AN ELEMENTARY TREATISE

ON

SURVEYING AND NAVIGATION

BY

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PREFACE.

This brief work on Surveying and Navigation is intended for those students who desire to supplement the study of Trigonometry with a brief course on its applications to those subjects.

No attempt has been made to treat the subjects fully. Special effort has, however, been made to have the work correct and accurate as far as it goes, and it is believed that the student who afterwards becomes a surveyor or navigator will have nothing to unlearn that he has learned from this work.

The author wishes to acknowledge his obligations to Messrs. W. & L. E. Gurley, Troy, N.Y., for the use of plates from which were made the cuts of the instruments found throughout the work.

ARTHUR G. ROBBINS.

Massachusetts Institute of Technology,
September, 1896.

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LAND SURVEYING.

Surveying is the art of measuring and locating lines and angles on the earth's surface.

Representation of these measurements on paper is called *plotting*.

By applying the rules of geometry and trigonometry to these measurements, distances and quantities may be computed.

In plane surveying all distances measured are horizontal distances.

The instruments for measuring distances are as follows:

Gunter's or surveyor's chain, made of iron or steel wire, contains

$$100 \text{ links} = 4 \text{ rods} = 66 \text{ feet.}$$
$$1 \text{ link} = 7\tfrac{92}{100} \text{ inches.}$$
$$10 \text{ sq. chains} = 1 \text{ Acre.}$$

Engineer's chain, 100 ft. long, contains 100 links, each one foot long.

The steel tape, which is made of a continuous ribbon of steel, generally in lengths of 50 or 100 ft., is graduated to feet, tenths and hundredths of a foot.

The metallic tape, generally 50 ft. in length, is made of cloth, into which is woven a number of fine wires to prevent stretching. It is graduated to feet, tenths and half-tenths.

Wooden rods, for measuring short distances, are graduated to read by a vernier to one-thousandth of a foot.

The inch is not used in surveying field work.

To measure a Straight Line with the Chain.

The work is done by two persons using a chain and a set of eleven marking-pins. Sometimes two lining-poles are used to mark the ends of the line.

The fore chainman takes ten of the eleven pins, and sets a pin at the end of each chain length, after the rear chainman has lined it in with the point to which the distance is to be measured. The rear chainman lines in the pin to be set by the fore chainman, holds the rear end of the chain against the last pin set by the fore chainman until the next one is set, and takes up each pin after the next forward one is set.

As soon as the fore chainman sets his last pin, he calls up and receives from the rear chainman his ten pins, taking care to count them to see that none are lost, records one tally, *i.e.* ten chains, and the work is continued till the end of the line is reached. The total distance is found by counting ten chains for each tally, one chain for each pin in the hand of the rear chainman, and the fractional part of a chain between the last pin set and the end of the line. The last pin should remain in the ground and not be counted as in the hand of the rear chainman.

When a line is being measured down hill, the forward end of the chain should be held at the same level as the rear end, and the point transferred to the ground by a plumb line, or, less accurately, by a lining-pole suspended between the thumb and finger. In all cases the chain must be held horizontal, whether the ground is level or not. The tendency, when learning to chain on a steep hill, is to hold the chain inclined at a less angle with the ground than the one between the horizontal and the surface of the ground.

Because of the large number of wearing surfaces, usually 600 in the surveyor's and engineer's chain, the length of every chain should be frequently compared with a standard.

A new steel tape may be used as a standard when there is none other convenient. The comparison should be made on a smooth and level surface, such as a level sidewalk or long hallway. Provision is made for shortening the chain by turning a nut at the end of either one of the end links.

If a line has been measured with a chain afterwards found to be too long or too short, the measured length should be corrected. In making this correction it should be remembered that the *true* length of a line, measured with a chain that is too long, is *greater* than the measured length, and *vice versa.*

The degree of precision obtained ought to depend upon the character of the work, and not upon the character of the ground. With ordinary skill in chaining, the error in measuring a line along the highways of a town or village should not exceed one part in four or five thousand. If no more care be used in measuring a line through the tangled undergrowth of some forest lands, an error of one in three or four hundred might be expected.

By applying the rules of geometry and trigonometry, it is practicable to measure angles, considerable areas, and distances to inaccessible objects by means of the chain.

Given the line ab, *to locate on the ground the line* be *perpendicular to* ab *at* b.

Select any convenient point as *c*, one chain length from *b*. With one end of the chain held at *c*, swing the other end to *d*, in the line *ab*, using

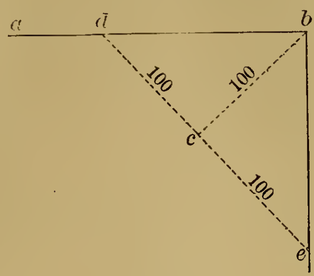

FIG. 1.

a lining-pole at *d* to aid in lining in from *a* if necessary. With one end of the chain still held at *c*, swing the other end to locate *e* in the line *dc* produced. *be* is the perpendicular required.

Another Method.

Measure $cb = 40$ ft. Hold one end of the chain at *c*, and the 80-ft. mark at *b*. Let a third person take the chain at the 50-ft. mark and pull both parts *cd* and *bd* taut. *bd* is the line required.

Other methods equally feasible will suggest themselves to the student.

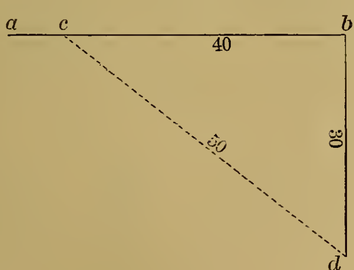

FIG. 2.

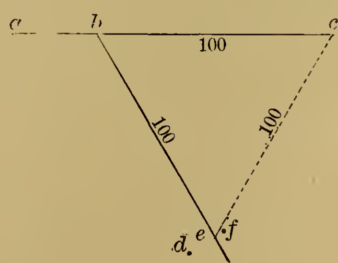

FIG. 3.

Given a line ab *on the ground, to locate a line starting at* b *and making an angle of* 60° *with this line.*

Prolong *ab* (Fig. 3) any convenient distance, as 100 ft., to *c*. Hold one end of the chain at *b*, and with the same length *bc* locate a number of pins at *d*, *f*, etc., a foot or two apart and at a distance from *c* as nearly equal to *bc* as can be estimated. Measure from *c* the distance $ce = cb$, lining in the point *e* between the two adjacent pins (*d* and *f* in this case). *be* is the line required.

To lay out Any Given Angle, as 22° 30′, with the Chain.

By trigonometry, twice the sine of one-half of an angle is equal to its chord, the radius being unity.

Given the line **ab** *on the ground to locate a line* **cd**, *making the angle* **bcd** = 22° 30′.

From *c* measure *ce* any convenient distance, as 100 ft. Twice the sine of one-half of 22° 30′ = .3902. Multiply this by distance *ce* (100 ft.,

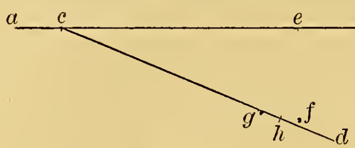

Fig. 4.

in this case). From *e*, with radius 39.02 ft., locate pins on arc *fg* one or two feet apart. From *c*, with radius 100 ft., locate the point *h* on the arc *fg*. *cd* passing through *h* is the line required.

By the reverse of the above process, the angle between any two lines located on the ground may be measured.

Figures 5, 6, 7, and 8 illustrate how lines may be prolonged through obstacles, and distances to inaccessible objects measured by application of these problems.

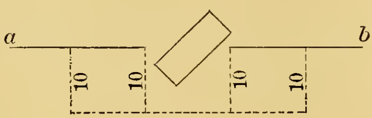

Fig. 5.

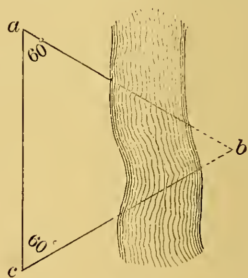

Fig. 6.

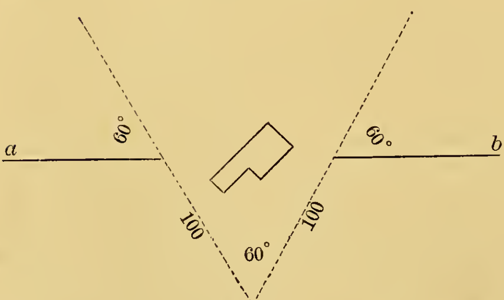

Fig. 7.

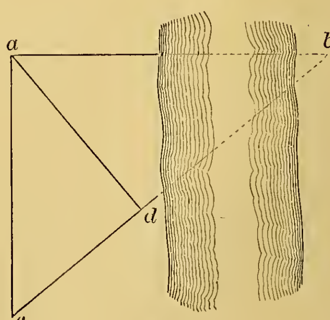

Fig. 8.
ab : *ac* : : *ad* : *cd*

Areas of fields may be determined by measuring the length of lines dividing the field into triangles and computing the area of each triangle separately. In dividing the field into triangles, much more accurate results will be obtained if the lines to be measured are so chosen as to divide the field into as few and as nearly equilateral triangles as possible.

The area of a narrow, irregular area such as that shown at *abc* (Fig. 9) can best be determined by measuring at regular intervals the lengths of

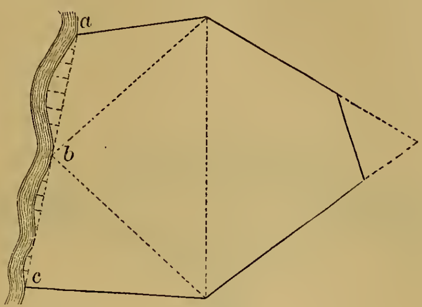

FIG. 9.

right-angle offsets to the lines *ab* and *bc* and then computing the area by the following rule, known as the trapezoidal rule,

$$A = \frac{d}{2}(h + 2\,\Sigma h_i + h'),$$

in which A is the area, d the common distance between the offsets, h and h' the two end offsets, and $2\,\Sigma h_i$ twice the sum of all the offsets between the two end offsets.

This rule assumes that the bounding line between any two adjacent offsets is a straight line.

THE COMPASS.

The compass, shown in Fig. 10, is an instrument for determining the angle that any line makes with the magnetic meridian.

It consists essentially of a circle, graduated to half-degrees, and numbered from 0° to 90° each way from the north and south points.

The sights are in the prolongation of the line N. S. Balanced on a steel pivot at the centre of the circle, is a magnetic needle, which points in the direction of the magnetic meridian. The compass is connected with the tripod by a ball and socket joint, which allows the compass to be turned in any direction for the purpose of levelling.

By setting the compass anywhere in a line and levelling it, by aid of the spirit bubbles shown on the compass box, then turning the sights till they point in the direction of the line, the angle which this line makes with the magnetic meridian may be read, by observing the number of degrees between the N. or S. point of the compass box and the N. or S. end of the needle. This angle is called the *bearing* of the line. Bearings

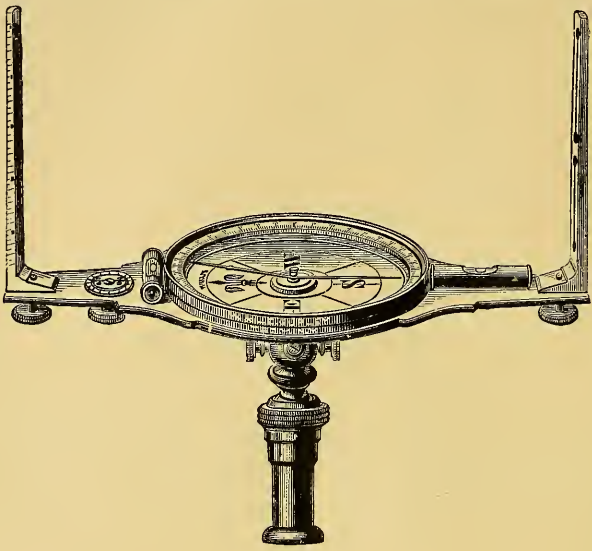

Fig. 10.

are read from the N. or S. point of the compass box, 90° in each direction. The letters which mark the E. and W. points on the compass box are reversed to aid in reading bearings, as is shown in Figs. 11, 12, 13, and 14.

The bearing of the line *ab*, Fig. 11, is N. 20° W.
The bearing of the line *ab*, Fig. 12, is N. 20° E.
The bearing of the line *ab*, Fig. 13, is S. 20° E.
The bearing of the line *ab*, Fig. 14, is S. 20° W.

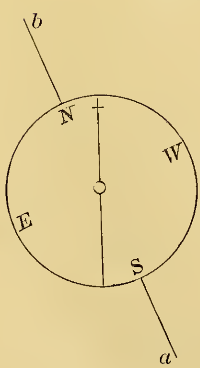

Fig. 11.

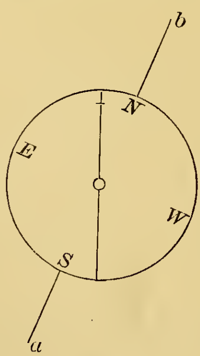

Fig. 12.

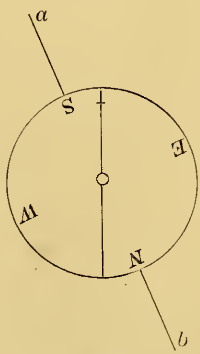

Fig. 13.

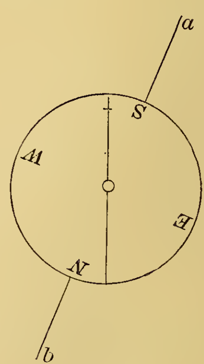

Fig. 14.

If the observer directs the N. point of the compass box in the direction of the line whose bearing is to be taken, and reads the N. end of the needle, the letters on the compass box between which that end of the needle lies are the ones to be used in recording the bearing. A careful observance of this rule will prevent the student from reading bearings incorrectly when learning to use the compass.

The magnetic needle does not point true north. Moreover, it does not always point in the same direction.

The angle which the magnetic needle makes with the true meridian is called the *declination* of the needle. Changes in the declination are called *variations* of the needle. The principal variations are, secular variation, daily variation, irregular variations.

The secular variation is a slow and continuous change in the pointing of the needle, which may be compared to the movement of a pendulum requiring centuries to make a single oscillation.

The daily variation is small, seldom exceeding six or eight minutes, and in ordinary surveying is neglected.

Irregular variations may occur at any time. They follow no known law.

In order to learn what the declination of the needle is at any time, the compass should be frequently set in a true meridian line[1] and the declination measured.

The United States Coast and Geodetic Survey publishes from time to time, charts of the United States, on which are drawn lines connecting places at which the declination at any given time is the same. These lines are called *isogonic lines.* The line connecting places where there is no declination is called the *agonic line.* In 1890 this line passed a little to the east of Charleston, S.C., through North Carolina, Virginia, Ohio, and Michigan. At all places to the east of this line, the declination is *west*, and at all places to the west of this line the declination is *east.*

At present the *west* declinations are increasing about two to four minutes a year, while the *east* declinations are decreasing; or, in other words, the agonic line is moving west.

From observations extending over a number of years it is possible to devise formulas by which the declination of the needle may be computed, with a considerable degree of precision, for some years in the future.

The following are a number of such formulas taken from tables in the United States Coast and Geodetic Survey Report for 1886:

$D =$ the declination, $+$ when east, and $-$ when west. $m =$ time $- 1850$, expressed in years and fraction of a year.

[1] A method of establishing a true meridian line is explained in the chapter on Practical Astronomy.

Name of Station and State.	Latitude.	West Longitude.	The Magnetic Declination expressed as a Function of Time.
	° ′	° ′	° ° °
Montreal, Can.	45 30.5	73 34.6	$D = +11.88 + 4.17 \sin(1.50m - 18.5)$
Eastport, Me.	44 54.4	66 59.2	$D = +15.14 + 3.90 \sin(1.20m + 31.7)$
Portland, Me.	43 38.8	70 16.6	$D = +11.26 + 3.16 \sin(1.33m + 5.8)$
Burlington, Vt.	44 28.5	73 12.0	$D = +10.81 + 3.65 \sin(1.30m - 20.5) + 0.18 \sin(7.00m + 132.0)$
Hanover, N.H.	43 42.3	72 17.1	$D = +9.80 + 4.02 \sin(1.40m - 14.1)$
Rutland, Vt.	43 36.5	72 55.5	$D = +10.03 + 3.82 \sin(1.50m - 24.3)$
Portsmouth, N.H. . . .	43 04.3	70 42.5	$D = +10.71 + 3.36 \sin(1.44m - 7.4)$
Boston, Mass.	42 21.5	71 03.9	$D = +9.48 + 2.94 \sin(1.30m + 3.7)$
Hartford, Conn.	41 45.9	72 40.4	$D = +8.06 + 2.90 \sin(1.25m - 26.4)$
Albany, N.Y.	42 39.2	73 45.8	$D = +8.17 + 3.02 \sin(1.44m - 8.3)$
Harrisburg, Pa.	40 15.9	76 52.9	$D = +2.93 + 2.98 \sin(1.50m + 0.2)$
Baltimore, Md.	39 17.8	76 37.0	$D = +3.20 + 2.57 \sin(1.45m - 21.2)$
Charleston, S.C.	32 46.6	79 55.8	$D = -2.14 + 2.77 \sin(1.40m - 3.1)$
Key West, Fla.	24 33.5	81 48.5	$D = -3.70 + 3.16 \sin(1.35m - 35.1)$
New Orleans, La. . . .	29 57.2	90 03.9	$D = -5.61 + 2.57 \sin(1.40m - 61.9)$
Cincinnati, O.	39 08.6	84 25.3	$D = -2.40 + 2.62 \sin(1.42m - 39.8)$
Pittsburgh, Pa.	40 27.6	80 00.8	$D = +1.85 + 2.45 \sin(1.45m - 28.4)$
Cleveland, O.	41 30.3	81 42.0	$D = +0.10 + 2.07 \sin(1.40m - 6.2)$
Detroit, Mich.	42 20.0	83 03.0	$D = -0.97 + 2.21 \sin(1.50m - 15.3)$
Buffalo, N.Y.	42 52.8	78 53.5	$D = +3.66 + 3.47 \sin(1.40m - 27.8)$
Sitka, Alaska	57 02.9	135 19.7	$D = -25.79 + 3.30 \sin(1.30m - 104.2)$
Port Townsend, Wash. T.,	48 07.0	122 44.9	$D = -18.84 + 3.00 \sin(1.45m - 122.1)$
San Francisco, Cal. . .	37 47.5	122 27.3	$D = -13.94 + 2.65 \sin(1.05m - 135.5)$
Monterey, Cal.	36 36.1	121 53.6	$D = -13.79 + 2.65 \sin(1.10m - 156.4)$
San Diego, Cal.	32 42.1	117 14.3	$D = -11.78 + 1.90 \sin(1.15m - 151.6)$

It is in relocating the lines of an old survey when the location of some of the corners has been lost that a knowledge of the changes in the declination of the needle is essential to the surveyor. For example, the declination of the needle at New Haven, Conn., was

5° 45′ W. in 1761,
5° 15′ W. in 1780,
4° 30′ W. in 1819,
5° 30′ W. in 1828,
6° 00′ W. in 1838.

Therefore, a line that had a bearing of N. 10° W. in 1761, had a bearing of

N. 10° 30′ W. in 1780,
N. 11° 15′ W. in 1819,
N. 10° 15′ W. in 1828,
N. 9° 45′ W. in 1838.

At a place in Illinois the declination of the needle in 1821 was 8° 00′ E., and in 1843 it was 7° 15′ E.; therefore a line that in 1821 had a bearing of S. 20° W., had, in 1843, a bearing of S. 20° 45′ W.

When the bearing of a line is being read, see that there is no iron or steel near, to turn the needle from its normal position. Some of the things most liable to cause this are, the chain or pins, hatchet, covered steel buttons on coat, steel-bowed spectacles, keys or knife in vest pocket. In fact, any bit of iron held near the needle may attract it appreciably.

To measure the bearing of a line when local attraction cannot be avoided, as, for example, when the line is an iron fence or a railroad track, measure the bearing in the usual way, and also, from the same place, measure the bearing to a point where no local attraction exists. Next set the compass over this latter point and measure the bearing to the first point. This bearing being correct, the bearing of the fence or track may be computed.

Example.—Compass at *a* (Fig. 15), bearing *ab* = N. 20° E.; bearing *ac* = N. 85° E.; therefore angle *cab* = 65°.

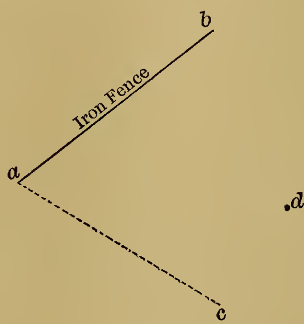

FIG. 15.

Compass at *c*, bearing *ca* = N. 83° W., which is the true bearing of *ca*, provided there is no local attraction at *c*; therefore the true bearing of *ac* is S. 83° E.; and since angle *bac* = 65°, the bearing of *ab* is N. 32° E. instead of N. 20° E.

In order to prove that there is no local attraction at *c*, select a fourth point *d* and measure the bearing *cd*; then at *d* measure the bearing *dc*. If one is exactly the reverse of the other, there is no local attraction at either *c* or *d*.

To make a compass survey of a field, the bearing and reverse bearing of all the sides should be taken, and the length of all the sides measured.

The following are samples of field notes of compass surveys:

	Station.	Bearing.	Reverse Bearing.	Distance.	Station.	Bearing.	Reverse Bearing.	Distance.	
		°	°	chains		°	°	chains	
(1)	1	N. 37 E.	S. 37 W.	15.32	1	N. 75¼ W.	—	5.22	(3)
	2	N. 46½ W.	S. 46¾ E.	4.53	2	S. 77 W.	—	10.60	
	3	S. 43½ W.	N. 43¼ E.	13.75	3	S. 74¾ W.	—	4.57	
	4	S. 26 E.	N. 26 W.	5.00	4	N. 86 E.	—	3.84	
	5	S. 57 E.	N. 57 W.	1.60	5	S. 50 E.	—	4.00	
					6	S. 27½ E.	—	3.93	
(2)	1	N. 67 E.	S. 67 W.	3.66	7	N. 65½ E.	—	7.90	
	2	S. 24½ E.	N. 24½ W.	0.95	8	N. 23 E.	—	2.17	
	3	S. 36¼ E.	N. 36¼ W.	1.34	9	N. 33 E.	—	1.00	
	4	S. 53½ E.	N. 53½ W.	2.00	10	N. 46½ E.	—	1.84	
	5	S. 42¼ E.	N. 42 W.	1.14	11	N. 60½ E.	—	1.40	
	6	S. 35¼ E.	N. 35½ W.	2.52					
	7	S. 74¼ W.	N. 74¼ E.	3.20	1	N. 89¼ W.	S. 89 E.	4.74	(4)
	8	N. 33 W.	S. 32¾ E.	3.30	2	N. 17½ W.	S. 17½ E.	12.50	
	9	N. 50¾ W.	S. 50¾ E.	1.77	3	S. 73¾ E.	N. 73½ W.	15.36	
	10	N. 61¾ W.	S. 61½ E.	1.14	4	S. 38½ W.	N. 38½ E.	9.87	
	11	N. 47½ W.	S. 47¼ E.	1.53					

(2) Area 2 A. 29 Rds. (3) Area 7 A. 155 Rds. (4) Area 9 A. 127 Rds. *Ans.*

If the readings of the two ends of the needle are not alike, the trouble is probably due to a bent needle or a bent pivot. If the needle is bent, the difference in the readings of the two ends will be the same, whatever the direction of the compass sights. If the difference in the readings of the two ends of the needle changes with the direction of the compass sights, the pivot is bent, and the needle may or may not be straight.

To straighten the pivot, turn the compass box till the difference in the readings of the two ends of the needle is greatest, then bend the pivot at right angles to the direction of the needle. After the pivot is straightened, to straighten the needle, bend till the readings of the two ends are the same.

To remagnetize the Needle.

Remove the glass cover from the compass box and take the needle from the pivot. Place the needle on a flat surface and draw an ordinary bar magnet from the centre of the needle to the end, repeating the operation a number of times. In returning the magnet from the end of the needle to the centre, it should be carried several inches away from the needle and not moved back close to its surface. Use the *north* pole of the magnet on the *south* end of the needle, and *vice versa*.

Sometimes when the surface of the glass cover to the compass has been rubbed with a dry cloth, electricity will be developed and cause the needle to adhere to the glass when lowered to the pivot. A touch on the glass with the moistened finger will remove the difficulty and cause the needle to swing free.

Always keep the *needle raised from the pivot except when taking a bearing*, otherwise the sharp point of the pivot will become blunt, and the pointing of the needle much less precise.

CALCULATION OF THE AREA.

The *latitude* of a line, or its northing or southing, is the distance that one end of the line is north or south of the other end.

It is equal to the length of the line, multiplied by the natural cosine of its bearing.

The *departure* of a line, or its easting or westing, is the distance that one end of the line is east or west of the other end.

It equals the length of the line multiplied by the natural sine of its bearing.

Obviously in a closed field the sum of the northings should equal the sum of the southings, and the sum of the eastings equal the sum of the westings.

It is customary to consider north latitudes and east departures *positive*, and south latitudes and west departures *negative*.

Compute all the latitudes and departures. The difference between the sum of the northings and the sum of the southings is the error in latitude. The difference between the sum of the eastings and the sum of the westings is the error in departure. The square root of the sum of the squares of these errors is the *error of closure*. This should not exceed a certain percentage of the perimeter, the amount depending upon the precision required for the work in hand.

After determining the error in latitude and in departure, this error must be distributed among the several courses, in order that the latitudes and departures shall balance exactly. This may be done by the following rule:

The sum of all the latitudes, or departures, is to any latitude, or departure, as the total error in latitude, or departure, is to the correction to be applied to that latitude, or departure.

This simply distributes the error throughout the whole length of the survey. It would be more precise, if not scientific, to make the correction to the lines or bearings that were incorrectly measured. The surveyor, from his knowledge of the survey, should distribute the error among those lines and bearings in which the natural difficulties to doing correct work were the greatest.

The surveyor should be cautious about increasing the length of any of the lines, in order to balance the survey, since any chainman, however careful, is much more likely to make the measured length of a line too *long* rather than too *short*, because of the difficulty in drawing the chain perfectly straight and horizontal.

After balancing the latitudes and departures, compute the *double meridian distances.*

The double meridian distance of any line is equal to twice the distance of its centre from any chosen meridian.

It will be found convenient to choose the meridian passing through the extreme east or west point of a survey, from which to compute the double meridian distances, in order that they may all have the same algebraic sign. This point may generally be easily determined by a simple inspection of the field notes. Having selected this point, call the line starting from it the *first course.*

Rule. — *The double meridian distance of the first course is equal to its departure.*

The double meridian distance of the second course is equal to the double meridian distance of the first course, plus its departure, plus the departure of the second course.

The double meridian distance of any course is equal to the double meridian distance of the preceding course, plus its departure, plus the departure of the course itself.

The double meridian distance of the last course should equal its departure, which checks the computation.

Multiply the double meridian distance of each course by its corrected northing or southing, observing that the product of a northing multiplied by a positive double meridian distance gives a positive result, and that the product of a southing multiplied by a positive double meridian distance gives a negative result.

The algebraic sum of all these products is twice the area of the field.

The following is the computation of the area from the field notes given in Example 1, page 10:

Station.	Bearing.	Distance.	Latitude. N+	Latitude. S−	Departure. E+	Departure. W−	Balanced. Latitude.	Balanced. Departure.	D.M.D.	+Areas.	−Areas.
1	N. 37° E.	15.32	12.24	—	9.23	—	+12.23	+9.22	16.28	199.104	—
2	N. 46½ W.	4.53	3.11	—	—	3.28	+ 3.11	−3.28	22.22	69.104	—
3	S. 43½ W.	13.75	—	9.97	—	9.46	− 9.98	−9.47	9.47	—	94.511
4	S. 26 E.	5.00	—	4.49	2.19	—	− 4.49	+2.19	2.19	—	9.833
5	S. 57 E.	1.60	—	0.87	1.34	—	− 0.87	+1.34	5.72	—	4.976

15.35 15.33 12.76 12.74 268.208 109.320
15.33 12.74 109.320
error in southing, 0.02. 0.02, error in westing. 2)158.888
79.444 sq. chains
= 7 acres 151.1 rods.

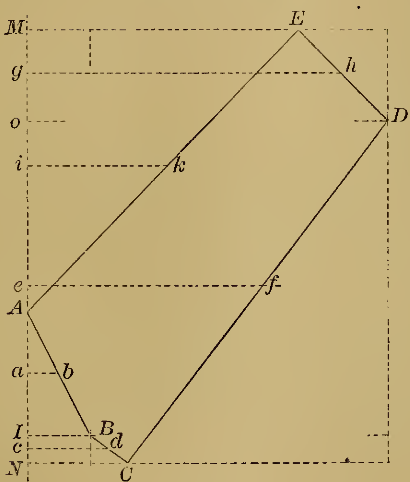

FIG. 16.

ABCDE (Fig. 16) shows a plot of the field, the area of which is computed above. *MN* is a meridian through *A*, the most westerly point. (Sta. 4 in field notes.)

Course.	Latitude.	D.M.D.	+Areas.	− Areas.
AB	*AI*	2 *ab*	——	2 *ABI*
BC	*IN*	2 *cd*	——	2 *IBCN*
CD	*No*	2 *ef*	2 *NCDo*	——
DE	*oM*	2 *gh*	2 *oDEM*	——
EA	*MA*	2 *ik*	——	2 *AME*

It is clear from Fig. 16 that each area given in the last table is equal to the product of the latitude of the course forming one of its limits, into the double meridian distance of that course. It is also evident that the difference between the + areas and the − areas is equal to twice the area *ABCDE*.

Plotting.

A survey may be plotted by laying off the angles with a protractor, and the distances, to the desired scale, with any convenient form of scale; or the corners of the field may be plotted by means of total latitudes and departures.

The protractor, one form of which is shown in Fig. 17, is generally made of a circular or semi-circular piece of brass or German silver, graduated to degrees on its circumference.

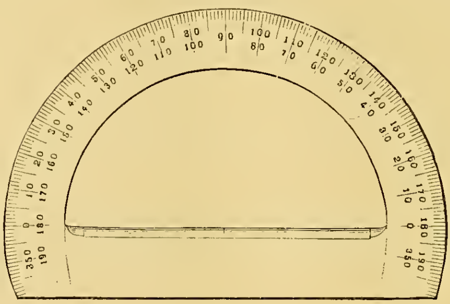

Fig. 17.

To lay off an angle with the protractor, place the centre mark over the vertex of the angle, with the diameter through the 0° mark along the given line. Mark off the number of degrees contained in the angle, and connect this point with the vertex.

In every case the line from which the angle is laid off should extend beyond the circumference of the protractor in both directions, to insure an accurate result.

To plot by Total Latitudes and Departures.

The total latitude, or departure, of any station is equal to the algebraic sum of all the latitudes, or departures, of all the preceding courses.

The following are the total latitudes and departures of the survey computed on page 13:

Station.	Total Latitude.	Total Departure.
A	0	0
B	− 4.49	+ 2.19
C	− 5.36	+ 3.53
D	+ 6.88	+ 12.75
E	+ 9.98	+ 9.47
A	0	0

Draw a meridian line, and locate at any convenient point along this line the point having 0 for its total latitude and departure. Lay off above this point the greatest positive total latitude, and below this point the greatest negative total latitude. At each of these latter points erect perpendiculars, and give each a length equal to the greatest total departure. Draw a line connecting the ends of these perpendiculars, and a rectangle will be formed, inside of which the plot is to be located. From these lines locate each station of the survey, using the total latitudes as ordinates and the total departures as abscissas. Draw lines connecting each adjacent station, and the survey is plotted. To check the work, scale each line and see if it agrees with the measured length. (*ABCDE*, Fig. 16, is plotted by total latitudes and departures.)

Every plot should show, in addition to the outline of the field, the length and bearing of each line, the date of the survey, the declination of the magnetic needle at the time the survey was made, the scale of the plot, the location of the field, and the name of the surveyor.

When it is not practicable to measure one line of a survey, the missing data may be supplied by the computation, as in that case the difference between the northings and the southings is the latitude of the missing line, and the difference between the eastings and westings is its departure. Divide the departure by the latitude, and the quotient is the natural tangent of the missing course. If practicable, all of the courses and distances should be measured, as whenever one is omitted the whole error of closure is, of necessity, thrown into the missing line, and the surveyor has no knowledge of the magnitude of the error.

PARTING OFF LAND.

Suppose that in the field plotted on page 13, it is required to locate from a point F (Fig. 18) on the line CD, eight chains from C, a line FX

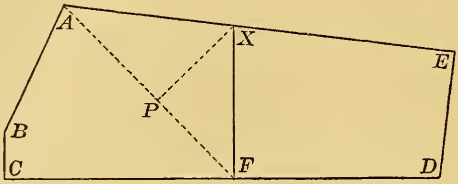

Fig. 18.

such that the area $ABCFX$ shall be four acres. It is evident that the point X will fall somewhere on the line AE.

Draw AF. Compute the area $ABCF$.

The latitude, departure, and bearing of AF may be determined as in the case of the missing bearing and distance of any line. Subtract the area $ABCF$ from four acres, and there remains the area of the triangle AFX. Compute the length of PX, perpendicular to AF ($\frac{1}{2} AF \times PX$ $=$ area AFX). With the angle XAF and the length PX given, AX may be computed, and X located by measuring this distance from A.

RUNNING A RANDOM LINE.

If for any reason, in running from one station of a survey to another, the two points are not intervisible, and there is no dividing fence, a random line is first run in as nearly the right direction as can be judged, till a point is reached, at which an offset at right angles to the random line

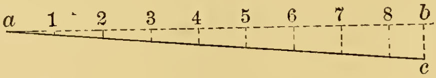

Fig. 19.

will pass through the other end of the true line. Stakes should be set at regular intervals, say one chain apart, along the random line, from which points on the true line may be located as shown in Fig. 19.

$ab = 8.75$ ch., $bc = .35$ ch.; then to locate a point on the line ac opposite the stake at 8 the offset is $\frac{8}{8.75} \times .35 = .32$ ch. The offset from the stake at 7 is .28 ch., and so on.

If the line divides two wood lots, one of which is to be cut, the line is marked by blazing adjacent trees on the side nearest the line, as shown in Fig. 20.

FIG. 20.

To replace a broken boundary line by a single straight line, without changing the area of the adjoining fields.

Let *ab* (Fig. 21) be a curved or broken line separating the two fields. It is required to locate a line, extending from *a* to the line *cd*, such that the area on either side of this line will be the same as that on either side of the line *ab*.

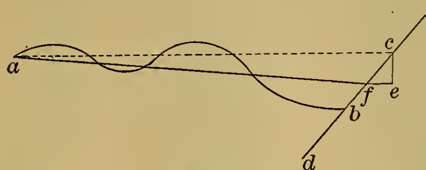

FIG. 21.

From *a* run a random line *ac*. From this random line measure offsets to the curved line *ab* and compute the area on either side of *ac* between it and *ab*. Divide the difference of these areas by half of *ac* and measure this distance, *ce* in the figure, from *c*, perpendicular to *ac*; through *e*, parallel to *ac*, lay off *ef* to *cd*. *af* is the line required.

THE TRANSIT.

The transit, two forms of which are shown in Fig. 22, is an instrument for measuring angles independent of the magnetic needle and with much greater accuracy.

The telescope differs from an ordinary telescope in that there is placed between the object-glass and the eye-piece, a ring carrying two fine spider threads called "cross-hairs." These are used to define the line of sight through the telescope, in the same way as do the compass sights on the compass.

The telescope carries with itself an index, which moves about a graduated circle and marks the number of degrees and minutes contained between two successive sightings of the telescope.

The circle is graduated to half-degrees, or sometimes into twenty-minute spaces. The index carries a vernier, which is a device for reading accurately fractional parts of a degree. The single-minute vernier is generally made as follows:

A space on the vernier, equal to twenty-nine half-degrees on the circle, is divided into thirty equal parts. This makes each division on the vernier one-thirtieth of half a degree shorter than a half-degree, or one minute shorter.

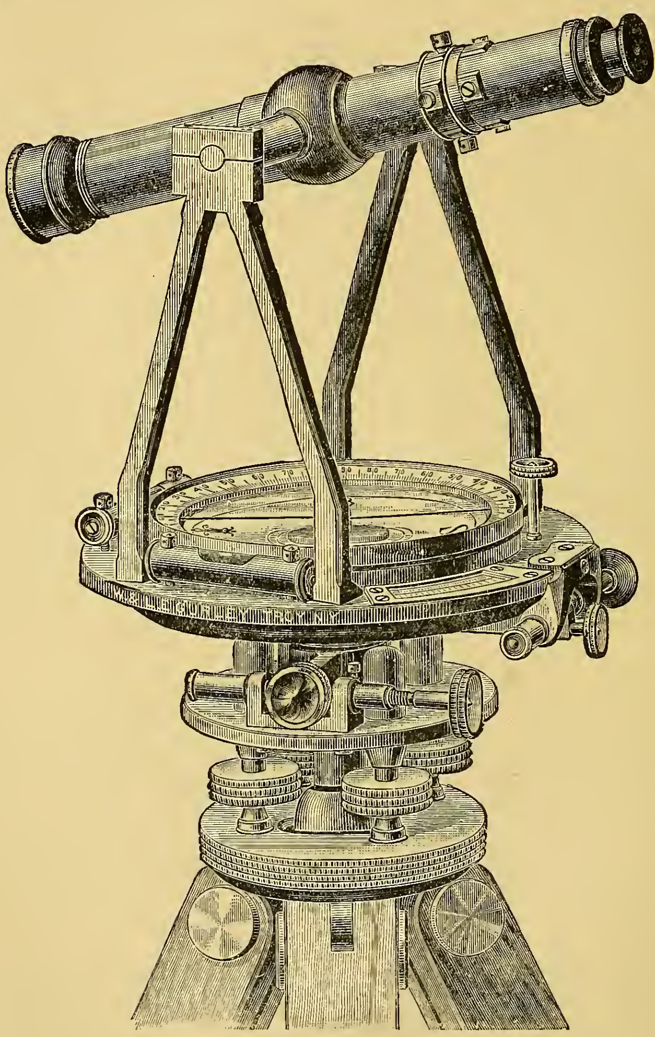

FIG. 22.—PLAIN TRANSIT.

If the zero of the vernier is set opposite the zero of the circle, the first division of the vernier falls one minute short of the first division of the circle, the fifth division five minutes short, and so on, till the thirtieth division on the vernier is reached. This division falls thirty minutes short of the thirtieth division on the circle, and coincides with the twenty-ninth half-degree division.

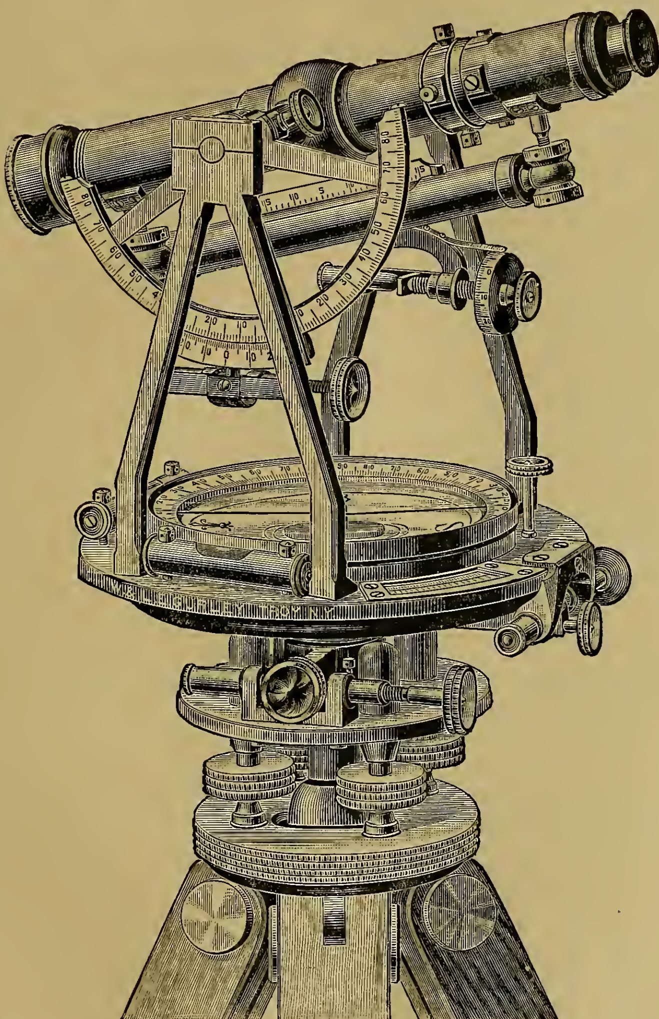

Fig. 22. — Engineer's Transit.

By moving the vernier till the first division from the zero coincides with the first division of the circle, the telescope is turned through an angle of one minute. By turning the vernier till its fifth division coincides with the fifth division of the circle, the telescope is moved through an angle of five minutes, and so on.

To read the angle for any setting of the vernier, count, on the circle, the number of degrees and half-degrees between the zero of the circle and the zero of the vernier; then look along the *vernier,* till a division is found which coincides exactly with one of the divisions on the circle; the number of this division *on the vernier* is the number of minutes to be added to the degrees and half-degree, if any, to give the true reading.

The following is a general rule for determining the smallest reading of any vernier:

Divide the value of the smallest division on the circle by the number of divisions on the vernier.

For example, thirty divisions on vernier and circle divided to thirty-minute spaces,

$$\frac{30'}{30} = 1'. \tag{1}$$

Forty divisions on vernier; circle divided to twenty-minute spaces,

$$\frac{20'}{40} = 30''. \tag{2}$$

Sixty divisions on vernier; circle divided to twenty-minute spaces,

$$\frac{20'}{60} = 20''. \tag{3}$$

Sixty divisions on vernier; circle divided to ten-minute spaces,

$$\frac{10'}{60} = 10''. \tag{4}$$

Ten divisions on vernier; rod divided to hundredths of a foot,

$$\frac{.01 \text{ ft.}}{10} = .001 \text{ ft.} \tag{5}$$

Ten divisions on vernier; scale divided to tenths of an inch,

$$\frac{.1 \text{ in.}}{10} = .01 \text{ in.} \tag{6}$$

Twenty-five divisions on vernier; scale divided to .05 in.,

$$\frac{.05 \text{ in.}}{25} = .002 \text{ in.} \tag{7}$$

(1), (2), and (3) are different forms of transit verniers, (4) is a sextant vernier, (5) is the form of vernier used on Boston and New York levelling-rods, and (6) and (7) are two forms of vernier used on mercurial barometers.

To measure an Angle with the Transit.

Bring the plumb-bob, hung from the transit, exactly over the point at which the angle is to be measured. Place the plate bubbles parallel to opposite levelling-screws and level, by grasping opposite levelling-screws between the thumb and forefinger of each hand and turning both thumbs in or out, as is necessary. (The bubble will move in the same direction as the left thumb in turning the levelling-screws.)

Set the zero of the vernier opposite the zero of the circle; focus the eye-piece on the cross-hairs, and the object-glass on the object to be sighted; tighten the lower clamp and bring the image of the object, defining one line of angle, in exact coincidence with the vertical cross-hair, by means of the lower tangent screws. Loosen the upper clamp; sight the telescope to the object defining the second line of the angle; tighten the upper clamp and bring the image of the object in exact coincidence with the vertical cross-hair, by means of the upper tangent screw. Read on the circle the number of degrees passed over by the index and on the vernier opposite the line in coincidence with a line on the circle the number of minutes to be added to the circle reading, to give the correct reading.

Generally there are four verniers on a transit, one on each side of the zero of each of the two opposite indices. The student, when learning to use the transit, should read that vernier which lies in the same direction from the zero set, as that in which the zero of the vernier was turned in sighting the second object.

In making a survey of a closed field with the transit, either the interior angles or the deflection angles may be measured. If the interior angles are measured, their sum should equal twice as many right angles as the field has sides, less four right angles. If the deflection angles are measured, the sum of all the right deflections should differ from the sum of all the left deflections by 360°.

In either case the bearing of each line should be measured with the needle. The bearing of each line should also be computed by assuming the computed bearing of one line (generally the first) equal to its observed bearing, and then computing the bearing of each succeeding line from the deflection angle. If the computed bearing does not agree approximately with the observed bearing, both should be remeasured to discover if any mistake has been made.

In Fig. 23, *a*, *b*, *d*, *e*, and *f* are right deflection angles; *c* and *g* are left deflection angles.

Areas may be determined from the computed bearings and the distances in the same way as with compass surveys.

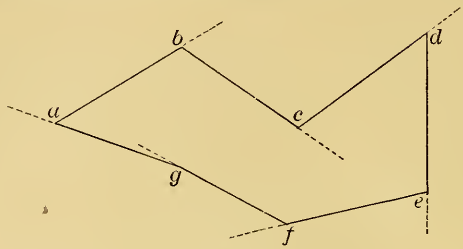

FIG. 23.

Adjustments of the Transit.

First. To make the plate bubbles parallel to the horizontal circle. Level the bubbles in the ordinary way; then revolve the instrument half-way round. If either bubble runs toward one end of its tube, turn the adjusting screw at the end of the bubble tube till the bubble moves half-way back to the centre. Repeat the operation till the bubble will remain in the centre during a whole revolution of the instrument.

Second. To adjust the line of collimation. This consists in placing the vertical cross-hair in the optical axis of the telescope, so that a straight line may be prolonged by revolving the telescope on its horizontal axis.

Sight the telescope on some point *A* 200 to 300 ft. away, and clamp the spindle; then revolve the telescope on its horizontal axis and locate a point *B* in the line of sight. Loosen the clamp and revolve the instrument on its vertical axis till *A* is sighted. Clamp the spindle and again revolve the telescope on its horizontal axis, and note whether the line of sight strikes the point *B*. If it strikes to one side, call this point *C*. To adjust, move the cross-hair ring till the line of sight strikes a point one-fourth of the distance from *C* toward *B*. Test the correctness of the adjustment by repeating. To move the cross-hair ring, loosen the capstan-headed screw on one side of the telescope tube and tighten the opposite one. Unless the telescope is "inverting," the cross-hair ring must be moved in the direction opposite to what appears to be correct.

Third. To make the axis of the telescope horizontal, so that the line of sight will move in a vertical line. Set up the instrument near some high object, as a steeple; sight the high point and clamp the spindle. Depress the telescope and locate a point, in the line of sight, nearly on a level with the telescope. Loosen the clamp; revolve the instrument on its vertical axis and the telescope on its horizontal axis; sight the low point and clamp the spindle. Raise the telescope and note if the line of sight strikes the high point. If the line of sight is to one side, the standard on the opposite side is too high, and the axis of the telescope must be moved till the line of sight moves half-way back toward the high point first sighted. To move the axis of the telescope, turn the screw under the bearing at one end of the axis. Repeat the operation in order to see if the adjustment has been correctly made.

STADIA SURVEYING.

In addition to the centre horizontal cross-hair in the telescope, most modern engineers' transits have two other horizontal hairs, situated at equal distances above and below the centre one. These are called *stadia hairs.*

They are placed at such a distance apart that the distance intercepted by them, on a rod held vertical, is one-hundredth of the distance to the rod, from a point in front of the object-glass equal to the focal length of the telescope; or, expressed in formula,

$$d = 100\,s + f + c.$$

d = distance from the centre of the instrument.

s = space on rod intercepted by stadia hairs.

f = focal length of telescope, *i.e.* distance from the object-glass to the cross-hairs = from three-fourths to one foot in most transits.

c = distance from the object-glass to the centre of the instrument (= one-half foot, about).

If the rod is not at the same level as the transit, and is still held vertical, the horizontal distance and the difference in elevation may be found by the following formulas:

$$\text{Horizontal distance} = s\cos^2 v + (c + f)\cos v.$$

$$\text{Difference in elevation} = s\,\tfrac{1}{2}\sin 2\,v + (f + c)\sin v.$$

v = vertical angle (either elevation or depression).

The difference in elevation given, is the vertical distance between the centre of the telescope and the point on the rod intercepted by the centre horizontal cross-hair. By sighting at a point on the rod equal to the height of the telescope, the difference in elevation between the surface of the ground at the instrument and at the rod is given.

The tables computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania, give values of $s \cos^2 v$ and $\frac{1}{2} s \sin 2v$ for angles from 0° to 30° with $s = 1$. By use of these tables the reduction of stadia notes is made quite easy.

Example in Use of Tables.

Vertical angle $= + 4° 28'$ $(c + f) = 1.25 = c$ in table.

Rod reading $= 4.42$ ft.

Difference in elevation $= 4.42 \times 7.76 + .10 = 34.40$ ft.

Horizontal distance $= 4.42 \times 99.39 + 1.25 = 440.55$ ft.

The stadia furnishes a very rapid method of measuring distances when an error of one part in four or five hundred is admissible.

PUBLIC LAND SURVEYS.

The larger part of the land north of the Ohio River and west of the Mississippi is laid out according to law, by lines running north and south, and east and west.

The method is as follows:

A *principal meridian* is run due north. At intervals of twenty-four miles north of latitude 35° and at intervals of thirty miles south of this latitude, *standard parallels* are run due east and west. At intervals of forty-two miles, lines are run north and south, connecting standard parallels.

This divides the surface into figures shaped as shown in Fig. 24.

Fig. 24.

Each of these approximately rectangular plots, or *checks*, as they are called, is divided into townships which are six miles square, "as nearly as may be." The bounding lines of the townships are run as follows:

Starting at *a*, six miles from *o* (Fig. 25), run a line due north six miles to *b*, establishing section and quarter-section corners every one-half mile. From *b* run a random line toward *c*. *c* has been already established on the prin-

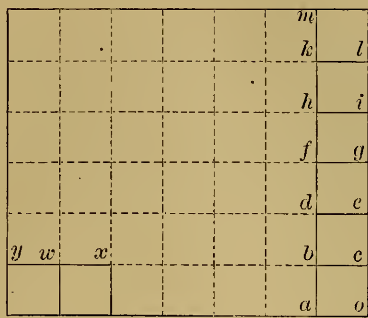

Fig. 25.

cipal meridian. In all probability the random line will not strike *c* exactly, but somewhere north or south of it. In this case the true line must be run from *c* to *b* by correcting each one-half-mile point established on the random line. In the same way run from *b* to *d* due north; then a random line from *d* toward *e*; then correcting this from *e* to *d*.

When the line between the sixth and seventh township lines is run, a random line is run both east and west from *w* toward *x* and *y*, and so on till the whole check is divided into townships.

Each township is subdivided into sections numbered as shown in Fig. 26.

Each section is divided into quarter-sections called respectively the N.E., N.W., S.E., and S.W. quarter-section.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. 26.

The lines of townships running east and west from a principal meridian are called *townships.* The lines of townships running north and south of a standard parallel are called *ranges;* thus

N.E. $\frac{1}{4}$ sec. 18 T. 3 N. R. 5 W.

is the northeast quarter of section eighteen, in the township in the third line of townships north of a standard parallel, and the fifth range west of a principal meridian.

Burt's solar compass, shown in Fig. 27, is the instrument commonly used in running lines on government surveys.

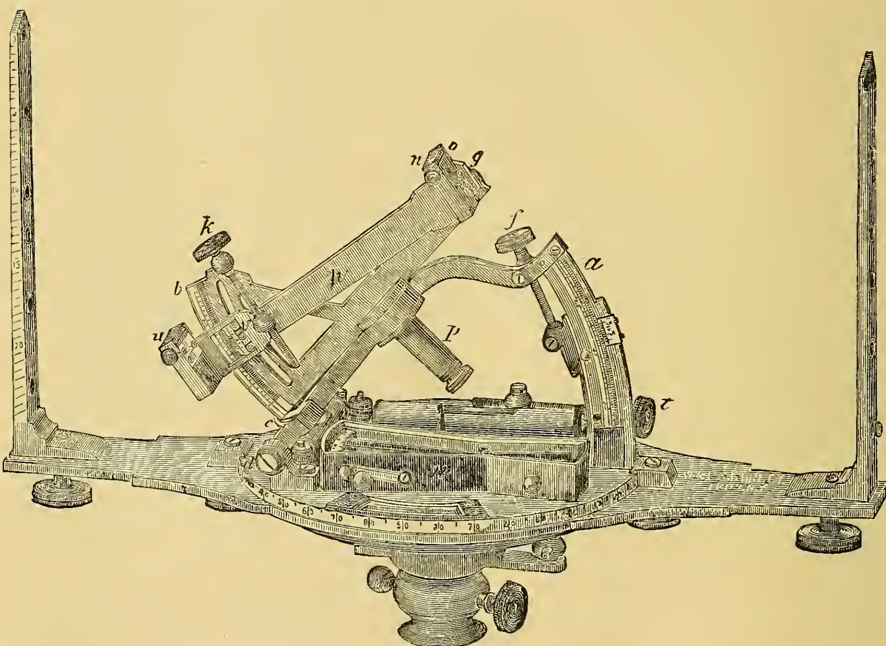

FIG. 27.

This instrument is devised to determine a true north and south line by means of an observation on the sun.

At *a* is the latitude arc, on which is laid off the latitude of the place; *b* is the declination arc, on which is laid off the declination of the sun (corrected for refraction) at the time when the observation is made.

The compass is then turned about on its vertical axis, and at the same time the arm *h* about the polar axis *p,* till an image of the sun, formed by a lens at *g,* falls between the four lines on the silvered plate at *u.* The polar axis *p* is then parallel to the axis of the earth, and the compass sights are in a true north and south line. Do not confuse the false image, sometimes reflected from the arm *h*, with the true image formed by the sunlight passing directly from the lens to the silvered plate. The false image is much less bright, and its outline is less clearly defined than that of the real image.

To determine the Declination of the Sun at Any Time and Place.

The Nautical Almanac gives in advance the declination of the sun for apparent and for mean noon at Greenwich for each day in the year. It also gives the change in declination for one hour. By multiplying this change by the number of hours after noon and adding the result to or subtracting it from the declination at noon, according as the declination is increasing or decreasing, the declination for any time may be determined.

Having the declination at Greenwich, the time when the declination is the same at any other place is known, if the difference between local and Greenwich time is known; or, in other words, if the longitude is known. For example, the declination in New England at 7 A.M. is the same as it is at mean noon, of the same day, at Greenwich, since the longitude of New England is five hours west of Greenwich. The declination in the Middle States at 10 A.M. is the same as that at Greenwich at 4 P.M. of the same day, since their longitude is six hours west of Greenwich.

When the declination is *north*, the correction for refraction is to be *added* to the declination obtained from the Nautical Almanac, and when it is *south*, the refraction correction is to be *subtracted*, to give the angle to be laid off on the declination arc of the solar compass.[1]

Tables giving the correction for refraction for different hours of the day, for different latitudes, and for different declinations, have been computed. Those published by Messrs. W. & L. E. Gurley in their manual were prepared especially for use in connection with the solar compass.

THE LEVEL.

This instrument, shown in Fig. 28, is used to determine the difference in elevation of different stations.

It consists essentially of a telescope, attached to the under side of which is a delicate spirit level. The whole, mounted on a tripod, may be freely revolved about a vertical axis.

The instrument is levelled by bringing the bubble into the centre of the tube when successively parallel to one set of opposite levelling-screws

[1] This is true for places north of the equator; for places south of the equator the words *north* and *south* should be interchanged.

and then parallel to the other set. The levelling-rod, two forms of which are shown in Fig. 29, is used to measure the distance down from the level line of sight to any point the elevation of which is desired.

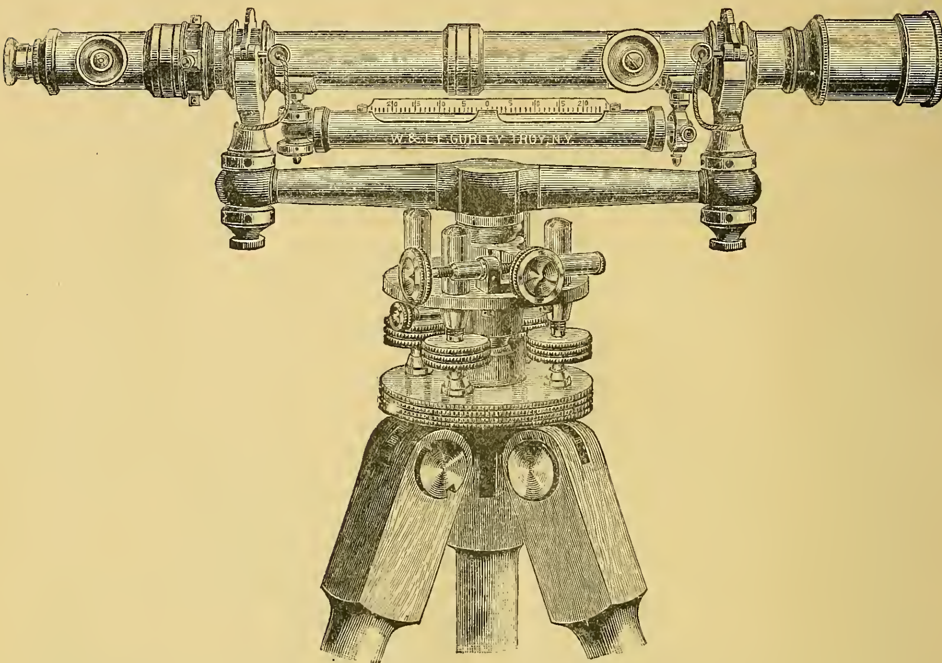

FIG. 28.

To determine the difference in elevation between two points, the instrument is set between them and levelled, and the distance of each point below the line of sight determined. The difference between these distances is the difference in elevation.

To determine each of these distances, the rodman holds the rod on one of the points, and moves the target up, or down, as directed by the leveller, till the horizontal line on the target exactly coincides with the line defined by the horizontal cross-hair in the telescope. He then records the rod reading. The rod is then taken to the second point, the telescope is revolved on its vertical axis till the rod is sighted, the target is again set, and the rod reading taken. The leveller should see to it that the bubble is in the centre of the tube at the instant of sighting, as the fact that the instrument has been once levelled will not insure its remaining so. If, for any reason, the difference in elevation of the two points cannot be determined by a single setting of the instrument, determine successively the difference in elevation of a number of intermediate points, as shown in Fig. 30.

The algebraic sum of these differences is the difference in elevation.

Profile Levelling.

In order to make a profile of the surface of the ground along any given line, elevations are taken at equal (generally 100 ft.) intervals. The elevations are measured above an assumed horizontal plane called the *datum plane.* This plane should be below the lowest point on the surface, to avoid negative readings.

A *bench-mark* (B. M.) is some permanent mark, the elevation of which has been determined and recorded together with a description, by which it may be found at any time.

A *height of instrument* (H. of I.) is the elevation of the line of sight through the level above the datum plane. It is found by adding a back-sight to the elevation of a bench-mark or turning-point. A *back-sight* or + *sight* (B. S.) is a rod reading taken on a bench-mark or turning-point, and is used to determine the height of instrument. A *fore-sight,* or −*sight* (F. S.) is a rod reading taken on a point, the elevation of which is to be determined. This elevation is determined by subtracting the fore-sight from the height of instrument.

A *turning-point* (T. P.) is a point on which both a back-sight and a fore-sight are taken. It is used to determine a new height of instrument. A turning-point should be solid, and not one whose elevation can be changed by pressure of the rod while the sights are being taken.

To determine the elevations for a profile, stakes are first set along the line at intervals of 100 ft., and numbered 0, 1, 2, 3, etc., to the end of the line.

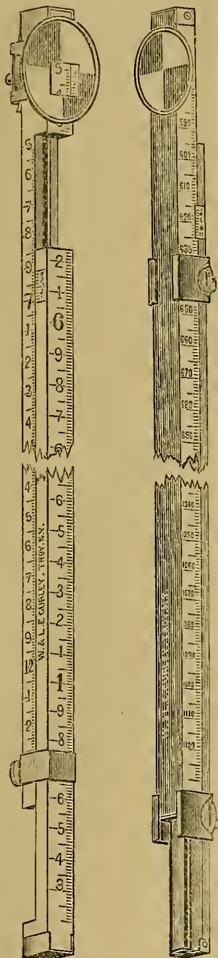

FIG. 29.

The level is set near the 0 station, and a back-sight is taken on the nearest bench-mark. Fore-sights are then taken on as many stations as can be reached from that setting of the level. A turning-point is then chosen and a fore-sight taken on it. The level is then carried forward along the line and set up again and levelled. A back-sight is then taken on the turning-point, a new height of instrument determined, and then fore-sights are taken on as many more stations as may be sighted from this new position of the level. In this way the work is carried on till the end of the line is reached.

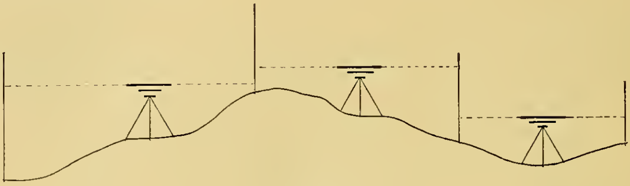

Fig. 30.

The following is a set of profile level notes:

Station.	B. S.	H. of I.	F. S.	Elevation.
B. M.	7.206	114.900		107.694
0			4.2	110.7
1			5.1	109.8
2			6.3	108.6
3			4.9	110.0
T. P.	5.182	116.210	3.872	111.028
4			4.2	112.0
+ 40			5.7	110.5
5			2.7	113.5
6			1.8	114.4
B. M.			0.987	115.223

Figure 31 shows a profile plotted from the above notes.

In this way profiles for railroad lines, water-pipe lines, highways, etc., are determined.

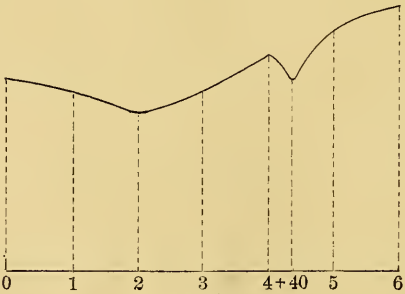

Fig. 31.

Whenever excavation or filling is to be made over considerable areas, as in excavating for reservoirs, filling marshes, etc., the amount of material to be moved may be determined by what is known as cross-sectioning the area. This is done by running transit lines at right angles to each other to divide the field into squares, and then determining the elevation at each intersection. From these elevations, and the area of the total number of squares, the amount of material above any plane may be computed by multiplying the area of each square by the mean of the heights of the four corners above the given plane.

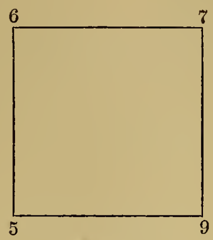

Fig. 32.

Let Fig. 32 represent a piece of land 100 ft. square, and the figures at each corner the elevations in feet; then the cubic contents above a two-foot horizontal plane are $\frac{4+5+3+7}{4 \times 27} \times 100 \times 100$ cu. yds.

When there are a large number of squares, take the sum of the heights at all of the corners common to one square, plus twice the sum of the heights at all the corners common to two squares, plus three times the sum of the heights at all the corners common to three squares, plus four times the sum of the heights at all of the corners common to four squares;

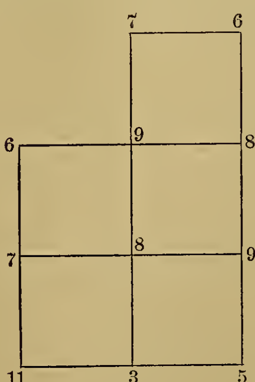

Fig. 33.

divide the result by 4, multiply by the area of one square, and the result is the contents in cubic feet. Divide by 27 to reduce to cubic yards.

Figure 33 represents a plot of land, the area of each square being 100 ft. The figures at the corners are the elevations in feet. The volume in cubic yards above a zero plane is:

$$100 \times 100 \times \frac{11+6+7+6+5+2(8+9+3+7)+3\times 9+4\times 8}{4\times 27}=13703.7 \text{ cu. yds.}$$

Adjustments of the Wye Level.

First. The adjustment of the line of collimation. This consists in placing the intersection of the cross-hairs in the centre of the wye-rings. Loosen the clips that hold the telescope in the wyes. Sight the vertical cross-hair on some well-defined line 200 to 300 ft. away, and clamp the spindle. Turn the telescope half-way round in the wyes and note if the vertical cross-hair still coincides with the line first sighted. If it does not, bring the line of sight half-way back to this position by moving the cross-hair ring as is done in adjusting the line of collimation in the transit. Adjust the horizontal cross-hair in the same way. When this adjustment is made, the intersection of the cross-hairs will remain on a point while the telescope is turned through a whole revolution in the wyes.

Second. To make the bubble parallel to the line of sight. Bring the telescope parallel to two opposite levelling-screws and clamp the spindle. Bring the bubble into the centre of the tube by turning the levelling-screws, and then turn the telescope a few degrees in the wyes. Should the bubble move toward one end of the tube, it would show that a vertical plane through the axis of the telescope is not parallel to a vertical plane through the bubble tube. To correct this, move the bubble to the centre by turning the two capstan-headed screws, giving a horizontal motion to one end of the bubble tube.

Repeat if necessary, till the bubble will remain in the centre when the telescope is revolved in the wyes five or ten degrees either side of its normal position. Next reverse the telescope in the wyes, and note if the bubble moves towards one end of the tube. If it does, that end is too high and must be lowered, or the opposite one raised, till the bubble takes a position half-way back toward the centre. This is done by turning the capstan-headed screws, giving a vertical motion to one end of the bubble tube.

Third. To make the line of sight and the bubble at right angles to the vertical axis of the instrument. Fasten the clips holding the telescope in the wyes; bring the bubble parallel to two opposite levelling-screws, and level; revolve the instrument 180° on the vertical axis, and if the bubble moves toward one end of the tube, turn the capstan-screws at the end of the wyes till the bubble moves half-way back to the centre. Relevel, and repeat the operation to test the adjustment.

Contour lines are lines connecting points of equal elevation.

A shore line of a pond represents a contour line. If the water in the pond should rise a given amount, as 10 ft., then that shore line would represent another contour line 10 ft. higher than the first.

Contour lines furnish an easy and accurate way of representing on a map the relief of a region.

Such maps are valuable for many engineering purposes, among which are, location of routes for roads, railroads, water-pipes, aqueducts, military movements, etc.

Contour maps on a small scale are often made by interpolating the contour lines on a plot that has been cross-sectioned as shown in Fig. 34.

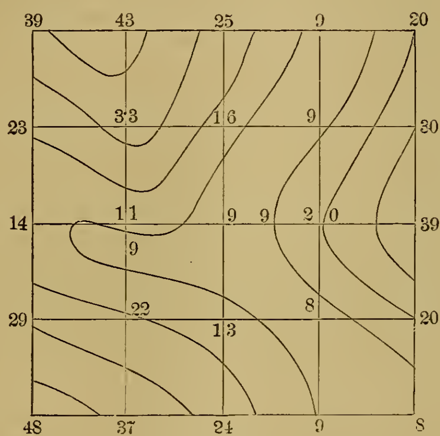

FIG. 34.

PRACTICAL ASTRONOMY AND NAVIGATION.

THE navigator, and sometimes the surveyor, needs a knowledge of astronomy sufficient to enable him to determine his position (latitude and longitude) on the surface of the earth. In the case of the navigator the error should not exceed the distance from the ship's deck to the horizon line, a distance seldom exceeding five or six miles.

The *zenith* is the point vertically above the observer, and is 180° from the direction taken by a plumb-bob.

The *nadir* is the point vertically underneath the observer, and lies in the direction taken by a plumb-bob at rest. The zenith and nadir change with every change in the observer's position.

The *plane of the horizon* is a plane passing through the observer's position, and everywhere at right angles to the direction of the zenith. Planes at right angles to the plane of the horizon, and containing the line connecting the zenith and nadir, are called *planes of altitude.*

The two *celestial poles* are the points where the axis about which the earth revolves, would, if produced, pierce the heavens. The one which is above the horizon in the northern hemisphere is called the *north celestial pole ;* the other is called the *south celestial pole.*

The two poles are independent of the observer's position.

The circle in which a plane passing through the earth's centre, and at right angles to the line connecting north and south poles, cuts the surface of the earth, is called the *terrestrial equator.* If this plane be extended, the circle in which it cuts the heavens is called the *celestial equator.*

Hour circles are circles containing the line connecting the poles and at right angles to the plane of the equator.

The plane containing the hour circle passing through the zenith intersects the horizon in its north and south points.

The *altitude* of a point is its angular distance above the horizon, measured in a vertical circle passing through the point. The zenith distance is the complement of the altitude.

The *azimuth* of a point is the angle at the zenith between the meridian and a vertical circle passing through the point. Azimuth is generally measured from the south through the west, north, and east, from 0° to 360°.

The declination of a point is its angular distance from the equator measured on an hour circle passing through the point. Declination is considered positive when the point is north of the equator, and negative when south of it. Polar distance is the complement of the declination.

The hour angle of a point is the angle at the pole between the meridian and the hour circle passing through the point, or it may be defined as the arc of the equator intercepted by these two circles. Hour angles are usually measured from the south point of the equator from 0° to 360°, or from zero hours to twenty-four hours in the direction of the motion of the hands of a watch.

The *latitude* of a point on the earth's surface is equal to the altitude of the pole. The observer's latitude is considered positive when he is north of the equator, and negative when south of it.

Longitude is the distance east or west of any assumed meridian. The meridian passing through the Greenwich, England, observatory is the one commonly used. Longitude is measured 180° or twelve hours east and west of the assumed meridian.

The Nautical Almanac is a book, published by the government, which contains, among other data, the right ascensions and declinations of the sun, moon, planets, and certain fixed stars, the semi-diameters of the sun and moon, and the equation of time for Greenwich apparent and mean noon of each day in the year — all computed several years in advance.

The following is taken from the tables giving data in regard to the sun:

JULY, 1896, AT GREENWICH APPARENT NOON.

Day of the Week.	Day of the Month.	The Sun's					Equation of Time to be added to Apparent Time.	Difference for One Hour.
		Apparent Right Ascension.	Difference for One Hour	Apparent Declination.	Difference for One Hour	Semi-diameter.		
		h. m. s.	s.	° ′ ″	″	′ ″	m. s.	s.
Wednesday,	1	6 43 49.95	10.332	N 23 4 19.8	−10.81	15 46.16	3 40.69	0.474
Thursday .	2	6 47 57.80	10.321	22 59 48.2	11.82	15 46.15	3 51.95	0.463
Friday . .	3	6 52 5.38	10.310	22 54 52.5	12.82	15 46.15	4 2.94	0.452
Saturday .	4	6 56 12.68	10.297	22 49 32.7	−13.82	15 46.15	4 13.64	0.439
SUNDAY.	5	7 0 19.65	10.284	22 43 49.1	14.81	15 46.16	4 24.03	0.426
Monday. .	6	7 4 26.29	10.269	22 37 41.7	15.80	15 46.17	4 34.09	0.412

The **sextant** shown in Fig. 35 is a hand instrument for measuring angles. Since it does not require a rigid support, it is particularly adapted to measuring angles from a ship at sea.

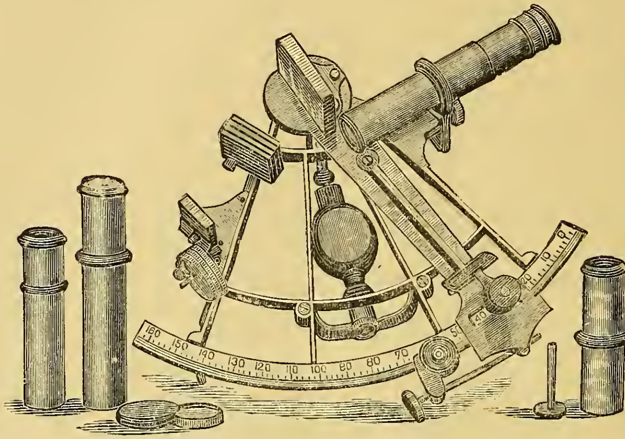

FIG. 35.

The construction of the sextant depends upon the principle of optics, that if a ray of light be twice reflected from two plane mirrors, its angular change in direction is equal to twice the angle of the mirrors.

At I (Fig. 36) is the index glass which rotates with the arm IV. This arm carries at V a vernier which moves along the graduated arc AB. At H is the horizon glass, one half of which is silvered and the other half clear.

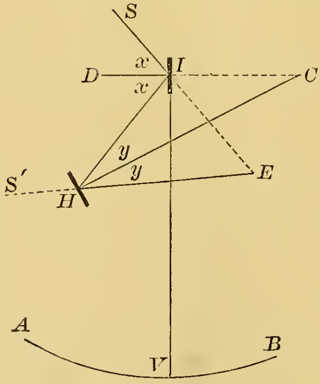

FIG. 36.

A ray of light coming from the direction S is reflected from I and H, and enters the eye at E. As the arm IV is moved along the arc, the image of S

travels across the mirror at H. When the image is seen exactly in the line $S'E$, which is the path of a ray of light from S' through the clear part of the glass at H, the angle of the mirrors is one-half the angle SES'. The arc AB is so graduated that any angle, as SES', may be read directly by the vernier at V.

To prove that $SES' =$ twice the angle between the mirrors, draw DC perpendicular to the mirror at I, and HC perpendicular to the mirror at H. Then $DCH =$ angle between the mirrors.

$$\angle SES' = 2x - 2y;$$

$$\angle DCH = x - y;$$

$$\therefore\ SES' = 2\,DCH.$$

If the two mirrors are not parallel when the vernier reads zero there is an *index error*. This error is a constant correction which must be made to all angles measured with the instrument.

To determine the index error, measure the angular diameter of the sun by bringing the discs of the two images in contact.

Call the diameter of the sun d, the sextant reading r, and the index error e.

Then $d = r + e$.

Now move the vernier till the discs are again in contact, the image that was first above being now underneath.

The zero of the vernier will now probably be back of the zero of the circle.

Call this reading $-r'$.

Then $$-d = -r' + e.$$

Hence $$e = \frac{r' - r}{2}.$$

In order to obtain the true altitude of a heavenly body, the observed altitude measured with the sextant must be corrected for dip, parallax, refraction, and, in the case of the sun or moon, for semi-diameter.

Dip.

Altitudes at sea are measured from the visible horizon, which is below the true horizon an amount depending upon the height of the observer above the surface of the water.

Let OH (Fig. 37) be the true horizon, and OH' the visible horizon. $OA = a =$ the observer's height above the water. $AC = BC = R =$ mean radius of the earth $= 20{,}900{,}000$ ft. approximately. $HOH' = ACB = D =$

the dip of the horizon. Tan $D = \frac{OB}{BC} = \frac{\sqrt{2\,Ra + a^2}}{R} = \sqrt{\frac{2\,a}{R} + \frac{a^2}{R^2}}$. $\frac{a^2}{R^2}$ is very small and may be neglected. The angle D is also small, and may be taken as $D \tan 1'$. Substituting, we have

$$D \text{ in minutes} = \frac{1}{\tan 1'} \times \frac{1}{3233} \sqrt{a}$$

$$= .94 \sqrt{a},$$

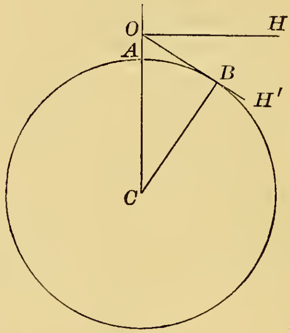

Fig. 37.

or the dip in minutes is approximately the square root of the observer's height in feet above the water.

The correction for dip must be subtracted from the observed altitude to give the true altitude.

Parallax.

The difference in direction of a heavenly body as seen by an observer on the earth's surface and as it would be seen from the earth's centre is called *parallax.*

The magnitude of the parallax depends upon the altitude of the heavenly body and the ratio of the earth's radius to the distance of the heavenly body. The horizontal parallax is the parallax when the heavenly body is in the horizon. It may be found as follows:

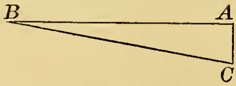

Fig. 38.

Let P (Fig. 38) = horizontal parallax, AC = the earth's radius, BC = the distance of the heavenly body from the earth's centre.

$$\text{Sin } P = \frac{AC}{BC}.$$

The parallax of a heavenly body in any other position may be found as follows: Let ZAB (Fig. 39) be the observed zenith distance. $ABC = p$ = parallax;

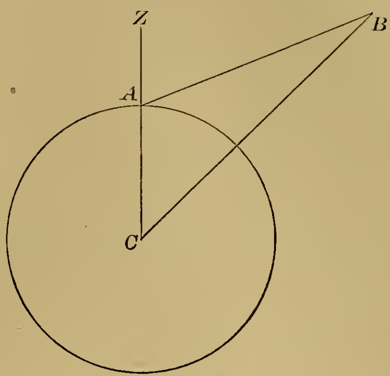

Fig. 39.

then
$$BC : AC :: \sin BAZ : \sin ABC,$$
$$\sin ABC = \frac{\sin BAZ \times AC}{BC}.$$

$\frac{AC}{BC}$ = horizontal parallax. Substituting this, we have
$$\sin p = \sin P \sin BAZ.$$

Both p and P are small; therefore we may without excessive error assume
$$p = P \sin BAZ,$$

or the parallax in any position is equal to the horizontal parallax into the sine of the observed zenith distance.

The correction for parallax must be added to the observed altitude to give the true altitude. The parallax of the sun is small, never exceeding $9''$, and in this work is neglected.

Refraction.

A ray of light coming obliquely through the atmosphere is bent out of a straight line so that the observer sees a heavenly body *above* its true position.

Let SO (Fig. 40) be the direction of the ray before refraction, and OS'' that of the refracted ray; then, from the laws of refraction, OS, OZ, OS'' lie in the same plane, and $\frac{\sin ZOS}{\sin Z'OS''}$ is constant for the same media, whatever the angle SOZ. Call this ratio i; then $\sin(x+y) = i \sin x$,
$$\sin x \cos y + \cos x \sin y = i \sin x.$$

The angle of refraction y is small, and $\sin y = y$ approximately, and $\cos y = 1$ approximately. Substituting these values,

$$\sin x + y \cos x = i \sin x, \quad y = (i - 1) \tan x.$$

Let Y equal the refraction when $ZOS = 45°$; then $\tan x = 1$ and $Y = i - 1$. Substituting these values we have,

$$y = Y \tan x;$$

or the refraction for any zenith distance is equal to the product of refraction at 45° into the tangent of the observed zenith distance.

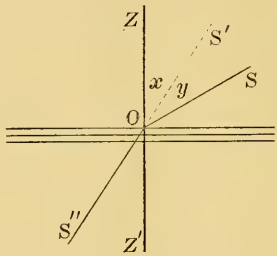

FIG. 40.

With the barometer at 30 in. and the thermometer at 50° F., the refraction at 45° is 58.2″.

For all altitudes of 20° or over measured with a sextant or engineer's transit it is permissible to take the *refraction correction in minutes equal to the natural tangent of the observed zenith distance.* For altitudes less than 20° the correction obtained in this way will be too large.

If the altitude of a heavenly body is measured with a sextant, the true altitude = observed altitude − dip + parallax − refraction.

If the transit is used, there is no correction for dip.

When the body observed is the sun or moon, the altitude of one limb is measured, and the angular semi-diameter must be added or subtracted according as the lower or the upper limb is observed.

To find Latitude.

First. By observation on a circumpolar star at upper or lower culmination.

Let P (Fig. 41) represent the pole, S' a circumpolar star at upper and S a circumpolar star at lower culmination.

Let $h' = S'H'$ = the true altitude of S', and h the true altitude of S. Let $d = QS'$ = the declination; then latitude $= h + (90° - d)$ for lower culmination, and latitude $= h' - (90° - d)$ for upper culmination.

The latitude may also be found by observing, when possible, the altitude of a circumpolar star at both upper and lower culmination, and taking one-half the sum.

Double latitude $= h + (90° - d) + h' - (90° - d)$, or latitude $= \frac{1}{2}(h + h')$. In this case it is not necessary to know the declination.

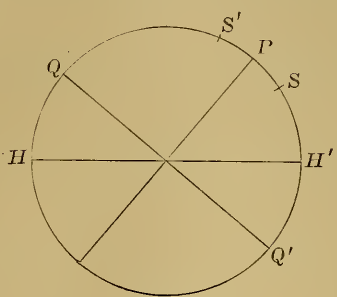

FIG. 41.

For the greater part of the year this observation cannot be taken, because one of the culminations occurs during daylight.

Example. — The altitude of Polaris at upper culmination, as observed with an engineer's transit, was 45° 28′. Declination = 88° 45′ 15″; what was the latitude?

$$\begin{array}{rl} 45° \; 28' & = \text{obs. alt.} \\ 01' & = \text{refr. cor.} \\ \hline 45° \; 27' & = \text{true altitude.} \\ 88° \; 45' \; 15'' & \\ \hline 134° \; 12' \; 15'' & \\ 90° & \\ \hline 44° \; 12' \; 15'' \text{ N.} & = \text{latitude.} \end{array}$$

Second. By observing the meridian altitude of any star.

In the northern hemisphere the meridian altitude of any star that culminates south of the zenith, minus its declination, is equal to the altitude of the equator. This is the complement of the latitude. If the star culminates north of the zenith, $(180° - h)$ must be substituted for the altitude.

Example. — The observed meridian altitude of the sun's lower limb, as measured from the visible horizon at sea, was 33° 22′ 30″; sun's semidiameter, 16′; height of eye above surface of water, 16 ft.; sun's declination south, 13° 03′. Required latitude.

33° 22′ 30″	obs. alt.
1′ 30″	refr. cor.
33° 21′	
04′	cor. for dip.
33° 17′	
16′	sun's diam.
33° 33′	true altitude of sun's centre.
− 13° 03′	sun's declination.
46° 36′	= co. latitude.
43° 24′ N.	= latitude.

Third. By the altitude of the sun or a star in any position, the time being known.

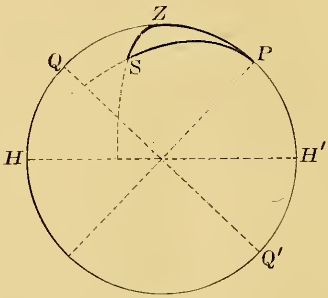

Fig. 42.

This requires the solution of a spherical triangle. Let S (Fig. 42) represent the position of the sun or a star whose altitude has been measured. In the triangle ZSP,

$$ZS = 90° - \text{altitude},$$
$$SP = 90° - \text{declination},$$
$$ZP = 90° - \text{latitude},$$

and the angle $ZPS = 360° -$ time expressed in degrees. Therefore in the triangle ZSP we have two sides, and the angle opposite one of them given, to find the third side, which is the complement of the latitude required.

Time.

Apparent time is the time of the true sun.

An apparent solar day is the interval of time between two successive transits of the sun over the same meridian.

Because the motion of the earth about the sun is not uniform, and also because it moves in the plane of the ecliptic instead of that of the equator, solar days are not of the same length, the greatest variation being about sixteen minutes.

To obviate this difficulty, an imaginary sun is supposed to revolve in the equator, about the earth, in the same time that the real sun appears to revolve in the ecliptic: this latter gives uniform motion.

A mean solar day is the interval of time between two successive transits of this imaginary sun over the same meridian. The equation of time is the time that has to be added algebraically to apparent time to give mean time.

The civil day begins at midnight, and is divided into two parts of twelve hours each. The astronomical day begins at noon of the civil day, and is divided into twenty-four hours.

To change from civil time to astronomical time: If it is A.M. civil time, subtract one day and add twelve hours; if it is P.M., omit the P.M.

To change apparent time to mean solar time: Add algebraically the equation of time to apparent time, and the result is mean solar time.

To determine local time by observing the altitude of the sun in the morning or afternoon, the latitude being known: In Fig. 42, let S represent the sun, the altitude of which has been measured, Z the zenith, and P the pole. In the triangle ZSP, $SZ = 90° -$ altitude, $PS = 90° -$ declination, and $PZ = 90° -$ latitude.

Let $h =$ altitude, $l =$ latitude, $d =$ declination, and $z =$ zenith distance;

$$\text{then } \sin \tfrac{1}{2} SPZ = \sqrt{\frac{\sin \frac{1}{2}[z + (l - d)] \sin \frac{1}{2}[z - (l - d)]}{\cos l \cos d}}.$$

The angle SPZ is the hour angle if the altitude is measured in the afternoon. If the altitude is measured in the morning, the hour angle is 360° minus the angle obtained by the solution of the triangle. The hour angle changed to hours, minutes, and seconds is local apparent time, which may be changed to mean local time by adding the equation of time.

Example. — Aug. 11, 1894, A.M. In latitude 42° 30′ N. the observed altitude of the sun's lower limb was 38° 19′; height of eye, 25 ft.; sun's declination, 15° 12′ N.; semi-diameter, 16′; equation of time, $+ 5^{m.} 01^{s.}$. Required local time.

	$42° 30' = l$	colog cos =	0.1324
	$15° 12' = d$	colog cos =	0.0155
38° 19′	$27° 18' = (l - d)$		
5′ dip.	$51° 32' = z$		
1′ refr. cor.	2)78° 50′		
38° 13′	$39° 25' = \frac{1}{2}[z + (l - d)]$	log sin =	9.8027
16′ semi-diam.	2)24° 14′		
$38° 28' = h$.	$12° 07' = \frac{1}{2}[z - (l - d)]$	log sin =	9.3220
			2)9.2726
		log sin $\frac{1}{2}$ SPZ =	9.6363
		$\frac{1}{2}$ SPZ =	25° 38′.9
		SPZ =	51° 17′.8.

$360° - SPZ = 308° 42'.2 = 20$ h. 34 m. 49 s.

$= 8^{h.}\ 34^{m.}\ 49^{s.}$ A.M. apparent time
$\quad 5\ \ 01$ eq. of t.

$8^{h.}\ 39^{m.}\ 50^{s.}$ A.M. mean local time.

Ans.

Aug. 6, 1894, P.M. In latitude 42° 30′ N. the observed altitude of the sun's upper limb was 32° 55′; height of eye, 16 ft.; sun's declination, 16° 32′ N.; semi-diameter, 16′; equation of time, $+ 5^{m.}\ 39^{s.}$. Required mean local time. $4^{h.}\ 08^{m.}\ 27^{s.}$. *Ans.*

To establish a Meridian Line by an Observation on the North Star (Polaris) at Elongation.

Polaris appears to revolve about the pole in a small circle in a little less than twenty-four hours. When at its extreme east or west point, it is said to be at elongation. Fifteen or twenty minutes before the star gets to its elongation, set up the transit at one end of the line to be established and sight on the star. The star will move toward the east if the elongation is east, and toward the west if the elongation is west. When the star reaches its greatest elongation, it will move vertically along the cross-hair for some time, and then leave it in a direction opposite to its former motion. As long as the star moves toward the point of its elongation, the cross-hair must be kept sighted on it, by turning the tangent screw. When the point of elongation is reached, the line defined by the transit must be established on the ground, by driving a stake or spike in this line at some point 100 to 200 ft. from the transit.

To establish a meridian, lay off from this line the azimuth of Polaris, to the *right* if west elongation was observed, and to the *left* if east elongation was observed, the transit being set over the south end of the line.

To find Azimuth.

$$\text{sin azimuth} = \frac{\text{sin polar distance}}{\text{cos latitude}}.$$

It is necessary to luminate the cross-hairs when observing at night. This is done by means of a lantern, and a reflector made to fit over the object-glass end of the telescope.

A temporary reflector may be made by fastening over the object-glass end of the telescope a piece of oiled paper, leaving a hole one-fourth to one-half inch in diameter over the centre of the object-glass.

The times at which the elongations of Polaris occur are given in Bulletin No. 14, published by the United States Coast and Geodetic Survey, at Washington.

NAVIGATION.

There are two ways by which the navigator determines the position of his vessel at sea: by *dead reckoning* and by *observation.*

In navigation by dead reckoning the courses and distances run are measured by the compass and log.

In navigation by observation the position is determined by observing the altitude of the sun or some other heavenly body.

The mariner's compass, shown in Fig. 43, consists of a circular card divided into thirty-two equal parts called points. Attached to the under

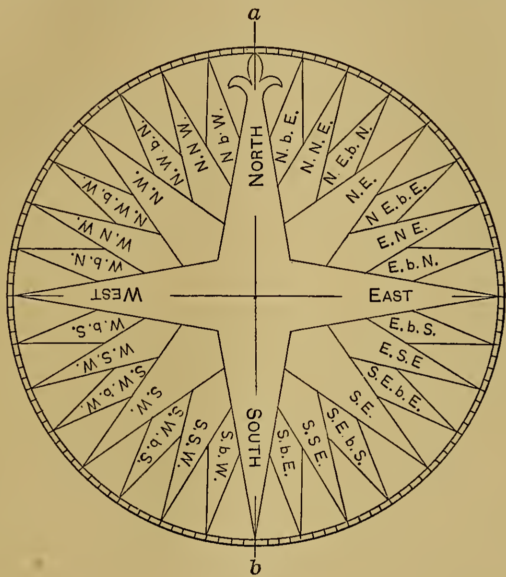

Fig. 43.

side of the card, in the direction of its north and south line, is a magnetic needle, supported at its centre on a pivot. Both swing in gimbals in a box, so that the card remains horizontal whatever the inclination of the ship. On the inside of the box are two points, *a* and *b*, the line connecting which is parallel to the ship's keel; so that if *a* is toward the bow, the point on the card opposite it will give the ship's course.

Each of the thirty-two points are subdivided into quarter-points. Naming the points in order from the north through the east, south, and west, is called *boxing the compass.*

The compass gives the magnetic course. In order to determine the true course, the magnetic course must be corrected for *variation* and *deviation.*

Variation of the mariner's compass is the angle that the needle makes with the true meridian. It is the same as *declination* of the surveyor's compass.

Deviation is the change in the direction of the pointing of the needle caused by the iron and steel used in constructing the ship. Deviation changes with the direction in which the ship is headed. It is generally counteracted as much as possible by placing permanent magnets near the compass.

Leeway is the angle which a vessel's course makes with her keel. It is caused by the wind sliding the vessel to leeward.

Although leeway is not a compass error, the effect is the same as if it were, and the compass course must be corrected for it, to give the true course of the ship.

When the variation is $\frac{\text{west}}{\text{east}}$, the true course is to the $\frac{\text{left}}{\text{right}}$ of the compass course. The same rule applies in correcting for deviation.

The correction for leeway on the $\frac{\text{starboard}}{\text{port}}$ tack is the same as for $\frac{\text{west}}{\text{east}}$ variation.

A ship is on the $\frac{\text{starboard}}{\text{port}}$ tack when the wind blows on the $\frac{\text{right}}{\text{left}}$ side of the ship.

Compass Course.	Variation.	Deviation.	Leeway.	True Course.
W.S.W.	$\frac{1}{4}$ pt. W.	$\frac{1}{2}$ pt. W.	$\frac{1}{4}$ pt. port.	S.W. by W. $\frac{1}{2}$ W.
N. by E.	1 pt. E.	$\frac{1}{4}$ pt. W.	$\frac{1}{2}$ pt. starb.	N. by E. $\frac{1}{4}$ E.
N.W. $\frac{3}{4}$ N.	$2\frac{1}{2}$ pts. W.	$\frac{1}{4}$ pt. E.	$\frac{1}{2}$ pt. starb.	W.N.W.

The common log consists of a reel, line, and chip. The chip (Fig. 44) is a flat piece of wood shaped like a sector of a circle, weighted with lead on its curved edge to make it stand upright in the water. A jerk on the line

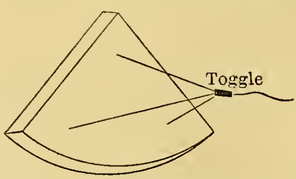

Fig. 44.

will free the two lower cords at the toggle, causing the chip to lie flat, so that it may be easily hauled aboard. With this log a sand-glass is used to measure the time. The line is graduated to the same part of a knot as the part of an hour measured by the sand-glass. This makes the knot 47′ 4″ long when a twenty-eight-second glass is used. Vessels going at high speed use a fourteen-second glass.

The patent log is an instrument fitted with curved blades, shaped somewhat like a ship's propeller, which cause it to revolve when drawn through the water. The number of revolutions are registered on a dial, which is graduated so as to record the revolutions per minute in knots per hour.

The log gives the speed of the vessel in relation to the water. If there are currents, then the speed indicated by the log must be corrected; *i.e.* if sailing against a current, deduct its speed from that given by the log; if sailing with the current, add its speed.

In a head wind the log is apt to overrate, and *vice versa.*

A knot is equal to one minute (1′) of latitude, or to one minute of longitude at the equator. Thus if a vessel sails due N. 60 knots from latitude 40° S., she will arrive in latitude 39° S. Since all meridians converge, a knot is equal to a minute of longitude only at the equator.

The difference in longitude in minutes of angle may, however, be found when sailing due E. or W., by dividing the distance in knots by the cosine of the latitude.

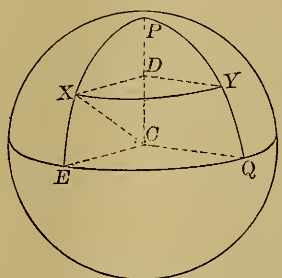

Fig. 45.

Let P (Fig. 45) be the pole, EQ an arc of the equator, XY a parallel in latitude EX; then $\angle ECX = \angle CXD =$ latitude $= L$. Cos $CXD = \frac{DX}{CX}$; or $\cos L = \frac{DX}{R}$. $R =$ radius.

Since similar arcs are to each other as their radii,

$$EQ : XY :: R : R \cos L.$$

Hence $EQ = \frac{XY}{\cos L}$. But EQ is the difference in longitude, and XY is the distance sailed E. or W.

A ship sails 55 knots due E. from latitude 40, longitude 65 W. Find her longitude.

$$\frac{55}{.766} = 72' = 1^\circ\ 12'. \quad 65^\circ - 1^\circ\ 12' = 63^\circ\ 48'\ \text{W.} \qquad \textit{Ans.}$$

Middle Latitude Sailing.

If a vessel sails on any course other than a meridian or parallel the new position may be found by computing the latitude and departure in the same way that they are computed in compass surveys. The latitude gives the distance sailed N. or S. of the starting-point, and the departure the distance E. or W. In like manner, if a vessel sails on several courses, the algebraic sum of all the latitudes gives the distance sailed N. or S., and the algebraic sum of all the departures gives the distance sailed E. or W. If the distances are in knots, the algebraic sum of the latitudes is the difference of latitude in minutes of angle. To determine the difference in longitude, the algebraic sum of the departures must be divided by the cosine of the latitude; but the course sailed was not on any one parallel of latitude. We may, however, determine the difference in longitude by dividing the departure by the cosine of the mean of the latitudes of the two ends of the course; or, in other words, by dividing the departure by the cosine of the middle latitude. This will give the difference in longitude somewhat less than the true difference. In low latitudes, where the difference in latitude of the two ends of the course is not great, the error is small.

A vessel sails from lat. 40° 28′ N., long. 74° 01′ W., as follows:

E. by N.	10 knots.
E.S.E.	20 "
S.E. by E.	26 "
S.S.W.	15 "

Require her lat. and long.

Course.	Angle.	Distance. Knots.	Latitude.		Departure.	
			N.	S.	E.	W.
E. by N.	78° 45′	10	1.95	——	9.81	——
E.S.E.	67° 30′	20	——	14.44	21.62	——
S.E. by E.	56° 15′	26	——	7.65	18.48	——
S.S.W.	22° 30′	15	——	13.86	——	5.74
			1.95	35.95	49.91	5.74
				1.95	5.74	
			Diff. in lat. in min. =	34.00	44.17	

$$40° 28' - 34' = 39° 54' \text{ N.} = \text{latitude.}$$

$$40° 11' = \text{middle latitude.} \quad \cos = .7640.$$

$$\frac{44.17}{.7640} = 58' = \text{diff. in long. in minutes.}$$

$$74° 01' - 58' = 73° 03' \text{ W.} = \text{longitude.}$$

A chronometer is a well-made and nicely adjusted timepiece swung in gimbals and set to Greenwich time.

The *rate* of a chronometer is the amount of time that it gains or loses in a day.

The difference between the chronometer time and Greenwich time and the rate of the chronometer are determined when in port. Knowing these, the Greenwich time may be determined, when desired, during the voyage.

On June 12 a chronometer compared with Greenwich time was $2^{m.} 14^{s.}$ fast; its rate was $1.1^{s.}$ losing. What was the Greenwich time corresponding to $4^{h.} 40^{m.} 17^{s.}$ June 20?

$$8 \times 1.1^{s.} = 9^{s.} -. \quad 2^{m.} 14^{s.} - 9^{s.} = 2^{m.} 5^{s.},$$

$$4^{h.} 40^{m.} 17^{s.} - 2^{m.} 5^{s.} = 4^{h.} 38^{m.} 12^{s.} \text{ Greenwich time.}$$

Since position determined by dead reckoning is always liable to error, it is customary to determine the position of a ship by observation, daily if possible.

In determining the position of a ship by observation, the latitude is found by measuring the meridian altitude of the sun, as explained in the chapter on astronomy.

The longitude is determined by measuring the altitude of the sun when it bears nearly E. or W., and computing local time. The ship's chronometer gives Greenwich time. The difference between these is the difference in longitude. The longitude is W. if the Greenwich time is later than the local time, and E. if it is earlier.

The change in latitude during the interval of time between the observation for latitude and that for time is found by dead reckoning. It is necessary to compute this in order to get the latitude to be used in solving for time.

On Feb. 28, 1896, at $21^{h.} 40^{m.}$ approximate Greenwich time, the observed meridian altitude of the sun's lower limb was $33° 36' 30''$, the sun bearing south. Height of eye above the sea 20 ft.; sun's declination $7° 41' 30''$ S.; semi-diameter $16'$. Required the latitude. $48° 32'$ N. *Ans.*

On May 20, 1896, at $22^{h.}$ $20^{m.}$ approximate Greenwich time, the observed meridian altitude of the sun's lower limb was $68° 12' 30''$, the sun bearing south. Height of eye 25 ft.; sun's declination 20° 20′ N.; semi-diameter 16′. Required the latitude. 41° 57′ N. *Ans.*

On Sept. 29, 1896, at 0 h. 20 m. approximate Greenwich time, the observed meridian altitude of the sun's upper limb was 63° 25′, the sun bearing north. Height of eye 20 ft.; sun's declination 2° 43′ S.; semi-diameter 16′. Required the latitude. 29° 39′ S. *Ans.*

On March 11, 1896, A.M., in latitude 47° 28′ N., the observed altitude of the sun's lower limb was 26° 25′. The corresponding Greenwich time was 23 h. 31 m. 14 s.. Height of eye 25 ft.; sun's declination 3° 24′ S.; semi-diameter 16′; equation of time + 9 m. 59 s.. Required the longitude.

2 h. 14 m. 30 s. W. *Ans.*

On Sept. 6, 1896, in latitude 36° 18′ N., the observed altitude of the sun's lower limb was 33° 14′. The corresponding Greenwich time was 17 h. 20 m. 41 s.. Height of eye 20 ft.; sun's declination 6° 16′ N.; semi-diameter 16′; equation of time − 1 m. 51 s.. Required the longitude.

10 h. 08 m. 08 s. E. *Ans.*

STADIA TABLES.[1]

M.	0°		1°		2°		3°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$c = 1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

[1] These tables were computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania.

M.	4°		5°		6°		7°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c = 0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$c = 1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$c = 1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

M.	8°		9°		10°		11°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$c = 0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$c = 1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$c = 1.25$	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

M.	12°		13°		14°		15°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	25.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c = 0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$c = 1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$c = 1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

M.	16°		17°		18°		19°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.32	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
$c = 0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$c = 1.00$	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$c = 1.25$	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42

M.	20°		21°		22°		23°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87 66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85 19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c = 0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$c = 1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$c = 1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

M.	24°		25°		26°		27°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c = 0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$c = 1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$c = 1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

M.	28°		29°		30°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
$c = 0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$c = 1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$c = 1.25$	1.10	0.60	1.09	0.62	1.08	0.64

A TABLE OF MEAN REFRACTIONS IN DECLINATION.

To apply on the declination arc of Solar Attachment of either Compasses or Transits.

Computed by Edward W. Arms, C. E., for W. & L. E. Gurley, Troy, N. Y.

HOUR ANGLE.	DECLINATIONS.								
	For Latitude 15°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h.	−05″	0″	+05″	10″	15″	21″	27″	33″	40″
2	−03	+02	07	12	18	23	29	36	43
3	+01	05	11	16	22	28	34	41	49
4	08	12	19	24	30	37	44	53	1′04
5	29	34	41	49	59	1′10	1′24	1′43	2 08
	For Latitude 17° 30′.								
0 h.	−02″	+02″	08″	13″	18″	24″	30″	36″	44″
2	0	05	10	15	21	27	33	40	48
3	+02	10	15	21	27	33	40	48	57
4	13	18	23	29	35	43	51	1′01	1′13
5	34	41	49	58	1′10	1′23	1′41	2 06	2 42
	For Latitude 20°.								
0 h.	0″	05″	10″	15″	21″	27″	33″	40″	48″
2	03	07	13	18	24	30	36	44	52
3	06	13	18	24	30	36	44	52	1′02
4	17	22	28	35	42	50	1′00	1′11	1 26
5	39	47	57	1′07	1′20	1′37	2 00	2 32	3 25
	For Latitude 22° 30′.								
0 h.	02″	08″	13″	18″	24″	30″	36″	44″	52″
2	06	11	15	21	27	33	40	48	57
3	11	15	21	27	33	40	48	57	1′08
4	20	26	32	39	46	56	1′07	1′19	1 37
5	45	53	1′03	1′16	1′31	1′52	2 21	3 07	4 28
	For Latitude 25°.								
0 h.	05″	10″	15″	21″	27″	33″	40″	48″	57″
2	08	14	19	25	31	38	46	54	1′05
3	12	18	24	30	37	44	53	1′04	1 18
4	23	29	35	45	53	1′03	1′16	1 31	1 52
5	49	59	1′10	1′24	1′52	2 07	2 44	3 46	5 43

HOUR ANGLE.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
	DECLINATIONS.								
	For Latitude 27° 30′.								
0 h.	08″	13″	18″	24″	30″	36″	44″	52″	1′02″
2	11	16	22	28	34	41	49	1′00	1 10
3	17	22	28	35	42	50	1′00	1 11	1 26
4	28	35	42	50	1′00	1′11	1 26	1 43	2 09
5	54	1′05	1′18	1′34	1 54	2 24	3 11	4 38	8 15
	For Latitude 30°.								
0 h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06
	For Latitude 32° 30′.								
0 h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′03	1′15	1′31	1 53	2 20	3 05	4 25	7 36	
	For Latitude 35°.								
0 h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 00
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	
	For Latitude 37° 30′.								
0 h.	18″	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″
2	22	28	35	42	50	1′00	1 12	1 26	1 45
3	29	36	43	52	1′02	1 14	1 29	1 49	2 16
4	43	51	1′01	1′13	1 27	1 49	2 14	2 54	4 05
5	1′11	1′26	1 54	2 10	2 49	3 55	6 15	14 58	
	For Latitude 40°.								
0 h.	21″	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″
2	25	32	39	46	52	1′06	1 19	1 35	1 57
3	33	40	48	57	1′08	1 21	1 38	2 02	2 36
4	47	55	1′06	1′19	1 36	1 58	2 30	3 21	4 59
5	1′15	1′31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 42° 30′.								
0 h.	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″
2	28	35	39	50	1′00	1 12	1 26	1 45	2 11
3	36	43	52	1′02	1 13	1 29	1 49	2 17	2 59
4	50	1′00	1′11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′16	1 36	1 58	2 30	3 22	5 00	9 24		

HOUR ANGLE.	DECLINATIONS.								
	For Latitude 45°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h.	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″
2	32	39	46	52	1′06	1 19	1 35	1 57	2 29
3	40	47	56	1′07	1 21	1 38	2 00	2 34	3 29
4	54	1′04	1′16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′23	1 41	2 05	2 41	3 40	5 40	12 02		
	For Latitude 47° 30′.								
0 h.	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″
2	35	42	50	1′00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′27	1 46	2 12	2 52	4 01	6 30	16 19		
	For Latitude 50°.								
0 h.	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″
2	38	46	55	1′06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′03	1′14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		
	For Latitude 52° 30′.								
0 h.	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″	3′05″
2	43	50	59	1 11	1 26	1 42	2 23	2 49	3 55
3	50	1′00	1′11	1 25	1 45	2 11	2 51	2 58	6 22
4	1′05	1 18	1 35	2 10	2 28	3 19	4 53	8 42	
5	1 34	1 56	2 27	3 16	4 47	8 52			
	For Latitude 55°.								
0 h.	40″	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″	3′33″
2	46	55	1′05	1 18	1 34	1 56	2 30	3 15	4 47
3	55	1′06	1 19	1 35	1 58	2 30	3 21	4 58	9 19
4	1′10	1 23	1 42	2 06	2 43	3 44	5 49	12 41	
5	1 37	2 01	2 34	3 28	5 15	10 18			
	For Latitude 57° 30′.								
0 h.	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″	3′05″	4′27″
2	50	59	1 11	1 25	1 43	2 09	2 47	3 51	6 04
3	58	1′10	1 24	1 42	2 07	2 43	3 45	5 50	12 47
4	1′11	1 25	1 43	2 10	2 50	3 55	6 14	14 49	
5	1 41	2 06	2 42	3 42	5 46	12 26			
	For Latitude 60°.								
0 h.	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″	3′33″	5′23″
2	54	1′04	1 17	1 33	1 54	2 24	3 12	4 38	8 15
3	1′03	1 15	1 30	1 51	2 20	3 04	4 24	7 31	24 44
4	1 18	1 34	1 56	2 28	3 18	4 50	8 53		
5	1 45	2 11	2 50	3 57	6 21	15 32			

AN ELEMENTARY TREATISE

ON

SURVEYING AND NAVIGATION

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