MACHINERY'S REFERENCE SERIES

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No. 15—SPUR GEARING

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CHAPTER I.

FIRST PRINCIPLES OF GEARING.

Gear wheels are such common objects about the machine shop, and are manufactured with such rapidity and ease by the aid of the modern automatic gear cutter, that many seldom stop to think what they really are, why the teeth must be constructed with certain curves, and what it is desired that they shall accomplish. In a following chapter we shall take up some of the practical questions, touching upon the calculations that come up in the shop, but will here deal chiefly with a few of the theoretical points of the subject that are seldom explained in a simple manner for the benefit of those who have had neither the time nor the opportunity to look into matters of this kind.

Suppose there are two wheels arranged as in Fig. 1 with their faces in close, frictional contact, and that both are exactly the same size, so

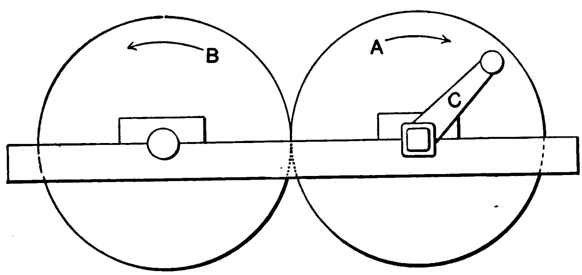

Fig. 1.

that when the crank is turned around once, wheel *B* will turn exactly once also, provided, of course, there is no slipping between the two wheels. It must be noticed, moreover, that if the crank be turned uniformly, wheel *B* will not only make the correct number of revolutions relative to *A*, but it will revolve uniformly, as well; that is, both its total motion and the motion from point to point will be correct.

Now there are many places in machine construction where the slipping inseparable from friction wheels cannot be tolerated, and this difficulty might be overcome by fastening small projections to one of the wheels, as on *A* in Fig. 2, and cutting grooves in the other wheel, *B*. Then, if the crank were turned, wheel *B* would always make just the right number of turns, even if considerable power were transmitted. It is probable, however, that these projections and grooves would not fulfill the purpose of gear teeth. What is wanted of gear teeth is that they shall give exactly the same kind of motion as corresponding friction wheels, running without slipping. They must not only keep

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the number of revolutions right, but they must give a perfectly even and smooth motion from point to point or from tooth to tooth.

Fig. 3 will show clearly how such a result is obtained. It represents the friction wheels with teeth fastened to them, the teeth, of course, extending all the way around instead of part way as shown. These teeth are set so as to be partly without and partly within the edges of the two wheels, as obviously they will give better results thus arranged than with all the projections on one wheel and all the grooves or depressions on the other, as in Fig. 2.

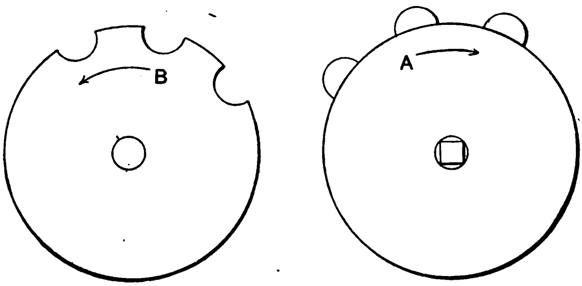


Fig. 2.

With the wheels fitted in this way it can be proved that the only conditions which must be fulfilled in order that the teeth shall give wheel *B* the same motion that it would have if it were driven by frictional contact with wheel *A* is that a line drawn from the point *O*, where the two wheels meet, to the point where the tooth curves touch shall be at right angles to both tooth curves at this point, whatever the

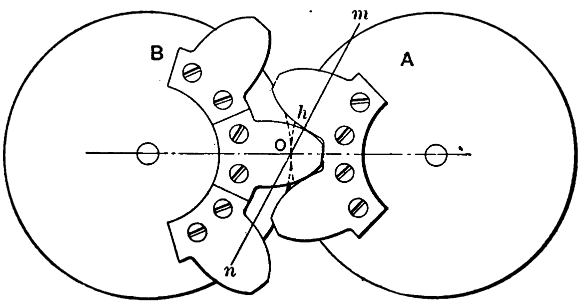


Fig. 3.

position of the gears. For example, in Fig. 3, two of the teeth touch at *h*. If the curves are of the right shape, a line *m n*, drawn through *h* and *O*, will be at right angles to both curves at point *h*. This is the law of tooth curves, and it makes no difference what the shape of the teeth is, so far as their correct action is concerned, if this law holds true for every successive point where the teeth come in contact.

In technical language the "friction wheels" mentioned are known as "pitch cylinders," and they are always represented on a gear drawing by a line—usually a dash and dot line—called the "pitch line." As teeth are generally proportioned, this line falls nearly, but not quite, midway between the tops and bottoms of the teeth, the inequality being due to the space left at the bottom of the teeth for clearance. The diameter of the pitch cylinder is called the "pitch diameter."

Involute System.

We are now ready to consider the particular forms of teeth most often used. The one that is at present most in favor is the involute tooth, the term "involute" being the name of a curve described by the

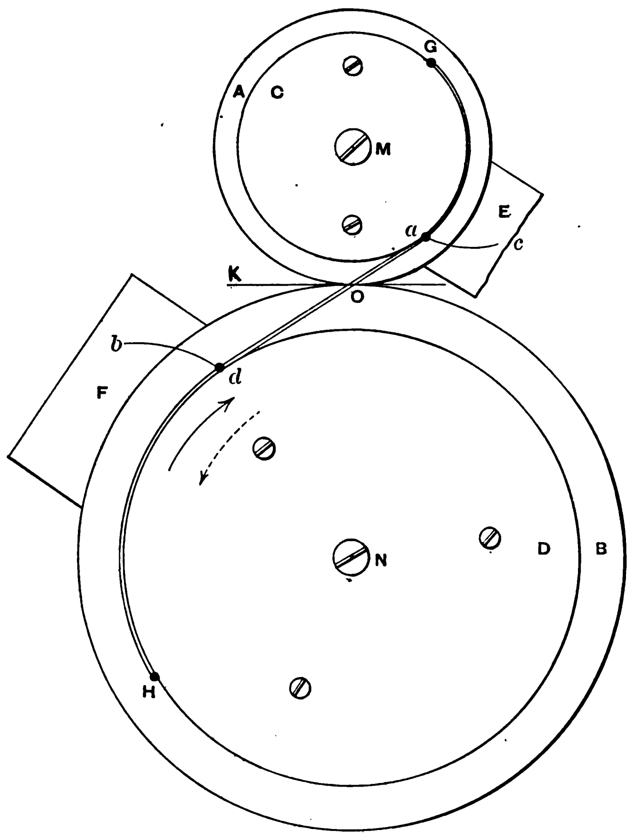


Fig. 4.

end of a cord as it is unwound from another curve. For example, to draw an involute, wind a cord around a circular disk of any convenient material, and make a loop in the outer end of the cord. Lay the disk flat on a piece of paper, and with a pencil in the loop, unwind the string, keeping it drawn tight, and let the point of the pencil trace a curve, which will then be an involute.

In Fig. 4 is shown how the same principle is applied to forming tooth curves. *A* and *B*, with centers at *M* and *N*, are two disks which serve the purpose of pitch cylinders. *C* and *D* are two smaller disks fastened to the larger ones and around which a cord is stretched and fastened at points *G* and *H*. When either disk is turned, the cord is supposed to pull the other one around at the same speed that it would go if moved solely by frictional contact between disks *A* and *B*. To do this, it is simply necessary to have the disks *C* and *D* in the same ratio as *A* and *B*. If *A*, for example, is half as large as *B*, then *C* must be half as large as *D*.

To make room for drawing the curves, let pieces *F* and *E* be fastened to the large and small wheels, respectively. With a pencil fixed at point *d* on the cord, turn the wheels in the direction of the solid

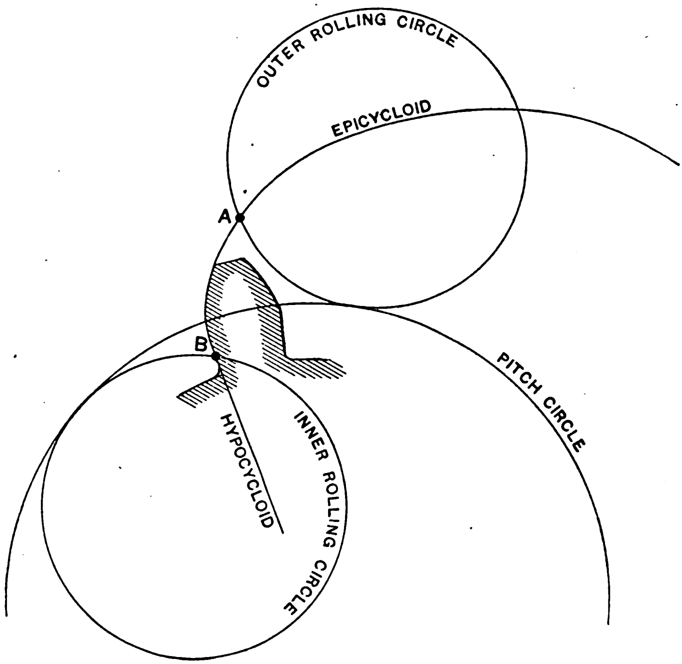


Fig. 5.

arrow, meanwhile moving the pencil outward, and the curve *db* will be described, which will be a suitable tooth curve for the larger wheel, and which it can be proved will answer the requirements of the general law. Starting again with the pencil at *a*, and turning the wheels in the direction of the dotted arrow, and moving the pencil outward, a similar curve, *ac*, for the smaller wheel will be traced.

The circles representing the disks *C* and *D* are called "base circles," and in practice are drawn at a distance from the pitch circle of about one-sixtieth of the pitch diameter. This brings the angle, *KOd*, called the angle of obliquity, in Fig. 4, about 14½ degrees; and although it is not by any means certain that this is the best angle, it is the one commonly used.

Cycloidal System.

Take a silver dollar and roll it along the edge of a ruler, holding the point of a pencil at the rim of the dollar, so that as the latter rolls, the pencil will trace a curve. This curve is a cycloid. Should the dollar be rolled on the edge of a circular disk, however, the curve traced would be an epi-cycloid, and should it be rolled on the inside of a hoop, it would be called a hypo-cycloid. These curves are employed for the teeth of the cycloidal system of gears.

In Fig. 5 it is shown how the face or the outer portion of the tooth is rolled up by the point *A* on the outer rolling circle, and how the flank or inner portion is generated by point *B* on the inner rolling circle. In this case the hypo-cycloid and flank are straight lines, the reason for this being that, as drawn, the diameter of the rolling circle

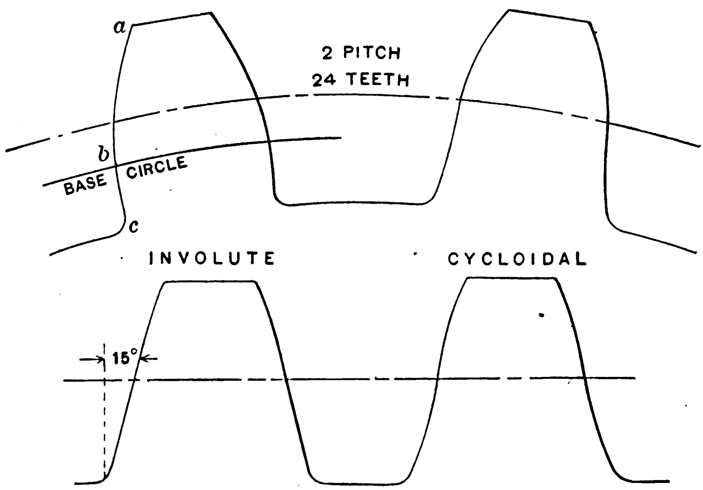


Fig 6.

is one-half the diameter of the pitch circle of the gear, and the hypo-cycloid generated under these conditions becomes a straight line.

The involute and cycloidal systems are the only two that are used to any extent, and in Fig. 6 a gear tooth and rack tooth of both are shown for comparison. The involute gear tooth has the involute curve from point *a* to point *b* on the base circle, and from *b* to *c* at the bottom of the tooth the flank is a straight, radial line. One difficulty with the involute system is that with the standard length of tooth the point *a* will interfere when running with gears or pinions having a small number of teeth. To avoid this, the point is rounded off a little below the involute curve. In general appearance the tooth seems to have a broad, strong base, and a continuous curve from *a* to *c*. A strong feature of the involute gearing is that it will run correctly even if the distance between the centers of the wheels is not exactly right. This will be evident by referring to Fig. 4, where it will appear that the relative velocities of the two wheels will be the same however far apart they may be, and if involute teeth are used in place of the string connection there shown, the action will be just the same. The involute rack tooth has straight sides at an angle of 14½ degrees, with the points rounded off.

Of the cycloidal teeth but little need be said except that they have two distinct curves above and below the pitch line, as previously explained, and that in the rack tooth the two curves are just alike, but reversed.

Whatever system is used, it is essential that all the wheels of a given pitch should be capable of running together. To make this possible with the involute, all the wheels must have the same angle of obliquity; and with the cycloidal system the same size rolling or describing circle must be employed for all sizes. The circle generally chosen is one having half the diameter of a 12-tooth pinion, which makes the flanks of this pinion radial. In Fig. 5, if the diameter of the rolling circle had been either greater or less than half the diameter of the pitch circle, the flank of the tooth would have been curved, and in the case of the greater circle, the curve would have fallen inside of the radial flank drawn in the figure, causing a weak, under-cut tooth. With the smaller circle, the curve would fall outside, making a strong tooth.

CHAPTER II.

FORMULAS FOR DIMENSIONS OF SPUR GEARS.

When we consider the number of gears used in machinery, and the number of men employed in the manufacture of machines using gears, it is rather surprising to find men who are unable to find the outside diameter, having given the pitch diameter and pitch, or to find the distance between centers of two gears, having given the number of teeth and pitch, and similar problems. The object of this chapter is to explain in as clear and practical a manner as possible the underlying principles of gearing, and to give concise rules or formulas for the solution of problems which arise in our everyday work upon gears.

Pitch Diameters.

Two shafts *A* and *A′* (Fig. 7) carry rollers *B* and *B′*. By having pressure on the shafts as indicated by the arrows, and revolving *A*, the

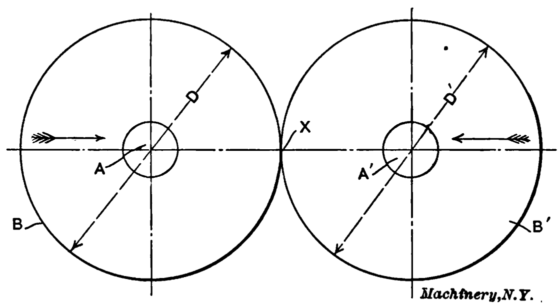


Fig. 7.

friction of the rolls at the point of contact, *X*, will cause *A′* to revolve, but we can readily see that if any great amount of power is to be transmitted, the rolls are liable to slip at the point of contact *X*, which will not give a positive motion; that is, it will require more than one revolution of the shaft *A* to produce one revolution of the shaft *A′*.

Suppose, as shown in Fig. 8, that we put projections on the surface of the roller *B* and cut recesses in the roller *B′*, making them of such shape that the sides of the projections on roller *B* will slide with as little friction as possible upon the sides of the projections caused by cutting the recesses in roller *B′*. Then, when shaft *A* is revolved, shaft *A′* must also revolve. The identity of the rollers *B* and *B′* is not lost, for we have simply added a number of projections to one, and cut the same number of recesses in the other, and the point of contact of the two rollers is still at *X*, but in this case there is no special pressure required to keep the rollers together as in the preceding case, nor is there any slip, and consequently shaft *A′* will make one revolution in the same time that shaft *A* does.

In Fig. 9 we have changed Fig. 8 by adding projections between recesses in roller *B′*, and by cutting recesses on roller *B* between projections, and we have the regular gear tooth. We have now no visible part of the original rollers *B* and *B′* left, but we have in their places imaginary rollers, the diameters of which are the pitch diameters of

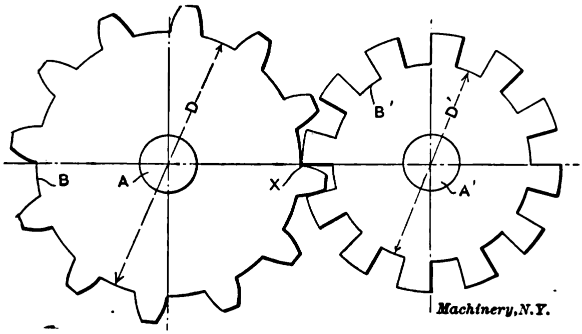


Fig. 8.

the gears. Thus we might have called our original rollers pitch rollers, and then proceeded to put on our projections and cut our recesses, which would have given us the gear wheel. This has already been explained in a general way in Chapter I.

Of course, in practice gears are never made in this way; the gear blank is first turned up to the correct diameter, and then the space

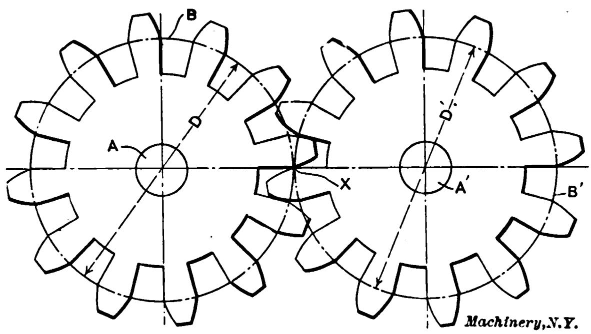


Fig. 9.

between the teeth is cut. The method of finding the outside diameter will be given later, this illustration being used simply to show the evolution of the gear wheel from the friction disks or pitch rollers.

Pitch.

When we speak of the pitch of a gear, the diametral pitch is generally referred to. The gear really has two pitches, diametral and circular. The *diametral pitch* of a gear is the number of teeth for each inch of pitch diameter. If a gear has 20 teeth and the pitch diameter is 2 inches, the diametral pitch would equal 20 ÷ 2, or 10; or there are 10 teeth in the gear for each inch of pitch diameter which it contains, and we would call it a 10-pitch gear. The *circular pitch* of a gear is the distance from the center of one tooth to the center of the next adjacent tooth, measured on the pitch lines. It is very seldom that circular pitch is used in describing cut gears.

It can readily be seen that the circular pitch being equal to the distance from the center of one tooth to the center of the next, must be the result of dividing the circumference of the pitch circle by the number of teeth in the gear. Should an occasion arise where it would be necessary to obtain the circular pitch, having the diametral pitch given, divide 3.1416 by the diametral pitch, and the quotient will be the circular pitch, or, expressed in its simplest form,

$$\frac{3.1416}{P} = P_1 \qquad (1)$$

in which P = diametral pitch; P_1 = circular pitch.

Example.—If the diametral pitch of a gear is 4, and it is required to find the circular pitch, divide 3.1416 by 4, and the quotient, 0.7854, is the circular pitch of the gear.

If the circular pitch be given, to find the diametral pitch, we can readily see that formula (1) would have to be transposed and would read thus:

$$\frac{3.1416}{P_1} = P \qquad (2)$$

P and P_1 representing the same as before.

Now, having given the rules, we will proceed to explain how they were obtained. We know that the distance around the circumference of a circle is equal to 3.1416, multiplied by the diameter of the circle; consequently, for every inch of diameter we have 3.1416 inches of circumference. If the diametral pitch of a gear is equal to the number of teeth for each inch of pitch diameter, and each inch of diameter is represented by 3.1416 inches of circumference, then the diametral pitch equals number of teeth for each 3.1416 inches of circumference. As the circular pitch is the distance from the center of one tooth to the center of the next, then the circular pitch must be equal to 3.1416 divided by the number of teeth in that 3.1416 inches of circumference, and, as we have shown that the diametral pitch is equal to the number of teeth in each 3.1416 inches of circumference, then the circular pitch must equal 3.1416 divided by the diametral pitch, which proves formula (1).

It may not be actually necessary to show how we obtain the diametral pitch from the circular pitch, but we will endeavor to explain everything as we go along. As in the preceding case, we begin with the ratio of the circumference of the circle to its diameter, which is 3.1416. In each 3.1416 inches of circumference we have a certain number of teeth, which is the diametral pitch of the gear. Now, having given the circular pitch, if we divide 3.1416 by that, we obtain the number of teeth for 3.1416 inch of the circumference, which is the diametral pitch of the gear, which proves formula (2).

The accompanying tables will facilitate the finding of corresponding diametral and circular pitches. Table I gives the even diametral pitches with the corresponding circular pitches, while Table II gives the even cixcular pitches with the corresponding diametral pitches.

TABLE I. DIAMETRAL PITCH CONVERTED INTO CIRCULAR PITCH.

Diametral Pitch.	Circular Pitch.	Diametral Pitch.	Circular Pitch.
2	1.571 inch.	12	0.262 inch.
2¼	1.396 "	14	0.224 "
2½	1.257 "	16	0.196 "
2¾	1.142 "	18	0.175 "
3	1.047 "	20	0.157 "
3½	0.898 "	22	0.143 "
4	0.785 "	24	0.131 "
5	0.628 "	26	0.121 "
6	0.524 "	28	0.112 "
7	0.449 "	30	0.105 "
8	0.393 "	32	0.098 "
9	0.349 "	36	0.087 "
10	0.314 "	40	0.079 "
11	0.286 "	48	0.065 "

TABLE II. CIRCULAR PITCH CONVERTED INTO DIAMETRAL PITCH.

Circular Pitch.	Diametral Pitch.	Circular Pitch.	Diametral Pitch.
2 inches.	1.571	7/8 inch.	3.590
1 7/8 "	1.676	13/16 "	3.867
1 3/4 "	1.795	3/4 "	4.189
1 5/8 "	1.933	11/16 "	4.570
1 1/2 "	2.094	5/8 "	5.027
1 7/16 "	2.185	9/16 "	5.585
1 3/8 "	2.285	1/2 "	6.283
1 5/16 "	2.394	7/16 "	7.181
1 1/4 "	2.513	3/8 "	8.378
1 3/16 "	2.646	5/16 "	10.053
1 1/8 "	2.793	1/4 "	12.566
1 1/16 "	2.957	3/16 "	16.755
1 "	3.142	1/8 "	25.133
15/16 "	3.351	1/16 "	50.266

Pitch Diameter.

Having given the diametral pitch and number of teeth in a gear, to find tne pitch diameter, divide the number of teeth by the pitch, and the quotient will be the pitch diameter, which, expressed in its simplest form, is:

$$\frac{N}{P}=D \qquad (3)$$

in which $N=$ number of teeth; $P=$ pitch (diametral); $D=$ pitch diameter.

Example.—A 10-pitch gear has 35 teeth, what is the pitch diameter? Divide 35 (the number of teeth) by 10 (the pitch), and the quotient 3½ is the pitch diameter of the gear.

The definition of diametral pitch proves this formula. If the diametral pitch equals the number of teeth to each inch of pitch diameter, then dividing the number of teeth in the gear by the diametral pitch will give the number of inches of the pitch diameter. If the circular pitch and number of teeth are given, first find the diametral pitch, and proceed as given above.

Addendum.

The addendum of a gear tooth is the distance from the pitch circle to the outside circumference of the gear. This distance is always equal to the reciprocal of the diametral pitch, or 1 divided by the diametral pitch, and, expressed as a formula, is:

$$A = \frac{1}{P} \qquad (4)$$

in which A = addendum; P = diametral pitch.

Outside Diameter.

When we start to make a gear, we first wish to know the outside diameter. If we have the pitch and number of teeth given, this may easily be found by the following rule: Add 2 to the number of teeth, and divide by the pitch. This, expressed as a formula, is:

$$\frac{N + 2}{P} = D_1 \qquad (5)$$

in which N = number of teeth; P = diametral pitch; D_1 = outside diameter.

Example.—Given a gear of 20 teeth and 4 pitch, to find the outside diameter. The number of teeth, 20, plus 2 equals 22, and 22 divided by 4 (the pitch of the gear) equals 5½, the outside diameter of the gear.

This formula is simply a combination of formulas 3 and 4, for we first find the pitch diameter, and then add the addendum twice, for it must be added on each side of the pitch diameter. The mathematical solution is as follows:

$$\frac{N}{P} = D; \quad D + \frac{1}{P} + \frac{1}{P} = D_1$$

$$D_1 = D + \frac{2}{P}; \; D_1 = \frac{N + 2}{P} \qquad (5)$$

Dedendum and Clearance.

The dedendum is the working depth of the tooth below the pitch line, and must be equal to the addendum or $\frac{1}{P}$, for the pitch circles of two gears are tangent (touching), so the addendum of one will give the working depth of the other below the pitch line. The clearance is the distance from the end of the dedendum to the bottom of the space between the teeth. There is no common standard for this distance, different gear makers using different distances, yet the difference between them is very slight.

The Brown & Sharpe formula for this distance is:

$$F = \frac{0.157}{P} \qquad (6)$$

in which F = clearance; P = diametral pitch.

The Geo. B. Grant formula is:

$$F = \frac{A}{8} \qquad (7)$$

in which F = clearance; A = addendum.

Thickness of Tooth.

The thickness of tooth and width of the space of a gear are always equal at the pitch line, and if the circular pitch is the distance from the center of one tooth to the center of the next tooth measured on the pitch line, tooth and space being equal, then the thickness of tooth must be equal to one-half the circular pitch, or

$$T = \frac{P_1}{2} \qquad (8)$$

in which T = thickness of tooth at pitch line; P_1 = circular pitch.

We know by formula (1) that

$$P_1 = \frac{3.1416}{P} \qquad (1)$$

and substituting this value for P_1 in formula (8) we have:

$$T = \frac{\frac{3.1416}{P}}{2}$$

and this formula resolved to its simplest form is:

$$T = \frac{1.5708}{P} \qquad (9)$$

in which T = thickness of tooth at pitch line; P = diametral pitch.

Example.—Given a gear 1 3/16 circular pitch, what is the thickness of tooth at the pitch line? 1 3/16 (the circular pitch) divided by 2 gives 19/32, the thickness of tooth at the pitch line.

Example.—Given a 6-pitch gear to find the thickness of tooth at the pitch line. 1.5708 divided by 6 (the diametral pitch of the gear) gives 0.262, the thickness of tooth at the pitch line.

Table III gives the thickness of tooth at the pitch line for the different diametral pitches.

Depth of Tooth.

After we get the gear blank turned up, we next want to know how deep to run the gear cutter in order to get a perfect tooth. The working depth of the tooth we have shown to be equal to the sum of the addendum and dedendum, or $\frac{1}{P}+\frac{1}{P}=\frac{2}{P}$, and the whole depth of the tooth must equal $\frac{2}{P}$ plus the clearance.

Using the Brown & Sharpe standard, we have $\frac{2}{P}+\frac{0.157}{P}=$

$$E=\frac{2.157}{P} \qquad (10)$$

in which $E=$ full depth of tooth; $P=$ diametral pitch.

Example.—Given a gear of 6 diametral pitch, to find the depth of cut to be taken to get a perfect gear tooth.

Divide 2.157 by 6 (diametral pitch) and the quotient 0.359 is the depth to be cut in the gear.

If we had the circular pitch given, to find the depth of tooth, we

TABLE III. THICKNESS OF TOOTH AT PITCH LINE.

Diametral Pitch.	Thickness of Tooth at Pitch Line.	Diametral Pitch.	Thickness of Tooth at Pitch Line.
2	0.785 inch.	12	0.131 inch.
2¼	0.697 "	14	0.112 "
2½	0.628 "	16	0.098 "
2¾	0.570 "	18	0.087 "
3	0.523 "	20	0.079 "
3½	0.448 "	22	0.071 "
4	0.393 "	24	0.065 "
5	0.314 "	26	0.060 "
6	0.262 "	28	0.056 "
7	0.224 "	30	0.052 "
8	0.196 "	32	0.049 "
9	0.175 "	36	0.044 "
10	0.157 "	40	0.039 "
11	0.143 "	48	0.033 "

could substitute in formula (10) the value of P as given in the formula (2), and we would have

$$E=\frac{2.157}{3.1416 \div P_1}$$

which, reduced to its simplest form, is:

$$E=0.6866\,P_1 \qquad (11)$$

in which $E=$ depth to be cut in gear;
$P_1=$ circular pitch.

Example.—Given a gear 1½ inch circular pitch, to find the depth to be cut.

Multiply 0.6866 by 1½ (circular pitch), and the product 1.030 is the depth to be cut in gear.

Table IV gives the depth to be cut in a gear for different diametral pitches.

Distance Between Centers.

Having given the number of teeth and diametral pitch of two gears, to find the distance between centers, add the number of teeth together, and divide by twice the diametral pitch, or

$$\frac{N+n}{2P}=C \qquad (12)$$

in which N = number of teeth in one gear.
n = number of teeth in other gear.
P = diametral pitch.
C = distance between centers.

This formula is obtained from formula (3):

$$\frac{N}{P}=D.$$

This formula gives us the pitch diameter of one gear, and, if we get the pitch diameters of two gears and add them together, we have twice the distance between centers, for the sum of the pitch diameters is twice the sum of the pitch radii, which is the distance between centers.

TABLE IV. DEPTH OF TOOTH.

Diametral Pitch.	Depth to be cut in gear.	Diametral Pitch.	Depth to be cut in gear.
2	1.078 inch.	12	0.180 inch.
2¼	0.958 "	14	0.154 "
2½	0.863 "	16	0.135 "
2¾	0.784 "	18	0.120 "
3	0.719 "	20	0.108 "
3½	0.616 "	22	0.098 "
4	0.539 "	24	0.090 "
5	0.431 "	26	0.083 "
6	0.359 "	28	0.077 "
7	0.308 "	30	0.072 "
8	0.270 "	32	0.067 "
9	0.240 "	36	0.060 "
10	0.216 "	40	0.054 "
11	0.196 "	48	0.045 "

We have now traced, by the aid of a few "rules," the proportions of a gear tooth, having given the pitch and number of teeth, through pitch diameter, addendum, dedendum, clearance, width of tooth and depth to be cut, up to the distance between centers. We now give some formulas for the solution of problems in which some of the quantities which were known in preceding problems are unknown.

Pitch.

1. To find the pitch, having given the pitch diameter and number of teeth. Divide the number of teeth by the pitch diameter, and the quotient will be the pitch. The proof of this assertion is derived from the formula:

$$D=\frac{N}{P} \qquad (3)$$

If the pitch diameter equals the number of teeth divided by the pitch, then the pitch diameter multiplied by the pitch must equal the number of teeth; therefore the pitch must equal the number of teeth divided by the pitch diameter, and this, expressed in its simplest form, is:

$$P = \frac{N}{D} \tag{13}$$

in which $P =$ pitch (diametral); $N =$ number of teeth in gear; $D =$ pitch diameter.

Example.—A gear, 3 inches pitch diameter, has 36 teeth. Find the diametral pitch.

Divide 36 (the number of teeth) by 3 (the pitch diameter), and we have 12, the diametral pitch of the gear.

2. Having given the outside diameter and number of teeth, to find the diametral pitch. Add 2 to the number of teeth, and divide by the outside diameter, and the quotient will be the pitch of the gear.

In formula (5) we have:

$$\frac{N + 2}{P} = D_1 \tag{5}$$

If the number of teeth + 2 divided by the pitch equals the outside diameter, then the outside diameter multiplied by the pitch must equal the number of teeth + 2, and then the pitch must equal the number of teeth + 2 divided by the outside diameter, which, expressed as a formula, is:

$$\frac{N + 2}{D_1} = P \tag{14}$$

in which $N =$ number of teeth in gear; $D_1 =$ outside diameter; $P =$ diametral pitch.

Example.—Given a gear of 36 teeth and 3 1/6-inch outside diameter; to find the diametral pitch.

36 (the number of teeth) + 2 = 38.

38 ÷ 3 1/6 = 12, the diametral pitch of the gear.

Pitch Diameter.

1. Having given the outside diameter and the pitch, to find the pitch diameter. The distance from the pitch diameter to the outside diameter is $\frac{1}{P}$, as explained in formula

$$A = \frac{1}{P} \tag{4}$$

and as this is to be added on each side of the center, the outside diameter of the gear must be equal to the pitch diameter plus $\frac{2}{P}$. If this is so, then $\frac{2}{P}$ subtracted from the outside diameter will give the pitch diameter, or

$$D = D_1 - \frac{2}{P} \qquad (15)$$

in which D = pitch diameter; D_1 = outside diameter; P = diametral pitch.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch; to find the pitch diameter.

3 1/6 inches (the outside diameter) — 2/12 = 3 inches, the pitch diameter of the gear.

2. Having given the outside diameter and number of teeth, to find the pitch diameter. Multiply the outside diameter by the number of teeth, and divide by the number of teeth plus 2.

We have shown in formula (5) that the outside diameter equals the number of teeth + 2 divided by pitch, or

$$D_1 = \frac{N + 2}{P} \qquad (5)$$

and in formula (13) that pitch equals the number of teeth divided by the pitch diameter, or

$$P = \frac{N}{D} \qquad (13)$$

Now, if the outside diameter equals the number of teeth plus 2 divided by the diametral pitch (and the diametral pitch equals the number of teeth divided by the pitch diameter), then the outside diameter must be equal to the number of teeth plus 2, divided by a fraction with the number of teeth as numerator and the pitch diameter as denominator. This is simply substituting the value of the pitch as shown in formula (13) for the pitch in formula (5), and expressed as a formula, is:

$$D_1 = \frac{N + 2}{N \div D}$$

Multiplying both sides of the equal sign by $\frac{N}{D}$ we have

$$D_1 \times \frac{N}{D} = N + 2, \text{ or } \frac{D_1 \times N}{D} = N + 2,$$

and now, multiplying both sides by D, we have

$$D_1 \times N = (N + 2) \times D$$

and dividing both sides by $N + 2$ we get

$$\frac{D_1 \times N}{N + 2} = D, \text{ or } D = \frac{D_1 \times N}{N + 2} \qquad (16)$$

in which D = pitch diameter; N = number of teeth; D_1 = outside diameter.

Example.—Given a gear 3 1/6 inches outside diameter and 36 teeth. To find the pitch diameter.

3 1/6 (the outside diameter) multiplied by 36 (the number of teeth) equals 114. 36 (the number of teeth) + 2 = 38. 114 ($D_1 \times N$) divided by 38 ($N + 2$) = 3 inches, the pitch diameter of the gear.

Number of Teeth.

1. Having given the pitch diameter and pitch, to find the number of teeth. Multiply the pitch diameter by the pitch, and the product will be the number of teeth in the gear.

The diametral pitch of a gear equals the number of teeth for each inch of pitch diameter; hence, if we multiply the pitch by the number of inches of pitch diameter we will have the number of teeth in the gear, which, expressed as a formula, is:

$$N = P \times D \qquad (17)$$

in which P = diametral pitch; D = pitch diameter.

Example.—Given a gear 3 inches pitch diameter and 12 diametral pitch, to find the number of teeth. 3 (pitch diameter) multiplied by 12 (diametral pitch) = 36, the number of teeth in the gear.

2. To find the number of teeth, having given the outside diameter and pitch. Multiply the outside diameter by the pitch and subtract 2, or

$$N = (D_1 \times P) - 2 \qquad (18)$$

in which N = number of teeth; D_1 = outside diameter; P = diametral pitch.

This formula is simply the reverse of formula

$$\frac{N + 2}{P} = D_1 \qquad (5)$$

If the outside diameter equals the number of teeth + 2 divided by the pitch, which we have already proved, then the number of teeth plus 2 must equal the outside diameter multiplied by the pitch, and subtracting 2 from this result we have the number of teeth in the gear.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch, to find the number of teeth. Multiply 3 1/6 (outside diameter) by 12 (the pitch) and we have 38, and subtracting 2 from this result we have 36, the number of teeth in the gear.

Outside Diameter.

To find the outside diameter having given the pitch diameter and pitch. Divide 2 by the pitch and add to the pitch diameter, or

$$D_1 = D + \frac{2}{P} \qquad (19)$$

in which D_1 = outside diameter.
D = pitch diameter.
P = pitch.

The addendum of a gear is $\frac{1}{P}$ [formula (4)] and this, added on each side of the pitch diameter, gives the outside diameter.

Example.—Given a gear 3 inches pitch diameter and 12 pitch; to find the outside diameter.

3 (pitch diameter) plus 2/12 $\left(\frac{2}{P}\right)$ = 3 1/6 inches, the outside diameter of the gear.

Summary of Formulas.

Having given the general principles of the proportions of gear teeth, we will now group the formulas (which we have proved to be correct) under one head, so that they may be more easily found when wanted.

In the following formulas,

P = diametral pitch.
P_1 = circular pitch.
D = pitch diameter.
D_1 = outside diameter.
N = number of teeth in one gear.
n = number of teeth in mating gear.
A = addendum.
T = thickness of tooth at the pitch line.
E = full depth of tooth.
C = distance between centers.
F = clearance.

$$P_1 = \frac{3.1416}{P} \qquad (1)$$

$$P = \frac{3.1416}{P_1} \qquad (2)$$

$$D = \frac{N}{P} \qquad (3)$$

$$A = \frac{1}{P} \qquad (4)$$

$$D_1 = \frac{N+2}{P} \qquad (5)$$

$$F = \frac{0.157}{P} \text{ or } \frac{A}{8} \qquad (6 \text{ and } 7)$$

$$T = \frac{P_1}{2} \qquad (8)$$

$$T = \frac{1.5708}{P} \qquad (9)$$

$$E = \frac{2.157}{P} \qquad (10)$$

$$E = 0.6866\,P_1 \qquad (11)$$

$$C = \frac{N+n}{2P} \qquad (12)$$

$$P = \frac{N}{D} \qquad (13)$$

$$P = \frac{N+2}{D_1} \qquad (14)$$

$$D = D_1 - \frac{2}{P} \qquad (15)$$

$$D = \frac{D_1 \times N}{N+2} \qquad (16)$$

$$N = P \times D \qquad (17)$$

$$N = (D_1 \times P) - 2 \qquad (18)$$

$$D_1 = D + \frac{2}{P} \qquad (19)$$

CHAPTER III.

DESIGN AND CALCULATION OF GEAR WHEELS.

The complete calculations required for the design of a *pair* of gears according to the usual shop practice are few and simple, and it is proposed to put these calculations into easily-understood form in the present chapter. The calculations should be made in the following manner: First—Find out if the gears are intended to give a certain *velocity ratio* between two shafts, or a certain *power ratio* between the shafts; and, assuming the number of teeth in one gear, make the number of teeth in the other gear such as to have the *required ratio* to the number of teeth in the first gear. Second—Assume the pitch of the gears, and calculate the pitch diameters of the two gears, and the distance between the centers of the shafts. Third—Calculate the width of face required to give the gears proper strength. Fourth—Lay out the gears and the tooth forms. The relations of these several steps one to another are such as to make some assumptions necessary, and these depend upon the judgment and experience of the designer, especially when the distance between the shafts is approximately settled, and certain ratios are to be obtained without materially changing the shaft centers. In the case of the younger designers, however, these assumptions are all made beforehand and given to them with instructions to lay out the gears. The several steps will now be taken up and each explained.

Speed Ratios.

Fig. 10 represents two shafts connected by a pair of spur gears, A being the driven shaft, and B, the driving shaft. If shaft A is required to revolve half as fast as shaft B, it is easily seen that the gear on A must be twice as large, and, being of the same pitch, must have twice as many teeth as the gear on B. If n and n_1 represent the number of teeth in each gear, respectively, we have the proportion,

$$y : x = n : n_1 \text{ or } \frac{x\,n}{n_1} = y.$$

If, now, a third shaft were to be driven by gears from shaft A, we would assume A to be the driver revolving y times a minute, and by the above proportion determine the revolutions of the third shaft, and so continue indefinitely for as many shafts as are geared together in any one train. Thus follows the rule: *The speed of the last shaft equals the speed of the first shaft multiplied by the product of the number of teeth in the driving gears and divided by the product of the number of teeth of the driven gears.*

Power Ratios.

In case a certain ratio of power is wanted, we shall find some sort of a crank or pulley, the radius R of which is known, upon the power shaft B, and upon the driven shaft there will also be some sort of a crank, pulley or drum, the radius r of which is known. We can now make the equation (referring to Fig. 10):

$$\frac{P\,R\,n_1}{n\,r} = Q$$

This expression may be made general by following through as before from shaft to shaft, and may be given as the following rule: *The power, multiplied by the power arm, times the product of the number of teeth in the driven gears, and divided by the weight arm times the product of the number of teeth in the driving gears, equals the weight.*

Fig. 11 shows in tabular form several different forms of gear trains with their formulas for speed and power ratios. It is to be noted that

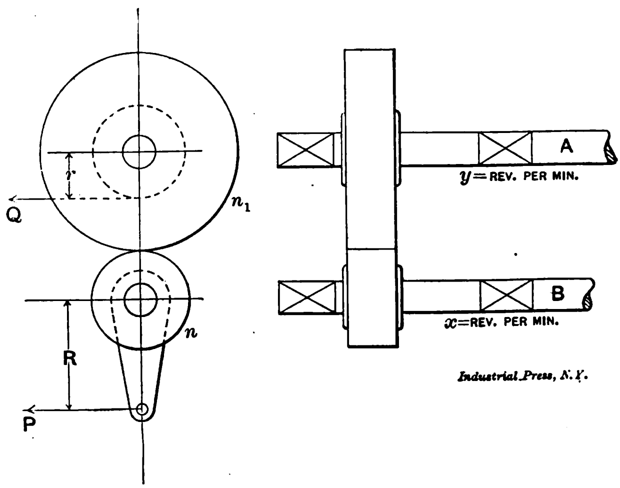
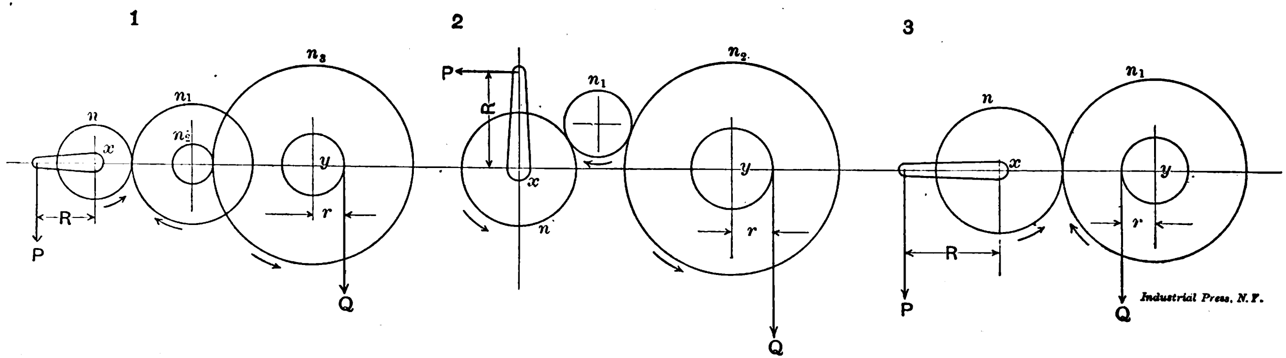


Fig. 10. Speed and Power Ratios.

in place of using the numbers of teeth in these ratios we may use the pitch diameters of the gears, but as these diameters are very often expressed in fractional parts of an inch, while the number of teeth is always a whole number, it is found more convenient to use the latter. Idlers are often used, as shown in the sketch in section 2, Fig. 11, and they have no effect upon either the speed or power ratios. They are introduced either to connect two shafts where the great distance between centers would involve very large gears if geared directly together, or to effect a change in direction of motion, as may be seen by the arrows in Fig. 12. An inspection of this engraving proves the rule that an even number of idlers does not change the direction of motion between two shafts, while an odd number of idlers reverses the direction of motion.



First Illustration.

Speed ratio:

$$\frac{x\,n\,n_2}{n_1\,n_3} = y$$

Power ratio:

$$\frac{P\,R\,n_1\,n_3}{n\,n_2\,r} = Q$$

Second Illustration.

Speed ratio:

$$\frac{x\,n\,n_1}{n_1\,n_2} = \frac{x\,n}{n_2} = y$$

Power ratio:

$$\frac{P\,R\,n_1\,n_2}{n\,n_1\,r} = \frac{P\,R\,n_2}{n\,r} = Q$$

Note.—As the number of idlers cancels out they do not affect the result.

Third Illustration.

Speed ratio:

$$\frac{x\,n}{n_1} = y$$

Power ratio:

$$\frac{P\,R\,n_1}{n\,r} = Q$$

NOTATION.—$n\ n_1\ n_2\ n_3$ = number of teeth in gears. x = rev. per min. of power shaft. y = rev. per min. of driven shaft. R – rad. of power arm. r = rad. of load arm.

Fig. 11. Diagrams Illustrating Speed and Power Ratios of Gearing.

Pitch Diameters.

Having determined upon the velocity or power ratio required of our gears, the next step is to determine the two pitch diameters of the gears. To do this it is necessary to assume the pitch of the gears, and this assumption depends upon the judgment and experience of the

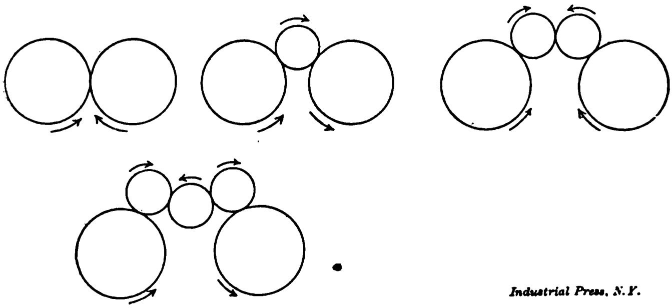


Fig. 12. Illustration of the Effect of Idlers.

designer, although very often it may be confirmed by comparison with gears of about the same size and doing about the same work as those to be designed. After assuming the pitch and finding the pitch diameters, a calculation for the strength of the gears will show whether

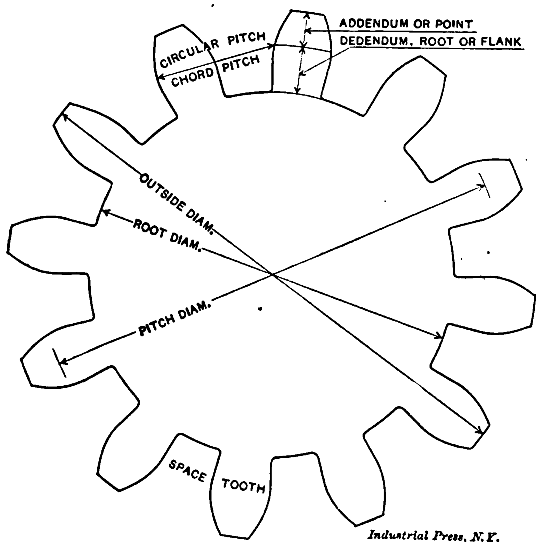


Fig. 13. Terms used to denote Gear Quantities.

the assumed pitch is right, and if it then proves to be too small or too large, the calculation may be repeated with another assumed pitch.

Pitch of Gears.

It is first necessary to understand what is meant by the pitch of a gear, and its relation to the diameter. This has been explained in the previous chapter, but in order to make the present discussion a complete whole by itself, the most important definitions and formulas have here been repeated. Fig. 13 shows a gear of twelve teeth (such a small gear is often called a pinion), and the names of the different parts are clearly indicated. As will be seen, the circular pitch is the distance on the pitch circle from a point on one tooth to the corresponding point on the next tooth. The circumference of the pitch

TABLE V. CONSTANT FOR DETERMINING CHORD PITCH.

Number of Teeth.	Constant K.	Number of Teeth.	Constant K.	Number of Teeth.	Constant K.
12	.258	31	.102	52	.059
13	.239	32	.097	54	.057
14	.222	33	.094	56	.055
15	.207	34	.093	58	.053
16	.195	35	.089	60	.052
17	.184	36	.087	62	.049
18	.173	37	.084	64	.048
19	.165	38	.082	66	.045
20	.156	39	.080	68	.044
21	.148	40	.078	70	.043
22	.141	41	.076	75	.041
23	.136	42	.075	80	.039
24	.130	43	.073	85	.036
25	.125	44	.071	90	.034
26	.120	45	.069	95	.032
27	.115	46	.067	100	.031
28	.112	47	.066	125	.025
29	.107	48	.065	150	.019
30	.104	49	.063	175	.017
		50	.061	200	.015

circle is equal to the pitch multiplied by the number of teeth and dividing this by 3.1416 gives the diameter of the pitch circle, or

$$D = \frac{P_1 N}{3.1416}$$

when, D = the diameter of the pitch circle,

N = the number of teeth,

P_1 = the circular pitch.

After having determined the pitch diameter and drawn the pitch circle, we must divide the pitch circle into as many equal parts as the number of teeth, or, what is the same thing, lay off the circular pitch upon the pitch circle. In the case of a small pinion, such as Fig. 13, this may be most easily done by trial with a pair of dividers. It very often happens, however, that a gear is so large as to make this method impracticable, because only a portion of the gear, showing a few teeth, will be drawn. It thus becomes necessary to have some method of accurately laying off the circular pitch upon the pitch circle when only a portion of the circle is drawn. From Fig. 13 it is evident that if we set our dividers to the circular pitch, and attempt to step off the spaces, what we shall actually be stepping off will be chords instead of circular arcs, and the resulting arcs will be greater than the circular pitch. In very large gears this error is very small, but in ordinary gears it is quite appreciable, and the dividers should be set to the chord pitch. Table V has been computed to enable the chord pitch to be easily determined, as the pitch diameter multiplied by the constant K, opposite the number of teeth in the table, equals the chord pitch.

Relation between Number of Teeth, Pitch, and Pitch Diameter of Gears.

Molded or rough-cast gears are usually designed by circular pitch, but cut gears are designed by what is known as diametral pitch. Since the number of teeth bears a fixed relation to the pitch circumference, and the pitch diameter bears a fixed relation to the pitch circumference, it follows that the number of teeth bears a fixed relation to the pitch diameter. This being so, we may divide the pitch diameter expressed in inches by the number of teeth, and the result will be what is termed the diametral pitch. It is also evident that if the number of teeth bears a fixed relation to the pitch circumference and pitch diameter, the circular pitch and diametral pitch must have some fixed relation to each other. These different relations are most conveniently given for use as follows:

Circular pitch = P_1

$$D = \frac{P_1 N}{\pi}$$

$$P_1 = \frac{D \pi}{N}$$

$$N = \frac{D \pi}{P_1}$$

$$D_1 = D + 0.6 P_1$$

$$C = \frac{D + D_2}{2} = \frac{P_1 N_1}{2 \pi}$$

Diametral pitch = P

$$D = \frac{N}{P}$$

$$P = \frac{N}{D}$$

$$N = P D$$

$$D_1 = \frac{N + 2}{P}$$

$$C = \frac{D + D_2}{2} = \frac{N_1}{2 P}$$

Relation of Circular and Diametral Pitch.

$$P_1 P = \pi \qquad P_1 = \frac{\pi}{P} \qquad P = \frac{\pi}{P_1}$$

π = 3.1416,
P_1 = circular pitch,
P = diametral pitch,
D = pitch diameter,
D_2 = pitch diameter of mating gear,
D_1 = outside diameter,
N = number of teeth,
N_1 = number of teeth in a pair of gears = sum of the teeth in each gear,
C = distance between centers of shafts.

When designing cut gears it is not necessary to lay out the form of the teeth, as these are formed by the gear-cutting process, and it is only necessary for the designer to calculate the pitch diameters that will give the required ratios, and then to find the outside diameter of the blank from which the gear is to be cut. For such gears diametral

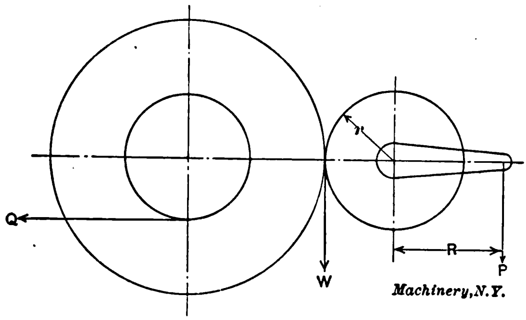


Fig. 14.

pitch is a great convenience, as the relations of pitch, diameter and number of teeth are so simple.

Strength of Teeth and Width of Face.

Before proceeding further, it is well to know if our assumed pitch for the gears will give strong enough teeth without requiring a wider face than is practical, and it becomes necessary to know the force or power transmitted by the gears. The most convenient way to do this is to get the force in pounds which is carried by the gear at the pitch

TABLE VI. FACTOR FOR CALCULATING WIDTH OF FACE OF TEETH.

Radial Flanks.		Involute 15°.	
Number of Teeth.	Factor Y.	Number of Teeth.	Factor Y.
12	052	38	.107
13	053	43	.110
14	.054	50	.112
15	.055	60	.114
16	.056	75	.116
17	.057	100	.118
18	.058	150	.120
19	.059	300	.122
20	.060	Rack	.124
21	.061		
23	.062		
25	.063		
27	.064		
30	.065		
34	.066		

line. In Fig. 14, let it be supposed that the power ratio is such as to make it just possible to move the load Q with a force P upon the crank. Then the force W at the pitch line of the gears will be,

$$W = \frac{P R}{r}.$$

Having found W we may calculate the required width of face for our gears having the assumed pitch, and if the required face proves to be too wide, we shall have to assume a larger pitch, in order to get stronger teeth. The most widely used formula for the strength of gears is that proposed by Mr. Wilfred Lewis and given in Kent's handbook, page 901, as

$$W = SPFY,$$

in which W = force on pitch line in pounds,
P = circular pitch,
S = allowable fiber stress for the material used,
F = width of face of gear,
Y = variable from Table VI.

S may be assumed as 3,500 pounds for cast iron, and 8,000 pounds for cast steel, and as the pitch has been already assumed, the formula may be changed to give the required width of face thus,

$$F = \frac{W}{SPY}.$$

Substituting in this formula the values already obtained for W, S, P, and Y, we find the required face for the gears, and if this is too great, a larger pitch must be used.

Chart for Strength of Spur Gears.

The accompanying chart, Fig. 15, for the strength of spur gears, enables problems to be solved easily and quickly, and the result of any changes in pitch or face to be quickly seen. The heavy line with the arrows shows the method of working out the problem for a 50-tooth gear as stated in the upper right-hand corner of the chart. The chart also shows the safe working loads for different speeds as given by Mr. Lewis. Enter the chart on the left at the number of teeth; then follow over to the diagonal line for the pitch of the gear, then up or down to the diagonal line for the allowable fiber stress for the material of which the gear is to be made, then over to the diagonal line for the face of the gear, then down, and read the load in pounds that the gear will carry at the pitch line. If the load to be carried and the number of teeth required are known, the chart may be entered at each end, that is, at the number of teeth and at the load, and by then following each way, the face required for a certain pitch, or the pitch required for a certain face are easily seen, and thus the best combination of face and pitch for any case is easily determined without any calculations.

Laying out the Tooth Outlines.

Having gone through the steps described previously, we shall know the pitch diameter, pitch, number of teeth, and face of the gear to be designed, and are ready to lay out the gear and tooth outlines. As has been mentioned in Chapter I, two forms of teeth are in general use, the cycloidal and the involute, each having its champions among able designers. The involute form has the advantage of being the more easily ground to an approximately correct tooth form in the case of

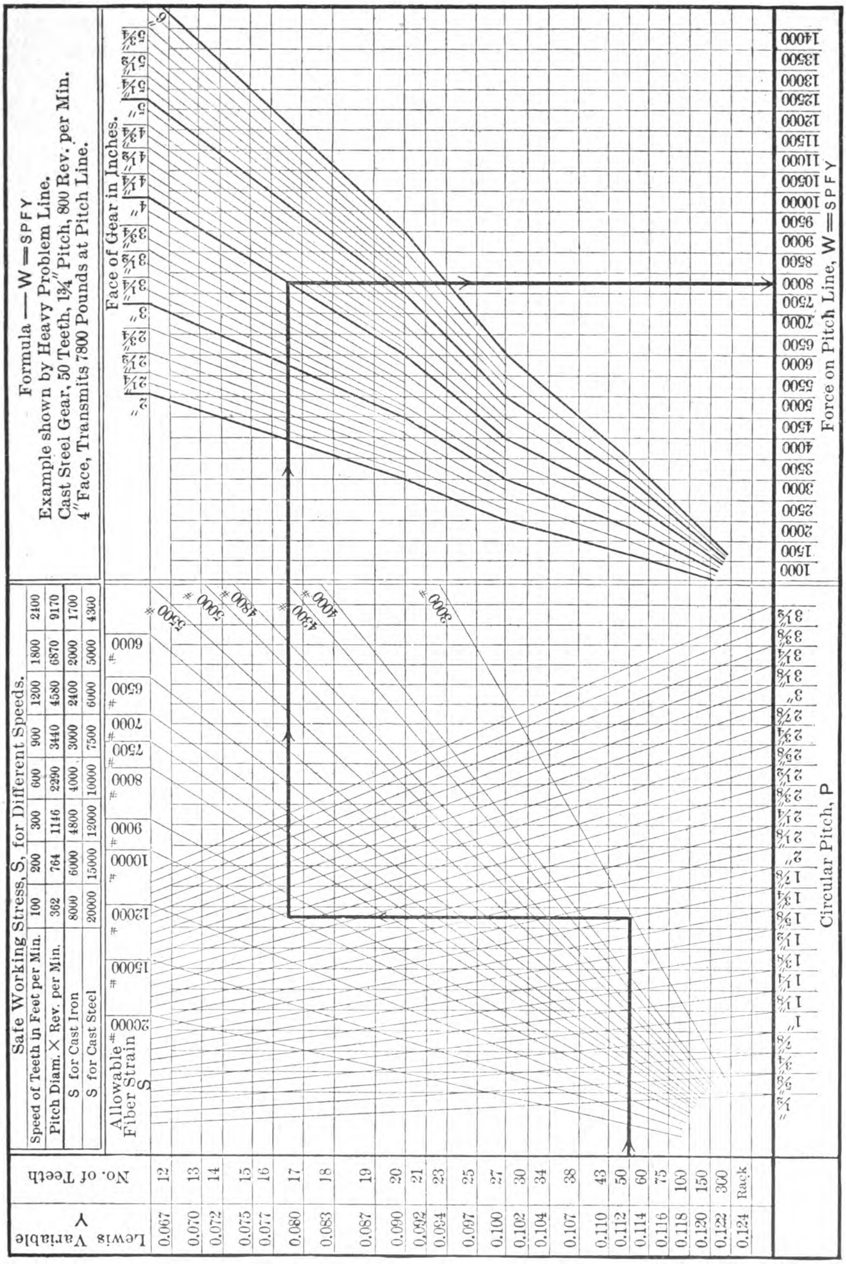


Fig. 15. Chart for Proportioning Spur Gears.

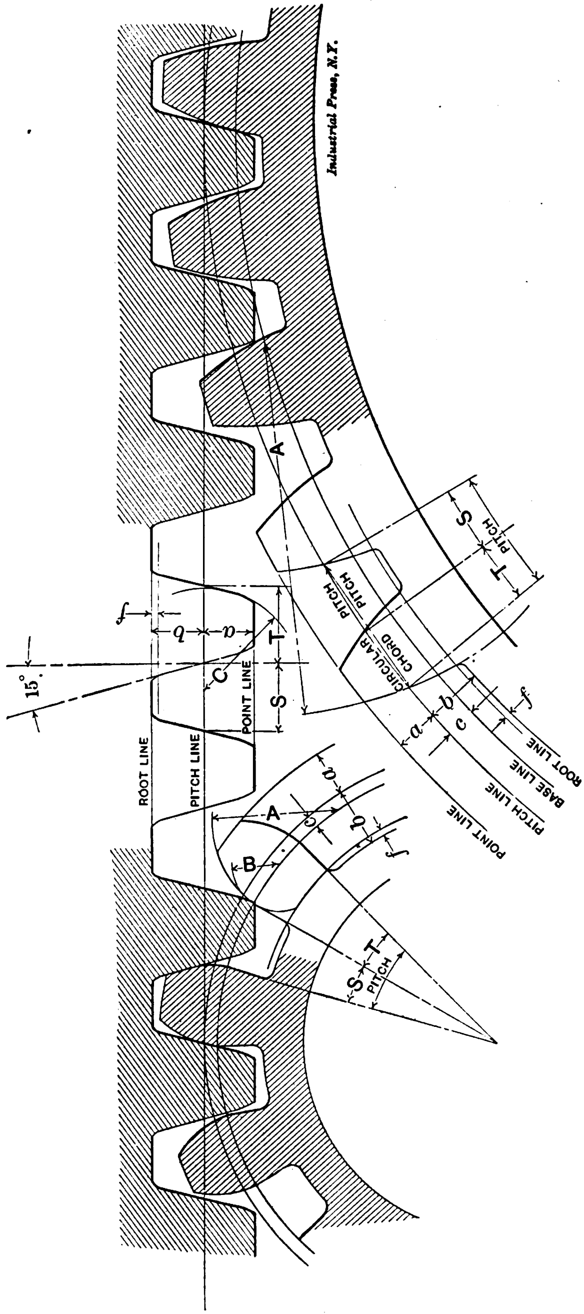


Fig. 16. Two Gears in mesh with a Rack. The Gear to the Left shows the Method of laying out a Gear having twelve to thirty-six Teeth; the View to the Right, a Gear having thirty-six Teeth or more.

molded gears, and of running well with a small deviation from the true distance between centers. The involute form only will be considered, as it is believed that this form is becoming very much the more common of the two. The tooth outlines are most conveniently laid out by the use of an odontograph table (Table VII), in which the dimensions shown in Fig. 16 are given in terms of the pitch. Two gears are shown in mesh with a rack—that on the left showing the method of laying out a gear having from 12 to 36 teeth, and that on the right showing the method for a gear having 36 teeth or more.

TABLE VII. DIMENSIONS AND CONSTANTS FOR LAYING OUT GEAR TEETH.

Circular Pitch.		Diametral Pitch.
0.3 × pitch.	*a*	1 ÷ pitch.
0.4 × pitch, below 1″.	*b*	1.15 ÷ pitch.
0.375 × pitch, above 1″.	*b*	
0.016 × pitch diameter.	*c*	
0.53 × pitch.	*S*	
0.47 × pitch.	*T*	
0.1 × pitch.	*f*	

Multiply by Number of Teeth.			Multiply by Number of Teeth.		
Pitch.	A.	B.	Pitch.	A.	B.
¾″	0.03	0.015	1″	0.125	0.062
⅞″	0.035	0.0175	1¼″	0.100	0.050
1″	0.04	0.02	1½″	0.083	0.041
1⅛″	0.045	0.0225	1¾″	0.071	0.035
1¼″	0.05	0.025	2″	0.062	0.031
1⅜″	0.055	0.0275	2¼″	0.056	0.028
1½″	0.06	0.03	2½″	0.050	0.025
1⅝″	0.065	0.0325	2¾″	0.045	0.022
1¾″	0.07	0.035	3″	0.042	0.021
1⅞″	0.075	0.0375	3½″	0.036	0.018
2″	0.08	0.04	4″	0.031	0.015
2⅛″	0.085	0.0425	5″	0.025	0.012
2¼″	0.09	0.045	6″	0.021	0.010
2⅜″	0.095	0.0475			
2½″	0.100	0.05			

When laying out a molded gear, first draw the pitch circle of a diameter equal to the pitch diameter previously determined. Draw the point circle outside the pitch circle, and a distance from the pitch circle of 0.3 times the pitch, and draw the root circle inside the pitch circle and a distance from the pitch circle of 0.4 times the pitch. These two distances are given as *a* and *b*, respectively, in Fig. 16, and in Table VII it will be seen that as the point is 0.3 of the pitch outside the pitch circle, while the root is 0.4 of the pitch inside the pitch circle, the teeth of the two meshing gears will have a clearance between point and root of 0.1 of the pitch. For gears having greater than 1-inch pitch, this clearance will be greater than necessary even for rough gears, and will not look well; so for gears above 1-inch pitch, 0.375 instead of

0.4 may be the multiplier for the pitch, as will be found noted in the table. The base line is now drawn at a distance 0.016 times the *pitch diameter* from the pitch circle. This base line may sometimes come inside the root circle. The pitch is then laid off upon the pitch circle as before described, and the spaces thus made upon the pitch circle are to be divided into tooth and space parts. The tooth part will be 0.47 times the pitch, and the space part will be 0.53 times the pitch, thus giving the tooth a small clearance in the space. We are now ready to draw in the tooth outlines, which are circular arcs drawn from centers on the base line. In the case of a gear of less than 36 teeth the tooth outline will be composed of two arcs, while for all gears of 36 teeth or more the tooth outline is only one arc. The radii for these arcs are found by multiplying the number of teeth in the gear by the constants found in the odontograph table opposite the pitch of the gear. These radii are designated in Fig. 16 and Table VII as *A* and *B*. In the case of gears of less than 36 teeth the tooth outline is completed by radial lines as shown in Fig. 16.

CHAPTER IV.

STRENGTH OF GEAR TEETH.

In considering the strength of gear teeth we shall, in the first place, neglect the actual shape of the tooth and assume it to be rectangular in every section, as shown in Fig. 17. Further, for the sake of simplicity, it will be assumed that the load, *P*, acts on the outer circumference of the gear, and hence at the extremity of the tooth. If we consider the conditions under which the load is assumed to be applied in actual practice, we shall find that: (1) A rough cast gear may have the whole load concentrated upon one corner of a tooth; or the gear itself may be well made, but may be out of alignment, due to springing of shaft, bad workmanship, or other causes; in this case also the load may be concentrated upon the corner of one tooth. (2) The gear may

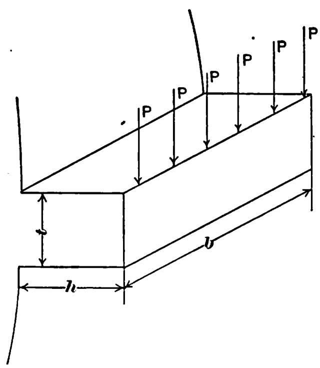


Fig. 17.

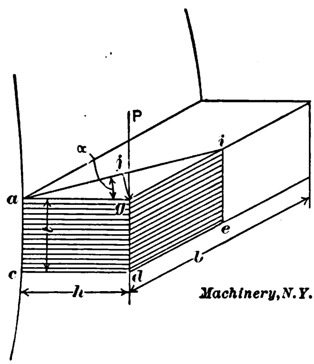


Fig. 18.

be well made, with accurately cut or cast teeth, well mounted upon heavy shafts having proper alignment, in which case the load may be distributed over the entire width of a single tooth. In determining a formula, then, for the pitch of a gear tooth, we must know the kind of gears we have to deal with, and what considerations affect their strength.

In the first case, if the gear is rough cast or placed so that the load may come upon one corner of a tooth, we must consider not only that the tooth is a cantilever, as in Fig. 17, but that the corner of the tooth is itself a cantilever, as in Fig. 18, in which the whole load, *P*, is concentrated at the point, *g*, so that the moment tending to break the corner off from the tooth is *P* multiplied by the perpendicular distance from *P* to the line, *a i;* hence $P \times g\,j = P\,h \sin a =$ bending moment. Now, the resistance which the tooth offers depends upon the character of the metal, as well as its actual section, which in the present case is the rectangle $a c e i$. It can be shown that the resistance of this corner to breaking across is equal to

$$1/6\, f\, t^2 \times a i = 1/6\, f\, t^2 \frac{h}{\cos a} \qquad (20)$$

in which f = stress in the metal. Equating this to the bending moment, we have

$$Ph \sin a = 1/6\, f\, t^2 \frac{h}{\cos a} \qquad (21)$$

or

$$f = \frac{6\, P \sin a \cos a}{t^2} \qquad (22)$$

Noting that $\sin a \cos a = ½ \sin 2\, a$, and substituting in (22) we have:

$$f = \frac{3\, P}{t^2} \sin 2\, a$$

When $a = 45$ degrees, we have $\sin 2\, a$ = a maximum = 1; therefore

$$f = \frac{3\, P}{t^2};\ \text{or}\ P = f \frac{t^2}{3};\ \text{and}\ t = \sqrt{\frac{3\, P}{f}} \qquad (23)$$

For new teeth, rough cast, in which $t = 0.48\ p_1$ (p_1 being the circular pitch) we have:

$$0.48\, p_1 = \sqrt{\frac{3\, P}{f}};\ \text{or}\ p_1 = 3.6 \sqrt{\frac{P}{f}} \qquad (24)$$

For wooden cogs working with cast iron teeth, in which $t = 0.6\, p_1$, we have:

$$p_1 = 2.9 \sqrt{\frac{P}{f}} \qquad (25)$$

If in any case the width of face, b, is less than the height of tooth, the above formulas do not apply. In this case we have used the thickness of tooth without considering loss of strength, due to wear; but we have also assumed the whole load as concentrated upon the corner, whereas, as wear occurs, although the tooth itself is reduced in section, yet the load is better distributed, and the tooth may be actually stronger.

In the case of rough cast iron and mortise gears in which the whole load may come upon the corner of one tooth, the width of gear does not affect its strength, and there is no advantage, as far as strength is concerned, in making such a gear wider than $h \times \frac{1}{\cos 45\ \text{deg.}}$, or $1.41\ h$; as h, the height of the tooth, is frequently made equal to $0.7\, p_1$, we have the safe width of tooth, $b = 1.41 \times 0.7\, p_1 = p_1$, very nearly.

In order to prevent excessive wear, such gearing is frequently made so that its breadth of face $b = 2\, p_1$. So far we have assumed that the tooth section was a rectangle, but in practice the tooth may have the shape shown in Fig. 19, or that in Fig. 20, which represent a wide range in the strength of two gears, when accurately made and aligned, but for a load concentrated upon one corner the difference is not so marked as the shape of the teeth would indicate.

Investigation shows that if the smaller of two gears has at least 18 teeth, and not more than 50, the strength of the tooth will be practically the same as that determined by the above formulas. Since these values may be said to represent fairly well the limits of the number of teeth in the smaller of two gears in those cases where rough gearing is permissible, we shall not at the present time discuss the influence of shape upon the strength of this kind of gearing.

Of more importance is the consideration of that case in which the load is supposed to be equally distributed between two gear teeth.

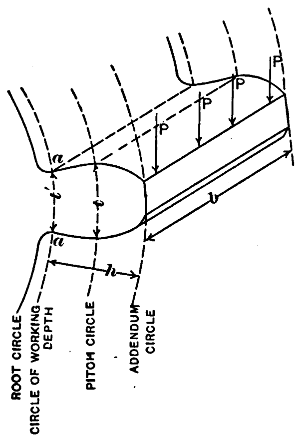


Fig. 19.

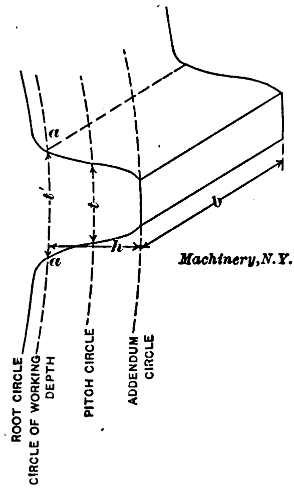


Fig. 20.

This matter relates to the strength of all gear teeth, and we shall discuss its effects in general.

The point of contact between tooth surfaces in correct gearing can readily be determined when the system upon which the gears have been designed is known. Thus in the involute system, the point of contact is located on the straight line passing through the line of centers at the point of intersection of the pitch circles, and making an angle with the horizontal of 14½ degrees to 22½ degrees, as shown by $P_1 T P'$ in Fig. 21.

On the other hand, the point of contact between two tooth surfaces of the cycloidal system will always be upon a reverse curve, which is a portion of the describing circle for the faces and flanks of the tooth; thus, in the interchangeable system of 12-tooth base, *i. e.*, in that system where the smallest pair of gears in the set is assumed to have 12 teeth and these have radial flanks, the diameter of rolling circle will evidently be equal to the radius of the 12-tooth pinion; and since the tooth profiles are generated by rolling this circle upon the pitch circles of any pair of gears in the set, the point of contact will always fall upon the circumference of this describing circle when the center of the latter lies upon the line joining the centers of the two gears, as shown in Fig. 22. If in Figs. 21 and 22 $M' N'$ and $R' S'$ represent the addendum circles, or the circles drawn through the tops of the teeth of the two gears, we shall have the two loci $P' T P_1$, of the points of contact, as shown in heavy lines; that is, contact will begin in each case at P' and, as the teeth move around, the point of contact between the surfaces will travel along the line or reverse curve, $P' T P_1$, until contact ceases. Now, considering Fig. 22, if the pitch were equal to the arc $P' T P_1$, one tooth would just be beginning action at P', while another would be quitting at P_1. In practice, in order to obtain smoothness of action the effective height of tooth is usually taken at about six-tenths of the circular pitch in cast gears, and about 0.64 of the

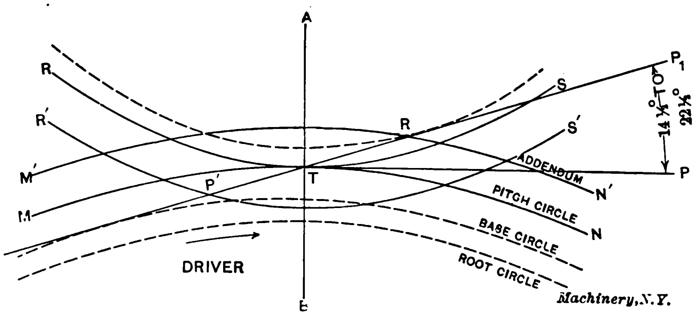


Fig. 21.

circular pitch in cut gears. If $p =$ diametral pitch, $p' =$ circular pitch, and $h =$ effective height of tooth, we have: $h = 0.6\,p'$ for cast gears; and

$$h = 0.64\,p' = \frac{2}{p} \text{ for cut gears.}$$

We would here note that the effective height of tooth is not the total height, as an additional amount, frequently $0.1\,p'$ for cast gears, is allowed for clearance between the root circle and the tops of the teeth of the mating gear. With an addendum equal to half the above height of tooth, it will be found that the average arc of contact for gears between 12 teeth and a rack, in both the involute and cycloidal systems, has a length equal to about one and two-thirds times the circular pitch. In any case, by stepping off the pitch on $P' T P_1$, Fig. 22, from both P' and P_1, we shall obtain the points a and b, which indicate that with perfect gearing the two pairs of teeth will be simultaneously in contact at P' and at a and will remain in contact until b and P_1 are reached; the distance from b to a will be traversed with only one pair of teeth in contact; however, in this latter case, it will be noticed that the leverage of the tooth is very much reduced. For a 12-toothed pinion (cycloidal system), this leverage, shown in Fig. 22 as h', is $0.41\,p'$; and for a rack it is $0.35\,p'$. Under these conditions, then, the dimensions of the tooth would be determined upon the supposition that the whole working load is carried by one tooth, and that the force is applied with a leverage of $0.35\,p'$ to $0.41\,p'$, the constant depending upon the number of teeth. For the 15-degree involute system the effective height for a 12-toothed pinion is the same as for the cycloidal system, *viz:* $0.41\,p'$, but for a 15-degree involute rack there are always two teeth in contact.

This determination is correct for all gear teeth properly shaped and spaced, but as Mr. Wilfred Lewis has so ably pointed out, it must be admitted that mechanical perfection in forming and spacing has not

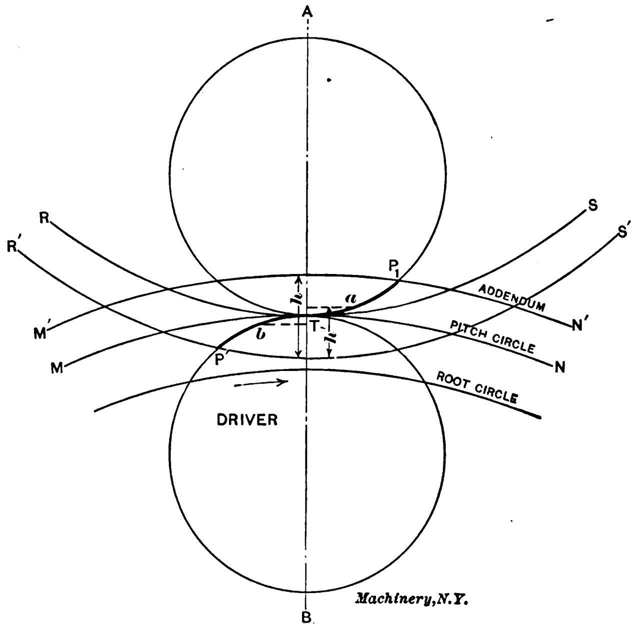


Fig. 22.

yet been reached, and that the slightest deviation in either respect is sufficient to concentrate the entire load at the end of one tooth. Even with cut gears the same conditions obtain, for the cutters ordinarily employed are correct only for a single gear, although they are used within certain equidistant limits for various other gears. To what extent the elasticity of bronze, copper, and steel influences the distribution of pressure on cut gears is not known, and it is, therefore, unsafe to consider its effect. As the teeth become worn, the concentration of the load may be reduced, but until this wear takes place, the whole load should be assumed as acting at the extreme end of the tooth.

For rough cast iron and mortise gears, as we have shown, it is possible for the whole load to be concentrated on the corner of one tooth, and this may occur even with the best cut gears, owing to careless alignment, or lack of stiffness in the shafting and supports; but when iron patterns are used with accurately cut teeth, especially if the mold is baked, or when the gear is machine molded, or the teeth are cut, the contact should be fairly uniform, and the load evenly distributed across the face of the tooth. It must be understood, however, that in many cases circumstances will arise when it is impossible to secure these favorable conditions; particularly is this true when the gearing is subjected to shock and variable loads. However, with careful work, stiff shafts and not too wide a face, the assumption of fairly uniform distribution of pressure across the tooth may be considered as satisfactory for general practice.

The pressure which comes upon the tooth in the direction of the line of thrust (the common normal to the tooth surface) is greater

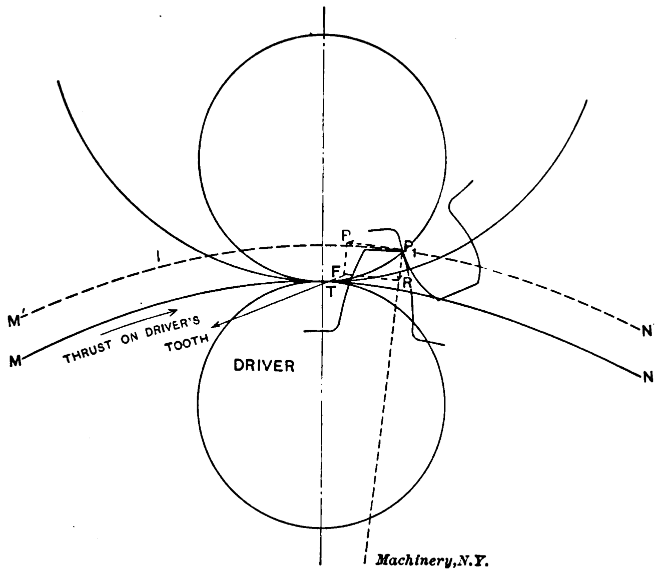


Fig. 23.

than that which transmits motion, for if we resolve the force, F, along and perpendicular to the radius at the point of contact, P_1, Fig. 23, the radial component, R, does not tend to produce rotation; it will, however, exert a pressure on the bearings, and tends to crush the tooth; as the compressive force has little effect upon the strength of the tooth, especially with cast iron gears, its influence may be neglected. The other component, or P, acts at right angles to the radius and produces rotation by its pressure on the tooth, and thus may be considered as the effective working load.

Now considering the tooth as a rectangular cantilever with the load P uniformly distributed across the outer edge, as in Fig. 17, the thickness, t, assumed equal to $0.49\ p' - 0.02$ inch for well-made cast gears, may be obtained by equating the bending moment to the moment of resistance of the tooth; that is

$$P h = f \times \frac{1}{6} b t^2 \qquad (26)$$

Assuming the effective height of tooth $h = 0.6\, p'$ (circular pitch); breadth of face, $b = x p'$; thickness of tooth, $t = 0.49\, p' - 0.02$, therefore $t^2 = 0.24\, (p')^2 - 2 \times 0.0098\, p' + 0.0004$, in which we may neglect the last two terms, as they will not appreciably affect the result.

Substituting these values in (26) we have:

$$P \times 0.6\, p' = \frac{f x p' \times 0.24\, (p')^2}{6}$$

$$P = f \frac{x \times 0.24\, (p')^2}{3.6} = 0\ 066 f x\, (p')^2 \qquad (27)$$

$$p' = \sqrt{\frac{P}{0.066 f x}} = \sqrt{\frac{15 P}{f x}} \qquad (28)$$

For cut gears the addendum is usually made equal to the reciprocal of the diametral pitch, or $1/p$; but since $p p' = \pi = 3.14$ we have $1/p = 0.32\, p'$, hence the effective height of tooth now becomes $0.64\, p'$. The thickness of tooth in this case is $0.5\, p'$; therefore the load, from (26) is:

$$P = 0.065 f x\, (p')^2 \text{ and } p' = \sqrt{\frac{15.4\, P}{f x}},$$

results practically the same as those just obtained. For cast gears, if the breadth of face equals twice the pitch

$$\text{or } x = 2, \quad \text{then } p' = 2.74 \sqrt{\frac{P}{f}};$$

$$\text{if } x = 2\tfrac{1}{2}, \text{ then } p' = 2.45 \sqrt{\frac{P}{f}};$$

$$\text{if } x = 3, \quad \text{then } p' = 2.24 \sqrt{\frac{P}{f}};$$

$$\text{if } x = 3\tfrac{1}{2}, \text{ then } p' = 2.07 \sqrt{\frac{P}{f}};$$

It will be noticed that the formulas thus deduced apply only to rectangular teeth, as in Fig. 17, whereas in practice we have to deal with such forms as given in Figs. 19 and 20. In Fig. 19, the width of tooth is considerably less measured on the working depth circle than on the pitch circle, and fracture would occur along the line $a\,a$, where the thickness is t'.

With a uniformly distributed pressure, P, acting at the extremity of the tooth as shown, the relative strength of this tooth compared with one of rectangular section having the uniform thickness t varies as $\left(\frac{t'}{t}\right)^2$. Thus if a gear tooth of 1 diametral pitch ($p' = 3.14$) measures 1.52 inch on the pitch circle and 1.34 inch on the working depth circle between the points *a* and *a*, Fig. 19, the strength of the tooth will be only $\left(\frac{1.34}{1.52}\right)^2 = 0.78$ of that which it would have if the tooth had the constant thickness 1.52 inch. In the same way, if the thickness of tooth at *a a*, Fig. 20, measures 1.76 inch, then its relative strength will be $\left(\frac{1.76}{1.52}\right)^2 = 1.34$.

This suggests a convenient method not only of ascertaining the load which a given gear will sustain, but also one which will give a suitable pitch when the load and number of teeth are known.

If in the first case the pitch had been determined from formula (28), the tooth would have only about three-fourths of its assumed strength for a given load *P*, whereas in the second case, the formula would give a tooth about a third stronger than necessary for the same load *P*.

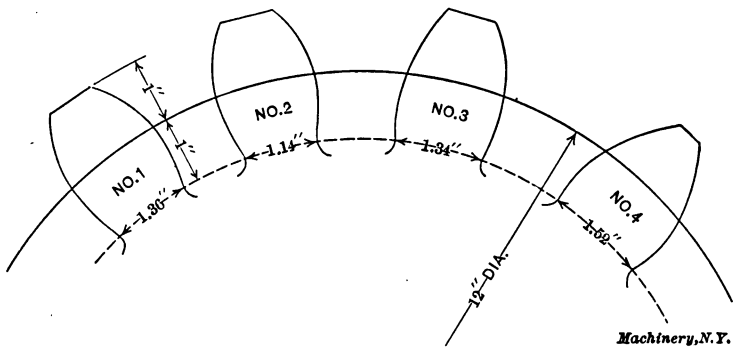


Fig. 24.

Since the pitch varies as the square root of the pressure, it will be evident that if we multply the pitch in each case by the inverse ratio of $\frac{t'}{t}$ we shall obtain a pitch giving the actual strength required in both cases. Therefore, since

$$\frac{1.52}{1.34} = 1.14, \text{ and } \frac{1.52}{1.76} = 0.86,$$

the respective pitches will be 1.14 p' and 0.86 p', in which

$$p' = \sqrt{\frac{15\,P}{f\,x}}$$

from formula (28).

It is evident that the method of laying out the gear tooth will have some influence upon its strength; this is very clearly shown in Fig. 24, which represents a twelve-tooth pinion of 3.14 inches pitch accurately drawn for the four systems represented, *viz*: (1) cycloidal, 12 base, *i. e.*, the smallest pair of gears is assumed to have twelve teeth, and these have radial flanks, so that the diameter of rolling circle is one-half of the diameter of pitch circle of a twelve-tooth pinion; (2) cycloidal, 15 base; (3) involute, 15 degrees line of action, and (4) involute, 22½ degrees lines of action. By laying out a series of different gears, according to the different systems represented, the exact thickness of t' and t may be determined graphically, from which the values of the ratios $\frac{t}{t'}$ and $\left(\frac{t'}{t}\right)^2$ may be readily ascertained.

This has been done, and the results are given in Tables VIII and IX, in which the ratio is presented in the form of a coefficient corresponding to the number of teeth in the gear.

TABLE VIII. VALUES OF COEFFICIENT $C_p = \frac{t}{t}$

No. of Teeth in Gear		Value of Coefficient C_p			
		Involute System		Cycloidal System	
Exact	Intervals	15 deg.	22½ deg.	12 Tooth Base	15 Tooth Base
12	12	1.14	1.00	1.12	1.33
13½	13 – 14	1.10	0.96	1.08	1.19
15½	15 – 16	1.07	0.92	1.04	1.10
17½	17 – 18	1.04	0.88	1.00	1.03
20	19 – 21	1.01	0.84	0.96	0.98
23	22 – 24	0.98	0.80	0.92	0.95
27	25 – 29	0.95	0.76	0.88	0.92
33	30 – 36	0.92	0.73	0.85	0.89
42	37 – 48	0.88	0.70	0.82	0.86
58	49 – 72	0.84	0.68	0.79	0.83
97	73 – 144	0.80	0.66	0.76	0.80
290	145 – Rack	0.76	0.65	0.73	0.77

Combining these coefficients with formulas (27) and (28) we obtain

$$P = 0.066\, f\, x\, C_s\, (p')^2 \qquad (29)$$

and

$$p' = C_p \sqrt{\frac{15\, P}{f\, x}} \qquad (30)$$

In the same way formulas may be deduced for wooden cogs, working with cast iron, by assuming the width of tooth equal to 0.6 p'.

In these formulas, as previously stated, the effective or working height of tooth was used instead of the total height, which is always somewhat greater, but as the strength of the tooth varies with the square of its thickness, and as this value increases below the working depth circle more rapidly than the increased height of tooth on account of the fillet, we are justified in neglecting that portion of the tooth between the working depth and root circles, assuming that a suitable fillet has been used at the base of the tooth.

The formula here presented will give a close approximation to the working strength of a gear tooth when the pitch and number of teeth are given, provided we know the working stress in the material of which the tooth is composed. This is, however, such a variable factor that it is possible only to suggest limitations covering general conditions; and the judgment of the designer, based on a knowledge of the conditions under which the gear is to work, must modify the general values, as occasion may require. Indeed, it is much more satisfactory to the designer to use a general formula involving a choice of stress than to employ an entirely empirical rule in which the stress is unknown, and which offers no opportunity for variation under varying conditions.

Professor Unwin assumes that for cast iron f may have the three values 9,600, 6,100, and 4,300 pounds per square inch for "little shock," "moderate shock," and "excessive shock," respectively, for those gears

TABLE IX. VALUES OF COEFFICIENT $C_s = \left(\frac{t'}{t}\right)^2$

No. of Teeth in Gear		Value of Coefficient C_s			
		Involute System		Cycloidal System	
Exact	Intervals	15 deg.	22½ deg.	12 Tooth Base	15 Tooth Base
12	12	0.77	1.00	0.79	0.56
13½	13 – 14	0.83	1.08	0.85	0.70
15½	15 – 16	0.87	1.18	0.92	0.83
17½	17 – 18	0.92	1.29	1.00	0.94
20	19 – 21	0.98	1.42	1.08	1.04
23	22 – 24	1.02	1.56	1.18	1.11
27	25 – 29	1.10	1.72	1.29	1.18
33	30 – 36	1.18	1.87	1.38	1.26
42	37 – 48	1.29	2.03	1.48	1.35
58	49 – 72	1.42	2.16	1.60	1.45
97	73 – 144	1.56	2.29	1.73	1.56
290	145 – Rack	1.72	2.39	1.87	1.69

in which, from inaccuracy of form or mounting the pressure may come on a corner of the tooth. For carefully fitted gearing, however, in which the pressure is assumed to be distributed along the whole width of the tooth, he suggests 4,350 and 2,780 pounds per square inch, "the latter to apply to cases where there is some vibration and shock."

In selecting these values he assumed that the load is divided between two pairs of teeth in all cases, so that the pressure on each tooth is 2-3P. Multiplying these several values of f by 2-3, we obtain the stress in pounds per square inch, assuming the total load to be carried on one tooth.

TABLE X. VALUES OF STRESS f FOR CAST IRON (UNWIN).

	Little Shock	Moderate Shock	Excessive Shock
Inaccurate gearing	6,300	4,000	2,850
Well formed gearing........	2,875	....	1,850

It will be noted that the real stress in the well formed gearing may be considerably less than that given, on account of the influence of increased thickness of tooth at the base. Professor Reuleaux, in his "Constructor," states that the dimensions of gear wheels must, for the same pressure on the teeth, be increased to meet shock in proportion to the increase in initial velocity. For slow-running gears, however, he neglects this action and divides gears into two classes, *viz.*, hoisting gears, and transmission gears, and includes under hoisting gears all those having a linear velocity at the pitch circle of not more than 100 feet per minute, and under transmission gears all those running at a higher velocity. For hoisting gears he recommends a fiber stress of about 4,200 pounds per square inch for cast iron, and states that an increase of one-fourth in the permissible stress would reduce the pitch only 7 per cent, but on the other hand, he notes that too low a value of f causes unnecessary increase in the size and weight, not only of the gears, but also of the bearings, frame work and other parts of the machine. In assuming this value of f, Reuleaux does not consider any increase in strength due to increased section at the base of the tooth, although he states that "the actual stress is properly somewhat less, because the thickness of the tooth at the base is usually more than one-half the pitch as assumed in the formula."

For transmission gears Reuleaux states that the fiber stress should be taken smaller for a given force P, as, when the circumferential velocity increases, the dynamic action of shock and vibration also increases.

For cast iron he recommends

$$f = \frac{9{,}600{,}000}{V + 2{,}164}$$

in which V is the velocity of the pitch circle in feet per minute. For steel he states that f may be taken 3 1/3 times, and for wood, 6/10 times the value thus obtained.

Arranged in tabular form this gives results as shown in Table XI.

TABLE XI. VALUES OF SAFE STRESS f FOR TRANSMISSION GEARS (REULEAUX).

Velocity in Feet per Minute.	100	200	400	600	800	1000	1500	2000	2500
For cast iron f =	4240	4060	3744	3473	3238	3034	2620	2302	2068
For steel f =	14112	13020	12467	11565	10782	10103	8725	7665	6886
For wood f =	2544	2436	2246	2083	1943	1820	1572	1381	1240

Mr. Wilfred Lewis in his paper on Gearing, read before the Engineers' Club of Philadelphia, in 1893, makes the following pertinent remarks:

"What fiber stress is allowable under different circumstances and conditions cannot be definitely settled at present, nor is it probable that any conclusions will be acceptable to engineers unless based upon carefully made experiments. In the article referred to, certain factors are given as applicable to certain speeds, and in the absence of any later or better lights upon the subject. Table XII has been constructed to embody in convenient form the values recommended.

It cannot be doubted that slow speeds admit of higher working stresses than high speeds, but it may be questioned whether teeth running at 100 feet a minute are twice as strong as at 600 feet a minute, or four times as strong as the same teeth at 1,800 feet a minute. For teeth which are perfectly formed and spaced, it is difficult to see how there can be a greater difference in strength than the well-known difference occasioned by a live load or a dead load, or two to one in extreme cases. But, for teeth as they actually exist, a greater difference than two to one may easily be imagined from the noise sometimes produced in running, and it should be said that this table is submitted for criticism rather than for general adoption. It is one which has given good results for a number of years in machine design, and its faults, such as they may be, are believed to be in the right direction."

TABLE XII. SAFE WORKING STRESS f FOR DIFFERENT SPEEDS (LEWIS).

Speed of Teeth in Feet per Minute	100 or less	200	300	600	900	1,200	1,800	2,400
Cast iron	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Steel	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

If the formulas and coefficients presented by the writer are followed in the determination of the proportions of gear teeth, the permissible stress may be considerably greater than the real, as determined from the thickness of tooth near the base; this accounts in a large measure for the high stresses assumed to be carried by some gears. Thus, for instance, a gear having sixty-two teeth, 3.6-inch pitch, and 15-inch face, runs at 280 feet per minute and carries a load of 22,240 pounds. The working stress f is assumed to be 7,250 pounds, but if the increased section on the working depth circle be taken into consideration the working stress is reduced to 5,200 pounds.

While high rim speed does not necessarily imply shock, yet the effect of shock is more liable to be disastrous if high speed obtains, and for this reason the stress should diminish as the speed increases. Another reason for this is found in the fact that the stress is increased, due to the centrifugal force set up in the metal itself, which varies as the square of velocity; while this is not very great at ordinary speeds, it is still of sufficient importance to be considered for high speeds.

The influence of change of load is well understood in engineering construction, and usually a factor of safety is adopted for such loads from two to three times greater than would be the case for a steady or dead load, the value depending largely upon the range of stress involved. There seems good reason to believe that a similar relation modified by the effect of speed should obtain in selecting a suitable value for the working stress in gear teeth. From the foregoing considerations, then, it would appear that the working value of f should be chosen both with reference to the velocity and to the character of the acting force. With this in view the author of the present chapter has deduced the following formula for working stress:

$$f = \frac{50,000}{k + \sqrt[3]{V}}$$

in which f = the allowable stress in pounds per square inch;

V = velocity at pitch circle in feet per minute;

k = a constant having the values: 5 for little shock, 10 for moderate shock, and 15 for violent shock.

From this formula Table XIII has been calculated, and is offered as

TABLE XIII. VALUE OF STRESS f FOR CAST IRON (FLATHER).

Velocity in Feet per Minute	Value of Stress in pounds per square inch. Little Shock	Moderate Shock	Violent Shock
100	5,200	3,400	2,550
200	4,600	3,150	2,400
400	4,050	2,900	2,250
600	3,700	2,700	2,150
800	3,500	2,600	2,050
1,000	3,300	2,500	2,000
1,400	3,100	2,350	1,900
1,800	2,900	2,250	1,840
2,400	2,700	2,150	1,770
3,000	2,600	2,050	1,700

a guide to the designer in selecting suitable values of f under varying conditions.

While the previous discussion and the formulas given are of interest to the student of the subject of strength of gear teeth, as indi-

TABLE XIV. OUTLINE FACTORS FOR USE WITH LEWIS FORMULA.

No of Teeth.	Factor for Strength, Y.		No. of Teeth.	Factor for Strength, Y.	
	Involute 20° Obliquity.	Involute 15° and Cycloidal.		Involute 20° Obliquity.	Involute 15° and Cycloidal.
12	.078	.067	27	.111	.100
13	.083	.070	30	.114	.102
14	.088	.072	34	.118	.104
15	.092	.075	38	.122	.107
16	.094	.077	43	.126	.110
17	.096	.080	50	.130	.112
18	.098	.083	60	.134	.114
19	.100	.087	75	.138	.116
20	.102	.090	100	.142	.118
21	.104	.092	150	.146	.120
23	.106	.094	300	.150	.122
25	.108	.097	Rack.	.154	.124

cating what has been done and proposed along this line, it should be mentioned here that the Lewis formula given on page 28 is the formula now almost exclusively used. Table XIV gives the factor Y necessary for use with this formula for a complete range of number of teeth, and for both involute and cycloidal tooth-forms.

CHAPTER V.

VARIATION OF THE STRENGTH OF GEAR TEETH WITH THE VELOCITY.

The generally accepted formula for calculating the strength of gear teeth is that proposed by Mr. Wilfred Lewis, first published in the Proceedings of the Engineers' Club of Philadelphia, January, 1893, and referred to in the preceding chapter.

The merit of this formula lies in the great number of variables taken into account as compared with other rules in more or less common use, and in the fact that these variables are rationally considered. The effect of each of them can be calculated with some assurance, with

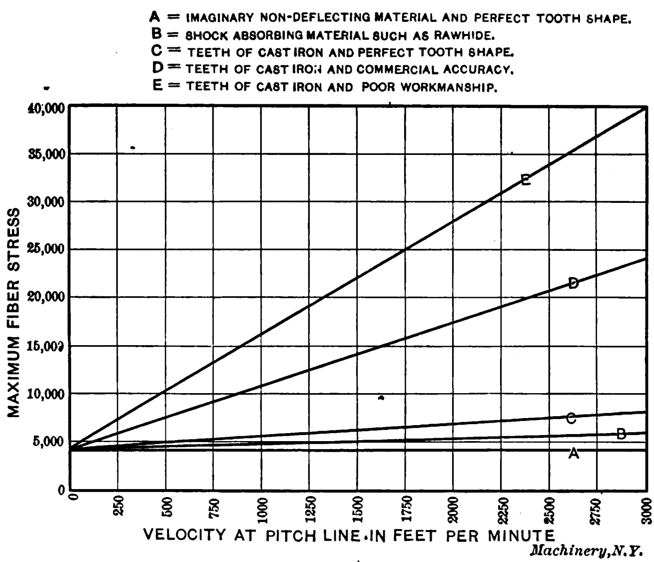


Fig. 25. Hypothetical Diagram showing the Relation of the Velocity to the Fiber Stress.

the single exception of the influence of the velocity on the safe stress. In the fifteen years since the formula was first proposed, the original values for the stress as affected by the velocity have been largely used. Many designers, however, have felt that these values are rather unsatisfactory, although most of them will agree that they err rather on the side of safety than otherwise. By referring to Mr. Lewis' original paper it will be seen that these values were not given as being definitely determined, but merely as agreeing well with successful cases met with in his own practice. The following is a general analysis of the conditions involved.

A variation in the strength of the teeth of a gear, due to a variation in the velocity, can be due, of course, to but one thing—impact. To illustrate this idea, and to show the cause of the impact, we will study the action of gearing under three different conditions.

1. *Gears of an imaginary undeflectable material.*—In Fig. 25 is a diagram in which the horizontal distances give velocity in feet per minute, and vertical distances give stresses in pounds per square inch, starting in this case at 4,000, which is assumed to be the maximum fiber stress in the gear we are considering, due to the load at the pitch line, which is supposed to be constant at all speeds. If the teeth of this

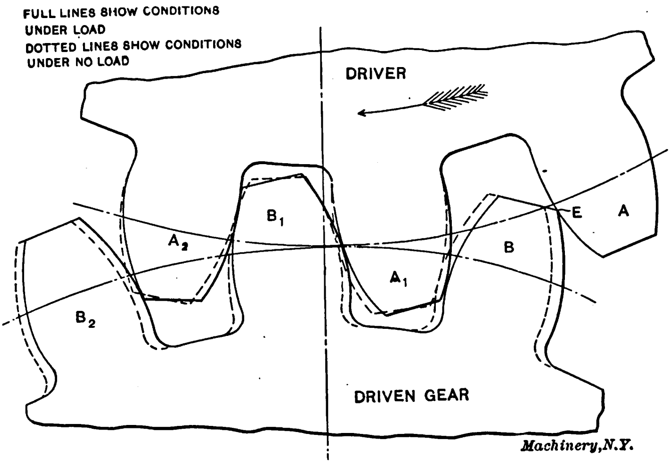


Fig. 26. The Action of Gear Teeth under Load, Greatly Exaggerated.

gear are perfectly formed and well fitted together, so that there is no back lash, if the power is delivered to them steadily and smoothly, and the mechanism they drive runs without shock, any disturbance of the even movement will be impossible, and impact will be entirely absent. In the diagram in Fig. 1, then, there will be no rise of maximum fiber stresses with the velocity, so that the horizontal line *A* will show the conditions for this imaginary case.

2. *With commercial material and theoretically accurate workmanship.* The conditions in this case are shown in Fig. 26, with all the phenomena greatly exaggerated. The full lines show the conditions under load, while the dotted outlines show the conditions when the load is removed from the driven gear. The teeth A_1, B_1, and A_2, B_2, carrying the load, are deflected by it, as shown. Tooth *B*, just about to come into contact with tooth *A*, is on that account shifted from its normal position; it should be located as shown by the dotted lines. If it were in this position, it would come in contact with tooth *A* under mathematically perfect conditions, and there would be no shock of engagement. As it is, the two come suddenly into action as shown at *E*, under different conditions than those contemplated by the design, thus the contact takes place in the form of a slight blow, after which the

teeth are deflected more and more, until they have taken up their share of the load, as shown later at A_1 and B_1. If the gears are moving very slowly, the deflection takes place very slowly, and the problem is practically a static one. If the gears are running at a high velocity, the problem becomes essentially a dynamic one, and the stresses are greater than with the slow speed. The increase in stress with the increase in speed for this second case could probably be represented by a line something like *C*, in Fig. 25.

3. *With commercial materials and commercial accuracy.* This is, of course, the practical case to consider. A line to show the relation of the velocity to the maximum fiber stress for a given gear, would very probably look something like *D* in Fig. 25. This is, in fact, approximately the line which embodies the conclusions of the Lewis tables for a static stress of 4,000 pounds. It is considerably higher than line *C*, because impact due to irregular tooth outlines is added to the impact due to the deflection.

Practical Considerations Affecting Design.

The fact that the variation of the strength with the velocity is due to impact, suggests also a number of points relating to design.

1. *Value of accuracy.* It is evident that this theory of impact puts a premium on accuracy in workmanship for gears that are to run at high speed under a heavy load. It is probable that the strength of a given pair of gears may be cut in two if the tooth outlines are not carefully determined, and if the cutter is not set centrally. This suggests the desirability of a greater sub-division of the standard cutter series for work of this kind.

2. *Resilience of design and materials.* In high-speed gearing it is evident that the shock due to the impact should be absorbed as quickly and as fully as possible. This suggests the use, at abnormally high speeds, of rawhide, wood, etc., for one of the members of the pair of gears. The introduction of spring couplings or similar devices may also be desirable, especially where the other parts of the mechanism are liable to transmit shock to the gearing.

3. *Easing off the points of the tooth.* There has always been a sort of superstition that the points of the tooth should be eased off to make the action smoother. This is done, of course, in standard involute gears, though for another reason, that of avoiding interference with the flanks of the pinions. It can now be seen that there is a solid basis for this practice in all cases where gears are to run at such speeds that severe impact is liable to take place. Referring to Fig. 26, teeth *A* and *B* are taking up the load very suddenly, owing to the fact that they are out of step, due to the deflection of the other teeth momentarily carrying the load. Easing away the points of *A* and *B* would mitigate this sudden reception of the load, allowing the inevitable deflection to take place more slowly, with a consequent gain in the strength of the gear at high speeds.