Supervised Study Series

EDITED BY

ALFRED L. HALL-QUEST

SUPERVISED STUDY IN MATHEMATICS AND SCIENCE

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SUPERVISED STUDY

IN

MATHEMATICS AND SCIENCE

BY

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To the Memory of

MY FATHER

WHO BELIEVED AN EDUCATION WAS THE
RICHEST HERITAGE A PARENT COULD
BEQUEATH TO HIS CHILDREN

PREFACE

In attempting a book on supervised study which will cover even approximately the subjects of mathematics and science, it is impossible to do more than give suggestive lessons. This, therefore, has been my plan — to give only one or two typical outlines of a topic or subject, but to leave an intimation of its application whenever or wherever the teacher may elect. Thus, only one Red Letter Day lesson is presented in Algebra, but the teacher will undoubtedly desire to use many such plans during the year. The material in the lessons mentioned may be suggestive for the planning of others.

I have not tried to add another learned book in pedagogy to the many already on the market. It has rather been my aim to write a book that may be of explicit and direct value to the teacher or principal who is daily striving to teach his children *how to study* and *how to learn*. I have tried to write it in simple language, so that the reader may get the meat, if there be any, without too much stuffing.

It is needless to say that I am a firm believer in supervised study. It has done much for our children; I am sure it will do more as we progress in the proficiency of its administration. It is not a panacea for all pedagogical ills, but it is valuable for what it claims to be, and it holds great promise for the future.

I am greatly indebted to Professor Alfred L. Hall-Quest of the University of Cincinnati, who, as editor of this series, has not only made it possible for this volume to be, but who,

through the reading and criticism of the manuscript and through innumerable other suggestions, has been of inestimable help to me.

Deep appreciation is also here expressed to Professor Charles M. Rebert of St. Lawrence University, for valuable suggestions and advice; to Mr. A. E. Breece of the Hughes High School, Cincinnati, Ohio, who made a very careful and valuable critical review of the manuscript as relating to mathematics; to my teachers at Canton, N. Y., who made it possible to actually try out many of the lessons; and to my wife, for her constant counsel and encouragement.

In addition, I wish to acknowledge my thanks for the courtesy of The Macmillan Company, the American Book Company and the Charles E. Merrill Company for permission to quote more or less extensively from their publications.

S. Clayton Sumner.

Palmyra, N. Y.
January 31, 1922.

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INTRODUCTION BY THE EDITOR

SUPERVISED STUDY A MORAL IMPERATIVE

MILLIONS of words have been written about education. Theories have abounded and still are fertile. The visitor at educational conventions, especially in the department of school superintendents, is impressed, however, with the rapid multiplication of devices for visualizing educational practice. A rich variety of moving picture machines and already voluminous catalogues of educational films witness to the dawn of a new era in the technic of teaching. Later we shall no doubt find boards of censors passing upon these films — boards composed of theorists, critic teachers, educational scientists, *et al.*, — but at present the field is open for all. Doubtless many teachers will find in this form of visual education an opportunity for enlargement of income as well as for the demonstration of teaching skill.

Increasing Emphasis on Demonstration. Demonstration and description are rapidly coming to the front in discussions of methods of teaching. One carefully prepared and successfully performed demonstration is of more value than many verbal descriptions, however clear these may be. A series of vivid verbal descriptions makes definite and concrete a volume of abstractions and theorizings on educational practice. Theory is important; it must not be discounted; but here, as elsewhere, an illustration turns on light and makes

objective and easily understood the necessarily vaguer discussions of abstract theory.

This series of volumes on Supervised Study attempts to visualize one form of study supervision. Each book is written by a teacher who has had considerable experience in this type of work. The emphasis in each discussion is to make concrete in as detailed description as possible, what the author has actually done in his own classroom. Very briefly each author states the theory underlying his practice, but beyond this brief statement he refrains from a discussion of principles. Teachers desire to see how theory is applied. One cannot be too clear and too definite in describing the mode of procedure in supervision.

At Present No Generally Accepted Meaning of Supervised Study. In answer to those who believe that Supervised Study as described in this and other volumes of the series is different from the general understanding of the term, it should be emphasized that at present there is no generally accepted form of Supervised Study. It is the conviction of the editor of this series that a standardized form is undesirable. The main objective is teaching children — all children — how to study and guiding them while they apply the principles of correct studying. It is of comparatively little importance how this is done, providing it is done effectively. If the teacher makes this type of teaching superlatively significant, it follows that the management of the class and the method of presenting subject matter will change accordingly. But each teacher must be the final judge of how to adapt this new point of view to local needs.

The Imperative Need of Preventing Failures in School. It should be said, however, that any plan which seeks *to prevent*

failures and which aims to train *all pupils* to study as effectively as native ability permits is superior to plans that simply correct improper methods of work and that are concerned only with the retarded pupils. If school work is limited to the assigning and hearing of lessons, only a few — the highly endowed — will permanently profit by such experience. There are well-meaning people who sincerely believe that the school is the place for eliminating society's mentally unfit, and that the surest way of such elimination is to assign lessons, long and hard. Those who can will; those who cannot will not. Those who will and can are the fit!

Some there are who learn to swim by the "sink or swim" method; they are destined, forsooth, to be swimmers if they do not sink. But how many of you who read these pages learned to swim by this fatalistic method? There are children who early judge themselves incapable of school work. Nobody cares! They either can or cannot study. By means of the hard, soulless machinery of assigning and hearing lessons they are cast out. We call this a safe test and out they go labeled mentally weak, unfit to partake in a world of thrilling knowledge, unfit to climb to altitudes of self-revelation and social worth. If, however, they could have been taught how to use their minds, how to partake in the feast of knowledge, who knows but that many of them would have found a new meaning of their destiny!

Supervised Study Is Not Only an Intellectual Necessity; It Is a Moral Imperative. As teachers it is our plain duty to teach children how to study. The whole class period must be conducted in this spirit. The specific aim of every class period must be to so direct the pupils that their grasp of the new work is adequate for independent application. The teacher

is preëminently a director of study and not primarily a dispenser of subject matter.

The Point of View in This Volume. The author of this volume is convinced of the effectiveness of Supervised Study. He and his teachers have tried it long enough to know its advantages. The subjects of mathematics and science are especially favorable to this method. A comprehensive view of the courses in the high school is given in a series of typical lessons describing in great detail how children may be directed in beginning, continuing, and reviewing their study of particular units of subject matter. The author is well aware of the movement for reorganization of courses especially in ninth grade mathematics, but inasmuch as such revision is not likely to be possible in all schools for some time to come the usual division of courses is considered in this volume. It is believed also that even where general mathematics is taught, not a few pupils will elect additional special courses in the field of mathematics. Inasmuch as general science is at present little more than a combination of various special sciences the separate treatment used in this volume seems preferable. It is hoped that general science will evolve increasingly along the lines of natural correlations through which the pupil will be able to understand the intimate relationships that exist among the phenomena of nature.

PART ONE

MATHEMATICS

SUPERVISED STUDY IN MATHEMATICS AND SCIENCE

CHAPTER ONE

THE MANAGEMENT OF THE SUPERVISED STUDY PERIOD IN MATHEMATICS

Causes of Failures in Mathematics. — There are a number of contributory causes which, together or separately, might account for the high mortality in mathematics classes. That it is high is so commonly accepted among the profession, that a large percentage of failures has almost come to be an established expectation. In eleven high schools near Chicago, the percentage of failures in algebra and geometry was found to be greater than in any other subject.[1] In the report of the New York State Education Department on statistics for Regents Academic Examinations, the failures in mathematics for the past five years have been between thirty-three per cent and forty per cent.[2] The nearest competitors for scholastic dishonors are the commercial subjects which are largely mathematical in content.

The causes of these failures are psychological, pedagogical, and physical. Psychologically, mathematics has been by

[1] *School Review*, June, 1913, p. 415.

[2] *Annual Report of the State Department of Education* (10th to 14th inclusive), New York State.

almost common opinion accorded the position of being the hardest subject in the school curriculum. This estimate of the subject, persisted in by pupils, teachers, and the laity, has inevitably resulted in a state of mind that predetermines a large percentage of failures. Until we teachers succeed in dispelling this opinion, pupils in many instances will expect to fail, and they will fail. There is no sane reason why mathematics should be so considered, and with the new vision of teaching the subject and with the readjustment of the course of study, combined with its scientific treatment (which will emphasize the functional and practical side instead of the formal aspect), this view of the severity of mathematics doubtless will gradually disappear.

Mathematics Taught with Deliberate Unattractiveness. — It is repeating a platitude to refer to the fact that mathematics has been very poorly taught in the public school. There has been no serious lack of scholarship and of emphasis on the acquirement of knowledge of subject matter, but this very emphasis has tended toward the serious neglect of training pupils to apply mathematical rules and formulas to practical reasoning. Too much emphasis has been laid on the formal examination, the " spectacular " effects according to Schultze.[1] Too much is attempted in the time allotted, with insufficient assimilation of the matter studied. Pupils are not taught *how to study* mathematics. They are only drilled on abstract formulas. The result is an *overdeveloped* memory and *undeveloped* powers of reasoning.

Because of the above noted unsound pedagogical methods, with the resulting formal examinations, and because the

[1] Arthur Schultze, "The Teaching of Mathematics in Secondary Schools"; The Macmillan Company, 1912.

pupils are graded chiefly on mechanical ability, their progress in mathematics can be determined to a highly refined nicety. They have failed to " do " a certain number of problems. Ergo, they are just that much deficient in ability and improvement. There is no leeway for difference of opinion, for the exercising of the reasoning faculty, for the training of individual characteristics and differences. Being largely a fact subject, as now taught, it resolves itself mainly into a question of " yes " or " no," and this accentuates the probability of failure. Individuals differ vastly in their ability to memorize, and therefore the poor memorizer is placed at a disadvantage. The pupil who can reason out a new demonstration in geometry knows infinitely more geometry than he who can transcribe on paper every one of the prescribed demonstrations in a book on this subject.

The Value and Place of Supervised Study. — This leads us logically to a discussion of the value of supervised study in mathematics. Unsupervised study is inefficient study because much time and energy are lost in misdirected effort. Pupils do not know how to attack a lesson any more than they know how to perform the mechanical processes, until they are carefully taught. Class exercises avail little for the majority of the pupils because no two minds react in the same way. To clinch the class exercise individual guidance is required. The unsupervised recitation as a rule does not provide for this. Problems in algebra and originals in geometry are entirely dependent upon the characteristics of the individual mind, which can be developed and trained only through the individual himself. To quote from an article by the author,[1] " the school must teach its pupils not to be perfect

[1] *Journal of the New York State Teachers' Association*, November, 1918.

automatons, responding with machine-like accuracy to the whim of the examiner, but to become thinkers, with power and knowledge of how to attack and study out a problem, how to form personal opinions, how to get results, by themselves. This, then, is the function of supervised study: to properly direct the pupil in his work so that he may develop the best methods of attacking problems; that he may avoid wrong methods of reasoning; that he may most efficiently employ his time; and that he may eventually acquire a power of skill that will classify him as a finished thinker, an educated man."

Supervised study is only one of the several methods that need to be employed in bringing about a closer relationship between teacher and pupil and in the development of the pupil's native endowment in the field of mathematics. Such relationship might be illustrated as spokes of a wheel. Just as every spoke (Figure I) is necessary in the connection between the rim which represents the pupil and the hub which represents the teacher, so supervised study should be given its proper position in the devices of the schoolroom. The other spokes, each with its peculiar evaluation, might be the recitation, the assignment, the equipment, tests and quizzes, standard tests and measurements, inspiration and sympathy.

Division of the Course into Units of Instruction, Recitation, and Study. — In our discussion of the technic of the supervised study of mathematics, let us first agree on our use of terms, as formulated by Professor Hall-Quest in his pioneer book, "Supervised Study" [1] and followed out in the other books in this series. In *program of studies*, let us include all the work offered in a school; by *curriculum*, let us understand a

[1] Hall-Quest, "Supervised Study"; The Macmillan Company, 1916.

group of subjects leading to a special end, as college preparatory curriculum, domestic science curriculum, etc.; and by *course*, any single subject as algebra, civics, etc. Then, as a means of evaluating the course and giving it a definite and

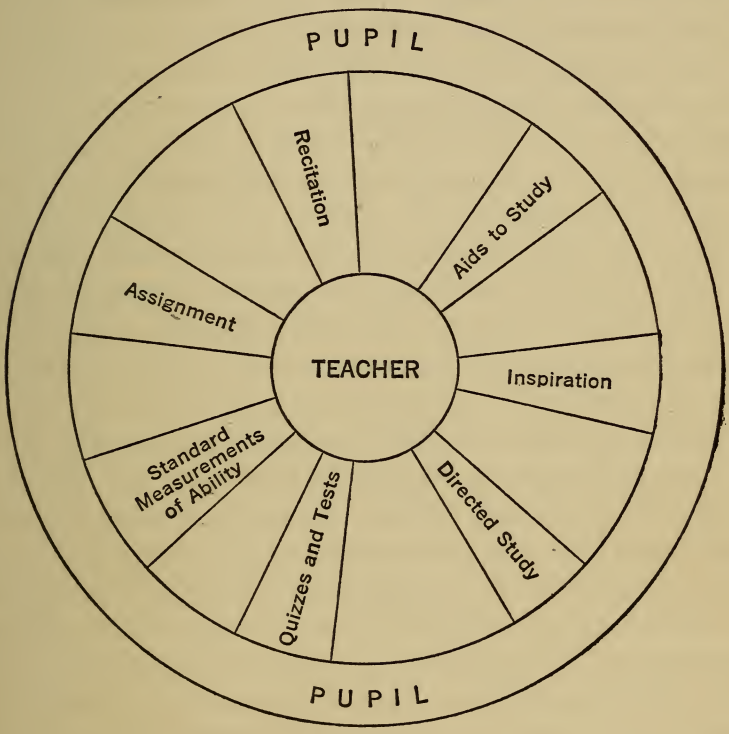


FIGURE I

comprehensive development, we shall separate the subject matter into various units. By *units of instruction*, we shall mean the large topics around which the material revolves. In many cases these are the divisions noted in the table of contents; they are the divisions of the subject into " type

lessons." [1] Such general topics as percentage, banking, factoring, graphs, would thus become units of instruction. Then the subdivisions of these larger units into smaller ones, around which one or more recitations would revolve, may be called *units of recitation.* These units may again be subdivided into the work planned for a single day, or *units of study.*

Types of Recitation. — In addition to this analysis of any course of study into its various units, the careful teacher will further decide on the technical form of presentation of each unit of study, or the *type of lesson.* Following the treatment of this phase as detailed by Strayer [2] and Earhart,[3] we may employ, as occasion prompts, the *deductive*, *inductive*, *habituation*, *expository*, *how to study*, *socialized*, or *review* lessons. Since each type, however, has its peculiar aim and technic, the teacher will do well to make a careful study of them and of their application to the subject in hand.

In general terms, the deductive lesson aims to draw forth new conceptions from our present knowledge. It is based on the process by which we think. The inductive lesson, on the other hand, leads up to new concepts by a series of successive steps, each definite and complete in itself. It is the process by which we accumulate knowledge. In the drill or habituation lesson, the mechanical side of learning is stressed. By drill, needful automatic reactions are established. The expository lesson seeks to make the new assignment as clear as demonstration and analysis make it possible. It is usually employed as a connecting link between the old and new mate-

[1] McMurry, "How to Study"; Houghton Mifflin Co., 1909.

[2] Strayer, "Brief Course in the Teaching Process"; The Macmillan Company, 1912.

[3] Earhart, "Types of Teaching"; Houghton Mifflin Co., 1915.

rial of a unit of study. The how-to-study lesson is self-explanatory. Under review are usually included written or oral examinations. In a larger sense the review should cover the bringing together for periodic consideration, the clinching of a unit of recitation or unit of study. The socialized lesson may be poorly named from a standpoint of nomenclature; possibly a better term at the present time would be the democratized lesson. At any rate, it is that type of lesson which introduces the human element into the school work. By it the work outside and that inside of the schoolroom are correlated. Although this type of lesson may be used sparingly, it is none the less important, and the teacher should feel that his greatest opportunity for reaching the consciousness of the child is presented through this form of lesson organization. Group assignments, dramatic productions, class programs, dual projects, mathematical clubs, and like devices for impressing the interdependence of individuals in solving social or economic problems, will tend to vitalize and democratize the subject matter.

A more elaborate classification of exercise types as applied to the teaching of high school mathematics has been evolved in an illuminating article by Professor G. W. Myers, of the College of Education of the University of Chicago, in *High School Mathematics and Science*, June, 1921. In this article, the following types of mathematical class exercises are discussed, and specifications or norms are given for judging each:

I. The conceptual type.
II. The expressional type.
III. The associational type.
IV. The assimilational type.
V. The review type.
VI. The drill type.
VII. The problem type.
VIII. The topic type.
IX. The applicational type.
X. The test type.
XI. The research type.
XII. The appreciational type.

While space does not permit a detailed review of these, the article in question is commended to the reader for careful study. To treat it adequately here would be to quote it as a whole.

The Time Schedule. — In order that the plan of supervised study may be carried out in its finest application, the period should be long enough for the pupil to do most if not all of his studying in school. This would mean a period of from ninety minutes to two hours in length and would also involve an extension of the school day in most cases. Superintendent L. M. Allen[1] remarks that a shorter time than the above is "neither hay nor grass," and that less than forty-five minutes for the study period itself will not suffice. The author of this book will grant that the longer the period the better the results, but from the experience in his own school for the past five years, he is constrained to disagree with the above conclusion of Superintendent Allen. In the school at Canton the periods are all one hour in length, the first thirty-five minutes being devoted to the recitation and the assignment, and the last twenty-five minutes being given up to the study of the lesson for the next day. Realizing that a longer study period would be very desirable, the author knows from experience that even this length of time will justify itself by increased and better work, as shown from the statistics as applied to the Canton school.

It seems, therefore, only fair to conclude that, when it is impossible to increase the length of the periods to the limits suggested above, the installation of supervised study is still feasible and good results may be secured from the shorter period. In any case, the period will be divided into three

[1] *School Review*, June, 1917.

parts: the review, the assignment, and the silent study. When the periods are sixty minutes in length, the approximate division of time among these three parts should be as follows:

The Review	15 minutes
The Assignment	20 minutes
The Silent Study	25 minutes

With a longer period, the study section will be increased more than the others. A sixty minute period means, of course, that only part of the study will be done in class; a ninety or one hundred twenty minute period should make home work unnecessary.

The Review. — The review will take the place of the old recitation and, while its length has been decreased, by intensive and well-applied questions the work ought to be thoroughly covered in this time. The class review should essentially be a *re-view* of the difficult parts of the day's lesson; and, while the weaker pupils will get the most attention, it will be a sort of summing up for all. It is unnecessary to review every minute step; half of the usual recitation is spent in reciting on perfectly well-known and understood things. *The review is not the time to show off what we know but to clear up the things we do not know or know only indistinctly. It should always be a real step forward, a sort of clearing house for the previous day's assignments.* Again, the usual type of recitation is apt to be a kind of monologue with the teacher taking the leading part. As a matter of fact, the teacher should remain in the background. In war, the generals give orders but the rank and file does the fighting. The review, then, should be incisive, intensive, and conclusive. The entire class should not be held back by a few backward pupils. On the other

hand, the bright pupils should not be conciliated by insipid questions. The teacher should address the review to the weaker ones but by methods that will appeal to all.

The Assignment. — The assignment is always the most important part of the recitation period. It should include, in addition to a definite allotment of new work, very clear explanations. The advance lesson should be carefully planned beforehand, so that there will be a definite amount of ground to be covered, a definite objective gained, and a definite advance made. If this is slurred over, the pupil will have no clear idea of what the lesson is about or of what he is to do. The assignment to be prepared should also be made with due consideration of its difficulty, the varying abilities of the members of the class, and the often overlooked fact that the pupils have lessons also in other departments. If all these elements have been taken into consideration and given thoughtful planning, the time allotted to this particular section of the period ought to be sufficient. But there must be plenty of time to cover the assignment fully and thoroughly; therefore, it is better to assign too little than too much. In any quota of problems or examples, special difficulties likely to be encountered should be pointed out and possible methods of attack suggested. The assignment is perfect only when every pupil knows exactly what is the aim of the new work, what is the best method for its solution, and just what ground he is expected to cover before the next recitation.

The Study Period. — The study section is the part devoted by the pupil to the study of the advance lesson, under the direct and sympathetic supervision of the teacher. In the sixty minute period used in the author's school, the pupils are not all expected to complete the assignment during the

period. Some will, however, and it will be an incentive to all to strive to complete the work during the time allotted to study. At any rate, all will have been able to get a start and a start in the right direction.

The teacher will have two classes of pupils to look after during the silent study period. One will be those who have some little technical difficulty with the new lesson. A well-directed question will usually set them on the right path. The other division will be those who are commonly considered failures, but who in many instances are simply pupils with some individual characteristics which react unfavorably for maximum efficiency. These pupils should be carefully studied by the teacher and their cases diagnosed. The problems thus presented to the teacher should awaken all of his determination to solve them. The silent study period thus gives the teacher an opportunity for a study of the individual personnel of his class. *The elimination of pupils from the list of failures should become the predominant effort rather than the elimination of pupils from class and school.* Often a pupil, who would ordinarily have failed, has found himself through a little attention and study on the teacher's part during the study period. When the teacher can sit down with him, note his manner of work, detect his deficiences and weaknesses, and by a little tactful and sympathetic guidance, lead him into the paths of success, such a pupil will gain confidence and later economical independence. He must be taught how to walk — how to study.

The Assignment Sheet. — The assignment sheet used in Canton, which is similar to the one described by Miss Simpson in her companion book in this series,[1] is reproduced at the end

[1] "Supervised Study in History"; The Macmillan Company, 1918.

of this chapter. The object of the sheet is to induce the teacher to have a definite plan for each day's work. Work not carefully planned is apt to be poorly done. Nothing begets carelessness and indifference on the part of the class so much as a lack of purpose and plan on the part of the teacher. These sheets need take scarcely any time; in fact, they will save time, because the teacher will know exactly what he is going to do, what material he is going to use, and where it is to be found. In addition thereto, he will have at the close of the term a complete record of work accomplished.

How to Make a Lesson Plan Sheet. — Under *Review* note exactly the things that need to be reëmphasized. The references to other books for supplementary material may be noted under *Memoranda.* A good scheme is to write on the back of the assignment sheet the names of the pupils who should receive especial attention during this review.

The Threefold Assignment. — As Professor Hall-Quest explains in his book, the assignment should be in three parts: one to take care of the inferior pupil, one to take care of the average pupil, and one to take care of the superior pupil. Hence we have the three part assignments, or the minimum-average-maximum plan.

The *minimum assignment* should cover the minimum essentials, *i.e.* the work that all must do at the very least, and that the majority can easily do within the twenty-five minutes allotted to study. It should be so planned that pupils who never do any more than this amount will be able to pass the final examination, which is or should be the *minimum* requirement, but too often becomes the *only* aim of the teacher. These pupils will not obtain a high mark, but they will have mastered enough of the subject to get a passing grade. The

object of the average and maximum assignments is to produce pupils who will not only pass but pass *high*.

The *average assignment* should include more examples and different kinds of examples but not necessarily of a much harder nature. However, the more problems a pupil can solve correctly in a given time, the more skilled he will become, up to a certain limit.

The *maximum assignment* is to take care of the brighter pupils, — those who are able to do more than the average pupil and who should have some incentive to do advanced work. Usually this assignment will be given from other texts and will consist of more difficult material. This section should be so limited in amount as not to discourage but to incite to a desire to cover it. It will incidentally keep a disturbing pupil out of mischief. But the tasks should be constructive and not simply the old fashioned " busy " work which employs but does not develop.

Under *Study* note any points to be kept in mind while the pupils are working, especially in regard to mistakes they are likely to make. This section of the sheet may be made very effective, if the teacher is fully alive to the function of the supervised study idea. In the course of a year this plan will have become a real series of methods.

The loose-leaf notebook makes an excellent method of preserving the sheets. They will be found of inestimable value another semester.

How to Use the Assignment and Study Sheet. — Do not become a slave to the sheet; make it your servant. If it means omitting your daily recreation, eliminate it, not your recreation. It ought not to take much time and, as has been suggested, it will eventually be found to save time.

Strive always to inspire the class to reach the maximum assignment, to raise the maximum and lower the minimum end.

Determine the proper proportions of pupils who should get the various sections. The following is a normal distribution: 80 per cent should complete the average assignment, 10 per cent the minimum only, and 10 per cent the maximum. When these percentages vary greatly from the established norm, the nature and length of the assignments should be modified proportionately. That is, in a class of 30 pupils, 24 should do all the average assignment before the end of the period. Three will be behind and will need special attention next day; three will be working some of the examples in the maximum assignment. This of course is subject to variation from day to day, but may serve as a guide.

At the close of the period, the teacher should ascertain how many of the class have completed the minimum, the average, and the maximum assignments. This may be done by having all hand in their papers a few seconds before the period ends. The pupil may note on his paper which assignments he has completed. What remains of the assignment may be required of the pupils the next day. Various schemes may be evolved; some will be touched upon in connection with the illustrative lessons in Part One.

The assignment should be written each day upon the board and numbered I, II, III to correspond with the different assignment groups. The pupils need not be told the reason for this as it will be better if they do not understand the reason for the differentiation. The pupils should form the habit of noting down the assignment, thus getting practice in keeping a written record of important facts and engagements.

The Management of the Supervised Study Period. — As soon as the class meets, take the roll call and at once start the review. The time spent in taking the roll should be reduced to a minimum; certainly not over two minutes at the most. An excellent method is to have on a sheet of paper a diagram representing the seats and in each space, which represents a seat, put the name of the pupil who occupies it each day. Then those absent can be quickly noted through the vacant spaces, and their names checked.

The time allotted to the review and the assignment during the first thirty-five minutes should be held to as closely as possible; a schedule or time table is absurd unless adhered to conscientiously. At the end of the thirty-five minutes, a bell may be rung simultaneously in all rooms from the study hall or some other central place, and the next two minutes devoted to physical drills, setting-up exercises, etc.

Then, for the next twenty-five minutes or fraction thereof, the pupils should be required to work on their new lesson. The teacher should insist that no other work be done during the study period, until the lesson in hand is finished. At first, some may try to study the succeeding lesson but a little firmness will soon bring desired results.

As soon as possible after the class has been organized, it may be expedient to reseat the pupils according to their abilities; those of minimum ability on one side of the room, those of maximum ability on the opposite side, and the average pupils between. This allows the teacher to come into closer contact with the weaker pupils, and he can give them additional attention. This classification must be done with tact so as not to hurt any child's feelings; therefore, the earlier in the term it is done the better. It is not necessary to name

the groups. Calling them *A*, *B*, *C* without any further characterization will suffice.

Some method of checking the results of the work accomplished during the period should be worked out. Various schemes will be mentioned in connection with the illustrative lessons; but a few general remarks may be made here. Any plan which would depend entirely on the amount of work done in class must take into consideration the fact that some children are accurate but slow; these must not be discouraged by low marks. While rapidity is desired, accuracy is more important, and these pupils should be encouraged to make an effort to maintain accuracy and secure greater rapidity. On the other hand, it is very important that as much of the work be done in class as practicable so that the teacher may know that the work is the pupil's own. Thus there is presented to the teacher a fine question for analysis: to discover why a pupil does not accomplish so much as is expected. The teacher should be untiring in his effort to solve such a problem. Theoretically the pupils who habitually solve only the first or minimum assignment should receive a mere passing grade, those doing the average assignment should receive marks between 80 and 90, and those doing the maximum amount should receive honors. But this must eventually be determined by the ability of the pupil to work similar exercises in the review, by his lack of dependence upon the teacher during the study period, and to a lesser extent by the periodic test.

ASSIGNMENT AND STUDY SHEET

SUBJECT..... PERIOD..... DATE.....

UNIT OF INSTRUCTION .....

UNIT OF RECITATION.....

UNIT OF STUDY .....

LESSON TYPE.....

REVIEW:	MEMORANDA
ASSIGNMENT:	
1. MINIMUM	
2. AVERAGE	
3. MAXIMUM	
STUDY:	

Number of pupils solving minimum assignment.....

Number of pupils solving average assignment.....

Number of pupils solving maximum assignment.....

Total .....

FIGURE II

FIRST SECTION. ALGEBRA

CHAPTER TWO

DIVISIONS OF ELEMENTARY ALGEBRA

It is advisable in any course of study that the teacher have a definite outline of the successive stages in the development of the subject, and that these be further subdivided into their smaller units. These general topics may be called *Units of Instruction* and the subtopics, *Units of Recitation.* The course of study here outlined is that suggested by the New York State Education Department in its 1910 syllabus.

A. UNITS OF INSTRUCTION.

I. Introduction.
II. Positive and negative numbers.
III. Addition.
IV. Subtraction.
V. Multiplication.
VI. Division.
VII. Factoring.
VIII. Common factors and multiples.
IX. Fractions.
X. Simple equations.
XI. Graphic representation.
XII. Involution.
XIII. Evolution.
XIV. Radicals.
XV. Quadratic equations.

B. UNITS OF RECITATION.

The subdivisions do not imply that only one recitation is to be given to each topic, but rather that all the recitations for the length of time needed will center around this special topic.

I. INTRODUCTION. The material given under this head varies in different texts, but it will at least contain:

Units of Recitation:

1. Symbols of algebra.
2. Literal numbers.
3. Historical notes.
4. Definitions and notation.

II. POSITIVE AND NEGATIVE NUMBERS.

Units of Recitation:

1. Explanation and illustration of signed numbers.
2. The addition, subtraction, multiplication, and division of signed numbers.

III. ADDITION.

Units of Recitation:

1. Addition of monomials.
2. Addition of polynomials.

IV. SUBTRACTION.

Units of Recitation:

1. Subtraction of monomials.
2. Subtraction of polynomials.
3. The parenthesis.

V. MULTIPLICATION.

Units of Recitation:

1. Multiplication of monomials by monomials.
2. Multiplication of polynomials by monomials.
3. Multiplication of polynomials by polynomials.
4. Special cases.

VI. DIVISION.

Units of Recitation:

1. Division of monomials by monomials.
2. Division of polynomials by monomials.
3. Division of polynomials by polynomials.

VII. FACTORING.

Units of Recitation:

1. To factor a monomial.
2. To factor a polynomial.
3. To factor a polynomial whose terms may be grouped to show a common polynomial factor.
4. To factor a trinomial which is a perfect square.
5. To factor the difference of two squares.
6. To factor a trinomial in the form of x^2+px+q.
7. To factor a trinomial in the form of ax^2+bx+c.
8. To factor the sum or difference of cubes.
9. To factor the sum or difference of the same odd powers of two numbers.
10. To factor the difference of the same even powers of two numbers.
11. To factor by the factor theorem.
12. To factor by special devices.

VIII. COMMON FACTORS AND MULTIPLES.

Units of Recitation:

1. Highest common factor.
2. Least common multiple.

IX. FRACTIONS.

Units of Recitation:

1. Reduction to higher or lower terms.
2. Reduction to an integral or mixed expression.
3. Reduction to similar fractions.

4. Addition of fractions.
5. Subtraction of fractions.
6. Multiplication of fractions.
7. Division of fractions.
8. Complex fractions.

X. Simple Equations.

Units of Recitation:

1. Equations with one unknown.
2. Equations involving fractions.
3. Literal equations.
4. Problems.
5. Simultaneous equations.
   a. Elimination by addition or subtraction.
   b. Elimination by comparison.
   c. Elimination by substitution.
6. Literal simultaneous equations.
7. Problems.
8. Equations with three or more unknowns.
9. Problems.

XI. Graphic Representation.

Units of Recitation:

1. Graphs of statistics.
2. Graphic representation of linear equations.

XII. Involution.

Units of Recitation:

1. Involution of monomials.
2. Involution of polynomials.
3. Involution by the binomial theorem.

XIII. Evolution.

Units of Recitation:

1. Evolution of monomials.
2. To extract the square root of polynomials.

XIV. Radicals.

Units of Recitation:

1. To reduce radicals to their simplest form.
2. To reduce a mixed surd to an entire surd.
3. To reduce radicals to the same order.
4. Addition and subtraction of radicals.
5. Multiplication of radicals.
6. Division of radicals.
7. Involution and evolution of radicals.
8. Rationalization.
9. Square root of a binomial quadratic surd.
10. Radical equations.

XV. Quadratics.

Units of Recitation:

1. Pure quadratic equations.
2. Affected quadratic equations.
3. Solution by
   - *a.* Factoring.
   - *b.* Completing the square.
   - *c.* The formula.
4. Literal quadratic equations.
5. Radical equations in quadratics.
6. Problems.
7. Simultaneous equations involving quadratics.
   - *a.* One simple equation, one involving the second degree.
   - *b.* Two homogeneous equations of the second degree.
   - *c.* Symmetric equations of third or fourth degree, readily solvable by dividing the variable member of one by the variable member of the other.
8. Problems.

Time Table for the Term. — Below is a suggested time table for the year's work:

ALGEBRA: 40 weeks

FIRST TERM

1st week	Introduction
2d week	Addition
3d, 4th, 5th weeks	Subtraction and parenthesis
6th and 7th weeks	Multiplication
8th and 9th weeks	Division and review
10th to 15th week	Factoring
16th week	Common factors and common multiples
17th to 20th week	Fractions

SECOND TERM

21st to 24th week	Simple equations
25th week	Graphs
26th and 27th weeks	Involution
28th and 29th weeks	Evolution
30th to 34th week	Radicals
35th and 36th weeks	Quadratics
37th to 40th week	Review

Factors Modifying the Foregoing Arrangements. — Local conditions will necessarily determine the emphasis to be placed on the respective units of subject matter. If a course in general mathematics is followed, it is obvious that considerable modifications will be required. Some classes are more mathematically-minded than others and this fact should affect emphasis. Whatever units are found to be rarely of value even in advanced studies of mathematics should be practically ignored. There is not time in public school work for the elaboration of the useless. Many textbooks in mathematics are now so arranged that certain designated sections may be omitted without affecting the continuity.

LESSON I

THE INSPIRATIONAL PREVIEW

Purpose. — The purpose of such a lesson is to arouse in the child the will to learn, to awaken interest in the study of algebra, and to outline in a general way some of the things of interest which will be studied during the course.

Need. — Coming from the grades with no very clear idea of what high school means and assigned to new subjects, the very names of which are often strange, it is no wonder that the young pupil is not only lacking in any particular interest in his new work, but may even have a natural antipathy toward it from the start, unless interest be aroused through this inspirational preview.

It is important in meeting any class for the first time that the teacher get *en rapport* with the pupil as quickly as possible. It is well in place to give a simple talk on the history, practical value, and general content of the subject, — a sort of advertising or "selling" talk. The careful traveler will always plan his trip ahead in order that he may be prepared to note all the important and interesting things that may lie in store for him; otherwise, many things would escape his attention. So the preview is a sort of bird's-eye view of the course, a cranking up of the pupil's interest, preparatory to a good start and a run.

Method. — Simple language should be used. The class is composed of immature boys and girls to whom big words and phrases mean little; the talk should be more to the child than about the subject. The essential thing is to make the children feel at home, to arouse enthusiasm for the subject, and to make them look forward to their work in algebra with pleasure.

When a strange word or important date is given, it will have more effect upon the class if it is written upon the blackboard at the time that it is mentioned. Prearranged work upon the board, simply referred to in passing, does not rivet their attention so well.

When the preview is completed, it might be well to ask a few questions concerning what has been said and give, if necessary, any further details needed to make the subject clear. Ask the pupils to jot down the data you have placed upon the board and tell them to hand in the next day a simple statement of what has been said. This is not so important from the standpoint of the material as it is from the standpoint of inculcating at once the necessity of paying attention, of being specific regarding facts; and of the implication that mathematics is closely correlated with English in the clearness of its exposition.

Historical. — A little of the history of the subject will arouse the pupils' interest in the age and romance of algebra. It is best not to go much into detail because of the complexity of its historical development. It will be better also to introduce each new topic, when studied, by its individual history. Many of the recent texts in algebra have historical notes scattered through the book as a help to humanizing the subject. Pictures of famous mathematicians add to the interest of these historical references. A framed portrait of one or two mathematicians of note, or a statuette, placed in the room, will give an added atmosphere to the study of the subject.

Origin of the Word "Algebra." Its name is derived from the title of a book which the Arabs introduced into Europe in the ninth century. The full title of the book was "Al-jebr w' al muqābalah," of which the first two syllables have been cor-

rupted into the present term, algebra. In the original tongue it referred to the process of transposing terms in the manipulation. Hence the solving of problems was early considered the main business of this subject.

Contributors to the Subject. Modern algebra has not come down to us in its present composition, but like the automobile and every other invention, it is the result of years of growth and of contributions of many minds. The early Egyptians and the ancient Greeks of the "golden age" had some conception of the equation and have left their imprint upon its development. Heron of Alexandria about 100 B.C. made the greatest advance in its development up to that time, but Diophantus, a fourth-century Greek, was the first to write an entire book upon the subject. He emphasized indeterminates, which are even now called Diophantine after him. But as stated above, the book whose title has given the subject its modern name was the first general treatise of importance. The modern founder of algebra was a Frenchman, named Vieta, who, in 1591, gave the science the technical symbolism which is in use to-day. Among other modern contributors were Wessel of Norway, Gauss of Denmark, and Sir Isaac Newton of England.

Interesting Incidents. The history of the subject abounds in many interesting episodes. It is said[1] that Sir William Hamilton, an Englishman, who had been working for years on a certain problem, was one day taking a stroll with his wife, when the solution suddenly flashed into his mind, and he at once engraved upon a stone in Brougham Bridge, which he was crossing at the time, one of the fundamental formulas of

[1] "Science-History of the Universe," VIII; *Mathematics*, Current Literature Publishing Company, 1912.

modern algebra, called quaternions. This bridge has ever since been called Quaternion Bridge.

An Italian named Tartaglia, who claimed a certain algebraic discovery, was challenged by another famous mathematician by the name of Fiori. The contest was to see which one could solve the greatest number of a collection of thirty problems within thirty days. Tartaglia, by using his new discovery, which by the way is now a matter of common knowledge, *i.e.* the cubic equation, solved the entire thirty within two hours' time. He celebrated his triumph by composing some verses, but, according to the custom of the times, he kept his discovery a secret for many years.

Practical Value of Mathematics in General and Algebra in Particular. — From the very earliest times, the study of mathematics has been considered primarily practical. Mathematics has been, in some form or other, the bulwark of the education of the Chinese, the Arabian, the Assyrian, the Jew, the Greek, the Roman, and every modern race. One might as well try to conceive of life without atmosphere as to try to separate the influence of mathematics from human life and endeavor.

Dr. Eugene Smith of Columbia University says that "if all mathematical knowledge were eliminated, civilization would be demoralized, factories would stop for want of machinery, and life would revolt to chaos." [1]

This is the era of machinery. A machine, mathematically wrong, is a failure. Long before the machine is put on the market, however, it has been the subject of painstaking effort on the part of the inventor, the draftsman, the pattern-

[1] "Mathematics in Training for Citizenship"; *Teachers College Record*, May, 1917.

maker, the mechanic, and the promoter. Each one has applied his knowledge to his labor, and the finished product is the result of the accumulated researches and experiences of these men in turn. Imagine the invention of the steam engine, the sewing machine, the typewriter, the adding machine, the lathe, the printing press, the automobile, the airplane, without an expert mathematical training on the part of the inventors.

The recent war with its wonderful though terrible inventions, which sprang into being from all sides, illustrates most forcibly the value of mathematical ability, because every machine from the gas mask to the submarine involved the exercise of mathematical genius. From the moment of its first conception in the mind of the inventor to the actual firing of the first shot on the battlefield, the giant field gun is a product of mathematics. The battle of Messines Ridge was won by engineers who skillfully tunneled the hills. Dr. Nichols, of the University of Virginia, contributed to our wonderful shipbuilding exploit by successfully solving an equation to the ninth degree.[1]

Moritz shows our commercial dependence on mathematics. Thales, the ancient mathematician, was by profession a merchant, and yet he studied mathematics for the intrinsic value it held for him. Such modern business problems as equation of payments, theory of interest, valuation of debenture bonds, amortization of interest-bearing notes, life insurance mortality tables, distribution of dividends, casualty insurance, and a thousand others, have their solution through the application of pure mathematics. Statistical work, which is used in hundreds of transactions and enterprises of every kind, is *all* mathematics. Business executives say they prefer

[1] C. E. White in *School Science and Mathematics*, January, 1919.

mathematically trained men because they are more methodical, exact, and resourceful, and, therefore, efficient.[1]

Algebra Has an Important Position. From a strictly utilitarian standpoint, algebra has a firm claim for an important position in our program of studies. It offers the only means of solving many of the problems connected with engineering, architecture, navigation, surveying, meteorology, geology, physiology, and even psychology, if we accept the Weber-Fechner law,[2] astronomy, physics, chemistry, and many other branches. No mechanic or artisan can read intelligently his trade journal, a technical book, an article in the encyclopedia, without a knowledge of the universal language of algebra. There are 27,000 volumes in the Naval Observatory at Washington which absolutely require a mathematical training for their perusal.[3] Statistics of all kinds are given in equations or mathematical formulas. The graphical treatment is employed by the economist, the business expert, the physician, the dietitian, and men of hundreds of other professions. The handling of pig iron in the modern foundry is a result of long continued analytical experiments based on algebraic formulas.

Algebra Is Necessary to Secure a Higher Education. It is indispensable to the future student of astronomy, physics, chemistry, and higher mathematics. The modern engineering world and all technical schools are forever closed to him without a mastery of this subject. Just the other day the writer had an illustration of this. A young man who had been

[1] R. E. Moritz, "Our Relation of Mathematics to Commerce"; *School Science and Mathematics*, April, 1919.

[2] C. E. White in *School Science and Mathematics*, January, 1919.

[3] Schultze, "The Teaching of Mathematics"; The Macmillan Company, 1912.

recently graduated from high school found that it was possible for him to go to college but he had been allowed to go through this school without algebra and geometry, and he found to his dismay that he was unable to matriculate because of this deficiency in his education. Many a young man seems forever doomed to a nominal wage and a position near the foot of the ladder because he failed to fit himself for higher positions by the careful study of mathematics.

A noted inventor once said that he had no use for algebra or higher mathematics, but in the next breath he admitted that he hired experts to work out for him all problems involving advanced mathematics. Mathematical physics and mathematical chemistry are important branches of science to-day, and every phase of electrical engineering is intimately bound up with algebra and its manipulations.

The writer once knew a man who lost a fine position as assistant contractor because he did not know algebra, although he was an expert workman. His would-be employer felt he could not afford to have a man in this responsible position who was ignorant of this subject, as occasions sometimes arose which demanded a knowledge of it.

A man must know algebra to advance in the automobile business. A modern blue print with its mechanical and mathematical symbols looks like a Chinese puzzle to the layman, but to the trained mind of the skilled mechanic it is as plain as the printed page.

And so we might multiply the examples of the practical value of algebra indefinitely, but enough has been said to arouse in the mind of the pupil the strong suggestion that here is a subject which has a bearing, both directly and indirectly, upon his future as well as upon the progress of the world.

Bird's-eye View of the Course. — A few minutes might profitably be spent in outlining the semester's work. Tell the class that there will be a certain amount of formal work, similar to that done in arithmetic, such as learning how to add, subtract, multiply, and divide algebraic terms and quantities, finding factors, reducing fractions, and manipulating equations, in order that we may arrive at the solution of problems. Explain that this is necessary in order to become familiar with the tools of algebra, just as the carpenter must learn how to use the hammer, the saw, the square, and the compass, before he can build a house.

Explain that from time to time, as a topic is finished, we shall have a sort of field day, when we may exhibit our work, give special reports of certain phases of the subject, and invite our friends to see the things we have accomplished.

Tell the pupils also that we shall occasionally have contests, in which we shall choose sides to see which side can win. This may be done with fine results, as will be illustrated later, upon the completion of the work in factoring.

Again, explain that during the year the class will learn how to solve various problems which will be drawn from business, agriculture, the vocations, etc. Tell them that they themselves will also be expected to make up and solve problems, and that data drawn from current events, like an election or a ball game, will be used.

Conditions for a Successful Preview. — Too much must not be attempted in this first lesson. The word "inspirational" very graphically illustrates the underlying motive for this lesson. The preceding material may well be added to or in part eliminated, as seems best in the judgment of the teacher himself. It is given simply to suggest some of the things that

may make this first meeting of the class an inspiration and a forward look.

Illustrations concerning the practical value will have more force if they have come under the actual observation of the teacher. Indeed, the teacher of algebra may well keep a notebook in which such material and experiences may be collected and added to from time to time. Such a teacher will soon accumulate a valuable set of illustrations, and one that will have variety and modern application. The historical notes will also vary with the teacher; he must ever be on the alert for interesting incidents to use in this connection. Such material, aside from the cases which come under his own observation, will be found in magazine articles, newspaper articles, and the experiences of others and will be drawn from the pupils themselves.

LESSON II

UNIT OF INSTRUCTION I. — INTRODUCTION

LESSON TYPE. — A LESSON IN CORRELATION

Program or Time Schedule

The Review	5 minutes
The Assignment	30 minutes
The Study of the Assignment	25 minutes

Purpose. — The purpose of this lesson is threefold: (1) to gain the confidence and free expression of the pupil by getting him to do something he knows how to do; (2) to review and reëmphasize some of the fundamental operations; and (3) to link together arithmetic and algebra by showing their interrelationship.

The Review. — Since this is the first review work of the year and there has been no assignment, a few questions like the following might be asked, suggested by the " Inspirational Preview ":

How did we get the word *algebra?*

Name some countries which have contributed to the development of this subject.

For what is Vieta important?

Relate an interesting incident connected with the history of this subject.

Name some professions which presuppose a knowledge of algebra.

The Assignment.

1. Information given by the teacher regarding the (*a*) function and (*b*) applications of algebra.

2. Review of the fundamental processes in arithmetic.

3. Recognition of the interdependence of algebra and arithmetic.

The Function of Algebra. Sir Isaac Newton called algebra " Universal Arithmetic." Comte defined arithmetic as the science of *values* and algebra as the science of *functions*. Algebra deals historically and primarily with number. Primeval man, desiring to count his possessions, used various forms of tallies. The ancient Roman used the pebble, setting aside one for each article counted. The ten fingers or digits were early used in counting, thus giving us the term used in numeration, and later evolving into the ten digit or decimal (*i.e. decem*, ten) system. Gradually numbers became of interest because they allowed combinations. We therefore use symbols to stand for or represent things, later substituting the thing itself. Thus we are enabled to derive a general statement which may be applied to all similar cases.

We have done something of this in our arithmetic when we used the statement $B \times R$ equals P. Any problem in percentage may be reduced to this formula or some variant of it. When the substitutions are made, the problem may be solved. As in the case of percentage we have used the first letter of the word indicated, so in algebra we also represent the unknown by letters. But in that case, not knowing what the words may be, we do not try to use the initial letter. It has become customary to employ the least used or last letters of the alphabet, x, y, and z.

We find, therefore, that algebra becomes a general science while arithmetic remains a particular science, and, though they may be said to resemble each other in some respects, in reality algebra becomes a new and more general method of manipulating quantities.

Applications of Algebra. The fundamental processes of addition, subtraction, multiplication, and division of whole numbers and fractions form the foundation stone of the formal work in algebra as well as in arithmetic, but we shall find that their functions and applications in this new science reach a breadth which is impossible in arithmetic. In solving problems in arithmetic we are always limited to things concrete, but in algebra, through the use of the unknown, we are at liberty to sweep the whole field in our solution. It is for this reason that algebra assumes a universal value of its own.

Review of the Fundamental Processes in Arithmetic. — A half hour's time may well be spent in reviewing the fundamental operations in arithmetic and in bringing out the errors which are very common to most first year pupils, such as the product when multiplying a number by unity or zero, the manipulation of simple fractions, mixed numbers, reduction

of a fraction to unity in cases like $\frac{4}{4}=1$, cancellation, etc. It will amaze one to find out the number of pupils who, supposedly ready for algebra, cannot do correctly some of the simplest fundamental operations in arithmetic. It will surprise not only the teacher but the pupil as well, since he is apt to feel that, having passed the final preacademic examination in this subject, he is somehow endowed with a supreme and unfailing knowledge of arithmetic for all time.

The teacher may apply some of the operations of arithmetic to algebra, but it is best not to do too much of this at first. For instance, after the fraction $\frac{4}{4}$ has been reduced to 1, we might take $\frac{4x}{4}$ and show that it reduces to $1x$, or after a review of the multiplication of fractions, as $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$, we might show that $\frac{1}{2}\times\frac{a}{3}=\frac{a}{6}$, keeping in mind that we are not at present interested so much in the teaching of algebra as we are in showing that our present knowledge of arithmetic will be of constant help to us; and in emphasizing that we are not going to work in a strange land but will have with us old friends.

Interdependence of Algebra and Arithmetic. The nomenclature is similar to that of arithmetic and even the mechanical method of setting down the problem, as in division, will be the same.

We have already used the simple equation in the lower grades when in teaching addition we give such problems as "two and what make five?" In arithmetic we call this the Austrian method, but in algebra it becomes finding the unknown.

We have learned that we cannot add together 5 apples and 3 oranges except to say that we have 5 apples plus 3 oranges. So in algebra we cannot add $5a$ and $3b$ except by indicating $5a$ plus $3b$.

There is an intimate relation between exercises in removing the parentheses in arithmetic and algebra, such as $(18-2 \div 4 \times 2)$, in which the order of performing the operations and rules for removing the signs of aggregation are identical.

In the use of the question mark to indicate what is desired, we have in arithmetic simply anticipated the use of the unknown x in algebra, in such an example as this:

$$\frac{3}{8} \times \frac{5}{8} = \frac{?}{8}$$

The graph is used in many exercises in arithmetic to show data which are used in the problem. These statistical graphs are a very important phase of elementary algebra. (See Unit of Instruction XI, Chapter Two.)

We have already referred to the use of the formula in arithmetic. In addition to $B \times R$ equals P, we have a number of others relating to problems in interest, as P equals $I \div (\$1 \times r \times t)$. Again, in measurement we have A equals $b \times h$; circumference of a circle, or O, equals $\pi \times D$.

Many texts in arithmetic also to-day have a section covering simple linear equations of one unknown which are solved algebraically.

The Study of the Assignment. — *I or Minimum Assignment.*

Ten or more examples to illustrate the common errors made in arithmetic as suggested in the paragraph on fundamental processes.

1. $48 \times 0 =$
2. $0 \times 10\frac{1}{2} =$
3. $4\frac{3}{8} \times 5\frac{1}{16} =$
4. $\frac{4}{7} + \frac{3}{5} + 2\frac{1}{2} =$
5. $\frac{5}{8} \div \frac{2}{3} =$
6. $\frac{5}{6} \times \frac{1}{3} \times \frac{3}{5} =$
7. $\frac{1}{2} + \frac{1}{4} + \frac{2}{8} =$
8. $4\frac{1}{5} - 2\frac{1}{3} =$
9. $12 \div 4 \times 3 + 6 =$
10. $48 \div 3\frac{1}{2} =$
11. $16 \times \frac{1}{8}$ of $\frac{1}{2} =$

II or Average Assignment.

Eight or ten examples to illustrate the similarity of arithmetic and algebra as explained in the paragraph on interdependence.

12. 32 and what make 50?

13. 17 and (x) make 25, what is x?

14. 23 and 32 give?

15. 67 times 7 give (x)?

16. 7 books plus 3 chairs plus 2 books plus 6 chairs equal (?) books plus (?) chairs.

17. $5b+3c+2b+1c=(?)\ b+(?)\ c$.

18. $3x+7x=$how many x?

19. If $b=5$ and area $=120$, find the altitude in formula, area $= b \times h$.

III or Maximum Assignment.

20. Mention some other similar cases of algebraic applications in arithmetic.

21. Give some formulas besides those stated, which you have had in arithmetic.

22. Bring to class some illustration of the use of the graphic means of showing statistics.

23. Report on the origin of symbols of operation. (Slaught and Lennes' "Elementary Algebra," p. 7, and Hawkes, Luby, and Touton's "First Course in Algebra," pp. 4, 5.)

The Silent Study Period. — As soon as the time for the study period arrives, the pupils should commence work upon the lesson assigned for the next day. This assignment in three sections has been fully explained in Chapter One. The complete assignment should always be placed upon the board before the class assembles, so that there may be no delay in commencing to work.

Since the exercises above suggested are simple, the pupils will probably not have much trouble, but they should understand that, in case anyone finds difficulty with the work, he has the privilege and is expected to raise his hand for aid. The teacher may then step quietly to his desk and find out what the difficulty is.

It is important that the teacher and the pupil realize that the teacher is not expected to do the pupil's work for him. The teacher's part is to *direct* attention to his difficulty. The obstacle should be skillfully cleared up through the *redirected* effort of the pupil along the right path.

If the point raised seems likely to be a stumblingblock to others, the teacher may step to the board and, calling the attention of the class to the difficulty, make necessary explanations which will at once be a help to all. It often happens that some little point was overlooked in the explanation before the class, which may now be made clear.

Encourage the pupils to do as much of the assigned lesson as possible during the study period. The more the pupil does in the classroom, the better will he understand his work.

Summary on the Review. — Each day should provide some definite review of the preceding day's work, some definite advance in the mastery of the subject, and some definite work assigned. The pupil then becomes conscious of something positive having been accomplished. Nothing is more detrimental to the morale of the pupil than for him to feel that a day's work or a recitation period has been wasted or, at least, has passed without some definite advance. When our pupils realize that nothing will be allowed to interfere with the day's work, we shall find that they will be more anxious to eliminate their absences. Friday or the day before or after

a vacation or some circumstance is allowed often to break up the routine of steady, purposeful work. With the proper attitude and evaluation of each day's importance, however, the teacher can make every meeting of the class, no matter under what disadvantages, a distinct step forward. The morale of the allied army was at high pitch until the Rhine was reached, and then it became a matter of anxiety to the commanders, because the soldiers felt that their object had been attained and that, with no further advance being made, they were merely marking time.

The review should be short, snappy, and purposive. It really takes the place of the old recitation, as such, and since it is much shorter in time, there must be intensive work done. It should be a *re-view* of the previous day's work, a clearing house for all the difficulties encountered in the study of the assignment, and an opportunity for the pupils to view from a new and broader angle the work studied the preceding day.

Summary on the Assignment. — This is the portion of the period devoted to the explanation of the new lesson. The teacher should do most of the board work, the pupils following the operations at their seats. As far as possible the work should be developed through the pupils, since they will then become active participants and their interest peculiarly acute. Except on special occasions, all board work should be eliminated, so far as the pupils are concerned. Much time, chalk, and patience are lost when all or part of the class are working at the board. The inequality of time needed by various pupils, and the ease with which the brighter ones, out of a job temporarily, turn to things not connected with the subject, cause dismay to the teacher and an undesirable diversion for the rest of the class.

But when the teacher himself develops the work on the board, and the class follows the operations on paper, every pupil is of necessity alert and attentive, because he may be called upon at any time for some point. The entire class is therefore kept up to a high pitch of intensive work. The blackboard as a means of visualizing a demonstration before the whole class has a distinct value, but as a common working ground it is open to criticism.

Summary on the Study of the Assignment. — As has already been stated, the various assignments should be placed upon the board before the meeting of the class. Let them be definite, concise. In the case of references, the exact title and page of the book referred to should be given. The use of the assignment sheets has been fully explained in Chapter One and need not be repeated here.

Besides giving the pupils something definite to do at once, this first assignment of work will enable the teacher almost immediately to single out the poorer pupils and those who are likely to be the workers of maximum ability. Plans must be made at once to take care of both classes,—something to which the supervised study period is especially well adapted. A rearrangement of the seating of the class will be made after a few days, as bad results sometimes come from the premature announcement that permanent seats have been assigned. A workable plan is to put the slow workers on one side of the room, the rapid workers on the other side, and the rest of the class between them. If the class can be approximately so divided early in the term, no especial attention will be called to this classification, and the reasons therefor may remain a secret with the teacher. This arrangement will make it much easier to reach the two extremes of the class and give to them

special aid. It will also be a distinct help in administering the details of board work, individual instruction, and the use of supplementary material.

Summary on the Silent Study. — The importance of getting to work at once without loss of time should be explained in the beginning of the term. The pupil should be made to feel that every minute is valuable, and that waste of time will not be countenanced.

Various devices for keeping track of the completion of the different sections of the assignment will be given in succeeding lessons. The object of these sectional assignments is solely to aid the teacher in developing to the utmost each individual pupil in the class. This section of the supervised study period should be the most important part of the hour, because it is here that the pupil comes into personal contact with the teacher and receives first hand the kind of aid he needs. At the same time a certain amount of sympathetic relationship is developed on the part of both teacher and pupil.

LESSON III

UNIT OF INSTRUCTION I. — INTRODUCTION

LESSON TYPE. — A HOW TO STUDY LESSON

Program or Time Schedule

The Review	5 minutes
The Assignment	30 minutes
The Study of the Assignment	25 minutes

Purpose. — The crux of the supervised or directed study period consists in the definite directions for the proper study of the subject in question, and careful explanations of just *how* to study rather than *what* to study. It is, therefore,

valuable during the course and especially at the beginning, that very definite rules for study should be outlined and insisted upon.

The Review. — *Subject Matter.* A summary of the previous day's assignment.

Method. Call for questions on the exercises assigned. Ask for a few leading facts brought out by this assignment, such as:

1. What was Newton's definition of algebra? Comte's definition of arithmetic? (Show their pictures to the class.)
2. How did we get the word *decimal?*
3. Mention some formulas used in arithmetic.
4. What makes algebra a broader subject than arithmetic?

Give a few examples like those in assignments I and II. Call on someone who completed the maximum assignment of yesterday's lesson to give the answers to the questions in this assignment.

The Assignment. — 1. Methods of study.

2. The system of supervised study explained by the teacher.
3. The technic of the textbook in algebra.
4. Definite instructions in *how to study* algebra.

Methods of Study. — (*a*) *External conditions.* In the first place a correct physical environment is necessary. One must be in a comfortable seat, with good light coming over the left shoulder, breathing fresh air, and in a room of proper temperature. The pupil must have the necessary tools, as paper, well-sharpened pencil, ruler, textbook, etc. Next, the pupil must put himself in the proper attitude toward the subject and concentrate his mind upon his work. And then, with a determination and expectation to succeed, he is ready to commence his work. All these prerequisites will in time become

automatic if the teacher frequently calls the pupil's attention to them. It is easier to talk about concentration of mind than it is to achieve it, but the pupil should be carefully shown that the better control one has over his ability to keep his mind from wandering, the better student he will be and the sooner will he be able to master the task in hand. It takes will power and practice, but the teacher cannot emphasize too strongly the inestimable value of this acquirement, if once formed even to a limited degree. As McMurry suggests,[1] one of the best methods of acquiring concentration is through the employment of time tests, which require undivided attention. When we must do a certain thing within a definite time, we concentrate upon it. With sufficient practice, this may become habituated.

(*b*) *Technical factors.* 1. The first technical factor in proper studying is the sensing of the problem. If the pupil simply goes at his work with a view of covering a certain amount of prescribed ground, without a realization of the problem involved, his study will degenerate into a mechanical grind. Every assignment, as stated before, should have some definite object or problem, around which the lesson will revolve. In the first lesson, as outlined in these pages, it was "Is algebra practical?" In the second lesson it was "Is algebra entirely new?" and to-day it is "What is the best way in which to study algebra?"

2. The second factor, after the recognition of the motive of the lesson, is bringing to bear upon it all our present knowledge of the problem and then supplementing the problem from our text and possibly other books and sources.

3. We next seek to master the supplementary material by

[1] F. M. McMurry, "How to Study"; Houghton Mifflin Co., 1909.

constantly referring to our present knowledge and associating our new facts with data already known. This may mean the employment of the memory, which, if rightly used, will become a means and not an end. Too much of our studying resolves itself into memorizing alone, and it then becomes a detriment. But as Miss Earhart well says, " Memory must not be substituted for thought but be based on thought."[1]

There are three kinds of memorizing: purely arbitrary memorizing, memorizing based on reasoning, and remembering the sequence rather than the things themselves.

4. The fourth and last factor is the application of our data and material in the solving of our problems or in making our new power a part of ourselves.

The Supervised Study Organization. A few words about the supervised study period may now be given. Each day's work will be divided into three parts: the review, the assignment, and study of the assignment. During the review, the previous day's work will be *re-viewed* in summarized form and any difficulties cleared up. This will usually take about 15 minutes. The new work will be *explained* during the second time division, *i.e.* that of the assignment. This will take about 20 minutes but may vary in amount. The *study* of the new lesson will take place the last 25 minutes, and during this period the pupil will be expected to do as much of the new work as possible. Explain that he may feel free to raise his hand if he finds need of help, that he must not expect to have the teacher do his work, but only redirect him to find his own trouble, or lead him to see wherein his line of procedure is erroneous. The study period is not an opportunity for

[1] Lida B. Earhart, "Teaching Children to Study"; Houghton Mifflin Co., 1909.

getting someone else to do the pupil's task but an opportunity for getting proper directions so he may be able to do it himself.

Explain that the assignment will be placed daily upon the board, that it will be in three parts, and that each pupil will be expected to do the first two parts and as many pupils as possible to do the third. The assignments should be so arranged that much of the work may be done during the period itself. The exact amount, of course, will depend on the pupil. It has been found in work of this kind that pupils are proud to excel their classmates. There are only the rarest instances of pupils deliberately doing only the minimum assignment.

The Open Book. The pupils are asked to open the textbook in algebra while the teacher explains the structure. Comparison is made between title-page and the cover. A few words regarding the position or personality of the author, the name of the publisher, and the date of the book's publication may arouse some interest in the author as a second teacher of the class; for, of course, the author of a textbook is to be regarded as a teacher.

Have a pupil read the preface and then ask him a few questions which will bring out the reason for such an introduction to the book.

Turn to the table of contents and note the various topics and subtopics of the subject. Compare this with the table of contents of some other text in algebra. Note the number of pages one or two units of instruction cover in your book and in some other text. If any topics are to be omitted from study, mention the fact.

If your text has answers, give a few words as to their use and abuse. Teachers differ in their opinions regarding the

value of the printed answers, and, if it is impossible for the teacher to train the class to make them a side issue and not the most worn portion of the book, it is clear that their publication is a serious mistake.

Now open the book at the first page, calling attention at the same time to the fact that the paging of the book proper commences at this point. Announce that the work for the next day will begin here.

This review of the make-up of the book proper may seem irrelevant to the study of algebra and a waste of time, but aside from the value of the general knowledge thus gleaned, it serves as an introduction to the text with which the pupils are to have intimate acquaintance during the year. It is well that they know something of the nature of the tool with which they are going to work. It often happens that things learned incidentally in connection with a subject will be of greater educational value than the subject matter itself. After all, our children are coming to school primarily to be educated and secondarily to learn algebra, Latin, or any other particular subject. It is *through* these subjects that we hope to *attain* the ends of education.[1]

Instructions in How to Study. A few mimeographed directions may now be distributed to the class, and, after necessary explanations, the pupils may be told to insert them in their books for future reference. Explain that you may add other directions from time to time as the class progresses, and suggest that each pupil should feel free to make any suggestions for the enlargement of the list.

[1] "The Textbook — How to Use and Judge It" by Hall-Quest, The Macmillan Company, 1918, gives a full discussion of what might well be covered in teaching pupils how to learn to use academic tools.

The list which follows is by no means perfect or complete; it is simply suggestive:

SUGGESTIONS FOR EFFECTIVE STUDYING

Be sure you understand the assignment.

Study the meaning of the type of problem you are to solve, as suggested in its name, *i.e.* highest common factor, addition of radicals, etc.

Recall the teacher's explanation of the new work.

Study again the type form or example of the new problems.

Understand thoroughly what is wanted before you begin to use your pencil.

Avoid guesswork.

Take time to think. Do not rush into an exercise trusting to luck you will strike it right. Be *sure* you are right; then go ahead.

Be sure you set the exercise down correctly on your paper.

Work carefully; it is easier to avoid mistakes than it is to find them.

When you find a new application, study it until you master it. Expect each new problem to be different from the one preceding; else, we would never advance.

In case you cannot proceed, raise your hand. Do not expect the teacher to find your mistake but to direct you to find it yourself.

Be neat in your work. A good workman is known by his neat performance.

Slovenly habits of work lead to slovenly habits of thought.

The Study of the Assignment. — The assignment for to-morrow will be in one section only. It will consist of some questions on the points of to-day's lesson, in regard to the attitude of study, factors of study, the technic of the text-book and the list of directions on how to study. These questions will help to focus the study on the essential features and to prevent wrong conclusions. A few sample questions are given:

What is the proper temperature for a living room?

Why should the light come over the left shoulder?

Suggest a good method of practice to attain concentration of mind.

What was the problem of to-day's lesson in biology?

Which of the three kinds of memorizing do you use in relating the incidents of a ball game?

Name some sources of supplementary material aside from the textbook.

Is the preface necessary in every book?

Which do you think the author compiled first — the table of contents or the index? Give your reasons.

Suggest any other directions than those given to you on the printed list.

Which one of those given do you think would save you the most work, if carefully carried out?

BRING PAPER AND PENCIL TO-MORROW

LESSON IV

UNIT OF INSTRUCTION I. — INTRODUCTION

LESSON TYPE. — AN INDUCTIVE AND HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* The questions on the previous lesson.

Method. Write the various questions assigned yesterday upon slips of paper and have these in a loose pile, face down, upon the teacher's desk. Call on some pupil to come to the front of the room, to pick up one of the slips at random and, after reading it aloud, to proceed to answer it. If the class accepts this answer as correct and complete, ask someone else to repeat the process, and so on until all the questions have

been answered. If any question is not answered acceptably, replace the slip in the pile. This method of review will further help to break down the barrier between pupil and teacher, to accustom the pupils to talking before the class, to teach clearness and accuracy of expression, and to test the judgment of the value of the answer, thus giving all something to do. Furthermore it helps to review thoroughly the essential points of the preceding day's lesson.

NOTE. — The lessons in algebra will not be based upon any special textbook, but the directions will be found applicable to any textbook upon the market. Copies of all the modern texts are upon the teacher's desk, and constant reference is made to these either for supplementary examples or other material. Inasmuch as the present writer is interested chiefly in presenting a variety of schemes for teaching and training pupils in economical and effective methods of study, it is hoped that the point of view herein developed will be comprehensive enough to include the situations that may arise in the use of any text in algebra.

The Assignment. — 1. Instructions in how to study the printed page.

2. Treatment of illustrative material.

3. Treatment of class exercises (*a*) oral, (*b*) written.

Instructions in How to Study. The pupils will open their texts and have their attention directed to the " definitions." It will be noted that this is the first unit of instruction as listed in the table of contents. (In the divisions of algebra outlined in Chapter Two, it is given as a unit of recitation under *introduction;* authors differ in the arrangement of the material.)

The first paragraph is read carefully and the central or important point or problem discussed. The pupils will readily select the essential point of this paragraph. If there are any words which are new or not clearly understood, they should

be immediately defined by the teacher. Before we can comprehend the sentence, we must know the meaning of its component parts. Explain the use of italics and heavier type. These take the place of the emphases in oral speech. These mechanical means call the attention of the reader to the importance of the word or phrase and should be specially noted by the pupil.

If the sentence or paragraph is not clear at the first reading, reread it until the thought is mastered. Insist on the importance of making reading thought-producing, and not simply a mechanical pronunciation of words. The language of mathematics is absolute and therefore cannot be read rapidly or slurringly; every word means something.

Now have someone reproduce the paragraph in his own words. Call upon a number of pupils to do the same thing, thus bringing out in various degrees of perfection the meaning of the assignment, and setting up a little rivalry for the best work. Emphasize the facts that we know what we can reproduce in our own words and that, when reproduced word for word like the text, we are thinking more of the mechanical reproduction than we are of the thought to be reproduced.

The above outlined study of the paragraph might well be applied to the printed page of any book which is a subject of study, although some authorities strongly advise that the first reading of the page or section be made as a whole in order to get the general sense of the material. In algebra, however, since the textual matter is localized in its meaning, the prereading might be dispensed with. The reader will note also that the first three steps, mentioned in the preceding lesson as the order in which a subject should be studied, have been

followed, *i.e.* the point of view or problem, the data or material, and its mastery. Its application, as is often the case, will be made later. It often happens that we accumulate material through these steps for some length of time before we finally bring it together in the fourth or concluding step.

Treatment of Illustrative Material. When we come to illustrative material, the example should be reworked on paper. Otherwise the pupil will mechanically read the operation, think he understands it, and in a short time find that it has slipped away from his consciousness. This reworking of the example on paper will also help to fix it firmly in his mind and to establish each step thoroughly as it is written down, provided always that the pupil does the work with the motive of understanding the operations as they are evolved. Since this is an illustrative or model lesson, the teacher will also do the work which he will expect the pupils to do for themselves in their future study.

For instance, suppose this formula is given:

$$a = b \times h.$$

The meaning of the symbols is studied and then the value of the letters in a specific case is given and put upon the board.

Thus, a equals 20;
h equals 5.

The question is, what is the value of b? The substitutions are now made in this formula and the solution performed. This operation should be repeated a number of times with varying values for the letters.

Two things are being done in this operation: the pupil is learning how to interpret the printed word by his clarified perception, and he is also learning the fundamental character-

istics of algebra, the broad application of the algebraic function. It might be well at this point to request some pupil to turn to the introduction of Milne's Standard Algebra [1] and read what that author has to say concerning it:

> The basis of algebra is found in arithmetic. Both arithmetic and algebra treat of number, and the student will find in algebra many things that were familiar to him in arithmetic. In fact, there is no clear line of demarcation between arithmetic and algebra. The fundamental principles of each are identical, but in algebra their application is broader than it is in arithmetic.
>
> The very attempt to make these principles universal leads to new kinds of numbers, and while the signs, symbols and definitions that are given in arithmetic appear in algebra, with their arithmetical meanings, yet in some instances they take on additional meanings. . . .
>
> In short, *algebra* affords a more general discussion of number and its laws than is found in arithmetic.

Since with this introduction the pupil has an idea of the manner in which he should study, the teacher should further encourage him to proceed alone in his study. Questions need to be asked from time to time, however, to make sure that the pupil is following the directions and getting the right ideas. To illustrate, after the class has studied some paragraph or section, ample time having been allowed for the use of the dictionary, etc., ask some questions about it, and then call upon someone to state the problem involved; someone else to restate it in his own words; and others to supplement it from their own knowledge if possible. Thus we more and more throw the pupil upon his own resources but always with the proper methods of procedure before him, and careful supervision on the part of the teacher to see that he gets the correct interpretation. He will eventually acquire the habit of study as outlined

[1] American Book Co., 1914.

above, which may be of more lasting value to him than the algebra itself.

Treatment of Class Exercises. The treatment of exercises to be worked in class will differ somewhat from that of illustrative material. Suppose we wish to take up such exercises as the following:

Read and explain:

1. $a+b$.
2. $a-b$.
3. $a\times b$.
4. $a\div b$.

We have now accumulated our material and are ready for its application, or the fourth step. Here are definite examples of what we have been studying about up to this point. All study is for an end. As the final end of algebra is the solution of problems, so an intermediate step in the attainment of this end is the ability to perform the mechanical processes which will later be involved in their solution.

Class exercises will therefore be of two kinds, (*a*) oral and (*b*) written.

(*a*) *Oral exercises.* Some pupil is told to rise and read the first example. He is then asked to analyze or tell the meaning of it, which should be something like this: Two general members of different values are added together by indication, *a* added to *b*. The teacher should insist on complete answers, told in technical terms and in simple English. Clear thinking and clear expression will thus be unconsciously habituated by the pupil.

(*b*) *Written exercises.* Exercises like the following, however, may preferably be treated in a wholly different manner. Some such procedure as outlined here may be used or some modification of it:

If $a=3$, $b=2$, and $c=\frac{1}{2}$, find the value of each of the following:

1. $\frac{6a}{b}$.
2. $\frac{2a^2}{c}$.
3. $\sqrt{2c}$.
4. $3b^a$.
5. $\frac{a-b}{a+b}$.
6. $a\times\frac{b}{c}$.

Most of the board work should be done by the teacher himself. The pupils should remain at their seats and either work on paper or tell the teacher what to write upon the board. This elimination of board work by the pupils will result in a more efficient use of the time of the period, as all members of the class will be either at work or on the lookout for possible questions. Every mark put upon the board should first be supplied by some member of the class and accepted by all as correct. Thus each member becomes personally interested in the operations and alert to give directions or to detect errors. The class is thus kept up to a high tension and intensive work may be accomplished. The board work becomes a check and not a key, and the pupils feel that they have had a real part in its development.

To illustrate, the work on the first example given above will proceed like this: the teacher will ask someone to read the example and to explain how to make the substitutions. He will then write it upon the board, directing the pupils to do likewise on their papers. Another pupil will then be called upon to tell what is to be done next. As the pupil states the various steps, the teacher will place them upon the board, the class meanwhile doing the same on their papers. The pupil reciting will say something like this: The expression, $6a$, means that the literal number a is taken 6 times, or that a multiplied by

6 constitutes the term in the numerator, and that the product is to be divided by b. Unless we give these literal numbers some values, the actual division can only be indicated as in the example. But if we assign some arbitrary values to the literals, we may substitute these values in the expression, perform the necessary operations, and reduce to its simplest term. In this case, since a is given the value 3 and b the value 2, we find that $6a$ is equivalent to 18, and this divided by 2, or the value of b, gives us the result, or 9. The work on the blackboard will appear as follows:

$$\frac{6a}{b}=\frac{6\times 3}{2}=\frac{18}{2}=9. \quad \textit{Ans.}$$

In this way, it would be well for the teacher to work on the board, with the assistance of the pupils, these six examples, in order that the pupils may learn how to handle written exercises. When written work is next required, it will be safe to assume that they will know how to go about their work, after one or two typical demonstrations have been given by the teacher. Work of this kind is *oral or coöperative studying and should characterize every general assignment.*

The Study of the Assignment. — Assuming a list of 30 graded exercises in the textbook in use, assign as

I or Minimum Assignment. Exercises 1–20.

II or Average Assignment. Exercises 21–30.

III or Maximum Assignment. Exercises 15–20 on page 46 of Ford and Ammerman's First Course in Algebra,[1] or exercises on page 7 of Slaught and Lennes' Elementary Algebra.[2]

Value of Outside Work. Many of the introductory lessons will be along the line of the foregoing, the majority of the

[1] The Macmillan Company. [2] Allyn and Bacon.

exercises being worked in the class under the supervision of the teacher. Each day a short assignment should be made, based on the ground covered and *preferably taken from outside texts*, especially the maximum assignment. This ought to be in the nature of a review of the work done in class and should be short enough to allow the majority of the class to complete all three assignments. These examples may be handed in the next day, but the best way for the teacher to make sure that the principles are thoroughly understood is to work out on the board through the minimum workers, or those who only completed the minimum assignment during the study period, a few typical examples during the review.

The pupil must realize that the teacher is interested not so much in what the pupil has done as in what he can do now. *If pupils could be made to know that work done outside of class is important only in so far as it makes them capable of doing something the next day in class, the incentive for getting other people to do their work would be greatly diminished.* The outside work must be insisted upon, unless the periods are long enough to have all the work done in class, but the credit should always be allowed largely upon the ability to do similar work again in class. It is the same rule that applies through the walks of life. The stenographer, the carpenter, the printer, the dentist, the worker of every sort is not paid for the record he has made in speed or the house he has built or the books he has printed or the bridge work he has done, but for his *ability to do* similar work again. To be sure, the experience has made him proficient, but we pay for results and not for the practice that has made the results possible. The one is indispensable, but the other is the criterion by which all of us are judged.

The next two or three lessons in the textbook may be worked out in a manner similar to the above. The amount of time spent on the work will of course depend on the book used and on the teacher. He may condense it into a shorter period or take even longer. The material given is merely suggestive and no teacher is expected to follow it verbatim. In fact, such a procedure would probably predetermine the failure of supervised study, because more than anything else its successful operation depends on the *originality and individuality of the teacher himself*. No system has been or ever will be evolved which automatically may be operated by someone and without change or adaptation be a success for everyone else. All that may be hoped for any method is that it be suggestive; its final application and adaptation, in the last analysis, lies with the teacher himself. In the words of Miss Simpson, author of a companion book in this series, "it is imperative that teachers *adapt* rather than *adopt* the methods suggested in these lessons." [1]

LESSON V

UNIT OF INSTRUCTION III. — ADDITION

LESSON TYPE. — AN INDUCTIVE LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — Upon the completion of each unit of instruction, it is advisable to *re-view* the unit *in toto*. This may take the form of an oral or written review. Various methods

[1] "Supervised Study in History"; The Macmillan Company.

should be used. In reviewing Positive and Negative Numbers, the following is suggested:

Method. Write upon the board a large number of exercises covering the various phases of this topic, and call on different members of the class for the answers. These should be written upon the board in the proper place. Send a pupil to the board and as the answers are given have him write them down if he considers them correct. If he calls one correct when it is wrong, he must take his seat and another be sent to the board in his place. Thus the pupils are tested on their ability to solve the problems and, also, on their ability to judge correct results. It might be well to select someone to call on the different pupils to recite, the teacher noting, however, that all or most of them are given a chance. When the pupil at the board makes a mistake, then the leader should take his place, and so on. Thus the review becomes socialized. It will provide interest for work that too often is needlessly wearisome.

The Assignment. — 1. The arithmetic preview.

2. Recognition of the problem.

3. Explanation of the type form.

The Arithmetic Preview. As intimated in the second lesson (page 36), the pupils' present knowledge of arithmetic should always be drawn upon when possible to illustrate the new work. A few examples in adding numbers are followed by the implied addition of literal terms. The induction should be made by the class and the Commutative Law of Addition deduced. After a few attempts, a workable law will be developed by some such questioning as this: how much is three and five? five and three? a and b? b and a? Ask whether it makes any difference in what order numbers or

letters are added. If the answer is " no," ask someone to state this principle in a sentence. Answer: Numbers can be added in any order. Tell the class that this is a law of order or the Commutative Law.

Then broaden this principle when two or more numbers are grouped, as (5 plus 6) plus 4. What is the sum? (5 plus 4) plus 6? The sum is the same. Then we broaden the above law to include groups. Ask someone to state the revised principle. Answer: Numbers may be added in any order or group. Tell them that this is the Associative Law of Addition.

Recognition of the Problem. Ask the pupils what is meant by a term, a monomial. The above illustrations are all monomials. Therefore the first problem under Addition will be addition of monomials, which becomes our first problem.

Explanation of the Type Form. Place these examples upon the board:

Add:

1.	**2.**	**3.**
3	3 boys	$3a$
5	5 boys	$5a$

There will be no difficulty with the first two. Ask in 3, what a stands for. Someone will say " boys." But might it not stand for girls or houses or almost anything? The class will readily see that a may stand for anything and therefore the answer to the example will be $8a$.

Repeat with other simple monomials, all positive. Then put these examples on the board:

Add:

1.	**2.**	**3.**
5	5 dollars	$5a$
$-$ 2	$-$ 2 dollars	$-$ $2a$

With their previous knowledge of positive and negative numbers, the class will see that in each case the coefficient is 3.

ASSIGNMENT AND STUDY SHEET

SUBJECT Elementary Algebra PERIOD 2d

DATE September 7, 1921

UNIT OF INSTRUCTION Addition (III)

UNIT OF RECITATION Addition of monomials (I)

UNIT OF STUDY Examples 1–25, text

LESSON TYPE Inductive

REVIEW: Positive and Negative numbers. Exercises from Wheeler's Examples in Algebra, pages 4–7.	MEMORANDA Examples on board. One writes answers which others give. Change when mistake is made. Work out laws.
ASSIGNMENT: 1. Arithmetic preview. 2. Recognition of new problems. 3. Explain type form.	What are monomials? Add: 3 3 boys $3a$ 5 5 boys $5a$
1. MINIMUM Exercises 1–20	In text.
2. AVERAGE Exercises 21–25	In text.
3. MAXIMUM Exercises 43–50, Wells and Hart's New High School Algebra, page 38.	Involve fractions and decimals.
STUDY: See that the signs are correctly copied.	

Number of pupils solving minimum assignment	7
Number of pupils solving average assignment	22
Number of pupils solving maximum assignment	1
Total	30

FIGURE III

Now call on some pupil to stand and solve the first example, which may be:

$$\text{Add:} \quad \begin{array}{r} 2a \\ \underline{5a} \end{array}$$

Similar exercises may be given orally. They may well be supplemented with others from the board until the principle is well understood.

Then a typical example like the following may be developed on the board and the class set to work on the study of the new assignment:

$$\text{Add:} \quad 2x,\ \tfrac{1}{2}x,\ -x.$$

The Study of the Assignment. — Assuming the textbook in use gives a list of 25 similar exercises, make the following assignments:

I or Minimum Assignment. Exercises 1–20.

II or Average Assignment. Exercises 21–25.

III or Maximum Assignment. Exercises 43–50, page 38, in Wells and Hart's New High School Algebra.[1] These exercises are similar but a little more difficult, involving fractions and decimals.

The Silent Study Period. — *The Assignment Sheet.* The division of the assignment into three parts, as suggested in Hall-Quest's book on Supervised Study, has been fully explained in Chapter One of the present volume, which should be reread. The assignment numerals only should be used when designating the sections in placing the assignment upon the board and this should always be done before the class assembles. A sample sheet, made out to conform to this lesson, is given in full on page 62. It will aid the teacher materially

[1] D. C. Heath and Co.

if he will make these sheets out conscientiously during the term. There will then be no confusion or waste of time if additional exercises are needed during the class period. Many precious moments are saved by a little foresight and planning. A lack of prearranged plans may also break down the morale of the class. Pupils are quick to respond to fine or poor executive ability when either is exhibited by the teacher. Napoleon was one of the world's greatest generals because he had the absolute confidence of every soldier under him. Teachers are generals of a little school army and the morale of the one is analogous to that of the other.

The Completion of the Assignment. The minimum and average assignment should cover the amount of work that the teacher would ordinarily give under the old method of only one assignment for all. That is, the ordinary lesson for the next day would be about fourteen exercises in the text. But these have been broken up into two sections, the first of which should be worked by all within the 25 minute study period; if not, then the pupils who fail to complete this part need special attention.

All the class is expected to have completed the average assignment before the next day and some pupils will do so before the end of the period. Those having trouble will have their work taken up during the review.

The *maximum assignment* is designed primarily for the brighter and quicker pupils, those who are capable of doing more than the average amount of work. They should be given an opportunity of trying more difficult applications of the day's work. Not many will complete this part and it should not be demanded from all; but when done, some system of giving extra credit should be used. A good method

is to add half a credit to the monthly grade for every day that the maximum assignment was done correctly. In case a pupil did this correctly every day for a month, it would only mean ten extra credits, which might raise the grade from 80 to 90. Very few would attain this maximum advance grade, however. But the teacher must be careful not to give too much credit to this advance work and so discourage the slower worker; it should be the aim always of the teacher to encourage each one to do his best all the time.

The Teacher's Duty during the Study Period. As soon as the study period begins, all start to work on the next day's assignment. The rate of speed will soon become uneven. Some will experience no difficulty and will advance rapidly; others will be in trouble at the outset. For the latter, the upraised hand will quickly bring the teacher with aid. Thus help comes when it is needed and at the time that the correct direction or word of helpful explanation will do the most good. The teacher must, of course, ever be on the alert to see that his help is corrective or suggestive and not simply a crutch. It should be directive and not simply finding the mistake for the pupil. The teacher, in quietly moving among the pupils, will note many wrong methods and incorrect habits of work which he can tactfully correct. Many small but important things, such as legible handwriting, neatness, careful arrangement, accuracy in copying the example, may be brought to the attention of the child at the time that he is working. It is a case of striking when the iron is hot.

Occasionally a glaring error and its possible results may be called to the attention of the class; for instance, the danger of mistaking a poorly formed 6 for a o if the loop is not carefully attached to the bend of the figure below the top. Draw

the attention of the class to the fact that this little error invalidates the whole later process. This leads excellently to an explanation of the value of carefully checking the work as one proceeds. If each step is carefully gone over and checked for errors of omission or commission, before the next step is taken, valuable time may be saved. It is easier to avoid mistakes than it is to find them. (See rule, page 49.)

Verification. — A minute or so before the close of the period, check up the work done in class by the pupils. Various methods may be employed, two or three of which will here be explained. Others will readily suggest themselves to the teacher.

First Method. Have some printed slips like this:

Name_____

Class_____ Period_____

Examples completed_____

Assignment completed_____

Time spent outside of class on to-day's lesson_____

Teacher's check_____

FIGURE IV

Such a form can be filled out by the pupil in a minute's time; the teacher can collect them and the next day file them with the papers handed in. It can then be noted how much of the lesson was done in class, and how that amount corresponds with the examples done outside of the class period. This method will give the teacher the names of the pupils,

who failed to complete the minimum assignment in class, and these should have special attention the next day. This checking will also serve to tell the teacher whether his assignments are too long, too short, or about right. If the class does not approximately conform to the percentages mentioned in Chapter One, page 16, something is wrong in the assignment and an analysis of the situation should be made. Incidentally, this plan will take care of the roll call.

Second Method. Have the pupils hand in all exercises completed at the end of the period. Then the data may be computed by the teacher. The next day the examples worked outside of class may be handed in and filed with the others, which will thus constitute the completion of the assignment.

Third Method. Just before the close of the period, call on all who have completed the minimum assignment to stand or raise their hands; the teacher can either take down the names or have the pupils hand in their names on slips of paper. Then in like manner the names of those who have done the average and maximum assignments may be obtained.

Fourth Method. Have the pupils check on their papers the point they had reached when the period terminated; then when these papers are handed in next day, the teacher may compile his own lists.

LESSON VI

UNIT OF INSTRUCTION III. — ADDITION

LESSON TYPE. — AN INDUCTIVE LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The division of the time of the class period, as stated at the beginning of each lesson, is that followed in the Canton High School, Canton, N. Y., where supervised study has been in operation since 1915. The day is divided into five periods of one hour each. Longer periods, which would allow the pupil to do all his studying in school, would be ideal, but in many schools such a program could not be administered. If possible, the time schedule should be amplified, but the above proportion of time seems very adaptable, where different arrangements cannot be made.

The Review. — *Subject Matter.* Addition of monomials. *Method.* The object of to-day's review is (*a*) to assist those pupils who had trouble with the assignment and (*b*) to give additional drill in this work to those who had no special difficulty but who were unable to complete the work.

Those who have completed the threefold assignment and who have mastered the addition of monomials should be allowed to proceed at once with the advance assignment in polynomials. This will serve as an inducement for intensive work and will encourage the brighter pupils to work harder and to solve all the exercises during the period if possible. This number will be small if the assignments have been carefully planned. Those who have done part of the maximum assignment should be instructed to complete it.

Now that we have the more advanced pupils working on the next lesson, or on the more difficult examples of the maximum assignment, we can turn our attention to those who did not complete the minimum assignment or who had more or less difficulty. These pupils may be treated in different ways. If several failed on the same problem, they may be sent to the board to work on it under the supervision of the teacher. The

teacher can then watch their work and soon note the trouble. As soon as a pupil finishes one, he should commence on the next with which he experienced difficulty, and so on. This method is not advised, however, for reasons already stated against board work.

A better method would be for the teacher himself to work out the problem, with the pupil directing him what to do, the others meanwhile following the process at their seats.

Pupils should never be sent to the board, however, to work, for the benefit of others, examples which they solved themselves. Board work has been inordinately stressed in mathematics. When a pupil can do a thing, he should not be asked to do it again; it is his ability to do something which he could not do before which will make him advance. On the other hand, such exercises, written out on the board and afterwards read for the benefit of those who could not do them, will be of practically no value to them. Pupils can learn best by doing the work for themselves.

As soon as the problems giving trouble have thus been solved, the remainder of the review period should be devoted to working additional ones of like nature, until this difficulty has been mastered.

If there are still those who do not seem to be able to understand the problem, they should be given individual attention during the remainder of the period. The teacher must feel that his special task is to help the less capable; children have varying degrees of ability and it is the peculiar province of the supervised study scheme that the backward ones are thus given special attention and brought up to the standard of the class as quickly as possible. The old process of the elimination of the dull pupil must give way to the new idea of reaching him

through a study of his particular difficulties and applying the proper stimulus which will enable him to " find himself."

The Assignment. — 1. Explanation of the method of adding polynomials.

2. Recognition of the new problem and its attendant rule.

Explanation of How to Add Polynomials. As in arithmetic we can only add or subtract terms of the same kind, so in algebra like must come under like, before we can add or subtract. In the example, Add: $a+4y$ and $2a-5y$, we write it as follows:

$$\begin{array}{r} a+4y \\ 2a-5y \\ \hline 3a-\ y \end{array}$$

and add each term separately. Thus a plus $2a$ equals $3a$, and $4y$ plus *minus* $5y$ equals *minus* y. If the order were different in the example, we would be obliged to rearrange the terms so that the a's would come under the a's, and the y's under the y's.

Write another similar example on the board and ask someone to direct the work. Put on another and send someone to the board to work it. If all claim to understand the operations, pass on to the development of the problem involved in this lesson.

Recognition of the New Problem and Its Rule. Ask what kind of expressions these are that the pupils have been manipulating. After you get the right term, polynomials, ask what is being done with them. Then ask someone to state the problem of to-day's lesson. The answer should be " Addition of Polynomials."

It must not be forgotten that every day's lesson should have an object or problem; to-day it is Addition of Poly-

nomials. Various schemes may be used to emphasize it. One which has been used with success is writing it upon the board with yellow crayon. Thus it stands emblazoned in the mind of the child; and, noticing it a number of times during the hour, he cannot forget that there is a definite object in view, and that the work in hand leads up to its understanding and solution. Again, if the special problem under consideration has not been thoroughly mastered, it remains upon the board and in this way the aim of the lesson is even more deeply imprinted upon the pupils' understanding. Pupils like to advance and if they find that another day must be spent upon some topic, — say addition of monomials — because they did not master it, renewed efforts will be made to move on to something new.

Develop the rule for adding polynomials by some such analytical method as this: ask why we put the various terms involving a in one column, and what kind of terms these are. Develop the definition of similar terms. When the example has been set down, what do we do? Draw a line and add each column, connecting them with their signs. The rule has thus been developed. Have some pupils state it in full. Repeat with different ones until you have something like this:

RULE. — *Arrange the similar terms in the same column, add each, and connect the resulting terms by their proper signs.*

Work two or three examples on the board, asking pupils to apply this rule by specific reference to the terms in the example thus solved. Such mechanical drill is necessary in all study of mathematics but, after all, there is only one object of drill, *i.e.* to grasp the principle involved, — and if by any means this may be done quickly, it ought in all justice to be employed. Mathematics should not become a master but a servant.

The Study of the Assignment.—*I or Minimum Assignment.* Exercises 2–23, in textbook.

II or Average Assignment. Exercises 24–27, in textbook.

III or Maximum Assignment. Exercises 14–20, on page 41, Durell's School Algebra.[1]

Verification. — Especial attention should be given to the pupils who were yesterday on the minimum list. For expediency, the back of the assignment sheet used yesterday might be employed to record the names of these pupils. It might be a good plan, as soon after the organization of the class as possible, to reseat the pupils, placing those habitually in the minimum classification at the front of the room where they may be easily watched. Care should be taken, however, not to name the groups in such a way as to embarrass any pupil. The teacher will use tact under all circumstances. Certainly a gain in facility of class management should not be achieved at the loss resulting from humiliating or embarrassing any member of the class.

If the teacher is unable to locate a pupil's particular difficulty on account of illegible figures or general confusion of data, such a pupil may be sent to a side blackboard and there given a private lesson in some of the fundamentals of study. If he seems to have no conception of the problem, let him analyze it for the teacher, telling him what each term is and what it signifies. Make him comprehend the make-up of each term of the expression; ask him why he puts it in a certain place, why he draws the line under it, how he treats signs in adding, etc.

Such individual instruction takes time but is well worth while if thereby some boy or girl is saved from failure. One or two such private lessons like this each day, while the class

[1] Charles E. Merrill Co.

is at work upon the assignment, will serve to keep the teacher pretty busy, and yet the intensified effort will well repay the expenditures of time and trouble. The satisfaction of joy over seeing the weak pupil become strong is a great reward in itself. Such a case is like the physician's. It needs special study, painstaking oversight. But surely the restoration to health and the happy development of a "case" is a deep professional satisfaction.

LESSON VII

UNIT OF INSTRUCTION X. — THE EQUATION AND PROBLEMS

LESSON TYPE. — AN EXPOSITORY AND HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Simple equations.

Method. Give a speed test in reviewing simple equations. Have a large assortment of exercises on the board, or on mimeographed sheets; see to it that the class is provided with paper and pencils; set all at work on the minute. See how many exercises can be worked correctly in some definite length of time, say five minutes. When the time is up, have all stop immediately. Read the answers and ascertain how many attained a mark of 100. Collect all the papers and put the names of those having all correct upon the board. In looking over the papers which fall under 100, the teacher can note just where the trouble lies with the individual pupils and can remedy it the first chance he has.

Such a review, based on the time element, serves to strengthen the pupil's ability to concentrate, puts snap into the work, and lends an element of interest, as all young people like anything that savors of a contest.

There are a number of excellent standardized algebra tests now on the market, which the teacher may use to advantage in this work. These tests were primarily constructed that there might be given an opportunity to teachers to compare the work done in their classes with that in other school systems. Since it is a fact that teachers will differ more or less markedly in their ordinary grading of examination papers and in their judgment of pupils' ability, the employment of tests, which have been used by a large number of teachers and the results of which have been standardized, gives an excellent means of evaluating the work in any class. But aside from this, these tests have other values for the teacher. They point out beyond doubt where weaknesses exist and allow a scientific basis for constructive work. Again, they arouse a vital interest in the results among the pupils tested because they like to know how their progress compares with that of other schools. The avidity with which pupils will strive to raise the standard of the school and of themselves offers the best inducement for intensified work in the classroom.

(*a*) *The Rugg and Clark tests.* These consist of booklets, containing a series of sixteen tests on the various types of algebraic operations from one on collecting terms to one on quadratic equations. These may be given at one time at the completion of the work in algebra, but the author has found them of greater value in checking up his pupils on the completion of each type process and noting wherein the pupils are weak, thus affording an opportunity for his diagnosing each

individual case and permitting him to give more drill in those processes that seem to need it.

(*b*) *The Hotz scales.* These are in the form of sheets covering all the processes in algebra, including problems. The sheet on addition and subtraction gives examples in adding and subtracting terms, expressions, fractions, and radicals. The one on equations and formulas gives examples in simple equations, simultaneous equations, fractional, radical, and quadratic equations, and equations involving the manipulation of formulas. There are other sheets treating multiplication and division, problems, and graphs. These tests are primarily useful in testing a class at the close of the work in algebra, as a means of comparison with standard scores. Used in connection with the *Rugg and Clark tests*, they form a valuable system of accurately and scientifically testing the progress of the class.

The Assignment. — 1. Explanation of algebraic representation.

2. The algebraic equations applied to a concrete problem.
3. Definite rules for studying and solving problems.
4. Analyses of several simple problems.

The Representation of Concrete Things Algebraically. Some such questions as the following lead up very logically to the study of the problem by means of algebraic representation:

1. Express the sum of five and three; of *a* and *b*.
2. Express the difference of five and three; of *a* and *b*.
3. What number increased by three is equal to eight?
4. What number diminished by three is equal to two?
5. How do the last two questions differ from the first two?

The pupils will see that in the last two questions something is lacking which is to be found. Tell them to indicate this

unknown by some letter, as x. Then the third question stated in terms of the known values and the unknown values, will read:

$$x+3=8,$$

and, after solving by transposition of terms,

$$x=5;$$

the fourth question will read:

$$x-3=2,$$

or, after solving,

$$x=5.$$

6. If a pencil costs five cents, what will three cost?

7. If a pencil costs n cents, what will three cost?

8. Express the fact that a tablet costs five cents more than a pencil in both 6 and 7.

9. Express the fact that two pencils and a tablet cost fifteen cents. *Ans.* $2n+5=15$.

10. Solve and find the cost of one pencil. *Ans.* $n=5$.

These questions and others of like nature may be read to and be answered by the class; the result will be that the pupils will gradually sense the fact that by using literal numbers, we are able to represent many things in a manner that we could not do otherwise. When definite values are assigned to letters, so that they will for the moment stand for something concrete, they take on an entirely different meaning. Very strange "$4x$ plus $5x$" may sound to a boy, but when we let x stands for dollars it assumes a very sensible and familiar form. Bring out the fact that the mechanical work preceding the study of problems has aimed at enabling the class to manipulate the resulting algebraic representation of something concrete.

Application of the Equation to a Concrete Problem. Let us take this simple equation, x plus 5 equals 12. Have someone analyze it. It means that 5 added to some number unknown will give us 12. By the law of transposition of terms in equations we solve, and x equals 7.

Now suppose we have this problem: What number increased by 5 will be equal to 12? What are we trying to find? A *certain* number. Then since this is unknown, we will for the moment let x stand for it or equal it. How do we represent increased value? By adding. Then how may we indicate the expression "number increased by 5"? Since x stands for the number, it will be $x+5$. But according to the remainder of the statement, it is equal to 12; then $x+5=12$. And solving, we find that x equals 7, or what we wanted to find.

In like analytical manner take up several similar problems, such as: What number diminished by or increased by or exceeded by, etc., equals something? Lead the pupil in each case to see, through a prior arithmetical representation if necessary, the algebraic representation of the same.

Definite Directions for Solving Problems. At this point either give the pupils the following definite steps, previously mimeographed, or have them written upon the board and copied by the pupils.

a. Read the problem very carefully; study it until you know its every meaning. Close the book to see whether you can state it to yourself, silently. Then reopen the book to see whether you were right.

b. Decide what is the thing wanted and represent it by x.

c. If more than one unknown is involved, represent them by some other letters.

d. Express in algebraic language each of the conditions mentioned in the problem.

e. Make an equation of the two statements that express the same conditions.

f. Solve for the unknown.

g. There should be as many equations as there are unknown quantities.

Illustration of the Directions. Given this problem: What number diminished by 8 is equal to 12?

By *a*: We mentally analyze this to be: What number is there which will be equal to 12 or become 12 after 8 has been taken away?

By *b*: *Number* is the thing wanted; therefore let x equal the number.

By *c*: Only one unknown is implied in this problem.

By *d*: x *minus* 8, and 12 are two expressions concerning the unknown.

By *e*: They are equal, therefore,

$$x-8=12.$$

By *f*: $x=20$, or what was desired.

Analyses of Several Problems. The teacher, through different pupils, will then work out in similar analytical form, the analyses of several related problems. Insist on the above mentioned steps being followed each time; pupils must be taught *how to study* problems and not merely *how to solve* them. The pith of the whole thing lies in the ability of the pupil to read the problem intelligently and to understand it so thoroughly that he can tell it in his own words. Pupils are apt to commence work before they fully comprehend what is given and what is wanted.

The Study of the Assignment. — *I or Minimum Assignment.* Exercises 1–16, in text. (All of these should have been analyzed in class but *not* worked.)

II or Average Assignment. Exercises 17–21, in text. (These have not been analyzed.)

III or Maximum Assignment. Exercises 26–30, page 176, Vosburgh and Gentleman's Junior High School Mathematics, Second Course.

After this preliminary lesson on how to study problems, all of which should be very similar and not too difficult, it is advisable to take up problems in the following manner:

Have each day a typical problem on the board; as soon as the class assembles, let all read it over carefully and study it for a few minutes. Then call on someone to state what is wanted, someone else to state the expressions, someone to make the equation, and someone to solve it. Call on a number of different members to explain various phases of it, making the problem an object of class study and analysis. If the problem is simple in principle, it is often found profitable to have someone make up a similar problem. This method of having the pupil make his own problem and then solve it will be found an excellent means of getting him interested in this kind of work. Before the end of the year the problems which pupils will make up by themselves will astonish the most experienced teacher. Data may be supplied from current events, such as elections, ball games, business statistics, etc.

The author cannot recommend too highly this method of spending each day a few minutes on a problem and then proceeding with the regular work. It serves to keep the principles of solving problems ever before the class rather than for short, intermittent periods. Pupils do not tire of them but will really look forward to this phase of the day's work. Solving problems becomes a habit and what is quite generally considered the hardest feature of algebra loses this aspect, because the children have become so used to problem solving that it has become "second nature." Occasionally, an advance

lesson may be given on problems only; but the above plan has been found, after careful trial, to cover their treatment adequately and well.

LESSON VIII

UNIT OF INSTRUCTION VII. — FACTORING

LESSON TYPE. — A SOCIALIZED LESSON

Program or Time Schedule

The Review	30 minutes
The Assignment	30 minutes

The Review. — *Subject Matter.* Factoring; all cases.

Method. Have the boy and girl, who received the highest mark on the last grade card in algebra, choose sides. When this has been done, the lines should be placed as in the old fashioned "spelling-down bee." Commencing with the leaders and taking them alternately from the two sides, the teacher sends the pupils in turn to the board to work an exercise in factoring. Excellent material will be found in the numerous textbooks on the teacher's desk. If the exercise is worked correctly, the pupil returns to his place in the line and one from the opposite side goes to the board. If a pupil misses an exercise, he must take his seat and during the remainder of the contest work out all the exercises on paper and hand them in later to the teacher.

In this way a large number and variety of exercises may be worked, and the pupils be tested for their skill in recognizing correct answers, for it is evident that the teacher should not take too active a part in reviews of this kind. The pupils know that they are expected to pass judgment on what is worked at the board. If serious confusion results, the teacher is resorted to as judge of the court of appeals.

When one side wins, *i.e.* has *factored down* its opponent, the assignment is taken up.

The Assignment. — The following miscellaneous exercises in factoring, taken from various texts, will have been written upon the board. The first eleven will illustrate the different type forms of factoring, and a question or remark is set opposite each to direct the pupil in analyzing it.

Factor:

1.	$3x^3-3x$.	(Look out for a common factor.)
2.	a^3-1.	(Difference of what? Rule?)
3.	$x^2+17x+72$.	(What type form have you here?)
4.	$ac-ax-4bc+4bx$.	(Be careful in your grouping.)
5.	$(x+a)^2-(x-a)^2$.	(The parentheses indicate what?)
6.	$4a^2+4ab+b^2$.	(Is this a perfect square?)
7.	m^5+n^5.	(Note the powers; odd or even?)
8.	$a^4+b^4+a^2b^2$.	(How may this be made a perfect square?)
9.	x^3-7x+6.	(As a last resort, use factor theorem.)
10.	r^6-s^6.	(How may such an example be best treated?)
11.	$x^{3n}-a^{3b}$.	(Keep in mind what $3n$ and $3b$ are.)

Then give the pupils an equal number of examples illustrating all the various phases of factoring but in a different order, of course, and with no remarks. Tell the pupils to indicate, in addition to the answer, the type form which each illustrates, as:

$a^3+b^3=(a+b)(a^2-ab+b^2)$. *Ans.* Sum of cubes.
$x^{n+1}+x=x(x^n+1)$. *Ans.* Common term.

Supply the pupils with copies of Wheeler's Algebra and refer them to page 70. Tell them to select all the exercises that illustrate some type form which is mentioned, such as

the difference of squares, and note them on their papers without working them. For example, Nos. 3, 6, 10, etc.

Then make a list of all the examples which illustrate some other case in factoring, and so on, covering all the principal cases. This may be carried out to any degree the teacher wishes, the idea being to acquaint the pupils thoroughly with the different type forms, to practice judgment in associating the example with its type form, and therefore in selecting the method which must be applied for its solution.

LESSON IX

UNIT OF INSTRUCTION IX. — FRACTIONS

LESSON TYPE. — A DEDUCTIVE AND HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Multiplication of fractions.

Method. Write a number of exercises on the board, illustrating the previous lesson on fractions. Send two pupils to the board, telling the others to work the examples on their papers. Then let the two at the board work the same example simultaneously. The one who solves it first may take his seat. As soon as the second pupil has solved it, the teacher sends a third pupil to the board to work the next exercise with him. In this way the second pupil, who had trouble, gets additional drill. If this method is continued the poorer ones will remain the longest and therefore get the most practice. Meanwhile all the others are busy. The pupils are expected

to solve the problems as rapidly as possible, the teacher during the meantime helping those at the seats who are experiencing difficulty. To vary this method, it is sometimes well to allow the one remaining at the board to choose his next opponent.

The Assignment. — 1. Definitions of complex fractions.

2. Directions for their solution.

Complex Fractions Defined. A fraction containing one or more fractions in the numerator or denominator, is called a complex fraction. For example:

$$\frac{\frac{x}{y}}{\frac{a}{b}}.$$

In other words, it means that the quotient obtained by dividing x by y is divided by the quotient obtained by dividing a by b. It may be set down like this:

$$\frac{x}{y} \div \frac{a}{b},$$

and it then becomes similar to fractions in to-day's lesson. But sometimes the numerator of the complex fraction may itself be a mixed number or a series of fractions or another complex fraction, in which case it becomes necessary to follow out certain definite directions. These are:

a. Simplify the numerator.

b. Simplify the denominator.

c. Divide the first result or quotient by the second.

Illustration:

$$\frac{x+\frac{y}{3}}{a+\frac{b}{3}}.$$

By a: $$x+\frac{y}{3}=\frac{3x+y}{3}.$$

By b: $$a+\frac{b}{3}=\frac{3a+b}{3}.$$

By c: $$\frac{3x+y}{3}\div\frac{3a+b}{3} \text{ or } \frac{3x+y}{3}+\frac{3}{3a+b}=\frac{3x+y}{3a+b}.$$ *Ans.*

The Study of the Assignment. — *I or Minimum Assignment.* Exercises 2–11, in text.

II or Average Assignment. Exercises 12–17, in text.

III or Maximum Assignment. Make up and solve five complex fractions.

The Silent Study. — The three steps as outlined above should be written upon the board with colored crayon so that the pupils may have them plainly in view. If they will carefully follow out each step as applied to each exercise, the class will have no difficulty. If they do have trouble, it will be from carelessness. Most of the difficulty in fractions comes from the pupil's own illegible figures. The exercises on account of the awkward shape of their graphic representation are easily confused unless care be taken to set them down in good form and adhere to logical order in their solution. The teacher, by passing around among the pupils, will be able to note any such errors and he should avail himself of the opportunity to correct them.

In case someone has difficulty and calls upon the teacher for directions, unless the difficulty is easily found, it will be better for him to start anew, with the teacher overseeing that application is made of the three successive steps. If any help is given, it should be only to direct properly the application of these rules to the exercises under consideration.

LESSON X

A Red Letter Day Program in the Nature of a Field Day

I. Parade. — Each pupil in the class is to be assigned some rule covered in algebra during the first term, which he is to recite upon being called to the front of the room. For instance, the teacher, or *judge* of the parade as he might be designated, will announce: *To add two algebraic numbers.* The pupil who has been assigned this rule will come forward and answer: "If they have like signs, add the absolute values and prefix the common sign; if they have unlike signs, find the difference of the absolute values and prefix the sign of the numerically greater." (Milne.)

When all have been called on, the judge might award a prize, of no intrinsic value, to the pupil who made the best appearance and recited the rule in the most distinct voice.

II. Races. — (Select three qualified pupils to act as judges.)

1. *Multiplication Race.*

Method. Have two exercises in multiplication, exactly alike, upon the board. Send two pupils to work on them; the one getting his done first and correctly wins.

2. *Division Race.*

Method. Similar to above, but using different pupils.

3. *Championship Race in a Multiplication and Division Contest.*

Method. Give each of the winners in the first two races the same exercise, which will be a combination of multiplication and division. The one solving it correctly is considered the champion, and if deemed advisable may be awarded some prize, such as a colored ribbon.

4. *Grand Relay Race in Removing Parentheses.*

Method. Put two exercises involving the removal of a number of signs of aggregation, upon the board. Pick out a relay team of at least as many pupils for a side as there will be complete operations. Start one from each side at the same instant and as soon as one operation is complete, let the next in order take his place. The side that first gets the exercise done correctly wins. It is suggested that two pupils be allowed to choose sides for this, members going to the board in the order chosen.

III. Game of Factoring (modeled after baseball).

Method. Select two teams of nine pupils each, preferably of pupils who have not taken part yet in the program, except the parade. These again may be chosen as noted above for the relay race. Also select an umpire. Each team will be composed of a pitcher, catcher, etc., as in a ball game. These positions will probably be best selected or assigned by the teacher, who will also explain the duties of the players and the rules of the game.

The *pitcher* will read the exercises in factoring which will have been handed to him by the teacher.

The *catcher* will try to tell the type form of the exercise before the batter can do so.

The *batter* will tell the type form of the exercise as soon as he can. For instance, if the example is: $a^2+2ab+b^2$, he will say: " a perfect square."

Each *baseman* and *fielder* will have been assigned some type form and his duty will be to solve the exercise by its application as soon as the batter refers it to him. For instance, the *shortstop* may have been assigned " a perfect square " as his position, so that as soon as the batter gave the exercise this classification,

the shortstop will solve it. If he correctly solves it, the batter is out; if he cannot solve it, the batter makes a home run.

If, however, the catcher gives the correct form before the batter does, and the fielder can solve it correctly the batter is also out; if the fielder fails in this case, it is called a strike and the batter has another chance. Three such strikes will put him out.

Again, if the batter gives the wrong type form, it counts a strike. Failure to understand the exercise at the first reading constitutes a foul; the first two count as strikes, as in baseball.

The game may be varied as to number of innings, according to the length of time that is available, but probably three will suffice for this program.

As already mentioned, there should be an umpire to call strikes, fouls, etc. The teacher himself may act as referee in case of dispute. One or two score keepers may also be selected.

IV. Picnic. — *Method.* Each of the following typical exercises in fractions may be considered to represent different articles of food, and the ability to solve them correctly will give the pupil a helping of each kind. Inability to solve the last one, for instance, which represents ice cream, would deprive him of this dish. All pupils are given paper and told to solve the exercises which are placed upon the board. In parentheses is indicated what each exercise represents. The picnic may be held after school, if the period is not long enough, which probably will be the case. It would also be difficult to determine the amount of food needed before then.

EXERCISES

Simplify: $\frac{1}{s}+1+\frac{2s}{1+s}-2.$ (Sandwiches)

Simplify: $\frac{c-4}{c+2}\times\frac{4-c^2}{16-c^2}.$ (Cabbage salad)

Simplify: $\frac{d^3+27}{d^3-27}\div\frac{d+3}{d^2+3d+9}.$ (Doughnuts)

Simplify: $\frac{\frac{l+s}{ho}}{\frac{l^2+s^2}{h^2o}}.$ (Lemonade)

Simplify: $\frac{c^2-c-12}{3c^2-9c-54}.$ (Cake)

Reduce to mixed number: $\frac{i^2-6i+15}{i-1}.$ (Ice cream)

LESSON XI

UNIT OF INSTRUCTION XIV. — RADICALS

LESSON TYPE. — A SOCIALIZED REVIEW

Program or Time Schedule

The Review . . . . . 60 minutes

The Review. — *Subject Matter.* Reduction of radicals to the same order.

Method. The idea of an occasional socialized lesson is to keep up the interest, throw the responsibility of failure upon the pupils, and impress them with the fact that algebra may be treated from the standpoint of practical application to the world's work.

The class is divided into two groups, one of which will constitute a sort of court and the remainder the witnesses. The teacher will select a judge, a lawyer, a jury of three and a court crier. They may be chosen on the basis of scholarship, the best pupil being judge, etc.

The judge will take the teacher's chair. His business is to determine the fairness of procedure and to see that the rule of reducing radicals to the same order is carefully carried out. The lawyer is to question the witness on the exercise under consideration. The jurors are to decide on the correctness of the solution as given by the witnesses. The court crier is to assign the exercises to the various members of the class. The remaining pupils in the class constitute the witnesses who are to testify to their ability to solve the exercises which are under review.

Procedure. The teacher swears in each officer by demanding his duty, thus giving all officials the opportunity to display their absolute grasp of the principles involved in the work.

Answers to the following or similar questions might be required —

Of the *judge:*

"State the rule for reduction of radicals to the same order."

"Will you allow the lawyer to ask questions which might be misleading?"

"Will your attitude toward the witness be austere or sympathetic?"

"Will you keep order in the court room?"

Of the *lawyer:*

"What is your duty?"

"Will you ask helpful questions or try to confuse the witness?"

"Will you act on the principle that the witness is innocent of

the charge that he cannot solve the exercise until he is proven guilty?"

Of the *jurors:*

" Will you promise to render a fair decision as you understand the rules of algebra relating to this subject?"

Of the *court crier:*

" Will you state the exercise in a clear and distinct voice?"
" Will you be impartial in your assigning of exercises?"

If all the above questions are answered satisfactorily, the officers take their respective stations and the court is declared open by the teacher and the program turned over to the judge, always subject, however, to recall by the teacher.

The court crier calls on some pupil, who goes to the board and is given this exercise:

Which is greater, $\sqrt{3}$ or $\sqrt[3]{6}$?

The witness then solves the exercise. The lawyer may ask him any question on his work or the method of solution, the object being to ascertain whether said witness thoroughly understands the exercise and is able to acquit himself of any inference as to guilt of the lack of such knowledge. The lawyer, for instance, might ask him why he cannot compare them as they stand, or what is the significance of the small figure 3.

When the questions and the solution are completed, the jury passes judgment on the pupil's work with the word " correct " or " incorrect " as the case may be If the verdict is " correct," another witness is called and the proceedings repeated. If, however, the verdict is " incorrect," the judge must pass sentence. He may order a new trial immediately or he may impose some penalty, such as isolation in some part

of the room to work out at his seat some extra exercises selected by the teacher. When all the class have been examined in turn, the failing pupil, or pupils, may be given a new trial and thus offered an opportunity of redeeming his standing in the class. If he again proves unable to clear himself of the charge of " guilty of error," the judge may appoint someone to give him individual aid. The court may remain open until all the class have been cleared of any imputation of lack of ability to handle these exercises, or it may be terminated at any time by the teacher and court declared adjourned *sine die.*

Many changes will occur to the teacher who tries out this plan, but the author believes that, aside from the added interest given to the solution of these exercises, the pupils are meanwhile unconsciously learning invaluable lessons in court procedure and social responsibility as well as lessons in self-expression and self-control.

It is sometimes advisable to devote the entire period to this kind of work. Hence the assignment of exercises for further study will be given in one assortment only and it will be done outside of class and handed in the next day. In this case, twenty-five exercises to review further this type of problems may be assigned, taken from some supplementary textbook. In order to carry out the idea of the three assignments, the first ten might be considered a minimum number to be solved, the first fifteen the average number and the entire twenty-five the maximum quota.

The Study of the Assignment. — Twenty-five similar exercises taken from Ford and Ammerman's First Course in Algebra,[1] page 256.

[1] The Macmillan Company.

LESSON XII

UNIT OF INSTRUCTION XV. — QUADRATIC EQUATIONS

LESSON TYPE. — AN EXPOSITORY AND HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Quadratic equations solved by factoring.

Method. Have an example of this type written on each panel of the blackboard and send the minimum workers, as developed from the previous day's work, to solve them. Also place over them as monitors the maximum pupils, admonishing them not to solve the exercises, but to see that they are worked correctly. The teacher may meanwhile ask the other pupils necessary questions concerning such exercises. As soon as the exercises on the board have been completed and pronounced correct by the monitors, take up the new work, the work on the board being left for future inspection.

The Assignment. — 1. Explanation that all quadratic equations cannot be easily factored and that even those which can be factored may also be solved by other methods.

2. Exposition of the method of completing the square.

3. Statement and explanation of the rule for completing the square.

Other Methods Sometimes Necessary. Write a quadratic equation upon the board which cannot be factored as it stands. For example:

$$x^2 - 2x - 6 = 0.$$

Method of Completing the Square. Take this example:

$$x^2-x-2=0.$$

It is in a form that can be factored at once into $(x-2)(x+1)$; but let us solve it as though it could not be so factored.

a. First, transpose terms so that the unknowns will be on one side and the knowns on the other side of the equation.

$$x^2-x=2.$$

b. Then, unless, as in this case, the coefficient of the unknown to the second degree is unity, divide the equation through by the coefficient so that it will be unity.

c. Now take one half the coefficient of the unknown to the first degree, in this case 1, and square it, *i.e.* $\frac{1}{2}$ squared becomes $\frac{1}{4}$.

d. Add this to both sides of the equation, as

$$x^2-x+\tfrac{1}{4}=2+\tfrac{1}{4}=\tfrac{9}{4}.$$

e. Extract the square root of both members, and we have

$$x-\tfrac{1}{2}=\pm\tfrac{3}{2}.$$

f. Solve for x; $x=2$ or -1. *Ans.*

g. Verify by substitution, $4-2-2=0$, or $4=4$, and $1+1-2=0$, or $2=2$.

Statement of Rule for Completing the Square. This is already given above, but divided into separate steps; it may now be given entirely.

Take a similar exercise and have someone tell what should be done, following each step by the directions above. Repeat until the class understands this new lesson.

The Study of the Assignment. — *I or Minimum Assignment.* Exercises 3–20.

II or Average Assignment. Exercises 21–25.

III or Maximum Assignment. Exercises 18, 19, 21, on page 269 of Ford and Ammerman's First Course in Algebra.[1]

The Silent Study. — Tell the pupils to apply each step as in the above set of rules, and to letter the result of each step with its corresponding letter. Their solutions should appear as follows:

EXERCISE: $2x^2+3x-27=0$.

a. $2x^2+3x=27$. (Transposition)

b. $x^2+\frac{3x}{2}=\frac{27}{2}$. (Division by coefficient)

c. $\frac{1}{2}$ of $\frac{3}{2}=\frac{3}{4}$.
$(\frac{3}{4})^2=\frac{9}{16}$. (Squaring)

d. $x^2+\frac{3x}{2}+\frac{9}{16}=\frac{27}{2}+\frac{9}{16}=\frac{216}{16}+\frac{9}{16}$. (Addition)

$x^2+\frac{3x}{2}+\frac{9}{16}=\frac{225}{16}$.

e. $x+\frac{3}{4}=\pm\frac{15}{4}$. (Extracting square root)

f. $x=\frac{12}{4}$, or $-\frac{18}{4}$. (Solving for x)
$x=3$, or $-4\frac{1}{2}$. *Ans.*

g. $2(3)^2+3(3)=27$.
$18+9=27$. (Verification)

If the pupil has trouble and it proves to be a matter of committing small errors of computation, let him go over his work again and carefully check each step.

If, on the other hand, the pupil seems to have failed to grasp the principles involved and to be unable to apply the steps as suggested, let him go to the board and work there under the teacher's supervision. Let him read the rule over carefully and tell just what each step means. Let him study

[1] The Macmillan Company.

it until he does understand thoroughly just what it implies. A little direction from the teacher should relieve any misunderstanding of the method.

If the pupil still fails to grasp the procedure, the teacher should solve the problem, explaining in detail each step as he proceeds. Then erase and have the pupil go over the same process, step by step. Many pupils fail to grasp thoroughly directions when given to a class at large, either because of inattention or failure to comprehend each step as it is developed. This individual supervision will usually bring the desired results. When the pupil finds that you are not going to work his problem for him but will direct *him* how to do it, he will increase his efforts and try to master the technic.

LESSON XIII

TESTS

LESSON TYPE. — AN EXAMINATION

Program or Time Schedule

Examination . . . . . 60 minutes

Real Tests. — The daily review is in fact a continuous test of the ability of the pupil to do his work. As has already been emphasized, the pupil's outside work is important only in so far as it prepares him to do something similar again. After a pupil has worked thirty problems in addition of fractions, and handed them in to the teacher, and reported no difficulties, the teacher is certainly within his rights when he expects that pupil to be able to do similar examples under his observation in class. If he cannot, then his outside work is valueless and can be safely assumed not to be his own. The pupil should be made to realize that his assigned work is to make him pro-

ficient in a certain line and he must feel that the real test of his proficiency is not measured through his practice but through his ability to repeat the process. The *review*, therefore, while being a part of the recitation participated in by all, should be especially directed toward the weaker pupils.

Then *the ability to understand the new work from day to day* and to follow directions and get results is another real test of the pupil's advancement. It takes no written examination to demonstrate that a pupil is failing, when he continually comes to class unprepared and is unable to comprehend the new work as it is developed. In fact, the rapidity with which some grasp the new topic marks them specifically as unusual pupils in that line.

Problems are in many ways real tests of the pupil's ability. The very statement of the problem in algebraic terms is indicative of logical reasoning; after this process becomes familiar, then the ability to solve problems so stated is further indicative of the pupil's mentality and mental growth. These processes characterize the whole field of this subject and readily and accurately measure the pupil's ability. Some pupils claim they can never master applied problems, and some teachers display a sympathetic attitude to this position by stating that, except for problems, their pupils could pass the final examination. While it is a fact that this is true in many cases, yet this is no reason why problems should be slighted; in fact, it seems to the author almost an unanswerable argument that problems *should* comprise the major part of the final examination. But there are problems and problems; all preposterous, ultra complex, and catch problems should, of course, be omitted. They should be fairly simple, fully reasonable, and straightforward in their applications. Some

texts, such as Ford and Ammerman,[1] have exceedingly well-selected, practical problems in accord with the best modern conceptions of instruction in algebra. The ability to handle such problems constitutes an almost ideal test of the pupils' knowledge of algebra.

Besides these three daily records which themselves constitute *real tests* of the ability of the pupil to grasp the subject, there remains *the formal written examination* and its treatment.

Written Tests. — 1. *Object.* The object of the written examination is twofold: (*a*) to find the points the pupils do not understand, in order to drill on them, and (*b*) to obtain a definite grade or per cent. The former is the really valuable thing; the latter is immaterial.

2. *Testing for Weak Points.* The teacher wishes to know definitely what points the class has mastered and what points have mastered the class. He is interested in the former only from an academic standpoint, but in the latter he is vitally interested because it is his business not only to diagnose but to treat and cure.

Hence, after each unit of instruction has been completed, a review test is given to find out what the class has failed to grasp. These questions should be in the nature of drill exercises, similar in nature to those already studied and given solely for the reason stated above. The writer has found it expedient and trustworthy to use some standardized test in which the results may be compared with a standard of excellency, rather than to use exercises of his own making. Rugg's tests[2] are exceedingly good for this work. There are others on the market, mentioned in Lesson VII. When the class

[1] "First and Second Course in Algebra"; The Macmillan Company, 1919.

[2] University of Chicago Press.

fails to come up to the standard set in these tests, the pupils who fall below should be noted and special attention given to them on the principles in which they are found to be weak. Meanwhile those who have measured up to standard should be given more advanced work along the same line.

The principle of the minimum-average-maximum classification should ever be kept in mind, and the teacher should strive not only to get all to qualify on the minimum requirements but also should assist those who are capable to master more difficult applications and get higher grades. Teachers as well as pupils are very apt to be satisfied with the former.

3. *Testing for a Final Grade.* The ideal method of giving grades is the average class mark. The written final examination, however, is bound to be with us for some time to come, in at least some modified form. It should be along the line of questions which test reasoning ability and not the memory alone. As already noted, problems offer the most ideal method of formally testing the ability to "do algebra," but questions of other kinds may be asked which will approximate, at least, the standard sought in the above statement. A sample examination of such nature follows, by way of illustration or suggestion, covering the work of the first term.

A SUGGESTED EXAMINATION

DIRECTIONS: Answer all the questions under Part I. Necessary to pass, 3 correct from Part I and 2 correct from Part II. If all of Part I and Part II are correct, the grade will be 85%. If, in addition, 2 from Part III are correct, the pupil will receive honors.

Part I

(*Answer all the questions*)

1. Explain what you understand by positive and negative numbers, using some illustration, such as the thermometer, a bank account, etc. Give the rules and explanations of how to add, subtract, multiply, and divide signed terms.

2. Make up and solve an example involving the removal of parentheses, containing at least twelve terms and at least three groups under signs of aggregation.

3. Divide x^5+y^5 by $x+y$ giving reasons for each step, and prove your answer.

4. Solve the following exercise in cancellation, stating what typical case in factoring is exemplified in each step, and explaining the process involved in each step:

$$\frac{a^4-b^4}{a^2-2ab+b^2} \div \frac{a^2+b^2}{a^2-ab}.$$

5. Replace the question marks with values for x and y, and solve the finished example:

$$\left(\frac{?-?}{?+?}\right)\left(\frac{?}{?-\frac{?}{?}}\right) \div \left(\frac{?+?}{(?+?)^2}\right).$$

Part II

(*Answer at least two*)

6. Simplify: $a^3-[a^2-(3-a)-\{2+a^2-(1-a)+a^3\}]$.

7. Factor: (*a*) $1-m^8$; (*b*) $10x^2-7xy+y^2$; (*c*) $a^4+b^2y^2+y^4$.

8. Simplify: $\dfrac{ac-\frac{a^2}{4}}{\frac{c^2}{4}-ac}$.

PART III

9. Expand: $(.3x^2-.4x-1.4)\ (.2x^2+.2x-2.5)$.

10. Divide: $a^{2x+2}-a^{x+1}b^4-2b^{2x}+3b^xc^{x-1}-c^{2x-2}$ by $a^{x+1}+b^x-c^{x-1}$.

11. Factor: a^3-7a+6.

12. Simplify: $\dfrac{1}{a-\dfrac{1}{a-\dfrac{1}{a-x}}} \div \dfrac{1}{a}$.

It will be noted that each part successively comprises exercises of an increasing difficulty, more technical and assuming advanced knowledge and ability on the part of the pupil.

Value of This Form of Examinations. — This type of examinations seems to be a fairer test of the pupils' ability than the conventional one which comprises a series of exercises, any combination of which may be selected and the grade found by marking the paper. In such an examination there is really no testing of the ability of the brighter pupils to do superior work since all the questions are aimed at the average student. On the other hand, there is no minimum requirement for the pupils of lesser ability, which should be demanded of all. In other words, the pupils tested may attempt all and possibly salvage enough to get a passing grade, while their mastery of any one phase may be below standard.

In the type above suggested, it is hoped that these maladjustments may be at least partially eliminated. In the first place, a certain number of exercises must be done correctly in order to receive a passing grade. These may be selected from Parts I and II, which give a choice in each part. If, therefore, the pupil cannot solve five eighths of these without error, he

will fail. If, in addition to these five, he succeeds in getting all of the first part and all of the second part correct, he will get a higher grade, or 85. And if there be anyone who can further work correctly two from the third part, he will receive honors. Thus each type of pupils, whether of minimum, average, or maximum ability, is given an opportunity of receiving higher credit for additional work and work of a more advanced kind. But none can get honors by solving *more* of those of the simpler type or by carefully selecting *all* the easy exercises on the examination paper. So it would seem that this type of examinations definitely tests everyone in the class.

SECOND SECTION

PLANE GEOMETRY

CHAPTER THREE

DIVISIONS OF PLANE GEOMETRY

Practically all texts in Plane Geometry agree on the Euclidian arrangement of material under the FIVE-BOOK system. These, with the introduction, thus become the units of instruction.

A. UNITS OF INSTRUCTION.

I. Introduction.
II. *Book I.* Rectilinear figures.
III. *Book II.* The circle.
IV. *Book III.* Proportion. Similar polygons.
V. *Book IV.* Areas of polygons.
VI. *Book V.* Regular polygons and circles.

B. THE DIVISION OF EACH UNIT INTO UNITS OF RECITATION.

I. INTRODUCTION. The material in this unit will vary in different books but will probably comprise:

Units of Recitation:

1. Definitions.
2. Axioms, postulates, etc.
3. Oral exercises.
4. Historical notes.

II. *Book I.* RECTILINEAR FIGURES.

Units of Recitation:

1. Triangles.
2. Parallel lines.
3. Loci.
4. Quadrilaterals.
5. Polygons.
6. Exercises and problems.

III. *Book II.* Circles.

Units of Recitation:

1. Theorems on the circle.
2. Problems on the circle.
3. Exercises.

IV. *Book III.* Proportion. Similar Polygons.

Units of Recitation:

1. Theorems on proportion.
2. Similar polygons.
3. Exercises and problems.

V. *Book IV.* Areas of Polygons.

Units of Recitation:

1. Areas of equivalent and similar figures.
2. Exercises and problems.

VI. *Book V.* Regular Polygons and Measurement of Circles.

Units of Recitation:

1. Regular polygons.
2. Measurement of the circle.
3. Maxima and minima.
4. Symmetry.
5. Exercises and problems.

LESSON I

THE INSPIRATIONAL PREVIEW

Meaning of Inspirational Preview. — As has already been emphasized in the companion introduction to algebra, the purpose of the preview is to arouse the child's desire to learn geometry. To this end it is important to skillfully advertise the subject. Such advertising should include flashes from the history of geometry, well proved values of its application and

a brief survey of the contents of the course; just enough of each to whet the appetite for more. The pupils are likely to feel that it is going to be dry or hard or futile, but such misconceptions may be quickly removed by a clear preview.

History of Geometry. The word *geometry* — meaning in the Greek language, *to measure land or earth* — indicates that the science developed from the early practice of the modern science of surveying. It is not known with what people the science originated but certainly the Egyptians had acquired a considerable understanding of the subject as is attested by their pyramids, which are built in strictly geometric designs. Recently discovered tablets have proved that also the Babylonians were acquainted with this subject.

But the first practical study of geometry for its own sake was made by the Greeks. Pythagoras, about 560 B.C., discovered many new propositions and added to the popularity and inspired increased study of the subject. Euclid was the first to make a successful attempt to write a book which would contain in an orderly manner all the known proofs, and so well did he do his work that all subsequent texts have been modeled after his book. He lived between 330 and 275 B.C.

So we find that geometry, like algebra, is the combined product of many minds of many ages. Contributions are being made to its content at the present time.

Practical Value. Geometry has a vital connection with many important phases of life. Indeed it is difficult to comprehend how our modern civilization, with its machinery, buildings, bridges, ships, and other marvelous engineering accomplishments, could exist without the contributions which this science has made. Without a knowledge of geometry we would know nothing about the size of the earth, about our

solar system, about the universe as we to-day conceive it. The principles of geometry are used by engineers in constructing bridges, trestle work of all kinds, arches, etc. The employment of formulas developed through use of geometry is universal in the application of mathematical knowledge to all kinds of mechanical construction. Such structures as the Brooklyn Bridge, the Eiffel Tower, the Capitol at Washington, the Ferris Wheel, the Roosevelt Dam, and countless others, are all the result of the application of geometric principles to practical engineering accomplishments.

Again, geometry is made use of in designing mosaics, vaultings, tile patterns, church windows, parquet flooring, steel ceilings, oilcloth, iron grilles, embroidery, lace work, etc.

Although a study of mensuration begins in arithmetic, it is nevertheless only fair to say that its derivation is purely geometrical, and this science should be credited for its great contribution to this practical aspect of mathematics.

If we believe in formal discipline, then geometry certainly deserves much credit for its contributions along this line. Plato believed explicitly in the mental value of this subject and it is said that he had a sign over his school of philosophy, reading, "He who knows not geometry may not enter here." Abraham Lincoln studied geometry to cultivate a logical mind. Geometry is practically the only subject in the school program which gives practice in the use of pure deductive logic. The concentration of mind and the method of logical steps required to prove original problems in geometry combine to give one of the best mental exercises offered by any subject in school. The pupil learns to think clearly, logically, concisely, along mathematical lines. Its study offers more possibilities for correct and incisive thinking than any other branch of

mathematics. If more of the method of geometrical proof were applied in situations demanding clear-cut thinking, it is not unlikely that many of the present-day issues would be better understood.

A Bird's-eye View of the Course. Call the attention of the pupils to the fact that they have already had some geometry in arithmetic since they have studied about areas of plane and solid figures, such as rectangles, triangles, circles, cylinders, pyramids, spheres, and cones. In algebra, also, some of the facts of geometry have been employed in data stated in the problems. But now we come to a study of these truths from a new point of view, that of their value in themselves and not so much their application to the affairs of the world.

Note with the class the division of the text into so-called books or chapters, each book devoted to some particular phase of the subject. Explain that these are *the books* of Euclid, and while they are now collected in *one textbook*, they still retain the old classification. Call the pupils' attention to the numerous drawings and devote a few words to an explanation of the value of neat, accurate figures.

In conclusion and for a review of the above introductory work, put the following questions upon the board and require the answers to be written out and handed in the following day. Those given below are only suggestive; better ones will no doubt occur to the teacher.

1. The first syllable of geometry, or *ge*, means earth. Can you think of any other science which begins with *ge*? If so, of what is it the study?

2. What natural phenomenon occurring regularly in Egypt caused the early development of surveying?

3. Find out, by reference to other texts in geometry, some other men than Pythagoras and Euclid who contributed to the development of this subject.

4. State the Pythagorean theorem which you studied in arithmetic.

5. How does geometry help us to a knowledge of the size of the earth? Of what value is this knowledge to us?

6. Mention some decorative design you have noticed which is composed of geometric figures.

7. What do you understand by *deductive* reasoning?

8. You have learned in your arithmetic that the area of a rectangle is the product of the length and the width. How did the Egyptians make use of this rule? Has it any value in modern outdoor sports?

9. Into how many books is the work in geometry divided?

10. Why must we have neatly drawn figures? Are they more necessary in geometry than in arithmetic? Why?

LESSON II

UNIT OF INSTRUCTION II. — *Book I*
RECTILINEAR FIGURES

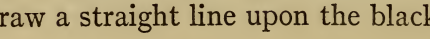
LESSON TYPE. — A DEDUCTIVE AND HOW TO STUDY LESSON

Program or Time Schedule

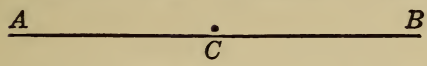
The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Properties of angles.

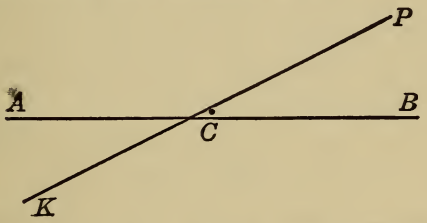
Method. Draw a straight line upon the blackboard, as



Ask what a straight line is. Designate a point by C, as



Ask what an angle is. Have someone go to the board and draw a number of angles, lettering each by ACB in which C is the vertex. Ask what kind of angle the above ACB is. What is the value in degrees of such an angle? Draw a line to C from some other point outside, as P.



What kind of angles are ACP and BCP? What is the value of the two?

Extend PC through C to K. What kind of angles are PCA and KCA? KCA and KCB?

What kind of line is PK? What kind of angle is PCK? Then what is the value in degrees of angle PCK? of PCA plus ACK? of PCB plus BCK? of ACK plus KCB? What kind of angles are they? If angle PCA is equal to 100°, what is angle ACK equal to? Repeat with the various combinations, giving them different values.

When the class has grasped the above, take up the assignment.

The Assignment. — 1. Statement of the problem.

2. Deduction of its proof.
3. Explanation of steps in proving a geometric proposition.
4. Rules to govern the study of a proposition.

The Statement of the Problem. Explain that " proposition " is a general term for either a theorem or a problem. A theorem is a geometric proposition requiring proof; a problem is a geometric proposition requiring construction. The proposition for to-day's consideration is in the form of a theorem, thus:

Theorem. — ***If two straight lines intersect, the resulting vertical angles are equal.***

Deduction of the Proof. Using the figure already drawn by the teacher and referred to above, ask what vertical angles may be considered. Since there are two groups, we may study either pair, or ACK and PCB.

Ask someone to state all the properties we know about the various angles involved in this figure. Have a pupil write these properties in algebraic form upon the board, as:

I. $PCA+PCB=180°$.
II. $PCA+ACK=180°$.
III. $ACK+KCB=180°$.
IV. $KCB+PCB=180°$.

What peculiar thing is noticeable about these equations? That they are all equal to 180° or the same thing. What can be said about things that are equal to the same thing? That they are equal to each other. Let us see if this will be of any value to us.

What further significance can we note in the first two equations besides the fact that they are both equal to 180°? That each contains the same angle, PCA. What similar observation can be made with reference to the second and third equations? That angle ACK is common. And in the third and fourth that the angle KCB is common. In our

consideration of the above proposition, which of these three common angles might be of interest to us? The class may select *ACK*. We shall take the second and third equations containing this angle, and combine them into a new equation since they are equal to each other:

$$PCA + ACK = ACK + KCB.$$

Have someone state why this is true. How can algebra be of service here?

But since we have a common term on both sides of the equation, what can we do with it? We can subtract this common angle from both sides without changing the value of the equation. Why? Then we have:

$$PCA = KCB.$$

What kind of angles are these? Then we have proved that the two vertical angles are equal. But since these are not the two that we started out to prove equal, we will try another set of equations.

Ask what angles we want left after solving the equation. *ACK* and *PCB*. Then let us take two equations which have these angles and also a common angle. The class will readily select the third and fourth, or

$$ACK + KCB = KCB + PCB. \quad \text{(Why?)}$$
$$ACK = PCB, \quad \text{(Why?)}$$

or what we wished to prove.

Now, draw two other intersecting lines upon the board, and call on someone to designate them by other letters, to find the various equations, to pick out the desired ones and prove the proposition. The pupil will tell what to do and the teacher will write the necessary operations on the board.

Then, have all the class draw two intersecting straight lines upon their papers, and go through the same process themselves, calling you to their aid if they get into difficulty. The pupils will respond to this with avidity because all boys and girls like to achieve things themselves.

Next, have them open their textbooks, which have been closed up to this time, and follow out the similar proof there developed. They will be very pleased to learn that they have already mastered the new lesson, and they have incidentally been given a lesson in how to study.

An Explanation of the Steps to Take in Solving a Geometric Proposition. Explain that every demonstration of a geometric proposition is divided into definite, logical steps, as

a. Statement of the theorem.
b. Drawing of the figure.
c. Stating data given in the theorem.
d. Stating what is given to prove.
e. The proof, consisting of steps and reasons.
f. Conclusion (*Q.E.D.* or *quod erat demonstrandum*).

Rules on How to Study a Proposition. The following set of rules may, with excellent results, be mimeographed and distributed to the class, as was suggested in algebra.

Suggestions For Effective Studying

Theorem. — *a*. Read and reread the theorem very carefully.
b. Note what is given and what is wanted.
c. Review in your mind the properties of all geometric terms occurring therein.
d. Have a blank card, the size of the printed page of your book; call this card *No. 1*. Have another card the length of the page but only one half as wide; call this card *No. 2*. With card

No. 1 cover up all the page except the theorem. Do not uncover any more of the page until you have mastered the above directions.

Figure.—*e.* On blank paper make drawings to conform to the data given in the theorem.

f. Push down the large card to disclose the drawing in the book and compare with yours. Do not put in any auxiliary lines until later; letter your figure.

Data.—*g.* Write on your paper under the head *Data*, the things you have stated concerning the theorem. Again push down the large card to compare. If your statement of data does not agree with the book, note wherein it differs and thoroughly understand it as given by the author before you proceed.

To Prove.—*h.* Repeat this operation with the statement *To Prove.* Under this heading is given the thing desired by the theorem. (See suggestion *b*.)

i. Divide the remainder of your paper into two equal parts by a vertical line. Label the first column *Steps* and the second column *Reasons.*

Proof.—*j.* Slip down your leading card *No. 1* to uncover the *first step*, keeping card *No. 2* over the corresponding *reason.*

k. If this statement incurs auxiliary lines, make them on your figure and compare for correctness with the figure in the book.

l. Try to state a reason why this may be done. Then slip down card *No. 2* to disclose the author's reason and note whether you were right. If not, master the correct reasoning for the operation before proceeding.

m. Repeat with the next step in the demonstration, disclosing first the step and then the reason for it after you have attempted to discover it for yourself.

n. Repeat the process until you have completed the demonstration. You will then have the complete work upon paper, also.

o. Now close your book and after destroying your paper, try to write the complete demonstration upon new paper. When you are unable to proceed, refer to your text, but never memorize the steps, or you will never really understand geometry.

In General.—*p.* Understand each step absolutely before proceeding any further.

q. Take time. Thoroughly studying the theorem once should be enough.

r. Always ask yourself *why* after each step.

s. Number the steps and the reasons to correspond.

t. Master the work from day to day; do not let it master you. If you rely on memorization, you will become the slave instead of the master of this subject.

The Study of the Assignment. — *I or Minimum Assignment.* Proposition I.

II or Average Assignment. Exercises on Proposition I.

III or Maximum Assignment. Prove the theorem informally by referring to Slaught and Lennes' Plane Geometry,[1] pp. 33, 38, 79, 103.

The Silent Study. — Pass from pupil to pupil to see that they are following out the directions as given above for the study of the lesson. Explain that a little practice with this method will soon make it automatic, and the pupil will find that before long he will master a demonstration in a short time.

Insist on carefully drawn figures, neat and clearly lettered. Show the pupils how to make the cards suggested. Each one should be labeled and kept permanently in the book. They may be made of paper if desired; their value lies in keeping from the pupil's observation all except that which is being studied at the moment. Where they have been used, they have given good results. Their use should be encouraged by the teacher until the pupils *realize* their value as an aid to study.

[1] Allyn and Bacon.

LESSON III

UNIT OF INSTRUCTION II. — *Book I*
RECTILINEAR FIGURES

LESSON TYPE. — A DEDUCTIVE LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* **Theorem.** ***If two straight lines intersect, the vertical angles are equal.***

Method. Send two or three pupils, who completed the maximum assignment, to the board to write out the demonstration of the theorem in full. Assign to those completing the average assignment the examples of the text to be worked upon the board. Review the demonstration of the theorem with the others.

Have the figure already drawn upon the front board. Call on someone to state the theorem, someone else to give the data, and so on through the proof. When the demonstration has been completed in this way, have someone stand and go through the whole demonstration orally. After two or three have done this, erase the figure and have someone prove it, carrying the figure in his mind. This is good practice if carefully supervised, for it keeps the minds of all concentrated on the development of the proof. Occasionally break in on the demonstration to call on another to proceed with it. It is a good plan to have the class know that anyone is likely to be called upon at any time; they will then give better attention to what is going on.

After the theorem has been thoroughly reviewed, ask for

the answers to the examples of the assignment, noting those that are correct upon the board. Also have the pupils who completed the maximum assignment tell how the theorem was proved informally according to the references given. We are now ready to take up the new lesson.

The Assignment. — 1. Study of definitions.

2. Explanation of the new theorem.

The Study of Definitions of Triangles. Read the definitions over with the class, showing the pupils how to study them. Tell the pupils that the main points are to master each word as they read it; to follow out all references to figures; to look up the meaning of all the words they do not know, and the proper pronunciation of words they cannot pronounce. Then take up the new proposition.

Explanation of the New Theorem. Follow the plan given in the preceding lesson. As new propositions occur from time to time, much of this work may be shortened as the pupils are able to do more of the deductive work themselves, following out the suggestions of the preceding rules. The teacher should determine, from day to day, what will probably constitute the real difficulties and should clear these up during the assignment period. It is better, however, to leave as much as possible to the silent study period and give those capable of solving the problems unaided a chance to do so. The class demonstration assumes that all are equally unable to study it out for themselves, which is not only *not* a fact but is stultifying to the more capable pupils.

The Study of the Assignment. — *I or Minimum Assignment.* Proposition II on triangles.

II or Average Assignment. Simple exercises on Proposition II.

III or Maximum Assignment. Examples on page 28, Wentworth and Smith's Plane Geometry.[1]

The Silent Study Period. — The above assignment having been placed in advance upon the board, the pupils are ready to begin their silent study of the lesson. Tell them to open their books at the new proposition, and to study it according to the directions given them yesterday. With the large card over all the page except the theorem, they should begin with rule *a*. For a day or two, or until they have acquired the correct method of studying the demonstration as given in the book, it will be best for the teacher to direct orally this study, guiding the pupils to apply correctly these directions. Since one purpose of supervised study is to develop eventually in the pupil a knowledge of how to study, too much emphasis cannot be placed upon this practice. In time, if such study is directed and insisted upon, the pupils will find it unnecessary to rely upon the teacher's guidance and will be able to study the lesson unaided. This should be, of course, the ideal sought for, but it will take more or less time and the acquisition will only come through patience and perseverance.

After the pupils have been given time to digest the stated theorem, ask someone to state what is given and what is wanted. Not until it is clearly understood that two triangles are given, each of which has two sides of one equal to two sides of the other, and the included angle of the one equal to the corresponding included angle of the other, and that we desire to prove that under these conditions the two triangles are equal, are we ready to pass to the third rule. Call on various pupils to state these things. A clear comprehension of these

[1] Ginn and Co.

two elements of every proposition is absolutely necessary before we proceed.

Now ask what we know concerning the properties of the geometric terms involved in this proposition. Bring out that all triangles have three sides; that there are also three angles and that each angle is included between two sides. Such an angle is called an included angle. Why? Ask when geometric figures may be said to be equal. If the pupils are unable to tell, refer them to the axiom. Having now analyzed all of the properties, turn to rule *d*. While they keep the large card over all of the page except the statement of the proposition, they will next draw figures upon their papers to conform to the facts of the theorem. When this has been done, tell them to lower the card to disclose the figure and to compare, lettering it to conform with the lettering of the drawing in the textbook.

Next, tell them to write after the heading *Given* the facts that have been stated in the theorem as data. Before the card is pushed down again, ask someone to tell you what he has written under this head. Discuss it with the class to see if it is complete. Then tell them to uncover this section of the book to compare with the statement of the author.

Repeat the process with the heading *To Prove.* In order to expedite the work, the teacher may discuss each of these points orally rather than have the pupils write them down, possibly incorrectly. The thing desired is the deduction of the various steps, as far as possible, by the pupils themselves without referring to the book, except for verification.

The same procedure may be followed throughout the study of the demonstration. The use of the divided card, as explained in Lesson II, will tend to make the pupil study each

step out himself before referring to the author. This should especially be insisted upon in giving the reasons.

After the first step in the proof has been studied, ask the pupils, preferably individually, to give some reason why this step has been taken. The teacher will thus get all the class to thinking, and they will review mentally all the facts which they have learned up to this time. This is a point which cannot be overemphasized. Ability to solve originals or in fact to do any part of the work in geometry requires a continual revolving in the mind of all our previous knowledge with a view of applying it to the specific case under consideration. The difficulty with originals often results from this very inability or failure to practice.

When the complete demonstration has been studied in this way, the teacher may tell the class to take a sheet of paper and try to rewrite the proof, or better, to *reconstruct* the proof. The only thing which will at all necessitate the use of the memory will be the order of the steps and the continual review mentally of previously learned facts concerning geometric terms.

As the pupils begin this work, the teacher will pass among them to note any special difficulties. Occasionally a leading question will set the pupil aright but definite answers should be avoided; it is our duty to lead the pupil to see his trouble rather than simply to find it for him.

The Study of the Advance Assignments. — As soon as a pupil has mastered the new proposition, he should go to work upon the second assignment. It will be best at first to demand that the answers to these exercises be written out. It will take more time and later may be dispensed with, but at first it will cause the pupil to concentrate his attention more

definitely upon his work. These first exercises are simple and will not take much time or paper, but the pupil by this method will soon acquire the habit of being accurate, comprehensive, and neat.

In case there are any exercises that might involve special difficulty, a leading question concerning it may be written upon the board, numbered to refer definitely to such exercises.

The examples assigned for the maximum assignment may either be written upon the board or the pupil allowed to take the book referred to. In case more than one should reach these examples during the period, the difficulty arising from having only one copy would be eliminated by the use of the board. On the other hand, the actual use of supplementary textbooks by the pupil is an excellent practice and, whenever it is feasible, such additional books should be provided and in as large a variety as possible.

Encourage the pupils who are capable to attempt the maximum assignment but not to the detriment of their other work. Make it clear, too, that it is only to be studied after the mastery of the first and second parts. Its purpose is to keep employed those in the class who are especially capable and thus further to develop their powers. As a rule, these examples should be of a more difficult nature. This method, carried out to its ideal administration, would permit these pupils to proceed with new propositions, and so finish the book ahead of the others; but this involves too much difficulty for the average school to attempt and should only be done under exceptional circumstances. For a further discussion of this important phase of supervised study as relating to higher mathematics, see Chapter Four.

LESSON IV

UNIT OF INSTRUCTION II. — *Book I*
RECTILINEAR FIGURES

LESSON TYPE. — A HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* The first three propositions of *Book I.*

Method. Send three pupils to the board to draw the figures of the propositions which are to be reviewed. While these are being put on the board, quickly review the class on the leading facts already studied in regard to angles, triangles, etc. Such questions as the following are suggestive:

What kind of angles are equal?
State under what two conditions, already studied, triangles are equal.
What method of proof is employed to prove triangles equal?
When is a line perpendicular to another line?
What do you mean by *bisecting a line? an angle?*

The questions asked should always review the knowledge acquired *recently*, and facts that might be useful in studying *new* propositions and originals should be gradually introduced by this means.

The figures having been drawn upon the board, call on some pupil to state the theorem about vertical angles and to prove it, using the figure on the board. See that he omits nothing important. It is best to prompt the one reciting as little as possible, because, if the pupil learns to depend on the teacher

for his approval or disapproval of every statement, he will never be able alone to complete a demonstration. The teacher must avoid aiding him too much; if the instructor breaks in with " why " every time the pupil fails to give a reason, he will early learn to expect it and will not trouble himself to give it unaided. A better way is to stop him when he makes an error and let someone else proceed with the proof. He will soon learn that he must rely on himself, and will exert himself to be thorough. After another pupil has completed the demonstration successfully, tell the one who failed, unless he himself senses it, wherein he made his mistake, and let him try again.

Proved orally, many demonstrations can be covered in the allotted time. As soon as this proposition is given correctly and the pupil who failed has been able without help to give it, call on someone to prove the second proposition, and so on until the subject matter of the review has been covered. It is well to review each day not only the propositions which were the immediate subject of study for that day, but also to review continually others, dwelling especially on those which have given the most difficulty. In cases of complicated figures, let the pupil go to the board and use the pointer, and in the case of many equations being necessary, it will help the pupil to allow him to write them on the board.

If there are some in the class who still have difficulty and are unable to go through a demonstration correctly, entirely alone, let them go to the board during the study period and write it out. This method, however, although much used, is of special value only in the case of pupils who are unable to prove the proposition orally before the class. It ordinarily takes too much time to be of much worth.

When a pupil is demonstrating a proposition, the teacher

must bear in mind that if the time thus spent is not to be wasted by the rest of the class, the pupil must talk loud enough for all to hear him clearly. Let him stand at one side of the figure and talk to the class. The pupil should be made to feel that he is taking their time and they are entitled to all the benefits that may accrue to them from his work.

A few minutes might also be spent in running over the exercises, which are based on the day's lesson. These may be given orally or written upon the board, depending on their nature and their value. When possible, similar exercises with different values may well be given by the teacher; and the pupils may be encouraged to make up others of a like nature.

The Assignment. — The new lesson will be on originals; and since it is the first lesson on these, a few words as to how to study them properly will not be out of place.

Notebooks. The author advises the use of notebooks; one to use in class for original exercises and to be handed over to the teacher at the close of the period; the other for use outside of class for additional exercises which were not done under the immediate observation of the teacher. The former will contain beyond peradventure the pupil's own work; the latter may be assumed to be such and assessed according to the ability of the pupil to work similar examples in class.

The pupils are told to open their textbooks and to read the first exercise. Opening their notebooks, they will draw the figure, state what is given, and what is to be found. Ask various questions, taking pains to see that all understand the data given and the results desired. The questions that you will ask to-day will illustrate how the pupil is later to question himself when studying similar problems. The following illustrations may serve to make the method clear.

Example. Given lines AB and CD bisecting each other at O. Draw straight lines connecting A and C, and D and B. Prove that $\triangle ACO = \triangle OBD$.

Questions. When lines bisect each other, what results? Which segments, then, are made equal in this example?

To prove that triangles are equal, we must try them by three conditions; what are they?

In this figure, what do we know about each triangle?

Does this give us enough knowledge of each to throw them into one of the conditions under which triangles may be proved equal?

Then, since we know that each triangle has two sides and the included angle equal respectively to two sides and the included angle of the other, what must follow?

Is this what we wish to prove?

Solution. Tell the pupils to write out later the entire proof according to the method of the demonstrations in the textbook, using the divided page, with the steps on the left of the line and the reasons on the right.

Example. Given $\angle ABC$ bisected by BY, P is any point in BY, equal lines are dropped from P to the sides of the angle, as PM and PK. Prove that $\triangle BPM = \triangle BPK$.

Questions. First ask the pupils what questions they think they should ask themselves concerning this example. Naturally many questions will come to the mind of the individual pupil which will be found to have no bearing on the problem under consideration, but it will be best to exhaust all possible conditions even at the risk of some matter which is not pertinent. He will learn that in the investigation of any new subject, much of the effort exerted fails to be of consequence, but it is necessary to bring to bear all the known facts so that we may study their relationships.

Explain that, in the first place, the pupils must study the examples by passing over in their minds all possible related facts, and, in the second place, by eliminating all except those that will bear directly on the problem involved. We know such and such a thing. What ought we to know if we solve

the problem? Do these facts help us and how? If not, are there any others that we have omitted that we could possibly use?

In this way analyze five or six examples in to-morrow's assignment, and then require the pupils to write out in full in their notebooks the demonstration of each in the way previously indicated.

The Value of Definite Rules for the Study of Different Phases of the Subject. Give each pupil, on mimeographed sheets if possible, the following rules for attacking and solving original exercises. If he will adhere rigidly and conscientiously to these directions he will be able to solve any original exercise he may meet. And not only that, he will save himself the time that is often spent in aimless study — study that brings to bear no intelligent and directed effort. It is comparable to the way the expert machinist looks for " trouble " and the way the unskilled layman looks for it. The expert does not aimlessly take off nuts, loosen joints, and dislocate couplings; before he touches a thing, he studies his problem. How should the machine function; and, is it so functioning? If not, what might cause such trouble? And so on, until he finally eliminates many improbable or impossible causes and reduces it to something which is probable or possible, and then he goes after that thing. The layman, on the other hand, not studying it out beforehand, will do something here and something there, until the chances are he will add to his trouble instead of removing it. So in solving exercises in geometry, a little careful analyzing of the data, the problems involved, the things known and the things desired, and how one may affect the other, will lead eventually and logically to the correct solution.

SUGGESTIONS FOR STUDYING ORIGINALS

a. Digest every word in the problem.

b. Make the figure carefully and go over it to see that it follows directions.

c. Ask yourself what you know about every word in the data.

d. Ask yourself how this knowledge may be applied to the question under consideration.

e. State carefully what is to be proved.

f. Review mentally under what conditions the proof may be made.

g. Write out the proof in full, giving a reason for each step.

The Study of the Assignment. — *I or Minimum Assignment.* Exercises 1–6, in text.

II or Average Assignment. Exercises 7–9, in text.

III or Maximum Assignment. Exercises 10 and 11, in text.

Method of Manipulating the Notebooks. As noted above, have all the pupils write out in their notebook, or *Notebook A*, all the exercises worked during the class period; work in the other notebook, or *Notebook B*, those done outside of class. At the close of the period, stand at the door and collect the former notebooks. It will take only a few minutes to look them over and check those found to be incorrect. Make a note of these failing pupils on the back of the day's assignment sheet, and take up the unsolved problems with these pupils the next day.

The notebook is easily kept in order, it indicates the pupil's work and progress from day to day and becomes an efficient reminder of poor work. Such remarks as "too slovenly," "don't guess," "figures inaccurate," etc., might well call the attention of the pupil to the reason for his failures. Rubber stamps with such phrases could be used to advantage with a view of saving time and energy. Written work taken up but never returned with criticism is inefficient and does not repay

the teacher for the time he spends looking it over. Papers handed in and simply marked with a grade or per cent are of little help to teacher or pupil. The real benefits are derived only when the errors are noted and the attention of the pupil called to them so that he may avoid making similar mistakes.

The notebooks containing the work done outside of the class period may be looked over during the study of the assignment and errors brought at once to the notice of the pupil. They may be returned to the pupils at the close of the period, or collected only as the teacher is able to look them over. Hence the value of notebooks; they are always ready for inspection, and their administration is very easily effected.

All pupils ought not to be required to do the same amount of original work; all, however, should do the minimum assignment, which should cover all the simpler applications. The more efficient workers should be encouraged to solve more difficult exercises, and so all may be kept up to the limit of their respective capacities.

LESSON V

UNIT OF INSTRUCTION II. — *Book I*
RECTILINEAR FIGURES

LESSON TYPE. — A DEDUCTIVE LESSON

Program or Time Schedule

The Review	30 minutes
The Assignment	5 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Original exercises.

Method. The figures for the exercises which were assigned in yesterday's lesson and which are to be reviewed to-day

should be placed upon the blackboard prior to the assembling of the class. By a survey of the *A* notebooks, handed in at the close of the period the day previous, the teacher will have noted those examples which have given trouble. This may be easily kept in mind by a simple method, that of noting on the back of the assignment sheet, or any piece of paper, the examples by number and beside the numbers the names of the pupils having difficulty with them. The exact trouble might also be indicated. These memoranda might read as follows:

No. 1. O.K.
No. 2. O.K.
No. 3. John Jones (does not understand meaning of midpoint).
No. 4. O.K.
No. 5. Ben Ayers.
No. 6. John Jones.
Ethel Clare.
Mollie Pond.

Assuming, then, that all the pupils solved the first two examples correctly, and that one pupil had trouble with the third, begin with that one. When all have solved a certain exercise, there is no object in giving it any more attention.

Ask John to rise and read the third example, which is as follows:

Given O, the midpoint of the line AB, $CO \perp AB$, P any point in CO. Prove $\triangle APO = \triangle OPB$.

Then tell John to close his book and tell you the example in his own words. Until he can do this he is unable intelligently to consider its solution. Glancing at your notes, you will see that he seemed not to sense the meaning of the word "mid-

point." After he has correctly stated the example, ask him what is given. Then ask him what he understands by the word "midpoint" and upon his failure to tell you, call on someone else to explain its meaning and significance in connection with this example. When John realizes that O divides the line AB into two equal parts or segments, he may be able to proceed. If not, direct him by skillful questioning until he understands just wherein the key to the situation lies. It is probably in the fact that CO being perpendicular to AB makes equal angles at O because they are right angles, and all right angles are equal. John is not told this, of course, but it is brought out either through questioning or is answered by some other pupil. Thus by guiding him to see significant facts concerning the example, which he failed to see by himself, he is being taught how to study and will feel within himself a growing power which will stimulate him to greater efforts.

Now that he has been guided through the solution, tell him to start over and go through it again. This may take longer than the teacher feels ought to be devoted to one pupil, but it should be of value to the rest of the class at the same time, and that a few are directed to find their difficulties and solve the problem that gave them trouble is worth much more than many exercises hastily done and nothing positive accomplished. It is not necessarily the number of exercises that one does which counts but the ability to do them by one's self. It is of more value to a class to conquer a few problems, even if done painstakingly and with toil, than it is to work many which require practically no effort. The teacher may feel well repaid if each day he can help a small number to make a definite advance in the mastery of the subject.

Repeat the above procedure with the fifth exercise, the next one to give trouble, always giving the problem or proposition to the pupil who experienced difficulty with it. Those who had no difficulty may at the same time feel that they also are advancing for they have the assurance that, through their additional work with the maximum assignments and their work as shown by the *B* notebooks, they are acquiring a real grasp of the subject.

When the examples giving trouble have thus been disposed of, if time remains, a few similar examples from other texts may be given to those who successfully completed the minimum and average assignments. If there are some who have in addition completed part or all of the maximum assignment, they might be given one of special difficulty and either sent to the board or told to work at their seats. Excellent supplementary material may be found in Schultze and Sevenoak's Plane Geometry.[1] Special attention is called to the practical applications of geometry in the back of this book.

The Assignment. — 1. Study of the notebooks.

2. Explanation of the assignment.

Directions for Study of the Returned Notebooks. Return the *A* notebooks to the class and explain that you have checked the exercises which have been done correctly with a "C" and that you have marked errors made in those done incorrectly, calling their attention to the thing that caused them trouble. Tell them to look these over first and to note your remarks. If they do not understand the remarks, remind them that they may call you to them and you will explain in more detail. If their work shows no errors, they may proceed at once with the advance assignment.

[1] The Macmillan Company.

Explanation of the Nature of the New Assignment of Exercises. Call the attention of the class to their new assignment upon the board. Explain that the exercises are similar to those of to-day, but that some lack the figures. Call their attention also to the rules you gave them yesterday for studying originals and explain that these should be applied to the study of each item in the assignment.

The Study of the Assignment. — *I or Minimum Assignment.* Exercises 94, 95, 96 (with figure), 97, 98, Schultze and Sevenoak's Plane Geometry, page 26.

II or Average Assignment. Exercises 99, 100, 101, 102 (same source).

III or Maximum Assignment. Exercises 108, 112, and 115 (same source).

The Silent Study. — As soon as all are at work on the new lesson, look over as many of the outside or *B* notebooks as possible, making needed criticisms as before. These errors may be called to the attention of the pupils at once, or at the close of the period. The amount of time that the teacher will be able to devote to this will depend on the amount of time he will be called upon to give to pupils with their new work. But since the pupils will resume the study of propositions to-morrow, it is not necessary to review all the work at this time. The work may be collected to-morrow and checked as soon as convenient. Sometimes the *B* notebook may not be required and the work done outside of class simply handed in on loose paper. This may be found to be a more satisfactory way under some conditions, but the notebook that includes the work done in class is felt by the writer to be very important and the work so preserved is always available for purposes of comparison or review.

LESSON VI

UNIT OF INSTRUCTION II. — *Book I*
RECTILINEAR FIGURES

LESSON TYPE. — SOCIALIZED REVIEW LESSON

Program or Time Schedule

The Review . . . . . 60 minutes

The Review. — *Subject Matter.* All propositions of *Book I.*

Method. Have the various theorems of *Book I* written upon small cards about two by three inches in size. Shuffle these and, after sending as many pupils to the board as you have panels, let each one draw a card and proceed to draw the figure and write out the demonstration. After the pupils have worked awhile, send the one at the extreme left to his seat and request the others to move one panel to the left, and to go on with the demonstration where the other left off. A new pupil is sent to take the place of the one seated and to work the proposition at the extreme right. This process may be repeated at intervals of a few minutes or done only once, depending on the judgment of the teacher. When an exercise is completed, send another pupil with a new card to the board to start a new proposition.

This method of reviewing a number of propositions will entail considerable work on the part of the teacher, but by careful planning it may be very readily administered. Pupils at their seats should in the meantime have been given the same kind of work to do on paper, which may or may not be interchanged from time to time.

Some Advantages of This Kind of Reviews Are:

1. All are working.
2. Each pupil is reviewed on a number of propositions.

3. The pupil's work is under the scrutiny of teacher and pupil.

4. It emphasizes the necessity for neatness, thoroughness, and legibility.

5. It develops judgment on the part of the pupil who takes up the unfinished work as to the correctness of that already done.

6. It socializes the work of all the class; each one is dependent not only on what he has done but on what the preceding pupil has done. Many modifications will occur to different teachers. For instance, it may be better to have the pupils finish their demonstrations and then assign others to look them over and report; if any are incorrect, have these erased and reworked by the pupil, or he may be directed to make the corrections himself.

Again, two pupils may be assigned to each proposition, one to write down the steps and the other to write down the corresponding reasons in the second column. These two may change places on a given signal.

LESSON VII

AN EXHIBITION OR RED LETTER DAY LESSON

Object. The object of a lesson of this nature is threefold:

1. To bring out the practical nature of geometry.
2. To arouse in the class the desire to do superior work.
3. To further interest in the subject.

Place. This type of lesson, although not to be overemphasized, may with profit be given at the close of the work on each book.

Preparation. Like all stated programs, the lesson should be

planned a week or more in advance, the different assignments carefully made, and the purpose well explained, and it should be carried out according to plan.

Method. The program may consist of three parts:

1. Work to be put on display.
2. Papers and oral proofs.
3. An exposition of some particular phase by the teacher, principal, or superintendent.

Explain early in the study of the subject that you are going to have an exhibition at the completion of this book of the best work that has been handed in during the period covered. This may consist of the best notebooks, best construction problems, best designs, etc. Tell the pupils that from time to time you will select the best drawings for special attention, such as having them traced in India ink on high-grade paper. Encourage the pupils to do their work from day to day with this exhibition in mind.

Assign someone to prepare a paper on some geometrician's life or some interesting phase in the history of the subject, or some equally interesting topic. Naturally the display work will be done by the best pupils but encourage all to do their best to get on the program, and emphasize this as being an honor.

The third part of the program will be taken care of by the teacher.

Program. Keep the day of the special program before the class by skillful advertising, so that all will be on the *qui vive* for its approach. The teacher has had it in mind for some time and will have formed a clear outline of just what he wants done. With careful planning, the following program will be found of interest and at the same time instructive:

Part I. Display

1. Two or three of the best notebooks.
2. A few of the best construction problems, drawn on high-grade drawing paper and traced in India ink.
3. A few geometric designs,[1] done by maximum pupils and traced in colored inks.

Part II. Papers and Oral Proofs

4. Paper on Thales, who enunciated the first proposition, as well as many others.
5. A recreation problem[2] proving that every triangle is isosceles, by the pupil with the highest grade.
6. Model demonstration of some proposition in *Book I*, by some pupil, who will give it orally and without a figure.
7. Paper on the use of the protractor for measuring angles.

Part III. An Exposition

8. Talk on the application of the truths learned in *Book I* in the problems of life, as surveying, architecture, and designing.

[1] Excellent material may be found in Sykes' "Source Book of Problems in Geometry"; Allyn and Bacon.

[2] Wentworth and Smith, "Plane Geometry"; Ginn and Co.

THIRD SECTION

ADVANCED MATHEMATICS

CHAPTER FOUR

SPECIAL METHODS OF SUPERVISED STUDY IN HIGHER MATHEMATICS

Intermediate and Advanced Algebra. — The subjects considered in this chapter are usually offered to pupils in the last two years of their high school program. These pupils, through the study of elementary algebra and plane geometry, which it is assumed have been taught on the supervised study plan, should have mastered by this time the technic of *how to study*, and should be able to handle this advanced work without the detailed directions heretofore necessary. Consequently, the inspirational preview and how to study lessons will only rarely be required. The general plan outlined in the preceding lessons may be followed if preferred, but, since the ideal method would be for the pupils to advance individually and as rapidly as they are capable of doing, the suggested lessons herewith given will be based on this plan. With this in mind, and recognizing that pupils electing these courses will be those having more or less marked mathematical ability, we shall naturally expect that some will be able to cover part if not all the subject matter of intermediate and advanced algebra during the semester.

In the same manner and with the same expectation we shall treat solid geometry and plane trigonometry as a unit. The time spent on each course being assumed to be twenty weeks, the scheme will probably work out about as follows: If at the end of the first ten weeks, some have covered at least three

fourths of the work in intermediate algebra or the same amount in solid geometry, they will be expected to complete both courses by the end of the term. Those who have not advanced so far will finish the original course and spend the remaining time in review. This plan provides, therefore, for each pupil to proceed as rapidly as he is capable of doing and yet with no detriment to himself if he finds he is unable to complete both courses. The work will thus resolve itself largely into individual instruction, with a minimum of class demonstration.

Method. Since a large part of intermediate algebra is a review of elementary algebra, and since some pupils will need but little review and drill while others will require more, after a preliminary explanation of the plan for individual advancement, a test or series of tests covering the work of the elementary course should be given to enable the teacher to judge just what preliminary or review study is necessary. This will naturally vary with the individual pupils and therefore resolve itself at once into personal direction. Those showing a perfect knowledge of elementary algebra will immediately be set at work on the first advanced topic; those displaying a mastery of all except a particular topic (for example, completing the square) should be assigned work on this unit of instruction, and so on. Some may have forgotten many of the details of the preceding course and, if there be enough of such pupils, they may easily be separated from the others and given some class instruction. In other words, each pupil should have his knowledge of elementary algebra analyzed, and he should be set to work upon the things he has forgotten or knows imperfectly. By intensive work with the individual pupils, such deficiencies soon may be overcome.

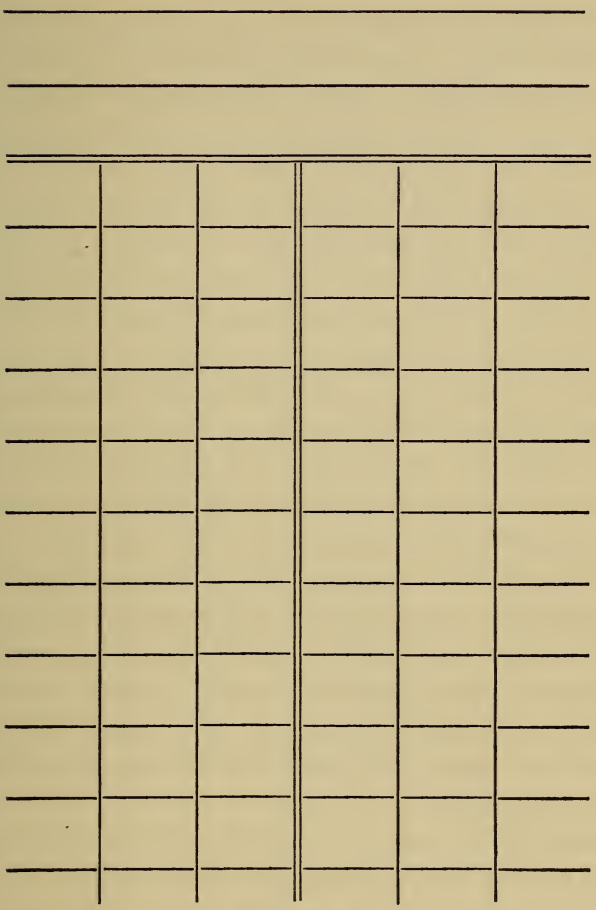


FIGURE V

Keeping a Card Index of the Pupil's Advancement. The teacher should keep a record of each pupil, preferably on the card index plan, noting thereon what each pupil is doing, when he has mastered each unit, and other pertinent data. If the units of instruction of the courses in algebra have been eval-

uated in some such manner as was done prefatory to the illustrative lessons in elementary algebra in Section I, this record may be kept in a very efficient way. The six column charging card, used by librarians, has been found very satisfactory in this connection. (See Figure V.) In the first column is checked the number of the unit of instruction, in the second column is noted the date it is begun, and in the next column the date it has been completed to the satisfaction of the teacher. The next three columns may be utilized for memoranda concerning the difficulties experienced, test marks, etc. One of these cards for each pupil is easily administered and will effectually enable the teacher to "keep tab" on each pupil.

When the class assembles, each pupil will immediately begin work on his individual task. He will request the teacher to render assistance if it is needed. The teacher, on the other hand, when not so employed, will pass from pupil to pupil, noting progress, checking results, and outlining advanced work. The schoolroom thus becomes a busy workshop, with everyone engaged on his particular problems, and no time is wasted. The pupils will respond eagerly to this method, because they will feel that they must proceed just as rapidly as they can with thoroughness.

The mastery of each topic by the pupils may be effectually checked by the time-honored test, but it should be given by units and as soon as the pupil considers himself proficient enough to take it. The efficiency of the correspondence school is thus realized with the additional benefits derived from personal contact with the teacher and the momentum and inspiration of the classroom and its environment. Whatever additional work it may cause the teacher is counterbalanced by the high rate of efficiency and the elimination of ordinary

routine of the recitation. This will offset any possible hardship of added labor. *It is not work that wears out the teacher, but worry;* if the results are satisfactory, very few teachers will be found to complain of the effort demanded.

Solid Geometry and Plane Trigonometry. — It is again assumed that pupils electing these courses have had those subjects already treated. Through the process of the survival of the fittest, the class will undoubtedly be composed of those pupils with natural mathematical ability. The plan, as already outlined, will therefore prevail with some modifications, due to the nature of these subjects.

With all superfluous work eliminated, pupils of natural ability ought to be able to master both subjects in twenty weeks' time. The ability to comprehend and master the textbook demonstrations having been acquired in plane geometry, the study of these in solid geometry should be very easy. A few class explanations of the use of the third dimension might be necessary at first, until the pupils realize and are able to visualize the figures. The method of proof is practically identical with that of the demonstrations in plane geometry and the number employed is much less; and the amount of work in originals is greatly reduced. Since not all pupils are likely to complete both solid geometry and trigonometry, the work in the latter may well be treated individually as each pupil reaches it.

Method. As soon as a pupil considers himself master of a demonstration, he may call the teacher to his side and either prove the proposition orally or on paper as the teacher elects. For the work on circles, a large spherical blackboard should be used by the pupils and his figures thereon should be explained in detail. The use of stereoscopic views for showing

the more complicated figures will do much to enable the pupil to grasp their construction. These may be studied by the pupil whenever there is need to encourage him to work them out for himself. When called on for aid, the teacher should as far as possible confine his help to directing questions, which will enable the pupil to discover the answer to his own inquiry.

Summary. — The success of the above plan for the advancement of each pupil as rapidly as he is able to master the subject rests on two essentials: (1) a thorough mastery of the technic of how to study, developed through a system of supervised study from the early grades, and (2) a teacher, thoroughly acquainted with this system and with an intelligent grasp of his subject, who is sympathetic and alert to the possibilities of the effective administration of this system. With such qualifications of the teacher and normal intelligence and application on the part of pupil, the results will be highly successful, because they are the logical and inevitable culmination of the system of *supervised study.*

PART TWO

SCIENCE

CHAPTER FIVE

THE MANAGEMENT OF THE SUPERVISED STUDY PERIOD IN SCIENCE

Supervised study in its last analysis is essentially the method of the laboratory, and therefore science should and does lend itself ideally to its application. The study of a textbook is more or less subordinated to the study of actual materials, individual experimentation supplanting to a marked degree the assumption of certain facts and conclusions because stated as such in some book. In other words, the pupil is taught to verify what the author has said.

The study of science, largely perhaps because of its peculiar method of approach, has always been more or less popular with young people. Growing boys and girls are enabled through the actual handling of and experimentation with real things to give expression to their very active desire to do things, and an interest is aroused which can never be imparted through the mere reading of the printed page. It is for this very reason that manual training, domestic art, and allied subjects are especially attractive. The adolescent youth yearns to deal with "objective realities"[1] and, when they assume in addition some practical aspect, their study seems peculiarly valuable.

[1] Lloyd and Bigelow, "The Teaching of Biology"; Longmans, Green and Co., 1914.

Laboratory work, however, needs to be carefully supervised if the student is to acquire real benefits from it, as aimless tinkering with apparatus and equipment may become simply a waste of time. It is for this reason that the supervised study period in science is particularly well suited to show the benefits of the system. Pupils must be directed in their scientific study even more carefully than in the case of mere book study and with correspondingly larger results. They must be taught how to test and how to properly draw conclusions; how to check their personal observation with that of others; how to use various kinds of supplementary material and how to judge it; how to interpret printed directions and to form unprejudiced opinions; in fine, how to study.

Much that has been written in Chapter One is equally applicable in connection with the work in science.

The Time Schedule. — The length of the period being assumed to be sixty minutes, the division of the time for the regular recitation periods may be made as follows:

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

When laboratory work is desired, however, the entire sixty minutes may well be devoted to the experimental work and its recording. If five or ten minutes be taken at the opening of the hour, however, to explain what the object of the experiment is, it will be time well spent. And, again, much will be gained if a few minutes at the close of the period be used to clinch the essential results of the laboratory work. These variations will be more fully explained in connection with the illustrative lessons.

The Assignment Sheet. — The use of the assignment sheet, as explained in the foregoing chapter, is strongly recommended for the science teacher in order that he may have a clear outline of just what he expects to accomplish within the hour.

The threefold assignments are also urged if the highest efficiency of the class is to be developed. While there is certain work that must of necessity be demanded of all, there should always be some arrangement made by which those who are particularly gifted may be led to do advanced work and thus proceed at a maximum rate and so become more proficient in their knowledge of the subject. This may take the line of extra study, supplementary reading, use of special apparatus, making charts, classifying museum materials, attending to equipment, making of simple apparatus, etc. Obviously the teacher who is satisfied with a minimum or average amount of work being done alike by all is failing to realize that the schools should develop each pupil according to his individual ability. This possibility of leading all pupils to do their best, of inspiring them to strive always for maximum results, is indeed one of the strongest arguments in favor of supervised study in science as well as in all departments. There is no other method of conducting classes which so nearly approaches the ideal of scientifically educating *all* the children to the limit of the individual capacity of *each*.

Mimeographed Sheets. — The use of laboratory manuals is not recommended since they naturally contain many things that the live teacher will find it necessary to change, on account of the apparatus at hand or because he will find that many things he wishes to incorporate are not mentioned. Mimeographed sheets, which allow the individuality of the teacher to be displayed, are much to be preferred. These may be

changed from time to time as the teacher develops new ideas. In lieu of these, the blackboard may be used, but less advantageously.

Too much should not be stated on these sheets. One of the gravest faults with laboratory manuals is their completeness; too often they leave nothing for the pupil to do but to "press the button." If the pupil is really to grow from day to day in his ability to perform experiments and draw correct conclusions, much must be left to his own ingenuity and resourcefulness. In cases where the pupil seems to have this ability undeveloped, the necessary explanations may better be made to him individually; thus it becomes possible for each pupil to rely to a large extent on his own endeavors. In other words, the directions should be mainly suggestive, and details should be supplied by the teacher or pupil as the case requires.

The pupils should also be encouraged to construct as much of their apparatus as they are able to make. Every well-equipped laboratory should have a workbench, fully supplied with ordinary tools, materials, etc., so that the pupil may have an opportunity to develop whatever ingenuity he possesses.

The teacher should also be careful not to do too much himself. His ingenuity in constructing apparatus and performing experiments may best be displayed by his ability to develop ingenuity and performance in his pupils. The pupil should be led to draw the proper conclusions and not told what conclusions he should draw. The latter is rather dogmatic teaching but the former may well be called *superb leadership*.

FOURTH SECTION

BIOLOGY

CHAPTER SIX

DIVISIONS OF BIOLOGY

In evaluating a course in biology, since, from the very nature of the vast amount of available material, it is impossible to cover all the phases, it is best to select certain topics in botany, zoölogy, and physiology which illustrate the fundamental principles of life. This method at the same time will give the boys and girls some important first-hand knowledge of plant and animal life, with special emphasis on the economic and hygienic sides. Since authorities will naturally differ on the topics that should be included, the divisions as here made may be open to criticism, but for the sake of concreteness those outlined by the State Department of Education of the State of New York are followed, with a few modifications.

UNITS OF INSTRUCTION

I. Introductory topics.

PLANT BIOLOGY

II. Seeds and seedlings.
III. The cellular structure of living plants.
IV. Roots.
V. Stems.
VI. Leaves.
VII. Flowers and fruits.
VIII. Forests and forest products.

ANIMAL BIOLOGY

IX. Insects.
X. Crustaceans.
XI. Fishes.
XII. The frog.
XIII. Birds.
XIV. Mammals.

HUMAN BIOLOGY

XV. Foods, stimulants, narcotics.
XVI. Bones and muscles.
XVII. Organs of digestion and their functions.
XVIII. Blood and circulation.
XIX. Respiration.
XX. Excretion.
XXI. Bacteria and sanitation.
XXII. Additional topics, including the nervous system, special senses, first aid.

Below is given a suggestive time table for the year's work in biology. This will of course vary with local conditions but is given merely for its possible suggestive value:

First twelve weeks	Plant biology
Next twelve weeks	Animal biology
Next twelve weeks	Human physiology
Last four weeks	Review

LESSON I

THE INSPIRATIONAL PREVIEW

Purpose. — The purpose of such a lesson is to arouse the interests of the child in the study of biology. There is no subject in the whole program of studies which may have as many practical applications to the welfare of the pupil as does the

study of biology. But arousing the pupils to an appreciation of these values and incidentally to its many pleasures and interesting phases necessitates that the teacher make some effort to predispose the child's mind to receive its valuable lessons. This first meeting with the class presents this opportunity.

Method. Naturally, the method of conducting an inspirational lesson will vary with the teacher, since it will reflect his personal ingenuity, but for the purpose of possible suggestion, the following scheme, which has been successfully used by the author, is presented herewith.

Previous to the assembling of the class for the first time, place a small quantity of grass and some grasshoppers in a fine-screened cage. As soon as the class is assembled, call attention to the cage and state that it contains illustrations, in a way, of the purpose of the study of biology. Explain that biology is the study of life, animal and vegetable, and its practical applications to the existence of man. Man is not only dependent on these two kingdoms of nature for his sustenance, but his raiment, his health, his vocations, and his avocations are largely biologic in their nature. Draw attention to the grasses and plants in the cage and explain that the vegetable kingdom, of which they are examples, furnishes us with food, directly or indirectly, clothing, homes in which to live, and determines many of our most important occupations; that some plants cause sickness and others offer the means of preventing or curing our ills, others give us pleasures, etc. Explain that in order to know just how these things come about, it behooves us to know how the plant grows, how it lives, and how it reacts upon our lives. Tell the pupils that there is within this cage also a representative of the animal kingdom

— a large group of living things, on which in a large way we are likewise dependent for food, raiment, transportation, good health, and also many of the pleasures of life. Our work in biology will lead us into a systematic study of animal life and its part in the scheme of living.

Now have someone come forward to open the cage and find a grasshopper within. The difficulty of finding him in his camouflaged natural habitat will give the teacher an opportunity to speak of the insect's environment, his protective coloration or adaptation, his dependence upon the plant kingdom, and, therefore, the close connection between plants and animals.

Place the grasshopper upon the floor and let him jump. Measure the length of the jump and compare it with his size. Ask some pupil how high he can jump and make a like comparison with his height. The wonderful adaptability of the hind legs of the grasshopper for this activity will appeal to the pupils as extraordinary, as it indeed is. Mention the fact that all the various classes of animals we shall study have like wonderful adaptations for their modes of life and also that they are beautifully made for the part they are to play in the scheme of nature.

Now after a few words briefly explaining the various functions of living matter, write upon the board the following captions which may be said to form the background of our work:

Classification.
Habitat.
Structure.
Life history.
Adaptations.
Functions.

If now the teacher will read some interesting incident showing the wonderful intelligence of some insect, as the ant from Romanes' "Animal Intelligence,"[1] or the fly from Fabre's "The Life of the Fly,"[2] or the wasp from Morey's "Wasps and Their Ways,"[2] he will arouse an intense desire to know more about these wonderful little creatures, and the year's work in biology will begin auspiciously and interest can be easily maintained.

Then briefly tell the story of the Panama Canal and its gigantic failure until General Goethals attacked the biologic problems and made the region safe for man to inhabit, changing failure to success. Tell about the work of Dr. Koch and his discovery of the cause of tuberculosis, which has resulted in its subsequent treatment; the wonderful work of Luther Burbank in producing new varieties of plants; the investigations of Dr. Wiley in regard to pure food; the work of the Federal Bureau of Entomology and its aid to the farmer. Explain that in every case it was the application of biologic facts that has made life more enjoyable, more healthful, and more successful.

With such a bird's-eye view of the contributions of biology, the pupils will not fail to realize that here is a subject full of the practical and the interesting; and they will actually feel the desire to master this subject in so far as they are able.

If time permits, a rapid survey of the course may be made. Explain that first we shall make a study of the elements that enter into all life, both animal and plant; then we shall study plants from the seedling to the matured plant; animals from the simple one-celled species, which can only be seen under the

[1] D. Appleton and Co. [2] Dodd, Mead and Co.

microscope, up through the crustaceans, the insects, the fishes, birds, and mammals; and finally we shall make a study of man, the most wonderful and highest creation of all.

The pupils should be informed that they will be expected to bring specimens to class, from time to time, to add to the school's collection of natural history materials. Tell them to be on the lookout for articles relating to biology, such as are appearing constantly in newspapers, magazines, and books. Tell them that in addition to the study of their textbook we shall do more or less experimentation and investigation of actual specimens in the laboratory and in their natural environment.

The teacher must be careful not to make this preview too technical or too stilted. It is very important to make the pupils forget that they are about to become formal students of a new subject which may contain a good deal of dry, uninteresting drill on facts. Young people at this age are naturally more or less inquisitive and something which will make keener this instinct for the novel or the unknown will be worth while if at the same time it arouses within them a sincere desire to learn.

The above suggested plan may be objected to on the ground that it smacks too much of the idea of nature study and is too childish. Are we not dealing with young children of immature minds and is not biology in fact nature study with the stress on its applications to life? In the best sense biology is or should be the study of natural history with the underlying thought of the continuity of life processes and functions, and its applications to the whole scheme of nature. Both experience and pedagogical considerations seem to justify this viewpoint.

LESSON II

UNIT OF INSTRUCTION I. — INTRODUCTORY TOPICS

Lesson Type. — A Lesson on How to Study

Program or Time Schedule

The Assignment	10 minutes
The Study of the Assignment	50 minutes

Purpose. — The purpose of this lesson is to introduce the pupil to the study of biology through some preliminary experiments with chemical elements and compounds. The textbook is at once to be supplemented with the actual handling and examination of biological materials, thus illustrating the laboratory method to be followed more or less throughout the course. It is desirable to make clear at the outset that the textbook is in the nature of a guide, to explain and organize our study. Just as guidebooks in travel may not only serve as a prospectus of what we may expect to find and see, but also as a means of interpreting those things which we see and wish to know more about, so our textbook in biology will guide us in our work to discover certain things and will supplement what we see with data concerning which we would otherwise remain in ignorance.

The Review. — This will usually take the form of clinching firmly in the mind of the child the lessons learned previously, and the process should be pointed, short, and clarifying. As this is the first lesson besides the inspirational preview, the review may be dispensed with to-day.

The Assignment. — In all laboratory work the assignment may well take but a few minutes and will be used to explain the nature of the new work. In the present case, it will suffice

to explain that we are going to spend the period examining some material as to its physical characteristics and as to what it will do under certain manipulations.

Notebooks. Explain that the use of notebooks, or some means of recording our investigations, is necessary in all laboratory work in order that we may set down scientifically the facts we learn and that we may have them for future verification and review. Data so recorded must be accurate and as clear as we know how to make them. Explain that some system should be followed which will help to make the data clear. Each of the following heads should be employed:

1. Date.
2. Object.
3. Material.
4. Manipulations.
5. Results.

All pupils should be supplied with some form of notebook, the kind necessarily varying with the individual taste of the teacher. The end-open loose-leaf notebook, size about 4×6 inches, is recommended. This makes a compact little book which may be carried easily in the pocket or bag on field excursions. State that these books will be collected from time to time and examined by the teacher, and emphasize the importance of keeping them as neat as possible and of always having them ready for use.

The Study of the Assignment. — There should be placed upon the desk or table of each pupil all the materials which are to be studied. A little careful planning will obviate wasting of time during the period. In the present instance, each pupil

should be supplied with small pieces of carbon (charcoal), iron, sulphur, and phosphorus (in water).

Procedure. Tell the pupils to open their texts and read the paragraph on elements and compounds. As soon as they have had time to read it through understandingly, have them close their books and question them on what they have read. The teacher may well at this point expand somewhat the statements of the author, being careful not to confuse chemical with physical compounds.

Then tell the pupils to examine the piece of charcoal without referring to the book, and after a few minutes question them as to its characteristics. Try to get them to tell what they have found out about the charcoal without your asking too many questions. Different pupils may be called upon to add something new to the peculiarities or characteristics already given, until all the important ones have been mentioned. If this method fails to elicit all the desired information, it may become necessary to ask definite questions, such as, What is its taste?

Repeat the process with the piece of iron, and then have the pupils compare the two elements for similarities and differences. Insist on the results being stated in sentence form with due regard to the English used. Ask them to tell in what forms each of the two may be found and draw out therefrom, if possible, the significance of these characteristics. For instance, they have discovered that charcoal is soft and leaves a mark when drawn across paper. This quality has given rise to charcoal pencils used by artists. A similar characteristic of graphite has been utilized in the making of so-called lead pencils. Iron is found to be tough; how has this characteristic been utilized in making iron stoves, rails, etc.?

After they have done this, let them open their books and read over what the author has to say concerning these two elements. Someone might take another text in biology and read aloud to the class what the author has to say concerning this subject. Step to the board and write a list of things made from each element, items being suggested by the pupils themselves. It is extremely important to have the pupils feel that they are furnishing the data and that the teacher is merely leading the way and correcting any wrong deductions.

Recording the Experiments. The next step is to record in the notebooks the results of this exercise. Have the pupils open their notebooks and, following the order noted above, make their records. The one on charcoal should appear something like this:

PLAN OF NOTEBOOK

Date: September 4, 192–.

Object: To find the characteristics of carbon.

Materials: Piece of charcoal, a dish of water, a match, a knife, paper.

Experiment: I took the charcoal in my hand, noted its weight, color, feel, taste, odor, and texture. I put it in some water to see whether it would dissolve, I tried to make it burn, I tried to cut it with a knife, and I rubbed it upon some paper.

Results: I found that it was light in weight, black in color, felt rough, had no taste or odor, and looked to be rather porous. It will not dissolve in water, burns with a glow, and makes a mark on paper.

Some other forms of carbon are: graphite (used in lead pencils); diamonds.

Carbon results from burning wood, as the match, and is sometimes found free in nature.

Next, the pupil should take up the study of sulphur in a similar way. The work on phosphorus will need to be guided more definitely by the teacher, due to its peculiar characteristics. After the four elements have thus been studied and written up in the notebooks, if time permits, the pupils should study the paragraph on oxygen and the air, after which the same order of study may be followed.

It is very important to make haste slowly in this new work. The pupils are immature and cannot grasp too many new principles at one time. Each phase should be covered thoroughly, with all the variations that it is possible to make. The actual material in the textbook should be supplemented by other books, by the pupils' experience and knowledge and, if necessary, by the teacher. The illustrations and applications should be as varied and comprehensive as possible, for this is the way by which the pupils will learn to study.

LESSON III

UNIT OF INSTRUCTION I. — INTRODUCTORY TOPICS

LESSON TYPE. — AN INDUCTIVE LESSON

Program or Time Schedule

The Review	20 minutes
The Assignment	15 minutes
The Study of the Assignment	25 minutes

The Review. — The work to date has been on the preliminary experiments. Since we are now to start on a new topic or unit of recitation, it may be best to spend a little longer time than is given usually to review, in clinching the lessons already studied. This may take the manner of a short, snappy quiz.

In addition, we might make a formal testing of some phase of the work previously studied. These twenty minute reviews occurring every four or five days, partly oral and partly written, are much better than formal written examinations of length and given at longer intervals. These reviews test the knowledge of the class on many of the details; and through the short written test, they cover with definiteness some particular feature of the work and make it possible for the teacher to know just how completely the pupils have grasped important points. The question or questions — one is usually sufficient — should be broad and should be based upon some definite or underlying principle. For example, the class to-day, after the oral quiz, might be asked to write a paragraph on the importance of reducing all compounds to their elements.

Allow ten minutes for this paragraph and then collect the papers. These should be carefully examined and errors noted, and two or three of the best answers read aloud to the class the next day. If none is satisfactory, the reasons for this fact should be summarized and possibly a model answer written upon the board. Pupils will thus learn how to give satisfactory written answers. Since written examinations as a means of judging proficiency in a subject are bound to be with us for some time to come, this is an effective method of preparing the pupils for them. By practice and through criticism each pupil will learn the kind of complete and definite statements that will be expected from him in these written tests. As already noted, fact questions need not be emphasized in written tests since the oral quiz is just as satisfactory and is a time saver.

The Assignment. — Explain that many of the lessons in biology will be in the nature of problems. To-day's problem

is: Are two plants or parts ever alike? Our study will be based upon illustrative material and the textbook.

Tell the pupils to make a list of articles, found in nature, which are similar in form. After a few minutes ask someone to read the name of the first article on his list. It might be "man." Ask whether two men are ever exactly alike. The pupils will readily answer that they are not. Repeat with others on the lists, as dog, potato, stone, apple, etc. The pupils will agree that all seem to differ in some particular from each other. Ask whether things in the artificial world so differ from one another. They are more likely to be seemingly alike but, even in the case of the textbooks in use in class, each book may differ somewhat from its neighbor. Some may have pages uncut, some may have defects in binding, some may have pages poorly printed, etc.; yet they differ, if at all, in degrees of perfection. Then direct the pupils to study the maple leaves which will have been placed in advance upon their desks.

The Study of the Assignment. — Each pupil having been supplied with a number of leaves, ask them to find two alike if they can. They may exchange with each other if they wish. If some pupil thinks he has found two exactly alike, it may be necessary for the teacher to point out some difference in veining, hairiness, markings, color, etc. It would be an excellent thing if a quantity of leaves of a different kind could also be passed around for examination. The greater the variety of specimens for examination the more scientific and impressive will be the lesson learned. A quantity of twigs from the same tree should also be secured for a like investigation. After ample time has been allowed for all to satisfy themselves that no two specimens are entirely alike, have the

pupils open their textbooks and read what the author says concerning this problem (Bailey and Coleman's First Course in Biology, pp. 1–3).[1] Draw the attention of the class to the cut which also justifies the same conclusion, noting, however, that a study of the specimen itself is to be preferred always to a picture.

Tell them to read carefully the last sentence and then explain in simple manner what the author means. This word *variation*, then, is the key word of our lesson to-day.

The pupil may never have appreciated so fully before the wonderful ways of nature as he will now when he comes to the realization that, of all the billions of maple leaves in the world to-day, as well as in all the years gone by, there are never two exactly alike. Here is an opportunity for the teacher to impress the child with an appreciation of nature through scientific observation.

The observations listed on page 3 of the text with regard to size may well be taken up next and each pupil told to answer these questions concerning one of the leaves he has before him. If time permits, it would also be beneficial for him to note these facts in his notebook.

Before this is done, however, the result of the day's investigations should be noted and cast into a form of statement; and it may with good effect be written upon the board with colored crayon. Ask someone to tell what conclusion he has drawn from the day's lesson. If the pupil has been alive to the problem under discussion and examination, he will say, "No two plants or parts are ever alike."

(N.B. Note the title of the chapter referred to.)

[1] The Macmillan Company.

LESSON IV

UNIT OF INSTRUCTION I. — INTRODUCTORY TOPICS

LESSON TYPE. — AN INDUCTIVE LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — On each of five panels of blackboard, have these directions:

Write a list of points in which leaves may vary from each other.
Write a list of various kinds of leaves.
Make a list of flowers which do not at all resemble each other.
How do people resemble each other?
How do people differ from each other?

Assign a pupil to each of these sets of instructions, directing him to write the answers upon the board. Meanwhile, ask someone for the conclusion reached in the preceding day's lesson, and ask different pupils for proof of its accuracy.

The lists having now been completed, read each exercise and the answers given. Remarks may be necessary to amplify or correct mistakes; have other pupils supply this material where possible.

The Assignment. — Explain that the new lesson is to be based upon a study of a continual warfare that is being waged in the plant and animal world; namely, that of a struggle for life. Remark, while one may go to the mountains for the summer and meditate upon the tranquillity of nature, that in fact a very serious strife is being waged by every tree and flower and living thing. Each is making a desperate struggle

for a chance to live, and, just as each is successful or unsuccessful, will the existence of a particular flower or bird be determined.

Ask the pupils whether they can relate any experience in observed animal or plant life which would go to prove that this is so. Many examples in the animal world will come to their minds of one animal's living upon another. After a few illustrations have been given, ask whether anyone can give instances from the vegetable world. This may be a poser, but if you suggest *weeds*, the class will follow the clue.

After this factor in plant life has been quite fully covered, explain that these plants and animals solve their problems by various means, such as the bird's escaping from the cat by rapid flight. This instance will set the pupils thinking, and the teacher will need the next few minutes to give them an opportunity to state similar cases of adaptation to conditions.

State finally that these conditions, to which a plant or animal is obliged to adapt its habits of life or die, is called its *environment*, and bring out the various conditions of environment, such as climate, food, habitat, etc.

The Study of the Assignment. — The advance lesson will be the chapter on " The Struggle for Life." The essential features of this chapter have been already covered in our study, but without any reference to the book itself. Now through the study of the assignment or the textbook, the pupils are to organize this material into a clear, concrete increment of knowledge. Some words may need explanation; pupils should early feel that the dictionary is to be a constant help and should be referred to frequently. If any pupil finds some-

thing in the lesson which he cannot understand, he should raise his hand and call the teacher to his aid.

An added exercise should be the requirement to write out a list of at least ten cases of struggle for existence, ten adaptations to conditions, and ten varieties of environment.

LESSON V

UNIT OF INSTRUCTION II. — SEEDS AND SEEDLINGS

LESSON TYPE. — A HOW TO STUDY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	15 minutes
The Study of the Assignment	35 minutes

The Review. — Each pupil having been assigned some plant for special report to-day on the essential points of classification, the review may be spent on this work. Each pupil in turn may be called upon to name the plant assigned to him for study, and to classify as annual, pseud-annual, plur-annual, biennial, etc., and to state clearly his reasons for his answer. (A suggestive list is given in Bailey and Coleman's First Course in Biology, page 19.)[1]

The Assignment. — *Preparatory Work of the Teacher.* Each pupil should be supplied with a specimen of the bean and a kernel of corn, which have been previously soaked in water. Each pupil should also have a bottle containing a solution of iodine. A large chart showing the essential parts of these seeds should also be in full view of the class. In lieu of a chart, the teacher might make large drawings upon the blackboard.

[1] The Macmillan Company.

The teacher should explain in the beginning that the lesson to-day concerns seeds, their essential parts and their functions. He should then explain how to open a seedling, point out the embryo, and indicate how the test for starch should be made. He should also explain that these seeds have been soaked in water in order to study them more easily and to show the effect water has on them.

The class should then be instructed to open the books and to study the text carefully, verifying each statement from examination of the specimens. When they have opened their books, but before they start their work, it will be well to explain the correct pronunciation of the names of the various parts of the seed. These are new words and it is best to make sure that the pupils learn from the first just how they are to be pronounced. The derivation of some of the words might well be given, as **plumule** from the French word *plume* meaning to *ascend* and hence given to that part of the seed which will rise or ascend. Explain that **monocotyledon** is formed from two Greek words, *mono*, meaning *one*, and *cotyledon*, meaning *a cup-shaped hollow*, and signifies a seed having one cotyledon. Note that *monoplane* is an airship having one wing or plane. Again, **endosperm** is formed from the Greek adverb meaning *around*, and *sperm*, which signifies *the life-giving element*, hence the word denotes that which is around or encloses the embryo or sperm. The pupils thus having learned the reasons for the new names will find them not simply unfamiliar and unmeaning terms but living words, and they will experience no particular difficulty in learning to use them correctly.

All of the words which are likely to give trouble should be marked in the teacher's book and definitions or explanations of them prepared prior to the meeting of the class. It is this

careful preparedness on the part of the teacher that will do more than anything else to dispel confusion and that will result in the interested and undivided attention of the assembled class.

The Study of the Assignment. — The pupils will now begin the study of the new work. The first sentence presupposes the opening of the seed and the discovery of the embryo. This then should be done, the bean being used first. The next sentence, we shall assume, describes the three essential parts of this embryo. With the direction of the teacher and the aid of the large charts, these should be found in the specimen and the proper name for each learned. If any pupil has difficulty in finding these parts in his specimen, he should call the teacher at once to his side. Let it be definitely understood that no one is to proceed until he has covered thoroughly each individual point of the text.

After this has been done with the bean seed, tell the pupils to open up the kernel of corn and trace its embryo through a like investigation. The teacher should follow the actual work of the pupils throughout, keeping pace with them in their study and supplying any details necessary to make the work clear. Ninety per cent of the textual work in a science must be studied in this painstaking and critical manner, nothing being passed by until mastered and, if possible, verified from a study of the specimen or material itself. We simply cannot read scientific literature over rapidly, looking for the high spots, as is sometimes done in some subjects, but each statement must be closely and carefully studied. It is for this reason that supervised study in science becomes very important, and why study without it often results in superficial knowledge or utter failure.

Not too much should be assigned for a lesson. Better a little well studied and principles well grounded than to attempt too much. An excellent method of varying the order suggested in the above time schedule would be to use the last five or ten minutes of the period for a review of the essential points studied in the new lesson, thus firmly clinching them. Step to the chart, for instance, and call on someone to name the different parts of the seed, someone else to tell the function of each, and someone else to spell the various names.

Before the class is excused, some of the unopened soaked seeds should be planted in a suitable box and placed in a sunny place; some similar seeds which have not been soaked should also be planted in another box and the two boxes labeled, one as *A* and the other as *B*. It is suggested that some pupils be assigned to this duty, preferably someone who has been a little more industrious than the others and who may have completed the assignment. This will serve to keep all busy and will give some recognition to the more rapid workers. Speed should never be tolerated in place of thoroughness, however.

If there be time for any more work during the period, the pupils may be instructed to make a drawing of the opened seed in their notebooks, labeling each part. Under the title of *Seed Germination*, the pupils should also commence a notebook record of the planting of the soaked and unsoaked seeds. The record will be added to later. If the work has been carefully planned in advance, all of the required details can be completed within the hour; the exact amount, however, will necessarily vary with teachers and classes. Sufficient work should be planned by the teacher for an emergency, always keeping the time limit of the period in mind so that there may

be no undue hurry and no possibility of leaving some task half done. As has already been emphasized, it is very desirable to have sufficient time before the close of the period for a rapid review and the clear affirmation in some sort of summary of the important points of the lesson.

LESSON VI

UNIT OF INSTRUCTION II. — SEEDS AND SEEDLINGS

LESSON TYPE. — AN INDUCTIVE LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — Ask some general review questions on the preceding lesson, such as:

Name some dicotyledons, monocotyledons.
Where is the food stored in each?
Of what use is the seed coat?
Are all seeds alike? Are all bean seeds alike?
Why is it poor policy to plant old seeds?
What care should be taken of seeds before they are planted?

A few such questions, combined with a quick review of the parts of the seed from the chart or drawing, will suffice to cover the review.

The Assignment. — The new lesson will be similar to the preceding one in that it will be a continuation of the study of the textbook, exemplified in every possible particular by the use of real specimens. There will be also the starting of several experiments which from their very nature must take

three or four days for their completion. These experiments will be along the line suggested by the paragraph on germination in the new lesson. The author states that "when supplied with moisture, warmth, and oxygen (air), it (the embryo) grows."[1] We wish to verify these facts. We have already started an experiment to discover the value of moisture as a factor of rapid growth by our planting of soaked and unsoaked seeds. We shall now make another experiment. Tell the pupils that every time we make an experiment, it is necessary that we have a so-called check or control experiment, the one to show that certain things happen under certain conditions and the other to show that without these conditions they will not happen.

The pupils should be told to open their notebooks and, under separate heads, note what is done to prove the three things stated in the above quotation.

Experiment I: Do seeds need moisture?

Procedure: Let each pupil plant a seed, bean or corn, in a plant dish. Designate two pupils to keep it watered. Tell all the pupils to consider this *C* in their notebooks, and so label the dish. In another dish, marked *D*, plant some of the same seeds, remarking that this dish will not be watered.

Experiment II: Do seeds need warmth?

Procedure: Have the pupils plant seeds in two dishes, both to be kept watered, one to be set in the sunlight where it is warm, the other set aside in some cool place. Label them *E* and *F* respectively.

Experiment III: Do seeds need oxygen?

[1] Bailey and Coleman, "First Course in Biology"; The Macmillan Company, 1908.

Procedure: Make the double planting of moistened seeds in dishes labeled *G* and *H*, one to be left uncovered and the other securely corked.

As soon as the pupils have made the requisite records in their notebooks, pass on to the study of the assignment.

The Study of the Assignment. — Study carefully the assigned pages in the text which we shall assume cover the germination of seeds. Although the seeds planted yesterday of course have not sprouted yet, the teacher should have some beans in the various stages of growth ready for study, having planted them at various intervals previous to this time. When those planted by the class have begun to come up, the class should use them for review and for their drawings. They will thus serve as a verification of the textbook and also as the models for the drawings.

It will be seen that the teacher must be continually planning for the future: by having seeds at the right stages of germination; by having plants at the right stages of development; by having plenty of illustrative material ready at the proper time; by keeping note of the progress of various experiments; and by taking care that all phases of the subject are studied in their right sequence. A great deal of this may be done by the pupils themselves, outside of school hours. The teacher will find that they will be glad to do this work. As far as possible pupils should be rotated in these preparations, so that all may feel a corresponding responsibility. The biology laboratory should be at all times full of growing plants and flowers; there should be an aquarium, cages for living animals, herbariums, etc. The pupils should feel that their biology room is essentially a place of living things, and they should be encouraged to contribute as much material as they can from time to

time. The pupils should also be encouraged to make similar experiments at home and to compare their results with those obtained in the laboratory. Incidentally it might serve to arouse a coöperative interest in their school work on the part of their parents.

There should also be plenty of cabinet room for permanent specimens. The pupils should be encouraged to add to this collection either temporarily or permanently. In this way a very adequate museum may be accumulated which may be used for demonstration purposes. The teacher should emphasize at all times, however, the greater importance of living things since biology is essentially the study of life. The author has little use for biological specimens purchased from commercial houses; some specimens, which are foreign to the locality, must necessarily be purchased, such as starfish, etc. These, however, have only a subordinate value; the pupils' environment should furnish the mass of illustrative material. If it is a rural community, the specimens should be those of the farm; if an urban community, those of the locality are to be emphasized.

More can be learned about birds from some pupil's bringing his pet canary to school or from a trip to the woods and studying the birds as they are in their natural habitat, than from cases filled with mounted specimens of birds, native to foreign localities. At best, all the latter can do is to serve as means of identification, and it is doubtful if a child will ever learn to know the birds except from the study of the live birds themselves. Let us inculcate a love for wild life in its natural setting, even if the specimens be limited in number; at least, let us attempt to make biology a study of what it is generically intended to be, that of life and living things.

LESSON VII

UNIT OF INSTRUCTION II. — SEEDS AND SEEDLINGS

LESSON TYPE. — A SOCIALIZED LESSON

Program or Time Schedule

At least 60 minutes

Purpose. — The purpose of this lesson is to study nature in its own setting, to learn some of the lessons from nature's great laboratory, the out-of-doors. Field excursions are absolutely essential to the proper evaluation of biology, and the more of them the better; but let them always have some specific object in view, some particular lesson to be learned. Incidentally many other lessons will be learned, but some outstanding purpose for each trip should govern the excursion.

As soon as the class assembles, explain that you are going to take a field trip for the main purpose of studying the ways in which seeds are scattered or dispersed. Tell the pupils to have their eyes open for the observation of anything else which pertains to botany, especially any illustrations that would seem to verify the lessons already learned in class. But the main object of the expedition is to study seed dispersal. All the pupils should have their notebooks handy and all examples of this phase of plant study should be noted therein.

A few words on the proper conduct of a field trip may not be out of place. Foremost, as in all laboratory work, and this is but a phase of laboratory work, the teacher should actually take the trip himself a day or so previously so that all the illustrative material looked for may be sure to be found within the time allotted. The teacher must be fairly well acquainted with the surrounding country and should know exactly where

to go, how to get there, about the time it will take, etc. If it is impossible to complete the trip during a regular period then some other hour should be arranged, but it is better if it can be completed within the hour allotted.

The discipline of a field trip will never give one any trouble, if the trip is well planned and if the class realize that you hold them as accountable for their conduct as in the regular classroom. Of course there will be some freedom not allowed ordinarily, but there should be no waste of time, no wandering from the class, and no boisterousness. The pupils are out for a specific purpose, and they should be made to realize that you will tolerate no infraction of discipline.

The excursion should take one through the fields, the woods, and the pastures. Many examples of seed dispersal may be found, some that have already been noted in the text and some that perchance have been omitted. The pupils should exchange experiences as they go along; and if one finds a good example, the others should have their attention called to it. Let it be more or less in the form of a game or contest to see who can find the most examples. Many other illustrations of their work in botany will also be observed, such as plant societies, the struggle for life, etc.

The teacher will need to be alert every minute, and he need not feel abashed if the pupils ask many questions which he cannot answer. It would take a very wise man indeed to know all the secrets of nature. It is a much better sign to have pupils ask questions which you cannot answer than to have them ask no questions or show no interest in the trip.

If seeds of plants whose names you do not know are being dispersed, make a record in the notebook and take back specimens to the schoolroom for later identification. Properly

planned, the trip should be full of valuable experiences and should open up a new vista to the pupils. As you proceed in the study of this subject and later expeditions are undertaken, the pupils will find an endless amount of corroborative material for the work covered.

Possibly the first seeds being dispersed may be the dandelions'. Some late specimens of this weed may be found in the school yard, yet how many will ever have sensed before the meaning of the tiny hairlike wings of this seed? Ask the pupils to blow some to see how far the wind will carry them. Require drawings of one of the winged seeds to be made in the notebooks. The next specimen may be a milkweed, and as the pupils get into the open country they may find the snapdragon, beggar's-lice, etc. A cow may pass with her tail tangled with burdocks, a tumbleweed may blow merrily down the field, scattering its seeds right and left, and so on indefinitely may nature give an illustrated lecture of one of her many phases.

LESSON VIII

UNIT OF INSTRUCTION IX. — INSECTS

LESSON TYPE. — A HOW TO STUDY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject.* Crustaceans.

Method. Ask the questions found in the text, supplying others of a like nature, such as:

Name some fresh-water crustaceans.
Name some salt-water crustaceans.
Name some crustaceans used as food.
What is the general weapon of defense of all crustaceans?

Then call on someone to go to the chart and point out and name the various parts of the crayfish.

The Assignment. — Tell the pupils that we are now about to start the study of another *arthropod*, the insect. Tell them we shall take as our type form the grasshopper, since our study of it will answer anatomically for all insects. Ask someone to state the prominent characteristics of crustaceans. Then have the pupils open their books and call on someone to read the characteristics of insects. These are found to be: three body divisions and six legs. They may now be told to close their books again while you proceed to give them a little *inspirational preview* on insects in general. Many pupils may have a predetermined distaste or even horror for *bugs* and this will serve to put them in the proper attitude toward insects in general. Tell them that all insects are wonderfully made, that they have a complete, although in many insects, a very minute nervous system. This high-strung nervous system is one of the reasons why certain insects, like bees, are very apt to resent and resent immediately any rapid movement which may be interpreted by them as indicative of harm to them. Tell them a few of the wonderful things about ants, how they keep their "cows" (aphids) and milk them, how they organize for battle, etc.; tell them something about the wonderful life history of the butterfly; tell them that many scientists have spent their lives studying the life histories of various insects and have written some very interesting books about them, as did Romanes, Lord Avebury, Darwin, Fabre, and

others. Tell them that some insects are very valuable to mankind, as the ladybug, the ichneumon fly, the honeybee; and others of course are very harmful, some carrying diseases, as the mosquito, some destroying clothing, as the clothes moth, and so on. Then remark that in addition to a specific study of insects in general, we shall spend some time on the characteristics and classifications of insects, learn how to mount them, how to destroy the harmful ones, and how to protect the useful ones. In all, our study of insects is likely to be one of the most interesting and fascinating phases of our work in zoölogy.

Then ask the class to name the insects they know, at the same time writing their names upon the board. This will serve to ascertain just how extensively they know insects and will concentrate their interest at once upon the subject, for they will feel that their present knowledge is of importance at the outset.

The Study of the Assignment. — The study of insects and of the grasshopper in particular will follow closely the suggestions outlined in the study of seeds. All pupils should supply themselves, if possible, with live specimens of native grasshoppers. If it is possible for the class to go out into the fields and collect some live specimens, it would be an excellent thing to do, but the season of the year in which this topic is reached may preclude this possibility, in which case the insects must be purchased or have been kept in cages and raised for this purpose. With the specimen before him, the pupil should accompany the text with the actual handling and study of the insect itself. Some textbooks, as Bailey and Coleman,[1] are themselves guides for laboratory work and, if the teacher elects,

[1] "First Course in Biology"; The Macmillan Company, 1908.

may be made the basis for the notebook work, the pupil making the drawings and recording the facts learned from the actual observation of the grasshopper. Not all the facts observed should be noted therein, of course; this might be left to the judgment of the pupil, or the teacher might indicate on the board certain facts which are to be recorded. The notebook must never be so autocratic as to be petty; it loses its essential value unless it is written up in the expectation that it shall serve as a check on the individual work.

As soon as the pupils have completed the first two or three pages or that part of the book covering the external characteristics, the teacher should direct them to close their books, and he should review their work, asking questions corresponding to those of the text. This will help to emphasize the study, will clarify any doubtful points or rectify any mistakes in observation, and will further serve as an opportunity for the teacher to supplement with any additional data or material which he may feel necessary or expedient.

In the few minutes remaining, the teacher may unfold a chart on the grasshopper and rapidly review the essential details already studied regarding the external characteristics of the insect.

The Silent Study. — As far as it is possible, insist that the pupils find out the answers to the questions themselves. Discourage their asking you direct questions. If they make incorrect deductions, you can find that out later, either in the quiz period or from their notebooks. The pupils should be trained to feel that they are the ones to do the work and that the teacher's duty is simply to direct that work, correct wrong impressions, and supplement their own observations. Facts that the pupils ascertain for themselves will remain with

them always, while information you give them will be but transitory. The teacher should always be ready to offer suggestions, however, to help the pupil interpret the text if he finds him unable to do it for himself; but the skillful instructor will avoid direct answers and will strive to lead the pupil to form his own answer through careful questioning. For instance, a pupil might call the teacher to his side to ask him what is the general shape of the grasshopper. Instead of saying outright that it is cylindrical, ask him what shape it resembles, whether it is round or square, circular, or rectangular, etc., whether it is uniform throughout or only suggests some form as a whole. If he admits that it is more or less irregular but that it is somewhat curved, ask him with what sort of curved figure it compares favorably. In other words, get him to make the final decision himself. If he is obviously wrong, try by various questions to get him to see that he is wrong without directly telling him so. Remember that the pupils are learning judgment of fact as well as the facts themselves. The pupil must learn self-reliance, a lesson much more important than any biological fact he may acquire. Indeed we may safely say that all study should develop the powers of observation and the training of judgment, the ability to see and to interpret correctly what we see. It is one great fault with many teachers that they often tell too much. The teacher must know the facts, not to recite them but to make sure that the pupils sense them aright.

If the teacher finds that several pupils are becoming confused over some question or statement, it might be well to take the matter up with the class as a whole and develop through them the correct answer or understanding, just as has been

suggested before for individual pupils. However, the individual method is much to be preferred.

The Outside Work. — Necessarily much of the work in biology must be done during the class period. The more this can be done, the better will be the results. Outside work should consist largely of supplementary reading, home experiments, field observations, special reports, tabulations, collecting materials, and the like. Tables of comparative study like that on page 85, Bailey and Coleman, give excellent outside work and from their nature are quite adaptable for such assignments. The biology library should have a rich assortment of natural histories, bird books, flower guides, nature readers, etc., to which the pupils may be given assignments for outside reading and reports. Most textbooks furnish complete bibliographies, and the teacher should be reasonably acquainted with them. Many of the farm bulletins may be used, such as deal with the fly, the toad, raising bees, the codling moth, obnoxious weeds, etc.

An excellent plan is to assign each day special topics for reports either written or oral, and to make them as varied as possible. They should be correlated as far as possible with the interest of the child, those living on farms being given some topic relating more or less to their environment. If some pupil happens to have chickens, for instance, he may be given a topic: "Chicken Lice and How to Eradicate Them"; or if he has a pet dog, give him a pamphlet on fleas and let him report; if he is the son of a physician, he may be told to look up and discuss the relation of the mosquito and malarial fever, and so on.

Encourage the pupils to bring to class magazine or newspaper articles on some phase of animal life, anecdotes con-

cerning animal intelligence, bird stories, etc. Get the pupils to feel that their study of biology is vital, practical, and full of rich interest. They will look through the papers for interesting articles, will question other people for experiences, and will learn to observe and interpret many of the interesting things that are of everyday occurrence.

LESSON IX

UNIT OF INSTRUCTION IX. — INSECTS

LESSON TYPE. — A LABORATORY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — The few minutes of the review may be spent in the ordinary question method, quizzing rapidly various members of the class on the preceding day's work. The kind of questions will depend on the teacher, but they should at least be as thought-producing as possible, although the nature of the subject necessitates more or less dogmatic answers. The best method in asking fact questions is always to state the question first and assign it to some pupil afterwards. This intensifies the attention of all, as no one knows but he may be called upon.

The Assignment. — The internal dissection of the grasshopper is too complicated and minute to be required of the pupils. Either the teacher should make the dissection as a demonstration or he should use charts or blackboard drawings and explain the general characteristics of the anatomy from

them. We shall assume that the charts will be used and that they will resemble very closely those given in the book.

Method. Have the pupils open their books at the paragraph describing the mouth parts. Have someone who is a good reader stand and read slowly the description of the mouth parts. As he proceeds, the teacher will point them out on the chart, interposing any additional facts or explaining the text in more detail as he may elect. When this has been completed, ask some questions covering the ground studied.

Then pass on to the next paragraph which may be that on respiration, and this may well be covered in the same way. In this way the new lesson on internal structure may be gone over entirely, the teacher acting as an interpreter.

The Study of the Assignment. — *I or Minimum Assignment.* The three or more pages in the text covering the anatomy of the grasshopper.

II or Average Assignment. Drawings, copied, of course, of the mouth parts, nervous system, and digestive system (in notebook).

III or Maximum Assignment. Study of two or three microscopic slides, such as show the facets of the eye, the veining of the wings, and an abdominal segment containing a spiracle.

The Silent Study. — With the assignment explained as suggested above, the pupils may not require much help during the study period. There may be some, however, who even now have not thoroughly grasped all that the author has to say, and the teacher will be ready to lend any additional aid if necessary. Particular attention should be given to the pupils who are not doing satisfactory work and an effort made during this period to aid them. This may take the method of sitting down beside such a pupil and giving him personal supervision in his

study. Let him read a few sentences and then ask him questions on it, thus making sure that he is getting a thorough understanding of the meaning. Too much emphasis cannot be placed on the importance of the teacher's selecting each day someone who is falling behind in his work and through a study of his method of study trying to put him on his feet. If the teacher will do this at every opportunity, he will be well repaid. It is trite to say that the teacher should never feel satisfied with his work until he has exhausted every possible avenue of assistance to enable a pupil to master his work. Indeed it seems to the writer that more real good can be done in these individual "first aid" sections than in the regular class recitation periods. It is our business as teachers to pay particular attention to the extremes of our classes, those doing minimum and those doing maximum work.

LESSON X

UNIT OF INSTRUCTION IX. — INSECTS

LESSON TYPE. — A CORRELATION AND RESEARCH LESSON

Program or Time Schedule

The Review	20 minutes
The Assignment	15 minutes
The Study of the Assignment	25 minutes

The Review. — The object of this review is to emphasize graphically the entire order of insects as to (*a*) general characteristics common to all insects, (*b*) outstanding characteristics of each order, *i.e.* wings, (*c*) classification according to mouth parts, *i.e.* biting or sucking insects, and (*d*) economic importance, *i.e.* harmful or beneficial.

Method. Write in yellow crayon, at the top of each of four panels of the blackboard, one of the four suggested topics. When the class assembles, the teacher should develop through questioning the data which he will write under each caption. When completed, the four panels should be a correlated review of the essential characteristics of insects as a class, and it will serve forcibly to impress upon the pupils the integrity of the work on the study of insects.

Under the first caption, ask someone to name the characteristics which all insects have in common. If the pupil should mention some point which is not common to all, instead of ruling it out point-blank, mention some insect which does not possess it and thus direct the pupil how to draw upon his own knowledge before he answers. If the pupil should state correctly the two characteristics desired, *i.e.* three pairs of legs and three body divisions, mention some others and ask whether they should be allowed. Exhaust, in other words, all possible items that might be included, making clear that these are the only ones that may be considered in this general summary.

Under the next caption, write the suffix *-ptera* ten times, calling on various pupils to supply the prefix and to state the meaning, as *diptera*, two wings, *coleoptera*, sheath wings; etc. Also call for an example under each order.

Under the third caption, divide the board into two sections, one with the heading *Biting Insects* and the other *Sucking Insects*. Call on someone to name all those given under the preceding classification, which may be placed in the first group and those which may fall into the second group; or possibly a better way would be to state each one separately and call on someone to tell in which group it should be placed. If a pupil

makes a wrong classification, instead of simply declaring it incorrect, direct him to judge for himself whether he is right or wrong. A few questions concerning the characteristics illustrative of this order, which have been noted on the second board, will help him to do this.

On the fourth board, ask first for insects which are beneficial, and insist in each case that some simple explanation be given to characterize its value, as *Bees: make honey; Ichneumon fly: destroys harmful larvæ of deep-boring insects through deposition of its eggs in their tunnels.* Then under harmful insects, do the same. Make the questions terse, scatter them among the pupils, keep all on the alert for illustrations and for proper classifications.

The Assignment. — As soon as the four panels have been completed, turn to the new work, which is a research lesson. Explain that you are going to give each pupil a card which will have written upon it a problem concerning some insect, and that the book or bulletin containing the information will be found to be indicated upon the same card. Direct them to study the problem, and then search for its answer in the references given. Tell them, when they are satisfied that they have found the correct answer, to write upon a sheet of paper the statement of the problem, its answer as they have decided it should be given, and then the reference by title and page. These papers may be collected at the close of the period or handed in the next day if the period does not suffice for its completion.

The Study of the Assignment. — In this case there will be only one general assignment, but the questions may be so graded that the harder problems may be given to those showing more marked ability, and so on down to the easier ones

which are assigned to those who ordinarily are able to do but the minimum amount.

Below is a list of suggested topics and references:

1. When should apple trees be sprayed to kill the codling moth, and what is used? See "The Control of the Codling Moth"; Farmers' Bulletin No. 171.
2. How did the cotton-boll weevil get into the United States? See Weed's "Farm Friends and Farm Foes"; D. C. Heath and Co.
3. What is the estimated value of a toad to the farmer? See "The Usefulness of the Toad"; Farmers' Bulletin No. 196.
4. What economic value has the cochineal bug? See any encyclopedia.
5. What is parthenogenesis? See Bigelow's "Applied Biology"; The Macmillan Company.
6. Why is the tachina fly beneficial? See Smallwood-Reverly-Bailey's "Biology for High Schools"; Allyn and Bacon.
7. What is Bordeaux mixture and for what is it used? See p. 263, Warren's "Elements of Agriculture"; The Macmillan Company.
8. How is the Rocky Mountain locust destroyed? See p. 15, Linville and Kelly's "Textbook in Zoology"; Ginn and Co.
9. Where do flies spend the winter? See p. 81, Hegner's "Practical Zoology"; The Macmillan Company.
10. What is the estimated yearly economic loss from insects? See Hunter's "Essentials of Biology"; American Book Co.

LESSON XI

UNIT OF INSTRUCTION IX. — INSECTS

LESSON TYPE. — A SOCIALIZED LESSON

Program or Time Schedule

The Review	25 minutes
The Assignment	10 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Harmful insects and how to get rid of them.

Method. Previous to the assembling of the class, write upon the board the names of as many harmful insects as there are members in the class. Such a list would probably include the mosquito, house fly, codling moth, carpet moth, bedbug, cotton-boll weevil, potato bug, squash bug, tent caterpillar, etc. Also write upon separate slips of paper the names of the various members of the class, and have these in a box.

Explain to the class that they are going to take part in a sort of game which might be called, " Ridding the Community of Obnoxious Insects." Each pupil will be called upon to tell why the insect assigned to him is harmful and how to get rid of it. Each time the pupil answers correctly concerning the insect assigned to him, the name of that insect will be erased from the board. The object will be to try to erase from the board or the community all of the names, *i.e.* the pests.

Allow a few minutes for the pupils to look up in their text-book or any other books available any of the insects mentioned on the board on which they wish to be better posted. Then at the end of ten minutes, call someone to the front of the room, who might be designated *class entomologist* for the nonce, and direct him to draw some slip from the box and read the name upon it. The pupil whose name is read must rise and without help tell why this particular insect, taking it in the order in which it has been written upon the board, is harmful and how it may be destroyed. If he can recite successfully, the name of that insect is erased; otherwise, it is left on the board, and as a penalty for failing, he may be required to look it up and hand in a written answer before he leaves. The process is repeated until all the pupils have been called

upon, and the board has been cleaned more or less completely, according to the successful answers.

A little more spirit may be injected into this game if sides are chosen and a contest developed to see which side can eradicate the larger number of the harmful insects.

The Assignment. — Tell the class that the assignment for the next day will be in the nature of a series of short illustrated lectures by the various members of the class. Tell them they are to bring to class some insect which they have found on some of their field trips, or in lieu of that, the picture of some insect, and that you will ask them, when called upon, to come forward and, holding the insect or picture in their hands, describe to the class its characteristics, habits, order, etc. Also, that other members of the class may question them further concerning their specimens and that you will expect them to be posted fully on each particular insect. Suggest that they follow the outline below in describing their specimen:

Name:
Order:
Characteristics of form:
" " life history:
" " habitat:
Food:
Economic importance:

The Study of the Assignment. — *I or Minimum Assignment.* Each pupil to be prepared to describe according to the outline given above, some insect to be selected by himself.

II or Average Assignment. Each pupil to bring to class a specimen of the insect he is to describe, or at least a picture of it.

III or Maximum Assignment. Be able to quote some source of information other than the textbook in use.

LESSON XII

A RED LETTER LESSON IN ZOÖLOGY

Time Schedule

Program . . . . . 60 minutes

Purpose. — The object of a red letter lesson is to review vividly all the important phases of the work covered in zoölogy and also to introduce an incentive to superior work during the weeks devoted to the study of some particular section of the work in biology. It is recommended that at least one such lesson follow the completion of the work in botany, zoölogy, and physiology.

Method. The program should be outlined some time in advance, so that all may be making their plans for this **Grand Review.**

The program may be divided into three sections, the first or major part of the program being devoted to a review of the various orders of zoölogy, the other two sections of the program being a display of notebooks, drawings, charts, etc. Printed or mimeographed programs will add to the enjoyment of the occasion and will serve as a souvenir, something which young people always treasure.

PROGRAM

SECTION ONE: GRAND REVIEW

A five minute talk on each of the type forms studied during the work in zoölogy:

The Paramecium
The Crayfish
The Grasshopper
The Perch
The Frog
The English Sparrow
The Rabbit

SECTION TWO: DISPLAY

1. Two or three of the best notebooks.
2. Two or three of the best written descriptions of some bird, in prepared booklets.
3. A display of all the specimens which have been collected during the year.
4. Charts, done in India ink, showing the fly nuisance.
5. Bird houses, flytraps, animal snapshots taken by the pupils, etc.

SECTION THREE: ILLUSTRATED LECTURE

A stereopticon lecture, showing slides of birds common to the neighborhood, by the instructor.

Procedure. If there are pupils who are especially gifted in the ability to draw, let them the day prior to the rendition of the program, make drawings of the animals suggested in section one, on the blackboard, using colored crayon if they choose. These of course may be prepared by the instructor if preferred, or charts may be utilized for this purpose. Every well-equipped biology room will necessarily be supplied with these charts, which should be on display at all times on the front wall of the room. The pupils designated to give these talks should be selected beforehand and they should have their reports well planned.

The objects for display in section two should be selected in advance, and some pains taken to exhibit them in an attractive way. The attention of the class will have been drawn to this red letter lesson some time before the actual date, and the pupils encouraged to prepare something for it. Some of the charts on the fly nuisance, for instance, may easily be arranged with the teacher of drawing and may take the forms suggested in a little pamphlet issued by the International

Harvester Company of New Jersey, entitled, "Trap the Fly."

Pupils in the manual training department may also be encouraged to make bird houses, flytraps, etc., or these may be borrowed from some of the pupils in the grades where this work is done in connection with nature study. Some very interesting kodak pictures of birds, mammals, and other animals will be forthcoming from the announcement of this feature. It may also serve the double purpose of interesting the pupils in that sport of hunting with the camera which is so much more to be commended than hunting with a gun.

A short illustrated lecture on birds will also be easily arranged if the school is supplied with a lantern. In New York State slides are furnished without charge by the Division of Visual Instruction, and in case they are not accessible from some free source, there are a number of firms which will rent or sell suitable slides very reasonably.

LESSON XIII

UNIT OF INSTRUCTION XVI. — BONES AND MUSCLES

LESSON TYPE. — A HOW TO STUDY LESSON

Program or Time Schedule

The Review	15 minutes
The Assignment	20 minutes
The Study of the Assignment	25 minutes

The Review. — *Subject Matter.* Bones.

Method. Let one of the pupils go to the skeleton or manikin and point to the various bones and call on individual members of the class to describe them as to name, kind, and use. He

is to be the judge of the correctness of the recitation given; and if he accepts an erroneous answer, let him take his seat and assign another to act as temporary teacher. Pupils like occasionally to have the responsibility of playing teacher and it is excellent practice, calling as it does for accurate knowledge and for display of judgment.

The Assignment. — Have some pupil step to the front of the room, open his book at the chapter on muscles, and read the first paragraph, the rest of the class meantime reading with him silently. Ask the one who read the paragraph if there are any statements of the author he does not understand. If there are, either bring out the meaning through careful questioning or explain it in simple language. If there are any words which are new and are likely to give trouble, have someone look them up in the dictionary and report. Now have all the pupils close their books, and ask the one who has been reading to state the substance of the paragraph in his own words. Call on one or two others to do likewise. Then proceed in a similar manner with the next paragraph. In case the paragraph has necessitated quite a bit of explaining, it might be well to have the pupils reread it very carefully before reproducing it. In this way the pupils will be taught how they should study their new assignment.

The principal object of this intensive study of the assignment will be to inculcate a method of study which shall be at once exhaustive, intelligent, and comprehensive. Too much so-called study is simply mechanical reading of the printed page without assimilating what the author has to say. The habit of retrospection of each paragraph before proceeding to the next is an invaluable acquisition to the pupil who desires to master the subject matter.

The Study of the Assignment. — *I or Minimum Assignment.*

Four or five pages of the text in the new chapter which deals with muscles.

II or Average Assignment.

Draw diagrammatically a group of involuntary muscle cells.

III or Maximum Assignment.

The following or similar thought questions:

a. Why is it important that some muscles are voluntary? Name two or three.

b. Mention some involuntary muscle which may be made voluntary at the desire of the owner.

c. What makes a muscle red? tough? elastic?

The Maximum Assignment. — When the maximum assignment consists of auxiliary thought questions, they should be questions that are not treated in the textbook in use, but should either be of such a nature as to be inferred with a little thought or to necessitate the use of other books. Usually the answers should be carefully written out and handed in the next day. If other books are used for authority, their titles and the names of the authors should be stated as references. It should be made emphatic that answers should be authoritative and should be substantiated by concrete references to the authorities. If this method becomes habituated, many loose statements common to-day will in time be done away with, for as the child learns to act and think and make statements during his school days when his mind is being trained to react along definite lines, these habits are likely to become the outstanding characteristic of his more mature attitude as a student.

LESSON XIV

UNIT OF INSTRUCTION XVI. — MUSCLES

LESSON TYPE. — A LABORATORY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	10 minutes
The Study of the Assignment	40 minutes

The Review. — Picking out some of the important things to be reviewed, quiz the class rapidly and intensively on the things deemed essential. Many of the questions ordinarily asked during the recitation period may well be dispensed with since they are of minor importance and have presumably been drilled on sufficiently during the regular work on this chapter. This short review may be more in the light of summarizing some of the more essential facts about muscles. Ten minutes, used intensively, and thoroughly planned as to the objects desired, may well suffice for to-day as the laboratory work will take the greater part of the period.

The Assignment. — Explain that the work to-day will be the study of a few microscopic slides. Have the slides that you wish examined carefully selected and arranged beforehand so that there may be no loss of time. The compound microscope should be adjusted and the first slide to be examined in place. A large diagrammatic drawing of what will be seen under the microscope should also have been drawn upon the board and the attention of the class called to the essential things that the pupils are to try to see in the specimen. While the class is examining the drawing on the board, call someone to the microscope and have him examine the slide,

referring occasionally to the blackboard drawing for comparison and verification. Then let him return to his seat, copy the drawing on the board and note under it any remarks that seem to be necessitated by his verified examination of the slide. As soon as the first pupil has used the microscope, have another pupil ready to take his turn. In this way, with a little careful executive ability, the entire class will be enabled to examine a half dozen or more slides.

It is here assumed that the pupil has already had experience with the compound microscope, but if not, then a few minutes should be taken at the outset to explain the workings of the instrument and to give directions as to how one should look through it. Do not allow a pupil to make any adjustments of the instrument as the inexperienced are not to be trusted to turn the adjustment screws. The slides may easily be broken, or the objective, and adjustment is too complicated a proceeding for the ordinary pupil to attempt. If he finds that his eyes are such as to require a readjustment, the teacher should always give the needed assistance. At some other time, if the instructor sees fit, individual instruction in adjustment may be given to the pupils but always with slides made for the occasion and not with those forming the equipment of the biological laboratory.

On Previous Arrangement of Blackboard Material. It will possibly have been noted in these illustrative lessons that much emphasis has been placed on the diagrams and data to be placed on the blackboard by the instructor previous to the assembling of the class. The science teacher above all else should be able to make good diagrammatic drawings. The use of the board should be almost wholly employed by the teacher. Colored crayons do much to emphasize the essential

features of the drawings, which should be carefully executed and labeled; they should also be fairly large. The good teacher is a good executive and one of the first rules of executive ability is careful planning, with strict attention to all details and possible complications.

The Study of the Assignment. — Give as a general assignment for all a study of the drawings the pupils have made and the verification of their authenticity through references to other books which give illustrations of similar drawings. Have them look carefully to see whether their drawings compare favorably with these book drawings and if not, to find out why.

Some of the slides recommended for this lesson are:

a. Involuntary muscle cells or fibers.
b. Voluntary muscle cells or fibers.
c. Heart muscle cells.
d. Motor nerve fibers ending among fibrils of voluntary muscle.
e. Capillaries among fibers of voluntary muscle.

LESSON XV

UNIT OF INSTRUCTION XVI. — MUSCLES

LESSON TYPE. — A DEDUCTIVE LESSON

Program or Time Schedule

The Review	20 minutes
The Assignment	15 minutes
The Study of the Assignment	25 minutes

The Review. — Assign to as many pupils as you had different slides under examination the preceding day the task of going to the board and making from memory a rough sketch of the slide assigned. While they are at the board, quiz the class on the essential features which they found in their mi-

croscopic study of these slides. When the drawings have been completed, ask the class for criticisms. If any are made which are well justified, have the pupil making the proper criticism go to the board and make the alterations.

The Assignment. — Tell the class that the work to-day will be in the nature of the problem: How does muscular activity aid the health of the individual? The class will agree that muscular activity does improve one's health, but the question is — just how? All textbooks will have something on this topic, but none will deal with it exhaustively and the teacher will be able to make definite references to various authorities for more advanced research. Encourage the pupils to add to the references noted by discovering others.

The Study of the Assignment. — *I or Minimum Assignment.*

a. Cite cases where lack of muscular activity has resulted in poor health.

Reference: p. 48, Bailey and Coleman, "First Course in Biology."[1]

b. Cite cases where regular muscular activity has resulted in improved health.

Reference: Chapters IV and V, O'Shea and Kellogg, "Making the Most of Life."[1]

c. Mention some muscular activities which might be good for improving the health.

Reference: Chapter VII, O'Shea and Kellogg, "Health Habits."[1]

II or Average Assignment.

a. What becomes of muscles which are not exercised?

Reference: Chapter II, Jewett, "The Body and Its Defenses."[2]

[1] The Macmillan Company. [2] Ginn and Co.

b. What happens to muscles when they are exercised?

Reference: Chapter III, Jewett, "The Body and Its Defenses."[1]

c. Explain the biological result of exercise on muscles.

Reference: Lagrange, "Physiology of Bodily Exercise."[2]

III or Maximum Assignment.

a. Discuss the following exercises as to their specific value: walking, swimming, chopping wood, playing tennis, setting-up exercises, use of gymnastic apparatus.

Reference: Chapter XXII, Hutchinson, "Handbook of Health."[3]

b. List some cautions to keep in mind while exercising.

Reference: Chapter XIX, Eddy, "Textbook in General Physiology and Anatomy."[4]

c. Discuss: A healthy mind needs a healthy body.

Reference: Lee, "Play in Education"[5]; Groos, "The Play of Man."[2]

LESSON XVI

UNIT OF INSTRUCTION XVI. — MUSCLES

LESSON TYPE. — A LESSON IN CORRELATION

Program or Time Schedule

The Review	20 minutes
Program	40 minutes

The Review. — Call on different pupils to stand and tell all they can to substantiate the truth of the proposition that muscular activity tends to good health. Let each recitation

[1] Ginn and Co. [2] D. Appleton and Co. [3] Houghton Mifflin Co.
[4] American Book Co. [5] The Macmillan Company.

be as complete as possible, being in the nature of an exposition of this particular problem. At the conclusion of each recitation, make any suggestions that will tend to direct the next pupil to make his contribution logical and clear. Make the review not only scientific in nature but an exposition in good English, thus directly correlating the work with the oral English. The teacher should never accept as final any extended recitation which does not follow the lines of correct English, logical sequence of thought and well-rounded sentences. Habituate the pupils in the use of short sentences, pure English, and definite statements. Explain that the mere reciting of scientific data without regard to its recital in the best structural form causes the loss of much of its inherent value. It is a pet theory of the author that aside from a short course in technical English, the place to teach rhetoric and English composition efficiently is in the various subjects of the school program. The teacher should feel that primarily he is to develop the power of the pupil to express himself and to sense that science, history, etc. are important largely for the fact that they supply the pupil with the material for his conversation and composition. A man may be very proficient in his knowledge of bird life, but if he is unable to express himself in a clear, logical, and pleasing manner, his knowledge is apt to be of but little use to himself or to others. Teachers are too often prone to be satisfied with the mere acquisition of facts and to pass over as of little or slight importance the exposition of those facts.

The Assignment. — *The Object of a Lesson in Correlation.* All school work should have more or less interrelationship. The correlation of English and the work in biology has just been noted. The work of muscles and their development

through athletic and other activities have been for the past few lessons the object of investigation. Systematic physical training has been mentioned as one of the best means of developing the muscles and thus improving the health of the individual. Explain that you have asked the physical instructor to give the class a series of exercises, with proper explanations, which will tend to develop the various muscles of the body. Ask them to take notes on the various exercises and to be able to demonstrate each to-morrow, with a few words regarding their special functions.

The physical instructor may now be introduced, and he will proceed to demonstrate various setting-up exercises which have as their essential object the strengthening of various sets of muscles. If the plan of the demonstration has been talked over with the physical instructor prior to the class period, a very helpful and interesting lesson will result. The physical instructor on his side will welcome this opportunity to demonstrate the scientific basis for his work, and the pupils will come to have a new realization of the specific value of the setting-up exercises which they have been doing from day to day, and physical training will have a new meaning for them. The value of the demonstration will depend, of course, upon the exposition of the reasons for each exercise and its hygienic and physiologic function.

LESSON XVII

AN EXAMINATION LESSON

PART I

(*Answer all five questions*)

1. Name four food nutrients other than water, and name a food in which each nutrient predominates.

2. Describe the different kinds of teeth and their special adaptation for their respective functions.
3. Compare arteries with veins as to structure and function.
4. Name three organs of excretion, and name a waste product given off by each.
5. Describe the effects of alcohol on the nervous system; on digestion.

PART II

(*Answer any three*)

6. Why should we masticate thoroughly? take systematic exercise? clean the teeth regularly? drink only pure water?
7. Compare the human body with an engine in three particulars.
8. Why must athletes abstain from the use of alcohol and tobacco?
9. How may a knowledge of biology help us to live longer? (Touch on at least three phases.)

PART III

(*Answer any two; reference to library or other books allowed*)

10. Discuss some contagious disease as to source of contagion, symptoms, treatment, after effects.
11. By the use of the table on food values, compute the food values and calories of a meal consisting of: one grapefruit, one boiled egg, two Vienna rolls, one pat of butter, one baked apple, a glass of milk, and one doughnut.
12. Look up in some textbook on physiology other than the one you use, one of the following topics and report in detail in your own words: gross structure of the eye; gross structure of the heart; gross structure of a kidney.

An Analysis of the Suggested Examination. The object of such an examination, divided into sections, is to test the pupil on his knowledge of certain facts, on his power to answer thought questions, and on his ability to look up topics in outside reference books and to reproduce this knowledge in his own

words. It will be noted that the questions in Part I are essentially fact questions and may be said to have been selected from certain minimum essentials which should be required of all. The second part is composed of questions which will require the exercise of more or less thought and yet are graded to reach pupils of only average ability. The third group requires special effort on the part of the pupil and shows his ability to use books other than the one he has been studying. It tests his power of assimilating and reproducing the author's material and of judgment as to the selection he shall make.

It is expected that all the pupils will answer the questions in the first two groups, but that only pupils of more than average ability and resourcefulness will attempt the last group. It is further planned that the grading of the paper be along these lines: four correct answers in group one and two in group two are necessary for a passing grade; if all the answers in the first group and the three in the second group are correct, the grade will be 80; in addition, each of two correct answers in the third group will add ten more credits, thus giving an honor, and if two are correct in the third group, the result will be perfection, or 100.

The advantage claimed for this kind of examination papers is that it will allow pupils to secure a passing grade through a definite mastery of certain facts of minimum requirement, it will give added value to the grade through the ability of the pupils to answer correctly simple thought-producing questions, and will enable the candidate to secure honor marks only through his power to answer correctly more advanced questions after giving correct answers to the fact and thought questions. It is held that the common form of examinations, such as the Regents Examinations of New York State, which allow pupils

to range from 60 to 100 in their grades, according to their ability to answer more or less correctly ten or more questions from a larger collection — all of which are of about the same difficulty and nature — is not a scientific method of estimating the pupil's knowledge or judgment. In order to receive high marks, the pupil should answer questions of recognized severity, in addition to others requiring less breadth of view but which are more precise in nature. Such a plan seems to encourage not only the mastery of certain minimum requirements but also advanced work during the year, for only through such extra research and effort may the pupil be trained to secure the coveted high grades. The old method seems to the writer too much like paying the artisan according to the number of times he can do a certain simple task well instead of according to his ability to do more complicated, skilled, and technical work. As a matter of fact the foreman in a shoe factory is not paid for his ability to turn out a great number of heels in a day, necessary as this is and a thing he is capable of doing, but for his ability to supervise the work of all the employees, manage men, and keep up production. This higher degree of skillfulness on his part is the criterion by which he is able to secure higher wages; so it should be in the examination, — the pupil receiving the higher wage or grade should be the pupil who in addition to having mastered the less technical yet important knowledge can prove himself capable of performing additional work of a higher and more advanced order.

FIFTH SECTION

PHYSICS

CHAPTER SEVEN

FURTHER LESSONS IN SCIENCE

Space precludes any extended elaboration of illustrative lessons in physics, chemistry, physiography, and other advanced sciences. It is assumed that the suggestions in biology will serve to illustrate possible lessons in these subjects. Naturally, as the pupil advances in his school life and learns more definitely how to study, the need of extensive directed study will be of less importance and necessity. For this reason, only a few typical lessons are given, and these in physics. The illustrative lessons in this subject may well serve, however, as suggestive of similar work in the other sciences.

For the same reasons, it is deemed unnecessary to take space for the evaluation of the content of these subjects as has been done in algebra, geometry, and biology. Such an evaluation of each subject into units of instruction and units of recitation is nevertheless important, and the teacher will do well to prepare such a prospectus or syllabus if he desires to cover the work systematically and with proportionate thoroughness.

There is a concerted effort on the part of educational authorities to-day toward reorganizing the courses in all sciences in secondary schools along the line of the general needs of pupils

and society [1] rather than the specialization of the content matter. The progressive teacher will keep abreast of these new investigations and will evaluate his courses according to the latest and best suggestions. Any organization of science courses will therefore be more or less temporary since science is by nature a subject of growth and change. It is, indeed, this quality of progress and development which makes the study of science so peculiarly fascinating and vital.

LESSON I

UNIT OF INSTRUCTION. — FLUIDS

LESSON TYPE. — AN EXPOSITORY AND HOW TO STUDY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — Take up the problems on pressure of liquids, assigned for to-day, as follows: Call on some pupil to read the first example, to explain what is called for, and to tell orally how he solved it, giving his answer. If the answer is correct and all consent to his solution, pass to the next one. If there is some question about it, however, raised either by the teacher or some member of the class, direct these queries to the pupil reciting, making him substantiate his method or see his mistake if he is in error. Avoid telling him where he made his mistake as he will never gain the power of problem solving unless he is directed in the finding of his own error and also directed how

[1] "Reorganization of science in secondary schools," Bulletin No. 26, 1920, Federal Bureau of Education.

to correct the same. Each problem explained should be the means of clarifying and forcibly demonstrating the principles involved. Explaining them singly and orally concentrates the attention of the entire class and brings out any wrong impressions or false methods of solution that may have been practiced by anyone. It will also serve to clarify in the pupil's mind the process of reasoning which must mark all attempts at solving problems in physics.

If the time for the review is limited as in the time schedule noted above, it will expedite matters to take up only those exercises which gave trouble. As a general thing this is the best way at all times, for there is little value in explaining problems which all have successfully solved, unless it be to make sure that they all solved them by correct methods.

The Assignment. — *Method.* Put a fresh egg in some fresh water, so that the class may clearly see that the egg sinks. Then put it in a glass of a saturated saline solution. Ask the class why it does not sink now. In the same manner put a marble in the glass of fresh water and also in some mercury. Ask someone if in swimming he has ever noticed how he will with difficulty keep his feet if he wades out slowly into the water up to his mouth. Ask someone who may have lived or been near the ocean, if he experienced the same effect in salt water. These simple experiments will serve to emphasize the fact that bodies submerged in a liquid will be forced upward, the degree depending upon the nature of the liquid and the nature of the body submerged. Ask someone to state this fact in the form of a simple statement or rule. If the statement as given is not complete, draw out the nature of its incompleteness by skillful questioning. In other words, teach the pupils how to gather the results of experimentation and study and

make deductions therefrom. Then when they read in their textbook similar conclusions they will feel a thrill of power in their ability to conclude results for themselves. This idea should be the predominating object of the explanation of the assignment — to develop through the pupils' own observation and research the sensing of the laws and facts of physics.

Ask if anyone knows who Archimedes was. If none knows anything about this great scientist, direct someone to secure the story of his life in an encyclopedia. While he is doing this, explain the meaning of the word *buoyancy*, using as an illustration the word *buoy*, which all will probably understand. Ask for illustrations of the application of buoyant force, which might well include the floating of logs down a river, the floating of a boat, the fact that a drowning person will come to the surface two or three times, etc. Referring to the report of the pupil concerning Archimedes and the story of his life explain that he was the first person to discover certain laws which govern buoyancy. Mention the fact that we shall try to discover these laws for ourselves.

Weigh some metal, as a piece of lead, in air and then weigh it when submerged in water. Call on some pupil to come forward and announce aloud the readings of the scales in each case. Ask what we understand by weight. If, then, it is the force of attraction between the body and the earth, ask if the piece of metal has really lost any weight. The pupils will readily see that this is impossible according to the definition. Ask them what has caused the apparent loss of weight. The class will be quick to see that there has been some readjustment of the weight rather than any loss. Now perform the same experiment, using in this case a cylindrical dish which will hold exactly a solid of similar form. Then if we allow the

weight to be submerged in water and fill the receptacle with the displaced water, the equilibrium will be restored. It will be a slow class that will not at once see that the weight lost in water an amount equal to the weight of the water it displaced. Now have someone give this in terms of a statement. After two or three have done this, each time making the statement more complete and clear, tell them they have evolved one of the laws of Archimedes.

Referring to the previous experiment of putting the egg in the brine, ask them what it did in the solution. They will tell you it floated. Ask them what caused the egg to sink in the fresh water but not in the brine. Perform the experiment, similar to the one above, for sinking bodies. It is clearly given in all textbooks and need not be stated here. The principle for floating bodies may also be deduced as was done with the above principle of buoyancy.

Ask someone to tell what is meant by mass. Stating that the quantity of matter or the mass in a unit volume measures the density or comparative density of a solid, ask if we could tell the density of a piece of iron, for instance, by weighing it. Presumably not, for it is possibly irregular in character and its weight, depending on its size, will vary, while its density would always be the same. Therefore, as in all measurements, we must take some unit and we shall find that the density may be found by dividing the mass by the volume. Put this on the board, first as represented by words, as

$$\text{density} = \frac{\text{mass}}{\text{volume}},$$

and then as a formula:

$$d = \frac{m}{v}.$$

Weigh a piece of iron which may be measured and the volume of which may be computed in c.c., and then divide the one by the other. When this is done, ask someone to turn to the table of density or specific gravity in his book to compare the result with that given. They will be very much gratified to find their result tallies with that given in the book. Let some other pupils repeat the operation with another piece of iron; then with some lead, marble, etc.

Noting that both of these substances are regular in surface, ask how they would suggest proceeding with some irregular substance, as a piece of quartz. Someone may be quick enough to suggest that we might get its weight by the method of water displacement, and then substitute in the formula as before.

Again note that these substances are heavier than water, and ask someone to suggest a method for finding the specific gravity of something lighter than water, as paraffin. Again someone will probably suggest that we attach a known weight to the paraffin and submerge both in water, then subtract the known weight from the gross weight and proceed as before. Thus we have outlined the methods of ascertaining the specific gravity of solids.

The Study of the Assignment. — *I or Minimum Assignment.* The textbook work on the subject.

II or Average Assignment. The questions found at the end of the chapter covering this subject.

III or Maximum Assignment.

1. Tabulate in order of their density: tin, ice, gold, zinc, glass, and butter.

   References: any chemistry.

2. What is the principle of the submarine?
3. Look up about Descartes and his Cartesian diver.

LESSON II

UNIT OF INSTRUCTION. — FLUIDS

LESSON TYPE. — A LABORATORY LESSON

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — Conduct a short, snappy recitation on the textbook work on density and specific gravity of solids. This review should serve as a preliminary review and preparation for the individual experiment.

The Assignment. — During the assignment the instructor should outline how the experiment is to be made and what computations and deductions must enter into the written record.

Method. The instructor should write on the blackboard with yellow crayon the object of the experiment as follows:

Object. To compare the buoyant effect on a solid submerged in the water with the weight of the water which is displaced.

Under the caption *Apparatus* for the present no data are to be placed. It is best to leave this to be filled after the actual experiment has been made, so that the pupil may sense the real object of these data, which is to be the recording of the apparatus used rather than some arbitrary listing of equipment. It is as premature to list the articles of apparatus before the experiment is made as it would be to attempt to count the votes before an election. How many mechanics could tell prior to doing some repair work on an automobile just what tools were going to be used?

Under *Procedure*, explain to the class in brief outline how

the actual experiment is going to be made, but by no means dictate any directions for its performance. This must be written up in the language of the pupils and is to follow the experiment. Explain that the method employed will be to weigh the solid in air and then in water, and that the best method of doing this will be to allow the scales to project over the table a little so that the solid may be attached to the under side of one scale and weighed. Then we shall take an overflow can and fill it with water, holding the finger over the spout. Now after allowing all superfluous water to run out, we shall place it under the projecting solid and then allow the solid to drop into the bucket, catching the overflow of water in a catch bucket. When the body is entirely submerged, weigh the solid in this position. Explain that care must be taken to catch every drop of water and also to be as accurate as possible in every weighing operation. Also suggest that other methods of arranging the scales for this experiment may be made as the pupils desire, such as supporting the scales on a pile of books, a box, etc.

Then ask some pupil to state what data will have been collected up to this point. Step to the board and, as these are stated, write them down. They will be:

1. Weight of the solid in air.
2. Weight of the solid submerged in water.
3. Weight of the empty catch bucket.
4. Weight of the catch bucket and the displaced water.

If the four mentioned above are given in different order, write them down as given and afterward ask whether any would change the order, according to logical steps. The above order will probably be suggested. It will be noted by the reader that at every step the pupil is thrown on his own judgment as

to the order of procedure and not merely told these steps arbitrarily. He will thus be trained to exercise his own resources and judgment.

When these tabulations have been arranged for satisfactorily, ask some pupil to tell you what computations must be made in order to draw a conclusion. With a little skillful leading he will see that we must compute from the figures found: (*a*) the loss of weight of the solid in water, and (*b*) the weight of the water displaced by the submerged solid. Using some fictitious numbers for data, have him tell you these two computations.

Suppose for illustration, you assign these amounts to his list of data:

1. Weight in air . . . . . 15 gm.
2. Weight in water . . . . . 10 gm.
3. Weight of bucket . . . . . 5 gm.
4. Weight of bucket with the displaced water . . 10 gm.

Then our computations will be:

1. Loss of weight of solid in water: 15 gm. − 10 gm. = 5 gm.
2. Weight of water displaced: 10 gm. − 5 gm. = 5 gm.

What then may we conclude from these computations? How do the two results compare? The pupils will of course see that they are the same.

Ask someone to express this conclusion in the form of a statement. How does this agree with Archimedes' principle?

Now ask someone to state the rule or formula for density. It is

$$d = \frac{m}{l},$$

or density equals the mass divided by the loss of weight in water.

Using the data of the above, have some pupil compute the density of the solid, thus, $15 \div 5 = 3$.

The Study of the Assignment. — Now let each pupil get the apparatus he needs, make the experiment with various solids, and write up his experiment in the form suggested. Insist that each pupil get his own apparatus, and clean it up and put it away after the work is completed. The author has no sympathy with the method by which some teachers conduct a laboratory experiment, where all of the apparatus is set out before the class and blanks are distributed to the pupils, who participate only in the experiment by inserting the data in the blank spaces. This method is too much along the line of " press the button and we do the rest " photography. Pupils taught in this manner may perform experiments until doomsday and they will know no more physics at the end than they did at the beginning. The main value of the individual laboratory experiment is in the fact that the actual procedure makes a more lasting impression on the pupil than the mere reading of the experiment out of some book or witnessing a demonstraation made by the instructor.

Drawings. When drawings may further explain just how the experiment was done, they are important, but when the process is self-evident, or easily explained, drawings are a waste of time. When required they should be done freehand, with the fewest lines possible and they should be clearly explained by the accompanying legend. Copied drawings are valueless as they do not function as they ought; the pupil is thinking more of the technic of the figure than of the object for which it is drawn; namely, graphically to illustrate and supplement the written description.

During the performance of the experiment by the pupil, the

teacher should pass about to see that everything is being done correctly, ready with suggestions but never actually telling the pupil how he may better his work. In other words, the work of the laboratory should be used to develop the pupil's ability to understand the textbook, to appreciate the value of careful manipulation of apparatus, and to make possible the logical drawing of conclusions as a result of the work performed.

LESSON III

UNIT OF INSTRUCTION. — FLUIDS

LESSON TYPE. — A HOW TO STUDY LESSON IN PROBLEMS

Program or Time Schedule

The Review	10 minutes
The Assignment	25 minutes
The Study of the Assignment	25 minutes

The Review. — Call on some pupil to step to the front of the room and briefly review his entire work in the laboratory yesterday without referring to his notebook except for figures.

The Assignment. — The best way to take up a set of new problems in physics is to have the class orally analyze a number of simple exercises, such as will be found in every textbook, thus giving a large number of pupils practice in interpreting what is wanted. It is also well to have a number of supplementary problems from other sources, which may be used in this way, if the instructor finds that the pupils do not readily grasp the principles involved. A large assortment of applied problems may be found in Lynde's Physics of the Household.[1]

[1] The Macmillan Company, 1918.

It is found that the greatest difficulty pupils have with problems is their inability to interpret correctly the language of the particular problem; the actual mathematics is usually very simple. It is for this reason that the oral analysis is an exceedingly good method of training pupils to make the proper interpretation.

It will be found expedient to have some pupil read a certain problem aloud; then after giving him a few seconds for thought, ask him to shut his book and repeat the essential points of the problem. If he can state the problem in his own words, the instructor may feel assured that he understands what it is about, unless it be memorized. Now have him state what is to be found. With each step of his solution, insist on his giving a physical reason for the step. The teacher should never accept such bald statements as "multiply so and so and divide the result by —," unless the pupil gives as his reason for so doing some rule or physical fact. Also, the answer should be given in some denomination, so that the instructor may know that the pupil has understood the purpose of the problem. Thus the analyses of the exercises become an excellent review of the principles of physics.

The exercises which are assigned for the next lesson may be some of those taken up thus analytically in class, as well as some others of similar and possibly more severe character. These should be worked out on paper neatly and with logical steps, omitting of course much of the reasoning that has been done in the oral work.

The Study of the Assignment. — *I or Minimum Assignment.* The first ten of the fifteen problems in the textbook.

II or Average Assignment. The remaining five problems.

III or Maximum Assignment. Problems 18–22, page 48, Lynde's Physics of the Household.[1]

The Silent Study. — During this part of the period the instructor will find that he will be needed by some of the weaker pupils who have still failed to master the solution of the problems, either through a faulty understanding of the laws or of the principles involved. The teacher will find this an opportunity for individual help and as has been emphasized many times throughout these lessons, he must ever be on the alert not to tell how to do them but to lead the pupil to solve them himself.

All exercises should be collected at the close of the hour, with notice that the remainder will be collected to-morrow. Thus the teacher will know that the ones handed in are indeed the work of the pupils themselves, and by examining them he may get a clear idea of just how proficient they are becoming in handling this work. The exercises done outside of class are of doubtful value, except as they reflect the pupil's mastery of the principles as he later shows by his ability to do others of similar nature. They must be required, however, in schools which do not have periods lasting more than sixty minutes, and they should be carefully checked and as far as possible carefully examined. They are of doubtful value in awarding grades, but the failure to hand them in or incorrect solutions should be reflected in the grades awarded.

LESSON IV

RED LETTER DAY LESSONS

From time to time throughout the course, there should be provided special programs or red letter day lessons. Instead

[1] The Macmillan Company.

of limiting this lesson to any one program, a number of suggestive lessons will be mentioned, with a few words of explanation concerning each. The progressive teacher of physics will think of many others, of course, and he should choose the kinds that seem to be of most value and interest to his particular class.

1. A stereopticon lecture on some phase of physics. Suitable slides are sold by a number of firms, as L. E. Knott Apparatus Co., Boston; Central Scientific Co., Chicago.

2. If the school owns a moving picture machine, many valuable films may be secured from various sources. See Extension Leaflet No. 2, Department of the Interior, for list (December, 1919).

3. Cuts from magazines and books, photos, etc. may be used to advantage in giving a review of some unit of instruction through the use of an opaque projector.

4. A talk to the class by some college professor of physics; by the city electrician on some practical phase of his work; by some physician on the X-ray in surgery; by a musician on the pipe organ; etc.

5. A trip to the electric power plant, or to the pumping station, or to a plant using a hydrostatic press, etc.

6. A wireless apparatus may be set up in the laboratory and the hour spent in sending and receiving messages.

7. An examination of various kinds of vacuum cleaners, either expository by the teacher or the actual examination of samples which may be collected from different sources and loaned for the day. Excellent cuts may be found in Lynde's Physics of the Household. [1]

[1] The Macmillan Company.

8. Some pupil may give an exposition of some article on an appropriate subject taken from a current magazine, as *Scientific American* or *Popular Science Monthly*.

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