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OCULAR DEFECTS

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THE THEORY

OF

OCULAR DEFECTS

AND OF

SPECTACLES.

TRANSLATED FROM THE GERMAN OF

DR. HERMANN SCHEFFLER

BY

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With PREFATORY NOTES and a Chapter of PRACTICAL INSTRUCTIONS.

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PREFACE.

Early in the past year, I received from Dr. Scheffler a copy of his *Theorie der Augenfehler und der Brille*, then just published, with a request that I would endeavour to make arrangements for the appearance of an English version.

The labours of Professor Donders, which were first made known in this country in 1862, and which were fully laid before the profession in 1864, in his great work on *The Anomalies of Refraction and Accommodation of the Eye*, produced so great an advance in this department of scientific inquiry as to lead to the impression, among many, that little more remained to be achieved. The contrast between the results afforded by the empirical employment of spectacles in former times, and those afforded by their proper adaptation subsequently, was certainly so great as to explain, if not to justify, the general prevalence of such a belief.

In common, however, I think, with many other ophthalmic surgeons, I had gradually become convinced that the subject was not exhausted, and that the distinguished Utrecht Professor had left some room for others, even in the field that he had cultivated with such signal success. Cases every now and then presented themselves in which the facts would not perfectly harmonize with the doctrines, and in which there was manifestly present some additional unknown quantity, some disturbing factor, of which the doctrines took no account. Spectacles that seemed properly adapted to the requirements of the eyes were found to become sources of discomfort; and a fresh and more careful testing of vision failed to furnish an explanation of its cause.

Under these circumstances I was prepared to receive gladly any additional information upon the subject; and a first perusal of Dr. Scheffler's treatise assured me that, even if he had not removed my difficulties, he had at least pointed out the way in which their removal should be attempted. The separate examination and complete analysis of the several elements of the visual act, and the introduction of the function of 'application,' were steps that seemed likely to explain the conditions actually present in obscure cases, and to supply the unknown factor of which I had felt the need. Especially I believed that I saw through them a clue to the comprehension of some perplexing questions arising out of the manifold varieties of squint; and the view put forth in the chapter on the 'Opponency of the Fields of Vision,' by which the nervous fibres of the two eyes are regarded as having functions in some sense analogous to those of the positive and negative poles of a galvanic circuit, and as being able, by a sort of commutation, to reverse the action and relieve each other by becoming, as it were, anodes and cathodes by turns, seemed likely, if confirmed by further investigation, not only to explain many important visual phenomena that are at present imperfectly understood, but even to throw light upon the general purpose of the duality of nervous organs. For a complete account of these theories, and of the facts upon which they are founded, the reader must be referred to Dr. Scheffler's earlier works, the *Physiologische Optik*, and the *Gesetze des raümlichen Sehens*, to which reference is so frequently made in the text of the present treatise.

As regards the combination, in spectacles, of lenses and prisms, I was aware that steps in this direction were taken, several years ago, by Dr. Giraud-Teulon, of Paris; and that his 'decentred' spectacles, as they were called, were sold in London. On referring to his writings upon the subject I find that his work was, to a certain point at least, parallel with that of Dr. Scheffler; and to Dr. Giraud-Teulon belongs, as far as I know, the merit of having originated the idea. To Dr. Scheffler belongs, I believe, the merit of having worked out the particular 'orthoscopic' combination; and of having investigated the forces that must be taken into account before the combinations required for special cases can be correctly determined.

The original work, as published in Germany, was carried down only to page 167 of that now in the hands of the reader. I suggested to Dr. Scheffler that it would be desirable to supply detailed practical instructions for the examination of eyes and the selection of spectacles; and this task he was good enough to undertake. The last seventy-three pages are devoted to the subject, and were translated from the author's manuscript.

The test case, or the collection of lenses and prisms, required for carrying out the investigations, has demanded careful consideration. As regards the primary element, the lenses, it is well known that much dissatisfaction has been felt with the irregular intervals between those in the test cases commonly sold; and that confusion has been produced by the varieties of the so-called 'inches' in which focal lengths have been expressed. At the last International Ophthalmological Congress, held at Paris in 1867, a committee was appointed to investigate these questions. Their report is not yet made public, but I have reason to believe that they will advise the general adoption of a metrical scale, and of a series of which the unit, and the interval between each two steps, shall be a lens having a focal length of 240 centimetres. This would give a series of 48 convex and 48 concave lenses, proceeding in regular gradation from 240 to 5 centimetres of focal length. These lenses might be designated by their numbers, according to the plan suggested by Dr. Zehender. The lens of 240 cm. focal length would be No. 1, the unit of the series. The lens of 120 cm. focal length would be No. 2, and would be equal to two such units. The lens of 24 cm. focal length would be No. 10, and would be equal to ten such units. The lens of 5 cm. focal length would be No. 48, and would be equal to forty-eight such units.

As a practical question, however, it would nct be necessary to possess the whole of such a series; because, if the lenses were made with one plane side, it would be easy to supply any breaks by combinations of two, put together in the testing frame. A set of twenty pairs of convex lenses, and twenty pairs of concave lenses, would be sufficient for every purpose; and the interrupted series given in the following table is that which would, I believe, satisfy all requirements. The first column contains the number of the lens, the second its focal length in centimetres, the third its focal length in Paris inches, the fourth its focal length in Prussian inches, and the fifth the refracting angle (i.e. the measured angle between the sides), expressed in English degrees, of the prism that, for eyes separated by the ordinary interval, would form with it an orthoscopic combination. The convex and the concave series would, of course, be the same; and the prisms could be used with either, by reversing the direction of their bases in the testing frame. In the series suggested it will be seen, by a glance at the table, that the breaks can be supplied, between 6 and 24, by the addition of No. 1 to the lower number; between 24 and 30, by the addition of No. 1 or No. 2; between 30 and 34, by one of the first three numbers; between 34 and 40, by one of the first five; and between 40 and 48, by one of the first six, whose number, when added to the next lower number in the table, will make up the number required.

Number of each Lens in the Series	Focal Length in Centimetres	Focal Length in Paris Inches	Focal Length in Prussian Inches	Angular Measurement of the Prism that forms with it an 'Orthoscopic Combination'
1	240	88·66	91·75	1° 29′ 10″
2	120	44·33	45·88	2 58 23
3	80	29·55	30·58	4 27 37
4	60	22·16	22·94	5 56 54
5	48	17·73	18·35	7 26 14
6	40	14·78	15 29	8 55 39
8	30	11·08	11·47	
10	24	8·86	9·17	
12	20	7·38	7·65	
14	17·14	6·33	6·55	
16	15	5·54	5·73	
18	13·$\dot{3}$	4·93	5·10	
20	12	4·43	4·59	
22	10·90	4·03	4·17	
24	10	3·69	3·82	
27	8·8	3·28	3·40	
30	8	2·95	3·06	
34	7·05	2·61	2·70	
40	6	2·22	2·29	
48	5	1·85	1·91	

The angular measurement of the prisms, in degrees, minutes, and seconds, as given in the table, has been calculated for each focal length f, upon the index of refraction (1·53) of the glass actually used by Messrs. Paetz and Flohr, of Berlin, for making testing prisms, and upon the supposition that d, the interval between the centres of the eyes, has its average value of 66 millimetres, according to the formula on page 6, $m = \frac{d}{2(n-1)} \cdot \frac{1}{f}$, and in the manner following:—If ϕ be the refracting angle of the prism,

$$m = \text{the chord (to radius 1) of the subtending arc.}$$

$$= 2 \sin. \frac{\phi}{2}.$$

$$\therefore \sin. \frac{\phi}{2} = \frac{d}{4(n-1)} \cdot \frac{1}{f};$$

whence the value of sin. $\frac{\phi}{2}$ can be determined, and thence $\frac{\phi}{2}$, and therefore ϕ, be found from a table of sines.

Ex. 1. By natural numbers.

Let $d = 66$ millimetres; $f = 2400$ millims.;

$$n = 1{\cdot}53 \quad \therefore n - 1 = 0{\cdot}53.$$

$$\therefore \frac{d}{4(n-1)} = \frac{66}{4 \times 0{\cdot}53} = 31{\cdot}1320755;$$

$$\therefore \sin. \frac{\phi}{2} = \frac{31{\cdot}1320755}{2400} = 0{\cdot}0129717,$$

which will be found in the tables as sin. 44′ 35″.

$$\therefore \frac{\phi}{2} = 44' \; 35'', \quad \therefore \phi = 1^\circ \; 29' \; 10''.$$

Ex. 2. The same by logarithmic computation.

$$\text{Log. sin.} \frac{\phi}{2} = 10 + \log. 66 - \log. 4 - \log. 0{\cdot}53 - \log. 2400.$$

$10 + \log. 66 =$	. . . . .	11·8195439
$\log. 4 = 0{\cdot}6020600$		
$\log. 0{\cdot}53 = \bar{1}{\cdot}7242759$	. .	0·3263359
$\therefore 10 + \log. 66 - (\log. 4 + \log. 0{\cdot}53)$.		= 11·4932080
and log. 2400 .		= 3·3802112
$\therefore \log. \sin. \frac{\phi}{2}$.		= 8·1129968;

whence from the table of logarithmic sines—

$$\frac{\phi}{2} = 44' \; 35'', \text{ and } \phi = 1^\circ \; 29' \; 10'' \text{ as before.}$$

Hence, to find the refracting angle required:

1. By natural numbers; divide 31·1320755 by the proposed focal length of the lens in millims., look for the quotient in a table of natural sines, and double the angle there given; or

2. By logarithms; subtract the logarithm of the focal length in millims. from 11·4932080, look for the remainder in a table of logarithmic sines; double the angle thus found will be the angle required.

If it be desired to find the angle for any other breadth between the eyes than 66 millimetres, or for glass of any other refractive index than 1·53, this can be done from the commencement, after the manner of either of the preceding examples.

It is found in practice that the difficulties arising from aberration and dispersion become insuperable, excepting by achromatic prisms, when an angular measure of more than 8° is attained; and therefore, for the purpose of orthoscopic combinations, and for the required testing, only the first five prisms given in the table will be required, and it has not been thought necessary to calculate the rest. But for other and ordinary purposes, the test case should contain, of course, the usual full set of prisms, from 1° to 24°, the special ones given in the table forming a separate series. Messrs. Weiss and Son have designed an instrument for holding glasses, in which the lenses and prisms may be combined; and will be prepared to supply the complete cases, from Paetz and Flohr, and also the orthoscopic or other combination spectacles.

The phrase 'proportion of deviation,' used to denote the quantity h or tan ψ, would have been better translated 'ratio of deviation.' It is, in the original, '*Ablenkungsverhältniss*,' and is simply the goniometric function or ratio which expresses the amount of deviation caused by the prism.

I put forth this little book with much hopefulness, fully believing that practical use will demonstrate its value, and that it will be found to contain the key to many difficulties. The complete application of the author's principles is at present only possible within a limited range, on account of the impediments mentioned above. If the progress of science should furnish us with glass of higher refractive power, without a corresponding increase of the dispersive power, these impediments would be proportionately removed.

In conclusion, I have to thank the author for the care with which he has replied to my inquiries upon various questions that have arisen from time to time; and also to say that I am indebted to the great kindness of the Rev. W. de Lancey Lawson, of Fishponds, for the verification of all the formulæ in the work, as well as for many valuable suggestions and much general assistance. Mr. Lawson has also favoured me with the following explanatory notes, calculated to remove difficulties that might otherwise perplex the reader at the very threshold of his undertaking.

§ *On the Optical Formulæ in pages* 6, 7, *and* 8.

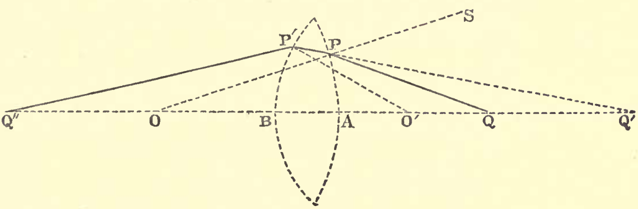
The investigations in this work are founded on a few mathematical formulæ, presented in its early pages, which are either supposed to be already known to the reader, or for which he is referred to the author's earlier writings, which have not been translated into English. For some of these, reference might be made to well-known English books, where, however, they would be found expressed in other characters, and, in some cases, with other algebraical signs, than those used by Dr. Scheffler; while others are his own deductions from them, of which we are here presented only with the results. It has therefore been thought that it would be a boon to the English reader if a short investigation of these were prefixed to the translation, so as to render it intelligible, without reference to other books, to any one possessed of an elementary knowledge of algebra, geometry, and trigonometry.

These investigations are necessarily founded on the well-known law of refraction, that the ratio of the sines of the angles which the incident and refracted rays make with the normal, or perpendicular to the surface, at the point of refraction, is always constant for the same substance, and the same kind of light. This ratio is called the index of refraction, and is represented by n in the following pages, but by English writers is usually expressed by μ. It should also be observed that the refracting angles of the prisms, and therefore the angles of total deviation of the rays of light, are supposed to be small; so that in some cases their sines, tangents, and the corresponding arcs and chords, to radius unity, may be substituted for each other without sensible error.

1. To find the focus of a pencil of rays after refraction through a thin lens.

Let Q′ Q″ be the axis of a lens A P P′ B, and also that of a pencil of rays proceeding from a point Q; let Q P be another ray of the pencil refracted by the first surface at P into a

Fig. *a*.



direction P P′, produced backwards to Q′; O, O′ the centres, and r', r'' the radii of the first and second surfaces; and let A P be supposed small, so that Q A, and Q′ A are nearly equal to Q P, Q′ P respectively.

$$\text{Let } PQ = QA = a; \quad \therefore OQ = a + r'.$$
$$PQ' = Q'A = a', \quad \therefore OQ' = a' + r'.$$

Then $\frac{\sin. \text{OPQ}}{\sin. \text{OPQ}'} = \frac{\sin. \text{SPQ}}{\sin. \text{SPQ}'} = n$ $\left\{\begin{array}{l}\text{since these are the angles of} \\ \text{incidence and refraction.}\end{array}\right.$

But, in the triangles POQ, POQ′,

$$\sin. \text{POQ} = \frac{\text{PQ}}{\text{QO}} \sin. \text{OPQ} = \frac{a}{a + r'} \sin. \text{OPQ},$$

$$\sin. \text{POQ}' = \frac{\text{PQ}'}{\text{Q}'\text{O}} \sin. \text{OPQ}' = \frac{a'}{a' + r'} \sin. \text{OPQ}';$$

Whence, since the angles POQ, POQ′ are the same, we have

$$\frac{a}{a + r'} \sin. \text{OPQ} = \frac{a'}{a' + r'} \sin. \text{OPQ}';$$

$$\text{or } n \cdot \frac{a}{a + r'} = \frac{a'}{a' + r'}; \text{ or } n\left(1 + \frac{r'}{a'}\right) = 1 + \frac{r'}{a'}.$$

$$\therefore \frac{1}{a} - \frac{n}{a'} = \frac{n-1}{r'}. \quad . \quad . \quad . \quad . \quad . \quad [a]$$

Then let the ray Q′PP′ be refracted from the lens at P′ in the second surface; it will pass out of the glass with an index of refraction $\frac{1}{n}$; let it meet the axis in Q″.

Let P′Q″ = BQ″ = x; ∴ O′Q″ = $x + r''$; O′Q′ = $a' - r''$, since the thickness of the lens is supposed small.

It may be shown, by the same steps as in the former case, that

$$\frac{1}{n} \cdot \frac{a'}{a' - r''} = \frac{x}{x + r''}; \text{ or } n \cdot \frac{a' - r''}{a'} = \frac{x + r''}{x}$$

$$\therefore n\left(1 - \frac{r''}{a'}\right) = 1 + \frac{r''}{x}.$$

$$\therefore \frac{1}{x} + \frac{n}{a'} = \frac{n-1}{r''}. \quad . \quad . \quad . \quad . \quad . \quad [b]$$

Hence, by adding [a] and [b] together, we have

$$\frac{1}{a} + \frac{1}{x} = (n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right) \quad . \quad . \quad [1].$$

This expression, though found for a bi-convex lens, is perfectly general, and can be adapted to concave surfaces by making one or both of the radii negative; to plane surfaces by making one or both of them infinite; to parallel rays, before or after refraction, by making a or x infinite; to convergent incident rays by making a negative; to divergent emergent rays by making x negative.*

To adapt it to rays parallel before refraction;

Let $a = \infty$, $\therefore \frac{1}{a} = 0$; and for x substitute f, as it will now become the *principal* focal length of the lens;

$\therefore \frac{1}{f} = (n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right)$ for a bi-convex lens.

$\frac{1}{f} = -(n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right)$, for a bi-concave lens, since in the latter case r' and r'' are both negative.

From the former $f = \frac{r'\,r''}{(n-1)(r'+r'')}$, as in page 8; and, by substituting this value in formula [1], we have

$\frac{1}{a} + \frac{1}{x} = \frac{1}{f}$, for converging rays after refraction;

$\frac{1}{a} - \frac{1}{x} = \frac{1}{f}$, for diverging rays.

If r'' is negative, and $> r'$, then we have the convexo-concave lens mentioned in p. 230, for which the formula becomes

$$f = \frac{r'\,r''}{(n-1)(r''-r')}.$$

Again, from the formula $\frac{1}{a} + \frac{1}{x} = \frac{1}{f}$, we have

$$x = \frac{af}{a-f} = f + \frac{f^2}{a-f} = f + \frac{f^2}{a'},$$

* Various assumptions have been made by different writers about the signs of these quantities; that of Mr. Coddington (*Reflexion and Refraction*, § 85) has been here adopted, as it makes the formulæ harmonize in this respect with those in the following pages.

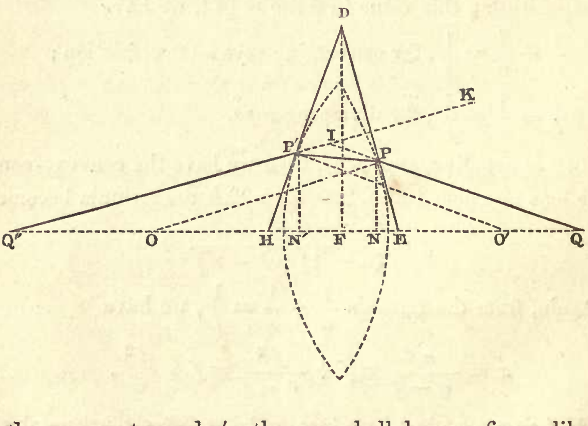
where a' is substituted for $a - f$; that is, the distance of the original luminous point is measured from a point which lies at a distance f from the front of the lens, or of the cornea of the eye, to which this last formula, though with different characters, is applied in pages 23 and 35.

The expression [1] is the foundation for the whole theory of the lens; but, where great accuracy is required, it admits of a second and nearer approximation, and also a correction for the central thickness of the lens, where that is too considerable to be neglected. It is, however, sufficient for our present purpose.

2. To find the total deviation of a ray, or axis of a pencil of rays, passing through a thin prism, in terms of the base and sides of the prism.

In Fig. *a* in the last article, suppose tangents to be drawn

Fig. *b*.



to the curves at P and P′; then we shall have a figure like *b*, representing a section of a prism by a plane perpendicular to its edges; and since at the points P, P′ the faces of the prism coincide with the curved surfaces, the direction of a ray Q P P′ Q″ passing through these points will be the same for the prism and the lens, and the equation

$$\frac{1}{a} + \frac{1}{x} = (n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right) \text{ will still hold good.}$$

Let P N, D F, P′ N′ be drawn perpendicular to Q″Q, and Q P, Q″ P′ be produced to K and I, meeting in I. Since the angles at Q, Q″ and D are small, we may consider approximately that Q N = Q P = a; Q″ N′ = Q″ P′ = x; PN = P′N′ = e; and D H = D E. Then, the total deviation ψ = the exterior angle Q I K = angles Q + Q″. ∴ tan. ψ = tan. (Q + Q″) = tan. Q + tan. Q″ (since the angles are small),

$$= \frac{\text{PN}}{\text{QN}} + \frac{\text{P}'\text{N}'}{\text{Q}''\text{N}'} = e\left(\frac{1}{a} + \frac{1}{x}\right) = e\,(n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right). \quad [a]$$

But $\frac{e}{r'} = \frac{\text{P N}}{\text{P O}} = \frac{\text{F E}}{\text{D E}}$; and $\frac{e}{r''} = \frac{\text{P}'\text{N}'}{\text{P}'\text{O}'} = \frac{\text{F H}}{\text{D H}}$, by similar triangles;

$$\therefore e\left(\frac{1}{r'} + \frac{1}{r''}\right) = \frac{\text{FE} + \text{FH}}{\text{DE}} = \frac{\text{HE}}{\text{DH}} = m. \quad . \quad . \quad [b]$$

$$\therefore h = \text{tan. } \psi = e\,(n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right) = m\,.\,(n-1), \text{ as in p. 6.}$$

where m = the ratio of the base to the side of the prism, or = the chord, to radius unity, of ϕ the refracting angle of the prism.

3. To find the relation between the lens and the prism which form an orthoscopic glass as in p. 6.

Let the reader, after considering the explanation given in pages 5 and 6, suppose b and b', the centres of the glasses (Figs. 1 and 2), to be joined by a line $b\,m\,b'$ cutting the frontal axis $a\,f$ at right angles at m.

$$\text{Then } b\,m = b'\,m = \frac{b\,b'}{2} = \frac{d}{2} \text{ nearly.}$$

Let $a = b\,a = b'\,a = m\,a$ nearly.

$x = b\,e = b'\,e = m\,e$ nearly.

$\alpha =$ the angle $b\,a\,m = b'\,a\,m$.

$\beta =$ the angle $b\,e\,m = b'\,e\,m$.

Then, in Fig. 1 for the convex lens,

$$\tan.\ \alpha = \frac{b\,m}{a\,m} = \frac{d}{2\,a};\ \tan.\ \beta = \frac{b\,m}{e\,m} = \frac{d}{2\,x}.$$

And in order that the prism may be adapted to the lens, we have the angle of deviation $\psi = \alpha - \beta$,

$$\therefore h = \tan.\ \psi = \tan.(\alpha - \beta) = \frac{\tan.\ \alpha - \tan.\ \beta}{1 + \tan.\ \alpha\ \tan.\ \beta} = \tan.\alpha - \tan.\beta$$

$$= \frac{d}{2}\left(\frac{1}{a} - \frac{1}{x}\right);\ \text{since the angles are small.}$$

But, from formula [1] for the lens, we have $\frac{1}{a} - \frac{1}{x} = (n - 1)\frac{1}{r'}$; where, since the emergent rays diverge, the negative sign is given to x; and, since the second surface is plane, $r'' = \infty$, $\therefore \frac{1}{r''} = 0$.

$$\therefore h = \frac{d}{2}\left(\frac{1}{a} - \frac{1}{x}\right) = \frac{d}{2}(n - 1)\cdot\frac{1}{r'} = \frac{d}{2}\cdot\frac{1}{f};$$

or, if the angle of the prism be required,

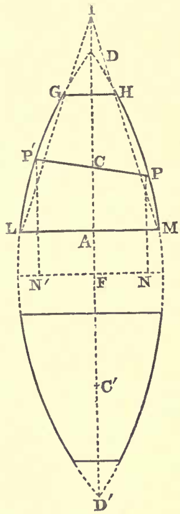
$$m = \frac{h}{n - 1} = \frac{d}{2\,(n - 1)}\cdot\frac{1}{f};\ \text{as in pages 6 and 7.}$$

The same investigation would apply to figure 2 for the concave lens, except that f would then be negative, which would make m also negative, which would imply that the base of the prism must be turned in the contrary direction *from* the frontal axis.

4. To find the distance from the centre of a larger lens, at which smaller prismatic lenses may be cut eccentrically.

In Fig. *b*, if C be the point where the ray P P′ crosses the central line, and G H, L M be drawn at equal distances from C and parallel to N′ N, and the chords L G, M H be produced to meet in I, we have the figure *c*, which represents a smaller prismatic lens G H P M L P′ cut out eccentrically from a larger lens D D′.

Fig. *c*.



Then, since P N is supposed approximately equal to P′ N′, the chords H M, L G may be considered as parallel to the tangents at P and P′; the prism L I M will have the same refracting angle at I as that in § 2, and the deflection of the ray Q P P′ Q″ will not be altered.

Hence it may be shown, as in that section, that, if F C $= e$,

$$e \cdot \left(\frac{1}{r'} + \frac{1}{r''}\right) = m\,; \quad \therefore\, e = \frac{r'\,r''}{r' + r''} \cdot m\,;$$

as in p. 8; and, the other two formulæ in that page having been already proved, we have

$$e = \frac{d}{2}\,.$$

This lens, though made of a continuous piece of glass, evidently consists, *in form*, of a prism interposed between two plano-convex lenses. The reader can easily conceive, or draw for himself, the figure representing a prism interposed between two plano-concave lenses, where the base of the prism will be turned *from* the centre of the larger lens, and its sides will be tangents to its surfaces at P and P′.

It will be seen from this investigation, that any eccentric portion of a lens must always combine the deflecting properties of a prism, with the focalizing properties due to its own curved surfaces.

§§ *To express the prism, required in any case, in degrees of its refracting angle.*

If, instead of adapting a prism to a lens of given focal length f, it be required to find a prism of the convergence distance g, since

$$m = \frac{h}{n-1}, \text{ and } h = \frac{d}{2g}, \text{ (page 70)}$$

$$\therefore\ m = \frac{d}{2\,(n-1)} \cdot \frac{1}{g} \ \therefore\ \sin. \frac{\phi}{2} = \frac{d}{4\,(n-1)} \cdot \frac{1}{g};$$

or, by substituting g for f, the same method may be applied, or the same table used, to find the angle of the prism.

If ψ, the required angle of deviation of the prism, be given in degrees, as in page 77, and be small, we may, without considerable error, find the refracting angle ϕ from the equation

$$\phi = \frac{\psi}{n-1}, = \frac{\psi}{0{\cdot}53} \text{ if } n = 1{\cdot}53;$$

so that ϕ, the refracting angle of the prism, will be very nearly equal to double the angle of total deviation.

8 Princes Street, Hanover Square:
January 1869.

CONTENTS.

THEORY

OF

OCULAR DEFECTS.

INTRODUCTION.

The determination of the proper spectacles, for any defect of the eyes, is undoubtedly a matter of the greatest importance in practical ophthalmology. This importance is increased, on the one hand, by the number of patients requiring such assistance—a number that, throughout the civilised world, may be reckoned by millions—on the other, by the actual injury inflicted by unsuitable glasses; and, lastly, by the customary sale of most spectacles by persons who have no adequate knowledge of the conditions of their utility.

Since the visual process is not an absolutely simple action, but is compounded of several acts which are originally independent, and which, through the unity of the sensorium, come secondarily to influence one another, it follows that, in defects of vision, the determination of the necessary spectacles can never be a perfectly simple procedure. It requires a due consideration of all the ocular functions; and the neglect of any one of them may produce a strain which will act injuriously, and will impair the eyes; while the proper spectacles, adapted to correct the defect, would be preservative and beneficial. For example, it is possible that a given pair of eyes, in order to see clearly at a given distance, may exert only a moderate effort of accommodation, together with an excessive effort of convergence; and this discord between accommodation and convergence may produce a painful strain, may prevent continued exertion of the eyes at the particular distance, and may, by prolonged effort, aggravate the original defect. Spectacles which remove this discord, and which in moderate tension of the accommodation call for only a moderate degree of convergence, would relieve all the inconvenience, and would, consequently, exert a beneficial influence upon the organs of vision.

But although we may set down the removal of disproportion between the several elements of the visual act as an essential principle in the selection of spectacles, yet in the practical application of this principle it is easy to fall into error, unless that which we call a disproportion be thoroughly understood. In any particular case we must not affirm the existence of disproportion between accommodation and convergence, because their relative tension differs from that of a perfectly normal pair of eyes, or from the absolutely normal proportion between the two functions; since that proportion which is most suitable for a given pair may be determined by their special physiological and pathological conditions, or by the nature of their defects. We have, therefore, generally speaking, to consider an individual or relative normal proportion, which cannot be deduced from completely normal eyes alone, but only by the proper estimation of the influences of certain abnormal conditions.

It is well known that perfectly normal eyes, by unusual methods of exertion, may be brought into a state in which the absolutely normal proportion between the elements of the visual function becomes itself a cause of strain.

This state may even become more lasting; and then the conditions for the suitable or relative normal proportion are already modified; the eye has become accustomed to an abnormal proportion between its functions. Most defective eyes are found to be permanently in such a condition; in which a proportion, more or less deviating from the normal, is the one most comfortable to them.

If it be necessary to quote any ophthalmological authority for these facts, I may adduce the words of Professor Donders, in his work on the 'Anomalies of Refraction and Accommodation.' He says, on pages 124, 125, 'The myopic eye has learnt to converge in a certain degree, without bringing its power of accommodation into action in the same proportion as the emmetropic eye. The hypermetropic eye, on the contrary, found itself obliged, even with parallel visual lines, to put its power of accommodation on the stretch, and it has brought itself so far in that respect, that it is no longer in a position to become completely relaxed; that at least on every effort to see, the act of accommodation takes place involuntarily. The use of positive or negative glasses has, even after the lapse of a few hours, an influence on the range of accommodation of the emmetropic eye. The relative range of accommodation is displaced in ametropic individuals when they have for a long time worn correcting glasses.'

It therefore depends very much upon circumstances how far an abnormal proportion between the functions, when once established, should be respected; and how far it should be corrected by glasses. In my *Physiologische Optik*, and in the supplement thereto, entitled *Die Gesetze des räumlichen Sehens*, I have entirely assented to the principle of latitude in reference to the modification of existing functional proportions; and I have given calculations and conclusions chiefly for those ocular defects in which the functional proportion actually present during unaided vision is the most suitable, and is to be maintained. As far as the theory is concerned, it is immaterial whether the number of such cases be large or small; but the application of the theory in practice requires a careful study of the cases in which the contrary will hold good, and the more so, the more numerous they are. The most recent observations, and especially those of Donders and von Graefe, seem to leave no doubt that cases of the latter kind, in which it is necessary to combat the actually existing proportion in defective eyes, occur in considerable numbers. I purpose, therefore, in the following pages, to place before ophthalmologists a general mathematical analysis of ocular defects, and of the principles on which spectacles for them should be selected. I believe that the data thus furnished, together with the admirable works of Donders and of von Graefe, will not only contribute to an elucidation of the nature of the visual processes, but will also furnish useful aids to practical treatment.

ORTHOSCOPIC SPECTACLES.

Before entering upon the special subject of my inquiry, it seems desirable to justify, by a short description, the commendation that I have bestowed in my earlier writings upon *orthoscopic spectacles.* I believe that these glasses, although they may require modification in many instances, are yet the most useful for large classes of ocular defects; and that they rest upon the only rational basis of spectacle construction for binocular vision; so that modifications of them should be considered exceptions, rendered necessary by the peculiarities of individual cases.

When any normal or abnormal pair of eyes is furnished with ordinary convex or concave centric spectacles, no change takes place in the direction of the visual lines, and none, therefore, in the convergence of the eyes; although the object looked at changes its apparent distance on the visual line of each eye. There is therefore a change in the accommodation, while the convergence remains unaltered; and it follows that the proportion between the two functions is modified. There is no longer a single object presented to the two eyes, but to each eye its own object: the two no longer fixing a single point in space, but two separated points. (It is evident that no double vision would be thus produced, since the two images fall upon identical portions of the retinæ.) By this the eyes are placed in a condition which does not correspond with their state during unassisted vision without glasses, since a single object is then always presented to them. Under certain circumstances the modification so produced may be beneficial;

Fig. 1.

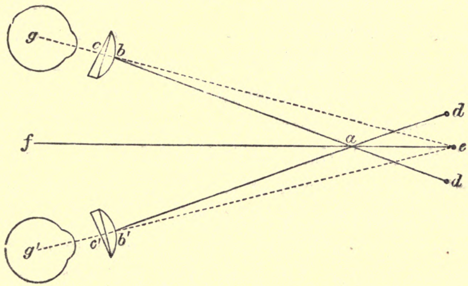
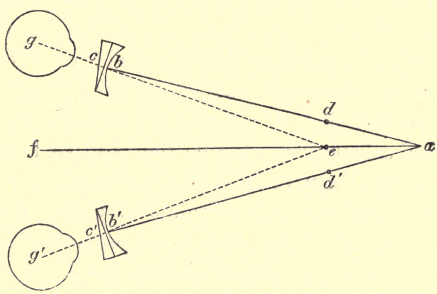


Fig. 2.



but, in all cases in which the functional proportion maintained during unassisted vision is the most convenient, the use of common spectacles produces an undesirable visual strain.

The spectacles that I have called orthoscopic consist of a combination of lenses and prisms, so arranged that the deviation of the rays of light occasioned by the prism corresponds accurately with the change in the visual distance occasioned by the lens. Through such spectacles (Fig. 1 or Fig. 2), the simple actual object *a* is displaced, by means of the lenses, to the distances *g d*, *g′d′*; and, by means of the prisms, in the directions *g e*, *g′e*. There is presented to the eyes, therefore, through the orthoscopic spectacles, instead of the single actual object *a*, a single apparent object *e*; and the eyes, looking at *a* through the spectacles, are in precisely the same condition as if they were looking at *e* without spectacles. Vision through these spectacles is therefore, for every pair of eyes, whether normal or abnormal, completely free vision. The spectacles produce only an apparent change in the distance of the object, and a consequent displacement of the region of accommodation; with the result that the limits of the clearly visible space are, according to the power of the glasses and the state of the eyes, extended, contracted, or moved.

From the foregoing it follows that the orthoscopic spectacles never produce strain. Although they may not diminish the strain sometimes associated with the unaided use of the eyes, they never increase it; and usually, indeed, leave it wholly unchanged.

It is of special importance that the proportion of the prism to the lens, in an orthoscopic glass, should be determined not on physiological, but solely on physical principles, and by a purely mathematical calculation. If f be the positive focal length of a convex, or the negative focal length of a concave lens, d the distance between the central points of the two eyes, n the index of refraction of the glass, m the proportion of the base of a prismatic or wedge-shaped glass to the length of its sides, ψ the angle of total deviation of the prism, $h = \text{tang. } \psi$, the proportion of deviation of the prism, then the prism that should be combined with the lens f to form an orthoscopic glass is shown by the formula:—

$$m = \frac{d}{2(n-1)} \cdot \frac{1}{f}$$

or, since we have $h = m(n-1)$, by the formula,

$$h = \text{tang.}\ \psi = \frac{d}{2} \cdot \frac{1}{f}.$$

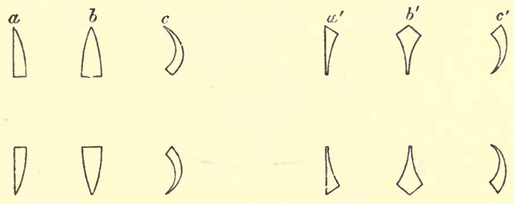
For a convex lens the base of the prism must be turned inwards, that is, towards the nose; and for a concave lens it must be turned outwards.

The above formula gives for every lens a determinate prism, which forms with it an orthoscopic glass. This prism is wholly independent of the distance of the object; an orthoscopic glass being orthoscopic not only for a given distance, but for all distances. Its determination does not rest upon physiological grounds, and it is therefore orthoscopic for any eye, whether normal or abnormal. From hence it follows that the orthoscopic glass is the rational or absolute basis for the construction of spectacles for binocular vision.

A spectacle lens composed of a spherical and a prismatic glass may be called, as a general term, a combination lens. Such a lens becomes orthoscopic by reason of the proper value of its prism.

It is evident that the prism and the convex or concave lens need not be formed from separate pieces, but may be ground from a single glass, and that the combination may be either plano-convex and plano-concave, *a* and *a′* (Fig. 3), bi-convex and bi-concave, *b* and *b′*, or periscopic, *c* and *c′*.

Fig. 3.



In § 50 of the *Physiologische Optik*, I have also shown that these glasses may be regarded as eccentric portions of larger lenses, and may therefore be called eccentric glasses.

This is of special importance as regards their fabrication, since the larger lenses, from which the eccentric portions are cut, may be made in the ordinary way. It is shown in p. 99 of

Fig. 4.

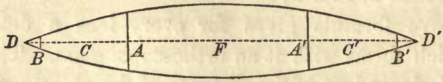
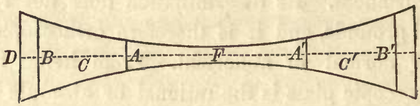


Fig. 5.



the second part of the *Physiologische Optik*, that the distance F C $= e$ (Figs. 4 and 5) of the centre C of the eccentric glass A B from the centre F of the larger lens, and that, when we take r' and r'' as the radius respectively of the upper and under surface,

$$e = \frac{r' \, r''}{r' + r''} \cdot m.$$

If we substitute for the radii r'' and r' the focal length of the lens f, after the formula,

$$f = \frac{r' \, r''}{(n-1)\,(r' + r'')}$$

and for the prism m the already ascertained value

$$m = \frac{d}{2(n-1)} \cdot \frac{1}{f}$$

we obtain, for the distance F C, the value

$$e = \frac{d}{2}.$$

This result is highly important. It teaches that the distance of the centre C of the eccentric spectacle lens A B, from the centre F of the large lens out of which it is cut, must in all cases be equal to the distance of the centre of the eye from the axis of the nose, whatever may be the focal length of the lens.

Furthermore, it shows that two companion eccentric lenses, A B and A′ B′, may be cut from the single large lens B B′, since in such a lens there are two portions of which the centres C and C′ measure exactly the distance apart $C C' = 2e, = d$, the distance between the centres of the eyes.

If we consider further, that spectacle lenses have a breadth of 1·3 Paris inch, and that therefore the large lens, for a medium distance of 2·44 inches between the eyes, should have a breadth of 3·74 inches, or, on account of waste in grinding, of 4 inches, the diagram in Fig. 6 will show that a large lens of such dimensions would furnish three pairs of similar eccentric lenses, or three pairs of orthoscopic spectacles.

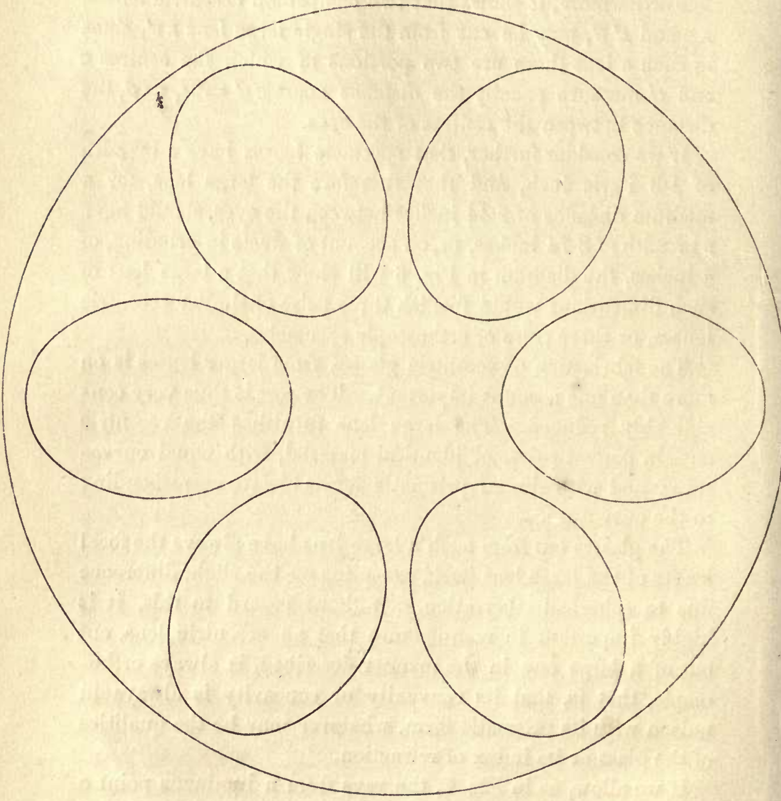
The fabrication of eccentric glasses from larger lenses is on more than one account important. The cost is thus very considerably reduced. The large lens furnishes the eccentric ones in perfect pairs, of identical material, with equal curvatures, and with similar prismatic forms always corresponding to the curvature.

The glasses cut from such a large lens have always the focal length of the large lens itself, excepting for the slight difference due to spherical aberration. Without regard to this, it is highly important in manufacture that an eccentric lens, cut out of a large one in the manner described, is always orthoscopic, that is, that its convexity or concavity is always in unison with its prismatic form, whatever may be the qualities of the glass or its index of refraction.

If we allow, as in Fig. 7, the rays from a luminous point G to pass through the symmetrically situated parts A B and A′ B′ of the large lens D D′, these rays will be reunited in a point H of the axis G H; and this reunion in a single axial point is the characteristic of orthoscopic spectacles.

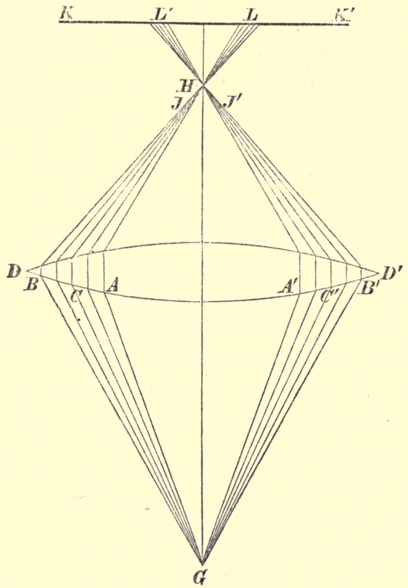
The defect from spherical aberration, above referred to, amounts to this, that the two bundles of rays, passing from the point G through the two portions A B and A′ B′ of the lens

Fig. 6.



D, do not unite accurately in one axial point H, but in two lateral points J and J′. This defect increases with the curvature of the lens, and may easily be shown in convex lenses by covering the surface D D′ by a diaphragm perforated so as to admit light only to the portions A B and A′ B′. The rays from G, passing through these portions, must be received upon a screen, placed at the focal length of A B and A′ B′, as shown by the sharp definition of the images J and J′.

Fig. 7.



If these two images do not exactly coincide, the portions A B, A′ B′, in the same relative position that they occupy in the large lens, will not be perfectly orthoscopic.

If the distance of the two foci, J and J′, from the axis, be only small, the error may be corrected by setting the two glasses A B and A′ B′, after they are cut out, nearer together or farther apart, until the foci unite. In this way we obtain orthoscopic glasses for eyes of which the centres are separated by a distance a little greater or less than the original distance C C′.

If the distance between the two foci be more considerable, or if the glasses are required for eyes separated by precisely the distance C C′, the foregoing method is not available; and it becomes necessary to modify the refracting angle in each glass. It is, notwithstanding, always possible to cut the pair of lenses required for orthoscopic spectacles from a single lens of the given curvature. For when the distance of the centres C C′, of the portions to be cut out, is increased, the two foci J J′, when situated as in Fig. 7, also become still wider apart; but the distance between them does not increase in the same degree as the distance between the openings A B, A′ B′. Accordingly, the distance between the openings in the diaphragm may be increased until the distance apart of their central points, C C′, is too great by the distance apart of the foci J J′. Then the lenses A B, A′ B′, when cut out, may be again approximated, in the spectacle frame, by the same distance J J′; and in this way orthoscopic glasses are obtained for the given distance C C′ of the centres of the eyes.

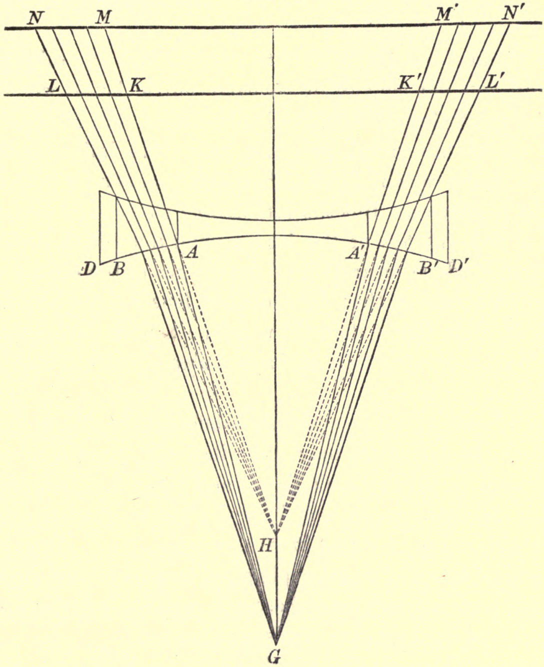
As regards manufacture, it is highly important that this accuracy can always be secured, without calculation, by simple experiment with the diaphragm and screen upon the large lens.

For concave spectacles, which have no actual, but only virtual foci, the proceeding must be somewhat modified. In order to avoid repetitions, I will first refer to the means by which glasses are to be tested.

It is desirable to have a simple method by which the orthoscopic character of any given pair of spectacles can be readily proved. This method is furnished, for convex spectacles, by holding them between any source of light and a screen, and by moving them to and fro until they throw sharply-defined images of light upon the screen. When these images are perfectly defined, they should exactly cover each other. If with perfect definition their coincidence is incomplete, the spectacles are not completely orthoscopic.

This method of testing can only be applied to convex glasses. For concaves, which furnish only virtual images, the proceeding given on p. 187 of *Die Gesetze des räumlichen Sehens*, is to be recommended. In this, the rays from the most distant possible luminous point G (Fig. 8), after passing through the glasses A B, A′ B′, are received on two screens, L L′

Fig. 8.



and N N′. The resulting images are bright spots, having respectively the diameters K L, K′ L′, and M N, M′ N′. These bright spots are the bases of cones M H N, M′ H′ N′, having for their apex the virtual focus H of the concave lenses. Beside these bright spots there are two shadow spots, which form the bases of cones having for their apex the luminous point G; and which, therefore, partly cover the bright spots when the screens are near together, but are separate from them when the screens are farther apart. If the spectacles are orthoscopic, the diameter of the bright spots must increase, on the hinder screen, in the same proportion as the breadth of the interval between them; that is, M′ N′, M N, must be larger than L′ K′, L K, in the same proportion as M M′ is greater than K K′: since the rays B L N, A K M, A′ K′ M′, B′ L′ N′, have one and the same virtual focus H as their point of convergence.

It would be a grave error to suppose that an oblique position of common convex or concave centric lenses would give to the spectacles an orthoscopic character. An oblique position across the visual line has very little influence on the course of the rays of light, either with common or with orthoscopic spectacles. But the orthoscopic glasses are very sensitive to any *rotation* of the lenses upon the visual line, since this will produce separation of the images of the object. It is therefore necessary that the excentric lenses should be so fastened in their frame that their thinnest and thickest points should lie accurately in a horizontal plane, passing through the centres of the eyes.

On account of the inconsiderable effect of an oblique position of spectacle lenses, we may unhesitatingly place a meniscus, or a concavo-convex glass (Figs. 9 and 10) C D, which has been cut for an orthoscopic lens from the larger meniscus B B′, in the position C D, in which it is also periscopic. Moreover, as the middle curve of a periscopic glass may be of any radius, so two companion lenses for orthoscopic-periscopic spectacles may be supplied by a smaller meniscus than B B′.

Orthoscopic spectacles, whether convex or concave, whether of high or low power, have the remarkable peculiarity that they may be used by any pair of eyes, normal, short-sighted, or far-sighted, without aggravation.

It is self-evident that no pair of eyes can use these spectacles at all distances, since the accommodation does not admit of this. The rational use is confined within the limits of clear vision dependent upon the eyes themselves and upon the power of the lenses. At page 183 of *Die Gesetze* *des räumlichen Sehens*, I have given these limits for normal eyes, and for lenses of various power. For example,

Fig. 9.

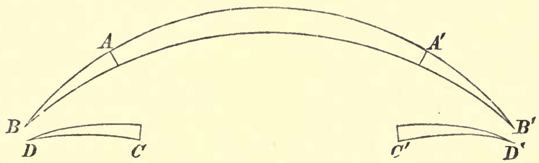
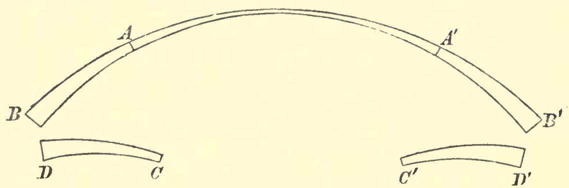


Fig. 10.



normal eyes with orthoscopic convex spectacles of 100 inches focal length, would not see clearly at a greater distance than 100 inches; with spectacles of 7-inch focal length, at a greater distance than seven inches. And just as convex spectacles curtail the farthest distance of the range of vision to the limit of their own focal length, so they also curtail the middle distance, or the range of *clearest* vision. Convex spectacles of 7-inch focal length would, for example, curtail the middle distance to four inches: that is to say, a pair of normal eyes, in order to get the clearest possible image through such spectacles, would require the object to be as near as four inches to them.

The concave orthoscopic spectacles have the contrary effect; and, for normal eyes, they increase the distance of the middle visual region, and leave distant vision wholly unlimited. Concave spectacles limit the region of vision, therefore, towards the near point. Concave spectacles of 100 inches focal length remove the point of clearest vision to a distance of ten inches;

and those of 15 inches focal length, to a distance of $22\frac{1}{2}$ inches. Stronger concaves, of less than ten inches focal length, merge the middle distance in infinity, and are no longer convenient for any distance whatever.

This limitation of the range of vision is, however, not an effect of the prismatic form of the orthoscopic lenses, but solely of their convexity or concavity. With common spectacles a normal pair of eyes can scarcely see clearly at any distance; but such eyes, with orthoscopic spectacles, and within the limits determined by their focal length, can see clearly and without strain.

By convex spectacles the farther boundary of the range of vision of normal eyes is contracted; and this contraction occurs in a higher degree when they are placed before short-sighted eyes. By concave spectacles the nearer boundary of the range of vision of normal eyes is contracted, that is, it is put farther away; and this effect is more marked when they are put before far-sighted eyes. It is obvious that convex spectacles can seldom be useful to short-sighted eyes, concaves seldom to far-sighted eyes; but yet for these, as well as for normal eyes, the statement remains entirely true that within the limits of vision prescribed by the form and power of the lenses, every normal, or short-sighted, or far-sighted pair of eyes may use any pair of convex or concave orthoscopic spectacles with clearness and without strain.

The limitation of the range of vision which the use of orthoscopic spectacles causes in normal eyes, must not be regarded as if these spectacles were only adapted for vision at some given distance. The limitation concerns only the *boundaries* of the range; and, within these boundaries, the spectacles are perfectly comfortable for *every* distance; while common spectacles, before normal eyes, are not comfortable for *any* distance.

That orthoscopic spectacles can be worn comfortably by normal eyes, is a fact important to many classes of workmen. Watchmakers, engravers, and others, who require to use convex glasses often and long at their occupation, now employ a single magnifier. Without considering that the magnifier occupies one hand—because this evil can be obviated by fixing it in the margin of the orbit—it requires the use of one eye only, and the suppression of the images received by the other. Both conditions act injuriously upon the eyes. If two lenses of the strength of the magnifier are set in a frame orthoscopically, the above-named evils will be removed. A normal pair of eyes can work with them as easily and conveniently as with unaided vision at the middle distance.

As with convex spectacles for near objects, so the orthoscopic concave spectacles may be used for distance, as by military men, and others, who wish to see the most remote objects as clearly as possible, although diminished; and thus to include the deepest and widest range of vision in one view, or to change quick movements over a large field into slower movements over a small field. For example, nine-inch concave spectacles bring infinitely distant objects into the middle range, and therefore permit a continuous and exact fixing of the most distant objects, and of the whole circle of the field of vision, with as much comfort as in ordinary reading without glasses.

THE INDEPENDENCE AND THE INTERDEPENDENCE OF ACCOMMODATION AND CONVERGENCE.

After we have in the foregoing learnt the principle of the orthoscopic spectacles, and have convinced ourselves that they in no way change the individual functional proportion between accommodation and convergence, whether this proportion be normal or abnormal, we may proceed to cases in which the proportion is such that the eye does not feel permanently comfortable; and in which, therefore, the ordinary orthoscopic glasses require a modification, or to be replaced by combination glasses suited to the condition.

A rational selection of spectacles requires a rational classification of ocular defects; and I will introduce this by first considering the physiological purposes of accommodation and of convergence, as well as the independence, and the contrasted interdependence, of these two acts of the visual process.

The accommodation is primarily governed by the effort to obtain clearness of the visual impression, or of the retinal images; the eyes accommodate for the object (if not prevented by insurmountable impediments) exactly so much as to concentrate the cone of rays upon the retina. With regard to the primary object of convergence, different views may be held. In the present state of science the most prominent of these is the assumption that convergence depends upon the effort to obtain a single visual impression, or single vision; since the eyes, in the fixation of any object, take the degree of convergence that is necessary in order to project the images upon identical or corresponding portions of the retinæ. I reserve to myself to prove, in the sequel, that this assumption is erroneous; and at present only make the statement, which I will hereafter show to be well founded, that the principal purpose of convergence is to *fix* the object, and that single vision depends chiefly upon a wholly different and independent process, not yet sufficiently estimated in ophthalmology, and to which, in my *Gesetze des räumlichen Sehens*, I have given the name of 'Application.' I postpone also the definition of the idea of *fixation* until I come to the special consideration of convergence; and will at present only state as a fact that the accommodation and the convergence are two primitive independent visual faculties, directed to the attainment of two wholly distinct and definite purposes, through efforts after wholly distinct and definite sensory impressions. By reason of the original independence of these faculties, every pair of eyes possesses the power to accommodate more or less with the same convergence, and to converge more or less with the same accommodation; as also to exert accommodation and convergence voluntarily, without the stimulus of light or of an external object of vision, and to increase the degree of this voluntary exertion by practice.

Notwithstanding this original independence, the two faculties pass into a secondary dependence upon one another; caused either by congenital organization, or in consequence of their usually coincident exercise, or as the result of an induction framed by the sensorium, as a higher central apparatus dominating over all the visual processes. From the secondary dependence (which in my opinion is the foundation of the law of identity, when this is extended as set forth in Section XXI. of my *Gesetze des räumlichen Sehens*) it follows that, in voluntary convergence, even when the eyes do not fix any object, the accommodation, if unstrained, involuntarily keeps pace with the convergence; and, inversely, that in voluntary accommodation without the stimulus of light, the eyes involuntarily assume a corresponding degree of convergence.

So long as this secondary proportionate dependence between accommodation and convergence, in regarding an object, is such as corresponds to the normal relative proportion of the functions in the pair of eyes concerned, the visual process is carried on without strain and with entire completeness. So soon, however, as the eyes are called upon for an abnormal relative proportion, the visual process is attended by strain. If this strain be small, it may be overcome without detriment to the completeness of the visual impression; and, in many respects, the shifting of the bacillary layer, so often demonstrated in my *Physiologische Optik*, and in the *Supplement*, plays the part of a regulating power for the correction of the disproportion. If, however, the strain exceeds a certain degree, then one or other of the two functions, or both, will fail in their duty; in such a way that the eyes may accommodate correctly, but converge falsely, so as not to see with their poles, an effect that generally produces double vision; or they may converge rightly, so as to see with their poles and singly, but accommodate falsely, so as to see indistinctly.

In most cases, under such circumstances, the convergence prevails, in consequence of the greater desire for single vision; so that most pairs of eyes see indistinctly, but singly, rather than double, and clearly. The contrary may, however, occur. In either case there is the indication to regulate the functional proportion in order to relieve the visual strain. This may be done by proper combination spectacles, which I shall describe in the following special researches into the different forms of ocular defect.

It has been already observed that the combination spectacles, formed of united lenses and prisms, are modified orthoscopic spectacles; and that they have the effect of dividing the actual object into two apparently distinct images, which, however, do not produce diplopia, since they fall upon identical portions of the retinæ. No spectacles—neither the common spherical (which are combination spectacles with the refraction of the prisms$=0$), nor the common prismatic (which are combination spectacles with lenses of infinite focal length), nor any possible combination spectacles—produce double vision directly. If double vision occur during the use of spectacles it is but as a consequence of increased disproportion between the efforts of the independent ocular functions, and especially as a consequence of transgression of the limit of the faculty of convergence.

With regard to the limits of the faculty of accommodation or of convergence, the researches of Donders and of von Graefe have established that every pair of eyes possesses, for any given degree of convergence, a certain play of accommodation (the relative range of accommodation); and also for a given degree of accommodation, a certain play of convergence (the relative power of squinting). Within the limits of such play the pair of eyes can endure the already mentioned disproportion, even if not without strain, or for a continuance.

Moreover, it is certain that every pair of eyes has absolute limits for the accommodation (corresponding to the near- and the far-point) and absolute limits to the convergence (corresponding to the maximum and minimum convergence angle).

In the former results of the observations of Donders and von Graefe, I find a direct confirmation of the opinion stated in my *Physiologische Optik*, and there based upon different grounds, that the accommodation and the convergence may on the one hand be regarded as two independent acts of the visual process; and on the other hand as acts materially dependent on, or materially influenced through, a law of induction dominating them both. If we grasp this as a fundamental principle of vision, the above described results follow as necessary consequences. On account of the independence of the two faculties, either may vary while the other remains constant; and, on account of their interdependence, every change in one produces an effort to change the other also, and thus causes visual strain. When this effort, or this strain, reaches a certain degree, the independence of the two functions can no longer be maintained; that is, every independent variation of the one function, with immutability of the other, is restrained within definite limits.

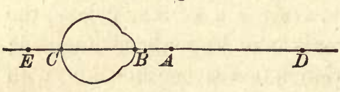
While the observations of Donders and von Graefe furnish confirmation of this principle of vision, the principle must itself be regarded as the physiological ground of the phenomena, and as conveying the explanation of them.

It is obvious that the condition of continuous vision without strain is that the limits of the above relative ranges should be avoided in both directions, and that nearly the middle point between them should be preserved. This *à priori* conclusion is entirely confirmed by the observations of ophthalmologists, and forms an important element in the selection of proper spectacles.

DEFECTS OF ACCOMMODATION,* OR ANOMALIES OF REFRACTION AND OF ACCOMMODATION.

In order to specialize the ocular defects that are to be discussed, I commence with those of accommodation; that is, with the abnormal conditions which impair the *distinctness* of the visual impression. We consider first the changes of accommodation which an eye must undergo in order to accommodate in succession for all points along a straight line of infinite length. For this purpose, let c be the focal length, for parallel rays, of the refracting system of an eye, measured from B (Fig. 11) the anterior pole or apex of the cornea. By a normal eye we mean one in which the length of the axis B C, for parallel rays, or in fixation of an infinitely distant object, is exactly $= c$: so that the cone of light is united exactly upon the retina C. We call this position of the retina the state of

Fig. 11.



* It will be observed that the author frequently uses the word accommodation in its ordinary or restricted sense, to express the action by which the eye adapts itself for different distances; and frequently in a wider or more general sense, to express the actual refractive condition at any given time. For although accommodation is an action, not a condition, yet we speak of the condition of accommodation to express the result of that action. The refraction itself is also a condition of accommodation, that namely in which the action of accommodation is *nil*. Hence the word accommodation may be used as a general term to include the refraction; and we may, for the sake of brevity, describe the anomalies of refraction and accommodation as 'defects of accommodation.' The context will always clearly show in which sense the word is used in any given passage, and I have thought it better to retain the author's language than to alter it.—TRANS.

rest, in which the accommodation $= 0$. Let x be the distance of any object D from the eye; or, more accurately, the distance A D of the object D from the point A, which lies in front of the eye at a distance A B $=$ B C $= c$ the focal length of the eye itself. The distance B E of E, the crossing point of the rays of light, from the cornea B, will be very nearly $= c + \frac{k}{x}$, in which the nearly constant coefficient k is determined by the refractive indexes and the curvatures of the ocular media. The distance C E of this crossing point E from C, the point of rest of the retina, is $= \frac{k}{x}$. Accurately to accommodate for D the eye must undergo changes, which if the curvatures and thickness of its media remained unaltered would be expressed by an elongation of its axis $= \frac{k}{x}$. In actual fact the accommodation depends upon a simultaneous change in the curvatures, the thickness, and the axial diameters of the refracting media; in which change the alteration of the curvatures of the crystalline lens plays the chief part. For the better display of these changes we may reduce them all to an elongation of the axis; that is, when we speak of an elongation of the ocular axis, we mean thereby the sum of all the elementary changes of curvature, thickness, and dimensions, which, in respect of the position of the crossing point of the rays of light, have the same effect that would follow a simple elongation or simple shortening of the axis of the eye, if all its curvatures and thicknesses remained unchanged. Hence it is possible to take the value $\frac{k}{x}$ as the measure of the accommodation effort required.

Let y be the effort of accommodation, which is necessary to accommodate accurately for the distance x; so that $y = \frac{k}{x}$. This formula will apply not only to all positive values of x, from $x = 0$ to $x = +\infty$, that is, for the distances of all objects in front of the point A, but also for all negative values of x. A negative value of x indicates an object lying behind the eye, and thus seems to involve an absurdity. This, however, is not the case. The position in front or behind, or the geometric place of the object, or generally the existence of an external object, is of no importance to the eye as a visual organ. The eye sees only by virtue of physiological processes excited within it; and these depend entirely upon the nature of the ether undulations or rays of light, which fall upon the retina and call forth the sensory nervous process; or, therefore, upon the conditions which the rays of light assume in the vitreous body. An anteriorly situated object D, corresponding to a positive distance $x = AD$, is to the eye only a bundle of rays, which converge towards the retina (Fig. 12). An object lying in the point A, which corresponds to the distance 0 or the value $y = \frac{k}{0} = \pm \infty$, is to the eye only a bundle of rays which pass through the vitreous in a parallel course towards the retina (Fig. 13). An object D′ lying behind the point A, and therefore behind the eye (Fig. 14), and corresponding to a negative distance $AD' = -x'$, is to the eye a bundle of rays which in the vitreous body diverge towards the retina; that is, a bundle of rays which, where they are outside the eye and uninfluenced by its refracting media, converge towards a point D′ lying behind A.

Fig. 12.

Fig. 13.

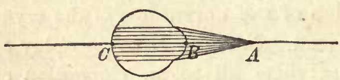
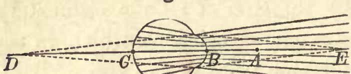


Fig. 14.



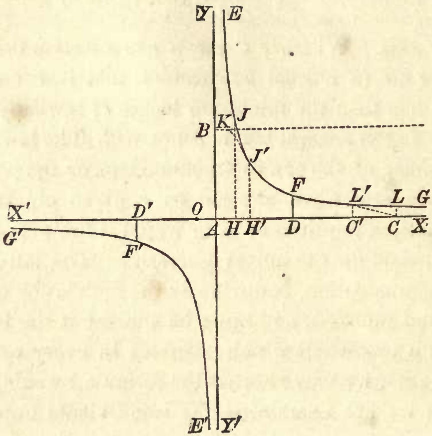
Hence both the positive and the negative values of x have a decided bearing upon the function of the eye. A positive value of x, that is, a luminous point lying in front of A in space, requires an accommodative function $\frac{k}{x}$ which tends to an elongation of the ocular axis. A negative valve of $x = -x'$, that is, a luminous object which sends forth converging rays, tending to meet in the distance x' behind A, requires a negative accommodative function $\frac{k}{x} = -\frac{k}{x'}$, which tends to a shortening of the ocular axis. Whether the eye can exercise this negative function or not (a normal eye cannot, that is, cannot shorten its axis to less than the minimum length c) is wholly unimportant as far as the general law is concerned. The law expresses, not the faculty of the eye to accommodate, or the power which it actually exerts when affected by a given object, but that which would be required of it, or which it must exert, in order to accommodate for the object correctly. The latter, the required accommodation, is obviously the foundation of the ideal law of visual function, and must be studied at the beginning.

As k is a coefficient which possesses in every eye a nearly constant value, we may shorten the formula by substituting 1, so long as we are examining the same visual organ, or are comparing the accommodative power required under different circumstances; so that simply $y = \frac{1}{x}$ may be used as the measure of the accommodation required for an object lying at the distance x. If we take x as the abscissa, and y as the ordinate, of a curve, the law of required accommodation can with the greatest clearness be graphically expressed.

We call the curve $y = \frac{1}{x}$ the curve of requirement for the accommodation. This presents itself as an equilateral hyperbola (Fig. 15). The previously indicated point A, which lies before the eye O at the focal length A O $= c$, is the point of origin of the co-ordinates; the axis of abscissæ X′ X is one, and the axis of ordinates Y′ Y is the other asymptote of the hyperbola. The branch E F G corresponds to the positive distances

A D $= x$ and to the positive accommodation requirements D F $= y = \frac{1}{x}$; and the branch E′ F′ G′ corresponds to the negative distances A D′ $= x = -x'$, and to the negative accommodation

Fig. 15.



requirements D′ F′ $= y = -y' = -\frac{1}{x'}$. Since for a normal eye the focal length A O $= c$, the length of the axis of the eye itself, or about = one inch, it follows that A, the point of origin of the co-ordinates, must lie very near to and in front of the eye O.

If the eye were organized with absolute completeness, so that it could perfectly realize the ideal plan of vision, and accommodate exactly for every possible distance, it would possess an accommodative faculty corresponding with the hyperbolic curve of requirement. But no earthly organism, no material system, and certainly not the eye, possesses such an ideal completeness in the realization of a natural law. The actual accommodation is only accurate for a certain distance, the middle range of vision, or for a short departure from that distance in either direction; and the actual accommodative faculty coincides with the accommodative requirement only over a certain short portion in the vicinity of the point D, which is determined by the middle visual range A D, and may be called the point of middle vision. For this point D only, the ordinate D F is the measure of the actual accommodation of the eye. On this and on that side of the middle visual range, the actual accommodation does not attain to what is required. The curve which illustrates the actual accommodation—and hence the accommodative effort or faculty, and which we may term the *facultative curve of accommodation*—has the form of the dotted line B F C. The character of this facultative curve consists in this, that it at first, over a certain distance B K, runs parallel with the axis of abscissæ, then touches the curve of requirement at F, and at a point C falls tangentially upon the axis of abscissæ, and coincides with that axis thenceforward.

For the visual distance A H, or the point H, which is the near-point, the actual accommodation attains, therefore, its maximum H K, which deviates only a little from the requirement, H J; but for still nearer objects remains almost the same, and hence deviates from the requirement more and more. Between the points H and C the deviation of the actual from the required accommodation is inconsiderable. At the point C the actual accommodation attains its minimum, which is = 0, and which remains constantly = 0 for all greater distances. The deviation of the accommodation from the requirement is inconsiderable for these greater distances, although the former remains unchanged; since the requirement itself is very inconsiderable over this portion of the visual range, on account of the approach of the hyperbola to its asymptote. The point C, at the distance A C, is the far-point.

The prevailing doctrine is, that for a normal (emmetropic) eye, the far-point is at infinite distance. This doctrine is not accurate. It arose from the fact that for all great distances only a small degree of accommodation is required, and hence only a small degree is exerted. The principal criteria for the near- and for the far-point are these, that the actual accommodation is at its maximum for the former, and at its minimum ($=0$) at the latter. The accommodation is extinct for still more distant objects, or for infinite distance; for which the eye undergoes no further accommodative change, but remains settled at its minimum axial length; just as, for points within its near-point, it undergoes no further change, and remains settled at its maximum axial length.

It is hence not only my definition of the far-point, but my definition of the near-point also, that differs from those commonly accepted. As a rule, people understand by the near-point a point H′, lying somewhat farther from the eye than H, and for which the accommodation is yet completely clear and sharp; while I mean by the near-point H, the point at which the accommodation attains its constant maximum. If, according to common use, we accept for the near-point H′, for which the actual accommodation still coincides accurately with the required, we ought to take as the far-point C′, for which also the actual and the required accommodation coincide. While H′ lies farther off than my near-point H, C′ lies nearer than my far-point C. The prevailing definition of near- and far-point not only expresses, therefore, a different test from mine, but it is for both points untrue; a fault which, as well as those above explained, causes error with regard to the limits of the weakest accommodation.

I leave to practical ophthalmologists the accurate determination of the near- and far-points for the maximum and minimum of accommodation, or for the limits of the accommodative faculty. My near-point for a normal eye would be at a distance A H = to about 3″; and my far-point at a distance of some hundred feet, or possibly A C = 2000″; while the middle visual distance A D = about 8″.

To avoid misunderstanding I will observe that the incapacity of the eye to accommodate for shorter distances than three inches, and for longer distances than two thousand inches, must not be supposed to imply incapacity to *recognize* shorter and longer distances. The eye distinguishes such shorter and longer distances very well, although it cannot accommodate for them. As I have stated in my *Physiologische Optik* and in the *Supplement*, and have shown by numerous facts, the recognition of distance has nothing to do with the accommodation; and the accommodative faculty is not, as it is erroneously assumed to be in the physiology of the day, a measure for this recognition. The accommodation has no other object than the clearness of the visual impressions; and the recognition of distance rests upon the proportion of axial and of lateral components in the bundle of rays, the undulations of which strike upon and penetrate the bacilli of the retina: a proportion which is directly felt, and the sensory impression from which produces the conception of distance. In consequence, the eye falsely accommodated may yet perceive correctly the distances of objects; and, since the nerve substance of the retinal bacilli is more sensitive than the motor apparatus of accommodation, the former reacts for the determination of distance beyond the limits of the accommodation as fixed by the near-point and the far-point: that is, the eye recognises the distance of nearer or of more distant objects, although it can undergo no further change of accommodation with regard to them.

If any phenomenon whatever is calculated directly to confirm my theory of the recognition of distance, and of the independence between the judgment about distance and the accommodation, it is the fact just stated, that the eye can actually distinguish distances which lie on this side of its near-point, and beyond its far-point, without undergoing any change of accommodation with regard to them.

The range between the two limits of the strongest and the weakest accommodation is measured by the length of the ordinate A B. This ordinate forms a measure of the whole accommodative faculty of the eye; which is called, by Donders, the range or breadth of the accommodation. For the assumption of the near-point at H′, the actual accommodation, H′ J′, coincides accurately with the required accommodation, and gives the range H′ J′. For my near-point H, the actual range of accommodation A B = H K, and is somewhat less than the required accommodation H J.

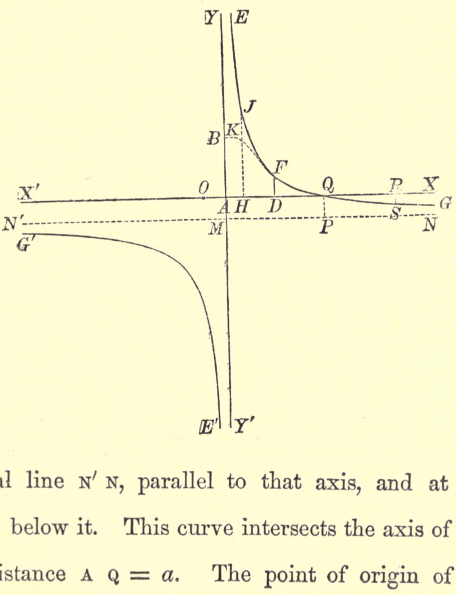
The two points H and C, which indicate the extreme limits of the accommodative faculty, are important for many purposes; and, for others, the points H′ and C′ which indicate the extreme limits of the coincidence of the actual with the required accommodation, are of still greater importance. Where it is necessary to distinguish these points from each other, the former will be called the near- and far-point of the accommodative faculty; the latter, the near- and far-point of accurate accommodation.

Having shown that the normal eye is characterized by the above described hyperbolic curve of required accommodation, and by the dotted facultative curve, we have next to deal with the characteristics of abnormal or defective eyes. At the first glance one would be inclined to assume that the essential character of an eye with abnormal accommodation would be shown by the special form of its facultative curve. This, however, is not the case. The facultative curve of an abnormal eye has always an abnormal form; but the most important alteration is shown by the curve of requirement.

For example, a myopic or short-sighted eye has an axis too long for its refraction. This shows a dioptric system which, quite independently of its accommodative faculty, in the state of rest of the retina, does not concentrate upon the retina parallel rays, or those coming from infinite distance, but only rays coming from a certain finite distance a. The required accommodation, therefore, has not the value of 0 when $x = \infty$, but when $x = a$; that is, the curve of requirement has not the axis of abscissæ for its asymptote, but cuts this axis at the distance a. If we indicate the minimum axial length of the short-sighted eye, which corresponds to the visual distance a, by c; then the distance from the cornea of the point of convergence of a cone of rays, issuing from a luminous point at the distance x, after refraction by the eye, will be $= c + \frac{k}{x} - \frac{k}{a}$; and hence the required accommodation $y = \frac{k}{x} - \frac{k}{a} = k \left(\frac{1}{x} - \frac{1}{a}\right)$; or, when we assume, as before, $k = 1$, $y = \frac{1}{x} - \frac{1}{a}$.

If we make $y + \frac{1}{a} = y'$; so we perceive from the equation $y' = \frac{1}{x}$ that the curve of requirement of a short-sighted eye is accurately congruent with that of an emmetropic eye, but is vertically depressed by the depth $\frac{1}{a}$, so that it is an equilateral hyperbola E G, E′ G′ (Fig. 16), the horizontal asymptote of which is not the axis of abscissæ X′ X, but a

Fig. 16.



horizontal line N′ N, parallel to that axis, and at a depth A M $= \frac{1}{a}$ below it. This curve intersects the axis of abscissæ in the distance A Q $= a$. The point of origin of the co-ordinates, A, lies in front of the eye O in the distance $OA = c - \frac{1}{a}$; and therefore not so far distant from the short-sighted eye as the whole length of its axis, supposing that c is $> \frac{1}{a}$.

The facultative curve, which depicts the actual accommodation for different visual distances, has a form B K Q, like the preceding one: that is, it shows at its commencement B K, a maximum of accommodation A B, which is attained for the near-point H, and remains unchanged for all nearer distances. It shows the most complete accommodation D F for the middle distance A D, and this sinks to its minimum, 0, for the far-point Q. The near-point H lies nearer to the short-sighted than to the emmetropic eye; and so does the far-point Q, the latter being so near that it falls upon a proportionately strongly curved portion of the hyperbola. For an object R, lying beyond the far-point, the required accommodation $RS = y = \frac{1}{x} - \frac{1}{a}$ would be negative, since $x > a$, and therefore $\frac{1}{x}$ is $< \frac{1}{a}$. An infinitely distant object would require the negative accommodation $y = \frac{1}{\infty} - \frac{1}{a} = - \frac{1}{a} = AM$; that is, the cone of refracted rays would converge to a point at a distance $= \frac{1}{a}$ in front of the retina.

For the near- or far-point of the accommodative faculty, and the near- or far-point of the exact accommodation, the same difference holds good in the short-sighted, as in the emmetropic eye. The two far-points, however, fall more nearly upon the same point Q, the more short-sighted the eye, or the more abruptly the hyperbolic curve inclines to this point.

It is manifest that the short-sighted eye is distinguished from the emmetropic by the position, rather than the form, of the curve of requirement. They are also distinguished by the form of the facultative curve : since, as a rule, both the near-point and the far-point are nearer to the eye. It may, however, very well happen that the near-point and the far-point of a short-sighted eye are at distances corresponding to those of an emmetropic eye. If such be the case, the different position of the curve of requirement would always serve to establish the essential difference between them ; which consists in this, that the short-sighted eye, on account of its great axial length, requires a negative accommodation for distances beyond its far-point, a requirement that it is unable to fulfil.

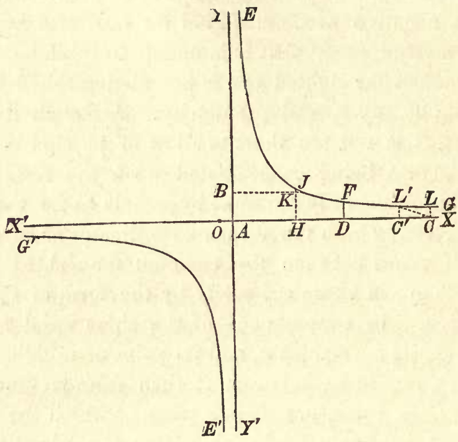
The common far-sighted eye is not distinguished from the emmetropic by any special peculiarity. Although its axis is very short, it is not too short to allow of parallel rays from infinite distance being concentrated upon the retina. The curve of requirement is the same hyperbola as for the emmetropic eye, having both the axes of co-ordinates as asymptotes.

The difference between the common far-sighted and the emmetropic eye is expressed solely by the form of the facultative curve. In consequence of the short axial length of such an eye, its near-point H, and its point of middle distance D (Fig. 17), recede farther from it than from an emmetropic eye. The latter point, D, falls in a place at which the curve of requirement has but a slight inclination towards the axis of abscissæ. The far-point, C, lies self-evidently far away, and assumes proportions as in emmetropic eyes. Generally the near- and far-point are much nearer together than in a normal eye.

From the foregoing observations upon the peculiarities of a normal, a short-sighted, and a far-sighted eye, it appears that the signs indicated do not convey any rational characteristic of either ; that is, one based upon general laws. Essential and non-essential, or specific and gradual differences, are mingled together, and among them no principle appears. A character based on principle can only be based upon the position of the curve of requirement, and upon the form of the facultative curve, and thus we have two separate points of view from which the eyes may be considered.

We will turn first to the curve of requirement. This, for every eye, is one and the same equilateral hyperbola. The eyes are therefore to be distinguished, as regards the required accommodation, only by the position of the hyperbola. This

Fig. 17.

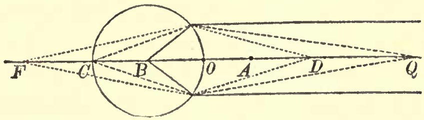


position is determined by the position of the middle point M of the hyperbola, with regard to the eye O, that is, to the cornea of the eye (Fig. 16). It therefore has reference to two definite distances: the vertical distance, A M, of the horizontal asymptote, and the horizontal distance, O A, of the vertical asymptote from the eye. Instead of the vertical distance A M of the horizontal asymptote, we may take the abscissa M P = A Q as the determining distance. The latter is the clearer method; and we say that the position of the curve of requirement, with regard to the eye O, is determined by the distance A Q of its point of intersection Q with the visual line O X, and by the distance O A of the vertical asymptote from the eye O.

These two elements are determined in the following way, through the refracting system of the eye O, or of any eye (Fig. 18): We take—

b = O B the focal length, or the distance of the focal point B from the cornea O, for parallel rays, therefore, for rays proceeding from infinite distance.

Fig. 18.



A a point in front of the cornea O, in the focal length O A = O B.

a = A Q the distance from the point A to any luminous point Q, from which the rays are accurately focussed upon the retina, C, when in its state of rest: a distance that must correspond with the visual distance, or, yet more precisely, with the accommodation distance.

c = O C the length of the axis of the eye in a state of rest, therefore the minimum length of axis.

x = A D the distance of any desired luminous point D from A; or any desired visual distance.

y = C F the distance from C, the retina in a state of rest, of the point F, at which rays coming from D are focussed; therefore the required accommodation for the visual distance x.

ξ = O F the distance of the point F from the cornea O.

k a nearly constant coefficient, determined by the refraction of the eye.

According to general optical laws, we have:

$$c = b + \frac{k}{a} \qquad b = c - \frac{k}{a}$$

$$\xi = b + \frac{k}{x} \qquad \xi - b = \frac{k}{x}$$

$$y = \xi - c = \frac{k}{x} - \frac{k}{a} = k\left(\frac{1}{x} - \frac{1}{a}\right).$$

If we assume the value k for the unit of length by which all the magnitudes, a, b, c, x, y, ξ, are measured, we may then, in all the formulæ, suppress the quantity k.

The curve of required accommodation is always one and the same equilateral hyperbola, $y = k\left(\frac{1}{x} - \frac{1}{a}\right)$ whatever may be the values of the magnitudes a, b, c. According to these last values, the eyes are classified in the manner following:

1. When Q, the point at which the curve of requirement cuts the visual line, lies at infinite distance; and when, therefore, $a = \infty$, $b = c$, that is, the focal point of the eye falls on the retina in its state of rest, and the vertical asymptote lies at the distance c, the length of the ocular axis, in front of the cornea; so that an object at the distance c, that is, very near the eye, would require the infinitely strong accommodation $y = \infty$. The equation of the curve of requirement would here be simply $y = \frac{k}{x}$ (Fig. 15).

This eye is the so-called *emmetropic*; and the condition $a = \infty$, that is, an infinite visual distance, is the distinguishing sign of emmetropia.

2. When Q, the point of intersection of the curve of requirement, is situated in positive finite distance, that is, at the finite distance a in front of the eye; when, therefore, the rays from an object at the distance a are united upon the retina in a state of rest, then the focal length of the eye is

$$b = c - \frac{k}{a}.$$

The focal point falls, consequently, within the eye; and an infinitely distant object requires a negative accommodation $y = -\frac{k}{a}$. The vertical asymptote falls in the distance $b = c - \frac{k}{a}$; and is therefore very near the eye. The equation of the curve of requirement is $y = k\left(\frac{1}{x} - \frac{1}{a}\right)$. This gives an equilateral hyperbola, the horizontal asymptote of which lies at the depth $\frac{k}{a}$ below the eye; that is, an hyperbola depressed to the extent $\frac{k}{a}$ (Fig. 16.)

I call this eye the *hypometropic*; and the condition of a positive finite value for a, or a positive visual distance, is the criterion of hypometropia. (Hypometropia is not, as assumed by Donders, in his work on the anomalies of refraction and accommodation, the equivalent of short sight; and we shall see in the sequel that brachymetropia, the expression used by Donders, is not so expressive as hypometropia.)

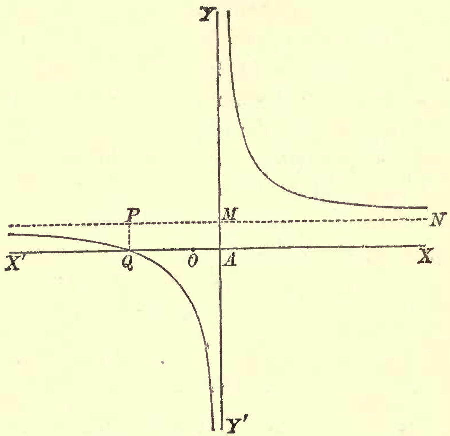
3. When Q, the point of intersection of the curve of requirement, is situated in negative finite distance, and therefore behind the eye at the distance $a = -a'$; and when, therefore, rays which converge towards this point behind the eye, are so refracted in the eye, as to be united on the retina in its state of rest, then the focal length of the eye is $b = c + \frac{k}{a}$. The focal point falls, therefore, behind the eye; and an infinitely distant object, lying in front of the eye, requires the positive accommodation $y = k\left(\frac{1}{\infty} + \frac{1}{a'}\right) = \frac{k}{a'}$. The vertical asymptote falls at the distance $b = c + \frac{k}{a'}$; and therefore in front of the eye, at a distance which is greater than the ocular diameter c. The equation of the curve of requirement is

$$y = k\left(\frac{1}{x} - \frac{1}{a}\right) = k\left(\frac{1}{x} + \frac{1}{a'}\right).$$

This describes an equilateral hyperbola, the horizontal asymptote of which lies above the eye at the height $\frac{k}{a'}$; that is, an hyperbola raised to the extent $\frac{k}{a'}$ (Fig. 19).

This eye is the so-called hypermetropic, and the condition of a negative finite value for $a = -a'$, that is, a negative visual distance, is the characteristic sign of hypermetropia.

Fig. 19.



The hypermetropic eye has the peculiarity that it may positively accommodate for rays of light which converge towards points behind the eye; since $y = k\left(\frac{1}{x} + \frac{1}{a'}\right)$ would be positive for all negative values of $x = -x'$ for which $x' > a'$.

Since both hypometropia and hypermetropia are deviations from emmetropia; so they both fulfil the condition which Donders has designated ametropia.

The curve of required accommodation for the emmetropic, the hypometropic, and the hypermetropic eye, that is, for any given eye, is therefore shown by one and the same equation, $y = k\left(\frac{1}{x} - \frac{1}{a}\right)$ in which the quantity a is for the first infinite, for the second positive, and for the third negative.

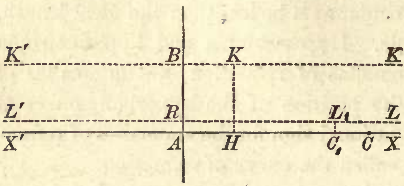
It follows, from the foregoing, that emmetropia, hypometropia, and hypermetropia, are determined exclusively by the refractive power of the eye; or, if we so please, by the length of the ocular axis; or, generally, by the structure of the eye, and not by its innervation or accommodative function. If the length of the ocular axis, in the state of rest, is equal to the focal length for parallel rays, the eye is emmetropic. If the ocular axis be greater than the focal length, the eye is hypometropic. If the ocular axis be less than the focal length, the eye is hypermetropic. Hypometropia and hypermetropia are therefore true anomalies of refraction, not anomalies of accommodation; and the position of the hyperbolic curve of requirement forms the rational sign for the anomalies of refraction, so that it may be called the curve of refraction.

The power to vary the accommodation, which we call the accommodative faculty, and which depends upon innervation, furnishes totally different criteria, which depend not upon the curve of requirement, but upon the facultative curve. The essentials which come into consideration with regard to this power are, first, the maximum of accommodation, which corresponds to the greatest attainable axial length of the eye, and determines the near-point, since the eye, for all greater requirements of accommodation, retains this maximum of axial length, or remains insensitive to the greater requirement; secondly, the minimum of accommodation, which corresponds to *c*, the smallest attainable axial length, and determines the far-point, since the eye remains insensitive to, or retains its shortest axial length notwithstanding, all demands for a weaker accommodation.

The interval between this maximum and minimum corresponds to the absolute range of accommodation. The peculiarity of an eye in question is therefore determined by two things, which we may express either by the maximum and the minimum axial length, or the near-point and the far-point, or the near-point and the range of accommodation, or the far-point and the range of accommodation.

The general form of the facultative curve, which hereafter may also be called the accommodative curve, is, between the near-point and the far-point, nearly the same as that of the hyperbolic curve of requirement. On this side of the near-point it changes abruptly into a horizontal; and at the far-point it falls into the horizontal visual line, having gradually left the curve of requirement at some distance before. If we take the sudden bending at the near-point to be an acute angle, and assume, therefore, that the facultative curve is at first, and until it enters the curve of requirement, horizontal, that it follows the curve of requirement from hence to near the far-point, and there blends with the horizontal visual line, so we may exhibit the accommodative faculty of the eye, as in Fig. 20, by two horizontal lines K′K and L′L, which both lie above the visual line X′X. The first, K′K, which corresponds to the maximum of the accommodative faculty, gives the near-point, H, by its intersection, K, with the hyperbolic curve of requirement. The second, L′L, which usually lies close above the visual line, gives, by its intersection, L_1, with the curve of requirement, the far-point, C_1, of exact accommodation; which deviates but little from C, the far-point of the accommodative faculty. (See definition before given.)

Fig. 20.



On this basis we may classify the anomalies of accommodation, which are expressed by the form of the facultative curve, in the manner following:

1. When the total accommodative faculty, and therefore the distance R B between the maximum and the minimum of accommodation, corresponds to the normal range of accommodation, we call the eye *broad-sighted*, or *europic* (Fig. 21).

2. When the total accommodative faculty or the range of accommodation R B is small, the eye is *narrow-sighted* or *stenopic*. Under this head there are two forms to be distin-

guished. First, that in which the maximum of accommodation A B is small as compared with the normal standard, while the minimum A R corresponds to the normal minimum, and is small (Fig. 22). This stenopic eye, of which the narrow range of accommodation lies deep, is *deep-sighted* or *bathopic.*

Fig. 21.

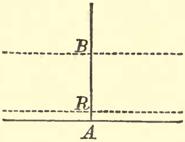


Fig. 22.

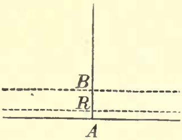
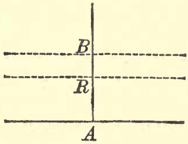


Fig. 23.



Bathopia exists in an eye in which the power of reaction soon ceases, when the focus of the cone of rays is distant from the position of rest of the retina. The chief character of this eye is *weakness*, with sufficient irritability or sensitiveness.

3. When the maximum A B of the stenopic eye is as large as the normal maximum, and the minimum A R is large as compared with the normal, so that the range of accommodation R B is narrow, but placed high, then the eye is *high-sighted* or *hypsopic* (Fig. 23).

Hypsopia exists in an eye which is not sufficiently sensitive to react on a cone of rays the focal point of which is only a little distant from the retina in a state of rest; which therefore requires a considerable stimulus in order to set its apparatus of accommodation in action. This eye is strong, or powerful, but insensitive.

If the eye be emmetropic as regards its refraction, we may characterize europia, bathopia, and hypsopia by saying that for the europic eye the near-point will be near and the far-point distant, for the bathopic eye the near-point will be remote, and for the hypsopic eye the far-point will be near.

In general it must be stated that neither the anomalies of refraction alone, nor the anomalies of accommodation alone, furnish a sufficient criterion for short-sightedness or far-sightedness; and we cannot say either of a hypotropic or hypertropic, of a bathopic or a hypsopic eye, that it is short or far-sighted without further data. Short-sightedness and far-sightedness are not simple or primitive defects; but are the results of combinations of different states of accommodation and refraction; and we may convince ourselves that both these defects may be occasioned in several ways.

Since both for refraction and accommodation there are two chief anomalies, besides the normal state; that is, three chief conditions of each, so the combinations between each state of refraction and each state of accommodation produce on the whole the following nine cases:—

I. 1. Emmetropia, combined with europia, gives a normal eye, with near near-point and remote far-point (Fig 24). The character of this eye we may call *full-sightedness.*

Fig. 24.

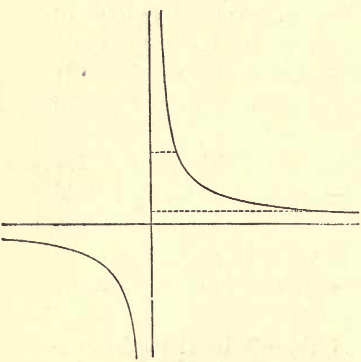
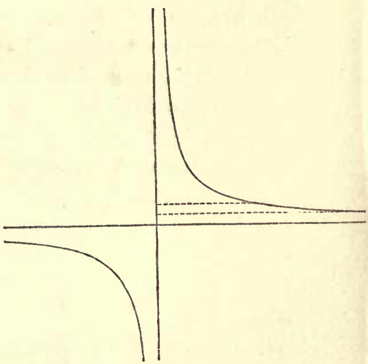


Fig. 25.



I. 2. Emmetropia, combined with bathopia, gives common far sight or presbyopia (Fig. 25), in which the eye can accommodate for infinite distance.

I. 3. Emmetropia, combined with hypsopia, gives a short-sightedness (Fig. 26) that differs essentially from the common form, since the power of accommodation is gradually lost beyond the far-point, but returns again for greater distances; so that the eye can see very distant objects with sufficient distinctness, and therefore can see at once at very short and at very long, but not at middle distances.*

Fig. 26.

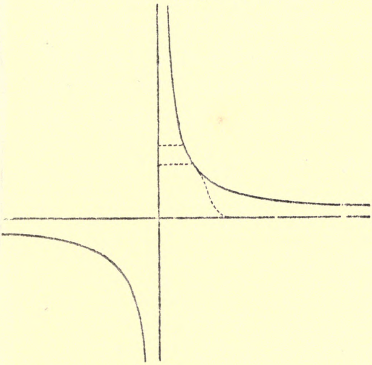


Fig. 27.

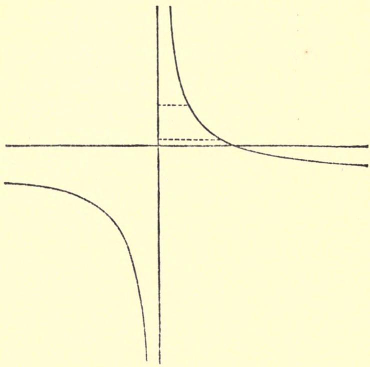


Fig. 28.

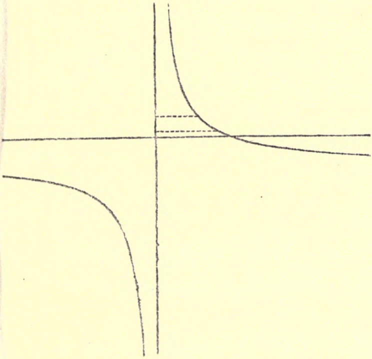
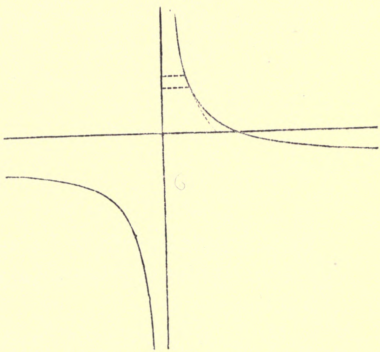


Fig. 29.

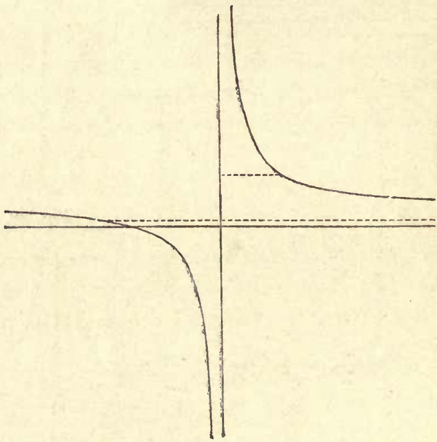


II. 1. Hypometropia, combined with europia, gives common short sight (Fig. 27).

* The peculiarities of the German language have permitted the author to coin German names for all these nine combinations, and also for the analogous defects of convergence presently to be described. These names, of which *Klammsichtigkeit* and *Strengwinkligkeit* may be taken as examples, are not translatable into English to any useful purpose, and they are therefore omitted.—Trans.

II. 2. Hypometropia, combined with bathopia, produces, according to circumstances, either short sight or presbyopia (Fig. 28). If the hypometropia be of high degree, there will be myopia with a narrow range of accommodation. If the hypometropia be of small degree, there will be presbyopia. Such presbyopia is distinguished from the common form by this, that the eye cannot accommodate for infinite, but only for a finite distance; and that therefore it can only see objects that

Fig. 30.

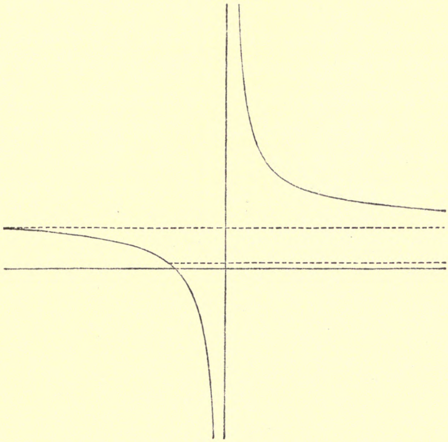


lie between two definite finite limits. This peculiarity is well known as the characteristic of a particular class of over-sightedness.

II. 3. Hypometropia, combined with hypsopia, gives a short-sightedness (Fig. 29) which differs from the common form in this, that the power of vision is gradually lost beyond the far-point, and returns again at a certain definite farther distance, without being extended from this last to greater distances; so that the eye can see at a very small distance, and also at some greater finite distance.

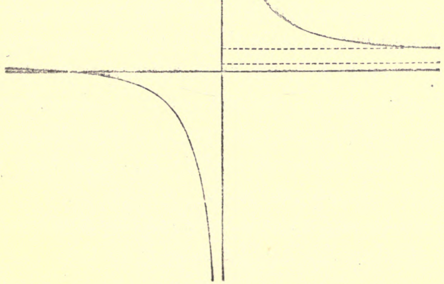
III. 1. Hypermetropia, combined with europia, gives accord-

Fig. 31.



ing to circumstances far-sightedness or common over-sightedness. If the hypermetropia be slight in degree, there will be far-sightedness, distinguished from the common form by this, that the eye uses yet some effort of accommodation in looking

Fig. 32.



at infinite distance. If the hypermetropia be of high degree, there will be over-sightedness or hyperopia (Fig. 31), in which the eye sees clearly no object in front of it, but is able to react on rays which converge towards points lying behind it.

III. 2. Hypermetropia, combined with bathopia, gives, when of very slight degree, that form of far-sightedness which is described in the foregoing paragraph. When the hypermetropia is more considerable, it produces over-sightedness, as in

Fig. 33.

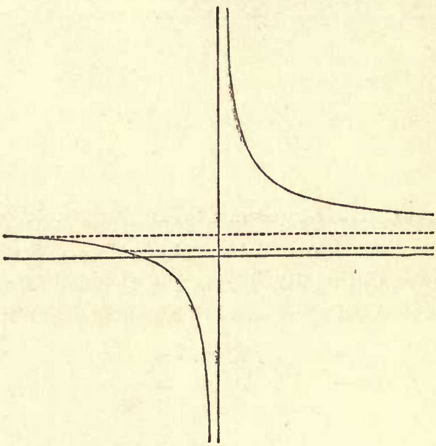
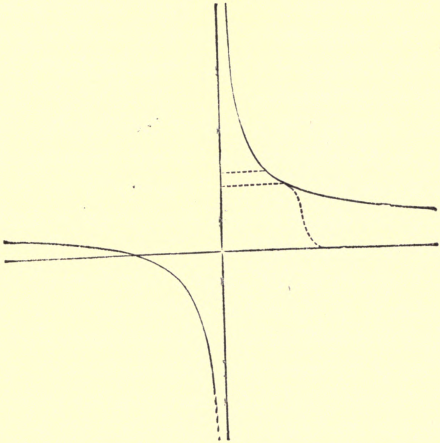


Fig. 33, in which the eye cannot see clearly at any positive distance.

III. 3. Hypermetropia, combined with hypsopia, gives, when slight, short-sightedness, as in Fig. 34; and, when considerable, far-sightedness, as in Fig. 35, distinguished from the common form by this, that the eye cannot see in infinite distance, but only between two definite boundaries. In both cases the eye may only be able to accommodate for rays that converge towards points situated behind it.

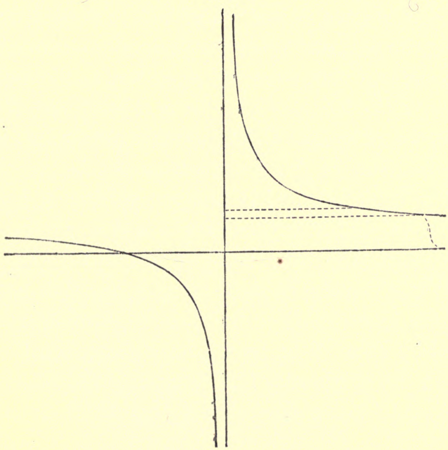
It must here be remarked that, when the stenopia or small-sightedness reaches the highest degree, so that the range of

Fig. 34.



accommodation R B is reduced to nothing, or the two lines L′L and K′K (Fig. 20) coincide, the eye can then only accommodate for a single distance, and therefore has either only a near-point or only a far-point, which, according to circumstances, may be

Fig. 35.



situated either before or behind it. Such an eye may yet see with sufficient clearness an object that corresponds with the null point of its accommodative power, and therefore that is situated at the distance at which the visual line is intersected by the curve of requirement.

The state of an eye deprived of its lens, the aphakia of Donders, fulfils this condition of extreme stenopia; since such an eye has almost entirely lost its power of accommodation, and can only see clearly at one determinate distance, or has its range of accommodation *nil*.

We may, however, remark, that a narrowing of the range of accommodation is in general associated with its displacement. Fig. 35 shows that by a narrowing of the range of accommodation the points of intersection of the facultative curve with the curve of requirement, that is, the near- and the far-point, or one of them, must be displaced.

Which among the foregoing ocular defects will be the most common in actual practice, and which the most unusual, can only be determined by practical ophthalmology. The information will be interesting; but the principles of our classification will be in no way affected by it.

The description of these defects by Donders, in his most recent work upon the anomalies of the refraction and accommodation of the eye, contains a much smaller number of characteristic states, and moreover puts together, as identical, defects between which I recognise an entire difference. Hypsopia is not described as a special failure of accommodation; and hence there is no account of the defects thence arising. Moreover, Donders has been led, by his non-recognition of hypsopia, to describe short sight and hypometropia as equivalent conditions; and to declare that myopia is the opposite to hypermetropia, and presbyopia not the opposite to myopia, but a normal condition of an emmetropic eye with its range of accommodation diminished by age. I believe I have shown in the foregoing that only in the single case II. 1 is myopia the defect described by Donders; that as a general thing myopia does not coincide with hypometropia, and does not depend upon a simple anomaly of refraction, but may be a result of very different anomalies of refraction and accommodation. Presbyopia, too, is the defect described by Donders only in the single case I. 2; and does not generally depend upon a simple anomaly of accommodation, but may arise from different anomalies of refraction and of accommodation. I do not doubt that an unprejudiced estimate would admit the foregoing general principles to a share of recognition.

SPECTACLES FOR DEFECTS OF ACCOMMODATION AND REFRACTION.

After the foregoing exposition of fundamental principles, the question about the optical aids for the defects described can be rationally discussed.

It is at once manifest that no such aids can be given to the anomalies of accommodation. Defects depending upon innervation, or upon irritability or insensitiveness, cannot be set aside; and the range of accommodation of the eye cannot be altered. We can by external means, especially by the addition of refracting media, modify the optical state of the eye, and therefore its refracting power, but not its nerve power, or its sensitiveness to rays of light. Doubtless diminution of light will diminish, and increase of light will increase, the refraction of the eye; but these means would not alter the limits of sensitiveness, and, even if under certain circumstances they might do so, the effect would be feeble, not capable of definition by measure and numbers, only very imperfectly attainable by optical means, such as clouded glasses, and best to be realized by placing the eyes in a brighter or darker visual space, so that we may wholly disregard them in the mathematical determination of spectacles.

A stenopic eye therefore is and remains stenopic, and cannot be changed into one that is europic; we can never confer upon it a great range of accommodation.

As regards, on the contrary, the anomalies of refraction, the state of refraction of an eye may obviously be changed in either direction and may be rendered normal. By combination with a fitting lens the focal length of any eye may be altered to any desired condition, and may thus be normalized. In the figures 36 and 37:

Fig. 36.

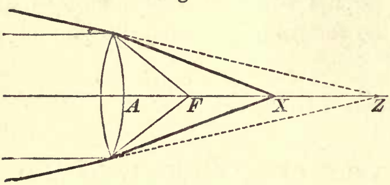
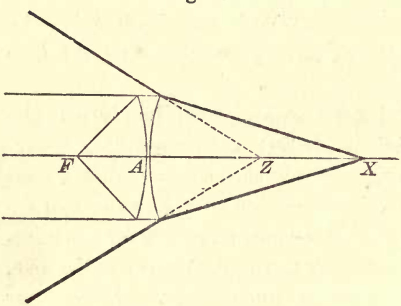


Fig. 37.



Let f be the focal length A F of a glass lens, and let it be positive for a convex and negative for a concave. We call this lens shortly the lens f.

Let x be the actual distance A X of a luminous point from this lens.

Let z be the distance A Z of the point to which the rays converge after having been refracted in passing through the lens, and therefore the distance of the optical image of the former point, or of the visible object, likewise measured from the lens. The distances x and z must be positive when they are before the lens, and negative when they are behind it.

Between the quantities f, x, z, there exists the relation

$$\frac{1}{z} = \frac{1}{x} - \frac{1}{f}, \text{ or } \frac{1}{x} = \frac{1}{z} + \frac{1}{f}.$$

If we imagine the lens to be exactly in the point already designated A, and therefore at the same distance in front of the eye as the length of its natural focal distance b, it follows, since we have measured the distance of the object from this point, that the optical effect of the lens will be to transform the actual visual distance x into the apparent visual distance z. Hereby therefore the eye is in no way changed in its function; the change affects solely the point of convergence of the external rays, which in a physiological sense is identical with the object, since the eye does not react on the actual space, but on the incident rays; and the space is but our objective conception of the subjective light affection. Hence the formulæ already given for the curve of required accommodation retain their applicability, since z may be put in the place of x, or $\frac{1}{z} = \frac{1}{x} - \frac{1}{f}$, in the place of $\frac{1}{x}$. This gives for the curve of requirement, when the ordinate is expressed by y', the equation

$$y' = k \left(\frac{1}{x} - \frac{1}{f} - \frac{1}{a}\right).$$

This equation gives an equilateral hyperbola which is completely congruent with the former, obtained from $y = k\left(\frac{1}{x} - \frac{1}{a}\right)$, and differs from it only in position, while the ordinate of the new curve, $y' = y - \frac{k}{f}$, differs from the ordinate of the former curve only by the constant term $-\frac{k}{f}$.

We call the former curve of requirement the natural, and this one the artificial.

If we take the constant quantity $\frac{1}{f} + \frac{1}{a} = \frac{1}{a'}$, we have $y' = k\left(\frac{1}{x} - \frac{1}{a'}\right)$. The quantity $a' = \frac{1}{\frac{1}{a} + \frac{1}{f}} = \frac{af}{a+f}$ indicates, therefore, the distance of that luminous point Q which, by the intervention of the lens f, has its rays accurately united upon the retina in its state of rest; while a is the natural visual length, a' must be the artificial visual length.

From the preceding it is clear that a convex or positive lens f depresses the hyperbolic curve of requirement by the depth $\frac{k}{f}$, while a concave or negative lens $f = -f'$ elevates this curve by the height $\frac{k}{f'}$; or that the convex lens moves the terminal point Q of the visual distance a in a direction towards the eye, since it changes this visual distance to $a' = \frac{1}{\frac{1}{a} + \frac{1}{f}}$; while a concave lens moves the terminal point in a direction away from the eye, since it changes the visual distance to $a' = \frac{1}{\frac{1}{a} - \frac{1}{f'}}$.

The above-named terminal point Q of the visual distance is the point of intersection of the hyperbolic curve of requirement with the visual axis. The hypometropia, which is the fundamental basis of common short sight, and through which the hyperbola lies too low, or the terminal point of the visual length at a finite distance in front of the eye, is therefore removed by a concave lens; while the hypermetropia, which forms the basis of common over-sightedness, and in which the hyperbola is too high, or the terminal point of the visual length is behind the eye, is neutralized by a convex lens.

Complete removal of the hypometropia or hypermetropia requires the placing of the terminal point Q of the visual length in infinite distance, or that $a' = \infty$. This requirement is fulfilled by a lens of which the focal length $f = -a$; therefore by a lens of which the focal length is equal to the negative value of the natural visual distance a. For a hypometropic eye, for which a is positive, f must therefore be negative, that is, the lens must be concave; but for a hypermetropic eye, for which a is negative, f must be positive, that is, the lens must be convex.

We see that the convexity or concavity of the lens is the means by which the hypermetropia or the hypometropia, the anomaly of refraction, may be set aside completely. By means of a proper lens every eye without exception can be rendered completely emmetropic; since the hyperbolic curve of requirement can be restored to its normal position, in which the horizontal asymptote is the visual line.

It should still be observed that it is a necessary condition for this result that the lens should be placed exactly in the point A, and therefore at the distance of the natural focal length b of the eye. This distance is nearly equal to the diameter c of the eye; although for hypometropic eyes it is somewhat less, and for hypermetropic eyes somewhat more, than this diameter. In the construction and fitting of spectacle frames this requirement should be remembered.

It is manifest that the convexity or the concavity of the lens is no general or essential means of aid as against the short, the far, or the over-sightedness, since these defects do not depend unconditionally upon a simple anomaly of refraction, but, as a rule, upon a combination of an anomaly of refraction with an anomaly of accommodation; combinations which may, as we have seen, be of various kinds.

It is also clear, since spherical lenses and external aids generally act by changing only the refraction, and not the accommodation, that as a rule a short-sighted, or a far-sighted, or an over-sighted eye cannot be rendered normal, or changed into one that is at once emmetropic and europic. When we arrive at the question how to improve the vision of an abnormal eye for actual use, we shall find that this can only be done within certain limits of distance, and in a degree more or less complete.

It is first, however, to be observed that every europic or broad-sighted eye, that is, every eye with a normal range of accommodation, may be rendered completely normal by a spherical lens, since in such a case we have only to remove the hypometropia or the hypermetropia. Hence the common short sight or myopia, II. 1, of a hypometropic europic eye, and the common over-sight or hyperopia, or that far-sightedness which occurs in both the cases under III. 1, may respectively be completely neutralized by a concave or a convex lens.

In every other ocular defect which has stenopia or small-sightedness as its foundation, we can only displace the hyperbolic curve of requirement in such a manner that it will fall within the range of accommodation for the desired visual distance, or will cover a certain portion of the facultative curve. For this the above figures (24 to 35) furnish the necessary guidance. The nine conditions given require the following remarks:—

I. 1. The full-sighted eye requires no lens.

I. 2. If the emmetropic far-sighted eye, with small deep lying accommodation, requires to use near vision, its hyperbola must be depressed. This is done by a convex lens. But with this the eye loses its faculty of seeing at infinite distance, since the depressed hyperbola cuts the visual line at a finite distance. Such an eye sees best near at hand with a convex lens, and best at distance without a lens.

I. 3. The emmetropic short-sighted eye, with small but high lying range of accommodation, requires, for vision at moderate distance, that its hyperbola should be considerably raised by a comparatively strong concave lens. With this, however, it can see only within narrow fixed limits, and loses the power peculiar to it of vision at infinite distance.

II. 1. The common short-sighted eye, which is hypometropic and europic, is rendered completely normal by elevation of its hyperbola by a concave lens.

II. 2. The weak-sighted eye, which is hypometropic, with narrow deep lying accommodation, and suffers, according to circumstances, from short or from far sight, requires, when it is short-sighted, the elevation of its hyperbola by a concave lens for vision at a distance; and when it is far-sighted, the depression of its hyperbola by a convex lens for near vision. In both cases it only acquires the power of vision between certain limits, determined by the power of the lens.

II. 3. The short-sighted eye, which is hypometropic, and has a narrow high lying range of accommodation, requires for vision at greater distance the elevation of its hyperbola by a concave lens, but obtains thus only the power of vision at certain distances, which the power of the lens determines.

III. 1. The common over-sighted eye, which is hypermetropic and europic, and cannot see clearly at any positive distance, may be rendered completely normal by depression of its hyperbola by a convex, usually by a rather strong convex, lens. Also when such an eye is far-sighted, it can be rendered normal by a convex lens.

III. 2. The hypermetropic eye with narrow deep lying range of accommodation, which according to circumstances may be far-sighted or over-sighted, requires for vision at a positive distance a depression of its hyperbola, and therefore a convex lens, but thus obtains vision only between certain limits which the power of the lens determines.

III. 3. The hypermetropic eye with narrow high lying accommodation, requires, according as it is near-sighted or far-sighted, for vision respectively at a distance and near, a concave or a convex lens. It thus obtains vision only between certain limits, which the power of the lens determines.

The preceding results are interesting, as showing that short-sightedness is always relieved by a concave, far-sightedness by a convex, and over-sightedness by a proportionately strong convex lens; and further that the short-sightedness II. 1, and the far-sight and over-sight III. 1 of an europic eye, can be removed completely, but all other defects only within certain limits, determined by the power of the lens; so that the eyes II. 1 and III. 1 are enabled to see clearly at every distance by one and the same lens, while all other eyes require different lenses for different distances.

For the ophthalmic surgeon it is important to know the most simple methods which are adapted to discover, first, the kind and degree of any occurring anomaly, and, secondly, the lens that will produce any given effect. The foregoing inquiries point out the way to this end.

With regard to any given ocular defect, the anomaly of refraction has first to be ascertained; and it is necessary to discover whether the eye be emmetropic, and, if not, in what degree it is hypometropic or hypermetropic. If the eye receives the luminous impression of an infinitely distant object without any exercise of accommodation, and sees the object clearly, then it is emmetropic. If it receives the rays from an infinitely distant object certainly without effort of accommodation, but without obtaining a clear image, then it is hypometropic. If it requires some effort of accommodation in order to obtain a clear image of an infinitely distant object, then the eye is hypermetropic, or possesses the power to accommodate for rays that converge towards a point lying behind it.

The degree of the hypometropia or hypermetropia is most easily determined by the experimental application of convex or concave lenses of different powers. If we find by such experiments that l is the lens (i.e. the focal length of the lens) with the aid of which the eye sees an infinitely distant object with clearness and without accommodative effort; so that the lens l makes the artificial visual distance a' infinitely large, we have, from the formula,

$$a' = \frac{1}{\frac{1}{a} + \frac{1}{l}} = \infty \text{ or } \frac{1}{a} + \frac{1}{l} = 0$$

$$\frac{1}{a} = -\frac{1}{l} \text{ or } a = -l.$$

The natural visual distance a is therefore equal to the negative value of the focal length l of the above indicated lens. If the eye requires a concave lens $l = -l'$, it is then hypometropic in the degree $\frac{1}{a} = \frac{1}{l'}$. If it requires a convex lens l, it is then hypermetropic in the degree $\frac{1}{a} = -\frac{1}{l}$.

This lens, $l = -a$, which makes the artificial visual distance infinitely long, and serves to determine the degree of hypometropia or of hypermetropia, is also that which completely removes this hypometropia or hypermetropia, if such complete removal be in contemplation. We call this lens l, briefly the emmetropic lens.

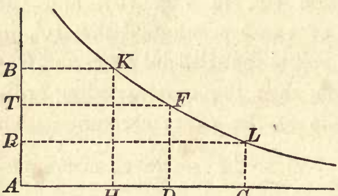
To determine the anomaly of accommodation the question arises whether the eye is europic; and, if not, whether the stenopic eye is bathopic or hypsopic, and in what degree. The determination is most readily accomplished by the aid of the above described emmetropic lens l. Furnished with this, the eye is to be regarded as equivalent to one that is emmetropic; so that its curve of requirement has the equation $y = \frac{k}{x}$. If we determine, in this condition, that is to say, with the aid of the emmetropic lens, the distance n of its near-point of exact accommodation (H′, in Fig. 15), and the distance w of its far-point of exact accommodation (C′, in Fig. 15), and if we call these points the artificial near- and far-points of exact accommodation, then the corresponding ordinates which indicate the limits of the exact accommodation will be respectively $\frac{k}{n}$ and $\frac{k}{w}$, and the range of accommodation, which shows the actual accommodative power of the eye, will be $k\left(\frac{1}{n} - \frac{1}{w}\right)$. The coefficients $\frac{1}{n}$, $\frac{1}{w}$, $\frac{1}{n} - \frac{1}{w}$, can be taken as measures for the degree of the anomaly of accommodation. If $\frac{1}{n} - \frac{1}{w}$ be smaller than the normal range, then the eye is more or less stenopic. If we indicate the normal range of accommodation of an europic eye by $\frac{1}{\beta}$, since β is the distance of its near-point; then the degree of stenopia of the given eye is expressed by the quantity $\frac{\frac{1}{\beta}}{\frac{1}{n} - \frac{1}{w}} = \frac{nw}{\beta(w-n)}$.

If $\frac{1}{w}$ be nearly null, then the eye is as deep-sighted as a normal eye. The greater $\frac{1}{w}$, the more high-sighted is the eye; so that $\frac{1}{w}$ expresses the degree of the hypsopia.

We know already that the europic eye, for which $\frac{1}{w}$ is $= 0$, and $\frac{1}{n} = \frac{1}{\beta}$, can be rendered completely normal by a lens. For such an eye, therefore, the already described emmetropic lens *l* is the means by which its defect of refraction is wholly set aside.

For every stenopic eye only a certain movement of the curve of refraction (hyperbolic curve of requirement) can be accomplished, by which another part of the visual line comes into the territory of accommodation. The condition for the required lens may therefore be formulated by saying that an object lying at a given distance must be brought into a given part of the range of accommodation. In order that the eye may be exerted continuously, this given part must fall nearly in the middle of the range of accommodation $\frac{1}{n} - \frac{1}{w}$, and must, therefore, nearly correspond to the value

Fig. 38.



$$\frac{1}{w} + \tfrac{1}{2}\left(\frac{1}{n} - \frac{1}{w}\right) = \tfrac{1}{2}\left(\frac{1}{n} + \frac{1}{w}\right).$$

If, in order to make the condition somewhat more general, we state the requirement to be that an object lying in the visual distance *s* shall correspond with that portion of the range of accommodation which makes the part *i* of the whole, so that, therefore, when, in Fig. 38, for the near-point H, A B $= \frac{1}{n}$, and for the far-point C, A R $= \frac{1}{w}$, so that the range of accommodation R B $= \frac{1}{n} - \frac{1}{w}$, an object lying in the distance A D $= s$ shall correspond with the portion T of the range of accommodation, for which

$$\text{R T} = i \,.\, \text{R B} = i\left(\frac{1}{n} - \frac{1}{w}\right),$$

consequently,

$$\text{A T} = \frac{1}{w} + i\left(\frac{1}{n} - \frac{1}{w}\right) = \frac{i}{n} + \frac{1-i}{w}.$$

The latter condition involves that the abscissa x in the curve of refraction corresponds to the ordinate $y = k\left(\frac{1}{n} + \frac{1-i}{w}\right)$.

Suppose the eye furnished with any lens f, the equation of the curve of refraction is $y = k\left(\frac{1}{x} - \frac{1}{f} - \frac{1}{a}\right)$. In this, a shows the natural visual distance, which is $= -l$, when l expresses the emmetropic lens. The equation is, therefore

$$y = k\left(\frac{1}{x} - \frac{1}{f} + \frac{1}{l}\right).$$

Our last condition requires that $x = s$, whence $y = k\left(\frac{1}{s} - \frac{1}{f} + \frac{1}{l}\right) = k\left(\frac{i}{n} + \frac{1-i}{w}\right)$; it is therefore fulfilled by

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{l} - \left(\frac{i}{n} + \frac{1-i}{w}\right).$$

If the power of accommodation desired for the given object be taken as one half, so that $i = \frac{1}{2}$, then $\frac{1}{f} = \frac{1}{s} + \frac{1}{l} - \frac{1}{2}\left(\frac{1}{n} + \frac{1}{w}\right)$.

Hence the lens f, adapted to produce the desired effect, may be determined from the easily observed magnitudes l, n, and w, and the given values s and i, by a very simple calculation, in any case however complicated.

We must repeat that in these formulæ n, the distance of the near-point, and w, the distance of the far-point, are taken not from the naked eye, but from the eye furnished with the emmetropic lens $l = -a$. If n_1, and w_1, indicate the distances of the near- and far-points from the naked eye, of which the curve of requirement has the equation $y = k\left(\frac{1}{x} - \frac{1}{a}\right) = k\left(\frac{1}{x} + \frac{1}{l}\right)$, there will obviously be, between the magnitudes n, w, and n_1, w_1, the relation

$$\frac{1}{n} = \frac{1}{n_1} - \frac{1}{a} = \frac{1}{n_1} + \frac{1}{l}, \text{ and } \frac{1}{w} = \frac{1}{w_1} - \frac{1}{a} = \frac{1}{w_1} + \frac{1}{l}.$$

If, therefore, there should be an opportunity of learning the values of n_1, w_1, and a without the application of the emmetropic lens, the values of $\frac{1}{n}$ and $\frac{1}{w}$ for the formulæ may be easily calculated. The difference $\frac{1}{n_1} - \frac{1}{w_1}$ is always $= \frac{1}{n} - \frac{1}{w}$, that is, equal to the absolute range of accommodation.

Example. Given an eye which obtains an infinite visual length by a concave lens $l = -40$ (Paris inches); it is therefore hypometropic, and has a natural visual length $a = -l = 40$ inches. With this lens the artificial near-point is found at a distance $n = 3$ inches; and the artificial far-point at a distance $w = 15$ inches. The range of accommodation is therefore $\frac{1}{n} - \frac{1}{w} = \frac{1}{3} - \frac{1}{15} = \frac{4}{15} = \frac{1}{3\frac{3}{4}}$; and the eye is moderately stenopic, and $\frac{1}{w}$ being $= \frac{1}{15}$, it is somewhat high-sighted. If it be desired that this eye should clearly see objects at a distance of 20 feet, or $s = 240$ inches, with the use of half its accommodation, therefore, with $i = \frac{1}{2}$, it will require a lens which is determined by

$$\frac{1}{f} = \frac{1}{240} - \frac{1}{40} - \tfrac{1}{2}\left(\frac{1}{3} + \frac{1}{15}\right) = -\frac{53}{240} = -\frac{1}{4\frac{1}{2}},$$

and which is, therefore, the strong concave lens $f = -4\frac{1}{2}$.

DEFECTS OF CONVERGENCE, OR ANOMALIES OF DIRECTION AND OF MOTILITY.

The preceding chapters refer exclusively to monocular vision. The common function of the two eyes certainly in no way changes the fundamental principles of refraction and accommodaticn for each eye alone; but yet, in binocular vision, the function of convergence has to be taken into consideration as an especial factor which, on account of the reciprocal interdependence of all the visual functions in the general visual plane, so far influences the effect as to determine the degree of strain, or even the very possibility of using the vision at all.

In principle the convergence is independent of the accommodation, but each function exerts an acquired influence over the other. Hence this influence first obtains a definite degree in a more or less considerable deviation from the normal actions. We leave this degree, and generally the influence of the two functions, for a time out of account; and consider first the convergence, in its original independence, as the act which is performed solely for the sake of fixation, or in order to guide the two retinal images to the poles of the retinæ.

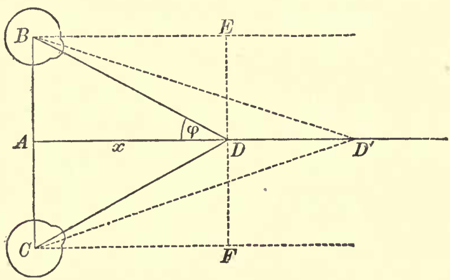
The physiological stimulus, which excites this function of convergence, depends entirely upon the production of the equilibrium, to which the nerve affections of the two retinal images forcibly constrain the spherical organs of the binocular visual apparatus; as mechanical forces, which act upon the circumference of a sphere, rotate the sphere and cause it to assume a definite position.

In monocular vision the single eye is constrained to place itself with its pole in the optical centre of intensity of the retinal image; that is, upon that point of this image, in which lies the centre of intensity of the affection of the nerve produced by the objective stimulus of light. In binocular vision the two retinæ, connected through the sensorium into an united physiological apparatus, concur in such a manner that the pole of each strives to attain to the centre of intensity of its retinal image; and this concurrence produces primarily the convergence which is only secondarily influenced through other functions. (See the *Gesetze des räumlichen Sehens.*)

The convergence has therefore the object of guiding the optical centre of intensity of the luminous image, or the centre of intensity of the objective nerve affection, to the physiological pole of each eye, lying in the yellow spot. This pole forms the most sensitive point of the retina, and is the sensational centre of the eye. With the affection of the pole is therefore united the most perfect definition and the most complete sensibility, and therefore also the clearest consciousness of the visual impression. I believe that this subjective straining after definition and clear consciousness of the sensory impression is identical with the previously described tendency to turning of the eyes; that this tendency constitutes the fundamental element of fixation; and that, therefore, the function of convergence is directed principally to the fixation, not to the single vision, and that the single vision is, in a normal pair of eyes, only secondarily united with correct convergence, or dependent upon it; and that the singleness of the visual impression rests primarily upon a perfectly distinct function, of which I shall say more in the sequel. The fully conscious vision with the pole of the eye, is also the so-called direct vision; and we may therefore describe direct vision as the purpose of convergence.

If the two eyes B and C (Fig. 39), of which the distance apart of their central points B C $= d$, are directed to a point lying at the distance A D $= x$, then each eye must describe an arc which corresponds to the angle E B D $=$ B D A $= \frac{1}{2}$ B D C $= \phi$, that is, the angle of inclination of the ocular axis B D to the

Fig. 39.



axis of the face A D. In two eyes of equal convergence this angle ϕ is equal to half of B D C, the angle of convergence of the two eyes. At first sight, therefore, the magnitude of the angle ϕ would appear to be a measure for the effort of convergence of one eye. If we consider, however, that this effort would be infinitely great when the angle $\phi = 90°$, it will appear that not the angle ϕ itself, but its tangent, furnishes this measure. Consequently we may express the necessary effort of convergence, or the convergence requirement y by the value of tan. ϕ, therefore by $\frac{d}{2x}$. If now κ indicate a certain constant factor, given by the physiological nature of the eye, we obtain for the convergence required from each eye, for binocular vision at the distance x, simply the formula

$$y = \kappa \tan. \phi = \frac{\kappa d}{2x};$$

or, if we make,

$$\frac{\kappa d}{2} = k',$$

$$y = \frac{k'}{x}.$$

If we consider the visual distance x as the abscissa, and the required convergence y as the ordinate of a curve which we call the curve of required convergence, this curve, as well as the curve of required accommodation, will form an equilateral hyperbola which differs from the latter only by the constant coefficient k', therefore by the length of its axes, and in the fact of A, the point of commencement for the axes of co-ordinates of the curve of convergence, lying in the centre of the line B C that unites the centres of the two eyes, or at the root of the nose; while the point of commencement of the axes of co-ordinates for the accommodation curve lies about the length of its own diameter in front of each eye. This difference of position of the point of commencement, from which the visual distance x is measured, is so trifling that it may be neglected; and we may assume that the magnitude x represents both the hyperbolic curve of accommodation and the hyperbolic curve of convergence for the same visual distance from the face.

A normal pair of eyes requires no effort of convergence for infinite distance, or for parallel position, when for $x = \infty$, y is $= 0$. I call such a pair of eyes *engonic* (caring more, in this expression, for analogy with 'emmetropic' than for grammatical logic). The criterion for such a pair of eyes is, that they are parallel in the absence of effort, or in the state of rest of their converging apparatus. When we do not consider a pair, but one eye only, then the criterion of engonia is that the eye, during the state of rest of its convergence, shall be parallel to the axis of the face.

The anomalies of convergence, like those of refraction and of accommodation, fall into two great classes, of which the first rests upon the condition or structure of the converging apparatus, and the second upon its irritability, sensitiveness, innervation, or functional state generally. We will, in the first place, consider the former—the anomalies depending upon the structure of the converging apparatus. They are such as affect the direction of convergence of the eyes in the state of rest of the converging apparatus, and they may be called the anomalies of ocular direction. The following three forms require to be considered.

1. The normal state of ocular direction, or engonia, has been already characterized. In it the curve of requirement is $y = \frac{k'}{x}$, which we call the curve of direction. There are two anomalies to be considered.

2. The first anomaly of ocular direction affects a pair of eyes which, in the state of rest of the converging apparatus, converge towards a positive finite distance e, or which are convergent in the state of rest. We may call such eyes *hypogonic*. For them the curve of required convergence is expressed by the equation

$$y = k'\left(\frac{1}{x} - \frac{1}{e}\right);$$

since, when the eye B leaves the state of rest of the convergence, in order to fix, or to see directly, an object at the distance x, it describes the angle D B D′ (Fig. 39) which forms the difference between the angles A D B and A D′ B; the tangent of which, on account of the presumed smallness of this angle, is with sufficient nearness proportional to the value $\frac{1}{x} - \frac{1}{e}$. This curve, like the curve of required accommodation of a hypometropic eye, forms a depressed hyperbola, which cuts the visual line in the distance A Q $= e$ (Fig. 16). In order to accommodate for the infinite distance $x = \infty$, or in order to assume a direction parallel to the frontal axis, this eye requires a negative converging effort $y = -\frac{k'}{e}$.

We call the distance e the converging distance of a pair of eyes. While the infinite converging distance is the sign of an engonic pair, the positive converging distance is the sign of a hypogonic pair.

3. The second anomaly of ocular direction is represented by a pair of eyes which are divergent in the state of rest, and are directed to a point Q lying behind them in a negative distance $e = -e'$; so that they have a negative converging distance. We may call them *hypergonic*. Their equation is

$$y = k'\left(\frac{1}{x} - \frac{1}{e}\right) = k'\left(\frac{1}{x} + \frac{1}{e'}\right).$$

This, like the curve of required accommodation of a hypermetropic eye, gives an elevated hyperbola, which cuts the visual line behind the head in the distance A Q $= e = -e'$ (Fig. 19). To be directed to infinite distance, $x = \infty$, such eyes require a positive effort of convergence $y = \frac{k'}{e'}$.

Hypogonia and hypergonia, being both deviations from engonia, may be called anengonia.

It is apparent that the curve of requirement of engonic, of hypogonic, and of hypergonic eyes, that is to say, the curve of convergence requirement of any eyes whatever, is expressed by one and the same equation, $y = k'\left(\frac{1}{x} - \frac{1}{e}\right)$ in which the quantity e is for the first infinite, for the second positive, and for the third negative.

Like the faculty or power of accommodation, so also the faculty of convergence is for every eye restrained within two limits which determine the facultative curve. The anomalies of the faculty of convergence are in truth anomalies of motility, wherefore the facultative curve of convergence may also be called the curve of motility. The eye has a near-point and a far-point of convergence, and is not able to attain to any stronger or to any weaker degree; so that it sees objects in front of its near-point and behind its far-point, not with its pole; that is to say, not directly or with complete fixation. A consequence of this is deviation of the ocular axis from the visual line, or squinting; and this squinting in the use of both eyes, or in binocular vision, is attended by diplopia, since the retinal images in the two eyes fall upon different portions of the retinæ.

In Figs. 21 to 23, let the ordinate A B represent the near-point, and the ordinate A R the far-point of the convergence; then by the difference R B of these two ordinates is given the quantity called by von Graefe the squint capacity; and which, by analogy to the range of accommodation, may be called the range of convergence. On the basis of the different values of A R and A B there are the following three cases to be considered.

1. The normal state of motility, which may be called *eurogonia* (Fig. 21). In this state the range of convergence R B is of normal breadth, and A R is so small that it may be considered nearly $= 0$.

The near-point of the convergence of the eurogonic may nearly coincide with the near-point of the accommodation of the europic eye; but in general it seems to me to be somewhat nearer.

The two anomalies of motility are characterized by a small range of convergence R B (Figs. 22 and 23), and may be described as *stenogonia;* and also,

2. As *bathogonia*, when, as in Fig. 22, A R is very small.

3. As *hypsogonia*, when, as in Fig. 23, A R is proportionately large.

If the eye, as regards its direction, is engonic, then eurogonia, bathogonia, and hypsogonia are thus characterized. In the eurogonic eye the near-point of convergence is near and the far-point distant; in the bathogonic eye the near-point is distant; and in the hypsogonic eye the far-point is near. In general, however, no single one of these anomalies of motility, and no single one of the anomalies of direction, determines the possible defects of convergence.

We may find all the defects of convergence by combining the three states of direction with the three states of motility. This gives a system of nine conditions, accurately corresponding with the foregoing system of defects of accommodation. The figures 24 to 35 display the corresponding defects of convergence, if we put the hyperbola for the curve of direction, and the dotted lines for the limits of motility. It will here be sufficient to indicate the nine conditions briefly. They are as follows:

I. 1. Engonia with Eurogonia, or the condition of the normal eye (Fig. 24).

I. 2. Engonia with Bathogonia (Fig. 25).

I. 3. Engonia with Hypsogonia (Fig. 26).

II. 1. Hypogonia with Eurogonia (Fig. 27).

II. 2. Hypogonia with Bathogonia (Fig. 28).

II. 3. Hypogonia with Hypsogonia (Fig. 29).

III. 1. Hypergonia with Eurogonia (Fig. 30 or Fig. 31).

III. 2. Hypergonia with Bathogonia (Fig. 32 or Fig. 33).

III. 3. Hypergonia with Hypsogonia (Fig. 34 or Fig. 35).

The immediate optical effect of the defects of convergence consists in this, that the defective eye, of which the convergence is abnormal, does not assume the position of fixation as towards the object, but squints at it, so that the retinal image of the object does not fall on the pole of the eye. If the convergence be too great, the squint is convergent, so that the image of the object falls on the inner side of the pole. The eye, therefore, projects the object too far outwards. In binocular vision, with two similar eyes in this condition, there will be diplopia with homonymous images. If, on the other hand, the convergence be too feeble, the squint will be divergent, the retinal image will fall on the outer side of the pole, and the eye will project the image too far inwards. In binocular vision with two similar eyes in this condition, there will be diplopia with crossed images.

If we have to deal with two similar eyes, then the kind and the degree of the defect of convergence, namely, the convergence distance e, and the range of convergence, may be determined by the double images. But with unlike eyes the defective projection of each must be observed singly, which may be done as well in binocular as in monocular use.

SPECTACLES FOR ANOMALIES OF DIRECTION AND MOTILITY.

With regard to the optical means of affording relief to the defects of convergence, it is manifest that by such means only the anomalies of ocular direction can be remedied, and that those of motility cannot even be influenced. The rational resource against the anomalies of direction is the use of the common plane prismatic spectacles. Their action is shown by the figures 40 and 41. We call a prism, as in Fig. 40,

Fig. 40.

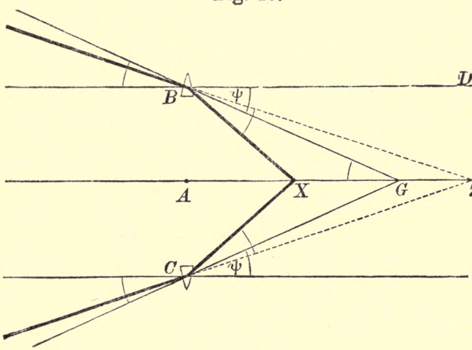
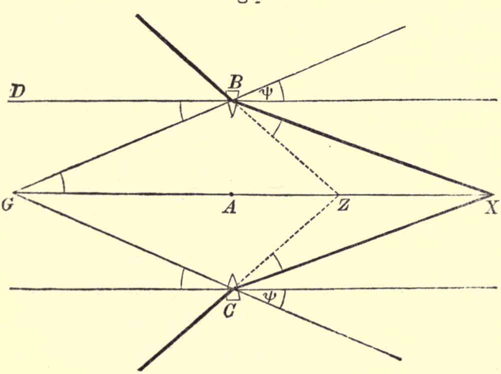


Fig. 41.



positive, which has its base inwards, or to the nasal side, and which, therefore, refracts the rays inwards, and consequently produces an apparent displacement of the object outwards, or homonymous double images. We call a prism negative, when, as in Fig. 41, its base is turned outwards, so that it refracts rays outwards, and produces an apparent displacement of the object inwards, or crossed double images. If

ψ be the angle of total deviation D B G = B G A of the prism, so that ψ is positive for a positive prism and negative for a negative,

g the convergence distance A G of the prism, that is, the distance from the eye A, in which a ray parallel with the visual line strikes that line after being refracted by the prism; so that g is positive for a positive prism and negative for a negative, and ψ is equal to the angle of inclination of the refracted ray towards the visual line,

$h = \frac{d}{2g} = \tan. \psi$ the refracting or deviating proportion of the prism (since d = B C, as before, the distance between the centres of the eyes, or $\frac{d}{2}$ = A B the distance of one centre from the median line) so that h is positive for a positive prism and negative for a negative,

x the actual distance A X of a luminous point X from the eye (or rather from the point A, in which the prism is placed),

z the distance A Z of the point Z in which the ray refracted by the prism cuts the visual line, so that B Z is the line of sight of the apparent object.

The refracted ray Z B inclines towards the unrefracted ray X B at the angle Z B X = ψ, and consequently the angle Z B X = A G B = D B G = ψ.

The angle B X A = D B X, which the ray B X, proceeding from the actual object at the distance x, forms with the visual line A X, is consequently equal to the sum of the angles Z B X = ψ, and D B Z = A Z B, formed by the ray proceeding from the apparent object at the distance z, and the visual line. Hence we have B X A = B G A + B Z A. Since now the tangents of these three angles are respectively $\frac{d}{2x}$, $\frac{d}{2g}$, $\frac{d}{2z}$, we obtain a known trigonometrical formula.

$$\frac{d}{2x} = \frac{\frac{d}{2g} + \frac{d}{2z}}{1 - \frac{d}{2g} \cdot \frac{d}{2z}}.$$

Considering that the quantity $\frac{d}{2g} \cdot \frac{d}{2z}$ is always very small in practice, much less than 1, we may omit it from the denominator on the right side of the equation. This gives the simple expression $\frac{1}{x} = \frac{1}{z} + \frac{1}{g}$, or $\frac{1}{z} = \frac{1}{x} - \frac{1}{g}$, which accurately coincides with the analogous formula for lenses.

When we now, in our equation for the requirement curve of the convergence, have put z in the place of x, or $\frac{1}{z} = \frac{1}{x} - \frac{1}{g}$ in the place of $\frac{1}{x}$, we obtain as the equation for the curve of required convergence for eyes furnished with prismatic spectacles, when we take y' as the ordinate,

$$y' = k'\left(\frac{1}{x} - \frac{1}{g} - \frac{1}{e}\right).$$

This equation again gives an equilateral hyperbola which is congruent with the former one expressed by $y = k'\left(\frac{1}{x} - \frac{1}{e}\right)$, and is only distinguished from it by position: while the ordinate of the new curve, $y' = y - \frac{k'}{g}$, differs from the ordinate of the former curve only by the constant term $-\frac{k'}{g}$.

Here also we call the former curve of requirement the natural, and the present the artificial.

If we take the constant quantity $\frac{1}{g} + \frac{1}{e} = \frac{1}{e'}$; we have $y' = k'\left(\frac{1}{x} - \frac{1}{e'}\right)$. The quantity $e' = \frac{1}{\frac{1}{e} + \frac{1}{g}} = \frac{eg}{e+g}$ indicates, therefore, the distance of that luminous point Q which, with the aid of the prismatic spectacles g, sends its rays in the state of rest of the eyes to corresponding portions of the retinæ. While e was the natural convergence distance, e' must be the artificial convergence distance.

From the foregoing it is clear that a positive prism g depresses the hyperbolic curve of requirement by the depth $\frac{k'}{g}$, while a negative prism $g = -g'$ raises this curve by the height $\frac{k'}{g}$; or, that positive prismatic spectacles displace the terminal point Q of the convergence distance e in a direction towards the eyes, since they change this convergence distance into $e' = \frac{1}{\frac{1}{e} + \frac{1}{g}}$, and that negative prismatic spectacles displace the terminal point of the convergence distance away from the eyes, since they change it into $e' = \frac{1}{\frac{1}{e} - \frac{1}{g'}}$.

Hence hypogonia, in which the hyperbola lies too deep, is to be corrected by negative glasses (bases outwards as in Fig. 41), and hypergonia, in which the hyperbola is too high, by positive glasses (bases inwards as in Fig. 40). A complete removal of these anomalies requires the placing of the terminal point of the convergence distance at infinity, or that $e' = \infty$. This condition is fulfilled by the prismatic spectacles $g = -e$, therefore by those spectacles of which the convergence length is equal to the negative value of the convergence distance e of the pair of eyes. For a hypogonic eye, for which e is positive, the prism g must be negative; and for a hypergonic eye, for which e is negative, the prism g must be positive.

A prism g is best determined by its proportion of deviation h, which, as above stated, is $= \frac{d}{2g}$. The prism which entirely removes the hypogonia or hypergonia must, therefore, have the proportion of deviation

$$h = \frac{d}{2g} = -\frac{d}{2e}.$$

By prismatic spectacles, therefore, the anomalies of ocular direction can be completely neutralized, and every eye rendered completely engonic. The anomalies of motility, on the contrary, cannot be altered by any optical means. In practice we can therefore only displace the curve of requirement of a pair of eyes not completely eurogonic, by means of prismatic spectacles, so far as the limits of the curve of motility (the squint range) will allow, with reference to the part of the range of vision which it is desired to render accessible. For this purpose, in the already specified nine cases, the methods to be observed are so completely analogous to those for the anomalies of refraction and accommodation, and will be so readily apparent from the Figs. 24 to 35, that any detail would be superfluous.

Upon the determination of the degree of an occurring anomaly of position or motility, in a concrete case, and upon the determination of the prismatic spectacles for a given effect, we have to remark as follows.

If the pair of eyes receives the luminous impression from an infinitely distant object without any exertion of convergence, and sees the object singly, then it is engonic. If the pair of eyes receives the impression of an infinitely distant object without any effort of convergence, but in a convergently squinting position, so that it sees homonymous double images, or projects the object too far outwards, then it is hypogonic. If the pair of eyes requires a certain effort of convergence for single vision of an infinitely distant object, and therefore, without such effort, sees with a divergent squint and with crossed double images, then it is hypergonic, or possesses the faculty of converging towards points lying behind it, and therefore of diverging forwards.

If it be found by experiment that p is the convergent distance, or q the proportion of deviation, of the prismatic spectacles with the aid of which the pair of eyes, without any effort of convergence, in the position of rest, sees infinitely distant objects singly; if therefore the prism $p = \frac{d}{2q}$, or $q = \frac{d}{2p}$, renders the artificial convergence distance infinite; then, from the formula

$$e' = \frac{1}{\frac{1}{e} + \frac{1}{p}} = \infty \text{ or } \frac{1}{e} + \frac{1}{p} = 0, e = -p = -\frac{d}{2q}.$$

The natural convergence distance e is therefore equal to the negative value of the convergence distance p of the above-named prism. Hence, if the pair of eyes requires negative spectacles $p = -p'$, it is hypogonic in the degree $\frac{1}{e} = \frac{1}{p'} = \frac{2q'}{d}$. If it requires positive spectacles p, then it is hypergonic in the degree $\frac{1}{e} = -\frac{1}{p} = -\frac{2q}{d}$.

These spectacles, $p = -e$, which render the artificial convergence distance infinite, and which serve to determine the degree of hypogonia or of hypergonia, are precisely those which remove this hypogonia or hypergonia completely, when such removal is desired. We call the spectacles p briefly the engonic spectacles.

An anomaly of motility is most easily discovered by the aid of these engonic spectacles. When furnished with them, the eyes are to be regarded as engonic; so that their curve of requirement has the equation $y = \frac{k'}{x}$. In this condition we determine the distance n of the near-point of accurate convergence (H′, in Fig. 15), and the distance w of the far-point of accurate convergence (C′, in Fig. 15), and call these points the artificial near- and far-points of the accurate convergence. Then the corresponding ordinates $\frac{k'}{n}$ and $\frac{k'}{w}$ are the limits of the accurate convergence; and, for the range of convergence or squint range, which expresses the true convergence faculty of the pair of eyes, we have $k'\left(\frac{1}{n}-\frac{1}{w}\right)$. The coefficients $\frac{1}{n}$, $\frac{1}{w}$, $\frac{1}{n}-\frac{1}{w}$, may be taken as measures for the degree of anomaly of motility. If $\frac{1}{n}-\frac{1}{w}$ be smaller than the normal range of convergence, then the eyes are stenogonic. If $\frac{1}{w}$ is nearly 0, then the eyes are as deep-angled as a normal pair. The greater $\frac{1}{w}$ the more high-angled the eye; so that $\frac{1}{w}$ displays the degree of hypsogonia.

Only an eurogonic pair of eyes can be rendered completely normal by prismatic spectacles. For a stenogonic pair the displacement of the curve of requirement can only produce a more or less favourable effect. In order to give the eyes endurance in vision, their motility must only employ half of the squint range. In general the conditions of endurance may be formulated thus, the part i of the squint range being employed, and the ordinate of the curve of requirement for the point of fixation having, therefore, the value

$$k'\left[\frac{1}{w}+i\left(\frac{1}{n}-\frac{1}{w}\right)\right]=k'\left(\frac{i}{n}+\frac{1-i}{w}\right).$$

Let us imagine a pair of eyes furnished with any prismatic spectacles g; then the equation of the curve of requirement is

$$y=k'\left(\frac{1}{x}-\frac{1}{g}-\frac{1}{e}\right).$$

In this e represents the natural convergence distance, which is $= -p$, when p represents engonic spectacles. The equation is therefore,

$$y = k'\left(\frac{1}{x} - \frac{1}{g} + \frac{1}{p}\right).$$

If it be desired that through these spectacles an object lying at the distance s should be seen with the indicated degree of effort of convergence, then, x being $= s$,

$$y = k'\left(\frac{1}{s} - \frac{1}{g} + \frac{1}{p}\right) = k'\left(\frac{i}{n} + \frac{1-i}{w}\right).$$

The condition is fulfilled by the spectacles

$$\frac{1}{g} = \frac{1}{s} + \frac{1}{p} - \left(\frac{i}{n} + \frac{1-i}{w}\right).$$

If it be desired to express the prisms not by their convergence lengths g and p, but through their proportions of deviation h and q; since $g = \frac{d}{2h}$ and $p = \frac{d}{2q}$, we have

$$h = q + \frac{d}{2s} - \frac{d}{2}\left(\frac{i}{n} + \frac{1-i}{w}\right).$$

If for the given object one half of the faculty of convergence is to be called into play, so that $i = \frac{1}{2}$, then

$$h = q + \frac{d}{2s} - \frac{d}{4}\left(\frac{1}{n} + \frac{1}{w}\right).$$

Hence the prismatic lens h, for the effect desired, may be determined, for any given case, by the easily observed magnitudes q, n, and w, and the given values of s, i, and d.

The distance d between the centres of the eyes has in adults a mean value of $66^{\text{mm.}}$ or 2·44 Paris inches; so that, in inches, $\frac{d}{2}$ is $= 1{\cdot}22$.

If instead of the proportion of deviation h, it is desired to determine the prisms by the angle of deviation ψ in degrees, we have then only to consider that

$$\psi = \frac{180}{\pi} h = 57{\cdot}306 \ . \ h \text{ degrees,}$$

$$h = \frac{\pi}{180} \psi = 0{\cdot}01745 \ . \ \psi$$

Between the values n and w, which indicate the position of the near- and far-points of convergence for the eyes furnished with the engonic spectacles p, and the values of n_1, and w_1, which indicate the position of the near- and far-points for the naked eyes, the proportions hold good which are given on page 60 for the near- or far-points of accommodation.

Example.

Suppose a pair of eyes that obtains an infinite convergence distance by positive prismatic spectacles with a proportion of deviation $q = 0{\cdot}01525$, or a convergence distance $p = \frac{d}{2q} = \frac{1{\cdot}22}{0{\cdot}01525} = 80$; then such eyes are hypergonic, and have the natural convergence distance $e = -p = -80$; that is, they diverge in the state of rest of the converging apparatus. Under the application of these spectacles they have an artificial near-point of convergence $n = 10$ inches, and an artificial far-point $w = 100$ inches. The convergence or squint range is therefore $\frac{1}{n} - \frac{1}{w} = \frac{1}{10} - \frac{1}{100} = \frac{9}{100} = \frac{1}{11\frac{1}{9}}$; and the eyes are stenogonic and deep-angled.

If it be desired that these eyes should see an object singly at a distance of 20 feet, or $s = 240$ inches, with the exercise of half their convergence, or with $i = \frac{1}{2}$, then they will require spectacles,

$$h = 0{\cdot}01525 + \frac{1{\cdot}22}{240} - \frac{1{\cdot}22}{2}\left(\frac{1}{10} + \frac{1}{100}\right) = -\,0{\cdot}04677$$
$$= -\frac{1}{21\frac{1}{3}}.$$

Hence negative prismatic spectacles (with bases outward) of the proportion of deviation $-\,0{\cdot}04677 = -\frac{1}{21\frac{1}{3}}$ or of the convergence distance $g = \frac{d}{2h} = -\frac{1{\cdot}22}{0{\cdot}04677} = -\,26$ inches, will be necessary.

CONCURRENCE OF DEFECTS OF ACCOMMODATION WITH DEFECTS OF CONVERGENCE. — COMBINATION SPECTACLES.

By inclusion of the normal conditions we have now become acquainted with nine chief states of accommodation, and with nine chief states of convergence. Any one of the former may be united with any one of the latter. This gives a total of 81 possible states, one of which is normal, while the other 80 constitute defects. All these may be grouped, according to what has gone before, into a synoptical system, in which the chief anomalies of refraction, of accommodation, of direction, and of motility, form the leading points. It is wholly unnecessary to devise names for the possible cases, since the preceding mathematical analysis shows that, together or separately, they require only special combinations of general and independent formulæ, of which one expresses the accommodation and the other the convergence. These formulæ, and the magnitudes contained in them, will determine the individual special cases, both in kind and in degree, with far more certainty than any verbal descriptions, and will also allow the limits between the two chief anomalies to be unequivocally defined. I shall therefore content myself by setting forth some important general results.

An entirely normal pair of eyes is at the same time emmetropic, europic, engonic, and eurogonic; that is, has its visual and converging distances infinite, and its range of accommodation and convergence of normal extent and of the lowest depth.

A pair of eyes with only a defect of accommodation, but no defect of convergence, can only use common spherical or centric spectacles. If, therefore, a short-sighted, or far-sighted, or over-sighted pair of eyes possesses a normal position and convergence, they can only be aided by common convex or concave spectacles; the use of prisms would be irrational and directly injurious.

A pair of eyes with only a defect of convergence, can only use common plane prismatic spectacles. The combination with lenses would be ineffectual.

An europic pair of eyes can be rendered completely normal by spherical glasses; and an eurogonic pair by prismatic glasses.

A pair of eyes at once europic and eurogonic, can be rendered completely normal by combination spectacles; that is, by the union of the required lenses with the required prisms.

If for an europic and eurogonic pair of eyes, the visual distance is equal to the converging distance, that is, if the eyes are in the same degree hypometropic as hypogonic, or in the same degree hypermetropic as hypergonic, the necessary combination spectacles will be normal or orthoscopic. In any other case this will not be so; and it may happen that either concave or convex lenses may have to be combined with positive or with negative prisms.

An europic pair of eyes would be rendered normal by the emmetropic lenses $l = -a$, and an eurogonic pair by the engonic prisms $p = -e$, or $q = -\frac{d}{2e}$. An europic and also eurogonic pair would therefore be rendered normal by the combination of $l = -a$, and $p = -e$. If the visual distance a of these eyes be equal to their converging distance e, then we have $l = p = -a$, or $q = -\frac{d}{2a} = \frac{d}{2} \cdot \frac{1}{l}$. The latter proportion, however, is that for normal orthoscopic spectacles. (See *Gesetze des räumlichen Sehens*, s. 177, z. 1.)

Whatever may be the combination of accommodation defect with convergence defect, the necessary combination spectacles may always be determined through the lens f and the prism g, according to the above two formulæ. In order, namely, that the pair of eyes may see clearly, singly, and continuously, and therefore with the corresponding portion i of accommodation and of convergence effort, an object lying at the distance s, it is necessary that,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{l} - \left(\frac{i}{n} + \frac{1-i}{w}\right),$$

$$\text{and } \frac{1}{g} = \frac{1}{s} + \frac{1}{p} - \left(\frac{i'}{n'} + \frac{1-i'}{w'}\right).$$

In both these formulæ s has the same given value: l expresses the emmetropic lens, and p the engonic prism, i the accommodation effort, and i' the convergence effort. The two quantities n and w in the first formula are the artificial near- and far-point of accommodation; and the two quantities n' and w' in the second formula are the artificial near- and far-point of convergence. If the near- and far-point of accommodation have respectively the same position as the near- and far-point of convergence, so that $n = n'$ and $w = w'$, and if we desire equal accommodation and convergence effort, so that $i = i'$, then subtraction of the two foregoing equations gives the proportion

$$\frac{1}{f} - \frac{1}{g} = \frac{1}{l} - \frac{1}{p}.$$

This proportion between the lens and the prism is independent of the distance s of the object, and also of the near- and far-points, and is conditioned only by the visual distance l and the converging distance p. If these two are infinite, the eyes being emmetropic and also engonic, we have $\frac{1}{f} - \frac{1}{g} = 0$, therefore $f = g = \frac{d}{2h}$. The latter condition characterizes normal orthoscopic spectacles, and teaches that an emmetropic and engonic eye, with an equal anomaly of accommodation and of motility, or in an equal degree stenopic and stenogonic, has need of a normal orthoscopic lens f, g, determined by the degree of the anomaly and by the given visual distance s.

If a common short-sighted or hypometropic eye, which possesses a somewhat depressed accommodation hyperbola, has long worked without glasses, it will have its hyperbola of convergence also more or less depressed, and the spectacles required will more or less approach the orthoscopic. But, if exertion has not yet brought about this condition, then only common concave lenses will be required.

A common far-sighted eye, the defect of which consists in a small deep-lying range of accommodation, while the hyperbolic curves of requirement, of accommodation, and of convergence, and also the range of convergence, are entirely normal, requires, for near vision, not only a depression of the hyperbolic accommodation curve, but also an equal depression of the hyperbolic convergence curve, since, if the latter be not also lowered, the eye will exert for near vision a feeble effort of accommodation with a strong effort of convergence, which will occasion strain. Consequently, far-sighted eyes almost always require normal orthoscopic spectacles, not common convex spectacles.

The two last results, namely, that a common far-sighted (i. e. bathopic) eye requires an orthoscopic glass, and a common short-sighted (i.e. hypometropic) eye requires, on the contrary, a common spherical glass, are of special importance on account of the great frequency of these conditions. For the far-sighted eye, this rule has no exception when the eye has become bathogonic by use, that is, has acquired a small range of convergence. For the short-sighted eye, however, an exception occurs continually, as soon as the eye has become hypogonic by use, that is, has acquired a shortened convergence distance; since then the spherical lens must more approach the orthoscopic.

The great difficulty which exists, according to Donders, in the treatment of myopia, seems to me to depend essentially upon the mischievous application of common spherical glasses, and to admit of removal by the addition of properly determined prisms.

It should be observed that, by the foregoing arrangement, the visual process is so regulated as to place the effort of accommodation at the part indicated by i of the *absolute* range of accommodation $\frac{1}{n} - \frac{1}{w}$, and the effort of convergence at the part i' of the *absolute* range of convergence $\frac{1}{n'} - \frac{1}{w'}$. To do this seems very important when the sharpest possible vision is required, as in the use of the eyes about common employments, such as reading, drawing, and the like. If, however, spectacles are required for other purposes, as for the most continuous possible vision at great distances, then the *relative* accommodation and convergence range come in the place of the *absolute*. They are given by the course of the two facultative curves, and I shall speak more fully of them in the sequel.

DETERMINATION OF VISUAL STRAIN.

If we furnish a pair of eyes with any given combination spectacles, f, g; then the two foregoing formulæ enable us to determine the strain of accommodation and of convergence with which the eyes see, clearly and singly, an object situated at the distance s. This strain is to be estimated by the values of i and i'. For these we have, through the first and second formula,

$$i = \frac{\frac{1}{s} + \frac{1}{l} - \frac{1}{f} - \frac{1}{w}}{\frac{1}{n} - \frac{1}{w}}, \text{ and } i' = \frac{\frac{1}{s} + \frac{1}{p} - \frac{1}{g} - \frac{1}{w'}}{\frac{1}{n'} - \frac{1}{w'}}.$$

If the tension of the accommodation and the convergence be equal, the equality of these values of i and i' gives a relation between the lens f and the prism g.

From the same formulæ it may also be ascertained with what tension the unaided pair of eyes will see at the distance s. Unaided eyes are equivalent to the lens $f = \infty$, and the prism $g = \infty$, so that we obtain,

$$i = \frac{\frac{1}{s} + \frac{1}{l} - \frac{1}{w}}{\frac{1}{n} - \frac{1}{w}}, \text{ and } i' = \frac{\frac{1}{s} + \frac{1}{p} - \frac{1}{w'}}{\frac{1}{n'} - \frac{1}{w'}}.$$

If we imagine a pair of short-sighted and broad-sighted eyes, with complete faculty of accommodation and no defect of convergence, so that $w = \infty$, $p = \infty$, and $w' = \infty$; then,

$$i = \frac{\frac{1}{s} + \frac{1}{l}}{\frac{1}{n}} = n\left(\frac{1}{s} + \frac{1}{l}\right), \text{ and } i' = \frac{\frac{1}{s}}{\frac{1}{n'}} = \frac{n'}{s}.$$

For these eyes the artificial near-point of accommodation coincides very nearly, and the artificial near-point of convergence exactly, with the natural; so that the two quantities n and n' have about the normal value. The quantity $l = -a$ expresses the negative value of the visual distance; and has therefore a negative and finite value, so that, since n is nearly $= n'$,

$$i = n\left(\frac{1}{s} - \frac{1}{a}\right) = i' - \frac{n}{a}, \text{ and } i' = \frac{n}{s} = i + \frac{n}{a}.$$

For a completely normal pair of eyes $a = \infty$, and therefore $i = i' = \frac{n}{s}$.

From this it follows that a pair of common short-sighted eyes, in order to see at the distance s, must make the same convergence as a normal pair; and, at the same time, a smaller effort of accommodation. If the object be in front of the middle visual distance of a normal eye, s is $< 2\,n$, and therefore $\frac{n}{s} > \frac{1}{2}$. In this case, where $i' > \frac{1}{2}$, the short-sighted eye must therefore exceed the measure of convergence that is adapted for continuous vision; while its effort for accommodation, $i = \frac{n}{s} - \frac{n}{a}$, has not yet reached this measure.

The quantity i is the proportion of the faculty of accommodation actually exerted, to the range of accommodation shown by $\frac{1}{n} - \frac{1}{w}$; and the quantity i' has the same significance with regard to the range of convergence. When i is nearly $= \frac{1}{2}$, the eye has the greatest accommodation endurance, and the visual strain is therefore as small as possible. We will assume generally that the least visual strain corresponds to the amount of accommodation $i = j$, and to the amount of convergence $i' = j'$. The measure of this visual strain is then the difference respectively between i and j, and between i' and j'. If we indicate this exact visual strain respectively by ζ and ζ', then we have $\zeta = i - j$, and $\zeta' = i' - j'$.

If ζ or ζ' be positive, then too strong an effort is demanded from the eye; and, if ζ or ζ' be negative, too weak an effort is demanded. When i has the value 1, and therefore $\zeta = 1 - j$, the highest endurable positive tension is attained; and, when $i = 0$ and hence $\zeta = -j$, the highest endurable negative tension is attained.

The quantities i, j, and ζ are proportional numbers, which give a certain proportional part of the range of accommodation $\frac{1}{n} - \frac{1}{w}$ $\left(\text{or convergence } \frac{1}{n'} - \frac{1}{w'}\right)$. For many considerations this proportional part is of less importance than the *absolute* value of the portion of the range of accommodation expressed by it. If in a small-sighted eye (of small range of accommodation) i is actually $= j$, and therefore $\zeta = 0$, and when therefore actually no visual tension exists, yet such an eye does not possess the endurance of an europic eye, since, in the former the required effort of accommodation lies nearer to its limits on either side than in the latter. A normal endurance requires that the effort of accommodation should be sufficiently far above its lower limit, and sufficiently far below its higher limit. Hence the difference between the actual effort of accommodation, and the limits of accommodation, appears to be a measure for the endurance of the eyes; and we have to consider this endurance, which we express by a, in two opposite relations. The difference between the maximum of accommodation and the actual accommodation is the endurance a_1, in the exertion, the difference between the actual and the minimum accommodation is the endurance a_2, in the relaxation, of the accommodating apparatus. We have, for the eye furnished with the lens f,

$$a_2 = i\left(\frac{1}{n} - \frac{1}{w}\right) = \frac{1}{s} + \frac{1}{l} - \frac{1}{f} - \frac{1}{w}$$

$$a_1 = (1-i)\left(\frac{1}{n} - \frac{1}{w}\right) = \frac{1}{n} - \frac{1}{s} - \frac{1}{l} + \frac{1}{f}$$

and for the naked eye, or for $\frac{1}{f} = 0$,

$$a_2 = i\left(\frac{1}{n} - \frac{1}{w}\right) = \frac{1}{s} + \frac{1}{l} - \frac{1}{w}$$

$$a_1 = (1-i)\left(\frac{1}{n} - \frac{1}{w}\right) = \frac{1}{n} - \frac{1}{s} - \frac{1}{l}$$

Complete endurance requires that the two values a_1 and a_2 should have values corresponding to those in an europic eye. This condition is obviously not generally to be fulfilled; and the foregoing formulæ enable us to estimate the difference between the endurance of a defective eye and that of a normal eye, or between the endurance of an eye in normal and in abnormal use, since for normal use i will be $= j$ in the formulæ, and for a normal, that is, broad-sighted or europic eye, n and w have the normal values.

In the foregoing we have treated the quantities n and w, which belong to the near- and far-points, as constants, since we have taken $\frac{1}{n}$ as the absolute maximum, $\frac{1}{w}$ as the absolute minimum, and $\frac{1}{n} - \frac{1}{w}$ as the absolute range of the accommodation. If it be wished to pursue more exactly the facts of visual strain and endurance, then we may understand by $\frac{1}{n}$ the relative maximum, by $\frac{1}{w}$ the relative minimum, and by $\frac{1}{n} - \frac{1}{w}$ the relative range of accommodation which corresponds with the convergence for the visual distance s, under the spectacles l, and which we shall hereafter consider more closely. In this way we obtain the formulæ of the relative visual strain, and the relative endurance for a determinate distance of vision.

Besides these relative values, the absolute values of n and w always retain their special importance; since they enable us to ascertain what absolute effort the eye must make for vision at this distance.

ASTHENOPIA.

The last preceding inquiries into visual strain and endurance lead to the knowledge that a stenopic eye cannot have the endurance of one that is europic; since the actual effort of accommodation, even when so regulated by spectacles that i has its most favourable value j, lies too near the limits of the range.

This inconvenience is especially likely to occur in a hypermetropic eye, since such a one, even when europic, cannot employ the lower portion of its range of accommodation, on account of the elevation of the curve of requirement. But the europic eye may always be supplied with the normal endurance by proper spectacles, since these will depress the curve of requirement. It is only the stenopic eye that cannot be relieved of the infirmity, although we can raise its endurance to the maximum attainable, by the spectacles that render the value $i = j$.

The above described deficiency in endurance, which, as an insuperable defect, arises only from stenopia, but which may occur also in other cases, as in hypermetropia, constitutes the essence of the abnormalities described in ophthalmology under the name of asthenopia.

If the defect be in the accommodation, we have the so-called accommodative asthenopia. If the defect be in the convergence, for which i, j and ζ, in the foregoing formulæ, must be replaced by i', j' and ζ', we have the so-called muscular asthenopia.

From the foregoing it appears, therefore, that asthenopia does not, as assumed by Donders, arise solely from hypermetropia; that it is in principle the result of too small a range of accommodation; but that hypermetropia must necessarily tend to produce asthenopia, even when the eye possesses a large range of accommodation.

It appears, further, that the asthenopia of an europic eye, and hence that of most hypermetropes, may be completely relieved by spectacles; but that this can never be done when the evil depends upon a narrow range of accommodation; although in these cases a regulation of the accommodation between its extreme limits may be attempted by spectacles, and will be useful.

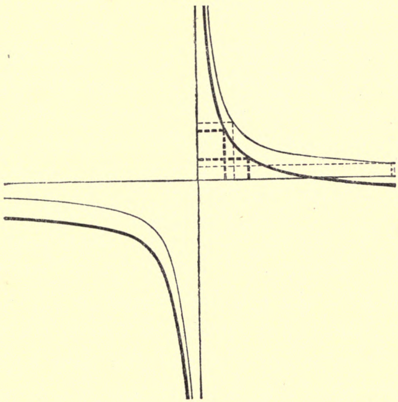
Lastly, it follows that the common far-sighted eye, since it possesses too small a range of accommodation, must constantly experience more or less asthenopia from near vision through convex spectacles; and hence that a far-sighted person, with even the best spectacles, cannot have the endurance for near vision of a short-sighted person or an emmetrope.

If, for example, in a far-sighted emmetropic eye, for which the emmetropic lens $l = \infty$, or $\frac{1}{l} = 0$, the near-point be at the distance $n = 50''$, and the far-point so distant that without much error we may make $w = \infty$ or $\frac{1}{w} = 0$, then the absolute range of accommodation $\frac{1}{50}$ is only about the sixteenth part of the range $\left(\frac{1}{3}\right)$ of a very europic eye. If, therefore, the eye in question be enabled to see near at hand by convex spectacles, and if the visual strain be made as small as possible, still the difference between its actual accommodation and the extreme upper or lower limit of its accommodative power, or the value of α_1 and α_2, is only about 1-16th of the corresponding difference for an europic eye. The endurance of the far-sighted eye supposed will therefore be much less than that of the europic eye.

GRAPHIC DELINEATION OF THE COMBINED DEFECTS OF ACCOMMODATION AND OF CONVERGENCE.

The concurrence of the different defects of accommodation and convergence may be displayed by a single graphic delineation, which contains the two curves of requirement and the two facultative curves. For this purpose, we may make the two coefficients, k and k' each $= 1$, and may therefore represent the hyperbolic curve of refraction by the equation $y = \frac{1}{x} - \frac{1}{a}$, and the hyperbolic curve of direction by the equation $y = \frac{1}{x} - \frac{1}{e}$; while the ordinates, limiting the curves of accommodation and of motility, will be respectively indicated by $\frac{1}{n}$ and $\frac{1}{w}$, $\frac{1}{n'}$ and $\frac{1}{w'}$. Thus, in Fig. 42, the hyperbola, drawn with a dark line, represents the curve of required accommodation, and the dark dotted lines represent the limits

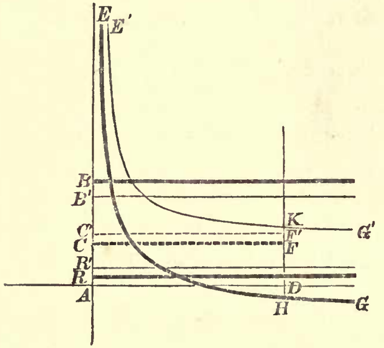
Fig. 42.



of the accommodative faculty. The hyperbola drawn with a faint line represents the curve of required convergence, and the faintly dotted lines represent the limits of the converging faculty, in both cases for any given pair of eyes. If the two hyperbolas coincide, then the eyes are in equal degrees hypometropic and hypogonic, or hypermetropic and hypergonic, and therefore require normal orthoscopic spectacles, which in any other case they do not.

This simultaneous graphic delineation of the requirement and the facultative curves of accommodation and of convergence furnishes a very complete picture of the peculiarities of the eye, and a guide to the determination of the proper spectacles, when we consider that the requirement curves, that is, the hyperbolas, may be elevated or depressed in any required degree, respectively by spherical or by prismatic glasses; while the horizontal limits of the facultative curves, which express the absolute ranges of accommodation and of convergence, are immovable, and that from these fixed limits the values of i and i' in the foregoing formulæ are to be determined.

Fig. 43.



With the help of the graphic delineation, the spectacles, to produce any desired effect, may very easily be geometrically constructed in the following manner:

Suppose that we have to deal with the eye represented in Fig. 43, in which the dark hyperbola E G represents hypometropia with the range of accommodation R B, and the faint hyperbola E′ G′ represents hypergonia, with the range of convergence R′ B′. Both curves are laid down according to the formulæ for $k = 1$ and $k' = 1$, and respectively according to the formulæ $y = \frac{1}{x} - \frac{1}{a}$, and $y = \frac{1}{x} - \frac{1}{e}$, in which a indicates the (in this case positive) accommodation distance, and e the (in this case negative) convergence distance.

It is required that the eye shall see clearly an object at the distance A D, with the accommodation effort A C, and the convergence effort A C′.

If we draw through D the vertical ordinate D K, and through C and C′, the horizontals C F, C′ F′, to F, F′, the points of intersection with the ordinate D K, then the hyperbolic curve of accommodation must pass through the point F, and the hyperbolic curve of convergence through the point F′; so that the former curve E G must be elevated by the height H F, and the latter curve E′ G′, depressed by the depth K F′, which is to be accomplished respectively by a negative or concave lens, and by a positive prism.

If $HF = \frac{1}{f}$ and $KF' = \frac{1}{g}$, then $-f$ is the focal length of the required concave lens, and g the converging length of the required positive prism, of which the proportion of deviation $h = \frac{d}{2g}$, and the angle of deviation $\psi = 57{\cdot}306 \,.\, h$ degrees.

It is evident that this method is very simple; and it may be much facilitated by using, for the equilateral hyperbola, which, in all cases, has the same outline for accommodation and for convergence, a form which shows also the position of the rectangular asymptotes, and which can easily be so arranged that the vertical asymptote remains in the line A B, while the anterior or posterior branch of the curve passes through the experimentally determined terminal point of the accommodation or convergence distance.

The arrangement of the spectacles will then only be dependent upon the measured determination of the points C and C′, which indicate the exertion that is to be demanded from the accommodation and the convergence. For determining these, the limits of the absolute range of accommodation R B, and of the absolute range of convergence R′ B′, are not exclusively authoritative, but serve for general guidance.

THE RECIPROCAL INFLUENCE OF THE ACCOMMODATION AND THE CONVERGENCE.

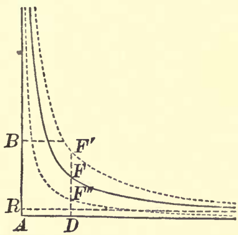
It has been observed in an earlier chapter, that accommodation and convergence are faculties originally independent, but that they influence each other secondarily, or by induction. This influence is displayed, when the normal proportion between accommodation and convergence is either fallen short of or exceeded, in the first instance as *visual strain.* So long as the disproportion does not exceed a certain degree, the pair of eyes will see clearly and singly, but with strain. When this certain degree is exceeded, either one or the other function will no longer be discharged; and there will be either single vision with an imperfect image, or double vision with clear images, so long as the apparatus, still in activity, does not reach the absolute limit of its power.

To what degree an eye can bear this visual strain between accommodation and convergence, for the different degrees of each, that is, between what limits, for a given convergence, the accommodation can be raised above, or lowered below, the normal standard, or between what limits, for a given accommodation, the convergence may be increased or diminished, might be expressed after a series of observations, in the form of a general law, which, in particular cases, would be more or less modified by individual conditions. I believe that such an average of the relative accommodation and convergence limits would lead to the result, when y is a given convergence, and therefore the ordinate of the hyperbolic convergence curve for a definite convergence distance x, that the limits of accommodation with visual strain may be shown by two ordinates of the values $m\,y$ and $\frac{y}{m}$, and that the coefficient m will, for all values of y, be nearly constant. If, for example, we find that for any given convergence the normal accommodation may, with strain, be increased by one-sixth; so that $m = 1 + \frac{1}{6} = \frac{7}{6}$; then the accommodation may, for any convergence, range from $\frac{7}{6}\,y$ to $\frac{6}{7}\,y$.

If this supposition were confirmed, the limits of the relative accommodation would be shown by two equilateral hyperbolas of which one, with the ordinates $m\,y$, would be above the requirement curve of the convergence, while the other, with the ordinates $\frac{y}{m}$, would be below it. Inversely, the limits of relative convergence would be shown by two similar hyperbolas, one of them above, and the other beneath, the requirement curve of the accommodation.

Hence, for an emmetropic and engonic eye, for which the requirement curve of the accommodation coincides with that of the convergence, as shown, in Fig. 44, by the hyperbola drawn with a continuous line, the limits of the relative accommodation (or convergence) are shown by the two dotted hyperbolas. The figure teaches that the eye, in convergence for the distance A D, may exert its power of accommodation from a minimum D F″ to a maximum D F′. The distance F″ F′ denotes the relative range of accommodation, or of convergence respectively, for the distance of accommodation or of convergence A D.

Fig. 44.



The absolute limits, A R and A B, of the accommodation or convergence, or the absolute range of accommodation or convergence, R B, would be very little, or not at all, influenced by the foregoing, since they correspond to the extreme limits of either faculty.

The foregoing purely theoretical doctrine finds its confirmation in the observations of Donders. The graphic delineation given in his work, *On the Anomalies of Refraction and Accommodation*, of the relative range of accommodation of an emmetropic eye, for different conditions of convergence, shows that, so long as the object is not too near either to the near- or the far-point, the positive part of the relative range of accommodation may increase to double what is normally required, and the negative part may sink to one half of this amount. Hence $m = 2$. The normal accommodation, y, may, with strain, be intensified to $2\,y$ or relaxed to $\frac{y}{2}$.

In Fig 44, in the neighbourhood of the near-point, the upper boundary line of the relative accommodation passes, earlier than the lower boundary line, into the horizontal corresponding to the limit of the absolute accommodation. Near the far-point, on the other hand, the lower boundary of the relative accommodation reaches, earlier than the upper boundary, the horizontal corresponding to the lower absolute limit. Hence the boundary lines of the relative accommodation for an emmetropic eye follow the course of the dark line in Fig. 45, which accurately coincides with the observations of Donders.

For a hypometropic eye, the figure (46) is nearly the same as that for an emmetropic eye. For a hypermetropic eye,

Fig. 45.

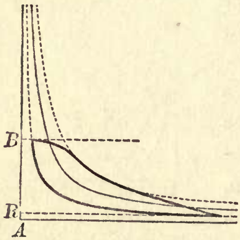


Fig. 46.

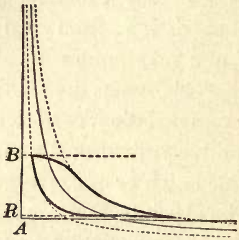
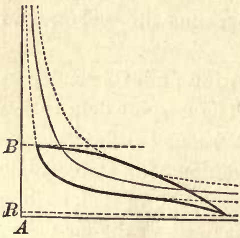


Fig. 47.



however, we obtain the essentially changed Fig. 47. These results of theory again correspond to the observations of Donders.

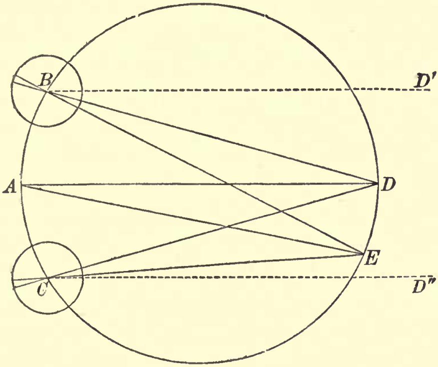
The form of the limiting curves of the relative convergence, in given accommodation, is precisely like that of the limiting curves of relative accommodation in given convergence.

For an abnormal position of the curve of requirement of one ocular faculty, e.g. for a depressed accommodation hyperbola, the form of the limiting curves of the other faculty, that is, of the relative convergence, would be more or less modified. But since the absolute limits of the other faculty are immovable, so it would always be possible, by the associated course of the requirement curves, and the absolute limits of the facultative curves, for both faculties, to decide, according to page 91, upon the localisation to be effected by spectacles.

APPLICATION.

According to the prevailing doctrine, when an eye B (Fig. 48) fixes upon an object D, it looks at this object in the direction of its visual line B D (the slight deviation of the visual line, B D, going to the physiological pole, or yellow spot, from the geometrical axis of the eye or of the cornea, is in this respect unimportant). This doctrine rests upon an error.

Fig. 48.



If the left eye saw the object in the direction B D, and the right eye the same object in the direction C D, that is, if the geometric lines B D and C D were the sources of the sensory impression which we call *direction*, the two eyes would receive two totally different impressions, which must necessarily lead to two different conceptions or opinions of the direction. For since the object would appear to each eye in a different direction, it could not appear to the two together in the same direction. It could not therefore be seen as a single object; but must necessarily appear double.

But, in reality, an object fixed by the two eyes appears single, and therefore in one single direction. This fact is quite sufficient for the inference that the right and left eyes do not see the object in the directions C D and B D, or that the visual lines are not the apparent directions.

The circumstance that the two visual lines, C D and B D, meet in one and the same point of space, is, for the visual organ, wholly without significance. All the demonstrations of physiologists, which refer to the coincidence of the relations of absolute space with the phenomena of visual appearances, are brain cobwebs. That two straight lines intersect in a point, and that hence a point can be determined by the direction of two straight lines, is a pure reflection, or exercise of the understanding. But the eye is not an organ of intelligence; it is an organ of sense. The eye does not think, does not reflect, does not conclude, does not judge; the eye discharges a totally different function; it sees, that is, it reacts upon the ether affections of the rays of light, or it feels the vibrations of these rays. There is no absolute space, and no mathematical extension for the eye, but only for the understanding; the conditions of vision rest only in the affection by the rays of light of the nervous apparatus of the retina: what is actually present external to the eyes, whether there is an actual object or not, whether space exists as something objective, and whether points and lines exist therein, and behave in such or such a manner, whether there is an above and a below, whether the retinal image is erect or inverted, of all this the eye knows nothing; since knowing is not an affair of the sensorium, but of the understanding, whence, on the other hand, seeing and feeling are not the business of the understanding but of the sensibility. Certainly the sensational faculty, by means of the organic unity of the brain, arouses an abstract faculty; and inversely the abstract faculty, as attention, intelligence, inquiry, can influence the sensational faculty, just as the will also can influence, call forth, and sustain this faculty; but this influence is something merely accessory, which does not in the least affect the distinctness, in principle, of the sensational and the abstracting processes. At present there prevails, with regard to the sensational and the intellectual processes, much needless confusion, arising from the use of the same words, such as bulk, distance, light, colour, and so forth, to express both the direct sensory impressions, and the intellectual judgments founded upon them.

The first condition for single vision is, that the apparent directions for the two eyes should have no deviation from each other, that they should therefore be parallel. But this alone would not suffice for single vision; and its fulfilment would allow the left and right eye to see the object separated, as at D′ and D″. There is yet a second condition to be fulfilled, that both apparent directions should have a common point of origin. In fact, in vision with both eyes we see the object D in a direction A D proceeding from the forehead or root of the nose A, and therefore in the frontal axis.

If A B D C be the circle of the horopter, then the centre A of the arc between the two eyes will be the exact point of origin of the frontal axis; and, for an object E, placed obliquely in front of the face, A E shows the oblique frontal axis, and the common apparent direction in which both eyes see the object.

These facts about the apparent direction I have, as I believe, in §§ 5–8 of the *Gesetze des räumlichen Sehens*, conclusively established. It follows therefrom that the apparent direction A E undergoes a deviation from the actual direction B E and C E for either eye, in which the angle B E A = C E A, and therefore, in a normal pair of eyes, is equal to half of the angle of convergence B E C. Generally, for a normal or abnormal pair, the deviation of the apparent direction from the actual corresponds to the angle of inclination of B E or C E, the visual line of the eye concerned, towards the frontal axis A E.

In the place referred to I have also shown that this deviation of the apparent from the actual direction for each single eye is not dependent upon the participation of the other eye in the visual act; but that it occurs even in a greater degree when the other eye is excluded. Hence the deviation appears to be a sensational process in every eye, analogous to accommoda-tion and to fixation, which may certainly be in some degree influenced by these functions, just as they reciprocally influence one another, but which is originally independent. Hence I believe that the visual faculty which is subservient to the knowledge of the direction of objects requires to be distinguished by a special name; and I have called it the *Application.*

The Application, since it affords a definite sensational feeling, must rest upon a definite nervous process, and must therefore have a definite material basis. In the *Gesetze des räumlichen Sehens* I have not declared this basis; but now it is so clear to my mind, that I may do so. I believe that the *mobility of the bacillary layer of the retina*, of which, from the deductions in my *Physiologische Optik* and *Gesetze*, there can no longer be any doubt, forms the basis of the Application.

The bacilli, as terminal points of the optic nerve fibres, are the organs in which the physiological process excited by the luminous ether vibrations occurs; the various concussions and other affections which excite this process being felt in a manner wholly analogous to that in which sensitive organs feel mechanical concussions, pressure, traction, shock, caloric, and other effects, and producing conditions of the organism which we, by means of the consequent affection of the brain through the sensorium, call by such names as distance, direction, luminosity, colour, and so forth. We call the axial (or parallel with the axis of the bacillus) components of the luminous concussion the direction of the object. Since these components lie in the direction of the bacillary axis, the direction of the percipient bacillus is also the direction in which we feel the luminous impression, that is, in which we see objects. If we assume that the bacilli are perpendicular to the retina, and that the retina is spherical, we must see the object in a line of direction perpendicular to the retina, and this line must pass through the central point of the sphere. It has been hitherto believed that the bacilli stand perpendicular to the retina, and that the retina, at least in the vicinity of the pole, is spherical. Both these suppositions are only true approximatively and at times.

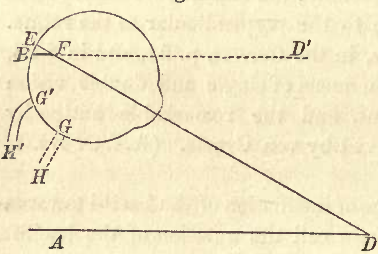
In § 23 of the *Physiologische Optik* I have already remarked on the deviation of the retina from the spherical outline at the blind spot, where the entrance of the optic nerve forms a papillary elevation. Here the direction of the bacilli is not through the centre of the sphere; and this deviation from the spherical outline affords an explanation of the hitherto erroneously interpreted phenomena produced by the incidence of rays of light upon the blind spot. This coincidence of the phenomena with our theory is a strong confirmation of the latter; especially for this, that we feel the direction of the object in the direction of the bacilli.

The mobility of the bacilli has two forms; first, the shifting of the bacilli in the retina, from which may be explained many remarkable phenomena that have led to a variety of contradictory opinions; secondly, the change of inclination of the bacilli with reference to the perpendicular to the retina. From this I have already, in the *Gesetze*, p. 54, and in § 28, explained the peculiar phenomena of single and double vision after strabismus operations, and the remarkable antipathy against single vision observed by von Graefe. (A. f. O. Bd. 1, p. 117 *et seq.*)

It is the latter, the change of inclination of the bacilli towards the retina, which we will now call the nutation of the bacilli, on which the application, and therefore the recognition of apparent direction, depends. It constitutes a faculty, and is associated with an effort, precisely as the accommodation and the convergence or fixation. The striving after single vision, or after union of the visual impressions of the two eyes, is the immediate stimulus to the exercise of this faculty, precisely as the striving after clearness of impression, or after concentration of the cone of rays upon the retina, is the occasion of accommodation, and as the striving after a definite and certain recognition of the impression, by guiding its optical centre of intensity to the pole of the retina, is the occasion of convergence. No ophthalmologist can have failed to notice that, in many cases, such as in looking through prismatic spectacles or into a stereoscope, on account of the abnormal proportion of functions demanded from the eyes, the object at first appears double, but that the two images, after first being approximated to a certain degree, and when an effort is made to fix their corresponding parts, rush together with remarkable force to cover and blend with one another. In this movement of the double images, during complete rest of the eyes, I see a direct proof of the assumption that the apparent direction is variable, that it depends upon material changes in the eyes, and that it requires a definite measure of nervous power to be exerted. The process I judge to be as follows.

When the eye B (Fig. 49) has fixed and accommodated for the object D, the retinal image is sharp, and falls upon the physiological pole; so that the object is seen accurately and clearly. If the polar bacillus retains its original direction E F, towards the centre of the eye, the object D will appear to be in the direction E D. If the same condition obtains in the other eye, the object will appear to the second eye in another direction, and a double object will be seen. The tendency to union of the two visual appearances produces a movement and a nutation of the bacillus E F, so that it assumes a position B F parallel to the frontal axis A D. By means of the nutation by the angle E F B = E D A the object now appears in the direction B F D′ of the bacillus B F, and therefore in a direction parallel to the frontal axis. By means of the movement of the basal extremity E of the bacillus to B, over the arc E B, the point of origin of the line in which the eye sees the object, appears displaced towards the forehead in a direction towards A. The nutation and the movement together have therefore the effect that

Fig. 49.



the object D appears to the one eye in the frontal axis A D. Since the same applies also to the other eye, so, by means of the application, the object, in a binocular visual act, is seen as a single object, notwithstanding the convergence of the two eyes.

If the second eye does not participate in the visual act, then certainly the striving after the union of two visual impressions would not be present. A person born with only one eye would have less demand for application, and would perceive the object more in the direction of the visual line E D of his single eye. But in monocular vision the faculty of application is not wholly in abeyance. It is highly probable, when the eye turns from its state of rest, in which the optic nerve has the position G H, to assume the indicated position of convergence, in which the nerve necessarily acquires the position G′ H′, that this change is attended by a traction upon all the nerve fibres, which must produce in all of them, and therefore in all points of the retina, an affection which influences the position and direction of all the bacilli. This affection, since it is a common angular rotation, must be nearly the same for all bacilli, and must displace each for nearly the same distance E B, and incline each to nearly the same angle B E F with the normal position. In consequence of the equal displacement, the forehead A remains the common point of origin for the lines of direction of all objects seen at once,

Fig. 50.

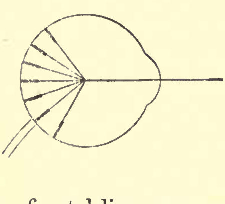
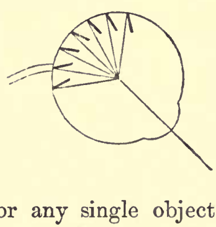


Fig. 51.



and the frontal line A D remains for any single object D the line of common apparent direction for that object. When the eye rightly completes the application, all the bacilli retain, for every convergence, the same direction B F that they assume in the state of rest of the eye; if therefore Fig. 50 shows the direction of the bacilli when at rest, Fig. 51 will represent their direction for any state of convergence.

In so far as the second eye is inactive in this convergence, the tendency to the union of two different visual impressions is wanting, the application will therefore not follow in a complete degree; it will rather exist only as a secondary or induced action of the convergence and the accommodation, and will consequently produce a more or less false projection. The eye exerting insufficient application will see the object too far towards the inner side; that is, the left eye would project the object too much to the right, the right eye too much to the left. An excessive application would produce the contrary effect—a false projection to the outer side.

The foregoing views certainly bear a hypothetical character, but yet they are the necessary fruits of the theory, and since this theory is based upon simple and rational considerations, and since it explains, without being strained, the most complicated and peculiar phenomena, I hold them to be correct, even although I cannot adduce experimental proof of the material changes, for which the means of a clinique and a physiological laboratory are wanting to me. But the above-mentioned part of my theory derives an essential confirmation from the work of Max Schultze, recently come to my knowledge, *Zur Anatomie und Physiologie der Retina.*

According to his new and extremely careful microscopical examination of the retina, the arrangement of the cylindrical bacilli and of the flask-shaped cones, as well as their union with the choroid and with the external granular layer of the retina, are of such a kind that a movement of these, the nerve elements perceptive of luminous impressions, seems to be practicable and as if designed. The wrinkles on the external attachments of the cones signify that these parts have to undergo actual changes of form during life, and in Plate III. of that work, in Fig. 12, there is a representation of the bacillary layer of a diseased eye, in which the bacilli have assumed a curvilinear form.

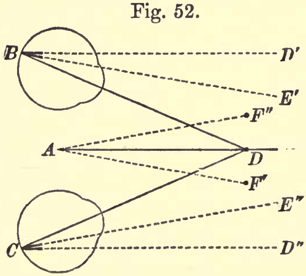
I purpose, in a later portion of this treatise, after the theory of application has been completely laid down, to make some further comments upon Schultze's observations.

DEFECTS OF APPLICATION, OR ANOMALIES OF POSITION AND OF NUTATION.

We proceed now to a more close examination of the mathematical theory of Application and its anomalies. The basis of this theory rests upon the following positions.

In the normal eye, the bacilli in a state of rest stand perpendicular, or in a radial direction to the retina, and are directed towards the central point of its sphere. They possess the power of nutation outwards, as shown in Fig. 51, and they perform this nutation, when the eye is made to converge inwards, so that they form the same angle outwards as the eye itself inwards. The angle of nutation is therefore equal to the angle of convergence of the axis of the eye towards the frontal axis; and hence, in two normal eyes, is equal to half of the angle of convergence of the ocular axes towards each other. In consequence of this, each bacillus, in convergence, retains its original direction with regard to the frontal axis.

Fig. 52.



The angle of nutation, or the unilateral angle of convergence, or rather the tangent of this angle, is the measure of the effort of application. The application or nutation has, like the accommodation and the convergence, its upper and lower limits, which correspond respectively to its near-point and its far-point. Above the upper limit or maximum of the application, which is the near-point, the eye cannot apply; and, when an object is brought within this limit, it is seen double, and with crossed images, which we call negative double images. When, in the convergence of the left eye B upon the object D (Fig. 52), the bacillus B does not perform a complete nutation, so as to attain the position B D′ parallel to the frontal axis A D, but only reaches the direction B E′, then the line A F′, drawn from the forehead parallel to B E′, will be the apparent direction of the object D for the left eye. Hence this eye will see the object D at F′, or too far inwards, or to the right; and the right eye under the same conditions would see the object D in the direction C F″ at F″, too far inwards or to the left.

Beyond the lower limit or minimum of application, which gives the far-point, the stimulus to application is too weak to excite reaction. The bacilli remain, therefore, in their radial position as regards the retina, and this produces the effect that the apparent direction deviates inwards, since it remains parallel to the line B D. When under this weak stimulus no displacement of the bacillus B occurs, the point of origin of the apparent direction does not reach the centre A of the forehead; and hence there follows a small degree of diplopia, which depends upon the difference of the points of origin of the apparent directions for the two eyes, and which gives crossed images very near together.

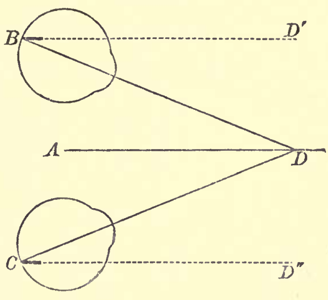
According to these fundamental considerations, the defects of application, like those of accommodation and convergence, fall naturally into two chief classes. The first class depends upon the structure of the eye; that is, upon an abnormal, or non-radial, position of the bacilli upon the retina in a state of rest. We call these the anomalies of position. The second class depends upon the innervation, or upon the applicative faculty, the power to apply accurately, and we call these the anomalies of application or of nutation.

1. We take first the anomalies of position, and we call an eye of normal structure, in which the bacilli have a radial direction in the state of rest, and, therefore, in vision at infinite distance, an *enklitic* eye.

Since the measure of the effort required for accurate application is the tangent of the angle of inclination of the nutation of the bacilli, and since this angle coincides with the angle of convergence of the ocular axis towards the frontal axis, the formulæ for the required application y, in a given visual distance x, have the same form as those already given for the convergence or fixation requirement; since in the new formulæ only the constant coefficient k', which here also may be taken as unity, would have a different value corresponding to the application. The curve of requirement of the application is therefore an equilateral hyperbola $y = \frac{k'}{x}$.

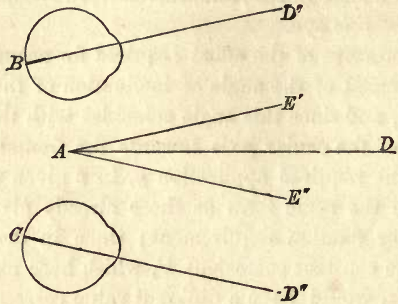
2. The anomalies of position, which are characterized by deviation of the bacilli from the radial position in a state of rest, stand opposed to enklisis as *aklisis*. Two opposite forms require consideration.

Fig. 53.



If the bacilli in the state of rest deviate outwards from the radial position, as in Fig. 51, we call the eye *hypoklitic*. Such an eye, in convergence upon an object D (Fig. 53) lying at a given finite distance, A D $= e$, requires no effort of application, since the apparent direction B D′ is already parallel to the frontal axis A D. For such an eye the requirement curve, $y = k'\left(\frac{1}{x} - \frac{1}{e}\right)$, is a depressed hyperbola, as for the hypometropic and hypogonic eyes. This curve cuts the axis of abscissæ in the positive finite distance *e*, which we call the application distance. The application for infinite

Fig. 54.



distance would require a negative application, which the eye cannot accomplish. The left eye B (Fig. 54) would therefore see the infinitely distant object D in the direction A E′, parallel to B D′, and the right eye C would see the same object in the direction A E″; that is, the two eyes would see double, with homonymous or positive double images, when accurately fixed and accurately accommodated. As a rule, the tendency to single vision would prevail over the straining after sharpness and clearness, and the eyes would forsake the parallel direction and converge so far as to see singly, although less clearly.

The hypoklitic eyes, therefore, for visual distances greater than their application distance, will have convergent strabismus in binocular vision, but will see singly and will project correctly. In monocular vision, on the contrary, they will fix the object, and therefore will not squint, but will project falsely to the outer side.

Strabismus with single vision hence appears to be the immediate consequence of the defect of application, and the action of the anomaly of position.

The application distance *e* of a hypoklitic eye is that at which the eye, without squinting or strain, sees singly and clearly; supposing that it does not suffer from any defect of accommodation or of convergence.

We must observe that a hypoklitic eye, in the fixation of objects which lie nearer than its far-point of application, does not squint; and that it sees these objects clearly, with less effort of application than an enklitic eye.

3. If the bacilli in the state of rest deviate inwards from the radial position, we call the eye *hyperklitic*. Such an eye would require no effort of application for rays which, as in Fig. 55, were convergent towards a point D, placed at a given

Fig. 55.

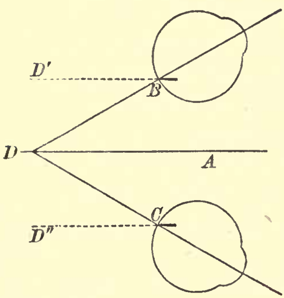
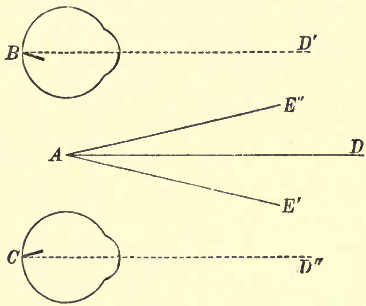


Fig. 56.



negative distance A D behind the head. For such an eye the curve of requirement has the same form as for the hypoklitic eye, when we place $AD = e = -e'$ as a negative application distance.

The application for infinite distance requires from the hyperklitic eye (Fig. 56) a positive application, which the eye may accomplish if the requirement be not too great, which will depend upon the degree of the hyperklisis. If the eyes can accomplish nutation to the directions B D′ and C D,″ the apparent directions will be parallel to the frontal axis. If the eyes cannot accomplish so much, there will result, in binocular vision, if the tendency to single vision preponderates, a divergent strabismus; so that the apparent directions A E′

and A E″ fall into the frontal axis. Hence we obtain divergent strabismus, with single vision and incomplete clearness, as effects of a high degree of hyperklisis in binocular vision at any distance. In monocular vision the hyperklitic eye will fix any object lying before it, and therefore will not squint, but will falsely project the object inwards.

Like the faculties of accommodation and of convergence, the faculty of application is confined within two boundaries, which correspond to the absolute near- and far-points of application, and inclose the absolute range of application.

1. The normal state of the application or nutation faculty may be called *euroklisis.* In the euroklitic eye the range of application is of normal extent. The near-point of application lies very near, and the far-point distant. For visual distances which lie between the two, the eye applies correctly, and therefore does not squint. For objects within the near-point, the application does not attain the required degree, and is therefore too weak ; the eye will therefore project too far inwards, and the pair of eyes will receive crossed double images. If the tendency to single vision prevails, the eye must incline outwards, in divergent strabismus.

The same must also occur in looking at objects beyond the far-point; since here, also, the application is too weak. In general the divergent strabismus for very distant objects can be only inconsiderable.

The anomalies of application form *stenoklisis*, or the anomalies of a small range of application, and they fall into two kinds.

2. The first anomaly of nutation is *bathoklisis*, in which the small range of application lies deep.

3. The second anomaly of nutation is *hypsoklisis*, in which the small range of application lies high.

The combination of these three forms of nutation faculty, with the already described three states of position dependent upon the structure of the eye, furnishes once more nine varieties, of which one is the normal state, and the other eight are defects of application.

These defects of application form exact parallels to the already described defects of accommodation and convergence. They combine the different states of the infinite, the positive finite, and the negative finite application distance e, with the different states of the range of application $\frac{1}{n} - \frac{1}{w}$, and the position of the near- and far-points of application, that is, the values of $\frac{1}{n}$ and $\frac{1}{w}$.

The specializing of these nine cases, after the analogy of the foregoing, is easy, and may be left to the reader. These defects produce manifold phenomena of strabismus with single vision, and with greater or less clearness of the visual impression. By their concurrence with defects of convergence is naturally produced strabismus with double vision. The direction which the eye will assume in any given case, or for any particular visual distance, is easily to be determined, under the supposition that the tendency to single vision will preponderate over the tendency to fixation and to accommodation, and by remembering that this position must be of such a kind that the direction of the bacilli shall be parallel to the frontal axis, without the bacilli being called upon for a negative application, or for a positive application transcending the boundary of the range of application. Hence it may befall that the same eye may have convergent squint for some visual distances and divergent squint for others; and that the degree of squint may vary with the visual distance.

We must further mention that the squinting of an eye with defective application will occur only during binocular vision, since it can only exist in consequence of a preponderance of the effort after single vision over the efforts after clearness and after direct vision, and under the supposition of a sufficient motility. In vision with one eye only the effort after single vision, and therefore the above concurrence, ceases entirely; the eye then fixes rightly (sees directly with its pole), and accommodates rightly, but projects falsely. Since the squinting with single vision in binocular vision is dependent upon several contingencies, which sometimes may not be fulfilled, it is best to determine the defects of application not in binocular, but in monocular vision, when there will be no squinting but a false projection, and by observation of the latter.

It is to be considered that, in binocular vision, there is the tendency always present to fix with one eye and to see indirectly with the other.

Between the squinting and the projection arising from defects of convergence and from defects of application there exists an essential difference. For a defect of convergence the squint, since there is a defect of fixation present, is independent of binocular or of monocular vision, and is always combined with the same false projection. In monocular vision, however, the squint may be avoided by a corresponding oblique position of the head (see *Gesetze des räumlichen Sehens*, § 25). In binocular vision the squint is always associated with double images. In defects of application, on the contrary, the squint occurs only during binocular vision, and is then not combined with false projection, and therefore not with diplopia, but only with indirect vision. In monocular vision the squint ceases, and there is fixation or direct vision associated with false projection.

The manner in which an association of defects of application and of convergence would modify the degree and kind of the strabismus, the projection, and the diplopia is easy to be perceived.

SPECTACLES FOR ANOMALIES OF POSITION AND OF NUTATION.

The optical aids for the diminution of the defects of application are prismatic spectacles. Since these deflect the rays of light, and therefore change the actual into an artificial visual direction, they elevate or depress the hyperbolic requirement curve of the application, and may therefore completely counteract hypoklisis and hyperklisis. By prismatic spectacles, therefore, anomalies of direction are to be relieved. The peculiar facultative or nutation anomalies are not to be corrected by optical means. With these, as with those of motility and accommodation, spectacles can only act by raising or depressing the hyperbolic curve of requirement, so that the power exerted shall be made to fall at a fitting mean between the boundaries of the range.

The determination of application spectacles is based upon the same formulæ that have been given for convergence spectacles. The spectacles $p = -e$, which render the artificial application distance infinitely great, are the enklitic spectacles. By the help of these may be found by experiment, for the above formulæ, the values of $\frac{1}{n}$ and $\frac{1}{w}$, the near- and far-points, and of $\frac{1}{n} - \frac{1}{w}$, the range of the application.

THE CONCURRENCE OF DEFECTS OF ACCOMMODATION, CONVERGENCE, AND APPLICATION.

Each of the nine conditions of the accommodation may be associated with any one of the nine conditions of convergence, and with any one of the nine conditions of application, in any eye. This gives $9 \times 9 \times 9 = 729$ different cases; of which one is the normal condition, and the others form an assemblage of ocular defects, which, according to the foregoing methods, may be divided into synoptical groups, and every one of which may be experimentally determined.

A graphic delineation of the three curves of requirement, and of the three facultative curves, on the same diagram, gives a convenient and complete view of the combined peculiarities of the eye.

The visual strain, which may also thus be recognised, and which follows the fixation of any object, has three sources: first, a misproportion between accommodation and convergence; secondly, a misproportion between accommodation and application; and, lastly, a misproportion between convergence and application.

The graphic delineation shows most easily the way in which the curves of requirement may be raised or depressed, so as to bring the ocular functions to convenient places between their absolute limits.

The relative faculty or range of each single function comes into consideration in relation to each of the other two; but in the determination of spectacles it is the absolute limits of each faculty with which we are in most cases concerned.

Since for the defects of convergence and of accommodation there exists only a single optical means of help, the prismatic spectacles, it is manifest that, when these two defects exist together, in different degree and kind, only one of them can be relieved by spectacles. Under such circumstances, therefore, if we were limited to optical means, we must either entirely neglect one of the defects, or take some mean between their requirements. In either way would be proved the inefficiency of art to regulate the visual processes, were there not a third and independent means within our reach. This third means is furnished by a surgical operation.

The direction of convergence of the eyes can be regulated by the displacement, forwards or backwards, of the ocular muscles. The operation upon the muscles fulfils, therefore, the same purpose as the prismatic spectacles, as regards the defects of convergence; and I am of opinion that it should be considered as the special or proper remedy for such defects. While accommodation and application are really sensory processes, since they require the direct activity of the sensory apparatus of the eye, convergence is essentially a motor function, proper to the ocular muscles. We have, therefore, for the two defects of sensational function, the two optical remedies, the lens and the prism; and for the defect of motility the mechanical remedy of the operation.

It is highly interesting to observe that this mechanical remedy cannot be otherwise applied, in principle, than the two optical remedies. Just as, by the lens and the prism, we can only remove an anomaly depending upon structure, that is, one of refraction or of position, and not one that depends on innervation or function, that is, not one of accommodation or of nutation; so only an anomaly of direction, and not an anomaly of motility, can be removed by operation. By the lengthening or shortening of a muscle, or by its displacement backwards or forwards, we can in general only adjust a defective direction of the eye; that is, we change a hypogonic or hypergonic eye into one that is engonic, and thus restore the hyperbola to its normal place. But, generally, we cannot in this way enlarge a small range of convergence, or increase the motility of the eye; that is, we cannot change a bathogonic or hypsogonic eye into one that is eurogonic; although in exceptional cases effects of such a kind may be produced.

Although the range of convergence cannot generally be altered by operation, yet still the operation, for stenogonic eyes, fulfils the same purpose as the lens or the prism for eyes of small range of accommodation or of application. This depends upon the fact that every elevation or depression of the curve of requirement produces a displacement of the range of convergence; since, with every such elevation or depression, the points of intersection of the requirement and the facultative curves are altered. The operation is therefore a remedy for defects of convergence in the same way that a lens or a prism is a remedy for defects of accommodation or of nutation.

I think it is important, in practical ophthalmic surgery, to lay especial stress upon the different object of the surgical operation according as it is intended to relieve an abnormal convergence distance e, or an abnormal range of convergence $\frac{1}{n'} - \frac{1}{w'}$; since a clear appreciation of this difference must essentially modify the proceeding in the two cases. As a hypometropic or hypermetropic eye, notwithstanding that it has full power of accommodation, requires a lens in order to see at certain distances; so a hypogonic or hypergonic eye, notwithstanding its large ránge of convergence, or complete motility, requires an operation in order to raise or depress its hyperbolic curve of requirement. And as, farther, an emmetropic, but stenopic eye, that is, an eye with a narrow range of accommodation, requires a lens to bring its accommodation effort to a convenient part of this narrow range; so an engonic eye, one therefore with no defect of direction, when it has a narrow range of convergence, requires an operation in order to bring its convergence effort to a convenient part of this narrow range. By consideration of the special object, in either case, the kind of operation must be determined.

The aim of the operator should always be a definite elevation or depression of the curve of requirement. For elevation it is necessary that the eye should be turned more outwards, or rendered more divergent; for depression, that it should be turned inwards, or rendered more convergent. The degree may be learned from the graphic delineation of the requirement and the facultative curves.

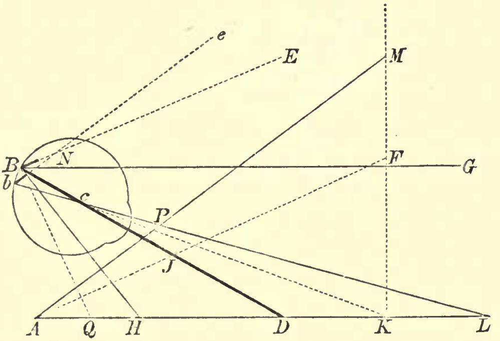
Self-evidently, in concurrence of defects of convergence with defects of accommodation and of application, the conjoined conditions must all be taken into account; which may be done, most readily, by examining the three requirement and three facultative curves in the figure already recommended.

It is manifest that, in complicated cases, it may be necessary to use all three remedies, the operation, the lens, and the prism; and that the drawing in question will show in what way to combine them.

The effort which, in the combined exercise of the three chief faculties, any one of them has to make, we will endeavour, by the aid of Fig. 57, to set forth in the following manner.

Let L be an object lying in the frontal axis A L, and looked at by the left eye B; and let B G be parallel to the frontal axis. If the eye accommodate for the distance A K, the condition corresponds to a case in which the object is placed in the line K M, perpendicular to A K. A K is the distance of accommodation, and B K A is the angle of accommodation. If the eye converge towards the point D on the frontal axis, then A D is the convergence distance, and B D A = G B D is the angle of convergence.

Fig. 57.



The eye therefore does not fix the object, but squints inwards; does not see the object with the polar bacillus, or directly, but with an inner bacillus *b*, which is in a line L *c* *b*, leading from L through the centre *c*. If the eye applies in such a degree that the polar bacillus B makes the angle of nutation E B D with the ocular axis or the frontal axis, or that the percipient bacillus *b* makes the same angle with the luminous ray *b* L; then if A F be drawn parallel to B E, and the angle D B H be made = E B G, we have angle B H A = G B H = E B D = B J A, and this latter angle, which is equal to the angle of nutation E B D, is the angle of application, and the distance A H is the application distance.

The tangents of the angles of accommodation, of convergence, and of application, or the reciprocals of the accommodation, the convergence, and the application distances, measure the respective functional efforts.

If the luminous object be situated in the visual line or ocular axis B D, that is, if the light be received by the polar bacillus B, the object would appear at F, in the direction A F, parallel to B E. But, since the laterally situated bacillus *b* is affected, the apparent direction is in the line A M, which forms with A F the angle M A F = L *c* D. The deviation of the apparent direction A M from the frontal axis A L, or the angle M A L, is therefore the sum of the angles F A L = E B G, and M A F = L *c* D. The directions of the polar bacillus B, and of the affected bacillus *b*, or the directions B E and *b e*, intersect under the angle B N *b* = B *c b* = L *c* D; since these bacilli, as regards the radii of the eye *c* B and *c b*, have equal nutation *e b* L = E B D. Hence the apparent direction A M is parallel to the direction *b e* of the affected bacillus *b*; or the direction *be* of the bacillus *b*, affected by the object L, shows the direction, commencing from the forehead, in which this object appears to the eye. The effort of application is determined either by the nutation of the polar bacillus B to the ocular axis or visual line B D, or by the nutation of the affected bacillus *b* to the ray of light *b* L. The apparent direction A M deviates from the ray of light *b* L by the angle of nutation or application M P L = *e b* L = E B D. Since this angle of application is also equal to the angle B H A, so is the effort of application of the affected bacillus *b* to be measured by the same angle of application B H A, or by the reciprocal of the application distance A H, as the effort of application of the polar bacillus B; which also is manifest from the nature of the case.

The eye under consideration squints inwards, or with convergent squint, as regards the object L, and therefore does not see it directly, but with a lateral bacillus. This defect of convergence or fixation does not determine a false projection or double vision. The latter occurs, but by reason of the deviation of the apparent direction A M from the actual A L. The application is here in fault. Since the direction *b e* of the affected bacillus is not parallel to the frontal axis, but deviates outwards, so the eye falsely projects the object outwards, towards M, or experiences in binocular vision an outward (positive or homonymous) double image.

It is apparent that, notwithstanding false convergence and false application, a correct projection, and therefore single vision, may occur. This is the case whenever the direction of the affected bacillus is parallel to the frontal axis. The squinting eyes then see singly.

Inversely, it is apparent that a correctly converging or fixing eye may project falsely, and may therefore receive a double image; and that this happens whenever the direction of the affected bacillus deviates from that of the frontal axis.

In monocular vision with the eye B, and therefore when the other eye is covered, the object L affords a stimulus only to accommodation and fixation, not to application, since no double images are present, and consequently there is no tendency to unite them. The eye will then accommodate and fix as well as possible, and the application will only follow the accommodation by induction. But these two chief faculties are united by so energetic an inductorial tension that, when there is no objective stimulus to the application, the accommodation, that is, the effort of accommodation, seems to correspond to one.

I believe that, even in anomalies of accommodation and application, the induced tension between the two functions is very strong; so that both, in common vision, even when an objective stimulus would call upon them for different efforts, and therefore also in binocular use of the eyes, in general differ but little from each other: that is to say that the application under given accommodation, or the accommodation under given application, will vary only within narrow limits. These relative limits for accommodation and application are certainly far more narrow than the relative limits for accommodation and convergence, or for application and convergence.

The last proposition is of special importance with regard to the artificial regulation of the visual processes. It does not forbid that the absolute range of accommodation in an eye may be essentially different from its absolute range of application; or that the eye may accommodate in a degree totally different from that in which it applies. It only expresses that the accommodation effort and the application effort stand in a seemingly close relation, so that each is powerfully influenced by the other.

In the binocular use of the eye B the objective stimulus to accommodation and fixation is associated with the objective stimulus to single vision, or to the union of two images. This stimulus is in general very strong. But, since the induced tension between accommodation and application is likewise very energetic, so the tendency to single vision, as a rule, obtains its fulfilment not by a change in the application or nutation of the bacilli, which would involve also a change in the accommodation and impaired clearness, but by a change in the convergence, so that the fixation or direct vision with the polar bacillus is abandoned. The great range of the muscular function of convergence facilitates this deviation, and the result is clear and single, but indirect, vision. For this purpose the eye B becomes more or less convergent, or moves so far inwards or outwards that the affected bacilli of the two eyes have a common direction, and therefore for two similar eyes the direction of the frontal axis A L, while the accommodation and the nutation, and therefore the angle E B D or *e b* L, remain unchanged. This requires, for the proportions given in the figure, a farther rotation inwards by the angle M A L = *e b* G.* Hence the ocular axis B D comes into the direction B Q, which deviates from the direction B H by the angle Q B H = *e* N E = B *c b* = D *c* L, and for which the angle of convergence B Q A is = G B Q = *e* B D = E B D + *e* N E = E B D + D *c* L, the sum of the nutation angle E B D, and of the distance from the pole B *c b* = D *c* L of the first affected bacillus. After this convergence the affection falls naturally upon a more internally situated bacillus.

If we proceed from the condition in which the eye B, in

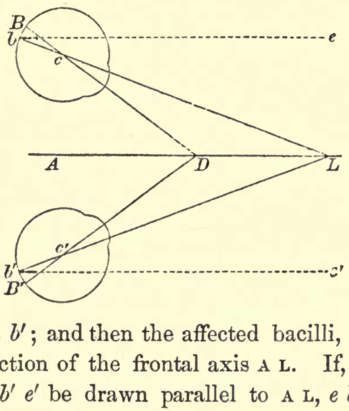
* Strictly speaking, there is no angle *e b* G in the construction. The angle intended is that between the lines *e b* and B G. If another letter, as *r*, had been set at their point of meeting, the angle would have been *e r* G, but the figure would have been inconveniently crowded. The same may be said of the angle *e* B D.—TRANS.

monocular vision, fixes the object L, then direct vision occurs when the bacillus *b* coincides with the polar bacillus, so that the point D turns towards L, and the angle D *c* L = B *c* *b* = *e* N E = 0. The angle of nutation is then equal to E B L, and we have for the angle of application the value B H A = E B L. In order to unite the homonymous double images in binocular vision, the eye turns by the angle E B G; that is, so far inwards that its axis moves from the direction B L to the direction B Q, for which the angle of convergence B Q A = E B L, that is, is equal to the angle of nutation. Hence follows single, but indirect vision, with a bacillus which has from the pole the angular distance E B G = E B L — G B L = E B L — B L A, which is equal to the difference between the angle of nutation or convergence E B L and the angle of fixation B L A.

The preceding examination gives also a simple means of determining the actual state of application of an eye. This can be accomplished in several ways.

Let L be the point in space (Fig. 58) which appears single to the two eyes B and B′, when they converge towards the point D. Draw from L, through the centres of the eyes *c* and *c*′, the

Fig. 58.



rays L *b* and L *b*′; and then the affected bacilli, *b* and *b*′, must have the direction of the frontal axis A L. If, therefore, the lines *b* *e* and *b*′ *e*′ be drawn parallel to A L, *e* *b* L and *e*′ *b*′ L will show the angles of application or nutation, since these angles correspond to the inclination of the affected bacilli, *b* and *b′*, to the radii *b c* and *b′ c′*. Every point in the lines *b* L and *b′* L would appear to be in the frontal axis A L, and the points lying in each of the two lines would therefore coincide.

Fig. 59.

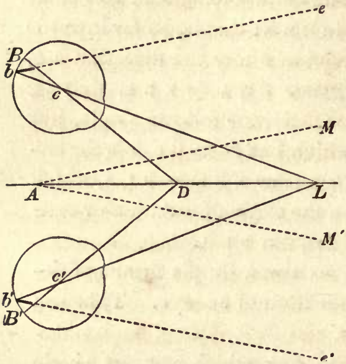
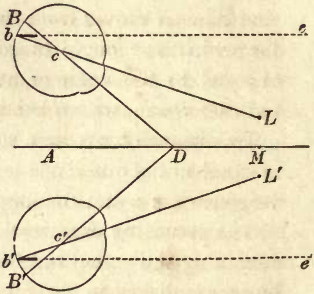


Fig. 60.



Next, let the object L appear double to a pair of eyes convergent towards the point D; and let it be seen (Fig. 59) by the eye B at M, and by the eye B′ at M′. If we draw from L, through the ocular centres *c* and *c′*, the rays L *b* and L *b′* and through the affected points of the retinæ *b* and *b′*, the lines *b e* and *b′ e′* parallel to A M and A M′; then these lines show the directions of the affected bacilli, and *e b* L and *e′ b′* L are the angles of application. All points on the line *b* L would appear to be in the direction A M; and all points on the line *b′* L would appear in the direction A M′.

Again, let the two objects L and L′ (Fig. 60) appear blended, at the point M of the frontal axis, to the pair of eyes convergent upon the point D. If we draw from L and L′ through the ocular centres *c* and *c′*, the rays L *b* and L′ *b′*, and through the affected points of the retinæ *b* and *b′*, the lines *b e* and *b′ e′* parallel to the frontal axis A M, then *e b* L and *e′ b′* L′ will be the angles of application. All points on the lines *b* L and *b′* L′ would appear to be in the frontal axis, and would coincide.

UNILATERAL OCULAR DEFECTS.

With regard to any of the already described defects of accommodation, convergence, and application, it may happen that both eyes are not alike; so that any one of the already-mentioned 729 conditions of the left eye may coexist with any one of these conditions in the right. This gives $729 \times 729 = 531{,}441$ possible states of the binocular organs of vision, of which one is normal, and the others form over half a million of different defects.

The condition of each single eye may be determined from the foregoing for each of the three visual functions. As regards the convergence and the application, the frontal axis bears the same relation to each eye that is borne, in the case of two similar eyes, by the line bisecting the angle of convergence. We may, therefore, by covering one eye, determine for the other the degree of hypometropia or hypermetropia, of hypogonia or hypergonia, of hypoklisis or hyperklisis, or its accommodation distance, its convergence distance, and its application distance; as also the degree of bathometropia or hypsometropia, of bathogonia or hypsogonia, of bathoklisis or hypsoklisis, or the accommodation range, the convergence range, and the application range, in magnitude and in position; and may thus obtain a complete and clear picture of the condition of the whole organ, which, by the often recommended graphic delineation of the requirement hyperbolas and the facultative curves, is not only clear of comprehension, but contains all the necessary data for a rational conclusion upon the kind and degree of the necessary aids.

It is then only requisite to consider whether it is desirable to treat each of such eyes as an independent organ, without reference to the other; or whether the interdependence of the two eyes has any influence upon the selection of remedies.

Donders objects, in general, to the use of two different glasses for two different eyes (p. 565 of his work), and he generally employs like glasses for such eyes, and considers any departure from this rule to be exceptional. As a reason for his practice he asserts that two unlike eyes are accustomed to their dissimilarity, and are therefore brought, by dissimilar glasses, into a new and inconvenient state. If this rule were well founded, it would not do to supply any myope of equal eyes with ordinary concave spherical spectacles; but, on principle, with orthoscopic spectacles only. For we must in that case consider that two equally short-sighted eyes have become accustomed to their abnormal functional proportion between accommodation and convergence, and that they require glasses which respect this abnormal proportion. Such glasses are not the ordinary, but the orthoscopic only.

He, therefore, who attaches so much importance to habitual abnormal proportion that he prescribes, on principle, similar glasses for dissimilar eyes, must also, on principle, prescribe orthoscopic glasses for similar eyes; and, *vice versâ*, he who prescribes, on principle, ordinary spectacles for similar eyes, must also prescribe dissimilar glasses for unlike eyes. The general recommendation of ordinary glasses for like eyes, and of similar glasses for unlike eyes, appears to me to be a contradiction in principle.

Generally, however, I think that the question is not to be decided by general principles, but solely by actual facts. Where any given proportion of function has become completely habitual, it is self-evident that it should be respected; although whether the habit has attained the presumed degree is always *a priori* doubtful, and cannot be determined by a bare assumption.

The actually existing condition of the eyes, and the functional proportion suitable to them, are the essential questions in every rational endeavour for the determination of the suitable remedies. When these data are determined by experiment, the solution of the problem follows without any further hypothesis.

If the function of one eye were wholly uninfluenced by the other, then the proper aid for each single eye could be determined by the foregoing. But the functions of the two eyes, although originally independent, influence each other as much as any two chief functions in a single eye—as much as accommodation and convergence. A binocular vision without strain corresponds, therefore, to a certain definite functional proportion between the two eyes, which is individual, and can be determined by experiment. A deviation from this proportion produces strain; and, when the deviation oversteps a certain degree, binocular vision becomes impossible, and the active visual function becomes a burden to one or to the other eye, so that the general incompleteness of monocular vision is produced.

According to the observations of Donders, the reciprocal influence exerted between the two eyes appears to be very considerable. Donders says 'we are unable to equalize even a small difference in the refraction of the two eyes by accommodation, so inseparably is the accommodation effort of one eye bound up with that of the other.' This result, however, was chiefly obtained by experiments upon normal eyes, different glasses being applied to each; and, according to my judgment, it does not therefore follow that there is any necessity for applying like glasses to unlike eyes. While the unlike glasses produce, for like or normal eyes, a disproportion which the visual organs resist, and by which the foregoing observations are explained; like glasses would maintain, in unlike eyes, the already existing disproportion, which unlike glasses would set aside.

Before proceeding further, I should observe that the explanation given by Donders of the resistance of the two eyes to unequal efforts does not seem to me to be correct. Donders explains that unequal glasses are less comfortable than similar ones, chiefly on the ground that, when the distances for clear vision are made equal, the retinal images of the two eyes are not equal, but of different sizes. But, with any lens, the visual angle of the apparent object is so nearly equal to that of the actual, and the distance of the crossing point of the principal rays from the retina varies so little, that the size of the retinal image does not change in any marked degree; and the size cannot therefore be the source of difficulty. The fact mentioned by Donders, that through one lens we may see an object large and small, and also that without spectacles the same object sometimes appears larger or smaller to the left eye than to the right, when the two eyes are unlike, does not establish his conclusion about the presence of unequal retinal images. As I have shown in my *Physiologische Optik*, and in the *Gesetze*, the apparent size of an object does not depend entirely upon the size of the retinal image, but is the result of several processes, especially of those which impart a knowledge of distance.

We shall obtain a sure guide to the determination of the fit spectacles for dissimilar eyes, when the part taken by each eye in the visual act, at any visual distance, is discovered by a series of trials, and graphically delineated. The institution of such complicated and extended experiments may be difficult in practice; and would, I believe, generally be unnecessary. I believe that the establishment of the absolute lengths and ranges of the three chief functions, in monocular vision, will be sufficient for the rational selection of spectacles, when the following points are recognized:

a. Is the exertion or relaxation of a chief function of one eye united, in binocular vision, with exertion or relaxation of the same function in the other?

b. Does the absolute maximum or minimum of a chief function of one eye correspond with the absolute maximum or minimum of the same function in the other?

c. Does a definite degree of a chief function of one eye, measured by the position, or ordinate, of a given point in the absolute range of this function, correspond to the same degree of the same function in the other eye?

The first two points need but little comment; the third, however, since it concerns the binocular visual strain, and affects the comfort of the two eyes in their common action, may require more careful consideration; although this strain, even in cases of marked inequality of the two eyes, may easily be of small amount, and hence affords only an approximative standard for the regulation of vision.

In consideration of the difficulty, ascertained by ophthalmologists, of combining visual impressions of unequal size, it seems to me that an essential condition for the proper spectacles is that they shall make the object appear of equal size to each eye. According to the above remarks upon the apparent size of objects, the fulfilment of this condition requires not only equal retinal images, but something more also, to the consideration of which we will proceed.

APPARENT SIZE AND DISTANCE.

In the *Physiologische Optik*, and in the *Gesetze des räumlichen Sehens*, I have shown that apparent distance rests principally upon the perception of the components of the ether undulations in the conical pencil of rays that penetrates the bacillary layer; that this apparent distance is, therefore, primitively independent of the accommodation and of all other ocular functions, especially of binocular vision; that it is recognized in false accommodation and in monocular vision; and that we can, at the same time, and in the same state of accommodation, recognize very different distances, and therefore see stereoscopically; and that also each single eye sees stereoscopically.

But, as all the ocular functions secondarily influence one another, so the other functions have a certain influence over the recognition of apparent distance. The influence arises thus: that any effort of accommodation, or of convergence, or of application, modifies, in a certain degree, the sensitiveness of the perceptive nerve-substance of the bacilli, and produces a modified perception.

I have also shown that it is one and the same fundamental process which leads to a judgment of apparent distance, and to a judgment of the size or width of the arc of the radial elementary unit of the nervous substance in the retina; and that therefore the induced or secondary influences, which modify the perception of distance, change, in the same degree, the perception of the size of the latter elementary arc. Hence the apparent size of an object is proportional to the product of the size of the retinal image (the sum of all the affected retinal units) and of the apparent distance.

As regards the secondary influence of the ocular functions upon the apparent size and distance, this influence is exerted first by the accommodation, and secondly by the application.

In the *Gesetze* I have ascribed the latter influence chiefly to the convergence, since I have there supposed the application to be normal, or coincident with the convergence. In a more rigid separation of the two functions, which in abnormal conditions is generally necessary, I think the influence must be ascribed entirely to the application, which is a nerve-function, and not to the convergence, which is a muscular function. In Fig. 57, for the position A D of the ocular axis or visual line as regards the frontal axis A K, let

a be the distance A L of the actual object L, or of the optical image of this object given by a lens, since such an image always plays the part of an actual object;

α the visual angle of this object;

$r = a\,\alpha$, the actual diameter of the object;

b the distance A K, for which the eye is accommodated, or the accommodation distance;

β the tangent of the angle of accommodation B K A;

c the distance A H, for which the eye is applied, or the application distance;

d the distance between the centres of the eyes;

$\gamma = \frac{d}{2c}$, the tangent of the angle of application or nutation EBD = BJA = BHA = GBH; then according to the *Gesetze* (p. 117), we have the apparent distance of the object

$$a' = \frac{\beta}{\gamma} . a = \frac{c}{b} . a,$$

and the apparent size or the apparent diameter of the object

$$r' = \frac{\beta}{\gamma} . r = \frac{c}{b} . r.$$

(If instead of the angle with the frontal axis, we have the double angle, that is the angle of the convergence and accommodation directions of two eyes supposed to be alike, this does not affect the result; and, moreover, under the supposition of the angles being small, we have put the angles themselves for their tangents.)

From these formulæ it appears that too strong an accommodation increases the apparent size and distance, that too weak an accommodation diminishes them; that too strong application (eventually convergence) diminishes the apparent size and distance, too weak application increases them.

Moreover, the apparent size and distance are not determined by one of these two efforts alone, but by the ratio of one to the other, or by the quotient $\frac{\beta}{\gamma} = \frac{c}{b}$. If, therefore, the eyes are rightly accommodated for an object (which may be known by the clearness of the image), while at the same time they are falsely convergent, and consequently applying falsely (which may be known by double vision), so in too strong a convergence the object appears too small, and in too weak convergence too large. False accommodation without false application, and therefore without double vision, is much more difficult with naked eyes. Too strong an accommodation usually entails a yet stronger application, so that the object does not appear enlarged, but diminished.

When the eye is rightly accommodated, and therefore $b = a$, but falsely applied, the apparent distance a' is $= c$, that is, equal to the application distance.

When the accommodation and application efforts are equal, so that $b = c$, whether they are both too strong or both too weak, the object always appears of right size and at the right distance.

It is important to mention that the quantities b and c or β and γ refer to the efforts of accommodation or application actually exerted; which have their insurmountable limits of value in the boundaries of the range of the accommodation or the application.

I believe that these formulæ, with their consequences, which I have found to be in unison with the actual appearances (§ § 19 and 20 of the *Gesetze*), deserve the attention of physiologists; since they afford a clue to many remarkable phenomena.

An important result of the second formula is that the apparent size of an object is wholly independent of a its distance; and that hence an object of a given size, r, always appears equally large, at whatever distance it may be placed, so long as the accommodation and the application (or convergence) are normal, or equally strong. Only when the accommodation or the application, or both, become abnormal, as is the case on this side of the near point or beyond the far point, does the apparent size vary.

If, in the formula for apparent size, we substitute for the diameter of the object r, its value $\alpha\ a$, then the apparent size will be

$$r' = \frac{\beta}{\gamma}\ \alpha\ a = \frac{c}{b}\ \alpha\ a = \alpha\ a'.$$

If therefore an object retains a constant visual angle α, its apparent size r' will vary with its apparent distance a'.

If therefore we look at an object through orthoscopic concave spectacles, which change the values of b and c in equal proportions, but make α smaller, and therefore diminish the apparent distance a', while they leave the visual angle a unchanged, the object will appear nearer and smaller. Orthoscopic convex spectacles cause the object to appear more distant and larger.

If we look at the object through common centric concave spectacles, which diminish b and a, and also b and r, in equal proportions, but leave c and a unchanged, then the object appears in unaltered size and distance. The same applies to vision through common convex spectacles. But here, however, the express condition must be asserted, that the supposed abnormal proportion between accommodation and application actually exists; that therefore b and c have actually the assumed values. Generally, for reasons immediately to be stated, the supposed conditions are not accurately fulfilled; and we can therefore only say that the common spectacles respectively magnify or diminish less than the orthoscopic. We may further assert that such spectacles magnify or diminish less in binocular vision than in monocular; as may easily be shown by trial.

The grounds on which the above described effects, as a rule, are only observed in a slight degree, rest on the observation already made, p. 117, that accommodation and application have between them a very energetic induced concord, which keeps them nearly coincident, both in monocular and binocular vision. On account of this concord, which makes itself felt as strained vision as soon as the proportion between the two functions becomes abnormal, the quotient $\frac{c}{b}$ retains a nearly constant value. Since, under these conditions, a and r may vary, the apparent size and distance change almost as much as in using orthoscopic spectacles, but somewhat less, on account of the smaller change of $\frac{c}{b}$. The special process is as follows. On first looking through the concave spectacles the accommodation distance b is smaller, while the application distance c remains nearly the same.

Hence the fraction $\frac{c}{b}$ is too large. But, since the distance a and the diameter r are diminished in proportion to the power of the spectacles, the values of $\frac{c}{b}a$ and $\frac{c}{b}r$ will be diminished notwithstanding the small enlargement of $\frac{c}{b}$; that is, the common concave spectacles will diminish, but not in the same degree as the orthoscopic. Inversely, the common convex spectacles magnify somewhat less than the orthoscopic. These results coincide accurately with those attained by observation.

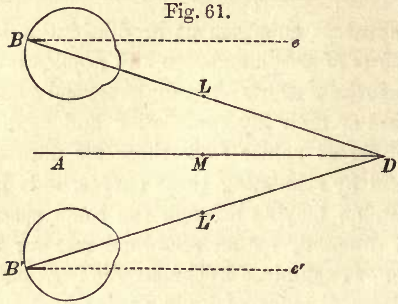
If an object be looked at through common positive prismatic spectacles (bases inwards) there will immediately be positive or homonymous double images. To unite these images the eyes turn inwards, or converge more strongly, so that they give up fixation with their poles, and see indirectly. If, at the same time, the accommodation b remains unchanged, and the application is increased, so that c is smaller; then, since a and r remain unchanged, the object must appear nearer and smaller. But, as the induced link between accommodation and application keeps those functions nearly in accordance, the proportion $\frac{c}{b}$ really changes but little, and thus no considerable difference in apparent size and distance is observed. The special process is as follows. Since, on first looking through the glasses, the accommodation distance b remains the same, but the application distance c is larger, the fraction $\frac{c}{b}$ will be somewhat too large; that is to say, the homonymous images will be not quite so far apart as in a normal relation between accommodation and application. The subsequent convergence does not change the value of $\frac{c}{b}$, and therefore leaves the object by so much too large and too remote as $\frac{c}{b}$ is too great. Inversely, negative prismatic glasses (with bases external), produce a slight diminution and approximation of the object. This result also is in accordance with observation.

Since the effect of apparent enlargement or diminution is dependent upon the efforts of accommodation and application actually exerted, and since these efforts have, for abnormal eyes, according to the standard of their accommodation and application curves, totally different values from those for normal eyes, and can be changed by regulating these curves, so it is manifest that this effect, on looking through the same spectacles, will be totally different, for an abnormal and for a normal pair of eyes. For example, a hypometropic (myopic) eye requires, for the finite distance of its far point, scarcely any effort of accommodation, but a considerable effort of application. With its concave lens the effort of accommodation is increased, or b rendered smaller, and consequently $\frac{c}{b}$ larger, since the application c remains unchanged. Since without the lens a strain exists between accommodation and application, which is relieved by the lens, so in fact the fraction $\frac{c}{b}$ is considerably larger, nearly in the degree by which b is diminished. In the same degree is increased the distance a of the object, and hence $\frac{c}{b}a$ retains its former value. For the myopic eye, therefore, concave spectacles do not diminish objects as they do for the normal eye, a result which is familiarly known in practice.

An important confirmation of this theory of apparent size and distance is furnished by the appearances of stereoscopic pictures and of tapestry patterns.

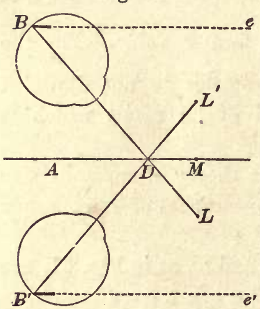
Let us assume that the two eyes B and B′, as in Fig. 61, are looking at the objects L and L′. The eye B fixes the object L and the eye B′ the object L′, so that both see with the polar bacillus, B or B′, and assume a position of convergence towards the point D lying behind L and L′. If now the two objects appear in one place M, and therefore in the frontal axis A M, the parallels to A M, B e and B′ e', will be the directions of the affected polar bacilli, and therefore e B D will be the angle of application γ, and A D, the application distance c. On account of the rays coming from L, the bacillus B seeks to accommodate for the distance B L = A M = a of the object L, therefore to make $b = a$. But the strong induced link between accommodation and application produces a deviation of the accommodation towards the value of the application, c; that is, b will be greater than a, but smaller than c. The expression $\frac{c}{b}\,a$, for the apparent distance of the object M, in which L and L′ coincide, will therefore, on account of $c > b$, produce an apparent removal and enlargement of the object L.

Fig. 61.



If, as in Fig. 62, the left eye B fixes the right-hand object L, and the right eye B′ the left hand object L′; then, since both objects are seen singly, the construction remains the same. The application distance c is then A D, and therefore much smaller than before. The accommodation distance b deviates from B L or A M = a, towards c, and is therefore less than a, but remains greater than c. The expression $\frac{c}{b}\,a$ for the apparent distance of the object M, in which L and L′ coincide, shows therefore, on account of $c < b$, that this object will appear nearer and smaller than the object L.

Fig. 62.



Since the vision as in Fig. 61 produces an enlargement, and the vision as in Fig. 62 a diminution, the diminution in transition from the former to the latter must be still more remarkable, and this is experimentally found to be the case.

From the foregoing it follows that the apparent size and distance may be regulated, for each eye singly, by the formulæ

$$a' = \frac{c}{b}\,a,\ r' = \frac{c}{b}\,r = \frac{c}{b}\,\alpha a;$$

and that we may therefore always obtain an equality of the visual impression in the two eyes, which, for binocular vision, is certainly of importance.

PERCEPTION OF SIZE AND DISTANCE.

In the foregoing we have established the formula $a' = \frac{c}{b}a$ for the apparent distance a'. In this $\frac{1}{b}$ is the actually exerted accommodation effort, and $\frac{1}{c}$ the actually exerted application effort. From the circumstance that the formula does not explicitly contain the convergence effort, it does not follow that the apparent distance is wholly uninfluenced by the convergence. Since all the ocular functions act more or less upon each other by induction, so has the convergence also more or less influence upon the rest. We say, however, that the convergence influences the process on which the recognition of distance rests, not directly, but only through its effect upon the accommodation or application. An abnormal convergence would modify the values of b and c, and is therefore contained, by implication, in the formula $\frac{c}{b}a$.

An increased effort of any one visual function tends always to produce a more or less increased effort of all the rest, or to arouse a tendency to such effort. This tendency, even when not followed by any external consequences, is to be considered as an increase of the particular effort, with which we are here concerned.

Since an abnormal convergence influences not only the accommodation b, but also, and in a nearly equal degree, the application c, it only tells, in the above formula, upon the proportion $\frac{c}{b}$ of these two functions; and thus the general influence of the convergence upon the apparent distance is, in most cases, very inconsiderable.

It has been observed that the induction between accommodation and application is far more energetic than that between either of these faculties and convergence. In whatever degree, therefore, one of the two magnitudes, b or c, is influenced, the other will nearly follow. Accommodation and application can more easily emancipate themselves from convergence than from each other.

While b and c, in the expression $\frac{c}{b}a$ for the apparent distance, indicate two functions or efforts, a, as the actual distance, indicates a mathematical quantity. For the eye, however, such a quantity, as a pure matter of the understanding, has no significance. The eye recognizes, or feels, only sensory nerve processes; and what the understanding, by its special function, abstracts from such impressions, is no affair of the visual function, but only of the power of abstraction. Hence, in any physiological formula, the quantity a must have its physiological significance, and this must rest upon a material process.

In the *Physiologische Optik*, and in the *Gesetze*, I have shown that a is the proportion of the rectangular components of the luminous concussions given by the conical pencil of rays to the nervous substance of the bacilli. Distance is therefore immediately felt by the bacilli, from the concussions that they sustain. If the sensibility of the bacilli were altered, the sensation itself would be altered also. We may assume that this sensibility may so far be influenced by the accommodation and the application, that the elastic state of the bacilli may, through those functions, undergo a change. Since, therefore, accommodation and application may modify the state of elasticity of the bacilli, or their sensitiveness to luminous oscillations, they may influence the sensation itself, or the apparent distance.

It is obvious that the sensitiveness of the bacilli, like the functional reaction of every other part of the visual apparatus, must have its upper and its lower limit. A proportion of components, which exceeded a certain amount, would still be only attended by the maximum of sensation; and a proportion which fell below a certain amount, would no longer produce any reaction of the bacilli. No reaction implies no function, and no sensation. It would be erroneous, however, to assume that such a state answers to a null or infinite value. The null, the nothing, and the infinite, are pure ideas, not perceptions of sense. The perceptions of sense, as such, can exist only in a perceptible degree, as a definite maximum or minimum. In so far, therefore, as the bacilli experience a sensation of distance, it can be only of finite distance, lying between a certain maximum and minimum. In other words, all objects nearer to the eye than a certain point, the near point of perception, must appear to be at the distance of this near point; and all objects beyond a certain point, the far point of perception, must appear to be at the distance of this far point, not at infinite distance.

If we, for brevity, call the quantity a the perception of distance, then this will have a minimum a_1 for the near point, and a maximum a_2 for the far point. By means of the influence of the accommodation and the application, the apparent perception will be $a' = \frac{c}{b}a$. The accommodation and the application cannot alter the absolute limits of perception, so that a_1 and a_2 indicate also the minimum and the maximum of a'.

In normal accommodation and application, for which we have $\frac{c}{b} = 1$, the apparent distance, a', corresponds always to the perception a. Within the imagined limits, this perception is the true expression of the actual proportion of the components of the luminous vibrations, and the object therefore appears at the proper distance; while within the near point the object appears at the distance a_1, farther than it is, and beyond the far point, at the distance a_2, nearer than it is.

On approaching the boundaries a_1 and a_2, the apparent distance deviates a little gradually from the actual, until the former becomes stationary at a_1 or at a_2. Hence we have for the perception a facultative curve, which, like those already considered, is parallel to the axes of abscissæ at its extremities, and accurately corresponds to the curve of requirement at its central part, especially near the terminal points of the middle visual distance, deviating gradually from the requirement curve elsewhere.

The expression for the apparent size of the object, $r' = \frac{c}{b} r$ is not purely physiological, since r represents a mathematical quantity, namely, the diameter of the object. It first assumes a physiological form as $r' = \alpha \frac{c}{b} a$, in which b and c represent the accommodation and application, a the perception, and α the quantity of the sensitive nerve points lying in the diameter of the retinal image, corresponding to the visual angle of the object. Hence we may call α the perception of the visual angle.

Since $\frac{c}{b} a$ is equal to the apparent distance, we have also $r' = \alpha a'$; that is, in abnormal accommodation and application, when the apparent distance deviates from the real distance, the apparent size always corresponds to the product of the visual angle and the apparent distance; or, in other words, by the influence of accommodation and application, size and distance are increased or diminished in equal proportion.

Between the boundaries of perception, but not too near to these boundaries, an object appears, in normal accommodation and application, of its proper size, $r' = \alpha a = r$. An object of given size, r, appears, therefore, within any moderate distance equally large. Hence the remarkable fact is explained that an object, lying at moderate visual distance, does not appear to change its size, even when moved considerably nearer or farther.

On this side of the near point of perception an object of constant diameter, r, appears of the size $r' = \alpha a_1$; and therefore too large. Beyond the far point it appears of the size $r = \alpha a_2$; and therefore too small. Beyond these limits, since a_1 and a_2 remain constant, the apparent size varies as the visual angle α. This result also is supported by experience.

An object which everywhere retains the same visual angle, α, and which is therefore increased by distance, and diminished by approximation, since it always fills a definite visual angle, appears of proper size at moderate distances, because $r' = \alpha a = r$, and therefore increases proportionately as the distance. But on this side of the near, and on that side of the far point of perception, the same object appears always of the constant size αa_1 or αa_2, because $r' = \alpha a$, or $r' = \alpha a_2$, while α, a_1, and a_2, all remain the same.

Since α, as a physiological quantity, shows the magnitude of the retinal image, so it is influenced by the distance from the retina of the crossing point of the chief rays. If we call this distance $= \rho$, and the visual angle of the object $= \phi$, we have $\alpha = \rho \phi$. So long as the object (for the naked eye the actual object, and for an eye with a lens the optical image of the object) is not too near, that is, not nearer than four inches, the variation of the visual angle ϕ, measured from the crossing point of the chief rays, that can be produced by displacing this crossing point, is so trivial that it may be left out of account; and, therefore, ϕ may be assumed to be constant. With the distance of the object, and, therefore, also with the use of a concave or convex glass, only the distance ρ of the crossing point varies; and because $\alpha = \rho\ \phi$, this variation produces proportionate enlargement or diminution of the retinal image. The displacement of the crossing point, in consequence of a change in the distance of the object, or of the luminous point, is not considerable, even in the case when the eye maintains its refraction. But the change of accommodation that attends upon the change of distance, so nearly equalizes this small displacement of the crossing point that it may be disregarded, and ρ also may be assumed to be nearly constant. Hence it follows that the retinal image does not change its size α, by the use of spectacles, in any noteworthy degree; and that we may always substitute the visual angle of the object as a measure for this size.*

ASYMMETRICAL OCULAR DEFECTS.

In all the abnormal states hitherto mentioned, it has been supposed that the individual eye is symmetrical with regard to its axis. We meet, however, with defects that are asymmetrical. They may depend either upon structure or upon innervation or function, and they may affect any of the three chief functions, the accommodation, the application, or the convergence. This consideration furnishes the scheme for the systematic division of asymmetrical defects.

We will consider first the asymmetrical defects of the accommodation. These consist of an incomplete concentration of the pencil of rays proceeding from a luminous point; that is, of an aberration. Aberration may arise from two different causes—first, from a want of symmetry of the eye in the different meridional planes passing through its axis, which is astigmatism; and secondly, from a want of symmetry of the eye in its conical radial surfaces, that is, in the different conical sur-

* The remaining paragraph of this chapter in the original is devoted to the correction of an error in the *Physiologische Optik*; and has no interest except in relation to a passage quoted from that work.—Trans.

faces of which the centre of the eye is the apex; and this formation I have called '*Aspherismus*' in the *Gesetze des räumlichen Sehens.*

Astigmatism may have its basis in the structure of the eye; and it is this form which has been especially investigated by Donders. It may also be functional; that is, the eye may in accommodation become asymmetrical in the different meridians, or astigmatic.

Astigmatism may arise from defects of different organs, from an asymmetrical curvature of the cornea or of the lens, from an asymmetrical density of the lens or of the vitreous body, and so forth. According to the researches of Donders, the asymmetric curvature of the cornea is the most frequent cause; and in this there are almost always two meridians, of maximal and minimal curvature, at right angles to each other.

The optical means of relief in astigmatism is the cylindrical convex or concave lens, which equalizes the refraction or accommodation in all the meridians.

With such a cylindrical lens a spherical lens may be combined, in so far as there is any defect of accommodation beyond the astigmatism, and with these two a prism also may be united, if there be any defect of convergence or of application. The prism must be in the middle; so that it may have on one side a plano-cylindrical lens, on the other side a plano-spherical.

Aspherism also may depend either upon structure or function. The curvature of the cornea is the chief cause of this defect. If the marginal part of the cornea or of the lens is too strongly curved (or the marginal part of the lens too dense), there is an *over-curved* eye; but if these marginal parts are too little curved (or wanting in density), there is an *under-curved* eye.

For the relief of aspherism there is the aspheric lens, described in the *Gesetze*, p. 144, which is convex-concave, with a hyperbolic form for the over-curved eye, and an ellipsoid form for the under-curved eye. The characteristic of this lens is that its external and internal curvatures are equal at the apex, while the section of the internal curve differs from that of the external.

The asymmetrical defects of convergence may be either anomalies of position or of motility. They include the cases of oblique and distorted ocular direction, and the abnormal muscular efforts which move the eyes in other than the natural direction of convergence.

The optical remedy for these defects is a prism placed obliquely. But in general we know, from the foregoing, that the rational means of relief is not optical, but a surgical operation on the ocular muscle.

For this operation a knowledge of the laws of normal ocular movement, and a correct judgment about the normal and abnormal action of the various muscles is highly important. In § 18 of the *Gesetze* I have shown that the hitherto received laws of ocular movement are erroneous, and I have there, as well as previously in the *Physiologische Optik*, given the actual laws of movement. Moreover, in the *Physiologische Optik*, §§ 14 and 15, I have calculated the effect of every normal or abnormal muscular action, and have described, under the name of 'ocularium,' a very convenient instrument for teaching a knowledge of these actions.

The asymmetrical defects of application, again, may be either structural or functional, or may be anomalies either of position or of nutation.

From these defects arise the most various cases of false projection to the left, to the right, upwards, downwards; and also the most various cases of abnormal strabismus. Moreover, an abnormal direction or nutation of the bacilli towards different points of the retina may produce apparent distortion of the object, apparent obliquity of position, and apparent changes of size, as well as other deformities. (See *Phys. Opt.* § 75.)

If the defect consist of a uniform deviation towards a meridian, it may be corrected by a straight or oblique prism. Irregular defects would require glasses of irregular form.

Defects depending upon a displacement of the retina (so-called incongruence) upon a displacement of the lens, upon an obliquity of the lens to the ocular axis, upon a curvature of the retina deviating from the spherical form, which give a false direction to the chief rays, or change their apparent direction, may, on account of this change of direction, be numbered with the defects of application; and may be corrected as far as practicable, partly by glasses, partly by operations.

It does not form part of the plan of this work to follow farther the asymmetric and irregular ocular defects. If we consider that any of the former symmetrical defects may in any eye be combined with any of the asymmetrical defects, and that in each of the two eyes any such combination may exist, we see that the number of the possible conditions of the binocular visual apparatus amounts to an incalculable legion. But the systematic classification arranges this legion in groups, and allows a rational treatment of all to appear possible. In reality, there is no absolutely normal pair of eyes; and we may say that each one possesses all the defects that can be combined, only in a greater or less degree.

The mathematical formula suffices for the general expression of any condition, the normal as well as the abnormal. The formula knows no specific, but only gradual distinctions; and the special values, which are assumed by the different quantities between the limits of 0 and ∞, not only characterize the above-mentioned legion of cases, but the practically infinite number of intermediate steps and transitions with more certainty than could be attained by any nomenclature.

Moreover, the general formula points out the remedy for each defect. A glass formed by measurement, or a measured surgical operation, is able to equalize every ocular defect based upon the structure of the organ; and it is only the functional or facultative limits, which depend upon individual nerve force, that cannot be altered by external means. These limits form, therefore, definite and immovable halting-places for the artificial regulation of the visual processes.

In conclusion, we should observe that defects of both eyes, symmetrical in each and alike in both, and which therefore constitute symmetrical defect of the combined organs, yet do not always occasion symmetrical function. The tendency to the most complete possible satisfaction of the three chief functions in any eye preponderates, not uncommonly, over the tendency to an equal participation of both eyes in the visual act; and produces an asymmetrical discharge of function by the entire apparatus. Consequently a pair of eyes, compelled to squint by some defect of convergence or of application, does not usually look symmetrically at the object, so that both eyes may squint in an equal degree. More frequently the object is fixed by one eye, which therefore completely satisfies the desire for fixation by seeing with the polar bacillus; while the other eye deviates from the point of fixation by so much the more.

THE ASSISTANCE TO FIXATION BY THE DISPLACEMENT OF THE BACILLI.

The fixation, which fulfils the purpose of bringing the pole of the eyeball to the affecting ray of light, and thus of producing direct vision with the polar bacillus lying in the yellow spot, is accomplished by the ocular muscles, and is, if we consider only the acting apparatus, a motor function, the function of convergence. But the impulse to this function manifestly proceeds from the optic nerve; that is, the cause of the muscular movement must be a sensational process. In fact, we have seen already (pp. 61–2) that the tendency to preserve the optical equilibrium directly occasions the ocular rotation.

When this is recognized it is not difficult to assume that, when the convergence, for any reason, does not attain the end towards which it is directed, and therefore does not bring the pole of the eyeball to the luminous ray, a tension may continue on the retina, and may constantly strive to move the polar bacillus into the desired position. This tendency would have more or less result; that is, the polar bacillus, and after it in a diminishing degree the surrounding bacilli of the yellow spot, would be moved towards the ray of light; thus, notwithstanding the false convergence or the strabismus, direct vision, or fixation by the polar bacillus, being brought about, so long as too great a movement of this bacillus is not required.

We obtain hence, for further consideration, the very important conclusion that in convergent strabismus the polar bacillus tends inwards, towards the entrance of the optic nerve; and that in divergent strabismus it tends outwards, away from the entrance of the nerve.

Just as the two polar bacilli in binocular vision strive after the points that receive the light, so the corresponding bacilli of the two retinæ strive after equal affections, and, in vision in material space, they move in the manner explained in §§ 12 and 13 of the *Gesetze*, so that, notwithstanding the incidence of the rays of light upon different points of the retina, in consequence of this movement identical bacilli are influenced by similar rays.

The movement of the corresponding or identical bacilli to similar rays of light arises, as in the polar bacilli, from the effort after optical equilibrium in the two eyes, under the stimulus of the images formed on both retinæ by the luminous vibrations and the nervous processes.

That, notwithstanding this co-operation of identical retinal fibres, we need not imagine any specific function of single vision, and that therefore the common belief in a law of identity is founded on error, is fully shown in § 21 of the *Gesetze*. The singleness of the binocular visual impression is no special quality, but only a result of the coincidence of two impressions. For the coincidence of the apparent visual directions of these two impressions there is nothing necessary, according to our theory of application, but similarity of direction of the affected bacilli; whether these are identical or different is of no moment; but, by means of the force subservient to the tendency to optical equilibrium, identical bacilli are drawn towards the spots equally affected.

THE REACTION OF OCULAR DEFECTS UPON ONE ANOTHER.

Although, as already frequently set forth, the three chief functions of the eyes, accommodation, application, and convergence, are originally independent; yet it is necessary, for the easy exertion of any one of them, that the other two should bear a certain proportion to it. This proportion forms that fundamental law of the visual function, or of the scheme of vision, which, in § 21 of the *Gesetze*, I have declared as the proper law of identity. The consensus of the ocular muscles, and the law of the ocular movements, form only a special portion of this law, since in free movement the exertion of individual muscles must bear a definite proportion to each other.

If this normal proportion does not exist, visual strain is the result. The several ocular functions, in like manner, secondarily influence one another; but this influence does not display itself in any external effect, that is, in any irregularity of the individual functions, until the visual strain exceeds by a certain degree the relative limits of function. Hence the originally independent chief functions come to stand in a relation of acquired dependence upon one another.

The earlier chapters contain all the materials necessary for the experimental determination of the actual condition of the individual organs and their functions, and of the means by which the act of vision may be regulated, and the visual strain (for given conditions) relieved. They contain also the key to the explanation of many phenomena, interesting in themselves and important to ophthalmic surgeons, which depend upon the acquired influence, or upon the mutual reaction of the respective functions in states of abnormal proportion. Of this, several illustrations have already been given, and some others must be briefly noticed.

The general basis for the explanation of this reaction is furnished by the three following positions:—

1. Too great an increase or diminution of any single function produces an abnormal increase or diminution of the others.

2. The interdependence between the functions of the ocular nerves—that is, between accommodation and application, is far closer than that between either of these and the convergence, which is a function of the muscles.

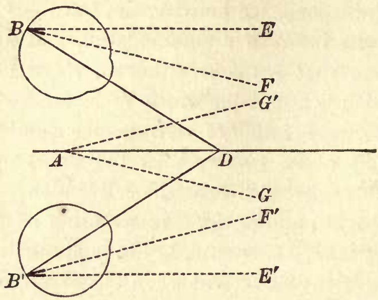
3. If the motor function of convergence should fall short of the attainment of its object, then the effort after fixation, which occasions convergence, produces a tendency to movement of the polar bacilli, and of the corresponding bacilli generally. This tendency is very energetic, but only produces the extreme result of a sufficient movement when the requirement does not exceed a certain degree. If this degree be greater than the power of movement of the bacilli, there arises a sensible tension or visual strain, which more or less influences the accommodation or the application.

Let us examine, on these data, the frequent divergent strabismus of the hypometropic, and the very frequent convergent strabismus of the hypermetropic. Donders has found these forms of squint so frequently in the respective anomalies of refraction, that he cannot doubt the causal connection between them. He explains them, however, upon different grounds; the divergent squint of the short-sighted by insufficiency of the muscles, or of motility; and the convergent squint of the hypermetropic by abnormal effort of the accommodation. I believe these explanations to be erroneous; that the cause of the strabismus must be sought elsewhere, and that for both forms it is the same.

The myope must certainly, in near vision and with small effort of accommodation, exert a strong convergence; and it is possible that for this his muscles may be insufficient. If this were only so, and no other cause were present, his squint would necessarily be united with double vision. Such, however, is not the case; and, therefore, the insufficiency cannot produce the strabismus.

The same applies to the convergent squint of the hypermetropic, who, with slight convergence, is called upon for a

Fig. 63.



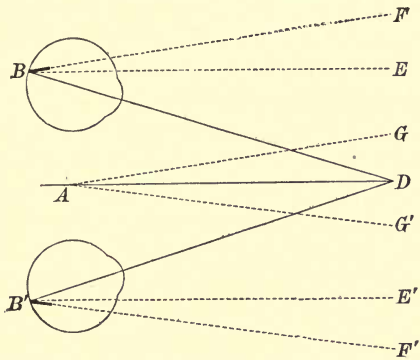
considerable effort of accommodation. If the latter induced the squint by occasioning too much convergence, the single vision would be inexplicable.

According to my conviction, the strabismus depends, in both cases, not upon the comparatively lax interdependence of the accommodation and the convergence, but upon the far closer interdependence of the accommodation and the application.

When the myope fixes the near object D (Fig. 63), he exerts only a small amount of accommodation. For single vision he would require the great nutation E B D, = B D A, in order to render the bacillus B parallel to the frontal axis A D. If the function of application be in its normal state, then so extreme an application is impossible under a feeble accommodation; and the bacilli B and B′ must therefore apply in less degree, as in the directions B F and B′F′. If we draw A G parallel to B F, and A′G′ parallel to B′F′, it becomes manifest that diplopia with crossed images must be produced. The craving after single vision compels the eyes, with small effort of accommodation and a corresponding degree of application, to roll outwards and to assume the position of divergent squint.

When the hypermetrope fixes the distant object D (Fig. 64), his eyes are called upon for a strong effort of accommodation. With this the feeble application E B D, = B D A, cannot be combined. The bacilli B and B′ apply, therefore, in a greater degree, and assume the directions B F and B′F′. If we draw A G parallel to B F, and A′G′ parallel to B′F′, it becomes mani-

Fig. 64.



fest that diplopia with homonymous images must be produced. Hence the craving after single vision compels the eyes, with strong accommodation and a corresponding degree of application, to roll inwards, and to assume the position of convergent squint.

In this way the very common divergent squint of the short-sighted, and the convergent squint of the hypermetrope, may be explained upon one and the same principle, and without disregard of the single vision; while the explanation affords also an important support to my theory of the application.

It is also manifest that, when a defect of application coincides with a defect of accommodation, strabismus may be absent, or may even occur in a direction contrary to the usual one; of which latter condition v. Graefe has seen many examples in the short-sighted. If the hypometropic or hypermetropic eye be in the same degree hypoklitic or hyperklitic, no strabismus will be produced.

The common concave spectacles remove the disproportion between accommodation and application in the myope; and common convex spectacles remove this disproportion in the hypermetrope. By the use of proper spectacles, therefore, the strabismus of myopic and of hypermetropic eyes is relieved.

The relief of divergent strabismus is equivalent to an increase of the convergence, and the relief of convergent strabismus is equivalent to a decrease of the convergence. A normal pair of eyes must assume a convergent squint on looking through common concave spectacles, and a divergent squint on looking through common convex spectacles. In the former case the increased accommodation, produced by the apparent approximation of the object, is attended by increased application, that is, by diplopia with homonymous images, which are united by the convergent squint. In the latter case the exact reverse will hold good.

Hence it follows that experiments with common centric spectacles, in order to determine the limits of relative accommodation in a given convergence, can lead to no accurate results, since the supposed degree of convergence is not accurately maintained.

When a pair of normal eyes squint in consequence of looking through common centric spectacles, such eyes no longer fix accurately, or see directly with the polar bacillus, but indirectly. The same thing occurs in the unaided vision of myopic and hypermetropic eyes. These cease to see directly, since they squint; or, rather, when they squint.

In all these cases of incomplete fixation, and, therefore, in strabismus generally, the already described movement of the bacilli becomes of very great importance. If the degree of strabismus required to afford single vision be such as not to deviate too far from the position of fixation, the polar bacillus may so move as to come into the visual line, and may receive the incident ray. We have then fixation, or direct vision with the displaced polar bacillus, notwithstanding the strabismus; a result that certainly merits special consideration. It affords the seeming paradox of fixation under false convergence.

It must be expressly remarked that the movement of the polar bacillus, which takes place for the sake of fixation, has no influence or effect upon the strabismus, which takes place for the sake of the application, or of single vision. The polar bacillus, that moves into the incident ray, assumes the same nutation as the bacillus that its movement displaces.

The above-described movement of the polar bacillus naturally occurs only in binocular vision; since, in the use of one eye only, there can be no obstacle to fixation.

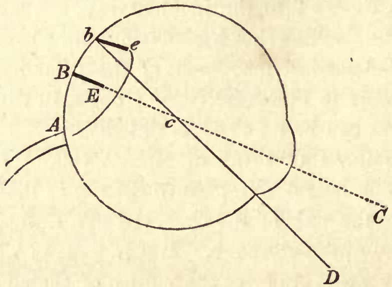
Hence it follows that the movement cannot be observed through the ophthalmoscope; since the use of this instrument requires that the axis of the observed eye should come into the visual line of the observer. But in eyes in which, on account of their structure or of the nature of their employment, the displacement of the bacilli takes place frequently and continuously, this movement comes to be observed by the production of certain permanent conditions.

Upon this ground the question of the movement of the polar bacilli, and of corresponding bacilli generally, acquires much importance in many ocular defects, and especially in short sight.

We have seen that an eye, which is short-sighted by reason of hypometropia, makes an effort of application or nutation of the bacilli outwards, that is disproportionately strong with regard to its accommodation, and that thus, in order to preserve single vision, it assumes a divergent squint; while, for the sake of preserving fixation notwithstanding the squint, the polar bacillus moves outwards until it either comes into the visual line, or attains the limits of its range in that direction. We have here a ready explanation of all the abnormal conditions of the retina and of the choroid by which short sight is so frequently attended (see Donders); such as the following :—

1. The atrophy, stretching, and thinning of the choroid. Since the pole B of the eye, Fig. 65, lies at a small distance from, and external to, the entrance of the optic nerve, and since the polar bacillus B dips by its external extremity into the pigment of the choroid, the movement of the bacillus to *b* must produce an unnatural stretching of the choroid between A and *b*.

Fig. 65.



2. The separation of the choroid from the optic nerve, and the degeneration of the choroid chiefly around the nerve, especially on the outer side A, that is, the establishment of the atrophic 'crescent.'

3. The extension of the retina and the stretching of its vessels between A and *b*.

4. The diminution of the diameter of the optic papilla in the direction of *b* A; since a displacement of the bacillus B away from A must produce a secondary pressure against the papilla.

5. The separation of the retina from the choroid in consequence of the great nutation of the bacilli, by which the latter may be torn from the choroid.

6. The gradual increase of distance between the yellow spot and the papilla.

7. The general morbid condition into which all the membranes and the whole eye may be brought by such abnormal strivings.

8. The increase of the short sight, as an effect of the morbid condition, in this way, that the thinning and tearing of the membranes promotes a further elongation of the ocular axis. This action takes place more rapidly in slight myopia, where the membranes still retain considerable extensibility, than in more extreme cases, when these membranes have already been reduced by atrophy to hard tissues.

Hence, we must assume, not, as Donders supposes, that myopia depends upon a condition which itself determines the development of atrophy; but, that the atrophy is the direct result of the myopic refraction, in consequence of the employment of the myopic eyes without sufficient preservative means.

It is further manifest that any means by which the above-mentioned forms of tension, and therefore the tendency to movement and excessive nutation of the bacilli, can be relieved, will be preservative against the progressive increase of the disease, and of the shortness of sight. Such relief is chiefly to be sought from proper spectacles. Orthoscopic spectacles are insufficient for this purpose, since their only action is to bring distant objects within the near range of vision, and they do not correct the disproportion between accommodation and application. If there be only an anomaly of refraction present, so that the shortness of sight depends solely upon hypometropia (which may be determined in the manner formerly shown), the disproportion in question may be relieved by centric concave spectacles.

It seems, however, to be indicated, for the relief of this evil, to unite with the concave lens a prism with its base turned inwards, by which the bacilli will be caused to move and bend in the opposite direction, or towards the papilla.

It is self-evident that hypermetropia will exert upon the membranes an effect precisely opposite to that of short sight. The bacilli will be pressed towards the papilla. From this can arise only a thickening of the membrane, which does not appear to be a source of disease. In consequence of such a change, atrophy could only be expected at the *ora serrata*;

but, as the distance from the pole to this region is considerable, and as only the terminations of the nerve fibres and of the vessels would be affected, it is not easy to imagine that any hurtful influence could be exerted. Possibly, however, the thickening of the ocular membranes may explain the diminished acuteness of vision so frequently seen in the hypermetropic. (See Donders.)

If we were to follow into all details the action of one abnormal ocular function upon the remaining functions, we should, doubtless, arrive at an explanation of many other phenomena.

Among such we might thus elucidate the degeneration of many ocular muscles; and, in turn, the reaction of this degeneration, and of abnormal motility, upon the sensory functions.

By going a step farther, and including in the inquiry the position and movements of the head, we should come to understand the tendency to assume certain attitudes, which have the effect of diminishing, more or less, a visual strain that is felt when the head is erect. (See *Gesetze*, &c., § 25.)

THE OPPONENCY OF THE FIELDS OF VISION.

In conclusion I desire to set forth briefly the doctrine developed in the *Phys. Optik*, and in the *Gesetze*, and, as I believe, completely established by actual phenomena, with regard to the character of the opponency between the fields of vision of the two eyes.

Of two corresponding nervous fibres only one is always seeing; that is, only one of the two immediately evokes the visual phenomena caused by the incident ray. The other, however, is not devoid of function, for the visual process is carried on in the chiasma and the sensorium by both; but one takes in a certain sense the active part of a positive pole, and the other the passive part of a negative pole. Even when one eye is covered or blinded, this process is not interrupted, but is only so far limited for the useless eye, that the action of the sensorium is propagated only more or less deeply along the corresponding fibre.

In binocular vision two corresponding fibres or bacilli take the active part alternately, each for a time, and as a consequence of the fatigue of its fellow. Consequently only one of the two fixes, in the full sense of the word, and with complete functional activity; and this one is in a certain degree relieved from the visual act when the other takes it up. Hence the negative fibre, in consequence of the continuance of the visual process, remains accommodated and applied with more or less exactness.

It does not therefore follow that, of the fibres receiving a particular retinal image, all those of one eye shall be active, and those of the other passive, at the same time, so that any extended object would always be seen and completely fixed by only one eye at a time. Rather, of the sum total of the fibres, a certain portion of those of one eye and a certain portion of those of the other are in simultaneous activity; so that both eyes see at the same time, but each a different part of the object. The active fibres of each eye vary incessantly, so that sometimes these, sometimes those, now more, and now less, are called into play.

I believe that the proofs I have advanced leave no room for doubt with regard to these facts. For such evidence I must refer to the works already cited, and will only here remark that the doctrine laid down explains the instances observed by ophthalmologists, in which one eye evidently did not originate any visual impression, but in which its participation in binocular vision essentially aided this impression.

Moreover we may conclude that under abnormal conditions, where there is a strain of the convergence or of the application, such as would be produced by strabismus or by the lack of fixation, and where, therefore, all the bacilli of the retinal image of each eye are striving immoderately after fixation, all the bacilli of one eye may in such an effort obtain a preponderance over all the corresponding bacilli of the other eye, and consequently first the one eye, and then the other, will exclusively assume the fixation of the whole object.

Hence, of two similar eyes, it is usual for only one to squint, while the other is rightly directed; and the two fix alternately. Especially is this the case in high degrees of strabismus, when a correction by movement and nutation of the bacilli is no longer possible.

In dissimilar eyes, moreover, it is usual for the same one always to assume the fixation, and the other the negative part.

In fixation with one eye, if the deviation of the other should exceed a certain degree, so that very different bacilli receive the image, then the participation of the latter becomes so small that it is scarcely more functionally active than an eye that is covered.

In the *Gesetze*, §§ 24 and 25, I have entered more fully into the varieties of strabismus; but the views there stated, in §§ 23 to 28, upon squinting and upon ocular defects in general, require modification and correction in many respects, through the investigations laid down in the present work, and in consequence of a more strict separation of the convergence from the application.

THE MICROSCOPIC EXAMINATION OF THE RETINA.

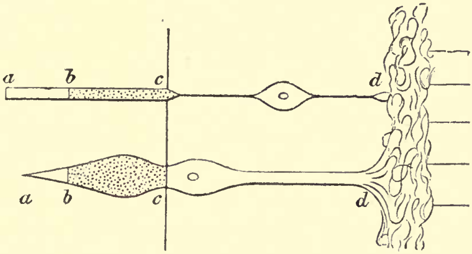
By Max Schultze.

The already-mentioned researches of Max Schultze upon the retina are not only of great anatomical interest, but they afford also many points of support to my theory of vision, and these I purpose to set forth.

The bacillary layer of the human eye, and of all examined eyes of vertebrata, consists of cylindrical rods, Fig. 66, and of flask-shaped cones, Fig. 67. The distribution of these rods and cones forms, in every zone encircling the pole of the eye, a regularly designed figure, and varies with the place of the zone, so that the rods preponderate towards the *ora serrata*,

Fig. 66.

Fig. 67.



and the cones towards the pole, while the human eye, in its yellow spot, contains cones only. But the nearer they are to the pole so much the more do the cones (while still having the general anatomical characters of cones) lose their flask shape, and approach the cylindrical outline of the rods.

The eyes of animals living in a very subdued light, as of owls, bats, moles, rats, rabbits, and fish, contain almost exclusively rods; and the eyes of animals living in bright light, as birds, lizards, and serpents (which, to use Schultze's expression, play in the sunshine) contain almost exclusively cones.

Schultze assumes that the rods and cones are nerve elements, and the terminal portions of nervous fibres, as may be anatomically proved by the structure of their extremities that are turned towards the retina, as at *c*. He assumes also, as I myself have done, that they are the perceptive organs of luminous impressions, and upon the above described distribution of the two kinds, and upon the exclusive presence sometimes of one, sometimes of the other, he founds the conclusion that rods and cones alike possess the faculty of receiving these impressions.

It is of great importance that Schultze confirms, in every case, without exception, the discovery previously made by Krause, that every rod or cone is sharply divided into two portions by a perfectly plane surface, as at *b*. The outer portion, *a b*, penetrates the choroidal pigment, and the inner portion, *b c*, is in contact with the *membrana limitans externa*. The homogeneous substance of the outer portion is distinguished from the substance of the inner portion by a higher degree of refractive power and also by chemical peculiarities, shown by treating it with reagents. Under the action of acids the previously homogeneous substance of the inner portion is coagulated into a finely granular mass, while that of the outer portion is not affected. The outer portion of a rod is cylindrical, that of a cone is conical.

On these facts Schultze rests the hypothesis that the plane *b* is the surface on which the luminous rays are concentrated in exact accommodation, and on which they form, by reflexion, the retinal image, and that through this reflexion the visual impression exists. To this hypothesis I cannot assent. Reflexion, as the turning back of luminous rays from a smooth surface, is, on the one hand, a purely geometrical conception, which has no bearing upon a physiological nervous function. On the other hand, if we suggest a functional participation of the reflecting surface, then reflexion, as an action perpendicular to this surface, is not adapted to produce an affection with which a sensation of the system of ether oscillations represented by the cone of rays must be combined. Upon sensibility to these oscillations, and especially upon sensibility to the proportion between the lateral and the axial components of the luminous affections, depends, in my opinion, the recognition of the relations of objects in space, a recognition for which accurate accommodation is required. In order to feel this system of vibrations accurately and completely, the nervous substance must itself be regularly and thoroughly agitated by the luminous cone. This is accomplished by the penetration of the luminous cone into the outer portion of the retinal rod or cone, and the different refractive power of the outer portion is an important aid to the sensation. For a body to be sensitive to a system of luminous oscillations, it must in the first place be accessible to light, that is, it must be transparent, as the outer portion of the rod or cone actually is. A transparent substance, however, by reason of its very transparency, would only be feebly stimulated by rays of light passing through it, but this stimulation would be increased if the substance had a different refractive power from its limiting medium (see the Exposition of the Action of Light in § 2 of the *Phys. Opt.*). The stimulus would be still more increased if the portion *b a* consisted of a substance the refractive power of which varied from *b* to *a*, although the investigations of Schultze do not show whether this is actually the case. The insertion of the outer portion *a b* into a layer of dark pigment would have chiefly the effect of preventing the passage of the penetrating rays of light into neighbouring rods, and thus of preventing disturbing luminous action in those rods. Hence this pigment does not form the foil of a reflecting surface, but surrounds the whole outer portion of the rod or cone as an enclosing sheath, which, in many animals assumes the form of a tuft of hairs increasing in a direction forwards from the choroid.

Schultze has observed that the conical outer portion, *a b*, of every cone, is more or less wrinkled together. I conclude from this that such portion possesses, during life, the power of lengthening and shortening. Such a power, according to my theory of the method of recognising distance, would be of great importance, if we assume that the division at *b* forms the elementary nervous unit, and that this division, by prolongation or by shortening of the outer portion, is so arranged that the point of the luminous cone falls accurately upon the point *a* of this portion (see especially the 19th section of the *Gesetze*).

If, in this prolongation and shortening of the outer portion, the plane *b* retains nearly its fixed position, there must in strong accommodation be an extension, and in weaker accommodation a drawing together, of the outer portion. The wrinkling when accommodation is relaxed, and therefore after death, would thus be a natural consequence.

The variation in the size of the nervous unit, in consequence of accommodation, that is required by my theory, need not, since we are concerned with sensations only, be an external or mechanical variation of the section *b*. The size in question can be changed, as regards our sensation, only by the varying internal tension of different states of accommodation; although a mechanical change of the thickness of the cone at *b* would conduce to accuracy of sensation. According to the above cited section of the *Gesetze*, a diminution of size at *b* must be associated with increased accommodation, or with a prolongation of the outer portion of the cone; and an increase of size at *b* with diminished accommodation or shortening of the outer portion; since such changes would correspond with the unchanged quantity of the contents of the outer portion. For such changes, moreover, the flask-like shape of the cones would be favourable, if we assume that the point *a* of the outer portion remains fixed in the choroidal pigment; and the plane *b* moves forward towards the vitreous body in stronger accommodation, and backwards away from it in weaker accommodation.

The latter hypothesis, however, of the mobility of the plane *b*, requires the surrender of the former one, of the mobility of the point *a* of the conical outer portion; since we cannot assume that both extremities of this portion are movable. It is possible, and even most probable, that the point *a* is firmly fixed in the pigment; and that therefore the place in which the bundle of rays must be concentrated in exact accommodation remains nearly the same; while the outer portion *a b* is lengthened in consequence of accommodation effort by the movement forwards and the contraction of the surface *b*, and shortened as the accommodation relaxes by the movement backwards and the expansion of the surface.

Whether the cylindrical rods, which show no marks of wrinkling, possess the same extensibility, cannot at present be determined. If they are not extensible and contractile, this would not imply that the recognition of distance by them is impossible, but only that it is incomplete. In fact, the recognition of the distance of objects seen indirectly, and the rays from which fall upon peripheral parts of the retina, where rods predominate, is incomplete; and it is very probable that animals living in twilight, and possessing chiefly rods, do not require so exact a recognition of distance as those living in a bright light, and possessing chiefly cones. Some of the latter class, as birds, are called upon to scan a considerable range of space.

What then is the significance of the difference between the rods and the cones? In order to reply to this question it is necessary further to describe the peculiarities that have been brought to light by Schultze's investigations.

The difference in question has reference partly to the union of the outer portion with the retina, partly to the union of the inner portion with the choroid. The outer portion of a rod penetrates deeply into the choroidal pigment, and appears to pass almost through it; while the pointed outer portion of a cone does not penetrate so deeply, and appears completely to terminate in the pigment. The inner portion of a rod sends a single, extremely fine, nervous filament *c d*, through the *membrana limitans* to the intergranular layer *d d*, upon which this filament terminates by a small enlargement *d*. The inner portion of a cone sends a larger nervous cord, consisting of a bundle of single fibres, to the intergranular layer; and this bundle there divides into its constituent filaments, which penetrate into the tissue of the layer.

From the compound structure of the cones, Schultze forms the conjecture that they chiefly subserve sensations of colour; and that the rods subserve sensations of proportionate intensity and position in space. The former conjecture rests upon the hypothesis of Young and Helmholtz, that particular filaments are sensitive to red light, others to yellow, and others again to blue. Such an hypothesis certainly requires bundles of fibres for the recognition of different colours; since a single fibre would react to a single colour only.

The above-named hypothesis stands upon the same level with the hitherto generally diffused doctrine of the identity of corresponding retinal elements, so far that they both ascribe to the elements in question a mysterious quality not explicable by a natural law. In the 'law of identity' this quality consists of the faculty of single vision; and in the hypothesis of Young and Helmholtz, it consists of the faculty of reacting only on a single colour, and certainly in a perfectly inexplicable manner. The latter hypothesis is, indeed, nothing else than the substitution, for the problem of colour sensation, of a problem still more difficult; since it does not come a step nearer to explain the material process of sensation as regards that colour for which a particular filament is supposed to be adapted, and gives, therefore, no help as regards the principal phenomenon; while it adds to the chief problem two new problems, by requiring an explanation of the specific differences of the red, yellow, and blue colour processes, and an explanation also of the strange circumstance that these three colours should each require a specific process, while the intermediate colours requires only a simple mingling of the specific processes.

After the microscopic researches of Schultze, however, we need no longer seek for the refutation of this hypothesis in principles; it is best refuted by the anatomy of the eye. For while the cones consist of bundles of filaments, the rods are single, and could, therefore, according to the hypothesis, perceive only a single colour, and that a simple one, as red; but could never perceive a mixture, such as white. But beyond the area of the yellow spot the rods greatly predominate. Notwithstanding this, we recognise all colours in every part of the field of vision; and hence it follows irresistibly that a single nervous filament may conduct each and every colour process as well as a compound one; and that the hypothesis referred to is devoid of foundation.

In § 65 of my *Phys. Opt.* I have ascribed the colour process to a chemical or material oscillation, which may take place in every single nerve filament. This view seems hitherto to have escaped observation; but I do not doubt that it will compel assent, and that the conception of chemical oscillation will play a great part in natural science. The observations of Schultze afford much support to this conception. For the inner portion, *b c*, of a rod or cone is shown, by its becoming a granular mass when treated by acids, to consist of a chemically compound substance, such as my notion of the nature of the colour process requires. This circumstance suffices to lead me to the assumption that the inner portion of each rod or cone is the nerve element by which the feeling of colour is received.

According to the foregoing, either the rod or the cone is a percipient organ for the recognition of relations in space and of colour. The former faculty is seated in the outer portion, *a b*, and depends upon the relative elasticities of the ether and the nerve substance; and the latter is seated in the inner portion, *b c*, and depends upon the chemical constitution of its substance, which is irritated by the rapid vibrations of the light. (*Phys. Opt.*, § 65.)

A third function, which Schultze, following Aubert, has called the sense of light, has no existence. This assumed sense of light was to measure the intensity of the oscillations. But for this purpose no other organ would be required than those which habitually react on these oscillations; and their intensity would be felt through the sensitiveness or irritability of these organs.

We have thus arrived at no explanation of the different terminations of the outer and inner portions of the rods and the cones, as regards space and colour. Upon this I venture to express the following conjecture. I conceive that the blood and the nervous substance, like the copper and zinc of a battery, furnish the two poles necessary to afford a physiological nervous current. The deeper penetration of the rods towards the bloodvessels of the choroid appears to me to produce a more intimate contact with these vessels. The more intimate the contact, the more sensitive or excitable would be the nervous element. An increased sensitiveness would appear to be desirable for the elements more remote from the yellow spot, since these receive a weaker and less complete luminous impression; and hence in such situations the rods prevail. For the same reason animals living in a feeble light are furnished chiefly with rods. In the yellow spot, the irritability given by a copious blood supply might be hurtful and perplexing; since the external stimulation by light is more intense, and an excess of blood might diminish the accuracy of perception. On that account the less deeply penetrating cones prevail in this portion, and cones alone are found in animals living in bright sunshine; in which also these cones sometimes contain turbid corpuscles, adapted to diminish the external light.

Furthermore the less firm insertion of the cone points into the choroid would increase the mobility of the cones on every side. This is more important for the elements lying in the yellow spot, and for the percipient elements of animals living in sunshine generally, on account of the need of more exact accommodation, than for elements that are feebly irradiated, and therefore less adapted for accurate vision. For such the exact accommodation is less essential, and may be sacrificed in order to obtain the firmness of position that is on other grounds desirable.

As regards the continuation of the inner portions in nervous filaments, the intergranular layer *d d*, into which these filaments penetrate, plays an important part. This layer forms a fibrous network, to which the single rod fibres are loosely applied by a bulbous termination; while the bundles of cone fibres, after their division into single filaments, appear to extend into it. From this intergranular layer other single fibres penetrate inwards, towards the vitreous body, as far as the so-called molecular layer. The latter forms a second inextricable network of nerve fibres, unequally thicker than the intergranular layer. From the molecular layer spring, lastly, the fibres of the optic nerve layer, which proceed to the papilla. This layer consists of thicker cords, and of fine filaments. The thick cords are produced by several fibres of the molecular layer, uniting in a ganglion cell, and proceeding forwards from this cell as a compound fibre; but the fine filaments appear to pass through no ganglion cell, and to proceed directly from the molecular layer to the optic nerve layer as single threads.

The two fibrous webs, of the intergranular layer and of the molecular layer, next attract our attention. By these webs is accomplished a manifold contact of all the nerve filaments, and also the formation of two concentric organs, by which the nervous function discharged by a single perceptive element may communicate certain actions over the whole of the retinal cup, to the *ora serrata*, and to all points of this circumference. In my earlier works I have already expressed the opinion that every rod or cone tries to accommodate for itself. But such an accommodation, on account of the spherical shape of the eye, would not be practicable unless all parts of the spherical mass, lens, vitreous, iris, and cornea, were affected by it. Hence every element must stand with the rest in a state of organic union; and such a union may be accomplished by the above-named two layers, in which the filaments entwine. I further conjecture that the first or intergranular layer, the more external, controls the changes of the pupil; and that the second, the thicker molecular layer, which is more internal or nearer the vitreous, controls the communication for accommodative purposes between the lens and the vitreous humour. This conception would very well explain the account given by Schultze of the development of the eye of the fowl.

The elements which are adapted for the most exact accommodation, which would be injured in their function by too much light, and hindered by too little, that is to say, the cones, must hence require an especially extended contact with all the filaments, and must be able to act in the most energetic possible way upon the pupil. We here obtain a plausible ground of explanation for the division of each cone fibre into several filaments in the intergranular layer. The single filaments of the rods would act far less energetically upon the pupil.

The union between the intergranular and the molecular layers appears to be effected by independent filaments, which are not direct continuations of the filaments of either the rods or the cones. But the cones would, notwithstanding, act upon the molecular layer, and hence upon the accommodation of the lens and the vitreous body, more energetically than the rods, in consequence of possessing more numerous points of contact with the intergranular layer.

The division, interruption, contact, reunion, and intertwining of all the nervous filaments in the different layers, appears to me to give important support to my doctrine of all nervous sensations: which is that these sensations occur not from a bare external excitation, but also from a subjective perception in the peripheral nerve elements; and that therefore the form or way of nervous conduction to the brain may be immaterial, since the peripheral nervous system is itself nothing else than an extension of the brain over the organism.

According to this conception, the corresponding or identical nervous fibres of the two eyes need not necessarily be in direct union with one another. The account of the corresponding retinal points, according to my view of the law of identity, as given in § 21 of the *Gesetze*, retains, nevertheless, its entire applicability. By the reciprocal contact of the fibres an organic union of all fibres is produced. In this way an influence upon the fibres of one eye by those of the other is already rendered possible. An external luminous stimulus, which falls upon a fibre of one eye, produces certain optical forces or tensions, determined by the direction of the nervous point concerned. From the participation of the brain, as the common central organ, a reflex action upon the other eye may be occasioned, and would be concentrated upon the corresponding point therein; thus leading to normal binocular vision, as much as when the second eye is equally impressed by the external light, and the affection of the corresponding point is produced. This reaction between the two eyes may also very suitably illustrate the positive character of the nervous current in the fibre of one eye, and the negative character of the current in the fibre of the corresponding point of the other, as my theory of opponency requires. Furthermore, the unfulfilment of the law of identity would entail the impulse to the movement of the fibres of the corresponding point for the attainment of single vision.

If we think of the system of all the sensory fibres of the visual organs, which have their peripheral extremities in the rods and cones, and their common central organ in the brain, and which on their way to the brain unite, divide, and touch, at many points, and if, for the sake of a clearer conception, we identify the nervous with a galvanic current, it will be manifest that such a current, when passing between a given rod and the central point in the brain, must take a perfectly determinate course through the maze of fibres, namely, the course of the smallest resistance to its conduction. We may regard this course as the direct nervous union between the rod and the brain; and by means of this union two corresponding rods form an anatomically united whole.

As regards the movement and the nutation of the rods and cones, required for single vision and for fixation, the observations of Schultze do not enable us to construct these movements; but that they are possible may be inferred from many considerations.

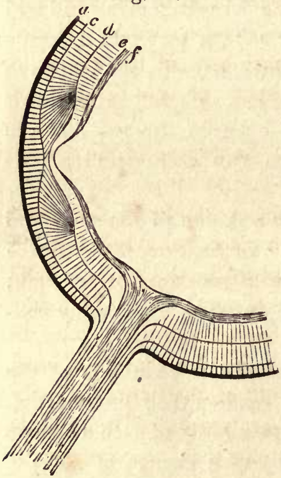
Among these may first be remarked the wrinkling of the outer portions of the cones, which renders their prolongation and contraction probable. But the possibility of such a change of length alone implies the possibility of a change of place of the moveable part. If the point *a* remains fast in the pigment, and the base *b* moves; the latter cannot simply move forwards and backwards, but must also be displaced along the periphery, so that the plane of the important section *b* may obtain a different inclination to the radius of the eye. The *membrana limitans* would offer no impediment to such a movement, since it is not a closed membrane, but is perforated by all the cones.

The flask shape of the cones seems to allow of their expansion and contraction, and thus permits the movement of the cones, and of the rods leaning against them.

The connecting tissue found by Schultze in all the retinal layers, and affording an arboriform support to all the filaments and cells, appears to render possible a movement and change of shape of all nerve filaments without mechanical strain upon them.

The Fig. 12 in Plate III. of Schultze's drawings, shows that the rods themselves can bend. This figure is taken from a diseased eye, in which the bending has become permanent; but it allows us to conclude that bending is possible in a sound eye also.

Fig. 68.



The form of the several elements and layers at the yellow spot is also of special importance. According to Fig. 68 the cones in the yellow spot, where they are nearly cylindrical, are so much longer than elsewhere that they cause the *membrana limitans externa*, *c*, to curve inwards towards the vitreous body. And since all the other layers of the retina are here very thin, so that the *membrana limitans interna* curves outwards towards the choroid, and forms the *fovea*, the cone filaments *c d*, which unite the cones with the intergranular layer *d*, assume a very oblique direction with regard to the radius of the eye. It is plain that the elongation of the cones, and the oblique direction of their fibres, must both considerably facilitate the movement and nutation of the cones. The same arrangement, with the inclination of the rods outwards, would also render a movement outwards easily possible, such as is required for the normal visual act. For this purpose we must consider the point of the outer portion of the rod or cone to be firmly fixed in the choroidal pigment, and the inner base to be moveable. Such a movement would then be accomplished by the contractility of the retina, especially of its intergranular layer.

Moreover the observation that the outer portion of a rod or cone may be easily separated from the inner portion, in the plane *b*, by mechanical injury, has a significance for my theory. Since, according to this theory, myopia occasions an abnormal movement and inclination of the rods, it is unquestionable that a stronger tendency to separation at *b* would be occasioned by such movement if the outer portion were fixed in the choroidal pigment and only the inner portion were moveable. Hence the theory explains the frequent detachment of the retina in myopia; and, inversely, the frequency of the detachment confirms the theory.

If, in conclusion, we reflect that the degree of movement and of inclination necessary to an accurate visual impression, depends upon the distance of the object; and that the visual act is not directed to a single point in space, but to an extended object, or to a great part of the visual field, which, at the same time, presents very different degrees of depth, it will be manifest that complete vision of a large object will not require an equal movement and nutation of all rods; but that there must rather be a power for different points of the retina to apply differently. If the bacillary layer were furnished in all parts with similar and easily moveable elements, especially with cones, a tendency to movement manifested in any part, as, for instance, at the yellow spot, and in any direction, would be propagated over the whole layer, and would carry with it the elements which for accurate vision were required to assume a weaker or a stronger movement, or to remain at rest. I conceive that the division of the elements of the bacillary layer into rods and cones, is an important aid to the regulation of the application over the whole retina. It would seem as if the cones, which are easily moveable, and liable through their strong fibres to be directly moved from the intergranular layer, would find in the rods, which are firmly inserted into the pigment, and incapable of being moved from the intergranular layer, a passive impediment which certainly, on account of its elasticity, would yield somewhat to the lateral push of the cones; but yet which would prevent the unchecked propagation of this push over large portions of the retina.

Hence the difference in the structure of the two kinds of perceptive elements, and also the regular distribution of the two kinds in the retina, may be reasonably explained with reference to the regulation of the function of application.

PRACTICAL RULES FOR THE DETERMINATION OF PROPER SPECTACLES.

The determination of the spectacles necessary for any defective eye presupposes a knowledge of its optical condition depending upon its structure, and of its functional condition depending upon its innervation. The discovery of these conditions is effected by suitable physical experiments. It is manifest that these experiments may be conducted after different methods, and by the aid of different kinds of apparatus; and that the practical value of each procedure will not entirely depend upon the objective peculiarities of the method itself, but also upon the subjective habitude and skill of the observer; so that many ophthalmic surgeons may maintain the superior value of certain tests and accessories, on no other ground than that furnished to themselves by their own custom and experience. In this book, which has for its chief object to lay down the fundamental principles of a rational spectacle construction, it would lead us too far if we attempted to describe, and to follow into the smallest details, all the possible methods of examination; but it may be useful to commence the description of selected methods by some general observations, intended practically to elucidate the theories set forth in the earlier chapters, to remove, as much as possible, certain sources of error, and to assist the observer, who wishes to apply these theories, in the conduct of a rational method of inquiry.

1. *Anomalies of Refraction and Accommodation.*

We will, in the first place, consider the Anomalies of Refraction and Accommodation.

The anomalies of refraction require for their comprehension the determination of the refraction of the eye in the state of repose of the accommodation. This refraction may be such as to constitute emmetropia, hypometropia, or hypermetropia. The distinction rests upon a knowledge of the visual distance, or, more definitely, of the accommodation distance a; that is, in the knowledge of the distance of the luminous point, the rays from which are accurately united upon the retina, when the accommodation of the eye is at rest. In the determination of this distance a there are, therefore, two conditions to be fulfilled: first, the accurate concentration of the rays upon the retina; and, secondly, that the accommodation of the eye should be at rest. Of the fulfilment of the first condition, the *clearness* or *sharpness* of the visual impression of the patient affords a direct, and, generally speaking, a sufficient criterion. When, however, the patient can form no adequate judgment upon such a point, it is evident that only those methods of investigation can be used which are independent of his testimony; for example, the examination with the ophthalmoscope. As regards the second condition, the sensation which attends upon an accommodation effort, and which, in the experiment, should sink to *nil*, affords some evidence. This evidence, however, is in general uncertain, and hence the point must be determined without the direct co-operation of the patient. To paralyse the accommodation by means of atropine affords to the observer the most trustworthy proof of complete repose of the function. In examination with the ophthalmoscope, since the eye of the patient does not fix any object, we may generally assume that its accommodation is nearly relaxed; but this assumption is not entirely trustworthy, since the stimulus of the light reflected from the mirror, or some object in the patient's field of vision, or even an involuntary effort, may produce some exertion of the accommodation. Every method of investigation, whatever it may be, must be so arranged that the question whether the accommodation be in repose shall not be left to the decision of the patient, but to that of the observer.

That for the determination of the state of the refraction two conditions must be fulfilled, is a requirement of the theory. The physiological possibility of such fulfilment is quietly presupposed. It may be questioned, however, whether the co-existence of the two conditions is always attainable in practice; and whether physiological laws do not, under certain circumstances, render such a co-existence impossible. We may imagine that the exercise of the conscious visual function, that is, the co-operation of the intellectual power of perception with the sensational power of seeing may, in many persons, never permit complete repose of the accommodation to occur; for that, in such persons, when they see with consciousness, and therefore clearly, a certain degree of action of the accommodation is always exerted, as a result of the induced influence of the brain upon the visual apparatus; and that when their accommodation is completely relaxed, they no longer possess the power of seeing with consciousness, and therefore clearly. The phenomena of hypermetropia certainly render it probable that in many persons such an influence as between seeing and perception is exerted by the brain; and it is essential not to lose sight of the probability.

Where such an influence prevails, and we assume that it is more or less present in all persons, the eye, in repose of its accommodation, does not possess the faculty of conscious or clear vision. If, therefore, it were desired to experiment in this state of repose, any proceeding would be futile which required the testimony of the patient with regard to the clearness of the visual impression, in the degree in which the apparatus of accommodation had not its full sensitiveness. For such a purpose the use of atropine is an indispensable accessory. By it the accommodation is *paralysed*; and therefore the patient is enabled to receive a clear visual image in complete quiescence of the function. Moreover, the examination with the ophthalmoscope, during which, as the eye is not engaged in seeing, the accommodation may be relaxed, will in many cases answer the purpose with sufficient accuracy.

It must be asked, however, for the special purpose of determining the required spectacles, which is the more important, the absolute repose of the accommodation, in which the eye can no longer see consciously, or the state of minimal accommodation which is necessary to clear vision. For many purposes a complete knowledge of the eye, and therefore a knowledge of both these conditions, is to be desired; but for the purpose of prescribing spectacles it appears to me that the condition of absolute repose is wholly unimportant (since, in this condition the eye is unable to see); and that the exercise of a certain minimal accommodation is a much more essential requirement. Hence, in my opinion, the spectacles actually required for the present time can only be determined under the supposition that the eye can exert no smaller degree of accommodation than the presumed minimum. And, since it is possible, and therefore probable, that this minimum, when it is unusually large, may be gradually reduced by the continued use of suitable glasses, so, after the lapse of an uncertain time, the investigation may require to be repeated, and fresh glasses to be prescribed, according to the results then yielded. The knowledge of the refraction of the eye in absolute repose of its accommodation is, therefore, for the immediate purpose, not an essential requirement; and the knowledge of the refraction in the state of minimal accommodation is much more important.

Since, for certain phenomena of hypermetropia, we have generalized the customary expressions, *manifest*, *latent*, and *total*, we may call the refraction of the state of minimal accommodation the manifest refraction, and that of absolute repose of the accommodation the total refraction. Further, we may call the distance of the luminous point, the rays from which are accurately united upon the retina in the state of minimal accommodation, so that therefore the point is clearly seen, the *manifest accommodation distance*; and the distance of the point the rays from which are united upon the retina in absolute repose of the accommodation, and which therefore cannot be clearly seen, the *total accommodation distance*. The difference between the maximum and the minimum accommodation is the *manifest accommodation range*; the difference between the minimum and *nil* is the *latent accommodation range*; and the difference between the maximum and *nil* is the *total accommodation range*. It is self-evident that the total range is the sum of the manifest and of the latent.

Moreover it follows that, as well in hypometropia as in hypermetropia, the manifest part is to be distinguished from the latent part, and either of them from the total hypometropia or hypermetropia; since, for example, manifest hypometropia of the degree $\frac{1}{a_1}$, is the defect which corresponds to a positive manifest accommodation distance a_1; while total hypometropia of the degree $\frac{1}{a}$ is the defect which corresponds to a positive total accommodation distance a; and between them the degree $\frac{1}{a} - \frac{1}{a_1}$ of hypometropia is latent.

Manifest emmetropia is displayed by an eye which unites parallel rays upon its retina in the state of minimal accommodation; and which therefore is the subject of total hypermetropia. Again, an eye is totally emmetropic which unites parallel rays upon its retina in absolute repose of its accommodation; and which therefore displays manifest hypometropia. Since no eye in common use exhibits its total defect, but only the manifest part, so every eye which appears to be emmetropic is in a certain degree hypermetropic; and the actual or totally emmetropic eye appears to be hypometropic. An apparently hypometropic eye is, totally, less hypometropic than it seems, and may even actually be hypermetropic; and a totally hypometropic eye appears more hypometropic than it is. An apparently hypermetropic eye is totally more hypermetropic; while a totally hypermetropic eye appears less hypermetropic than it is, and may even appear to be hypometropic.

The most simple means of determining the manifest accommodation distance a_1 is afforded by causing the patient to look with the naked eye at an object at various distances. The greatest distance at which the eye sees the object clearly and sharply is the value of a_1. This procedure is only applicable to hypometropes, who have a positive accommodation distance. For manifest hypermetropes, who in minimal accommodation can only unite convergent rays upon the retina, and who have, therefore, a negative accommodation distance, the same method may be made available by the employment of a concave mirror, since the curvature of such a mirror, and the distance of the actual object which the hypermetrope sees clearly reflected, permit of a conclusion as to the backwards situated or negative distance of the apparent object.

In surgical practice this simple method is often actually employed, by the aid of Snellen's or similar test objects. There are, notwithstanding, many objections to it; among others, the difficulty of arranging for observation, at great distances, objects of suitable size and illumination, and of measuring these distances. A trial by the aid of glass lenses, and of objects lying within the moderate range of distance afforded by a consulting room, is in most cases to be desired;

particularly as it is equally applicable to hypometropia and hypermetropia.

The method by furnishing the eye of the patient with lenses may be conducted according to two different plans. We may proceed either by causing a variable object to be looked at through the same lens, or the same object through various lenses. The first of these methods is in reality nothing more than the already mentioned fixation procedure with the naked eye; since the case of the naked eye is only a special form of the addition of a constant lens. While, therefore, under certain circumstances, the constant lens with varying objects may render good service, yet the method is in the same way and degree incomplete as the trial with the naked eye.

It is a more useful and convenient method, and applicable in a far larger number of cases, to cause the eye to fix a constant object, placed at a known distance, through different lenses of known focal length. Such an object retains, notwithstanding the change of lenses, a constant retinal image, occasions therefore no considerable change in the acuteness of vision, requires no measurement of distances, and can be used as well for the investigation of hypermetropia as of hypometropia. It is self-evident that, in certain cases and for special purposes, a change of object may be desirable, either as regards its size, shape, colour, or distance, in order accurately to investigate all the peculiarities of an eye, or in order to prescribe spectacles for some particular employment, as for reading, or for out-door use. This, however, involves no departure from the general principle of the method, the fixing a constant object with varying lenses.

In general, even with similar eyes, it is necessary to examine each alone, not both together, since the binocular visual act, by the co-operation of the convergence and of the application, may exert a considerable induced influence upon the accommodation, and may thus much disturb the state of the refraction.

To carry out the investigation it is necessary to possess a set of lenses from the most powerful concave to the most powerful convex. We consider these lenses as forming a regular series, commencing at the left hand with the highest concave, having a plane glass in the middle, and the highest convex at the right. The focal length f of these lenses commences on the left with the smallest negative value, and passes through that of the plane glass, $\pm \infty$, to the positive values, thus representing a double series of numbers decreasing from left to right, as -10, -50, -100, $\pm\infty$, 100, 50, 10; and the inverted values forming a simple increasing series of numbers, as $-\frac{1}{10}, -\frac{1}{50}, -\frac{1}{100}, 0, \frac{1}{100}, \frac{1}{50}, \frac{1}{10}$; so that we can call every lens of the whole series, which is placed farther to the right than another, the more powerful of the two. The object for fixation may be placed at any desired distance x. When the whole series of lenses, beginning with the concave of the greatest focal distance have been tried, the lens of highest power f (that is, the lens of the focal length f which lies farthest towards the right hand end of the series of convex lenses) through which the eye still sees the object clearly, is to be selected. The apparent distance z of the object is then determined (page 50) by the formula

$$\frac{1}{z} = \frac{1}{x} - \frac{1}{f} \quad \ldots\ldots \quad (1)$$

and it is plain that this value of z shows the *manifest accommodation distance.* For example, if the object be placed at a distance $x = 10$ inches, and if it be found that $f = 15$ inches is the most powerful convex lens through which the object is yet clearly seen, we have $\frac{1}{z} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$; and therefore a hypometropic eye of the manifest accommodation distance of 30 inches. If $f = 8$ inches were the most powerful convex lens, then $\frac{1}{z} = \frac{1}{10} - \frac{1}{8} = -\frac{1}{40}$; and the eye would therefore be hypermetropic, with a manifest accommodation distance ot -40 inches. If it were found that $f = -50$, the concave lens lying nearest to the convex series, and therefore the weakest of the concave lenses, was the strongest through which the object could still be clearly seen, then $\frac{1}{z} = \frac{1}{10} + \frac{1}{50} = \frac{3}{25} = \frac{1}{8\frac{1}{3}}$; and the eye would be strongly hypometropic, with a manifest accommodation distance of $8\frac{1}{3}$ inches.

The procedure is of the simplest kind when the fixation object is placed at infinite distance, or, what is sufficient for all practical purposes, at very great distance. Then $x = \infty$, and consequently $\frac{1}{z} = -\frac{1}{f}$, or $z = -f$, so that the manifest accommodation distance z is equal to the negative value of the focal length of the strongest lens through which the object can still be clearly seen. It is self-evident that, according to the acuteness of vision of the eye, an object that subtends a suitable visual angle must be selected.

The accommodation distance, or the value of z, has been indicated in the earlier chapters by a, and the lens f by l, and we have called this lens, $l = -a$, the emmetropic lens, since it renders the accommodation distance of the eye infinite, and therefore entirely neutralizes the hypometropia or hypermetropia. In this the *total* accommodation distance was referred to, and we called the manifest accommodation distance a_1; the lens f found by the method just described, $= l_1 = -a_1$ is therefore that which renders the *manifest* accommodation distance infinite, and it may be called the *manifest* emmetropic lens. We have already observed that the manifest accommodation distance is that which has to be decided for the selection of the spectacles immediately required, and it is therefore very important that this distance should be ascertained, independently of the latent, in every case. The *total* accommodation distance a may be discovered in the same manner, when desired, by first paralysing the accommodation by the action of atropine.

We may now proceed to consider the *anomalies of accommodation.* The function of accommodation, which may constitute *Europia*, *Bathopia*, or *Hypsopia*, is determined by the positions of the natural near point H and the natural far point C_1 (Fig. 38), or by the position and extent of the natural range of accommodation R B, or by the position and extent of the natural territory of accommodation H C_1. The accommodative function of the eye corresponds to the curve K L of the hyperbola in Fig. 38; the territory of the accommodation is the difference of the abscissæ, and the range of accommodation is the difference of the ordinates, of the same portion of the curve.

The extent and position of the range of accommodation R B, meaning by this as we do the difference between the maximum and the minimum of the accommodation effort, is completely independent of the hyperbolic curve of requirement. But since when this curve varies, the segment K L of the curve, which corresponds to the near and far points, and to the territory of accommodation H C_1, also varies, the relation between the range and the territory of the accommodation changes. In order to establish a fixed relation between these two quantities and to simplify the formula for calculating the range of accommodation, R B, from the distance between the ends H and C_1 of the territory of accommodation, we have in Fig. 38 furnished the eye with its emmetropic lens, so as to consider the horizontal asymptote of the hyperbola as situated in the fundamental axis A C_1. When thus H and C_1 indicate the artificial near and far points, or the near and far points for the eye furnished with the lens, and A H is taken $= n$, and A $C_1 = w$, we have A B $= \frac{1}{n}$, and A R $= \frac{1}{w}$, and therefore, for the range of accommodation, R B $= \frac{1}{n} - \frac{1}{w}$.

The distances n and w of the artificial near and far points bring, therefore, by means of the values $\frac{1}{w}$ and $\frac{1}{n} - \frac{1}{w}$, the existing degree of stenopia, or of bathopia or hypsopia, to immediate recognition.

This determination of the range of accommodation requires the putting back the horizontal asymptote of the requirement hyperbola to the basic axis of the system of co-ordinates, from which the accommodation effort y is measured as a vertical ordinate. For this putting back, the totally emmetropic lens is required. If this has been discovered by the aid of atropine, and thus the total accommodation distance a been made known, the necessary elevation or depression by the distance $\frac{1}{a}$ can be accomplished. But if the observer confines himself to experimenting upon an eye in its natural condition, a remains unknown, and then only the manifest accommodation distance a_1, and the manifest emmetropic lens $l_1 = -a_1$ can be employed. By the employment of this lens the horizontal asymptote is not replaced in the basic axis A C_1, but in the horizontal R L, which lies higher than the basic axis by the unknown value A R of the minimal accommodation. In fact, with this lens, the manifest accommodation distance is rendered infinite. We obtain $w = \infty$, or $\frac{1}{w} = 0$; and for A H $= n$, the ordinate R B, $= \frac{1}{n}$, is equal to the range of accommodation.

When the apparatus of accommodation works in such a manner, that the tension of the accommodation quickly sinks to *nil* when the manifest accommodation distance is exceeded, the facultative curve of accommodation will assume, beyond the far point, a form like that of the dotted line in Figs. 26, 29, or 34. When, however, as will probably most frequently be the case, the eye in looking beyond its far point continues over a certain space to exert its minimal accommodation, the facultative curve of the accommodation will cut the curve of requirement, at the far point, in a horizontal line, as in Fig. 20 in the portion L_1 L. The last supposition, namely, that in vision beyond the far point the eye still exerts for a space the minimal accommodation, requires, as a general supposition, to be considered in the experiments.

It will easily be seen from the foregoing that, in this book, the manifestness, the latency, and the totality, which have been regarded by practical ophthalmologists as peculiarities of hypermetropia, are considered in a more general way, and as appertaining to all ocular defects; since they are referred to the simple general principle of the maximum, the minimum, and the *nil* of the accommodation. In Figs. 20 to 23, and in Fig. 38, R B is the manifest, A R is the latent, and A B is the total accommodation distance.

In all cases, the operation with the manifest emmetropic lens $l_1 = -a_1$, is completely sufficient for the determination of the spectacles then immediately necessary; since the judgment thereupon (as, in any visual act, the accommodation may be assumed to be within its possible range from A R to A B), may be as well founded upon the hyperbola which has its horizontal asymptote in the line R L, as upon that which has its asymptote in the line A C_1.

We have called the near and far points of the unaided eye the *natural*, and the near and far points of the eye furnished with a lens, the *artificial*. It is in many cases impossible to investigate the unaided eye; and we must generally, therefore, conduct our experiments with the help of lenses. A lens f (of the focal distance f) changes the natural into the artificial near and far points in such a way that when n_1 and w_1 are the distances of the natural, and n and w the distances of the artificial points, according to page 60,

$$\left.\begin{array}{l}\frac{1}{n}=\frac{1}{n_1}+\frac{1}{f}\\[4pt]\frac{1}{w}=\frac{1}{w_1}+\frac{1}{f}\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \; (2);$$

or inversely,

$$\left.\begin{array}{l}\frac{1}{n_1}=\frac{1}{n}-\frac{1}{f}\\[4pt]\frac{1}{w_1}=\frac{1}{w}-\frac{1}{f}\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \; (3).$$

By the first two formulæ (2) we find the artificial near and far points for the lens f, when the natural points have been discovered by direct experiment. By the next two formulæ (3) we find the natural points when the artificial, with the lens f, have been noted. Hence we can, from the one, easily arrive at a conclusion about the other. Under the supposition that, as a rule, all experiments must be conducted with the aid of a lens, it will be found a matter of great convenience to use always that lens which has already served to determine the anomaly of refraction; so that, therefore, its corresponding artificial near and far points may establish the values of n and w. If then it be desired to ascertain the natural near and far points, or the values of n_1 and w_1, the formulæ (3) will suffice for this purpose. For the selection of the necessary spectacles, however, such a calculation is not needed; and, for the formulæ of this book, it is sufficient to know the artificial near and far points with the manifest emmetropic lens, since the thus observed magnitudes n and w enter immediately into these formulæ.

As regards, first, the natural far point, or that point for the fixation and clear vision of which the minimal accommodation is required, its distance, w_1, is equal to the manifest accommodation distance a_1. If, therefore, as in the foregoing, l_1 be the manifest emmetropic lens, we have $w_1 = a_1 = -l_1$, and from the formulæ (2), we find, for the distance of the artificial far point, the already stated result,

$$\frac{1}{w} = \frac{1}{w_1} + \frac{1}{l_1} = 0, \text{ or } w = \infty;$$

that is to say, this lens renders the manifest accommodation distance infinitely great, or entirely corrects the manifest ametropia.

The distance n of the artificial near point for the eye furnished with this lens, that is, of the point at which the eye, through this lens, sees still clearly, but with the extreme effort of accommodation, may be easily ascertained by simple approximation of an object. If, however, instead of such approximation, and direct measurement of the distance n of the near point, it be preferred still to fix an infinitely distant object, then the value of n may be determined by the aid of lenses. For a *concave* lens l', added to the emmetropic lens

$l_1 = -a_1$ produces an apparent approximation of the object; since it forms an optical image of this object at the distance $-l'$ in front of the emmetropic lens. If therefore l' be the strongest concave lens, or, in the whole series of concave and convex lenses, the weakest lens, through which the eye, furnished with the emmetropic lens, can see clearly an infinitely distant object, then $-l_1 = n$ is the distance of the artificial near point, and $\frac{1}{n}$ is the range of accommodation.*

If it be wished in these experiments to employ only a single lens l_2, which is certainly to be preferred; then, since this one lens has to supply the two lenses l_1 and l', $\frac{1}{l_2} = \frac{1}{l_1} + \frac{1}{l'}$, and, consequently, $\frac{1}{l'} = -\frac{1}{n} = \frac{1}{l_2} - \frac{1}{l_1}$.

If, therefore, l_2 be the weakest lens, in the combined series of concave and convex lenses, with which the eye can still see clearly an infinitely distant object, then the distance n gives the artificial near point by the formula $\frac{1}{n} = \frac{1}{l_1} - \frac{1}{l_2}$, which shows directly the range of accommodation.

It may also be remarked that, with the manifest emmetropic lens, every eye appears completely bathopic, since $\frac{1}{w} = 0$; and any hypsopia present remains latent. The degree of the actual stenopia is indicated by the value of $\frac{1}{n}$, which shows

* The words *strongest* and *weakest* in the text are exactly translated. The *power* of a lens is expressed by $\frac{1}{f}$, the reciprocal of its focal length. The strongest lens, therefore, is that in which $\frac{1}{f}$ is greatest or f least; the weakest lens has f greatest. But, when speaking of a whole series of lenses, convex and concave, the author calls that convex lens the strongest which has $+\frac{1}{f}$ the greatest; and that concave lens the weakest which has $-\frac{1}{f}$ greatest.—TRANS.

also the range of accommodation. If $\frac{1}{n}$ has the normal value of $\frac{1}{3}$ or $\frac{1}{4}$, then the eye is europic.

The formula

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{l}-\left(\frac{i}{n}+\frac{1-i}{w}\right) \quad . \quad . \quad (4),$$

from page 59, is manifestly applicable to a hyperbola that has its horizontal asymptote in the line R L, when we make $l = l_1$ the manifest emmetropic lens, n the distance of the artificial near point for this lens, and therefore $\frac{1}{n}$ the range of accommodation R B, and assume the distance w of the artificial far point to be infinitely great, so that $\frac{1}{w} = 0$. Hence this formula assumes the simple aspect,

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{l_1}-\frac{i}{n} \quad . \quad . \quad . \quad . \quad . \quad (5),$$

and serves to determine the lens f which enables the eye to see objects at the given distance s, with the degree of accommodation effort indicated by i.

When a normal eye, having its near point at a distance of three or four inches, reads by preference at a distance of nine or ten inches, it has $i = \frac{3}{10} = 0{\cdot}3$, or $i = \frac{4}{9} = 0{\cdot}44$ of its accommodation power. Under the supposition that a certain part of the power of accommodation remains latent, or that the eye cannot relax its effort below a certain minimum, the overplus R T of the middle effort above the minimal effort A R, forms a yet somewhat smaller part of the manifest range of accommodation R B, that is, of the difference between the maximum and the minimum of the accommodation. For the purpose of reading we may therefore take i to be nearly equal to $\frac{1}{3}$; and, when we make $s=10$ inches, the formula becomes,

$$\frac{1}{f}=\frac{1}{10}+\frac{1}{l_1}-\frac{1}{3n} \quad . \quad . \quad . \quad (6).$$

The formula (5) serves for the determination of the lens f, under the supposition that the eye should see at a *given distance* s, with a *given accommodation effort* i. The limits of the range of vision, to which the eye may attain with this lens, are found from the same formula, by making at one time $s = 0$, and then $s = 1$. The first substitution, $s = 0$, gives the greatest visual distance s_1 by the form

$$\frac{1}{s_1} = \frac{1}{f} - \frac{1}{l_1} \quad . \quad . \quad . \quad . \quad (7),$$

and the second substitution, $s=1$, gives the smallest visual distance, s_2, by the form

$$\frac{1}{s_2} = \frac{1}{f} - \frac{1}{l_1} + \frac{1}{n} \quad . \quad . \quad . \quad (8).$$

It is worthy of note that the difference $\frac{1}{s_2} - \frac{1}{s_1}$ is always $= \frac{1}{n}$, which is also the state of refraction l_1 of the eye, with whatever lens f it may be supplied.

If the lens f be required not for a middle distance s, and a middle accommodation i, but in order to give vision at a given maximal distance s_1 with a minimal accommodation effort $i = 0$, then from formula (7) we have

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{l_1} \quad . \quad . \quad . \quad . \quad (9);$$

and, if the lens f be determined under the supposition that the eye shall see with the maximal accommodation effort $i = 1$, in the given minimal distance s_2, then, from equation (8),

$$\frac{1}{f} = \frac{1}{s_2} + \frac{1}{l_1} - \frac{1}{n} \quad . \quad . \quad . \quad (10).$$

If it be desired to know what part i of its manifest range of accommodation an eye, supplied with any lens f, exerts in looking at any distance s, or with what accommodation effort it sees at the distance s, the equation (5) furnishes the value of i. It gives,

$$i = n\left(\frac{1}{s} + \frac{1}{l_1} - \frac{1}{f}\right) \quad . \quad . \quad . \ (11).$$

The naked eye, which is equivalent to a lens $f = \infty$, sees therefore at the distance s with the effort

$$i = n\left(\frac{1}{s} + \frac{1}{l}\right) \quad . \quad . \quad . \quad . \ (12).$$

From the foregoing it appears that, for the determination of the anomalies of refraction, only a single value need be experimentally ascertained; namely, the manifest emmetropic lens l_1; that is, the strongest lens through which the eye is able to see clearly an infinitely distant object. For the anomalies of accommodation, likewise, only a single observation is required; namely, the distance n of the artificial near point for this manifest emmetropic lens.

When these two magnitudes, l_1 and n, are found, we have all the data required to calculate, by equation (5), the power of the lens f that will give clear vision at the given distance s, with the prescribed accommodation effort i; or, by equation (9), the lens that will enable the eye to see at the given extreme distance s_1; or, by equation (10), the lens that will give clear vision at the given smallest distance s_2. The quantity $-l_1$ indicates the manifest visual distance, therefore the degree of manifest ametropia, and the quantity $\frac{1}{n}$ indicates the manifest range of accommodation, or the degree of stenopia from which the eye suffers.

If, for example, we find that an eye sees clearly in the distance with the concave lens $l_1 = -30$, and that with this lens it sees also at a near point of which the distance $n = 5$ inches, then, because $-l_1 = 30$, the eye is hypometropic, and because $\frac{1}{n} = \frac{1}{5}$, it is stenopic. The lens with which this eye will see at a distance $s = 9$ inches, with an effort i equal to one-third of its accommodation, is, according to equation (5),

$$\frac{1}{f}=\frac{1}{9}-\frac{1}{30}-\frac{1}{3\times 5}=\frac{1}{90};$$

therefore the concave lens f required is $=\frac{1}{90}$. If the eye requires vision at the maximal distance of $s_1 = 240$ inches, then, by equation (9),

$$\frac{1}{f}=\frac{1}{240}-\frac{1}{30}=-\frac{7}{240}=-\frac{1}{34} \text{ nearly,}$$

and therefore the concave lens $\frac{1}{34}$ is required. If it be desired to see with the maximal accommodation effort at the distance $s_2 = 3$ inches, then, from equation (10),

$$\frac{1}{f}=\frac{1}{3}-\frac{1}{30}-\frac{1}{5}=\frac{1}{10},$$

and a convex lens $f = 10$ would be required.

If the eye be furnished with the convex lens $f = 20$, then it will see, at a distance $s = 8$ inches, according to equation (11), with the effort,

$$i = 5\left(\frac{1}{8}-\frac{1}{30}-\frac{1}{20}\right)=\frac{5}{24}.$$

At the same distance, according to equation (12), the naked eye would see with the effort,

$$i = 5\left(\frac{1}{8}-\frac{1}{30}\right)=\frac{11}{24}.$$

If it were found by experiment that an eye could see clearly in distance with a convex lens $l_1 = 30$, and that, when furnished with this lens, its near point was at a distance $n = 5$ inches, such an eye, because $-l_1 = -30$, would be hypermetropic, and, because $\frac{1}{n}=\frac{1}{5}$, it would be stenopic. Vision at $s = 9$ inches, with $i = \frac{1}{3}$ of its accommodation, would require, by equation (5),

$$\frac{1}{f}=\frac{1}{9}+\frac{1}{30}-\frac{1}{3\times 5}=\frac{7}{90}=\frac{1}{13} \text{ nearly,}$$

a convex lens $f = 13$; vision at the maximal distance $s_1 = 240$ inches, by equation (9),

$$\frac{1}{f} = \frac{1}{240} + \frac{1}{30} = \frac{3}{80} = \frac{1}{27} \text{ nearly,}$$

a convex lens $f = 27$; and vision, at the minimal distance $s_2 = 3$ inches, by equation (10),

$$\frac{1}{f} = \frac{1}{3} + \frac{1}{30} - \frac{1}{5} = \frac{1}{6},$$

a convex lens $f = 6$.

If the same eye were furnished with a concave lens $f = -24$, it would see, by equation (11), at the distance $s = 40$ inches, with the effort,

$$i = 5\left(\frac{1}{40} + \frac{1}{30} + \frac{1}{24}\right) = \frac{1}{2}.$$

The naked eye, by equation (12), would see at the same distance with the effort,

$$i = 5\left(\frac{1}{40} + \frac{1}{30}\right) = \frac{7}{24}.$$

It has already been shown that the distance n of the artificial near point may be deduced from observation of the *weakest* lens, l_2, through which the eye is still able to see clearly an infinitely distant object. Hence, the anomaly of refraction is determined by the *strongest* lens l_1, and the anomaly of accommodation by the *weakest* lens l_2, through which the eye can see clearly at distance. The quantity $-l_1$ expresses the manifest visual distance, that is, the degree of manifest ametropia; and the quantity $\frac{1}{n_1} = \frac{1}{l_1} - \frac{1}{l_2}$, the manifest range of accommodation, or the degree of stenopia, from which the eye suffers. By means of the two lenses, l_1 and l_2, the five formulæ, (5), (9), (10), (11), and (12), assume the valuable forms,

$$\frac{1}{f} = \frac{1}{s} + \frac{1-i}{l_1} + \frac{i}{l_2} \quad . \quad . \quad (13),$$

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad (14),$$

$$\frac{1}{f} = \frac{1}{s_2} + \frac{1}{l_2} \quad . \quad . \quad . \quad . \quad . \quad (15),$$

$$i = \frac{\frac{1}{s} + \frac{1}{l_1} - \frac{1}{f}}{\frac{1}{l_1} - \frac{1}{l_2}} \quad . \quad . \quad . \quad (16),$$

$$i = \frac{\frac{1}{s} + \frac{1}{l_1}}{\frac{1}{l_1} - \frac{1}{l_2}} \quad . \quad . \quad . \quad . \quad . \quad (17),$$

which will be at once understood when it is observed that, in Fig. 38, $i = \frac{\text{R T}}{\text{R B}}$, and $1 - i = \frac{\text{T B}}{\text{R B}}$.

If, for example, $l_1 = 15$ were the strongest, and $l_2 = -15$ the weakest lens, with which an eye could see clearly in distance, such an eye, because $-l_1 = -15$, is hypermetropic; and, because $\frac{1}{n} = \frac{1}{l_1} - \frac{1}{l_2} = \frac{1}{15} - \left(-\frac{1}{15}\right) = \frac{2}{15} = \frac{1}{7\frac{1}{2}}$, it is strongly stenopic.

In order that it may see at nine inches distance, with an accommodation effort $i = \frac{1}{3}$, by equation (13),

$$\frac{1}{f} = \frac{1}{9} + \frac{2}{3 \times 15} - \frac{1}{3 \times 15} = \frac{2}{15} = \frac{1}{7\frac{1}{2}},$$

a convex lens $f = 7\frac{1}{2}$ will be required.

In order to see without effort at a maximal distance $s_1 = 240$ inches, by equation 14,

$$\frac{1}{f} = \frac{1}{240} + \frac{1}{15} = \frac{17}{240} = \frac{1}{14} \text{ nearly},$$

a convex lens $f = 14$ will be required.

In order to see with the full effort of accommodation at a minimal distance $s_1 = 3$ inches, by equation (15),

$$\frac{1}{f} = \frac{1}{3} - \frac{1}{15} = \frac{4}{15} = \frac{1}{3\frac{3}{4}},$$

a convex lens $f = 3\frac{3}{4}$ must be employed.

If this eye be supplied with a concave lens $f = -30$, it will see, by equation (16), at a distance $s = 100$ inches, with the effort

$$i = \frac{15}{2} \cdot \left(\frac{1}{100} + \frac{1}{15} + \frac{1}{30}\right) = \frac{33}{40}.$$

Without a lens, at the same distance, by equation (17), it would see with the effort

$$i = \frac{15}{2} \cdot \left(\frac{1}{100} + \frac{1}{15}\right) = \frac{23}{40}.$$

If $l_1 = 10$ were the strongest, and $l_2 = 30$ the weakest lens, with which an eye could still see clearly in distance, such an eye, because $-l_1 = -10$, is hypermetropic. Its range of accommodation $\frac{1}{n} = \frac{1}{l_1} - \frac{1}{l_2} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}$. It is therefore highly stenopic.

If $l_1 = -30$ were the strongest, and $l_2 = -10$ the weakest lens, then the eye, because $-l = 30$, is hypometropic. It has the range of accommodation $\frac{1}{n} = \frac{1}{l_1} - \frac{1}{l_2} = -\frac{1}{30} - \left(-\frac{1}{10}\right) = \frac{1}{15}$; and is therefore stenopic in the same degree as the preceding.

We may next observe that, when it is not desired to ascertain all the peculiarities of the eye, but only to select the necessary spectacles for a given distance s, this may be done immediately by the following method.

The object being placed at the given distance s, ascertain by trials the strongest and the weakest lens, l' and l'', through which it can still be clearly seen in monocular vision. Then choose the required lens by the formula

$$\frac{1}{f} = \frac{1-i}{l'} + \frac{i}{l''},$$

in which i may have a value between $\frac{1}{3}$ and $\frac{1}{2}$. The smaller the difference between the two lenses l' and l'', the more important is it to make $i = \frac{1}{2}$, and therefore

$$\frac{1}{f} = \frac{1}{2}\left(\frac{1}{l'} + \frac{1}{l''}\right).$$

It is manifest that, in the latter cases it is often practicable to determine the lens f, that produces the smallest visual strain at the given distance, by direct experiments.

If it were desired, for example, to discover a suitable lens f, for reading a particular writing at a given distance s, nothing could be more simple and satisfactory than such experiments.

Lastly, it should be remarked that, when, in these observations, no infinitely distant object of sufficient size and illumination is attainable, so that some object at a known finite distance x is employed, the trials can be conducted in the following manner. We ascertain the strongest and the weakest lens, λ_1 and λ_2 through which the eye still sees this object clearly. Since the lens $\frac{1}{x}$ would apparently displace the object to infinite distance, the lens λ_1 produces precisely the same effect as the two lenses x and l_1 and therefore

$$\frac{1}{\lambda_1} = \frac{1}{x} + \frac{1}{l_1}, \text{ whence}$$

$$\frac{1}{l_1} = \frac{1}{\lambda_1} - \frac{1}{x},$$

$$\frac{1}{l_2} = \frac{1}{\lambda_2} - \frac{1}{x}.$$

If we substitute these values for $\frac{1}{l_1}$ and $\frac{1}{l_2}$, in the formulæ (13) to (15), we shall obtain the equations from which the lens f is to be determined for the several conditions. For example, the equation (13), for determining the lens with which the eye will see at the given distance s, with the effort i, would become

$$\frac{1}{f} = \frac{1}{s} - \frac{1}{x} + \frac{1 - i}{\lambda_1} + \frac{i}{\lambda_2}.$$

When, however, the object is distant at least twenty feet, = 240 inches, $\frac{1}{x}$ is less than $\frac{1}{240}$, and therefore so small that it may be disregarded, and the distance treated as infinite, unless the most extreme accuracy of calculation be desired.

2. *Anomalies of Direction and Motility.*

It has already been shown that the object of Convergence is *fixation*, or *direct vision* with the poles of the eyeballs; and that the object of application is *single vision*, or the production of similar apparent visual directions. It has also been shown that the defects of convergence, which constitute the anomalies of direction and motility, are rationally treated not by optical, but by surgical means; and that the defects of application, on the contrary, which constitute the anomalies of position and nutation, are to be relieved by prismatic spectacles. But as, notwithstanding, the prismatic spectacles are also applied to diminish defects of convergence, it must be borne in mind that they can only completely fulfil this purpose when they do not come into conflict with the application. And, since the present object is to determine the prism required for any defect of convergence, it is well to simplify the examination by the following preliminary assumptions:

a. That the two eyes are alike in their refraction and accommodation.

b. That the accommodation is not modified either by the convergence or the application.

c. That the application is undiminished and unimpeded.

All fundamental principles, as regards manifestation, latency, and totality, which have been laid down under Refraction, apply also to ocular direction. There is a minimum of motility, below which the action of convergence, or the motility in conscious fixation, does not descend. The motility lying between this minimum and the state of absolute rest remains latent; and only that manifests itself which lies between the maximum and the minimum, and constitutes the *range*, or, when we consider the distance of the corresponding far and near points of the convergence, the *territory* of convergence. If we possessed a means of paralysing the motility, as atropine paralyses the accommodation, we should then be in a position to study the state of rest of the convergence. In the absence of such means, it is possible that we might attain to such a state of rest by the suppression of all visual function; but we abstain from the attempt, because it is not necessary for the practical determination of the spectacles actually required. For this purpose we remain at the stand-point assumed with regard to the lens required for the accommodation.

If the defective convergence of the two eyes were unequal, each eye would certainly require its especial prism. If, however, the sum of the two prisms be divided equally between the two eyes, there will be produced only a false projection. If we overlook this, or if we have it in view to correct the false projection by a measured division of the prisms, we may always operate (in examining) with equal prisms before the two eyes; and this is desirable in order to facilitate the experiments.

For conducting these experiments a series of negative and positive prismatic spectacles are required. At the extreme left of the series is the strongest negative pair, with bases turned outwards; and which, therefore, deflect the rays outwards, and, apparently, displace the object inwards. At the extreme right of the series is the strongest positive pair, with bases turned inwards; and which, therefore deflect the rays inwards, and apparently displace the object outwards. The left hand (strongest negative) spectacles, we call the weakest of the whole series; and the right hand (strongest positive) the strongest. We define each prism, not directly by its angle of total deviation ψ, but by its total proportion of deviation h, which is the goniometric tangent of this angle, or $= \tan \psi$; and which, when d is the distance between the central points of the eyes, and g is the convergence distance, that is, the distance of the point of convergence of two parallel rays when refracted by the prisms, has the value $h = \tan \psi = \frac{d}{2g}$: g being positive for positive, and negative for negative prisms. We also often use the convergence distance g to denominate the corresponding prismatic spectacles. In the supposed connected series, the values of g form, therefore, a series of numbers, as -10, -50, -100, $+\infty$, 100, 50, 10; and the values of $\frac{1}{g}$, which varies as the proportion of deviation h, form a steadily increasing series of numbers, as $-\frac{1}{10}$, $-\frac{1}{50}$, $-\frac{1}{100}$, 0, $\frac{1}{100}$, $\frac{1}{50}$, $\frac{1}{10}$.

For every degree of accommodation the eye is permitted a certain play of the convergence, and, therefore, a certain relative minimum and maximum of convergence, corresponding to the relative range of convergence. If g' indicate the strongest, and g'' the weakest prismatic spectacles through which a pair of eyes can see a given object clearly and singly; so that the eyes, in using the strongest prisms, exert their relative minimum, and in using the weakest exert their relative maximum of convergence, it follows, from considerations like those already stated with regard to accommodation, that $\frac{1}{m} = \frac{1}{g'} - \frac{1}{g''}$ will be the relative range of convergence for the given degree of accommodation.

If j indicate that portion of the relative range of convergence with which the pair of eyes exerts the given degree of accommodation most pleasantly, then the formula

$$\frac{1}{g} = \frac{1-j}{g'} + \frac{j}{g''} \quad . \quad . \quad . \quad (18)$$

will give the prismatic spectacles required; and this, by making $j = \frac{1}{2}$, will be reduced to the arithmetic mean

$$\frac{1}{g} = \frac{1}{2}\left(\frac{1}{g'} + \frac{1}{g''}\right).$$

If, therefore, it be desired to determine the prismatic spectacles that should be combined with given spherical spectacles, the most rational method is to furnish the eyes with the spherical spectacles, and then, while an object placed at the distance for which the spherical spectacles are adapted is being regarded, to determine the strongest and the weakest prismatic spectacles, g' and g'', with the addition of which the object can still be seen clearly and singly; in order to determine by equation (18) the prisms necessary to be combined with the spherical glasses: prisms of which the proportion of deviation would be

$$h = \frac{d}{2g} = \frac{d}{2}\left(\frac{1-j}{g'} + \frac{j}{g''}\right) \quad . \quad . \quad (19).$$

When the necessary spherical glasses have not been fixed upon, or when it is intended afterwards to prescribe these for different conditions; so that it is not practicable to work at once with these glasses, the determination of the prismatic spectacles is of a more general kind; since it then furnishes an index of the condition of the faculty of convergence of the pair of eyes. Under such circumstances the following method may be adopted.

The eyes being first of all furnished with their manifest emmetropic spectacles l_1, an infinitely distant object is used as a point of fixation. The eye is thus placed in the condition of minimal accommodation. In this condition must be determined the strongest and the weakest prismatic spectacles, g' and g'', with which the object can be seen clearly and singly. Then $-g_1'$ shows the absolute manifest convergence distance, or $-\frac{d}{2g'}$ the minimum of the ratio of convergence (the tangent of half the angle of convergence of the two ocular axes in the minimal convergence effort), and characterizes the pair of eyes as hypogonic or as hypergonic, according as $-g_1'$ is positive or negative. The prismatic spectacles adapted for the state of minimal accommodation will then be

$$\frac{1}{g_1} = \frac{1-j}{g_1{}'} + \frac{j}{g_1{}''} \quad . \quad . \quad . \quad (20);$$

or, if it be desired to express them by the proportion of deviation,

$$h_1 = (1-j)\, h_1{}' + j h_1{}''.$$

The pair of eyes must next be furnished with the weakest spherical spectacles, l_2, with which each single eye in its maximal accommodation can clearly see the infinitely distant object. In this condition must again be determined the strongest and the weakest prismatic spectacles, $g_2{}'$ and $g_2{}''$, through which the object can still be seen clearly and singly. The latter, $g_2{}''$, corresponds to the maximum of convergence action, so far as this can be exerted with the given accommodation. We may therefore say,

$$\frac{1}{m} = \frac{1}{g_1{}'} - \frac{1}{g_2{}''} \quad . \quad . \quad . \quad (21);$$

or, expressed by the proportion of deviation,

$$\frac{d}{2m} = h_1{}' - h_2{}'',$$

as the absolute manifest range of convergence, according to the extent of which the degree of stenogonia is to be estimated. The prismatic spectacles needed for the maximal accommodation would be,

$$\frac{1}{g_2} = \frac{1-j}{g_2{}'} + \frac{j}{g_2{}''}. \quad . \quad . \quad . \quad (22);$$

or, by proportion of deviation,

$$h_2 = (1-j)\, h_2{}' + j h_2{}''.$$

We have expressed by the letter i that part of the range of accommodation which is employed in a visual act. According to the formulæ (20) and (22), in the accommodation $i = 0$ the prism g_1, and in the accommodation $i = 1$ the prism g_2, is to be applied. Hence we may deduce the result that it is reasonable, in the accommodation i, to use the prism determined by

$$\frac{1}{g} = \frac{1-i}{g_1} + \frac{i}{g_2} \quad . \quad . \quad . \quad . \quad (23);$$

of which the proportion of accommodation is

$$h = (1-i)\, h_1 + i h_2.$$

Hence, therefore, with the spherical spectacles f (which, by equation (5) or (13), confer upon the pair of eyes the power to see at the distance s, and with the accommodation effort i), we must combine the prismatic spectacles g, found by equation (23), and having the proportion of deviation h.

With the spherical glasses (9) or (14), which are to be used at the maximal distance s_1, must be combined the prismatic glasses g, which are found by equation (23) for $i = 0$, and therefore with $\frac{1}{g} = \frac{1}{g_1}$, or $g = g_1$; that is, the prismatic glasses found by equation (20).

With the spherical glasses (10) or (15), which are to be used at the minimal distance s_2, must be combined the prismatic glasses g_1, given by equation (23) when $i = 1$, and therefore $\frac{1}{g} = \frac{1}{g_2}$, or $g = g_2$; that is, the prismatic glasses found by equation (22).

We may yet observe that, following these conclusions, the strongest and the weakest prisms, g' and g'', which can be overcome in the accommodation i, will correspond respectively to the formulæ—

$$\frac{1}{g'} = \frac{1-i}{g_1'} + \frac{i}{g_2'} \quad . \quad . \quad . \quad . \quad (24),$$

$$\frac{1}{g''} = \frac{1-i}{g_1''} + \frac{i}{g_2''} \quad . \quad . \quad . \quad . \quad (25);$$

and that the most suitable prism g, which is determined by equation (23), will correspond equally to the formula given in equations (20) and (22), and contained in equation (18),

$$\frac{1}{g}=\frac{1-j}{g'}+\frac{j}{g''} \quad . \quad . \quad . \quad . \quad (26),$$

when we substitute for g' and g'' the values taken from equations (24) and (25); since, then, the right sides of the two formulæ (23) and (26) become identical. Furthermore, it follows, from equation (24) and (25) that the relative range of convergence for the accommodation i has the value,

$$\frac{1}{g'}-\frac{1}{g''}=(1-i)\left(\frac{1}{g_1'}-\frac{1}{g_1''}\right)+i\left(\frac{1}{g_2'}-\frac{1}{g_2''}\right) . \ (27).$$

In the preceding formulæ, which serve for the determination of the prismatic spectacles g, necessary to be combined with the spherical spectacles f, we have introduced a quantity j, which shows the proportion in which it is desirable to employ the relative range of convergence for a given accommodation. The average value of j can only be deduced from the function of normal eyes in actual use; that is, by observing in what degree a pair of normal eyes, in constant accommodation, can exert or relax convergence, short of the production of defined double images. In the absence of certain data, we may assume $j=\frac{1}{2}$, which would lead us to take the arithmetic mean between the strongest and the weakest prism which the eye in a given accommodation, was able to bear. We have no observations upon the play of the convergence in constant accommodation, or upon the relative range of convergence; but only upon the play of accommodation in constant convergence, or upon the relative range of accommodation. The latter may be easily instituted by the help of common spherical spectacles, since the change of such spectacles during vision of one and the same object changes the accommodation only, and leaves the convergence unaltered. We may assume that, as a general rule, applicable to all near and remote objects, the effort of accommodation which a pair of eyes is able to exert continuously, without strain, in a given convergence, is about the mean between the greatest and least effort possible in this convergence. Proceeding from this assumption, we will endeavour to arrive at a conclusion upon the value of j, or upon the greatest and least convergence effort which the pair of eyes is capable of making under unchanged accommodation.

According to the foregoing formula we have the pair of eyes with the accommodation effort i, bearing a strongest prism g', and a weakest g''. If we suppose these eyes to assume a greater accommodation i_2, then the strongest prism applicable would be, by equation (24),

$$\frac{1-i_2}{g_1{}'}+\frac{i_2}{g_2{}'}.$$

If we further suppose the pair of eyes to assume the smaller accommodation i_1, then the weakest prism applicable would be, by equation (25),

$$\frac{1-i_1}{g_1{}''}+\frac{i_1}{g_2{}''}.$$

Assuming also, that the above-named two prisms are together equal to the prism g, with which the eye sees without strain in the accommodation i_1, it will be obvious that i_1 and i_2 indicate the weakest and the strongest accommodation that the pair of eyes are capable of assuming in the constant convergence brought about by the prismatic spectacles g. According to the above-mentioned experiments, therefore, we should obtain as a general rule, nearly,

$$i_2-i=i-i_1 \text{ or } i=\tfrac{1}{2}(i_1+i_2);$$

and, in order, for the present, to keep the calculation free from the hypothesis that $i_2-i=i-i_1$ we may assume, generally, that the ratio k exists between i_2-i and $i-i_1$, and that we have

$$\frac{i_2-i}{i-i_1}=k.$$

The first two conditional equations are now

$$\frac{1-i_2}{g_1{}'}+\frac{i_2}{g_2{}'}=\frac{1}{g},$$

$$\frac{1-i_1}{g_1''} + \frac{i_1}{g_2''} = \frac{1}{g};$$

and from these it follows that—

$$i_1 = \frac{\frac{1}{g_1''} - \frac{1}{g}}{\frac{1}{g_1''} - \frac{1}{g_2''}},$$

$$i_2 = \frac{\frac{1}{g_1'} - \frac{1}{g}}{\frac{1}{g_1'} - \frac{1}{g_2'}}.$$

If we insert these values of i_1 and i_2 in the third conditional equation $i_2 - i = k(i - i_1)$, and substitute for $\frac{1}{g}$ its value in equation (26), substituting also for $\frac{1}{g'}$ and $\frac{1}{g''}$ their values in equations (24) and (25) we obtain the highly interesting result that the quantity i, which exhibits the assumed accommodation effort, wholly disappears: since the equation may be divided by a factor which contains only the quantity i. The remaining equation then affords for j the solution

$$j = \frac{k}{k + \frac{\frac{1}{g_1''} - \frac{1}{g_2''}}{\frac{1}{g_1'} - \frac{1}{g_2'}}} \quad . \quad . \quad . \quad (28).$$

This formula teaches, *that the most suitable portion of the range of convergence is wholly independent of the special value i of the accommodation; and that j indicates a proportion that is of one and the same constant value for every degree of accommodation.* This value of j is found by the four prisms g_1', g_2', g_1'', g_2''; and especially by the range of convergence $\frac{1}{g_1'} - \frac{1}{g_2'}$, which corresponds to the two strongest prisms for the least and greatest accommodation, and by the range of convergence $\frac{1}{g_1''} - \frac{1}{g_2''}$, which corresponds to the two weakest prisms for the least and greatest accommodation. The ratio of these two ranges does not, on an average, deviate much from the value 1; and, if we call it approximatively = 1, we obtain

$$j = \frac{k}{k+1} \quad . \quad . \quad . \quad . \qquad (29);$$

and, when we substitute for k its average value 1, we obtain $j = \frac{1}{2}$.

We are thus led to the very noteworthy result that, *while it appears reasonable, in a given convergence, to exert half of the manifest relative range of accommodation, it is also reasonable, as a consequence of this disposition, in a given accommodation to exert half of the manifest relative range of convergence.*

By the value $j = \frac{1}{2}$, all the foregoing formulæ are simplified; and the process for determining the prismatic spectacles g will be the following:—

Determine the strongest and the weakest prismatic spectacles, g_1' and g_1'', with which the eyes, furnished with the strongest spherical spectacles l_1, can yet see an infinitely distant object clearly and singly. Determine also the strongest and weakest prismatic spectacles, g_2' and g_2'', with which the eyes, furnished with the weakest spherical spectacles l_2, can yet see the same object clearly and singly. The prismatic spectacles g, proper to be combined with the spherical spectacles f—which are adapted for use at the distance s, and with the accommodation effort i, and are found by equation (5) or (13),—may be found by equation (23) or (26) by the formula

$$\frac{1}{g} = \frac{1-i}{2}\left(\frac{1}{g_1'} + \frac{1}{g_1''}\right) + \frac{i}{2}\left(\frac{1}{g_2'} + \frac{1}{g_2''}\right) \quad . \ (30).$$

If the prismatic spectacles are to be combined with spherical spectacles adapted for use at the maximal distance s_1, as by equation (9) or (14), when i is = 0, therefore

$$\frac{1}{g} = \frac{1}{2}\left(\frac{1}{g_1'} + \frac{1}{g_1''}\right) \quad . \quad . \quad . \; (31).$$

If they are to be combined with spherical spectacles adapted for the minimal distance s_2, by equation (10) or (15), when $i = 1$, then

$$\frac{1}{g} = \frac{1}{2}\left(\frac{1}{g_2'} + \frac{1}{g_2''}\right) \quad . \quad . \quad . \; (32).$$

If the prisms be designated by their proportions of deviation h_1', h_1'', h_2', h_2'', we have, instead of equation (30),

$$h = \frac{1-i}{2}(h_1' + h_1'') + \frac{i}{2}(h_2' + h_2'') \; . \quad (33);$$

instead of equation (31),

$$h = \frac{1}{2}(h_1' + h_1'') \; . \quad . \quad . \quad . \; (34);$$

and, instead of equation (32),

$$h = \frac{1}{2}(h_2' + h_2'') \; . \quad . \quad . \quad . \; (35).$$

It is also manifest that, in these formulæ, either the angle of deviation, or the refracting angle of the glass, may be substituted for h at pleasure.

If, for example, it were found in a given case that the strongest and weakest prismatic spectacles that could be combined with the strongest spherical spectacles had proportions of deviation $h_1' = 0{\cdot}015$, $h_1'' = -0{\cdot}012$; and that the strongest and weakest prismatic spectacles that could be combined with the weakest spherical spectacles had proportions of deviation $h_2' = 0{\cdot}045$, $h_2'' = -0{\cdot}051$; then, according to equation (23), we should combine with the spherical spectacles f, adapted for use with the accommodation effort i, prisms having the proportion of deviation $h = \frac{1-i}{2}(0{\cdot}015 - 0{\cdot}012) + \frac{i}{2}(0{\cdot}045 - 0{\cdot}051) = 0{\cdot}0015(1-i) - 0{\cdot}003i$. For $i = \frac{1}{2}$, $h = -0{\cdot}00075$; that is, there would be required very weak negative prisms (with bases outwards), having a proportion of deviation 0·00075, or, by page 77, an angle of deviation 0·00075 . 57·3 = 0·043°, in order to produce the most comfortable vision. If $i = \frac{1}{3}$, then $h = 0$; that is, no prisms would be required at all.

3. *Anomalies of Position and Nutation.*

While the convergence has for its sole purpose *fixation*, or direct vision with the poles of the eyeballs, the application produces single vision, or vision of the object in similar directions. The two functions are thoroughly independent and essentially distinct. Defects of convergence produce, therefore, phenomena wholly different from those of defects of application.

In a case of failure of convergence, direct vision with both eyes is impossible, and squinting is necessarily produced. In defective fixation the object seen no longer appears the chief object, or the concentration point of the attention and consciousness; but as something accessory or collateral which only engages the attention secondarily, or to which it cannot be completely devoted: the incomplete fixation is therefore united with a feeling of imperfect satisfaction. This feeling of imperfect satisfaction of the visual organs, from deficient attractive power of the object, is at its greatest, and hence the value of the object as such is at its least, when neither eye fixes. Generally, in a defective convergence, the conflict between the two eyes terminates by one of them fixing, while the other deviates so much the more, and sees only indirectly, that is, not with the polar bacillus, and consequently squints. As a rule, the angle of convergence of the two ocular axes does not correspond to the angle of convergence of the visual lines, and rays of light impinge upon different parts of the two retinæ. With this squint, arising from a defect of convergence, there is on principle absolutely no double vision united. If the power of application be sufficient, the affected bacilli assume, notwithstanding the squinting, the indirect vision, and the difference of the nerve fibres, parallel positions; so that the object is seen in like directions, and therefore singly. Double vision first occurs when the application is insufficient.

If there be a defect of application, the affected bacilli do not assume a parallel position; but exhibit the object in different directions, and produce double vision. This double vision, produced by insufficient application, is not in principle combined with squinting or indirect vision: the eyes may perfectly fix the object, and therefore see with their polar bacilli; although, nevertheless, these bacilli, from not assuming a parallel direction, may produce double images; that is, two fixed double images. The striving after single vision is, however, generally greater than that after fixation. Consequently, as a rule, the former preponderates over the latter, and compels the eyes to abandon their fixation, and to squint just so much that the different bacilli (one polar and one eccentric) which are impressed, have the power to become parallel, and thus to afford a single image; in so far as the faculty of convergence may be able to accomplish the required degree of squint. It is only when the convergence is insufficient for this purpose, that there remains a permanent diplopia caused by the defect of application, and only moderated by the convergence effort.

We learn from this that we have, as a consequence of defects of convergence, *squinting with single vision*; and, as a consequence of defects of application, *double vision without squinting*, which may give way to *squinting with single vision*; and also that, from either defect, *squinting with double vision* may be produced; in so far as in defects of convergence the application, and in defects of application the convergence, may be insufficient to render single vision possible by means of increased squinting.

Squinting with double vision is, as a rule, the result of the complication of a defect of convergence with a defect of application, and results from the tendency to fixation and to single vision. Generally speaking, that is, when the attainment of single vision is not too difficult, or when the images are not too far apart, the struggle between fixation and single vision will end in the victory of the latter tendency; that is, the eyes will so place themselves, that the affected bacilli, by the employment of a corresponding degree of application, will become parallel, and furnish a single visual impression; although by this the fixation may be sacrificed, or, more frequently, limited to a single eye. In general the direction of the parallel bacilli will deviate from the direction of the middle visual line, drawn to the centre of the forehead from the object; and the object will therefore appear, although single, in a false position, or will be *falsely projected.* This false projection will vary, accordingly as one eye or the other takes up the fixation.

Since either of the two faculties will the more easily be insufficient, the more it is already overtaxed by its use in normal vision, and as this strain will vary with the distance of the object, or with the accommodation, it may very well happen that a pair of eyes may behave very differently in near and in distant vision, and may in the one or the other case squint or not squint, see singly or double, project falsely or truly.

When it is desired to obtain a clear and complete knowledge of the peculiarities of the visual organs in all their functions, which depend partly upon formation and partly upon innervation, a variety of suitable experiments must be instituted. Of these some will require optical aids, some, instruments for geometrical measurement, some, mechanical apparatus, such as an ocularium, and graphic delineations and diagrams. Moreover, these experiments must be partly during monocular, partly during binocular vision. The theory of ocular defects already given in this volume, as well as the theory of ocular movements contained in my earlier writings, especially in the *Gesetze des raümlichen Sehens*, and in the *Physiologische Optik*, furnish the necessary data for the conduct of these experiments. Whenever practical ophthalmology succeeds in discovering two anæsthetic agents, which, as atropia paralyses the accommodation, shall paralyse respectively the convergence (the motility or muscular apparatus), and the application (the nutation of the bacilli), the use of such agents would be of great value in establishing the defects of ocular direction during rest of the convergence, and the defects of position during rest of the application. In the absence of such means, the accurate determination of all sources of defect would be too laborious; and, even when they are eventually discovered, would be a proceeding by no means without difficulty.

But when the sole object of the examination is to fix upon the most appropriate spectacles, and since defects both of convergence and of application are to be combated by the same optical means—the prismatic spectacles—we may be content to forego the accurate determination of every defect. The method already described will be fully sufficient, when the strongest and weakest prisms are selected with reference to the obtaining of an adequate visual effect: that is, an effect by which the requirements of single vision and of fixation are both sufficiently provided for.

It should also be mentioned that the combination spectacles f, g, selected in the manner described, since they are adapted only to the manifest, and not to the total, anomaly of refraction, direction, and position, are specially fitted only for the actually present time. It is possible that they may admit of improvement after a while; because, under the influence of their use, latent defect may possibly become gradually manifest.

It is even possible that the addition of the prisms, determined in the manner described, may render manifest a part of the defect that had remained latent under the application of centric lenses. This would show itself when the eyes were furnished with the plano-prismatic spectacles g, and if then, in binocular vision, the strongest spherical spectacles l_1, with which an infinitely distant object could still be clearly seen, were ascertained by experiment.

4. *Spectacles for dissimilar eyes.*

Let us first suppose that the two eyes of a pair are alike in respect of their refraction and accommodation, although not normal, and that they have dissimilar defects of convergence or of application. However considerable the dissimilarity may be, it will always be practicable, by the methods described under section No. 2, to find prismatic spectacles, with two similar glasses, which shall produce at pleasure one of two effects: either that both eyes shall see with their polar bacilli, and therefore fix their apparent objects, although they may not blend them into a single one; or that the two eyes shall see only one object, although in a false direction, and not with both alike by the polar bacilli.

Direct vision with both eyes at the expense of double images can scarcely ever, or at least very rarely, be the goal of a rational selection of spectacles; and in almost every case the second result would be the one aimed at; namely, so to select the prismatic glasses that single vision should be the result. The fulfilment of the first condition—direct vision with both eyes—is not thus unconditionally excluded, and can in many cases be accomplished together with the other, when the state of the convergence does not oppose too great difficulties. If these difficulties should be insurmountable—and their being so would soon become evident in the course of the above-described experiments—we must aim at the attainment of single vision, which will, however, be subject to the two following imperfections. In the first place, one eye will see directly with the polar bacillus, and the other indirectly with a lateral bacillus; and both eyes will therefore periodically lose their direct vision. Secondly, the object will appear in a wrong direction, or *falsely projected.* The first of these two deficiencies, namely, the unilateral fixation, cannot be remedied by any optical means, but only by an operation on the muscle. The second, however, the false projection, may be corrected by the use of dissimilar prisms. The determination of these is very simple.

Assuming that the combination spectacles, selected according to the rules already laid down, have the effect of apparently displacing an object, at a distance of a inches, b inches too far to the left; then for the restoration of the right projection, it would be necessary to place before each eye a prism with its base turned to the left, and having a proportion of deviation $q = \frac{b}{a}$. This prism, according to the foregoing definition, would be negative for the left eye, since it would displace the object inwards, and positive for the right eye, since it would displace the object outwards. Hence, the two equal prisms h, determined as in the last section, must be exchanged for two unequal prisms, which must have, for the left eye and the right eye, the proportion of deviation of $h - q$, and $h + q$, respectively.

If the object were projected towards the right, then the two prisms for the left and for the right eye would be $h + q$, and $h - q$.

We see, therefore, that a pair of eyes, alike in their refraction and accommodation, may always be furnished with prismatic spectacles, that will afford them a single and rightly projected apparent object, although it be not fixed with both eyes. It is important to observe that the apparent size of the object will be the same, as a rule, for both eyes; and that, therefore, both eyes can participate, without strain, in binocular vision. The proof of this assertion rests upon my theory of apparent size and distance. According to this theory, the apparent size and distance is determined by the quotient $\frac{\beta}{\gamma}$ (p. 127) in which β represents, under normal conditions, the tangent of the angle of accommodation, and under abnormal conditions, or generally, the exerted accommodation effort; and γ, under normal conditions, the tangent of the angle of application, and under abnormal conditions, or generally, the exerted application effort. If we consider that, when the object appears single, that is, to both eyes in the same direction, the two affected bacilli are parallel; and that they, therefore, make nearly equal angles with the visual lines, or with the lines drawn from them through the central points of the eyes; therefore, the application effort, γ, is also nearly the same for both eyes, unless they suffer from anomalies of position of different degrees. Moreover, it has been supposed that there is similarity of refraction and accommodation, and hence the accommodation effort, β, will also be alike for both eyes. Since, therefore, the quotient $\frac{\beta}{\gamma}$ has for each eye an equal value, and since the distance of the actual or apparent object is the same for each, so it must appear to each of equal magnitude.

Only in those cases in which the state of the position in the two eyes, and therefore the angle of nutation of the bacilli in the state of rest of the application is different, would the quantity γ be different for the two, since it indicates the effort of application, not the angle of application or nutation. The object would then appear to one eye larger, in the same degree as the effort of application, γ, of that eye, was smaller than the same effort of the other.

Equality of apparent size is an important, or even an indispensable condition of binocular joint vision; so that to it correct projection must undoubtedly be sacrificed. If it be found, therefore, that the tests prescribed in the second section of this chapter, which have for their object to discover prisms adapted for single vision, and, whenever possible, for direct vision also, have the effect of showing the object, single, indeed, but of unequal magnitude for the two eyes, then we must conclude that there exists a defect of application, either unilateral or unequal in the two eyes. In such a case we must no longer endeavour to remove the defect of projection produced by the given combination spectacles, but the inequality of the apparent size of the two visual impressions received through them. The necessary method of procedure is based upon the following considerations.

When, as in Fig. 58, the two polar or eccentric bacilli, b and b', are impressed, it is necessary for single vision that $b\ e$ and $b'\ e'$, the directions of these bacilli, shall be parallel. If there were an equal defect of position in both eyes, the equality of the application effort, γ, would determine the equality of the angles of nutation, $e\ b\ c$ and $e'\ b'\ c'$. If, however, either one eye only be aklitic, or each eye in a different degree; that is, if the bacilli of one eye, in the state of rest of the application, form, with its radii, different angles from the bacilli of the other, then equality of application effort, γ, manifestly implies inequality of the two angles $e\ b\ c$ and $e'\ b'\ c'$. This inequality may be attained, in a pair of eyes that, on account of an unilateral defect of application, strive after it—that is, after the exercise of an equal application effort in both eyes—by a unilateral prism, or by two prisms of half the strength, and with similar directions of deviation. Experiment with a single prism upon one eye only leads directly to the result.

If, for example, the object appears too large to the left eye, and therefore requires to be diminished to this eye, then the application effort γ, and hence the angle of nutation, $e\ b\ c$, must be increased, and consequently the object displaced, apparently, inwards, or to the right. For this purpose, it is necessary to furnish the left eye with a negative (base towards the left) prism, $-q$. This prism will show, experimentally, when the eyes are first furnished with the combination spectacles determined as already shown, and then the negative prism $-q$ is placed before the left eye, or a positive (base also towards the left) prism q is placed before the right eye, or a negative $-\frac{q}{2}$ before the left, and a positive $\frac{q}{2}$ before the right, that the visual impressions of the two eyes will assume equal magnitudes. If the impression upon the left eye required to be enlarged, then prisms with their bases to the left would be necessary. Generally, therefore, prisms with their bases to the left produce diminution of the image of the left eye, or enlargement of the image of the right; and prisms with their bases to the right produce diminution of the image of the right eye, or enlargement of the image of the left. We must observe, however, that this effect is only to be produced with a pair of eyes having the supposed defect of position: since it is only such eyes that strive after the inequality of the nutation angle in question. Normal eyes, in which equality of the angles of nutation is a necessary condition, only undergo, by the use of unilateral prisms, a common deviation of the position of both, with undisturbed equality of the angles $e\ b\ c$ and $e'\ b'\ c'$.

The final result would be, that with the lenses f must be combined prisms, of the proportions of deviation $h - \frac{q}{2}$, and $h + \frac{q}{2}$.

We now proceed to the case of two eyes of unequal anomalies of refraction or accommodation, a case of especial practical importance, on account of its frequent occurrence, and also because the want of clearness of visual impression which it produces is, of all defects, that to which the eye is most sensitive.

If the total state of refraction of both eyes were alike, and only their accommodation different, if, therefore, both eyes were either emmetropic, or in equal degree hypometropic or hypermetropic, but in different degrees stenopic, and, therefore, of larger and smaller, higher and deeper lying, ranges of accommodation, it is manifest that both eyes would be made emmetropic by the same lenses. If, then, a part of the range of accommodation of one eye was coincident with that of the other, so that, for example, with the emmetropic lenses for both eyes, for the far point $\frac{1}{w'}$ were the greatest of the two common values, and for the near point $\frac{1}{n'}$ were the greatest of the two common values, then complete binocular vision would require spectacles with equal lenses which should place the accommodation effort between $\frac{1}{w'}$ and $\frac{1}{n'}$; since the portion of the range of accommodation $\frac{1}{n} - \frac{1}{w'}$ is common to both eyes. If it be desired, also, that this portion should be divided in a proportion i, the accommodation must correspond to the ordinate $\frac{1}{w'} + i\left(\frac{1}{n'} - \frac{1}{w'}\right) = \frac{i}{n'} + \frac{1-i}{w'}$. Instead of equation (5), we have, therefore, to determine the spherical spectacles, by which the pair of eyes will see at the distance s, and with the accommodation effort i, by

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{l_1} - \left(\frac{i}{n'} + \frac{1-i}{w'}\right) \quad . \quad (36).$$

If w' were equal for both eyes, then, instead of the total emmetropic lens, the manifest emmetropic lens l_1 should be employed, for which $w' = \infty$, or $\frac{1}{w'} = 0$. In the formula

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{l_1} - \frac{i}{n'} \quad . \quad . \quad . \quad (37),$$

n' represents the distance of the most remote of the two near points. Generally, the determination must be guided by the weakest of the two eyes.

Since, in the determination of the glasses immediately necessary in any case, we are not interested in the total, but only in the manifest defect, the larger $\frac{1}{w'}$ corresponds to the smaller w', or to the eye with the smallest accommodation distance. If the emmetropic spectacles, l_1 are determined by this eye, we may retain formula (36); since we make $\frac{1}{w'} = 0$, and use for $\frac{1}{n'}$ the above mentioned smallest value.

If it should happen that the minimum of accommodation effort of one eye were greater than the maximum accommodation effort of the other, then binocular vision, with similar lenses, would be manifestly impossible. This condition is characterized by the weaker of the two strongest lenses, l_1 through which the left and the right eye can see in the distance, being weaker than the stronger of the two weakest lenses l_2.

Without further reference to special cases, we may proceed to the most general kinds of defect of one single eye. In all, the problem is, how to furnish each eye with an image that is clear, single, fixed, and of equal size with that of the other; and, when all four conditions cannot be fulfilled, how to combine the first, second, and fourth, with direct vision of one, and indirect vision of the other eye. In the practical solution of this problem it is necessary to realize, practically, the following fundamental principles of the law of the scheme of vision.

In every individual eye, the three chief functions of accommodation, convergence (as regards the frontal axis), and application, so influence one another that a given exertion of one of them is alone sufficient to produce a given exertion of the others, when they are free to act. This relation between the three efforts, which varies with the absolute degree of one of them, is the most pleasant to the eye. Thus, for example, a single eye (the other being covered) does not accommodate for a very near object perfectly, if this object be placed straight before the eye, so that convergence and application are not called into play, but only if the object be in the frontal axis so that convergence and application are exerted in the same degree as accommodation. Moreover, this relation, on account of the primitive independence of the three faculties, permits to each of them a certain range, between definite minimal and maximal values. Any departure from this favourable relation produces visual strain, which interferes with the duration of visual effort. And just as the three chief functions of the same eye influence one another in monocular vision, so the functions of one eye influence those of the other, when the two are acting together in binocular vision: in so much that the visual apparatus is not comfortable unless the two eyes are exerting their functions in a certain definite proportion to each other (in similar eyes in an equal degree). The functional relation between the two eyes also permits of a certain range, short of the production of binocular visual strain. In general we may say that the minimal or maximal exertion respectively of one function, in monocular or in binocular vision, requires the minimal or maximal exertion of the others, as the condition of the most perfect vision with the smallest degree of strain. But as regards the possible play of each single function, and the visual strain produced by a given deviation of one or the other of them from the normal relation, there exist many different conditions, due to the complication of the visual organs. These again may, in defective eyes, be in many respects modified. Generally there is, between the accommodation and the application of the same eye, between the accommodation of the two eyes, and also between the accommodation of the pair and their convergence, the strongest apparent union. Moreover, in binocular vision, each function modifies the others very powerfully, in such a way as to conduce to single vision. It is especially difficult to exert different degrees of accommodation with the two eyes; and rest of accommodation of one eye very powerfully maintains rest of the other. On the other hand, convergence of one eye towards the frontal axis has a very great range; that is, it is variable within wide limits without the production of much strain; but it guides very energetically the direction of the other eye. In other words, the ocular direction plays only a subordinate part in monocular, but a very important part in binocular vision.

Under the guidance of these general principles, we are led to the following very simple and practical method for determining by experiment the spectacles necessary for a pair of eyes, the individuals of which suffer from any given different, but symmetrical, defects of the three chief functions.

We first select for each eye its manifest emmetropic lens l_1, that is, the strongest lens through which each eye alone can see at infinite distance. Each of these lenses, in monocular vision, produces the minimal accommodation of the eye to which it belongs. We may assume, therefore, that both eyes together, with these two lenses, exert, in binocular vision, the same minimal effort that they each exert singly; or that both eyes, through a pair of spectacles composed of these separate manifest emmetropic lenses, receive together a visual impression that is *clear*, although it may not be single, or composed of images of equal size. This assumption is supported by experience in eyes with similar defects; and must be general, applying to dissimilar defects also. If, however, the minimal accommodations in binocular vision should be unexpectedly found to have different values from the monocular, this must be established by the simultaneous application of two glasses.

We must next ascertain for each eye singly the weakest lens l_2, with which it can still see clearly in the distance, exerting its maximal accommodation.

The supposition that both the eyes in binocular vision exert, not only their minimum, but also their maximum of accommodation together, and also that degree of accommodation in the interval between the minimum and the maximum, which is proportionately expressed by i, leads to the conclusion that, for binocular vision at the distance s, with the accommodation i, we may calculate a lens for the left eye, as well as for the right, by formula (13)

$$\frac{1}{f} = \frac{1}{s} + \frac{1-i}{l_1} + \frac{i}{l_2} \qquad (38);$$

since we substitute, for l_1 and l_2, their respective values for each eye singly.

Eyes with dissimilar refraction must therefore be furnished with spectacles containing dissimilar lenses. It has, however, already been remarked that, in binocular vision, the proportion between the lenses calculated by the foregoing formula may possibly require to be somewhat varied, in order to arrive at the most beneficial effect. To this end it is useful to know, for each eye, the range of the lens f, which renders vision possible at the distance s. This range is given by the variations of the magnitude i, from 0 to 1, and therefore by the two formulæ,

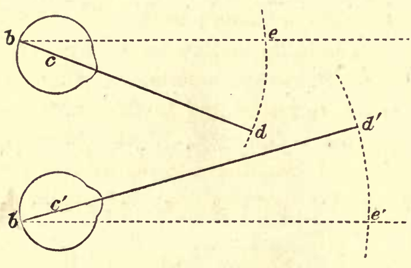
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad (39),$$

$$\text{and } \frac{1}{f} = \frac{1}{s} + \frac{1}{l_2} \quad . \quad . \quad . \quad . \quad . \quad (40).$$

If these two formulæ are calculated for each eye, the strongest and the weakest lens for each eye will indicate the range within which it possibly may be used. If the weaker of the two strongest lenses is stronger than the stronger of the two weakest lenses, it will be possible to furnish the two eyes with similar lenses for use within this range, and naturally with different values of the coefficient i.

When the two eyes are furnished with the unequal lenses f, above indicated, it must not be expected that these eyes will be able forthwith to see the object clearly, with binocular vision, at the distance s. It is possible that the convergence and the application may yet offer impediments, requiring to be set aside by prisms. If, for example, we suppose that each eye is furnished with its emmetropic lens, and fixes an infinitely distant object, the lenses have the effect of placing the apparent object at e and e' (Fig. 69). In order that the eyes may fix the objects e and e', they must be able, in their minimal accommodation, to assume a parallel direction, making the angle of convergence *nil*. In order that they may see the objects e and e' in the same direction, and therefore singly, they must be able to make the angle of

Fig. 69.



application *nil*, so that the polar bacilli *b* and *b′* assume at the same time a parallel position, and coincide in direction with the radii of the eyeballs. Generally it would be required that absolute rest of the convergence and the application should be united with the minimal accommodation. This is usually impossible, the minimal accommodation generally requiring the minimal convergence and the minimal application. Hence it follows that the eyes cannot assume the above-supposed condition. They must, according to their organization, either fix in the parallel direction, and then see double, on account of the divergence of the bacilli *b* and *b′*; or they must so turn themselves, for the sake of single vision, that the affected bacilli, being parallel, form the application angles *e b c* and *e′ b′ c′*, corresponding to the minimal application, so that there will be single vision with squint; or they must stop short both of complete fixation and of complete single vision, so that they see double and squint also; or they must limit themselves to unilateral accommodation, so as to surrender binocular vision. If now we suppose, by means of a prism, the object *e* displaced to *d*, and the *e′* to *d′*, it is manifest that the angle of nutation *e b c* will correspond to the minimal application of the left eye, and that the angle of nutation *e′ b′ c′* will correspond to the minimal nutation of the right eye. Accommodation and application will thus be brought into harmony; so that, by the aid of this prism, a clear and single impression becomes possible in binocular vision. It only remains to ascertain whether the convergence of the two eyes, corresponding to the directions *b d* and *b′ d′*, may yet present a difficulty. If this were so, the prisms would certainly be insufficient to regulate at the same time the application and the convergence; and the result would generally be as follows:—

If the discordance were of so slight a degree that it could be overcome by visual strain, then the arrangement shown in Fig. 69, with its complete optical effects, but with the visual strain produced by a forced convergence, would be brought about. If the discordance were so great as to overpass the relative range of each single function, then, when the impulse after direct vision exceeded the impulse after single vision, direct vision in both eyes, after the position in Fig. 69, would be brought about, but with false application; that is, with a deviation from parallelism of the polar bacilli *b* and *b′*, and with double vision of fixed images. In case, however, as would most commonly happen, that the impulse after single vision preponderated, and that binocular fixation was surrendered, only one eye, for example the left eye *c*, would remain in the position delineated, and would see directly. The right eye *c′*, on the contrary would turn itself and see indirectly, so that its affected bacillus would continually be parallel to the bacillus *b* of the left eye, and single vision with indirect vision would be the result.

What has been said of the action of the emmetropic lenses applies also, in principle, to every other pair of lenses; a certain degree of accommodation of any eye always requires a corresponding exertion of its application and of its convergence. The convergence of the single eye towards the frontal axis has always a large range; and the other eye is turned with more energy to a corresponding position when the convergence of the two axes towards each other is induced by a common effort of accommodation.

We have yet to remark that, when a pair of eyes of like or of unlike accommodative faculty, either without glasses, or furnished with spherical centric glasses, are called upon to exert different degrees of accommodation effort, they are involuntarily made to squint; and this squint presents the following peculiarities. If the left eye fixes, and exerts more accommodation than the right, there is a stronger application induced in the former than in the latter. The homonymous double images thus occasioned can only be fused by the right eye assuming a convergent squint. If the right eye take up the fixation it will induce, since it accommodates less strongly than the left, in the left a weaker application. The resulting crossed double images can only be fused by the left eye assuming a divergent squint.

Hence is explained the singular fact that, when two eyes, either in the unaided or the spectacled condition, work at certain distances with unequal accommodation, one of them will fix, and the other will squint; and that, according as one or the other eye fixes, the squint of its fellow will be convergent or divergent; so that sometimes by changing the visual distance from a near to a far point, the convergent squint will change to a divergent one, or *vice versa.* The explanation of these phenomena rests upon my theory of application.

Furthermore, this theory affords us the means of bringing two unequal eyes to a common visual action. If with the two lenses f, determined by equation (38) for each eye, two prisms are united, which act in such a manner that the apparent objects are placed in the points d and d' (Fig. 69) from whence they require an application represented by the angles $e b d$ and $e' b' d'$, and harmonizing with the accommodation for the distances $b d$ and $b' d'$, the eyes will be placed in a position to see clearly and singly without strain. Whether both eyes would be enabled by these combination spectacles to see directly, that is, with their polar bacilli, would be dependent upon the state of the apparatus of convergence, and could not be determined by anticipation. If fixation with both eyes were not attainable, one eye alone would fix, and the other see indirectly. We have said already that by fixation is always intended direct vision with the polar bacillus, and, therefore, the direction of the axis of the eye $b c$, to the *apparent object* d, not to the actual object. Hence direct vision may very well require a squinting position.

As regards the determination of the prisms g, to be combined with the lenses f, this is accomplished in the most simple manner by direct experiment with two equal prisms. We seek, when the eyes are already furnished with the two lenses f, the strongest and the weakest prismatic spectacles, g' and g'', with which the eyes can see the object at the distance s clearly and singly, and then take for practical application, prisms of the mean value between them :

$$\frac{1}{g} = \frac{1}{2}\left(\frac{1}{g'} + \frac{1}{g''}\right).$$

Instead of by g, we may also define these prisms by their proportion of deviation h, and may say

$$h = \frac{1}{2}(h' + h'');$$

using h also to express either the angle of total deviation or the refracting angle.

In this determination of the prisms h, we have considered only the single vision. For the binocular functions it is also of great importance that the visual impressions of the two eyes should be of equal magnitude. It has already been explained that equality of apparent magnitude can be attained by variation of the two prisms before the left and the right eye, since this equality depends upon the ratio of the accommodation to the application effort in each single eye. If, therefore, on the addition of two equal prisms h to the two dissimilar lenses f, the object at the distance s should appear larger to the left eye than to the right, it would be necessary to ascertain what single prism q, or what two prisms $\frac{q}{2}$, with their bases to the left, would produce equality of apparent magnitude. When the value of the proportion of deviation q is found by actual experiment, we can either give to one eye the prism $h + q$, and to the other the prism h, or to one the prism $h + \frac{1}{2}q$, and to the other $h - \frac{1}{2}q$. Generally we may apply the two unequal prisms $h + mq$ and $h - (1 - m)q$; and in many cases it will be necessary to use dissimilar prisms, in order to obtain equality of the visual impressions.

For example, if it has been found that positive prismatic spectacles (with bases inwards), of equal proportion of deviation $h = 0{\cdot}015$, in combination with the lenses f, produce single vision, but that the object appears too large to the left eye or too small to the right, and that to equalize this difference of apparent magnitude a prism with its base to the left is required, of the proportion of deviation $q = 0{\cdot}02$, we may, since this prism is negative for the left eye and positive for the right, employ at pleasure, or according to circumstances, any of the following pairs :—

$$0{\cdot}015 - 0{\cdot}02 = -0{\cdot}005 \text{ and } 0{\cdot}015 = 0{\cdot}015$$
$$\text{or } 0{\cdot}015 - \tfrac{1}{2} . 0{\cdot}02 = 0{\cdot}005 \text{ and } 0{\cdot}015 + \tfrac{1}{2} . 0{\cdot}02 = 0{\cdot}025$$
$$\text{,, } 0{\cdot}015 - \tfrac{1}{3} . 0{\cdot}02 = 0{\cdot}008 \text{ ,, } 0{\cdot}015 + \tfrac{2}{3} . 0{\cdot}02 = 0{\cdot}028$$
$$\text{,, } 0{\cdot}015 = 0{\cdot}015 \text{ ,, } 0{\cdot}015 + 0{\cdot}02 = 0{\cdot}035$$

Herewith it appears that the practical rules for the selection of lenses and prisms for any abnormal pair of eyes, each eye of which labours under a symmetrical defect, are exhausted in all essentials. I think that, upon the basis of these rules, the contemplated effect of obtaining binocular vision, with common action of both eyes, clear, single, as much as possible direct, and with the smallest possible amount of strain, is to be attained by very simple experiments.

With regard to the technical manufacture of a combination glass, when the necessary strength of the lens and the prismatic deviation are determined, some remarks will be made at the close of the volume.

5. *Periscopic Combination Spectacles.*

Whether any combination glass A or B is regarded as an eccentric portion of a common flat lens A B (Fig. 70 or 71),

Fig. 70.

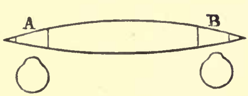
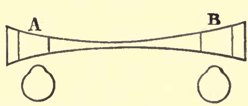


Fig. 71.



that is, of a lens with a straight chord in section, or as an eccentric portion of a periscopic lens (Fig. 72 or 73), that is, of a lens with a curved chord in section, is, as far as concerns its essential nature, a matter of indifference. Since the peri-

Fig. 72.

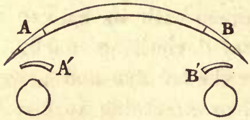
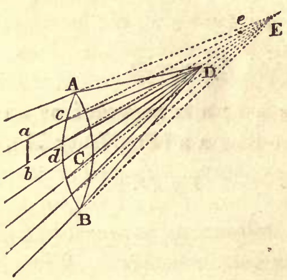


Fig. 73.

scopic glasses are not distinguished from flat glasses by any fundamental considerations, I have not devoted any special attention to the former in the *Theory of Ocular Defects.* By later observations, however, I have found that the aberration in the flat combination glasses produces inconvenient effects, which may be either remedied or very much diminished by periscopic combination glasses. Upon this ground, and because the disturbing phenomena produced by flat combination glasses are of the highest physiological interest, I seek to draw the attention of ophthalmologists to the periscopic combination spectacles.

The optical illusions produced by aberration in monocular vision through a centric lens are well known. In order to explain them it is generally necessary to depart from the purely physical view, which, in the passage of rays from a luminous point D, through a lens A B (fig. 74) considers the lens to be acting in its entire aperture A B, and takes into account the entire cone of rays A D B. On this supposition, the rays of such a cone, by their refraction in the lens, converge towards the relative focal point E, which lies nearly in the direction of the principal ray C D; and, by the aberration, are spread over a *focal surface.* This conception has, for an eye looking through the lens, with a pupil considerably smaller than the lens, only

Fig. 74.



a very limited value: and scarcely any value at all for the explanation of the physiological phenomena of aberration. An eye so placed is acted upon only by rays actually passing through its small pupil *a b*, and all others are non-existent for it. These efficient rays form only a small pencil on the base of the pupillary opening *a b*, and therefore an almost elementary part of the whole cone passing through the lens A B. We will call the former briefly the pupillary pencil. To such a pupillary pencil, D *c a*, the three following essential laws apply: first, that its rays *a c*, *b d*, converge nearly in a *point* *e*, of the focal surface of the whole cone; secondly, that this point of convergence *e* is for a convex lens nearer to the lens, and for a concave lens farther from the lens, than the point of convergence E of the principal rays of the whole cone, passing through the centre of the lens; thirdly, that the central ray of the pupillary pencil, and therefore the whole elementary pencil, is caused to deviate by the refraction in the lens, and is turned outwards by a convex lens, and inwards by a concave lens, since the ray *a c* is not directed towards D, but towards the laterally placed point *e*.

From these three laws, in monocular vision through a lens over an extended field, and especially over a vertical plane, arise the following phenomena. Aberration of the rays of the pupillary pencil is inconsiderable; so that an extended object, notwithstanding its boundaries, is seen only by means of marginal rays, yet sharply, in so far as the eye can accommodate for the apparent distance of the marginal points. By the second law, the points of a vertical plane lying laterally to the ocular axis, in vision through a convex lens appear nearer to, and in vision through a concave lens farther from, the spectator, than the point of the same plane lying in the axis; that is, the vertical plane appears to be curved, and through a convex lens to be sunken or concave, through a concave lens to be convex. By the third law the field of vision, in looking through a convex lens appears externally enlarged or expanded, and a rectilinear square assumes the form shown in Fig. 75, of a quadrangle with its sides concave in a direction outwards. On the contrary, on looking through a concave lens the field of vision appears externally diminished

Fig. 75.

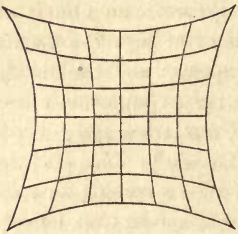
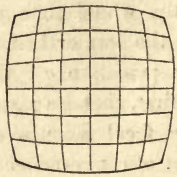


Fig. 76.



or compressed, so that a rectilinear square assumes the form shown in Fig. 76, of a quadrangle with its sides convex in a direction outwards.

If the lens be small, about the size of a spectacle glass, so that it does not permit a view over a large field of vision, all the above deformities or distortions occur only in a proportionately slight degree, and are hence but little noticed.

But if the lens is eccentric, or a combination glass, these distortions, even if the lens be small, are observable in a much higher degree ; since then the centric middle portion is either absent or much curtailed, and hence marginal rays come chiefly into action. Moreover, since in consequence of the removal of the middle portion, these marginal rays are all from one side, a square vertical surface is caused to lose its symmetric form, and appears through a convex lens, something like fig. 77, and, through a concave lens, something like fig. 78.

Fig. 77.

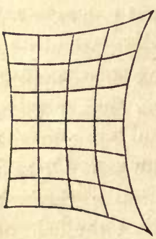
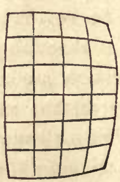


Fig. 78.



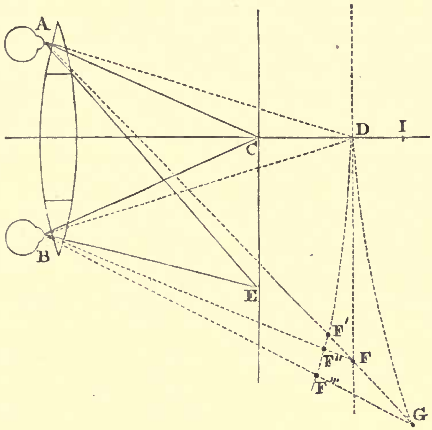
This considerable and unsymmetrical distortion of the field of vision is manifestly an objection to the eccentric glasses. In order to estimate it rightly, however, we must not be guided by the effect on monocular vision through a single glass, but by the effect on binocular vision through a pair of spectacles. For centric spectacles this effect does not much deviate from that on monocular vision; but it does so deviate for eccentric, and therefore also for orthoscopic spectacles. In looking through the latter the field of vision, as in monocular use, appears extended or condensed at its boundaries, and a vertical square exhibits curved sides, as in Figs. 75 or 76. But the apparent curvature of a vertical plane is reversed, that is, through orthoscopic convex spectacles it appears no longer concave, but convex, and through orthoscopic concave spectacles no longer convex, but concave. In vertical lines this curving becomes especially manifest.

These appearances in monocular vision, and the change produced by changing from monocular to binocular vision through combination glasses, are highly worthy of note, and of great importance in the physiology of the eye. The apparent arching of a vertical plane in monocular vision affords a remarkable confirmation of my theory of the recognition of distance: since it shows, first, that the faculty of recognizing distance appertains to each single eye; and, secondly, that the single eye can simultaneously recognize different distances, so that the recognition does not depend upon some general function of the eye as a whole organ, but upon an elementary function of the particular nerve-fibre or bacillus that is impressed. The inversion of the arching in the transition from monocular to binocular vision, and therefore, the seeming approximation of the more distant images, and the seeming recession of the nearer, teaches further that the judgment of distance does not depend exclusively upon the actual distance of the apparent object or of its optical image, and therefore not exclusively upon the accommodation for this image, nor exclusively upon the position of convergence of the two eyes: since first, in this transition from monocular to binocular vision, the actual distance of the optical images of all points of the vertical plane remains unchanged, and since this distance is solely determined by the physical refraction of the rays; since, secondly, the accommodation remains unchanged, the object being as sharply defined afterwards as before; and since, thirdly, the position of convergence of the two eyes for all points of the visual field must be the same. The appearances on binocular vision can therefore certainly not be explained by the doctrines that have hitherto prevailed in physiology; but readily by my theory of the recognition of distance. The phenomena consequently afford to that theory an important support, which I will now proceed to explain.

In the *Gesetze dès raümlichen Sehens*, s. 117, and in the earlier part of the present work (p. 127), I have shown that, when a is the actual distance of an optical image, b the accommodation distance, that is, the distance for which the retinal bacillus that is acted upon accommodates, and c the application distance, that is, the distance for which this bacillus is applied, or which represents the angle of convergence of the corresponding bacilli of the two eyes, the apparent distance of the object has the value $a' = \frac{c}{b} a$. From this it follows that, when the actual distance a of the optical image, and the accommodation distance b of the impressed bacillus, both remain unchanged, but the application distance c, or the distance of the point towards which the two corresponding bacilli converge, changes, then the apparent distance a' will become greater or smaller, in proportion as c increases or diminishes. We have therefore to inquire what changes the convergence distance of the optical images of the different points of the vertical plane undergo in binocular vision through combination spectacles; and whether these changes are in unison with the alteration in the observed appearances.

Let us take a pair of normal orthoscopic spectacles A, B, shown in diagram in Fig. 79. If C is an object in the horizontal frontal axis, its optical image will be formed at D, since the rays C A and C B are refracted outwards through the glasses in the directions A D and B D. If E be a point in a horizontal drawn through C, the rays E A and E B, coming to the eyes,

Fig. 79.



would likewise be refracted outwards; so that they, when D F is parallel to C E, would converge towards F, in so far as no aberration interfered. Aberration would displace the optical image of E somewhat in front of the line D F, somewhat towards F′ for the left eye, and somewhat towards F″ for the right. Hence the horizontal line C E, in monocular vision through either the left or the right eye, would appear concave like the curve D F′.

If the two glasses A and B were centric, in vision with the left eye the angular deviation F′ A E would be equal to D A C, and in vision with the right eye the angular deviation F″ B E=D B C; and, therefore, the angle of convergence A F B= A D B. But, since each glass is eccentric, or prismatic, this equality is not produced; the deviation of the ray going from E to the right eye is somewhat greater than the deviation of the ray going to the left eye, since the former passes through a part of the glass B, which, on account of its nearer approximation to the margin of the large lens, A B, increases the deflection in greater measure than the part of the glass A, through which passes the ray for the left eye: since this part, by reason of its being less distant from the centre of the large lens, A B, produces a less degree of deflection. Consequently, the corresponding rays for the two eyes converge towards a point G, which is situated behind the line D F; the angle of convergence A G B is less than A D B; or the distance of bacillary convergence or application C, which is determined by the point G, is increased. The actual distance a of the optical images F′ and F‴ for the two eyes, and also the corresponding accommodation distance b, which, on account of the sharpness of the visual impression, may be taken to be equal to a, certainly remain somewhat less than the distance of the point F; but, since the apparent distance of this point F is determined by $a'=\frac{c}{b}a$, then, because $a=b$, $a'=c$, that is, the point E, or its optical image, which is F′ for the left eye and F‴ for the right, appears to be situated at G. Hence it is established that a horizontal line, C E, in binocular vision, through orthoscopic convex spectacles, would appear not concave, but convex.

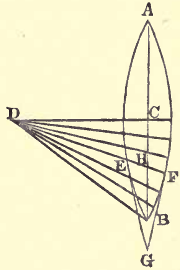
For the orthoscopic concave spectacles the foregoing conditions are so reversed, that the horizontal C E will appear concave.

If we now investigate the appearance of a vertical line, erected upon the point C, let H be any point in this vertical line. The rays which proceed from the point H to enter the two eyes, incline symmetrically towards the glasses and towards the eyes. If we follow the course of such rays, falling obliquely on the refracting surfaces, and consider that the refraction of such surfaces occurs in a plane which passes through the incident ray and through the radius leading to the centre of curvature of the refracting surface, we shall perceive that the rays, after refraction in the glasses, and, therefore as they reach the eyes, when we place vertical planes through them, are projected not upon the lines A D and B D, but upon two lines A I and B I, which enclose an angle A I B, that is less than the angle A C B; or that the corresponding vertical meridians of the two eyes, in which the higher lying point H is seen, correspond to a greater convergence distance *c*, than the meridians for the point C. And, although the actual distance *a* of the optical image of H, and also the accommodation *b* for this image, are less, on account of the aberration, than the distance of the image D, so that, therefore, in monocular vision the vertical line C H will appear concave, yet in binocular vision, because $a' = \frac{c}{b} a = c$, and because *c* is greater than *a*, it will appear convex.

This appearance also will be reversed by the use of orthoscopic concave spectacles, in such a manner that the vertical line, C H, will then appear concave.

The foregoing results afford a satisfactory explanation of the peculiar appearances observed in binocular vision through combination spectacles. They teach that these appearances essentially depend upon the application of the bacilli, and, therefore, furnish a further confirmation of the existence and of the physiological effects of this function of the visual organs.

Fig. 80.



When we inquire into the geometrical conditions which impart to the eccentric form of lens the peculiarity of producing such an aberration that apparent curvature of the field of vision necessarily ensues, we find on more exact investigation that they are the following. If we follow, as in Fig. 80, the part E F of the ray that lies within the lens A B, and which, since it becomes constantly more convergent towards the point D, penetrates the lens more and more near to its margin, we find that for such a ray, e.g. for that which belongs to the portion E F, the lens has only the physical value of a prism

E G F, of which the plane containing surfaces E G and F G are the tangential planes of the lens at the points of entrance and exit E and F. With the deviation of the ray from the axial position D C towards the outer marginal ray D B, the angle E G F of the supposed prism varies, and in the same degree the deviation of the emerging portion of the ray from the direction of the entering portion. The essential character of this variation is that it is not uniform with the turning of the ray towards the point D; but that the angle E G F, and hence the deviation of the ray by its refraction in the lens, is at first small, and at last undergoes a great increase, for one and the same angular bending of the ray.

The knowledge of the external cause of the apparent curvature of the field of vision, gives at once the means of removing, or of greatly diminishing, this inconvenience; and, since it teaches for what forms of lens it occurs in the greatest and in the least degree, affords a rational basis for judgment about the different forms of spectacle glasses. It is manifest that every form of lens which increases the above-described want of uniformity of the variation of the deviation of the ray of light towards its margin, will increase the apparent curvature of the field; while every form which diminishes the want of uniformity will also diminish the apparent curvature; so that the form which produced a uniform variation of the deviation, would set aside the apparent curvature entirely.

If we name a lens with regard to its sphericity by the forms of its anterior and posterior spherical surfaces, calling the anterior surface that which is external, or away from the eye, and the posterior surface that which is internal, or towards the eye, we find the following gradation from the most prejudicial to the most beneficial form.

a. For the centric or eccentric convergence-lens (Fig. 81) the concavo-convex, A, is the most prejudicial. The plano-convex, B, is better, the biconvex, C, is better still; and then follows the convexo-plane lens D. The best of all is the convexo-concave E.

b. For the centric or eccentric divergence-lens (Fig. 82) the concavo-convex lens A is the most prejudicial. The concavo-

Fig. 81.

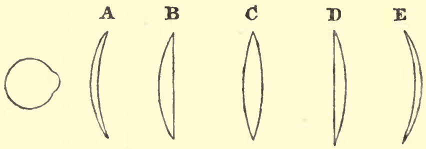
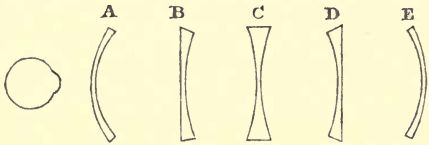


Fig. 82.



plane lens, B, is better, the biconcave, C, is better still; and then follows the plano-concave lens D. The best of all is the convexo-concave E.

The most beneficial lens, in both cases, has the convexo-concave form E, that is, it is curved towards the eye. This form, when it is used for monocular vision, and therefore as a centric lens, is more advantageous, the more its central curvature approaches a circle described about the central point of the eye. For binocular vision, or for two eccentric glasses, the proper curvature must be determined by the considerations stated below. A convexo-concave convergence- or divergence-lens is *periscopic.* This name in no way distinguishes the one lens from the other, by the fundamental peculiarities of each; it is only another expression for a lens with a convex anterior, and a concave posterior surface. We are therefore led to the result that periscopic spectacles are adapted to reduce the apparent curvature of the visual field to a minimum.

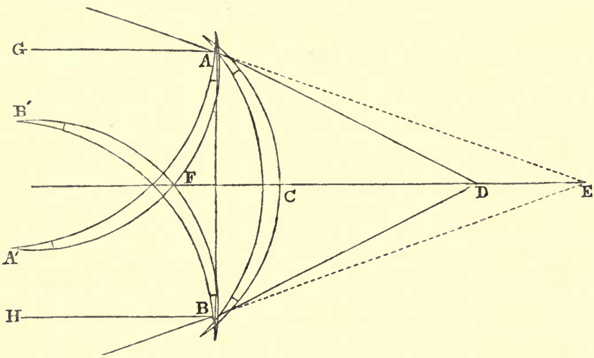
As regards the distortion of the field of vision, namely, the compression or extension of its outer portions, and the apparent bending of straight lines, these defects, since they arise from the inevitable, and, on account of the accommodation, directly designed refraction of the rays of light, cannot be set aside by any form of lens, and cannot even be essentially diminished.

By all this we are induced, as well for centric as for eccentric glasses, to turn the attention to the convexo-concave form.

When r is the radius of the convex anterior surface, r' the radius of the concave posterior surface, and n the index of refraction (1·55) of glass, the focal distance of a periscopic lens, measured from the apex of the convex surface of the complete centric lens, with a supposed small thickness of glass, and for parallel incident rays, will be $f = \frac{r\,r'}{(n-1)(r'-r)}$ $= 1{\cdot}82\,\frac{r\,r'}{r'-r}$; and the crossing-point of the chief rays, that is, of the straight lines which unite the points of the object with the corresponding points of its optical image, lies nearly in the apex of the centric lens. Hence the optical effects of such a lens are easy of demonstration.

In order to determine the most proper curvature for periscopic combination lenses we will first consider the case of normal orthoscopic spectacles. From these spectacles is required their characteristic property, that the rays from any luminous point shall be so refracted through both lenses as to appear to both eyes to proceed from one and the same point. This is only fulfilled when the apices of the complete lenses, of which the two eccentric lenses, A and B, form parts (Fig. 83), coincide. This occurs when these apices lie in the frontal axis F D. The simplest solution of the problem is to cut the eccentric glasses, A and B, out of the same large centric lens, A C B, at the distance A B of the ocular centres. This arrangement supposes, strictly taken, that the two eccentric glasses shall be put before the eyes in the same positions that they occupied in the lens A C B. But in this oblique position they would not be practically applicable, and they must be so turned as to assume a straight position in front of the eyes. Such turning, when it is not too considerable, has scarcely any appreciable influence upon the refraction and deviation of the rays of light; and

Fig. 83.



even this influence can be essentially diminished by the following disposition.

It is obvious that the fundamental property of the orthoscopic spectacles is also realized when the two glasses A and B belong to the two centric lenses, A A′ and B B′, which are equal to the lens A C B, but so placed, that their apices may coincide in any desired point F of the frontal axis. Hence we may give to the two glasses A and B any desired position before the eyes; and we can also still cut them from one and the same large centric lens A C B, since, in general, the chord A F is not equal to the chord A C, so that the glasses for this position may be cut in distances A A′ or B B′, other than those already considered, from the large lens. These changed distances may be deduced from Fig. 83 by an easy drawing.

The case is of chief interest in which the two spectacle lenses are to be placed perfectly straight; that is, so before the eyes that the central points G and H of their middle curvatures fall into the lines A G and B G, drawn through the eyes A and B, parallel to the frontal axis. This, indeed, is the position in which the use of glasses is most advantageous, and the external view is the least embarrassed. For this position the chord A F is in a remarkable way exactly equal to the chord A C; so that the eccentric glasses to be placed straight before the eyes are precisely the same which are cut out of the large lens A C B at the distance A B.

The large lens A C B, to which the lenses of periscopic spectacles belong as eccentric portions, is conditioned by the focal distance $f = \frac{r\,r'}{(n-1)(r'-r)}$, required by the accommodation. Since the value of f depends upon two quantities, r and r', lenses of very different curvature satisfy every requirement. In order to fulfil, as completely as possible, the purpose of removing the apparent curvature of the field of vision, the middle curvature of the periscopic lens must, when practicable, be so great, that the central point of each eye corresponds to the central point, G or H, of the middle curvature of each glass. But it is manifestly impossible to give to the large lens A C B, from which the spectacle lenses are to be cut, a smaller radius than F A, which is the half of the distance A B $= d$, of the centres of the eyes from one another. Since d about equals 66 millimetres, or 2·44 Paris inches, the radius R of the middle curvature of the periscopic lens, or the value of $R = \frac{1}{2}(r' + r)$, may be assumed to be 33 millimetres, or 1·22 Paris inches. In practice we shall seldom obtain this outside value, and R will generally equal 1·5 or 2 Paris inches.

It is very noteworthy that a spectacle lens, the middle curvature of which has a radius of from 1·25 to 1·5 inch, when it is placed straight before an eye, has the central point of its middle curvature nearly coincident with the central point of the eye; so that periscopic spectacles of this kind fulfil, as much as possible, the requirements stated.

When the middle curvature and the focal length of the periscopic lens A C B are given, we have the two equations of quality,

$$f = \frac{r\,r'}{(n-1)(r'-r)} \qquad R = \frac{1}{2}(r' + r).$$

Hence the two radii r and r' are completely determined, since we find

$$r = R + (n-1)f - \sqrt{R^2 + (n-1)^2 f^2},$$
$$r' = R - (n-1)f + \sqrt{R^2 + (n-1)^2 f^2};$$

or, placing for n, 1·55; and for R, 1·5 inches,

$$r = 1{\cdot}5 + 0{\cdot}55\,f - \sqrt{2{\cdot}25 + 0{\cdot}3025\,f^2} \text{ inches},$$
$$r' = 1{\cdot}5 - 0{\cdot}55\,f + \sqrt{2{\cdot}25 + 0{\cdot}3025\,f^2} \text{ inches}.$$

Again, making $n = 1{\cdot}55$, and R $= 2$ inches,

$$r = 2 + 0{\cdot}55\,f - \sqrt{4 + 0{\cdot}3025\,f^2} \text{ inches},$$
$$r' = 2 - 0{\cdot}55\,f + \sqrt{4 + 0{\cdot}3025\,f^2} \text{ inches}.$$

When, according to these formulæ, we calculate the semi-diameters r and r' for different focal lengths f, we find only a little difference between these semi-diameters. Hence it follows that a strongly curved periscopic glass for a given focal distance must be ground with far greater accuracy than a flat glass; and, therefore, it is the more important that the two portions required for eccentric periscopic spectacles should be obtained from one and the same original lens.

6. *The Manufacture of Combination Glasses.*

The ophthalmic surgeon must make his experiments with single lenses and with single prisms, in order to determine the combination that is required. For the greatest possible accuracy it would be desirable to use plano-convex and plano-concave lenses, and to place the additional prisms, when possible, in direct contact with the plane sides of the lenses, taking care also that the prisms shall not be rotated. This would probably be best secured by quadrangular lenses and prisms, instead of round or oval ones; and a set of such glasses, adapted to all the experiments with binocular vision, would be very easily manufactured.

The business of the optician will then be to provide the desired combination lens, in the form of a prismatic convex or concave glass, made out of a portion of a larger spherical lens.

The larger spherical lens, out of which the combination lens is cut, must always have the same focal length as the lens f that is required for the combination glass. The eccentric place, from which the piece must be cut, is shown most easily and certainly by the following simple geometric diagrams. In Fig. 84, A B C is the anterior surface, that is, the surface turned to the object, of a large positive lens of the desired focal distance f, and D is its actual focus for the rays I E coming from a

Fig. 84.

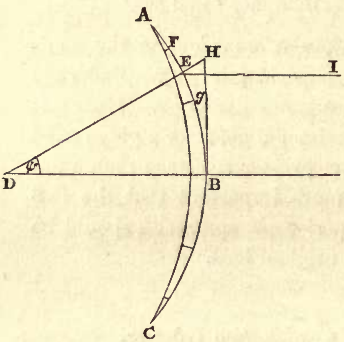
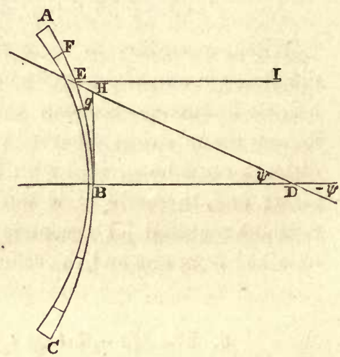


Fig. 85.



point in front of it. If the combination with a positive prism is desired, of which the angle of deviation is ψ, or the proportion of deviation is tan. $\psi = h$, the line D E must be drawn through the focal point D in such a way that it may form, with the axis of the lens, an angle B D E $= \psi$, or we make the ordinate B H $= h \times$ B D. If the focal length f is measured from the anterior surface of the lens, so that B D $= f$, then the point H, through which the line D H must be drawn, will be at the height B H $= h \,.\, f$. The line D H, so drawn, cuts the large lens in the point E, which must be the central point of the spectacle glass F g.

If the object be to make a negative lens A C, of the focal length $-f$, in combination with a negative prism $-\psi$, for which tan. $(-\psi) = -h$, then Fig. 85 furnishes the necessary diagram; since D is the focal point, and therefore B D $= f$, and the angle B D H $= \psi$, or B H $= h$ and B D $= h \cdot f$.

The combination of a positive lens f with a negative prism $-h$ is accurately shown by Fig. 84, with the change that the glass F g, which has its prismatic base turned towards the right, would then be applied to the right eye instead of to the left.

In the same way the combination of a positive prism h with a negative lens $-f$ is shown by Fig. 85, with the change that the glass F g, which has its prismatic base turned towards the left, would then be applied to the right eye instead of to the left.

The length of the curve B E, or of the chord B E from the axis B of the large lens to the central point E of the spectacle glass, will be nearly equal to the line B H or $= h \cdot f$. When, therefore, the most extreme accuracy is not required, we may say that the central point E of the spectacle glass F g is at a distance $= hf$ from the axis B of the large lens. By this direct measurement, or by the foregoing more exact construction, the place is determined at which the combination glass is to be cut out of its parent lens.

If we indicate the conventional diameter F g of a spectacle lens by c, then the distance B $g = h \cdot f - \frac{c}{2}$, and the distance B F $= h \cdot f + \frac{c}{2}$, and therefore the diameter or the aperture A C, which the larger lens must possess, in order that the combination glass F g may be cut from it, is $= 2$ B F $= 2h \cdot f + c$.

It is important in practice that the large lens, from which the combination glasses are cut, should not itself be much larger than is absolutely necessary. Not only does increase ct size increase the quantity of glass and the work of the grinder, thus increasing the cost, but further, the desired combination glass should be made as thin as possible, so as to preserve an appearance not unbecoming, and to diminish the disturbing optical effect of any great thickness of glass. On the other hand, the requirements of the manufacturer, and the prompt supply of any desired combination, demand that he should be placed in a position to keep in stock only a small number of the large lenses, from which combination lenses may be cut as needed. Lastly, it is of advantage as regards price, in the fabrication of spectacles containing two symmetrical combination glasses, that both of them should be cut symmetrically from one and the same lens; and this is also beneficial from an optical point of view, since two glasses thus made will coincide more accurately than any two cut out of different lenses.

All these advantages may be secured by the following considerations. When B E $<$ g E, or when $h\,f < \frac{c}{2}$, there can be cut from the large lens, whatever its size, only *one* combination glass. The diameter A C of the large lens need not, therefore, be larger than the double of F g. When, on the contrary, B E $>$ g E, but $< \frac{4}{\pi} \cdot g$ E, then from the large lens can be cut *two* symmetrical circular combination glasses of the diameter F g. When B E $> \frac{4}{\pi} \cdot g$ E, but $< \frac{6}{\pi} \cdot g$ E, then four such glasses can be cut, and so on.

If we thus assume that we are dealing with a convex or concave lens of definite focal length f, the manufacturer would require to keep only four large lenses, of the diameters

$$2\,c, \qquad 2{\cdot}273\,c, \qquad 2{\cdot}909\,c, \qquad 3{\cdot}546\,c.$$

From the first of these, $2\,c$ in diameter, he would be able to cut only one combination glass, and one for which h is $< 0{\cdot}5 \times \frac{c}{f}$. From the second, of diameter $2{\cdot}273\,c$, he might cut *two* combination glasses, for which $h > 0{\cdot}5 \times \frac{c}{f}$, but $< 0{\cdot}637\,\frac{c}{f}$. From the third, of diameter $2{\cdot}909\,c$, he might cut *four*, for which $h > 0{\cdot}637 \times \frac{c}{f}$, but $< 0{\cdot}955\,\frac{c}{f}$. From the fourth, of diameter 3·546 c, he might cut *six*, for which $h > 0{\cdot}955\ \frac{c}{f}$, but $< 1{\cdot}273\ \frac{c}{f}$.

If we take 1·3 inch as the diameter c of the spectacle-glass, or, allowing for loss in working, 1·5 inch, then the diameters of the large lenses required are respectively 3, 3·41, 4·36, and 5·32 inches. From the first is obtained *one* glass, for which $h < \frac{0{\cdot}75}{f}$; from the second, *two*, for which $h > \frac{0{\cdot}75}{f}$, but $< \frac{0{\cdot}955}{f}$; from the third, *four*, for which $h > \frac{0{\cdot}955}{f}$, but $< \frac{1{\cdot}432}{f}$; and from the fourth, *six*, for which $h > \frac{1{\cdot}432}{f}$, but $< \frac{1{\cdot}91}{f}$.

If circular glasses are not desired, but only oval ones, having a horizontal diameter of $c = 1{\cdot}3$, or, with allowance for loss in working, $= 1{\cdot}5$, and a vertical diameter $c' = 1$, or, with allowances, $= 1{\cdot}2$ inches, then we may obtain two, four, and six glasses from lenses of smaller aperture. The four large lenses, from which could be cut respectively one, two, four, and six combination glasses, with the above-stated values of h, may then be reduced to three, with diameters of 3, 3·42, and 4·36 inches, since from the first can be cut all single and double glasses for which $h < \frac{0{\cdot}75}{f}$.

When the glass-cutter is skilful, and therefore does not require so much space for working between any two of the portions that are to be cut out, then the diameters of the large lenses may be still further reduced, which is always much to be wished.

The advisability of keeping in stock a certain quantity of large lenses of definite focal lengths f, from which any desired combination glasses can be quickly cut, is associated with the disadvantage that many glasses must then be cut from lenses larger than are absolutely necessary to yield them, and that these glasses are therefore thicker than they need be. If we attach to the greatest possible thinness of the spectacle-glasses the great importance which, from the point of view of the most complete fulfilment of their uses, would be justifiable, then the large lens, from which the glasses are cut, should itself only be made when these glasses have been determined. It would then be made no larger than would be absolutely necessary; and it would therefore have only the diameter $2\,h\,f + c = 2\,h\,f + 1{\cdot}3$, or, on account of the necessary loss in rounding the glasses, the diameter $2\,h\,f + 1{\cdot}5$ inch, so that every combination glass would be cut close to the margin of the large lens yielding it.

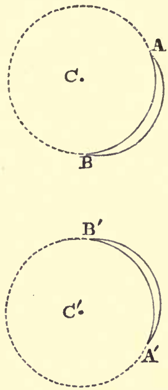
In photographic and other optical instruments, which consist of several lenses, convex and concave glasses are often united by means of a transparent cement. In the same way it is possible that a plano-convex or plano-concave lens might be united to a plane prism. If to such a proceeding there are no practical impediments, the circumstance that it would involve a less sacrifice of glass would plead in its favour. The optician would then require to keep only common lenses and common prisms, of definite numbers, and to unite them according to his needs.

This procedure would, however, involve the abandonment of the periscopic form of the spectacle-glasses. Since this form is essential to the completeness of the spectacles, the method by cementing could not give the best instruments.

When it is the object to obtain periscopic glasses of so strong a curvature that the central point of the eye is coincident with the centre of the sickle-shaped curvature of the glass, we may wholly lay aside the experiments with prisms to determine the peculiarities of the eye, and the calculation for the necessary combination-glasses, and may proceed in the manner following.

We take an apparatus which, as in Fig. 86, has the form of a spectacle-frame, in which on both sides can be placed periscopic lenses of the already-mentioned size, with their middle curvature C A = R, and which can be rotated at pleasure round their central points C and C′. After these lenses have been placed by rotation in the position in which they best content the eyes, we obtain immediately from the angle of rotation the necessary eccentricity of the spectacle-glass. If the judgment of the patient upon the minimum of visual strain is uncertain, we may discover, by turning the glasses into the extreme position inwards and outwards, the outside limits of the eccentricity, corresponding to the strongest and the weakest prisms, with which clear and single vision is still possible. The mean between these limits will then be the proper eccentricity.

Fig. 86.



Lastly, we cannot forbear to mention, as a disadvantage of all eccentric glasses, arising from their prismatic form, the unilateral chromatic aberration which fringes the object with the colours of the spectrum. Although this disadvantage is not considerable, it is greater than in centric glasses. It can only be obviated by lenses composed of two different kinds of glass—flint glass and crown glass—united together.

With regard to the care that should be exercised in the manufacture of spectacles, it must be observed that the eccentric require far more than the centric, and that the chief matter of attention would be to secure that the horizontal diameters of both glasses, which would point towards the common apex of their parent lens, should be placed as accurately as possible in a horizontal plane, or that any rotation of the glasses about the ocular axes should be prevented. Such rotation, which for centric glasses would be immaterial, in eccentric glasses would cause the rays from the same point of the object to fall upon different points of the two retinæ. So long as the rotation does not exceed a certain degree, we find, and this is of much physiological interest, that single vision is preserved; but only by an upward and downward movement of the corresponding bacilli, and therefore with an unusual dragging of the retina, which is at once recognisable by a feeling of strain. Hence it is advisable that on every spectacle-glass the terminations of the horizontal diameter should be marked by a notch, so that the wearer would immediately be able to rectify any displacement that might be produced by accidental bending of the frame.

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