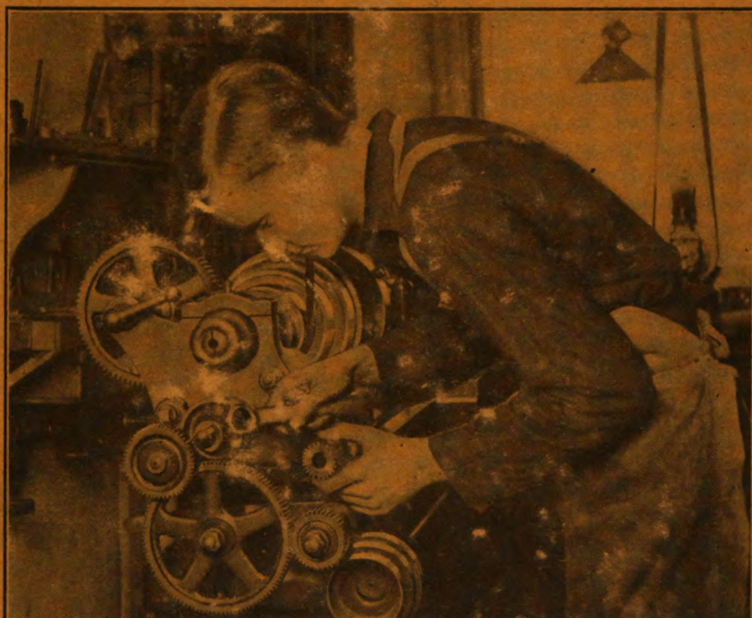
5 CENTS

ADVANCED

SHOP ARITHMETIC FOR THE MACHINIST

BY ERIK OBERG

THIRD EDITION



MACHINERY'S REFERENCE BOOK NO. 52

PUBLISHED BY MACHINERY, NEW YORK

MACHINERY'S REFERENCE SERIES

EACH NUMBER IS A UNIT IN A SERIES ON ELECTRICAL AND STEAM ENGINEERING DRAWING AND MACHINE DESIGN AND SHOP PRACTICE

NUMBER 52

ADVANCED SHOP ARITHMETIC FOR THE MACHINIST

By ERIK OBERG

THIRD EDITION

CONTENTS

Students whose knowledge of elementary arithmetic and its application to simple problems is too limited for intelligent study of this treatise, are advised to first study MACHINERY'S Jig Sheets 5A to 15A, inclusive, Common Fractions and Decimals, and MACHINERY'S Reference Book No. 18, "Shop Arithmetic for the Machinist." Not until the principles of elementary arithmetic and their application to simple shop problems are well understood, can the student expect to derive the full benefit from the study of the present book.

CHAPTER I

SQUARE ROOT

The square of a number is the product of that number multiplied by itself. The square of 2 is $2 \times 2 = 4$, and the square of 10 is $10 \times 10 = 100$; similarly the square of 177 is $177 \times 177 = 31{,}329$. Instead of writing 2×2 for the square of 2, it is often written 2^2, which is read *two square*, and means that 2 is multiplied by 2. In the same way 128^2 means 128×128. The small figure (^{2}) in these expressions is called *exponent*.

The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number. Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. We may say that the square root is the reverse of the square, so that if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{\ }$, but the *index figure* (^{2}) is generally left out, making the square-root sign simply $\sqrt{\ }$, thus:

$\sqrt{4} = 2$ (the square root of four equals two),

$\sqrt{100} = 10$ (the square root of one hundred equals ten).

The operation of finding the square root of a given number is called *extracting* the square root.

Assume that the square root of 119,716 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will, of course, have only one figure.

11'97'16 | Space for root.

Now find the greatest whole number the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure of the root. In the example this number is 3, the square of which is 9. Subtract this square from the left-hand period, and move down the next period of two figures and annex it to the remainder, thus:

$$\begin{array}{rl} & 11'97'16 \mid \underline{3\quad} \\ 3 \times 3 = & \underline{9\qquad} \\ & 297 \end{array}$$

Now multiply the figure of the root obtained by the constant 20 which is always used when extracting the square root by this method ($3 \times 20 = 60$), and find how many times this product is contained in the number 297. This gives us a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

$$\begin{array}{rl|l} & 11'97'16 & 34 \\ 3 \times 3 = & 9 & \\ \hline 3 \times 20 = 60 & 297 & \end{array}$$

Now subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); $(60 + 4) \times 4 = 256$. If this product were larger than 297 it would indicate that the trial figure is too large, and a figure one unit smaller should be used.

Then move down the next period of two figures and annex it to the remainder.

$$\begin{array}{rl|l} & 11'97'16 & 34 \\ 3 \times 3 = & 9 & \\ \hline 3 \times 20 = 60 & 297 & \\ (60 + 4) \times 4 = & 256 & \\ \hline & 4116 & \end{array}$$

Now multiply the figures of the root thus far obtained by 20; $(34 \times 20 = 680)$, and find how many times this product is contained in 4116. This gives us a trial figure for the third figure of the root; 680 is contained 6 times in 4116, and 6 is therefore placed as the third figure of the root. Then subtract from 4116 the product of 680 plus the figure of the root just obtained (6), multiplied by the same figure (6).

$$\begin{array}{rl|l} & 11'97'16 & 346 \\ 3 \times 3 = & 9 & \\ \hline 3 \times 20 = 60 & 297 & \\ (60 + 4) \times 4 = & 256 & \\ \hline 34 \times 20 = 680 & 4116 & \\ (680 + 6) \times 6 = & 4116 & \\ \hline \end{array}$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716.

If there is a remainder when the last period of figures has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, multiply the number so far obtained in the root by 20, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

$$\begin{array}{rl|l} & 1'25 & 11.18 \\ 1 \times 1 = & 1 & \\ \hline 1 \times 20 = 20 & 25 & \\ (20 + 1) \times 1 = & 21 & \\ \hline 11 \times 20 = 220 & 400 & \\ (220 + 1) \times 1 = & 221 & \\ \hline 111 \times 20 = 2220 & 17900 & \\ (2220 + 8) \times 8 = & 17824 & \\ \hline \end{array}$$

It will be seen from the calculation that when multiplying by the constant 20, the decimal point is disregarded, and the figures obtained in the root considered as a whole number. The decimal point must, however, be placed in the root as already explained before annexing the two first ciphers (not in the given number) to the remainder, in order to give a correct value to the root.

When extracting the square root of a decimal fraction, or when the square root of a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of two figures each, *beginning at the decimal point*, thus:

2'17'63.56'78'5

If the number of decimal places is not an even number, the period to the right will have only one figure instead of two. By placing a cipher after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

2'17'63.56'78'50

It should be borne in mind that the pointing off of periods of two figures each should always be begun at the decimal point, both for the whole numbers and for the decimals. Thus, for instance, the pointing off in the first line below is correct, while the pointing off in the second line is incorrect:

Correctly pointed off:	0.76'34'5	3'26.75'4
Incorrectly pointed off:	0.7'63'45	32'6.7'54

When extracting the square root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

Example:

$$\begin{array}{rl|l}
 & 5.71'21 & 2.39 \\
2 \times 2 = & 4 & \\
\hline
2 \times 20 = 40 & 171 & \\
(40 + 3) \times 3 = & 129 & \\
\hline
23 \times 20 = 460 & 4221 & \\
(460 + 9) \times 9 = & 4221 & \\
\hline
\end{array}$$

When it is found that the next figure in the root is a cipher, place it as usual in the root, and move down the next period of two figures, in all other respects following the procedure already explained.

Example:

$$\begin{array}{rl|l}
 & 9'12'04 & 302 \\
3 \times 3 = & 9 & \\
\hline
\left.\begin{array}{r} 3 \times 20 = 60 \\ 30 \times 20 = 600 \end{array}\right\} & 1204 & \\
(600 + 2) \times 2 = & 1204 & \\
\hline
\end{array}$$

The square root of a common fraction may be obtained by extracting the square root of both numerator and denominator, thus:

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$$

When the terms of the fraction are not perfect squares (squares of whole numbers), it is preferable to change the common fraction to a decimal fraction, and extract the square root of this.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by multiplying the root by itself, in which case the product must equal the number given, of which the square root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation; if this approximate root is multiplied by itself, the product should *very nearly* equal the given number; if not, an error has been made.

CHAPTER II

CUBE ROOT

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1{,}728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read "two cube." In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in these expressions is called *exponent*. An expression of the form 18^3 may also be read the "third power of 18."

In the same way as square root means the reverse of square, so cube root means the reverse of cube; that is, the cube root of a given number is the number which, if repeated as factor three times, would give the number given. Thus the cube root of 27 is 3, because $3 \times 3 \times 3 = 27$. If the cube of 15 is 3,375, then the cube root of 3,375 is, of course, 15. The mathematical sign for the cube root is $\sqrt[3]{}$, thus:

$\sqrt[3]{64} = 4$ (the cube root of sixty-four equals four),

$\sqrt[3]{4096} = 16$ (the cube root of four thousand ninety-six equals sixteen).

In the case of all roots, except the square root, the index, or the small figure in the radical sign ($\sqrt{}$), must be given.

Assume that the cube root of 80,621,568 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of *three* figures each. According to the total number of figures in the given number, the last period to the left will, of course, have one, two or three figures.

80'621'568 | Space for root.

Now find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root. The cube of 4 is 64 ($4 \times 4 \times 4 = 64$), and the cube of 5 is 125 ($5 \times 5 \times 5 = 125$). Hence 4 is the greatest whole number, the cube of which does not exceed 80, and 4, therefore, is the first figure of the root. Subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder, thus:

$$\begin{array}{rr|l} & 80'621'568 & 4 \\ 4 \times 4 \times 4 = & 64 & \\ \hline & 16621 & \end{array}$$

Now multiply the square of the figure in the root by the constant 300, which is always used when extracting the cube root by this method ($4^2 \times 300 = 4 \times 4 \times 300 = 4{,}800$), and find how many times this product is contained in the number 16,621. This gives us a trial figure for the second figure of the root; 4,800 is contained three whole times in 16,621, and 3 is therefore placed as the next figure of the root:

$$\begin{array}{rr|l} & 80'621'568 & 43 \\ 4 \times 4 \times 4 = & 64 & \\ \hline 4^2 \times 300 = 4{,}800 & 16621 & \end{array}$$

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root, excepting the last one, multiplied by 300, and this product multiplied by the figure just obtained in the root, thus:

$$4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14{,}400.$$

2. The figure or figures already obtained in the root, excepting the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained, thus:

$$4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1{,}080.$$

3. The cube of the last figure obtained, thus:

$$3^3 = 3 \times 3 \times 3 = 27.$$

The method followed will be understood by studying the example and comparing the different quantities with the worded explanations just given. If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used.

Now move down the next period of three figures, and annex it to the remainder.

$$\begin{array}{rr|l} & 80'621'568 & 43 \\ 4 \times 4 \times 4 = & 64 & \\ \hline 4^2 \times 300 = 4{,}800 & 16621 & \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & 15507 & \\ \hline & 1114568 & \end{array}$$

Multiply the square of the figures of the root thus far obtained by 300 ($43^2 \times 300 = 43 \times 43 \times 300 = 554{,}700$), and find how many times this product is contained in 1,114,568. This gives a trial figure for the third figure of the root; 554,700 is contained two times in 1,114,568, and 2 is therefore placed as the third figure of the root. Now subtract from 1,114,568 a sum made up of the three products previously given, and shown in the example below:

	80'621'568	432
$4 \times 4 \times 4 =$	64	
$4^2 \times 300 = 4{,}800$	16621	
$4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 =$	15507	
$43^2 \times 300 = 554{,}700$	1114568	
$43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 =$	1114568	

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 432 is the exact cube root of 80,621,568.

If there is a remainder when the last period of three figures has been moved down, place a decimal point after the figures already obtained in the root, annex three ciphers (000) to the remainder, multiply the square of the number thus far obtained in the root by 300, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

	1'816	12.2
$1 \times 1 \times 1 =$	1	
$1^2 \times 300 = 300$	816	
$1^2 \times 300 \times 2 + 1 \times 30 \times 2^2 + 2^3 =$	728	
$12^2 \times 300 = 43{,}200$	88000	
$12^2 \times 300 \times 2 + 12 \times 30 \times 2^2 + 2^3 =$	87848	

It should be noted in these calculations that when squaring the figures thus far obtained in the root, and multiplying by the constant 300, the decimal point is disregarded and the figures obtained in the root considered as a whole number. The decimal point, must, however, be placed in the root as already explained, before annexing the first three ciphers (not in the given number) to the remainder, in order to give a correct value of the root.

When the cube root of a number containing a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of three figures each, *beginning at the decimal point*, thus:

83'675'731.563'75

If the number of decimal places is not evenly divisible by three, the period to the right will have only one or two figures instead of three. By placing one or two ciphers after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

83'675'731.563'750

It should be borne in mind that the pointing off of periods of three figures each should always be begun at the decimal point, both for the whole number and for the decimals. Thus, for instance, the pointing off in the first line below is correct while the pointing off in the second line is incorrect:

Correctly pointed off:	0.765'354'3	2'765.354'2
Incorrectly pointed off:	0.7'653'543	27'65.3'542

When extracting the cube root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

When it is found that the next figure in the root is a cipher, place it as usual in the root and move down the next period of three figures, in all other respects following the procedure already explained.

The cube root of a common fraction may be obtained by extracting the cube root of both the numerator and denominator, thus:

$$\sqrt[3]{\frac{27}{1000}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1000}} = \frac{3}{10}$$

When the terms of the fraction are not perfect cubes (cubes of whole numbers), it is preferable to change the common fraction to a decimal fraction and then extract the cube root.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by repeating the root as a factor three times, in which case the product must equal the number given, of which the cube root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation. If this approximate root is repeated as a factor three times the product should *very nearly* equal the given number; if not, an error has been made.

CHAPTER III

THE USE OF FORMULAS

In mathematical and mechanical books and treatises, as well as in articles containing calculations published in the engineering journals, formulas are used to a great extent instead of rules expressed in words. In these formulas, signs and symbols are used in order to condense into a small space the essentials of what would otherwise be long and cumbersome rules. The symbols used are generally the letters in the alphabet, and the signs are simply the ordinary signs for arithmetical calculations, with some additional ones necessary for special purposes. Letters from the Greek alphabet are commonly used to designate angles, and the Greek letter π (pi) is always used to indicate the proportion of the circumference of a circle to its diameter; π, therefore, is always, in formulas, equal to 3.1416. The most commonly used Greek letters, besides π, are α (alpha), β (beta), and γ (gamma).

Knowledge of algebra is not necessary in order to make possible the successful use of formulas for the solving of problems such as occur in ordinary shop practice; but a thorough understanding of the rules and processes of arithmetic is very essential. The symbols or letters used in the formulas simply stand in place of the actual figures or numerical values which are inserted in the formula in each specific case, according to the requirements of the problem to be solved. When these values are inserted, the result required may be obtained by simple arithmetical processes.

There are two main reasons why a formula is preferable to a rule expressed in words. Firstly, the formula is more concise, it occupies less space, and it is possible for the eye to catch at a glance the whole meaning of the rule laid down; secondly, it is easier to remember a short formula than a long rule, and it is, therefore, of greater value and convenience, as it is not always possible to carry a handbook or reference book about, but the memory must be relied upon to store up a number of the most frequently occurring mathematical and mechanical rules.

The use of formulas can be explained most readily by actual examples. In the following, therefore, a number of simple formulas will be given, and the values will be inserted so as to show, in detail, the principles involved.

Example 1.—When the diameter of a circle is known, the circumference may be found by multiplying the diameter by 3.1416. This rule, expressed as a formula, is:

$$C = D \times 3.1416$$

in which C = circumference of circle,

D = diameter of circle.

This formula shows at a glance that no matter what the diameter of the circle be, the circumference is always equal to the diameter times 3.1416. Let it be required to find, for example, the circumference of a circle 24 inches in diameter. If, then, we insert 24 in place of D in the formula, we have:

$$C = 24 \times 3.1416 = 75.3984 \text{ inches.}$$

Hence, our formula gives, by means of a simple multiplication, the result required.

Assume that the diameter of a circle is 5.13 inches. The circumference of this circle is found by inserting this value instead of D in the formula:

$$C = 5.13 \times 3.1416 = 16.1164 \text{ inches.}$$

Example 2.—In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the diametral pitch. Then the formula would be:

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter (D) of the gear equals 2 added to the number of teeth (N), this sum divided by the pitch (P).

If the number of teeth in a gear is 26 and the diametral pitch 4, then simply put these figures in the place of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{26 + 2}{4} = \frac{28}{4} = 7.$$

D, or the outside diameter, then, is 7 inches.

In another gear the number of teeth is 62 and the pitch 8; find the outside diameter of the gear.

$$D = \frac{62 + 2}{8} = \frac{64}{8} = 8 \text{ inches.}$$

From the examples given it will be seen that in formulas, each letter stands for a certain dimension or quantity. When using a formula for solving a problem, replace the letters in the formula by the equivalent figures given in a certain problem, and find the result by means of regular arithmetical calculation.

Example 3.—The formula for the horsepower of a steam engine is as follows:

$$\text{H. P.} = \frac{P \times L \times A \times N}{33{,}000}$$

in which H. P. = indicated horsepower of engine,
P = mean effective pressure on piston in pounds per square inch,
L = length of piston stroke in feet,
A = area of piston in square inches,
N = number of strokes of piston per minute.

Assume that $P = 120$, $L = 2$, $A = 320$ and $N = 160$; what would be the horsepower?

If we insert the given values in the formula we have:

$$\text{H. P.} = \frac{120 \times 2 \times 320 \times 160}{33{,}000} = 372.36$$

In formulas the sign for multiplication ($\times$) is often left out between letters, the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$\frac{P \times L \times A \times N}{33{,}000} \text{ can also be written } \frac{PLAN}{33{,}000}$$

Thus, if $A = 6$ and $B = 7$, then:

$$AB = A \times B = 6 \times 7 = 42.$$

If $A = 9$, $B = 6$ and $C = 7$, then:

$$ABC = A \times B \times C = 9 \times 6 \times 7 = 378.$$

It is only the multiplication sign ($\times$) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic.

A parenthesis ( ) or bracket [ ] in a formula means that the expression inside the parenthesis or bracket should be considered as one single symbol, or in other words, that the calculation inside the parenthesis should be carried out by itself, before other calculations are carried out.

Examples:

$5 \times (8 + 4) = 5 \times 12 = 60.$

$7 \times (18 - 6) + 6\ (4.52 - 1.95) = 7 \times 12 + 6 \times 2.57 = 84 + 15.42 = 99.42.$

In the last example above it will be seen that 7 is multiplied by 12, and 6 by 2.57, and then the products of these two multiplications are added. From the order of the numbers $7 \times 12 + 6 \times 2.57$, one might have assumed that the calculation should have been carried out as follows: 7 times 12 = 84, plus 6 = 90, times 2.57 = 231.3. This latter procedure, however, is not correct, as the following rule should be applied:

When several numbers or expressions are connected by the signs +, —, $\times$ and $\div$, the operations are carried out in the order written, except that *all multiplications should be carried out before the other operations.* The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. Divisions should be carried out before additions and subtractions, if the division is indicated in the same line with these other processes.

Examples:

$4 \times 7 + 9 - 2 \times 9 = 28 + 9 - 18 = 37 - 18 = 19.$

$6 + 7 \times 4 = 6 + 28 = 34.$

$72 \div 3 \times 8 = 72 \div 24 = 3.$

$8.5 + 16.4 \div 4.1 - 2.5 = 8.5 + 4 - 2.5 = 10.$

But $4 \times (7 + 9) - 2 \times 9 = 4 \times 16 - 18 = 64 - 18 = 46.$

$(6 + 7) \times 4 = 13 \times 4 = 52.$

$(72 \div 3) \times 8 = 24 \times 8 = 192.$

$(8.5 + 16.4) \div (4.1 - 2.5) = 24.9 \div 1.6 = 15.56.$

In Chapters I and II the meaning of square and cube, and square root and cube root have already been explained. The squares and square roots as well as the cubes and cube roots of all numbers up to 1,000 (sometimes up to 1,600) are generally given in all standard handbooks.

Example:—Find the value of A in the formula

$$A = \sqrt{B^2 + C^2}$$

if $B = 16$ and $C = 12$.

If we insert the given values in the formula, we have

$$A = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20.$$

In the same way as we write $2^2 = 2 + 2$, and $2^3 = 2 \times 2 \times 2$, we can write $2^4 = 2 \times 2 \times 2 \times 2$; and the expression 2^5 would mean that 2 is repeated as a factor five times, or

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

The expression 2^4 is read "the fourth power of 2" and 6^5, "the fifth power of 6," etc.

In the same way as we may say that the square root means the reverse of square, and the cube root the reverse of cube, so we may say that the fourth root is the reverse of the fourth power; that is, if we want the number which repeated as a factor four times gives a given number, we must obtain the fourth root, or $\sqrt[4]{\ }$. Thus $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Similarly we write the fifth root $\sqrt[5]{\ }$; and $\sqrt[5]{32} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

The examples given indicate the principles involved in the use of formulas, and show also how easily formulas may be employed by anyone who has a general understanding of arithmetic. While it would be possible to express in words all the rules required in ordinary

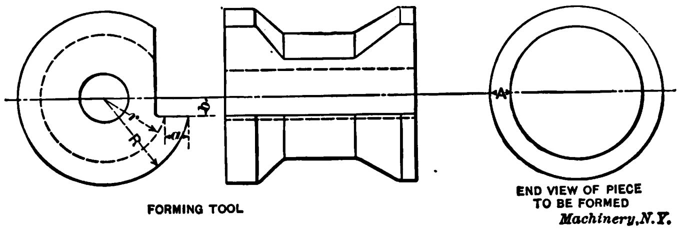


Fig. 1

shop problems, it is very much simpler to make use of formulas; and in the following, formulas will be employed wherever required, and their use in practical work thus made clear.

A useful application both of the use of formulas and of the square and square root of numbers, is found in the problems occurring when figuring forming tools.

Formulas for Circular Forming Tools

When laying out circular forming tools, such as shown in Fig. 1, the cutting edge, as is well known, must be located a certain amount below the horizontal center line of the tool, in order to provide for sufficient clearance for the cut. On account of this, the actual differences of diameters in the piece of work to be formed cannot be directly copied in the forming tool. The distance A in the piece to be formed must equal the distance a on the forming tool, but as this latter distance is measured in a plane a certain distance b below the horizontal plane through the center of the forming tool, it is evident that the differences of diameters in the tool and the piece to be formed are not the same. A general formula may, however, be deduced, by the use of elementary geometry, by means of which the various diameters of the forming tool may be determined if the largest (or smallest) diameter of the tool, the amount that the cutting edge is below the center, and, of course, the diameters of the piece to be formed, are known.

If R = the largest radius of the tool,

a = difference in radii of steps in the work, and

b = amount cutting edge is below center,

then, if r be the radius required,

$$r = \sqrt{(\sqrt{R^2 - b^2} - a)^2 + b^2}$$

If the smaller radius r is given and the larger radius R sought, the formula takes the form:

$$R = \sqrt{(\sqrt{r^2 - b^2} + a)^2 + b^2}$$

Suppose, for an example, that a tool is to be made to form the piece in **Fig.** 2. Assume that the largest diameter of the tool is to be 3

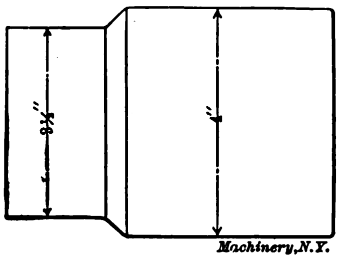


Fig. 2

inches, and that the cutting edge is to be ¼ inch below the center of the tool. Then the diameter next smaller to 3 inches is found from the formulas given by inserting the given values: $R = 1\frac{1}{2}$ inch, $b = \frac{1}{4}$ inch, and $a = \frac{1}{4}$ inch (half the difference between 4 and 3½ inches; see **Fig.** 2).

Then

$$r = \sqrt{(\sqrt{(1\tfrac{1}{2})^2 - (\tfrac{1}{4})^2} - \tfrac{1}{4})^2 + (\tfrac{1}{4})^2} = \sqrt{(\sqrt{\tfrac{35}{16}} - \tfrac{1}{4})^2 + \tfrac{1}{16}} = \frac{5.017}{4}$$

$= 1.254$ inch.

While the formula looks complicated, by means of a table of squares the calculations are easily simplified and can be carried out in three or four minutes. The value of r being 1.254 inch, the diameter to make the smaller step of the forming tool will be 2.508 inches, instead of 2½ inches exact, as would have been the case if the cutting edge had been on the center line.

CHAPTER IV

TIME REQUIRED FOR DRILLING, MILLING AND PLANING

In MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, a rule is given for calculating the time required for turning in the lathe, with a given feed. In this chapter, rules and formulas will be given for calculating the time required for drilling, milling and planing.

The feed of a drill in the drill press is the downward motion of the drill per revolution. The feed of a milling cutter is the forward movement of the milling machine table for each revolution of the cutter. Sometimes the feed is expressed as the distance which the drill or the milling machine table moves forward in one minute. In order to avoid confusion, it is, therefore, always best to state plainly in each case whether feed per revolution or feed per minute is meant.

Time Required for Drilling

In order to calculate the time required for drilling a given depth of hole, the number of revolutions per minute of the drill, and the feed per revolution (or the cutting speed, the diameter of the drill and the feed per revolution) must be known.

Assume that a 1⅞-inch drill makes 80 revolutions per minute and that the feed per revolution is 0.008 inch. How long a time will it require to drill a hole 5½ inches deep? To find the number of revolutions required to drill the full depth of the hole, divide 5½ by 0.008, obtaining the quotient 687.5 or approximately 690 revolutions. As the drill makes 80 revolutions in one minute, we find the total number of minutes required by dividing 690 by 80, the quotient 8.6 being the number of minutes required to drill a hole 5½ inches deep under the given conditions. If, in the foregoing,

T = time required for drilling, in minutes,
L = depth of drilled hole, in inches,
N = number of revolutions per minute of the drill,
F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}.$$

Expressed as a rule, this formula would be:

To find the time required to drill a hole to a given depth when the feed per revolution of the drill, the depth of the hole, and the number of revolutions per minute are given, divide the depth of the hole by the number of revolutions per minute multiplied by the feed per revolution.

If the cutting speed of the drill and its diameter are given instead of the number of revolutions, the number of revolutions must first be found before applying the formula given.*

If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

The feed of drills should be about 0.004 inch per revolution for a 1/16-inch drill, 0.005 inch for a ¼-inch drill, 0.008 inch for a ½-inch drill, 0.010 inch for a 1-inch drill, and 0.015 inch for a 2-inch drill. If the drill breaks or chips at the cutting edges, the feed should be reduced.

Time Required for Milling

The time required for milling may be found if the number of revolutions per minute of the cutter, and the feed per revolution (or the cutting speed, the diameter of the cutter and the feed per revolution) are known. If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

If the length of the cut taken in a milling machine is 8⅜ inches and the feed is 1/64 per revolution, how long a time will it take for a cutter making 20 revolutions per minute to traverse the work? As the feed per revolution is 1/64 inch and the cutter makes 20 revolutions per minute, the feed per minute is 20/64 or 5/16 inch. To find the time required for the cutter to traverse the full length of the work, divide the length of the cut, 8⅜ inches, by the feed in one minute; thus:

$$8\frac{3}{8} \div \frac{5}{16} = \frac{67}{8} \times \frac{16}{5} = \frac{134}{5} = 26\frac{4}{5} = 26.8.$$

The time required would thus be 27 minutes, approximately.

If T = time required for the cutter to traverse the work, in minutes,
L = length of cut, in inches,
N = revolutions per minute of the cutter,
F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}.$$

It will be seen that the form of this formula is the same as that of the formula for the time required for drilling.

If the cutting speed and the diameter of the cutter are given instead of the number of revolutions, the latter number must first be found before the formula above is applied.*

The average feed of milling cutters per minute should vary from about 4 inches for a ½-inch mill cutting cast iron, and 1½ inch for the same mill cutting steel, to 1¼ inch for a 6-inch cutter on cast iron and ½ inch for the same cutter on steel. Of course, these feeds must be varied with the depth of the cut.

---

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition.

Feed of Planer Tools

The feed of a planer tool is its sidewise motion for each cutting stroke of the table or platen. If for each cutting stroke the tool-carrying head moves 1/16 inch along the cross-rail, we say that the feed is 1/16 inch. Each cutting stroke necessitates a return stroke, and in the following, when the expression "number of strokes" is used, it means number of cutting strokes.

Time Required for Planing

The time required for planing a piece of work can be calculated if the feed per stroke, the number of strokes of the planer table per minute, and the width of the work, are known.

Assume that a planer makes 6 strokes per minute, that the feed per stroke is 3/32 inch, and that the width of the work is 22 inches. Find the time required for planing the work.

As the planer makes 6 strokes per minute and the feed per stroke is 3/32 inch, the feed per minute is 6 × 3/32 or 9/16 inch. The tool must traverse 22 inches to plane the complete work; the traverse in one minute being 9/16 inch, the total number of minutes required to traverse the work is found by dividing 22 by 9/16.

$$22 \div \frac{9}{16} = \frac{22}{1} \times \frac{16}{9} = \frac{352}{9} = 39\frac{1}{9} \text{ minutes.}$$

The time required for planing the work is thus 40 minutes, approximately.

This calculation may be summed up in the following formula, applicable to any case where the feed per stroke, the number of strokes per minute, and the width of the work are known:

$$T = \frac{W}{F \times N}.$$

In this formula

T = time required for planing, in minutes,

W = width of work, in inches,

F = feed per stroke, in inches,

N = number of strokes per minute.

The formula expressed as a rule would be as follows:

To find the time required for planing when the width of the work, the feed per stroke and the number of strokes per minute, are known, divide the width of the work by the feed times the number of cutting strokes per minute.

CHAPTER V

PULLEY AND GEAR DRIVES

In MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, the calculations for simple and compound gear drives and simple pulley drives are treated. In this chapter some special cases of compound pulley drives and combined pulley and gear drives will be considered.

Compound Pulley Speeds

In Fig. 3 are shown four pulleys of which the two pulleys *B* and *C* are keyed to the same shaft. Pulley *A* is the driving pulley and drives pulley *B*; pulley *C*, on the same shaft as *B*, is also a driving pulley, and pulley *D*, a driven pulley. The rules and formulas for compound

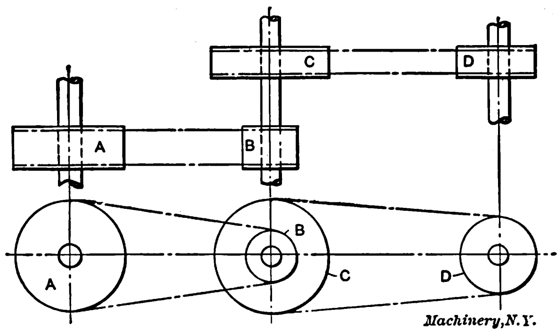


Fig. 3. Compound Pulley Drive

gearing can be directly applied to pulleys arranged in this manner by simply substituting in the formulas the diameters of the pulleys, in inches, for the numbers of teeth in the gears. Thus, to find the revolutions per minute of the driven pulley *D* when the diameters of all the four pulleys and the number of revolutions of pulley *A* are given, the formula below is used:

$$\begin{array}{c}\text{rev. per min.}\\ \text{of driven pulley}\end{array} = \begin{array}{c}\text{rev. per min.}\\ \text{of driving pulley}\end{array} \times \frac{\text{product of diameters of driving pulleys}}{\text{product of diameters of driven pulleys}}$$

If the numbers of revolutions of the shafts on which pulley *A* and pulley *D* are mounted, are given, and it is required to find the diameters of four pulleys which will transmit motion from pulley *A* to pulley *D* at the given speed ratio, we proceed in the same way as when finding the number of teeth in gears for transmitting a given motion.*

Find the speed ratio by writing the number of revolutions of the

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 30.

driving pulley as the numerator and the number of revolutions in the driven pulley as the denominator of a fraction, and reduce this fraction to its lowest terms. Then divide both the numerator and denominator in the fraction giving the ratio in two factors, and multiply each "pair" of factors by the same number until pulleys with suitable diameters are found. **(One factor in the numerator and one in the denominator are considered as "one pair.")**

Assume that the number of revolutions per minute of the shaft with pulley *A* is 260, and that it is required to drive the shaft on which pulley *D* is mounted at 720 revolutions. What diameters of pulleys can be used? The fraction $\frac{260}{720}$ reduced to its lowest terms is

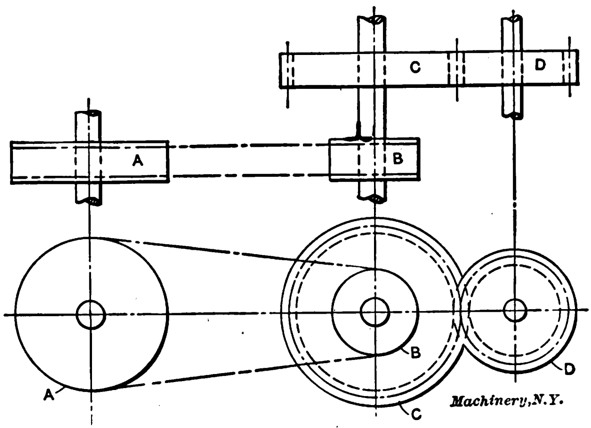


Fig. 4. Combined Pulley and Gear Drive

$\frac{13}{36}$; $\left(\frac{260}{720}=\frac{26}{72}=\frac{13}{36}\right)$. The speed ratio, therefore, is $\frac{13}{36}$. Now, following the rule given above:

$$\frac{13}{36}=\frac{1\times13}{2\times18}=\frac{(1\times12)\times(13\times1)}{(2\times12)\times(18\times1)}=\frac{12\times13}{24\times18}$$

The pulleys in the numerator, with 12 and 13 inches diameter, are the driven pulleys *B* and *D*, and the pulleys in the denominator, with 18 and 24 inches diameter, are the driving pulleys. The rule above reduced to a formula would be:

$$\text{ratio of speed of the first driving pulley to the last driven pulley}=\frac{\text{product of diam. of driven pulleys}}{\text{product of diam. of driving pulleys}}$$

Combined Belt and Gear Drive

In Fig. 4 is shown a combined belt and gear drive, where pulley *A* drives pulley *B*, and gear *C*, which is mounted on the same shaft as pulley *B*, drives the gear *D*. Calculations for numbers of revolutions and numbers of teeth and diameters of pulleys are carried out exactly

as in the examples where we have dealt exclusively with gears or exclusively with pulleys. When dealing with the pulleys we use the diameter of the pulley in inches, and when dealing with the gears, the number of teeth in the gears.

Assume that the diameter of pulley *A* is 54 inches and that of pulley *B*, 18 inches, that gear *C* has 112 teeth, and that gear *D* has 78 teeth. If pulley *A* makes 39 revolutions per minute, how many revolutions per minute does gear *D* make? Using the formula for finding the revolutions per minute previously given, we have:

$$\text{rev. per min.} = 39 \times \frac{54 \times 112}{18 \times 78} = 168.$$

If the number of revolutions of the shaft on which pulley *A* is mounted is 60, and the number of revolutions required for the shaft on which gear *D* is mounted, is 110, what diameter pulleys and what size of gears could we employ to transmit the required motion? The speed ratio is $\frac{60}{110} = \frac{6}{11}$. Proceeding as before, we have:

$$\frac{6}{11} = \frac{2 \times 3}{1 \times 11} = \frac{(2 \times 16) \times (3 \times 8)}{(1 \times 16) \times (11 \times 8)} = \frac{32 \times 24}{16 \times 88}$$

The numbers 32 and 24 in the numerator of the last fraction give the diameter of the driven pulley *B* and the number of teeth of the driven gear *D*, respectively, and the numbers 16 and 88 in the denominator of the fraction give the diameter of the driving pulley *A*, and the number of teeth in the driving gear *C*. In this case, then, pulley *A* would be 16 inches in diameter, pulley *B*, 32 inches, gear *C* would have 88 teeth, and gear *D*, 24 teeth.

CHAPTER VI

HORSEPOWER OF BELTING

The horsepower which a belt of a given size can transmit depends on the speed with which the belt travels and the working stress advisable to permit in the belt. The speed with which the belt travels, of course, depends on the diameter and number of revolutions per minute of the pulley over which it travels, it being assumed that there is no appreciable slip between the belt and the pulley. If we are to find the horsepower a belt can safely transmit, we must, therefore, consider in our formulas the diameter of the pulley, its number of revolutions per minute, and the permissible working stress in the belt.

Let d = diameter of driving pulley in inches,
v = velocity of belt in feet per minute,
n = number of revolutions of pulley per minute,

S = working stress of belt per inch of width, in pounds,
w = width of belt in inches.

Then:

$$v = \frac{\pi d n}{12} = \frac{3.1416\, d n}{12} = 0.2618\, d n$$

$$\text{H. P.} = \frac{S v w}{33,000} = \frac{0.2618\, S d n w}{33,000}$$

A commonly used value for the safe working stress per inch of width of single belts is 33 pounds. When this value is adopted, a belt one inch wide, traveling at a rate of 1,000 feet per minute, will transmit one horsepower.

Example:—How many horsepower will a single belt 2½ inches wide, traveling over a pulley 12 inches in diameter, transmit, if the pulley makes 200 revolutions per minute? Assume the working stress at 33 pounds per inch of width of belt.

In this example $d = 12$, $n = 200$, $S = 33$ and $w = 2\frac{1}{2}$. If these values are inserted in the horsepower formula given, we have:

$$\text{H. P.} = \frac{0.2618 \times 33 \times 12 \times 200 \times 2.5}{33,000} = 1.57.$$

A working stress up to 45 pounds per inch of width of belt is permissible for single belts in good condition. If we adopt this latter value for the stress, how many horsepower would the given belt transmit?

We only need to change 33 in the expression above to 45, and then we have:

$$\text{H. P.} = \frac{0.2618 \times 45 \times 12 \times 200 \times 2.5}{33,000} = 2.14.$$

If the horsepower to be transmitted is known, the width of belt required may be found by a transposition of the given formula, as follows:

$$w = \frac{\text{H. P.} \times 33,000}{S v} = \frac{\text{H. P.} \times 33,000}{0.2618\, S d n}$$

in which formula the letters denote the same quantities as previously given.

Example: Find the width of single belt required to transmit 20 horsepower with a belt velocity of 1,800 feet per minute?

In this example H. P. $= 20$, $v = 1,800$, and S may be assumed to be 45. If we insert these values in the given formula for width of belt, we have:

$$w = \frac{20 \times 33,000}{45 \times 1,800} = 8.15 \text{ or, say, } 8\tfrac{1}{4} \text{ inches.}$$

In order to reduce the width of a single belt when it becomes too wide, a double belt may be used. The working stress of a double belt per inch of width may be assumed at from 65 to 90 pounds, the latter value being only for belts kept in good condition.

Assume that in the example just given, we use a double belt instead of a single, and assume a working stress of 80 pounds per inch of width of belt. How wide, then, would the belt be?

Substituting 80 for 45, we have:

$$w = \frac{20 \times 33{,}000}{80 \times 1{,}800} = 4.58 \text{ or, say, } 4\tfrac{3}{4} \text{ inches.}$$

As the working stress is an assumed quantity, always somewhat uncertain, it is, of course, not necessary to retain in our formulas so exact a quantity as 0.2618. If this number is given in round figures as 0.25 or ¼, we could simplify the given formulas as follows:

$$\text{H. P.} = \frac{S\,d\,n\,w}{4 \times 33{,}000}$$

$$w = \frac{\text{H. P.} \times 33{,}000 \times 4}{S\,d\,n}$$

As a final example, find the horsepower transmitted by a 5-inch wide double belt, working stress 75 pounds per inch width of belt, if the belt transmits power from a 4-foot pulley running at 200 revolutions per minute.

In this example $w = 5$, $S = 75$, $n = 200$, and $d = 4 \times 12 = 48$ inches. If we insert these values in our simplified formula, we have:

$$\text{H. P.} = \frac{75 \times 48 \times 200 \times 5}{4 \times 33{,}000} = 27.3.$$

CHAPTER VII

CHANGE GEARS FOR CUTTING METRIC THREADS

The metric system of length measurement is in use in practically all countries except in the United States, Great Britain and the British colonies. The unit of length in the metric system is the meter, which equals nearly 39.37 inches (or practically 39⅜ inches). The subdivisions of the meter are given below:

1 meter = 10 decimeters,
1 decimeter = 10 centimeters.
1 centimeter = 10 millimeters.

In medium and small machine design the unit employed is almost always the millimeter. One millimeter equals 0.03937 inch; one inch equals $\frac{1}{0.03937}$, or 25.4 millimeters, almost exactly.

When screws are made in accordance with the metric system it is not the usual practice to give the number of threads per millimeter or centimeter in the same way as the number of threads per inch is given in the English system. Instead, the lead of the thread in millimeters is given. A screw thread is said to have 2 millimeters lead, 3 millimeters lead, 4.5 millimeters lead, etc.

Change Gears for Cutting Threads with Metric Pitch

It often happens that screws and taps having threads according to the metric system are required. This thread can be cut on a lathe having an English lead-screw, provided change gears with the required number of teeth are used.

The first step in finding the change gears is to find how many threads per inch there are in the screw to be cut, when the lead is given in millimeters. Assume that a screw is required with 3 millimeters lead. How many threads per inch are there in this screw? As there are 25.4 millimeters in one inch, we can find how many threads there would be in one inch, if we find how many times 3 is contained in 25.4; in other words, we divide 25.4 by 3. It is not necessary to carry out the division; simply write it as a fraction in the form $\frac{25.4}{3}$, which implies that 25.4 is to be divided by 3. This fraction now gives the number of threads per inch to be cut. When this fraction has been obtained, proceed as if change gears were to be found for cutting threads with English pitches.* Place the lathe screw constant in the numerator of a fraction and the number of threads per inch to be cut in the denominator. If the screw constant of a lathe is 6 and the number of threads to be cut $\frac{25.4}{3}$, as previously found, the ratio of the change gearing is

$$\frac{6}{\frac{25.4}{3}} = \text{ratio.}$$

This seems complicated, but remembering that the line between the numerator and denominator in a fraction means that the numerator is to be divided by the denominator, we get, by carrying out this division:

$$6 \div \frac{25.4}{3} = 6 \times \frac{3}{25.4} = \frac{6 \times 3}{25.4}.$$

The fraction $\frac{6 \times 3}{25.4}$ is the ratio of the change gearing required, and all we have to do now is to multiply numerator and denominator of this fraction by the same number until we find suitable numbers of teeth for the change gears. By trial we find that the first whole number by which we can multiply 25.4 so as to get a whole number as a result, is 5. Multiplying 25.4 by 5 gives us 127. Thus we must have one gear with 127 teeth whenever we cut a metric thread by means of

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 81.

an English lead-screw. The other gear required in this case has 90 teeth, because 5 × 6 × 3 = 90. The calculation would be carried out as shown below:

$$\frac{6 \times 3 \times 5}{25.4 \times 5} = \frac{18 \times 5}{127} = \frac{90}{127}.$$

What has just been said can be expressed in the following rule:

To find the change gears for cutting metric pitches with an English lead-screw, place the lathe screw constant multiplied by the number of millimeters lead of the thread to be cut multiplied by 5, in the numerator of the fraction, and 127 as the denominator. The product of the numbers in the numerator give the number of teeth in the gear on the spindle stud, and 127 is the number of teeth in the gear on the lead-screw.

Written as a formula this rule would be:

$$\frac{\text{lathe screw constant} \times \text{lead of thread to be cut, in millimeters} \times 5}{127} = \frac{\text{teeth in spindle stud gear}}{\text{teeth in lead-screw gear}}$$

As an example, assume that a screw with 2.5 millimeters lead is to be cut on a lathe having a screw constant 8. By placing the given figures in the formula we have:

$$\frac{8 \times 2.5 \times 5}{127} = \frac{100 \ldots\text{spindle stud gear}}{127 \ldots\text{lead-screw gear}}$$

Compound Gearing

Sometimes it is necessary to compound the gears because the gear on the spindle stud would have too many teeth, that is, it would be too large to be used in simple gearing. It may also happen that the product of the screw constant × the lead in millimeters × 5, is not a whole number, in which case it would be necessary to compound the gears to get whole numbers of teeth.

The method for finding the change gears is exactly the same as the method for compound gears for cutting regular English pitch threads.*

Assume that a screw of 6 millimeters lead is to be cut on a lathe with a screw constant 8. By first applying the formula just given, and then dividing the numerator and denominator into factors, each "pair" of which are multiplied by the same number, we find the change gears as follows:

$$\frac{8 \times 6 \times 5}{127} = \frac{240}{127} = \frac{60 \times 4}{127 \times 1} = \frac{(60 \times 1) \times (4 \times 25)}{(127 \times 1) \times (1 \times 25)} =$$

$$\frac{60 \times 100 \ldots\text{driving gears}}{127 \times 25 \ldots\text{driven gears.}}$$

In a case when the lead of the metric screw to be cut is not a whole number but a fraction, it sometimes causes difficulty in dividing up the numerator in two factors that can be multiplied by whole num-

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 38.

bers so as to give numbers of teeth for gears which are available. Several trials must often be made.

Assume that the lathe screw constant is 6, and that a screw with 1.25 millimeters lead is to be cut. In this case we would find the change gears as below:

$$\frac{6 \times 1.25 \times 5}{127} = \frac{37.5}{127} = \frac{30 \times 1.25}{127 \times 1} = \frac{(30 \times 1) \times (1.25 \times 40)}{(127 \times 1) \times (1 \times 40)} = \frac{30 \times 50}{127 \times 40}.$$

It would not be necessary to write "30 × 1" and "127 × 1" as has been done in the example above, but these numbers have been multiplied by 1 simply to preserve a systematic appearance.

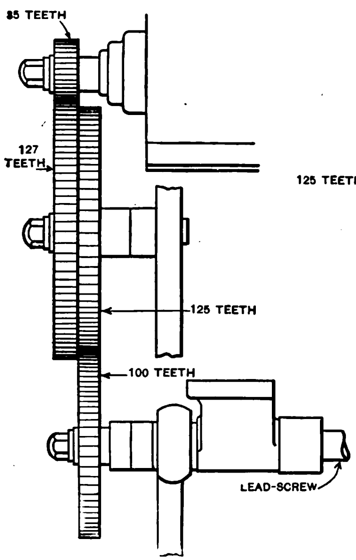


Fig. 5. Cutting a Metric Thread with an English Lead-screw

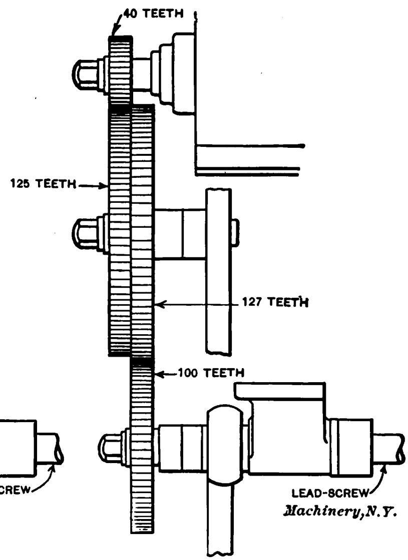


Fig. 6. Cutting an English Thread with a Metric Lead-screw

In Fig. 5 is shown the arrangement of the gearing when cutting a screw of 1.25 millimeters lead on a lathe with a screw constant 7.

$$\frac{7 \times 1.25 \times 5}{127} = \frac{43.75}{127} = \frac{35 \times 1.25}{127 \times 1} = \frac{(35 \times 1) \times (1.25 \times 100)}{(127 \times 1) \times (1 \times 100)} = \frac{35 \times 125}{127 \times 100}.$$

Cutting an English Thread with a Metric Lead-screw

If the lathe has a lead-screw having metric pitch, and it is required to cut a screw with a given number of threads per inch, we must find the "metric screw constant" of the lathe. This is found by placing gears (in simple gearing) with the same number of teeth on the spindle stud and the lead-screw of the lathe, and an idler with any number of teeth, between them, and then cutting a thread on a piece in the lathe. The lead of the thread thus cut, in millimeters, is the metric screw constant of the lathe. Now the method of figuring the change gears when a screw with a given number of threads per inch is to be cut with a lead-screw of metric pitch, is simply the reverse of the method already explained for cutting a metric thread with an English lead-screw.

To find the change gears for cutting English threads with a metric lead-screw, place 127 *in the numerator, and the threads per inch to be cut multiplied by the metric screw constant of the lathe multiplied by* 5 *in the denominator of the fraction;* 127 *is the number of teeth in the gear on the spindle stud, and the product of the numbers in the denominator gives the number of teeth in the gear on the lead-screw.*

This rule expressed as a formula would be:

$$\frac{127}{\text{metric screw constant} \times \text{threads per inch to be cut} \times 5} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}$$

Assume that 5 threads per inch are to be cut in a lathe having a metric screw constant of 4 millimeters. The gears are found directly by using the formula given:

$$\frac{127}{4 \times 5 \times 5} = \frac{127 \ldots\text{spindle stud gear}}{100 \ldots\text{lead-screw gear}}$$

It is sometimes necessary to compound the gears in order to obtain gears which are found in the set of change gears provided with the lathe.

Assume that 10 threads per inch are to be cut in a lathe with a metric screw constant of 4 millimeters. To find the gears we would proceed as follows:

$$\frac{127}{4 \times 10 \times 5} = \frac{127}{200} = \frac{127 \times 1}{100 \times 2} = \frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (2 \times 40)} =$$

$$\frac{127 \times 40 \ldots\text{driving gears}}{100 \times 80 \ldots\text{driven gears.}}$$

In Fig. 6 is shown the arrangement of the gearing when cutting a screw, having 12½ threads per inch in a lathe with a metric lead-screw, the metric screw constant being 5 millimeters.

$$\frac{127}{5 \times 12\frac{1}{2} \times 5} = \frac{127}{312.5} = \frac{127 \times 1}{100 \times 3.125} =$$

$$\frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (3.125 \times 40)} = \frac{127 \times 40}{100 \times 125}$$

CHAPTER VIII

AREAS OF PLANE FIGURES

Squares

The square, Fig. 7, has four sides of equal length, and each of the four angles between the sides is a right or 90-degree angle.

The area of the square equals the length of the side multiplied by itself, or the square of the length of the side. If the side of a square is 14 inches, then the area equals $14 \times 14 = 196$ square inches. If the side is 14 feet, then the area is 196 square feet.

If the area of a square is known, the length of the side equals the square root of the area. Assume that the area of a square equals 1,024 square inches. Then the side equals $\sqrt{1{,}024} = 32$ inches.

Rectangles

The rectangle, as shown in Fig. 8, has four sides, of which those opposite each other are of equal length, and the four angles between the sides are right or 90-degree angles.

The area of a rectangle is found by multiplying the height or altitude by the length or base. In Fig. 8, B is the altitude and C the base,

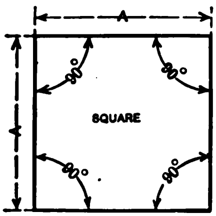


Fig. 7. Square

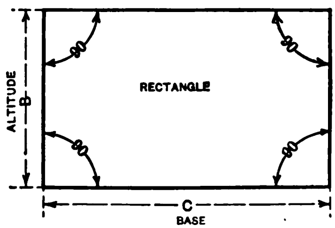


Fig. 8. Rectangle

and the area equals $B \times C$. If $B = 6$ inches, and $C = 11$ inches, then the area equals $6 \times 11 = 66$ square inches.

If the area of a rectangle and the length of its base are known, the height is found by dividing the area by the length of the known base. Either the longer or the shorter side may be considered as the base, the altitude being the side at right angles to the base. If, in Fig. 8, the area of the rectangle is 96 square inches and the side C is 12 inches, then the side $B = 96 \div 12 = 8$ inches.

One square foot equals $12 \times 12 = 144$ square inches. If the area is given in square feet, it can, therefore, be transformed into square inches by multiplying by 144. If the area is given in square inches, it can be transformed into square feet by dividing by 144.

Parallelograms

Two lines are said to be parallel when they have the same direction; when extended, they do not meet or intersect, and the same distance is maintained between the two lines at every point.

Any figure made up of four sides, of which those opposite are parallel, is called a parallelogram. The square and rectangle are parallelograms in which all the angles are right angles. In Fig. 9 is shown a parallelogram where two of the angles are less and two more than 90 degrees. A line drawn from one side of a parallelogram at right angles to the opposite side is called the height or altitude of the parallelogram. In Fig. 9, *D* is the altitude, and *E* is the length or base.

The area of a parallelogram equals the altitude multiplied by the base. The area of the parallelogram, in Fig. 9, equals $D \times E$. If *D* is 16 inches, and *E*, 22 inches, then the area equals $16 \times 22 = 352$ square inches.

If the area and the base are given, the altitude is found by dividing the area by the base.

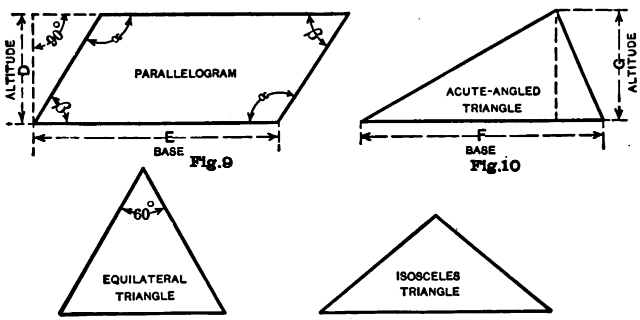


Fig. 11 Fig. 12

Figs. 9 to 12. Parallelogram and Triangles

In parallelograms the angles opposite each other are alike, as indicated in Fig. 9, where the two angles α are equal, and the two angles β also are equal.

Triangles

Any figure bounded by three straight lines is called a triangle. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle. In Fig. 10, if the side *F* is taken as the base of the triangle, then *G* is the altitude.

If all three sides of a triangle are of equal length, as in the one shown in Fig. 11, the triangle is called *equilateral*. Each of the three angles in an equilateral triangles equals 60 degrees.

If two sides are of equal length, as shown in Fig. 12, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 13; the side opposite the right angle is called the *hypotenuse*.

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown in Fig. 10. If one of the angles is larger than 90 degrees, as shown in Fig. 14, the triangle is called an *obtuse* or *obtuse-angled* triangle.

tudes of the two triangles into which the trapezium has been divided. If the dimensions of the base and height of the one triangle are R and S, respectively, and of the other T and V, as shown in Fig. 16, then the area of the whole trapezium would be $(\frac{1}{2} \times R \times S) + (\frac{1}{2} \times T \times V)$. Assume that $R = 20$ feet, $S = 17$ feet, $T = 23$ feet, and $V = 9$ feet, then the area of the trapezium $= (\frac{1}{2} \times 20 \times 17) + (\frac{1}{2} \times 23 \times 9) =$ 273.5 square feet.

The Circle

The circle is a plane surface bounded by a curved line called the *periphery* or *circumference*, which is at all points at an equal distance from a point within the circle called the center. The distance from the center of the circle to the periphery is the *radius*, and the distance across the circle through the center is the *diameter*. (See Fig. 17.) It is evident that the radius is one-half of the diameter. If a line is drawn from one point on the periphery to another point, so that it does not pass through the center, it is called a *chord*.

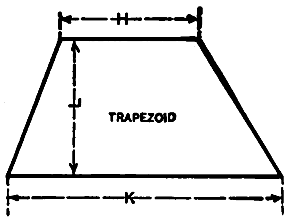


Fig. 15. Trapezoid

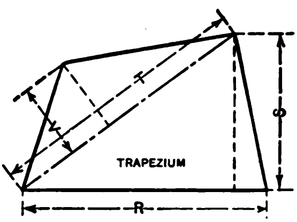


Fig. 16. Trapezium

If the diameter of a circle is known, the circumference is found by multiplying the diameter by 3.1416. Assume that the circumference of a circle is stretched out into a straight line by the circle rolling upon a flat surface and unfolding itself, as shown in Fig. 20, then the length of the straight line would be three times the diameter plus a distance equal to 0.1416 times the diameter; or the whole length of the circumference would be 3.1416 times the diameter. As the diameter equals 2 × radius, the circumference equals 2 × radius × 3.1416.

If the circumference of a circle is known, the diameter is found by dividing the circumference by 3.1416; the radius is found by dividing the circumference by 2 × 3.1416.

If D = diameter, R = radius, C = circumference, then the previous rules can be written as formulas, thus:

$$D = 2 \times R,$$
$$C = 2 \times R \times 3.1416,$$
$$C = D \times 3.1416,$$
$$R = \frac{C}{2 \times 3.1416}, \qquad D = \frac{C}{3.1416}.$$

Instead of writing out the number 3.1416, the Greek letter π (pi) is often used; thus, for example, $3\pi = 3 \times 3.1416$.

Example: The diameter of a circle is 6 inches; find its circumference.

Using the formula given, we have:
Circumference = 6 × 3.1416 = 18.8496 inches.
The circumference of a circle is 13.509 inches; find its radius.

$$\text{Radius} = \frac{13.509}{2 \times 3.1416} = 2.150 \text{ inches.}$$

The area of a circle equals the square of the radius multiplied by 3.1416; or, the square of the diameter multiplied by 0.7854.

If the area of a circle is known, the radius is found by extracting the square root of the quotient of the area divided by 3.1416.

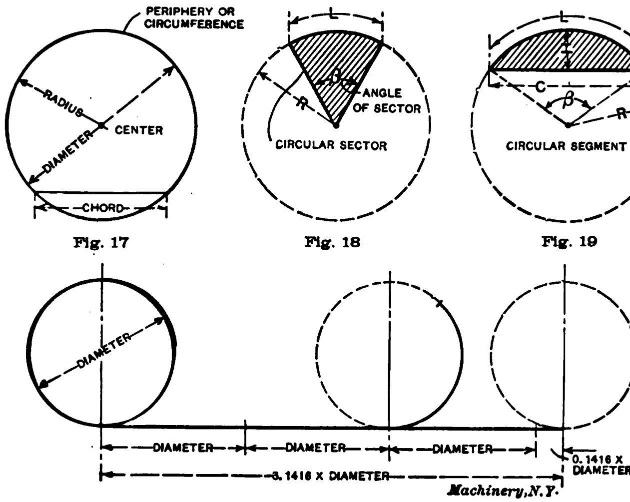


Fig. 17 Fig. 18 Fig. 19

Fig. 20

Figs. 17 to 20. Circles, Sectors and Segments

If D = diameter, R = radius, A = area,
then $A = R^2 \times 3.1416$,

$$A = \frac{D^2 \times 3.1416}{4} = D^2 \times 0.7854,$$

$$R = \sqrt{\frac{A}{3.1416}}.$$

Examples: The diameter of a circle is 6 inches, find the area.
Using the formula given, we have:
Area = $6^2 \times 0.7854 = 6 \times 6 \times 0.7854 = 28.2744$ square inches.
The area of a circle is 95.033 square inches, find the radius.
Using the formula given, we have:
Radius = $\sqrt{95.033 \div 3.1416} = 5.5$ inches.

Circular Sectors

A figure bounded by a part of the circumference of a circle and two radii, as shown in Fig. 18, is called a circular *sector*. The angle

β (beta) between the radii is called the angle of the sector, and the length L of the circumference of the circle is called the arc of the sector.

If R = radius of circle of which the sector is a part,
β = angle of sector, in degrees,
L = length of arc of sector,
A = area of sector,
then the formulas below are used:

$$L = \frac{R \times \beta \times 3.1416}{180} = \frac{2 \times A}{R},$$

$$\beta = \frac{180 \times L}{R \times 3.1416},$$

$$A = \frac{L \times R}{2},$$

$$R = \frac{2 \times A}{L} = \frac{180 \times L}{\beta \times 3.1416}.$$

If the radius of a circle is 1½ inch, and the angle of a circular sector is 60 degrees, how long is the arc of the sector?

Using the given formula, we have:

$$L = \frac{1\frac{1}{2} \times 60 \times 3.1416}{180} = 1.5708 \text{ inch.}$$

What is the area of the same sector?

From the formula given, we have:

$$A = \frac{1.5708 \times 1\frac{1}{2}}{2} = 1.1781 \text{ square inch.}$$

Circular Segments

A figure bounded by a part of the circumference of a circle and a chord, as shown in Fig. 19, is called a circular *segment*. The distance H from the chord to the highest point of the circular arc is called the height of the segment.

If R = radius, C = length of chord, L = length of arc of segment, H = height of segment, A = area of segment, then the following formulas are used:

$$C = 2 \times \sqrt{H \times (2 \times R - H)},$$

$$R = \frac{C^2 + 4 \times H^2}{8 \times H},$$

$$A = \frac{L \times R - C \times (R - H)}{2}.$$

If the angle, β, Fig. 19, is given, instead of the length of arc L, the length of the arc is found by the previously given formula:

$$L = \frac{R \times \beta \times 3.1416}{180}.$$

Assume that the radius of a segment is 5 feet and the height 8 inches. How long is the chord of this segment?

First transform 5 feet into inches; $5 \times 12 = 60$ inches. Then apply the formula given:

$C = 2 \times \sqrt{8 \times (2 \times 60 - 8)} = 2 \times \sqrt{896} = 2 \times 29.93 = 59.86$ inches.

The length of the chord of a segment is 16 inches and the height 6 inches. How long is the radius of the circle of which the segment is a part?

Applying the formula given, we have:

$$R = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = 8\ 1/3 \text{ inches.}$$

Regular Polygons

Any plane surface or figure bounded by straight lines is called a *polygon*. If all the sides are of equal length and the angles between the sides are equal, the figure is called a *regular* polygon.

A regular polygon having five sides is shown in Fig. 21. The length of each of the five sides equals S, and each of the angles between the sides equals β.

A regular polygon with five sides is called a *pentagon;* one with six sides (Fig. 22), a *hexagon;* one with seven sides (Fig. 28), a *heptagon;* and one with eight sides (Fig. 23), an *octagon*. When a regular polygon has only three sides (Fig. 24), it becomes an equilateral triangle, and when it has four sides (Fig. 25) a square.

A circle may be drawn so that it passes through all the angle-points of a regular polygon, as shown in Figs. 24 to 29 inclusive; such a circle (with the radius R) is said to be *circumscribed* about the polygon. The smaller circle in the same illustrations (with the radius r) which touches or is tangent to the sides of the polygon, is said to be *inscribed* in the polygon. The centers of the circumscribed and inscribed circles are located at the same point. If the angle-points of the polygon are connected by lines with this center, as shown by the dotted lines in Figs. 21, 22 and 23, the polygon is divided up into a number of triangles of equal size and shape. The number of triangles equals the number of sides in the polygon.

The angle α (alpha) of each of these triangles at the center (see Fig. 21) can be determined for any polygon when the number of sides is known. This angle, in degrees, equals 360 divided by the number of sides in the regular polygon, or expressed as a formula, if N equals the number of sides:

$$\alpha = \frac{360}{N}.$$

The angle β between two adjacent sides of the polygon (see Fig. 21) equals α subtracted from 180, or:

$$\beta = 180 - \alpha.$$

The area of a polygon can be found by dividing it into triangles, as shown in Figs. 21, 22 and 23. After having measured the base and height of one triangle and calculated its area, the area of the whole polygon is found by multiplying the area of one triangle by the number of triangles or sides.

For the more commonly used regular polygons, the formulas in the following give the area directly when the length of the side is known.

Equilateral Triangles

The sum of the three angles in any triangle equals 180 degrees, as already mentioned. Each of the angles in an equilateral triangle, therefore, equals 1/3 of 180 degrees, or 60 degrees.

The radius r of the circle inscribed in an equilateral triangle equals the side multiplied by 0.289.

The radius R of the circumscribed circle equals the side multiplied by 0.577.

If the radius of the circumscribed circle is known, the side is found by multiplying the radius by 1.732.

If the radius of the inscribed circle is known, the side is found by multiplying the radius by 3.464.

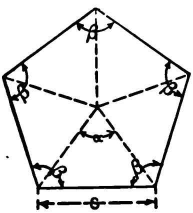


Fig. 21

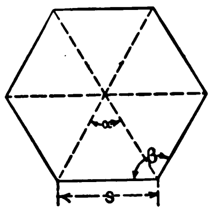


Fig. 22

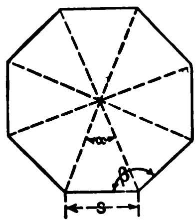


Fig. 23

Regular Polygons

The area of an equilateral triangle equals the square of the side multiplied by 0.433; or, the square of the radius of the circumscribed circle multiplied by 1.299; or, the square of the radius of the inscribed circle multiplied by 5.196.

If r = radius of inscribed circle,

R = radius of circumscribed circle,

S = length of side,

A = area of equilateral triangle,

then the previous rules may be expressed in formulas as follows:

$r = 0.289 \times S,$

$R = 0.577 \times S,$

$S = 1.732 \times R = 3.464 \times r,$

$A = 0.433 \times S^2 = 1.299 \times R^2 = 5.196 \times r^2.$

The Square

Each of the angles between the sides of a square is a 90-degree or right angle.

The radius of the inscribed circle equals one-half of the side.

The radius of the circumscribed circle equals the side multiplied by 0.707.

The side of a square equals twice the radius of the inscribed circle, or 1.414 times the radius of the circumscribed circle.

The area equals the square of the side. The area also equals the square of the radius of the circumscribed circle multiplied by 2; or, the square of the radius of the inscribed circle multiplied by 4.

Using the same meaning for the letters as before, the previous rules may be expressed in formulas as follows:

$r = 0.5 \times S,$

$R = 0.707 \times S,$

$S = 1.414 \times R = 2 \times r,$

$A = S^2 = 2 \times R^2 = 4 \times r^2.$

The Pentagon

In the pentagon (Figs. 21 and 26) the angle β between the sides equals 108 degrees. This is found by the formulas previously given as shown below:

$N =$ number of sides $= 5.$

$$\alpha = \frac{360}{N} = \frac{360}{5} = 72 \text{ degrees.}$$

$\beta = 180 - \alpha = 180 - 72 = 108$ degrees.

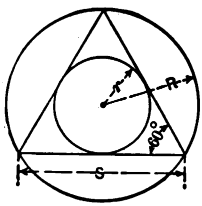


Fig. 24. Equilateral Triangle

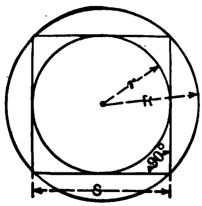
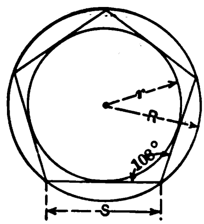


Fig. 25. Square



Machinery,N.Y.

Fig. 26. Regular Pentagon

The following formulas are used for finding the radii of the circumscribed and inscribed circles, the side and the area of regular pentagons:

$r = 0.688 \times S,$

$R = 0.851 \times S,$

$S = 1.176 \times R = 1.453 \times r,$

$A = 1.720 \times S^2 = 2.378 \times R^2 = 3.633 \times r^2.$

The Hexagon

In the hexagon (Figs. 22 and 27) the length of the side S equals the radius R of the circumscribed circle so that each of the six triangles formed, when lines are drawn from the center to the angle-points, are equilateral triangles. The angle β between two adjacent sides equals the sum of two angles in two of the equilateral triangles and, consequently, equals $60 + 60 = 120$ degrees.

Using the same letters as previously given in the formulas, we have for the hexagon:

$r = 0.866 \times S,$

$R = S,$

$S = R = 1.155 \times r,$

$A = 2.598 \times S^2 = 2.598 \times R^2 = 3.464 \times r^2.$

The Heptagon

The heptagon, Fig. 28, has seven sides, and the angle between two adjacent sides is found by the formulas already given, as shown below:

$N =$ number of sides $= 7$.

$$\alpha = \frac{360}{N} = \frac{360}{7} = 51\ 3/7 \text{ degrees.}$$

$\beta = 180 - 51\ 3/7 = 128\ 4/7$ degrees.

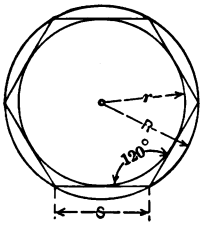


Fig. 27. Regular Hexagon

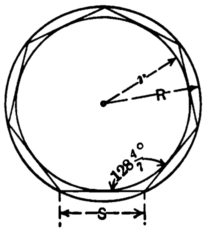
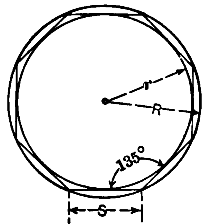


Fig. 28. Regular Heptagon



Machinery, N.Y.

Fig. 29. Regular Octagon

Using the same letters as in the formulas previously given, we have for the heptagon:

$r = 1.038 \times S$,

$R = 1.152 \times S$,

$S = 0.868 \times R = 0.963 \times r$,

$A = 3.634 \times S^2 = 2.736 \times R^2 = 3.371 \times r^2$.

The Octagon

The angle β between two adjacent sides of the octagon, as shown in Figs. 23 and 29, is 135 degrees.

Using the same meaning for the letters as previously given, the formulas for the octagon are:

$r = 1.207 \times S$,

$R = 1.307 \times S$,

$S = 0.765 \times R = 0.828 \times r$,

$A = 4.828\ S^2 = 2.828 \times R^2 = 3.314 \times r^2$.

CHAPTER IX

VOLUMES OF SOLIDS

Volume of a Cube

The cube, Fig. 30, is a solid body having six surfaces or faces, all of which are squares; as all the faces are squares, all the sides are of equal length.

If the side of a face of a cube equals S, the volume equals $S \times S \times S$ or, as it is commonly written, S^3.

Assume that the length of the side of a cube equals 3 inches; then the volume equals $3 \times 3 \times 3 = 27$ cubic inches.

When the volume of a cube is known, the length of the side is found by extracting the cube root of the volume.

Assume that the volume of a cube equals 343 cubic inches. If we extract the cube root of 343, we find that the side of the cube is 7 inches.

One cubic foot equals $12 \times 12 \times 12 = 1728$ cubic inches; therefore, a volume given in cubic feet can be transformed into cubic inches by

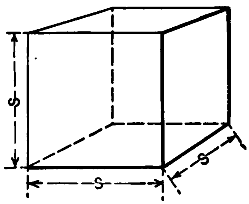
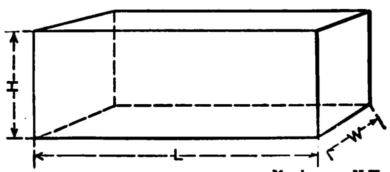


Fig. 30. Cube



Machinery,N.Y.

Fig. 31. Square Prism

multiplying by 1728; if the volume is given in cubic inches it can be transformed into cubic feet by dividing by 1728.

Volume of Prisms

A solid body, the sides of which are all rectangles, and the ends of which are either rectangles or squares is commonly called a *square prism.* Opposite surfaces or faces are parallel, and all the angles are right angles. A square prism is shown in Fig. 31, where L is its length, W its width, and H its height. The volume of a square prism equals the length times the width times the height, or, expressed as a formula, if V = volume,

$$V = L \times W \times H.$$

Assume that $L = 20$ inches, $W = 4$ inches, and $H = 5$ inches, then volume $= 20 \times 4 \times 5 = 400$ cubic inches.

A solid body having the end faces parallel, and the lines along which the other faces intersect or meet parallel, is called a *prism.* The two parallel end faces are called *bases.* The length, height, or altitude L, Fig. 32, of a prism is the distance between the bases, measured at right angles to the base surfaces.

The volume of a prism equals the area of the base multiplied by the length or height of the prism. The area of the base must, therefore, first be found before the volume can be obtained. If the base is a triangle, parallelogram, trapezoid, trapezium or a regular polygon, its area is found by the rules given in Chapter VIII. If it is a polygon which is not regular, it can always be divided into triangles, and the area of each of the triangles can be calculated, and these areas added together to obtain the area of the whole polygon.

Assume that it is required to find the volume of a prism, the base of which is a regular hexagon having a side S; the length of the prism is L. The volume of this prism is

$$2.598 \times S^2 \times L$$

[See page 35 for formula for area of hexagon.]

If, in this example, S equals 1½ inch, and L equals 9 inches, then the volume equals

$$2.598 \times 1\tfrac{1}{2}^2 \times 9 = 2.598 \times 1.5 \times 1.5 \times 9 = 52.6095 \text{ cubic inches.}$$

Volume of a Pyramid

A solid body having a polygon for the base and a number of triangles all having a common vertex for the sides is called a *pyramid*. In Fig. 33 a pyramid is shown where the base has four sides and the side surfaces are made up of triangles having two equal sides. If a line is drawn from the vertex of the pyramid at right angles to the base, the length of this line is the altitude or height H of the pyramid.

The volume of a pyramid equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the base area before the volume can be found.

Assume that it is required to find the volume of a pyramid, the base of which is a regular pentagon, having a side S; the height of the pyramid is H. The volume of the pyramid equals

$1.720 \times S^2 \times 1/3 \times H$ (area of base $\times$ one-third the height).

[See page 35 for formula for area of pentagon.]

If $S = 2$ inches and $H = 9$ inches, then the volume equals

$1.720 \times 2^2 \times 1/3 \times 9 = 1.720 \times 2 \times 2 \times 3 = 20.640$ cubic inches.

A *frustum of a pyramid* is shown in Fig. 34. It is a pyramid from which the top has been cut off, the top surface being parallel to the base. The height of a frustum of a pyramid is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a pyramid can be found when the height, the top area, and the base area are known.

If V = volume of frustum of a pyramid,
H = height of frustum,
A_1 = area of top,
A_2 = area of base,
then

$$V = \frac{H}{3} \times (A_1 + A_2 + \sqrt{A_1 \times A_2}).$$

Assume, for example, that the base of a frustum of a pyramid is a square, and that the side of the square is 5 inches. The top area is, of course, also a square; assume the side of this to be 2 inches. The height of the frustum is 6 inches. By first calculating the base and top areas and then inserting the values in the formula given, the volume is obtained.

$$\text{Volume} = \frac{6}{3} \times (5^2 + 2^2 + \sqrt{5^2 \times 2^2}) = 2 \times (25 + 4 + \sqrt{25 \times 4})$$

$$= 2 \times (25 + 4 + 10) = 78.$$

The Prismoidal Formula

The prismoidal formula is a general formula by which the volume of any prism, pyramid or frustrum of a pyramid, and the volume of any solid body bounded by regular curved surfaces may be found.

If A_1 = area at one end of the body

A_2 = area at other end,

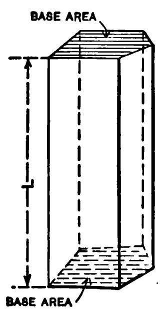


Fig. 32. Prism

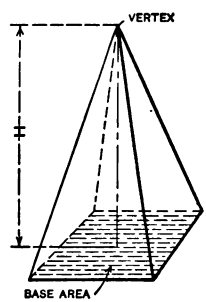


Fig. 33. Pyramid

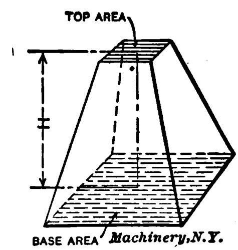


Fig. 34. Frustum of Pyramid

A_m = area of a middle section between the two end surfaces,

H = height of the body,

V = volume of body,

then

$$V = \frac{H}{6} \times (A_1 + 4\,A_m + A_2).$$

As this formula applies to all regular solid bodies, it is useful to remember. For ordinary calculations, however, the formulas given on the two previous pages, for each kind of solid, should be used because of greater simplicity.

Volume of a Cylinder

A solid body, as shown in Fig. 35, having circular and parallel end faces of equal size, is called a *cylinder*. The two parallel faces are called *bases*. The height or altitude H of a cylinder is the distance betwen the bases measured at right angles to the base surfaces.

The volume of a cylinder equals the area of the base multiplied by the height. The area of the base, must, therefore, first be found before the volume can be obtained. If the diameter of the base is D, the area of the base equals $0.7854D^2$. The volume of the cylinder then equals:

$$0.7854 \times D^2 \times H$$

If $D = 3$ inches and $H = 5$ inches, then the volume equals:

$$0.7854 \times 3^2 \times 5 = 0.7854 \times 3 \times 3 \times 5 = 35.343 \text{ cubic inches.}$$

Volume of a Cone

A solid body having a circular base and the sides inclined so that they meet at a common vertex, the same as in a pyramid, is called a cone. (See Fig. 36.) If a line is drawn from the vertex of the cone at right angles to the base, the length of this line is the altitude or height H of the cone.

The volume of a cone equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the area of the base circle before the volume can be found. If the diameter of the base

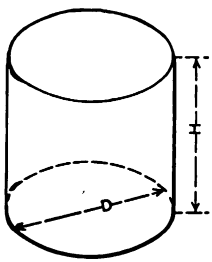


Fig. 35. Cylinder

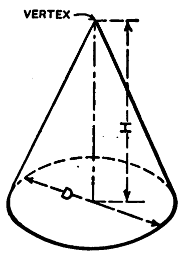
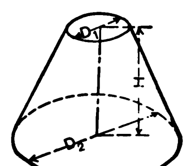


Fig. 36. Cone



Machinery,N.Y.

Fig. 37. Frustum of Cone

area equals D, then the area equals $0.7854D^2$, and this multiplied by one-third of the height H gives us the volume:

$$0.7854 \times D^2 \times 1/3 \times H = 1/3 \times 0.7854 \times D^2 \times H = 0.2618 \times D^2 \times H.$$

If the diameter of the base of a cone equals 4 inches and the height 6 inches, then the volume equals:

$$0.2618 \times 4^2 \times 6 = 0.2618 \times 4 \times 4 \times 6 = 25.1328 \text{ cubic inches.}$$

A frustum of a cone is shown in Fig. 37. It is a cone from which the top has been cut off, the top surface being a circle parallel to the base. The height H of a frustum of a cone is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a cone can be found when the diameters of the top and base circles, and the height are known.

If V = volume of frustum of a cone,
H = height of frustum,
D_1 = diameter of top circle,
D_2 = diameter of base circle,

then

$$V = 0.2618 \times H \times (D_1^2 + D_2^2 + [D_1 \times D_2]).$$

Assume, for example, that the diameter of the base of a frustum of a cone is 5 inches, and that the diameter of the top circle is 2 inches. The height of the frustum is 6 inches. By inserting these values in the formula given we have:

$$V = 0.2618 \times 6 \times (2^2 + 5^2 + [2 \times 5]) = 0.2618 \times 6 \times (4 + 25 + 10)$$
$$= 0.2618 \times 6 \times 39 = 61.2612 \text{ cubic inches.}$$

Volume of a Sphere, Spherical Sector, Segment and Zone

The name *sphere* is applied to a solid body shaped like a ball or globe, that is, bounded by a surface which at all points is at the same distance from a point inside of the sphere called its center. The diameter of a sphere is the length of a line drawn from a point on the surface through the center to the opposite side.

The volume of a sphere equals 3.1416 multiplied by four-thirds of the cube of the radius, or 3.1416 multiplied by one-sixth of the cube of the diameter.

If R = radius of the sphere, D = diameter, and V = volume, this rule given can be written in the form of formulas thus:

$$V = 3.1416 \times 4/3 \times R^3 = 4.1888 \times R^3,$$
$$V = 3.1416 \times 1/6 \times D^3 = 0.5236 \times D^3.$$

If the volume of a sphere is known, the radius can be found by extracting the cube root of the quotient of the volume divided by 4.1888; the diameter can be found by extracting the cube root of the quotient of the volume divided by 0.5236.

Written as formulas, these rules are:

$$R = \sqrt[3]{\frac{V}{4.1888}} \qquad D = \sqrt[3]{\frac{V}{0.5236}}$$

A *spherical sector* is a part of a sphere bounded by a section of the spherical surface and a cone, having its vertex at the center of the sphere, as shown in Fig. 39. The volume of a spherical sector can be found if the radius R and the height H, Fig. 39, are known.

The formula for the volume V is

$$V = 2.0944 \times R^2 \times H.$$

Assume that the length of the radius of a spherical sector is 15 inches and the height is 4 inches. Then the volume equals

$$2.0944 \times 15^2 \times 4 = 2.0944 \times 15 \times 15 \times 4 = 1884.96 \text{ cubic inches.}$$

A *spherical segment* is a part of a sphere bounded by a portion of the spherical surface and a plane circular base, as shown in Fig. 40.

The volume of a spherical segment can be found when the radius of the sphere and the height H of the segment, or the diameter C of the base of the segment and its height H, are known.

If V = volume of segment,
H = height of segment,
R = radius of the sphere of which the segment is a part,
C = diameter of the base of the segment,

then,

$$V = 3.1416 \times H^2 \times \left(R - \frac{H}{3}\right)$$

$$V = 3.1416 \times H \times \left(\frac{C^2}{8} + \frac{H^2}{6}\right)$$

Assume that the height of a spherical segment is 6 inches and the radius 8 inches, then the volume is

$$3.1416 \times 6^2 \times (8 - 6 \div 3) = 3.1416 \times 6 \times 6 \times (8 - 2) =$$
$$3.1416 \times 6 \times 6 \times 6 = 678.5856 \text{ cubic inches.}$$

A *spherical zone* is bounded by a part of a spherical surface, and by two parallel circular bases, as shown in Fig. 40, where C_1 and C_2 are the diameters of the circular bases of the zone, and H its height.

The volume of a spherical zone can be found when the height of the segment and the two base diameters are known.

If $V =$ volume of zone,

$C_1 =$ diameter of the smaller base circle,

$C_2 =$ diameter of the larger base circle,

$H =$ height of zone,

then

$$V = 0.5236 \times H \times \left(\frac{3\,C_1^2}{4} + \frac{3\,C_2^2}{4} + H^2 \right)$$

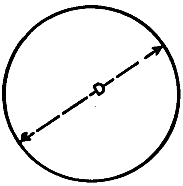


Fig. 38. Sphere

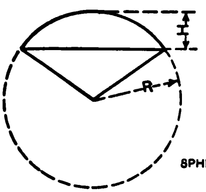


Fig. 39. Spherical Sector

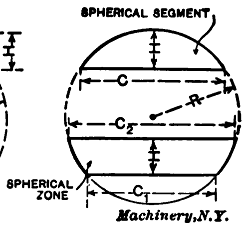


Fig. 40. Spherical Segment and Zone

Assume that the diameter $C_1 = 3$ inches, the diameter $C_2 = 4$ inches, and the height of the segment equals 1 inch, then the volume is

$$0.5236 \times 1 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1^2 \right) =$$

$$0.5236 \times 1 \times \left(\frac{27}{4} + \frac{48}{4} + 1 \right) = 0.5236 \times 1 \times 19.75 = 10.3411 \text{ cubic inches.}$$

[If a plane parallel with the end faces and passing through the center of the sphere intersects the zone, consider the zone as two zones, one zone being on each side of the center. Calculate the volume of each, and add these to find the total volume.]

CHAPTER X

SPECIFIC GRAVITY AND WEIGHTS OF BAR STOCK AND CASTINGS

The expression "specific gravity" indicates how many times a certain volume of a material is heavier than an equal volume of water. If it is found, for example, that one cubic inch of steel weighs 7.8 times as much as one cubic inch of pure water, the specific gravity of steel is 7.8.

As the density of water differs slightly at different temperatures, it is usual to make comparisons on the basis that the water has a temperature of 62 degrees F. The weight of one cubic inch of pure water at 62 degrees F. is 0.0361 pound. If the specific gravity of any material is known, the weight of a cubic inch of the material can, therefore, be found by multiplying its specific gravity by 0.0361.

TABLE OF SPECIFIC GRAVITY AND WEIGHT PER CUBIC INCH OF VARIOUS METALS AND ALLOYS

Metal	Specific Gravity	Weight in Pounds per Cubic Inch
Aluminum	2.56	0.092
Antimony	6.71	0.242
Bismuth	9.80	0.354
Brass	8.00	0.289
Copper	8.82	0.318
Gold	19.32	0.697
Iron, cast	7.20	0.260
" pure	7.77	0.280
" wrought	7.70	0.278
Lead	11.37	0.410
Manganese	8.00	0.289
Mercury	13.58	0.490
Nickel	8.80	0.318
Platinum	21.50	0.776
Silver	10.50	0.379
Steel, machine and tool	7.85	0.283
Tin	7.29	0.263
Tungsten	17.60	0.635
Vanadium	5.50	0.199
Zinc	7.15	0.258

The specific gravity of cast iron, for example, is 7.2. The weight of one cubic inch of cast iron is found by multiplying 7.2 by 0.0361. The product, 0.260, is the weight of one cubic inch of cast iron.

As there are $12 \times 12 \times 12 = 1{,}728$ cubic inches in one cubic foot, the weight of a cubic foot is found by multiplying the weight of a cubic inch by 1,728.

If the weight of a cubic inch of a material is known, the specific gravity is found by dividing the weight per cubic inch by 0.0361.

The weight of a cubic inch of gold is 0.697 pound. The specific gravity of gold is then found by dividing 0.697 by 0.0361. The quotient, 19.32, is the specific gravity of gold.

If the weight per cubic inch of any material is known, the weight of any volume of the material is found by multiplying the weight per cubic inch by the volume expressed in cubic inches. If brass weighs 0.289 pound per cubic inch, 16 cubic inches of brass, of course, weigh $0.289 \times 16 = 4.624$ pounds. In an example of this kind, if the specific gravity is known, instead of the weight per cubic inch, this latter weight is first found by the rule previously given for finding the weight per cubic inch from the specific gravity.

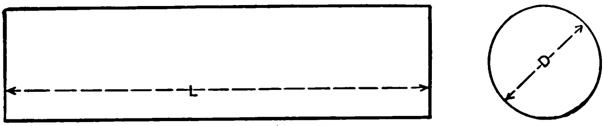


Fig. 41

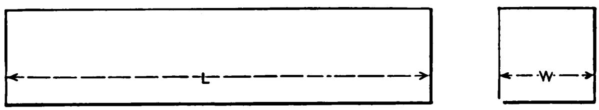


Fig. 42

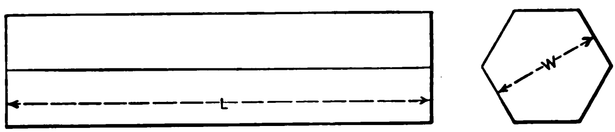
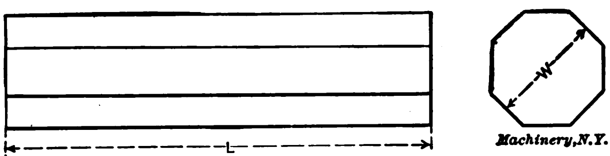


Fig. 43



Machinery,N.Y.

Fig. 44

Figs. 41 to 44. Round, Square, Hexagonal and Octagonal Bar Stock

If the specific gravity of tool steel is 7.85, what is the weight of 12 cubic inches of tool steel? The weight of one cubic inch is found by multiplying 7.85 by 0.0361. The product, 0.283, is then multiplied by 12 to find the weight of 12 cubic inches; $0.283 \times 12 = 3.396$ pounds.

Weight of Bar Stock

The weight of a piece of round bar stock, as shown in Fig. 41, can be found by first calculating the volume of the piece. When the volume is found in cubic inches, the weight is found by multiplying the volume by the weight of the material per cubic inch, as already explained.

If the diameter D, Fig. 41, of a piece of round tool steel bar is 2 inches, and the length L is 7 inches, the volume of this piece equals

0.7854 × square of diameter × length, or $0.7854 \times 2^2 \times 7 = 0.7854 \times 2 \times 2 \times 7 = 21.991$ cubic inches. The volume in cubic inches having been found, it is multiplied by the weight of tool steel per cubic inch, which is 0.283 pound, as given in the accompanying table of specific gravity and weight per cubic inch of various metals and alloys. The weight of the bar is then $21.991 \times 0.283 = 6.2235$ pounds.

If the specific gravity is given instead of the weight per cubic inch, find the weight per cubic inch as explained on page 43.

The weight of a square bar, as shown in Fig. 42, can be calculated when the width across flats, W, the length of the bar, L, and the weight of one cubic inch of the material from which the bar is made, are known.

Assume that the width across flats is 2½ inches, that the length is 11 inches, and that the bar is made from brass; the volume of this bar equals the area of its end section multiplied by its length, or, in this case, 2½ × 2½ × 11 = 68¾ cubic inches. The weight of one cubic inch of brass is 0.289 pound, and the weight of the given bar is, therefore, 68¾ × 0.289 = 19.869 pounds.

In order to find the weight of a hexagonal bar, as shown in Fig. 43, when the width across flats, W, the length L, and the weight per cubic inch of the material from which the bar is made, are known, the area of its end section must first be found so that the volume can be determined by multiplying this area by the length; when the width across flats, W, is given, this area equals 0.866 × the square of the width across flats.

Assume that the weight is to be found of a hexagonal piece of machine steel bar stock 3 inches across flats, and 6 inches long. The volume of this piece equals then $0.866 \times 3^2 \times 6 = 0.866 \times 3 \times 3 \times 6 = 46.764$ cubic inches, and the weight equals $46.764 \times 0.283 = 13.234$ pounds. The factor 0.283 is the weight of one cubic inch of machine steel, as given in the table on page 43.

In order to find the weight of a piece of octagonal stock, as shown in Fig. 44, it is first necessary to find the area of the end section; when the width across flats, W, is given, this area equals 0.828 × the square of the width across flats.

Assume that the weight of an octagonal piece of tool steel 4 inches across flats and 15 inches long is to be found. The volume of this piece then equals $0.828 \times 4^2 \times 15 = 0.828 \times 4 \times 4 \times 15 = 198.72$ cubic inches, and the weight equals $198.72 \times 0.283 = 56.238$ pounds. The factor 0.283 is the weight of one cubic inch of tool steel as given in the table on page 43.

The Weight of Castings

The weight of a casting can be calculated when the volume of the casting and the specific gravity or the weight per cubic inch of the material from which the casting is made, are known. If the volume is known in cubic inches, the volume is simply multiplied by the weight per cubic inch to obtain the weight of the casting.

The specific gravity of cast iron is 7.2 and the weight per cubic inch is 0.260; the specific gravity of brass is 8 and the weight per cubic inch is 0.289; the specific gravity of cast zinc is 6.86, and the weight per cubic inch 0.248; the specific gravity of gun metal bronze is 8.7 and the weight per cubic inch is 0.314.

With the constants above given, the problem of finding the weight of castings reduces itself to finding the volume of the casting. The multiplication by the weight per cubic inch of the material is then a simple matter.

Assume that it is required to find the weight of a hollow cast iron cylinder, as shown in Fig. 45, where the outside diameter is A, the inside or core diameter B, and the length L. To find the volume, first calculate the volume of a cylinder with the diameter A and the length L and then subtract from this the volume of the cylinder forming the core.

Assume that in a hollow cylinder as shown in Fig. 45, $A = 3$ inches, $B = 2$ inches, and $L = 8$ inches. The volume of a cylinder = 0.7854 × the square of the diameter × the height. The volume of a cylinder with 3 inches diameter and a height of 8 inches = $0.7854 \times 3^2 \times 8 =$

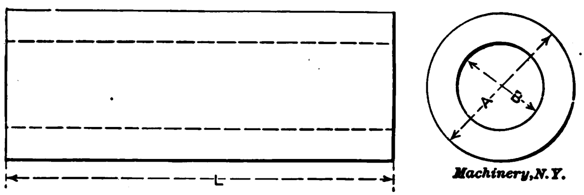


Fig. 45. Hollow Cylinder

$0.7854 \times 3 \times 3 \times 8 = 56.5488$ cubic inches. From this is subtracted the volume of the cylinder forming the core, which has a diameter of 2 inches. The volume of this cylinder is $0.7854 \times 2^2 \times 8 = 25.1328$ cubic inches. This last volume subtracted from the volume 56.5488 gives us 31.416 cubic inches as the volume of the hollow cylinder ($56.5488 - 25.1328 = 31.416$). As the weight per cubic inch of cast iron is 0.260 pound, the total weight of the hollow cylinder will be $31.416 \times 0.260 = 8.168$ pounds.

If the cylinder had been cast from gun metal bronze instead of cast iron, the volume should be multiplied by 0.314, in order to find the weight.

If the outside diameter of a hollow cylinder is A, the inside diameter B, and the length L, the following formula may be used for finding the volume of the cylinder:

$$\text{Volume} = 0.7854 \times (A^2 - B^2) \times L.$$

In Fig. 46 is shown a knee made from cast iron, all the necessary dimensions for calculating the weight being given. To calculate the volume of a casting of this shape, it is divided into prisms or other simple geometric shapes, and the volume of each of the parts is found, after which these volumes are added together to find the total volume of the casting. The piece shown in Fig. 46 can be divided up into three parts, the volume of each of which can be calculated by simple means. One part has for base the rectangle *HMLK*, another the rectangle *PFMN*, and the base of the third is bounded by two straight lines *EF* and *FG* and the circular arc *EG*. The length of all the parts in this case equals the length of the casting, or 5 inches, as shown.

The area of the rectangle *HMLK* equals $6 \times 2 = 12$ square inches. This area multiplied by 5 gives us the volume of this part in cubic inches; $12 \times 5 = 60$ cubic inches.

The length of the line *NM* is 4 inches ($6 - 2 = 4$), and, therefore, the area of the rectangle *PFMN* is $4 \times 2 = 8$ square inches. This area multiplied by 5 gives us the volume of this section of the casting in cubic inches; $8 \times 5 = 40$ cubic inches.

It now remains to find the volume of the section having for base the area bounded by the two straight lines *EF* and *FG* and the circular arc *EG*. The area of the base is found by first finding the area of the square *DEFG* and subtracting from this area the area of the circular

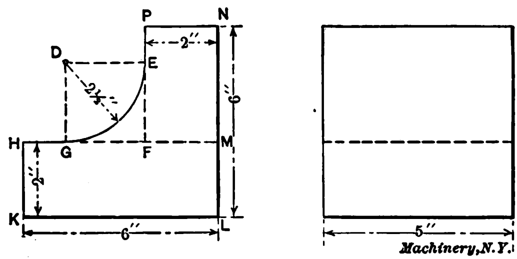


Fig. 46. Bracket or Knee

sector *DEG*. The area of the square is $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$ square inches. The area of the circular sector which is one-fourth of a complete circle is $\frac{2\frac{1}{2}^2 \times 3.1416}{4} = 4.909$ square inches. This subtracted from the area of the square equals 1.341 square inch ($6.25 - 4.909 = 1.341$). This is then the area of the base of the third part into which the casting is divided, and this area multiplied by 5 gives the volume of the third part of the casting ($1.341 \times 5 = 6.705$). Now adding the volumes of the three parts together we have $60 + 40 + 6.705 = 106.705$ cubic inches. This total volume multiplied by the weight per cubic inch of cast iron gives us the total weight: $106.705 \times 0.260 = 27.743$ pounds.

When the pattern for a casting contains no core-prints, but is in all respects an exact duplicate of the casting to be made, the weight of the casting may be approximately found by multiplying the weight of the pattern by a constant which varies for different kinds of woods used for the pattern. When the pattern is made from white pine, multiply the weight of the pattern by 13 to obtain the weight of a cast iron casting; if the pattern is made from cherry, multiply by 10.7; if made of mahogany, multiply by 10.28. When an aluminum pattern is used, the weight of the aluminum pattern may be multiplied by 2.88 to obtain the weight of a cast iron casting.

Assume that the weight of a cast iron bracket, as shown in Fig. 47, is required. All the required dimensions are here given by the letters *A*, *B*, *C*, *D*, *E*, *F*, and *G*. The casting is divided up into sections, and the volume of each section is calculated separately; then the volumes are added together and the total volume multiplied by the weight per cubic inch of cast iron. Very small fillets, like those shown at *N* and *R*, are not considered, and the area *NRST* is regarded as a perfect rectangle.

In the example given, the casting is divided up in five parts; one is a hollow cylinder with an outside diameter *A*; two parts have for bases the rectangles *NRST* and *KMTU*; and two parts have for bases the areas *HKL* and *OML*, respectively, each being bounded by two straight lines and a circular arc.

For an example, assume that in Fig. 47, $A = 7$ inches, $B = 4$ inches, $C = 3$ inches, $D = 4$ inches, $E = 12$ inches, $F = 10$ inches, $G = 8$ inches.

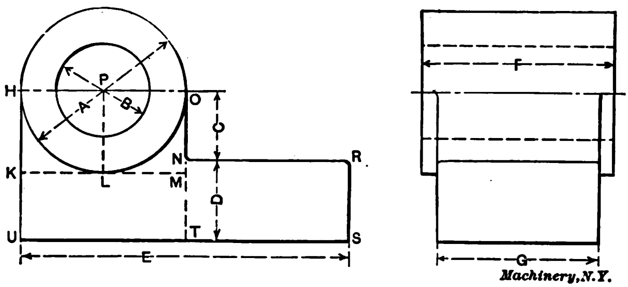


Fig. 47. Bearing Bracket

The volumes of the different parts will then be found as follows:

Volume of hollow cylinder having an outside diameter of 7 inches, an inside diameter of 4 inches, and length of 10 inches:

$$0.7854 \times (7^2 - 4^2) \times 10 = 0.7854 \times (49 - 16) \times 10$$
$$= 0.7854 \times 33 \times 10 = 259.18 \text{ cubic inches.}$$

Volume of section having for base the rectangle *NRST*:

$$4 \times 5 \times 8 = 160 \text{ cubic inches.}$$

Volume of section having for base the rectangle *KMTU*:

$$3\tfrac{1}{2} \times 7 \times 8 = 196 \text{ cubic inches.}$$

Volume of section having for base the area *HKL*:

$$\left(3\tfrac{1}{2} \times 3\tfrac{1}{2} - \frac{3\tfrac{1}{2}^2 \times 3.1416}{4}\right) \times 8 = (12.25 - 9.62) \times 8$$
$$= 2.63 \times 8 = 21.04 \text{ cubic inches.}$$

The volume of the section having for base *OML* equals the volume of the section having for base *HKL* and is consequently 21.04 cubic inches.

The total of the five sections then equals

$$259.18 + 160 + 196 + 21.04 + 21.04 = 657.26 \text{ cubic inches.}$$

The total weight of the casting equals $657.26 \times 0.260 = 170.89$ pounds.

CHAPTER XI

USE OF TABLES OF SINES, COSINES, TANGENTS AND COTANGENTS

The figuring of angles the average mechanic usually looks upon as something above his capacity; but simple cases of the figuring of angles from given formulas are often much easier than many ordinary arithmetical problems in the shop which he successfully solves. All that is necessary is a table of sines, cosines, tangents, and cotangents; after having found the figures corresponding to a given angle from the table, the whole thing resolves itself into a case of simple multiplication or division.

Often, in technical papers, the reader will find himself confronted by such formulas as, for instance,

$$A = \frac{27}{\cos 36 \text{ deg.}}.$$

Of course, it is impossible to figure out how much A is from this formula, unless the expression "cos 36 deg." (read: cosine of 36 degrees) can be transformed and expressed in plain figures. But if we know how much "cos 36 deg." is expressed in plain figures, then we can immediately divide 27 by this value, and thus find the value of A. Suppose that A stands for the length of one side in a triangle and that the expression "cos 36 deg." equals 0.80901. Then,

$$A = \frac{27}{0.80901} = 33.37.$$

The tables of sines, cosines, tangents, and cotangents simply serve the purpose of giving in figures the values of these expressions for different angles. The four expressions: sine, cosine, tangent, and cotangent, which are used to designate certain numerical values, to be found from the tables, are called the *functions of the angle*. These functions or numerical values equal a definite amount for each different angle. On pages 52, 53, 54 and 55 will be found tables giving the values referred to for all degrees and for every ten minutes (1/6 of a degree). From these tables, when the angle is given, the angular function can be found, and when the function is given, the angle can be determined. The four expressions, sine, cosine, tangent, and cotangent are abbreviated "sin," "cos," "tan," and "cot," respectively.

The tables of sines, cosines, etc., are read in the same way as a railroad time-table. At the top of Tables I and II the heading reads "Table of Sines," and at the bottom is the legend "Table of Cosines." At the top of Tables III and IV the heading reads "Table of Tangents," and at the bottom is the legend "Table of Cotangents." At the top of all the tables the heading of the extreme left-hand column reads

"Deg." and the following columns are headed 0′, 10′, 20′, etc. At the bottom of the tables the same legends are placed under the columns, but reading from right to left.

When the sine or tangent of a given angle is to be found, first find the number of degrees in the extreme left-hand column in the respective tables, and then locate the number of minutes at the top of the table. Then follow the column, over which the number of minutes is given, downwards until arriving at the figure in line with the given number of degrees. This figure is the numerical value of the sine or tangent, as the case may be, for the given angle. If the angle is given in even degrees with no minutes, the corresponding function will be found opposite the number of degrees in the column marked 0′ at the top.

The cosines and cotangents of angles are found in the same tables as the sines and tangents, but the tables in this case are read *from the bottom up.* The number of degrees is found in the extreme right-hand column and the number of minutes at the bottom of the columns.

If the number of minutes given is not an even multiple of 10, as 10′, 20′, 30′, etc., but 27′, for example, it is, for nearly all shop calculations near enough to take the figures for the nearest number of minutes given, being in this case, for 30′.

Examples of the Use of the Tables

Example 1.—Find from the tables given the sine of 56 degrees, or, as it is written in formulas, sin 56°.—The "sines" are found by reading Tables I and II from the top; the number of degrees, 56, is found in Table II in the left-hand column, and opposite 56 in the column 0′, read off 0.82903.

Example 2.—Find sin 56° 20′.—In the column marked 20′ at the top, follow downwards until opposite 56 in the left-hand column. The value 0.83227 is the sine of 56° 20′.

Example 3.—Find cos 36° 20′.—To find the cosines, read the tables from the bottom, and locate 36 in the right-hand column in Table II. Then follow the column marked 20′ at the bottom upwards until opposite 36, and read off 0.80558.

Example 4.—Find tan 56° 40′.—The tangents are found in Tables III and IV by locating the number of degrees in the left-hand column and reading the value in the column under the specified number of minutes. In Table IV then we find tan 56° 40′ to be 1.5204.

Example 5.—Find the cotangent of 56° 40′.—Read the tables from the bottom, locating 56 in the right-hand column, and find the required value in line with this figure in the column marked 40′ at the bottom. Thus, cot 56° 40′ = 0.65771.

Example 6.—Find sin 20° 48′.—For shop calculations it is almost always near enough to find the value of the angular functions for the nearest 10 minutes. Therefore in this case find sin 20° 50′, which is 0.35565.

Example 7.—The sine for a certain angle, which may be called *a*, equals 0.53238. Find the angle.—In the body of the tables of sines find the number 0.53238. It will be seen that this number is opposite

32 degrees and in the column headed 10′ at the top. The angle α, therefore, equals 32° 10′.

Example 8.—Cot β = 0.77195. Find β.—The cotangents are read from the bottom in Tables III and IV. The value 0.77195 is located opposite 52 in the right-hand column and in the column marked 20′ at the bottom. Angle β, then, is 52° 20′.

Example 9.—Sin β = 0.31190. Find β.—It will be found that the value 0.31190 is not given in the table of sines; the nearest value in the table is 0.31178. For shop calculations it is near enough to consider the angle β equal to the angle corresponding to this latter value; the angle then is 18° 10′.

Right-angled Triangles*

In right-angled triangles the remaining sides and angles can be found when either two sides, or one side and one of the acute angles, are known. As previously mentioned, the side opposite the right angle, or side a in Fig. 48, is called the *hypotenuse*. Side b is the side

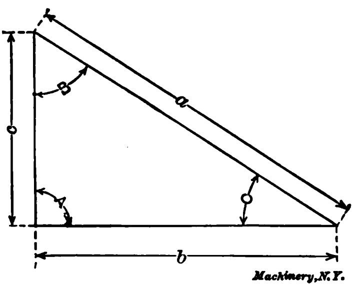


Fig. 48. Right-angled Triangle

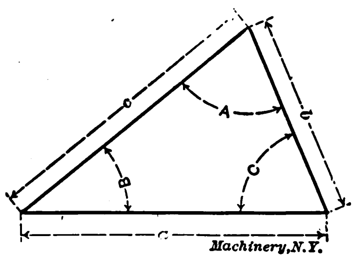


Fig. 49. Oblique-angled Triangle

adjacent to angle C and side c is the side *opposite* to the same angle. In the same way, c is the side *adjacent* to angle B, and b is the side *opposite* angle B.

The problems in right-angled triangles may be divided into five classes, for which the following formulas are given:

1. The hypotenuse and one of the sides forming the right angle are given. Call the hypotenuse a, and the other given side b. Then (see Fig. 48):

$$c = \sqrt{a^2 - b^2} \qquad \sin B = \frac{b}{a} \qquad C = 90° - B$$

2. The two sides forming the right angle are given. Call these two sides b and c. Then (see Fig. 48):

$$a = \sqrt{b^2 + c^2} \qquad \tan B = \frac{b}{c} \qquad C = 90° - B$$

3. The hypotenuse and one of the acute angles are given. Call the hypotenuse a and the given angle B. Then (see Fig. 48):

$$C = 90° - B \qquad b = a \times \sin B \qquad c = a \times \cos B$$

* For a more complete treatment of the solution of triangles, see MACHINERY'S Reference Series No. 54, Solution of Triangles.

I. TABLE OF SINES

Read degrees in left-hand column and minutes at top
Example: sin 7° 10′ = .12475

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom
Example: cos 56° 20′ = .55436

II. TABLE OF SINES

Read degrees in left-hand column and minutes at top

Example: sin 56° 20′ = .83227

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom

Example: cos 7° 10′ = .99218

III. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top
Example: tan 7° 10′ = .12573

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom
Example: cot 56° 20′ = .66607

IV. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top
Example: tan 56° 20′ = 1.5013

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1 2349	39
51	1.2349	1.2422	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom
Example: cot 7° 10′ = 7.9530

4. One acute angle and its adjacent side are given. Call the given angle B and its adjacent side c. Then (see Fig. 48):

$$C = 90° - B \qquad a = \frac{c}{\cos B} \qquad b = c \times \tan B$$

5. One acute angle and its opposite side are given. Call the given angle B and the side opposite it b. Then (see Fig. 48):

$$C = 90° - B \qquad a = \frac{b}{\sin B} \qquad c = b \times \cot B$$

Formulas for Solving Oblique Triangles

Below are given a summary of all the generally required formulas, and the methods of procedure in solving oblique triangles. In all the formulas reference is made to Fig. 49, in which the sides and angles are given the same names as in the formulas below.

1. When two angles and one side are given, call the given side a, the angle opposite it A, and the other angle B. Then if A is known:

$$C = 180° - (A + B) \qquad b = \frac{a \times \sin B}{\sin A} \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being as above.

2. When two sides and the included angle are given, call the given sides a and b and the given angle between them C. Then:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} \qquad B = 180° - (A + C) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

3. When two sides and the angle opposite one of the sides are given, call the given angle A, the side opposite it a and the other given side b. Then:

$$\sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

4. When the three sides of a triangle are given, call them a, b and c and the angles opposite them A, B and C respectively. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c} \qquad \sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B)$$

$$\text{Area} = \frac{a \times b \times \cos C}{2}$$

MACHINERY'S DATA SHEET SERIES

MACHINERY'S Data Sheet Books include the well-known series of Data Sheets originated by MACHINERY, and issued monthly as supplements to the publication; of these Data Sheets over 500 have been published, and 6,000,000 copies sold. Revised and greatly amplified, they are now presented in book form, kindred subjects being grouped together. The price of each book is 25 cents (one shilling) delivered anywhere in the world.

CONTENTS OF DATA SHEET BOOKS

No. 1. Screw Threads.—United States, Whitworth, Sharp V- and British Association Threads; Briggs Pipe Thread; Oil Well Casing Gages; Fire Hose Connections, Acme, Worm and Metric Threads; Machine, Wood, Lag Screw, and Carriage Bolt Threads, etc.

No. 2. Screws, Bolts and Nuts.—Fillister-head, Headless, Collar-head and Hexagon-head Screws; Standard and Special Nuts; T-nuts, T-bolts and Washers; Thumb Screws and Nuts; Machine Screw Heads; Wood Screws; Tap Drills.

No. 3. Taps and Dies.—Hand, Machine, Tapper and Machine Screw Taps; Taper Die Taps; Sellers Hobs; Screw Machine Taps; Straight and Taper Boiler Taps; Stay-bolt, Washout, and Patch-bolt Taps; Pipe Taps and Hobs; Threading Dies.

No. 4. Reamers, Sockets, Drills and Milling Cutters.—Hand Reamers; Shell Reamers and Arbors; Pipe Reamers; Taper Pins and Reamers; Brown & Sharpe, Morse and Jarno Taper Sockets and Reamers; Drills; Wire Gages; Milling Cutters; Setting Angles for Milling Teeth in End Mills and Angular Cutters, etc.

No. 5. Spur Gearing.—Diametral and Circular Pitch; Dimensions of Spur Gears; Tables of Pitch Diameters; Odontograph Tables; Rolling Mill Gearing; Strength of Spur Gears; Horsepower Transmitted by Cast-iron and Rawhide Pinions; Design of Spur Gears; Epicyclic Gearing.

No. 6. Bevel, Spiral and Worm Gearing.—Rules and Formulas for Bevel Gears; Strength of Bevel Gears; Design of Bevel Gears; Rules and Formulas for Spiral Gears; Diagram for Cutters for Spiral Gears; Rules and Formulas for Worm Gearing, etc.

No. 7. Shafting, Keys and Keyways.—Horsepower of Shafting; Strength of Shafting; Forcing, Driving, Shrinking and Running Fits; Woodruff Keys; Standard Keys; Gib Keys; Milling Keyways; Duplex Keys.

No. 8. Bearings, Couplings, Clutches, Crane Chain and Hooks.—Pillow Blocks; Babbitted Bearings; Ball and Roller Bearings; Clamp Couplings; Flange Couplings; Tooth Clutches; Crab Couplings; Cone Clutches; Universal Joints; Crane Chain; Crane Hooks; Drum Scores.

No. 9. Springs, Slides and Machine Details.—Formulas and Tables for Spring Calculations; Machine Slides; Machine Handles and Levers; Collars; Hand Wheels; Pins and Cotters; Turn-buckles.

No. 10. Motor Drive, Speeds and Feeds, Change Gearing, and Boring Bars.—Power required for Machine Tools; Cutting Speeds and Feeds for Carbon and High-speed Steel; Screw Machine Speeds and Feeds; Heat Treatment of High-speed Steel Tools; Taper Turning; Change Gearing for the Lathe; Boring Bars and Tools.

No. 11. Milling Machine Indexing, Clamping Devices and Planer Jacks.—Tables for Milling Machine Indexing; Change Gears for Milling Spirals; Angles for setting Indexing Head when Milling Clutches; Jig Clamping Devices.

No. 12. Pipe and Pipe Fittings.—Pipe Threads and Gages; Cast-iron Fittings; Bronze Fittings; Pipe Flanges; Pipe Bends; Pipe Clamps and Hangers.

No. 13. Boilers and Chimneys.—Flue Spacing and Bracing for Boilers; Strength of Boiler Joints; Riveting; Boiler Setting; Chimneys.

No. 14. Locomotive and Railway Data.—Locomotive Boilers; Bearing Pressures for Locomotive Journals; Locomotive Classifications; Rail Sections; Frogs, Switches and Cross-overs; Tires; Tractive Force; Inertia of Trains; Brake Levers.

No. 15. Steam and Gas Engines.—Saturated Steam; Steam Pipe Sizes; Steam Engine Design; Volume of Cylinders; Stuffing Boxes; Setting Corliss Engine Valve Gears; Condenser and Air Pump Data; Horsepower of Gasoline Engines; Automobile Engine Crankshafts, etc.

No. 16. Mathematical Tables. — Squares of Mixed Numbers; Functions of Fractions; Circumference and Diameters of Circles; Tables for Spacing off Circles; Solution of Triangles; Formulas for Solving Regular Polygons; Geometrical Progression, etc.

No. 17. Mechanics and Strength of Materials.—Work; Energy; Centrifugal Force; Center of Gravity; Motion; Friction; Pendulum; Falling Bodies; Strength of Materials; Strength of Flat Plates; Strength of Thick Cylinders, etc.

No. 18. Beam Formulas and Structural Design.—Beam Formulas; Sectional Moduli of Structural Shapes; Beam Charts; Net Areas of Structural Angles; Rivet Spacing; Splices for Channels and I-beams; Stresses in Roof Trusses, etc.

No. 19.—Belt, Rope and Chain Drives.—Dimensions of Pulleys; Weights of Pulleys; Horsepower of Belting; Belt Velocity; Angular Belt Drives; Horsepower transmitted by Ropes; Sheaves for Rope Drive; Bending Stresses in Wire Ropes; Sprockets for Link Chains; Formulas and Tables for Driving Chain.

No. 20. Wiring Diagrams, Heating and Ventilation, and Miscellaneous Tables.—Typical Motor Wiring Diagrams; Resistance of Round Copper Wire; Current Densities for Various Contacts and Materials; Centrifugal Fan and Blower Capacities; Hot Water Main Capacities; Decimal Equivalents, Metric Conversion Tables, Weights and Specific Gravity of Metals, Drafting-room Conventions, etc.

MACHINERY, the leading journal in the machine-building field, the originator of the 25-cent Reference and Data Books. Published monthly. Subscription, $2.00 yearly. Foreign subscription, $3.00.

The Industrial Press, Publishers of MACHINERY,

140-144 Lafayette Street, New York City, U. S. A.