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THE SLIDE RULE

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GIVE INSTRUCTION TO A WISE MAN . . .

. . . . AND HE WILL BE YET WISER *Proverbs 9 9*

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THE SLIDE RULE

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THE SLIDE RULE

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FOREWORD

The present era is sometimes termed the mechanical age because so many operations which, in earlier days, were carried out slowly and often painfully by hand, are now performed by machines with an enormous saving in time and effort.

The slide rule cannot be regarded as a modern invention since the first design dates from the early part of the seventeenth century, but every year sees additions and variations made and the up-to-date instrument has, as might be expected, advanced greatly beyond the earlier types. Every year a considerable number of Patent Specifications are lodged in the British Patents Office to give protection to the latest inventions in slide rule technique, and we may say fairly that the slide rule in its own field is keeping pace with modern mechanical advance.

This Teach Yourself Book is published to increase the popularity of the slide rule. It is hoped that it may help to remove the fallacy that there is something difficult or even mysterious connected with a simple instrument with which everybody who has calculations to make should be acquainted.

Among the technicians and artisans upon whom we so much depend for the maintenance and improvement of national prosperity, are many who have frequently to make calculations. They would be hampered in their activities if the improvements we have referred to had not extended to expediting their work. The slide rule and other instruments which give the same facilities for rapid calculations, are covered by the term "Mechanical Calculation".

It is unfortunate that, for some reasons not easy to see, the slide rule is sometimes regarded as a difficult instrument with which to become proficient. There is a tendency for some people to become facetious in their references to this simple instrument. Journalists and broadcasters are great offenders in this respect and some of their references are unbelievably absurd and show a lack of elementary knowledge.

The clumsy and unscientific system of monetary units and measures in weights, lengths, areas, etc., which have grown up and are still used in this country, give some slight difficulty in applying the slide rule to calculations in which they are involved and since the scales of slide rules are, in most cases, subdivided in the decimal system, any notations of weights and measures which are similarly designed, such as the metric system, lend themselves readily to calculation by slide rule.

We shall find, however, that when working in our monetary system of pounds, shillings and pence, or in lengths in miles, yards and feet, or in any of our awkward units, the slide rule can be employed to simplify our work and give results quickly, and with a degree of accuracy sufficient for practical requirements.

We have said that, in general, slide rule scales are subdivided in decimal fractions, and since there are still some people who cannot easily calculate in the decimal system, we give at an early stage a simple explanation of the principles of this system. Perhaps we need hardly add that any sections of this book which deal with matters with which the reader is quite familiar may be glanced at and passed over.

We shall find that the underlying principle of the slide rule is calculation by logarithms. Just as a man may be an expert motor-car driver without understanding the principles of the internal-combustion engine and the mechanism of his car, so can a slide rule be used without the slightest knowledge of logarithms. In fact, we hesitated at including the section on logarithms, in case the mention of the term might cause discouragement and increase the sense of awe with which some people regard the slide rule. The section on logarithms may be disregarded entirely, and indeed we ask that it should be, on the first reading of this book, but when the rudiments of the slide rule have been mastered—and again we stress the simplicity of these—it may be that some readers will find

interest and advantage in learning something of the first principles of "logs"—one of the most fascinating parts of elementary mathematics.

Another factor which has contributed to the reluctance of people to purchase a slide rule lies in the erroneous impression that it is a costly instrument. Naturally enough, people are averse to paying two or three pounds for an instrument which they fear may be of little use to them. Inexpensive slide rules have been available in this country for over thirty years, and their makers claim that for accuracy and utility they are equal to the more expensive varieties which have been manufactured for a much longer period. The first "Unique" slide rule, the 10″ log-log model, was produced at a popular price for students. Its introduction was welcomed, and it met with success. There are now about a score of different slide rules in the "Unique" range, and sales have progressively increased, and it may fairly be said that this make of rule is now the "best seller" in this country. "Unique" slide rules carry all the useful scales, including the log-log scale, in most models. In the expensive type of rule the inclusion of the log-log scale means a much higher-priced instrument than the "standard" or ordinary models. The log-log scale in the "Unique" range is included at no increase in the cost of the rule. The makers of "Unique" slide rules introduced a new technique in manufacture by printing the scales and coating them with transparent plastic material. This important change allowed of a great reduction in the manufacturing cost as compared with the older method of separately dividing the scales.

This book, however, is not published primarily to boost any particular make of slide rule. All slide rules are difficult to manufacture, and in most cases are honestly worth the prices charged for them. Some shopkeepers charge more than the recognised retail prices fixed by the manufacturers, and purchasers should be vigilant and resist any attempt at this sort of imposition.

We would, with all respect, urge members of the teaching profession to make more effective efforts to introduce the slide rule into schools. The proper place to become acquainted with this invaluable time-saver is in the classrooms of the primary schools; normal boys and girls of the age of 13 or 14 years are able to attain proficiency in its use. More often than not the student does not become acquainted with the slide rule until he or she reaches a technical school or college, and even in such institutions the slide rule is by no means the universal and everyday instrument it deserves to be.

The writer has had long experience of teaching in a technical college, and has never had the least difficulty in arousing interest in the application of the slide rule to practical problems. There was never any necessity to urge students to adopt the rule; directly a slide rule appeared in a classroom and was demonstrated, students expressed the desire to acquire one, and within a week or two the majority had done so. A few minutes devoted to instruction were sufficient to teach the fundamentals. We know that the slide rule is used in a number of primary and central schools by teachers who think as we do. Unfortunately, we also know that even in some grammar and secondary schools a slide rule is almost unknown.

We would particularly direct attention to Section 8, which deals with slide rules designed for commercial calculations. We say, without fear of being proved wrong, that every individual who has to make calculations can, at times, use a slide rule to great advantage, and this statement applies to the commercial man. The slide rule costs but a few shillings, and takes little time to master. To refuse to investigate the potentialities of the instrument is to adopt a non-possumus attitude.

The commercial rule can be recommended also for technical work since it incorporates the ordinary C and D scales, which deal with the bulk of the work, and four other scales, which automatically multiply or divide by 12 or 20, without using the slide, and a reciprocal scale. For many purposes this rule is more adaptable than the usual type with A, B, C and D scales.

The monetary slide rule is scaled directly in £, s. d., and for some purposes, for example in checking invoices, is more convenient to use than the commercial rule. It carries also the C and D scales and so provides facilities for straightforward multiplication and allied operations.

We hope we shall not be accused of unduly stressing the advantage of slide rules which depart from the standard type. We can only attribute to the conservatism with which most of us are endowed the fact that the large majority of slide rules in use are of the standard type whose salient features are the A, B, C and D scales. Men who have used a slide rule for years have never handled any other than the standard type; to them we would suggest a change to a more efficient instrument, several of which we mention later. The standard type slide rule, except for the beginner, is moribund.

For the tyro we advise first the reading of Sections 1 and 3, making sure he can read the scales. He should read also Section 2 if he is likely to have any difficulty with decimals.

He should then *study* Section 4 thoroughly, since this is the most important part of the early instruction. This section deals with C and D scales, which are by far the most used scales of the standard slide rule.

He should use his slide rule for the examples and problems given in the text and work through additional simple examples he can make up for himself, using numbers which can be easily reduced mentally. Such examples as $\frac{6\times 9\times 16}{2\times 4}$, which gives 108 as the result. He may say that it is not worth while using a slide rule to calculate a result he can obtain mentally, and he would be quite right, but we are here advising how to approach the slide rule and to gain experience and confidence in using it.

The student may then use these figures $\frac{6{\cdot}42\times 9{\cdot}35\times 16{\cdot}7}{2{\cdot}04\times 4{\cdot}41}$, which he cannot cope with mentally. He should obtain as a result 111·5 and he will see how the position of decimal point has been fixed.

For the experienced reader we recommend the dualistic slide rule as being the best available for general purposes. This rule is discussed in Section 11. It is quicker in action, more accurate in its results and the irritating necessity of traversing the slide when using the more usual type of slide rule is eliminated.

We include in the explanatory sections of the book examples in respect of which the movements of slide and cursor are indicated. Often there are alternative ways of selecting various factors, giving rise to alternative ways of moving the slide and cursor. For practice some of these should be worked out by the reader.

Problems are inserted for the student to solve, and a check can be made by comparing results with those given at the end of the book. To reduce typography in the worked examples, abbreviations are used; these are mentioned in Section 4.

CONTENTS

ILLUSTRATIONS

SECTION ONE

THE PRINCIPLE OF THE SLIDE RULE

The reader will agree that the arithmetical operations of addition and subtraction are less tedious to carry out than those of multiplication and division. When very simple numbers are involved, none of these four operations gives trouble, and it is just as simple to multiply 4 by 2 as to add together 4 and 2. When the numbers are larger, this is no longer the case. It is still comparatively easy to add together, say 492 and 374; most people could perform the operation mentally, and give the result as 866, without resorting to the aid of pencil and paper. If, however, these two numbers have to be multiplied together, only a few people who have the unusual gift of being able to cope with such computations mentally could give the answer with confidence. As a test of memory and concentration write down these numbers with a view to multiplication, namely,

374; now lay down your pencil and try to complete the multi-
492
———

plication, memorising the figures as they emerge, and then mentally add up the three resulting lines. It is clear that to obtain a correct result one must possess exceptional powers; in fact, to perform mentally the operations correctly with two figure factors is commendable. Addition or subtraction remain comparatively simple, and can be quickly performed with even a dozen or any number of factors, but multiplication and division when carried out by ordinary arithmetical means become progressively tedious as the number of factors increases.

There are now in use, for office and industrial purposes, ingenious calculating machines. These are, comparatively, of recent origin, and are designed rapidly to deal with the masses of routine computations which have to be dealt with in large offices, banks and industrial organisations. For their particular purposes they stand supreme, and it is no part of this treatise to deal with them. To some extent these elaborate and expensive machines execute the same operations as the simple slide rule, but many operations which can be effected by slide rule are impossible with the calculating machine, and the converse applies also.

Reverting to our simple addition of 4 and 2, it is clear that we could perform the operation with the use of two scales placed as shown in Fig. 1. These scales might be divided in inches or in centimetres, or in any arbitrary unit, and if each

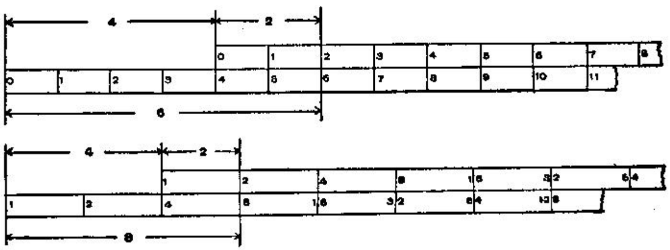

FIG. 1 and FIG. 2

unit was subdivided into tenths we could effect the addition of such quantities as 3·6 + 7·8. Further, if we had a convenient way of marking the result of the first addition by means of an index sliding along the lower scale, we could proceed to add or subtract as many factors as we desired. The reader will have no difficulty in seeing how subtraction would be effected, and he need not be much concerned at the possibility of requiring an absurdly long lower scale, since we are not suggesting that anyone would indulge in this unpracticable demonstration, we are leading up to the slide rule method of multiplication. We have, however, occasionally seen a mechanic adopt this method of adding together dimensions shown in a drawing of some engineering product. He may wish to add $6\frac{1}{16}''$ to $2\frac{1}{8}''$ and subtract $1\frac{3}{32}''$. The fractions give him a little trouble, so he takes his steel rule and places his thumb nail against the line marking

$6\frac{1}{16}''$. With a finger of his other hand he counts off a further $2\frac{1}{8}''$, and moves his thumb to register the new position further along the rule, and finally counts back $1\frac{3}{32}''$. The last mark gives him the result he is seeking.

Now please examine the scales indicated in Fig. 2. You will first notice that while the end divisions are marked 0 in Fig. 1 they are marked 1 in Fig. 2. You will also observe that whereas in Fig. 1 the graduations are marked 0, 1, 2, 3, etc., in Fig. 2 they are marked 1, 2, 4, 8, 16, etc., each number being twice the value of the preceding one. Clearly, if all the graduations were shown in Fig. 2, there would be a great crowding together as we move in the right-hand direction along the scale. For instance, the distance lying between graduations 1 and 2 is about $\frac{1}{2}''$. In this same space between graduations 16 and 32, it would be necessary to crowd in 16 smaller spaces.

The scales of Fig. 2 are logarithmic, and you will understand their properties when you have perused the section on logarithms. (There is no necessity to break off at this stage to read about logarithms, and we recommend that you read on without concern for them.) Fig. 1 gave us the sum of 4 and 2. Fig. 2 gives us the product of 4 and 2, i.e. 8, and it becomes evident that one of the rules of logarithms is that by adding them together we are effecting multiplication of numbers, and when we subtract one from another we are dividing.

In these simple facts lies the principle of our slide rule, which, in effect, is the equivalent of a table of logarithms arranged in a convenient form for rapid working.

Further study of Fig. 2 will show that the scales are set so that we can at once read off $4 \times 2 = 8$; $4 \times 4 = 16$; $4 \times 8 = 32$, and if the scale had been extended and subdivided we should have been able to read off many other results.

The procedure for multiplication of two factors is:

(*a*) Select any one of the factors and note its position in the lower scale.

(*b*) Slide the upper scale to the right, to bring the 1 of this scale opposite the factor noted in the lower scale.

(c) Find the second factor in the upper scale.

(d) Directly below the second factor, read the number which you will see in the lower scale. This number is the answer to the multiplication of these two factors.

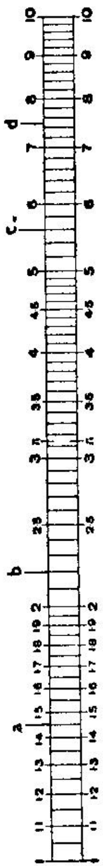

FIG. 3

We know that these instructions sound rather forbidding, but they are quite simple and if the reader will follow them through carefully he will, in the course of a few minutes, learn to use the two scales for multiplication. It will certainly assist if the scales are drawn out on two strips of card-board, so that they can be moved along one another into the different positions.

Division is effected by using the scales to subtract the logarithm of the divisor from the logarithm of the dividend. Referring once more to Fig. 2, we see at once that in order to divide, say, 128 by 32, we slide the upper scale along until the 32 in it stands immediately above the 128 in the lower scale. Then opposite the 1 of the upper scale we read the answer, 4, in the lower scale.

Fig. 3 shows you a logarithmic scale which has been subdivided between the primary numbers. You will notice that between graduations 1 and 2 it has been possible to show twenty smaller spaces, whereas between 9 and 10 only 5 subdivisions have been made. This change in the distance between consecutive lines is a feature of all logarithmic scales; it is the crowding together we have mentioned earlier.

We now come to what may prove a difficulty for some readers to whom scales are not familiar. We refer to what is generally termed "reading the scales". We will, therefore, spend a little time in studying this difficulty since it is quite certain that ability to read the scales easily and with certainty is essential. The difficulty—if there is any—lies in the fact that the graduations of the scale alter. In Fig. 3, the distance between 1 and 2 is subdivided into 20 parts. If the subdivision continued in the same way, the spaces would soon become inconveniently small. At the division 2 a change in the dividing occurs, and the space between 2 and 3 is subdivided into only 10 parts, and this subdividing continues from 3 to 4 and again from 4 to 5. At 5 another change becomes necessary and the main divisions from 5 to 10 are now subdivided each into five parts only.

In reading any position of the scale, the graduations on either side of that position must be examined. Look along the scale to the nearest main figure, then note whether the subdivisions are tenths or fifths or any other fractions of the main division. With a very little practice you will quickly develop the faculty of reading the positions in the scale with a high degree of accuracy.

As examples let us attempt to read the positions in the scale of the four lines marked *a*, *b*, *c* and *d* of Fig. 3. Line *a* exactly coincides with a division of the scale and appears to be about midway between 1·4 and 1·5. The position of the line *a*, therefore, is 1·45. Line *b* also coincides with a division of the scale and lies between main numbers 2 and 3. A glance shows that there are ten sub-divisions between 2 and 3. The graduation immediately to the right of 2 is 2·1, and the next to the right is 2·2, and it is at this position that line *b* stands. Line *c* lies between numbers 5 and 6, and now we find there are only 5 subdivisions in this space. We will write down fully the readings of the lines at this part of the scale; they are 5·0, 5·2, 5·4, 5·6, 5·8 and 6·0. Line *c* clearly stands at 5·6. Line *d* does not coincide with any graduation in the scale and now our ability to estimate fractions must be exercised. Line *d* lies between 7·4 and 7·6. Let us try to visualise the small distance between these two lines being further subdivided into five smaller spaces. There would now be four lines very close to one another in between the lines at 7·4 and 7·6, and the

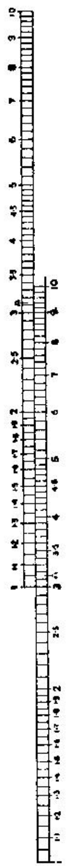

FIG. 4

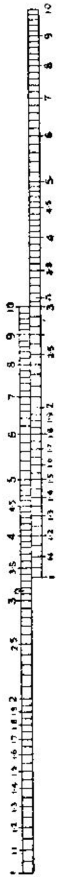

FIG. 5

readings of these four lines would be 7·44, 7·48, 7·52, 7·56, and we estimate that line *d* is very near to 7·48.

The observant reader may object with some justification that there is a fundamental error involved in the method we

have adopted in arriving at the value 7·48. He will have noticed that in effect we have estimated the position of the line *d* as being 2/5ths of the distance between 7·4 and 7·6, and he will point out that even if our estimate is quite correct the true position of the imaginary line 7·48 is not exactly at this point, because the scale being logarithmic the five small spaces between 7·4 and 7·6 are not equal to one another. Actually the imaginary 7·48 line is slightly to the right of the position we have assigned to it. This is typical of the errors we invariably make when we estimate the positions of points which do not coincide with any real lines in the scale. We shall return to this matter when we consider later on the degree of accuracy possible when using a slide rule, but the inquisitive reader may care to know that the true scale reading of a point *exactly* 2/5ths of the distance along 7·4—7·6 of the scale is 7·476. Our estimate has involved us in the small error of 4 parts in 7000. To obtain an idea of what this error means, imagine you are asked to measure the length of the table at which you are working. By means of a rule or tape and measuring carefully you find the length is, say, 56·4 inches, whereas when measured with more precise apparatus the length is found to be 56·43 inches; the error you have made is, therefore, three hundredths of an inch, and proportionately these two errors are nearly equal.

Fig. 4 illustrates two scales set so that we can multiply 3 by various numbers. Notice that directly under any number in the upper scale, three times that number appears in the lower scale, e.g. $3 \times 11 = 33$, $3 \times 12 = 36$, $3 \times 2 = 6$ and several others. This same setting of the scales shows how we can divide 9 by 3, or 6 by 2, etc. Now, in Fig. 4 the upper scale projects to the right beyond the lower, and we cannot read results directly under the projecting part. This difficulty is surmounted by sliding the upper scale to the left a distance equal to its own length. Fig. 5 shows these new positions of the scales and now we can perform multiplications such as $4 \times 3 = 12$, $6 \times 3 = 18$, $9 \times 3 = 27$, and we can also divide 18 by 6 or 15 by 5, etc.

We do not wish to weary the reader by pursuing unduly this very elementary conception of the slide rule. This book is primarily designed to assist readers who have had no previous acquaintance with the slide rule, and we think that those who have persevered so far will, by now, realise that there is nothing difficult to learn and that the manipulation of a slide rule is very simple indeed.

We feel that we should now pass on to examine the slide rule in its modern practical form.

(The next section deals with fractions and decimals. It is included to assist readers who may have difficulty in reading the scales in the decimal system. It should be ignored by others.)

SECTION TWO

FRACTIONS—DECIMALS

THE logarithmic scales of slide rules are, with a few exceptions, subdivided into decimal fractions, or, as we more often say, into decimals. It is impossible to take practical advantage of the slide rule without a working knowledge of the decimal system. We believe a brief note of explanation may assist those readers who think that the slide rule is useless to them because they cannot easily work in decimals. This section is not intended for readers who are familiar with the decimal system, and can use it without difficulty.

We propose to start with a short reference to ordinary fractions. The word fraction means "a part". Thus when we speak of $\frac{1}{2}$ an inch—which is sometimes called an ordinary fraction, as distinct from a decimal fraction—we think of a length of 1″ being divided into two equal parts, of which we take one part. When we mention $\frac{3}{4}$ as a fraction, we think of something, say, a yard, or an hour, or a shilling, being divided into four equal parts, of which we take three. The upper figure of an ordinary fraction is called the numerator, and the lower figure the denominator. A fraction in which the numerator is smaller than the denominator is always less than 1, and is sometimes called a proper fraction. A fraction such as $\frac{7}{5}$, in which the numerator is larger than the denominator, is called an "improper" fraction. These terms proper and improper, when referred to fractions, are of no practical importance.

The value of a fraction is not altered if we multiply or divide numerator and denominator by any number. For instance,

$$\frac{3}{5}=\frac{3\times 4}{5\times 4}=\frac{12}{20}=\frac{12\times 2}{20\times 2}=\frac{24}{40}.$$

The fractions $\frac{3}{5}$, $\frac{12}{20}$ and $\frac{24}{40}$ are all exactly equal to one another.

but we may say that the $\frac{3}{5}$ is the simplest form, and in general this is the way it is written. You will see that the fraction $\frac{24}{40}$ can be reduced to $\frac{3}{5}$ by dividing both numerator and denominator by 8. This kind of simplification is called cancelling.

Addition and Subtraction

To add together two or more fractions, we express them in terms of a common denominator, and then add together the numerators.

Example: Add together $\frac{2}{3}$ and $\frac{4}{5}$.

$$\frac{2}{3}+\frac{4}{5}=\frac{(2\times 5)}{(3\times 5)}+\frac{(4\times 3)}{(5\times 3)}=\frac{10}{15}+\frac{12}{15}=\frac{22}{15}.$$

Result is $\frac{22}{15}$ or $1\frac{7}{15}$.

Problem 1. Add together $\frac{1}{6}+\frac{1}{4}+\frac{2}{5}$.

Subtraction of one fraction from another is effected in a similar manner.

Example: Find the result of taking $\frac{1}{6}$ from $\frac{3}{8}$.

The smallest number which is a multiple of 6 and 8 is 24.

$$\frac{3}{8}-\frac{1}{6}=\frac{(3\times 3)}{(8\times 3)}-\frac{(1\times 4)}{(6\times 4)}=\frac{9}{24}-\frac{4}{24}=\frac{5}{24}.$$

Multiplication and Division

To multiply together two or more fractions it is only necessary to multiply together all the numerators to form the numerator of the result and to multiply all the denominators to obtain the denominator of the result.

Example: Evaluate $\frac{2}{3}\times\frac{4}{5}\times\frac{2}{7}=\frac{2\times 4\times 2}{3\times 5\times 7}=\frac{16}{105}.$

Cancellation of numbers common to both numerator and denominator should be effected whenever possible since this leads to simplification.

Example: Evaluate $\frac{1}{3}\times\frac{3}{5}\times 1\frac{1}{4}$.

This may be written $\frac{1}{3}\times\frac{3}{5}\times\frac{5}{4}=\frac{1}{4}$ the 3's and 5's cancelling out leaving only the 1 in the numerator and the 4 in the denominator.

Problem 2. Evaluate $1\frac{3}{5}\times\frac{3}{4}\times 2\frac{1}{6}$.

Division may be regarded as a special case of multiplication. To divide a number by a fraction you may interchange the numerator and denominator of the divisor, and then multiply by the inverted factor. An easy example is that of dividing by 2, which is exactly the same as multiplying by a $\frac{1}{2}$.

Example: Divide $\frac{3}{4}$ by $\frac{2}{5}$.

This should be written $\frac{3}{4}\times\frac{5}{2}=\frac{15}{8}=1\frac{7}{8}$.

Problem 3. Divide the product of $\frac{3}{8}$ and $2\frac{1}{5}$ by $\frac{3}{4}$.

Decimals

The word decimal is derived from the Latin word meaning ten, and the decimal system is based on 10. Consider, for example, the number 8888, it is built up of 8000 + 800 + 80 + 8. It is evident that the 8's are not all of equal value and importance. The first 8 expresses the number of thousands, the second the number of hundreds, the third the number of tens, and the last the number of units.

We have used the number consisting of the same figure 8 used four times; this was done because we wished to emphasise that the same figure can have different values attached to it, depending upon its position in the group. The number might have included any or all of the figures from 0 to 9 arranged in an infinite number of ways.

Let us consider a simpler number, say 15. In this the 1 actually means 10 units, and the 5 represents 5 units. Now we might desire to add a fraction to the 15 making it, say, $15\frac{1}{2}$, and it seems feasible to do so by extending beyond the units figure this system of numbering by 10's. To indicate the end of a whole number we write a dot, called the decimal point, and any figures on the right-hand side of it represent a part or fraction of a unit.

We have seen that any figure in the fourth place to the left, counting from the units figure, represents so many thousands, the next to the right so many hundreds, and next so many tens, and the next so many units. If we continue we shall here pass the decimal point, and the next figure to the right must represent so many tenths of a unit. Still moving to the right the next figure will represent so many hundredths, the next so many thousandths, and so on indefinitely.

Now, $\frac{1}{2}$ is $\frac{5}{10}$, and remembering that the figure immediately to the right of the decimal point represents so many tenths, we can express $15\frac{1}{2}$ by 15·5. Instead of saying fifteen and a half, we should say fifteen decimal five, or as is more usual, fifteen point five. It would not be incorrect to express 15·5 by 15·50, or by 15·5000; the final noughts in both these cases are unnecessary but not actually wrong. In the form of a common fraction, the ·50 means $\frac{50}{100}$ which cancels to $\frac{5}{10}$, and finally to $\frac{1}{2}$, and similarly ·5000 as a common fraction becomes $\frac{5000}{10000}$ which also cancels to $\frac{1}{2}$.

We sometimes see one or more noughts preceding a whole number, e.g. 018. The nought has no significance, and is only used when for some reason we wish to have the same number of figures in a series of numbers. 018 means 18, and 002 means 2. We must understand that one or more noughts at the beginning of a whole number, and noughts following the decimal part of a number do not alter the value of the number.

Conversion of Decimal Fractions into Ordinary Fractions

It is easy to convert a decimal fraction into an ordinary fraction. Take as an example the number 46·823, which means

46 units and a fraction of a unit. Earlier we have said that the first figure to the right of the decimal point indicates so many tenths of a unit, the next to the right so many hundredths, and the next so many thousandths of a unit. We have, therefore,

$46 + \frac{8}{10} + \frac{2}{100} + \frac{3}{1000}$ which may be written $46 + \frac{800}{1000} + \frac{20}{1000} + \frac{3}{1000}$ which reduces to $46\frac{823}{1000}$.

From this we deduce the simple rule for converting a decimal fraction into an ordinary fraction. As the numerator of the fraction write all the figures following the decimal point, and for the denominator write a 1, followed by as many noughts as there are figures in the numerator.

Example: $152{\cdot}61 = 152\frac{61}{100}$. $9{\cdot}903 = 9\frac{903}{1000}$.

Problem 4. Convert the following into numbers and common fractions expressed in the simplest forms: 6·8, 13·08, 19·080, 20·125, 41·0125, 86·625.

There is a different rule for recurring decimals which will be given later.

Addition and Subtraction

When numbers include fractions it is easy to effect addition or subtraction in the decimal notation. It is only necessary to write down the numbers so that their decimal points are in a vertical line, then add or subtract in the usual manner, and insert the decimal point in the answer immediately below the decimal points of the original figures.

Example: Add together 16·26, 8·041 and 186·902.

```
 16·26
  8·041
186·902
-------
211·203
```

Subtract 108·694 from 423·47.

```
   423·470
   108·694
   -------
   314·776
```

Problem 5. Add together 12·801, ·92, 5·002 and 11·0. Subtract 82·607 from 96·2.

Multiplication and Division

A number expressed in the decimal system is very easily multiplied by or divided by 10 or 100, etc. To multiply by 10, move the decimal point one place to the right; to multiply by 100, move the decimal point two places to the right, and so on. When dividing move the decimal point to the left one place for each division by 10.

Examples:

$61{\cdot}24 \times 10 = 612{\cdot}4$

$61{\cdot}24 \times 100 = 6124$

$61{\cdot}24 \times 1000 = 61240$

$61{\cdot}24 \div 10 = 6{\cdot}124$

$61{\cdot}24 \div 100 = {\cdot}6124$

$61{\cdot}24 \div 1000 = {\cdot}06124$

Multiplication, when neither of the factors is 10 (or an integral power of 10, i.e. 100, 1000, etc.) should be carried out in the usual way, and the position of the decimal point ignored until the product is obtained. The number of decimal figures in the answer is easily obtained; it is equal to the sum of the numbers of figures after the decimal points of the factors.

Example: Multiply 62·743 by 8·6.

```
 62·743
    8·6
 -------
501·944
 37·6458
 --------
539·5898
```

Here there are $3 + 1 = 4$ decimal figures in the two factors. Starting from the last figure in the product we count off 4 decimal figures and insert the decimal point.

Problem 6. Multiply 9·274 by 82·6.

When dividing in the decimal notation it is advisable to convert the divisor into a whole number by moving the decimal point. If the decimal point of the dividend is moved the same number of places and in the same direction, the result will not be affected by these changes.

The following example will make this procedure clear.

Example: Divide 896·41 by 22·5.

Here the result is 39·8. The next figure in the answer would be a 4, so that if the result is required to only one decimal place it is 39·8.

```
       39·8
225)8964·1
    675
    ----
    2214
    2025
    ----
     1891
     1800
     ----
      910
```

Had the next figure been 5 or over 5, the result would then be given as 39·9, since this result would have been nearer to the exact answer than 39·8. When a numerical result which does not divide out exactly is to be expressed to a stated number of places of decimals, the division should be carried to one further decimal place. If this additional figure is less than 5 the figure preceding it should be left unaltered, but if the additional figure is 5 or over the preceding figure should be increased by 1.

Contracted Methods

When the factors which enter into the operations of multiplication or division are large, contracted methods should be used. This section is not intended to deal with all arithmetical rules and processes, but the reader will find a chapter dealing with contracted methods in books on elementary mathematics.

Conversion of Ordinary Fractions into Decimal Fractions

An ordinary fraction can be converted into a decimal expression by dividing the numerator by denominator. If we desire to change $\frac{3}{4}$ into decimals we divide 3·00 by 4. We generally add noughts to the 3 as shown. This is a case of simple division which we should often work mentally, but for the sake of clarity we will write it out in full.

```
4)3·00
   ·75
```

Now 4 will not divide into 3 so we include with the 3 the 0 which follows it, and divide 4 into 30. This gives 7 with 2 over and the 2 with the next 0 makes 20, which divides by 4 and gives 5 with no remainder. We insert the decimal point immediately below the decimal point in the original number and so obtain ·75 as the decimal equivalent of $\frac{3}{4}$.

Examples: Express as decimals $\frac{5}{8}$ and $\frac{17}{25}$.

```
                      ·68
                  25)17·00
8)5·000              150
   ·625              ---
                      200
                      200
```

Problem 7. Express as decimals $\frac{7}{8}$ and $\frac{13}{16}$.

The reader will see that we can convert an ordinary fraction into a decimal fraction by converting the fraction into a form in which the denominator is 10 or 100 or 1000, as the mathematicians say, into a positive integral power of 10.

Reverting to the $\frac{17}{25}$ considered a little earlier, we can convert the 25 into 100 by multiplying by 4, but to maintain the value of the fraction unaltered we must also multiply the 17 by 4. We have, therefore,

$$\frac{17}{25} = \frac{17 \times 4}{25 \times 4} = \frac{68}{100} = \cdot 68.$$

This method of conversion is sometimes quicker and easier than dividing denominator into numerator.

Recurring Decimals

If we attempt to convert the fraction $\frac{1}{3}$ into decimals by division, we obtain a result which is unending.

```
3)1·0000
  ·3333 . . .
```

This result is said to be a recurring decimal and is often written $\cdot\dot{3}$. The dot over the 3 indicates that the 3 is repeated indefinitely.

A number such as $24{\cdot}82\dot{1}\dot{6}$, means 24·82161616—the 16 being repeated indefinitely.

Conversion of Recurring Decimals into Ordinary Fractions

The rule to which we refer earlier is:

Subtract the figures which do not repeat from the whole of the decimal expression and divide by a number made up of a 9 for each recurring figure, and a 0 for each non-recurring figure.

Example: Convert $14{\cdot}6\dot{4}\dot{2}$ into an ordinary fraction.

```
642
  6
---      Result 14 636/990 = 14 106/165.
636
```

$$\text{Result } 14\tfrac{636}{990} = 14\tfrac{106}{165}.$$

Problem 8. Convert $2{\cdot}83\dot{1}\dot{3}$ into an ordinary fraction. Check the result by dividing denominator into numerator to see if $2{\cdot}83\dot{1}\dot{3}$ results.

SECTION THREE

THE MODERN SLIDE RULE

THE simple slide rule, consisting of two logarithmic scales drawn on strips of cardboard mentioned in Section 1 would, in actual practice, be inconvenient to use. Clearly the two scales should be linked together by some means, so that whilst they could be made to slide to and fro along one another, they would, when set, retain their positions and not fall apart. In order to mark any point in a scale when desired, a movable index would be a useful adjunct to the scales. We shall find that these points have not been overlooked in the slide rule as we find it to-day.

We do not propose to write a long description of the modern slide rule. We assume that the reader possesses a slide rule, or, at least, has access to one, and the mechanical construction of the instrument is so straightforward that we would not presume to enter into superfluous details.

There are a few points which we believe may be mentioned with advantage, and we think illustrations of a de-luxe instrument, and also an inexpensive type, should be included in this section. These are shown in Figs. 6 and 7 respectively.

Protection of Slide Rule

Whatever type of slide rule you decide to buy we ask you to take great care of it. The manufacture of slide rules is a technical and highly skilled craft, and much painstaking effort goes into their production. Your slide rule should be protected from exposure to heat and damp. You should particularly avoid leaving it lying exposed to the direct rays of the sun in warm weather. The majority of slide rules are constructed in part of celluloid; this material discolours and shrinks if unduly

exposed. When not in use please replace the rule in the protective case supplied with it, and put away in a cool, dry place, preferably in the drawer of your desk.

Component Parts

Since we frequently refer to them, we think we should mention the names of the component parts of a slide rule. The body of the rule is usually termed the "stock". The smaller part which can be moved to right or left, is called the "slide", and the movable index is known as the "cursor".

If you will examine the stock you will find it is built up of several parts which give it a degree of flexibility. If the stock was just a solid strip of wood with the necessary grooves machined in it to accommodate the slide and the cursor, it would invariably in the course of time warp sufficiently to grip the slide tightly and make the manipulation of the rule difficult or impossible. We have seen such rules with the slides so tight that it has been necessary to use a hammer or something similar to drive the slides out.

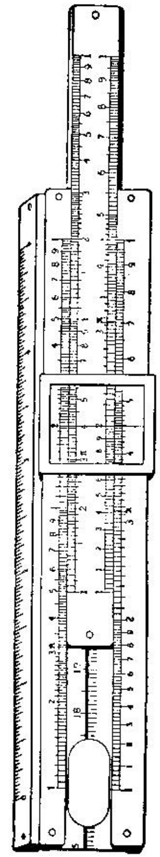

FIG. 6

Sizes of Slide Rules

The 10″ rule is the popular size. In this the scales are 10″, or sometimes 25 cm., in length, and the overall length of the rule 11″ or 12″. More convenient to carry in the pocket is the 5″ rule. There are also available rules of lengths 15″ or 20″ or more.

Cylindrical and circular instruments are made which employ the logarithmic principles and these are commonly called slide rules, although the term is certainly not appropriate. We shall make a brief mention of these instruments at a later stage.

C and D Scales

You will notice that there are several scales on the rule. The layout of scales is varied to adapt the rule to different requirements. If your rule is one of the general-purpose type it will be equipped, among others, with two scales usually denoted by the letters C and D. Scale C lies along the bottom edge of the slide, and scale D is on the stock adjacent to scale C. These two scales are identical in their graduations and are, in reality, one single scale which has been cut through lengthwise. The main graduations of scales C and D are numbered 1, 2, 3, etc., up to 10. Subdivisions should be numbered as fully as possible without carrying the process to the extent of causing confusion. The scales of some slide rules are numbered in a very confusing manner. We contend that when subdivisions are numbered the figure marked on them should be exact and not abbreviated. In some of the higher-priced rules the principal subdivisions between main divisions 1 and 2 of scales C and D are marked 1, 2, 3, etc., up to 9. These figures should be 1·1, 1·2, 1·3, etc., up to 1·9. We recommend the reader to avoid purchasing a rule in which the scale numbering is abbreviated, as it will inevitably involve him in errors due to misreading the scales.

Scales C and D are those most frequently used of all; we have mentioned them first and shall return to them in Section 4.

A and B Scales

Scales A and B lie adjacent to one another, A on the stock and B along the upper edge of the slide.

The numbering of the main divisions of scales A and B, starting from the left-hand end, should be 1, 2, 3, etc., up to

10, then 20, 30, etc., up to 100. The figure 10 marks the line mid-way along the length of the scale. The principal subdivisions should also be numbered as far as conveniently possible. Abbreviated figures should be avoided for the reason mentioned earlier.

At this stage we would ask you in all seriousness not to acquire the very bad habit of using Scales A and B for multiplication and division. The objection to this practice lies in the fact that when the A and B scales of a 10″ rule are so used, the instrument, in effect, becomes a 5″ rule, and results cannot be obtained with the same degree of accuracy as when the C and D scales are used. It is true that when scales A and B are employed, the results need never be "off the scale", but accuracy should not be sacrificed for a doubtful gain in convenience.

Until comparatively recently scales A, B, C and D were often all that appeared on the face of the rule. As a result of the change in manufacturing technique referred to earlier, it became possible to include other scales without increasing production costs to any great extent. "Unique" slide rules, almost since their inception, have carried log-log scales in many models, and these scales are now taken for granted. Their inclusion certainly adds value to a slide rule. They are not difficult to understand as will be shown presently.

In the absence of log-log scales the combination of the A, B, C and D scales is probably the best that could be devised, but if a slide rule is equipped with log-log scales we think the provision of the A and B scales is unnecessary, and that other scales can be substituted for them which increase the usefulness of the rule. Section 10 deals with rules designed on these lines.

Scales A and B in conjunction with scales C and D give a quick means of extracting square and cube roots, and of squaring and cubing numbers. Since scale D is twice the length of each of the indentical halves of scale A, it follows that in moving along scale A, you will be passing the logarithmic "milestones", twice as fast as when moving along scale D.

Now, if you double the logarithm of a number, you will arrive at the logarithm of the square of that number. Please examine your rule and with the aid of a cursor project readings in scale D to scale A—or from scale C to scale B on the slide. You will see that opposite 2 in D appears 4 in A, and for every number in D the square of that number appears in A.

Square roots are very quickly obtained by reversing the process and projecting from scale A into scale D. The problems of cubing numbers and extracting cube roots are in like manner facilitated by using the four scales A, B, C and D, and we shall return to this problem at a later stage.

If log-log scales are included in your slide rule, *all* powers and roots of numbers may be evaluated easily, and it is for this reason we say that the A and B scales are of doubtful value, since their uses in the processes of evolution and involution are very limited. Log-log scales give the means in conjunction with scale D of evaluating any power or any root of any number, whereas scales A and B will only deal with powers and roots of 2 or 3, and multiples of 2 or 3.

Log-log Scales

We have mentioned log-log scales several times. When included in a slide rule these scales are often placed along the top and bottom edges of the stock. Please refer to Fig. 7 and

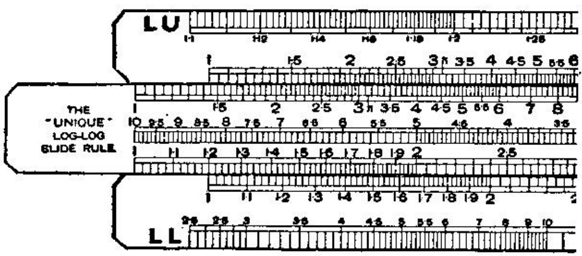

FIG. 7

you will see the log-log scales; they are marked LU and LL, at the left-hand end of the rule. Log-log scales are very useful in dealing with certain technical problems. We have heard the opinion expressed that log-log scales give the appearance of complexity to the face of the slide rule, and, since they are seldom used, they should not be included. We do not agree. The log-log scales are not obtrusive, and one quickly learns to ignore them when not required, and we have not found them inconvenient or confusing. They are sometimes to be found on the reverse of the slide as mentioned later on.

Section 6 deals with the problems which demand the provision of log-log scales and which cannot be solved by the slide rule without the aid of them.

The primary object of this book is to attempt to remove the impression that the slide rule is a difficult instrument to use. If, therefore, any reader feels that he prefers the very simplest slide rule with only the A, B, C and D scales included, we agree that he may be well advised to use this type, especially if his work is of a straightforward nature, and not likely to involve the use of log-log or trigonometrical scales.

Sine and Tangent Scales

If you will look at the undersurface of the slide of your rule you will probably see two or perhaps three scales.

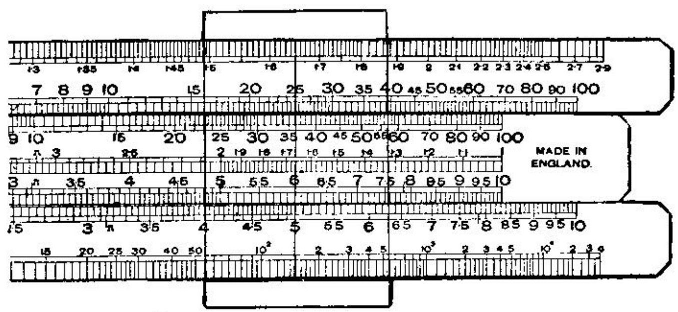

FIG. 7

The scale marked S is a scale of sines. Its graduation will probably commence at a value of 35 minutes—marked 35′—and finish at 90°. This scale is used in conjunction with scale A.

The tangent scale will be labelled T. Graduations may start just below 6° and proceed to 45°. Alternatively, the tan scale may start at 34′ and finish at 45°. In the former case the T scale is used in conjunction with the D scale; in the latter case it is used with the A scale.

In some rules the S and T scales appear on the face of the rule. Some people prefer this arrangement of scales, and the manufacture of the rule is simplified when it is adopted. The disadvantage lies in the fact that the face of the rule becomes somewhat congested with these additional scales.

If you will look carefully at the S and T scales you may see that unlike the A, B, C and D, and log-log scales, they are not subdivided consistently in tenths, fifths, etc. Below 20° on the S scale, and throughout the T scale, the unit divisions are subdivided into sixths, twelfths, etc. This system of subdivision is adopted because we do not always work in decimals of a degree, but we use the corresponding number of minutes, and as you will see subdivision into sixths, etc., is more convenient for this purpose, there being 60 minutes in one degree.

In recent years manufacturers have adopted the practice of subdividing in decimals to the S and T scales. The reader may encounter slide rules in which the minutes' graduations have given place to decimals of degrees.

Section 7 deals with examples of trigonometrical work employing scales S or T.

Log Scale

The third scale on the reverse side of the slide is an evenly divided scale, usually marked L, which enables us in conjunction with scale D to read off logarithms of numbers. If the scales on your rule are 10″ long, you will see that the log scale is subdivided into tenths and fiftieths, and it is, in effect, a 10″ measuring rule. As a matter of fact, you can obtain logarithms of numbers with the aid of an ordinary rule used in conjunction with scale C or D.

The scales mentioned in this section are, with the exception of the log-log scales, those you will find in the ordinary or standard type of slide rule; the type which seems to be preferred by the large majority of users. In later sections we shall deal with slide rules provided with different arrangements of scales.

The Cursor

In closing this section we would add a note of warning concerning the cursor. We strongly recommend the reader to purchase a slide rule which is fitted with a "free-view" cursor. The best type of cursor is that which has supports on only its top and bottom edges for engaging with the grooves in the stock. Some types have a light rectangle frame into which the glass or celluloid window is fitted. The edges of the frame lie across the face of the rule and obliterate to some extent the figures and graduations of the scales, and create an element of uncertainty and add to the possibility of making errors. One form of cursor, now only occasionally seen, has fitted on one side of it a small index and scale, designed to assist in fixing the position of the decimal point in the numerical result. This form of cursor hides a considerable part of the scales, and generally is a source of annoyance.

On some cursors you may find two or three hair lines. The additional lines give assistance in calculations concerning areas of circles, etc. Confusion may arise when multiple-line cursors are used, and we prefer the simple free-view cursor with a single hair line.

Linear Scales

We would say a word concerning the linear scales which are often fitted to slide rules. These have no connection with the rule as a calculating device. They add to the appearance of a rule, but we think they are entirely superfluous. A slide rule should be always handled carefully, and it is one of the minor annoyances in life to see it used for ruling lines or to take measurements when a wooden office ruler or a steel rule should be used.

SECTION FOUR

C AND D SCALES

IN this section appears the first examples involving the aid of a slide rule. In the condensed instructions, we shall adopt abbreviations, namely: C for scale C; D for scale D; 12C means line 12 in scale C; X refers to the index line of the cursor.

Examples are worked to assist the student. **Problems** are inserted for the student to solve. Answers to problems are given at the end of the book.

This section is devoted to those operations most often effected by slide rules; those which every student must first learn, multiplication and division.

We have in Section 3 advised the reader to refrain from using scales A and B for multiplication and division, and we shall confine our attention to scales C and D. Throughout this section no mention will be made of the other scales, which, for the time being, you may ignore.

Scales C and D are subdivided in decimals, and we must now assume that you are able to read them without difficulty. Fig. 8 illustrates the C and D scales as you should find them in practically all 10″ rules. In order to illustrate them full size, we show in the upper part the left-hand half, and in the lower part the right-hand half of the scales. To show the scales complete in one length would necessitate them being reduced in size in order to print them on a page of this book; this would make some of the divisions inconveniently small and difficult to read.

Problem 9. You are asked to read the positions of the lines marked in Fig. 8 and compare your reading with those we give in the answers to problems. If you feel confident that

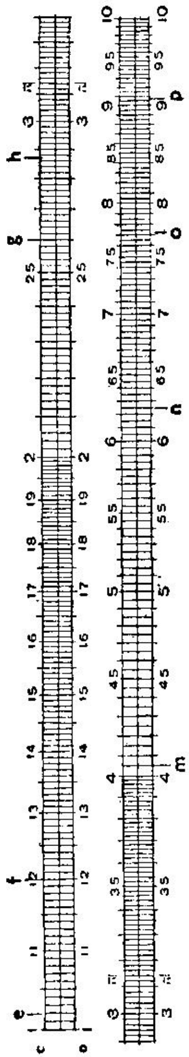

Fig. 8

you are able to read the scales we can safely proceed. If you have any difficulty we ask you to turn back to Section 1 and study that part dealing with reading scales, or better, to enlist the aid of someone conversant with scales. A few minutes of oral explanation will assist more than pages of written notes which would be too tedious to be endured.

Multiplication

Let us examine these four simple examples:

$$12 \times 32 = 384$$
$$1{\cdot}2 \times 3{\cdot}2 = 3{\cdot}84$$
$$\cdot012 \times \cdot032 = \cdot000384$$
$$120 \times 320 = 38400$$

In every case, if we ignore the position of the decimal point and the noughts which precede or follow the significant figures, we are concerned only with the multiplication of 12 × 32. You will notice that the answer in each case has only the three significant figures, 384. If we use a slide rule to carry out these four multiplications the operations would be identical.

The line marked *f* in Fig. 8 is drawn to coincide with the 1·2 graduation. In some rules this line may be marked 2, and if such is the case, you will be using a rule with the abbreviated marking to which we have referred earlier. Try to secure a rule in which the C and D scales are numbered as in Fig. 8.

1·2 graduation may, for our purpose, be read as 1·2, or 12, or ·012, or 120, or any combination of figures in which 12 stand together followed or preceded by any number of noughts. If, however, we read this line as 102 we shall be making a fundamental error, and our result would be incorrect. You will, no doubt, have found the line 1·02 in connection with Problem 9. It is the line marked *e*.

We will now see how the result 384 is derived from the two factors 12 and 32. You will understand that we are dealing with simple numbers for the purpose of instruction. Obviously there is no other point in using a slide rule to compute results which could easily be obtained without its aid.

If at any time you are in doubt as to whether you are using your slide rule correctly, always work out an elementary example with simple figures, so that you can check the slide rule result. We tender this advice more particularly when complicated examples may arise, or when the reader is using scales which he seldom needs.

Example: Multiply 12 by 32.

Find 12 in scale D, move the slide to the right to bring the 1 of scale C coincident with the 12 of scale D. Directly under the 32 in scale C you will find the result, 384, in scale D.

We have described the two operations fully, but we repeat them below in the condensed form we shall hereafter use. If you will get familiar with the condensed form, you will find it much less tedious to follow than a wordy description, and typography is reduced. The description started by saying, find the 12 in scale D. You may just note the 12 in scale D by eye, or, if you prefer it, place the cursor index over it. If the cursor is used, an additional mechanical operation is involved, but we think it is the easier method to adopt; this is a matter of opinion, and you may please yourself. We shall make no reference to using the cursor for picking up the first or final readings.

In the condensed form the operations would be:

Set 1C to 12D. Under 32C read result, 384, in D.

If we had been multiplying 32 by 10 the answer would have been 320. Our answer must be rather greater than 320 since we are really multiplying by 12, and we, therefore, write it as 384. The answer could not be 38·4, nor could it be 3840. We hope you will understand that having obtained the figures 384 from our slide rule, we must determine the "order" of the result, or, in other words, find the position of the decimal point.

Example: Find the area in square inches of a rectangular sheet of paper measuring 4·8″ × 6·4″.

Set 10C to 48D. Under 64C read 307 in D.

Position of the decimal point is determined by inspection of the two factors. We obtain an approximate answer by taking the factors as 5 and 6; in doing this you will notice that we have increased 4·8 to 5 and reduced 6·4 to 6. The product of 5 times 6 = 30 must be fairly near the true result, and we may, therefore, insert the decimal point, making our answer 30·7 sq. in.

There are other methods of determining the position of the decimal point, but we think at this stage you would find them very difficult to understand. We mention them at the end of this section, but we advise you always to adopt the approximation method of finding the position of the decimal point.

In the example 4·8 × 6·4 you will notice that if we set the 1 of C to 48 of D, then the 64 of C is "off the scale of D", and we must move the slide to bring the 10 of C to coincide with the 48 of D to obtain a reading. The necessity of re-setting the slide does sometimes occur when we are using scales C and D, but when we have a little experience of using a slide rule, we seem to acquire an instinct which warns us when we are using the wrong end of scale C. When setting the 1 or 10 of C,

you should move the slide roughly into position and then take a quick glance at the factor in C which you wish to use. If this factor lies over some part of scale D you can proceed accurately to adjust the slide and obtain your result. Occasionally the factor in C is only slightly outside the scale of D. If you will examine Fig. 9 in Section 7, you will find there are a few graduation lines on the left-hand side of the 1 of scale C and D, and also a few graduations on the right-hand of the 10 of these scales. These extensions of the scales are sometimes useful for picking up a result which otherwise would be just off the scale. You will no doubt notice that the graduations to the right of the 10 are identical with those immediately to the right of the 1, and those which lie to the left of the 1 are the same as those which precede the 10. These extensions are short additions of the C and D scales.

Example: Calculate the weight of a cast-iron plate $40\frac{1}{2}$″ long × 28·7″ wide × $\frac{5}{8}$″ thick. (1 cu. in. of C.I. weighs ·26 lb.)

We have four factors to evaluate. 40·5 × 28·7 × ·625 × ·26.

Set 10C to 405D. X to 287C. 1C to X. X to 625C. 10C to X.

Result 189D under 26C.

Approximation: ·26 is slightly more than a $\frac{1}{4}$, and $\frac{1}{4}$ of 40 is 10. $\frac{5}{8}$ of 28 is somewhere near 18. 10 times 18 = 180. The answer must be 189·0 lb. weight. (The positions of the four necessary readings are marked in Fig. 8 to assist those who may still have difficulty in reading the scale.)

You are now requested to repeat the foregoing example by taking the factors in different orders. Start say with the $\frac{5}{8}$ and multiply by $40\frac{1}{2}$, then by ·26, and finally by 28·7. The result should be the same irrespective of the order in which the factors are selected. With four factors there are possible 24 different sequences in which the operation of multiplication may be effected, and we think it is an excellent exercise for the reader who is just becoming familiar with the slide rule to work through a few of these sequences. 1891 should result from every attempt, and it is a matter of interest to see how little variation there is in the results obtained by taking the factors in different orders. With a little care the reader will find the correct result emerging time after time.

The student may like to know how the 24 sequences referred to are derived. Let us for ease of expression denote the four factors by the letters a, b, c and d. Here are six different sequences, $a \times b \times c \times d$, $a \times b \times d \times c$, $a \times c \times b \times d$, $a \times c \times d \times b$, $a \times d \times b \times c$, $a \times d \times c \times b$. Each of these six starts with a. Now there are six others each starting with b, and similarly six commencing with c and with d. You will, no doubt, be able to complete the whole of the series without difficulty. We are not suggesting the example need be solved in all 24 ways, but to carry out a few will serve as good practice.

Problem 10. A rectangular water tank has dimensions 2′ 3″ × 18″ × 4′ 6″. Calculate the weight of water this tank will contain when it is three-quarters full. (1 cu. ft. of water weighs 62·3 lb.)

Example: Calculate the area of a circle 2·8″ radius. (If r is the radius of a circle, and d its diameter, then area $= \pi r^2 = \frac{\pi d^2}{4}$. π, pronounced pi, is the Greek letter which is always used to denote the value 3·14. It is the ratio of circumference to diameter of a circle, and it enters into all our calculations concerning circles, spheres and other associated forms. π is generally denoted by a special "gauge mark" in scales C and D, see Fig. 8.)

Area $= \pi(2{\cdot}8)^2 = \pi \times 2{\cdot}8 \times 2{\cdot}8$.

Set 1C to 28D. X to 28C. 10C to X. Result 246 in D under π in C.

Approximation: $3 \times 3 \times 3 = 27$. Result is 24·6 sq. in.

Problem 11. Calculate the volume of a cylinder 8·2″ radius and 12·6″ long. (Volume = $(\pi r^2)\,L$.)

Division

If you have now understood the rules for multiplication of two or more factors, you should have no difficulty in using your slide rule for dividing. We will, however, consider a few examples in order to make sure.

If your rule is set to multiply together two numbers, then it is also set for division. Will you adjust the slide so that the 1 in scale C is coincident with 2·5 of scale D. Immediately under the 3 of C you will find 7·5 in D. This setting of the slide enables us to multiply 2·5 by 3. Now, working backwards from this result, we see that in order to divide 7·5 by 3, we need only adjust the slide to bring the 3 of scale C opposite the 7·5 of scale D; exactly under the 1 of C we find the 2·5 of D and we have effected the division of 7·5 by 3 and obtained the result 2·5.

We repeat our advice, if in doubt concerning method, work out an easy case with simple figures so that a mental check can be obtained easily, such as the following:

Example: How long will it take a man walking at the rate of 4 miles an hour to cover 14 miles? To 14D set 4C; under 10C read 35D; the answer is, therefore, 3·5 hours.

Please note that henceforward we shall often write the significant figures to be selected in the scales without inserting the decimal points. In the example above the numbers 14 and 35 in scale D are those marked 1·4 and 3·5. We hope by now the reader has appreciated that we take no notice of the positions of the decimal points in the various numbers while we are manipulating the slide rule. When we have obtained a numerical result we insert the decimal point by inspection if the numbers are simple; if the numbers are too complicated to allow of a mental approximation to be made, we shall write them down, then simplify and cancel them sufficiently to enable us to obtain an approximate result.

Example: Find the radius of a circle which has an area of 161 sq. ft.

Now $\pi r^2 = 161$. $r^2 = \dfrac{161}{\pi}$.

Set π in C to 161 in D. Under 10C read 512. Insert decimal point by inspection giving $51{\cdot}2 = r^2$. To obtain the radius we must find the square root of 51·2. There are several ways of finding square roots by slide rule, but remember we are restricted in this section to the use of scales C and D only. We can easily obtain our result: Place X over 51·2 in D and move the slide to bring 7 in C under X. ($7 \times 7 = 49$, and it is clear that our square root is a little greater than 7.) Now move the slide slowly to the left until the reading in C under X is the same as the reading in D under the 10 in C. We make the answer 7·15.

You will see that our endeavour has been to set the slide so that we are multiplying a number by itself and obtaining 51·2 as the result. Please study very carefully this method of obtaining square root.

Problem 12. A sample of coal weighing 13·4 grammes, on analysis was found to contain 9·6 grammes of carbon. Calculate the percentage of carbon contained in the sample.

Example: Find the value of $\dfrac{182}{6{\cdot}2 \times 808 \times {\cdot}029}$.

To 182D set 62C; X to 10C; 808C to X; X to 10C; 29C to X. Answer 1252 in D under 1C.

To find position of decimal point we may write the expression in a simpler form $\dfrac{182}{6{\cdot}2 \times 8{\cdot}08 \times 2{\cdot}9}$; you will notice that we have moved the decimal point in the 808 two places to the left. This is equivalent to dividing 808 by 100 and reducing it to 8·08. At the same time we have multiplied the ·029 by 100 by moving the decimal point two places to the right. Now, in an expression such as we are dealing with, we do not alter its numerical value if we multiply and divide the denominator (or numerator) by 100, or any other number, but by this device we alter the terms of the expression in such a manner that we can more easily see the order of the result. If you now look at the denominator you will see that we have approximately $6 \times 8 = 48$, and $48 \times 2{\cdot}9$ is somewhat less than 150. 150 divided into 182 is clearly greater than 1 and less than 2. Our result is, therefore, 1·252.

Multiplication and Division Combined

Frequently calculations involve a combination of multiplication and division. We shall find that our slide rule is particularly well designed to cope with them.

First consider the simple case $\dfrac{8 \times 3}{4}$. We can easily obtain the answer mentally as 6. Using the slide rule we might multiply 8 by 3 and then divide by 4. Alternatively, we might divide the 8 by 4 and then multiply by 3. The answer would be 6 by either method, but we shall find that there are less slide rule operations if we adopt the second.

By first method: Set 10C to 8D, X to 3C, 4C to X, under 10C read answer 6 in D.

Second method: Set 4C to 8D under 3C read answer 6 in D.

The first method involves us in four slide rule operations, whereas the second method demands only two. *You are parti-*

cularly advised to cultivate the habit of using the second method in all calculations which involve combined multiplications and division. Compared with the first method you will in general reduce the number of operations by about one-half, and you will often be nearer the exact result.

A little thought will show how the saving is effected. Please set your rule so that 4C is over 8D. This is the setting for dividing 8 by 4 and the answer, 2, is in D immediately under 1 in C. Now, to multiply 2 by 3 we must set the 1 of C to the 2 of D and read the answer, 6, in D under the 3 in C. We find, however, that having set the slide for dividing 8 by 4, we have also, with the same setting, prepared for the multiplication of 2 by 3. We do not even take the trouble to read the intermediate answer, 2, but go direct to the final one, 6.

Example: A farmer is asked to make for a Government Department a return showing, as percentages, the acreages he has under wheat, oats, barley, root crops, grass and fallow. After check up he finds that he has sown wheat 37½ acres, oats 29 acres, barley 17½ acres, root crops 42 acres, has grass 19½ acres and lying fallow 7½ acres. Total acreage is 153.

We could divide each of the separate acreages by 153 and so obtain the percentages, but we find it much easier first to divide 1 by 153 and then effect the multiplications with one setting of the rule.

Set 153C to 1D.

Under	37½C	read in	D	24·5%	wheat
,,	29 C	,, ,,	D	19·0%	oats
,,	17½C	,, ,,	D	11·5%	barley
,,	42 C	,, ,,	D	27·5%	roots
,,	19½C	,, ,,	D	12·8%	grass
,,	7½C	,, ,,	D	4·9%	fallow
	Total			100·2%	

If we have made our calculations correctly, the total of the crop percentages will be 100%. As you see, our individual percentages give 100·2% as the total. The slight discrepancy is due to the small errors we make when using a slide rule, but the total is so close to 100% that we may assume we have made no error of importance and we need not check through the calculations. If you try to read the percentages to the second place of decimals you will probably get results which are even nearer to the 100%, but it is futile to express your slide rule result to a degree of accuracy greater than that of the original data. It is quite certain that the farmer's estimation of acreage under the different crops will contain errors much greater than 2 in 1000.

We think the foregoing example gives excellent practice, and we give a similar one for the reader to work through.

Problem 13. The manufacturing costs of a certain article were estimated as follows: Direct Labour £68. Drawing Office £6. Materials £91. Works Overheads £4 10s. 0d. General Office Overheads £3 10s. 0d. Express these items as percentages of the total cost.

We will finish this dissertation with a typical example of combined multiplication and division, since this type of problem arises very frequently in the course of practical work.

Example: $$\frac{8{\cdot}2 \times 14{\cdot}7 \times 29{\cdot}1 \times 77{\cdot}6 \times 50{\cdot}2}{18{\cdot}6 \times 32{\cdot}7 \times {\cdot}606 \times 480}.$$

Set X to 82D, 186C to X, X to 147C, 327C to X, X to 291C, 606C to X, X to 776C, 480C to X, X to 502C.

Read the result 772 in D under X.

Approximation gives 70; therefore, result is 77·2.

Position of a Decimal Point

We have stated earlier in this section that we would give rules for the determination of decimal points in numerical results. In many cases the positions of decimal points are known from the nature of the problem; in many others the decimal point may be inserted by making mentally rough approximations. In cases in which the figures are numerous and diverse so that it is unsafe to attempt to approximate mentally, the data should be written down in round numbers and then reduced to simple forms by cancellation and other means, so that approximation can be made.

Our advice to the reader always to fix the position of decimal points by inspection or approximation is, we believe, quite sound; in the course of a long acquaintance with slide rules and users of slide rules, we have met only one individual who consistently adopted any other method.

Digits

When we speak of the number of digits in a factor we refer to the number of figures lying before the decimal point when the factor is 1 or more. When the factor is less than 1, the number of digits is the number of noughts immediately following the decimal point, and this number of digits is negative. In the following factors given as examples, the numbers of digits are given in brackets: 6 (1); 81 (2); 508 (3); ·45 (0); ·026 (−1); ·0048 (−2); ·0007 (−3).

Rule for Multiplication

Please set your rule for multiplying 3 by 4. The result is 12, and the slide is protruding at the left-hand end of the stock. In this example the number of digits in the product is 2, which is equal to the sum of the digits of the two factors which contain one each.

The following examples in which 3 and 4 are the significant figures show how the index rule works. In each case you will see the sum of the digits (which are shown in brackets) of the two factors, is equal to the digits in the product.

·3 (0) × 4 (1) = 1·2 (1). 400 (3) × 3000 (4) = 1,200,000 (7).
·03 (−1) × ·004 (−2) = ·00012 (−3).

Now if you will set the rule for multiplication of 2 by 4, the slide will protrude at the right-hand end of the stock. In this case the sum of the digits of the two factors is 2, whereas there is only one digit in the product.

The rule for a product which emerges from these simple examples, and which is true for all is:

> If the rule is set with the slide protruding at the left-hand end of the stock, the number of digits in the answer is the sum of the digits of the factors. If the slide is protruding at the right-hand end, the number of digits in the product is one less than the sum of the digits of the factors.

Example: Multiply 61·3 × ·008 × ·24 × 9·19 × 18·6.

> There are four settings of slide necessary, and we shall find that in three the slide protrudes to the left, and in one to the right. We must, therefore, find the sum of the digits of the five factors and subtract 1, shown in the square bracket, i.e. 2 − 2 + 0 + 1 + 2 − [1] = 2. The final reading in scale D is 201, the result is 20·1.

Problem 14. Multiply ·068 × 1200 × 1·68 × ·00046 × 28·3.

Rule for Division

If you have understood the rule for multiplication you will have no difficulty with the corresponding rule for division, which may now be stated:

> If, when dividing, the slide protrudes at the left-hand end of the stock, the number of digits in the result is found by subtracting the number of digits in the divisor from the number in the dividend. If the slide protrudes to the right, the number of digits in the result will be one greater than the difference between the numbers of digits in the dividend and divisor respectively.

Example: $\dfrac{6{\cdot}1}{128 \times {\cdot}039 \times 18}$.

Set 128C to 61D	Slide to right	Digit adjustment	+ 1
X to 1C			
39C to X	,, ,,	,, ,,	+ 1
X to 1C			
18C to X	Slide to left	,, ,,	0

Result 679 in D under 10C.

Since the slide protruded twice at the right-hand end we must add 2, to the number of digits derived from the factors.

$$\text{Digits in answer} = 1 - 3 - (-1) - 2 + [2]$$
$$= 1 - 3 + 1 - 2 + 2 = -1.$$

Result is ·0679.

Problem 15. Evaluate $\dfrac{864}{917 \times {\cdot}0028 \times 46{\cdot}1 \times 8{\cdot}9}$.

Example: Let us now examine an example such as the following, which consists of simple numbers:

$$\frac{2 \times 3 \times 4}{1{\cdot}5 \times 8}.$$

We can see at a glance that the answer is 2, since the $1{\cdot}5 \times 8$ in the denominator cancels with the 3×4 in the numerator, leaving only the 2 as the result.

If you will use your slide rule to find this result, you will see that commencing with the 2 you can divide by 15 and multiply by 3 with one setting of the slide, and then divide by 8 and multiply by 4 with another setting of the slide. When it is possible to carry out two operations at one slide setting you may disregard the position of the slide, i.e. whether to right hand or left hand of the stock, since if digits

have to be added or subtracted they will be equal and of opposite signs, and will consequently cancel out. It is only when the slide protrudes to the right and either multiplication or division is effected separately that the number of digits in the result is affected.

We will write down the operations involved in this simple exercise:

Set 15C to 2D	Slide to right	Digit adjustment	+ 1
Set X to 3C	,, ,,	,, ,,	− 1
Set 8C to X	Slide to left	,, ,,	0
Result 2 in D under 4C	Slide to left	,, ,,	0

The first and second operations are performed at one setting of the slide. They represent division by 15 followed by multiplication by 3. The slide protrudes to the right hand of the stock, and if we consider these two operations as quite distinct from one another, the digit adjustment will be + 1 for the division and − 1 for the multiplication. These cancel one another. In the third and fourth operation since the slide protrudes to the left-hand end, the digit adjustment is 0 in either case. You will see, therefore, that in cases of combined multiplication and division, you can reduce the check, on the digits to be added or deducted, if you select the factors so that two operations may be performed with the single setting of the slide as often as possible.

Example: Find the value of the following expression:

$$\frac{8{\cdot}1 \times 143 \times {\cdot}0366 \times 92{\cdot}8 \times 238}{62 \times 188 \times {\cdot}450 \times 85{\cdot}5}.$$

We will first multiply the five factors in the numerator and follow with the divisions by the four factors in the denominator.

Set 10C to 81D	Slide to left	Digit adjustment	0
X to 143C			

1C to X	Slide to right	,,	,,	− 1
X to 366C				
10C to X	,, left	,,	,,	0
X to 928C				
1C to X	,, right	,,	,,	− 1
X to 238				
62C to X	,, right	,,	,,	+ 1
X to 1C				
188C to X	,, left	,,	,,	0
X to 10C				
45C to X	,, right	,,	,,	+ 1
X to 1C				
85·5C to X	,, left	,,	,,	0
Result 209 in D under 10C.		Total		0

Digits in numerator $= 1 + 3 - 1 + 2 + 3 = 8$

,, denominator $= 2 + 3 + 0 + 2 = 7$

Diff. . . 1

Collecting the digits gives a total of 1 from the two sources, therefore, the answer is 2·09.

Let us re-work this example by dividing and multiplying alternately to see if the digits rule gives the same result.

To 81D set 62C Slide to right X to 143C ,, ,,	}	Digit adjustment 0
188C to X ,, left X to 366C ,, ,,	}	,, ,, 0
45C to X ,, left X to 928C ,, ,,	}	,, ,, 0
85·5C to X ,, ,, Result 209D under 238C ,,	}	,, ,, 0

Take note of the great saving in manipulation of the rule, as compared with doing all the multiplication first and the division afterwards. The digit adjustment is 0 and the result is still the same, 2·09. The operations bracketed together in pairs are those which are effected at one setting of the slide. The first operation of each pair is always a division effected by setting the slide, and the second a multiplication, made by moving the cursor.

As an exercise we suggest you work through the problem for a third time by dividing and multiplying alternately, but taking the factors in a different order from that we have adopted above. The result is quite independent of the order selected, and the digit rule will give the same position for the decimal point.

We leave the decision to you whether you will use these rules for fixing the position of the decimal point or to adopt the approximation method. Apart from slide rule considerations, you will find that to develop the faculty for making quick approximate estimations is useful in many other ways. In a long computation there is a risk that we may overlook a factor and omit it from our slide rule calculation. The chance of doing this is perhaps not great, but if we have several factors in both numerator and denominator we try to select them in pairs, one in the denominator and one in the numerator so that we can use them together in one setting of the slide, and further, we try to select a pair of factors which are near to one another in values, so that the movement of the cursor is small. Now, in making selection of factors to best suit the manipulation of the slide rule lies the risk of omitting a factor. If we subsequently make an approximation to fix the position of decimal point, the omission of a factor may be disclosed.

Additional Examples

We mentioned earlier that we regard the C and D scales as most important in the early stages of our acquaintance with the slide rule. We therefore now give some additional examples, graded in difficulty, illustrating the use of these scales. The solutions are given in each case, but we recommend the reader to work these examples independently and to refer to the solutions only when in doubt. He will understand that there are alternative ways of selecting the various factors involved and he should repeat some of the examples by using different sequences of operations.

Example: A train journey of 437 miles occupies $8\frac{1}{4}$ hours. What is the average speed?

To 437D set 825C

Under 10C read 53 (53 m.p.h.).

Example: A student obtained $47\frac{1}{2}$ marks out of a possible 78. What is the percentage marks obtained?

To 475D set 78C

Read 609D under 10C (60·9%).

Example: A group of students obtained the following numbers of marks, in all cases out of a possible 78. Calculate the percentages. $47\frac{1}{2}$, 63, $51\frac{1}{2}$, 72, 65, 23, $37\frac{1}{2}$.

Set 78C to 10D

Read 609D under 475C (60·9%).
„ 808D „ 63C (80·8%).
„ 66D „ 515C (66%).
„ 923D „ 72C (92·3%).
„ 833D „ 65C (83·3%).
„ 295D „ 23C (29·5%).
„ 482D „ 375C (48·2%).

The reader will note that if only one percentage is required, it is best to divide the marks obtained by the marks possible. If a series of results is to be dealt with, it is much quicker to proceed as indicated in this example.

Example: Calculate the weight of water which can be carried in tank $10\frac{1}{2}$ feet long by $3\frac{1}{2}$ feet diameter (1 cu. ft. of water weighs 62·3 lb.).

This computation is

$$\frac{\pi}{4}(3\tfrac{1}{2})^2 \times 10{\cdot}5 \times 62{\cdot}3 \text{ lb.}$$

To 35D set 4C
X to 35C
1C to X
X to πC
10C to X
X to 105C
1C to X

Read 63D under 623C (6300 lb.). Approximation: $3\frac{1}{2}$ squared is about 12, dividing by 4 gives 3 and $3 \times \pi$ is nearly 10. We have then $10 \times 10 = 100$ times 62 is 6200. In this example you may shorten the work by squaring $3\frac{1}{2}$ mentally. $(\frac{7}{2})^2 = \frac{49}{4}$ and combining the 4 under the π we start with $\pi \times \frac{49}{16} \times 10{\cdot}5 \times 62{\cdot}3$. This idea of reducing the factors is valuable provided the mental operations are simple. It is a habit all slide rule users soon acquire.

Example: A job takes $8\frac{1}{2}$ days to complete by 19 men working $12\frac{1}{2}$ hours per day. How long would the same job take if the number of men is increased to 23 and the working day reduced to 8 hours?

Result is obtained from $8\frac{1}{2} \times \frac{19}{23} \times \frac{12\frac{1}{2}}{8}$.

To 85D set 23C
X to 19C
8C to X

Read 10·9D under 125C (11 days).

Example: Compute $\dfrac{4{\cdot}2 \times 71 \times 6{\cdot}76 \times {\cdot}382}{7{\cdot}24 \times 2{\cdot}5 \times {\cdot}855}$.

This example is typical of a large range of problems which give rise to a string of figures which has to be reduced to a numerical answer. The reader will notice no useful cancellation can be made nor is it possible to combine any of the factors mentally. The result can quickly be obtained by combined multiplication and division. A check on the result should be obtained by repeating the slide rule manipulation with alternative factor sequences. In the solution below the factors have been selected in such a manner that the movements of cursor have been reduced to a minimum. This is a desirable practice and should be cultivated by the student.

To 42D set 724C
X to 71C
25C to X
X to 382C
855C to X
Read 498D under 676C.

Result is 49·8, the decimal point being fixed by approximate cancellation.

SECTION FIVE

A AND B SCALES

WITHOUT doubt the most frequently used scales of the standard slide rule are the C and D scales we have just studied, and we might, with justification, say that these are the most important scales in our slide rule equipment. It is impossible to say which scales stand next in importance. It depends upon the nature of the work to be done; if trigonometrical problems loom prominently in our work, then the sin and tan scales will frequently be used. Work of a different nature may demand frequent recourse to the log-log scale, and again electrical or commercial calculations may bring into service scales particularly designed to deal with them.

The reader will notice that we do not suggest the A and B scales possess a high degree of priority in the scheme of things. We are of the opinion that these scales are of little importance and that others could, with advantage, be substituted for them.

Since, however, the large majority of slide rules are equipped with A and B scales, we must spend a little time in studying them.

The A and B scales are adjacent to one another, the B scale lying along the top edge of the slide and the A scale on the stock. The reader will see them in Figs. 6 and 7. Each of these scales consists, so far as its graduations are concerned, of two identical halves, and we speak of the right-hand half, or the left-hand half, when we desire to make a distinction.

Scales A and B should carry 1 at the extreme left-hand end, 10 at the middle point where the two halves abut, and finish with 100 at the right-hand end, with the corresponding intermediate figures. In many slide rules the left-hand and right-hand halves are numbered exactly alike, with the figure 1 at the beginning and end of each half. Whilst this arrangement is not a great disadvantage to those familiar with slide rules, and expert in the use of them, we think the scale should be completely numbered as shown in Fig. 7. In subsequent notes we shall refer to the numbers as they are depicted in Fig. 7.

Each half of scales A or B is similar to scales C and D inasmuch as it is logarithmic. It is only half the length, and has only about half the graduations, and herein lies the disadvantage of using A and B for multiplication or division.

The reader is by now quite well aware that a slide rule will not give results with absolute accuracy. If we multiply together two numbers using ordinary arithmetical procedure we should obtain a result accurate to the last figure, but with a 10″ slide rule we know that we can never be certain of the fourth figure and must often regard the third figure with suspicion. A 5″ rule is less precise, and if one uses the A and B scales of a 10″ rule for ordinary multiplication or division, he is in effect using a 5″ rule. We have heard sarcastic criticism of the slide rule arising from the fact that results are not always completely accurate, but the thoughtful reader will, of course, realise that in our practical problems, the data we use are derived generally from measurements or observations which are suceptible to considerable error, in comparison with which the errors made in computation by slide rule are permissible.

We would, however, warn the reader carefully to consider whether the slide rule is likely to introduce errors which might seriously impair the result of some work or investigation he is pursuing. In the course of a chemical analysis, we might, using a good balance, determine the weight of a sample as 13·562 grammes, and we should be fairly certain that the last figure, the 2, was correct, and not 1 or 3. If this weight had to be multiplied by some other number which could equally be relied upon, we should hesitate at using a 10″ slide rule, which might introduce an error many times as great as any error in the original figures. Some physical measurements can be made to a high degree of accuracy, and computations must, of course, be made with the same precision. When necessary we must discard the slide rule and use other means of reaching the result, but for most of our practical work the C and D scales of a 10″ slide rule give results to an acceptable degree of accuracy.

When great accuracy of results is not important, and we are working to approximate figures, there is no harm in using scales A and B for multiplication and division, but we do ask the reader to avoid making this a practice, or soon he will find himself by habit using A and B when he should be working with C and D.

With very little modification, the instructions we have given in respect of scales C and D for multiplication and division apply to A and B. For scale D read A, for C read B, and remember that due to the duplication of the scales all the numbers in C and D appear twice in A and B, e.g. the 2 in C and D appears as 2 and 20 in A and B.

We have seen that when using C and D it occasionally happens that after carefully setting the slide we find that the next factor is "off the scale", and the slide has to be moved its own length and then re-set to obtain the required reading. When we use scales A and B, we find that if a factor in scale B is off the A scale at one end of the rule, the result can still be found by looking for the factor in the other half of scale B. It *is* possible to set the slide so that no result can be found. To avoid this, refrain from moving the slide so that more than half its length protrudes from the stock, remembering there are two alternative settings. Cultivate the habit, when setting the slide, of keeping it near the centre of the stock. It is natural to do this, and if persisted in for a time becomes a habit. We do not propose to say anything further concerning multiplication and division with scales A and B.

Squares and Square Roots

In Section 3 we mentioned that squares and square roots of numbers are easily obtained by using scales A and D in conjunction, and this, we suggest, is the most useful feature arising from the inclusion of scales A and B in our slide rule.

Immediately above any number in scale D appears its square in scale A. Look at your slide rule and you will find 4 in A over 2 in D, 9 in A over 3 in D, 25 in A over 5 in D, and similarly throughout the length of the scales. Conversely, the square roots of numbers in A lie directly below in D. When projecting from A to D or *vice versa*, we may use the cursor index line, or, if preferred, the index lines of the slide. The index lines of the slide are the end lines (excluding extensions if any), the 1 and 100 of scale B, and 1 and 10 of C. It will be clear that these lines give a means of striking across from A to D, and sometimes these are preferred to the cursor index, since there is no possibility of slight error due to parallex.

The student will find no difficulty in squaring numbers:

Example: Find the square of 4·55.

Set X to 455D. Under X read 207 in A. Result 20·7.

If we use the slide to project across the rule, the procedure is:

Set 10C to 455D. Read result 20·7 in A over 100B.

We shall, in subsequent notes, refer to X, the cursor index for projecting from A to D, but we advise the reader to use the slide when it is convenient to do so. In many calculations involving squares and square roots, the slide cannot be used for projecting across since it is required for other operations. In such cases the X must be employed.

Problem 16. Find the squares of 8·75 and 167.

Evaluation of square roots is the reverse operation and is just as easy, but there is one point we must mention in passing.

If using scales A and B we desire to multiply 2 by some other factor, we may use the 2 in the left-hand half of A, or the 20 in the right-hand half, taking the figure most convenient, but if we require the square root of 2, we may not use the 20. The reader will see that under 2 of A the reading in D is 1·414, whereas under 20 in A appears 4·47 in D. We know of no more prolific source of slide rule error than this one of using the wrong half of scale A when extracting square roots.

There should be no difficulty in finding the square root of any number lying between 1 and 100. We know the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100, and we should make no mistake with any number within this range.

Assume we require the square root of 45·2. We place X over 452 in the left-hand half of A and note the corresponding value in D; it is 213. The square root of 49 is 7 and our answer should be just less than 7. 213 does not agree, and we see immediately that we have in error taken the square root of 4·52, which is 2·13. If we move the cursor to 452 in the right-hand half of scale A we find the corresponding reading in D is 6·72; this is the square root of 45·2.

If the scales of your slide rule are comprehensively numbered as in Fig. 7, the problem of extracting square roots is simplified, as there will be no difficulty with numbers lying between 1 and 100. When extracting square roots, it is advisable to multiply or divide the original number by even powers of 10 to bring it into the range of 1 to 100, and after taking the square root, to make the necessary adjustment in the result.

Examples: Find the square roots of (i) 1462; (ii) ·0000227 and (iii) ·000227.

(i) $\sqrt{1462} = \sqrt{14{\cdot}62 \times 100} = \sqrt{14{\cdot}62} \times 10 = 3{\cdot}82 \times 10 = 38{\cdot}2$.

(ii) Starting with ·0000227, we tick off pairs of figures as shown ·00′00′22′7; we thus obtain 22·7 as the figure whose square root we must find on the slide rule. This root is 4·76, but we must now move the decimal point three places to the left to correct the alteration made when earlier we ticked off three pairs of figures to the right.

Result is ·00476.

(iii) $\sqrt{\cdot 00'02'27}$
$\sqrt{2\cdot 27} = 1\cdot 505$ Result ·01505.

We could give the reader other rules for the determination of position of decimal point in the root, but we are confident that the method we have adopted above is the best. It is easily understood, but because of its importance we will enumerate the steps:

(1) Examine the number whose square root is required. If it lies between 1 and 100 its square root will lie between 1 and 10. Find the number in A and project with X to D where the root will be found. Insert the decimal point to the right of the first figure of the result.

(2) If the number does not lie between 1 and 100 move the decimal point in steps of two figures at a time until the number falls in this range; now take the square root of the number so altered and insert the decimal point as at (1) above.

(3) Finally, move the decimal point in the result obtained at (2), one place for each step of two figures made when altering the number, moving in the opposite direction.

Problem 17. Find the square roots of 814, 8140, ·0166 and ·0000166.

Example: Calculate the volume of a cylinder 11·2″ diameter, 19·6″ long.

In terms of diameter *d* and length *l* the volume is

$$\frac{\pi}{4} d^2 l.$$

Set 1C to 112D, X to 196B, 10B to X.

Result 193 in A above 785B. $\left(\cdot 785 = \frac{\pi}{4}\right)$.

Approximation 11 × 11 = 132. $\frac{3}{4}$ of 132 is near 100. 100 × 19·6 = 1960.

Result 1930 cu. inches.

To reduce the result to cu. feet:

To 1930A set 1728B. Read 1115 in A above 1 (or 10) in B.

Result 1·115 cu. feet.

The symbols *c* and *c′* which appear in some slide rules near the left-hand end and near the middle of the C scale are provided to assist in calculations involving volumes of cylinders. The special lines are termed gauge points; they are referred to in Section 15.

Problem 18. Calculate the diameter of a pipe which will discharge 3 cu. ft. of water per second at a rate of flow of 8 ft. per sec.

There are methods of finding square roots without using scales A and B. We used one in the example preceding Problem 4. Other methods will be mentioned in the sections dealing with log-log and reciprocal scales.

Cubes and Cube Roots

To find the cube of any number set the 1 or 10 of scale C to the number in D. Above the number in B read the cube in A.

Example: Find the cube of 2·44.

Set 1C to 244D. Over 244 in B read in A the cube which is 145. Insert the decimal point by inspection, making the answer 14·5.

There are rules which may be used for the determination of the position of the decimal point in the result, but they are confusing, and we cannot recommend the reader to use them. It is simpler to obtain an approximate result.

Problem 19. Cube 16·8.

Cube roots may be extracted by several different methods using scales A, B, C and D. The method we now describe is, we think, the best.

When extracting square roots we converted the number whose root was required to one lying between 1 and 100. In the case of cube roots we step off figures, in groups of three, until the number whose cube root we are finding lies between 1 and 1000. The cube root will then lie between 1 and 10.

The slide rule manipulation is as follows:

> Place the cursor index X over the original number in scale A. Adjust the slide so that the number in scale B under X is exactly the same as the number in D opposite 1 (or 10) of C. The number so found is the cube root.

If the reader will use his slide rule and set X over 8 in A he will find that when the slide is set so that 2B lies under X, 2D will be opposite 1C; 2 being the cube root of 8.

If the number whose cube root is sought lies between 1 and 100, use the appropriate reading in scale A for setting X, but if the original number lies between 100 and 1000, select it in the left-hand half of A, which, for our present purpose must be regarded as a continuation of the A scale and stretching from 100 to 1000.

These instructions may seem rather complicated, but if the reader will take his slide rule to find the cube roots of say, 6, 60 and 600, using the 6 in the left-hand part of scale A for the 6 and 600, and the right-hand half of the scale for the 60, he will find no difficulty in reading the three roots, 1·82 in D under 1C, 3·91 in D under 1C and 8·44 in D under 10C.

In extracting cube roots it helps considerably in setting the slide if a mental estimation of the root is made. If we require the cube root of 450, we try, say, 6. $6 \times 6 = 36$, which we call 40; now $6 \times 40 = 240$, and this is well below 450. Try 7; 7 squared is 49, say 50, and $7 \times 50 = 350$. Still too small, so try 8. $8 \times 8 = 64$; and $8 \times 60 = 480$. We have passed the 450, so our cube root lies between figures 7 and 8. We therefore set X to 45 in the right-hand part of scale A and our slide so that 10 in C is near 8D, and if we now move the slide slowly to the left we shall find that when 10C is over 7·66D, X is over 7·66B. $\therefore \sqrt[3]{450} = 7{\cdot}66$.

Example: Find the cube root of ·000′012′64.

> First move the decimal point to the right in steps of three figures, as shown by the ticks, until a number lying between 1 and 1000 is found. This number is 12·64. The cube root of 12 is between 2 and 3. Set X to 12·64 in A. Set 1C to 2D. Now move the slide to the right. When 1C reaches 233D, 233B will be under X. The cube root of 12·64 is 2·33, but we must now move the decimal point two places to the left to compensate for the stepping off of two groups of figures in the original number. The required cube root is ·0233.

Problem 20. Find the cube roots of 8, 80, 800, 9481, ·0213 and ·00046.

Cube Scale

Slide rules equipped with a special scale for evaluating cubes and cube roots of numbers are available. Unless the reader is concerned with work which involves the necessity of frequently finding cubes or cube roots—we cannot think of any work which does—he will find little use for the scale. The cube scale usually lies along the top or bottom edge of the face of the stock, and if the reader will inspect it, he will see that the complete scale is made up of three identical scales placed end to end. Each of these three parts is one-third of the length of the C or D scales, and is divided logarithmically. The left-hand third of the scale starts at 1 and finishes at 10, the middle third stretches from 10 to 100 and the right-hand third from 100 to 1000.

To cube a number it is only necessary to project it from scale D to the cube scale, and cube roots are found by projecting numbers from the cube scale to the D scale.

After reading the instructions we have given for extracting cube roots by the A, B, C and D scales, the reader should have no difficulty when he is using a rule with a scale of cubes.

The stepping off of groups of three figures to bring the original number within the limits of 1 and 1000 should be effected. This makes it easy to select the number in the appropriate section of the cube scale. After the cube root is found in D the adjustment of the position of the decimal point follows the rules we have given earlier.

SECTION SIX

LOG-LOG SCALES

ASSOCIATED with slide rules, we occasionally see quite unfamiliar scales which have been evolved for special purposes by people who have many calculations of a peculiar nature to make. If a complete collection of such special scales could be made, it would, no doubt, furnish an interesting study, and occasion surprise on account of its diversity.

Log-log scales are less frequently used than the scales we have so far studied, but they are not in any sense "special" scales. Many people who have possessed a slide rule for years are not familiar with log-log scales, since these are not included in the scale equipment of the standard slide rule.

Popular models of inexpensive slide rules sold in large quantities in this country include log-log scales, and as there are now a very large number of these rules in use, it may be assumed that the use of the log-log scales is extending. Apart from considerations of utility, the inclusion of log-log scales adds to the pleasure which may be derived from the use of a slide rule.

In Section 13 we shall see that in order to raise a quantity to a power or to extract a root we must look out the logarithm of the quantity, then multiply by the index, and so obtain the logarithm of the result. The evaluation of $5{\cdot}6^{1{\cdot}8}$ involves finding the log of 5·6, multiplying it by 1·8. This gives the log of the answer, i.e. (log 5·6) $\times$ 1·8 = log of answer. We cannot perform these operations entirely with the ordinary scales, since we must consult a table of logarithms. (It is true that if a slide rule is equipped with a log scale—which must not be confused with the log-log scale—we can find the log of a quantity, and after multiplying by the index, find the anti-log of the product, and so obtain the result. Regarded from a practical standpoint, the slide rule is used to save time, and on these grounds there is nothing to be gained by using the log scale in preference to a table of logs; the latter is certainly more accurate.)

If we have a means of finding the log of (log 5·6) we can proceed thus: log (log 5·6) + log 1·8 = log (log answer).

As its name signifies, the log-log scale is designed so that its graduations represent values of the logarithms of logarithms of numbers; the graduations of the ordinary scale represent logarithms of numbers.

It is important to understand that the numbers marked along a log-log scale cannot be varied. We know that the 2 in the C or D scales may be used as 2 or 20 or 2000 or ·0002, but the number 2 in the log-log scale can have no other value except 2. The reader will, therefore, realise that the range of the log-log scale selected for use in any rule is fixed and limited. It is for the designer of the slide rule to decide what is the best range to include in any particular type of rule.

The log-log scale frequently lies along the top and bottom edges of the face of the stock, and if the reader will examine Fig. 7 he will see that the scale lying along the upper edge of the stock starts at 1·1 and finishes at 2·9. The lower scale starts at 2·6 and finishes at 40,000. These two scales are, in fact, one scale only, divided into two parts. The upper portion should be regarded as the part of the complete scale lying in front of the lower section. There are small overlaps on both scales. Strictly speaking, the upper scale finishes at 2·7183, immediately above 10D, and the lower starts at the same value directly under 1D. The log-log scale is used in conjunction with scale C.

As we have said, the limits of the complete scale can be varied, and you may find that the scales of your slide rule, assuming it has a log-log scale, may be different from the one shown in Fig. 7. In some rules you may find more than two sections of log-log scales, but whatever type of rule you possess the examples given below are typical of the calculations which can be made with it.

Evaluation of Powers and Roots

The most useful feature associated with the log-log scale is the ease with which all powers and roots can be calculated. (Abbreviations LU and LL = Upper and Lower log-log scales, respectively.)

Example: Evaluate (i) $6{\cdot}4^{2{\cdot}7}$ and (ii) $\sqrt[2{\cdot}7]{6{\cdot}4}$.

(i) To 6·4 LL set 1C. X to 2·7C.
Result under X in LL = 150.

(ii) To 6·4 LL set 2·7C. X to 10C.
Result under X in LU = 1·99.

Example: Evaluate $6{\cdot}4^{-2{\cdot}7}$.

$$6{\cdot}4^{-2{\cdot}7} = \frac{1}{6{\cdot}4^{2{\cdot}7}} = \frac{1}{150} \text{ (From Ex. (i) above)} = {\cdot}00667.$$

Problem 21. Evaluate $21{\cdot}5^{1{\cdot}66}$; $\sqrt[1{\cdot}66]{21{\cdot}5}$; $21{\cdot}5^{-1{\cdot}66}$.

Example: Evaluate (i) $21^{4{\cdot}5}$; (ii) $\sqrt[9]{2}$.

If the reader will attempt to effect these evaluations by the methods adopted in the preceding examples, he will find the answers "off the scale", in both cases. Result can be found quite easily as now shown.

(i) $21^{4{\cdot}5} = 7^{4{\cdot}5} \times 3^{4{\cdot}5}$ (or the factors 2·1 and 10 might be taken)
$= 6300 \times 140$ (evaluate separately as first example)
$= 882000$ (multiplication by C and D scales)

(ii) $$\sqrt[9]{2} = \frac{\sqrt[9]{20}}{\sqrt[9]{10}} = \frac{1{\cdot}395}{1{\cdot}292} = 1{\cdot}08.$$

In this example the reader will find that when the 20 is found in LL and 9C is brought into coincidence with it, the 1C index is off the LL scale. If he imagines the LU scale to lie in front of the LL scale, he will see that 1C would then be directly over 1·395 LU. Actually, the 1·395 LU is found, using X directly over 10C, since in effect we have moved the LU scale from its imaginary position in front of the LL scale a distance to the right equal to the length of the C scale, i.e. 10″.

Problem 22. Evaluate $1{\cdot}2^{80}$ and $\sqrt[3]{1{\cdot}2}$.

Common Logarithms

The log-log scale gives a means of finding common logarithms. Using X, set 1 of C to 10 in LL and project with X, the number whose log is required, from LU or LL into scale C. The logarithm so found will be complete with characteristic and mantissa. When 10 of C is set to 10LL the result obtained is 10 times the true figure.

Example: Find the common log of 150.

Set 1C to 10LL.

Above 150 in LL read in C 2·178.

Example: Find the common log of 3.

Set 10C to 10LL.

Above 3 in LL read in C 4·77, one tenth of which is 0·477.

The reader will see that the logs to any base may be found in a similar way. The 1C or 10C being set to the base in LL or LU.

Natural Logarithms

If the reader will examine Fig. 7 he will see that the 1 of D lies immediately above 2·7183 (the base of the Naperian system of logarithms) in LL. The scales are positioned so that

the natural or Naperian logs of all numbers in the log-log scales appear directly opposite in D. When projecting from LU, however, the result obtained is 10 times the true figure.

Example: Find the natural logs of 1·8 and 250.

Use X to project from 1·8 in LU, and 250 in LL into scale D.

Logs so found are ·588 and 5·51.

We do not recommend that logarithms should be found as above except when other means of finding them are not to hand. Logarithms should be taken from tables. Naperian logs are derived from common logs by multiplying by 2·303.

The tenth powers of all numbers in LU lie immediately below in LL, and the tenth roots of all numbers in LL lie directly above in LU. The reader will appreciate these facts if he remembers that LU and LL together form one continuous scale with LU preceding LL.

Example: Find the tenth power of 21.

21 lies in LL, but 2·1 is in LU, and we can use $2{\cdot}1 \times 10$, and raise each factor to the tenth power. Projecting 2·1 from LU to LL we obtain 1670, and the result, therefore, is 1670×10^{10}.

Example: Find the tenth root of 200.

Set X to 200 in LL. Read in LU 1·7.

Problem 23. Evaluate $3{\cdot}2^{10}$ and $\sqrt[10]{13}$.

We do not suggest that tenth powers and roots are likely to be required often in practical work.

Since we can use the log-log scales to evaluate *all* powers and roots, we can find square roots and cube roots by the same means, and frequently with a higher degree of accuracy than when using the A and D scales. The reader will now understand our contention that the A and B scales are of little value in a slide rule equipped with a log-log scale.

Example: Find the square root of 1·28 using (i) scales A and D; (ii) log-log scale.

(i) Use X or the index lines of the slide to project 1·28A into scale D. The result appears to be a shade greater than 1·13.

(ii) Set X to 1·28LU, 2C to X, X to 1C. Result 1·1313 under X in LU.

The reader will find scope for the display of his ingenuity in obtaining results which cannot be directly taken from the log-log scales, and we think he will find a good deal of pleasure in using a slide rule equipped with these scales. (Several examples, which involve the use of log-log scales, appear in Section 16.)

The Dualistic rule and the Brighton rule which are dealt with in Sections 11 and 12 respectively are equipped with *three*-section log-log scales which are carried on the reverse of the slides. The additional sections of these scales increase the range from 1·01 to 40,000 as against 1·1 to 40,000 of the two-section LL scales.

If the reader has comprehended the explanation given above in respect of log-log calculations, he will have no difficulty with a slide rule which has its LL scale on the back of the slide. He will find an explanatory note, under reference log-log scales, in Section 11.

SECTION SEVEN

THE TRIGONOMETRICAL SCALES

The scale equipment of what may be termed the standard 10″ slide rule comprises scales A, B, C and D on the faces of the slide and the stock, together with three scales on the back of the slide. These three scales are (i) a logarithmic scale of sines of angles usually denoted by S; (ii) a logarithmic scale of tangents of angles usually denoted by T; (iii) a scale equal in length to the D scale divided into 10 equal parts and each of these parts subdivided into fiftieths; this scale is generally designated by L and is designed for reading common logarithms.

In those models of the "Unique" range of slide rules which are equipped with trigonometrical scales the S and T scales will be found on the faces of the rule, or as in the Brighton rule on the edge of the stock. This layout of scales facilitates manufacture, reduces the cost of production and contributes to the possibility of supplying slide rules at what are often regarded as absurdly low prices. In some models the S and T scales are carried by the stock, in others by the slide, and it will be found that some calculations are made more easily with one arrangement and others more easily with the alternative. The logical step is to equip both stock and slide with S and T scales; this has been done in the Navigational rule which is illustrated, reduced in size, in Fig. 9. For trigonometrical work we believe this rule is superior to any other obtainable, and in examples which follow we shall be using it.

If you possess a slide rule in which the S and T scales are on the undersurface of the slide, you will find no great difficulty in making the necessary modification to these instructions. You will find the slide can be withdrawn and turned over so that what is generally the undersurface is brought uppermost.

In this inverted position some problems can be quickly solved; others may be dealt with when the slide is fitted in its normal position, and you will find fixed index marks in the slots at the ends of the rule which are used in conjunction with the trigonometrical scales.

In Fig. 9 the reader will recognise the A, B, C and D scales. In addition there are two identical sin scales designated by S_1 and S_2, and two tangent scales denoted by T_2 and T_1.

The reader should first carefully examine the manner in

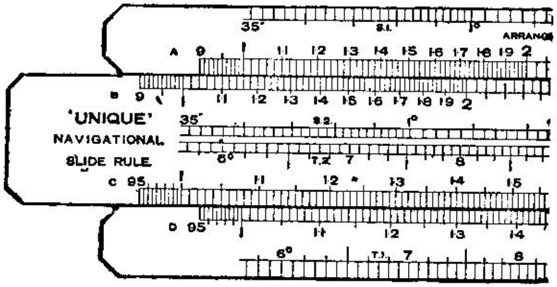

FIG. 9

which the S and T scales are graduated, since these do not altogether follow the principle of decimal subdividing. The wider spaces of the S scale and the whole of the T scale are subdivided in terms of minutes. The smaller spaces towards the right-hand end of the S scales represent unit degrees.

To find the sin of any angle we project with the aid of the cursor direct from S_1 to A.

Example: Find the sines of 2° and 40°.

Using X, we find in A under 2 and 40 of S_1 the values 348 and 643. The actual sines are ·0348 and ·643.

We assume that all readers remember that sin 30° is ·5. Sin 40 must be, therefore, somewhat greater than ·5. If when

projected from S_1 we find the result within the 1 to 10 part of scale A, it must be prefixed by the decimal point and one cypher. If the result falls within 10 to 100 part of the scale, the decimal point only should be inserted before the actual figures. For finding sines of angles less than 35′, other means must be adopted, and for sines of angles approaching 90° the slide rule is unreliable. We recommend the use of a table of sines in preference to the indirect methods which can be employed.

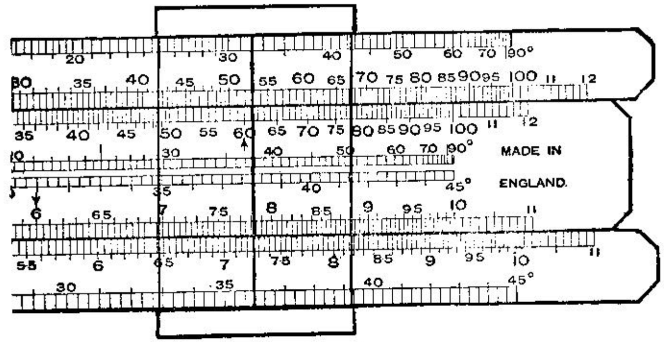

FIG. 9

To find the tangent of any angle, project from T_1 to D. We bear in mind that tan 45° is 1, and we, therefore, insert the decimal point immediately before the reading obtained in D.

Example: Find tan 20°.

Set X to $20T_1$, and read in D 364; result: ·364.

Tangent of angles between 45° and 90° should be obtained by finding the reciprocals of the tangents of the complementary angles.

Example: Find the tan of 55°.

Complementary angle is 35°. Set X to $35T_1$ and in D read ·7. The reciprocal of ·7 is 1·43, which is the tan of 55°.

(NOTE.—If your slide rule is provided with a reciprocal scale you may read the 1·43 direct without having to divide 1 by ·7.)

The sines and tangents of small angles differ so little from one another that tangents of angles less than 6° may be taken as sines without appreciable error. For example, sin 4° = ·0698, and tan 4° = ·0699; the difference between these two values is too small to be observed in slide rule calculations.

Cosines of angles are obtained by finding the sines of complementary angles.

Example: Cos 54° = sin 36° = ·588.

Cosecants, secants and cotangents should be obtained when required as reciprocals of the corresponding sines, cosines and tangents respectively. The following exercises illustrate the uses of the slide rule for the solutions of triangles. (To avoid conflict with the letters A, B and C used to denote the scales, we shall use K, L and M to represent angles, and k, l and m to represent the opposite sides of the triangle; also $2s$ to represent the perimeter, namely $k + l + m$.

Example: Given $M = 25°$, $K = 90°$ and $k = 5''$. Find the remaining angles and sides of the triangle.

$$L = 90° - 25° = 65°.$$

$l = 5 \sin 65°$. $m = 5 \sin 25°$.

Set 100B to 5A.

X to $65S_2$. Read in A under X 4·53 the value of l.

X to $25S_2$. Read in A under X 2·11 the value of m.

Example: Given $K = 90°$, $m = 4{\cdot}2$ and $l = 5{\cdot}6$. Find k, L and M.

$\tan M = \frac{4{\cdot}2}{5{\cdot}6}$. Set 5·6C to 4·2D, X to 10C.

$M = 37°$ in T_1, under X.

$k = \frac{4{\cdot}2}{\sin 37}$. Set X to 4·2A, $37S_2$ to X.

Read $k = 7$ in A above 100B.

Set 7B to 5·6A; X to 100B.

Read $L = 53°$ in S_1 under X.
(Check 37° + 53° = 90°.)

Problem 24. Given $K = 90°$, $l = 5{\cdot}5$ and $k = 12{\cdot}9$. Find L, M and m.

Problem 25. Given $M = 21°$, $K = 90°$ and $m = 1{\cdot}8$. Find k, L and l.

The examples and problems cited above refer to right-angled triangles. We continue with a few typical problems relating to triangles which are not right angled.

Example: Given $l = 4{\cdot}4$, $m = 5{\cdot}2$ and $K = 64°$. Find the remaining side and angles.

$$k^2 = l^2 + m^2 - 2lm \cos 64°$$
$$= 19{\cdot}4 + 27{\cdot}1 - 20{\cdot}1 = 26{\cdot}4.$$

Set X to 26·4A. Read 5·13 in D under X = k.
(l and m are squared using scales A and D.)

To find the third term which is $2 \times 4{\cdot}4 \times 5{\cdot}2 \times \sin 26°$.

Set X to $26S_1$, 100B to X, X to 88B, 100B to X.

Read 20·1 in A over 52B.

(There is no point in using the slide rule for operations which can easily be done mentally, e.g. 2 × 44 = 88, but only simple factors should be combined in this way as it is easy to make slips when using the slide rule and making mental calculations simultaneously.)

Now use the sine rule: $\frac{\sin L}{l} = \frac{\sin M}{m} = \frac{\sin K}{k}$

Set X to $64S_1$, 513B to X.

Read in S_1 over 44B, $L = 50°$; and over 52B, $M = 66°$.

(Check $64° + 50° + 66° = 180°$.)

Results: $k = 5{\cdot}13$; $L = 50°$; $M = 66°$.

There are alternative methods of solving this problem; we might have used $\cos M = \frac{k^2 + l^2 - m^2}{2kl}$, then L would have been found by $180° - K - M$, or the cosine rule used again to find L. We would point out that if both L and M are found by sine or cosine rules, we get a very good check by seeing if the sum of the three angles is 180°. If the third angle is found by subtracting the sum of the other two from 180°, any error previously made is not disclosed. When using the sine rule the reader is reminded that the sines of supplementary angles are equal in both sign and magnitude, and it is necessary to determine which to take. A diagram drawn roughly to scale is the best means of selecting the appropriate angle.

Problem 26. Given $k = 5{\cdot}3$, $l = 7{\cdot}1$ and $m = 3{\cdot}1$. Find K, L and M.

(Use the cosine rule to find L, then the sine rule to find K and M. Check against 180°.)

Example: Given $K = 80°$, $L = 43°$ and $m = 4{\cdot}5$. Find k, l and M.

$M = 180 - 80 - 43 = 57°$.

Set X to 45A; $57S_2$ to X;

X to $43S_2$; read $l = 3{\cdot}65$ under X.

X to $80S_2$; read $k = 5{\cdot}3$ under X.

Problem 27. Given $K = 30°$, $L = 118°$ and $l = 10{\cdot}5$. Find k, M and m.

Example: Given $k = 53$, $l = 35$ and $L = 28°$. Find K, M and m.

This is an example of the well-known ambiguous case with which the reader may be acquainted. If he is not, we advise him to construct a diagram with the data given, and he will find that the side l can be drawn in alternative positions giving rise to two triangles. We must remember that side l is opposite angle L.

$\sin K = \frac{k \sin L}{l}$.

Set X to $28S_1$, 35B to X, X to 53B.

Under X read 45 in S_1.

K is therefore 45° or 135°.

M is 107° or 17°, i.e. $180 - 28 - 45$ or $180 - 28 - 135$.

$m = l \frac{\sin M}{\sin L}$.

Set X to 35A. $28S_2$ to X.

X to $73S_2$ (= 180 − 107) and read in A 7·1 under X.

X to $17S_2$ and read in A 21·8 under X.

Results: $K = 45°$ or $135°$; $M = 107°$ or $17°$; $m = 7{\cdot}1$ or $2{\cdot}18$.

Problem 28. Given $k = 8$, $l = 10$ and $m = 14$. Find the angles and area of the triangle using the formulæ:

$$\sin \frac{K}{2} = \sqrt{\frac{(s - l)(s - m)}{lm}}$$

$$\cos \frac{L}{2} = \sqrt{\frac{s(s - l)}{km}}$$

$$\tan\frac{M}{2} = \sqrt{\frac{(s-l)(s-k)}{s(s-m)}}$$

$$\text{Area of } \Delta = \sqrt{s(s-k)(s-l)(s-m)}.$$

Navigational Problems

The reader will understand that it is not the function of this book to teach the principles of any branch of science or to establish formulæ. The book embraces examples dealing with energy, friction, heat, deflection of Beams, strength of shafts, electricity, building, etc., and in no case do we deal with the underlying principles of any of these subjects. We are attempting to show how the slide rule can be employed quickly and easily to cope with the numerous computations which arise. For the principles and formulæ involved, the reader must consult the textbooks which are available if he needs such assistance. He will find several such textbooks in the Teach Yourself series.

One of the really fascinating characteristics of the slide rule is the ease with which it deals with some of the problems in trigonometry which are encountered during the navigation of a coasting or sea-going ship, or of an aircraft.

There are many proportional problems which arise in connection with aircraft or ships which have no bearing on navigation. For example, the time taken to cover a given distance for a known speed of craft, fuel consumption, conversion from one set of units to another, stowage and loading calculations. Problems of draught and trim and stability.

These problems require no use of trigonometry but can be solved with the aid of the C and D scales or with the use of the other scales we have already examined. We shall not include any examples of these types of problems, but proceed to examine some of those which demand trigonometrical treatment.

We shall, for the remainder of this section, use the Naviga-

tional rule which, in the 10″ size, gives a satisfactory degree of accuracy for most problems. We would, however, mention that, if used, the 10/20 Precision rule or the Precision scales of the 10″ Dualistic rule will invariably give a higher degree of accuracy. The disadvantages of using these rules lies in the fact that the values of sines and tangents must be taken from tables, since the trigonometrical scales are not included in the scale equipment of these two rules.

It will be noticed that more space is devoted to navigational problems in Section 7 than to any other single subject. This is because the Navigational rule which is the subject of this section, was designed to deal with the trigonometrical work involved in navigation and we felt that some additional exercises should be given. This rule was introduced in the early days of World War II for the benefit of air navigators, and it will be seen that the data given on the back of the rule, and the scales on the edges, are more concerned with air navigation than with ship navigation, but the problems met with in the two branches of navigation are generally similar and sometimes identical and the Navigational slide rule can be just as easily used for problems arising in ship navigation. The notes on air navigation were contributed by an experienced air navigator.

The slide rule may often be used instead of the traverse table.

Example: An $8\frac{1}{2}'$ pole standing vertically on a horizontal plane casts a shadow 18′ 3″ long. Calculate the altitude of the sun at the time of observation.

Over 85D set 1825C.

Move X to 1C.

Under X read $25T_1$.

Answer: altitude is 25°.

Example: A ship steering 10° S. of E. observes a light bearing 40° N. of E. After steaming 8 miles the light bears 15° E. of N. Calculate the distances to the light at the two observations.

1st angle from the bow = 50°.
2nd ,, ,, ,, ,, = 85°.
Over $35S_2$ set 8A.
Over $50S_2$ read 107A.
Over $85S_2$ read 139A.
Distances 13·9 miles and 10·7 miles.

Problem 29. A ship sailing due N. sights two gas buoys bearing 020° and 035°. After sailing 8 miles the two marks were in line dead abeam. Calculate the distance between the two buoys.

Example: From a vessel steaming on a straight course a lighthouse was observed 38° forward from the beam. The light was observed to be exactly abeam after the ship had steamed a further 8·3 miles. Calculate the distance at which the light was passed abeam.

Over 8·3D set $38T_2$.
Under 1C read 106D.
Answer: 10·6 miles.

Note that this answer is obtained by dividing the distance run by the tangent of angle forward from the beam. If this angle is above 45°, the result is obtained by multiplying the distance run by the tangent of the complementary angle.

Problem 30. Find the answer to the foregoing example if the angle forward of the beam had been (*a*) 58°; (*b*) 45°.

The *Haversine Formula* is sometimes useful in dealing with problems which involve the solution of triangles, given the three sides.

(*Note.*—Versine $A = 1 - \cos A$

$$\text{Haversine } A = \text{Hav. } A = \frac{1-\cos A}{2}.$$

Employing the usual symbols for plane triangles, viz. A, B, C, a, b, c, $2's = a + b + c$,

$$\text{Hav. } A = \frac{(s-b)(s-c)}{bc}.$$

$$\text{Hav. } B = \frac{(s-a)(s-c)}{ac}.$$

$$\text{Hav. } C = \frac{(s-a)(s-b)}{ab}.$$

Problem 31. If $a = 7$, $b = 5$, $c = 4$, find A, B and C.

Navigational Units and Formulae

Length of nautical mile at Equator, 6046 ft.
,, ,, ,, ,, ,, Poles, 6108 ft.
Standard nautical mile (used in practice), 6080 ft.
1 knot = 1 nautical mile per hour.
1 cable = 600 feet.

dep. = D. long. × cos mid. lat.
= D. lat. × tan course.
= dist. × sin course.
D. lat. = dist. × cos course.
= dep. × cot course.
dist. = dep. × cosec course.
= D lat. × sec course.

$$\text{tan course} = \frac{\text{dep.}}{\text{D. lat.}}.$$

D. long. = dep. × sec mid. lat.

Example: A ship steering a course S. 28° E. is making a speed of 14 knots. Calculate the D. lat. and dep. over a 3-hour run.

D. lat. = dist. × cos course.
Set 42A over $90S_2$

Over $28S_2$ read 197A
Over $62S_2$ (compl. course) read 370A.
Answer: D. lat. 37′; departure 19·7′ E.

Problem 32. Given lat. 24° N. and departure 32 miles, find D long.

Example: A ship steamed on a course of 055° for 4 hours at a speed of 18 knots. Calculate the departure and D. lat.

Distance is 72 miles.
Course angle is 55°.
use dep. = dist. × sin course
D. lat. = dist. × cos course
Set $90S_2$ under 72A.
Over $55S_2$ read 59A.
Over $35S_2$ read 41·2A.
Answer: Departure 59 miles E.; D. lat. 41·2′ N.

Problem 33. A ship steamed on a course of 300° for a distance of 400 miles. Calculate D. lat. and departure.

Example: Calculate the distance from a light known to be 110′ above sea-level at the time when the light first appears above the horizon. Height of observer's eye 50′.

Use the formula on page 106 for distance of sea horizon $1{\cdot}15\sqrt{H}$.

Set 1B under 50A.
Under 115C read 8125D.
Set 1B under 110A.
Under 115C read 1207D.
Distance 8·125 + 12·07 = 20·195 miles.

Problem 34. The Spurn Light is 120 ft. high. At what distances will it be just visible to an observer on the bridge of a ship, at height of eye of (i) 20 ft.; (ii) 40 ft.; (iii) 60 ft.?

Wind and Drift Problems

To find the course to steer and ground speed along given track.

Using scales S_2 and A. Under the airspeed set the angle on the bow or quarter of the track that the wind is blowing. Under the wind speed read the drift angle. The drift angle, added or subtracted to the track, will give the course to steer. To find the ground speed: if the wind is a head wind, subtract the drift angle from the wind angle on the bow; if a tail wind, add the drift angle to the wind angle on the quarter. Above the resulting angle read the ground speed.

Example: Airspeed 126 m.p.h. Track 040° T. Wind velocity 20 m.p.h. from 090° T. (50° on the bow).

To 126A set $50S_2$.
Under 20A read $7S_2$.
Over $43S_2$ (50° − 7°) read 112A.
Result: Course to steer 043° T.; ground speed 112 m.p.h.

Example: Airspeed 97 knots. Track 352° T. Wind velocity 15 knots from 110° T (62° on the starbd. quarter).

To 97A set $62S_2$.
Under 15A read $8S_2$ (drift).
Over $70S_2$ (62° + 8°) read $103S_2$.
Result: Course to steer 360° T.; ground speed 103 knots.

Note that when the drift is less than 1 degree, the ground speed should be found by adding or subtracting the wind speed and the airspeed.

To find the wind velocity, knowing the track and ground speed, course and airspeed.

On scale A mark the airspeed and ground speed with the cursor and a light pencil mark. Adjust the slide until the number of degrees read between the airspeed and the ground speed markings equals the drift angle, i.e. the difference between the course and the track. Above the drift angle on scale S_2 read the wind speed on scale A. Under the airspeed read the wind direction as an angle on the bow or quarter of the track, or under the ground speed as an angle on the bow or quarter of the course. Note that if the G/S is less than the A/S, the angle is on the bow, and if the A/S is less than the G/S, an angle on the quarter.

Example: Course 137° T. Airspeed 150 m.p.h. Track 142° T. Ground speed 130 m.p.h.

Mark the scale A at 130 and 150. Adjust the slide until a difference of reading of 5 degrees on scale S_2 is obtained between the above markings. In this example 29° and 34° will be found to correspond. On scale A above 5° read 23·4 m.p.h. (wind speed). The wind direction is 34° on the bow of the track, and is therefore 108° T. as the drift is to starboard.

To find the wind speed and ground speed, knowing the course and airspeed, drift angle and wind direction.

This method is particularly useful when a flight is being made over the sea and it is desired to know the ground speed. In such cases the drift angle can nearly always be found by a drift sight or back-bearings of an object dropped from the aircraft, but it is not such a simple matter to determine the ground speed. It is a known fact, in a steadily moving air mass, the difference between the wind direction at the surface and the wind direction at a reasonable height remains nearly constant with the changes of surface wind direction.

This difference can be ascertained at the departure point from meteorological information available and applied to the

direction of the surface wind obtained during the flight by bearings of the wind lanes on the sea surface. It has also been found that in practice a reasonably accurate forecast of the direction of the upper winds can be given by a meteorologist, whereas difficulty is sometimes experienced in forecasting the speed. The wind direction can also be ascertained by noting the direction of movement of cloud shadows on the surface. With these various sources at the navigator's disposal little difficulty is usually encountered in finding the wind direction.

Example: Airspeed 110 knots. Course 136° T. Track 142° T. Drift 6° to starbd. Wind direction 348° T.

The difference between the track and wind direction is 206°. The wind angle is therefore 26° on the quarter, a tail wind. Therefore 26° added to 6° (the drift) will give the angle to use to find the ground speed.

To 110A set $26S_2$.

Over $6S_2$ read 26·2 (wind speed).

Over $32S_2$ read 133 (ground speed).

Result: Wind speed 26·2 knots; ground speed 133 knots.

To find the new track and ground speed after an alteration of course.

This method is not mathematically correct, but is sufficiently accurate when it is desired to know the track and ground speed immediately after an alteration of course. In practice, drift should be checked by observation as soon as possible after any alteration of course, so it should never be really necessary to calculate the D.R. track.

Example: Before alteration of course the track was 045° T. Wind velocity 20 m.p.h. from 095° T. True airspeed 140 m.p.h. Drift $6\frac{1}{4}$°. Course steered 051° T. Course is altered to 120°. The wind is now 25° on the port bow of the aircraft, previously it was 44° on the starbd. bow.

To 6·25A set $44S_2$.

Over $25S_2$ read 3·8A.

Result: The new drift is 3° 48′, and the new track is 124° to the nearest degree.

To find the new ground speed:

To 20A set 3° 48′ S_2 (wind speed).

Move X to 140A.

If the angle under the cursor is not an even degree or half degree, adjust the slide accordingly, to bring the nearest half or whole degree under the cursor. Then move the cursor 3° 48′ to the left (as the wind is ahead) and read on scale A the new ground speed. 122½ m.p.h.

If course is being altered frequently, as it would be if a search of some kind were being carried out, it would be far easier and decidedly more accurate to keep a plot of air courses on the chart. When it is desired to know the D.R. position, the total windage affecting the aircraft during the search can be applied to the air position. This can most effectively be accomplished by plotting a wind scale, subdivided into intervals of, say five minutes. The distance that the aircraft has been blown downwind can then be conveniently stepped off with dividers from the air position.

Thus if the air courses flown have totalled 55 minutes from the last fix, then 55 minutes of wind is used. The wind scale is, of course, constructed to the same scale as the distance scale of the chart or map.

It should always be borne in mind that the errors of D.R. navigation are accumulative; therefore the more observations of drift, ground speed or position that can be made, the more accurate will be the final result.

It will be seen in all these problems that the drift angle is always subtracted from the wind angle to the track for head

winds, and added for tail winds. Little difficulty will be found in remembering this, for it will always be readily seen when using the slide rule, for a head wind will always reduce the ground speed, and a tail wind will increase it.

Interception Problems

In theory, the most accurate method of determining the course to steer to intercept a moving surface-vessel is by plotting, and for examination purposes this is the safest method to adopt. In practice, however, this very seldom works out, because changes of wind or weather *en route* often necessitate an alteration of course, and consequently the time and labour spent in solving the original problem is wasted. Again, the problem often arises: When the ground speed during the flight is not what it was estimated to be, how much has course to be altered? Theoretically, a new interception problem should be worked out, but this is a rather lengthy procedure. The following method, using the slide rule, gives a simple solution to this type of problem.

Example: Bearing and distance of ship 012° 282′. Ship's course and speed 125° 17 knots. Airspeed 120 knots. Wind velocity 22 knots, from 247°.

Find the angular difference between the relative bearing and the ship's course, i.e. 125° − 12° = 113° or 67° on the "quarter" of the relative bearing. Estimate the ground speed of the aircraft. This can be done quite roughly, as a few knots either side of the correct ground speed is negligible. Ground speed is therefore estimated to be 125 knots.

Under 125A (G/S) set $67S_2$.

Traverse slide.

Under 17A (ship's speed) read 7° 11′ in S_2.

This will give the angle the track out makes with the relative bearing. The track is therefore 019° T.

Find now the angular difference between the wind direction and the track. 019° + 180° = 199° – 247° = 48° on the quarter.

Under 120A set 48S_2.

Under 22A read 7° 50′S_2.

Over 55° 50′ S_2 read 133½A.

As the wind is a tail wind, 7° 50′ and 48° are added, and the true ground speed out is 133½ knots. If the first part of the problem is re-checked, using the correct G/S of 133½ knots, it will be seen that the angle between the relative bearing and the track is still approximately 7°.

The drift has been found to be 7° 50′ starboard. The course to steer is therefore 011°.

If, after the course has been set, an alteration in the estimated ground speed is discovered, the amount which course has to be altered (to maintain the relative bearing of approach) can be found by carrying out the procedure adopted in the first part of this example, using the new ground speed, and thus finding the new track to intercept. This track can then be maintained by drift observations and slight alterations to course. During the latter stages of the flight, if large changes of drift or ground speed are found, a new relative bearing should be measured between the calculated D.R. positions of the ship and the aircraft, at the same instant of time. If this is done for a few minutes ahead, and a change of relative bearing is discovered, the new course to steer can be determined by using the new angle between the new relative bearing and the ship's course.

The estimated time of interception should be calculated by measuring the distance along the track, and applying the measured ground speed. If the speed of closing is used along the line of relative bearing, some difficulty will been countered in the calculation of a new E.T.I. when it is found that the G/S is not what it was estimated to be.

The Calculation of True Track and Distance

The true track and distance by the middle latitude formula.

Formula:

dep. = D. long. × cos mid. lat.
tan tr. = dep. ÷ D. lat.
dist. = dep. × cosec tr.
D. lat. × sec tr.

To find the rhumb line track and distance from Calais to Heligoland.

Calais	lat. 50° 58′ N.	long. 1° 51′ E.
Heligoland	54° 11′ N.	7° 53′ E.
	D. lat. 3° 13′ N.	D. long. 6° 02′ E.
	193′	362′
	mid. lat. 52° 34′ 5	

To find the departure.

Set 362B to 90S_1.

Under 37° 25½S_1 (comp. of mid. lat.).

Read 220B.

Departure 220′.

To find the true track.

Rule: Always set the larger value of D. lat. and dep. on scale C, and the smaller value on scale D.

Over 193D set 220C.

Set X to 10C.

Under X read 41° 15′ in T_1.

As the dep. is greater than the D. lat. the track is obviously greater than 45°. Therefore the complement of the angle 41° 15′ is used. The track is always named the same as the D. lat. and the D. long.

True track = N. 48° 45′ E. or 049° T. to the nearest half degree.

To find the rhumb line distance.

Set X to 48° 45′ S_1 and move 220B to X.

Under 90S_1 read 293B.

Rhumb line distance 293 nautical miles.

A check on the answer can be obtained by setting X to 41° 15′ (complement track) and reading 293′ (D. lat.) on scale B.

To find the rhumb line track and distance from Calais to Gibraltar.

Calais	lat. 50° 58′ N.	long. 1° 51′ E.
Gibraltar	36° 04′ N.	5° 26′ W.
	D. lat. 14° 54′ S.	D. long. 7° 17′ W.
	894′	437′
	mid. lat. 43° 31′	

To find the departure.

To 90S_1 set 437B.

Under 46° 29′S_1 read 317B.

Departure 317′.

To find the true track.

Over 317D set 894C.

Set X to 10C.

Under X read 19° 30′T_1.

True track S. 19½° W. or 199½° true.

To find the rhumb line distance.

Set X to 19° 30′S_1.

Set 317B to X.

Under 90S_1 read 951B.

Rhumb line distance 951 nautical miles.

Distances of over 200 miles should always be calculated in preference to measuring the distance by dividers on the map or chart. Again, it is always easier and more accurate to calculate the track and distance between places which are not on the same map or chart sheet. A little practice with the slide rule will enable this problem to be solved more accurately, and in a shorter time than it would be if traverse tables were employed.

The great circle track and distance, etc.

In air navigation, the great circle track has other uses than the saving in distance during a long flight. It can be used to avoid ranges of high mountains or prohibited areas without increasing the distance flown, and in flights, when it is desired to bring the track of the aircraft within visibility distance of some landmark, to assist navigation, when by flying the rhumb line track the landmark would have been missed altogether. It is, of course, not always possible to utilise the great circle track in this manner, but these advantages should not be forgotten when planning a flight of even moderate distance.

To calculate the problems involved, by spherical trigonometry, is a very tedious procedure. When, in order to shorten the work, short tables are employed, an added disadvantage is encountered, namely the necessity of having to choose two points near the departure point and destination to obtain angles to fit the tables.

By using the slide rule all these difficulties are overcome, for the solution is both rapid and accurate, and the actual positions of the chosen places can be used.

To calculate the great circle distance.

Formula:

tan. lat. $A \times$ tan lat. $B +$ cos D. long.

(If lat. A and B are in the same hemisphere.)

tan lat. $A \times$ tan lat. $B -$ cos D. long.

(If lats. A and B are in different hemispheres.)

The result is called C.

$C \times$ cos lat. $A \times$ cos lat. $B =$ cos distance.

N.B.—The rules regarding the plus and minus to D. long. are reversed if the D. long. exceeds 90°.

Example: Position A, lat. 38° 45′ N., long. 9° 30′ W.
Position B, lat. 40° 25′ N., long. 73° 15′ W.
Difference of longitude is 63° 45′ W.

Over 38° 45′T_1 set 45T_2.
Under 40° 25′ T_2 read 683D.
i.e. tan lat. $A \times$ tan lat. $B = \cdot 683$.
Under 26° 15′S_1 (comp. 63° 45′).
Read 442A i.e. cos D. long. = ·442.
·683 + ·442 = 1·125 (as A and B are in the same hemisphere).
Under 51° 15′S_1 set 90S_2 (comp. lat. A).
Move X to 49° 35′S_2 (comp. lat. B).
Set 1B to X.
Read 48° 03′S_1 over 1·125B.

The angle 48° 03′ is read as a complementary angle to that indicated on scale S_1, as the answer is the cosine of the distance.

Great circle distance 48° 03′ or 2883 nautical miles.

In the latter process the formula is 1·125 × cos lat. $A \times$ cos lat. B = cos distance.

Note that the scales S_1 and S_2 are sine scales, and to use these scales for cosines the complementary angles must be used. After becoming acquainted with the use of the trigonometrical scales, the beginner will be able to select the complements of angles quite easily from the sine scale by counting the degrees from the right-hand index, or from an easily recognised complement, e.g. 60°, 45° or 30°.

The initial track.

Formula:

sec. lat. $A \times$ sin lat. $B \times$ cosec distance = A (opposite name to lat. A).
tan lat. $A \times$ cot dist. = B (same name as lat. B).

The algebraic sum of A and B is the cosine of the initial track. Lat. A 38° 45′ N. Lat. B 40° 25′ N. Distance 48° 03′.

To find A.

Under 51° 15′S_1 (comp. 38° 45′) set 40° 25′S_2.
Move X to 48° 03′S_2.
100B to X.
Under 90S_1 read 112B.

To find B.

Over 38° 45′T_1 set 45T_2.
Under 41° 57′T_2 (comp. dist.) read 721D.
1·12 − ·721 = ·399 (see rules above)
(A is South, B is North).

·399 is the cosine of 66° 30′. This is read from the scales A and S_1 by setting the cursor over the required figures.

Initial track N. 66½° W. or 293½° T.

To find the latitude of the vertex.

Formula:

sin init. tr. × cos lat. dep. = cos lat. vertex.
init. course 66° 30′. lat. dep. 38° 45′ N.
Set 90S_2 under 66° 30′S_1.
Over 51° 15′S_2 read 44° 20′S_1 (as a cosine).
Lat. of vertex 44° 20′ N.

To find the longitude of the vertex.

Formula:

cosec lat. dep. × cot init. tr. = tan difference of longitude between point of departure and longitude of vertex.
lat. of dep. 38° 45′ N. Init. tr. 66° 30′.
Under 38° 45′S_1 set 100B.
Under 1A read 16B.
Over 16D set 1C.
Under 23° 30′T_2 (comp. 66° 30′) read 34° 50′T_1.

The longitude of the vertex will be 34° 50′ W. of the point of departure, and will therefore be in long. 44° 20′ W.

To find the latitudes in which the great circle track will cut given meridians.

Formula:

tan lat. = tan lat. vertex × cos D. long. between the longitude of the vertex and the given meridian.

Meridian 19° 30′ W. Lat. vertex 44° 20′ N. D. long. is 24° 50′.

From scales A and S_1 find cos D. long. 24° 50′ equals ·907.

Over 907D set 45T_2.

Under 44° 20′T_2 read 41° 33′T_1.

Lat. of tr. in meridian 19° 30′ W. = 41° 33′ N.

To find the track in any latitude.

Formula:

sin tr. = sec lat. × cos lat. vertex. Chosen lat. 40°.

Lat. of vertex 44° 20′ N.

Under 90S_1 set 50S_2 (comp. of 40°).

Over 45° 40′S_2 (comp. 44° 20′) read 69S_1.

Tr. in lat. 40° N. is N. 69° W. or 291° T.

For calculation of a D.R. or air position.

Formula:

dep. = dist. × sin tr.
D. lat. = dist. × cos tr.
D. long. = dep. × sec mid. lat.

To find the D.R. position after a run of 3 hours along a track of 242° T. at a G/S of 138 knots, from a position lat. 50° 30′ N., long. 9° 20′ W.

242° T. is a bearing of S. 62° W. 3 hours at 138 knots represents a run of 414 nautical miles.

To find the departure.

Under 414A set 90S_2.

Over 62S_2 read 365A.

Departure 365′.

To find the difference of latitude.

Under 414A set 90S_2.

Over 28S_2 (comp. of 62°) read 194·5A.

D. Lat. 194′·5 or 3° 14′·5 S. (named S. because the tr. is S.).

(Note that both these problems can be solved together with one setting of the slide by moving the cursor from the course on scale S_2 to the complement of the course, thus multiplying by the sine in the first case, and by the cosine in the second.)

To find the difference of longitude.

The middle latitude is obtained by applying half the D. Lat. found to the latitude of the departure position. Thus the Mid. Lat. is 48° 53′.

Under 365A set 41° 07′S_2 (comp. mid. lat.).

Over 90S_2 read 556A.

D. Long. 556′ or 9° 16′ W. (named W. because the tr. is W.). The D.R. position is therefore lat. 47° 15′·5 N. Long. 18° 36′ W.

This process of calculating the D.R. position is very useful when changing from one chart to another, especially if the charts are of a different scale, or when greater accuracy is required than can be obtained by measuring a long distance run on a chart with a varying scale of latitude.

If it is desired, the course and airspeed of the aircraft can be used to calculate an air position. When the geographical position is required the wind velocity is applied as an additional track and distance. This "track" will represent the direction of the wind (downwind), and the distance will be the distance the aircraft has been blown downwind, during the time of flight at the given wind speed.

The use of pre-computed lines of position.

When a long flight is being carried out above layers of clouds, or over the high seas, and astronomical navigation is being employed, it is very convenient sometimes to compute the calculated altitude before the observed altitude is taken. This saves time if course has to be altered as a result of the observation made, and it enables most of the work to be done when the navigator is fresh and not flurried.

Knowledge of the estimated track and ground speed of the aircraft will enable its D.R. position to be plotted for every hour or half-hour of the flight. Using these positions and the G.M.T. of each E.T.A., calculate the individual altitudes of the chosen heavenly body.

When in flight, and the time approaches for the first observation, take the series of sights as near as possible to the G.M.T. which was used for the calculated altitudes. The pre-computed altitude can be corrected for any slight difference of time by the following formula:

$$\text{Correction} = \cos \text{lat.} \times \sin \text{az.} \times \frac{\text{diff. in secs.}}{4}.$$

Example: Pre-calculated altitude 36° 47′, azimuth S. 47° W. G.M.T. 15 hrs. 06 mins. 00 secs. D.R. lat. 48° N. G.M.T. of observation 15 hrs. 07 mins. 06 secs. Obs. alt. 36° 23′. Difference 66 secs.

Under $42S_1$ set $90S_2$.
Move X to $47S_2$.
Set $90S_2$ to X.
Move X to 66B.
Set 40B to X.
Over 100B read 8·1A.

The correction to pre-computed altitude is 8·1 minutes. As the hour angle is westerly, the body is decreasing in altitude. The correction is therefore subtracted and the calculated altitude to use is 36° 39′ and the intercept will be 16 miles away.

The fix by horizontal angles.

The horizontal angle between two points results in a circle of position, the radius of which is obtained by the formula:

$$\frac{\cdot 5d}{\sin \theta}$$

where d = the distance between the points and θ = the horizontal angle.

The intersection of two circles of position will give the position of the observer.

Although this problem can be solved by using a station pointer or a protracter, these instruments are not always convenient to use on all charts. By using the slide rule to solve the problem of finding the radii of the circles of position, and determining the centres of these circles by construction, an accurate and simple method is always at hand.

Example: *A* and *B* are two objects 6 miles apart, and *C* a third object 8·6 miles from *B*. The horizontal angle between *A* and *B* is 68° and between *B* and *C* 56°. Required the radii of the two circles of position.

Under $68S_1$ set 6B.
Under 5A read 3·23B.
Under $56S_1$ set 8·6B.
Under 5A read 5·19B.

The respective radii are 3·23 and 5·19 miles respectively. Arcs made, using each pair of objects, and the radius applicable will determine the centres of the required circles of position, and the intersection of arcs from these centres will give the position of the observer.

Although a fix by this method is not very practicable when airborne, it is invaluable when a survey of an advanced landing-ground or anchorage is being carried out.

To find conversion angle.

Formula:

$\frac{1}{2}$ D. long. × sin mid. lat. = conversion angle.

D. long. 6° 40′. Mid. lat. 62°.

Under 6° 40′S_1 set 90S_2.

Move X to 62S_2.

Set 90S_2 to X.

Over 5B read 2° 56′S_1.

Conversion angle = 2° 56′.

Numerous rules have been proposed which are purported to aid the navigator in remembering how conversion angle should be applied. If it is remembered that no matter whether the latitude is North or South, or whether the bearing is from a shore station or from the aircraft, the conversion angle correction is always applied towards the Equator, no difficulty will be encountered.

Radius of action.

The problem of finding the radius of action of an aircraft along a given track for a known endurance is a very simple matter if the slide rule is always employed.

Formula:

$$\frac{\text{G/S out} \times \text{G/S home}}{\text{G/S out} + \text{G/S home}} \times \text{fuel hours} = \text{radius of action.}$$

Example: Airspeed 230 m.p.h. Track out 040° T. Wind velocity 20 m.p.h. from 102° T. Fuel hours 35.

Wind is 62° on the bow.

Set X to 230A. 62S_2 to X. X to 20A.

Under X in S_2 read 4$\frac{1}{2}$° (drift).

Over 57$\frac{1}{2}$ S_2 read 220A = G/S out (against wind).

Over 66$\frac{1}{2}$ S_2 read 239A = G/S home (with wind)

220 + 239 = 459.

To 22D set 1C. X to 239C. 459C to X.

Read in D under 35C radius of action, 4010 nautical miles.

The time to turn is found from the time required to fly the 4010 miles at the outward G/S.

To find the error in track from a position line parallel to the D.R. track.

When navigating by astronomical observations, every advantage should be taken to observe heavenly bodies abeam of the aircraft, as the resulting position lines will give a good indication of the true track of the plane.

If no terrestrial observations have been possible since the departure, it will be desired to know the error in the course being steered. This can be easily found as follows.

Example: After flying 186 miles an observer obtained a position line placing the aircraft 38 miles to starboard of the D.R. track. Find the error in the track.

Over 38D set 186C.

Under 1C read 11$\frac{1}{2}$$T_1$.

The Track error is 11$\frac{1}{2}$°.

This method can also be employed when navigating over land, or when D/F W/T bearings are being used. It can also be used to give the alteration of course required to reach a certain destination, when of course the distance of the aircraft from the destination should be used instead of the distance run since departure.

It might be noted here that when sights are being taken to make a running fix, a sight of a heavenly body on the beam should invariably be taken first, as it will not then be necessary to transfer this position line for the "run" between it and the next observation. It will be found sufficiently accurate in practice to extend the position line until it cuts the line resulting from the second sight, thus giving a fix and saving one the labour of transferring the observation for the time interval between sights.

Correction of refraction.

Altitudes above 8°. Multiply the cotangent of the altitude by ·96.

Altitude 36°, required the correction for refraction.

Over 36T_1 set 96C.

Over 1D read 1·3C (to nearest first place of decimals).

Correction is 1·3 minutes minus to the apparent altitude.

Correction for dip of the sea horizon.

Formula: $\sqrt{\text{Height}} \times {\cdot}98$.

Example: An observer flying at 840 ft. above sea-level requires the dip correction.

Set X to 840A.

Set 10C to X.

Under 98C read 284D.

Dip Correction is 28·4 minutes minus to the observed altitude.

To find the distance to the sea horizon.

Formula: $\sqrt{\text{height}} \times 1{\cdot}15$. The resulting distance is given in nautical miles.

To find the distance of an object from the angle of depression.

For distances up to ten miles.

Formula:

$$\frac{{\cdot}565\,H}{\theta} = \text{distance in nautical miles} = D_1.$$

H = height of observer in feet above the object.

θ = angle of depression in minutes.

For distances over ten miles or for greater accuracy.

Formula: $$\frac{{\cdot}565\,H}{\theta - {\cdot}4D_1} = D_2.$$

D_1 and D_2 = distances in nautical miles.

D_1 should be found, using the first formula, and then when $\cdot 4D_1$ is obtained the second formula can be used to calculate the final result.

To find D_1.

Over 565D set θ in scale C.

Under H in scale C read D_1 in scale D.

Evaluate $\theta - {\cdot}4D_1$.

To find D_2.

Over 565D set $\theta - {\cdot}4D_1$ in scale C.

Under H in scale C read D_2 in scale D.

SECTION EIGHT

THE COMMERCIAL RULE

We have expressed the opinion that to most people the slide rule is not the vade-mecum it should be. The engineer, the architect, the draughtsman and many others would, at times, find his work irksome and tedious without the aid of his slide rule, which helps him to cope with a mass of detailed calculations.

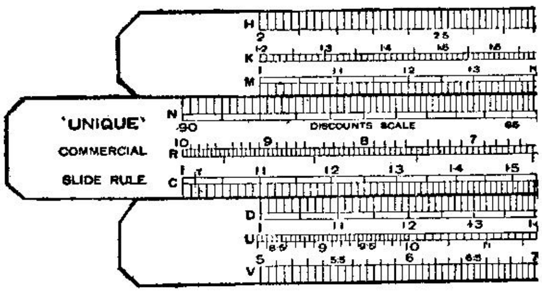

Fig. 10

Unfortunately, there is a deep-rooted impression that the slide rule is of little use for commercial calculations, and especially for those which involve monetary values. It is true that if a result is required correct to the last penny, the slide rule may fail to give it, but there are many commercial calculations in connection with which the slide rule will give valuable assistance. When the slide rule will not give results with the necessary precision, it can always be used as a check, with the necessary precision, it can always be used as a check, and in this service alone it is worth a place on the desk. Should this section meet the eye of any individual who, whilst dealing

with accounts, does not use a slide rule, we ask him to keep an open mind on the subject and to spend a few minutes in investigating the possibilities.

In Fig. 10 we illustrate a slide rule designed to meet the requirements of the commercial user. It will at once be seen that C and D scales, which we have studied earlier, and which we regard as being the most important scales in any slide rule, occupy their usual positions, C lying along the lower edge of the slide, and D on the stock adjacent to it. The M scale just above the slide is identical with D, but is used in conjunction with scale N. This combination of scales allows us quickly to calculate net amounts, after taking off discounts.

The scales H, K, U and V are all graduated in the same

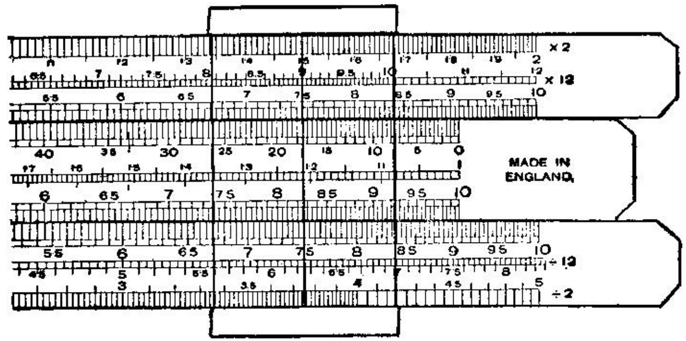

Fig. 10

manner as C and D, but they are so placed that results appearing in D are automatically multiplied or divided by 12 or 20, when read directly above or below in H, K, U or V. Other ratios may be obtained by projecting directly from one scale to another, e.g. readings in K are 144 times those in U and 6 times those in H. Readings in H are 4 times those in V and 24 times those in U. Now all the numbers mentioned appear frequently in our calculations. There are 12 in a dozen, 12 pence in a shilling, 12 months in a year, 12 inches in a foot, 144 in a gross, 60 minutes in an hour, 20 shillings in a pound, 20 cwt. in a ton, 24 hours in a day. Many simple calculations can be made therefore by the simple process of projecting from one scale to another.

For easy reference we now show the relationships between the various scales in tabulated form.

Scale H. This scale is positioned, relative to other scales, so that any value in scale M or scale D is multiplied by 2 by projecting, by means of the cursor index X, from M or D into H.

These relationships are more easily indicated in the abbreviated form which we now employ:

Scale H. H = 2 × M
H = 2 × D
6 × H = K
H = 24 × U
H = 4 × V

Scale K. K = 12 × M
K = 12 × D
K = 144 × U
K = 24 × V

Scale M. M = D

Scale N. This is a special scale which enables discounts to be quickly deducted. Scale M is adjacent to scale N for convenience in such calculations.

Scale R. The reciprocal scale lying along the centre of the slide. We do not regard this scale as an important part of the scale equipment of a slide rule, but we know that some people consider its inclusion an asset. Its uses and limitations are discussed in Section 10 dealing with the Electrical rule.

Scales C and D. The scales we have studied in Section 4.

Scale U. 12 × U = D = M
144 U = K

24 U = H
6 U = V

Scale V. V = 6 U
4 × V = H
24 × V = K

We have purposely stated the ratios in terms of multiplication. The reader will readily appreciate that we could just as easily have stated them in terms of division. There is no necessity to memorise these scale relationships. It is sufficient to remember that 12, 2, 6 and 144 appear among them. With a little practice one soon becomes familiar with the most useful combinations.

Example: A certain type of coiled spring requires 8·25 inches of wire to make it. What length of wire will be required to manufacture 50 springs?

Over 825D set 10C.

Move X to 5C.

Under X read 4125 in D or 344 in U.

Also 4U is opposite 48D.

The answer may therefore be read as
412·5 inches, or
34·4 feet, or
34 feet 4·8 inches.

Problem 35. The cost of 1000 articles in U.S.A. is 48 dollars. Find the cost of one gross in G.B. (rate of exchange \$2·8 = £1). Use scales U, C and K.

Example: Convert 655 kilos into tons (454 grammes = 1 lb.).

Over 655D set 224C.

Move X to 1C.

Set 454C to X.

Read 645D under 10C.

Result: ·645 tons.

Alternatively

Over 655D set gauge point T (1014) in scale C.
Read 645D under 1C.
(This method makes use of the special gauge point marked T in scale C. 1016 kilos = 1 ton.)

Example: Below is part of a workman's time-sheet. His rate of pay for a 44-hour week is £7, 1s. 0d. For costing purposes, each job has to be charged with the labour cost.

Job No. 904 .	. $8\frac{1}{2}$ hours
,, ,, 918 .	. $2\frac{1}{4}$ hours
,, ,, 721 .	. 45 mins.
,, ,, 800 .	. $3\frac{3}{4}$ hours
,, ,, 856 .	. 50 mins.

Over 141D (141 = rate in shillings) set 44C.
Under 85C read 27·2s. in D or 327d. in K.
Over 45C read 29d. in H.
Under 375C read 12s. in D or 144d. in K.
Over 50C read 32d. in H.
For the second Job No. 918, the slide must be traversed, so
Move X to 10C.
Move 1C to X.
Under 225C read in D 7·2s. or in K $86\frac{1}{2}$d.

The last example shows how more accurate results may sometimes be obtained by reading values in different scales. In the Job No. 918 the answer may be obtained in shillings If scale D and appears to be 7·2s. with a possible error of + 1d. in the answer is read in scale K we obtain $86\frac{1}{2}$d. without any doubt as to the last penny.

In Job Nos. 721 and 856 the answers are read off in pence in scale H and the reader may perhaps have some difficulty in seeing why we change to this scale.

Consider Job No. 721. The computation expressed fully is

$\frac{141}{44}$. This gives the man's rate per hour in shillings; $\frac{141}{44} \times \frac{45}{60}$ gives the man's rate for the job in shillings; and $\frac{141}{44} \times \frac{45}{60} \times 12$ is the cost of this job in pence.

Now the 12 divided into 60 leaves 5 in the denominator and dividing by 5 is equivalent to multiplying by 2, and this gives the reason for reading the result in scale H.

For practice in this important type of calculation we ask the reader carefully to work through the following problems.

Problem 36. A woman's rate for 42 hours is £4, 11s. 6d. Calculate the labour costs to the nearest penny to be set against the following jobs.

Job No. 121 .	. 3 hours
,, ,, 129 .	. $4\frac{1}{4}$ hours
,, ,, 167 .	. $6\frac{3}{4}$ hours
,, ,, 188 .	. 35 mins.
,, ,, 196 .	. 22 hours
,, ,, 219 .	. 55 mins.

Example: Goods bought at 22% below list prices are to be sold at 15% above list prices. Calculate the selling prices corresponding to list prices of 8s. 6d., 9s. 4d., 15s., 17s. 6d. and 22s. 3d.

Now $100 - 22 = 78$
and $100 + 15 = 115$

these calculations involve multiplying the original list prices by $\frac{115}{78}$.

Over 115D set 78C.
Under 85C read 12s. 6d. in D or 150d. in K.
Under 933C read 13s. 9d. in D or 165d. in K.
Now traverse the slide, i.e.
set X to 10C and move slide to bring 1C under X.
Under 15C read 22s. 2d. in D or 266d. in K.
Under 175C read 25s. 9d. in D or 309d. in K.
Under 2225C read 32s. 9d. in D or 393d. in K.

Problem 37. A man's rate of pay for 40 hours is £8, 10s. 0d. Calculate his rate per hour and the pay for $6\frac{1}{2}$, 29 and 60 hours.

Example: A Hydraulic Power Co. charges 2s. 11d. per 1000 gallons of water at a pressure of 950 lb. per square inch. Calculate the cost per horse-power hour. (1 gallon water = 10 lb. 1 cu. ft. water = 62·3 lb., 1 h.p. = 33,000 ft.-lb./min.)

The relevant figures are:

$$\frac{33000 \times 60 \times 62{\cdot}3}{10000 \times 950 \times 144} \times 35\text{d.}$$

which cancels down to

$$\frac{33 \times 62{\cdot}3 \times 35}{950 \times 24}.$$

Over 33D set 95C.

Move X to 623C.

Set 24C to X.

Under 35C read 315D.

Answer: 3·15d.

On the back of the rule a conversion table for decimalising is given; it is the equivalent of the table Fig. 11.

The reader is advised carefully to study the examples which now follow and to work out the problems. Other examples are given in Section 16.

Example: An invoice price of £43, 16s. 6d. is subject to $33\frac{1}{3}$%, less 15%, less $2\frac{1}{2}$%. What is the net amount?

From the table on the back of the rule or from Fig. 11 read 16s. 6d. = £·825.

Set 0 of scale N to 43·825M.

X to 333N. (Note special gauge point at $33\frac{1}{3}$%.)

0 of N to X. X to 15M. 0 of N to X.

Above 2·5N read in M 24·2 = £24, 4s.

(The reader will please note that scale N reads backwards from right to left.)

TABLE FOR CONVERTING SHILLINGS AND PENCE INTO POUNDS AND HUNDREDWEIGHTS AND QUARTERS INTO TONS.

PENCE	QUARTERS	SHILLINGS OR CWTS.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0			·050	·100	·150	·200	·250	·300	·350	·400	·450	·500	·550	·600	·650	·700	·750	·800	·850	·900	·950
1		·004	·054	·104	·154	·204	·254	·304	·354	·404	·454	·504	·554	·604	·654	·704	·754	·804	·854	·904	·954
2		·008	·058	·108	·158	·208	·258	·308	·358	·408	·458	·508	·558	·608	·658	·708	·758	·808	·858	·908	·958
3	1	·012	·062	·112	·162	·212	·262	·312	·362	·412	·462	·512	·562	·612	·662	·712	·762	·812	·862	·912	·962
4		·017	·067	·117	·167	·217	·267	·317	·367	·417	·467	·517	·567	·617	·667	·717	·767	·817	·867	·917	·967
5		·021	·071	·121	·171	·221	·271	·321	·371	·421	·471	·521	·571	·621	·671	·721	·771	·821	·871	·921	·971
6	2	·025	·075	·125	·175	·225	·275	·325	·375	·425	·475	·525	·575	·625	·675	·725	·775	·825	·875	·925	·975
7		·029	·079	·129	·179	·229	·279	·329	·379	·429	·479	·529	·579	·629	·679	·729	·779	·829	·879	·929	·979
8		·033	·083	·133	·183	·233	·283	·333	·383	·433	·483	·533	·583	·633	·683	·733	·783	·833	·883	·933	·983
9	3	·037	·087	·137	·187	·237	·287	·337	·387	·437	·487	·537	·587	·637	·687	·737	·787	·837	·887	·937	·987
10		·042	·092	·142	·192	·242	·292	·342	·392	·442	·492	·542	·592	·642	·692	·742	·792	·842	·892	·942	·992
11		·046	·096	·146	·196	·246	·296	·346	·396	·446	·496	·546	·596	·646	·696	·746	·796	·846	·896	·946	·996

PENCE EXPRESSED AS DECIMALS OF A SHILLING.

½d. is ·042/- 1d. is ·083/- 2d. is ·167/- 3d. is ·250/- 4d. is ·333/- 5d. is ·417/-

6d. is ·500/- 7d. is ·583/- 8d. is ·667/- 9d. is ·750/- 10d. is ·833/- 11d. is ·917/-

FIG. 11

We would here point out that we cannot add percentage amounts on by just reversing the operation defined above. The reader will easily see why this cannot be done: 50% off 40 leaves 20. If we add 50% to 20 we obtain 30, not 40. However, if we wish to add on percentage amounts we can do so easily by using scale C. Increasing a quantity by, say, 10% is equivalent to multiplying by 1·1. Calling the 1 of C a 100, and the 1·1 of C 110, we see that the space between 1 and 1·1 represents an increase of 10%. The 1·1 graduation, which we will call 110, is our 10% increase mark, the 1·3, called 130, is the 30% increase mark, and so on. If we use the graduation 2, which we must now call 200, we shall be adding 100%, and if we use graduation 3, we shall be adding 200%, and likewise. It will seldom be necessary to go to these high percentages.

Example: To 35 add 10%, then add 60%. (Please note that this is not the same as adding 70% to 35.)
Set X to 35M. 1C to X. X to 1·1C.
1C to X. X to 1·6C.
Read the result in M under X = 61·6.

Problem 38. Calculate £8, 4s. 2d. + 15% − 30% − 12½% − 7½% − 2½% + 10%.

Example: Calculate 17% of £108, 10s. 0d.

Please note that this is a different type of calculation from those immediately above. We now use scales C and D.
Over 1085D set 1C.
Under 17C read the result, 1842, in D.

Now we can see that 17% is approximately one-sixth, and our result must be £18·42 = £18, 8s. 5d. (Scale on back of rule shows £·42 = 8s. 5d.)

If you will work out this result precisely, you will find that to the nearest penny the correct answer is £18, 8s. 11d., and our slide rule has given us a result in error to the extent of 6d. 6d. in £18, 0s. 0d. is about 1 in 720, or less than one-seventh

of 1%, and it is the sort of error we must expect with a 10″ slide rule. We can do better than this, and we hope the reader will note how to make the best use of his slide rule.

17% of £100 is £17, 0s. 0d., and we can write this down without using a slide rule. We now find 17% of the remaining £8, 10s. 0d.

Set 10C to 85D. Under 17C read £1·445.

Taking the nearest figure from the table we see that ·446 = 8s. 11d. and our result now appears as

£17	0	0
1	8	11
£18	8	11

By breaking up the original figures into a large part which we can deal with mentally, leaving a smaller odd amount for slide rule calculation, we generally can get very near the exact result as we have done in this example. If the reader will work through the above, he will obtain the same result and satisfy himself that it is genuine.

Instead of using the table for converting £·445 to shillings and pence the reader may prefer to use the H and K scales.

Set X to 445D. Read in H under X 8·9s.
Set X to 9D. Read in K under X 10·8d.
Thus £·445 is equal to 8s. 11d. to the nearest penny.

Problem 39. What is 22½% of £1662, 11s. 7d.?
(Work mentally 22½% of £1000, and of £600, and use the slide rule for the odd amount, then obtain a check by using the slide rule to take 22½% of £1663. This will give the result to the nearest pound.)

Example: In a factory time-sheet the following hours of overtime appeared:

Mrs. A . . .	3½ hours
,, B . . .	4½ ,,
Miss C . . .	4¾ ,,
,, D . . .	2¾ ,,
,, E . . .	7¼ ,,

All these women are paid 78s. 0d. for 44 hours, and are paid $1\frac{1}{2}$ times day rate for all overtime. The amount of pay due to each is required. We shall calculate payments for overtime and add them to the 78s. 0d. since more accurate results will be obtained.

To 78D set 10C. X to 1·5C. 44C to X.
Set X to 45C.
Under X read 11s. 11d. in D or 143d. in K.
Set X to 475C.
Under X read 12s. 8d. in D. or 152d. in K.
Set X to 725C.
Under X read 19s. 4d. in D or 232d. in K.

Now traverse the slide to compute the overtime for Mrs. A and Miss E.

Move X to 10C. Set 1C to X.
Set X to 35C.
Under X read 9s. 4d. in D or 112d. in K.
Set X to 275C.
Under X read 7s. 4d. in D. or 88d. in K.

The overtime payments are Mrs. A 9s. 4d., Mrs. B 11s. 11d., Miss C 12s. 8d., Miss D 7s. 4d. and Miss E 19s. 4d. These amounts added to 78s. give the amounts to be credited to the various workers. This example shows that for small amounts of a few shillings the results may be read in scale K to the nearest penny.

This example shows how quickly pay for different hours worked can be calculated. There are alternative methods of working as the reader will realise, but all the workers on any rate, in the above 78s. 0d. should be dealt with together, since all the payments can be obtained at one or two settings of the slide. When the 78s. 0d. group has been exhausted, re-set the slide to, say, 82s. 0d. The slide rule will generally give results quite accurate enough for the purpose. When the method of working is appreciated, the slide rule will be preferred to the ready reckoner.

Problem 40. Calculate the simple interest on £59, 8s. 2d. for 182 days at $4\frac{3}{4}$% per annum.

Example: Convert $341 to sterling. Rate of exchange £1 = 2·86$.
Over 341D set 286C.
Under 1C read £119·2 = £119, 4s. 0d. in D.

Problem 41. A workman receives $8\frac{3}{4}$d. per piece. In the course of a day of $8\frac{1}{2}$ hours he turns out 61 pieces. Calculate his earnings per day and per hour.

A great variety of examples could be given, but this book must be kept to moderate dimensions. Any calculation which involves the operations of multiplication and division may be effected by slide rule, and the reader will have no difficulty in finding opportunities for exploiting it. Further examples will also be found in Section 16.

The Monetary Rule

The idea of constructing a scale in terms of pounds, shillings and pence will, by this time, have occurred to those readers who have persevered in the perusal of this book, more particularly to those who have studied the instructions immediately above concerning the Commercial rule.

The illustration of the Monetary slide rule, Fig. 12, shows such a scale.

The monetary scale, lying on the stock of the rule, is in three sections designated by M_1, M_2 and M_3. M_1 lies immediately above the slide, and M_2 and M_3 along the top and bottom edges respectively. M_1 commences at 1d. and in logarithmic intervals increases to 10s. at the right-hand extremity. 10s. is the starting value in M_2 and this section of the scale extends to £50. M_3 begins at £50 and proceeds to the maximum value of £5000. The complete scale, therefore, covers a range of 1d. to £5000.

(The reader will understand that the illustrations of the different slide rules are reduced in size and that a part has been omitted from each, further to reduce the lengths, so that the diagrams can be printed within the limits of the pages.)

The remaining scales in this rule are the B, C and D scales with which we are now familiar. In all the scales "legible" dividing has been adopted (legible dividing means that the spaces along the scales are wider and there are less graduations. If the reader will compare Figs. 10 and 12, the wider dividing in the latter will be apparent.)

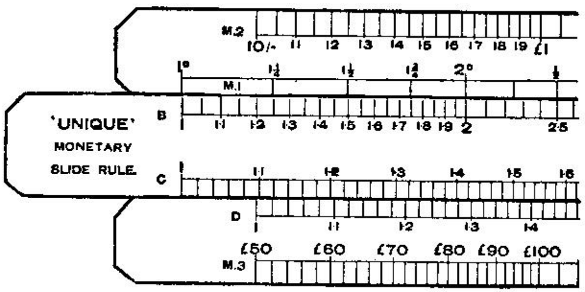

FIG. 12

The C and D scales have no connection with M_1, M_2 and M_3. C and D are included so that this rule may be used for ordinary numerical calculations. They have been dealt with adequately in Section 4 and we shall make no further reference to them here.

Scale B is used in conjunction with M_1, M_2 and M_3 for multiplication, division and discounts off.

First examine the three sections of this monetary scale. Place the cursor index X over the 1s. graduations in M_1. Immediately above in M_2 will be seen £5 and below in M_3 £500. Thus values in M_2 are 100 times those in M_1, values in M_3 are 100 times those in M_2.

These relationships apply throughout the three sections of the M scale.

Now set the slide so that 1B is coincident with 4d. in M_1.

In line with	2B	will be found	0s. 8d. in M_1
,, ,, ,,	3B	,, ,, ,,	1s. 0d. ,, ,,
,, ,, ,,	10B	,, ,, ,,	3s. 4d. ,, ,,
,, ,, ,,	30B	,, ,, ,,	10s. 0d. ,, ,,

In effect, with this setting of the slide the 4d. selected for illustration, is multiplied by any selected number in scale B by reading the results in M_1. Division is effected in the reverse

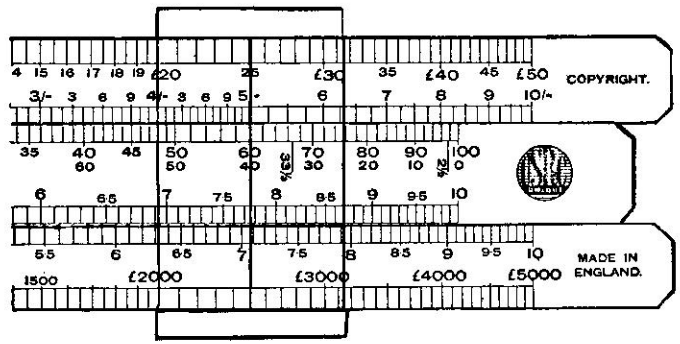

FIG. 12

manner. The rule setting we have adopted will effect division of 10s. 0d. by 30, 6s. 0d. by 18 and many other examples. Now traverse the slide, i.e. push it to the left and set it so that 100B is brought into coincidence with 4d. in M_1. Set X to 60B and read in M_2 the result of multiplying 4d. by 60, viz. £1. In the first setting of the slide, some of the numbers in B were off the M_1 scale. It is, we think, obvious that these missing values would have given results in M_2 if M_2 had formed a continuation of M_1, but since M_2 has, in effect, been moved to the left a distance equal to the length of the scales, the slide must be moved similarly. This complication need not give any concern since it is only necessary to remember "*If when multiplying, the* 100 *of scale B is used in setting the slide, the result will be found in the next higher M section.*" The corresponding rule for division is fairly obvious, and we leave it to the reader to put into words.

When we examined the A and B scales in Section 5 we saw that in multiplication or division if we desired to use say 25 in some computation, we could select the 2·5 in the left-hand half or the 25 in the right-hand half of the scale without affecting the final result. It must be remembered when using this Monetary rule, that we must use the appropriate figure in scale B. Reverting to the earlier setting of the slide with 1B set to 4d. in M_1, we find opposite 2·5B the result 10d. and opposite 25B the result 8s. 4d. Both results are correct, but we shall be in error if we use 25 when 2·5 is called for.

It will be sufficient now if we give a few examples illustrating the uses to which this ingenious rule may be put.

Example: Calculate the invoice value of 72 pieces at 3s. 9d. each.

Set 100B to 3s. 9d. M_1.

X to 72B.

Result in M_2 under X is £13, 10s. 0d.

If this amount is subject to less 40% less 10% and less 2½% find the nett amount by setting 100B to X. X to 40%. 100B to X. X to 10%. 100B to X. X to 2½%.

Result: £7, 2s. 0d. in M_2 under X.

Example: Multiply 1s. 4d. by 5·8, 580 and by 58,000.

Set 1B to 1s. 4d. M_1. X to 5·8B.

Under X read 7s. 9d. in M_1, £38, 15s. 0d. in M_2 and £3,875 in M_3.

The reader will appreciate that towards the end of the M scale, values cannot be determined with accuracy. This end of the scale will seldom be used in normal calculations but will always provide a check.

Example: The pay rate for a certain operation is 1s. 9d. per piece. Calculate the pay due for 3½, 18½, 22, 39 and 42 pieces.

Set 1B to 1s. 9d. in M_1.

Against 3½ B read 6s. 2d. M_1.

Set 100B to 1s. 9d. in M_1.

X to 18½B; amount in M_2 under X is £1, 12s. 5d.
X „ 22 B; „ „ M_2 „ X „ £1, 18s. 6d.
X „ 39 B; „ „ M_2 „ X „ £3, 8s. 3d.
X „ 42 B; „ „ M_2 „ X „ £3, 13s. 6d.

We can think of no more valuable aid for the costing, invoicing and purchasing staff in a busy office than this slide rule. For many purposes results obtained with it will be sufficiently accurate, and in those calculations where the "last penny" must be correct the rule is a great aid in checking.

In weekly wages calculations, where the amounts are of the order of a few pounds, the rule will give reliable results.

SECTION NINE

THE PRECISION RULE

THE reader will appreciate that the degree of accuracy with which computations may be made with a slide rule depends primarily upon the length of the scales employed, and because of this we have consistently stressed the importance of using C and D scales in preference to the A and B scales for ordinary work.

Obviously, if we used a slide rule with 20″ scales, our result should be more precise than when we use a 5″ or 10″ rule. 20″ slide rules are frequently to be seen in drawing-offices, and occasionally even larger rules, up to 40″ length, are encountered. These rules are rather awkward to use, but in operation they are no different from the 10″ rule.

Precision Rules

The type of slide rule which employs scales twice the length of the rule, i.e. 20″ scales on a 10″ stock, or 10″ scales on a 5″

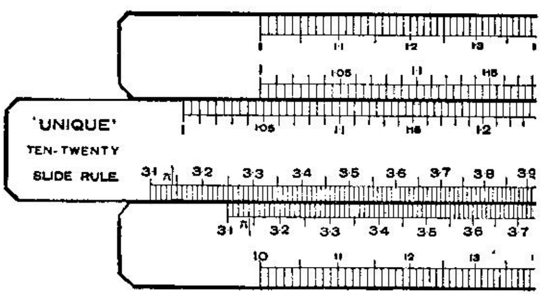

FIG. 13

stock, is not so well known as it should be. The 10/20 rule illustrated (reduced size) in Fig. 13 gives the same degree of accuracy as the 20″ slide rule, and is more convenient to use. The illustration shows the additional dividing rendered possible by the longer scale. The C and D scales are divided into two halves, the parts running from 1 to 3·2 occupy the usual positions of the A and B scales and the remainder from 3·1 to 10 lie in the normal C and D scale positions. The 1 to 10 part of the A scale, doubled in length, is moved to the top edge of the stock, and the 10 to 100 half to the lower edge of the stock.

Multiplication and division are effected by the C and D scales, employing the principles which apply to the ordinary type of rule, but since the graduations on either edge of the slide cannot be brought directly into contact with graduations on the opposite part of the stock, the cursor must be used to bridge across the slide when necessary, and herein lies the only disadvantage of this form of rule. The expert user of a slide rule should find no difficulty. Any uncertainty in deciding on which part of the stock the result lies will be removed, if the following simple maxim is remembered: *If when setting the slide it is necessary to use the cursor to cross the slide, it will also be necessary to use the cursor to re-cross the slide, when reading the result.*

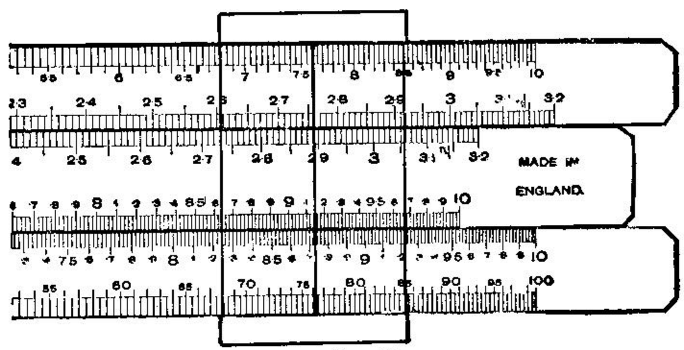

FIG. 13

This maxim is easy to apply when two factors only are involved, e.g. a simple multiplication, or a simple division, and it can be extended to more complex operations.

We have the feeling that the Precision rule suffers in popularity because of the little extra trouble involved in keeping check on the position of the result, but if our work is of such a nature that we can profitably take advantage of the higher degree of accuracy of which the rule admits, we think any prejudice should be removed, and we will therefore give a few examples of the methods we use for keeping check.

For reference purposes we designate that part of the C scale which lies along the upper edge of the slide by c, and the remainder of this scale lying along the lower edge of the slide by C. Similarly, the upper and lower parts of the D scale will be denoted by d and D respectively. These reference letters do not appear in Fig. 13. They should be added as shown in Fig. 14.

We will first take a simple example, $11 \times 12 \times 2$. The result, which we can calculate mentally, is 264.

Set 1c to 11d. X to 12c. 1c to X.

Result: 264 in d above 2c.

In this example all the readings lie in the c and d scales, and there is no complication, the operations following one another as with the ordinary C and D scales. The reader will see that in multiplying, say, $9 \times 8 \times 5 = 360$, the operations are all effected in the C and D parts of the scales, and again there is no complication.

Now let us multiply $2 \times 7 = 14$. If we set 1c to 2d the result is off the scale, so we must set 10C to 2d, and in doing so we must use X to cross the slide. We move X to 7C and we find the result 14 in d, if we use the cursor to cross the slide again. If we read the result in D we obtain an erroneous answer, and we must guard against this. The reader will see that if we read the result on the wrong side of the slide, it will be about 3 times too large or too small, and often this will disclose the error.

In a longer computation, the Precision rule would be a source of danger if we had no easy way of keeping check on which side of the slide to read the result. We should, of course, always make an approximation, and by doing so determine position of decimal point, or we can adopt the method we will now explain.

Example: Evaluate $\dfrac{1{\cdot}8 \times 6{\cdot}1 \times 108}{4{\cdot}09 \times 32{\cdot}1}$.

Set X to 18d	and jot down	T.
409C to X	,, ,, ,,	B.
X to 61C	,, ,, ,,	B.
321C to X	,, ,, ,,	B.
X to 108c	,, ,, ,,	T.

Now the cursor index X in its final position registers 286 in d and 904 in D, and we must determine which is the appropriate result. The T we first jotted down indicates that we commence in the top c and d scales by selecting the 18 in d, and the B's indicate that in the second, third and fourth operations the relevant factors were found in the lower C and D scales; the final T means that the last factor was found in the top scale.

In jotting down these letters we should write them horizontally, and after the first T write the remainder in pairs, thus: T BB BT. Each pair of different letters indicates that the result moves from one side of the slide to the other. When the letters of a pair are the same, i.e. two TT's or two BB's, the operations they govern do not move the result across the slide, and in checking off we ignore such pairs. In our example the first factor appeared in the upper scales, the BB we ignore, and the BT indicates we must cross the slide to the bottom scales, and the result, 9·04, lies there.

A longer example might result in the following sequence: T BB TB BT TT BT BB TB TT TB, and to reduce this we first strike out the BB and TT pairs; we are then left with T TB BT BT TB TB. Now since each TB or BT indicates crossing the slide, we may cancel these out in pairs, and this will reduce the symbols to T TB, indicating the result is on the opposite side of the slide from the first factor, that is to say, in D.

If the reader will work through a few examples he will find it an easy matter to make the check. There is no necessity to trace the result in its transition from side to side of the slide as the operations are made; it is difficult to do so. Having set X to the first factor, we move the slide or cursor to the other factors in turn, noting the T or B for each operation, and finally we cancel out as shown above to ascertain if the result lies in D or d.

We soon find that it is unnecessary to write down all the T and B signs we have used above. Every operation of multiplication or division, or combined division-multiplication, involves two factors and two movements:

For multiplication—Setting 1 or 10 of the slide scale, followed by setting X to a factor.

For division—Setting factor in slide scale, followed by setting X to 1 or 10 of slide scale.

For division-multiplication—Setting factor in slide scale to X followed by moving X to another factor in the slide scale.

If in any operation both factors lie on the same edge of the slide scale—and directly we use the first factor we can see if such is the case—we know that we have a TT or BB to sum up, and we do not even trouble to write them down. We are left with the necessity of recording cases in which the two factors lie on opposite edges of the slide scale. The first time this occurs we write a stroke thus: /, and the second time we add a stroke to complete a ×. At the end of the sequence, if we

disregard all the ×'s, we shall be left only with the initial T or B, or with the T or B followed by a /. If the T (or B) stands alone, the result of the computation will be read on the T (or B) side of the stock, but if a / follows, the result must be taken from the opposite side of the stock.

Problem 42. Find the value of

$$\frac{43{\cdot}8 \times 8{\cdot}28 \times 284 \times {\cdot}332 \times 719}{2{\cdot}32 \times 192 \times 505 \times {\cdot}266}.$$

(i) Start with the first factor in the numerator then divide by the first factor in the denominator, next multiply by the second factor in the numerator and proceed in this manner until all factors have been used.

(ii) Write down and reduce the T and B symbols. Repeat taking the factors in different orders to see if the rules for finding in which scale d or D the result must be read give consistent results.

(iii) Also work through the problem by first multiplying together the factors of the numerator and then divide by the factors of the denominator, taking them in the order in which they are printed. Write down the T and B symbols, reduce them, and compare with the answer given at the end of the book.

We hope the reader has persevered in this matter of using the long scale of the Precision rule. We are confident he will not regret the time he has spent, and we recommend him to purchase a Precision rule should he not already possess one and if his work is of such a nature that it will benefit by a higher degree of accuracy. We know one man who uses a 10/20 rule for all his ordinary calculations, and, although we do not think it desirable to advocate such a practice, we should be sorry to be deprived of its use when it is applicable; we definitely prefer it to the rather clumsy 20″ model we occasionally use.

Squares and square roots are obtained easily by projecting from the d or D scales, to the scales lying along the outer edges of the stock, and *vice versa*. These operations are so simple that they require no explanation.

The Precision rule is not recommended as a "first" slide rule; it is intended for those who are conversant with the standard type rule, and whose work demands its use.

Further examples, designed to illustrate the advantages of the 10/20 Rule will be found in Section 11, which explains the operation of the Dualistic slide rule. This rule is equipped with 10/20 scales. They occupy different positions on the rule from those we have just been studying, but the principle of working is the same.

SECTION TEN

THE ELECTRICAL RULE

THE rule illustrated in Fig. 14, is designed for general purposes, but has some special features particularly related to electrical calculations.

The two scales of temperature, Fahrenheit on the upper part of the stock, and centigrade on the lower part, give a ready means of converting from either thermometric scale to the other by projection, and in addition they are designed so that the variation in resistance of copper conductors due to change of temperature may be determined quickly. (These two scales lie in the part of the rule cut out in Fig. 14.)

Example: A copper wire has a resistance of 2·8 ohms at 20°C. Find its resistance at 5° C. and 200° F.

Set X to 20° Cent. 28C to X. X to 5° Cent.

Read 2·63 ohms under X in C.

Set X to 200° Fahr.

Read 3·6 ohms under X in C.

Dynamo and Motor Efficiencies

In some makes of slide rules special 5″ scales are fitted for calculating efficiencies of dynamos and motors. In the rule illustrated the same end is achieved with the aid of gauge points, and the efficiencies are found in 10″ scales. These efficiencies are always of the order of 80% and 90%, and, therefore, they are found in the crowded parts of the scales. The advantage of using a 10″ scale in place of the 5″ scale employed in other rules, is obvious.

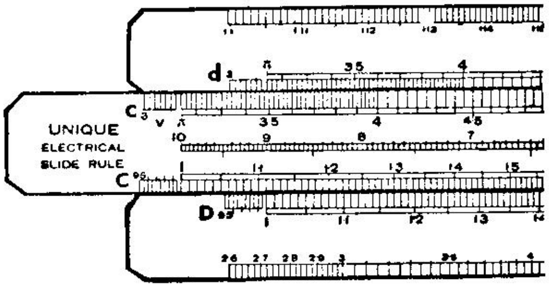

FIG. 14

Example: Calculate the efficiency of a dynamo which gives an output of 33·4 kw. for 51·6 h.p.

To 334D set 516C.

Read the efficiency, 86·6% in d opposite the gauge point N in c.

Problem 43. Calculate the efficiency of a motor which develops 161 h.p. for 137 kw.

(To h.p. in D set kw in C. Read efficiency in D or d opposite gauge point W in C or c.)

Volt Drop

In the case of direct current or induction free alternating current, the drop in potential along a copper conductor is obtained easily. Volt drop is given by the formula $\frac{I \times l}{c \times a}$ in which I is current in amperes, l is length of conductor in yards, c is conductivity of copper, and a is section of conductor in circular mils. (Circular mils = diameter of wire in thousandths of an inch squared.) The point V shown near each end of the c scale is used; it is the reciprocal of the conductivity

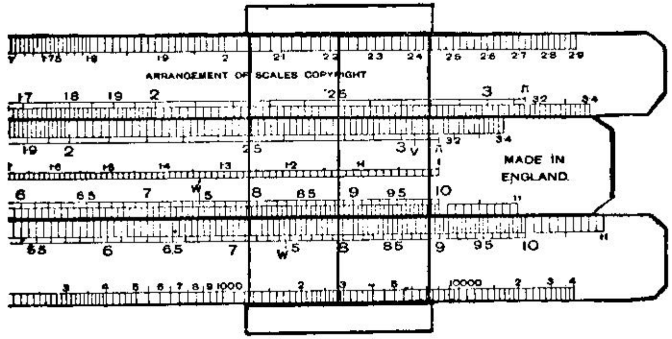

FIG. 14

of copper at 60° F. If the temperature differs much from 60° F., a correction should be made using the Fahr. scale as explained above.

Example: Calculate the volt drop in a copper conductor 208 yards long, ·18″ diameter, carrying a current of 20·4 amps. $180^2 = 32400$.

Set 1C to 204D. X to 208C. 324C to X.

Read 39·9 in d above V in c.

Volt drop: 39·9.

Duplicate C and D Scales

The illustration shows that the A and B scales, which we have pointed out earlier are of little value, have been omitted, and in their place 10″ scales identical in dividing and numbering with the C and D scales are substituted. These two scales are designated by c and d, and they are so positioned on the rule that π in d is immediately over 1 and 10 in D. This principle of displacing one scale relative to a similar one has been explained in Section 8. In the rule now under review all values in D are multiplied by π, by the simple process of projecting by means of the cursor from D to d, and conversely, values in d are divided by π when projected across to D.

There is a great number of practical problems in which π appears. Calculations relating to areas of circles, volumes and surfaces of spheres and cylinders, etc., necessitate the inclusion of π, and this arrangement of scales facilitates the manipulative operations of the rule. $\pi = 3{\cdot}14$.

Example: Calculate the area of millboard required to make a cylindrical tube $2\frac{1}{4}''$ diameter, 14″ long.

Set 1C to 14D. X to 2·25C.

Read in c under X.

Result: 99 square inches.

We have seen that when using the C and D scales of a standard slide rule the result sometimes is off the scale and it becomes necessary to re-set the slide by traversing it through its own length. This need never happen with the duplicated scales, since if the result is off D scale, it will be found on the d scale.

Another valuable characteristic of this rule is the additional facility it gives for setting quickly the slide and cursor, which need never be moved more than half the length of the rule for any operation.

If we remember that multiplication is effected by using the scales to add together the logarithms of the factors, the manipulation of the rule will quickly be appreciated. We are confident that many people who for years have used a slide rule equipped with the A, B, C and D scales would discard it for one giving greater facilities, if they would investigate the possibilities of other types. We will, therefore, carry our discussion of the duplicated C and D scales further and give a tyoical example of combined multiplication and division.

We will first take the simple case of 4×3. If we elect to perform this multiplication by using C and D only, we set 10C to 4D and read 12 in D under 3C. With the same setting of the slide, we also find the answer 12 in d above 3 in c, and very close to the 4 we commenced with. Now if we used the cursor in setting 10C to 4D, and we had other factors in our multiplication, we should need only to move the cursor about a quarter of an inch to pick up the 3 in c, whereas we must move it several inches if we work on the C and D scales only.

This simple exercise illustrates the saving in movement of slide and cursor when the two portions of the scales are used. We can go a step further and show how even shorter movements of the slide and cursor are possible, but this involves a complication which we prefer to avoid at this juncture. We refer to it again at a later stage.

If the reader will work through, step by step, the example following this paragraph, he will find no difficulty in using these duplicated scales, and provided he has had some previous experience of slide rules, we predict that he will prefer this type of rule to the more usual form.

Example: Evaluate $\frac{4 \times 8 \times 6 \times 9}{3 \times 2 \times 16}$.

To 4D set 3C.

Set X to 8c. 2c to X. X to 6C. 16c to X. X to 9c.

Result: 18 in D under X.

It is unimportant whether the first factor is selected in D or d, but we prefer to work as much as possible near the middle part of the rule; we choose our scales accordingly and adopt the following methods:

First method.—Having selected the first factor in D or d, and marked its position with the cursor, we move the slide to bring the second factor under X. The second factor lies in C and c, and we take the one nearest to the cursor. We next move the cursor to the third factor *in the same C or c scale in which the second factor was selected.* If we proceed in this manner of always taking the factors in pairs and in the same C or c scale, the result will lie in the scale in which we selected the first factor.

Second method.—There is an alternative procedure which may be adopted. We may start with the first factor in D or d, and bring into coincidence with it the second factor *in the adjacent C or c scale*. We then move the cursor to the third factor in C or c, selecting that which necessitates the least movement of cursor. The intermediate result will lie in the D or d scale which is adjacent to C or c scale in which the third factor was selected.

These instructions sound difficult, and in fact it is not easy to express them in words, but there is nothing complex to learn. The best way is to work through a few easy examples, and we think the reader will then agree that the duplicated C and D scales allow for more rapid working, and lead to greater accuracy.

The two methods of working we have defined may be used in conjunction with one another. We have adopted this procedure in the worked example above; the reader will see that we used the second method when using the factors 3 and 8, the second method for factors 2 and 6, and the first method for factors 16 and 9.

The reader will soon discover what appears to be a difficulty. Let us revert to the multiplication of 4×3. Set X to 4D. 1c to X. Result is 12 in D under 3c. Close to the 4 in D lies 3 in C, but if we project 3C across the slide to d we notice the answer is apparently in error, the cursor line falling a little below 12d. We also find in a similar manner the 2C falls below 8d, and 5C below 20d. In fact, all the values in C when projected into d give readings slightly below 4 times the values in the C scale. These discrepancies are not errors in the rule, but arise as a result of the manner in which the scales are placed relative to one another.

If the cursor has two hair lines drawn on it, at a distance of ·058″ apart, the apparent departures we have observed may be allowed for. Returning to our simple 4×3 example, we first set the cursor X to 4D then brought the 1c to X. If now we place the left-hand cursor line over 3C, the right-hand line will give the correct result 12, in d. We must, therefore, bear

in mind, in any operation in which we cross the slide to select our second factor and re-cross to select the third factor, we must cross the slide again, using the double line when reading the result. We do not recommend the use of the double-line cursor, as it is liable to lead to errors, especially when we are involved in a series of operations. If the multiple-line cursor is used, it is advisable to use one which has a staggered line quite separate from the central index line. The latter can be used in the normal way, and the former for the special purpose.

At the end of this section we make a brief mention of the type of slide rule with duplicated C and D scales which does not require a double-line cursor for the operations we have just discussed.

Reciprocal Scale

This scale lies along the middle of the slide and inspection of it discloses that it is divided in the same way as the C and D scales, but it is reversed and reads backwards, from right to left. We will designate this scale by R, as in the commercial rule.

By projecting direct from C to R, or *vice versa*, we obtain reciprocals. The reciprocal of any number being the result obtained by dividing 1 by the number, e.g. the reciprocal of 5 is one-fifth or ·2.

Square roots are conveniently obtained with the aid of this scale. We set the 1 or 10 of C to the number whose square root is required in D, and then slide the cursor along until we find a position in which the readings under the cursor index in scales R and D are identical. These readings are the square root of the number.

If the original number lies between 1 and 10, we shall set 1C to it; if between 10 and 100, we use the 10C index. For any number outside the 1—100 range we shift the decimal point in steps of two places to bring the number between 1 and 100, and after finding the square root, move the decimal point in the opposite direction one place for each step of two places originally made. This procedure is more fully described in Section 5.

There is another way of determining which index of Scale C should be used, or when scales A and D are being employed which half of scale A should be selected:

The rule is: If the original number has an odd number of digits preceding its decimal point, or, when less than unity, has an odd number of ciphers immediately following its decimal point, the left-hand half of scale A must be used, or, if the reciprocal scale is being employed, 10C should be set to the number in D. When the number of digits preceding, or the ciphers immediately following the decimal point in the original number is even, the right-hand half of scale A or the 1 of C must be used.

We mention this method of extracting square roots only as a matter of interest. We do not recommend it in practice. It is always better to use A and D scales, or if these are not available, the log-log scale or the method explained in Section 4 using C and D.

In conjunction, the C, D and R scales give a means of multiplying together three factors at one setting of the slide. Some of the standard rules, i.e. those supplied with A, B, C and D scales, are equipped with a reciprocal scale, and the property of multiplying three factors at one setting is usually claimed for this type of rule. We will investigate this feature.

Multiplication of three factors is effected by: setting the cursor to one factor in D; moving the slide to bring the second factor in R to X; reading the result in D (or d) opposite the third factor in C (or c).

Take the simple example of 4 × 5 × 6, the result of which, as we can see without using the rule, is 120.

Set X to 4D. 5R to X. The result, under 6C in this example, is off the D scale, but if we are using a rule with duplicate C and D scales, we find the answer, 120, in d opposite 6c. If, in addition to the R scale, we have only the C and D scales available, it is necessary to traverse the slide after the first setting in

order to obtain a reading, and there is no advantage in adopting this method. In a rule equipped with duplicate C and D scales, the result will always be obtainable at one setting of the slide, but occasionally it will be necessary to select the first factor in the d scale, and when this procedure is followed, the third factor must be projected across the slide to obtain the final reading on the opposite side of the stock (in d).

Dividing by two factors with a single setting of the slide, e.g. $\frac{4{\cdot}26}{{\cdot}035 \times 2{\cdot}88}$ can be effected with scales C, D and R. The cursor is used to mark the numerator in D (or d), one factor of the denominator in C (or c) is placed under X by adjusting the slide, and the result is read in D (or d) opposite the remaining factor in R. In this type of calculation, when using the ordinary rule, we find the same limitations, the result fairly frequently being off the scale. When this occurs, a second setting becomes necessary, and again there is no saving in time over and above using the C and D scales in the usual manner. The rule with duplicated C and D scales is much more convenient, the result always being obtainable at one setting of the slide. The method we invented in connection with the precision rule, in Section 9, for determining in which scale, D or d, the result lies, applies in exactly the same manner to our present problem; we ask the reader to turn back and study this method again; it is very simple.

In applying the method, regard scale R as being part of scale C, and if the two factors used lie in R and C, imagine them as being in one scale. This is easy to remember for, although the scales are in reality quite separate, they are intimately connected by virtue of their reciprocal relationship.

Example: We will now work an example with a number of factors to illustrate the time-saving effected, by employing scales C, D and R, and using the type of rule in which 1c lies immediately over $\sqrt{10}$C, as this will not necessitate the use of the staggered-line cursor.

Evaluate $3{\cdot}42 \times {\cdot}722 \times 5{\cdot}08 \times 13{\cdot}5 \times 2{\cdot}12 \times {\cdot}38 \times {\cdot}0818$.

Set X to 342D	B	The symbols for check on final scale reading are: B BT BT BT, or in the more condensed form—
722R to X	B	$B \times /$
X to 508c	T	
135R to X	B	indicating that the result must
X to 212c	T	be read in the top scale, i.e. in
38R to X	B	d, and we read the answer
X to 818c	T	there as 112. Approximation gives a result about 10. Our answer, therefore is 11·2.

The reader will see that there is a considerable saving in the movements of slide and cursor as compared with those necessary if multiplication is effected by C and D scales alone, and if we frequently have to make computations of this type, the method we have just used is worth adopting.

There is no saving in using the R scale in combined multiplication and division, since in such cases we can use the method of dividing by one factor and multiplying by another at one setting of the slide, as fully explained in Section 4.

Time saving can be effected when there are several factors in the divisor, and we will leave the following example for the reader to solve:

Problem 44. Evaluate

$$\frac{166}{2{\cdot}1 \times 3{\cdot}2 \times {\cdot}85 \times {\cdot}196 \times 4{\cdot}2 \times 34{\cdot}2}.$$

We would sum up a rather controversial subject in this way. If our work only occasionally involves these calculations, we would use the C and D scales in the normal way and not resort to the use of the R scale, since we may make errors by occasionally changing our way of working. But our work may involve a long list of three-factor calculations, all alike in form, only

differing in the actual numbers used. We think it may now be an advantage to use the C, D and R scales. Even with the standard type of rule, some results will appear at one setting of the slide, and this will effect a saving of time. With the C and D scales in duplicate the result will be always obtained at one slide setting, resulting in still further time saving.

We prefer to let the reader make his own decision in cases similar to the preceding example and problem. Time saving is effected by using the C, D and R scales in conjunction; it is a question of whether, when an isolated computation involving several factors in either numerator or denominator arises, it is worth while changing our method of working.

Expressing an opinion, with which some slide rule users will disagree, we would say there is very little advantage to be derived from the reciprocal scale when used in conjunction with the normal C and D scales.

If the C and D are duplicated, the reciprocal scale is perhaps worth its place.

SECTION ELEVEN

THE DUALISTIC RULE

IN Section 10 we examined the Electrical rule, which includes as part of its scale equipment duplicated C and D scales. We attempted to show that a saving of time can be effected when this type of rule is used. We explained that the c and d scales are identical with the C and D scales, but that the former are positioned so that the π in c is immediately over the 1 and 10 of C and the π in d over the 1 and 10 of D. This arrangement of scales enables the user to multiply or divide by π by merely projecting by means of the cursor index X from D to d or *vice versa*. We pointed out that a special 2-line Cursor is necessary if full advantage is to be taken of the duplicated scales.

We propose now to study a type of rule which, although closely resembling the Electrical rule in respect of the duplicated scales, is different in some respects.

In this particular rule, shown in Fig. 15, the 1 of c is

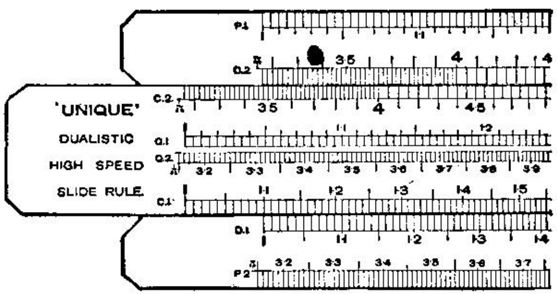

FIG. 15

immediately above $\sqrt{10}$ in C, and similarly, the 1 of d is directly over $\sqrt{10}$ in D. Now $\sqrt{10}$ (which is exactly midway between 1 and 10 of any scale, as will be agreed if a little consideration is given to the logarithms of the numbers) has a value very near to 3·16 and thus is not far removed from $\pi(= 3{\cdot}14)$ so at first sight the two rules we are comparing may appear identical in the layout of their duplicated C and D scales.

The slight difference in the relative positions of the "folded" scales means that we cannot with the Dualistic rule multiply or divide by π by simple projection, but as a compensation, the 2-line cursor is not required for the comprehensive use of the Dualistic rule.

(The reader will readily appreciate that an additional broken line can easily be added to the cursor to provide the facility of multiplying and dividing by π, but we do not recommend such an addition. In any event, the point is of no great importance since the other features of the duplicated C and D scales are predominant.)

Apart from work of a specialised nature, probably 90% of the computations effected by slide rule involve the use of the C and D scales only. In the Dualistic rule these scales occupy their usual positions. They are designated by the symbols C_1

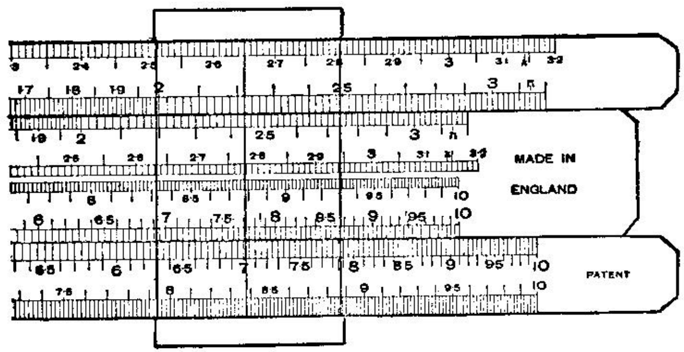

FIG. 15

and D_1 and lie along the lower edge of the slide, and on the adjacent edge of the stock, respectively, as seen in Fig. 15

The upper margin of the slide, and the edge of the stock adjacent to it, are equpped with modified C and D scales. These are designated by C_2 and D_2 respectively, and are used in conjunction with the C_1 and D_1 scales, as described below.

The extreme margins of the stock and the centre of the slide are provided with a pair of 20" scales. These will be recognised as the principal scales of the 10/20 rule. They may be used separately; they are equivalent to a 20" rule and give the same high degree of accuracy. Scale references P_1, P_2, Q_1 and Q_2. These scales may be used in conjunction with the C_1 and D_1 scales, as will be demonstrated presently.

On the reverse of the slide three scales, LL_1, LL_2 and LL_3, will be found; these are three sections of a continuous log-log scale, extending from 1·01 to 40,000, and are used with the slide inverted in conjunction with the D_1 scale.

C_1 and D_1 Scales

As stated above, these are the C and D scales of the standard type slide rule. The instruction given in Section 4, dealing with the operations of multiplication and division, apply without modification, except that C and D should be read as C_1 and D_1 respectively. In all cases when a calculation involves the use of the C and D scales only, the standard slide rule, or the Dualistic rule, may be used without any discrimination.

C_2 and D_2 Scales

On inspection, it will at once be seen that these two scales are divided in the same manner as the C_1 and D_1 scales, but they are placed differently on the rule. The 1 of the scales C_2 and D_2 is in the middle of the length of the rule. The scales commence at π at the left-hand end of the rule; the readings increase, reaching 10 (or 1) at the mid-point, and then increase, reaching π at the right-hand extremity of the rule.

Scales C_2 and D_2 should not be used alone for multiplication and division. A few simple examples will at once show that, although multiplication or division may be effected with their aid, frequently the result is off the scale at the first setting, and cannot be obtained without traversing the slide through its own length. The same dilemma sometimes arises with the C_1 and D_1 scales when multiplying, but the traversing of the slide is rather more easily effected. In any case, there is no advantage gained by using C_2 and D_2 in preference to C_1 and D_1, and since all slide rule users are familiar with the C_1 and D_1 scales, it is advisable to adhere to them. To illustrate this point the reader is asked to compute 6×4 using C_2 and D_2 scales. On setting 1 C_2 to 4 D_2 it will be found that the 6 C_2 lies beyond the D_2 scale at the left-hand end. The result may be obtained by traversing the slide. Set the cursor index X to the π near the right-hand end of C_2, and move the slide to bring the π at the left-hand end of C_2 under X. Immediately above 6 C_2 will be found the result, 24, in D_2.

Scales C_1, D_1, C_2 and D_2 used in Conjunction

Computations involving multiplication and/or division are more rapidly effected when using these four scales than when C_1 and D_1 only are employed. The following example is designed to illustrate this feature:

Compute the value of $\dfrac{3{\cdot}1 \times 6{\cdot}4 \times 9{\cdot}2}{1{\cdot}5 \times 11{\cdot}2}$.

Using C_1 and D_1 only, and performing division and multiplication alternately since this saves time, the following operations are required:

To $31D_1$ set $15C_1$.

Set X to $1C_1$. $10C_1$ to X. X to $64C_1$. $112C_1$ to X. X to $1C_1$. $10C_1$ to X.

Result: 109 in D_1 immediately under $92C_1$.

Approximation, performed mentally, shows the answer is of the order 10, and the result therefore is 10·9.

Using the four scales:

To $31D_1$ set $15C_1$.

Set X to $64C_2$. $112C_2$ to X.

Result: 109 in D_2 above $92C_2$.

If the rule is used to carry out these two series of operations, it will be found that by using all four scales the number of movements of slide and cursor is greatly reduced and the actual distances through which the slide and cursor are moved in these various operations are very much smaller. Greater accuracy will be attained, because in the course of time all slide rules develop small errors in their scales due to shrinkage or other distortion, and scales which originally were identical differ slightly in length. Critical inspection will almost invariably show that in slide rules which have been in use for some time, the overall lengths of scales on slide and stock differ slightly. With such a rule, imagine multiplication of 12×4 is being effected using C_1 and D_1 scales. Set $1C_1$ to $12D_1$ and the result, 48, appears in D_1 under $4C_1$. If the slide scale has, through shrinkage, become, say, slightly shorter than the stock scale, a small error will be seen, the $4C_1$ falling just below the $48D_1$. Using the Dualistic rule, set $1C_1$ to $12D_1$, and read the result, 48, in D_2 over $4C_2$. With this setting the length of slide scale used is only about 1″, and the error will be only about one-sixth of that involved in using the C_1 and D_1 scales, where the length of slide employed is about 6″. The same argument applies to any series of operations.

The principle involved in using the four scales of the Dualistic rule is the same as that employed in slide rules generally. Multiplication and division are effected by adding or subtracting logarithms, but with two sets of scales available there are alternative scale readings provided, and the manipulation of the rule is easier and speedier than in the case of the standard slide rule.

In using the Dualistic rule the first factor is selected in the D_1 or D_2 scales. The choice of scales is unrestricted, but it is an advantage to start with that scale in which the first factor lies near the middle of the length of the rule. If the first factor lies between 2 and 6, use the D_1 scale, but if it lies between 6 and 2, start in the D_2 scale. For the factors used subsequently there are alternative scale readings, and the one lying nearest should be used. An example will make this selection of factors clear.

Example: Evaluate $3 \times 1{\cdot}2 \times \frac{9}{4} \times 2{\cdot}5$.

Set $1C_2$ to $3D_1$ (using X). X to $12C_2$. $4C_1$ to X. X to $9C_2$. $1C_2$ to X. X to $25C_1$.

Result in D_1 under X is 20·2.

It will at once be noticed that the movements of slide and cursor are small compared with those necessary if the C_1 and D_1 scales are used alone.

Whether the final result appears in D_1 or D_2 depends upon which scales, C_1 or C_2, were used for the intermediate factors; the determination presents no difficulty. Very often, especially in short computations, the order of the result is already known, and the slide rule is used to obtain an accurate figure. In such cases it is only necessary to glance at the results lying in D_1 and D_2 under X. These values differ in the ratio of $\sqrt{10}$ to 1, i.e. about 3·16 to 1. In such a case the appropriate reading will be obvious.

In longer computations, when the result is not obvious, a rough approximation should be made to determine the position of the decimal point. This approximation will also disclose in which of the two scales D_1 or D_2 the result lies.

The following method for determining in which scale the result lies may be preferred, and the reader is advised to spend a few minutes making himself familiar with it, since it applies also to the 20″ scales which will be dealt with later. The method may, at first reading, sound complicated. It is, in fact, very easy of application and has earlier been explained in Section 9, but we think some repetition here may be desirable. We have in mind also the fact that an experienced slide rule user may be reading this section without having perused the earlier notes.

Every multiplication, or division, or combined multiplication and division, involves using two factors in the C scale. In a multiplication the 1 (or 10) C is set to some value in D, and the result found in D opposite the multiplying factor in C. In division the divisor in C is first set, and the result read opposite the 1 (or 10) C, and in multiplication/division, the divisor in C is set and the quotient obtained opposite another factor in C. In applying the method—which we believe to be original—it is only necessary to observe whether the two factors are both in the same C scale or whether one is in C_1 and the other in C_2. If the two factors are selected in different sections of the C scale, the result is obtained by crossing from D_1 to D_2, or *vice versa*; if both factors lie in the same part of the C scale, the result will be found in that part of the D scale in which the number being multiplied or divided appeared.

A simple example may assist. Suppose multiplication of 8×3 is desired. There are six different ways of obtaining the result, 24, they are:

(*a*) Set 1 C_2 to 8 D_2. Result in D_2 opposite 3 C_2.
(*b*) „ 1 C_2 to 8 D_2. „ „ D_1 „ 3 C_1.
(*c*) „ 10 C_1 to 8 D_1. „ „ D_1 „ 3 C_1.
(*d*) „ 10 C_1 to 8 D_1. „ „ D_2 „ 3 C_2.
(*e*) „ 1 C_2 to 8 D_1. „ „ D_2 „ 3 C_1.
(*f*) „ 10 C_1 to 8 D_2. „ „ D_1 „ 3 C_2.

the cursor index X being used in setting where necessary.

In the first and third settings of the slide the two factors 1 and 3 lie in the same section of the C scale, namely, both in C_2 in the first, and both in C_1 in the third method. In both the result lies in the section of the D scale in which the factor 8 was chosen. In the other four methods the factors 1 and 3 lie in opposite sections of the C scale, and the result is always in the opposite section of the D scale from that in which the first factor 8 was selected.

In the simple example just cited it is easy to determine in which part of the D scale the result will be found, but in a longer one it is advisable to record the various operations as now suggested. When the first slide setting is made, note which section of the D scale is used and jot down D_1 or D_2 as the case may be. If in the next operation the two factors used are in the same section of the C scale, take no further notice of them, but if they are in different sections of the C scale write a stroke thus, /, following the D_1 or D_2. Proceed in this way, making a stroke each time scale C_1 and C_2 are both used in any one setting of the slide, the second stroke cancelling the first by changing it into a $\times$, so the record starting with, say D_1, would next become D_1/, and then $D_1 \times$. At the end of the computation the record will finish either with D_1, or D_1 /, or $D_1 \times$. If the last symbol is a stroke, the final result will lie in the D_2 scale; in other cases it will lie in D_1.

Example: Evaluate $\frac{2{\cdot}8 \times 93 \times 107 \times 46}{18 \times 52 \times 29}$.

Set X to $28D_1$ and jot down D_1

„ $18C_2$ to X
„ X to $93C_1$ } „ „ /

„ $52C_1$ to X
„ X to $107C_2$ } „ „ \

„ $29C_1$ to X
„ X to $46C_1$ } No symbol necessary here.

Under X read 472 in D_1, and 1495 in D_2.

The symbols when written down in line result in $D_1 \times$; the indication is that the result is in D_1. Approximation gives 46 and the result is 47·2.

The 20″ Scales

The scales lying along the top and bottom edges of the face of the stock designated by the symbols P_1 and P_2 respectively, together form a 20″ logarithmic scale, and in combination with a similar pair of scales placed in the middle of the slide and designated by Q_1 and Q_2 form the equivalent of a 20″ slide rule.

When a higher degree of accuracy than can be derived from the 10″ C and D scales is desired, the P and Q scales should be used. Inspection of the illustration will show the additional dividing which has been made possible by the use of these long scales.

Multiplication and division are effected by using the P and Q scales and the cursor index X. The method given earlier for determining whether the final result should be read in D_1 or D_2 may be adopted when there is any doubt as to whether the result appears in P_1 or P_2. This method has already been dealt with fully and need not be repeated. Two examples are now given to illustrate the use of the 10/20 scales.

Example: Evaluate $\frac{13{\cdot}65 \times 23{\cdot}4}{39{\cdot}6}$.

Set X to $1365P_1$. $396Q_2$ to X. X to $234Q_1$.
Result is 807 in P_2.

The value in P_1 under X is 255 and it is obvious that this result is incorrect. 396 appeared in Q_2 and 234 in Q_1, therefore the result must be in P_2, since the first factor, 1365, is in P_1. It is quite unnecessary to write down the symbols, but if, for illustration only, we do so, they will be P_1 /. The stroke at the end indicates the final result is in the opposite scale to that in which the first factor, 1365, was found: 13 into 39 is 3, and 3 into 24 gives 8 as an approximate result. Now, with two values under X, 807 and 255, there is no difficulty in selecting the correct one and at the same time inserting the decimal point. Result: 8·07.

A longer example is now given:

Evaluate $\frac{4{\cdot}4 \times 69{\cdot}2 \times 24{\cdot}6 \times 1{\cdot}246 \times 36}{15{\cdot}1 \times 82{\cdot}2 \times 18{\cdot}6 \times 28{\cdot}1}$.

Set X to $44P_2$. Note down P_2

$151Q_1$ to X X to $692Q_2$	,, ,, /
$822Q_2$ to X X to $246Q_1$	,, ,, \
$186Q_1$ to X X to $1246Q_1$	No symbol required here.
$281Q_1$ to X X to $10Q_2$	Note down /
$1Q_1$ to X X to $36Q_2$	,, ,, \

The symbols written in line should appear P_2 × ×, showing the result is in P_2. It is 519.

Approximation.—4·4 into 15 is slightly over 3, which divides into 69 about 20: 20 into 82, say, 4, 4 into 24 gives 6, 6 into 18 is 3, and 3 into 36 gives 12: 12 times 1·2 is 14 approximately and we are left with $\frac{14}{28}$ = ·5 as the approximate result. The actual result is, therefore, ·519; it lies in P_2, as indicated by the symbols.

The procedure explained in Section 4 for determination of position of decimal point, may be used if desired, but we strongly recommend the approximation method as being easier and safer. In a long computation there is a risk that a factor may be inadvertently omitted in the slide rule manipulation. The approximation if carefully made will disclose the error—another sound reason for making it.

The reader is now advised to practise the use of this new rule by working through a few simple examples, the results of which may easily be checked. It is confidently predicted that when familiarity with the scales is attained the rule will make an appeal as being superior to the standard type. The difficulties—if there are any—have now been dealt with and the remaining instruction deals with simple points.

Squares and Square Roots

The relative positions of the 10″ and 20″ scales give a ready means of evaluating squares and square roots. The square of any number is obtained by projecting by means of X, the number from either Q_1 or Q_2 into C_1. For example, 1·6, when projected from Q_1 into C_1, gives 2·56. When projecting from Q_2 to C_1, the squares are 10 times the actual values of the numbers engraved along the C_1 scale. 5 in Q_2 lies immediately above 2·5 in C_1, and this value must be read as 25. Readers now familiar with the A, B, C and D scales of a standard rule will notice the similarity in procedure. They will also notice the higher degree of accuracy possible with the longer scales.

Square roots are obtained by the reverse process of projecting from C_1 into Q_1 or Q_2. Square roots of numbers from 1 to 10 are obtained by projection from C_1 into Q_1, and square roots of numbers from 10 to 100 by projection from C_1 into Q_1. When a number whose square root is desired lies outside the range 1 to 100, the procedure outlined in Section 5 should be used, reading Q_1 for the left-hand half of scale A, and Q_2 for the right-hand half.

Scales P_1, P_2 and D_1 may be used for squares and square roots if preferred; the procedure will be obvious from the instructions given above.

Cube and Cube Roots

Cubes are easily obtained by setting the 1 (or 10) of C_1 to the number in D_1; the cube lies in D_1 immediately below the number in Q_1 or Q_2. To cube 2·2 set $10C_1$ to $2{\cdot}2D_1$; set X to $2{\cdot}2Q_1$ and read in D_1 under X the result, 10·65, the decimal point being inserted by inspection.

Cube roots are evaluated by setting X to the number in D_1 and then moving the slide until the value in D_1 coincident with 1 (or 10) C_1 is the same as the number in Q_1 or Q_2 under X. Suppose the cube root of 2 is required. Set X over 2 in D_1; now move the slide about an inch to the right of its mid-position, and then carefully adjust it until the value in D_1 coincident with $1C_1$ is the same as the value in Q_1 under the cursor index X; this value will be found to be 1·26, which is the cube root of 2 (1·25992).

It may assist to observe that:

If the number lies within the range 1-10 its cube root will be found in Q_1 and coincident with $1C_1$.

If the number lies within the range 10-31 (π^3) its cube root will be found in Q_1 and coincident with $10C_1$.

If the number lies within the range 31-100 its cube root will be found in Q_2 and coincident with $1C_1$.

If the number lies within the range 100-1000 its cube root will be found in Q_2 and coincident with $10C_1$.

Numbers from 1 to 1000 have cube roots from 1 to 10. If the number whose cube root is required is not within the range 1-1000 it should first be altered by moving the decimal point three, or multiples of three, places to right or left to bring the number within that range. The cube root should then be found as detailed above, and finally the decimal point of the result should be moved *back* one place for each step of three places made in the original number.

Example: Find the cube root of 116,300.

Moving the decimal point three places to the left alters the figure to 116·3, which lies within the 1 to 1000 range. The cube root of 116·3 is 4·88. The decimal point must now be moved one place to the right, giving the actual result as 48·8.

In evaluating cube roots it is a good plan to find the nearest integral result mentally.

Example: Find the cube root of ·682. First move the decimal point three places to the right, so that the number becomes 682. The cube of 5 is 125, which is well below 682. Try the cube of 7; $7 \times 7 = 49$ (say 50); $7 \times 50 = 350$, still too small; try 9; $9 \times 9 = 81$; and $9 \times 80 = 720$. The required cube root is less than 9. Set X over 682D_1; and the slide so that 10C_1 is over 9D_1. Now move the slide slowly to the left until the reading in D_1 below 10C is the same as that in Q_2 under X. These identical values are 8·8. The cube root of 682 is 8·8, and of ·682, ·88.

Squares, square roots, cube and cube roots, may be evaluated easily with the aid of the log-log scales, often with a higher degree of accuracy than can be attained with the P and Q scales.

Log-log Scales

When the log-log scale is used the slide should be inverted so that the surface which generally is underneath is brought uppermost, or, if the log-log scale is fitted to a separate slide, the slides should be interchanged. The log-log scale provides a means of effecting unusual computations. *Its most useful property is the ease with which powers and roots may be evaluated, even when the power root is a mixed number.*

Suppose the value of $8{\cdot}4^{1{\cdot}79}$ is required. Set 8·4LL_3 to 1D_1, then immediately above 1·79D_1 will be found the result, 45·1, in LL_3. If the index of the power is negative, e.g. $8{\cdot}4^{-1{\cdot}79}$, the value of $8{\cdot}4^{1{\cdot}79}$ should first be evaluated and the reciprocal of this be found, using the C and D scales;

$$\text{i.e.} \quad 8{\cdot}4^{-1{\cdot}79} = \frac{1}{45{\cdot}1} = {\cdot}0222.$$

To evaluate $\sqrt[4{\cdot}15]{1{\cdot}31}$; to 415$D_1$ set 1·31LL_2, and then use X to project 10D_1 into LL_1 and read the result 1·067.

Results outside the range of the log-log scale may be obtained by the methods suggested in Section 6.

Logarithms to any base may be obtained by setting the base in LL to 1 of D_1, or 1 of D_2 and projecting the number whose log is required from the log-log scale into D_1 or D_2. The log so obtained will be complete, comprising characteristic and mantissa. Common logarithms are found by setting the 10LL_3 to 1D_1. It will be seen that immediately below 100LL_3 stands 2D_1, 2 being the log of 100. Below 1000 stands 3 and below 10,000 stands 4. To obtain the logs of numbers in LL_1 and LL_2 the cursor index must be used to project into D_1. If the number whose log is required lies towards the left-hand end of the log-log scale, 10LL_3 should be set to 10D_1, or 1D_2.

Natural logarithms are obtained by setting the value 2·7183 near the left-hand end of the LL_3 scale to the 1 of D_1. This setting will enable all the natural logs of numbers within the range of the log-log scale to be read without moving the slide; the numbers in LL_1 and LL_2 being projected into D_1 by using the cursor index X.

It is useful to remember that the 10th powers of numbers in LL_1 lie immediately below in LL_2, and the 10th power of numbers in LL_2 lie immediately below in LL_3.

SECTION TWELVE

THE BRIGHTON RULE

THE manufacturers of slide rules are confronted with a variety of difficulties quite separate from those connected with production.

Production problems are almost confined to the difficulty of securing first-class materials.

During the war period and for several years after, timber and plastics, which are the principal raw materials of manufacture of most types of slide rules, were of very poor quality and often gave rise to difficulties in production.

Apart from the initial seasoning of materials, slide rules undergo a further period of seasoning when partly made.

Final inspection is the last process carried out by every reputable manufacturer of slide rules. Each rule is subjected to a critical examination just before despatch from the factory, and any which shows a defect is returned to the production line for rectification. However, it may be six months or much more before the rule reaches the user, in the meantime having been in transit to a distant overseas customer or having lain in stock or some wholesale or retail establishment. The rule may have been displayed in the retailer's shop window exposed to sunlight or damp conditions and then passed on to the purchaser.

A slide rule is unlike most manufactured products in so far as it can be quickly checked against itself. The first test to apply is to line up the slide in its mid-position and to check the lengths of the C and D scales, which should be identical. Next, if the rule is equipped with A, B, C and D scales, the slide should be set so that the 1 of B is coincident with the 10 of A, then check to see if the 10 of B lines up accurately with the 100 of A. The observant reader will notice a number of similar tests. If there are perceptible discrepancies the rule has

suffered some deterioration since it left the factory. If complaint of inaccuracy is made to the source of supply, the rule will almost always be returned to the manufacturer for replacement. All manufacturers are from time to time called upon to make replacements, but in the normal course of business this is not a very serious item. It has the good effect of keeping a high standard of production in the factory and some control over the activities of the distributing agents.

Another kind of difficulty sometimes facing the producer is to satisfy the requirements of the overseas purchasing organisations in respect of the arrangement of scales on a slide rule. Frequently an overseas merchant will refuse a particular type of slide rule because the scales are not quite the same as those with which he is familiar. Sometimes the difference is quite unimportant and may only be a matter of a different arrangement of the same set of scales. Purchasing agencies in the United States of America are usually most insistent that the rule they require must have certain scales arranged in a particular manner, and although there is no difficulty involved to redesigning a slide rule, an undue variety of rules is apt to cause confusion and to hamper production.

The Brighton slide rule, Fig. 16, which is the subject of this section, was designed to meet the requirements of one of the Continental countries whose purchasing agents demanded a specific arrangement of scales. There is nothing which makes it more suitable for one country than for any other.

The scale equipment is very extensive. All the well-known scales are included, viz:

Scales A, B, C and D.

Reciprocal scale. Reference $\frac{1}{X}$.

Cubes and Cube Roots scale. Reference X^3.

Logarithmic scale. Reference L.

Sine scale. Reference S } These scales are subdivided
Tan scale. Reference T } in decimals.

Three-section log-log scale. References LL_1, LL_2, LL_3.
Pythagoras scale. Reference $\sqrt{1 - X^2}$, or Py.
Linear scale in cms.

Fig. 16 shows the face of the rule. The linear scale and the sin and tan scales are on the edges of the rule and the log-log scale on the reverse of the slide.

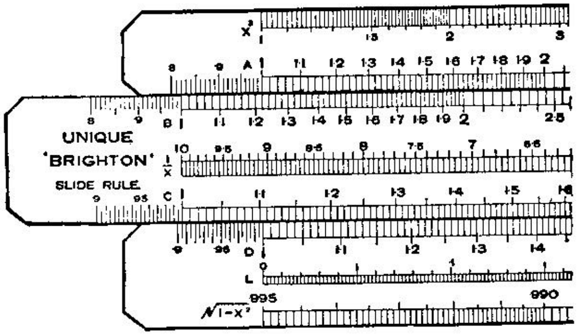

FIG. 16

The data slip on the back of the rule gives all the useful trigonometrical formulae and there is a decimal equivalent table which shows when the slide is withdrawn.

Log-log Scale

The three-section log-log scale in the Brighton rule has been extensively dealt with in Section 11 and needs no further mention.

Sine and Tangent Scales

These appear on the edge of the rule and are read by means of a small index attached to the cursor. Instead of being subdivided in minutes, these two scales are subdivided in decimals of degrees. Both sine and tan scales are used in conjunction with scale D.

Sine Scale

Since $\sin^2 A + \cos^2 A = 1$, it follows that the values of sin A and cos A will be found in D and Py respectively if the cursor index is set to A in the sin scale. (Example—set cursor to 30° and find ·5 in D and ·866 in Py.)

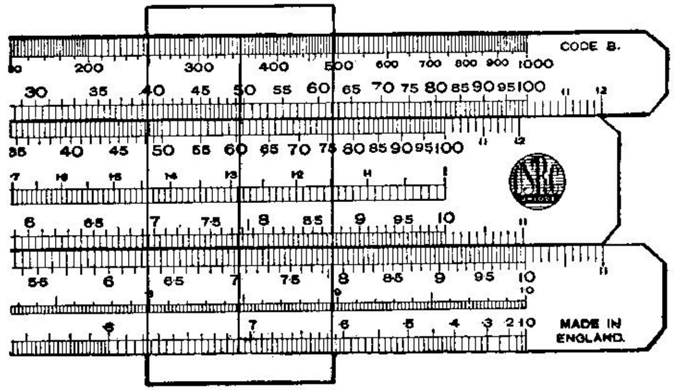

FIG. 16

Tan Scale

Line up the slide so that scales C and D coincide. By projecting from the T scale, the tangent and cotangent appear in D and $\frac{1}{X}$ respectively. (Example—tan 13·4° = ·238 and cotangent 13·4 = 4·2.)

X³ Scale

Numbers in D are cubed when projected into X^3. Cube roots of numbers are found by projecting from X^3 into D.

L Scale

The mantissae of common logs of the numbers in D appear directly below in L. (Example—log 2 = ·3010.)

All these scales have been dealt with in earlier sections of this book, with the exception of the Pythagoras scale, and very brief mentions are made here.

Scales A, B, C and D are the subjects of Sections 4 and 5.

The reciprocal scale is fully discussed in Section 10.

The cubes and cube roots scale is dealt with in Section 5. It will be seen in Fig. 16 as the scale lying along the upper edge of the stock.

The log scale is mentioned in Section 3. With its aid the mantissae of common logarithms are obtained by projecting the numbers in scale D into scale L, e.g. 2D projected by the cursor index into scale L gives ·3010.

The Pythagoras scale is an important addition. With its aid, time is saved in solving right-angled triangles, and there are many examples in technical problems and in dealing with vector quantities when it can be used with advantage.

As a matter of interest let us time ourselves in solving a simple problem in the usual way and by using the slide rule equipped with the Pythagoras scale. We will call this scale Py for ease of expression.

Example: In a right-angled triangle the hypotenuse is 21·7″ long and one other side is 18·3″ long. We desire to find the remaining side and angles. (See Fig. 17.)

Square 21·7 using C and D scales = 470

" 18·3 " " " = 335

Subtract 135

Extract sq. root of 135 = 11·6.

$$\tan x = \frac{11 \cdot 6}{18 \cdot 3} = \cdot 633.$$

$x = 32{\cdot}4°$, $y = 90 - 32{\cdot}4° = 57{\cdot}6°$.

Results: Side = 11·6

Angles = 32·4° and 57·6°.

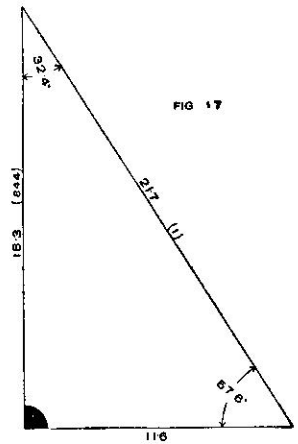

FIG. 17

If the reader will work through this first method, writing down only the necessary figures, he will probably require about 2 minutes. We used scales C and D in squaring and extracting sq. root for the sake of accuracy. A little time could perhaps be saved by using scales A and D, but the employment of the 5″ scales would be a disadvantage.

Solutions using the Py scale:

Divide 18·3 by 21·7 using C and D, which gives ·844.

Move X to 844 in Py.

Set 10C to X.

Under 217C read 1165D.

Under X read 32·4° in S.

Other angle 90 − 32·4 = 57·6°.

Time taken, under 1 minute.

SECTION THIRTEEN

INDICES AND LOGARITHMS

In the foreword we said that for the benefit of those who desire to learn something of logarithms we would give a simple explanation of the nature and qualities of this intriguing chapter of mathematics. We shall not take the reader into deep waters, but shall confine our remarks to the elementary parts of the subject.

Indices

We must first learn a few simple rules concerning the raising of numbers to different powers.

We prefer to let our numbers or quantities be represented by letters, such as a, or b, or m, or n, but we shall sometimes use actual figures. The reader will have seen in technical books, or in newspapers, such expressions as a^2, b^3, m^5 or n^8, and he may have speculated as to what the small figures printed on the right-hand side above the letters signify. The numbers, 2, 3, 5 and 8 are termed indices of a, b, m or n respectively. a^2 means, $a \times a$, and is read as a squared; b^3 means $b \times b \times b$ and is read b cubed. Similarly for m^5, which is read m to the 5th power, or simply m to the fifth, and n to the eighth. The index of a number, therefore, indicates how many times 1 is to be multiplied by the number. The indices in the above examples are all positive integers, i.e. whole numbers. Indices may be positive (+) or negative (−). They may be numbers or fractions, e.g., a^2, a^{-4}, $a^{\frac{1}{2}}$, $a^{-\frac{5}{3}}$.

a^{-4} is a different way of writing $\dfrac{1}{a^4}$.

$a^{\frac{1}{2}}$,, ,, ,, $\sqrt{a}$.

$a^{-\frac{5}{3}}$,, ,, ,, $\dfrac{1}{a^{\frac{5}{3}}}$.

$a^{\frac{5}{3}} = \sqrt[3]{a^5}$, expressed in words is the cube root of the fifth power of a.

There are three simple rules for indices which must be understood:

First rule.—To multiply together powers of a quantity, add the indices. To divide, subtract the index of the divisor from the index of the dividend.

Example: Multiply $a \times a^2 \times a^4$ (a means a^1, but the index 1 is always omitted).

$$a \times a^2 \times a^4 = a^{1+2+4} = a^7.$$

Divide a^6 by a^2.

$$\frac{a^6}{a^2} = a^{6-2} = a^4.$$

The student will, we hope, recognise the connection between these examples and the method of working of the slide rule.

Problem 45. Multiply $a^{\frac{3}{4}} \times a^3 \times a^{-\frac{1}{4}}$.
Divide $a^{\frac{1}{2}}$ by a^2.

Second rule.—To calculate the power of a power of a quantity, multiply the two indices together.

Example: $(b^3)^2 = b^6$.

$$(b^4)^{-\frac{1}{2}} = b^{-2} = \frac{1}{b^2}.$$

Third rule.—To raise to any power, a quantity which consists of several factors, raise each factor to the given power.

Example: Raise $a^2bm^{\frac{1}{2}}n^{-3}$ to the 5th power.

$$(a^2bm^{\frac{1}{2}}n^{-3})^5 = a^{10}b^5m^{\frac{5}{2}}n^{-15}.$$

Problem 46. Reduce to a simple form: $(x^{-\frac{1}{4}}y^{-\frac{1}{2}})^{-8}xy^{-4}$.

Logarithms

By using logarithms we can often calculate much more rapidly than by ordinary arithmetic. There are some calculations which may readily be made with the aid of logs which would be almost impossible otherwise. The slide rule is the mechanical method of computing by logs.

A logarithm is a number. It is the power to which one quantity must be raised to make it equal to another quantity.

For instance, if $a^b = N$, then we might say that the quantity a (called the base) must be raised to the power b in order to be equal to the quantity N, or, expressed in the orthodox manner, b is the logarithm of N to the base a.

Example: $5^3 = 125$, therefore the log of 125 to the base 5 is 3.

Problem 47. What is the log of 32 to base 2?

There is a system of logarithms termed Naperian—after the inventor of logarithms—or natural or hyperbolic logarithms, which has as base the number 2·7183, always denoted by e. This system of logs is of great importance in some parts of mathematics; it is never used for ordinary calculations such as we are concerned with, and the reader need not be in the least concerned with it at present; a brief reference to it was made in Section 6.

In all our numerical work we use a system of logarithms to base 10, generally called common logarithms. In the following notes we shall be dealing with common logs to the base 10 exclusively:

$$10 = 10^1 \quad \therefore \log 10 = 1.$$

$$100 = 10^2 \quad \therefore \log 100 = 2.$$

$$10{,}000 = 10^4 \quad \therefore \log 10{,}000 = 4.$$

$$1{,}000{,}000 = 10^6 \quad \therefore \log 1{,}000{,}000 = 6.$$

Multiplication and Division

To multiply together 100 and 10,000 we might apparently take the log of 100, which is 2, the log of 10,000, which is 4, and add them together, $2 + 4 = 6$. If we now look for the number whose log is 6, we see it is 1,000,000, which is the product of $100 \times 10{,}000$.

From the foregoing example we deduce the simple rule that the log of a product is the sum of the logs of the individual factors.

The reader will see that we have merely used the first index rule mentioned a little earlier in this section, and he will no doubt be able to anticipate the rule for division, which is: The log of the quotient is obtained by subtracting the log of the divisor from the log of the dividend.

If we divide $10{,}000 \div 1000$ mentally, we obtain 10 as the result. The log of 10,000 is 4, the log of 1000 is 3, and the difference between these two logs is 1, which is the log of 10. Our rule for division gives a correct result.

So far we have dealt only with numbers which are integral powers of 10. We can write down the logs of all such numbers by noticing that the log is the number of noughts following the 1. Between 10 and 100 there are 90 whole numbers, 11, 12, 13—97, 98, 99, and it is clear that if we desire to use a number, say, 43, as a factor of multiplication or division, we must be able to write down its logarithm: 43 lies between 10 and 100, and its log must lie between 1 and 2, since $\log 10 = 1$, and $\log 100 = 2$. Tables of logarithms have been calculated for our use. There are four-figure logs, in which the decimal part is given to four places of decimals, and these are sufficiently accurate for many purposes, and the whole range of figures is printed on four pages of this book. For more accurate work there are five-figure and seven-figure logs, the latter would fill perhaps 200 pages of this book and cannot, of course, be given. They require a book to themselves, and are to be found in books of Mathematical Tables, which are devoted to logs and other tables.

If we consult the table on page 174 and run our eye down the first column of figures, we find 43 a few lines from the bottom. In the second column we find the figures 6335. This is the decimal part of the log of 43, and it is called the mantissa of the log. Now we know that the log of 43 lies between 1 and 2, and with the aid of the table we can now write it down; it is 1·6335. The integer, or whole number part of the log—the 1 in this case—is termed the characteristic of the logarithm. The characteristic is always one less than the number of integers which precede the decimal point in the number whose log we are seeking.

We will write down one further log for practice, but this part of the work is so easy that we need not pause long at this stage.

Write down the log of 876·4. On page 175 we find 87 in the first column of figures, and place our pencil over it. Now we move the pencil horizontally to the right until it reaches the column of figures which has 6 in its top line—this is the eighth column from the left. The number we find at this point is 9425. Keeping this number marked with a finger of the left hand, we carry the pencil still further to the right until it reaches the column headed 4 in the differences section of the table on the right, it is the sixth column from the right-hand side of the table. In this column we find the number 2; we have now marked with the left hand the number 9425, and with the pencil the number 2. Adding these together we obtain 9427 as the mantissa of the log we are seeking.

There are three figures before the decimal point of our number 876·4, so that characteristic of the log is 2.

Log 876·4 is 2·9427.

In the top part of the table on page 174 you will notice that opposite numbers 10 to 19 in the left-hand column there are two series of figures in the columns of differences on the right. At this part of the table the mantissae alter rather quickly, and the differences have to be changed. Notice that the figures in the main part of the table step down after the column headed 5. Now, if you are using figures in the main part of the table in columns 0 to 4, which are printed high, you must use the upper figures in the difference columns. From column 5 to column 9 in the main table, the figures are printed lower; if you are using any of these you must also use the lower line of figures in the differences columns.

Problem 48. Write down the logs of the following numbers: 10, 110, 21·6, 942·3 and 1865.

The log of 1 is 0.

We can see that this must be so because $1 = \frac{10^1}{10^1} = 10^{1-1} = 10^0$, a result which follows from the first index rule. The reader will see that the result will be the same if we took any power of any quantity and divided it by itself, e.g. $1 = \frac{a^n}{a^n} = a^{n-n} = a^0$. If log 1 had some value other than nothing, the reader will see that our logarithmic rule for multiplication would be invalid. The logs of numbers between 1 and 10 lie between 0 and 1.

Example: Using logs, compute the value of 18·63 × 7·644.

From the table log 18·63 = 1·2702
log 7·644 = ·8833

Adding 2·1535

2·1535 is the log of 142·4, the result.

Looking in the table for the number whose log is 2·1535, we first ignore the characteristic 2 and find the decimal part ·1535. Opposite 14 in the first column and under the 2 in the top line we find 1523. The difference of 12 is found in the column of differences headed 4. The number whose log we are seeking is 142·4; the position of the decimal point is governed by the characteristic 2.

On pages 176-7 will be found tables of figures called antilogarithms. While it is quite easy to find a number corresponding to a given logarithm by using the table of logs, it is just a little easier to use the table of antilogs. Using only the mantissa of the log and the same method as when looking out a logarithm, the reader will have no difficulty in finding the number 1424 corresponding to the mantissa 1535.

Problem 49. Using logs, evaluate $124{\cdot}0 \times 50{\cdot}63 \times 1{\cdot}2 \times 8{\cdot}69$.

Example: Using logs, evaluate $\frac{4299}{67}$

$$\begin{aligned} \text{Log } 4299 &= 3{\cdot}6334 \\ 67 &= 1{\cdot}8261 \\ \text{Difference} \quad & \ 1{\cdot}8073 \\ \text{Antilog } 1{\cdot}8073 &= 64{\cdot}16. \end{aligned}$$

Problem 50. Using logs, evaluate $\frac{18{\cdot}92 \times 104 \times 7{\cdot}22}{50{\cdot}18 \times 19{\cdot}6}$.

Powers and Roots

We could, if necessary, find the value of $1{\cdot}63^{18}$ by direct multiplication, but we should feel aggrieved if the necessity arose. We should probably proceed to cube 1·63, then cube the result so obtained and then square, i.e., find $[(1{\cdot}63^3)^3]^2$.

Working to only four places of decimals throughout, the work would take half an hour with a chance of making a slip.

Observe the work involved when using logs.

$$\begin{aligned} \text{Log } 1{\cdot}63 &= {\cdot}2122 \\ \text{(multiply by 18)} \quad & \ 18 \\ & \ 2122 \\ & \ 16976 \\ & \ 3{\cdot}8196 \\ \text{Antilog } 3{\cdot}8196 &= 6601 \text{ Result.} \end{aligned}$$

If we had to find the eighteenth root of 1·63 the calculation would be just as simple. Dividing ·2122 by 18 we obtain ·0118, which is the antilog of 1·028. $\therefore \sqrt[18]{1{\cdot}63} = 1{\cdot}028$.

The reader will notice that we might have selected a much higher power or root than 18 as our example, and the work of multiplying by ordinary methods might then become impossible. We leave the reader to speculate on the other difficulties of extracting the root.

We can imagine the reader objecting that the evaluation of high powers or roots is never a practical necessity. Calculation of compound interest over long periods is just such a problem however. Now let us evaluate what at first sight appears to be a simple quantity, such as $2^{1{\cdot}41}$, which is of practical importance in problems concerning internal-combustion engines. The reader will quickly discover, without the aid of logs or slide rule, he is faced with a real difficulty. The ease with which powers and roots of numbers can be calculated with logs is perhaps the most valuable property of this branch of mathematics.

Example: Evaluate $2^{1{\cdot}41}$ and $\sqrt[1{\cdot}41]{2}$.

Log 2 = ·3010. ·3010 ÷ 1·41 = ·2135.
·3010 × 1·41 = ·4244. Antilog ·2135 = 1·635.
Antilog ·4244 = 2·657.

Problem 51. Evaluate $8{\cdot}7^{5{\cdot}2}$ and $\sqrt[5]{243}$.

Logarithms with Negative Characteristics

We hope that up to this stage any reader hitherto unacquainted with logs has been able to follow these notes without much difficulty. The remainder of this section is, perhaps, a little more complex, and if the reader feels that he is getting out of his depth, we recommend him to stop at this stage for a time. If he works more examples of a simple type—which he can easily make up for himself—the pause will give time for the knowledge he has absorbed to sink in, and he will find the difficulties diminish.

Ability to work with logs is a great asset. Some of the simple examples and problems we have earlier examined could not have been worked out by other means, except a slide rule, which we now know is the quick and easy way of using logs. It may be that the reader is taking this section in his stride. If so, he may safely proceed.

We have seen that the log 1 is 0. While numbers increase from 1 to 10, the corresponding logs increase from 0 to 1. By the time the number has reached 100, the log has grown to 2. It is quite clear that the logs do not increase at a steady rate, by which we mean that the difference between the logs, say, of 10 and 20, is not the same as the difference between logs of 20 and 30, or 80 and 90. It is because of this variable rate of increase that we see the divisions crowd together more and more as we look along the scales of our slide rule.

Now we must learn something of the logarithms of numbers less than unity. Take a simple fraction, say, ·5. Use the fraction $\frac{1}{2}$ and proceed to divide 2 into 1 using logs.

Log $1 = 0$ and log $2 = \cdot 3010$ $\therefore$ log $\frac{1}{2} = 0 - \cdot 3010$. If to $-\cdot 3010$ we add 1 and subtract 1 we shall not alter the value. $-1 + (1 - \cdot 3010)$ gives $-1 + \cdot 699$.

If we consult our table of logs we find the log of 5 is ·6990; we know also that the log of 50 is 1·6990, and of 500, 2·6990, and now we find the log of ·5 is $-1 + \cdot 6990$. This number is usually written $\bar{1}\cdot 6990$, and expressed in words by "Bar 1 point 6990". The reader should have no difficulty in showing that log ·05, which is $\frac{1}{20}$, is $\bar{2}\cdot 6990$, and the log of ·005 is $\bar{3}\cdot 6990$. The bar denotes that the characteristic is negative while the mantissa remains positive. By adopting this method of expressing the values of logs of numbers less than 1, we need only one table of logs and antilogs.

The rule for finding in the tables the log of a fraction is quite simple. Express the fraction in its decimal form and find in the table the mantissa of the log of the significant figures. Write the mantissa down with the decimal point immediately preceding it. There is always a negative characteristic for the log of a number less than 1. This negative characteristic is always one greater than the number of noughts which lie between the decimal point and the first significant figure in the number whose log we are seeking. In ·005 there are two noughts between the decimal point and the 5. The characteristic is $\bar{3}$. Similarly for ·05, the characteristic is $\bar{2}$. The reader will see that the rule also applies to ·5, the characteristic being $\bar{1}$.

The following table may help at this stage:

Number	*Logarithm*
$5000 = 5 \times 10^3$	$\cdot 6990 + 3 = 3\cdot 6990$
$500 = 5 \times 10^2$	$\cdot 6990 + 2 = 2\cdot 6990$
$50 = 5 \times 10^1$	$\cdot 6990 + 1 = 1\cdot 6990$
$5 = 5 \times 10^1$	$\cdot 6990 + 0 = \cdot 6990$
$\cdot 5 = 5 \times 10^{-1}$	$\cdot 6990 - 1 = \bar{1}\cdot 6990$
$\cdot 05 = 5 \times 10^{-2}$	$\cdot 6990 - 2 = \bar{2}\cdot 6990$
$\cdot 005 = 5 \times 10^{-3}$	$\cdot 6990 - 3 = \bar{3}\cdot 6990$

Example: Using the log tables, write down the logs of ·0123 and ·006009. From the tables we see the log of 123 is 0899, and of 6009 is 7788.

The required logs are $\bar{2}\cdot 0899$ and $\bar{3}\cdot 7788$.

Problem 52. Using the tables, find the logs of ·802 and ·001176. Also find the numbers whose logs are $\bar{1}\cdot 6261$ and $\bar{3}\cdot 4710$.

We will end this section with two examples which involve negative characteristics.

Example: Find the square root of ·00591.

$\sqrt{\cdot 00591} = (\cdot 00591)^{\frac{1}{2}}$

Log $\cdot 00591 = \bar{3}\cdot 7716$.

We must divide the log by 2 and to do this we increase the negative part by 1 to make it exactly divisible by 2, and to maintain the log unaltered we must also increase the positive part by 1.

We then have $\bar{4} + 1{\cdot}7716$, and this divided by 2 is $\bar{2} + {\cdot}8858$ or $\bar{2}{\cdot}8858$.

Antilog $\bar{2}{\cdot}8858 = {\cdot}07688$.

The square root of ·00591 is $\pm{\cdot}07688$.

The $\pm$ sign indicates that the root may be either positive or negative.

Example: Using logs, evaluate $\dfrac{5{\cdot}722 \times \sqrt[5]{72{\cdot}6}}{({\cdot}0122)^{1\cdot2} \times \sqrt{82{\cdot}8}}$.

$$\begin{array}{lll}
\text{Log } 5{\cdot}722 & = {\cdot}7576 & {\cdot}7576 \\
\frac{1}{5}\log 72{\cdot}6 & = \frac{1}{5} \times 1{\cdot}8609 & {\cdot}3722 \\
 & \text{Adding} & 1{\cdot}1298\ (1) \\
1{\cdot}2 \log {\cdot}0122 & = 1{\cdot}2 \times \bar{2}{\cdot}0864 & \\
 & = -2{\cdot}4 + {\cdot}1037 & \\
 & = -2{\cdot}2963 & \\
 & = \bar{3}{\cdot}7037 & \bar{3}{\cdot}7037 \\
\frac{1}{2}\log 82{\cdot}8 & = \frac{1}{2} \times 1{\cdot}918 & {\cdot}9590 \\
 & \text{Adding} & \bar{2}{\cdot}6627\ (2) \\
 & \text{Subtracting (2) from (1)} & 2{\cdot}4671
\end{array}$$

Antilog $2{\cdot}4671 = 293{\cdot}2$.

Problem 53. Find the values of

$\dfrac{29{\cdot}2 \times {\cdot}0826}{{\cdot}1945}$ and $\dfrac{\sqrt[3]{1{\cdot}82} + \sqrt{{\cdot}0043}}{\sqrt[4]{{\cdot}0986} + \sqrt[5]{186{\cdot}9}}$ using logs.

We suggest the reader now works through the examples and problems in this section using his slide rule. It will be necessary to use the log-log scale and we think the task will prove both interesting and valuable.

LOGARITHMS
AND
ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
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·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

SECTION FOURTEEN

OTHER CALCULATING INSTRUMENTS

In order to accommodate long logarithmic scales, and with a view to securing a higher degree of accuracy, various devices are employed. We cannot proceed with a lengthy description of the instruments which are available; the reader will see them illustrated in catalogues of mathematical instruments, but we will make a short reference to the principles employed.

Cylindrical Calculators

Let us visualise a logarithmic scale in the form of the diagram illustrated in Fig. 18. The graduations commence at *A* and run

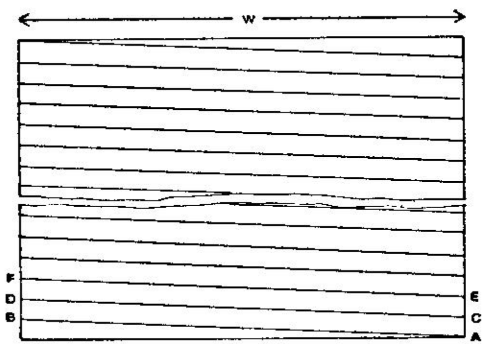

Fig. 18

in a sloping direction to *B*, continue from *C* to *D*, and then from *E* to *F*, and so on. *B* and *C* are identical points in the scale, as also are *D* and *E*. The total length of the scale is the

sum of all the sloping lines. This flat scale is glued on to a cylindrical stock whose circumference exactly equals the width *W* of the rectangle, and the sloping lines now form a continuous helix, the point *B* joining up with *C*, *D* with *E*, etc.

In some instruments the total length of the scale is 500″ i.e. 50 times the length of scale of a standard slide rule, and the degree of accuracy attainable is high. The instrument is operated by means of adjustable pointers, which may be set to any desired points on the scale, and then moved together to other positions. If the reader will take a pair of dividers and set them to, say, the distance between 1 and 3 on the D scale of his slide rule, then move them so that the left-hand leg is placed at 2, the right-hand leg will register with the 6. This is the fundamental method of multiplication of $2 \times 3 = 6$, and in principle it is the method used in the cylindrical calculator employing the helical scale.

In other cylindrical instruments the scale runs in sections in the axial direction, and is used in conjunction with a grid surrounding the main cylinder.

Circular Calculators

The reader will understand that it is easy to set out the C and D scales of his slide rule in circular instead of rectilinear form. Fig. 19 shows, slightly reduced in size, a simple form of circular calculator in which the C and D scales are still 10″ in length, but being in circular form, result in a more compact instrument.

The reference letters A, B, C, D and E mentioned in the description, allude to the five circular scales taken in order from outer to inner.

Scale E is for evaluating squares and square roots; scale B deals similarly with cubes and cube roots, and the outer scale A gives a means of finding common logarithms. The cursor takes the form of a transparent sector, rotating about the centre, on which is drawn a radial hair line.

Multiplication is effected by rotating a knob at the back of the instrument, to bring the 1 of D into coincidence with one of the factors in C; opposite the second factor in D the product will be found in C. These operations are exactly analogous to

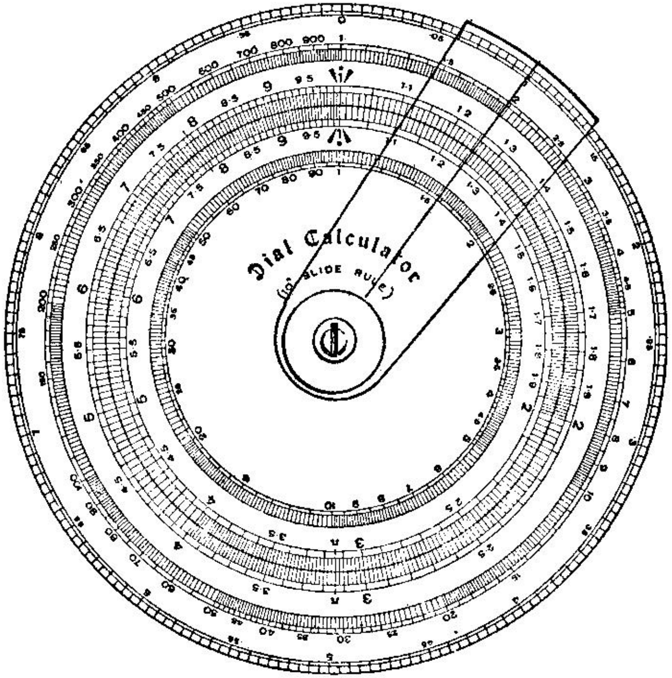

FIG. 19

the movement of the slide of the ordinary slide rule. Division, and combined multiplication and division, follow the same general rules applicable to slide rules. Squares and square roots, cubes and cube roots, and logarithms, are found by projecting by means of the rotating cursor from the appropriate scale, to the C or D scales, and *vice versa*.

Fig. 20 shows a circular calculator in which the main scale is 50″ long. This scale occupies five concentric circles; if the numbering is followed progressively round the five circles

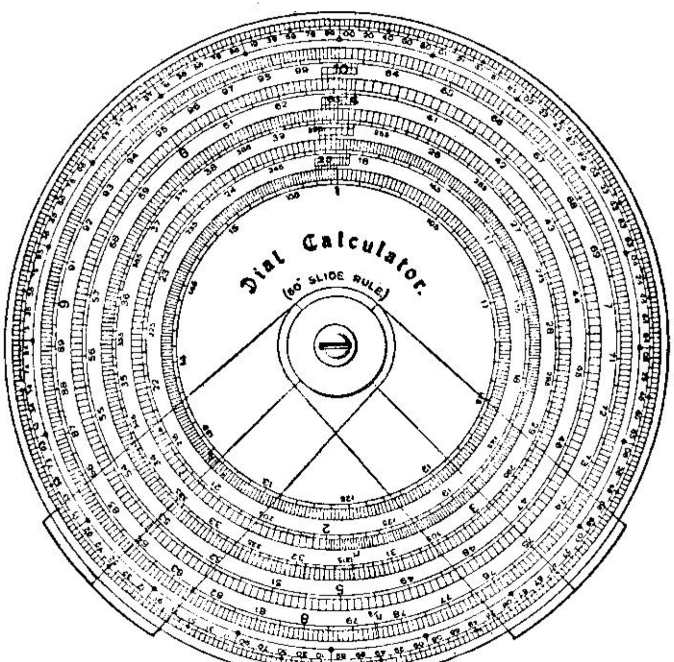

FIG. 20

starting at 1 in the smallest circle, and proceeding clockwise until the 10 in the fifth circle is reached, no difficulty in passing from one circle to the next will be encountered. Short lengths of bridging scales are provided to assist. The outermost scale is evenly divided and this, in conjunction with the main scale, gives mantissae of common logarithms.

Multiplication is effected by setting the first cursor to the 1 of the main scale and the second cursor to one of the factors in the main scale. By means of a knob at the back of the instrument the two cursors are *moved together* to bring the first cursor to the second factor, and the product appears coincident with the second cursor. The cursors intersect all five circles, and care is necessary in selecting the correct scale in which to read the result.

Watch-type Calculators

Circular calculators resembling pocket watches in size and shape are available. In principle they resemble the two calculators illustrated in Figs. 19 and 20, but they are operated by small spindles equipped with milled heads, similar to the winding mechanism of a watch.

Circular calculators have one advantage over the ordinary type of slide rule; the result is never off the scale, and there is nothing equivalent to traversing the slide to change over from one index to the other; they are also more convenient for carrying in the pocket.

For general convenience in working, and for speed in operation, the ordinary type of slide rule is altogether superior to the circular or cylindrical types, and when the novelty of working with the latter has worn off, the user almost invariably discards the instrument and reverts to the use of his slide rule.

Other Rules

The rules we have described, and the various combinations of scales we have dealt with, do not exhaust our subject. We have attempted to deal with two aspects only; firstly, to teach the rudiments of the simple slide rule to those who previously were unacquainted with them, and to stress the argument that proficiency can be attained easily; secondly, to convince those who use only the standard type of rule that they are employing an instrument of limited utility, and that other rules are available which, whilst retaining the best points of the standard rule, have other features which increase the efficiency of the instrument.

It is quite probable that the reader will occasionally see a slide rule which is equipped with one or more scales which we have not mentioned. As stated above, we do not claim to have covered completely the subject in this small book, but we think we have dealt with the most useful scales. We have made no attempt to deal with the large variety of special slide rules designed for effecting computations related to specific industries. Such slide rules (which are often used for advertising purposes) shorten the work connected with specific problems, but computations effected with their aid may usually be carried out by the ordinary types of slide rule.

SECTION FIFTEEN

HISTORICAL NOTE

NATURAL or hyperbolic logarithms were invented by Napier of Merchiston in 1614, and the system is frequently known by the name of Naperian logarithms. The base of the Naperian system of logs is 2·7183; this number usually is denoted by *e*. Common logarithms, namely those to the base 10, are sometimes called Briggsian logarithms; this system is invariably used for ordinary computations.

The first practical application of logs in the form of scales was produced by Professor Gunter in 1620. His instrument consisted of one scale only, and was used in conjunction with a pair of dividers. The slide rule in its modern form was first devised by Wingate in 1626, and the cursor was added by Mannheim in 1851.

Degree of Accuracy

The only criticism we hear advanced against the slide rule is that results obtained with its aid are not always exact. Speaking now of the 10″ C and D scales, errors should not much exceed ·1 to ·2%. Accuracy will depend upon the care taken in manipulating and reading, and upon the accuracy of the instrument itself. All slide rules exhibit small errors in the dividing of the scales if examined critically, but in a good instrument such errors are small, and often difficult to detect. In the course of time, defects develop due to shrinkage or distortion of the rule itself. An old rule frequently displays discrepancies in the lengths of the scales which originally were identical. The effective life of a rule will be considerably lengthened by careful treatment and protection from unnecessary exposure in a hot or moist atmosphere.

The negligence of the shopkeeper who displays for sale slide rules in his window, in the direct rays of the sun, is reprehensible, and indicates ignorance of the merchandise he handles. No slide rule, except the all-metal types which are seldom seen, will retain accuracy and easy movement after prolonged exposure in direct sunlight.

We have attempted to advance the claims of slide rules fitted with duplicate scales, such as the types described in Sections 10 and 11. With this type of rule it is possible to obtain results while using much shorter lengths of scales, and, of course, it follows that the errors, due to discrepancies in the scales, will be smaller.

Interpolated readings are certain to introduce small errors in results, since all we can do in assessing values which do not coincide with an actual graduation of the scales is to estimate their position as though the scales are evenly divided instead of being logarithmic. These errors are smaller perhaps than would be expected. The widest space in the C or D scale of the 10″ rule is that lying between 4 and 4·05. If we set the cursor index exactly in the middle of this space we should no doubt read its position as *4·025*. Its true reading should be 4·0249. We are, however, likely to make larger errors when we estimate other fractions of spaces, since the half-way position is the easiest of all to assess correctly.

All instruments when used are susceptible to errors of varying degrees. If we are asked to name a simple instrument possessing a high degree of accuracy, we immediately think of the engineer's micrometer. In using the instrument, we may, as a result of error in the thread, or zero error, or faulty execution, obtain an error of ·0005, i.e. "half a thou". If we are measuring a rod of about ½″ diameter, the ·0005″ error is of the order of 1 in 1000, not far removed from the degree of error we may encounter in a slide rule. If we are measuring the thickness of a sheet of foil of the order of ·005″, our micrometer error is 10%, something much worse than our slide rule inaccuracies. Again, a work's accountant might criticise the slide rule because it may not give him quite accurately the cost of 4 tons 2 cwt. 3 qrs. of material at £1, 4s. 6d. per ton, forgetting that his weight may be in error to the extent of 1% or more, a larger error than the slide rule will introduce.

When discussing accuracy of slide rule results, points such as those we have mentioned should be remembered.

Gauge Points

In addition to the scale graduations, a few other lines appear in the majority of slide rules. These additional lines, termed gauge points, represent the positions of factors commonly used in calculations.

In nearly all rules the value of $\pi = 3{\cdot}14159$ is marked in the principal scales, π being the constant which enters into calculations relating to circles, spheres, etc. $\frac{\pi}{4} = {\cdot}7854$, is sometimes shown by a gauge point, $\frac{\pi}{4}d^2$ being the area of a circle of diameter d.

Gauge points, denoted by c and c^1, appear at 1·13 and 3·57, respectively, in the C scale of many slide rules. The volume of a cylinder is $\frac{\pi}{4}d^2l$; it may be written in the form $\left(\frac{d}{\sqrt{\frac{4}{\pi}}}\right)^2 l$.

The value of $\sqrt{\frac{4}{\pi}}$ is 1·13 approximately. If the gauge point c is set to the value of the diameter of a cylinder on D, the volume of the cylinder may be read on A coincident with the length, l, on B. For some values of d and l, the result will be off the scale when c is set to diameter. In such cases if c^1 is used, the result will be obtainable.

The gauge point M is seen in scales A and B in some makes of rule. Its virtual value is $\frac{1}{\pi} = {\cdot}3183$. To find in one setting of the slide the area of the curved surfaces of a cylinder, we set

M to diameter in A, and read over the length in B the area of curved surface in A.

Other gauge points may be found, their inclusion or omission being dependent upon the decision of the manufacturers or designers of the rule. We mention the following, which are the commonest:

ρ' at 3438, and ρ'' at 206255, in scale C, give the numbers of minutes and seconds in a radian respectively. These gauge points may be used for finding the values of trigonometrical functions of small angles. For any small angle, say, less than 2°, the sin and the tan may be taken as identical. If we set the ρ' mark to the graduation in scale D, representing one-tenth of the number of minutes in the angle, the sin or tan may be read in D under the 1C or 10C.

Example: Find the sin or tan of 22′.

Set ρ' to 2·2D. Read sin or tan in D under 10C = ·0064.

If the angle is expressed in seconds, the ρ'' is used in a similar manner.

A third gauge point ρg occasionally may be seen between 6·3 and 6·4 on scale C; this is used in the same way when the angle is expressed in the centesimal system.

If we remember that the sin 1° = tan 1° = ·0174, we shall have no difficulty in inserting the decimal points in results obtained when using these gauge points.

A gauge point is sometimes placed between division 114 and 116 in scales A and B; this is called the gunner's mark, and is used in certain calculations relating to artillery.

The value of g = 32·2′ (per sec.)^2, the gravitational acceleration imposed on freely falling bodies near the earth's surface, is occasionally indicated by a gauge point. g is used frequently by engineers in problems concerning dynamics. A gauge point at 746—the number of watts equivalent to one horse-power—is sometimes to be found.

The inclusion of many gauge points in a slide rule is to be deprecated. The only one we think deserves its place is π and possibly $\frac{\pi}{4}$.

If any number enters frequently into our calculations, it is fairly easy to add a gauge point to register its position. The mark should be scribed with a razor blade broken so as to provide a sharp corner, and a square should be used to ensure the line lies at right angles to the length of the rule. It is exasperating to add a gauge mark and then find its position is not quite correct, and we have found a safe method to adopt is first to paste a small piece of paper on the scale, and very lightly pencil the mark on the paper. The position of the mark should be very carefully checked, and, if necessary, corrected. The mark can now be cut through the paper into the scale, care being exercised to avoid cutting too deeply, the paper removed, and a trace of printer's ink rubbed into the cut impression, after which the scale may be polished. If neatly executed, a fine black line will result. Lines registering gauge points should stand slightly off the scales with which they are associated, in order to avoid confusion with the divisions of the scales.

The signs $\frac{\text{Quot.}}{+1}$ and $\frac{\text{Prod.}}{-1}$ which appear at the left and right-hand ends respectively of certain makes of rules, are of little consequence, and we would prefer not to mention them. They are designed to assist in ascertaining the numbers of digits in a product or quotient. We have, in Section 4, given rules for determination of the position of decimal points, based on the position of slide relative to the stock. We have shown that if when multiplying the slide is set so that it protrudes to the right of the stock, the number of digits in the product is one less than the sum of digits in the two factors. Another way of expressing the same rule is to say: If when multiplying, the result lies to the right of the first factor the digits in the product are one less than those of the two factors. The sign $\frac{\text{Prod.}}{-1}$ at the right-hand end of the stock is a reminder of the rule when expressed in this manner. The sign $\frac{\text{Quot.}}{+1}$ similarly reminds us that the quotient will contain one more digit than the difference between the digits of the dividend and divisor if the result appears to the left of the dividend. When the result in a multiplication lies to the left of the first factor, the number of digits of the product is equal to the sum of the numbers of digits in the two factors, and in division the number of digits in the quotient is equal to the difference between the digits of dividend and divisor when the result is found on the right of the dividend.

SECTION SIXTEEN

EXERCISES

We give in this section a selection of examples and problems which illustrate the various classes of work in which the slide rule may be used. The number of such examples could be increased indefinitely.

We trust that no reader will think that we are suggesting he should search for his particular type of problem and then merely memorise the movements of slide and cursor and repeat them for his own calculations. To follow such a course would be futile. The only way to become proficient with the slide rule is to understand its fundamental principles, and to work out simple exercises. When a practical problem presents itself the relevant numbers should be written down; figures which cancel out completely should be eliminated, and simple factors should be combined mentally to reduce the slide rule operations to a minimum.

Take as a simple case $\frac{4}{6} \times 34{\cdot}2$. There is no saving in cancelling this to $\frac{2}{3} \times 34{\cdot}2$. There may be a saving in cancelling the $\frac{4}{6}$ to $\frac{1}{1{\cdot}5}$, but it is doubtful whether it is worth while doing this. A case such as $\frac{3 \times 12 \times 78{\cdot}3}{6 \times 4 \times 5{\cdot}7}$ should be cancelled down to $\frac{3}{2} \times \frac{78{\cdot}3}{5{\cdot}7}$, or better, to $1{\cdot}5 \times \frac{78{\cdot}3}{5{\cdot}7}$. When two or three simple factors appear as in $8 \times 3 \times 16{\cdot}3$, they should be combined mentally, and the figures treated as $24 \times 16{\cdot}3$. We would warn the reader against attempting to cancel or combine anything beyond very simple factors.

In some of the examples which follow, the rule best suited for use is mentioned. If scales C and D are to be used, any rule will meet the case. If trigonometrical work is involved, use the Navigational rule every time; the reader will soon see why we recommend this rule.

The time occupied in making acquaintance with the duplicated C and D scales of Dualistic and Electrical rules will be a good investment.

In studying the practical examples we give below, the reader should write down the essential figures arising from the problem. He will find we have cancelled out or combined simple factors when they occur, but only in the obvious cases.

If the reader has worked through the exercises in the earlier sections, and solved some of the problems, probably there will be no need to study all those given in this section, but if he feels he needs still more practice with his slide rule, the following examples are suitable for the purpose.

Commerce

Example: £56, 8s. 0d. is invested at 5½% per annum compound interest. Calculate the value after 8½ years.

£1 at the end of one year becomes £(1 + ·055) = 1·055; at the end of two years becomes £(1·055) (1·055) = £$(1{\cdot}055)^2$, and at the end of 8½ years becomes £$(1{\cdot}055)^{8\frac{1}{2}}$.

Use log-log scale, and if 1·055 is not within the range of the scale, treat as $\frac{2{\cdot}11}{2}$.

Set X to 211LU. 10C to X. X to 85C.

Read in LL under X 570.

Set X to 2LU. 10C to X. X to 85C.

Read in LL under X 360.

Over 57D set 36C.

Under 1C read 1·58D.

Now £56, 8s. 0d. = £56·4.

Under 564C read 891 in D. £89·1 = £89, 2s. 0d.

Problem 54. In costing a job it was found that 55 operations in a certain machine took 450 minutes to complete. The operator's rate of pay being £6, 7s. 0d. for 48 hours. Calculate the wages cost per operation.

Example: One gross articles weigh 84 lb. What is the weight of one piece?

Set X to 84K. Under X read ·583 lb. in U.

No setting of slide is required, the conversion being effected by projecting direct from K to U, using Commercial rule.

Problem 55. A time sheet for a group of workers credits:

A	with $6\frac{1}{2}$	hours'	overtime.	Worker's	rate	92s.	for	48 hrs.
B	,, $5\frac{3}{4}$	,,	,,	,,	,,	88s.	,,	,, ,,
C	,, $8\frac{1}{4}$	,,	,,	,,	,,	85s.	,,	44 ,,
D	,, $2\frac{3}{4}$	,,	,,	,,	,,	84s.	,,	,, ,,
E	,, $11\frac{1}{2}$	,,	,,	,,	,,	84s.	,,	,, ,,

Calculate the wages due, time and a quarter being paid for all overtime. (Use Commercial rule.)

Example: Material costs 2s. $2\frac{1}{2}$d. per cwt., the equivalent price per ton is required. 2s. $2\frac{1}{2}$d. = 2·209s.—taken from conversion table.

Set X to 2209D.

Under X in H read 44·2s. = £2, 4s. $2\frac{1}{2}$d.

Problem 56. The price of a certain kind of foil is $3\frac{3}{4}$d. per sq. foot. Calculate the cost of 100 sheets of foil 21″ × $15\frac{1}{2}$″. (Use scales C and K for multiplication, but read result in U which automatically divides by 144.)

Energy and Power

Example: A gas engine uses 93·1 cu. ft. of Dowson gas per i.h.p. per hour. Calorific value of this gas is 123,000 ft. lb. per cu. ft. Calculate the efficiency of the engine.

Over 33D set 931C.

Set X to 6C. 123C to X.

Result: 17·25% in D under 1C.

Problem 57. The average heights of indicator diagrams taken from both ends of a cylinder are: 1·52″ and 1·42″. Spring 1″ = 60 lb. weight. Piston 8″ dia. Stroke 16″. Speed 220 r.p.m. Calculate i.h.p.

Example: A cut of depth ·11″ is being made in a lathe. Feed is ·03″ per rev. Speed 50 r.p.m. Dia. of bar 4″; pressure on tool 910 lb. weight. Find the h.p. expended at the tool, and weight of metal removed per minute. (Density of steel ·28 lb. per cu. inch.)

To 91D set 3C.

Set X to πC. 33C to X. X to 1C. 10C to X.

Read in D under 5C 1·44 h.p.

To 44D set 10C.

Set X to πC. 1C to X. X to 15C. 1C to X.

Result in D under 28C = ·58 lb.

Problem 58. Calculate the overall efficiency of a steam engine and boiler using 1·6 lb. of coal per h.p per hour. (Calorific value of coal 12,200 B.T.U. per lb.)

Friction

Example: A horizontal shaft, 8″ dia., carries a load of 5 tons. Calculate the h.p. absorbed in friction at 200 r.p.m. $\mu = \cdot 05$.

To 25D set 33C.

Set X to 224C. 12C to X. X to πC. 1C to X.

In D under 16C read h.p. = 7·1.

Problem 59. Calculate the h.p. required to drive a motor-car at 60 miles per hour along a level road. Rolling resistance 30 lbs. per ton. Assume air resistance accounts of four-fifths of the total h.p. at this speed. Weight of car $1\frac{1}{4}$ tons.

Example: To lower a load of 1 ton, a rope is wrapped three times round a horizontal post. Diameter of post 4″. $\mu = \cdot 25$. Calculate the pull necessary at the free end of the rope to lower steadily.

Formula: $\frac{T_1}{T_2} = e^{\mu\theta} (e = 2\cdot 7183)$.

Set 1C to πD. Under 15C read 4·72 in D. (4·72 is $\mu\theta$.)

Set 1C to 2·72LL (log-log scale). Under 472C read in LL, 111.

To 224D set 111C. Under 1C read 20·2 lb. in D.

(Provided the post is strong enough its diameter does not enter into the calculation.)

Problem 60. A belt laps 180° round a flat pulley, and is just on the point of slipping. Tension in the slack side is 400 lb. Calculate tension in the tight side. $\mu = \cdot 3$.

Heat

Example: A railway line is laid in 40-ft. lengths on a day when the temperature is 10° C., and gaps of $\frac{1}{4}$″ are left between adjacent lengths. What will be the gaps when the temperature is 40° C. and −10° C.?

Coefficient of linear expansion of steel ·000012 per degree C.

Set 10C to 48D.

Read in D under 36C increase in length ·173.

„ „ „ 24C decrease „ ·115.

Gap at 40° C. = ·25 − ·173 = ·077″.

„ −10° C. = ·25 + ·115 = ·365″.

Problem 61. Gas has a volume of 188 c.c. at a pressure of 76 cm. of Hg. and temperature 20° C. What will be the volume at 40 cm. of Hg. and 80° C.?

Example: 5·2 grm. of ice added to 54·8 grm. of water at 20° C. give a final temperature of 12° C. Calculate the latent heat of fusion (water equivalent of calorimeter 5·1 grm.).

Over 599D set 52C. Under 8C read 92·1 in D.

Latent heat = 92·1 − 12 = 80·1 C.H.U.

Problem 62. 152·5 grm. of Hg. at 100° C. were mixed with 87·5 grm. of water (including water equivalent of calorimeter) at 10·1° C. The final temperature being 15·2° C. Calculate the specific heat of Hg.

Strength and Deflection of Beams

Example: A cantilever, 50″ long, carries a load of 4000 lb. at its free end. The section is rectangular of breadth 3″. Calculate the depth at distances of 10″, 20″, 30″, 40″ and 50″ from the free end if the maximum stress is to be 3000 lb. per square inch.

Formula: $\frac{f}{y} = \frac{M}{I}$.

Under 80A set 3B.

Read results in D:
5·15″ under 1A. 7·3″ under 2A. 8·94″ under 3A.

Now traverse slide and read in D:
10·3″ under 4A. 11·5″ under 5A.

Problem 63. A beam of uniform cross-section, breadth = 2·2″, depth = 4·6″, is simply supported at its ends. It is 30″ long and carries a load of 1000 lb. 18″ from one end. Calculate the maximum stress induced by the load.

Example: A uniform beam, 3″ diameter, 4′ 0″ long, simply supported at ends, carries a load of 4000 lb. uniformly distributed. Find the maximum deflection. E = 30 × 10⁶ lb. per sq. inch. (Use Electrical or Dualistic rule.)

Formula: $\frac{5\ W.L^3}{384\ E.I}$.

Under 48d set 81c.

Set X to 64c. πC to X.

Result: 484 in d or D coincident with 4 in c or C.

Approximation gives ·04. Result: ·0484″.

Problem 64. Calculate the maximum stress induced in the beam of the above example.

Strength of Shafts—Deflection of Springs

Problem 65. Calculate the h.p. which may be safely transmitted by a circular shaft 4″ diameter at 200 r.p.m. Stress to be limited to 9000 lb. per sq. inch.

Example: Calculate the angle of twist per foot of length of the shaft in Problem 65. Modulus of rigidity = C = 13 × 10^6 lb. per sq. inch.

Formula: Angle in radians $= \frac{2\,f.l}{\text{C. dia.}} = \frac{32\ T.l}{\pi\text{C. (dia.)}^4}$.

After cancelling, over 54D set 13C.

Under 1C read in D 416.

Approximation gives ·004. Result: ·00416 radians.

X to 416D. πC to X. Under 180C read ·238° in D.

Example: Calculate the number of coils necessary in a helical spring to give an extension of ·5″ for an axial load of 12 lb. Dia. of wire ·21″, dia. of coil 2·5″. Modulus of rigidity = C = 11 × 10^6 lb. per sq. inch.

Formula:

$$\text{Number of coils} = \frac{\text{C} \times (\text{dia. of wire})^4 \times \text{extension}}{8 \times \text{Load} \times (\text{dia. of coil})^3}.$$

Over 5D set 25C. Set X to 21C. 25C to X. X to 21C. 25C to X. X to 21C. 1C to X. X to 21C. 96C to X. X to 10C. 1C to X. Result in D under 11C 7·15 coils.

Problem 66. A coiled spring has 50 turns of wire ·11″ dia. Dia. of coil 1·5″. C = 10 × 10^6 lb. per sq. inch. Find the extension caused by a load of 2 lb.

Electricity

Example: The specific resistance of platinum is 8·96 microhms at 0° C., and its temperature coefficient is ·0034. What length of platinum wire of 32 s.w.g. (dia. ·0274 cm.) will have a resistance of 5 ohms. at 50° C.? What will be its resistance at 100° C.?
Over 5D set 4C.
Set X to πC. 117C to X. X to 274C. 896C to X.

Read in D under 274C, the length, 282 cm.

Over 5D set 117C.

Read in D under 134C, 5·73 ohms. at 100° C.

Problem 67. A generator feeds 1500 75-watt lamps at 230 volts; find the current supplied.

Example: Calculate the h.p. required to drive a dynamo generating 40 kw. Efficiency 85%. (Use Electrical rule.)

Set N in c to 85d.

Above 4D read 63 in C. Result: 63 h.p.

Problem 68. The output of an electric motor is 65 h.p. and its efficiency is 82%. Find the power required to drive it.

Problem 69. A copper conductor is 500 yards long and carries a current of 21 amps. Calculate the diameter of wire if the volt drop is to be limited to 5·2. (Specific resistance of copper 1·7 microhms.)

Building

Example: Imported scantlings cost £25 per standard (165 ft. cube) to which must be added £2 per standard for delivery and £4 per standard for planing. Find the cost per foot cube wrought.

£25 + 2 + 4 = £31.

Over 31D set 165C.

Under 1C read in D £·188 or under 2C read 3·75s. = 3s. 9d.

Problem 70. Calculate the cost per foot run to excavate, fill and ram a trench for a 4″ drain; depth of trench 3′ 0″. Concrete bed 18″ wide, 6″ thick. Benching up and displacement of pipe taken as half volume of bed. Excavating at 2s. 3d. per yard cube. Returning, filling and ramming at 4s. per yard cube.

Example: Calculate the cost of tiles per square (100′ super), tiles $10\frac{1}{2}$″ × $6\frac{1}{2}$″ gauge (i.e. exposed part o tile) 4″; tiles at £8, 15s. 0d. per 1000.

Over 144D set 4C.

Set X to 875C. 65C to X.

Result in D under 10D = £4·85 = £4, 17s. 0d.

Problem 71. Find the cost of 450 ft. run of timber 9″ × 6′ at £84, 7s. 6d. per standard.

Surveying

Problem 72. A tower subtends an angle of 10° 4′ at a point 627 ft. horizontally from its base. Calculate the height of the tower.

Example: A hill subtends an angle of 8° 30′ at a point A. At B, 1750 ft. nearer along a horizontal line, the hill subtends 12° 50′. Find the height of the summit above the points of observation.

12° 50′ − 8° 30′ = 4° 20′.

To 1750A set 4° 20′ S_2.

Set X to 12° 50′ S_2. 90 S_2 to X.

Result: 760 ft. in A over 8° 30′ S_2.

Problem 73. A plot of land measured by a planimeter on a plan gave an area of 34·36 sq. inches. The scale of the plan being 1 chain = 1 inch; find the actual area of the plot.

Example: A survey line, *XYZ*, crosses a river too wide to be chained. *X* is on one bank, *Y* on the opposite bank, and *Z* is 100 ft. beyond *Y*. At a point *P* 200 ft. from *Y*, and on a line through *Y* at right angles to *XYZ*, sights are taken on *X* and *Z*. The angle *XPZ* was observed to be 78°. Calculate the width of the river.

Set X to 5D. In T_1 read 26° 40′ under X.
78° – 26° 40′ = 51° 20′. 90° – 51° 20′ = 38° 40′.

Set X to 2D. 38° 40′ T_2 to X.

Result: 250 ft. in D under 10C.

Navigation

Example: A ship steaming at 12 knots runs into a 2-knot current setting S. 10° W. Ship's true course is to be N. 50° W. Calculate course to steer, and find true speed.

Angle between true course and current is 120°
sin 120° = sin 60°.

Set X to $60S_1$. 12B to X. Over 2B read 8° 20′ in S_1.
60° – 8° 20′ = 51° 40′.

Course to steer is N. 41° 40′ W.

Set X to 12A. $60S_2$ to X. X to 51° 40′ S_2.

In A over X read 10·9 knots, the true speed.

Problem 74. From a vessel a lightship was observed 65° forward of the beam on the port side; after steaming 12 miles the light was abeam. Calculate the distance at which the vessel passes the light, and the distance from the light when the first observation was made. If it was desired to pass the light at 2 miles, what alteration to course would have been necessary at the time of first observation?

Example: A ship steaming at 15½ knots will dock in 11¾ hours; it is imperative to dock earlier. What must be the speed to dock in 9½ hours?

To 155D set 95C.

Set X to 10C. 1C to X.

Result: 19·1 knots in D under 1175C.

(This result could have been obtained in one setting of the slide with the duplicated C and D scales—see Section 10.)

Problem 75. From a ship steering N. 60° E., two observations on a lightship were made. First bearing was due E. After steaming 10·5 miles, the second bearing was S. 28° W. Find the distances from the light when the observations were made.

(This problem is: Given $K = 30°$, $L = 118°$, $l = 10·5$. Find k and m.) (A diagram should be drawn.

Miscellaneous

Example: 5¾% stock is purchased at (i) a premium of 8%; (ii) a discount of 8%. Calculate the interest yields in these two cases.

Over 5·75D set 108C.

Read 5·32D under 1C. Result £5, 6s. 5d.%.

Set 92C over 5·75D.

Read 6·25D under 10C. Result £6, 5s. 0d.%.

Example: 68 kilos of material cost 2450 francs. Calculate the cost per lb. and per ton in sterling. (Rate of exchange 960 francs = £1. 453·6 grammes = 1 lb.)

Over 245D set 96C.

Move X to 4536C.

Set 68C to X.

Move X to 10C.

Set 1C to X.

Under 24C read 408D.

Under 224C read 382D.

Answer 4·08 pence per lb.
£38·2 = £38, 4s. 0d. per ton.

Example: Calculate the combined resistance of a divided circuit in which the four branches have separate resistances of 18·6, 24·2, 8·7 and 2·56 ohms.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \cdots.$$

Over 1D set 186C.	
Read 5375D under 10C.	·05375
Over 1D set 242C.	
Read 4135D under 10C.	·04135
Over 1D set 87C.	
Read 115D under 10C.	·115
Over 1D set 256C.	
Read 391C under 10.	·391
	$\frac{1}{R} = \cdot 60110$

Over 1D set 6011C.

Read 1664D under 10C.

Answer 1·664 ohms.

Example: Calculate with one setting of the slide rule the percentages of the various elements in potassium chlorate ($KClO_3$).

$$KClO_3 = 39 + 35{\cdot}5 + 48 = 122{\cdot}5$$

Over 1D set 1225C.	
Under 39C read 318D	i.e. K = 31·8%
Under 355C read 29D	Cl = 29·0
Under 48C read 392D	O = 39·2
Total	100%

Problem 76. Repeat the foregoing exercise for (i) sodium bicarbonate ($NaHCO_3$); (ii) sulphuric acid (H_2SO_4).

Problem 77. A body falls freely under the action of gravity. Calculate how far it will fall in the 1st second, the 5th second and the 10th second after release. Also find the velocity after falling 200 feet and its velocity after 10 seconds from rest.

Example: A cubic foot of water weighs 62·3 lb. Calculate the weight of a sphere of copper 8½″ dia. Specific gravity of Cu 8·8. Repeat for lead, cast iron and silver. (Sp. gravities 11·4, 7·2 and 10·5 respectively.

Over 425D set 10C
Move X to 425C
Set 1C to X
Move X to 425C
} This completes the cubing of 4·25

Set 3C to X.

Move X to πC.

Set 1728C to X.

Move X to 1C.

Set 10C to X.

Move X to 4C.

Set 10C to X.

Move X to 623C.

Set 1C to X.

Under 88C read 102D.

Under 114C read 132·5D.

Under 72C read 836D.

Under 105C read 122D.

Answers 102, 132·5, 83·6 and 122 lb.

This exercise involves an unusual number of operations, all of which have been effected with the C and D scales of the 10″ rule. The cubing of 4¼ at the outset might have been carried

ANSWERS TO PROBLEMS

Problem

1. $\frac{49}{80}$. **2.** $2\frac{3}{5}$. **3.** $1\frac{1}{10}$.

4. $6\frac{4}{5}$, $13\frac{2}{25}$, $19\frac{2}{25}$, $20\frac{1}{8}$, $41\frac{1}{80}$, $86\frac{5}{8}$.

5. 29·723. 13·593. **6.** 766·0324.

7. ·875. ·8125. **8.** $2\frac{823}{990}$.

9. e = 1·02, f = 1·2, g = 2·6, h = 2·87.
m = 4·05, n = 6·25, o = 7·72, p = 9·075.

10. 709 lb. **11.** 2660 cu. inches. **12.** 71·7%.

13.

Direct Labour	39·30%
Drawing Office	3·48
Materials	52·60
Works Overheads	2·60
Gen. Office	2·02
Total	100·00%

14. 1·785.

15. ·819. Digits 3 – 3 – (– 2) – 2 – 1 + [1] = 0.

16. 76·6. 27900.

17. 28·5. 90·2. ·1287. ·00407.

18. 8·3 inches. **19.** 4740.

20. 2. 4·31. 9·28. 21·2. ·277. ·0772.

21. 163. 6·35. ·00615.

22. 2168000. 1·063. **23.** 112500. 1·292.

24. $L = 25° 15'$. $M = 64° 45'$. $m = 11·7$.

25. $L = 69°$. $l = 4·7$. $K = 5·03$.

26. $L = 112·6°$. $M = 23·8°$. $K = 43·6°$.

27. $M = 32°$. $k = 5·6$. $m = 5·9$.

28. $K = 34°$. $L = 44°$. $M = 102°$. 39·2 sq. inches.

29. 2·69 miles. **30.** (*a*) 5·19 miles. (*b*) 8·3 miles.

31. $A = 102°$. $B = 44°$. $C = 34°$.

32. D. long. 35′. **33.** 347 miles W. 200′ N.

34. (i) 17·74 miles. (ii) 19·88. (iii) 21·51.

35. £2, 9s. 4d.

36. 6s. 6d. 9s. 3d. 14s. 8d. 1s. 3d. 48s. 2s.

37. 4s. 3d. £1, 7s. 7d. £6, 3s. 3d. £12, 15s. 0d.

38. £5, 15s. 0d. **39.** £374, 1s. 7d.

40. £1, 8s. 2d. **41.** £2, 4s. 6d. 5s. 3d.

42. 410. **43.** 87·5%. **44.** 1·032.

45. $a^{\frac{13}{4}}$. $a^{-\frac{3}{2}}$. **46.** x^3. **47.** 5.

48. 1. 2·0414. 1·3345. 2·9742. 3·2706.

49. 65480. **50.** 14·44. **51.** 76810. 3.

52. $\bar{1}$·9042. $\bar{3}$·0704. ·4228. ·002958.

53. 12·4. ·378. **54.** 4·33d.

55. (*a*) £5, 7s. 7d. (*b*) £5, 1s. 2d. (*c*) £5, 4s. 11d.
(*d*) £4, 10s. 7d. (*e*) £5, 11s. 6d.

56. £3, 10s. 8d. **57.** 78·5 i.h.p. **58.** 13·1%.

59. 30 h.p. **60.** 1028 lb. **61.** 430 c.c.

62. ·0346. **63.** 928 lb. per sq. inch.

64. 9060 lb. per sq. inch. **65.** 360 h.p.

66. 1·67″. **67.** 489 amps. **68.** 59 kw. **69.** ·248″.

70. 10½d. per foot run. **71.** £86, 6s. 0d.

72. 111 feet. **73.** 3·43 acres.

74. 5·6 miles. 13·3 miles. 16° 21′ to port.

75. 5·9 and 6·3 miles.

76.

(i)		(ii)	
Na	27·4%	H	2·05%
H	1·2%	S	32·65%
C	14·3%	O	65·3%
O	57·1%		

77. 16·1. 145. 306 feet. 113·3 and 322 feet per sec.

78. 178 lb. 4450 lb. **79.** 54%.

80. 14 volts. 78·3 watts. ·105 h.p.