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No. 20—SPIRAL GEARING.

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CHAPTER I.

CALCULATING SPIRAL GEARS.

In taking up the subject of spiral gears with students at the Worcester Polytechnic Institute, some difficulty was experienced over the formulas relating to their construction. If these trouble men accustomed to the use of trigonometry, they must certainly be confusing to shop men. As the explanation of the method of figuring these gears, which has been arrived at for the students' use, involves a minimum amount of mathematics, it may be of value to others. At the same time, the formulas given will be thoroughly explained and mathematically proved for the benefit of those who wish to fully study the subject.

As it is most convenient to adapt these gears to the standard diametral pitch cutters used for spur gears, we will consider the subject only

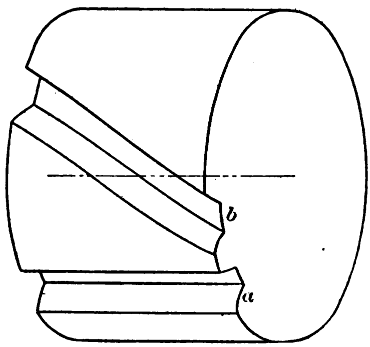


Fig. 1.

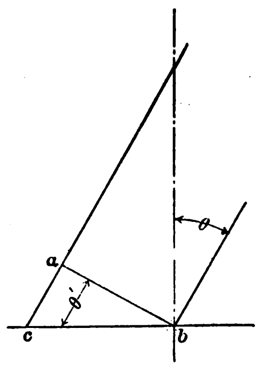


Fig. 2.

from that point of view. This gives us at once the normal pitch, that is, the pitch measured perpendicular to the face of the tooth, and also the shape and depth of the tooth.

In the case of a spur gear, we cut the teeth at right angles with the base of the cylinder on which the gear is cut, as at *a* in Fig. 1. The space appears in its true size and shape on the base of the blank. If, now, we cut the teeth at some other angle, say at 30 degrees with a line parallel to the axis of the gear blank, as at *b* in Fig. 1, we see that the width of the space measured on the base is greater, and it will be greater still if the angle is increased. It is thus evident that the number of teeth that can be cut on a given cylinder decreases as the angle of the teeth with a line parallel with the axis of the gear increases.

Number of Teeth and Diameter of Blank.

Referring to Fig. 2, suppose the line $b\,c$ to be a part of the base of the cylinder, and the two parallel lines making the angle θ with a line parallel with the axis to represent the center lines of two adjacent teeth. Then, $a\,b$ will represent the normal pitch, and $b\,c$ the circumferential pitch; but, as $\theta' = \theta$,

$$a\,b = b\,c \cos\theta, \text{ whence}$$

$$b\,c = \frac{a\,b}{\cos\theta}.$$

The number of spur-gear teeth that can be cut in a blank of pitch radius r is expressed by the formula:

$$N_c = 2\,r\,P, \qquad (1)$$

where P is the diametral pitch and N_c the number of teeth.

From this we see that the number of teeth in a spiral gear of this pitch and pitch radius, and of angle θ, will be

$$N = 2\,r\,P \cos\theta. \qquad (2)$$

Take as an example a gear to be cut 6-pitch, with teeth at an angle of 60 degrees to a line parallel with the axis of the gear, and a pitch diameter of about 2½ inches. Then, $r = 1\frac{1}{4}$; $P = 6$; $\cos\theta = 0.5$. Hence,

$$N = 2 \times 1\tfrac{1}{4} \times 6 \times 0.5 = 7\tfrac{1}{2}.$$

As a gear of 7½ teeth is impossible for continuous rotation, we must make the number of teeth either 7 or 8. Suppose we make it 8. Then, to find the pitch diameter of the gear we use the same formula, but transposed as follows:

$$r = \frac{N}{2\,P\cos\theta},$$

from which we get, after substituting 8 for N,

$$r = \frac{8}{2 \times 6 \times 0.5} = 1\ 1/3.$$

The pitch diameter of our blank must, therefore, be 2 2/3 inches, the same as for a spur gear of 16 teeth. As we are using diametral pitch cutters, the addendum will be the same as for a spur gear of the same pitch. Adding $\frac{1}{P}$ to the pitch diameter on each side will make the whole diameter 3 inches in this case.

Milling Spiral Teeth.

In order to mill the teeth, we must be able to set up the machine so as to make, approximately, the correct advance per revolution of the work. This advance will be equal to the circumference of the blank, measured on the pitch line, multiplied by the cotangent of the angle of the teeth. As this usually presents no difficulty, we pass it over with simply saying that gears run together quite nicely, even if the lead as figured is not exactly obtainable on the milling machine.

After having set our machine to cut the desired spiral, we next wish to select the proper cutter. This will be, unless the angle θ is very small, quite a different cutter from that used for a spur gear of the same diameter, or of the same number of teeth. Brown & Sharpe Co. advises turning up a blank of the size of the pitch diameter and laying out on it a helix at right angles to the helix of the teeth of the gear to be cut, as in Fig. 3, fitting a cardboard templet to the face of the cylinder along this curve, and then finding the diameter of the circle corresponding to this templet.

The cutter should be such as will be suitable for a gear of this diameter and the given normal pitch. This is a sufficiently close method for gears of a large number of teeth, but requires considerable care for gears of 12 or less teeth. Moreover, we require a method

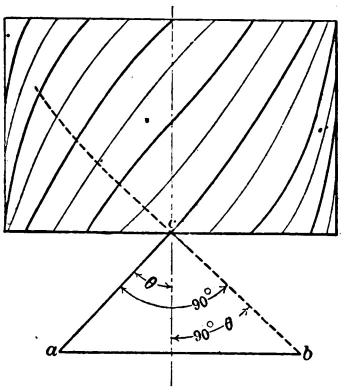


Fig. 3.

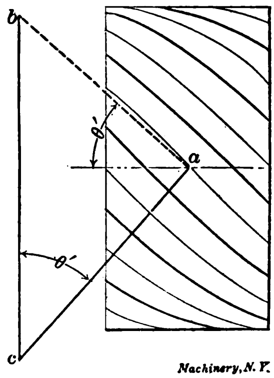


Fig. 4.

that can be worked out entirely in the drafting room. Grant says that the cutter should be right for a spur gear having a number of teeth equal to the number of teeth in the spiral gear, divided by the cube of the cosine of the angle of the teeth. This gives an exact result, but he offers no explanation of his statement. The following, we hope, will seem a clear demonstration.

Demonstration of Grant's Formula.

It will be seen that what we wish to find at the start is a circle having the same radius as the helix which is drawn on our pitch cylinder perpendicular to the teeth, as in Fig. 3. The angle of this helix will be $90 - \theta$ degrees. If $R =$ radius of curvature of this helix, then from the well-known formula in analytic geometry for the radius of curvature of a helix, we have

$$R = \frac{r}{\sin^2 (90 - \theta)} = \frac{r}{\cos^2 \theta}. \qquad (3)$$

The demonstration of this formula is as follows:

Assume that $r =$ radius of the cylinder on which the helix is drawn, and θ is the angle of the helix with a line parallel to the axis of the cylinder. In Fig. 5 is a cylinder of radius $m'c' = r$, on which is drawn a helix. We have assumed three points, a, b, and c equidistant on the helix, the middle point, b, being taken at the extreme front of the helix, for convenience only.

We wish to draw a circle passing through the three points, a, b, and c. To do this we have revolved the two outside points into the same horizontal plane as b, placing a at g and c at f. We represent these points in the top view by g' and f'. Through g', b', and f' we draw a circle having its center at k' and radius k' f', which we will call R_2.

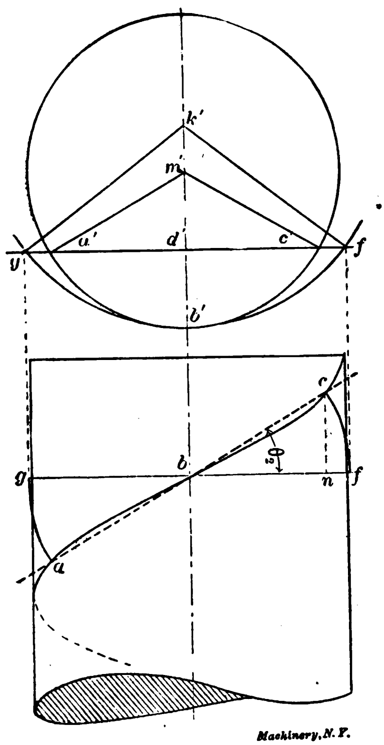


Fig. 5.

This circle will be represented in the front view by the horizontal line g to f. The original position of this circle in the front view is represented by the straight line a to c. The angle between these two lines we call θ_2. Remember that this is not the angle of the helix with the base, but is the angle of the original plane of the circle through a, b, and c with the horizontal. Now,

$$b\,n = d'\,c' = b\,c \cos \theta_2 \tag{4}$$

$$b\,c = b\,f = d'\,f'. \tag{5}$$

Then,

$$d'\,c' = d'\,f' \cos \theta_2. \tag{6}$$

Squaring,

$$(d'\,c')^2 = (d'\,f')^2 \cos^2\theta_2 \tag{7}$$

$$(d'\,c')^2 = (m'\,c')^2 - (m'\,d')^2 = r^2 - (m'\,d')^2 \tag{8}$$

$$(d'\,f')^2 = (k'\,f')^2 - (k'\,d')^2 = R^2{}_2 - (k'\,d')^2. \tag{9}$$

Substituting (8) and (9) in (7), we have,

$$r^2 - (m'\,d')^2 = [R^2{}_2 - (k'\,d')^2] \cos^2\theta_2 \tag{10}$$

$$m'\,d' = r - d'\,b'$$

$$(m'\,d')^2 = r^2 - 2\,r\,(d'\,b') + (d'\,b')^2 \tag{11}$$

$$k'\,d' = R_2 - d'\,b'$$

$$(k'\,d')^2 = R_2{}^2 - 2\,R_2\,(d'\,b') + (d'\,b')^2. \tag{12}$$

Substituting from (11) and (12) in (10) we get:

$$r^2 - r^2 + 2\,r\,(d'\,b') - (d'\,b')^2 =$$

$$[R^2{}_2 - R^2{}_2 + 2\,R_2\,(d'\,b') - (d'\,b')^2] \cos^2\theta_2. \tag{13}$$

Cancelling we have,

$$2\,r - d'\,b' = (2\,R_2 - d'\,b') \cos^2\theta_2. \tag{14}$$

This expression is true for any three points equidistant on the helix. Let us remember that the radius of curvature for any curve is the radius of the circle passing through any three *consecutive* points. We will accordingly consider points a and c moved up so that they become consecutive points with b, and see what the effect is on equation (14). Then

r will remain constant,

$d'\,b'$ will become practically zero on each side of the equation and may be neglected,

R_2 becomes R, the radius of curvature of the helix, and

θ_2 becomes θ, the angle of the helix.

Substituting these values in (14), we have,

$$2\,r = 2\,R \cos^2\theta \tag{15}$$

$$\text{or, } R = \frac{r}{\cos^2\theta}$$

Referring, now, to formula (1), and applying it to a gear of radius R, we have

$$N_c = 2\,R \times P = \frac{2\,r}{\cos^2\theta} \times P. \tag{16}$$

For our spiral gear we found, by formula (2), that:

$$N = 2\,r\,P \cos\theta.$$

Dividing (16) by (2), we have

$$\frac{N_c}{N} = \frac{2\,r\,P}{\cos^2\theta} \times \frac{1}{2\,r\,P\cos\theta} = \frac{1}{\cos^3\theta}$$

or,

$$N_c = \frac{N}{\cos^3\theta}.$$

Another derivation of the same formula, which may be of interest to some, was presented by H. W. Henes in MACHINERY, April, 1908. The following notation is used in this derivation:

N_c = number of teeth in spur gear for which cutter is intended.

N = number of teeth in the desired spiral gear.

α = the angle which the direction of the spiral makes with the axis of the gear.

Let P_n be the perpendicular distance between two consecutive teeth on the spiral gear, and let D_1 be the diameter of the spiral gear. Let the gear be represented as in Fig. 6, and pass a plane through it perpendicular to the direction of the teeth. The section will be an ellipse as shown in *CEDF*. Designate the semi-major and semi-minor axes by a and b respectively.

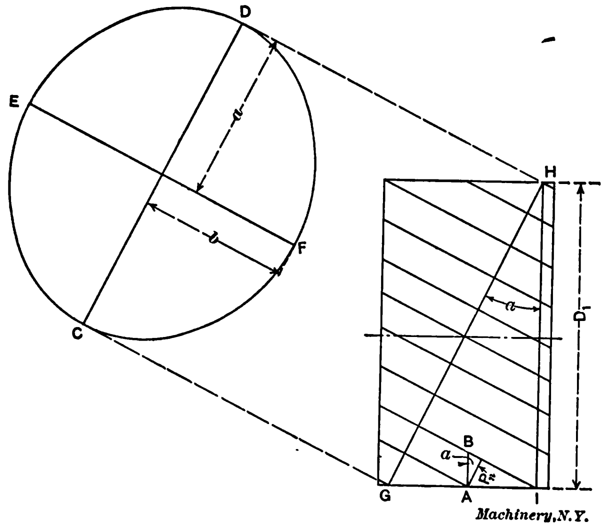


Fig. 6. Diagram for Deriving the Formula for Determining Spur Gear Cutter for Cutting Spiral Gears.

Now N_c is the number of teeth which a spur gear would have if its radius were equal to the radius of curvature of the ellipse at E. Therefore, it is required to determine the radius of this curvature of the ellipse. This is done as follows:

From the figure we have:

$$2b = \text{axis } EF = D_1 \tag{17}$$

$$2a = \text{axis } CD = GH = \frac{HI}{\cos \alpha} = \frac{D_1}{\cos \alpha} \tag{18}$$

From (17) and (18) we have for a and b,

$$b = \frac{D_1}{2} \tag{19}$$

and

$$a = \frac{D_1}{2 \cos \alpha} \tag{20}$$

It is known, and shown by the methods of calculus, that the minimum curvature of an ellipse, that is, the curvature at E or F, equals

$\frac{b}{a^2}$. Taking the values of a and b found in (19) and (20), we have the curvature at E:

$$\text{Curvature} = \frac{b}{a^2} = \frac{\frac{D_1}{2}}{\frac{D_1^2}{4\cos^2 a}} = \frac{4\,D_1\cos^2 a}{2\,D_1^2} = \frac{2\cos^2 a}{D_1} \qquad (21)$$

It is also shown in calculus that the *curvature* is equal to $\frac{1}{R}$, where R is the *radius of curvature* at the point E. Therefore from (5) we have:

$$\frac{1}{R} = \frac{2\cos^2 a}{D_1}, \text{ whence } R = \frac{D_1}{2\cos^2 a} \qquad (22)$$

Formula (22) can also be arrived at directly, without reference to the minimum curvature of the ellipse, by introducing the formula for the radius of curvature in the first place. The curvature is simply the reciprocal value of the radius of curvature, and is only a comparative means of measurement. The radius of curvature of an ellipse at the end of its short axis is $\frac{a^2}{b}$, from which formula (22) may be derived directly by introducing the values of a and b from equations (19) and (20).

Having now found the radius of curvature of the ellipse at E, we proceed to find the number of teeth which a spur gear of that radius would have. From Fig. 6 we have:

$$A\,B = \frac{P_n}{\cos a} \qquad (23)$$

Now, if $A\,B$ be multiplied by the number of teeth of the spiral gear, we shall obtain a quantity equal to the circumference of the gear; that is:

$$A\,B \times N = \pi\,D_1, \text{ and since } A\,B = \frac{P_n}{\cos a} \text{ from (23)}$$

$$\frac{P_n}{\cos a} \times N = \pi\,D_1 \qquad (24)$$

Since N_c is the number of teeth which a spur gear of radius R would have, then,

$$N_c = \frac{2\,\pi\,R}{P_n} \qquad (25)$$

In equation (25) the numerator of the fraction is the circumference of the spur gear whose radius is R, and the denominator is the circular pitch corresponding to the cutter.

From equation (22) we have:

$$R = \frac{D_1}{2 \cos^2 \alpha}$$

Substituting this value of R in (25), we have:

$$N_c = \frac{2 \pi D_1}{P_n \times 2 \cos^2 \alpha} \tag{26}$$

From equation (24) we have:

$$D_1 = \frac{N P_n}{\pi \cos \alpha} \tag{27}$$

Substitute this value of D_1 in equation (26) and we have:

$$N_c = \frac{2 \pi N P_n}{2 P_n \pi \cos^3 \alpha}$$

or

$$N_c = \frac{N}{\cos^3 \alpha} \tag{28}$$

Since N is the number of teeth in our spiral gear and N_c is the number of teeth in a spur gear which has the same radius as the radius of curvature of the helix above referred to, this is the equivalent of saying that the cutter to be used should be correct for a number of teeth which can be obtained by dividing the actual number of teeth in the gear by the cube of the cosine of the tooth angle. Since the cosine of angle θ (or α, as it was denoted in the derivation of the cutter formula last given) is always less than unity, its cube will be still less, so N_c is certain to be greater than N, which will account for the fact that spiral gears of less than 12 teeth can be cut with the standard cutters. The getting of the cube of $\cos \theta$ may bother some, as the cubing of any fraction is apt to do, but a graphical method is given later in this chapter which, even if roughly laid out, will give sufficiently accurate results for this purpose. For the other uses of this graphical method, care must be used, or the results are not to be depended on.

Calculation of Velocity Ratio.

Now we are able to cut the gear, once having decided on the number of teeth, pitch (or pitch diameter), and angle of teeth, but in designing we almost always wish to transmit motion with some definite velocity ratio. If we were dealing with spur gears we would know that the ratio of speeds would be inversely proportional to the pitch diameters or the number of teeth. If the teeth were twisted or cut spiral on the surface and the axes still were left parallel, this same velocity ratio would obtain, but the moment we move the axes out of the same plane, this convenient ratio ceases to exist. Then there can be but one point of contact of the pitch cylinders, consequently all motion must be transmitted as if through this one point, if smooth running is to be attained. The actual motion of the tooth at this point must be at right angles to the axis of the gear, but it may be considered as the resultant of two motions, one of sliding parallel to the teeth, which we can see must happen since the two gears do not run in the same plane, and the other perpendicular to the teeth, which is the effective, or driving motion. This latter motion normal to the teeth must be the same for both gears.

In the case of a driving gear of radius r and angle θ, the velocity of this point in a plane perpendicular to the axis will be $2\pi r n$, where n is the number of revolutions per minute of the driving gear. Let us consider the point c of the gear in Fig. 3. Assume the line $a\,b$ to represent the linear velocity and to be equal to $2\pi r n$. The line $c\,b$ is perpendicular to the tooth, and $a\,c$ parallel to the tooth. These three lines complete the triangle $a\,b\,c$, and therefore $a\,c$ will represent the sliding component of the point c, and c b the motion perpendicular to the tooth. Then,

$$b\,c = 2\pi r n \cos\theta,$$

since in the triangle $a\,b\,c$ the angle $a\,b\,c$ is equal to θ.

This, also, is the velocity of the contact point of the driven gear in the same direction, or in a direction normal to the teeth of the driven gear. We will assume this gear to have a radius r' and angle θ'. Considering the gear in Fig. 4 to be the driven gear, with axis at right angles to the axis of the driving gear, we have

$$a\,c\ (\text{Fig. } 4) = b\ c\ (\text{Fig. } 3) = 2\pi r n \cos\theta.$$

The resulting motion perpendicular to the axis of the gear will then be

$$c\,b\ (\text{Fig. } 4) = \frac{a\,c}{\cos\theta'} = \frac{2\pi r n \cos\theta}{\cos\theta'}$$

This is the linear velocity of the point a; to get the number of revolutions of the driven gear we divide by the circumference of the driven gear, which is $2\pi r'$, giving

$$n' = \frac{2\pi r n \cos\theta}{2\pi r' \cos\theta'},$$

$$\text{whence}\quad \frac{n'}{n} = \frac{r\cos\theta}{r'\cos\theta'}$$

That is, the relative motion of the two gears is inversely proportional to the product of their diameters and the cosines of the angles of their teeth.

If both are 45-degree gears, this last factor becomes inoperative, and the gears produce motion in the same ratio as spur gears of the same sizes. The same is also true if the axes are parallel, for θ and θ' then become equal.

If the axes are at right angles, $\theta - 90 = \theta'$, and $\dfrac{\cos\theta}{\cos\theta'} = \dfrac{\cos\theta}{\sin\theta} =$ $\cot\theta$, whence: $\dfrac{n'}{n} = \dfrac{r}{r'} \times \cot\theta$.

This property of spiral gears, of having a varying velocity ratio for both size and angle, is valuable, in that it enables one to obtain varying velocity ratios with the same size gear. For example, suppose we have two gears, one of 8 teeth and one of 16 teeth, both 45-degree gears, on axes at right angles. The velocity ratio is 2 to 1. If, now, we want a velocity ratio of 3 to 1 on the same axes with the same size gears, we use the formula last arrived at,

$$\frac{n'}{n} = \frac{r}{r'} \cot\theta, \text{ or, } \frac{1}{3} = \frac{1}{2} \cot\theta,$$

$$\cot\theta = \frac{2}{3} = 0.6666.$$

θ will then be 56° 19′ and θ' will be 33° 41′.

If we use cutters of the same pitch as before, the number of teeth become fractional numbers, thus making impossible conditions for practical use. It will, then, be necessary to use a fractional pitch cutter. To find what this cutter should be, decide on the number of teeth to be used in each of the two gears to give the desired new velocity ratio of 3 to 1; then solve formula (2) for P, substituting the required data from either of the two gears.

The relation between the angles of the shafts and gear teeth will be readily understood by a little thought. In gears whose axes are at right angles we have seen that the sum of the angles of the gear teeth is equal to 90 degrees, the angle of the shafts. This is true for any gears whose spirals are both right-hand or both left-hand. Carrying this to an extreme, we find that if the tooth angles become zero degrees (as in spur gears), the shaft angle becomes 180 degrees, or the shafts are parallel. If one gear is right-hand and the other left-hand, then the angle of the shafts will be equal to the difference of the tooth angles. If the gears have their teeth at equal angles, but one right-hand and one left-hand, then the shaft angle will be zero; that is, the shafts are parallel and the gears are twisted gears, or Hooke's gears.

Fig. 7, while it is innocent looking enough, contains a solution of all the bothersome points of the figuring of the spiral gears to be cut with the usual diametral pitch cutters.

To illustrate the use of the figure, we will take as an illustration a 24-tooth gear of 30-degree spiral angle, to be cut with an 8-pitch spur-gear cutter.

Lay off $a\ b = 3$ inches, the diameter of a spur gear of 24 teeth, 8-pitch. Lay off the angle θ 30 degrees as shown, and erect a perpendicular at b to $a\ b$, intersecting at $a\ c$ at c. The line $a\ c$ will be the pitch diameter of the required spiral gear (3.46 inches). The outside diameter will be equal to this diameter plus $\frac{2}{P}$, as in spur gears (3.71 inches). The depth of tooth will be the same as for a spur gear of the same pitch.

Extend $a\ b$ and $a\ c$. At c erect a perpendicular to $a\ c$, meeting $a\ b$ at d. At d, in turn, erect a perpendicular to $a\ d$, meeting $a\ c$ at e; $a\ e$ will be the diameter of a spur gear having the correct number of teeth from which to choose a cutter to cut our spiral. In this case the diameter is 4⅝ inches, corresponding to a 37-tooth gear. So we will use the same cutter to cut our 24-tooth spiral gear as that we would use to cut a 37-tooth spur gear.

Extend, in turn, $a\ d$ and $a\ e$ till a line, of length equal to $a\ c$, drawn perpendicular to $a\ d$ will meet $a\ e$, as $f\ g$; then $f\ a \times \pi$ will be the

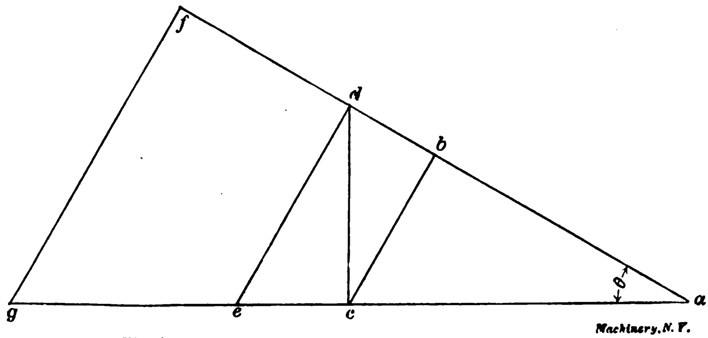


Fig. 7. Graphical Solution of Spiral Gear Problems.

lead of the spiral to which we should set the milling machine (in this case 18.85 inches). The diagram depends on the following facts relating to spiral gears.

$$\frac{b\ a}{a\ c} = \cos\theta = \frac{\text{diam. spur gear}}{\text{diam. spiral gear}}$$

$$a\ c = \frac{b\ a}{\cos\theta}$$

$$a\ d = \frac{a\ c}{\cos\theta} = \frac{b\ a}{\cos^2\theta}$$

$$e\ a = \frac{a\ d}{\cos\theta} = \frac{b\ a}{\cos^3\theta}.$$

$$\frac{\text{Lead of helix}}{\text{Circumference of pitch cylinder}} = \tan\ (90° - \theta)$$

Divide by π and transpose

$$\frac{\text{Lead}}{\pi} = \text{diameter of pitch cylinder} \times \frac{f\ a}{f\ g}$$

But pitch diameter $= a\ c = f\ g$; therefore

$$\text{Lead} = \frac{f\ a}{f\ g} \times f\ g \times \pi = f\ a \times \pi$$

Therefore lead of helix $= f\ a \times \pi$.

CHAPTER II.

RULES AND FORMULAS FOR DESIGNING SPIRAL GEARS.

In accordance with time-honored custom, this contribution to the art of designing helical or "spiral" gears opens with an apology. The subject is one which, from its very nature, can be approached from any one of a number of different ways, and it has been approached from so many of these possible different ways that perhaps the subject has become quite confused in the minds of many readers of technical literature. The writer does not offer the excuse of novelty in the methods presented in the following paragraphs, since some of the details which were independently worked out by him have been described by others. His reason for adding one more to the series of solutions of helical gear problems is that the method described appears to reduce the more serious of this class of problems to its most simple elements. The method of procedure will be described without proof or comment.

Two terms will be used which may require some explanation. In using the expression "tooth angle," the angle made by the teeth with the axis of the gear is meant, not the angle of the tooth with the face of the gear, an unfortunate use sanctioned by some writers. Fig. 8 shows α_a as the tooth angle of gear a, and α_b as the tooth angle of gear b, used in the sense in which we will use them. The angle between the shafts, γ, is 90 degrees in all the examples which will be considered in this chapter. The first rule to be used in the design of helical gears relates to the tooth angles.

Rule 1. *The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle.*

That is to say, in Fig. 8, angle α_a added to angle α_b equals angle γ, as is self-evident from the cut.

The second term which requires explanation is the "equivalent diameter." The quotient obtained by dividing the number of teeth in a helical gear by the diametral pitch of the cutter used gives us a very useful factor for figuring out the dimensions of helical gears, so the writer has ventured to give it this name "equivalent diameter," an abbreviation of the words "diameter of equivalent spur gears," which more accurately describe it. This quantity cannot be measured on the finished gear with a rule, being only an imaginary unit of measurement. The next rule deals with this term.

Rule 2. *The equivalent diameter of a helical gear is found by dividing the number of teeth in the gear by the diametral pitch of the cutter with which it is cut.*

For instance, in a 20-tooth gear, cut with an 8 diametral pitch cutter, the equivalent diameter will be 2½ inches. The actual diameter of the gear will vary widely from this, depending on the tooth angle.

The process of locating a railway line over a mountain range is divided into two parts; the preliminary survey or period of exploration, and the final determination of the grade line. The problem of designing a pair of helical gears resembles this engineering problem in having many possible solutions, from which it is the business of the designer to select the most feasible. For the exploration or preliminary survey the diagram shown in Fig. 9 will be found a great convenience. The materials required are a ruler with a good straight edge, and a piece of accurately ruled, or, preferably, engraved, cross-section paper. If a point, *O*, be so located on the paper that *B O*, the distance to one margin line, be equal to the equivalent diameter of gear *a*, while *B′ O*, the distance to the other margin line, be equal to

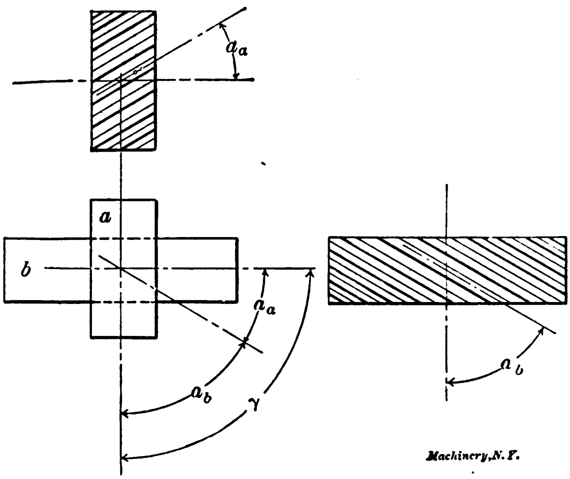


Fig. 8. Diagram Showing Notation used for Tooth Angles.

the equivalent diameter of gear *b*, then (when the rule is laid diagonally across the paper in any position that cuts the margin lines and passes through point *O*) *D O* will be the pitch diameter of gear *a*, *D′ O* the pitch diameter of gear *b*, angle *B O D* the tooth angle of gear *a* and angle *B′ O D′* the tooth angle of gear *b*. This simple diagram presents instantly to the eye all possible combinations for any given problem. It is, of course, understood that in the shape shown it can only be used for shafts making an angle of 90 degrees with each other.

The diagram as illustrated shows that a pair of helical gears having 12 and 21 teeth each, cut with a 5-pitch cutter, and having shafts at 90 degrees from each other and 5 inches apart, may have tooth angles of 36° 52′ and 53° 8′ respectively, and pitch diameters of 3 inches and 7 inches.

Suppose it were required to figure out the essential data for three sets of helical gears with shafts at right angles, as follows:

1st. Velocity ratio 2 to 1, center distance between shafts 2¼ inches.
2d. Velocity ratio 2 to 1, center distance between shafts 3⅜ inches.
3d. Velocity ratio 2 to 1, center distance between shafts 4 inches.

We will take the first of these to illustrate the method of procedure about to be described.

We have a center distance of 2¼ inches and a speed ratio between driver and driven shafts of 2 to 1. The first thing to determine is the pitch of the cutter we wish to use. The designer selects this according to his best judgment, taking into consideration the cutters on hand and the work the gearing will have to do. Suppose he decides that 12-pitch will be about right. In Fig. 9 it will be remembered

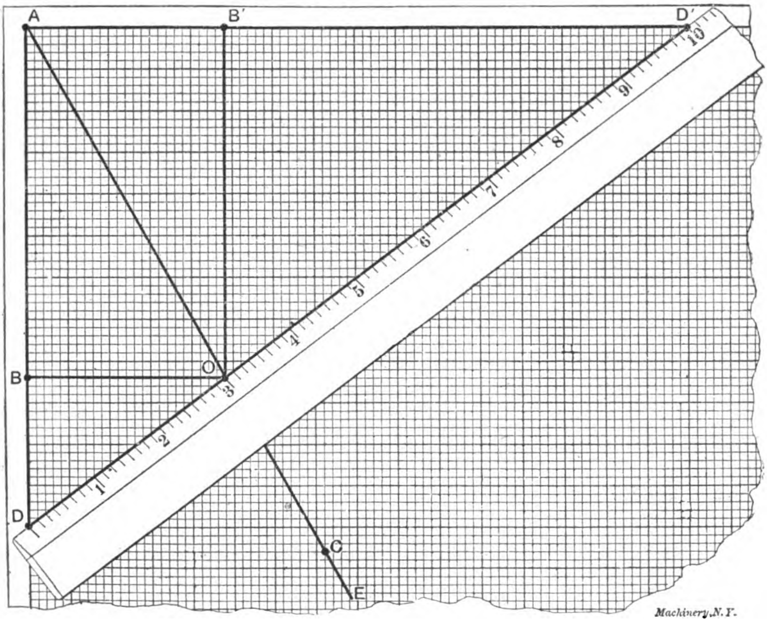


Fig. 9. Preliminary Solution with Rule and Cross-section Paper.

that *D O* was the pitch diameter of gear *a*, while *D' O* was the pitch diameter of gear *b*. That being the case *D O D'* is equal to twice the distance between the shafts. In the problem under consideration this will be equal to 2 × 2¼, or 4½ inches. Fig. 10 is a skeleton outline showing the operation of making the preliminary survey with rule and cross-section paper. *A G* and *A G'* represent the margin lines of the sheet, while *D D'* represents the graduated straight edge. By the conditions of the problem the distance between points *D* and *D'*, where the ruler crosses the margin lines, must be equal to 4½ inches. There has next to be determined at what angle of inclination the ruler shall be placed in locating this line. To do this, we will first find our "ratio line." Select any point *C* such that *C F'* is to *C F* as 2 is to 1, which is the required ratio of our gears. Draw through point *C*, so located, the line $A\ E$. Line $A\ E$ is then the ratio line, that is, a line so drawn that the measurements taken from any point on it to the margin lines will be to each other in the same ratio as the required ratio between the driving and driven gear. Now, by shifting the ruler on the margin lines, always being careful that they cut off the required distance of 4½ inches on the graduations, it is found that when the rule is laid as shown in position No. 1, cutting the ratio line at O', the distance from the point of intersection to corner A is at its maximum. For the minimum value, the tooth angle is the limiting feature. For a gear of this kind, 30 degrees is, perhaps, about as small as would be advisable, so when the ruler is inclined at an angle of about 30

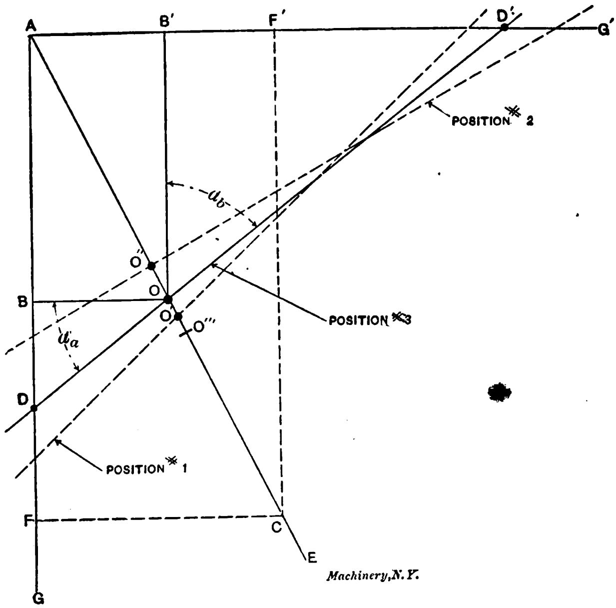


Fig. 10. Preliminary Graphical Solution for Problem No. 1.

degrees with margin line $A\ G'$, and occupies position No. 2 as shown, it will cut line $A\ E$ at O'', and the distance cut off from the point of intersection to corner A will be at its minimum value. The ruler must then be located at some intermediate position between No. 1 and No. 2.

Supposing, for example, 14 teeth in gear *a* and 28 teeth in gear *b* be tried. According to Rule 2, the equivalent diameter of gear *a* will then be $14 \div 12$, or 1.1666 inch; the equivalent diameter of *b* will be $28 \div 12$, or 2.3333 inches. Returning to the diagram to locate the point of intersection, it will be found that point O''' is so located that lines drawn from it to $A\ G$ and $A\ G'$ will be equal to 1.1666 inch and 2.3333 inches respectively, but this is beyond point O', which was found to be the outermost point possible to intersect with a 4½-inch line, $D\ D'$. This shows that the conditions are impossible of fulfillment.

Trying next 12 teeth and 24 teeth, respectively, for the two gears, the equivalent diameters by Rule 2 will be 1 inch and 2 inches. Point O is now so located that $O\ B$ equals 1 inch and $O\ B'$ equals 2 inches. Seeing that this falls as required between O' and O'', stick a pin in at this point to rest the straight edge against, and shift the straight edge about until it is located in such an angular position that the margin lines $A\ G$ and $A\ G'$ cut off $4\frac{1}{2}$ inches, or twice the required distance between the shafts, on the graduations. This gives the preliminary solution to the problem. Measuring as carefully as possible, $D\ O$, the pitch diameter of gear a, is found to be about 1.265 inch diameter, and $D'\ O$, the pitch diameter of gear b, about 3.235 inches. Angle $B\ O\ D$, the tooth angle of gear a, measures about 37° 50′. Angle $B'\ O\ D'$, the tooth angle of gear b, would then be 52° 10′ according to Rule 1. To determine angle $B\ O\ D$ more accurately than is feasible by a graphical process, use the following rule:

Rule 3. *The tooth angle of gear a in a pair of mating helical gears, a and b, whose axes are 90° apart, must be so selected that the equivalent diameter of gear b plus the product of the tangent of the tooth angle of gear a by the equivalent diameter of gear a, will be equal to the product of twice the center distance by the sine of the tooth angle of gear a.*

That is to say, in this case, $O\ B' + (O\ B \times$ the tangent of angle $B\ O\ D) = D\ D' \times$ the sine of angle $B\ O\ D$. Perform the operations indicated, using the dimensions which were derived from the diagram, to see whether the equality expressed in this equation holds true. Substituting the numerical values;

$$2 + (1 \times 0.77661) = 4.5 \times 0.61337,$$
$$2.77661 = 2.76016,$$

a result which is evidently inaccurate.

The solution of the problem now requires that other values for angle $B\ O\ D$, slightly greater or less than 37° 50′, be tried until one is found that will bring the desired equality. It will be found finally that if the value of 38° 20′ be used as the tooth angle of gear a, the angle is as nearly right as one could wish. Working out Rule 3 for this value:

$$2 + (1 \times 0.79070) = 4.5 \times 0.62024,$$
$$2.79070 = 2.79108.$$

This gives a difference of only 0.00038 between the two sides of the equation. The final value of the tooth angle of gear a is thus settled as being equal to 38° 20′. Applying Rule 1 to find the tooth angle of gear b we have: $90° - 38°\ 20' = 51°\ 40'$. The next rule relates to finding the pitch diameter of the gears.

Rule 4. *The pitch diameter of a helical gear equals the equivalent diameter divided by the cosine of the tooth angle; or the equivalent diameter multiplied by the secant of the tooth angle.*

If a table of secants is at hand, it will be somewhat easier to use the second method suggested by the rule, since multiplying is usually easier than dividing. Using in this case, however, the table of cosines, and performing the operation indicated by Rule 4, we have for the pitch diameter of gear *a*:

$$1 \div 0.78442 = 1.2748, \text{ or } 1.275 \text{ inch, nearly;}$$

and for the pitch diameter of gear *b*:

$$2 \div 0.62024 = 3.2245, \text{ or } 3.225 \text{ inch, nearly.}$$

To check up the calculations thus far, the pitch diameter of the two gears thus found may be added together. The sum should equal twice the center distance, thus:

$$1.275 + 3.225 = 4.500,$$

which proves the calculations for the angle.

Rule 5. *The outside diameter of a helical gear equals the pitch diameter plus the quotient of* 2 *divided by the diametral pitch of the cutter used.*

Applying this rule to gear *a:*

$$1.2748 + (2 \div 12) = 1.2748 + 0.1666 = 1.4414 = 1.441 \text{ inch, nearly.}$$

For gear *b:*

$$3.2245 + (2 \div 12) = 3.2245 + 0.1666 = 3.3911 = 3.391 \text{ inches, nearly.}$$

In cutting spur gears of any given pitch, different shapes of cutters are used, depending upon the number of teeth in the gear to be cut. For instance, according to the Brown & Sharpe system for involute gears, eight different shapes are used for a gear from 12 teeth to a rack. The fact that a certain cutter is suited for cutting a 12-tooth spur gear is no sign that it is suitable for cutting a 12-tooth helical gear, since the fact that the teeth are cut on an angle alters their shape considerably. To find out the number of teeth for which the cutter should be selected, use the following rule:

Rule 6. *The number of teeth for which the cutter should be selected to cut a helical gear is found by dividing the number of teeth in the gear by the cube of the cosine of the tooth angle.*

Applying this rule to gear *a:*

$$12 \div 0.784^3 = 12 \div 0.4818 = 25-,$$

and for gear *b:*

$$24 \div 0.620^3 = 24 \div 0.2383 = 100+,$$

giving, according to the Brown & Sharpe catalogue, cutter No. 5 for gear *a* and cutter No. 2 for gear *b*.

In gearing up the head of the milling machine to cut these gears it is necessary to know the lead of the helix or "spiral" required to give the tooth the proper angle. To find this, use Rule 7.

Rule 7. *The lead of the helix or "spiral" of a helical gear is equal to the product of the cotangent of the tooth angle by the pitch diameter by* 3.14.

In solving problems by this rule, as for Rule 6, it will be sufficient to use trigonometrical functions to three significant places only, this being accurate enough for all practical purposes. Solving by Rule 7 to find the lead to set up the gearing for in cutting *a:*

$1.275 \times 1.265 \times 3.14 = 5.065$, or 5 1/16 inches, nearly;

for gear *b:*

$3.225 \times 0.791 \times 3.14 = 8.010$, or 8 3/32 inches, nearly.

The lead of the helix must be, in general, the adjustable quantity in any spiral gear calculation. If special cutters are to be made, the lead of the helix may be determined arbitrarily from those given in the milling machine table; this will, however, probably necessitate a cutter of fractional pitch. On the other hand, by using stock cutters and varying the center distance slightly, we might find a combination which would give us for one gear a lead found in the milling machine table, but it would only be chance that would make the lead for the helix in the mating gear also of standard length. It is then generally better to calculate the milling machine change gears according to the usual methods to suit odd leads, rather than to adapt the other conditions to suit an even lead. It will be found in practice that the lead of the helix may be varied somewhat from that calculated without seriously affecting the efficiency of the gears.

The remaining rules relating to the proportions of the teeth do not vary from those for spur gears and are here set down for the sake of completeness only.

Rule 8. *The thickness of the tooth of a standard gear at the pitch line is equal to* 1.5708 *divided by the diametral pitch of the cutter.*

For gears *a* and *b* of our problem this gives:

$$1.5708 \div 12 = 0.1309 \text{ inch.}$$

Rule 9. *The addendum of a standard gear is equal to* 1 *divided by the diametral pitch of the cutter.*

For gears *a* and *b* this will give:

$$1 \div 12 = 0.0833 \text{ inch.}$$

Rule 10. *The whole depth of the tooth of a standard gear is equal to* 2.1571 *divided by the diametral pitch of the cutter.*

This gives for gears *a* and *b:*

$$2.1571 \div 12 = 0.1797 \text{ inch.}$$

This completes all the calculations required to give the essential data for making our first pair of helical gears. To illustrate the variety of conditions for which these problems may be solved, the other cases will be worked out somewhat differently. In the case just considered no allowance was made for possible conditions which might have limited the dimensions of the gears, and the problem was solved for what might be considered general good practice. Gear *a*, however, might have been too small to put on the shaft on which it was intended to go, while gear *b* might have been too large to enter the space available for it. If, as we may assume, these gears are intended to drive the camshaft of a gas engine, the solution would probably be unsatisfactory. Case No. 2 will therefore be solved for a center distance of 3⅜ inches as required, but the two gears will be made of about equal diameter. Fig. 11 shows the preliminary graphical solution of this problem, the reference letters in all cases being the same as in Fig. 10. With a 10-pitch cutter, if this suited the judgment of the designer, 15 teeth in gear *a* and 30 teeth in gear *b* would require that the point of intersection on the ratio line *A E* be located at *O* where *B O* equals the equivalent diameter of gear *a*, which equals 1½ inch, while *B′ O* equals

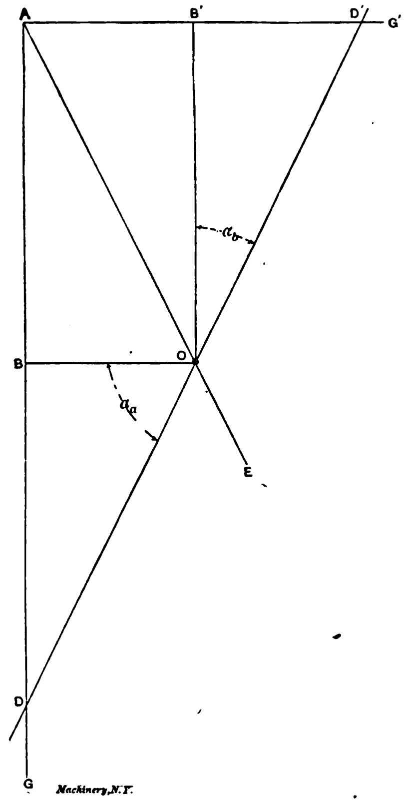


Fig. 11. Solution of Problem No. 2 for Equal Diameters.

the equivalent diameter of gear *b*, or 3 inches, both calculated in accordance with Rule 2. The required condition now is that *D O* be approximated to *D′ O*; that is to say, that the pitch diameters of the two gears be about equal. After continued trial it will be found impossible to locate *O*, using a cutter of standard diametral pitch, so that *D O* and *D′ O* shall be equal, and at the same time have *D D′* equal to twice the required center distance, which is 2 × 3⅜ inches or 6¾ inches. If this center distance could be varied slightly without harm, *B D* could be taken as equal to *A B*; then it would be found that a line drawn from *D* through *O* to *D′*, though giving a somewhat shortened center distance, would make two gears of exactly the same pitch diameter.

Drawing line $D\,O\,D'$, however, as first described to suit the conditions of the problem, and measuring it for a preliminary solution the following results are obtained: The tooth angle of gear $a =$ angle $B\,O\,D = 63°\ 45'$; and the tooth angle of gear $B =$ angle $B'\,O\,D' = 90° - 63°\ 45' = 26°\ 15'$, according to Rule 1. Performing the operations indicated in Rule 3 to correct these angles, it is found that when the tooth angle of gear a is 63° 54′, and that for gear b is 26° 6′, the equation of Rule 3 becomes

$$3 + (15 \times 2.04125) = 6.75 \times 0.89803$$
$$6.06187 = 6.06170$$

which is near enough for all practical purposes. The other dimensions are easily obtained as before by using the remaining rules.

To still further illustrate the flexibility of the helical gear problem, the third case, for a center distance of 4 inches, will be solved in a third way. It is shown in MacCord's Kinematics that to give the least amount of sliding friction between the teeth of a pair of mating helical gears, the angles should be so proportioned that, in our diagrams, line $D\,D'$ will be approximately at right angles to ratio line $A\,E$. On the other hand, to give the least end thrust against the bearings, line $D\,D'$ should make an angle of 45° with the margin lines $A\,G$ and $A\,G'$, in the case of gears with axes at an angle of 90°, as are the ones being considered. The first example worked out in detail was solved in accordance with "good practice," and line $D\,D'$ was located about one-half way between the two positions just described, thus giving in some measure the advantage of a comparative absence of sliding friction, combined with as small degree of end thrust as is practicable. To illustrate some of the peculiarities of the problem, Case 3 will now be solved to give the minimum amount of sliding friction, neglecting entirely the end thrust, which is considered to be taken up by ball thrust bearings or some equally efficient device. On trial it will be found that, with the same number of teeth in the gear and with the same pitch as in Case 2, giving in Fig. 12, $B\,O$, the equivalent diameter of gear a, a value of 1½ inch, and $B'\,O$, the equivalent diameter of gear b, a value of 3 inches as in Fig. 11, line $D\,D'$ which is equal to twice the center distance, or 8 inches, can then lie at an angle of about 90° with $A\,E$, thus meeting the condition required as to sliding friction. Thus this diagram, while relating to gears having the same pitch and number of teeth as Fig. 11, yet has an entirely different appearance, and gives different tooth angles and center distances, solving the problem as it does for the least sliding friction instead of for equal diameters of gears.

Measuring the diagram as accurately as may be, the following results are obtained: Tooth angle of gear $a = B\,O\,D = 28°$; tooth angle of gear $b =$ angle $B'\,O\,D' = 90° - 28° = 62°$. This is the preliminary solution. After accurately working it out by the process before described, we have as a final solution, tooth angle of gear $a = 28°\ 28'$;

tooth angle of gear $b = 61° 32'$. From this the remaining data can be calculated.

For designers who feel themselves skillful enough to solve such problems as these graphically without reference to calculations, the diagram may be used for the final solution. The variation between the results obtained graphically and those obtained in the more accurate mathematical solution is a measure of the skill of the draftsman as a graphical mathematician. The method is simple enough to be readily copied in a note book or carried in the head. If the graphical method is to be used entirely, it will be best not to trust to the cross-section paper, which may not be accurately ruled; instead skeleton diagrams like those shown in Figs. 10, 11, and 12 may be drawn. For rough solutions however, to be afterward mathematically corrected, as in the examples considered in this article, good cross-section paper is accurate enough. It permits of solving a problem without drawing a line. Point O may be located by reading the graduations; a pin inserted here may be used as a stop for the rule, from which the diameter and center distance are read directly; dividing AD, read from the paper, by DD', read from the rule, will give the sine of the tooth angle of the gear a.

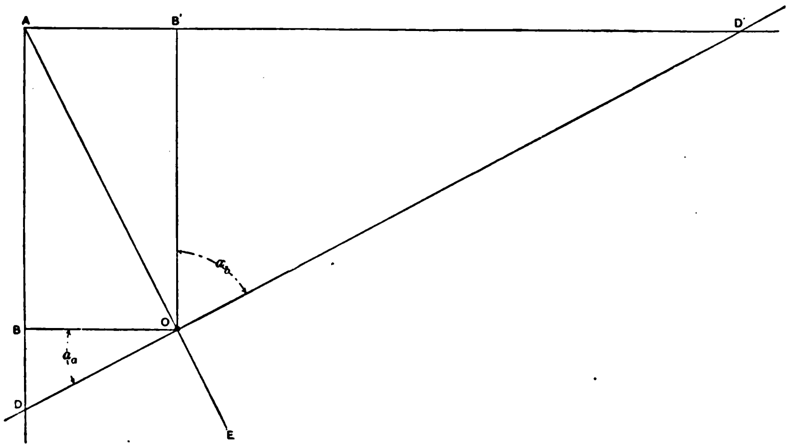


Fig. 12. Solution of Problem No. 3 for Minimum Sliding Friction.

Formulas for Spiral Gearing.

For sensible people, who prefer their rules to be embodied in formulas, the appended list has been prepared, using the following reference letters, which agree in general with the nomenclature of the Brown & Sharpe gear books.

N_a = No. of teeth in gear a.
N_b = No. of teeth in gear b.
R = Velocity ratio $= N_b \div N_a$.
P'' = Normal diametral pitch or pitch of cutter.
E = Equivalent diameter (explained above).

$D =$ Pitch diameter.
$C =$ Center distance.
$B =$ Blank or outside diameter.
$T =$ No. of teeth for which cutter is selected.
$L =$ Lead of spiral.
$\gamma =$ Angle of axes.
$\alpha =$ Angle of tooth with axis.
$t =$ Thickness of tooth on pitch line.
$S =$ Addendum.
$D'' + f =$ Whole depth of tooth.

Where subscript letters $_a$ and $_b$ are used, reference is made to gears a and b, as for instance, "N_a" and "N_b," where the letter N refers to the number of teeth in gears a and b, respectively, of a pair of gears a and b.

$$\gamma = \alpha_a + \alpha_b. \tag{29}$$

$$E = \frac{N}{P''}. \tag{30}$$

$$E_b + (E_a \times \tan \alpha_a) = 2\,C \times \sin \alpha_a. \tag{31}$$

$$D = \frac{E}{\cos \alpha} = E \times \sec \alpha. \tag{32}$$

$$B = D + \frac{2}{P''} \tag{33}$$

$$T = \frac{N}{(\cos \alpha)^3} \tag{34}$$

$$L = \cot \alpha \times D \times \pi. \tag{35}$$

$$t = \frac{1.5708}{P''}. \tag{36}$$

$$S = \frac{1}{P''}. \tag{37}$$

$$D'' + f = \frac{2.1571}{P''}. \tag{38}$$

CHAPTER III.

DIAGRAMS FOR DESIGNING SPIRAL GEARS.

Great difficulties are usually experienced in designing spiral gears, and these difficulties are greatly accentuated when one has to design them for two shafts whose center distance cannot be altered to suit the gears, and also when the angle between the shafts is not a right angle, and the speed ratio is not equal. The general practice is to work out the gears by lengthy mathematics, and should the answer not come out as desired, then a new trial is made, varying either one or the other factor, until the angles and diameters are correct. This method of "cut and try" entails a great deal of work and waste of time. The following method, together with the diagrams used with it, will remove some of the difficulties, and enable one to arrive at the data required in a very short time. The method adopted is graphical, but the results may be checked by simple figuring.

As the pitch diameter, spiral angle, and circular pitch are interdependent, they cannot be considered as a starting point in solving the problem, because they are not known. The starting point, therefore, must be the speed ratio, and some idea of the strength required, together with the center distance. These factors, as a rule, can easily be ascertained. As it is common usage to employ ordinary spur gear cutters for regular diametral pitch to cut spiral gears with, the normal pitch, or distance from one tooth to the next measured at right angles to the tooth, must be the same as the pitch of a spur gear for which the cutter to be used is intended; therefore the corresponding diametral pitch and the speed ratio must be the initial data, all others being obtained afterwards.

Three diagrams are given for the graphical solution of spiral gears. The diagram in Fig. 13 shows the relation between the quotient of number of teeth ÷ diametral pitch, spiral angles, and pitch diameters. The diagram in Fig. 14 shows the relation between the diametral pitch, the number of teeth, and the quotient of the number of teeth ÷ diametral pitch. Finally, the diagram in Fig. 15 shows the relation between the pitch diameter, the spiral angle, and the lead of the helix. We will now proceed to give some typical examples illustrating the use of the diagrams.

Example 1. Given a gear having 24 teeth, 6 diametral pitch, and a spiral angle of 40 degrees. Find the pitch diameter.

First obtain the value of the ratio, number of teeth ÷ diametral pitch, which, in this case, can be obtained without referring to diagram Fig. 14, being simply 24 ÷ 6 = 4. Locate 4 on the horizontal line in diagram Fig. 13, and project vertically until the line from figure 4 intersects the line for 40 degrees spiral angle. Then follow the circular arc from this point, either to the right or downward, reading off 5.22 on the corresponding scale, this being the pitch diameter. Should the diameter be required accurately, we can figure it by the formula:

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\text{cos spiral angle}}$$

$$= 4 \times \frac{1}{\text{cos 40 deg.}} = 5.222 \text{ inches.}$$

This also gives a check of the result obtained by means of the diagram. The lead of the helix is now obtained from Fig. 15, by pro-

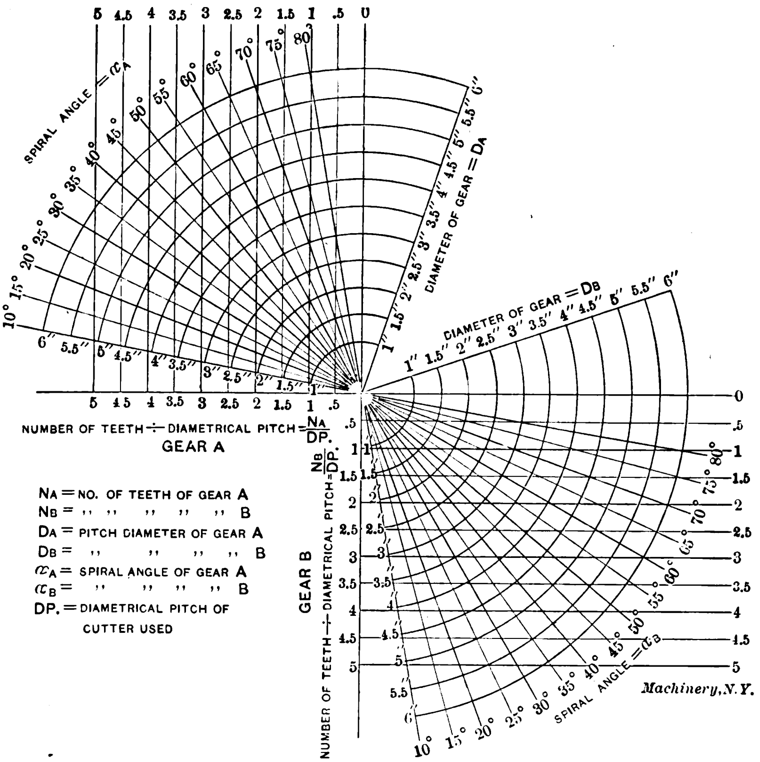


Fig. 13. Diagram of Relation between Number of Teeth, Diametral Pitch, Spiral Angles, and Pitch Diameters.

jecting the pitch diameter 5.22 horizontally to the radial line for the spiral angle, and then, following the vertical line to the lead scale at the bottom of the diagram, we find, in this case, a lead of 19.6 inches. Of course, the outside diameter of the blank would be $5.222 + 2 \times 1/6 = 5.555$ inches, which is the pitch diameter + 2 times the addendum.

Example 2. Required two gears which are to be equal in all respects, the diametral pitch being 8, and the centers to be approximately 4 inches apart.

As the centers are not fixed, the gears in this case may be made with 45 degrees spiral angle, and the center distance may be slightly adjusted to suit the pitch diameters. Referring to Fig. 13, follow the circular arc from diameter of gear = 4 inches, until it intersects the radial line for 45 degrees spiral angle; then follow the vertical line down to the scale of the ratio between the number of teeth and diametral pitch, which is found to be 2.82. Then, from Fig. 14, we find that with this ratio and 8 diametral pitch, the number of teeth is not a whole number, but the nearest number is 23, giving a ratio of 2.875 instead of 2.82, which, by reversing the process and referring to dia-

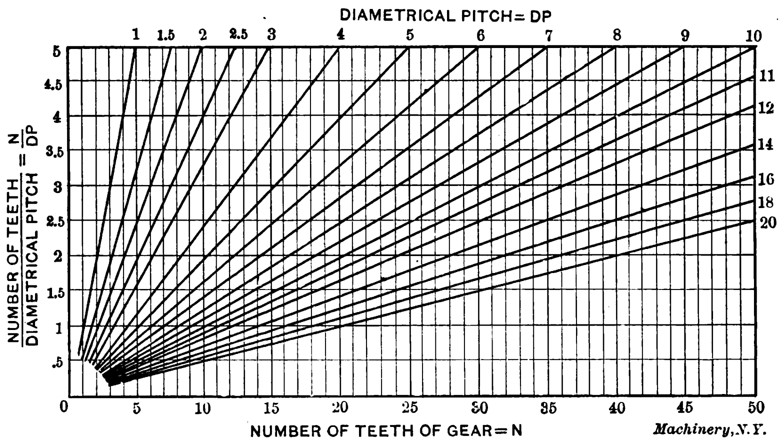


Fig. 14. Relation between Diametral Pitch, Number of Teeth, and Quotient of Number of Teeth divided by Diametral Pitch.

gram Fig. 13, gives a pitch diameter of 4.07 inches. These results may be checked as follows:

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\cos 45 \text{ deg.}}$$

$$= 2.875 \times \frac{1}{0.707} = 4.07 \text{ inches.}$$

The outside diameter is $4.07 + 2 \times 0.125 = 4.32$. The lead, as obtained from diagram Fig. 15, in the same way as in Example 1, is 12.79 inches.

Example 3. Required a pair of spiral gears having a normal pitch corresponding to 10 diametral pitch, having a given center distance of 2½ inches approximately, the sum of the spiral angles being 90 degrees, and the speed ratio equal to 5 to 1.

In this case both portions of diagram Fig. 13 are used, the upper part being employed for one gear and the lower part for the other, the easiest way being to get a strip of paper with two lines marked on its edge 5 inches (twice the center distance) apart, drawn to the same scale as the diagram. Move this strip of paper on the diagram (so that the edge of the strip passes through the center), as indicated at *A*, Fig. 16, until the lines marked coincide with points where the ratios of number of teeth ÷ diametral pitch equal 5 ÷ 1, and then determine from Fig. 14 that these diameters also give whole numbers of teeth with 10 diametral pitch. We find that 0.5 and 2.5 at 78 degrees and 12 degrees are two such positions, and also 0.6 and 3.0 at 70 degrees and 20 degrees. If we use the latter values, we will have 6 teeth and 30 teeth at 70 and 20 degrees angle respectively. The exact diameters can now be determined, as in our previous problem, and are 1.75 and 3.19 inches, respectively, the outside diameters being 0.2 inch

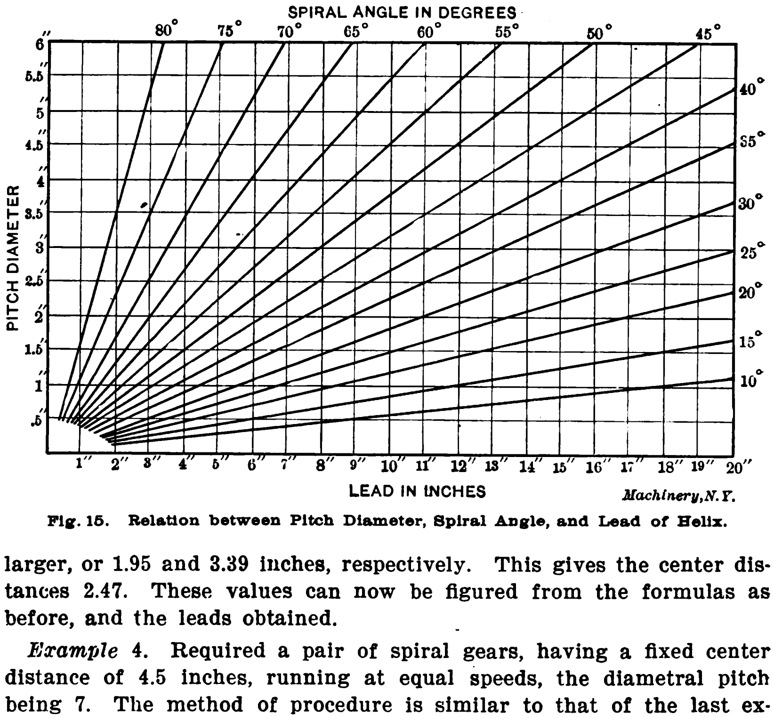


Fig. 15. Relation between Pitch Diameter, Spiral Angle, and Lead of Helix.

larger, or 1.95 and 3.39 inches, respectively. This gives the center distances 2.47. These values can now be figured from the formulas as before, and the leads obtained.

Example 4. Required a pair of spiral gears, having a fixed center distance of 4.5 inches, running at equal speeds, the diametral pitch being 7. The method of procedure is similar to that of the last example, using a strip of paper having a distance of 9 inches marked on the edge in the proper scale, as indicated at *B* in Fig. 16. At about 40 degrees spiral angle we find in Fig. 13 the ratio of number of teeth to diametral pitch to equal 3.14. This ratio must be adjusted on diagram 14, as previously shown, so as to enable one to get a whole number of teeth with 7 diametral pitch, this number being in this case 22. The ratio is then 3.143, and following from this in Fig. 13 to the 40-degree line, one obtains a pitch diameter of about 4.1 inches for one gear, and at 50 degrees about 4.9 inches for the other.

The spiral angles should now be carefully checked mathematically as follows:

$$\text{Cos spiral angle (first gear)} = 3.143 \times \frac{1}{4.1} = 0.766; \text{ spiral angle} = 40 \text{ deg.}$$

$$\text{Cos spiral angle (second gear)} = 3.143 \times \frac{1}{4.9} = 0.642;$$
$$\text{spiral angle} = 50 \text{ deg., nearly.}$$

Now obtain the leads from diagram Fig. 15 in the same way as before, giving the leads of the gears 15.4 and 12.9 inches, respectively.

Example 5. Required a pair of spiral gears, the axes of which are at an angle of 120 degrees; center distance 4.125; the ratios between number of teeth and diametral pitch should be to each other as 2 to 3, and the diametral pitch equals 5.

We require first of all two numbers representing the ratios of number of teeth to diametral pitch, these two numbers bearing the ratio to

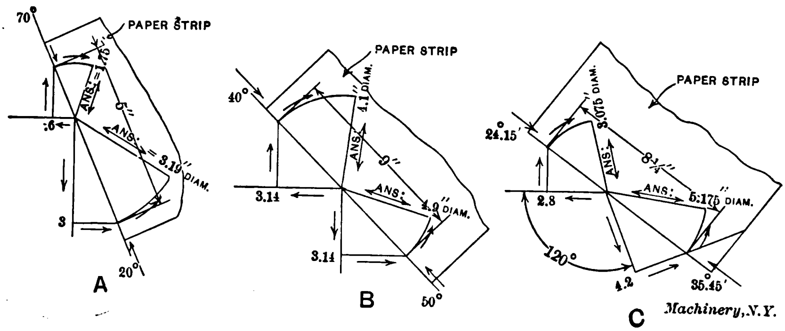
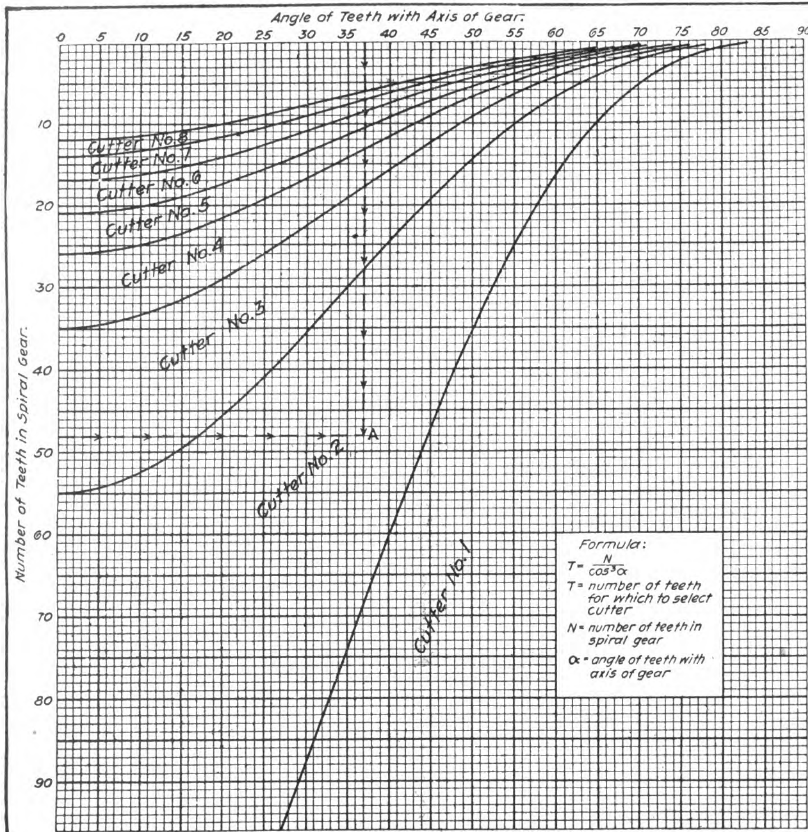


Fig. 16. Separate Diagrams for the Solution of some of the Problems Presented.

each other of 2 to 3, and giving a whole number of teeth with 5 diametral pitch. These two numbers, when projected onto two spiral angle lines in a diagram made up as in Fig. 13, the sum of the angles of which equals 120 or 60 degrees, give two diameters whose sum equals the center distances multiplied by 2, or 8.25. In this case we cannot use both parts of the diagram Fig. 13, as it is made up for shafts at 90 degrees angle, and for this reason we must take the two readings from the same part of the diagram. The ratios 3 and 4.5 at 30 degrees give corresponding diameters of 3.5 and 5.2, the sum being 8.7. The ratios 2.8 and 4.2 giving 14 and 21 teeth at 25 and 35 degrees, respectively, have diameters of 3.1 and 5.15 (equals 8.25). From this we see that we must use 14 and 21 teeth and the ratios 2.8 and 4.2. The diameters and spiral angles can now be obtained graphically and more accurately in this manner:

Draw two radial lines, as shown at *C* in Fig. 16, at 120 degrees angle on a separate piece of paper, and lay off on these to same scale 2.8 and 4.2. From these points draw lines at right angles to the radial lines. It is now necessary to find the position of a line 8.25 inches long,



The cutters used for milling spiral or helical gears are standard spur gear cutters, the number of a cutter and its pitch for a given case being defined by the angle (with axis) and normal pitch. This diagram gives the numbers of the cutters only, the pitch having been previously determined.

The selection of the cutter is fixed by the formula given in the lower right-hand corner of the diagram. The delimiting curves thereon were plotted by the formula, the area between the curves being the field of intersection of the combinations of angles and numbers of teeth covered by each designated cutter number.

For example, suppose the angle of the teeth of a gear is 37 degrees with its axis, and the number of teeth is 48. The point A, at which the horizontal line (representing the tooth number), and the vertical line (representing the angle) intersect, falls within the area marked "Cutter No. 2" Therefore, a No. 2 cutter is required to cut a 48-tooth spiral gear having the teeth at an angle of 37 degrees with its axis.

Fig. 17. Diagram for Determining Cutter to Use for Milling Spiral Gears.

terminating upon these lines, and passing through the center. A strip of paper is used in the same manner as before, and upon careful measuring of the respective distances from the center to the lines, one obtains the distances 3.075 and 5.175 inches, which represent the respective diameters, the sum being 8.25. The spiral angle is obtained by measuring or calculating as follows:

$$\text{Cos spiral angle of first gear} = 2.8 \times \frac{1}{3.075} = 0.910;$$

$$\text{spiral angle} = 24 \text{ deg. } 15 \text{ min.}$$

$$\text{Cos spiral angle of second gear} = 4.2 \times \frac{1}{5.175} = 0.812;$$

$$\text{spiral angle} = 35 \text{ deg. } 45 \text{ min.}$$

The above examples will show the careful student the manner of working out each kind of gear required, and if the directions are properly followed, this method will be found to be a great time-saver. It may be mentioned that it is advisable to keep the spiral angle as nearly equal in the two gears as possible in order to obtain the greatest efficiency of transmission. It should be noted that when diagrams of this type are to be used for practical calculation of spiral gears, they should be laid out in a much larger scale than is possible to show in these pages, and it would be advisable to lay out radial lines in Fig. 13 for every degree, and vertical and horizontal lines for every tenth of an inch, and circular arcs for equally fine subdivisions. The same is true of the diagrams in Figs. 14 and 15. In Fig. 14, horizontal lines should be laid out for every tenth of an inch, and vertical lines should be laid out for all whole numbers of teeth. In Fig. 15, the horizontal lines should be laid out for every tenth of an inch, vertical lines for at least every 0.2 of an inch, and radial lines for every degree. This diagram should also be laid out so that leads over 20 inches may be read off, as well as those below this figure.

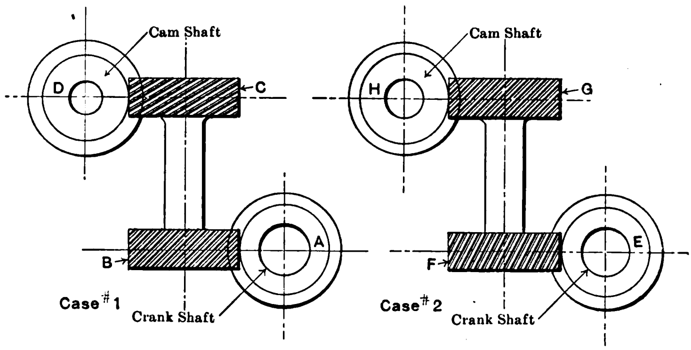
In Fig. 17 is given a diagram for determining the cutter to use when milling the teeth of spiral gears. The instructions for the use of the diagram are given directly on the chart itself, so that no other explanation is necessary. This diagram was contributed to MACHINERY by Elmer G. Eberhardt, and appeared in the September, 1907, issue.

CHAPTER IV.

COMPARISON OF EFFICIENCY OF SPIRAL GEARS.

Suppose a problem, accompanied by the data shown in Fig. 18, were presented as follows: "Given two different sets of spiral gears for gas engines. In each case the cam shaft runs at half the speed of the crank-shaft; to be decided which is the better arrangement for efficiency and wearing qualities, taking into consideration the nature of the work the drive has to perform, *viz.*, a single cylinder gas engine working on the 'Otto' cycle."

The solution of this problem involves a little work along the line



	Angle of Teeth with Axis	Diameter of Pitch Circle	Number of Teeth
A	45°	4"	20
B	45°	4"	20
C	63° 26′	4"	10
D	26° 34′	4"	20

	Angle of Teeth with Axis	Diameter of Pitch Circle	Number of Teeth
E	63° 26′	4"	10
F	26° 34′	4"	20
G	45°	4"	20
H	45°	4"	20

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Fig. 18. The Two Arrangements to be Investigated.

of resolution of forces and the calculation of efficiency; it is entirely elementary, but interesting nevertheless, as a practical illustration of the working of well-known principles in mechanics. For the sake, then, of their value as illustrations of the principles involved, these calculations are here given in detail.

The problem requires us to find which of the two arrangements, that in Case 1 or Case 2, is superior in efficiency and wearing qualities. It may be roughly stated that, other things being equal, the more efficient of two mechanisms is the more durable. We will consider this to be true in this case, so will examine the two arrangements for efficiency in the transmission of power. The power losses in the various journals we cannot estimate, because we do not know enough about the arrangement and design of the bearing surfaces. We can easily make an estimate for the power lost in the thrust bearings, and we may also get a comparative idea, at least, of the power lost in the rubbing of the teeth on each other, so to these losses, which are the principal ones, we will confine ourselves.

When two bodies are sliding under pressure, the power lost is equal to the continued product of the normal pressure between the surfaces, the linear velocity of the rubbing, and the coefficient of friction. To estimate the power lost at the various bearing points we are to consider, we have then to estimate these three factors for each case.

We will first estimate the relative bearing pressures at the different places where friction is met with in Case 1. To be logical, we will commence our calculations at the driven end of the train of gears, since the forces in the mechanism are due to the resistance offered by the driven members. Fig. 19 is another view of Case 1 as shown in Fig. 18. Gears *A* and *B* make contact on line *Y Z*, which represents the direction of the teeth at the point of contact; *W X* represents the position of the teeth of gears *C* and *D* in contact.

As gear *C* revolves in the direction shown, its teeth, set at the angle

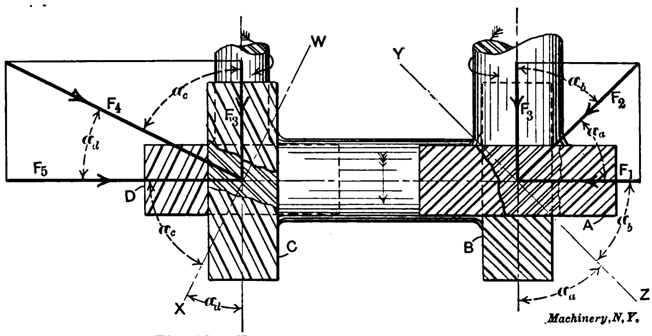

Fig. 19. Force Diagrams for Case 1.

of the line *W X*, have a wedging action on those of gear *D* which revolves them in the direction shown. The action and the forces involved can best be understood by referring to Fig. 20. Here *C* is a slide moving upward. Its beveled edge, representing the tooth surface of gear *C*, forces to the left on the beveled edge on slide *D*, which represents the tooth surface of gear *D*. If slide *D* offers a resistance to this movement, of a magnitude represented by the length of line F_5 in the parallelogram of forces shown, slide *C* will evidently have to exert a force equal to F_3 to overcome this resistance. The resulting normal pressure on the inclined bearing surface of contact will evidently be F_4. The end thrust or pressure against its abutment of slide *D* will be F_3, while that of slide *C* against its abutment will be F_5.

Understanding the method of applying the parallelogram of forces in Fig. 20, we may transfer the construction to gears *C* and *D* in Fig. 19. Having F_5 given, we can find F_4 and F_3 as there shown. F_3 is the tangential pressure at the pitch line, required to be given by gear *C* to move the mechanism against the resistance F_5 offered by gear *D*. Since gears *B* and *C* have the same diameter, F_3 must likewise be the tangential pressure applied at the pitch line to gear *B*. Constructing a second parallelogram of forces for gears *A* and *B*, as shown, we find that F_2 is the normal pressure between the faces of the teeth in contact, and F_1 is the tangential force which has to be brought to bear at the pitch line of gear *A* to move the mechanism. Consider that F_5 equals unity (since we require comparative results only) and measure the other forces to this scale. This can be done fully as well by calculation as by measurement. An elementary knowledge of trigonometry will give us the following results:

$$F_5 = 1$$
$$F_4 = F_5 \div \sin a_c = 1 \div 0.894 = 1.118$$
$$F_3 = F_5 \times \tan a_d = 1 \times 0.500 = 0.500$$
$$F_2 = F_3 \div \sin a_a = 0.500 \div 0.707 = 0.707$$
$$F_1 = F_3 \times \tan a_b = 0.500 \times 1.000 = 0.500.$$

We have next to find the rubbing velocities of the various bearing points. Fig. 21 will assist us in this. Here we have the same slides

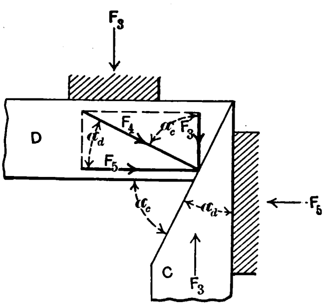

Fig. 20. Illustration of Principle Involved in Fig. 19.

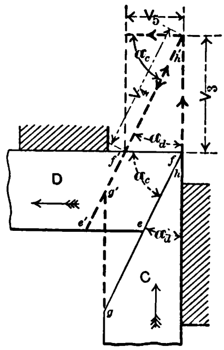

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Fig. 21. Illustration of Principle Involved in Fig. 22.

C and *D*, representing gears *C* and *D* in Fig. 19 or Fig. 22. If we consider that slide *C* is moved upward at a uniform velocity, in a unit of time it will traverse a distance equal to V_3, moving from position *g h* to *g′ h′*. This evidently forces slide *D* to the left at a uniform velocity, moving it in a unit of time from *e f* to *e′ f′*, a distance measured by dimension V_5. The beveled surface of slide *D* has meanwhile slipped on that of slide *C* so that corners *f* and *h*, which were in contact, have reached positions *f′* and *h′*, a distance measured by dimension V_4. It is evident then that V_3, V_4, and V_5 may be taken as measures of relative velocities of the parts in question.

Since the mechanism shown in Fig. 21 represents, in principle, conditions existing between gear *C* and *D* in Fig. 22, we may transfer the velocity diagram of Fig. 21 to Fig. 22, where V_5 rerpesents the pitch velocity of gear *D*, V_4 the rate of rubbing at the pitch line between gears *C* and *D*, and V_3 the circumferential velocity at the pitch line of gear *C*. The circumferential velocity at the pitch line of gear *B* is evidently the same as that of gear *C*, since they are of the same diameter and move together. V_3 being thus known, a similar velocity diagram may be drawn for gears *A* and *B*, in which V_1 equals the velocity at the pitch line of gear *A*, and V_2 equals the velocity of sliding between the teeth of gears *A* and *B*.

We may, if we wish, measure these lines to the scale $V_1 = 1$ to obtain the relative velocities desired, or, better, we may derive formulas from these velocities, thus making unnecessary the drawing of diagrams for subsequent examples of this kind. By a simple use of trigonometrical functions, after carefully examining the diagrams, it is plain that the following relations hold true:

$V_1 = 1$
$V_2 = V_1 \div \sin a_a = 1 \div 0.707 = 1.414$
$V_3 = V_1 \times \tan a_b = 1 \times 1.000 = 1.000$
$V_4 = V_3 \div \sin a_c = 1 \div 0.894 = 1.118$
$V_5 = V_3 \times \tan a_d = 1 \times 0.500 = 0.500$

The power lost in any bearing is equal to the continued product of the total pressure on that bearing, the velocity of sliding, and the

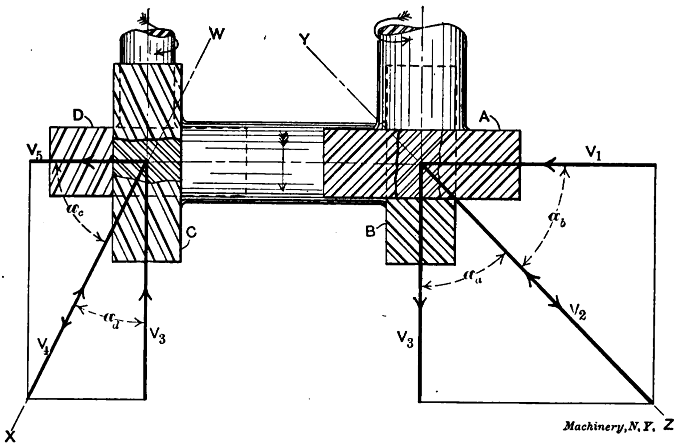

Fig. 22. Velocity Diagrams for Case 1.

coefficient of friction. We will first find the power lost in end thrust. Since our calculation is being made for comparison only, and not for positive results, we will consider the coefficient of friction as being equal to 1. We will make the assumption that the mean diameter of the end thrust bearings of the various shafts is equal to half the pitch diameter of the gears. The mean velocity of rubbing will then be half the velocity of the gears at the pitch line. For the loss of power in the thrust bearing of shaft *A* we have:

$$F_3 \times \frac{V_1}{2} \times 1 = 0.500 \times 0.500 \times 1 = 0.250.$$

The end thrust on the intermediate shaft is that due to the difference between the opposing forces F_1 and F_5 in Fig. 19. For lost work in the end thrust of the intermediate shaft we then have:

$$(F_5 - F_1) \times \frac{V_3}{2} \times 1 = (1 - 0.500) \times 0.500 \times 1 = 0.250.$$

The loss in the thrust bearing of shaft D equals

$$F_3 \times \frac{V_5}{2} \times 1 = 0\ 500 \times 0.250 \times 1 = 0.125.$$

Adding these three losses together we get a total value of 0.625 as the power loss in end thrust.

For the power loss in tooth friction, we had better use a somewhat higher coefficient; perhaps 1.5 would be about right. The velocity of sliding between gears A and B is V_2, the normal pressure of the surfaces of contact is F_2. We have then for the lost power at this point:

$$F_2 \times V_2 \times 1.5 = 0.707 \times 1.414 \times 1.5 = 1.500.$$

Similarly, the work lost between gears C and D equals

$$F_4 \times V_4 \times 1.5 = 1.118 \times 1.118 \times 1.5 = 1.875.$$

The total loss due to tooth friction is then equal to the sum of these two or 3.375, which, added to the loss in the thrust bearings, gives us $3.375 + 0.625 = 4.0$, the total loss with this form of gearing.

It will not be necessary to draw new diagrams, like those in Figs. 19 and 22, for the second case, since we may use the formulas already derived for obtaining the various forces and velocities, making, however, the following substitutions. This change is in accordance with the data in Case 2.

Change α_a to $\alpha_e = 63° 26'$ Change α_c to $\alpha_g = 45°$
" α_b to $\alpha_f = 26° 34'$ " α_d to $\alpha_h = 45°$

Solving these formulas for velocities, we obtain the following quantities:

$$V_1 = 1$$
$$V_2 = V_1 \div \sin \alpha_e = 1 \div 0.894 = 1.118$$
$$V_3 = V_1 \times \tan \alpha_f = 1 \times 0.500 = 0.500$$
$$V_4 = V_3 \div \sin \alpha_g = 0.500 \div 0.707 = 0.707$$
$$V_5 = V_3 \times \tan \alpha_h = 0.500 \times 1.000 = 0.500$$

and for pressures we have the following:

$$F_5 = 1$$
$$F_4 = F_5 \div \sin \alpha_g = 1 \div 0.707 = 1\ 414$$
$$F_3 = F_5 \times \tan \alpha_h = 1 \times 1.000 = 1.000$$
$$F_2 = F_3 \div \sin \alpha_e = 1.000 \div 0.894 = 1.118$$
$$F_1 = F_3 \times \tan \alpha_f = 1.000 \times 0.500 = 0\ 500$$

The work lost with the thrust bearing on shaft E equals

$$F_3 \times \frac{V_1}{2} \times 1 = 1 \times 0.5 \times 1 = 0.5.$$

That lost in the intermediate shaft equals

$$(F_5 - F_1) \times \frac{V_3}{2} \times 1 = (1 - 0.5) \times 0.25 \times 1 = 0.125.$$

The loss in power due to end thrust in shaft H equals

$$F_3 \times \frac{V_5}{2} \times 1 = 1 \times 0.25 \times 1 = 0.25$$

These three losses added together equal 0.875.

The loss of power due to tooth friction between *E* and *F*, assuming a coefficient of friction of 1.5 as before, equals

$$F_3 \times V_3 \times 1.5 = 1.118 \times 1.118 \times 1.5 = 1.875.$$

Friction loss between *G* and *H* equals

$$F_4 \times V_4 \times 1.5 = 1.414 \times 0.707 \times 1.5 = 1.5.$$

The tooth friction loss in the tooth surfaces then equals $1.875 + 1.500 = 3.375$. For Case 2 the total lost work due to tooth friction and end thrust friction equals $3.375 + 0.875 = 4.250$. The difference between this quantity and the 4.000 obtained for Case 1 is scarcely large enough to be of any practical importance. There is but one consideration, in fact, we can think of for preferring one construction to the other. The 45-degree gears have teeth of slightly smaller size than those of the other pair in each case, and they are therefore somewhat weaker. In Case 1, these teeth are subjected to a normal pressure F_2 of 0.707. In Case 2 they are subjected to a normal pressure F_4 of 1.414, twice as great. In Case 1, then, the strongest teeth are bearing the greatest strain, which is as it should be.

CHAPTER V.

SETTING THE TABLE WHEN MILLING SPIRAL GEARS.

In cutting a spiral gear in a milling machine as ordinarily arranged, it is necessary to set the table to the helix angle in order that the sides of the cutter may not interfere, or drag in the cut. But the helix angle varies with the depth, being greatest at the top of the tooth, less at the pitch line, and still less at the bottom of the cut. In fact, if the cut were deep enough to reach all the way to the center of the piece being operated on, the helix angle would become zero, or parallel to the center line. If the general run of mechanics were asked what would be the proper angle at which to set the table, they would say that the helix angle at the pitch line would be the one to determine the setting. This setting has the effect of making the width of the cut exactly right at the pitch line, but it does so at the expense of undercutting and weakening the teeth. For quite some time the writer has thought that the helix angle at or near the bottom of the cut should be the one to set the table to in order to get a strong tooth, and to convince other mechanics of the correctness of this view the following experiments were made:

The piece *G*, Fig. 23, is a cast iron taper stem with a flange cast on, and fits in the dividing head of the milling machine. *H* is a brass chuck that was made for another job, and is fastened to *G* with four screws. *K* is a piece of 1⅝-inch round brass, with the ends faced true, while *L* is a piece of scrap brass with two faces machined at an angle of 30 degrees, one of these faces being tapped for the screw *M*, and also containing the two dowel pins *N*. *H*, *K*, and *L* are sweated together with soft solder.

The six pieces of sheet brass, *A*, *B*, *C*, *D*, *E*, and *F*, Fig. 24, were drilled so they would fit onto the dowel pins on the 30-degree face of the improvised chuck, *F* being shown in place in the line cut, Fig. 23. These six pieces were placed in succession on this chuck and the curved edges of all turned to a diameter of 1.23 inch. The object was to make a spiral cut in each of these six pieces, varying the setting of the table angle, and also the shape of the cutter, and to compare the shape of the cut with that of the cutter that made it.

The lead used was 5.33 inches to one turn, and the depth of cut ¼

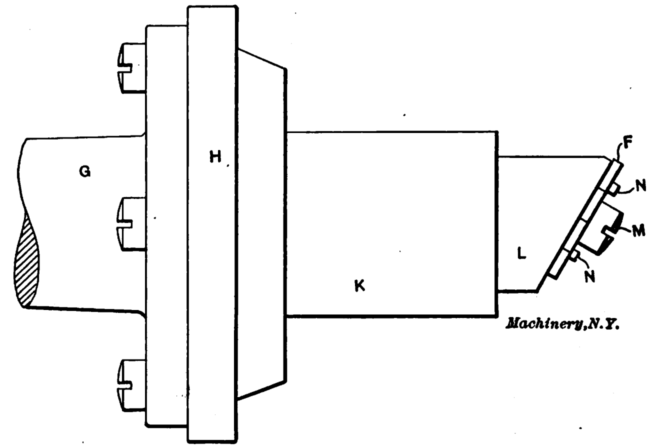

Fig. 23. Chuck for Mounting the Pieces shown in Fig. 24.

inch, both these elements being alike in all six cases. The pieces of sheet brass were intended to stand at right angles with the cut, but, of course, this was impossible, as the helix angle varied with the depth, so they were set to stand at right angles with the helix at half depth. Assuming this helix angle to be 30 degrees, we can find the diameter of the imaginary cylinder whose surface is at half the depth of the cut by multiplying the lead, 5.33 inches, by the tangent of 30 degrees, and dividing by 3.1416, which gives 0.98 inch. Adding 0.25 inch to this, we get 1.23 inch for the outside diameter, and also, by subtracting 0.25 from 0.98, we get 0.73 inch for the diameter at the bottom of the cut. Knowing the outside diameter to be 1.23 inch, we multiply this by 3.1416 and divide by 5.33 to get the tangent of the helix angle at the top, which we find to be 35 degrees 56 minutes. In a similar manner we multiply the bottom diameter, 0.73, by 3.1416 and divide by 5.33 to get the tangent of the helix angle at bottom of cut, which we find to be 23 degrees 17 minutes.

The cutter used in this job was a fly-cutter, the holder being shown in the half-tone illustration Fig. 25, while its blades, *R* and *S*, are shown in the line illustration Fig. 24. *R* was used to cut *A*, *B*, and *C*, while *S* was used to cut *D*, *E*, and *F*. The table setting was nearly 36 degrees for *A* and *D*, 30 degrees for *B* and *E*, and about 23¼ degrees for *C* and *F*.

The shapes of the cuts show that the width of the cutter is accurately reproduced only at the particular depth where the helix angle is the same as the table setting, this point being shown by the arrow heads at the sides of the various cuts. The shapes of the cuts also show that the departure from the true form of the cutter due to faulty

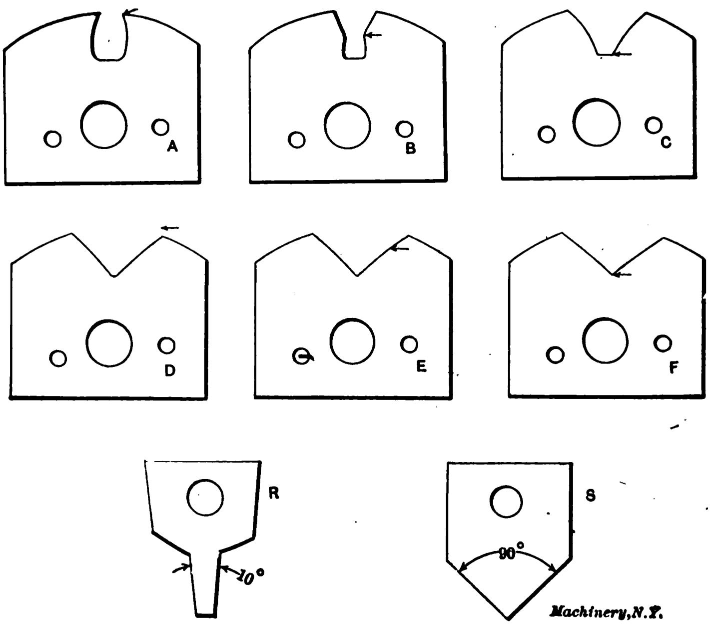

Fig. 24. Pieces Milled on the Chuck, Fig. 23, with Fly-cutters R and S, showing the Effect of varying the setting of the Milling Machine Table.

table setting is less in the case of the more flaring cutter *S* than in the case of cutter *R*, whose sides come nearer to being parallel.

This demonstrates that the table setting for a spiral gear should be the same as the helix angle at or near the bottom of the cut, because at this point the sides of the cutter come closer to parallelism, while at the top of the cut they are more flaring, and the table setting not being correct for the helix angle at this point would produce a comparatively slight error. This also suggests a slight modification in selecting a cutter to do the job, as the tops of the teeth would be rounded off somewhat more than in the case of a spur gear cut with the same cutter. Therefore, the writer suggests that it would be well to select a cutter for a greater number of teeth than the spiral gear formula

$$T = \frac{N}{\cos^3 a}$$ calls for.

Setting the table for a less angle than that of the pitch line helix also has the effect of slightly increasing the width of the cut at the pitch line, but not to the extent that a comparison of C and R would seem to indicate, as in the experiments here described the depth of the cut was purposely made a very large percentage of the diameter in order to accentuate the errors due to faulty setting of the table, and if the table is to be set correct for the bottom of the cut, it might be well to consider the normal circular pitch as slightly greater than

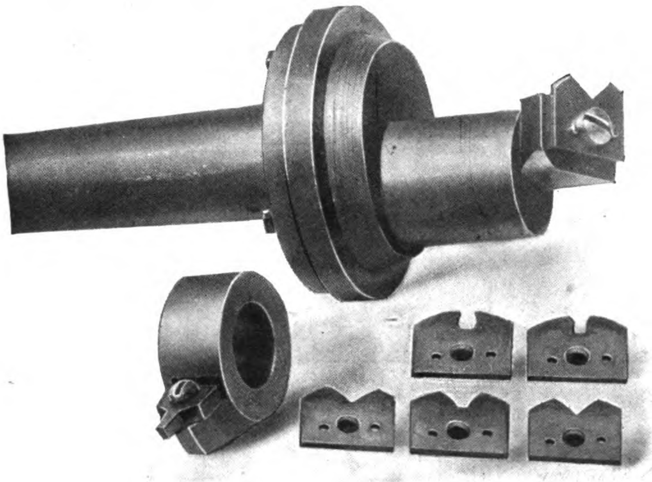

Fig. 25. Chuck, Fly-cutter, and Pieces Milled in Experiments Described.

that rightfully belonging to the cutter in use, and size the blank accordingly. These experiments were entirely of a qualitative nature, and were only intended to guide the judgment of the designer and the man who puts the design in material form in cold metal.

Those not thoroughly familiar with universal milling machine use should carefully distinguish between the table angle and the helix angle produced by the gearing of the dividing head. The dividing head is geared to produce the required helix angle, measured on the pitch line the same as usual, of course. What is advocated, in order to reduce interference, is simply setting the table to some helix angle between the pitch line and the dedendum or root circle rather than to the helix angle indicated by the pitch line. A point worth attention, also, is that the interference of a cutter increases with increase of diameter. Small cutters, therefore, tend to reproduce their outlines more accurately than large cutters, other things being equal.