A Manual
—OF—
Navigation
For the Lakes.

A
Manual of Navigation
FOR THE LAKES.

By H. C. PEARSONS.

CLEVELAND:
BISSELL & SCRIVENS,
1891.

PREFACE.

It is probably not apprehended by more than a very few, that in the recent adoption of "Standard Time," we have the greatest aid to navigation, on the coast and lakes, that could by any means have been devised.

The experienced seaman knows that "direction" is the ultimate object of all the labored astronomical observations by navigators at sea, and that a knowledge of "place" is precedent to that of "direction."

The determination of place is the office of nautical astronomy, which question alone not infrequently draws largely upon the highest powers of the trained mathematician and astronomer.

But, fortunately, we are relieved of nautical work on the lakes. "The U. S. Lake Survey" has determined the co-ordinates of place for all the light-houses on the whole chain of lakes, and every ship-master on those waters is furnished with the results of this survey, in the "List of Lights of the Northern Lakes and Rivers," for the asking. And as the "standard time," now available in nearly every town in the United States, to the fraction of a minute, relieves us of the "chronometer," as used at sea, we have only our "lead" and "log" and "compass," to look after, but which, as we will find, will give us something to do, for the rapid growth in the magnitude of our vessels, resulting from the enormous growth in the volume of our commerce, make it imperative that those who are to have the care and navigation of them, keep pace in technical and scientific attainment with those who design and build them.

Accordingly, to help in this matter, I have prepared the following **Manual of Navigation for the Lakes.**

Among those features that may be considered novel, are the articles on the Dumb Compass, with its application to the finding of compass errors; also a method of deducing a table of Time Azimuths of the Sun, from an amplitude when the latitude and the declination are of the same name. This table is available for from one to three hours, requiring but ordinary arithmetical knowledge on the part of the ship-master.

With the admirable charts in use, and with a precise knowledge of the place of every light, **the finding of compass errors is the one problem that suffices for the navigation of the lakes.**

The custom of compensating deviation by means of magnets, is becoming general, the idea being that a needle brought to its place, will remain there indefinitely, and that when it is right on the cardinal points, it is also right on all other points. But this is a sorry mistake. All compasses, are in error, in some parts of the card, even after the most careful adjustment.

The faces that give position to the needle are so many, and so mutable, that it is not safe to depend on the adjustment for more than a short time, particularly in a new vessel.

The remedy lies in **adjusting the ship-master.**

To attain a confidence in his compass, a living confidence, born of certainty, to meet the growing responsibility laid upon him, the ship-master must be educated up to a full understanding of this question.

This subject is not, as too often thought, beyond the reach of very moderate attainments, any more than is book-keeping, telegraphy, or photography. The diligent application that masters the one, will master the other.

Accordingly, I have dwelt on this topic at some length, giving a number of methods of finding compass errors, including that by time azimuths of the sun, which, though costing perhaps a little more time to master, will in the end be found the most efficient and satisfactory of all the methods in use.

It is believed that among the several methods given, of finding compass errors, any ship-master of fair capacity will find some method that he can readily master, so as to utilize it.

The article on the "correction," or compensation of ship's compasses, is based on the method taught by such scientists as Prof. G. B. Airy, Astronomer Royal; Frederick J. Evans, and Archibald Smith, of the Liverpool Compass Committee.

While I cannot claim this little work to be free from fault or imperfection, I yet have a confidence that it will be found of sufficient merit to claim respectful consideration from those interested in the maritime affairs of the lakes, and to be particularly helpful to the young student of navigation, in which last event, my highest ambition will have been gratified.

H. C. PEARSONS.

Ferrysburg, Mich., Oct. 27, 1891

CONTENTS.

A MANUAL OF NAVIGATION FOR THE LAKES.

CHAPTER I.

TRIGONOMETRY.

Signs and Symbols.—The student not already acquainted with algebraic notation, should make himself familiar with the following **Signs** and **Symbols**, which are among the many used in mathematical language:

$=$ This is the sign of **equality;** it implies that the quantities between which it is used are equal, as 12 inches=1 foot, 3 miles =1 league, etc.

$+$ This is called the **plus** sign, implying an increase or addition, as $7+3=10$.

$-$ This is called the **minus** sign, implying diminution or subtraction, as $7-3=4$.

The plus and minus signs are also used to imply relative direction, as to the right or left, up or down, heat or cold, forward or backward, etc.

$\times$ This is the sign of **multiplication,** signifying that the quantities between which it is written are to be multiplied together, as $4\times5=20$.

$\div$ This is the sign of **division,** implying that the number prceding it is to be divided by that one following, as $9\div3=3$.

( ) { } [ ] These are called **brackets.** Their use is to indicate that the quantities embraced within them are to be considered as one quantity, thus, $5(5+3)-6(3+2)=10$.

——— This is another form of bracket, and called a **vinculum** or **bar,** and is used when two or more quantities already connected by brackets, are to be regarded as one quantity, as $\overline{5\,(5+3)-6}$ $\overline{(6-1)}\div\overline{5\,(8-6)}\div 2=5$.

: :: : These are the signs of **proportion.** They are a modification of the signs of division ÷, and equality =, and may be written thus, ÷ = ÷. They signify that the first term divided by the second=the third term divided by the fourth. Quantities having this relation are called proportional quantities or numbers, as 4:8::3:6,, or better, $4\div8=3\div6=\frac{1}{2}$. The value in this case, $\frac{1}{2}$, is called the **ratio** of the proportion.

A **power** of a number is indicated by a small figure above and to the right of the number, thus, $4^2=16$ indicates that the number 4 is used twice as a factor. $4^3=64$.

A **root** is indicated by the symbol $\sqrt{\ }$, which alone implies the second or square root. . A small figure written over the symbol, thus, $\sqrt[3]{\ }$, implies the degree of the root, thus, $\sqrt[3]{64}=4$.

Symbols of Quantity.—To abreviate mathematical investigations, quantities or numbers are represented by some symbol, as a letter,—usually the initials of the quantity; and the symbols of operation already explained, apply to them the same as to the numerical quantities.

Lines are separated by letters at different points in their extent, usually at their extremities. Thus, the line between the points A———B is called the line AB.

Angles are defined by giving one letter in each of the lines containing the angle, together with the letter at their intersection, thus, the angle between the two lines 

AB and BC, is written ABC, the middle letter indicating the angular point.

Numerals are represented by single letters, the leading letters of the alphabet representing known quantities and the final letters unknown quantities.

The use of an equation is, generally, to bring out the value of some unknown quantity, which is the object of inquiry, and which is involved in the equation. Thus, in a proportion, one of the terms is usually an unknown or required quantity represented by one of the terminal letters (usually x) of the alphabet, as

A:B::C:X.

From this, by the principle well known in arithmetic, that the product of the extremes equals the product of the means, we have,

AX=BC,

in which X is the quantity desired.

Then, from the well known axiom, that we may multiply or divide equal quantities by the same number without destroying the equality between those quantities, we may divide both sides of this equation by the factor, when we have the equation,

$$X = BC \div A,$$

in which the quantity represented by X is equal to the product of the two quantities B and C divided by the quantity or factor A. This operation is called solving the equation for X, and the expression, $X = BC \div A$, is called a **formula.**

Algebraic Addition is the aggregation into one sum, of quantities affected by the opposite signs of +, —, or plus and minus. Thus, if a man has $100 in bank to his credit, or +, and he has bills payable, —, $90, his estate is only $10—that is to say, the sum of Dr. and Cr. sides of his % is + $10, the two signs cancelling each other to the extent of the smaller number.

Algebraic Difference is found by changing the sign of the quantity to be subtracted, then combining the quantities as in addition. Example: The difference of latitude between 10° N. and 5° S., or +10° and —5° = $\pm$15°. By changing either sign, both terms become "like," i. e., both + or both —, and they come together by addition, so that we may have +15° or —15° as the difference of latitude of the two places.

Trigonometry is that branch of geometry that treats of the relations of the sides and the angles of triangles, both plane and spherical. In this work, only plane triangles will be considered.

In all the wide range of mathematical science, there is, perhaps, no one branch so generally useful or of so wide an application in general affairs of life, as trigonometry.

To the navigator, the astronomer, surveyor, geographer and the civil engineer, it is simply indispensible. Natural philosophy, mechanics, optics and geology could not be treated without this branch of mathematics.

It is hoped the little I may introduce of the subject, and which is as little as we can do with, will induce the student not already familiar with it, to pursue it further in some of the many excellent standard works on that science.

Whatever we may give of trigonometry, we shall treat without the use of logarithms,—the object being to make the subject as elementary as possible.

There are two kinds of magnitude considered in triangles, viz., **angles and sides or lines.**

For the purpose of measuring angles, a circle is introduced. The circumference of this circle is supposed to be divided into 360 equal parts, called degrees; these are further divided into 60 equal parts, called minutes; and these, again, into 60 equal parts called seconds.

The degrees are indicated by a small circle ° over and to the right of the figure; minutes by a dash ′, and seconds by two dashes ″ above and to the right of the number of minutes or seconds. Thus, 15 degrees, 28 minutes and 46 seconds would be written,

15°, 28′, 46″, etc.

A degree or a minute, then, is not a magnitude of length, but merely a certain part of the whole circumference, without regard to lineal dimensions; it is merely ratio.

In measuring an angle, the center of the measuring circle is located at the intersection of the two lines that contain the angle to be measured. The arc intercepted by the two lines is the measure of the angle.

But, as degrees, etc., and lines are of different kinds of magnitude, they cannot be directly compared. Moreover, the magnitude of an angle does not vary with the size of the triangle. So, to make these elements comparable, **auxilliary lines** have been introduced in and about the arc to be measured, or considered, in such a manner as to make them depend not only on the magnitude of the arc, but on that of the triangle to be considered.

These auxilliary lines are called **functions** of the arc, and have received different names according to the different positions or relations they have to the arc.

Names of the Functions. These are eight in number, as follows (see Fig. 1):

In the triangle ADC, let the arc AB be the measure of the angle ACD. Then (1) **the Sine** is the perpendicular BG drawn from one extremity, B, of the arc, to a diameter or radius drawn through the other extremity of the arc.

(2). **The Tangent, AD,** is drawn perpendicular to the diameter through one end of the arc, to meet a line drawn from the center of the circle through the other end of the arc, as at D.

(3). **The Secant, CD,** is a right line from the center through one end of the arc, to meet the tangent, as in D.

(4). **The Cosine, CG,** is that part of the radius intercepted between the foot of the sine and the angular point C.

(5). **The Versine, AG,** is that part of the radius intercepted between the foot of the sine and the extremity of the arc, or the foot of the tangent.

The **Compliment** of an arc or angle is what the angle lacks of being a full quadrant, or 90°. Thus, the angle ACE, being a right angle or 90°, the arc BE is the compliment of the arc AB. The prefix "co" being used to show that the function before which it is used pertains to the **compliment** of the arc under consideration. Thus:

(6). **The Co-Tangent, EF,** is the tangent of the compliment of the arc AB.

(7). **The Co-Secant, CF,** is the secant of the compliment of the arc AB.

(8). **The Co-Versed Sine, E H,** is the versed sine of the arc AB.

The **Supplement** of an arc or angle, is what the arc lacks of 180°. Thus, the arc BCI, embracing the angle DCI, is the supplement of the arc AB. And it has the same sine BG as the arc AB.

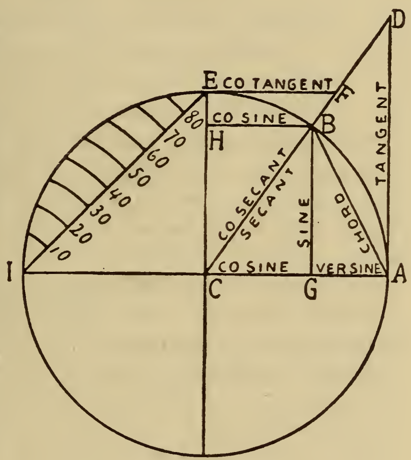


Fig. 1.

The **Sine and Cosine** connect the two arcs AB and BE, so that when one is known the other is also known, for it will be observed that the sine of one of the arcs is the cosine of the other. The student will observe that the **sine** and **tangent** of an angle are always **opposite** the angle.

Relations Between the Parts of a Triangle.—It is shown in geometry that the sum of the two acute angles of a right plane triangle **is equal to two right angles,** so that when one of the two is known the other is also known.

The **Radius** is any right line drawn from the center to the circumference of the circle. The radius diminished by cosine is

versine (see AG in Fig. 1), and diminished by sine, is **co-versine,** EH.

In tables of trigonometrical functions, radius being unity, all the internal functions, as sine, cosine, etc., are **decimal fractions,** as also the tangents of arcs less than 45°, and the co-tangents of arcs greater than 45°. This is more particularly shown in the following table of the

Limits of Trigonometrical Functions:—Let AB be any arc (see Fig. 1) varying from 0 at B to 90° at E; then when

AB=0, the sine=0, the versine=0 and cosine=1.
" tan.=0, " secant=1 and cotan.=infinity.
AB=90°, the sine=1, the versine=1 and cosine=0.
the tan.=infinity, the secant=infinity and cotan.=0.
AB=45°, the tan.=1 and cotan.=1.
AB=60°, " cosine=$\frac{1}{2}$ and versine=$\frac{1}{2}$.

The student should make himself familiar with this table, so that he can tell the limits of any function on hearing it called. He should also observe the effect of any change in the arc AB. Thus, if AB be increased, the radius remaining the same, then all the primitive or direct functions will be **increased,** while the co-functions will be **diminished.** Again, if the radius AC be increased, the angle or arc BA remaining the same, then **all the functions** will be **increased** in the same proportion.

Thus it is seen that no change can be made in the magnitude of the angle or in the radius of the measuring circle, without a corresponding change in the value of all these auxilliary lines. So it is the introduction of these lines that has brought the solution of triangles within the pale of arithmetic.

The two sides containing the right angle are called the **legs.** Sometimes **base** and **perpendicular.** But any side may be made the **radius,** i. e., the side by which the others are measured, when the other sides take names according to their relation to the angles of the triangle: Thus, in the following triangle, ABC, if BC is made the unit for measuring the other sides, then AB is sine and AC is the cosine of angle C.

Again, if in Fig. 3, AC be made the radius, or measuring unit, then AB becomes the tangent and BC the secant of angle C.

Or if, as in Fig. 4, the perpendicular AB be made radius, then the base AC becomes tangent and BC the secant of the angle B.

The student should make himself familiar with these changes.

The relations existing between the functions of arcs, are those resulting from the comparison of **similar triangles,** and consist of **equality of ratios,** which is a fundamental principle of trigonometry.

Fig. 2.

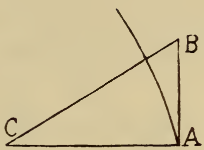


Fig. 3.

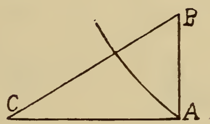


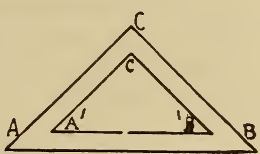
Fig. 4.

Thus, in Fig. 5, the triangles ABC and $A^1 B^1 C^1$, having their sides opposite the equal angles A, A^1; B, B^1, etc., parallel, are **similar.**

Also the triangles ABC and $A^1 B^1 E$ (Fig. 6), and ADE of same Fig., having their **"like" sides** parallel, are all **similar triangles.** **"Like Sides"** are those that are opposite equal angles. They are also called **homologous sides;** thus, BC and DE, Fig. 6, are homologous sides, being opposite the angle A, with the sides BC and DE parallel. Again, the sides AD, $A^1 B^1$, being parallel, and opposite the same angle E, are homologous.

The ratios of these **homologous sides is the same** between any two "like" sides of similar triangles, i. e., if any side of one of the triangles be divided or measured by a "like" side in the other triangle, the quotient will be the same as that of any other side in the same triangle, measured by its corresponding "like" in the other.

Fig. 5.



This relation for a right triangle is expressed thus, for any angle, as ACB

$$\frac{BC}{DE} = \frac{AC}{AE} = \frac{AB}{AD} \text{ or}$$

$$\frac{\text{Cosine}}{\text{R}} = \frac{\text{Sine}}{\text{Tan.}} = \frac{\text{R}}{\text{Secant}}$$

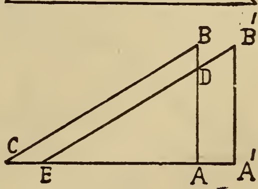


Fig. 6.

These relations are the foundation of the several rules for the solution of right triangles, and when stated in the form of a proposition, are as follows :

Cos.: R:: Sin.: Tan. :: R: Sec.	Sin.: Cos.:: R: Cotan. :: R: Cosec.

Here, it will be seen, are five quantities besides the right angle, —three sides and two angles—any three of which being given,—one being a side,—the other two may be found. Hence,

In a right angled triangle,—the two acute angles being compliments of each other,—**there are only four parts to be considered,** viz., three sides and one angle.

Some two of the six ratios of the preceding article will always contain the two given quantities and the required quantity, And we have the four following cases, viz.:

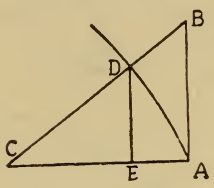


Fig. 7.

(I). Given, the **Hypothenuse and Angles,** to find the two sides containing the right angle.

(II). Given, the **Hypothenuse and one Side,** to find the two angles and the other side.

(III). Given, the **two Angles and one Side,** to find the hypothenuse and the other side.

(IV). Given, the **two Sides,** to find the angles and the hypothenuse.

To Solve a Right Triangle, for any of its parts, it is only necessary to select any two of the six ratios that contain the two given elements or parts and the required part of the problem, and **arrange them in the form of a proportion,** then

Solve the proportion precisely as in arithmetic, by multiplying together the extremes and the means of the proportion in two products, and divide both products by the factor that is found connected with the required quantity.

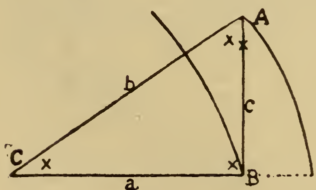


Fig. 8.

Example. Suppose we wish to find the hypothenuse of a right triangle, the angles and one side, —say the perpendicular,—being given.

Solution. Let ABC be the triangle whose side b is required,—the angles and side AB being given.

It will be found convenient to represent the angles by the capital letters and the sides by the small letters of the same name as the opposite angle.

Making either side, say a, the radius, the given side c is the tangent of the angle C. Also, the required side is the secant of angle C. Thus, our two **given parts** are **R** and **tan.,** and the required part is **sec.** of angle C.

Then, as in arithmetic, form a proportion, so that the ratio of the functions will form the first couplet, and that of given and required side the last couplet, thus:

$$\text{Tan. } C : \text{Sec. } C :: c : b.$$

Multiplying extremes and means, as in arithmetic,

$$\text{Tan. } C\, b = \text{Sec. } C\, c.$$

Dividing both products by first term, as in arithmetic,

$$b = \frac{\text{Sec. } C\, c}{\text{Tan. } C.} \qquad \text{———}(1).$$

Again, making b the radius, then,

$$\text{Sin. } C : R :: c : b, \text{ and, as before,}$$

$$\text{Sin. } C\, b = R\, c, \text{ then dividing and R being unity,}$$

$$b = \frac{C}{\text{Sin. } C.} \qquad \text{———}(2).$$

Again, making AB or c the radius,

$$R : \text{Cosec. } C :: c : b, \text{ or}$$

$$R\, b = \text{Cosec. } C\, c.$$

Whence, by dividing, and remembering that R is unity, we have,

$$b = \text{Cosec. } C\, c$$
$$= \text{Sec. } A. \qquad \text{———}(3).$$

From the above, it will be seen that we may make any side of the triangle the measuring side, but to make the given side, as c, or the required side, as b, in the above example, the measuring unit will give the **simple solutions,** as seen in equations (2) and (3), above.

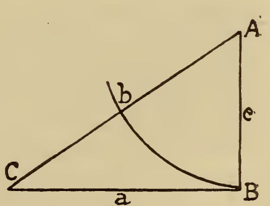


Fig. 9.

Thus, to find the side BC or a, we would in the same manner have,

$$a = c \text{ tan. } A,$$

or, if we make a the radius, then,

$$a = \frac{c}{\tan. c}$$

In this manner many formulas have been constructed for the solution of right plane triangles. From them I have selected a few of the most useful, and which are sufficient for the solution of such questions as are likely to come before the navigator in **"dead reckoning."**

Preparatory to the solution of practical questions, the student must be informed of some of the Trigonometrical Tables and their uses.

Table of the Natural Functions.—These are the Sines, Cosines, Co-tangents, the Secants and Co-secants, Versines and Co-versines of arcs varying by one minute, for a whole quadrant.

They are called natural functions to distinguish them from the same functions when given by their logarithms.

This table gives all the above elements to radius unity,—thus, we have all the three sides and their corresponding angles for 5400 right triangles; and by considering the angles to vary by parts of a minute, as by 10″ or 20″ we could readily deduce all the elements for many thousand more triangles.

So, having the elements of any proportioned triangle to radius unity, that can come before us in practice, we have only to institute a proportion between the known parts of one triangle and the corresponding parts of the tabular triangle, and reduce the proportion to find the parts desired.

These sines, tangents, secants, etc., are arranged in columns, under their respective names.

The degrees for arcs of less than 45° are found at the top of the page, with the minutes in a column at the left. For arcs greater than 45°, the degrees are found at the foot of the page, with the minutes in a column at the right. (See table II).

Let the student look out the following functions. (Our table gives them only to intervals of 5°) :

The natural		sine	of	25°, 40′	= .4331
"	"	cosine	"	"	= .9013
"	"	tangent	"	"	= .4805
"	"	cotangent	"	"	=2.0809
"	"	secant	"	"	=1.1095

The natural cosecant af 25°,40′=2.3087				
"	"	sine	of 74°, 25′=	.9632
"	"	cosine	" " =	.2686
"	"	tangent	" " =	3.5856
"	"	cotangent	" " =	.2789
"	"	secant	" " =	3.7224
"	"	cosecant	" " =	1.0382

The equations spoken of in a former article, as being selected for this work, are the following:

Formulas for the Angles of a Right Triangle.

1. { Sin. C=c÷b=perpendicular÷hypothenuse.
   Cos. C=a÷b=base÷hypothenuse.

2. { Tan. C=c÷a=perpendicular÷base.
   Cotan. C=a÷c=base÷perpendicular.

3. { Sec. C=b÷a=hypothenuse÷base.
   Cosec. C=b÷c=hypothenuse÷perpendicular.

It will be observed that the preceding equations are in pairs, each pair finding an angle by a function with its co-function.

It will be observed, too, that each angle is found indirectly by some function, as sine, cosine, etc., so that the angle correspoding to the function must be found from the table of sines, cosines, etc.

It will be observed, also, that any function is found by dividing one side by some other side of the triangle, which is for the purpose of reducing the sides of the triangle in question, to radius unity, for the purpose of making them comparable with a tabular triangle.

It will also be observed, in the second member of each of the equations in the preceding article, that some one side of the triangle is divided by some one of the other sides. Then, turning this **divisor** over to the other member of the equation, as a **multiplier.** we find a side of the triangle. In this manner we derive the following:

Formulas for the Sides of Right Triangles.

1. { a=b cosine C=hypothenuse×cos. C, or sin. A.
   =c cotan. C=perpendicular×cotan. C, or tan. A.

2. { b=a sec. C=base×sec. C, or cosec. A.
   =c cosec. C=perpendicular×cosec. C, or sec. A.

3. { c=b sin. C=hypothenuse×sin. C, or cosine A.
   =a tan. C=base×tan. C, or cotan. A.

(See Fig. 10).

Geometry gives the following equations for the sides of a triangle, viz.:

$$a=\sqrt{b^2-c^2}=\sqrt{\text{rad.}^2-\sin.^2 C} \text{ ——(1).}$$
$$b=\sqrt{a^2+c^2}=\sqrt{\sin.^2 C+\cos.^2 C} \text{——(2).}$$
$$c=\sqrt{b^2-a^2}=\sqrt{\text{rad.}^2-\cos.^2 C} \text{ ——(3).}$$

The student should make himself so thoroughly familiar with the three sets of equations in each of the three preceding articles, that he can readily select any one wanted for the occasion.

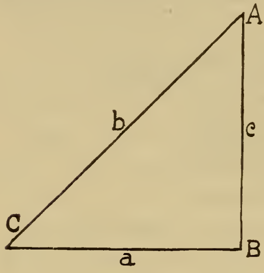


Fig. 10.

With regard to numerical work, the student, having made himself familiar with the preceding articles, is prepared for the solution of plane right triangles, but preparatory to a numerical solution, he should be able to solve them by **construction,** which involves the use of a

Table of Chords of Arcs, or a Protractor.—A scale of chords is sometimes engraved on the ordinary drafting scales. But it is to a particular radius given on the scale,—and therefore of but limited use,—moreover, it is not sufficiently precise for good work, though a good protractor will do.

By a principle of geometry, **the chord of an arc is twice the sine of half the arc.** Whence, a table of chords can be constructed directly from a table of natural sines. Thus,

Required to find the chord of 26°, 28′. The sine of the half of this arc, or 13°, 14′, is .22892. Twice this decimal, retaining four places of figures, is .4578, which is the chord of 26°, 28′ to radius unity. (Our tables, varying by 10′, will give the chord for 26°, 30′=.4584).

In this manner was the table of chords computed. It is arranged with the degrees at the head of the columns and the minutes in columns at the margin of the page. And it is computed for 90°, varying by 10′. (See table VIII).

Thus, the chord of 43°, 10′= .7357
" " 60°, 00′=1.0000
" " 84°, 40′=1.3469
" " 4°, 05′= .0713
" " 15′= .0044

In using a table of chords for the construction or the measurement of angles, the student should have a scale **divided decimally.** Then, by removing the decimal point one or two places to the right, he may have a large scale to set off his decimals with, and thus attain great precision.

He will also want compasses for ink and pencil, a T square and triangle or parallel rule. If not already provided with such instruments, he should send twenty or thirty cents to some dealer in drawing instruments, for an Illustrated Catalogue.

Solution of Plane Triangles.—Examples:

1. Sailed S. 48° W., 126 miles. How far did I sail south, and how far west?

By Construction. Draw a vertical line AB (see Fig. 11) to represent the meridian from which the course is reckoned, A representing the starting point.

Look in the table for the chord of 48°, which we find to be .8135 to radius unity. Removing the decimal point one place to the right, we have 8.135 as the chord of 48° to radius 10.

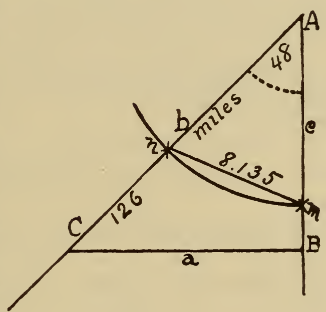


Fig. 11.

Take 10 units to any convenient scale and set them off from A to m. Then take the chord 8.135 to the same scale and set them off from m to n.

Through the points A and n, draw an indefinite line, and on it set off the distance AC, 126 miles, to any convenient scale.

Through C, draw a line at right angles to the line AB, meeting that line in B.

Measure the lines AB and BC by the same scale as that by which AC was set off, and we have the measure desired of the two sides.

AB or c=southings=84.31 miles.
BC or a=westings=93.63 miles.

By Computation. In this problem, it is seen that we have the angles and hypothenuse from which to find the sides a and c, containing the right angle.

By reference to the first equation of the third pair of equations, (page 9), we see that the perpendicular c is found by multiplying the hypothenuse b into the cosine of the angle A, adjacent to c, or into the sine of C, the angle opposite, or

$$c = b \times \cos. (A = 48^\circ)$$
$$= 126 \times .6691 = 84.31 \text{ miles southings.}$$

In the same manner from the first equation of the first pair, we have,

$$a = b \times \text{sine } (A = 48^\circ)$$
$$= 126 \times .7431 = 93.63 \text{ miles westings.}$$

It will be seen that the numerical work is only indicated here. The student should look the functions from the tables and perform the multiplication; and he may also construct the angles with a protractor, though the method by "chords" should be learned.

2. Example: Sailed due north 146 miles, then due east 76 miles. Required the course and distance back to the place sailed from.

By Construction. Draw vertical line on the page to represent the meridian, and set off on it, to any convenient scale, the distance sailed north, 146 miles, C to B.

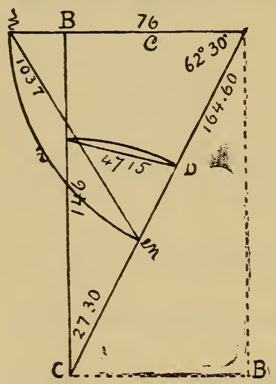


Fig. 12.

Then turn off a line at right angles to the right, and set off on it the distance sailed east, 76 miles, B to A, then join A with C.

AC, measured on the same scale, will be the distance required. We are now to find the angles at A and C. We could apply a protractor and read the angle directly from it, but it is better to measure it by means of its chord to some radius—say 100. Thus,

Take 100 in the compass, to any convenient scale, and sweep an arc from the side a to the side b, with C as the center. Also from the side b to the side c produced, from A as a center, sweep the arc m, n, for the purpose of measuring the angle A.

The object of measuring or computing both angles is to have a **check on our work.** The student will soon see abundant reason for this in the non-precision of his first efforts. A and C together should=90°.

The chord measuring the angle C to radius 100, will be found =47.5, and that for A=103.7.

Removing the decimal point, in each case, two places to the left, for the purpose of finding the chord for radius unity, we have for C, .405, and for A, 1.098. Taking these chords to the table for the angles, we find A=62°, 30′, and C=27°, 30′, and A C=164.6.

As courses are always measured from the meridian, we must take the compliment of the angle A for our return course, which is S. 27°, 30′ W. and distance=164.6 miles.

By Computation. In this case we have the two sides containing the right angle to find the angles and the hypothenuse. By equation 2 (page 9), we have,

Tan. C=perpendicular÷base,
=76÷146=.5205=tan. (27°, 30′=C).

Thus, dividing the perpendicular by the base, we get the tangent of the angle opposite the perpendicular,—in this case, the decimal .5205. Then, looking in the table of natural sines, tangents, etc., we find this decimal in column of tangents, under 27° at the head, and in line of 30′ on the left.

One of the two angles being found, we may subtract it from 90° for the other=62°, 30′.

But, as a check, it is better to compute it in the same manner, by the same rule, thus:

146÷76=1.92105=tan. (A=62°, 30′).

From equation (2) of (page 9), we have,

b=a × secant (C=27°, 30′)
=146 × 1.1274=164.6=distance.

Examples:

3. Given the hypothenuse 26. The angle at the base=26°, 20′. Required base and perpendicular. (See eq. 3 of page 9).

Ans. Base=23.37.
Perp.=11.38.

NOTE.—Let the student construct graphically, with care. He should use a sharp lead pencil, making a line as small as can be clearly seen, and using a needle point for pricking off distances.

4. Given the hypothenuse 146. Angle at the base, 72°, 30′. Find perpendicular and base.

Ans. Base=43.95.
Perp.=89.28.

5. Given hypothenuse 84, base 46. Required the perpendicular and the angles. (See pages 8–9 for angles).

Ans. Angle at base=56°, 50′
" " vertex=33°, 10′
Perpendicular=70.29.

6. Given hypothenuse 218, one of the sides contain the right angle=46. Required the angles and other side.

Ans. Angle opposite the greater leg=77°, 50′
" " " smaller " =12°, 10′
The other leg=213.13.

7. One leg or side=76, angle opposite=43°, 28′. Required the hypothenuse and other side.

Ans. Hypothenuse=110.5.
Other Side, =80.18.

8. One side=243, adjacent angle=18°, 40′. Required the hypothenuse and other side of the triangle.

Ans. Hypothenuse=256.49.
Other Side, = 82.08.

9. Given hypothenuse 180.3, legs 176, and 39.04. Required the angles of the triangle. See group of equations for angles, (page 8).

Ans. Angles, 12°, 37′, and 77°, 23′.

Let the student find an angle, say the smaller one, from each of the three sides, thus:

Hypoth.÷perp.=cot. angle at base=12°, 37′.
Perp.÷base=tan. angle at base=12°, 37′.
Hypoth.÷base=sec. angle at base=12°, 37′.

Note.—As our tables vary by 5′, the nearest minute given by them for this case will be 12°, 35′, and as our compasses are not graduated to appreciate angles smaller than 1°, the practice of computing, using 5 or 6 place decimals with intervals of 1′ and finding all angles to the nearest 1′, is simply **a waste of time,**—accordingly, we have abridged the tables, and thereby the work.

Oblique Plane Triangles.—The solution of oblique plane triangles will require the use of a few formulas different from those required for right plane triangles.

Proposition 1. In any plane triangle, the sides are proportioned to the sines of the opposite angles.

In the oblique triangle ABC, if we make the two sides AC and BC radius, the perpendicular CD is the sine of each of the angles A and B. Then,

AC : CD : : R : sin. A, or AC sin. A=R CD.
And BC : CD : : R : sin. B, or BC sin. B=R CD.

But the second members in each of the above equations are the same, whence the other two are proportioned, and we have,

AC sin. B=BC sin. B, or
AC : BC : : sin. A : sin. B,

according to proposition.

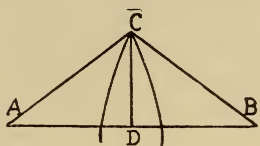


Fig. 13.

Solution by the Properties of Right Triangles,—three cases.—As all cases of right triangles may be solved by the properties of right triangles, we give no more rules for their direct solution, though a number are known.

I. The three parts given may all be adjacent, as when two sides with their included angle are given.

In this case, demit a perpendicular from the extremity of the smaller side onto the other side, as from C to D in the triangle ABC.

In the right triangle A C D, we have hypothenuse and one side, with the angles from which to find the perpendicular C D, which may be found by equations on page 9, as also may the base A D.

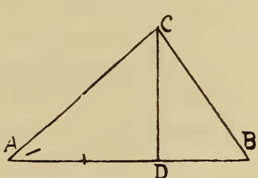


Fig. 14.

In the triangle BCD, subtracting AD from AB, we have BD. Then, CD having been computed, we have the two sides containing the right angle from which to find the angles.

The angles A and B, being now known, we have only to take their sum from 180°. when we have the angle at C, on the principle that the sum of the three angles of any plane triangle is 180°.

The side BC remains to be found. We may find it by Prop. 1, just explained, and called the **sine proportion**, or we may find it by means of the equations for the sides of a right triangle, page 9. By the "sine proportion," the solution is,

Sin. B : AC : : Sin. A : BC. (Page 12).

II. **When the Angles and One Side are Given.**—From one extremity of the given side, demit a perpendicular onto one of the other sides, or onto one of them produced, as from C to D on the side AB produced, of the triangle ABC. (Mark with a dash, —, the given parts).

In the right triangle ACD, we have all the angles and the hypothenuse AC given, from which we may find AD and DC, from rules already explained.

Then, in the right triangle DBC, the angle at B is known, because it is the supplement of B in the oblique triangle ABC, whence both angles of the triangle BDC are known, and DC having been computed, BC can be found from equations, page 9.

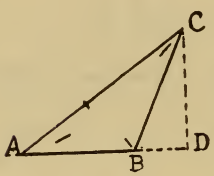


Fig. 15.

Now, AB may be found from the "sine proportion," Prop. 1, page 12, when the triangle is determined.

The proportion for BC is,

Sin. B : AC : : Sin. A : BC,

and for AB it is,

Sin. B : AC : : Sin. C : AB.

Thus the case may be solved either by the rules for right triangles or by the sine proportion.

III. **When the Three Sides are given, to find the Angles.** The solution of this will depend on the following:

Proposition 2. In any plane triangle, as the greater side
: The **sum** of the other two sides,
: : The **difference** of the other two sides,
: The difference of the segments of the greater side made by a perpendicular from the opposite angle.

In the triangle ABC, let AB be the greater side. With AC as a radius, and from C as a center, sweep the arc EAF and produce BC to E. Then BE is the sum of the two smaller sides, and BF is their difference.

AB is the sum of the two segments of the greater side, made by the perpendicular CD, and BG is their difference.

Then, because the two lines BA and BE are two secants, cutting the circle and meeting outside the circle, the whole secants and their external segments are inversely proportional, and we have the proportion,

$$AB : BE :: BF : BG,$$

which is the proposition.

Then, half of this difference **added** to half the base, or greater side=**the greater segment.** And half this difference **subtracted** from half the base, gives the smaller segment.

This being done, the triangle is divided into two right triangles, in each of which the base and hypothenuse are known, from which the angles are determined by the equations on pages 8–9.

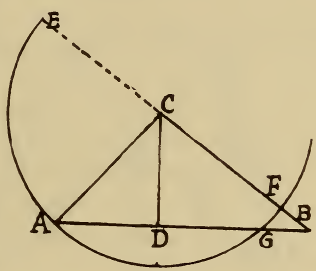


Fig. 16.

Example: In the oblique triangle ABC, given the three sides, 64, 56, 34, to find the angles.

Solution: Demit a perpendicular from C onto the greater side at D, dividing that into the two segments m and n, then by the last proposition we have,

$$\begin{aligned}\text{As}\quad & AB\ (=m+n)=64\\ & : AC+CB=90\\ & :: AC-CB=22\\ & : m-n=90\times 22\div 64=30.9416.\end{aligned}$$

$$\begin{aligned}\text{Then,}\quad & \tfrac{1}{2}(m-n)=15.4708,\\ & \tfrac{1}{2}(m+n)=32.0000.\\ \text{Whence,}\quad & m\quad =47.4708,\\ \text{and}\quad & n\quad =16.5292.\end{aligned}$$

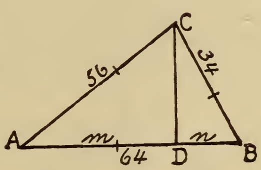


Fig. 17.

The oblique triangle is now divided into two right triangles, in each of which the base and hypothenuse are given to find the angles, which can be done by means of the equations on page 8.

Then, the sum of the angles A and B being taken from 180°, as before explained, we have angle C.

Examples of the three cases of oblique triangles for solution.

1. Given the two sides, 91 and 60, with their included angle, 42°, 30′. Required the other side and the other two angles.

Solution: **First, by Construction.** In the triangle ABC, lay off one side, preferably the longer side AB, to any convenient scale=to the given side, 91. From each point, A and B, sweep an arc on a radius of 10 or 100 units,—say 100, to any convenient scale, for the purpose of measuring the angles at B and C.

On the arc opposite A, set off the chord of 42°, 30′, from n to c, and draw the line A c. On this line, set off from A, the distance AC=60, and join C with B. Then is the triangle constructed.

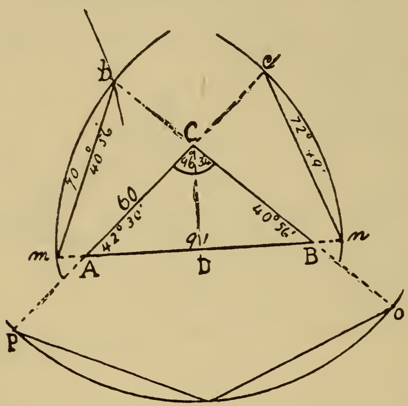


Fig. 18.

Measure BC by the same scale, and that side is determined=(61.8).

Produce BC to b, and measure the chord mb=70, which corresponds to (40°, 56′) for the angle at B. Then 180°—($\overline{B+C}$=83°, 26′)=(96° 34′)=angle at C.

Verify by constructing C thus: With the same radius as for the other angles, sweep an arc from C as center and produce, if necessary, the CB and CA to it, as at o and p. We find the chord o p greater than the chord of 90, the limit of the table of chords, so we measure the arc in two parts, and find it to be (96°, 34′), as before, or verify with a protractor.

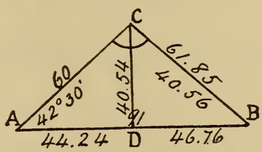


Fig. 19.

By Computation. In the triangle ACD, we have the angles and one side (the hypothenuse) to find the other sides.

Then multiplying AC by sine of A, we have DC or .6756×60=(40.54=DC). Eq. 3, page 9. And AC× cos. 42°, 30′,=AD; or .7373×60=(44.24=AD). Eq. 1, page 9. And AB—AD=91—44.54=(46.76=BD).

The angle at B is found by its tangent, by dividing DC by DB, or 40.54÷46.76=(.8670=tan. 40°, 56′). Eq. 2, page 8.

The side BC is found by multiplying the side or base BD by sec. of angle at B, or 46.76×1.324=(61.85=BC). Eq. 2, page 9.

The angle at C is found by taking the supplement of the sum of the angles A and B, which gives (96°, 34′=C).

The student should complete the multiplications indicated in the above computations, looking the functions out of the tables, and he should make careful constructions to scale, of all his problems of triangles, as the geometrical figure will always help to a clear conception of the numerical work required.

There are many other theorems for the solution of oblique triangles, but those given are deemed sufficient for all of that class of questions likely to occur.

Examples of Oblique Triangles.—

1. Given one side b of a triangle=32 rods. The angle A 56°, 20′, and the angle at C=49°, 10′, to find the sides a and b.

Ans. a=27.5 rods,
b=25.0 "

2. Given the angle A=63°, 35′, the side b=32, and the side a =36. Required the other side and the other two angles.

Ans. Angle B=52°, 45½′,
" C=63°, 39½′,
Side AB=36, nearly,

3. Given angle A=26°, 14′, the side b=78, the side c=106. Required the angles B and C, and the side a.

Ans. B=43°, 14′; C=110°, 2′; side a=50.

4. Given the side a=54, c=78 and b=70. Required the angles.

Ans. A=42°, 22′; B=60°, 52⅔′; C=76°, 45⅓′.

CHAPTER II.

THE MARINER'S COMPASS.

Concerning the early history of this instrument, we have but little reliable information.

We first hear of it in China, where the needle or "loadstone" was used to give direction, by floating it on a piece of cork, on water,

Then we hear of it suspended at the middle by a string, when the magnetic force of the earth would give it direction.

Early in the 14th century, the compass was improved by placing the needle on a "pivot post;" and in 1608, the Rev. Wm. Barlow further improved it by applying the "gimbal joint" to the bowl carrying the needle. But we must consider it as at present constructed.

The Needle is made of steel, hardened and magnetized. It is then surmounted by a card graduated into 32 points, and sometimes into quadrants of 90° each, and sometimes into degrees, continuously, from 0 at the north, around by the east, south, west, to 360° at the north.

The better class of compasses have two to four needles under the card.

Names of the Points.—The line passing through the point on which the needle swings, and the north point of the card, is called the **Zero, or Meridian Line,**—the intersection of this line with the north part of the card being marked with a "fleur-de-lis," by way of distinction; the south side being marked by the letter S.

The line at right angles to this meridian, is called the **"Equatorial,"** from its analogy to the equator,—its extremities being marked with the initials of the directions they represent,—E for east, and W. for west.

The four letters, N., S., E., W., are called the cardinal letters, and the points they represent are called the four **Cardinal Points.** Sometimes the **Equatorial** is called the **Prime Vertical.**

Subdividing the Card.—The points midway between the cardinal points are called the **Inter-Cardinal Points,** sometimes the **Quadrantal Points.**

A quadrantal point takes the name of the two cardinal points between which it lies. Thus, the point midway between the north and east, is called the N. E. point, and so of the other quadrants.

In the same manner, the point midway between the cardinal point and a quadrantal point, is indicated by calling the letters between which it lies,—as N., N. E., indicates the point midway between the north and the northeast, and so for the other octants.

The remaining sixteen points are found adjacent to the cardinal letters and the quadrantal letters, one on each side of each letter, the adjacency being indicated by the word "by."

In defining one of those points, the reference letter is first named, then the point immediately to the right or left of the reference letter, as the case may be. Thus, the first point immediately to the right of east, is called E. by S.; that to the left of north, N. by W., etc. Thus, the plan of naming the points being understood, the name of any point of the compass is easily brought to mind.

The naming of the points of the compass in consecutive order around the card, is called **"Boxing the Compass,"** and the student should make himself so familiar with it that he can name them readily in either direction.

Reading the Compass Card by Numerals.—In giving the course on which the ship is to be steered, the officer of the deck usually refers the same to the nearest cardinal or quadrantal letter. But in making computations for bearings, or for reduction of course, or the finding of compass errors, reference is always made to the nearest meridian letter, N. or S., and the bearing is reckoned in points and parts of a point, or in degrees, toward the east or west, as the case may be.

This is made necessary by the fact that all trigonometrical tables used in calculating courses, are arranged in that manner.

The student must therefore make himself familiar with reading the card in that manner. Thus,

E. by N.	would be read,		N. 1 p. E.
N. E.	"	"	N. 4 p. E.
E. $\frac{1}{4}$ N.	"	"	N. $7\frac{3}{4}$ p. E., etc.

The student should also be able to change a course given by its numerical value to its cardinal name. Thus,

N. 3 p. E.	would be	called	N. E. by N.
S. 6 p. W.	"	"	W. S. W.
N. 5 p. W.	"	"	N. W, by W.
N. 45° E.	"	"	N. E.
S. 33¾° E.	"	"	S. E. by S., etc.

Table III will facilitate this reduction of courses, and it is important that the student make himself expert in these reductions. Some compass cards are graduated both in degrees, and in points and ¼ points. Such a card is then, in itself, a table for the reduction of course from one denomination to the other.

The Use of the Compass, is to give the bearing of a line, or to point out the direction in which a ship is to be steered. All courses are ultimately referred to the astronomical meridian, when they are called **True or Astronomical Courses,** sometimes they are called **Chart Courses.**

But the compass seldom points to the true north. In most places it is turned away from that direction by the earth's magnetism, more or less.

It is also disturbed from the position which it assumes under the earth's magnetism, by the magnetism of the iron in the ship. This is the most serious disturbance that can come to the compass needle,—for as the ship heads to different courses, the needle also takes different directions,—so that the change in the readings of the card is no indication of the change made in the course of the ship. Yet, at sea, in dark weather, the navigator is obliged to refer his course to the disturbed needle,—whence it is of the first importance that the navigator know the error of his compass.

The Agonic Line, or line of no variation, is a line on which the magnetic needle, when not disturbed by local causes, **points to the true north.**

This line, in the United States, commencing in South Carolina, takes a course a little to the west of north, crossing the head of Lake Erie near the mouth of the Detroit River, thence through the eastern part of the State of Michigan, crossing Lake Huron a little west of Mackinac Island, thence across the eastern part of Lake Superior to a little east of the Copper Islands.

From this chart it will be seen that east of the agonic line, the north end of the needle stands to the left. or west of the astronomical north; and to the west of this line, the north end of the needle points to the east of the true meridian. Thus, in the Gulf of St. Lawrence, the needle points 20° to the **left** of north, while on the Pacific coast, near the mouth of Columbia River, the north end of the needle stands to the **right** of the true north, 22°. Thus, in crossing the continent, the north end of the needle swings toward the east nearly four points.

Preparatory to finding the error of the needle, we give a few definitions:

Astronomical Meridian.—This is the north and south line given on all charts, and to which all courses are eventually referred. The meridian of any place is in the plane containing the earth's axis, and that place.

Magnetic Meridian.—This is the line pointed out by the magnetic needle, when acted upon by the earth's magnetism alone.

Compass Meridian.—This is the line pointed out by the magnetic needle, acting under the combined influence of the earth's magnetism and that of the iron in the ship,

Variation, is the difference between the directions of the astronomical meridian and the magnetic meridian. The north end of the needle may stand either to the right (east) or to the left (west) of the true meridian, in which case the variation is called east or west, as the case may be. Easterly variation is considered + (plus) and westerly — (minus), and is usually given in degrees.

Deviation, is the difference between the directions pointed out by the magnetic and the compass meridians. The compass, when aboard ship, may be disturbed by the ship's magnetism, so as to take position to the right or left of the magnetic meridian, precisely as the magnetic meridian takes position to the right or left of the true meridian. Whence,

Deviation is east or west, + or —, precisely as in variation

Variation and deviation are thus seen to be strictly analogous, but with this difference, viz.: Variation is **constant** for all headings of ship, but deviation is **different** for all headings of ship.

Three Kinds of Bearing.—Bearing being the direction of an object or place, with regard to some line of reference, as a meridian line, it follows from the above definition that we have three kinds of bearing, viz.:

Astronomical, or True Bearing, which is the direction of an object, with regard to the true meridian.

Magnetic Bearing is the direction of an object or place, with regard to the magnetic meridian; and

Compass Bearing is the direction of an object or place, as given with the deviated needle. Whence, we have at sea:

Variation and Deviation combined into one error called **Correction** or **Total Variation,** and which is the algebraic sum of variation and deviation, for all compasses at sea are under the influence of both,—the earth's magnetism and that of the ship.

That part of compass error due to variation, is found at the time of making a survey of the coast, or of the lake, or of any locality, and recorded in the charts of the same; and is regarded, for the time, as constant, though it changes slowly by the slow change in the "agonic" line. Thus, in 1840, Prof. Elias Loomis informed us, in the American Journal of Science, that the agonic line, commencing at a point near Wilmington, N. C., went northwesterly, crossing Lake Erie east of Cleveland; thence, through the middle of Lake Huron and entirely east of Lake Superior. The variation in 1890, or rather the agonic line, was from $1\frac{1}{2}°$ to $2°$ to the east of what it was in 1840, i. e., places that then had 0 variation, have variation of $1\frac{1}{2}°$ to $2°$ west, now. As a consequence, the variation that was given on charts that were made from early surveys, are in error, according to the age of their surveys—say $1°$ to $1\frac{1}{2}°$—westerly variation increasing and easterly variation decreasing.

But not so with that part of compass error due to deviation. This must be found for the compass of each individual ship, and for compasses in different parts of the same ship. Nor can we tell from the behavior of a compass in one ship, what may be looked for in the compass of another ship.

Azimuth, is the angle or course by which an object is referred to a meridian. In geodesy, it is usually reckoned from the south part of the meridian (in north latitude), and from 0 to the right, by the west to the north, and east 360° back to the south. In navigation and surveying, it is reckoned from both parts of the meridian, 90° each way to the east and west.

Amplitude, is the bearing of an object when referred to the east or west point of the compass.

Azimuth and Amplitude are **compliments** each to the other, i. e., each is what the other lacks of eight points, or 90°.

Swinging Ship for Compass Errors.—That part of the compass error due to deviation, is found by "swinging ship," and comparing the reading of the ship's compass with the simultaneous readings of a "shore compass" on successive headings, as the ship turns around in azimuth.

The "shore compass" is the ordinary compass used by surveyors, but sent on shore to get it away from the influence of the ship's magnetism to where it is acted on only by the earth's magnetism. The line between the ship's compass and the "shore" compass is a **reference line.** The bearing of it, for successive headings of ship, compared with the bearing given by the "shore" compass, gives the error (deviation) of the ship's compass on the several headings.

Standard Compass.—The above method of finding the error of ship's compass, involves the use of some appliance on board the ship for measuring the angle between the ship's heading and the reference line.

At sea, this appliance is found in the movable ring and movable sights attached to the standard compass, which is placed on the open deck, and high enough so that bearings can be taken over the rail, even when the ship is heeled over a few degrees.

But on our lake vessels there are no such appliances. The compasses are not only not provided with the ring and sights, but they are boxed up in the pilot house where such things could not be of any avail if we had them. As a consequence, our lake vessels, as a class, are utterly without the means of finding their compass error.

The Dumb Compass has been prepared to meet this deficiency. It is the movable ring and sights of the standard compass, as used at sea, but in another form. It is not necessarily expensive,—a good one may be made by the ship's carpenter,—though better by the instrument maker.

The essential feature of the dumb compass is **that it may be read by two indexes.** It has no needle. The card is graduated like that of the ship's compass.

The lower index corresponds precisely to the "lubber's" mark of the ordinary compass bowl,—the line joining it with the center of the compass to be fixed parallel with the eenter line of ship, as is that of ship's compass.

The plate on which the card is fixed is movable to any position about a vertical spindle, and may be clamped in any position.

The sights are on a bar that turns in azimuth about the same spindle as that carrying the card, and which can be clamped to the card in any position.

It should be mounted on a tripod and be provided with a socket joint, and a level vial should be fixed in the plate carrying the card, for adjusting the card to a horozontal position; and it should be set up high enough so that a clear view can be had from it of the whole horizon.

(NOTE.—The ideal instrument would be on "gimbals," but this is not necessary, or even important, for, as the instrument is not used for a steering compass, but merely as an auxilliary compass, it is quite sufficient if it be mounted on a tripod provided with a ball and socket joint, and with cross levels in the card).

The Use of the Dumb Compass, is to find the angle between the heading of the ship and any line that is used as a reference line in finding compass errors. Thus, in port, when it is desired to find compass errors by reference to a line of known magnetic bearing, the process is as follows:

1. Send the shore compass ashore and set it up at any convenient point where it is away from any local disturbance of the needle, and take the bearing to the "dumb compass" on board ship.

2. The dumb compass being set up on deck in a suitable place for observation, the zero line and lubber line being parallel with ship's center line, and "looking forward," the same as the ship's compass.

3. Set the upper index, or sights, to the same reading as that given by the shore compass, the north side of the card being to the north.

4. The card and sights being clamped together, turn them together till the sights "back.sight" on the shore compass. Then is the card of the dumb compass oriented to the magnetic meridian. The reading of it by the lower index is the magnetic head-

ing of the ship. And the difference between the magnetic heading and "compass" heading is the **deviation,** if any, of the ship's compass, for that heading. This is called the method by **Reciprocal Bearings,** and is used in harbors or other places where a distant azimuth mark cannot be found.

Finding Compass Errors.—The illustration of the use of the dumb compass in the preceding article, gives the method of finding compass errors in port, but the office of that compass is the same for all methods of finding compass errors, viz., to refer ship's head to the reference line,—which may be any line of known bearing,—the results only being different, as when the difference line is known by its magnetic bearing or by its astronomical bearing.

Direct Method, by Reference to a Distant Object.—In this case, the work of finding compass errors is somewhat simplified. The azimuth mark may be a distant building, headland or shore line, but it must be so distant that the change in the place of the compass, in swinging ship, will not cause any appreciable parallax, —say two or three miles, or not less than about 100 times the diameter of the circle made by the compass in swinging ship.

The bearing of this line is found as in the preceding article, or it may be found from a chart, but when found, the subsequent work is that of the former case, except there are no reciprocal bearings to make with the shore compass,—whence, it is called the direct method.

Method of Recording Observations of Compass Errors and deriving therefrom the Deviation.—The method of recording the observations for finding deviation of our compasses, is somewhat different from that at sea, for the reason that our compasses are differently mounted and differently equipped from those at sea.

There, the bearing of an azimuth mark or reference line can be directly compared with the bearing of the same line, as indicated by the ship's compass, and on the same instrument. But our compasses, as before seen, are so arranged that the bearings of objects cannot be taken with them, they can only show the apparent bearing of ship's head, hence the necessity of a different form of record.

Example: The table on the following page, gives the observations of the Steamer Huron, swung for deviation, at South Haven, by the author, June 26, 1876. (See form I). The first column gives the bearings of ship's head, as indicated by her compass.

The second column gives the bearings of ship's head, as indicated by the "shore compass" (or rather by solar transit with index correcting for variation). The third column gives the difference of these two columns, which is the deviation for the respective bearings or headings of ship.

It will be seen by referring to columns 1 and 2, form I, that the first course in the second quadrant of the first column, corresponds to the last course of the first quadrant in the second column. Now, in finding their difference, we must reckon both courses from the same zero point,—no matter which—say the north, in this case. Then, instead of S. 6 p. E., we would have N. 10 p. E., i. e., we must take the **supplement** of the course. Then we can find the difference of the two bearings, 2½ p.

	I. Deviations of Steamer Huron.			II. Deviations of Steamer Huron.		
	Head by Ship's Compass.	Head by "Shore" Compass.	Deviations.	Head by Ship's Compass.	Head by "Shore" Compass.	Deviations.
	North.	N. ⅝p.E.	⅝p.E.	0	⅝	⅝ p. E.
1	N.1¼p.E.	N.1⅛ E.	⅛ W.	1¼	1⅛	⅛ W.
	N.2¾ E.	N.1⅞ E.	⅞ W.	2¾	1⅞	⅞ W.
	N.4¼ E.	N.2⅝ E.	1½ W.	4⅛	2⅝	1½ W.
	N.5 E.	N.3¼ E,	1¾ W.	5	3¼	1¾ W.
	N.6 E.	N.3⅞ E.	2⅛ W.	6	3⅞	2⅛ W.
	N.7⅞ E.	N.5½ E.	2⅜ W.	7⅞	5½	2⅜ W.
	S.6 E.	N.7½ E.	2½ W.	10	7½	2½ W.
	S.5 E.	S.7½ E.	2¼ W.	11	8¾	2¼ W.
2	S.3¾ E.	S.5⅞ E.	2⅛ W.	12¼	10⅛	2⅛ W.
	S.3 E.	S.4⅞ E.	1⅞ W.	13	11⅛	1⅞ W.
	S.2 E.	S.3½ E.	1½ W.	14	12½	1½ W.
	S.1 E.	S.2 E.	1 W.	15	14	1 W.
	S. ¼ W.	South.	¼ W.	16¼	16	¼ W.
	S. ¾ W.	S. ¾ W.	0	16¾	16¾	0
	S.1⅜ W.	S.1⅝ W.	¼ E.	17⅜	17⅝	¼ E.
3	S.2½ W.	S.3⅛ W.	⅝ E.	18½	19⅛	⅝ E.
	S.3 W.	S.3⅞ W.	⅞ E.	19	19⅞	⅞ E.
	S.3⅜ W.	S.4¾ W.	1⅛ E.	19⅝	20¾	1⅛ E.
	S.4½ W.	S.5⅞ W.	1⅜ E.	20½	21⅞	1⅜ E.
	S.5½ W.	S.6⅞ W.	1⅜ E.	21½	22⅞	1⅜ E.
	S.6½ W.	N.7¾ W.	1¾ E.	22½	24¼	1¾ E.
	S.7¾ W.	N.6¾ W.	1⅞ E.	23¾	25⅝	1⅞ E.
	N,6⅞ W.	N.4¾ W.	2⅛ E.	25⅛	27¼	2⅛ E.
4	N.3¼ W.	N.1½ W.	1¾ E.	28¾	30½	1¾ E.
	N.1⅝ W.	N. ½ W.	1⅛ E.	30⅜	31½	1⅛ E.

And so in the third and fourth quadrants.

Writing the Courses by Azimuth.—This method is given in form II of the preceding table, which consists in writing them continuously in points, from zero at the north, around by the east, etc., to 32 points.

This method is very simple, when the card is graduated continuously around the circle, and greatly facilitates the finding of the difference of the courses given by the two compasses. On that account, it is desirable to reduce the courses, as ordinarily given, to azimuth readings, as follows:

1. In the **first** quadrant, i. e., from the north to the east, write the courses as they are given from the card,—the courses being given by their numerals.

2. In the **second** quadrant, E. to S., write the **supplement** of the compass reading.

3. In the **third** quadrant, S. to W., add 16 p. to the compass reading.

4. Add 24 p. to the **compliment** of the compass reading, in the fourth quadrant.

Thus, the azimuth reading for S. 2 p. E. is 14 p.; for S. 2 p. W., it is 18 p.; for N. 7 p. W., it is 25 p.

We remind the student that **supplement** is what the course lacks of 16 p. or 180°, and **compliment** is what it lacks of 8 p. or 90°.

Naming the Deviations, will require a little careful attention, from the fact that each two adjacent quadrants are read in opposite directions; which may cause the young student to stumble in this matter, if he is not on his guard.

He should first go over the table and write the difference of the courses given in the first and second columns, without regard to name or sign **in the third column.** Then consider which way (right or left) the card of the "shore" compass must be turned to make its readings agree with those of the ship's compass. That direction is the name to be applied to the differences for the deviations.

Thus, in the first quadrant, the readings of the "shore" compass for any given course, are **less** than those of the ship's compass for the same course, and if we turn the card of the "shore" compass to the **left,** the two readings may be made to agree,—i. e., the deviation is **west** by the amount of the difference of the readings.

Again, in the second quadrant, the reading of the "shore" compass for any particular course, is **greater** than that of the ship's compass, and yet the card of the "shore" compass must be turned to the **left** to make the readings of the two compasses agree,—so that there, also, we have **westerly deviation.**

The same results will be found between the third and fourth quadrants. This comes from reading the alternate quadrants in opposite directions.

Deviation Curve.—We are told by the books, to take the deviations on the consecutive points, when swinging ship. But this, while possible, is at times totally impracticable, and we take such as we can get. Thus, it will be seen that the headings given of the Huron, in the preceding table, are very irregular as to their intervals. Still, it is important to have the deviation for regular intervals, as by points, so that the deviation for intermediate courses, as for parts of a point, may be interpolated for steering purposes.

Many schemes have been devised for the attainment of this object,—and mostly of the **graphical** type. Among the best of these methods, are those of J. R. Napier, Esq., F. R. S., and Archibald Smith, M. A., of the Liverpool Compass Committee. (See Admiralty Manual, for 1874).

Some sixteen years ago (1891 now), the author of this Manual devised a scheme for the representation of deviation and the conversion of compass courses, which on being submitted to Prof. J. H. C. Coffin and Commodore A. W. McCormick, U. S. N., is endorsed and recommended by both of these eminent men, as being a decided improvement on that of Mr. Napier, accordingly I give the method of constructing it and of using it.

Pearson's Diagram, was invented for the representation of deviation, and for the conversion of compass errors.

This diagram (see plate I) shows two scales,—one vertical and one horizontal.

The vertical scale represents azimuth, and is to be read downwards,—such reading corresponding to the reading of the compass card from the left to the right, as we read a watch dial. It is simply the circumference of the card, developed by rolling it down the page and marking the points.

This scale may be read in points and parts, or in degrees, as desired,—the horozontal lines across the page being the graduation marks.

The horizontal scale,—or rather the two scales, one at the head, the other at the foot of the page,—is for setting off variation and deviation.

The thick line drawn vertically through the middle of the page, represents the **magnetic meridian,** to which deviation is referred.

The thin lines drawn obliquely across the page, from the left, upward and to the right, are for the purpose of performing addition and subtraction graphically, of variation and deviation.

A thin line drawn vertically through the page, parallel with the magnetic meridian, and cutting the variation scale at any point, is called the **local meridian,** corresponding to that variation.

Any two convenient scales may be taken, at pleasure, for the variation and the azimuth scales,—the condition being that the oblique lines for performing addition and subtraction, pass through points in each scale, having **the same numerical value.** Thus, if an oblique line pass through the azimuth line at, say N. 1 p. E., then it must also pass through the variation or horizontal scale at $11\frac{1}{4}°$ from the zero of that scale, etc.

It is observed that easterly variation is placed to the **left** of the magnetic meridian, and westerly variation to the **right.** This is for the purpose of locating the magnetic meridian to the **right** of the true meridian for easterly variation, and to the **left** of the true meridian for westerly variation, as it should be.

But, deviation being referred to the magnetic meridian, must be set off from that line, according to its name,—to the right for easterly and to the left for westerly deviation.

Construction of the Deviation Curve.—The deviations having been satisfactorily found, as on page 29, set them off to the right or left of the magnetic meridian, according to their name (see plate II). Thus, when ship's compass needle indicates north, it is known to be out of place to the **right** by $\frac{5}{8}$ p. Take $\frac{5}{8}$ p. in the dividers from the variation scale at head of page, and set it off to the right on the line of north, from the magnetic meridian, and make a dot surrounded with a small circle, thus ⊙.

Again, when ship's head is $1\frac{1}{4}$ p. E. of N., the needle is out $\frac{1}{8}$ p. to the left. As before, take $\frac{1}{8}$ p. in the dividers and set it off on azimuth, N. 1¼ p. E. from the magnetic meridian, making the mark (⊙) as before.

In this manner, set off all the deviations, according to their name,—to the right for easterly deviations and to the left for westerly deviations, on the horizontal lines corresponding to their azimuth, and from the magnetic meridian as zero point.

Through the points set off, draw a fair freehand curve, giving and taking a little at times to make the curve fair, and you have the desired deviation curve.

It will be observed that about half of the curve is on either side of the magnetic meridian. On this account, this deviation is called **semi-circular.**

It will be observed, too, that the two parts are about equal, as the curve crosses the meridian when ship heads N. by E., also when it heads S. by W., but this is not always the case. And when they are not equal, it is not always that good reversals can be had in compensating the compass for deviation, as will be seen hereafter.

Use of the Deviation Curve, Reduction of Courses.—The deviation curve being constructed, we are prepared to solve many questions that vex the ship-master who is compelled to work with a deviated compass. A few examples will illustrate:

Example 1. At Grand Haven, Mich., mean var. 3° E. It is desired to sail to Milwaukee, with Steamer Huron. Required steering course for ship's compass, also the return steering course.

Solution: First, draw a line through var. 3° E., to represent the true or chart meridian. On this line, take up the chart course required to be made, viz., W. ⅛ S., as at a, aud project it onto the curve at b, at W. by S. ⅞ S., which is the compass course desired. —Ans. S. 6⅛ p. W.

To find the return course: Reverse the given chart course, W. ⅛ S., to E. ⅛ N., and take it up on the true meridian as before, as at c, and project the same onto the curve at d, when will be found the required compass course for the return, viz., E. S. E. ⅛ S., or S. 5 ⅞ p. E.

Example 2. On the same diagram will be seen the deviation curve of the U, S. S. Monadnoc. Required the outward and the return steering courses for the compass of this vessel for the same voyage.

Solution: As with Example 1, take up the chart course, W. $\frac{1}{8}$ S., on the true meridian, as at a, and project the same onto the curve of the Monadnoc, as seen at b^1, in azimuth W. $\frac{1}{2}$ N.=N. $7\frac{1}{2}$ p. W.

For the return course. Reverse the chart course, W. $\frac{1}{8}$ S., to E. $\frac{1}{8}$ N., and as before, project the same onto the curve, as at c^1, where we find the required return course, viz., E. by N. $\frac{3}{4}$ N., or N. $6\frac{1}{4}$ p. E.

Discussion of the Preceding Problem.—It will be remembered that when the card is turned to the right of its normal place, the readings for the bearing of a given object **are too small,** when regarded as azimuth (as all courses are eventually regarded), and consequently we must take up a course **smaller** than the compass would indicate by the amount of disturbance, which deduction is made by moving up the page of the diagram; otherwise it would take us too far to right.

For the same reason, when the card is out of place to left, its readings for the bearing of an object are **too large,** and without reduction, the card would take us too far to left. So that in this case we must **increase** the readings by the card by the amount of disturbance, to attain any given course. This is attained by moving **down** the page.

Variation, is seen to be the distance between the magnetic meridian and the true meridian.

Deviation, is the distance between the magnetic meridian and the curve, at any point measured on the horizontal line passing through the point, and by the variation scale.

"Correction," or Total Error, is the distance between the true meridian and the curve.

These quantities always come together algebraically, westerly deviation and variation being called minus (—), and easterly are called plus (+).

Correction for Leeway, can also be made by means of this diagram.

Example 3. At Grand Haven, Mich., mean var. 3° E. Required the outward and the return compass courses for each of the steamers Huron and Monadnoc, taking into consideration a port-hand leeway of $\frac{3}{8}$ p.—the chart course to Chicago being S. W. $\frac{1}{4}$ S.

Solution: Take up chart course, S. W. $\frac{1}{4}$ S., on the true meridian (3° to left of magnetic meridian) and move **down** the page

$\frac{3}{8}$ p. to compensate leeway. Then project the course thus found, viz., S. W. $\frac{1}{8}$ W., onto the curve at e, when we find S. W. by S. for the Huron; and projecting onto the curve of the Monadnoc, we find S. W. $\frac{1}{8}$ W. for that vessel's compass.

Returning, with the same wind, our leeway will be to starboard, then moving **up** the page $\frac{3}{8}$ p. to N. E. $\frac{3}{4}$ N. (the course having been reversed) and going to the curves, we find for the Huron, N. E. $\frac{3}{4}$ E., and for the Monadnoc, N. N. E. $\frac{1}{2}$ E., for their respective compass courses for the return voyage.

Example 4. At Duluth; var. 8° E.; wish to make a chart course N. E. by E. $\frac{1}{2}$ E., and looking for a port-hand leeway of $\frac{1}{2}$ p. What is the compass course of the Huron and for the Monadnoc, as deduced from their respective deviations?

Solution: On a true meridian drawn through var. 8° E., take up the desired chart course and move **down** the page $\frac{1}{2}$ p., to compensate the port leeway. Project the point thus found, onto the curve for each vessel, and we find for the Huron, E. $\frac{1}{4}$ N. at g; and for the Monadnoc, N. E. $\frac{3}{8}$ E. at g^1, as their respective steering courses.

DEVIATIONS OF STEAMER HURON.

Head by Ship's Compass.	Deviation.	Head by Ship's Compass.	Deviation.
North.	7½° E.	**South.**	5 ° W.
N. by E.	0	S. by W.	0
N. N. E.	6 W.	S. S. W.	5½ E.
N. E. by N.	11¼ W.	S. W. by S.	11 E.
N. E.	17 W.	S. W.	13½ E.
N. E. by E.	21 W.	S. W. by W.	14 E.
E. N. E.	24 W.	W. S. W.	18¼ E.
E. by N.	26½ W.	W. by S.	21 E.
East.	28 W.	**West.**	23 E.
E. by S.	28½ W.	W. by N.	24½ E.
E. S. E.	28 W.	W. N. W.	24½ E.
S. E. by E.	27 W.	N. W. by W.	24 E.
S. E.	25 W.	N. W.	22 E.
S. E. by S.	21½ W.	N. W. by N.	19½ E.
S. S. E.	18 W.	N. N. W.	15 E.
S. by E.	12 W.	N. by W.	11½ E.
South.	5 W.	North.	6 E.

It is believed the above illustrations will fully explain the operation of reducing compass courses.

Another use of the curve, besides the reduction of courses, is found in the facility with which it gives the **Deviation on Consecutive Points** of the card when observations have been made at irregular intervals, thus making the deviations available for a steering card. To do this, we have only to measure the deviations on each point, by taking the distance between the curve and the magnetic meridian, with a pair of dividers, refer them to the variation scale at the head of the page, and tabulate the results. Thus, we find at north, the deviation curve to the **right** of the magnetic meridian. Applying the distance to the scale at the head of the diagram, we find it $7\frac{1}{2}°$, which we write in the deviation column, with its name, E.

Shaping the Course.—The four questions just discussed constitute the problem of "shaping the course," and assumes the following form: With a compass that is deviated, what course shall be taken to attain a given chart course?

Rule, using the diagram: Project the given chart course from the true meridian onto the curve. At the intersection, is found the compass course desired, to attain the required chart course.

To Find the Course "made good," is to find what chart course has been actually attained, by sailing with a compass that is deviated.

Rule. Project the compass course from the curve onto the true meridian. At the intersection will be found the equivalent chart course.

This problem, which is seen to be the reverse of the first question, is wanted at sea, in making up the "day's work" for finding place of ship, but is seldom or never wanted on the lakes, for the reason that the place of ship, when outside, is rarely, if ever, looked for, as it is looked for at sea.

The runs being short,—only a few hours at most—points, as lighthouses, are seen so frequently, and their co-ordinates of place being known from ihe list of lights, there is no necessity for "reducing a traverse," as at sea, to find place of ship, and no necessity for finding the "course made good."

The Log-Line and Time-Glass.—These are also essential parts of the equipment for keeping account of the place of ship at sea.

The log-line is adapted to the sea-mile of 6080 feet, with a time-glass of 28 seconds.

Theoretically, the time-glass was supposed to run 30 seconds and the sea-mile to be 6087 feet, but as the circle of the equator is larger than any other great circle of the earth, vessels always found themselves **ahead of their reckoning,** by using the equatorial mile as the unit of distance.

To correct this inconvenience, the time-glass has been reduced to 28 seconds, and the sea-mile has been reduced to 6080 feet, which corresponds now nearly with the minute of a circle on the mean diameter of the earth.

These values are now adopted by nearly all maritime people. Then taking the same part of 6080 feet, that 28 seconds are of one hour, the length of the "knot" on the log-line would be 47 feet. Some navigators make it more or less, accordingly as they think their ship over-runs or under-runs her reckoning.

The U. S. navy make the length of the log-line 45 feet, for a 28 second glass.

On and about the great lakes, the statute mile of 5280 feet is regarded as the unit of measure for distance. Accordingly, for a 28 second glass, the knot on the log-line should be 40 feet, or 43 feet for a 30 second glass.

If the patent log, adapted to the sea-mile be used, its readings may be reduced to indicate statute miles, by multiplying the indicated distance by 1.15, which is adding 15% to the indicated distance.

As the modern patent log depends on the "pitch" of its screw to give it the right number of turns for indicating the distance, the labor of getting the right pitch is somewhat tedious, it being a tentative operation.

But it is not necessary that it should show the number of turns for the correct distance. It is quite enough to know the number of turns made for a known distance.

Suppose, for a distance known to be 76 miles, the dial indicates 80 miles,—that is, it indicates 80–76ths of the distance. **Invert** this fraction and make 76–80ths of 80 miles, for the correct distance, i. e., multiply the indicated distance by the reciprocal of the screw's rate.

Thus, suppose the log shows only 13 miles when it should show 15 miles. Then, inverting the ratio 13–15ths to 15–13ths and multiplying by 15, we get the correct distance. Whence, 15–13ths or 1.154 is the factor or co-efficient by which to multiply all indications of this log. This is called **finding the co-efficient of the log.**

Distance by Propeller Wheel.—Another method of measuring the ship's rate of sailing, is that by means of the propeller wheel.

The vessel, when in her usual trim, is run over a known distance, noting the total time and the number of revolutions per minute.

The known distance being reduced to feet and divided by the total number of revolutions in the given time, gives the "net pitch" of the wheel in feet, i. e., the distance made good by one revolution.

Example: A steamer making with her wheel 106 revolutions per minute. for 64 minutes, makes a known distance of 12.4 miles. Required net pitch of wheel.

Solution. $\frac{12.4 \times 5280}{64 \times 106} = 9.66$ feet. Net pitch of wheel.

The net pitch being known, and multiplied by the number of turns per minute, and by 60, the number of minutes in an hour, then divided by the number of feet in one mile, 5280, gives the rate of ship in miles per hour,—or if he divide by 6080 feet, he will have the rate in nautical miles per hour.

In this manner the ship master can readily log his ship for different speed of engine, and thus learn his rate of sailing to a good degree of certainty, and better than by a log whose "rate" is not known.

CHAPTER III.

The Sailings.

Navigation implies the conducting of a ship from one port to another, and in its broadest sense, implies the solution of many problems involving a knowledge of geography, nautical astronomy and mathematics, to a large extent.

The terms Plane Sailing, Traverse Sailing, Parallel Sailing, Middle Latitude Sailing, Mercator's Sailing, Current Sailing, etc., indicate different methods of finding place of ship, rather than any peculiarity in the manner of sailing, as the name might seem to imply.

The result of any of these methods is called the place of ship by **dead reckoning,** or the method of finding place of ship from the course, the distance, the rate, time sailed, and a knowledge of the place sailed from. It is also called the place of ship by account.

Plane Sailing, is the method of finding place of ship with regard to place sailed from, by means of the co-ordinates, course and distance, from the properties of the plane triangle.

The method of plane sailing has been denounced as being inaccurate, but when we recollect that the course or "rhumb line" makes a constant angle with the several meridians which it crosses, we see that distance sailed, the difference of latitude, and the departure, are correctly represented by the hypothenuse and sides of a right triangle, in which the course is the angle opposite the departure, and the hypothenuse is the distance. We give a few definitions:

Equator.—A plane through the center of the earth and at right angles to the earth's axis, is called the plane of the equator; and the intersection of this plane with the surface of the earth, is called the equator.

Zero, or Prime Meridian.—Any plane through the earth, and containing the earth's axis, is called a meridian. Any and every point on the surface of the earth has its meridian. The meridian passing through the observatory of Greenwich, is now assumed as the prime meridian, or meridian from which longitude is reckoned by most maritime people.

Geographic Latitude, is the distance in arc, measured on the meridian of any point, from the equator,—or the angle between the plane of the equator and a vertical or plumb line passing through the point; but,

Geocentric Latitude, is the angle included between the plane of the equator and a line joining any point on the surface of the earth with the center of the earth. Geographic latitude is always slightly greater than geocentric latitude, in consequence of the unequal diameters of the earth, this excess amounting to about 11¼ minutes at lat. 45°. Geocentric latitude is wanted in finding longitude from the place of the moon, but geographic latitude is always used in giving the place of any point, with regard to the equator.

Longitude, is distance in arc on the equator, east or west. Its use is to refer any point on the surface of the earth to the prime or zero meridian. Latitude and longitude together, are called the geographical co-ordinates of place.

Course, is the angle between the meridian and the track of ship, and is usually represented by its initial, C. **Distance,** is the length of line sailed; and course and distance together, are called the polar co-ordinates of place,—the course referring to the meridian, and the distance referring to the place sailed from.

But this is special. It concerns only the two points mentioned, —the place of ship and the place sailed from,—whence, the method by geographical co-ordinates, is general. It not only refers the places to each other, but to the equator and the prime meridian, and thence to any other point on the surface of the earth.

Whence, before we can compare the location of places as found by "course and distance," with that as determined by latitude and longitude, we must reduce our polar co-ordinates to geographical co-ordinates. This reduction is made by means of a Traverse Table, which may be called a table of right triangles.

In sailing on any oblique course, we make two components,— one with regard to distance on the meridian, and one with regard to distance east and west, on the prime vertical.

The component, north and south, is called the difference of latitude; and that made on the east and west line, is called departure. Just what these components are for any course and distance, is shown by table I.

This table is computed for courses varying by $\frac{1}{4}$ point, and for distances varying by unity up to 10.

By removing the decimal point one place to the right, in any column, we multiply the sum by 10; moving two places, we multiply by 100, etc.

By this device, the table is made to give the distance likely to be sailed at any one run, and to occupy but a small part of the space it otherwise would.

The courses are given up to four points in the left hand column, —increasing from 0 at the head to four points at the foot.

On the right hand. they are given from four points at the foot, to eight points at the head.

The components are abbreviated at the head of the columns, to diff. lat. and dep., for the courses in the left hand column, and at the foot of the column for the courses given in the right hand column.

It will be observed that the column that is marked **lat.** at the head, has **dep.** at the foot.

The Manner of Using the Table, will be seen from the following examples:

1. Sailed N. N. E. $\frac{1}{4}$ E., or N. $2\frac{1}{4}$ E., 57 miles. Required the components of the course, i. e., the northings and the eastings,—or diff. lat. and dep.

Opposite $2\frac{1}{4}$ p. in the left hand column and under 5 in the lat. column, we find 4.52. Removing the decimal point one place to the right, we have 45.2 miles as the northings for 50 miles.

Under 7 in the column of lat., we find 6.3, which added to the 45.2 miles, make 51.5 miles as the total northings for the run of 57 miles.

In the same manner, we find under 5 in the dep. column, 2.14. Removing the decimal point one place to the right, and we have 21.4 miles as the dep. or eastings due to 50 miles. Under 7 in the dep. column, we find 2.9 miles, which added to the 21.4, wc have 24.3 miles as the total dep. or eastings due to 57 miles on that course.

2 Required the diff. lat. and dep. for the following courses and distances, viz.:

		N.	S.	E.	W.
1.	S. W. by W.=S. 5 p. W. 46 miles,		21.7		40.6
2.	S. ¼ E. =S. ¼ p. E. 53 "		52.9	2.6	
3.	S. by W. ¼ W.=S. 1¼ p. W. 40 "		38.8		9.7
Ans.	Diff. lat. or southings=113.4 miles.		113.4	2.6	50.3
	Dep. or westings = 47.7 "				2.6
					47.7

Let the student prepare a table with a column for each of the components and a line for each of the courses, as in the example.

Having taken the components for each of the several courses from the traverse table and arranged them in their respective columns, **add the numbers in each of the several columns,** writing their sum at the foot of the column.

Then the difference latitude will be the algebraic sum of the northings and southings, and

The departure will be the algebraic sum of the eastings and westings, whence,

Take the difference of the northings and southings for the difference of latitude, and

Take the difference of the eastings and westings for departure.

Thus, in the example, the sum of the southings is 113.4 miles, and as there are no northings, this is all difference of latitude.

And the difference between the eastings and the westings is found to be 47.7 miles westings.

NOTE.—The table of **Natural Sines and Cosines,** elsewhere explained, is a **traverse table** for distance unity for all courses varying by *1′ from 0 up to 90°. The column of cosines corresponding to difference of latitude, and the column of sine, to departure. This table should be used when precision in regard to the course is wanted.

The tabular components of the course, being multiplied by the distance, gives the actual components.

Rhumb Line.—In consequence of the convergence of the meridian, as we approach the pole, or recede from it, in sailing on any

*The intervals of our table vary by 5′.

oblique course, the ship makes a curved line or track, called a Rhumb Line. The course of ship is frequently called a Rhumb. This curve is always convex toward the equator.

The constancy of the angle, or course, in crossing the several meridians, while they are all inclined to each other, except on the equator, is what gives curvature to the course, and it is this property of the rhumb that makes the results of plane sailing rigorously correct, notwithstanding a spherical surface cannot be developed on a plane.

Difference of Latitude, is the distance between two parallels of latitude.

As given from a course, by means of the traverse table, it is miles or minutes and is called northings or southings, accordingly as ship has made northings or southings. But when reduced to degrees by dividing by 60 or 69½, it is called Difference of Latitude.

Difference of Longitude of two places, is the arc of the equator intercepted by the two meridians that pass through the two points or places.

Longitude, is distance in arc (degrees) measured on the equator, east or west, from the zero meridian (Greenwich).

Reduction of Departure to Longitude.—It will be observed that eastings or westings called Departure, as found by the preceding article, is really difference of longitude,—it being the distance between two meridians,—though on a small circle instead of the equator.

And when we recollect that there are just as many degrees in a small circle as in a large one, we see that some reduction must be made before we can measure longitude on a small circle with the same unit (the sea mile) that is used in measuring it on a large circle.

Departure, though strictly difference of longitude, is not called such till this reduction is made.

Two methods of converting departure to difference of longitude are the following:

First. We may reduce the measuring unit in the same proportion as that by which the small circle has been reduced from the large one; or

Second. We may expand or enlarge the departure in the same proportion as that by which the equator is greater than the parallel, so as to make it embrace as many degrees on the equator as it otherwise would on the parallel.

Bearing in mind that the circumferences of circles vary as their diameters, and that the cosine of any latitude is the radius of that parallel, we have the following proportion:

$$R : \cos.\ L :: \left\{\begin{array}{c}\text{length of any arc}\\ \text{on the equator}\end{array}\right\} :: \left\{\begin{array}{l}\text{length of correspond-}\\ \text{ing arc on the parallel.}\end{array}\right.$$

Then, bearing in mind that R is unity, and multiplying extremes and means, we have the

$$\textbf{Arc of any parallel} = \left\{\begin{array}{l}\text{the corresponding equatorial arc}\\ \text{multiplied by cosine of parallel.}\end{array}\right.$$

The use of this equation is seen in the following problem:

Required the length of a degree of longitude in any latitude, L, say 43°, that on the equator being 60 miles.

$$\text{Ans.}\quad \text{Arc} = 60 \times \cos.\ 43^\circ$$
$$= 60 \times 7314 = 43.88 \text{ miles.}$$

Or suppose we wish to find the number of feet in 1′ of longitude on parallel 43°, we would have to multiply the equatorial minute, 6087 feet, by the cosine of 43°, thus,

$$5087 \times .7314 = 4452 \text{ feet.}\quad \text{Ans.}$$

Thus we see that any unit of measure for longitude on the equator, may be used for measuring longitude on any parallel, by first multiplying that unit by the cosine of the parallel.

Second method. By inverting the terms of the proportion in the preceding article, we have,

$$\text{Cos. } L : R :: \left\{\begin{array}{c}\text{any arc on the}\\ \text{parallel of L.}\end{array}\right\} : \left\{\begin{array}{c}\text{corresponding arc on}\\ \text{the equator.}\end{array}\right.$$

Whence,

$$R \left\{\begin{array}{c}\text{any arc on the}\\ \text{parallel of L.}\end{array}\right\} = \cos.\ L \left\{\begin{array}{c}\text{corresponding arc on}\\ \text{the equator.}\end{array}\right\}$$

Or,

$$\frac{R}{\text{Cos. } L} \left\{\begin{array}{c}\text{any arc on the}\\ \text{parallel of L.}\end{array}\right\} = \left\{\begin{array}{c}\text{corresponding arc on}\\ \text{the equator.}\end{array}\right\}$$

But,

$$\frac{R}{\text{Cos.}} = \text{Secant.}$$

Whence, we have the following **Rule** for converting departure into difference of longitude, viz.:

Multiply the departure into the secant of the latitude.

Example: How much longitude will 60 miles of departure embrace on parallel L=40°, 60°, and 80°?

Solution:

Secant	40°=1.305,	then	60×1.305= 78.3 miles.	Answers.
"	60°=2.000,	"	60×2.000=120.0 "	
"	80°=5.759,	"	60×5.759=345.5 "	

The student will observe that the longitude or difference of longitude found above, is minutes, which must be divided by 60 to reduce them to degrees.

We could divide by the cosine of the L and get the same results, but it is easier to multiply by secant.

Middle Latitude Sailing.—The methods of plane sailing, though correct in their results, are incomplete in that they do not determine the particular parallel on which departure shall be reduced to difference of longitude, but,

Middle Latitude Sailing has for its object to determine on what paralled the departure corresponding to any **rhumb-line,** shall be converted into diference of longitude.

The methods of findiug the components of a course, is identical with that for Plane Sailing, and indeed, for all the sailings.

It is the practice among seamen, to assume the middle paralled of any rhumb-line, as the parallel on which to make this reduction. This is well for short runs, but only roughly approximate for small courses, in high latitudes, with large distances.

The true parallel, on which to make this reduction is somewhat above the middle of latitude,—just how much is shown by a table of corrections given at page 76 of Bowditch's Navigation, called **Workman's Table.** The construction of this table is fully explained in Prof. Coffin's Navigation, Problems 5 to 10 of Plane Sailing, but too abstruse for an elementary work like this.

It is table XI. of this work.

Preparatory to the discussion and solution of problems, it is well to introduce and explain some notation, for the purpose of shortening our work.

C=the course.	L=latitude left.
d=the distance.	L^1=latitude arrived at.
l =the difference of latitude.	p=departure.
D=the difference of longitude.	L^2=corrected middle latitude.

Thus, in the triangle A B C.

C B being the meridian, the side.

A C=the distance d.
C B=the difference latitude l.
A B=the departure p.

And when the departure is expanded from A B into m n, it is called **difference of longitude** and represented by D.

Let the student make himself familiar with this notation, as it will help him greatly in getting clear apprehension of his work.

Example: Sailing from L=65° N. on a course C=N. 42° E, distance d=648 miles. Required difference of latitude l and difference of longitude D.

Let the student construct the problem carefully to scale.—constructing C by its chord.

Solution: As in Plane Sailing, find difference of latitude and departure.

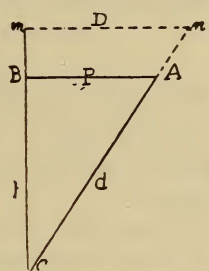


Fig. 20.

1. Dep. $p = d \times \sin C$ (=42°)=648×.6691=433.6′ or miles.
2. Diff. lat. $l = d \times \cos C$ (=42°)=648×.7431=481.5′ or miles.
3. Or in degrees =481.5÷60=8°.1½′ "
4. Lat. attained, L^1=65°+8°.1½′=73°.1½′ "
5. Middle lat. =(65°+73°.1½)÷2=69°.¾′ "
6. Correction to Middle lat. (see table) = 16′ "
7. Corrected middle lat. L^2=69°.¾+16′=Say 69°.17′ "
8. Then, by the rule of the preceeding article

Diff. of long. D=Sec. $L^2 \times p$, or

2.8263×433.6=1225.5′, or 20°.25½′=D.

Thus we have our answer in equation (2) and in equation (8.)

l =481.2 miles, or 8°.1½′
D= 20°.25½′

Let the student go carefully over this work, looking all the numbers out of the tables and performing all the indicated work. By this means he will very quickly gain command of the problems.

The student will observe that in equation (4) we added the difference of latitude to L to find L^1. If we had made southings instead of northings, with our course, we should have used the— instead of the + sign.

Instead of multiplying the departure p. by the sec. of L, we could have devided by cosine of L^2 thus:

433.6÷.3538=1225.5′ or 20°.25½′, as before.

A table for reducing departure to difference of longitude is given (table X.) that will give results slightly **less** than the preceeding solution, **yet more nearly correct**, as it is adapted to the periodical or actual form of the earth, rather than that of the sphere, as is the table of cosines.

It is calculated for parallels 30′ apart, from 0 to 80°. When it is desired to be more precise, the divisor for internediate minutes may be readily interporlated. The divisors will be found to be **slightly greater** than the cosines of the corresponding parallels. Thus in the preceeding case, the divisor for L^2= 69°,17′=.355, instead of .3538 and gives D=20°,21½′, instead of 20°,25½′.

In using this table, we have merely to divide the departure by the value of one minute for the parallel, as given by the table.

Parallel Sailing, is the finding of the place of ship where it sails on a parallel of latitude.

In this case, the entire distance is departure, the converting of which into difference of longitude is already explained, and therefore requires no further attention.

Mercator's Sailing.—To devise a means of representing correctly, large areas of the earth's surface, and at the same time to simplify the reduction of departure to difference of longitude, Girard Mercator, in 1566, invented a chart called **Mercator's Chart.** This chart, from its many merits, has come into general use, the world over, by maritime people.

To construct this chart, he first expanded all the parallels as we have done, thereby making the meridianal distance on all parallels equal between any two meridians, and thereby making the **meridians all parallel** instead of convergent toward the poles, as on the sphere. Then, to preserve the relative positions and the relative magnitude of objects, he expanded the meridians in the same proportion.

Each minute of the meridian, from the equator up to the limit of the chart,—usually 80° of latitude,—was multiplied by the secant of its middle latitude, precisely as we expanded our departure to difference of longitude (see page 47) and the sum of these augmented minutes, up to any parallel, was called the **meridianal**

parts for that parallel,—i. e., the **expanded meridian.** Thus, the actual distance from the equator to the parallel 42° would be,

$$42 \times 60 = 2520 \text{ miles, geographic,}$$

but when each mile is multiplied by the secant of its middle latitude and added into one sum, we have for parallel 42°, 2782 meridianal parts.

To distinguish between the two measurements, the expanded distance on the meridian lines is called **Augmented Latitude,** while that on the sphere, as usually measured, is called **Proper Latitude.**

The difference of two augmented latitudes is called **Meridianal Difference of Latitude.** Thus, the meridianal parts for 42° (abbreviated to M. P.) is 2781.7. The M. P. for 44° is 2945.8, and their difference is 2945.8—2781.7=164.1, which is the **meridianal** difference of latitude between 42° and 44°, whereas their proper difference of latitude would be but 120. But, if the two parallels be on opposite sides of the equator, then their **sum** is the meridianal difference of latitude.

The **course** on a Mercator's chart is represented by a **straight line.** This is a consequence of parallel meridians.

The relation of the parts is seen in the following figure:

In the triangle ABC,

CB=Proper Diff. Latitude =l.
CA=Distance =d,
BA=Departure =p.
C l =Augmented Diff. Latitude=m.
a l =Augmented Departure =Difference of Longitude } =D.
Course=angle ACB =C.

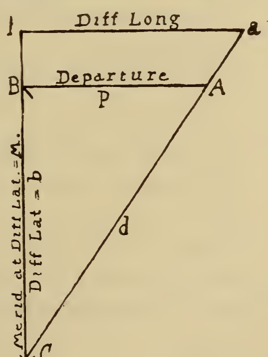


Fig. 21.

From the properties of similar triangles, we have,

C l : l a :: R : tan. C, whence,

l a × R = C l × tan. C,

but l a is diff. long. D; R is unity and c l is the augmented difference of latitude, whence,

$$D = c\,l \times \tan. C = m \tan. C.$$

That is to say,

The difference of longitude is found by multiplying the augmented difference of latitude by the tangent of the course.

Thus we find the difference of longitude without the necessity of first finding the departure. This is one of the advantages of Mercator's sailing. But we cannot measure distances by scale on this chart.

Problem: Given the course C=N 26° E, the distance d=142 miles, L=28° N. Required L^1 and D.

Solution: Preparatory to finding L^1, we must find l, as in plane sailing. Then, augmenting l, we have m.

$$l=d\times\cos. C=142\times.8988=127.6'=2°, 7.6'$$
$$L^1=L+l=28+2°, 7.6'=30°, 7.6'. \text{ Ans.}$$

Meridianal parts (M. P.) of L^1=1897.1
M. P. of L =1751.2

Meridianal difference of lat.=m=145.9′ Then,
Diff. long. $D=m\times\tan. C$ (=26°)
$=145.9\times.488=71.2'=1°. 11.2'$. Ans.

Second Solution. Working this example by middle latitude sailing, will show the agreement of the two methods.

As in plane sailing,

$l=2°, 7.6'$ and L^1	$=30°, 7.6'$
Middle lat.$=(L+L^1)\div2=(28°+30°, 7.6')\div2$	$=29°, 3.8'$
Correction to middle latitude (see table)	$=$ 1′
Whence, L^2	$=29°, 5'$ say,
$p=d\times\sin. C=142\times.4384$	$=$ 62.24 miles.
$D=p\times$secant L^2 $(=29°, 5')=62.24'\times1.144$	$=$ 71.2′
	$=1°, 11.2'$ Ans.

Or the same result as before.

The student must keep fresh in mind the notation of pages 46 and 47.

Note.—As the tables of sines, tangents, etc., given in this work, vary by 5′ of arc, the results obtainable with them will not correspond strictly with the results given here.

Problem: Given two places by their geographical co-ordinates of place.

$$L=46°, 10' \text{ S.} \quad \text{Long. } 46°, 30' \text{ E.}$$
$$L^1=52°, 15' \text{ S.} \quad \text{Long. } 51°, 10' \text{ E.}$$

Required C and d from L to L_1.

Solution: From the co-ordinates of place we have the base and perpendicular of a right triangle from which to find the hypothenuse, which is the d, and the angle opposite the departure, which is C.

M. P. of L^1 (=52°, 15′) =3689.6
M. P. of L (=46°, 10′) =3130.0

Meridian difference of latitude=m = 559.6 miles.
Diff. long. D=(51°, 10′—46°, 30′)=4°, 40′=280 miles.

By equation 2, page 9,

Tan. C=D÷meridian difference of latitude
" "=280′÷559.6=.5003=tan. 26°, 35′. Ans.

Proper difference of lat.=(52°, 15′—46°, 10′)=6°, 5′=365 miles.
Distance d=l ×secant C (=26°. 35′)
=365×1.1182=408.1 miles. Ans.

Ans. { C=S. 26°, 35′ E.
d=408.1 miles.

It will be observed that d is found from the proper difference of latitude l. This is necessary because oblique distances cannot be measured on Mercator's chart.

Solution by middle latitude sailing:

Proper difference of latitude=52°, 15′—46°, 10′=6°, 5′=365=l.
Middle latitude=(52°, 15′+46°, 10′)÷2=49°, 12½′.
Correction for middle latitude (see table)=5′.
Corrected middle latitude L^2=49°, 17½′.
Difference of longitude D=(51°, 10′—46°, 30′)=4°, 40′=280 miles.
Departure p=cos. L^2×D.
=.6522×280=182.6 miles.
Tan. C=p÷l=182.6÷365=.5003=tan. 26°, 35′. Answer, as before. Also,
Distance d=l×sec. C (=26°, 35′)
=365×1.1182=408.1 miles. Ans., as before.

Examples for exercise:

1. Sailed from L=42°, 30′ N., and long. 58°, 51′ W. S. W. by S., 591 miles. Required the latitude and longitude in.

Ans. Latitude 34°, 19′ N.
Longitude 65°, 51′ W.

2. A ship sailed from L=49°, 57′ N., and long. 30°, 00′ W., on course C=S. 39° W., till she arrives at L^1=45°, 31′ N. Required the distance sailed and the longitude in.

Ans, Distance=342.3 miles.
Longitude in=35°, 21′ W.

Let the student construct these problems to scale, carefully, before attempting numerical solution.

Traverse Sailing.—When a vessel sails on a number of courses in making a run, she is said to make a **traverse** or irregular track. And the finding of the equivalent of the traverse in one course and distance, is called traverse sailing. While the operation or the solution of the question is called the **reduction** of the traverse, or finding the course and distance "made good," or **reducing a day's work.**

For moderate distances, the method of plane sailing is satisfactory. But on longer voyages, some one of the other sailings must be combined with it for the purpose of keeping account of the longitude. It is one of the most useful of the several methods. An example will illustrate:

Sailed N. N. E.=N. 2 p. E.,	140 miles,	
Thence, N. E. by E.=N. 5 p. E.,	48	"
Thence, east, - -	26	"
Thence, S. by W.=S. 1 p. W.,	36	"
Thence, S.S.W. $\frac{1}{2}$ W.=S. $3\frac{1}{2}$ p. W.,	76	"

Required the equivalent **single course and distance,** or course and distance "made good."

First, prepare a table in which to arrange the several courses, with their distances and their several components,—providing a column also for changing the courses into degrees, as follows:

	Course.	Degrees.	Dist.	North'gs	South'gs	Eastings	West'gs.
1	N. 2 p. E.	22°, 30′	140	.924 129.4		.383 53.6	
2	N. 5 p. E.	56°, 15′	48	.555 26.6		.831 39.9	
3	East.	90°, 00′	26			1.000 26.0	
4	S. 1 p. W.	11°, 15′	36		.981 35.3		.195 7.0
5	S. $2\frac{1}{2}$ p. W.	28°, 07′	76		.882 67.0		.471 35.8
				156.0	102.3	119.5	42.8
				102.3		42.8	
				53.7		76.7	

Having arranged the courses in order, with their degrees and distances in their proper columns, take the **cosines of the courses** from a table of natural sines and cosines, and write them in the upper left hand of the space for the northings or southings for that course, as the case may be.

And write the **sines of the courses** in the column for departure —eastings or westings, as the case may be; thus,

In the upper left hand of the space for northings, is written the **cosine of 22°, 30′;** and in the column for eastings, is written the **sine** of 22°, 30′.

These are the factors with which to multiply the distance, 140 miles, for the **components** of course; and so for all the courses.

Having multiplied each distance into the cosine and sine of its course, and arranged the products in their respective columns. write the sum of the components of each column at the foot of the same, and

Find the difference of the northings and southings and the difference of the eastings and westings, thus,

In this case, we find the northings in excess of the southings 53.7 miles; and the eastings exceed the westings 76.7 miles.

Thus far, the problem is merely another form of plane sailing. We have now our difference of latitude l=53.7 miles, and departure 76.7 miles, from which to find the course C and the distance d, whence,

$$p \div l = 76.7 \div 54.7 = 1.4021 = \tan. 54°, 30' = \text{course},$$

and as the course takes its name from its components, we have the course,

$$\text{N. } 54°, 30' \text{ E,, or N. E. } \tfrac{7}{8} \text{ E.,}$$

and multiplying northings by secant of C, we have,

$$\text{Distance} = 54.7 \times 1.722 = 94.2 \text{ miles.}$$

Instead of taking our components from a traverse table, we have taken the cosines and sines to distance unity, and multiplied by their respective distances.

This is for the purpose of showing a more precise way of computing the components of a course, as the ordinary traverse tables are not computed for anything lower than degrees,—though this is not designed to supercede the use of the traverse table for ordinary work.

If, now, the question of longitude is involved, we must introduce **middle latitude sailing,** as in the following

Example: From latitude 41°, 12′ N., longitude 20° W., make the following traverse: Required the latitude and longitude attained, and the course and distance "made good,"

1. S. W. by W. 21 miles. 2. S. W. ½ S, 31 miles.
3. W. S. W. ½ S. 16 " 4. S. ¾ E. 18 "
5. S. W. ¼ W. 14 " 6. W. ½ N. 30 "

As in the former case, rule seven columns for courses, etc.

Angle	Dist.	North'gs	South'gs	Eastings	West'gs.
1. S. W. by W.=5 p.	21		11.7		17.5
2. S. W. ½ S. =3½ p.	31		24.0		19.7
3. W.S.W. ½ S.=5½ p.	16		7.5		14.1
4. S. ¾ E. = ¾ p.	18		17.8	2.6	
5. S. W. ¼ W.=4¼ p.	14		9.4		10.4
6. W. ½ N. =7½ p.	30	2.9			29.8
		2.9	70.4	2.6	91.5
			2.9		2.6
		Southings	=67.5	Westings	=88.9

We find we have made difference of latitude southlngs 67.5 miles, and departure westings 88.9 miles. Dividing the southings by 60, we have,

67.5÷60=1°, 7½′ southings in latilude, or,

latitude attained is

41°, 12′—1.7½=40°, 4½′ N.=L^1.

Then, dividing departure p by difference of latitude l, we have,

p÷l=88.9÷67.5=1.317=tan. 52°, 48′=course,

and because the components of our course are southings and westings, the **name** of our course is S. 52°, 48′ W.

Multiplying difference of latitude by secant of C, we have distance d=67.5×1.655=111.7 miles.

By middle latitude sailing, we have,

D=p×sec. L^2=88.9×1.318=117.2 miles,
=1°, 57′ diff. long. westings.

Then the longitude in

=20°+1°, 57′=21°, 57′ W.

Collecting our results, we have,

L^1=40°, 4½′ N.
C=S. 52°, 48′ W.
d=111.7 miles.
Longitude in=21°, 57′ W.

It will be observed that we have in this problem taken half the sum of the extreme latitudes for the middle latitude L^2. This is the usual practice for small distances. But for large distances, or with small C, it is necessary to apply the correction of table XI.

Or the following rule may be used for finding the parallel on which the departure p must be augmented for difference of longitude.

Take the parallel whose cosine is half the sum of the cosines of the extreme parallels, for the middle parallel. (This rule is original).

This, although not rigorously correct, is practically so for all ordinary cases. It gives the L^2 very slightly too large for L less than 45°, and slightly too small for L larger than 45°.

The student should solve all his questions by at least two methods, as a means of checking against mistakes,—one of which should be by **construction**, for fixing the problem clearly in the mind; another is by **inspection** for some of the parts.

Thus, after we have found the l and p for a traverse, we can search the latitude and departure columns of a traverse table till we find these two components **in the same line.** The corresponding **distance** will be found in the "distance" column on the left, and the course will be found at head or foot of page or column.

But this plan. though rigorously correct in principle, will not generally be found satisfactory.

The traverse tables are not computed generally to arcs varying by less than one degree, so that the two components can rarely be found precisely,—hence some interpolation will be required.

A better method is to divide both the components by the difference of latitude l, Then, by a principle of trigonometry, we have the tangent of the course C, which may be found from the table of Natural Sines, Tangents, etc. And from the same table, the secant of C is found, which gives us the distance d for unity. Then, multiplying sec. C by l, we have the d desired, as in the preceding examples.

Current Sailing.—The effect of a current on a vessel is the same as that of another course and distance, the course being the direction of the current, and the distance being the rate per unit of time,—as an hour,—multiplied by the time sailed in the same.

If a vessel sails with a current, she will be ahead of her reckoning, by the amount of the motion of the current, during the time of sailing in it. And if she sail against the current, she will be behind her reckoning by the same amount.

If she sail across the current, she will be carried with it through the distance the current moves while the ship is in it.

The **direction** of the current, with regard to the meridian, is called the **set;** and the rate at which it runs per hour is called the **drift,**

From the above conditions, it is seen that in all cases, when sailing with a current, the set and drift must be regarded as an independent course and distance. Also that **time** is an element to be considered.

Example: A ship made the following traverse in a current setting N. by W., at the rate of two miles per hour.

1. S. W. ½ W., 2 hours, 8 miles per hour=16 miles,
2. W. ½ S. 3 " 7 " " =21 "
3. W. by N. 3 " 6 " " =18 "

Required the course and distance "made good."

		Dist.	N.	S.	E.	W.
S. W. ½ W.	50°, 37′	16		10.1		12.4
W. ½ S.	84°, 22′	21		1.1		20.9
W. by N.	78°, 45′	18	3.5			17.6
N. by W.	11°, 15′	16	15.7			3.1
Current.			19.2	11.2		54.0
			11.2			miles.
			= 8.0 miles.			

Course=N. 81°, 34′ W. Distance, 54.7 miles.

Example 2: A ship sails S. 17° E. for two hours, at two miles per hour, as indicated by ship's log; thence S. 18° W. four hours. at the rate of seven miles per hour; and during the whole time, the current sets N. 76°, at the rate of two miles per hour. Required the course and distance "made good."

Ans. C=S. 21°, 49′ W. Distance=42 miles.

NOTE.—It will be observed that this is merely the application of **traverse sailing,** with an extra course and distance introduced into the traverse.

Oblique Sailing.—It will be observed that, up to the present, all our determinations for place of ship have been made by means of the properties of the **right plane triangle.** Some questions require the use of the properties of **oblique triangles.** The finding of place of ship by this means, or the determining of questions in navigation by this means, is called **oblique sailing.**

It is used chiefly in the survey of harbors,—in the location of shoals, with regard to their bearing from other objects,—or in finding compass errors. A few examples will illustrate.

But these questions will involve the use of some means on board ship, and as but very few of our lake vessels are provided with such appliances, I give the following method for the

Construction of the Dumb Card, which should be on every ship on the lakes:

Let the ship's carpenter describe a circle, say six or eight feet in diameter, with center C in center line of ship.

Through C, and at right angles to center line, draw the line o o o.

Divide each ¼ of the circle into eight equal parts, and by means of chalk-line or straight-edge, transfer these points onto the rail, and make a clean, deep mark at the intersection. Number these marks from 0 to 4, from forward and from abeam, on each side of the head of ship; and number from 0 to 1, 2, 3, etc., points abaft the beam.

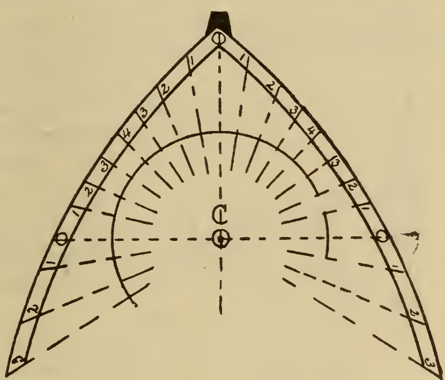


Fig. 22.

Let the marks on the rail be numbered with the number of the point, by driving brass or tin headed nails to indicate the number.

Let the center of the circle be marked, as by driving some nails at the intersection of the fore-and-aft and 'thwart-ship lines.

Then, to use this card, the eye being over the center and looking over the rail, the bearing of any object from ship is readily seen; and the bearing is referred to the **nearest zero line,**— thus we would say an object appears 1½ points abaft the port beam, 2 points forward of the starboard beam, or 3 points off the port bow, as the case may be.

Problem: A ship-master being about to sail, wishes to examine his compass, as to its accuracy. He observes from his chart that when 30 miles out on his proposed voyage, he will be 4 miles to the left of a certain lighthouse that stands on a headland. When nearing the light, he observed it to be $1\frac{3}{4}$ points forward of the starboard beam. After sailing 6 miles, as indicated by the log, and on the same course, the light appeared $1\frac{1}{2}$ points abaft the beam. Having shaped his course, on the supposition that his compass was correct, he wishes to know from the above observations if it is so. If not, which way is it out, and how much.

Solution: Construct the problem as per figure. Draw any right line AB to represent course of ship, and on it take AB=6 miles, to any convenient scale, At A draw the line AC, forward of the beam $1\frac{3}{4}$ p. And at B, draw BC, abaft the beam $1\frac{1}{2}$ p. They will intersect at C.

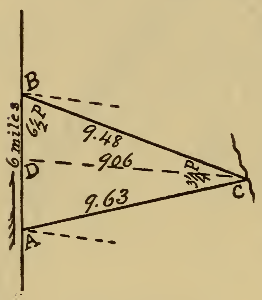


Fig. 23.

It will be observed that each of the angles B and A, in the triangle ABC, is the compliment of the observed angle, i, e., the angle at A=8 p.—$1\frac{3}{4}$ p.=$6\frac{1}{4}$ p., etc.

Then, bearing in mind that the sum of the three angles of any plane triangle is 16 p., we have only to take the compliment of the sums of the angles A and B, to know C=$3\frac{1}{4}$ p.

Then, in the triangle ABC, knowing one side and the three angles, we have a case for the "sine proportion," by which we have,

Sin. C : AB : : sin. B : AC
: : sin. A : BC, or

Sin. $3\frac{1}{4}$ p. (=.5958):6 miles :: sin. $6\frac{1}{2}$ p. (=.9569):AC (=9.63 miles)
:: sin. $6\frac{1}{4}$ p. (=.9416):BC (=9.48 miles).

See table V, for the sines, etc., of points and parts, and let the student verify by careful construction, and let him perform the numerical work here indicated.

Our question requires us to know the height CD of the triangle. This can be measured by scale, or computed numerically.

By equation 3, page 9,

$$CD = AC \times \sin. A \ (=6\tfrac{1}{4} \text{ p.})$$
$$= 9.63 \times .9416 = 9.06 \text{ miles.}$$

By the conditions of our question, we should be 4 miles to the left of C, but the above work shows us to be 9.06 miles to left,—that is to say, our compass has taken us to left of our true course 5.06 miles in 30. Then, by equation 2, page 9, we have,

$$\text{Tan. course} = 5 \div 30 = .1666 = \tan.\ 9°,\ 29'.$$

So that our compass is out to the left $9\frac{1}{2}° = \frac{7}{8}$ p.

In the preceding solution, it would have been sufficient to make one proportion for one side of the triangle ABC; but finding both sides, gave means of checking our work. Thus, either side, AC or BC, multiplied into the sine of its adjacent angle, should produce the perpendicular CD=9.06 miles.

Problem. The following problem is of the same character as the preceding, except the bearings are referred to the meridian instead of to center line of ship:

A port, n, bears N. E. $\frac{1}{2}$ N. from a port, m. At 26 miles out from m, toward n, directly abreast the port beam, is a light, distant 6 miles, when the ship is on the right course. The master having shaped his course on the supposition that his compass was correct, when nearing the light, took its bearing, N. by W. $\frac{1}{2}$ W.=N. $1\frac{1}{2}$ p. W. After sailing on same course 8 miles further, as indicated by ship's log, he took a second bearing, W. by N. $\frac{3}{4}$ N. =N. $6\frac{1}{4}$ p. W. Required to know the compass error, if any,—how much and which way.

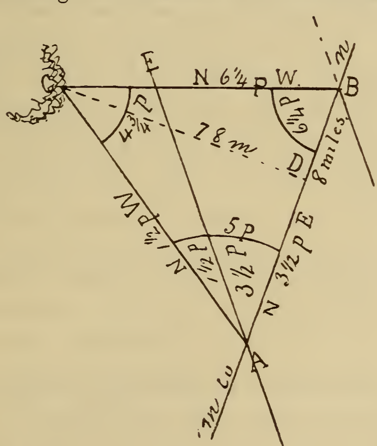


Fig. 24.

Solution by Construction: Draw any line, as AE, through the page, vertically, for the meridian. From any point on this line, as at A, set off the course of ship, $3\frac{1}{2}$ p. to right of the meridian and 8 miles long. to any convenient scale. From the same point A, set off the bearing of the light C, $1\frac{1}{2}$ p. to left of meridian. From B set off BC $6\frac{1}{4}$ p. to the left of the meridian, or, what is the same thing, make the angle ABC=to the supplement of the bearings AB and AC, viz., $6\frac{1}{4}$ p, Then by scale, we find,

CD=7.8 miles. Answer.

Solution by Oblique Trigonometry: In the triangle ABC, the

Angle at A=5 p. or $3\frac{1}{2}+1\frac{1}{2}$,
Angle at B=$6\frac{1}{4}$ p. or 16 p—$(3\frac{1}{2}+6\frac{1}{4})$,
Angle at C=$4\frac{3}{4}$ p. or 16 p.—$(5+6\frac{1}{4})$.

Then, by the sine proportion:

Sin. C : AB :: sin. A : BC,
:: sin. B : AC, or,

Sin. $4\frac{3}{4}$ p. (=.8032) : 8 :: sin. 5 p. (=.8315) : BC (=8.28 miles),
:: sin. $6\frac{1}{4}$ p. (=.9416): AC (=9.38 miles).

Then, either side, AC or BC, multiplied by the sine of its adjacent angle, gives the perpendicular, thus,

AC (=9.38 miles)×sin. 5 p. (=.8315)=CD (=7.79 miles),
and BC (=8.28 miles)×sin. $6\frac{1}{4}$ p. (=.9416)=CD (=7.79 miles).

But, by the conditions of our problem, CD should be 6 miles, that is to say, our compass has taken us to the right, say 1.8 miles in 26, which, by right trigonometry, corresponds to an angle of 4°.

Answer. Compass out to the right 4°.

Many other questions could be proposed for solution by oblique trigonometry, but they would be of a class that seldom or never occur in practice,—are more curious than useful,—so we spend no time with them. Following are a few miscellaneous questions:

Distance of an Object by two Bearings.—The two preceding problems are at the foundation of table XII. They are deemed of so much importance to ship masters who have occasion to round headlands in the night, that a table has been prepared from which the distance of a light, or other object from a ship,—as also the line of the ship's course, at right angles from the light or object, may be told in advance, or before reaching the vicinity of the "danger line."

But the table here presented is quite different from that used at sea, and for the following reason: There, the bearings of the object or light are referred to the meridian, by means of the standard compass, which is furnished with a movable ring and sights for the purpose.

But, on the lakes, our compasses being boxed up in the pilot house, are not available for such work, even if they had the ring and sights. We can only take the bearing of the object from the ship's center line, by means of the ship's dumb compass,—reading the bearing directly from the card. when she is fortunate enough to have one,—which, indeed, is the better way.

The table is constructed generally by solving a number of triangles, varying in their angles by $\frac{1}{4}$ or $\frac{1}{2}$ point, through such limits as would embrace all the cases likely to occur, both for the side opposite the first bearing and for the perpendicular distance of the base or line of the ship's bearing, produced, from the light or object, for a distance of unity.

The results are tabulated in column under the first bearings taken, and in line of the second bearings as factors by which to multiply the distance run between the times of observation for the distances sought,—the larger product giving the distance from ship to light at the time of the second observation; the smaller product giving the distance to light or object when it comes abeam,—or the height of the triangle, as it is technically termed.

In the table which I give (table XII) I introduce the factor, as above, for the distance of the light from ship at the time of second observation. But, instead of the second factor, I introduce the sine of the second bearing, to be multiplied by distance of ship from light, for the perpendicular distance of object, or height of the triangle.

An example will illustrate the use of the table:

Being about to pass a headland in the night, the track by which, as given by the chart, lies two miles to the right, and not knowing whether my compass is correct or whether I was in the proper track when shaping course, I wish to know if my course will take me the proper distance to the right of the light. Soon after making the light, I found it to bear 2 points to the left of the ship's heading. After sailing $3\frac{1}{2}$ miles, as indicated by ship's log, it bore $4\frac{1}{2}$ points to left. Required the distance of ship from the light at time of second observation, also the perpendicular distance of the track of ship from the light. (See Fig. 25).

Solution: Draw the indefinite line A m to represent the ship's course. From any point A in this line, draw the line AC two points to the left of A m, and from A set off to B the distance sailed, $3\frac{1}{2}$ miles. From B draw BC $4\frac{1}{2}$ p. to the left of ship's course, meeting AC in C. Then is C the place of the light.

From table XII, in column of 2 p. and in line of $4\frac{1}{2}$ p., will be found the decimal .81. This, multiplied by the distance, $3\frac{1}{2}$ miles, gives us BC. the distance of light from ship=2.83 miles. This again multiplied by the sine of $4\frac{1}{2}$ p. (=.773)=2.19 miles=DC, the distance of ship's track from the light. This problem is also available in finding compass errors.

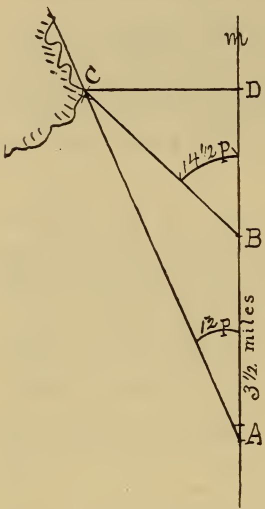


Fig. 25.

Course and Distance from the Co-ordinates of Place.—The wonderful clearness and fullness of detail in our lake charts are purchased at the expense of one convenience, and that is the finding of course and distance, in some cases.

The scale of the chart is so large as to make it impracticable to give us a full or entire lake on one sheet; as a consequence, they are given in sections, which must be combined before courses can be ohtained from them in all cases.

This combination is inconvenient at times, nevertheless the course between ports represented on different sections can be readily obtained from their "co-ordinates of place," as given in the list of lights. An example will illustrate:

At Sturgeon Bay Canal. Required the course and distance to Michigan City.

Solution: From the U. S. List of Lights for the Lakes, we find Sturgeon Bay in latitude 44°, 47′, longitude 87°, 18′. Michigan City in latitude 41°, 43′, longitude 86°, 54′.

The difference of latitude is 3°, 04′=212 miles,
The difference of longitude is 24′=20 miles,
for the mean latitude 43°, 15′.

Whence, in sailing from Sturgeon Bay to Michigan City, we must make sôuthings 212 miles, and eastings 20 miles. The southings are found by multiplying the difference of latitude in degrees by the value of 1 degree=69.15 miles, giving 212 miles. The eastings are found by multiplying the difference of longitude, 24′ by the value of 1 minute of longitude, for the mean latitude, as given by table X, giving us 20 miles.

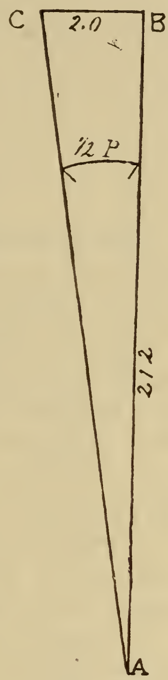


Fig. 26.

Constructing a triangle, as in Fig. 26, with base 212 and with perpendicular 20, and measuring the angle at A with protractor or scale of chart, we find the angle say ½ p. That is to say, our course is S. ½ E.

Then, by plane trigonometry, multiplying the base 212 by the secant of course (=1.0048, table III), we have 213 miles for distance.

By plane trigonometry, also, the course may be found, for

$$20 \div 212 = .0943 = \tan.\ 5°,\ 25' = \text{say } \tfrac{1}{2}\ \text{p.}$$

This, it must be remembered, is the chart course, which must be modified by variation, —in this case ¼ p. to the right,—making the magnetic course S. ¾ E.; and this must again be modified by deviation, if any, also by leeway and for current, if any.

CHAPTER IV.

Construction of Charts.

Construction and Use of Mercator's Chart.—The principles underlying the construction of this chart have been examined under the head of Mercator's Sailing. It only remains to make the practical application.

Suppose we were to construct a Mercator's chart of the territory embracing the great lakes,—say from latitude 40° to 50°, and from longitude 75° to 95° W. We must prepare the skeleton or blank, as follows: (See table on following page).

1. Write down in a column the degrees and parts of a degree that are required to be represented on the sides of the map,—as the whole degrees, ½° or the ¼°. (See table IX).

2. From a table of meridianal parts (abbreviated to M. P.) write the M. P's corresponding to each degree and part of a degree, in an adjoining column.

3. From the M. P's of the highest latitude, subtract those of the lowest latitude. Thus, the M. P's of 50° (=3474.5), the M. P's of 40° (=2622.7), their difference is 891.8, which is the depth of our map in latitude, in the scale units that represent 1°-

4. From the M. P's of 50°, take the M. P's of the whole degrees, 49°, 48°, 47°, etc., setting down the differences opposite their respective latitudes. Thus each difference will represent the distance between the consecutive parallels that are one degree apart, and the place of each parallel can be marked on the side of the map at one placing of the scale, after we

5. Add the several differences consecutively. for the height up to the consecutive parallels. Thus,

The M. P's for 50°=3474.5
" " " " 49°=3382.1

92.4, seen at foot of 3d column,

M. P. for 49°—M. P. for 48°=90.6, as seen in 3d column, etc.

In this manner will the widths of single degrees be found and recorded in the 3d column. Then, adding consecutively, as per (5), we have the height of the first parallel from base of map= 78.9 scale units. Then 78.9+80.1=159=height of 42° parallel from base of map; 159+81.4=240.4 M. P's for the height of the 43d parallel, etc. In this manner was the fourth column produced, that shows the height of each parallel from base of map.

6. In the widths for the ½ degrees, subtract the M. P's of 49½° from those of 50°, and we get 46.5 M. P's, found at the foot of the 5th column. The M. P's for 49½°—those for 49°=45.9 M. P's, seen in the 5th column. And in this manner was the width of each consecutive half degree found.

Table of Elements for a Mercator's Chart.

Latitude 40° to 50°.

Lat.	Meridianal Parts.	Widths of Degrees.	Heights in Latitude.	Widths of Half Deg.
40	2622.7			39.3
½	2662.0			39.6
41	2701.6	78.9	78.9	39.9
½	2741.5			40.2
42	2781.7	80.1	159.0	40,6
½	2822.3			40.8
43	2863.1	81.4	240.4	41.2
½	2904.3			41.5
44	2945.8	82.7	323.1	41.9
½	2987.7			42.3
45	3030.0	84.2	407.3	42.6
½	3072.6			43.0
46	3115.6	85.6	492.8	43.4
½	3159.0			43.7
47	3202.7	87.1	580.0	44.2
½	3246.9			44.6
48	3291.5	88.8	668.8	45.1
½	3336.6			45.5
49	3382.1	90.6	759.4	45.9
½	3428,0			46,5
50	3474.5	92.4	851.8	

The M. P's for half degrees were found by taking the difference of the consecutive half degrees in columns 1 and 2.

The table for the values of the degrees of latitude, being prepared, we are ready to construct the framework or skeleton of our chart.

1. Assume any convenient scale.—preferably one that is decimally divided, and set off on a horizontal line, at the foot of the chart (the south side,) the amount of longitude required,—in this case, 20° or 120 miles (1 mile being the scale unit), marking the points for the meridians,—each degree or each alternate degree, as wanted.

2. At such extremity of the line representing the width of the map in longitude, erect a perpendicular for the extreme meridian or sides of the chart.

3. On each of these meridians, set up the distances given in the preceding table, for the places of the several latitudes, commencing at the bottom of the sheet, thus,

Set up 78.9 M. P's for the place of the 41° parallel,
" 159.0 " " " " 42° "
" 240.4 " " " " 43° " etc.

Thus we have all the parallels located at their proper places, to represent the augmented latitude.

4. We can now locate the places for the half degrees from their scale values, in the column for the widths of half degrees.

The framework or skeleton is now ready to take the location of places,—as towns, coast-lines, rivers, islands, boundaries, etc., which are located from their known places of latitude and longitude, by means of two T squares,—one locating the latitude, the other the longitude,—and their intersection being marked by a needle-point or sharp pencil. Thus, a number of points in the boundary of a lake or bay being located and a fair line traced from one to another, the shore line is located, etc.

It is to be remarked that the **"meridianal parts"** given in our table IX, are those that have been in use a long time, as first constructed, on the supposition that the earth is a **sphere.** But more modern works compute them for the earth regarded as a **spheroid,**—making them slightly **smaller** than for the sphere, for any given latitude.

Bearings and Distances.—The **bearing** between any two points on a Mercator's chart, is very readily found.

It is only necessary to draw a straight line between the two points and apply a protractor to any meridian crossed by this line, to read the bearing. This, and the straight "rhumb line," are two of its conveniencies. But not so with **distance.** This **cannot be measured** by scale, on a Mercator's chart, except in the direction of longitude; and this measurement must be multiplied by the cosine of the latitude before it is available for use, for it will be remembered that the whole map, away from the equator, has been expanded.

Scale measurements **cannot be made in any oblique direction,** because the scale varies from the equator toward the pole. Thus, at 40° of latitude, a half degree is only 39.3 M. P., while at 50°, it is 46.5 M. P.,—and this is an objection.

Construction of the Plane Chart.—In the plane or rectangular chart, the meridians are parallel lines at a uniform distance apart.

This distance, for one degree, is found by multiplying the equatorial distance, 60 miles geographic or 69.15 statute, by the cosine of the latitude for which the map is made.

The following considerations will show the amount of error for such a chart:

First. It must be remembered that the meridians at the equator are parallel, and at the poles they have their maximum inclination, which is the whole difference of longitude, and that between these limits, the inclination varies as the sine of the latitude.

Example: Suppose we wish to make a rectangular or plane chart for the latitude of 42°, for an area of one or two counties, or say 30 miles square. Multiplying 60, the number of geographical miles in one degree, by the sine of 42° (=.6691) we have 40.14 geographic miles for one degree of longitude, or 46.28 statute miles, and the error resulting from their being parallel, would be $60' \times \sin. 42°$ (=.6691)=40′ for 1° of longitude, i. e., in a block 30 miles square, the south side would be about 55 rods **too small,** while on the north it would be that amount **too large.**

The Conical Projection.—In the conical projection, the meridians are all right lines, but they are inclined by the amount due to the central latitude of the map. There are two methods for this projection,—the **tangent** and the **secant.** The latter being the more accurate, is the one we will illustrate.

In Fig. 27, let ABC be the arc of latitude to be embraced by the map. Set off from each end of the arc of latitude, one fourth of its length, to a and a^1, and through these points draw the secant intersecting the earth's axis produced in D. Then, the cone, of which $a a^1 D$ is an element, will, when rolled out or developed, be a portion of a circle and the radials through it, and of which the secant a a^1 D is one, will represent the meridians of the sphere, and circles described through it from D as a center, will be the parallels of latitude. The points whose latitude is a or a^1, would be correctly represented in magnitude, while those near A and C would be too large and those near B would be represented too small.

The following example will illustrate, viz., to construct the lines for a map embracing latitude 40° to 50°, and 75° to 95° of longitude, i. e., the vicinity of the great lakes.

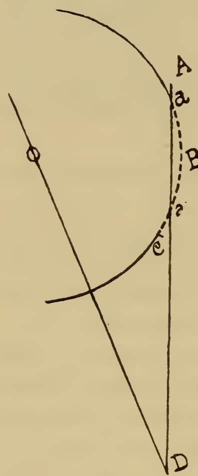


Fig. 27.

Dividing the difference of latitude, 10°, into four parts ,we find the two parallels, a and a_1, to be five degrees apart, and two and one-half degrees from the extremity of the map; and a=42½°, and a^1=47½°.

The length of one degree of longitude on the parallel a

=cos. 42½ (=.7373)×60=44.24 miles,

and on a^1,

=cos. 47½ (=.6756)×60=40.54 miles,

and their distance apart on the meridians,

=5×60=300 miles.

And the radius of the developed parallel a, is the cotangent of 42½° multiplied by the radius of the earth; also that of a^1 is cotangent of 47½° multiplied by the earth's radius. These being too large to be used as sweeps with which to describe the parallels, we must devise some other means of sweeping arcs of circles.

Draw a line vertically through the middle of the map, to represent the middle meridian. From head of the map, set down 150 miles for the place of the parallel 47½°. Below this, set down 300 miles more for the place of the parallel 42½°, and below that, set down 150 for the parallel 40°, limiting the south side of the map.

From a, (see Fig. 28) with one or two times the width of one degree of longitude, 44.24 miles, as computed above, accordingly as a meridian is wanted for every alternate degree, describe the arc of a circle, both to right and left.

In the same manner, from a^1, with the same multiple of the width, 40.54 miles, describe an arc on each side of the meridian, and through these arcs draw the meridians, as for 83° and 87° of longitude.

With twice their distances. sweep again from same centers, for the meridians 81° and 89°, and three times their distances, sweep arcs for 79° and 91°, etc.

Produce the two outside meridians of the map till they meet the middle meridian, from which point sweep the parallels of latitude 40°, 42°, 44°, etc.

The above is the most practical and readily available of the many methods in use for representing areas of several hundred miles square. A more accurate method, called the **Polyconic Projection,** is in use by the U. S. Coast and Geodetic Survey, but as it is somewhat abstruse in its construction, we do not illustrate it.

Orienting Ship.—Two methods, the Direct and Reciprocal, of finding compass errors, have been treated, together with the appliances for the work. But the subject is of such importance, that we give a number more, including the Orienting of Ship, for the purpose of compensating ship's compass.

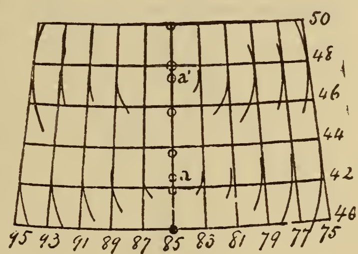


Fig. 28.

There are many opportunities of finding lines of "known bearing," as between intervisible lights, or between two headlands, or between a light and a headland. Such lines are said to have a port-hand or a starboard-hand bearing, accordingly as the line is to the left hand or to the right of the meridian, as the observer looks to one of the objects.

Thus, the line Skilligille-Waugoshanee, bears N. 2½ p. E. The observer standing in this line and looking either to the north or the south, will see one of the lights to the **right** of the meridian, whence, the light is said to have a **starboard-hand** bearing. This notation will be found very convenient for defining the bearing of lines.

Example: The ship crosses a line known from the chart, to have a port-hand bearing of $2\frac{1}{2}$ p. At the moment of crossing, the line is seen $\frac{1}{2}$ p. abaft the port beam, as indicated by the dumb card, described on page 57; and the ship's compass indicated ship to be headed E. by N. $\frac{1}{2}$ N., and the chart gave the variation E. $\frac{1}{4}$ p. Required whether the compass is in error, which way and how much.

Solution: Draw two lines, N. S. and E. W., to represent the meridian and the prime vertical, at right angles to each other. Through the intersection of these lines, draw the reference line, to the left of the meridian $2\frac{1}{2}$ p., and draw the magnetic meridian $\frac{1}{4}$ p. to the right of the true meridian and the E. W. line. Then, by observation, the ship's head (indicated by the arrow-point) was $8\frac{1}{2}$ p. to the right of the reference line. From this, subtract $2\frac{3}{4}$ p., the distance of the magnetic meridian to the right of the true meridian, and find ship's head to be $5\frac{3}{4}$ p. to the right of magnetic meridian, leaving $2\frac{1}{4}$ p. between ship's head and magnetic E.; but the compass shows only $1\frac{1}{2}$ p., whence compass is out to left $\frac{3}{4}$ p. Thus the dumb card gives great facility and clearness to this class of questions.

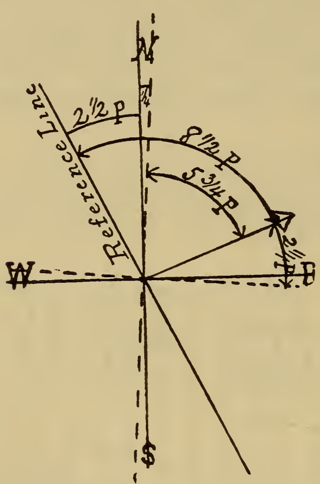


Fig. 29.

Example 2: The line St. Helena–McGulpin's Pt,, has a port-hand course of $44\frac{1}{2}°$, or say 4 p. A steamer, in crossing this line, observed it $6\frac{1}{2}$ p, off the starboard bow. The compass indicated ship's head at S. W. by W., variation $\frac{1}{8}$ p. to the right. Required compass error, if any.

Solution: As in the former example, draw the cardinal lines and the line representing the magnetic meridian. Then, by observation, the ship's head was $6\frac{1}{2}$ p. to the left of the reference line, or $10\frac{1}{2}$ p. to the left of the N., or $5\frac{1}{2}$ p. to the right of the S. But by ship's compass, the ship's head was only 5 p. to the right of

S., whence, by deviation and variation, it is $\frac{1}{2}$ p. out to right Deducting variation $\frac{1}{8}$ p., the needle is out to right $\frac{3}{8}$ p.

These two problems will show the method of treating this class of questions, and the great utility of the dumb card.

With the dumb compass, described elsewhere, there would be no computation whatever. The sights being set to the bearing of the reference line and then turned into that line, the true heading of ship is read from the card.

Orienting Ship by Amplitudes.—The bearing of the sun at sunrise or sunset, when referred to the prime vertical, i. e., to the E. or W., is called the **amplitude** of the sun.

This angle or bearing depends on the **latitude** of the observer and on the **declination** of the sun. It is a problem of spherical trigonometry, and is solved by the following equation:

Sin. amplitude=
sec. lat.×sin. dec., or
=sin. dec.÷cos, lat.

Example: In latitude 43° N., with sun's declination 21° N., what is the bearing of the sun at sunset or sunrise.

Solution:

Sec. L (=43°)=	1.367
Sin. D (=21°)=	.3584
	5448
	10936
	6835
	4101

Amplitude
=sin. 29°, 20′=.4899308

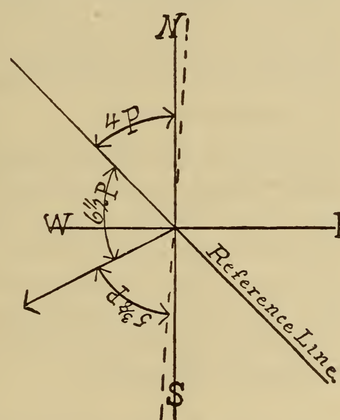


Fig. 30.

As the bearing takes its name from the declination, the sun will bear E. 29°, 20′ N. at sunrise, and W. 29°, 20′ N. at sunset,—or better, N. 60°, 40′ E. and N. 60°, 40′ W.

The dumb compass makes this reference line available for orienting ship, with great facility. Thus,

1. Set the index to the reading of the amplitude and turn the card under the index, to the right or to the left, by the amount

of variation for the locality. This reduces the true amplitude to a magnetic amplitude, which is what is wanted when we are looking for compass deviation or when we are compensating for deviation.

An example will best illustrate, though at the risk of some repetition:

It is required to orient ship at Buffalo. Lat. 43° N., sun's dec. 18° N., var. 5° W.

Solution: From the table of amplitudes (table IV), select the amplitude 25°. corresponding to the given latitude and declinaation. The dumb compass being in position, with the zero line of the lower index, or lubber line, set toward the head of the vessel, and the north side of the card approximately to the north, set the index of the right bar to read the amplitude, viz., E. 25° N. for sunrise, or W. 25° N. for sunset.

Turn the card under the index 5° to the left. Clamp the index and card together and sight to the sun. Then is the card oriented to the magnetic meridian.

Hold the card and sights pointing to the sun and give ship the "wheel" till ship's head is at the desired cardinal point, as indicated by the reading of the lower index. Then will ship be oriented to the magnetic meridian.

The two preceding methods of finding true bearing of ship's course, are eminently practical and of easy attainment, readily available and requiring but little or no mathematical work; but each has its inconvenience.

In the first, the ship must recover her place in the reference line after each observation, in order to avoid parallax.

In the second, by amplitudes, according to the usual practice, the time for work is too **limited.** The sun soon leaves the place defined by the amplitude, when observations for compass error must cease. But there is a property in the rate of the change of the sun's place, by which this inconvenience may be avoided, at least for some time, and that is,

A Constancy in the Rate of Change of the Sun's Azimuth. It is found that, with latitude and declination of the same name, the rate of change of the sun's azimuth, between sunrise and the time of his crossing the prime vertical, is practically constant for any latitude above 40°. Thus, with latitude 37°, declination 0°, the rate of change is one degree in 6 minutes, 40 seconds. With declination 12°, the rate is 1 degree in 6 minutes, 20 seconds. Again, in latitude 45°, with declination 0°, the change of the sun's azimuth is 1 degree in 5 minutes, 50 seconds. With declination 12°, it is 1 degree in 5 minutes, 30 seconds, and with declination 20°, it is 1 degree in 5 minutes, 30 seconds. And this rate will hold for a few minutes after passing the prime vertical, after which it increases rapidly.

This property in the rate of the change of the sun's azimuth, between sunrise and the prime vertical, or between the prime vertical and sunset, makes it easy to construct a table that is in all respects a veritable table of the sun's time azimuth.

To facilitate this work, I have computed the rate of change for each latitude, and written it immediately under the latitude of each of the columns of table IV.

An example will best illustrate. With latitude 43° and declination 15°, alike, the amplitude 20°, 44′,—say 21°. Observe the watch time of the sun's rising and change this amplitude to azimuth by taking the compliment. Then, say at 5 A. M., the sun rises N. 69° E.; at 5:06 A. M., he is at N. 70° E.; at 5:12 A. M., he is N. 71° E., etc., till we get 90° of azimuth, when the sun is in the prime vertical and the rate of change begins to increase. The evening would do as well, but then the observer would have to know the time of sunset and the error of his watch on apparent time.—things not always known.

By Time Azimuths of the Sun,—The relations of latitude and declination, and local apparent time, are such that these elements being known, the bearing of the sun can be readily deduced. These bearings are called Time Azimuths.

They are deemed of such importance at sea that tables of them for all declinations up to 80°, and for time up to the length of the longest day, have been computed. Among the best of these azimuth tables, are those of Major General R. Shortrede, F.R.A.S., a part of which we give in this work, with explanations for their use. (See table VI).

These tables require a precise knowledge of the local apparent time, and, as a consequence, at sea, require daily observations for that element.

But with us on the lakes, or along the coast, now that we have Standard time established, and the latitude and longitude of all the lights along the coast and lakes being known, the case is very different. The astronomical work required at sea for local apparent time, is dispensed with on the lakes,—for the reason that it is done for all time, and to a degree of precision that we could not hope to attain,—in the U. S. List of Lights.

This with the equation of time as furnished in the **Nautical Almanac**, and our **Standard** or **Mean Time**, which is now available in nearly every R. R. Station, gives us the means of knowing the apparent time at any locality, with great precision, whence the Azimuth tables are available to us without the astronomical instruments, and without the nautical service, requisite for their use at sea.

Local Apparent Time.—As our watches are supposed to be regulated to Standard or Mean time for some particular meridian, and as we want the local apparent time, it is necessary to find the **error of our watch** on such time.

And as we are to find this error from the difference of longitude between the meridian of the observer, and that to which our watch is regulated,the reduction of arc or longitude to time, and the reverse, will be involved in the question. (See table VIII.)

This **reduction of Arc to Time**, and the reverse is made by means of margined tables to the tables of natural Sines Tangents, etc. which see. The outer column on the **left** contains the time for the degrees at the top of the page, and the minutes of arc on the left. (See large works on Navigation.)

The outer column on the **right**, gives the time for the degrees at the foot of the page, and the minutee of arc on the right.

In the time columns the hours under 3, will be found at the **head** of the column. The minutes in **black faced** figures, and the seconds in common figures in the column.

The hours from 3 to 6 will be found at the foot of the column.

Thus, for 34°, 28′ of arc, the corresponding time=2 h. 17 m. 52 s.
Thus, for 76°, 16′ of arc, the corresponding time=5 h. 5 m. 4 s.

The first step in this problem, is to find that part of the error of our watch due to difference of longitude.

This is found by taking the difference of longitude of different places, as given in the List of Lights, on the lakes, and reducing the same to time at the rate of one hour for 15° of longitude, one minute of time for 15′ of longitude, etc. or 4 m. to 1°,

Thus the longitude of Chicago is given as 87°, 37′, while the longitude of the standard time watch is 90°,—the difference of longitude is 2°, 23′. This, reduced to time by table XIII, is 9 m. 32 s. That is to say the difference between standard time, and the local mean time of Chicago. is 9 m. 32 s.

And because the meridian of the standard time 90°, is West of that of Chicago, the watch is **slow**, and requires the (+,) plus correction to reduce it to the local mean time of Chicago.

Again,—the longitude of Buffalo is given as 78°, 54′, while the longitude of the standard time watch,—to which Bnffalo mean time is referred, is 75°,—and their difference is 3° 54′, which reduced to time, is 15 m. 36 s. But in this case, the standard meridian is East of the local meridian,—whence the time correction must be **subtractive** to reduce the standard to the local mean time,—and the minus sign is prefixed to the time correction. In this manner was table VII prepared for most places on the lakes.

Having found the error of our standard time watch on the mean time of any locality, it remains to find the error on **"Apparent Time,"** i. e., the error on the time indicated by the sun.

Apparent time does not progress uniformly as does mean time. As a consequence, it is sometimes **fast** and sometimes **slow** of the mean time clock. Just how much fast or slow is shown by the Nautical Almanac, under the heading "Equation of Time," technically called the Eq. of time.

The sign prefixed to this correction of time,—or equation of time,—or eq. of time,—is that for reducing apparent time to mean time. Thus, when the sun is slow, the eq. of time has the + sign prefixed to reduce apparent time to mean time, and when he is fast, the — sign is applied.

But in our case, we wish to reduce our mean time to apparent, i. e., we wish to find the error of our mean local time on apparent local time, whence the above signs must be reversed. Accordingly, in table V, we have prefixed to the time correction, or eq. of time, the sign requisite to reduce mean time to apparent time, whence,

To find the error of the standard time watch on local apparent time, we have only to take the algebraic sum of the two errors. or corrections of tables V and VII, observing the signs.

A few examples will illustrate this problem:

1. At Chicago, June 15th (no matter what year), required the error of the standard time watch on local apparent time, for the purpose of finding azimuth from the sun.

Solution :

Time correction for diff. long. (table VII),	+ 9 m., 32 s.
Eq. of time (table V), - -	— 10 s.
Error of watch on apparent time, -	+ 9 m., 22 s.

That is to say, we must count the time forward 9 m., 22 s., on the watch, to get apparent local time.

2. At Chicago, November 1, required the error of standard time on local apparent time.

Solution:

Time correction for diff. of long. (table VII),	+ 9m., 32s.
Eq. of time (table V), - - -	+16m., 20s.
Error of watch on local apparent time,	+25m., 52s.

As before, we must read our watch forward, say 26 m., to get local apparent time.

3. At Cleveland, May 24, required the error of the standard time watch on local apparent time.

Solution:

Time correction for diff. long. (table VII),	—26 m., 48 s.
Eq. of time (table V), - -	+ 3 m., 26 s.
Error of watch on local apparent time,	—23 m., 22 s.

Here we must count **backwards** on our watch to get local apparent time, 23 m., 22 s.

It will be observed we have not given the year of the date in our examples. This is not necessary for purposes of azimuth, for the corrections being the mean corrections,—equations, as they are called,—for four years, the error cannot be more than the change of declination for $\frac{1}{2}$ day, and cannot, therefore, appreciably vitiate the resulting azimuth.

Explanation of the Tables of Tlme Azimuths.—The tables are arranged with the declinations in a column on the left of the page, and azimuths in a line at the head of the column, the hour angle corresponding to any declination and azimuth, being found in the line of the one and column of the other.

The declination 0, is written in the middle of the column, and it increases upwards and downwards to 24°, varying by intervals of 2°. The upper part of the column is marked + to correspond with latitude of the same name; the lower part is marked — to correspond with latitude of different name.

Definite azimuths are assumed, and the corresponding H. A. (hour angles) are computed for each degree of latitude and for each second degree of declination, and written in columns under their respective azimuths (Z), and in lines opposite their respective declinations, thc degrees of the latitude being written at the head of the page.

The table commenees with the sun at apparent noon, giving on the first page, azimuths (Z) for intervals of 5°, and on the second page, at intervals of 3°.

Azimuths, as well as declinations, might have been given at smaller intervals, but these intervals were assumed to keep the tables from being too voluminous, while they are yet small enough to make interpolation easy. The following definitions will help to a more ready apprehension of the manner of using the tables:

Upper and Lower Pole.—The upper pole is that one having the same name as the latitude of the observer. Thus, with us, the north pole is the upper pole, while the south is the lower pole.

North and South Meridian.—That part of the observer's meridian between himself and the north pole, is called the north meridian, while that part between himself and the south pole, is called the south meridian. The meridian opposite in longitude, is called the nether meridian.

Supplement of H. A., is what the hour angle lacks of 12 hours, or 180°.

P. M. and A. M.—In popular language, these initials imply the after part or the fore part of the day. But in this question, A.M. implies that part of the forenoon within 90° of the meridian; while P. M. implies that part of the afternoon within 90° of the meridian.

Morning, is that part of the forenoon between sunrise and the prime vertical, or 90° from noon; and **Evening** is that part of the day between the prime vertical and sunset. These terms do not occur, except with latitude and declination of the same name.

Azimuths are reckoned from that pole that is on the same side of the prime vertical as the sun. Thus, with the sun in N. declination. we reckon azimuth from the north, **morning and evening,** i. e., before the sun passes the prime vertical in the morning, and after he passes it in the P. M.

Zenith and Nadir.—Zenith is the pole of the observer's horizon, while Nadir is the pole of the nether horizon. So that when the sun is more than six hours from the meridian of the observer, it is less than six hours from the nether meridian, and consequently above the nether horizon; and the supplement for the H. A. for the evening is the H. A. for the morning.

As the terms latitude, azimuth, declination and H. A., do not, in themselves, show immediately when the sun is above or below the horizon, this information is given by the **black line** in the several columns. The terms below the black line belong to the sun when below the horizon, and as a consequence, to the evening and the morning.

To Use the Tables.—The tables answer such questions as the following:

At what time in the day will the sun be at a given azimuth Z from the observer? Or, at a given time in the day, what is the bearing of the sun? In reply to the latter question, we will say:

If the latitude and declination are of the same name, i. e., both N. or both S.. find the declination in the upper part of the page, on the left, with the latitude at the head of the page. Then follow the line of the declination till the H. A., or time nearest to it, is found. Then, over the column containing the H. A. thus found, will be the azimuth Z.

1. Thus, in latitude 43, with sun in declination 20°, both N., what will be the bearing Z of the sun at 34 m. P. M.?

The latitude and declination being of the same name, we look for the declination on the upper left of the page, with the given lat tude at the head. Then, following the line till we find the given H. A., 34 m., and looking to the head of the column, we find Z=20°. That is to say, the sun will then bear S. 20° W. Or at 11 h., 26 m. A. M., the sun would bear S. 20° E.

2. In latitude 46°, with declination 18°, both N., what will be the bearing of the sun at 7 o'clock in the morning?

Taking up the declination in the upper left of the page and following the line of H. A. to the right, we find that the sun does not cross the prime vertical till it is 4 h., 56 m., from noon, so the sun is still on the morning side of the prime vertical. Looking in the line of 16° declination, in the lower part of the page, we find at 7 h,, 04 m., the sun is in the prime vertical, i. e., due east.

3. What will be the bearing of the sun at 8 o'clock A. M., at 10 and 11, apparent time?

As the H. A. in these are the times from noon, we must look for the **supplement** of A. M. time. Thus, at H, A. 4 h., 08 m., Z=81°; for H. A. 2 h., 01 m., Z=50°; for H. A. 1 h., 06 m., Z=30°.

With the preceding general information concerning the table, the following more concise instructions. adapted to our particular latitude, viz,, the coast and lakes of the U, S., will be permissable:

To find the H. A. for Morning, with the sun in N. declination. Look in line of the declination for the given latitude, and under the black line, where will be found the H. A. from sunrise to the prime meridian, with the corresponding Z at the head of the column.

The H. A. will be found increasing, as also the Z, from left to right, till the sun crosses the prime vertical. At the crossing, the morning changes to A. M., and the supplement of the clock time becomes the H. A. which is found in the line of the declination in the upper part of the page, then

The A. M. H. A's will be found in the upper part of the page, with the corresponding Z at the head of the column, both the Z and H. A. diminishing from right to left, from prime vertical to noon, where they vanish.

NOTE.—It must not be forgotten that while the sun is north of the prime vertical, the Z must be measured from the north meridian, or, if read from the south meridian, the Z will be found in the corresponding column at foot of page.

Thus, for latitude 43° N.. declination 20° N., in the morning at 4 h., 46 m., apparent time, the sun is 63° from the north meridian, i. e., its Z is N. 63° E. At 7 h., 32 m., it is due east, i. e., it is in the prime vertical, and its Z is 90°, as seen both at the foot and the head of the right hand column. The supplement of this time, or 4 h., 28 m., shows the same thing, in the same column, in the upper part of the page, in line of the given declination, 20°.

With the Sun in South Declination, we shall never have use for the H. A's under the black line.

The **supplement** of the clock time will be the H. A. from sunrise to noon, where it is 0.

These commence in the line of the given declination, immediately to the left of the black line, and diminish to the left,—as do also the Z's.

In the P. M., the clock time is the H. A., which will increase from left to right,—from noon to sunset,—which will be found at the black line or immediately above it.

Example: Thus, with latitude 43° N. and declination 8° S. the last P. M. H. A. given is 5 h., 24 m., with corresponding Z=78°. The H. A. and Z would be the same for A. M., but the clock time would be the supplement of the H. A., or 6 h., 36 m.

Interpolation.—We may want Z for intervals, or at times intermediate between those given in the table. These are easily obtained.

The difference between two consecutive H. A. in the same line, is the difference or interval between the two corresponding Z's. This, divided by the difference of the Z's, gives the time due to a change of 1° in the Z.

Thus, for latitude 36°, declination 20°, both + the H. A. for Z=60, is 1 h., 38 m. For Z=63°, the H. A. 1 h., 47 m., i. e., in 9 m. Z has changed 3°, or 1° in 3 m.

Whence, having found the H. A. for Z, we may set the index of our dumb compass, or azimuth ring forward 1 degree for each 3 minutes, or for each 4 minutes, or 5 minutes, as the case may be. By forward, is meant the motion from left to right, as we read the dial of a watch,—supposing the eye of observer at the center of card.

Interpolation for Declination.—This is done with the same facility. Thus in latitude 43°, declination 19°, with names alike, or +. The H. A. for 60° azimuth is required. Here, the H. A. for declination 20° is 2 h., 10 m. For declination 22°, the H. A. is 2 h., 02 m. The difference is 8 m., whence the H. A. corresponding to declination 19° is 2 h., 6 m., for Z=60°. This can all be done mentally.

The rate of change of azimuth is not alike in all parts of the day. It is slowest at the prime vertical and most rapid at the meridian, as before shown.

Preparation in Advance.—The ship-master or compass-adjuster who contemplates to swing ship for compass errors, particularly in the A. M. part of the day, should find the error of his watch on local apparent time in **advance.** Take the supplement of the H. A. in advance and reduce them to his watch time,—thereby saving time by being ready for any favorable moment, and avoiding liability to mistake by not having too many things on the mind at once.

A convenient way of interpolating is to find the rate at the azimuth, Z changes, between two consecutive H. A., as given in the tables, precisely as given in the methods by amplitudes (page 71). Thus, with latitude 43° N., declination 18° N., the

H. A. for Z=66° is 2 h., 40 m.
" " "=69° is 2 h., 52 m.

3° 12 m., or Z changes 1° in 4 m.

H. A. for Z=15° is 27 m.
" " "=20° is 37 m.

5° 10 m., or Z changes 1° in 2 m., etc.

The error of the watch on local apparent time being found, and the H. A. for any particular Z, or azimuth, being looked out of the tables, the work of orienting ship is precisely that explained in former articles.

The method of orienting ship by means of time azimuths will cost the student more study, more diligent and patient application than any other of the several methods enumerated for orienting ship, but it is also the most useful, the most satisfactory and the most generally available; and now that we have standard time established, it is peculiarly adapted to use on the lakes and coast, where ii is available without the astronomical work requisite for its use at sea.

CHAPTER V.

Terrestrial Magnetism and the Magnetism of Iron in Vessels, as Affecting the Compass Needle; and the Correction of Compass Errors.

Of magnetism we know little more than that it is one of the many imponderable agents by which the material universe is actuated. Some of the laws governing its mode of action have been discovered by observation.

Prof. Bartlett, formerly of the Military Academy of West Point, gives us to understand that what is called **Terrestrial Magnetism** is generated by a thermal wave constantly flowing from east to west, caused by the constant change in temperature of that portion of the earth's surface exposed to the sun's rays, together with the axial motion of the earth.

And electricians tell us that one of the properties of a magnetized needle is to take position directly across or at right angles to a current of electricity, when brought in proximity to such current.

As a consequence, if this current were strictly parallel with the geographical equator, the magnetic needle would coincide with the geographical meridian and there would be no variation, but unfortunately, this is not the case.

For some reason, supposed to be the nonsymmetry of the topographical conditions contiguous along the track, or to inequality of conducting power of different parts of the earth's surface, this current is diverted from a parallelism with the earth's equator,

—resulting in what is called the "variation" of the needle,—the current being sometimes to the right, and again to the left of the equator, in its motion from east to west.

A little to the west of Africa, in longitude about 0, this current, or rather the center of this current, crosses the equator, coming from the north and taking a southwesterly direction, touches South America near latitude 18° S. From this point in its westerly course, it tends gradually northward, reaching the equator near longitude 70° W. from Greenwich, and giving **westerly** variation in the Indian and Atlantic Oceans, Africa, most of Europe, and that part of North America east of the great lakes, wtth a small portion of eastern South America. And **easterly** variation to the entire Pacific Ocean, most of the two Americas, and Asia.

But this terrestrial magnetism is not the only force that is active in giving position to the needle. The magnetism in the iron of the ship is the principal disturbing element that makes trouble tor navigators, with regard to their compasses.

Now that the practice is becoming general, on the lakes, of compensating ships' compasses for deviation, by the use of opposing magnets, it is important that those who are to do this work should have a little idea of the manner this local magnetism is deduced from terrestrial magnetisms, and of some of the laws of its action. Accordingly I offer the following few elementary facts and definitions :

Magnetic Equator.—There is a point in the current spoken of, in which the magnetic needle assumes a **horizontal position** when suspended in the magnetic meridian. To the north of this point the north end of the needle **dips**, i. e., inclines downward, while to the south, the south end dips. The line on which the needle is horizontal is called the magnetic equator, in consequence of proximity to, and analogy to the geographical equator. The Dip is the angle of inclination to the plane of the horizon and increases from the magnetic equator to the magnetic pole, each way,—but at an irregular rate,—when it is vertical, or 90°.

Magnetic Latitude, is distance either north or south, defined by the dip of the locality. Thus, along the south shore of Lake Erie, or the south part of Lake Michigan, the north end of the unbalanced magnetized needle would dip about 73°. These localities would then be said to be in 73° of magnetic latitude, though their geographic latitude is only abont 42° north.

Line of Force.—The opposite poles of a magnet being at its extremities, the direction which the needle takes shows the direction of the action of the forces residing in the two ends, or opposite poles.—such direction being called the line of force.

It must not be supposed that, as between the forces of the two opposite poles, there is any force of translation, but merely that of direction. The maximum intensity of magnetism in any locality being in this line of force.

Neutral Plane.—If a plane be conceived to intersect the line of force at right angles, it will show the position of minimum force for that locality, or rather the position of no magnetic force. It is in this plane that magnetic forces change sign in passing from one pole to the other,—whence it has come to be called the neutral plane.

If any object susceptible to the earth's influence, as a bar of soft iron, make any angle with the plane, the intensity of the magnetic action will be proportioned to the sine of this angle. Thus, at the points before mentioned, in latitude 43°, if a bar of soft iron be laid in a horizontal position and in the plane of the magnetic meridian, it will be 17° from the neutral plane. Then as the sine of 17° is .2923, it will be acted on by less than .3 of the earth's magnetism at that place.

If the rod is up-ended to the vertical, it will be 73° from the plane and will therefore be acted upon by .956 of the earth's magnet force,—this being the sine of 73°. While if the rod be placed horizontally in an east and west line, it will be in the neutral plane, and therefore will not be acted on at all. This variable intensity of the earth's magnetic action makes great trouble for navigators.

Components of Magnetic Force.—The needles in use in all compasses are horizontal, while, as we have seen, the earth's magnetic force acts in an inclined direction. It is therefore necessary to regard the earth's magnetic force as the resultant of two other forces,—one horizontal and one perpendicular, or rather, vertical.

The Horizontal Component, is that part of the earth's magnetic force that orients the needle, while the vertical component acts constantly in a vertical direction, in the magnetic meridian, —varying the dip with the change of magnetic latitude, and, as we shall see hereafter, indirectly disturbing the needle.

The Relative Magnitude of these two forces is shown on Plate II, Figs, 2 and 3, by a plan which I have devised from a chart by Mr. R. J. Evans.

Fig. 2 shows the varying intensity of the total force between the magnetic equator and the magnetic pole, for the meridian of about 170° W. and passing near Behring's Strait,—that at the equator being unity.

It is known to be about 2.3 times as great at the magnetic pole as at the equator,—and between these limits it has been found to vary nearly as the square root of 1 plus three times the square of the sine of the magnetic latitude.

The curve bounding the section of Fig. 2 is formed by the preceding law, and shows that the intensity of the earth's magnetism increases faster on first leaving the equator than when nearing the pole.

Fig. 3 represents the corresponding **horizontal component.** This is a maximum at the equator, where the whole force is horizontal. At the magnetic pole, where the line of force is vertical, this horizontal component vanishes. Between these limits it varies as the total force at the locality multiplied by the cosine of the dip.

The curve bounding the section of Fig. 3, is made by the above proportion,—the result showing that for some distance from the equator, toward the pole, the horizontal or directive component holds its own with but little diminution, after which it falls off rapidly on nearing the pole, where it vanishes.

Those who have read Dr. Kane's report of his expedition in the Arctic Ocean, will remember the trouble he had with his compass needle, for want of directive force.

Fig. 4 represents the **Vertical Component.** This is nothing at the magnetic equator, where the needle is horizontal, and has its maximum,—which is the total force,—at the magnetic pole, where the unbalanced magnetic neeedle stands vertical. Between these limits it varies as the total force of any locality multiplied by the sine of the dip for that locality.

Induced Magnetism.—In consequence of the constant stream of magnetism flowing around the earth, all fuliginous matter within its influence becomes charged with magnetism by induction. Whence, in our (north) latitude, the lower portions of all masses of iron, such as columns, vertical shafts, smoke funnels, vertical rods of any kind,—being below the neutral plane,—are on the polar side of that plane and therefore have the same magnetism that is in the north end of the needle. While the upper parts being on the equatorial side of this plane, have that kind of magnetism that is in the south end of the needle.

Hard and Soft Iron.—The influence of this induced magnetism on different kinds of iron, is seen in different effects on the compass needle.

Hard Iron is slow to receive magnetism by induction, and as slow to part with it. If a bar of hard iron, having its longer dimension approximately parallel with the "line of force," be violently treated, as by bending, twisting, chipping, filing, or in putting lathe-work upon it, its magnetism will not only be increased, but comparatively fixed.

Soft Iron on the contrary, becomes magnetic almost immediately when its longer dimension is placed in a position approximately parallel with the "line of force." It also has its intensity increased and comparatively fixed by violent treatment.

Fixed or Permanent Magnetism, is that which remains constant, or practically so, in the body to which it pertains, as in a hardened steel magnet, or in the so-called loadstone. Magnetism in vessels is not permanent. Hard iron retains its magnetism some time, but not so permanently as steel.

The plates of vessels, boilers, smoke funnels, tubes, etc., being worked cold, then hammered in riveting and caulking, have their magnetism partially fixed, producing a magnetism called **sub-permanent,** i. e., composed partly of permanent and partly of transient magnetism. This transient part of mechanism passes away rapidly from a new vessel, by use, making much trouble for the navigator.

Different Effects of the Different Magnetisms.—The difference in the effects produced by a fixed magnet, as that of a bar of hard steel magnetized, and that produced by a bar of soft iron that is made magnetic by induction, is seen in the following experiment:

Apply the end of a soft iron rod to a compass needle, the rod being in a level position, at the same height of the compass and in an east and west direction. The rod, in such position, being in the neutral plane, is not acted upon by the earth's magnetism, and as a consequence, cannot affect the needle. Now, up-end the rod to a vertical position, keeping one end,—say the lower end,—on a level with the compass. The lower end of the rod, in this position, is below the neutral plane, and therefore is charged by induction with the magnetism that is in the north end of the needle, as will be shown by its repelling the north end of the needle.

If now, we lower the rod away, canting it till the upper end comes below the level of the compass,—still keeping it vertical,—we shall bring the south magnetism of the rod to act on the needle, which will be shown by its attraction of the needle, thus giving the opposite result, and showing that it is not any peculiarity of the rod that produces this change, but the almost instantaneous effects of the earth's magnetism on the rod,

Effect of Direction of Ship's Head, while Building.—It has been seen from observation that iron ships receive a permanent magnetic character from the direction in which the head is located during the building. In a vessel built with head to the magnetic north, the magnetic meridian would coincide with the ship's plane symmetry,—or with her center line, as it is called.

The forward lower part of the vessel, being below the neutral plane (in north latitude), would have that kind of magnetism that is in the north end of the needle, while the after end of the vessel would have south magnetism,—and so for other directions of the ship's head.

Semi-circular Deviation.—In swinging ship, the fixed magnetism of the head iron retains its relative position with the other forces and is therefore on one side of the compass needle in one-half of the revolution, and on the other side during the other half, thereby producing deviation through 180° of azimuth on each side of the magnetic meridian,—whence the name semi-circular deviation.

The deviation due to the effects of hard iron, is far from being constant. It falls off rapidly with the use of the vessel during the first few months of service. As a consequence, a new iron ship cannot be depended on to carry her adjustments long, without some change.

The magnetism resulting from vertical component, being of constant sign and constant intensity in the same latitude, also produces a semi-circular deviation. And this is always combined with that produced by the hard iron, and is made less in amount than the latter.

These two parts, which always come together by addition, have different properties.

That depending on hard iron is not constant in its intensity, because of the inability of the iron to hold its magnetism permanently. Hence the term, "sub-permanent magnetism." While that depending on the vertical component of induced magnetism, is constant in the same latitude.

Quadrantal Deviation, is the result of magnetic action induced by the horizontal component in the soft iron in the ship and it is constant in amount, or nearly so,—being proportional to horizontal component of the locality. And, as will be presently seen, it changes sign in each quadrant. Commencing in the first quadrant, N. to E., the signs are +, —, +, —, whence the name "quadrantal deviation."

Graphical Illustration.—The several parts of which deviation is composed, are illustrated by Fig. 1, of Plate II,

The two segmental areas, +B, —B, represent that part of the semi-circular deviation due to hard iron, and which is generally the larger part.

The curve bounding this arc, is a **curve of sines**, i. e., if the deviation at east or west be multiplied by the sine of the azimuth of any point in the curve, measured from the Node of the curve, or the place where the curve crosses the magnetic meridian, the product will be the deviation at that point.

Thus, if a deviation at E. be 20°, that for N. N. E. would be (see table III)

sin. 2 p. (=.384)×20°= 7.7°; and for N. E.
would be sin. 4 p. (=.707)×20°=14.1°; and for E. N. E.
would be sin. 6 p. (=.924)×20°=18.5°.

These distances set off from the magnetic meridian, will give points in the curve for the semi-circular deviation.

The Lunular Segments, +C, —C, show the part of the semi-circular deviation due to the effects of the vertical component. The boundry of this area is also a curve of sines.

The four lunular segments, +D, —D, +D, —D, show how quadrantal deviation is combined with semi-circular deviation to produce the total error of the compass. These, also, are curves of sines.

The Law of Change of Signs will be seen from the following considerations:

When the head of the ship is to the magnetic north, the magnetism of the ship and that of the earth conspire in one direction on the needle, so that, though the directive force be increased, there is no deviation,—whence the magnetic north is a nodal point of this curve, or nearly so.

When the ship's head is east magnetic, the horizontal component is in the neutral plane and thus does not act, and as a consequence, there is no deviation,—whence, the east is also a nodal point for this curve.

In the same manner the south and west may be shown to be nodal points.

The several parts of deviation above enumerated, are called **"Co-efficients."** They are A, B, C, D, E^1, of which we have shown B, C and D.

"A" is that part of compass error due to what may be called **Index error,** as when the zero line of compass is not parallel with center line of ship or when the card is not cemented onto the needle in the proper place,—and is usually too small to notice.

"E" is a small error remaining in the octants after the quadranted deviation D, has been corrected. It is seldom seen except with very long compass needles, and is generally so small as not to be worth noticing.

The preceding illustrations are on the supposition that the several causes of disturbance are symmetrically arranged to each other on the two sides of the ship's plane of symmetry, called also her "midship plane;" and that the vertical magnetic plane that coincided with the plane of the magnetic meridian during the building of ship, coincides also with this 'midship plane,—and that the ship's compass is in this plane. But all these conditions seldom or never prevail at the same time.

And this trouble is aggravated by the habit of masters and builders of placing their compasses to one side of the center line of ship,

The result is, these several parts of deviation, which always come together by addition, algebraically, are shifted in azimuth with regard to each other, so that the nodes of the curve are not at the magnetic cardinal points, as they otherwise would be, nearly.

This is illustrated in the deviation curves of the "Trident" and "Warrior," Plate I.

In the case of the "Trident," it will be seen that the curve crosses the magnetic meridian at N. ½ E. and at S. ⅞ W.—thus making one semi-circle ¾ point longer than the other.

The curve of the "Warrior," crosses the meridian at N. 3 p. W. and at S. ⅝ p. E., thus making the distance between nodes, 2⅜ p. greater on one side than on the other;—which makes it impossible to bring the needle to place at both North and South or East and West as the case might be, by means of opposing magnets. And this often brings the compass adjuster into disrepute by people who are ignorant of the above facts, and who think that because he cannot get good "reversals" in all cases, that therefore he does not know his business.

A Vivid Illustration of the effect of combining quadrantal, with semi-circular deviation, is seen also in the deviation curves of the "Trident" and "Warrior" above referred to.

It will be remembered that the signs of B and C, when west, are —, and that the sign of D is +, —, +, —, commencing in the first quadrant, and going around by the East, South and West, to the North.

Constructing the curve bounding the two parts B and C, (as on Page 88),—taking the deviation at West, 23°, as the height of the curve, we find for intervals of 2 p. of azimuth, the height of curve as follows, viz: (See table III.)

$$\begin{aligned} \text{Sin. 2 p. } (=.383) & \quad = 8°.8. \\ \text{Sin. 4 p. } (=.707) & \times 23° = 16°.3. \\ \text{Sin. 6 p. } (=.924) & \quad = 21°.3. \end{aligned}$$

Setting off these distances, by means of the scale at head or foot of the plate, we have the place of the curve shown by the dotted line, thus showing the area embraced by the dotted or broken line, to be **reduced** in the first quadrant, and making the deviation curve slightly concave at N. E.

At S. E. the deviation curve is sharply convex, because the two signs being alike (—), their parts come together by addition.

At S. W. the two signs being alike (+), show the deviation curve to be sharply convex.

At the N. W. the two signs being unlike, (×, —), the two parts come together by subtraction, and we have a lean concave curve, as in the first quadrant. Thus:

The total deviation in the two northern quadrants, is greater than the deviation in their diagonally opposite quadrants by twice the mean quadrantal deviation of those quadrants.

The above property affords a ready means of separating quadrantal deviation from the total deviation. Rule: Take half the arithmetical difference of the diagonally opposite quadrantal deviations, for the mean quadrantal deviation.

And in constructing the curve, observe that the heights, or ordinates of the curve vary as the sine of **twice the azimuth of ship's heading,** measured from the point midway between the magnetic meridian and the compass meridian.

The Variation of the Intensity of Magnetism, for distance is known to be **Inversely as the Square** of the distance. But when two magnetic forces act on each other, the intensity of their mutual or joint force varies **Inversely** as the **third course** of their distance.

Thus—if two magnetic bodies acting on each other at a given distance produce a given deviation, then at twice the distance, the deviation will be only $\frac{1}{8}$ as much; at three times the distance, it will be only $\frac{1}{27}$ as much; at ten times the distance, $\frac{1}{1000}$ as much, etc.

This law shows us how a small element of disturbance near by, can make more trouble than a shipload of iron a little ways off.

Heeling Deviation.—So far, our consideration of compass errors have been considered with regards vessels on an "even beam. But there is an error resulting from the lifting or heeling of the vessel, called **Heeling Deviation.**

When the vessel is heeled to starboard or port, the vertical longitudinal plane, containing the compass, is shifted to leeward of that containing the general center of magnetic effort of the ship. As a consequence, the relations existing with ship on an even beam are disturbed.

But, fortunately, our lake vessels have so much more beam for their tonnage and draft of water than sea-going vessels, that this disturbance will now give us much trouble.

This error is at its **maximum** when the ship's center line is parallel with the compass needle, as with ship's head N. or S. and **nothing** with head E. or W. magnetic.

Mechanical Correction of Deviation, by Means of Magnets.—Because the two parts B and C, of deviation are semicircular, or nearly so, they may be compensated, or neutralized mechanically by introducing magnets acting in contrary direction.

Having oriented the vessel by any of the methods heretofore explained, with ship's head to any one of the cardinal points,—say to the North,—mark on the deck under the compass, and on the wall of the pilot house, if near, the intersection of the magnetic meridian plane with them, that passes through the compass. This line will be parallel to ship's center line.

Also, at right angles to this line, draw the intersection of the transverse vertical plane passing through the centre of compass,—marking the trace of same on the side of pilot house, when near,—showing a possible place for a magnet.

Place a magnet with its center on the fore-and-aft line of the deck, moving from or towards compass,—changing ends if necessary,—**till the compass point correctly,**—and fasten temporarily, keeping the magnet perpendicular to the meridian plane.

If the deck is too far off from the compass, for the strength of the magnet, apply a larger magnet, or use two magnets,—or fasten one to the side or front of the pilot house.

Then, ship's head being swung to magnetic East or West, again bring the needle to its normal place, as before,—being careful to keep the magnet truly fore-and-aft, with its center on the transverse line on the deck,—and fasten temporarily.

If, now the fixed magnetism of the ship represented by $\times$B and —B in Fig. 1, of Plate II, be in its proper place, with its zero points at the North and South, coinciding with the nodal points of the other two parts C and D, then **compensation for the semi-circular deviation will be complete,** and ship will "reverse bearings" on the cardinal points, and in the quadrants, except for the error due to quadrantal deviation.

But this is too often not the case. This part B may be shifted in azimuth with regard to the other parts C and D, so as to give a material error of a $\frac{1}{2}$ or possibly a whole point on reversal of ship. In this case no amount of "fixing" or "fudging" will avail us. If we bring needle to place on one cardinal point, it will be out on the other,—and if we bring it to place on the other, it will be out on the one.

In this case we can do nothing more in the way of correction, but resort to a **Deviation Card.** Swing ship again to verify the work, and secure the magnets permanently.

If the quadrantal deviation (for there is some in all ships), is to be corrected by deviation card, which is the better way, swing ship again, putting ship's head carefully to all the alternate points of the card, magnetic by means of the desired compass, and note the reading carefully of the ship's compass, **and record them.**

If there was no error at the cardinal points, the errors found in the quarters, will rarely exceed a $\frac{1}{2}$ point. But if these courses at the cardinal letters, are reversed, these errors will affect the quadrantal errors.

The following is a deviation card from actual practice, the original error being over $2\frac{1}{4}$ p.

The first and third columns giving the magnetic course desired; the second column giving the correction, which must be read with the course in the magnetic column, to get the compass course desired.

Thus, to get S. E. magnetic, we take S. E. $\frac{1}{4}$ S. per compass; and for N. W. magnetic, we take N. W. $\frac{1}{4}$ N. per compass, etc.

STEERING CARD.

Magnetic Course.	Correction.	Magnetic Course.	Correction.
Compass Course.		Compass Course.	
North.		South.	
N. N. E.		S. S. W.	
N. E.		S. W.	
E. N. E.		W. S. W.	
East.		West.	
E. S. E.	$\frac{1}{8}$ S.	W. N. W.	$\frac{1}{8}$ N.
S. E.	$\frac{1}{4}$ S.	N. W.	$\frac{1}{4}$ N.
S. S. E.	$\frac{1}{8}$ S.	N. N. W.	$\frac{1}{8}$ N.
South.		North.	

Mechanical Correction of Quadrantal Deviation, is made by means of soft iron, or cast iron. Experience has shown that **cylinders of cast iron, 3 to $3\frac{1}{4}$ inches in diameter, and 9 to 12 inches long,** with hemispherical ends, and placed on a level with the compass card, and with their ends pointing radially to the center of the card, give the best results.

Nails and chain in boxes have been used; cast iron balls, also, have been used with satisfactory results.

The Correction.—The semi-circular deviation having been corrected, set ship's head to one of the inter-cardinal points,—say N. E.—magnetic.

Place one of the cylinders to the north of the card, on a level with the card, and with end pointing to the center of the compass. This corrector should be directly in line of the needle when brought to its normal place.

Place another cylinder to the east or west side of the compass, in the same manner, as may be necessary to make the compass point correctly.

Now, keeping the ends of the correctors at the same distance from the card,—move them both outward or inward till the compass points correctly,—and the work is done,—secure correctors.

Theoretically, this correction should be nearly perfect, but practically it may be very imperfect, as the result of a non-symmetrical arrangement of soft iron in the ship.

Correction of the Heeling Error.—This is made by means of a **Vertical Magnet,** under the center of the compass.

Small vessels may be readily heeled 8° to 10°, when a magnet may be applied in a line that would be vertical when ship is on an even beam,—with that end up that brings the needle to place,—and varying the distance, by moving it up or down, till the needle shall point correctly, when it may be secured.

Large vessels cannot be readily heeled. In this case resort must be had to a **Magnetic Survey** of the ship. But the discussion of such a survey is beyond our purpose. For this information the student is referred to the **Admiralty Manual for 1874.**

Practical Conclusions.—The following are some of conclusions drawn from the scientific investigations and long practical experience of Messrs. Smith & Evans, of the Liverpool Compass Committee.

(I.) All mechanically corrected compasses, should have compound needles,—or two parallel needles, whose extremities are 60° apart.

(II.) If single needle compasses are to be corrected, the needles should not be over six or seven inches in length.

(III.) A correcting magnet on the same level of the compass, should not be nearer to the center of the needle, than six times the length of the needle.

(IV.) In making corrections for quadrantal deviation, the soft iron correctors should not be brought nearer to the center of the compass, than two times the length of the needle.

(V.) No compass that is to be corrected by magnets, should be placed where the original deviation is more than two points.

(VI.) The compass should not be near either end of an iron ship. And if the decks are of iron, they should be provided with a hatch immediately below the compass, of a width $1\frac{1}{2}$ times the height of compass above deck.

(VII.) If the compass of an iron ship is to be carried amidships, the direction of the head of ship, during the building, is not important.

Remark.—The preceding is but a meager outline of the subject, "Terrestrial Magnetism," with ships, and their compasses, which alone would require a large volume, but it is believed to be enough to give the shipmaster some idea of the forces that disturb his compass, and to put him on his guard in the care of that valuable instrument.

For further information on this topic, he is referred to "Magnetism of Ships" and "Deviation of Compasses," published by the Bureau of Navigation,—Navy Department, 1867.

CHAPTER VI.

THE PROPELLER WHEEL.

In view of the great importance of the propeller wheel in promoting the vast and growing commerce of the lakes, it is desirable that a more general, and in some respects, a more correct knowledge of that indispensable agent be had by our maritime people. Accordingly I offer the following concerning it.

Deffinitions.—The **Length** of the wheel is the distance from the forward edge to the after edge of the blade, measured in a direction parallel with the shaft, and usually is about $\frac{18}{100}$ of the diameter. In the early history of the wheel, it was much longer.

The **Pitch** is the distance made, by one revolution of a point, with regard to the shaft, and is usually given in terms of the wheel,—as a pitch of $1\frac{1}{4}$ or $1\frac{1}{3}$ diameters. It is sometimes given in feet, but that method gives no idea of the pitch—angle.

The Net Pitch is the distance made by the vessel, by one revolution of the wheel. This is usually given in feet, for the purpose of deducing the speed of ship from the number of revolutions of the wheel, in a given time, though when used for comparing the value of different wheels, it should be given in terms of the diameter. It is the pitch as usually defined, diminished by slip.

Slip is the difference between the pitch of the wheel, and the distance made by the vessel at one turn of the wheel, and is expressed in feet, or as a per cent of the pitch, as wanted.

Example. If a wheel of 9 feet diameter have a pitch of $1\frac{1}{3}$ diameters, propel a vessel 10 feet with one revolution, it is said to have a pitch of 12 feet,—a net pitch of 10 feet, and slip of 2 feet, or of $2 \div 12 = .166$, or say $\frac{17}{100}$.

Negative Slip.—Sometimes the vessel shows a speed **greater** than that due to the pitch of her wheel. This apparent paradox,—for it is only apparent,—comes from measuring the pitch at the end of blade instead of measuring it at a point called the **Center of Effort**, situated about $\frac{10}{100}$ to $\frac{15}{100}$ of the length of blade inboard from the end, as will be hereafter explained.

It is pitiful, yet laughable, to see the absurd theories that have been advanced, even by learned professors, to account for this apparent anomaly.

True Screw.—In this screw, the working face is a warped surface, generated by a line having two motions,—one angular at a uniform rate, and the other, a motion of translation at a uniform rate, along a right line, and always making the same angle with that line.

Illustration.—In Fig. 31, let m. n. represent the center line of shaft, or hub of wheel, and let the line n. a. A. represent the generating line.

Then if this line be moved uniformly along the line n. m., and with a uniform angular motion along A. C. as indicated by the arrow points, it will generate the **helicoidal surface,** A. c. m. n. forming one surface of the blade of a true screw,—A. C. being the **Helix.**

Pitch Angle.—In Fig. 31, let A. B. be the arc of a circle whose plane is at right angles to the line m. n. or shaft,—and the arc A. C. the helix generated by the point A. in its revolution about the line m. n.—then is B. A. C. the **pitch angle,** and B. C., parallel with and equal to n. m., is the pitch corresponding to width of blade.

Center of Effort is that point in the length of the blade, having the same amount of work on either side of it, and, in the true screw, is at $\frac{84}{100}$ of length of blade from center of wheel,—and not at $\frac{17}{100}$ as told us in engineering books.

If the wheel were quiessent, supporting a weight uniformly distributed over the disk area of the same,—then the center of effort would be at $\frac{71}{100}$ of radius from center, as said. But the circular motion throws this center outboard.

The disc area of the wheel varies as the square of the diameter. Also the intensity of percussion varies as the square of velocity of the several points, in the radius of the wheel. And the amount of work done varies with the product of these two factors,—whence the total work up to any point in the radius of the wheel, varies as the **fourth power** of the distance of such point from the center of wheel.

As a consequence, the center of effort is at a point equal to the fourth root of $\frac{1}{2} = \frac{84}{100}$ of R., that is, in the helix a. b. $\frac{84}{100}$ of radius from n. m.

The angle which this helix a. b. makes with the plane of the arc A. B. and which is at right angles to the shaft, is the angle that should always be used in any calculations concerning the efficiency of wheels, or the amount of power wanted for propulsion.

When the pitch is measured at that point, we shall never hear of negative slip.

Expansion of Pitch, Radial and Axial, is a change of the pitch from that of the true screw.

If the blade (Fig. 31) be made narrower at the inboard end, by lifting the point n. towards m. the pitch of the blade at that end will be reduced.

This change is called **Radial Expansion,** and is introduced by many wheel makers, for the purpose of throwing the work of this end of blade out to where the pitch-angle is smaller, to avoid part of the loss from oblique action.

But there is a limit to this reduction, it must not exceed the prospective slip. The object is to relieve the inboard end entirely of work, which is attained by making the pitch the same as the net pitch. If this is made less than the net pitch, the blade takes water on the forward face,—doing negative work.

Axial Expansion, is an increment of pitch from the forward edge to after edge of blade, making the working face concave.

The object is to take hold of the water gradually, and to increase the pressure by increasing the pitch gradually, in conse--quence of which the helix AC. (Fig. 31) is concave towards AB.

This expansion is all right in the hands of those who know how to use it, for it has its limit, which is the prospective slip. If the expansion exceed this limit the blade takes water on the forward face, doing negative work. Besides, the propelling of ship is done with the after part of blade, where the pitch angle is greater,—and where, consequently loss from oblique action is greater.

This is probably the most disasterous leak the coal-bunkers of the lakes have ever been called on to pay. This mistake has been general.

Fortunately, our best wheel makers know better now than to exceed this limit,—while others, perhaps afraid of it, do not give their blades any axial expansion at all,—which is perhaps as well, for the smaller the slip, the more nearly must the blade be a true screw.

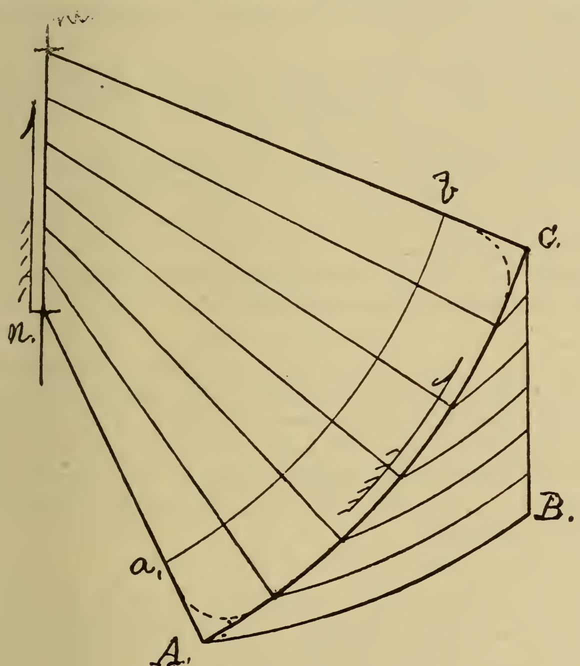


Fig. 31.

By means of radial expansion, the center of effort is moved outward from its normal place in the true screw, but just how much, owing to the different ways of doing it, and the different degrees to which it is done, it would be difficult to say. The probability is we may locate the center of effort between $\frac{85}{100}$ and $\frac{90}{100}$ of the distance from center to end of blade.

To Measure the Pitch, is frequently required for the purpose of comparing different wheels. It is of importance for every engineer, master and owner of a steamer, to know the pitch of his wheel, so as to know, in the event of a break, how to order a new one.

This involves a knowledge of the angle BAC. (Fig. 31.) or of the sides of that triangle.

The angle may be found by applying a carpenter's bevel, so as to have one arm of the bevel on the helix AC. and the other on the plane AB. and both at right angles to the radial at the point of application, as at A.

The angle being found, construct on paper, as BAC. From any point B. set up the perpendicular BC. Then is BC. the pitch due to the developed helix BC.

Divide the base AB. by 3.14, then we have the diameter of a circle whose circumference is AB. Divide BC. by the quotient thus found, when we have the pitch in terms of the diameter of the wheel. Example.

Suppose we find AB. in (Fig. 32.) by scale measure to be $25\frac{1}{8}$ inches, and BC. to be 12 inches, then

$25\frac{1}{8} \div 3.14 = 8$ and

$12 \div 8 = 1\frac{1}{2}$. That is, our wheel has a pitch of $1\frac{1}{2}$ diameters. It is the pitch at the end of blade, and is the pitch by which wheels are usually defined. But wheels must be compared by means of the angle at the center of effort, called

The Working Pitch Angle, which is always greater than that found at end of blade, by $\frac{10}{100}$ to $\frac{12}{100}$, depending on the amount of radial expansion.

This angle is readily found in degrees,—thus

Divide the ratio of pitch to diameter, as 1, $1\frac{1}{4}$, $1\frac{1}{2}$, etc., by 3.1416, and we have the tangent of pitch angle at end of blade.

Increase this tangent by $\frac{10}{100}$ or $\frac{12}{100}$, as may be required and we have **tangent** of the **Working Pitch Angle.**

Example for a wheel whose pitch is $1\frac{1}{2}$ diameters:

$$1.5 \div 3.1416 = .4774 = \text{Tan. } 25.^\circ\ 31'$$

which is angle at end of blade.

Increasing this tangent by its $\frac{12}{100}$, we have

$$.4774 \times \frac{12 \text{x} .4774}{100} = .5374 = \text{Tan. } 28.^\circ\ 08'$$

for angle at center of effort.

Or measure by protractor, or scale of chords.

The "Holding Power," or the ability of the wheel to furnish inertia for the engine to work upon, depends on the disc area of the wheel, and the depth of water drawn by it. So that when the wheel runs "awash," **the Holding Power varies as the cube of the diameter,** or

If the draft of the wheel is greater than the diameter, then the Holding Power will vary as the **square of the diameter, multiplied by the draft.**

Illustration.—If a wheel of given diameter give a certain amount of inertia, running "awash," then a wheel twice as large under the same condition, will give **eight** times as much; and a unit of area in the large wheel, as a circular foot of the disc area, will have twice as much holding power as the same unit in the small wheel, for it has twice the weight of water on it.

The Holding Power per I. H. P., furnished by different wheel makers, is widely different in the amount,—varying from 1 to 5 cylindric feet.

The minimum furnished by the best designers at present, is the inertia to be derived from about 1 cylindric foot of water per I. H. P., though the average of wheels on the lakes, is considerable above that.

The wheels of the steamers "Alaska," "Spree," and the U. S. Steamship "Maine," are so proportioned as to give about 1 cylindric foot per I. H. P.,—whence, **to determine the diameter** of the wheel, in feet for that amount of inertia, or holding power, we have only to **take the cube root of the I. H. P.**

Or, if the wheel be deeply submerged, we may divide the I. H. P. by the draft, and take the **square root** of the quotient. Example:

The wheels of the U. S. Steamship "Maine," are calculated to work off 9,000 Horse Power, or 4,500 each. And the vessel is to draw 21½ feet of water. Dividing the 4,500 by the draft, which is say 20 feet, we have 225, as the square of the diameter,—the square root of which is 15 feet, as provided by her designers.

Difficulty in Providing Holding Power for small vessels. We know from Geometry, that similar solids or volumes vary as the **third** power of any of their similar dimensions. Also, that surface of similar figures are as the **second** power of their similar sides.

And we know that when two vessels are similar, and loaded to the same per cent. of their depth of hold, that their displacement volumes are similar, and therefore that their wet surfaces are similar, and have the relation of the **squares** of any of their similar dimensions. Whence, the power required for them will be as the **squares** of those dimensions, while as we have seen, the "holding power" varies as the **cube** of those dimensions. Hence we see that **the holding power changes twice as fast** as the requirements for propelling power. **Illustration.**

Say a vessel with 8 feet draft, is provided with a satisfactory power, and wheel,

A vessel of 16 feet draft, would have **four times as much power,** if provided in proportion to increase of surface, while the inertia available, **is eight times** as much, i. e. the inertia available, varies twice as fast as the power required.

This law is not generally known, and what is worse, it cannot be repealed, nor circumvented.

If it were better understood, people would know why a ton of displacement in a small vessel, requires so much more power for a given speed, than it does in a large one,—which is simply because the inertia required, is not obtainable.

With this rate of change, between the power required, and the inertia available, there is some point, of course, where the inertia to be had, will meet the requirements, when the wheel is running "awash."

This condition will be found with vessels drawing 7 to 8 feet of water, i. e. with such vessels provided say with 7 feet wheels, all the inertia required, is available, **but not more.**

With vessels drawing more water, the supply increases faster than the demand,—i. e. as the cube of the draft, while the amount wanted is only as the square of the draft.

With vessels of smaller draft, the supply falls short of the requirements, by the same ratio. Example:

With a vessel of only 4 feet draft, the power required, to be in the proportion of their surfaces if the vessels were similar, would be as the **square** of $\frac{1}{2}$, which is $\frac{1}{4}$, while the inertia available would the **cube** of $\frac{1}{2}=\frac{1}{8}$, that is, the supply **falls short** of the demand, in this case, in the ratio of $\frac{1}{8}$ to $\frac{1}{4}$, or 1 to 2, i. e. we can only get half of what we want.

Many plans have been devised to surmount this difficulty,—chief among which is the increase of the pitch. This introduces two other troubles, loss by oblique action of blade, and a serious lateral pull, by the great amount of lost work in the lower blade, resulting in loss by great slip.

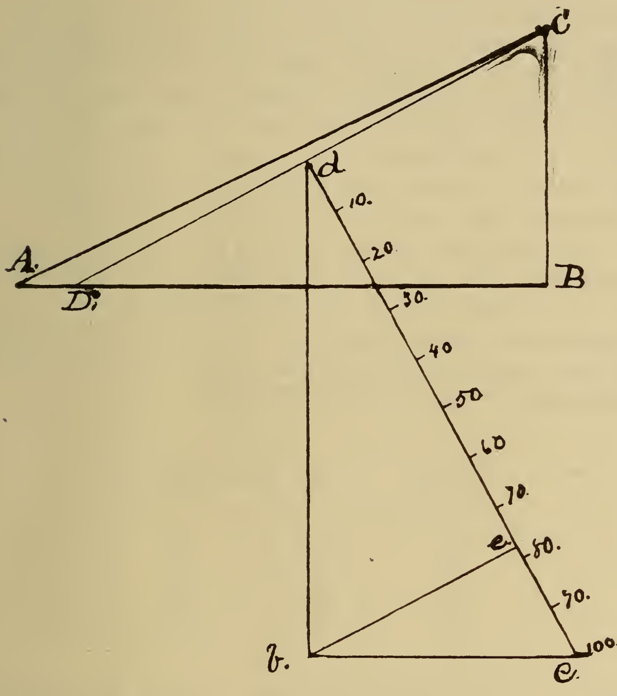


Fig. 32.

Two wheels have been introduced, but this is very expensive, besides it increases the exposure to injury. The expense is seen when we multiply the diameter of the one wheel, say 8 feet, by the cube root of $\frac{1}{2}$=.794, and find that we must use two wheels, $6\frac{1}{3}$ feet diameter, to get the holding power afforded by one 8 feet diameter, when the wheels run "awash."

Useful and Lost Work.—To represent these parts, let the line DC. in Fig. 32, be the helix at center of effort.

From any point in this line, and at right angles to it, draw the line c. d. to represent the total work of blade. Divide into 100 equal parts. Through d. draw d. b. parallel with BC., and through c. draw b. c. parallel with D. B.

Then will b. d. represent the amount and direction of the useful work, and b. c. those of the lost work.

The total, or 100 parts of work must be divided on these lines, in proportion to their squares. This is done by demitting a perpendicular from b. onto d. c., showing in this case, the useful work to be about $\frac{78}{100}$ of the total work. The lost work, represented by b. c. is expended entirely in lateral work, like a paddle wheel turned fore-and-aft and which is very much increased by increasing the pitch angle, is seen to be $\frac{22}{100}$, in this example.

The triangles BCD. and b. c. d. being alike, the side b. c. corresponding to pitch of blade, is the **Sine** of the Pitch Angle, while d. c. is cosine, whence from the preceding, we have

Cosine^2 of the working pitch angle, measures the **useful work,** and **sine**^{2} measures the **lost work,** in per cent of the whole. In this manner was the following table made:

Pitch in Diameter.	Angle.	Per Cent. Loss.
1	19°	.11
$1\frac{1}{4}$	24°	.16
$1\frac{1}{2}$	28°	.22
$1\frac{3}{4}$	32°	.28
2	36°	.35

Thus, it is seen from the table that we more than treble the loss from oblique action of blade, by doubling the pitch.

Advantage of a Large Wheel.—It is not infrequently desirable to make the wheel larger in diameter than the preceding considerations would indicate, not necessarily for the purpose of getting more inertia for the engine to work upon,—but to keep the pitch-angle within economic limits.

When it is desired to obtain great speed, we must have great pitch in some form. We may either **increase the piston-speed,** by which more revolutions of the wheel are attained in a given time, or we **increase the diameter of the wheel,**—keeping the pitch-angle constant,—till the desired pitch is attained. Another method, practiced by some,—who have yet their first lesson to learn concerning the propeller wheel,—**is to increase the pitch-angle.** But as this increases the loss from oblique action of blade very fast, after a pitch of about $1\frac{1}{3}$ diameters has been attained, it is not to be tolerated, above that limit.

It is true that increase of diameter is objectionable on some accounts,—as increased first cost,—increased exposure to injury,—increase of friction;—but reduction of loss by oblique action of blade, i. e. a saving of power, gained thereby; is greater than all.

EXPLANATION OF THE TABLES.

I. Is a traverse table, giving the differences of latitude and departure for distances up to 10, and by courses varying by $\frac{1}{4}$ point up to 8 points. For illustration of their use, see pages 41 to 43.

II. Is a table of the natural functions,—Sine and Cosine,—Tangent and Cotangent,—Secant and Cosecant, for every 5 of the quadrant, and to four places of decimals,—more decimals, or shorter intervals of arc, for purposes of azimuth, with our lake compasses being entirely unnecessary.

For further explanations of their use, see pages 11 to 13.

III. Is a table of rhumbs, varying by $\frac{1}{8}$ point, for converting points to degrees, and the reverse;—with their Sines and Cosines, Tangents and Secants,—useful in making computations for angles in points, etc., instead of in degrees,—as in Table II.

IV. Is a table of amplitudes of the sun, for the latitude of the lakes, with the time of his rising and setting. Also the rate of the change of his azimuth from sunrise, to the time of his crossing the prime vertical, when the latitude and declination are alike.

This rate being practically uniform for all declinations in any latitude above about 40°, affords the means for deducing a table of time azimuth, good for one to three hours, from a single amplitude;—thereby vastly enlarging the utility of amplitudes.

For further explanations see pages 71 to 73.

V. Is a table of the sun's declination to the nearest minute, for the current year (1891), with the corresponding equation of time,—the sign prefixed, being that for reducing mean time to apparent time;—wanted with the table of time azimuth, when orienting ship.

This table is never precisely correct, except for the particular date for which it was computed; but as it is the mean between two leap-years, it can never be far wrong,—the change of declination for 12 hours, being the maximum error that can occur during any four years,—after which interval, the table is very nearly correct again.

Ignoring this small error,—as we may for all purposes of azimuth, this table will be available for a great many years.

VI. Is a table of time azimuths of the sun, for the latitude of the lakes. Useful in orienting ship for compass errors.—See pages 74, 77, 78.

VII. Is a table of time corrections for reducing standard time to local mean time. It is computed by reducing the difference of longitude between any locality, and the longitude of the standard time, to time, and applying the sign that reduces the standard time to local mean time. Thus:

The difference of longitude between the light-house at Chicago, and the longitude of central standard time, is 90°—87°, 37′=2°, 23′, or 9^M, 32^S of time. Then because the standard meridian is west of Chicago, this difference must be applied with the plus sign, i. e. we must add 9^M, 32^S to central standard time, to obtain the mean local time for Chicago. In this manner was the table constructed. It is wanted with the equation of time in using the tables of time azimuths. See page 74.

VIII. Is a table of chords for setting off, or measuring angles. For particulars see pages 12–13.

IX. Is a table of meridianal parts, for the construction of Mercator's chart. It is given for every 2 minutes of latitude from 0 to 75°. The intermediate minute is readily found by interpolation, when wanted. See pages 48 to 52.

X. This table gives the value of 1 minute of longitude for each degree of latitude up to 30°, and for each half degree thence up to 75°. It is wanted in many questions of dead reckoning for reducing departure to difference of longitude. See pages 44 to 46.

XI. Is a table for the correction of middle latitude, in middle latitude sailing. See pages 46–47.

XII. Is a table for finding distance of objects from ship, from two bearings, and distance sailed between them. Pages 59 to 61.

XIII. Is a table for reducing difference of longitude to time, wanted in deducing local apparent time of a locality from the longitude of the locality and standard time.

Variation Chartlet.—Wishing to amplify the subject of variation somewhat, I give the following as an explanation of the chartlet of "**Magnetic Variations.**"

First, to explain the terms **right** and **left** as applied to the magnetic card. When the north end of the needle is to the right of the true north, it is in east variation, and in west variation when the north end is to the left of the true north. This is well when the ship has northings in her course, but it is a more general expression to say that the card is out to the **right** or **left**, as the case may be, by variation.

It will be observed that in sailing either up or down the lakes, we are continually crossing the isogonic curves. In going west the card swings to the **right** 18½° between the foot of Lake Ontario and the head of Lake Superior,—thereby leading the vessel to the **right** or north.

In the same manner, in going from Duluth to Buffalo, the card swings to the **left**, taking the vessel to the **left**, or north, as before.

That is to say,—in going any way across these lines, **we are taken to the north** by **change** of the variation.

This "change" of variation, is a very different thing from variation itself. When we go west, with a westerly variation, we are led to the **South**, and to the north with an easterly variation,—unless correction is made,—and conversely.

But with the "change," we are led to the north in any case. And the track made by the ship as the result of this "change," is a **curve**, like a railroad curve, precisely. And the straight line joining the end of this curve, changes its direction, with a change in the length of the curve, just **half as fast**, as the curve changes its direction. Whence,

Correction is made for this "change," by taking half the sum of the variation at the two extremities of a run for the **mean variation** to be applied at either end of the run.

Thus:—Toledo to Buffalo,—Variation at Toledo,=0°. Variation at Buffalo,=$4\frac{1}{2}$° W. The $\frac{1}{2}$ sum=$2\frac{1}{4}$° W. is the mean variation to be applied in going either way between these ports.

In "Shaping Course," the following rule will always apply for mean variation.

With variation to the right, apply correction to the **left.**

With variation to the left, apply correction to the **right.**

There is scarcely another place in the world where the "change" of variation must be taken into account as an independent factor. This results from the fact that there is no other place in the world where the isogonic curves are so close together, or are crossed so directly by vessels, as on our lakes.

TRAVERSE TABLE. I.

Distance	1		2		3		4		5		6		7		8		9		10		
Course. P. ¼ p.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	0 8
1	.999	.049	1.99	.098	2.99	.147	3.99	.197	4.99	.246	5.99	.295	6.91	.344	7.99	.393	8.99	.442	9.98	.492	3
2	.995	.098	1.99	.196	2.98	.249	3.98	.392	4.98	.490	5.97	.588	6.97	.686	7.96	.784	8.96	.883	9.95	.985	2
3	.989	.147	1.98	.294	2.97	.440	3.95	.587	4.95	.734	5.94	.880	6.92	1.03	7.91	1.17	8.90	1.32	9.89	1.47	1
1 0	.981	.195	1.96	.390	2.94	.585	3.92	.780	4.90	.975	5.88	1.17	6.87	1.37	7.85	1.56	8.83	1.75	9.81	1.95	0 7
1	.970	.243	1.94	.486	2.91	.729	3.88	.972	4.85	1.22	5.82	1.46	6.79	1.70	7.76	1.94	8.73	2.19	9.70	2.43	3
2	.957	.290	1.91	.580	2.87	.871	3.83	1.16	4.78	1.45	5.74	1.74	6.70	2.03	7.66	2.32	8.61	2.61	9.57	2.90	2
3	.942	.337	1.88	.674	2.83	1.01	3.77	1.35	4.71	1.68	5.65	2.02	6.59	2.36	7.53	2.69	8.47	3.03	9.42	3.37	1
2 0	.924	.383	1.85	.765	2.77	1.15	3.69	1.53	4.62	1.91	5.54	2.30	6.47	2.68	7.39	3.06	8.32	3.44	9.24	3.83	0 6
1	.904	.428	1.81	.855	2.71	1.28	3.62	1.71	4.52	2.14	5.42	2.57	6.33	2.99	7.23	3.42	8.14	3.85	9.04	4.28	3
2	.882	.471	1.76	.943	2.65	1.41	3.53	1.89	4.41	2.36	5.29	2.83	6.17	3.30	7.05	3.77	7.94	4.24	8.82	4.71	2
3	.858	.514	1.71	1.03	2.57	1.54	3.43	2.06	4.29	2.57	5.14	3.08	6.00	3.60	6.86	4.11	7.72	4.63	8.58	5.14	1
3 0	.831	.556	1.66	1.11	2.50	1.67	3.33	2.22	4.16	2.78	4.99	3.33	5.88	3.89	6.65	4.44	7.48	5.00	8.31	5.56	0 5
1	.803	.596	1.61	1.19	2.41	1.79	3.21	2.38	4.02	2.98	4.82	3.57	5.62	4.17	6.43	4.77	7.23	5.36	8.03	5.96	3
2	.773	.634	1.55	1.27	2.32	1.90	3.08	2.54	3.86	3.17	4.63	3.81	5.40	4.44	6.18	5.07	6.96	5.71	7.73	6.34	2
3	.741	.671	1.48	1.34	2.22	2.01	2.96	2.69	3.70	3.36	4.45	4.03	5.19	4.70	5.93	5.37	6.67	6.04	7.41	6.71	1
4 0	.707	.707	1.41	1.41	2.12	2.12	2.83	2.38	3.53	3.53	4.24	4.24	4.95	4.95	5.66	5.66	6.36	6.36	7.07	7.07	0 4
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.
Distance.	1		2		3		4		5		6		7		8		9		10		

TABLE II A,

Natural Sines and Cosines.

′	0°		1°		2°		3°		4°		′
	Sine,	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.0000	Unit.	.0174	.9998	.0349	.9994	.0523	.9986	.0698	.9976	60
5	.0014	Unit.	.0189	.9998	.0363	.9993	.0538	.9985	.0712	.9975	55
10	.0029	Unit.	.0120	.9098	.0378	.9993	.0552	.9985	0727	.9974	50
15	.0044	.9999	.0218	.9998	.0393	.9992	.0567	.6984	.0741	.9973	45
20	.0058	.9999	0233	.9997	.0407	.9992	.0581	.9983	.0756	.9971	40
25	.0073	.9999	.0247	.9997	.0422	.9991	0596	9982	.0770	.9970	35
30	.0087	.9999	.0262	.9996	.0436	.9990	.0610	.9985	.0785	.9969	30
35	.0102	.9999	.0276	.9996	.0451	.9989	.0625	.9980	.0799	.9968	25
40	.0116	.9999	.0291	.9996	.0465	.9989	.0639	.9979	.0814	9968	20
45	.0131	.9999	.0305	.9995	.0480	.9988	.0654	.9978	.0828	.9966	15
50	.0145	.9998	.0399	.9995	.0494	.9988	.0668	.9977	.0843	.9964	10
55	.0160	.9998	.0334	.9994	.0509	.9987	.0683	.9976	.0857	.9963	5
60	.0174	.9998	.0349	.9994	.0523	.9986	.0697	.9975	.0872	.9962	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.0872	.9962	.1045	.9945	.1219	.9925	.1392	.9903	.1564	.9877	60
5	.0886	.9961	.1060	.9944	.1233	.9924	.1406	.9901	.1579	.9875	55
10	.0900	.9959	.1074	.9942	.1248	.9922	.1420	.9899	.1593	.9872	50
15	.0915	.9958	.1089	.9941	.1262	.9920	.1435	.9896	.1607	.9870	45
20	.0929	.9957	.1103	.9939	.1276	.9918	.1449	.9894	.1622	.9868	40
25	.0944	.9955	.1118	.9937	.1291	.9916	.1464	.9892	.1636	.9865	35
30	.0958	.9954	.1132	.9936	.1305	.9914	.1478	.9890	.1650	.9863	30
35	.0973	.9953	.1146	.9934	.1320	.9912	.1492	.9888	.1665	.9860	25
40	.0987	.9951	.1161	.9932	.1334	.9911	.1507	.9886	.1679	.9858	20
45	.1002	.9950	.1175	.9931	.1348	.9909	.1521	.9884	.1693	.9856	15
50	.1016	.9948	.1190	.9929	.1363	.9907	.1535	.9881	.1708	.9853	10
55	.1031	.9947	.1204	.9927	.1377	.9005	.1550	.9879	.1722	.9851	5
69	.1045	.9945	.1219	.9925	.1392	.99 3	.1564	.9877	.1765	9848	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.1736	.9848	.1908	.9816	.2079	.9781	.2249	.9744	.2419	.9703	60
5	.1751	.9845	.1922	.9813	.2093	.9778	.2264	.9740	.2433	.9699	55
10	.1765	.9843	.1937	.9811	.2108	.9775	.2278	.9737	.2447	.9696	50
15	.1779	.9840	.1951	.9808	.2122	.9772	.2292	.9734	.2461	.9692	45
20	.1794	.8438	.1965	.9805	.2136	.9769	.2306	.9730	.2476	.9689	40
25	.1808	.9835	.1979	.9802	.2150	.9766	.2320	.9727	.2490	.9685	35
30	.1822	.9832	.1994	.9799	.2164	.9763	.2334	.9724	.2504	.9681	30
35	.1837	.9830	.2008	.9696	.2179	.9760	.2349	.9720	.2518	.9678	25
40	.1851	.9827	.2022	.9793	.2193	.9757	.2363	.9717	.2532	.9674	20
45	.1865	.9824	.2036	.9790	.2207	.9753	.2377	.9713	.2546	.9670	15
50	.1879	.9822	.2051	.9787	.2221	.9750	.2391	.9710	.2560	.9667	10
55	.1894	.9819	.2065	.9784	.2235	.9747	.2405	.9706	.2574	.9663	5
60	.1908	.9816	.2079	.9781	.2249	.9744	.2419	.9707	2588	.9659	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

TABLE II A,

Natural Sines and Cosines—Continued.

'	15°		16°		17°		18°		19°		'
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.2588	.9659	.2756	.9613	.2924	.9563	.3090	.9511	.3256	.9455	60
5	.2602	.9655	.2770	.9608	.2938	.9559	.3104	.9506	.3269	.9450	55
10	.2616	.9652	.2784	.9605	.2951	.9554	.3118	.9501	.3283	.9446	50
15	.2630	.9648	.2798	.9600	.2965	.9550	.3132	.9497	.3297	.9441	45
20	.2644	.9644	.2812	.9596	.2979	.9546	.3145	.9492	.3311	.9436	40
25	.2658	.9640	.2826	.9592	.2993	.9541	.3159	.9488	.3324	.9431	35
30	.2672	.9636	.2840	.9588	.3007	.9537	.3137	.9483	.3338	.9426	30
35	.2686	.9632	.2854	.9584	.3021	.9533	.3187	.9479	.3352	.9421	25
40	.2700	.9628	.2868	.9580	.3035	.9528	.3201	.9474	.3365	.9417	20
45	.2714	.9625	.2882	.9576	.2049	.9524	.3214	.9469	.3379	.9412	15
50	.2728	.9621	.2896	.9571	.3062	.9519	.3228	.9465	.3393	.9407	10
55	.2742	.9617	.2910	.9567	.3076	.9515	.3242	.9460	.3406	.9402	5
60	.2756	.9613	.2924	.9563	.3090	.9511	.3255	.9455	.3420	.9397	0
'	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	'
	74°		73°		72°		71°		70°		

'	20°		21°		22°		23°		24°		'
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.3420	.9397	.3584	.9336	.3746	.9272	.3907	.9005	.4067	.9135	60
5	.3433	.9392	.3597	.9331	.3759	.9266	.3921	.9199	.4081	.9127	55
10	.3447	.9387	.3611	.9325	.3773	.9261	.3934	.9194	.4094	.9124	50
15	.3461	.9382	.3624	.9320	.3786	.9255	.3947	.9188	.4107	.9118	45
20	.3475	.9377	.3638	.9315	.3800	.9250	.3961	.9182	.4120	.9111	40
25	.3488	.9372	.3652	.9309	.3813	.9244	.3974	.9176	.4134	.9106	35
30	.3502	.9367	.3665	.9304	.3827	.9239	.3987	.9171	.4147	.9100	30
35	.3516	.9362	.3678	.9299	.3840	.9233	.4001	.9165	.4160	.9094	25
40	.3529	.9356	.3692	.9293	.3854	.9228	.4014	.9159	.4173	.9087	20
45	.3543	.9351	.3706	.9288	.3867	.9222	.4027	.9153	.4187	.9081	15
50	.3556	.9346	.3719	.9283	.3881	.9216	.4041	.9147	.4200	.9075	10
55	.3570	.9341	.3733	.9277	.3891	.9211	.4054	.9141	.4213	.9069	5
60	.3584	.9336	.3746	.9272	.3907	.9205	.4067	9135	.4226	.9063	0
'	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	'
	69°		68°		67°		66°		65°		

'	25°		26°		27°		28°		29°		'
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.4226	.9063	.4384	.8988	.4540	.8910	.4695	.8829	.4848	.8746	60
5	.4239	.9057	.4397	.8982	.4553	.8903	.4708	.8823	.4861	.8739	55
10	.4252	.9051	.4410	.8975	.4566	.8897	.4720	.8816	.4873	.8732	50
15	.4266	.9045	.4423	.8969	.4579	.8890	.4733	.8809	.4886	.8725	45
20	.4279	.9038	.4436	.8962	.4592	.8883	.4746	.8802	.4899	.8718	40
25	.4292	.9032	.4449	.8956	.4605	.8877	.4759	.8795	.4912	.8711	35
30	.4305	.9026	.4462	.8949	.4617	.8870	.4772	.8788	.4924	.8704	30
35	.4318	.9020	.4475	.8943	.4630	.8863	.4784	.8781	.4936	.8696	25
40	.4331	.9013	.4488	.8936	.4643	.8857	.4797	.8774	.4949	.8689	20
45	.4344	.9007	.4501	.8930	.4656	.8850	.4810	.8767	.4962	.8682	15
50	.4357	.9001	.4514	.8923	.4669	.8843	.4823	.8760	.4975	.8675	10
55	.4371	.8994	.4227	.8917	.4682	.8836	.4835	.8753	.4987	.8667	5
60	.4384	.8988	.4540	.8910	.4695	.8829	.4848	.8746	.5000	.8660	0
'	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	'
	64°		63°		62°		61°		60°		

TABLE II A,

Natural Sines and Cosines—Continued.

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.5000	.8660	.5150	.8572	.5299	.8480	.5446	.8387	.5592	.8290	60
5	.5013	.8653	.5163	.8564	.5311	.8473	.5459	.8379	.5604	.8282	55
10	.5025	.8646	.5175	.8557	.5324	.8465	.5471	.8371	.5616	.8274	10
15	.5038	.8638	.5188	.8549	.5336	.8457	.5483	.8363	.5628	.8266	45
20	.5050	.8631	.5200	.8542	.5348	.8449	.5495	.8355	.5640	.8258	40
25	.5063	8624	.5213	.8534	.5361	.8442	.5507	.8347	.5652	.8249	35
30	.5075	.8616	.5225	.8526	.5373	.8434	.5519	.8339	.5664	.8241	30
35	.5088	.8609	.5237	.8519	.5385	.8426	.5531	.8331	.5676	.3233	25
40	.5100	.8601	.5250	.8511	.5397	.8418	.5544	.8323	.5688	.8225	20
45	.5113	.8594	.5262	.8503	.5410	.8410	.5556	.8315	.5700	.8216	15
50	.5125	.8587	.5274	.8496	.5422	.8402	.5568	.8307	.5712	.8208	10
55	.5138	.8579	.5287	.8488	.5434	.8395	.5580	.8298	.5724	.8200	5
60	.5150	8572	.5299	.8480	.5446	.8387	.5592	.8290	.5736	.8191	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.5736	.8191	.5878	.8090	.6018	.7986	.6157	.7880	.6293	.7771	60
5	.5747	.8183	.5890	.8082	.6030	.7978	.6168	.7871	.6304	.7762	55
10	.5760	.8175	.5901	.8073	.6041	.7969	.6179	.7862	.6316	.7753	50
15	.5771	.8166	.5913	.8064	.6053	.7960	.6191	.7853	.6327	.7744	45
20	.5783	.8158	.5925	.8056	.6064	.7951	.6202	.7844	.6338	.7735	40
25	.5795	.8158	.5936	.8047	.6076	.7942	.6214	.7835	.6350	.7725	35
30	.5807	.8141	.5948	.8039	.6088	.7933	.6225	.7826	.6361	.7716	30
35	.5819	.8133	.5960	.8030	.6099	.7925	.6236	.7817	.6372	.7707	25
40	.5831	8124	.5972	.8021	.6111	.7916	.6248	7808	.6383	.7698	20
45	.5842	.8116	.5983	.8012	.6122	.7905	6259	.7799	.6394	.7688	15
50	5854	.8107	.5995	.8004	.6134	.7898	.6271	.7790	.6406	.7679	10
55	.5866	.8099	.6006	.7995	.6145	.7889	.6282	.7781	.6417	.4670	5
60	.5878	.8090	.6018	.7986	.6157	.7880	.6293	.7771	.6428	.7660	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.6128	.7660	.6561	.7547	.6691	.7431	.6820	.7313	.6947	.7193	60
5	.6439	.7651	.6572	.7537	.6702	.7422	.6831	.7304	.6957	.5183	55
10	.6450	.7642	.6582	.7528	.6713	.7412	.6841	.7294	.6967	.7173	50
15	.6461	.7632	.6593	.7518	.6724	.7402	.6852	.7284	.6978	.7163	45
20	.6472	.7623	.6604	.7509	.6734	.7392	.6862	.7274	.6988	.7153	40
25	.6483	.7613	.6615	.7499	.6745	.7383	.6873	.7364	.6999	.7143	35
30	.6494	.7604	.6626	.7490	.6756	.7363	.6883	.7254	.7009	.7132	30
35	.6505	.7595	.6637	.7480	.6766	.7373	.6894	.7244	.7019	.7122	25
40	.6517	.7585	.6648	.7470	.6777	.7353	.6905	.7234	7030	.7112	20
45	.6528	.7576	.6659	.7461	.6788	.7343	.6915	.7224	.7040	.7092	15
50	.6539	.7566	.6690	.7451	.6790	.7333	.6926	.7214	.7050	.7092	10
55	.6550	.7557	.6680	.7441	.6809	.7323	.6936	.7203	.7061	.7081	5
60	.6561	.7547	.6691	.7431	.6820	.7313	.6947	.7193	.7071	.7071	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

TABLE II B,

Natural Tangents and Cotangents.

′	0° Tang.	0° Cotan.	1° Tang.	1° Cotan.	2° Tang.	2° Cotan.	3° Tang.	3° Cotan.	4° Tang.	4° Cotan.	′
0	.0000	Infinit.	.0175	57.29	.0349	28.64	.0524	19.08	.0699	14.301	60
5	.0014	687.55	.0189	52.88	.0363	27.49	0589	18.56	.0714	14.008	55
10	.0029	343 77	.0204	49.10	.0378	26.43	.0533	18.07	.0728	13.727	50
15	.0044	229.18	.0218	45.83	.0393	25.45	.0568	17.61	.0743	13.456	45
20	0058	171.88	.0233	42 66	.0407	24.54	.0582	17.17	.0758	13.197	40
25	.0073	137.51	.0247	40.44	.0422	23.69	.0597	16.75	.0772	12.947	35
30	0087	114.59	.0262	38.19	.0437	22.90	.0612	16.35	.0787	12.706	30
35	.0102	98.22	.0276	36.18	.0451	22.16	.0626	15.97	.0802	12.474	25
40	.0116	85.94	.0291	34.37	.0466	21.47	0641	15.60	.0816	12.251	20
45	.0131	76.39	.0305	32.73	.0480	20.82	.0655	15.26	.0831	12.035	15
50	.0145	68.75	.0320	31.24	.0495	20.21	.0670	14.92	.0846	11.826	10
55	.0160	62.50	.0335	29.88	.0509	19.63	.0685	14.61	.0860	11.625	5
60	.0175	27.29	.0349	28.64	.0524	19.08	.0699	14.30	.0875	11.430	0
′	89° Cotan.	89° Tang.	88° Cotan.	88° Tang.	87° Cotan.	87° Tang.	86° Cotan.	86° Tang.	85° Cotan.	85° Tang.	′

′	5° Tang.	5° Cotan.	6° Tang.	6° Cotan.	7° Tang.	7° Cotan.	8° Tang.	8° Cotan.	9° Tang.	9° Cotan.	′
0	.0875	11.430	.1051	9.514	.1228	8.144	.1405	7.115	.1584	6.314	60
5	.0889	11.242	.1066	9.383	.1243	8.047	.1420	7.041	.1600	6.255	55
10	.0904	1.053	.1080	9.255	.1257	7.953	.1435	6.968	.1614	6.197	50
15	.0719	10.883	.1095	9.131	.1272	7.860	.1450	6.897	.1629	6.140	45
20	.0933	10.712	.1109	9.010	.1287	7.770	.1465	6 827	.1644	6.084	40
25	.0948	10.546	.1125	8.892	.1302	7.682	.1480	6.758	.1658	6.030	35
30	.0963	10.385	.1139	8.777	.1316	7.596	.1494	6.691	.1673	5.976	30
35	.0978	10.229	.1154	8.665	.1331	7.511	.1509	6.625	.1688	5.923	25
40	.0992	10.078	.1169	8.555	.1346	7.429	.1524	6.561	.1703	5.871	20
45	.1007	9.931	.1184	8.449	.1361	7.348	.1539	6.497	.1718	5.820	15
50	.1022	9.788	.1198	8.345	.1376	7.269	.1554	5.435	.1733	5.769	10
55	.1036	9.649	.1213	9.243	.1391	7.191	.1569	6.374	.1748	5.720	5
60	.1051	9.514	.1228	8.144	1405	7.115	.1584	6.314	.1763	5.671	0
′	84° Cotan.	84° Tang.	83° Cotan.	83° Tang.	82° Cotan.	82° Tang.	81° Cotan.	81° Tang.	80° Cotan.	80° Tang.	′

′	10° Tang.	10° Cotan.	11° Tang.	11° Cotan.	12° Tang.	12° Cotan.	13° Tang.	13° Cotan.	14° Tang.	14° Cotan.	′
0	.1763	5.671	.1944	5 144	.2126	4.705	.2308	4.331	.2493	4.011	60
5	.1778	5.623	.1959	5.104	.2141	4.671	.2324	4.303	.2509	3.986	55
10	.1799	5.576	.1974	5.066	.2156	4.638	.2339	4.275	.2524	3.962	50
15	.1808	5.530	.1989	5.027	.2171	4.606	.2355	4.247	.2540	3.937	45
20	.1823	5.484	.2004	4.989	.2186	4.574	.2370	4.219	.2555	3.914	40
25	.1838	5.440	.2019	4.952	.2202	4.542	.2385	4.192	.2571	3.890	35
30	.1853	5.395	.2034	4.915	.2217	4.511	.2401	4.165	.2586	3.867	30
35	.1868	5.352	.2050	4.879	.2232	4.480	.2416	4.139	.2602	3 844	25
40	.1883	5.309	.2065	4 843	.2247	4.449	.2432	4.113	.2617	3.821	20
45	.1899	5.267	.2080	4.808	.2263	4.419	.2447	4.087	.2633	3.798	15
50	.1914	5.226	.2095	4.773	.2278	4.390	.2462	4.061	2648	3.776	10
55	.1929	5.185	.2110	4.738	.2293	4.360	.2478	4.036	.2664	3.754	5
60	.1944	5.144	.2126	4.705	.2309	4.331	.2493	4.011	.2679	3.732	0
′	79° Cotan.	79° Tang.	78° Cotan.	78° Tang.	77° Cotan.	77° Tang.	76° Cotan.	76° Tang.	75° Cotan.	75° Tang.	′

TABLE II B,

Natural Tangents and Cotangents—Cont'd.

′	15° Tang.	15° Cotan.	16° Tang.	16° Cotan.	17° Tang.	17° Cotan.	18° Tang.	18° Cotan.	19° Tang.	19° Cotan.	′
0	.2679	3.732	.2867	3.487	.3057	3.271	.3249	3.078	.3443	2.904	60
5	.2695	3.710	.2883	3.468	.3073	3.254	3265	3.062	.3460	2.890	55
10	.2711	3.689	.2899	3.449	.3089	3.237	.3281	3.047	.3476	2.877	50
15	.2726	3.668	.2915	3.431	.3105	3.220	.3297	3.033	.3492	2.864	45
20	.2742	3.647	.2930	3.412	.3121	3.204	.3314	3.018	.3508	2.850	40
25	.2758	3.626	.2946	3.394	.3137	3.188	.3330	3.003	.3525	2.837	35
30	.2773	3.606	.2962	3.376	.3153	3.172	.3346	2.989	.3551	2.824	30
35	.2789	3.586	.2978	3.358	.3169	3.156	.3362	2.974	.3558	2.811	25
40	.2805	3.566	.2994	3.340	.3185	3.140	3378	2.960	.3574	2.798	20
45	.2820	3.546	.3010	3.323	.3201	3.124	.3394	2.946	.3590	2.785	15
50	.2836	3.526	.3025	3.305	.3217	3.108	.3411	2.932	.3607	2.772	10
55	.2852	3.507	.3041	3.288	.3233	3.093	.3427	2.918	.3623	2.760	5
60	.2867	3.487	.3057	3.271	.3249	3.078	.3443	2.904	.3640	2.747	0
′	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	′
	74°		73°		72°		71°		70°		

′	20° Tang.	20° Cotan.	21° Tang.	21° Cotan.	22° Tang.	22° Cotan.	23° Tang.	23° Cotan.	24° Tang.	24° Cotan.	′
0	.3640	2.747	.3839	2.605	.4040	2.475	.4245	2.356	.4452	2.246	60
5	.3656	2.735	.3855	2.594	.4054	2.467	.4258	2.348	.4470	2.237	55
10	.3673	2.723	.3872	2.583	.4074	2.454	.4279	2.337	.4487	2.229	50
15	.3689	2.711	.3889	2.571	.4091	2.444	.4296	2.328	.4505	2.220	45
20	.3706	2.698	.3905	2.560	.4108	2.434	.4314	2.318	.4522	2.211	40
25	.3722	2.686	.3922	2.549	.4125	2.424	.4331	2.309	.4540	2.201	35
30	.3739	2.675	.3939	2.539	.4142	2.414	.4348	2.300	.4557	2.194	30
35	.3755	2.663	.3956	2.528	.4159	2.404	.4365	2.291	.4575	2.186	25
40	.3772	2.651	.3973	2.517	.4176	2.394	.4383	2.282	.4592	2.177	20
45	.3789	2.639	.3989	2.501	.4193	2.385	.4400	2.273	.4610	2.169	15
50	.3805	2.628	.4006	2.496	.4210	2.375	.4417	2.264	.4628	2.161	10
55	.3822	2.616	.4023	2.485	.4228	2.365	.4435	2.255	.4645	2.153	5
60	.3839	2.605	.4040	2.475	4245	2.356	.4452	2.246	.4663	2.144	0
′	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	′
	69°		68°		67°		66°		65°		

′	25° Tang.	25° Cotan.	26° Tang.	26° Cotan.	27° Tang.	27° Cotan.	28° Tang.	28° Cotan.	29° Tang.	29° Cotan.	′
0	.4663	2.144	.4877	2.050	.5095	1.962	.5317	1.881	.5543	1.804	60
5	.4681	2.136	.4895	2.043	.5114	1.956	.5336	1.874	.5562	1.798	55
10	.4698	2.128	.4913	2.035	.5132	1.949	.5354	1.868	.5581	1.792	50
15	.4716	2.120	.4931	2.028	.5150	1.942	.5373	1.861	.5600	1.786	45
20	.4734	2.112	.4949	2.020	.5169	1.935	.5392	1.855	.5619	1.779	40
25	.4752	2.104	.4968	2.013	.5187	1.928	.5411	1.848	.5638	1.773	35
30	.4770	2.096	.4986	2.005	.5206	1.921	.5429	1.842	.5658	1.767	30
35	.4788	2.089	.5004	1.998	.5224	1.914	.5448	1.835	.5677	1.761	25
40	.4805	2.081	.5022	1.991	.5243	1.907	.5467	1.829	.5696	1.755	20
45	.4823	2.073	.5040	1.984	.5261	1.901	.5486	1.823	.5715	1.750	15
50	.4841	2.065	.5059	1.977	.5280	1.894	.5505	1.816	5735	1.744	10
55	.4849	2.058	.5077	1.970	.5298	1.887	.5524	1.810	.5754	1.738	5
60	.4877	2.050	.5095	1.963	.5317	1.881	.5543	1.804	.5773	1.732	0
′	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	′
	64°		63°		62°		61°		60°		

TABLE II B,

Natural Tangents and Cotangents—Cont'd.

′	30°		31°		32°		33°		34°		′
	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	
0	.5773	1.732	.6008	1.664	.6249	1.600	.6494	1.540	.6745	1.483	60
5	.5793	1.726	.6028	1.659	.6269	1.595	6515	1.535	.6766	1.478	55
10	.5812	1.720	.6048	1.653	.6289	1.590	.6535	1.530	.6787	1.473	50
15	.5832	1.715	.6068	1.648	.6309	1.585	.6556	1.525	.6809	1.469	45
20	5851	1.709	.6088	1 643	.6330	1.580	.6577	1.520	.6830	1.464	40
25	.5871	1.703	.6108	1.637	.6350	1.575	.6598	1.516	.6851	1.460	35
30	5891	1.698	.6128	1.632	.6371	1.570	.6619	1.511	.6873	1.455	30
35	.5910	1.692	.6148	1.626	.6391	1.565	.6640	1.506	.6894	1.450	25
40	.5930	1.686	.6168	1.621	.6412	1.560	6661	1.501	.6916	1.446	20
45	.5949	1.681	.6188	1.616	.6432	1.555	.6682	1.497	.6937	1.441	15
50	.5969	1.675	.6208	1.611	.6453	1.550	.6703	1.492	.6959	1.438	10
55	.5989	1.670	.6228	1.606	.6473	1.545	.6724	1.487	.6980	1.433	5
60	.6009	1.664	.6249	1.600	.6494	1.540	.6745	1.483	.7002	1.428	0
′	Cotan	Tang.	Cotan.	Tang	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	
0	.7002	1.428	.7265	1.376	.7535	1.327	.7813	1 288	.8098	1.235	60
5	.7024	1.424	.7288	1.372	.7558	1.323	.7836	1.276	.8122	1.231	55
10	.7045	1.419	.7310	1.368	.7581	1.319	.7860	1.272	.8146	1.228	50
15	.7067	1.415	.7332	1.364	.7604	1.315	.7883	1.268	.8170	1.224	45
20	.7089	1.411	.7355	1.360	.7627	1.311	.7907	1.265	.8195	1.220	40
25	.7111	1.406	.7377	1.355	.7650	1.307	.7931	1.261	.8219	1.217	35
30	.7133	1.402	.7400	1.351	.7673	1.303	.7954	1.257	.8243	1.213	30
35	.7155	1.398	.7422	1.347	.7696	1.299	.7978	1.253	.8268	1.209	25
40	.7178	1.393	.7445	1.343	.7719	1.295	.8002	1.250	.8292	1.206	20
45	.7199	1.389	.7467	1.339	.7742	1.291	.8026	1.246	.8317	1.202	15
50	.7221	1.385	.7490	1.335	.7766	1.287	.8050	1.242	.8341	1.199	10
55	.7243	1.381	.7513	1.331	.7789	1.284	.8074	1.239	.8366	1.195	5
60	.7265	1.376	.7535	1.327	7813	1.280	.8098	1.235	.8391	1.192	0
′	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang,	Cotan.	Tang	Cotan.	Tang.	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang.	Cotan.	Tang.	Cotan.	Tang.	Cotan.	Tang,	Cotan.	Tang	Cotan.	
0	.8391	1.192	.8693	1 150	.9004	1 111	.9325	1.072	.9657	1.036	60
5	.8416	1.188	.8718	1.147	.9030	1.107	.9352	1.069	.9685	1.032	55
10	.8441	1.185	.8744	1.144	.9057	1.104	.9380	1.066	.9713	1.029	50
15	.8466	1.181	.8770	1.140	.9083	1.101	.9407	1.063	.9742	1.026	45
20	.8491	1.178	.8795	1.137	.9101	1.098	.9434	1.060	.9770	1.023	40
25	.8516	1.174	.8821	1.134	.9137	1.094	.6462	1.057	.9798	1.021	35
30	.8541	1.171	.8847	1.130	.9163	1.091	.9490	1.054	.9827	1.018	30
35	.8566	1.167	.8873	1.127	.9190	1.088	.9517	1.051	.9856	1 015	25
40	.8591	1.164	.8899	1 124	.9217	1.085	.9545	1.048	.9884	1.012	20
45	.8617	1.161	.8925	1.120	.9244	1.082	.9573	1.045	.9913	1.009	15
50	.8642	1.157	.8951	1.117	.9271	1.079	.9601	1.042	.9942	1.006	10
55	.8667	1.154	.8978	1.114	.9298	1.075	.9629	1.039	.9971	1.003	5
60	.8693	1.150	.9004	1.111	.9325	1.072	.9657	1.036	1.000	1.000	0
′	Cotan.	Tang.	Cotan.	Tang	Cotan.	Tang.	Cotan.	Tang,	Cotan.	Tang.	′
	49°		48°		47°		46°		45°		

TABLE II C,

Natural Secants and Cosecants

'	0°		1°		2°		3°		4°		'
	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	
0	1.0000	Infinit	1.0001	57.300	1.0006	28.654	1.0014	19.107	1.0024	14.335	60
5	1.0000	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	1.0025	14.043	55
10	1.0000	343.77	1.0002	49.114	1.0007	26.150	1.0015	18.103	1.0026	13.763	50
15	1.0000	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.639	1.0027	13.494	45
20	1.0000	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	1.0029	13.255	40
25	1 0080	137.51	1.0003	40.448	1.0099	23.716	1.0018	16.779	1.0030	12.985	35
30	1.0000	114 59	1.0003	38.201	1.0009	22.925	1.0019	16.380	1.0031	12.745	30
35	1.0000	98.22	1.0004	36.191	1.6010	22.186	1.0019	16.000	1.0032	12.514	25
40	1.0001	85.94	1.0004	34.382	1.0011	21.494	1.0020	15.637	1.0033	12.291	20
45	1.0001	76.39	1.9005	32.745	1.0011	20.843	1.0021	15.290	1.0034	12.076	15
50	1.0001	68.76	1.0005	31.257	1.0012	20.230	1.0022	14.958	1.0036	11.868	10
55	1.0001	62.51	1.0005	29.899	1.0013	19.653	1.0023	14.640	1.0037	11.668	5
60	1.0601	57.30	1.0006	28.654	1.0014	19.107	1.0024	14.335	1.0038	11.474	0
'	Cosec.	Secant	Cosec	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	'
	89°		88°		87°		86°		85°		

'	5°		6°		7°		8°		9°		'
	Secant	Cosec.	Secant	Cosec.	Secant	Cosec	Secant	Cosec	Secant	Cosec	
0	1.0038	11.474	1.0055	9.567	1.0075	8.205	1.0098	7.185	1.0125	6.392	60
5	1.0039	11.286	1.0057	9.436	1.0077	8.109	1.0100	7.112	1.0127	6.334	55
10	1.0041	11.104	1.0058	9.309	1.0079	8.016	1.0102	7.039	1.0129	6.277	50
15	1.0042	10.929	1.0060	9.185	1.0080	7.924	1.0104	6.969	1.0132	6.221	45
20	1.0043	10.758	1.0061	9.065	1.0082	7.834	1.0107	6.900	1.0134	6.166	40
25	1.0045	10.593	1.0063	8.948	1.0084	7.747	1.0109	6.832	1.0136	6.112	35
30	1.0046	10.433	1.0065	8.834	1.0086	7.661	1.0111	6.765	1.0139	6.059	30
35	1.0048	10.278	1.0066	8.722	1.0088	7.577	1.0113	6.700	1.0141	6.007	25
40	1.0049	10.127	1.0068	8.614	1.0090	7.496	1.0115	6.636	1.0144	5.955	20
45	1.0050	9.981	1.0070	8.508	1.0092	7.416	1.0118	6.574	1.0146	5.905	15
50	1.0052	9.839	1.0071	8.405	1.0094	7.337	1.0120	6.512	1.0149	5.855	10
55	1.0053	9.701	1.0073	8.304	1.0096	7.260	1.0122	6.452	1.0152	5.807	5
60	1.0055	9.567	1.0075	8.205	1.0098	7.185	1.0125	6.392	1.0154	5.759	0
'	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	'
	84°		83°		82°		81°		80°		

'	10°		11°		12°		13°		14°		'
	Secant	Cosec.	Secant	Cosec	Secant	Cosec	Secant	Cosec.	Secant	Cosec.	
0	1.0154	5.789	1.0187	5.241	1.0223	4.810	1.0263	4.445	1.0306	4.134	60
5	1.0157	5.712	1.0190	5.202	1.0226	4.777	1.0266	4.417	1.0310	4.109	55
10	1.0159	5.665	1.0193	5.164	1.0230	4.745	1.0270	4.390	1.0314	4.086	50
15	1.0162	5.620	1.0196	5.126	1.0233	4.713	1.0273	4.363	1.0317	4.062	45
20	1.0165	5.574	1.0199	5.088	1.0236	4.682	1.0277	4.336	1.0321	4.039	40
25	1.0167	5.531	1.0202	5.052	1.0239	4.651	1.0280	4.310	1.0325	4.016	35
30	1.0170	5.487	1.0205	5.016	1.0243	4.620	1.0284	4.284	1.0329	3.994	30
35	1.0173	5.445	1.0208	4.980	1.0246	4.590	1.0288	4.258	1.0333	3.972	25
40	1.0176	5.403	1.0211	4.945	1.0249	4.560	1.0291	4.232	1.0337	3.949	20
45	1.0179	5.361	1.0214	4.911	1.0253	4.531	1.0295	4.207	1.0341	3.828	15
50	1.0181	5 320	1.0217	4.876	1.0256	4.502	1.0300	4.182	1.0345	3.906	10
55	1.0184	2.280	1.0220	4.843	1.0260	4.474	1.0302	4.158	1.0349	3.845	5
60	1.0187	5.241	1.0223	4.810	1.0263	4.445	1.0306	4.134	1.0353	3.864	0
'	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	'
	79°		78°		77°		76°		75°		

TABLE II C,

Natural Secants and Cosecants—Continued.

′	15° Secant	15° Cosec.	16° Secant	16° Cosec.	17° Secant	17° Cosec.	18° Secant	18° Cosec.	19° Secant	19° Cosec.	′
0	1.0353	3.864	1.0403	3.628	1.0457	3.420	1.0515	3.236	1.0576	3.071	60
5	1.0357	3.843	1.0407	3.610	1.0461	3.404	1.0520	3.222	1.0581	3.059	55
10	1.0361	3.822	1.0412	3.591	1.0466	3.388	1.0525	3.207	1.0587	3.046	50
15	1.0365	3.802	1.0416	3.574	1.0471	3.372	1.0530	3.193	1.0592	3.033	45
20	1.0369	3.782	1.0420	3.556	1.0476	3.356	1.0535	3.179	1.0598	3.021	40
25	1.0374	3.762	1.0425	3.538	1.0480	3.341	1.0540	3.165	1.0603	3.008	35
30	1.0377	3.742	1.0429	3.521	1.0485	3.325	1.0545	3.151	1.0608	2.996	30
35	1.0382	3.722	1.0434	3.504	1.0490	3.310	1.0550	3.138	1.0614	2.983	25
40	1.0386	3.703	1.0438	3.487	1.0495	3.295	1.0555	3.124	1.0619	2.971	20
45	1.0390	3.684	1.0443	3.470	1.0500	3.280	1.0560	3.111	1.0625	2.959	15
50	1.0394	3.665	1.0448	3.453	1.0505	3.265	1.0566	3.098	1.0630	2.947	10
55	1.0399	3.646	1.0452	3.437	1.0510	3.251	1.0571	3.085	1.0636	2.935	5
60	1.0403	3.628	1.0457	3.420	1.0515	3.236	1.0576	3.071	1.0642	2.924	0
′	Cosec. 74°	Secant 74°	Cosec. 73°	Secant 73°	Cosec. 72°	Secant 72°	Cosec. 71°	Secant 71°	Cosec. 70°	Secant 70°	′

′	20° Secant	20° Cosec.	21° Secant	21° Cosec.	22° Secant	22° Cosec.	23° Secant	23° Cosec	24° Secant	24° Cosec.	′
0	1.0642	2.924	1.0711	2.790	1.0785	2.669	1.0864	2.559	1.0946	2.459	60
5	1.0647	2.912	1.0717	2.780	1.0792	2.660	1.0870	2.551	1.0953	2.451	55
10	1.0653	2.901	1.0723	2.769	1.0798	2.650	1.0877	2.542	1.0961	2.443	50
15	1.0659	2.889	1.0729	2.759	1.0804	2.641	1.0884	2.533	1.0968	2.435	45
20	1.0664	2.878	1.0736	2.749	1.0811	2.632	1.0891	2.525	1.0975	2.427	40
25	1.0670	2.867	1.0742	2.738	1.0817	2.622	1.0897	2.516	1.0982	2.419	35
30	1.0676	2.855	1.0748	2.728	1.0824	2.613	1.0904	2.508	1.0989	2.411	30
35	1.0682	2.844	1.0754	2.718	1.0830	2.604	1.0911	2.499	1.0997	2.404	25
40	1.0688	2.833	1.0761	2.708	1.0837	2.595	1.0918	2.491	1.1004	2.396	20
45	1.0694	2.822	1.0766	2.698	1.0844	2.586	1.0925	2.483	1.1011	2.389	15
50	1.0699	2.812	1.0773	2.689	1.0850	2.577	1.0932	2.475	1.1019	2.381	10
55	1.0705	2.801	1.0779	2.679	1.0857	2.568	1.0939	2.467	1.1026	2.374	5
60	1.0711	2.790	1.0785	2.669	1.0864	2.559	1.0946	2 459	1.1034	2.366	0
′	Cosec. 69°	Secant 69°	Cosec. 68°	Secant 68°	Cosec. 67°	Secant 67°	Cosec. 66°	Secant 66°	Cosec. 65°	Secant 65°	′

′	25° Secant	25° Cosec	26° Secant	26° Cosec	27° Secant	27° Cosec.	28° Secant	28° Cosec.	29° Secant	29° Cosec.	′
0	1.1034	2.366	1.1126	2.281	1.1223	2.203	1.1326	2.130	1.1433	2.063	60
5	1.1041	2.359	1.1134	2.274	1.1231	2.196	1.1334	2.124	1.1443	2.057	55
10	1.1049	2 351	1.1142	2.267	1.1240	2.190	1.1343	2.118	1.1452	2.052	50
15	1.1056	2.344	1.1150	2.261	1.1248	2.184	1.1352	2.113	1.1461	2.047	45
20	1.1064	2.337	1.1158	2.254	1.1257	2.178	1.1361	2.107	1.1471	2.041	40
25	1.1072	2.330	1.1166	2.248	1.1265	2.172	1.1370	2.101	1.1480	2.036	35
30	1.1079	2.323	1.1174	2.241	1.1274	2.166	1.1379	2.096	1.1489	2.031	30
35	1.1087	2.316	1.1182	2.235	1.1282	2.160	1.1388	2.090	1.1499	2.026	25
40	1.1095	2.309	1.1190	2.228	1.1291	2.154	1.1397	2.085	1.1508	2.020	20
45	1.1102	2.302	1.1198	2.222	1.1299	2.148	1.1406	2.679	1.1518	2.015	15
50	1.1110	2.295	1.1207	2.215	1.1308	2.142	1.1415	2.073	1.1528	2.010	10
55	1.1118	2.288	1.1215	2.209	1.1317	2.136	1.1424	2.068	1.1537	2.005	5
60	1.1126	2.281	1.1223	2 203	1.1326	2.130	1.1433	2.063	1.1547	2.000	0
′	Cosec. 64°	Secant 64°	Cosec. 63°	Secant 63°	Cosec. 62°	Secant 62°	Cosec. 61°	Secant 61°	Cosec. 60°	Secant 60°	′

TABLE II C,

Natural Secants and Cosecants—Continued.

′	30°		31°		32°		33°		34°		′
	Secant	Cosec	Secant	Cosec	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	
0	1.1547	2.000	1.1666	1.942	1.1792	1.887	1.1924	1.836	1.2062	1.788	60
5	1.1557	1.995	1.1676	1.937	1.1802	1.883	1.1935	1.832	1.2074	1.784	55
10	1.1566	1.990	1.1687	1.932	1.1813	1.878	1.1946	1.828	1.2086	1.781	50
15	1.1576	1.985	1.1697	1.928	1.1824	1.874	1.1958	1.824	1.2098	1.777	45
20	1.1586	1.980	1.1707	1.923	1.1835	1.870	1.1969	1.820	1.2110	1.773	40
25	1.1596	1.975	1.1718	1.918	1.1846	1.865	1.1980	1.816	1.2122	1.769	35
30	1.1606	1.970	1.1728	1.914	1.1857	1.861	1.1992	1.812	1.2134	1.765	30
35	1.1616	1.965	1.1739	1.909	1.1868	1.857	1.2004	1.808	1.2146	1.762	25
40	1.1626	1.961	1.1749	1.905	1.1879	1.853	1.2015	1.804	1.2158	1.758	20
45	1.1636	1.956	1.1760	1.900	1.1890	1.848	1.2027	1.800	1.2171	1.754	15
50	1.1646	1.951	1.1770	1.896	1.1901	1.844	1.2039	1.796	1.2183	1.751	10
55	1.1656	1.946	1.1781	1.891	1.1912	1.840	1.2050	1.792	1.2195	1.747	5
60	1.1666	1.942	1.1792	1.887	1.1924	1.836	1.2062	1.788	1.2208	1.743	0
′	Cosec.	Secant	Cosec	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Secant	Cosec.	Secant	Cosec,	Secant	Cosec	Secant	Cosec	Secant	Cosec	
0	1.2208	1.743	1.2361	1.701	1.2521	1.662	1.2690	1.624	1.2867	1.589	60
5	1.2220	1.740	1.2374	1.698	1.2535	1.658	1.2705	1.621	1.2883	1.586	55
10	1.2233	1.736	1.2387	1.694	1.2549	1.655	1.2719	1.618	1.2898	1.583	50
15	1.2245	1.733	1.2400	1.691	1.2563	1.652	1.2734	1.615	1.2913	1.580	45
20	1.2258	1.729	1.2413	1.688	1.2577	1.649	1.2748	1.612	1.2929	1.578	40
25	1.2270	1.726	1.2427	1.684	1.2591	1.646	1.2763	1.609	1.2944	1.575	35
30	1.2283	1.722	1.2440	1.681	1.2605	1.643	1.2778	1.606	1.2960	1.572	30
35	1.2296	1.718	1.2453	1.678	1.2619	1.640	1.2793	1.603	1.2975	1.569	25
40	1.2309	1.715	1.2467	1.675	1.2633	1.636	1.2807	1.600	1.2991	1.567	20
45	1.2322	1.712	1.2480	1.671	1.2647	1.633	1.2822	1.598	1.3006	1.564	15
50	1.2335	1.708	1.2494	1.668	1.2661	1.630	1.2837	1.595	1.3022	1.561	10
55	1.2348	1.705	1.2508	1.665	1.2676	1.627	1.2852	1.592	1.3038	1.558	5
60	1.2364	1.701	1.2521	1.662	1.2690	1.624	1.2867	1.589	1.3054	1.556	0
′	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Secant	Cosec	Secant	Cosec	Secant	Cosec	Secant	Cosec	Secant	Cosec.	
0	1.3054	1.556	1.325	1.524	1.346	1.494	1.367	1.466	1.390	1.439	60
5	1.3070	1.553	1.327	1.522	1.347	1.492	1.369	1.464	1.392	1.437	55
10	1.3086	1.550	1.328	1.519	1.349	1.490	1.371	1.462	1.394	1.435	50
15	1.3102	1.548	1.330	1.517	1.351	1.487	1.373	1.459	1.396	1.433	45
20	1.3118	1.545	1.332	1.514	1.353	1.485	1.375	1.457	1.398	1.431	40
25	1.3134	1.542	1.333	1.512	1.354	1.483	1.377	1.455	1.400	1.429	35
30	1.3151	1.539	1.335	1.509	1.356	1.480	1.379	1.453	1.402	1.427	30
35	1.3167	1.537	1.337	1.507	1.358	1.478	1.381	1.450	1.404	1.425	25
40	1.3184	1.534	1.339	1.504	1.360	1.476	1.382	1.448	1.406	1.423	20
45	1.3200	1.532	1.340	1.502	1.362	1.473	1.384	1.446	1.408	1.420	15
50	1.3217	1.529	1.342	1.499	1.364	1.471	1.386	1.444	1.410	1.418	10
55	1.3233	1.527	1.344	1.497	1.365	1.469	1.388	1.442	1.412	1.416	5
60	1.3250	1.524	1.346	1.494	1.367	1.466	1.390	1.439	1.414	1.414	0
′	Cosec	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	Cosec.	Secant	′
	49°		48°		47°		46°		45°		

Trigonometrical and Conversion Table. III.

BEING A TABLE OF RHUMBS.

Name of Course.	Points.	Degrees and Minutes.	Sine or Departure	Co Sine or Diff. Lat.	Tangent.	Secant.
N.	0.	0 1	.0000	1.0000	.0000	0.0000
	⅛	1 24½	.0246	.9997	.0245	1.0003
and	¼	2 49	.0491	.9988	.0492	1.0012
	⅜	4 13	.0735	.9973	.0737	1.0027
S.	½	5 37½	.0980	.9952	.0985	1.0048
	⅝	7 2	.1224	.9925	.1234	1.0076
	¾	8 26	.1467	.9892	.1483	1.0109
	⅞	9 50½	.1709	.9853	.1736	1.0150
N. by {E. W.}	1.	11 15	.1951	.9808	.1989	1.0196
	⅛	12 39½	.2191	.9757	.2246	1.0249
and	¼	14 4	.2430	.9700	.2506	1.0309
	⅜	15 28	.2667	.9638	.2767	1.0376
S. by {E. W.}	½	16 52½	.2903	.9569	.3034	1.0450
	⅝	18 17	.3137	.9495	.3304	1.0532
	¾	19 41	.3368	.9417	.3577	1.0629
	⅞	21 5½	.3599	.9330	.3857	1.0718
N. N. {E. W.}	2.	22 30	.3827	.9239	.4142	1.0824
	⅛	23 54½	.4053	.9142	.4433	1.0939
and	¼	25 19	.4276	.9040	.4730	1.1062
	⅜	26 43	.4496	.8932	.5033	1.1195
S. S. {E. W.}	½	28 7½	.4714	8819	.5345	1.1339
	⅝	29 32	.4929	.8701	.5665	1.1493
	¾	30 56	.5140	.8578	.5992	1.1658
	⅞	32 20½	.5350	.8448	.6332	1.1836
N. {E. by N. W. by N.}	3.	33 45	.5556	.8315	.6682	1.2025
	⅛	35 9½	.5758	.8176	.7043	1.2231
and	¼	36 34	.5958	.8032	.7418	1.2451
	⅜	37 58	.6152	.7884	.7803	1.2684
S. {E. by S. W. by S.}	½	39 22½	.6344	.7730	.8207	1.2936
	⅝	40 47	.6532	.7572	.8627	1.3207
	¾	42 11	.6715	.7410	.9062	1.3495
	⅞	43 35½	.6895	.7243	.9520	1.3807

Trigonometrical and Conversion Table. III.

BEING A TABLE OF RHUMBS.

Name of Course.	Points.	Degrees and Minutes.	Sine or Departure	Co Sine or Diff. Lat.	Tangent.	Secant.
N. {E. W. and S. {E. W.	4.	45	.7071	.7071	1.0000	1.4142
	⅛	46 24½	.7242	.6895	1.0504	1.4502
	¼	47 49	.7410	.6715	1.1035	1.4892
	⅜	49 13	.7572	.6532	1.1592	1.5309
	½	50 37½	.7730	.6344	1.2185	1.5763
	⅝	52 2	.7884	.6152	1.2815	1.6255
	¾	53 26	.8032	.5958	1.3481	1.6785
	⅞	54 50½	.8176	.5758	1.4198	1.7366
N. {E. by E. W. by W. and S. {E. by E. W. by W.	5.	56 15	.8315	.5556	1.4966	1.7999
	⅛	57 39½	.8449	.5350	1.5793	1.8694
	¼	59 4	.8578	.5140	1.6687	1.9454
	⅜	60 28	.8701	.4929	1.7651	2.0287
	½	61 52½	.8819	.4714	1.8708	2.1214
	⅝	63 17	.8932	.4497	1.9868	2.2243
	¾	64 41	.9040	.4476	2.1139	2.3385
	⅞	66 5½	.9142	.4053	2.2558	2.4675
E. N. E. W. N. W. and E. S. E. W. S. W.	6.	67 30	.9239	.3827	2.4142	2.6131
	⅛	68 54½	.9330	.3599	2.5926	2.7788
	¼	70 19	.9415	.3368	2.7954	2.9689
	⅜	71 43	.9495	.3137	3.0267	3.1876
	½	73 7½	.9569	.2903	3.2966	3.4449
	⅝	74 32	.9638	.2667	3.6140	3.7498
	¾	75 56	.9700	.2430	3.9910	4.1144
	⅞	77 20½	.9757	.2191	4.4525	4.5634
E. W. by N. and E. W. by S.	7.	78 45	.9808	.1951	5.0273	5.1258
	⅛	80 91	.9853	.1709	5.7647	5.8505
	¼	81 34	.9892	.1467	6.7448	6.8186
	⅜	82 58	.9925	.1224	8.1054	8.1668
	½	84 22½	.9952	.0980	10.154	10.2023
	⅝	85 47	.9973	.0735	13.563	13.6002
	¾	87 11	.9988	.0491	20.325	20.3809
	⅞	88 35½	.9997	.0246	40.688	40.6889
East—West.	8.	90 00	1.0000	.0000	Infinite.	

The Sun's Amplitude. IV.

With rate of change of Azimuth, from Sunrise to the time of his crossing the Prime Vertical, and with the time of his Rising and Setting.

Lat.	41°		42°		43°		44°		45°	
Dec.	Change of Azimuth 1° in 6¼ m.		Change of Azimuth 1° in 6¼ m.		Change of Azimuth 1° in 6 m.		Change of Azimuth 1° in 6 m.		Change of Azimuth 1° in 6 m.	
	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.
1°	1°.20′	R5.57 S 6.03	1°.21′	R5.56 S 6.04	1°.22′	R5.56 S 6.04	1°.23′	R5.56 S 6.04	1°.25′	R5.56 S 6.04
2	2.39	5.53 6.07	2.42	5.53 6.07	2.44	5.53 6.07	2.47	5.52 6.08	2.50	5.52 6.08
3	3.59	5.50 6.10	4.02	5.49 6.11	4.06	5.49 6.11	4.10	5.48 6.12	4.15	5 48 6.12
4	5.18	5.46 6.14	5.23	5.46 6.14	5.28	5.45 6.15	5.34	5.45 6.15	5.40	5.44 6.16
5	6.38	5 43 6.17	6.44	5.42 6.18	6.51	5.41 6.19	6.58	5.41 6.19	7.05	5.40 6.20
6	7.58	5.39 6.21	8.05	5.38 6.22	8.13	5.38 6.22	8.21	5.37 6.23	8.30	5 36 6.24
7	9.18	5.35 6.25	9.26	5.35 6.25	9.36	5.34 6.26	9.45	5.33 6.27	9.55	5.32 6.28
8	10.38	5.32 6.28	10.48	5.31 6.29	10.58	5.30 6.30	11.09	5.29 6.31	11.21	5.28 6.32
9	11.58	5.28 6.32	12.09	5.27 6.33	12 21	5.26 6.34	12.34	5.25 6.35	12.47	5.24 6.36
10	13.18	5.25 6.35	13.31	5.23 6 37	13.44	5.22 6.38	13.58	5.21 6.39	14.13	5.19 6.41
11	14.39	5.21 6.39	14.53	5.20 6.40	15.07	5.18 6.42	15.23	5.17 6.43	15.39	5.15 6.45
12	15.59	5.17 6.43	16.15	5.16 6.44	16.31	5.14 6.46	16.48	5.13 6.47	17.06	5 11 6.49
13	17.20	5.14 6.46	17.37	5.12 6.48	17.55	5.10 6.50	18.13	5.08 6.52	18.33	5.07 6.53
14	18.42	5.10 6.50	19.00	5.08 6.52	19.19	5.06 6.54	19.39	5.04 6.56	20.00	5.02 6.58
15	20.03	5.06 6.54	20.23	5.04 6.56	20.44	5.02 6.58	21.05	5.00 7.00	21.28	4.58 7.02
16	21.25	5.02 6.58	21.46	5.00 7.00	22.08	4.53 7.07	22.32	4.56 7.04	22.57	4.53 7.07
17	22.48	4.58 7.02	23.10	4.56 7.04	23.34	4.54 7.06	23.59	4.51 7.09	24.25	4.49 7.11
18	24.10	4.54 7.06	24.34	4.52 7 08	25.00	4.49 7.11	25.26	4.47 7.13	25.55	4.44 7.16
19	25.33	4.50 7.10	25.59	4.48 7.12	26.26	4.45 7.15	26.55	4.42 7.18	27.25	4.49 7.21
20	26.57	4.46 7.14	27.24	4.43 7.17	27.53	4.41 7.19	28.23	4.38 7.22	28.56	4.35 7.25
21	28.21	4.42 7.18	28.50	4.39 7.21	29.20	4.26 7.34	29.53	4.33 7.27	30.27	4.30 7.30
22	29.46	4.28 7.22	30.16	4.35 7.25	30.49	4.31 7.29	31.23	4.28 7.32	31.59	4.25 7.35
23	31.11	4.33 7.27	31.43	4 30 7.30	32.18	4.27 7.33	32.54	4.23 7.37	33.33	4.20 7.40
23½	31.51	4.31 7.29	32.24	4.28 7.32	32.59	4.24 7.36	33.37	4.21 7.39	34.16	4.17 7.43

R. and S. are applied for Lat. and Dec. of the same name.

The Sun's Amplitude.—Continued.

With rate of change of Azimuth, from Sunrise to the time of his crossing the Prime Vertical, and with the time of his Rising and Setting.

Lat.	46°		47°		48°		49°		50°	
Dec.	Change of Azimuth 1° in 5¾ m.		Change of Azimuth 1° in 5¾ m.		Change of Azimuth 1° in 5½ m.		Change of Azimuth 1° in 5⅓ m.		Change of Azimuth 1° in 5⅓ m.	
	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.	AMP.	H. M.
1°	1°.26′	R5.56 S 6.04	1°.28′	R5.56 S 6.04	1°.30′	R5.56 S 6.04	1°.31′	R5.55 S 6.05	1°.33′	R5.55 S 6.05
2	2.53	5.52 6.08	2.56	5.51 6.09	2.59	5.51 6.09	3.03	5.51 6.09	3.07	5.50 6.10
3	4.19	5.48 6.12	4.24	5.47 6.13	4.29	5.47 6.13	4.35	5.46 6.14	4.40	5.46 6.14
4	5.46	5.43 6.17	5.52	5.43 6.17	5.59	5.42 6.18	6.06	5.42 6.18	6.14	5.41 6.18
5	7.12	5.39 6.21	7.21	5.38 6.22	7.29	5.38 6.22	7.38	5.37 6.23	7.48	5.36 6.24
6	8.39	5.35 6.25	8.49	5.34 6.26	8.59	5.33 6.27	9.10	5.32 6.28	9.22	5.31 6.29
7	10.06	5.31 6.29	10.18	5.30 6.30	10.30	5.29 6.31	10.42	5.28 6.32	10.56	5.26 6.34
8	11.33	5.27 6.33	11.46	5.25 6.35	12.00	5.24 6.36	12.15	5.23 6.37	12.30	5.21 6.39
9	13.01	5.22 6.38	13.16	5.21 6.39	13.31	5.19 6.41	13.48	5.18 6.42	14.05	5.16 6.44
10	14.29	5.18 6 42	14.45	5.16 6.44	15.02	5.15 6.45	15.21	5.13 6.47	15.40	5.11 6.49
11	15.57	5.14 6.46	16 15	5.12 6.48	16.34	5.10 6.50	16.54	5.08 6.52	17.16	5.06 6.54
12	17.25	5.09 6.51	17.45	5.07 6.53	18.06	5.05 6.55	18.29	5.03 6.57	18.52	5.01 6.59
13	18.54	5.05 6.55	19.16	5.03 6.57	19.39	5.01 6.59	20.03	4 58 7.02	20.29	4.56 7.04
14	20.23	5.00 7 00	20.47	4.58 7.02	21.12	4.56 7.04	21.38	4.53 7.07	22.07	4.51 7.09
15	21.53	4.56 7.04	22.18	4.53 7.07	22.45	4.51 7.09	23.14	4.48 7.12	23.45	4.46 7.14
16	23.23	4.51 7.09	23.50	4.48 7.12	24.20	4.46 7.14	24.51	4.43 7.17	25.24	4.40 7.20
17	24.53	4.56 7.14	25.23	4.53 7.17	25.55	4.41 7.19	26.28	4.38 7.22	27.03	4.35 7.25
18	26.25	4.41 7.19	26.57	4.38 7.22	27.30	4.35 7.25	28.06	4.32 7.28	28.44	4.29 7.31
19	27.57	4.36 7.24	28.31	4.33 7.27	29.07	4.30 7 30	29.45	4.27 7.33	30.26	4.23 7.37
20	29.30	4.31 7.29	30.06	4 28 7.32	30.44	4.25 7.35	31.25	4.21 7.39	32.09	4.17 7.43
21	31.03	4.26 7.34	31.42	4.23 7.37	32.23	4.19 7.41	33.07	4.15 7.45	33.53	4.11 7.49
22	32.38	4.21 7.39	33.19	4.17 7.43	34.03	4.13 7.47	34.49	4.09 7.51	35.39	4.05 7.55
23	34.14	4.16 7.44	34.57	4.12 7.48	35.44	4.07 7.53	36.33	4.03 7.57	37.26	3.58 8.02
23½	34.59	4.13 7.47	35.43	4.09 7.51	36.31	4.05 7.55	37.22	4.00 8.00	38.17	3.55 8.05

When Lat. and Dec. are unlike, R. and S. must change places with each other.

TABLE V.

Sun's Declination for every 2nd day, with corresponding Equation of Time, for 1891.

	January.		February.		March.		April.		May.		June.		
	South.		South.		South. / North.		North.		North.		North.		
Day	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Day
	S. ° ′	m.	S. ° ′	m.	S. ° ′	m.	N. ° ′	m.	N. ° ′	m.	N. ° ′	m.	
1	23.01	— 4	17.06	—14	7.35	—13	4.33	—4	15.05	+3	22.04	+2	1
3	22.50	— 5	16.31	—14	6.49	—12	5.19	—3	15 40	+3	22.19	+2	3
5	22.37	— 6	15.55	—14	6.03	—12	6.04	—3	16.15	+4	22.33	+2	5
7	22.23	— 6	15.18	—14	5.16	—11	6.50	—2	16.49	+4	22.46	+1	7
9	22.06	— 7	14.40	—14	4.29	—11	7.35	—2	17.21	+4	22.56	+1	9
11	21.48	— 8	14.01	—14	3.42	—10	8.19	—1	17.53	+4	23.06	+1	11
13	21.29	— 9	13.21	—14	2.55	—10	9.03	—1	18.23	+4	23.13	+0	13
15	21.07	—10	12.40	—14	2.08	— 9	9.46	—0	18.52	+4	23.19	—0	15
17	20.44	—10	11.58	—14	1.20	— 9	10.29	+0	19.20	+4	23.24	—1	17
19	20.20	—11	11.16	—14	S 0.33	— 8	11.10	+1	19.46	+4	23.26	—1	19
21	19.54	—12	10.33	—14	N 0.15	— 7	11.52	+1	20.11	+4	23.27	—1	21
23	19.26	—12	9.49	—14	1.02	— 7	12.32	+2	20.35	+4	23.27	—2	23
25	18.57	—13	9.05	—13	1.49	— 6	13.11	+2	20.57	+3	23.24	—2	25
27	18.27	—13	8.20	—13	2 36	— 6	13.50	+2	21.18	+3	23.20	—3	27
29	17.55	—13	7.35	—13	3.23	— 5	14.28	+3	21.37	+3	23.15	—3	29
31	17.23	—14	......	.....	4.09	— 4	......	.....	21.55	+3	......	.....	31

NOTE.—The sign prefixed to the Equation of Time, in the above table, is that for reducing **mean time to apparent time.**

TABLE V.—Continued.

Sun's Declination for every 2nd day, with corresponding Equation of Time, for 1891.

	July.		August.		September.		October.		November.		December.		
	North.		North.		North. / South.		South.		South.		South.		
Day	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Dec.	Eq.	Day
	N. ° ′	m.	N. ° ′	m.	N. ° ′	m.	S. ° ′	m.	S. ° ′	m.	S. ° ′	m.	
1	23.08	—4	18.03	—6	8.18	+ 0	3.11	+10	14.26	+16	21.49	+11	1
3	22.59	—4	17.32	—6	7.34	+ 1	3.57	+11	15.04	+16	22.07	+10	3
5	22.48	—4	17.00	—6	6.50	+ 1	4.44	+12	15.41	+16	22.23	+ 9	5
7	22.36	—5	16.27	—6	6.05	+ 2	5.30	+12	16.17	+16	22.37	+ 8	7
9	22.23	—5	15.53	—5	5.20	+ 3	6.16	+13	16.52	+16	22.50	+ 8	9
11	22.08	—5	15.18	—5	4.35	+ 3	7.01	+13	17.26	+16	23.01	+ 7	11
13	21.51	—5	14.42	- 5	3.49	+ 4	7.46	+14	17.59	+16	23.10	+ 6	13
15	21.33	—6	14.05	—4	3 03	+ 5	8.31	+14	18.30	+15	23.17	+ 5	15
17	21.13	—6	13.27	—4	2.16	+ 5	9.15	+15	19.00	+15	23.22	+ 4	17
19	20.52	—6	12.48	—4	1.30	+ 6	9.59	+15	19.29	+15	23.25	+ 3	19
21	20.30	—6	12.08	—3	N.43	+ 7	10.42	+15	19.56	+14	23.27	+ 2	21
23	20.06	—6	11.28	—3	S. 4	+ 7	11.24	+16	20.21	+14	23.27	+ 1	23
25	19.41	—6	10.47	—2	.50	+ 8	12.06	+16	20.46	+13	23.25	— 0	25
27	19.14	—6	10.05	—2	1.37	+ 9	12.47	+16	21.08	+12	23.20	— 1	27
29	18.47	—6	9.23	—1	2.24	+10	13 27	+16	21.30	+12	23.14	— 2	29
31	18.18	—6	8.40	—0	......	.....	14.07	+16	......	.....	23.07	— 3	31

Note.—The sign prefixed to the Equation of Time, in the above table, is that for reducing **mean time to apparent time.**

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 42°.

	Declination.	AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.21	.28	.35	.43	.52	1.01	1.11	1.22	1.35	1.49
	22	.22	.30	.29	.47	.57	1.06	1.17	1.29	1.43	1.57
	20	0.24	0.33	0.42	0.51	1.01	1.12	1.23	1.36	1.50	2.06
	18	.26	.35	.45	.55	1.05	1.17	1.29	1.42	1.57	2.13
	16	.28	.37	.48	.58	1.09	1.21	1.34	1.48	2.04	2.21
	14	.29	.40	.50	1.02	1.13	1.26	1.40	1.55	2.11	2.28
	12	.31	.42	.53	1.05	1.17	1.31	1.45	2.00	2.17	2.36
	10	0.33	0.44	0.56	1.08	1.21	1.35	1.50	2.06	2.24	2.43
	8	.34	.46	.59	1.12	1.25	1.40	1.55	2.12	2.30	2.50
	6	.36	.48	1.01	1.15	1.29	1.44	2.00	2.18	2.36	2.57
	4	.38	.51	1.04	1.18	1.33	1.49	2.05	2.23	2.42	3.03
	+ 2	.39	.53	1.07	1.21	1.37	1.53	2.10	2.29	2.49	3.10
	0	0.41	0.55	1.09	1.24	1.40	1.57	2.15	2.34	2.55	3.17
Unlike Latitude.	— 2	.42	.57	1.12	1.28	1.44	2.02	2.20	2.40	3.01	3.24
	4	.44	.59	1.15	1.31	1.48	2.06	2.25	2.45	3.07	3.30
	6	.45	1.01	1.17	1.34	1.52	2.10	2.30	2.51	3.13	3.37
	8	.47	1.03	1.20	1.37	1.56	2.15	2.35	2.57	3.19	3.44
	10	0.49	1.05	1.23	1.41	1.59	2.19	2.40	3.02	3.26	3.51
	12	.50	1.08	1.25	1.44	2.03	2.24	2.45	3.08	3.32	3.58
	14	.52	1.10	1.28	1.47	2.07	2.28	2.51	3.14	3.39	4.05
	16	.54	1.12	1.31	1.51	2.11	2.33	2.56	3.20	3.46	4.13
	18	.55	1.14	1.34	1.54	2.16	2.38	3.01	3.26	3.53	4.20
	20	0.57	1.17	1.37	1.58	2.20	2.43	3.07	3.33	4.00	4.28
	22	.59	1.19	1.40	2.02	2.24	2.48	3.13	3.39	4.07	4.36
	—24	1.01	1.22	1.43	2.06	2.29	2.53	3.19	3.46	4.15	4.45
										125°	120°

Table VI.—Continued.

LATITUDE 42°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
24°	1 58	2.08	2.19	2.31	2.42	2.57	3.12	3.27	3.44	4.01
22	2.07	2.18	2.29	2.41	2.54	3.08	3.23	3.39	3.56	4.13
20	2.16	2.27	2.39	2.51	3.05	3.19	3.34	3.50	4.07	4.25
18	2.24	2.36	2.48	3.01	3.14	3.29	3.44	4.01	4.18	4.35
16	2.32	2.44	2.56	3.10	3.24	3.39	3.55	4.11	4.28	4.46
14	2.40	2.52	3.05	3.19	3.33	3.48	4.04	4.21	4.38	4.56
12	2.47	3 00	3.13	3.27	3.42	3.57	4.14	4.30	4 48	5.05
10	2.55	3.08	3.22	3.36	3.51	4.06	4.23	4.40	4.57	5.15
8	3.02	3.16	3.30	3.44	3.59	4.15	4.32	4.49	5.06	5.24
6	3.10	3.23	3.37	3.52	4.08	4.24	4.41	4.58	5 15	5.33
4	3.17	3.31	3.45	4.00	4.16	4.33	4.49	5.07	5.24	5.42
+ 2	3.24	3.38	3.53	4.08	4.24	4.41	4.58	5.16	5.33	5.51
0	3.31	3.45	4.01	4.16	4.33	4.50	5.07	5.24	5.42	6.00
— 2	3.38	3.53	4 08	4.24	4.41	4.58	5.15	5.33	5.51	6.09
4	3.45	4.00	4.16	4.32	4.49	5.06	5.24	5.42	6.00	6.18
6	3.52	4.08	4.24	4.41	4.58	5.15	5.33	5.51	6.09	6.27
8	3.59	4.15	4.32	4.49	5.06	5.24	5.42	6.00	6.18	6.36
10	4.07	4.23	4.40	4.57	5.15	5.33	5.51	6.09	6.27	6.45
12	4.14	4.31	4.48	5.05	5.23	5.42	6 00	6.18	6.37	6.55
14	4.22	4.39	4.56	5.14	5.32	5.51	6.09	6.28	6.46	7.04
16	4.30	4.47	5.05	5.23	5.41	6.00	6.19	6.38	6.56	7.14
18	4.38	4.58	5.14	5.32	5.51	6.10	6.29	6.48	7.06	7.25
20	4.46	5.04	5.23	5.42	6.01	6.20	6.39	6.58	7.17	7.35
22	4.55	5.13	5.32	5.52	6.11	6 31	6.50	7.10	7.28	7.47
24	5.04	5.23	5.42	6.02	6.23	6.42	7.02	7.21	7.40	7.59
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 43°.

Declination.		AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.22	.29	.37	.46	.55	1.04	1.15	1.26	1.39	1.54
	22	.24	.32	.40	.49	.59	1.09	1.21	1.33	1.47	2.02
	20	0.25	0.34	0.43	0.53	1.03	1.14	1.26	1.39	1.54	2.10
	18	.27	.37	.46	.57	1.08	1.19	1.32	1.46	2.01	2.17
	16	.29	.39	.49	1.00	1.12	1.24	1.37	1.52	2.07	2.23
	14	.30	.41	.52	1.03	1.16	1.29	1.43	1.58	2.14	2.32
	12	.32	.43	.55	1.07	1.20	1.33	1.48	2.03	2.20	2.39
	10	0.34	0.45	0.57	1.10	1.23	1.38	1.52	2.09	2.27	2.46
	8	.35	.47	1.00	1.13	1.27	1.42	1.58	2.15	2.33	2.52
	6	.37	.50	1.03	1.17	1.31	1.46	2.03	2.20	2.39	2.59
	4	.38	.52	1.05	1.20	1.35	1.51	2.07	2.26	2.45	3.06
	+ 2	.40	.54	1.08	1.23	1.38	1.55	2.12	2.31	2.51	3.12
	0	0.41	0.56	1.11	1.26	1.42	1.59	2.17	2.36	2.57	3.19
Unlike Latitude.	— 2	.43	.58	1.13	1.29	1.46	2.03	2.22	2.42	3.03	3.26
	4	.45	1.00	1.16	1.32	1.50	2.08	2.27	2.47	3.09	3.32
	6	.46	1.02	1.18	1.35	1.53	2.12	2.32	2.53	3.15	3.39
	8	.48	1.04	1.21	1.39	1.57	2.16	2.37	2.58	3.21	3.46
	10	0.49	1.06	1.24	1.42	2.01	2.21	2.42	3.04	3.27	3.52
	12	.51	1.08	1.26	1.45	2.05	2.25	2.46	3.09	3.34	3.59
	14	.52	1.11	1.29	1.48	2.09	2.30	2.52	3.15	3.40	4.06
	16	.54	1.13	1.32	1.52	2.13	2.34	2.57	3.21	3.47	4.13
	18	.56	1.15	1.35	1.55	2.17	2.39	3.03	3.27	3.53	4.21
	20	0.58	1.17	1.38	1.59	2.21	2.44	3.08	3.33	4.00	4.28
	22	.59	1.20	1.41	2.03	2.25	2.49	3.14	3.40	4.07	4.36
	—24	1.01	1.22	1.44	2.06	2.30	2.54	3.20	3.47	4.15	4.44

Table VI.—Continued.

LATITUDE 43°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2 03	2.13	2.24	2.36	2.49	3.02	3.17	3.32	3.49	4.06
22	2.12	2.22	2.34	2.46	2.59	3.13	3.28	3.44	4.00	4.17
20	2.20	2.31	2.43	2.56	3.09	3.23	3.38	3 54	4.11	4.28
18	2.28	2 40	2.52	3.05	3.19	3.33	3.48	4.04	4.21	4.38
16	2.36	2.48	3.00	3.14	3.28	3.43	3.58	4.14	4.31	4.48
14	2.43	2 56	3.09	3.22	3.37	3.52	4.07	4.24	4.41	4.58
12	2.51	3 03	3.17	3.31	3.45	4.00	4.16	4.33	4 50	5.07
10	2.58	3.11	3.25	3.39	3.54	4.09	4.25	4.42	4.59	5.16
8	3.05	3.18	3.32	3.47	4.02	4.18	4.34	4.51	5.08	5.25
6	3.12	3.26	3.40	3.55	4.10	4.26	4.42	4.59	5.17	5.34
4	3.19	3.33	3.48	4.03	4.18	4.34	4.51	5.08	5.25	5.43
+ 2	3.26	3.40	3.55	4.10	4.26	4.43	4.59	5.16	5.34	5.51
0	3.33	3.47	4.03	4.18	4.34	4.51	5.08	5.25	5.42	6.00
— 2	3.40	3.55	4 10	4.26	4.42	4.59	5.16	5.33	5.51	6.09
4	3.47	4.02	4.17	4.34	4.50	5.07	5.24	5.42	6.00	6.17
6	3.54	4.09	4.25	4.41	4.58	5.15	5.33	5.51	6.08	6.26
8	4.01	4.16	4.33	4.49	5.06	5.24	5.41	5.59	6.17	6.35
10	4.08	4.24	4.40	4.57	5.15	5.32	5.50	6.08	6.26	6.44
12	4.15	4.31	4.48	5.06	5.23	5.41	5.59	6.17	6.35	6.53
14	4.22	4.39	4.56	5.14	5.32	5.50	6.08	6.26	6.44	7.02
16	4.30	4.47	5.05	5.23	5.41	5.59	6.17	6.36	6.54	7.12
18	4.38	4.55	5.13	5.31	5.50	6.08	6.27	6.46	7.04	7.22
20	4.46	5.04	5.22	5.41	5.59	6.18	6.34	6.56	7.14	7.32
22	4.54	5.13	5.31	5.50	6.09	6.28	6.48	7.06	7.25	7 43
—24	5.03	5.22	5.41	6.00	6.20	6.39	6.59	7.18	7.36	7.54
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 44°.

	Declination.	AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.23	.31	.39	.48	.57	1.07	1.18	1.30	1.43	1.58
	22	.24	.33	.42	.52	1.02	1.12	1.24	1.36	1.51	2.06
	20	0.26	0.36	0.45	0.55	1.06	1.17	1.29	1.43	1.58	2.14
	18	.28	.38	.48	.59	1.10	1.22	1.35	1.49	2.04	2.21
	16	.30	.40	.51	1.02	1.14	1.27	1.40	1.55	2.11	2.28
	14	.31	.42	.54	1.05	1.18	1.31	1.45	2.00	2.17	2.35
	12	.33	.44	.56	1.08	1.22	1.35	1.50	2.06	2.23	2.42
	10	0.35	0.46	0.59	1.12	1.25	1.40	1.55	2.12	2.30	2.49
	8	.36	.49	1.02	1.15	1.29	1.44	2.00	2.17	2.36	2.55
	6	.38	.51	1.04	1.18	1.33	1.48	2.05	2.22	2.42	3.02
	4	.39	.53	1.07	1.21	1.36	1.53	2.10	2.28	2.47	3.08
	+ 2	.41	.55	1.09	1.24	1.40	1.57	2.14	2.33	2.53	3.15
	0	0.42	0.57	1.12	1.27	1.44	2.01	2.19	2.38	2.59	3.21
Unlike Latitude.	− 2	.44	.59	1.14	1.31	1.47	2.05	2.24	2.44	3.05	3.27
	4	.45	1.01	1.17	1.34	1.51	2.09	2.29	2.49	3.11	3.34
	6	.47	1.03	1.19	1.37	1.55	2.14	2.33	2.54	3.17	3.40
	8	.48	1.05	1.22	1.40	1.58	2.18	2.38	3.00	3.23	3.47
	10	0.50	1.07	1.25	1.43	2.02	2.22	2.43	3.05	3.29	3.53
	12	.51	1.09	1.27	1.46	2.06	2.26	2.48	3.11	3.35	4.00
	14	.53	1.11	1.30	1.49	2.10	2.31	2.53	3.16	3.41	4.07
	16	.55	1.13	1.33	1.53	2.14	2.35	2.58	3.22	3.47	4.14
	18	.56	1.16	1.36	1.56	2.18	2.40	3.03	3.28	3.54	4.21
	20	0.58	1.18	1.38	2.00	2.22	2.45	3.09	3.34	4.01	4.28
	22	1.00	1.20	1.41	2.03	2.26	2.50	3.14	3.40	4.08	4.36
	−24	1.02	1.23	1.44	2.07	2.30	2.55	3.20	3.47	4.15	4.44
										125°	120°

Table VI.—Continued.

LATITUDE 44°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2.08	2.18	2.29	2.41	2.54	3.07	3.22	3.37	3.53	4.10
22	2.16	2.27	2.38	2.51	3.04	3.18	3.32	3.48	4.04	4.21
20	2.24	2.35	2.47	3.00	3.13	3.27	3.42	3.58	4.14	4.31
18	2.32	2.44	2.56	3.09	3.22	3.37	3.52	4.08	4.24	4.41
16	2.40	2.51	3.04	3.17	3.31	3.46	4.01	4.17	4.34	4.51
14	2.47	2.59	3.12	3.26	3.40	3.55	4.10	4.26	4.43	5.00
12	2.54	3.07	3.20	3.34	3.48	4.03	4.19	4.35	4.52	5.09
10	3.01	3.14	3.28	3.42	3.56	4.12	4.28	4.44	5.01	5.18
8	3.08	3.21	3.35	3.49	4.04	4.20	4.36	4.52	5.09	5.26
6	3.15	3.28	3.42	3.57	4.12	4.28	4.44	5.01	5.18	5.35
4	3.22	3.35	3.50	4.05	4.20	4.36	4.52	5.09	5.26	5.43
+ 2	3.28	3.42	3.57	4.12	4.28	4.44	5.01	5.17	5.34	5.52
0	3.35	3.49	4.04	4.20	4.36	4.52	5.09	5.26	5.43	6.00
— 2	3.42	3.56	4.12	4.27	4.43	5.00	5.17	5.34	5.51	6.08
4	3.48	4.03	4.19	4.35	4.51	5.08	5.25	5.42	5.59	6.17
6	3.55	4.10	4.26	4.42	4.59	5.16	5.33	5.50	6.08	6.25
8	4.02	4.18	4.34	4.50	5.07	5.24	5.41	5.59	6.16	6.34
10	4.09	4.25	4.41	4.58	5.15	5.32	5.50	6.07	6.25	6.42
12	4.16	4.32	4.49	5.06	5.23	5.41	5.58	6.16	6.33	6.51
14	4.23	4.40	4.57	5.14	5.31	5.49	6.07	6.25	6.42	7.00
16	4.30	4.47	5.05	5.22	5.40	5.58	6.16	6.34	6.52	7.09
18	4.38	4.55	5.13	5.31	5.49	6.07	6.25	6.43	7.01	7.19
20	4.46	5.03	5.21	5.40	5.58	6.16	6.35	6.53	7.11	7.29
22	4.54	5.12	5.30	5.49	6.08	6.26	6.45	7.03	7.21	7.39
—24	5.02	5.21	5.40	5.58	6.18	6.37	6.55	7.14	7.32	7.50
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 45°.

Declination.	AZIMUTHS.									
	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude. +24°	.24	.32	.41	.50	1.00	1.10	1.21	1.34	1.47	2.02
22	.26	.35	.44	.54	1.04	1.15	1.27	1.40	1.54	2.10
20	0.27	0.37	0.47	0.57	1.08	1.20	1.33	1.46	2.01	2.18
18	.29	.39	.50	1.01	1.12	1.25	1.38	1.52	2.08	2.25
16	.31	.41	.52	1.04	1.16	1.29	1.43	1.58	2.14	2.32
14	.32	.43	.55	1.07	1.20	1.33	1.48	2.03	2.20	2.39
12	.34	.46	.58	1.10	1.24	1.38	1.53	2.09	2.26	2.45
10	0.35	0.48	1.00	1.14	1.27	1.42	1.58	2.14	2.32	2.52
8	.37	.50	1.03	1.17	1.31	1.46	2.02	2.20	2.38	2.58
6	.38	.52	1.05	1.20	1.35	1.50	2.07	2.25	2.44	3.04
4	.40	.54	1.08	1.23	1.38	1.55	2.12	2.30	2.50	3.11
+ 2	.41	.56	1.10	1.26	1.42	1.59	2.16	2.35	2.55	3.17
0	0.43	0.58	1.13	1.29	1.45	2.03	2.21	2.40	3.01	3.23
Unlike Latitude. − 2	.44	1.00	1.16	1.32	1.49	2.07	2.26	2.46	3.07	3.29
4	.46	1.02	1.18	1.35	1.52	2.11	2.30	2.51	3.13	3.35
6	.47	1.04	1.21	1.38	1.56	2.15	2.35	2.56	3.18	3.42
8	.49	1.06	1.23	1.41	2.00	2.19	2.40	3.01	3.24	3.48
10	0.50	1.08	1.26	1.44	2.03	2.23	2.44	3.07	3.30	3.54
12	.52	1.10	1.28	1.47	2.07	2.28	2.49	3.12	3.36	4.01
14	.54	1.12	1.31	1.50	2.11	2.32	2.54	3.17	3.42	4.08
16	.55	1.14	1.34	1.54	2.15	2.36	2.59	3.23	3.48	4.14
18	.57	1.16	1.36	1.57	2.19	2.41	3.04	3.29	3.55	4.21
20	.58	1.19	1.39	2.00	2.23	2.46	3.10	3.35	4.01	4.29
22	1.00	1.21	1.42	2.04	2.27	2.50	3.15	3.41	4.08	4.36
−24	1.02	1.23	1.45	2.08	2.31	2.55	3.21	3.47	4.15	4.44
									125°	120°

Table VI.—Continued.

LATITUDE 45°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2.12	2.23	2.34	2.46	2.59	3.12	3.26	3.42	3.58	4.14
22	2.20	2.31	2.43	2.55	3.08	3.22	3.37	3.52	4.08	4.25
20	2.28	2.39	2.51	3.04	3.17	3.31	3.46	4. 02	4.18	4.35
18	2 36	2. 47	3.00	3.13	3.26	3.41	3.56	4.11	4.27	4.44
16	2.43	2.55	3.08	3.21	3.35	3. 49	4.05	4.20	4.37	4.53
14	2.50	3 02	3.15	3. 29	3.43	3.58	4.13	4.29	4.45	5.02
12	2.57	3 10	3.23	3.37	3.51	4.06	4.22	4.38	4 54	5.11
10	3.04	3.17	3.30	3.44	3.59	4.14	4.30	4.46	5.03	5.19
8	3.11	3.24	3.38	3.52	4.07	4.22	4.38	4.54	5.11	5.28
6	3.17	3.31	3.45	3.59	4.14	4.30	4.46	5.02	5.19	5.36
4	3.24	3.38	3.52	4.07	4.22	4.38	4.54	5.10	5.27	5.44
+ 2	3.30	3.44	3.59	4.14	4.30	4.45	5.02	5.18	5.35	5.52
0	3.37	3.52	4.06	4.21	4.37	4.53	5.09	5.26	5.43	6.00
— 2	3.43	3.58	4.13	4.29	4.44	5.01	5.17	5.34	5.51	6.08
4	3.50	4.05	4. 20	4.36	4.52	5.08	5.25	5.42	5.59	6.16
6	3.56	4.12	4. 27	4.43	5.00	5.16	5.33	5.50	6.07	6.24
8	4.03	4.19	4.34	4.51	5.07	5.24	5.41	5.58	6.15	6.32
10	4.10	4.26	4.42	4.58	5.15	5. 32	5.49	6.06	6.24	6.41
12	4.17	4.33	4.49	5.06	5.23	5.40	5.57	6.15	6.32	6.49
14	4.24	4.40	4.57	5.14	5.31	5.48	6.06	6.23	6.41	6. 58
16	4.31	4.47	5.04	5.22	5.39	5.57	6.14	6.32	6.50	7.07
18	4.38	4.55	5.12	5.30	5. 48	6.06	6.23	6.41	6.59	7.16
20	4.46	5.03	5.21	5.39	5.57	6.15	6.33	6.51	7.08	7.25
22	4.53	5.11	5.29	5.47	6.06	6 24	6.42	7.00	7.18	7.35
—24	5.02	5.20	5 38	5.57	6.15	6.34	6.53	7.11	7.28	7.46
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 46°.

	Declination.	AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.25	.34	.43	.52	1.02	1.13	1.25	1.38	1.51	2.07
	22	.27	.36	.46	.56	1.07	1.18	1.30	1.44	1.58	2.14
	20	0.28	0.38	0.49	0.59	1.11	1.23	1.36	1.50	2.05	2.21
	18	.30	.40	.51	1.03	1.14	1.27	1.41	1.56	2.11	2.28
	16	.32	.43	.54	1.06	1.18	1.32	1.46	2.01	2.17	2.35
	14	.33	.45	.57	1.09	1.22	1.36	1.51	2.07	2.23	2.42
	12	.35	.47	.59	1.12	1.26	1.40	1.55	2.12	2.29	2.48
	10	0.36	0.49	1.02	1.15	1.29	1.44	2.00	2.17	2.35	2.54
	8	.38	.51	1.04	1.18	1.33	1.48	2.05	2.22	2.41	3.01
	6	.39	.53	1.07	1.21	1.36	1.52	2.09	2.28	2.46	3.08
	4	.41	.55	1.09	1.24	1.40	1.56	2.14	2.33	2.52	3.13
	+ 2	.42	.57	1.12	1.27	1.43	2.00	2.18	2.38	2.58	3.19
	0	0.44	0.59	1.14	1.30	1.47	2.04	2.23	2.43	3.03	3.25
Unlike Latitude.	— 2	.45	1.01	1.17	1.33	1.50	2.03	2.27	2.48	3.09	3.31
	4	.47	1.03	1.19	1.36	1.54	2.12	2.32	2.53	3.14	3.37
	6	.48	1.05	1.22	1.39	1.57	2.16	2.37	2.58	3.20	3.42
	8	.50	1.07	1.24	1.42	2.01	2.21	2.41	3.03	3.25	3.49
	10	0.51	1.09	1.27	1.45	2.05	2.25	2.46	3.08	3.31	3.56
	12	.53	1.11	1.29	1.48	2.08	2.29	2.50	3.13	3.37	4.02
	14	.54	1.13	1.32	1.51	2.12	2.33	2.55	3.19	3.43	4.08
	16	.56	1.15	1.34	1.55	2.16	2.37	3.00	3.24	3.49	4.15
	18	.57	1.17	1.37	1.58	2.19	2.42	3.05	3.30	3.55	4.22
	20	0.59	1.19	1.40	2.01	2.23	2.46	3.10	3.36	4.01	4.29
	22	1.01	1.21	1.43	2.04	2.27	2.51	3.16	3.42	4.08	4.36
	—24	1.02	1.24	1.46	2.08	2.32	2.56	3.21	3.48	4.15	4.43
										125°	120°

Table VI.—Continued.

LATITUDE 46°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2 16	2.27	2.38	2.50	3.03	3.17	3.31	3.46	4.02	4 18
22	2.25	2.35	2.47	3.00	3.12	3.27	3.41	3.56	4.12	4.28
20	2.32	2.43	2.55	3.08	3.21	3.36	3.50	4 05	4.21	4.38
18	2 39	2.51	3.03	3.16	3.30	3.44	3.59	4.14	4.30	4.47
16	2.47	2.58	3.11	3.24	3.38	3.53	4.08	4.23	4.39	4.56
14	2.53	3 06	3.19	3.32	3.46	4.01	4.16	4.32	4.48	5 04
12	3.00	3 13	3.26	3.40	3 54	4.09	4.24	4.40	4 56	5.13
10	3.07	3.20	3.33	3.47	4.02	4.17	4.32	4.48	5.04	5.21
8	3.13	3.26	3.40	3.54	4.09	4.25	4.40	4.56	5.12	5.29
6	3.20	3.33	3.47	4.02	4.16	4.32	4.48	5.04	5 20	5.37
4	3.26	3.40	3.54	4.09	4.24	4.40	4.55	5.11	5.28	5.45
+ 2	3.32	3.46	4.01	4.16	4.31	4.47	5.03	5.19	5.36	5.52
0	3.39	3.53	4.03	4.23	4.38	4.55	5.10	5.27	5.43	6.00
— 2	3.45	4.00	4 14	4.30	4.46	5.02	5.18	5.34	5.51	6.08
4	3.51	4 06	4.21	4.37	4.53	5.09	5.25	5.42	5.59	6.15
6	3.58	4.13	4.28	4.44	5.00	5.17	5.33	5.50	6.07	6.23
8	4.04	4.19	4.35	4.51	5.08	5.24	5.41	5.58	6.14	6.31
10	4.11	4.26	4.42	4.58	5.15	5.32	5.49	6.06	6.22	6.39
12	4.17	4.33	4.49	5.06	5.23	5.40	5.57	6.14	6.30	6.47
14	4.24	4.40	4.57	5.13	5.30	5.48	6.05	6.22	6.39	6.56
16	4.31	4.47	5.04	5.21	5.38	5.56	6.13	6.30	6.47	7.04
18	4.38	4.55	5.12	5.29	5.47	6.05	6.22	6.39	6.56	7.13
20	4.45	5.03	5.20	5.37	5.55	6.13	6.31	6.48	7.05	7 22
22	4.53	5.10	5.28	5.46	6.04	6 22	6.40	6.58	7.15	7.32
—24	5.01	5.19	5.37	5.55	6.13	6.32	6.50	7.07	7.25	7.42
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 47°.

	Declination.	AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.26	.35	.45	.54	1.05	1.16	1.28	1.41	1.55	2.11
	22	.28	.37	.47	.58	1 09	1.21	1.33	1.47	2.02	2.18
	20	0.29	0.40	0.50	1.01	1.13	1.25	1.38	1.53	2.08	2.25
	18	.31	.42	.53	1.04	1.17	1.30	1.44	1.58	2.14	2.32
	16	.33	.44	.56	1.08	1.20	1 34	1.48	2.04	2.20	2.38
	14	.34	.46	.58	1.11	1.24	1.38	1.53	2.09	2.26	2.45
	12	.36	.48	1.01	1.14	1.28	1.42	1.58	2.14	2.32	2.51
	10	0.37	0.50	1.03	1.17	1.31	1.46	2.02	2.19	2.38	2.57
	8	.38	.52	1.06	1.20	1.35	1.50	2.07	2.25	2.43	3.03
	6	.40	.54	1.08	1.23	1.38	1.54	2.11	2.30	2.49	3.09
	4	.41	.56	1.11	1.26	1.42	1 58	2.16	2.34	2.54	3.15
	+ 2	.43	.58	1.13	1.29	1.45	2.02	2.20	2.39	3.00	3.21
	0	0.44	1.00	1.15	1.32	1.48	2.06	2.25	2.44	3.05	3.27
Unlike Latitude.	— 2	.46	1.02	1.18	1.34	1.52	2.10	2.29	2.49	3.10	3.33
	4	.47	1.03	1.20	1.37	1.55	2.14	2.34	2.54	3.16	3.39
	6	.49	1.06	1.23	1.40	1.59	2.18	2.38	2.59	3.21	3.44
	8	.50	1.07	1.25	1.43	2.02	2.22	2.42	3.04	3.27	3.50
	10	0.52	1.09	1.27	1.46	2.06	2.26	2.47	3 09	3.32	3.57
	12	.53	1.11	1.30	1.49	2.09	2.30	2.52	3 14	3.38	4.03
	14	.55	1.13	1.33	1.52	2.13	2.34	2 56	3.19	3.44	4.09
	16	.56	1.15	1.35	1.55	2.16	2.38	3.01	3.25	3.50	4.15
	18	.58	1.18	1.38	1.59	2.20	2.43	3.06	3.30	3.56	4.22
	20	0.59	1.20	1.40	2.02	2.24	2.47	3.11	3.36	4.02	4.29
	22	1.01	1.22	1.43	2.05	2 28	2.52	3.16	3.42	4.08	4.36
	—24	1.03	1.24	1.46	2.09	2.32	2.56	3.31	3.48	4.15	4.43
										125°	120°

Table VI.—Continued.

LATITUDE 47°.

Declination.	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2 21	2.32	2.43	2.55	3.08	3.21	3.35	3.50	4.06	4.22
22	2.29	2.40	2.51	3.04	3.17	3.30	3.45	4.00	4.15	4.31
20	2.36	2.47	2.59	3.12	3.25	3.39	3.54	4 09	4.24	4.41
18	2 43	2 55	3.07	3.20	3.33	3.48	4.02	4.17	4.33	4.49
16	2.50	3.02	3.14	3.28	3.41	3.56	4.10	4.26	4.42	4.58
14	2.57	3 09	3.22	3.35	3.49	4.04	4.18	4.34	4.50	5.06
12	3.03	3.16	3.29	3.42	3 57	4.11	4.26	4.42	4 58	5.14
10	3.10	3.22	3.36	3.49	4.04	4.19	4.34	4.50	5.06	5.22
8	3.16	3.29	3.43	3.57	4.11	4.26	4.42	4.57	5.14	5.30
6	3.22	3.35	3.49	4.04	4.18	4.34	4.49	5.05	5 21	5.37
4	3.28	3.42	3.56	4.11	4.25	4.41	4.56	5.12	5.29	5.45
+ 2	3.34	3.48	4.03	4.17	4.33	4.48	5.04	5.20	5.36	5.53
0	3.41	3.55	4.09	4.24	4.40	4.55	5.11	5.27	5.44	6.00
— 2	3.47	4.01	4 16	4.31	4.47	5.02	5.18	5.35	5.51	6.07
4	3.53	4.07	4.22	4.38	4.54	5.10	5.26	5.42	5.59	6.15
6	3.59	4.14	4.29	4.45	5.01	5.17	5.33	5.50	6.06	6.23
8	4.05	4.20	4.36	4.52	5.08	5.24	5.40	5.57	6.14	6.30
10	4.12	4.27	4.43	4.59	5 15	5.32	5.48	6.05	6.21	6.38
12	4.18	4.34	4.50	5.06	5.22	5.39	5.56	6.13	6.29	6.46
14	4.25	4.40	4.57	5.13	5.30	5.47	6.04	6.21	6.37	6.54
16	4.31	4.47	5.04	5.21	5.38	5.55	6.12	6.29	6.45	7.02
18	4.38	4.55	5.11	5.28	5.46	6.03	6.20	6.37	6.54	7.11
20	4.45	5.02	5.19	5.36	5.54	6.11	6.29	6.46	7.03	7.19
22	4.53	5.10	5.27	5.45	6.02	6 20	6.37	6.55	7.12	7.29
—24	5.00	5.18	5.36	5.53	6.11	6.29	6.47	7.04	7.21	7.38
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

Azimuth and Hour Angle, for Latitude and Declination.—Table VI.

LATITUDE 48°.

	Declination.	AZIMUTHS.									
		15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
Like Latitude.	+24°	.27	.37	.46	.57	1.07	1.19	1.31	1.44	1.59	2.15
	22	.29	.39	.49	1.00	1.11	1.24	1.35	1.50	2.05	2.22
	20	0.30	0.41	0.52	1.03	1.15	1.28	1.41	1.56	2.12	2.29
	18	.32	.43	.54	1.06	1.19	1.32	1.46	2.01	2.18	2.35
	16	.33	.45	.57	1.10	1.23	1.37	1.51	2.07	2.23	2.42
	14	.35	.47	1.00	1.13	1.26	1.41	1.56	2.12	2.29	2.48
	12	.37	.49	1.02	1.16	1.30	1.45	2.00	2.17	2.35	2.54
	10	.38	0.51	1.05	1.19	1.33	1.48	2.05	2.22	2.40	3.00
	8	.39	.53	1.07	1.21	1.37	1.52	2.09	2.27	2.46	3.06
	6	.41	.55	1.09	1.24	1.40	1.56	2.14	2.32	2.51	3.11
	4	.42	.57	1.12	1.27	1.43	2.00	2.18	2.37	2.56	3.17
	+ 2	.44	.59	1.14	1.30	1.47	2.04	2.22	2.41	3.02	3.23
	0	0.45	1.01	1.16	1.33	1.50	2.08	2.26	2.46	3.07	3.29
Unlike Latitude.	— 2	.46	1.02	1.19	1.36	1.53	2.12	2.31	2.51	3.12	3.34
	4	.48	1.04	1.21	1.39	1.57	2.15	2.35	2.56	3.17	3.40
	6	.49	1.06	1.24	1.41	2.00	2.19	2.39	3.01	3.23	3.46
	8	.51	1.08	1.26	1.44	2.03	2.23	2.44	3.05	3.28	3.52
	10	0.52	1.10	1.28	1.47	2.07	2.27	2.48	3.10	3.33	3.57
	12	.53	1.12	1.31	1.50	2.10	2.31	2.53	3.15	3.39	4.03
	14	.55	1.14	1.33	1.53	2.14	2.35	2.57	3.20	3.44	4.09
	16	.57	1.16	1.36	1.56	2.17	2.39	3.02	3.26	3.50	4.16
	18	.58	1.18	1.38	1.59	2.21	2.43	3.07	3.31	3.56	4.22
	20	1.00	1.20	1.41	2.03	2.25	2.48	3.12	3.36	4.02	4.29
	22	1.01	1.22	1.44	2.06	2.29	2.52	3.18	3.42	4.08	4.35
	—24	1.03	1.25	1.47	2.09	2.33	2.56	3.22	3.48	4.15	4.42
									130°	125°	120°

Table VI.—Continued.

LATITUDE 48°.

Declination.	AZIMUTHS.									
	63°	66°	69°	72°	75°	78°	81°	84°	87°	90°
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
+24°	2.25	2.36	2.47	2.59	3.12	3.25	3.40	3.54	4.10	4.25
22	2.32	2.44	2.55	3.08	3.20	3.34	3.48	4.03	4.19	4.35
20	2.40	2.51	3.03	3.16	3.28	3.43	3.57	4.12	4.28	4.43
18	2.46	2.58	3.10	3.23	3.36	3.51	4.05	4.20	4.36	4.52
16	2.53	3.05	3.18	3.31	3.44	3.59	4.13	4.29	4.44	5.00
14	3.00	3.12	3.25	3.38	3.51	4.06	4.21	4.36	4.52	5.08
12	3.03	3.18	3.32	3.45	3.59	4.14	4.29	4.44	5.00	5.16
10	3.12	3.25	3.38	3.52	4.06	4.21	4.36	4.52	5.07	5.23
8	3.18	3.31	3.45	3.59	4.13	4.28	4.43	4.59	5.15	5.31
6	3.24	3.38	3.51	4.06	4.20	4.35	4.51	5.06	5.22	5.38
4	3.30	3.44	3.58	4.12	4.27	4.42	4.58	5.14	5.29	5.46
+ 2	3.36	3.50	4.04	4.19	4.33	4.49	5.05	5.21	5.37	5.53
0	3.42	3.56	4.11	4.26	4.40	4.56	5.12	5.28	5.44	6.00
− 2	3.48	4.02	4.17	4.32	4.47	5.03	5.19	5.35	5.51	6.07
4	3.54	4.09	4.24	4.39	4.54	5.10	5.26	5.42	5.58	6.14
6	4.00	4.15	4.30	4.45	5.01	5.17	5.33	5.49	6.06	6.22
8	4.06	4.21	4.37	4.52	5.08	5.24	5.40	5.57	6.13	6.29
10	4.12	4.28	4.43	4.59	5.15	5.31	5.48	6.04	6.20	6.37
12	4.19	4.34	4.50	5.06	5.22	5.39	5.55	6.11	6.28	6.44
14	4.25	4.41	4.57	5.13	5.29	5.46	6.03	6.19	6.36	6.52
16	4.31	4.47	5.04	5.20	5.36	5.54	6.10	6.27	6.44	7.00
18	4.38	4.54	5.11	5.28	5.44	6.01	6.18	6.35	6.52	7.08
20	4.45	5.02	5.18	5.35	5.52	6.10	6.27	6.44	7.00	7.17
22	4.52	5.09	5.26	5.43	6.00	6.18	6.35	6.52	7.09	7.25
−24	4.59	5.17	5.34	5.52	6.09	6.27	6.44	7.01	7.18	7.35
	117°	114°	111°	108°	105°	102°	99°	96°	93°	90°

TABLE VII.

A table of differences of local and standard time, on the great lakes, for reducing standard time to the mean local time of the places mentioned. The sign prefixed, indicates the manner of applying the correction, to the standard time, the + sign meaning addition to, and the — sign meaning subtraction from standard time to get local mean time.

LAKE ONTARIO.

75° Long.

EASTERN STANDARD.

	M. S.
Sackett's Harbor	— 4.36
Stony Point	— 5.12
Fair Haven	— 6.52
Big Sodus	— 7.56
Genesee	—10.24
Oak Orchard	—12.48
30 Mile Point	—13.56
Fort Niagara	—16.16
Kingston	— 5.54
Desoronto	— 8.08
Bellville	— 9.32
Weller's Bay	—10.44
Coburg	—12.56
Port Hope	—13.20
Frenchman's Bay	—16.08
Toronto	—17.56
Fort Dalhousie	—17.24

LAKE ERIE.

75° Long.

EASTERN STANDARD.

	M. S.
Buffalo	—15.36
Dunkirk	—17.24
Erie	—20.16
Ashtabula	—23.12
Fairport	—25.08
Cleveland	—26.48
Black River	—28.44
Cedar Point	—30.48
Sandusky Bay	—30.52
Green Island	—31.28
Turtle Island	—33.32
Detroit River (mouth)	—33.32
Detroit City	—27.52
Fort Colbourne	—17.16
Fort Maitland	—18.40
Long Point (east end)	—20.36
Port Burwell	—23.36
Fort Stanley	—24.52
Pelee Spit	—30.32
Middle Island	—30.40
Kingsville	—30.56
Amherst Bay	—32.52

TABLE VII.—Continued.

LAKE HURON.

90° Long.

CENTRAL STANDARD.

	M. S.
Belle Isle (Saint Clair)	+28.12
Fort Gratiot	+30.20
Fort Sanilac	+29.52
Sand Beach	+29.40
Point of Barques	+29.00
Charity Island	+26.16
Saginaw River (mouth)	+24.36
Sturgeon Point	+26.56
Thunder Bay Island	+27.24
Presque Isle	+26.08
Spectacle Reef	+23.28
Cheboygan	+22.20
Detour	+24.24

CANADIAN SIDE.

75° Long.

	M. S.
Goderich	—26.12
South Hampton	—25.32
Michael's Point	—27.44
Great Duck Island	—31.44
Owen Sound	—23.44
Collingwood	—20.08
Whiskey Island	—19.32
French River	—23.40
Manitowaning	—27.12
Thessdon River	—23.40

LAKE MICHIGAN.

90° Long.

CENTRAL STANDARD.

	M. S.
McGulpin's Point	+20.56
Skilligallee	+19.20
Beaver Island	+17.44
Little Traverse	+20.08
South Fox Island	+16.40
Grand Traverse	+17.52
South Manitou	+15.40
Manistee	+14.36
White River	+14.20
Muskegon	+14.40
Grand Haven	+15.00
St. Joseph	+14.04
Michigan City	+12.24
Chicago	+ 9.32
Kenosha	+ 8.44
Racine	+ 8.52
Milwaukee	+ 8.32
Sheboygan	+ 9.12
Sturgeon Bay Canal	+ 9.44
Pilot Island	+12.20
Escanaba	+11.52
Chamber's Island	+10.32
Menominee	+ 9.40

LAKE SUPERIOR.

90° Long.

CENTRAL STANDARD.

	M. S.
St. Mary's Falls	+22.32
White Fish Point	+20.12
Grand Island	+13.16
Marquette	+10.28
Stanard's Rock	+11.08
Copper Harbor	+ 8.32
Ontonagon	+ 2.40
Devil's Island	— 3.12
St. Louis River	— 8.04
Duluth	— 8.20
Grand Marias	— 1.20
Isle Royal	+ 5.00

CANADA.

	M. S.
Agate Island	+15.52
Lamb Island	+ 6.32
Thunder Cape	+ 3.20
Port Arthur	+ 3.08
Victoria Island	+ 2.36

Table of Chords to Radius Unity, for Protracting.—VIII.

	0	1°	2°	3°	4°	5°	6°	7°	8°	9°	
0′	.0000	.0175	.0349	.0524	.0698	.0872	.0147	.0	.1395	.1569	0′
10	.0027	.0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	.1598	10
20	.0058	.0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	.1627	20
30	.0087	.0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	.1656	30
40	.0116	.0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	.1685	40
50	.0145	.0320	.0494	.0669	.0843	.1018	.1192	.1366	.1540	.1714	50
	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	
0′	.1743	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	0′
10	.1772	.1946	.2119	.2293	.2466	.2639	.2812	.2985	.3157	.3330	10
20	.1801	.1975	.2148	.2322	.2495	.2668	.2841	.3014	.3186	.3358	20
30	.1830	.2004	.2177	.2351	.2524	.2697	.2870	.3042	.3215	.3387	30
40	.1859	.2033	.2206	.2380	.2553	.2726	.2899	.3071	.3244	.3416	40
50	.1888	.2062	.2235	.2409	.2582	.2755	.2927	.3100	.3272	.3444	50
	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	
0′	.3473	.3645	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	0′
10	.3502	.3673	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	10
20	.3530	.3702	.3873	.4044	.4215	.4386	.5556	.4725	.4895	.5064	20
30	.3559	.3730	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	30
40	.3587	.3759	.3930	.4101	.4272	.4442	.4612	.4782	.4951	.5120	40
50	.3616	.3788	.3959	.4130	.4300	.4471	.4641	.4810	.4979	.5148	50
	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	
0′	.5176	.5345	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	0′
10	.5204	.5373	.5541	.5708	.5875	.6042	.6208	.6374	.6539	.6704	10
20	.5233	.5401	.5569	.5736	.5903	.6070	.6236	.6401	.6566	6731	20
30	.5261	.5429	.5597	.5764	.5931	.6097	.6263	.6429	.6594	.6778	30
40	.5289	.5457	.5625	.5792	.5959	.6125	.6291	.6456	.6621	.6786	40
50	.5317	.5485	.5652	.5820	.5986	.6153	.6318	.6484	.6649	.6813	50
	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	
0′	.6840	.7004	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	0′
10	.6868	.7031	.7195	.7357	.7519	.7681	.7841	.8002	.8161	.8320	10
20	.6895	.7059	.7222	.7384	.7546	.7707	.7868	.8028	.8188	.8347	20
30	.6922	.7086	.7249	.7411	.7573	.7734	.7895	.8055	.8214	.8373	30
40	.6950	.7113	.7276	.7438	7600	.7761	.7922	.8082	.8241	.8400	40
50	.6977	.7140	.7303	.7465	.7627	.7788	.7948	.8108	.8267	.8426	50

Table VIII.—Continued.

	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	
0′	.8452	.8610	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	0′
10	.8479	.8636	.8794	.8950	.9106	.9261	.9415	.9569	.9722	.9874	10
20	.8505	.8663	.8820	.8976	.9132	.9287	.9441	.9594	.9747	.9899	20
30	.8531	.8689	.8846	.9002	.9157	.9312	.9466	.9620	.9772	.9924	30
40	.8558	.8715	.8872	.9028	.9183	.9338	.9492	.9645	.9798	.9950	40
50	.8584	.8741	.8898	.9054	.9209	.9364	.9518	.9671	.9823	.9975	50
	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	
0′	1.0000	1.0157	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	0′
10	1.0025	1.0176	1.0326	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	10
20	1.0050	1.0201	1.0351	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	20
30	1.0075	1.0226	1.0375	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	30
40	1.0101	1.0251	1.0400	1.0549	1.0697	1.0844	1.0990	1.1136	1.1280	1.1424	40
50	1.0126	1.0276	1.0425	1.0574	1.0721	1.0868	1.1014	1.1160	1.1304	1.1448	50
	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	
0′	1.1472	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	0′
10	1.1495	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	10
20	1.1519	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	20
30	1.1543	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	30
40	1.1567	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	40
50	1.1590	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	50
	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	
0′	1.2856	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0′
10	1.2878	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
20	1.2900	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
30	1.2922	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
40	1.2945	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
50	1.2967	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	
0′	1.4142	1.4265	1.4386	1.4508	1.4627	1.4745	1.4863	1.4979	1.5094	1.5208	0′
10	1.4162	1.4285	1.4406	1.4528	1.4647	1.4765	1.4883	1.4998	1.5113	1.5227	10
20	1.4183	1.4305	1.4426	1.4548	1.4667	1.4785	1.4902	1.5017	1.5132	1.5246	20
30	1.4203	1.4325	1.4446	1.4568	1.4686	1.4804	1.4921	1.5037	1.5151	1.5265	30
40	1.4224	1.4346	1.4467	1.4588	1.4706	1.4824	1.4940	1.5056	1.5170	1.5283	40
50	1.4245	1.4366	1.4487	1.4608	1.4726	1.4843	1.4960	1.5074	1.5189	1.5302	50

TABLE IX.

MERIDIANAL PARTS.

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	M.
0	0	60	120	180	240	300	361	421	482	542	603	664	725	0
2	2	62	122	182	242	302	363	423	484	544	605	666	727	2
4	4	64	124	184	244	304	365	425	486	546	607	668	729	4
6	6	66	126	186	246	306	367	427	488	548	609	670	731	6
8	8	68	128	188	248	308	369	429	490	550	611	672	734	8
10	10	70	130	190	250	310	371	431	492	552	613	674	736	10
12	12	72	132	192	252	312	373	433	4 4	554	615	676	738	12
14	14	74	134	194	254	314	375	435	496	556	617	678	740	14
16	16	76	136	196	256	316	377	437	498	558	619	680	742	16
18	18	78	138	198	258	318	379	439	500	560	621	682	744	18
20	20	80	140	200	260	320	381	441	502	562	623	684	746	20
22	22	82	142	202	262	322	383	443	504	565	625	687	748	22
24	24	84	144	204	264	324	385	445	506	567	627	689	750	24
26	26	86	146	206	266	326	387	447	508	569	629	691	752	26
28	28	88	148	208	268	328	389	449	510	571	632	693	754	28
30	30	90	150	210	270	331	391	451	512	573	634	695	756	30
32	32	92	152	212	272	333	393	453	514	575	636	697	758	32
34	34	94	154	214	274	335	395	455	516	577	638	699	760	34
36	36	96	156	216	276	337	397	457	518	579	640	701	762	36
38	38	98	158	218	278	339	399	459	520	581	642	703	764	38
40	40	100	160	220	280	341	401	461	522	583	644	705	766	40
42	42	102	162	222	282	343	403	463	524	585	646	707	768	42
44	44	104	164	224	284	345	405	465	526	587	648	709	770	44
46	46	106	166	226	286	347	407	467	528	589	650	711	772	46
48	48	108	168	228	288	349	409	469	530	591	652	713	774	48
50	50	110	170	230	290	351	411	471	532	593	654	715	777	50
52	52	112	172	232	292	353	413	473	534	595	656	717	779	52
54	54	114	174	234	294	355	415	476	536	597	658	719	781	54
56	56	116	176	236	296	357	417	478	538	599	660	721	783	56
58	58	118	178	238	298	359	419	480	540	601	662	723	785	58
M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	M.

TABLE IX.—Continued.

M.	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	M.
0	787	848	910	973	1035	1098	1161	1225	1289	1354	0
2	789	851	913	975	1037	1100	1164	1227	1291	1356	2
4	791	853	915	977	1039	1102	1166	1229	1293	1358	4
6	793	855	917	979	1042	1105	1168	1232	1296	1360	6
8	795	857	919	981	1044	1107	1170	1234	1298	1362	8
10	797	859	921	983	1046	1109	1172	1236	1300	1364	10
12	799	861	923	985	1048	1111	1174	1238	1302	1367	12
14	801	863	925	987	1050	1113	1176	1240	1304	1369	14
16	803	865	927	989	1052	1115	1178	1242	1306	1371	16
18	805	867	929	991	1054	1117	1181	1245	1308	1373	18
20	807	869	931	994	1056	1119	1183	1247	1311	1375	20
22	809	871	933	996	1058	1121	1185	1249	1313	1377	22
24	811	873	935	998	1060	1123	1187	1251	1315	1380	24
26	813	875	937	1000	1063	1126	1189	1253	1317	1382	26
28	816	877	939	1002	1065	1128	1191	1255	1319	1384	28
30	818	879	942	1004	1067	1130	1193	1257	1321	1386	30
32	820	882	944	1006	1069	1132	1195	1259	1324	1388	32
34	822	884	946	1008	1071	1134	1198	1261	1326	1390	34
36	824	886	948	1010	1073	1136	1200	1264	1328	1393	36
38	826	888	950	1012	1075	1138	1202	1266	1330	1395	38
40	828	890	952	1014	1077	1140	1204	1268	1332	1397	40
42	830	892	954	1016	1079	1142	1206	1270	1334	1399	42
44	832	894	956	1019	1081	1145	1208	1272	1336	1401	44
46	834	896	958	1021	1084	1147	1210	1274	1339	1403	46
48	836	898	960	1023	1086	1149	1212	1276	1341	1406	48
50	838	900	962	1025	1088	1151	1215	1278	1343	1408	50
52	840	902	964	1027	1090	1153	1217	1281	1345	1410	52
54	842	904	966	1029	1092	1155	1219	1283	1347	1412	54
56	844	906	969	1031	1094	1157	1221	1285	1349	1414	56
58	846	908	971	1033	1096	1159	1223	1287	1352	1416	58
M.	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	M.

TABLE IX.—Continued.

M.	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	M.
0	1419	1484	1550	1616	1684	1751	1819	1888	1958	2028	0
2	1421	1486	1552	1619	1686	1753	1822	1891	1960	2031	2
4	1423	1488	1554	1621	1688	1756	1824	1893	1963	2033	4
6	1425	1491	1557	1623	1690	1758	1826	1895	1965	2035	6
8	1427	1493	1559	1625	1693	1760	1829	1898	1967	2038	8
10	1430	1495	1561	1628	1695	1762	1831	1900	1970	2040	10
12	1432	1497	1563	1630	1697	1765	1833	1902	1972	2043	12
14	1434	1499	1565	1632	1699	1767	1835	1905	1974	2045	14
16	1436	1502	1568	1634	1701	1769	1838	1907	1977	2047	16
18	1438	1504	1570	1637	1704	1772	1840	1909	1979	2050	18
20	1440	1506	1572	1639	1706	1774	1842	1912	1981	2052	20
22	1443	1508	1574	1641	1708	1776	1845	1914	1984	2054	22
24	1445	1510	1577	1643	1711	1778	1847	1916	1986	2057	24
26	1447	1513	1579	1645	1713	1781	1849	1918	1988	2059	26
28	1449	1515	1581	1648	1715	1783	1852	1921	1991	2061	28
30	1451	1517	1583	1650	1717	1785	1854	1923	1993	2064	30
32	1453	1519	1585	1652	1720	1787	1856	1925	1995	2066	32
34	1456	1521	1588	1654	1722	1790	1858	1928	1998	2069	34
36	1458	1524	1590	1657	1724	1792	1861	1930	2000	2071	36
38	1460	1526	1592	1659	1726	1794	1863	1932	2002	2073	38
40	1462	1528	1594	1661	1729	1797	1865	1935	2005	2076	40
42	1464	1530	1596	1663	1731	1799	1868	1937	2007	2078	42
44	1467	1532	1599	1666	1733	1801	1870	1939	2010	2080	44
46	1469	1535	1601	1668	1735	1803	1872	1942	2012	2083	46
48	1471	1537	1603	1670	1738	1806	1875	1944	2014	2085	48
50	1473	1539	1605	1672	1740	1808	1877	1946	2017	2088	50
52	1475	1541	1608	1675	1742	1810	1879	1949	2019	2090	52
54	1477	1543	1610	1677	1744	1813	1881	1951	2021	2092	54
56	1480	1546	1612	1679	1747	1815	1884	1953	2024	2095	56
58	1482	1548	1614	1681	1749	1817	1886	1956	2026	2097	58
M.	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	M.

TABLE IX.—Continued.

M.	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	M.
0	2100	2171	2244	2318	2393	2468	2545	2623	2702	2782	0
2	2102	2174	2247	2320	2395	2471	2548	2625	2704	2784	2
4	2104	2176	2249	2323	2398	2473	2550	2628	2707	2787	4
6	2107	2179	2252	2325	2400	2476	2553	2631	2710	2790	6
8	2109	2181	2254	2328	2403	2478	2555	2633	2712	2792	8
10	2111	2184	2257	2330	2405	2481	2558	2636	2715	2795	10
12	2114	2186	2259	2333	2408	2484	2560	2638	2718	2798	12
14	2116	2188	2261	2335	2410	2486	2563	2641	2720	2801	14
16	2119	2191	2264	2338	2413	2489	2566	2644	2723	2803	16
18	2121	2193	2266	2340	2415	2491	2568	2646	2726	2806	18
20	2123	2196	2269	2343	2418	2494	2571	2649	2728	2809	20
22	2126	2198	2271	2345	2420	2496	2573	2651	2731	2811	22
24	2128	2200	2274	2348	2423	2499	2576	2654	2733	2814	24
26	2131	2203	2276	2350	2425	2501	2578	2657	2736	2817	26
28	2133	2205	2279	2353	2428	2504	2581	2659	2739	2820	28
30	2135	2208	2281	2355	2430	2506	2584	2662	2742	2822	30
32	2138	2210	2283	2358	2433	2509	2586	2665	2744	2825	32
34	2140	2213	2286	2360	2435	2512	2589	2667	2747	2828	34
36	2143	2215	2288	2363	2438	2514	2591	2670	2750	2830	36
38	2145	2217	2291	2365	2440	2517	2594	2673	2752	2833	38
40	2147	2220	2293	2368	2443	2519	2597	2675	2755	2836	40
42	2150	2222	2296	2370	2445	2522	2599	2678	2758	2839	42
44	2152	2225	2298	2373	2448	2524	2602	2680	2760	2841	44
46	2155	2227	2301	2375	2451	2527	2604	2683	2763	2844	46
48	2157	2230	2303	2378	2453	2530	2607	2686	2766	2847	48
50	2159	2232	2306	2380	2456	2532	2610	2688	2768	2849	50
52	2162	2235	2308	2383	2458	2535	2612	2691	2771	2852	52
54	2164	2237	2311	2385	2461	2537	2615	2694	2774	2855	54
56	2167	2239	2313	2388	2463	2540	2617	2696	2776	2858	56
58	2169	2242	2316	2390	2466	2542	2620	2699	2779	2860	58
M.	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	M.

TABLE IX.—Continued.

M.	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	M.
0	2863	2946	3030	3116	3203	3292	3382	3474	3569	3665	0
2	2866	2949	3033	3118	3206	3295	3385	3478	3572	3668	2
4	2869	2951	3036	3121	3209	3298	3388	3481	3575	3672	4
6	2871	2954	3038	3124	3212	3301	3391	3484	3578	3675	6
8	2874	2957	3041	3127	3214	3303	3394	3487	3582	3678	8
10	2877	2960	3044	3130	3217	3306	3397	3490	3585	3681	10
12	2880	2963	3047	3133	3220	3309	3400	3493	3588	3685	12
14	2882	2965	3050	3136	3223	3312	3403	3496	3591	3688	14
16	2885	2968	3053	3139	3226	3316	3407	3499	3594	3691	16
18	2888	2971	3055	3142	3229	3319	3410	3503	3598	3695	18
20	2891	2974	3058	3144	3232	3322	3413	3506	3601	3698	20
22	2893	2976	3061	3147	3235	3325	3416	3509	3604	3701	22
24	2896	2979	3064	3150	3238	3328	3419	3512	3607	3704	24
26	2899	2982	3067	3153	3241	3331	3422	3515	3610	3708	26
28	2902	2985	3070	3156	3244	3334	3425	3518	3614	3711	28
30	2904	2988	3073	3159	3247	3337	3428	3521	3617	3714	30
32	2907	2991	3075	3162	3250	3340	3431	3525	3620	3717	32
34	29[illegible]0	2993	3078	3165	3253	3343	3434	3528	3623	3721	34
36	2913	2996	3081	3168	3256	3346	3437	3531	3626	3724	36
38	2915	2999	3084	3171	3259	3349	3440	3534	3630	3727	38
40	2918	3002	3087	3173	3262	3352	3443	3537	3633	3731	40
42	2921	3005	3090	3176	3265	3355	3447	3540	3636	3734	42
44	2924	3007	3093	3179	3268	3358	3450	3543	3639	3737	44
46	2926	3010	3095	3182	3271	3361	3453	3547	3643	3741	46
48	2929	3013	3098	3185	3274	3364	3456	3550	3646	3744	48
50	2932	3016	3101	3188	3277	3367	3459	3553	3649	3747	50
52	2935	3019	3104	3191	3280	3370	3462	3556	3652	3750	52
54	2937	3021	3107	3194	3283	3373	3465	3559	3655	3754	54
56	2940	3024	3110	3197	3286	3376	3468	3562	3659	3757	56
58	2943	3027	3113	3200	3289	3379	3471	3566	3662	3760	58
M.	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	M.

TABLE IX.—Continued.

M.	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	M.
0	3764	3865	3968	4074	4183	4294	4409	4527	4649	4775	0
2	3767	3868	3971	4077	4186	4298	4413	4531	4653	4779	2
4	3770	3871	3975	4081	4190	4302	4417	4535	4657	4784	4
6	3774	3875	3978	4085	4194	4306	4421	4539	4662	4788	6
8	3777	3878	3982	4088	4197	4309	4425	4543	4666	4792	8
10	3780	3882	3985	4092	4201	4313	4429	4547	4670	4796	10
12	3784	3885	3989	4095	4205	4317	4433	4551	4674	4801	12
14	3787	3889	3992	4099	4208	4321	4436	4555	4678	4805	14
16	3790	3892	3996	4103	4212	4325	4440	4559	4682	4809	16
18	3794	3895	3999	4106	4216	4328	4444	4564	4687	4814	18
20	3797	3899	4003	4110	4220	4332	4448	4568	4691	4818	20
22	3800	3902	4006	4113	4223	4336	4452	4572	4695	4822	22
24	3804	3906	4010	4117	4227	4340	4456	4576	4699	4826	24
26	3807	3909	4014	4121	4231	4344	4460	4580	4703	4831	26
28	3811	3913	4017	4124	4234	4347	4464	4584	4707	4835	28
30	3814	3916	4021	4128	4238	4351	4468	4588	4712	4839	30
32	3817	3919	4024	4132	4242	4355	4472	4592	4716	4844	32
34	3821	3923	4028	4135	4246	4359	4476	4596	4720	4848	34
36	3824	3926	4031	4139	4249	4363	4480	4600	4724	4852	36
38	3827	3930	4035	4142	4253	4367	4484	4604	4728	4857	38
40	3831	3933	4038	4146	4257	4370	4488	4608	4733	4861	40
42	3834	3937	4042	4150	4260	4374	4492	4612	4737	4865	42
44	3838	3940	4045	4153	4264	4378	4499	4616	4741	4870	44
46	3841	3944	4049	4157	4268	4382	4495	4620	4745	4874	46
48	3844	3947	4052	4161	4272	4386	4503	4625	4750	4879	48
50	3848	3951	4056	4164	4275	4390	4507	4629	4754	4883	50
52	3851	3954	4060	4168	4279	4394	4511	4633	4758	4887	52
54	3854	3958	4063	4172	4283	4398	4515	4637	4762	4892	54
56	3858	3961	4067	4175	4287	4401	4519	4641	4766	4896	56
58	3861	3964	4070	4179	4291	4405	4523	4645	4771	4901	58
M.	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	M.

TABLE IX.—Continued.

M.	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	M.
0	4905	5039	5179	5324	5474	5631	5795	5966	6146	6335	0
2	4909	5044	5184	5328	5479	5636	5800	5972	6152	6341	2
4	4914	5049	5188	5333	5484	5642	5806	5978	6158	6348	4
6	4918	5053	5193	5338	5489	5647	5811	5984	6164	6354	6
8	4923	5058	5198	5343	5495	5652	5817	5989	6170	6361	8
10	4927	5062	5203	5348	5500	5658	5823	5995	6177	6367	10
12	4931	5067	5207	5353	5505	5663	5828	6001	6183	6374	12
14	4936	5071	5212	5358	5510	5668	5834	6007	6189	6380	14
16	4940	5076	5217	5363	5515	5674	5839	6013	6195	6387	16
18	4945	5081	5222	5368	5520	5679	5845	6019	6201	6394	18
20	4949	5085	5226	5373	5526	5685	5851	6025	6208	6400	20
22	4954	5090	5231	5378	5531	5690	5856	6031	6214	6407	22
24	4958	5095	5236	5383	5536	5695	5862	6037	6220	6413	24
26	4963	5099	5241	5388	5541	5701	5868	6043	6226	6420	26
28	4967	5104	5246	5393	5546	5706	5874	6049	6233	6427	28
30	4972	5108	5250	5398	5552	5712	5879	6055	6239	6433	30
32	4976	5113	5255	5403	5557	5717	5885	6061	6245	6440	32
34	4981	5118	5260	5408	5562	5723	5891	6067	6252	6447	34
36	4985	5122	5265	5413	5567	5728	5896	6073	6258	6453	36
38	4990	5127	5270	5418	5573	5734	5902	6079	6264	6460	38
40	4994	5132	5275	5423	5578	5739	5908	6085	6271	6467	40
42	4999	5136	5280	5428	5583	5745	5914	6091	6277	6473	42
44	5003	5141	5284	5433	5588	5750	5919	6097	6283	6480	44
46	5008	5146	5289	5438	5594	5756	5925	6103	6290	6487	46
48	5012	5151	5294	5443	5599	5761	5931	6109	6296	6494	48
50	5017	5155	5299	5448	5604	5767	5937	6115	6303	6500	50
52	5021	5160	5304	5454	5610	5772	5943	6121	6309	6507	52
54	5026	5165	5309	5459	5615	5778	5948	6127	6315	6514	54
56	5030	5169	5314	5464	5620	5783	5954	6133	6322	6521	56
58	5035	5174	5319	5469	5625	5789	5960	6140	6328	6528	58
M.	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	M.

TABLE X.

Showing the length of **1 minute** of longitude on different latitudes.

Lat.		Statute Miles.	Naut. Miles.	Lat.		Statute Miles.	Naut. Miles.	Lat.		Statute Miles.	Naut. Miles.
0°	′	1.1527	1.0000	27°	—′	1.0277	.8916	54°	—′	.6790	.5891
	30	1.1526	.9999		30	1.0232	.8877		30	.6708	.5820
1	—	1.1525	.9998	28	—	1.0185	.8836	55	—	.6626	.5749
	30	1.1523	.9996		30	1.0138	.8795		30	.6544	.5678
2	—	1.1520	.9994	29	—	1.0089	.8753	56	—	.6460	.5605
	30	1.1516	.9991		30	1.0041	.8711		30	.6377	.5533
3	—	1.1511	.9986	30	—	.9991	.8667	57	—	.6293	.5459
	30	1.1506	.9981		30	.9940	.8624		30	.6208	.5416
4	—	1.1499	.9976	31	—	.9889	.8579	58	—	.6123	.5312
	30	1.1491	.9969		30	.9837	.8534		30	.6048	.5238
5	—	1.1483	.9962	32	—	.9784	.8488	59	—	.5951	.5161
	30	1.1475	.9954		30	.9731	.8442		30	.5865	.5088
6	—	1.1464	.9945	33	—	.9676	.8395	60	—	.5778	.5013
	30	1.1453	.9936		30	.9622	.8348		30	.5691	.4938
7	—	1.1441	.9926	34	—	.9566	.8299	61	—	.5602	.4861
	30	1.1428	.9915		30	.9509	.8250		30	.5514	.4785
8	—	1.1415	.9903	35	—	.9452	.8200	62	—	.5425	.4707
	30	1.1401	.9891		30	.9394	.8150		30	.5337	.4630
9	—	1.1386	.9877	36	—	.9336	.8099	63	—	.5247	.4552
	30	1.1348	.9863		30	.9277	.8048		30	.5157	.4474
10	—	1.1352	.9849	37	—	.9217	.7996	64	—	5066	.4395
	30	1.1335	.9833		30	.9157	.7943		30	.4976	.4317
11	—	1.1316	.9817	38	—	.9095	.7890	65	—	.4885	.4237
	30	1.1297	.9800		30	.9033	.7837		30	.4794	.4158
12	—	1.1276	.9783	39	—	.8970	.7782	66	—	.4701	.4079
	30	1.1254	.9764		30	.8907	.7727		30	.4610	.3999
13	—	1.1233	.9745	40	—	.8842	.7671	67	—	.4517	.3918
	30	1.1211	.9726		30	.8778	.7615		30	.4424	.3838
14	—	1.1187	.9705	41	—	.8712	.7558	68	—	.4330	.3757
	30	1.1163	.9682		30	.8646	.7501		30	.4238	.3676
15	—	1.1137	.9661	42	—	.8579	.7443	69	—	.4143	.3594
	30	1.1111	.9638		30	.8512	.7385		30	.4049	.3513
16	—	1.1083	.9615	43	—	.8443	.7325	70	—	.3954	.3430
	30	1.1056	.9571		30	.8375	.7266		30	.3859	.3358
17	—	1.1026	.9566	44	—	.8305	.7205	71	—	.3764	.3265
	30	1.0997	.9540		30	.8235	.7143		30	.3668	.3183
18	—	1.0966	.9513	45	—	.8164	.7080	72	—	.3572	.3099
	30	1.0935	.9486		30	.8093	.7020		30	.3476	.3016
19	—	1.0903	.9458	46	—	.8021	.6960	73	—	.3380	.2933
	30	1.0870	.9430		30	.7949	.6891		30	.3284	.2850
20	—	1.0836	.9401	47	—	.7875	.6831	74	—	.3187	.2765
	30	1.0802	.9372		30	.7802	.6769		30	.3090	.2681
21	—	1.0766	.9340	48	—	.7727	.6704	75	—	.2993	.2596
	30	1.0728	.9308		30	.7652	.6639		30	.2895	.2512
22	—	1.0692	.9276	40	—	.7576	.6573	76	—	.2797	.2427
	30	1.0654	.9243		30	.7500	.6507		30	.2699	.2343
23	—	1.0616	.9209	50	—	.7422	.6440	77	—	.2601	.2256
	30	1.0577	.9175		30	.7345	.6347		30	.2503	.2171
24	—	1.0536	.9140	51	—	.7268	.6306	78	—	.2404	.2085
	30	1.0495	.9105		30	.7190	.6238		30	.2305	.2000
25	—	1.0453	.9069	52	—	.7111	.6169	79	—	.2206	.1914
	30	1.0411	.9032		30	.7032	.6101		30	.2107	.1823
26	—	1.0367	.8994	53	—	.6952	.6031	80	—	.2008	.1742
	30	1.0323	.8956		30	.6872	.5962				

TABLE XI.

This table contains the correction, in minutes, to be added to the middle latitude, to obtain the correct middle latitude.

Middle Latitude.	DIFFERENCE OF LATITUDE.										
	3°	4°	5°	6°	7°	8°	9°	10°	12°	14°	16°
15°	2′	3′	5′	7′	9′	12′	15′	18′	26′	36′	47′
18	1	3	4	6	8	10	13	16	23	32	41
21	1	2	4	5	7	9	12	15	21	29	37
24	1	2	3	5	7	9	11	14	20	27	35
30	1	2	3	5	6	8	10	13	18	25	32
35	1	2	3	4	6	8	10	12	18	24	32
40	1	2	3	5	6	8	10	13	18	25	32
45	1	2	3	5	6	8	10	13	18	25	32
50	1	2	4	5	7	9	11	14	20	28	36
55	1	3	4	6	8	10	13	16	22	31	40
58	2	3	4	6	8	11	14	17	24	33	43
60	2	3	4	6	9	11	14	18	26	35	46
62	2	3	5	7	9	12	15	19	27	37	49
64	2	3	5	7	10	13	16	20	29	40	52
66	2	4	5	8	11	14	18	22	32	43	57

This table is to be entered at the top with the **Difference** of the two latitudes, and at the side with the **Middle Latitude;** under the former, and opposite the latter, is the correction, in minutes, to be added to the middle latitude, to obtain the corrected middle latitude.

TABLE XII.

Distance of Objects by Two Bearings.

Useful in rounding headlands in the night.

Second Bearings.	Sines of Bearings.	FIRST BEARINGS FROM HEADING OF VESSEL.—POINTS.														
		2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8	8½	9
P.																
3	.556	1.96														
3½	.634	1.32	2.42													
4	.707	1.00	1.62	2.85												
4½	.773	.81	1.23	1.91	3.25											
5	.831	.69	1.00	1.45	2.19	3.62										
5½	.882	.60	.85	1.18	1.66	2.44	3.96									
6	.924	.54	.74	1.00	1.35	1.85	3.66	4.26								
6½	.957	.50	.67	.88	1.14	1.50	2.02	2.86	4.52							
7	.981	.47	.61	.79	1.00	1.27	1.64	2.17	3.04	4.74						
7½	.995	.43	.57	.72	.90	1.11	1.39	1.76	2.30	3.18	4.91					
8	1.000	.41	.53	.67	.82	1.00	1.22	1.50	1.87	2.41	3.30	5.03				
8½	.995	.40	.51	.63	.76	.91	1.09	1.31	1.59	1.96	2.50	3.38	5.10			
9	.981	.39	.49	.60	.72	.85	1.00	1.18	1.39	1.66	2.03	2.57	3.43	5.13		
9½	.957	.38	.48	.58	.69	.80	.93	1.08	1.25	1.46	1.72	2.08	2.60	3.44	5.10	
10	.924	.38	.47	.57	.66	.77	.88	1.00	1.14	1.31	1.51	1.77	2.11	2.61	3.43	5.03

Rule.—The number in column of first bearing, and in line of second bearing, multiplied by the distance seen between the times of taking the bearings, is the distance of object from the vessel at time of taking the second bearing, in units of the distance seen.

And this distance multiplied by the sine of the second bearing, is the distance of the line of ship's course from the object, at right angles.

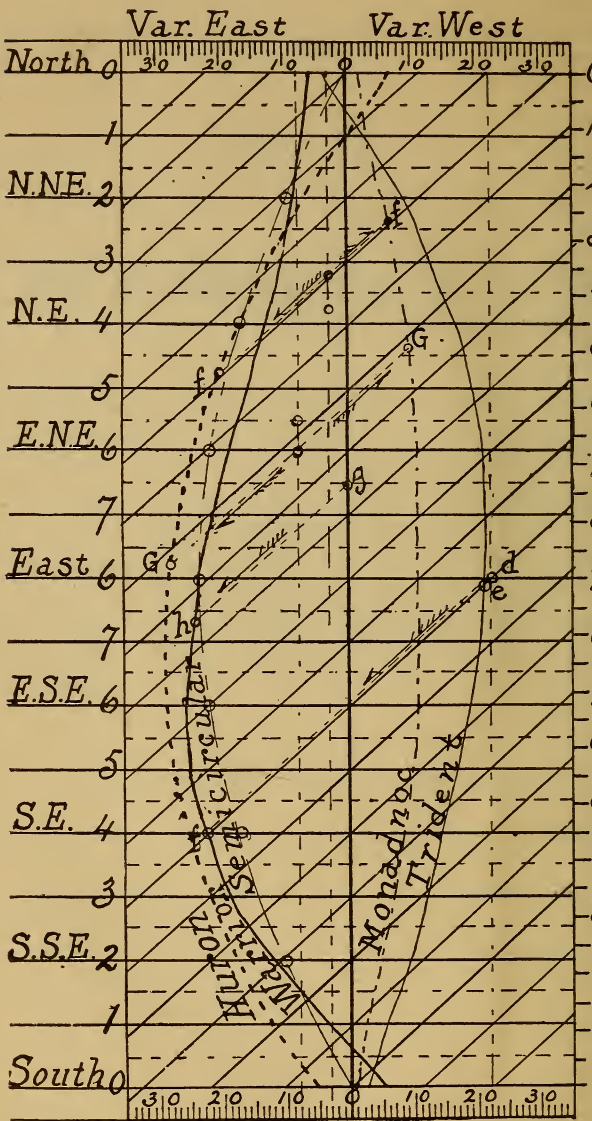
TABLE XIII.

Table for Reducing Longitude to Time.

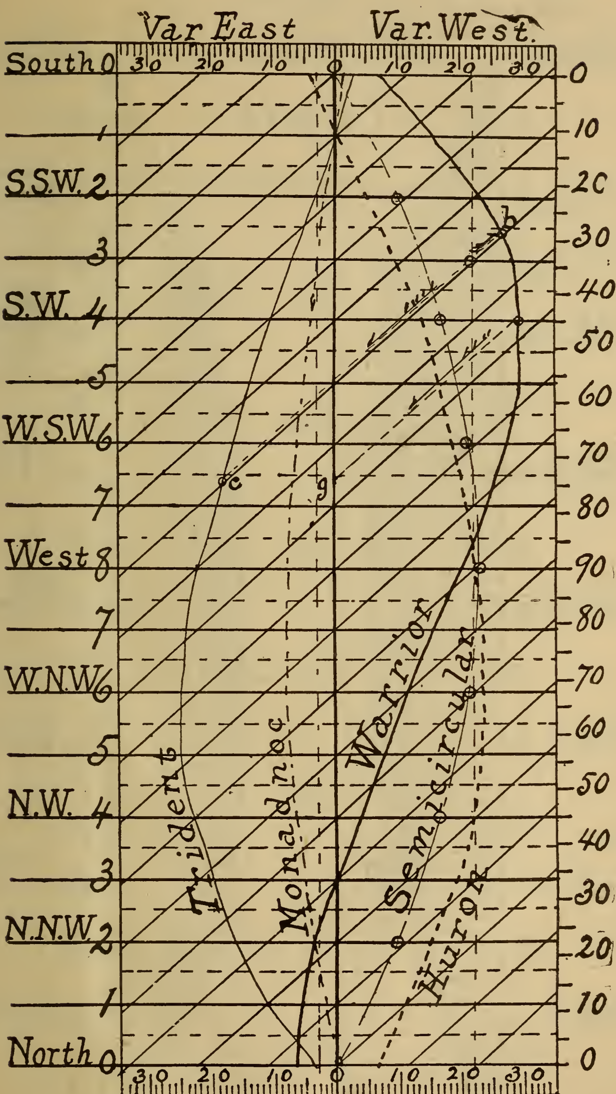
°	M.	′	M. S.	′	M. S.	′	M. S.	′	M. S.	′	M. S.	′	M. S.
1	4	1	4	11	.44	21	1.24	31	2.04	41	2.44	51	3.24
2	8	2	8	12	.48	22	1.28	32	2.08	42	2.48	52	3.28
3	12	3	12	13	.52	23	1.32	33	2.12	43	2.52	53	3.32
4	16	4	16	14	.56	24	1 36	34	2.16	44	2.56	54	3.36
5	20	5	20	15	1.00	25	1.40	35	2.20	45	3.00	55	3.40
6	24	6	24	16	1.04	26	1.44	36	2.24	46	3.04	56	3.44
7	28	7	28	17	1.08	27	1.48	37	2.28	47	3.08	57	3.48
8	32	8	32	18	1.12	28	1.52	38	2.32	48	3.12	58	3.52
9	36	9	36	19	1.16	29	1.56	39	2.36	49	3 16	59	3.56
10	40	10	40	20	1.20	30	2.00	40	2.40	50	3.20	60	4.00

The above table is wanted for finding the error of the standard time watch, on local **mean time,** or on local **apparent time,** when expanding an amplitude into a table of time azimuths of the sun, for the purpose of finding compass errors.

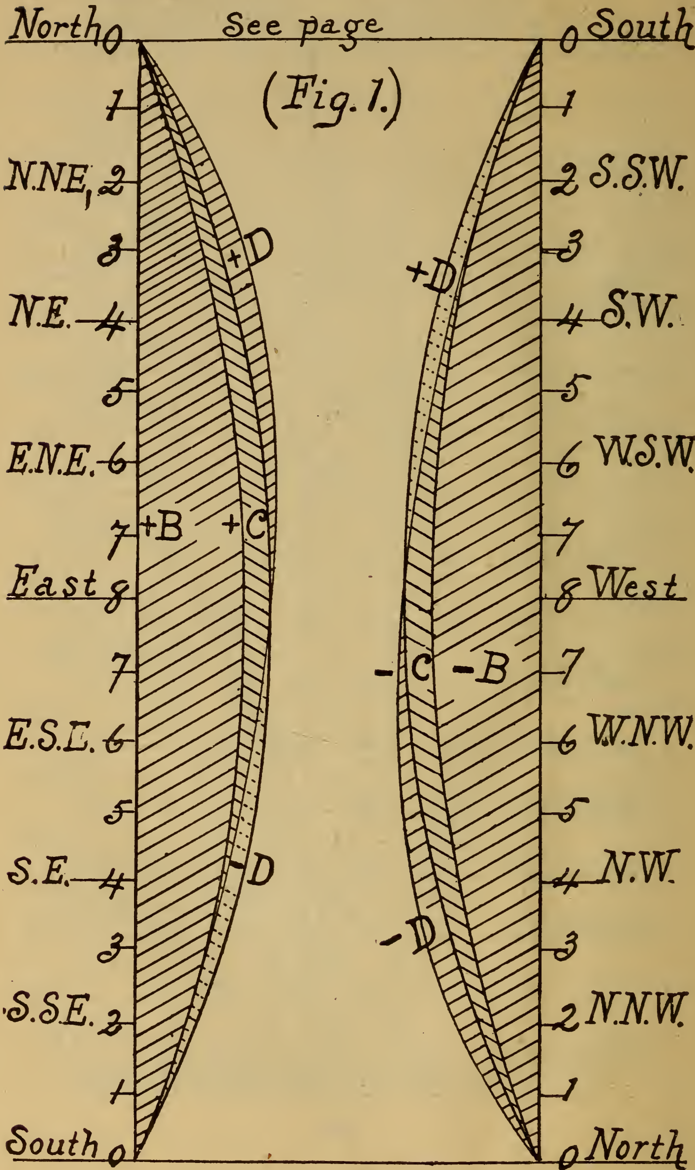
PLATE I.—PEARSON'S DIAGRAM.



PEARSON'S DIAGRAM.—PLATE I.



Semicircular and Quadrantal Deviation combined.



Relative Magnitude of the Magnetic Components.

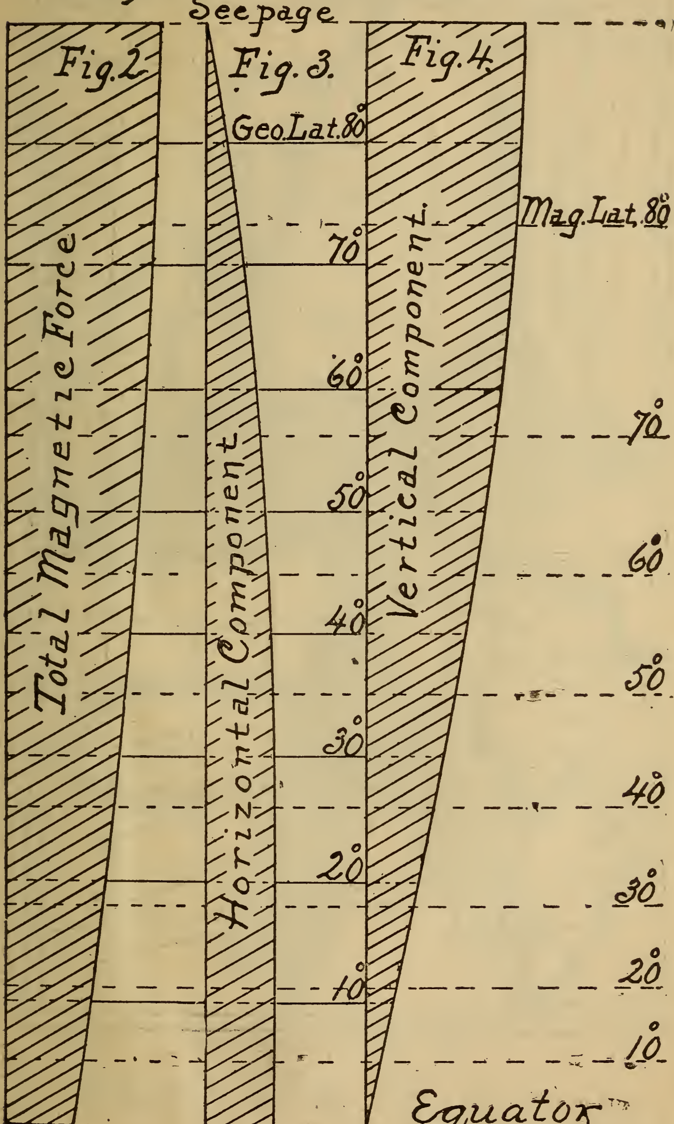
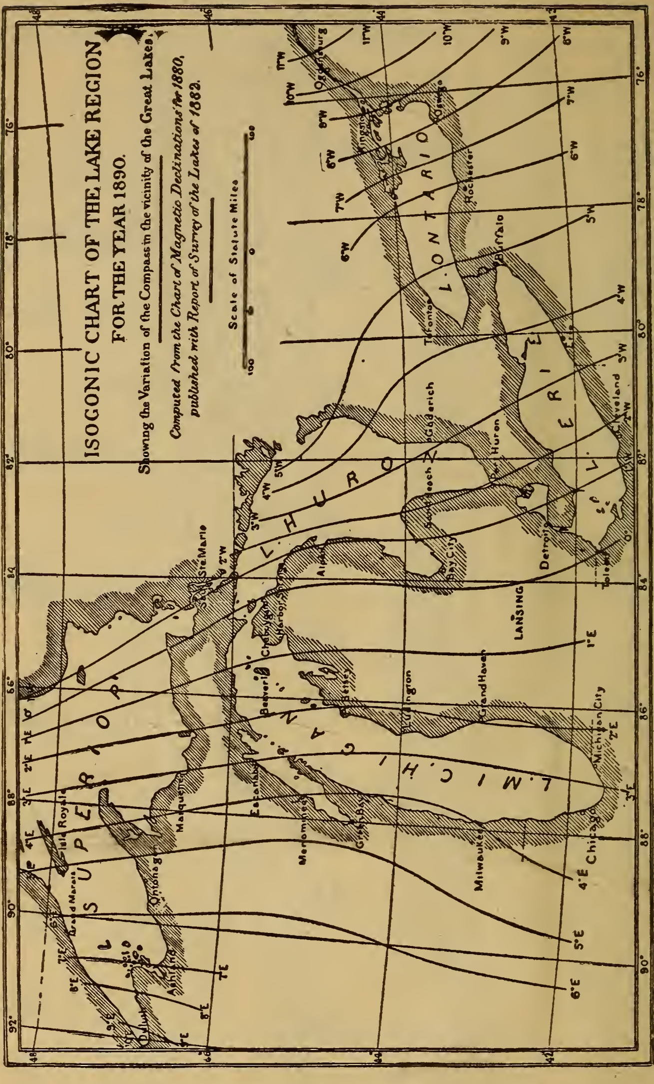


PLATE II.

PLATE III.



Prepared by the Engineer of the 9th & 11th L H Districts