ADVANCED

MECHANICAL DRAWING

A TEXT FOR ENGINEERING STUDENTS

BY

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PREFACE.

Having in charge the preparation of all of the engineering students in Purdue University in Mechanical Drawing for their Course in Engineering Design, the writer has compiled a series of progressive notes on the subject calculated to impart a working knowledge of the principles of graphic representation, and offering such examples as will acquaint the student with the conventions of the art. The work is divided into two parts, Part I being "A Course in Elementary Mechanical Drawing," administered in the Freshman year, and Part II a course in "Advanced Mechanical Drawing," administered in the Sophomore year as a course in drawing, and in connection with the classroom and lecture work in Descriptive Geometry.

The work is purely elementary, dealing with methods of representation alone, manipulations of construction, and does not treat of Design, being preliminary to that subject.

This part, Advanced Drawing, is offered to students and draughtsmen who have a working knowledge of the principles of the art, such as is offered in Part I, and who have, also, some knowledge of the principles of Descriptive Geometry.

The discussions have been made as brief as was thought consistent with clearness, and are intended simply to suggest such lines of thought as will render the figures, the illustrations—an engineer's "description"—self-explanatory.

In selecting a "Course in Drawing" from the examples offered, it is suggested that, in so far as possible, the Practical

Problems be made to follow the Theoretical Problems delineating the principle involved; such an arrangement, for example, as is given by Plate 27, page 177.

The writer has enjoyed the advice and co-operation of Prof. M. J. Golden, Mr. A. M. Wilson, Mr. E. B. Smith, and Mr. O. E. Williams in the preparation of the manuscript and illustrations, and wishes to thank them for their many courtesies and valued assistance.

A. P. JAMISON.

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ADVANCED MECHANICAL DRAWING.

PART I.

THEORY, DEFINITIONS, ETC.

CHAPTER I.

ISOMETRIC DRAWING.

1. **Definition.**—Isometric drawing is that branch of mechanical drawing which enables one to represent an object in such a manner as to present three sides or faces in a single drawing or view. That is, such a drawing serves the same end as an ordinary three-view mechanical drawing; furthermore, it pictures, after a fashion, the object as it would appear if placed before the observer, and because of this characteristic is legible to one not able to read a mechanical drawing.

Isometric drawing is also called "isometric perspective" and "practical perspective"; it is called isometric perspective because it pictures an object as a whole, and practical perspective because of its greater simplicity as compared with perspective drawing.

It is really a joint between ordinary mechanical drawing and perspective drawing, since it contains features of each. For example, lines which are drawn parallel in the mechanical drawing of an object are also drawn parallel in the isometric drawing of the object, and since the isometric drawing shows three faces it is a pseudo perspective.

2. **Theory.**—If a cube be held in a position such that one of its diagonals is perpendicular to one of the planes of projection (Fig. 1), its projection on that plane is said to be an isometric projection (since all the lines of the cube are equally foreshortened in the projection because of their uniform inclination to the plane)—the term "isometric" meaning "in equal parts."

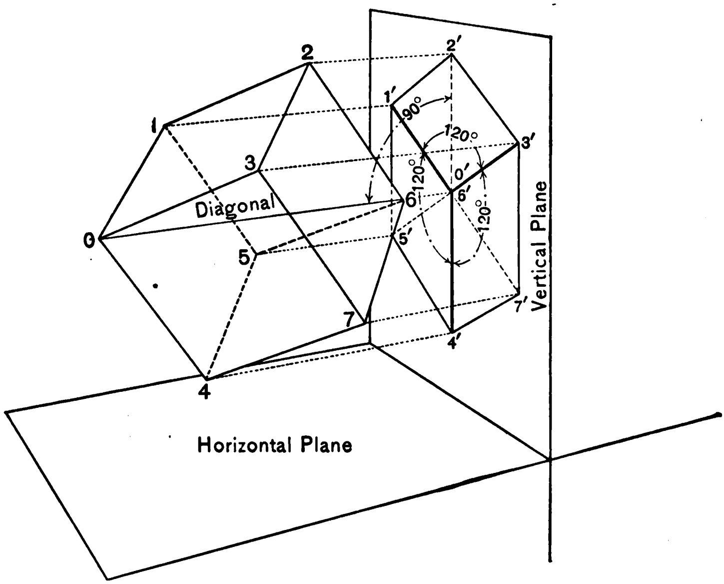


FIG. 1.

The figure shows the projection on the vertical plane (either plane of projection may be used) and shows three visible faces. This is sufficient for all practical purposes, and it is customary to disregard the other projection entirely and thus eliminate all reference to the planes of projection.

Now any object may be considered as inclosed within a cube (a side of the cube would equal the greatest dimension of the object) and, thus considered, it can be projected with the cube.

3. Explanation of Terms.—In Fig. 2, which is a mechanical drawing of the arrangement shown in Fig. 1, note the central point of the front elevation: this point is called the ***origin;*** the three full lines radiating from it, the ***isometric axes;*** the planes

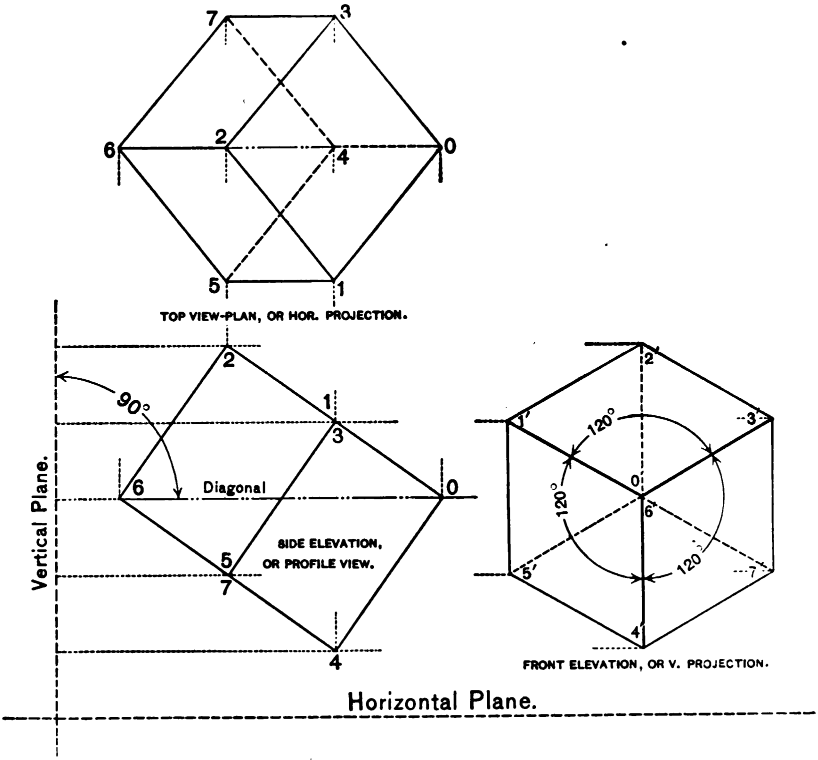


FIG. 2.

determined by the isometric axis are called *isometric planes*, and all lines in these planes drawn parallel to the isometric axes are called *isometric lines*.

The isometric axes—three lines 120° apart (Fig. 3)—form the basis for all isometric drawing.

4. Distinction between Isometric Projection and Isometric Drawing.—The lines of the projection of the cube in Figs. 1 and 2 are about .8 of the length of the original; that is, the size of the projection is about .8 of the size of the cube. If the projection is drawn the same size as the cube, it will represent the *projection* of a cube about 1.25 times the size of the original. Since it would require special scales to obtain .8 of the several dimensions of an object, it is common practice to construct the drawing to represent the projection of an object 1.25 larger than the original; that is, dimensions are taken as in ordinary mechanical drawing —full size, one-half size, one-quarter size, etc. The drawing so constructed is called an isometric *drawing* to distinguish it

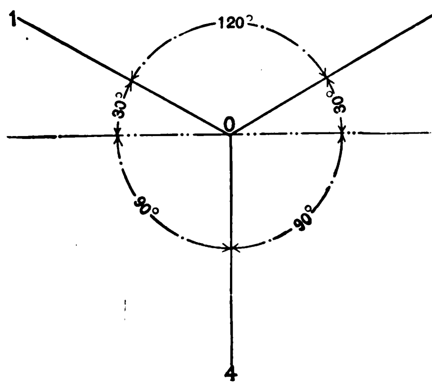


FIG. 3.

from the isometric *projection*, the dimensions of which would be .8 of those of the isometric drawing; in short, an isometric drawing is one-fourth larger than an isometric projection.

5. Isometric Scales.—The uniform angle made by the lines of the cube with the plane of projection in Fig. 1 is an angle of 35° 16′; now, if a scale with full-size divisions—one inch equal to one inch—be placed at such an angle with another scale, as is shown by Fig. 4, the divisions on the inclined scale can be projected onto the upright scale the same as the lines of the cube are projected onto the plane of projection, and a scale of foreshortened divisions obtained—an isometric scale. In the figure the left scale of *B* is projected onto the right side of scale *A*.

On the other hand, if full-size divisions be projected from the upright scale onto the inclined scale, as is shown in the figure by the projections from the left side of scale *A* to the right side of *B*, a series of divisions will be obtained which are 1.25 larger—a second form of isometric scale.

Such scales as these would be used to construct isometric *projections;* using scale *A*, the object would be measured with the scale on the left and the projection constructed from the scale on the right; or, using scale *B*, the object would be measured

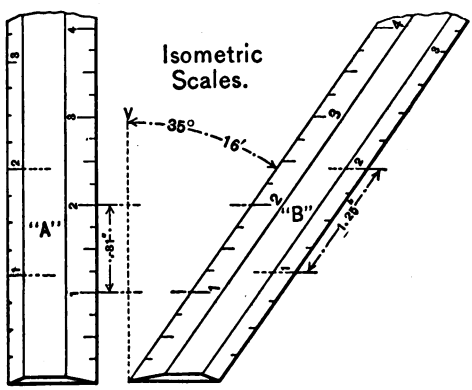


FIG. 4.

with the scale on the right and the projection constructed from the scale on the left.

As already stated these scales are special and are not used in ordinary practice, the usual architect's or engineer's scale being used, and the *projection* made full size or to scale, as already described—an *isometric drawing.*

6. Practical Application of the Theory.—To consider an object as inclosed within a cube, a side of which would equal the greatest dimension of the object, would involve the draughting of a number of lines in the execution of the isometric representation which would be longer than necessary and require greater space for the drawing. In practice the inclosing figure is a rectangular "box," the three dimensions of which conform with the three dimensions—length, breadth, and thickness—of the object; the application of the theory is seen by taking a side of the inclosing box and on it constructing a cube, then arranging the cube with the correct reference to a plane of projection and projecting the cube and the rectangular box together with its contents, as witness Fig. 5: the figure 0-1-2-3-4-

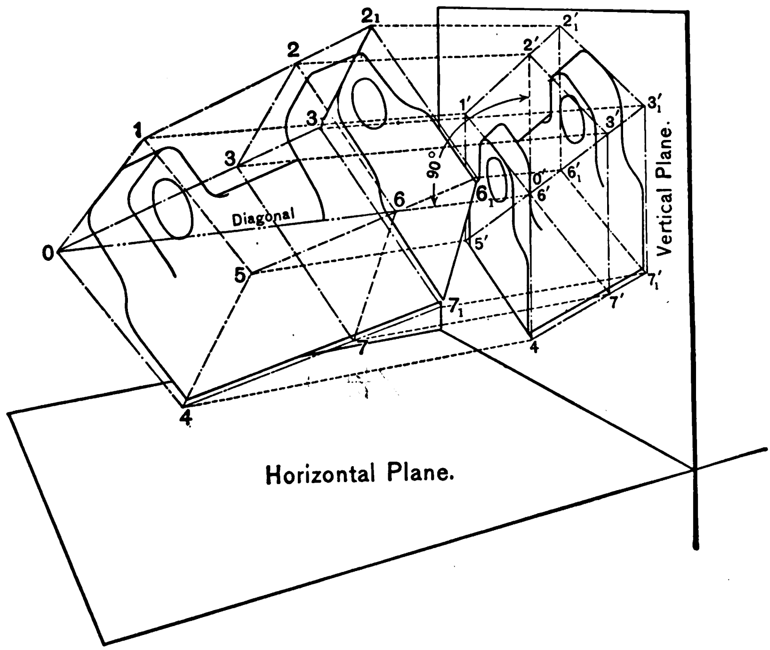


FIG. 5.

5-6-7 representing the cube, the corners of which are numbered the same as the corners of the cube in Fig. 1.

7. **Method of Procedure.**—The above is carried out in practice as follows:

Let it be required to construct an isometric drawing of the object, the mechanical drawings for which are shown in *A*, Fig. 6. The first step is to draw the isometric axes—three lines 120° apart, as shown, and so taken (one line perpendicular and the other two at 30° with the horizontal) for convenience in draw-

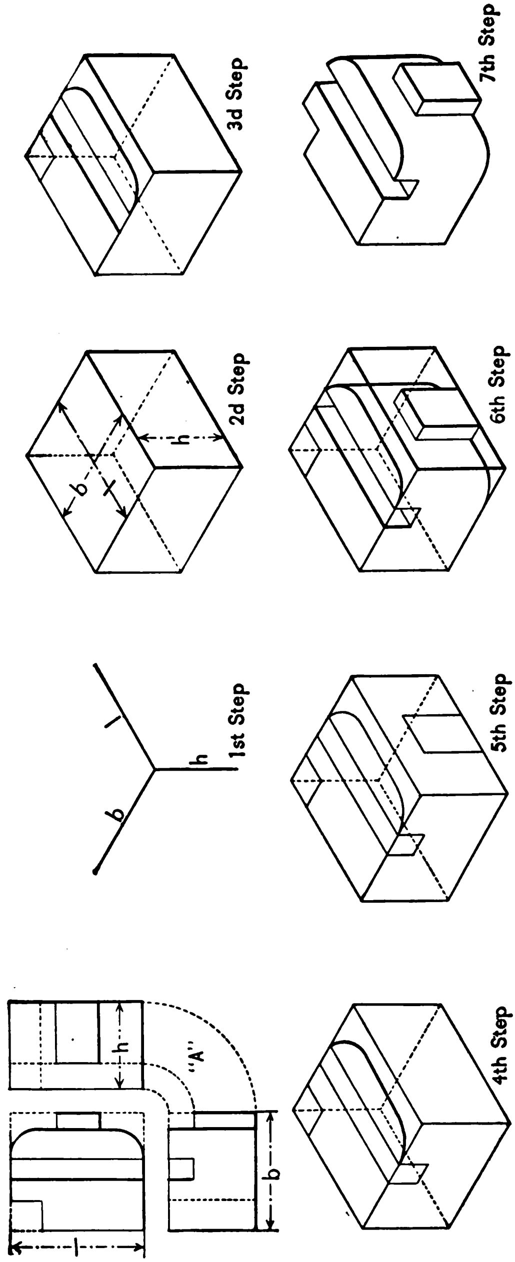

"A"
l
h
b
1st Step
2d Step
3d Step
4th Step
5th Step
6th Step
7th Step

FIG. 6.

ing—and select (arbitrarily) one axis to represent length, one for breadth, and one for thickness or height—*l*, *b*, and *h*; the second step is to ascertain the three dimensions of the inclosing box for the object by inclosing the mechanical drawings within rectangles, then lay off these dimensions on the isometric axes, and draw the isometric drawing of the inclosing box; the third step is to select one face of the box and draw in it all the lines of the inclosed object showing there; the fourth step is to similarly treat a second face; and so on, until all of the faces have been treated, and all of the lines of the object drawn; the last step is to erase all construction lines.

8. Flexibility.—The assumption of the isometric axes is the first step in all isometric drawing; they may be assumed in any

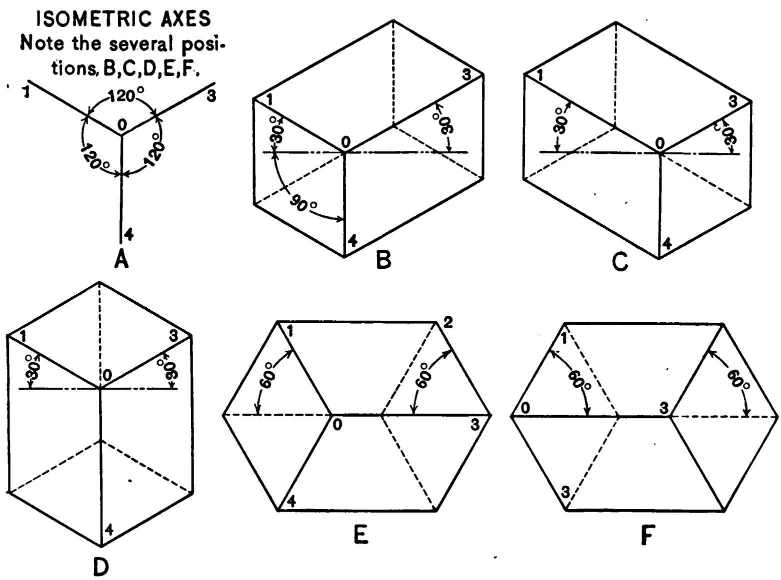


FIG. 7.

position so long as their proper relation, one to another, is maintained. It is obvious, of course, that for practicability a position must be assumed to suit the drawing instruments, that is, one axis should always be upright, horizontal, at 30° or 60° with the horizontal, etc., and the other two axes drawn at 120° with it.

In practice the position of the axes is determined, in a measure by the object itself; that is, the isometric axes determine the isometric planes, and the planes are determined by the faces of the object which are to be illustrated.

As evidence of the above, assume that an object is to be drawn which can be inclosed in a rectangular box, and note the several positions which the box may assume, as shown in Fig. 7; also note the other isometric drawings of the text.

9. **Practical Examples.**—As a further exposition of the subject, it is proposed to discuss the execution of a number of representative examples in drawing.

Plane figures.—A square. Let it be required to represent a 2″ square, Fig. 8, in "isometric," and, furthermore, let it be required to represent it with its plane horizontal.

Now, in isometric drawing a horizontal plane is determined by lines which are at 30° with the horizontal; hence to draw the square, draw the two lines *A-B* and *A-D*, as shown, each making, an angle of 30° with the horizontal, and from their point of intersection lay off on each line a length equal to 2″—the length of a side of the square; through the point thus determined on *A-B*, point *B*, draw a line *B-C* parallel to the line *A-D*, and through the point *D* of *A-D*, similarly determined, draw a line *D-C* parallel to *A-B* to an intersection with *B-C*. The line *B-C* is, clearly, equal to *A-D*, which is equal to a side of the square, being parallel lines comprehended between parallels; also, for the same reason, the line *D-C* is equal*A-B*, a second side of the square; therefore, the figure *A-B-C-D* is the required isometric.

The term "horizontal plane" as applied to the plane determined by lines at 30° with the horizontal is an arbitrary one, and is used to distinguish it from planes determined by a vertical line and a line at 30° with the horizontal, which for the purpose of discussion will be termed "vertical planes." Both terms, however, are misnomers, as the planes are neither horizontal nor vertical, but are oblique planes, as witness Fig. 1: the horizontal plane corresponds to the plane of the top or bottom of the cube (0-1-2-3 or 4-5-6-7), and the vertical plane to the side faces of the cube (0-3-7-4, etc). In like manner, a plane which is determined by a horizontal line and a 60° line or by two 60° lines will be termed an "oblique plane." (See *E* and *F*, Fig. 7.)

A circle. Let it be now required to draw a circle on a horizontal plane. This is accomplished by first assuming the circle to be inclosed within a square, then drawing the square in isometric, and proceeding as follows:

If the circle is inclosed within a square, it is obvious that it will be tangent to each side of the square at its middle point; therefore,

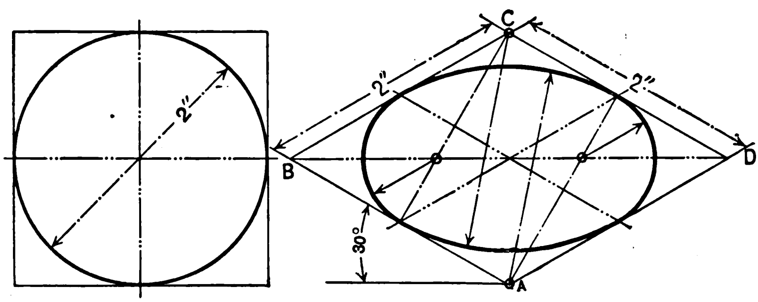


Fig. 8.

locate these points by drawing the center lines of the square,* as shown in Fig. 8, then with a corner of the square as a center and a radius equal to the distance from this point to the middle point of an opposite side of the square, draw a circular arc between the two opposite middle points; next reverse the operation, that is, use the opposite corner of the square as a center, etc.; these two arcs drawn, draw the diagonal of the square and draw the lines determining the above radii: the points where these lines cut the diagonal will be new centers, and with new radii represented by the distance from one of these points to the nearest middle point of a side of the square, other circular arcs may

* The isometric drawing of the square is not a square, of course; the terms square, rectangle, etc., are used, however, the same as in ordinary mechanical drawing.

be drawn and a closed curve obtained representing the circle in isometric. The construction is clearly shown by the figure.

An isometric view being an oblique view, it is obvious that the representation is an ellipse. The above is not a true representation, but is an approximation, and is the usual practice in isometric because of its ease of execution. The figure shows its application in the horizontal plane; it is applied in exactly the same manner in all isometric planes. (See Fig. 9.)

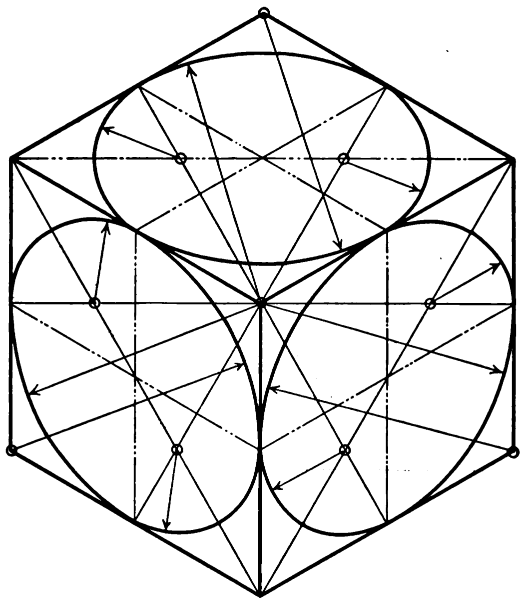


FIG. 9.

A triangle. Fig. 10 shows a triangle, *A* being the mechanical drawing and *B* the isometric drawing. To construct the figure inclose the mechanical drawing within a rectangle, then draw the rectangle in isometric; now the figure shows the triangle inclosed in such a manner that one of its sides forms a side of the inclosing rectangle, and the triangle is such that the apex of the angle opposite this side is at the middle point of the opposite side of the rectangle; hence, to draw the triangle within the isometric rectangle, draw the coincident side and join the extremities with the middle point of the opposite side of the rectangle.

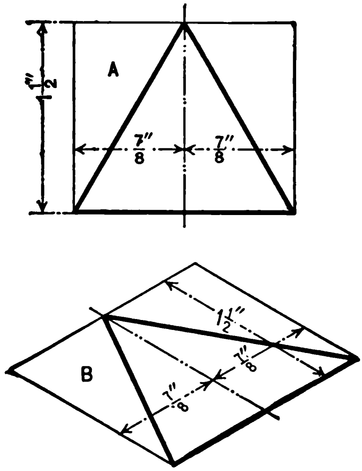


FIG. 10.

It will be remarked that the figure is drawn on a horizontal plane; the construction is the same for any plane.

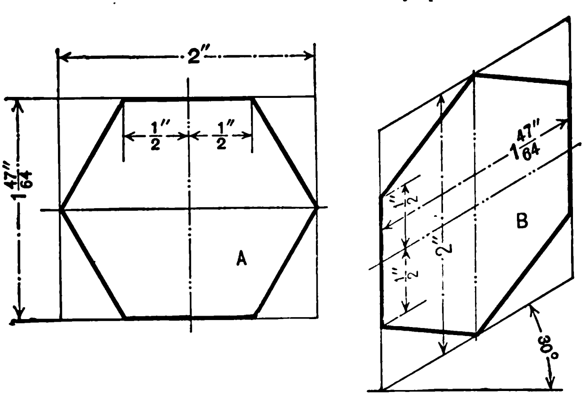


FIG. 11.

A hexagon. In Fig. 11, *A* is a mechanical drawing of a hexagon; to draw it in isometric, inclose the hexagon within a rectangle, then draw the rectangle as shown in *B*; now if the diameters of the hexagon be drawn, the extremes of the long diameter—the "diagonal" of the hexagon—will be two points in the hexagon, and the extremes of the short diameter—the distance between "flats"—will be the middle points of two opposite sides of the hexagon; these two sides are coincident with two of the sides of the inclosing rectangle, and by laying off from these middle points distances on the sides equal to one-half of the length of a side, as shown, the remaining four corners of the hexagon are obtained; the hexagon is then drawn by connecting the six points.

The isometric plane in this example is a vertical plane, and corresponds to the 0-3-7-4 plane of the cube in Fig. 1.

An octagon. A method for drawing an octagon in isometric is shown by Fig. 12. The inclosing rectangle is drawn as in

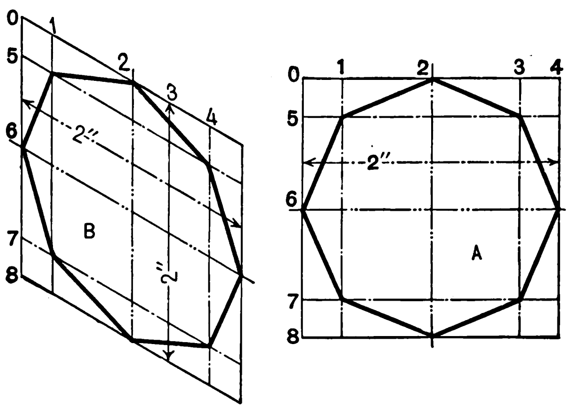


FIG. 12.

the other examples, and the four points of the figure which touch the sides of the rectangle are found by drawing the center lines as shown; the remaining four points are found by projecting (horizontally and vertically—parallel with the sides of the inclosing rectangle) the points of the mechanical drawing onto the sides of the inclosing rectangle, then transferring these divisions to the sides of the isometric rectangle, and through them drawing the isometric lines as shown. The intersections will define the location of the points. The octagon is then drawn by connecting the eight points as shown.

This method of locating points by intersecting lines corresponds to the method of locating points by ordinate and abscissa of analytic geometry, and is much used in isometric construction; for the purpose of discussion it will be referred to as *plotting*.

The plane of this figure is a vertical plane and corresponds to the plane 0-1-5-4 of Fig. 1.

A star. Fig. 13 shows a method of plotting applied in laying out a star.

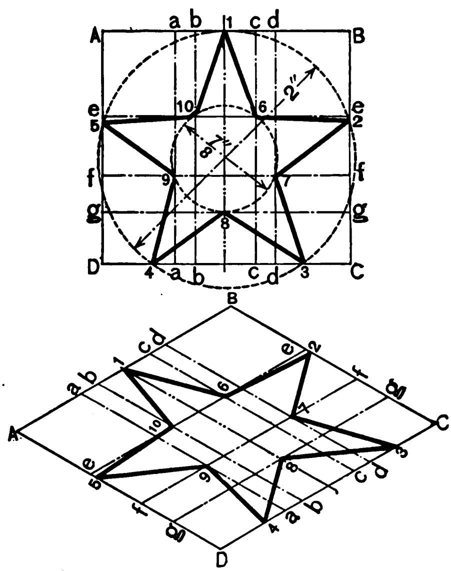


FIG. 13.

An ellipse. Fig. 14 shows the layout of an ellipse.

A parabola. Fig. 15 illustrates a parabola.

Any irregular figure. Fig. 16.

A special irregular figure.—Circular arcs. Fig. 17 illustrates a method for use when the figure is made up of circular arcs. The inclosing rectangle is obtained and drawn in the

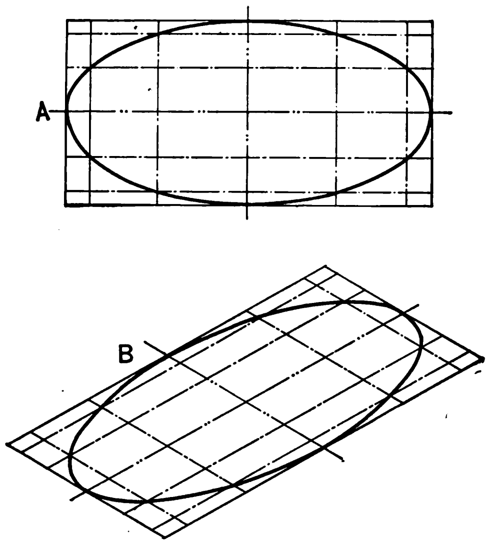


FIG. 14

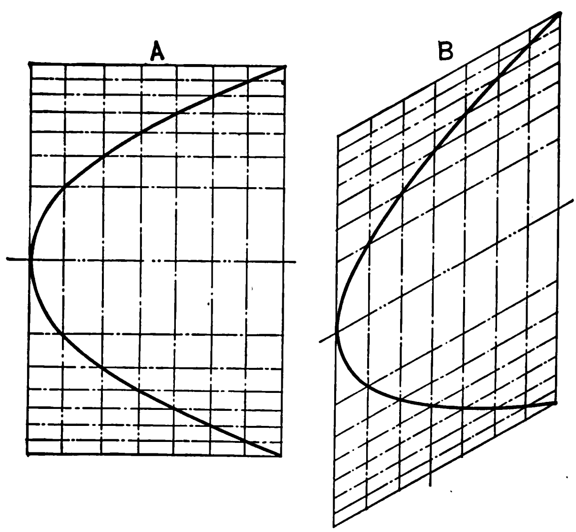


FIG. 15

usual manner; in this case the plane is an oblique plane and corresponds to the plane 0-1-2-3 of *E*, Fig. 7. The semicircle at the left end is found by laying out an inclosing rectangle

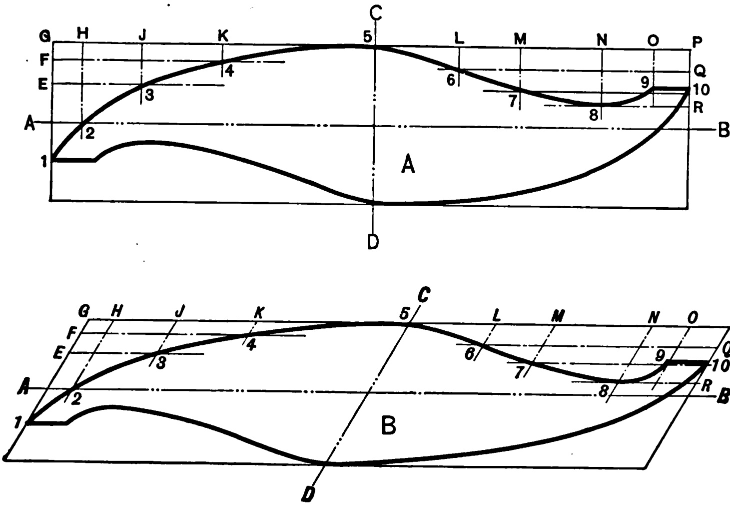


FIG. 16.

A-H-P-D, each side of which is drawn $1\frac{1}{4}''$ long, and the semicircle *E-U-E* drawn by means of two circular arcs, found as in the approximate method given for drawing a circle (page 10).

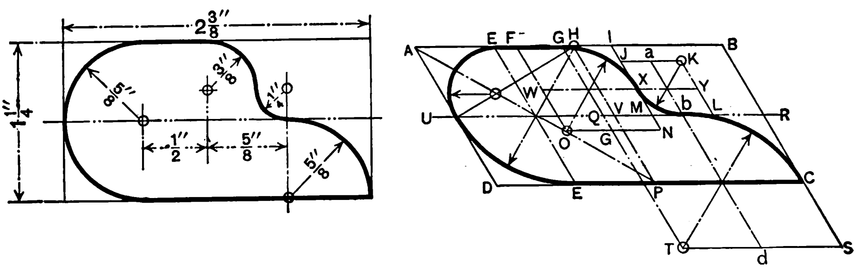


FIG. 17.

The $\frac{3}{8}''$ arc is obtained by drawing the inclosing rectangle *F-I-N-O*, each side of which is $\frac{3}{4}''$ long, from which the radius is found and the arc drawn as shown. The other arcs are drawn in a similar manner, as illustrated by the drawing.

From the above the following rule is deduced: *To draw any part of a circle, consider it as a whole circle; draw the inclosing rectangle and find the centers and radii as if going to draw the entire circumference, then use only the center and radius necessary for the desired portion.*

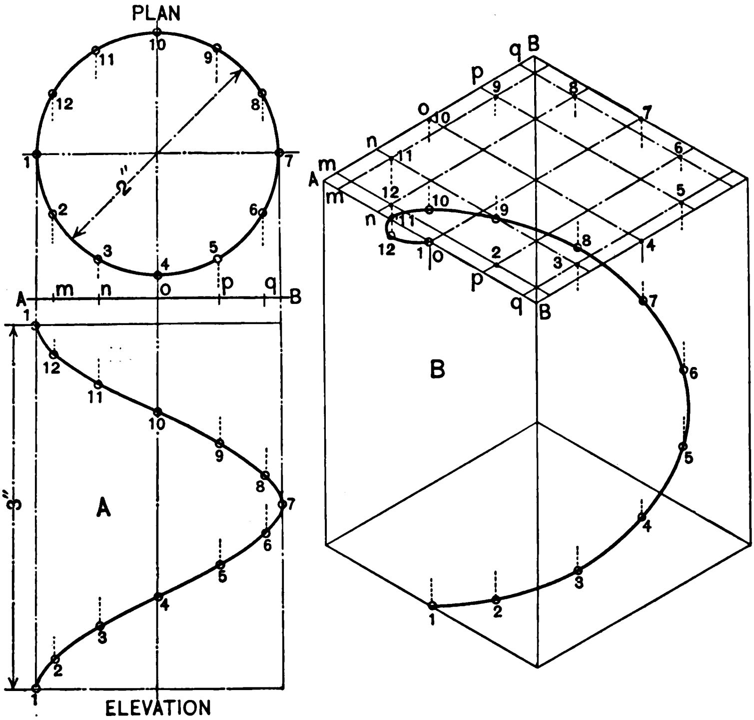


FIG. 18.

Curves.—A helix. A helix or any curve which is not a plane curve may be drawn as follows:

Plot the curve as shown by the mechanical drawing *A* of Fig. 18 and obtain the dimensions of the inclosing box; next, draw the box in isometric, then plot the plan of the curve in the proper isometric plane, as is indicated by the numbered dots; this done, draw lines parallel to the remaining isometric axis (two have been used to define the plane of the plan) through each point or dot, and on these lines lay off lengths to correspond to the plotting of the mechanical drawing. The points thus defined will be the locus of the required curve.

Solids.—A sphere. A sphere is drawn in isometric as a true circle, the radius of which is equal to one-half of the major axis of the isometric representation of a circle of the same diameter, as witness Fig. 19. The figure shows the layout for three great circles of a sphere; now it is evident that the isometric

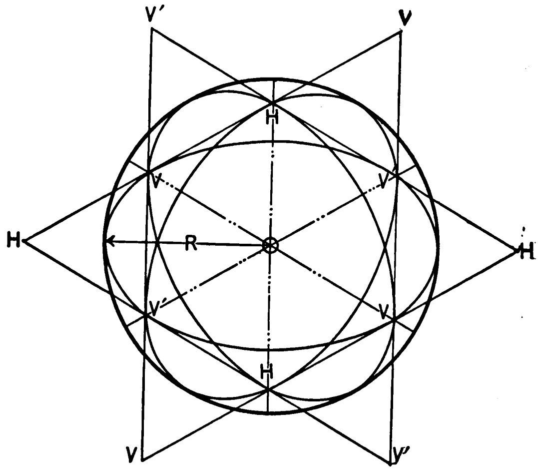


FIG. 19.

drawing of the sphere must contain all of its great circles, hence the center and radius as shown.

A solid with straight lines only. When the lines of an object are all straight lines, the execution of the drawing is very simple, and particularly so when most of the lines are parallel with the isometric axes, as is the case in the dovetail shown in Fig. 20. To draw the pieces forming the joint, draw the inclosing boxes and in these lay out the lines of the figure as shown.

Solids with curved lines. Fig. 21 illustrates the layout for a small collar. The inclosing box is obtained in the usual manner, and the drawing constructed in it as follows:

Draw the center lines of one end face, and on these construct

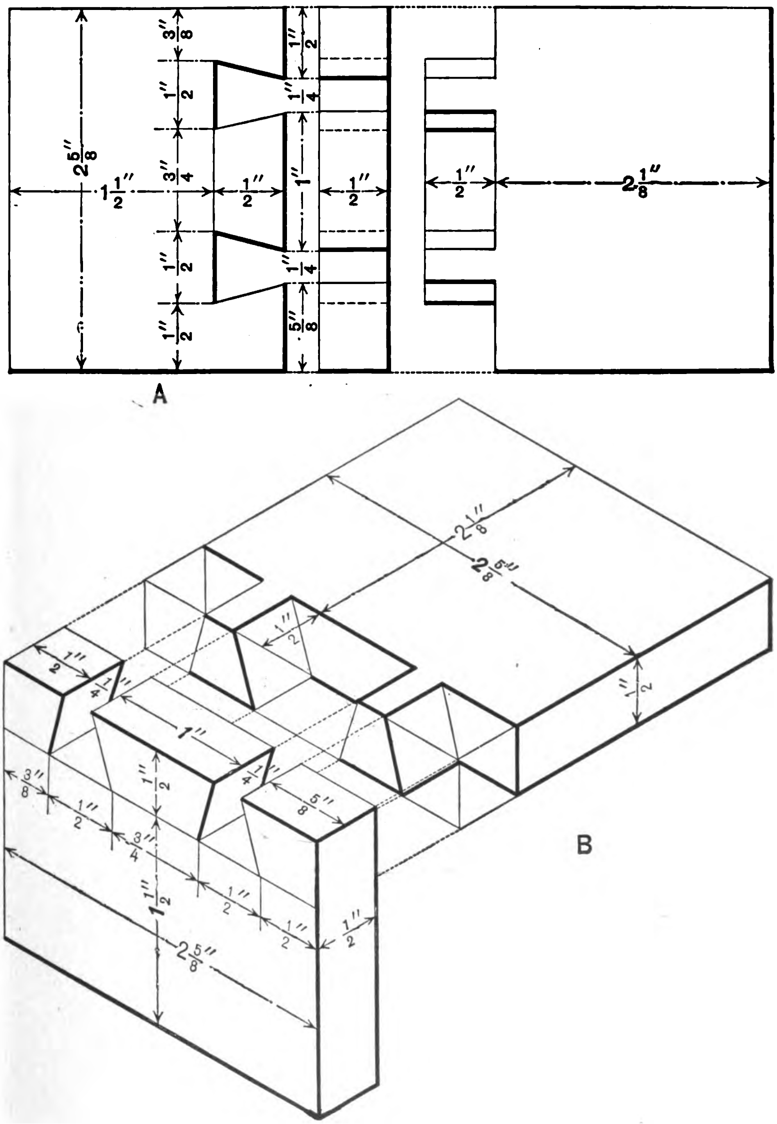


FIG. 20.

the inclosing rectangles of the circles as shown; the ellipses are then drawn as described on page 10 and as indicated by the drawing. The other end face may be laid out in a similar manner, or by a shorter method as follows: Having the centers of the arcs for the ellipse in one end face, and knowing the distance (2″) between the planes of the end faces, move the centers in the proper direction, and parallel with the axis of the collar,

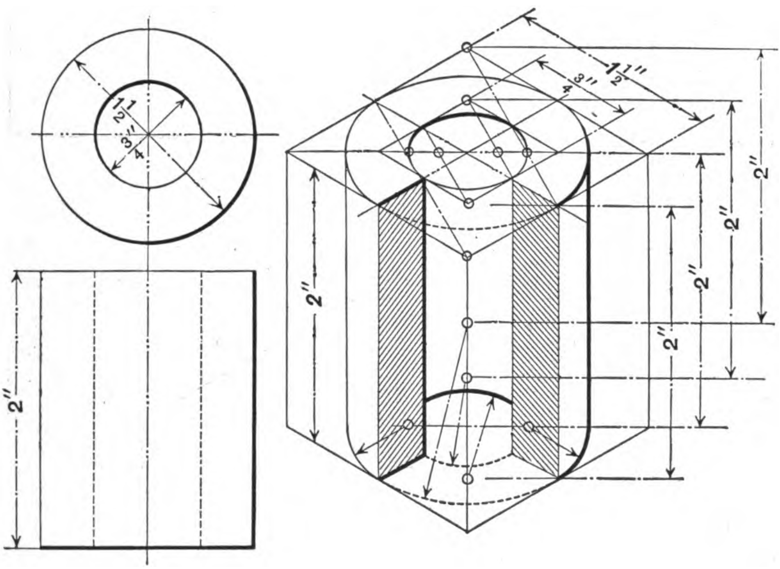


FIG. 21.

a distance equal to this dimension, then with the new centers and the same radii as used for the other end describe the necessary arcs. The ends drawn, the figure is completed by drawing tangent lines as shown. The figure shows the construction clearly, also a method of showing a half-section in isometric.

Fig. 22, depicting all of the lines of construction, illustrates a method of executing a figure when the circles are in different planes. The characteristic of the method is the use of a longitudinal center line on which the elevation or position of the several planes is made manifest by the location of their center points;

these points determined, the center lines of the rectangles, the rectangles, and ellipses are drawn substantially as already described, and as shown in the drawing.

An object with both straight and curved lines. Fig. 23

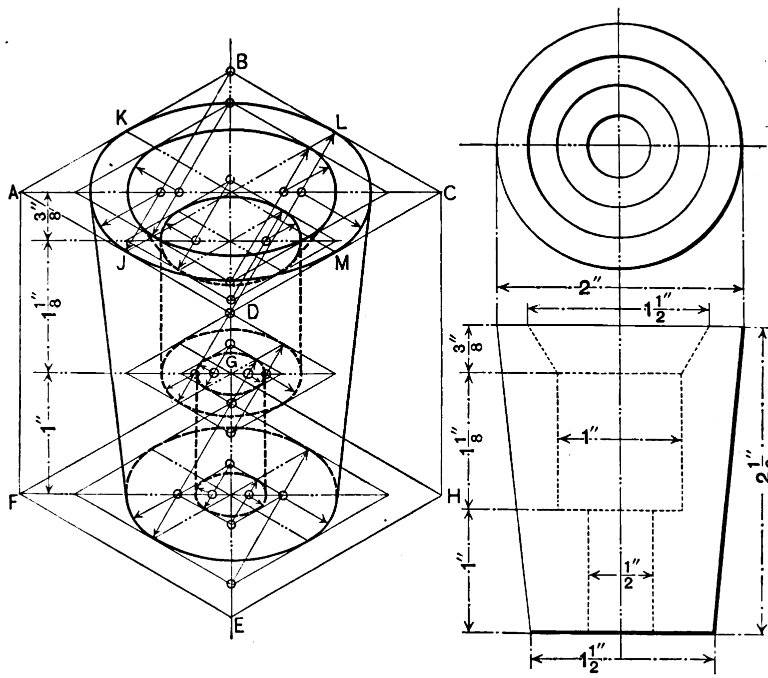


FIG. 22.

shows a method of representing an object involving the drawing of both straight and curved lines; the construction is obvious from the figure.

The drawing is a conventional representation of Fig. 5, and also a further exposition of section 2, the cube *A-B-C-D-E-F-G-H* being laid out on the $2\frac{11}{16}''$ dimension of the inclosing box. The extra lines shown are not necessary, of course, but are given for the above purpose.

The representation of screw-threads. The true representation of screw-threads is quite complicated, and is rarely done; the usual practice is the convention illustrated by Fig. 24. The threads in the nut are drawn by first constructing an ellipse in the top plane representing a circle of the diameter of the bolt at the root of the thread, then projecting the centers used in the direction of the axis of the nut, and drawing a series of parallel circular arcs as shown. For example, if the thread be an eight-pitch thread—eight threads to one inch—lay off a series of divisions

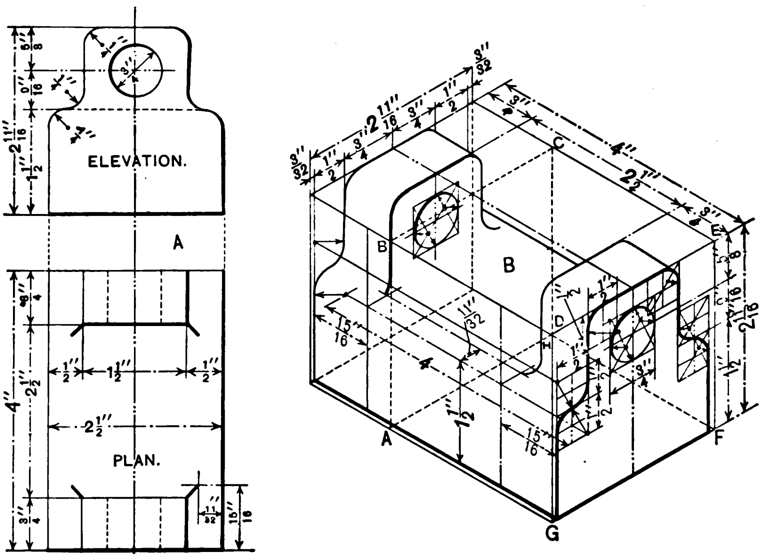


FIG. 23.

on the line of centers which are $\frac{1}{16}''$ apart; the points thus obtained will be the necessary center points. It is obvious that the distance between centers will be one-half of the distance between the points of the threads, there being a center necessary for both the top and bottom of the thread. (Understand, this is not a true representation, but a convenient convention.)

The thread on the bolt is drawn in a similar manner.

Fig. 25 illustrates a true representation of a thread, the curve (a helix) of both top and bottom of the "V" having been plotted.

It is well to note the layout of the nut in this example, as the construction is a typical one. The inclosing solid is drawn as usual, and the hexagon of the base laid out as described on page 12; this done, at each corner of the hexagon erect a line perpendicular to the plane of the base and on it lay off a

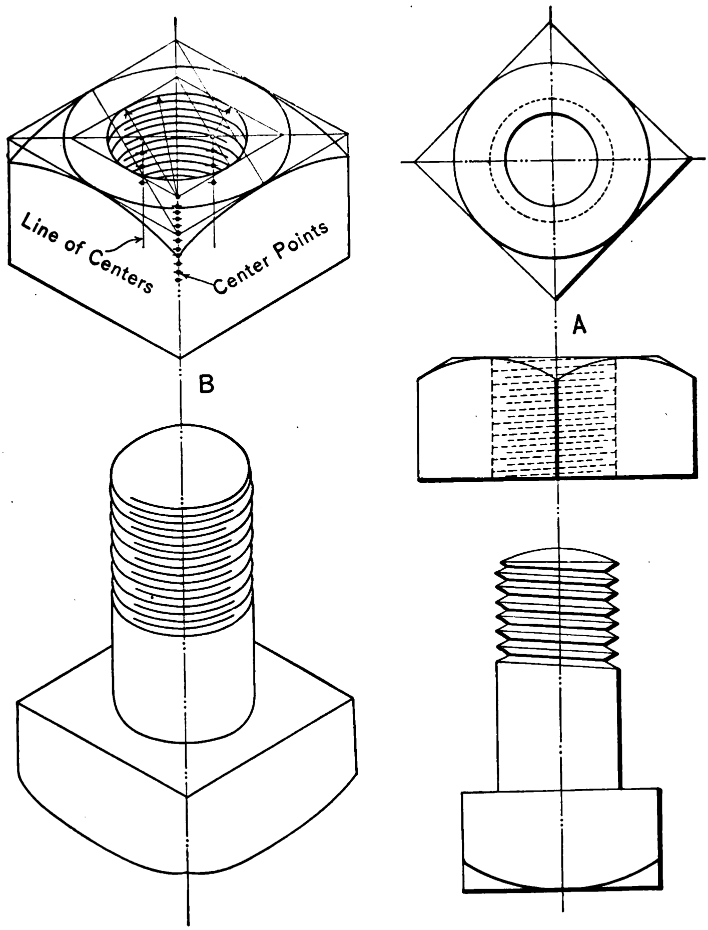


Fig. 24.

length equal to an edge of the nut; next, bisect each side of the hexagon and at the points of bisection erect other perpendiculars and on them lay off lengths equal to the length of the face of the nut at this point; this will give three points of the curved edge of each face, and through these a curve may be drawn with the irregular curve; the layout in the top plane is obvious.

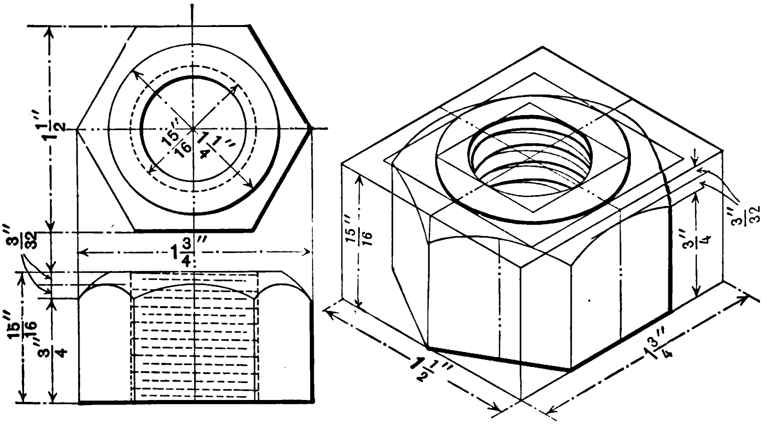


FIG. 25.

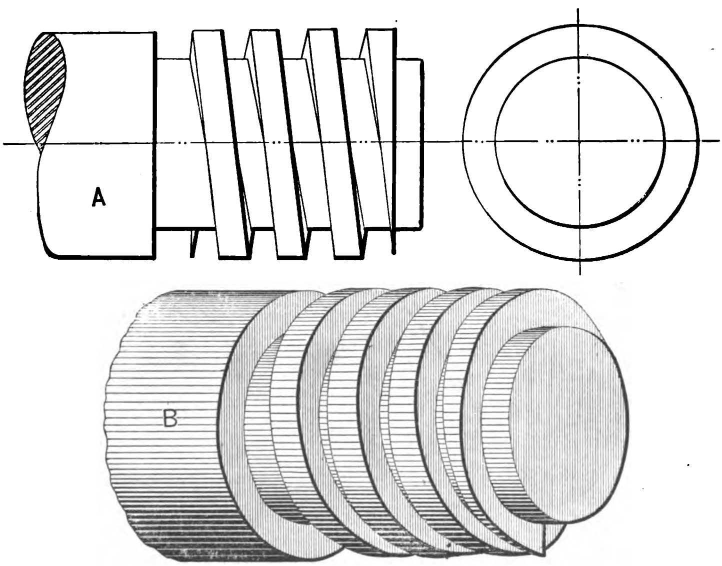


FIG. 26.

Fig. 26 illustrates a single, square thread.

10. Dimensioning.—In ordinary orthographic projection the drawings are dimensioned in two directions, horizontally and vertically; in isometric drawing the drawings are dimensioned in three directions—parallel with the three isometric axes.

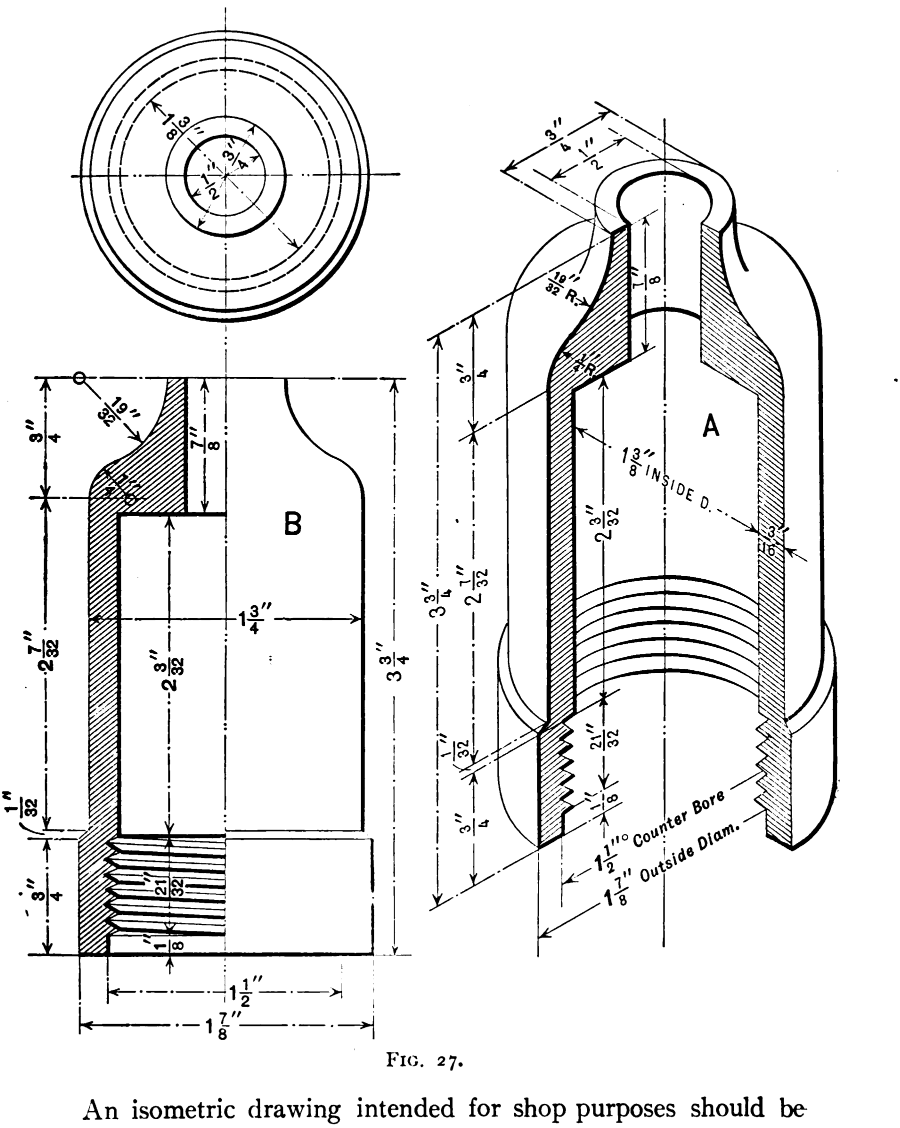


FIG. 27.

An isometric drawing intended for shop purposes should be well and completely dimensioned, as such a drawing is difficult to scale; if it is scaled, however, the scaling should be done in directions parallel with the axes.

The planning of the dimensioning is a matter of some moment, as the legibility of the drawing is dependent directly upon it.

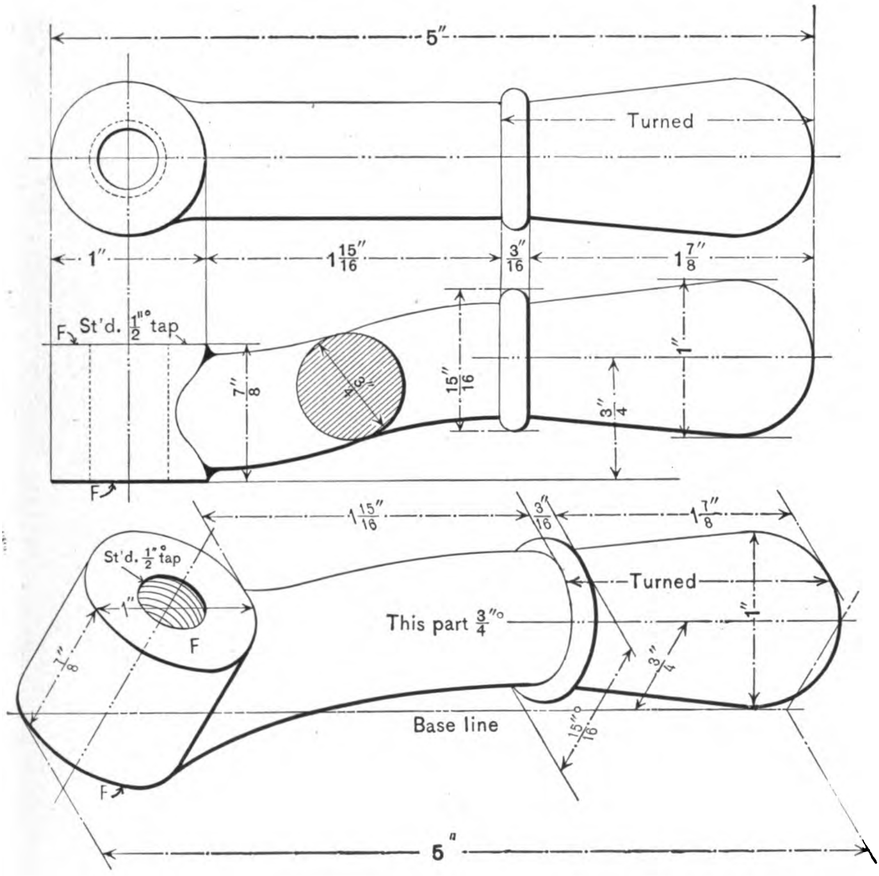


FIG. 28.

Where possible the dimension lines and figures should be so arranged as to appear to lie flat on the plane containing the representation of the part dimensioned. A number of the preceding figures are dimensioned to illustrate this point, also, that they may serve as a copy for practice in acquiring the art; it is

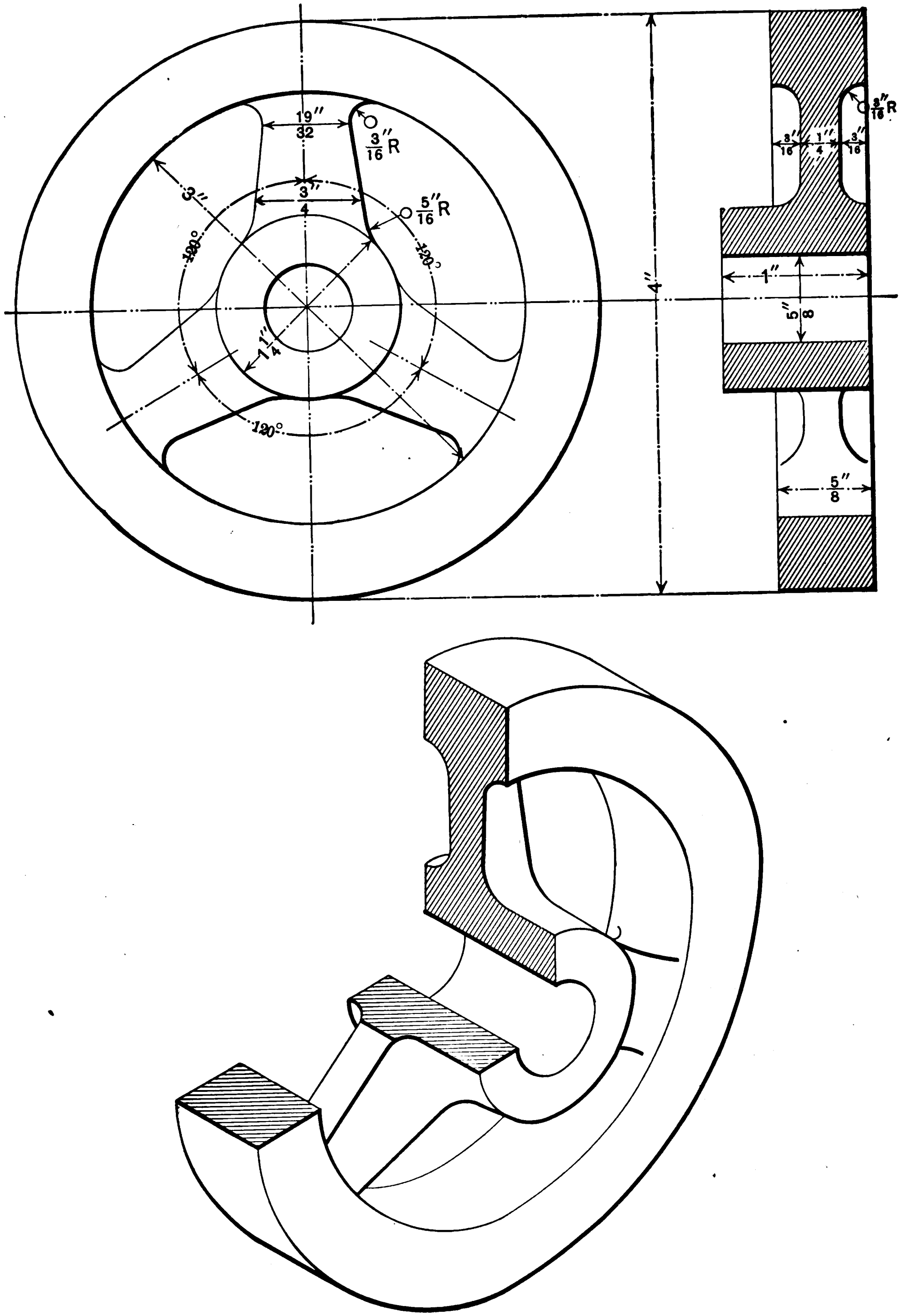


Fig. 29.

not always possible or convenient, however, to place the dimensions in the proper plane, and Figs. 27 and 28 are given as examples to be followed in such cases. Fig. 29 is given as an example for the student to dimension.

11. **Remarks.**—In mechanical drawing one finds coordinate ruled paper a great convenience for sketch-work; it may be used, also, for isometric sketches, as witness Fig. 30, though for this purpose a specially ruled paper (sometimes called "Iso" paper) is to be had of the trade. (See Fig. 31.)

In viewing Figs. 23 and 26 the eye seems to recognize that receding lines should converge, and the drawing appears distorted. This feature of isometric drawing—distortion because of parallel lines—is one of the objectionable features of the art, and one should exercise his judgment in its use, using some other method of representation in cases where the object is of such character as to produce marked distortion.

The examples given have necessarily referred directly to the mechanical drawing of the object under consideration; in practice it is often necessary to first execute a mechanical drawing of the object before the isometric drawing can be drawn; there are, however, many cases where the drawing can be executed directly from the object. In such cases the use of an inclosing box is dispensed with and the drawing is laid out with reference to a center or base line as in ordinary drawing, the dimensions, of course, being taken parallel with the isometric axes. It is well, however, to always use the inclosing box, as its outline will furnish convenient lines for reference.

The actual work of executing an isometric drawing is much less than one would suppose from a perusal of these notes, which deal with theoretical fundamentals; with a thorough knowledge of these, however, and with some little practice, many short cuts are obvious which materially shorten the process.

Cavalier Projection.

12. **Introductory.**—There is a kind of drawing closely resembling isometric drawing and isometric projection, which is

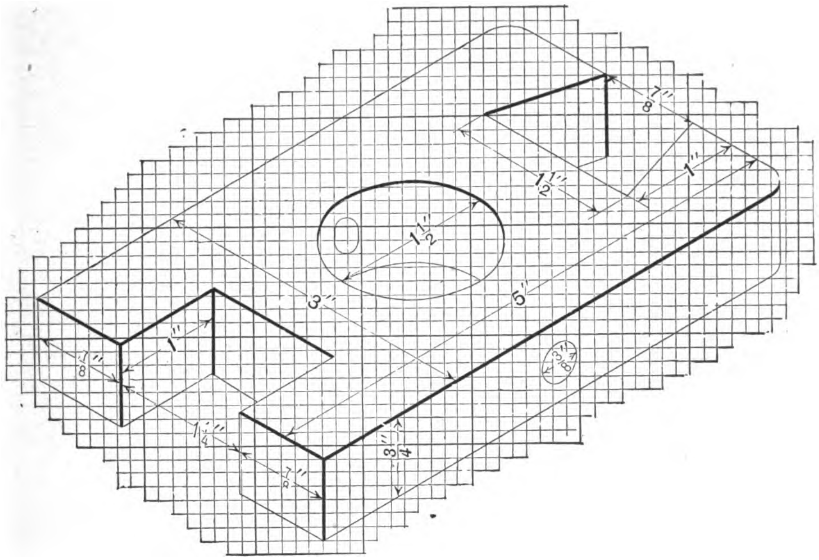


FIG. 30.

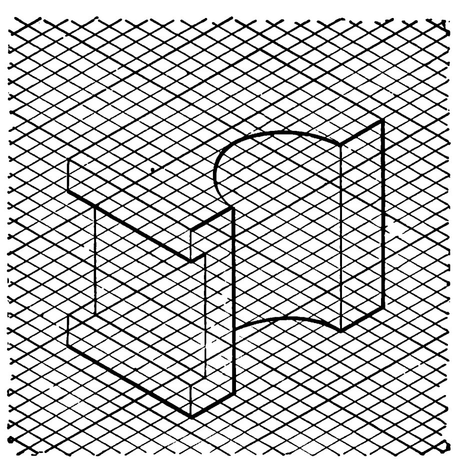


FIG. 31.

a true oblique projection, and is so drawn in practice. This class of drawing is called "Cavalier Projection"; it is more flexible than isometric drawing, it portrays three faces in a single view, is easily and readily constructed, and, altogether, is well adapted to the representation of small machine parts, rectangular objects, etc.

13. Theory.—In Fig. 32 let *E-F-G-H* be a transparent plane of projection—say a portion of the vertical plane, let *X-Y* be a line perpendicular to the plane, and let the line *A-a-Y* represent a line of sight directed at the plane from the direction *A*, and making an angle of 45° with the plane; now, it is evident that the point *a* in which the line *A-a-Y* pierces the plane *E-F-G-H* is the projection of the point *Y* on the plane; it is also evident that the point *X* of the perpendicular *X-Y* being in the plane is its own projection on the plane; therefore the line *a-X* is the projection of the perpendicular *X-Y*, and, furthermore, is equal to *X-Y*, since the line *A-a-Y* makes an angle of 45° with the plane.

Assuming the point of sight *A* to be at infinity, as in ordinary mechanical drawing, the lines of sight all become parallel, and the projections of all lines perpendicular to the plane parallel to *a-X* and equal to the lines themselves.

Assuming any direction for the point of sight, as *B*, *C*, *D*, etc., and maintaining the 45° angle with the plane, it is seen that the line *a-X*—the projection of *X-Y*—may have any desired inclination, the true length of the projection being dependent upon the oblique projection of 45°, and the angularity upon the assumed direction of sight.

14. Application of the Theory.—In Fig. 33 let *E-F-G-H* be a plane of projection, let the dashed figure 1-2-3-4-5-6-7-8 represent a rectangular block with its face 1-2-3-4 in the plane *E-F-G-H* and its side faces, 3-4-8-7, etc., perpendicular to the plane of projection, and let the lines *a*, *a*, *a*, etc., represent lines of sight from the direction *A* directed against the object and making an angle of 45° with the plane of projection. Now, as in the previous example, it is evident that the points in which

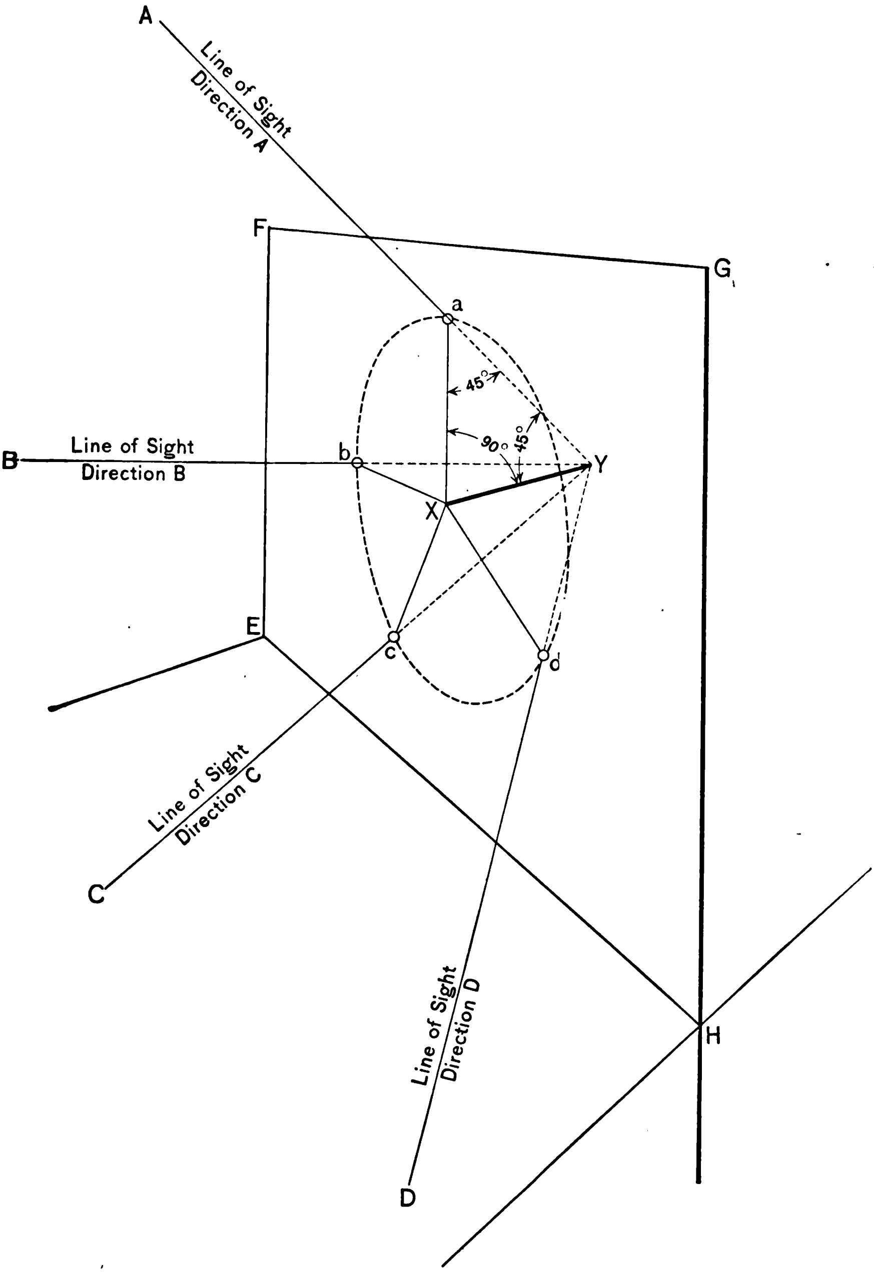

A
Line of Sight
Direction A
F
G
a
45°
90°
45°
B
Line of Sight
Direction B
b
Y
X
E
c
d
Line of Sight
Direction C
C
Line of Sight
Direction D
H
D

FIG. 32.

the lines of sight to the corners of the rear base pierce the plane of projection are the projections of the corners on the plane, and when properly joined by right lines represent the projection

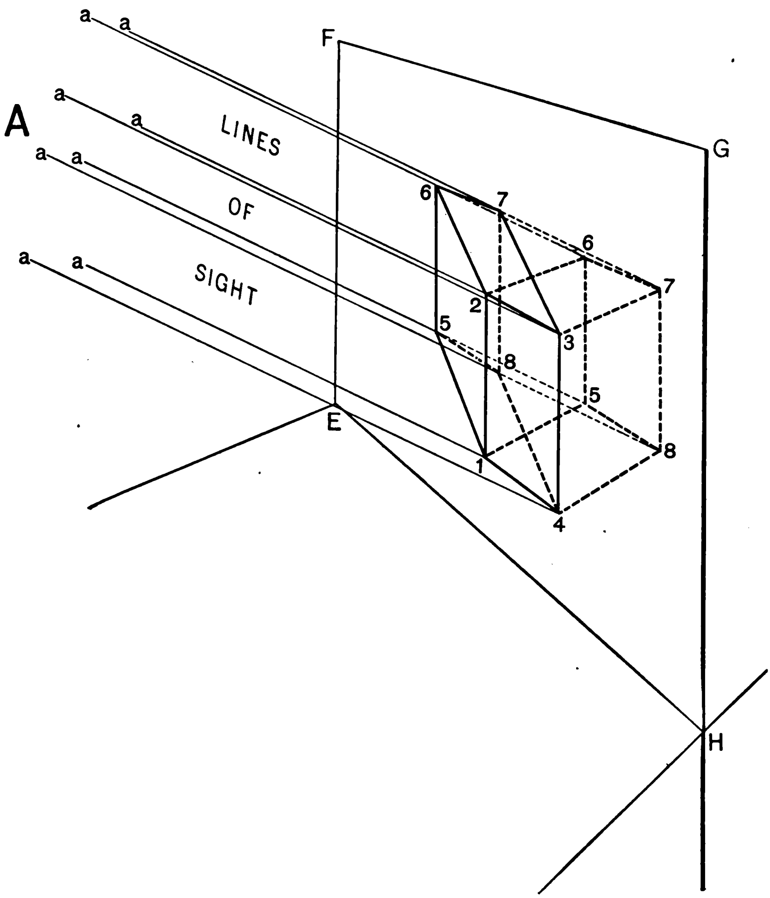


Fig. 33.

of the rear base; the front base, or face, being in the plane is its own projection, and when properly connected with the rear base the resulting figure (the full-line figure 1-2-3-4-5-6-7-8) represents the projection of the block, and the lines of the projection are equal to the lines of the object.

15. **Method of Procedure.**—The above reduced to the conventional method of representation is shown by Fig. 34, and is drawn as follows: First draw the face which is its own projection to any desired scale—full size, half-size, etc.—exactly as in ordinary orthographic projection, then, having decided upon a direction of sight, draw those lines which represent the projections of lines perpendicular to the plane of projection at an inclination (A°) corresponding to the direction of sight, and equal to the true length of the line projected.

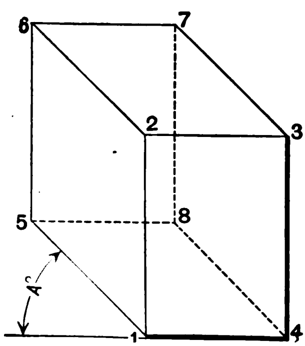


FIG. 34.

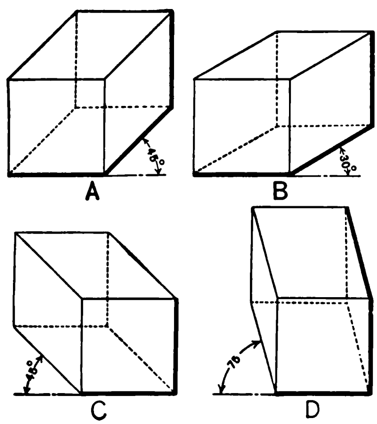


FIG. 35.

16. **Flexibility of the Art.**—Fig. 35 shows the projection of a cube viewed from four directions, and illustrates the flexibility of the art. The angle of inclination may be any angle, but usually is one which can be conveniently drawn, as with the 45° triangle, the 60° triangle, etc., and is determined in practice by the faces of the object to be pictured.

17. **Practical Examples.**—This method of representation being so like an isometric representation is readily understood and acquired by one well versed in the principles of isometry. As in isometric drawing, an inclosing box may be used and the location of points and lines obtained by means of offsets—plotting.

The method, however, has two advantages over the isometric method of representation, in that the front face of an object is drawn a true orthographic projection, and in the delineation of circles the planes of which are parallel with the plane of the front face of the object; circles other than these are drawn as ellipses.

Fig. 36 is a cavalier projection of one-half of a "split brass" (a bearing) obtained from a 60° direction of sight. Note the

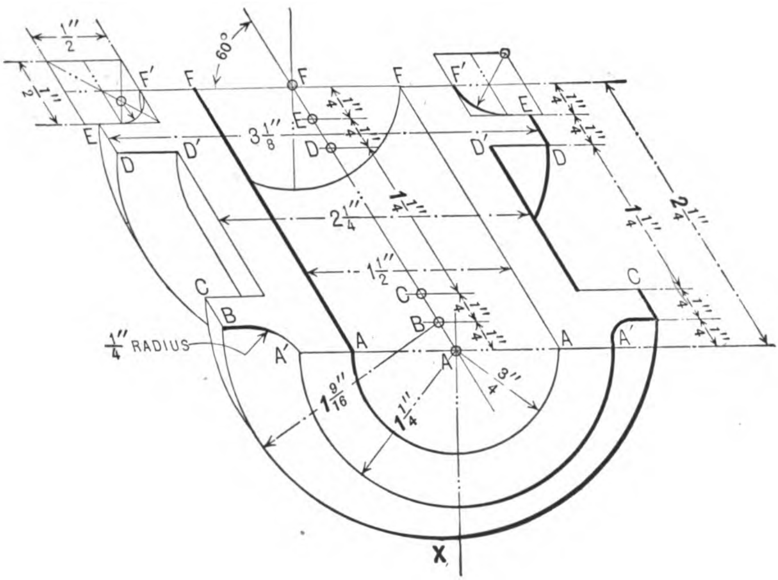


FIG. 36.

dimensioning of the figure, which is similar to that used in isometric drawing, then the construction of the drawing, which is as follows:

In cases such as this, where the object is symmetrical with certain center lines, it is well to draw the center lines first and use them as the basis of construction, as in ordinary mechanical drawing; therefore, draw the center lines *A′-A′* and *A-X* for the front face, and the 60° center line *A-F* for the top face; these lines drawn, draw the two circles of the front face, *A-A* and *A'-A'*, then locate the centers *B, C, D, E,* and *F* on the line of centers (center line) *A-F* in accordance with the dimensions of the object, and draw the other circles of the figure. The method of drawing the other lines of the projection is similar to that used in isometric drawing, and is clearly shown in the figure.

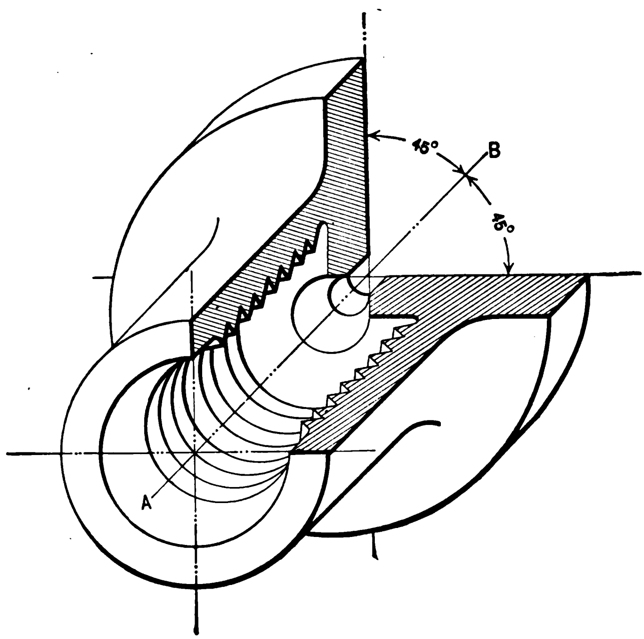


FIG. 37.

It will be remarked that ¼″ fillets are drawn exactly as in isometric; this feature is a convenience of the 60° direction of sight, and does not occur when the direction of sight is other than this; where the direction is 15°, 30°, 45°, etc., circles and arcs of circles appearing in either the top or side planes may be drawn by drawing the major and minor axes of the ellipse, the directions for which are known, as is also their extent (equal to the diameter of the circle, or double the radius of the arc), then the ellipse by any standard method; however, with the axes of the ellipse known, it is common practice to draw a rhombus and, with this for reference, to pencil in the ellipse or arc free-hand, and, when satisfactory, to ink it in with the drawing instruments.

Screw-threads. Since the real advantage of cavalier projection over isometric drawing is in the delineation of circles, it follows that the representation of screw-threads is greatly sim-

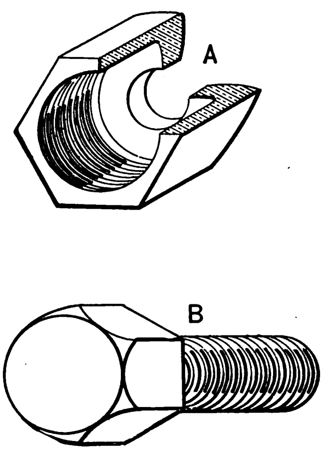


FIG. 38.

plified. Fig. 37 is a cavalier projection of a small face-plate for a wood-turning lathe, and illustrates the representation of screw-threads, the arcs representing the points of the threads being arcs of circles (the centers for which are all on the center line *A*-*B*) and all parallel.

The representation of the V's of the thread is a laborious and time-consuming construction, and can be expedited by simply indicating the thread as in ordinary mechanical drawing, as shown by Fig. 38, the distance between the arcs corresponding, approximately, to the pitch of the thread.

18. Distortion.—A reference to some of the figures shows an unpleasant distortion, as is present, also, in isometric draw-

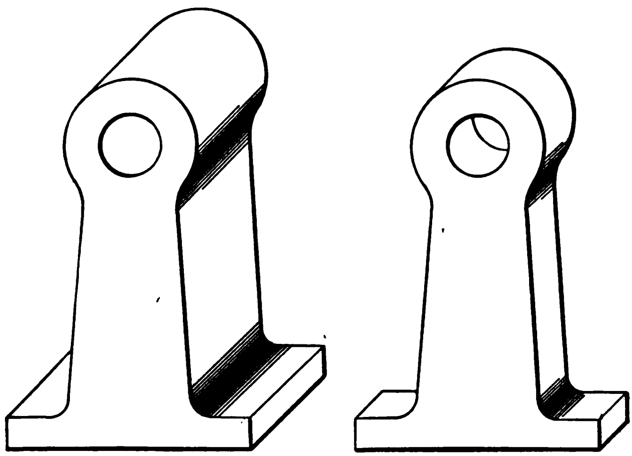


FIG. 39.

ings of certain character; where the drawing will permit of it, this feature may be eliminated to a degree, by shortening the width or length of the projection, as shown by Fig. 39.

CHAPTER II.

SHADOWS.

19. Introductory.—Without light and shade a drawing is merely a flat outline. A simple outline drawing, shade- or back-lined, answers for usual shop purposes; for catalogue and show purposes it is sometimes desirable to have a drawing depicting the light and shade. The preparation of such drawings is a trade in itself; however, the engineer, at times, may desire to produce a handsome, shaded drawing, and having a knowledge of shading to convey form (cylindrical, inclined, concave and convex surfaces, etc.), he may enhance his work by the addition of shadows.

It is the purpose of these notes to impart a working knowledge of the finding of cast shadows—a practice seldom resorted to in ordinary commercial mechanical drawing, though used to some extent in architectural work.

20. Theory of Shadows.—The rays of light are commonly assumed as emanating from the sun, and coming from such a distance they are assumed to be parallel and usually at 45° to the planes of projection, as shown by Fig. 40. These rays, unobstructed, illuminate the planes of projection; however, should a ray be intercepted, it would not reach the planes of projection and there would be a spot thereon unilluminated—a shadow. That is, if *P*, Fig. 41, be a point in space, it will intercept a ray of light and will cast a shadow on the plane of projection first reached by the intercepted ray if unobstructed—the shadow being, clearly, the point in which the ray would pierce the plane.

From the above it is evident that to find the shadow of a point one has but to pass a ray of light through it and find the point in which the ray pierces the planes of projection. For the purpose of finding cast shadows the planes of projection are assumed to be opaque and to stop the rays of light; from this it

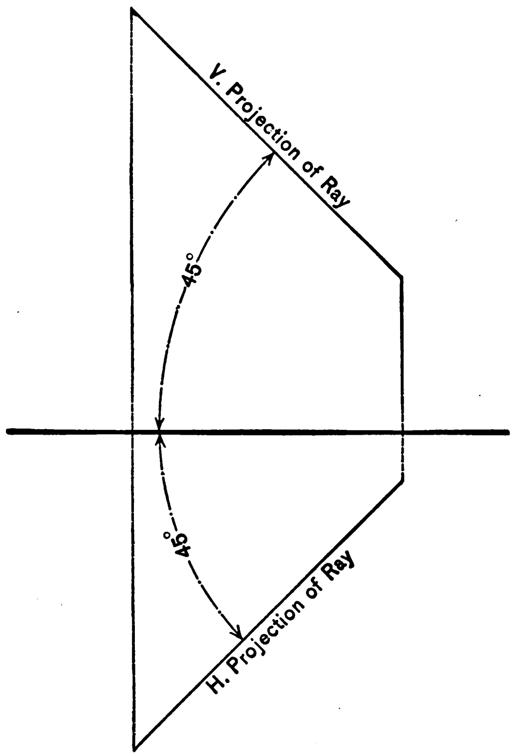


FIG. 40.

is seen that the shadow is cast on that plane first pierced by the light.

To find the shadow cast by lines and objects one has but to find the shadow of a number of points and then join the shadow-points in the proper manner.

Since the light is commonly assumed to be at 45° with the planes of projection, it is evident that the first quadrant is the only one in which the rays will cast shadows on both planes;

and since these are both visible here, it is usual to assume the object situated in the first quadrant.

21. The Shadow of a Point.—As already stated, to find the shadow of a point, pass a ray of light through the point and find the point in which it pierces the planes of projection—the shadow being the point of piercing the plane first reached.

It will be remarked that this is a practical application of the elementary problem in the Descriptive Geometry, "Find

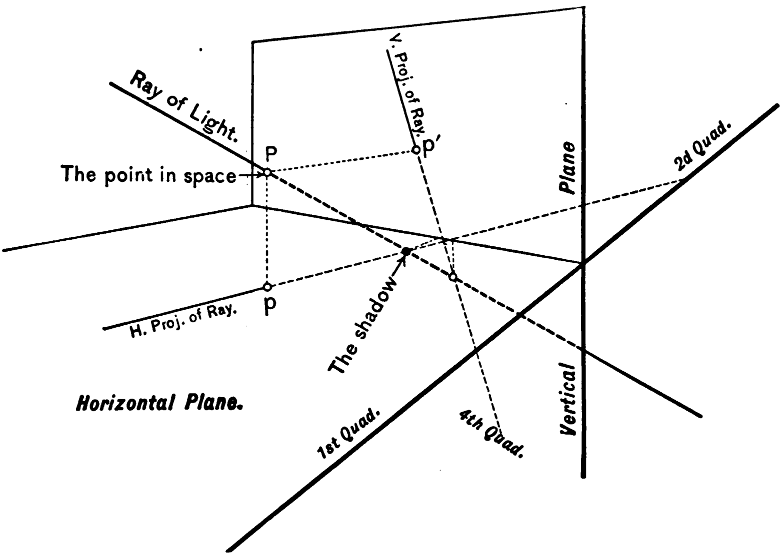


FIG. 41.

the point in which a line pierces the planes of projection." The procedure is a simple one, but, as it is the basis of all shadow-work, the student should understand it thoroughly; he must remember that a point has *two* projections; also, that a line has *two* projections, and that in passing a ray of light through a point by the convention used in drawing, he must draw the *vertical projection of the ray through the vertical projection of the point, and the horizontal projection of the ray through the horizontal projection of the point,* and that the point on the plane of projection first reached by the ray is the shadow-point.

To illustrate the above, consider Fig. 42: *A* shows the projections *p* and *p'* of a point *P* in space through which a ray of light is passed; this ray is seen to pierce the vertical plane first, hence the shadow of *P* falls on the vertical plane. *B* shows

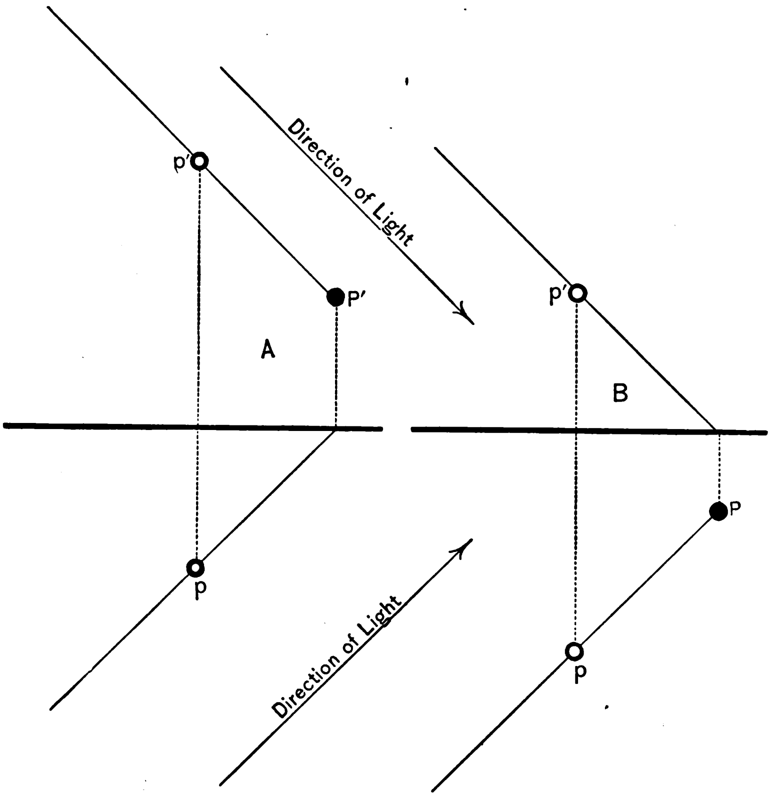


FIG. 42.

tne projections of a point *P* so situated with reference to the planes of projection that the shadow falls on the horizontal plane.

22. The Shadow of a Right Line.—Since a line is made up of points, any two points in the line may be taken (provided the same points are shown in the two projections, that is, provided the points are properly projected) and the shadow of these points found as directed above, and the shadow of the line obtained by joining the shadow-points by a right line. If the line be a definite line, the two points taken are, clearly, the two extremes of the line.

The shadow falling on one plane only. *A*, Fig. 43, shows the projections of a definite line *M*-*N*, through the extremes of which rays of light are passed and the shadow (*M'*-*N'*) found in accordance with the above explanation. It will be

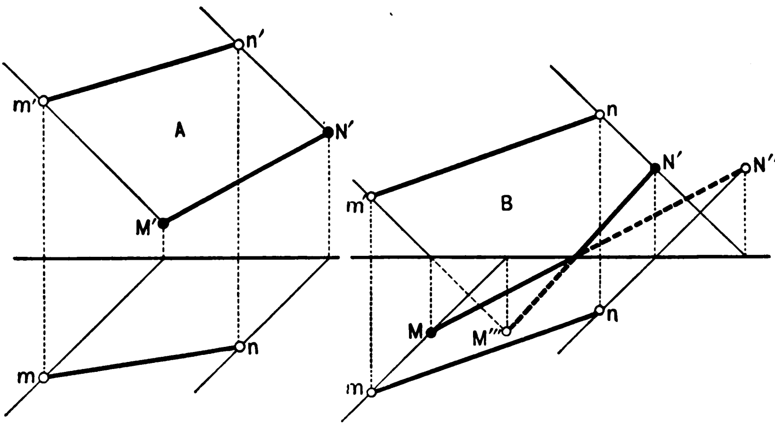


FIG. 43.

noted that both of the rays used pierce the vertical plane first, and that of a consequence the shadow falls on this plane.

The shadow falling on both V and H.* When a line is so situated that its shadow falls on both of the planes of projection, it is necessary to find the points in which the rays pierce both *V* and *H*, the points on the planes first reached by the rays not being sufficient, as is apparent from an inspection of *B*, Fig. 43. Here is depicted the projections of a line *M*-*N*, through the extremes of which the projections of rays of light are drawn; it is seen that the ray through the point *M* pierces the horizontal plane first, and that the point *M* on this plane is the shadow of the *M* extreme of the line. Now, it is evident that to

* In this discussion the letter H stands for the horizontal plane of projection and the letter V for the vertical plane of projection.

find the shadow of the line on the horizontal plane, a second shadow-point must be found on it, and, naturally, one finds the shadow of the other extreme of the line. The ray through this point is seen to pierce the vertical plane first, then to continue on and pierce the horizontal plane in the second quadrant. Joining the two shadow-points on the horizontal plane gives the shadow of the line on *H*, but since the first quadrant only is considered, that part of the connecting line which is in the first quadrant is the only part of the shadow visible.

Having the shadow of the *N* extreme on the vertical plane,

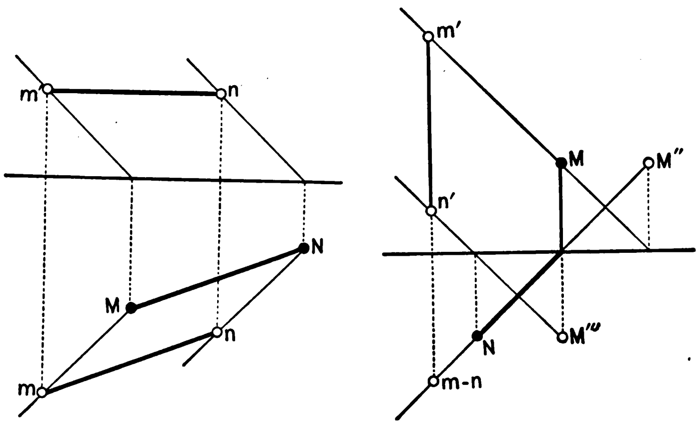


FIG. 44.

the shadow of the line may be completed by joining the point in which the shadow on the horizontal plane crosses the ground-line with this point, or, by finding the shadow of the *M* extreme on the vertical plane (this is seen to be in the fourth quadrant) and joining this point with the *V* shadow of the *N* extreme. If this latter method is used the *H* and *V* shadows of the line should cross at the ground-line.

The shadow of a line when parallel to one of the planes of projection. *A*, Fig. 44, shows the projections of a line *M-N* which is parallel to the horizontal plane of projection, together with its shadow, *M-N*, on this plane (the line is so assumed that its shadow falls entirely on *H*). Now, since the line is parallel to H, and since the rays of light are assumed to be parallel, it follows that the length of the lines m-M and n-N are equal and the line M-N therefore parallel and equal to m-n.

B, Fig. 44, shows the projections of a line which is perpendicular to H, and hence parallel to V, together with its shadow, which is seen to fall on both H and V (the line being so assumed). The method of finding the shadow is the same method as is described on page 42.

An inspection of the shadow shows that that part of it falling on H is in the direction of the H projection of the rays of light, and that portion falling on V is parallel to the V projection of the line.

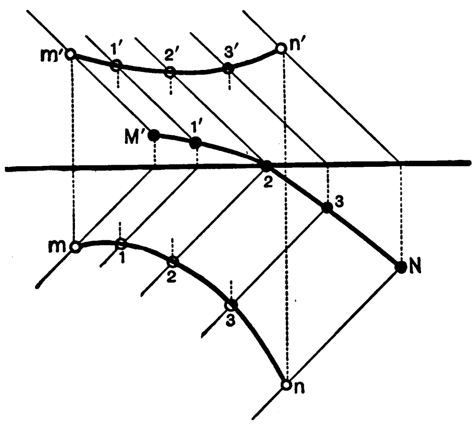


FIG. 45.

The above examples demonstrate two points which should be well fixed in mind, as a knowledge of them will greatly expedite the work of finding cast shadows. These points are:

(1) *The shadow of a line on a parallel plane is parallel and equal to the line.*

(2) *The shadow of a line on a plane to which it is perpendicular lies in the direction of the projection of the rays on that plane.*

23. The Shadow of a Curved Line.—Fig. 45 shows the projections of a curved line M-N, together with its shadow

M'-1'-2-3-N. The line is divided into a number of points, the shadow of each point found, and the shadow-points joined with a curved line. Therefore,

To find the shadow of a curved line, find the shadow of a number of points in the line and join the shadow-points with a curved line.

Since the shadow of a line on a parallel plane is parallel and equal to the line, it follows that

The shadow of a plane curve on a parallel plane is parallel and equal to the curve.

For example:

To find the shadow of a circle on a plane to which its plane is parallel, find the shadow of the center of the circle, and with this point as a center and a radius equal to the radius of the circle draw a circle; this circle will be the shadow of the given circle.

24. **The Shadow of Solids. Plane Surfaces.**—*A*, Fig. 46, shows the projections of a small, rectangular block, the eight corners of which are numbered and the shadow cast by each corner shown. The shadow cast by the lines or edges of the object are shown by the right lines joining the shadows cast by the corner points; thus the shadow-line 8-4 is the shadow of the edge 8-4, etc.

An inspection of the figure shows conditions such that the shadow falls entirely on the horizontal plane; also, that the top and bottom planes or bases of the block are parallel to *H*, and that the lines or edges joining the two bases are perpendicular to *H*. It is interesting to note, then, that the shadows of the lines of the bases are parallel and equal to the same lines on the object, and that the shadow-lines joining the shadow bases lie in the direction of the H projection of the rays of light.

A further inspection of the figure shows a number of the shadow-points and lines to fall within the outline—the limits—of the shadow. This presents the real problem in shadows, i.e., to find the outline of the shadow cast by an object without finding the shadow of all of its points and lines. To find the shadow of every point and line of an object, many of which fall within the limits of the shadow, is, obviously, both time-consuming and laborious and to be avoided in so far as practicable. To this end, then, the student must be able to read his drawing—the presentation of the object by the two projections—in such a manner as to conceive of the object as occupying the position of the projection on the paper, to actually stand out from the

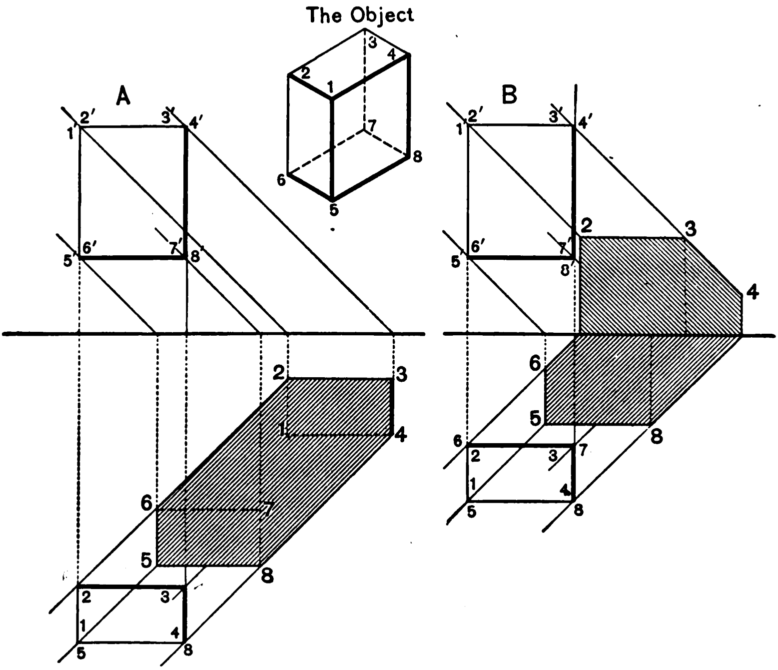


FIG. 46.

surface as if the object rested there, and with the object thus in mind carefully study it and select those points and lines which will cast the desired outline. These points and lines known, it is then a very simple matter to find the shadow, as witness *B*, Fig. 46.

Having considered *A* of the same figure, it is known that the lines 5-8 and 5-6 of the lower base, the edges 6-2 and 8-4 of the side faces, and the lines 2-3 and 3-4 of the upper base cast the outline of the shadow of the block. *B* shows a new position of the object with reference to the planes of projection, but not with reference to the rays of light, and hence the same lines 6-2, 8-4, etc., will cast the required outline. This known, note the method of finding the shadow (*B*, Fig. 46). The correctness of the shadow may be recognized and the labor of execution further reduced if one will remember that the shadow of a line on a parallel plane is parallel and equal to the line; thus, having found the shadow-point 8, the shadow-line 8-5 may be drawn parallel and equal to the edge 8-5 of the block, the shadow-line 5-6 may be drawn parallel and equal to the edge 5-6 of the block, etc.

The shadow on the object. Fig. 47 illustrates an object of

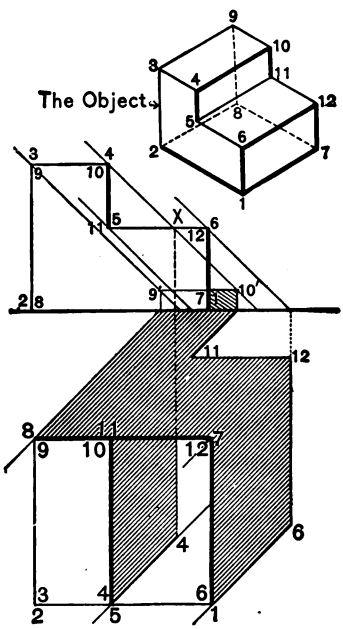


FIG. 47.

such form and position with reference to the planes of projection that, in addition to the shadow cast on the planes of projection, it casts a shadow on itself.

The shadow cast on the planes is found as in the preceding example, and as shown by the drawing (note the shadow cast by those lines which are parallel and perpendicular to *V* and *H*). The shadow cast by the object on itself is found as follows:

As already explained (page 45) the drawing must be studied and those lines selected which will cast the outline of the shadow. Such an inspection of the figure, then, shows that the lines 5-4 and 4-10 are the only lines concerned in the shadow, and that the shadow will fall on the plane 5-11-12-6; note, also, that this plane is parallel to *H* and that the line 5-4 is perpendicular and the line 4-10 parallel to this plane. Now the line 5-4 being perpendicular to the plane 5-11-12-6, its shadow will lie in the direction of the projection of rays on this plane, and the point 5 being in the plane will be its own shadow; to find the shadow of point 4, pass a ray of light through it and find the point in which it pierces the plane 5-11-12-6; this point is seen to be point 4 on the plane 5-11-12-6, and the shadow of the line 5-4 to be the line joining the point 5 and this point. (Note how the shadow of point 4 is found: how the V projection of a ray through the V projection of the point strikes the V projection of the plane at *X*, and the projection of this point to the horizontal projection of the ray through the horizontal projection of point 4; that is, how the V projection of the plane 5-11-12-6 serves as a groundline, as it were.) The shadow-point 4 determined, and knowing that since the line 4-10 is parallel to the plane 5-11-12-6, its shadow thereon will be parallel and equal to the line itself, the shadow of this line may be found by drawing a line through the shadow-point 4 parallel to the edge 4-10, and of a length equal to the length of the edge; such a length, however, carries the shadow off of the object and onto the horizontal plane, and falling there within the limits of the shadow is disregarded.

Fig. 48 shows a second example of a shadow falling on the object. The method of finding the shadow on the planes of projection and on the horizontal projection of the object is the same as has already been described and illustrated, and as is shown by the figure. However, the object is of such form and position with reference to the light that a new feature is introduced in that a shadow is cast by the object on itself on a plane which is visible in its vertical projection. This shadow is seen to be the shadow of the lines—edges—2-6 and 6-5 on the plane 1-2-3-4. These lines being perpendicular and parallel respectively to the plane, the shadow is found as in the preceding example, and as is shown by the drawing.

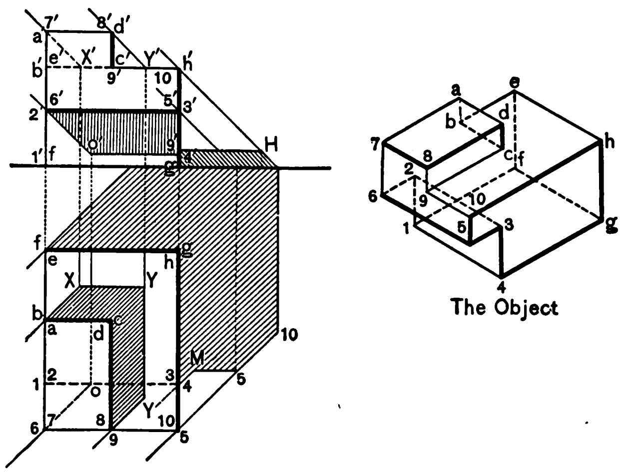


FIG. 48.

Single-curved surfaces. Fig. 49 illustrates the plan and elevation of a section of a hollow cylinder and introduces the finding of cast shadows on single-curved surfaces.

A study of the figure shows that it casts a shadow on itself and on the planes of projection; also, that the limits of the shadow on the planes is cast by elements *C-C*, *A-A*, and *B-B*, and the curve *B-A* of the top base of the cylinder, and the shadow on itself by the element *C-C*, and the lines *C-E* and *E-D* of the upper base. The shadows of the straight lines are found as already described, and those of the curved lines as directed on page 44, and as shown on the drawing by the points *a* and *b*.

Fig. 50 is a second example of a single-curved surface and illustrates a method for finding the shadow of a right line on the surface.

The shadow on the planes of projection is found in the usual way, and the shadow on the figure as follows:

A study of the object shows that the shadow on the object is cast by the projection of the top or cap, and that the shadow will show in the vertical projection of the figure. Now it is evident that when viewing the vertical projection one sees but

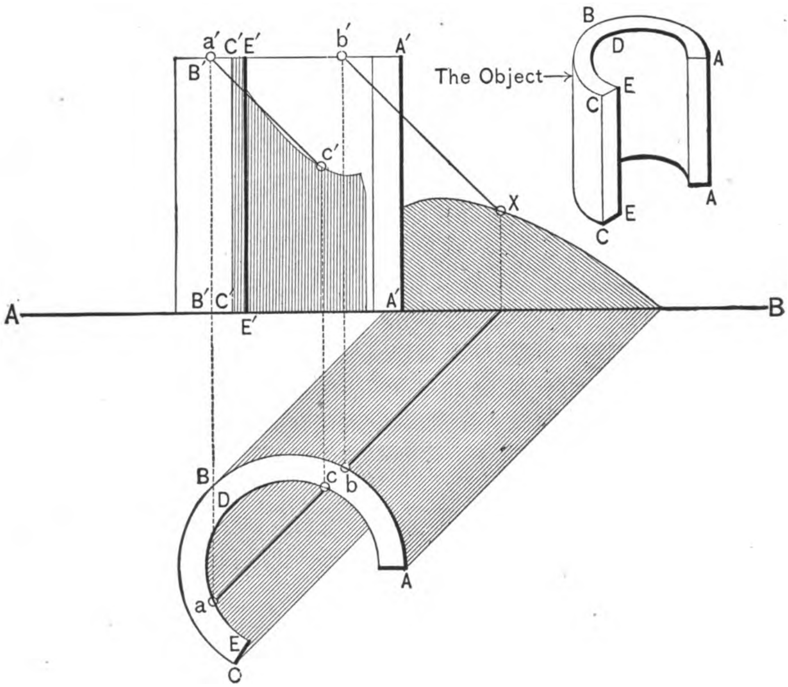


FIG. 49.

one-half of the figure, this half being that half in front of a vertical plane parallel with *V* passing through the center of the object. The point *a*, then, in the line 1-8 is the first point on the left to cast a visible shadow. To find the shadow cast by the line *a*-8, divide it into a number of points, as *b*, find the shadows of these points, and join them with a curved line, as shown; the shadow of the line-edge 8-7 is obtained in a similar manner.

A further inspection of the figure shows that the ray of light through the point *d* of the line or edge 7-6 is tangent to the single-curved surface—the base of the figure—at the point *k*, and is the last point on the right to cast a shadow on the object, the remainder of the line 7-6, *d*-6, casting its shadow on the horizontal plane.

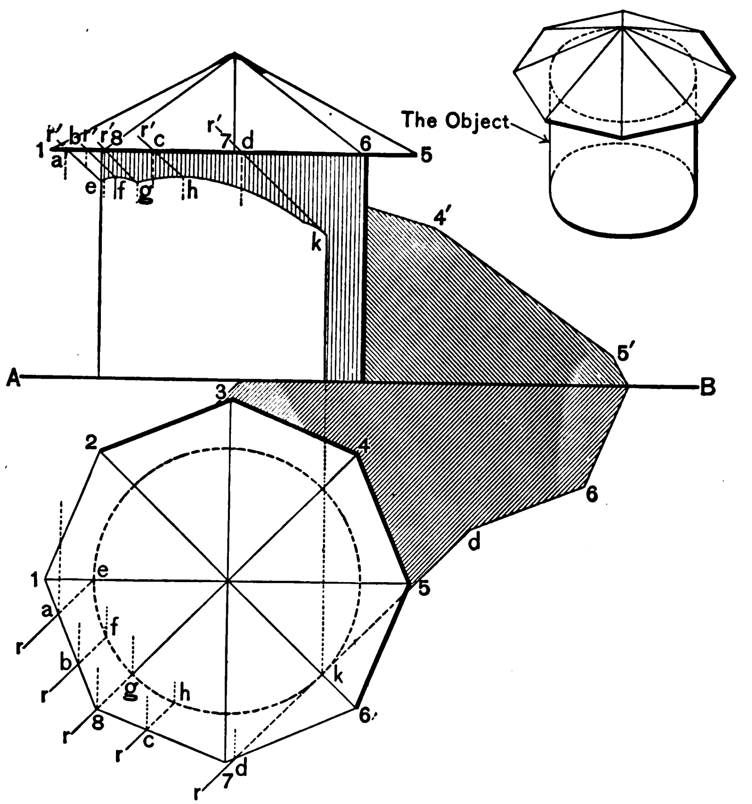


FIG. 50.

(It is interesting to note that this shadow is parallel and equal to the line *d*-6.) Now, the ray *d-k* being the tangent ray it is obvious that all of that portion of the surface beyond the element through the point of tangency, *k*, is in the shadow, and is so shown in the vertical projection.

Double-curved surfaces. Fig. 51 represents the projections of a block so hollowed out as to present a surface part of which is of single curvature and part of double curvature, and is typical of the architectural niche designed to house a statue.

The drawing shows the shadow within the niche only; it is found as follows:

The shadow on the single-curved portion of the recess is

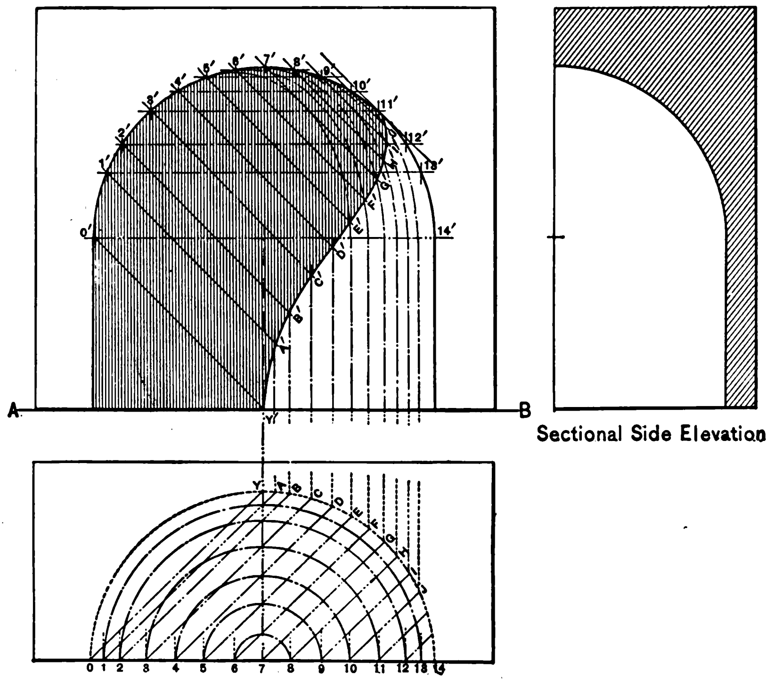


FIG. 51.

found as has already been described, and as is shown by the drawing. To find the shadow on the double-curved surface, pass a series of vertical projecting planes through the surface and parallel with H, as shown by the horizontal lines 0′-14′, 1′-13′, 2′-12′, etc., of the vertical projection. These planes will intersect the surface in semicircles which are visible on the H projection as the curves 0-14, 1-13, 2-12, etc.; next, through any point casting a shadow on the surface, as point 6, pass a ray of light, then through the ray pass its horizontal projecting plane, and by projecting from the points of intersection of the H trace of this plane with the H traces of the above auxiliary planes to the V traces of the auxiliary planes find the V trace of the plane of rays on the surface; this trace is the curve $6'$-F'. Now it is evident that the shadow of the point 6 will lie in the trace of the plane of the ray—the curve $6'$-F', also, that the shadow will lie in the projection of the ray, therefore, the intersection of the line $6'$-F'—the V projection of the ray—with the curve $6'$-F'—the trace of the plane of the ray on the surface—is the required shadow. The shadow is completed by selecting a number of points and finding their shadows in a similar manner, then joining these shadows by a curved line, as shown.

The feature here introduced is the use of a plane of rays, and is to be employed whenever the surface is such that the two projections of a ray are not sufficient to indicate the point in which the ray pierces the surface.

Fig. 52 illustrates the shadow cast by a sphere. The shadow may be found by locating the great circle of contact of the rays of light by the method used in the dome of the niche in the previous example, or as follows:

Pass a cylinder of rays about the sphere: this will define the great circle of contact, and this found, select a number of points in it, find their shadows, then join these with a curved line.

In executing the above analysis, one meets with a practical application of the problem in Descriptive Geometry, "Pass a circle through three points," as witness the figure:

It is obvious that the horizontal projection corresponds with a horizontal section of the sphere, vertically represented as the line X-Y, and that the vertical projection corresponds with a vertical section of the sphere horizontally projected as the line M-N; it is evident, then, that the tangent-points A and B defined by the horizontal projection of one set of limiting rays of light, and the tangent-points E and D defined by the vertical projection of a second set of limiting rays, may be readily projected, and will represent the projections of points in the great circle of contact of the cylinder of light with the sphere.

With four points of the circle of contact known, it is an easy matter to pass a plane through any three of them, to revolve the plane of the points into coincidence with one of the planes of projection, then, while in this position, to pass a circle through

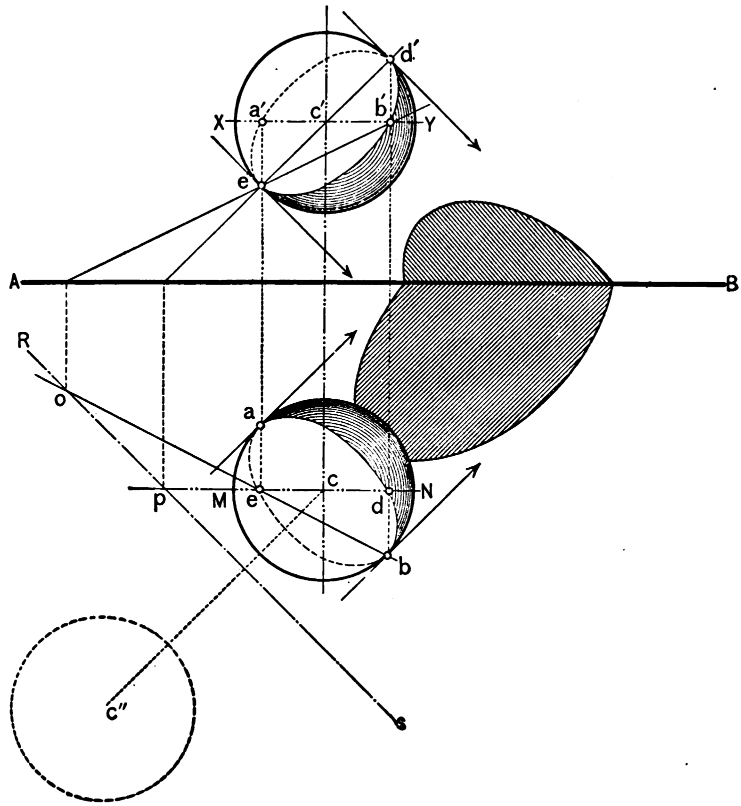


FIG. 52.

the points, and then to revolve the circle back to the original position of the points; the projections of the circle will then represent the projections of the great circle of contact of the cylinder of light. The circle of contact defined, the shadow on the object is obvious, and the shadow on the planes found by finding the shadows cast by a number of points in the circle of contact, then joining these shadow-points with a curved line.

25. Remarks.—Since the ordinary engineer draughtsman is so little concerned with the finding of cast shadows, it is thought that this brief discussion of the subject is sufficient; the examples given are typical ones, and carefully studied show that there is but one principle involved, i.e., that principle of orthographic projection enabling one to find the points in which a given line pierces a plane or surface of projection. The subject, however, is of importance in the training of the student engineer in that it shows a practical application of the principles of descriptive geometry, and affords excellent practice in the reading of drawings.

CHAPTER III.

PERSPECTIVE.

26. Definition.—Perspective drawing, or Linear Perspective, commonly called "Perspective," is the art of representing an object or objects on paper or other plane surface in such a manner as to present the object as it would appear when viewed from a definite viewpoint.

27. Perspective and Mechanical Drawing Compared.—Perspective differs from mechanical drawing, which presents an object in detail—each face or side separately, and as it really is, and not as it appears to the eye—in that it presents the object as a whole, showing several faces or sides in a single drawing, and as it would appear if viewed from a given standpoint.

28. Mechanical and Free-hand Perspective.—These terms are used relatively. By "mechanical" perspective is meant that perspective drawing of an object which is drawn—said to be "found"—from the mechanical drawings of it; that is, to execute a mechanical perspective of an object, one has to first prepare mechanical drawings of the faces or sides it is desired to picture and then use these drawings to find the perspective.

By free-hand perspective is meant that perspective drawing which is drawn directly from the object or scene; that is, the artist, or draughtsman, prepares this perspective by simply looking at the thing to be pictured and then draws—free-hand—his conception of it. The work of the newspaper artist, the landscape- and portrait-painter, etc., are examples of "free-hand perspective."

The student of mechanical drawing is primarily concerned with mechanical perspective, and it is this class only which is here discussed.

29. Perspective as Applied by the Engineer.—The art of perspective drawing is of minor importance to the engineer, since his conceptions are best expressed by simple mechanical drawings. An exception, however, is met with in the case of the architectural engineer. To him perspective drawing is equally as important as mechanical drawing, since he "pictures" his proposed work.

While it is true that the art is mostly applied by the architectural engineer, it is equally true that all engineers, sometime, find it desirable to picture a conception, and therefore it is well to have a working knowledge of the principles of the art.

The following examples and remarks, then, are not designed to produce expert perspective draughtsmen, but as the fundamentals are given, when these are understood one may become quite expert in the art with practice.

30. Theory of Perspective.—If an object is viewed from a finite point of sight, the lines of sight will converge at the eye (one sees with two eyes but in perspective a single point of sight is assumed, and to make the analogy correct the observer is supposed to close one eye), and if they be intersected by a plane —usually assumed to be between the object and the point of sight for reasons explained later—and the points in which the several lines of sight pierce it be properly connected, a drawing is obtained which represents the object, decreased in size, exactly as it appears to the observer. (Fig. 53.)

The intersecting plane assumed in practice is the vertical plane of orthographic projection, being assumed because of its position, which enables one to place objects to be pictured with a large number of their principal lines either parallel or perpendicular to the plane, thus expediting the work of constructing the perspective; it is called the "plane of the picture or picture plane," while the orthographic projection of the point of sight on this plane is called the "principal point of the picture."

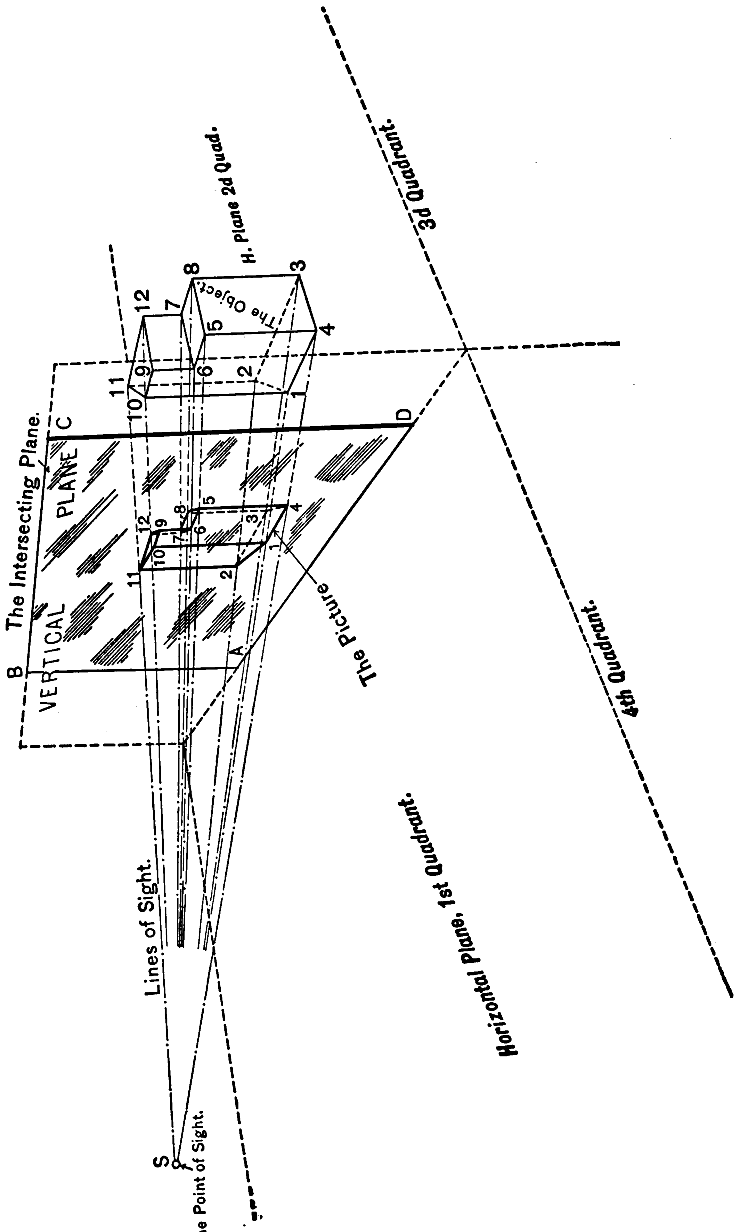

The Intersecting Plane.
B
C
D
A
VERTICAL PLANE
The Picture
The Object.
H. Plane 2d Quad.
3d Quadrant.
4th Quadrant.
Horizontal Plane, 1st Quadrant.
Lines of Sight.
S
The Point of Sight.
1
2
3
4
5
6
7
8
9
10
11
12

FIG. 53.

From the above it is seen that the theory of the art is very simple, that is, to find the perspective of a point one has but to find the point in which the line of sight to the point pierces the picture plane, and since the picture plane is assumed to be the vertical plane of orthographic projection, and the point of sight and the given point are assumed by their two projections, and the projections of the line of sight as lines joining the projections of the two points, the entire procedure becomes a practical application of the elementary problem in Descriptive Geometry, "Find the point in which a line pierces the planes of projection."

31. **The Perspective of a Point.**—With the theory of perspective well in mind, the finding of the perspective of a point should be a very easy matter. However, the writer has observed that students have some difficulty in applying the theory even here, due to the fact that they forget that a point or line is assumed by its *two* projections. In the example to follow, note that the point of sight has a horizontal and a vertical projection, that the given point has two like projections, and that the *horizontal* projection of the line of sight is a line joining the *horizontal* projection of the point of sight and the *horizontal* projection of the given point; also, that the *vertical* projection of the line of sight is a line joining the two *vertical* projections. (Do not join a horizontal and a vertical projection, or vice versa.)

Example. In Fig. 54 let P be a point in the second quadrant, and let p be its horizontal projection and p' its vertical projection, and let S be the point of sight situated in the first quadrant, and let s be its horizontal projection and s' be its vertical projection.

By definition a line of sight is a line joining a point and the point of sight, from which it is seen that the line S-P represents the line of sight, and the lines s-p and s'-p' its horizontal and vertical projection respectively. This line of sight pierces the vertical or picture plane at P', which point, according to the theory of perspective, is the perspective of point P.

To check the above, and to apply the principles of geometry, consider the two projections of the line of sight and note that the line pierces the vertical plane at P'.

To discuss the example on practical and familiar ground, consider Fig. 55*: The point P being assumed in the second

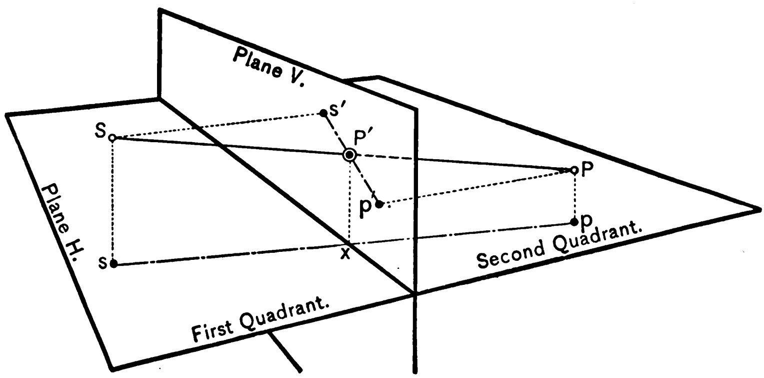


FIG. 54.

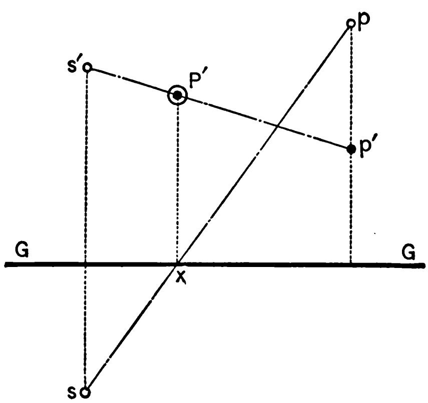


FIG. 55.

quadrant has its *two* projections above the ground-line G-G, and the point of sight being in the first quadrant has its vertical

* The student will remark a violation of the conventions of second quadrant projection in this and other figures in this chapter, in that the vertical projection of lines is invisible and should be represented by a dashed line. It was thought, however, that such notation would result in confusion in certain cases, and that used is arbitrarily taken for the occasion in the belief that it makes the figures more clear.

projection above the ground-line and its horizontal projection below. Joining the *horizontal* projection of the point of sight with the *horizontal* projection of the given point gives the *horizontal* projection of the line of sight, and joining the *vertical* projection of the point of sight with the *vertical* projection of the given point gives the *vertical* projection of the line of sight; the line thus determined is found to pierce the vertical plane at P', the perspective of the point.

32. The Perspective of a Right Line.—To find the persepctive of a right line one has but to find the perspective of any two points in the line and then join these two perspectives with a right line. If the given line be a definite line the two points taken are the extremes of the line.

To illustrate, consider Fig. 56, which shows a line M-N in the second quadrant, together with its projections m-n (horizontal) and m'-n' (vertical); also a point of sight, S, situated in the first quadrant together with its two projections s (horizontal) and s' (vertical). The given line is a definite one, and to find its perspective, find the perspective of the extremes M and N and join them by a right line. By section 31 these perspectives are found to be M' and N', and the perspective of the line to be M'-N'.

Fig. 57 illustrates the conventional orthographic projection of the above example.

33. The Perspective of a Curved Line. — The perspective of a curved line is found by finding the perspective of a number of its points and then joining these perspectives by a curved line.

34. Why Objects are Assumed in the Second Quadrant.—The student may have remarked that in both of the foregoing examples the thing given is situated in the second quadrant and the point of sight in the first quadrant. This is the usual practice, the reason for which is as follows: In Fig. 53 the lines of sight are seen to converge at the point of sight, and it is clearly evident that as the position of the picture plane is changed, the size of the picture or perspective is changed, becoming smaller as the plane is moved toward the point of sight and larger as it is moved

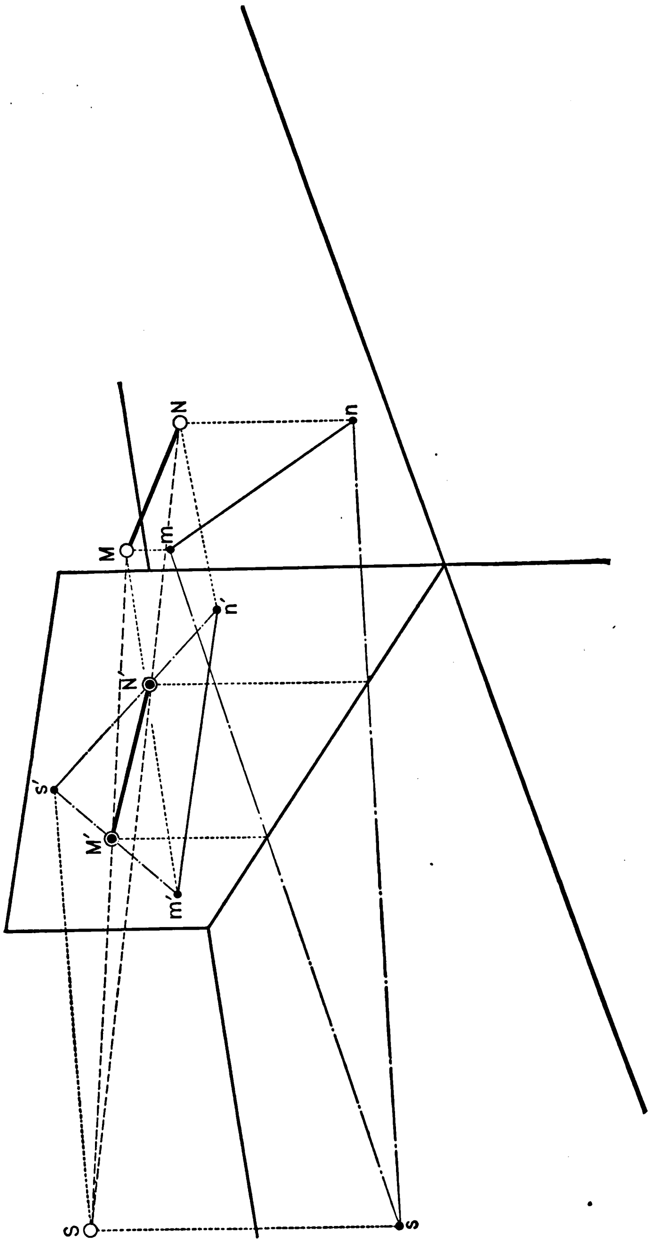


FIG. 56.

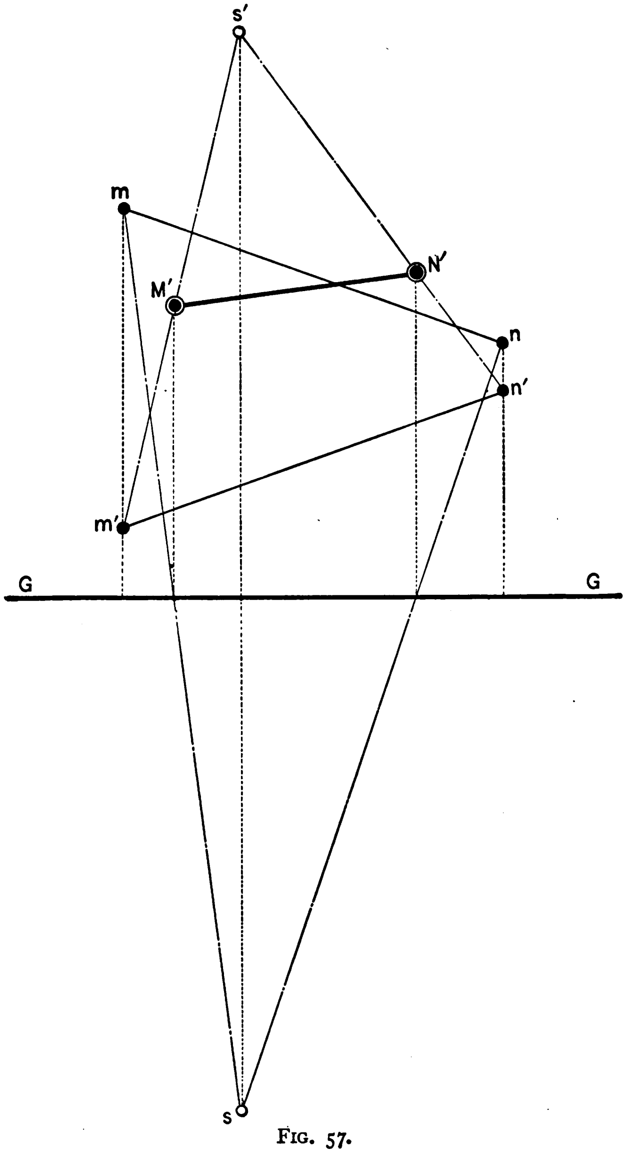


FIG. 57.

toward the object. It is evident also that so long as the intersecting plane is between the object and the point of sight the perspective will be smaller than the object, and that if the plane be placed beyond the object the perspective will be larger than the object.

In nearly every case a picture which is smaller than the object is desired, hence the assumption of the object in the second quadrant and the point of sight in the first quadrant, or, in other words, the assumption of the picture plane *between* the object and the point of sight.

35. **The Perspective of an Indefinite Right Line.**—To find the perspective of any right line, proceed as in section 32, or, since all right lines may be considered as indefinite right lines (definite right lines may be produced or extended) a second method for finding the perspective of the line may be used. This method is as follows:

If a line is of indefinite length it will pierce the picture plane at some point, unless, of course, the line is parallel to the plane. This point is clearly a point in the perspective of the line. Having, then, one point in the perspective of the line determined, one has to find the perspective of but one other point by the usual method (section 31) and then join these two perspectives by an indefinite right line.

Example. Consider Fig. 58, which shows an indefinite right line A-B together with its projections a-b (horizontal) and a'-b' (vertical) situated in the second quadrant, and a point of sight, S, together with its projections s (horizontal) and s' (vertical) in the first quadrant. The line thus determined is found (by the usual orthographic method) to pierce the vertical plane at P'; this point, then, according to the above is a point in the perspective of the line. (Note that the correctness of the point P' is established by extending the line A-B on through the planes of projection.)

Having one point in the perspective of the line thus determined, select any second point, as C [do not forget that a point is fixed by its *two* projections, c (horizontal) and c' (vertical)], and in accordance with section 31 find its perspective. This point is

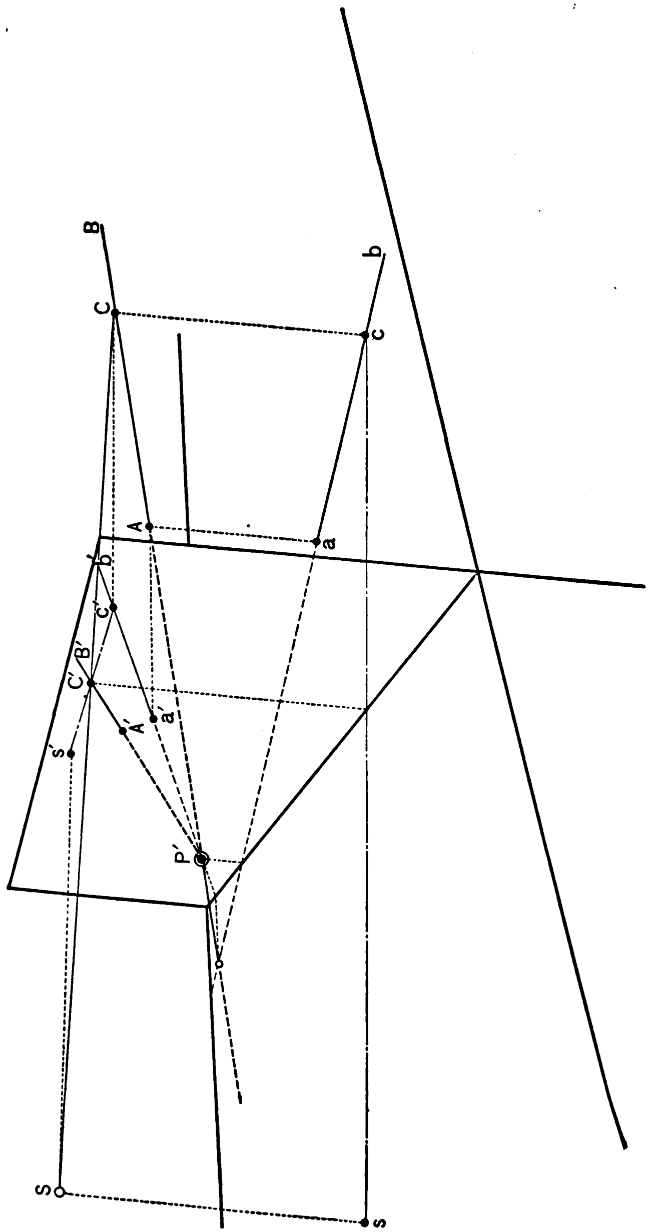


Fig. 58.

found to be C', which connected with P' by a right line determines the perspective of the indefinite right line A-B.

When the indefinite right line is parallel to the picture plane, two points in the line must be selected and their perspectives found in the usual way, and the perspective of the line drawn through the perspective of the points.

36. The Vanishing-point of a Line.—Lines in the same plane which are not parallel will meet, intersect; parallel lines in the same plane are said to meet at infinity. In the science of perspective, when two parallel lines thus meet they are said to vanish. Assuming two parallel lines, then, (1) a line in space and (2) a line parallel to it through the point of sight, the lines being no exception to the rule will meet—vanish—at infinity; now, assume the parallel line through the point of sight to be the line of sight connecting this meeting or vanishing-point of the two lines with the point of sight, the point in which this line of sight pierces the picture plane is the perspective of the vanishing-point; therefore, *the vanishing-point of a line is where a line parallel to it through the point of sight pierces the picture plane.*

From the above it is evident that a system of parallel lines has a common vanishing-point, for the line through the point of sight parallel to one line is parallel to all of them, hence the common vanishing-point.

37. Rule for Finding the Vanishing-point of a Line.—To find the vanishing-point of a line pass a parallel line through the point of sight and find where it—the parallel line—pierces the picture plane; that is, in orthographic projection, through the *horizontal* projection of the point of sight draw a line parallel to the *horizontal* projection of the given line, and through the *vertical* projection of the point of sight draw a line parallel to the *vertical* projection of the given line; the line thus determined in its two projections is the parallel line through the point of sight, and the point in which it pierces the picture plane is the vanishing-point for the given and all parallel lines.

Example. In Fig. 59 let M-N be a line in the second quadrant, and let S be a point of sight in the first quadrant. To find the vanishing-point of M-N, through the horizontal projection of the point of sight, s, draw a line parallel to the horizontal

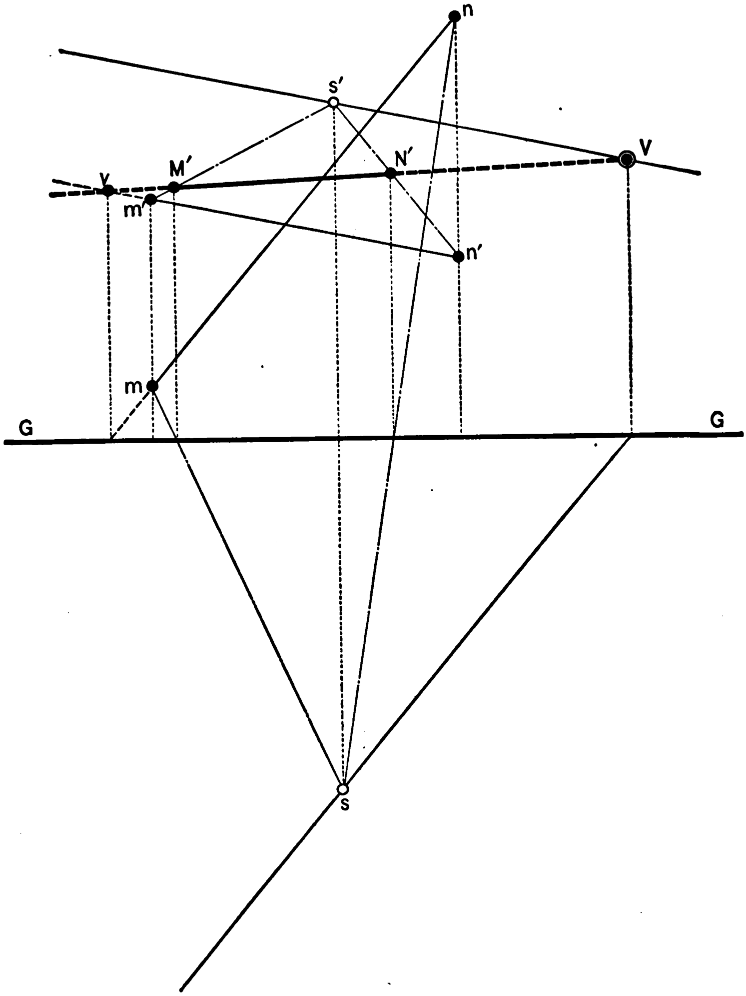


Fig. 59.

projection of M-N, m-n, and through the vertical projection of the point of sight, s', draw a line parallel to the vertical projection of the given line, m'-n'; the point V in which the line thus shown in its two projections pierces the vertical or picture plane is the required vanishing-point.

38. Rule for Finding the Perspective of a Line.—Since all lines may be extended and then considered as of indefinite length, by combining section 35 (which shows that the point in which a line pierces the picture plane is a point in the perspective of the line) with section 36 (which shows that the vanishing-point of a line is a point in the perspective of the line) the following definitive rule is obtained:

The perspective of a line is a line joining the point in which it (the line) pierces the picture plane and its vanishing-point. (See Fig. 59.)

39. The Diagonal and Perpendicular.—There are two special cases of the right line much used in perspective, (1) a diagonal line, and (2) a perpendicular line.

The diagonal. A diagonal line is a line which is *parallel* with the *horizontal* plane of projection and which makes an angle of *forty-five degrees* (45°) with the *vertical* plane.

To find the perspective of a diagonal, first find the point in which it pierces the picture plane; second, find its vanishing-point, and third, join these two points with a right line; this line will be the required perspective.

Fig. 60, in which s and s' are the horizontal and vertical projections, respectively, of the point of sight, and m-n and m'-n' the like projections of a diagonal, illustrates the above procedure. For example, to find the vanishing-point of the diagonal, draw a line through the *horizontal* projection of the point of sight parallel to the *horizontal* projection of the diagonal (note that this line makes an angle of 45° with the ground-line), and through the *vertical* projection of the point of sight draw a line parallel to the *vertical* projection of the diagonal (note that this line is parallel to the ground-line), and find the point in which the thus determined line pierces the vertical plane; this point, V, will be the vanishing-point of the diagonal.

To find the perspective of the diagonal, in addition to the above, find the point P', in which the line itself pierces the vertical

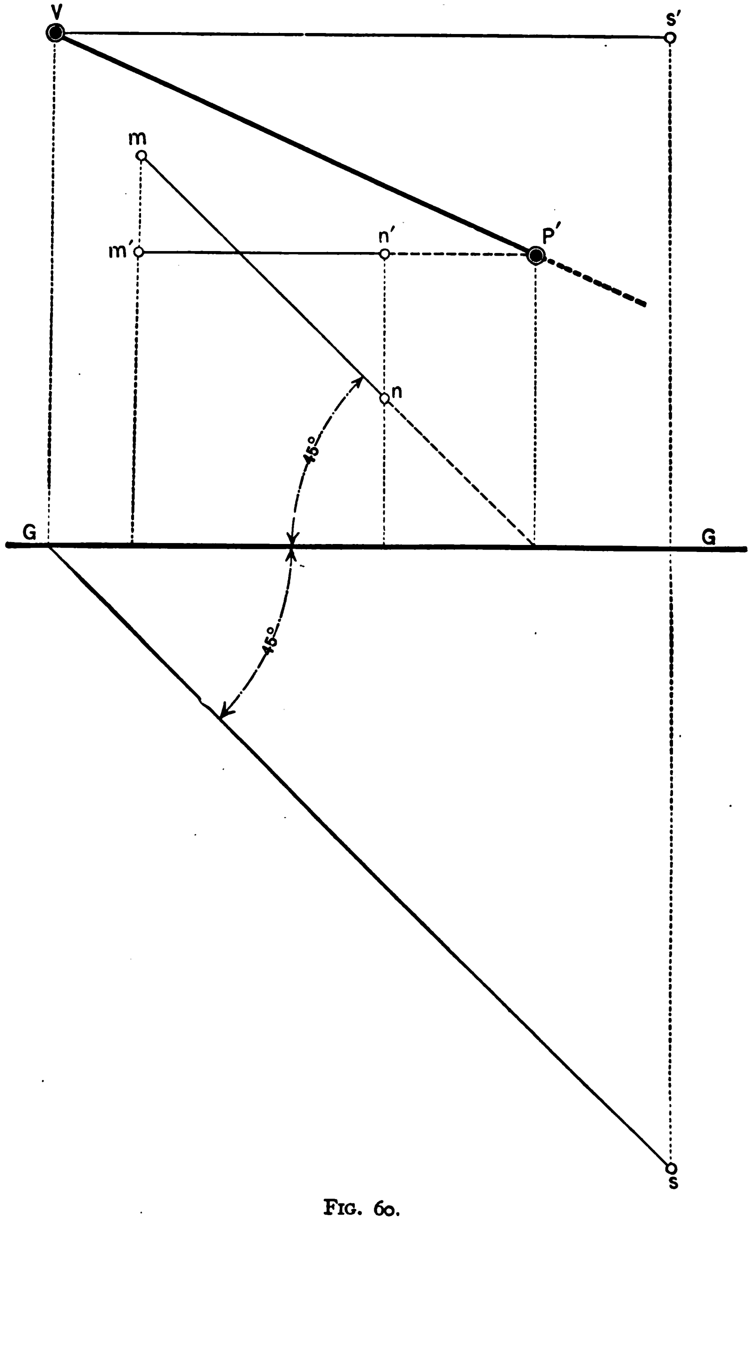


FIG. 60.

plane, then join it—the point—with the vanishing-point V—the line will be the required perspective. (Section 38.)

The perpendicular. A perpendicular is a line which is *parallel* to the *horizontal* plane of projection and which makes an angle of *ninety degrees* (90°) with the *vertical* plane, that is, the line is parallel to H and perpendicular to V.

Referring to Fig. 61, it is obvious that such lines vanish at the vertical projection of the point of sight (section 36), and, since the line itself is seen to pierce the picture plane in the point representing the vertical projection of the line, the perspective of the perpendicular is the line m'-n'-s' joining this point and the vanishing-point of the line. (Section 38.)

40. Conventional Method for Finding Perspectives.—In addition to the elementary method already given for finding the perspective of a point—that of finding the point in which the line of sight to it pierces the picture plane—a second method is available. This method is as follows: Pass two lines through the point, then find the perspectives of these lines; the intersection of the perspectives will be the perspective of the point.

The two lines used for the above purpose are a diagonal and a perpendicular. These two lines are used because of their convenience; any two lines may be used, however.

Example. In Fig. 62 let s and s' be the projections of the point of sight and p and p' the projections of the given point, and let V' (found as in section 36) be the vanishing-point for all diagonals inclined to the left; the vanishing-point for all perpendiculars is s' (section 36). To find the perspective of the point P by the perpendicular-diagonal method, first pass a perpendicular (P-O) through the point. This is found to pierce the picture plane in o' (the same point as p', which represents the vertical projection of the given point), and by section 38 its perspective is found to be the line o'-s' joining this point and its vanishing-point; next pass a diagonal line (R-N) through the point, find the point n' in which it pierces the vertical plane, then join this point with the vanishing-point V'; this line (n'-V') will be the perspective of the diagonal. The intersection of the

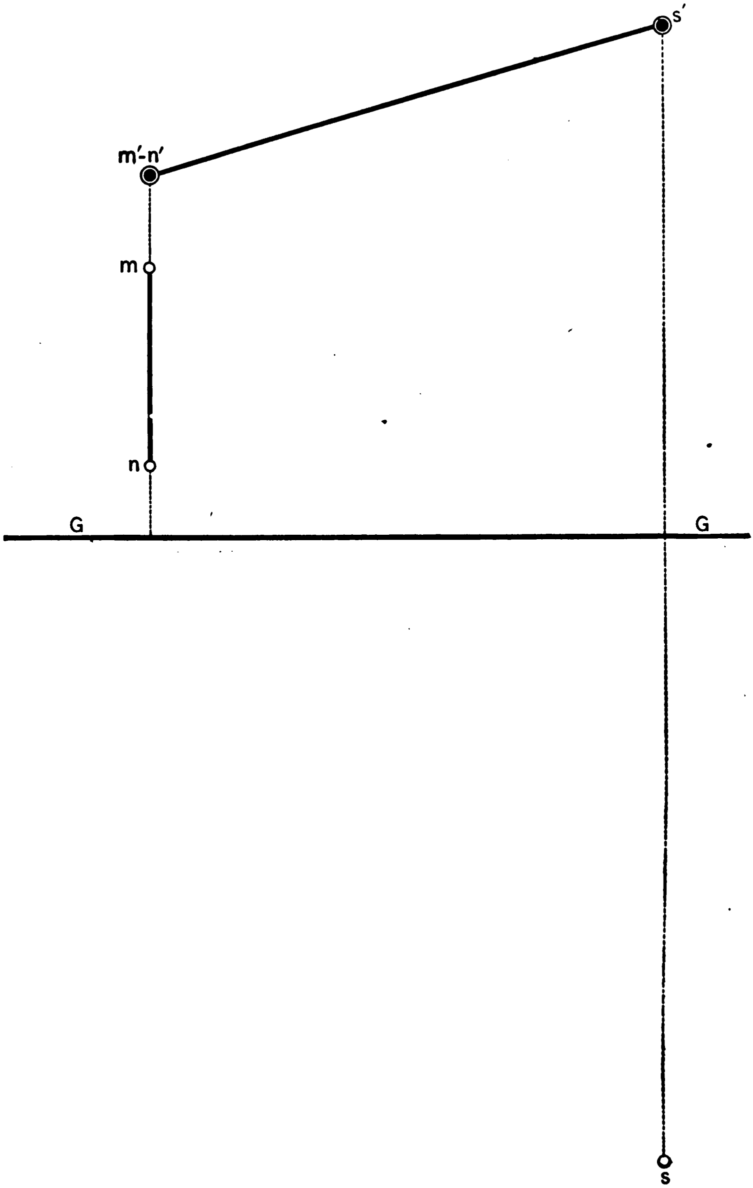


FIG. 61.

perspective of the perpendicular and the perspective of the diagonal, *P′*, is the perspective of the point.

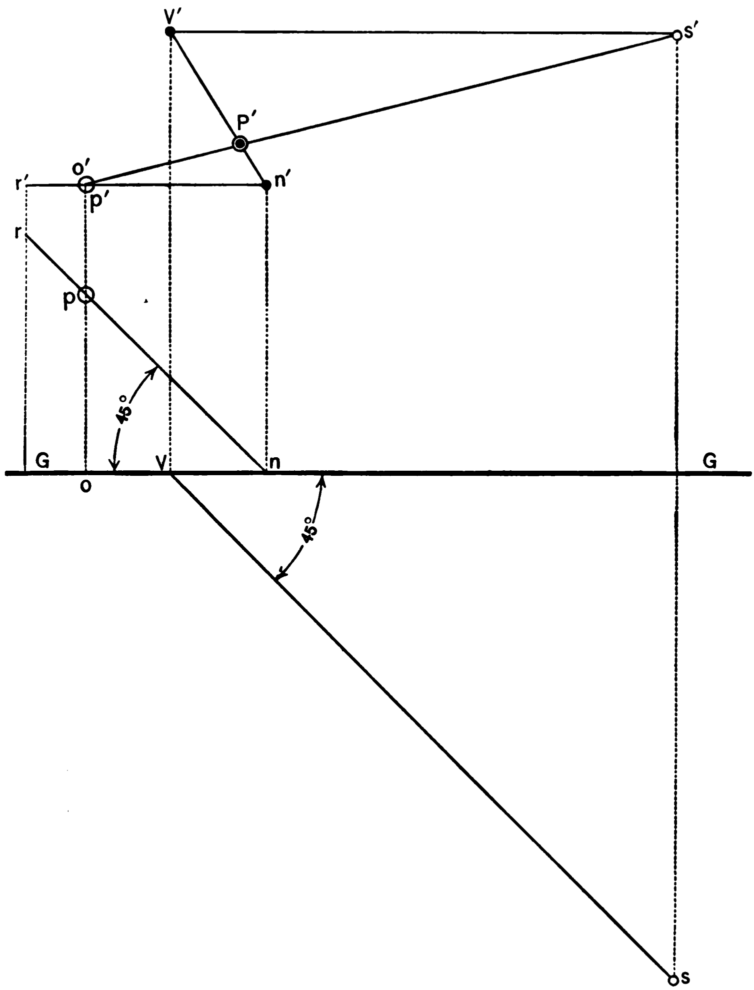


Fig. 62.

Fig. 63 illustrates the finding of the perspective of a line by the perpendicular diagonal method, two points in the line being found in perspective, then joined by a line.

This seemingly roundabout method—the perpendicular-diagonal or two-intersecting-lines method—is the method adopted

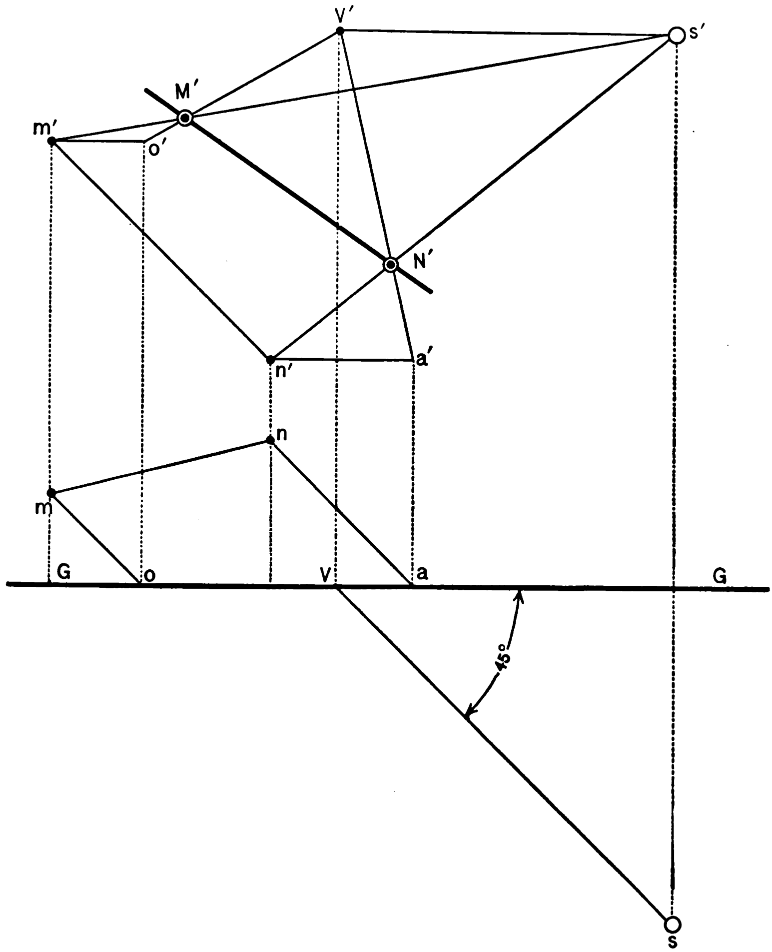


FIG. 63.

in practical perspective; the reasons for its adoption will become apparent as the discussion progresses.

41. The Horizon-line.—The horizon at sea is that bounding circle of vision where the sea seems to meet the sky; on land, barring obstructions, it is that line where the sky and earth seem to meet; the horizon in perspective is a circle the plane of which is parallel to the horizontal plane of projection and which passes through the point of sight. This plane is of infinite extent,

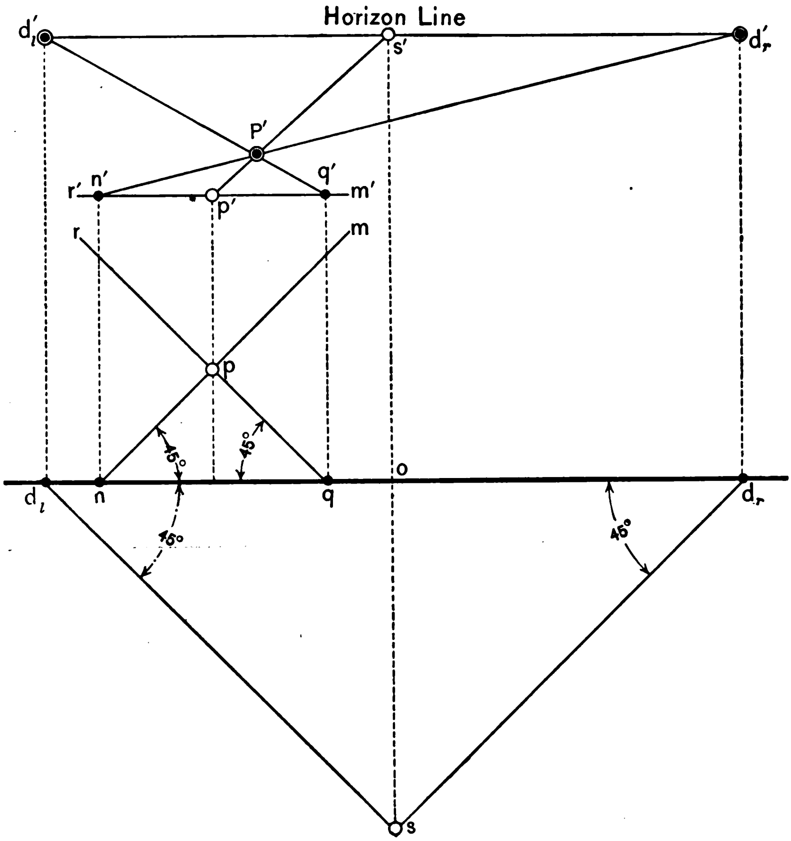


Fig. 64.

and, it is obvious, will intersect the picture plane in a straight line parallel to the ground-line passing through the vertical projection of the point of sight. (See Fig. 64.) This line of intersection is called the horizon-line of the picture.

It is evident that all lines which are parallel to the horizontal plane will vanish at some point in the horizon-line. (Section 36.)

42. Distance Points.—From an inspection of the several figures used thus far to illustrate the discussion it will be seen that most of the drawing is done above the ground-line, and that the horizontal projection of the point of sight is only used to find the vanishing-point for diagonals. Since the horizontal projection of the point of sight is so little used, the space necessary for its depiction is saved in practice by omitting it entirely and assuming a vanishing-point for diagonals at will; this point, of course, is always a point in the horizon-line.

The point thus assumed may be either to the right or to the left of the vertical projection of the point of sight; in fact it is customary to assume two such points—one right and one left—for convenience in executing the perspective. The two points thus assumed are called "distance points," and fix the position of the horizontal projection of the point of sight, as witness Fig. 64: Since the line s-d_l makes an angle of 45° with the ground-line, and since the horizon-line is parallel to the ground-line, the line s'-$d_l' = O$-$d_l = O$-s. Hence with the vertical projection of the point of sight and the distance points known, to find the horizontal projection of the point of sight, draw an indefinite vertical line through the vertical projection of the point of sight and make that part of it below the ground-line equal in length to the distance between the vertical projection of the point of sight and either distance point; the lower extreme of this line will be the required horizontal projection. That is, the horizontal projection of the point of sight is as far in front of the vertical plane as the distance points are to the right or to the left of the vertical projection of the point of sight.

To illustrate the application of the distance points, again consider Fig. 64: For this explanation disregard the horizontal projection of the point of sight, and let it be required to find the perspective of the point P by the perpendicular-diagonal method, and let the two projections of the given point and the vertical projection of the point of sight be the known conditions. To find the perspective of the point, first draw the horizon-line and assume the distance points, then (using the distance point on the left) find the perspective of the point as usual (section 40); this is seen to be P'.

Suppose one finds it more convenient to use a diagonal which is inclined to the right instead of one inclined to the left as in the above case, by definition (section 36) such diagonals are found to vanish at d' on the right, and carrying the perspective through with such a diagonal line the perspective of the point is found to check with that found by using a diagonal of opposite inclination.

From the above, then, it is evident that it is optional which distance point is used.

43. The Perspective of a Plane Figure.—In Fig. 65 let 1-2-3-4-5 represent the horizontal projection of a five-sided polygon and let 1′-2′-3′-4′-5′ represent its vertical projection, and let s' be the point of sight, X-Y the horizon-line, and d' and d' the distance points.

To find the perspective of the figure, find the perspective, separately, of each of the five corner points, using the perpendicular-diagonal method, and join these five perspectives in the same order as they occur in the original with right lines, i.e., point 1 to point 2, 2 to 3, etc.; the resulting figure will be the required perspective.

For example, consider point 1: the perpendicular through it is shown in perspective as the line $1'$-s', the diagonal through it is shown in perspective as the line o'-d', and the perspective of the point as the point 1′, the intersection of the two above perspectives; the perspective of point 5 is found in a similar manner and is seen to be point 5′; the perspective of the line 1-5, then, is clearly the line 1′-5′ joining the points 1′ and 5′.

It will be remarked that but one of the distance points has been used in the figure; the perspective may be found by using either one or both.

44. Special Cases of the Right Line.—Before proceeding further with the discussion, it is desired to call the attention of the student to a number of special cases of the right line, as these well fixed in mind enable one to recognize the accuracy of a perspective at a glance, and in the execution of a drawing to

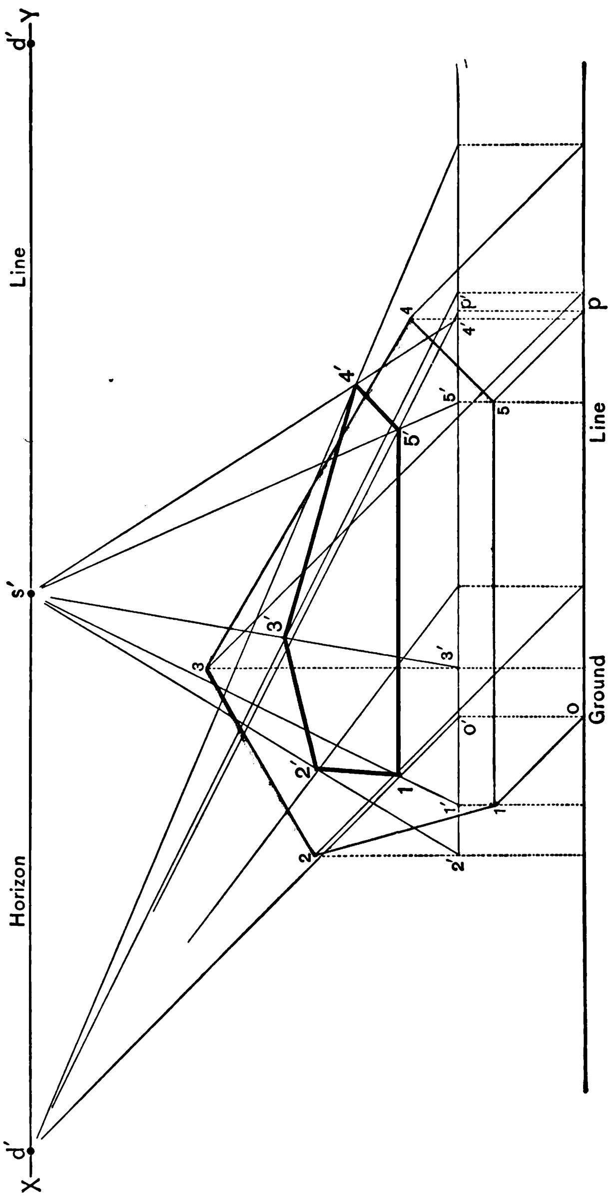


FIG. 65.

greatly expedite the labor of construction. These "helps" or "aids" are as follows:

1. Every system of parallel lines, unless parallel to the picture plane, has a common vanishing-point. (Section 36.)

2. The perspective of a line which is parallel to the picture plane is parallel to the line itself. (See Fig. 66.) If the line is parallel to both planes of projection—a horizontal line parallel with the picture plane—its perspective is a line which is parallel

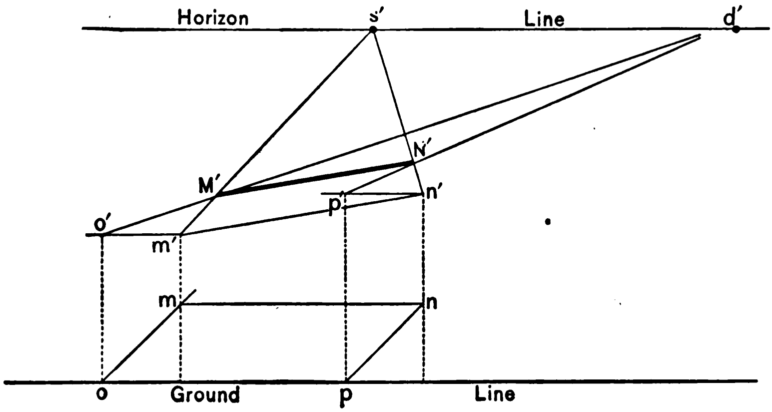


FIG. 66.

with the ground-line, as witness the line 1-5 in Fig. 65. If the line is perpendicular to the horizontal plane and parallel to the picture plane, its perspective is a line which is perpendicular to the ground-line. (See Fig. 67.)

3. The perspective of a line or point which is in the picture plane is the line or point itself; that is, such a perspective would be "full-size."

45. The Plan and Elevation Moved from the Field of the Picture.—By referring to Fig. 65, which shows the plan and elevation in projection and in their assumed position with reference to the picture plane, one remarks the apparent confusion of lines, and can readily imagine the real confusion which would occur in a more complex perspective, as the perspective of a house if it were drawn in a similar manner. To minimize this confusion of lines the plan and elevation are separated—no longer in projection—and are removed from the field of the picture and arranged and used as follows:

Let Fig. 68, *A*, represent the conditions for a certain example in perspective, p and p' being the projections of a point in the second quadrant, *A* distance above the horizontal plane and *B* distance away from the vertical plane, and s and s' the projections

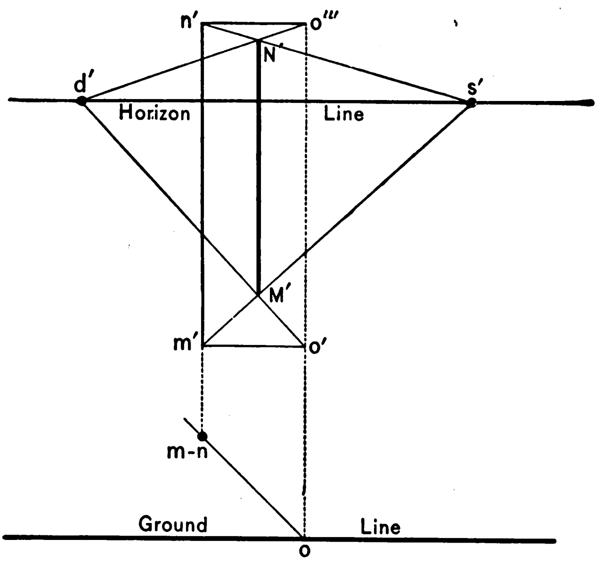


FIG. 67.

of a point of sight, *C* distance in front of the vertical or picture plane, and *D* distance above the horizontal plane, and *E* distance to the right of the assumed point.

To find the perspective of the point under the above conditions, and at the same time to keep the projections of the point out of the field of the picture, it is necessary to employ an auxiliary ground-line, Fig. 68, *B*, delineating the usual arrangement. To execute this figure, with the field of the picture (that portion of the drawing surface to receive the picture) known, draw the ground-line and locate the point s' (the vertical projection of the point of sight), *D* distance above it; through *s′* draw the horizon-line, and on it locate the distance point *d′* [note that this is *C* distance to the left (may be either right or left) of *s′* and represents or corresponds to the distance the point of sight is in front of or away from the picture plane]; next draw an auxiliary ground-line at any convenient point above the field of the picture and locate the horizontal projection or plan of the point *B* distance

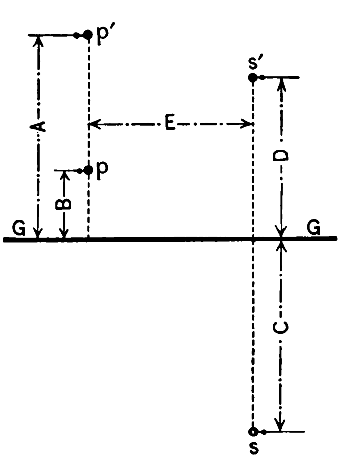


FIG. 68, A.

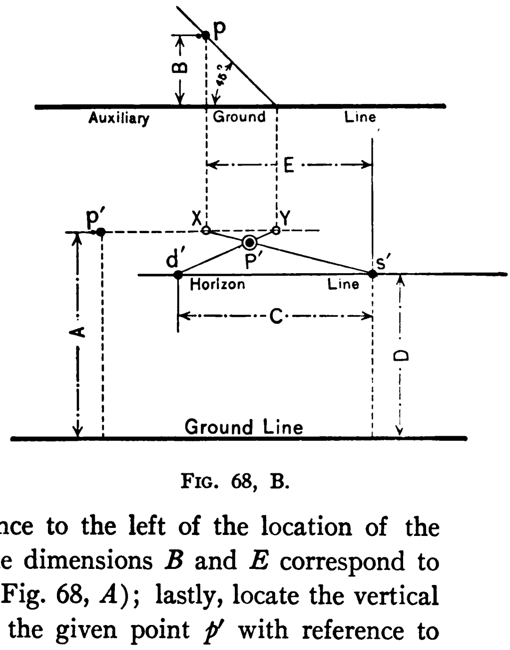


FIG. 68, B.

away from it, and *E* distance to the left of the location of the point of sight (note that the dimensions *B* and *E* correspond to the dimensions *B* and *E* of Fig. 68, *A*); lastly, locate the vertical projection or elevation of the given point *p′* with reference to the ground-line (*A* distance above it), and at some convenient point to the left (may be right or left) of the field of the picture.

That is, the plan or H projection is located with reference to one ground-line, and the elevation or V projection with reference to a second ground-line, and while the principle of conventional projection is violated (since the two projections do not lie in a perpendicular to a common ground-line) the location of the point is, nevertheless, absolutely and accurately fixed.

To find the perspective of the point under this new arrangement, first draw an elevation line *X-Y* across the field of the picture (obtained by drawing a horizontal line through the elevation of the point, said to be "projected in") representing the elevation or distance above the ground-line of the given point, then pass a perpendicular (section 39) through the point; this pierces the picture plane at X, and is shown in perspective as the line X-s' (section 38); next pass a diagonal (section 39) through the point; this pierces the picture plane at Y, and is shown in perspective as the line Y-d'; the intersection of these two perspectives, P', is the required perspective. (Section 40.)

Fig. 69, *A*, depicts the conditions for a second example in perspective, and shows the projections of a line M-N and a point of sight S. Fig. 69, *B*, shows the conventional arrangement of the above conditions and delineates the finding of the perspective of the line.

46. Practical Perspective.—The arrangement of the plan and elevation—the projections—of an object, and of the point of sight, the two ground-lines, etc., shown in Figs. 68, *B*, and 69, *B*, is the one adopted for practical purposes. The statement has been made that the plan and elevation are removed from the field of the picture to minimize the confusion of the lines; in the two examples just discussed, it will be remarked that this new arrangement of the plan and elevation has not simplified matters at all; rather, since one has to project down from the plan and in from the elevation, it has added new lines to the drawing and increased the confusion of lines. This is true in the examples given, and if all examples were as simple as these, it would be better to use the method described in section 40, wherein the plan and elevation are in projection and occupy the field of the picture. In practice, however, one deals with many points and lines in a single object, and with the plan and elevation out of the field of the picture, the points of the object are projected into it by means of the T square and triangles and the perspective of lines obtained by many "short cuts"; thus one finds it necessary to draw but few lines, and the field of the picture is kept comparatively free from confusion.

Figs. 68, *B*, and 69, *B*, delineate all that is fundamental in practical perspective, and should be carefully studied and thoroughly understood, as the examples discussed from this point on

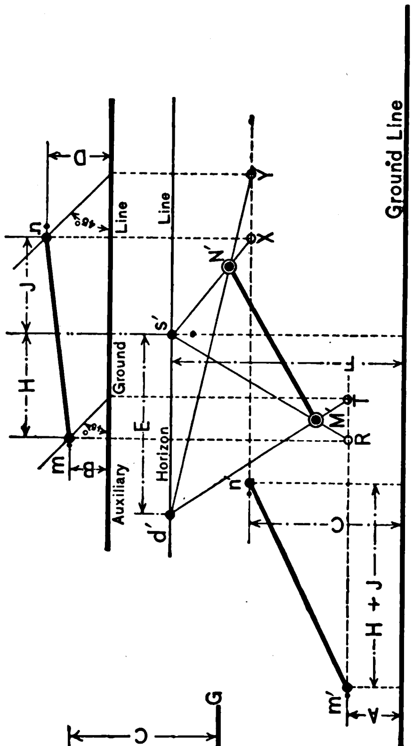


FIG. 69, B.

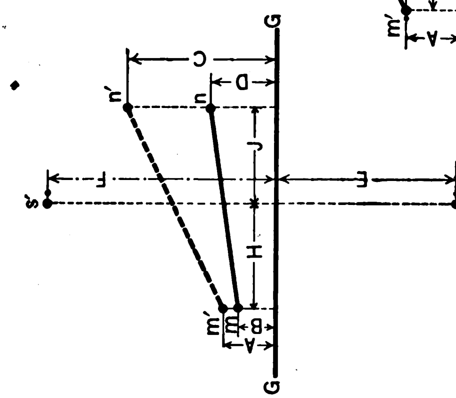


FIG. 69, A.

cannot be understood without a first understanding of the above.

As a first real example in practical perspective—the above examples being very elementary—let Fig. 70, *A*, be the projections of a small oilstone mounted in a wooden box, and let it be required to picture it as resting on the horizontal plane with

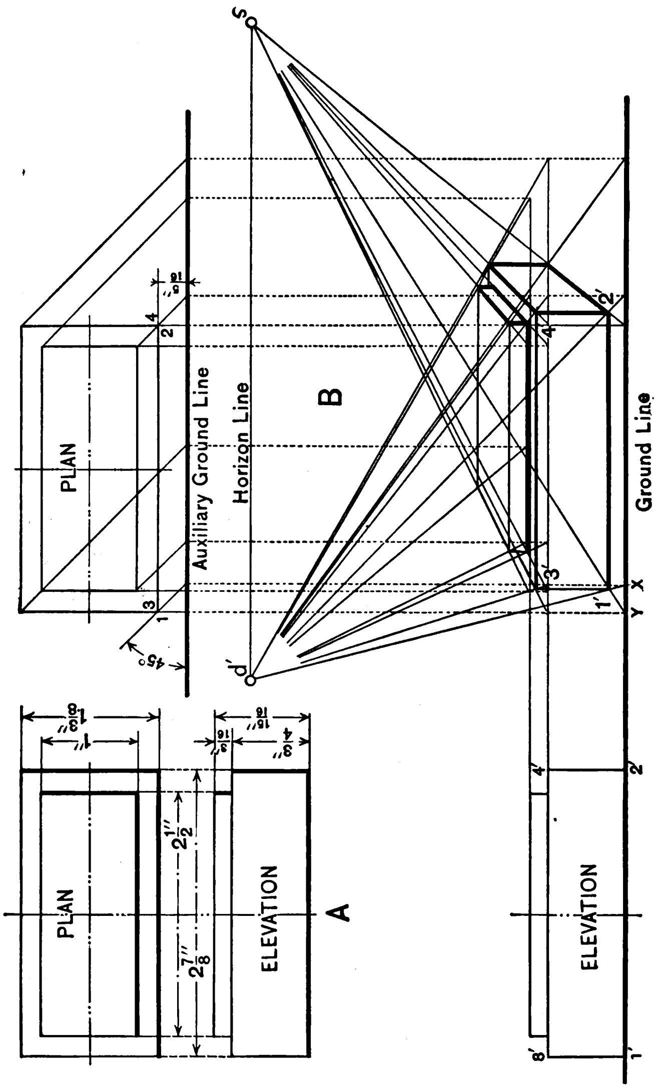


Fig. 70.

its long sides parallel with the vertical plane and its nearest side $\frac{5}{16}''$ away from it.

Fig. 70, *B*, shows the disposition of the plan and elevation in accordance with the previous explanation, the assumption of a viewpoint, *s'*, the horizon-line, and a distance point, *d'*.

These preliminaries arranged, the perspective is found as follows:

Through point 1 pass a perpendicular; this is seen to pierce the vertical plane at *Y* (*Y* being the vertical projection of point 1 obtained by projecting in from the elevation at the side parallel with the ground-line to an intersection with the perpendicular from the plan), and since all perpendiculars vanish at *s'*, its perspective is the line *Y-s'*; next pass a diagonal through the point 1 (through the horizontal projection—plan—draw a line at 45° with the ground-line, and through the vertical projection—elevation—draw a line parallel with the ground-line); this is seen to pierce the picture plane at *X*, and since it vanishes at *d'* its perspective is the line *X-d'*; the intersection of the two perspectives—the perpendicular and diagonal—is the required perspective; this is seen to be point 1'. Proceeding in this manner the perspective of all of the corner points may be obtained—one at a time—and when properly joined by right lines will give the required perspective.

The labor of construction can be reduced by applying section 44. For example, having found the perspective, 1', of the corner point marked 1, the perspective of the line 1-2 may be found as follows: The line is known to be parallel with both planes of projection, and hence, that its perspective will be parallel to the ground-line, therefore, since one point and direction will determine a line, draw a horizontal line through point 1' and terminate it where it crosses the perspective of the perpendicular through point 2. By this procedure not only is point 2 found in perspective but the perspective of the line 1-2 is obtained at the same time, and this, too, without the use of both a perpendicular and diagonal. (The above mentions the use of a perpendicular to terminate the perspective of the line 1-2; either a perpendicular or a diagonal may be used—it is not necessary to employ both—that one being used which will give the sharper intersection;

that is, if the given perspective and the perspective of the perpendicular are so nearly parallel that it is difficult to determine the exact intersection of the two, it is well to use a diagonal, as it is probable that it will give a sharp intersection—will cross the other line more nearly at right angles.)

The student should analyze the disposition made of the plan and elevation in the foregoing explanation and see for himself that it is nothing new, and that all of the principles of orthographic projection are observed in that a second vertical projection—elevation—is obtained, a point at a time, which is in projection with the plan by projecting, horizontally, in from the elevation at the side as needed to a point in projection, vertically, with the same point of the plan. Furthermore, that the auxiliary ground-line used gives the same results as would the use of the real ground-line.

47. Parallel Perspective.—When an object is so situated that a large number of its principal lines are parallel to the picture plane, the perspective obtained is said to be a "parallel" perspective. Fig. 70, *B*, is an example of parallel perspective.

Parallel perspective is very simple and convenient, involving the use of only the point of sight and distance points, and since most of the lines of the object are either parallel or perpendicular to the picture plane, after one or two principal points are found by the perpendicular-diagonal method the perspective can be quickly finished by the "one-point-and-direction" method.

In this class of perspective it is clearly evident that the perspective of a circle the plane of which is parallel to the picture plane will be a true circle; the perspective of a hexagon, a square, etc., similarly situated will be true figures—points which facilitate the drawing.

48. Oblique or Angular Perspective.—If an object is so situated relative to the picture plane that a large number of its principal lines are not parallel to the plane but make a known angle with it, the perspective obtained is said to be an "angular perspective"; Fig. 71 is an example of this class.

Angular perspective involves the use of a point of sight, distance-points, and vanishing-points for all systems of parallel lines. In Fig. 71 it will be seen that all of the lines of the object are either at 60° or 30° with the picture plane and require two vanishing-points. These are found as follows:

The ground-line and the point of sight assumed, draw an indefinite perpendicular, *M-M*, through the vertical projection of the point of sight *s'*, and draw the horizon-line; next, assume one of the vanishing-points, in this case say that for the 30° lines, the point *Y'*, and from this vanishing-point drop a perpendicular *Y'-Y* to the ground-line, and from the point of intersection, *Y*, draw a line making an angle of 30° with the ground-line and inclined opposite in direction to the inclination of the 30° lines of the object, until it intersects the perpendicular *M-M* through the point of sight; from this point, *s''*, draw a 60° line, *s''-X*, opposite in direction to the 60° lines of the object, to the ground-line, and from the point *X* in which it intersects the ground-line erect a perpendicular, *X-X'*, to an intersection with the horizon-line; this point (point *X'*) will be the vanishing-point for all 60° lines.

An analysis of the above will show that it is the same as the procedure given in section 37, if one will but consider the horizontal projection of the point of sight as having been revolved, for convenience, over into the second quadrant.

To find the perspective in this case, use a perpendicular and either a 30° or 60° line method. That is, to find the perspective of point 1, for example, pass a perpendicular *H-K* through the point and find where it pierces the picture plane (point *K'*) by projecting in from the elevation, and then draw its perspective (*K'-s'*) by joining this point with *s'*, the vanishing-point for perpendiculars; next, in place of passing a 45° diagonal through the point, pass either a 30° or 60° diagonal line [a line at 30° or 60° with the picture plane and parallel to the horizontal plane (a special diagonal), as the line *C-D* or *E-F*], find where it pierces the picture plane (point *F'* or *D'*), and draw its perspective (*F'-Y'* or *D'-X'*) by joining this point with its vanishing-point. The intersection of this perspective with the perspective of the perpendicular (*K'-s'*) is the required perspective.

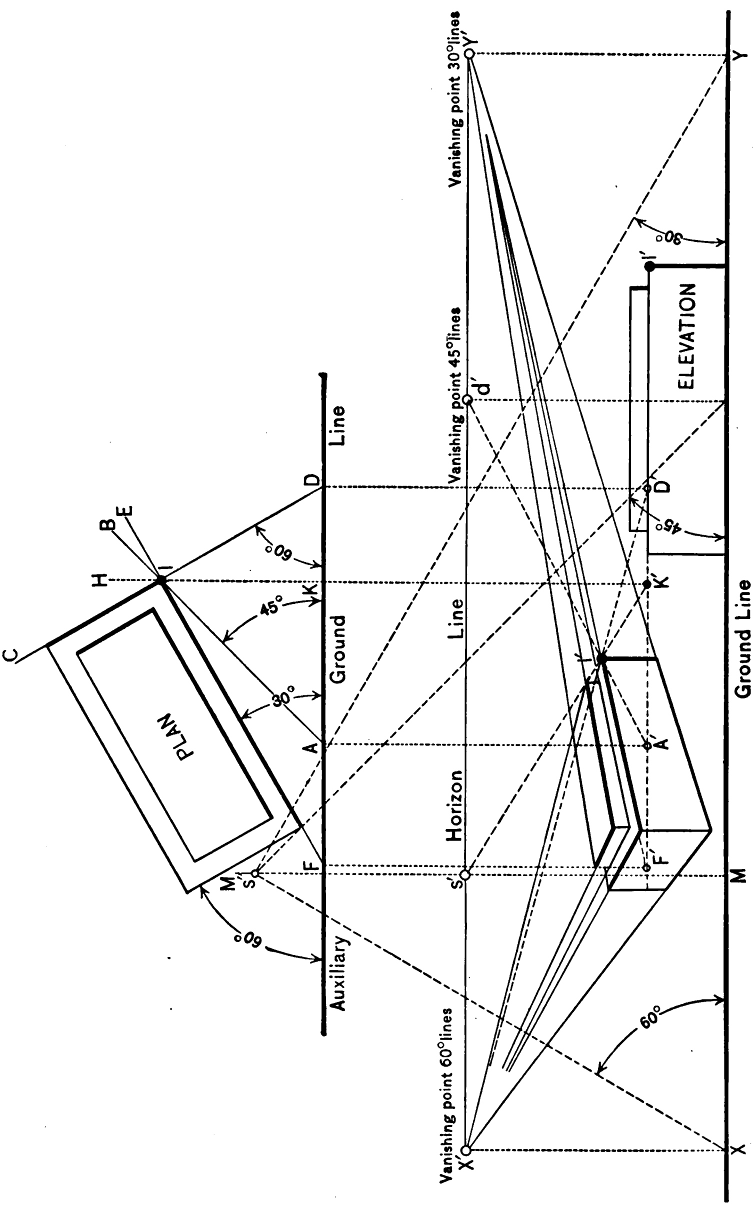

PLAN
Auxiliary Ground Line
Horizon Line
Vanishing point 30° lines
Vanishing point 45° lines
Vanishing point 60° lines
ELEVATION
Ground Line
60°
45°
30°

FIG. 71.

The same point can be found by using the 45° diagonal, as shown by the drawing, or in fact any degree diagonal provided the proper vanishing-point is used, the advantage in this particular case being that the use of either the 30° or 60° diagonal not only gives the point desired, but at the same time gives the lines of the object.

49. How to Assume Conditions.—As has been explained in the early part of these remarks, it is customary to assume the object as situated in the second quadrant, and at the same time it was shown that the nearer the object was to the picture plane the larger its perspective would be, and that the farther away from the picture plane, the smaller the perspective of the object would become. It is obvious, then, that the position of the plan with reference to the auxiliary ground determines the size of the perspective.

For obvious reasons most objects are assumed to rest on the horizontal plane. This, of course, places the elevation on the ground-line.

The point of sight is taken with reference to those faces of the object it is desired to depict. That is, if the top face is desired in connection with certain side faces, the point of sight must obviously be above the elevation of the top face—made manifest on the drawing by the elevation of the vertical projection of the point of sight referred to the elevation of the top of the elevation of the object. The side faces desired are made manifest on the drawing by the disposition of the plan and the horizontal projection of the point of sight; the sides of the plan which are nearest the picture plane are the ones that will be seen, and how much of each is determined by the position of the horizontal projection of the point of sight—whether to one side or not, and its distance from the picture plane.

For economy of space it is well to assume the first quadrant horizontal projection of the point of sight to be revolved over into the horizontal plane, second quadrant, as shown in Fig. 71, and when so assumed will reverse the direction of inclination of the H projection of any lines inclined to the picture plane passing through the point of sight.

It is obvious that the farther away the point of sight is from the picture plane, the greater the distance between the vertical projection of the point of sight and any vanishing-points used, and, of a consequence, the slower convergence of the lines of the picture. From this, then, it is seen that to keep the convergence from becoming conspicuous, the vanishing-points should be as "wide" as possible, in which case it is well to apply the foregoing principles as follows:

In Fig. 71, when two vanishing-points are required, let the line X'-X represent one horizontal extreme of the available drawing-surface, say the left-hand side or end of the drawing-board, and let Y'-Y represent the other horizontal extreme, as the right end of the drawing-board, and let the horizontal line X-Y represent the lower vertical extreme of the drawing-surface, say the bottom of the drawing-board.

From the lower left-hand corner, X, draw the 60° line X-s'', and from the lower right-hand corner draw the 30° line Y-s'', and produce these lines to an intersection s''; this point will be the revolved position of the horizontal projection of the point of sight which will give the "widest" vanishing-points possible on the drawing-surface used.

The projections of the point of sight assumed, it is then an easy matter to arrange the plan and elevation, to give the desired perspective or view, and to locate all vanishing-points, the horizon-line, etc. That is, it is sometimes convenient to assume the conditions "backwards," so to speak, to fit the drawing-board.

In cases where the elevation of a point coincides with the elevation of the point of sight, or in cases where it is difficult to obtain a sharp intersection of the perpendicular and diagonal used because of their inclination, as the lines A'-s' and B'-d', Fig. 72, it is well to assume an auxiliary elevation for the point, then find its perspective at this elevation and project it (vertically) onto either the perpendicular or diagonal drawn at the correct elevation.

50. The Perspective of Shadows.—Before attempting to digest this section the student should have a working knowledge of the principles of cast shadows—such as may be derived from a perusal of Chapter II; this knowledge, together with an understanding of the elementary principles of perspective, renders the finding of the perspective of the shadow cast by an object a very simple procedure.

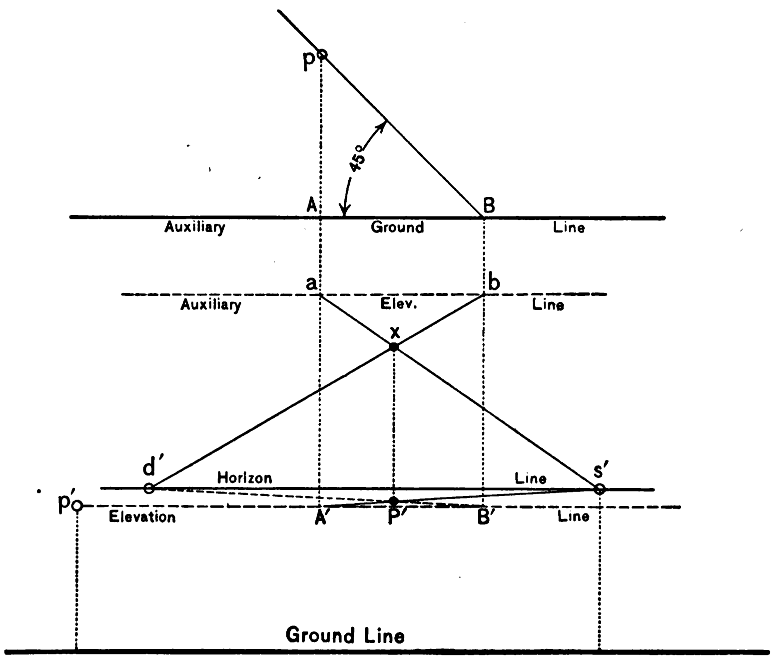


FIG. 72.

Shadows are rarely shown in their true outline in practical draughting, but are approximated in accordance with the draughtsman's conception of them; that is, from his experience and observation he is able to "guess" in a shadow which is approximately true to life. For this reason, and since the art is seldom applied by the usual engineer-draughtsman, it is proposed to treat the subject very briefly, explaining the fundamentals and giving a few elementary examples only.

Theory. The theory is the same as that for all shadows, i.e., every surface is uniformly covered with light except in so far as the rays of light are intercepted by a point, line, or object, and of a consequence leaving a portion of the surface unilluminated representing the shadow of the point, line, or object.

The theory is applied exactly as in the orthographic projection of shadows.

Application of the theory. To find the perspective of the shadow cast by the perspective of a point, pass the perspective of a ray of light through the perspective of the point and find the point in which the perspective of the ray pierces the perspective of the planes of projection.

Now, the object (point in this case) being always assumed in the second quadrant, and the light conventionally assumed as making an angle of 45° with the planes of projection (first quadrant) as shown in Fig. 73, it is evident that the shadow always falls on the horizontal plane, except, of course, that part of it which falls on the object itself. The finding of the perspective of the shadow cast on the horizontal plane is the principal point in the perspective of shadows, as in most objects there are planes which are parallel with the horizontal plane, and with the shadow located on these and on H, with a knowledge of shadows (such as the shadow of a line on a parallel plane is parallel and equal to the line, etc.) the shadow on vertical and other planes is easily located.

As in all perspective work where there are parallel lines it is necessary to locate a vanishing-point for the lines, it is necessary to locate a vanishing-point for the rays of light and for the horizontal projection of rays In Fig. 73, then, which represents the perspective of a small rectangular block, let s and s' be the projections of the point of sight, let the light be at 45° to the planes of projection, and note how these vanishing-points are obtained.

Through the vertical projection of the point of sight, s', draw the line s'-r_1 parallel to the vertical projection of the rays of light, and through the horizontal projection of the point of sight, s, draw the line *s-r* parallel with the horizontal projection of the rays; the point r_1 in which this line pierces the vertical or picture

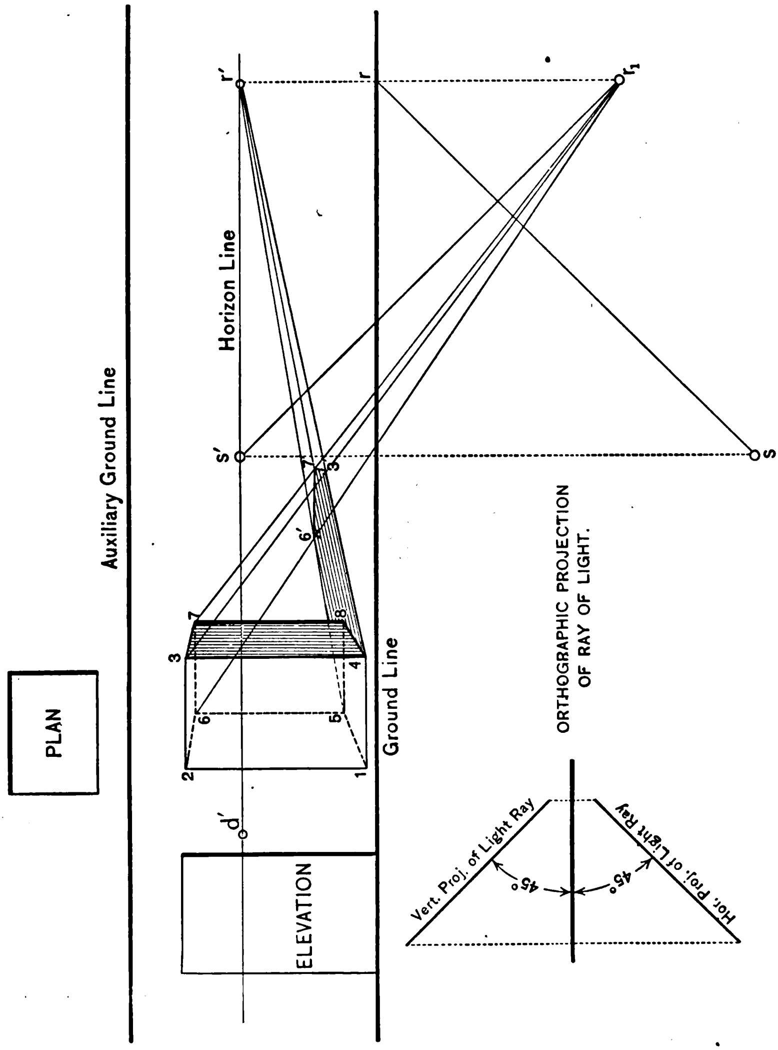


FIG. 73.

plane, is the vanishing-point for all rays of light. (Section 37.) In a similar manner the point *r′* is found to be the vanishing-point for all horizontal projections of rays.

To apply these two vanishing-points to find the shadow of a point, say point 3, Fig. 73, proceed as follows:

It is evident that the point 3 itself is one point in the perspective of the ray of light through the point, and since all rays vanish at r_1, the line 3-r_1 is the perspective of a ray; in like manner, it is evident that the point 4 is the horizontal projection of the point 3 (the object being assumed to rest on H, and the line—edge—3-4 being perpendicular to H), and being the horizontal projection is one point in the perspective of the horizontal projection of the ray of light through point 3, and since all horizontal projections of rays vanish at r', the line 4-r' is the perspective of the horizontal projection of the ray. Now, it is evident that the shadow of the point must necessarily lie in the perspective of the ray, and also in the perspective of the H projection of the ray, therefore, the shadow of the point is (3′) the intersection of these two lines.

Examples. After selecting the points and lines which cast the outline of the shadow, a series of shadow-points are found in the above manner, and these properly joined together give the desired shadow, as witness the points 3′-7′-6′, etc., Fig. 73. The shadow on the H plane drawn, it is an easy matter to determine those faces of the object which are in the shadow, as the face 4-3-7-8.

Fig. 74 represents the perspective of a carriage-block. In this example note the position of the horizontal projection of the point of sight, s (revolved into the second quadrant for convenience), and the construction used in locating the vanishing-points r_1 and r'. The figure shows the perspective of the shadow on the H plane, showing the work, point by point, and also the shadow cast on the object itself. (In the shadow on the H plane note the point 9′; how the point 9 is projected onto H at X, that the perspective of the H projection of the ray through point 9 may be drawn.)

The figure introduces the finding of the perspective of the shadow on a plane other than the H plane. The plane in this

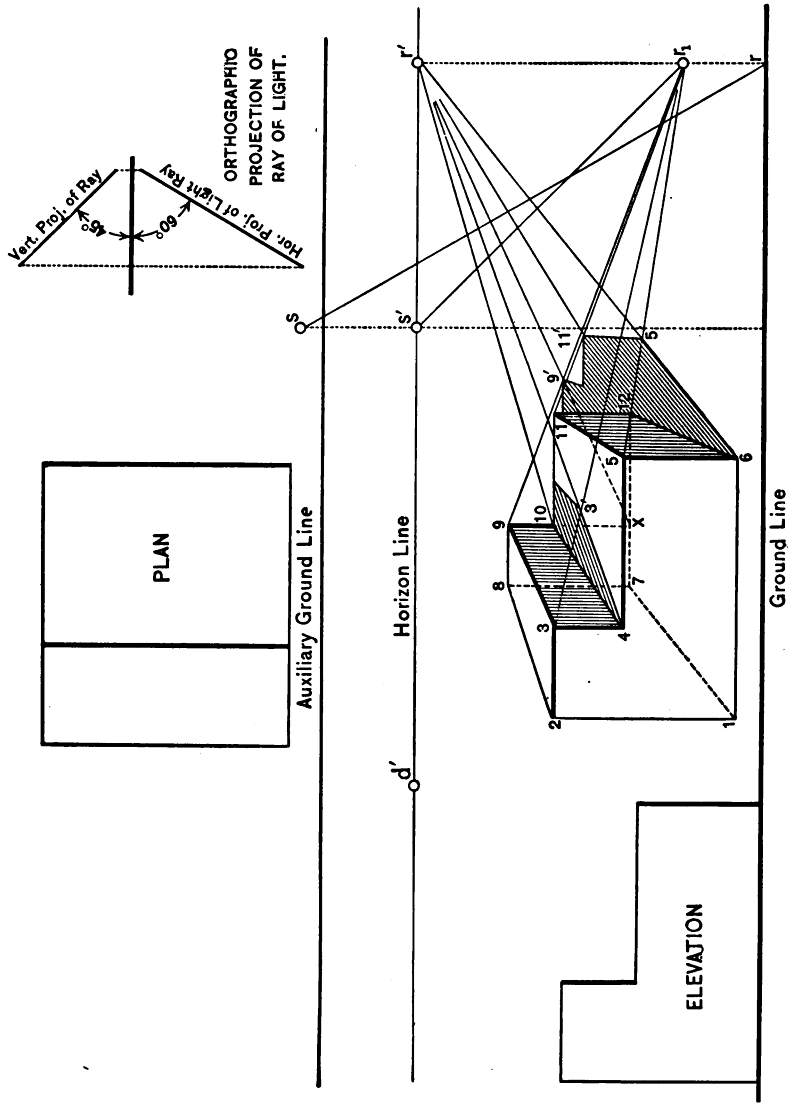

ORTHOGRAPHIC
PROJECTION OF
RAY OF LIGHT.
Vert. Proj. of Ray
Hor. Proj. of Light Ray
45°
60°
PLAN
ELEVATION
Auxiliary Ground Line
Horizon Line
Ground Line

FIG. 74.

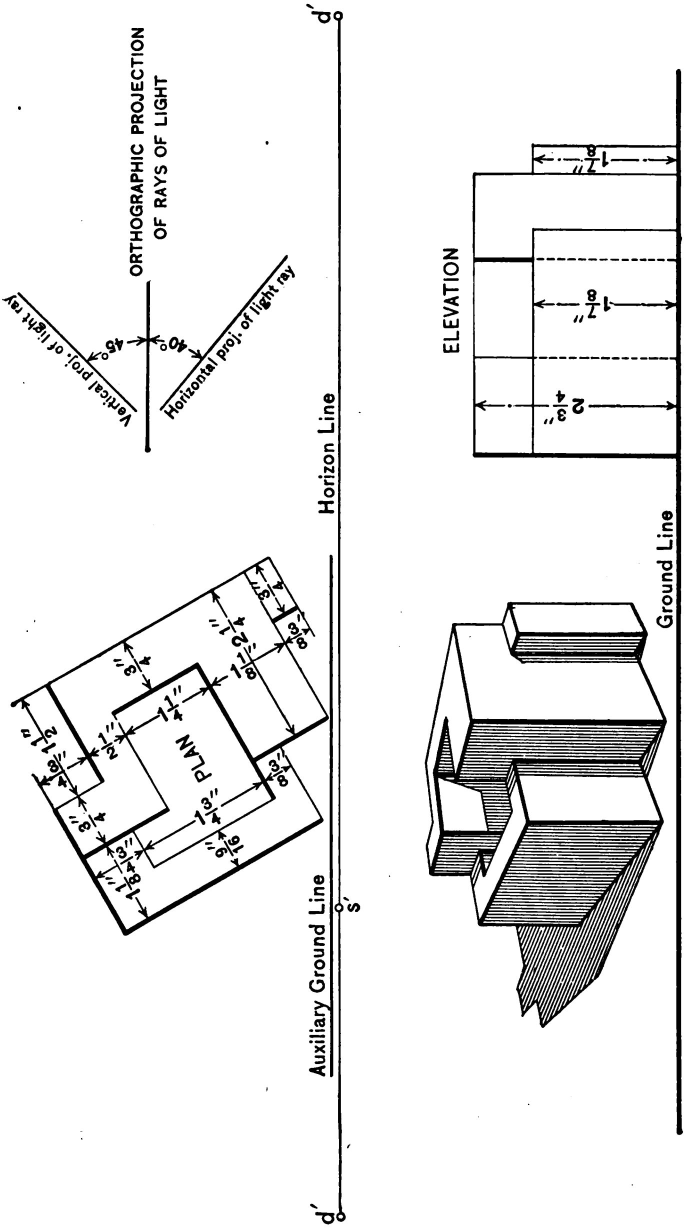

ORTHOGRAPHIC PROJECTION
OF RAYS OF LIGHT
Vertical proj. of light ray
Horizontal proj. of light ray
45°
40°
Horizon Line
Auxiliary Ground Line
Ground Line
ELEVATION
PLAN
d′
s′
d′

FIG. 75

case being a plane parallel with H the procedure is very simple and is as follows:

Through the point 3 draw the line 3-r_1, the perspective of a ray, and through the point 4, the H projection of point 3 on the plane of the shadow, draw the line 4-r', the perspective of the H projection of the ray on the new plane; the intersection of these two lines, point 3′, is the shadow of point 3, and since the line 3-9 is parallel to the plane its shadow on the plane will be parallel to itself. The shadow is finished by drawing a line through the point 3′ parallel to the line 3-9, i.e., to the same vanishing-point.

The foregoing examples, while being specific, are illustrative of the method for finding the shadow of a point on any plane, the rule for which is as follows:

To find the perspective of the shadow of a point on any plane, through the perspective of the point draw the perspective of a ray, and through the perspective of the projection of the point on the plane draw the perspective of the projection (on this plane) of the ray; the intersection of these two lines will be the required shadow.

Remarks. In all shadow work, orthographic or scenographic (perspective) projection, it is necessary that the draughtsman be able to read his drawing well, and thus be able to select those lines and points which cast the outline of the shadow; with this ability and a knowledge of the principles of cast shadows and perspective many short cuts are open to the draughtsman whereby the execution of the work is greatly expedited. This skill of execution can only be acquired by practice, and it is recommended that the student analyze all of his constructions and see wherein they might have been shortened. Such a practice in a comparatively few examples will suffice to give a knowledge of the shortest construction to use.

In all of the examples discussed the light was assumed to make an angle of 45° with the planes of projection. This assumption is simply a conventional one; the light may be assumed at will, as witness Fig. 75.

PART II.

EXERCISES.

CHAPTER IV.

THEORETICAL PROBLEMS.

51. **Explanatory.**—The following problems are given as exercises in drawing calculated to perfect a working knowledge of the principles of Descriptive Geometry, and, together with the problems given in Chapter V, to form a series of exercises for a course in advanced drawing.

The work is so designed that the executed solution of one or more examples constitutes an exercise, and the exercises so finished and lettered as to form a drawing sheet or plate.

52. **General Directions.**—The paper used for the course should be a good quality of drawing-paper, and should be 9″×12″, or a little greater, in dimensions; the border-line will be a rectangle 8″×11″ in dimensions, and the finished sheet should have a ½″ margin on all four sides outside of this. (See Fig. 76.)

The notation used to be that shown in Fig. 76, and to be used in strict accordance with the conventions of Descriptive Geometry, i.e., a vertical projection of the second quadrant is a hidden line, etc.

The conditions for the problems are to be those given, when such is the case, otherwise, they are to be assumed at the draughtsman's discretion. (Always assume a figure of sufficient size to afford a clean-cut, legible solution.)

All lettering to be free-hand, single-line Gothic letters; the letters may be upright or inclined. The title letters to be capitals or upper-case letters, initial letters $\frac{3}{16}''$ high, other letters $\frac{1}{8}''$ high. The name and date, and all other lettering, to be of the lower-case or small letters, initial letters excepted, and to range

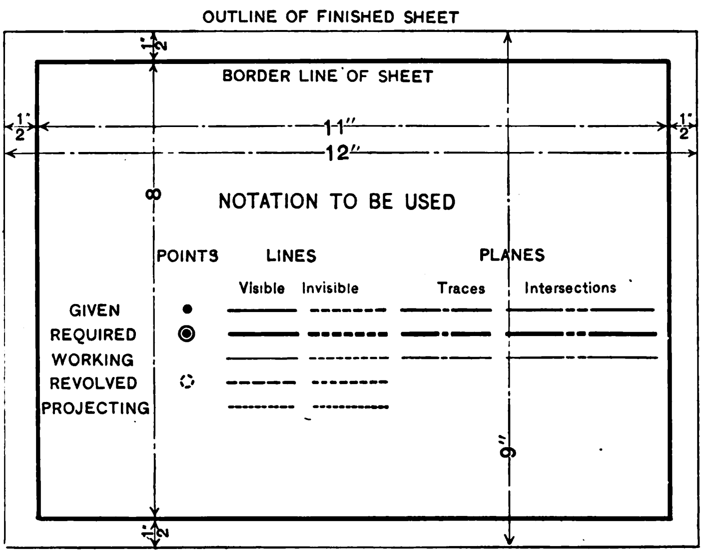


FIG. 76.

in size from $\frac{1}{8}''$ to $\frac{1}{16}''$ high. Any lettering may be condensed, square, or extended.

The sheets must be well balanced, the figures with reference to one another, and the whole with reference to the border-line. (Do not fail to reserve a space at the top for the title of the sheet, and a space at the lower right-hand corner for the signature.) (See sample sheets, Plates 1 and 2, pages 99 and 100).

Each sheet is to be neatly executed in pencil, in accordance with the foregoing directions, then submitted for approval, and when approved, inked in, and lettered, cleaned and trimmed to size ($9'' \times 12''$), then offered for acceptance, grading, and filing.

PLATE No. 1.

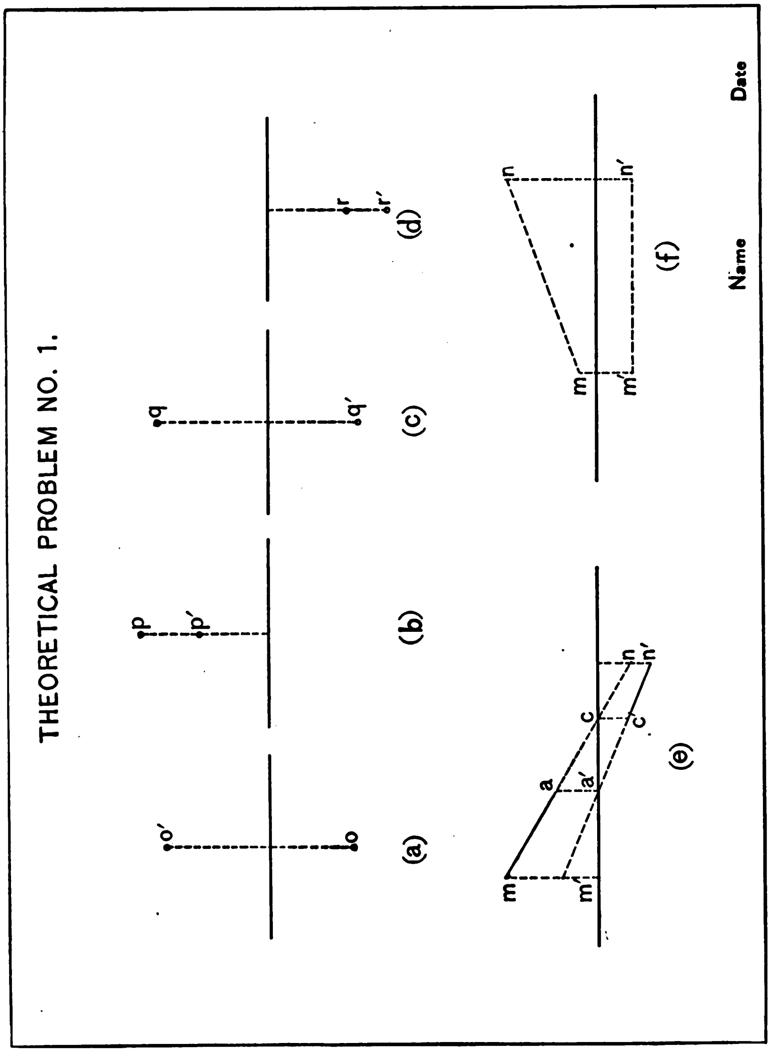
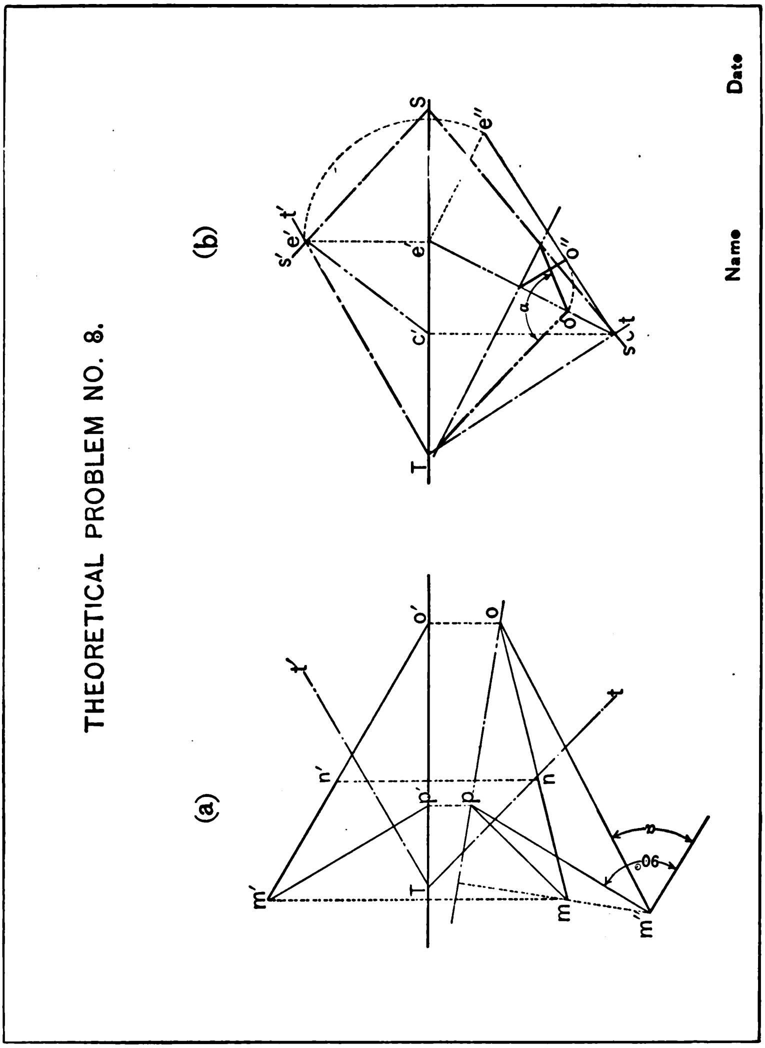


PLATE No. 2.



POINTS, LINES, AND PLANES.

53. **Problem 1:**

(**a**) *Locate the point P in the first quadrant ¾″ from V and 1″ from H.*

(**b**) *Locate the point Q in the second quadrant ½″ from V and ¾″ from H.*

(**c**) *Locate the point R in the third quadrant ¾″ from V and ½″ from H.*

(**d**) *Locate the point S in the fourth quadrant 1″ from V and ½″ from H.*

(**e**) *Show a line in the first quadrant, passing through the fourth quadrant into the third quadrant.*

(**f**) *Show a line in the first quadrant 2″ long, 30° to V, and parallel to H.*

Suggestion:

(*a*), (*b*), (*c*), (*d*). Since the location of a point with reference to *H* is shown by its V projection, and its position with reference to *V* is shown by its H projection, and since the two projections of a point always lie in the same perpendicular to the ground-line, draw a line perpendicular to the ground-line, and on it lay off the dimensions of the point.

(*e*) Assume one end of the line in the first quadrant (a point), and a point in the fourth quadrant; join the like projections of these two points, and produce the line obtained into the third quadrant: this line will be the required line.

(*f*) Since the position of a line with reference to *V* is shown by its H projection, and since a line is projected in its true length on a parallel plane, draw a line in the H plane 2″ long and making an angle of 30° with the ground-line: this line will be the H projection of the line; to locate its vertical projection, draw perpendiculars to the ground-line through the extremes of the H projection, and at any convenient point above the ground-line draw a horizontal line between the perpendiculars: this line will be the required V projection.

54. PROBLEM 2:

(a) *Show a line in the first quadrant, 2″ long, the projections of which are 30° to the ground-line.*

(b) *Find the projections of a line (in any quadrant), 2″ long, 45° to V, and 30° to H.*

(c) *Show a plane, T, oblique to the ground-line, and draw a line in the plane.*

(d) *Find the traces of a plane, S, that will contain the line drawn in plane T* **(c)**.

Suggestion:

(*a*) Assume the projections of any line, the projections of which are at 30° to the ground-line, then revolve the line into coincidence with one of the planes of projection; in this position the line will appear in its true length, therefore lay off a 2″ length on the revolved line, then revolve the line back to its original position.

(*b*) Assume a line 2″ long and parallel with the ground-line (in this position the projections will each be 2″ long and parallel to the ground-line); now, revolve the H projection of this line about one extreme until it makes the V angle with the ground-line; the new position of the other extreme defines the locus of the point in the required projection with reference to the vertical plane. Next, revolve the V projection of the parallel line about the same extremes as used above, until it makes the H angle with the ground-line; this position defines the locus of the other extremes of the line in the required projection with reference to the horizontal plane; revolve the H projection of this extreme in this position until it intersects a horizontal line drawn through the point defining the locus of the point with reference to *V*; this intersection will be the required H projection of one extreme of the line; the distance of the required vertical projection of the point being defined by the point showing the distance of the required projection from *H*, the V projection is obtained by projection; this extreme projected, join the projections with the projections of the other extreme (this point has remained stationary), and the resulting lines will be the required projections.

(*c*) Since a plane is shown by its intersection with *H* and *V* (its traces), a plane that is oblique to the ground-line will have its traces oblique to the ground-line. To locate a line in any given plane it is only necessary to assume a point in each trace and to then join these two points with a line.

(*d*) To pass a plane through any given line, find where the line pierces the planes of projection. These points will be points in the respective traces, and since the traces of a plane always meet in a point in the ground-line, assume any point in the ground-line and join it with the points in which the line pierces *V* and *H*.

55. **PROBLEM 3:**

(**a**) *Revolve the point O about the line M-N into V.* (Fig. 77.)

(**b**) *Pass a plane through three points, M-N-O.* (Fig. 78.)

(**c**) *Find the intersection of two planes, T and S.* (Fig. 79.)

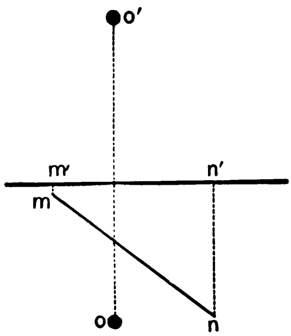


FIG. 77.

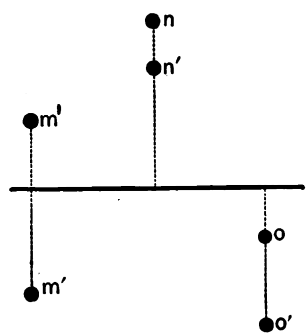


FIG. 78.

Suggestion:

(*a*) Pass a plane through the point perpendicular to the line; find the intersection of this plane with *V*, and with the point at which the line pierces this plane as a center, revolve the point into the V trace of the auxiliary plane.

(*b*) Draw a line through any two of the points, and through any point in this line and the remaining point draw a second line;

find where these lines pierce H and V, and draw the V trace of the required plane through the two points in which the lines pierce V, and the H trace through the two points in which the lines pierce H.

(*c*) Since the V traces intersect in a point, and the H traces intersect in another point, there will be two points in the intersection, which is a right line. Draw the line of intersection through these two points.

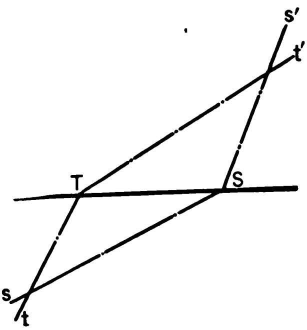


FIG. 79.

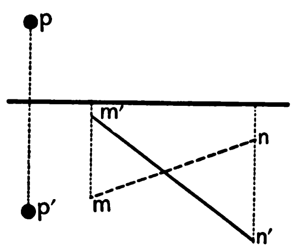


FIG. 80.

56. PROBLEM 4:

(**a**) *Pass a plane through the point P perpendicular to the line M-N.* (Fig. 80.)

(**b**) *Find the perpendicular distance from the point P to the line M-N.* (Fig. 81.)

Suggestion:

(*a*) The traces of the required plane are perpendicular to the corresponding projections of the given line; therefore, through the point draw a line parallel to either trace, and find the point in which it pierces H or V; this is one point in the trace on this plane; through this point draw a line perpendicular to the corresponding projection of the given line, and from the point in which it crosses the ground-line draw a line perpendicular to the other projection of the lines. These lines will be the traces of the required plane.

(*b*) *First method.*—Pass a plane through the point and the given line, and revolve the plane into one of the planes of projection about the corresponding trace, then draw a line from the revolved position of the point perpendicular to the revolved position of the line; this is the required distance.

Second method.—Pass a plane through the point perpendicular to the line; find where the line pierces this plane and join this point with the given point. The true length of this line is the required perpendicular distance.

57. **Problem 5:**

(**a**) *Find where a line pierces a given plane.*

(**b**) *Project the line M-N on the plane T.* (Fig. 82.)

(**c**) *Project the ground-line on the plane T.* (Fig. 82.)

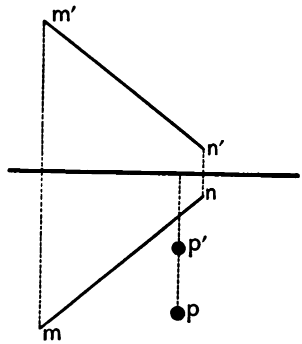


Fig. 81.

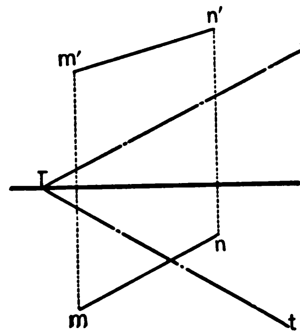


Fig. 82.

Suggestion:

(*a*) Pass a plane through the given line perpendicular to one of the planes of projection, and find the intersection of the plane with the given plane. The point in which the line pierces the plane must evidently be a point in the intersection of these two planes, also, the point must lie in the projection of the line; therefore, the intersection of these two lines is the required point.

(*b*) Drop a perpendicular from each end of the given line to the given plane, and find where these lines pierce the plane; join the points by a right line.

(*c*) Where the traces of the given plane meet in the ground-line will be one point in the required projection; from any other point in the ground-line drop a perpendicular to the given plane and find where it pierces it; join this point with the intersection of the traces on the ground-line.

58. Problem 6:

(**a**) *Find the angle between the line M-N and the plane T.* (Fig. 83.)

(**b**) *Through the point P draw a line making an angle of* 45° *with the plane T.* (Fig. 83.)

Suggestion:

(*a*) *First method.*—The angle between a line and a plane is the angle between the line and its projection on the plane; therefore, project the given line on the given plane, pass a plane

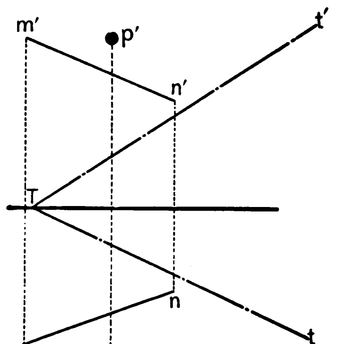


Fig. 83

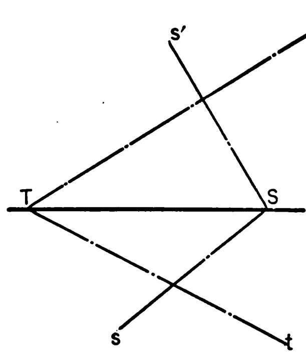


Fig. 84.

through the line and its projection, and revolve this plane into one of the planes of projection. The angle between the lines in the revolved position is the required angle.

Second method.—Drop a perpendicular from any point in the given line to the given plane; the angle between the perpendicular and the given line will be the complement of the required angle.

(*b*) Pass a plane through the given point perpendicular to the given plane, and find its intersection with the given plane; revolve the perpendicular plane about one of its traces, and draw a line through the revolved position of the point making the required angle with the revolved position of the intersection of the two planes; now revolve this line back to the original position of the point, this will give the projections of the required line.

59. **PROBLEM 7:**

(a) *Find the angle between the planes T and S.* (Fig. 84.)

(b) *Find the traces of a plane T making an angle of* 60° *with H and* 45° *with V.*

Suggestion:

(*a*) Through any convenient point in the line of intersection of the given planes pass a plane perpendicular to the line of intersection; this plane will cut a line from each plane; the angle between these two lines is the required angle. To show the true size of this angle, revolve the auxiliary plane about one of its traces into the corresponding plane of projection.

(*b*) Consider the problem solved: now, through any point, *P*, in the ground-line pass a plane, *R*, perpendicular to the H trace of *T*—this plane will give the H angle; next pass a plane through *P* perpendicular to the V trace of *T*—this gives the V angle. The perpendicular distance from *P* to the intersection of either planes *R* and *T* or *S* and *T* is the perpendicular distance from *P* to plane *T*; therefore, to find the traces of a plane making given angles with *H* and *V* assume any point in the ground-line as point *P*, and with this as a center and any convenient radius, as the perpendicular distance from *P* to *T*, describe a circle; tangent to this and above the ground-line draw a line making the given H angle with the ground-line; the point where this line cuts a perpendicular to the ground-line through point *P* will be one point in the V trace, and the distance from *P* to where this line cuts the ground-line will be the radius of a circle to which the H trace will be tangent; now, tangent to the first circle and below the ground-line draw a line making the V angle with the ground-line; where this line cuts the perpendicular through *P* will be one point in the H trace; through this point and tangent to the second circle draw the H trace, and from where it cuts the ground-line draw the V trace through the V point in the trace.

60. PROBLEM 8:

(**a**) *Find the perpendicular distance between two lines, M-N and O-P.* (Fig. 85.)

(**b**) *Show two parallel planes that are ½″ apart.*

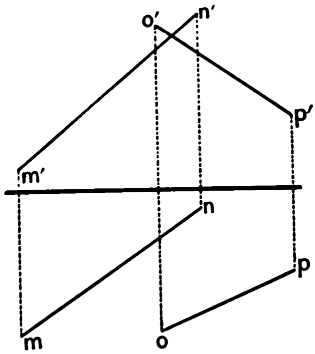


FIG. 85.

Suggestion:

(*a*) *First method.*—Through one line pass a plane parallel to the second line, and project the second line on the plane; at the point where this projection intersects the first line erect a perpendicular to the plane, and produce it until it intersects the second line. The true length of this perpendicular is the required distance.

Second method.—Revolve the two lines about a point in one of the lines (usually one extreme) until that line is parallel to *V*; next, revolve the lines in this position about the same point until the line parallel to *V* is perpendicular to *H*. This line will now project on *H* as a point, and the second line as a line. Now draw a line through the H projection of the point perpendicular to the H projection of the line; this will be the required perpendicular distance between the two lines.

(*b*) Assume one of the planes as a given plane, and pass an auxiliary plane perpendicular to it. This plane will cut a line from the given plane and a parallel one from the required plane, and the distance between the lines will be the perpendicular distance between the planes; therefore, revolve the auxiliary plane about one of its traces into the corresponding plane of projection, and draw the line of the required plane the given distance from the line cut from the given plane. The points in which this line pierces *H* and *V* are points in the corresponding trace of the required plane; through these points draw traces parallel to the respective traces of the given plane.

61. PROBLEM 9:

Pass a circle through three points. (Fig. 86.)

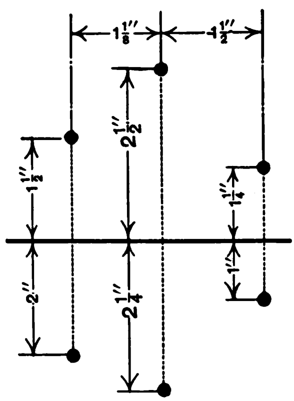


FIG. 86.

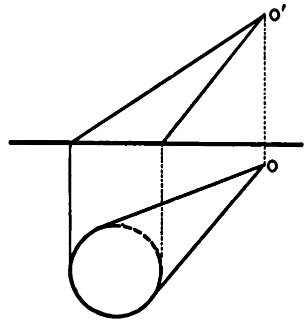


FIG. 87.

Suggestion:

Pass a plane through the three points and revolve the plane into *H* or *V* about the corresponding trace; the points will then appear in their true position with respect to one another; therefore, draw a circle through the three points while in this position, assume a number of points, other than the three given points on the circle, and revolve the plane back to its initial position.

TANGENT PLANES.

62. **PROBLEM 10:**

(**a**) *Pass a plane tangent to a cone at a point on the surface.* (Fig. 87.)

(**b**) *Pass a plane parallel to a line M-N and tangent to a cylinder.* (Fig. 88.)

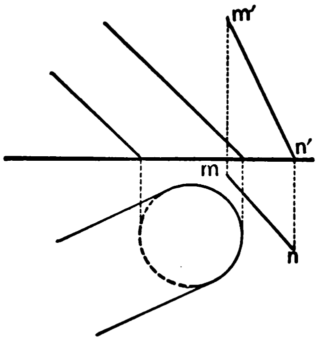


FIG. 88.

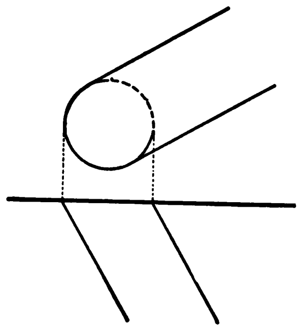


FIG. 89.

Suggestion:

(*a*) Through the point on the cone draw an element of the cone; at the point where this element cuts the base of the cone draw a tangent to the base. These two intersecting lines will determine the tangent plane.

(*b*) Through any point in the line draw a line parallel to the elements of the cylinder; these two lines will determine a plane parallel to the required plane; this plane will cut a line from the plane of the base of the cylinder; parallel to this line draw a line tangent to the base of the cylinder, and at the point of tangency draw an element of the cylinder. These two lines will determine the required plane.

63. **PROBLEM 11:**

(**a**) *Pass a plane tangent to a cylinder at a point on the surface.* (Fig. 89.)

(**b**) *Pass a plane through a point without the cone tangent to the cone.* (Fig. 90.)

Suggestion:

(*a*) Through the point on the cylinder draw an element; at the point where the element cuts the base of the cylinder draw a tangent to the base. These two lines will determine the tangent plane.

(*b*) Draw a line through the given point and the apex of the cone; find where the line pierces the plane of the base of the cone, and through this point draw a line tangent to the base. These two lines will determine the required plane.

64. Problem 12:

(**a**) *Pass a plane tangent to a sphere at a point on the surface.*

(**b**) *Pass a plane parallel to a line and tangent to a cone.* (Fig. 91.)

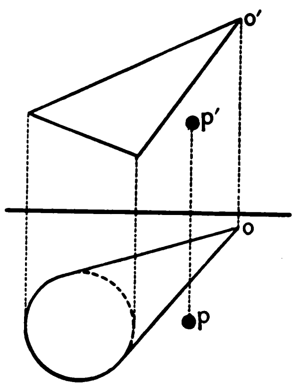


Fig. 90.

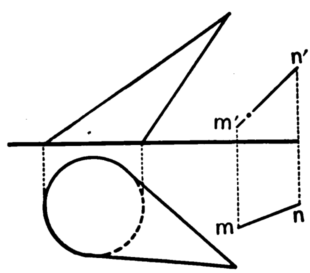


Fig. 91.

Suggestion:

(*a*) Pass a plane through the point parallel to either *H* or *V*, and draw a tangent to the curve cut from the surface at the given point. This line will pierce one of the planes of projection, and through this point draw a trace perpendicular to the corresponding normal to the surface at the point of tangency; from where this trace cuts the ground-line draw the other trace perpendicular to the other projection of the normal.

(*b*) Draw a line through the apex of the cone parallel to the given line; produce this line to an intersection with the plane of the base of the cone, and through the point of intersection draw a line tangent to the base. These two lines will determine the required plane.

65. PROBLEM 13:

(**a**) *Pass a plane tangent to a hyperbolic paraboloid at a point on the surface.* (Fig. 92.)

(**b**) *Pass a plane tangent to a cylinder and through a point P outside of the cylinder.* (Fig. 93.)

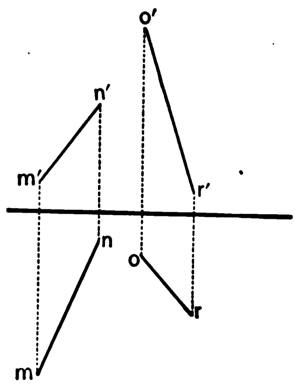


FIG. 92.

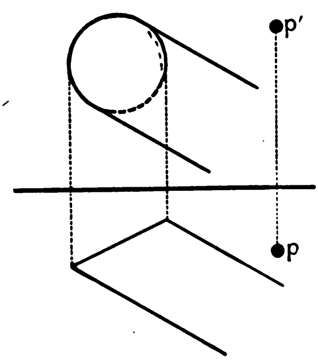


FIG. 93.

Suggestion:

(*a*) Pass a projecting plane through the point. This plane will cut a curve from the surface; a line drawn tangent to this curve at the given point will be one line in the required plane. Pass some other projecting plane and get a second curve and tangent line at the point on the surface; the two tangent lines will determine the required plane.

(*b*) Draw a line through the point and parallel to the axis of the cylinder and find where it pierces the plane of the base of the cylinder; through this point draw a line tangent to the base. This line and the line through the point will determine the required plane.

66. **Problem 14:**

(**a**) *Pass a plane through a line and tangent to a sphere.* (Fig. 94.)

(**b**) *Pass a plane perpendicular to a line and tangent to a sphere.* (Fig. 95.)

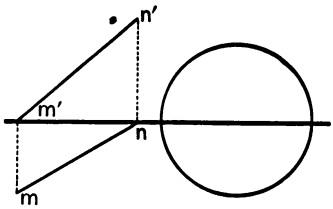


Fig. 94.

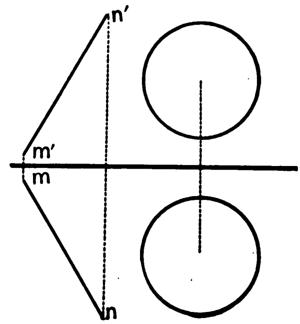


Fig. 95.

Suggestion:

(*a*) *Single-cone method.*—Take a line through the center of the sphere and parallel to either *H* or *V*, and produce this line to an intersection with the given line. This line will be the axis of a tangent cone, and the point where it intersects the given line will be the apex of the cone. The base of the cone will be perpendicular to the axis. Find where the given line pierces the plane of the base, and from this point draw a line tangent to the base. This line and the given line will determine the required plane.

(*b*) Draw a line through the center of the sphere parallel to the given line, and find where it pierces the surface of the sphere; pass a plane through this point perpendicular to the given line.

67. **Problem 15:**

(**a**) *Pass a plane tangent to a helical convolute and parallel to a line.* (Fig. 96.)

(**b**) *Pass a plane tangent to a helicoid and perpendicular to a line.* (Fig. 97.)

Suggestion:

(*a*) With any point in the line as the apex draw a cone the elements of which make the same angle with the H plane as

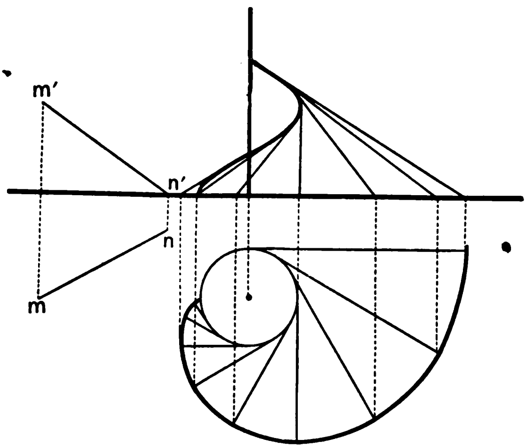


FIG. 96.

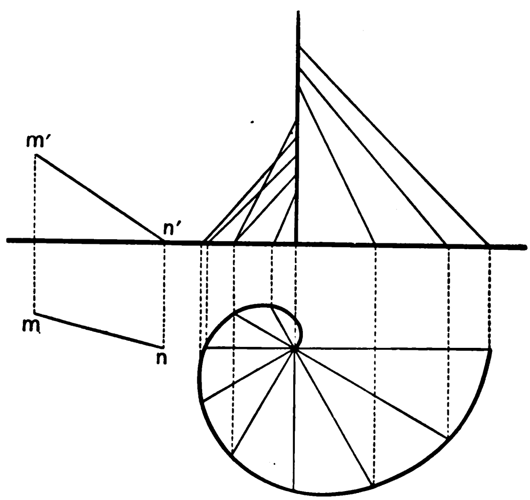


FIG. 97.

do the elements of the convolute. Pass a plane through the line tangent to this auxiliary cone, and find the element of tangency; now find an element of the convolute parallel to this element of the cone, and find where it pierces H and V; draw the required traces through these points parallel to the respective traces of the plane tangent to the cone.

(*b*) With any point in the vertical projection of the axis of the helicoid as the apex construct a cone the elements of which make the same angle with H as do the elements of the helicoid, and pass a plane through the apex of this cone parallel to the required plane, that is, perpendicular to the given line. This plane will, in most cases, cut two elements from the cone; now find an element in the helicoid parallel to either element cut from the cone, and find the points in which it pierces H and V; through these points draw traces parallel to the respective traces of the auxiliary plane. This plane will be the required plane.

68. PROBLEM 16:

Pass a plane through a line and tangent to any double curved surface of revolution. (Fig. 98.)

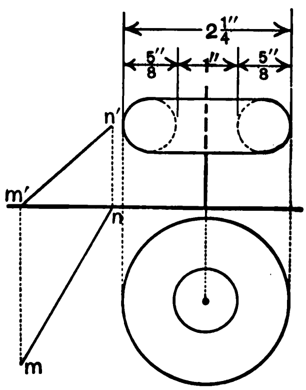


FIG. 98.

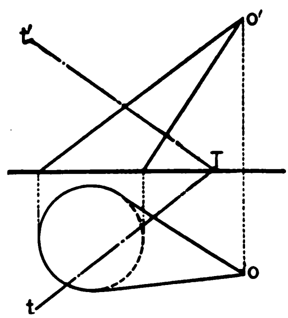


FIG. 99.

Suggestion:

Revolve the line around the axis of the surface and generate a rolling hyperboloid an auxiliary surface of revolution; now pass a meridian plane through the axis parallel to the plane of projection to which the axis is parallel; this plane will cut a meridian curve from each surface; next draw a common tangent to these two curves, then revolve it about the axis until it intersects the given line. These two intersecting lines will determine the required plane.

INTERSECTIONS.

(a) *Find the intersection of a cone and a plane.* (Fig. 99.)

69. Problem 17:

(b) *Find the intersection of a cone and a cylinder.* (Fig. 100.)

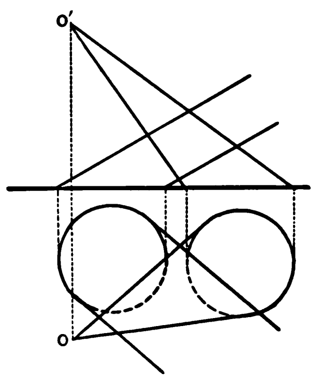


Fig. 100.

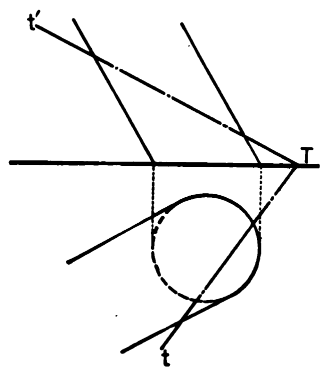


Fig. 101.

Suggestion:

(*a*) Pass a series of projecting planes through the apex of the cone that will cut elements from the cone and right lines from the plane. The intersection of these lines will be points in the required curve of intersection.

(*b*) Draw a line through the apex of the cone parallel to the axis of the cylinder, and pass planes through it that will cut elements from both the cone and cylinder. The intersection of these lines will be points in the required intersection.

70. Problem 18:

(a) *Find the intersection of a cylinder and a plane.* (Fig. 101.)

(b) *Find the intersection of two cones.* (Fig. 102.)

Suggestion:

(*a*) Pass a series of projecting planes through the elements of the cylinder; these planes will cut right lines from the given plane. The intersection of these lines with the elements of the cylinder will be points in the required intersection.

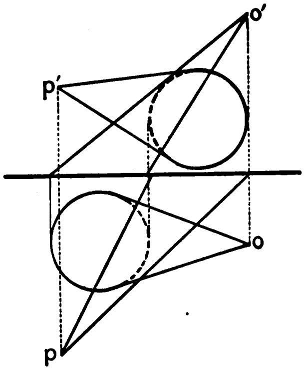


FIG. 102.

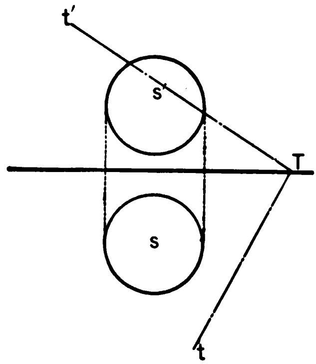


FIG. 103.

(*b*) Draw a line through the apices of the two cones, and pass a series of planes through it that will cut elements from both cones. The intersections of the elements will be points in the required intersection.

71. PROBLEM 19:

(**a**) *Find the intersection of a sphere with a plane.* (Fig. 103.)

(**b**) *Find the intersection of two cylinders.* (Fig. 104.)

Suggestion:

(*a*) Pass a series of projecting planes through the sphere; these planes will cut circles from the sphere and right lines from the given planes. The intersection of these lines with the circles will be points in the required intersection.

(*b*) At some convenient point assume a point, and through it draw a line parallel to the elements of each cylinder; these two lines will determine a plane director; now pass a series of planes parallel to this auxiliary plane that will cut elements from both cylinders. The intersections of these elements will be points in the required intersection.

DEVELOPMENTS.

72. **PROBLEM 20:**

Develop the oblique cone, Fig. 105.

Suggestion:

Draw a number of elements of the cone, and find the true length of each. To begin, select some particular element, usually the longest or the shortest, and lay this off in its true length; next, take the distance from the point where this element cuts the base of the cone to the point where the next element cuts the base, as a radius, and with one end of the line already laid off

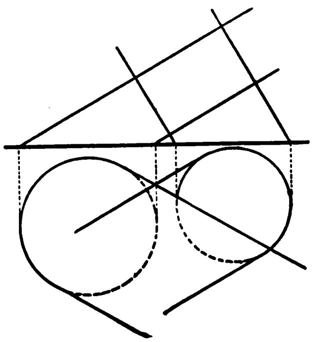


FIG. 104.

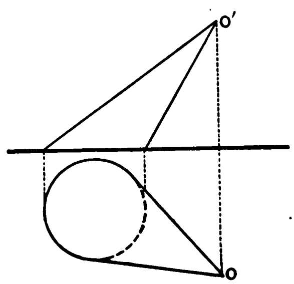


FIG. 105.

as a center, describe an arc; now, with the other end of the developed element as the developed apex of the cone, as a center, and a radius equal to the true length of the second element, strike a second arc intersecting the first one, the line joining this point and the apex will be the developed position of the second element; proceed in this manner, taking one element at a time, until all of them have been laid out, then draw a curved line through the free end of the elements. The figure obtained will be the developed cone.

73. **PROBLEM 21:**

Develop the oblique cylinder, Fig. 106.

Suggestion:

Revolve the cylinder until it is parallel to either *H* or *V*, and while in this position pass a plane perpendicular to the cylinder; this plane will cut a right section from the cylinder that will develop as a right line; next, assume a number of elements, and find the true size of the right section; this will show the true distance between the elements; now lay off the right section in its true development, and locate the points where the elements cut it; draw lines through these points perpendicular to the right line of the developed section, and on these lay off the true lengths of the elements on each side of the section; finish the development by drawing curved lines through the ends of the elements.

74. **PROBLEM 22:**

Develop the cylinder between the horizontal plane and the hyperbolic paraboloid, Fig. 107.

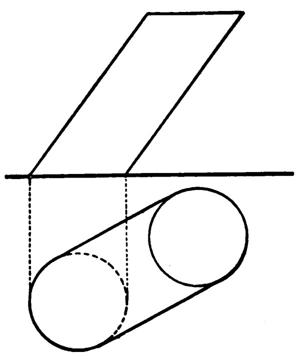


FIG. 106.

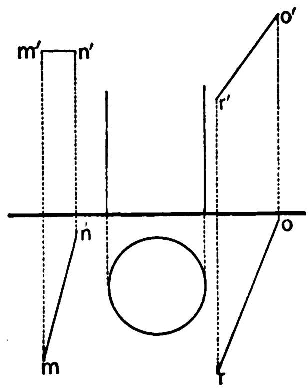


FIG. 107.

Suggestion:

In this case the cylinder is perpendicular to H. First find the intersection of the cylinder with the surface; this will give the upper base of the cylinder; now the horizontal base of the cylinder is a right section, and will develop as a right line, and the vertical projection of the cylinder shows the true lengths of the elements; therefore, proceed as in Problem 21.

75. PROBLEM 23:

Develop the general case of the convolute surface. (Fig. 108.)

Suggestion:

Since the convolute surface is a single curved surface a plane can be passed through any two consecutive elements; therefore, assume a number of elements of the surface as close together as is practicable, and thus divide the surface into a number of small sections; now, beginning with any section, revolve it about a trace drawn through the points in which the two limiting elements pierce the horizontal plane until each element is coincident with H; this will give the approximate size of the section; next, repeat this process for all of the small sections, then add these developments together, and draw a curved line through each end of the elements. The figure thus obtained is the required development, approximately.

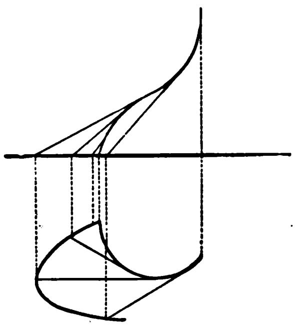


FIG. 108.

76. PROBLEM 24.

Develop a 2″ sphere, approximately.

Suggestion:

Pass a series of meridian planes cutting the sphere into a number of equal meridian sections, and develop one section carefully; repeat for the number of sections in the surface.

CHAPTER V.

PRACTICAL PROBLEMS.

77. Explanatory.—The student having acquired a working knowledge of the principles of Descriptive Geometry, the following examples are given to illustrate their practical application. The examples offered are typical of the problems confronting the engineering draughtsman in every-day practice, with the exception, however, that they are more or less hypothetical in that all Design is omitted (no allowance is made for lap, fastenings, etc.), the work being preliminary to that subject.

78. General Directions.—The general directions for the execution of the exercises are the same as those given in Chapter IV, page 97, the specific differences being that here each exercise has all necessary dimensions given, either on the plate or in the instructions. Some of the exercises are to be drawn full size, others to some proportional scale. For some exercises dimensions for balancing the drawing on the sheet are given. In every case these figures represent full-size lengths, and are to be omitted on the finished drawing.

In place of the notation given in Chapter IV, use the conventions of ordinary drawing, and make all working lines very light, full lines.

TRUE LENGTHS, TRUE ANGLES, INTERSECTIONS, DEVELOPMENTS, ETC.

79. **Problem 1:**

To lay out the cutting lines for getting out the wreath starting from a newel post.

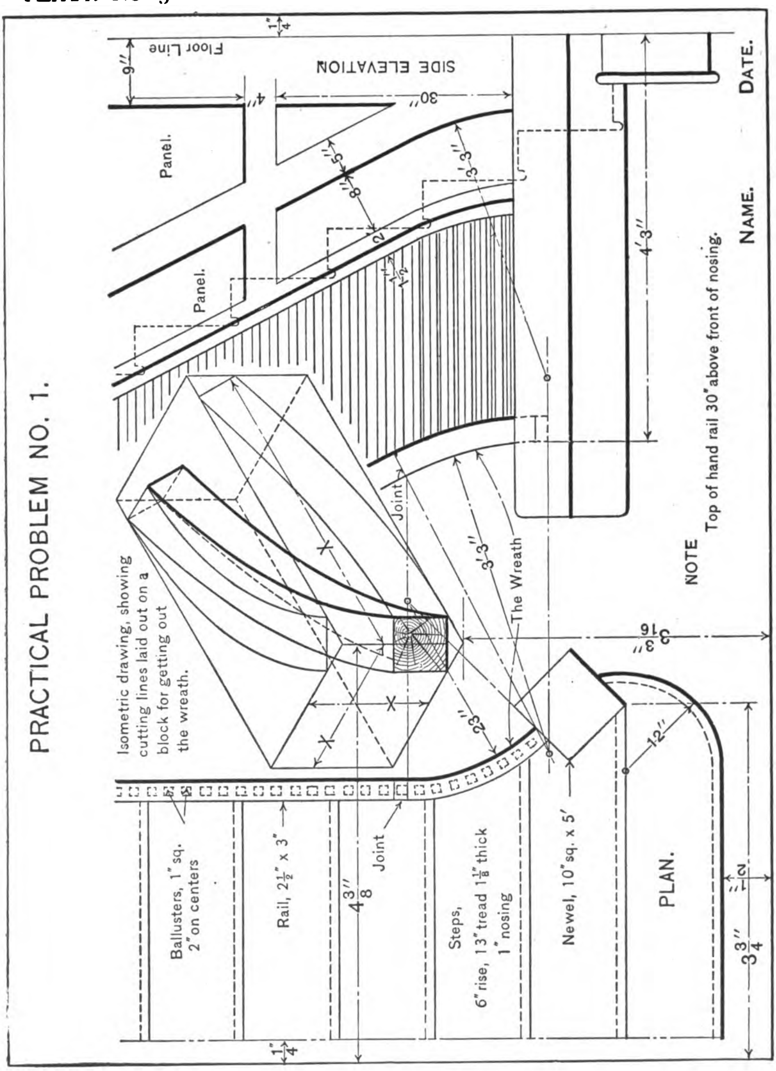
Let the problem be that presented by Plate 3, and let it be required to show the layout in isometric. As may be seen by the plate, the wreath is the curved portion of the hand-rail. Such a piece is usually cut from a block; to find the size of the block, inclose the mechanical drawings—plan and elevation—of the wreath within rectangles; the length of the block will equal the length of either rectangle—the lengths are the same—the width will equal the width of the rectangle inclosing the plan drawing, and the thickness of the block will equal the width of the rectangle inclosing the elevation drawing. In laying out the cutting lines on the block, the top face will contain the plan of the wreath, and the right side the elevation drawing; these lines laid out, the wreath may be sawed out by cutting along them. The drawing shows a rectangular rail; in practice the rail is of a section calculated to be ornamental, and useful as a grip for the hand; such a section is carved in the wreath after cutting out as above.

The isometric drawing is constructed according to the principles of isometric drawing as set forth in Chapter I, by first drawing the outline of the inclosing block, then locating the wreath within it.

Directions for Drawing.

Execute a scale drawing ($1'' = 1''$) according to the dimensions given, drawing the plan first, then the elevation, then lay out the block. Draw all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate, give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

PLATE No 3


PRACTICAL PROBLEM NO. 1.
Isometric drawing, showing cutting lines laid out on a block for getting out the wreath.
SIDE ELEVATION
Floor Line
Panel.
Panel.
Joint
The Wreath
Ballusters, 1" sq. 2" on centers
Rail, 2½" x 3"
Joint
Steps, 6" rise, 13" tread 1⅛" thick 1" nosing
Newel, 10" sq. x 5'
PLAN.
NOTE
Top of hand rail 30" above front of nosing.
NAME.
DATE.

80. PROBLEM 2:

To show the layout for the shop, for a wrought-iron corner support.

Let the problem be that presented by the mechanical drawing of Plate 4, illustrating a plan and elevation of a wrought-iron corner support for an all-steel fence. An inspection of the drawing shows the support to be in one piece, and of a shape including both plane and single curved surfaces, and therefore developable.

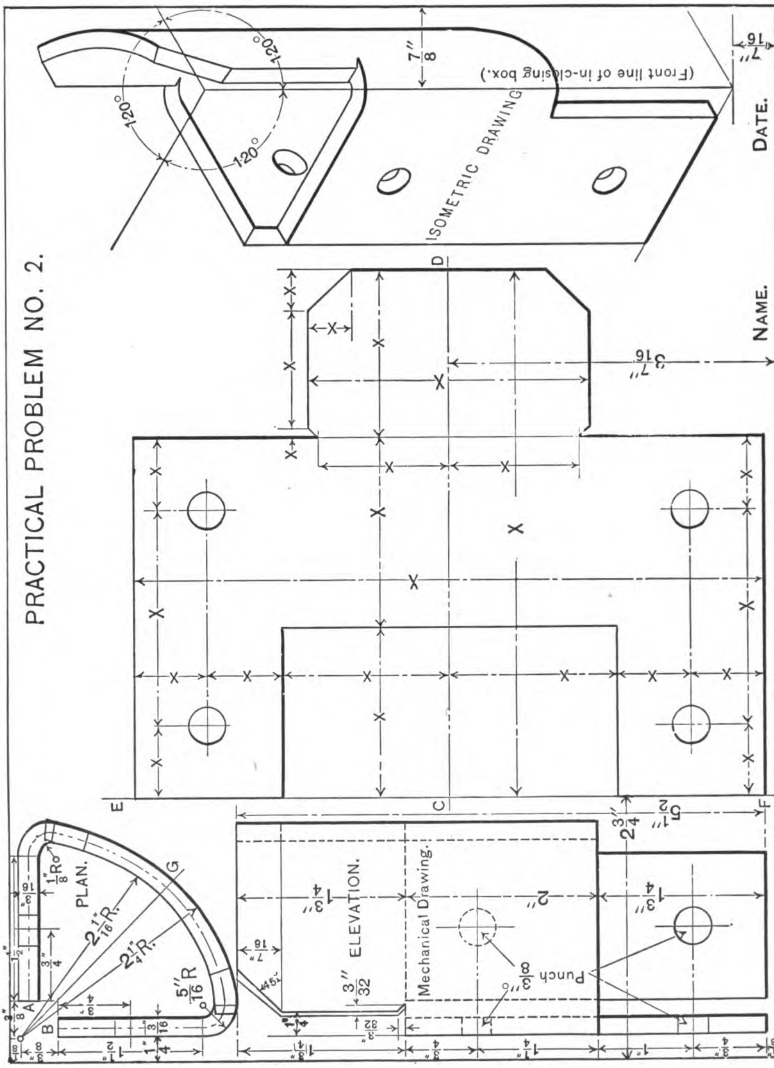
To develop the piece, draw the center line *A-B* of the plan drawing, then draw the center line *C-D* of the development and on it lay off horizontal lengths corresponding to the vertical dimensions of the elevation drawing—these lengths being there shown in their true dimensions since the lines are parallel to that plane of projection; next, draw the indefinite base line *E-F* perpendicular to *C-D*, and working from the center-point *G* of the plan, take dimensions by stepping along the center line *A-B*—both ways—and lay them off on *E-F* symmetrical with the center line *C-D*; through this last set of points, draw a series of horizontal lines, and through the first set of points—those on the line *C-D*—draw a series of vertical lines; these two series of lines will intersect in a series of points from which the outline of the development may be obtained.

In addition to the above statement, let it be required to elucidate the mechanical drawing with an isometric drawing of the support. This is done by first assuming the mechanical drawing to be inclosed within a rectangular box, then draw the box as suggested by the light lines of the isometric drawing, and in accordance with the principles of isometric drawing as set forth in Chapter I, construct the piece within the box, then erase the working lines.

Directions for Drawing.

Execute a full size drawing according to the dimensions given, drawing the plan first, then the elevation, then lay out the development; these drawn, execute the isometric drawing.

PLATE No. 4.


PRACTICAL PROBLEM NO. 2.
ISOMETRIC DRAWING
(Front line of in-closing box.)
PLAN.
ELEVATION.
Mechanical Drawing.
Punch
NAME.
DATE.

Draw all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate; give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

81. PROBLEM 3:

To locate, and to find the length of guy-wires for a smoke-stack.

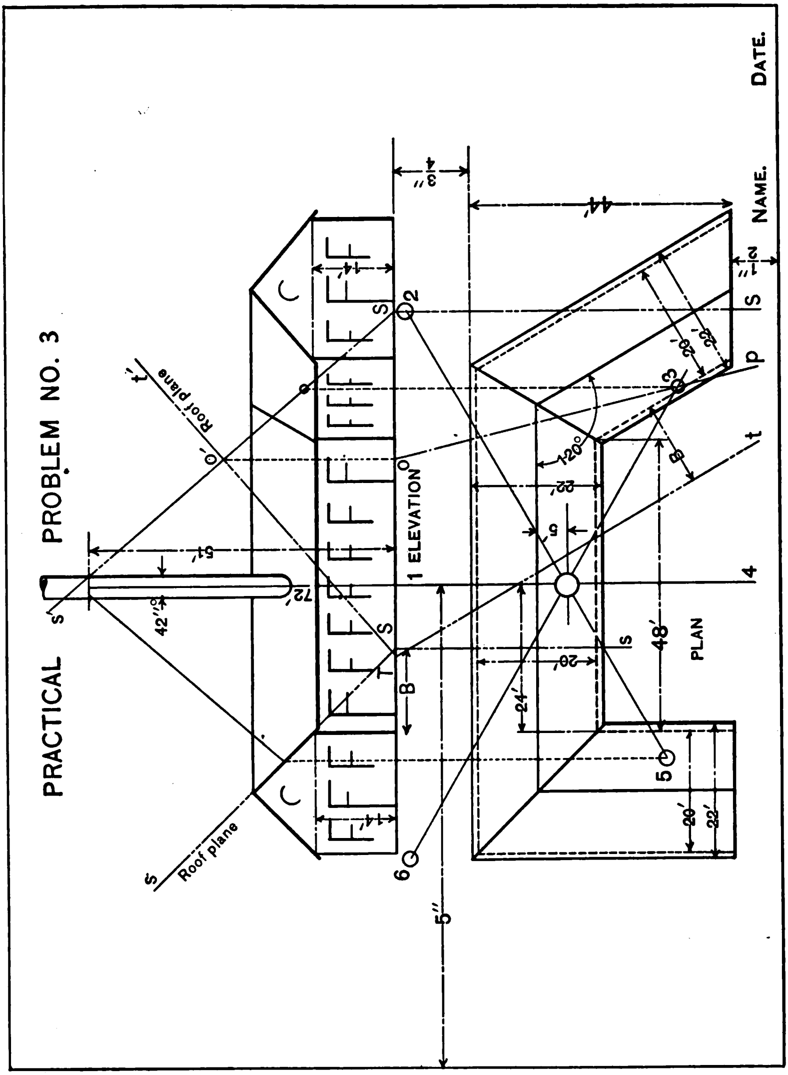
The principles of geometry involved in the solution of this problem are, "To find the point in which a given line pierces a given plane, and to find the true length of a line."

Let the problem be that presented by Plate 5. A power-house of the shape—roof half pitch (45°)—and size given, is to have a 42″° stack, guyed by six guy-wires, arranged as shown, and making an angle of 45° with the stack; the conditions are such that two of the guys will strike the roof plane, and let it be required to locate the ground end of each wire, and to find the length of each.

A cable suspended as in this example would not assume a straight line as shown, but would assume a curve, however for the problem the hypothetical case of the straight line is to be taken.

To find the points in which the guys pierce the ground, revolve one of them parallel to the vertical plane and note the distance of the point in which the vertical projection of the guy strikes the ground-line from the point in which the vertical projection of the center line of the stack intersects the ground-line; with this length as a radius and the horizontal projection of the center line of the stack—the center of the circle—as a center, describe a circle intersecting the horizontal projections of the guys; these points of intersection will be the horizontal projection of the points in which the guys pierce the ground,—they may be located by referring them to the foundation of the building. The true length of the guys reaching the ground, is, evidently, he length of the 45° line on the vertical plane—the vertical protection of a guy when parallel to *V*.

PLATE No. 5.


PRACTICAL PROBLEM NO. 3
1 ELEVATION
PLAN
Roof plane
Roof plane
NAME.
DATE.

To find the point in which the guy on the right pierces the roof plane (the example on the left is a case of simple projection, the plane of the roof, there, being perpendicular to *V*), note that the plane of the roof strikes the ground *B* distance from the foundation, which enables one to draw the trace of the roof plane (*t*-*T*-*t'*), and with the projections of the guy given, to find the point in which the guy pierces the plane *T*. The true length of the guys are found by revloving them into parallelism with *V*.

Directions for Drawing.

Execute a scale drawing ($\frac{1}{16}'' = 1'$) according to the dimensionsgiven, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines, give all dimensions—supplying the required lengths—and finish the sheet by lettering it as shown.

82. PROBLEM 4:

To find the shape, size, and bevels of the three pieces forming the triangular object shown in Plate 6, and to find the angles between the pieces.

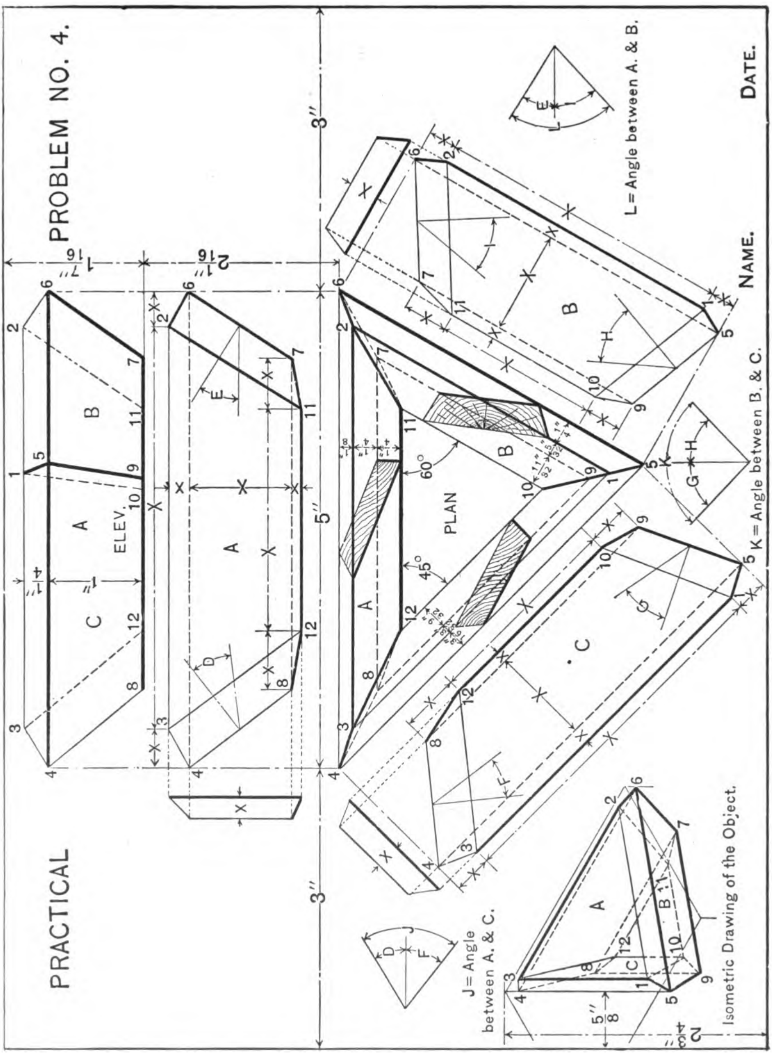
This problem is a typical one, and involves the principles of geometry, "To find the true length of a line, and to find the true size of an angle." It is met with in practice in many different forms.

Sufficient dimensions are given that the section of each piece may be found, then each piece revolved about one of its edges into parallelism with the horizontal plane—a position from which the bevels may be found.

Directions for Drawing.

Execute a full-sized drawing according to the dimensions given, drawing the plan drawing first, then the elevation, then draw the sections; taking one piece at a time, assume it to be removed to one side, then revolve it into parallelism with *H*; scale this drawing and supply the dimensions marked X. To find the angles *D*, *E*, *F*, *G*, *H*, and *I*, pass a horizontal projecting plane perpendicular to one side of the bevel; this will cut

PLATE No. 6.


PRACTICAL
PROBLEM NO. 4.
ELEV.
PLAN
L = Angle between A. & B.
K = Angle between B. & C.
J = Angle between A. & C.
Isometric Drawing of the Object.
NAME.
DATE.

the required angle, which may be shown by revolving the plane parallel to H. The angles between the pieces are found by adding the bevels of each piece, as shown by the plate.

In addition to the above statement, let it be required to draw a half-size isometric drawing of the object, taking the dimensions from the mechanical drawing by scaling it. This is done in accordance with the principles of isometric drawing as given in Chapter I.

Draw all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate; give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

Plate 7, showing a skeleton drawing of a type of locomotive, illustrates a number of typical problems in intersections and developments met with in sheet-metal work. Six plates—those numbered—are chosen for example, and a mechanical drawing of each is given at the bottom of the plate.

PLATE No. 7.

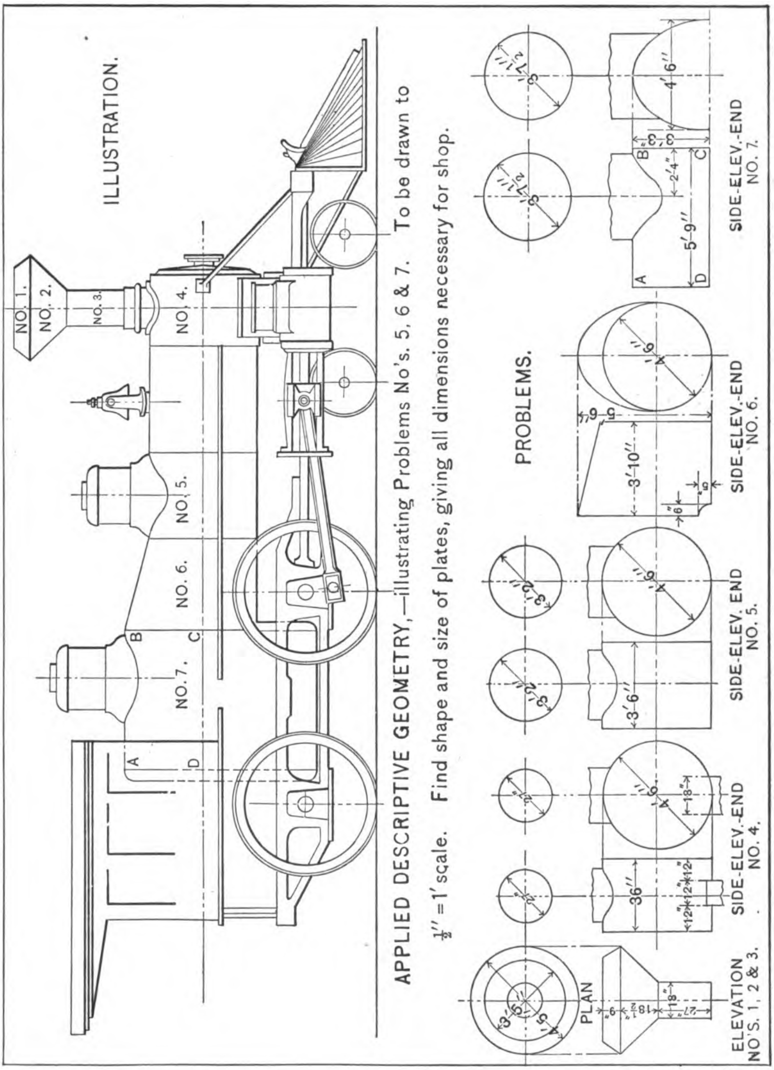


ILLUSTRATION.
NO. 1.
NO. 2.
NO. 3.
NO. 4.
NO. 5.
NO. 6.
NO. 7.
APPLIED DESCRIPTIVE GEOMETRY,—illustrating Problems No's. 5, 6 & 7. To be drawn to
1/2″ = 1′ scale. Find shape and size of plates, giving all dimensions necessary for shop.
PROBLEMS.
PLAN
ELEVATION NO'S. 1, 2 & 3.
SIDE-ELEV.-END NO. 4.
SIDE-ELEV. END NO. 5.
SIDE-ELEV.-END NO. 6.
SIDE-ELEV.-END NO. 7.

83. **PROBLEM 5:**

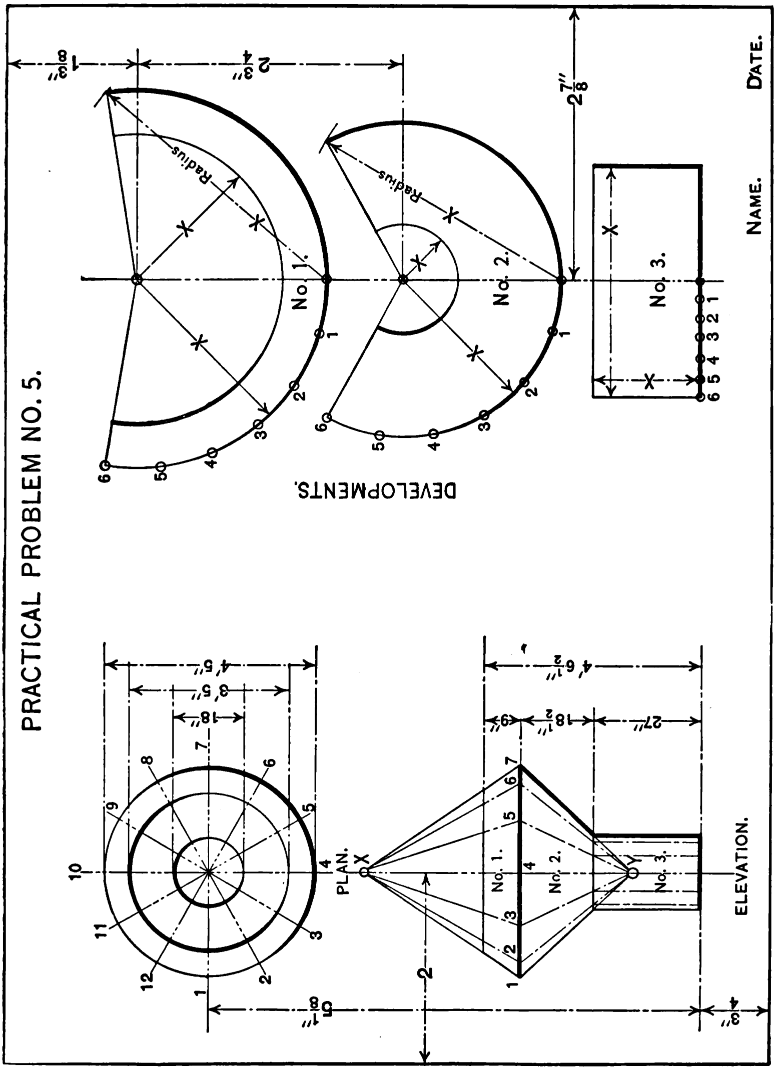
To show the layout for the sheets forming a locomotive stack. Plate 8 (see Plate 7, also).

An analysis of the stack shows it to be made up of a right cylinder (No. 3) and parts of two right cones (Nos. 1 and 2); therefore, produce the sides of the conical parts to complete the cones, and then lay out the developments as suggested by the lines of the plate.

Directions for Drawing.

Execute a scale drawing ($\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines; give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

PLATE No. 8.


PRACTICAL PROBLEM NO. 5.
DEVELOPMENTS.
No. 1.
No. 2.
No. 3.
Radius
PLAN.
ELEVATION.
NAME.
DATE.

84. **Problem 6:**

To lay out the sheet for the smoke-box (No. 4) and the second ring (No. 5) of the barrel of a locomotive. Plate 9 (see Plate 7, also).

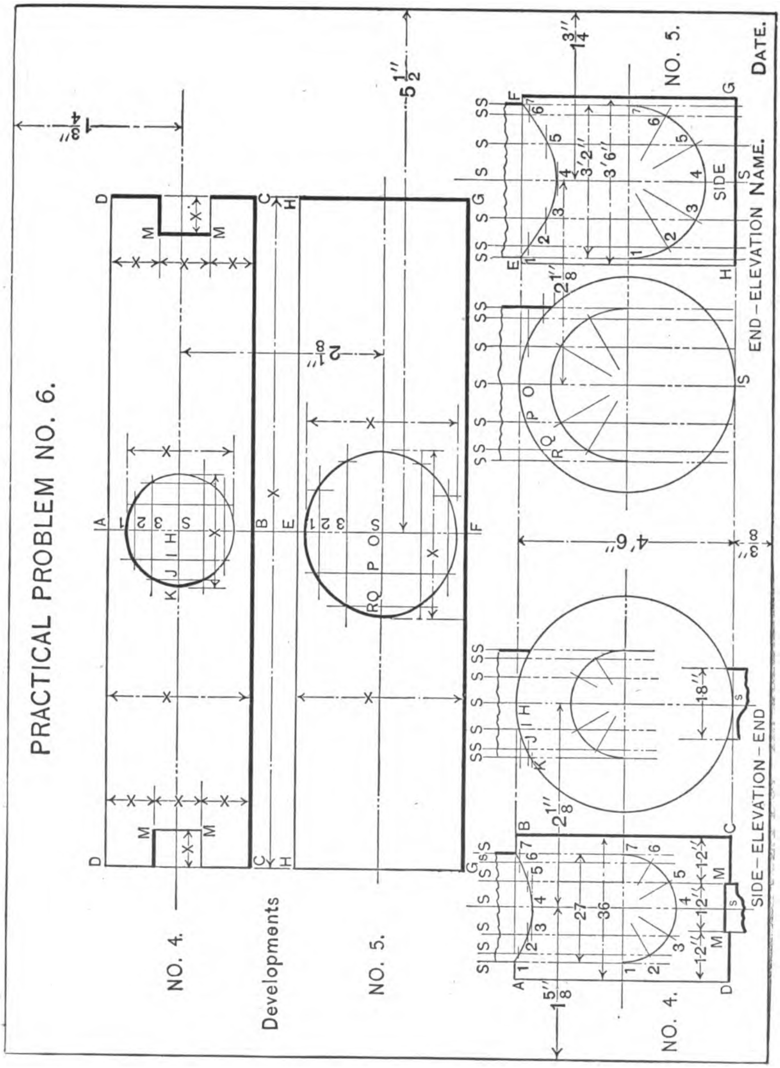
An inspection of the plate shows these two sheets to be right cylinders, in the case of No. 5 intersecting another right cylinder (the sand-dome), and in No. 4 intersecting a right cylinder (the stack) and a rectangular solid (the exhaust-nozzle in the smoke-box).

To develop the sheets, find the above intersections, then proceed as is suggested by the lines of the plate.

Directions for Drawing.

Execute a scale drawing ($\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines; give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

PLATE No. 9.


PRACTICAL PROBLEM NO. 6.
NO. 4.
Developments
NO. 5.
SIDE-ELEVATION-END
END-ELEVATION NAME.
SIDE
DATE.

85. PROBLEM 7:

To lay out the slope-sheet (No. 6) and the outside sheet (No. 7) of a locomotive boiler. Plate 10 (see Plate 7, also).

An analysis of the sheets shows that No. 6, the slope-sheet, is one half a conoid and one half a right cylinder, and that No. 7 is an elliptical right cylinder.

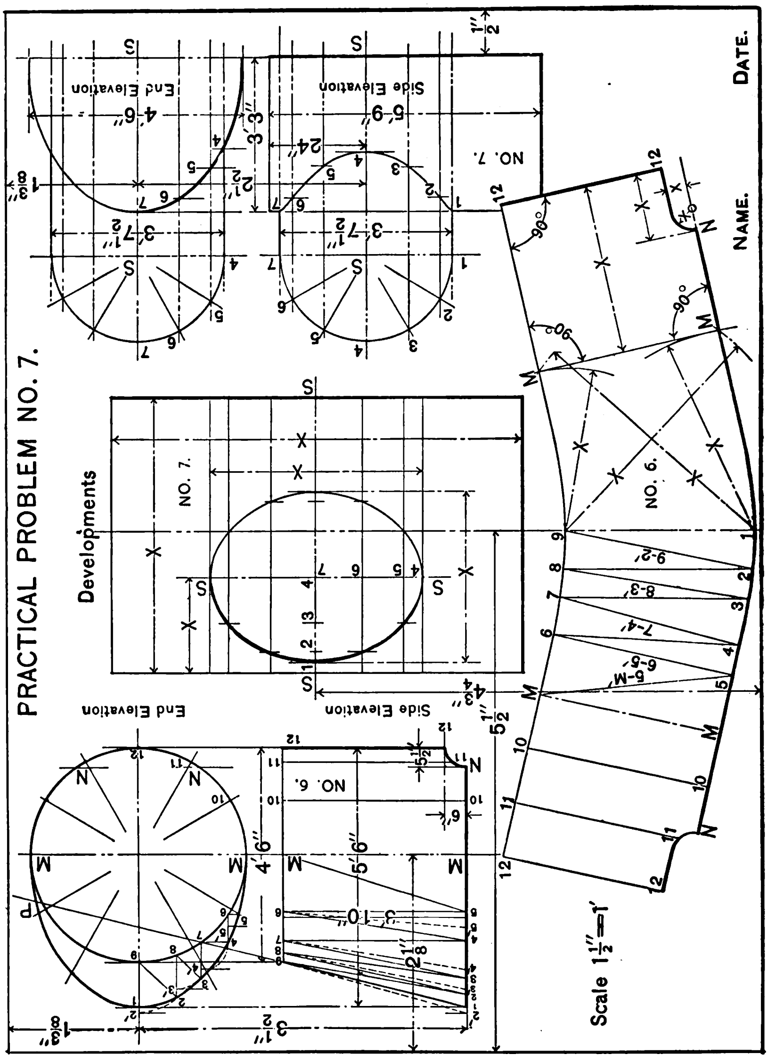
To develop sheet No. 7 find the intersection with the steam-dome, and proceed as suggested by the lines of the plate.

To develop sheet 6, draw a number of elements of the conoid, as the lines 1-9, 2-8, etc., of the end elevation drawing, and project them to the side elevation; next, begin at the center line 9-1 of the development, and lay off a length 9-1 equal to the true length of the element 9-1 (taken from the side elevation); now find the true length of the arc 1-2, end elevation, and with this length as a radius and the point 1 of the developed element 9-1 as a center, strike an arc; now find the true length of the diagonal line 9-2 (end elevation)—the line 9-2′ of the side elevation—and with this length as a radius and a center at the point 9 of the developed element 9-1 strike a second arc intersecting the first arc; this will give the point 2 of the development, and complete the development of the triangle 9-1-2—a small portion of the surface. Proceed in this manner (dividing the surface into a number of small sections), and lay the several sections out with the common side of each two sections—the diagonal lines as the line 9-2′—common to the developed two sections, and draw a curved line through the corner-points of the sections; as shown, and a figure, *M-M-M-M*, will be obtained, which represents the development of the conoid part of the sheet. The development of the remainder of the sheet is that of a right cylinder, and the procedure is clearly indicated by the plate.

Directions for Drawing.

Execute a scale drawing ($\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines, give all dimensions—supplying those marked X—and finish the sheet by lettering it as shown.

PLATE No. 10.


PRACTICAL PROBLEM NO. 7.
Developments
End Elevation
Side Elevation
NO. 7.
NO. 6.
Scale 1½″=1′
NAME.
DATE.

86. PROBLEM 8:

To find the shape and size of the plates used to form an elbow.

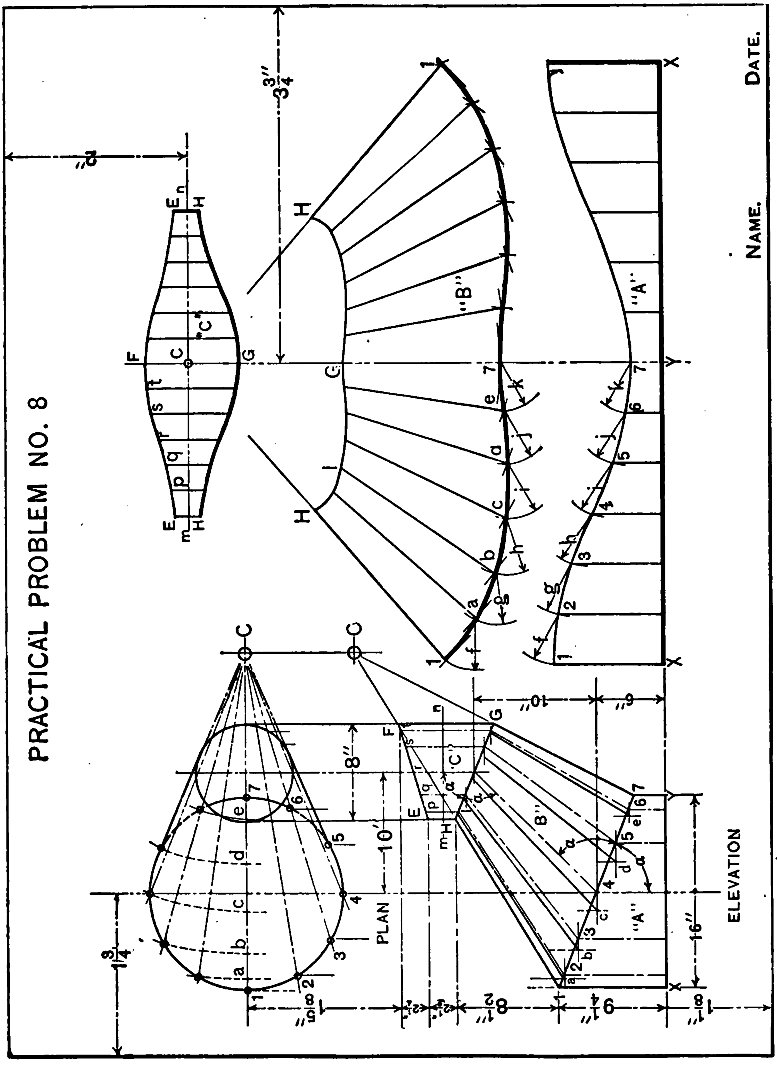
Let the problem be that presented by Plate 11, and let it be required to lay out templets for the three plates forming the elbow, the allowances for lap at the joints being disregarded.

An inspection of the elbow shows it to be made up of three figures, "*A*" and "*C*" being right cylinders of circular section, and "*B*" an oblique cone, and the principles of geometry involved in the solution of the problem to be the development of these three figures, as a drawing of the developments executed on heavy paper, cut out and duplicated in wood or thin sheet metal, or the paper laid directly on a sheet of metal for laying out the plates, would be called a set of templets for the job.

To develop the plates, first pass a number of intersecting planes intersecting the three figures in elements (for convenience, it is suggested that these planes be made to divide the bases into an equal number of equal arcs, twelve being a good working number); then, beginning with cylinder "*A*," lay off the line *X*-*Y*-*X* equal to the circumference of "*A*" and erect the perpendiculars representing the elements cut by the intersecting planes (these lengths are taken directly from the elevation drawing of "*A*"), and through their extremes draw the curve *l*-*l* representing the line of the upper base of the cylinder.

To develop the cone "*B*," the true length of each element must first be obtained by revolving it into parallelism with the horizontal plane (this is a third angle projection) and with the length of the lower base of the cone known—for, since the cone is fitted to cylinder "*A*," the circumference of the bases are equal—proceed to construct development "*B*" as follows· Select a center-point, *C*, and draw the center line *C*-*G*-7 equal in length to the true length of element 7-*C*. (It should be noted that the figures are cut along the outside element—elevation drawing—cut by the intersecting plane 1-*C* of the plan drawing.) With *C* as a center and a radius equal to the true length of element 6-*C* describe an arc; then, with point 7 as a center, and a radius *k*, taken from the development of cylinder "*A*," describe an arc

PLATE No. 11.



intersecting the first arc—the point of intersection, *e*, will be the locus of the lower base end of element 6-*C*; this element may then be drawn by connecting points *e* and *C*; similarly combining the true length of each element with the proper distance between elements, taken from development "*A*," obtain a series of points 7-*e*-*d*-*c*, etc., through which the line 1-7-1, representing the developed circumference of the lower base of the cone, is drawn. With these points, and the center-point *C*, known, it is a simple procedure to lay off the true length of each element and through the extremes to draw the developed line of the upper base, and thus complete the development.

Fig. "*C*" is a right cylinder neither base of which is at right angles to the elements. To develop this cylinder, assume an intermediate base, *m*-*n*, the plane of which is perpendicular to the elements and which will develop as the straight line *m*-*n*; this line is used as a base line for drawing the development, the method of procedure being similar to that used for developing cylinder "*A*."

Directions for Drawing.

Execute a scale drawing ($1\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines, give all dimensions, and finish the sheet by lettering it as shown.

87. PROBLEM 9:

To lay out a reducing breeching for a hot-air conduit in which the main is 26″ in diameter, reducing in one leg to 20″ in diameter, and in the other to 14″ in diameter; the angle between the legs and the main to be 135°.

Let the problem be that presented by Plate 12 and illustrated by Fig. 109, and let it be required to develop sheets *A* and *B*.

To lay out the breeching, first draw the center lines making the required angles with one another, then draw the end view of the 26″ main; next, at the intersection of the center lines.

PLATE No. 12.

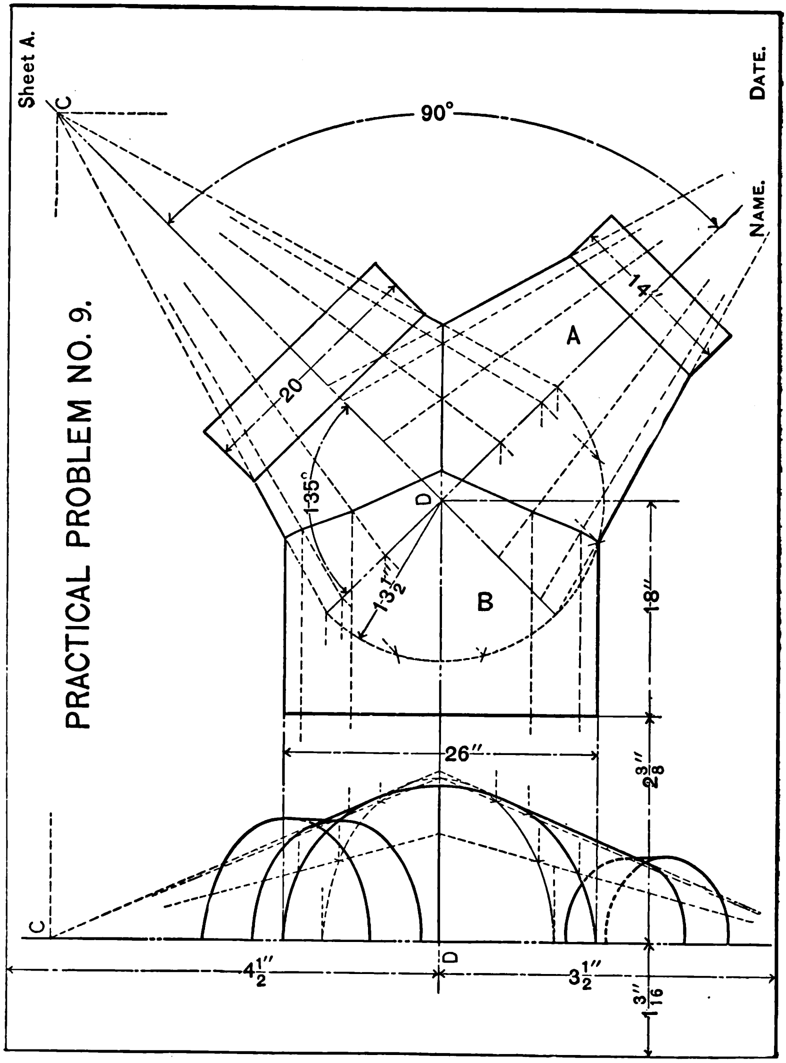
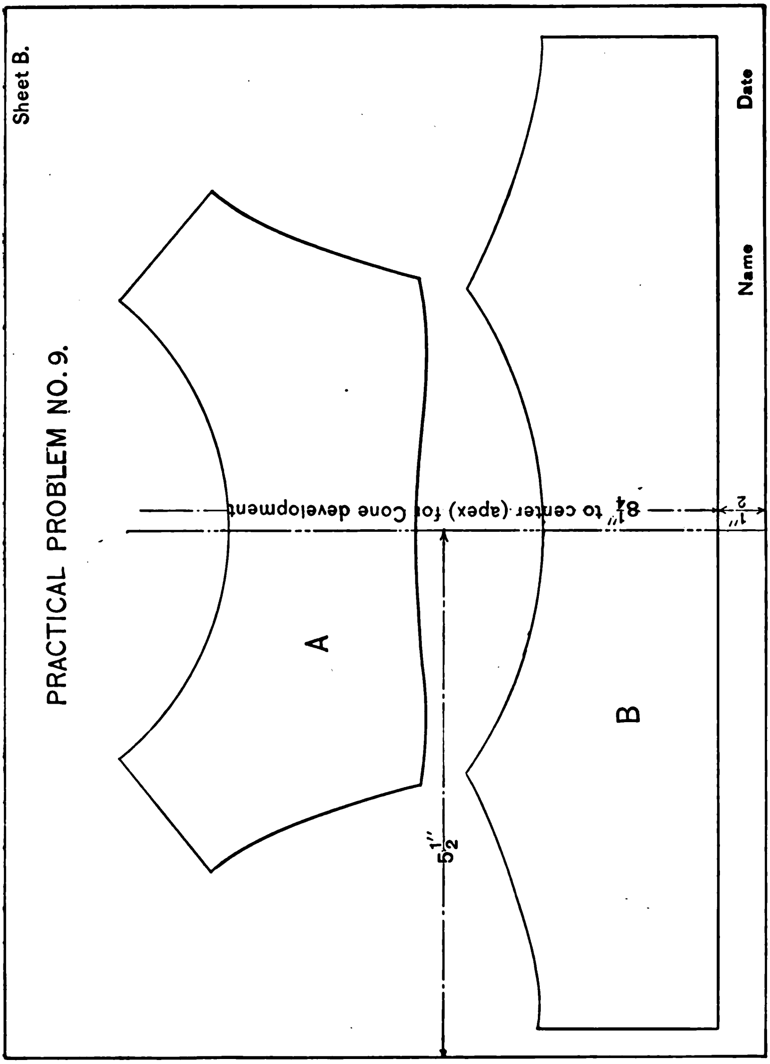

Sheet A.
DATE.
NAME.
PRACTICAL PROBLEM NO. 9.
90°
C
A
B
D
20
14
135°
$13\frac{1}{2}$"
18"
26"
$2\frac{3}{8}$"
$4\frac{1}{2}$"
$3\frac{1}{2}$"
$1\frac{3}{16}$"

PLATE No. 12A.


Sheet B.
PRACTICAL PROBLEM NO. 9.
A
B
8¼" to center (apex) for Cone development
5½"
½"
Name
Date

or axes, point *D*, draw bases of right cones, the diameters of which are a little greater than the diameter of the main (in this case 27″), and the planes of which are perpendicular to the respective axes of the reducing legs of the breeching. This done, draw lines tangent to the bases of the cones and the outline of the cylindrical main (end view), and produce them to an intersection with the plane of the axes—the points of intersection

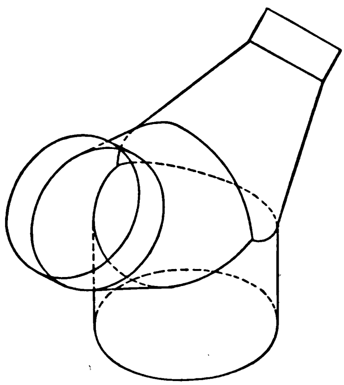


FIG. 109.

will define the apices of the cones assumed; next, find the intersection of the cylindrical main with the conical reducing legs, and the intersection of the legs themsleves, then cut the legs off at the proper point to give the required diameter.

Sheets *A* and *B* forming a right cylinder and a right cone, respectively, the developments are readily obtained, and are to be laid out in accordance with Plate 12, A.

Directions for Drawing.

Execute a scale drawing ($1\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawings for inspection. In inking, ink only those lines shown on the Plates, give all dimensions, and finish the sheets by lettering them as shown.

88. Problem 10:

To lay out the plates for a screw-grain conveyor.

Let the problem be that presented by Plate 13, which illustrates a portion of a form of grain-conveyor (small model), and let it be required to lay out the blade in the most economical manner for punching from sheet metal.

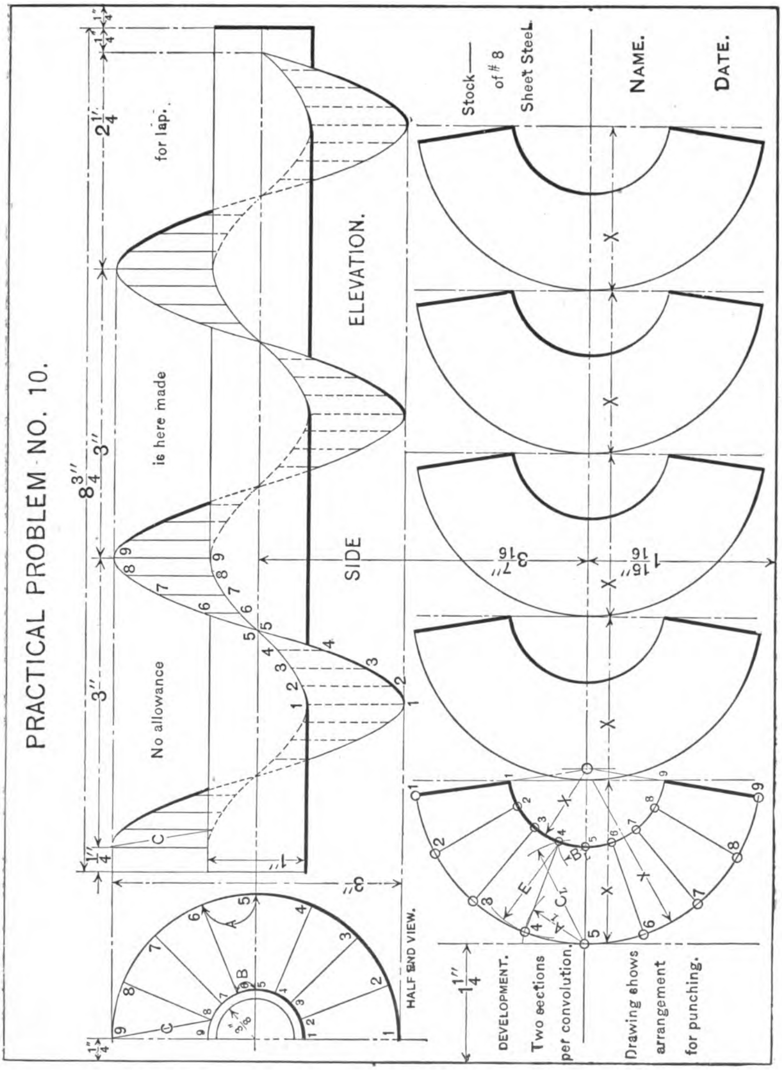
The problem is typical of a form of conveyor of a wide range of usefulness; post-hole augers, the helical, inclined plane up which the circus performer a-foot of a ball or astride a wheel wends his way, are other examples of the surface. For punching, it is evident that by laying out the blade in two sections for each convolution one can effect a saving of material, and, disregarding the question of lap and method of fastening to the central core —items to be considered in actual manufacture—let it be required to develop the blade.

Inspecting the figure one recognizes a practical application of the right helicoid, and the principle of geometry involved in the solution of the problem is, "The development of a right helicoid." This being a surface of double curvature—a warped surface—theoretically it cannot be developed, though practically it can be very closely approximated.

To develop the figure, draw the straight line 5-5 equal to the true length of an element—it is evident that all of the elements are of the same length—then, find the true distance between elements, inner and outer ends—these lengths are also uniform; this distance is the true length of the cord of arc *B* and of arc *A*, respectively. With these lengths as radii, and the extremes of the line 5-5 as centers, describe arcs as shown. Next, find the true length of the diagonal *C*, which gives the last of the lengths required for the development; C_L being this length, and dimension *E* the length of an element, the various lengths are combined as shown, and the points 4, 3, 2, etc., obtained; these points form the locus of the curves forming the outline of the development.

In a carefully executed drawing these curves will be found to be concentric circles, and by using any three points of either

PLATE No. 13.


PRACTICAL PROBLEM NO. 10.
No allowance
is here made
for lap.
ELEVATION.
SIDE
HALF END VIEW.
DEVELOPMENT.
Two sections
per convolution.
Drawing shows
arrangement
for punching.
Stock——
of # 8
Sheet Steel.
NAME.
DATE.

extreme the center of the circles may be found and the arcs drawn.

Directions for Drawing.

Execute a full-sized drawing according to the dimensions given, arranging the punchings as shown; draw all necessary lines in light pencil, then submit the drawing for inspection. In inking, omit all construction lines, give all dimensions—supplying dimensions marked X—and finish the sheet by lettering it as shown.

89. PROBLEM 11:

To find the shape and size of certain plates forming part of a positive-feed mechanism.

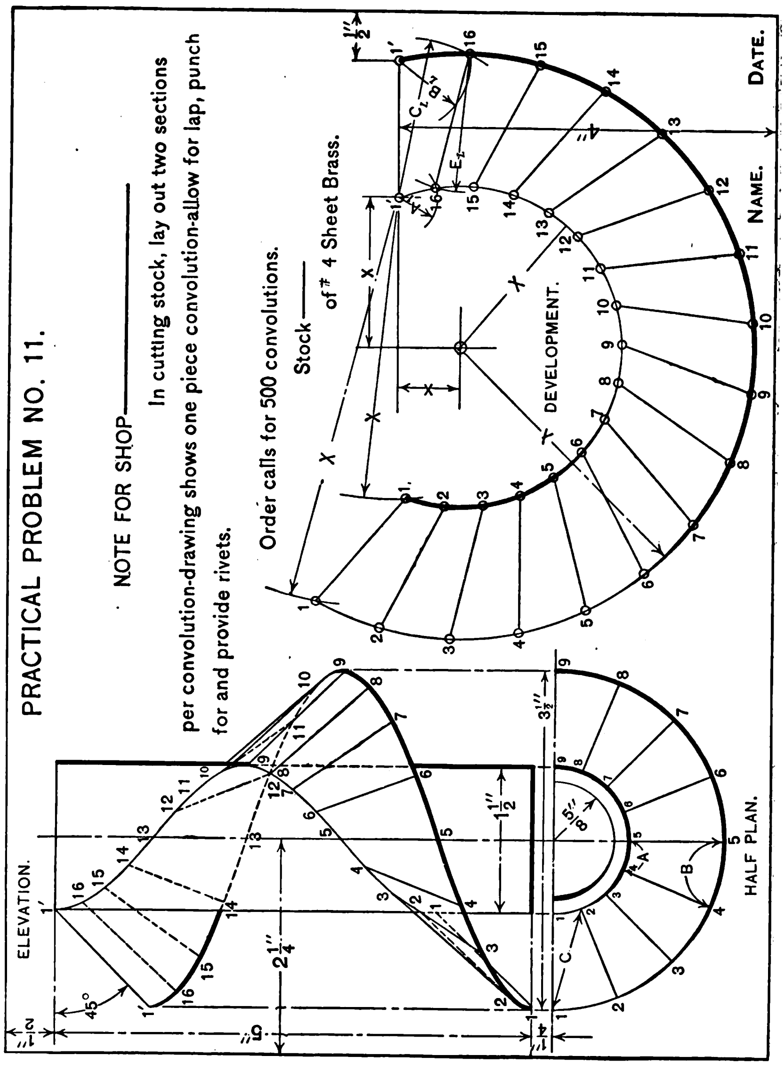
In Problem 10 is found a practical application of the right helicoid; the oblique helicoid is also often met with in practice. It is obvious that the sides of a square thread are right helicoids; the sides of a V thread are examples of the oblique helicoid.

Let it be required to construct a V-threaded screw of plates of metal to form a positive feed for some such mechanism as the housewife's food-grinder—this is named as an example because of its familiarity, though in this apparatus the "screw" is usually of cast iron. The more usual application of the oblique helicoid is in certain forms of screw-propellers for boats, vanes for water-wheels, in positive pressure blowers, etc.

Assuming that if one side of the screw-thread can be laid out, the other side may be readily obtained, the problem deals with a single face of the thread and is presented by Plate 14 illustrating one convolution of this face, the problem being to develop the plate shown.

The figure is a warped surface, hence the development can only be approximated. The method of procedure is exactly as given for developing the right helicoid of Problem 10; dimensions A_L, B_L, C_L, and E (see development) being equal to the true distance between elements—inner and outer extremes—the true length of a diagonal, and the true length of an element, respectively; it is evident that these dimensions are uniform

PLATE No. 14.


PRACTICAL PROBLEM NO. 11.
NOTE FOR SHOP—
In cutting stock, lay out two sections per convolution-drawing shows one piece convolution-allow for lap, punch for and provide rivets.
Order calls for 500 convolutions.
Stock—
of # 4 Sheet Brass.
ELEVATION.
HALF PLAN.
DEVELOPMENT.
NAME.
DATE.

throughout the development, and when laid out will give limiting curves, which are circular arcs.

Directions for Drawing.

Execute a full-sized drawing according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate, give all dimensions, supplying dimensions marked X—these would be necessary for laying out on a sheet of metal—and finish the sheet by lettering it as shown.

90. PROBLEM 12:

To find the angle necessary for the section of an angle-iron for framing the corners of a metal coal-hopper.

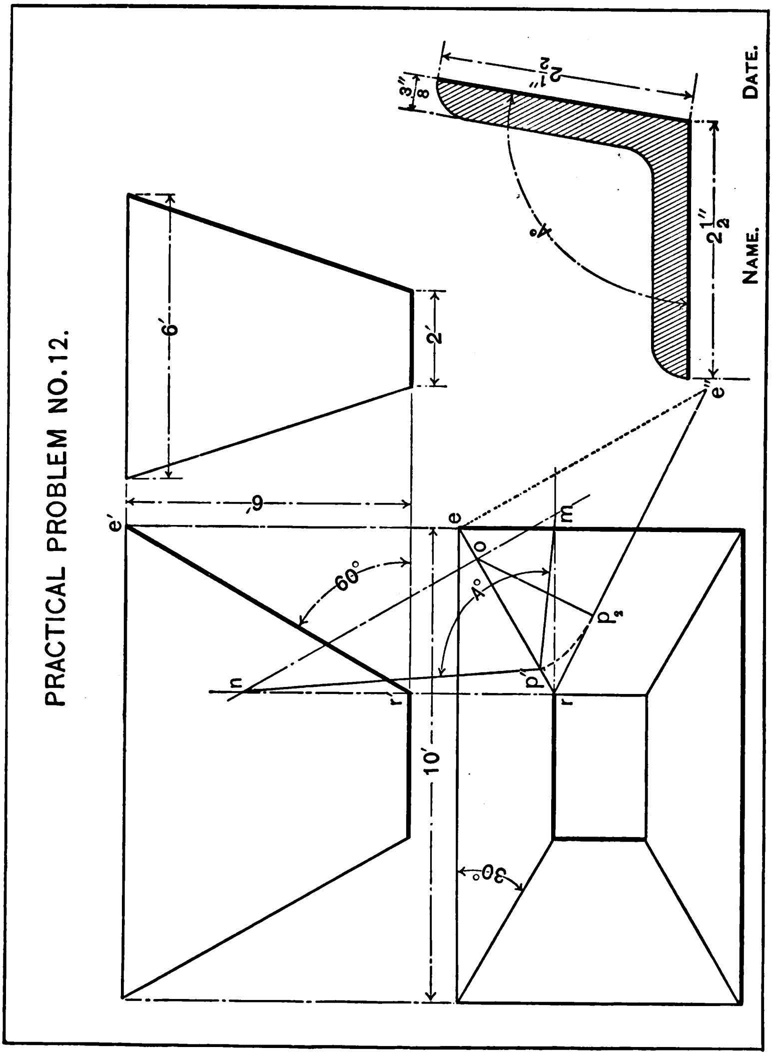
Let the problem be that presented by Plate 15, an inspection of which shows the hopper to be formed by four inclined planes, and the principle of geometry involved to be, "To find the angle between two planes."

To find the required angle pass an auxiliary plane perpendicular to the intersection of the side planes of the hopper, and find the line cut from each side plane by the auxiliary plane—the angle between these two lines will be the required angle. The construction is clearly shown on the plate.

Directions for Drawing.

Execute a scale drawing ($\frac{1}{2}'' = 1'$) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate, give all dimensions—supplying the angle *A*—and finish the sheet by lettering it as shown.

PLATE No. 15.


PRACTICAL PROBLEM NO. 12.
NAME.
DATE.
6'
2'
9'
10'
60°
30°
4°
e'
e
e''
n
r'
r
o
m
p'
p_2
3/8"
2 1/2"
2 1/2"
4°

SHADOWS.

91–93. PROBLEMS 13, 14, and 15:

To find some elementary shadows.

Let the problems be those presented by Plates 16, 17, and 18, respectively, and let it be required to find all of the shadows cast by the figures. The principle of Descriptive Geometry involved in these problems is, "To find the point in which a line pierces the planes of projection."

Before attempting these exercises the student should read Chapter II, and should then study the figures given, and select those points and lines which cast the limiting lines of the shadow, and thus find the shadows with as few working lines as possible.

In Problem 15, Plate 18, in addition to finding the shadow, let it be required to illustrate the problem with an isometric drawing of it.

Directions for Drawing.

Execute a full-sized drawing of each problem (three separate exercises) according to the dimensions given, drawing all necessary lines in light pencil, then submit the drawing for nspection. In inking, ink only those lines shown on the plate, and the outline of the shadow, rule the shadow in as shown by the examples of Chapter II, omit all dimensioning, and finish the sheet by lettering it as shown.

PLATE No. 16.

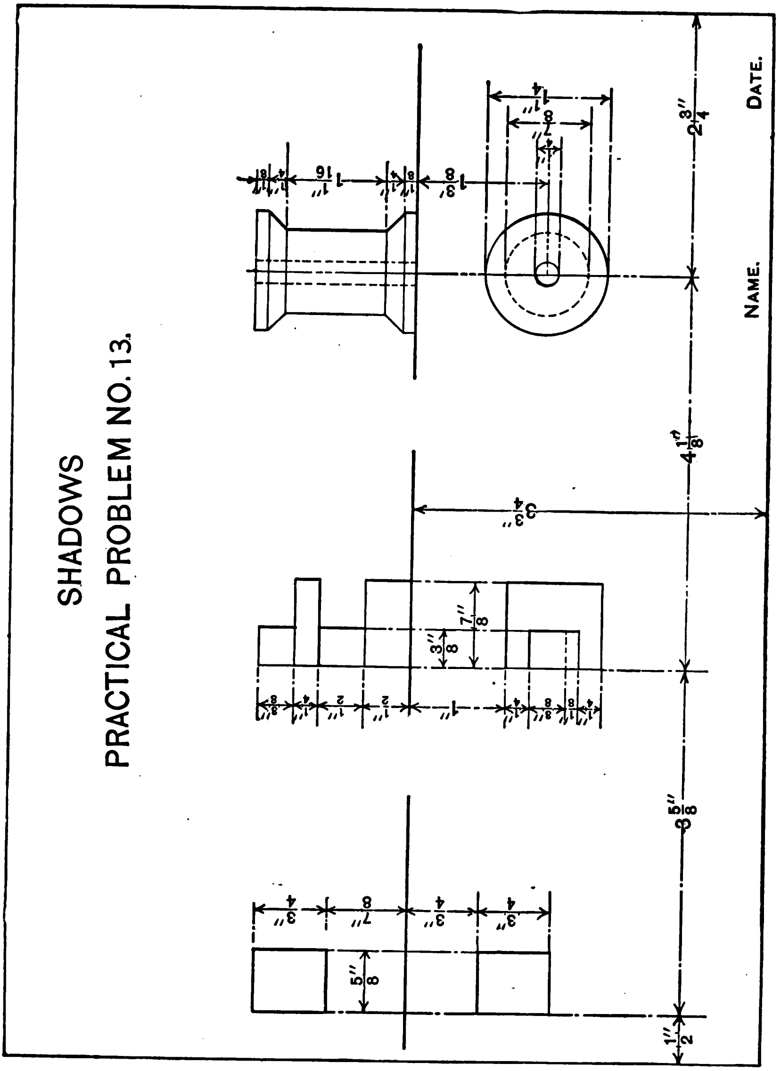


PLATE No. 17.

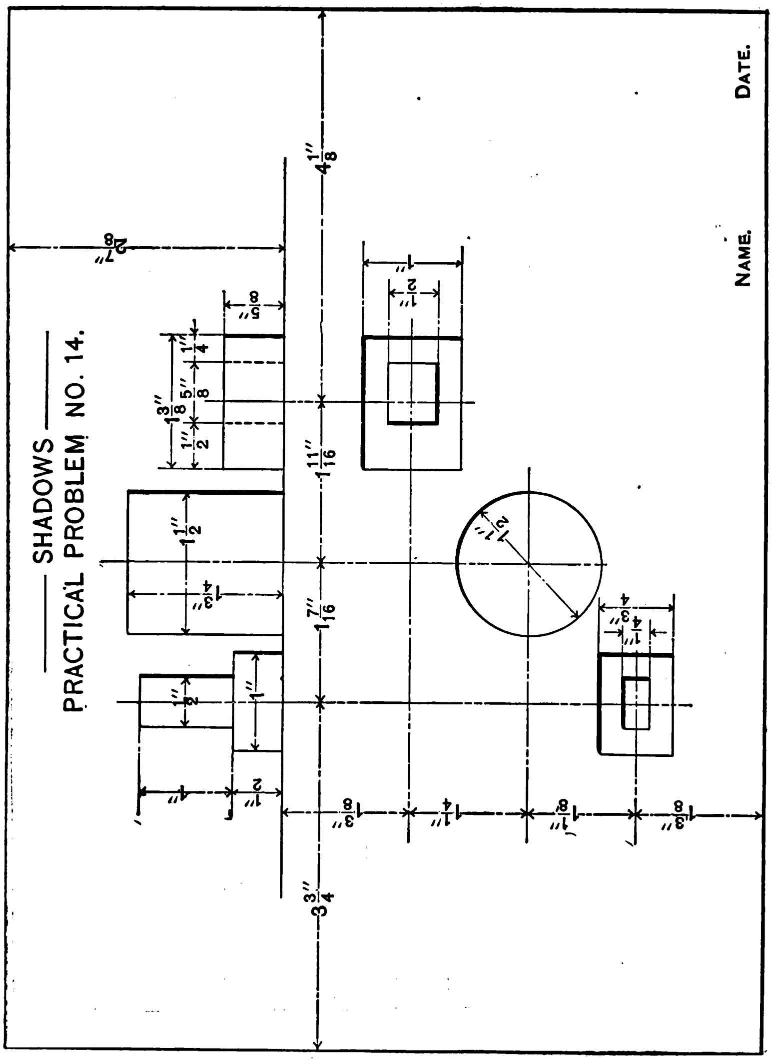
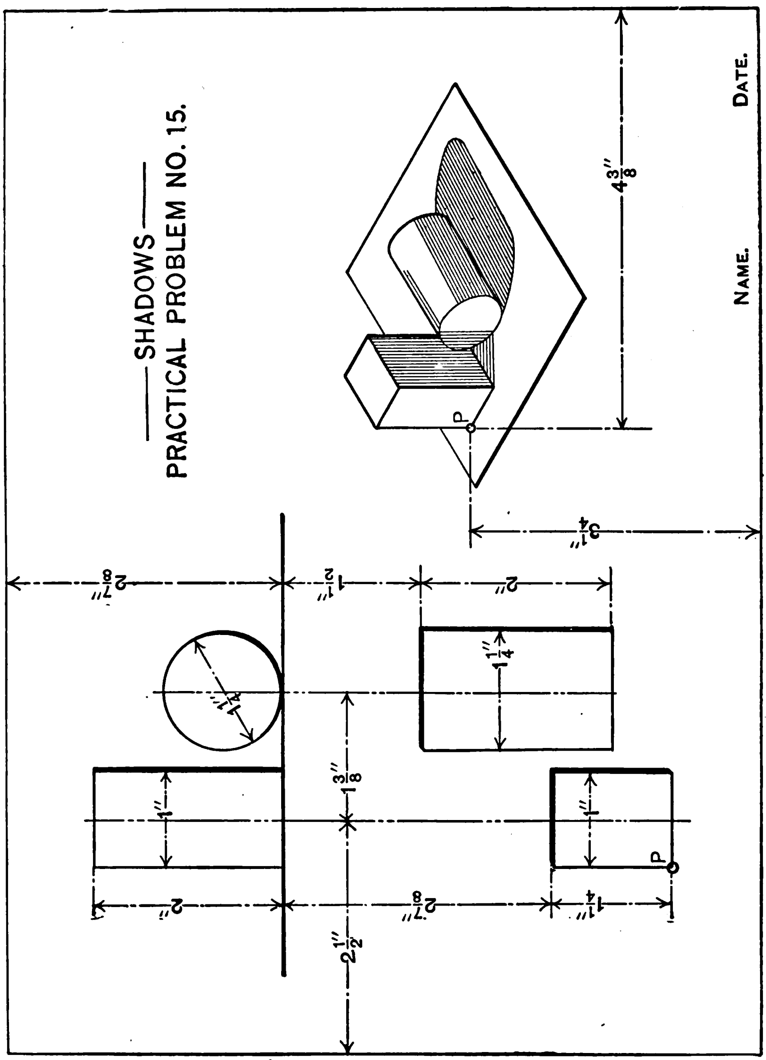


PLATE No. 18.



94. PROBLEM 16:

To find the shadow cast by a taboret.

Let the problem be that presented by Plate 19, and let it be required to find all of the shadows cast by the object.

Directions for Drawing.

Execute a scale drawing to a scale of $2'' = 1'$, drawing all necessary lines in light pencil, then submit the drawing for inspection. In inking, ink only those lines shown on the plate, and the outline of the shadow, rule in the shadow, omit all dimensioning, and finish the sheet by lettering it as shown.

95–96. PROBLEMS 17 and 18:

To find the shadow cast on a double curved surface.

Problem 17:

Find the shadow of the niche shown on page 52 by either assuming dimensions and drawing a similar figure, or by enlarging the cut; in *either* case, draw a figure that will nearly fill the sheet, reserving, however, a space for the title and signature.

Problem 18:

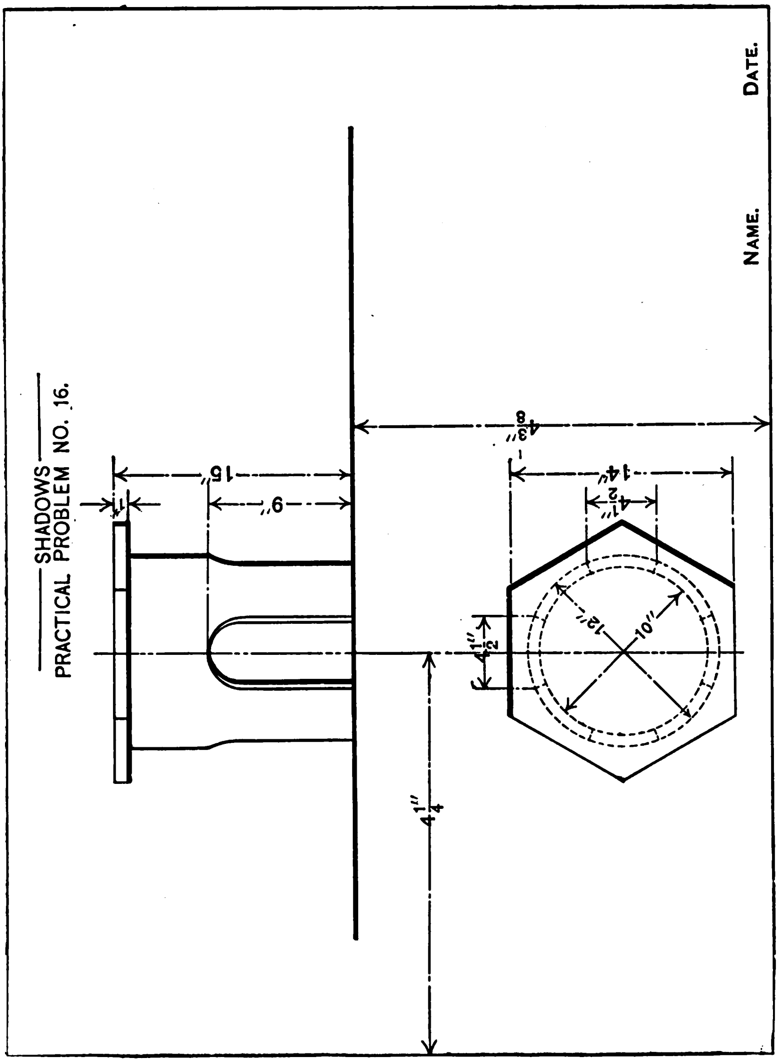
Find the shadow of a sphere.

Let the sphere be $2\frac{1}{2}''$ in diameter; let it be equidistant —say $1\frac{3}{4}''$—from both of the planes of projection; let the ground-line be $3\frac{3}{4}''$ above the lower border line of the sheet and let the center of the sphere be $4\frac{1}{2}''$ in from the left border line, and let it be required to find the shadow on the sphere and on the planes of projection.

Directions for Drawing.

Each problem is to constitute an exercise and is to occupy an entire sheet. Construct the drawings in accordance with the instructions given, find the shadows, then submit the sheets (one at a time) for inspection. In inking, ink only the outline of the figures and shadows, rule in the shadow, omit all dimensioning, and finish the sheets by lettering them in accordance with Plate 19.

PLATE No. 19.


SHADOWS
PRACTICAL PROBLEM NO. 16.
NAME.
DATE.
15"
9"
1"
14"
4½"
4⅜"
12"
10"
4½"
4¼"

PERSPECTIVE.

97. **PROBLEM 19:**

To find some elementary perspectives.

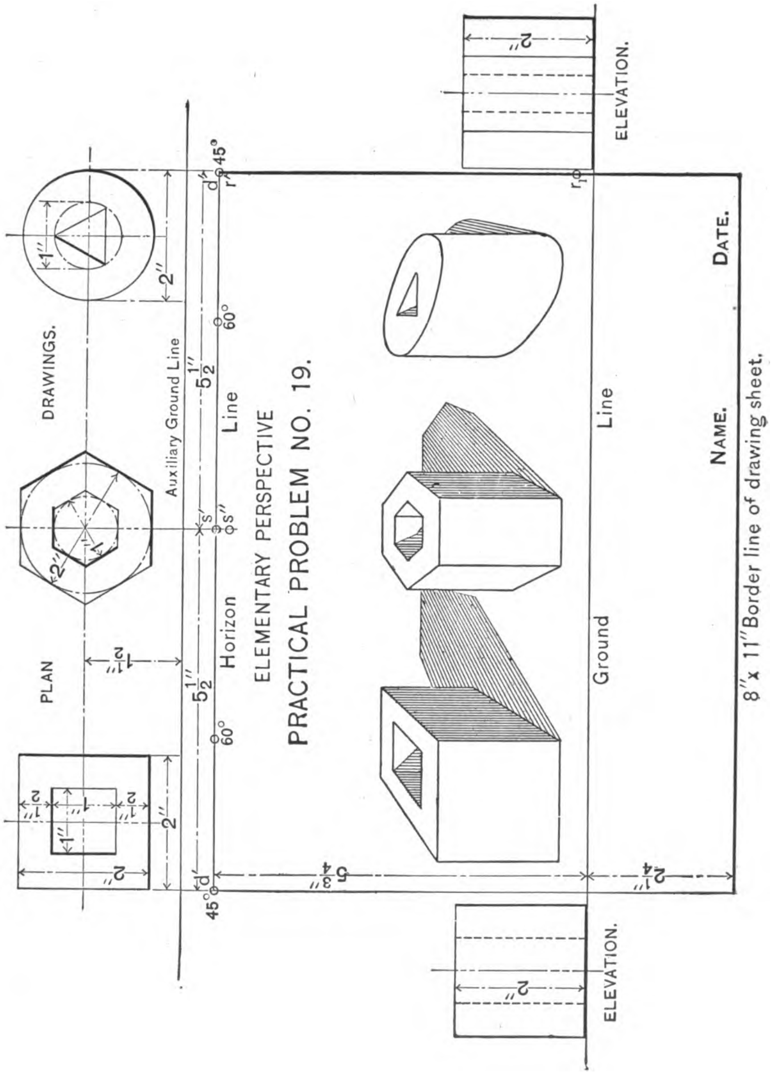
Let the problem be that presented by Plate 20, showing the mechanical drawings for a hollow cube, a hollow, hexagonal prism, and a hollow cylinder, also, the conditions for the perspective, and the required perspectives, and let it also be required to show all of the shadows cast by the objects.

Before attempting the execution of the problem, the student should read Chapter III.

Directions for Drawing.

Execute a full-sized mechanical drawing of the objects, according to the dimensions given, on a sheet of paper other than that to receive the perspective—the finished sheet—then cut the paper, separating the plans from the elevations, and arrange them about the sheet to receive the perspective—the field of the picture—as shown; find the perspectives, then submit the drawing for inspection. In inking, ink only the perspectives, and the 8″×11″ border line of the sheet, and finish the sheet by lettering the title and signature only.

PLATE No. 20.


PLAN
DRAWINGS.
ELEVATION.
ELEVATION.
Auxiliary Ground Line
Horizon Line
ELEMENTARY PERSPECTIVE
PRACTICAL PROBLEM NO. 19.
Ground Line
NAME.
DATE.
8″ x 11″ Border line of drawing sheet.

98. **PROBLEM 20:**

To find the perspective of a flight of stone steps.

Let the problem be that presented by Plate 21, showing the plan and elevation of the steps, the conditions for the perspective, and the required perspective, and let it also be required to show all of the shadows cast.

The conditions given are intended more as an example than as specific instruction, and it is suggested that the student assume other, similar conditions.

Directions for Drawing.

Execute a mechanical drawing of the steps to a scale of $1'' = 1'$ according to the dimensions given, on a sheet of paper other than the sheet to receive the perspective, then cut the paper, separating the plan and elevation, and arrange these views and the sheet to receive the perspective as suggested by the plate; find the perspective, then submit the sheet for inspection. In inking, ink the perspective and the $8'' \times 11''$ border line only, and finish the sheet by lettering it as follows:

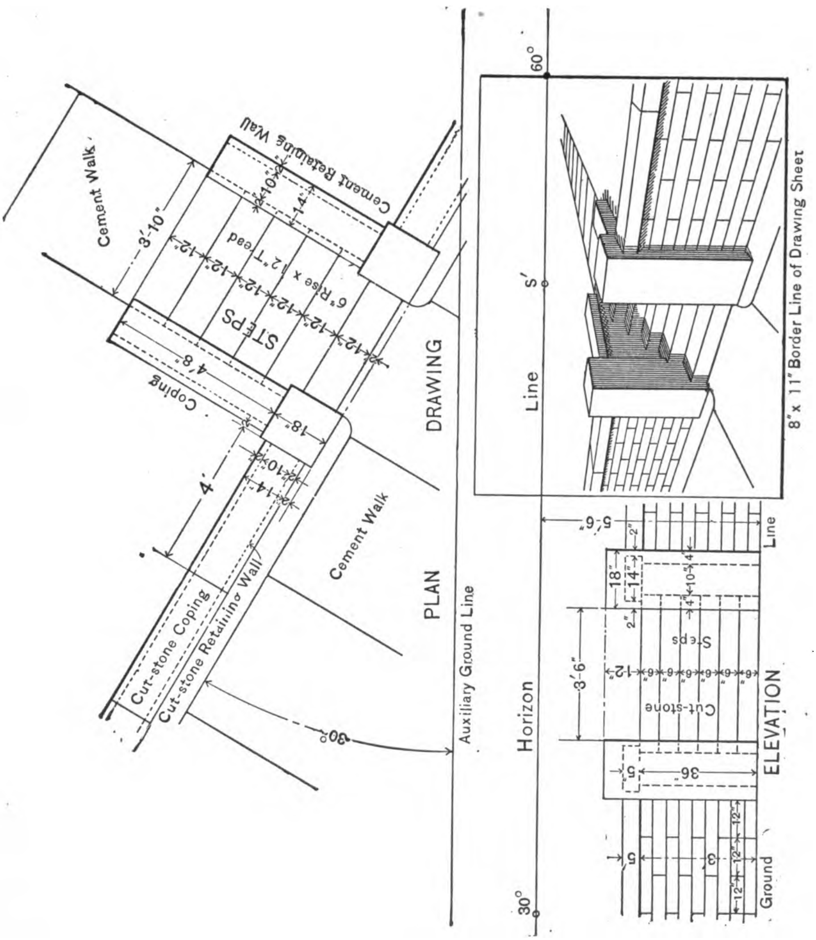
TITLE,

PERSPECTIVE.

Practical Problem No. 20.

NAME AND DATE, to be in the lower right-hand corner of the sheet.

PLATE No. 21.


Cement Walk
3'-10"
Cement Retaining Wall
2'-10"
1'-4"
STEPS
6" Rise x 12" Tread
4'-8"
Coping
18"
4'
10"
14"
Cement Walk
Cut-stone Coping
Cut-stone Retaining Wall
30°
PLAN
DRAWING
Auxiliary Ground Line
Horizon
Line
S'
60°
30°
8" x 11" Border Line of Drawing Sheet
5'-6"
3'-6"
Steps
Cut-stone
36"
ELEVATION
Ground
Line

99. PROBLEM 21:

To find the perspective of a small railway station-house.

Let the problem be that presented by Plate 22, illustrating a set of conditions (these or other conditions may be used).

Directions for Drawing.

Execute a mechanical drawing of the building to a scale of $\frac{1}{8}'' = 1'$ on a sheet of paper other than the sheet to receive the picture, then cut the paper, separating the views, and arrange them about the field of the picture as suggested by the plate; find the perspective, then submit the sheet for inspection. In inking, ink the lines of the perspective and the border line only, and finish the sheet by lettering it as follows:

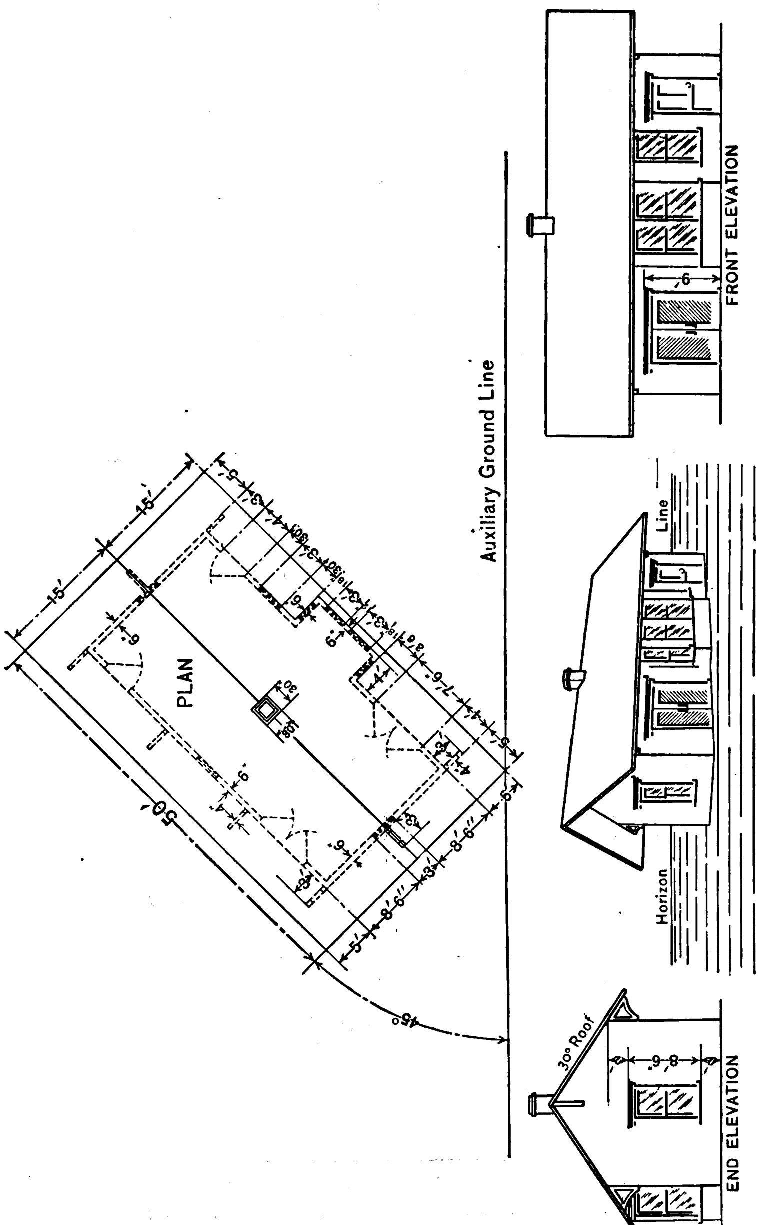
TITLE,

PERSPECTIVE.

Practical Problem No. 21.

NAME AND DATE, to be in the lower right-hand corner of the sheet.

PLATE No. 22.


PLAN
Auxiliary Ground Line
FRONT ELEVATION
END ELEVATION
Horizon
Line
30° Roof
50'
15'
15'
45°

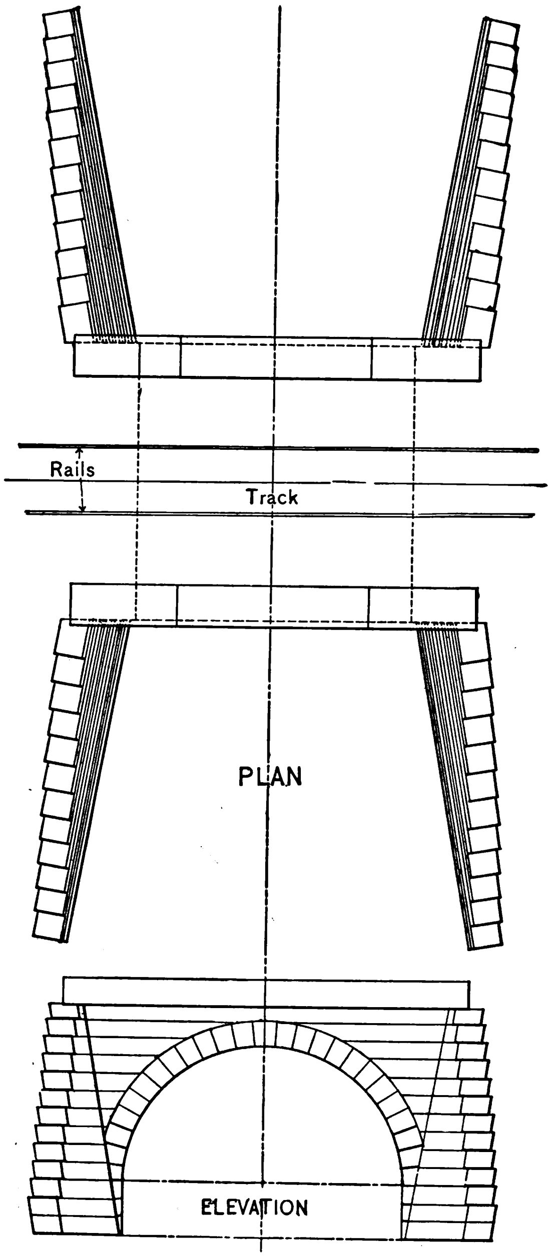


FIG. 110.

100. PROBLEM 22:

To find the perspective of a railway arch.

Let the problem be that presented by Fig. 110, and illustrated by Fig. 111, and let the student assume his own conditions, such that the perspective will look well on a standard 8″×11″ sheet of paper.

Directions for Drawing.

Execute the mechanical drawings of the arch by enlarging the copy, say two times; then separate the views and arrange

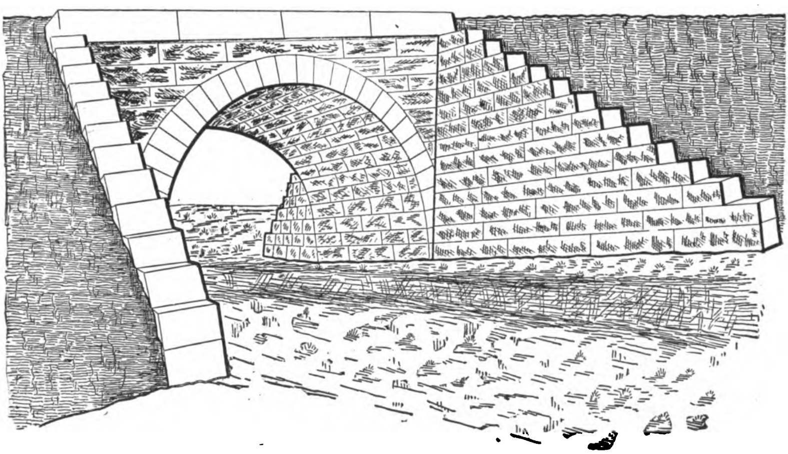


FIG. 111.

them about the sheet to receive the picture in accordance with the conditions assumed, then submit the arrangement for approval; next, find the perspective, then submit the drawing for inspection. In inking, ink the lines of the perspective and the border line only, and finish the sheet by lettering it as follows:

TITLE,

PERSPECTIVE.

Practical Problem No. 22.

NAME AND DATE, to be in the lower right-hand corner of the sheet.

101. PROBLEM 23:

To find the perspective of an architectural arch.

Let the problem be that presented by Fig. 112 (the mechanical drawings), and illustrated by Fig. 113, and let the student assume his own conditions, such that the perspective will look well on a standard 8″×11″ sheet of paper.

Directions for Drawing.

Execute the mechanical drawings of the arch by enlarging the copy, say two times; then separate the views and arrange them about the sheet to receive the picture in accordance with the conditions assumed, then submit the arrangement for approval; next, find the perspective, then submit the drawing for inspection. In inking, ink the lines of the perspective and the border line only, and finish the sheet by lettering it as follows:

TITLE,

PERSPECTIVE.

Practical Problem No. 23.

NAME AND DATE, to be in the lower right-hand corner of the sheet.

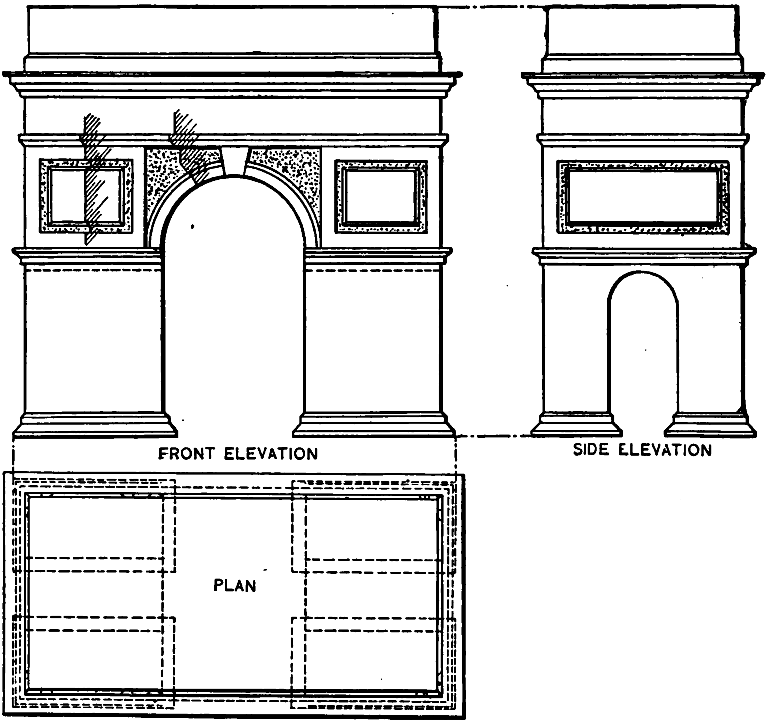


FIG. 112.

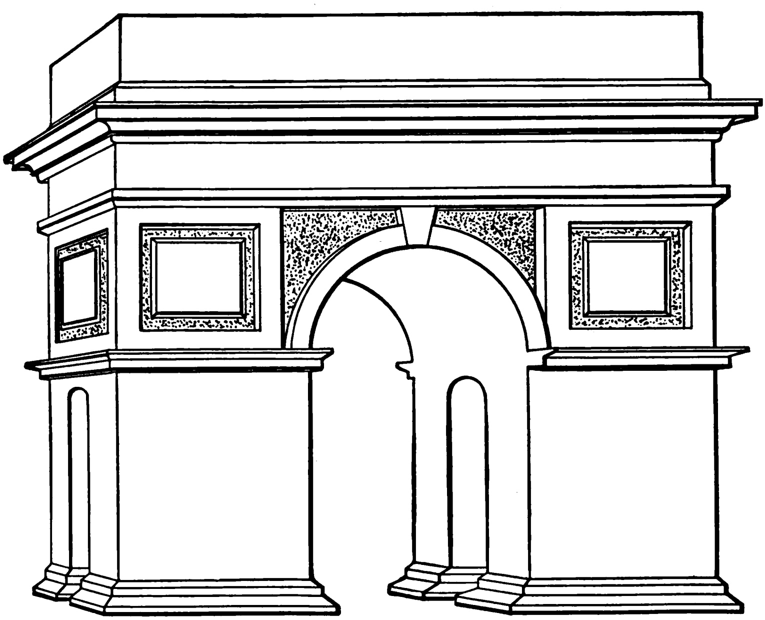
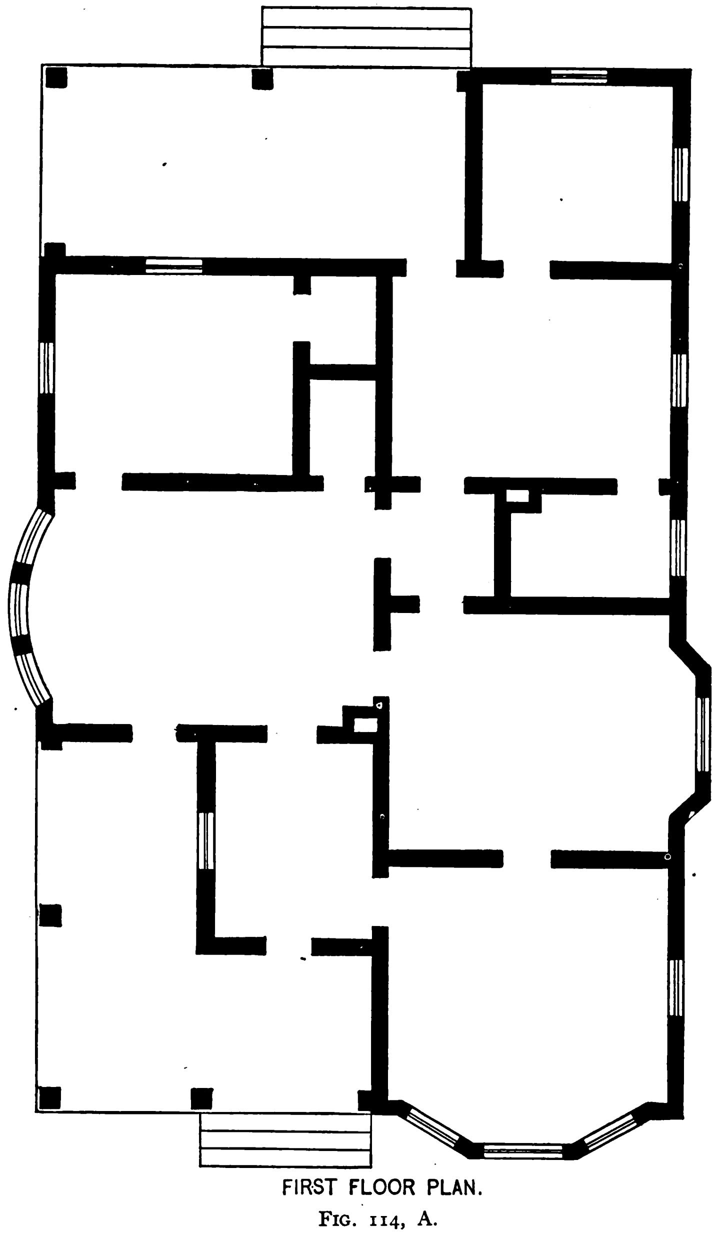


FIG. 113.



FIRST FLOOR PLAN.

FIG. 114, A.

102. PROBLEM 24.

To find the perspective of a small dwelling-house.

Let the drawings of the house be those presented by Fig. 114, *A*, *B*, *C*, *D*, and *E*, and the problem that illustrated by

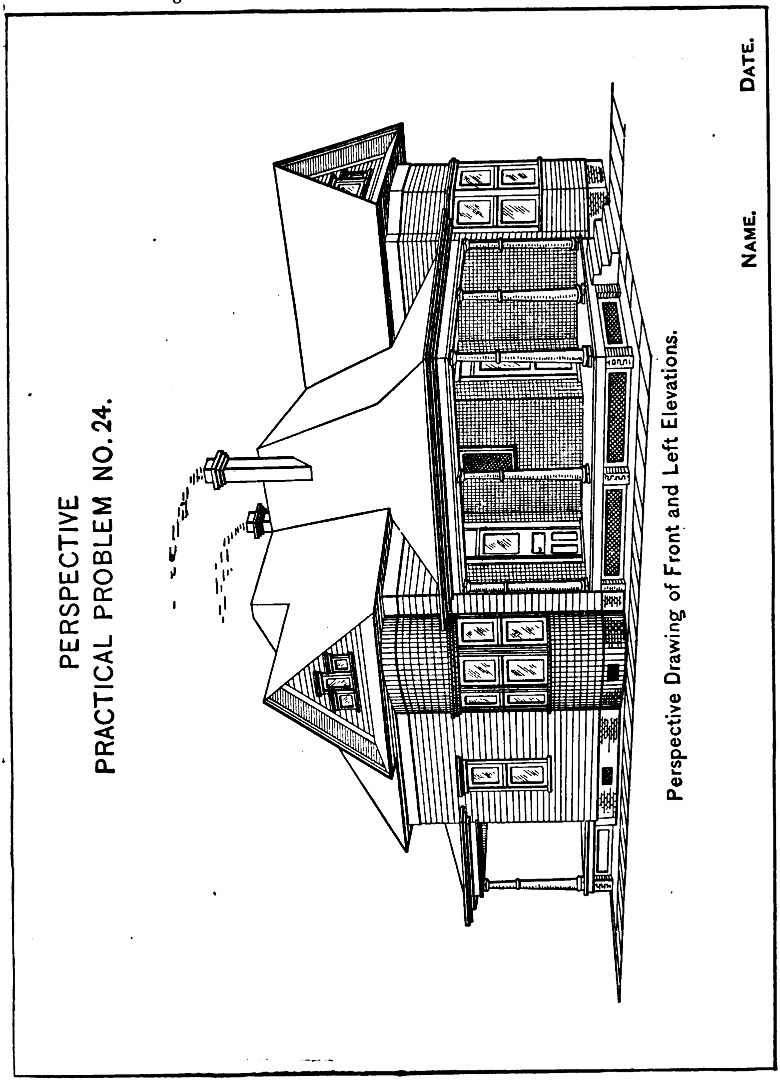
FIG. 114, B.

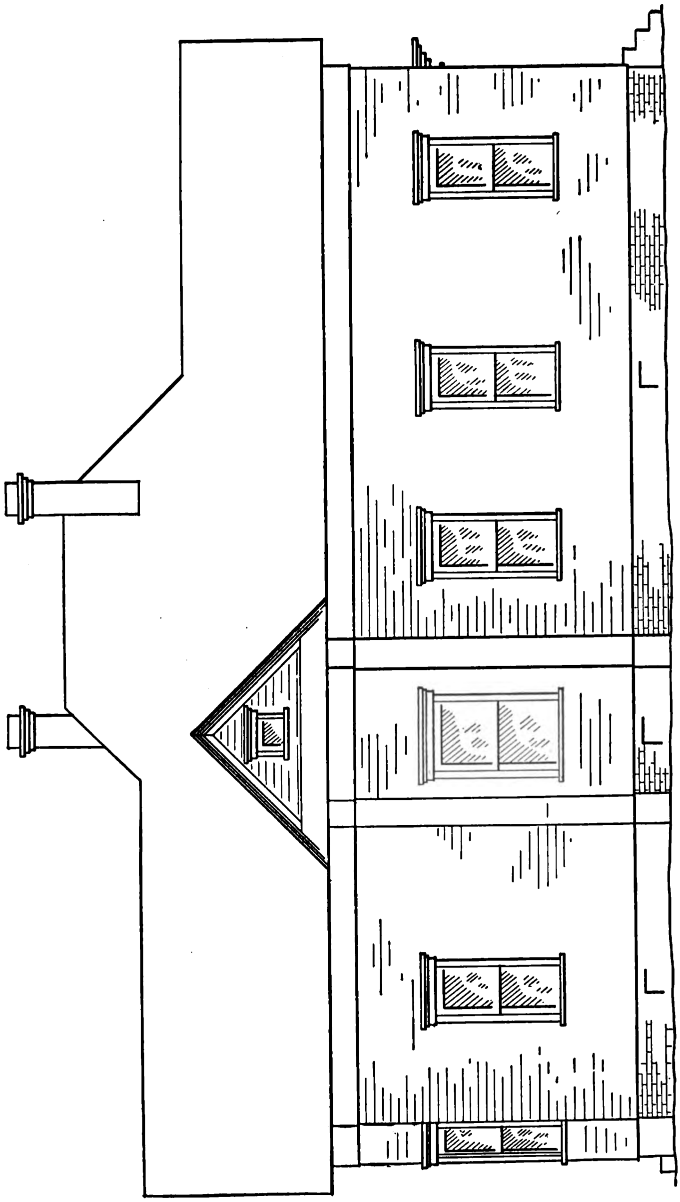
Plate 23, and let the student assume his own conditions, such that the perspective will look well on a standard $8'' \times 11''$ sheet of paper.

Directions for Drawing.

Execute the mechanical drawings of the house (a plan and left elevation is sufficient) by enlarging the copy, say two times; then separate the views and arrange them about the sheet to receive the picture in accordance with the conditions assumed; then submit the arrangement for approval; next, find the perspective, then submit the drawing for inspection. In inking,

PLATE No. 23.


PERSPECTIVE
PRACTICAL PROBLEM NO. 24.
Perspective Drawing of Front and Left Elevations.
NAME.
DATE.

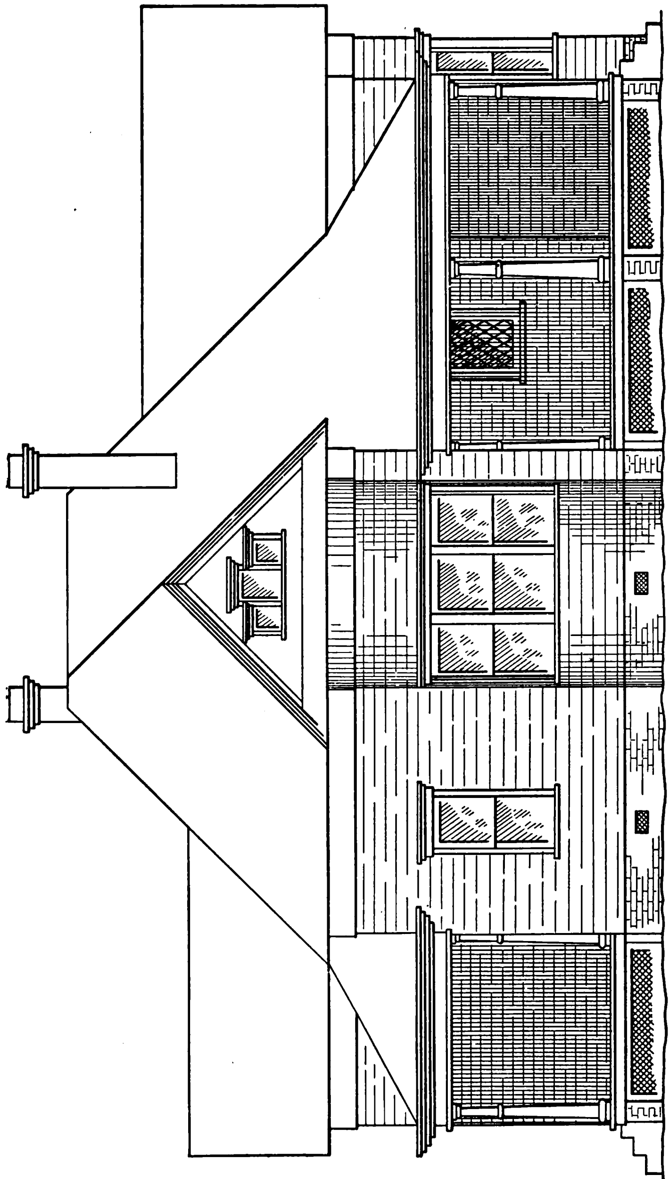


RIGHT SIDE ELEVATION.
FIG. 114, C.

ink the lines of the perspective and the border line only, and finish the sheet by lettering it as shown.


REAR ELEVATION.

FIG. 114, D.



LEFT SIDE ELEVATION.

FIG. 114, E.

103. PROBLEM 25:

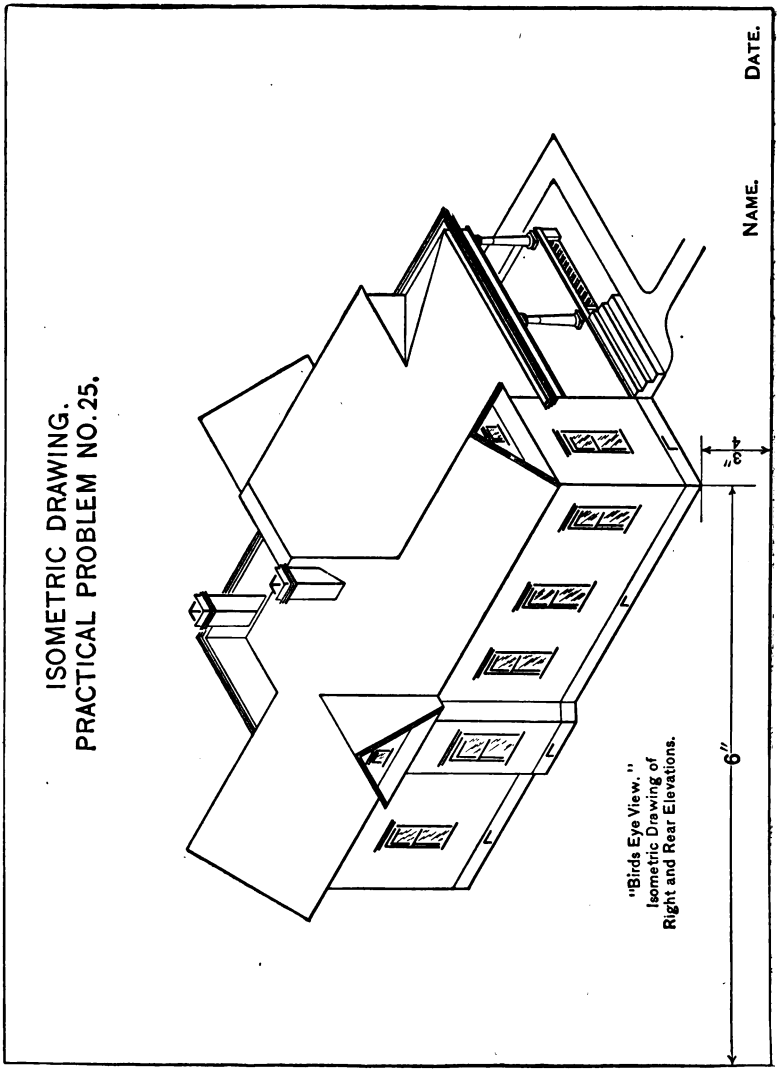
To find the "isometric perspective" of a house.

Let the drawings of the house be those presented by Fig. 114, *A*, *B*, *C*, *D*, and *E*, and let it be required to picture the right and rear elevations in isometric. (See Plate 24.)

Directions for Drawing.

Let the dimensions for the drawing be obtained by scaling the copy, and then construct the drawing one and one half times the size of the copy. In accordance, then, with these conditions and the principles of isometric drawing as set forth in Chapter I, execute the drawing, then submit it for inspection. In inking, ink only the lines shown on the plate, and finish the sheet by lettering it as shown.

PLATE No. 24.


ISOMETRIC DRAWING.
PRACTICAL PROBLEM NO. 25.
"Birds Eye View."
Isometric Drawing of
Right and Rear Elevations.
NAME.
DATE.
6"

SUPPLEMENTAL.

Exercises in Free-hand Lettering.

104. Explanatory.—Since there is so little free-hand work—lettering and dimensioning—on the Plates of the Course, the student is apt to permit his hand to lose the dexterity acquired in the execution of the Plates of the Course in Elementary Mechanical Drawing, and the following exercises are offered as affording examples for practice, and it is suggested that they be inserted at regular intervals in the course.

In all of the exercises the letters are to be executed, free-hand, in ink directly, with top and bottom guide-lines in pencil as the only guide, except, in cases where the letters are to balance with reference to something, as with the border line, when they should be first pencilled in to insure balance, then inked in. The student is to make a choice of the size of letters and spacing by comparison with the copy. All of the sheets are to be the standard, 8″×11″ border, 9″×12″ outside dimension, sheet of the course.

Plate 25 illustrates some types of simple, plain, easily executed free-hand letters; Plate 26, the sometimes practice of tabulating standard information for use in the shop, and Plate 27, a cover-sheet for a folio for a set of drawings, such as the exercises of this Course in Drawing.

PLATE No. 25.

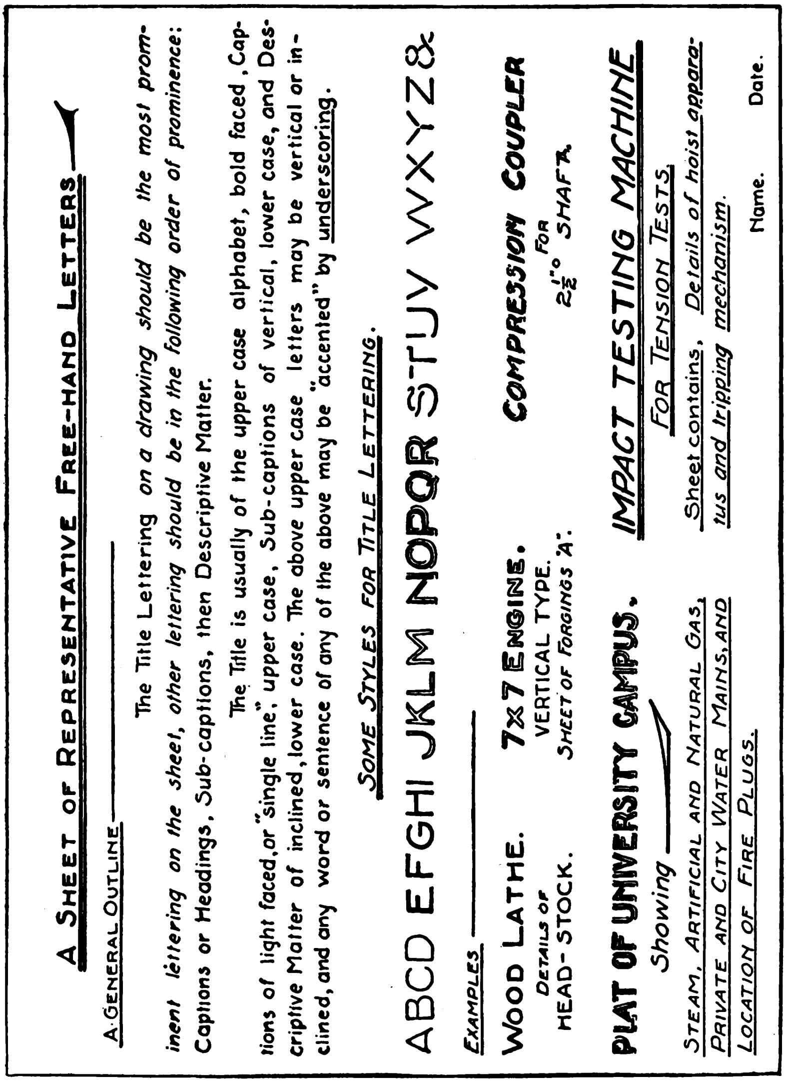

A SHEET OF REPRESENTATIVE FREE-HAND LETTERS
A GENERAL OUTLINE
The Title Lettering on a drawing should be the most prominent lettering on the sheet, other lettering should be in the following order of prominence: Captions or Headings, Sub-captions, then Descriptive Matter.
The Title is usually of the upper case alphabet, bold faced, Captions of light faced, or "single line", upper case, Sub-captions of vertical, lower case, and Descriptive Matter of inclined, lower case. The above upper case letters may be vertical or inclined, and any word or sentence of any of the above may be "accented" by underscoring.
SOME STYLES FOR TITLE LETTERING.
ABCD EFGHI JKLM NOPQR STUV WXYZ&
EXAMPLES
WOOD LATHE.
DETAILS OF
HEAD-STOCK.
7x7 ENGINE.
VERTICAL TYPE.
SHEET OF FORGINGS "A".
COMPRESSION COUPLER
FOR
2½" SHAFT.
PLAT OF UNIVERSITY CAMPUS.
Showing
STEAM, ARTIFICIAL AND NATURAL GAS,
PRIVATE AND CITY WATER MAINS, AND
LOCATION OF FIRE PLUGS.
IMPACT TESTING MACHINE
FOR TENSION TESTS.
Sheet contains, Details of hoist apparatus and tripping mechanism.
Name.
Date.

PLATE No. 26.

SHOP CARD, NO. 12.

STOCK, FOR SIZES AND LENGTHS, TO CUT BAR STEEL

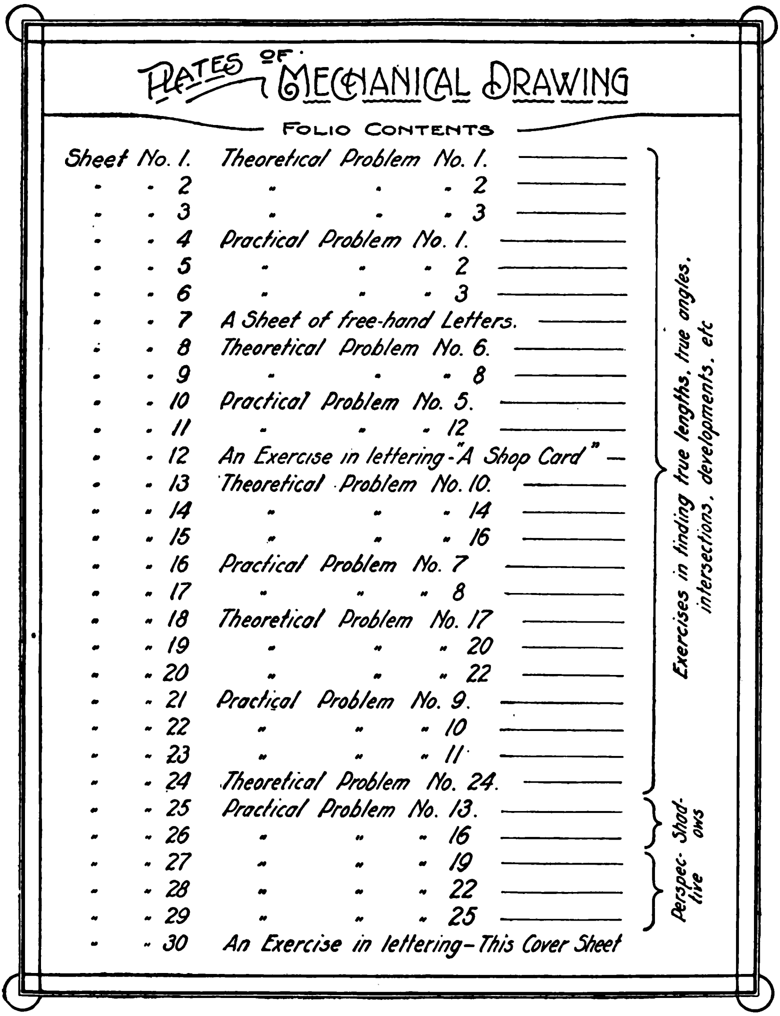
REGULAR EXERCISES:

Name of piece.	No.	Kind of steel.	Section.	Diameter.	Length of stock, finished	+ allowance.
Stud bolt.	1	*Machine.*	●	$\frac{11}{16}$"	5"	$\frac{1}{32}$"
Tail-stock screw.	1	,,	●	$\frac{3}{4}$"	$6\frac{5}{8}$"	$\frac{1}{32}$"
Tail-stock clamp scr.	1	,,	●	$\frac{9}{16}$"	$5\frac{1}{2}$"	$\frac{1}{32}$"
Shaft coupling.	2	,,	●	$3\frac{1}{8}$"	$5\frac{5}{8}$"	$\frac{1}{16}$"
Head-stock spind'l.	1	*W.*	●	$1\frac{3}{4}$"	$12\frac{3}{16}$"	$\frac{1}{16}$"
Tail-stock spindle.	1	*Machine.*	●	$1\frac{1}{4}$"	$7\frac{1}{8}$"	$\frac{1}{16}$"
Live spur center.	1	,,	●	$\frac{3}{4}$"	$4\frac{1}{4}$"	$\frac{1}{4}$"
Dead cup center.	1	,,	●	$\frac{3}{4}$"	$4\frac{1}{4}$"	$\frac{1}{4}$"
Cone center.	1	*R.*	●	$\frac{3}{4}$"	$4\frac{1}{2}$"	$\frac{1}{4}$"
Counter-shaft.	1	*Machine.*	●	$1\frac{1}{4}$"	24"	$\frac{1}{32}$"
Collar screw.	1	,,	●	$\frac{3}{4}$"	5"	$\frac{1}{32}$"
1" Arbor	1	*Y.*	●	$1\frac{1}{8}$"	6"	$\frac{1}{32}$"
$1\frac{1}{8}$" Arbor	1	*Y.*	●	$1\frac{1}{4}$"	6"	$\frac{1}{32}$"
Key for shaft cp'ling.	1	*Cold rolled.*	■	$\frac{3}{16}$	$1\frac{3}{4}$"	0
Bolts for shaft cp'ling	2	*Machine*	⬢	$\frac{5}{8}$"	$2\frac{1}{4}$"	0

R = Red stripe tool steel, high grade.
Y = Yellow stripe tool steel, medium grade.
W = White stripe, crucible spindle steel.

Name. Date.

PLATE No. 27.


PLATES OF MECHANICAL DRAWING
FOLIO CONTENTS
Sheet No. 1. Theoretical Problem No. 1.
" " 2 " " " 2
" " 3 " " " 3
" " 4 Practical Problem No. 1.
" " 5 " " " 2
" " 6 " " " 3
" " 7 A Sheet of free-hand Letters.
" " 8 Theoretical Problem No. 6.
" " 9 " " " 8
" " 10 Practical Problem No. 5.
" " 11 " " " 12
" " 12 An Exercise in lettering-"A Shop Card"
" " 13 Theoretical Problem No. 10.
" " 14 " " " 14
" " 15 " " " 16
" " 16 Practical Problem No. 7
" " 17 " " " 8
" " 18 Theoretical Problem No. 17
" " 19 " " " 20
" " 20 " " " 22
" " 21 Practical Problem No. 9.
" " 22 " " " 10
" " 23 " " " 11
" " 24 Theoretical Problem No. 24.
" " 25 Practical Problem No. 13.
" " 26 " " " 16
" " 27 " " " 19
" " 28 " " " 22
" " 29 " " " 25
" " 30 An Exercise in lettering-This Cover Sheet
Exercises in finding true lengths, true angles, intersections, developments, etc
Shad-ows
Perspec-tive