## The Scales of the Slide Rule - Doing the Math

In the first column is the scale name. In the second column is the formula used for that scale. Any simplification is left as an exercise for the reader. The convention used is that $\mathbf{R}$ denote the length of the rule and \# denotes the number on the scale whose position is being calculated. The final column contains notes about the particular scale.
Derivation of the scales was not always easy, and I have not shown here how it was done. Essentially, I went in knowing that the whole thing was based on logarithms, and then played around until I came up with something that worked. Generally I started out quite close -- it became mainly a matter of playing with constants. Jason.

| Scale | Formula | Comments |
| :---: | :---: | :---: |
| A/B | (R/2)* $\log (\#)$ | Used to calculate squares and square roots with the D scale, used to calculate the sine of an angle with the S scale on a Mannheim slide rule |
| C/D | $\mathrm{R} * \log (\#)$ | Used in multiplication and division, and also used with many other scales in various operations |
| CF/DF | $(\log \#-\log \mathrm{PI}) * \mathrm{R}$ if \# less than R then add R | The folded scales used as a shortcut in multiplication and division |
| CI | $a b s[\mathrm{R} * \log (10 / \#)-\mathrm{R}]$ | The inverse of the C scale, often used as a shortcut in division |
| $\begin{aligned} & \quad \mathrm{CIF} \\ & \operatorname{abs}[\mathrm{R} *(\log (1 \\ & / \#)- \\ & \log (1 / \mathrm{PI}))] \\ & \text { if } \#<(10 / \mathrm{PI}) \\ & \operatorname{abs}[\mathrm{R} *(\log (1 \\ & / \#)- \\ & \log (1 / \mathrm{PI}))- \\ & 25] \\ & \text { if \#>(10/PI) } \end{aligned}$ |  | The inverse of the CF scale |
| K | (R/3)* $\log (\#)$ | Used with the D scale to find the cube or cube root of a number |
| L | \#*R | Used with the D scale to calculate the logarithm $\log _{10}(\#)$ of a number |
| LL0 | $\log (\ln (\#)) * \mathrm{R}+3 * \mathrm{R}$ | Contains all numbers greater than or equal to 1.001 and less than or equal to 1.01 ; these scales (LL0-LL3) are used for logarithms, roots, and powers |
| LL1 | $\log (\ln (\#)) * \mathrm{R}+2 * \mathrm{R}$ | Contains all numbers greater than or equal to 1.01 and less than or equal to 1.105 |
| LL2 | $\log (\ln (\#))^{*} \mathrm{R}+\mathrm{R}$ | Contains all numbers greater than or equal to 1.105 and less than or equal to $e$ |


| LL3 | $\log (\ln (\#)) * \mathrm{R}$ | This contains all numbers greater than or equal to |
| :---: | :---: | :---: |
| LL/0 | $\log (\ln (1 / \#)) * \mathrm{R}+3 * \mathrm{R}$ | This contains all numbers greater than or equal to $e^{-0.01}$ and less than or equal to $e^{-0.001}$ |
| LL/1 | $\log (\ln (1 / \#)) * \mathrm{R}+2 * \mathrm{R}$ | This contains all numbers greater than or equal to $e^{-0.1}$ and less than or equal to $e^{-0.01}$ |
| LL/2 | $\log (\ln (1 / \#)) * \mathrm{R}+\mathrm{R}$ | This contains all numbers greater than or equal to $e^{-1.0}$ and less than or equal to $e^{-0.1}$ |
| LL/3 | $\log (\ln (1 / \#)) * \mathrm{R}$ | This contains all numbers greater than or equal to $e^{-10.0}$ and less than or equal to $e^{-1.0}$ |
| $\mathrm{R}_{1}$ | $\log (\#) * 2 * \mathrm{R}$ | Used with the D scale to find squares and square roots; those numbers greater than about 3.13 are on the $\mathrm{R}_{2}$ scale |
| $\mathrm{R}_{2}$ | $[\log (\#) * 2 * \mathrm{R}]-25$ | Used with the D scale to find squares and square roots; those numbers greater than about 3.13 are on the $\mathrm{R}_{2}$ scale |
| $\mathrm{S}_{\text {mannheim }}$ | $(\mathrm{R} / 2) *[2+\log (\sin (\#))]$ | Used with the A scale to calculate the sine of a number, or the tangent of a number less than 5.7 degrees |
| S,T | $[\log (100 * \sin (\#))]^{*} \mathrm{R}$ | Used with the C scale to calculate the sine or the tangent of a number less than 5.7 degrees |
| S | $[\log (10 * \sin (\#))]^{*} \mathrm{R}$ | Used with the C scale to calculate the sine of a number greater than 5.7 degrees |
| T | $\mathrm{R} * \log \left[10^{*} \tan (\#)\right.$ | Used with the D scale to calculate the tangent of angles greater than 5.7 degrees |

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