ELEMENTARY TREATISE

ON

NAVIGATION

AND

NAUTICAL ASTRONOMY

BY

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W. P. 2

PREFACE

The following pages are the outcome of the author's own teaching. To understand the principles set forth in them a knowledge of elementary Plane and Spherical Geometry and Trigonometry is all that is needed.

The author wishes to acknowledge special obligations to Martin's "Navigation" and Bowditch's "Navigator." To either of these works the present book might serve as an introduction.

Most of the examples have been worked by means of Bowditch's "Useful Tables," published by the United States Government. The corrections to Middle Latitude have been taken from the table (pages 172, 173) prepared by the author.

References to *Elements of Plane and Spherical Trigonometry* by the author and to *Elements of Geometry* by Phillips and Fisher are indicated by (Trig.) and (P. and F.) respectively.

CONTENTS

NAVIGATION AND NAUTICAL ASTRONOMY

CHAPTER I

PLANE SAILING — MIDDLE LATITUDE SAILING — MERCATOR'S SAILING

In navigation the *earth* is regarded as a *sphere*. Small parts of its surface (as in surveying) are considered as *planes*.

Art. **1.** The *axis* of the earth is the *diameter* about which it revolves. The extremities of this axis are called *poles*, one being named the North Pole and the other the South Pole.

2. The *meridian* of any point, or place, on the earth is the *great* circle arc passing through the point, or place, and through the poles of the earth.

The meridian of a point, or place, may be said to be the intersection of a plane with the surface of the earth, the plane being determined by the axis and the point (Phillips and Fisher, *Elements of Geometry*, 526, 807).

(*a*) Meridians are, therefore, *north* and *south* lines.

3. The earth's *equator* is the circumference of the great circle, whose plane is *perpendicular* to the axis.

(*a*) The equator is perpendicular, therefore, to the meridians (P. and F., 837).

4. *Parallels* of latitude on the earth are circumferences of *small* circles, whose planes are *perpendicular* to the *axis*.

The planes of these parallels are parallel to each other and to the plane of the equator (P. and F., 559).

(*a*) *Parallels* of *latitude* are *east* and *west* lines.

5. The *longitude* of a point, or place, is the *angle* between the plane of the meridian of the point, or place, and the plane of some fixed meridian. This angle is measured by the arc of the equator intercepted between these planes, since this arc measures the plane angle of the dihedral angle of the planes (P. and F., 836). This *arc*, intercepted between the two meridians, is spoken of as *the longitude*, as its degree measure is the same as that of the dihedral angle.

One assumed meridian from which longitude is reckoned is the meridian of the Observatory of Greenwich, England; another is the meridian of the Observatory of Washington. The French, also, have a fixed meridian from which longitude is reckoned.

(*a*) Longitude is reckoned, on the *arc* of the equator, east and west of the assumed meridian, from 0° to 180°.

(*b*) The *difference of longitude* of two places is the angle between the planes of their meridians, and is measured by the arc of the equator intercepted between these meridians.

This arc is evidently the difference of the two arcs, which measure the longitudes of the two places, if the places are either both E. or both W. of the assumed meridian.

(*c*) If we give to E. longitudes the sign +, and to W. longitudes the sign −, the arc which measures the difference of longitude of two places will always be the *algebraic* difference of the longitudes of the places.

(*d*) To find, then, the *difference* of longitude of two places whose longitude is given, we *subtract* the *less from the greater if both are E. or both are W., but add the two if one is E. and the other W.*

6. The *latitude* of a point, or place, is the *angle* made with the plane of the equator by a line drawn from this point, or place, to the center of the earth. The latitude is measured by the arc of the meridian (of the point) which subtends the angle. This subtending *arc* is spoken of as *the latitude*, as its degree measure is the same as that of the inclination of the line to the plane of the equator.

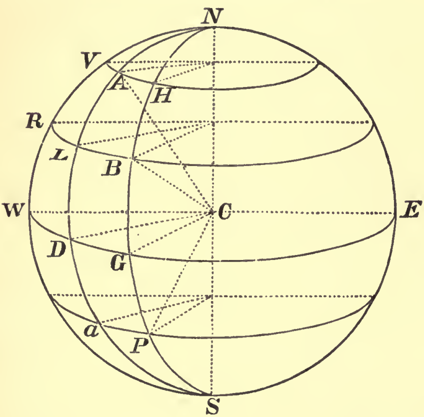
Latitude is reckoned from 0° to 90°, north and south of the equator.

7. The *difference of latitude* of two places is the difference between the latitudes of the two places, difference being understood as *algebraic*, and north latitudes having the sign + and south latitudes the sign −.

(*a*) To find, then, the *difference of latitude* of two places whose latitude is given, *take the less from the greater if both are N. or both are S., but add the two if one is N. and the other S.*

(*b*) The *difference of latitude* of two places is measured on *any arc of a meridian intercepted between the parallels of latitude of the places.*

Let the figure represent a hemisphere of the earth. Let *N* and *S* be the poles; *C* the center; and *WDE* the equator. Suppose *A* and *B* to be two points on the surface; *VAH* to be the parallel of latitude of *A*, and *NAS* to be its meridian; *RLB* to be the parallel of latitude, and *NBS* the meridian of *B*. Then it is to be proved that the difference of latitude of *A* and *B* is measured by *AL*, *HB*, *VR*, or any other meridian arc intercepted between *VAH* and *RLB*.



Let the meridian *NAS* intersect the parallel *RLB* in the point *L*, and the equator in the point *D*. Also, let the meridian *NBS* intersect the parallel *VAH* in the point *H*, and the equator in the point *G*. Draw the straight lines *CA*, *CD*, *CB*, and *CG*.

The plane of the meridian *NAS* is perpendicular to the plane of the equator (Arts. 2 and 3), and is, therefore, the plane which projects the line *CA* upon that plane; *CD* is the intersection of these two planes (P. and F., 528), and contains (as a part of it) the projection of the line *CA*. The angle *ACD* is, therefore, the angle made by the line *AC* with the plane of the equator (P. and F., 586), and is, consequently, the latitude of the point *A* (Art. 6). Also, the plane of the meridian, *NGS*, is perpendicular to the plane of the equator, and by its intersection with that plane determines the projection of the line *CB* upon the plane. Therefore, *BCG* is the latitude of the point *B*.

Now, *ACD*, or the latitude of *A*, is measured by *AD*, and *BCG* is measured by *BG*; therefore, the difference of latitude of *A* and *B* is measured by the difference between *AD* and *BG*; that is, by $AD - BG$.

$$AD = ND - NA = NG - NH = NW - NV \text{ (P. and F., 817).}$$

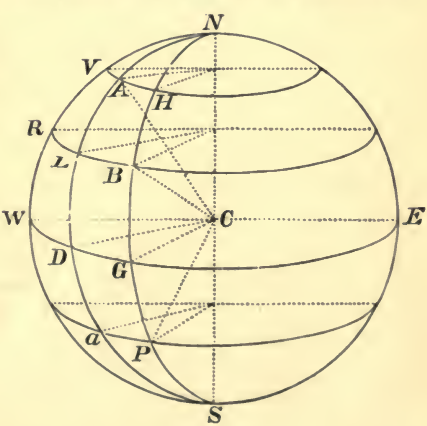
$$\text{Also,} \quad BG = ND - NL = NG - NB = NW - NR.$$

$$\therefore\ AD - BG = NL - NA = NB - NH = NR - NV;$$

$$= AL = HB = VR, \text{ etc.}$$

If the point *P* be taken on a parallel of latitude south of the equator, the difference of latitude would be measured by *HP* or by *Aa*, an arc of a meridian intercepted between the parallels.

8. It is evident that *the position of any point or place* on the earth's surface *is determined if the latitude and longitude of the point, or place, are known.*



Thus, suppose *NWSE* to represent a hemisphere of the earth; *NWS* to be the meridian from which longitude is reckoned; *WDE* to be the arc of the equator; *RLB* and *VAH* to be parallels of latitude; and *NDS*, *NBS* to be meridians.

Suppose the latitude of the point to be 30° N., and the longitude to be 40° E. If, now, *RLB* be a parallel of latitude, of which the polar distance *NR*, *NL*, or *NB* is 60°, since *NW, ND,* or *NG* is 90°, *WR, DL,* or *GB* is 30°; therefore, *RLB* is a parallel of *latitude, every point* of which is 30° N. of the equator. Consequently, the point whose latitude is given must be found somewhere on this arc *RLB.* Again, if *NDS* be a meridian, whose plane *NDS* makes with the plane *WNS* an angle of 40° (measured by the arc *WD* of the equator), the *longitude* of every point on *NDS* is 40° E. Therefore, the point whose longitude is given must be somewhere on the meridian *NDS.* Since the point is on the arc *RLB,* and at the same time on the arc *NDS,* it must be at their intersection, *L.* Therefore, the point is *determined* when its latitude and longitude are given.

It might be said that two circles intersect twice, and therefore that the point of the circle *RLB* diametrically opposite to *L* would be indicated by lat. 30° N., long. 40° E. This is evidently false, since the other half of *NDS* and the other half of *RLB,* which, by their intersection, determine this second point, are on the other hemisphere. The *latitude* of this second point *is* 30° N., but its *longitude* is 140° *W.* of the assumed meridian (Art. 5, (*a*)).

9. As charts of the earth's surface are constructed for the use of navigators with meridians and parallels of latitude either drawn on them, or indicated, if a ship's *latitude* and *longitude* are known, the position of the ship is determined. It is important that this position should be determined from day to day, and therefore it is important that the ship's latitude and longitude should be known. Latitude and longitude are best obtained by observations of the heavenly bodies. This is a department of navigation which belongs to astronomy. It is necessary to have other methods of determining a ship's position when it is impossible to resort to the methods of astronomy. These other methods are now to be considered.

10. (*a*) The *distance* sailed by a ship, in going from one point to another, is the *length* of the line traversed by the ship between the two points.

(*b*) The *bearing* or *course* of a ship, at any point, is the *angle* which the line traversed by the ship (that is, the *distance*) makes with the meridian passing through that point.

If a ship cuts every meridian at the *same angle*, she is said to continue on the *same course.*

If a ship is said to sail a given distance on a given course, it is assumed that in that distance she continues on the *same course.*

The path made by a ship continuing on the same course is called a *rhumb line*, or simply a *rhumb.*

(*c*) The *departure* of a ship, in sailing from one point to another, is the whole east or west distance she makes measured from the meridian from which she sails, and is an easting or westing according as she sails in an easterly or westerly direction.

If the distance sailed is *small*, it may be considered a straight line, and the departure might also be regarded as a *straight* line measuring the perpendicular distance between the meridians of the two points. In this case the *meridians* may be considered parallel straight lines (as in surveying), since they are lines on a small portion of the earth's surface, and are perpendicular to the same line.

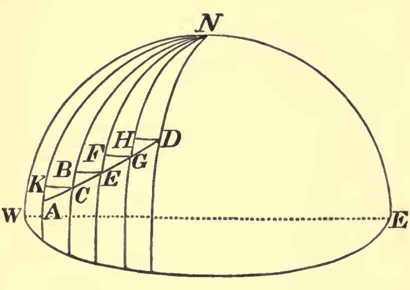
If the distance is *not* small, it may be divided up into such a number of small parts that each of them may be considered as a straight line. The departure of each of these small distances will then also be a *straight* line, and the departure of the *whole* distance will be the sum of the departures of the parts.

(*d*) *Difference of latitude* of two points has already been defined (Art. 7).

If a given distance sailed by a ship is small, it may be regarded as a straight line, and then the difference of latitude of the two extremities of the line, representing this distance, is measured by a line, which may be also regarded as a straight line. The difference of latitude is then a northing or southing (as in surveying).

If the distance is *not* small, it may be divided into such a number of parts that each part may be small enough to be considered a straight line. The difference of latitude of each part will then be a straight line, and the difference of latitude of the whole distance will be the sum of the differences of latitude of the parts.

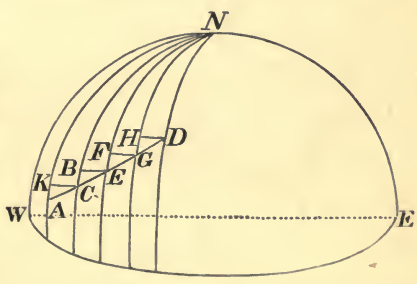
Thus, suppose *AC* to be a small distance on the earth's surface. Let *AK* and *BC* be meridians of the points *A* and *C*, and let these lines be considered parallel. If *CK* be a perpendicular to *AK* drawn from *C*, it will be the departure of *AC*, and *AK* will be the difference of latitude of *A* and *C*, or the difference of latitude for the distance *AC*.



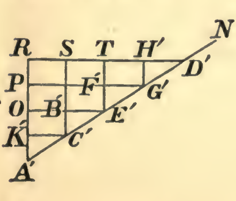
If the distance be a *long* distance, as from *A* to *D*, then it can be divided into such a number of short distances — as, for instance, *AC*, *CE*, *EG*, and *GD* — that each one of them can be considered as a straight line. If *AK*, *CB*, *EF*, and *GH* be the meridians of the points A, C, E, and G, and if CK be the perpendicular from C to AK, EB be perpendicular to CB, GF to EF, and DH to GH, then the *departure* for AD will be $KC + BE + FG + HD$, and the *difference of latitude* will be $AK + CB + EF + GH$.

11. *Plane sailing* is the art of determining the position of a ship at sea by means of a right-angled plane triangle. Of this triangle the hypotenuse is the distance, the base is the difference of latitude, the perpendicular is the departure, and the angle between the base and the hypotenuse is the course.

When the distance sailed is short, it is evident from the figure that the four quantities mentioned are the parts of a right-angled triangle; for then AC is the distance, AK is the difference of latitude, KC at right angles to AK is the departure, and CAK is the course.



If the distance sailed is not short, — as, for instance, the distance AD, — then divide it into such a number of small distances, AC, CE, EG, and GD, that each may be considered a straight line. Complete the figure as in the preceding article. Suppose the ship's course to be the same in sailing from A to D, then the angles CAK, ECB, GEF, and DGH are equal (Art. 10, (*b*)).



Now, take any straight line $A'N$, and on it lay off $A'C'$, $C'E'$, $E'G'$, and $G'D'$, equal respectively to AC, CE, EG, and GD, and on these lines $A'C'$, $C'E'$, $E'G'$, and $G'D'$ construct right-angled triangles $A'K'C'$, $C'B'E'$, $E'F'G'$, and $G'H'D'$ equal respectively to AKC, CBE, EFG, and GHD, then $A'D' = AD$, the distance, and

$A'K' + C'B' + E'F' + G'H' = AK + CB + EF + GH =$ difference of latitude;

$K'C' + B'E' + F'G' + H'D' = KC + BE + FG + HD =$ departure.

Since the ship sails on the same course, the angles $K'A'C'$, $B'C'E'$, $F'E'G'$, and $H'G'D'$ are all equal, and, therefore, the lines $A'K'$, $C'B'$, $E'F'$, $G'H'$ are parallel; also, the lines $K'C'$, $B'E'$, $F'G'$, and $H'D'$ are parallel (P. and F., 44). Produce $A'K'$ and $D'H'$ to meet at R; produce $C'B'$ and $E'F'$ to meet $D'R$ at S and T; and produce $E'B'$ and $G'F'$ to meet $A'R$ at O and P. R is a right angle, since it is equal to K'. $A'RD'$ is consequently a right-angled triangle. $A'D'$ represents distance sailed. $D'A'R$ represents the course. $A'R$ represents the distance of latitude, for

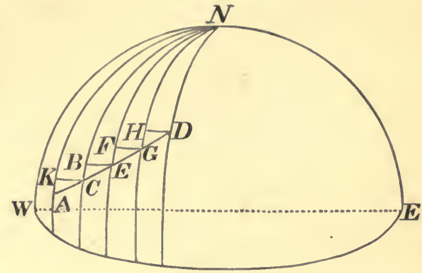
$$A'R = A'K' + K'O + OP + PR = A'K' + C'B' + E'F' + G'H'.$$

RD' represents the departure, for

$$RD' = RS + ST + TH' + H'D' = K'C' + B'E' + F'G' + H'D'.$$

12. Any two parts of a right-angled triangle being given, in addition to the right angle, the other parts may be found; therefore, of the four quantities, the *distance*, the *course*, the *departure*, and the *difference* of latitude, any two being given, the other two may be found, since these quantities may be represented by the parts of a right-angled triangle, as has been shown in the preceding article, and will, therefore, have the same relation to one another as the corresponding parts of the right-angled triangle.

When the distance is small, this is evident. If the distance is great, it may be divided, as before, into such a number of small distances, AC, CE, EG, and GD, that each may be considered a straight line. Let the differences of latitude for these small distances be AK, CB, EF, and GH, and let the departures be KC, BE, FG, and HD. As the course is supposed to be the same for the whole distance AD, the angles CAK, ECB, GEF, and DGH are all equal.



In the right-angled triangle AKC, $\frac{AK}{AC} = \cos CAK = \cos$ course;

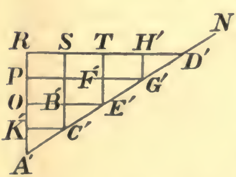
In the right-angled triangle CBE, $\frac{CB}{CE} = \cos BCE = \cos$ course;

In the right-angled triangle EFG, $\frac{EF}{EG} = \cos FEG = \cos$ course; and

In the right-angled triangle GHD, $\frac{GH}{GD} = \cos HGD = \cos$ course.

Therefore (P. and F., 265),

$$(1)\quad \frac{AK + CB + EF + GH}{AC + CE + EG + GD} = \cos \text{course}.$$



Now, if we construct the right-angled triangle $A'RD'$, as in the preceding article, having $A'D' = AD$, then, as in Art. 11, it may be shown that

$A'R = AK + CB + EF + GH =$ dif. of latitude, and

$$A'D' = AC + CE + EG + GD = AD = \text{dist.}$$

Substituting these values in (1), we have:

$$\frac{A'R}{A'D'} = \frac{\text{difference of latitude}}{\text{distance}} = \cos \text{course; or,}$$

(2) difference of latitude = distance × cos course.

In the same manner it can be shown

(3) departure = distance × sine of course;

(4) departure = difference of latitude × tan course;

and that the other relations shown to hold between the parts of a right-angled plane triangle hold between the quantities in navigation represented by these parts.

13. If at a given time it is required to find the position of a ship by *plane sailing*, the rate of speed per hour at which she is sailing is first ascertained. This rate, multiplied by the number of hours elapsed since the last ascertained position, will give the *distance* from that position. The angle made by the direction in which the ship is headed, and the N. and S. line of the mariner's compass (with correction, if necessary), will furnish the *course*. From these data the *difference of latitude* and the *departure* are found (Art. 12, (2) and (3)), and thus the position of the ship is known.

For example, suppose the average rate of sailing is ascertained to be 9 miles an hour, and that 12 hours have elapsed since the last ascertained position, then the distance is 108. If the course is observed to be N. 30° E., the ship's position N. of her last position will be, in miles, 108 × cos 30°, or 93.5 miles, and her position E. will be 108 × sin 30°, or 54 miles.

14. The rate of sailing is ascertained by means of the *log*.

The *log*, in one of its simplest forms, is a triangular piece of wood, so weighted as to assume, when attached to its line and placed in water, a position calculated to oppose the most resistance to force applied to the line. The *line* is a rope knotted at regular intervals.

When the log is thrown overboard and the line is reeled out by the forward motion of the ship, the number of knots passing over a given point in a given period of time will give the rate of sailing for that period of time. Moreover, *if the interval between the knots be the same part of a mile that the period of time is of an hour, the number of knots passed out during the period of time will give the number of miles per hour* sailed by the ship.

For instance, let the period of time be $\frac{1}{2}$ minute or $\frac{1}{120}$ hour, then the interval between the knots must be $\frac{1}{120}$ mile. Suppose, then, 4 such knots (counting the intervals by the knots) should be reeled out by the forward motion of the ship during $\frac{1}{2}$ minute, we should find the distance sailed per hour (that is, the rate per hour) by the proportion

$\frac{1}{2}$ min. : 60 min. : : $\frac{4}{120}$ mile : x (the distance per hour).

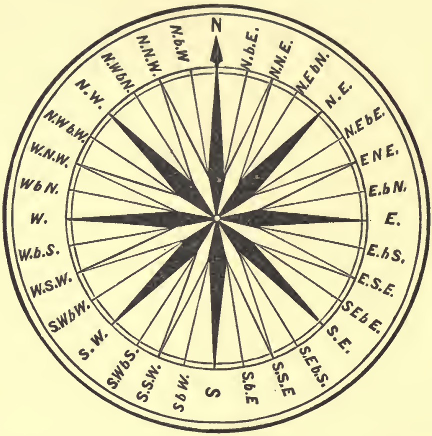
$\therefore x = 2 \times 60 \times \frac{4}{120} = 4$ miles, the same number of miles per hour as knots per half minute.

15. The *mariner's compass* consists of a circular card attached to a magnetic needle, which generally points N. and S.* Each quadrant of this card is divided into eight equal parts, called points, to which names are given as represented in the accompanying figure.†

* The magnetic needle does not at all places on the earth's surface point N. and S. Charts for the use of navigators, however, give the amount of variation for places where the needle is subject to variation, so that for such places a correction can be applied to the direction indicated by the needle, so as to obtain a true N. and S. line.

† The naming of the points in each quadrant will be seen to be not without method. Thus, in the quadrant between N. and E., the point midway between N. and E. takes its name from both these points; then the point midway between N. and N.E., and the point midway between

Also, to express courses between the points, the points are subdivided into half points and quarter points.



The points are read (taking the quadrant between the N. and E. points), North by East, North North East, North East by North, North East, etc., etc.

The angle between two adjacent points is $\frac{90}{8}°$, or $11° 15'$

16. Distance, departure, and difference of latitude are all expressed in *nautical miles*.

E. and N.E., take their names respectively from the two points between which each is situated, as one of these is north and the other is east of N.E.

The remaining points are named from the nearest *main* point (calling N., E., and N.E. main points), with the addition of N. or E. as the point to be named is north or east of this nearest point, with the word *by* placed between the two. Thus, the point between N. and N.N.E. is N. *by* E.; the point between N.N.E. and N.E. is N.E. *by* N.; the point between N.E. and E.N.E. is N.E. *by* E.; and the point between E.N.E. and E. is E. *by* N.

The points of the other quadrants may be shown to be named on the same method.

A *nautical mile* is equal to a minute of an arc of the circumference of a great circle of the earth.

As there are 69.115 *common* miles in a degree of such an arc (Trig., Art. 173, Ex. 5), a nautical mile is *longer* than the common statute mile.

Differences of latitude expressed in degrees and minutes is, therefore, easily converted into miles, or, when expressed in nautical miles, is easily changed into degrees and minutes.

Thus, 5° 33′ difference of latitude = 333 miles; and 656 miles difference of latitude = 10° 56′.

Ex. 1. A ship sails N.E. b. N. a distance of 70 miles. Required her departure and difference of latitude at the end of that distance. The course is 3 points from N. toward E., and is, therefore, $3 \times (11° 15')$ or 33° 45′.

Ans. Dep. = 38.89 miles; dif. lat. = 58.2 miles.

Ex. 2. A ship from lat. 33° 5′ N. sails S.S.W. 362 miles. Required her departure and the latitude arrived at.

Ans. Dep. = 138.5 miles; lat. 27° 30.6′ N.

Ex. 3. A ship, leaving port in lat. 42° N., sails S. 37° W. till her departure is 62 miles. Required the distance sailed and the latitude arrived at. *Ans.* Dist. = 103 miles; lat. 40° 38′ N.

Ex. 4. A ship sails S. 50° E. from lat. 7° N. to lat. 4° S. Required her distance and departure.

Ans. Dist. = 1026.78 miles; dep. = 786.56 miles.

Ex. 5. A ship sails from the equator on a course between S. and W. to lat. 5° 52′ S., when her departure is found to be 260 miles. Required her course and the distance sailed.

Ans. Course = S. 36° 27′ W.; dist. = 437.6 miles.

Ex. 6. A ship sails from lat. 3° 2′ N. on a course between N. and W. a distance of 382 miles, when her departure is found to be 150 miles. Required her course and the latitude arrived at.

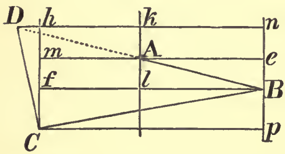
Ans. Course = N. 23° 7¼′ W.; lat. 8° 53′ N.

17. A *traverse* is the path described by a ship which changes its course from time to time.

The object of *traverse sailing* is to find the position of a ship at the end of a traverse; the *distance* sailed from the position left to the position reached; and the *course* for this distance.

The method of accomplishing this object will best be seen by means of an example.

Suppose a ship to start from A and sail to B, then from B to C, and then from C to D. It is required to find her position at D; that is, to find the difference of latitude and the departure made in going from A to D. These quantities being found, the distance AD, and the course DAk, can be calculated.



(REMARK. — The distances AB, BC, and CD are all supposed, in traverse sailing, to be short distances, and therefore are to be treated, like similar distances in plane sailing, as straight lines.)

Through B, A, and C suppose meridians pn, lk, and Ch, and through B, A, C, and D parallels of latitude Bf, me, Cp, and Dkn to be drawn.

Ak is the difference of latitude of AD, and kD is the departure of AD.

$$(1)\quad Ak = mh = Ch - Cm = Ch - (Cf + fm) = Ch - (Bp + eB).$$

Now, Ch is a north latitude, and Bp and eB are south latitudes, therefore, the difference of latitude of AD is equal to the difference between the $N.$ latitude of one distance of the traverse and the sum of the $S.$ *latitudes* of the other distances.

(2) $kD = nD - kn = nh + hD - Ae = pC + hD - Ae.$

But pC and hD are west departures, and Ae is an east departure; therefore, the departure for AD is equal to the difference between the sum of the *west* departures of two distances of the traverse and the *east* departure of the third distance.

In the case given above, the number of the parts of the traverse is only three, but if a fourth distance on a course between N. and E. were given, a second north latitude and a second east departure would enter our figure, so that the *difference of latitude* between the *first* and *last* position of the ship would, in that case, be *equal to the difference between* the *sum of the north* latitudes and the *sum of the south* latitudes; and the departure, in passing directly from the first to the last position, would be *equal to the difference between the sum of the east* departures and the *sum of the west* departures. The same principle would hold true for a traverse of any number of distances greater than four. The proof would be similar to that given above for a traverse of three distances.

The principle stated may, therefore, be taken as a *general* one.

Ex. 1. Suppose a ship sailing on a traverse makes courses and distances as follows: from A to B, E. b. S. 16 miles; from B to C, W. b. S. 30 miles; and from C to D, N. b. W. 14 miles. Required the distance from A to D and the course for that distance. Before solving these examples the student is advised to plot the figures for them by means of a protractor and a plane scale.

	Course	Distance	N.	S.	E.	W.
1	S. 78° 45′ E.	16		3.12	15.69	
2	S. 78° 45′ W.	30		5.85		29.42
3	N. 11° 15′ W.	14	13.73			2.73
Sum			13.73	8.97	15.69	32.15
			8.97			15.69
		dif. of lat. =	4.76 N.		dep. =	16.46 W.

∴ (In the figure, page 22) $Ak = 4.76$, and $kD = 16.46$.

$$\text{Course} = kAD. \quad \frac{kD}{Ak} = \frac{16.46}{4.76} = \tan.\ 73°\ 52'\ 15'' = \tan.\ kAD$$

∴ *Course* = N. 73° 52′ 15″ W., or N. 73° 52′ W., as the result is generally given only to the nearest minute.

$$Dist. = AD = \frac{kD}{\sin DAk} = \frac{16.46}{\sin 73°\ 52'} = 17.13 \text{ miles.}$$

Ex. 2. A ship sails on a traverse, making the following courses and distances: S.E., 25 miles; E.S.E., 32 miles; E., 17 miles; N. b. W., 63 miles. Required the distance from her first to her last position, and the course.

Ans. Dist. = 60.94 miles; course = N. 58° 29′ E.

Ex. 3. A ship sails on a traverse, making the following courses and distances: N.E., 25 miles; E.S.E., 40 miles; E. b. N., 35 miles; N. b. W., 33 miles. Required the course and distance from her first position to her last position.

Ans. Course = N. 63° 16′ E.; dist. = 92.41 miles.

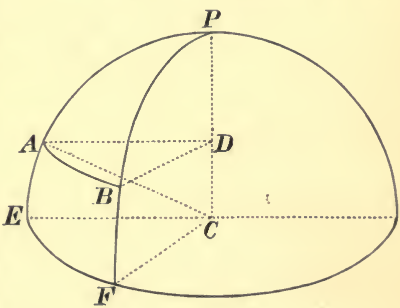
18. *Parallel sailing* is sailing on a parallel of latitude. In parallel sailing, therefore, a ship sails east or west (Art. 4, (*a*)). The *distance* is the same as her *departure*, and the *difference* of *latitude* disappears.

The problem in parallel sailing is to convert *dis-*

tance on a parallel into *difference of longitude;* that is, given a distance between two meridians measured on a parallel of latitude, to find from it the distance between the same meridians measured on the equator (Art. 5, (*b*)).

The method of solving this problem will be understood by means of the accompanying figure, which represents a part of the earth.

In this figure let C represent the center of the earth; P be one of the poles; EF a part of the equator, CE and CF its radii; AB a part of a parallel of latitude intercepted between two meridians, PAE and PBF; and let DA and DB be the radii of this parallel.



Draw the radius AC.

$$\frac{AB}{EF} = \frac{AD}{EC} = \frac{AD}{AC} = \cos DAC = \cos ACE.$$

But ACE is the latitude of A (Art. 6), or of the parallel AB.

$$\therefore \frac{\text{distance on parallel between two meridians}}{\text{distance on equator between same meridians}}$$

$$= \text{cos lat. of parallel.}$$

Or, expressing this in other terms,

$$(1)\quad \frac{\text{dist. on a parallel}}{\text{dif. of longitude}} = \text{cos lat. of parallel.}$$

$$(2)\quad \therefore \text{dif. of longitude} = \frac{\text{dist. on parallel}}{\text{cos lat. of parallel}}$$

$$= \text{dist. on parallel} \times \text{sec lat.}$$

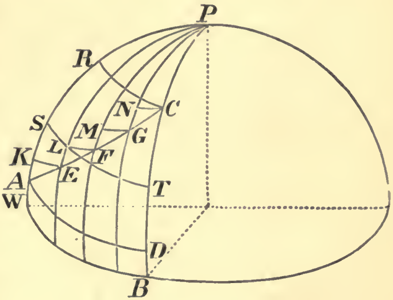
19. Since for a short distance departure is measured on a parallel of latitude (Art. 10, (*c*)), in (1) of last article substituting departure for distance on a parallel, we have

$$(1)\quad \text{departure} = \text{dif. of long.} \times \text{cos lat.; and}$$

$$(2)\quad \text{dif. of long.} = \frac{\text{departure}}{\text{cos lat.}} = \text{departure} \times \text{sec lat.}$$

20. In plane sailing, when the distance sailed is short, the departure can be converted into difference of longitude by formula (2) of the preceding article, or, when the difference of longitude is given, it can be changed into departure by formula (1); in both cases the parallel of latitude being supposed to be known. But if the distance is not short, there is danger of error, since the latitude varies from point to point of the distance, and the departure is neither the distance on a parallel through the point from which the ship sails, nor on a parallel through the point arrived at. This will be understood from the figure.

Suppose the figure to represent a part of the earth's surface, and that AC represents the distance sailed by a ship. The departure for that distance would be $KE + LF + MG$, etc. (Art. 10, (*c*)), which is, evidently, equal to neither AD nor RC, since, as the meridians PE, PF, etc., meet at the pole P, the distance between them measured on a parallel diminishes as we proceed from the equator. The total departure is consequently *less* than AD and



greater than RC. It would, also, be incorrect to convert this departure into difference of longitude by using the latitude of RC or the latitude of AD, as we really ought to use the latitude of the part departure, KE for KE, the latitude of LF for LF, etc., and then take the sum of the differences of longitude corresponding to these departures for the whole difference of longitude. If this method were practicable, and we could make the distances AE, EF, etc., small enough, we should find the difference of longitude without appreciable error. As this method is not practicable, two other methods are used for changing departure into difference of longitude. One is the method of *middle latitude* sailing, the other is the method of *Mercator's sailing*.

21. In *middle latitude* sailing, departure is converted into difference of longitude by using, in Art. 19, (2), the latitude, whose parallel is midway between the parallel of the point sailed from and the parallel of the point arrived at.

This latitude is equal to the *half sum* of the latitude sailed from and the latitude arrived at, if both latitudes are on *the same side* of the equator, but to the half *difference*, if *one* is *north* and the *other south* of the equator.

Thus, suppose ST is the parallel midway between RC and AD; that is, suppose $AS = SR$, A and C being both north of the equator. WS is the measure of the latitude of the parallel ST (Art. 6).

$$WS = \frac{2(WA + AS)}{2} = \frac{WA + WR}{2}.$$

In a similar manner it may be shown that, in case one place is north and the other south of the equator, the middle latitude is half the difference of the latitudes of the two places.

The method of middle latitude sailing is not perfectly exact, but is made nearly so by applying corrections taken from a table prepared for that purpose.* For short distances or for sailing near the equator it is practically correct.

22. By Art. 19,

(1) $$\text{dep.} = \text{dif. of long.} \times \cos \text{lat.},$$

and (2) $$\text{dif. of long.} = \frac{\text{dep.}}{\cos \text{lat.}} = \text{dep.} \times \sec \text{lat.}$$

In middle latitude sailing, for latitude we substitute mid. lat., and (1) becomes

(*a*) $$\text{dep.} = \text{dif. of long.} \times \cos \text{mid. lat.},$$

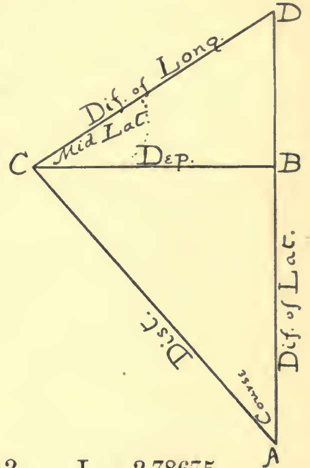
and (2) becomes

(*b*) $$\text{dif. of long.} = \frac{\text{dep.}}{\cos \text{mid. lat.}} = \text{dep.} \times \sec \text{mid. lat.}$$

Equations (*a*) and (*b*) can be represented in terms of base and hypotenuse of a right-angled triangle.

* Table of Corrections to Middle Latitude, pages 172, 173.

This triangle can be combined in one figure with the triangle for plane sailing, as will be seen by the accompanying diagram.



Ex. 1. From lat. 40° N. and long. 50° W. a vessel sails on a course N.W. b. N. to lat. 50° 12′ N. Required distance sailed, and the longitude of point of arrival.

$$AB = 10° \, 12' = 612.$$

$$\text{Angle } A = 33° \, 45'.$$

Angle DCB

$$= \text{mid. lat.} = \frac{40° + 50° \, 12'}{2}$$

$$= 45° \, 6' + \text{cor}^{*} \text{ of } 2' = 45° \, 8'.$$

$$AC = \text{dist.} = \frac{AB}{\cos A} = \frac{612}{\cos 33° \, 45'} \qquad \begin{aligned} \text{L.} &= 2.78675 \\ \text{L.} &= 9.91985 \\ \log 736.3 &= 2.86690 \end{aligned}$$

$$\text{dist.} = 736.3 \text{ miles.}$$

$$BC = \text{dep.} = AB \tan A = 612 \tan 33° \, 45'.$$

$$CD = \text{dif. of long.} = \frac{BC}{\cos DCB} = \frac{612 \tan 33° \, 45'}{\cos 45° \, 8'}.$$

$$\begin{aligned} \log 612 &= 2.78675 \\ \log \tan 33° \, 45' &= 9.82489 \\ \text{colog.} \cos 45° \, 8' &= 0.15153 \\ \log 579.7 \quad & 2.76317 \end{aligned}$$

$$\begin{aligned} \text{dif. of long.} = 579'.7 \text{ W.} &= 9° \, 39'.7 \text{ W.} \\ \text{long. of pt. of departure} &= 50° \qquad \text{W.} \\ \text{long. of pt. of arrival} &= 59° \, 39'.7 \text{ W.} \end{aligned}$$

* Table of Corrections to Middle Latitude, pages 172, 173.

Ex. 2. From lat. 32° 22′ N., long. 64° 38′ W., a ship sails S.W. by W. a distance of 375 miles. Required the latitude and longitude of point of arrival.

Ans. 28° 53′.7 N.; 70° 38′.6 W.

Ex. 3. From lat. 40° 28′ N., long. 74° 1′ W., a ship sails S.E. b. S. a distance of 450 miles. Required the latitude and longitude of point of arrival. *Ans.* 34° 13′.8 N.; 68° 47′.5 W.

Ex. 4. From lat. 40° 28′ N., long. 74° 1′ W., a ship sails S.E. b. E. to lat. 31° 10′ N. Required the distance sailed and longitude of point of arrival. *Ans.* 1004 miles; 56° 53′.5 W.

Ex. 5. From lat. 32° 28′ N., long. 64° 48′ W., a vessel sails on a course between S. and W. to lat. 28° 54′ N., making a distance of 475 miles. Required the course and the longitude of the point of arrival. *Ans.* S. 63° 13′ 22″ W.; 72° 59′ W.

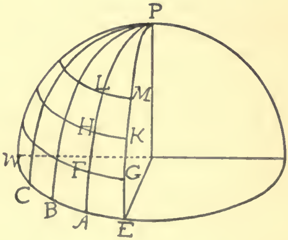
Ex. 6. If from lat. 46° 40′ N., long. 53° 7′ W., a ship sails to lat. 32° 38′ N., long. 16° 40′ W., required the course and distance sailed. *Ans.* S. 63° 23′ E.; 1879 miles.

23. In *Mercator's sailing* departure is converted into difference of longitude by means of the principles of Mercator's chart.

As the meridians all pass through the poles, a chart, in order to represent correctly the earth's surface, should make the meridian lines curved and approaching one another toward either pole. The parallels of latitude being circles smaller and smaller the nearer they are to the poles, should, on a correct chart, be shorter and shorter curves the farther they are from the equator.

On Mercator's chart the equator, the meridians, and parallels of latitude are all represented as *straight* lines. Meridian lines are all drawn at right angles to the equator, and are, therefore, parallel to each other. Parallels of latitude are made parallel to the equator, and therefore, like parts of any parallel, are equal to like parts of the equator. On Mercator's chart, therefore, the east and west dimensions of any part of the earth's surface are made too large, except near the equator. To preserve the true proportion existing between the dimensions of any particular part of the earth, the north and south dimensions are lengthened in proportion to the lengthening of the east and west dimensions. The method of accomplishing this will be understood by means of the accompanying figures.

On a globe representing the earth, the meridians *PE*, *PA*, etc., make with the equator and with parallels of latitude a number of quadrilaterals, all of whose sides are curved lines. Thus, in the figure, if the equator, represented by *WE*, be supposed to be divided into a number of parts of 10°, each equal to *AE*, and on the meridian *PE*, we lay off *EG*, *GK*, *KM*, etc., each

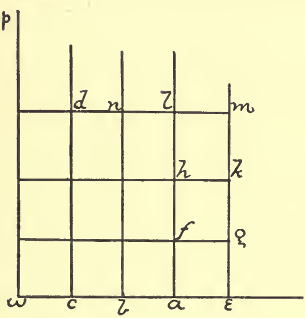


also equal to 10°, drawing parallels of latitude through the points of division *G*, *K*, *M*, etc., we should divide the surface of the globe into several tiers of quadrilaterals; one tier composed of quadrilaterals each equal to *FGAE*, a second tier of quadrilaterals each equal to *HFGK*, a third tier of figures each equal to

LHKM, etc. Supposing the earth to be a sphere, on the globe representing it, *AE*, *EG*, *GK*, and *KM* would all be equal, as they are equal parts of equal great circles. Also, the ratio of *EG* to *GF* $=\sec 10°$ (Art. 18, (2)), and

$$\frac{GK}{KH}=\sec 20°;\quad \frac{KM}{ML}=\sec 30°.$$

If we desire to represent these various tiers of quadrilaterals on Mercator's chart, with the features of the earth which they inclose, we draw a straight line of the same length as the curved line representing the equator on the globe; that is, we make *we* equal to *WE*, and divide it into parts *wc*, *cb*, *ba*, and



ae, each equal to *AE*, and at the points *w*, *c*, *b*, *a*, and *e* erect perpendiculars *wp*, *cd*, *bn*, etc., to represent the meridians *PW*, *PC*, *PB*, etc. The lines *wp*, *cd*, *bn*, etc., being at right angles to *we*, are parallel. If we draw a series of lines parallel to *we* to represent parallels of latitude, as *dm*, *hk*, and *fg*, we form tiers of rectangles; one tier of rectangles each equal to *fgea*, a second tier of rectangles each equal to *hkgf*, and so on. By this construction *fg*, *hk*, and *bn* are all made equal to *ae*. To make the quadrilaterals, like *fgea*, represent the corresponding quadrilaterals, like *FGEA*, we must lengthen eg as much as we have lengthened fg. We have made

$$fg = ae = AE = FG \sec 10° \text{ (Art. 18, (2))},$$

therefore we must make

$$eg = EG \sec 10° \text{ (or, } ae \sec 10°),$$

so that

$$\frac{eg}{gf} = \frac{EG}{GF}.$$

Consequently, if on the line em we take a point g so that $eg = ae \sec 10°$, and through g draw a straight line parallel to wae, we shall form a tier of quadrilaterals each equal to $afge$, whose sides af and eg have the same ratio to fg which AF and EG bear to FG. In like manner, if we make $gk = ae \sec 20°$, and through k draw another straight line parallel to wae, we shall form a second tier of quadrilaterals each equal to $fhkg$, whose sides fh and gk have the same ratio to hk which FH and GK bear to HK. Through m, if $km = ae \sec 30°$, we draw another straight line parallel to wae, making a third tier of quadrilaterals, and so on for the rest of the chart.

If, instead of taking the parts, like ae, equal to 10° of the equator we make them 1° or 1′, then the parallels of latitude will be drawn at smaller intervals on the meridian me.

If

$$ae = 1',$$

then

$$em = 1' (\sec 1' + \sec 2' + \sec 3').$$

In the same way the length of Mercator's meridian up to 30° would equal the sum of

$$\sec 1' + \sec 2' + \sec 3' \cdots + \sec 29° 59' + \sec 30°,$$

or the sum of the series of secants, from sec 1′, increasing by intervals of 1′ up to sec 30°.

Mercator's chart is, therefore, a chart of the earth's surface on which the unit of the scale of representation is continually changing. Near the equator the parts of the earth's surface are correctly represented. As we go north or south to any distance from that line, the parts of the earth are enlarged, as compared with the parts near the equator.

As the earth is not a perfect sphere, but a spheroid with its shorter diameter connecting the poles, the meridians are all smaller curves than the equator, so that in the later Mercator's charts, and in the tables of the lengths of Mercator's meridians for different latitudes (called Tables of Meridional Parts), this fact is taken into account. However, with this modification, the method of construction of a Mercator's chart just given is substantially correct.

In Mercator's sailing the unit of measure, or the nautical mile, is 1′ of the equator. Tables of Meridional Parts accordingly give in minutes, or nautical miles, the length of Mercator's meridian from the equator to any point of latitude denoted by the table.

24. The path of a ship continuing on the same course is, on Mercator's chart, a *straight* line, since to continue on the same course the ship must cut each of the meridians at the same angle, and the meridians are parallel straight lines.

As Mercator's meridian is longer than the true meridian on a chart representing a curved surface, and is continually lengthening, the *number of parts* in a certain number of degrees and minutes of the table will generally be greater than the number of *minutes* in the corresponding number of degrees and minutes of true meridian.

Thus, for example, the number of parts of 16° of the table of meridional parts is 966.4, while the number of minutes of 16° of true meridian is 960.

Near the equator the number of minutes of true meridian is greater than the number of meridional parts of the same degree measure.

Thus, 4° of true meridian = 240°, while meridional parts of 4° = 238.6.

(*a*) *Meridional difference of latitude* is the distance on Mercator's meridian between two parallels of latitude.

Where the latitudes of two places are given, the *meridional difference* of *latitude* is found by taking the *meridional parts* of the less latitude *from* the *meridional parts* of the greater, if *both* are *north*, or *both* are *south* latitudes; but, by *adding* the meridional parts of the two latitudes, if *one* is *north* and the *other south* latitude.

The rule is the same as for finding the true difference of latitude, except that *meridional parts* of latitude are used instead of latitude.

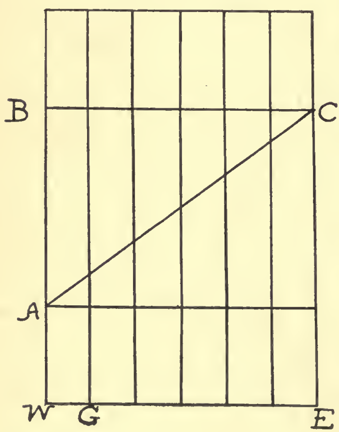
Ex. 1.

lat. of Newport, R.I., is	41° 29′ N.	merid. parts	2725.0
lat. of Savannah, Ga., is	32° 5′ N.	merid. parts	2022.1
dif. of lat.	9° 24′	merid. dif. lat.	702.9

Ex. 2.

lat. of Pato Island is	10° 38′ N.	merid. parts	637.5
lat. of Cape St. Roque is	5° 29′ S.	merid. parts	327.3
dif. of lat.	16° 7′	merid. dif. lat.	964.8

If one latitude is given, and the meridional difference of latitude is found, the latitude required is found by *adding* the meridional parts of the given latitude to the meridional difference of latitude, if the place whose latitude is required is farther from the equator than the place whose latitude is given, and if both places are on the same side of the equator; but, by *subtracting* the meridional difference of latitude from the meridional parts of the given latitude if the place whose latitude required is nearer the equator than the place whose latitude is given; the degrees and minutes, answering to the result as found in the table of meridional parts, will be the latitude required.



This will be evident from the figure, in which *WE* represents the equator on Mercator's chart.

If the latitude of *A* is given, the meridional parts, or the distance *AW*, can be found from the table. *AB* being the meridional difference of latitude, the meridional parts of C or the distance, $EC = WB = WA + AB$.

If the latitude C is given, then from the table $CE (= WB)$ is found; then, $AW = WB - AB = CE - AB$.

Ex. 1. lat. of place left is 25° 6′ N. merid. parts = 1546.9
ship sails northerly till she makes merid. dif. of lat. 750.0
meridional parts of place arrived at = 2296.9
Therefore, from table, latitude arrived at is (nearly) 35° 54′ N.

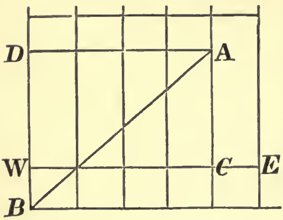
Ex. 2. lat. of place left is 46° 10′ S. merid. parts = 3113.4;
ship going northerly, her merid. dif. of lat. is found
to be . . . . = 825.6
merid. parts of lat. arrived at = 2287.8
Therefore, latitude arrived at is 35° 46′.4 S.

If a ship starting from one side of the equator sails to a point on the other side, the latitude of the point arrived at is found by subtracting the meridional parts of the given latitude from the meridional difference of latitude; the result will be the meridional parts of the required latitude.

Ex. The meridional difference of latitude is . . . 1805.8
which is made by a ship going *north*, starting from lat.
8° 41′ S. . . . . merid. parts = 519.5
Therefore, latitude arrived at is 21° 5′ N. Merid. parts 1286.3

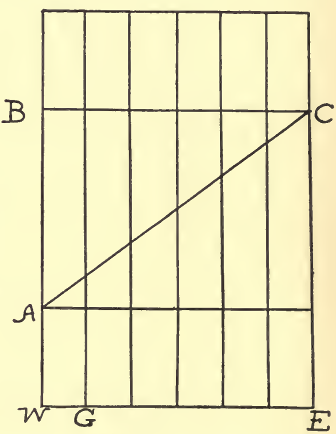
It is evident, therefore, if the meridional difference of latitude made by a ship sailing *from* a point on either side of the equator *toward* a point on the opposite side, exceeds the meridional parts of the latitude left, that the ship has crossed the line and has arrived at a N. latitude, if the latitude left was S., but has arrived at a S. latitude if the latitude left was N.

Thus, on the figure, WE representing the equator, a ship sails from B toward A. BW represents the meridional parts of the latitude left, BD is the meridional difference of latitude, AC represents the meridional parts of the latitude arrived at.



$$AC = WD = BD - BW.$$

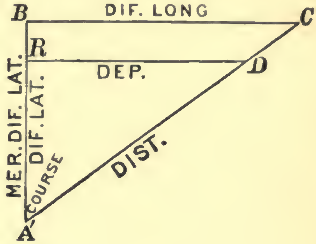
25. *Combining Mercator's sailing with plane sailing.* Let the figure represent a part of Mercator's chart, on which WE represents the equator, and AC is the lengthened distance between two points A and C. CAB is the course for that distance. If, from C, CB be drawn perpendicular to the meridian WB, AB will be the meridional difference of latitude, and BC will be the lengthened departure (Art. 10, (c)).



Now, $BC = WE$; that is, *departure*, on Mercator's chart, *equals difference of longitude* (Art. 5, (b)).

If, in plane sailing, the same course and distance were represented by the hypotenuse and acute angle of a right-angled triangle, $A'RD$, $A'R$ would be true difference of latitude, RD would be departure.

Now, ABC and $A'RD$ are similar triangles, since the angles A' and A' are equal, as they both represent the same course, and the angles R and B are right angles. Placing the angle A upon A', the two triangles may be combined, as in the figure; then



$$\frac{A'R}{A'B}=\frac{RD}{BC};\ \text{that is,}$$

$$(1)\quad \frac{\text{dif. of lat.}}{\text{merid. dif. of lat.}}=\frac{\text{dep.}}{\text{dif. of long.}}.$$

Also, $\qquad BC = A'B \times \tan A'$; that is,

(2) dif. of long. = merid. dif. of lat. × tan course.

By means of these two triangles all cases of Mercator's sailing may be solved, and the position of a ship at sea may be determined from the usual data.

The latitude of one position of the ship, either of the point left or the point arrived at, must always be known in order to use Mercator's sailing.

The line $A'C$ is not required in calculations. $A'D$ represents the *true* distance.

Ex. 1. A ship starting from lat. 37° N., long. 10° W., sails on a course between N. and E. to lat. 41° N., making a distance of 300 miles. Required the course and the longitude arrived at.

lat. 41° N.	merid. parts = 2686.5
lat. 37° N.	merid. parts = 2378.8
dif. of lat. = 4° = 240′	merid. dif. of lat. = 307.7

Taking the figure of the preceding article, $A'D = 300$, $A'R = 240$, and $A'B = 307.7$.

$$\frac{A'R}{A'D} = \frac{240}{300} = \cos A' = \cos \text{course}. \qquad BC = A'B \times \tan A'.$$

$$\text{dif. long.} = 307.7 \times \tan 36° \, 52' \, 12''$$

$$\begin{array}{ll} \log 240 = 2.38021 & \log 307.7 = 2.48813 \\ \log 300 = \underline{2.47712} & \log \tan 36° \, 52' \, 12'' = \underline{9.87506} \\ \log \cos 36° \, 52' \, 12'' \quad 9.90309 & \log 230.8 \quad 2.36319 \end{array}$$

course = N. 36° 52′ E. dif. long. = 3° 50′.8 E.

$$\begin{array}{rl} \text{long. left,} & 10° \text{ W.} \\ \text{dif. of long.} & \underline{3° \, 50'.8 \text{ E.}} \\ \text{long. arrived at} = & 6° \quad 9'.2 \text{ W.} \end{array}$$

Ex. 2. A ship leaving lat. 50° 10′ N., long. 60° E., sails E. S. E. till her departure is 957 miles. Required latitude and longitude arrived at, and the distance sailed.

$$\frac{957}{\sin 67° \, 30'} = \text{dist.*} \qquad \frac{957}{\tan 67° \, 30'} = \text{dif. of lat.}$$

$$\begin{array}{ll} \log 957 = 2.98091 & \log 957 = 2.98091 \\ \log \sin 67° \, 30' = \underline{9.96562} & \log \tan 67° \, 30' = \underline{10.38278} \\ \log 1035.8 \quad 3.01529 & \log 396.4 = 2.59813 \end{array}$$

dist. = 1035.8 miles.

$$\begin{array}{rl} \text{dif. lat.} = & 6° \, 36'.4 \text{ S.} \\ \text{lat. left} = & \underline{50° \, 10' \text{ N.}} \\ \text{lat. reached} = & 43° \, 33'.6 \text{ N.} \end{array}$$

$$\begin{array}{rl} \text{merid. parts of } 50° \, 10' = & 3472.4 \\ \text{merid. parts of } 43° \, 33'.6 = & \underline{2893.4} \\ \text{merid. dif. of lat.} = & 579 \end{array}$$

$$\text{dif. long.} = 579 \times \tan 67° \, 30'.$$

$$\begin{array}{ll} \log 579 = 2.76268 & \text{dif. long.} = 23° \, 17'.8 \text{ E.} \\ \text{long. } \tan 67° \, 30' = \underline{10.38278} & \text{long. left} = \underline{60° \qquad \text{E.}} \\ \log 1397.8 = 3.14546 & \text{long. reached} = 83° \, 17'.8 \text{ E.} \end{array}$$

* No figure is given for this example, but the student is advised to plot the figure for it, and the figure for each of the examples which follow.

Ex. 3. From a point in lat. 49° 57′ N., long. 5° 14′ W., a vessel sails on a course S. 39° W. to a point in lat. 45° 31′ N. Required the distance sailed and the longitude reached.

Ans. Dist. = 342.28 miles; long. = 10° 33′.5 W.

Ex. 4. From a point in lat. 49° 57′ W., long. 5° 14′ W., a vessel goes to lat. 39° 20′ N., making a W. departure of 789 miles. Required the course sailed, the distance made, and the longitude reached.

Ans. Course = S. 51° 5′ W.; dist. = 1014 miles; long. = 23° 43′.8 W.

Ex. 5. From a point in lat. 14° 45′ N., long. 17° 33 W., a vessel sails S. 28° 7½′ W. to a point in long. 29° 26′ W. Required the latitude reached and the distance sailed.

Ans. Lat. reached = 7° 26′.5 S ; dist. = 1509.8 miles.

Ex. 6. From a point in lat. 20° 22′ N., long. 45° 24′ W. to a point in lat. 40° 30′ N., long. 20° 10′ W., it is required to find the course and distance.

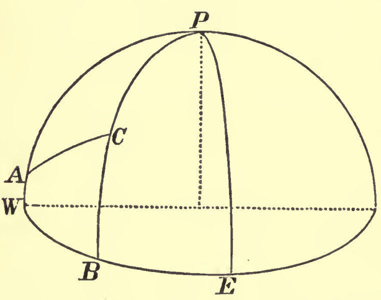
Ans. Course = N. 47° 6½′ E. ; dist. = 1774.9 miles.

CHAPTER II

GREAT CIRCLE SAILING

26. *To find the distance on the arc of a great circle between two points on the earth, the latitude and longitude of each point being given.*

Suppose A and C to represent the two points. If P represents the pole of the earth, WE a part of the equator, PE the meridian from which longitude is reckoned, and PW and PB meridians through A and C, then, WB will be the difference of longitude between A and C; WA will measure the latitude of A, and BC will measure the latitude of C.



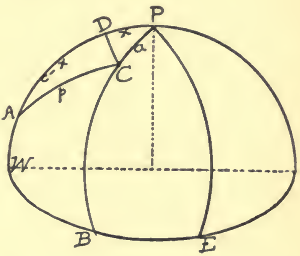
In the spherical triangle APC,

$$AP = PW - AW = 90° - \text{lat. of } A;$$

$$PC = PB - BC = 90° - \text{lat. of } C;$$

angle APC is measured by arc WB, or, degrees of APC = degrees of difference of longitude; therefore, we have given two sides and included angle of a spherical triangle to find the third side.

27. If it is required to find the distance only, we may proceed in the following manner:



Denote the sides opposite A, P, and C by a, p, and c, respectively. From C draw an arc, CD, perpendicular to PA at D. Denote the segment PD by x. Then the segment AD will be $c-x$, if D falls within the triangle; if D falls on PA produced, AD will be $x-c$.

(1) Take the case where the perpendicular falls within the triangle. Applying Napier's Rule of the Circular Parts to triangle CDP, we find

$$\tan x = \frac{\cos P}{\cot a}; \qquad (a)$$

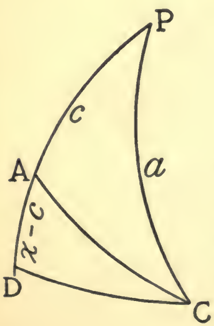
also,

$$\cos CD = \frac{\cos a}{\cos x}. \qquad (b)$$

In the triangle CDA, from Napier's Rule,

$$\cos p = \cos AD \cos CD$$

$$= \frac{\cos (c-x) \cos a}{\cos x}. \qquad (c)$$



(2) If the perpendicular falls without the triangle, then $PD = x$, and equations for $\tan x$ and $\cos CD$ remain the same; but for $\cos AD$ we have $\cos (x-c)$, so that the equation for P becomes

$$\cos p = \frac{\cos(x-c)\cos a}{\cos x}. \qquad (d)$$

To find p, therefore, it is necessary only to compute x from equation (a), and to substitute its value in (c) or (d).

Ex. 1. It is required to find the distance, on the arc of a great circle, between a point in lat. 40° 28′ N., long. 74° 8′ W., and a point in lat. 55° 18′ N., long. 6° 24′ W. Let the first point be represented by A and the second point by C in a figure similar to the first figure of the preceding article.

Then, $c = PA = 90° - 40° 28' = 49° 32'$,

$$a = PC = 90° - 55° 18' = 34° 42',$$

$$\text{angle } APC = P = WB = 74° 8' - 6° 24' = 67° 44'.$$

$$\tan x = \frac{\cos 67° 44'}{\cot 34° 42'} \quad \begin{array}{l} \log = 9.57855 \\ \log = 10.15962 \end{array}$$

$$\log \tan 14° 42' 7'' = 9.41893$$

$$\begin{aligned} c &= 49° 32' \\ x &= 14° 42' \ 7'' \\ c - x &= 34° 49' 53'' \end{aligned}$$

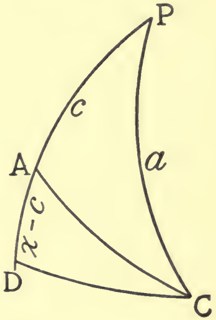
$$\cos p = \frac{\cos 34° 49' 53'' \cos 34° 42'}{\cos 14° 42' 7''}$$

$$= \cos 34° 49' 53'' \cos 34° 42' \sec 14° 42' 7''.$$

$$\begin{aligned} \log \cos 34° 49' 53'' &= 9.91425 \\ \log \cos 34° 42' &= 9.91495 \\ \log \sec 14° 42' \ 7'' &= 10.01445 \\ \log \cos 45° 45' 37'' & \quad 9.84365 \end{aligned}$$

$$p = 45° 45'\tfrac{37}{60} = 2745.6 \text{ nautical miles.}$$

Ex. 2. It is required to find the distance, on the arc of a great circle, between a point in lat. 32° 44′ N., long. 73° 26′ W., and a point in lat. 8° 14′ S., long. 14° W.



$$c = 90° - 32° 44' = 57° 16',$$

$$a = 90° + 8° 14' = 98° 14',$$

$$P = 73° 26' - 14° = 59° 26'.$$

$$\tan PD = \tan x = \frac{\cos 59° 26'}{\cot 98° 14'}$$

$$\begin{aligned} \log &= 9.16046 \\ \log &= 9.70633 \\ \log \tan 105° 52' 57\tfrac{1}{2}'' &= 10.54587 \end{aligned}$$

$$\tan x = \text{minus quantity.}$$

$$\therefore x \text{ or } PD \text{ is} > 90°.$$

$$\begin{aligned} x &= 105° 52' 57\tfrac{1}{2}'' \\ c &= 57° 16' \\ x - c &= 48° 36' 57\tfrac{1}{2}'' \end{aligned}$$

$$\cos AC = \cos p = \frac{\cos 48° 36' 57\tfrac{1}{2}'' \cos 98° 14'}{\cos 105° 52' 57\tfrac{1}{2}''}.$$

$$\begin{aligned} \log \cos 48° 36' 57\tfrac{1}{2}'' &= 9.82027 \\ \log \cos 98° 14' &= 9.15596 \\ \log \sec 105° 52' 57\tfrac{1}{2}'' &= 10.56278 \\ \log \cos 69° 45' 37'' &= 9.53901 \end{aligned}$$

$$p = 69° 45\tfrac{37}{60}' = 4185.6 \text{ nautical miles.}$$

Ex. 3. Required to find the distance, on the arc of a great circle, between a point in lat. 41° 4′ N., long. 69° 55′ W., and a point in lat. 51° 26′ N., long. 9° 29′ W. *Ans.* 2507.5 miles.

Ex. 4. Required to find the distance, on the arc of a great circle, between a point in lat. 37° 48′ N., long. 122° 28′ W., and a point in lat. 6° 9′ S., 8° 11′ E. *Ans.* 7516.3 miles.

28. When a ship sails between two points, making the shortest distance between these points, it sails on the arc of a great circle.

To do this, it cannot continue on the *same course*, as an arc of a great circle between two points, of different latitudes and longitudes, does not cut the meridians at the same angle.

Thus taking Ex. 1 of the previous article and solving by Napier's Analogies, we have:

$$\frac{c+a}{2}=\frac{155^\circ\ 30'}{2}=77^\circ\ 45'; \qquad \frac{P}{2}=29^\circ\ 43'.$$
$$\frac{c-a}{2}=\frac{40^\circ\ 58'}{2}=20^\circ\ 29';$$

$$\tan\tfrac{1}{2}(C+A)=\cos 20^\circ\ 29' \times \cot 29^\circ\ 43' \sec 77^\circ\ 45',$$

and $\tan\tfrac{1}{2}(C-A)=\sin 20^\circ\ 29' \cot 29^\circ\ 43' \operatorname{cosec} 77^\circ\ 45'.$

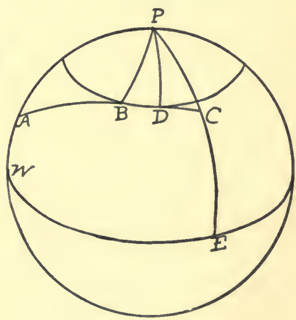
log cos 20° 29′ =	9.97163	log sin 20° 29′ =	9.54399
log cot 29° 43′ =	10.24353	log cot 29° 43′ =	10.24353
log sec 77° 45′ =	10.67330	log cosec 77° 45′ =	10.01000
log tan 82° 38′	10.88846	log tan 32° 6′ 10″ =	9.79752

$$\tfrac{1}{2}(C+A)=82^\circ\ 38'$$
$$\tfrac{1}{2}(C-A)=32^\circ\ \ 6'\ 10''$$
$$A=50^\circ\ 31'\ 50''$$
$$C=114^\circ\ 44'\ 10''$$

We see, therefore, that the distance AC makes an angle with the meridian PA of 50° 31′ 50″, and with the meridian PC, of 114° 44′ 10″. Consequently, the vessel starts on a course N. 50° 31′ 50″ E., and ends with a course N. 65° 15′ 50″ E. (the supplement of 114° 44′ 10″). Between the points A and C the course would be continually changing. In practice, the course is altered at certain intervals, as, for instance, at points

10° in longitude apart, for which the new course is calculated, and the distance between the points is run by Mercator's Sailing.

29. In great circle sailing, the arc of the circle might lead to too high a latitude, or to some obstacle like land or ice, which it would be necessary to avoid. In such cases *composite sailing* is adopted, or a combination of sailing on the arcs of great circles and on a parallel of latitude.



Thus, suppose it were desired to sail from A to C by composite sailing, and that BD were the parallel of highest latitude to be reached. The great circle starting from A and tangent to the parallel is first found; then the great circle through C and tangent to BD at D is found. Since these circles are tangent to BD, AB is perpendicular to the meridian PB,* and CD is perpendicular to the meridian PD.

We have, therefore, two right-angled spherical triangles, APB and CDP, in each of which an hypotenuse and a side are given; PA from the latitude of A and PC from the latitude of C are known; PB and PD, since each is the complement of the latitude of the highest parallel to be reached, are also known. Conse-

* PB is the least line which can be drawn from P to arc AB, and therefore passes through the pole of AB. Consequently, by geometry, PB cuts AB at right angles.

quently, the other parts of these triangles can be computed by Napier's Rule of the Circular Parts. We can thus ascertain the courses at A and C and the angles APB and DPC. The angles APB and DPC will give us the difference of longitude between A and B, and between C and D. Since the longitudes of A and C are known, the longitudes of B and D are also known. By this method of sailing the vessel goes on the arc of a great circle from A to B, on a parallel of latitude from B to D (in the figure due E.), and then on a great circle from D to C.

Ex. 1. A ship sails on a composite track from lat. 37° 15′ N., long. 75° 10′ W. to lat. 48° 23′ N., long. 4° 30′ W., not going north of lat. 49° N. Required, the longitude of the point of arrival on the parallel of 49° N., the longitude of the point of departure from the parallel, the initial and final courses, and the total distance sailed.

In the triangle ABP right-angled at B, $PA = 52° 45'$, $PB = 41°$.

$$\cos APB = \cot 52° 45' \tan 41°$$

$$\begin{aligned} \log \cot 52° 45' &= 9.88105 \\ \log \tan 41° &= 9.93916 \\ \hline \log \cos 48° 37' 21'' &= 9.82021 \end{aligned}$$

In the triangle PDC right-angled at D, $PC = 41° 37'$, $PD = 41°$.

$$\cos DPC = \cot 41° 37' \tan 41°$$

$$\begin{aligned} \log \cot 41° 37' &= 10.05141 \\ \log \tan 41° &= 9.93916 \\ \hline \log \cos 11° 53' 40'' &= 9.99057 \end{aligned}$$

$$\sin A = \frac{\sin 41°}{\sin 52° 45'} \quad \begin{aligned} \log &= 9.81694 \\ \log &= 9.90091 \end{aligned}$$

$$\log \sin 55° 30' 27'' = 9.91603$$

$$\sin C = \frac{\sin 41°}{\sin 41° 37'} \quad \begin{aligned} \log &= 9.81694 \\ \log &= 9.82226 \end{aligned}$$

$$\log \sin 81° 3' = 9.99468$$

$$\cos AB = \frac{\cos 52°\,45'}{\cos 41°} \quad \begin{array}{l} \log = 9.78197 \\ \log = \underline{9.87778} \end{array}$$

$$\log\cos 36°\,40'\,33'' = 9.90419$$

$$\cos CD = \frac{\cos 41°\,37'}{\cos 41°} \quad \begin{array}{l} \log = 9.87367 \\ \log = \underline{9.87778} \end{array}$$

$$\log\cos 7°\,52' = 9.99589$$

long. of $A = 75°\,10'$ W.
dif. of long. $= \underline{48°\,37'.4}$ E.
long. of $B = 26°\,32'.6$ W.

long. of $C = 4°\,30'$ W.
dif. of long. $= \underline{11°\,53'\tfrac{2}{3}}$ W.
long. of $D = 16°\,23'.7$ W.

Course at $A =$ N. $55°\,30'\,27''$ E. Course at $C =$ S. $81°\,3'$ E.

long. of $B = 26°\,32'.6$ W.
long. of $D = \underline{16°\,23'.7}$ W.
dif. of long. $= 10°\ \ 8'.9$ W.
$= 608.9$ log $= 2.78455$
log cos 49° $= \underline{9.81694}$
log 399.5 $= 2.60149$

dist. $AB = 2200.55$
dist. $BD = 399.5$
dist. $CD = \underline{472.0}$
total dist. $= 3072.05$ miles

Ex. 2. A vessel sails on a composite track from a point in lat. 46° 10′ S., long. 45° E. to a point in lat. 43° 40′ S., long. 71° 15′ W., not going S. of parallel of 50° S. Required the longitude of the point of arrival on the parallel of 50° S., the longitude of the point of departure from that parallel, the initial and final courses, and the total distance sailed.

Ans. 15° 55′.4 E., 34° 27′.9 W.; S. 68° 8′ 48″ W.; N. 62° 42′ W.; 4663.2 miles.

CHAPTER III

COURSES

The magnetic needle of the compass is supposed to give a north and south line, but in point of fact it rarely points north and south. It is subject to influences which deflect it from a north and south line; so that the north point of the magnet is sometimes east and sometimes west of a true north and south line. The most important deflecting influences cause two errors, as they are called; namely, an error of *Variation*, and an error of *Deviation*.

The error of *Variation* is due to the magnetic action of the earth. The error is greater or less, or even nothing, according to the position of the compass at various points on the earth's surface. Variation may therefore be called a geographical error. It is known and calculable, and allowance can be made for it at any point on the earth.

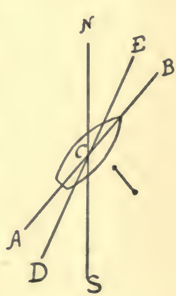
The error of *Deviation* is due to the magnetic action of the ship and its cargo, and changes according to the direction in which the ship is headed. Each ship has its own error of Deviation. This error can be known, and, to a certain extent, can be counteracted by proper arrangements, but must always be taken into account.

30. The *True Course* of a ship is the angle between the distance, or the line traversed by the ship, and the meridian or true north and south line.

31. The *Magnetic Course* of a ship is the angle between the distance and a north and south line, as indicated by the *magnet of a compass which is not affected by the error of Deviation.*

32. The *Compass Course* of a ship is the angle between the distance and a north and south line, as indicated by the *compass of a ship.*

33. For a steamship, in calm weather, or in a sailing vessel with a wind directly astern, the Compass Course, when corrected for variation and deviation, will give the True Course; but when the wind blows from any direction, except from right ahead or astern, it pushes the vessel aside from the course on which she is headed, so that her track is not in the direction in which she is headed, but makes an angle with that direction. This angle is called *leeway*, because the push of the wind on the vessel is to *leeward*.



Thus, in the figure *NS* is a true meridian; *NCB* is the *apparent* course; *NCE* is the *true* course; the wind, shown by the direction of the arrow, diverting the vessel from the track *AB*, in which she is headed, to the true track *DE*. The *angle between these tracks, ECB, leeway;* is given in points; and is estimated by the eye.

Although leeway is not an error of the compass, the effect of it is the same as if it were, and allowance must be made for it in order to determine the true course of a ship.

In navigation it is important to be able to convert a true course into a compass course and also a compass course into a true course, by applying corrections for the various errors, which have been mentioned.

The method of doing this will be best ascertained by applying the errors one by one.

In expressing, or converting courses, the observer is supposed to be at the *center* of the compass card.

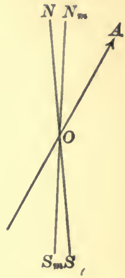
34. To convert a *true course into a magnetic course*, the *variation* being given.*

Both variation and deviation are given in terms which are applied to the *north* point of the compass needle. For instance, if the variation is given as 8° E., the *north* point of the needle points 8° east of a true N. and S. line, or, looking from the center of the compass, 8° to the *right* of that line.

Looking *south* from the center, the variation would still be 8° to the *right*.

In works on Navigation it is customary to give rules for converting courses, but it is best to draw a diagram, which will illustrate the example given, and after a little practice, rules can be derived by the learner himself.

* Variation charts are published by the Government Coast Survey.

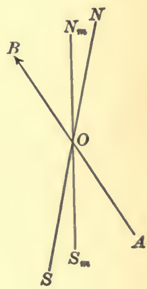


Ex. 1. Let the true course be N.E. b. N. and the variation be 8° E. Required the magnetic course.

Suppose the observer to be at O; the line NS = true N. and S. line; N_mS_m = magnetic N. and S. line.

true course $= NOA = 33° 45'$ to right of N.
variation $= NON_m = 8°$ to right of N.

mag. course $= N_mOA = 25° 45'$ to right of N.
or N.N.E. $\frac{1}{4}$ E., nearly.



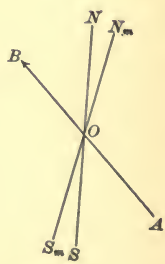
Ex. 2. Let the *true* course be N.W., and the variation be 12° W.

NS = true N. and S. line; N_mS_m = magnetic N. and S. line.

true course $= BON = 45°$, or 4 pts. left of N.
variation $= N_mON = 12°$, or 1 pt., nearly, left of N.

mag. course $= N_mOB = 33°$,
or 3 pts. left of N. = N.W. b. N.

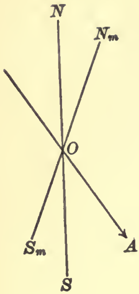
In converting courses sometimes the work is expressed in degrees, and sometimes to the nearest points, half points, or quarter points.



Ex. 3. If the course is N.W., and the variation is 12° E., to obtain the magnetic course we *add* the 12° to the true course.

true course $= NOB = 45°$, or 4 pts. left of N.
variation $= NON_m = 12°$, or 1 pt. right of N.

mag. course $= N_mOB = 57°$, or 5 pts. left of N.



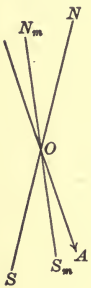
Ex. 4. Let true course be S.E. b. S. and variation be 22° E.

$$NS = \text{true N. and S. line.}$$

$$N_mS_m = \text{magnetic N. and S. line.}$$

Suppose observer to be at O.

$$\begin{aligned} \text{true course } SOA &= 3 \text{ pts. left of S.} \\ &= 33°\,45' \text{ left of S.} \\ \text{variation } SOS_m &= 22° \text{ right of S.} \\ \hline \text{magnetic course} = S_mOA &= 55°\,45' \text{ left of S.} \\ &= \text{nearly 5 pts. left of S.} \end{aligned}$$



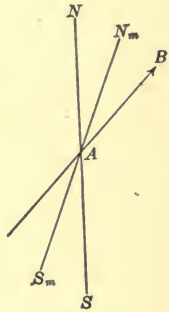
Ex. 5. Let true course be S.E. b. S., but variation be 22° W.

$$\begin{aligned} \text{true course } SOA &= 33°\,45' \text{ left of S.} \\ \text{variation } SOS_m &= 22° \quad \text{ left of S.} \\ \hline \text{magnetic course} = S_mOA &= 11°\,45' \text{ left of S.} \end{aligned}$$

From these examples and by an inspection of the figures, supposing the observer to be at center of compass, it will be seen that *when the true course and the variation are both to the right* or *both to the left* of either the N. or S. points, the magnetic course is *the difference* of the two; but when *one is to the right and the other to the left* of the N. or S. points, the magnetic *course is the sum of the two.*

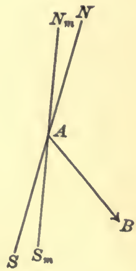
35. To change a *magnetic course* into a *true course:* if the given course and variation are *both to the right or both to the left of either N. or S. points, add* the *two;* if *one is to the right and the other to the left, take the difference.*

This rule for changing magnetic to true courses would naturally follow, from what has been said of converting true courses into magnetic courses, as the processes are reversed, and we should, therefore, reverse the former rule. We will illustrate by examples.



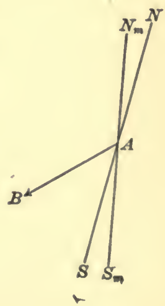
Ex. 1. Magnetic course is N.N.E. Variation is 22° E. Find true course.

$$\begin{aligned} \text{mag. course} &= N_mAB = 22°\,30' \text{ right of N.} \\ \text{variation} &= N_mAN = 22° \quad \text{right of N.} \\ \hline \text{true course} &= NAB = 44°\,30' \text{ right of N.} \\ &= 4 \text{ pts., nearly.} \\ &= \text{N.E., nearly.} \end{aligned}$$



Ex. 2. Let the magnetic course be S.E. b. S., and the variation be 11° W. Find true course.

$$\begin{aligned} \text{mag. course} &= S_mAB = 33°\,45' \text{ left of S.} \\ \text{variation} &= S_mAS = 11° \quad \text{left of S.} \\ \hline \text{true course} &= SAB = 44°\,45' \text{ left of S.} \\ &= \text{S.E., nearly.} \end{aligned}$$



Ex. 3. Let the magnetic course be S.W. b. W., and the variation be 11° 15′ W. Find true course.

$$\begin{aligned} \text{mag. course} &= S_mAB = 5 \text{ pts. right of S.} \\ \text{variation} &= S_mAS = 1 \text{ pt. left of S.} \\ \hline \text{true course} &= SAB = 4 \text{ pts. right of S.} \\ &= \text{S.W.} \end{aligned}$$

36. To convert magnetic courses into compass courses, or compass courses into magnetic courses, it is necessary to have a list of deviations corresponding to the different directions in which the ship heads. This is determined before the ship leaves port. Deviation acting on the compass needle to deflect it from a *magnetic* N. and S. line, tables of deviation give the amounts of deviation E. and W. of the magnetic north point.

Though each ship has its own Deviation Table, the table here given will serve to illustrate the subject.

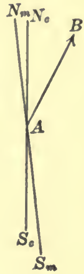
DEVIATION TABLE

I. Direction in Degrees and Minutes.	I. Course by Ship's Compass.	II. Deviation of the Compass.	I. Course by Ship's Compass.	II. Deviation of the Compass.
0	North	3° 10′ W.	South	3° 10′ E.
11° 15′	N. b. E.	2 35 E.	S. b. W.	0 5 E.
22 30	N.N.E.	8 10 E.	S.S.W.	3 0 W.
33 45	N.E. b. N.	13 10 E.	S.W. b. S.	6 30 W.
45	N.E.	16 50 E.	S.W.	9 40 W.
56 15	N.E. b. E.	19 30 E.	S.W. b. W.	13 0 W.
67 30	E.N.E.	20 30 E.	W.S.W.	16 10 W.
78 45	E. b. N.	21 5 E.	W. b. S.	19 15 W.
90	East	20 20 E.	West	21 10 W.
78 45	E. b. S.	19 15 E.	W. b. N.	23 20 W.
67 30	E.S.E.	18 5 E.	W.N.W.	24 0 W.
56 15	S.E. b. E.	16 30 E.	N.W. b. W.	23 35 W.
45	S.E.	14 40 E.	N.W.	22 0 W.
33 45	S.E. b. S.	12 5 E.	N.W. b. N.	19 0 W.
22 30	S.S.E.	9 40 E.	N.N.W.	14 50 W.
11 15	S. b. E.	6 0 E.	N. b. W.	9 15 W.
	South	3 10 E.	North	3 10 W.

37. To find the *magnetic course*, having *given the compass course* and *the deviation.*

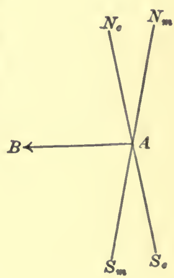
Ex. 1. Let the compass course be N.N.E. By the table the deviation is 8° 10′ E.

Let N_mS_m be magnetic N. and S. line; N_cS_c be compass N. and S. line; and AB be ship's track.



$$\begin{aligned} \text{com. course} &= N_cAB &&= 22°\,30' \text{ right of N.}\\ \text{deviation} &= N_cAN_m &&= 8°\,10' \text{ right of N.}\\ \hline \text{mag. course} &= N_mAB &&= 30°\,40' \text{ right of N.}\\ & &&= \text{N.N.E. } \tfrac{3}{4} \text{ E.} \end{aligned}$$

Ex. 2. Let the compass course be N. 80° W. This is 1¼° W. of W. b. N. The deviation for W. b. N. is 23° 20′ W. The deviation for N. 80° W. will be a little less. As in steering a vessel it is impossible to hold her head to a minute of correction, if we call the deviation 23° W. we shall not be much out of the way.



$$\begin{aligned} \text{com. course} &= N_cAB &&= 80° \text{ left of N.}\\ \text{deviation} &= N_cAN_m &&= 23° \text{ left of N.}\\ \hline \text{mag. course} &= N_mAB &&= 103° \text{ left of N.}\\ & &&= 77° \text{ right of S.} \end{aligned}$$

mag. course = S. 77° W.

38. To find the *compass course*, the *magnetic course* and *deviation being given.*

Ex. 1. Let the magnetic course be E.N.E.

magnetic course = 6 pts. right of N.	or N. 67° 30′ E.
deviation from page 56 = 1¾ pts. right of N.	or N. 20° 30′ E.
approximate compass course = 4¼ pts. right of N.	or N. 47° E.

This is only an approximate answer, as will be evident; for if we steer by *compass* N. 47° E., the deviation for that course is nearly 17° 30′. Thus:

compass course =	47° right of N.
deviation =	17° 30′ right of N.
magnetic course would then =	$64\frac{1}{4}$° right of N.
	or $3\frac{1}{4}$° less than the given course.

But, if we apply to the given magnetic course the correction due to deviation for the *approximate* compass course, the example will prove. Thus:

	magnetic course =	6 pts. or 67° 30′ right of N.
	deviation for N. $4\frac{1}{4}$ E. =	$1\frac{1}{2}$ pts. or 17° 30′ right of N.
	compass course N.E. $\frac{1}{2}$ E. =	$4\frac{1}{2}$ pts. or 50° right of N.
Proof:	compass course =	$4\frac{1}{2}$ pts. or 50° right of N.
	deviation =	$1\frac{1}{2}$ pts. or 17° 30′ right of N.
	magnetic course =	6 pts. or 67° 30′ right of N.

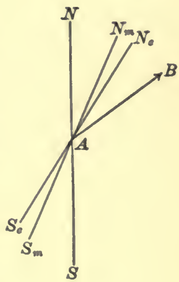
Courses and deviations, when given in points, are given to *nearest* points, half points, or quarter points.

Since the Deviation tables are made for angles indicated by the compass courses, we get only an *approximate* result by applying the deviation corresponding to the *magnetic course*. Hence, to be accurate, we first find this approximate compass course, and then apply the correction, which corresponds to this approximate course in the table, to the original magnetic course.

We have considered the applications of variation and deviation separately, for the sake of clearness; but in practice, their action on the magnet of the compass is combined. We have to convert compass courses into true courses, and also true courses into compass courses.

In changing a compass course into a true course the result is the same, whether we apply corrections for variation and deviation separately, or together; but *in converting a true course* into a *compass course* we must *apply* correction for *variation first,* and *then* correction *for deviation.*

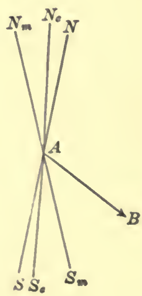
Ex. 1. Find *true course;* variation being 25° E.; compass course being N.N.E.; and deviation being taken from table on page 56. In figure let notation of lines be the same as in preceding figures.



compass course $= N_cAB = 22° 30'$ right of N.
variation $= NAN_m = 25°$ right of N.
deviation $= N_mAN_c = 8° 10'$ right of N.
sum $= NAN_c = 33° 10'$ right of N.

true course $= NAB = 55° 40'$ right of N.
= nearly N.E. b. E.

Ex. 2. Find true course, variation being 25° W.; compass course being S.E. b. E.; and deviation being taken from table.

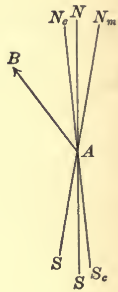


variation $= NAN_m$
$= SAS_m = 2\frac{1}{4}$ pts. left of S.
deviation $= S_cAS_m = 1\frac{1}{2}$ pts. right of S.

difference $= SAS_c = \frac{3}{4}$ pt. left. of S.
compass course $= S_cAB = 5$ pts. left of S.

true course $= SAB = 5\frac{3}{4}$ pts. left of S.
= E.S.E. $\frac{1}{4}$ S.

Ex. 3. Let the true course be N. 35° W. and the variation be 10° E. Find the compass course.



true course = NAB = 35° left of N.
variation = NAN_m = 10° right of N.

---

magnetic course = N_mAB = 45° left of N.
deviation (approximate) = 22° left of N.

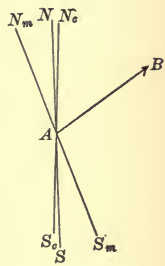
---

approx. compass course = 23° left of N.

---

deviation = 14° 50′ left of N.
compass course N_cAB = 30° 10′ left of N.
= N. 30° 10′ W.

Ex. 4. Let the *true course* be N.E. b. E. and the variation be 20° W. Find the compass course.



true course = NAB = 5 pts. right of N.
variation = NAN_m = $1\frac{3}{4}$ pts. left of N.

---

magnetic course = N_mAB = $6\frac{3}{4}$ pts. right of N.

By table, page 56:

approximate deviation = $1\frac{3}{4}$ pts. right of N.

---

approx. compass course = 5 pts. right of N.

---

deviation = $1\frac{3}{4}$ pts. right of N.

---

compass course = 5 pts. right of N.

Same examples by degrees:

true course = NAB = 56° 15′ right of N.
variation = NAN_m = 20° left of N.

---

magnetic course = N_mAB = 76° 15′ right of N.
approximate deviation = 21° right of N.

---

approximate compass course = 55° 15′ right of N.

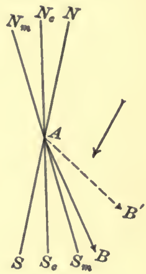
---

deviation (to be taken from 76° 15′) = 19° 30′ right of N.

---

compass course = N_cAB = 56° 45′ right of N.

Ex. 5. Find the true course; the compass course being S.E., the variation being 28° W., leeway being 2 pts., and the wind blowing E.N.E. Take deviation from table on page 56.



compass course $= S_cAB' = 45°$ left of S.
deviation $= S_cAS_m = 14° 40'$ right of S.
variation $= S_mAS = 28°$ left of S.

dif. $= S_cAS = 13° 20'$ left of S.

appar. true course $= SAB' = 58° 20'$ left of S.

But the influence of the wind, whose direction is shown by arrow in figure, changes this apparent true course to the leeward by two points, represented by the angle BAB'.

Thus:

apparent true course $= SAB' = 58° 20'$ left of S.
leeway $= BAB' = 22° 30'$ toward S., or right of S.

true course $= SAB = 35° 50'$ left of S., or S.E. $\frac{3}{4}$ S.

Ex. 6. Compass course is S.W. $\frac{1}{4}$ S. Variation is 6° E.; wind is S.S.E., and leeway $1\frac{3}{4}$ pts. Deviation being taken from table on page 56. Find true course. Example can be worked without figure thus:

course by compass $= 3\frac{3}{4}$ pts. right of S.
variation $= 6°$ right of S.
deviation $= 9°$ left of S.

dif. $= 3°$ left of S. $= \frac{1}{4}$ pt. left of S.

apparent true course $= 3\frac{1}{2}$ pts. right of S.
leeway $= 1\frac{3}{4}$ pts. right of S.

true course $= 5\frac{1}{4}$ pts. right of S.
W.S.W. $\frac{3}{4}$ S.

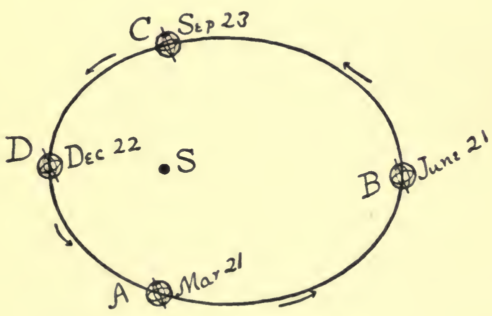
The preceding examples could all have been worked without figures, but, until the learner has become familiar with the methods of applying the different corrections, it is best to check the numerical work by means of a diagram.

CHAPTER IV

ASTRONOMICAL TERMS

39. Before giving definitions of the terms used in Nautical Astronomy, we must first consider the effects of the earth's revolution around the sun, as they appear to an observer on the earth.

In the figure, let $ABCD$ represent the orbit in which the earth revolves about the sun, S; and

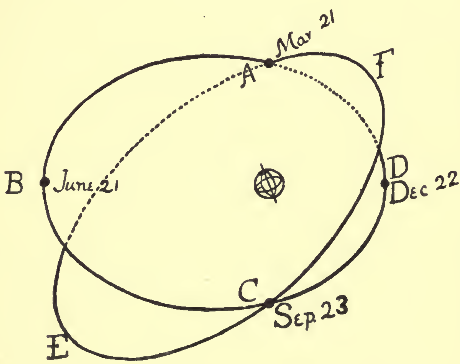


A, B, C, and D represent the positions of the earth at the beginning of the seasons of spring, summer, fall, and winter, respectively. If the figure represents the plane of the earth's orbit, the axis of the earth is not at right angles to that orbit, but makes an angle with it of about 66° 33′. The plane of the equator therefore makes an angle with it of 23° 27′.

To the observer on the earth the heavenly bodies, the sun included, appear to be on the interior surface of a very large sphere, of which the center is his own point of observation, or his own eye. This imaginary interior surface of a *sphere* is called the *celestial concave.* The poles of the heavens are the points of the celestial concave, toward which the axis of the earth is directed. The celestial pole *above* the horizon is called the *elevated* pole.

Considering the earth as motionless, to the observer on it, the *sun* appears to travel daily in the celestial concave from east to west. If from a standpoint on the earth we could watch the sun in the heavens during the whole year, it would appear to describe a circle on the celestial concave. This circle is called the ecliptic.

The plane of the earth's equator, being supposed produced, would cut the celestial concave in the celestial equator or *equinoctial.* The ecliptic and the equinoctial intersect in two points, known as the first point of Aries and the first point of Libra. About March 21 the center of the sun is at the first point of Aries, where the equinoc-



tial crosses the ecliptic: and about September 23 it is at the first point of Libra, where the equinoctial intersects the ecliptic a second time. These points of intersection are called equinoctial points, because, at the seasons of the year when the sun reaches them, the days and nights are of nearly equal length. Thus in the figure, *ABCD* is the ecliptic. *AECF* is the equinoctial. *A* is the first point of Aries, where the sun changes its declination from S. to N.; *C* is the first point of Libra, where the sun changes its declination from N. to S.

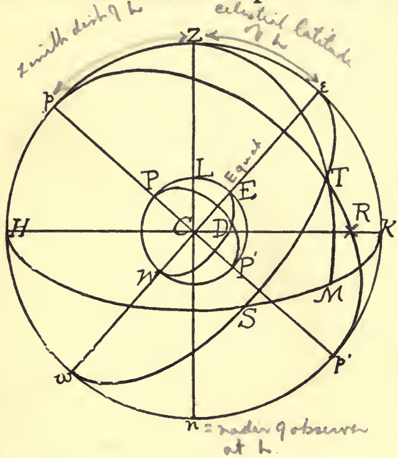
The equinoctial is a fixed circle on the celestial concave, and the first point of Aries is considered a fixed point,* as it is the point of intersection of the ecliptic and the equinoctial. The positions of heavenly bodies may therefore be expressed with reference to them, just as the positions of places on the earth's surface are expressed in latitude and longitude by reference to the equator and the meridian of Greenwich.

40. Let the accompanying figure represent the earth, *PWP′E*, surrounded by the celestial sphere, *pwp′e*.

If the axis of the earth, *PP′*, be produced to meet the celestial concave in the points *p* and *p′*, these points are called the *celestial poles*, and the line *pp′* is called the *axis* of the celestial sphere.

* First point of Aries moves yearly 50″ (nearly) to westward.

The plane of the equator, *WDE*, produced, intersects the celestial sphere in the *celestial equator*, *wSe*.



The planes of the meridians *PEP′*, *PDP′*, intersect the celestial sphere in great circles, *pep′*, *pTp′*, which are called *hour circles* and also circles of declination.

Since the earth revolves upon its axis once in 24 hours, every point on a celestial meridian would appear, to an observer on the earth's surface, to move through a complete circumference, or 360°, during that time. If, now, the celestial meridians are drawn at intervals of 15° (on the equator), there will be 24 such meridians. Since the time in which *all* these meridians pass by an observer is 24 hours, the interval of time of passage between two successive meridians will be one hour, since 24 of them pass by him in 24 hours. If meridians are drawn at intervals of 1°, the interval of time of passage of two such meridians will be $\frac{60 \text{ minutes}}{15}$, or 4 minutes. Thus, the passage of these meridians or of points on them being measured by time or degrees, we can convert one measure into the other.

The angles made by these meridians at p and p' are called hour angles, and these angles are measured by the arcs which they intercept on the arc of the celestial equator wSe.

The *celestial horizon* of any place, on the earth's surface, is the circle made by a plane passing through the center of the earth parallel to the plane of the horizon at that place, and intersecting the celestial sphere.

The celestial horizon of the point L is HSK.

If a straight line be drawn from the center, C, to L, and this line be produced through L to meet the celestial sphere at Z, Z will be the zenith of L; Zp will measure the *zenith distance* of L (*i.e.* the distance of the zenith of the point L from the pole), and Ze will measure the *celestial latitude* of L. The zenith distance is the complement of the celestial latitude. The *degree measure* of the *celestial* latitude is the same as that of the *terrestrial latitude*, since they both subtend the same angle at the center of the earth. Thus, Ze and LE both subtend the angle LCE.

Since Z is the extremity of the diameter perpendicular to the plane of the horizon HSK, Z is the pole of HSK, and therefore every point on HSK is $90°$ from Z. If the line CZ be produced to meet the surface of the celestial sphere again at n, n will be the *nadir* of the observer at L.

The *declination* of a heavenly body is the arc of the circle of declination, intercepted between the equinoctial and the position of the body.

Declination is measured in degrees, minutes, etc., N. or S. from the equinoctial, toward the pole.

Thus in the preceding figure, TR is the declination of R and is S. declination.

The *polar distance* of a heavenly body is the distance of that body from the *elevated* pole, and is $90° \mp$ the declination: the minus sign being taken if the declination of the body is of the same name with the pole, that is, both being N. or both S.; but the plus sign being used if the declination and the pole are not of the same name, that is, one being N. and the other S.

In the preceding figure, calling p the N. pole, and considering it the elevated pole, the polar distance of R is $90° + TR$. If p' were taken as the elevated pole, $p'R$ would be the polar distance and would be $90° - TR$.

The *altitude* of a heavenly body is the angle of elevation of the body above the plane of the horizon.

A distinction is made between an *observed* altitude of a body and its *true altitude.*

By an *observed altitude,* in Navigation, is generally understood the angle of elevation of a body above the *visible horizon,* as represented by the horizon line of the sea.

A *true altitude* is an observed altitude corrected, so as to represent the angle of elevation of the body above the *celestial horizon.*

Circles of altitude are great circles of the celestial sphere which pass through the zenith of the observer.

Circles of altitude are also called *vertical circles* because their planes are perpendicular or vertical to the plane of the horizon.

The *altitude* of a body is *measured* on the arc of a circle of altitude between the horizon circle and the position of the body. This *measure* is generally used in calculations as *the* altitude.

In the preceding figure, ZeK and ZTM are circles of altitude. MT is the altitude of T.

The *zenith distance* of a body is its distance from the zenith measured on a circle of altitude.

ZT is zenith distance of T and equals $90° - MT$ or $90° -$ altitude of T.

The *celestial meridian* of any place is the circle on the celestial concave in which the plane of the terrestrial meridian of that place produced cuts the concave.

It is the circle of altitude which passes through the celestial poles.

In the preceding figure, if L be a place on the earth's surface, and the plane of the meridian $PLEP$ be produced to cut the celestial concave in $HpZeK$, $HpZeK$ is the celestial meridian of L. It coincides with the circle of altitude through Z.

The points in which the celestial meridian cuts the horizon are the N. and S. points of the horizon.

H and K are the N. and S. points of the celestial horizon of the place L, supposing P and P' to be N. and S. poles.

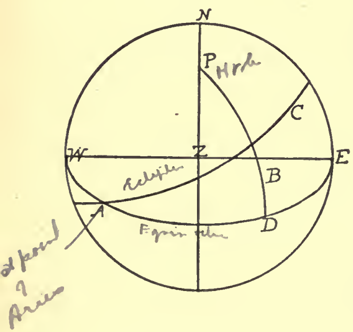
The prime vertical is the circle of altitude whose plane is at right angles to the plane of the celestial meridian. It intersects the horizon in the E. and W. points.

If, in the preceding figure, a plane be passed through *Cz* at right angles to the plane of *HpZeK*, the circle in which it cuts the celestial concave will be the prime vertical.

The *right ascension* of a heavenly body is the arc of the equinoctial intercepted between the first point of Aries and the circle of declination which passes through the center of the body.

Right ascension is measured eastward from the first point of Aries from 0° to 360°; or, in hours, from 0 h. to 24 h.

Let the figure represent the celestial sphere projected on the plane of the horizon *NWE*; *P* will represent the N. pole; *WDE* will represent the equinoctial; *AC* will represent the ecliptic; and *A*, the intersection of the ecliptic with the equinoctial, will represent the first point of Aries.



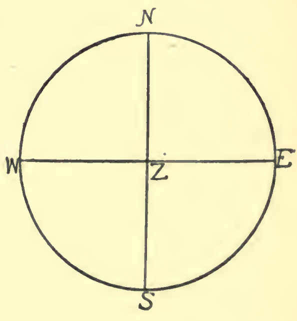
If *B* represent the position of a heavenly body, draw the arc of a circle of declination, *PB*, and produce the arc to meet the equinoctial at *D*. *AD* will represent the right ascension of *B*.

41. The earth being *inside* the celestial concave, the observer sees the heavenly bodies from the inside. Astronomical diagrams are drawn on the supposition that the observer is on the *outside* of the celestial concave, as the relations and positions of celestial bodies can best be represented on this supposition. The representations are made on different planes, according to the supposed different points of view.

Thus, if the *point of view* is directly *above* the *zenith*, the representation of the heavenly bodies is made on the *plane* of the *horizon*. This is a very useful mode of representation.

If the *point of view is at either* the *E.* or *W.* points, the representation is made on the *plane of the* celestial meridian.

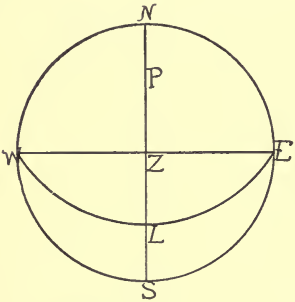
If the *point of view* is directly above the *celestial pole*, the representation is made on the *plane of the equinoctial or celestial equator*.



If *NWSE* represent the horizon, and if the point of view is directly above the zenith, the zenith will be projected on the center of the circle, and the *circles* of *altitude*, passing through the zenith, will be projected as straight lines. If *N*, *S*, *E*, *W* be the N., S., E., and W. points of the horizon, *NS* will be the celestial meridian of the observer whose zenith is *Z*. The *prime vertical*, or circle of altitude at right angles to the celestial meridian, in the figure will be *WE*.

42. To represent, *on the plane of the horizon,* the celestial *pole* and the *celestial equator* for a given latitude. Suppose the latitude to be 42° N.

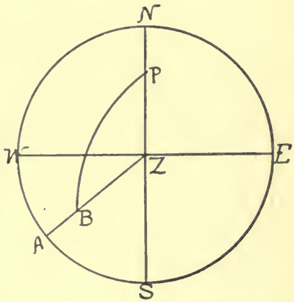
Let ZL represent 42°, and LP represent 90°. If an arc of a great circle WLE be drawn with P as a pole, it will pass through W, L, and E, and represent the celestial equator, or equinoctial. For, since by definition, the planes of the celestial meridian and prime vertical are at right angles to each other, the diameter joining E and W lies in the plane perpendicular to the plane of NS. Therefore, E and W are poles of NS. Consequently, E and W are each at a quadrant's distance from P, for the polar distance of a great circle is a quadrant. But PL is a quadrant by construction. Therefore, P representing the celestial pole, WLE will represent the equinoctial or celestial equator.



43. The *azimuth* of a heavenly body is the angle, at the zenith of the observer, between *the celestial meridian* and the *circle of altitude passing through the body.* It is measured by an arc of the horizon between the N. and S. points and the point in which the circle of altitude intersects the horizon. Azimuth is measured from the N. and S. points E. and W. from 0° to 90°.

Azimuth is sometimes called the *true* bearing of a heavenly body.

To represent on the plane of the horizon *the altitude, zenith distance*, and *azimuth* of a heavenly body.



Let $NWSE$ represent the plane of the horizon.

Let the azimuth be S. 50° W., and the altitude be 30°.

Measure $SA = 50°$; through A draw the circle of altitude, ZA. On ZA take $AB = 30°$ to represent *the altitude*. This will give B as the place of the heavenly body. ZB is the *zenith distance*. If P be supposed to be the celestial pole, PB will represent the *polar distance* of the body. SZB is the *azimuth*, measured by arc SA.

CHAPTER V

TIME

44. Time is measured by the intervals between the appearances of certain celestial bodies on the meridian of the observer.

Thus, *sidereal time* is measured by the successive appearances of the *first point of Aries* on the meridian. The period elapsing between two successive appearances of the first point of Aries on the same part of the meridian is called a *sidereal day.*

The *transit* of any heavenly body is its passage across the celestial meridian.

The instant when the first point of Aries, or when any heavenly body, is on the meridian is called the time of its transit.

As the celestial meridian passes through the zenith and nadir, the first point of Aries is really on the celestial meridian twice; but a *sidereal day* is measured by the *interval* between *two successive transits* on that part *of the meridian which contains the zenith.* Transits on this part of the meridian are called *upper transits*, while transits on the part of the meridian which contains the *nadir* are called *lower transits.*

The terms *meridian passage* and *culmination* are sometimes used in place of the term *transit.*

Besides *sidereal time*, we have *solar time.*

Apparent solar time is measured in terms of an *apparent solar day.*

An *apparent solar day* is the interval between two successive upper transits of the *center of the sun* over the meridian of the observer.

These successive returns of the real sun have not always equal intervals between them: first, because the sun does not move in the plane of the equinoctial, but in the ecliptic, which is inclined at an angle of 23° 27′ to the equinoctial; and, second, because the sun's movement in the ecliptic is not *uniform.* Thus, when the earth is nearest to the sun it moves in its orbit a little over 61′ daily, or, considering the earth as still, *the sun moves* in the *ecliptic* the same amount; but when the earth and sun are farthest from one another, the sun moves in the ecliptic about 57′ daily, and, at all other times, at rates varying between these two amounts.

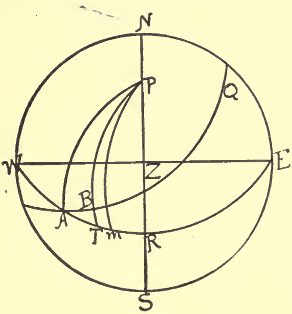
To secure an invariable unit of time, *mean solar* time is used, measured in terms of the *mean solar day*, which is equal in length to the *average* of all the apparent solar days of the year.

Mean solar time is supposed to be regulated by the movements of a fictitious sun, moving in the equinoctial or celestial equator, at a rate which is the average or *mean* rate of movement of the true sun in the ecliptic. If the imaginary or mean sun and the true sun are supposed to start from the same circle of declination, and return to the same circle at the end of the year, in the interval they are sometimes on the same circle of declination, but generally on different circles, the mean sun being sometimes ahead of the true sun and sometimes behind it.

The *equation of time* is the difference between time measured by the mean sun and time measured by the real sun. This equation of time for every day is always to be found in the Nautical Almanac on pages I and II of each month.

To illustrate, by a figure, the meanings of *sidereal time*, *apparent solar time*, *mean solar time*, and the *equation of time*.

Let *NWSE* represent the horizon; *P* the pole; *WRE* the celestial equator or equinoctial; *A* the first point of Aries; and *ABQ* the ecliptic.



Let *B* represent the place of the true sun on the ecliptic, and *m* the place of the mean sun on the equinoctial. Draw circles of declination, *PBT* and *Pm*.

Sidereal time is represented by the angle *RPA*, or by its measuring arc *RA*. *Apparent solar time* is the angle *RPB*, or its measuring arc *RT*. *Mean solar* time is *RPm*, or the arc *Rm*. *The equation of time* is *mPT*, or arc *mT*.

Thus we may define time by angles measured from the celestial meridian westward.

Sidereal time is the angle at the pole of the equinoctial between the meridian and a circle of declination passing through the first point of Aries.

Apparent solar time is the angle at the pole between the meridian and a circle of declination passing through the center of the *true* sun.

Mean solar time is the angle at the pole between the meridian and a circle of declination passing through the position of the *mean* sun.

A *sidereal clock* is adjusted so as to mark 24 hours between two successive transits of the first point of Aries.

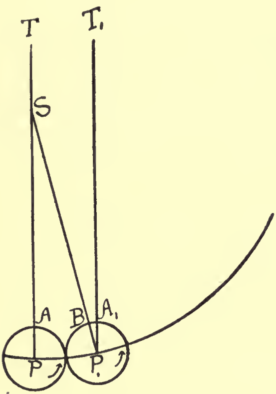
A *mean solar* clock is adjusted to mark 24 hours between two successive transits of the *mean* sun.

Clocks and watches in ordinary use are adjusted to *mean solar time*.

45. The daily motion of the mean sun, in the equinoctial, is found to be 59′ 8″.33. This is easily determined from the time it takes the true and the mean suns, starting from the meridian of any point, to return to the same meridian. This time is found to be 365.2422 mean solar days, during which the mean sun travels through a complete circle, or 360°. In one day, therefore, it would travel through $\frac{360}{365.2422}^{\circ}$, or 59′ 8″.33.

46. In order to find the arc described by a meridian of the earth in a mean solar day, let P and P_1 represent two positions of the center of the earth in its orbit, separated by an interval of time equal to a mean solar day.

Suppose a plane to be passed through the celestial equator; and that the small circles represent the terrestrial equator of the earth in its two positions; and S to be the position of the mean sun. PA and P_1A_1 will be the two projections of the same meridian. As the fixed stars are at such immense distances from the earth, rays of light from such a star, represented by TA and T_1A_1 would fall in parallel lines on the earth, in its two positions.



Thus, the meridian PA, having the light from the star on it, in its first position, would receive the same light in its second position P_1A_1, having in the interval made a complete rotation, or having gone through an arc of 360°.

Now if S, on the line TA, be supposed to be the position of the mean sun, we join SP_1. Since by Art. 45 PP_1 is 59′ 8″.33, the angle PSP_1 is also 59′ 8″.33. Therefore the alternate angle SP_1T_1 is an angle of 59′ 8″.33, and the arc AB is an arc of 59′ 8″.33; that is, the earth in passing from P to P_1 in its rotation on its axis, carries the meridian PA past its position P_1A_1 to the position P_1B, and, therefore, the meridian moves through an arc of 360° 59′ 8″.33 in a mean solar day, or 59′ 8″.33 more than in a sidereal day.

47. In a *sidereal day* of 24 hours the meridian of any place on the earth revolves through 360°. In *one hour* it passes through $\frac{360}{24}^\circ = 15^\circ$;

in one minute it passes through $\frac{15}{60}^\circ = \frac{1}{4}^\circ = 15'$;

in one second it passes through $\frac{15}{60}' = 15''$;

consequently, in passing through an arc of 59′ 8″.33, it takes an amount of time equal to $(\frac{59}{15})$ m. + $(\frac{8.33}{15})$ s., or equal to 3 m. 56.555 s.

In a *mean solar day* of 24 hours, the meridian of any place revolves through 360° 59′ 8″.33. A day of 24 hours of mean solar time is therefore longer than a day of 24 hours of sidereal time by the amount of time (sidereal) which it takes the meridian to pass through an arc of 59′ 8″.33; that is, 3 m. 56.555 s. Therefore, 24 h. mean solar time = 24 h. 3 m. 56.555 s. sidereal time. Thus the sidereal day is shorter than a mean solar day.

48. *To convert sidereal time into mean solar time, and mean sola time into sidereal time.*

Let S_t = any interval of sidereal time, and M_t = the same interval expressed in mean solar time.

As the sidereal day is shorter than the mean solar day, a given interval of time will have *more* sidereal hours in it than solar hours, and the ratio of the *hours sidereal* to the *hours mean solar* will be the ratio between the number of hours, minutes, and seconds in a sidereal day, and the 24 hours in a mean solar day.

Thus $\frac{S_t}{M_t} = \frac{24 \text{ h. } 3 \text{ m. } 56.555 \text{ s.}}{24 \text{ h.}} = 1.0027379,$

and $\frac{M_t}{S_t} = \frac{24 \text{ h.}}{24 \text{ h. } 3 \text{ m. } 56.555 \text{ s.}} = 0.9972697.$

$\therefore\ S_t = M_t \times 1.0027379 = M_t + .0027379\ M_t,$

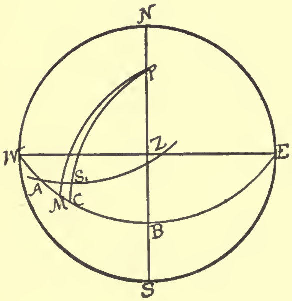
and $M_t = S_t \times 0.9972697 = S_t - .0027303\ S_t.$

By means of these formulæ the tables of the Nautical Almanac, and those in Bowditch's Tables, for converting sidereal into mean solar time or mean solar into sidereal time, can be computed.

49. To convert a given mean solar time into apparent solar time; and, conversely, to convert given apparent time into mean time; given also the equation of time.

Ex. 1. Let mean time be 3 h. 14 m.; and the equation of time be 3 m. 4 s., to be *subtracted.* Required apparent time.

mean time =	3 h. 14 m.
equation of time =	3 m. 4 s. −
apparent time =	3 h. 10 m. 56 s.



To illustrate this example by a figure, suppose in addition to the given terms, the declination of the sun is 15° N.

Let *NWSE* be the plane of the horizon; *Z* the zenith; *P* the pole; and *WBE* the celestial equator; AS_1 the ecliptic; S_1 the center of the true sun; M the position of the mean sun on the equinoctial.

Through S_1 draw the circle of declination PS_1C; and draw PM to M. $S_1C = 15°$.

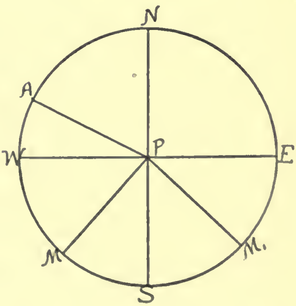
$$\begin{aligned} \text{Then } MPB &= \text{mean time} &&= 3 \text{ h. } 14 \text{ m.} \\ \underline{S_1PM} &= \text{equation of time} &&= \underline{\quad 3 \text{ m. } 4 \text{ s.}} \\ S_1PB &= \text{apparent time} &&= 3 \text{ h. } 10 \text{ m. } 56 \text{ s.} \end{aligned}$$

Ex. 2. Let apparent time be 4 h.; and equation of time be 2 m. 56 s., to be added; and declination of sun be 20° N. Required M_t. In figure above, $S_1C = 20°$.

$$\begin{aligned} \text{apparent time} &= S_1PB &&= 4 \text{ h.} \\ \text{equation of time} &= \underline{S_1PM} &&= \underline{\quad 2 \text{ m. } 56 \text{ s.}} \\ M_t &= MPB &&= 4 \text{ h. } 2 \text{ m. } 56 \text{ s.} \end{aligned}$$

Sometimes the equation of time is additive, and at other times subtractive. It is given for every day of the year, on pages I and II (for the month), in the Nautical Almanac, and whether additive or subtractive.

50. Given *mean time*, and the *right ascension* of the *mean* sun, to find *sidereal time* at any place; that is, *the right ascension of the meridian* of the observer.



Let $NWSE$ represent the plane of the equinoctial; NPS the projection on it of the celestial meridian; A the position of the first point of Aries; and M the position of the mean sun. (Defs. pages 76 and 77.)

(1) $S_t = SPA = MPA + SPM$

$= \text{right ascension of mean sun} + \text{mean time}.$

If M_1 be position of mean sun,

$$S_t = SPA = M_1PA - M_1PS.$$

But $M_1PS = 360°$ (or 24 h.) − angle measured by $SANM_1 = 24$ h. − mean time.

$\therefore$ S_t = R.A. mean sun − (24 h. − mean time), *i.e.*

(2) S_t = R.A. of mean sun + mean time − 24 h.

From equations (1) and (2) we see that sidereal time = R.A. mean sun + mean time, but that when the sum of R.A. mean sun and mean time is greater than 24 h., we subtract 24 h. from that sum.

Ex. 1. Given M_t = 7 h. 10 m. and R.A. mean sun = 2 h. 38 m. 42 s. Find sidereal time.

S_t = 2 h. 38 m. 42 s. + 7 h. 10 m. = 9 h. 48 m. 42 s.

Ex. 2. Given mean time 10 h. 32 m. 40 s. and R.A. mean sun = 18 h. 45 m. 35 s. Find sidereal time.

M_t	= 10 h. 32 m. 40 s.
R.A. mean sun	= 18 h. 45 m. 35 s.
Sid. time	= 29 h. 18 m. 15 s. − 24 h.
	= 5 h. 18 m. 15 s.

51. To convert *sidereal time* into *mean time;* given the right ascension of the mean sun.

Since by the preceding article *sidereal time* = R.A. mean sun + mean time, or = R.A. mean sun + mean time − 24 h.

Mean time = sidereal time − R.A. mean sun, or = sidereal time − R.A. mean sun + 24 h.

Ex. 1. Let	sidereal time =	15 h. 30 m. 12 s.
and	R.A. mean sun =	6 h. 24 m. 13 s.
Then	mean time =	9 h. 5 m. 59 s.

Ex. 2. Let	sidereal time =	4 h. 20 m. 18 s.
and	R.A. mean sun =	7 h. 50 m. 10 s.
Then	mean time =	20 h. 30 m. 8 s.

In this example we add 24 h. to 4 h. 20 m. 18 s. before subtracting R.A. mean sun.

Thus,	sidereal time =	4 h. 20 m. 18 s.
		24 h.
		28 h. 20 m. 18 s.
	R.A. mean sun =	7 h. 50 m. 10 s.
	mean time =	20 h. 30 m. 8 s.

52. Civil time and astronomical time.

The *civil day* begins at *midnight* and ends at *midnight*, after the lapse of 24 hours in two periods of 12 hours each, one period beginning at midnight, and the other at noon.

The *astronomical day* begins at *noon*, or 12 *hours later* than the civil day of the same date, and ends at the next noon, after a lapse of 24 hours.

The two periods of the civil day are distinguished from each other by placing, after the figures denoting time between midnight and noon, the letters A.M. (Ante Meridian); and, after the figures denoting the time between noon and midnight, the letters P.M. (Post Meridian).

Thus it will be seen that to convert civil time into astronomical time, *the* P.M. *is dropped if the given civil time is after noon;* but if the time is A.M., 12 *hours is added to the given civil time* and the date is changed to the preceding day.

Ex. 1. Given civil time = 3 h. 10 m. P.M., August 10. Astronomical time = 3 h. 10 m., August 10.

Ex. 2. Given civil time = January 8, 10 h. 15 m. A.M. Add 12 h., drop the A.M., and astronomical time = January 7, 22 h. 15 m.

Conversely, to convert *astronomical time* into *civil time.*

If the given time is *under* 12 *hours*, put on P.M.

If the given time is *over* 12 *hours*, subtract from it 12 hours, add A.M. to the remainder, and add *one day* to the *date.*

Thus, January 10, 4 h. 15 m. astronomical time = January 10, 4 h. 15. m. P.M. civil time.

February 11, 17 h. 16 m. astronomical time = February 12, 5 h. 15 m. A.M. civil time.

53. In every problem of Nautical Astronomy it is necessary to find either the apparent or mean time, at Greenwich, of the instant of taking an observation; since the calculated positions of the heavenly bodies are made for definite times at the meridian of Greenwich. These positions, with the definite times corresponding, are published in the Nautical Almanac.

54. The *hour angle of the sun*, at the celestial meridian of any place, is the *local time* of the place.

The hour angle of the sun, at the same instant, at the meridian of Greenwich is the *Greenwich* time.

55. As the earth makes one complete rotation on its axis in 24 hours, so that the same meridian, on its surface, is opposite the first point of Aries, or opposite the same fixed star at the beginning and end of this period of time, and as a complete rotation is measured by 360°, 24 *hours in time* corresponds to 360°, or we can say:

$$\begin{array}{rclcrcl} 24\text{ h.} &=& 360^\circ & \text{and} & 360^\circ &=& 24\text{ h.} \\ 1\text{ h.} &=& 15^\circ & & 15^\circ &=& 1\text{ h.} \\ 1\text{ m.} &=& 15' & & 1^\circ &=& 4\text{ m.} \\ 1\text{ s.} &=& 15'' & & 1' &=& 4\text{ s.} \\ & & & & 1'' &=& \tfrac{1}{15}\text{ s.} \end{array}$$

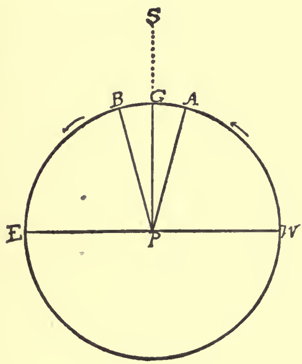
We can use the *first table* to convert *time* into *angular measure*, and the *second table to convert angular measure* into *time measure*.

$$\begin{array}{rrcr} \text{Thus } 3\text{ h. }10\text{ m. }30\text{ s.} = & 3 \times 15^\circ &=& 45 \\ & +\ 10 \times 15' &=& 2^\circ\ 30' \\ & +\ 30 \times 15'' &=& 7'\ 30'' \\ \hline & & = & 47^\circ\ 37'\ 30'' \end{array}$$

$$\begin{array}{rcr} \text{Again, } 48^\circ\ 15'\ 38'' &=& 3\text{ h. }13\text{ m. }2\tfrac{8}{15}\text{ s.} \\ \text{For } 48^\circ &=& 3\text{ h. }12\text{ m.} \\ 15 \times 4\text{ s.} &=& 1\text{ m.} \\ 38 \times \tfrac{1}{15}\text{ s.} &=& 2\tfrac{8}{15}\text{ s.} \\ \hline &=& 3\text{ h. }13\text{ m. }2\tfrac{8}{15}\text{ s.} \end{array}$$

56. In the case of a mean solar day, it was shown in Art. 46, that the meridian of any place moved through an arc of 360° 59′ 8.″33 during 24 mean solar hours. If we suppose 24 meridians drawn on the earth's surface, these meridians will be each 15° apart, and, in the rotation of the earth on its axis, will follow each other at an *hour's interval;* so that we can use the tables in the preceding article to convert mean solar *time* into angular *measure,* or angular measure into time measure.*

The same tables will give the relation of *apparent solar time* to *angular measure.*



57. These facts have an important bearing in the determination of longitude by means of time. This will be understood by means of a figure.

Let *GWE* be the plane of the earth's equator, *P* the projection of the pole on that plane, *PG* the projection of the meridian of

* As each meridian between two transits of the sun passes through an arc of 360° 59′ 8″33, on first thought it might seem that in order to make intervals correspond to hours, the space to equal *one* hour should be 15° 2′ +. The difficulty will be cleared by remembering that though it is true each meridian moves in *space* 15° 2′ + for an hour, before it comes to the position occupied by the meridian immediately preceding it, *all* the meridians here spoken of are 15° *apart* on *the earth*, corresponding to the division of a great circle of 360° by 24.

Greenwich, PA and PB the projections of the meridians of two places, each 15° from the meridian PG. If the sun is on the line PG produced at 12 noon, as the direction of the arrows shows the direction of the earth's rotation to be from W. to E., PA will be 15° *west*, and PB 15° *east* of PG. Consequently, when it is 12 noon at any place on the meridian PG, it will be 11 A.M. at any place on the meridian PA, and 1 P.M. at any place on the meridian PB; for there is an hour's interval of time required to bring PA to the place of PG and PG to the place of PB.

Now the *longitude* of any place on PA is 15° W., and the longitude of any place on PB is 15° E. of Greenwich:

consequently, 1 h. = 15° dif. of longitude;
1 m. = 15′ dif. of longitude;
1 s. = 15″ dif. of longitude;

or, 15° dif. of longitude = 1 h. dif. in time;
1° dif. of longitude = 4 m. dif. in time;
1′ dif. of longitude = 4 s. dif. in time;
1″ dif. of longitude = $\frac{1}{15}$ s. dif. in time.

CHAPTER VI

THE NAUTICAL ALMANAC

58. As the calculated positions of the heavenly bodies, recorded in the Nautical Almanac, are given in Greenwich time, the relations established in the preceding chapter between time and angular measure, and between time and difference of longitude, become important in determining the *Greenwich date* of any observation.

The *Greenwich date* is the *apparent or mean time* at *Greenwich*, corresponding to the time at which an observation of a heavenly body is taken at any other place on the earth.

Ex. 1. Given ship time June 8, 8 h. 16 m. P.M. (mean time), and longitude 40° 18′ W. Required the Greenwich date.

ship time June 8	8 h. 16 m.
long. 40° 18′ W. reduced to time =	2 h. 41 m. 12 s.
Ans. Greenwich, June 8	10 h. 57 m. 12 s.

The time of an observation is always expressed as *astronomical time* (Art. 52).

Ex. 2. Given ship time Jan. 18, 3 h. 20 m. A.M., and longitude 43° 25′ E. Required Greenwich date.

ship time = Jan. 17	15 h. 20 m.
long. in time =	2 h. 53 m. 40 s.
Ans. Greenwich, June 17	12 h. 26 m. 20 s.

59. *From the Nautical Almanac, to take the declination of the sun for any place and date, the longitude of the place being given.*

Ex. 1. Required sun's declination for Jan. 3, 1893, 8 h. 15 m. A.M., mean time, at a place in longitude 42° 18′ W.

ship, Jan. 2	20 h. 15 m.
long. in time	2 h. 49 m. 12 s.
Greenwich, Jan. 2	23 h. 4 m. 12 s. = 23.07 h.
	= Jan. 3 − 0.93 h.

Jan. 3, dif. for 1 h. = 15″.3	Jan. 3, sun's
.93	dec. at M.N. = 22° 46′ 46″ S.
459	14.2
1377	22° 47′ 00″.2 S.
14″.229 to be added.	

In this example, the correction for 0.93 h. we *add* to 22° 46′ 46″, because, as the declination is S. and *decreasing S.*, that is, tending N., it must be further S. 0.93 h. *before noon* than it is at noon.

Ex. 2. In longitude 72° 54′ W., on June 15, 1897, at 4.30 P.M., mean time, it is required to find the sun's declination.

ship, June 15	4 h. 30 m.
long.	5 h. 51 m. 36 s.
Greenwich, June 15	10 h. 21 m. 36 s. = 10.36 h.

sun's declination mean noon, June 15 = 23° 20′ 33″.7 N.
correction = 10.36 × 5″.66 = 58″.6 +
sun's declination at time of observation = 23° 21′ 32″.3 N.

difference for 1 h. 15th = 5″.87
difference for 1 h. 16th = 4″.84
decrease 24 h. = 1″.03

$$\begin{array}{rr} \text{decrease } 5\text{ h.} = \tfrac{5}{24} \times 1''.03 \\ \text{change for } 5\text{ h.} = \quad -0.21 \\ \text{hourly difference for 5 h. after noon} = 5''.66 \\ \underline{10.36} \\ 3396 \\ 1698 \\ \underline{566} \\ 58''.6376 \end{array}$$

As the difference per hour is changing, where great accuracy is required it is customary to find the change of difference for the hour *midway* between noon and the time of observation, and apply this change to the hourly difference, as in this example. For ordinary observations at sea, the hourly difference opposite the noon *nearest the time of observation* is used.

$$\begin{array}{rr} \text{Thus, } \odot\text{'s dec. June 15 noon} = & 23°\ 20'\ 33''.7\text{ N.} \\ \text{correction } 5''.87 \times 10.36 = & \underline{1'\ \ 0''.8\phantom{\text{ N.}}} \\ \odot\text{'s dec. at time of obs.} = & 23°\ 21'\ 34''.5\text{ N.} \end{array}$$

From these examples it is seen that, in order to obtain from the Nautical Almanac the sun's declination for any time and place, the longitude of the place being given, we *first:*

Find the Greenwich date; and, second, apply *the correction for time elapsed* since noon *to the declination* given opposite the nearest noon.

60. From the Nautical Almanac, to *find the equation of time* for *a given date,* the *longitude* of the place being given.

Ex. 1. In longitude 56° 10′ W., March 3, 1897, 6 h. 15 m. P.M., mean time, it is required to find the *equation* of time.

ship, March 3	6 h. 15 m.
longitude	3 h. 44 m. 40 s.
Greenwich, March 3	9 h. 59 m. 40 s. = 9.994 h.

dif. 1 h. = 0.541 s.	eq. of time =	12 m. 0.75 s.
9.99	correction	5.40
4869		11 m. 55.35 s. = eq. of time.
4869		
4869		
5.40458 to be subtracted.		

If it were required in this example to obtain apparent time, we subtract the 11 m. 55 s. from mean time. Thus:

March 3, 1897,	6 h. 15 m.	P.M. mean time
equation of time	11 m. 55 s.	
March 3, 1897,	6 h. 3 m. 5 s.	P.M. apparent time

Ex. 2. Given longitude 75° 18′ W., Sept. 13, 1897, 6 h. 30 m. A.M., apparent time. Required equation of time and corresponding mean time.

ship, Sept. 12	18 h. 30 m.
longitude	5 h. 1 m. 12 s.
Greenwich, Sept. 12	23 h. 31 m. 12 s.
Sept. 12	23.52 h. = Sept. 13 − 0.48 h.

eq. of time, Sept. 13, apparent noon =	4 m. 16.73 s.	
0.882×0.48 = correction	− .42	
eq. of time to be *subtracted* =	4 m. 16.31 s.	
apparent time	6 h. 30 m.	A.M.
Ans. Sept. 13, 1897	6 h. 25 m. 43.69 s.	A.M.

In all the foregoing examples the general method of arriving at the required result is:

1. *Express the ship time in astronomical time.*
2. *Find the* corresponding *Greenwich date.*
3. Take the required quantity *opposite the nearest Greenwich noon*, and *apply corrections* corresponding to the number of hours by which the given time exceeds or falls short of this nearest noon.

61. Given *mean solar time* and *the longitude;* by means of the Nautical Almanac, to find the corresponding *sidereal time* (Art. 50).

Thus, Jan. 20, 1895, 3 h. 19 m. P.M., mean time, in longitude 48° 40′ W., it is required to find the sidereal time.

ship, Jan. 20	3 h. 19 m.
longitude	3 h. 14 m. 40 s.
Greenwich, Jan. 20	6 h. 33 m. 40 s.

Jan. 20, 1895, Greenwich mean noon:

R.A. mean sun = 19 h. 58 m. 27 s.

Table 9, Bowditch:

correction for 6 h. 33 m. =	1 m. 4.56 s.
correction for 40 s. =	0.11
R.A.M. ⊙ =	19 h. 59 m. 31.67 s.
M.T.	3 h. 19 m.
sidereal time =	23 h. 18 m. 31.67 s.

62. Given *apparent solar time* and the *longitude;* from the Nautical Almanac, to obtain the *corresponding sidereal time.*

1. *Convert apparent into mean time.*

2. *Proceed as in previous article* to *convert mean time* into *sidereal time.*

Ex. July 15, 1895, 6 h. 14 m. A.M., apparent time, in longitude 20° 12′ E., required corresponding sidereal time.

ship apparent time, July 14	18 h. 14 m.
longitude	1 h. 20 m. 48 s.
Greenwich apparent time, July 14	16 h. 53 m. 12 s.

July 14, 16.887 h. = July 15 − 7.113 h.

July 15, noon, equation of time =	5 m. 41.34 s.
correction = 0.26 s. × 7.113 =	1.85 s.
eq. of time to be added to apparent time =	5 m. 39.49 s.
apparent time =	18 h. 14 m.
ship mean time =	18 h. 19 m. 39.49 s.
longitude =	1 h. 20 m. 48 s.
Greenwich mean time, July 14 =	16 h. 58 m. 51.49 s.
R.A.M. sun, July 14, noon =	7 h. 28 m. 24.34 s.
correction for 16 h. 58 m. =	2 m. 47.23 s.
correction for 51.5 s. =	0.14 s.
R.A.M. ⊙ =	7 h. 31 m. 11.71 s.
ship mean time =	18 h. 19 m. 39.49 s.
	25 h. 50 m. 51.2 s.
sidereal time =	1 h. 50 m. 51.02 s.

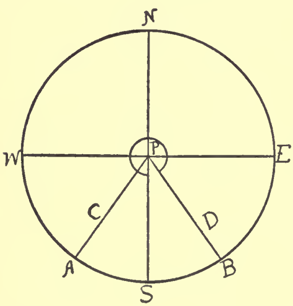
CHAPTER VII

THE HOUR ANGLE

The *hour angle* of any celestial body is *the angle*, at the nearer *celestial pole*, made *by the celestial meridian* of the place with *the circle of declination* which passes through the body.

Hour angles are *measured westward* from the meridian from 0 h. to 24 h.

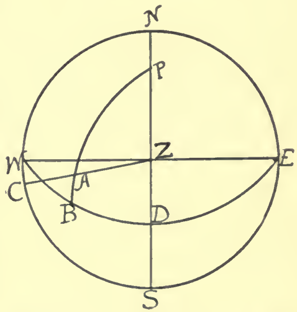
Let the figure represent the plane of the equinoctial, *P* the projection of the celestial pole, and *PA* and *PB* the projections of circles of declination, *PA* being to the W. and *PB* to the E. of the meridian *NPS*. If *C* and *D* represent the positions of two heavenly bodies, *SPA*, measured by the arc *SA*, is the hour angle of *C*, and the *salient angle* *SPB*, measured by the arc *SWNEB*, is the hour angle of *D*.



If *C* and *D* represent two positions of the sun, then *SPA* and *SPB* would be apparent *solar time*. *SPA* and *SPB* would be mean solar time if *A* and *B* represented the positions of the mean sun.

Also if A and B represented two positions of the first point of Aries, the angles SPA and SPB would be *sidereal* time (defs. pages 76, 77).

63. *Given the altitude*, the *declination of a heavenly body*, and *the latitude of the place* of observation; *to find the hour angle* of the body.



Let the figure represent the plane of the horizon; NS the projection on it of the meridian; and Z the projection of the zenith of the observer. Let P be the elevated or nearer celestial pole; A the position of a heavenly body; and let WDE be the equinoctial. Draw the circle of declination PAB, and the circle of altitude ZAC.

Then $AC =$ the altitude of A;

$AB =$ the declination of A;

$ZD =$ latitude of the observer.

Consequently, in the triangle APZ, in order to find the hour angle DPB, we have given:

$$ZA = 90° - AC = 90° - \text{altitude},$$

$$PA = 90° - AB = 90° - \text{declination},$$

and $$PZ = 90° - ZD = 90° - \text{latitude};$$

that is, to find P, in the triangle APZ, we have the three sides given.

Ex. 1. Given, in lat. 41° 24′ N., the declination of Venus = 24° 19′ N.; and the altitude = 24° 14′. Find the hour angle.

In the figure $ZA = 90° - 24° 14' = 65° 46'$. $PA = 90° - 24° 19' = 65° 41'$, $PZ = 90° - 41° 24' = 48° 36'$. Denoting the sides of the triangle by a, p, and z, $a = 48° 36'$, $p = 65° 46'$, $z = 65° 41'$; we can solve for P by the formula,

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\sin(s-a)\sin(s-z)}{\sin a \sin z}}$$

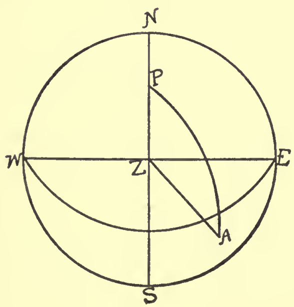
$$= \sqrt{\sin(s-a)\sin(s-z)\operatorname{cosec} a \operatorname{cosec} z}$$

$$\begin{array}{lll} a = & 48° 36' & \log \operatorname{cosec} = 10.12487 \\ z = & 65° 41' & \log \operatorname{cosec} = 10.04035 \\ p = & 65° 46' & \\ s = & 180° \ 3' & \\ & \quad 2 & \\ = & 90° \ 1' 30'' & \\ s - a = & 41° 25' 30'' & \log \sin = 9.82062 \\ s - z = & 24° 20' 30'' & \log \sin = 9.61508 \\ & & 2)\overline{19.60092} \end{array}$$

$$\log \sin 39° 10\tfrac{1}{5}' = 9.80046 = \log \sin \tfrac{1}{2} P$$

$$\therefore P = 78° 20\tfrac{2}{5}' = 5 \text{ h. } 13 \text{ m. } 21\tfrac{3}{5} \text{ s.}$$

Ex. 2. In lat. 41° 23′ N., the altitude of the sun was found to be 26° 38′ 44″, and its declination to be 19° 20′ 26″ S. Required the hour angle, supposing the sun to be east of the meridian; that is, that the observation was taken in the morning.



$$\begin{array}{ll} a = PZ = & 48° 37' \\ z = PA = & 109° 20' 26'' \\ p = ZA = & 63° 21' 16'' \\ s = & 221° 18' 42'' \\ & \quad 2 \\ = & 110° 39' 21'' \end{array}$$

$$s - a = 62^\circ\ \ 2'\,21''$$
$$s - z = 1^\circ\,18'\,55''$$
$$s - p = 47^\circ\,18'\ \ 5''$$

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\sin 62^\circ\,2'\,21'' \times \sin 1^\circ\,18'\,55''}{\sin 48^\circ\,37' \times \sin 109^\circ\,20'\,26''}}$$

$$\begin{aligned}
\log \sin\ 62^\circ\ \ 2'\,21'' &= 9.94609 \\
\log \sin\ 1^\circ\,18'\,55'' &= 8.36084 \\
\log \operatorname{cosec}\ 48^\circ\,37' &= 0.12476 \\
\log \operatorname{cosec}\ 109^\circ\,20'\,26'' &= \underline{0.02523} \\
2)\,&\underline{18.45692} \\
\log \sin \tfrac{1}{2}\ 1\text{ h. }17\text{ m. }56\text{ s.} &= 9.22846 \\
&= \log \sin \tfrac{1}{2}\ \textit{acute}\ \text{angle}\ ZPA,
\end{aligned}$$

but astronomical time = *salient* angle ZPA.

∴ hour angle = 24 h. − 1 h. 17 m. 56 s. = 22 h. 42 m. 4 s.,

or civil apparent time = 10 h. 42 m. 4 s. A.M.

Ex. 3. Suppose in addition to the data of the preceding example, the longitude of the place of observation was given as 72° 56′ W., and it was required to find the *mean* time at the instant of the observation on Nov. 19, 1894, at 10 h. 42 m. 4 s. apparent time.

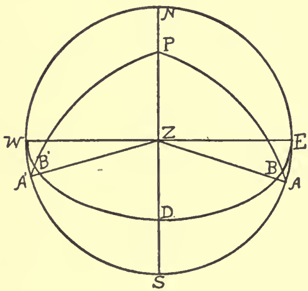
By definition on page 77 *apparent solar time* is the *angle, at the pole,* between the meridian and a circle of declination passing through the center of the true sun. *Consequently,* the answer in the *preceding* example is *apparent time,* and we have to apply the equation of time for the given date.

ship, Nov. 18,	22 h. 42 m. 4 s.
long. in time =	4 h. 51 m. 44 s.
Greenwich, Nov. 19 =	3 h. 33 m. 48 s. = 3.56 h.
eq. of time, Nov. 19, Green., noon =	14 m. 26.88 s. sub.
(dif. 1 h.) 0.574 s. × 3.56 =	2.04 s.
equation of time =	14 m. 24.84 s. sub.
apparent time =	10 h. 42 m. 4 s. A.M.
mean time =	10 h. 27 m. 39.16 s. A.M.

64. *To find the time of sunrise or sunset* for a given day, *at any place on the earth*, the *latitude and longitude* of the place, and *the sun's declination* for the day being given.

Let the figure represent, as in Art. 63, the projection of the celestial sphere on the plane of the horizon.

Suppose A to represent the position of the sun on the eastern horizon when it is first visible to an observer whose zenith is Z; and suppose A' to represent the position of the sun on the western horizon when it is last visible to the same observer.



$NPZS$ being the celestial meridian of the observer, when the sun is on that meridian, the time is *apparent noon*. The angle ZPA, expressed in time, would give the hours, minutes, and seconds which the sun, in its passage across the heavens, would take to go from its position at A to its position on *the meridian*. In other words, the angle ZPA gives the hours, minutes, and seconds of apparent time between sunrise and noon. In the same way, the angle ZPA' gives the apparent time between noon and sunset, or in common language, the apparent time of sunset. 24 h. − angle ZPA (expressed in time) would give the *astronomical* apparent time of sunrise. 12 h. − angle ZPA (expressed in time) would give the *civil* apparent time.

In the preceding figure, the declination BA is given as S. declination, while the elevated pole P is supposed to be N.

The zenith distance to A, a point on the horizon, is 90°. But as the time of sunrise is calculated from the instant when the *upper rim* of the sun is first visible, and as measurements are made to the center of the sun, 16′ is added to 90°, as the center of the sun is about that distance below the horizon. Moreover, as by refraction the sun, though *below* the horizon, is made to appear above it, 34′ is added also to 90° for refraction. Consequently, for problems in sunrise and sunset the distances ZA and ZA' are generally taken to be each 90° 50′.

Though the declination of the sun is continually changing, so that the declination is *not exactly the same* at sunrise and sunset, yet the change is so small that it is assumed to be the same both at those times and at noon. For convenience of calculation, therefore, the declination of the sun for *noon* is used in the solution of problems in sunrise and sunset.

Ex. 1. January 28, 1898, in lat. 42° 18′ N., long. 72° 55¾′ W., it is required to find the *apparent time* of sunrise and sunset.

local time at noon	= 0 h. 0 m. 0 s.
long. in time	= 4 h. 51 m. 43 s.
Greenwich, Jan. 28	= 4 h. 51 m. 43 s.
	= 4.86 h.

declination of sun, Greenwich noon January 28	= 18° 6′ 25″.8 S.
cor. = 39″.85 × 4.86	= 3′ 13″.7 N.
declination of sun at local apparent noon	= 18° 3′ 12″.1 S.

hourly difference of declination of sun = 39″.85 N.

$$\begin{array}{r} 4.86 \\ \hline 23910 \\ 31880 \\ 15940 \\ \hline 193''.6710 \end{array} = 3'\ 13''.7$$

In preceding figure,

$$\begin{aligned} PZ = a = 90° - 41°\ 18' &= 48°\ 42' \\ PA = z = 90° + 18°\ 3'\ 12'' &= 108°\ \ 3'\ 12'' \\ ZA = p = 90° + 50' &= 90°\ 50' \\ s &= \frac{247°\ 35'\ 12''}{2} \\ &= 123°\ 47'\ 36'' \\ s - a &= 75°\ \ 5'\ 36'' \\ s - z &= 15°\ 44'\ 24'' \\ s - p &= 32°\ 57'\ 36'' \end{aligned}$$

$$\sin \tfrac{1}{2} P = \sqrt{\sin(s-a)\sin(s-z)\operatorname{cosec} a \operatorname{cosec} z}.$$

$$\begin{aligned} \log \sin 75°\ \ 5'\ 36'' &= 9.98513 \\ \log \sin 15°\ 44'\ 24'' &= 9.43341 \\ \log \operatorname{cosec} 48°\ 42' &= 10.12421 \\ \log \operatorname{cosec} 108°\ 3'\ 12'' &= 10.02191 \\ &\quad 2)\underline{19.56466} \\ \log \sin 37°\ 17''\tfrac{3}{16} &= 9.78233 = \log \sin \tfrac{1}{2} P \end{aligned}$$

$P = 74°\ 34'\tfrac{6}{16} = 4$ h. 58 m. $17\tfrac{1}{2}$ s. = apparent time of sunset.

12 h. − 4 h. 58 m. $17\tfrac{1}{2}$ s. = 7 h. 1 m. $42\tfrac{1}{2}$ s. = apparent time of sunrise.

Ex. 2. In preceding example, required the *mean times* of sunrise and sunset; also eastern standard time of sunrise and sunset.

January 28, equation of time Greenwich noon = 13 m. 13.97 s.
difference for 1 h. = 0.457 s. × 4.86 = 2.22+
local equation of time at noon = 13 m. 16.19 s.

```
     0.457
      4.86
    ------
     2742
    3656
   1828
   -------
   2.22102
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local mean time of apparent noon = 12 h. 13 m. 16.19 s.
subtract hour angle = 4 h. 58 m. 17.5 s.
local mean time of sunrise = 7 h. 14 m. 58.69 s. A.M.
local mean time of sunset = 5 h. 11 m. 33.69 s. P.M.

eastern standard time = time of meridian of 75° W.
local meridian = 72° 55′$\frac{3}{4}$ W.
difference = 2° 4′$\frac{1}{4}$
= 8 m. 17 s.

taking 8 m. 17 s. from the mean times calculated above
eastern standard time of sunrise = 7 h. 6 m. 41.69 s. A.M.
eastern standard time of sunset = 5 h. 3 m. 16.69 s. P.M.

In this example we have used the noon equation of time to be applied to time of sunrise and sunset. A more exact calculation would apply the equation of time as derived for the instant of apparent time of sunrise or of sunset.

For sunrise.

Greenwich, 27th	19 h. 1 m. 42½ s.
longitude in time	4 h. 51 m. 43 s.
Greenwich, Jan. 27	23 h. 53 m. 25½ s.
or Jan. 28 −	0 h. 6 m. 34.5 s. = − .011
eq. of time, Greenwich, noon	13 m. 13.97 s.
correction 0.457 s × .011 h. =	0.01
equation of time for sunrise =	13 m. 13.96 s. +
apparent time of sunrise =	7 h. 1 m. 42.5 s.
exact mean time =	7 h. 14 m. 56.46 s. A.M.

For sunset. Jan. 28	4 h. 58 m. 17.5 s.	
longitude	4 h. 51 m. 43 s.	
Greenwich, Jan. 28	9 h. 50 m. 0.5 s.	= 9.83 h.
		0.457
		6881
		4915
		3932
correction =		4.49231 s.

eq. of time, Greenwich, noon =	13 m. 13.97 s.
correction	4.49
equation of time =	13 m. 18.46 s.
apparent time of sunset =	4 h. 58 m. 17.5 s.
exact mean time =	5 h. 11 m. 35.96 s. P.M.

Since the time of sunrise and the time of sunset are generally calculated to the *nearest minute only*, the first method of applying the local noon equation of time is generally used. By comparing the results by the two methods it will be seen that the difference in the answers does not much exceed two seconds.

Ex. 3. June 1, 1898, in latitude 41° 18′ N., longitude 72° 55′$\frac{3}{4}$ W., required the eastern standard times of sunrise and sunset.

local noon	0 h. 0 m. 0 s.
longitude	4 h. 51 m. 43 s.
Greenwich, June 1	4 h. 51 m. 43 s.
	= 4.86 h.

Declination of sun.

Greenwich, noon =	22° 6′ 0″.7 N.
correction 20″.12 × 4.86 =	137.8+
declination of sun =	22° 7′ 38″.5 N.
polar distance =	67° 52′ 22″

equation of time, Greenwich noon =	2 m. 24.55 s. −
correction 0.375 s. × 4.86 h. =	1.82 −
equation of time, local noon =	2 m. 22.73 s. −
apparent noon =	12 h.
mean time of apparent noon =	11 h. 57 m. 37.17 s. A.M.
deduct for eastern standard time	8 m. 17 s.
eastern standard time of apparent noon =	11 h. 49 m. 20.17 s. A.M.

Projecting the celestial concave on the celestial meridian.

$$PZ = a = 48^\circ 42' \qquad \log \operatorname{cosec} = 10.12421$$
$$PA = z = 67^\circ 52' 22'' \qquad \log \operatorname{cosec} = 10.03322$$
$$ZA = p = 90^\circ 50'$$
$$s = \frac{207^\circ 24' 22''}{2} = 103^\circ 42' 11''$$
$$s - a = 55^\circ \; 0' 11'' \qquad \log \sin = 9.91338$$
$$s - z = 35^\circ 49' 49'' \qquad \log \sin = 9.76744$$
$$2 \,)\, 19.83825$$
$$\log \sin 56^\circ 6' 33'' = 9.91912\tfrac{1}{2}$$
$$08$$
$$P = 112^\circ 13' \; 6'' \qquad 4\tfrac{1}{2}$$
$$= 7 \text{ h. } 28 \text{ m. } 52.4 \text{ s.}$$

eastern standard time of apparent noon = 11 h. 49 m. 20.2 s.

eastern standard time of sunrise = 4 h. 20 m. 27.8 s. A.M.

eastern standard time of sunset = 7 h. 18 m. 12.6 s. P.M.

Ex. 4. Jan. 10, 1898, in latitude 39° 57′ N., longitude 75° 9′ W., required mean time of sunrise and of sunset.

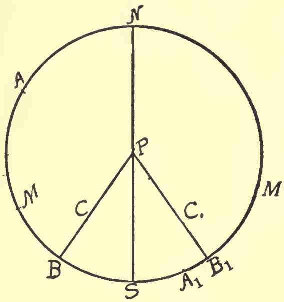
Ans. 7 h. 21 m. 38 s. A.M.; 4 h. 54 m. 16 s. P.M.

Ex. 5. May 16, 1898, in latitude 42° 36′ N., longitude 70° 40′ W., required eastern standard time of sunrise and of sunset.

Ans. 4 h. 18 m. 44 s. A.M.; 6 h. 58 m. 16 s. P.M.

65. *Given a star's hour angle, to find mean time.*

Let the figure represent the plane of the equinoctial; P the projection of the pole on the plane; C the position of the star; A the position of the first point of Aries; and M the position of the mean sun.



If NPS be the projection of the celestial meridian, and PCB be the projection of the circle of declination passing through C, SPC will be the hour angle of the star, and SB will measure that angle. Now $SM = SB + AB - AM$; that is, mean time = star's hour angle + R.A. of star − R.A. of mean sun. In the case just given the star is W. of the meridian.

Suppose the star is at C', and east of the meridian; that A_1 is first point of Aries, and M_1 is position of mean sun; then $SM_1 = SB_1 + A_1M_1 - A_1B_1$ or (24 h. − mean time) = (24 h. − star's hour angle) + R.A. mean sun − star's R.A.

∴ mean time = star's hour angle + R.A. of star − R.A. mean sun.

66. To find the *mean time at any place*, having given *the hour angle of a star*; the *longitude* of the place; the *date;* and the *approximate local mean time.*

By the previous article we have to add to the hour angle the *star's R.A.*, and from the sum subtract the

R.A. of the mean sun for the given date and approximate time.

Ex. 1. Nov. 22, 1891, 7 h. 15 m. P.M., approximate mean time in long. 87° 56′ W., the hour angle of Aldebaran (*a* Tauri), was 18 h. 55 m. 15 s. (E. of meridian). Star's R.A. = 4 h. 29 m. 41.5 s. Required mean time at the place.

ship, Nov. 22 =	7 h. 15 m.
longitude =	5 h. 51 m. 44 s.
Greenwich, Nov. 22 =	13 h. 6 m. 44 s.
Green., Nov. 22, noon, R.A. mean sun =	16 h. 4 m. 44.5 s.
correction for 13 h. 6 m. =	2 m. 9.1 s.
correction for 44 s. =	.1 s.
R.A. mean sun at time of observation =	16 h. 6 m. 53.7 s.
star's H.A. =	18 h. 55 m. 15 s.
star's R.A. =	4 h. 29 m. 41.5 s.
	23 h. 24 m. 56.5 s.
R.A. mean sun =	16 h. 6 m. 53.7 s.
Ans.	7 h. 18 m. 2.8 s. P.M.

Ex. 2. June 23, 1891, at 4 h. 12 m. A.M. mean time, nearly, in long. 50° 15′ W., the hour angle of *a* Lyræ was 3 h. 41 m. W. of meridian. Required mean time. Star's R.A = 18 h. 33 m. 15.8 s.

ship, June 22 =	16 h. 12 m.
longitude =	3 h. 21 m.
Greenwich, June 22 =	19 h. 33 m.
Green., June 22, noon, sid. time =	6 h. 1 m. 31.55 s.
correction for 19 h. 33 m. =	3 m. 12.69 s.
R.A. mean sun =	6 h. 4 m. 44.24 s.
star's H.A. =	3 h. 41 m.
star's R.A. =	18 h. 33 m. 15.8 s.
	22 h. 14 m. 15.8 s.
	6 h. 4 m. 44.2 s.
June 22	16 h. 9 m. 31.6 s. ast. time
June 23	4 h. 9 m. 31.6 s. A.M. m. t.

67. Given *mean*, or *apparent* time at place of *given longitude; to find what star of* 1*st or* 2*d magnitude* will *pass the meridian next after that time.*

The solution of this problem is simply to find the sidereal time corresponding to the given time, and then, from list of fixed stars in Nautical Almanac, to *choose* the star of required magnitude whose right ascension is the next greater than the sidereal time found.

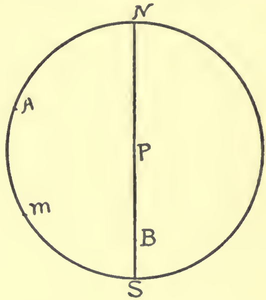
Ex. In long. 72° 56′ W., Dec. 7, 1897, at 11 h. 30 m. P.M. mean time, what star of 1st or 2d magnitude passed the meridian shortly after that time?

ship, Dec. 7, 1897 =	11 h. 30 m.
longitude =	4 h. 51 m. 44 s.
Greenwich, Dec. 7 =	16 h. 21 m. 44 s.
Dec. 7, mean noon R.A.M. ⊙ =	17 h. 6 m. 3.95 s.
correction for 16 h. 21 m. =	2 m. 41.15 s.
correction for 44 s. =	.12 s.
R.A.M. sun =	17 h. 8 m. 45.22 s.
ship, Dec. 7 =	11 h. 30 m.
=	28 h. 38 m. 45.2 s.
=	24 h.
sidereal time or R.A. of meridian =	4 h. 38 m. 5.8 s.

In catalogue of fixed stars (Capella), *α* Aurigæ has R.A. 5 h. 9 m. 4.8 s., and is, therefore, star required.

68. *To find at what mean time any star* will pass a *given meridian.*

Let the figure represent the plane of the equinoctial; *P* the pole: *NPS* the celestial meridian; *A* the first point of Aries; m the position of the mean sun; and B the position of the star at instant of crossing the meridian.



Then mS

$= \text{mean time} = AS - Am,$

or mean time

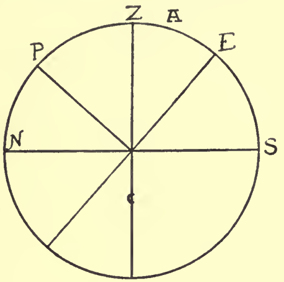
= sidereal time of star
− R.A. of mean sun
= R.A. of star
− R.A. of mean sun.

Ex. To find at what time Sirius passed the meridian in longitude 72° 56′ W., Dec. 8, 1897.

R.A. of Sirius =	6 h. 40 m. 39 s.
add	24 h.
	30 h. 40 m. 39 s.
R.A. of sun (noon) =	18 h. 10 m. 0.5 s.
ship approximate mean time =	13 h. 30 m. 38.5 s.
longitude =	4 h. 51 m. 44 s.
Greenwich, Dec. 8 =	17 h. 22 m. 22.5 s.
R.A. M.S. noon =	17 h. 10 m. 0.5 s.
correction for 18 h. 22 m. =	3 m. 1.03 s.
correction for 22.5 s. =	.06 s.
R.A. M. sun =	17 h. 13 m. 1.59 s. subtract
from R.A. Sirius =	30 h. 40 m. 39 s.
	13 h. 27 m. 37 s. ast. time
	12 h.
Ans. Dec. 8.	1 h. 27 m. 37 s. 3 A.M.

69. To find the *meridian altitude* of a heavenly body for a given place, and whether it will pass N. or S. of the zenith, the declination of the body and the latitude of the place being given.

Ex. 1. At a place in latitude 42° N., it is required to find the meridian altitude of a star whose declination is 25° N.; also whether it passes N. or S. of the zenith.



Let NZS represent the plane of the celestial meridian; P the upper or N. pole; Z the zenith; N and S the north and south points of the horizon; and E the point where the equinoctial intersects the meridian. Let $AE = 25°$, then A is the position of the star at transit. Let ZE = latitude 42° N.

$$ZA = ZE - AE = 42° - 25° = 17°.$$

∴ star's transit is south of zenith.

Again, altitude of star $= AS = ZS - ZA = 90° - 17° = 73°$.

Ex. 2. Dec. 9, 1897, at what time did α Orionis pass the meridian of longitude 72° 56′ W. in latitude 42° 18′ N.; and did it pass N. or S. of zenith? Required its altitude also.

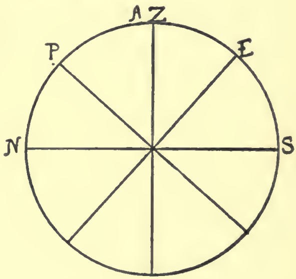
given the declination of star =	7° 23′ 16″ N.
R.A. of star = 5 h. 49 m. 40 s.; R.A. M.S. =	17 h. 13 m. 57 s.
R.A. of star + 24 h. =	29 h. 49 m. 40 s.
R.A. of sun (Greenwich noon) =	17 h. 13 m. 57 s.
mean time (approximately) =	12 h. 35 m. 43 s.
longitude =	4 h. 51 m. 44 s.
Greenwich mean time (approximately) =	17 h. 27 m. 27 s.
R.A. M. sun (Greenwich noon) =	17 h. 13 m. 57 s.
correction for 17 h. 27 m. =	2 m. 51.9 s.
correction for 27 s. =	.1 s.
R.A. M. sun =	17 h. 16 m. 49 s.
R.A. of star =	29 h. 49 m. 40 s.
star on meridian =	12 h. 32 m. 51 s.
=	32 m. 51 s. after midnight Dec. 10

$$\begin{array}{rl} \text{latitude} = & 42^\circ\ 18'\ \text{N.} \\ \text{declination of star} = & \underline{\ 7^\circ\ 23'\ 16''\ \text{N.}} \\ & 34^\circ\ 54'\ 44''\ \text{S. of zenith} \\ & \underline{90^\circ} \\ & 55^\circ\ \ 5'\ 16'' = \text{altitude} \end{array}$$

Ex. 3. At what time, Dec. 10, 1897, in latitude 42° 18′ N., longitude 72° 56′ W., did η Ursæ Majoris pass the meridian? Was the transit N. or S. of the zenith?

R.A. of star = 13 h. 43 m. 31 s.
declination of star = 49° 49′ 2″ N.

Let $NPZES$ be the meridian; P the pole; Z the zenith; A be the position of star at transit.



$$\begin{array}{rl} AE = & 49^\circ\ 49'\ \ 2'' \\ ZE = & \underline{42^\circ\ 18'\qquad} \\ ZA = & \ 7^\circ\ 31'\ \ 2'' \\ & \text{star N. of zenith} \\ ZN = & \underline{90^\circ\qquad\qquad} \\ \text{altitude} = AN = & 82^\circ\ 28'\ 58'' \end{array}$$

To find at what time the star passed the meridian *Dec. 10*, we must begin *one day back*, and take out the R.A. of M. ☉ for Dec. 9.

thus, R.A. of star + 24 h. =	37 h. 43 m. 31 s.
R.A. of M. sun, Dec. 9, noon =	17 h. 13 m. 57 s.
approximate mean time =	20 h. 29 m. 34 s.
longitude =	4 h. 51 m. 44 s.
Dec. 10, Greenwich mean time =	1 h. 21 m. 18 s.
" " R.A. M. ☉ noon =	17 h. 17 m. 54 s.
correction for 1 h. 21 m. =	13.3 s.
correction for 18 s. =	.05 s.
R.A. M. ☉ =	17 h. 18 m. 7.4 s.
R.A. star =	37 h. 43 m. 31 s.
Dec. 9	20 h. 25 m. 23.6 s. ast. time
Dec. 10	8 h. 25 m. 23.6 s. A.M.

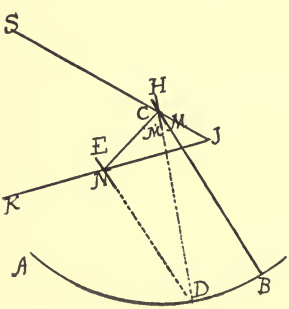
CHAPTER VIII

CORRECTIONS OF ALTITUDE

70. In order to obtain the *true* altitude of a heavenly body, a number of corrections must be applied to the *observed* altitude, namely:

Index correction, due to some error in the instrument used; and corrections for *dip*, *refraction*, *semi-diameter*, and *parallax*, corrections required by the fact that, to combine observations made at any place on the earth's *surface* with the elements from the Nautical Almanac, those observations must all be reduced to a common point of observation. This common point of observation is considered to be the *center* of the earth.

The sectant is an instrument for measuring angles in any plane. At sea it is used chiefly to measure the altitudes of heavenly bodies.



The accompanying figure will serve to explain the principles of the construction of the sextant.

AB is a circular arc a little longer than a sixth of the whole circumference. EN and CM are two glasses whose planes are perpendicular to the plane of the arc AB. EN is fixed in position, and its glass is silvered on the half next to the frame of the instrument. EN is called the *horizon* glass, because through it the horizon is viewed in taking observations. CM is called the *index* glass. It is entirely silvered (on one face). By means of the index bar, CB, it is movable about the point C, which is the center of the arc AB. When the index bar is at the zero point on the arc AB, the planes of the two glasses, EN and CM, are parallel.

If it is required to find the altitude of any body, S, above the horizon, the observer looks at the horizon line through the plain part of the glass EN, and moves the instrument and the index bar till an image of S reflected from CM upon EN appears to coincide with a point upon the horizon.

Let K be the point of the horizon with which S appears to coincide. Let CM' be the position of the index glass and CD be the position of the index bar when K and S appear in coincidence. Join SC, CN, and KN. Produce SC and KN to meet at J.

JK will represent the plane of the horizon, and the angle SJK will be the altitude of S.

Produce EN to meet CD (in this case) at D.

The arc DB measures the angle DCB. But $DCB = NDC$, since EN and CM are parallel.

When a ray of light is reflected from a plane surface, *the angle of incidence is equal to the angle of reflection:*

therefore $SCH = NCD$,

but $SCH = M'CJ$,

these being vertical angles; therefore,

$$NCJ = 2\,(NCD).$$

Also, since *angle of incidence is equal to angle of reflection*,

$$ENC = DNJ, \text{ but } DNJ = ENK;$$

therefore (1), $KNC = 2\,(ENC) = 2\,[(NCD) + D]$,

because ENC is exterior angle of triangle NCD.

Also (2), $KNC = NCJ + J = 2\,(NCD) + J$;

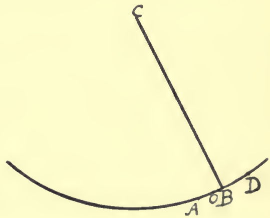
consequently, $2\,(NCD) + 2\,D = 2\,(NCD) + J$;

that is, $D = \frac{1}{2}\,J$;

but as $D = DCB$, and DB measures DCB, DB measures half of J, or half the altitude of S. The whole arc AB, however, is so graduated that each *half degree* counts as *a degree*, and the reading of the arc DB gives the measure of the whole angle J.

Index error. The planes of the index glass and horizon glass should be parallel when the index bar is at the zero point on the graduated arc AB. The distance, either *on* the arc (that is, to the left of the zero point), or *off* the arc (that is, to the right of the zero point), to which the index bar must be moved to make these planes parallel, is called the index error. This error demands a *correction* for every angle measured.

To determine the index error for any instrument, the simplest method is to measure at successive instants the angle subtended by the sun near the zero point. As the diameter of the sun is the same, these measurements should agree if there is no error, but if they do not agree, there is an error in the instrument. This will be understood by means of the figure.



Let AOB be a part of the arc of the sextant having the zero point at O. Suppose that in measuring the diameter of the sun *on the arc* the index bar is moved to A, and that in measuring the same diameter off the arc the index is moved to D. Then, denoting the measure of the diameter by d, $AD = 2d$; consequently B, the middle point of AD, should be the real zero point of the graduated arc. OB would represent the error, which is *off* the arc, in this case, and the *correction* for the error, called *index correction*, must be *added.*

Denote OB by ϵ; the reading OA by r; the reading OD by r'; then

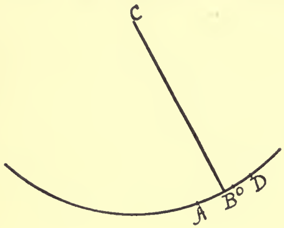
$$AB = BD,$$

or $$AO + OB = OD - OB;$$

that is, $$r + \epsilon = r' - \epsilon;$$

therefore, $$\epsilon = \frac{r' - r}{2}.$$

If the reading AO, on the arc, is greater than the reading OD, off the arc,



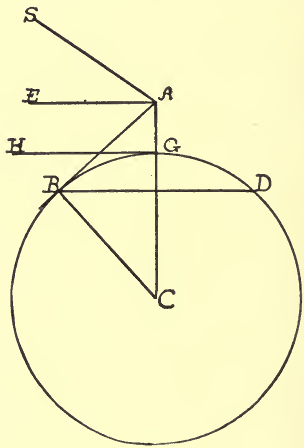
since $AB = BD$,

$$r - \epsilon = r' + \epsilon.$$

$$\therefore \epsilon = \frac{r - r'}{2}.$$

In this case the *index correction* must be *subtracted.*

71. The *dip* of the horizon is the angle of depression of the *visible* horizon below the horizontal plane of the observer. This depression of the visible horizon is due to the elevation of the eye of the observer above the level of the sea.



Let the figure represent a section of the earth by a plane passed through A, the point of observation, and C, the center of the earth.

The small circle, of which BD is the diameter, would represent the plane of the observer's visible horizon. If AE be the line in which the plane ABC intersects the horizontal plane through A, then EAB would be the *dip*, or angle of depression of the visible horizon, BD, below the horizontal plane of the observer at A. If S be a celestial body, the angle SAE would be its true altitude, SAB its measured or observed altitude. *Dip* must always be *subtracted* from the *observed* altitude to obtain the *true* altitude, for $SAB - EAB = SAE$.

AB is tangent at B. Join C and B by straight line, CB. EA is parallel to tangent at G, and therefore is perpendicular to CA.

Angles EAB and ACB are complements of BAC and therefore equal; that is, ACB = dip.

Let $AG = h$ and $CG = R$.

$$\text{Then } AB = \sqrt{AC^2 - CB^2} = \sqrt{(R + h)^2 - R^2}$$

$$= \sqrt{2\,Rh + h^2}.$$

$$\therefore \tan \text{dip} = \tan ACB = \frac{AB}{BC} = \frac{\sqrt{2\,Rh + h^2}}{R}$$

$$= \sqrt{\frac{2\,Rh + h^2}{R^2}}.$$

But since h is small compared with R, h^2 may be neglected, and

$$\tan \text{dip} = \sqrt{\frac{2\,h}{R}} \text{ nearly.}$$

But as the dip is usually a very small angle, and since for a very small angle the circular measure of the angle is approximately equal to the tangent of the angle, we can say

$$\textit{circular measure of dip} = \sqrt{\frac{2\,h}{R}}.$$

$$\text{Now circular measure of dip} = \frac{n\pi}{180}$$

where n = number of degrees in angle, n being integral or fractional; therefore reducing to minutes.

$$\frac{60\, n\pi}{180 \times 60} = \sqrt{\frac{2\,h}{R}};$$

$$\text{or, since } 60\,n = \text{dip in minutes},$$

$$\text{dip in minutes} = \frac{10800}{\pi} \sqrt{\frac{2\,h}{3960 \times 5280}},$$

reducing R to feet, R being 3960 miles.

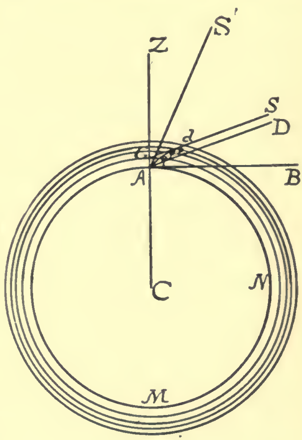
$$\text{Dip in minutes} = \frac{10800\,\sqrt{2}}{\pi\sqrt{3960 \times 5280}}\,\sqrt{h}.$$

$$\begin{aligned}
\log 10800 &= 4.03342 \\
\log \sqrt{2} &= 0.15051 \\
\text{colog } \pi &= 9.50285 - 10 \\
\text{colog } \sqrt{3960} &= 8.20115 - 10 \\
\text{colog } \sqrt{5280} &= \underline{8.13868 - 10} \\
\log 1.063 &= 0.02661
\end{aligned}$$

$$\therefore \text{dip in minutes} = 1.063\,\sqrt{h}.$$

This value of dip is diminished by refraction. The amount by which it is diminished is variously estimated. If we take that amount as $\frac{3}{40}$, we shall obtain the true value of dip; allowing for refraction, dip $= 1.063\sqrt{h} - \frac{3}{40}(1.063\sqrt{h}) = .984\sqrt{h}$, approximately.

72. *Refraction.* To understand the effect of refraction, we represent, by the figure, a great circle section of the earth *AMN*, made by a plane passing through *A*, the point of observation, and through the atmosphere surrounding the earth.



A ray of light from a distant object, as a star, *S*, entering the atmosphere obliquely at *d*, and passing through strata of varying density, is bent out of its course into a curve, *defgA*, concave to the earth's surface. The object itself appears at *A* on *AS′*, which is a line tangent to the curve *defgA* at *A*.

If we join the center *C* with *A* and produce the line to *z*, *z* will represent the zenith of the observer. Produce the line *Sd* (supposed to be a straight line before it enters the atmosphere at *d*) to meet *CZ* at *G*. If we draw *AD* parallel to *GS*, *DAB* would represent the true altitude of *S*; *DA* and *GS*, representing rays of light from an object so remote as one of the celestial bodies, may be regarded as parallel straight lines.

If there were no refraction, the light would come on the line *AD*. *S′AD* is the angle of refraction. The correction for refraction, therefore, is to be subtracted from the observed altitude to give the true altitude, for $S'AB - S'AD = DAB$.

Rays of light from an object in the *zenith*, falling on the strata of the atmosphere, are not *refracted*.

The more obliquely the light enters the atmosphere, the greater the refraction. Consequently, refraction increases, the nearer the body is to the horizon.

73. *Correction for semidiameter.* The positions of heavenly bodies indicated in the Nautical Almanac are given for their *centers*.

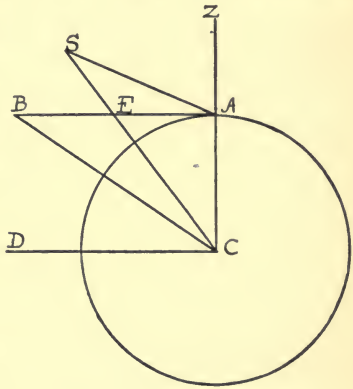
Observations of heavenly bodies of perceptible size are generally made to the upper or lower edge of the body, called respectively the *upper* or *lower limb*.

If an observed altitude is one of the *lower limb*, the semi-diameter expressed in minutes or seconds of the body must be *added* to give the altitude of the center. If an observation is taken of the *upper limb*, the semidiameter must be *subtracted* to give the true altitude.

74. *Parallax.* Altitudes of celestial objects are observed at the surface of the earth, or slightly above it. They are taken with reference to the *sensible horizon*, that is, with *a plane tangent to the earth's surface vertically below* the point of observation. But to these observed altitudes we have to apply corrections in order to obtain the altitudes of the same bodies if the observations were made at the center of the earth, and with reference to the *rational horizon*, that is, *a plane passed through the center of the earth parallel to the sensible horizon.*

Let the figure represent a section of the earth made by a plane passed through its center C, and through the point of observation at A.

Produce line CA to zenith Z. Let S be position of heavenly body. Its *altitude* with reference *to the sensible horizon*, represented by line AB drawn perpendicular to AC, is the *angle SAB*. Its *altitude* with reference to the *rational horizon*, represented by line CD, drawn parallel to AB, is the *angle SCD*.



Let E be the point where AB and SC intersect.

Since AB and CD are parallel,

$$(1)\quad SCD = SEB = SAB + ASC.$$

The angle ASC is called the *parallax in altitude* of S, or simply *parallax* of S. To obtain the true altitude of a heavenly body (in addition to the other corrections to be applied to the observed altitude), from equation (1) it is evident that parallax must be *added* to the *observed altitude*.

Let R denote AC, the radius of the earth; let d denote CS, the distance of the heavenly body from the center of the earth. Denote observed altitude SAB by h.

$$\frac{\sin ASC}{\sin SAC}=\frac{R}{d};\quad \text{that is,}\quad \frac{\sin \text{parallax}}{\sin(90^\circ+h)}=\frac{R}{d};$$

or (2) $\sin(\text{parallax})=\frac{R}{d}\sin(90^\circ+h)=\frac{R}{d}\cos h.$

Suppose the celestial body to be in the horizon at B; then

$$\sin \text{parallax}=\sin ABC=\frac{R}{d}.$$

In this case the parallax is called the *horizontal* parallax; that is,

$$(3)\quad \sin \text{horizontal parallax}=\frac{R}{d}.$$

Substituting in (2) this equivalent of $\frac{R}{d}$, we have

$$(4)\quad \sin \text{parallax}=\sin(\text{horizontal parallax})\cos h.$$

Since parallax and horizontal parallax are always small angles (except in the case of the moon), we may substitute for the sines the measures of these angles, at any altitude, and (4) becomes

$$\text{parallax}=\text{horizontal parallax}\times\cos h.$$

Both from the equation and from the figure it is evident that parallax is *greatest* when the heavenly body is in the *horizon*; *decreases* as the *altitude* of the body *increases*; and vanishes at the *zenith*.

Also, from the figure, if S be at a very great distance from the earth, d may be so large that the ratio $\frac{R}{d}$ approaches 0; in that case, sin parallax in (2) will vanish. For the *fixed stars*, which are supposed to be at such immense distances from the earth that rays of light from them fall on any two points of the earth in nearly parallel lines, no correction for *parallax* is applied.

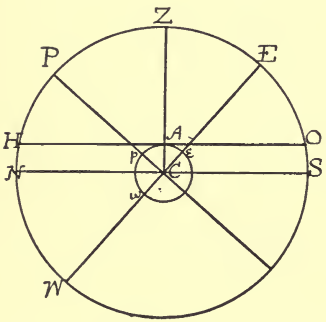
Again, the nearer S is to the earth, the greater the value of $\frac{R}{d}$, and consequently the greater the parallax. Of the heavenly bodies, the moon is the nearer to the earth and has the greatest parallax.

CHAPTER IX

LATITUDE

75. *Latitude.*

Let $wpAe$ represent a great circle section of the earth through the meridian of the observer at A; and let NPS be the celestial meridian of the same observer. wCe will then be the projection of the terrestrial equator, and WCE will be the projection of the celestial equator, or equinoctial on the same plane, viz. the plane of the terrestrial and celestial meridians. Let p be the pole of the earth, and P the corresponding elevated pole of the celestial concave. Join CA, and produce the line to meet the celestial concave at Z, the zenith of the observer.

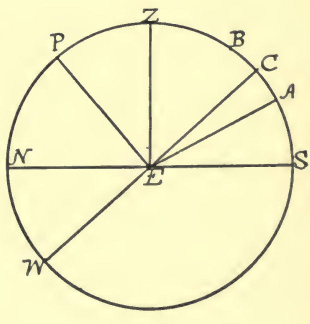


Through C at right angles to CA draw NCS, which will represent the projection of the *rational horizon* of the observer. If at A a line be drawn tangent to the circle pAe, cutting the celestial meridian at H and O, this line would represent the *sensible horizon* of the observer (Art. 74).

In consequence of the immense distances of the heavenly bodies on the celestial concave, O and S and H and N are supposed to coincide, and altitudes of objects are observed with reference to HAO. Where accuracy is required, such observations have to be *corrected* so as to equal the true altitude with reference to NCS (Art. 74).

Ae measures the latitude of A, viz. the angle ACe. This angle is also measured by ZE. $NZ = 90° = PE$. If from these equals we take away the common part PZ, we have $PN = ZE$; or, the *elevation* of the nearer *celestial pole* above the horizon of the observer is equal to his *latitude.*

76. To find the latitude. Latitude is found *by observing the altitude* of any heavenly body while *on the meridian, the declination* of the body *being given.*



The altitude of the body may be observed either at its *upper transit* or at its *lower transit,* in case it moves in a small circle on the celestial concave, and always above the horizon. Let $WPZS$ represent the celestial concave projected on the meridian of the observer; P will be the nearer (in this case N.) pole; Z the zenith; WEC the projection of the equinoctial; NES the projection of the horizon. ZC or PN will measure the latitude (Art. 75).

Suppose A to be the position of the heavenly body on the meridian at its *upper transit*.

If the angle AES is observed, the arc AS, which measures this angle, is known. CA is the declination, and in this figure is a S. declination.

$$(a)\ \text{lat.} = ZC = ZS - (AS + AC) = 90° - (\text{alt.} + \text{dec.}).$$

If the object observed is at B, and having a N. declination, BS is the measure of its altitude, and

$$(b)\ \text{lat.} = ZC = 90° - (BS - BC) = 90° - (\text{alt.} - \text{dec.}).$$

The observer is supposed to be in the N. hemisphere, and the latitude required is a N. latitude. In this case, therefore, it is easily seen that when the altitude of a body is taken at its *upper transit*, if the latitude required is N. and the declination is S.,

(a) lat. = the complement of the sum of the altitude and declination; but if the latitude required and declination are both N.,

(b) lat. = complement of the altitude diminished by the declination.

If the observer were in the S. hemisphere, since CA would then be a N. declination and CB a S. declination,

$$(c)\ \text{lat.} = 90° - (\text{alt.} + \text{dec.}),\ \text{if lat. is S. and dec. N.}$$

$$(d)\ \text{lat.} = 90° - (\text{alt.} - \text{dec.}),\ \text{if lat. is S. and dec. S.}$$

We can bring these four cases under one rule, viz.:

If latitude and declination are of the *same name* (either N. or S.),

$$(e)\ \text{the lat.} = 90° - (\text{alt.} - \text{dec.});$$

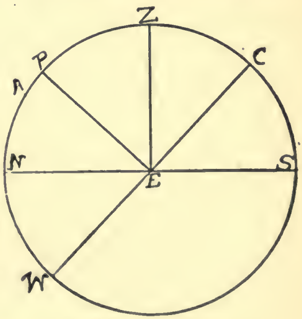
but, if of different names,

$$(f)\ \text{lat.} = 90° - (\text{alt.} + \text{dec.}).$$

Since the zenith distance of a heavenly body is the complement of its altitude,

$$(g)\ (e)\ \text{becomes lat.} = (90° - \text{alt.} + \text{dec.})$$
$$= \text{zenith dist.} + \text{dec.}$$
$$(h)\ (f)\ \text{becomes lat.} = \text{zenith dist.} - \text{dec.}$$

2. Considering now the case of the lower transit of a celestial body,



Let the figure represent, as before, the celestial meridian. Let A be the position of a heavenly body at its *lower transit*, and NA the measure of its altitude, and WA the measure of its declination.

Then lat. $= ZC = NP = NA + PA =$ alt. + (90 − dec.) or lat. = alt. + polar dist.

Ex. 1. June 10, 1895, in long. 87° 10′ W., the observed meridian altitude of the sun's lower limb was 69° 24′ (zenith N.); the index correction was + 2′ 20″; height of the eye above the sea was 20 ft. Required the latitude.

Local apparent time

June 10	0 h. 0 m.
longitude in time	5 h. 48 m. 40 s.
Gr. app. time	5 h. 48 m. 40 s.
	= 5.81 h.

sun's dec. at app. noon $23° 1' 27''$ N.

cor. $= 11''.5 \times 5.81 =$ $1'\ 6''.8+$

dec. at time of obs. $= 23° 2' 33''.8$ N.

$$\begin{array}{r} 5.81 \\ \underline{11.5} \\ 2905 \\ \underline{6391} \\ 66.815 \end{array}$$ or $1'\ 6''.8$

obs. alt.	$= 69° 24'$
in. cor.	$2' 20''+$
	$69° 26' 20''$
dip	$4' 23''-$
	$69° 21' 57''$
ref. $22''-$, par. $3''+$	$19''-$
	$69° 21' 38''$
sem. diam.	$15' 47''+$
true alt. =	$69° 37' 25''$
	$90°$
zen. dist.	$20° 22' 35''$
dec.	$23°\ 2' 34''$
latitude	$43° 25'\ 9''$ N.

In figure, p. 123, $BS = 69° 37' 25''$
$BC = 23°\ 2' 34''$
$SC = 46° 34' 51''$

$90° - SC =$ latitude $= ZC$
$= 43° 25'\ 9''$ N.

Ex. 2. In long. $85° 14'$ W., Feb. 10, 1897, the observed meridian altitude of the sun's upper limb was $36° 42'$ (zenith N.); index correction was $- 1' 40''$; height of eye above sea was 16 ft. Required latitude.

local time	0 h. 0 m.
longitude in time	5 h. 40 m. 56 s.
Gr. app. time	5 h. 40 m. 56 s.
	= 5.68 h.

sun's dec. at app. noon, $14° 9' 32''.6$ S.

cor. $= 49''.11 \times 5.68 =$ $4' 38''.9-$

dec. at time of obs. $= 14° 4' 53''.7$ S.

obs. alt.	$36° 42'$
in. cor.	$1' 40''-$
	$36° 40' 20''$
dip	$3' 55''-$
	$36° 36' 25''$
ref. $1' 18''-$, par. $7''+$	$1' 11''-$
	$36° 35' 14''$
sem. diam.	$16' 14''-$
true alt.	$36° 19'$

In figure, p. 123,

49.11	$SA = 36° 19'$		
5.68	$CA = 14° \ 4' 54''$		
39288	$SC = 50° 23' 54''$		90°
29466		true alt.	36° 19′
24555	$90° - SC = ZC = 39° 36' \ 6''$	zen. dist.	53° 41′
278.9448 = 4′ 38″.9.	latitude = 39° 36′ N.	dec.	14° 4′ 54″
			39° 36′ 6″

Ex. 3. March 22, 1898, the observed meridian altitude of Arcturus was 66° 42′ (zenith N.); index correction was 2′ 20″+; height of eye 16 ft. Declination of star was 19° 42′ 44″ N. Required latitude.

obs altitude	= 66° 42′		
index cor.	= 2′ 20″+		
	66° 44′ 20″		
dip	3′ 55″−		
	66° 40′ 25″		
ref. 25″−	25″−		
*true alt.	= 66° 40′		In figure, p. 123,
	90°		$SB = 66° 40'$
zen. dist.	= 23° 20′		$CB = 19° 42' 44''$
declination	19° 42′ 44″		$CS = 46° 57' 16''$
	43° 2′ 44″	latitude $= 90° - SC =$	43° 2′ 44″

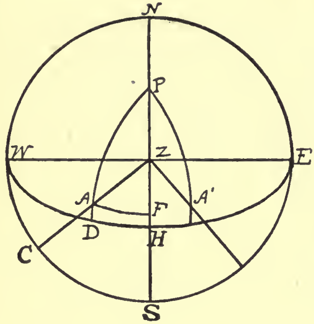
latitude = 43° 3′ N.

77. *To find the latitude* by an observation of *a heavenly body near the meridian,* the declination and the *time* of the observation *being known.*

Let $NWSE$ represent *the projection* of the celestial concave *on the plane of the* horizon; Z will be the

* For fixed star, parallax is 0.

zenith; P will be the pole; and WDE will be the equinoctial.



Suppose A to be the position of the object observed. Draw the circle of altitude ZAC, and the circle of declination PAD. From A draw the arc, AF, perpendicular to PH. NPH will represent the meridian. Denote the altitude of A, AC, by a, and the declination, AD, by d. In the figure, A is represented with N. declination. In this case, PA is $90° - d$. But if the object had a S. declination, A would be below D, and PA would be $90° + d$. $ZA = 90° - a$.

ZPA represents the time elapsed since noon. Denote this by t. If the object observed were at A', the time would be before noon, and the angle ZPA' would be $12 - t$, if the time given were civil time, or $24 - t$, if the given time were astronomical.

Let $PF = x$, and $ZF = y$; then $PZ = x - y$.

$$\text{Lat.} = ZH = PH - PZ = 90° - (x - y).$$

In right-angle triangle PAF, by Napier's rule,

$$(1) \qquad \tan PF = \frac{\cos ZPA}{\cot PA}$$

or,

$$\tan x = \frac{\cos t}{\cot (90° - d)} = \cos t \cot d.$$

$$(2)\quad \cos AF = \frac{\cos PA}{\cos PF} = \frac{\cos(90° - d)}{\cos x} = \frac{\sin d}{\cos x},$$

$$\cos ZF = \frac{\cos ZA}{\cos AF} = \frac{\cos(90° - a)\cos x}{\sin d};$$

that is, (3) $\cos y = \sin a \cos x \operatorname{cosec} d.$

By means of (1) we obtain the value of x, and by means of (3) we obtain y.

Then latitude $= 90° - (x - y)$.

As this method of obtaining latitude depends upon the time (before or after noon), an error in time introduces an error into the result, which is almost unavoidable, so that the method is not very reliable, when the object observed is far from the celestial meridian.*

Ex. 1. July 15, 1896, in long. 73° 45′ W. at 12 h. 45 m. P.M., mean time, the observed altitude of the sun's lower limb was 58° 42′ (zenith N. of sun); index correction was $+2' 20''$; height of eye was 15 ft. Required the latitude.

ship time, July 15 = 0 h. 45 m.
longitude = 4 h. 55 m.
Greenwich, July 15, M_t = 5 h. 40 m.
= 5.67 h.
equation of time = 5 m. 46.16 s.
correction (.245) × 5.67 = 1.39

5.67	5 m. 47.55 s. = equation of time
1715	45 m.
1470	39 m. 12.45 s. = time = apparent time
1225	39 m. = 9° 45′
1.38915	12.45 s. = 3′ 7″

apparent time = 38 m. 12.45 s. = 9° 48′ 7″

* Bowditch.

$$
\begin{array}{rl}
\text{observed altitude} = & 58^\circ\, 42' \\
\text{index correction} = & \underline{\quad 2'\, 20''+} \\
 & 58^\circ\, 44'\, 20'' \\
\text{dip} = & \underline{\quad 3'\, 48''-} \\
 & 58^\circ\, 40'\, 32'' \\
\left.\begin{array}{l}\text{ref. } 35''- \\ \text{par. } \ \ 4''+\end{array}\right\} = & \underline{\qquad 31''-} \\
 & 58^\circ\, 40'\, 01'' \\
\text{sem. diam.} = & \underline{\quad 15'\, 47''+} \\
\text{true altitude} = & 58^\circ\, 55'\, 48'' \\
 & \underline{90^\circ} \\
\text{zenith distance} = & 31^\circ\ \ 4'\, 12''
\end{array}
$$

$$
\begin{array}{rl}
\text{declination of sun at noon, Gr. mean time} = & 21^\circ\, 25'\, 17''.1 \text{ N.} \\
\text{correction} = 24''.33 \times 5.67 = & \underline{\qquad 2'\, 18''-} \\
\text{declination of sun at time of observation} = & 21^\circ\, 22'\, 59'' \text{ N.} \\
 & \underline{90^\circ} \\
\text{polar distance} = & 68^\circ\, 37'\ \ 1''
\end{array}
$$

In the preceding figure,

$$\angle APZ = 9^\circ\, 48'\, 7''; \quad PA = 68^\circ\, 37'\, 1''; \quad ZA = 31^\circ\, 4'\, 12''.$$

$$\tan x = \cos 9^\circ\, 48'\, 7'' \cot 21^\circ\, 22'\, 59''$$

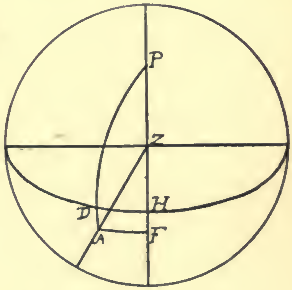
$$
\begin{array}{rll}
\log\cos 9^\circ\, 48'\, 7'' = & 9.99362 & \\
\log\cot 21^\circ\, 22'\, 59'' = & \underline{10.40721} & \\
\log\tan 68^\circ\, 19'\, 47'' = & 10.40083 & \qquad x = 68^\circ\, 19'\, 47''
\end{array}
$$

$$\cos y = \sin 58^\circ\, 55'\, 48'' \cos 68^\circ\, 19'\, 47'' \operatorname{cosec} 21^\circ\, 22''\, 59'$$

$$
\begin{array}{lrll}
\log\sin & 58^\circ\, 55'\, 48'' = & 9.93275 & \\
\log\cos & 68^\circ\, 19'\, 47'' = & 9.56734 & \\
\log\operatorname{cosec} & 21^\circ\, 22'\, 59'' = & \underline{10.43818} & \\
\log\cos & 29^\circ\, 49'\, 51'' = & 9.93827 & \qquad y = 29^\circ\, 49'\, 51''
\end{array}
$$

$$x = PF = 68° \, 19' \, 47''$$
$$y = ZF = 29° \, 49' \, 51''$$
$$x - y = PZ = 38° \, 29' \, 56''$$
$$PH = 90°$$
$$ZH = \text{lat.} = 51° \, 30' \ \ 4'' \text{ N.}$$

Ex. 2. Jan. 16, 1895, at 12 h. 42 m. 30 s. P.M., mean time, in long. 64° 20′ W., the observed altitude of the sun's lower limb was 17° 50′ 20″ (zenith N.); index correction was −2′ 10″; height of eye 12 ft. Required the latitude.



ship time = 0 h. 42 m. 30 s.
longitude = 4 h. 17 m. 20 s.
Greenwich, Jan. 16 = 4 h. 59 m. 50 s.
= 4.997 h.
= 5 h. nearly

declination of sun Jan. 16, noon = 20° 55′ 58″ S.
correction = $28''.68 \times 5 =$ 2′ 23″ −
declination of sun at time of observation = 20° 53′ 35″ S.

$$\therefore \ PA = 110° \, 53' \, 35''.$$

equation of time at noon = 9 m. 58.25 s.
correction = $0.849 \times 5 =$ 4.25 s.
equation of time for observation = 10 m. 2.5 s.
mean time = 42 m. 30 s.
apparent time = 32 m. 27.5 s.
$= 8° \, 6' \, 52\tfrac{1}{2}'' = \angle APF$

$$\begin{aligned}
\text{observed altitude} &= 17^\circ\ 50'\ 20''\\
\text{index correction} &= \underline{\quad\ \ 2'\ 10''}\\
&\ 17^\circ\ 48'\ 10''\\
\text{dip} &= \underline{\quad\ \ 3'\ 24''}\\
&\ 17^\circ\ 44'\ 46''\\
\left.\begin{array}{r}\text{ref. } 3'-\\ \text{par. } 8''+\end{array}\right\} &= \underline{\quad\ \ 2'\ 52''}\\
&\ 17^\circ\ 41'\ 54''\\
\text{sem. diam.} &= \underline{\quad\ 16'\ 18''}\\
\text{true altitude} &= 17^\circ\ 58'\ 12''\\
\therefore\ ZA &= 72^\circ\ \ 1'\ 48''
\end{aligned}$$

In triangle PAF,

$$\tan PF = \tan x = \frac{\cos 8^\circ\ 6'\ 52''\tfrac{1}{2}}{\cot 110^\circ\ 53'\ 35''}$$

$$\begin{aligned}
\log\cos 8^\circ\ 6'\ 52''\tfrac{1}{2} &= \ \ 9.99563\\
\log\cot 110^\circ\ 53'\ 35'' &= \ \ \underline{9.58175}\\
\log\tan 111^\circ\ 5'\ 10'' &= 10.41388
\end{aligned}$$

$$\cos AF = \frac{\cos 110^\circ\ 53'\ 35''}{\cos x}$$

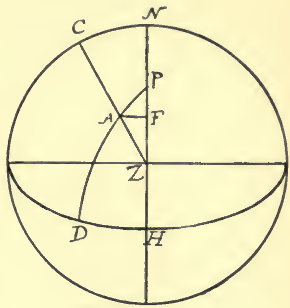
In triangle ZAF,

$$\cos ZF = \cos y = \frac{\cos ZA}{\cos AF},$$

or
$$\cos y = \frac{\cos 72^\circ\ 1'\ 48''}{\cos 110^\circ\ 53'\ 35''}\cos 111^\circ\ 5'\ 10''$$

$$\begin{aligned}
\log\cos\ \ 72^\circ\ \ 1'\ 48'' &= \ \ 9.48927\\
\log\cos 111^\circ\ \ 5'\ 10'' &= \ \ 9.55602\\
\log\sec 110^\circ\ 53'\ 35'' &= \underline{10.44778}\\
\log\cos\ \ 71^\circ\ 52'\ \ 1'' &= \ \ 9.49307
\end{aligned}$$

$$\begin{aligned}
x &= 111°\ 5'\ 10'' &&= PF\\
y &= 71°\ 52'\ 1'' &&= ZF\\
x - y &= 39°\ 13'\ 9'' &&= PZ\\
& \quad 90° &&= PH\\
\text{lat.} &= 50°\ 46'\ 51''\ \text{N.} &&= ZH
\end{aligned}$$



In case the perpendicular meets the meridian at F, a point between P and Z, as in the figure, then $PZ = x + y$ and $ZH = \text{lat.} = 90° - (x + y)$. In this case $PA = (90 - d)$.

Ex. 3. If in long. 60° 10′ W., on Jan. 3, 1895, at 5 h. 42 m. 13 s. P.M., mean time, the declination of a star was found to be 72° 12′ N., and its true altitude to be 58° 42′ 40″ (zenith N.), required the latitude.

ship time, Jan. 3 =	5 h. 42 m. 13 s.
longitude =	4 h. 0 m. 40 s.
Greenwich, Jan. 3. mean time =	9 h. 42 m. 53 s.
R.A. of mean sun 3d noon =	18 h. 51 m. 25.5 s.
Correction for 9 h. 42 m. 53 s. =	1 m. 35.7 s.
R.A. mean sun =	18 h. 53 m. 01 s.
mean time =	5 h. 42 m. 13 s.
	24 h. 35 m. 14 s.
	24 h.
sidereal time =	0 h. 35 m. 14 s. = APZ.

$$\begin{aligned}
AC &= 58°\ 42'\ 40'' & \qquad AD &= 72°\ 12'\\
& \quad 90° & PA &= 17°\ 48'\\
AZ &= 31°\ 17'\ 20''
\end{aligned}$$

$$\tan x = \frac{\cos 35\text{ m. }14\text{ s.}}{\cot 17°\ 48'} \qquad \begin{aligned} \log &= 9.99485\\ \log &= 10.49341 \end{aligned}$$

$$\log \tan 17°\ 36'\ 11'' \qquad = 9.50144$$

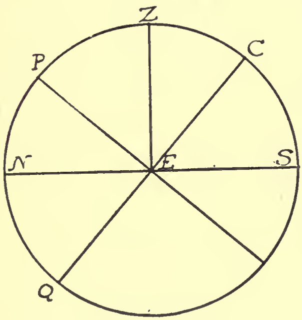
$$\cos y = \cos 31° \, 17' \, 20'' \cos 17° \, 36' \, 11'' \sec 17° \, 48'$$

$$\begin{aligned}
\log \cos 31° \, 17' \, 20'' &= 9.93174 \\
\log \cos 17° \, 36' \, 11'' &= 9.97917 \\
\log \sec 17° \, 48' &= \underline{10.02130} \\
\log \cos 31° \, 11' \, 15'' &= 9.93221
\end{aligned}$$

$$\begin{aligned}
PF &= 17° \, 36' \, 11'' = x \\
ZF &= \underline{31° \, 11' \, 15''} = y \\
PZ &= 48° \, 47' \, 26'' = x + y \\
& \underline{90° } \\
ZH = \text{lat.} &= 41° \, 12' \, 34'' \text{ N.} = 90° - (x + y).
\end{aligned}$$

78. To find the latitude by *observing* the *altitude* of the *Pole Star* (Polaris). This method is confined to northern latitudes.

Let the figure represent the projection of the celestial concave on the celestial meridian; P the N. pole; Z the zenith; QEC the projection of the equinoctial; NES the projection of the horizon.

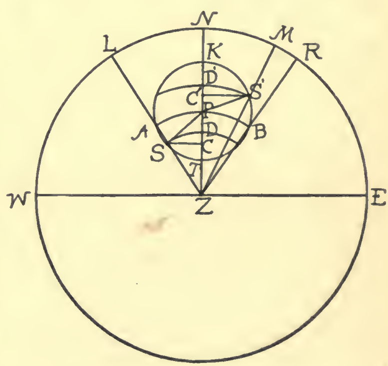


Since $PC = 90°$ and $ZN = 90°$, $PC = ZN$. If from these equals we take PZ, $PN = ZC$, but $ZC =$ the latitude of the observer; that is, PN, the altitude of the nearer pole above the horizon, is equal to the latitude (a principle already shown in Art. 75).

The star called Polaris is very near the N. pole of celestial sphere. It moves in a small circle about that pole. The polar distance of this circle is very nearly

1° 14′ (1898). By observing its altitude, at its upper and lower culminations, and subtracting or adding its exact polar distance, the latitude may be obtained. As this method is not always practicable, its altitude is observed at any moment, and to this altitude corrections are applied which are arranged in tables for the purpose of obtaining the true latitude.

Let the figure represent the projection of the celestial concave on the plane of the horizon. In order to understand the corrections required, draw $ASBS'$ to represent the circle in which Polaris moves each 24 hours (sidereal). If, with Z as a pole and a distance ZP we describe a circle, cutting $ASBS'$ in the points A and B, these points will be the points where the altitude of Polaris will be the same as the altitude of P. Since ZL, ZN, and ZR each equals 90°, and $ZA = ZP = ZB$, therefore $AL = PN = BR$. If we take any other position of the star, as S, on the arc ASB, its altitude will evidently be greater than that of the pole P, or if we take S' on the arc $AS'B$, its altitude will be *less* than that of the pole P.



If, with Z as a pole and polar distance ZS we describe a circle cutting the meridian ZN in D, the altitude of S will be the same as that of D; and if with polar distance ZS' we draw a circle cutting meridian at D', the altitude of S' will be the same as that of D'. Join PS and PS', and from S and S' draw SC and $S'C'$, perpendiculars to the meridian.

If we denote the hour angle of the star in any position by t, then at position S the angle SPC will be t, and at S' the salient angle $S'PC$ will be t. The triangles SPC and $S'PC'$ may be considered as plane triangles, since their sides are such small arcs. Consequently,

$$(1)\quad PC = PS \cos SPC \ = PS \cos t,$$

and $$(2)\quad PC' = PS' \cos S'PC' = PS \cos t.$$

Now, PS and PS' are the *polar distances* of the star, and therefore are the *complements* of *its declination*. As the declination is given in the Nautical Almanac, PS and PS' are known. Denote PS and PS' by p; then PC and PC' from equations (1) and (2) can both be expressed by one equation, viz.:

$$PC, \text{ or } PC' = p \cos t.$$

In this expression attention must be paid to the sign of $\cos t$. From 0 h. to 6 h. and from 18 h. to 24 h. the sign is $+$; between 6 h. and 18 h. the sign is $-$.

From the figure it is evident that for an observed altitude of the star in any position on the arc ATB, except at the points A, T, and B, the latitude,

$$PN = ND - DP = ND - (PC - CD)$$
$$= \text{altitude} - p \cos t + CD.$$

At A and B the latitude = altitude, since by construction ZA, ZP, and ZB are equal. At T the latitude $= NP = NT - PT =$ altitude $- p$.

For star observed in any position on arc AKB, except A, K, and B, latitude,

$$PN = ND' + D'P = ND' + (PC' + C'D'),$$

or latitude = altitude $+ p \cos t + C'D'$.

At K the latitude $= PN = NK + PK =$ altitude $+ p$.

The values of $p \cos t$ and of CD, for all positions of Polaris, are calculated and arranged in tables. When the latitude is desired within 2′ of the true latitude, the table for $p \cos t$ is used.* If, however, the correct latitude is required, the corrections for CD must also be applied.

The method of using the table for $p \cos t$, only, "is sufficiently precise for nautical purposes." †

Ex. April 1, 1898, 10 P.M. (mean time) nearly, in longitude 72° 56′ W., the altitude of Polaris was observed, and, corrected, was found to be 40° 22′. Required the latitude.

local time =	10 h. 0 m. 0 s.
longitude =	4 h. 51 m. 44 s.
Greenwich, April 1, mean time =	14 h. 51 m. 44 s.
Greenwich, April 1, R.A. mean sun =	0 h. 39 m. 27.9 s.
correction for 14 h. 51 m. 44 s. =	2 m. 26 s.
R.A. M. sun at time of observation =	0 h. 41 m. 54 s.
local mean time =	10 h.
local sidereal time =	10 h. 41 m. 54 s.
R.A. Polaris =	1 h. 21 m. 48 s. –
hour angle =	9 h. 20 m. 06 s.
for hour angle of	9 h. 20 m.
correction from page 170 is	+ 56′.9
approximate latitude =	41° 19′ N.

* Martin. † Bowditch.

CHAPTER X

LONGITUDE

79. By Art. 54 the *local time* was defined as the *hour angle* of the sun at the celestial meridian of the place; and the *Greenwich time* at the same instant was defined as the hour angle of the sun at the meridian of Greenwich, both angles being made at the pole by the hour circle passing through the sun with the respective meridians of the place and of Greenwich. The difference of these angles can be expressed either in degree measure or in time measure. Expressed in degree measure, it is called the longitude of the place. The *longitude* of *a place* can always be determined, therefore, by comparing the *local* time with the *Greenwich* time at the same instant.

All sea-going vessels are furnished with a fixed chronometer set to Greenwich time. Its *rate*, or the average amount of time which it loses or gains in a day, is ascertained, and applied to the time indicated.

The *error* of the clock is the amount of time by which it is *fast* or *slow*, as compared with true Greenwich time. Both the rate and error of the clock are kept on record, and taken into account in calculating longitude.

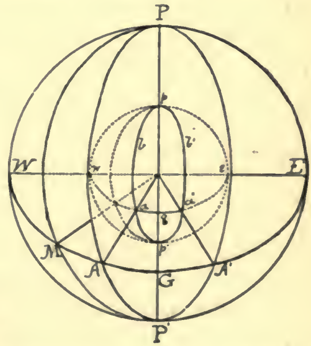
The *local*, or *ship* time, is determined by observing the altitude of some celestial body. When the object observed is not on the meridian of the observer, the latitude of the place of observation, and the declination of the object being known, the hour angle is calculated (Art. 63).

Observations for *latitude* are generally made when the object observed is on *the meridian*, or near it.

Observations for *longitude* are preferred to be taken at the time the object is near the *prime vertical.*

The latitude used in determining the hour angle for longitude is the latitude last observed, corrected for change due to the run of the ship in the interval between the two observations. This change of latitude is found by *dead reckoning.*

80. When the *Greenwich time* is *greater than* the *ship time*, the *longitude* of the ship is *West ;* when the *Greenwich time* is *less* than the *ship* time, the longitude of the ship is East.



Let the figure represent the earth, *pwp′e*, and the celestial concave, *PWP′E*, projected on the plane at right angles to the meridian of Greenwich. *pgp′* will represent the *terrestrial* meridian, and *PGP′* the *celestial* meridian of Greenwich. If *wge* represent the terrestrial equator, its plane when produced will intersect the celestial concave in the celestial equator, WGE.

If b be a place on the earth's surface west of Greenwich, the plane of its meridian $pbap'$ produced will intersect the celestial concave in the meridian PAP'. If b' be a place east of Greenwich, $pb'a'p'$ will be its terrestrial meridian, and $PA'P'$ its celestial meridian.

Now, if the meridian PMP' be the meridian passing through the mean sun at M, at the time of an observation,

GPM = Greenwich mean time, at that instant.

APM = mean time at b, at that instant.

$A'PM$ = mean time at b', at that instant.

$GPM - APM = GPA = gpb$; because GPA and gpb are two arc angles, which are each equal to the diedral angle of the same two planes. But gpb is measured by ga, and is the longitude of b west. Therefore, Greenwich mean time − local mean time = longitude west.

In the same way, $A'PM - GPM = gpb'$; but gpb' is measured by ga', and is the longitude of b' east. Therefore, local mean time − Greenwich mean time = longitude east.

Ex. 1. At 9.13 P.M. (mean time) nearly, June 24, 1898, in longitude 16° 18′ W. (by account), a ship's chronometer indicated 10 h. 11 m. 3 s. (Greenwich time). On June 14, at Greenwich mean noon, the chronometer was slow 1 m. 15.8 s., and its mean daily rate was 6.4 s., losing. Required the correct Greenwich mean time, corresponding to ship time.

ship time June 24	9 h. 13 m.
longitude	1 h. 5 m. 12 s.
Gr. June 24, M. time	10 h. 18 m. 12 s. approximately.

Interval of time between June 14 noon, and 10 h. 18 m June 24 = 10 d. 10 h. 18 m. = 10 d. 8 h. + 2 h. + 15 m.

daily rate	6.4 s.
10 d.	64.00
8 h. = $\frac{1}{3}$ d.	2.13
2 h. = $\frac{1}{12}$ d.	0.53
18 m. = $\frac{1}{80}$ d.	0.00
	66.7

accum. rate =	1 m. 6.7 s. slow. ∴ to be added.	
chronometer showed	10 h. 11 m. 3 s.	
	10 h. 12 m. 9.7 s.	
original error	1 m. 15.8 s.	
cor. Green. mean time =	10 h. 13 m. 25.5 s.	

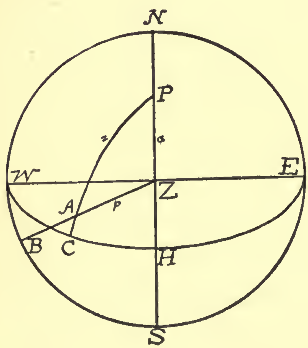
Ex. 2. April 19, 1898, 4 P.M. (mean time) nearly, in latitude 41° 19′ N., longitude (by account) 41° 18′ W., the altitude of the sun's lower limb was 29° 48′ 20″, when a chronometer showed 6 h. 49 m. 49 s. The index correction was − 2′ 30″; height of eye above sea level, 25 feet. On April 10 at noon, Greenwich mean time, the chronometer was fast 5 m. 10 s., and its daily rate was 2.5 s., gaining. Required the longitude.

ship, April 19	4 h. 0 m. 0 s.		eq. of time	0 m. 57.76 s.
longitude	2 h. 45 m. 12 s.		correction	3.69 s
Green. April 19	6 h. 45 m. 12 s.		eq. of time	1 m. 01.45 s.
	6.75 h.	0.546		*to be sub.*
		6.75		from ap. time.
		2730		
dec. of sun noon m. t.	11° 16′ 44″.2 N.	3822		
correction for 6.75 h.	5′ 49″.3+	3276		
declination of sun	11° 22′ 33″.5 N.	3.6855		

51″.75	observed altitude of sun	29° 48′ 20″
6.75	I. C.	2′ 30″−
25875		29° 45′ 50″
36225	dip	4′ 54″−
31050		29° 40′ 56″
349.3125	ref. 1′ 42″− par. 8″+	1′ 34″−
5′ 49″.3		29° 39′ 22″
	S. D.	15′ 57″+
	true altitude	29° 55′ 19″

Interval from April 10, noon, to date of observation, 9 d. 6.75 h.

daily rate	2.5 s.	
	9	
9 d.	22.5	
$\frac{1}{4}$ d.	.6	
$\frac{1}{32}$ d.	.1	
accum. gain . . .	23.2	to be subtracted.
chronometer	6 h. 49 m. 49 s.	
	6 h. 49 m. 26 s.	
original error	5 m. 10 s.	
correct Greenwich time	6 h. 44 m. 16 s.	



$PZ = 90° - 41° 19' = 48° 41'$

$PA = 90° - 11° 22' 33''.5$

$= 78° 37' 27''$

$AZ = 90° - 29° 55' 19''$

$= 60° 04' 41''$

$a = 48° 41'$

$z = 78° 37' 27''$

$p = 60° 04' 41''$

$s = \frac{187° 23' \ 8''}{2}$

$= 93° 41' 34''$

$$s - a = 45° \; 0' 34''$$
$$s - z = 15° 04' \; 7''$$
$$s - p = \underline{33° 36' 53''}$$

$$\sin \tfrac{1}{2} P = \sqrt{\sin(s-a)\sin(s-z)\operatorname{cosec} a \operatorname{cosec} z}$$

$$\begin{aligned}
\log \sin 45° 0' 34'' &= 9.84956 \\
\log \sin 15° 04' 7'' &= 9.41493 \\
\log \operatorname{cosec} 48° 41' &= 10.12432 \\
\log \operatorname{cosec} 78° 37' 27'' &= \underline{10.00862} \\
& 2)\underline{19.39743} \\
\log \sin \tfrac{1}{2} (3 \text{ h. } 59 \text{ m. } 44 \text{ s.} + 7 \text{ s.}) &= 9.69871\tfrac{1}{2} \\
& \underline{\qquad 53} \\
\text{correction } 7 \text{ s.} &= \qquad 18\tfrac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{ship apparent time} &= 3 \text{ h. } 59 \text{ m. } 51 \text{ s.} \\
\text{equation of time} &= \underline{\qquad 1 \text{ m. } 01 \text{ s.} -} \\
\text{ship mean time} &= 3 \text{ h. } 58 \text{ m. } 50 \text{ s.} \\
\text{Greenwich, mean time} &= \underline{6 \text{ h. } 44 \text{ m. } 16 \text{ s.}} \\
\text{longitude} &= 2 \text{ h. } 45 \text{ m. } 26 \text{ s.} \\
&= 41° 21' 30'' \text{ W.}
\end{aligned}$$

Ex. 3. Feb. 13, 1898, 6.30 A.M. (mean time) nearly, in lat. 45° 16′ S., and long. 28° 42′ E. (by account), a chronometer showed 4 h. 41 m. 48 s., when an observed altitude of the sun's upper limb was 14° 18′ 20″. Index correction was −1′ 13″, height of eye, 12 ft. Feb. 7, at noon (G.M.T.), the chronometer was slow 3 m. 6 s., and its daily rate was 1.4 s., losing.

ship time, Feb. 12	18 h. 30 m. 0 s.
longitude	1 h. 54 m. 48 s.
Greenwich, Feb. 12	16 h. 35 m. 12 s.
	16.59 h.
or Greenwich, Feb. 13	−7.41 h.

hourly difference of declination	50″.65
	7.41
	5065
	20260
	35455
	375″.3165, or 6′ 15″.3

declination of sun, Feb. 13, noon	13° 14′ 45″ S.
correction for − 7.41 h. =	6′ 15″
declination of sun =	13° 21′ S.
equation of time, Feb. 13, noon =	14 m. 24.2 s.
correction for − 7.41 h. =	0.6 s.
equation of time =	14 m. 24.8 s. to be *added* to ap. t.

hourly dif. of eq. of time	0.078
	7.41
	78
	312
	526
	.55798

Interval from Feb. 7 noon to time of observation 5 d. 16.59 h.

daily rate	1.4 s.
	5
5 d.	7.
½ d.	0.7
$\frac{1}{6}$ d.	0.2
accumulated loss	7.9 s.

chron. showed	4 h. 41 m. 48 s.
	4 h. 41 m. 56 s.
orig. error	3 m. 6 s.
cor. G.M.T.	4 h. 45 m. 2 s.

obs. alt. of sun	14° 18′ 20″
index cor.	1′ 13″−
	14° 17′ 07″
dip	3′ 24″
	14° 13′ 43″
ref. 3′ 46″− / par. 9″+	3′ 37″
	14° 10′ 06″
sem. diam.	16′ 14″
true alt. of sun	13° 53′ 52″

$$
\begin{aligned}
a &= 44^\circ\,44' \\
z &= 76^\circ\,39' \\
p &= \underline{76^\circ\,06'\ \ 8''} \\
s &= \frac{197^\circ\,29'\ \ 8''}{2} \\
&= 98^\circ\,44'\,34'' \\
s-a &= 54^\circ\ \ 0'\,34'' \\
s-z &= 22^\circ\,05'\,34'' \\
s-p &= 22^\circ\,38'\,26''
\end{aligned}
$$

$$\tan \tfrac{1}{2} P = \sqrt{\sin (s-a) \sin (s-z) \operatorname{cosec} s \operatorname{cosec} (s-p)}$$

	log cosec s =	10.00507
	log sin $(s-a)$ =	9.90801
	log sin $(s-z)$ =	9.57531
	log cosec $(s-p)$ =	10.41460
		2)19.90299
log tan ½	(6 h. 25 m. 34 s.)	9.95149½
equation of time	14 m. 25 s.	64
mean time of ship	6 h. 39 m. 59 s.	14½
Greenwich mean time	4 h. 45 m. 2 s.	
longitude	1 h. 54 m. 57 s. = 28° 44′ 15″ E.	

Ex. 4. Jan. 20, 1898, 8.30 A.M., (mean time) nearly, latitude 39° 58′ N., longitude, by account, 30° 15′ W., a chronometer showed 10 h. 53 m. 9 s., when an observed altitude of the sun's upper limb was 13° 2′ 30″. Index correction was − 3′ 50″, height of eye was 18 ft. Jan. 12, noon, Greenwich mean time, the chronometer was 10 m. 36 s. fast and its daily rate was 1.2 s., gaining. Required the longitude.

ship, Jan. 19	20 h. 30 m.	dec. of sun Jan. 20 noon	20° 3′ 9″.8 S.
longitude	2 h. 1 m.	correction for − 1.48	48″.6
Gr. Jan. 19	22 h. 31 m.	declination of sun	20° 3′ 58″.4 S.
	= 22.52 h. or Jan. 20 − 1.48 h.		

	32″.84		
	1.48		
interval from Jan. 12 noon	26272	eq. of time	11 m. 19.98 s.
to time of obs. 7 d. 22½ h.	13136	correction	1.07 s.
daily rate 1.2 s.	3284	eq. of time	11 m. 18.91 s.
7	48.6032	0.724	to be added to
7 d. = 8.4		1.48	apparent time.
½ d. = .6		5792	
⅓ d. = .4		2896	
$\frac{1}{12}$ d. = .1		724	
accum. gain 9.5 s.		1.07152	

accum. gain =	9.5 s.	obs. alt. of sun	13° 2′ 30″
chron. showed	10 h. 53 m. 9. s.	I.C.	3′ 50″
	10 h. 52 m. 59.5 s.		12° 58′ 40″
original error	10 m. 36 s.	dip	4′ 09″
Gr. M. time	10 h. 42 m. 23.5 s.		12° 54′ 31″
a =	50° 2′	ref. 4′ 9″−	4′ 00″−
z =	110° 3′ 58″	par. 9″+	
p =	77° 25′ 46″		12° 50′ 31″
s =	237° 31′ 44″	S.D.	16′ 17″−
	2	true alt. of sun	12° 34′ 14″−
=	118° 45′ 52″	log cosec =	10.05720
$s-a$ =	68° 43′ 52″	log sin =	9.96936
$s-z$ =	8° 41′ 54″	log sin =	9.17966
$s-p$ =	41° 20′ 06″	log cosec =	10.18016
		2)	19.38638
log tan ½ (8 h. 29 m. 52 s. + 3 s.)			9.69319
ship apparent time	8 h. 29 m. 55 s.		29
equation of time	11 m. 19 s.	3 s. cor. for	10
	8 h. 41 m. 14 s.		

Greenwich mean time =	10 h. 42 m. 23.5 s.
ship mean time	8 h. 41 m. 14 s.
longitude	2 h. 01 m. 9.5 s.
longitude	30° 17′ 22½″ W.

Ex. 5. April 9, 1898, 4 P.M. (mean time) nearly, in latitude 46° 52′ N., longitude (by account), 50° 35′ W., a chronometer showed 7 h. 28 m. 4 s., when the altitude of the sun's lower limb was 23° 58′ 40″. Index correction was + 2′ 48″; height of eye above sea level, 14 ft. April 1, noon, Greenwich mean time, the chronometer was slow 6 m. 35 s., and its daily rate was 1.2 s., losing. Required the longitude. *Ans.* 50° 39′ W.

Ex. 6. June 13, 1898, 6 P.M. (mean time) nearly, in latitude 42° 4′ N., longitude (by account), 36° 22′ W., the observed altitude of sun's lower limb was 15° 7′ 30″, when a chronometer showed 8 h. 16 m. 28 s. Index correction was − 3′ 14″; height of eye, 20 ft. June 1, noon, Greenwich mean time, chronometer was slow 8 m. 13 s., and its daily rate was 1.3 s., gaining. Required the longitude. *Ans.* 35° 57′ W.

Ex. 7. May 2, 1898, 5 P.M. (mean time) nearly, in lat. 50° 16′ N., longitude (by account) 40° 18′ W., the observed altitude of the sun's lower limb was 21° 16′ 50″, when a chronometer showed 7 h. 44 m. 2 s. Index correction was + 1′ 12″; height of eye above sea level was 15 ft. April 25, noon, G.M.T., chronometer was fast 6 m. 18 s., and daily rate was 0.6 s., losing. Required the longitude. *Ans.* 40° 16′ W.

Ex. 8. May 14, 1898, 6 A.M. (mean time) nearly, in lat. 44° 48′ N., longitude (by account) 33° 22′ W., the observed altitude of the sun's lower limb was 13° 5′ 40″, when a chronometer showed 8 h. 23 m. 28 s. Index correction was − 2′ 25″; height of eye above sea level was 18 ft. May 6, at noon, G.M.T., the chronometer was fast 12 m. 36 s., and its daily rate was 1.6 s., gaining. Required the longitude. *Ans.* 33° 24¼′ W.

Ex. 9. Feb. 28, 1898, 8 A.M. (mean time) nearly, in lat. 46° 22′ N., longitude (by account) 50° 42′ W., a chronometer showed 11 h. 30 m. 54 s., when the observed altitude of the sun's upper limb was 14° 25′ 30″. Index correction was +2′ 20″; height of eye above sea level was 20 ft. Feb. 20, noon, G.M.T., chronometer was slow 4 m. 30 s., and its daily rate was 0.8 s., gaining. Required the longitude.

Given dec. of sun, Feb. 28, Green., noon, 7° 50′ 24″ S.

Hourly dif. 56″.79 N.

Equation of time at Green., noon, 12 m. 40.7 s. to be added to mean time. Hourly dif. 0.479 s., decreasing from Feb. 28 to March 1. *Ans.* 50° 39′ 15″ W.

DEFINITIONS OF TERMS USED IN NAUTICAL ASTRONOMY

Altitude. The altitude of a heavenly body is the angle of elevation of the body above the horizon, and is measured on the circle of altitude passing through the body. This measured *distance* is generally used for the altitude.

Observed Altitude. The observed altitude of a heavenly body is the altitude of the body above the sea horizon taken with a sextant or other instrument.

True Altitude. The true altitude of a heavenly body is its observed altitude corrected for index error, dip, refraction, parallax, and semi-diameter.

First Point of Aries. The first point of Aries is the point on the celestial concave in which the ecliptic cuts the equinoctial, where the sun passes from the south to the north of the equinoctial.

Axis. The axis of the celestial sphere is the diameter about which the celestial concave appears to revolve from east to west. It is coincident with the earth's axis produced.

Azimuth. The azimuth or true bearing of a heavenly body is the angle at the zenith made by the celestial meridian and the circle of altitude passing through the body.

Celestial Concave. The celestial concave is the surface of a very large sphere of which the center is the center of the earth.

Apparent Solar Day. An apparent solar day is the interval of time between two successive transits of the sun over the same celestial meridian.

Mean Solar Day. A mean solar day is the interval of time between two successive transits of the mean sun over the same celestial meridian.

Sidereal Day. A sidereal day is the interval of time between two successive transits of the first point of Aries over the same celestial meridian.

Declination. The declination of a heavenly body is the arc of a circle of declination between the body and the equinoctial, or celestial equator.

Circles of Declination. Circles of declination are great circles of the celestial concave which pass through its poles. Circles of declination are also called hour circles.

Angle of Depression. The angle of depression of any body below the observer is the angle between a line drawn to it from the observer's eye, and the horizontal plane through the observer's eye.

Ecliptic. The ecliptic is the great circle in which the plane of the earth's orbit cuts the celestial concave.

Angle of Elevation. The angle of elevation of any body above the observer is the angle at the observer's eye, between a line drawn from it to the body and a horizontal plane through the eye.

Celestial Equator and Equinoctial. The equinoctial is the celestial equator and is the great circle of the celestial concave made by producing the plane of the terrestrial equator to cut the concave.

Greenwich Date. The Greenwich date is the astronomical time at Greenwich, when an observation is taken at any place on the earth.

Horizon. The celestial horizon or simply the horizon at any place is the great circle of the celestial concave, in which a plane tangent to the earth at that place meets the concave. This plane is known as the plane of the horizon.

Rational Horizon. The rational horizon is a plane passed through the center of the earth parallel to the sensible horizon.

Sensible Horizon. The sensible horizon is a plane tangent to the earth at a point vertically below the point of observation.

Visible Horizon. The visible horizon is the small circle which bounds the vision of the observer.

Hour Angle. The hour angle of any heavenly body is the angle at the pole between the celestial meridian of the observer and the hour circle passing through the body.

Hour Circles. Hour circles are circles of declination.

Celestial Meridian. The celestial meridian of any place is the great circle in which the plane of the terrestrial meridian cuts the celestial concave.

Apparent Noon. Apparent noon is the instant when the center of the real sun is on the celestial meridian.

Mean Noon. Mean noon is the instant when the mean sun is on the celestial meridian.

Poles of the Heavens. The poles of the heavens are the extremities of the axis of the celestial concave.

Prime Vertical. The prime vertical is the circle of altitude, whose plane is at right angles to the plane of the celestial meridian.

Right Ascension. The right ascension of a heavenly body is the arc of the equinoctial, or celestial equator, between the first point of Aries and the circle of declination passing through the body. Right ascension is *measured in time eastward* from 0 h. to 24 h.

Apparent Time. Apparent time is the hour angle of the real sun.

Mean Time. Mean time is the hour angle of the mean sun.

Equation of Time. The equation of time is the difference between apparent time and mean time.

Astronomical Time. Astronomical time is reckoned in periods of twenty-four hours, each period *beginning at noon.*

Civil Time. Civil time is reckoned in two periods of twelve hours, named A.M. and P.M. according as they come before or after noon of the day, which, in this method of reckoning time, *begins at midnight.*

Zenith. The zenith is the pole of the celestial horizon directly above the observer.

EXAMPLES

CHAPTER III

In the following examples, deviation is to be taken from table on page 56. Find the *true courses:*

Ex. 1. Compass course = N. 47° E.; variation = 9° W.; leeway = 0°. *Ans.* N. 56° E.

Ex. 2. Compass course = E. b. N. ¼ N.; variation = 21° E.; leeway = 1¼ pt. and wind N. *Ans.* S.E.

Ex. 3. Compass course = S. 51° E.; variation = 18° E.; leeway = 0. *Ans.* S. 17° E.

Ex. 4. Compass course = S. ¾ W.; variation = 21° W.; leeway = 1 pt.; wind E.S.E. *Ans.* S. ¼ E.

Ex. 5. Compass course = W. b. S. ¾ S.; variation = 11° W.; leeway = 1 pt.; wind S. *Ans.* S.W. ¾ W.

Ex. 6. Compass course = N.N.W. ¼ W.; variation = 30° W.; leeway = ¼ pt.; wind W. *Ans.* W.N.W.

Find the *compass course:*

Ex. 7. True course = N.N.E. ¼ E.; variation being 21° E. *Ans.* N. 5° E.

Ex. 8. True course = N. 62° E.; variation being 11° W. *Ans.* N. 54° E.

Ex. 9. True course = E. ¾ S.; variation being 12° W. *Ans.* East.

Ex. 10. True course = S. b. W. ¼ W.; variation being 19° E. *Ans.* S. 10° E.

Ex. 11. True course = N.W. ¼ W.; variation being 34° W. *Ans.* N. 9° W.

CHAPTERS V AND VI

Ex. 1. May 28, 1898, in long. 72° 55¾′ W., required mean time of apparent noon, and declination of sun at that time.
Ans. Mean time, 11 h. 57 m. 4.15 s. A.M.; dec. of sun, 21° 32′ 42″ N.

Ex. 2. May 28, 1898, in long. 72° 55¾′ W., given mean times 10.15 A.M. and 1.45 P.M., required corresponding sidereal times.
Ans. 14 h. 39 m. 42 s.; 6 h. 10 m. 17 s.

Ex. 3. May 27, 1898, in long. 72° 55¾′ W., given mean times 9.45 A.M. and 1.30 P.M., required corresponding sidereal times.
Ans. 2 h. 5 m. 41 s.; 5 h. 51 m. 18 s.

Ex. 4. May 27, 1898, in long. 72° 55¾′ W., required the mean time of apparent noon; also declination of sun at that time. *Ans.* 11 h. 56 m. 57 s. A.M.; 21° 23′ 2″ N.

Ex. 5. March 15, 1898, in long. 72° 55¾′ W., given apparent times, 6.30 A.M. and 5 P.M., to find corresponding mean times.
Ans. 6.39 A.M.; 5 h. 8 m. 52 s. P.M.

Ex. 6. In long. 72° 55¾′ W., March 19, 1898, 10.45 A.M. mean time, required apparent time, sidereal time, and declination of sun. *Ans.* Apparent time, 10 h. 37 m. 13 s.; sidereal time, 22 h. 33 m. 48 s.; declination of sun, 0° 22′ 10″ S.

CHAPTER VII

Ex. 1. In lat. 41° 18′ N., long. 72° 55¾′ W., May 2, 1898, 3.19 P.M. apparent time, nearly, the true altitude of the sun was 40° 14′; required its hour angle. *Ans.* 3 h. 18 m. 31 s.

Ex. 2. In lat. 41° 18′ N., long. 72° 55¾′ W., Jan. 10, 1898, 10 A.M. mean time approximately, the true altitude of sun was 20° 40′; required mean time. *Ans.* 10 h. 4 m. 53 s. A.M.

Ex. 3. In lat. 41° 18′ N., long. 72° 55¾′ W., Jan. 10, 1898, 11 A.M. mean time approximately, the true altitude of the sun was 24° 40′; required mean time. *Ans.* 10 h. 56 m. 23 s. A.M.

Ex. 4. April 1, 1898, at 7 P.M. mean time nearly, in long. 72° 55¾′ W., the hour angle of α Orionis was 1 h. 50 m. 56 s., W. of meridian. Required mean time.

Ans. 6 h. 59 m. 11 s. P.M.

Ex. 5. Nov. 22, 1898, 7.15 P.M. mean time nearly, in long. 87° 56′ W., the hour angle of Aldebaran (α Tauri) was 18 h. 55 m. 15 s. (E. of meridian). Nov. 22, noon Greenwich R.A. mean sun was 16 h. 5 m. 58.42 s. *Ans.* 7 h. 17 m. 14 s. P.M.

Ex. 6. Find at what time Procyon (α Canis Minoris) passed the meridian of 72° 56′ W., April 5, 1898.

Ans. 6 h. 36 m. 51 s. P.M.

Ex. 7. Find at what time Sirius passed the meridian 72° 55¾′ W., April 6, 1898. If the place is in lat. 41° 18′ N., required also its meridian altitude at transit.

Ans. 5 h. 39 m. 45 s. P.M.; 32° 7 27″.

Ex. 8. In lat. 41° 18′ N., long. 72° 55¾′ W., April 6, 1898, find at what time Regulus passed the meridian, and at what altitude. *Ans.* 9 h. 1 m. 29 s. P.M.; 61° 9′ 52″.

Ex. 9. In lat. 41° 18′ N., long. 72° 55¾′ W., April 5, 1898, 10 P.M. mean time nearly, the altitude of β Geminorum was 48° 17′, and its declination was 28° 16′ 19″ N. Required mean time. *Ans.* 9 h. 57 m. 33 s. P.M.

CHAPTER IX

Ex. 1. In long. 72° 55¾′ W., April 20, 1898, the observed meridian altitude of the sun's lower limb was 33° 22′ 30″ (zenith N.); index correction was −2′ 10″; height of eye above sea level was 25 ft. Required the latitude.

Ans. 68° 11′ 27″ N.

Ex. 2. April 21, 1898, in long. 72° 55¾′ W., the observed meridian altitude of the sun's lower limb was 56° 10′ 20″ (zenith N.); index correction was +2′ 25″; height of eye was 18 ft. Required the latitude. *Ans.* 45° 37′ 52″ N.

Ex. 3. Jan. 2, 1898, the observed altitude of Vega (α **Lyræ**) (zenith N.) was 70° 2′ 30″; index correction was +2′ 16″; height of eye above sea level was 14 ft. Required the latitude.
Ans. 58° 40′ 34″ N.

Ex. 4. April 20, 1898, the observed meridian altitude of Arcturus was 62° 40′ 30″; index correction was + 3′ 16″; height of eye above sea level was 20 ft. Required the latitude.
Ans. 47° 3′ 50″ N.

Ex. 5. March 14, 1898, at 2 A.M. (nearly), in long. 45° 40′ W., the observed altitude of Polaris was 43° 16′; index correction was − 2′ 22″; height of eye was 18 ft. Required the latitude.
Ans. 44° 22′ N.

Ex. 6. April 22, 1898, at 3 A.M. (nearly), in long. 50° 10′ W., the observed altitude of Polaris was 46° 38′; index correction was + 1′ 40″; height of eye was 13 ft. Required the latitude.
Ans. 47° 18′ N.

Ex. 7. In long. 16° 16′ W., June 16, 1898, 12 h. 12 m. 26 s. P.M. mean time, the observed altitude of the sun's upper limb (zenith N.) was 61° 40′ 10″; index correction was + 2′ 25″; height of eye above sea level was 17 ft. Required the latitude.
Ans. 51° 54′ 34″ N.

ASTRONOMICAL EPHEMERIS

FOR THE

MERIDIAN OF GREENWICH

JANUARY, 1898

I.

At Greenwich Apparent Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	THE SUN'S Semi-diameter	Equation of Time, to be Added to Apparent Time	Diff. for 1 hour
		° ′ ″	″	′ ″	m. s.	s.
Sat.	1	S. 22 59 1.6	+12.81	16 18.37	3 55.32	1.177
SUN.	2	22 53 40.7	13.94	16 18.37	4 23.36	1.160
Mon.	3	22 47 52.4	15.07	16 18.37	4 51.02	1.143
Tues.	4	22 41 37.1	+16.20	16 18.36	5 18.26	1.126
Wed.	5	22 34 54.8	17.32	16 18.35	5 45.08	1.108
Thur.	6	22 27 45.7	18.43	16 18.33	6 11.42	1.088
Frid.	7	22 20 10.2	+19.53	16 18.30	6 37.29	1.067
Sat.	8	22 12 8.3	20.62	16 18.26	7 2.64	1.045
SUN.	9	22 3 40.3	21.71	16 18.22	7 27.47	1.023
Mon.	10	21 54 46.4	+22.78	16 18.18	7 51.74	1.000
Tues.	11	21 45 26.9	23.84	16 18.13	8 15.46	0.975
Wed.	12	21 35 42.0	24.89	16 18.07	8 38.57	0.950
Thur.	13	21 25 32.0	+25.93	16 18.00	9 1.08	0.925
Frid.	14	21 14 57.1	26.96	16 17.93	9 22.96	0.899
Sat.	15	21 3 57.7	27.98	16 17.86	9 44.21	0.871
SUN.	16	20 52 34.1	+28.98	16 17.78	10 4.79	0.842
Mon.	17	20 40 46.5	29.97	16 17.70	10 24.69	0.814
Tues.	18	20 28 35.3	30.95	16 17.61	10 43.89	0.785
Wed.	19	20 16 0.9	+31.91	16 17.52	11 2.38	0.755
Thur.	20	20 3 3.6	32.86	16 17.42	11 20.12	0.724
Frid.	21	19 49 43.7	33.79	16 17.32	11 37.11	0.693
Sat.	22	19 36 1.6	+34.71	16 17.22	11 53.36	0.661
SUN.	23	19 21 57.8	35.61	16 17.11	12 8.82	0.628
Mon.	24	19 7 32.6	36.49	16 17.00	12 23.47	0.595
Tues.	25	18 52 46.4	+37.35	16 16.89	12 37.34	0.561
Wed.	26	18 37 39.6	38.20	16 16.77	12 50.37	0.526
Thur.	27	18 22 12.6	39.04	16 16.65	13 2.59	0.492
Frid.	28	18 6 25.8	+39.85	16 16.53	13 13.97	0.457
Sat.	29	17 50 19.7	40.65	16 16.40	13 24.53	0.422
SUN.	30	17 33 54.6	41.43	16 16.27	13 34.23	0.387
Mon.	31	17 17 10.9	42.20	16 16.13	13 43.10	0.352
Tues.	32	S. 17 0 9.0	+42.95	16 15.99	13 51.12	0.318

II.

JANUARY, 1898

At Greenwich Mean Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	Equation of Time, to be Subtracted from Mean Time	Diff. for 1 hour	Sidereal Time, or Right Ascension of Mean Sun
		° ′ ″	″	m. s.	s.	h. m. s.
Sat.	1	S. 22 59 2.4	+12.79	3 55.24	1.176	18 44 37.92
SUN.	2	22 53 41.7	13.93	4 23.27	1.160	18 48 34.48
Mon.	3	22 47 53.7	15.07	4 50.92	1.143	18 52 31.04
Tues.	4	22 41 38.5	+16.20	5 18.16	1.126	18 56 27.60
Wed.	5	22 34 56.4	17.31	5 44.97	1.108	19 0 24.15
Thur.	6	22 27 47.7	18.42	6 11.31	1.088	19 4 20.71
Frid.	7	22 20 12.4	+19.52	6 37.17	1.067	19 8 17.27
Sat.	8	22 12 10.7	20.61	7 2.52	1.045	19 12 13.83
SUN.	9	22 3 43.0	21.69	7 27.34	1.023	19 16 10.39
Mon.	10	21 54 49.4	+22.76	7 51.61	1.000	19 20 6.95
Tues.	11	21 45 30.2	23.83	8 15.32	0.975	19 24 3.50
Wed.	12	21 35 45.6	24.88	8 38.43	0.950	19 28 0.06
Thur.	13	21 25 35.9	+25.92	9 0.94	0.925	19 31 56.62
Frid.	14	21 15 1.4	26.95	9 22.82	0.899	19 35 53.18
Sat.	15	21 4 2.3	27.97	9 44.07	0.871	19 39 49.73
SUN.	16	20 52 39.0	+28.97	10 4.65	0.843	19 43 46.29
Mon.	17	20 40 51.8	29.96	10 24.55	0.814	19 47 42.85
Tues.	18	20 28 40.9	30.94	10 43.75	0.785	19 51 39.40
Wed.	19	20 16 6.8	+31.90	11 2.24	0.755	19 55 35.96
Thur.	20	20 3 9.8	32.84	11 19.98	0.724	19 59 32.52
Frid.	21	19 49 50.3	33.77	11 36.98	0.693	20 3 29.08
Sat.	22	19 36 8.6	+34.69	11 53.23	0.661	20 7 25.63
SUN.	23	19 22 5.1	35.59	12 8.69	0.628	20 11 22.19
Mon.	24	19 7 40.2	36.47	12 23.35	0.595	20 15 18.75
Tues.	25	18 52 54.3	+37.34	12 37.22	0.561	20 19 15.30
Wed.	26	18 37 47.8	38.19	12 50.26	0.526	20 23 11.86
Thur.	27	18 22 21.1	39.02	13 2.48	0.492	20 27 8.42
Frid.	28	18 6 34.7	+39.84	13 13.87	0.457	20 31 4.97
Sat.	29	17 50 28.8	40.64	13 24.43	0.422	20 35 1.53
SUN.	30	17 34 4.0	41.42	13 34.14	0.387	20 38 58.09
Mon.	31	17 17 20.6	42.19	13 43.02	0.352	20 42 54.64
Tues.	32	S. 17 0 19.0	+42.94	13 51.05	0.318	20 46 51.20

MARCH, 1898 I.

At Greenwich Apparent Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	THE SUN'S Semi-diameter	Equation of Time, to be Added to Apparent Time	Diff. for 1 hour
		° ′ ″	″	′ ″	m. s.	s.
Tues.	1	S. 7 27 25.8	+57.05	16 10.36	12 28.85	0.501
Wed.	2	7 4 33.4	57.30	16 10.12	12 16.55	0.522
Thur.	3	6 41 35.2	57.54	16 9.88	12 3.78	0.542
Frid.	4	6 18 31.4	+57.76	16 9.64	11 50.52	0.561
Sat.	5	5 55 22.6	57.97	16 9.39	11 36.80	0.580
SUN.	6	5 32 8.9	58.16	16 9.14	11 22.65	0.598
Mon.	7	5 8 50.8	+58.34	16 8.88	11 8.09	0.615
Tues.	8	4 45 28.7	58.50	16 8.62	10 53.11	0.631
Wed.	9	4 22 2.9	58.65	16 8.36	10 37.79	0.645
Thur.	10	3 58 33.7	+58.78	16 8.10	10 22.14	0.659
Frid.	11	3·35 1.5	58.90	16 7.84	10 6.14	0.672
Sat.	12	3 11 26.7	59.00	16 7.57	9 49.86	0.684
SUN.	13	2 47 49.6	+59.09	16 7.30	9 33.30	0.695
Mon.	14	2 24 10.5	59.16	16 7.03	9 16.48	0.705
Tues.	15	2 0 29.9	59.22	16 6.75	8 59.44	0.714
Wed.	16	1 36 48.2	+59.26	16 6.48	8 42.19	0.722
Thur.	17	1 13 5.6	59.28	16 6.20	8 24.76	0.730
Frid.	18	0 49 22.7	59.29	16 5.92	8 7.13	0.737
Sat.	19	0 25 39.7	+59.28	16 5.64	7 49.37	0.743
SUN.	20	S. 0 1 57.0	59.26	16 5.36	7 31.46	0.749
Mon.	21	N. 0 21 44.8	59.22	16 5.09	7 13.44	0.753
Tues.	22	0 45 25.6	+59.17	16 4.81	6 55.32	0.756
Wed.	23	1 9 4.8	59.10	16 4.54	6 37.13	0.759
Thur.	24	1 32 42.2	59.01	16 4.26	6 18.85	0.762
Frid.	25	1 56 17.2	+58.91	16 3.99	6 0.53	0.764
Sat.	26	2 19 49.6	58.79	16 3.72	5 42.19	0.765
SUN.	27	2 43 18.9	58.65	16 3.45	5 23.81	0.766
Mon.	28	3 6 44.8	+58.50	16 3.18	5 5.44	0.765
Tues.	29	3 30 7.0	58.34	16 2.91	4 47.10	0.763
Wed.	30	3 53 25.1	58.16	16 2.64	4 28.78	0.762
Thur.	31	4 16 38.8	57.97	16 2.37	4 10.54	0.760
Frid.	32	N. 4 39 47.7	+57.77	16 2.10	3 52.37	0.756

II.

MARCH, 1898

At Greenwich Mean Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	Equation of Time, to be Subtracted from Mean Time	Diff. for 1 hour	Sidereal Time, or Right Ascension of Mean Sun
		° ′ ″	″	m. s.	s.	h. m. s.
Tues.	1	S. 7 27 37.7	+57.06	12 28.95	0.501	22 37 14.73
Wed.	2	7 4 45.2	57.31	12 16.66	0.522	22 41 11.29
Thur.	3	6 41 46.8	57.55	12 3.89	0.542	22 45 7.84
Frid.	4	6 18 42.9	+57.77	11 50.63	0.561	22 49 4.39
Sat.	5	5 55 33.8	57.98	11 36.91	0.580	22 53 0.95
SUN.	6	5 32 20.0	58.17	11 22.76	0.598	22 56 57.50
Mon.	7	5 9 1.7	+58.35	11 8.20	0.615	23 0 54.05
Tues.	8	4 45 39.4	58.51	10 53.23	0.631	23 4 50.61
Wed.	9	4 22 13.3	58.66	10 37.91	0.645	23 8 47.16
Thur.	10	3 58 43.9	+58.79	10 22.25	0.659	23 12 43.71
Frid.	11	3 35 11.5	58.91	10 6.25	0.672	23 16 40.27
Sat.	12	3 11 36.4	59.01	9 49.97	0.684	23 20 36.82
SUN.	13	2 47 59.0	+59.10	9 33.41	0.695	23 24 33.37
Mon.	14	2 24 19.7	59.17	9 16.59	0.705	23 28 29.93
Tues.	15	2 0 38.9	59.23	8 59.55	0.714	23 32 26.48
Wed.	16	1 36 56.8	+59.27	8 42.30	0.722	23 36 23.03
Thur.	17	1 13 14.0	59.29	8 24.86	0.730	23 40 19.58
Frid.	18	0 49 30.8	59.30	8 7.23	0.737	23 44 16.14
Sat.	19	0 25 47.4	+59.30	7 49.47	0.743	23 48 12.69
SUN.	20	S. 0 2 4.5	59.28	7 31.56	0.749	23 52 9.24
Mon.	21	N. 0 21 37.7	59.24	7 13.53	0.753	23 56 5.80
Tues.	22	0 45 18.7	+59.18	6 55.41	0.756	0 0 2.35
Wed.	23	1 8 58.3	59.11	6 37.21	0.759	0 3 58.90
Thur.	24	1 32 35.9	59.02	6 18.93	0.762	0 7 55.46
Frid.	25	1 56 11.3	+58.92	6 0.61	0.764	0 11 52.01
Sat.	26	2 19 44.0	58.80	5 42.26	0.765	0 15 48.56
SUN.	27	2 43 13.6	58.66	5 23.88	0.766	0 19 45.12
Mon.	28	3 6 39.9	+58.51	5 5.51	0.765	0 23 41.67
Tues.	29	3 30 2.4	58.35	4 47.16	0.763	0 27 38.22
Wed.	30	3 53 20.8	58.17	4 28.84	0.762	0 31 34.78
Thur.	31	4 16 34.8	57.98	4 10.59	0.760	0 35 31.33
Frid.	32	N. 4 39 41.0	+57.78	3 52.42	0.756	0 39 27.88

APRIL, 1898 — I.

At Greenwich Apparent Noon

Day of the Week.	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	THE SUN'S Semi-diameter	Equation of Time, to be Added to / Subtracted from Apparent Time	Diff. for 1 hour
		° ′ ″	″	′ ″	m. s.	s.
Frid.	1	N. 4 39 47.7	+57.77	16 2.10	3 52.37	0.756
Sat.	2	5 2 51.5	57.55	16 1.82	3 34.29	0.751
SUN.	3	5 25 49.9	57.31	16 1.55	3 16.33	0.745
Mon.	4	5 48 42.5	+57.06	16 1.28	2 58.52	0.739
Tues.	5	6 11 29.1	56.81	16 1.01	2 40.86	0.731
Wed.	6	6 34 9.3	56.54	16 0.73	2 23.39	0.723
Thur.	7	6 56 42.8	+56.25	16 0.46	2 6.14	0.714
Frid.	8	7 19 9.3	55.95	16 0.18	1 49.09	0.705
Sat.	9	7 41 28.4	55.64	15 59.90	1 32.30	0.694
SUN.	10	8 3 39.9	+55.31	15 59.62	1 15.79	0.682
Mon.	11	8 25 43.4	54.97	15 59.34	0 59.56	0.669
Tues.	12	8 47 38.7	54.62	15 59.07	0 43.64	0.656
Wed.	13	9 9 25.2	+54.25	15 58.79	0 28.06	0.642
Thur.	14	9 31 2.8	53.87	15 58.52	0 12.81	0.628
Frid.	15	9 52 31.0	53.47	15 58.25	0 2.08	0.612
Sat.	16	10 13 49.5	+53.06	15 57.98	0 16.60	0.596
SUN.	17	10 34 58.0	52.64	15 57.71	0 30.71	0.580
Mon.	18	10 55 56.0	52.20	15 57.44	0 44.44	0.563
Tues.	19	11 16 43.4	+51.74	15 57.18	0 57.75	0.546
Wed.	20	11 37 19.6	51.27	15 56.92	1 10.64	0.528
Thur.	21	11 57 44.4	50.79	15 56.66	1 23.11	0.510
Frid.	22	12 17 57.3	+50.29	15 56.40	1 35.12	0.491
Sat.	23	12 37 58.1	49.77	15 56.15	1 46.70	0.472
SUN.	24	12 57 46.4	49.24	15 55.90	1 57.80	0.453
Mon.	25	13 17 21.8	+48.70	15 55.65	2 8.45	0.434
Tues.	26	13 36 44.1	48.15	15 55.41	2 18.63	0.414
Wed.	27	13 55 52.9	47.58	15 55.17	2 28.33	0.394
Thur.	28	14 14 47.8	+47.00	15 54.93	2 37.53	0.373
Frid.	29	14 33 28.7	46.40	15 54.70	2 46.24	0.352
Sat.	30	14 51 55.1	45.79	15 54.47	2 54.44	0.331
SUN.	31	N. 15 10 6.8	+45.18	15 54.24	3 2.13	0.310

II.

APRIL, 1898

At Greenwich Mean Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	Equation of Time to be Subtracted from / Added to Mean Time	Diff. for 1 hour	Sidereal Time, or Right Ascension of Mean Sun
		° ′ ″	″	m. s.	s.	h. m. s.
Frid.	1	N. 4 39 44.0	+57.78	3 52.42	0.756	0 39 27.88
Sat.	2	5 2 48.1	57.56	3 34.33	0.751	0 43 24.44
SUN.	3	5 25 46.8	57.33	3 16.37	0.745	0 47 20.99
Mon.	4	5 48 39.7	+57.08	2 58.56	0.739	0 51 17.54
Tues.	5	6 11 26.6	56.82	2 40.89	0.731	0 55 14.10
Wed.	6	6 34 7.0	56.55	2 23.42	0.723	0 59 10.65
Thur.	7	6 56 40.8	+56.26	2 6.16	0.714	1 3 7.20
Frid.	8	7 19 7.6	55.96	1 49.11	0.705	1 7 3.76
Sat.	9	7 41 27.0	55.65	1 32.32	0.694	1 11 0.31
SUN.	10	8 3 38.8	+55.32	1 15.81	0.682	1 14 56.86
Mon.	11	8 25 42.5	54.98	0 59.57	0.669	1 18 53.42
Tues.	12	8 47 38.0	54.63	0 43.65	0.656	1 22 49.97
Wed.	13	9 9 24.8	+54.26	0 28.07	0.642	1 26 46.52
Thur.	14	9 31 2.6	53.88	0 12.81	0.628	1 30 43.08
Frid.	15	9 52 31.0	53.48	0 2.08	0.612	1 34 39.63
Sat.	16	10 13 49.7	+53.07	0 16.60	0.596	1 38 36.19
SUN.	17	10 34 58.4	52.64	0 30.72	0.580	1 42 32.74
Mon.	18	10 55 56.7	52.20	0 44.45	0.563	1 46 29.30
Tues.	19	11 16 44.2	+51.75	0 57.76	0.546	1 50 25.85
Wed.	20	11 37 20.6	51.28	1 10.65	0.528	1 54 22.40
Thur.	21	11 57 45.5	50.79	1 23.12	0.510	1 58 18.96
Frid.	22	12 17 58.6	+50.29	1 35.13	0.491	2 2 15.51
Sat.	23	12 37 59.6	49.78	1 46.71	0.472	2 6 12.07
SUN.	24	12 57 48.0	49.25	1 57.82	0.453	2 10 8.62
Mon.	25	13 17 23.6	+48.71	2 8.47	0.434	2 14 5.18
Tues.	26	13 36 46.0	48.15	2 18.65	0.414	2 18 1.73
Wed.	27	13 55 54.9	47.58	2 28.35	0.394	2 21 58.29
Thur.	28	14 14 49.9	+47.00	2 37.55	0.373	2 25 54.84
Frid.	29	14 33 30.9	46.41	2 46.26	0.352	2 29 51.40
Sat.	30	14 51 57.4	45.80	2 54.46	0.331	2 33 47.95
SUN.	31	N. 15 10 9.1	+45.18	3 2.15	0.310	2 37 44.51

MAY, 1898 I.

At Greenwich Apparent Noon

Day of the Week	Day of the Month	The Sun's Apparent Declination	The Sun's Diff. for 1 hour	The Sun's Semi-diameter	Equation of Time, to be Subtracted from Apparent Time	Diff. for 1 hour
		° ′ ″	″	′ ″	m. s.	s.
SUN.	1	N. 15 10 6.8	+45.18	15 54.24	3 2.13	0.310
Mon.	2	15 28 3.4	44.55	15 54.01	3 9.29	0.288
Tues.	3	15 45 44.7	43.90	15 53.78	3 15.93	0.265
Wed.	4	16 3 10.4	+43.24	15 53.55	3 22.03	0.242
Thur.	5	16 20 20.2	42.57	15 53.32	3 27.56	0.219
Frid.	6	16 37 13.7	41.89	15 53.10	3 32.54	0.195
Sat.	7	16 53 50.8	+41.19	15 52.87	3 36.94	0.172
SUN.	8	17 10 11.0	40.48	15 52.65	3 40.78	0.148
Mon.	9	17 26 14.2	39.77	15 52.43	3 44.03	0.124
Tues.	10	17 42 0.0	+39.04	15 52.21	3 46.70	0.099
Wed.	11	17 57 28.2	38.30	15 51.99	3 48.78	0.075
Thur.	12	18 12 38.3	37.55	15 51.78	3 50.26	0.050
Frid.	13	18 27 30.2	+36.78	15 51.57	3 51.16	0.025
Sat.	14	18 42 3.6	36.00	15 51.37	3 51.46	0.000
SUN.	15	18 56 18.1	35.21	15 51.16	3 51.15	0.025
Mon.	16	19 10 13.4	+34.41	15 50.96	3 50.28	0.049
Tues.	17	19 23 49.4	33.59	15 50.76	3 48.82	0.073
Wed.	18	19 37 5.6	32.76	15 50.57	3 46.79	0.096
Thur.	19	19 50 1.8	+31.92	15 50.38	3 44.19	0.120
Frid.	20	20 2 37.8	31.07	15 50.20	3 41.05	0.143
Sat.	21	20 14 53.2	30.21	15 50.02	3 37.34	0.165
SUN.	22	20 26 47.9	+29.34	15 49.85	3 33.12	0.187
Mon.	23	20 38 21 5	28.46	15 49.68	3 28.38	0.208
Tues.	24	20 49 33.8	27.57	15 49.52	3 23.12	0.229
Wed.	25	21 0 24.7	+26.67	15 49.36	3 17.37	0.249
Thur.	26	21 10 53.8	25.76	15 49.20	3 11.16	0.269
Frid.	27	21 21 1.0	24.84	15 49.05	3 4.48	0.288
Sat.	28	21 30 46.0	+23.91	15 48.91	2 57.34	0.306
SUN.	29	21 40 8.6	22.98	15 48.77	2 49.77	0.324
Mon.	30	21 49 8.8	22.04	15 48.63	2 41.77	0.342
Tues.	31	21 57 46.2	21.08	15 48.49	2 33.36	0.359
Wed.	32	N. 22 6 0.7	+20.12	15 48.36	2 24.55	0.375

II.

MAY, 1898

At Greenwich Mean Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination		Equation of Time, to be Added to Mean Time	Diff. for 1 hour	Sidereal Time, or Right Ascension of Mean Sun
		Apparent Declination	Diff. for 1 hour			
		° ′ ″	″	m. s.	s.	h. m. s.
SUN.	1	N. 15 10 9.1	+45.18	3 2.15	0.310	2 37 44.51
Mon.	2	15 28 5.8	44.54	3 9.31	0.288	2 41 41.06
Tues.	3	15 45 47.1	43.89	3 15.94	0.265	2 45 37.62
Wed.	4	16 3 12.8	+43.24	3 22.04	0.242	2 49 34.18
Thur.	5	16 20 22.6	42.57	3 27.57	0.219	2 53 30.73
Frid.	6	16 37 16.2	41.89	3 32.55	0.196	2 57 27.29
Sat.	7	16 53 53.3	+41.20	3 36.95	0.172	3 1 23.84
SUN.	8	17 10 13.5	40.49	3 40.79	0.148	3 5 20.40
Mon.	9	17 26 16.7	39.77	3 44.04	0.124	3 9 16.95
Tues.	10	17 42 2.5	+39.04	3 46.71	0.099	3 13 13.51
Wed.	11	17 57 30.6	38.30	3 48.79	0.075	3 17 10.07
Thur.	12	18 12 40.8	37.54	3 50.26	0.050	3 21 6.62
Frid.	13	18 27 32.6	+36.77	3 51.16	0.025	3 25 3.18
Sat.	14	18 42 5.9	35.99	3 51.46	0.000	3 28 59.74
SUN.	15	18 56 20.4	35.20	3 51.15	0.025	3 32 56.29
Mon.	16	19 10 15.7	+34.40	3 50.28	0.049	3 36 52.85
Tues.	17	19 23 51.6	33.59	3 48.81	0.073	3 40 49.40
Wed.	18	19 37 7.7	32.76	3 46.78	0.096	3 44 45.96
Thur.	19	19 50 3.8	+31.92	3 44.18	0.120	3 48 42.52
Frid.	20	20 2 39.7	31.07	3 41.04	0.143	3 52 39.08
Sat.	21	20 14 55.1	30.21	3 37.33	0.165	3 56 35.63
SUN.	22	20 26 49.6	+29.34	3 33.11	0.187	4 0 32.19
Mon.	23	20 38 23.2	28.46	3 28.37	0.208	4 4 28.75
Tues.	24	20 49 35.4	27.56	3 23.11	0.229	4 8 25.30
Wed.	25	21 0 26.2	+26.66	3 17.36	0.249	4 12 21.86
Thur.	26	21 10 55.2	25.75	3 11.14	0.269	4 16 18.42
Frid.	27	21 21 2.3	24.83	3 4.46	0.288	4 20 14.98
Sat.	28	21 30 47.2	+23.91	2 57.32	0.306	4 24 11.53
SUN.	29	21 40 9.8	22.98	2 49.75	0.324	4 28 8.09
Mon.	30	21 49 9.8	22.03	2 41.75	0.342	4 32 4.65
Tues.	31	21 57 47.1	21.08	2 33.34	0.359	4 36 1.21
Wed.	32	N. 22 6 1.6	+20.12	2 24.53	0.375	4 39 57.76

JUNE, 1898 I.

At Greenwich Apparent Noon

Day of the Week	Day of the Month	THE SUN'S Apparent Declination	THE SUN'S Diff. for 1 hour	THE SUN'S Semi-diameter	Equation of Time, to be Subtracted from / Added to Apparent Time	Diff. for 1 hour
		° ′ ″	″	′ ″	m. s.	s.
Wed.	1	N. 22 6 0.7	+20.12	15 48.36	2 24.55	0.375
Thur.	2	22 13 52.2	19.16	15 48.23	2 15.36	0.390
Frid.	3	22 21 20.4	18.19	15 48.11	2 5.80	0.405
Sat.	4	22 28 25.3	+17.21	15 47.98	1 55.87	0.420
SUN.	5	22 35 6.6	16.23	15 47.86	1 45.59	0.435
Mon.	6	22 41 24.3	15.24	15 47.74	1 35.00	0.448
Tues.	7	22 47 18.1	+14.24	15.47.62	1 24.09	0.461
Wed.	8	22 52 48.0	13.24	15 47.51	1 12.88	0.473
Thur.	9	22 57 53.8	12.24	15 47.40	1 1.39	0.485
Frid.	10	23 2 35.3	+11.23	15 47.29	0 49.62	0.495
Sat.	11	23 6 52.6	10.21	15 47.19	0 37.63	0.504
SUN.	12	23 10 45.5	9.19	15 47.09	0 25.41	0.513
Mon.	13	23 14 13.8	+ 8.17	15 47.00	0 12.98	0.521
Tues.	14	23 17 17.6	7.14	15 46.91	0 0.38	0.528
Wed.	15	23 19 56.7	6.11	15 46.82	0 12.37	0.534
Thur.	16	23 22 11.0	+ 5.08	15 46.74	0 25.26	0.538
Frid.	17	23 24 0.6	4.05	15 46.67	0 38.25	0.542
Sat.	18	23 25 25.4	3.02	15 46.60	0 51.29	0.545
SUN.	19	23 26 25.4	+ 1.99	15 46.54	1 4.39	0.546
Mon.	20	23 27 0.6	+ 0.95	15 46.48	1 17.50	0.546
Tues.	21	23 27 10.9	− 0.09	15 46.43	1 30.61	0.545
Wed.	22	23 26 56.4	− 1.12	15 46.39	1 43.68	0.543
Thur.	23	23 26 17.1	2.16	15 46.35	1 56.68	0.540
Frid.	24	23 25 13.0	3.19	15 46.31	2 9.59	0.535
Sat.	25	23 23 44.2	− 4.22	15 46.28	2 22.37	0.530
SUN.	26	23 21 50.8	5.24	15 46.26	2 35.02	0.524
Mon.	27	23 19 32.7	6.26	15 46.24	2 47.49	0.516
Tues.	28	23 16 50.1	− 7.28	15 46.22	2 59.80	0.507
Wed.	29	23 13 43.0	8.30	15 46.21	3 11.87	0.498
Thur.	30	23 10 11.6	9.32	15 46.20	3 23.72	0.488
Frid.	31	N. 23 6 15.9	−10.32	15 46.19	3 35.32	0.478

II.

JUNE, 1898

At Greenwich Mean Noon

Day of the Week	Day of the Month	The Sun's Apparent Declination	The Sun's Diff. for 1 hour	Equation of Time, to be Added to / Subtracted from Mean Time	Diff. for 1 hour	Sidereal Time, or Right Ascension of Mean Sun
		° ′ ″	″	m. s.	s.	h. m. s.
Wed.	1	N. 22 6 1.6	+20.12	2 24.53	0.375	4 39 57.76
Thur.	2	22 13 53.0	19.16	2 15.34	0.390	4 43 54.32
Frid.	3	22 21 21.1	18.19	2 5.78	0.405	4 47 50.88
Sat.	4	22 28 25.9	+17.21	1 55.86	0.420	4 51 47.44
SUN.	5	22 35 7.1	16.22	1 45.58	0.435	4 55 43.99
Mon.	6	22 41 24.7	15.23	1 34.99	0.448	4 59 40.55
Tues.	7	22 47 18.4	+14.24	1 24.08	0.461	5 3 37.11
Wed.	8	22 52 48.2	13.24	1 12.87	0.473	5 7 33.67
Thur.	9	22 57 54.0	12.23	1 1.38	0.485	5 11 30.23
Frid.	10	23 2 35.5	+11.22	0 49.61	0.495	5 15 26.78
Sat.	11	23 6 52.7	10.21	0 37.62	0.504	5 19 23.34
SUN.	12	23 10 45.6	9.19	0 25.40	0.513	5 23 19.90
Mon.	13	23 14 13.9	+ 8.17	0 12.98	0.521	5 27 16.46
Tues.	14	23 17 17.6	7.14	0 0.38	0.528	5 31 13.02
Wed.	15	23 19 56.7	6.11	0 12.37	0.534	5 35 9.58
Thur.	16	23 22 11.0	+ 5.08	0 25.26	0.538	5 39 6.13
Frid.	17	23 24 0.6	4.05	0 38.24	0.542	5 43 2.69
Sat.	18	23 25 25.4	3.02	0 51.28	0.545	5 46 59.25
SUN.	19	23 26 25.4	+ 1.98	1 4.38	0.546	5 50 55.81
Mon.	20	23 27 0.6	+ 0.94	1 17.49	0.546	5 54 52.37
Tues.	21	23 27 10.9	− 0.09	1 30.60	0.545	5 58 48.92
Wed.	22	23 26 56.4	− 1.12	1 43.66	0.543	6 2 45.48
Thur.	23	23 26 17.2	2.15	1 56.66	0.540	6 6 42.04
Frid.	24	23 25 13.1	3.18	2 9.57	0.535	6 10 38.60
Sat.	25	23 23 44.4	− 4.21	2 22.35	0.530	6 14 35.16
SUN.	26	23 21 51.0	5.24	2 35.00	0.524	6 18 31.72
Mon.	27	23 19 33.0	6.26	2 47.47	0.516	6 22 28.28
Tues.	28	23 16 50.5	− 7.28	2 59.77	0.507	6 26 24.83
Wed.	29	23 13 43.5	8.30	3 11.84	0.498	6 30 21.39
Thur.	30	23 10 12.1	9.31	3 23.69	0.488	6 34 17.95
Frid.	31	N. 23 6 16.5	−10.32	3 35.29	0.478	6 38 14.51

FIXED STARS, 1898

Mean Places for the Beginning of 1898

Name of Star	Magnitude	Right Ascension	Annual Variation	Declination	Annual Variation
		h. m. s.	s.	° ′ ″	″
α Ursæ Min. (*Polaris*)	2	1 21 43.70	+24.832	+88 45 49.1	+18.79
α Tauri (*Aldebaran*)	1	4 30 4.02	3.438	+16 18 15.0	+ 7.48
α Aurigæ (*Capella*)	1	5 9 9.19	4.426	+45 53 38.7	+ 3.98
β Orionis (*Rigel*)	1	5 9 38.13	2.882	− 8 19 10.5	+ 4.36
α Orionis (*var.*)	1	5 49 38.96	3.247	+ 7 23 16.6	+ 0.91
α Canis Maj. (*Sirius*)	1	6 40 39.21	2.644	−16 34 34.6	− 4.74
α Canis Min. (*Procyon*)	1	7 33 57.77	3.143	+ 5 29 10.7	− 9.03
β Geminorum (*Pollux*)	1	7 39 4.53	3.678	+28 16 20.9	− 8.46
α Leonis (*Regulus*)	1	10 2 56.43	3.200	+12 27 56.5	−17.49
α Virginis (*Spica*)	1	13 19 49.10	3.155	−10 37 44.5	−18.89
α Bootis (*Arcturus*)	1	14 11 0.53	2.735	+19 42 48.1	−18.86
α Scorpii (*Antares*)	1	16 23 9.13	3.672	−26 12 20.5	− 8.26
α Lyræ (*Vega*)	1	18 33 29.12	2.031	+38 41 18.8	+ 3.19
α Aquilæ (*Altair*)	1	19 45 48.41	2.927	+ 8 35 55.7	+ 9.30

Table for Finding the Latitude by an Observed Altitude of Polaris

Reduce the observed altitude of Polaris to the true altitude.

Reduce the recorded time of observation to the local sidereal time.

If the sidereal time is
- less than 1 h. 21.8 m., subtract it from 1 h. 21.8 m.;
- between 1 h. 21.8 m. and 13 h. 21.8 m., subtract 1 h. 21.8 m. from it;
- greater than 13 h. 21.8 m., subtract it from 25 h. 21.8 m.;

and the remainder is the hour angle of Polaris.

With this hour angle, take out the correction from Table (next page), and add it to or subtract it from the true altitude, according to its sign. The result is the approximate latitude of the place.

Example. — 1898, Oct. 1, at 10 h. 40 m. 30 s. P.M., mean solar time, in longitude 29° east of Greenwich, suppose the true altitude of Polaris to be 43° 20′; required the latitude of the place.

	h. m. s.
Local astronomical mean time	10 40 30
Reduction for 10 h. 40. m. 30 s.	+ 1 45
Greenwich sidereal time for mean noon, Oct. 1 . .	12 40 58
Reduction for longitude (=1 h. 56 m. east, or minus),	− 0 19
Sum (having regard to signs) is equal to local sidereal time	23 22 54

	h. m. s.
	25 21 48
Subtract sidereal time	23 22 54
Remainder is equal to hour angle of Polaris . .	1 58 54

True altitude	+ 43 20
Correction from table (next page),	− 1 4
Approximate latitude . . .	+ 42 16

CORRECTIONS TO ALTITUDE OF POLARIS. — 1898

Hour Angle	0 h.		1 h.		2 h.		3 h.		4 h.		5 h.	
m.	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′
0	−1 13.9	0.0	−1 11.3	0.4	−1 3.8	0.8	−0 51.9	1.2	−0 36.4	1.4	−0 18.4	1.5
5	1 13.9	0.1	1 10.9	0.5	1 3.0	0.9	0 50.7	1.2	0 35.0	1.4	0 16.9	1.6
10	1 13.8	0.1	1 10.4	0.5	1 2.1	0.9	0 49.5	1.2	0 33.6	1.5	0 15.3	1.6
15	1 13.7	0.1	1 9.9	0.5	1 1.2	0.9	0 48.3	1.2	0 32.1	1.5	0 13.7	1.6
20	−1 13.6	0.1	−1 9.4	0.6	−1 0.3	1.0	−0 47.1	1.3	−0 30.6	1.5	−0 12.1	1.6
25	1 13.5	0.2	1 8.8	0.6	0 59.3	1.0	0 45.8	1.3	0 29.1	1.5	0 10.5	1.6
30	1 13.3	0.2	1 8.2	0.7	0 58.3	1.0	0 44.5	1.3	0 27.6	1.5	0 8.9	1.6
35	1 13.1	0.3	1 7.5	0.7	0 57.3	1.0	0 43.2	1.3	0 26.1	1.5	0 7.3	1.6
40	−1 12.8	0.3	−1 6.8	0.7	−0 56.3	1.1	−0 41.9	1.4	−0 24.6	1.5	−0 5.7	1.6
45	1 12.5	0.4	1 6.1	0.7	0 55.2	1.1	0 40.5	1.3	0 23.1	1.5	0 4.1	1.6
50	1 12.1	0.4	1 5.4	0.8	0 54.1	1.1	0 39.2	1.4	0 21.6	1.6	0 2.5	1.6
55	1 11.7	0.4	1 4.6	0.8	0 53.0	1.1	0 37.8	1.4	0 20.0	1.6	−0 0.9	1.7
60	−1 11.3		−1 3.8		−0 51.9		−0 36.4		−0 18.4		+0 0.8	

Hour Angle	6 h.		7 h.		8 h.		9 h.		10 h.		11 h.	
m.	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′
0	+0 0.8	1.6	+0 19.8	1.6	+0 37.6	1.4	+0 52.6	1.1	+1 4.2	0.8	+1 11.4	0.4
5	0 2.4	1.6	0 21.4	1.5	0 39.0	1.3	0 53.7	1.1	1 5.0	0.7	1 11.8	0.4
10	0 4.0	1.6	0 22.9	1.6	0 40.3	1.3	0 54.8	1.1	1 5.7	0.7	1 12.2	0.3
15	0 5.6	1.6	0 24.5	1.5	0 41.6	1.3	0 55.9	1.0	1 6.4	0.7	1 12.5	0.3
20	+0 7.2	1.6	+0 26.0	1.5	+0 42.9	1.3	+0 56.9	1.0	+1 7.1	0.7	+1 12.8	0.3
25	0 8.8	1.6	0 27.5	1.5	0 44.2	1.3	0 57.9	1.0	1 7.8	0.6	1 13.1	0.2
30	0 10.4	1.6	0 29.0	1.5	0 45.5	1.3	0 58.9	1.0	1 8.4	0.5	1 13.3	0.2
35	0 12.0	1.6	0 30.5	1.4	0 46.8	1.2	0 59.9	0.9	1 8.9	0.5	1 13.5	0.1
40	+0 13.6	1.6	+0 31.9	1.4	+0 48.0	1.2	+1 0.8	0.9	+1 9.4	0.5	+1 13.6	0.1
45	0 15.2	1.6	0 33.3	1.5	0 49.2	1.2	1 1.7	0.9	1 9.9	0.5	1 13.7	0.1
50	0 16.8	1.5	0 34.8	1.4	0 50.4	1.1	1 2.6	0.8	1 10.4	0.5	1 13.8	0.1
55	0 18.3	1.5	0 36.2	1.4	0 51.5	1.1	1 3.4	0.8	1 10.9	0.5	1 13.9	0.0
60	+0 19.8		+0 37.6		+0 52.6		+1 4.2		+1 11.4		+1 13.9	

CORRECTIONS

TO

MIDDLE LATITUDE

Mid. Lat.	DIFFERENCE OF LATITUDE																			Mid. Lat.
	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	
°	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	°
15	−79	−76	−79	−82	−79	−75	−71	−68	−64	−58	−53	−49	−44	−39	−32	−26	−20	−12	− 4	15
16	−74	−77	−74	−72	−70	−69	−66	−61	−57	−54	−50	−44	−39	−35	−29	−23	−16	− 8	− 2	16
17	−73	−70	−68	−67	−63	−61	−59	−57	−54	−50	−44	−41	−36	−31	−25	−19	−13	− 6	0	17
18	−67	−67	−63	−60	−59	−55	−54	−51	−48	−46	−42	−37	−32	−27	−22	−16	−10	− 4	+ 2	18
19	−58	−58	−58	−58	−55	−53	−50	−49	−45	−41	−37	−34	−30	−24	−19	−14	− 8	− 1	+ 4	19
20	−53	−53	−53	−53	−51	−51	−47	−44	−41	−38	−34	−29	−26	−21	−17	−11	− 4	0	+ 6	20
21	−53	−53	−49	−50	−48	−46	−43	−41	−38	−34	−31	−27	−23	−18	−14	−10	− 4	+ 2	+ 8	21
22	−49	−51	−49	−47	−44	−42	−40	−37	−35	−32	−28	−24	−21	−17	−12	− 6	− 1	+ 3	+ 9	22
23	−48	−48	−45	−43	−42	−39	−37	−34	−32	−29	−26	−23	−18	−14	− 9	− 5	0	+ 6	+11	23
24	−45	−41	−42	−40	−39	−37	−35	−32	−30	−27	−23	−19	−16	−12	− 8	− 3	+ 2	+ 7	+13	24
25	−40	−38	−40	−37	−36	−34	−33	−30	−27	−24	−21	−18	−14	−10	− 6	− 2	+ 3	+ 9	+14	25
26	−38	−35	−35	−34	−34	−32	−30	−27	−25	−22	−19	−16	−12	− 8	− 4	+ 1	+ 4	+10	+16	26
27	−35	−34	−35	−33	−31	−29	−28	−25	−23	−21	−17	−14	− 9	− 6	− 2	+ 2	+ 7	+12	+17	27
28	−34	−34	−32	−30	−29	−27	−26	−23	−22	−18	−15	−12	− 9	− 5	0	+ 4	+ 9	+13	+19	28
29	−32	−31	−30	−29	−28	−26	−24	−21	−19	−16	−14	−10	− 7	− 3	+ 1	+ 5	+10	+15	+20	29
30	−29	−29	−29	−28	−26	−24	−22	−20	−18	−14	−11	− 8	− 5	− 1	+ 3	+ 7	+12	+16	+21	30
31	−29	−27	−27	−26	−25	−22	−20	−18	−15	−13	−10	− 7	− 3	+ 1	+ 4	+ 9	+13	+18	+23	31
32	−28	−28	−26	−23	−22	−21	−19	−17	−13	−12	− 9	− 5	− 2	+ 2	+ 6	+10	+14	+19	+24	32
33	−26	−23	−24	−23	−21	−19	−17	−15	−13	−10	− 7	− 4	0	+ 3	+ 7	+12	+16	+21	+25	33
34	−23	−22	−21	−21	−19	−18	−16	−13	−11	− 8	− 6	− 2	+ 1	+ 5	+ 8	+13	+17	+22	+27	34
35	−20	−21	−20	−19	−18	−16	−14	−12	−10	− 7	− 4	− 1	+ 2	+ 6	+10	+14	+19	+23	+28	35
36	−20	−21	−19	−17	−16	−14	−13	−11	− 9	− 6	− 3	0	+ 4	+ 7	+11	+15	+20	+25	+31	36
37	−20	−18	−17	−16	−15	−13	−12	− 9	− 7	− 6	− 2	+ 2	+ 5	+ 9	+13	+17	+21	+26	+31	37
38	−17	−16	−17	−15	−14	−12	−10	− 8	− 6	− 3	0	+ 3	+ 6	+10	+14	+18	+22	+27	+32	38
39	−17	−15	−16	−14	−13	−11	− 9	− 7	− 4	− 2	+ 1	+ 4	+ 8	+11	+14	+19	+24	+29	+33	39
40	−17	−17	−14	−13	−12	−10	− 8	− 6	− 3	− 1	+ 2	+ 6	+ 9	+13	+17	+21	+25	+30	+35	40

Mid. Lat.	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	Mid. Lat.
41	−15	−15	−14	−12	−10	− 9	− 7	− 4	− 3	0	+ 3	+ 6	+10	+14	+18	+22	+26	+31	+ 36	41
42	−14	−13	−11	−11	−10	− 8	− 6	− 3	− 1	+ 2	+ 4	+ 8	+12	+15	+19	+23	+28	+33	+ 38	42
43	−11	−12	−11	−10	− 9	− 7	− 5	− 3	0	+ 3	+ 5	+ 9	+12	+16	+20	+24	+29	+34	+ 39	43
44	−11	−10	−10	− 9	− 8	− 6	− 4	− 2	0	+ 3	+ 7	+10	+14	+17	+21	+26	+30	+35	+ 41	44
45	−12	−10	− 9	− 8	− 6	− 5	− 3	− 1	+ 2	+ 5	+ 8	+11	+15	+19	+25	+27	+32	+37	+ 42	45
46	−11	− 9	− 9	− 7	− 6	− 4	− 2	0	+ 3	+ 6	+ 9	+12	+15	+19	+24	+28	+33	+38	+ 43	46
47	− 8	− 9	− 8	− 6	− 5	− 3	− 1	+ 1	+ 4	+ 6	+10	+13	+17	+21	+25	+29	+34	+39	+ 45	47
48	− 9	− 8	− 7	− 6	− 4	− 2	− 1	+ 2	+ 5	+ 7	+11	+14	+18	+22	+26	+29	+36	+41	+ 47	48
49	− 8	− 7	− 6	− 5	− 3	− 2	0	+ 3	+ 5	+ 8	+11	+15	+17	+23	+27	+32	+37	+42	+ 48	49
50	− 6	− 6	− 6	− 4	− 3	− 1	+ 1	+ 3	+ 6	+ 9	+12	+16	+20	+24	+29	+33	+38	+44	+ 49	50
51	− 7	− 6	− 5	− 4	− 2	0	+ 2	+ 4	+ 7	+10	+14	+17	+21	+25	+30	+35	+40	+45	+ 51	51
52	− 7	− 6	− 4	− 3	− 1	0	+ 3	+ 5	+ 8	+11	+15	+18	+22	+26	+31	+36	+41	+47	+ 53	52
53	− 5	− 5	− 5	− 3	− 1	+ 1	+ 4	+ 6	+ 9	+12	+15	+19	+23	+28	+32	+37	+43	+48	+ 55	53
54	− 6	− 4	− 3	− 2	0	+ 2	+ 4	+ 7	+ 9	+13	+16	+20	+24	+29	+34	+39	+44	+50	+ 56	54
55	− 5	− 4	− 2	− 2	0	+ 2	+ 4	+ 7	+10	+14	+17	+21	+25	+30	+35	+40	+46	+52	+ 58	55
56	− 3	− 4	− 3	− 1	+ 1	+ 3	+ 5	+ 8	+11	+15	+18	+22	+27	+31	+37	+42	+48	+54	+ 60	56
57	− 4	− 3	− 2	0	+ 1	+ 4	+ 6	+ 9	+12	+15	+19	+23	+28	+33	+38	+44	+49	+55	+ 62	57
58	− 4	− 2	− 1	0	+ 2	+ 4	+ 7	+10	+13	+16	+20	+25	+29	+34	+39	+45	+51	+58	+ 65	58
59	− 3	− 2	− 1	+ 1	+ 2	+ 5	+ 8	+10	+14	+17	+21	+26	+30	+36	+41	+47	+53	+60	+ 67	59
60	− 2	− 2	0	+ 1	+ 3	+ 5	+ 8	+11	+15	+18	+22	+27	+32	+37	+43	+49	+55	+62	+ 69	60
61	− 2	− 1	0	+ 2	+ 4	+ 6	+ 9	+12	+15	+20	+24	+28	+33	+39	+44	+52	+57	+64	+ 72	61
62	− 1	− 1	0	+ 2	+ 4	+ 7	+ 9	+13	+16	+20	+25	+29	+35	+40	+46	+53	+60	+67	+ 75	62
63	− 1	− 1	+ 1	+ 3	+ 4	+ 7	+10	+14	+17	+21	+26	+31	+36	+42	+48	+55	+62	+70	+ 78	63
64	− 1	0	+ 1	+ 3	+ 5	+ 8	+11	+14	+18	+22	+27	+32	+38	+44	+50	+57	+65	+73	+ 81	64
65	− 1	0	+ 1	+ 3	+ 6	+ 8	+12	+15	+19	+24	+29	+34	+40	+46	+53	+60	+68	+76	+ 85	65
66	− 1	+ 1	+ 2	+ 4	+ 6	+ 9	+12	+16	+20	+25	+30	+35	+42	+48	+55	+63	+71	+80	+ 89	66
67	− 1	0	+ 2	+ 4	+ 7	+10	+13	+17	+21	+26	+31	+37	+44	+50	+58	+66	+74	+83	+ 93	67
68	0	+ 1	+ 2	+ 4	+ 7	+10	+14	+18	+22	+28	+33	+39	+46	+53	+61	+69	+78	+88	+ 98	68
69	0	+ 1	+ 3	+ 5	+ 8	+11	+15	+19	+24	+29	+35	+41	+48	+56	+64	+73	+82	+92	+103	69
70	0	+ 1	+ 3	+ 5	+ 8	+12	+16	+20	+25	+30	+37	+43	+51	+59	+67	+77	+87	+97	+109	70