ROUND AND OVAL CONES

JOHN FULLER, Sr.

A New and Original Treatise

ON

The Geometrical Development

OF

Round and Oval Cones

With Easy Examples of their Application

FOR THE USE OF BEGINNERS AND
PRACTICAL SHEET IRON AND
TIN PLATE WORKERS

By

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DEDICATED

IN loving memory to my wife, Ann, to whose faithful encouragement and patient example in difficulties I owe whatever success I have achieved during our fifty years' companionship in an eventful and busy life.

Conical Vessels and Their Patterns.

When the writer was a boy the problems involved in conical work were quite a puzzle to him, as they have been to many others who were in the unfortunate position of having no opportunity of gaining the necessary education. There were then no books or papers of a practical nature on these subjects within my reach. A boy under such conditions must either make friends of the men with whom his lot is cast or acquire proficiency without their help. I tried both expedients. Perhaps it will interest my boy readers to tell of my first lessons in conical work, and show their application in a few articles of every day use, because they furnish valuable hints for further search in the mysterious but interesting science of sheet metal working, or, more properly, practical geometry. My first lessons in cones involved the making of common extinguishers and bedroom candlesticks. These two articles were of much interest to me, an ambitious boy, at the threshold of the sheet metal trade. After I had been working two or three years, diligently investigating all the problems that came in my way, I found that the old workmen used five primary standard fashions in cones, after which they patterned their work, so that they could be easily understood when giving directions in the various kinds of conical work.

These primary fashions were named and understood as follows: Extinguisher, muller, funnel, lantern-head and hood, and are illustrated in Figs. 1, 2, 3, 4, 5. The envelope of the extinguisher, Fig. 1, is formed of one-sixth of a circle, or 60 degrees, and when turned into shape forms at the apex an angle of 20 degrees very nearly. The muller, Fig. 2, is formed of two-sixths of a circle, or 120 degrees, which, when turned into shape, forms at the apex an angle of 40, nearly. The funnel, Fig. 3, requires three-sixths, or 180 degrees, and makes, when turned into shape, an angle of 60 at the apex. The lantern-head, Fig. 4, takes four-sixths, or 240 degrees, and forms an angle of about 80 when turned into shape; while the hood, Fig. 5, takes five-sixths, or 300 degrees, making an angle of 110 at the apex, approximately. Within these five standard fashions once lay all the principal varieties of conical shapes used by the old workmen in sheet metal working requiring flaring sides. To explain: First, in A, Fig. 1, is represented the pattern of an extinguisher, once used to put out the light of a candle. This first primary fashion gave the initial lesson in pattern cutting, and was an excellent boy's job. We were kept pretty busy in their manufacture. The writer made many a gross, and they afforded a good preparatory lesson in the art of turning by hand, as well as laying edges true and even, fit for soldering. To get the dimensions of this pattern we multiply the base by three; this gives the radius of the circle, of which the pattern is a part. Thus: Suppose an extinguisher, A, Fig. 6, or cone for any similar article, is required an inch and a half in diameter at the base *f*; then $1.5 \times 3 = 4.5$, and one-sixth of a circle whose radius is 4½ inches will make the extinguisher, without anything allowed for lap, the tin coming together edge to edge. Next, in the same figure, B is an old-fashioned quart

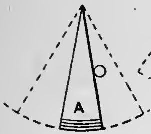


Fig. 1

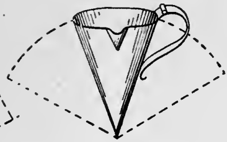


Fig. 2

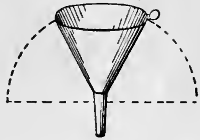


Fig. 3

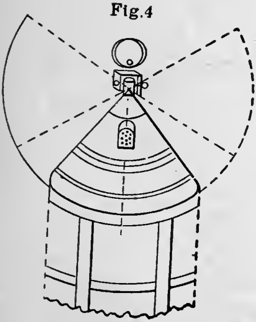


Fig. 4

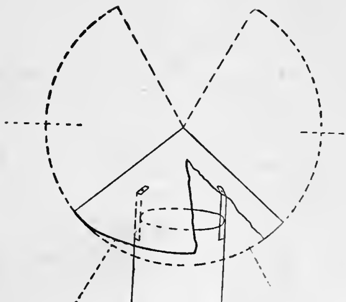


Fig. 5

cup. This cup is made, it will be seen, extinguisher fashion, and the radius of the pattern is obtained in the same way—namely, by multiplying the diameter of the bottom by three, which gives the radius of the circle of which the body of the cup is a part. Now, take for the length of the body one-sixth of the circumference of the circle, and for the depth of the cup one-third of the generating radius *a b,* thus: Suppose the bottom *b c* of the cup B is 4 inches in diameter; then $4 \times 3 = 12$, the radius *a b* of the circle, one-sixth of whose circumference will be the length and form the outer edge of our pattern for the cup B, and one-third of 12, or 4, will be the depth, also without edges for seam or wire. The pattern for the handle is shown in *d*, beside the cup. In C is shown a pretty milk can, cut in the same fashion as the quart cup, and the pattern, it will be seen, is obtained in the same manner, being one-third of the radius deep. The hasp *e* and catch for the cover are shown beside the can.

These are three illustrations of this fashion with the base down, two of which are in thirds—that is, a third of the generating radius as the depth of the vessel. The next, Fig. 7, D, illustrates the same fashion in halves—that is, the article is made one-half the generating radius deep; D represents a coffee pot with a strap handle and lip, and E is the same thing with a wood handle and spout. It can readily be seen that the same rule or fashion is used here, thus: If a coffee pot or any similar article measures 6 inches at the bottom, then $6 \times 3 = 18$, the radius of a circle of which one-sixth of the circumference is taken to form the body of the coffee pot, D, with

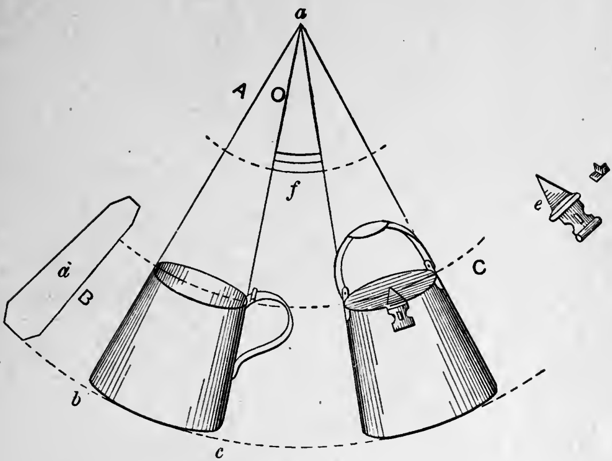


Fig.6

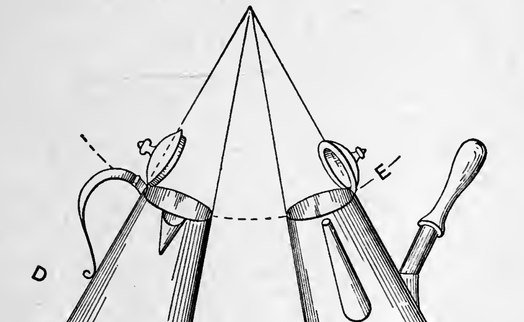


Fig.7

one-half the generating radius taken for the depth. Many other examples could be given, but these are deemed enough to show the principle. In Figs. 8 and 9 are illustrated two common pails, one an open milk or water pail, the other a slop pail. Here the order is reversed in the vessel, and the small end is made to serve for the bottom, but the law is unchanged. The bodies of both are extinguisher fashion, as before, which can be seen at a glance, the depth of them in these cases being one-fourth of the radius of the circle of which their bodies are a part. This slop pail affords a good example or lesson for careful study, as it embraces three primary fashions in one vessel; thus, the booge (or breast) *x* is cut lantern-head; the body *y* extinguisher, while the foot H is funnel fashion. Suppose now the pail, as shown in Fig. 8, is to be 10½ inches at the brim *a b;* then $10.5 \times 3 = 31.5$, the length of the radius *a e,* and if *e a* be divided into four equal parts we get in this instance the depth of the side, or $\frac{31.5}{4} = 7.875$ — that is, 7⅞ without edges. Now, the slop pail body, Fig. 9, is the same as the milk pail, hence for the body at the section *h g* it is 10½, and at the foot *e d* three-fourths of 10½, or 7⅞; then *e d,* or 7⅞, is the diameter of the inside or small end of the foot H, and also the inner radius of the semicircular ring from which the foot H is formed funnel fashion. The foot may be any depth to suit the taste, although one-fourth the depth of the body, or 2 inches in this case, is considered the right proportion. The booge (or breast) at *h g* is 10½ inches; then,

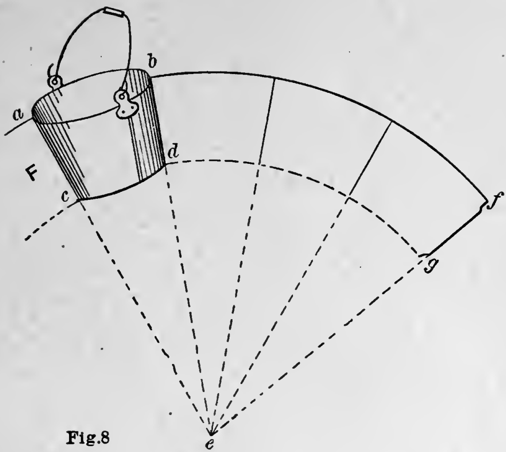


Fig.8

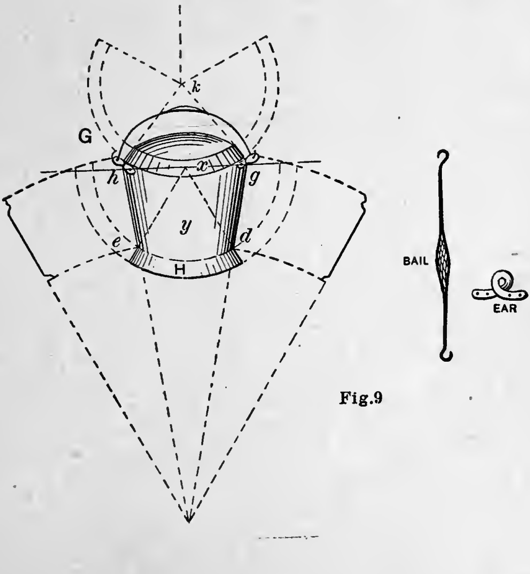


Fig.9

$\frac{10.5 \times 3}{*4} = 7.875$, the radius *k g*, of which circle four-sixths is required to make the booge, which is also made one-fourth the depth of the pail wide when wired, and hollowed in the hollowing block to give it the required curve and make it ready for the pail. From the foregoing explanation, and accompanying familiar illustration, the reader may soon become acquainted with the general method of forming similar objects.

MULLER FASHION.

In Fig 10 are shown three examples in the next, or muller fashion, which is formed of 120 degrees, or two-sixths of a circle. This fashion in an ordinary tin shop is used for dish pans, A, and deep basins or pudding pans, B, or some other articles, as the grease kettle, C, all of which were once made by hand. When the fashion or standard shape has been determined, and we wish to make a dish pan or any similar vessel of a given diameter, say, 12 inches, then we proceed thus: $\frac{12 \times 3}{*2} = 18$ inches, the radius of a circle, two-sixths of whose circumference will make the pan without edges. One-third the radius, or 6 inches, gives the proper depth. It should be noticed that many other vessels are made on exactly the same principle with the cone inverted. For example the grease kettle C, Fig. 10, is an exact pattern of a round bait kettle for fish-

* The rule for obtaining the generating radius of any of these standard fashions is: Multiply the diameter of the base of the cone to be formed by 3 and divide by the number of sixths of the circle used to form it.

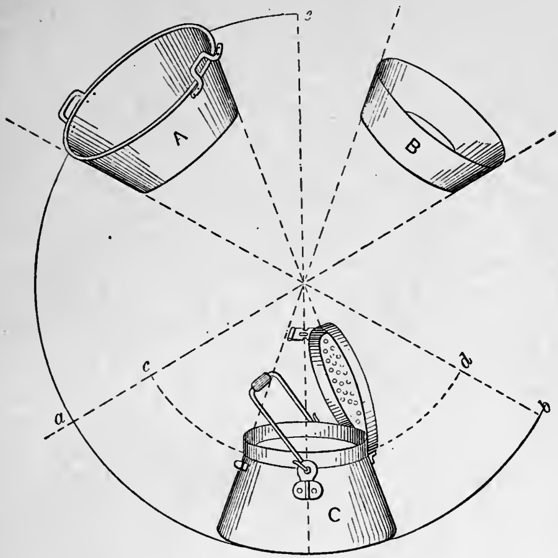


Fig.10

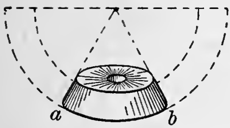


Fig.11

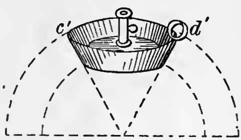


Fig.14

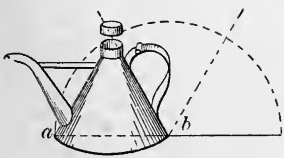


Fig.12

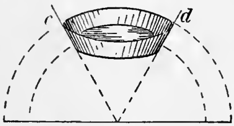


Fig.13

ing, and is the same fashion as the pans, but inverted, the base of the cone being turned down, while the depth is one-third of the generating radius, as before. The pan B may be made of any depth as may be required, its pattern being obtained in the same way, by multiplying the diameter of the brim by three and dividing the product thus obtained by two, when two-sixths of the circle of which the quotient is the radius will be the pattern required.

FUNNEL FASHION.

The funnel fashion, Fig. 3, is formed, as shown, of three-sixths, or one-half, of a circle, and is adapted especially to the funnel, Fig. 3, and a few pans and similar flaring articles. Several applications of this fashion are here shown. Fig. 11 shows a spittoon, with the cone base down. Fig. 12 is a lamp filler, also with the base down. Fig. 13 is a pan with the base turned up, forming the brim, and Fig. 14 is a candlestick, the pan of which was once made in pieces. To obtain this pattern we take the diameter c' d' of the brim, Fig. 14, as the radius, and three-sixths of a circle whose radius is equal to the bottom or brim diameter will make the pattern required, the depth of the pan being added or subtracted from the radius as the case requires. Many other examples could be given to illustrate where this fashion can be and is used in the construction of many articles called for and made in a country shop, where most people expect to get anything made from sheet metal.

LANTERN-HEAD FASHION.

The lantern-head is formed of four-sixths of a circle, and was used for the tops of old-fashioned horn and

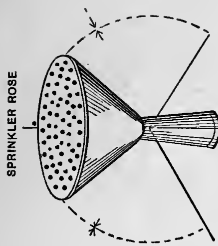


Fig.16

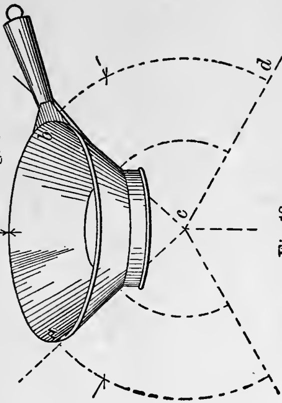


Fig.18

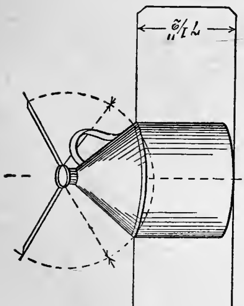


Fig.15

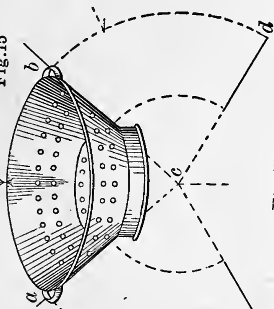


Fig.17

mica lanterns, shown in Fig. 4; also oil bottles, Fig. 15; colanders, Fig. 17; wash dishes, Fig. 18. To obtain this pattern we proceed thus: If it is required to make a colander, Fig. 17, 12 inches at the brim *a b*, then the rule is $\frac{12 \times 3}{4} = 9$, and four-sixths of a circle whose radius, *c d*, is 9 inches, will make the colander, which should be one-half the radius, or 4½ inches, deep—that is, without edges for seams or wire. Wash dishes, Fig. 18, sprinkler roses, Fig. 16, and all similar articles the same. Lantern-heads and oil bottle tops are examples with the base of the cone down. The application of this rule to oil bottles or cans will next be shown. When the author made oil cans the tops were all made lantern-head fashion, and if the reader will examine the best specimens that come under his notice he will see those that please the eye the best are still constructed in accordance with this standard fashion. A top made funnel fashion never looks well, because its form is not in harmony with the body. Let us make a gallon oil bottle, Fig. 15. We used to make gallon oil bottles from middle plates—that is, tin plates which measured 11 x 15 inches after squaring up. Take, then, a middle plate and cut it in two, making the pieces 7½ x 11. Notch the two ends of each piece, as shown, and make the notch ⅝ inch long the way of the seam and 3-16 the other, or one-half the groove, and fold them for grooving; then form them into cylindric shape by passing them through rollers, and groove down the seams; the two seams, it will be found, have taken up about ¾ inch, leaving us 21¼ inches in the circumference of the body. Then, $\frac{21.25}{3.1416} = 6.764$, or a little over 6¾ inches, the diameter of our oil can body. Now, burr both ends of the body a neat ⅛ inch and seam on the bottom. We are now ready for the top. Here we see the diameter of the base of our can top should be a little over 7 inches; then, proceeding in accordance with the rule for this fashion, $\frac{7 \times 3}{4} = 5.25$; that is to say, four-sixths of a circle whose radius is 5¼ inches will make the top, as shown in Fig. 15. Now add enough on each side for seam parallel with the edges, as shown, and the pattern is complete, ready for forming. After the top is formed and nicely rounded turn the edge at the base back twice the width of the burr on the body, and then turn up the edge for the seam. Next put in the neck for cork and rivet on the handle, snap on the top and pene it down for soldering.

Let us now see if the can will hold a gallon. The number of cubic inches in an imperial gallon (English Standard) is 277.274. The body of the can is cylindrical and is 6.75 inches in diameter by 7.25 inches high. The volume of a cylinder is equal to the area of the base multiplied by the hight, and the area is equal to the square of the radius of the base multiplied by 3.1416. Then $3.375 \times 3.375 \times 3.1416 \times 7.25 = 259.439$ = the contents of the body part. Next the top is 6.75 inches in diameter and 3.7 inches high. The volume of a cone is equal to the area of the base multiplied by one-third of the altitude or hight. Then $3.375 \times 3.375 \times 3.1416 \times$

$\frac{3.7}{3} = 44.134$ = the contents of the conical top and the whole volume of the can is

$$259.439 \times 44.134 = 303.573,$$

which shows margin enough over 277.274 so that the can is of full capacity. The same formula used with an American gallon (231 cubic inches) and a sheet 14 x 10 is a good example for practice.

HOOD FASHION.

The hood, or cap for stove pipe, is formed of five-sixths of a circle and is illustrated in Fig. 5. This fashion is used principally for making caps for stove pipe and flat covers, such as lard cans, spice boxes, bucket covers of different kinds, lamp crowns and many similar pieces used in the make-up of lamps and lanterns when they are made by hand. All these examples are instances of its use with the base down. Let it be required to make a hood for a stove pipe 11 inches in diameter, then, proceeding by the rule in a similar way as before, we have $\frac{11 \times 3}{5} = 6.6$, or 6 6-10 inches as the radius of a circle five-sixths of which will make the hood required, but without anything allowed for seam, which must be added parallel with the edge. The foregoing rules I have used for many years. They are very simple and practical, requiring but little thought or mathematical knowledge, and for the general run of work are sufficiently accurate. When the system is understood and fully grasped by the learner he will find it one of the most valuable systems for ready application in ordinary work. Sometimes, however, it happens that greater accuracy is necessary in

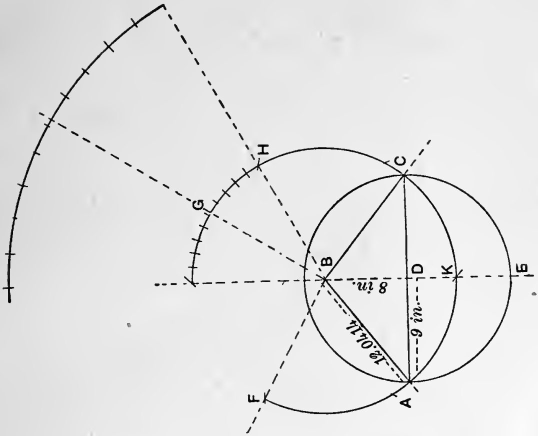


Fig. 20

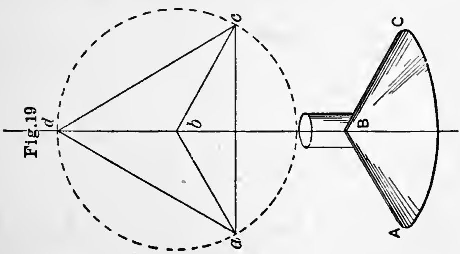


Fig. 19

the premises, and we have an example which calls for clear reasoning. For instance, in Fig. 19 is an example which once presented itself to the writer, and perhaps it has also to the reader. It was required to make a large hood to hang over a smith's fire, A B C, Fig. 19, having an angle of 120 degrees at the apex, or a rise of 30 degrees at or from the base. It was lined out on a board in a similar way to that for a longer taper, as in Fig. 19, and I was perplexed to find that the outlines *a b c*, *c b d*, *d b a*, or the three figures of the cone or hood required, completed the circle. The question then was what part of the circle it was necessary to take out to make the cone required. In the emergency I could only cut and try, and study the matter out afterward.

I waded through many books patiently searching for years for the required information, and finally found the key to it in an old mensuration book by John Bonnycastle, published in 1823, and here I present the result of my search, or its application to resolve cones of any given hight or rise from the base or angle at the apex accurately. To illustrate: Let it be required to make a cone, A B C, Fig. 20, 18 inches in diameter at the base, and any hight taken at random from the base to the apex, say, 8 inches perpendicularly, D B. Now we want to know how many degrees of a circle, whose radius is the slant hight, A B, of the cone A B C, it will take to make the cone. To do this it may be shown that where the radius of a circle is 1, half the circumference is 3.14159, and, therefore, $\frac{3.14159}{180^\circ} = .01745$, or the length of an arc of 1 degree. Hence, .01745 multiplied by the number of degrees in the arc will give the length of arc. In the example before us A D = 9, and D B = 8; then $\sqrt{9^2 + 8^2} = 12.0414$, the radius A B; then $12.0414 \times .01745 = .21012$, or the length of an arc of 1 degree of a circle whose radius is 12.0414. The diameter of the base circle A E C is 18; then $18 \times 3.1416 = 56.4488$, and $\frac{56.4488}{.21012} = 269.123$, the number of degrees, or the length of that part of the circumference F A K C H G whose radius, A B, is 12.0414. To measure or cut off 269.123 degrees from the circle F A K C H G, which is 24.0828 inches in diameter, divide 269⅛ degrees by 60 degrees, or $\frac{269.123°}{60°} = 4.485$—that is, four steps of 60 degrees and nearly one-half of another, which prove it to be half way between lantern-head and hood fashion. Now step off on the circumference with a pair of compasses four times the radius A B—that is, F A, A K, K C, C H, and $\frac{485}{1000}$ of another space, as shown by F A K C H, and on to G, which will be the number of degrees required for the cone A B C, Fig. 20.

One other example, Fig. 21. Let it be required to make a cone, A B C (or a frustum of a cone having the same slant or fashion), 18 inches in diameter at A B, and the slant hight, A C, 15 inches; then $D C = \sqrt{(A C)^2 - (A D)^2}$, or $\sqrt{15^2 - 9^2} = 12^2$; therefore, D C, or the perpendicular hight, will be 12 inches. Now, the slant hight A C of the cone C A B—that is, the radius of the circle *w x y z*—is 15; then $15 \times .01745 = .26175$, the length of an arc of 1 degree of a circle whose radius is 15 inches, and the circumference of A B or base of the cone is, $18 \times 3.1416 = 56.5488$, and $\frac{56.5488}{.26175} = 216.0332$, or the number of degrees of a 30-inch circle necessary to make the cone A B C, which proves it to be between funnel fashion and lantern-head. Dividing 216.0332 degrees by 60 degrees we get $\frac{216.0332^\circ}{60^\circ} = 3.6005$, or three steps of the generating radius A C around the circle *w x y z*, and six-tenths of another step on to F, which measures off 216.6 degrees, or a little more than half way between funnel fashion and lantern-head.

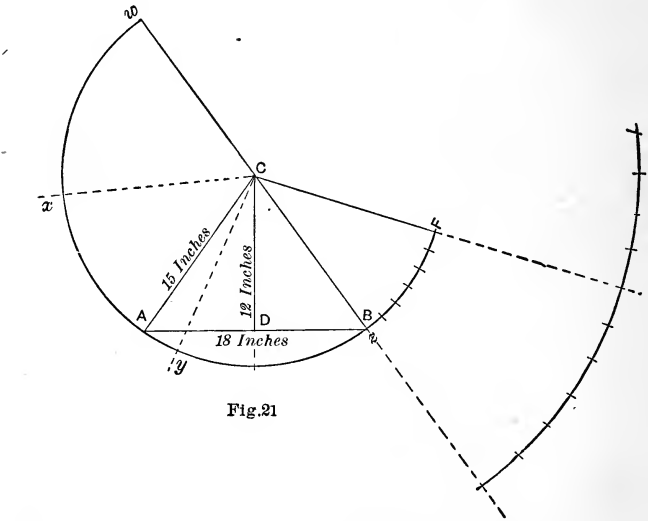


Fig.21

To Construct Ovals.

In the study of the properties of an oval or ellipse much time and labor are involved, which few men can spare or are willing to devote after a day's work. To know the ellipse one must inquire into the properties of an oblique section of a cylinder. In the following lessons treating of ovals and oval cones only those figures which can be drawn with a pair of compasses will be considered, the conical fashions coming in the same order as before in round cones and conical vessels. A few examples of ovals drawn with compasses, to introduce this curious but useful problem, will here be given. The last two examples shown are said to have been first proposed in 1774 by Rev. John Lawson, Rector of Swancombe, County Kent, England. All the other examples may be described with a pair of compasses, and are given as a prelude, the purpose being, as above stated, to treat only of those figures which may be drawn by this means. The first example, Fig. 22, is to construct an oval, the foundation of which is obtained from points on a square, thus: On the line A B construct the square A C D B, and divide the sides A B and C D into two

Fig.22

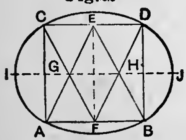
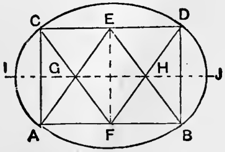


Fig.23



equal parts at the points F and E; join F C and F D, also A E and E B, and describe the arcs A B and C D, with E and F as centers; and with G A and H B as radii describe the arcs C I A and D J B, which complete the oval.

The second example, Fig. 23, is to construct an oval from points on a square and a half. On A B erect the parallelogram A C D B, making A B equal 3 and A C equal 2. Divide the sides A B and C D into two equal parts at the points F and E; join F C and F D, also E A and E B, and describe the arcs A B and C D, with E and F as centers, and with G A and H B as radii describe the arcs C I A and D J B, which complete the oval.

The third example, Fig. 24, is to construct an oval from points on two squares. On A B erect the parallelogram A C D B, making A B equal 4 and A C equal 2. Divide the sides A B and C D into two equal parts at the points F and E; join F C and F D, also E A and E B, and with F C and E B as radii describe the arcs A B and C D, and with G A and H B as radii describe the arcs C I A and D J B, which complete the oval.

In the fourth example, Fig. 25, the foundation of the figure is obtained by or from two circles, the centers of which lay in a line passing through the transverse axis, A B, and whose conjugate axis passes through the intersection of the two circles at C and D. To describe this oval, draw the line A B and divide it into three equal parts, A E, E F, F B. Now, with the radius F B describe the circle B C D, cutting the circle A C D in C and D; then with D G as radius from the points D and C describe the arcs G H and I J, and the oval is complete.

Fig. 24

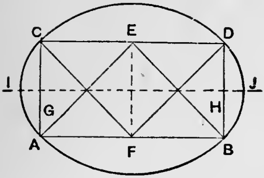


Fig. 25

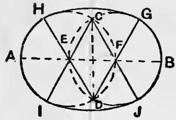


Fig. 26

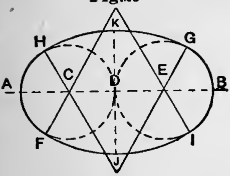
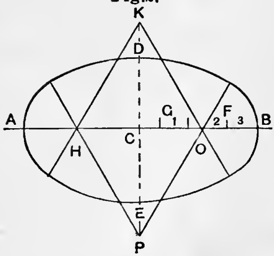


Fig. 27



In the fifth example, Fig. 26, the foundation of the figure is also obtained from two circles, the centers of which lay in the transverse axis of the figure, while the conjugate axis forms a tangent to each circle, at the center D of the transverse axis. To describe this oval draw the line A B and divide it into four equal parts, A C, C D, D E, E B. Now, with the diameter A D on C E as a common base, describe the two isosceles triangles C K E, C J E, extending the sides K C, K E and J C, J E. Then from C and E as centers with C D or E D as radius describe the arcs F A H and G B I, and from J and K with the radii J H and K F describe the arcs H G and F I, and the figure is complete.

The sixth example, Fig. 27, shows how to draw an oval when the length and width are given. Draw A B and D E, the length and width of the oval desired, at right angles to each other and cutting one another into two equal parts at the point of intersection, C. From A on A B mark off A G equal to D E, the width of the oval, and divide G B into three equal parts. With C as center and a radius equal to two of these parts, as G F, describe arcs cutting A B in the points H and O. With H and O as centers and H O as radius describe arcs cutting each other in the points K and P. Join P H, H K, K O, O P; in these lines extended the end curves will meet those of the sides. With H and O as centers and H A as radius describe the end arcs, and with P and K as centers and P D as radius describe the side arcs, and the oval is complete.

If these six figures are studied and understood a good foundation is laid by which any oval may be constructed that can be described by the aid of a pair of compasses. Further study of these figures will amply repay one for the time spent.

TO FIND THE CIRCUMFERENCE OF AN OVAL, THE LENGTH AND WIDTH BEING GIVEN.

Multiply the square root of half the sum of the squares of the two diameters by 3.1416, and the product will be the circumference, thus: Let the length be 8 and the width 6, then

$$3.1416\sqrt{\frac{8^2 + 6^2}{2}} = 3.1416\sqrt{50} = 3.1416 \times 7.071 = 22.214.$$

Rule 2. Multiply half the sum of the length and width by 3.1416 and the product will be the circumference near enough for most practical purposes.

THE CIRCUMFERENCE BEING GIVEN, TO FIND THE LENGTH AND WIDTH OF OVALS.

In Fig. 22 the ratio of the length to the width is as 1 to .76394—that is to say, if the length is 1 inch the width will be .76394 part of an inch. Let the circumference of Fig. 22 be 13.5 inches, then $13.5 \times 2 \div 5.5416$, or $(1 + .76394 \times 3.1416) = 4.8722$, the length, and $4.8722 \times .76394 = 3.722$, the width—that is, an oval as in Fig. 22, whose circumference is 13.5 inches, will be 4.8722 long and 3.722 wide. The proof is thus shown: $\frac{13.5}{3.1416} = 4.297$, and $\frac{4.8722 + 3.722}{2} = 4.297$.

In Fig. 23 the ratio of the length to the width is as 1 to .75—that is to say, if the length is 1 inch the width will be .75 part of an inch. Let the circumference be 13.5 inches, then

$$\frac{13.5 \times 2}{5.4978} = 4.911, \text{ or } \frac{13.5 \times 2}{1 + .75 \times 3.1416} = 4.911,$$

the length and $4.911 \times 75 = 3.6832$, the width—that is, an oval as in Fig. 23, whose circumference is 13.5 inches, will be 4.911 long and 3.6832 wide. The proof is thus shown: $\frac{13.5}{3.1416} = 4.297$ and $\frac{4.911 + 3.6832}{2} = 4.297$.

In Fig. 24 the ratio of the length to the width is as 1 to .7573—that is, the length is 1 inch and the width .7573 part of an inch. Let the circumference be 13.5 inches, then $\frac{13.5 \times 2}{5.5207} = 4.891$, or, $\frac{13.5 \times 2}{1 + .7573 \times 3.1416} =$ 4.891, the length, and $4.891 \times .7573 = 3.7049$, the width —that is, an oval as in Fig. 24, whose circumference is 13.5 inches, will be 4.891 long and 3.7049 wide. The proof is shown thus:

$$\frac{13.5}{3.1416} = 4.297 \text{ and } \frac{4.891 + 3.7049}{2} = 4.297.$$

In Fig. 25 the ratio of the length to the width is as 1 to .7565. Let the circumference in this example be 9 inches, then $9 \times 2 \div 5.182$, or $(1 + .7565 \times 3.1416$, the transverse axis added to the conjugate and multiplied by $\pi) = 3.2619$, the transverse axis, and $3.2619 \times .7565 = 2.4676$, the conjugate—that is, an oval as in Fig. 25, whose circumference is 9 inches, will be 3.2619 long and 2.4676 wide. The proof is shown as before, thus:

$$\frac{3.2619 + 2.4676}{2} = 2.8647 \text{ and } \frac{9}{3.1416} = 2.8647.$$

In Fig. 26 the ratio of the length to the width is as 1 to .63238. Let the circumference in this example be 13.5, then $13.5 \times 2 = 27$ and $27 \div 5.128$, or $(1 + .6323 \times$

3.1416) = 5.2652, the transverse axis, and 5.2652 × .6323 = 3.3291, the conjugate axis. The proof is $\frac{5.2652 + 3.3291}{2} = 4.297$ and $\frac{13.5}{3.1416} = 4.297$.

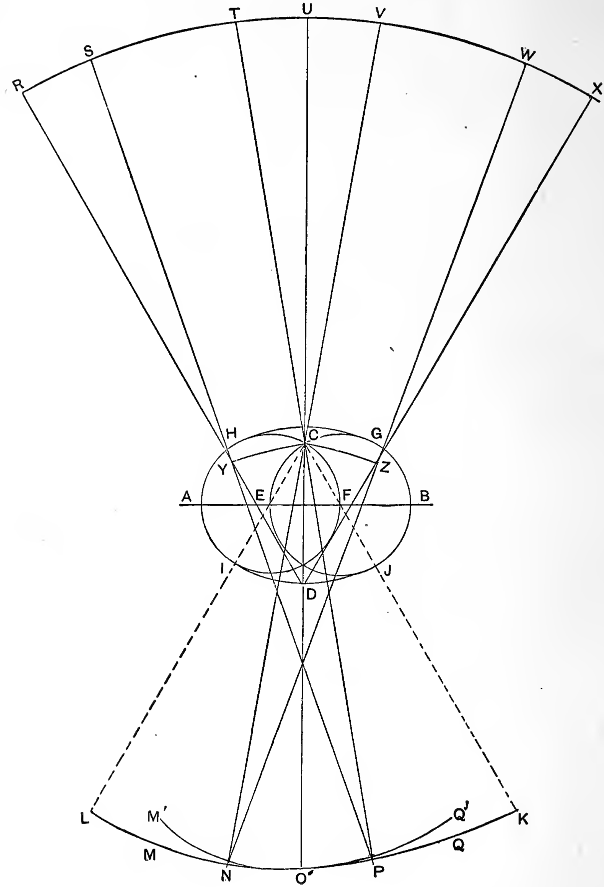
To Construct Oval Cones.

Now to show the application of the five fashions as given for round cones, in the development of oval cones, some of which examples must at some time present themselves to the active workman in his daily experience.

EXTINGUISHER FASHION.

Let it be required to make a cone extinguisher fashion, the base of which is an oval formed as in Fig. 28. First draw the oval as directed in Fig. 25, and let its transverse or longest diameter be 6 inches; then the diameter of the circle A F will be 4 inches, and by the rule give for extinguisher fashion $\frac{4 \times 3}{1} = 12$, or the radius of a circle one-sixth of which would make a cone extinguisher fashion, whose diameter at the base will be 4 inches. But it will be seen that the oval is made up of six parts of circles, the radius of the middle or two side parts being twice the length of the radius of the four parts forming the two ends, and we want in constructing this elliptic cone six sectors arranged side by side whose radii are six times longer than those of which the oval in Fig. 28 is composed, but having their arcs equal in length to those of the oval, and equal when taken together to its circumference. Now construct the oval, Fig. 28, and with a radius three times

Fig.28

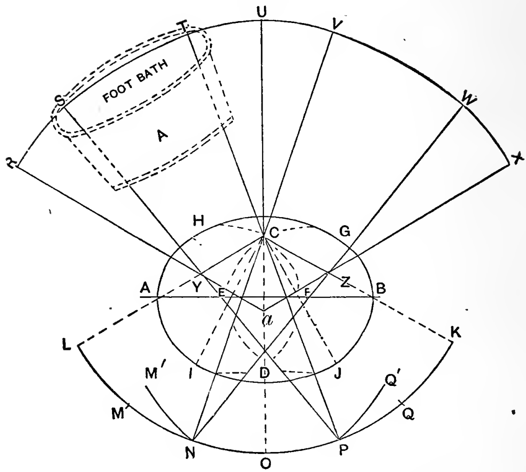


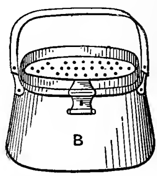
the length of the diameter A F, from the point C as center, describe the arc L O K, making it 60 degrees, or one-sixth of the circle of which C K is the radius; then from C through F and J draw the line C J to K, and from C through E I draw the line C I to L; now divide the arc L O K into six equal parts of 10 degrees each, L M N O P Q K, and draw the line U C O, making U C = C O; then draw N V and P T through C; with C as a center, describe the arc T U V equal to N O P; from P with P C as radius describe the arc C Y, and from N with N C as radius describe the arc C Z, making C Y and C Z equal to N M and P Q; then draw S P through Y, and W N through Z, and describe the arcs T S and V W with P and N as centers, and from the points Y and Z describe the arcs S R and W X, making them equal to L M and Q K. Then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, and a pattern cut as indicated by R D X will make when formed a true elliptic cone extinguisher fashion, which may be used for slightly flaring oval vessels.

MULLER FASHION.

The muller fashion contains 120 degrees, or six sectors of circles which, when laid side by side, will equal the circumference of a given oval whose transverse diameter is A B, Fig. 29. Let A H G B J I, Fig. 29, be an oval 6 inches long, and let it be required to make a foot bath the shape of this oval, muller fashion; then, by the rule given for muller fashion, three times A F divided by 2 equals the radius of a circle two-sixths of which would make a muller, Fig. 2, whose diameter at the brim would be 4 inches. But, as in the last example, we want six

Fig.29





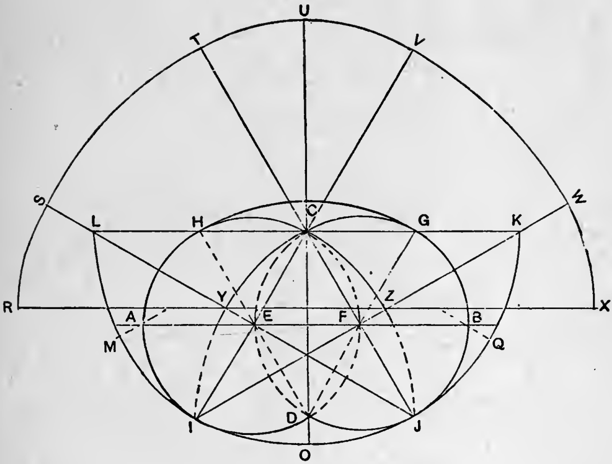
OVAL BAIT PAIL

sectors whose radii are three times longer than those of which the oval is composed, but having their arcs equal to that of the oval, and which when all are taken together will be equal to its circumference. Now, then, with a radius equal to $\frac{AF \times 3}{2}$ or 6 (that is, $\frac{4 \times 3}{2} = 6$), and from the point C as center describe the arc L O K, making it two-sixths of a circle of which six is the radius. Now divide the arc L O K into six equal parts of 20 degrees each, as L M N O P Q K, and draw the line U C O, U C being equal to C O; then draw N V and P T through the point C; with C as a center describe the arc T U V equal to N O P; now from P, with P C (that is, C T), describe the arc C Y, and from N, with N C (that is, C V), describe the arc C Z, making C Y and C Z equal to N M and P Q; then draw S P through Y and describe the arc T S, from P, and draw W N through Z and describe the arc V W, from N, and with Y S and Z W as radii, from the points Y and Z describe the arcs S R and W X, making them equal to L M and Q K. Then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, and a pattern cut out as indicated by *a* R U X will make when formed a true elliptic or oval cone muller fashion, which may be used for large pans, foot baths and oval bait kettles and many other vessels. As shown in A and B, Fig. 29, A has the base turned up, and B has the base turned down.

FUNNEL FASHION.

Funnel fashion contains 180 degrees, or six sectors of circles which, when laid side by side, will equal the circumference of a given oval whose transverse diameter is A B, Fig. 30. Let A H G B J I, Fig. 30, be an oval 6 inches long, and let it be required to make a conical top funnel fashion to fit it; then, by the rule given for this fashion, three times A F divided by three equals the radius of a circle three-sixths of which would make a funnel, Fig. 3, whose diameter at the brim would be 4 inches. But, similar to the two preceding examples, we want six sectors, whose radii are twice as long as those which compose the oval, but having their arcs equal to that of an oval, and which, when all taken together, will be equal to its circumference. Then, with a radius equal to $\frac{AF \times 3}{3} = 4$ (that is, $\frac{4 \times 3}{3} = 4$), from the point C as center describe the semicircle L O K; now divide the semicircle into six equal parts of 30 degrees each, L M N O P Q K, and draw the line O C U, O C being equal to C U; then draw I V and J T through C, and describe the arc T U V equal to L O K from C as a center; now from J with J C as radius (that is, C T) describe the arc C Y, and from I with I C as radius (that is, C V) describe the arc C Z, making them equal to I M and J Q; then draw S J through Y, and with J T as radius describe the arc T S, and draw W I through Z, and with I V as radius describe the arc V W, and with Y S and Z W as radii, from the points Y and Z describe the arcs S R and W X equal to M L and Q K; then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, as before, and a pattern cut as indicated by R U X will, when formed, make a true elliptic cone funnel fashion, which may be used for milk pans, dish pans and many other vessels for which it may be suited.

Fig.30

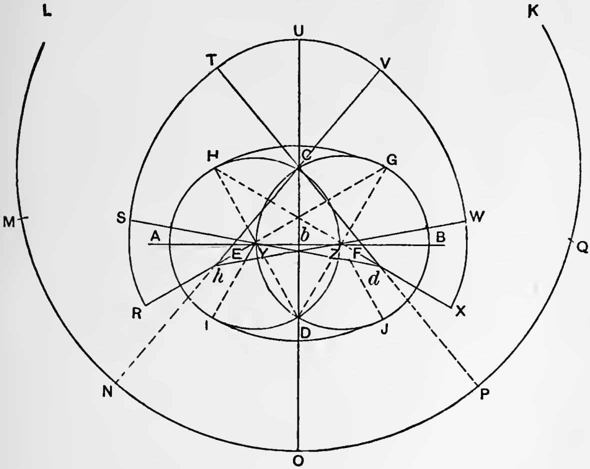


LANTERN-HEAD FASHION.

The lantern-head fashion is made up of 240 degrees and contains six sectors of circles which, taken together, will equal the circumference of a given oval whose transverse diameter is A B, Fig. 31. Construct the oval A H G B J I, making the length 6 inches, as in the preceding example, and divide the transverse axis A B into four equal parts, A E, E *b*, *b* F, F B, and draw U C D at right angles to A B; then from the point C, with any radius, C O, describe the part of a circle L O K, making the length of the arc O L two-sixths, and O K also two-sixths, or the whole part, L O K, 240 degrees. Now divide L O K into six equal parts, L M N O P Q K, and from N, through E and C, draw N E V, and from P, through F and C, draw P F T. By the rule for lantern-head $\frac{AZ \times 3}{4} = 3$, or $\frac{4 \times 3}{4} = 3$, the radius of a circle four-sixths of which would make a lantern-head, Fig. 4, whose diameter at the base would be 4 inches; then, with C T or 3 as radius, from the point C as center describe the arc T U V. Now on the line T C P from the point C lay off the distance C *d* equal to C T, and on the line V C N lay off the distance C *h* equal to C V, and from *d* and *h*, with the radii *d* C and *h* C, describe the arcs C Y and C Z, equal to T U and U V, and through the points Y and Z draw *d* Y S and *h* Z W; then from *d* as center with *d* T as radius describe the arc T S, and from *h* as center and *h* V as radius describe the arc V W; now from the points Y and Z, with the radii Y S and Z W (that is, C T), describe the arcs

S R and W X, equal to T U and U V, and curved line R S T U V W X is equal to the circumference of the oval A H G B J I, and a pattern cut as indicated by *b* R U X will, when formed, make a true elliptic cone lantern-head fashion, which may be used for shallow pans, low bottle tops or for pieced dish covers.

Fig.31

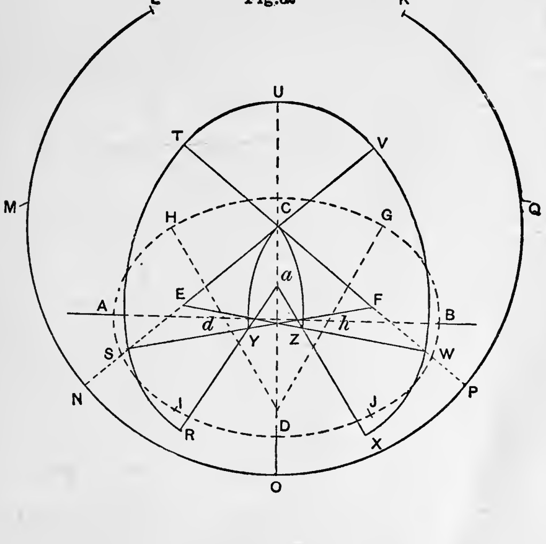


HOOD FASHION.

The hood is the lowest or flattest when formed of all the five primary fashions, and is not much used, although it would be quite useful for wash boilers if the boiler was made elliptical. It is made up of 300 degrees and contains six sectors of circles which, when taken together, will equal the circumference of a given oval whose transverse diameter is A B, Fig. 32. Construct the oval A H G B J I, shown by the dotted circumference, and make it 6 inches long, as in the preceding example. Then from the point C, with any radius, C O, describe the circle L O K, and having marked off five-sixths of its circumference, as L O K, divide it into six equal parts, as shown by L M N O P Q K. From N (which marks off one-sixth of these five-sixths from the point O), through C, draw N C V. By the rule for hood fashion $\frac{A\,h \times 3}{5} = 2.4$, or the radius of a circle five-sixths of which will make a cone hood fashion 4 inches in diameter at the base, or $\frac{4 \times 3}{5} = 2.4$, or the radius C T or C V. Then with C T as radius describe the arc T U V. With C T from C on the line T C P mark off the distance C F, and with C V from C on the line V C N mark the distance C E, and with E as center and E C as radius describe the arc C Z, equal to U V, and with F as center with F C as radius describe the arc C Y, equal to T U, and from E, through Z, draw E Z W, and with E V as radius the arc V W, and from F through Y draw F Y S, and from the point F as center and F T as radius describe the arc T S. From Y, with Y S as radius, describe the arc S R and make it equal to T U, and from Z with Z W describe the arc W X and make it equal to V U. Now join Y R and Z X, continuing them to their intersection *a*, and the curve line R S T U V W X will equal the circumference of the oval A H G B J I, and a metal pattern cut as indicated by *a* R U X will make a true elliptic cone hood fashion, which, when formed, will stand on the oval shown by the dotted lines in Fig. 32.

NOTE.—In the formation of elliptic cones, it should be observed that at the points C and Z where the foci vanish toward the apex *a* the pattern should be formed sharp to preserve the truth of the base of the cone.

Fig.32



OVAL CONES (CONTINUED).

We will now consider the five fashions of cones to suit the oval illustrated by Fig. 26. The foundation of this figure is two circles, the centers of which lay in the longest diameter, the shortest diameter forming a tangent to these circles in the center of the longest diameter of the oval, Fig. 26.

EXTINGUISHER FASHION.

Let it be required to make a cone extinguisher fashion, the base of which is an oval formed as in Fig. 33. First draw the oval A H G B J I as directed in Fig. 26, making the longest diameter, A B, 6 inches; then the diameter of the circle A *b* will be 3 inches, and the rule for extinguisher fashion is $\frac{A\,b \times 3}{1} = 9$, radius of a circle one-sixth of whose circumference would make a cone of this kind of taper or extinguisher fashion—that is, A *b* = 3 and $\frac{3 \times 3}{1} = 9$; then C U equals 9 inches. Now draw U C O, making C O equal to C U, and with C O describe the arc L O K, making it equal to one-sixth of the circle of which it is a part, and then divided into six equal parts, as L M N O P Q K. From P through C, draw T C P and extend it on to *d*, making T *d* three times the length of C U, or 27 inches long, and from V, through C, draw V C N and extend it on to *h*, making V *h* three times the length of C U, or 27 inches long. Then with C T describe the arc T U V. Through O draw the lines *d* O S and with *h* O W and with *d* T describe the arc T S and with *h* V the arc V W. From the point *h* with the radius *h* C describe the arc C Z, and

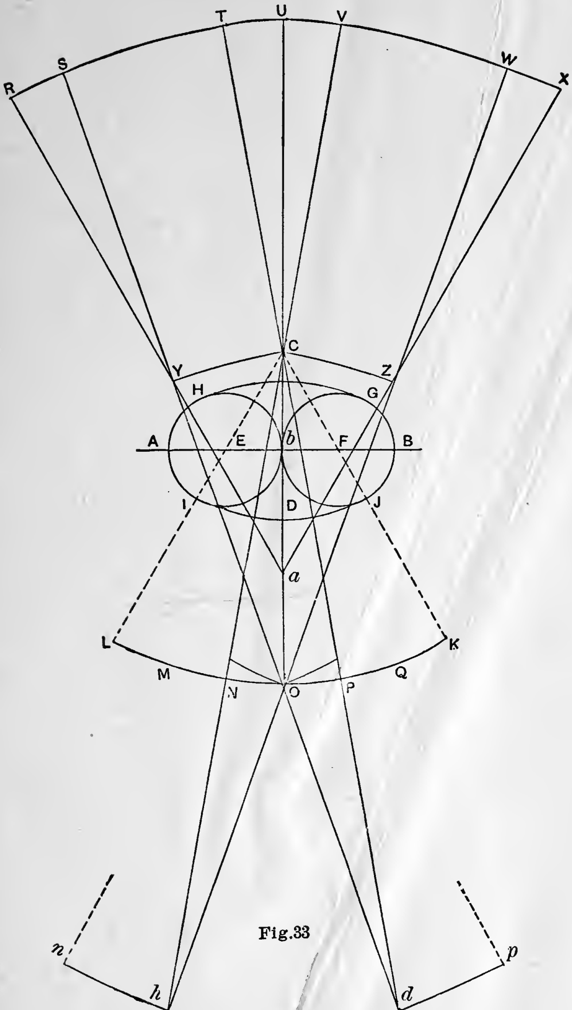


Fig.33

from the point *d* with the radius *d* C describe the arc C Y. From the points Y and Z with the radii Y S and Z W (that is, C U, or 9 inches) describe the arcs S R and W X, making them equal to T U and U V, and draw R *a* through Y and X *a* through Z. Then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, and a pattern cut as shown by *a* R U X will, when formed, make a true oval cone extinguisher fashion.

To further explain to the learner the foregoing principles: If a straight line be drawn from R to X, then R *a* X will be an equilateral triangle, and the angle at *a* will be 60 degrees; but the curved line is made up of sectors of two distinct circles, the radius of the larger one being three times the length of the other. The reason for this is because the radius of the two side arcs of the oval A B is three times longer than the radius of the arcs of the two ends, and, therefore, this is necessary to preserve the symmetry and proportion of the conic curve. Again, it will be seen by a little study that the angle contained in each sector of the pattern here developed is six times less than that of the oval from which the pattern has been evolved, while, on the other hand, the radii of the sectors are six times longer than those which compose the oval A B. In the next example, Fig. 34, it will be shown that in the development of the pattern the two radii forming it have been reduced by the rule one-half, while the angles of the sectors have been doubled—that is, when compared with the last example. This law must follow in proportion as the hight of the cone recedes, and having vanished has become a plane and the sectors coincide with those of the oval.

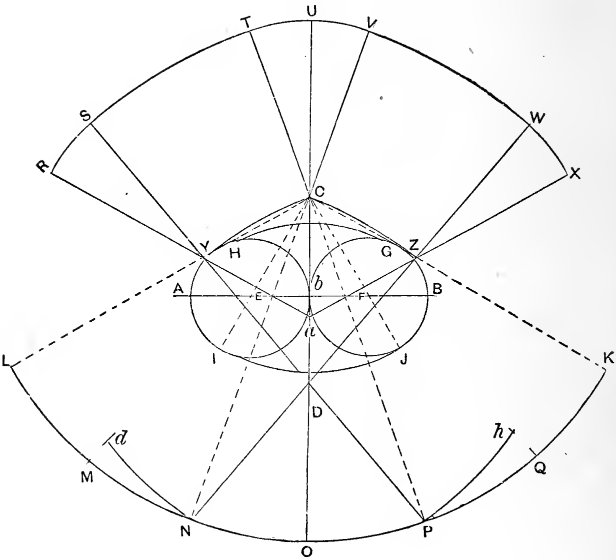
MULLER FASHION.

Let it be required to make a cone muller fashion to fit any size oval, as in Fig. 34. Construct the oval A H G B J I and let the length A B equal 6 inches. By the rule $\frac{A\,b \times 3}{2} = 4.5$, the radius of the circle two-sixths of which will make a cone muller fashion—that is, $\frac{3 \times 3}{2} = 4.5$; then C U equals 4.5, and by the same rule $\frac{{}^*C\,J + C\,J \times 3}{2} = 13.5$, or T P. Draw U C O at right angles with A B, and with C U as radius from the point C describe the arc T U V. Now, with C P as radius from the point C describe the arc L O K, making it 120 degrees, or two-sixths of the circle of which C P is the radius, and divide it into six equal parts, L M N O P Q K, three from O to L and three from O to K; then from P through C draw P C T and from N through C draw N C V. With P T as radius from the point T describe the arc T S, and with N V as radius from the point N describe the arc V W, and from the same centers the arcs C Y and C Z, making them equal to P O and N O; then from P through Y draw P Y S, and from N through Z draw N Z W. From the points Y and Z with the radii Y S and Z W equal to C U describe the arcs S R and W X, and draw R Y and X Z and continue them until they meet in *a;* then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, and a

* The diameter being twice the radius.

pattern cut as represented by R U X *a* will, when formed, make a true oval cone muller fashion.

Fig. 34



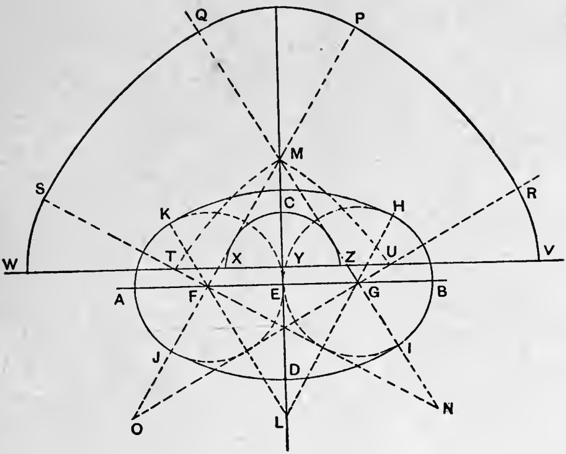
L. of C.

FUNNEL FASHION.

About 50 years ago we made what were called breakfast bottles—that is, oval tin bottles to carry tea or coffee in for breakfast. The shape of the bottle was an oval, as shown in Fig. 26. The body was mounted with an oval conical head or top, the pattern of which was supplied to us, as was also that of the neck and handle, all of which were kept together in the usual way on a ring, and we had no trouble. But after leaving the old shop I was called on to make some bottles where there were no patterns, and here it happened, as it has often happened with others, my trouble began. I, however, persevered through the job as best I could, and then spent many weary hours trying to find some correct way which could be relied on and easily retained in memory, ready for use at any time; at length I found what seemed the perfect plan. At this time I knew nothing of geometry; I had never seen "Euclid's Elements" even, or any other book of the kind, but was eagerly plodding along trying to add to the practical instruction my father was able to give me; and while I have somewhat extended the ideas then caught as they flitted by, I have never improved them in the main, because the first deductions were nearly geometrically true.

To illustrate my first discovery: Let it be required to make an oval bottle to hold a quart nominally; then, one-half a single plate (sheet 10 x 14 inches) cut lengthwise, with the seam deducted, or 13.5 long, will when formed into a cylinder hold a quart, and will measure 4.2971 in diameter and 5 inches deep. But the bottle is to be an oval form, as shown in Fig. 26, and the ratio of the length to the width of this oval or transverse axis is to the conjugate as 1 is to .63238, or if the length is 1 the width will be .63238. Now we must know what the length and width of an oval, as in Fig. 26, will be whose circumference is 13.5 inches. To do this we proceed as follows: Multiply the circumference by 2, and divide by 5.128 + (which is $1 + .63238 \times 3.1416$, or the transverse axis added to the conjugate and multiplied by 3.1416), thus: $13.5 \times 2 = 27$, and $27 \div 5.128 = 5.2652$, the transverse-axis, and $5.2652 \times .63238 = 3.3291$, the conjugate axis. The proof is shown by adding the length and width together and dividing by 2, which gives the circular diameter, thus: $\frac{5.2652 + 4.4291}{2} = 4.2971$. Then by turning the strip into a circle we have $\frac{13.5}{3.1416} = 4.2971$, which coincide near enough for all practical purposes. We now have the length and width of the oval, whose circumference is 13.5—namely, 5.2652, the length, and 3.3291, the width. Now lay off the length of the oval A B, Fig. 35, making it 5.2652 long, and draw C D at right angles to it at the point E. Now divide A E in F and E B in G, and with G B as radius and G and F as centers describe the arcs H B I and K A J and let them each contain two-sixths, or 120 degrees. From I through G draw I G M, and from J through F draw J F M, and with M I as radius from the point M describe the arc J I. From K through F draw K F L, and from H through G draw H G L, and with L K as radius from the point L describe the arc K H, and the oval

Fig.35

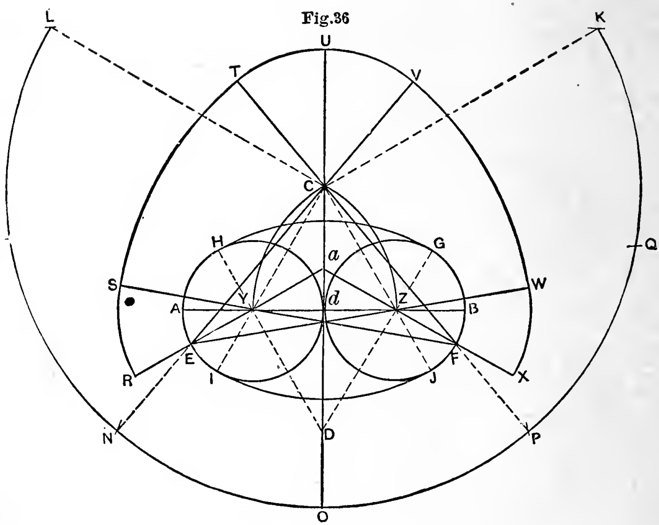


A J I B H K is complete and is the pattern of the bottom of the bottle without edges for seam, and from the lines of this bottom we develop the top or oval cone, proceeding as follows: Extend the line M F J both ways to P and O, and M G I to Q and N, and with the radius M P (that is, E B, which is twice the length of G B) describe the arc Q P; now with the distance E B from the point G lay off the distance G N, and with the distance A E from the point F lay off the distance F O, then from O through G draw the line O R, and from N through F draw the line N S, and with N S as radius (which is twice the length of L K) from the point N describe the arc Q S; also with O R as radius (which is twice the length of L H) from the point O describe the arc P R; now with O M describe the arc M U, and with N M describe the arc M T, then on the line N S with the distance T S (that is, A E) as radius from the point T describe the arc S W, and on the line O R with the distance U R (that is, E B) as radius from the point U describe the arc R V. Then through T and U draw the line W T U V, and the curve line W S Q P R V is equal to the circumference of the oval A K H B I J. The semicircle on X Y Z measures the hole for the neck of the bottle. The pattern here described as funnel fashion is true in every particular. If the curve was a complete semicircle instead of an irregular curve and drawn from the center Y it would be an exact pattern of a common funnel. All the radii by which it has been constructed are double the length of those which form the oval from which it has been developed.

LANTERN-HEAD FASHION.

Lantern-head fashion, as previously shown, is made up of 240 degrees. To make this pattern for any oval shaped as in Fig. 36 construct the oval A H G B J I and let A B equal 6 inches and draw U C D at right angles to A B. Then $\frac{A\,d \times 3}{4}$ equals the radius of a circle four-sixths of which would make a cone lantern-head fashion—that is, $\frac{3 \times 3}{4} = 2.25$, or C U. With any radius, C O, describe the part of a circle, L O K, and make the arcs O L and O K together equal to four-sixths of the circle of which C O is the radius. Now divide L O K into six equal parts, L M N O P Q K. From P through C draw P C T, and from N through C draw N C V. Then from the point C with C U as radius (that is, 2.25) describe the arc T U V. On T C P from C lay off the distance C F twice the length of C T, and on V C N from C lay off the distance C E twice the length of C V. From F through Y draw F Y S, and from F with F T as radius describe the arc T S. From E through Z draw E Z W, and from E with E V as radius describe the arc V W. With E C as radius from the point E describe the arc C Z, and with F C as radius from the point F describe the arc C Y. Now from the point Y with Y S as radius (that is, C T) describe the arc S R, equal to T U, and from the point Z with Z W (that is, C V) describe the arc W X, equal to V U, and draw R Y *a* and X Z *a*. Then the curve line R S T U V W X is equal to the circumference of the oval A H G B J I, and a pattern cut as shown by this curve line will, when formed, make a true oval cone lantern-head fashion.

Fig.36



HOOD FASHION.

Hood fashion, as before shown, is composed of six sectors which, when laid together, as in Fig. 37, will equal 300 degrees at the center *a*. To make this pattern fit any oval shaped as in Fig. 37 construct the oval A H G B J I and let the length A B equal 6 inches, and draw U C D at right angles to A B. Then A *b* equals 3, and by the rule $\frac{3 \times 3}{5} = 1.8$, or C U. With any radius, C K, describe the part of a circle, L O K, and make it equal to five-sixths, or 300 degrees. Now divide it into six equal parts, L M N O P Q K. From P through C draw P C T, and from N through C draw N C V; then from the point C with C U as radius (that is, 1.8) describe the arc T U V. On T C P from C lay off the distance C *h* twice the length of C T, and from the point *h* with *h* T as radius (that is, 1.8 × 3, or 5.4) describe the arc T S, making it equal to one-sixth of a circle of which D H is the radius, and draw *h* Y S; then with *h* C as radius describe the arc C Y. On N C V from C lay off the distance C *d* twice the length of C V (that is, 5.4), and from the point *d* with *d* V as radius, describe the arc V W, making it equal to one-sixth of a circle of which D H is the radius, and draw *d* Z W; then with *d* C as radius describe the arc C Z. Now from the point Y with Y S (that is, C T) describe the arc S R, making it equal to T U, and draw R *a* through Y, and from the point Z with Z W (that is, C V) describe W X, making it equal to V U, and draw X *a* through Z. Then the curve line R S T U V W X is equal to the circumference of the

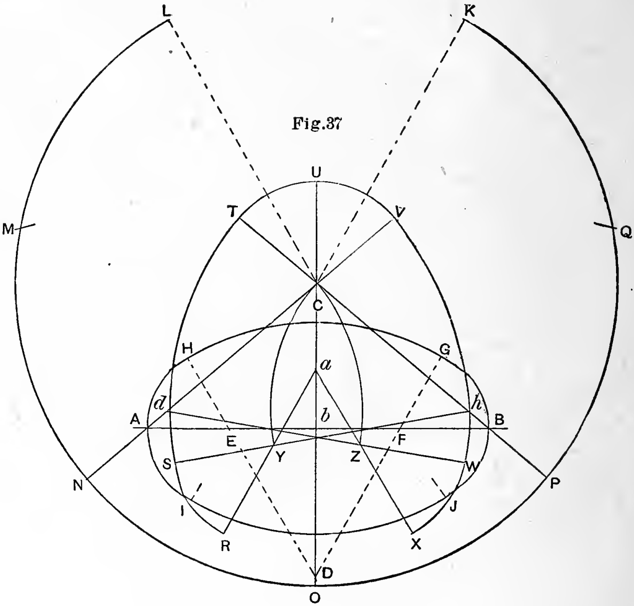


Fig.37
L
K
U
T
V
M
Q
C
H
G
a
d
h
A
b
B
E
F
S
Y
Z
W
N
P
I
J
R
X
D
O

oval A H G B J I, and a pattern cut as shown by the curve line will, when formed, make a true oval cone hood fashion.

In concluding the lessons on oval cones, a few words are necessary, as before noted, to show that in order to preserve for any special purpose the truth of the base when formed the pattern should be bent sharp along the lines *a* C and A Z, because the centers or the foci of the oval vanish at these points and meet in the center *a*, which will be readily seen by a few trials in practice.