same calculation to different values of the counterweight, we find that, for those different values, it is always the same expansion, very nearly, which produces the maximum of useful effect. This will therefore be a guide in the following research, namely, that of the counterweight which produces the absolute maximum of useful effect. For it will suffice to calculate at first the effects of divers counterweights with the expansion corresponding to $\frac{l'}{l} = \cdot 10$; and no more will be needed afterwards than a very slight correction of the expansion, in order to attain the absolute maximum of useful effect sought.

Proceeding thus, we find, as appears in the second table, that the most advantageous counterweight is 5 lbs. per square inch; and that, to produce the *absolute maximum of useful effect,* that counterweight is to be used, and at the same time the expansion $\frac{l'}{l} = \cdot 11$. If, in effect, we increase or diminish, either the value of the counterweight, or that of the expansion, we immediately find a diminution of the useful effect; for the calculation gives

$\frac{\Pi}{144} = 4{\cdot}75$	$\frac{l}{l} =$	·10.........u. E.=	3856200
		·11..................	3860020 max.
		·12..................	3854250
$\frac{\Pi}{144} = 5{\cdot}00$	$\frac{l'}{l} =$	·10.........u. E.=	3859220
		·11..................	3863940 ab. max.
		·12..................	3858430
$\frac{\Pi}{144} = 5{\cdot}25$	$\frac{l'}{l} =$	·10.........u. E.=	3853990
		·11..................	3862600 max.
		·12..................	3858160

The two preceding tables therefore show, either the load which produces the maximum of useful effect with a given counterweight; or the counterweight and load, which produce together the absolute maximum of useful effect. It is to be observed, moreover, that this research, though difficult in appearance, is not so in reality, first, because the same numbers always recur in the different cases, and again, because the first calculation having already made known the most favourable expansion, that expansion, as we have said before, varies but little in the second calculation.

It must be observed also that, on having derived from the formula, a table similar to the preceding, it is understood that the effects indicated can be produced by the engine, only on condition of there being no practical hindrance to the regulation supposed in the calculation. Thus, certain effects could not be attained, unless the motion of the piston were much more rapid, or much slower in one stroke than in the other; and though some difference may be admitted on this point, yet it would not be possible to carry it beyond certain limits. In this case then, a regulation must be adopted, as near as possible to the one which would produce the maximum useful effect. We shall so have the most advantageous *practical* regulation, and calculating with it the effects of the engine, we shall have the maximum of the effects which it is capable of producing.

We will finally remark that it is customary in these engines, to make use of the *cataract*. Under this circumstance the engine does not evaporate the full quantity of water, that its boiler would otherwise be capable of evaporating per minute; but on introducing into the formulæ the evaporation really effected, the formulæ will always give the corresponding effects of the engine.

The comparison which we have made between an engine of Watt and a Cornish engine, that would consume the same quantity of fuel per hour, proves the superiority of the latter engine to the former. It is evident also from the same comparison, that the effects attributed to their engines by the Cornish engineers, have been erroneously called in question;

since the example we have chosen shows that the useful effect arising from the combustion of 1 lb. of fuel is

909160 lbs. raised 1 foot,

which gives per imperial bushel, or 84 lbs. of fuel,

76369440 lbs. raised 1 foot.

If then we consider engines, wherein the steam should be employed at a total pressure of 80 lbs. for instance, instead of 50 lbs. per square inch, and more especially, whose boiler should be constructed in a more improved system for the application of the heat, still greater useful effects may be expected from them.

CHAPTER XII.

ATMOSPHERIC ENGINES.

ARTICLE I.

ATMOSPHERIC ENGINE WITH A CONDENSER.

SECT. I. *Regulation of the Engine.*

IN the atmospheric engines, the steam is first introduced under the piston, in order to raise it by the help of a counterweight attached to the opposite extremity of the beam; this steam is then condensed, and the piston pressed on the upper face by the weight of the atmosphere, re-descends to the bottom of the cylinder, causing the load, that is the water in the pumps, to rise to a corresponding height, and raising the counterweight at the same time. Then a fresh quantity of steam is admitted into the cylinder, the piston is raised again, and the action continues as before.

In this system then the motive power is successively, the pressure of the steam, aided by the counterweight, and afterwards the atmospheric pressure; and the useful effect, instead of being produced during the first of these periods, that is, at the moment of the application of the steam, is produced on the contrary during the action of the atmospheric pressure. Consequently these two forces act by turns; however, as the atmospheric pressure is here but an inert force, incapable of producing any effect, unless the means were first afforded to it by the production and application of the steam, it follows that the steam, after all, is the true force creative of the motion.

The mode of action of this engine may easily be reduced to that of a single-acting engine of Watt; for, since the atmospheric pressure is equivalent to a weight of 14·71 lbs. per square inch, we may suppose actual weights of 14·71 lbs. per square inch of the surface, to be placed on the upper part of the piston, and these weights will produce the same effect as the atmospheric pressure. We may then suppress the atmospheric pressure, and consider things as taking place in a vacuum; so, nothing in the system will be changed. In this manner the engine reduces itself to a material weight placed on the piston on the one hand, and on the other hand to the pressure of the steam introduced and suppressed under the piston by turns, to raise it first by the help of the counter-weight, and to let it afterwards descend by the effect of the weight which has replaced the atmospheric pressure. We are thus led precisely to the case of a single-acting engine of Watt, in which the useful effect, instead of being produced immediately, during the action of the steam, would be produced during the return of the piston; and consequently, in these engines, the steam performs the same office as in Watt's single-acting engines.

The atmospheric engines, having in general neither fly-wheel nor crank, that is, not being rotative, require like Watt's single-acting engines, a particular regulation; in order that the piston may stop of itself in the cylinder, after having performed the whole stroke assigned to it, without either going beyond or falling short. To this end, a moment before the piston reaches the top of the cylinder in the rising stroke, the flow of steam from the boiler is intercepted. So, through the space which yet remains to be traversed, the piston moves only by virtue of its acquired velocity, of the effort of the counterweight, and of the decreasing pressure of the steam during its expansion. But as soon as the communication from the boiler is intercepted, the resistance of the atmosphere above the piston presently becomes superior to the motive force; consequently the piston is brought to rest without shock and by insensible degrees.

In the descending stroke, on the contrary, the condensation of the steam is suspended somewhat before the end of the stroke, either by stopping the injection water when the steam is condensed in the cylinder itself, or by shutting in time the communication from the cylinder to the condenser, when the engine is furnished with a separate condenser. The steam thus confined under the piston without being condensed, opposes to it therefore, on being compressed, a greater and greater resistance, and at last brings it gently to rest. But it is to be remarked that, as this steam thus acquires all the labour developed by the piston in coming to rest, and as itself is to contribute to the next stroke of the piston, no loss of action is incurred.

By means of this management, the practical object of which is to preserve the bottom of the cylinder from the shocks of the piston, the atmospheric engine, like the single-acting engine of Watt, is regulated in its motion and brought to rest at each stroke without loss of *vis viva*. Consequently, referring to what has been said on this head, in treating of Watt's single-acting engines, it will be recognized that the formulæ suitable to the calculation of atmospheric engines may still be grounded on the same principles as the preceding, viz.: equality between the labour developed by the power and that performed by the resistance in the two strokes of the piston, and equality between the expenditure of steam by the cylinder and the useful evaporation of the boiler.

Sect. II. *Of the effects of the engine with a given counterweight and an indefinite load or velocity.*

Three cases are to be distinguished in the working of the atmospheric engines: that wherein they work with a given counterweight, and with a load or velocity *indefinite;* that in which they work with a given counterweight, and the load or velocity which produces the *maximum of useful effect for that counterweight;* and lastly, that wherein the counterweight having been previously regulated at the degree most advantageous for the working of the engine, the most favourable load for that counterweight be also adopted, which consequently produces the absolute *maximum of useful effect* of which the engine is capable.

We will first suppose the first case, and call as before:

Π The weight of the counterweight, supposed to be divided per unit of the surface of the piston;

P the total pressure of the steam in the boiler;

a the area of the cylinder;

l the stroke of the piston;

l' the portion of the ascending stroke, performed before the steam from the boiler is intercepted;

l'' the portion of the descending stroke, performed before the shutting of the injection cock, or of the communication from the cylinder to the condenser;

r the useful load of the piston, divided per unit of the surface of the piston;

f' the friction of the engine, then unloaded, during the ascending stroke of the piston;

f'' the friction of the unloaded engine, during the descending stroke of the piston;

δ the surplus accruing to the latter friction, per unit of the load r imposed on the engine in the descending stroke;

ϕ the atmospheric pressure;

p the mean pressure, subsisting under the piston, by reason of the imperfect condensation of the steam.

Finally, we will suppose, the engine furnished with a separate condenser.

This premised, the steam, during the ascending stroke of the piston, penetrates into the cylinder, as in Watt's single-acting engine, with the same pressure as in the boiler, and the quantity of action developed by its full pressure while coming directly from the boiler, and afterwards by its decreasing pressure during the expansion, has likewise for its expression

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}\right)\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right)-\frac{n}{q}al.$$

Again, during the same stroke, the work developed by the counterweight, in descending from the height l, is $\Pi a l$; that performed by the friction of the engine, when unloaded, is $f' a l$; and finally, that performed by the atmospheric pressure, is $\phi a l$. Consequently, the equality between the work developed by the power and by the resistance will furnish the equation

$$a(l'+c)\left(\frac{n}{q}+\mathrm{P}\right)\left(\frac{l'}{l'+c}+\log\frac{l+c}{l'+c}\right)-\frac{n}{q}al+\Pi al=\phi al+f'al.$$

And making, in order to simplify,

$$k' = \frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right),$$

the above equation may be written under the form

$$k' = \frac{\frac{n}{q} + \phi + f' - \Pi}{\frac{n}{q} + P} \quad .\ .\ .\ .\ (1).$$

This will then be the first of the three general relations sought.

To pass on to the descending stroke of the piston, the quantity of work applied by the atmospheric pressure, which is then the motive power, has for its value $\phi\, a\, l$. On the other hand, the quantity of action developed by the counterweight, is $\Pi\, a\, l$; that of the load is $r\, a\, l$, and that of the friction of the loaded engine is $(f'' + \delta\, r)\, a\, l$.

As to that which is developed by the pressure p, subsisting under the piston by reason of the imperfect condensation of the steam, it must be decomposed into two parts. The pressure p is at first exerted without any additional effect, while the piston is traversing the portion l'' of its stroke; that is to say, till the communication from the cylinder to the condenser is shut, or till the condensation of the steam is suspended, if the engine has no condenser. This pressure produces then, in this first interval, a quantity of action expressed by

$$p\, a\, l''.$$

But beyond this point, the condensation ceases; the steam which still subsists under the piston begins to be more and more compressed, and this effect prevails through the length $(l-l'')$, which the piston has yet to traverse to terminate its stroke. It remains then to determine the quantity of action developed by the steam during this compression.

Now, at the moment when this steam begins to be compressed, it is at the pressure p, and the volume it occupies below the piston is

$$a(l-l''+c).$$

If then we call π the pressure, measured per unit of surface, that it will have when the piston shall have performed the length λ of its stroke, and if we suppose that the piston perform besides an elementary space $d\lambda$, the corresponding elementary work, produced by the compression of the steam, will be

$$a\pi\, d\lambda.$$

But as it is the same steam which, after having occupied, in contact with the condensing water and under the pressure p, the space

$$a(l-l''+c),$$

now occupies under the pressure π, the space

$$a(l-\lambda+c),$$

without having lost, in the interval, any portion of its total heat, there exists between the volumes and the corresponding pressures of the steam, the relation (c), which we have demonstrated generally (Chap. III., Art. I., Sect. II.), viz.:

$$\pi = \left(\frac{n}{q} + p\right) \frac{a\,(l - l'' + c)}{a\,(l - \lambda + c)} - \frac{n}{q}.$$

Consequently, we thence conclude,

$$\pi\, a\, d\lambda = a \left(\frac{n}{q} + p\right) (l - l'' + c) \frac{d\lambda}{l - \lambda + c} - \frac{n}{q}\, a\, d\lambda.$$

Thus, proceeding as before, that is to say, taking the integral between the limits l'' and l, we have for the quantity of action developed by the gradual compression of the steam on the portion $(l - l'')$ of the stroke, the following expression:

$$a \left(\frac{n}{q} + p\right) (l - l'' + c) \log \frac{l - l'' + c}{c} - \frac{n}{q}\, a\, (l - l''),$$

wherein the term

$$\log \frac{l - l'' + c}{c}$$

expresses an hyperbolic logarithm.

Adding to this the work $p\, a\, l''$, produced during the portion of the stroke anterior to the compression of the steam, we shall have for the total quantity of action developed by the resistance of the uncondensed steam

$$a l \left(\frac{n}{q} + p\right) \left(\frac{l''}{l} + \frac{l - l'' + c}{l} \log \frac{l - l'' + c}{c}\right) - \frac{n}{q}\, a\, l.$$

If, to abridge, we make

$$k'' = \frac{l''}{l} + \frac{l - l'' + c}{l} \log \frac{l - l'' + c}{c},$$

the above expression may be written under the form

$$\left(\frac{n}{q} + p\right) k''al - \frac{n}{q}\, al.$$

Consequently, referring to what has been found for the quantity of action developed by the atmospheric pressure, the load, the counterweight, and the friction of the engine, we will express the equality between the work applied by the power and the work performed by the resistance, in the stroke now under consideration, by the following equation:

$$\phi al = ral + \Pi al + (f'' + \delta r)\, al + \left(\frac{n}{q} + p\right) k''al - \frac{n}{q}\, al,$$

which gives

$$k'' = \frac{\frac{n}{q} + \phi - (1 + \delta)\, r - f'' - \Pi}{\frac{n}{q} + p} \quad . \; . \; . \; (\mathrm{B})$$

This is, in consequence, the second relation between the data and the incognita of the problem.

Finally, the third relation sought will be obtained by expressing the equality between the production and the expenditure of the steam.

S being still the volume of water evaporated per minute in the boiler, and effectually transmitted to the cylinder, that volume of water, once converted into steam at the pressure P of the boiler, will become, as we have seen,

$$\frac{\mathrm{S}}{n + q\mathrm{P}}.$$

Moreover, the capacity of the cylinder which is filled with steam at every ascending stroke of the piston, is expressed by

$$a\,(l'+c).$$

But we have seen that at every descending stroke, a certain quantity of steam remains compressed under the piston, and serves for the next stroke; that is to say, is returned to the boiler, instead of being condensed. The pressure of this steam, at the moment of its separation in the cylinder, is p, and the volume it occupies under that pressure is

$$a\,(l-l''+c).$$

If it returned to the pressure P, without losing any of its total heat, its volume would change in the ratio

$$\frac{n+qp}{n+q\mathrm{P}};$$

that is to say, it would become

$$a\,(l-l''+c)\frac{n+qp}{n+q\mathrm{P}}.$$

This expression gives then the volume of steam, measured at the pressure of the boiler, which is given back at each descending stroke. So that the real expenditure of steam per double stroke of the piston, is but

$$a\,(l'+c)-a\,(l-l''+c)\,\frac{n+qp}{n+q\mathrm{P}}.$$

Consequently, representing by M the number of double strokes of the piston given by the engine in a minute, the expenditure of steam per minute will be

$$M\left[a(l'+c)-a(l-l''+c)\frac{n+qp}{n+qP}\right].$$

But, calling V the mean velocity of the piston, or the space it traverses per minute both in rising and falling, we shall have $V=2\,M\,l$; or, counting for the velocity, only the space traversed by the piston in producing the useful effect, that is, in the descending stroke alone, and calling that velocity thus measured v, we have

$$v=M\,l, \text{ or } M=\frac{v}{l}.$$

Thus, the volume of steam expended by the cylinder in one minute, will be

$$\frac{v}{l}a\left[l'+c-(l-l''+c)\frac{n+qp}{n+qP}\right].$$

The equation expressing the equality between the production and the expenditure of the steam, will be then

$$\frac{v}{l}a\left[l'+c-(l-l''+c)\frac{n+qp}{n+qP}\right]=\frac{S}{n+qP};$$

which gives

$$v=\frac{S}{a}\cdot\frac{1}{\frac{l'+c}{l}(n+qP)-\frac{l-l''+c}{l}(n+qp)}. \quad (1)$$

This is the third relation sought.

Consequently, bringing forward the two equations (A) and (B) precedently obtained, and resolving them with reference to the quantities r and S, we shall have the following formulæ:

$$k' = \frac{\frac{n}{q} + \phi + f' - \Pi}{\frac{n}{q} + \mathrm{P}} \dots\dots\dots\dots (\mathrm{A})$$

$$k'' = \frac{\frac{n}{q} + \phi - (1+\delta)\, r - f'' - \Pi}{\frac{n}{q} + p} \dots\dots (\mathrm{B})$$

$$v = \frac{\mathrm{S}}{a} \cdot \frac{1}{\frac{l'+c}{l}(n + q\,\mathrm{P}) - \frac{l - l'' + c}{l}(n + q\,p)} \dots (1)$$

$$a\,r = \frac{a}{1+\delta}\left[\frac{n}{q} + \phi - f'' - \Pi - \left(\frac{n}{q} + p\right)k''\right] \dots (2)$$

$$\mathrm{S} = a\,v\left[\frac{l'+c}{l}(n + q\,\mathrm{P}) - \frac{l - l'' + c}{l}(n + q\,p)\right] (3)$$

$$\text{u.E.} = a\,r\,v \dots\dots\dots\dots\dots\dots\dots\dots (4)$$

The expressions k' and k'' contained in these equations, have the following values:

$$k' = \frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right),$$

$$k'' = \frac{l''}{l} + \frac{l - l'' + c}{l}\log\frac{l - l'' + c}{c}.$$

Sect. III. *Of the velocity of the engine with a given load.*

The formulæ which we have just given, suffice for the solution of all the problems that can occur on the atmospheric engine. As, however, it is impossible to draw from the two equations (A) and (B) the direct values of $\frac{l'}{l}$ and $\frac{l''}{l}$, in terms of the data of the problem, in order to substitute them in the formulæ (1), (2), (3), (4), as would be requisite for these formulæ to give immediately the solution sought, it will be necessary to use a mode of calculation analogous to that which we have already indicated for Watt's single-acting engine, and to which we will recall attention here.

Suppose the load of the engine known, and that it be required to find what velocity the engine will assume with that load.

The given value of r must be substituted in the equation (B),

$$k'' = \frac{\frac{n}{q} + \phi - (1 + \delta)\, r - f'' - \Pi}{\frac{n}{q} + p},$$

which will consequently give the value of k''. Then, recurring to the Table which we are about to give, we will find the corresponding value of $\frac{l''}{l}$. On the other hand, by calculating the second member of the equation (A),

$$k' = \frac{\frac{n}{q} + \phi + f' - \Pi}{\frac{n}{q} + \mathrm{P}},$$

the value of k' will be obtained; and afterwards, by means of the same Table, the value of $\frac{l'}{l}$. Thus, therefore, will be had $\frac{l'}{l}$ and $\frac{l''}{l}$; and consequently by substituting the value of these two quantities in the equation (1)

$$v = \frac{S}{a} \cdot \frac{1}{\frac{l'+c}{l}(n+q\text{P}) - \frac{l-l''+c}{l}(n+qp)},$$

the required velocity will readily be deduced.

With respect to the latter calculation, we will remark that, if m and m' be put to express the relative volumes of the steam under the respective pressures P and p, there will result

$$m = \frac{1}{n+q\text{P}} \text{ and } m' = \frac{1}{n+qp}.$$

The value of v, then, may also be written

$$v = \frac{S}{a} \cdot \frac{1}{\frac{l'+c}{l} \cdot \frac{1}{m} - \frac{l-l''+c}{l} \cdot \frac{1}{m'}};$$

and under this form it will obviously be more easily calculated and more exactly obtained, for the quantities m and m' will be known with strict accuracy, and yet without calculation, by the Tables which we have given in Sect. III. of Chapter II.

Sect. IV. *Of the load of the engine with a given velocity.*

Suppose the velocity of the engine be known, and that it be required to find what load it can put in motion at that velocity.

The equation (A)

$$k' = \frac{\frac{n}{q} + \phi + f' - \Pi}{\frac{n}{q} + P}$$

will first furnish the value of k', and consequently that of $\frac{l'}{l}$, by means of the table we have mentioned. Then the value of $\frac{l'}{l}$ thus known must be substituted in equation (1), which will give the value of $\frac{l''}{l}$, namely

$$\frac{l - l'' + c}{l} = \frac{l' + c}{l} \cdot \frac{n + qP}{n + qp} - \frac{S}{av} \cdot \frac{1}{n + qp}.$$

Consequently, by recurring to the same table, the corresponding value of k'' may be known, and this substituted in equation (2), which is

$$ar = \frac{a}{1 + \delta}\left[\frac{n}{q} + \phi - f'' - \Pi - \left(\frac{n}{q} + p\right)k''\right],$$

will give definitively the value of $a\,r$.

SECT. V. *Of the evaporation of the engine, in order to produce desired effects.*

Suppose the load and the velocity of the engine to be fixed *à priori*, and that it be required to find the evaporation necessary to produce the desired velocity with the given load.

The equations (A) and (B) will first make known k' and k'' and subsequently $\frac{l'}{l}$ and $\frac{l''}{l}$; substituting these then, as well as the given velocity, in equation (3):

$$S = a v \left[\frac{l' + c}{l} (n + q P) - \frac{l - l'' + c}{l} (n + q p)\right],$$

we will thence conclude the value of S.

Sect. VI. *Of the useful effect of the engine.*

Finally, the load being given, let it be required to find what useful effect the engine will produce with that load.

The velocity corresponding to the load must first be calculated, as has just been said; then multiplying that velocity by the load, the product $a r v$ will be the corresponding useful effect.

If the velocity is known, instead of the load, we will calculate the load corresponding to that velocity, and their product $a r v$ will again give the corresponding useful effect.

Finally, the divers expressions of the useful effect, whether in horse-power, or measured by the force arising from a given expenditure of water or of fuel, will be determined, in terms of the product arv, by the formulæ of Sect. V. Art I. Chap. III., which suit all engines.

Thus, means are afforded of resolving all questions that can arise with respect to atmospheric engines.

SECT. VII. *Determination of the friction of the engine unloaded, and of its additional friction per unit of the load.*

As the use of the preceding formulæ supposes that there are means of determining the friction of the engines working without load, and the surplus accruing to that friction for each unit of the load imposed on the engine, it becomes necessary to dwell a moment on that determination.

The equation (A) viz.:

$$k' = \frac{\frac{n}{q} + \phi + f' - \Pi}{\frac{n}{q} + \mathrm{P}}$$

gives

$$f' = \left(\frac{n}{q} + \mathrm{P}\right) k' - \frac{n}{q} + \Pi - \phi.$$

To know the friction f' of the unloaded engine in the ascending stroke, it suffices then to determine by experiment or by direct observation, the quantities ϕ, Π, P, and k'.

Now, this is an easy matter: for ϕ is the atmospheric pressure, which is given by the barometer; Π is the counterweight of the engine, which is known; and P the total pressure of steam in the boiler, which is measured by means of a manometer. Thus, k' alone requires a particular determination. Suppose then an experiment to be made on an engine with any load whatever, and that the rising stroke be regulated by trials, as is always done in practice. Then the quantity $\frac{l'}{l}$, that is to say, the point at which the steam from the boiler should be intercepted, becomes given by the experiment itself. Consequently, recurring to the table which we shall give presently, or to the developed expression of k', namely:

$$k' = \frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right),$$

it will be easy to know the value of k' corresponding to that of $\frac{l'}{l}$. Thus, by substituting this value of k' in the preceding equation, with the observed values of ϕ, Π and P, we shall conclude without difficulty the value of f', or the friction of the unloaded engine, in the ascending stroke.

To find afterwards the similar friction f'', of the descending stroke, recourse must be had to equation (B)

$$k'' = \frac{\frac{n}{q} + \phi - (1+\delta)\,r - \Pi - f''}{\frac{n}{q} + p},$$

which gives

$$f'' = \frac{n}{q} + \phi - (1+\delta)\,r - \Pi - \left(\frac{n}{q} + p\right) k''.$$

This relation is general, and consequently must still subsist when the load of the engine is null, that is, when $r = 0$. But then it reduces itself to

$$f'' = \frac{n}{q} + \phi - \Pi - \left(\frac{n}{q} + p\right) k'';$$

in order, then, to have the value of f'', it will suffice to be able to find directly what values the quantities p and k'' assume, with no load; for as to ϕ and Π, they are constant quantities which may always be measured, as has been said above.

To have the values of p and k'' which correspond to the load zero, recourse must be had to experiment. The engine must be set to work without applying any load on it, but effecting the condensation very imperfectly, that is to say, retaining under the piston a high pressure for the uncondensed steam, and shutting with precaution and in time the cock of the condenser, or the injection cock; so that the engine shall be regulated in its descending stroke, by its friction alone and the resistance of the steam compressed under the piston. Then must be measured directly the portion $\frac{l''}{l}$ of the stroke, which is performed before suspending the condensation. Recurring afterwards to the developed expression of k'', viz. :—

$$k'' = \frac{l''}{l} + \frac{l - l'' + c}{l} \log \frac{l - l'' + c}{c},$$

it will be easy to know the value of k'' corresponding to the measured value of $\frac{l''}{l}$. At the same moment the quantities ϕ, Π and p, that is, the atmospheric pressure, the counterweight and the pressure of the uncondensed steam, will be taken by direct observation. Consequently, substituting these different values in the equation

$$f'' = \frac{n}{q} + \phi - \Pi - \left(\frac{n}{q} + p\right) k'',$$

we shall conclude the friction of the unloaded engine, in its descending stroke.

With respect to the direct measure of the quantity p, it must be effected with Watt's *indicator of the pressure*, if the engine has a condenser; because we have seen, in speaking of Watt's double-acting engines, that the pressure in the cylinder is always superior to that in the condenser. But if the engine has no condenser, it will suffice to take with a thermometer the temperature of the water coming out of the eduction pipe, that is, of the water which issues from the cylinder after the condensation. This temperature being also that of the steam with which the water was in contact, the corresponding pressure will be found in the tables which we have given in Chapter II. of this work, on the pressure and temperature of steam in contact with the liquid.

Knowing the quantity f'', or friction of the unloaded engine, it will be easy to obtain the surplus δ accruing to that friction per unit of the load imposed on the engine. To this end, the engine must be set to work with a known load r, and the stroke regulated by trials in the usual manner. Then, taking as above, the direct measure of the quantities ϕ, Π, $\frac{l''}{l}$ and p, and substituting them with the value of r, in the equation (B), which gives

$$1+\delta=\frac{\frac{n}{q}+\phi-\Pi-f''-\left(\frac{n}{q}+p\right)k''}{r},$$

the value of δ will be concluded immediately.

Thus the three constant quantities f', f'' and δ may be determined on several engines; and from the aggregate of these determinations, may be concluded a *mean* valuation of those quantities, which will afterwards be used in general calculations and applied to engines not yet constructed.

We will here, however, make an observation analogous to that which we have already made relative to Watt's single-acting engines: viz., that in the engines used for raising water, the friction of the engine in the ascending stroke is not strictly a constant quantity. That friction, in fact, comprises the resistance offered by the water to the pump piston or plunger, which resistance changes with the velocity of the motion; but as the velocity of atmospheric engines varies within very narrow limits, it will suffice to determine f' for the average usual velocity, and that determination may be considered as a mean applicable to the different cases.

SECT. VIII. *Tables for the numerical solution of the formulæ.*

As the use of the preceding formulæ requires a table, furnishing immediately the values of k' and

k'' corresponding to given values of $\frac{l'}{l}$ or $\frac{l''}{l}$, and *vice versâ*, we will here give that table calculated from hundredth to hundredth, for the values of $\frac{l'}{l}$ and $\frac{l''}{l}$.

When the result of a formula shall have made known k' or k'', the table will, on inspection, give $\frac{l'}{l}$ or $\frac{l''}{l}$; whence will immediately be concluded

$$\frac{l'+c}{l} \quad \text{and} \quad \frac{l-l''+c}{l},$$

since these fractions are nothing more than

$$\frac{l'}{l}+\frac{c}{l} \quad \text{and} \quad \frac{l+c}{l}-\frac{l''}{l},$$

and that the quantity c, which represents the clearance of the cylinder, is always known in terms of the useful length of the stroke, or of the quantity l.

We have, in the following table, taken the clearance of the cylinder equal to a tenth of the useful stroke of the piston, that is to say, we have made $c = {\cdot}1\, l$; because that is the proportion generally adopted in practice. However, as in atmospheric engines without a condenser, it often happens that the clearance of the cylinder amounts to 2 or 3 tenths of the stroke, we will add to the table No. II., which alone is required for calculating those engines, two other columns for the cases of $\frac{c}{l} = {\cdot}2$ and $\frac{c}{l} = {\cdot}3$.

No. I.—*TABLE for the numerical solution of the formulæ.*
(Atmospheric engines.)

Portion of the ascending stroke performed during the admission of the steam under the piston, or value of the fraction $\frac{l'}{l}$.	Corresponding value of k', or of the expression $\frac{l'+c}{l}\left(\frac{l'}{l'+c} + \log\frac{l+c}{l'+c}\right)$.
·50	·863
·51	·869
·52	·875
·53	·881
·54	·887
·55	·892
·56	·897
·57	·902
·58	·907
·59	·912
·60	·917
·61	·921
·62	·925
·63	·929
·64	·933
·65	·937
·66	·941
·67	·945
·68	·949
·69	·952
·70	·955
·71	·958
·72	·961
·73	·964
·74	·967
·75	·970
·76	·973
·77	·975
·78	·977
·79	·979
·80	·981
·81	·983
·82	·985
·83	·987
·84	·989
·85	·990
·86	·991
·87	·992
·88	·993
·89	·994
·90	·995
·91	·996
·92	·997
·93	·998
·94	·999
·95	·999

No. II.—*TABLE for the numerical solution of the formulæ. (Atmospheric engines.)*

Portion of the descending stroke performed before the shutting of the injection cock, or value of the fraction $\frac{l''}{l}$.	Corresponding value of k'', or of the expression $\frac{l''}{l} + \frac{l-l''+c}{l} \log \frac{l-l''+c}{c}$.		
	$\frac{c}{l}=\cdot1.$	$\frac{c}{l}=\cdot2.$	$\frac{c}{l}=\cdot3.$
·5	1·575	1·377	1·286
·51	1·557	1·364	1·275
·52	1·539	1·352	1·265
·53	1·522	1·340	1·255
·54	1·505	1·328	1·246
·55	1·488	1·316	1·237
·56	1·471	1·304	1·228
·57	1·454	1·293	1·219
·58	1·437	1·282	1·210
·59	1·421	1·271	1·201
·60	1·405	1·260	1·193
·61	1·389	1·249	1·185
·62	1·373	1·238	1·177
·63	1·357	1·227	1·169
·64	1·342	1·216	1·161
·65	1·327	1·206	1·153
·66	1·312	1·196	1·145
·67	1·297	1·186	1·137
·68	1·283	1·176	1·130
·69	1·269	1·167	1·123
·70	1·255	1·158	1·116
·71	1·241	1·149	1·109
·72	1·227	1·140	1·102
·73	1·214	1·131	1·096
·74	1·201	1·123	1·090
·75	1·188	1·115	1·084
·76	1·176	1·107	1·078
·77	1·164	1·099	1·072
·78	1·152	1·091	1·066
·79	1·141	1·084	1·060
·80	1·130	1·077	1·055
·81	1·119	1·070	1·050
·82	1·108	1·064	1·045
·83	1·098	1·058	1·041
·84	1·088	1·052	1·037
·85	1·079	1·046	1·033
·86	1·070	1·040	1·029
·87	1·061	1·035	1·025
·88	1·053	1·030	1·021
·89	1·046	1·026	1·018
·90	1·039	1·022	1·015
·91	1·032	1·018	1·012
·92	1·026	1·014	1·010
·93	1·020	1·011	1·008
·94	1·015	1·008	1·006
·95	1·011	1·006	1·004

SECT. IX. *Of the maximum useful effect with a given counterweight, and of the absolute maximum of useful effect.*

To arrive at the determination of the maximum useful effect with a given counterweight, and afterwards at that of the absolute maximum of useful effect of the engine, it would be necessary, first to form the general expression of the useful effect under a given load, in terms of the load only. Then putting equal to zero the differential of that expression, taken with reference to the load considered as variable, we should have the conditional equation, proper to determine the load suitable to the production of the *maximum of useful effect with a given counterweight.* Afterwards would be also concluded, in the same way, an equation for determining the *absolute maximum of useful effect,* as we have done with respect to the rotative engines, in Articles II. and III. of Chapter III. of this work.

Referring to equation (1),

$$v=\frac{S}{a}\cdot\frac{1}{\frac{l'+c}{l}(n+qP)-\frac{l-l''+c}{l}(n+qp)},$$

it is plain that on multiplying both members by ar, we obtain the general expression of the useful effect, viz.:

$$arv=\frac{Sr}{\frac{l'+c}{l}(n+qP)-\frac{l-l''+c}{l}(n+qp)}.$$

As however the quantity $\frac{l''}{l}$ varies with the load of the engine, it would be necessary, in order to continue the calculation, to be able to replace that quantity, in the expression of $a r v$, before or after differentiating, by its value in terms of the load r. This value would have to be concluded from the developed equation (B), viz.:—

$$\frac{l''}{l}+\frac{l-l''+c}{l}\log\frac{l-l''+c}{c}=\frac{\frac{n}{q}+\varphi-(1+\delta)r-f''-\Pi}{\frac{n}{q}+p};$$

but as the nature of this equation does not admit of resolving it directly with reference to r, we must here, as in Watt's and the Cornish single-acting engines, have recourse to the method of successive trials and approximations.

To this end, when the counterweight is given, the calculation must be made of the useful effect with various loads successively, taking care to vary the load on that side on which the useful effect shall be found to increase; and after a few trials, the load will be attained, which produces the maximum of useful effect with the given counterweight.

Afterwards the counterweight must be varied in the same manner, and the corresponding maximum useful effect determined for each of its values. Then, on comparing with each other the divers maxima useful effects, due to the different values of the counterweight, we shall determine that counterweight which produces the absolute maximum of useful effect in the engine.

This research, apparently complicated, simplifies itself considerably from the circumstance of the same numbers recurring constantly in the calculation. Besides, when the most advantageous load for a given counterweight is found, and that the counterweight is afterwards made to vary, it is observable that on the counterweight being increased a certain quantity, the corresponding most advantageous load will be lowered by a nearly equal quantity, relatively to the first load; and that if, on the contrary, the counterweight be diminished a certain quantity, the most advantageous load will augment nearly as much. By this observation, the research of the *absolute* maximum of useful effect is reduced to a very few trials, nor does it present any difficulty at all comparable to the importance of the object proposed, namely, to find the means of making an engine work in the most advantageous manner possible.

ARTICLE II.

ATMOSPHERIC ENGINE WITHOUT CONDENSER.

SECT. I. *Modifications to be made in the preceding formulæ, for the case of the engine not furnished with a separate condenser.*

In the theory of the atmospheric engine, which we have just explained, we have admitted that, till the moment of the communication from the boiler to the cylinder being intercepted, the steam acts in the cylinder at the same pressure as in the boiler. This does in fact take place, at least without noticeable error, in engines furnished with a separate condenser, and whose cylinder is properly protected by a double casing against all external refrigeration. But in very many atmospheric engines, the condensation after every stroke of the piston, takes place in the steam cylinder itself; so that the latter is found cooled to the temperature of condensation, at the moment when the steam arrives to cause a new ascending stroke. Immediately then on entering the cylinder, the steam must tend to establish an equilibrium of temperature with it, the effect of which tendency is to raise the temperature of the cylinder by lowering that of the steam. Hence, instead of retaining its original temperature, the steam necessarily acquires in the cylinder a certain temperature, and consequently, a certain pressure, intermediate between that of its formation in the boiler and that of condensation in the cylinder.

What is then this intermediate pressure which the steam assumes? That is the point which it will be proper to examine first.

To obtain this determination, it must be observed that on entering the cylinder, the steam comes immediately into contact with a cooled surface; that by degrees, as the piston is driven back in the cylinder, it lays open a new portion of surface to be heated; and finally, that the more the velocity of the piston increases, the more surface, in a unit of time, is exposed to the heating action of the steam. Now, the quantity of steam produced by the boiler in a unit of time is a fixed and determinate quantity. As that steam gradually penetrates into the cylinder, a portion of it will be condensed to effect the heating of the cylinder, and the rest will lower both in temperature and pressure; so that it can only be with a reduced pressure, that the steam will act on the piston to effect its motion in the cylinder. There may then occur three cases. If the pressure of the steam after refrigeration be found still superior to the resistance of the piston, its effect will be to create the velocity of the piston, or to augment it, if it be already produced. If the pressure of the refrigerated steam be merely equal to the resistance of the piston, its effect will be to maintain the motion of the piston in a state of uniformity, without increasing or diminishing its velocity. And finally, if the pressure of the steam after cooling be found inferior to the resistance of the piston, the motion of the latter will slacken and come at length completely to a stand.

This premised, at the first moment the steam is introduced into the cylinder, to produce the ascending stroke, there is already a considerable portion of surface to be heated, for it consists of the clearance of the cylinder, the bottom of the cylinder, and the lower face of the piston. The first effect of the contact of the steam with this large extent of surface will therefore be sensibly to lower the pressure of the steam. But the piston is loaded with a considerable resistance, amounting indeed very nearly to the pressure of the atmosphere, for the friction of the engine and the counterweight, which act contrariwise to each other, tend to produce an equilibrium. Thus the refrigerated steam will be found at first too weak for the resistance to be moved, and a moment will elapse before the starting of the piston can be effected.

However, by degrees, as the steam condensed by the contact of the cylinder is replaced by a fresh quantity of steam supplied from the boiler, and the temperature of the exposed portion of the cylinder rises, the steam in the cylinder will acquire a greater pressure. It will soon then attain the degree requisite to determine the starting of the piston; and the latter will assume a certain velocity, which will still increase as long as the pressure of the steam in the cylinder, that is to say of the refrigerated steam, shall exceed the resistance of the piston.

But since we have seen that, as the velocity of the piston increases, the surface to be heated in a given time increases also: it follows that the greater the velocity of the piston becomes, the more refrigeration the steam will undergo, and the more the pressure in the cylinder will be diminished. Now the original pressure of the steam in the boiler exceeds but little the atmospheric pressure, which represents very nearly the resistance on the piston. A velocity then will soon occur wherein the pressure of the steam, after its refrigeration in the cylinder, shall not exceed the resistance of the piston; and consequently, from that moment, the motion of the piston will become uniform, till the state of things be changed by entirely suppressing the admission of the steam from the boiler.

Finally, as soon as the communication from the boiler to the cylinder shall be intercepted, the steam contained in the cylinder will begin to expand, and consequently to diminish its pressure. But as the receding of the piston continues still to expose a new portion of the cylinder to be heated, the expansion of the steam will be attended with a continual condensation, which will itself contribute to lower the pressure more rapidly. Thus the resistance will promptly obtain a preponderance over the motive force, and the piston be brought to rest in a very short time.

Consequently, there obviously exists in the engine a rapid tendency to produce uniformity in the motion of the piston, that is to say, an equilibrium between the pressure of the steam after refrigeration in the cylinder, and the resistance of the piston; and save in a very short interval at the beginning and at the end of the stroke, that equilibrium must subsist during the whole ascending motion of the piston.

Now, on examining the motion of an atmospheric engine without condenser, it is observable, that at the moment when the steam penetrates into the cylinder to produce the rising stroke, the piston remains motionless an instant at first, before effecting its starting. Then, as soon as it begins to move, it acquires, in a very short time, a remarkably uniform motion; and this lasts without interruption till the moment when the steam from the boiler is intercepted, after which the velocity very rapidly decreases. We must then conclude from this fact, which all observers have remarked, that during nearly the whole of the ascending motion, the equilibrium which we have mentioned above, does effectually establish itself between the pressure of the steam in the cylinder and the resistance of the piston.

Now the resistance of the piston, in this stroke, consists of the atmospheric pressure, augmented by the friction of the engine and diminished by the action of the counterweight. Consequently, calling P′ the unknown pressure assumed by the steam in the cylinder, we shall have

$$P' = \phi + f' - \Pi \; . \; . \; . \; . \; . \; . \; . \; (C).$$

This equation makes known the pressure assumed by the steam, during the continuance of the uniform motion of the piston. And neglecting the beginning and the end of the stroke, this pressure may be taken as the mean pressure of the steam during the whole stroke of the piston. But if, moreover, it be observed that, at the beginning of the stroke, the pressure of the steam must exceed the resistance by a certain quantity, in order to produce the motion, and that at the end of the stroke, or during the expansion, the resistance must on the contrary exceed the pressure by an equal quantity, in order to destroy the velocity already acquired, it will be recognized that the variations which the pressure undergoes in these two extreme points compensate each other. Thus the value of P′ given by the equality between the two forces, that is to say, by the preceding equation, is at once the real pressure of the steam during the uniform motion of the piston, and the mean value of the pressure of the steam taken in the whole duration of the stroke; that is to say, as well before as during the expansion of the steam.

The circumstance of the cooling of the steam in the cylinder, first gives us then the preceding equation, relative to the ascending stroke of the piston. It will now be proper to examine whether the other analogies already obtained for the engines with condensers, ought still to subsist or undergo some modification.

Among those analogies, equation (A) is intended to make known the quantity

$$\frac{l'}{l},$$

that is, the point of the stroke at which the flowing of the steam to the cylinder should be intercepted, for the piston to stop of itself and without shock, after having performed the whole of the stroke assigned to it. The considerations which led to the establishing of that equation, suppose that the steam enters the cylinder with a pressure constant and equal to that of the boiler; that the velocity communicated to the piston by means of that force, continually increases till the moment of intercepting the steam; and that, from that moment, the steam expands in the cylinder without any loss of the total heat which it contains. Now these different circumstances no longer occur in the engines under consideration, and therefore equation (A) is not applicable to them.

To calculate now the point at which the flowing of the steam into the cylinder is to be intercepted, it would be necessary to establish, between the forces which act on the piston in the ascending stroke, a relation analogous to equation (A), introducing into it the variations incident to the pressure of the steam during the different periods of the stroke, and the partial condensation attendant on the expansion of the steam, after the communication from the boiler to the cylinder is intercepted. This relation would make known the value of the ratio

$$\frac{l'}{l}.$$

But as the introduction of these circumstances render the research extremely complicate, and as it will presently be shewn to be unnecessary for attaining the knowledge either of the velocity or of the useful effects of the engine, we deem it needless to pursue the enquiry.

As to equation (B), viz.

$$k'' = \frac{\frac{n}{q} + \phi - (1 + \delta)\, r - f'' - \Pi}{\frac{n}{q} + p},$$

since it expresses the circumstances of the motion during the descending stroke of the piston, and since these circumstances suffer no change from the fact of the cooling and reheating of the cylinder during the rising stroke, this relation evidently will undergo no modification.

Finally, to obtain the equation of the velocity of the piston, we will deduce it as before, from the equality which necessarily subsists between the expenditure of steam by the cylinder and the effective or useful evaporation in the boiler. Calling S′ the *total* volume of water evaporated per minute in the boiler, this water is first converted, in the boiler, into steam at the pressure P. This steam then passes into the cylinder. There a certain portion of it is condensed by the contact of the cylinder, and the rest, which answers to what we call the *effective* evaporation of the engine, passes, by cooling, to the pressure P′, indicated above. But, as the water formed in the cylinder by the partial condensation of the steam, subsists there till the end of the stroke, it follows that the steam, on assuming the pressure P′, will find itself in contact with a certain quantity of liquid, and that it will in consequence be at the maximum density for its pressure and temperature. Hence, expressing still by S the *effective* evaporation of the engine per minute, or the volume of water actually employed in working the machinery, that volume of water once passed to the state of steam at the pressure P′, will occupy (Chapter II., Sect. IV., V., VI.) a space expressed by

$$\frac{S}{n + q\,P'}.$$

This will, therefore, be the volume of steam resulting from the effective evaporation of the engine.

On the other hand, since P′ is the mean pressure of the steam in the cylinder, during the *entire* stroke, it follows that the capacity of the cylinder which, at every ascending stroke, is filled with steam at that pressure, is

$$a\,(l + c).$$

But at every descending stroke, a certain quantity of steam is found compressed under the piston and restored to the boiler. The pressure of this steam, at the moment of its separation, is p, and the volume it occupies under that pressure is

$$a\,(l - l'' + c).$$

On repassing to the pressure P′, it loses no portion of its total heat, since that action takes place during the descending stroke; that is to say, while the cylinder undergoes no refrigeration. In this mutation, therefore, its volume will vary in the ratio

$$\frac{n + qp}{n + qP'};$$

thus, under the pressure P′, this steam, restored to the boiler, will represent a volume expressed by

$$a\,(l - l'' + c)\,\frac{n + qp}{n + qP'}.$$

Therefore the real expenditure of steam per double stroke of the piston, will be

$$a\,(l+c)-a\,(l-l''+c)\,\frac{n+q\,p}{n+q\,\mathrm{P}'}\,.$$

Consequently, if the engine gives M double strokes of the piston per minute, the corresponding expenditure of steam will be

$$\mathrm{M}\,\left[a\,(l+c)-a\,(l-l''+c)\,\frac{n+q\,p}{n+q\,\mathrm{P}'}\right].$$

But, expressing by v the velocity of the piston, or the space it describes in producing the useful effect, we shall have

$$v=\mathrm{M}\,l,\ \text{or}\ \mathrm{M}=\frac{v}{l}\,.$$

The volume, therefore, of steam expended by the cylinder in one minute, will be

$$a\,v\left[\frac{l+c}{l}-\frac{l-l''+c}{l}\,.\,\frac{n+q\,p}{n+q\mathrm{P}'}\right].$$

Consequently, making it equal to the volume of steam resulting from the effective evaporation of the engine, and which has been shown above, the following equation will be deduced :

$$v=\frac{\mathrm{S}}{a}\,.\,\frac{1}{\frac{l+c}{l}\,(n+q\mathrm{P}')-\frac{l-l''+c}{l}\,(n+q\,p)}\,;\ .\ (1)$$

And replacing P′ by its value drawn from equation (C), it may equally be written under the form

$$v=\frac{\mathrm{S}}{a}\,.\,\frac{1}{\frac{l+c}{l}\,[n+q\,(\phi+f'-\Pi)]-\frac{l-l''+c}{l}\,(n+q\,p)}\,.$$

This equation, therefore, must now replace that which we have obtained for the case of engines with a condenser; and it will be remarked that it differs from that equation, only in the substitution of l and P' for l' and P. Thus the only modifications to be introduced into the equations obtained for the case of engines with a condenser, will be: first, the substitution just mentioned, and secondly, the suppression of equation (A), which is not applicable to engines without a condenser.

Sect. II. *Of the difference between the total and the effective evaporation of the engine, in consequence of the refrigeration of the cylinder at every stroke of the piston.*

To enable us to pass on to the application of the formulæ just given, there is yet a point which requires some development. We have made, in the engines under consideration, a distinction between the total evaporation and the effective or useful evaporation. The latter is that which figures in our formulæ, whereas the total evaporation is that alone which can be directly observed, or which can be deduced from the measure of the heating surface of the boilers. It is necessary then to furnish means of passing from the knowledge of one of these quantities to the knowledge of the other.

When the condensation of the steam is not effected in a separate condenser, but in the steam cylinder itself, the metal of the cylinder is, at each stroke of the piston, first cooled to the temperature of condensation, and then reheated by the steam of the boiler to the temperature corresponding to the pressure P′, which the steam definitively assumes in the cylinder. At each stroke then of the piston, there is occasioned, in mere loss, a condensation of steam depending on the quantity of metal to be heated, and on the difference of temperature between the two successive states of the cylinder; and it is this incidental loss which causes the difference between the total evaporation and the effective evaporation of the engine.

Now, in the same engine, the quantity of steam thus condensed at every stroke, is proportional to the quantity of heat absorbed by the metal of the cylinder, that is to say, to the difference between its two successive temperatures. Expressing these two temperatures by T′ and t, the quantity of steam condensed will be proportional to the difference

$$(T' - t).$$

Moreover, between two engines wherein these two temperatures were the same, but which should differ from each other as to the extent of surface exposed to the contact of the steam, the quantity of steam condensed will be in the ratio of the extent of surface and of the thickness of the metal; or, since the thickness of the metal may be taken as proportional to the diameter of the cylinder, the condensation in question would be proportional to the product of the surface exposed to refrigeration, by the diameter of the cylinder.

If we term d the diameter of the cylinder, and π the ratio of the circumference to the diameter, l and c still expressing the length of the stroke and the clearance of the cylinder, the surface exposed to refrigeration, including the lower surface of the piston and the bottom of the cylinder, will evidently be

$$\pi d\,(l+c) + \tfrac{1}{2}\pi d^2.$$

But as, in these engines, the diameter of the cylinder is usually made equal to two thirds of the stroke, or

$$d = \tfrac{2}{3}\,l,$$

the refrigerated surface will have for its measure, more simply

$$\tfrac{2}{3}\pi l\,(l+c) + \tfrac{2}{9}\pi l^2.$$

Thus, considering at once the difference of temperature, the extent of surface to be heated, and the thickness of the metal, the quantity of steam condensed at every stroke will be proportional to the product of those three quantities. If then we measure the steam under the pressure P′, which it assumes definitively in the cylinder, and express by H a constant coefficient, the volume of steam condensed at each stroke will be of the form

$$\mathrm{H}\,(\mathrm{T}'-t) \times [\tfrac{2}{3}\pi l\,(l+c) + \tfrac{2}{9}\pi l^2] \times \tfrac{2}{3}l = \tfrac{1}{9}\pi l^2 \times \tfrac{4}{3}\mathrm{H}\,(\mathrm{T}'-t)\,[3\,(l+c)+l].$$

But the steam turned to use at each stroke in the cylinder, occupies there, under the pressure P′, a volume expressed by

$$\frac{1}{4}\,\pi\,d^2\,(l+c) = \tfrac{1}{9}\,\pi\,l^2\,(l+c).$$

Therefore the steam condensed will be to the steam turned to use, in the ratio indicated by the number

$$\frac{4}{3}\,\mathrm{H}\left(\mathrm{T}'-t\right)\left(3+\frac{l}{l+c}\right);$$

and as the effective steam, plus the steam condensed, represents the total steam produced, the steam effectively turned to use in the cylinder will be definitively to the total production of steam, in the ratio

$$\frac{\mathrm{S}}{\mathrm{S}'} = \frac{1}{1+\frac{4}{3}\mathrm{H}\left(\mathrm{T}'-t\right)\left(3+\frac{l}{l+c}\right)}.$$

The factor H, contained in this expression, depends on the capacity of the steam and metal for heat, and ought necessarily to be determined, once for all, by direct experiment.

For this purpose, must be measured accurately in an engine, and during a sufficient length of time: 1st, the temperature of the water coming out of the eduction pipe, that is, issuing from the cylinder, after the definitive condensation of the steam which has produced its effect; 2dly, the mean temperature of the cylinder, taken by placing a thermometer in contact with it, for experience proves that a thermometer so placed will readily stop at a constant degree; 3dly, the quantity of water coming out of the cylinder after the definitive condensation of the steam; 4thly, the quantity of water injected into the cylinder to produce that condensation; and, finally, a reckoning must be kept of the number of strokes of the piston given by the engine during the continuance of the experiment.

This premised, 1st, the temperature of the water coming out of the cylinder, after the definitive condensation of the steam, will also be that of the steam in contact with that water; that is to say, it will be the temperature t of the steam imperfectly condensed in the cylinder.

2dly, The observed temperature of the cylinder will be a mean between the temperature of condensation and the temperature of the steam while it exerts its action in the cylinder; since, during every descending stroke of the piston, the cylinder tends to be cooled to the temperature of condensation, and during every rising stroke, on the contrary, it tends to be heated to the temperature of the steam during its effective action. Putting, therefore, T for the observed temperature of the cylinder, we have

$$T = \frac{T' + t}{2};$$

and consequently

$$T' = 2T - t.$$

Thus will readily be known T′, or the mean temperature of the steam during its action in the cylinder, that is, while its pressure there is P′.

3dly, The useful expenditure of steam at the pressure P′, made by the cylinder at each stroke, is expressed by

$$a\,(l+c).$$

Therefore, multiplying this quantity by the number of strokes of the piston during the experiment, we shall have the volume of steam effectively turned to use in the cylinder.

To conclude from this the corresponding volume of water, it suffices to observe that, since the water of condensation, which forms in the cylinder, is not withdrawn by degrees as it is formed, the steam effectively used in the cylinder is always found in contact with the liquid; that is to say, it is there, as has already been said, at the maximum density for its temperature. Now we have given (Sect. III. Chapter II.) a table of the *relative* volumes of the steam in contact with the liquid, under divers temperatures. Taking then in that table the relative volume corresponding to the temperature T′; since that *relative* volume is nothing more than the ratio of the volume of the steam to that of the same weight of water, it will suffice to divide by that ratio, the volume of the steam effectively turned to use, which has just been obtained, in order to conclude the corresponding volume of water, or the effective evaporation S of the engine.

4thly, The total quantity of water which comes out of the cylinder after condensation, being diminished by the injection water and the water turned to effective use, will make known the volume of water condensed by the contact of the cylinder.

Thus will readily be obtained the ratio between the volume of water condensed in consequence of the refrigeration of the cylinder, and the volume of water turned to useful account. Let M then be that ratio. Since we have seen that it has equally for its general expression the quantity

$$\frac{4}{3} \text{H} \left(\text{T}' - t \right) \left(3 + \frac{l}{l+c} \right),$$

we shall have

$$\frac{4}{3} \text{H} \left(\text{T}' - t \right) \left(3 + \frac{l}{l+c} \right) = \text{M};$$

and since the temperatures T′ and t are known, as well as the quantities l and c, it follows that this equation will definitively make known the value of the constant quantity H, which will be

$$\frac{4}{3} \text{H} = \frac{\text{M}}{(\text{T}' - t)\left(3 + \frac{l}{l+c} \right)}.$$

Experiments, such as those of which we have just pointed out the process, are as yet wanting; but till such experiments shall have been made with all proper care, we may make use of some practical observations, which have not indeed the degree of precision necessary to lead to the exact determination of the quantity H, but which, nevertheless, will authorize an approximate value being provisionally assigned to it.

Watt has recognised from a great number of direct observations, (*Watt on the Steam Engine*, pages 66 and 95,) that in the atmospheric engines least liable to the loss under consideration, the condensation which arises in consequence of the refrigeration of the cylinder amounts to ·75 of the efficient steam; and that in those which, on the contrary, are most liable to it, that condensation amounts to twice the efficient steam; and he found at the same time, that the temperature of condensation in these engines, varies between 174 and 142 degrees of Fahrenheit's thermometer.

The engines least liable to loss of steam by the cooling of the cylinder are evidently those in which the condensation takes place at the highest temperature, and in which the clearance of the cylinder is the smallest in use, since that clearance augments by so much the mass of metal to be heated. They are, therefore, the engines in which the condensation takes place at 174 degrees of temperature, and in which the clearance of the cylinder is but one tenth of the effective stroke of the piston.

The engines most liable to the same loss are, for the opposite reason, those in which the condensation is effected at the temperature of 142 degrees, and in which the clearance of the cylinder amounts to three tenths of the stroke.

On the other hand, a tolerably accurate valuation may be made of the pressure, and consequently of the temperature assumed by the steam in the cylinder of these engines, during the ascending stroke of the piston. It has, in effect, been seen that there is equilibrium between that pressure and the resistance of the piston. Now the resistance of the piston consists of the atmospheric pressure, augmented by the friction of the engine and diminished by the counterweight. If then the counterweight be considered as equi-balancing the friction of the engine, which experience indicates very nearly in Watt's engines, we see that the pressure P′ of the steam in the cylinder will be equal to the atmospheric pressure; and consequently, the temperature T′ corresponding to that pressure will generally be 212 degrees of Fahrenheit, or 100 degrees of the centigrade thermometer.

Making use then of this as a mean valuation, applicable without great error to the observations of Watt, we see that those observations will furnish, for the approximate value of the factor H, the two following equations:

$$\frac{4}{3}\,H = \frac{\cdot 75}{(212-174)\left(3+\frac{1}{1\cdot 1}\right)} = \cdot 0050,$$

$$\frac{4}{3}\,H = \frac{2}{(212-142)\left(3+\frac{1}{1\cdot 3}\right)} = \cdot 0076.$$

Between these two values the mean would be ·0063; but as the second case, that is, a loss so considerable as twice the efficient steam, is of much more rare occurrence than the first; and as it is an extreme case which may depend on an ill-contrived arrangement of the parts of the engine, or on some accidental cause of which no account can be taken in the calculation, we deem it more correct to hold to the first of the above determinations, viz.:

$$\frac{4}{3}\,\mathrm{H} = \cdot 0091.$$

Thus, till more precise researches be made on this subject, we will value, in these engines, the effective evaporation in terms of the total evaporation of the boiler, from the following analogy, in which the temperatures are taken from Fahrenheit's thermometer.

$$\mathrm{S} = \frac{\mathrm{S}'}{1 + \cdot 005\,(\mathrm{T}' - t)\left(3 + \dfrac{l}{l+c}\right)}.$$

In this equation, t expresses the temperature of the steam imperfectly condensed in the cylinder during the descending stroke of the piston, and T′ the mean temperature of the steam in the cylinder during the rising stroke of the piston. The latter temperature being, besides, that which corresponds to the pressure P′ determined by equation (C), or

$$\mathrm{P}' = \phi + f' - \Pi,$$

we see that, without any direct observation, it will readily be known from the tables given in Sect. III. Chapter II. In practice, however, in the greater number of cases, we may still more simply, take for T′ the temperature which answers to the atmospheric pressure, namely, 212 degrees of Fahrenheit's thermometer.

This estimate of the effective evaporation is, without doubt, but a very imperfect approximation; but till more precise researches be made, it may perhaps be found sufficient for engines in which so little regard is had to the exact employment of the motive force, that no scruple is made at losing from one half to two thirds of the evaporation effected in the boiler, that is, of the motive force produced by the engine.

The quantity S, determined by the above equation, is that which must be used to determine the effects of the engine; and referring to the modifications denoted in the preceding section of this article, we see that the formulæ proper to calculate the proportions or the effects of atmospheric engines without condenser, will be the following:

$$P' = \phi + f' - \Pi \quad \text{(C)}$$

$$\frac{S}{S'} = \frac{1}{1 + \frac{4}{3} H(T' - t)\left(3 + \frac{l}{l+c}\right)} \quad \text{(D)}$$

$$k'' = \frac{\frac{n}{q} + \phi - (1+\delta) r - f'' - \Pi}{\frac{n}{q} + p} \quad \text{(B)}$$

$$v = \frac{S}{a} \cdot \frac{1}{\frac{l+c}{l}(n + qP') - \frac{l - l'' + c}{l}(n + qp)} \quad (1)$$

$$ar = \frac{a}{1+\delta}\left[\frac{n}{q} + \phi - f'' - \Pi - \left(\frac{n}{q} + p\right) k''\right] \quad (2)$$

$$S = av\left[\frac{l+c}{l}(n + qP') - \frac{l - l'' + c}{l}(n + qp)\right] \quad (3)$$

$$\text{u. E.} = arv \quad (4)$$

To make use of these formulæ, it is obvious that the first thing to be done is to determine P′, or the pressure in the cylinder, by means of equation (C). Then, knowing P′, the corresponding temperature T′ will be found immediately, by the help of the tables given in Sect. III., Chapter II. Finally, that value of T′, substituted in equation (D), will make known the effective evaporation S. This done, in the different problems that may occur to resolve, we have only to proceed according to the mode explained in Sect. III., Article I., of the present chapter.

ARTICLE III.

PRACTICAL FORMULÆ FOR ATMOSPHERIC ENGINES, AND EXAMPLE OF THEIR APPLICATION.

To obtain the practical formulæ suitable to the calculation of atmospheric engines, the constant quantities in the algebraic equations developed above, must be replaced by their value deduced from experiment or from observation.

In these engines, the pressure P of the steam in the boiler, is usually from one pound and a half to two pounds per square inch above the atmospheric pressure; that is to say, we have in general

$$P = 16{\cdot}5 \times 144 \text{ lbs. per square foot.}$$

The atmospheric pressure varies according to the state of the atmosphere, and in delicate experiments, it is necessary to measure that pressure accurately by the barometer; but in the general calculations, it may be taken at its mean value, which is 14·71 lbs. per square inch. Referring it then to the square foot, we have

$$\phi = 14{\cdot}71 \times 144 \text{ lbs.}$$

The pressure p of condensation in the cylinder, ought to be observed directly in every case wherein the thing may be possible. For this purpose Watt's *indicator of the pressure* is to be used, if the engine has a condenser; and if not, it will suffice to take the temperature of the water issuing from the cylinder after condensation. This temperature being at the same time that of the steam with which the water was in contact, by consulting the tables of correspondence between the pressure and the temperature of the steam in contact with the liquid, (Chapter II. Sect. III.,) we shall have the pressure of condensation under the piston. The temperature of condensation thus observed in a great number of atmospheric engines, has been found to vary between 142 and 174 degrees of Fahrenheit's thermometer; which corresponds to the pressures of condensation comprised between 3 and 7 lbs. per square inch. Thus, referring this pressure to the square foot, we shall have most commonly

$$p = 4{\cdot}7 \times 144 \text{ lbs., and } t = 158°.$$

As to the value of the frictions f', f'' and δ, we want special experiments to give us sure notions on the subject; but in order to shew the process of the calculation in the applications, we will approximatively take these quantities, at the same valuation as in Watt's engines. The construction of these two kinds of engines being little dissimilar, gives us in fact room to think, that this valuation may be near enough to the truth, for the results obtained by this means to be of some utility in practice.

We will take then the friction of the unloaded engine, in either stroke, to be ·5 lb. per square inch of the surface of the piston, in engines which have a cylinder of about 48 to 50 inches diameter, and as varying with the proportions of the cylinder, according to what practice has indicated in Watt's engines; that is to say, amounting to 1·5 lb. in engines having a cylinder of only 17 inches diameter, or diminishing, on the contrary, in a similar way, when the cylinder has larger dimensions. We will also, till a special determination be obtained, take the additional friction δ, at the same rate as in the locomotive engines.

For the engines then which have a cylinder differing little from 50 inches diameter, we shall have

$f' = f'' = 72$ lbs. per sq. foot of the surface of the piston; and $\delta = \cdot 14$.

Finally, these engines being condensing engines, the value of the coefficients n and q, of the relative volume of the steam, will be

$$n = \cdot 00004227,$$
$$q = \cdot 000000258.$$

To obtain the definitive practical formulæ, suitable to the calculation of these engines, it would be necessary to substitute in the algebraic equations developed above, the value of each of the constant quantities which figure in them; but as the pressures P and p vary in different engines, and as the valuation which we have given of the frictions, varies likewise according to the diameter of the cylinder, we shall merely substitute the values of ϕ, n and q. The algebraic equations will then be replaced by the following:

Practical formulæ for atmospheric engines with a condenser.

$$k'=\frac{2282+f'-\Pi}{164+P}$$ .... Regulation of the ascending stroke of the piston.

$$k''=\frac{2282-(1+\delta)r-f''-\Pi}{164+p}$$ .... Regulation of the descending stroke of the piston.

$$ar=\frac{a}{1+\delta}\left[2282-f''-\Pi-(164+p)k''\right]$$ .... Useful load of the piston, in lbs.

$$v=\frac{S}{a}\cdot\frac{10000}{\frac{l'+c}{l}(\cdot 4227+\cdot 00258\,P)-\frac{l-l''+c}{l}(\cdot 4227+\cdot 00258\,p)}$$ .... Velocity of the piston, in feet per minute.

$$S = \frac{av}{10000}\left[\frac{l'+c}{l}(\cdot 4227 + \cdot 00258P) - \frac{l-l''+c}{l}(\cdot 4227 + \cdot 00258p)\right]$$

.... Effective evaporation, in cubic feet of water per minute.

$u.E. = arv$ .... Useful effect, in lbs. raised 1 foot per minute.

$u.HP. = \frac{u.E.}{33000}$ .... Useful horse-power.

$u.\ E.\ 1\ lb.\ co. = \frac{u.\ E.}{N}$ .... Useful effect of 1 lb. of coal, in pounds raised 1 foot.

$u.\ E.\ 1\ ft.\ wa. = \frac{u.\ E.}{S}$ .... Useful effect of 1 cubic foot of water evaporated, in pounds raised 1 foot.

$Q.\ co.\ for\ 1\ hp. = \frac{33000\ N}{u.\ E.}$ .... Quantity of coal, in pounds, producing 1 horse-power.

$Q.\ wa.\ for\ 1\ hp. = \frac{33000\ S}{u.\ E.}$ .... Quantity of water, in cubic feet, producing one horse-power.

$u.\ HP.\ 1\ lb.\ co. = \frac{u.\ E.}{33000\ N}$ .... Horse - power produced per lb. of coal.

$u.\ HP.\ 1\ ft.\ wa. = \frac{u.\ E.}{33000\ S}$ .... Horse - power produced per cubic foot of water evaporated.

Those formulæ suit the case in which the engine is furnished with a separate condenser; but for *engines without condenser,* the following must be used:

Practical formulæ for atmospheric engines without a condenser.

$P' = \varphi + f' - \Pi$ .... Total pressure of the steam in the cylinder, during the rising stroke.

$$\frac{S}{S'} = \frac{1}{1 + \cdot 005\,(T' - t)\left(3 + \frac{l}{l+c}\right)}$$ .... Ratio between the effective evaporation of the engine, and the total evaporation of the boiler.

$$k'' = \frac{2282 - (1+\delta)r - f'' - \Pi}{164 + p}$$ .... Regulation of the descending stroke of the piston.

$$ar = \frac{a}{1+\delta}\left[2282 - f'' - \Pi - (164+p)k''\right]$$ .... Useful load of the piston, in lbs.

$$v = \frac{S}{a} \cdot \frac{10000}{\frac{l+c}{l}(\cdot4227 + \cdot00258\,P') - \frac{l-l''+c}{l}(\cdot4227 + \cdot00258\,p)}$$ .... Velocity of the piston, in feet per minute.

$$S = \frac{av}{10000}\left[\frac{l+c}{l}(\cdot4227 + \cdot00258\,P') - \frac{l-l''+c}{l}(\cdot4227 + \cdot00258\,p)\right]$$ .... Effective evaporation, in cubic feet of water per minute.

u. E. .... $= a\,r\,v$. .... Useful effect, in lbs. raised one foot per minute.

u. HP............... $= \frac{\text{u. E.}}{33000}$ .... Useful horse-power.

u. E. 1 lb. co. ... $= \frac{\text{u. E.}}{N}$ .... Useful effect of 1 lb. of coal, in lbs. raised 1 foot.

u. E. 1 ft. wa. ... $= \frac{\text{u. E.}}{S'}$ .... Useful effect of 1 cubic foot of water evaporated, in lbs. raised 1 foot.

Q. co. for 1 hp.... $= \dfrac{33000\ N}{u.\ E.}$ .... Quantity of coal, in lbs., which produces one horse-power.

Q. wa. for 1 hp.... $= \dfrac{33000\ S'}{u.\ E.}$ .... Quantity of water, in cubic feet, which produces one horse-power.

u. HP. for 1 lb. co. $= \dfrac{u.\ E.}{33000\ N}$ .... Horse-power produced per lb. of coal.

u. HP. for 1 ft. wa. $= \dfrac{u.\ E.}{33000\ S'}$, ... Horse-power produced per cubic foot of water evaporated.

In the first of these formulæ, we leave the atmospheric pressure expressed by ϕ, in order that, whenever it may be more convenient, the pressure P' may be calculated according to the ordinary measure, that is, in pounds per square inch, instead of pounds per square foot; which will be done by assigning to ϕ, f' and Π their value referred to the same unit.

In all engines wherein there exists a difference between the total and the effective evaporation, it is plain that when the effects due to a determined evaporation are sought, the effects in question are always those of a *total* determined evaporation.

It is for this reason that in the formulæ relative to these determinations, we have introduced S' instead of S. If, however, it were required to know the effects arising from a given effective evaporation, it would suffice to restore in the formulæ the quantity S, and they would then give the effects sought.

Now to shew a numerical application of these formulæ, we will suppose an engine *without condenser*, offering the following dimensions and data:

Diameter of the cylinder, 52 inches; or surface of the piston $a = 14{\cdot}75$ square feet.

Stroke of the piston, $l = 7$ feet.

Clearance of the cylinder, ·29 of the useful stroke of the piston; or $c = {\cdot}29\ l$.

Total pressure of the steam in the boiler, 16·5 lbs. per square inch; or $P = 16{\cdot}5 \times 144$ lbs. per square foot.

Temperature of condensation, $t = 152$ degrees of Fahrenheit; which gives for the pressure of condensation, 4 lbs. per square inch, or $p = 4 \times 144$ lbs. per square foot.

Total evaporation of the boiler, 1·50 cubic feet of water per minute; or $S' = 1{\cdot}50$.

Consumption of coal in the same time, 11·9 lbs.; or $N = 11{\cdot}9$.

Counterweight, 1·25 lbs. per square inch of the surface of the piston; or $\Pi = 1{\cdot}25 \times 144$ lbs.

With these data, it is required to determine the effects to be expected from the engine. Adopting then first the counterweight such as is indicated, and supposing that different loads be given to the engine; trying afterwards divers values of the counterweight, to find that which will produce the most advantageous useful effect, and performing the calculation according to the mode indicated, Sect. III., IV., V., VI., and VIII., Art. I., and Sect. II., Art. II. of this Chapter, the following results will be obtained:

Effects of the engine with the counterweight given.

$$\frac{\Pi}{144} = 1{\cdot}25 \text{ lb.}$$

		Maximum of useful effect.	
$\frac{r}{144}$	= 7	7·65	7·80
$a\,r$	= 14866	16246	16565
v	= 81·03	76·29	74·48
S′	= 1·50	1·50	1·50
u. E.	= 1204510	1239370	1233670
u. HP.	= 36·50	37·56	37·38
u. E. 1 lb. co.	= 101220	104150	103670
u. E. 1 ft. wa.	= 803010	826250	822450
Q. co. for 1 hp	= ·326	·317	·318
Q. wa. for 1 hp	= ·0411	·0399	·0401
u. HP. for 1 lb. co.	= 3·07	3·16	3·14
u. HP. for 1 ft. wa.	= 24·33	25·04	24·92

Maxima effects of the engine with divers counterweights.

		Absolute max. of useful effect.	
$\frac{\Pi}{144}$	= 1·25	1·50	1·75
$\frac{r}{144}$	= 7·65	7·40	7·20
$a\,r$	= 16246	15715	15290

		Absolute max. of useful effect.	
v	= 76·29	79·09	80·92
S′	= 1·50	1·50	1·50
u. E	= 1239370	1242880	1237230
u. H. P	= 37·56	37·66	37·49
u. E. 1 lb. co.	= 104150	104440	103970
u. E. 1 ft. wa.	= 826250	828590	824820
Q. co. for 1 hp	= ·317	·316	·317
Q. wa. for 1 hp.	= ·0399	·0398	·0400
u. HP. for 1 lb. co.	= 3·16	3·165	3·15
u. HP. for 1 ft. wa.	= 25·04	25·11	25·00

The engine we have just submitted to calculation is that which Smeaton constructed at Long Benton, and which is well known. In evaporating the quantity of water which we have stated, its ordinary velocity, with a moderate load, was 84 feet per minute; and this result comes near enough to those which we have obtained, to serve as a practical verification of them. It will also be recognized, on examining the effects produced with different counterweights, that the counterweight attributed to that engine, was not the most advantageous possible, and that with a counterweight of 1·50 instead of 1·25 lb. per square inch of the surface of the piston, and an appropriate load, the engine would have produced a greater useful effect.

It is easily certified besides that the most advantageous load for the counterweight of 1·25 lb. per square inch, is in effect that of 7·65 lbs. per square inch of the piston, and that the most advantageous combination for the engine consists in giving it at once a counterweight of 1·50 lb. and a load of

7·40 lbs. per square inch; for, on effecting the necessary calculations, we obtain the three following tables:

$\frac{\Pi}{144} = 1\cdot25$	$\frac{r}{144} = 7\cdot60$ ....u. E.	=1238710	
	7·65	1239370	max.
	7·70	1237690	
$\frac{\Pi}{144} = 1\cdot50$	$\frac{r}{144} = 7\cdot30$ ....u. E.	=1236005	
	7·40	1242880	ab. max.
	7·50	1241910	
$\frac{\Pi}{144} = 1\cdot75$	$\frac{r}{144} = 7\cdot10$ ....u. E.	=1234840	
	7·20	1237230	max.
	7·30	1235805	

The atmospheric engines had never yet been calculated. The authors who have principally devoted their attention to these matters have renounced indicating any formulæ with respect to them, nor have they even attempted to apply a method analogous to that of coefficients. The facility with which these engines, as well as all others, are calculated by the theory which we have developed, affords then a final proof of the accuracy of that theory; and we consequently hope that the formulæ which we have deduced from it, will be of some service in the construction as well as in the calculation of every description of steam-engine.

APPENDIX.

Concise rules designed for persons not familiar with the algebraic signs, and intended to render clear and easy the use of the formulæ contained in the work.

AMONG the persons who are engaged either in the construction or in the working of steam-engines, and whom this work may consequently interest, there is a great number to whom the algebraic terms are little familiar, and who usually give up the reading of a book as soon as they perceive it step beyond the simple notions of arithmetic. When it is intended to make a work profitable to those persons, the usual practice is to annex to each of the definitive formulæ, an explanation, in full words, of the arithmetical operations which it represents.

With the number of formulæ contained in this work, such a proceeding would become almost impracticable, since the explanation of each series of formulæ would require a considerable number of pages. We think, however, that the want of such explanation may be very advantageously supplied here, by giving the signification of every sign employed in the formulæ; that is to say, by explaining what are the arithmetical operations represented by those signs. With the help of a very few rules on this subject, the persons whom this article may interest, will find that the reading of the formulæ is quite as easy in algebraic signs as if they were written in words; since, after all, it is but an abridged way of expressing the same things, and that, moreover, the operations to be performed in order to attain the result, are much more clear and more easy for the mind to seize. Again, a perfect acquaintance with the signification of the signs in common use can require but a few hours of attention, and when once a person shall have made himself master of them, he will be capable of reading not only the formulæ of this work, but all others that may present themselves in other works. We deem it then rendering a service to practical men, to add here the few concise rules which follow.

A, B, ... *a*, *b*, ... *l*, *m*, *n*, ... *α*, *β*, ... &c. The letters are an abridged manner of writing the numbers which those letters represent. Thus, when the stroke of the piston has been measured, and found, for instance, to be $17\frac{1}{2}$ inches, it would be inconvenient to write in all the formulæ, the number $17\frac{1}{2}$. But if the length of stroke, whatever it might be, has been represented by a letter, as *l*, for instance; then, every time the letter *l* occurs, there needs only to recollect that it represents the number $17\frac{1}{2}$, and performing with that number, the operations indicated in the formulæ, relative to the letter *l*, the result sought will be attained.

$=$ ....This sign signifies *equal to;* it expresses that a quantity sought is equal to the number resulting from certain operations performed on other quantities known. Thus, for instance, if we find the expression

$$V = 60\,v,$$

this will signify that the quantity V is equal to 60 times the quantity *v*. Consequently, if we know besides that the letter *v* represents the number 100, it will follow that the unknown quantity V will have for its value 60 times 100, or 6000.

+ ....This sign signifies *plus* (more). Placed between two letters or two numbers, it indicates that they are to be added together. If, for instance, there be in a formula an expression of the form

$$1+\delta,$$

it means that to the number 1 must be added the number δ. If, then, we know besides, that the letter δ represents the number ·14, it follows that the expression $1+\delta$ will have for its value

$$1+\delta=1+{\cdot}14=1{\cdot}14.$$

— ....This sign indicates *minus* (less). Thus, when an expression occurs of the form

$$P-f-2118,$$

the expression amounts to saying that, from the number P the numbers f and 2118 are to be successively subtracted. If, then, we know that the letter P represents the number 9360, and that the letter f represents the number 144, the expression will have for its value

$$P-f-2118=9360-144-2118=7098.$$

× ....This sign expresses *multiplied by*. Thus the expression

$$a\times v$$

indicates that the two numbers represented by the letters a and v, are to be multiplied one by the other; and the product of that multiplication will be the quantity expressed here by $a\times v$. This multiplication to be performed, is equally expressed by a point between the two letters, or by writing the two letters simply together without any sign interposed; so that the expressions

$$a\times v\ldots\ldots,\quad a\,.\,v\ldots\ldots,\quad a\,v,$$

amount to the same, all three expressing the result of the multiplication of the numbers represented by a and v. If, for instance an expression occur like the following,

$$a\,r\,v,$$

and it be known that the letter a expresses the number 1·57, the letter r the number 2640·96, and the letter v the number 300, the expression $a\ r\ v$ will have the value

$$a\ r\ v = 1{\cdot}57 \times 2640{\cdot}96 \times 300 = 1243800.$$

$\div$............This sign denotes *divided by*. Thus the expression

$$\frac{S}{a}$$

expresses S divided by a, or the quotient resulting from the division of the number expressed by S, by the number expressed by a.

For instance, if we have $S = {\cdot}67$ and $a = 1{\cdot}57$, it is plain that the term $\frac{S}{a}$ will have for its value

$$\frac{S}{a} = \frac{{\cdot}67}{1{\cdot}57} = {\cdot}4268.$$

A fraction may have its numerator or its denominator composed of several numbers, on which divers operations are indicated. In that case, those operations must first be performed; so as to reduce the numerator and the denominator to single numbers, before performing the division of the one by the other, as has just been said.

If, for example, we have the fraction

$$\frac{10000}{1{\cdot}492 + {\cdot}002415\ P},$$

and know besides that the letter P represents the number 9360; we shall first perform the multiplication of the number 9360 by the number ·002415, and then add to the product the number 1·492. The result will be the number 24·0964, which will therefore represent the denominator of the fraction. The fraction may then be written under the form

$$\frac{10000}{24{\cdot}0964},$$

and consequently it is reduced to the simple indication of the quotient of two numbers, as in the preceding case.

If two fractions occur, separated by the sign of addition, or that of subtraction, or that of multiplication, the meaning is that, after having sought separately the quotient indicated by each of those fractions, they are either to be added together, or one deducted from the other, or one multiplied by the other. Thus, the expression

$$\frac{S}{a} \cdot \frac{10000}{1{\cdot}492 + {\cdot}002415\,P}$$

signifies that, after having sought the quotient indicated by each of the two fractions, the first of these quotients is to be multiplied by the second. Supposing the letters to be of the same numerical value as in the preceding cases, the product of the two fractions would here be the definitive number 176.

It would be the same if we were to find one fraction divided by another. Each of them should be first reduced to a single number by finding the quotient they represent, and then the one of these quotients divided by the other.

( ) or [ ] or { }...Parentheses indicate that the different quantities contained between them, are to be reduced to a single number before performing the other operations indicated in the formula.

Thus, for instance, if we find in a formula the expression

$$(1+\delta)\,v,$$

this means that it is the expression $(1+\delta)$ entire, which is to be multiplied by v. The sum then of $1+\delta$ is first to be formed, and afterwards multiplied by the number v; whereas had we only

$$1+\delta\,v,$$

this would mean that the product δv is first to be formed, and afterwards the number 1 added to it.

There may occur several parentheses comprised one within the other, but their signification is always the same. The expression

$${\cdot}002415\,[(1+\delta)\,r+f]$$

denotes that the sum of $1+\delta$ is to be formed first, this to be multiplied by r, and the product added to the quantity f, which gives the number represented by the outer parenthesis; and finally that this number is to be multiplied by ·002415.

Lastly, when there occurs in the formulæ a letter with a small figure or *exponent* above it, it is the same thing as writing that letter as many times successively as there are units in the figure or exponent.

For instance, the expression

$$v^2$$

is equivalent to the expression $v \times v$, or v written twice; that is to say, it is the product of v by itself. If then v were known to be equal to 300, the quantity represented by v^2 would be

$$v^2 = 300 \times 300 = 90000.$$

These short explanations are all that is necessary, in order to read and perfectly understand all the practical formulæ contained in this work. Replacing each of the signs that are met with in a formula, by the periphrasis which the sign represents, you read the formula such as it ought to be expressed, and effecting the arithmetical operations indicated by those signs, you attain the result sought. A formula is then nothing more than an abridged manner of writing the series of operations to be performed, in order to arrive at the result which we want to obtain.

We will subjoin to this explanation some examples, taken from the practical formulæ of high-pressure engines (pages 151 and 153).

I. Suppose we have the formula

$$v = \frac{S}{a} \cdot \frac{10000}{6{\cdot}6075 + {\cdot}002415\,[(1+\delta)\,r + f]},$$

which is intended to determine the unknown value of v; and let it be supposed that we know, besides, that the other letters comprised in this formula have the following value (page 153):

$$S = \cdot 67$$
$$a = 1\cdot 57$$
$$\delta = \cdot 14$$
$$r = 2641$$
$$f = 144.$$

First form the sum $(1+\delta)$, indicated in the inner parenthesis, which will be

$$1+\delta=1\cdot 14.$$

Then multiply this number by r or 2641, and the result will be

$$(1+\delta)\,r=1\cdot 14\times 2641=3010.$$

Add to this f or 144, and the sum will consequently be the quantity indicated by the outer parenthesis, viz.

$$[(1+\delta)\,r+f]=3154.$$

Now multiply this sum by the number ·002415, and the product will evidently be

$$\cdot 002415\,[(1+\delta)\,r+f]=\cdot 002415\times 3154=7\cdot 6170.$$

Add to this last result the number 6·6075, and you obtain

$$6\cdot 6075+\cdot 002415\,[(1+\delta)\,r+f]=6\cdot 6075+7\cdot 6170=14\cdot 2245.$$

This is then the denominator of the fraction which forms the second member of the formula. Performing the division of the number 10000 by the number just obtained, the quotient will be

$$\frac{10000}{6\cdot 6075+\cdot 002415\,[(1+\delta)\,r+f]}=\frac{10000}{14\cdot 2245}=703\cdot 04.$$

On the other hand, dividing S by a, or the number ·67 by the number 1·57, you have the value of the fraction $\frac{S}{a}$, viz.

$$\frac{S}{a}=\frac{\cdot 67}{1\cdot 57}=\cdot 4268.$$

Finally, then, multiplying this latter quotient by that obtained immediately above, you have definitively

$$v=\frac{S}{a}\cdot\frac{10000}{6\cdot 6075+\cdot 002415\,[(1+\delta)\,r+f]}=\cdot 4268\times 703\cdot 04=300.$$

Thus, it is clear that, by effecting successively the series of calculations indicated by the few signs which we have explained, and proceeding gradually from the most simple terms to the more compounded ones, we arrive without difficulty at the definitive result.

We will give some other examples of these calculations; but instead of effecting the operations, we will merely express in words the signification of the formula, which amounts to the same.

II. Suppose we have the formula

$$ar = 4140750 \; \frac{S}{(1+\delta)\,v} - \frac{a}{1+\delta}\,(2736+f),$$

this signifies that the required value of ar will be obtained by performing the following arithmetical operations:

> Add 1 to the number represented by the letter δ, and multiply the sum by the number v.
>
> Then divide the number S, by the product thus obtained; multiply the quotient of this division by the number 4140750; and write apart this first partial result, which represents the first term of the formula.
>
> Add again to unity the number δ, and by that sum divide the number a.
>
> Similarly add to the number 2736 the number f, and multiply the sum by the last found quotient; and set apart this partial result, which represents the second term of the formula.
>
> Finally, from the first partial result, subtract the second, and the difference will be the quantity ar sought.

Performing these different operations with the values of S, a, δ, r and f, given above, and supposing the case wherein the letter v has the value $v=300$, you find that the quantity ar will have for its definitive value

$$ar = 4146.$$

III. If we have the formula

$$S = \frac{av}{10000} \left\{ 6{\cdot}6075 + {\cdot}002415 \, [(1+\delta)\,r + f] \right\};$$

it will amount to the following arithmetical explanation:

> To the number 1, add the number δ, and multiply the sum by the number r.
>
> To this product add the number f, and multiply the resulting sum by the number ·002415.
>
> To the latter product, add the number 6·6075, and keep apart this partial result, which expresses, in one number, what proceeds from all the operations comprised in the great parenthesis.
>
> Then multiply the number a by the number v, and divide the product by the number 10000, which will give you another partial result, expressing the portion of the formula situated beyond the parenthesis.
>
> Finally, multiply the former partial result by the latter, and the definitive product will be the required value of S.

For the values above attributed to the different letters contained in the formula, the result of the calculation will give S = ·67.

IV. If we have the formula

$$v' = \frac{S}{a} \cdot \frac{10000}{1{\cdot}492 + {\cdot}002415\,P}$$

it will be paraphrased as follows:

> Multiply the number ·002415 by the number P, and add to the product the number 1·492; divide the number 10000 by the sum thus obtained, and write the quotient apart.
>
> Then divide the number S by the number a, which will give a second quotient.
>
> Finally, multiply the former quotient by the latter, and the resulting product will be the required value of v'.

With the values already indicated for the letters, and moreover, for P = 9360, the result of the preceding formula will give $v' = 176$.

V. In fine, as a last example, we will suppose the formula

$$a\,r' = \frac{a}{1+\delta}\left(P - f - 2118\right).$$

It plainly will signify as follows:

> From the number P, deduct first the number f, and again from the remainder deduct the number 2118.
>
> Then to the number 1 add the number δ, and divide the number a by the sum thus obtained.
>
> Finally, multiply this quotient by the difference before obtained, and the definitive product thus formed will be the required value of $a\,r'$.

The operations thus indicated, would, for the case wherein the letters should have the values already given above, produce for the required value of $a\,r'$, the quantity 9777.

Thus we see how easy it is to replace all the formulæ by their expressions in words, and consequently the sight of algebraic formulæ ought in no wise to intimidate persons unfamiliar with algebra.

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