ELEMENTS OF PERSPECTIVE

SULLIVAN

ECLECTIC SYSTEM OF INDUSTRIAL DRAWING

ELEMENTS OF PERSPECTIVE

BY

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ELEMENTS OF PERSPECTIVE.

CHAPTER I.

PERSPECTIVE is the art of representing objects as they *appear* and not as they *really are*.

A perspective drawing is one in which we have the relative heights and distances of different parts according to the angles and distances at which they stand in reference to the observer. If the student understands the principles by which a square or cube is correctly placed in perspective, he can, with little difficulty, make a representation of more complicated objects in their true proportions.

Certain lines are supposed to exist by which we determine the direction of the lines in the object to be drawn; each one of these lines occupies a certain position with regard to the observer, and also with regard to the picture plane, and they are governed by rules, deduced from the sciences of Optics and Geometry. Before giving any of these rules, it will be necessary to define terms with which the student must become familiar—picture plane, ground plane, horizon, base, etc. The most important of these is the term *picture plane* or *plane of delineation* (page 12). The position of this plane with regard to the object to be represented determines whether the drawing is to be made in parallel, angular, or oblique perspective (page 11).

The picture plane or plane of delineation is an imaginary plane parallel to the observer and perpendicular to the ground plane. The rays proceeding from an object (visual rays) pass through this plane.

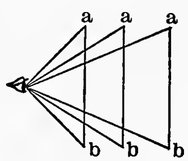


Fig. 1.

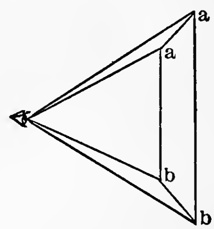


Fig. 2.

The visual angle is the angle formed by drawing lines from the extremities of any line to the eye; the angle varies with the size and distance of an object.

The visual angles are formed by lines extending from *a* and *b* to the eye.

If these rays proceed from a rectangular surface they form a pyramid of rays (Fig. 3), the vertex in the eye of the observer. If the rays proceed from a round surface, they form a cone of rays (Fig. 4).

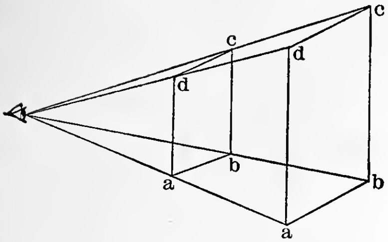


Fig. 3.

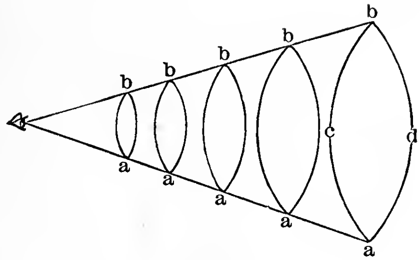


Fig. 4.

The base from which the rays proceed is termed the *field of view.* The distance *cd*, in Fig. 4, represents the cross diameter of the field of view. Lines drawn from these points to the eye form an angle of 60° (one-sixth of a circle). When looking at a point one can not see more than is included in an angle of 60°, and not more than one-half of this (30°) distinctly.

CHAPTER II.

THE apparent size of an object varies according to the distance at which it is situated from the eye of the observer. The size of an object appears to diminish as it recedes from the eye; an object at a distance of thirty feet from the observer appears one-half as large as the same object at a distance of fifteen feet from the observer. At a distance of one hundred feet, it will appear one-fourth as large as it will at a distance of twenty-five feet from the observer.

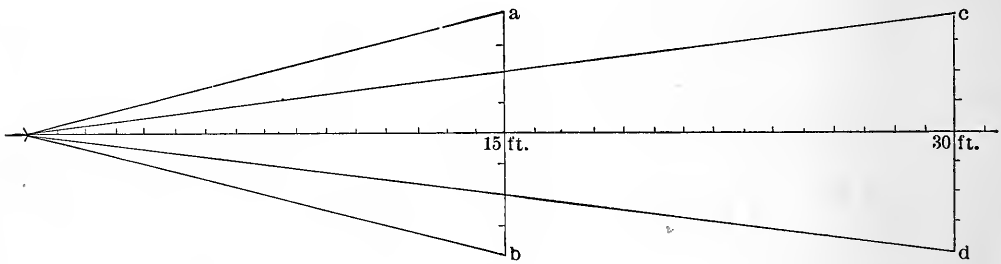


Fig. 5.

Figure 5 represents the line 8 feet long at a distance of 15 feet, and the same line again at a distance of 30 feet. At a distance of 30 feet, it appears one-half the length of the line at a distance of 15 feet.

If an object is at R. A. to the line of direction, (a line that extends from the eye of the observer to the object,) no matter how far removed from the eye, the apparent *shape* remains the same, the *size* only varies. A cube viewed in this position would be represented on a plane surface by a square, and a rectangular box by a rectangle, whether viewed at a distance of 5 feet or at a distance of 50 feet.

Fig. 6 represents the appearance of a cube when the line of direction is perpendicular to the point *a*.

Fig. 7 represents the appearance of a box when the line of direction is perpendicular to the point *b*.

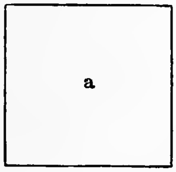


Fig. 6.

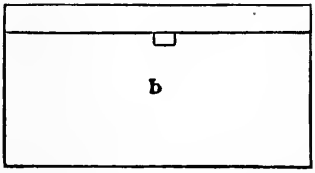


Fig. 7.

If an object is viewed at an angle, the apparent shape then varies. If we take a square figure and view it at a distance of 20 feet, the line of direction at right angles with the plane of the figure, the apparent shape is a *square*, and will be a square as long as it is viewed in that position. But let it occupy any position save that in which it is at right angles to the line of direction, and it will no longer appear of a square form. It will, if transferred to a flat surface, be represented by a figure that is not square.

As long as the square is parallel to the observer the opposite parallel lines do *not* appear to meet, but if we turn the square so that the right side is at a greater distance from the observer than the left side, then the upper and lower lines of the square, though parallel in reality, *appear* to approach. If a picture were made of it on paper, in this position, we should have the two vertical lines of the square, those that are not at an angle with, but parallel to the observer, represented by vertical lines. The line representing the right side, of course, is shorter than the line representing the left side, because it is farther from the observer; but those lines that are at an angle with the plane of delineation or picture plane (this is an imaginary plane between the observer and the object) are not represented as they are, but are represented in the drawing by two lines that will, if sufficiently prolonged, meet in a point.

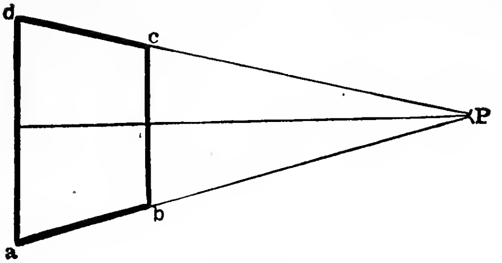


Fig. 8.

Fig. 8 represents a square *abcd* in perspective, the side *bc* at a greater distance from the observer than the side *ad*, and the lines *ab* and *dc* (parallel in reality) tending toward the point *P*.

If the square is *below* the level of the eye, the sides that recede from the observer tend to a point above the square.

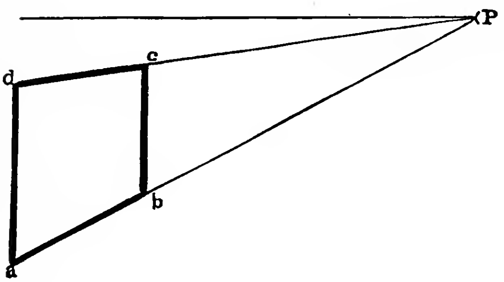


Fig. 9.

Fig. 9 represents a square below the level of the eye, which is here denoted by the horizontal line. In this the lines *dc* and *ab* at an angle to the observer, recede toward and appear to meet in the point *P*.

If the square is above the level of the eye, the lines that recede from the observer tend to a point below the square.

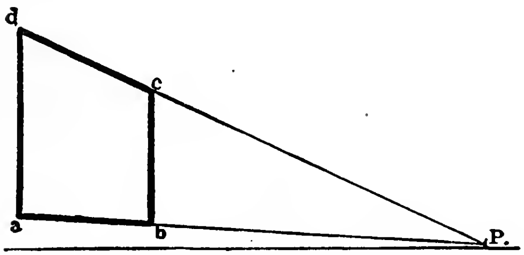


Fig. 10.

Fig. 10 represents a square in perspective above the level of the eye. The lines *ab* and *dc*, that are at an angle with the picture plane, recede and meet in the point *P*.

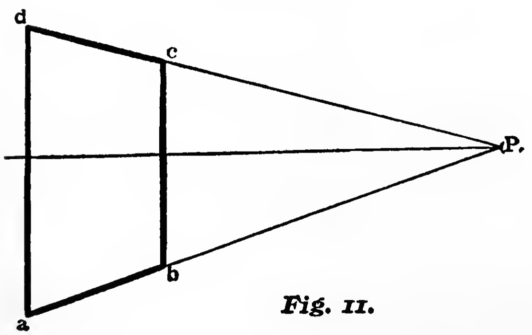


Fig. 11.

In Fig. 11 a very large square is represented, resting on the plane on which the observer stands, and extending above his head. The lines *dc* and *ab* that are at angles to the plane meet, if produced, in the point *P*. The line *dc* above the level of the eye tends *down* to the point *P*, and the line *ab* below the level of the eye tends *up* to the same point.

The point to which these receding lines recede, tend, or vanish, is called a *vanishing point;* it is on a level line, situated opposite the eye of the observer. In this case the vanishing point is on the right of the squares.

If these squares were so placed that the left side was farther from the observer than the right side, then the receding lines would vanish to a point on the left of the squares.

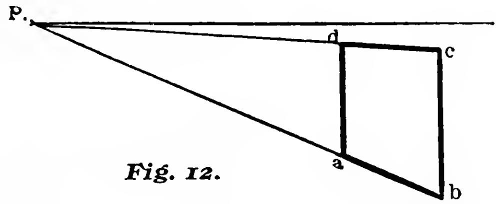


Fig. 12.

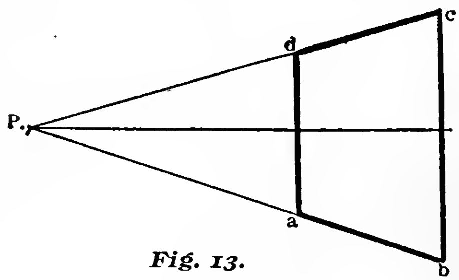


Fig. 13.

In Figs. 12 and 13 the vanishing points for the squares are on the left.

In representing a solid, such as a cube, if the front face is parallel to the observer, the receding lines vanish in one point (Fig. 14). If the cube is at an angle to the observer, then instead of one vanishing point there are two,—one on each side of the object (Fig. 15).

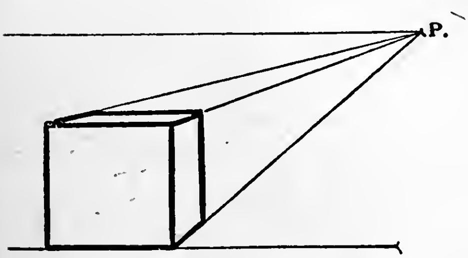


Fig. 14.

Fig. 14 represents a view of a cube parallel to the observer.

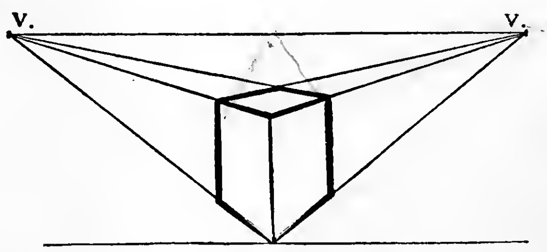


Fig. 15.

Fig. 15 represents a view of a cube at an angle to the observer.

The illustrations thus far show the direction of the lines when the objects are at right angles with the plane on which the observer is standing. We will now notice the direction of the lines in a square lying *flat* upon the same surface upon which the observer is standing.

Place the square at a distance of eighteen or twenty feet from the observer, so that a line drawn from the point where the observer is stationed, through the center of the square, will divide the front line into two equal parts; the front side of the square will, of course, appear longer than the far side, because it is nearer the observer. The line receding from the nearest right-hand corner, because it forms an angle with the picture plane, appears to tend to a point on the left of it, and the line extending from the nearest left-hand corner tends to a point on the right of it. These two receding lines approach nearer and nearer as they recede from the observer, and would, if produced, meet in a point opposite the eye of the observer (Fig. 16). This point is called the *point of sight*, and it locates the *horizon*, a horizontal line (page 8) to which all lines in a perspective drawing vanish, and on which are situated all vanishing points in parallel and angular perspective. All lines that are at *right angles* to the observer *vanish* in the *point of sight.*

All lines that are at an angle (greater or less than a right angle) to the observer, vanish in the horizon, but *not* in the point of sight.

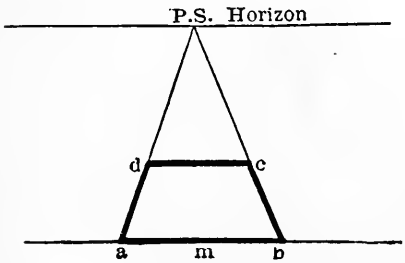


Fig. 16.

Fig. 16 represents a square lying flat on the surface upon which the observer is standing. The point *m* is opposite the observer.

CHAPTER III.

OBJECTS rest upon what is termed the ground plane in many different positions, which may be reduced to three general ones.

1. The objects may be so placed that their surfaces are at right angles and parallel with the picture plane. Objects viewed in this position are said to be in *parallel* perspective (Fig. 17).

abcd represents the picture plane; *abef* represents the ground plane. The face *hiln* and the face parallel to it, are parallel to the picture plane; the four remaining surfaces are at right angles to it.

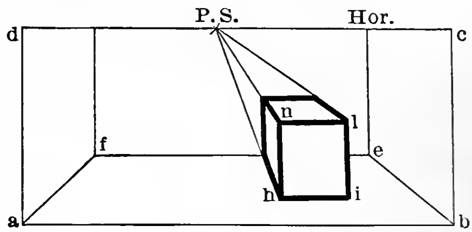


Fig. 17.

2. The objects may be so placed that some of the surfaces are at an angle with the spectator, and others parallel with the ground plane. Objects viewed in this position are said to be in *angular* perspective (Fig. 18).

abcd represents the picture plane; *abe* represents the ground plane. When the cube is in this position, the faces *ijk* and *ghn* are parallel to the ground plane, and the four remaining sides are at angles with the picture plane.

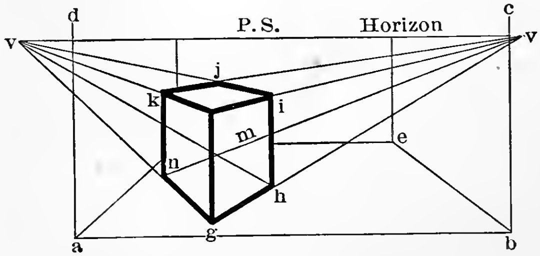


Fig. 18.

3. The objects may be so placed that the surfaces are not par-

allel with either the picture plane or the ground plane. Objects viewed in this position are said to be in *oblique* perspective (Fig. 19).

abcd represents the picture plane; *abef* represents the ground plane. The cube in this illustration is viewed in oblique perspective, and the faces are at angles with both the picture plane and the ground plane. In parallel and angular perspective, the lines vanish in the horizon; but in oblique perspective, the lines vanish on lines perpendicular to the horizon.

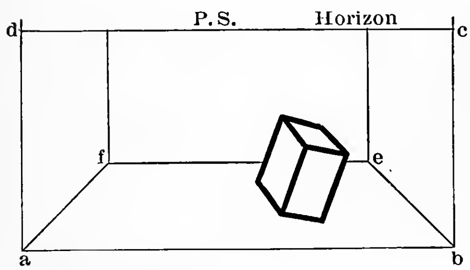


Fig. 19.

Oblique perspective is necessary in making drawings of roofs of houses, steeples of churches, gables, roads that are not level, etc.; in fact, in all views where there are oblique lines sloping from or toward the observer.

Before proceeding further, it is necessary to define the following terms:

(NOTE.—If these definitions are thoroughly understood, and committed to memory now, the after-work can be accomplished with very little difficulty.)

1. The *ground plane* is the plane or surface upon which the object or objects to be drawn are situated.

2. The *base* or *ground line* is a horizontal line that marks the nearest limit of the view to be taken; in a perspective drawing it is the lowest line of the picture.

3. The *picture plane* is an imaginary plane between the observer and the object, and is perpendicular to the ground plane (Fig. 17, page 11, and Fig. 20).

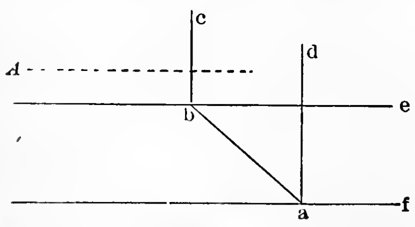


Fig. 20.

Fig. 20 represents the ground plane *abef*, and on the base line *ab*, the picture plane, *abcd*.

4. The *line of direction* is a horizontal line extending from the eye of the observer to the object; it is perpendicular to the picture plane.

In perspective drawings, the horizon is usually fixed at five feet above the base line; this distance is generally adopted (being about the distance from the ground plane to the line of direction) for the sake of uniformity.

In landscapes and sketches, this distance between the horizon and base line varies according to the nature of the scene to be represented.

1. In a representation of a low, flat, marshy desert, or of a prairie country, the horizon is quite low in order to give the idea of a *level landscape*.

2. If the picture is intended to convey the idea of *great height*, the horizon is placed low. In a sketch of a mountain gorge, cataract, mountain peak, or any scenery where an idea of great height is to be given, it is always low.

3. If an idea of *depth*, as of a scene viewed from a great height, is desired, the horizon is placed high—as the view of a city from a tower or steeple, or an expanse of country from a high hill.

4. In an ordinary view where no special height, depth, or distance is required, the horizon is placed about one-third the height of the picture above the base line.

5. The *horizon* is a horizontal line opposite the eye of the observer, parallel with the ground line, and distant from it five feet.

6. The *point of sight* is a point on the horizon opposite the eye of the observer; its position on the horizon varies as the spectator changes his point of observation.

7. The *prime vertical* is a vertical line passing through the *point of sight.*

8. The *point of station* is the point where the observer stands. The proper position from which to view the picture, is distant from the picture three times the height of the highest object in the scene; or, if the length exceeds the height, then the point of station will be distant from the picture three times the length of the object.

9. *Point of distance* is a point on the horizon to which all measurement lines vanish. In a drawing, it is as far from the point of sight as the point of station is from the point of sight (see page 16).

10. Vanishing points are points in which vanishing lines meet.

OBSERVATIONS.

1. Horizontal lines seen obliquely or at angles, if above the eye, appear to incline downward; if below the level of the eye, they appear to incline upward.

2. Straight lines may be drawn by finding the position of the extremities of the lines.

3. Curved lines may be drawn by finding their points of intersection.

4. The center of a perspective square is found by drawing its diagonals.

5. The center of a perspective rectangle is found by drawing its diagonals.

6. The perspective of any surface is determined by the perspective of its boundary lines.

CHAPTER IV.

THE following frame-work (Fig. 21) is necessary before a correct perspective drawing can be made; the points are placed and the lines are drawn in the following order:

1. Place the point of sight and mark it — P. S.

2. Through the P. S. draw a horizontal line of indefinite length, and mark it — Horizon.

3. Through the P. S. draw a vertical line, and mark this prime vertical — P. V.

4. Five feet below the P. S. on the prime vertical place a point through which draw a horizontal line of indefinite length, and divide it off into feet according to the scale. Mark this the base or ground line. Measure off on each side of the P. V.

5. Fix the point of distance on the horizon line, three times the height of the object from the point of sight, (if the length of the object exceeds the height, then take three times the length to avoid a violent fore-shortening) and mark it P. D.

In this case the supposed height of the object is 3½ feet; the point of distance is on the horizon three times 3½ feet from the point of sight.

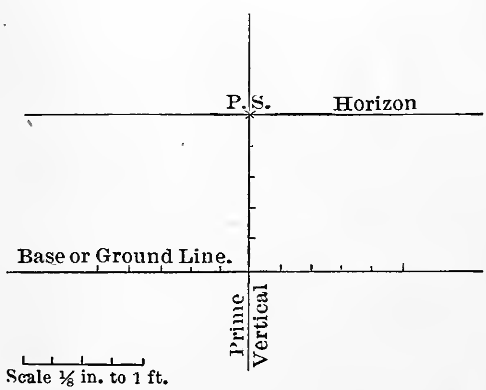


Fig. 21.

Another form is here given (Fig. 22) which is often used in preference to the one on the preceding page. In this form the points are placed and the lines drawn as in the form on page 15; but the points of distance are found by describing a half circle, with the point of sight as a center, and the line extending from the P. S. to the point of station as a radius. Where this half circle crosses the horizon, the points of distance are located.

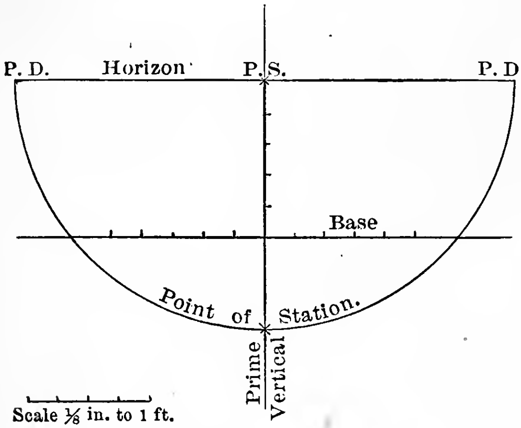


Fig. 22.

CHAPTER V.

RULES GOVERNING THE DIRECTION OF LINES AND SURFACES OF OBJECTS IN PARALLEL PERSPECTIVE.

RULE I. All measurements are taken on the *base* line, which is divided into feet according to a given scale.

RULE II. All lines *parallel* to the picture plane are represented in their actual position, and *do not vanish.*

RULE III. All lines *perpendicular* to the picture plane vanish in the point of sight. (P. S.)

RULE IV. All measurement lines vanish in the point of distance. (P. D.)

RULE V. All surfaces are governed by the lines that bound them, and their perspective is determined by these lines.

(NOTE.—After sketching an object or picture, the above rules may be applied to test the accuracy of the representation.)

In drawing a group of objects in still life or a landscape sketch, all the lines should be brought to the test of perspective calculations. It is impossible to place the different objects in their proper relation to each other, and in perfect harmony with the picture as to size and proportions, without a knowledge of the principles of perspective. All nature, animate and inanimate, is impressed on the sense of vision in accordance with the laws of perspective; consequently all art that represents in lines or masses must be in conformity to this most important branch of drawing.

Perspective admits of another division than that presented in this volume—aerial perspective. Aerial perspective has reference to atmospheric and other influences, by which objects more or less remote are effected in regard to light, shadow, color, gradation of tints, etc., according to their distances and relative positions. The principles for guidance in representations in aerial perspective are not reduced to systematic rules. A close observance of nature will, in time, enable the student to represent these effects with approximate accuracy.

CHAPTER VI.

REMARKS ON MODEL AND OBJECT DRAWING.

PRECISION is the basis of correct representation from the model. All questions are readily and satisfactorily settled by reference to the rules and principles of *perspective*, which are founded on the sciences, and are unchanging. (Geometry, Light and Color, and Optics.)

Practice in model drawing is necessary to satisfactory work in sketching from nature. It develops the power of observation, the ability to concentrate the attention, and the habit of noticing details—their relation to each other and to the whole, and the characteristic features of the object under consideration.

The objects used in sketching are geometrical solids, and objects based on these forms, supplemented by objects of beauty and utility. In drawing from the object, all work is *free-hand.*

In arranging objects for drawing, they should be placed so that the light comes from one side, thus giving the observer a view of the shaded side and the shadow.

The pupil should draw things as he sees them, and not as they really are.

To secure the right proportion in the drawing, the pencil may be held at arm's length, and the subtended amount of any dimension of the object may be marked on the pencil by moving the thumb-nail until the distance between it and the end of the pencil exactly covers the desired distance in the object.

By always viewing the object from the same point of station, and holding the pencil (when used as a measure) always at the same distance from the eye, and in the same plane, the copy will have the same relative proportions as the object.

SUGGESTIONS.

1. Place the model so that the light falls on it from *one* direction only.

2. Call attention to the facts and the representation of the surfaces.

3. Review rapidly the rules and principles of perspective governing the appearance of lines.

4. Note the distribution of light and shade — high light, shade, reflected light, shadow, and reflections, if any occur.

5. Make the drawing, and if you doubt the correctness of the representation, test by perspective. (Rules, page 17.)

6. The student may draw with board or stretcher at any angle that is convenient for him.

7. In making the drawing for illustration, if the object is not at hand, represent the light as falling from the upper left, falling on the object at an angle of 45°; the shade will then be on the opposite side.

8. The shade on objects bounded by plane surfaces is on the side away from the light.

9. When a plane is in shadow, the deepest shade is on that part nearest the observer.

10. Reflected lights appear on the farther side of the plane.

11. When a plane is in half-light, the highest light on it is on the part nearest the observer.

12. A plane partly illuminated has the deepest shade adjacent to the illuminated part.

13. A shadow cast by an object is darker than the shade on the object casting the shadow.

14. The darkest part of a shadow is near the object casting it.

NOTE.—Test your comprehension of these observations by drawing perspectives of a cube, on three different sheets of paper, showing:

(*a*) A cubical block.

(*b*) A cubical box without the lid.

(*c*) A cubical box, minus the front and left or right side.

15. If the light falls on a *sphere* from the left (7), illuminating the upper left-hand surface a little in from the apparent edge, the deepest shadow is on the lower right side a little in from the apparent edge; near this edge we find a medium tone that is called reflected light.

16. Reflected light on objects having curved surfaces is seen on the shaded side. The shaded surface is partially illuminated by lights cast from adjacent objects, and from the plane on which the object rests.

17. In *cylinders*, if the highest light falls on the left side a little way in from the apparent vertical boundary, the deepest shade comes on the right side a little from the apparent outline. This shade is darkest at the top, slightly modified in tone at the base by the reflected lights. The top of the cylinder is shaded slightly at the left; the right side is light. The shade and light do not extend to the limit of the outline.

NOTE.—Drawings may be finished by *hatching*, *stippling*, or shading with the stump.

CHAPTER VII

REFLECTIONS.

REFLECTIONS are produced (1) by polished surfaces that give back form and color, as mirrors, etc.; (2) by polished surfaces that give reflections somewhat distorted, as chinaware; (3) by liquids at rest — the smooth surface of the liquid serving as a mirror.

The rule that covers all reflections is, —

The angle of incidence equals the angle of reflection.

To place reflections in perspective, treat the reflections as realities. Such objects as rise or occupy a position perpendicular to the surface of the water, preserve their real proportions and relative positions. Thus posts, perpendicular cliffs, and masts of boats, throw their reflections to their full height, while surfaces that recede from us—foreshortened surfaces—although much higher in reality, and rising far above the objects mentioned, may scarce be seen at all in the reflection. If the point of sight is placed on a level with the water line, then the reflection will be a perfect repetition of the view, but the slightest elevation of the point of sight above the water line affects everything reflected that is not perpendicular to the water's edge. For instance, a roof jutting out over a boat-house on the bank will be much longer in the reflection than the vanishing lines in the original, and from the point of view we may see the top of the roof, while in the reflection we may see the under side of the projecting part of the roof. The reflection of an arch or culvert will be exactly like the original in the general face of the arch, but on account of the position of the observer he sees only the under side of the arch in the reflection.

The same rules that apply to linear perspective, apply to the perspective of reflections.

CHAPTER VIII

PROBLEM.—Place in perspective a 3-foot cubical box, the front line of the cube resting on the base line, and the nearest corner of the cube 3 feet to the left of the prime vertical.

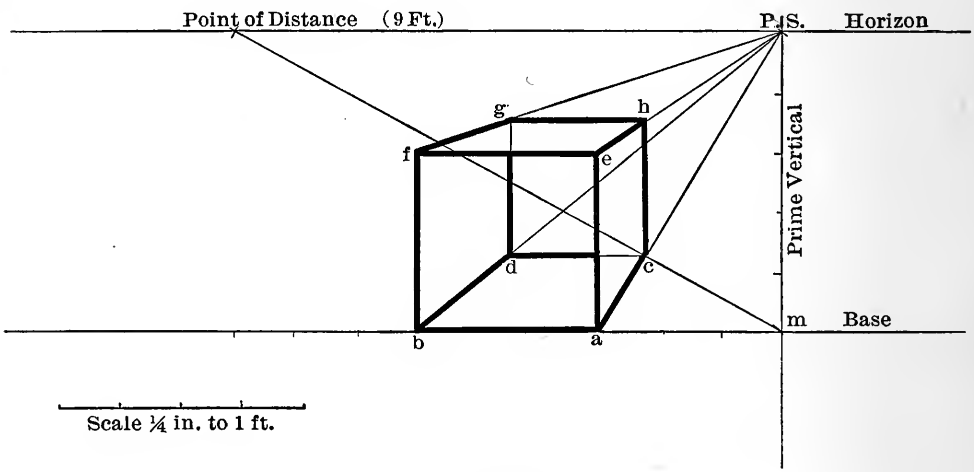


Fig. 23.

1. Locate the point of sight. If the object is on the left of the prime vertical, fix the point of sight on the right side of the paper. If the object is on the right of the prime vertical, then locate the point of sight near the left edge of the paper.

2. Draw the horizon, prime vertical, and base lines, marking the base line off according to the given scale.

3. Locate the point of distance. (Page 15.)

As the cube is 3 feet on the left of the prime vertical, move 3 feet to the left on the base line, and mark the point *a;* from *a* move 3 feet further to the left, and mark the point *b* (Rule I). From *a* and *b*, rule lines to the point of sight (Rule III). We now have the side *ab* and two sides of the base extending back from the front line; to determine the length of these receding lines is the next step.

As lines of the same length at different distances appear of different lengths, we know it is impossible to measure three feet on these lines, so we take the measurement on the base line (Rule I), and from the measurement point to the point of distance rule a line. Where this line crosses the line receding from *a*, the point *c* is located; from the point *c*, draw a line parallel to *ab*, mark the point *d*, and we have the base of the cube.

From *a* and *b*, draw vertical lines 3 feet in length, and draw *ef* (Rule II).

From *e* and *f*, draw lines to the point of sight (Rule III), erect vertical lines from *c* and *d*, and where these touch the lines receding from *e* and *f* locate the points *g* and *h*, and draw the line *gh* which completes the outline of the cube.

PROBLEM.—Place in perspective a box 6 feet high, with a base 3 feet square, the nearest corner of the base 7 feet to the right of the prime vertical on the base line.

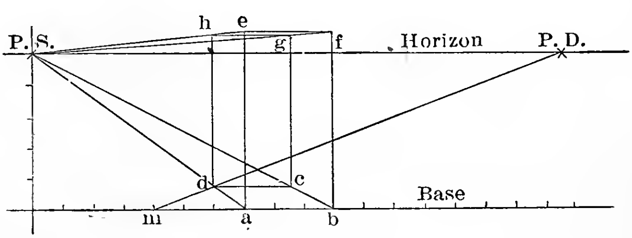


Fig. 24.

From the point where the base line is crossed by the prime vertical, move to the right on the base 7 feet, because the nearest corner of the box is 7 feet to the right of the observer, and locate the point *a*. From this draw the line *ab* on the base 3 feet long. From *a* and *b*, draw lines to the point of sight (Rule III). From the point *a*, move toward the point of sight 3 feet, and draw a line from the point of measurement to the point of distance. Where this line crosses the line receding from *a*, we have a point 3 feet from *a*. Mark this point *d*, and from it draw a line *dc* parallel to the front line of the base. This gives the square *abcd* in perspective, the base of the box. From *a* and *b* draw vertical lines, each 6 feet high, and draw the horizontal line *ef;* from *e* and *f* draw lines to the point of sight (Rule III). As the points *e* and *f* are above the level of the eye, the lines receding from them appear to incline to the horizon.

From the point *d*, draw a vertical line to the line receding from *e*, and mark this point *h;* and from *c*, draw a line to the line receding from *f*, and mark it *g*. Connect *h* and *g*, and we have the square forming the top of the box.

PROBLEM.—Place in perspective a box 5 feet long, 3 feet high, and 3 feet wide, 4 feet to the right of the observer, and 2 feet back from the base line.

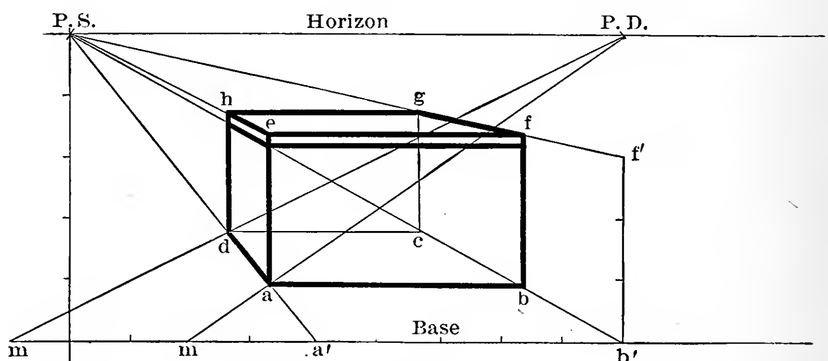


Fig. 25.

Locate the points and draw the lines as in the preceding problem. As the object is 4 feet to the right of the prime vertical line, count on the base line 4 feet to the right (Rule I), and place a point and mark it *a'*. As the object is 5 feet long, move 5 feet further to the right, and fix the point *b';* and from these two points draw the lines receding to the point of sight. The box is not on the base line, but 2 feet back from it; in order to find the points for the front line of the box, we measure 2 feet on the receding line *a*, according to Rule IV, and locate the point *a*. We then draw the line *ab;* according to the same rule, fix the point *d*, and draw the line *dc*. According to Rule II, draw the lines representing the front face of the box. Because the sides of the cover are perpendicular to the picture plane, the lines from *e* and *f* vanish in the point of sight; from the point *d*, draw a vertical line to the line vanishing from *e;* from this point *h*, draw a line parallel to *ef*, and draw the line *cg*.

CHAPTER IX.

PROBLEM.—Place in perspective a circle 4 feet in diameter, 7 feet to the left of a prime vertical, the circle tangent to the base line at that point.

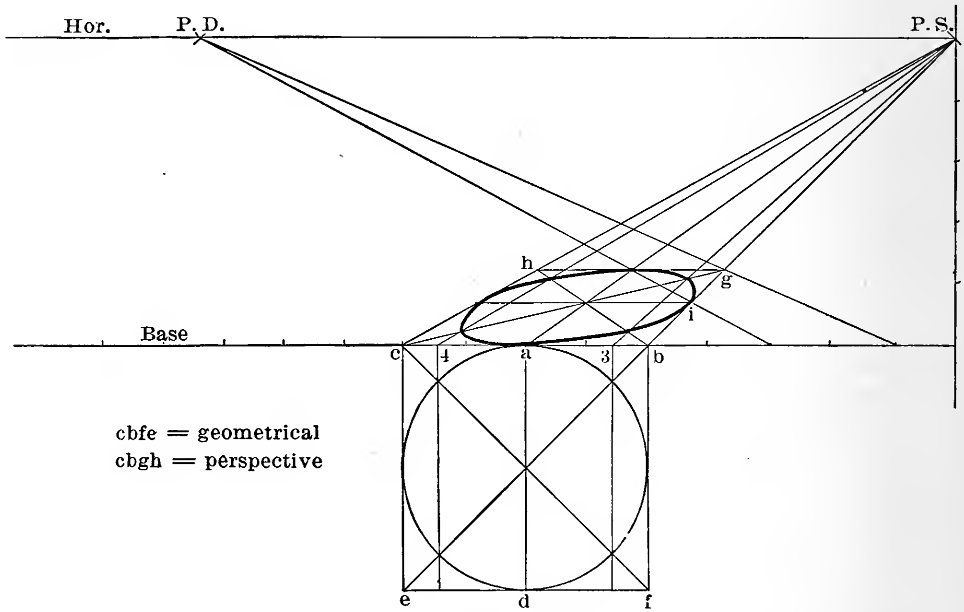


Fig. 26.

If a circle is placed opposite the eye, the diameter on a level with the horizon, it will be represented by a straight line; if it is directly opposite the observer at right angles with the ground plane, the circle is represented by a vertical line; if the center is directly opposite the eye, it will be represented by a perfectly round figure; but in any other position the circle will be represented by an ellipse (Fig. 26).

Now place the 4-foot square *cbef* in perspective (page 28), and we have the figure *cbgh;* then draw the diagonals *cg* and *bh.* From the point *b*, move toward the prime vertical 2 feet, and draw a line to the point of distance; this line divides the line *bg* into two parts. From *i* draw a line parallel to *cb*, and from *a*, 4, and 3, draw lines to the point of sight. When these lines are drawn, we have the eight points through which the circle passes located, and all that remains to be done is to pass a curved line through these eight points, and we have a circle, 4 feet in diameter, in perspective.

In order to place circles, triangles, hexagons, etc., in perspective, it is first necessary to draw a geometrical plan below the base line in order to locate certain points through which to draw the outline of these figures.

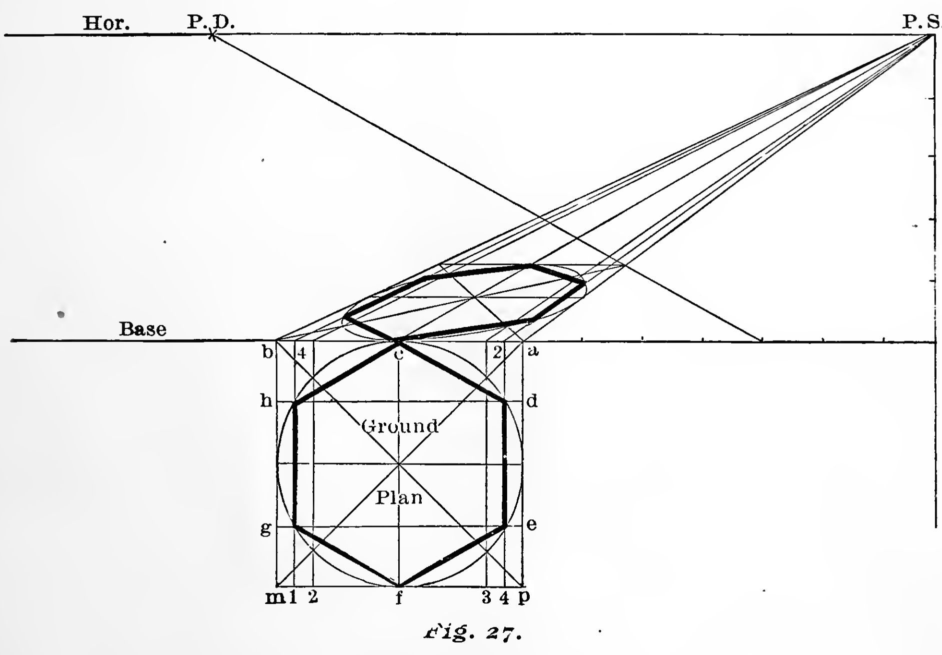


Fig. 27.

Before proceeding to place the circle in perspective, draw a 4-foot square below the base line (because the diameter of the circle is 4 feet), and in this draw a circle tangent to the four sides of the square; draw diagonals across the square, and through the points where the diagonals and circle intersect, draw the lines 3 and 4 to the base line; draw *ad*.

PROBLEM.—Place in perspective a hexagon with a diameter of 4 feet, touching the base line at a point 9 feet to the left of the *P. V.*

In this problem the square *abmp* is placed in perspective. Then the circle is drawn, and placed in perspective. In order to draw the hexagon, divide the vertical diameter into four equal parts, and from the first and third division points in this line draw horizontal lines to the points *h* and *d*, and *g* and *e;* from the point *c*, draw *ch* and *cd;* from *f* draw *fe* and *fg*. Connect the points *g* and *h* and *d* and *e*, strengthen these outlines, and the hexagon in the ground plane is completed.

Prolong the lines *gh* and *ed* until they reach the base line at the points 1 and 4, and from these points draw lines to the point of sight; in these lines the perspective of the sides *ed* and *gh* of the hexagon will be found. In the perspective circle locate the points corresponding to the points *cdefgh* in the ground plan, draw the lines connecting these points, stengthen them, and the hexagon is drawn in perspective.

PROBLEM.—Place in perspective 5 feet from the observer a circle whose vertical diameter is 3 feet, and 2½ feet from the base line, and whose horizontal diameter is at R. A. to the picture plane.

It is necessary to understand this problem before a box with an open cover can be drawn correctly.

If the horizontal diameter is at R. A. to the picture plane the perspective circle is in a square that is at R. A. to the picture plane. To locate this square it is necessary to first draw the ground plan.

As the circle is at a distance of 5 feet from the observer, move off on the base line 5 feet from the prime vertical, and from this point *e*, draw a line to the point of sight; in this line from *e* will be found a line parallel to the horizontal diameter, and one side of the perspective square.

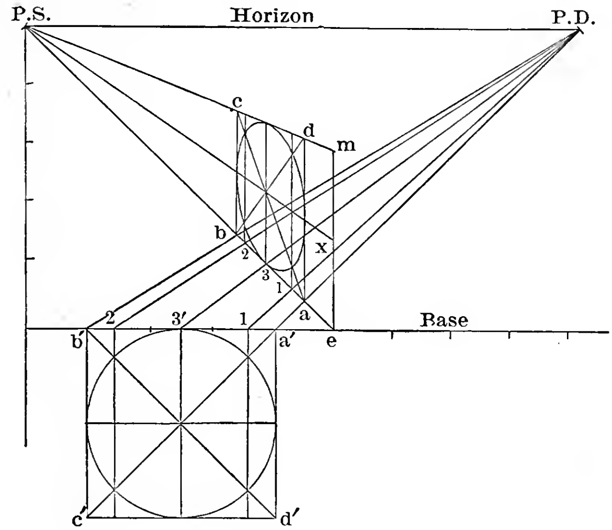


Fig. 28.

To find the perspective of the side of the square that is parallel to the horizontal diameter, erect from the point *e* a vertical line 3 feet high, and from this to the point of sight, draw a line; in this line we find the upper side of the perspective square. The horizontal diameter vanishes in the point of sight, and in the drawing is represented by a line from *x* (a point 1½ feet above *e*) to the point of sight.

The problem states that the vertical diameter is 2½ feet from the base line, on the line receding from *e*. In order to locate this point, move from *e* toward the prime vertical 2½ feet (Rule I), and fix the measurement point 3′; from this to the point of distance rule a line, and at the point where this line crosses the line receding from *e*, locate a point (3), and from this point erect a vertical line until it meets the line receding from *m*. This is the vertical diameter of the circle. Before proceeding, the ground plan must be drawn — the square, diagonals, circle, and the lines from the points where the diagonals and circle intersect. As the diameter is 3 feet, move from the point 3, 1½ feet each way, and locate *a′* and *b′* on the base line; finish the ground plan, and from *a′* and *b′*, 1 and 2, draw lines to the point of distance. Where these lines cross the line receding from *e*, points are located that correspond to points in the plan; from these points erect vertical lines until they meet the line receding from *m*, and draw diagonals. We now have the square *abcd* at R. A. to the picture plane; in this square, draw the circle through the points located by the ground plan, and strengthen the outlines.

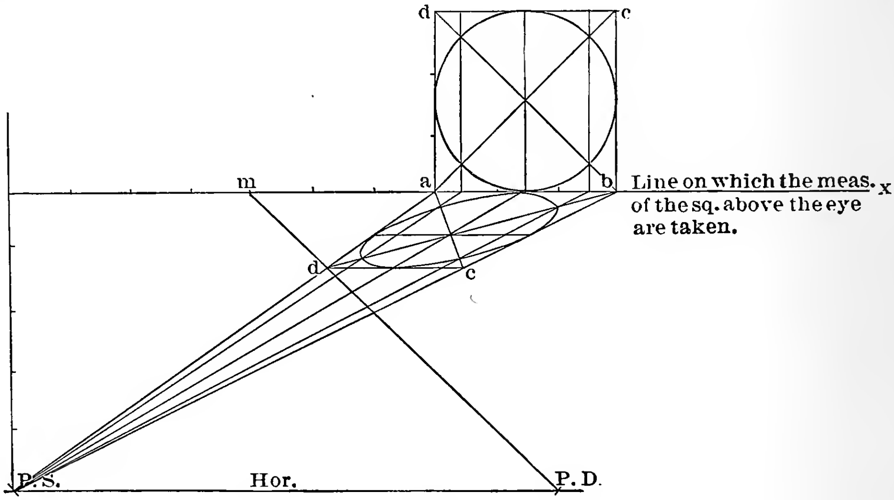


Fig. 29.

PROBLEM. — Place in perspective a circle 3 feet in diameter, 7 feet on the right, and 5 feet above the eye of the observer.

Draw the square *abcd* 7 feet on the right of the observer on the line *x*. From *a* and *b* draw lines to the point of sight (Rule IV). From *a* move off toward the prime vertical 3 feet, and draw a line to the point of distance. Locate the point *d*, and draw a line from *d* parallel to *ab*.

Locate the points in the perspective square, and draw the circle by points, located by the lines in the actual view.

CHAPTER X.

PROBLEM. — Place in perspective a pyramid 6 feet high, with a base 4 feet square; the front line of the base on the base line, the nearest corner 6 feet to the right of the observer.

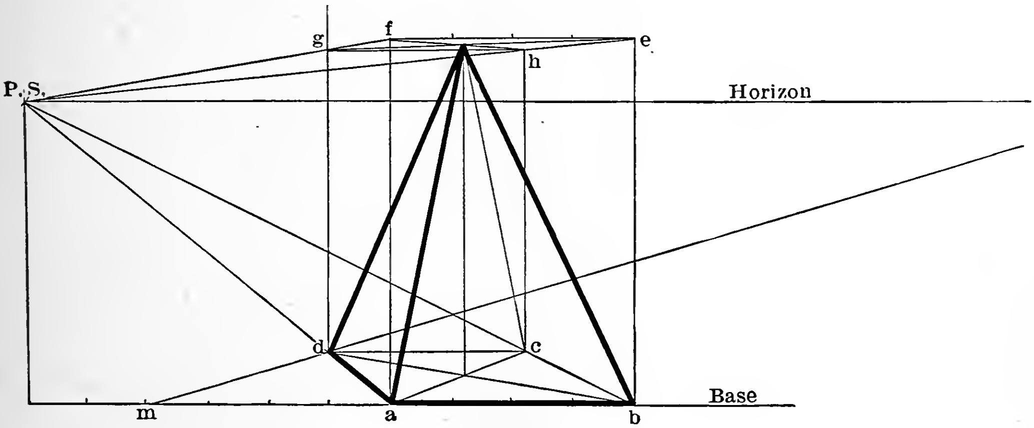


Fig. 30.

Place the base of the pyramid, the square *abcd*, in perspective, according to page 25.

The apex of the pyramid is directly over the center of the base, and its perspective height is found by erecting a vertical 6 feet from the point *b* to the point *e*.

Draw the square *efgh* above the horizon, draw diagonals, and at the point on the square where they cross, locate a point which marks the top of the pyramid.

Draw diagonals across the base *abcd*. The point where they cross marks the center of the base. A line drawn from this point to the center of the upper square marks the vertical height of the pyramid, and lines drawn from the corners *abcd* to the apex, form the sides of the pyramid.

PROBLEM.—(Figure 31.) A small two-story house in parallel perspective, situated about 45 feet on the right of the observer; and extending back from the picture plane about 28 feet. The front has three windows and a door. The windows in the upper story are 5 feet high and 2½ feet wide. The windows on the lower floor are 6 feet high and 2½ feet wide. The door is 7 feet high, and 3 feet wide. The vertical height of the roof is 5 feet.

Prepare the form according to page 15. Draw the lines representing the front of the house *abcd*. From *a* and *c*, draw lines to the point of sight; from a point 28 feet to the left of the point *a*, to a point on the horizon 60 feet from the point of sight, rule a line. Where this crosses the vanishing line *a*, fix the point *e*, and draw *ef*. From the center of *cd* erect a perpendicular 5 feet long; place the point *g*, and draw *cg* and *dg* in order to get the direction of the end of the roof parallel with *cg*. It is necessary to draw a vertical line *uv* across the horizon about 10 feet to the right of the point 2. Prolong the lines *cg* and *gd* until they meet this line; the line *cg* meets this vertical at a point not represented on this page. To this point, rule a line from *f*, and from *g* draw a line to the point of sight; this line is parallel to *fc* and *ae*, and vanishes in the same point. At the point where the line receding from *g* crosses the line from *f*, mark the point *h*—the point of the gable at the back of the house. All measurements for the windows and door must be taken on the base line, and on the line *ac*. The tops of the second story windows are 2 feet from the roof; so mark a point *m* 2 feet from *c*, and from this draw a line to *i* and another to the point of sight. As the upper windows are 5 feet high move down 5 feet to the point *n;* and draw *nj*, and *nr* to the point of sight. From *n*, move down 1 foot; draw *ok*, and *os* to the point of sight. As the lower windows are 6 feet high move down 6 feet, and from *p* draw *pt* and *os* to the point of sight. Now draw the lines representing the sides of the windows. Those in the front of the house are $2\frac{1}{2}$ feet apart, and the spaces between the sides of the front and the windows are $1\frac{1}{2}$ feet on each side. According to Rule II, draw the lines 3, 4, 5, and 6. The first window on the side is 4 feet from the corner of the house; to fix the line 7, 8, move from *a* 4 feet to the left, and rule a line to the point of distance. Where this line crosses the line receding from *a*, locate a point, and from this point erect a vertical line; this marks the right line of the windows. As the windows are $2\frac{1}{2}$ feet wide, move $2\frac{1}{2}$ feet further toward the point of sight, and locate a point; from this, rule a line to the point of distance. Where this crosses the line receding from *a*, erect a vertical; in this vertical 9, 10, we find the other side of the window. In this same manner draw the vertical sides of the remaining windows. As the door is 3 feet wide, measure three feet on the base, and draw the vertical lines representing the sides of the door. After all these general lines are drawn in, draw in the lines representing the panels of the door, and the sashes of the windows.

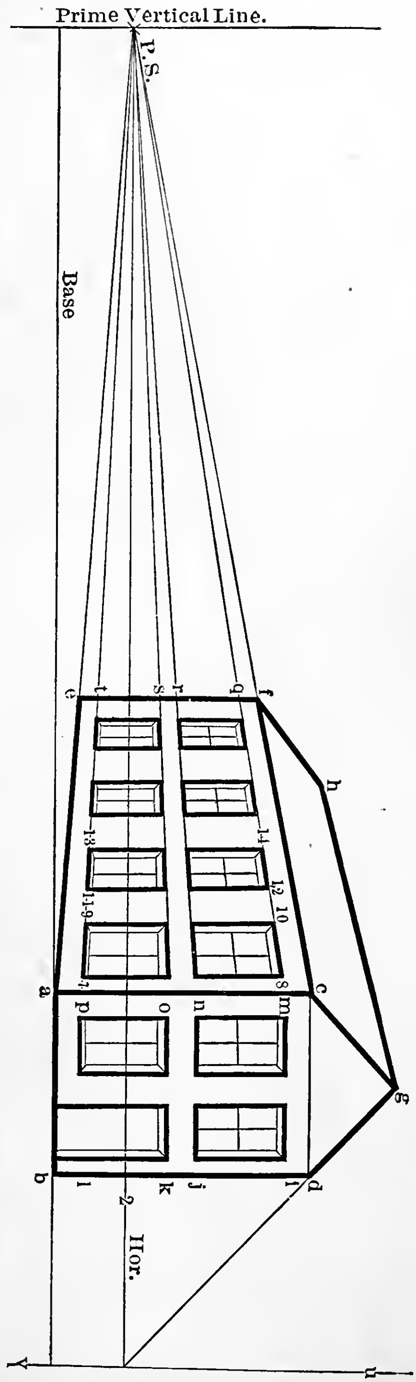


Fig. 31.

CHAPTER XI.

PROBLEM.—Place in perspective a room 14 feet wide, 15 feet deep, and 9 feet high, front line of floor being on a line with the base. (The observer is stationed at a point half way between the points *a* and *b*, and 5 feet from the base line.)

As the observer is at a point 5 feet from the base line, locate the point of distance on the horizon at a distance of 10 feet from the point of sight, having first drawn the prime vertical and base lines.

As the room is 14 feet wide, move 7 feet each way from the point where the prime vertical crosses the base, and locate the points *a* and *b*. From these points draw *af* and *be* 9 feet high to represent the side walls of the room; then draw *ef*, which represents the front line of the ceiling.

Then draw the lines from *abe* and *f* to the point of sight. To find the depth of the room, which is 15 feet, move from the point *b* 15 feet to the left, and locate the point *m*. From this, rule a line to the point of distance; where this crosses the line receding from *b*, locate the point *c*, and draw *cd* parallel to the base line. This line represents the line made by the floor and back wall of the room. From *c* and *d*, draw vertical lines to the lines receding from *e* and *f*, and where these lines meet, locate the points *g* and *h*. We now have *abcd*, representing the floor of the room, 15 ft. x 14 ft.

bche representing the *right* wall of the room, 9 ft. x 15 ft.
adgf representing the *left* wall of the room, 9 ft. x 15 ft.
cdgh representing the *back* wall of the room, 9 ft. x 14 ft.
efgh representing the ceiling of the room, 14 ft. x 15 ft.

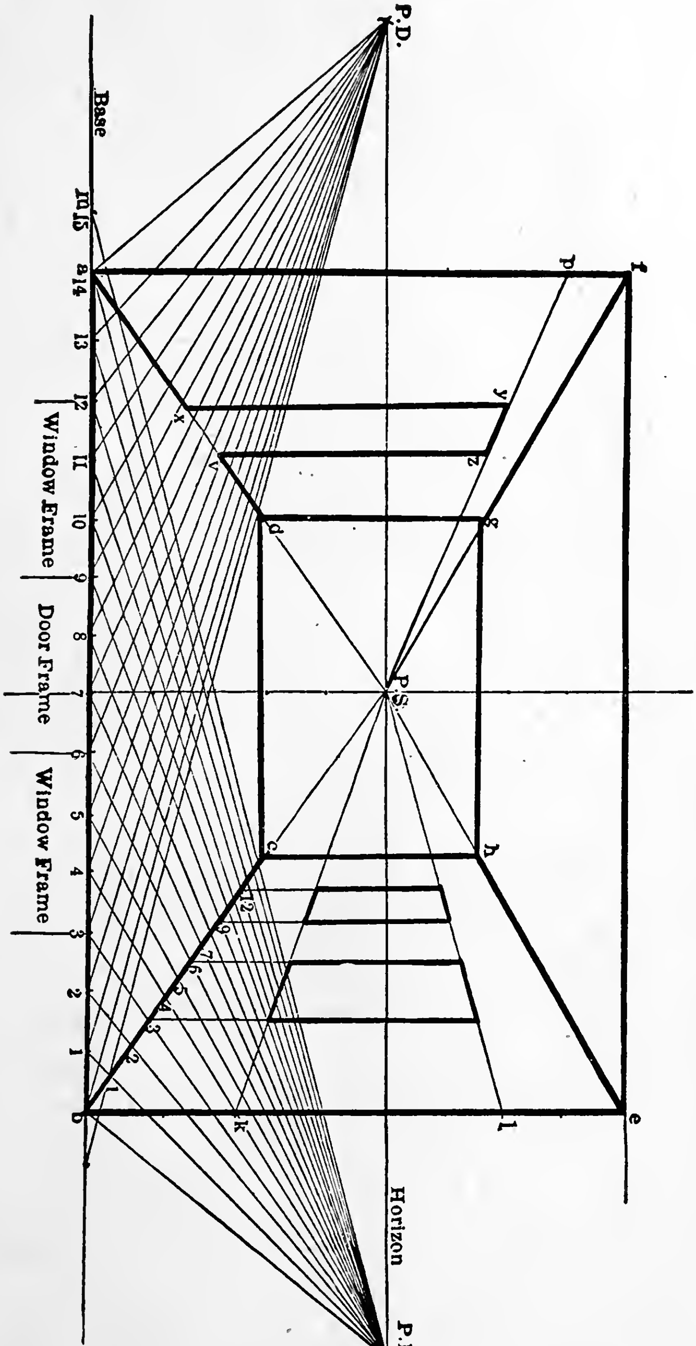


Fig. 32.

On the right side of the room are two windows, 3 ft. x 5 ft., 3 feet from the front line of the wall, and 3 feet apart. In the left side a door, 8 ft. x 3 ft., 5 feet from the nearest line of the left wall.

To find the perspective of the door and windows, it is necessary to take the measurement on the base line, and on the lines *af* and *be*. The width of the windows is found by drawing lines from each foot on the base line to the point of distance; where these lines cross *bc* they mark the line off into perspective feet. As the first window is 3 feet from the nearest line of the right wall, move 3 feet from *b*, and follow the line until it meets the line *bc;* mark this point 3, and erect a vertical line (Rule I). In the same manner, locate the point 6, leaving a space of 3 feet from the point 3—the width of the window—and erect a vertical line (Rule I).

Follow the lines 9 and 12 until they meet *cb*. Mark the points where they cross 9 and 12, and from these draw the vertical lines representing the sides of the second window.

As the windows are 5 feet high and 2 feet from the floor, move up the line *be* 2 feet, and locate the point *k;* then move 5 feet further, and locate the point *l*. From these points *k* and *l*, draw lines to the point of sight; where these lines cross the lines receding from 3, 6, 9, and 12, the points for the upper and lower lines of the windows are located. Strengthen the outlines of the windows in perspective, and proceed to draw the outlines of the door on the left side of the room

The door is 8 ft. x 3 ft., and 5 feet from the nearest wall of the room. The points *x* and *v* are found as the points 3, 6, 9, and 12 are found; the height is found by ruling a line from *p* to the point of sight; where this crosses the vertical lines from *x* and *v*, we find the top of the door. Strengthen the outlines *xvyz*, which form the perspective outlines of the door.

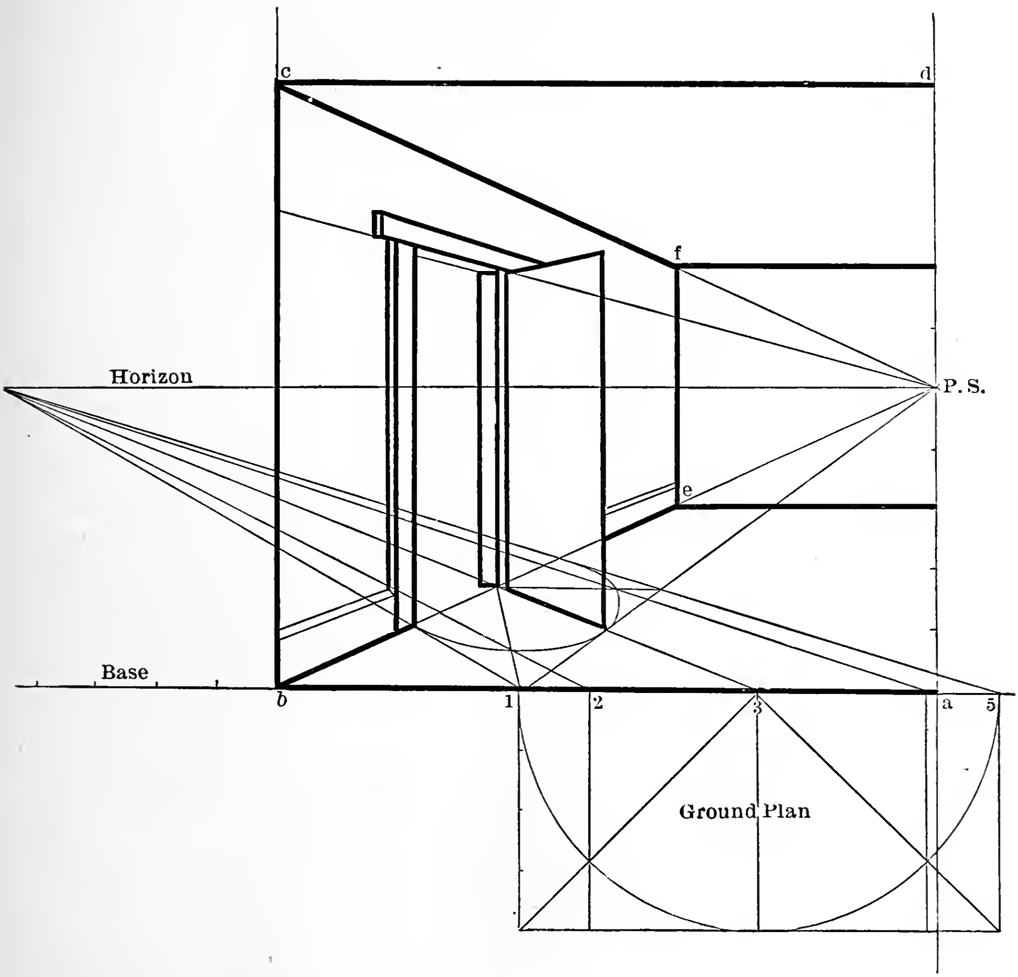
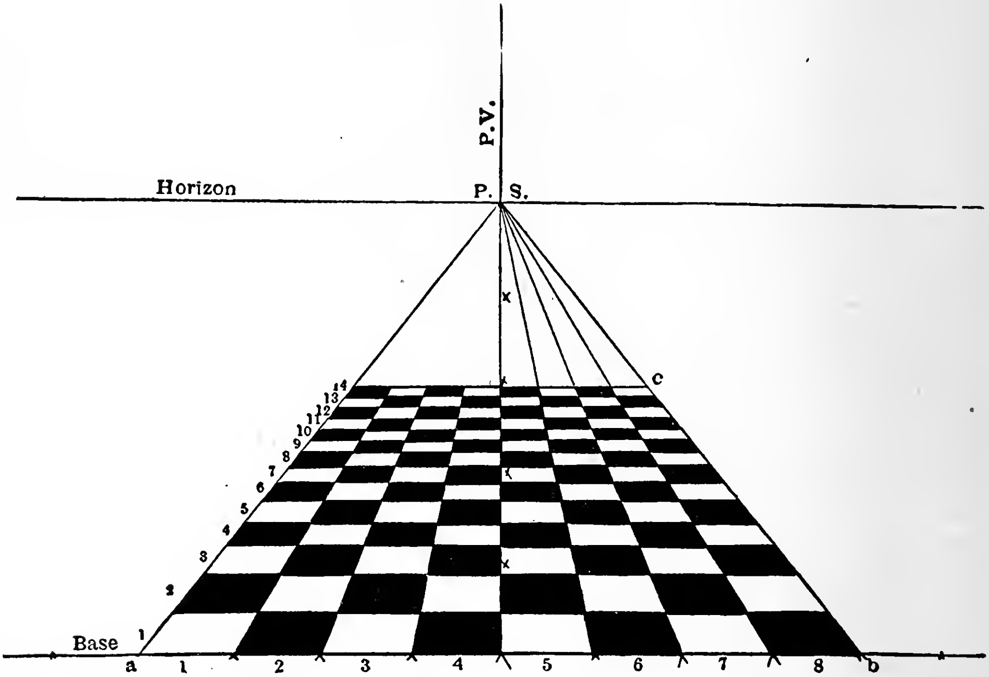


Fig. 33.

PROBLEM—(Figure 33).—A view of the left side of a room. The half-open door is 4 ft. x 8 ft. (frame 6 in.); the nearest corner 4 feet from the line *cb* (the drawing is made according to pages 17 and 28).

PROBLEM (Figure 34).—View of a tiled floor 8 ft. x 14 ft. The position of the observer is supposed to be half way between the two walls of the hall.



(Measurement point 10 feet to left of Prime V.)

Fig. 34.

In this figure, the point *c* is located by a line extending from *m* (a measurement point on the base line 14 feet from *b*) to the point of distance.

CHAPTER XII.

PROBLEM. — Find the perspective height of a 3-foot vertical line, 5 feet back from the base line, and 4 feet from the observer.

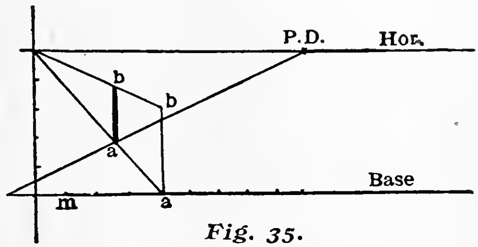


Fig. 35.

PROBLEM. — Find the perspective of a point 7 feet from the observer, and 3 feet from the base line — the point of station, 7 feet.

NOTE.—The required point is at *a*.

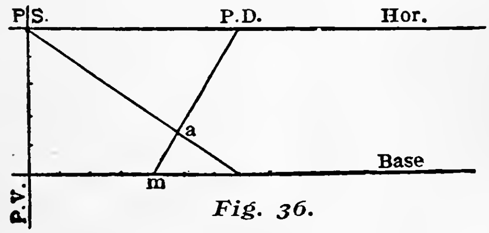


Fig. 36.

PROBLEM. — Place in perspective a horizontal line 4 feet long, at right angles with the picture plane, the nearest end 4 feet from the base line and 5 feet from the prime vertical — station point 7 feet.

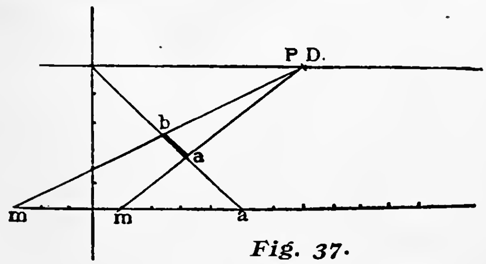


Fig. 37.

PROBLEM. — Find the center of a 4-foot square standing on the right of the observer at right angles to the picture plane at a point 3 feet from the prime vertical — point of station 8 feet.

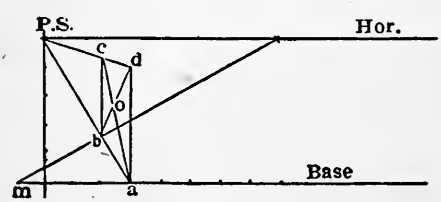


Fig. 38.

(*abcd* required square, *o* center.)

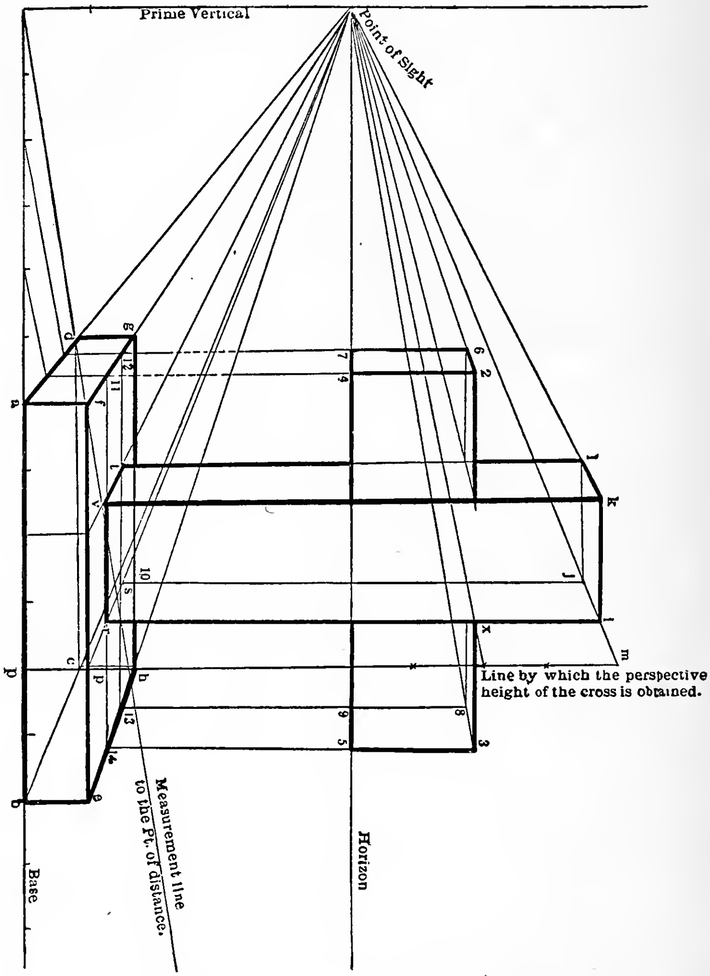


Fig. 39.

PROBLEM.—A skeleton cross 8 feet high, with a base 2 feet square. The cross piece is 2 feet below the top of the cross, and is formed by two 2-foot cubes, one on each side of the standard. The base is 6 feet square, and 1 foot high; the front line of the base rests on the base line, 6 feet to the right of the observer.

EXPLANATION OF FIGURE 39.

As the nearest corner of the base is 6 feet from the observer, move out on the base line to the right of the prime vertical 6 feet, and locate the point *a*. From this draw the line *ab* coinciding with the base line, and draw the base according to Rules I, II, III, and IV.

As the base of the standard is a 2-foot square, and is situated over the center of the base, it is necessary to divide the upper surface of the base into 2-foot squares; this is done by drawing lines to the point of sight, and to the point of distance, according to Rules III and IV.

As the standard is 8 feet high, and 2 feet back from the picture plane, its perspective height is measured by a line drawn from *p* to *m*. From *m* a line is drawn to the point of sight, and in this line we find the top of the standard (*i*) by drawing a vertical line from the point *r* until it meets the line receding from *m*.

Draw *ik* and *rv* parallel to *ab* (Rule II). From *v*, draw the vertical line *vk*, and from *k* draw a line to the point of sight (Rule III). From *t* draw *tl*. From *i* draw a line to the point of sight (Rule III), and draw *sj*.

As the cross piece is 2 feet from the top of the standard, move down from the point *m* 2 feet, and rule a line to the point of sight; where this crosses the line *ri* locate the point *x*, and through this draw a horizontal line of indefinite length.

The perspective length and width of the cross piece is determined by the base. The square of the base is divided into nine 2-foot squares, and the left cube of the cross piece is over the middle square in the left-hand row, and the right cube of the cross piece is over the middle square in the right-hand row.

To determine the length of the line from x, draw lines from 11 and 12 (middle square in left-hand row), and from 13 and 14 (middle square in right-hand row), to the line from x, and locate the points 2, 3, 4, and 5.

From 2 and 3, draw lines to the point of sight, and where these cross the lines from 12 and 13, locate the points 6 and 8.

As the cross pieces are 2-foot cubes, the length of 3, 5 is obtained by moving 4 feet from the point m, and drawing a line to the point of sight. As this line just drawn coincides with the horizon, the points 4, 5, 7, and 9 are on the horizon, and are determined by the lines from 11, 12, 13, and 14.

CHAPTER XIII.

PROBLEM. — Place in perspective a box 5 feet long, 3 feet wide, and 3 feet high, 7 feet to the right of the prime vertical. The cover of the box one-quarter open. (See page 31.)

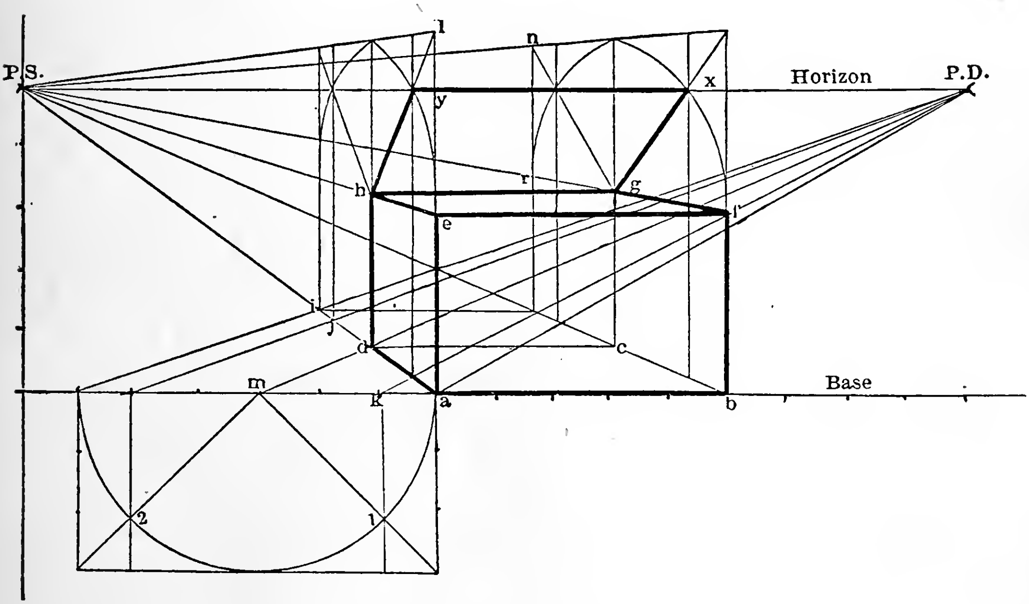


Fig. 40.

EXPLANATION OF FIGURE 40.

Draw the box according to rules governing the lines of objects in parallel perspective.

(A box-lid in opening all the way describes a half circle; in opening half way, one-fourth circle, and in opening one-quarter way, it describes one-eighth of a circle. Three-fourths open is defined by three-eighths of a circle).

In order to represent a lid open one-fourth of the way, it is necessary to construct a half circle in perspective. This is done by constructing a half circle below the base line extending from the point *a* to the left, a distance of 6 feet. (The box-lid is three feet wide, and represents the radius of the half circle.)

Draw the half circle, and from it locate the point for the perspective half circle; from *f* to *x*, and from *e* to *y*, represents one-eighth of a circle. If the points *x* and *y* are connected, and lines drawn from these points to *g* and *h*, we have the cover of the box one-quarter open.

CHAPTER XIV.

ANGULAR PERSPECTIVE.

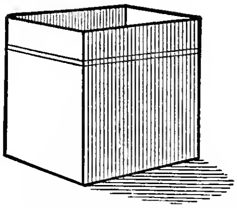


Fig. 41.

IN taking an angular view, we do not have a full view of any of the surfaces. The faces that are at angles to the picture plane recede from the spectator; sometimes at equal angles, and sometimes at unequal angles. The lines bounding the surface that is least foreshortened, vanish in a point further from the object than those of the surface that has a more decided foreshortening. When the two surfaces form equal angles with the picture plane, the vanishing points for the lines that bound these faces are equally distant from the point of sight; but as the object is turned, and its position with regard to the picture plane changed, the position of the vanishing points, with regard to the point of sight, changes. The surface that forms the greater angle with the picture plane has a more violent perspective, and the lines come to a point on the horizon nearer the point of sight, than those lines of the surface that makes a smaller angle with the picture plane. All objects in parallel perspective are at R. A. to the picture plane, or they stand at an angle of 90°. In angular perspective they are placed at any angle from 90° to 1°.

All those surfaces that are at an angle of 45° to the picture plane vanish in the point of distance. If the line vanishes from right to left, it vanishes in the point of distance on the left. If it recedes from left to right, it vanishes in the point of distance on the right. If an angle of *more* than 45° is formed by the surface and picture plane, the vanishing point is *between* the point of distance and point of sight. If an angle *of less* than 45° is formed by the object and picture plane, then the vanishing point is *beyond* the point of distance.

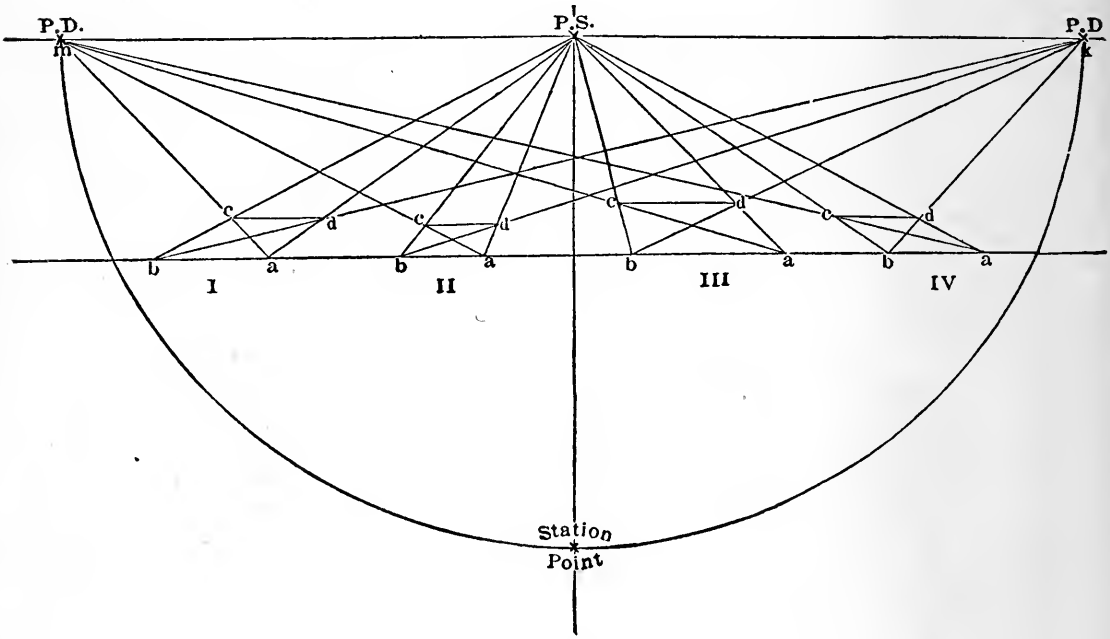


Fig. 42.

The squares *abcd* (Fig. 42) are in parallel perspective; the lines *ad* and *bc* recede from the picture plane at angles of 90°, and vanish in point of sight. (Each square has four right angles, and each R. A. measures 90°. By actual test, the student can satisfy himself that the diagonal of a square divides the angles into equal parts, each part measuring 45°.) If diagonals are drawn across these squares, they meet in the points of distance. In Figure 42, the angle *bad* measures 90°, and the receding side *ad* forms an angle of 90° with the picture plane, and vanishes in the point of sight. The diagonal *ac* divides the angle *bad* into two equal parts (this forms two angles of 45°), and vanishes in the point of distance.

Before a perspective drawing in angular perspective is attempted, it is well to have the following form understood.

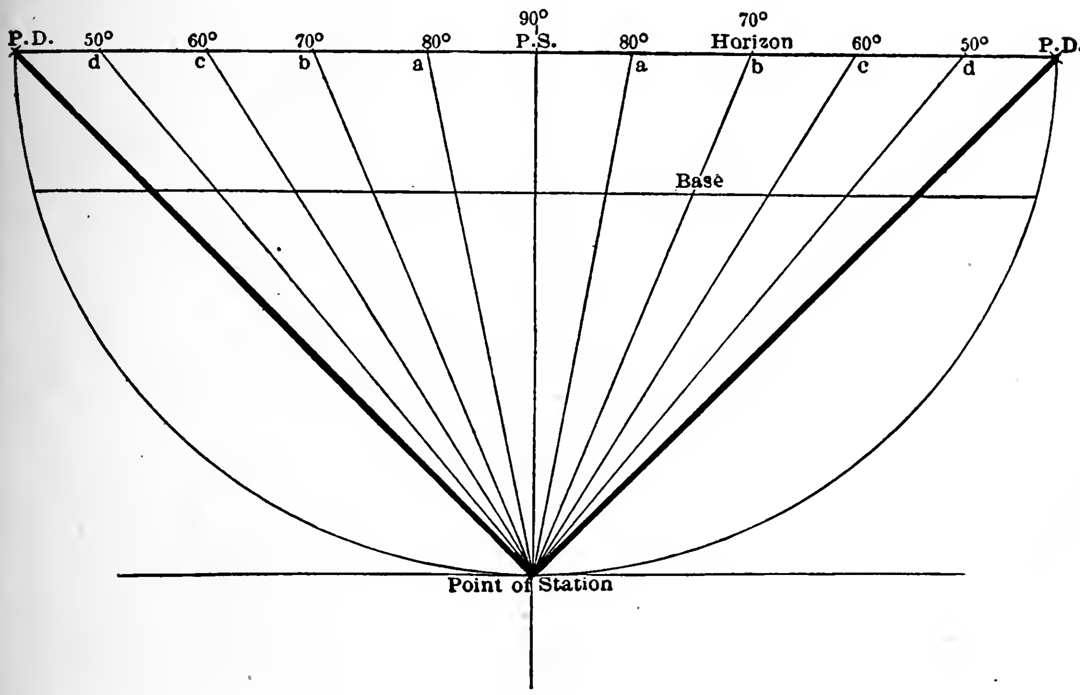


Fig. 43.

The points *abcd*, etc., on the horizon are the points to which lines that recede from the picture plane, at 80°, 70°, 60°, 50°, etc., vanish. If an object makes an angle of 80° with the picture plane, the vanishing point for this, and all receding lines parallel to this, will be found on the horizon at the point *a*; if the face forms an angle with the picture plane of 70°, 60°, 50°, the vanishing point for these lines will be on the horizon at the points 70°, 60°, 50°; at an angle of 45° in the point of distance, any angle formed by the receding lines less than 45° is beyond the point of distance on the horizon. These points in the horizon are found by constructing angles at the point of station. These angles are found in the following manner:

Through the point of station rule a line (parallel to the base line) forming two right angles or 180°. This space is divided into angles by the protractor, which is placed on the horizontal line, extending through the station point, with the point on the protractor marked 90° on the prime vertical; the points representing the angles 80°, 70°, 60°, etc., are marked on the paper by means of dots; then through these dots, from the point of station, lines are ruled to the horizon, where they locate the points, forming angles on the horizon equal to those at the point of station.

NOTE.—The angles at the station point are called the actual angles; the angles at the horizon are called perspective angles.

CHAPTER XV.

RULES GOVERNING THE DIRECTION OF LINES AND SURFACES OF OBJECTS IN ANGULAR PERSPECTIVE.

RULE I. All lines bounding surfaces that are at an angle of 45° to the picture, vanish in the *point of distance.*

RULE II. All lines bounding surfaces that make an angle with the picture plane *greater than 45°*, vanish *between* the *point of distance* and the *point of sight;* the greater the angle at the picture plane, the nearer the vanishing point to the point of sight.

RULE III. All lines bounding surfaces that make an angle of *less* than 45° with the picture plane, vanish in a point on the horizon *beyond* the *point of distance.* The less the angle at the picture plane, the further the vanishing point is from the point of distance.

RULE IV. All measurement points are located on the base line by means of the ground plan.

RULE V. All measurement lines vanish in the vanishing points.

RULE VI. The vanishing points in the horizon are found by means of angles at the station point. These angles are drawn, and points ascertained in the following manner:

Through the station point, rule a line parallel to the base line; place the protractor on this line, with the point of the protractor marked 90°, on the prime vertical; then mark the required angle on the paper, and from the station point draw a line through this dot to the horizon. This point on the horizon is a vanishing point for all lines that recede from the picture plane at the same angle as that which was drawn at the station point.

If an angle of 20° is measured, and a line drawn to the horizon, the point on the horizon marked by this line is the vanishing point for all lines receding at an angle of 20°.

CHAPTER XVI.

PROBLEM.—Place in perspective a 3-foot square, 5 feet to the right of the observer, the corner resting on the base line, and the nearest side receding from the picture plane at an angle of 45°.

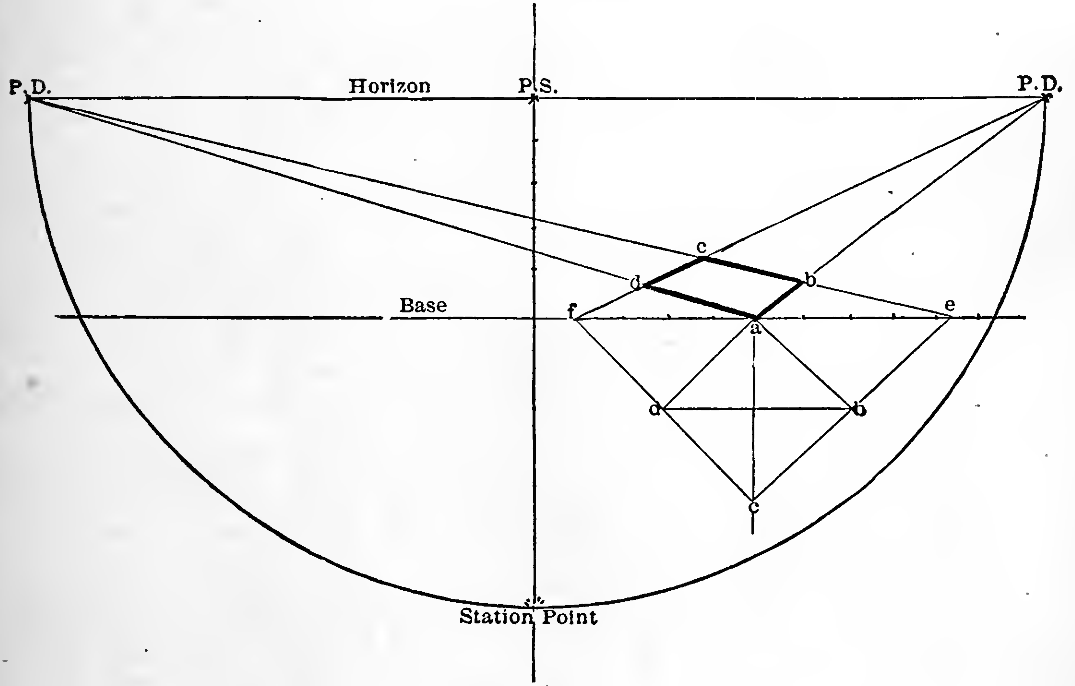


Fig. 44.

As the nearest corner touches the base line 5 feet to the right of the observer, move 5 feet to the right of the prime vertical and locate the point *a*. After locating this point, proceed to draw the ground plan in the following manner:

Through the point *a*, draw a vertical line of indefinite length, and at the base line construct a right angle with the protractor by placing the point marked 90° on the vertical, and the horizontal edge on the base line.

After the angle is constructed, draw the square in the ground plan; from *a*, draw lines to the points of distance. (Rule I.) To locate a point 3 feet from *a* on the right, and one the same distance toward the left on the receding lines, place the measurement points

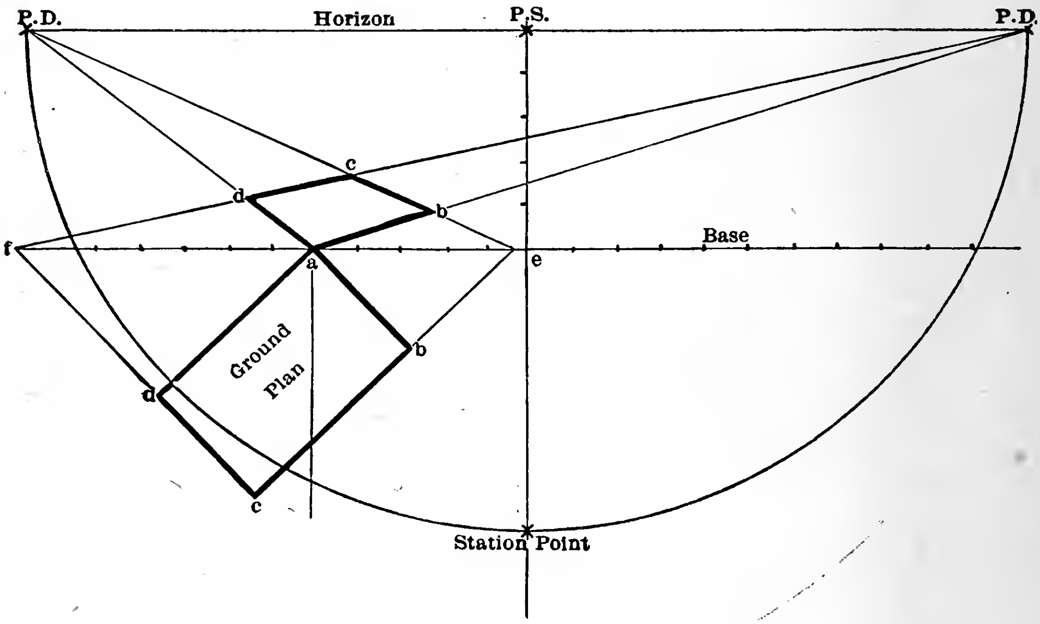


Fig. 45.

e and *f* located by means of the ground plan. (Rule IV.) From these draw lines to the points of distance; where these cross the lines from *a*, mark the points *d* and *b*, and where the lines from *e* and *f* to the points of distance intersect, mark the point *c*, and strengthen the outlines of the square *abcd*, at an angle of 45°.

PROBLEM.—Place in perspective a rectangle, 5 feet by 3 feet; the nearest corner 5 feet to the left of the prime vertical, at an angle of 45°; the longest diameter of the box extending from *a* toward the point of distance on the left.

Locate the point *a*, draw the lines to the points of distance, and draw the rectangle *abcd*, *geometrically*, with the longest diameter extending from *a* toward the left.

Prolong *cd* to the base line, and prolong *cb* to the point *e*. These

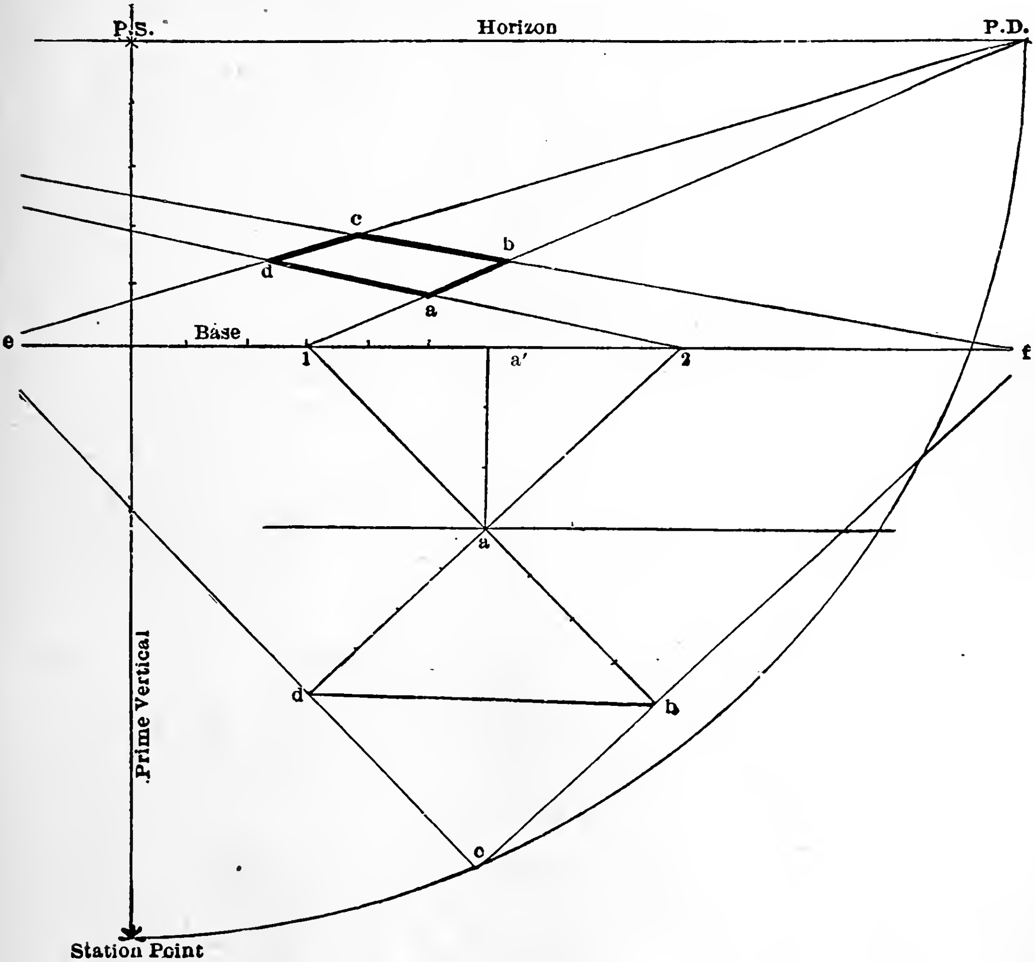


Fig. 46.

points locate the measurement points for the lines receding from *a*. From *e* and *f*, rule lines to the points of distance, and where these cross the lines receding from *a*, fix the points *b* and *d*. Strengthen the outlines of the rectangle *abcd* in perspective at an angle of 45°.

PROBLEM. — Place in perspective a 4-foot square, 6 feet to the right of the observer, and 3 feet back from the base line, at an angle of 45°.

Locate the point *a'*. In this case the square is 3 feet from the base line, so it is necessary to move from the point *a'* 3 feet down in a vertical direction, and fix the point *a*. Through this rule a line parallel to the base line; place the protractor on this line, and draw the right angle at *a* and complete the 4-foot square. Find the perspective of the point *a* in the following manner: locate the measurement points 1 and 2; prolong the lines *da* and *ba* until they meet the base line; from 1 and 2 rule lines to the points of distance; where these measurement lines cross, we have the perspective of the point *a*; from *a* rule the vanishing lines to the points of distance; prolong *cd* and *cb* until they meet the base line at *e* and *f* (measurement points). From the points *e* and *f*, draw measurement lines to the points of distance; where these lines cross the lines receding from *a*, mark the points *b* and *d*; from these points, rule lines to the points of distance; where they cross, mark the point *c*. Strengthen the outlines of the square *abcd*.

CHAPTER XVII.

PROBLEM.—Place in perspective, at an angle of 45° to the picture plane, a rectangular box, 10 feet long, 4 feet wide, and 3 feet high, 7 feet to the right of the observer. The longest diameter of the box is from the point *a* toward the point of distance on the right.

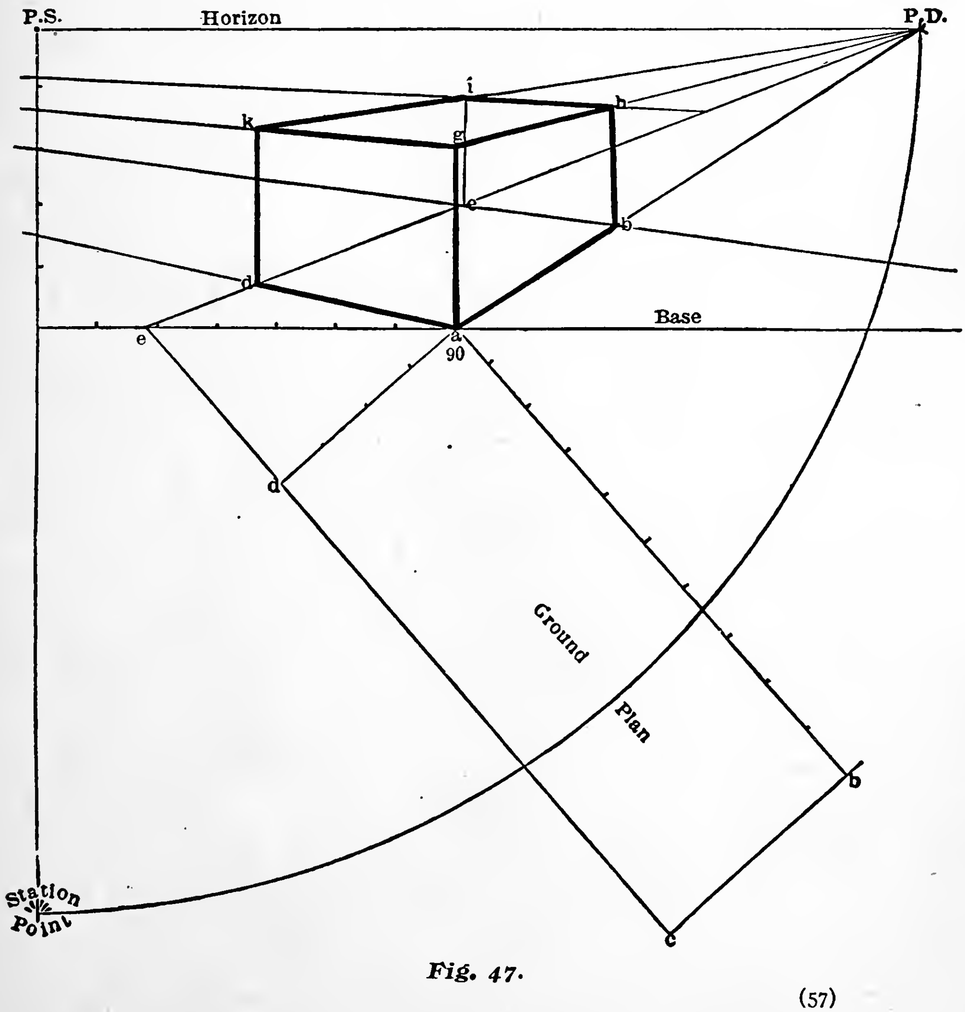


Fig. 47.

Locate the point *a*, draw the plan *abcd* geometrically, and from *a* to the points of distance, draw lines. (Rule I.) Prolong *cd* and *cb* to the base line, and where they touch, locate the points of measurement *e* and *f*. From these points draw lines to the points of distance, crossing the lines receding from *a* at the points *b* and *d*, and mark the point *e*, which completes the base. From *a* erect a vertical line 4 feet, and mark the upper point *g*; from *g* rule lines to the points of distance. (Rule I.) Erect vertical lines from *b* and *d*, and where these vertical lines touch the lines vanishing from *g*, mark *h* and *k*. From these points rule lines to the points of distance. Strengthen the outline of the perspective drawing of the box at an angle of 45°.

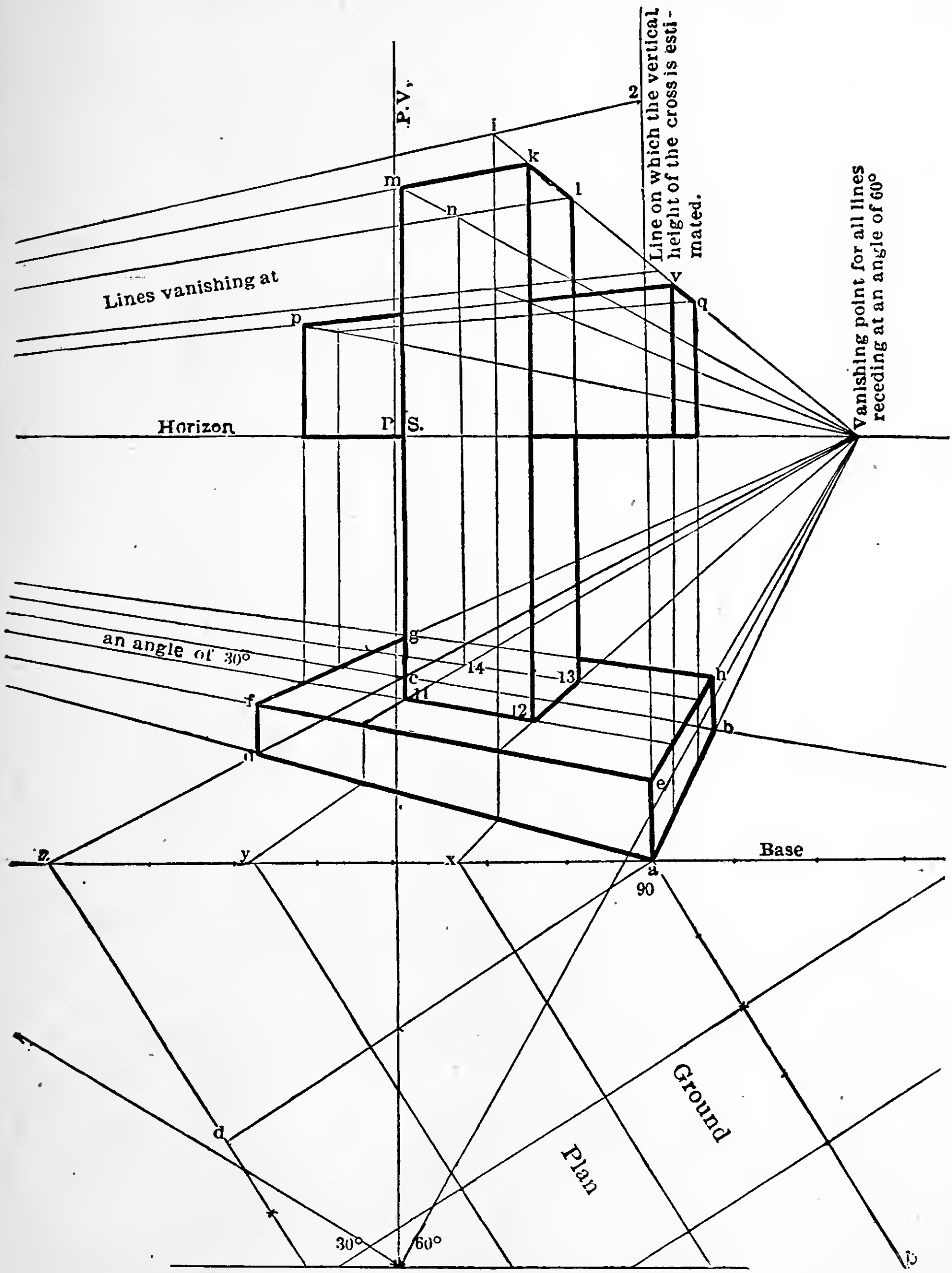


Fig. 48.

PROBLEM.—In this illustration (Fig. 48), the view is supposed to be taken from a point 3 feet to the left of the point where the cross touches the base line. The side near the observer recedes at an angle of 30°; dimensions same as in Fig. 39.

PROBLEM.—View of a tiled floor, 8 ft. x 14 feet. Viewed at an angle of 35°.

Locate the point *a*, and construct the ground plan *abcd* at an angle of 35° to the base line.

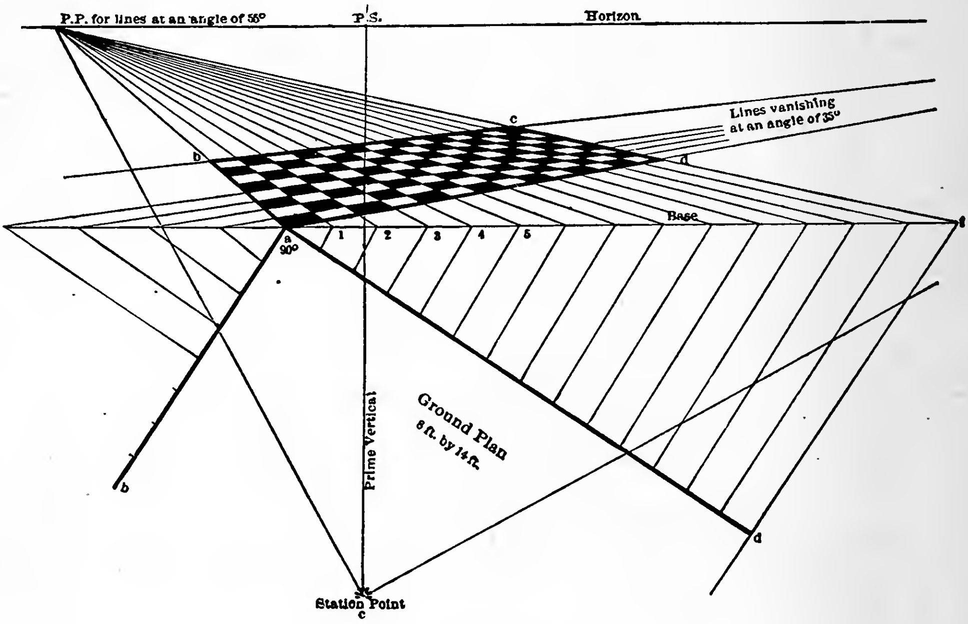


Fig. 49.

At the point of station, construct the angles of 55° and 35°; from these angles draw lines to the horizon, and locate the vanishing points for the receding lines.

Draw the lines from *b* and *d* of the plan until they meet the base, and mark these measurement points *e* and *f*. From these points, rule lines to the points of distance, and where they cross locate the point *c*.

The tiles in the floor are 1 foot square; to divide the perspective floor into these tiles, lines are ruled from each foot in the line *ad* of the plan, to the base line—to the points 1, 2, 3, etc. From these points, lines are ruled to the vanishing point on the left.

From each foot on the line *ab*, lines are ruled to the base, and from these measurement points lines are ruled to the vanishing point on the right. The lines vanishing to the left, and those vanishing to the right cross and divide the floor off into 1-foot tiles, which may be tinted according to the taste.

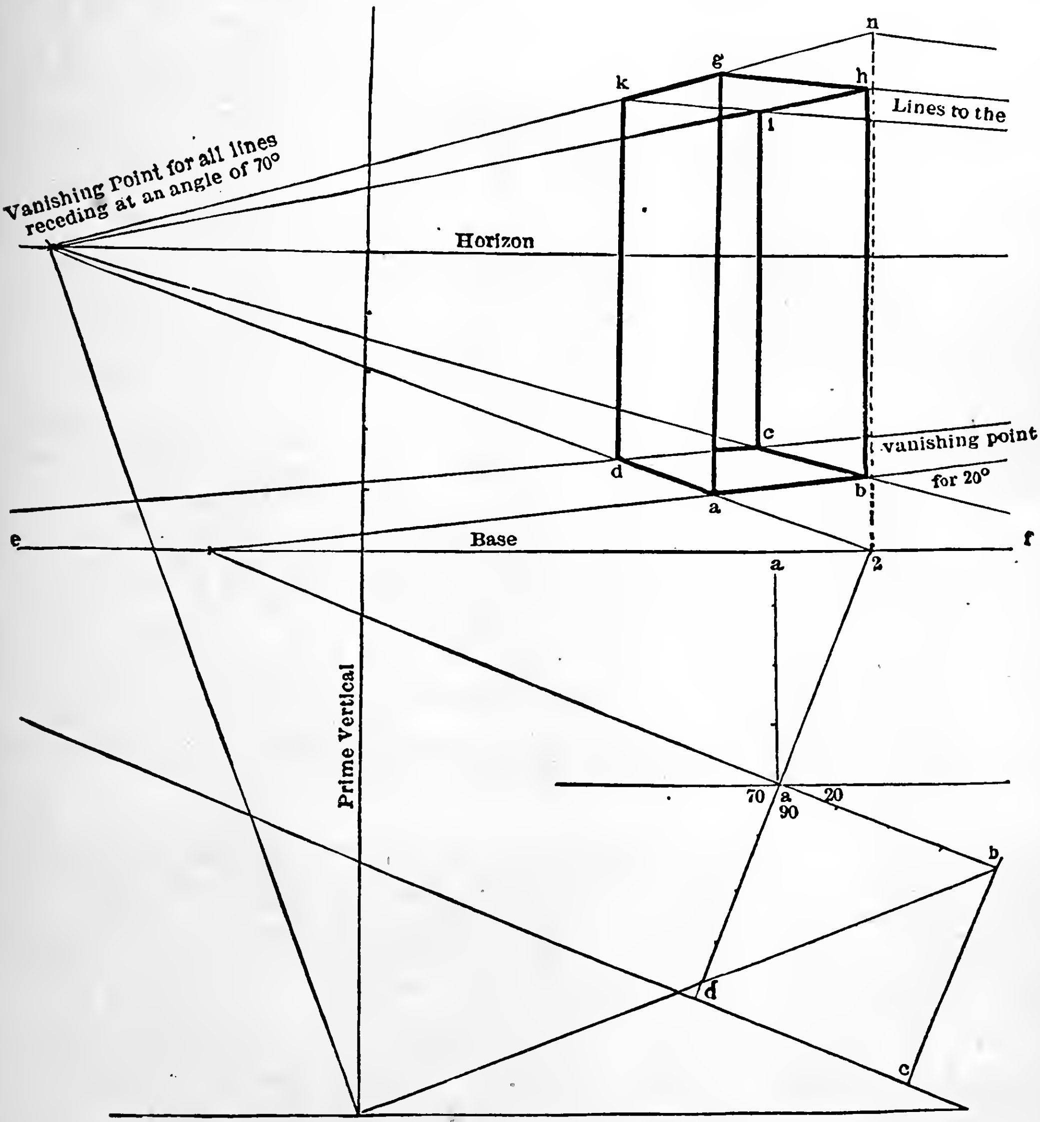


Fig. 50.

NOTE.—2 *n*, line by which the perspective height is determined. Explain Fig. 50.

CHAPTER XVIII.

SECOND METHOD FOR PLACING AN OBJECT IN ANGULAR PERSPECTIVE WITHOUT A GROUND PLAN, BY MEANS OF MEASUREMENT POINTS ON THE HORIZON.

RULE I. Construct the angles at the station point, and by these locate the vanishing points on the horizon. Determine the measurement points on the horizon by means of the points of distance, and from the base line to these points, draw lines to determine the width of the object.

RULE II. To find the measurement point on the *right* of the point of sight, move out on the horizon as far from the point of distance on the *left* as the distance from the *point of distance* to the *station point*.

RULE III. To find the measurement point on the *left* of the point of sight, move out on the horizon from the point of distance on the *right* as far as it is from the *point of distance* to the *station point*.

PROBLEM.—Place in perspective a 3-foot cube, touching the ground line at 5 feet to the right of the observer, the sides receding at an angle of 45°.

Draw the form as given on page 16; then, according to Rule I, draw the line *bc* through the station point *a*, and mark off at the point of station, an angle equal to the angle at which the side of the object recedes from the picture plane; and where this line cuts the horizon is the vanishing point for the faces of the cube that are at

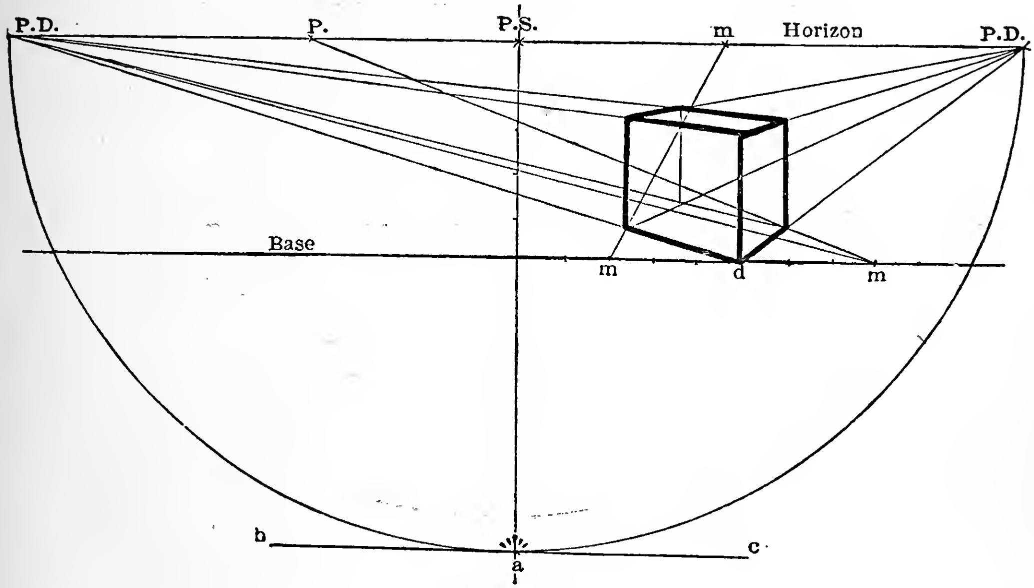


Fig. 51.

an angle of 45° to the picture plane. As the corner touches 5 feet to the right of the observer, move 5 feet to the right of the prime vertical, and mark the point *d;* as the two sides recede at an angle of 45° to the picture plane, draw lines from *d* to the points of distance.

The next step is to determine the length of these lines. As they are receding lines, we measure them on the base line (Rule I). Before the length of these lines can be determined, measurement points must be found, to which the measurement lines are drawn. To find these points, move out on the horizon from the point of distance on the left, as far as it is from point of distance to *a* (point of station), and mark the point *m;* and from the point of distance on the right, move out on the horizon as far as it is from the point of distance to *a*, and mark the point *p*, which is the measurement point for the line receding from *a*.

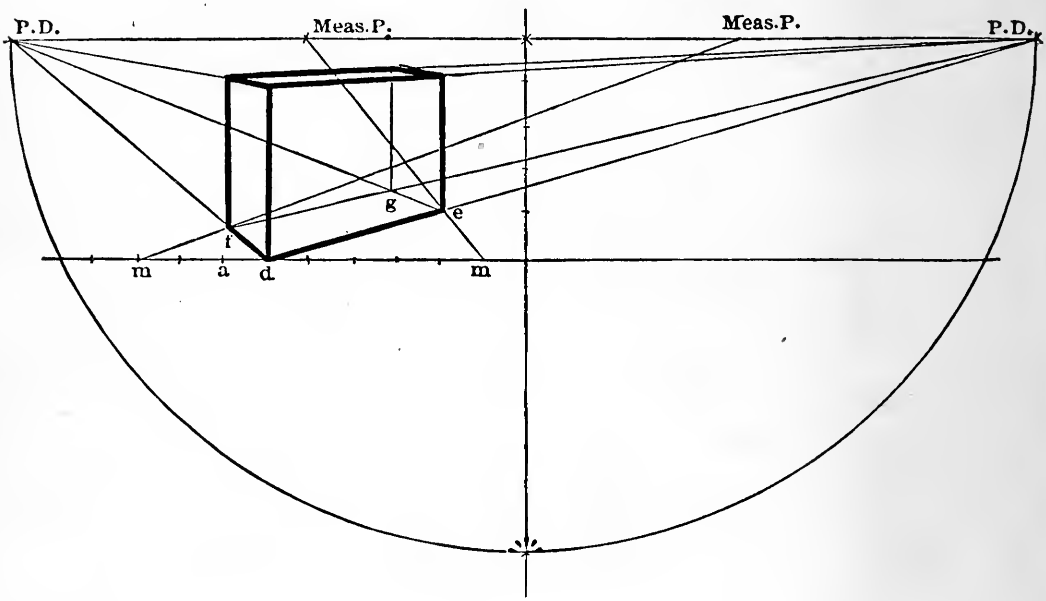


Fig. 52.

Give an explanation of Figure 52.

CHAPTER XIX.

AN EASY METHOD FOR DRAWING A PERSPECTIVE WHEN THE PLAN IS GIVEN (FIGURE 53, PAGE 67).

1. LOCATE on the sheet of paper a vertical line (the Prime Vertical — P. V.).

2. Draw the plan, *abcd*, either in a proper position on the paper, at the desired angle to the P. V., so that the nearest corner or angle of the plan comes on the P. V. Or, draw the plan on a separate piece of paper, and adjust same on the P. V. to the best or most desirable angle for viewing the most prominent features of the structure.

3. Locate the station point on the P. V., at a suitable distance from the nearest point of the plan (about four times the greatest dimension).

4. Locate a horizontal line (picture plane) on which all the measurements of the plan are projected by ruling lines to the station point from the features on the plan.

NOTE.—This line *ef* determines the size of the perspective, and is to be located at any point between the nearest point of the plan (*a*) and the station point, according to the judgment of the draughtsman. The nearer to the station point, the smaller the perspective.

5. Locate the base line.

6. Locate the horizon.

7. Locate the vanishing points (V. P.) by projecting lines from the station point, parallel to *ab* and *ac*, until they intersect the picture plane at *o* and *m*. Project vertical lines from these points of intersection, until they intersect the horizon. These latter intersections will form the vanishing points.

8. At the left edge of the paper, draw the vertical measurement line (for vertical heights), projecting, some above and below the horizon indefinitely. (The prime vertical may be used to serve the purpose of this measurement line.)

9. From *b* and *c* draw lines to the station point, intersecting the picture plane. Project these intersections, *e* and *f*, by vertical lines until they cross the horizon, prolonging indefinitely. (These lines establish the limits of the perspective.)

10. Lay off on the measurement line the different vertical heights of the structure, with reference to their relative positions to the horizon.

11. Project (horizontally) the points indicating the vertical heights over to the prime vertical.

12. From these points draw straight lines toward the vanishing points, outlining the foreshortened elevations.

13. Locate all doors, windows, and other features of the plan, on the picture plane, as described in 9.

14. Project, by vertical lines, to their relative positions in the perspective outline of the structure.

15. Establish heights of the above features on the measurement line; project to the prime vertical, and vanish toward vanishing points until these lines intersect the vertical (14).

16. Proceed according to previous directions until the picture is completed.

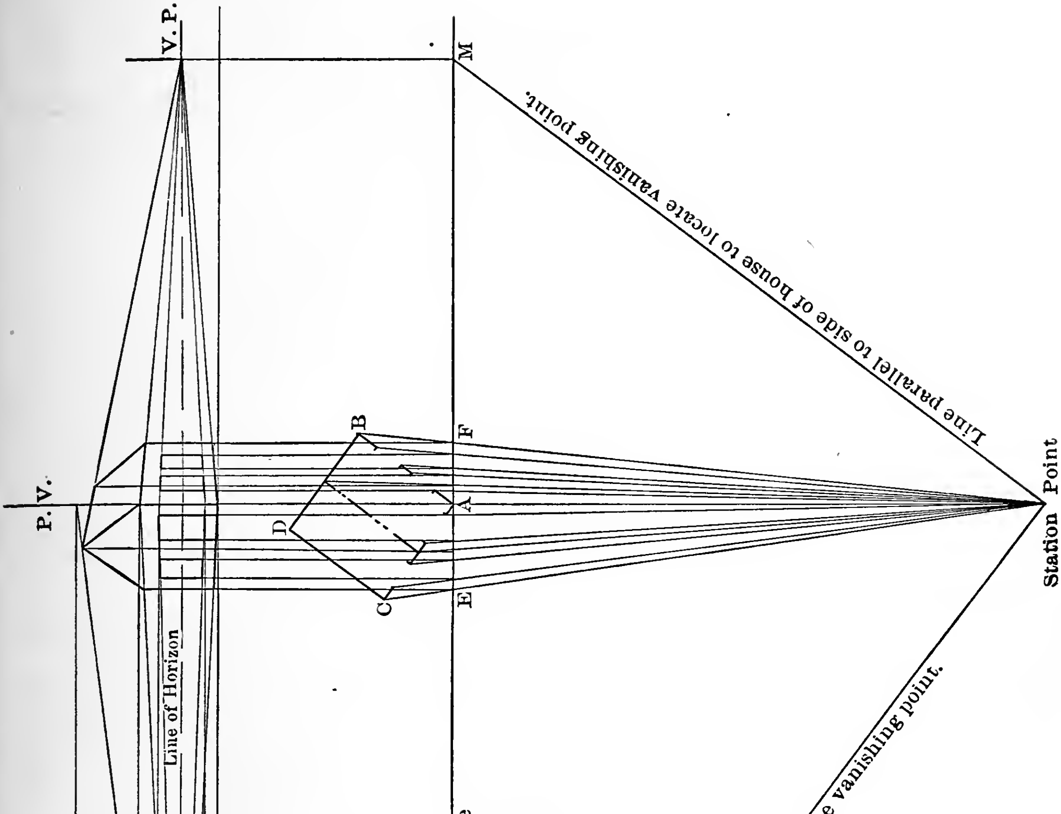

V. P.
M
Line parallel to side of house to locate vanishing point.
B
F
P. V.
D
A
Station Point
C
E
Line of Horizon
Base Line
Picture Plane
Line parallel to front of house to locate vanishing point.
V. P.
O
Measurement line for vertical heights.

Fig. 53.

CHAPTER XX.

ISOMETRIC PROJECTION.

Isometric projection differs from perspective and orthographic projection, inasmuch as it shows the view of the entire object, and all the lines in the drawing may be measured by a uniform scale. It is called the perspective of the workshop. This style of representing objects was first used by Professor Farish, of Cambridge, in 1820.

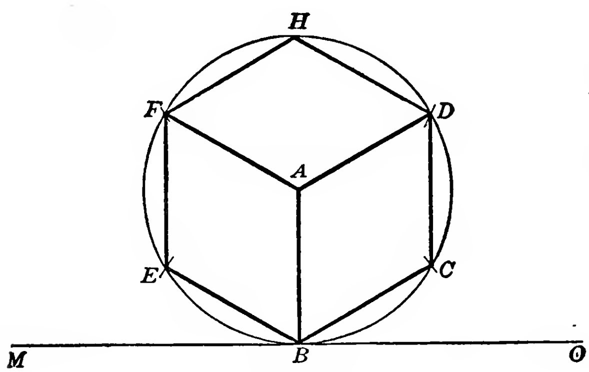


Fig. 54.

In perspective drawings objects diminish in size as they appear more distant, according to laws of optics, and it is difficult to measure their sizes. In orthographic projection two drawings are required, and the lengths of the lines are altered according to the angle at which the object may be placed. The whole system of isometric projection — meaning projections with equal measurements — is based on a cube so situated with relation to the horizontal plane that its projection on the vertical plane will be a hexagon *bcdhfe* (Fig. 54). The three visible faces of the cube are equal in the representation. The angles are not right angles, as in the actual cube, but are acute and oblique — two acute angles of 60°, and two oblique angles of 120°.

The line *bc* leaves the horizontal line *mo* at an angle of 30°, making the representation of the right angle an acute angle, *abc*, measuring 60°. The lengths of the lines are established by a scale. Vertical lines are represented by vertical lines. The angle at *a* measures 120°. The line *ad*, and all other lines of the object parallel to *bc*, are made parallel to *bc* in the representation. The faces *abef* and *adhf* are drawn in the same manner as the face *abcd*.

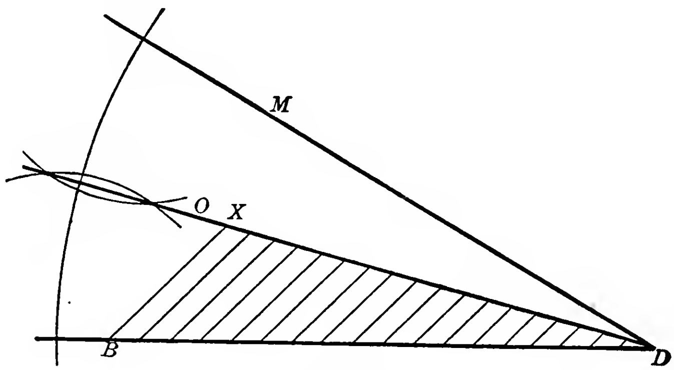


Fig. 55.

An isometric drawing unites plan, elevation, and projected view, in one.

To construct an isometric scale so that the object to be drawn may be one-twelfth of the real size, proceed as follows: — as the scale is one inch to the foot, and as an inch is one-twelfth of a foot, each of the twelfths will represent an inch.

Draw the line *bd* (Fig. 55) an inch and a half long, representing the *real* length of an object one foot and a half. Mark on *bd* the twelfths of inches which are to represent inches on the scale.

Make the angle *bdm* 30°, with the set square, and bisect it. This gives the angle *bdo*, 15°. From the point *b* draw an angle of 45°. From each point marked off on the line draw lines parallel to the line *bx*. The divisions on *dx* will represent inches, and the line *dx* is an isometric scale of $\frac{1}{12}$. Or, instead of making a scale, lay off on the lines which will represent the figure when completed, in isometric projection, the exact measurements. Thus, to draw a cube of 4′ 0″ in isometric projection, lay off the line *ab* (Fig. 54) vertical, 4′ 0″ long. From the point *b* project the lines *bc* and *be* at angles of 30° each with the base line, measuring 4′ 0″ along the lines to points *c* and *e*. At these points establish verticals 4′ 0″ long. Connect the point *a* with *f* and *d* by lines parallel to *be* and *bc*. From *d* and *f* project lines parallel to *af* and *ad*, until they meet in *h*. Represent the bottom of the cube by dotted lines from *e* and *c*, parallel to *fh* and *dh* respectively.

CHAPTER XXI.

OBLIQUE PERSPECTIVE.

A LINE is in *oblique perspective* when it is not parallel to either the ground plane or the picture plane—when it is neither horizontal nor vertical, but slanting. These lines occur in the steeples of churches, gables of houses, uneven roads, covers of open boxes, books, etc. These oblique lines do not vanish on the horizon, as do the vanishing lines in parallel and angular perspective, but on a line perpendicular to the horizon.

A line lying flat on the ground plane at the right or left of the observer, and at right angles to him, vanishes in the point of sight. If this line is inclined, that is, if one end is raised from the ground plane, the perspective inclination will not tend to the point of sight, but to a point above it in prolongation of the prime vertical (Fig. 57). The length of this oblique line is determined by a point on the prime vertical below the point of sight.

A line lying flat on the surface at an angle of 45° to the picture plane, vanishes in the point of distance. If this line is inclined, instead of vanishing in the point of distance, it will vanish in a point above the point of distance on a line perpendicular to the horizon at that point; and the oblique line will be measured by a point on this vertical line, prolonged below the point of distance (Fig. 56).

If a line is at an angle of 60° to the picture plane, it vanishes in a point on the horizon between the point of sight and the point of distance. If the end is elevated, the line will not vanish in this vanishing point, but in a vertical line drawn through the vanishing point for 60°, and the length will be determined by a point on the same line below the vanishing point on the horizon.

The vanishing point for an oblique line is always in a vertical above or below the point it would have vanished in, had it occupied a level position; and the measurement point for the oblique line is in the *same vertical* line.

In parallel perspective all lines that vanish in the point of sight are measured by lines that vanish in the points of distance, and these points are measurement points for all lines that are foreshortened in parallel perspective.

If the vanishing point for an oblique line is in the prime vertical above the point of sight, it is found by means of an angle at the point of distance (the measuring point for all lines that vanish at the intersection of the prime vertical and horizon), the side of which is prolonged until it meets the prime vertical, and locates the vanishing point for the oblique line.

The length of the oblique line is determined by a measurement point on the prime vertical as far from the vanishing point for the oblique line, as the distance from this vanishing point to the point of distance.

The point *P. P.* is a vanishing point for an oblique line; the length of this oblique line is determined by the measuring point *c*.

The point *c* is found by moving down on the prime vertical, the same line on which the vanishing point is located (Fig. 57), as far from the point *P. P.*, as it is from *P. P.* to *P. S.*, thus making the line *P. P.—c* equal to the line *P. P.—P. S.*

NOTE.—The vanishing points in oblique perspective are by some authors termed *accidental points*.

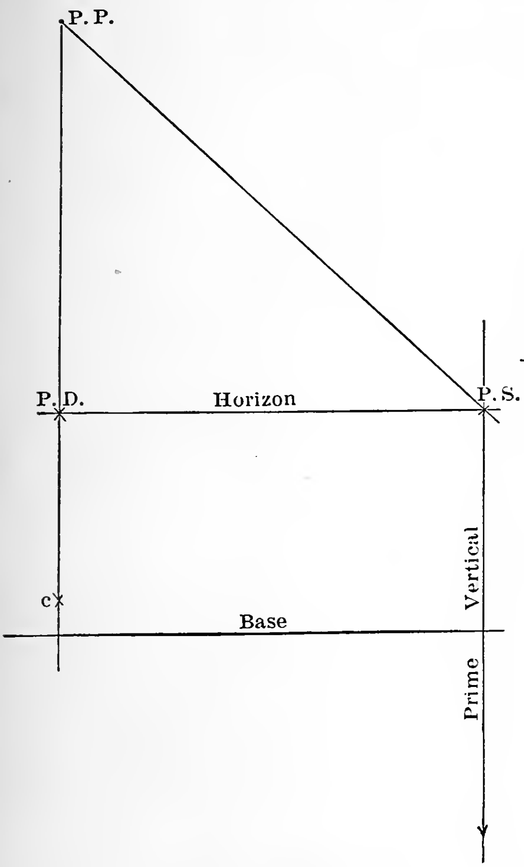

P. P.
P. D.
Horizon
P. S.
Vertical
c
Base
Prime

Fig. 56.

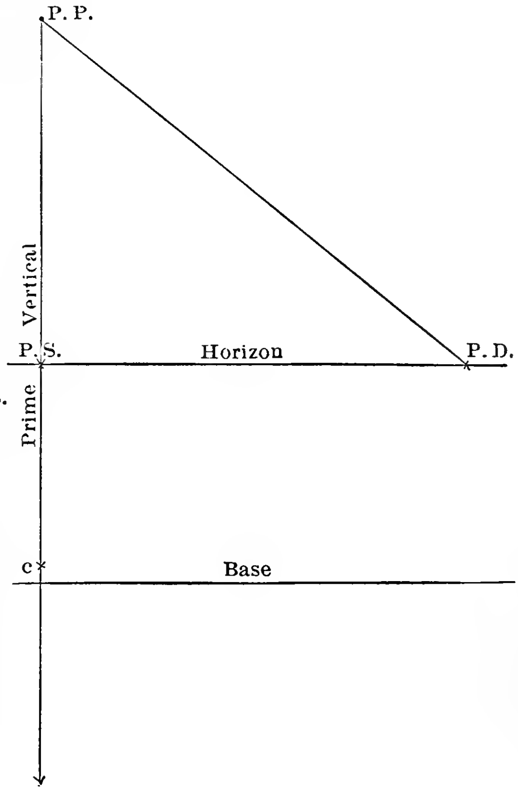

P. P.
Vertical
P. S.
Horizon
P. D.
Prime
c
Base

Fig. 57.

RULES

GOVERNING THE LINES AND SURFACES OF OBJECTS IN OBLIQUE PERSPECTIVE.

RULE I. To find a vanishing point for an oblique line, construct an angle at the *angular measuring* point equal to the angle made by the oblique line, and prolong the line forming the angle until it touches the perpendicular erected on the *angular vanishing* point; the point where these two lines meet is the vanishing point for the oblique line.

RULE II. To find the measuring point for the oblique line, move from the oblique vanishing point on the vertical line, downward to a point as far from the vanishing point as the vanishing point is from the point where the angle was measured off.

RULE III. Lines parallel in the object vanish in *one* point in the drawing.

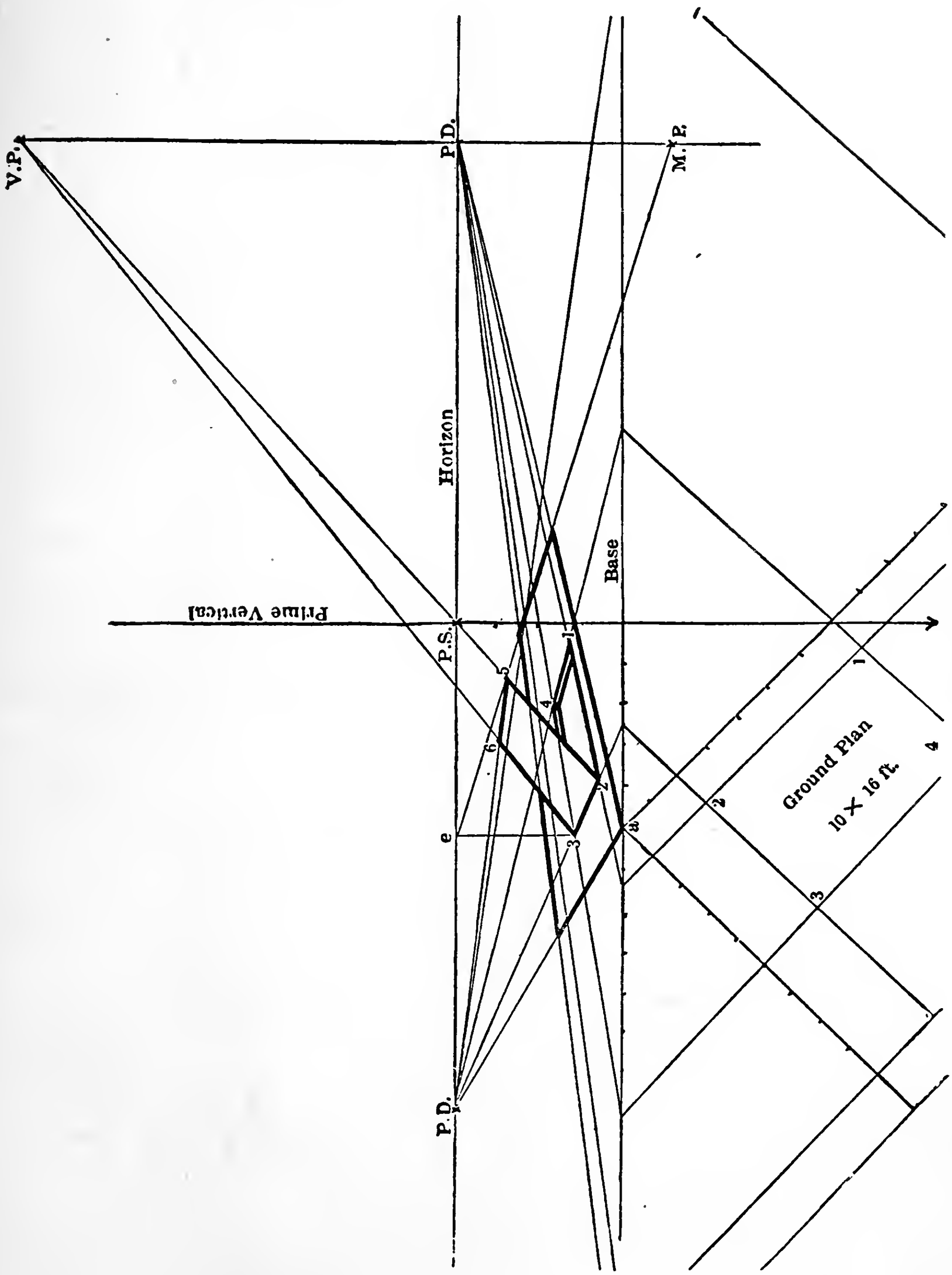

V.P.
P.D.
M.P.
Horizon
Base
Prime Vertical
P.S.
e
P.D.
Ground Plan
10 × 16 ft.

Fig. 58.

CHAPTER XXII.

EXPLANATION OF FIGURE 58.

PROBLEM.—Place in perspective a floor, 16 ft. x 10 ft., the nearest corner 5 feet to the left of the observer, receding at an angle of 45°; on this floor draw a trap door, 4 ft. x 5 ft., with the door one-fourth open. The nearest corner of the door is 2 feet from the left side, and 1 foot from the right side of the floor.

Draw the plan, and place the rectangular surface in perspective, and mark it *abcd* (Fig. 45). Locate the outlines of the door in the ground plan according to the problem, and mark them 1, 2, 3, 4. Prolong the lines from these points to the base line. Locate the measurement points on the base line, and from these draw lines to the points of distance, and where they cross, locate the points 1, 2, 3, 4, in the perspective view.

As the door is one-quarter open (according to Rule I) construct an angle of 45° at the point of sight, and prolong the side of this angle until it reaches a perpendicular over the point of distance. At this point locate the vanishing point for the oblique lines; then to this vanishing point, draw lines from 2 and 3; to find the length of these lines it is necessary to find the measurement point. This point is on the vertical through the point of distance as far from the vanishing point for oblique lines as it is from this vanishing point to the point of sight (Rule II). If the door were half-way open its perspective height would be determined by a 5-foot vertical line in perspective, from the point 3;

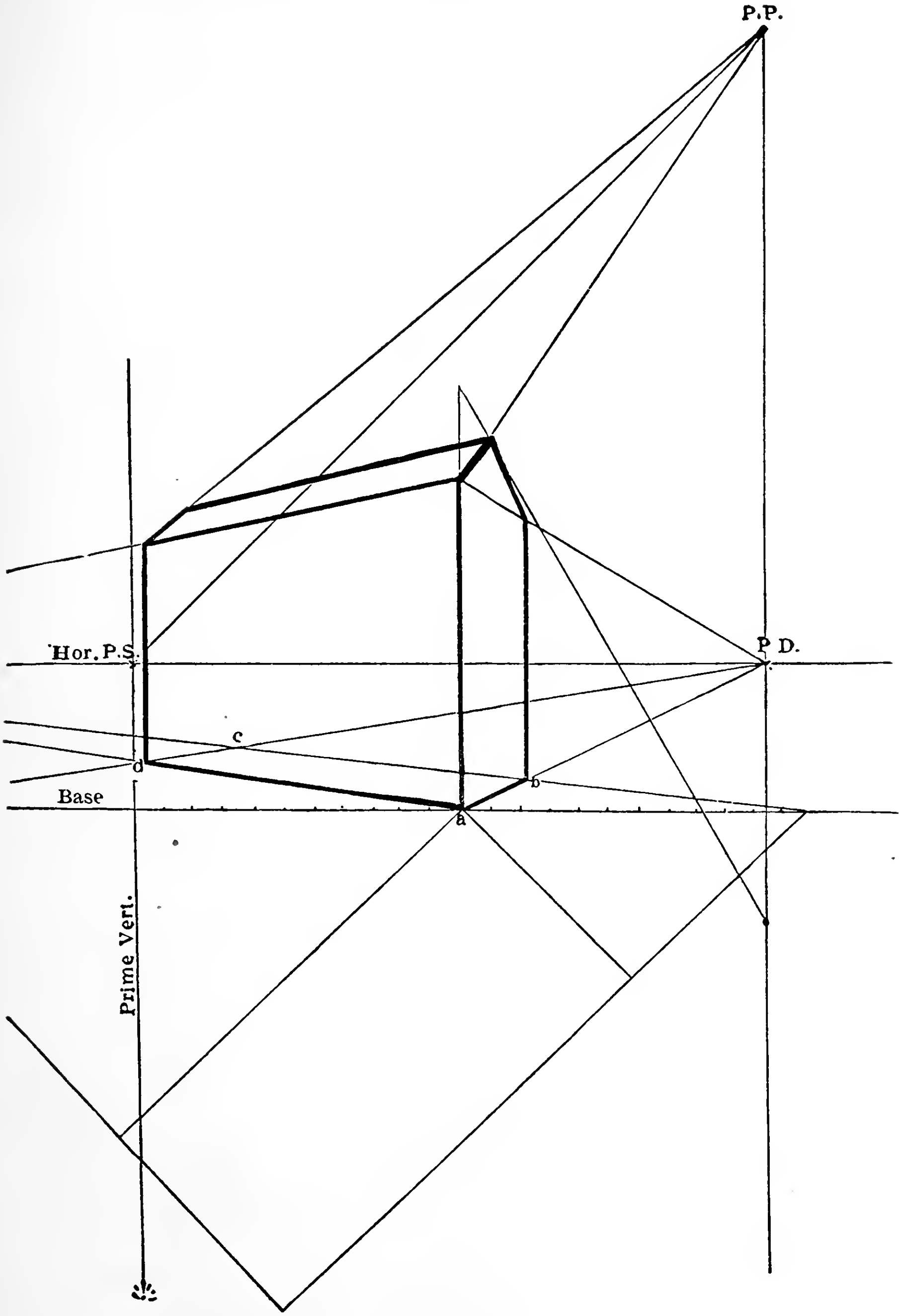


Fig. 59.

a 5-foot line from this point terminates at the point *e;* from this point to the measurement point draw a line. Where this line crosses

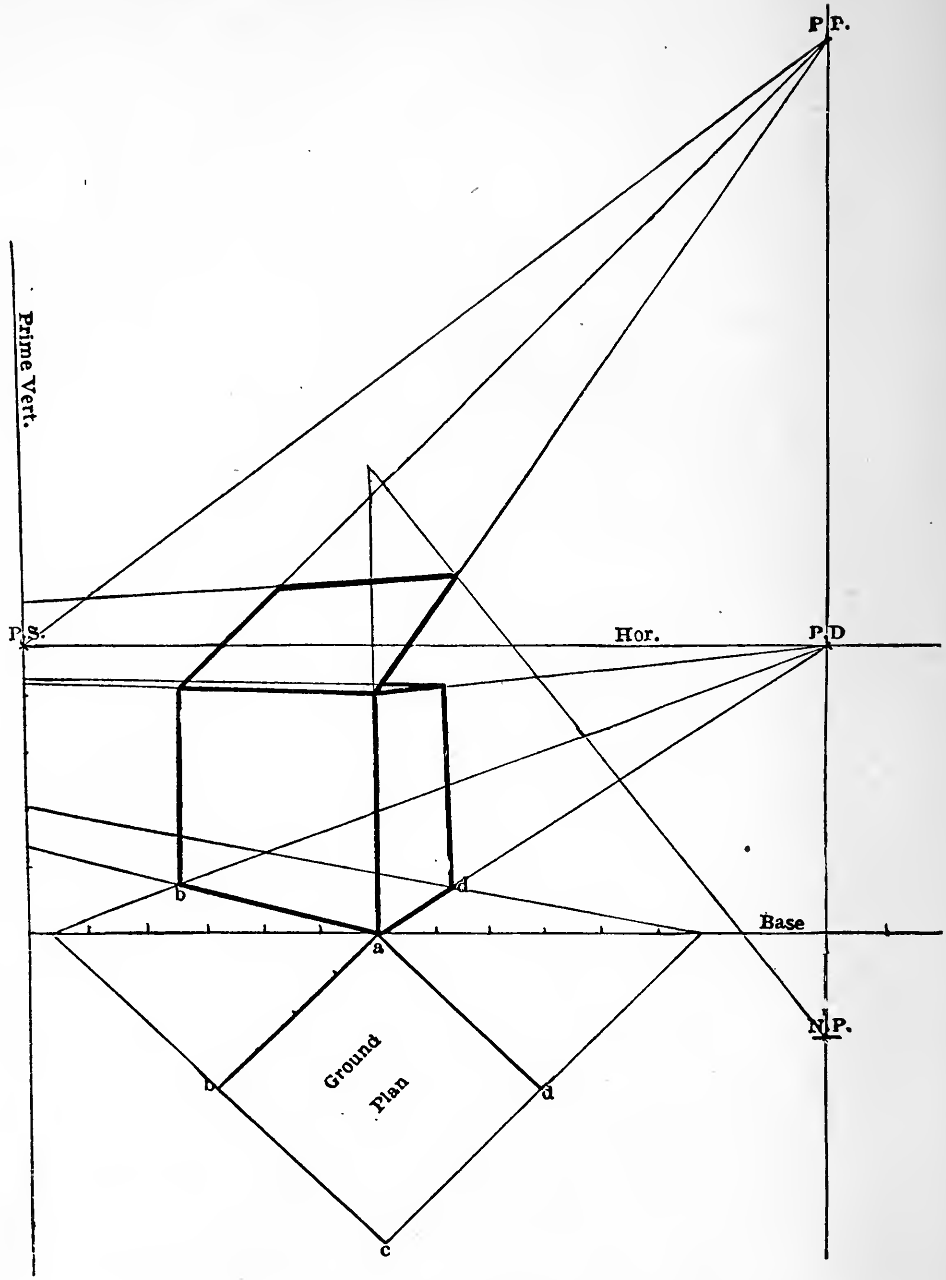


Fig. 60.

the oblique line from 2, locate the point 5, and from this to the point of distance on the left, draw a line (Rule III) to represent the upper edge of the door.

PROBLEM (Figure 59).—View of a rectangular box 8 ft. x 16 x 11 ft., surmounted by a triangle, with a base 8 ft. x 11 ft., and a vertical height of 3 feet. Nearest corner of box 11 feet on the right of the observer.

Construct the ground plan according to Figure 45, and draw the rectangular box according to Figure 47.

Find the vanishing and measurement points for oblique lines according to rules on page 74.

PROBLEM (Figure 60).—View of a 4-foot cubical box, the cover one-fourth open.

Construct the ground plan (Fig. 44), and draw the box according to Figure 47.

Find the vanishing and measurement points for the cover according to rules on page 74.

CHAPTER XXIII.

PROBLEMS.

1. PLACE in perspective a 4-foot square, 6 feet to the left of the prime vertical line, and a 3½-foot square, 5 feet to the right of the prime vertical line, both resting on the base line.

2. Place in perspective a series (4), of 2-foot squares, 2 feet apart. The first resting on the base, the nearest corner 5 feet to the left of the prime vertical.

3. Place in perspective a 3-foot cube, 2 feet back from the base line, 5 feet to the left of the prime vertical line.

4. Place in perspective a box (resting on the base line) 7 feet high, 3 feet wide, and 2 feet deep, the nearest corner 7 feet to the left of the prime vertical; and one, 4 feet to the right of the prime vertical, 8 feet long, 4 feet high, and 4 feet wide, 2 feet back from the base line.

5. Place in perspective a series of circles (2), 3 feet apart; the first on the base line, the diameter tangent to the base line, 6 feet to the left of the prime vertical.

6. Place in perspective a cylinder, 4 feet in diameter; the axis of the cylinder 6 feet to the right of the prime vertical, the base of the cylinder tangent to the base line.

7. Place in perspective an oval figure, 6 feet by 3 feet; the 3-foot diameter perpendicular to the base line, 9 feet from the prime vertical.

8. Place in perspective a pyramid, with a 4-foot square base,

vertical height 7 feet, nearest corner 3 feet to the left of the prime vertical, front line of the base 2 feet back from the base line.

9. Place in perspective a hexagon, one of the sides resting on the base line, 5 feet to the right of the prime vertical.

10. Place in perspective a pyramid, with a hexagonal base, and an altitude of 9 feet vertical; direction 7 feet to the right of the prime vertical.

11. Place in perspective a prism, 7 feet in altitude, with a base 4 feet wide by 6 feet long—front line resting on the base line.

12. Place in perspective a skeleton cross—standard 9 feet high on a base 6 feet square, and 1 foot high. The nearest corner 7 feet to the left of the observer, and 5 feet back from the base line. The cross is 1½ feet from the top of the cube, and is composed of 1½-foot cubes.

13. Place in perspective a cottage, 20 feet long, 10 feet wide, and 9 feet high, with a gable 5 feet high.

14. Place in perspective a cottage, 30 feet long, 12 feet wide, and 9½ feet high, with a gable 5 feet high, and two windows and a door, each 3 feet wide, and 7 feet apart.

15. Place in perspective a floor, 16 feet by 18 feet; the nearest corner 3 feet to the left of the observer, and 2 feet back from the base line.

16. Place in perspective the interior of a room, 12 feet by 16 feet, the point of sight in the middle of the back wall of the room; the front line of the floor on a level with the base; the height of the room 10 feet.

17. Place in perspective a half circle (the diameter 4 feet), 3 feet back from, and parallel to, the base; the nearest corner of the diameter 5 feet on the left of the observer.

18. Place in perspective a half circle (the diameter 5 feet),

at right angles to the picture plane, 8 feet to the left of the observer; the nearest point 4 feet back from the base.

19. Place in perspective a circle, vertical diameter 4 feet, at right angles to the ground plane, at a point 6 feet back from the base, and 7 feet on the right of the observer.

20. Place in perspective a 6-foot square, 4 feet above, and 3 feet to the right of the point of sight, and parallel with the ground plane.

21. Place in perspective, 6 feet above the level of the eye, and 5 feet to the right of the observer, a circle 7 feet in diameter.

22. Place in perspective a pyramid, 7 feet high — hexagonal base (six sides); the diameter, 5 feet, touches the base at right angles to it, at a point 11½ feet from the prime vertical.

23. Make perspective drawing, plan, and elevations of a 3-foot cubical box situated on the ground plane 4 feet to the left of the observer, the front face of the box lying in the picture plane.

24. Find a point on the ground plane 2 feet to the right of the observer and 5 feet back from the picture plane, 3 feet above the ground.

PLACE IN PERSPECTIVE THE FOLLOWING:

25. A line 4 feet long at right angles to the picture plane, and situated 6 feet to the right of the observer, with one end in the picture plane 7 feet above the ground.

26. A prism, having a base 1 foot square and an altitude of 3 feet, situated 2 feet to the left of the observer and 3 feet back from the picture plane.

27. A box 2 feet high, with base 6 feet by 3 feet, situated 3 feet to the right of the observer and 1 foot back from the picture plane — the longest side to be parallel to the picture plane.

28. A rectangular pyramid, base 2 feet square, altitude 4

feet, situated 2 feet to the right of the observer, and touching the picture plane.

29. A triangular pyramid, whose base is an equilateral triangle with sides of 3 feet, situated like No. 6.

30. A circle 5 feet in diameter, lying 6 feet to the left of the observer, and touching the picture plane.

31. A hexagon inscribed in above circle.

32. A cone, with a base 2 feet in diameter and 3 feet high, and situated 8 feet to the right of the observer, and touching the picture plane.

33. A horizontal line 5 feet long, lying on the ground plane at an angle of 45° to the picture plane, one end being in the picture plane and 4 feet to the right of the observer.

34. A 3-foot square lying on the ground plane side, at an angle of 30° to the picture plane, touching the base 2 feet to the left of the observer.

35. Inscribe an octagon in the above square.

36. A hexagonal prism, each side of whose base is 1 foot, and whose altitude is 6 feet, situated 4 feet to the right of the observer, and 2 feet back from the picture plane.

37. A cylinder 4 feet in diameter and 10 feet high, situated 3 feet to the left of the observer and 6 feet back from the picture plane. Draw plan and elevations.

38. A vertical plane 10 feet square, one side of which is in the picture plane, and which extends back at right angles to the picture plane, situated 12 feet to the right of the observer.

39. Inscribe a circle in above plane.

40. Inscribe a hexagon in above circle.

41. A 3-foot cubical box with the lid one-quarter open (hinge edge toward picture plane), situated 4 feet to the right of the observer and 2 feet back from the picture plane.

42. A 3-foot cube, one face in ground plane, and the vertical faces to be at angles of 60° and 30° with picture plane, the angle nearest the spectator to be 1 foot to his right and 2 feet back from the picture plane.

43. A right cone, 4 feet in diameter and 6 feet high, standing on the ground plane and touching the picture plane 2 feet on the right of the spectator.

44. A hexagonal pyramid, of which the sides of the base are 2 feet and the height 6 feet, standing on its base on the ground plane; the center of the base is 3 feet back from the picture plane and 1 foot to the left of the spectator.

45. A prism, whose base is an equilateral triangle, each side 5 feet, its height being 8 feet, with one side lying on the ground plane, its long edges being inclined at 60° to the picture plane, toward the right, and the angle nearest the spectator being 3 feet on his left and 2 feet from the picture plane.

46. A base-ball diamond (a square, 90 feet on a side), inclined 40° to the plane of the picture.

47. Find a point 2 feet to the left of the observer and 2 feet back from the picture plane; and another 5 feet to the left of observer and 7 feet back from the picture plane. Join these two points (*a* and *b*) and find the real distance from *a* to *b*.

48. A truncated cone, the lower base 2 feet in diameter, the upper base 1 foot in diameter, and its altitude 3 feet.

49. Surmount the above truncated cone with a cone whose base coincides with the upper base of the truncated cone, and whose altitude is 1 foot.

50. A 6-foot vertical square, standing at an angle of 60° to the picture plane, the nearest vertical side being 4 feet back from the picture plane.

51. Inscribe a circle in a square.

52. Inscribe a hexagon in a circle.

53. Locate a point in perspective, 3 feet to the left of the observer, and 2 feet back from the base line.

54. Locate a point in perspective, 5 feet to the right of the observer, and 3 feet back of the base line.

DRAW IN PERSPECTIVE THE FOLLOWING:

55. A 3-foot vertical line, 4 feet to the left of the observer, and 2½ feet back of the base line.

56. A 7-foot vertical line, 5 feet to the right of the observer and 2 feet back of the base line.

57. A 3-foot square, 5 feet to the left of the observer, the front side touching the base line.

58. A 4-foot square, 4 feet to the right of the observer, and 2 feet back of the base line. Draw its diagonals and its diameters.

59. Three rectangles, 4 feet long and 2 feet wide, one directly in front of the observer, one 5 feet to the left (nearest corner), and the other 5 feet to the right — front sides touching base line.

60. A 4-foot cube, the front edge resting on the base line 2 feet to the left of the observer.

61. An 8-foot cube, 4 feet to the right of the observer, the front edge touching the base line.

62. Three 6-foot cubes, faces parallel to picture plane, one directly in front of the observer, one 5 feet to the left (nearest corner), and the other 7 feet to the right, the front edges touching the base line.

63. Three 6-foot cubes, one directly in front, one 5 feet to the left, the other 5 feet to the right; the front line of each 2 feet back of base line, and front faces parallel to picture plane.

64. A rectangular box 7 feet long, 3 feet high, 4 feet wide,

and 5 feet to the left of the observer; front edge on the base line.

65. A rectangular box 8 feet high, 4 feet square at base, 6 feet to the right of the observer, and 2 feet back of base line.

66. Three rectangular boxes 8 feet high, 3 feet square at base; one directly in front, one 4 feet to the right, and the other 4 feet to the left of the prime vertical.

67. A table 5 feet long, 3 feet high, 2½ feet wide; 4 feet to the left of the observer.

68. A row of poles 15 feet high at right angles to ground plane, and 8 feet apart; nearest pole 6 feet to right or left of the observer.

69. Hitching posts, three on each side of the prime vertical, poles at right angles to ground plane 3½ feet high and 5 feet apart.

70. A view of a street running at right angles to the picture plane and crossed by one running parallel to the picture plane. Draw pavement 3 feet wide on each side of street, and fences, poles 3½ feet high, 3 inches wide, and ½ foot apart. Street 20 feet wide.

71. Draw lines indicating perspective of street; single car track and electric poles on either side, poles 15 feet high and 20 feet apart, observer equally distant from either side.

72. Three telegraph poles 15 feet high and three lamp-posts 8 feet high, alternating telegraph and lamp-posts 10 feet apart, poles parallel to picture plane and at right angles to ground plane.

73. Poles 10 feet high at right angles to ground, parallel to picture plane 6 feet apart, nearest poles on either side of the observer 4 feet.

74. Lamp-posts 10 feet apart, 8 feet high, posts parallel to picture plane, line of posts at right angles to picture plane, round globes, etc.

75. An iron fence and gate entrance, fence 3 feet high, single rod and gate 4 feet by 3 feet, finish as pupil or teacher suggests.

76. A room 20 feet long, 16 feet wide, 10 feet high, with windows 6 feet high, 4 feet wide, 4 feet apart, 2½ feet above floor. Windows in rear wall, floor 1 foot planks, middle of rear wall directly in front of observer. Place doors 7 feet high, 4 feet wide, in center of each side wall.

77. A circle, radius 4 feet, lying flat on ground plane, diameter at right angles to base line, 10 feet to the right of the observer.

78. A cone 8 feet to the left of the observer, diameter of base 6 feet, altitude 3 feet tangent to base line.

79. A cylinder, diameter 1 foot, perpendicular height 8 feet, situated 6 feet to the left, circumference touching base line.

80. An arch passage way perpendicular to ground plane, 10 feet high to arch, radius of arch 3 feet.

81. A pyramid that has for its base a square. Pyramid 7 feet to the right of the observer, front edge 2 feet back of base, perpendicular height 6 feet, base 4 feet square.

82. A monument, base 8 feet square and 1 foot high, 8 feet to the right of the observer, front edge touching base line, pyramid top directly over center of base, perpendicular height 10 feet to top square which is 2 feet. Place corners of the pyramid 1 foot from centers of the square base, front face parallel to picture plane (scale ½ inch to foot).

83. An ice chest 3 ft. long, 3 feet wide, 3 feet high, to the left of observer; 2 feet back of base line; front face parallel to picture plane; panel front and sides.

84. A pyramid with a base 9 feet square, and an altitude of 12 feet, the nearest corner being 6 feet to the left of the observer and 3½ feet back from the picture plane.

85. Three two-foot cubical boxes, lying in a straight line

parallel to, and 6 feet back from, the perspective plane, and to the right of the observer; the nearest edge of the nearest box being directly in front of the observer and the boxes being separated from each other by a space of 6 inches.

86. A rectangular box, 12 feet long, 3 feet wide, and 3 feet high, the nearest corner located in the base line 2 feet to the right of the observer; the box receding lengthwise toward the right, at an angle of 30°.

87. A box 7 feet high, with a base 3 feet square, 8 feet back from the base line, and 10 feet to the right of the observer, and a 2 ft. cube resting on the ground plane and the nearest corner lying 2 feet to the left of the observer and 4 feet back from the picture plane.

88. A stool, circular top, diameter of circle 1 foot, legs 2½ feet high, stool situated to right of observer 3 feet and back of base line 2 feet.

89. A chair, seat 1½ feet square, legs 1½ feet long, back 2 feet high, situated to left of observer 5½ feet, 2 feet back of base line, and the side of chair parallel to picture plane.

90. A chair, seat 2 feet square, legs 1½ feet long, back 2½ feet high, situated to right of observer 5 feet, and front of chair facing observer parallel to picture plane.

91. A bench 3 feet long, 1½ feet high and 1 foot wide; to right 2-foot bench resting on base line (scale ½ inch to 1 foot).

92. A table 2½ feet high, 2 feet wide, 5 feet long, situated to right of observer 3 feet, drawer in center of front of table 1½ feet long, ½ foot wide, front face parallel to picture plane and back of base line 2 feet.

93. A trunk 3 feet long, 2 feet wide, 2 feet high, 2 feet to left; handles and straps on base line parallel to picture plane (scale ½ inch to 1 foot).

94. A 3-foot cube, front edge resting on the base line, 5½ feet to the left of the prime vertical.

95. A 4-foot cube, 5 feet to the right of front edge, 3½ feet back of base line.

96. Two 3-foot cubes, the front edge of one resting on the base line. The front edge of the other to be 5 feet back of the base line (two feet from first cube), both cubes being 5 feet to the left of the prime vertical.

97. A box 7 feet long, 4 feet wide, and 4 feet high, so placed that the square ends are parallel to the picture plane. Front edge to rest on base line, nearest corner 5 feet to the right of the prime vertical. To the right of this box and adjoining it, construct a square-based pyramid. Base 4 feet square, altitude 7 feet, front edge resting on base line.

98. A wardrobe 9 feet high, 6 feet long, and 2 feet wide, in corner of room; back of base line 3 feet; to left of observer 6 feet; finish as desired.

99. A circle 8 feet in diameter, lying flat on the ground; the diameter at right angles to base line, and 7 feet to the left of the prime vertical.

100. A circle 10 feet in diameter standing upright, perpendicular to the picture plane, at a distance of 8 feet to the right of the prime vertical.

101. A 6-foot square, 7 feet to the left of the observer, the corner resting on the base line and the nearest side receding from the picture plane at an angle of 45°.

102. A rectangle 5 feet by 2 feet; the nearest corner 7 feet to the right of the observer and in the base line; the nearest long side receding toward the left, at an angle of 45° from the picture plane.

103. A 9-foot square, 2 feet to the left of the observer and 5 feet back from the base line, at an angle of 45°.

104. A rectangular box, 12 feet long, 3 feet wide, 3 feet high; the nearest corner located in the base line 2 feet to the right of the observer; the box receding (lengthwise) toward the right, at an angle of 30°.

105. A box 7 feet high with a base 3 feet square; 8 feet back from the base line, and 2 feet to the left of the prime vertical, at an angle of 70°.

106. A tesselated floor, 20 feet wide and 30 feet long, divided into 2-foot squares, the nearest corner lying in the base line 7 feet to the right of the observer and receding (lengthwise) toward the right, at an angle of 50°.

107. A pyramid 12 feet high, having for its base a 4-foot square, having its nearest corner in the base line, and 3 feet to the right of the observer, and inclined 45° to the picture plane.

108. A triangular prism 10 feet high, the base being a right-angled triangle with a base and perpendicular of 3 feet, the nearest angle lying 3 feet to the right of the observer and 4 feet back from the picture plane; the hypotenuse receding toward the right at an angle of 65° with the picture plane.

109. A piece of stovepipe 3 feet long and 6 inches in diameter, its nearest point lying in the picture plane and 3 feet to the right of the observer, the pipe receding toward the right, at an angle of 40°.

110. A circle 10 feet in diameter, inscribed in a vertical square, the nearest side of which is in the picture plane and 4 feet to the right of the observer, and receding toward the left at an angle of 60°.

111. Three 2-foot cubical boxes, lying in a straight line parallel to and 6 feet back from the picture plane; and to the right of the observer, the nearest edge of the nearest box being directly in front of the observer, the boxes being separated from each other by a space of 6 inches.

112. A pyramid with a base 9 feet square, and an altitude of

12 feet, the nearest corner being 6 feet to the left of the observer and 3½ feet back from the picture plane.

113. A post 1 foot square and 12 feet long, lying on the ground plane, extending toward the right, and running back, at an angle of 30° with the picture plane; the nearest corner of the post being 2 feet to the left of the observer and 2 feet back from the picture plane.

114. An octagon inscribed in a 9-foot square,—the square lying on the ground plane, inclined 45° to the picture plane, the nearest corner being 2 feet to the left of the observer and 6 feet back from the picture plane.

115. A tesselated pavement 10 feet square, divided into 1-foot squares; lying on the ground plane, inclined back toward the right at an angle of 20° with the picture plane, the nearest corner lying 6 feet to the left of the observer, and in the base line.

116. A board fence, running back at right angles to the picture plane, on level ground, and the nearest end lying in the picture plane, 10 feet to the right of the observer, the fence to be designed or selected by the pupil.

117. Draw isometric projections of:

(1.) Hollow cube.
(2.) Rectangular box.
(3.) Flight of four steps.
(4.) Newel post.
(5.) Hollow cylinder.
(6.) Section of a porch railing.
(7.) Brick chimney.
(8.) Wash bench.

118. A view of a 9-foot cube inclined 45° to the ground plane, and surmounted by a wedge-shaped solid with a base 9 feet square,

and an altitude of five feet, the nearest corner of the cube being in the base line 10 feet to the left of the observer.

119. A pyramid, with an altitude of 7½ feet, and a base 3 feet square, the nearest corner being 1 foot to the right of the observer and 1 foot back from the picture plane; the pyramid receding toward the right at an angle of 30° with the picture plane, and upward at an angle of 15° with the ground plane.

120. A prism, with an altitude of 10 feet, and having for its base a 5-foot square, the nearest corner of which is 3 feet to the right of the observer and 2 feet back from the picture plane; the base receding toward the left at an angle of 60° with the picture plane, and upward at an angle of 20° with the ground plane.

121. A 4-foot cubical box, inclined 45° to the picture plane, its nearest corner located in the base line 4 feet to the right of the observer; the lid of the box opened upward toward the right at an angle of 45°.

122. A floor, 15 feet by 12 feet, the nearest corner 4 feet to the left of the observer, receding at an angle of 45°. On this floor draw a trap-door 4 feet by 5 feet, with the door one-fourth open. The nearest corner of the door is 2 feet from the left side, and 1 foot from the right side of the floor.

123. A view of a 9-foot cube, inclined 45° to the ground plane, and surmounted by a wedge-shaped solid with a base 9 feet square and an altitude of 5 feet, the nearest corner of the cube being in the base line, 10 feet to the left of the observer.

124. A rectangular box, with a base 2 feet square and an altitude of 3 feet, surmounted by a pyramid with a base 2 feet square and an altitude of 2 feet, the box being inclined 45° to the picture plane, and inclined upward as it recedes toward the right, at an angle of 10° with the ground plane.

125. A pyramid, with an altitude of 7½ feet and a base of 3 feet square, the nearest corner being 1 foot to the right of the observer and 4 feet back from the picture plane, the pyramid receding toward the right at an angle of 30° with the picture plane, and upward at an angle of 15° with the ground plane.

126. A row of vertical posts, each 1 foot square and 7 feet high, receding to the right at an angle of 40° with the picture plane, and up a straight hill at an angle of 20° with the ground plane; the posts being 6 in number and 10 feet apart, the nearest corner of the nearest post being in the picture plane directly opposite the observer.

127. Draw an isometric projection of:

(1.) Watering trough, 10 feet by 2 feet base, 1 foot high (outside), and composed of boards 2 inches thick.

(2.) Anvil.

(3.) Wagon bed.

(4.) Carpenter's plane.

(5.) Hot-bed, showing glass covering.

(6.) Bench.

(7.) Grindstone.

(8.) Cord of Wood (4 feet by 4 feet by 8 feet).

(9.) Flight of steps.

(10.) Ice chest.

(11.) Office safe.

(12.) Stove.

(13.) Chair.

(14.) Small stone bridge.

(15.) Railroad signal tower.

128. A rectangular box, 12 feet long, 3 feet wide, and 3 feet high; the nearest corner located in the base line 2 feet to the right

of the observer, the box receding (lengthwise) toward the right, at an angle of 30°. Lid half way open.

129. A colonial clock, 7 feet high, the base 2 feet square, 8 feet back from the base line, the nearest corner 5 feet to the left of the observer.

130. A 4-foot square and inclined toward the left at an angle of 45° with the ground plane, directly opposite the observer, and 5 feet back from the picture plane. In this square inscribe a circle.

131. In the above circle inscribe a hexagon, and upon this hexagon as a base, construct a pyramid with an altitude of 8 feet.

132. A pyramid 12 feet high, having for its base a 4-foot square, having its nearest corner on the base line and 3 feet to the right of the observer, and inclined 45° to the picture plane.

133. Draw in perspective a triangular prism 13 feet high, the base being a right-angled triangle, with a base and perpendicular of 3 feet; the nearest angle lying 3 feet to the right of the observer, and 4 feet back from the picture plane; the hypotenuse toward the right at an angle of 65° with the picture plane.

134. A piece of stovepipe 5 feet long and 6 inches in diameter; its nearest point lying in the picture plane and 3 feet to the right of the observer, the pipe receding toward the right at an angle of 40°.

135. A board fence, running back at right angles to the picture plane, on level ground, and the nearest end lying in the picture plane, 13 feet to the right of the observer, the fence to be designed or selected by the pupil.

136. A circle 10 feet in diameter inscribed in a vertical square, the nearest side of which is in the picture plane and 11 feet to the right of the observer, and receding toward the left at an angle of 60°.

137. A box 3 feet wide, 10 feet long, and 4 feet high, the nearest lower corner of which is on the ground plane 5 feet to the

left of the observer and 4 feet back from the picture plane, the box receding toward the right at an angle of 25° with the picture plane and upward at an angle of 30° with the ground plane.

138. A prism with an altitude of 15 feet, and having for its base a 5-foot square, the nearest corner of which is 3 feet to the right of the observer and 2 feet back from the picture plane, the base receding toward the left at an angle of 60° with the picture plane and upward at an angle of 20° with the ground plane.

139. A 4-foot cubical box inclined 45° to the picture plane; its nearest corner in the base line 7 feet to the right of the observer; the lid of the box opened upward toward the right at an angle of 45°.

140. A floor 15 feet by 14 feet, the nearest corner 4 feet to the left of the observer, receding at an angle of 45°. On this floor draw a trap-door 4 feet by 5 feet, with the door one-fourth open. The nearest corner of the door is 12 feet from the left side, and 1 foot from the right side of the floor.

141. A view of a 12-foot square, inclined 45° to the picture.

142. A 4-foot square, inclined 45° to the picture plane, and inclined upward toward the left at an angle of 45° with the ground plane, the nearest corner being on the ground plane directly opposite the observer and 5 feet back from the picture plane. In this square inscribe a circle.

143. A tesselated floor, 20 feet wide and 50 feet long, divided into 2-foot squares—the nearest corner lying in the base line 7 feet to the right of the observer, and receding (lengthwise) toward the right, at an angle of 50°.

144. A 4-foot cube at 30° and 60° to the picture plane, one corner touching the base line 4 feet to the left of the prime vertical.

145. A box 2½ feet high, 5 feet long, and 2 feet wide, at 45° to the vertical plane, one corner touching the base line 5 feet to the right of the prime vertical.

146. A pyramid, the base a 5-foot square, altitude 8 feet, edges of base at an angle of 35° and 55° to picture plane, front corner being 6 feet from the prime vertical and 4 feet from the base line.

147. A tesselated pavement 30 feet square, blocks to lie square, and at an angle of 45° to the base line.

148. A section of wall 24 feet long, parallel with base line; 5 feet from line of direction and 3 feet back, 3 feet thick, 12 feet high, containing three arches 4 feet wide, and 9 feet to top of arches.

149. An ice chest whose base is 3 feet square and 4 feet high, lying upon one of its sides; the nearest angle being 4 feet on the left of the spectator and 4 feet from the picture plane, and the long sides receding at an angle of 60° with the ground line.

150. A cigar box in angular perspective at 45° to picture plane, the box 5 feet long, 3 feet wide, 1 foot high; to left — lid one-half open. (½ inch to 1 foot scale.)

151. Two arched windows. Windows in wall 15 feet long — distance between windows 4 feet. Windows 4 feet wide and 8 feet long at highest point of arch.

152. A piano, square grand, 4 feet high, 7 feet long, body 1 foot deep, 3 feet wide. Place pedals in center and legs 3 feet long (scale ½ inch to 1 foot); to left of observer 5 feet, at an angle of 45° to the picture plane.

153. A street scene. A street 20 feet wide, side-walks 2 feet. Car track, lamp, and telegraph poles 10 feet apart, at right angle to the ground; houses as designed by pupil, on each side of street, observer standing directly in center of street.

154. Tunnel in stone bridge, bridge wall 20 feet long; 12 feet high. Arch opening 8 feet, track with ties running into tunnels. Electric light pole with arm and lamp at entrance, observer directly in front of scene.