x

Serial No. 9

## DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIO SURVEY

14
E. LESTER JONES, SUPERINTENDENT

## GEODESY

## APPLICATION OF THE THEORY OF LEAST SQUARES TO THE ADJUSTMENT OF TRIANGULATION

BY

## OSCAR S. ADAMS

COMPUTER UNITED STATES (OAST AND GEODETIC SURYEI RET P






I-

$$
\begin{aligned}
& 1 \\
& 8
\end{aligned}
$$

$2+-2+2+2+2+2+2$
$\qquad$

## DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY
E. LESTER JONES, SUPERINTENDENT

## GEODESY

# application of the theory of least SqUARES TO THE ADJUSTMENT 0F triangulation 

BY<br>OSCAR S. ADAMS<br>COMPUTER<br>UNITED STATES COAST AND GEODETIC SURVEY

Special Publication No. 28



ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM THE SUPERINTENDENT OF DOCUMENTS GOVERNMENT PRINTING OFFICE WASHINGTON, D. C.
$\mathrm{AT}^{\mathrm{T}}$
25 CENTS PER COPY
$\nabla$

QB
321
4543
1915

## CONTENTS

Page.
General statement ..... 7
Station adjustment ..... 7
Observed angles ..... 8
List of directions ..... 8
Condition equations ..... 9
Formation of normal equations by differentiation ..... 9
Correlate equations ..... 11
Formation of normal equations ..... 11
Normal equations ..... 12
Discussion of method of solution of normal equations ..... 12
Solution of normal equations. ..... 13
Back solution ..... 13
Computation of corrections ..... 13
Adjustment of a quadrilateral ..... 14
General statement ..... 14
Lists of directions ..... 16
Figure ..... 16
Angle equations ..... 17
Side equation ..... 17
Formation of normal equations by differentiation ..... 17
Correlate equations ..... 18
Normal equations ..... 18
Solution of normal equations ..... 19
Back solution ..... 19
Computation of corrections ..... 19
Adjustment of a quadrilateral by the use of two angle and two side equations ..... 20
Angle equations ..... 20
Side equations ..... 20
Correlate equations ..... 20
Normal equations. ..... 20
Solution of normal equations ..... 21
Back solution ..... 21
Computation of corrections ..... 21
Solution of a set of normals including terms usually omitted ..... 22
Discussion of the solution ..... 22
Solution of triangles ..... 23
Position computations, secondary triangulation ..... 24
List of geographic positions ..... 26
Development of condition equations for latitude and longitude closures ..... 26
Equations in a net ..... 32
Adjustment of a figure with latitude, longitude, azimuth, and length closure conditions ..... 34
Figure ..... 34
Angle equations ..... 35
Azimuth equation ..... 35
Side equations ..... 35
Length equation ..... 37
Adjustment of a figure with latitude, longitude, azimuth, and length closure conditions-Continued. Page.
Figure for latitude and longitude equations. ..... 37
Formation of azimuth equation ..... 38
Preliminary computation of triangles ..... 38
Preliminary computation of positions, primary form ..... 40
Formation of latitude and longitude condition equations ..... 50
Latitude equation ..... 51
Longitude equation ..... 51
Correlate equations ..... 52
List of corrections ..... 55
Normal equations ..... 56
Solution of normals ..... 58
Back solution ..... 68
Computation of corrections ..... 69
Final solution of triangles. ..... 71
Final computation of positions ..... 76
List of geographic positions ..... 91
Adjustment of triangulation by the method of variation of geographic coordi- nates ..... 91
Development of formulas ..... 91
Adjustment of a quadrilateral with two points fixed ..... 94
Lists of observed directions ..... 94
Preliminary computation of triangles ..... 95
Preliminary computation of positions ..... 96
Formation of observation equations ..... 98
Table for formation of normals, No. 1 ..... 100
Table for formation of normals, No. 2 ..... 101
Normal equations ..... 101
Solution of normals ..... 101
Back solution ..... 102
Computation of corrections ..... 102
Adjusted computation of triangles ..... 103
Adjustment of three new points by variation of geographic coordinates ..... 103
General statement ..... 103
Figure ..... 104
First method ..... 105
List of directions. ..... 105
Lists of fixed positions ..... 106
Preliminary computation of triangles ..... 107
Preliminary computation of positions ..... 110
Formation of observation equations ..... 114
Table for formation of normals, No. 1 ..... 118
Table for formation of normals, No. 2 ..... 119
Normal equations ..... 119
Solution of normals ..... 120
Back solution ..... 121
Computation of corrections ..... 121
Final computation of triangles ..... 122
Second method ..... 125
Formation of observation equations ..... 126
Table for formation of normals, No. 1 ..... 127
Table for formation of normals, No. 2 ..... 127
Normal equations ..... 127
Solution of normals. ..... 128
Adjustment of three new points by variaion of geographic coordinates-Con. Second method-Continued. ..... Page.
Back solution ..... 129
Computation of corrections ..... 129
Final computation of triangles ..... 130
Final computation of positions ..... 134
Computation of probable errors ..... 138
Adjustment of a figure with latitude and longitude, azimuth, and length condi- tions by variation of geographic coordinates ..... 139
Table of fixed positions ..... 139
Preliminary computation of triangles ..... 140
Preliminary computation of positions ..... 144
Figure ..... 157
Formation of observation equations ..... 158
Table for formation of normals, No. 1 ..... 162
Table for formation of normals, No. 2 ..... 106
Normal equations. ..... 168
Solution of normals ..... 109
Back solution ..... 174
Computation of corrections ..... 175
Final computation of triancles. ..... 178
Final computation of positions ..... 182
Adjustments by the angle method ..... 196
Adjustment of verticals. ..... 197
General statement ..... 197
Figure ..... 197
Computation of elevations from reciprocal observations ..... 198
Computation of elevations from nonreciprocal observations. ..... 199
Fixed elevations. ..... 200
Assumed and adjusted elevations ..... 200
Formation of observation equations ..... 200
Table of formation of equations ..... 201
Computation of probable error ..... 202
Formation of normal equations by differentiatio:1. ..... 202
Table for formation of normal equations. ..... 204
Normal equations ..... 204
Solution of normal equations ..... 204
Back solution ..... 205
Development of formulas for trigonometric leveling. ..... 205
General statement ..... 205
Development of formulas ..... 207
Examples of computation by formulas. ..... 214
Recapitulation of formulas ..... 216
Notes on construction and use of tables. ..... 217
Tables. ..... 218
Notes on the developments ..... 219

# APPLICATION OF THE THEORY OF LEAST SQUARES TO THE ADJUSTMENT OF TRIANGULATION 

By Oscar S. Adams<br>Computer United States Coast and Geodetic Survey

## GENERAL STATEMENT

In this publication the aim has not been to develop the theory of least squares, but to illustrate the application of the method to the problems arising in the adjustment of triangulation. The general idea has been to collect material in one volume that will serve as a working manual for the computer in the office and for such other members of the Survey as may desire to make these special applications. It has not been deemed necessary to insert the derivation of formulæ except in the case of a few special ones that are not usually found in the textbooks on least squares.

For the general theory reference should be made to such books as the following:

> Crandall: Geodesy and Least Squares.
> Helmert: Die Ausgleichungsrechnung nach der Metode der kleinsten Quadrate. Jordan: Handbuch der Vermessungskunde, volume 1.
> Merriman: Textbook of Least Squares.
> Wright and Hayford: Adjustment of Observations.

Some of the simpler cases are treated first, such as the local adjustment at a station, the adjustment of a simple quadrilateral, etc. After these is given the development of the condition equations for latitude and longitude closures, followed by a sample adjustment including the condition equations for these closures, together with the equations for length and azimuth conditions.

A method of adjustment by the variation of geographic coordinates is then developed and applied first to a quadrilateral, then to a figure with a few new points connected with a number of fixed points. The same method is applied to the adjustment of a figure with latitude, longitude, length, and azimuth conditions. A sample adjustment of a vertical net is carried through and lastly there is given the development of the formulæ for the computation of vertical observations, together with examples of the method of computation.

## STATION ADJUSTMENT

The general rule followed by the observers of the Coast and Geodetic Survey is to measure the angles at each station in the order of azimuth, thus giving rise to no conditions except the horizon closure. Occasionally, however, sum angles are observed and, when this is done, other conditions are introduced in addition to the horizon closure making it necessary to adjust the angles at the station by the method of least squares. If all angles were observed in the same way, the weight of each would be unity and the adjustment would be made without regard to weights. In the adjustment given below the angles were measured by the usual Coast and Geodetic Survey repetition method; that is, six measures of the angles with the telescope direct and six with it reversed for each set. A station has been chosen at which there are angles measured with one, two, and three sets in order to illustrate the method of weighting.

Observed angles, Gray Cliff

| Observed stations | Angle |  |  | $\underset{p}{\text { Weight }}$ | $v$ | Adjusted seconds* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Boulder-Tower | $\stackrel{\circ}{65}$ | $06$ | $\left.\begin{array}{c} \prime \prime \prime \prime \\ 26.6 \\ 30.9 \\ 30.4 \end{array}\right\}^{\prime \prime}$ | 3 | $v_{1}$ | 29.9 |
| Tower-Tyonek. |  |  | $\left.\begin{array}{l} 27.7 \\ 25.0 \\ 28.0 \end{array}\right]^{26.9}$ | 3 | $v_{2}$ | 27.5 |
| Tyonek-Round Point. | 8 |  | $\left.\begin{array}{l} 14.6 \\ 18.4 \\ 14.8 \end{array}\right\} 15.9$ | 3 | $v_{3}$ | 15.9 |
| Round Point-Boulder. |  | 27 | 47.9 | 1 | $v_{4}$ | 46.7 |
| Round Point-Birch Hill. | 66 |  | $\left.\begin{array}{l} 20.6 \\ 23.0 \end{array}\right\}^{21.8}$ | 2 | $v_{5}$ | 22.4 |
| Birch Hill-Boulder. |  |  | $\left.{ }_{22.3}^{22.2}\right)^{22.2}$ | 2 | $v_{6}$ | 24.3 |
| Boulder-Tyonek. |  | 52 | 56.2 | 1 | $v_{7}$ | 57.4 |
| Tyonek-Birch Hill. |  | 02 | 35.0 | 1 | $v_{8}$ | 38.3 |
| Round Point-Moose Point. |  |  | $\left.\begin{array}{l} 12.9 \\ 09.8 \end{array}\right\}^{11.4}$ | 2 | $v_{9}$ | 11.3 |
| Moose Point-Birch Hill. |  | 51 | 11.2 | 1 | $v_{10}$ | 11.1 |

List of directions, Gray Cliff

| Observed station | Direction |  | Adjusted final seconds* |
| :---: | :---: | :---: | :---: |
|  | - , | " | " |
| Boulder. | $0 \quad 00$ | 00.0 | 00.0 |
| Tower.: | 6506 | $29.3+v_{1}$ | 29.9 |
| Tyonek ..... | 84 | 56. $2+v_{1}+v_{2}$ | 57.4 |
| Round Point. | ${ }^{93} 32$ | $12.1+v_{1}+v_{2}+v_{3}$ | 13.3 |
| Moose Point | 15804 | $2_{3.5}^{23.5}+v_{1}+v_{2}+v_{3}+v_{9}$ | 24.6 |
| Birch Hill.. | 15955 | $34.7+v_{1}+v_{2}+v_{3}+r_{9}+v_{10}$ | 35.7 |

[^0]There have been formed a complete list of directions without using five of the angles, each of which, then, gives rise to a condition, there being five conditions in all. The equations expressing these conditions are formed in the following manner:

\[

\]

In the same way the other condition equations are formed. $\Lambda$ s a result there are finally:

## Condition equations

1. $0=+0.0+v_{1}+v_{2}+v_{3}+v_{4}$
2. $0=+0.8-v_{5}+v_{9}+v_{10}$
3. $0=-3.1+v_{1}+v_{2}+v_{3}+v_{6}+v_{9}+v_{10}$
4. $0=+0.0+v_{1}+v_{2}-v_{7}$
5. $0=+3.5+v_{3}-v_{8}+v_{9}+v_{10}$

## FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

According to the theory of least squares, the most probable values will be determined by making the $\Sigma p_{\mathrm{n}} v_{\mathrm{n}}{ }^{2}$ a minimum, subject to the given conditions. By the method of Lagrangian multipliers the formation of the normal equations can be much simplified.

With the use of these the function $u$ that is to be made a minimum is

$$
\begin{aligned}
u=3 & v_{1}{ }^{2}+3 v_{2}{ }^{2}+3 v_{3}{ }^{2}+1 v_{4}{ }^{2}+2 v_{5}{ }^{2}+2 v_{6}{ }^{2}+1 v_{7}{ }^{2}+1 v_{8}{ }^{2}+2 v_{9}{ }^{2}+1 v_{10}{ }^{2}-2 C_{1}\left(+v_{1}+v_{2}+v_{3}\right. \\
& \left.+v_{4}+0.0\right)-2 C_{2}\left(-v_{5}+v_{9}+v_{10}+0.8\right)-2 C_{3}\left(+v_{1}+v_{2}+v_{3}+v_{6}+v_{9}+v_{10}-3.1\right)-2 C_{4} \\
& \left(+v_{1}+v_{2}-v_{7}+0.0\right)-2 C_{5}\left(+v_{3}-v_{8}+v_{9}+v_{10}+3.5\right) .
\end{aligned}
$$

The $C$ 's are merely undetermined multipliers, the values of which will be determined by the solution. The factor 2 is included to obviate later on the use of the fraction $\frac{1}{2}$; the minus sign is used for convenience. The function will be rendered a minimum if the partial differential coefficients with respect to $v_{1}, v_{2}$, etc., are equated to zero. By this means ten equations will be formed, giving the ten $v$ 's expressed in terms of the $C$ 's.

Differentiating with respect to $v_{1}, v_{2}$, etc., in succession and equating the results to zero, the following equations are obtained:

$$
\begin{aligned}
& 3 v_{1}-C_{1}-C_{3}-C_{4}=0 \\
& 3 v_{2}-C_{1}-C_{3}-C_{4}=0 \\
& 3 v_{3}-C_{1}-C_{3}-C_{5}=0 \\
& v_{4}-C_{1}=0 \\
& 2 v_{5}+C_{2}=0 \\
& 2 v_{6}-C_{3}=0 \\
& v_{7}+C_{4}=0 \\
& v_{8}+C_{5}=0 \\
& 2 v_{9}-C_{2}-C_{3}-C_{5}=0 \\
& v_{10}-C_{2}-C_{3}-C_{5}=0
\end{aligned}
$$

## Therefore

$$
\begin{aligned}
& v_{1}=+\frac{1}{3} C_{1}+\frac{1}{3} C_{3}+\frac{1}{3} C_{4} \\
& v_{2}=+\frac{1}{3} C_{1}+\frac{1}{3} C_{3}+\frac{1}{3} C_{4} \\
& v_{3}=+\frac{1}{3} C_{1}+\frac{1}{3} C_{3}+\frac{1}{3} C_{5} \\
& v_{4}=+C_{1} \\
& v_{5}=-\frac{1}{2} C_{2} \\
& v_{6}=+\frac{1}{2} C_{3} \\
& v_{7}=-C_{4} \\
& v_{8}=-C_{5} \\
& v_{9}=+\frac{1}{2} C_{2}+\frac{1}{2} C_{3}+\frac{1}{2} C_{5} \\
& v_{10}=+C_{2}+C_{3}+C_{5}
\end{aligned}
$$

Thus all of the $v$ 's are now expressed in terms of the $C$ 's. These can now be substituted in the condition equations forming. five normal equations containing five $C$ 's and these equations may then be solved for the $C$ 's. If the normals are formed from these values, fractions will occur in practically all of the coefficients. This can be avoided by replacing $C_{1}$ by $6 C_{1}^{\prime}, C_{2}$ by $6 C_{2}^{\prime}$, etc. This is equivalent to using $12 C_{1}^{\prime}, 12 C_{2}^{\prime}$, etc., in the original function instead of $2 C_{1}, 2 C_{2}$, etc., which, of course, is perfectly valid.

The equations will then stand as follows:

$$
\begin{aligned}
& v_{1}=+2 C_{1}^{\prime}+2 C_{3}^{\prime}+2 C_{4}^{\prime} \\
& v_{2}=+2 C_{1}^{\prime}+2 C_{3}^{\prime}+2 C_{4}^{\prime} \\
& v_{3}=+2 C_{1}{ }^{\prime}+2 C_{3}{ }^{\prime}+2 C_{5}^{\prime} \\
& v_{4}=+6 C_{1}^{\prime} \\
& v_{5}=-3 C_{2}^{\prime} \\
& v_{6}=+3 C_{3}{ }^{\prime} \\
& v_{7}=-6 C_{4}^{\prime} \\
& v_{8}=-6 C_{5}^{\prime} \\
& v_{9}=+3 C_{2}{ }^{\prime}+3 C_{3}{ }^{\prime}+3 C_{5}^{\prime} \\
& v_{10}=+6 C_{2}^{\prime}+6 C_{3}^{\prime}+6 C_{5}^{\prime}
\end{aligned}
$$

Dropping the prime and substituting these values in the first condition equation the following normal equation is obtained:

$$
\begin{aligned}
& 2 C_{1}+2 C_{3}+2 C_{4}+2 C_{1}+2 C_{3}+2 C_{4}+2 C_{1}+2 C_{3}+2 C_{5}+6 C_{1}+0.0=0 \\
& \quad+12 C_{1}+6 C_{3}+4 C_{4}+2 C_{5}+0.0=0
\end{aligned}
$$

In a similar manner the other normal equations are formed, giving in all the following five equations:

$$
\begin{array}{r}
+12 C_{1}+6 C_{3}+4 C_{4}+2 C_{5}+0.0=0 \\
+6 C_{2}+9 C_{3}+9 C_{5}+0.8=0 \\
+6 C_{2}+18 C_{3}+4 C_{4}+11 C_{5}-3.1=0 \\
+4 C_{1}+4 C_{3}+10 C_{4} \\
+2 C_{1}+9 C_{2}+11 C_{3}+17 C_{5}+3.5=0
\end{array}
$$

This manner of forming the normal equations is called the method of correlates and is most conveniently carried out by means of a table of correlates formed as on page 11.

After the determination of the $C$ 's by the solution of the normal equations, the $v$ 's may be computed from the equations of the $v$ 's
in terms of the $C^{\prime}$ 's. In the tabulated form below the first column is multiplied by $C_{1}$, the second by $C_{2}$, etc. The sum of the first line multiplied by the $\frac{6}{p}$ for that line gives $v_{1}$; so also for the other $v$ 's.

Correlate equations

|  | $\frac{6}{p}$ | 1 | 2 | 3 | 4 | 5 | $\Sigma$ | $v^{\prime}$ '* | $\begin{gathered} \text { Adopted } \\ v^{\prime} s \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | +1 |  | +1 | +1 |  | +3 | +0.618 | +0.6 |
| 2 | 2 | +1 |  | +1 | +1 |  | +3 | +0.618 | +0.6 |
| 3 | ${ }_{6}^{2}$ | +1 +1 |  | +1 |  | +1 | +3 +1 | -0.050 -1.152 | -0.0 |
| 5 | 3 |  | -1 |  |  |  | $\pm 1$ | -0.585 | +0.6 |
|  | 3 |  |  | +1 |  |  | +1 | +2.133 | +2.1 |
| 7 | 6 |  |  |  | -1 |  | -1 | +1.230 | +1.2 |
| 8 | 6 |  |  |  |  | -1 | -1 | +3.234 | +3.3 |
| 9 | 3 |  | +1 | +1 |  | +1 | +3 | -0.069 | -0.1 |
| 10 | 6 |  | +1 | +1 |  | +1 | +3 | -0.138 | -0.1 |

* These values result from the computation on p. 13.


## FORMATION OF THE NORMAL EQUATIONS

After the condition equations are tabulated in correlates as above, the next step is the formation of the normal equations. In forming these the various products must be multiplied by $\frac{1}{p}$ or by $\frac{a}{p}$ in which $p$ is the weight of the given $v$ and $a$ is some constant. (See the direct formation on p. 9.) It is most convenient to choose $a$ so as to make most of the values integers, if this can be done without making the quantities too large. In this case 6 is the L. C. M. of the $p$ 's, hence it is chosen for $a$. The normal equations are formed by taking the algebraic sums of $\frac{6}{p}$ times the products of the various columns. Normal No. 1 is, in symbols-

$$
\Sigma \frac{6}{p} \cdot 1 \cdot 1+\Sigma \frac{6}{p} \cdot 1 \cdot 2+\Sigma \frac{6}{p} \cdot 1 \cdot 3+\Sigma \frac{6}{p} \cdot 1 \cdot 4+\Sigma \frac{6}{p} \cdot 1 \cdot 5+\eta+\left(\Sigma \frac{6}{p} \cdot 1 \cdot \Sigma+\eta^{\dagger}\right)
$$

The algebraic sum of the sigma products in the formation checks or controls the formation of the normals. Each $\Sigma$ line in the correlates is the algebraic sum of that line in the table. As is easily seen, the sum of the products of this column in the formation of the normals should check the algebraic sum of the coefficients of the normal. On the first normal $+12+6+4+2=+24$, which is the same as the algebraic sum of the products in the correlates. The $\Sigma$ column in the normals also includes the constant term. In the third normal $+6+9$ $+18+4+11=+48$. In the $\Sigma$ columns of the normal $+48-3.1$ $=+44.9$.

Normal equations

|  | 1 | 2 | 3 | 4 | 5 | $\eta$ | $\Sigma$ | $C^{\prime} \mathrm{s}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | +12 | $+12$ | $+6$ | $+4$ | $+2$ | $+0.0$ | $+24$ | -0.19700 |
| 2 |  |  | +9 |  | +9 | +0.8 | +30.8 | -0.19531 |
| 3 |  |  | +18 | $+4$ | +11 | -3.1 | +44.9 | +0.71069 |
| 4 |  |  |  | $+10$ |  | $+0.0$ | +18 | -0.20547 |
| 5 |  |  |  |  | $+17$ | $+3.5$ | +42.5 | -0.53917 |
|  |  |  |  |  |  |  |  |  |

* These values result from the computation on p. 13.

DISCUSSION OF METHOD OF SOLUTION OF NORMAL EQUATIONS
In the normal equations the coefficients in each equation occurring before the diagonal term are omitted, as the equations are symmetrical with regard to the diagonal line. The set just given when written in full is as follows:

|  | 1 | 2 | 3 | 4 | 5 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | +12 |  | +6 | +4 | +2 | +0.0 | +24 |
| 2 |  | +12 | +9 |  | +9 | +0.8 | +30.8 |
| 3 | +6 | +9 | +18 | +4 | +11 | -3.1 | +44.9 |
| 4 | +4 |  | +4 | +10 |  | +0.0 | +18 |
| 5 | +2 | +9 | +11 |  | +17 | +3.5 | +42.5 |

It can be seen that the coefficients may be omitted to the left of the diagonal line and each equation may be read from the top down to the diagonal term and then across the page.

The Doolittle method of solution is used. Equation No. 1 is copied and then divided by the diagonal term ( +12 in this case), the signs being changed. Since No. 2 does not occur on No. 1, this also is divided at once by the diagonal term with a change of sign. No. 3 has +6 on No. 1 and +9 on No. 2 ; accordingly, the divided coefficients of No. 1 are multiplied by +6 and those of No. 2 by +9 and these give the two products on No. 3. These are then added algebraically and divided by the diagonal term with change of sign to give $C_{3}$ in terms of No. 4 and No. 5 plus a constant term. In a similar manner No. 4 and No. 5 are eliminated, the division on No. 5 giving the value of $C_{5}$. The back solution is then carried through $C_{4}=+0.17778 C_{5}-0.10962$. When the value of $C_{5}$ is substituted, $C_{4}=-0.09585-0.10962=-0.20547$. So also for the remaining $C$ 's. For an explanation of the omission of the terms before the diagonal term, see page 22. For a full discussion of the Doolittle method of solution, see Wright \& Hayford, Adjustment of Observations, page 114 et seq.

Solution of normal equations

| 1 | 2 | 3 | 4 | 5 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{C_{1}}^{+12}$ |  | $\begin{aligned} & +6 \\ & -0.5 \end{aligned}$ | +4 $\pm 0.33333$ | +2 -0.16667 | $\begin{aligned} & +0.0 \\ & +0.0 \end{aligned}$ | $\begin{aligned} & +24 \\ & -2 \end{aligned}$ |
|  | $+{ }_{C_{2}}^{+12}$ | $\begin{aligned} & +9 \\ & -0.75 \end{aligned}$ |  | $\begin{aligned} & +9 . \\ & -0.75 \end{aligned}$ | $\begin{aligned} & +0.8 \\ & { }_{-0.06667} \end{aligned}$ | $\begin{aligned} & +30.8 \\ & -\quad 2.56667 \end{aligned}$ |
|  | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & +18 \\ & +3 \\ & -6.75 \\ & +8.25 \\ & C_{3} \end{aligned}$ | $\begin{aligned} & +4 \\ & -2 \\ & +2 \\ & +0.24242 \end{aligned}$ | $\begin{aligned} & +11 \\ & \pm 1 \\ & -6.75 \\ & +3.25 \\ & -0.39394 \end{aligned}$ | $\begin{aligned} & -3.1 \\ & -0.6 \\ & -3.7 \\ & +0.44848 \end{aligned}$ | $\begin{aligned} & +44.9 \\ & -12 \\ & -23.1 \\ & +9.8 \\ & -1.18788 \end{aligned}$ |
|  |  |  | $\begin{aligned} & +10 \\ & +1.3333 \\ & -0.4848 \\ & +8.1819 \\ & C_{4} \end{aligned}$ | $\begin{aligned} & -0.6667 \\ & -0.7879 \\ & =1.4546 \\ & +0.17778 \end{aligned}$ | $\begin{aligned} & +0.0 \\ & +0.8969 \\ & +0.8969 \\ & -0.10962 \end{aligned}$ | $\begin{aligned} & +18 \\ & =8 \\ & -2.3758 \\ & +7.6242 \\ & -0.93184 \end{aligned}$ |
|  |  |  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{aligned} & +17 \\ & =0.3333 \\ & =6.75 \\ & =1.2803 \\ & -0.2586 \end{aligned}$ | $\begin{aligned} & +3.5 \\ & -0.6 \\ & +1.4576 \\ & +0.1595 \end{aligned}$ | $\begin{aligned} & +42.5 \\ & =4 \\ & -23.1 \\ & -3.8606 \\ & +1.3554 \end{aligned}$ |
|  |  |  |  | $+\underset{C_{5}}{8.3778}$ | $\begin{aligned} & +4.5171 \\ & -0.53917 \end{aligned}$ | $\begin{aligned} & +12.8949 \\ & -1.53917 \end{aligned}$ |

Back solution

| 5 | 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: |
| -0.53917 | $-0.10962$ | +0.44848 | $-0.06667$ |  |
| $-0.53917$ | -0.09585 | $+0.21240$ | $+0.40438$ | +0.08986 +0.06849 |
|  | -0.20547 |  |  | $-0.35535$ |
|  |  | . | 0.10531 | -0.19700 |

Computation of corrections

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.197 \\ & +0.711 \\ & +0.205 \end{aligned}$ | $\begin{aligned} & -0.197 \\ & +0.711 \\ & +0.205 \end{aligned}$ | $\begin{aligned} & -0.197 \\ & +0.711 \\ & +0.539 \end{aligned}$ | -0. 197 | +0.195 |
|  |  |  | $\begin{aligned} & -0.197 \\ & -1.182 \end{aligned}$ | $\begin{array}{r} +0.195 \\ 3 \\ +0.58 \end{array}$ |
| $\begin{array}{r} +0.309 \\ +0.618 \end{array}$ | $\begin{array}{r} +0.309 \\ +0.618 \end{array}$ | $\begin{array}{r} -0.025 \\ -0.050 \end{array}$ |  |  |
| 6 | 7 | 8 | 9 | 10 |
| +0.711 | +0. 205 | +0. 539 | $\begin{array}{r} -0.195 \\ +0.711 \\ -0.539 \end{array}$ | $\begin{aligned} & -0.195 \\ & +0.711 \\ & -0.539 \end{aligned}$ |
| +0.711 | +0.205 | $\begin{aligned} & +0.539 \\ & +3.234 \end{aligned}$ |  |  |
| +2.133 | +1.230 |  | $\begin{array}{r} -0.023 \\ 3 \\ -0.069 \end{array}$ | $\begin{array}{r} -0.023 \\ -0.138 \end{array}$ |

## ADJUSTMENT OF A QUADRILATERAL

## GENERAL STATEMENT

After the local conditions-that is, those arising from the relations of the angles to one another at each station-are satisfied there are general conditions arising from the geometrical relations necessary to form a closed figure which must be satisfied. To illustrate this, let the case of a quadrilateral be taken. The angles of each triangle should sum up to $180^{\circ}$ plus the spherical excess of the triangle. Except in rare cases this does not happen with the observed angles; therefore condition equations are needed to bring it about. There are four triangles in a quadrilateral, but if three of them close the other will also close. There will then be three angle equations in the quadrilateral. A fourth equation must be included to insure that the lines at the pole will pass through the same point. When this condition is satisfied, and the triangles are closed, the same values will be obtained for the various sides when the computation is carried through different triangles.

In the adjustment of triangulation in the United States Coast and Geodetic Survey the method of directions is used; that is, an angle is considered as the difference of two directions.* If $v_{1}$ is the correction to the first direction in order of azimuth at a given station and $v_{2}$ the correction to the second direction, the correction to the angle will be $-v_{1}+v_{2}$, or the algebraic difference of the $v$ 's applying to the directions. To avoid the use of so many $v$ 's, the custom is to write (1) instead of $v_{1}$; thus the angle given above will have the correction symbol $-(1)+(2)$, in which 1 and 2 are not quantities but the subscripts of the corresponding $v$ 's.

An angle equation simply states that the sum of the corrections to the angles of a given triangle is to equal the failure in the closure of the triangle. In the triangle $A_{3} A_{2} A_{1}$ (see fig. 1 on p. 16) the angle at $A_{2}$ is to be corrected by $-(1)+(2)$, the angle at $A_{1}$ by $-(4)+(6)$, and the angle at $A_{3}$ by $-(8)+(9)$. The sum of the angles needs to be increased by $2^{\prime \prime} .3$ to make up the sum of $180^{\circ}$ plus the spherical excess. (See triangle on p. 23.) Therefore $-(1)+(2)-(4)+(6)$ $-(8)+(9)=+2.3$, or, as it is usually written, $0=-2.3-(1)+(2)$ $-(4)+(6)-(8)+(9)$.
Three angle equations in a quadrilateral will bring about the closure of the four triangles, but it is possible to have all of the triangles close and still the sides fail to check when computed through different triangles. To make the computation of lengths consistent a side equation must be added to the three angle equations. In figure 1 on page 16 the sides can be made consistent in the following manner:

In the triangle $A_{1} A_{4} A_{2}$

$$
\frac{\text { side } A_{2} A_{4}}{\text { side } A_{1} A_{4}}=\frac{\text { sine angle } A_{1}}{\text { sine angle } A_{2}}=\frac{\text { sine }[-(5)+(6)]}{\operatorname{sine}[-(1)+(3)]}
$$

in the triangle $A_{1} A_{3} A_{4}$
$\frac{\text { side } A_{1} A_{4}}{\text { side } A_{3} A_{4}}=\frac{\text { sine angle } A_{3}}{\text { sine angle } A_{1}}=\frac{\text { sine }[-(7)+(9)]}{\operatorname{sine}[-(4)+(5)]}$
and in the triangle $A_{2} A_{3} A_{4}$

$$
\frac{\text { side } A_{3} A_{4}}{\text { side } A_{2} A_{4}}=\frac{\text { sine angle } A_{2}}{\text { sine angle } A_{3}}=\frac{\text { sine }[-(2)+(3)]}{\operatorname{sine}[-(7)+(8)]}
$$

If the sides are consistent, the product of these three equations gives

$$
\frac{\operatorname{sine}[-(5)+(6)] \text { sine }[-(7)+(9)] \text { sine }[-(2)+(3)]}{\operatorname{sine}[-(1)+(3)] \text { sine }[-(4)+(5)] \text { sine }[-(7)+(8)]}=1
$$

In a spherical triangle the same equation is obtained by using the sine of the side in place of the side. In the end the equation given above results, since the sines of the sides cancel out as did the sides above.

Passing to logarithms, we have
$\log$ sine $[-(5)+(6)]+\log$ sine $[-(7)+(9)]+\log$ sine $[-(2)+(3)]-\log$ sine $[-(1)+(3)]$ $-\log$ sine $[-(4)+(5)]-\log$ sine $[-(7)+(8)]=0$
As this will not be exactly true when the observed angles or angles adjusted only for closing errors of the triangles are used except in rare cases, a condition equation must be formed to accomplish this result. From the table of logarithms we find the amount of change of the log sine of the given angle for $1^{\prime \prime}$ change in the angle, and this multiplied by the $v$ 's applying to the angle will give the change in the $\log$ sine of that angle. It is customary to consider the log sines in six places of decimals, hence the change in the $\log$ sine for $1^{\prime \prime}$ will be taken as units in the sixth place of decimals.

(See tabulated form of this equation on p. 17.)
This condition requires the lines from $A_{2}, A_{1}$, and $A_{3}$ to pass through the same point at $A_{4}$. If $v$ 's are found that satisfy this equation, at the same time satisfying the three angle equations given on page 17, they will render the quadrilateral consistent in all respects.

In a full quadrilateral (see figure 1) there are four conditions. These can be put in as three angle equations and one side equation, or two angle and two side, or one angle and three side equations. (See article by C. A. Schott, Appendix No. 17, United States Coast and Geodetic Survey Report of 1875, p. 280.) To illustrate the fact, a quadrilateral is adjusted using two angle equations and two side equations. (See p. 20.) In order to hold the closure of the triangles, the logarithms in the side equations must be found at least to seven places to hold the closure to tenths of seconds. Of course this method would never be used in practice, as the side equations require much more work, but the fact is interesting as an illustration of what can be done in the method of adjustment. Four side equations or four angle equations could not be used, for the fourth is functionally related to the other three, and hence they would not be independent conditions.

In a set of equations, if an identical one is included, the diagonal


Fig. 1. term of the reduced normal will become zero with the possible exceptimon, of course, of a few units in the last place of solution due to accumulations. In any case, if the reduced diagonal term falls below unity, there may be danger of instability, since in this case any accumulations in the last place of the solution are increased when the normal is divided by this term.

Lists of directions

*These values result from the following computation.

Angle equations*

$$
\begin{aligned}
& 0=-2.3-(1)+(2)-(4)+(6)-(8)+(9) . \\
& 0=+3.6-(2)+(3)-(7)+(8)-(10)+(12) . \\
& 0=+2.2-(4)+(5)-(7)+(9)-(11)+(12) .
\end{aligned}
$$

Side equation

| Symbol | Angle | Logarithm | Tabular difference | Symbol | Angle | Logarithm | Tabular difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -7+9 \\ & -5+6 \\ & -2+3 \end{aligned}$ | $\circ$ $\prime$ $\prime \prime$ <br> 61 47 35.0 <br> 20 50 56.7 <br> 32 09 01.2 |  | $\begin{aligned} & +1.13 \\ & +5.53 \\ & +3.35 \end{aligned}$ | $\begin{aligned} & -4+5 \\ & -1+3 \\ & -7+8 \end{aligned}$ | $\begin{array}{rcc} \circ & \prime & \prime \prime \\ 26 & 40 & 23.5 \\ 133 & 53 & 46.3 \\ 31 & 03 & 42.5 \end{array}$ |  | $\begin{array}{r} +4.19 \\ -2.03 \\ +3.50 \end{array}$ |
|  |  | 9. 9450972 |  |  |  | 9. 6521506 |  |
|  |  | 9. 5513374 |  |  |  | 9. 8576926 |  |
|  |  | 9.7260280 |  |  |  | 9.7126180 |  |
|  |  | 9. 2224626 |  |  |  | 9. 2224612 |  |
|  |  |  |  |  |  |  |  |

$0=+1.4-2.03(1)-3.35(2)+5.38(3)+4.19(4)-9.72(5)+5.53(6)+2.37(7)-3.50(8)+$
$1.13(9)$.

## FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The function $u$ to be rendered a minimum is the sum of the squares of the $v$ 's, subject to the four given conditions.

$$
\begin{aligned}
u= & v_{1}{ }^{2}+v_{2}{ }^{2}+v_{3}{ }^{2}+v_{4}{ }^{2}+v_{5}{ }^{2}+v_{6}{ }^{2}+v_{7}{ }^{2}+v_{8}{ }^{2}+v_{9}{ }^{2}+v_{10}{ }^{2}+v_{11}{ }^{2}+v_{12}{ }^{2}-2 C_{1}\left(-2.3-v_{1}+v_{2}-v_{4}+\right. \\
& \left.v_{6}-v_{8}+v_{9}\right)-2 C_{2}\left(+3.6-v_{2}+v_{3}-v_{7}+v_{8}-v_{10}+v_{12}\right)-2 C_{3}\left(+2.2-v_{4}+v_{5}-v_{7}+v_{9}-v_{11}\right. \\
& \left.\quad+v_{12}\right)-2 C_{4}\left(+1.4-2.03 v_{1}-3.35 v_{2}+5.38 v_{3}+4.19 v_{4}-9.72 v_{5}+5.53 v_{6}+2.37 v_{7}-3.50\right. \\
& \left.v_{8}+1.13 v_{9}\right)
\end{aligned}
$$

Differentiating with respect to the $v$ 's in succession and equating to zero, there result after transposition the following equations:

$$
\begin{aligned}
& v_{1}=-C_{1}-2.03 C_{4} \\
& v_{2}=+C_{1}-C_{2}-3.35 C_{4} \\
& v_{3}=+C_{2}+5.38 C_{4} \\
& v_{4}=-C_{1}-C_{3}+4.19 C_{4} \\
& v_{5}=+C_{3}-9.72 C_{4} \\
& v_{6}=+C_{1}+5.53 C_{4} \\
& v_{7}=-C_{2}-C_{3}+2.37 C_{4} \\
& v_{8}=-C_{1}+C_{2}-3.50 C_{4} \\
& v_{0}=+C_{1}+C_{3}+1.13 C_{4} \\
& v_{10}=-C_{2} \\
& v_{11}=-C_{3} \\
& v_{12}=+C_{2}+C_{3}
\end{aligned}
$$

By the substitution of these values in the four condition equations the following normal equations result:

$$
\begin{aligned}
& +6 C_{1}-2 C_{2}+2 C_{3}+4.65 C_{4}-2.3=0 \\
& -2 C_{1}+6 C_{2}+2 C_{3}+2.86 C_{4}+3.6=0 \\
& +2 C_{1}+2 C_{2}+6 C_{3}-15.15 C_{4}+2.2=0 \\
& +4.65 C_{1}+2.86 C_{2}-15.15 C_{3}+206.0470 C_{4}+1.4=0
\end{aligned}
$$

These normal equations are formed most easily by means of the tabular form of the correlate equations given on page 18.

[^1]The sum of the squares of each column gives the diagonal term in that equation in the normals. All coefficients before the diagonal term are omitted; each equation is read by starting at the top of the tabular form below, reading down the column to the diagonal term, and then along the horizontal line. Compare the full normals given above with the tabular form below. After the diagonal terms are determined column No. 1 in the correlates is multiplied by column No. 2 and the algebraic sum of the products taken for the coefficient of normal No. 1 on No. 2; this is also the coefficient of No. 2 on No. 1. Column No. 1 times No. 3, with the algebraic sum of the products, gives the coefficient of No. 1 on No. 3 in the normals; also No. 3 on No. 1. Finally, the algebraic sum of the products of column No. 1 by column No. 4 gives the coefficient of normal No. 1 on No. 4. The algebraic sum of the products of column No. 1 by the $\Sigma$ column should check the algebraic sum of the coefficients of normal No. 1. To this should be added algebraically the constant term of normal No. 1 and the sum placed in the $\Sigma$ column of normal No. 1. (See the table of normals below.)

In the same way the sum of the products of column No. 2 times column No. 3 is determined for the second normal, and by continuing the process all of the normals are formed.

After the $C$ 's are determined by the solution of the normals the $v$ 's are most conveniently computed by multiplying column No. 1 in the correlates by $C_{1}$, column No. 2 by $C_{2}$, column No. 3 by $C_{3}$, and column No. 4 by $C_{4}$. Then the algebraic sum of line No. 1 gives $v_{1}$; of No. 2, $v_{2}$, etc. (See the computation of the $v$ 's on p. 19.)

Correlate equations

|  | 1 | 2 | 3 | 4 | $\Sigma$ | $v$ 's * | $\underset{v}{\text { Adspted }}$ | $v^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1 |  |  | -2.03 | $-3.03$ | -0. 503 | -0.5 | 0.25 |
| 2 | +1 | -1 |  | $-3.35$ | $-3.35$ | +1.004 | +1.0 | 1.00 |
| 3 |  | +1 |  | +5.38 | +6.38 | -0.501 | -0.5 | 0.25 |
| 4 | -1 |  | -1 | +4.19 | +2.19 | $-0.227$ | -0.2 | 0.04 |
| 5 |  |  | +1 | $-9.72$ | $-8.72$ | $-0.015$ | $-0.0$ | 0.00 |
| 6 | +1 |  |  | +5.53 | +6.53 | +0.242 | +0.3 | 0.09 |
| 7 |  | -1 | -1 | +2.37 | +0.37 | +0.663 | +0.7 | 0.49 |
| 8 | - 1 | +1 |  | $-3.50$ | $-3.50$ | $-0.493$ | $-0.5$ | 0.25 |
| 9 | +1 |  | +1 | +1.13 | +3.13 | $-0.170$ | -0.2 | 0.04 |
| 10 |  | -1 |  |  | -1 | +0.099 | +0.1 | 0.01 |
| 11 |  |  | $-1$ |  | -1 | +0.740 | +0.7 | 0.49 |
| 12 |  | +1 | +1 |  | +2 | $-0.840$ | $-0.8$ | 0.64 |
|  |  |  |  |  |  |  | $\Sigma v^{2}$ | 3.55 |

Normal equations

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma$ | C's * |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +6 | $\begin{aligned} & -2 \\ & +6 \end{aligned}$ | $\begin{aligned} & +2 \\ & +2 \\ & +6 \end{aligned}$ | $\begin{aligned} & +\quad 4.65 \\ & +\quad 2.86 \\ & -15.15 \\ & +206.0470 \end{aligned}$ | $\begin{aligned} & -2.3 \\ & +3.6 \\ & +2.2 \\ & +1.4 \end{aligned}$ | $\begin{aligned} & +\quad 8.35 \\ & +12.46 \\ & +\quad 2.95 \\ & +199.8070 \end{aligned}$ | $\begin{aligned} & +0.6547 \\ & -0.0994 \\ & -0.7401 \\ & -0.07461 \end{aligned}$ |

[^2]Probable error of an observed direction $= \pm 0.6745 \sqrt{\frac{3.55}{4}}= \pm 0.6$.
Solution of normal equations

| 1 | 2 | 3 | 4 | $\eta$ | s |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{C_{2}}^{+6}$ | $\begin{aligned} & -2 \\ & +0.33333 \end{aligned}$ | ${ }_{-0.33333}$ | $\begin{aligned} & +\quad 4.65 \\ & -\quad 0.775 \end{aligned}$ | $\begin{aligned} & -2.3 \\ & +0.38333 \end{aligned}$ | $\begin{aligned} & \mathbf{8 . 3 5} \\ & \mathbf{-} 1.39167 \end{aligned}$ |
|  | $\begin{aligned} & \pm 6 \\ & { }_{-0.6667} \\ & +5.3333 \\ & C_{2} \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.6667 \\ & +2.6667 \\ & +0.50001 \end{aligned}$ | $\begin{aligned} & +2.86 \\ & +1.55 \\ & +4.41 \\ & -0.82688 \end{aligned}$ | +3.6 ${ }_{-0.7667}$ +2.8333 ${ }_{-0.53125}$ | $\begin{aligned} & +12.46 \\ & +\quad 2.7833 \\ & +\quad 15.2433 \\ & +\quad 2.85814 \end{aligned}$ |
|  | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & +6 \\ & { }_{-0.6667} \\ & -1.3333 \\ & +4 C_{8} \end{aligned}$ | $\begin{aligned} & =15.15 \\ & =\quad 1.55 \\ & -\quad 2.205 \\ & =\quad 18.905 \\ & +\quad 4.72625 \end{aligned}$ | $\begin{aligned} & +2.2 \\ & +0.7667 \\ & { }_{-1.4167} \\ & +1.55 \\ & { }_{-0.3875} \end{aligned}$ | $\begin{aligned} & =\quad 2.95 \\ & =\quad 2.7833 \\ & = \\ & =1.6217 \\ & = \\ & \hline \end{aligned} \frac{3.335875}{}$ |
|  |  | $\begin{aligned} & \mathbf{1} \\ & \mathbf{2} \\ & \mathbf{3} \end{aligned}$ | $\begin{gathered} +206.0470 \\ =\quad 3.6038 \\ =\quad 3.6465 \\ -89.3498 \\ +109.4469 \\ C_{4} \end{gathered}$ | $\begin{aligned} & +1.4 \\ & +1.7825 \\ & +2.3423 \\ & +7.3257 \\ & +8.1654 \\ & { }_{-0.07461} \end{aligned}$ | $\begin{aligned} & \pm 199.8070 \\ & =\quad .4712 \\ & =12.6044 \\ & -63.1191 \\ & +117.6123 \\ & -\quad 1.07461 \end{aligned}$ |

Back solution

| 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: |
| $-0.07461$ | -0.3875 | $-0.5312$ | +0.3833 |
| -0.07461 | -0.3526 | $\begin{aligned} & +0.0617 \\ & +0.3701 \end{aligned}$ | $\begin{aligned} & +0.0578 \\ & +0.2467 \end{aligned}$ |
|  | -0.7401 | -0.0994 | -0.0331 |
|  |  |  | +0.6547 |

Computation of corrections

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.6547 \\ & +0.1515 \end{aligned}$ | $\begin{aligned} & +0.6547 \\ & +0.0994 \\ & +0.2499 \end{aligned}$ | $\begin{aligned} & -0.0994 \\ & -0.4014 \end{aligned}$ | $\begin{array}{r} -0.6547 \\ +0.7401 \\ -0.3126 \end{array}$ | $\begin{array}{r} -0.7401 \\ +0.7252 \end{array}$ | $\begin{aligned} & +0.6547 \\ & -0.4126 \end{aligned}$ |
| $\begin{aligned} & -0.5032 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & +1.0040 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -0.5008 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & -0.2272 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & -0.0149 \\ & -0.0 \end{aligned}$ | $\begin{aligned} & +0.2421 \\ & +0.3 \end{aligned}$ |
| 7 | 8 | 9 | 10 | 11 | 12 |
| +0.0994 +0.7401 | $\begin{aligned} & -0.6547 \\ & -0.0994 \\ & +0.2611 \end{aligned}$ | $\begin{array}{r} +0.6547 \\ -0.7401 \\ -0.0843 \end{array}$ | +0.0994 | +0.7401 | $\begin{aligned} & -0.0994 \\ & -0.7401 \end{aligned}$ |
| -0.1768 |  |  | $\begin{aligned} & +0.0994 \\ & +0.1 \end{aligned}$ | $\begin{aligned} & +0.7401 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & -0.8395 \\ & -0.8 \end{aligned}$ |
| $\begin{aligned} & +0.6627 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & -0.4930 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & -0.1697 \\ & -0.2 \end{aligned}$ |  |  |  |

ADJUSTMENT OF A QUADRILATERAL BY THE USE OF TWO ANGLE AND TWO SIDE EQUATIONS *
(See fig. 1 on p. 16.)
Angle equations

$$
\begin{aligned}
& 0=-2.3-(1)+(2)-(4)+(6)-(8)+(9) \\
& 0=+3.6-(2)+(3)-(7)+(8)-(10)+(12)
\end{aligned}
$$

Side equations

| Symbol | Angle |  |  | Logarithm | Tabular difference | Symbol | Angle |  |  | Logarithm | Tabular difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -7+9 \\ & -5+6 \\ & -2+3 \end{aligned}$ | $\begin{array}{lll}61 & 47 & 35.0 \\ 20 & 50 & 56.7 \\ 32 & 09 & 01.2\end{array}$ |  |  | $\begin{aligned} & \text { 9. } 9450972 \\ & 9.513374 \\ & 9.7260280 \end{aligned}$ | $\begin{aligned} & +1.13 \\ & +5.53 \\ & +3.35 \end{aligned}$ | $\begin{aligned} & =4+5 \\ & =1+3 \\ & -7+8 \end{aligned}$ |  | $\begin{array}{ll} 40 & 23.5 \\ 53 & 46.3 \\ 03 & 42.5 \end{array}$ |  | $\begin{aligned} & \text { 9. } 6521506 \\ & 9.8576926 \\ & 9.7126180 \end{aligned}$ | $\begin{array}{r} +4.19 \\ -2.03 \\ +3.50 \end{array}$ |
|  |  |  |  | 9. 2224626 |  |  |  |  |  | 9.2224612 |  |

$0=+1.4-2.03(1)-3.35(2)+5.38(3)+4.19(4)-9.72(5)+5.53(6)+2.37(7)-3.50(8)$

$$
+1.13(9)
$$

| $-2+3$ | 32 | 09 | 01.2 | 9.7260280 | +3.35 | $-7+8$ | 31 | 03 | 42.5 | 9.7126180 | +3.50 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $-11+12$ | 91 | 32 | 03.8 | 9.9998442 | -0.06 | $-4+5$ | 26 | 40 | 23.5 | 9.6521506 | +4.19 |
| $-8+9$ | 30 | 43 | 52.5 | 9.7084309 | +3.54 | $-1+2$ | 101 | 44 | 45.1 | 9.9908094 | -0.44 |
| $-5+6$ | 20 | 50 | 56.7 | 9.5513374 | +5.53 | $-10+11$ | 25 | 15 | 16.2 | 9.6300613 | +4.46 |
|  |  |  |  |  |  |  |  |  | 8.9856405 |  |  |
|  |  |  |  |  |  |  |  | 8856393 |  |  |  |

$0=+1.2-0.44(1)-2.91(2)+3.35(3)+4.19(4)-9.72(5)+5.53(6)+3.50(7)-7.04(8)$ $+3.54(9)+4.46(10)-4.40(11)-0.06(12)$

Correlate equations

|  | 1 | 2 | 3 | 4 | $\Sigma$ | $v$ 's $\dagger$ | $\underset{v ' s}{\text { Adopted }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1 |  | $-2.03$ | -0.44 | $-3.47$ | -0.495 | -0.5 |
| 2 | +1 | -1 | $-3.35$ | -2.91 | $-6.26$ | +0.996 | +1.0 |
| 3 |  | +1 | +5.38 | +3.35 | +9.73 | -0.502 | -0.5 |
| 4 | -1 |  | +4.19 | +4.19 | + 7.38 | $-0.227$ | $-0.2$ |
| 5 |  |  | $-9.72$ | $-9.72$ | $-19.44$ | $-0.013$ | -0.0 |
| 6 | +1 |  | $+5.53$ | +5.53 | $+12.06$ | +0.240 | $+0.3$ |
| 7 |  | $-1$ | +2.37 | +3.50 | + 4.87 | +0.659 | +0.7 |
| 8 | -1 | +1 | -3.50 | -7.04 | $-10.54$ | $-0.500$ | $-0.5$ |
| 9 | $+1$ |  | +1.13 | +3.54 | $+5.67$ | $-0.159$ | $-0.2$ |
| 10 |  | -1 |  | +4.46 | + 3.46 | +0.113 | +0.1 |
| 11 |  |  |  | -4.40 | - 4.40 | +0.717 | $+0.7$ |
| 12 |  | $+1$ |  | $-0.06$ | + 0.94 | -0.830 | $-0.8$ |

Normal equations

|  | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma$ | $C^{\prime}$ S $^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | +6 | -2 | + 4.65 | + 9.45 | $-2.3$ | $+15.80$ | +0.2328 |
| 2 |  | $+6$ | $\begin{array}{r} 2.86 \\ +\quad \end{array}$ | $\begin{array}{r} \top \\ -\quad 8.80 \end{array}$ | +3.6 | $\begin{array}{r} 10.66 \\ +\quad 1.6 \end{array}$ | $-0.8398$ |
| 3 |  |  | $+206.0470$ | $+208.2153$ | +1.4 | $+423.1723$ | $+0.16435$ |
|  |  |  |  | +276.0980 | +1.2 | +486. 1633 | -0.16302 |

[^3]Solution of normal equations

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{C_{1}}^{+6}$ | $\begin{aligned} & -2 \\ & +0.33333 \end{aligned}$ | +4.65 -0.775 | $\begin{aligned} & \mathbf{9 . 4 5} \\ & -1.575 \end{aligned}$ | $\begin{aligned} & -2.3 \\ & +0.38333 \end{aligned}$ | $\begin{aligned} & +\begin{array}{c} 15.80 \\ -\quad 2.63333 \end{array} \end{aligned}$ |
|  | $\begin{aligned} & +6 \\ & { }_{-0.6667} \\ & +5.3333 \\ & { }_{C_{2}} \end{aligned}$ | $\begin{aligned} & +2.86 \\ & +1.55 \\ & +\quad 4.41 \\ & -0.82688 \end{aligned}$ | $\begin{aligned} & \hline \quad 8.80 \\ & +\quad 3.15 \\ & \overline{+} \quad 1.65 \\ & \hline \end{aligned}$ | $\begin{aligned} & +3.6 \\ & { }_{-0.7667} \\ & +2.8333 \\ & \mathbf{- 0 . 5 3 1 2 5}^{2.5125} \end{aligned}$ | $\begin{aligned} & +1.66 \\ & +\quad 5.2667 \\ & +\quad 6.9266 \\ & -\quad 1.29875 \end{aligned}$ |
|  | $\stackrel{1}{2}$ | $\begin{gathered} +206.0470 \\ =\quad 3.6038 \\ -\quad 3.6465 \\ +198.7967 \\ C_{3} \end{gathered}$ | $\begin{aligned} & \hline+208.2153 \\ & +\quad 7.3238 \\ & +\quad 4.6719 \\ & +205.5634 \\ & +\quad 1.034038 \end{aligned}$ | $\begin{aligned} & +1.4 \\ & +1.7825 \\ & -2.3428 \\ & +0.8397 \\ & { }_{-0.004224} \end{aligned}$ | $\begin{aligned} & +423.1723 \\ & +12.2450 \\ & -\quad 5.7275 \\ & +405.1998 \\ & -\quad 2.038262 \end{aligned}$ |
|  |  | $\begin{aligned} & \mathbf{1} \\ & \mathbf{2} \\ & 3 \end{aligned}$ | $\begin{gathered} +276.0980 \\ =14.8838 \\ =5.9855 \\ -212.5604 \\ +42.6683 \\ C_{4} \end{gathered}$ | $\begin{aligned} & +1.2 \\ & +3.6225 \\ & +3.0015 \\ & -0.8683 \\ & +6.9557 \\ & +{ }_{-0.16302} \end{aligned}$ | $\begin{aligned} & +486.1633 \\ & +\quad 24.8550 \\ & +\quad 7.3379 \\ & -418.9920 \\ & +\quad 49.6240 \\ & -\quad 1.16302 \end{aligned}$ |

Back solution

| 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: |
| -0.16302 | -0.00422 | $-0.5312$ | +0.3333 |
| -0.16302 | +0.16857 | $\begin{aligned} & -0.1727 \\ & -0.1359 \end{aligned}$ | +0.2508 -0.1274 |
|  | +0.16435 | -0.8398 | -0.2799 |
|  |  |  | +0.2328 |

Computation of corrections

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.2328 \\ & -0.3336 \\ & +0.0717 \end{aligned}$ | $\begin{aligned} & +0.2328 \\ & +0.8398 \\ & +0.5506 \\ & +0.4744 \\ & +0.9964 \\ & +1.0 \end{aligned}$ | $\begin{array}{r} -0.8398 \\ +0.8842 \\ -0.5461 \end{array}$ | $\begin{aligned} & -0.2328 \\ & +0.6886 \\ & -0.6831 \end{aligned}$ | $\begin{aligned} & -1.5975 \\ & +1.5846 \end{aligned}$ | $\begin{aligned} & +0.2328 \\ & +0.9089 \\ & +0.9015 \end{aligned}$ |
| $\begin{aligned} & -0.4947 \\ & -0.5 \end{aligned}$ |  | $\begin{aligned} & -0.5017 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & -0.2273 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & -0.0129 \\ & -0.0 \end{aligned}$ | $\begin{aligned} & +0.2403 \\ & +0.3 \end{aligned}$ |
| 7 | 8 | 9 | 10 | 11 | 12 |
| $\begin{aligned} & +0.8398 \\ & +0.3895 \end{aligned}$ | $\begin{aligned} & -0.2328 \\ & =0.8398 \\ & \hline-0.5752 \\ & +1.1477 \end{aligned}$ | $\begin{aligned} & +0.2328 \\ & +0.1857 \\ & +0.5771 \end{aligned}$ | $\begin{aligned} & +0.8398 \\ & { }_{-0.7271} \end{aligned}$ | +0.7173 | $\begin{aligned} & -0.8398 \\ & +0.0098 \end{aligned}$ |
| $\begin{aligned} & +0.6587 \\ & +0.7 \end{aligned}$ |  | $\begin{aligned} & -0.1586 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & +0.1127 \\ & +0.1 \end{aligned}$ | +0.7 | $-0.8300$ |
|  | $\begin{aligned} & -0.5001 \\ & -0.5 \end{aligned}$ |  |  |  |  |

## SOLUTION OF A SET OF NORMALS INCLUDING TERMS USUALLY OMITTED

A set of four normal equations is solved below with inclusion of the terms omitted in the Doolittle method of solution.

Solution of normals

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{+6}{-1} C_{1}$ | $\begin{aligned} & \text {-2 } \\ & +0.33333 \end{aligned}$ | +2 -0.33333 | $+\quad 4.65$ $-\quad 0.775$ | $\begin{aligned} & -2.3 \\ & +0.38333 \end{aligned}$ | $\begin{aligned} & +8.35 \\ & -\quad 1.39167 \end{aligned}$ |
| $\begin{aligned} & -2 \\ & +2 \end{aligned}$ | $\begin{aligned} & +6 \\ & -0.6667 \end{aligned}$ | +2 +0.6667 | + 2.86 $+\quad 1.55$ | $\begin{aligned} & +3.6 \\ & -0.7667 \end{aligned}$ | $\begin{aligned} & +12.46 \\ & +\quad 2.7833 \end{aligned}$ |
|  | $\begin{aligned} & +5.3333 \\ & -1 \mathrm{C}_{2} \end{aligned}$ | $\begin{aligned} & +2.6667 \\ & -0.50001 \end{aligned}$ | $\begin{aligned} & +4.41 \\ & -0.82688 \end{aligned}$ | $\begin{aligned} & +2.8333 \\ & -0.53125 \end{aligned}$ | $\begin{gathered} 15.2433 \\ +\quad 2.85814 \end{gathered}$ |
| $\begin{array}{r} +2 \\ -2 \end{array}$ | $\begin{aligned} & +2 \\ & +0.6667 \\ & -2.6667 \end{aligned}$ | +6 $\pm 0.6667$ -1.3333 | $\begin{aligned} & -15.15 \\ & =\quad 1.55 \\ & -\quad 2.205 \end{aligned}$ | $\begin{aligned} & +2.2 \\ & +0.7667 \\ & -1.4167 \end{aligned}$ | $\begin{array}{ll} - & 2.95 \\ = & 2.7833 \\ - & 7.6217 \end{array}$ |
|  |  | $\begin{aligned} & +4 \\ & -1 C_{3} \end{aligned}$ | $\begin{aligned} & -18.905 \\ & +\quad 4.72625 \end{aligned}$ | $\begin{aligned} & +1.55 \\ & -0.3875 \end{aligned}$ | $\begin{aligned} & -13.355 \\ & +\quad 3.33875 \end{aligned}$ |
| $\begin{array}{r} +4.65 \\ -4.65 \end{array}$ | $\begin{aligned} & +2.86 \\ & +1.55 \\ & -4.41 \end{aligned}$ | $\begin{aligned} & -15.15 \\ & =1.55 \\ & -2.205 \\ & +18.905 \end{aligned}$ | +206.0470 $-\quad 3.6038$ $-\quad 3.6465$ -89.3498 | $\begin{aligned} & +1.4 \\ & +1.7825 \\ & -2.3428 \\ & +7.3257 \end{aligned}$ | $\begin{array}{r} +199.8070 \\ =\quad 6.4712 \\ -12.6044 \\ -63.1191 \end{array}$ |
|  |  |  | $\begin{aligned} & +109.4469 \\ & -\quad 1 C_{4} \end{aligned}$ | $\begin{aligned} & +8.1654 \\ & -0.07461 \end{aligned}$ | $\begin{aligned} & +117.6123 \\ & -\quad 1.07461 \end{aligned}$ |

## DISCUSSION OF THE SOLUTION

The quantities in heavy type are the ones omitted in the Doolittle method of solution of normal equations. They sum up to zero with the possible variation of a few units in the last place of the solution. This shows that the method is one of curtailed substitution. It can also be seen that the quantity in the $\Sigma$ column is the direct sum of all the quantities in each horizontal line including those in heavy type. All of the quantities in heavy type occur in the regular solution. This is of value in the control of the solution. If an equation fails to check the $\Sigma$ column after it is added up, the error can generally be located by adding back through noting that the coefficient is changed in sign because it is multiplied by -1 . Note the product of equation No. 1 on No. $4 ;-1.55$ and +1.55 are the products of No. 1 on No. 3 and No. 2, respectively; -4.65 is the coefficient of No. 4 on No. 1 with sign changed. The method is the same in all cases. Care should be taken with such coefficients as No. 2 and No. 3 on No. 1. They have the same value with opposite sign. If a mistake should be made on them the $\Sigma$ column control would not catch it. Care should be taken not to make a mistake in the $\eta$ column and a compensating one in the $\Sigma$ column. There is most danger of this in the addition. The control would not catch this and it would take much labor to correct it later.

After each equation is added, it should be added horizontally to check the $\Sigma$ column. If the check fails an error has been made and it must be found before proceeding. A slight variation in the last place of the solution is of course unavoidable. After the division of each equation by the reduced diagonal term, a horizontal addition should be made (including, of course, -1 ) to check the correctness of the division. No time is ever lost in using care in the solution of the equations. It takes so much time and labor to rectify a mistake later that every means should be employed to detect and correct it in the solution. The larger the set, the more important it is to be on guard against errors. It is possible to carry a set through with almost absolute assurance that the solution is correct.

If, in a given equation, the solution fails to check and the check of adding back through is satisfied, a mistake has been made somewhere in the solution columns and a compensating mistake in the $\Sigma$ column. This can be caught by building up the omitted columns to the left of the given equation. They should each sum up to zero. If any one does not, the mistake in addition has been made in that equation in the column of the one being eliminated.

Solution of triangles *


[^4]Position computation,
STATION $\mathrm{A}_{3}$


STATION $\boldsymbol{A}_{4}$


[^5]secondary triangulation *
STATION $A_{3}$

| $\begin{gathered} \alpha \\ \text { Third } \\ \text { angle }\} \\ \alpha \\ \Delta \alpha \end{gathered}$ | $A_{1}$ to $A_{2}$ <br> $A_{3}$ and $A_{2}$ <br> $A_{1}$ to $A_{3}$ |  |  |  |  |  |  | $\circ$ 336 $-\quad 47$ | 18 31 | $\begin{array}{r} 1 \prime \\ 08.4 \\ 20.7 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | + 288 | 46 10 | $\begin{aligned} & 47.7 \\ & 24.3 \end{aligned}$ |
| $\alpha^{\prime}$ | $A_{3}$ to $A_{1}$ |  |  |  |  |  |  | 180 108 | 00 57 | 00.0 12.0 -.1 |
| $\stackrel{\phi}{\phi}{ }_{\text {¢ }}$ | ${ }^{60}$ | 58 1 | 56.416 <br> 58.608 | $A_{1}$ |  |  | $\stackrel{\lambda}{\lambda}$ | ${ }^{149}$ | 36 11 | $\begin{aligned} & 57.360 \\ & 54.003 \end{aligned}$ |
| $\phi^{\prime}$ | 60 | 56 | 57.808 +1 |  | $A_{3}$ |  | $i^{\prime}$ | 149 | 25 | 03.357 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{ccc}\circ & \prime \\ 60 & 57 & 57\end{array}$ |  | $\cos _{B}^{s} \alpha$ | 4. 055122 <br> 9. 507767 <br> 8.509295$\|$ | $\begin{gathered} \sin ^{2} \\ \sin ^{2} \alpha \end{gathered}$ | $\begin{aligned} & 8.11024 \\ & 9.95248 \\ & 1.65837 \end{aligned}$ | ${ }^{h^{2}}$ | $\begin{aligned} & 4.1144 \\ & 2.3221 \end{aligned}$ | ${ }_{s^{2}}^{-h} \sin _{\mathrm{E}} \alpha$ | $\begin{aligned} & 2.072 \\ & 8.063 \\ & 6.640 \end{aligned}$ |
| $\left\{\begin{array}{c} \text { 1st term } \\ 2 \mathrm{~d}, 3 \mathrm{~d}, \text { and } \\ \text { th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{aligned} & \prime \prime \prime \\ & +115.0521 \\ & +\quad 0.5258 \end{aligned}$ |  | h | 2.072184 | 9. 72109 |  |  | 6. 4665 |  | 6. 775 |
|  |  |  | 0.52610.0003-0.0006 |  |  |  |  |  |
|  | +118.6079 |  |  |  |  |  |  |  |  |
|  | $\begin{gathered} \sin ^{8} \alpha \\ A^{\prime} \\ \sec \phi^{\prime} \end{gathered}$ |  | $\begin{aligned} & \text { 4. } 055122 \\ & 9.976241 \\ & 8.508600 \\ & \text { 8. } 313737 \end{aligned}$ |  | $\sin \frac{\Delta \lambda}{\frac{1}{2}\left(\phi+\phi^{\prime}\right)}$ | $\begin{aligned} & \text { 2. } 853700 \\ & 9.941676 \end{aligned}$ |  |  |  |  |  |
|  |  |  | 2. 833700 | 2. 795376 |  |  |  |  |  |
|  | 4 $\lambda$ |  | -714.0029 | $-\Delta \alpha$ |  |  | . 27 |  |  |  |  |

STATION $A_{4}$


List of geographic positions, Turnagain Arm, Alaska. Valdez datum

| Station | Latitude and longitude |  |  | Seconds in meters | Azimuth |  |  | Back azimuth |  |  | To station | Distance | Logarithm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{3}$ |  | , | " |  |  | 1 |  | $\bigcirc$ | , | " |  | Meters |  |
|  |  | 56 | 57.809 | 1789.4 | 78 | 13 | 19.1 | 258 | 05 | 13.2 | $A_{2}$ | 8552.6 | 3. 932100 |
|  |  | 25 | 03.357 | 50.5 |  | 57 |  |  |  |  | $A_{1}$ | 11353.3 | 4.055122 |
| $A_{4}$ | 60149 |  | 05.749 | 177.9 |  | 18 |  |  |  |  | $A_{2}$ | 4943.3 | 3. 694020 |
|  |  | 29 | 11.442 | 172.4 |  | 33 | 58.7 |  | 27 | 11.4 | $A_{1}$ | 10008.6 | 4.000374 |
|  |  |  |  |  |  |  |  |  |  | .37.8 | $A_{3}$ | 5098.3 | 3.707429 |

## DEVELOPMENT OF CONDITION EQUATIONS FOR LATITUDE AND LONGITUDE CLOSURES

After the conditions arising from the closure of triangles and from the equality of sides or lengths computed by different routes have been satisfied, cases frequently arise where azimuth, latitude, and longitude conditions must be satisfied. There is given now a development of a form of condition equations that will bring about a closure in geographic position.

Discrepancies in latitude and longitude arise whenever a chain of triangulation or a traverse closes on itself. The discrepancies may be distributed throughout the whole loop or in a selected portion of it, depending upon the circumstances. Of course the most rigid adjustment would require the discrepancies to be distributed throughout the whole chain. At times, however, this would require more labor than the importance of the work would justify. Also some parts of the loop may be much better determined than other parts, in which case the more poorly determined part should be required to make up the discrepancies.

The discussion of the form of equations to be employed to effect the closure without discrepancies will be based upon the position computation formulæ employed by the United States Coast and Geodetic Survey. (See United States Coast and Geodetic Survey Special Publication No. 8, p. 8.)

The amount to be distributed being, of course, small compared with the total change in latitude and longitude, the only term of the latitude computation formula that need be considered is the first one. No appreciable changes due to the adjustment will take place in the other terms.

The formation of the equations must always start from a line fixed in length and azimuth. If a scheme of triangulation should start from a fixed line and run to two points which are fixed in position but are not the ends of a single line, then the formation of the equations for each of the two points must start from the fixed line.
There are, of course, two elements that enter into the determination of the position of any point as computed from a known point; these
are the distance from the known point and the azimuth of the line from the known to the unknown point.

In the triangle 123 , let 1 and 2 be fixed in position, and let us consider what change in the position of 3 will be produced by small changes in angles $A, B$, and $C$. The length to be carried forward is 1 to 3 . Starting with the length 1 to 2 , we have $\log 1$ to $3=\log 1$ to $2-\log \sin B+\log \sin A$. The change in length, then, depends upon the changes in angles $A$ and $B$ and the change in azimuth of the line 1 to 3 depends upon the change in angle $C$. The angles $A$ and $B$, therefore, are called the length angles and angle $C$ the azimuth angle.

If we can derive a linear expression for the effect of each of these separately, the total effect will be the sum of the two.

Let $\delta_{\mathrm{A}}$ and $\delta_{\mathrm{B}}$ represent the change of the $\log \sin$ for a change of one second in the angles $A$ and $B ; v_{\mathrm{A}}$ and $v_{\mathrm{B}}$ the number of seconds change in angles $A$ and $B$, respectively. Then the change in $\log \sin$ $A$ will equal $\delta_{\Delta}\left(v_{\Delta}\right)$, and the change in $\log \sin B$ will equal $\partial_{B}\left(v_{B}\right)$; therefore, the change in $\log 1$ to 3 is $+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)$. Thechange in the logarithm of the first term of thelatitude due to the change in length is equal to $+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\hat{o}_{\mathrm{B}}\left(v_{\mathrm{B}}\right)$. This is the change in the logarithm, but for convenience of computation it is better to determine what change in the antilogarithm will be produced by this change; or, in other words, to determine what this logarithmic


Fig. 2. change will amount to in seconds of arc. From the nature of logarithms, if we know the number to which a given logarithm corresponds, the change in the number due to any small change in the logarithm can be found by multiplying the logarithm change by the number and dividing by $M$ (the modulus of the common system of logarithms). This can also be shown by differentiation:

$$
\text { Let } \begin{aligned}
y & =\log _{11} x \\
d y & =M \frac{d x}{x}
\end{aligned}
$$

Therefore $d x=\frac{x}{M} d y, d y$ being the small change in the $\log$ and $d x$ the corresponding small change in the number.
$+\hat{o}_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\hat{o}_{\mathrm{B}}\left(v_{\mathrm{B}}\right)$ must then be multiplied by ( $\phi_{\mathrm{B}}-\phi_{\mathrm{C}}$ ), (in which $\phi_{\mathrm{B}}$ is the computed latitude of 3 and $\phi_{\mathrm{C}}$ is the latitude of 1 ), and the product divided by $M$; this will give the change in seconds in the latitude of 3 due to the change in length of 1 to 3 .

Next must be considered the change in latitude due to the change in the azimuth angle $C$. If $s$ is the length in meters of 1 to 3 , the length of the small are through which 3 turns is equal to $s\left(v_{\mathrm{c}}\right)$ are $1^{\prime \prime}$ (as $d s=r d \theta$ for a circle about the origin in polar coordinates), $v_{c}$


Fig. 3. equals the number of seconds change in angle $C$, and arc $1^{\prime \prime}$ is included to reduce this angle to circular measure.

Let 3 be the original position of 3 and $3^{\prime}$ the position due to a small rotation of 1 to 3 about 1 .

$$
3 \text { to } 3^{\prime}=s v_{\mathrm{c}} \text { arc } 1^{\prime \prime}
$$

The azimuth of 3 to $3^{\prime}$ is $90^{\circ}$ $+\alpha$. The change in latitude due to $s\left(v_{\mathrm{c}}\right)$ arc $1^{\prime \prime}$ is equal to $-s\left(v_{\mathrm{c}}\right)$ arc $1^{\prime \prime} \cos \left(90^{\circ}+\alpha\right)$ times the $B$ factor in the position computation,

$$
\begin{aligned}
& =+s\left(v_{\mathrm{c}}\right) \text { arc } 1^{\prime \prime} \sin \alpha B \\
& =+\left(v_{\mathrm{c}}\right) \operatorname{arc} 1^{\prime \prime} B s \sin \alpha
\end{aligned}
$$

But $\lambda_{\mathrm{B}}-\lambda_{\mathrm{C}}=s \sin \alpha A^{\prime} \sec \phi^{\prime}$
Therefore $s \sin \alpha=\frac{\lambda_{\mathrm{B}}-\lambda_{\mathrm{c}}}{A^{\prime} \sec \phi^{\prime}}$
Therefore the change in latitude $=\frac{B \operatorname{arc} 1^{\prime \prime}}{A^{\prime} \sec \phi^{\prime}}\left(\lambda_{\mathrm{B}}-\lambda_{\mathrm{c}}\right)\left(v_{\mathrm{c}}\right)$.
In a similar way the change in longitude due to a change in length is,

$$
\frac{\lambda_{B}-\lambda_{\mathrm{C}}}{M}\left[+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right]
$$

and the change in longitude due to the change in azimuth is,
$s\left(v_{\mathrm{c}}\right)$ are $1^{\prime \prime} \sin \left(90^{\circ}+\alpha\right) A^{\prime} \sec \phi^{\prime}=+\left(v_{\mathrm{c}}\right)$ are $1^{\prime \prime} s \cos \alpha A^{\prime} \sec \phi^{\prime}$
$-s \cos \alpha \cdot B=\phi_{\mathrm{B}}-\phi_{\mathrm{c}}$ (neglecting the small terms)

$$
s \cos \alpha=-\frac{\phi_{\mathrm{B}}-\phi_{\mathrm{c}}}{B}
$$

Therefore change in longitude $=-\frac{A^{\prime} \sec \phi^{\prime} \text { arc } 1^{\prime \prime}}{B}\left(\phi_{\mathrm{B}}-\phi_{\mathrm{c}}\right)\left(v_{\mathrm{c}}\right)$.
The usage is to point off the log change for one second of arc as an integer in the sixth place of logarithms; therefore as a number the tabular difference $=\frac{\delta_{\mathrm{A}}}{10^{6}}$ and $\frac{\delta_{\mathrm{B}}}{10^{8}}$.

The total change in seconds of latitude in the triangle is

$$
\frac{\left(\phi_{\mathrm{B}}-\phi_{\mathrm{C}}\right)}{10^{\mathrm{B}} M}\left[+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \pm \frac{B \operatorname{arc} 1^{\prime \prime}}{A^{\prime} \sec \phi^{\prime}}\left(\lambda_{\mathrm{B}}-\lambda_{\mathrm{C}}\right)\left(v_{\mathrm{C}}\right) * *
$$

The total change in longitude is,

$$
\frac{\left(\lambda_{\mathrm{B}}-\lambda_{\mathrm{C}}\right)}{10^{8} M}\left[+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\partial_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \mp \frac{A^{\prime} \sec \phi^{\prime} \operatorname{arc} 1^{\prime \prime}}{B}\left(\phi_{\mathrm{B}}-\phi_{\mathrm{C}}\right)\left(v_{\mathrm{C}}\right) .^{*}
$$

[^6]In this way the change could be determined for each triangle in the chain and the sum placed equal to the discrepancy, but this would require a very great amount of work.

If any change takes place in the first triangle while the remaining triangles are for the moment supposed to remain fixed, this length change and azimuth change will affect not only this triangle, but will persist in each succeeding triangle. As a consequence the change of length and azimuth in the first triangle will be felt in the computation of every point after it in the chain. Let $\phi_{n}$ and $\lambda_{n}$ be the computed $\phi$ and $\lambda$ of the end point. The change in the first triangle will apply not merely to $\phi_{\mathrm{B}}-\phi_{\mathrm{c}}$, etc., but to $\phi_{n}-\phi_{\mathrm{c}}$, etc.
Therefore the change in the final position due to the changes in the first triangle is, for latitude,

$$
\frac{\left(\phi_{n}-\phi_{\mathrm{c}}\right.}{10^{\mathrm{B}} M}\left[+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\hat{\delta}_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \pm \frac{B_{\mathrm{C}} \operatorname{arc} 1^{\prime \prime}}{A_{n} \sec \phi_{n}}\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)\left(v_{\mathrm{c}}\right),^{*}
$$

and for longitude,

$$
\frac{\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)}{10^{\mathrm{B}} \mathrm{M}}\left[+\delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \mp \frac{A_{n} \sec \phi_{n} \operatorname{arc} 1^{\prime \prime}}{B_{\mathrm{C}}}\left(\phi_{n}-\phi_{\mathrm{C}}\right)\left(v_{\mathrm{c}}\right) .^{*}
$$

In the same way the change in the final position due to changes in the second triangle can be determined, and so on through the whole chain. Each triangle will have an $A, B$, and $C$ angle, $A$ being the length angle next to the known side; $B$, the one opposite the known side; and $C$, the azimuth angle.

The equations will finally stand

$$
\begin{aligned}
0=+\left(\phi_{n}-\phi_{n^{\prime}}\right) & +\Sigma\left[\frac{\left(\phi_{n}-\phi_{\mathrm{c}}\right)}{10^{\mathrm{B}} M} \delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\frac{\left(\phi_{n}-\phi_{\mathrm{c}}\right)}{10^{\mathrm{B}} M} \delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \\
& +\Sigma \pm \frac{B_{\mathrm{C}} \operatorname{arc} 1^{\prime \prime}}{A_{n} \sec \phi_{n}}\left(\lambda_{n}-\lambda_{\mathrm{c}}\right)\left(v_{\mathrm{c}}\right) .^{*} \\
0=+\left(\lambda_{n}-\lambda_{n^{\prime}}\right) & \left.+\Sigma\left[\frac{\lambda_{n}-\lambda_{\mathrm{c}}}{10^{\mathrm{s} M} M} \delta_{\mathrm{A}}\right)-\frac{\lambda_{n}-\lambda_{\mathrm{c}}}{10^{\mathrm{B}} M} \delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \\
& +\Sigma \mp \frac{A_{n} \sec \phi_{n} \operatorname{arc} 1^{\prime \prime}}{B_{\mathrm{c}}}\left(\phi_{n}-\phi_{\mathrm{c}}\right)\left(v_{\mathrm{c}}\right) .^{*}
\end{aligned}
$$

$\phi_{n}$ is the computed latitude of the final point and $\phi_{n^{\prime}}$, the fixed latitude; so also for $\lambda_{n}$ and $\lambda_{n^{\prime}}$.

It is exact enough to take $\phi_{n}-\phi_{\mathrm{c}}$ and $\lambda_{n}-\lambda_{\mathrm{c}}$ to minutes and tenths of a minute, so that it is advisable to divide the equations by 60 since, as they stand, $\phi_{n}-\phi_{c}$, etc., are in seconds. Also it is best to multiply through by $10^{6} M$ to remove this factor from the denominator of the first summation.

Then we have

$$
\begin{gathered}
0=+\frac{M}{60} 10^{6}\left(\phi_{n}-\phi_{n^{\prime}}\right)^{\prime \prime}+\Sigma\left[\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime} \delta_{\Lambda}\left(v_{\mathrm{A}}\right)-\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime} \delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \\
+\Sigma \pm 10^{6} M \frac{B_{\mathrm{C}} \operatorname{arc} 1^{\prime \prime}}{A_{n} \sec \phi_{n}}\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime}\left(v_{\mathrm{C}}\right) . * \\
0=+\frac{M}{60} 10^{\mathrm{B}}\left(\lambda_{n}-\lambda_{n^{\prime}}\right)^{\prime \prime}+\Sigma\left[\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime} \delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)-\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime} \delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right] \\
+\Sigma \mp 10^{6} M \frac{A_{n} \sec \phi_{n} \operatorname{arc} 1^{\prime \prime}}{B_{\mathrm{C}}}\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime}\left(v_{\mathrm{c}}\right) . *
\end{gathered}
$$

The $A$ and $B$ factors change so slowly that for any chain they can be taken for the mean $\phi$ and also sec $\phi_{n}$ can be used in the same way. A table can then be prepared for functions designated as $a_{1}$ and $a_{2}$ and defined as follows:

$$
\begin{aligned}
& a_{1}=+10^{6} M \frac{B \operatorname{arc} 1^{\prime \prime}}{A \sec \phi} \\
& a_{2}=-10^{6} M \frac{A \sec \phi \operatorname{arc} 1^{\prime \prime}}{B}
\end{aligned}
$$

the $A$ and $B$ factors and the $\phi$ being used at a convenient interval. A table has been computed for latitudes starting at $24^{\circ}$ for intervals of $4^{\circ}$ up to $56^{\circ}$. The minus sign is used with $a_{2}$ in order that the same sign can be used on the directions of the azimuth angle for both latitude and longitude equations. If the discrepancy to be made up by the adjustment is large, or if the chain extends over a great distance of latitude, it would be best to compute the values of $a_{1}$ and $a_{2}$ using $A_{n}$ and $\phi_{n}$ and the $B$ for the mean $\phi$.

If the chain to be adjusted extends principally east and west, in place of $\phi_{n}-\phi_{\mathrm{c}}$ a summation of the first terms ( $h$ ) in the position computations should be used. $\sum_{n}^{c} h$ would then replace $\phi_{n}-\phi_{\mathrm{c}}$, the sign being used that would conform to $\phi_{n}-\phi_{c}$. These quantities should then be used throughout in forming the equations.

If the latitude and longitude equations are to be included in the main adjustment and the equations all solved simultaneously, the computation of the positions through the chain must be made with one length carried through the figures by means of the observed plane angles; that is, the angles as observed each diminished by $\frac{1}{3}$ of the spherical excess of the triangle. This could be done by carrying the length through a selected chain of triangles and then computing each of the various positions over a single line. Both lines of the triangle could not be used because the observed plane angles must be used in carrying the length and, under ordinary circumstances, the triangle would not be closed. To obviate this difficulty, it is best to use only the observed $A$ and $B$ angles and to conclude the $C$ angle, using, of course, the concluded correction symbols on this

[^7]angle. This method gives a much more reliable determination of the discrepancy, as it furnishes a check on each position, and thus prevents a mistake being left in the computation. If the figure adjustment is carried out first, there is no need to follow this method as the triangles would then be closed. In this case it is the general custom of the United States Coast and Geodetic Survey to choose the best chain of triangles and to form the equations through them, using the angle method in place of the direction method. Equations with absolute terms equal to zero must be included for the various triangles in order to bold them closed; also, if a length equation is included in the figure adjustment, it must be retained with zero discrepancy to hold the length. If the figure ends on a fixed line and a length equation is not put in the figure adjustment, the discrepancy must be put on the length equation used with the latitude and longitude equations. After adjustment is made for these final discrepancies the cross lines are computed by two sides and the included angle.

The best results are probably obtained by the solution of all the equations at once, but this entails so much work that the angle method is often used in chains of minor importance.

We have finally:

$$
\begin{gathered}
\frac{M}{60} 10^{B}=7238.24 \\
0=+7238.24\left(\phi_{n}-\phi_{n^{\prime}}\right)^{\prime \prime}+\sum\left[\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime} \delta_{\mathrm{A}}\left(v_{\mathrm{A}}\right)\right. \\
\left.\quad-\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime} \partial_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right]+\Sigma \pm a_{1}\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime}\left(v_{\mathrm{C}}\right) . \\
0=+7238.24\left(\lambda_{n}-\lambda_{n^{\prime}}\right)^{\prime \prime}+\sum\left[\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime} \hat{\partial}_{\mathrm{A}}\left(v_{\mathrm{A}}\right)\right. \\
\left.\quad-\left(\lambda_{n}-\lambda_{\mathrm{C}}\right)^{\prime} \delta_{\mathrm{B}}\left(v_{\mathrm{B}}\right)\right]+\sum \pm a_{2}\left(\phi_{n}-\phi_{\mathrm{C}}\right)^{\prime}\left(v_{\mathrm{C}}\right) .
\end{gathered}
$$

In the equations $v_{\mathrm{A}}, v_{\mathrm{B}}$, and $v_{\mathrm{c}}$ would be replaced by their correction symbols, care being taken to use $v_{\mathrm{c}}=-v_{\mathrm{A}}-v_{\mathrm{B}}$, if the azimuth angle has been concluded in carrying the position computation through the chain.

If an azimuth equation occurs, the constant term must be corrected by $+\left(\lambda_{n}-\lambda_{n^{\prime}}\right) \times$ sine of the mean $\phi$, this being the amount that the azimuth will change from the changes in the back azimuths due to the changes in longitude.

It should be noted that whenever a discrepancy of position is adjusted into a section of a loop, an external condition is placed upon the chain, as at best only part of this discrepancy is due to errors in the chain, the rest being due to the remainder of the loop. It is necessary to hold some parts of the triangulation fixed; otherwise when a loop closure is put in it would frequently be necessary to readjust nearly all of the triangulation of the country. The result is, however, that some chains of triangulation, excellent in themselves, get some rather large corrections due to the position closure.

## EQUATIONS IN A NET

In the adjustment of a quadrilateral, use is made of the two kinds of condition equations that are necessary for the adjustment of any figure that does not contain external conditions such as length, azimuth, or loop closure. In fact most figures can be broken up into successive quadrilaterals. In forming the length equation, use is made of the two length angles in the various triangles passed through. In fig. 5, on page 37, the length angles are lettered $A$ and $B$. The angle omitted is the azimuth angle of the given triangle. The log $\sin$ of the $A$ angle is added to the first length and the $\log \sin$ of the $B$ angle to the final length. So with all of the triangles through which the length is carried. The discrepancy is found and the equation formed in the same way as in the case of an ordinary side equation. See the formation of the length equation on page 37. If the spherical angles are used a correction for are to sine must be applied to each length. (See the table of these corrections in Special Publication No. 8, p. 17.)

An azimuth equation is formed by adding algebraically to the first azimuth the various azimuth angles up to the second line fixed in azimuth. When passing from one end of a line to another, the azimuth difference due to convergence of the meridians, must be applied as determined in the computation of positions. The algebraic sum of the $v$ 's upon these angles must make up the discrepancy between the computed and fixed azimuths. See the computation on page 38.

The determination of the exact number of side and angle equations in a net and the manner in which they come in, is one of the difficulties encountered by a beginner in the adjustment of triangulation. This is especially true if the net is somewhat complicated. The best method for this determination is to plot the figure point by point. By plotting the triangle Tower, Turn, and Dundas, in the figure on page 34 one angle equation is determined. Add Lazaro by the lines Lazaro to Turn and Lazaro to Tower. This gives another angle equation, making two. Another angle equation and a side equation are obtained by putting in the line Lazaro to Dundas. This makes a total of three angle equations and one side equation for the quadrilateral, just as it should be. Next plot Nichols by the lines Nichols to Lazaro and Nichols to Tower; this is a closed triangle and gives a fourth angle equation. Put in Tow Hill by the lines Tow Hill to Nichols and Tow Hill to Lazaro; this does not give an angle equation as it is not a closed triangle. Draw the line Tow Hill to Tower; this gives a second side equation. In this way one can continue through the whole figure. If a full line Nichols to Turn were in the figure, it would give another angle and another side
equation. The angle equation added would have to include the directions on this line as would also the side equation. This method shows at once where the equations come in and what new $v$ 's must appear in the equations.

Lines sighted over in only one direction have no effect on the number of angle equations. If the closed part of the figure is plotted, omitting all of the extra lines-that is, putting in each station with only two lines from those already plotted, a closed framework of the figure will be formed. The first triangle requires three lines, those after the first require two lines. The number of angle equations in the framework of the figure is thus equal to the number of lines in the figure minus the number of stations plus one. Every full line added to this framework gives another angle equation. Therefore, the whole number of angle equations in a net is equal to the whole number of full lines minus the number of occupied stations plus one.

The lines sighted over in one direction have the same effect on the number of side equations that the full lines have. If the full framework of the figure is plotted with two lines to each station from those already determined, no side equation will as yet appear in the figure. Every extra line put in gives a side equation. The first triangle fixes three stations; the stations after these require two lines to be used in plotting them. Thus the number of lines needed to plot the framework is equal to twice the number of stations minus three. The full number of side equations will then be equal to the number of all the lines minus twice the number of all the stations plus three.

Let $n=$ total number of lines.
$n^{\prime}=$ number of lines sighted over in both directions.
$S=$ total number of stations.
$S^{\prime}=$ number of occupied stations.
Then
The number of angle equations in a net $=n^{\prime}-S^{\prime}+1$.
The number of side equations in a net $=n-2 S+3$.
These formulas should be used to check the number determined by directly plotting the figure.

In figure 4 on page 34,

$$
\begin{aligned}
n & =41 \\
n^{\prime} & =38 \\
S & =18 \\
S^{\prime} & =17
\end{aligned}
$$

Therefore number of angle equations $=38-17+1=22$.
number of side equations $=41-36+3=8$.
For convenience in solution it is best to use triangles with the larger angles for the angle equations, reserving the small angles to be used in the side equations. This will keep the large coefficients

## 34 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.

in the side equations from appearing on the same directions as are used in the angle equations and will aid in the solution of the normals.

* The small angles need to appear in the side equations, as their tabular differences are proportionally much less affected by the dropping of decimal places than are those of the larger angles.

ADJUSTMENT OF A FIGURE WITH LATITUDE, LONGITUDE, AZIMUTH, AND LENGTH CLOSURE CONDITIONS


Fig. 4.
In this figure, in addition to the angle, side, and length conditions, there are included conditions for azimuth, latitude, and longitude.

## Angle equations

$0=-5.5-(1)+(2)-(4)+(6)-(10)+(11)$
$0=+4.0-(1)+(3)-(5)+(6)-(12)+(13)$
$0=+12.0-(2)+(3)-(9)+(10)-(12)+(14)$
$0=-8.0-(8)+(9)-(14)+(16)-(25)+(26)$
$0=-2.9-(16)+(17)-(23)+(25)-(31)+(32)$
$0=-0.0-(16)+(18)-(24)+(25)-(33)+(34)$
$0=-3.2-(23)+(24)-(30)+(32)-(34)+(35)$
$0=-2.3-(17)+(19)-(29)+(31)-(36)+(37)$
$0=-3.2-(17)+(21)-(28)+(31)-(42)+(45)$
$0=-3.2-(19)+(21)+(36)-(38)-(42)+(44)$
$0=+5.0-(21)+(22)-(40)+(42)-(49)+(52)$
$0=-1.0-(20)+(22)-(47)+(48)-(49)+(51)$
$0=+3.7-(40)+(43)-(46)+(47)-(51)+(52)$
$0=+5.2-(40)+(41)-(50)+(52)-(56)+(58)$
$0=-2.5-(39)+(40)-(52)+(55)-(59)+(61)$
$0=+2.8-(50)+(55)-(57)+(58)-(59)+(60)$
$0=-5.1-(53)+(55)-(59)+(62)-(67)+(68)$
$0=-1.7-(54)+(55)-(59)+(63)-(71)+(72)$
$0=+2.1-(62)+(63)-(66)+(67)-(71)+(73)$
$0=+2.3-(63)+(65)-(70)+(71)-(74)+(75)$
$0=+6.1-(64)+(65)-(74)+(76)-(77)+(78)$
$0=+0.1-(69)+(70)-(75)+(76)-(77)+(79)$

## Azimuth equation

$$
\begin{aligned}
0=\left(-7.1^{*}-0.2\right)-(1)+(2)+(9)-(10)- & (14)+(21)+(40)-(42) \\
& -(52)+(55)-(59)+(63)+(70)-(71)-(75)+(76)
\end{aligned}
$$

Computation of correction to azimuth constant:

$$
\log 0.277=9.442
$$

$\log$ sin. mean $\phi=9.912$
$\log$ correction $=9.354$
correction $=-0.2$
Side equations

| Symbol | Angle |  |  | Logarithm | Tabular difference | Symbol |  | Ang |  | Logarithm | Tabular difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & =9+11 \\ & =12+13 \\ & -1+2 \end{aligned}$ | $\begin{array}{ccc} \circ & \prime & \prime \prime \\ 93 & 11 & 39.1 \\ 25 & 52 & 3 \times .1 \\ 24 & 17 & 25.4 \end{array}$ |  |  |  | $\begin{array}{r} -0.12 \\ +4.34 \\ +4.67 \end{array}$ | $\begin{aligned} & -13+14 \\ & =1+3 \\ & -10+11 \end{aligned}$ | $\begin{array}{rcc} 0 & \prime & 11 \\ 16 & 37 & 43.4 \\ 110 & 36 & 0.7 \\ 42 & 00 & 30.0 \end{array}$ |  |  |  | $\begin{aligned} & +7.05 \\ & -7.79 \\ & +2.34 \end{aligned}$ |
|  |  |  |  | 9.9993248 |  |  |  |  |  | 9. 4566222 |  |
|  |  |  |  | 9.6399291 |  |  |  |  |  | 9.9712965 |  |
|  |  |  |  | 9.6142236 |  |  |  |  |  |  |  |
|  |  |  |  | 9.2534775 |  |  |  |  |  | 9.2534997 |  |

$0=-22.2-5.46(1)+4.67(2)+0.79(3)+0.12(9)+2.34(10)-2.46(11)-4.34(12)+11.39(13)$ $-7.05(14)$

| $-7+9$ | $\begin{array}{llll}111 & 09 & 20.5\end{array}$ | 9.9696969 | -0.81 | $\left\{\begin{array}{l}+7-9 \\ +14-15\end{array}\right.$ | $\} \begin{array}{lll}21 & 40 & 38.8\end{array}$ | 9.5674745 | +5.30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & +15-16 \\ & +25-27 \\ & +25+26 \end{aligned}$ | $\} \begin{array}{lll}36 & 08 & 04.3\end{array}$ | 9.7706188 | +2.88 | $-25+27$ | $\begin{array}{llll}89 & 23 & 18.6\end{array}$ | 9.9999753 | +0.02 |
|  | $30 \begin{array}{llll}30 & 04 & 51.8\end{array}$ | 9.7000325 | $+3.64$ | $-8+9$ | $\begin{array}{lllll}48 & 16 & 10.2\end{array}$ | 9.8729041 | +1.83 |
|  |  | 9.4403482 |  |  |  | 9.4403539 |  |

$\begin{aligned} 0= & -5.7-4.49(7)+1.88(8)+2.61(9)-5.30(14)+8.18(15)-2.88(16)-0.74(25)+3.64(26) \\ & -2.90(27)\end{aligned}$

Side equations-Continued

| Symbol | Angle |  |  | Logarithm | Tabular difference | Symbol | Angle |  |  | Logarithm | Tabular difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -16+18 \\ & -23+24 \\ & -30+31 \end{aligned}$ | $\begin{array}{rrr} 38 & 29 & 18.8 \\ 27 & 51 & 39.2 \\ 9 & 31 & 16.8 \end{array}$ |  |  | 9. 7940405 <br> 9. 6696203 <br> 9.2185744 | $\begin{aligned} & +2.65 \\ & +3.98 \\ & +12.55 \end{aligned}$ | $\begin{aligned} & -24+25 \\ & -30+32 \\ & -17+18 \end{aligned}$ | $\begin{array}{rrr} 12 & 35 & 33.3 \\ 126 & 31 & 06.3 \\ 15 & 56 & 21.5 \end{array}$ |  |  | 9.3384902 <br> 9.9050754 <br> 9. 4387305 | $\begin{aligned} & +9.43 \\ & -1.56 \\ & +7.37 \end{aligned}$ |
|  |  |  |  | 8. 6822352 |  |  |  |  |  | 8.6822961 |  |

$0=-60.9-2.65(16)+7.37(17)-4.72(18)-3.98(23)+13.41(24)-9.43(25)-14.11(30)$
$+12.55(31)+1.56(32)$

| $-17+19$ | 35 | 17 | 43.3 | 9.7617712 | +2.98 | $-29+31$ | 16 | 21 | 23.9 | 9.4496564 |
| ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-28+29$ | 9 | 58 | 42.9 | 9.2387486 | +11.97 | $-44+45$ | 23 | 37 | 23.0 | 9.6028386 |
| $-42+44$ | 51 | 05 | 11.2 | 9.8910323 | +1.70 | $-19+21$ | 43 | 39 | 34.4 | 9.8390831 |

$0=-26.0-2.98(17)+5.19(19)-2.21(21)-11.97(28)+19.14(29)-7.17(31)-1.70(42)$
$+6.51(44)-4.81(45)$

| $-49+52$ | 89 | 47 | 52.8 | 9.9999973 | +0.01 | $-21+22$ | 26 | 22 | 55.0 | 9.6477280 | +4.25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| $-20+21$ | 26 | 37 | 18.2 | 9.6513730 | +4.20 | $-46+48$ | 105 | 41 | 48.0 | 9.9834943 | -0.59 |
| $-46+47$ | 33 | 20 | 40.5 | 9.7401044 | +3.20 | $-51+52$ | 35 | 09 | 14.0 | 9.7602524 | +2.99 |
|  |  |  |  | 9.3914747 |  |  |  |  |  | 9.3914747 | . |
|  |  |  |  |  |  |  |  |  |  |  |  |

$0=+0.0-4.20(20)+8.45(21)-4.25(22)-3.79(46)+3.20(47)+0.59(48)-0.01(49)$

$$
+2.99(51)-2.98(52)
$$

| $-39+41$ | 105 | 37 | 20.7 | 9.9836521 | -0.59 | $-60+61$ | 22 | 08 | 21.3 | 9.5761788 | +5.17 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $-59+60$ | 27 | 49 | 50.7 | 9.6691879 | +3.99 | $-50+55$ | 125 | 14 | 18.4 | 9.9120934 | -1.49 |
| $-50+52$ | 37 | 39 | 24.5 | 9.7859918 | +2.73 | $-40+41$ | 63 | 10 | 28.9 | 9.9505530 | +1.07 |
|  |  |  |  | 9.4388318 |  |  |  |  | 9.4388252 |  |  |

$0=+6.6+0.59(39)+1.07(40)-1.66(41)-4.22(50)+2.73(52)+1.49(55)-3.99(59)$

$$
+9.16(60)-5.17(61)
$$

| $-71+73$ | 59 | 25 | 24.7 | 9.9349784 |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $-59+62$ | 58 | 43 | 17.2 | 9.9317900 |  |  |  |  |  |  |  |
| $-53+54$ | 18 | 17 | 51.0 | 9.4968619 | +1.24 | $-62+63$ | 43 | 56 | 28.3 | 9.8413093 | +2.19 |
|  |  |  |  | 9.3636303 |  | $-53+55$ | 59 | 03 | 08.4 | 9.9333037 | +1.26 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

$0=-18.4-5.11(53)+6.37(54)-1.26(55)-1.28(59)+3.47(62)-2.19(63)-1.24(71)$

$$
+5.00(72)-3.76(73)
$$

| $\begin{aligned} & -74+76 \\ & -63+64 \\ & -69+70 \end{aligned}$ | $\begin{array}{lll} 94 & 10 & 29.2 \\ 28 & 13 & 32.1 \\ 61 & 25 & 13.8 \end{array}$ |  | $\begin{aligned} & \text { 9. } 9988462 \\ & \text { 9. } 6748099 \\ & \text { 9. } 9435708 \end{aligned}$ | $\begin{array}{r} -0.15 \\ +3.92 \\ +1.15 \end{array}$ | $\begin{aligned} & -64+65 \\ & -69+71 \\ & -75+76 \end{aligned}$ | $\begin{array}{rrr} 50 & 11 & 06.3 \\ 102 & 20 & 31.6 \\ 33 & 30 & 23.0 \end{array}$ |  |  | $\begin{aligned} & 9.8854273 \\ & 9.9898451 \\ & 9.7419627 \end{aligned}$ | $\begin{aligned} & +1.76 \\ & -0.46 \\ & +3.18 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 9.6172269 |  |  |  |  |  | 9.6172351 |  |

$0=-8.2-3.92(63)+5.68(64)-1.76(65)-1.61(69)+1.15(70)+0.46(71)+0.15(74)$

$$
+3.18(75)-3.33(76)
$$

## Length equation

| Symbol | Angle |  |  | Logarithm | Tabular difference | Symbol |  | Ang |  | Logarithm | Tabular difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Turn-Dundas |  |  |  | $4.266771{ }^{-6}$ |  | Ham-South Twin |  |  |  | $3.898371{ }^{-1}$ |  |
| $-4+6$ | 113 | 41 | 59.6 | 9.9617359 | -0.92 | -10+11 | 40 | 00 | 30.0 | 9.8255810 | +2.34 |
| $-2+3$ | 86 | 18 | 43.3 | 9.9990997 | +0.14 | $-12+14$ | 42 | 30 | 21.5 | 9.8297327 | +2.30 |
| $-7+9$ |  | 09 | 20.5 | 9.9696969 | -0.81 | $\left\{\begin{array}{l}+7-9 \\ +14-15\end{array}\right.$ | \} 21 |  | 38.8 | 9.5674745 | $+5.30$ |
| $\begin{aligned} & +15-16 \\ & +25-27 \\ & -23+25 \\ & -28+31 \\ & -21+22 \end{aligned}$ | ) 36 | 08 | 04.3 | 9.7706188 | +2.88 | $-25+27$ | 89 | 23 | 18.6 | 9.9999753 | +0.02 |
|  | 40 |  | 12.5 | 9.8121311 | +2.47 | $-31+32$ | 116 | 59 | 49.5 | 9.9498922 | -1.07 |
|  | 26 | 20 | 06.8 | 9.6470132 | +4.25 | $-42+45$ | 74 | 42 | 34.2 | 9.9843478 | +0.58 |
|  |  | 22 | 55.0 | 9.6477280 | +4.25 | $-49+52$ | 89 | 47 | 52.8 | 9.9999973 | +0.01 |
| $\begin{aligned} & -39+40 \\ & -54+55 \\ & -63+65 \\ & -69+70 \end{aligned}$ |  | $\begin{aligned} & 26 \\ & 45 \\ & 24 \\ & 25 \end{aligned}$ | 51.8 <br> 17.4 <br> 35.4 <br> 13.8 | 9.8292505 | $+2.30$ | $-59+61$ | 49 | 58 | 12.0 | 9. 8840631 | +1.77 |
|  |  |  |  | 9.8147959 | +2.44 | $-71+72$ | 36 | 34 | 55.5 | 9.7752272 | +2.84 |
|  |  |  |  | 9.9910544 | +0.43 | $-74+75$ | 60 | 40 | 06.2 | 9.9404164 | +1.18 |
|  |  |  |  | 9.9435708 | +1.15 | $-77+79$ |  | 04 | 23.4 | 9.9983924 | +0.18 |
|  |  |  |  | 2.6534656 |  |  |  |  |  | 2.6534708 |  |

$0=-5.2-0.14(2)+0.14(3)+0.92(4)-0.92(6)-4.49(7)+4.49(9)+2.34(10)-2.34(11)$
$+2.30(12)-7.60(14)+8.18(15)-2.88(16)-4.25(21)+4.25(22)-2.47(23)+5.37(25)$
$-2.90(27)-4.25(28)+3.18(31)+1.07(32)-2.30(39)+2.30(40)+0.58(42)-0.58(45)$
$+0.01(49)-0.01(52)-2.44(54)+2.44(55)+1.77(59)-1.77(61)-0.43(63)+0.43(65)$
$-1.15(69)+1.15(70)+2.84(71)-2.84(72)+1.18(74)-1.18(75)+0.18(77)-0.18(79)$


Fig. 5.

## Formation of azimuth equation



Preliminary computation of triangles


[^8]Preliminary computation of triangles-Continued


Preliminary position computation,
STATION TOWER


STATION LAZARO

primary triangulation
STATION TOWER


STATION LAZARO

| Third ${ }^{\boldsymbol{\alpha}}$ angle | Tower to Turn Lazaro and Turn |  |  |  |  | $\begin{array}{r} 0 \\ 201 \\ -\quad 51 \end{array}$ | $\begin{aligned} & 43 \\ & 10 \end{aligned}$ | $\begin{aligned} & 31.0 \\ & 57.1 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\alpha}{\Delta \alpha}$ | Tower to Lazaro |  |  |  |  | -150 | 32 14 | $\begin{aligned} & 33.9 \\ & 01.2 \end{aligned}$ |
| $\alpha^{\prime}$ | Lazaro to Tower |  |  |  |  | 180 330 | 18 | 32.7 |
|  | $\begin{array}{r} 0 \\ +\quad{ }^{54} \\ \hline \end{array}$ | $\begin{aligned} & 35 \\ & 17 \end{aligned}$ | $27.323$ $30.520$ | Tower | ${ }_{\lambda \lambda}^{\lambda}$ | ${ }^{131}$ | 04 17 | $\begin{aligned} & 48.045 \\ & 10.250 \end{aligned}$ |
| $\underset{\Delta \phi}{\phi}$ | 54 | 52 | 57.843 | Lazaro | $\lambda^{\prime}$ | 131 | 21 | 58. 295 |
| ${ }_{B}^{8} \cos _{B} \alpha$ | 4. 5722946 <br> 9. 9398800 <br> 8. 5097404 | $\begin{gathered} \sin ^{8^{2}} \alpha \\ \cos ^{2} \alpha \end{gathered}$ | $\begin{array}{\|l\|} 9.14459 \\ 9.38353 \\ 1.55122 \end{array}$ | ${ }_{(8 \phi)}^{\text {D }}{ }^{2}$ | $\begin{aligned} & 6.0428 \\ & 2.3683 \end{aligned}$ | $8^{2} \sin _{\mathrm{E}} \mathrm{~h} \alpha$ |  | $\begin{array}{r} 0219 \\ 5281 \\ .4516 \end{array}$ |
| h <br> 1st term 2d term | $\begin{array}{r} 3.0219150 \\ 11 \\ -1051.7559 \\ +\quad 1.2004 \end{array}$ | 3d term | $\begin{gathered} 0.07934 \\ \prime \prime \\ +0.0258 \\ +0.0100 \end{gathered}$ |  | 8. 4111 |  |  | . 0016 |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and } 4 \text { th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} -1050.5555 \\ +\quad 0.0358 \\ \hline-1050.5197 \\ 0, \quad \prime \prime \end{array}$ | $\begin{gathered} \sin ^{8} \alpha \\ A^{\prime}, \\ \sec ^{\prime} \phi^{\prime} \end{gathered}$ | -7 4.5722946 9.6917656 8.5087409 0.2401420 | $\begin{gathered} \text { Arg. } \\ \hline \quad \\ \Delta \lambda . \end{gathered}$ | $\begin{aligned} & -25 \\ & +18 \end{aligned}$ | $\begin{aligned} & \frac{\Delta \lambda}{\sin \frac{1}{2}\left(\phi+\phi^{\prime}\right)} \\ & \sec \frac{3}{2}(\Delta \phi) \end{aligned}$ |  | $\begin{aligned} & .0129424 \\ & .9119609 \end{aligned}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | 544412.6 | $4 \lambda$ | 3.0129424 $\prime \prime$ +1030.2498 | Corr. | $-7$ | $-\Delta \alpha$ |  | $\begin{array}{r} 9249033 \\ \prime \prime \prime \\ +841.20 \end{array}$ |

Preliminary position computation,
STATION TOW HILL


STATION NICHOLS

primary triangulation-Continued
STATION TOW HILL


STATION NICHOLS


Preliminary position computation,

## STATION KEN



STATION ROUND

primary triangulation-Continued
STATION KEN

| Third angle <br> $\stackrel{\alpha}{\Delta \alpha}$ | Nichols to Lazaro Ken and Lazaro |  |  |  |  | $\begin{array}{r} \circ \\ 251 \\ -\quad 40 \end{array}$ | $\begin{aligned} & 17 \\ & 27 \end{aligned}$ | $\begin{array}{r} \quad 11 \\ 13.5 \\ 12.5 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Nichols to Ken |  |  |  |  | 210 $+\quad$. | 50 9 | $\begin{aligned} & 01.0 \\ & 22.8 \end{aligned}$ |
| $\alpha^{\prime}$ | Ken to Nichols |  |  | Nichols | ${ }^{\lambda}{ }^{\lambda} \lambda$ | $\begin{array}{r} 180 \\ 30 \end{array}$ | 59 | 23.8 |
| ${ }_{\text {d }}{ }_{\text {d }}$ | $\begin{array}{r} 0 \\ +54 \\ +\quad 1 \end{array}$ | $\begin{aligned} & 43 \\ & 11 \end{aligned}$ | $\begin{gathered} \prime \prime \\ 30.831 \\ 04.068 \end{gathered}$ |  |  | $-^{132}$ | 11 | $\begin{aligned} & 12.693 \\ & 28.537 \end{aligned}$ |
| $\phi^{\prime}$ | 54 | 54 | 34.899 | Ken | $\lambda^{\prime}$ | 131 | 59 | 44. 156 |
| $\begin{gathered} 8 \\ \cos _{B}^{8} \alpha \end{gathered}$ | $\begin{aligned} & \text { 4. } 3790213 \\ & 9.9338209 \\ & \text { 8. } 5097307 \end{aligned}$ | $\begin{gathered} \sin ^{2} \\ \sin ^{2} \alpha \end{gathered}$ | 8. 75804 <br> 9. 41947 <br> 1. 55337 | $\left(\frac{(\delta \phi)^{2}}{D}\right.$ | 5. 6450 <br> 2. 3676 | $s^{2} \sin ^{-h} \alpha$ |  | $\begin{aligned} & 2.8226 \\ & 8.1775 \\ & 6.4553 \end{aligned}$ |
| h | $2.8225729$ | 3d tewn <br> 4th term | $\text { 9. } 73088$ |  | 8.0126 |  |  | 7. 4554 |
| 1st term 2d term | $\begin{array}{r} -664.6192 \\ +\quad 0.5381 \end{array}$ |  | $\begin{array}{r} +0.0103 \\ +0.0029 \end{array}$ |  |  |  |  |  |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} -664.0811 \\ +\quad 0.0132 \\ \hline-664.0679 \\ 0 \quad, \quad \prime \end{array}$ | $\begin{gathered} \sin _{A^{\prime}}, \alpha \\ \sec ^{\prime} \phi^{\prime} \end{gathered}$ | -2 4.3790213 9.7097334 8.508703 0.2404328 | $\begin{gathered} \text { Arg. } \\ 8 \\ f \lambda \end{gathered}$ | $\begin{aligned} & -10 \\ & +8 \end{aligned}$ | $\begin{gathered} \Delta \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}(\Delta \phi \phi) \end{gathered}$ |  | 2. 837928 <br> 9.912392 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | 544902.9 | $\Delta \lambda$ | 2.8379276 $\prime \prime$ -688.5375 | Corr. | -2 | $-\Delta \alpha$ |  | $\begin{gathered} 2.750320 \\ \prime \prime \prime \\ -562.76 \end{gathered}$ |

STATION ROUND

| Third ${ }^{\alpha}$ angle | Ken to Lazaro Round and Lazaro |  |  |  |  | $\begin{array}{r} \circ \\ 273 \\ -\quad 26 \end{array}$ | $\begin{aligned} & 59 \\ & 20 \end{aligned}$ | $\begin{gathered} 11 \\ 34.3 \\ 06.8 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\Delta \alpha}{\alpha}$ | Ken to Round |  |  |  |  | $+^{247}$ | $\begin{aligned} & 39 \\ & 29 \end{aligned}$ | $\begin{aligned} & 27.5 \\ & 17.8 \end{aligned}$ |
| $\alpha^{\prime}$ | Round to Ken |  |  |  |  | 180 | 08 | 45. 3 |
|  | $\begin{gathered} 0 \\ +\quad 54 \\ +\quad 1 \end{gathered}$ | $\begin{gathered} 1 \\ 54 \\ 8 \end{gathered}$ | $\begin{gathered} \prime \prime \\ 34.899 \\ 21.261 \end{gathered}$ | Ken | $\underset{\Delta \lambda}{\lambda}$ | $-^{131}$ | 59 35 | $\begin{aligned} & 44.156 \\ & 46.458 \end{aligned}$ |
| $\phi^{\prime}$ | 55 | 02 | 56.160 +1 | Round | $\lambda^{\prime}$ | 131 | 23 | 57.698 -1 |
| $\begin{gathered} \delta \\ \cos \alpha \\ B \end{gathered}$ | $\begin{aligned} & \text { 4. } 6149334 \\ & 9.5799436 \\ & 8.5097172 \end{aligned}$ | $\sin _{\mathrm{C}}^{\sin ^{2}} \alpha$ | $\begin{aligned} & \text { 9. } 22987 \\ & \text { 9. } 93222 \\ & \text { 1. } 55631 \end{aligned}$ | $\left(\begin{array}{c} (\delta \phi)^{2} \\ \hline \end{array}\right.$ | $\begin{aligned} & \text { 5. } 4091 \\ & \text { 2. } 3666 \end{aligned}$ | ${ }^{8}{ }^{-\frac{-h}{\sin ^{2}} \boldsymbol{L}}$ |  | $\begin{aligned} & \text { 2. } 7046 \\ & 9.1621 \\ & 6.4604 \end{aligned}$ |
| h <br> 1st term 2d term | $\begin{array}{r} 2.7045942 \\ \prime \prime \\ -506.5172 \\ +\quad 5.2288 \\ \hline \end{array}$ | 3d term 4th term | $\begin{gathered} 0.71840 \\ \prime \prime \\ +0.0058 \\ +0.0212 \end{gathered}$ |  | 7.7757 |  |  | 8.3271 |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} -501.2884 \\ +\quad 0.0270 \end{array}$ | $\begin{aligned} & \sin _{A^{\prime}}{ }^{\prime} \\ & \sec \phi^{\prime} \end{aligned}$ | $\begin{array}{r} + \\ +49 \\ 4.6149334 \\ 9.961083 \\ 8.5087369 \\ 0.2419389 \end{array}$ | $\begin{gathered} \mathrm{Arg} . \\ 8 \\ \Delta \lambda \end{gathered}$ | $\begin{aligned} & -30 \\ & +79 \end{aligned}$ | $\begin{gathered} \Delta \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \left.\sec \frac{1}{2}(\Delta \phi)\right) \end{gathered}$ |  | 3. 331722 <br> 9. 913254 |
| $\frac{1}{3}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{ccc} -501 . & 2614 \\ \circ & \prime & \prime \prime \\ 54 & 58 & 46.0 \end{array}$ | $\Delta \lambda$ | $\begin{array}{r} 3.3317224 \\ 1 \prime \\ -2146.458 \end{array}$ | Corr. | +49 | $-\Delta \alpha$ |  | $\begin{gathered} 3.244976 \\ 111 \\ -1757.83 \end{gathered}$ |

Preliminary position computation,
STATION CAT


STATION BEAVER

primary triangulation-Continued
STATION CAT


STATION BEAVER

| Third angle | Round to Cat Beaver and Cat |  |  |  |  | $\begin{array}{r} \circ \\ 289 \\ -\quad 42 \end{array}$ | $\begin{aligned} & 36 \\ & 26 \end{aligned}$ | $\begin{gathered} \prime \prime \prime \\ 58.5 \\ 51.8 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\alpha}{\Delta \alpha}$ | Round to Beaver |  |  |  |  | $\begin{array}{r}247 \\ +\quad \\ \hline\end{array}$ | 10 7 | $\begin{aligned} & 06.7 \\ & 40.0 \end{aligned}$ |
| $\alpha^{\prime}$ | Beaver to Round |  |  |  |  | 180 67 | 17 | 46.7 |
| $\phi_{\phi}$ | $\begin{array}{r} 0 \\ +55 \\ +\quad 1 \end{array}$ | $\begin{gathered} c_{1} \\ 02 \\ 2 \end{gathered}$ | $56.161$ $15.131$ | Round | $\underset{\Delta \lambda}{\lambda}$ | 131 | 23 9 | $\begin{aligned} & 57.697 \\ & 21.038 \end{aligned}$ |
| $\phi^{\prime}$ | 55 | 05 | 11.292 | Beaver | $\lambda^{\prime}$ | 131 | 14 | 36.659 |
| $\begin{gathered} \delta \\ \cos \alpha \\ \mathrm{B} \end{gathered}$ | 4. 0333437 <br> 9.5 <br> 8.5097071 | $\begin{gathered} 8^{2} \\ \sin ^{2} \alpha \\ C \end{gathered}$ | $\begin{aligned} & 8.0667 \\ & 9.9291 \\ & 1.5586 \end{aligned}$ | $\left(\begin{array}{c} (\delta \phi))^{3} \\ D \end{array}\right.$ | 4. 264 <br> 2.366 | ${ }_{8} \frac{-\mathrm{h}}{\sin ^{2}} \alpha$ |  | $\begin{aligned} & 2.132 \\ & 7.996 \\ & 6.464 \end{aligned}$ |
| h <br> 1st term 2d term | $\begin{array}{r} 2.1319070 \\ \prime \prime \\ -135.4899 \\ +\quad 0.3584 \end{array}$ | 3d term 4th term | $\begin{array}{r} 9.5544 \\ \quad \prime \prime \\ +0.0004 \\ +0.0004 \end{array}$ |  | 6. 630 |  |  | 6. 592 |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} -135.1315 \\ +\quad 0.0008 \\ \hline-135.1307 \\ 0 \quad, \quad \prime \end{array}$ | $\begin{gathered} \sin _{A^{\prime}} \alpha \\ \sec ^{\prime} \phi^{\prime} \end{gathered}$ | +3 4.0333437 <br> 9. 9645661 <br> 8.5087360 <br> 0.2423463 | $\begin{gathered} \text { Arg. } \\ 8 \\ 4 \lambda \end{gathered}$ | $\begin{aligned} & -2 \\ & +5 \end{aligned}$ | $\begin{gathered} \Delta \lambda \\ \sin \frac{3}{3}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}(\Delta \phi) \end{gathered}$ |  | 2. 748992 <br> 9.913723 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{gathered} -135.1307 \\ \circ \\ 55 \\ 50 \end{gathered} 0_{0}^{\prime \prime} 03.7$ | $\Delta \lambda$ | 2.7489924 $\prime \prime$ -561.0382 | Corr. | +3 | $-\Delta \alpha$ |  | $\begin{gathered} 2.662715 \\ \prime \prime \prime \\ -459.95 \end{gathered}$ |

$91865^{\circ}-15-4$

Preliminary position computation,
Station lim

| $\stackrel{\alpha}{\alpha}$ Second angle | Beaver to Cat Cat and Lim |  |  |  |  | $\begin{array}{r} 17 \\ +102 \end{array}$ | 19 39 | $\begin{gathered} 1 \prime \\ 34.7 \\ 47.2 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ $\Delta \alpha$ | Beaver to Linn |  |  |  |  | - 119 | 59 | 21.9 20.3 |
| $\alpha^{\prime}$ | Lim to Beaver F |  |  | irst angle of triangle |  | 180 299 36 | 54 34 | $\begin{aligned} & 01.6 \\ & 55.5 \end{aligned}$ |
| ${ }_{\dagger}^{\prime}{ }_{\phi}$ | $+{ }^{55}$ | $\begin{array}{r} 05 \\ 2 \end{array}$ | 11.292 08.973 | Beaver | $\stackrel{\lambda}{\lambda \lambda}$ | 131 + | 14 | $\begin{aligned} & 36.659 \\ & 30.479 \end{aligned}$ |
| $\phi^{\prime}$ | 55 | 07 | 20.265 | Lim | $\lambda^{\prime}$ | 131 | 21 | 07.138 |
| $\begin{gathered} s \\ \cos \alpha \\ \mathrm{~B} \end{gathered}$ | 3. 9025495 <br> 9. 6988310 <br> 8. 5097044 | $\sin ^{\boldsymbol{s}^{2}} \alpha$ | $\begin{aligned} & 7.8051 \\ & 9.8751 \\ & 1.5591 \end{aligned}$ | $\stackrel{(\delta \phi)}{ }{ }^{2}$ | 4. 222 2.366 | $s^{2} \sin _{\mathrm{E}} \sin ^{2} \alpha$ |  | $\begin{aligned} & 2.111 \\ & 7.680 \\ & 6.465 \end{aligned}$ |
| h | $\begin{array}{r} 2.1110849 \\ \prime \prime \\ -129.1471 \\ +\quad 0.1735 \\ \hline \end{array}$ | 3d term 4th term | 9. 2393 |  | 6.588 |  |  | 6. 256 |
| 1st term |  |  | +0.0004 +0.0002 |  |  |  |  |  |
| $\underset{\substack{\text { 3d and } 4 \text { th } \\ \text { terms } \\-\Delta \phi}}{ }\}$ | $\begin{array}{r} -128.9736 \\ +\quad 0.0006 \\ \hline \end{array}$ | $\begin{gathered} s \\ \sin ^{\prime} \alpha \\ \sec ^{\prime} \phi^{\prime} \end{gathered}$ | + 3. 9025495 9. 9375769 8. 5087351 0.2427356 | Arg. <br> 8 <br> $\Delta \lambda$ | -1 +3 | $\begin{aligned} & \Delta \lambda \\ & \sin \frac{1}{2}\left(\phi+\phi^{\prime \prime}\right. \\ & \sec \frac{1}{2}(\Delta \phi) \end{aligned}$ |  | $\begin{aligned} & 2.591597 \\ & 9.913918 \end{aligned}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{ccc} \hline-128.9730 \\ 0 & \prime & \prime \prime \\ 55 & 06 & 15.8 \end{array}$ | $\Delta \lambda$ | 2.5915973 <br> $\prime \prime$ <br> +390.4787 | Corr. | +2 | $-\Delta \alpha$ |  | $\begin{array}{r} 2.505515 \\ \prime \prime \\ +320.27 \end{array}$ |

STATION SOUTH TWIN


Discrepancy in latitude:
-0.035
$\times 7238.24 \div 100=$
-2.5334
Discrepancy in longitude:
-0.277 $\times 7238.24 \div 100=$
$\times 20.0499$
primary triangulation-Continued
STATION LIM


STATION SOUTH TWIN

Formation of latitude and longitude condition equations

| $\begin{gathered} 0 \\ <i_{0}^{0} \\ 0 \end{gathered}$ | $+\underbrace{\circ}$ |  |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{10} \\ & 1 \end{aligned}$ | $\begin{array}{r} \infty \\ \substack{0 \\ 1 \\ 1} \end{array}$ | $\underbrace{}_{\substack{0 \\ \\ 1 \\ \hline}}$ | $\begin{array}{r} 0 \\ 1 \\ 1 \end{array}$ | $\stackrel{\rightharpoonup}{\dot{\alpha}}$ | $\stackrel{\infty}{\underset{1}{1}}$ | $\stackrel{\square}{\infty}$ $1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 4 \\ & 1 \\ & 60 \\ & 3 \\ & + \\ & \hline \end{aligned}$ | $\begin{array}{r} 0 \\ 1 \\ 1 \\ + \\ +\quad 4 \\ \hline \end{array}$ | $\begin{aligned} & \infty \\ & \text { a } \\ & + \\ & + \\ & +1 \\ & 1 \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{2}{+} \\ & \stackrel{1}{+} \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \hline \\ & \stackrel{7}{2} \end{aligned}$ |  |  |  |  | $\begin{array}{r} i 8 \\ 18 \\ 1 \\ 1 \\ 1 \\ + \\ + \\ \hline \end{array}$ |  |  |
| $e_{1}^{0}$ | ＋\％${ }_{\text {® }}^{\text {¢ }}$ | $\begin{aligned} & 10 \\ & o \\ & + \end{aligned}$ | $\stackrel{0}{i}$ | $\begin{aligned} & 0 \\ & \stackrel{1}{1} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{1}{1} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{1}{1} \end{aligned}$ | $\stackrel{\rightharpoonup}{\oplus}$ | $\begin{aligned} & 0 \\ & +i \\ & i \end{aligned}$ | $\begin{gathered} \vec{i} \\ \text { i } \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & i \end{aligned}$ |  |
|  | ＋ | $\begin{aligned} & 10 \\ & \stackrel{0}{1} \\ & 1 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\dot{d}} \\ & + \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { + } \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ i \end{gathered}$ | $\begin{aligned} & \text { is } \\ & 10 \end{aligned}$ | $\begin{aligned} & \because- \\ & + \\ & + \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\circ} \\ & + \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & + \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & + \end{aligned}$ |  |
| $\propto$ | $\begin{aligned} & 7 \\ & + \\ & \mathbf{1} \\ & 1 \end{aligned}$ | $$ | $\begin{array}{r} 019 \\ 11 \\ +7 \\ +7 \end{array}$ |  | $\begin{aligned} & \text { 筑 } \\ & \stackrel{+}{\ldots} \\ & \hline \end{aligned}$ | $\begin{aligned} & 10 \\ & + \\ & + \\ & 1 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { on } \\ & + \\ & + \\ & \hline \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \stackrel{\rightharpoonup}{+} \\ & + \\ & \stackrel{\rightharpoonup}{i} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \stackrel{1}{+} \\ & \underset{i}{2} \end{aligned}$ | $\begin{aligned} & 18 \\ & \stackrel{18}{4} \\ & \stackrel{1}{4} \\ & \hline \end{aligned}$ |  |
| $\begin{gathered} \theta \\ 01_{0}^{2} \\ 0 \\ 0 \end{gathered}$ | ＋8\％సู ¢ | －180 | $\begin{aligned} & \underset{\dot{1}}{\sim} \\ & \stackrel{1}{1} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & i \end{aligned}$ | $\begin{aligned} & \stackrel{0}{0} \\ & + \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \hline \end{aligned}$ | $\stackrel{\rightharpoonup}{0}$ | $\begin{aligned} & \overrightarrow{\mathbf{d}} \\ & \overrightarrow{1} \end{aligned}$ | $\begin{aligned} & 10 \\ & \infty \\ & 1 \end{aligned}$ | $\begin{aligned} & \mathrm{i} \\ & i \end{aligned}$ |  |
| $<\overbrace{10}^{0}$ | ＋8） | $\begin{aligned} & \mathrm{I} \\ & + \end{aligned}$ | $\begin{aligned} & \stackrel{7}{5} \\ & + \\ & \hline \end{aligned}$ | $\overbrace{\substack{\infty \\ 1}}^{1}$ | $\begin{aligned} & \text { coㅕㅕ } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \end{aligned}$ | $\stackrel{\infty}{\text { Ci }}$ | $\begin{gathered} \infty \\ \infty \\ i \\ 1 \end{gathered}$ | $$ | $\begin{aligned} & \circ \\ & \text { i } \\ & 1 \end{aligned}$ |  |
| ＊ | $\begin{gathered} 0 \\ + \\ \vdots \\ \hline \end{gathered}$ | $\begin{gathered} \infty \\ + \\ + \\ 1 \end{gathered}$ | $\begin{gathered} \infty \\ \pm \\ 1 \\ 1 \end{gathered}$ | $\begin{aligned} & 0 \\ & 11 \\ & 1 \\ & \vdots \\ & +\quad+ \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \hline \stackrel{\rightharpoonup}{+} \\ & \stackrel{1}{\alpha} \\ & \underset{1}{1} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { ®̀ } \\ & \stackrel{1}{7} \\ & \text { I } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { il } \\ & + \\ & \hline \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 8 \\ & + \\ & +0 \\ & i \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{8}{8} \\ & + \\ & \vdots \\ & \hline \end{aligned}$ |  |
| $-e_{0}^{2}$ | ＋ 8 | $\begin{aligned} & 6 \\ & + \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { aid } \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & 7 \\ & \pm \\ & \hline \end{aligned}$ | $\stackrel{\infty}{\stackrel{\infty}{+}} \stackrel{+}{+}$ | $\begin{aligned} & 7 \\ & \stackrel{\rightharpoonup}{0} \\ & + \end{aligned}$ | $\begin{aligned} & \text { ה̀ } \\ & + \\ & + \end{aligned}$ | $\begin{aligned} & \stackrel{\leftrightarrow}{\circ} \\ & \stackrel{y}{+} \end{aligned}$ | $\begin{aligned} & \stackrel{?}{?} \\ & + \end{aligned}$ | $\begin{aligned} & \text { TH } \\ & + \\ & + \\ & \hline \end{aligned}$ |  |
| $i$ | $\begin{aligned} & \text { \#̈ } \\ & \text { di } \end{aligned}$ | $\begin{gathered} \text { O} \\ \text { ì } \\ \hline \end{gathered}$ | $\begin{aligned} & 8 \\ & 0 \\ & i \end{aligned}$ | $\begin{aligned} & \text { 으 } \\ & \text { i } \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{+} \\ & \ddagger \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & i \\ & i \end{aligned}$ | $\begin{aligned} & \text { io } \\ & 0 \\ & i \end{aligned}$ | $\begin{aligned} & \mathrm{N} \\ & \mathrm{i} \\ & \hline \end{aligned}$ | $\stackrel{\square}{\text { a }}$ | $\stackrel{\sim}{i}$ |  |
| ＋ | $\begin{aligned} & \text { Ơ } \\ & \text { O } \end{aligned}$ | $\begin{aligned} & \pm \\ & \stackrel{\rightharpoonup}{0} \\ & + \end{aligned}$ | $\begin{aligned} & \vec{\omega} \\ & \stackrel{1}{i} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{+} \\ & + \end{aligned}$ | $\begin{aligned} & \text { A } \\ & \text { à } \end{aligned}$ | $\begin{aligned} & \stackrel{\text { an }}{+} \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{4} \\ & + \\ & + \end{aligned}$ | $\begin{aligned} & \stackrel{\otimes}{\mathrm{H}} \\ & + \end{aligned}$ | $\begin{aligned} & \underset{~}{+} \\ & \underset{~}{+} \end{aligned}$ | $\begin{aligned} & \text { 哭 } \\ & + \end{aligned}$ |  |
| $\begin{aligned} & 0 \\ & 1 \\ & 1 \\ & 2 \\ & 0 \end{aligned}$ | $\begin{aligned} & i \\ & i \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { 冗o } \\ & \infty \\ & + \\ & \hline \end{aligned}$ | $\begin{gathered} \pm \\ \text { i } \\ 1 \end{gathered}$ | $\begin{gathered} \pm \\ \vdots \\ 1 \end{gathered}$ | $\begin{gathered} 3 \\ \vdots \\ i \\ i \end{gathered}$ | $\begin{aligned} & 7 \\ & 0 \\ & 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & \cong \\ & =1 \\ & 1 \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \infty \\ \hline \end{gathered}$ | $\begin{gathered} \infty \\ \stackrel{\infty}{1} \\ 1 \end{gathered}$ | $\begin{gathered} 8 \\ \stackrel{8}{6} \\ \infty \\ \hline \end{gathered}$ |  |
| $\begin{aligned} & 0 \\ & 10 \\ & -6 \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{1}{\mathbf{\alpha}} \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \stackrel{\circ}{0} \\ & \stackrel{2}{+} \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{+}{+} \end{aligned}$ | $\begin{aligned} & .{ }_{0}^{6} \\ & 1+ \\ & + \end{aligned}$ | $\begin{array}{r} \stackrel{\rightharpoonup}{6} \\ \stackrel{\rightharpoonup}{7} \\ \hline \end{array}$ | $\begin{aligned} & \vec{m} \\ & \stackrel{\rightharpoonup}{+} \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { \% } \\ & 10 \\ & + \\ & \hline \end{aligned}$ | $$ | $\begin{aligned} & \infty \\ & \infty \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\circ} \\ & \dot{0} \\ & \hline \end{aligned}$ |  |
| $\checkmark$ | 苍 | $\begin{aligned} & 8 \\ & \text { \& } \\ & \text { ti } \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\text { an }} \end{aligned}$ | $\begin{aligned} & \text { た } \\ & \text { ส่ } \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { ब่ } \end{aligned}$ | $\begin{aligned} & \text { N. } \\ & \text { से } \end{aligned}$ | $\begin{aligned} & \mathscr{\%} \\ & \underset{\text { ஷ }}{1} \end{aligned}$ | $\begin{aligned} & \stackrel{3}{6} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & \ddot{\circ} \\ & \pm \end{aligned}$ | $\stackrel{\text { In }}{\stackrel{1}{\dot{A}}}$ | $\stackrel{\text { ®® }}{\text { ® }}$ |
|  | －윾 |  |  |  |  | 퓩 | $\stackrel{\text { r }}{\sim}$ | 픽 | 翵 | 呺 | 픅 |
| $\because$ | －${ }_{\text {g }}^{\text {g }}$ |  | $\begin{aligned} & 8 \\ & \text { \&id } \\ & \hline \text { 웅 } \end{aligned}$ | $\begin{aligned} & 8 \\ & \text { \&id } \end{aligned}$ | $\begin{aligned} & \text { \& } \\ & \text { oin } \end{aligned}$ | $\begin{aligned} & \text { \& } \\ & \text { هiri } \end{aligned}$ | 艹̈ | $\stackrel{\tilde{O}}{\stackrel{\sim}{0}}$ | $\begin{aligned} & 9 \\ & 98 \\ & 88 \end{aligned}$ | $\begin{aligned} & \text { ざ } \\ & \stackrel{5}{\circ} \end{aligned}$ | \％ |
|  | －is | \％ | \％ | $\stackrel{3}{6}$ | \％ | \％ | 沼 | 滘 | is | 滘 | is |
|  | 否 | 产 |  |  | $\begin{aligned} & \text { O. } \\ & \text { ⿷匚山̆工二} \end{aligned}$ | $\begin{gathered} \text { ¿2 } \\ \text { W. } \\ \hline \end{gathered}$ | 菏 | \％ | H． $\stackrel{0}{む}$ M． | E |  |

## Latitude equation

$$
\begin{aligned}
0= & -2.5334-0.14(2)+0.14(3)+0.39(4)-0.39(6)-0.69(7)+0.69(9)+0.67(10)-0.67(11) \\
& -0.66(12)-1.36(14)+1.25(15)-0.55(16)-0.09(21)+0.09(22)-0.49(23)+0.93(25) \\
& +0.44(27)-0.76(28)+0.49(31)+0.27(32)-0.20(39)+0.20(40)-0.02(42)+0.02(45) \\
& +0.14(4)-0.14(52)-0.10(54)+0.10(55)+0.08(59)-0.08(61)+0.10(63)-0.10(65) \\
& +0.07(71)-0.07(72)+0.11(74)-0.11(75)
\end{aligned}
$$

## Longitude equation

$$
\begin{aligned}
0= & -20.0499+1.20(2)-1.20(3)-0.59(4)+0.59(6)+0.41(7)-0.41(9)-0.35(10)+0.35(11) \\
& +1.39(12)-0.34(14)-0.75(15)-0.30(16)+0.67(21)-0.67(22)-0.34(23)+0.07(25) \\
& +0.27(27)-0.17(28)-0.29(31)+0.46(32)-0.16(39)+0.16(40)-0.62(42)+0.62(45) \\
& +0.20(49)-0.20(52)-0.07(54)+0.07(55)-0.32(59)+0.32(61)+0.07(63)-0.07(65) \\
& -0.16(71)+0.16(72)-0.06(74)+0.06(75)
\end{aligned}
$$



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| -1 | -1 | $\cdots$ |
| :---: | :---: | :---: |
| +1 | $\cdots$ | -1 |
| $-\cdots$ | +1 | +1 |
| -1 | $\cdots$ | $\cdots$ |
| $\cdots \cdots$ | -1 | $\cdots$ |
|  |  | $\cdots$ |

$+1+1$


$\qquad$
+1
$\ldots . .$.
$\ldots .$.
1
+1
$\cdots$

| 1 | $\ldots$. | .- | -1 |
| :--- | :--- | :--- | :--- |
| .. | -1 |  |  |


equations

| 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | $\stackrel{*}{31}$ | $\stackrel{l}{l}$ | $\stackrel{\text { ¢ }}{ } \times$ | $\lambda$ 34 | $\Sigma$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.55 |  |  |  |  |  |  |  | -1 |  |  |  | - 3.55 |  |
| +0.47 |  |  |  |  |  |  |  | +1 | -0.14 | -0.14 | +1.20 | + 2.39 |  |
| +0.08 |  |  |  |  |  |  |  |  | +0.14 | +0.14 | -1.20 | +1.16 |  |
|  |  |  |  |  |  |  |  |  | +0.92 | +0.39 | -0.59 | - 0.28 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | -0.92 | -0.39 | +0. 59 | +1.28 |  |
|  | -4.49 |  |  |  |  |  |  |  | -4.49 | -0.69 | +0.41 | -9.26 |  |
| +0.01 | +1.88 +2.61 |  |  |  |  |  |  | +1 | +4.49 | +0.69 | -0.41 | +0.88 +8.39 +8.80 |  |
| +0.24 |  |  |  |  |  |  |  | -1 | +2.34 | +0.67 | -0.35 | +1.90 | 10 |
| -0.25 |  |  |  |  |  |  |  |  | -2.34 | -0.67 | +0.35 | - 1.91 | 11 |
| -0.43 |  |  |  |  |  |  |  |  | +2.30 | +0.66 | +1.39 | + 1.92 | 12 |
|  |  |  |  |  |  |  |  |  |  |  |  | + 2.14 |  |
| -0.71 | -5.30 |  |  |  |  |  |  | -1 | -7.60 | -1.36 | -0.34 | -16.31 | 14 |
|  | +8.18 |  |  |  |  |  |  |  | +8.18 | +1.25 | -0.75 | +16.86 |  |
|  | -2.88 | -0.27 |  |  |  |  |  |  | -2.88 | -0.55 | -0.30 | - 7.88 | 16 |
|  |  | +0.74 | -0.30 |  |  |  |  |  |  |  | -0.30 | -0.56 | 17 |
|  |  |  | +0. 32 |  |  |  |  |  |  |  |  | +0.53 <br> +0.52 | 19 |
|  |  |  |  | -4.20 |  |  |  |  |  |  |  | - 5.20 |  |
|  |  |  | -0.22 | +8.45 |  |  |  | +1 | -4. 25 | -0.09 | +0.67 | +6.56 | 21 |
|  |  |  |  | -4. 25 |  |  |  |  | +4.25 | +0.09 | -0.67 | + 1.42 | 22 |
|  |  | -0.40 +1.34 |  |  |  |  |  |  | -2.47 | -0.49 | -0.34 | - 5.70 | 23 |
|  | -0.74 | -0.94 |  |  |  |  |  |  | +5.37 | +0.93 | +0.07 | +1.34 +5.69 | 2 |
|  | +3.64 |  |  |  |  |  |  |  |  |  |  | + 4.64 |  |
|  | -2.90 |  |  |  |  |  |  |  | -2.90 | -0.4.70 | 70.27 | $\begin{array}{r}\text { + } \\ + \\ \hline\end{array}$ | 27 |
|  |  |  | -1.20 |  |  |  |  |  | -4.25 | -0.76 | -0.17 | - 7.38 | 28 |
|  |  | $-1.41$ | +1.92 |  |  |  |  |  |  |  |  | +0.92 +2.41 |  |
|  |  | +1. 25 | -0.72 |  |  |  |  |  |  |  |  | + 4.91 |  |
|  |  | +0.16 |  |  |  |  |  |  | +1.07 | +0.27 | +0.46 | +3.96 | 32 |
|  |  |  |  |  |  |  |  |  |  |  |  | -1.00 | 33 |
|  |  |  |  |  |  |  |  |  |  |  |  | +1.00 | ${ }_{35}^{34}$ |
|  |  |  |  |  |  |  |  |  |  |  |  | 0.00 | 36 |
|  |  |  |  |  |  |  |  |  |  |  |  | $+1.00$ | 37 |
|  |  |  |  |  |  |  |  |  |  |  |  | - 1.00 |  |
|  |  |  |  |  | +0.59 +1.07 |  |  |  | -2.30 +2.30 | -0.20 | -0.16 | -3.07 | 39 |
|  |  |  |  |  |  |  |  | +1 | +2.30 | +0.20 | $+0.16$ | +2.73 |  |
|  |  |  |  |  | -1.66 |  |  |  |  |  |  |  | 41 |
|  |  |  | -0.17 |  |  |  |  | -1 | +0.58 | -0.02 | -0.62 | -2.23 <br> +1.00 | ${ }_{4}^{42}$ |
|  |  |  | $\underline{+0.65}$ |  |  |  |  |  |  |  |  | +1.65 | 44 |
|  |  |  | -0.48 |  |  |  |  |  | -0.58 | +0.02 | $+0.62$ | + 0.58 | 45 |
|  |  |  |  | -3.79 |  |  |  |  |  |  |  | - 4.79 |  |
|  |  |  |  | +3.20 |  |  |  |  |  |  |  | +3.20 | 47 |
|  |  |  |  | ${ }_{+}^{+0.59}$ |  |  |  |  |  |  |  | + 1.59 |  |
|  |  |  |  |  | -4.22 |  |  |  | +0.01 | +0.14 | +0.20 | - 1.66 | ${ }^{49}$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | -2.98 | +2.73 |  |  | -1 | -0.01 | -0.14 | -0.20 | + 0.40 | 52 |
|  |  |  |  |  |  | 5. 11 |  |  |  |  |  | -6.11 |  |
|  |  |  |  |  |  | 6. 37 |  |  | -2.44 | -0.10 | -0.07 | + 2.76 | 54 |
|  |  |  |  |  | +1.49 | -1.26 |  | +1 | +2.44 | $+0.10$ | +0.07 | + 7.84 | 55 |
|  |  |  |  |  |  |  |  |  |  |  |  | - 1.00 |  |
|  |  |  |  |  |  |  |  |  |  |  |  | -1.00 | 57 |
|  |  |  |  |  |  |  |  |  |  |  |  | +2.00 |  |
|  |  |  |  |  | $\begin{array}{r} \mathbf{3 . 9 9} \\ +9.16 \end{array}$ | -1.28 |  | -1 | +1.77 | +0.08 | -0.32 | -8.74 +10.16 | 59 60 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | -1.77 | -0.08 | +0.32 |  | ${ }_{62}^{61}$ |
|  |  |  |  |  |  | -2.19 |  | +1 | -0.73 | +0.10 | +0.07 | + | 63 |
|  |  |  |  |  |  |  | 68 |  |  |  |  | +4.68 | 64 |
|  |  |  |  |  |  |  |  |  | +0.43 | -0.10 | -0.07 | 0. 50 | 65 |

54 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.
Correlate

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 66 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -1 |  |  |  |
| $\begin{aligned} & 67 \\ & 68 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  | . |  |  |  | -11 |  | +1 |  |  |  |
| $\begin{aligned} & 68 \\ & 69 \\ & 70 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  | . |  |  |  |  |  |  |  |  | -1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -1 |  | +1 |
| 71 |  |  |  |  |  |  |  |  |  |  |  |  | . |  |  |  |  | -1 | -1 | +1 |  |  |
| 73 | $\ldots$ |  |  |  |  |  |  |  |  |  |  |  | ... |  |  |  |  | +1 | +1 |  |  |  |
| 74 | $\ldots$ | . |  | .. |  | .... | .... | .... | ... |  |  | . | .... |  |  | .. |  |  |  | -1 | -i |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | +1 |  | -1 |
| 76 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 78 79 | $\cdots$ |  | . | . | ... | .. | ... | . | . |  |  | .. | .. |  |  | .. |  |  |  |  | +1 | -1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | +i |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

equations-Continued


List of corrections

|  | v's.* | Adopted $v$ 's. | $v^{2}$. |  | r's.* | $\begin{gathered} \text { Adopted } \\ v \text { 's. } \end{gathered}$ | $v^{2}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | +0.699 | +0.7 | 0.49 | 41 | -0.789 | -0.8 | 0.64 |
| 2 | +2.448 | +2.4 | 5. 76 | 42 | -1.997 | -2.0 | 4.00 |
| 3 | -3.146 | -3.2 | 10.24 | 43 | +0.028 | +0.0 | 0.00 |
| 4 | -1.369 | -1.4 | 1.96 | 44 | +1.554 | +1.6 | 2.56 |
| 5 | -0.207 | -0.2 | 0.04 | 45 | +0.581 | +0.6 | 0.36 |
| 6 | +1.576 | +1.6 | 2.56 | 46 | +0.697 | +0.7 | 0.49 |
| 7 | +0.806 | +0.7 | 0.49 | 47 | -1.478 | $-1.5$ | 2.25 |
| 8 | -1.876 | -1.9 | 3.61 | 48 | +0.781 | +0.8 | 0.64 |
| 9 | +0.498 | +0.5 | 0.25 | 49 | +0.735 | +0.8 | 0.64 |
| 10 | -0.117 | -0.1 | 0.01 | 50 | +1.145 | +1.2 | 1.44 |
| 11 | +0.688 | +0.7 | 0.49 | 51 | +0.294 | +0.3 | 0.09 |
| 12 | +3.097 | +3.1 | 9.61 | 52 | -0.317 | -0.3 | 0.09 |
| 13 | +1.159 | +1.2 | 1.44 | 53 | -1.522 | -1.5 | 2.25 |
| 14 | -2. 691 | -2.7 | 7.29 | 54 | -0.138 | -0.1 | 0.01 |
| 15 | -1.472 | -1.4 | 1.96 | 55 | -0.197 | -0.2 | 0.04 |
| 16 | -0.728 | -0.7 | 0.49 | 56 | +0.741 | +0.7 | 0.49 |
| 17 | +0.755 | +0.8 | 0.64 | 57 | +0.525 | +0.5 | 0.25 |
| 18 | -0.168 | -0.1 | 0.01 | 58 | -1.266 | -1.3 | 1.69 |
| 19 | +0.945 | +1.0 | 1.00 | 59 | -0.592 | -0.6 | 0.36 |
| 20 | -0.090 | -0.1 | 0.01 | 60 | -0.262 | -0.2 | 0.04 |
| 21 | +0.102 | +0.1 | 0.01 | 61 | +0.524 | +0.6 | 0.36 |
| 22 | -0.910 | -0.9 | 0.81 | 62 | +1.193 | +1.2 | 1.44 |
| 23 | -1.665 | -1.6 | 2.56 | 63 | +0.065 | +0.1 | 0.01 |
| 24 | +0.614 | +0.7 | 0.49 | 64 | +0.364 | +0.4 | 0.16 |
| 2.5 | -1.570 | -1.5 | 2.25 | 65 | -1.294 | -1.3 | 1.69 |
| 26 | +2.090 | +2.1 | 4.41 | 66 | -0.190 | -0.2 | 0.04 |
| 27 | +0.530 | +0.5 | 0.25 | 67 | -0.898 | -0.9 | 0.81 |
| 28 | -0.966 | -1.0 | 1.00 | 68 | +1.088 | +1.1 | 1.21 |
| 29 | - +0.183 | +0.2 | 0.04 | 69 | -0.748 | -0.8 | 0.64 |
| 30 | -1.164 | -1.2 | 1.44 | 70 | -0.490 | -0.5 | 0.25 |
| 31 | +0.311 | +0.3 | 0.09 | 71 | +0.134 | +0.1 | 0.01 |
| 32 | +1.636 | +1.6 | 2.56 | 72 | +1.234 | +1.2 | 1.44 |
| 33 | -0.457 | -0.4 | 0.16 | 73 | -0.130 | -0.2 | 0.04 |
| 34 | +1.167 | +1.2 | 1. 44 | 74 | $+1.360$ | +1.3 | 1.69 |
| 35 | -0.710 | -0.7 | 0.49 | 75 | -0.203 | -0.2 | 0.04 |
| 36 | -0.494 | -0.5 | 0.25 | 76 | -1.158 | -1.2 | 1.44 |
| 37 | +1.484 | +1.5 | 2.25 | 77 | +0.445 | $+0.4$ | 0.16 |
| 38 | -0.990 | -1.0 | 1.00 | 78 | -1.482 | -1.5 | 2.25 |
| 39 40 | -0.319 | -0.3 | 0.09 | 79 | +1.037 | +1.0 | 1.00 |
|  |  |  |  |  | Total... |  | 103.76 |

* These values result from the computation on p. 69.

56 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.



* These values result from the computation on p. 68.

Solution of

| 1 | 2 | 3 | ${ }^{23}$ | 4 | 6 | 24 | 7 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{C 1}^{6}$. | ${ }_{-0.33333}^{+2}$ | $\begin{aligned} & -2 \\ & +0.33333 \end{aligned}$ | ${ }_{-0.08833}^{+0.53}$ |  |  |  |  |  |
|  | $\begin{aligned} & \hline{ }_{-0.6667}^{+6} \\ & +5.3333 \\ & C_{2} \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.6667 \\ & +2.6607 \end{aligned}$ | $\begin{aligned} & { }_{-0.1767}^{2.20} \\ & { }_{-0.2}^{2.0233} \end{aligned}$ |  |  |  |  |  |
|  |  | $\begin{aligned} & { }_{-0.6667}^{+6} \\ & -1.3333 \\ & { }^{+4}{ }_{C_{3}} \end{aligned}$ | $\begin{aligned} & \hline-0.44 \\ & +0.1767 \\ & -1.0117 \\ & -1.275 \\ & +0.31875 \end{aligned}$ | $\begin{aligned} & \hline-2 \\ & -2 \\ & +0.5 \end{aligned}$ |  | $\begin{aligned} & \hline-7.91 \\ & \mp \quad 7.91 \\ & \hline \quad 1.9775 \end{aligned}$ |  |  |
|  |  | $\begin{aligned} & 1 \\ & \frac{1}{2} \end{aligned}$ | $\begin{gathered} \hline+2.6386 \\ -0.0468 \\ -0.7676 \\ -0.4064 \\ +1.4178 \\ C_{23} \end{gathered}$ | $\begin{aligned} & \hline+0.72 \\ & -0.6375 \\ & +0.0825 \\ & { }_{-0.05819} \end{aligned}$ |  | $\begin{aligned} & +3.7891 \\ & -\quad 2.5213 \\ & +\quad 1.2678 \\ & \pm 0.89420 \end{aligned}$ |  |  |
|  |  |  |  | $\begin{aligned} & \hline{ }_{-1}^{+6} \\ & -0.0048 \\ & +4.9952 \\ & C_{4} \end{aligned}$ | $\begin{aligned} & -2 \\ & -2.40038 \\ & +0.4 \end{aligned}$ | $-\quad .853$ <br> $+\quad 3.535$ <br> $-\quad 3.0738$ <br> $+\quad 3.5012$ <br> + |  | $\begin{aligned} & \hline-2 \\ & -2 \\ & +0.40038 \end{aligned}$ |
|  |  |  |  | ${ }^{4}$ | $\begin{aligned} & +6 \\ & { }_{-0.8008} \\ & +5.1992 \end{aligned}$ | $\begin{aligned} & \hline+\quad 2.14 \\ & +1.4018 \\ & +\quad 3.5418 \\ & +\quad 0.68122 \end{aligned}$ | -2 -2 +0.384675 | $\begin{aligned} & { }_{-0.8008}^{+2} \\ & +1.1992 \\ & { }_{-0.23065} \end{aligned}$ |
|  |  |  |  |  | $\begin{array}{r} 3 \\ 23 \\ 4 \\ 6 \end{array}$ | $\begin{gathered} \hline+156.0106 \\ -15.6420 \\ =\quad 1.1337 \\ =\quad 2.4540 \\ -\quad 2.4127 \\ +134.3682 \\ C_{24} \end{gathered}$ | $\begin{aligned} & +1.3624 \\ & { }_{-0.0101393}^{1.3624} \end{aligned}$ | +2.14 +1.4018 ${ }_{-0.8169}$ +2.7249 -0.0202794 |
|  |  |  |  |  |  | ${ }_{24}^{6}$ | $\begin{aligned} & +\begin{array}{l} +0.7693 \\ { }_{-0.0138} \\ +5.2169 \\ C_{7} \end{array} \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.4613 \\ & +0.0276 \\ & +2.4337 \\ & { }_{-0.46650} \end{aligned}$ |

normals

| 25 | 31 | 32 | 33 | 34 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & +3 \\ & -0.5 \end{aligned}$ | $-\quad 6.66$ $+\quad 1.11$ | $\begin{aligned} & -2.26 \\ & +0.37667 \end{aligned}$ | $\begin{aligned} & +3.08 \\ & -0.51333 \end{aligned}$ | $\begin{aligned} & -5.5 \\ & +0.91667 \end{aligned}$ | $\begin{array}{ll} -1.81 \\ +\quad 0.30167 \end{array}$ |
|  | +1 -1 | $\begin{array}{ll} - & 3.08 \\ + & 2.22 \\ - & 0.86 \\ + & 0.16125 \end{array}$ | $\begin{aligned} & -0.91 \\ & +0.7533 \\ & -0.1567 \\ & +0.02938 \end{aligned}$ | $\begin{aligned} & -2.00 \\ & -1.0267 \\ & -3.0267 \\ & +0.56751 \end{aligned}$ | $\begin{aligned} & +4.0 \\ & +1.8333 \\ & +5.8333 \\ & -1.09375 \end{aligned}$ | $\begin{aligned} & +11.21 \\ & +\quad 0.6033 \\ & +\quad 11.8132 \\ & -\quad 2.21499 \end{aligned}$ |
|  | $\begin{aligned} & -4 \\ & +1 \\ & \\ & -3 \\ & +0.75 \end{aligned}$ | $\begin{array}{r} \text { - } 11.77 \\ -\quad 2.22 \\ +\quad 0.43 \\ -\quad 13.56 \\ +\quad 3.39 \end{array}$ | $\begin{aligned} & -1.76 \\ & -0.7533 \\ & +0.0783 \\ & -2.435 \\ & +0.60875 \end{aligned}$ | $\begin{aligned} & -4.07 \\ & +1.0267 \\ & +1.5133 \\ & -1.53 \\ & +0.3825 \end{aligned}$ | $\begin{aligned} & +12.0 \\ & -1.8333 \\ & -2.9167 \\ & +7.25 \\ & -1.8125 \end{aligned}$ | $\begin{aligned} & -13.95 \\ & =\quad 0.6033 \\ & -\quad 5.9067 \\ & -\quad 20.4600 \\ & +\quad 5.115 \end{aligned}$ |
|  | $\begin{aligned} & \pm_{-0.2650}^{1.50} \\ & -0.9562 \\ & +0.2788 \\ & { }_{-0.19664} \end{aligned}$ | $\begin{aligned} & +\quad 5.5439 \\ & +0.5883 \\ & +\quad 0.3263 \\ & -4.3222 \\ & +\quad 2.1363 \\ & -1.50677 \end{aligned}$ | $\begin{aligned} & +0.9624 \\ & +0.1996 \\ & +0.0594 \\ & -0.7762 \\ & +0.4452 \\ & -0.31401 \end{aligned}$ | $\begin{aligned} & -0.0639 \\ & -0.2721 \\ & +1.1482 \\ & -0.4877 \\ & +0.3245 \\ & -0.22888 \end{aligned}$ | $\begin{aligned} & -2.22 \\ & +0.4858 \\ & +2.2130 \\ & +2.3109 \\ & -1.6363 \\ & +1.15411 \end{aligned}$ | $\begin{aligned} & +15.1601 \\ & \pm \quad 0.1599 \\ & =\quad 4.4816 \\ & -\quad 6.5216 \\ & +\quad 4.3166 \\ & -\quad 3.04458 \end{aligned}$ |
| $+0.67$ | $\begin{aligned} & +2 \\ & -1.5 \\ & -0.0162 \end{aligned}$ | $\begin{aligned} & \pm \quad 3.84 \\ & =6.78 \\ & -0.1243 \end{aligned}$ | $\begin{aligned} & +0.57 \\ & -1.2175 \\ & -0.0259 \end{aligned}$ | $\begin{aligned} & -0.44 \\ & -0.7650 \\ & -0.0189 \end{aligned}$ | $\begin{aligned} & -8.0 \\ & +3.6250 \\ & +0.0952 \end{aligned}$ | $\begin{aligned} & \pm \quad 6.89 \\ & -\quad 10.23 \\ & -\quad 0.2512 \end{aligned}$ |
| $\begin{aligned} & +0.67 \\ & -0.13413 \end{aligned}$ | $\begin{aligned} & +0.4838 \\ & -0.09685 \end{aligned}$ | $\begin{aligned} & -3.0643 \\ & +\quad 0.61345 \end{aligned}$ | $\begin{aligned} & -0.6734 \\ & +0.13481 \end{aligned}$ | $\begin{aligned} & -1.2239 \\ & +0.24502 \end{aligned}$ | $\begin{aligned} & -4.2798 \\ & +0.85678 \end{aligned}$ | $\begin{array}{ll} - & 3.5912 \\ + & 0.71893 \end{array}$ |
| $\begin{aligned} & -2.48 \\ & +0.2683 \end{aligned}$ | +0.1937 | $\begin{array}{ll} + & 8.25 \\ - & 1.2269 \end{array}$ | $\begin{aligned} & +1.48 \\ & -0.2697 \end{aligned}$ | $\begin{aligned} & +0.37 \\ & -0.4900 \end{aligned}$ | $\begin{aligned} & =0.0 \\ & -1.7135 \end{aligned}$ | $\begin{aligned} & +13.76 \\ & -\quad 1.4378 \end{aligned}$ |
| $\begin{aligned} & -2.2117 \\ & +0.42539 \end{aligned}$ | $\begin{aligned} & +0.1937 \\ & -0.03726 \end{aligned}$ | $\begin{array}{ll} + & 7.0231 \\ - & 1.35080 \end{array}$ | $\begin{aligned} & +1.2103 \\ & -0.23279 \end{aligned}$ | $\begin{aligned} & -0.12 \\ & +0.02308 \end{aligned}$ | $\begin{aligned} & -1.7135 \\ & +0.32957 \end{aligned}$ | $\begin{aligned} & +12.3221 \\ & -\quad 2.37000 \end{aligned}$ |
| $\begin{aligned} & +1.4732 \\ & -0.4696 \\ & +1.5067 \end{aligned}$ | $\begin{aligned} & +7.91 \\ & -5.9325 \\ & -0.2493 \\ & -0.3391 \\ & -0.1320 \end{aligned}$ | $\begin{array}{r} +151.8020 \\ -\quad 26.8149 \\ +\quad 1.9103 \\ +\quad 2.1478 \\ -\quad 4.7843 \end{array}$ | $\begin{array}{r} +24.5038 \\ -4.8152 \\ +0.3981 \\ +0.4720 \\ -0.8245 \end{array}$ | $\begin{aligned} & -7.2148 \\ & -3.0256 \\ & -0.2902 \\ & +0.8578 \\ & +0.0817 \end{aligned}$ | $\begin{aligned} & -5.7 \\ & +14.3369 \\ & +1.4632 \\ & +2.9998 \\ & +1.1673 \end{aligned}$ | $\begin{array}{r} +336.4739 \\ -\quad 40.4597 \\ +\quad 3.8599 \\ +\quad 2.5171 \\ -\quad 8.3941 \end{array}$ |
| $\begin{aligned} & +2.5103 \\ & -0.0186822 \end{aligned}$ | $\begin{aligned} & +1.2571 \\ & -0.0093556 \end{aligned}$ | $\begin{aligned} & +120.4403 \\ & -\quad 0.8963453 \end{aligned}$ | $\begin{aligned} & +18.9380 \\ & -\quad 0.1409411 \end{aligned}$ | $\begin{aligned} & -9.5911 \\ & +0.0713792 \end{aligned}$ | $\begin{aligned} & +14.2672 \\ & -0.1061799 \end{aligned}$ | $\begin{aligned} & +286.2773 \\ & -\quad 2.1305435 \end{aligned}$ |
| $\begin{aligned} & +3.31 \\ & -0.8508 \\ & -0.0255 \end{aligned}$ | $\begin{array}{r} +0.0745 \\ -0.0127 \end{array}$ | $\begin{aligned} & +\quad 3.54 \\ & +\quad 2.7016 \\ & -\quad 1.2212 \end{aligned}$ | $\begin{aligned} & +0.76 \\ & +0.4656 \\ & -0.1920 \end{aligned}$ | $\begin{aligned} & +0.80 \\ & -0.0462 \\ & +0.0972 \end{aligned}$ | -3.2 -0.6591 -0.1447 | $\begin{aligned} & +11.21 \\ & +\quad 4.7400 \\ & -\quad 2.9027 \end{aligned}$ |
| $\begin{aligned} & +2.4337 \\ & -0.46650 \end{aligned}$ | $\begin{aligned} & +0.0618 \\ & -0.01185 \end{aligned}$ | $\begin{aligned} & +5.0204 \\ & -\quad 0.96233 \end{aligned}$ | $\begin{aligned} & +1.0336 \\ & -0.19813 \end{aligned}$ | $\begin{aligned} & +0.8510 \\ & -0.16312 \end{aligned}$ | $\begin{aligned} & -4.0038 \\ & +0.76747 \end{aligned}$ | $\begin{aligned} & 13.0473 \\ & +\quad 2.50097 \end{aligned}$ |

Solution of

|  | 5 | 25 | 8 | 9 | 26 | 10 | 12 | 27 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 4 \\ \cdot \quad 6 \\ 24 \\ \hline \end{array}$ | $\begin{aligned} & +6 \\ & -0.8008 \\ & -0.2766 \\ & -0.0553 \\ & -1.1353 \\ & +3.7320 \\ & C_{5} \\ & \\ & \\ & 4 \\ & 6 \\ & 24 \\ & 7 \\ & 5 \end{aligned}$ | $\begin{aligned} & -0.62 \\ & +0.2683 \\ & +0.501 \\ & -0.0509 \\ & -1.1353 \\ & -1.0278 \\ & +0.27540 \end{aligned}$ | $-2$ $\begin{aligned} & -2 \\ & +0.53591 \end{aligned}$ | $-2$ $\begin{aligned} & -2 \\ & +0.53591 \end{aligned}$ | $+0.42$ ${ }_{-0.11254}^{+0.42}$ |  |  |  |
|  |  | $\begin{gathered} +7.2568 \\ -0.089 \\ -0.9408 \\ -0.0469 \\ -1.1353 \\ -0.2831 \\ +4.7605 \\ C_{25} \end{gathered}$ | $\begin{aligned} & \hline+0.51 \\ & -0.5508 \\ & -0.0408 \\ & +0.00857 \end{aligned}$ | $\begin{aligned} & +0.51 \\ & -0.5508 \\ & -0.0408 \\ & +0.00857 \end{aligned}$ | $\begin{aligned} & -1.1220 \\ & +0.1157 \\ & -1.0063 \\ & +0.21137 \end{aligned}$ |  |  |  |
|  |  | $\begin{array}{r} 5 \\ 25 \end{array}$ | $\begin{aligned} & +6 \\ & { }_{-1.0718}^{-1.0003} \\ & +4.9279 \\ & C_{8} \end{aligned}$ | $\begin{aligned} & \hline+2 \\ & -1.0718 \\ & -0.0003 \\ & +0.9279 \\ & -0.18830 \\ & \hline \end{aligned}$ | -1.82 +0.2251 -0.0086 -1.6035 +0.32539 | $-2$ $\begin{aligned} & -2 \\ & +0.40585 \end{aligned}$ |  |  |
|  |  |  | $\begin{array}{r} 5 \\ 25 \\ 8 \end{array}$ | $\begin{aligned} & +6 \\ & -1.0718 \\ & -0.0003 \\ & -0.1747 \\ & +4.7532 \\ & C_{9} \end{aligned}$ | $\begin{aligned} & +0.25 \\ & +0.2251 \\ & -0.0086 \\ & +0.3019 \\ & +0.7684 \\ & -0.16166 \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.3766 \\ & +2.3766 \\ & -0.5 \end{aligned}$ |  | $\begin{aligned} & +8.45 \\ & \\ & +8.45 \\ & -1.77775 \end{aligned}$ |
|  |  |  |  | $\begin{array}{r} 5 \\ 25 \\ 8 \\ 9 \end{array}$ | $\begin{gathered} +6.7354 \\ -0.0473 \\ -0.2127 \\ -0.5218 \\ -0.1242 \\ +5.8294 \\ C_{26} \end{gathered}$ | $\begin{aligned} & +0.08 \\ & -0.6508 \\ & -0.3842 \\ & -0.9550 \\ & +0.16382 \end{aligned}$ |  | $\begin{aligned} & -1.8590 \\ & -1.3660 \\ & -3.2250 \\ & +0.55323 \end{aligned}$ |
|  |  |  |  |  | $\begin{array}{r} 8 \\ 9 \\ 26 \end{array}$ | $\begin{aligned} & { }^{+6} \\ & -0.8117 \\ & -1.1883 \\ & -0.1564 \\ & +3.8436 \\ & C_{10} \end{aligned}$ |  | $\begin{aligned} & +8.45 \\ & -4.2250 \\ & -0.5283 \\ & +3.6967 \\ & { }_{-0.96178} \end{aligned}$ |
|  |  |  |  |  |  |  | ${ }^{+6} C_{12}$ | $\begin{aligned} & { }_{-0.05667}^{+0.34} \end{aligned}$ |

normals-Continued

| 11 | 13 | 31 | 32 | 33 | 34 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | +0.1937 -0.0447 -0.0255 -0.0288 +0.0947 ${ }_{-0.02538}$ | $\begin{aligned} & +8.61 \\ & -1.2269 \\ & -1.6199 \\ & -2.4425 \\ & -2.3420 \\ & +0.9787 \\ & -0.26225 \end{aligned}$ | $\begin{aligned} & +1.75 \\ & -0.2697 \\ & -0.2792 \\ & -0.3841 \\ & -0.4822 \\ & { }^{+0.3348} \\ & -0.05971 \end{aligned}$ | $\begin{aligned} & +1.46 \\ & { }_{-0.4900}^{+0.4277} \\ & +0.1945 \\ & { }_{-0.3970} \\ & +0.7952 \\ & { }_{-0.21308} \end{aligned}$ | $\begin{aligned} & -2.9 \\ & -1.7135 \\ & +0.3952 \\ & -0.2893 \\ & +1.8678 \\ & -2.6398 \\ & +0.70734 \end{aligned}$ | $\begin{aligned} & +14.86 \\ & =1.4378 \\ & =2.8421 \\ & =5.8055 \\ & -6.0566 \\ & -1.3122 \\ & +0.35161 \end{aligned}$ |
|  |  | $\begin{aligned} & -0.0649 \\ & +0.0824 \\ & -0.0235 \\ & -0.0288 \\ & +0.0261 \\ & -0.0087 \\ & +0.00183 \end{aligned}$ | $\begin{aligned} & +0.8640 \\ & +0.4110 \\ & +2.9876 \\ & -2.2501 \\ & -2.3420 \\ & +0.2695 \\ & -0.0600 \\ & +0.01260 \end{aligned}$ | $\begin{aligned} & +0.1260 \\ & +0.0003 \\ & +0.5148 \\ & -0.35388 \\ & -0.4822 \\ & +0.0922 \\ & -0.0127 \\ & +0.00267 \end{aligned}$ | $\begin{aligned} & -0.1377 \\ & +0.1642 \\ & -0.0510 \\ & +0.1792 \\ & -0.3970 \\ & +0.2190 \\ & -0.0233 \\ & +0.00489 \end{aligned}$ | $\begin{aligned} & -6.09 \\ & +0.5740 \\ & =0.7289 \\ & -0.2665 \\ & +1.8678 \\ & -0.7270 \\ & -5.3706 \\ & +1.12809 \end{aligned}$ | $\begin{aligned} & +4.2703 \\ & +0.417 \\ & +5.2417 \\ & +5.3483 \\ & -6.0866 \\ & -0.3614 \\ & -1.8024 \\ & +0.37859 \end{aligned}$ |
|  |  | $\begin{aligned} & +0.0508 \\ & { }_{-0.0001}^{0} \\ & +0.0507 \\ & { }_{-0.01029} \end{aligned}$ | $\begin{aligned} & +3.18 \\ & { }^{+3.5245} \\ & { }_{-0.0005} \\ & +3.7040 \\ & { }_{-0.75164} \end{aligned}$ | $\begin{aligned} & +0.49 \\ & +0.1794 \\ & -0.0001 \\ & +0.6693 \\ & -0.13582 \end{aligned}$ | $\begin{aligned} & -0.29 \\ & { }_{-0.42002}^{+0.42} \\ & +0.1360 \\ & { }_{-0.02760} \end{aligned}$ | $\begin{aligned} & =2.3 \\ & =1.4147 \\ & -0.0460 \\ & -3.7607 \\ & +0.76314 \end{aligned}$ | $\begin{aligned} & +3.77 \\ & +0.7032 \\ & -0.0154 \\ & +3.0516 \\ & -0.61925 \end{aligned}$ |
| $\begin{aligned} & -2 \\ & -2 \\ & +0.42077 \end{aligned}$ |  | $\begin{aligned} & +2.0508 \\ & +0.0508 \\ & -0.0001 \\ & -0.0095 \\ & +2.0412 \\ & +0.42944 \end{aligned}$ | $\begin{aligned} & +2.02 \\ & +0.5245 \\ & { }_{-0.0005} \\ & -0.6975 \\ & +1.8465 \\ & +0.38548 \end{aligned}$ | $\begin{aligned} & +1.20 \\ & +0.1794 \\ & -0.0001 \\ & -0.1260 \\ & +1.2533 \\ & +-0.263675 \end{aligned}$ | $\begin{aligned} & +1.79 \\ & +0.4262 \\ & -0.0002 \\ & -0.0256 \\ & +2.1904 \\ & { }_{-0.46083} \end{aligned}$ | $\begin{aligned} & -3.2 \\ & -1.4147 \\ & -0.0460 \\ & +0.7081 \\ & -3.9526 \\ & +0.83157 \end{aligned}$ | $\begin{aligned} & +19.02 \\ & =0.7032 \\ & =0.0154 \\ & -0.5746 \\ & +17.7270 \\ & { }^{2} \quad 3.72949 \end{aligned}$ |
| $\begin{aligned} & +0.05 \\ & +0.3233 \\ & +0.3733 \\ & -0.06404 \end{aligned}$ |  | $\begin{aligned} & -0.05 \\ & -0.0107 \\ & -0.0018 \\ & +0.0165 \\ & -0.3300 \\ & -0.3760 \\ & +0.06450 \end{aligned}$ | $\begin{aligned} & +3.9252 \\ & { }_{-0.1101} \\ & -0.0127 \\ & { }_{-0.2052}^{+1.2052} \\ & +4.7091 \\ & { }_{-0.80782} \end{aligned}$ | $\begin{aligned} & +0.5728 \\ & { }_{-0.0377}^{-0.0027} \\ & { }^{+0.2178} \\ & { }_{-0.2026} \\ & +0.5476 \\ & { }_{-0.09394} \end{aligned}$ | $\begin{aligned} & +0.0732 \\ & -0.0895 \\ & -0.0049 \\ & +0.0443 \\ & -0.3541 \\ & -0.3310 \\ & +0.05678 \end{aligned}$ | $\begin{aligned} & -2.60 \\ & +0.2971 \\ & -1.1352 \\ & -1.2237 \\ & +0.6390 \\ & -4.0228 \\ & +0.69009 \end{aligned}$ | $\begin{aligned} & +4.6556 \\ & +0.1477 \\ & +0.3810 \\ & +0.9930 \\ & -2.8657 \\ & +2.5496 \\ & -0.43737 \end{aligned}$ |
| $\begin{aligned} & -2 \\ & +1 \\ & +0.0612 \\ & -0.9388 \\ & +0.24425 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & +2 \\ & +0.0206 \\ & { }_{-1.0206} \\ & -0.0616 \\ & +0.9384 \\ & { }_{-0.24415} \\ & \hline \end{aligned}$ | $\begin{aligned} & -4.83 \\ & +1.5033 \\ & -0.9233 \\ & +0.7714 \\ & -3.4786 \\ & +0.90504 \end{aligned}$ | $\begin{aligned} & \hline-0.07 \\ & +0.2716 \\ & -0.6266 \\ & +0.0897 \\ & -0.3353 \\ & +0.08724 \end{aligned}$ | $\begin{aligned} & +1.29 \\ & +0.0552 \\ & -1.0952 \\ & -0.0542 \\ & +0.1956 \\ & -0.05089 \end{aligned}$ | $\begin{aligned} & \hline-3.2 \\ & -1.5263 \\ & +1.9763 \\ & -0.6590 \\ & -3.4090 \\ & +0.85693 \end{aligned}$ | $\begin{aligned} & +7.72 \\ & +1.2385 \\ & +8.8335 \\ & +0.4177 \\ & +0.5126 \\ & -0.13336 \end{aligned}$ |
| ${ }_{-0.33333}$ | $\begin{aligned} & -2 \\ & +0.33333 \end{aligned}$ |  | $\begin{aligned} & +4.24 \\ & { }_{-0.70667} \end{aligned}$ | $\begin{aligned} & -0.05 \\ & +0.00833 \end{aligned}$ | $\begin{aligned} & -0.87 \\ & +0.145 \end{aligned}$ | $\begin{aligned} & -1.0 \\ & +0.16667 \end{aligned}$ | $\begin{aligned} & +8.66 \\ & +1.44333 \end{aligned}$ |

Solution of

| $\begin{array}{r} 9 \\ 26 \\ 10 \\ 12 \end{array}$ | 27 | 11 | 13 | 14 | 15 | 28 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{r} +149.8778 \\ =15.0220 \\ =\quad 1.7842 \\ =\quad 3.5554 \\ =\quad 0.0193 \\ +129.4969 \\ C_{27} \end{array}$ | $\begin{aligned} & -15.67 \\ & +3.5555 \\ & +0.2065 \\ & +0.9029 \\ & -0.1133 \\ & -11.1184 \\ & +0.0858584 \end{aligned}$ | $\begin{aligned} & +1.02 \\ & +0.1133 \\ & +1.1333 \\ & -0.0087516 \end{aligned}$ | $-2.98$ $\begin{aligned} & -2.98 \\ & +0.0230121 \end{aligned}$ | $+2.98$ $\begin{aligned} & +2.98 \\ & { }_{-0.0230121} \end{aligned}$ | $\begin{aligned} & -8.1354 \\ & \\ & -\quad 8.1354 \\ & +\quad 0.0628231 \end{aligned}$ |  |  |
|  | $\begin{array}{r} 9 \\ 26 \\ 10 \\ 12 \\ 27 \end{array}$ | $+6$ <br> $-0.8415$ <br> $-0.0239$ <br> $-0.2293$ <br> $-0.6667$ <br> $-0.9546$ <br> $+3.2840$ | $+2$ $\begin{aligned} & +0.6667 \\ & +0.0973 \end{aligned}$ $+2.7640$ $-0.84166$ | $\begin{aligned} & +2 \\ & \\ & -0.2559 \\ & +1.7441 \\ & -0.53109 \end{aligned}$ | $\begin{aligned} & -2 \\ & +0.2559 \\ & -1.7441 \\ & +0.53109 \end{aligned}$ | $\begin{aligned} & +1.66 \\ & -\quad 0.6985 \\ & +\quad 0.9615 \\ & -\quad 0.29278 \end{aligned}$ |  |  |
|  |  | $\begin{aligned} & 12 \\ & 27 \\ & 11 \end{aligned}$ | $\begin{aligned} & +6 \\ & { }_{-0.6667} \\ & -0.0099 \\ & -2.3263 \\ & +2.9971 \\ & C_{13} \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.0261 \\ & -1.4679 \\ & +0.5582 \\ & -0.18625 \end{aligned}$ | $\begin{aligned} & -2 \\ & -0.0261 \\ & +1.4679 \\ & -0.5582 \\ & +0.18625 \end{aligned}$ | $\begin{aligned} & +1.66 \\ & \pm 0.0712 \\ & -0.8093 \\ & +0.9219 \\ & -0.30760 \end{aligned}$ |  |  |
|  |  |  | $\begin{aligned} & 27 \\ & 11 \\ & 13 \end{aligned}$ | $\begin{aligned} & +6 \\ & { }_{-0.0686} \\ & -0.9263 \\ & -0.1040 \\ & +4.9011 \\ & { }_{C_{14}} \end{aligned}$ | $\begin{aligned} & -2 \\ & +0.0686 \\ & +0.9263 \\ & +0.1040 \\ & -0.9011 \\ & +0.18386 \end{aligned}$ |  0.3022 <br> $\pm$ 0.1872 <br> $=$ 0.5106 <br> - 0.1717 <br> + 3.3505 <br> - 0.68362 | $+2$ ${ }^{+}{ }_{0.40807}$ |  |
|  |  |  |  | $\begin{aligned} & 27 \\ & 11 \\ & 13 \\ & 14 \end{aligned}$ | +6 -0.0686 -0.9263 -0.1040 -0.1657 +4.7354 $C_{15}$ | $\begin{aligned} & \mathbf{-} 1.94 \\ & +0.1872 \\ & +0.5106 \\ & +0.1717 \\ & +0.6160 \\ & -\quad 0.4545 \\ & +\quad 0.09598 \end{aligned}$ | $+2$ $\begin{aligned} & +0.3677 \\ & +2.3677 \\ & -0.5 \end{aligned}$ | $+2$ ${ }_{-0.42235}^{2}$ |
|  |  |  |  |  | $\begin{aligned} & 27 \\ & 11 \\ & 13 \\ & 14 \\ & 15 \end{aligned}$ | $\begin{gathered} +158.2846 \\ =\quad 0.5111 \\ =\quad 0.2815 \\ =\quad 2.2836 \\ =\quad 0.0436 \\ +154.8743 \\ C_{28} \end{gathered}$ | $\begin{aligned} & +18.86 \\ & \\ & +1.3672 \\ & +0.2273 \\ & +17.7201 \\ & -0.114416 \end{aligned}$ | $+5.48$ $\begin{aligned} & +0.1920 \\ & +5.6720 \\ & -0.036623 \end{aligned}$ |
|  |  |  |  |  |  | $\begin{aligned} & 14 \\ & 15 \\ & 28 \end{aligned}$ | $\begin{aligned} & +6 \\ & =0.8161 \\ & =1.1839 \\ & -2.0275 \\ & +1.9725 \end{aligned}$ | $\begin{aligned} & \hline+2 \\ & -1 \\ & -0.6490 \\ & +0.3510 \\ & -0.17795 \end{aligned}$ |

normals-Continued

| 18 | 29 | 31 | 32 | 33 | 34 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - |  | $\begin{aligned} & +11.43 \\ & =3.6287 \\ & -0.2000 \\ & -0.9025 \end{aligned}$ $\begin{aligned} & +6.6008 \\ & -0.0516676 \end{aligned}$ | $-53.9453$ <br> $-3.2826$ <br> $+2.6052$ <br> $+3.3456$ <br> $-0.2403$ <br> $-51.5174$ <br> $+0.3978273$ | $\begin{aligned} & -0.7272 \\ & -2.2281 \\ & +0.3029 \\ & +0.3225 \\ & +0.0028 \\ & -2.3271 \\ & +0.0178703 \end{aligned}$ | $\begin{aligned} & +9.1030 \\ & -3.8940 \\ & -0.1831 \\ & -0.1881 \\ & +0.0493 \\ & +4.8871 \\ & { }_{-0.0377391} \end{aligned}$ | $\begin{aligned} & +0.0 \\ & +7.0267 \\ & +2.2255 \\ & +3.2787 \\ & +0.0567 \\ & +8.1366 \\ & { }_{-0.0628324} \end{aligned}$ | $\begin{aligned} & +108.3339 \\ & +\quad 31.5142 \\ & \pm \quad 1.4105 \\ & -0.49300 \\ & +\quad 77.2464 \\ & -\quad 0.5965116 \end{aligned}$ |
|  |  | $\begin{aligned} & -4 \\ & +0.8589 \\ & +0.0241 \\ & +0.2292 \\ & +0.5745 \\ & -2.3133 \\ & +0.70442 \end{aligned}$ | $\begin{aligned} & +6.76 \\ & +0.7700 \\ & =0.3016 \\ & =0.8496 \\ & =1.4133 \\ & -4.4232 \\ & +0.5493 \\ & -0.16727 \end{aligned}$ | $\begin{aligned} & -0.32 \\ & +0.5274 \\ & -0.0351 \\ & -0.0819 \\ & +0.0167 \\ & -0.1998 \\ & -0.0927 \\ & +0.02823 \end{aligned}$ | $\begin{aligned} & -2.52 \\ & +0.9217 \\ & +0.0212 \\ & +0.0478 \\ & +0.2900 \\ & +0.4196 \\ & -0.8197 \\ & +0.24960 \end{aligned}$ | $\begin{aligned} & +5.0 \\ & +1.6631 \\ & +0.2576 \\ & +0.8326 \\ & +0.3333 \\ & +0.6956 \\ & +3.7938 \\ & +1.15524 \end{aligned}$ | $\begin{aligned} \hline & 3.04 \\ + & 7.4530 \\ \hline & 0.1633 \\ \pm & 0.1252 \\ + & 2.8866 \\ + & 6.6323 \\ + & 8.1269 \\ - & 2.47470 \end{aligned}$ |
|  |  | $\begin{aligned} & \hline-2 \\ & -0.0556 \\ & +1.9470 \\ & -0.116 \\ & +0.03724 \end{aligned}$ | $\begin{aligned} & \hline-2.31 \\ & +1.4133 \\ & +0.4509 \\ & -0.4623 \\ & -0.9081 \\ & +0.30299 \end{aligned}$ | $\begin{aligned} & -0.34 \\ & -0.0167 \\ & +0.0204 \\ & +0.0780 \\ & -0.2583 \\ & +0.08618 \end{aligned}$ | $\begin{aligned} & -0.36 \\ & =0.2900 \\ & -0.0428 \\ & +0.6899 \\ & -0.0029 \\ & +0.00097 \end{aligned}$ | $\begin{aligned} & { }_{-0.3333}^{3.7} \\ & -0.0712 \\ & -3.1931 \\ & +0.1024 \\ & { }_{-0.03417} \end{aligned}$ | $+\quad 7.37$ $+\quad 2.8866$ -0.6760 -6.8401 $+\quad 2.7405$ $-\quad 0.91438$ |
|  |  | $\begin{aligned} & -2 \\ & +0.1540 \\ & +1.2226 \\ & +0.0208 \\ & -0.5966 \\ & +0.12173 \end{aligned}$ | $\begin{aligned} & -2.31 \\ & =1.1855 \\ & +0.2917 \\ & +0.691 \\ & -3.6181 \\ & +0.73822 \end{aligned}$ | $\begin{aligned} & -0.34 \\ & -0.0536 \\ & +0.0492 \\ & +0.0481 \\ & -0.2963 \\ & +0.06046 \end{aligned}$ | $\begin{aligned} & -0.36 \\ & +0.1125 \\ & +0.4353 \\ & +0.0005 \\ & +0.1883 \\ & +0.03842 \end{aligned}$ | $\begin{aligned} & +5.2 \\ & +0.1872 \\ & -2.0148 \\ & -0.0191 \\ & +3.3533 \\ & +-0.6 \times 419 \end{aligned}$ |  |
| $+2$ | $+0.02$ | $\begin{aligned} & \pm 4 \\ & =0.1540 \\ & =1.2286 \\ & =0.0208 \\ & -0.1097 \end{aligned}$ | $\begin{aligned} & +3.51 \\ & +1.1855 \\ & +0.2917 \\ & -0.1691 \\ & -0.6652 \end{aligned}$ | $\begin{aligned} & +0.48 \\ & +0.0536 \\ & -0.0492 \\ & -0.0481 \\ & -0.0545 \end{aligned}$ | $\begin{aligned} & +1.23 \\ & -0.1125 \\ & -0.4353 \\ & -0.0005 \\ & +0.0346 \end{aligned}$ | $\begin{aligned} & -2.5 \\ & -0.1572 \\ & +2.0148 \\ & +0.0191 \\ & +0.6165 \end{aligned}$ | $\begin{aligned} & +13.78 \\ & +\quad 1.7776 \\ & +\quad 4.3161 \\ & +\quad 0.5104 \\ & +\quad 1.5409 \end{aligned}$ |
| ${ }_{-0.42235}^{2}$ | $\begin{aligned} & +0.02 \\ & { }_{-0.00422} \end{aligned}$ | $\begin{array}{r} +2.4869 \\ -0.52517 \\ \hline \end{array}$ | $\begin{aligned} & +4.1529 \\ & -0.87699 \end{aligned}$ | $\begin{gathered} +0.3818 \\ { }_{-0.08063} \end{gathered}$ | $\begin{aligned} & +0.7163 \\ & -0.151265 \end{aligned}$ | $\begin{aligned} & -0.0368 \\ & +0.00777 \end{aligned}$ | $\begin{array}{r} 18.3697 \\ +\quad 3.87923 \end{array}$ |
| $\begin{aligned} & +5.4 \varepsilon \\ & +0.1920 \end{aligned}$ | $\begin{aligned} & +3.2298 \\ & +0.0019 \end{aligned}$ | $\begin{aligned} & +3.82 \\ & +0.4203 \\ & +0.673 \\ & +0.0343 \\ & +0.4078 \\ & +0.2357 \end{aligned}$ | $\begin{aligned} & +6.8009 \\ & =3.2365 \\ & -0.1609 \\ & +0.2793 \\ & +\quad 2.4734 \\ & +0.3986 \end{aligned}$ | $\begin{aligned} & -0.0428 \\ & -0.1462 \\ & +0.0271 \\ & +0.0795 \\ & +0.2026 \\ & +0.0366 \end{aligned}$ | $\begin{aligned} & -0.7425 \\ & +0.3070 \\ & +0.2400 \\ & +0.0009 \\ & +0.1287 \\ & +0.0688 \end{aligned}$ | $\begin{aligned} & +6.6 \\ & +0.5112 \\ & -1.1107 \\ & -0.0315 \\ & -2.2924 \\ & -0.0035 \end{aligned}$ | +205. 2346 <br> $+\quad 4.8529$ <br> $-\quad 2.3794$ <br> $=\quad 0.3430$ <br> $\quad 5.295$ <br> $+\quad 1.7631$ |
| $\begin{aligned} & +5.6720 \\ & { }_{-0.036623} \end{aligned}$ | $\begin{aligned} & +3.2317 \\ & { }_{-0.020867} \end{aligned}$ | $\begin{aligned} & +5.5984 \\ & -0.036148 \end{aligned}$ | $\begin{aligned} & \pm 6.5549 \\ & -0.042324 \end{aligned}$ | $\begin{aligned} & { }_{-0.001012}^{+0.156 \mathrm{~S}} \end{aligned}$ | $\begin{aligned} & -0.2545 \\ & +0.001643 \end{aligned}$ | $\begin{aligned} & +3.6731 \\ & { }_{-0.023717} \end{aligned}$ | $\begin{aligned} & +202.8988 \\ & -\quad 1.310087 \end{aligned}$ |
| $\begin{aligned} & +2 \\ & -1 \\ & -0.6490 \end{aligned}$ | $+0.02$ <br> $-0.01$ <br> $-0.3698$ | $\begin{aligned} & +2 \\ & +0.2435 \\ & =1.2435 \\ & -0.6405 \end{aligned}$ | $\begin{aligned} & +0.67 \\ & +1.4764 \\ & =2.0764 \\ & -0.7500 \end{aligned}$ |  | +0.39 -0.0768 <br> -0.3582 +0.0291 | +2.8 +1.3684 $+0.0184$ $-0.4203$ | $\begin{aligned} & +38.76 \\ & -\quad 3.4201 \\ & -\quad 9.1849 \\ & -\quad 23.2149 \end{aligned}$ |
| $\begin{aligned} & +0.3510 \\ & { }_{-0.17795} \end{aligned}$ | $\begin{aligned} & -0.3598 \\ & +0.18241 \end{aligned}$ | $\begin{aligned} & +0.3595 \\ & -0.18226 \end{aligned}$ | $\begin{aligned} & -0.6500 \\ & +0.34474 \end{aligned}$ | $\begin{aligned} & -0.0679 \\ & +0.03442 \end{aligned}$ | $\begin{aligned} & -0.0159 \\ & +0.00506 \end{aligned}$ | $\begin{aligned} & +1.0297 \\ & -0.52203 \end{aligned}$ | $\begin{aligned} & 2.9401 \\ & -\quad 1.45054 \end{aligned}$ |

$91865^{\circ}-15-5$

64 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.
Solution of

|  | 17 | 18 | 29 | 19 | 20 | 21 | 22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 15 \\ & 23 \\ & 16 \end{aligned}$ | $\begin{aligned} & +6 \\ & -0.8447 \\ & -0.2077 \\ & -0.0625 \\ & +4.8851 \\ & C_{17} \end{aligned}$ | $\begin{aligned} & { }^{+2} \\ & -0.8447 \\ & -0.2077 \\ & -0.0625 \\ & +0.8851 \\ & -0.18118 \end{aligned}$ | $\begin{array}{ll} + & 8.60 \\ \pm & 0.0084 \\ + & 0.1184 \\ + & 0.0640 \\ + & 8.5372 \\ - & 1.74760 \end{array}$ | $-2$ $\begin{aligned} & -2 \\ & +0.40941 \end{aligned}$ |  |  |  |
|  | $\begin{aligned} & 15 \\ & 28 \\ & 16 \\ & 17 \end{aligned}$ | $\begin{aligned} & +6 \\ & -0.8447 \\ & -0.2077 \\ & -0.0625 \\ & -0.1604 \\ & +4.7247 \\ & { }^{18} \\ & \\ & \\ & \\ & 15 \\ & 28 \\ & 16 \\ & 17 \\ & 18 \end{aligned}$ | $\begin{array}{ll} = & 2.30 \\ = & 0.0084 \\ = & 0.1184 \\ \mathbf{~} & 0.0640 \\ & 1.5468 \\ & 3.9096 \\ + & 0.82748 \end{array}$ | $\begin{aligned} & +2 \\ & +0.3624 \\ & +2.3624 \\ & -0.50001 \end{aligned}$ | $-2$ $\begin{aligned} & -2 \\ & +0.42331 \end{aligned}$ |  |  |
|  |  |  | $\begin{gathered} +127.4272 \\ =\quad 0.0001 \\ =0.0674 \\ =0.0656 \\ =\quad 14.9196 \\ -\quad 3.2351 \\ +109.1394 \\ C_{20} \end{gathered}$ | $\begin{aligned} & -8.18 \\ & \\ & +3.4952 \\ & +1.9548 \\ & -2.73 \\ & +0.025014 \end{aligned}$ | $\begin{aligned} & +0.95 \\ & -1.6550 \\ & -0.7050 \\ & +0.006460 \end{aligned}$ |  | 1 |
|  |  |  | $\begin{aligned} & 17 \\ & 18 \\ & 29 \end{aligned}$ | $\begin{aligned} & +6 \\ & -0.8188 \\ & -1.1812 \\ & -0.0683 \\ & +3.9317 \\ & C_{19} \end{aligned}$ | $\begin{aligned} & \hline-2 \\ & +1 \\ & -0.0176 \\ & -1.0176 \\ & +0.25882 \end{aligned}$ |  |  |
|  |  |  |  | $\begin{aligned} & 18 \\ & 29 \\ & 19 \end{aligned}$ | $\begin{aligned} & +6 \\ & -0.8466 \\ & =0.0046 \\ & -0.2634 \\ & +4.8854 \\ & C_{20} \end{aligned}$ | $+2$ ${ }_{-0.40938}^{+2}$ | $-2$ $\begin{aligned} & -2 \\ & +0.40938 \end{aligned}$ |
|  |  |  |  |  | 20 | $\begin{aligned} & +6 \\ & -0.8188 \\ & +5.1812 \\ & C_{21} \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.8188 \\ & +2.8188 \\ & -0.54404 \end{aligned}$ |
|  |  |  |  |  |  | $\begin{aligned} & 20 \\ & 21 \end{aligned}$ | $\begin{aligned} & { }^{+6} \\ & -0.8188 \\ & -1.5335 \\ & +3.6477 \\ & C_{22} \end{aligned}$ |

## normals-Continued

| 30 | 31 | 32 | 33 | 34 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | +2 ${ }^{+1.0503}$ -0.2050 -0.0640 +0.6507 -0.13934 | $\begin{aligned} & +0.67 \\ & \pm 1.7540 \\ & -0.2401 \\ & +0.1210 \\ & -1.2031 \\ & +0.24628 \end{aligned}$ | $\begin{aligned} & { }^{+0.002} \\ & -0.1613 \\ & -0.0057 \\ & +0.0121 \\ & -0.1349 \\ & +0.02761 \end{aligned}$ | $+0.39$ <br> $-0.3025$ <br> +0.0093 +0.0028 <br> $+0.0996$ <br> $-0.02039$ | $\begin{aligned} & -5.1 \\ & +0.0155 \\ & -0.1345 \\ & -0.1832 \\ & -5.4022 \\ & +1.105 .55 \end{aligned}$ | $\begin{aligned} & +22.06 \\ & =7.7584 \\ & =7.4300 \\ & -0.5232 \\ & +6.3475 \\ & -1.29936 \end{aligned}$ |
| - 4.3 | $\begin{aligned} & { }_{-1.0503}^{4} \\ & -0.2050 \\ & -0.0060 \\ & -0.1233 \end{aligned}$ | $\begin{aligned} & 1.7540 \\ & \mathbf{0 . 2 4 0 1} \\ & +0.1210 \\ & +0.2180 \end{aligned}$ |  | ${ }_{-0.3025}^{+0.85}$ +0.003 <br> $+0.0093$ <br> ${ }_{-0.0150}^{+0.0028}$ | $\begin{aligned} & -1.7 \\ & \pm 0.0155 \\ & \pm 0.1345 \\ & =0.1132 \\ & +0.9788 \end{aligned}$ | $\begin{aligned} & \pm 11.03 \\ & \pm 7.754 \\ & =7.4303 \\ & =0.5332 \\ & =1.1500 \end{aligned}$ |
| $\begin{aligned} & -4.33 \\ & +0.92704 \end{aligned}$ | $\begin{aligned} & +2.5574 \\ & { }_{-0.542 S} \end{aligned}$ | $\begin{aligned} & -4.6551 \\ & +0.98527 \end{aligned}$ | $\begin{aligned} & -0.0505 \\ & +0.01069 \end{aligned}$ | $\begin{aligned} & +0.5416 \\ & { }_{-0.11463} \end{aligned}$ | $\begin{aligned} & 1.0234 \\ & +0.21661 \end{aligned}$ | $\begin{aligned} & -5.53 .5 \\ & +1.23447 \end{aligned}$ |
| +8.0144 -3.6244 | $\begin{aligned} & -0.93 \\ & -0.0105 \\ & -0.1163 \\ & +0.0656 \\ & +1.1196 \\ & +2.1162 \end{aligned}$ | $\begin{aligned} & -0.017 \\ & =0.136 \\ & =0.124 \\ & +2.120 \\ & \hline-3.859 \end{aligned}$ | $\begin{aligned} & -1.5212 \\ & -0.0016 \\ & -0.0033 \\ & -0.0034 \\ & +0.0258 \\ & -0.0418 \end{aligned}$ | $\begin{aligned} & -0.0030 \\ & +0.0053 \\ & +0.0029 \\ & -0.174 \\ & +0.4482 \end{aligned}$ | $\begin{aligned} & -18.4 \\ & +0.0002 \\ & +0.066 \\ & +0.188 \\ & \pm .9 .479 \\ & \hline 0.8468 \end{aligned}$ | +79.9581 $+\quad 0.0776$ -4.239 +0.5393 -01.029 -4.8263 |
| $\begin{aligned} & +4.3900 \\ & -0.040224 \end{aligned}$ | $\begin{aligned} & -0.0051 \\ & +0.000596 \end{aligned}$ | $\begin{aligned} & -39.6905 \\ & +0.36366 \end{aligned}$ | $\begin{aligned} & -1.3445 \\ & +0.012319 \end{aligned}$ | $\begin{aligned} & +0.9941 \\ & { }_{-0.009109} \end{aligned}$ | $\begin{aligned} & \text { + 9. } 69 \leqslant 5 \\ & +0.0 \leqslant \leqslant \$ 27 \end{aligned}$ | $\begin{aligned} & +60.2939 \\ & -0.559449 \end{aligned}$ |
| $\begin{aligned} & -4.39 \\ & +2.19 \\ & +0.1098 \end{aligned}$ | $\begin{aligned} & +2 \\ & +0.2 \pi 87 \\ & =1.2 \pi 77 \\ & -0.0016 \end{aligned}$ | $\begin{aligned} & -3.27 \\ & \mathbf{0 . 4 9 2 6} \\ & +2.3727 \\ & +0.9928 \end{aligned}$ | $\begin{aligned} & +0.03 \\ & -0.0525 \\ & { }_{-0.0252}^{+0.0252} \end{aligned}$ | $\begin{aligned} & \hline+0.23 \\ & +0.0208 \\ & +0.2008 \\ & +0.0249 \end{aligned}$ | $\begin{aligned} & \pm 2.1117 \\ & \pm .2 .2117 \\ & \mathbf{0 . 5 1 7} \end{aligned}$ | $\begin{aligned} & \hline 7.47 \\ & +2.598 \\ & +2.9963 \\ & +1.5082 \end{aligned}$ |
| $\begin{aligned} & -2.0002 \\ & +0.52908 \end{aligned}$ | $\begin{aligned} & +0.9984 \\ & { }_{-0.25394} \end{aligned}$ | $\begin{aligned} & -2.4278 \\ & +0.61749 \end{aligned}$ | $\begin{aligned} & -0.0336 \\ & +0.00555 \end{aligned}$ | $\begin{aligned} & +0.000633 \end{aligned}$ | $\begin{aligned} & 0.1575 \\ & +0.04006 \end{aligned}$ | $\begin{aligned} & \quad 0.4467 \\ & +0.11361 \end{aligned}$ |
| $\begin{aligned} & \hline+4.50 \\ & \pm .1 .8511 \\ & +0.0514 \\ & -0.0354 \end{aligned}$ | $\begin{aligned} & -4.0 \$ 2 \\ & +0.0 .004 \\ & +0.0004 \end{aligned}$ | $\begin{aligned} & \pm 0.19 \\ & \mathbf{1} .906 \\ & =0.2066 \\ & -0.6254 \end{aligned}$ | $\begin{aligned} & -0.3 .5 \\ & -0.0214 \\ & -0.0077 \\ & -0.0087 \end{aligned}$ | $\begin{aligned} & \hline-0.18 \\ & +0.2293 \\ & +0.0064 \\ & +0.0064 \end{aligned}$ | $\begin{aligned} & \pm 2.3 \\ & \pm 0.4332 \\ & 0.0 .066 \\ & +0.0408 \end{aligned}$ | $\begin{aligned} & \pm 5.411 \\ & \pm 2.490 \\ & \pm 0.3195 \\ & \hline 0.3157 \end{aligned}$ |
| $\begin{aligned} & +2.1359 \\ & -0.43720 \end{aligned}$ | $\begin{aligned} & -2.6594 \\ & +0.5436 \end{aligned}$ | $\begin{aligned} & -2.6654 \\ & +0.5455 \mathrm{~s} \end{aligned}$ | $\begin{aligned} & -0.3888 \\ & +0.07958 \end{aligned}$ | $\begin{aligned} & { }_{-0.01271}^{+0.0621} \end{aligned}$ | $\begin{array}{r} 1.5450 \\ \pm 0.37766 \end{array}$ | $\begin{aligned} & +3.2148 \\ & -0.65804 \end{aligned}$ |
| $\begin{aligned} & -10.92 \\ & =0.8744 \end{aligned}$ | $\begin{aligned} & +1.0857 \\ & +1.058 \end{aligned}$ | $\begin{aligned} & -0.93 \\ & +1.0912 \end{aligned}$ | $\begin{aligned} & -0.21 \\ & +0.1592 \end{aligned}$ | $\begin{aligned} & -0.0254 \\ & -0.021 \end{aligned}$ | $-0.75$ | +5.03 <br> 1.3161 |
| $\left\|\begin{array}{c} -11.7944 \\ +2.27638 \end{array}\right\|$ | $\begin{array}{\|} +2.0887 \\ -0.40313 \end{array}$ | $\begin{aligned} & 0.1612 \\ & -0.03111 \end{aligned}$ | $\begin{aligned} & -0.0508 \\ & +0.009 \mathrm{~s} 0 \end{aligned}$ | $\begin{aligned} & -0.0054 \\ & +0.006 i s 3 \end{aligned}$ | $\begin{array}{r} +5.3447 \\ -1.03156 \end{array}$ | $\begin{aligned} & +3.7140 \\ & -0.71682 \end{aligned}$ |
| $\begin{aligned} & -3.75 \\ & +0.874 \\ & +6.4166 \end{aligned}$ | $\begin{aligned} & \hline{ }_{-1.0857}^{3} \\ & -1.1363 \end{aligned}$ | $\begin{aligned} & \pm .12 \\ & \pm .1 .0912 \\ & -0.0687 \end{aligned}$ | $\begin{aligned} & { }_{-0.1592}^{+0.11} \\ & +0.0276 \end{aligned}$ | $\begin{aligned} & -0.06 \\ & +0.025 \\ & +0.0193 \end{aligned}$ | $\begin{aligned} & +0.1 \\ & +0.7503 \\ & +2.9077 \end{aligned}$ | $\begin{aligned} & +8.52 \\ & +1.5161 \\ & +2.0206 \end{aligned}$ |
| $\begin{aligned} & +3.5410 \\ & -0.97075 \end{aligned}$ | ${ }_{-0.21246}^{+0.7750}$ | $\pm \begin{aligned} & 1.9411 \\ & -0.53214 \end{aligned}$ | $\begin{aligned} & -0.0216 \\ & +0.00592 \end{aligned}$ | $\begin{aligned} & -0.0153 \\ & +0.00419 \end{aligned}$ | $\begin{array}{r} -2.0524 \\ +0.56266 \end{array}$ | $\begin{aligned} & +7.8155 \\ & +2.14258 \end{aligned}$ |

Solution of normals-Continued


Solution of normals-Continued


Back solution


Computation of corrections

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.007 \\ & -0.207 \\ & -0.459 \\ & +1.372 \end{aligned}$ | $\begin{aligned} & +0.007 \\ & +0.792 \\ & +0.303 \\ & -1.372 \\ & -0.003 \\ & +0.038 \\ & +2.593 \end{aligned}$ | $\begin{aligned} & +0.207 \\ & -0.792 \\ & +0.067 \\ & +0.003 \\ & -0.003 \\ & -2.593 \end{aligned}$ | $\begin{aligned} & -0.007 \\ & +0.019 \\ & -0.106 \\ & -1.275 \end{aligned}$ | -0.207 | $\begin{aligned} & +0.007 \\ & +0.207 \\ & -0.019 \\ & +0.106 \\ & +1.275 \end{aligned}$ | $\begin{aligned} & -0.174 \\ & -0.094 \\ & +0.188 \\ & +0.886 \end{aligned}$ | $\begin{array}{r} -1.949 \\ +0.073 \end{array}$ |
|  |  |  |  | $\begin{aligned} & -0.207 \\ & -0.2 \end{aligned}$ |  |  | $\begin{aligned} & -1.876 \\ & -1.9 \end{aligned}$ |
| $\begin{aligned} & +0.699 \\ & +0.7 \end{aligned}$ |  |  | $\begin{aligned} & -1.369 \\ & -1.4 \end{aligned}$ |  | $\begin{aligned} & +1.576 \\ & +1.6 \end{aligned}$ | $\begin{aligned} & +0.806 \\ & +0.7 \end{aligned}$ |  |
|  | $\begin{aligned} & +2.448 \\ & +2.4 \end{aligned}$ | $\begin{aligned} & -3.146 \\ & -3.2 \end{aligned}$ |  |  |  |  |  |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $\begin{aligned} & +0.792 \\ & +1.949 \\ & +0.008 \\ & +0.101 \\ & +1.372 \\ & +0.094 \\ & -0.188 \\ & -0.886 \end{aligned}$ | $\begin{aligned} & -0.007 \\ & -0.792 \\ & +0.200 \\ & +1.372 \\ & +0.049 \\ & -0.183 \\ & -0.756 \end{aligned}$ | $\begin{aligned} & +0.007 \\ & -0.209 \\ & -0.049 \\ & +0.183 \\ & +0.756 \end{aligned}$ | $\begin{aligned} & -0.207 \\ & +0.792 \\ & -0.359 \\ & +0.048 \\ & -0.180 \\ & +3.003 \end{aligned}$ | $\begin{aligned} & +0.207 \\ & +0.952 \end{aligned}$ | $\begin{aligned} & -0.792 \\ & -1.949 \\ & -0.593 \\ & -0.205 \\ & +1.372 \\ & -0.160 \\ & +0.371 \\ & -0.735 \end{aligned}$ | $\begin{aligned} & +0.317 \\ & +0.172 \\ & +0.341 \\ & -1.620 \end{aligned}$ | $\begin{aligned} & +1.949 \\ & -1.190 \\ & -0.457 \\ & -0.112 \\ & -0.359 \\ & -0.061 \\ & +0.150 \\ & -0.648 \end{aligned}$ |
|  |  |  |  |  |  |  |  |
|  |  |  |  | $\begin{aligned} & +1.159 \\ & +1.2 \end{aligned}$ |  |  |  |
|  |  |  |  |  |  | $\begin{aligned} & -1.472 \\ & -1.4 \end{aligned}$ |  |
|  |  | $\begin{aligned} & +0.688 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & +3.097 \\ & +3.1 \end{aligned}$ |  |  |  |  |
| $\begin{aligned} & +0.498 \\ & +0.5 \end{aligned}$ | $\begin{aligned} & -0.117 \\ & -0.1 \end{aligned}$ |  |  |  | $\begin{aligned} & -2.691 \\ & -2.7 \end{aligned}$ |  | $\begin{aligned} & =0.72 S \\ & -0.7 \end{aligned}$ |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| $\begin{aligned} & +1.190 \\ & -1.484 \\ & +0.325 \\ & +0.984 \\ & -0.260 \end{aligned}$ | $\begin{array}{r} +0.457 \\ -0.625 \\ \hline \end{array}$ | $\begin{aligned} & +1.484 \\ & -0.990 \\ & +0.451 \end{aligned}$ | $\begin{aligned} & -0.894 \\ & +0.804 \end{aligned}$ | $\begin{aligned} & -0.325 \\ & +0.990 \\ & +1.233 \\ & -0.191 \\ & -1.617 \\ & -1.372 \\ & -0.089 \\ & +0.025 \\ & +1.448 \end{aligned}$ | $\begin{aligned} & -1.233 \\ & +0.894 \\ & +0.813 \\ & +0.089 \\ & -0.025 \\ & -1.448 \end{aligned}$ | $\begin{aligned} & -1.190 \\ & +0.710 \\ & -0.532 \\ & -0.052 \\ & +0.134 \\ & { }^{0} 0.735 \end{aligned}$ | $\begin{aligned} & -0.457 \\ & -0.710 \\ & +1.781 \end{aligned}$ |
|  |  |  |  |  |  |  |  |
|  | $\begin{aligned} & -0.168 \\ & -0.1 \end{aligned}$ | $\begin{aligned} & +0.945 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -0.090 \\ & -0.1 \end{aligned}$ |  |  |  | $\begin{aligned} & +0.614 \\ & +0.7 \end{aligned}$ |
| $\begin{aligned} & +0.755 \\ & +0.8 \end{aligned}$ |  |  |  |  | $\begin{aligned} & -0.910 \\ & -0.9 \end{aligned}$ | $\begin{aligned} & -1.665 \\ & -1.6 \end{aligned}$ |  |
|  |  |  |  | $\begin{aligned} & +0.102 \\ & +0.1 \end{aligned}$ |  |  |  |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| $\begin{aligned} & -1.949 \\ & +1.190 \\ & +0.457 \\ & -0.029 \\ & -1.249 \\ & +0.113 \\ & -0.254 \\ & +0.151 \end{aligned}$ | $\begin{aligned} & +1.949 \\ & +0.141 \end{aligned}$ | $\begin{aligned} & -0.112 \\ & -0.061 \\ & +0.120 \\ & +0.583 \end{aligned}$ | $\begin{aligned} & +0.325 \\ & -1.042 \\ & -0.089 \\ & +0.207 \\ & -0.367 \end{aligned}$ | $\begin{aligned} & -1.484 \\ & +1.667 \end{aligned}$ | $\begin{array}{r} +0.710 \\ -1.874 \end{array}$ | $\begin{aligned} & -1.190 \\ & +1.484 \\ & +0.325 \\ & +1.661 \\ & +0.625 \\ & +0.067 \\ & +0.134 \\ & -0.627 \end{aligned}$ | $\begin{aligned} & +1.190 \\ & -0.710 \\ & +0.213 \\ & +0.023 \\ & -0.074 \\ & +0.994 \end{aligned}$ |
|  | $\begin{aligned} & +2.090 \\ & +2.1 \end{aligned}$ |  |  | +0.183+0.2 | $\begin{aligned} & -1.164 \\ & -1.2 \end{aligned}$ |  |  |
|  |  |  |  |  |  |  |  |
|  |  | $\begin{aligned} & +0.530 \\ & +0.5 \end{aligned}$ | $\begin{aligned} & -0.966 \\ & -1.0 \end{aligned}$ |  |  |  |  |
|  |  |  |  |  |  |  | $\begin{aligned} & +1.636 \\ & +1.6 \end{aligned}$ |
| $\begin{aligned} & -1.570 \\ & -1.5 \end{aligned}$ |  |  |  |  |  | $\begin{aligned} & +0.311 \\ & +0.3 \end{aligned}$ |  |
| 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| -0.457 | $\begin{aligned} & +0.457 \\ & +0.710 \end{aligned}$ | -0.710 | $\begin{aligned} & -1.454 \\ & +0.990 \end{aligned}$ | +1.484 | -0.990 | $\begin{aligned} & +0.003 \\ & +0.017 \\ & -0.048 \\ & +0.0 .05 \\ & -0.346 \end{aligned}$ | +1.233 |
| $\begin{aligned} & =0.457 \\ & -0.4 \end{aligned}$ |  | $\begin{aligned} & -0.710 \\ & -0.7 \end{aligned}$ |  | $\begin{aligned} & +1.484 \\ & +1.5 \end{aligned}$ | $\begin{aligned} & -0.990 \\ & -1.0 \end{aligned}$ |  | -0.741 |
|  | $\begin{aligned} & +1.167 \\ & +1.2 \end{aligned}$ |  | $\begin{aligned} & -0.494 \\ & -0.5 \end{aligned}$ |  |  |  | +0.003 +0.031 |
|  |  |  |  |  |  | -0.319-0.3 | $\begin{aligned} & -1.372 \\ & +0.048 \\ & -0.055 \\ & +0.346 \end{aligned}$ |
|  |  |  |  |  |  |  | $\begin{aligned} & +0.941 \\ & +0.9 \end{aligned}$ |

Computation of corrections-Continued


Final solution of triangles


Final solution of triangles-Continued


Final solution of triangles-Continued

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Symbol \& Station \& Observed
angle \& Correc-
tion \& Spherical angle \& Spherical excess \& \[
\begin{aligned}
\& \text { Plane an- } \\
\& \text { gle }
\end{aligned}
\] \& Logarithm \\
\hline \multirow[t]{2}{*}{\[
\begin{aligned}
\& -42+45 \\
\& -17+21 \\
\& -28+31
\end{aligned}
\]} \& Lazaro-Ken Round Lazaro Ken \& \begin{tabular}{lll}
\multicolumn{2}{c}{\(\circ, " \prime\)} \\
74 \& 42 \& 34.2 \\
78 \& 57 \& 17.7 \\
26 \& 20 \& 06.8
\end{tabular} \& \[
\begin{array}{r}
+2.6 \\
+0.7 \\
+1.3
\end{array}
\] \& \(\prime \prime\)

36.8
17.0
08.1 \& $\prime \prime$

0.6
0.7
0.6 \& - $\quad 11$
-
36.
16.3

07.5 \& $$
\begin{aligned}
& \text { 4. } 607407 \\
& 0.015051 \\
& 9.99189 \\
& 9.647016
\end{aligned}
$$ <br>

\hline \& Round-Ken Round-Lazaro \& \& $+3.2$ \& \& 1.9 \& \& | 4. 614937 |
| :--- |
| 4. 270074 | <br>

\hline \multirow[t]{2}{*}{\[
$$
\begin{aligned}
& -44+45 \\
& -37+38 \\
& -23+29
\end{aligned}
$$

\]} \& | Mid-Ken |
| :--- |
| Round |
| Mid |
| Ken | \& \[

$$
\begin{array}{rll}
23 & 37 & 23.0 \\
146 & 23 & 56.9 \\
9 & 55 & 42.9
\end{array}
$$
\] \& -1.0

-2.5
+1.2 \& 22.0
54.4

44.1 \& $$
\begin{aligned}
& 0.2 \\
& 0.1 \\
& 0.2
\end{aligned}
$$ \& 21.8

54.3

43.9 \& $$
\begin{aligned}
& \text { 4. } 474720 \\
& 0.397167 \\
& 9.743050 \\
& 9.238,60
\end{aligned}
$$ <br>

\hline \& Round-Ken Round-Mid \& \& $-2.3$ \& \& 0.5 \& \& $$
\begin{aligned}
& 4.614937 \\
& 4.110647
\end{aligned}
$$ <br>

\hline \multirow[t]{2}{*}{$$
\begin{aligned}
& -49+52 \\
& -21+22 \\
& -40+42
\end{aligned}
$$} \& Lazaro-Round Cat Lazaro Round \& \[

$$
\begin{array}{lll}
89 & 47 & 52.8 \\
26 & 22 & 55.0 \\
63 & 49 & 17.0
\end{array}
$$
\] \& -1.1

-1.0
-2.9 \& 51.7
54.0

14.7 \& $$
\begin{aligned}
& 0.2 \\
& 0.1 \\
& 0.1
\end{aligned}
$$ \& 51.5

53.9
14.6 \& 4. 270074
0.000003
9.64723
9. 952595 <br>

\hline \& Cat-Round Cat-Lazaro \& \& $-5.0$ \& \& 0.4 \& \& | 3. 917800 |
| :--- |
| 4. 223072 | <br>

\hline \multirow[t]{2}{*}{$$
\begin{aligned}
& -46+47 \\
& -40+43 \\
& -51+52
\end{aligned}
$$} \& \[

$$
\begin{aligned}
& \text { Round-Cat } \\
& \text { Spur } \\
& \text { Round } \\
& \text { Cat }
\end{aligned}
$$

\] \& \[

$$
\begin{array}{rrr}
33 & 20 & 40.5 \\
111 & 30 & 09.4 \\
35 & 09 & 14.0
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& -2.2 \\
& =0.9 \\
& -0.6
\end{aligned}
$$
\] \& 38.3

08.5

13.4 \& $$
\begin{aligned}
& 0.1 \\
& 0.0 \\
& 0.1
\end{aligned}
$$ \& 38.2

03.5
13.3 \&  <br>

\hline \& Spur-Cat Spur-Round \& \& $-3.7$ \& \& 0.2 \& \& | 4. 146374 |
| :--- |
| 3. 937953 | <br>

\hline \multirow[t]{2}{*}{$$
\begin{aligned}
& -46+48 \\
& -42+43 \\
& -20+21
\end{aligned}
$$} \& \[

$$
\begin{aligned}
& \text { Round-Lazaro } \\
& \text { Spur } \\
& \text { Round } \\
& \text { Lazaro }
\end{aligned}
$$

\] \& \[

$$
\begin{array}{rrr}
105 & 41 & 48.0 \\
47 & 40 & 51.8 \\
26 & 37 & 15.2
\end{array}
$$

\] \& | +0.1 |
| :--- |
| $+\quad 2.0$ |
| +0.2 | \& 48.1

53.8
18.4 \& 0.1
0.1
0.1 \& 48.0
53.7

18.3 \& $$
\begin{aligned}
& \text { 4. } 270074 \\
& 0.016050 \\
& 9.86858 \\
& 9.651373
\end{aligned}
$$ <br>

\hline \& Spur-Lazaro Spur-Round \& \& $+2.3$ \& \& 0.3 \& \& $$
\begin{aligned}
& 4.155468 \\
& 3.937953
\end{aligned}
$$ <br>

\hline \multirow[t]{2}{*}{\[
$$
\begin{aligned}
& -47+48 \\
& =49+51 \\
& -20+22
\end{aligned}
$$

\]} \& | Cat-Lazaro |
| :--- |
| Spur |
| Cat |
| Lazaro | \& \[

\left|$$
\begin{array}{ccc}
72 & 21 & 07.5 \\
54 & 38 & 33.8 \\
53 & 00 & 13.2
\end{array}
$$\right|
\] \& $\begin{array}{r}+2.3 \\ -0.5 \\ -0.8 \\ \hline\end{array}$ \& 09.8

38.8

12.4 \& $$
\begin{aligned}
& 0.1 \\
& 0.2 \\
& 0.2
\end{aligned}
$$ \& 09.7

38.1

12.2 \& $$
\begin{aligned}
& 4.223072 \\
& 0.020394 \\
& 9.911462 \\
& 9.902368
\end{aligned}
$$ <br>

\hline \& \[
$$
\begin{aligned}
& \text { Spur-Lazaro } \\
& \text { Spur-Cat }
\end{aligned}
$$

\] \& \& + 1.0 \& \& 0.5 \& \& | 4. 155468 |
| :--- |
| 4. 146374 | <br>

\hline \multirow[t]{2}{*}{\[
$$
\begin{aligned}
& -59+61 \\
& -52+55 \\
& -39+40
\end{aligned}
$$

\]} \& | Cat-Round Beaver Cat |
| :--- |
| Round | \& \[

$$
\begin{array}{lll}
49 & 58 & 12.0 \\
87 & 34 & 53.9 \\
42 & 26 & 51.8
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& +1.2 \\
& +0.1 \\
& +1.2
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 13.2 \\
& 54.0 \\
& 53.0
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
0.1 \\
0.1 \\
0.0 \\
0.1
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& 13.1 \\
& 54.0 \\
& E 2.9
\end{aligned}
$$

\] \& | 3. 917800 |
| :--- |
| 0. 115935 |
| 9. 999613 9.829253 | <br>

\hline \& $$
\begin{aligned}
& \text { Beaver-Round } \\
& \text { Beaver-Cat }
\end{aligned}
$$ \& \& $+2.5$ \& \& 0.2 \& \& \[

$$
\begin{aligned}
& \text { 4. } 033348 \\
& 3.862988
\end{aligned}
$$
\] <br>

\hline
\end{tabular}

Final solution of triangles-Continued

| Symbol | Station |  | Observed angle | $\begin{gathered} \text { Correc- } \\ \text { tion } \end{gathered}$ | Spherical | Spherical excess | Plane an- gle | Logarithm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Round-Beaver Snipe Round Beaver |  | - , " | " | ; | " | " |  |
| $\begin{aligned} & -56+57 \\ & -39+41 \\ & -60+61 \end{aligned}$ |  | $\left\lvert\, \begin{array}{rrr} 52 & 14 & 18.1 \\ 105 & 37 & 20.7 \\ 22 & 08 & 21.3 \end{array}\right.$ |  |  | $\begin{aligned} & 17.9 \\ & 20.2 \\ & 22.1 \end{aligned}$ |  |  | 4. 033348 |
|  |  |  |  | -0.2 -0.5 |  | 0.1 0.0 | 17.8 20.2 | 0.102063 9.983652 |
|  |  |  |  | +0.8 |  | 0.1 | 22.0 | 9. 576182 |
|  |  |  |  | + 0.1 |  | 0.2 |  |  |
|  | Snipe-Beaver Snipe-Round |  |  |  |  |  |  | 4. 119063 <br> 3.711593 |
| $\begin{aligned} & -56+58 \\ & =40+41 \\ & -50+52 \end{aligned}$ | $\begin{aligned} & \text { Round-Cat } \\ & \text { Snipe } \\ & \text { Round } \\ & \text { Cat } \end{aligned}$ |  | 79 10 11.9 <br> 63 10 28.9 <br> 37 39 24.5 | $-2.0$ | $\begin{aligned} & 09.9 \\ & 27.2 \\ & 23.0 \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0.0 \\ & 0.1 \end{aligned}$ | $\begin{aligned} & 09.9 \\ & 27.2 \\ & 22.9 \end{aligned}$ | 3.917800 <br> 0.007806 <br> 9. 950551 |
|  |  |  |  | - 1.0 |  |  |  |  |
|  |  |  |  | -1.5 |  |  |  |  |
|  |  |  |  | -5.2 |  | 0.1 |  |  |
|  | Snipe-Round |  |  |  |  |  |  | 3. 876157 <br> 3.711593 |
| $\begin{aligned} & -57+58 \\ & =59+60 \\ & -50+55 \end{aligned}$ | Beaver-Cat <br> Snipe <br> Beaver <br> Cat | $\begin{array}{rrr} 26 & 55 & 53.8 \\ 27 & 49 & 50.7 \\ 125 & 14 & 18.4 \end{array}$ |  | $-1.8$ | $\begin{aligned} & 52.0 \\ & 51.1 \\ & 17.0 \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0.0 \\ & 0.1 \end{aligned}$ | $\begin{aligned} & 52.0 \\ & 51.1 \\ & 16.9 \end{aligned}$ | 3. 862988 <br> 0. 343980 <br> 9. 669189 <br> 9. 912095 |
|  |  |  |  | 1.8 +0.4 |  |  |  |  |
|  |  |  |  | -1.4 |  |  |  |  |
|  |  |  |  | - 2.8 |  | 0.1 |  |  |
|  | Snipe-Cat Snip-Beaver |  |  |  |  |  |  | 3. 876157 <br> 4. 119063 |
| $\begin{aligned} & -67+68 \\ & -59+62 \\ & -53+55 \end{aligned}$ | Beaver-Cat Khwain Beaver Cat | $\begin{array}{lll} 62 & 13 & 29.4 \\ 58 & 43 & 17.2 \\ 59 & 03 & 08.4 \end{array}$ |  | +2.0 | $\begin{aligned} & 31.4 \\ & 19.0 \\ & 09.7 \end{aligned}$ | $\begin{aligned} & 0.1 \\ & 0.0 \\ & 0.0 \end{aligned}$ | $\begin{aligned} & 31.3 \\ & 19.0 \\ & 09.7 \end{aligned}$ | 3. 862988 <br> 0.053161 <br> 9.931792 <br> 9. 933305 |
|  |  |  |  | +1.8 |  |  |  |  |
|  |  |  |  | +1.3 |  |  |  |  |
|  | Khwain-Cat <br> Khwain-Beaver |  |  | $+5.1$ |  | 0.1 |  |  |
|  |  |  |  |  |  |  |  | 3. 847941 <br> 3. 849454 |
| $\begin{aligned} & -71+72 \\ & =59+63 \\ & -54+55 \end{aligned}$ | $\begin{aligned} & \text { Beaver-Cat } \\ & \text { Lim } \\ & \text { Beaver } \\ & \text { Cat } \end{aligned}$ | $\left\|\begin{array}{rrr} 36 & 34 & 55.5 \\ 102 & 39 & 45.5 \\ 40 & 45 & 17.4 \end{array}\right\|$ |  | $+1.1$ | $\begin{aligned} & 56.6 \\ & 46.2 \\ & 17.3 \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0.0 \\ & 0.1 \end{aligned}$ | $\begin{aligned} & 56.6 \\ & 46.2 \\ & 17.2 \end{aligned}$ | $\begin{aligned} & 3.862988 \\ & 0.224770 \\ & 9.98906 \\ & 9.814795 \end{aligned}$ |
|  |  |  |  | +0.7 |  |  |  |  |
|  |  |  |  | -0.1 |  |  |  |  |
|  |  |  |  | + 1.7 |  | 0.1 |  |  |
|  | $\begin{aligned} & \text { Lim-Cat } \\ & \text { Lim-Beaver } \end{aligned}$ |  |  |  |  |  |  | 4.077064 <br> 3.902553 |
| $\begin{aligned} & -71+73 \\ & -62+63 \\ & -66+67 \end{aligned}$ | Beaver-Khwain Lim <br> Beaver <br> Khwain | $\begin{array}{lll} 59 & 25 & 24.7 \\ 43 & 56 & 28.3 \\ 76 & 38 & 09.2 \end{array}$ |  | -0.3 | $\begin{aligned} & 24.4 \\ & 27.2 \\ & 08.5 \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0.0 \\ & 0.1 \end{aligned}$ | $\begin{aligned} & 24.4 \\ & 27.2 \\ & 08.4 \end{aligned}$ | 3. 849454 <br> 0.065022 <br> 9. 841307 <br> 9. 988077 |
|  |  |  |  | - 1.1 |  |  |  |  |
|  |  |  |  | $-0.7$ |  |  |  |  |
|  |  |  |  | $-2.1$ |  | 0.1 |  |  |
|  | Lim-Khwain Lim-Beaver |  |  |  |  |  |  | $\begin{aligned} & 3.755783 \\ & 3.902553 \end{aligned}$ |
| $\begin{aligned} & -72+73 \\ & -53+54 \\ & -66+68 \end{aligned}$ | Cat-Khwain <br> Lim <br> Cat <br> Khwain <br> Lim-Khwain <br> Lim-Cat | $\begin{array}{rrr} 22 & 50 & 29.2 \\ 18 & 17 & 51.0 \\ 138 & 51 & 38.6 \end{array}$ |  | -1.4 | $\begin{aligned} & 27.8 \\ & 52.4 \\ & 39.9 \end{aligned}$ | 0.1 | $\begin{aligned} & 27.7 \\ & 52.4 \\ & 39.9 \end{aligned}$ | 3. 8479410.4109729.4968709.818151 |
|  |  |  |  | +1.4 |  | 0.0 |  |  |
|  |  |  |  | +1.3 |  | 0.0 |  |  |
|  |  |  |  | +1.3 |  | 0.1 |  |  |
|  |  |  |  |  |  |  |  | 3. 755783 4.077064 |

Final solution of triangles-Continued


Final position computation,
STATION TOWER


STATION LAZARO

primary triangulation
STATION TOWER

| $\underset{\text { Third angle }}{ }{ }^{\alpha}$ | Dundas to Turn Tower and Turn |  |  |  |  | $\begin{array}{r} \circ \\ 177 \\ -113 \end{array}$ | 33 42 | $\begin{array}{r} 11 \\ 43.6 \\ 02.6 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ $\Delta \alpha$ | Dundas to Tower |  |  |  |  | - ${ }^{63}$ | 51 7 | 41.0 43.0 |
| $\alpha^{\prime}$ | Tower to Dundas |  |  |  |  | 180 243 | 43 | 58.0 |
| ${ }_{\Delta}{ }_{\phi}{ }^{\prime}$ | - ${ }^{54} 1$ | $\begin{gathered} \prime 2 \\ 38 \\ 2 \end{gathered}$ | 29, 4259 42.233 | Dundas | $\begin{gathered} \lambda \\ \Delta \lambda \\ \lambda^{\prime} \end{gathered}$ | $+^{130}$ | 55 9 | $\begin{aligned} & 20.042 \\ & 27.973 \end{aligned}$ |
| $\phi^{\prime}$ | 54 | 35 | 27.326 | Tower |  | 131 | 04 | 48.015 |
| $\cos _{B}^{s} \alpha$ | $\begin{aligned} & \text { 4. } 055419 \\ & \text { 9. } 6439895 \\ & \text { 8. } 5097372 \end{aligned}$ | $\sin _{\mathrm{C}}^{\mathrm{sin}^{2}}$ | 8.11087 <br> 9.90630 <br> 1. 55194 | $\left(\begin{array}{c} (\partial \phi)^{2} \\ \hline \end{array}\right.$ | $\begin{aligned} & \text { 4. } 4203 \\ & \text { 2. } 36 \$ 1 \end{aligned}$ | $s^{2} \frac{-\mathrm{h} \sin ^{2}}{\mathrm{E}} \boldsymbol{\alpha}$ |  | $\begin{aligned} & \text { 2. } 2091 \\ & \text { 8. } 0172 \\ & \text { 6. } 4528 \end{aligned}$ |
| h <br> 1st term 2d term | $\begin{array}{r} 2.2091457 \\ \prime \prime \prime \\ +161.8623 \\ +\quad 0.3708 \end{array}$ | 3d term <br> 4th term | $\begin{gathered} 9.56911 \\ \prime \prime \\ +0.0006 \\ -0.0005 \end{gathered}$ |  | 6. 7854 |  |  | 6. 6791 |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} +162.2331 \\ +0.0001 \\ \hline+162.2332 \end{array}$ | $\begin{gathered} \sin ^{8} \alpha \\ A^{\prime} \alpha \\ \sec ^{\prime} \phi^{\prime} \end{gathered}$ | +3 4. 0.55119 9.9531462 8.5087140 0.2370138 | $\begin{gathered} \operatorname{Arg} . \\ s \\ \Delta \lambda \end{gathered}$ | $\begin{aligned} & -2 \\ & +5 \end{aligned}$ | $\begin{gathered} \Delta \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}(\Delta \phi) \end{gathered}$ |  | $\begin{aligned} & 2.7543283 \\ & 9.9112981 \end{aligned}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{cc}54 & 3648.4 \\ \text { " }\end{array}$ | J $\lambda$ | $\begin{array}{r} 2.7543273 \\ +\quad \prime \prime \\ +567.9725 \end{array}$ | Corr. | +3 | - J |  | $\begin{gathered} \hline 2.6656264 \\ \prime \prime \\ +463.05 \end{gathered}$ |

STATION LAZARO


Final position computation,
STATION TOW HILL

| Second angle | Lazaro to Tower Tower and Tow Hill |  |  |  |  | $\begin{array}{r} 330 \\ +\quad 47 \end{array}$ | 18 10 | $\begin{gathered} \prime \prime \\ 17.4 \\ 08.5 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\Delta \alpha}{\alpha}$ | Lazaro to Tow Hill |  |  |  |  | - 17 | $\stackrel{28}{21}$ | $\begin{aligned} & 25.9 \\ & 07.5 \end{aligned}$ |
| $\alpha^{\prime}$ | Tow Hill to Lazaro First angle of triangle |  |  |  |  | $\begin{array}{r}180 \\ 197 \\ 21 \\ \hline\end{array}$ | 07 <br> 40 | $\begin{array}{r} 18.4 \\ 37.7 \end{array}$ |
| $\stackrel{\phi}{\phi}$ | ${ }^{\circ} 54 \mid$ | $\begin{aligned} & 1 \\ & 52 \\ & 48 \end{aligned}$ | $\begin{gathered} \prime \prime \\ 57.820 \\ 32.022 \end{gathered}$ | Lazaro | $\begin{gathered} \lambda \\ \Delta \lambda \end{gathered}$ | $+^{131}$ | 21 25 | $\begin{aligned} & 58.417 \\ & 57.250 \end{aligned}$ |
| $\phi^{\prime}$ |  | 04 | 25.798 | Tow Hill | $\lambda^{\prime}$ | 131 | 47 | 55.667 |
| $\begin{gathered} s \\ \cos \alpha \\ B \end{gathered}$ | 4. 974544 <br> 9. 9794818 <br> 8.5097191 | $\sin ^{\sin ^{2}} \boldsymbol{C}$ | 9.94909 <br> 8. 95504 <br> 1.55589 | $\stackrel{(\delta \phi)^{2}}{D}$ | $\begin{aligned} & \text { 6. } 9283 \\ & \text { 2. } 3667 \end{aligned}$ | $s^{2} \sin _{\mathrm{E}} \mathrm{sin}^{2} \alpha$ |  | $\begin{aligned} & \text { 3. } 4637 \\ & 8.9043 \\ & 6.4597 \end{aligned}$ |
| h $\substack{\text { 1st term } \\ \text { 2d term }}$ | $\begin{array}{r} 3.4637449 \\ \prime \prime \\ +2909.0080 \\ +\quad 2.8842 \end{array}$ | 3d term 4th term | $\begin{array}{r} 0.46002 \\ \prime \prime \\ +0.1972 \\ -0.0673 \end{array}$ |  | 9. 2950 |  |  | 8. 8277 |
| $\left.\begin{array}{c} \text { 3d and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{\|r} \hline+2911.8922 \\ +\quad 0.1299 \\ \hline+2912.0221 \\ \hline \end{array}$ | $\begin{gathered} \sin ^{8} \alpha \\ A^{\prime}, \\ \sec \phi^{\prime} \end{gathered}$ | -117  <br> 4.974544  <br> 9.4775129 Arg. <br> 8.5087606 8 <br> 0.2315526 $8 \lambda$ |  | $\begin{array}{r}-159 \\ +42 \\ \hline\end{array}$ |  |  | 3. 1923584 <br> 9. 9105687 108 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\left\lvert\, \begin{array}{ccc} +2912.0221 \\ \circ & \prime & \prime \prime \\ 54 & 28 & 41.9 \end{array}\right.$ | $\Delta \lambda$ | $\begin{array}{r} 3.1923584 \\ \prime \prime \\ +1557.2502 \end{array}$ | 4 Corr. | -117 | - - |  | $\begin{array}{r} 3.1029379 \\ \prime \prime \\ +1267.5 \end{array}$ |

## STATION NICHOLS


primary triangulation-Continued
STATION TOW HILL

| Third angle | Tower to Lazaro Tow Hill and Lazaro |  |  |  |  | $\begin{array}{r} \circ \\ 150 \\ -111 \end{array}$ | ' 32 09 | $\begin{gathered} \prime \prime \\ 18.7 \\ 20.3 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \alpha \\ \Delta \alpha \end{gathered}$ | Tower to Tow Hill |  |  |  |  | 39 | 22 | $\begin{aligned} & 58.4 \\ & 02.3 \end{aligned}$ |
| $\alpha^{\prime}$ | Tow Hill to Tower |  |  |  |  | 180 218 | 47 | 56.1 |
| $\begin{gathered} \phi \\ \Delta \phi \\ \phi^{\prime} \end{gathered}$ | $\begin{array}{r} 0 \\ -\quad 54 \\ - \\ \hline \end{array}$ | $\begin{array}{c\|} 1 \\ 35 \\ 31 \\ \hline \end{array}$ | $\begin{gathered} \prime \prime \\ 27.326 \\ 01.527 \\ \hline \end{gathered}$ | Tower | $\begin{array}{r} \lambda \\ \Delta \lambda \end{array}$ | $+^{131}$ | 04 43 | $\begin{aligned} & 48.015 \\ & 07.648 \end{aligned}$ |
|  | $54$ | 04 | $\text { 25. } 799$ | Tow Hill | $\lambda^{\prime}$ | 131 | 47 | $\begin{array}{r} 55.663 \\ +2 \end{array}$ |
| $\begin{gathered} 8 \\ \cos \alpha \\ B \end{gathered}$ | 4. 870159 <br> 9. 8881363 <br> 8. 5097404 | $\sin ^{s^{2} \alpha}$ | 9.74032 <br> 9.60486 <br> 1. 55122 | $(\delta \phi)^{2}$ | $\begin{aligned} & 6.5397 \\ & 2.36 \$ 3 \end{aligned}$ | $\frac{-\mathrm{h}}{s^{2} \sin ^{2} \alpha}$ |  | $\begin{aligned} & 2680 \\ & 3452 \\ & 4516 \end{aligned}$ |
| h <br> 1st term 2d term | 3.2680357 <br> $\prime \prime$ <br> +1853.6842 <br> $+\quad 7.8777$ | 3d term 4th term | $\begin{gathered} 0.89640 \\ \prime \prime \\ +0.0809 \\ -0.1160 \end{gathered}$ |  | 8.9050 |  |  | 0648 |
| $\left.\begin{array}{c} \text { 3d and 4th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{aligned} & +1861.5619 \\ & -\quad 0.0351 \\ & \hline+1861.5268 \end{aligned}$ | $\begin{gathered} s \\ \sin \alpha \\ \lambda^{\prime} \\ \sec \phi^{\prime} \end{gathered}$ | $\quad+16$ 4. 870159 9.8024314 8.5087606 0.2315526 | $\begin{gathered} \text { Arg. } \\ s \\ \Delta \lambda \end{gathered}$ | $\begin{array}{r} -98 \\ +114 \end{array}$ | $\begin{gathered} \Delta \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}\left(\Delta \phi^{2}\right) \end{gathered}$ |  | $\begin{array}{r} 4129052 \\ 9097770 \\ 44 \end{array}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{\|ccc} +1861 . & 5268 \\ \circ & , & \prime \prime \\ 54 & 19 & 56.6 \end{array}$ | $\Delta \lambda$ | $\begin{array}{r} 3.4129052 \\ \prime \prime \\ +2557.6482 \end{array}$ | Corr. | $+16$ | $-\Delta \alpha$ |  | $\begin{gathered} 3226866 \\ \prime \prime \\ -2102.3 \end{gathered}$ |

STATION NICHOLS

| Third angle <br> $\alpha$ $\Delta \alpha$ | Tow Hill to Lazaro Nichols and Lazaro |  |  |  |  | $\begin{array}{r} \circ \\ 197 \\ -\quad 36 \end{array}$ | ¢ 07 08 | $\begin{gathered} \prime \prime \\ 18.4 \\ 01.6 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tow Hill to Nichols |  |  |  |  | 160 | 59 18 | $\begin{aligned} & 16.8 \\ & 56.0 \end{aligned}$ |
| $\alpha^{\prime}$ | Nichols to Tow Hill |  |  |  |  | 180 340 | 40 | 20.8 |
| $\begin{gathered} \phi \\ \Delta \phi \\ \phi^{\prime} \end{gathered}$ | $\bullet$ +54 | 1  <br> 04  <br> 39  <br>   | $\prime \prime$ <br> 25.798 <br> 04.894 | Tow Hill | $\lambda$ $\Delta \lambda$ | ${ }_{+}^{131}$ | 47 23 | $\begin{aligned} & 55.665 \\ & 17.123 \end{aligned}$ |
|  | $54$ | 43 30 | 30. 692 | Nichols | $\lambda^{\prime}$ | 132 | 11 | $\begin{array}{r} \text { 12. } 788 \\ -1 \end{array}$ |
| $\begin{gathered} 8 \\ \cos \alpha \\ \mathrm{~B} \end{gathered}$ | 4. 885142 <br> 9. 9756388 <br> 8. 5097780 | $\sin _{\mathrm{C}}^{\boldsymbol{s}^{2}} \alpha$ | 9.77028 9.02582 1.54301 | $\left(\begin{array}{c} \partial \phi)^{2} \\ \hline \end{array}\right.$ | $\begin{aligned} & 6.7403 \\ & 2.3709 \end{aligned}$ | $s^{2} \sin _{\mathrm{E}}^{-\mathrm{h}} \alpha$ |  | $\begin{aligned} & 3706 \\ & 7959 \\ & 4373 \end{aligned}$ |
| h <br> 1st term 2d term | $\begin{array}{r} 3.3705588 \\ \prime \prime \\ -2347.2470 \\ +\quad 2.1833 \end{array}$ | 3d term 4th term | $\begin{array}{r} 0.33911 \\ \prime \prime \\ +0.1292 \\ +0.0402 \end{array}$ |  | 9.1112 |  |  | 6038 |
| $\left.\begin{array}{c} \left.\begin{array}{c} \text { 3d and } 4 \text { th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\} \end{array}\right\}$ | $\begin{array}{r} -2345.0637 \\ +\quad 0.1694 \\ \hline-2344.8943 \\ \hline \end{array}$ | $\begin{gathered} 8 \\ \sin ^{8} \alpha \\ A^{\prime} \\ \sec \phi^{\prime} \end{gathered}$ | -70 4.885142 9.5129060 8.5087447 0.2381490 | $\begin{gathered} \mathrm{Arg} . \\ 8 \\ A \lambda \end{gathered}$ | -104 +34 | $\begin{gathered} J \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}(\Delta \phi) \end{gathered}$ |  | $\begin{array}{r} 1452347 \\ 9101427 \\ 70 \end{array}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{ccc}  & 2011.0310 \\ \circ & \prime & \prime \prime \\ 54 & 23 & 58.9 \end{array}$ | J $\lambda$ | $\begin{gathered} 3.1452317 \\ \prime \prime \\ +1397.1232 \end{gathered}$ | Corr. | $-70$ | $-\int \alpha$ |  | $\begin{gathered} 0553844 \\ \prime \prime \\ 1136.02 \end{gathered}$ |

$91865^{\circ}-15-6$

Final position computation,
STATION KEN


STATION ROUND

primary triangulation-Continued
STATION KEN


STATION ROUND


Final position computation,
STATION CAT


STATION BEAVER

primary triangulation-Continued
STATION CAT


STATION BEAVER


Final position computation,
STATION LIM


STATION SOUTH TWIN

primary triangulation-Continued
STATION LIM


STATION SOUTH TWIN


Final position computation,
STATION SEAL


STATION MID

| Second anglo <br> ${ }_{\Delta \alpha}^{\alpha}$ | Round to Lazaro Lazaro and Mid |  |  |  |  | $\begin{array}{r} \circ \\ 353 \\ +\quad 51 \end{array}$ | 25 05 | $\begin{gathered} \prime \prime \\ 52.5 \\ 14.8 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Round to Mid |  |  |  |  | - 44 | 31 6 | 07.3 56.5 |
| $\alpha^{\prime}$ | Mid to Round |  | First angle of triangle |  |  | $\begin{aligned} & 180 \\ & 224 \\ & 85 \end{aligned}$ | 24 15 | $\begin{aligned} & 10.8 \\ & 12.2 \end{aligned}$ |
| ${ }_{\Delta}^{\phi}{ }_{\phi}$ | ${ }^{0} 55$ | $\begin{array}{r} 02 \\ 4 \end{array}$ | 56.147 57.777 | Round | ${ }_{\wedge}^{\lambda}$ | ${ }_{+}^{131}$ | 23 8 | $\begin{aligned} & 57.917 \\ & 28.436 \end{aligned}$ |
| $\phi^{\prime}$ | 54 | 57 | 58.370 | Mid | $\lambda^{\prime}$ | 131 | 32 | 26.353 -1 |
| $\begin{gathered} s \\ \cos \alpha \\ B \end{gathered}$ | $\begin{aligned} & \text { 4. } \\ & \text { 4. } 110647 \\ & 9.853103 \\ & 8.509707 \end{aligned}$ | $\begin{gathered} s^{s^{2}} \\ \sin ^{2} \alpha \\ C \end{gathered}$ | $\begin{aligned} & 8.2213 \\ & 9.6916 \\ & 1.5586 \end{aligned}$ | ${ }_{(8 \phi)}^{\text {D }}$ 2 | 4.947 2.366 | $s^{2} \sin _{\mathrm{E}} \mathrm{S}^{\text {¢ }} \boldsymbol{\alpha}$ |  | $\begin{aligned} & 2.473 \\ & 7.913 \\ & 6.464 \end{aligned}$ |
| h <br> 1st term 2 d term | $\begin{array}{r} 2.473457 \\ \prime \prime \\ +297.4795 \\ +\quad 0.2961 \end{array}$ | 3d term <br> 4th term | $\begin{array}{r} 9.4715 \\ \prime \prime \\ +0.0021 \\ -0.0007 \end{array}$ |  | 7.313 |  |  | 6.850 |
| $\left.\begin{array}{c} 3 \mathrm{~d} \text { and } 4 \text { th } \\ \text { terms } \\ -\Delta \phi \end{array}\right\}$ | $\begin{array}{r} +297.7756 \\ +\quad 0.0014 \\ \hline+297.7770 \end{array}$ | $\begin{gathered} s \\ \sin ^{s} \alpha \\ \operatorname{Aec}^{\prime} \phi^{\prime} \end{gathered}$ | $\begin{aligned} & 4.110647 \\ & 9.845406 \\ & 8.508740 \\ & 0.241043 \end{aligned}$ | $\begin{gathered} \text { Arg. } \\ \stackrel{1}{\Delta \lambda} \end{gathered}$ |  | $\begin{array}{r} \Delta \lambda \\ \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}(\Delta \phi) \end{array}$ |  | $\begin{aligned} & 2.706236 \\ & 9.913405 \end{aligned}$ |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | 550027.3 | $\Delta \lambda$ | $\begin{array}{r} 2.706236 \\ \prime \prime \prime \\ +508.4356 \end{array}$ | Corr. |  | $-\Delta \alpha$ |  | 2. 619641 <br> $+416.52$ |

primary triangulation-Continued
STATION SEAL


STATION MID


Final position computation,
STATION SPUR


## STATION SNIPE


primary triangulation-Continued
STATION SPUR

| Third ${ }^{\boldsymbol{\alpha}}$ angle | Cat to Round Spur and Round |  |  |  |  | $\begin{array}{r} \circ \\ 109 \\ -\quad 35 \end{array}$ | $\begin{aligned} & 42 \\ & 09 \end{aligned}$ | $\begin{array}{r} 1 \prime \prime \\ 37.4 \\ 13.4 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\Delta \alpha}{\alpha}$ | Cat to Spur |  |  |  |  | 74 | $\begin{aligned} & 33 \\ & 10 \end{aligned}$ | $\begin{aligned} & 24.0 \\ & 22.1 \end{aligned}$ |
| $\alpha^{\prime}$ | Spur to Cat |  |  |  |  | $\begin{aligned} & 180 \\ & 254 \end{aligned}$ | 23 | 01.9 |
| $\begin{gathered} \phi \\ \Delta \phi \end{gathered}$ | $\begin{array}{r} 0 \\ -55 \\ - \end{array}$ |  | 26.110 | Cat | ${ }_{\Delta \lambda}^{\lambda}$ | ${ }^{131}$ | $\begin{aligned} & 16 \\ & 12 \end{aligned}$ | $\begin{aligned} & 39.129 \\ & 39.355 \end{aligned}$ |
| $\phi^{\prime}$ | 54 | 59 | 24.827 | Spur | $\lambda^{\prime}$ | 131 | 29 | 18. 484 |
| $\begin{gathered} s \\ \cos \alpha \\ 13 \end{gathered}$ | 4. 146374 <br> 9. 425347 <br> 8. 5019709 | $\sin ^{\sin ^{2}} \alpha$ | $\begin{aligned} & \text { 8. } 2927 \\ & 9.96 \times 1 \\ & 1.5 \overline{25} 2 \end{aligned}$ | $\left(\begin{array}{c} (\delta \phi)^{2} \\ \hline \end{array}\right.$ | $\begin{aligned} & 4.163 \\ & 2.366 \end{aligned}$ | $8^{2} \sin _{\mathrm{E}}^{-\mathrm{h}} \alpha$ |  | $\begin{aligned} & \text { 2. } 081 \\ & \text { 8. } 261 \\ & \text { 6. } 464 \end{aligned}$ |
| h <br> 1st term <br> 2d term | $\begin{array}{r} 2.0 .51430 \\ +120.6230 \\ +\quad 0.6592 \end{array}$ | 3d term 4th term | $\begin{array}{r} 9.8190 \\ +0.0003 \\ +0.0006 \end{array}$ |  | 6.529 | $\begin{gathered} \Delta \lambda \\ \sin \frac{y}{y}\left(\phi+\phi^{\prime}\right) \\ \sec \frac{1}{2}\left(\phi \phi^{\prime}\right) \end{gathered}$ <br> $-\Delta \alpha$ |  | 6. 806 |
| $\begin{gathered} \left.\begin{array}{c} 3 \mathrm{~d} \text { and } 4 \mathrm{th} \\ \text { terms } \\ -\Delta \phi \end{array}\right\} \\ \hline \end{gathered}$ | $\begin{aligned} & +121.2 \times 32 \\ & -0.0003 \\ & \hline+121.2 \times 29 \end{aligned}$ | $\sin _{A^{\prime}}^{8} \alpha$ $\sec \phi^{\prime}$ | $\begin{array}{r} +1 \\ \text { 4. } 146374 \\ \text { 9. } 984029 \\ \text { 8. } 50873 . \\ 0.241303 \end{array}$ | $\begin{gathered} \text { Arg. } \\ \stackrel{8}{1 \lambda} \end{gathered}$ | +1 |  |  | 2. 880445 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | 550025.5 | 1才 | $\begin{array}{r} 2.580445 \\ \prime \prime \\ +759.3552 \end{array}$ | Corr. | +1 |  |  |  |

STATION SNIPE


Final position computation, primary triangulation-Continued
STATION KHWAIN


STATION KHWAIN*


* This is right-hand portion of computation above.

List of geographic positions-Felice Strait, Alaska, southeast Alaska datum

| Station | $\begin{aligned} & \text { Latitude } \\ & \text { and } \\ & \text { angitude } \end{aligned}$ |  |  | Seconds in meters | Azimuth |  |  | $\begin{aligned} & \text { Back } \\ & \text { azimuth } \end{aligned}$ |  |  | To station | Distance | Logarithn |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Tower } \\ 1907 \end{gathered}$ | - | , | " |  | 。 | , |  | 。 |  | " |  | Meters |  |
|  | 54 | 35 | 27.326 | 845.0 | 201 | 43 | 27.2 | 21 | 50 | 34.8 | Turn | 25288.4 | 4. 402921 |
|  |  | 04 | 48.015 | 862.3 | 243 | 43 | 58.0 | 63 | 51 | 41.0 | Dundas | 11361.1 | 4.055419 |
| $\begin{gathered} \text { Lazaro } \\ 1907 \end{gathered}$ | 131 | 5221 | $\begin{aligned} & 57.820 \\ & 58.417 \end{aligned}$ | 1788.0 | 287 | 48 | 01.7 | 108 | 09 | 12.5 | Turn | 29163.6 | 4. 464841 |
|  |  |  |  | 1041.5 | 313 | 40 | 37.9 | 134 | 02 | 23.3 | Dundas | 39641.7 | 4.598152 |
|  |  |  |  |  | 330 | 18 | 17.4 | 150 | 32 | 18.7 | Tower | 37351.0 | 4.572302 |
| $\begin{aligned} & \text { Tow Hill } \\ & 1908 \end{aligned}$ | 54 | 04 | 25.798 | 797.6 | 197 | 07 | 18.4 | 17 | 28 | 25.9 | Lazaro | 94307.0 | 4. 974544 |
|  |  | 47 | 55.665 | 1012.2 | 218 | 47 | 56.1 | 39 | 22 | 58.4 | Tower | 74158.2 | 4.870159 |
| $\underset{1907}{\text { Nichols }}$ | 54132 |  | $\begin{aligned} & 30.691 \\ & 12.787 \end{aligned}$ | $\begin{aligned} & 949.0 \\ & 228.9 \end{aligned}$ | 251 | 17 | 00.2 | 71 | 57 | 14.5 | Lazaro | 55612.1 | 4. 745169 |
|  |  |  |  |  | 281 | 21 | 55.6 | 102 | 16 | 06.1 | Tower | 72983.6 | 4.863225 |
|  |  |  |  |  | 340 | 40 | 20.8 | 160 | 59 | 16.8 | Tow Hill | 76761.2 | 4.885142 |
| $\mathrm{Ken}_{1907}$ | 54 | 54 | 34.785 | 1075.7 | 273 | 59 | 19.6 | 94 | 30 | 13.3 | Lazaro | 40495.5 | 4. 607407 |
|  | 131 | 59 | 44.326 | 789.7 | 30 | 59 | 10.4 | 210 | 49 | 47.6 | Nichols | 23934.3 | 4.379021 |
| $\begin{aligned} & \text { Seal } \\ & 1907 \end{aligned}$ | 54131 | $\begin{aligned} & 55 \\ & 35 \end{aligned}$ | $\begin{aligned} & 53.055 \\ & 38.377 \end{aligned}$ | 1640.6$6 \$ 3.4$ | 290 | 15 | 23.0 | 110 | 26 | 33.9 | Lazaro | 15582.6 | 4. 192639 |
|  |  |  |  |  | 59 | 10 | 33.9 | 238 | 41 | 29.1 | Nichols | 44485.1 | 4.648215 |
|  |  |  |  |  |  | 47 | 44.5 | 264 | 28 | 01.3 | Ken | 25868.5 | 4.412771 |
| $\underset{1914}{\mathrm{Mid}}$ | 54131 | 5732 | 58.37026.352 | 1805.0 | 309 | 39 | 23.0 | 129 | 47 | 56.8 | Lazaro | 14541.3 | 4. 162604 |
|  |  |  |  | 468.8 |  | 00 | 16.4 | 257 | 37 | 55.6 | Ken | 29834.6 | 4.474720 |
| $\begin{gathered} \text { Round } \\ 1914 \end{gathered}$ | 55131 | $\begin{aligned} & 02 \\ & 23 \end{aligned}$ | $\begin{aligned} & 56.147 \\ & 57.917 \end{aligned}$ | 1028.3 | 353 | 25 | 52.5 | 173 | 27 | 30.3 | Lazaro | 18624.0 | 4. 270074 |
|  |  |  |  |  | 44 |  | 07.3 | 224 | 24 | 10.8 | Mid | 12901.7 | 4.110647 |
|  |  |  |  |  | 68 | 08 | 29.3 | 217 | 39 | 11.5 | Ken | 41203.8 | 4.614937 |
| $\text { Spur }_{1914}$ | 54131 | 5929 | $\begin{aligned} & 24.827 \\ & 18.484 \end{aligned}$ | 767.7 | 221 | 02 | 23.6 | 41 | 06 | 46.3 | Round | 8668.7 | 3.937953 |
|  |  |  |  | 328.7 | 326 | 44 | 11.7 | 146 | 50 | 11.9 | Lazaro | 14304.3 | 4. 155468 |
| $\begin{aligned} & \text { Cat } \\ & 1914 \end{aligned}$ | 55131 | $\begin{aligned} & 01 \\ & 16 \end{aligned}$ | $\begin{aligned} & 26.110 \\ & 39.129 \end{aligned}$ | 807.4 695.2 | 19 | 54 | 45.7 | 199 | 50 | 24.3 | Lazaro | 16713.7 | 4. 223072 |
|  |  |  |  |  | 74 | 33 | 24.0 | 254 | 23 | 01.9 | Spur | 14007.9 | 4. 146374 |
|  |  |  |  |  | 109 | 42 | 37.4 | 289 | 36 | 37.8 | Round | 8275.6 | 3.917800 |
| $\begin{gathered} \text { Snipe } \\ 1914 \end{gathered}$ | 55131 | $\begin{aligned} & 00 \\ & 23 \end{aligned}$ | $\begin{gathered} 21.546 \end{gathered}$ | 340.4 | 172 | 47 | 34.8 | 352 | 47 | 05.0 | Round | 5147.5 | 3.711593 |
|  |  |  |  | 383.0 | 251 | 57 | 44.7 | 72 | 03 | 14.4 | Cat | 7518.9 | 3.876157 |
| $\begin{gathered} \text { Beaver } \\ 1914 \end{gathered}$ |  | $\begin{aligned} & 05 \\ & 14 \end{aligned}$ | $\begin{aligned} & 11.314 \\ & 36.898 \end{aligned}$ | $\begin{aligned} & 349.9 \\ & 654.5 \end{aligned}$ | 17 | 19 | 11.6 | 197 | 17 | 31.4 | Cat | 7294.4 | 3.862988 |
|  |  |  |  |  | 45 | 09 | 02.7 | 225 | 01 | 52.7 | Snipe | 13154.2 | 4.119063 |
|  |  |  |  |  | 67 | 17 | 24.8 | 247 | 09 | 44.8 | Round | 10798.1 | 4.033348 |
|  |  |  |  |  | 148 | 12 | 30.2 | 323 | 08 | 19.3 | Ham | 10275.3 | 4. 011793 |
|  |  |  |  |  | 198 | 23 | 34.8 | 18 | 25 | 02.5 | South Twin | 6003.4 | 3.778398 |
| Khwain1914 | 55131 | 0421 | $\begin{aligned} & 15.991 \\ & 03.579 \end{aligned}$ | 494.5 | 255 | 57 | 13.5 | 76 | 02 | 30.6 | Beaver | 7070.6 | 3.849454 |
|  |  |  |  | 63.5 | 318 | 10 | 44.9 | 138 | 14 | 21.7 | Cat | 7046.0 | 3.847941 |
| $\operatorname{Lim}_{1914}$ |  | 0721 | $\begin{aligned} & 20.262 \\ & 07.407 \end{aligned}$ | $\begin{aligned} & 626.6 \\ & 131.3 \end{aligned}$ | 197 |  |  | 17 |  | 14.5 | Ham | 4974.6 | 3. 696759 |
|  |  |  |  |  | 258 | 58 | 19.1 | 79 | 05 |  | South Twin | 8978.3 | 3.953192 |
|  |  |  |  |  | 299 | 53 | 37.5 | 119 | 58 | 57.8 | Beaver | 7990.1 | 3.902553 |
|  |  |  |  |  | 336 | 28 | 34.1 | 156 | 32 | 14.1 |  | 11941.6 | 4.077064 |
|  |  |  |  |  | 359 | 19 | 01.9 | 179 | 19 | 05.0 | Khwain | 5698.8 | 3.755783 |

## ADJUs'TMENT OF TRLANGULATION BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

## DEVELOPMENT OF FORMULAS

A scheme of triangulation may be adjusted not only by means of equations of condition * but also by means of observation equations in which the number of independent unknowns is just sufficient to

[^9]determine the entire triangulation. These independent unknowns may very conveniently be taken as the small corrections to the assumed approximate geographic coordinates (that is, the latitudes and longitudes) of the points in the triangulation. To form the observation equations the relation must be found that connects the small change in the direction of a line with the small arbitrary changes in the geographic coordinates of its ends. The following derivation of the formulas is based on the formulas for the computation of geographic positions given in U. S. Coast and Geodetic Survey Special Publication No. 8 and on the notation there used. A " $\delta$ " before the symbol of a quantity denotes a small arbitrary change in that quantity. $\phi$ and $\lambda$ are, respectively, the latitude and longitude of $A_{1}$, the initial point of the position computation, which may also be thought of as the occupied point, while $\phi^{\prime}$ and $\lambda^{\prime}$ are the latitude and longitude of $B_{1}$, the terminal point in the position computation, which may also be thought of as the point sighted on. By definition also,
\[

$$
\begin{gathered}
\Delta \phi=\phi^{\prime}-\phi \\
\Delta \lambda=\lambda^{\prime}-\lambda \\
h=s B \cos \alpha
\end{gathered}
$$
\]

$\alpha$ is the azimuth at $A_{1}$ of the line $A_{1} B_{1}$ reckoned from the south toward the west.

$$
\begin{aligned}
\Delta \lambda & =s A^{\prime} \sec \phi^{\prime} \sin \alpha \\
\cos \alpha & =\frac{h}{s B} \\
\sin \alpha & =\frac{\Delta \lambda}{s A^{\prime} \sec \phi^{\prime}} \\
\cot \alpha & =\frac{A^{\prime} \sec \phi^{\prime}}{B} \frac{\hbar}{\Delta \lambda}
\end{aligned}
$$

The meaning of $A^{\prime}$ and $B$ is explained in Special Publication No. 8 .
By differentiating the preceding equation and neglecting, the effects of changes in $A^{\prime}, B$, and sec $\phi^{\prime}$ there results:

$$
-\operatorname{cosec}^{2} \alpha d \alpha=\frac{A^{\prime} \sec \phi^{\prime}}{B}\left[\frac{\Delta \lambda \partial h-h \delta(\Delta \lambda)}{(\Delta \lambda)^{2}}\right]
$$

Multiplying by $-\sin ^{2} \alpha=-\frac{(\Delta \lambda)^{2}}{s^{2} A^{\prime 2} \sec ^{2} \phi^{\prime}}$
and dividing by are $1^{\prime \prime}$ in order to express $d \alpha$ in seconds instead of in radians gives,

$$
\begin{gathered}
d \alpha \text { in seconds }=\frac{1}{s^{2} B A^{\prime} \sec \phi^{\prime} \operatorname{arc} 1^{\prime \prime}}[h \partial(\Delta \lambda)-\Delta \lambda \delta h] \\
=\frac{s B \cos \alpha}{s^{2} B A^{\prime} \sec \phi^{\prime} \operatorname{arc} 1^{\prime \prime} \delta(\Delta \lambda)-\frac{s A^{\prime} \sin \alpha \sec \phi^{\prime}}{s^{2} B A^{\prime} \sec \phi^{\prime} \operatorname{arc} 1^{\prime \prime} \partial h}} \\
=\frac{\sin \alpha \cos \alpha}{s A^{\prime} \sec \phi^{\prime} \sin \alpha \operatorname{arc} 1^{\prime \prime \prime}} \delta(\Delta \lambda)-\frac{\sin \alpha \cos \alpha}{s B \cos \alpha \operatorname{arc} 1^{\prime \prime \prime}} \delta h \\
=\frac{\sin \alpha \cos \alpha}{\operatorname{arc} 1^{\prime \prime}}\left[\frac{\delta(\Delta \lambda)}{\Delta \lambda}-\frac{\delta h}{h}\right]
\end{gathered}
$$

By neglecting the variations in all the terms of the expression given for $\Delta \phi$ in Special Publication No. 8 except the first or principal term, $h$, there results,

$$
\delta(\Delta \phi)=-\delta h=\delta \phi^{\prime}-\delta \phi
$$

Evidently, also,

$$
\delta(\Delta \lambda)=\delta \lambda^{\prime}-\delta \lambda
$$

It thus appears that, to the degree of approximation here adopted, it is the difference in the changes of coordinates at the ends of a line that turns the line in azimuth. The formulas for computing $d \alpha$ become,

$$
\begin{aligned}
d \alpha \text { in sec. }= & \frac{1}{s^{2} B A^{\prime} \sec \phi^{\prime} \operatorname{arc} 1^{\prime \prime}}\left[\Delta \lambda\left(\delta \phi^{\prime}-\delta \phi\right)+h\left(\delta \lambda^{\prime}-\delta \lambda\right)\right] \\
& =\frac{\sin \alpha \cos \alpha}{\operatorname{arc} 1^{\prime \prime}}\left[\frac{\delta \phi^{\prime}-\delta \phi}{h}+\frac{\delta \lambda^{\prime}-\delta \lambda}{\Delta \lambda}\right]
\end{aligned}
$$

In practice $-\Delta \phi$ may be used for $h$, but if a position computation has been made over the line, $\log h$ will be immediately available. The change in the azimuth $\alpha^{\prime}$ at $B_{1}$ of the line $B_{1} A_{1}$ for given changes in the coordinates of $A_{1}$ and $B_{1}$ may usually be taken the same as the change in $\alpha$, the azimuth at $A_{1}$ of the line $A_{1} B_{1}{ }^{*}$ If the point $A_{1}$ is fixed $\delta \phi$ and $\delta \lambda$ are zero, and if $B_{1}$ is fixed $\delta \phi^{\prime}$ and $\delta \lambda^{\prime}$ are zero.

This formula will now be applied to three examples, first, the adjustment of a quadrilateral, next the adjustment of three new points connected with a number of fixed points, and, lastly, to a figure involving a closure in geographic position. The steps to be taken and the precautions to be observed will be explained as they arise in the course of the examples.

[^10]In all cases treated by this method, however complicated they may be, a start is made with the assumed positions of the points to be determined and the assumed azimuths and lengths of the lines sighted over. These positions, azimuths, and lengths must be consistent with each other and not too far from the final result so that the corrections to the assumed quantities are in fact small, as is implied in the development of the formulas. Otherwise it is not important how these preliminary quantities are found.

## ADJUSTMENT OF A QUADRILATERAL WITH TWO POINTS FIXED

As a simple example a quadrilateral, $A_{1}, A_{2}, A_{3}, A_{4}$, with two points, $A_{1}$ and $A_{2}$, fixed is adjusted. The coordinates of $A_{1}$ and $A_{2}$ and the length and direction of the line $A_{1}-A_{2}$ are fixed as shown in the first lines of the position computation that follows. The angles of the preliminary computation of the triangles are obtained from the list of directions. To obtain the preliminary positions, directions and lengths, the triangles $A_{1}, A_{2}, A_{3}$, and $A_{2}, A_{3}, A_{4}$ were made to close by correcting each angle by approximately one-third of the error of closure as indicated in the triangle computation. This determined the entire quadrilateral. In each of the other triangles two sides and an included angle became known and thus their remaining parts were computed.

List of observed directions *
AT $A_{2} \quad$ AT $A_{3}$

| Station | Direction $\dagger$ |  |  | Station | Direction $\dagger$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Initial |  | 00 | $00.0+z_{1}$ | Initial |  | 00 | $00.0+z_{3}$ |
| $A_{1}$ | 0 | 00 | $00.0+v_{1}$ | $A_{4}$ |  | 00 | $00.0+v_{7}$ |
| $A_{3}$ | 101 |  | 45. $1+v_{2}$ | $\boldsymbol{A}_{1}$ |  |  | $42.5+v_{8}$ |
|  |  |  | $46.3+v_{3}$ | $A_{1}$ |  |  | $35.0+v_{9}$ |
| AT $A_{1}$ |  |  |  | AT $A_{4}$ |  |  |  |
|  |  |  |  |  |  |  |  |
| $\boldsymbol{A}_{3}$ |  | 00 | $00.0+v_{4}$ | $\boldsymbol{A}_{2}$ | 0 | 00 | $00.0+v_{10}$ |
| $A_{4}$ |  | 40 | $23.5+v_{5}$ | $A_{1}$ | 25 | 15 | 16. $2+v_{11}$ |
| $A_{2}$ |  | 31 | $20.2+v_{6}$ | $\boldsymbol{A}_{3}$ | 116 | 47 | $20.0+v_{12}$ |

* See fig. 1 on p. 16.
$\dagger$ Each observed value has its symbolic correction aftixed.

Preliminary computation of triangles


* This triangle is computed from two sides and the included angle.
$91865^{\circ}-15-7$

Preliminary position computation,
STATION $\mathrm{A}_{\mathbf{3}}$


STATION A4

secondary triangulation.
STATION $A_{3}$

| $\underset{\text { Third angle }}{\alpha}$ | $\begin{aligned} & A_{1} \text { to } A_{2} \\ & A_{3} \text { and } A_{2} \end{aligned}$ |  |  |  |  |  | $\circ$ 336 -47 | 18 31 | $\begin{gathered} \prime \prime \\ 08.4 \\ 21.0 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ $\Delta \alpha$ | $A_{1}$ to $A_{3}$ |  |  |  |  |  | 288 $+\quad$ | $\begin{aligned} & 46 \\ & 10 \end{aligned}$ | 47.4 24.3 |
| $\alpha^{\prime}$ | $A_{3}$ to $A_{1}$ |  |  |  |  |  | 180 108 | 00 57 | 00.00 11.7 -.1 |
| ${ }_{\text {d }}{ }^{\text {d }}$ | ${ }^{\circ} \mathrm{60}$ | $$ | $\begin{gathered} " \prime \\ 56.416 \\ 58.607 \end{gathered}$ | $A_{1}$ |  | $\lambda$ <br> $\Delta \lambda$ | 149 | 36 11 | $\begin{aligned} & 57.360 \\ & 54.000 \end{aligned}$ |
| $\phi^{\prime}$ | 60 | 56 57. | 7. 809 | $A_{3}$ |  | $\lambda^{\prime}$ | 149 | 25 | 03.360 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{gathered} 60 \quad 57 \quad 57 \\ \quad \prime \prime \\ +118.0810 \\ +\quad 0.5258 \end{gathered}$ | $\begin{gathered} s \\ \cos \alpha \\ B \end{gathered}$ | 4.055120 <br> 9.507765 <br> 8.509295 <br> 2.072180 | $\left\|\begin{array}{c} s^{2} \\ \sin ^{2} \alpha \\ C \end{array}\right\|$ | 8. 11024 <br> 9.95248 <br> 1.65837 | $\underset{\mathrm{D}}{\mathrm{D}}$ | $\begin{aligned} & \text { 4. } 144 \\ & 2.322 \end{aligned}$ | $\underset{\mathrm{s}}{\mathrm{~s}^{2} \sin ^{2} \alpha}$ | $\begin{aligned} & 2.072 \\ & 8.063 \\ & 6.640 \end{aligned}$ |
| 1st term 2d,3d, and. 4th terms $-\Delta \phi$ |  | h | 2.072180 |  | 9.72109+0.5261+0.0003-0.0006 |  | 6. 466 |  | 6. 775 |
|  | $\begin{aligned} & \sin _{\Lambda^{8}} \alpha \\ & \sec _{\phi^{\prime}}^{\prime} \end{aligned}$ | 4. 055120 9.976241 8.50860 0.313737 | $\sin \frac{d \lambda}{2}\left(\phi+\phi^{\prime}\right)$ |  | $\begin{aligned} & \text { 2. } 853698 \\ & 9.941676 \end{aligned}$ |  |  |  |  |
|  |  | 2.853698 |  |  | 2. 795374 |  |  |  |  |
|  | $\Delta \lambda$ | $-713.9996$ | 6 - |  |  |  |  |  |  |

STATION $A_{4}$


## FORMATION OF OBSERVATION EQUATIONS

The observation equations used in making the adjustments are formed on the assumptions of the direction method.* Each pointing of the telescope is treated as an independent observation and the sum of the squares of the corrections to the separate pointings is to be made a minimum. A single pointing, however, taken by itself determines nothing, for if each of the pointings at a station be changed by the same amount the set of pointings has the same significance as before. The effect is simply a change in the zero direction, which is a purely arbitrary matter. If a set of corrections to directions at a point has been determined by any method and the mean of these corrections is not zero, the sum of the squares of these corrections can always be diminished by subtracting from each correction the mean of all of the corrections so that the algebraic sum of the reduced corrections is zero. Hence in any set of directions adjusted by the method of least squares the algebraic sum of the corrections at a point is zero. $\dagger$ To allow for this change of zero direction, or for the constant correction to all directions at a point, an unknown constant correction, " $z$," is introduced into all equations expressing the results of observations at a point, a different " $z$ " for each point where observations are taken.

The observation equation may be written,

$$
\text { Assumed azimuth }+d \alpha-\text { observed azimuth }+z-v=0 \text {. }
$$

The coefficients of the $\delta \phi$ 's and $\delta \lambda$ 's come from the last equation on page 93. As a sample, take those in the expression for $v_{9}$. $A_{1}$ corresponds to the $A_{1}$ and $A_{3}$ to the $B_{1}$ of the explanation of the formulas. Sin $\alpha, \cos \alpha, h$, and $\Delta \lambda$ come from the position computation on page 97.

| $\log \sin \alpha$ | 9. $9762 n$ | $\log h$ | 4. 7984n | $\log 4 \lambda$ | 4. $7984 n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\log \cos \alpha$ colog arc $1^{\prime \prime}$ | 9. 5078 |  | 2. 0722 |  | 2. $8537 n$ |
|  | 5. 3144 |  | 2. $7262 n$ |  | 1. 9447 |
|  | 4. $7984 n$ | Num | -532 | Numb | +88 |

The observed angles in the following formation of equations come from the list of directions on page 94.

| Azimuth $A_{2}$ to $A_{1}$ (initial direction) $\ldots \ldots$. | 156 | 20 | 26.6 |
| :--- | :--- | ---: | :--- |
| Observed angle initial direction to $A_{1} \ldots \ldots$ | 0 | 00 | $00.0-z_{1}+v_{1}$ |
| Observed azimuth $A_{2}$ to $A_{1} \ldots \ldots \ldots \ldots \ldots .156$ | 20 | $26.6-z_{1}+v_{1}$ |  |
| Assumed azimuth $A_{2}$ to $A_{1} \ldots \ldots \ldots \ldots \ldots$. | 156 | 20 | $26.6+d \alpha$ |
| Assumed azimuth - observed azimuth...... |  | $0=0.0+d \alpha+z_{1}-v_{1}$ |  |

[^11]Azimuth $A_{2}$ to $A_{1}$ is fixed. Therefore $d \alpha=0$ and $v_{1}=z_{1}$
Azimuth $A_{2}$ to $A_{1}$ (initial direstion) ........ $156 \quad 20 \quad 26.6$
Observed angle initial direction to $A_{3} \ldots \ldots .101 \quad 44 \quad 45.1-z_{1}+v_{2}$
Observed azimuth $A_{2}$ to $A_{3} \ldots \ldots \ldots \ldots \ldots .258 \quad 05 \quad 11.7-z_{1}+v_{2}$
Assumed azimuth $A_{2}$ to $A_{3} \ldots \ldots \ldots \ldots \ldots . .258 \quad 05 \quad 12.4+d \alpha$
Assumed azimuth -observed azimuth......
$0=+0.7+d \alpha+z_{1}-v_{2}$
$d \alpha=-730 \delta \phi_{3}-75 \delta \lambda_{3}$. Therefore $v_{2}=z_{1}-730 \delta \phi_{3}-75 \delta \lambda_{3}+0.7$.
Azimuth $A_{2}$ to $A_{1}$ (initial direction)
$\begin{array}{lll}156 & 20 & 26.6\end{array}$
Observed angle initial direction to $A_{4} \ldots \ldots .133 \quad 53 \quad 46.3-z_{1}+v_{3}$
Observed azimuth $A_{2}$ to $A_{4} \ldots \ldots \ldots \ldots \ldots .290 \quad 1412.9-z_{1}+v_{3}$
Assumed azimuth $A_{2}$ to $A_{4} \ldots \ldots \ldots \ldots . . . \begin{array}{ll} & 290 \\ 14 & 12.4+d \alpha\end{array}$
Assumed azimuth-observed azimuth.......
$0=-0.5+d \alpha+z_{1}-v_{3}$
$d \alpha=-1212 \delta \phi_{4}+218 \delta \lambda_{4}$. Therefore $v_{3}=z_{1}-1212 \delta \phi_{4}+218 \delta \lambda_{4}-0.5$
In the same way at $A_{1} \quad v_{4}=z_{2}-532 \quad \delta \phi_{3}+88 \quad \delta \lambda_{3}-0.8$

$$
v_{5}=z_{2}-447 \delta \phi_{4}+221 \delta \lambda_{4}-0.3
$$

$$
v_{6}=z_{2}
$$

Azimuth $A_{3}$ to $A_{4}$ (initial direction)........ $47 \quad 09 \quad 37.0$
Observed angle initial direction to $A_{4} \ldots \ldots .000000 .0-z_{3}+v_{7}$
Observed azimuth $A_{3}$ to $A_{4} \ldots \ldots \ldots \ldots \ldots$.......... $47 \quad 09 \quad 37.0-z_{3}+v_{7}$
Assumed azimuth $A_{3}$ to $A_{4} \ldots \ldots \ldots \ldots$.......... $47 \quad 0937.0+d \alpha$
Assumed azimuth -observed azimuth...... $0=0.0+d \alpha+z_{3}-v_{7}$
$d \alpha=+918\left(\delta \phi_{4}-\delta \phi_{3}\right)+414\left(\delta \lambda_{4}-\delta \lambda_{3}\right)$. Therefore $v_{7}=z_{3}-918 \delta \phi_{3}$ $-414 \delta \lambda_{3}+918 \delta \phi_{4}+414 \delta \lambda_{4}+0.0$
Similarly

```
\(v_{8}=z_{3}-730 \delta \phi_{3}-75 \delta \lambda_{3}-1.2\)
\(v_{9}=z_{3}-532 \delta \phi_{3}+88 \delta \lambda_{3}-0.4\)
\(v_{10}=z_{4}-1211 \delta \phi_{4}+218 \delta \lambda_{4}+0.0\)
\(v_{11}=z_{4}-447 \delta \phi_{4}+221 \delta \lambda_{4}+1.1\)
\(v_{12}=z_{4}-918 \delta \phi_{3}-414 \delta \lambda_{3}+918 \delta \phi_{4}+414 \delta \lambda_{4}-1.2\)
```

We have then the set of observation equations:

$$
\begin{aligned}
& v_{1}=z_{1} \\
& v_{2}=z_{1}-730 \delta \phi_{3}-75 \delta \lambda_{3}+0.7 \\
& v_{3}=z_{1}-1212 \delta \phi_{4}+218 \delta \lambda_{4}-0.5 \\
& v_{4}=z_{2}-532 \delta \phi_{3}+88 \delta \lambda_{3}-0.8 \\
& v_{5}=z_{2}-447 \delta \phi_{4}+221 \delta \lambda_{4}-0.3 \\
& v_{6}=z_{2} \\
& v_{7}=z_{3}-918 \delta \phi_{3}-414 \delta \lambda_{3}+918 \delta \phi_{4}+414 \delta \lambda_{4}+0.0 \\
& v_{8}=z_{3}-730 \delta \phi_{3}-75 \delta \lambda_{3}-1.2 \\
& v_{9}=z_{3}-532 \delta \phi_{3}+88 \delta \lambda_{3}-0.4 \\
& v_{10}=z_{4}-1211 \delta \phi_{4}+218 \delta \lambda_{4}+0.0 \\
& v_{11}=z_{4}-447 \delta \phi_{4}+221 \delta \lambda_{4}+1.1 \\
& v_{12}=z_{4}-918 \delta \phi_{3}-414 \delta \lambda_{3}+918 \delta \phi_{4}+414 \delta \lambda_{4}-1.2
\end{aligned}
$$

These equations contain $z$ 's which are of no particular interest in themselves. The normal equations might be formed and the $z$ 's eliminated in the regular way, but this work is made easier by the following mechanical rule, the effect of which is to form at once the
reduced normal equations with the $z$ 's eliminated. For the proof of the rule and further particulars see Jordan's Handbuch der Vermessungskunde, Vol. I, pages 151-171, of the third edition. Each direction is assumed to have equal weight. Write the observation equations dropping the $z$ 's and giving each unit weight. Add together as they stand all observation equations containing the $z$ for any particular point. Drop the $z$ term out and treat the resulting equation as a new observation equation with a negative weight equal to $-1 / r$, where $r$ is the total number of directions, both fixed and to be determined, that have been observed at the point in question. To reduce the new fictitious observation to unit weight it must be multiplied through by $\sqrt{\frac{-1}{r}}=\frac{1}{r} \sqrt{r} i$ where $i=\sqrt{-1}$.

Tabie 1 below shows the coefficients of the unknowns, the coefficients formed by adding the equations containing any particular $z$, and the weights. Table 2 shows these equations divided through by 100 for convenience. This has no effect on the relative weights. The table also contains the fictitious observation equations obtained by multiplying the sum equation by $\sqrt{\frac{-1}{r}}$. From
Table 2 the normal equations which do not contain the $z$ 's are formed in the ordinary way for observation equations of equal weight, using the $i$ 's strictly according to algebraic laws. Thus in the first line of Table $2(-4.23 i)^{2}$ contributes to the first diagonal coefficient not +17.8929 but $+17.8929 i^{2}$, or -17.8929 , and ( $-4.23 i$ ) $X+(1.26 i)$ contributes toward the side coefficient not -5.3298 , but $-5.3298 i^{2}$ or +5.3298 .

Table for formation of normals, No. 1

|  |  | $\delta \phi_{3}$ | $\delta \lambda_{3}$ | ¢ $\phi_{4}$ | $\delta \lambda_{4}$ | $l$ | $p$ | $\sqrt{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sum | $z_{1}$ |  |  |  |  |  | 1 |  |
|  | 1 | -730 | - 75 |  |  | +0.7 |  | 1 |
|  | ${ }_{3}^{1}$ |  |  | -1212 -1212 | +218 +218 | +0.5 +0.2 | - | 1 $0.58 i$ |
|  | 3 | - 730 | - 75 | -1212 | +218 | +0.2 | $-\frac{3}{3}$ | 0.58i |
|  | 2 1 | - 532 | $+88$ |  |  | -0.8 |  | 1 |
|  | 1 |  |  | - 447 | +221 | -0.3 | 1 | 1 |
| Sum | 3 | - 532 | $+88$ | - 447 | +221 | -1.1 | $-\frac{1}{3}$ | 0.58i |
|  | $2_{3}$ |  |  |  |  |  |  |  |
|  | 1 | -918 -730 | -414 -75 | + 918 | +414 | +0.0 -1.2 | 1 | 1 |
| Sum | 1 | - 532 | + 88 |  |  | -0.4 | 1 | 1 |
|  | 3 | -2180 | -401 | + 918 | +414 | -1.6 | $-\frac{1}{3}$ | 0. $58 i$ |
|  |  |  |  |  |  |  |  |  |
|  | 1 |  |  | -1211 | $+218$ | +0.0 | 1 | 1 |
| Sum | 1 |  |  | - 447 | +221 | +1.1 | 1 | 1 |
|  | 1 | -918 | -414 | +918 | +414 | -1.2 | -1 | ${ }_{0}^{1}$ |
|  | 3 | -918 | -414 | - 740 | +853 | -0.1 | $-\frac{1}{3}$ | 0.58i |

Table for formation of normals, No. 2

|  | $\delta \phi_{3}$ | $\lambda^{\prime} \lambda^{3}$ | $\partial \phi_{1}$ | $\delta \lambda_{4}$ | $l$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $-7.30$ | $-0.75$ |  |  | +0.007 | $-8.043$ |
| 3 |  |  | -12.12 | +2.18 | $-0.005$ | -9.945 |
| $z_{1}$ | $-4.23 i$ | $-0.44 i$ | $-7.03 i$ | +1.26i | $+0.00116 i$ | -10.43884i |
| 4 | $-5.32$ | +0.88 |  |  | $-0.008$ | $-4.448$ |
| 5 |  |  | $-4.47$ | $+2.21$ | $-0.003$ | - 2.263 |
| $z_{2}$ | $-3.09 i$ | +0.51i | $-2.59 i$ | +1.28i | $-0.00638 i$ | - $3.89638 i$ |
| 7 | - 9.18 | -4.14 | $+9.18$ | +4.14 | $+0.0$ | 0.0 |
| K | $-7.30$ | -0.75 |  |  | $-0.012$ | -8.062 |
| 9 | - 5.32 | +0.88 |  |  | -0.004 | - 4.444 |
| $z_{3}$ | $-12.64 i$ | $-2.33 i$ | $+5.32 i$ | +2.40i | -0.00928i | - 7.25928i |
| 10 |  |  | -12.11 | +2.18 | $+0.0$ | $-9.93$ |
| 11 |  |  | $-4.47$ | +2.21 | +0.011 | - 2.249 |
| 12 | $-9.18$ | -4.14 | +9.18 | +4.14 | -0.012 | $-0.012$ |
| $z_{4}$ | $-5.32 i$ | -2. $40 i$ | $-4.29 i$ | +4.95i | -0.0005si | - 7.06058i |

Normal equations

|  | $\delta \phi_{3}$ | $3 \lambda_{3}$ | $\partial \phi_{4}$ | $\delta \lambda_{4}$ | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 2 3 4 | $+116.2166$ | $\begin{aligned} & +35.0927 \\ & +25.3104 \end{aligned}$ | $\begin{aligned} & -161.8629 \\ & -75.6831 \\ & +399.2176 \end{aligned}$ | $\begin{aligned} & -10.0554 \\ & -16.9056 \\ & +24.0721 \\ & +20.0637 \end{aligned}$ | $\begin{aligned} & +0.0753 \\ & +0.0236 \\ & -0.0468 \\ & -0.01105 \end{aligned}$ | $\begin{aligned} & -20.5336 \\ & -32.1620 \\ & +185.6970 \\ & +17.16375 \end{aligned}$ |

The forward and back solution of the normals, conducted according to the Doolittle method, is next shown.

To compute the $v$ 's from the observation equations a knowledge of the $z$ 's is required. Substitute the $\delta \phi$ 's and $\delta \lambda$ 's in the right-hand side of the sum equation formed from the observation equations that contain the $z$ in question as if the $z$ were not there and divide the result of the substitution by the weight -r. As a check the sum of the $v$ 's about a point should equal zero. The computation of the $v$ 's is shown in the table on page 102. Below each $v$ as computed to 3 decimals is given its value as adopted and reduced to 1 decimal.

Following the computation of the $v$ 's there is given a computation of the triangles using the adjusted directions.

Solution of normals

| $\delta \phi_{3}$ | $8 \lambda_{3}$ | $\delta \phi_{4}$ | $8 \lambda_{1}$ | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $+\underset{\delta \phi_{3}}{+116.2166}$ | $\begin{aligned} & +35.0927 \\ & -0.301959 \end{aligned}$ | $\begin{aligned} & -161.8628 \\ & +\quad 1.392768 \\ & \hline \end{aligned}$ | $\begin{aligned} & -10.0544 \\ & +\quad 0.086523 \\ & \hline \end{aligned}$ | $\begin{aligned} & +0.0753 \\ & -0.000648 \end{aligned}$ | $\begin{aligned} & -20.5336 \\ & +\quad 0.176684 \\ & \hline \end{aligned}$ |
|  | $\begin{array}{r} +25.3104 \\ -10.5966 \end{array}$ | $\begin{array}{r} -75.6831 \\ +48.8759 \end{array}$ | $\begin{aligned} & -16.9056 \\ & +\quad 3.0363 \end{aligned}$ | $\begin{array}{r} +0.0236 \\ -0.0227 \end{array}$ | $\begin{array}{r} 32.1620 \\ +\quad 6.2003 \end{array}$ |
|  | $\begin{gathered} +14.7138 \\ \delta \lambda_{8} \end{gathered}$ | $\begin{aligned} & \\ & \hline \end{aligned} 26.8072$ | $\begin{aligned} & -13.8693 \\ & +\quad 0.942605 \\ & \hline \end{aligned}$ | $\begin{aligned} & +0.0009 \\ & -0.000061 \end{aligned}$ | $\begin{aligned} & 25.9618 \\ & +\quad 1.764452 \\ & \hline \end{aligned}$ |
|  | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{array}{r} +399.2176 \\ -225.4373 \\ -48.8403 \end{array}$ | $\begin{aligned} & +24.0721 \\ & -14.0048 \\ & -25.2686 \end{aligned}$ | $\begin{aligned} & -0.0468 \\ & +0.1049 \\ & +0.0016 \end{aligned}$ | $\begin{array}{r} +185.6970 \\ -28.5985 \\ -47.3000 \end{array}$ |
|  |  | $+\underset{\delta \phi_{4}}{+124.9400}$ | $\begin{aligned} & -15.2013 \\ & +\quad 0.121669 \end{aligned}$ | $\begin{aligned} & +0.0597 \\ & -0.000478 \end{aligned}$ | $\begin{aligned} & +109.7984 \\ & -\quad 0.878809 \end{aligned}$ |
|  |  | 1 2 3 | $\begin{array}{r} +20.0637 \\ -0.8700 \\ -13.0733 \\ -1.8495 \end{array}$ | $\begin{aligned} & -0.01105 \\ & +0.00652 \\ & +0.00085 \\ & +0.00726 \end{aligned}$ | $\begin{aligned} & +17.16375 \\ & -1.77663 \\ & +\quad 24.47172 \\ & +13.35904 \end{aligned}$ |
|  |  |  | $+4.2709$ | $\begin{aligned} & +0.00358 \\ & -0.000838 \end{aligned}$ | $\begin{array}{ll} + & 4.27448 \\ - & 1.000838 \end{array}$ |

Back solution

| $\delta \lambda_{4}$ | $\delta \phi_{4}$ | $\delta \lambda_{3}$ | $\delta \phi_{3}$ |
| :---: | :---: | :---: | :---: |
| -0.00084 | -0.00048 | -0.00006 | -0.00065 |
| -0.00084 | -0.00058 | $\begin{aligned} & -0.00079 \\ & -0.00106 \end{aligned}$ | $\begin{array}{r} -0.00007 \\ -0.00081 \end{array}$ |
|  |  | -0.00191 | +0.00058 |
|  |  |  | -0.00095 |

Computation of corrections

| $1=z_{1}$ | 2 | 3 | $z_{1}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.519 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & +0.6935 \\ & +0.132 \\ & +0.7 \\ & { }^{0} 5.519 \end{aligned}$ | $\begin{aligned} & +0.7030 \\ & -0.1831 \\ & -0.5 \\ & -0.519 \end{aligned}$ | $\begin{aligned} & +0.6935 \\ & +0.1432 \\ & +0.7030 \\ & -0.1831 \\ & +0.2 \end{aligned}$ |
|  | $\begin{aligned} & +1.018 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -0.499 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & +1.5566 \div-3 \\ & { }_{0.519}^{+} \end{aligned}$ |
| 4 | 5 | $6=z_{2}$ | $z_{2}$ |
| $\begin{aligned} & +0.5054 \\ & -0.1681 \\ & -0.8 \\ & +0.230 \end{aligned}$ | $\begin{aligned} & +0.2593 \\ & -0.1856 \\ & -0.3 \\ & +0.230 \end{aligned}$ | $\begin{aligned} & +0.230 \\ & +0.3 \end{aligned}$ | $\begin{aligned} & +0.5054 \\ & -0.1681 \\ & +0.2593 \\ & -0.1856 \\ & -1.1 \end{aligned}$ |
| $\begin{aligned} & =0.233 \\ & -0.2 \end{aligned}$ | $\begin{gathered} +0.004 \\ 0.0 \end{gathered}$ |  | $\begin{aligned} & -0.689 \div-3 \\ & +0.230 \end{aligned}$ |
| 7 | 8 | 9 | $z_{3}$ |
| $\begin{aligned} & +0.8721 \\ & +0.7907 \\ & -0.5324 \\ & -0.3478 \\ & -0.119 \end{aligned}$ | $\begin{aligned} & +0.6935 \\ & +0.142 \\ & -1.2 \\ & -0.119 \end{aligned}$ | $\begin{aligned} & +0.5054 \\ & -0.1681 \\ & =0.4 \\ & -0.119 \end{aligned}$ | $\begin{aligned} & +2.0710 \\ & +0.7659 \\ & -0.5324 \\ & -0.3478 \\ & -1.6 \end{aligned}$ |
| $\begin{aligned} & +0.664 \\ & +0.7 . \end{aligned}$ | $\begin{aligned} & -0.482 \\ & -0.5 \end{aligned}$ | $\begin{aligned} & -0.182 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & { }_{-0.119}^{0.3567 \div-3} \end{aligned}$ |
| 10 | 11 | 12 | $z_{4}$ |
| $\begin{aligned} & +0.7024 \\ & { }_{-0.1831} \\ & \hline 0.425 \\ & \hline+0.094 \end{aligned}$ | $\begin{aligned} & +0.2593 \\ & -0.1556 \\ & +1.1 \\ & -0.425 \end{aligned}$ | $\begin{aligned} & +0.8721 \\ & +0.7907 \\ & -0.5324 \\ & -0.3478 \\ & -1.2 \\ & -0.425 \end{aligned}$ | $\begin{aligned} & +0.8721 \\ & +0.7907 \\ & +0.4292 \\ & -0.7165 \\ & -0.1 \end{aligned}$ |
| +0.1 | $\begin{aligned} & +0.749 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & -0.425 \\ & \hline-0.842 \\ & -0.8 \end{aligned}$ | $+1.2755 \div-3$ |

Adjusted computation of triangles


ADJUSTMENT OF THREE NEW POINTS CONNECTED WTTH SEVERAL FIXED POINTS BY VARIATION OF GEOGRAPHIC COORDINATES

## GENERAL STATEMENT

The method of adjustment by geographic coordinates seems to be especially suitable for the adjustment of a few new points depending upon a number of fixed points. The number of normal equations in such case is $2 n, n$ being the number of new points. In the figure used the number of condition equations would be 15, which would form a very intricate set of normals. By the method of coordinates the number of normal equations is only six.

The adjustment of figure 6 is carried out in two different ways, the first one being more rigorous but a trifle longer than the second. The first method corresponds in its treatment of observed directions to the method developed in Jordan's Vermessungskunde, volume 1, pages $144-173$, of the third edition. The second method resembles somewhat the method given by Jordan on pages 173-179 for the approximate treatment of the $z$ 's and corresponds in its treatment of fixed directions to the ordinary practice of the Coast and Geodetic Survey for subsidiary triangulation as treated by the method of condition equations.


Fig. 6.

The solution by the method of condition equations was carried out for figure 6 and gave almost the same results, the greatest difference in the correction to a direction being $0.2^{\prime \prime}$. This difference was quite to be expected in view of the different formulas and the fact that the fixed positions, distances, and azimuths may not be strictly consistent with each other to the last figure given.

## FIRST METHOD

The first method is fundamentally the same as the method used in the adjustment of the quadrilateral previously given, but the greater complication of the figure, particularly the great number of fixed lines, brings to light points that need mention. The groundwork of the adjustment by either method is shown in the tables of observed directions and of fixed positions, azimuths, and lengths which follow. In the list of directions the names of stations that are sighted on over fixed lines are shown in heavy type. For these stations the directions corrected from a previous adjustment are also shown. In forming the table of triangles for the preliminary computation these corrected directions were taken with the directly observed directions of new points in order to obtain such of the angles in the column "Observed angle" as have a fixed line for one of its sides.* No particular procedure to obtain the consistent set of positions, azimuths, and lengths necessary to form the observation equations is essential to the method. In this particular case the corrections to the angles of the triangles Gunner-Larrabee-Mam, Cranberry Point-Gunner-Lubec Channel Lighthonse, and Telegraph-Cranberry PointGunner were arbitrarily assumed as shown in the table of preliminary computation of triangles. These assumptions, with the lines already fixed, determined enough parts in every one of the other triangles to make possible its solution with results as shown in the table.

Following the table of triangles the necessary preliminary computation of positions is included.

[^12]Lists of directions

AT INDIAN POINT

| Stations observed | Observed directions |  | Seconds after adjustment* |
| :---: | :---: | :---: | :---: |
|  | - , | " | " |
| Larrabee | $0 \quad 00$ | 00.0 | 01.5 |
| Mam | $66 \quad 55$ | 01.0 | 01.2 |
| Lubec Channel Lighthouse | $95 \quad 24$ | 38.4 | 41.2 |
| Gunner | 11400 | 37.5 |  |
| Lubec Church Spire | $117 \quad 33$ | 55.5 | 71.9 |

AT MAM

|  | 0 | 00 | 00.0 | 55.4 |
| :--- | ---: | ---: | ---: | ---: |
| Indian Point | 34 | 52 | 43.6 | 46.7 |
| Larrabee | 317 | 37 | 23.5 |  |
| Cranberry Point <br> Lubec Channel Light- <br> house | 318 | 56 | 54.1 | 55.2 |
| Gunner <br> Duck | 322 | 01 | 44.8 |  |

AT GUNNER

| Indian Point | 0 | 00 | 00.0 |
| :--- | ---: | ---: | ---: |
| Larrabee | 31 | 57 | 41.4 |
| Mam | 94 | 56 | 05.9 |
| Lubec Channel Light- | 101 | 48 | 54.9 |
| house | 168 | 28 | 59.0 |
| Telegraph | 186 | 30 | 29.8 |
| Lubec Church Spire | 191 | 54 | 53.6 |
| Cranberry Point | 346 | 14 | 12.0 |
| Buck |  |  |  |

AT TREAT 2

| Cranberry Point | 0 | 00 | 00.0 |  |
| :--- | ---: | ---: | ---: | ---: |
| Lubec Church Spire | 17 | 34 | 38.1 | 45.4 |

AT LARRABEE

| Stations observed | Observed directions |  |  | Seconds after adjustment* |
| :---: | :---: | :---: | :---: | :---: |
|  | - | , | " | / |
| Indian Point | 0 | 00 | 00.0 | 56.7 |
| Mam | 281 | 47 | 48.4 | 47.9 |
| Lubec Channel Lighthouse | 312 | 58 | 58.5 | 55.0 |
| Lubec Church Spire | 315 | 12 | 19.0 | 24.9 |
| Gunner |  | 58 | 24.9 |  |
| Duck | 336 | 12 | 51.0 | 50.6 |

AT DUCK

| Lubec Channel Light- <br> house | 0 | 00 | 00.0 | 59.9 |
| :--- | ---: | ---: | ---: | ---: |
| Gunner | 33 | 11 | 41.8 |  |
| Larrabee | 269 | 09 | 40.7 | 40.9 |
| Mam (computed) | 336 | 10 |  | 44.0 |

## AT CRANBERRY POINT

| Gunner Channel Light- | 0 | 00 | 00.0 |
| :--- | ---: | ---: | ---: |
| Lubec Ch | 45 | 12.8 |  |
| house | 78 | 37 | 05.9 |
| Mam | 153 | 39 | 19.7 |
| Telegraph | 174 | 08 | 32.3 |
| LubecChurch Spire | 191 | 52 | 35.2 |

## AT TELEGRAPH

| Cranberry Point | 0 | 00 | 00.0 |
| :--- | ---: | ---: | ---: |
| 2 | 54 | 53.2 |  |
| Gunner Channel Light- | 29 | 52 | 44.5 |
| Lubec Chase |  |  |  |
| house |  |  |  |
| Lubec Church Spire | 231 | 21 | 00.1 |

* Refers to final values of heavy lines in Fig. 6, p. 104, obtained from a previous adjustment.

List of fixed positions

| Station | Latitude and longitude |  | Azimuth |  |  | Back azimuth |  |  | To station | Loga rithm of dis- |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - | " | $\bigcirc$ | , | " | - | , | " |  |  |
| Lubec Church Spire | $\begin{array}{ll} 44 & 51 \\ 66 & 59 \end{array}$ | $\begin{aligned} & 38.470 \\ & 17.418 \end{aligned}$ |  |  |  |  |  |  |  |  |
| Lubec Channel Lighthouse | $\begin{array}{ll} 44 & 50 \\ 66 & 58 \end{array}$ | $\begin{aligned} & 31.652 \\ & 38.299 \end{aligned}$ |  |  |  |  |  |  |  |  |
| Treat 2 | $\begin{array}{ll}44 & 52 \\ 66 & 59\end{array}$ | $\begin{aligned} & 44.333 \\ & 25.919 \end{aligned}$ | 354 | 45 | 17.5 | 174 | 45 | 23.5 | Lubec Church Spire | 3.309982 |
| Indian Point | $\begin{array}{ll}44 & 50 \\ 66 & 57\end{array}$ | $\begin{aligned} & 03.537 \\ & 12.788 \end{aligned}$ |  | 48 | 33.8 | 294 | 47 | 33.5 | Lubec Channel Lighthouse | 3. 315762 |
|  |  |  | 136 | 58 | 04.6 | 316 | 56 | 36.7 | Lubec Church Spire | 3.603123 |
| Larrabee | $\begin{array}{ll}44 & 49 \\ 66 & 57\end{array}$ | $\begin{aligned} & 10.841 \\ & 38.857 \end{aligned}$ |  | 22 | 33.8 | 332 | 21 | 51.9 | Lubec Channel Lighthouse | 3.449573 |
|  |  |  |  |  | $\begin{aligned} & 03.7 \\ & \text { 35. } 7 \end{aligned}$ |  |  | $\begin{aligned} & 54.3 \\ & 54.1 \end{aligned}$ | Lubec Church Spire Indian Point | $\begin{aligned} & 3.702872 \\ & 3.236668 \end{aligned}$ |
| Duck | $\begin{array}{lll}44 & 50 & 33.886 \\ 66 & 57 & 47.822\end{array}$ |  | 355 |  | 23.1 | 175 | 36 | 29.4 | Larrabee | 3.410111 |
|  |  |  |  |  | 42.1 |  |  | 06.5 | Tubec Channel Lighthouse | 3. 045619 |
| Mam | $\begin{array}{ll}44 & 49 \\ 66 & 59\end{array}$ | $\begin{aligned} & 57.369 \\ & 26.892 \end{aligned}$ |  | 14 | 19.0 |  | 14 | 53.3 | Lubec Channel Lighthouse | 3. 176968 |
|  |  |  |  |  | 16.3 19.2 |  |  | 26.2 53.8 | Duck <br> Indian Point | 3. 3899282 3.470097 |
|  |  |  |  |  | 10.5 |  |  | 26.7 | Larrabee | 3.443126 |

Preliminary computation of triangles


## 108 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.

Preliminary computation of triangles-Continued


Preliminary computation of triangles-Continued


Preliminary position computation,
STATION GUNNER


STATION CRANBERRY POINT

secondary triangulation
STATION GUNNER


STATION CRANBERRY POINT


Preliminary position computation,
STATION TELEGRAPH

secondary triangulation-Continued.
STATION TELEGRAPH


## FORMATION OF OBSERVATION EQUATIONS

In the first column of the following table is given the assumed azimuths of the various lines. Under station Gunner the azimuth to Duck, $299^{\circ} 38^{\prime} 04^{\prime \prime} .3$, comes from the position computation on page 110, and the other azimuths are obtained by adding to this the corrected angles from the preliminary computation of triangles. Thus in the first triangle in that list, Gunner-Duck-Larrabee, the angle at Gunner is $45^{\circ} 43^{\prime} 34 .^{\prime \prime} 2$, which, added to the azimuth of Duck, gives $345^{\circ} 21^{\prime} 38^{\prime \prime} .5$, as shown in the table. For any one station the assumed and observed azimuths of some one station may be taken as identical. At Gunner they are identical on station Duck. The observed azimuths in the second column of the table have their symbolic corrections affixed. These azimuths are obtained by adding the observed angles as derived from the list of observed directions on page 106 to the azimuth of the line GunnerDuck.

At the fixed stations given in the lower part of the table the method of computing the assumed and observed azimuths is somewhat different. The assumed azimuths of the fixed lines come from the table of fixed positions on page 106. The assumed azimuths of the new lines are found by adding to one of these fixed azimuths the appropriate corrected angle from the computation of triangles on pages $107-109$. In the second column of the table the observed azimuth of one fixed line used as an initial line is taken identical with its assumed azimuth, and the other observed azimuths, whether of fixed lines or of new ones, are found by adding to this azimuth the observed angles between the initial line and each of the others as derived from the list of directions.

The coefficients of the $\delta \phi$ 's and $\delta \lambda$ 's are found from the formulas on page 93.
GUNNER

| Assumed azimuth |  |  | Observed azimuth |  |  | Equation | Station observed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | , | " | - | , | " |  |  |
| 299 | 38 | 04. 3 | 299 |  | 04.3-z $z_{1}+v_{1}$ | $v_{1}=2_{1}+8669{ }_{\delta \phi_{1}}-35098 \lambda_{1}+0.0$ | Duck |
| 313 | 23 | 50.1 | 313 |  | $52.3-z_{1}+v_{2}$ | $z_{2}=z_{1}+25388 \phi_{1}-17098 \lambda_{1}-2.2$ |  |
|  |  | 38.5 | 345 |  | 33. $7-z_{1}+v_{3}$ | $v_{3}=z_{1}+5418 \phi_{1}-14738 \lambda_{1}+4.8$ | Larrabee |
| 48 55 | 12 |  |  |  | $58.2-z_{1}+v_{4}$ $47.2-z_{1}+v_{3}$ | $v_{4}=z_{1}-2191 \delta \phi_{1}-13888 \lambda_{1}+3.6$ $v_{5}=z_{1}-7756 \delta \phi_{1}-3833 \lambda_{1}+7.1$ | Mam |
| 121 | 53 | 01.0 | 121 | 52 | 51.3-2 $z_{1}+v_{6}$ | $v_{6}=z_{1}-3643 \delta \phi_{1}+1612 \delta \lambda_{1}+3643 \delta \phi_{3}-1612 \delta \lambda_{3}+9.7$ | Telegraph |
| 139 | 54 |  |  |  | 22. $1-z_{1}+v_{7}$ | $v_{7}=z_{1}-1870 \delta \phi_{1}+15808 \lambda_{1}+25.5$ | Lubec church spire |
|  | 18 | 53.0 |  | 18 | $45.9-z_{1}+v_{8}$ | $v_{8}=z_{1}-21313 \delta \phi_{1}+219098 \lambda_{1}+21313 \delta \phi_{2}-219098 \lambda_{2}+7.1$ | Cranberry Point |


|  |  |  |  |  |  | CRANBERRY POINT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 325 | 18 | 49.9 | 325 | 18 | 49.9- $z_{2}+v_{9}$ | $v_{9}=z_{2}-213138 \phi_{1}+219098 \lambda_{1}+21313$ |
| 41 | 04 | 02.7 | 41 | 04 | $02.7-z_{2}+v_{10}$ | $v_{10}=z_{3}-6013 \delta \phi_{2}-49108 \lambda_{3}+0.0$ |
| 43 | 55 | 42.7 | 43 | 55 | $55.8-z_{3}+v_{11}$ | $v_{11}=z_{2}-2010 \phi_{2}-1485 \lambda_{2}-13.1$ |
| 118 | 58 | 06.9 | 118 | 58 | 09.6- $z_{2}+v_{19}$ | $v_{12}=z_{3}-11898 \phi_{2}+16508 \lambda_{2}+41898 \phi_{3}$ |
| 139 | 27 | 33.7 | 139 |  | 22.2-29+ ${ }_{13}$ | $v_{13}=z_{2}-2045 \delta^{2}+1701 \delta \lambda_{2}+11.5$ |
|  | 11 | 31.9 |  | 11 | $25.1-z_{2}+v_{14}$ | $v_{14}=z_{2}-6378 \phi_{2}+1078 \lambda_{2}+6.8$ |

TELEGRAPH


| TREAT |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 337 10 43.7 337 <br> 354 45 17.5 $39.4-z_{4}+v_{19^{\prime}}$  <br> 354 45 <br> $17.5-z_{1}+v_{19}$  | $v_{19}=z_{4}-6378 \phi_{2}+10788 \lambda_{2}+4.3$ <br> $v_{19}=z_{4}+0.0$ | Cranberry Point <br> Lubec church spire |

MAM

| Assumed azimuth | Obser | ved azimuth | Equation | Station observed |
| :---: | :---: | :---: | :---: | :---: |
| , " | - , |  |  |  |
| 22354533.7 | 22354 | $42.7-z_{3}+v_{20}$, | $v_{20}=z_{5}-2010 \delta^{\prime} \phi_{2}-14850 \lambda_{2}+11.0(+6.4)$ | Cranberry Point |
| $\begin{array}{lll}225 & 14 & 19.0\end{array}$ | 22514 | $13.3-z_{5}+v_{20}{ }^{\prime}$ |  | Lubec Channel L. H. |
| $\begin{array}{lll}228 & 19 & 09.7 \\ 242 & 36 & 16.3\end{array}$ | $\begin{array}{lll}228 & 19 \\ 242\end{array}$ | $04.0-z_{5}+v_{21}{ }^{\prime \prime}$ $11.3-z_{5}+v_{20}{ }^{\prime \prime}$ | $v_{21}=z_{5}-2191 \delta \phi_{1}-13888 \lambda_{1}+5.7(+1.1)$ | Gunner |
| $\begin{array}{llll}266 & 17 & 19.2\end{array}$ | 266 268 | ${ }_{19.2}{ }^{\text {a }}$ - $z_{5}+v_{20}{ }^{\prime \prime \prime}{ }^{\prime \prime}$ | ${ }^{2}{ }_{20}{ }^{\prime \prime}=z_{5}+5.0(+0.4)$ | Indian Point |
| $\begin{array}{llll}301 & 10 & 10.5\end{array}$ | 30110 | 02.8-z $z_{5}+v_{20^{\prime}}{ }^{\prime \prime \prime}$ | $v_{20}{ }^{\prime \prime \prime \prime \prime}=z_{5}+7.7(+3.1)$ | Larrabee |


INDIAN POINT

| 19 | 23 | 54.1 | 19 | 23 | $54.1-z_{7}+v_{23}{ }^{\prime}$ | $v_{23}{ }^{\prime}=z_{7}+0.0(-3.8)$ | Larrabee |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | 18 | 53.8 |  | 18 | $55.1-z_{7}+v_{23}^{\prime \prime}{ }^{\prime \prime}$ | $v_{23}{ }^{\prime \prime}=z_{7}-1.3(-5.0)$ | Mam |
| 11 | 48 | 33.8 | 114 | 48 | $3^{32.5-z_{7}+v_{23}{ }^{\prime \prime} \text {. }}$ | $v_{23}{ }^{\prime \prime \prime}=z_{7}+1.3(-2.4)$ | Lubec Channel L. H. |
| 133 | 24 | 32.6 |  |  |  | $v_{23}=z_{7}+25388 \delta_{1}-1709 \partial_{1} \lambda_{1}+1.0(-2.8)$ | Gunner |
| 136 | 58 | 04.6 |  |  | 49.6- $z_{7}+v_{23}{ }^{\prime \prime \prime \prime}$ | $v_{23}{ }^{\prime \prime \prime \prime}=z_{7}+15.0(+11.2)$ | Lubec church spire |

DUCK


Figure 6 taken with the following two tables shows that $z_{1}$ is for directions taken at Gunner, $z_{2}$ for directions at Cranberry Point, $z_{3}$ for directions at Telegraph, and $z_{5}$ for directions at Mam. The scheme for eliminating these $z$ 's by the use of the sum equations, as fictitious additional observation equations with negative weights, is used here in the same manner as in the previous example. Each weight is the negative reciprocal of the total number of observed lines in the adjustment that radiates from the point in question. The weights are, respectively, $-1 / 8,-1 / 6,-1 / 4$, and $-1 / 6$, as shown in Table 1.

At Treat ${ }_{2}$, Larrabee, Indian, and Duck where only one new line is to be determined the same process might be used, but the following method is identical in results and slightly shorter. Use is made of the fact that the directions taken at a point may each be changed by the same amount, a change equivalent to using merely a different zero point. Correct each of the directions by the averages of all the corrections necessary to reduce the observed results to the accepted results on the lines that have already been fixed. Then drop the $z$ from the observation equation of the new line and assign the equation a positive weight equal to $\frac{s}{s+1}$, where $s$ is the number of lines already fixed and therefore $s+1$ is the total number of lines. Thus at Larrabee the constant terms of the observation equations representing pointings on lines already fixed are $+2.6,-0.4,+9.0,-0.1$, and 0.0 , the mean of which is +2.8 . Subtracting this from each of the preceding numbers we have $-0.2,-3.2,+6.2,-0.1$, and -2.8 as the new constant terms, also -0.8 instead of +2.0 on Gunner, the new point. These new values are inclosed in parentheses añd are used in forming the normal equations. There are five fixed lines, so the weight of the new equation without $z$ that is used to replace the six equations containing $z$ is $5 / 6$ and the equation itself is

$$
v_{22}=+541 \delta \phi_{1}-1473 \delta \lambda_{1}-0.8
$$

which in Table 2 corresponds to the line No. 22,

$$
+0.49 \delta \phi_{1}-1.34 \delta \lambda_{1}-0.07
$$

The $z$ 's are computed from the sum equations as in the previous example, the result of substitution in the right-hand side being divided by $-1 / r, r$ being the total number of lines through the point to which $z$ applies. For fixed points where only one new line occurs, substitute $\delta \phi$ and $\delta \lambda$ in the right-hand side of the observation equation on the new line omitting the $z$, and divide the result by $-1 / r$. Thus at Duck (see p. 116),

$$
z_{8}=\left(8669 \delta \phi_{1}-3509 \delta \lambda_{1}-1.9\right) \div(-3)
$$

as shown in the computation below.

## 118 coast and geodetic survey spectal publication no. 28.

When the $z$ 's are known the $v$ 's or corrections are computed from the equations on pages 115 and 116. The details are shown in the table on page 121.

For convenience of solution in the normals it is best to divide the constant terms by 10 and the coefficients by 1000 . The solution will then give $100 \delta \phi_{\mathrm{n}}$ and $100 \delta \lambda_{\mathrm{n}}$.

Table for formation of normals No. 1

|  |  | 1 | 2 | 3 | 4 | 5 | 6 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\delta \phi_{1}$ | $\delta \lambda_{1}$ | $\delta \phi_{2}$ | $8 \lambda_{2}$ | $\delta^{\prime}{ }_{3}$ | $\delta \lambda_{3}$ | $\cdot \eta$ | $p$ | $\sqrt{p}$ |
|  | ${ }_{1}^{z_{1}}$ | + 8669 | - 3509 |  |  |  |  |  |  | 1 |
| 2 | 1 | + 2338 | - 1709 |  |  |  |  | + 2.2 | 1 | 1 |
| 3 | 1 | + 541 | -1473 |  |  |  |  | + 4.8 | 1 | 1 |
| 4 | 1 | - 2191 | - 1388 <br> -3833 |  |  |  |  | + 3.6 <br> $+\quad 7.1$ | 1 | 1 |
| 6 | 1 | - 3643 | - 3833 +1612 |  |  | +3643 | -1612 | $+\quad .9 .1$ $+\quad 9.7$ | 1 | 1 |
| 7 | 1 | - 1870 | +1580 |  |  |  |  | + 25.5 | 1 | 1 |
| Sum | 1 | -21313 | +21909 | ${ }_{+}^{+21313}$ | -21909 -21009 |  |  | + 7.1 | 1 |  |
| Sum | 8 | -25025 | +13189 | +21313 | -21909 | +3643 | -1612 | + 55.6 | $-\frac{1}{8}$ | 0.35355i |
| 9 | $z_{2}$ 1 | -21313 | +21909 | +21313 | -21909 |  |  | + 0.0 |  | 1 |
| 10 | 1 |  |  | -6013 | - 4910 |  |  | + 0.0 | 1 | 1 |
| 11 | 1 |  |  | - 2010 | - 1485 |  |  | -13.1 | 1 | 1 |
| 12 | 1 |  |  | - 4189 | +1650 | +4189 | -1650 | - 2.7 | 1. | 1 |
| 13 14 | 1 |  |  | - 2045 $-\quad 637$ | +1701 +1078 +1 |  |  | a +11.5 +6.8 | , | 1 |
| Sum | 6 | -21313 | +21909 | +6419 | -23875 | +4189 | -1650 | $+\quad 2.5$ $+\quad 15$ | $-\frac{1}{8}$ | 0.4083i |
| 15 | ${ }_{1}^{23}$ |  |  |  |  | -1181 | +4922 |  | 1 | 1 |
| 16 | 1 |  |  | - 4189 | $+1650$ | +4189 | -1650 | - 52.6 | 1 | 1 |
| 17 | 1 | - 3643 | + 1612 |  |  | +3643 +2413 | -1612 | - 54.8 | 1 | 1 |
| Sum | ${ }_{4}^{1}$ |  |  |  |  | +2413 +9064 | -2839 -1179 | -62.2 -169.6 | - ${ }^{\frac{1}{2}}$ |  |
| Sum | 4 | - 3643 | + 1612 | $-\quad 4189$ $-\quad 637$ | +1650 +1078 | +9064 | -119 | -189.6 $+\quad 4.3$ | - $\frac{1}{2}$ | 0. 0.707 |
| 20 | $2_{5}$ 1 |  |  | - 2010 | - 1485 |  |  |  |  |  |
| 21 | 1 | - 2191 | - 1388 |  |  |  |  | + 1.1 | 1 |  |
| Sum | 6 | - 2191 $+\quad 541$ | -1388 -1473 | - 2010 | - 1485 |  |  | $\begin{array}{r}+\quad 7.5 \\ +\quad 0.8 \\ \hline\end{array}$ |  |  |
| 22 23 |  | +541 $+\quad 2538$ | -1473 -1709 |  |  |  |  | $-\quad 0.8$ $-\quad 28$ | - | 0.9129 0.8944 |
| 24 |  | +8669 | - 3509 |  |  |  |  | - 1.9 | $\frac{8}{3}$ | 0.8165 |

Table for formation of normals No. 2

|  | $\delta \phi_{1}$ | $8 \lambda_{1}$ | ¢ $\phi_{2}$ | $8 \lambda_{2}$ | $8 \phi_{3}$ | $\lambda_{3}$ | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | + 8.67 | - 3.51 |  |  |  |  | +0.00 | + 5.16 |
| 2 | +2.54 | $-1.71$ |  |  |  |  | -0.22 | + 0.61 |
| 3 | + 0.54 | -1.47 |  |  |  |  | +0.48 | -0.45 |
| 4 | - 2.19 | -1.39 -383 |  |  |  |  | +0.36 | - 3.22 |
| 5 | - 7.76 | -3.83 |  |  |  |  | +0.71 | -10.88 |
| 6 | -3.64 | +1.61 |  |  | +3.64 | -1.61 | +0.97 +2.55 | +0.97 |
| 7 8 8 | -1.87 -21.31 | +1.68 +1.58 +21.91 |  |  |  |  | +2.55 +0.71 | +2.26 +0.71 |
| $\begin{array}{r}8 \\ 2_{1} \\ \hline\end{array}$ | -21.31 $-8.85 i$ | +21.91 $+4.66 i$ | +21.31 $+\quad 7.54 i$ | -21.91 $-7.75 i$ | +1.29i | -0.57i | +0.71 $+1.97 i$ | +0.71 $-1.71 i$ |
|  | -21.31 | +21.91 | +21.31 | -21.91 |  |  | $+0.00$ | + 0.00 |
| 10 |  | +21.01 | -6.01 | - 4.91 |  |  | +0.00 | -10.92 |
| 11 |  |  | $-2.01$ | -1.48 |  |  | -1.31 | - 4.80 |
| 12 |  |  | - 4.19 | +1.65 | +4.19 | -1.65 | -0.27 | -0.27 |
| 13 |  |  | - 2.04 | +1.70 |  |  | +1.15 | + 0.81 |
| 14 |  |  | -0.64 | +1.08 |  |  | +0.68 | +1.12 |
| 22 | $-8.70 i$ | $+8.95 i$ | + $2.62 i$ | - 9.75i | +1.71i | -0.67i | +0.10i | - $5.74 i$ |
| 15 |  |  |  |  | -1.18 | +4.92 | +0.00 | + 3.74 |
| 16 |  |  | - 4.19 | $+1.65$ | +4.19 | $-1.65$ | $-5.26$ | -5.26 |
| 17 | - 3.64 | $+1.61$ |  |  | +3.64 | -1.61 | -5.48 | - 5.48 |
| 18 |  |  |  |  | +2.41 $+4.53 i$ | -2.84 $-0.59 i$ | -6.22 $-8.48 i$ | - 6.65 $=6.82 i$ |
| 23 | - $1.82 i$ | $+0.81 i$ | - 2.09i | $+0.82 i$ | +4.53i | -0.59i | -8.48i | -6.82i |
| 19 |  |  | $-0.45$ | + 0.76 |  |  | +0.30 | + 0.61 |
| 20 |  |  | - 2.01 | $-1.48$ |  |  | +0.64 | - 2.85 |
| 21 | $-2.19$ | $-1.39$ |  |  |  |  | +0.11 | - 3.47 |
| 25 | - $0.89 i$ | $-0.57 i$ | - 0.82i | - 0.61i |  |  | +0.31i | - $2.58 i$ |
| 22 | + 0.49 | $-1.34$ |  |  |  |  | $-0.07$ | - 0.92 |
| 23 | + 2.27 | -1.53 |  |  |  |  | -0.25 | + 0.49 . |
| 24 | + 7.08 | $-2.86$ |  |  |  |  | -0.16 | +4.06 |

Normal equations

| 1 | 2 | 3 | 4 | 5 | 6 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +987.3526 | $\begin{array}{r} -852.5451 \\ +913.2320 \end{array}$ | $\begin{aligned} & -823.2428 \\ & +876.4443 \\ & +923.5619 \end{aligned}$ | $\begin{aligned} & +781.3412 \\ & +837.7306 \\ & -831.4401 \\ & +842.4946 \end{aligned}$ | $\begin{aligned} & +8.0389 \\ & -13.2644 \\ & -39.8513 \\ & +36.7824 \\ & +43.7023 \end{aligned}$ | $\begin{array}{r} -0.2265 \\ +3.9464 \\ +18.6471 \\ -15.9112 \\ -33.6441 \\ +41.7793 \end{array}$ | $\begin{aligned} & -8.9085 \\ & +6.5262 \\ & +4.1465 \\ & +2.6136 \\ & -18.8752 \\ & +30.2371 \end{aligned}$ | $\begin{aligned} & +91.8098 \\ & +96.6088 \\ & +128.2256 \\ & -21.8901 \\ & +17.1109 \\ & +44.8281 \end{aligned}$ |

Solution of normals

| 1 | 2 | 3 | 4 | 5 |  | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\delta \phi_{1}}{987.3526}$ | $\begin{aligned} & -852.5451 \\ & +\quad 0.8634657 \end{aligned}$ | $\begin{aligned} & -823.2428 \\ & +0.8337881 \end{aligned}$ | $\begin{gathered} +781.3412 \\ \mathbf{-}_{0.7913497} \end{gathered}$ | $\begin{aligned} & \pm 8.0389 \\ & { }_{0.0081419} \end{aligned}$ | $\begin{aligned} & -0.22665 \\ & +0.0002294 \end{aligned}$ | $\begin{aligned} & -8.9085 \\ & +0.0000226 \end{aligned}$ | $\begin{aligned} & \pm 91.8098 \\ & \mathbf{0 . 0 9 2 9 8 5 8} \end{aligned}$ |
|  | $\begin{aligned} & +{ }_{-736.143}^{+913.232} \\ & +177.088 \end{aligned}$ | $\begin{aligned} & +876.4443 \\ & -710.8419 \\ & +165.6024 \\ & -\quad 0.9351387 \\ & \hline \end{aligned}$ | $\begin{aligned} & -837.7306 \\ & +674.6613 \\ & -163.0693 \\ & +\quad 0.9208345 \end{aligned}$ | $\begin{aligned} & -13.2644 \\ & +6.9413 \\ & -6.3231 \\ & +0.0357059 \end{aligned}$ | +3.9464 <br> +0.1956 <br> +3.7508 <br> -0.0211804 | $\begin{aligned} & \pm 6.5262 \\ & -7.6922 \\ & -1.1660 \\ & +0.0665843 \end{aligned}$ | $\begin{aligned} & +96.6088 \\ & +79.2746 \\ & +175.8834 \\ & -\quad 0.9931944 \end{aligned}$ |
|  |  | $\begin{gathered} +923.5619 \\ -686.4101 \\ -154.8612 \\ +82.2906 \\ \delta \phi_{2} \end{gathered}$ | $\begin{aligned} & -831.4801 \\ & +651.4730 \\ & +152.4924 \\ & -\quad 27.5147 \\ & +\quad 0.334360 \end{aligned}$ | $\begin{aligned} & -39.8513 \\ & +6.7027 \\ & +5.9130 \\ & -27.2356 \\ & +0.330969 \end{aligned}$ | $\begin{aligned} & +18.6471 \\ & \pm 0.1889 \\ & -3.5075 \\ & +14.9507 \\ & -0.181682 \end{aligned}$ | $\begin{aligned} & +4.1465 \\ & +7.4278 \\ & +1.0904 \\ & -2.1909 \\ & +0.026624 \end{aligned}$ | $\begin{aligned} & +128.2256 \\ & +\quad 76.5499 \\ & -164.4754 \\ & +\quad 40.3001 \end{aligned}$ |
|  |  | $\begin{aligned} & \frac{1}{2} \\ & 3 \end{aligned}$ | $\begin{gathered} +842.4946 \\ -618.3141 \\ -150.1598 \\ -\quad 9.1998 \\ +\quad 64.8209 \\ \delta \lambda_{2} \end{gathered}$ | +36.7824 +6.3616 $=5.8225$ -9.1065 +15.4918 -0.238994 | -15.9112 $+\quad 0.1792$ +3.4539 +4.9989 -7.2792 +0.112297 | +2.6136 $\pm 7.0497$ -1.0737 -0.7326 $\pm 7.8570$ -0.121211 | $\begin{aligned} & \mathbf{-} 21.8901 \\ & \overline{-7.6236} \\ & +161.9595 \\ & +13.4747 \\ & +\quad 80.8905 \\ & -\quad 1.247908 \end{aligned}$ |
|  |  |  | $\begin{aligned} & 3 \\ & 4 \end{aligned}$ | +43.7028 +0.0655 $=0.2258$ $=3.0141$ -3.7024 +30.6950 $\delta \phi_{3}$ | $\begin{aligned} & -33.6441 \\ & +0.0019 \\ & +0.1339 \\ & +4.9482 \\ & +1.7397 \\ & -26.8204 \\ & +0.873771 \end{aligned}$ | -18.8752 +0.0725 $=0.0416$ $=1.7251$ -1.8778 -21.4472 +0.698720 | -17.1109 $+\quad 0.7475$ +6.2801 +13.3381 -19.3324 -17.5726 $+\quad 0.572491$ |
|  |  |  |  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 4 \\ & 5 \end{aligned}$ |  | $\begin{aligned} & +30.2371 \\ & +0.0020 \\ & +0.0247 \\ & +0.3980 \\ & +0.8823 \\ & -18.7399 \\ & +12.8002 \\ & +0.86892 \end{aligned}$ | +44.8281 <br> $+\quad 0.0211$ <br> $\pm \quad 3.753$ <br> 7.3218 <br> $\mathbf{+} .0837$ <br> 15.3544 <br> $+\quad 27.5314$ <br> $\quad 1.86892$ |

Back solution

| $\delta \lambda_{3}$ | $\delta \phi_{3}$ | $\delta \lambda_{2}$ | $\delta \phi_{2}$ | $\delta \lambda_{1}$ | $\stackrel{\text { ¢ }}{ }{ }_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $-0.86892$ | +0.69872 | $-0.12121$ | +0.02662 | $+0.00658$ | $+0.00902$ |
| $-0.86892$ | -0.75924 | -0.09758 | $+0.15787$ | +0.01840 | $-0.00020$ |
|  | -0.06052 | +0.01446 | -0.02003 -0.06832 | -0.00216 -0.18815 | +0.00049 +0.16170 |
|  |  | $-0.20433$ | $+0.09614$ | $-0.08990$ | +0.08016 |
|  |  |  |  | -0.25523 | -0.22038 |
|  |  |  |  | -0.25523 | $+0.03079$ |

Computation of corrections

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 +2.669 +8.956 -11.412 | $\begin{aligned} & +0.781 \\ & +4.362 \\ & -11.412 \\ & -2.2 \end{aligned}$ | +0.167 +3.760 -11.412 +4.8 | $\begin{aligned} & -0.675 \\ & +3.543 \\ & +11.412 \\ & +3.6 \end{aligned}$ | $\begin{aligned} & -2.388 \\ & +9.783 \\ & +11.412 \\ & +\quad 7.1 \end{aligned}$ | - 1.122 <br> $=4.114$ <br> 2.205 <br> +14.007 | $\begin{aligned} & -0.576 \\ & =4.033 \\ & -11.412 \\ & +25.5 \end{aligned}$ | $\begin{array}{r} -6.562 \\ -55.918 \\ +20.490 \\ +44.767 \end{array}$ |
| $\begin{aligned} & +0.213 \\ & +0.2 \end{aligned}$ | $\begin{gathered} -8.469 \\ =8.5 \end{gathered}$ | $\begin{aligned} & -2.685 \\ & -2.7 \end{aligned}$ | $\begin{array}{r} -4.944 \\ -4.9 \end{array}$ | $\begin{aligned} & +3.083 \\ & +3.1 \end{aligned}$ | -11.412 +9.7 | $\begin{aligned} & +9.479 \\ & +9.5 \end{aligned}$ | -11.412 +7.1 |
|  |  |  |  |  | +4.854 +4.9 |  | $\begin{aligned} & -1.535 \\ & =1.5 \end{aligned}$ |
| $z_{1}$ | 9 | 10 | 11 | 12 | 13 | 14 | $z_{2}$ |
| $\begin{aligned} & -7.705 \\ & -33.662 \\ & +20.490 \\ & +44.767 \\ & +2.205 \\ & +14.007 \\ & +55.6 \end{aligned}$ | $\begin{array}{r} -6.562 \\ -55.918 \\ +20.490 \\ +44.767 \\ -1.129 \end{array}$ | $\begin{array}{r} 5.781 \\ +10.033 \\ -1.129 \end{array}$ | $\begin{aligned} & -1.932 \\ & +3.035 \\ & -1.129 \\ & -13.1 \end{aligned}$ | $\begin{aligned} & =\quad 4.027 \\ & =\quad 3.371 \\ & =\quad 2.535 \\ & +14.337 \\ & =\quad 1.129 \\ & -\quad 2.7 \end{aligned}$ | $\begin{aligned} & -1.966 \\ & =3.476 \\ & =1.129 \\ & +11.5 \end{aligned}$ | $\begin{aligned} & -0.612 \\ & =22.203 \\ & =1.129 \\ & +6.8 \end{aligned}$ | $\begin{array}{r} -6.562 \\ -55.918 \\ +\quad 6.171 \\ +48.783 \\ +2.535 \\ +14.337 \\ +\quad 2.5 \end{array}$ |
|  | $\begin{aligned} & +1.648 \\ & +1.7 \end{aligned}$ | $\begin{aligned} & +3.123 \\ & +3.1 \end{aligned}$ | $\begin{aligned} & -13.126 \\ & -13.1 \end{aligned}$ |  | $\begin{aligned} & +4.929 \\ & +5.0 \end{aligned}$ | $\begin{aligned} & +2.856 \\ & +2.9 \end{aligned}$ |  |
| $\begin{array}{r} +91.292 \\ +11.412 \end{array}$ |  |  |  | $\begin{aligned} & +0.575 \\ & +0.6 \end{aligned}$ |  |  | $\begin{array}{r} +6.776 \\ -1.129 \end{array}$ |
| 15 | 16 | 17 | 18 | 23 | 19 | 24 | 19' |
| $\begin{aligned} & +0.715 \\ & -42.768 \\ & +44.369 \end{aligned}$ | -4.027-3.371-2.535+14.337+44.369-52.6 | $\begin{aligned} & \mathbf{- 1 . 1 2 2} \\ & =4.114 \\ & -22.205 \\ & +14.007 \\ & +44.369 \\ & -54.8 \end{aligned}$ | $\begin{aligned} & -1.460 \\ & +24.669 \\ & +44.369 \\ & -\quad 62.2 \end{aligned}$ | $\begin{aligned} & =1.122 \\ & =4.114 \\ & =4.027 \\ & =3.371 \\ & =5.486 \\ & +10.45 \\ & -169.6 \end{aligned}$ | $\begin{aligned} & -0.612 \\ & =2.203 \\ & =0.742 \\ & +4.3 \end{aligned}$ | $\begin{aligned} & -0.612 \\ & =2.203 \\ & +4.3 \end{aligned}$ | $\begin{aligned} & -0.742 \\ & -0.742 \\ & -0.7 \end{aligned}$ |
|  |  |  |  |  |  |  |  |
| $\begin{aligned} & +2.316 \\ & +2.3 \end{aligned}$ |  |  | $\begin{aligned} & +5.378 \\ & +5.4 \end{aligned}$ |  | $\begin{aligned} & +0.743 \\ & +0.8 \end{aligned}$ | $\begin{array}{r} 1.485 \\ -0.742 \end{array}$ |  |
|  | $\begin{aligned} & -3.827 \\ & -3.8 \end{aligned}$ | $\begin{aligned} & -3.865 \\ & -3.8 \end{aligned}$ |  | $\begin{array}{r} -177.475 \\ +44.369 \end{array}$ |  |  |  |
| 20 | 21 | $z_{5}$ | $20^{\prime}$ | $20^{\prime \prime}$ | $20^{\prime \prime \prime}$ | $20^{\prime \prime \prime \prime}$ | 22 |
| $\begin{array}{r} -1.932 \\ +3.035 \\ -1.912 \end{array}$ | $\begin{array}{r}-0.675 \\ +3.543 \\ \hline 1.912\end{array}$ | $\begin{aligned} & -0.675 \\ & +3.543 \\ & +1.932 \\ & +3.035 \\ & +7.5 \end{aligned}$ | -1.912 +1.1 | $\begin{aligned} & -\quad 1.912 \\ & +\quad 0.4 \end{aligned}$ | $\begin{aligned} & -1.912 \\ & -4.6 \end{aligned}$ | $\begin{aligned} & -1.912 \\ & +3.1 \end{aligned}$ | $\begin{aligned} & +0.167 \\ & +3.760 \\ & -0.521 \\ & -0.8 \end{aligned}$ |
| +6.4 | +1.1 |  | $\begin{aligned} & -0.812 \\ & -0.8 \end{aligned}$ | $\begin{aligned} & =1.512 \\ & =1.5 \end{aligned}$ | $\begin{aligned} & -6.512 \\ & -6.5 \end{aligned}$ | $\begin{aligned} & +1.188 \\ & +1.2 \end{aligned}$ |  |
| $\begin{aligned} & +5.591 \\ & +5.6 \end{aligned}$ | $\begin{aligned} & +2.056 \\ & +2.1 \end{aligned}$ | $\begin{array}{r} +11.471 \\ -\quad 1.912 \end{array}$ |  |  |  |  | $\begin{aligned} & +2.606 \\ & +2.6 \end{aligned}$ |
| $2_{6}$ | $22^{\prime}$ | $22^{\prime \prime}$ | $22^{\prime \prime \prime}$ | $22^{\prime \prime \prime \prime}$ | $22^{\circ}$ | 23 | 27 |
| +0.167 +3.760 | $\begin{aligned} & -0.521 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & -0.521 \\ & -3.2 \end{aligned}$ | $\begin{aligned} & -0.521 \\ & +6.2 \end{aligned}$ | $\begin{aligned} & =0.521 \\ & =0.1 \end{aligned}$ | $\begin{aligned} & -0.521 \\ & -2.8 \end{aligned}$ | $\begin{aligned} & +0.781 \\ & +4.362 \\ & -0.469 \\ & -2.8 \end{aligned}$ | $\begin{aligned} & +0.781 \\ & +4.862 \\ & -2.8 \end{aligned}$ |
|  | $\begin{aligned} & -0.721 \\ & -0.7 . \end{aligned}$ | $\begin{aligned} & -3.721 \\ & -3.7 \end{aligned}$ | $\begin{aligned} & +5.679 \\ & +5.7 \end{aligned}$ | $=\begin{aligned} & 0.621 \\ & 0.6 \end{aligned}$ | $\begin{aligned} & -3.321 \\ & -3.3 \end{aligned}$ |  |  |
| + +3.127 -0.521 |  |  |  |  |  | +1.874 +1.9 | $\begin{array}{r} +2.343 \\ +0.469 \end{array}$ |
| $23^{\prime}$ | $23^{\prime \prime}$ | $23^{\prime \prime \prime}$ | $23^{\prime \prime \prime \prime}$ | 24 | 28 | $24^{\prime}$ | $24^{\prime \prime}$ |
| - 0.469 -3.8 | $\begin{aligned} & -0.469 \\ & -5.0 \end{aligned}$ | $\begin{aligned} & -0.469 \\ & -2.4 \end{aligned}$ | $\begin{aligned} & -0.469 \\ & +11.2 \end{aligned}$ | $\begin{aligned} & +\quad 2.669 \\ & +\quad 8.956 \\ & -\quad 3.242 \\ & -\quad 1.9 \end{aligned}$ | $\begin{aligned} & +2.669 \\ & +\quad 8.956 \\ & -1.9 \end{aligned}$ | $\begin{aligned} & -3.242 \\ & +0.2 \end{aligned}$ | $\begin{aligned} & -3.242 \\ & -0.1 \end{aligned}$ |
| $\begin{aligned} & -4.269 \\ & -4.2 \end{aligned}$ | $\begin{aligned} & -5.469 \\ & -5.5 \end{aligned}$ | $\begin{aligned} & -2.869 \\ & -2.8 \end{aligned}$ | $\begin{aligned} & +10.731 \\ & +10.7 \end{aligned}$ | $\begin{array}{ll} = & 1.942 \\ \hline+ & 6.483 \\ + & 6.5 \end{array}$ | $\begin{array}{r} +9.725 \\ -3.242 \end{array}$ | $\begin{aligned} & -3.042 \\ & -3.0 \end{aligned}$ | $\begin{aligned} & -3.342 \\ & -3.3 \end{aligned}$ |

## Final computation of triangles



Final computation of truangles-Continued


Final computation of triangles-Continued


## SECOND METHOD.

The only difference between the second method of adjustment and the first is in the treatment of the directions taken at the fixed points. It these points observation equations are written for the directions of new points only and the $z$ 's are omitted. The observations taken over the fixed lines are not used, but the observed directions of the new lines are taken in connection with the adjusted direction of a fixed line, all directions being referred to a common initial line.

The equations with $z$ 's omitted are the same as if the angle method of adjustment were used. (See p. 196.) In this treatment these equations are given unit weight. Jordan (Vermessungskunde, vol. 1, p. 179, of the third edition) suggests that on some accounts it would be better to assign the equations for observations at fixed points only half weight.

The observation equations for directions taken at Gunner, Cranberry Point, and Telegraph are the same as for the first method given on page 115 and are not repeated here. Below are given the observation equations for the remaining points, formed according to the second method. The assumed azimuths are identical with those used in the first method. As an example, to illustrate the computation of the observed azimuths, take the line Mam-Gunner. Use the observed direction for the new line and the adjusted one for the fixed line. .

| Fixed azimuth Mam to Indian Point, page 106, | $=266$ | 17 | 19.2 |
| :--- | :--- | :--- | :--- | :--- |
| Angle Indian Point to Gunner, page $106\left(359^{\circ}\right.$ |  |  |  |
| $59^{\prime} 55^{\prime \prime} .4$ to $\left.322^{\circ} 01^{\prime} 44^{\prime \prime} .8\right)$, | $=322$ | 01 | 49.4 |
|  | $=228$ | 19 | 08.6 |

or by reckoning from any other fixed line through Mam the same result is reached, thus,


Note that the coefficients of the $\delta \phi$ 's and $\delta \lambda$ 's are exactly the same as for the first method and that the $z$ 's are omitted.
Formation of observation equations

## TREAT ${ }_{2}$

| Assumed azimuth | Observed azimuth | Equation | Station observed |
| :---: | :---: | :---: | :---: |
| $\begin{array}{ccc}\circ & \prime \prime \\ 337 & 10 & 43.7\end{array}$ | $\begin{array}{cccc} \circ 337 & 10 & 39.4+v_{19} \end{array}$ | $v_{19}=-6378 \phi_{2}+10788 \lambda_{2}+4.3$ | Cranberry Point |

## MAM

| $\begin{array}{llll}223 & 44 & 53.7 \\ 228 & 19 & 09.7\end{array}$ | $\begin{array}{lll}223 & 44 & 47.3+v_{20} \\ 228 & 19 & 08.6+v_{21}\end{array}$ | $\begin{aligned} & v_{20}^{20}=-20100 \partial_{2}-1455 \delta \lambda_{2}+6.4 \\ & v_{22}=2191{ }_{2}{ }_{21}-1388 \delta \lambda_{1}+1.1 \end{aligned}$ | Cranberry Point Gunner |
| :---: | :---: | :---: | :---: |

LARRABEE

| 165 | 22 | 02.6 | 165 | 22 | $03.7+v_{22}$ | $v_{22}=+541 \delta_{1}-1473 \delta \lambda_{1}-1.1$ | Gunner |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

indian point


The first part of each of tables 1 and 2 for the formation of normals according to the first method, pages 118 and 119, down to the line for $v_{19}$ will serve for the second method also and is not repeated here. The remainder of the tables according to the second method is given below.

In forming the normal equations the four $z$ 's that occur are eliminated by the device of the sum equation serving as a fictitious observation equation with negative weight. The other observations that do not contain $z$ 's enter into the formation of the normal equations in the usual way. After the normal equations have been solved the four $z$ 's are found from the sum equations in the way previously explained and enter into the computation of the $v$ 's, or corrections, from 1 to 18 and 20 and 21, but not into the others.

Table for formation of normals No. 1

|  | 1 | 2 | 3 | 4 | 5 | 6 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\delta \phi_{1}$ | $8 \lambda_{1}$ | $\delta \phi_{2}$ | $\delta \lambda_{2}$ | $\delta \phi_{3}$ | $8 \lambda_{3}$ | $\eta$ | $\boldsymbol{p}$ | $\sqrt{ } p$ |
| 19 |  |  | - 637 | $+1078$ |  |  | $+4.3$ | 1 | 1 |
| 20 |  |  | - 2010 | $-1485$ |  |  | + 6.4 | 1 | 1 |
| 21 | - 2191 | $-1388$ |  |  |  |  | + 1.1 | 1 | 1 |
| 22 | + 541 | - 1473 |  |  |  |  | - 1.1 | 1 | 1 |
| 23 | + 2538 | - 1709 |  |  |  |  | + 2.5 | 1 | 1 |
| 24 | + 8669 | $-3509$ |  |  |  |  | - 1.9 | 1 | 1 |

Table for formation of normals No. 2

|  | 1 | 2 | 3 | 4 | 5 | 6 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8 $\phi_{1}$ | d $\lambda_{1}$ | ग $\phi_{2}$ | $8 \lambda_{2}$ | $\delta \phi_{3}$ | $\delta^{2} \lambda_{3}$ | $\eta$ | $\Sigma$ |
| 19 20 21 22 23 24 | $\begin{aligned} & -2.19 \\ & +0.54 \\ & +2.54 \\ & +8.67 \end{aligned}$ | $=1.39$ $=1.47$ $=1.71$ -3.51 | -0.64 -2.01 | +1.08 |  |  | +0.43 +0.64 +0.11 -0.11 +0.25 -0.19 | +0.87 <br> +2.85 <br> +3.47 <br> 1.04 <br> +1.08 <br> +4.97 |

Normal equations

| 1 | 2 | 3 | 4 | 5 | 6 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +1014.5374 | $\begin{array}{r} -863.2282 \\ +918.6459 \end{array}$ | $\begin{aligned} & -822.5130 \\ & +886.9117 \\ & +924.4414 \end{aligned}$ | $\begin{aligned} & +781.8841 \\ & \mathbf{- 8 3 7 . 3 5 2 9} \\ & -831.3291 \\ & +843.4555 \end{aligned}$ | $\begin{aligned} & +8.0389 \\ & -13.2644 \\ & -39.8513 \\ & +36.7824 \\ & +43.7028 \end{aligned}$ | $\begin{array}{r} -0.2265 \\ +3.9464 \\ +1.6471 \\ -15.9112 \\ -33.6441 \\ +41.7793 \end{array}$ | $\begin{array}{r} \quad 8.5215 \\ +5.8167 \\ +3.7521 \\ +2.6609 \\ -18.8752 \\ +30.2371 \end{array}$ | $\begin{array}{r} +109.9712 \\ +91.4452 \\ +130.0589 \\ +19.8403 \\ +17.1109 \\ +44.8281 \end{array}$ |

$91865^{\circ}-15-9$
Solution of normals

| 1 | 2 | 3 | 4 | 5 | 6 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} +1014.5374 \\ \delta \phi_{:} \\ \\ 1 \end{array}$ | $\begin{aligned} & -863.2282 \\ & +\quad 0.8508589 \end{aligned}$ | $\begin{aligned} & -822.5130 \\ & +\quad 0.8107271 \end{aligned}$ | $\begin{aligned} & +781.8841 \\ & -\quad 0.7706804 \end{aligned}$ | $\begin{aligned} & +8.0389 \\ & -0.0079237 \end{aligned}$ | $\begin{array}{ll} -0.2265 \\ +0.0002233 \end{array}$ | $\begin{aligned} & -8.5215 \\ & +\quad 0.0083994 \end{aligned}$ | $\begin{aligned} & +109.9712 \\ & -\quad 0.1083954 \end{aligned}$ |
|  | $\begin{array}{r} +918.6459 \\ -734.4854 \end{array}$ | $\begin{array}{r} +876.9117 \\ -699.8425 \end{array}$ | $\begin{array}{r} -837.3829 \\ +665.2730 \end{array}$ | $\begin{aligned} & -13.2644 \\ & +6.8400 \end{aligned}$ | $\begin{array}{r} +3.9464 \\ -0.1927 \end{array}$ | $\begin{array}{r} +5.8167 \\ -7.2506 \end{array}$ | $\begin{aligned} & +91.4452 \\ & +93.5700 \end{aligned}$ |
|  | $\begin{gathered} +184.1605 \\ \delta \lambda_{3} \end{gathered}$ | $\begin{aligned} & +177.0692 \\ & -\quad 0.961494 \end{aligned}$ | $\begin{aligned} & -172.1099 \\ & +\quad 0.934565 \end{aligned}$ | $\begin{aligned} & -6.4244 \\ & +0.034885 \end{aligned}$ | $\begin{aligned} & +3.7537 \\ & -0.020383 \end{aligned}$ | $\begin{aligned} & -1.4339 \\ & +0.007786 \end{aligned}$ | $\begin{aligned} & +185.0152 \\ & -\quad 1.004641 \end{aligned}$ |
|  | 1 2 | $\begin{array}{r} +924.4414 \\ -666.8336 \\ -170.2510 \end{array}$ | $\begin{array}{r} -831.3291 \\ +633.8946 \\ +165.4826 \end{array}$ | $\begin{aligned} & -39.8513 \\ & +6.5174 \\ & +6.1770 \end{aligned}$ | $\begin{array}{r} +18.6471 \\ -0.1836 \\ -3.6092 \end{array}$ | $\begin{aligned} & +3.7521 \\ & +6.9086 \\ & +1.3787 \end{aligned}$ | $\begin{array}{r} +130.0589 \\ +\quad 89.1566 \\ -177.8910 \end{array}$ |
|  |  | $+\begin{gathered} 87.3568 \\ \delta \phi_{2} \end{gathered}$ | $\begin{aligned} & -31.9519 \\ & +\quad 0.365763 \end{aligned}$ | $\begin{aligned} & -27.1569 \\ & +0.310873 \end{aligned}$ | $\begin{aligned} & +14.8543 \\ & -0.170042 \end{aligned}$ | $\begin{aligned} & -1.7778 \\ & +0.020351 \end{aligned}$ | $\begin{aligned} & +41.3245 \\ & -\quad 0.473054 \end{aligned}$ |
|  |  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | $\begin{gathered} +843.4555 \\ -602.5828 \\ -160.8479 \\ -11.6868 \\ +68.3380 \\ \partial \lambda_{2} \end{gathered}$ | $\begin{aligned} & +36.7824 \\ & -6.1954 \\ & -6.0040 \\ & -9.9330 \\ & +14.6500 \\ & -0.214376 \end{aligned}$ | $\begin{aligned} & -15.9112 \\ & +0.1746 \\ & +3.5081 \\ & +5.4332 \\ & -6.7953 \\ & +0.099437 \end{aligned}$ | $\begin{aligned} & +2.6609 \\ & +6.5674 \\ & -1.3401 \\ & -0.6503 \\ & +7.2379 \\ & -0.105913 \end{aligned}$ | $\begin{aligned} & -19.8403 \\ & =84.7526 \\ & +172.9087 \\ & +15.1150 \\ & +83.4306 \\ & -\quad 1.220852 \end{aligned}$ |
|  |  |  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{array}{r} +43.7028 \\ =0.0637 \\ =0.2241 \\ -8.4423 \\ -3.1406 \\ +31.8321 \\ \delta \phi_{3} \end{array}$ | $\begin{aligned} & -33.6441 \\ & +0.0018 \\ & +0.1309 \\ & +4.6178 \\ & +1.4567 \\ & -27.4369 \\ & +0.861926 \end{aligned}$ | $\begin{aligned} & -18.8752 \\ & +0.0675 \\ & -0.0500 \\ & -0.5527 \\ & -1.5516 \\ & -20.9620 \\ & +0.658518 \end{aligned}$ | $\begin{array}{r} 17.1109 \\ =\quad 0.8714 \\ +\quad 6.4543 \\ +12.8467 \\ -17.8855 \\ -16.5668 \\ +\quad 0.520443 \end{array}$ |
|  |  |  |  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{aligned} & +41.7793 \\ & =0.0001 \\ & =0.0765 \\ & =2.5259 \\ & -0.6757 \\ & -23.6486 \end{aligned}$ | $\begin{aligned} & +30.2371 \\ & +0.0019 \\ & +0.0292 \\ & +0.3023 \\ & +0.7197 \\ & -18.0677 \end{aligned}$ | +44.8281 <br> $+\quad 0.0246$ <br> $-\quad 3.7712$ <br> $+\quad 8.0269$ <br> $+\quad 14.2961$ |
|  |  |  |  | - | $+14.8525$ | $\begin{aligned} & +13.2187 \\ & -0.89000 \end{aligned}$ | $\begin{aligned} & +28.0712 \\ & -\quad 1.89000 \end{aligned}$ |

Back solution

| $\delta \lambda_{3}$ | $\partial \phi_{3}$ | $\delta \lambda_{2}$ | $\delta \phi_{2}$ | $\delta \lambda_{1}$ | $\delta \phi_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $-0.89000$ | $+0.65852$ | $-0.10591$ | $+0.02035$ | $+0.00779$ | $+0.00840$ |
|  | $-0.76711$ | $-0.08850$ | +0.15131 | +0.01814 | $-0.00020$ |
| -0.89000 | -0.10859 | $+0.02328$ | $-0.03376$ | -0.00379 | $+0.00086$ |
|  |  | -0.17113 | -0.06259 | $\begin{aligned} & -0.15993 \\ & -0.07244 \end{aligned}$ | +0.13189 +0.06108 |
|  |  |  | +0.07534 | -0.21023 | -0.17888 |
|  |  |  |  | -0.21023 | +0.02315 |

Computation of corrections

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} +2.007 \\ +7.377 \\ +10.753 \end{array}$ | $\begin{aligned} & +0.588 \\ & +3.593 \\ & -10.753 \\ & -2.2 \end{aligned}$ | $\begin{aligned} & +0.125 \\ & +3.07 \\ & +10.753 \\ & +4.8 \end{aligned}$ | $\begin{aligned} & -0.507 \\ & +2.918 \\ & +10.753 \\ & +3.6 \end{aligned}$ | $\begin{aligned} & -1.796 \\ & +8.058 \\ & -10.753 \\ & +\quad 7.1 \end{aligned}$ | -0.843 $=3.389$ $=3.956$ +14.317 | $\begin{aligned} & -0.433 \\ & =3.322 \\ & -10.753 \\ & +25.5 \end{aligned}$ | $\begin{array}{r} -4.934 \\ -46.059 \\ +16.057 \\ +37.493 \end{array}$ |
| $\begin{aligned} & -1.369 \\ & -1.4 \end{aligned}$ | $\begin{aligned} & =8.772 \\ & =8.8 \end{aligned}$ | $\begin{aligned} & -2.731 \\ & -2.8 \end{aligned}$ | $\begin{aligned} & =4.742 \\ & =4.8 \end{aligned}$ | $\begin{aligned} & +2.609 \\ & +2.6 \end{aligned}$ | $\begin{aligned} & -10.753 \\ & +9.7 \end{aligned}$ | $\begin{aligned} & +10.992 \\ & +11.0 \end{aligned}$ | $\begin{aligned} & -10.753 \\ & +\quad 7.1 \end{aligned}$ |
|  |  |  |  |  | $\begin{aligned} & +5.106 \\ & +5.1 \end{aligned}$ |  | $\begin{aligned} & -1.096 \\ & -1.1 \end{aligned}$ |
| $z_{1}$ | 9 | 10 | 11 | 12 | 13 | 14 | $x_{2}$ |
| -5.793-27.727+16.057+37.493-3.956+14.347+55.6 | $\begin{array}{r} -4.934 \\ -46.059 \\ +16.057 \\ +37.493 \\ -1.223 \end{array}$ | $\begin{array}{r} -4.530 \\ \pm 8.402 \\ -1.223 \end{array}$ | $\begin{aligned} & -1.514 \\ & \pm 2.541 \\ & -1.223 \\ & -13.1 \end{aligned}$ | $\begin{aligned} & =3.156 \\ & =2.824 \\ & =4.549 \\ & +14.685 \\ & =1.223 \\ & -2.7 \end{aligned}$ | $\begin{aligned} & -1.541 \\ & =2.911 \\ & =1.223 \\ & +11.5 \end{aligned}$ | $\begin{aligned} & -0.480 \\ & =1.845 \\ & =1.223 \\ & +6.8 \end{aligned}$ | $\begin{aligned} & -4.934 \\ & -46.059 \\ & +4.836 \\ & +40.857 \\ & +4.549 \\ & +14.685 \\ & +2.5 \end{aligned}$ |
|  | $\begin{aligned} & +1.334 \\ & +1.3 \end{aligned}$ | $\begin{aligned} & +2.649 \\ & +2.7 \end{aligned}$ | $\begin{aligned} & -13.296 \\ & -13.3 \end{aligned}$ |  | $\begin{aligned} & +5.825 \\ & +5.8 \end{aligned}$ | $\begin{aligned} & +3.252 \\ & +3.2 \end{aligned}$ |  |
| $\begin{aligned} & +86.021 \\ & { }_{-10.753} \end{aligned}$ |  |  |  | $\begin{aligned} & +0.233 \\ & +0.2 \end{aligned}$ |  |  | $\begin{array}{r} 7.336 \\ \pm 1.223 \end{array}$ |
| 15 | 16 | 17 | 18 | $z_{3}$ | 19 | 20 | 21 |
| $\begin{aligned} & +1.282 \\ & +43.806 \\ & +44.790 \end{aligned}$ | $\begin{aligned} & -3.156 \\ & =2.824 \\ & \hline 4.549 \\ & +14.685 \\ & +\quad 44.790 \end{aligned}$ | $\begin{aligned} & -0.843 \\ & =3.389 \\ & =3.956 \\ & +14.347 \\ & +44.790 \\ & -54.8 \end{aligned}$ | $\begin{aligned} & -2.620 \\ & +25.267 \\ & +44.790 \\ & -62.2 \end{aligned}$ | $\begin{aligned} & =0.843 \\ & =3.889 \\ & =3.156 \\ & =2.824 \\ & =9.843 \\ & \pm 16.493 \end{aligned}$ | $\begin{aligned} & =0.480 \\ & =1.845 \\ & +4.3 \end{aligned}$ | $\begin{aligned} & \mathbf{- 1 . 5 1 4} \\ & +2.541 \\ & +6.4 \end{aligned}$ | $\begin{aligned} & -0.507 \\ & +2.918 \\ & +1.1 \end{aligned}$ |
| $\begin{aligned} & +2.266 \\ & +2.2 \end{aligned}$ |  |  | $\begin{aligned} & +5.237 \\ & +5.3 \end{aligned}$ |  | $\begin{aligned} & +1.975 \\ & +2.0 \end{aligned}$ | $\begin{aligned} & +7.427 \\ & +7.5 \end{aligned}$ | $\begin{aligned} & +3.511 \\ & +3.5 \end{aligned}$ |
|  | $\begin{aligned} & -3.654 \\ & -3.7 \end{aligned}$ | $\begin{aligned} & -3.851 \\ & -3.9 \end{aligned}$ |  | $\begin{array}{r} -179.162 \\ +\quad 44.790 \end{array}$ |  |  |  |
| 22 | 23 | 24 |  |  |  |  |  |
| $\begin{aligned} & +0.125 \\ & +3.097 \\ & -1.1 \end{aligned}$ | $\begin{aligned} & +0.588 \\ & +3.593 \\ & +2.5 \end{aligned}$ | $\begin{aligned} & +2.007 \\ & \pm 7.377 \\ & \pm 1.9 \end{aligned}$ |  |  |  |  |  |
| $\begin{aligned} & +2.122 \\ & +2.1 \end{aligned}$ | $\begin{aligned} & +6.681 \\ & +6.7 \end{aligned}$ | $\begin{aligned} & +7.484 \\ & +7.5 \end{aligned}$ |  |  |  |  |  |

Final computation of triangles


Final computation of triangles-Continued


Final computation of triangles-Continued


Final computation of triangles-Continued

| Symbol | Station. | Observed angle | Correction | Spherical angle | Spherical excess | Plane angle | Logarithm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -16+18 \\ & -10+12 \end{aligned}$ | Cranberry Point-Lubec Channel Lighthouse <br> Telegraph | •, " 295244.5 | $+9.0$ | $\prime \prime$ 53.5 |  | - , " | 2.842384 0.302589 |
|  | Cranberry Point | $\begin{array}{rr} 29 & 52 \\ 77 & 54.5 \\ & 06.9 \\ & 08.6 \end{array}$ | $\begin{array}{r} +9.0 \\ -2.5 \end{array}$ | $\begin{aligned} & 53.5 \\ & 04.4 \\ & 02.1 \end{aligned}$ |  | 721302.1 | 9.990245 |
|  | Lubec Channel Lighthouse <br> Telegraph-Lubec Channel <br> Lighthouse |  |  |  |  |  | 9. 978738 <br> 3. 135218 |
| $\begin{aligned} & -17+18 \\ & -5+6 \end{aligned}$ | Gunner-Lubec Channel Lighthouse | $\begin{array}{rrr} 26 & 57 & 51.3 \\ 66 & 40 & 04.1 \\ & 64.6 \end{array}$ | $\begin{aligned} & +9.2 \\ & +2.5 \end{aligned}$ | $\begin{aligned} & 60.5 \\ & 06.6 \\ & 52.9 \end{aligned}$ |  | 862152.9 | 2. 828820 |
|  | Telegraph |  |  |  |  |  | 0.343447 |
|  | Lunner Channel Lighthouse |  |  |  |  |  | 9.962951 9.999125 |
|  | Telegraph-Lubec C'hannel <br> Lighthouse |  |  |  |  |  | 3. 135218 |

## STATION GUNNER

Final position computation,


STATION CRANBERRY POINT

secondary triangulation
STATION GUNNER


STATION CRANBERRY POINT


Final position computation,
STATION TELEGRAPH

secondary triangulation-Continued.
STATION TELEGRAPH


Computation of probable errors

|  | Adopted $v_{\text {'s }}$ first solution | $v^{2}$ |  | $\begin{gathered} \text { Adopted } \\ v \mathrm{~s} \\ \text { second } \\ \text { solution } \end{gathered}$ | $v^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | + 0.2 | 0.04 | 1 | - 1.4 | 1.96 |
| 2 | -8.5 | 72.25 | 2 | - 8.8 | 77.44 |
| 3 | - 2.7 | 7.29 | 3 | - 2.8 | 7.84 |
| 4 | - 4.9 | 24.01 | 4 | - 4.8 | 23.04 |
| 5 | + 3.1 | 9.61 | 5 | + 2.6 | 6.76 |
| 6 | + 4.9 | 24.01 | 6 | + 5.1 | 26.01 |
| 7 | + 9.5 | 90.25 | 7 | +11.0 | 121.00 |
| 8 | $-1.5$ | 2.25 | 8 | -1.1 | 1.21 |
| 9 | + 1.7 | 2.89 | 9 | + 1.3 | 1.69 |
| 10 | +3.1 | 9.61 | 10 | +2.7 | 7.29 |
| 11 | $-13.1$ | 171.61 | 11 | $-13.3$ | 176.89 |
| 12 | + 0.6 | 0.36 | 12 | + 0.2 | 0.04 |
| 13 | + 5.0 | 25.00 | 13 | + 5.8 | 33.64 |
| 14 | +2.9 | 8.41 | 14 | +3.2 | 10.24 |
| 15 | +2.3 | 5.29 | 15 | + 2.2 | 4.84 |
| 16 | -3.8 | 14.44 | 16 | -3.7 | 13.69 |
| 17 | - 3.8 | 14.44 | 17 | - 3.9 | 15.21 |
| 18 | + 5.4 | 29.16 | 18 | + 5.3 | 28.09 |
| 19 | + 0.8 | 0.64 | 19 | +2.0 | 4.00 |
| $19^{\prime}$ | -0.7 | 0.49 | 20 | + 7.5 | 56. 25 |
| 20 | + 5.6 | 31.36 | 21 | $+3.5$ | 12.25 |
| ${ }_{21}^{21}$ | + 2.1 | 4.41 | 22 | + 2.1 | 4.41 |
| $20^{\prime \prime}$ | - 0.8 | 0.64 | 23 | + 6.7 | 44.89 |
| $20^{\prime \prime \prime}$ | - 1.5 | 2.25 | 24 | + 7.5 | 56.25 |
| $20^{\prime \prime \prime \prime}$ | +1.2 | 1.44 |  |  | 734.93 |
| 22 | + 2.6 | 6.76 |  |  |  |
| ${ }_{2}^{22^{\prime \prime}}$ | - 0.7 | 0.49 |  |  |  |
| 22'ı | -3.7 +5.7 | 13.69 32.49 |  |  |  |
| $22^{\prime \prime \prime \prime}$ | a +5.7 -0.6 | 32.49 0.36 |  |  |  |
| $22^{2}$ | -3.3 | 10.89 |  |  |  |
| ${ }^{23}$ | + 1.9 | 3.61 |  |  |  |
| ${ }_{23} 3^{\prime \prime}$ | -4.2 | 17.64 |  |  |  |
| $23^{\prime \prime \prime}$ | - 5.5 -2.8 | 30.25 7.84 |  |  |  |
| $23^{\prime \prime \prime \prime}$ | +10.7 | 114.49 |  |  |  |
| 24 | +6.5 | 42.25 |  |  |  |
| ${ }_{24 \prime}^{24^{\prime \prime}}$ | - 3.0 -3.4 | 9.00 |  |  |  |
| 24 | -3.4 | 11.56 |  |  |  |
|  |  | 895.72 |  |  |  |

In the first method of adjustment, there are 40 equations to determine 14 unknown quantities, namely, $6 \delta \phi$ 's and $\delta \lambda$ 's and $8 z$ 's. The probable error of an observed direction is therefore,

$$
0.6745 \sqrt{\frac{895.72}{40-14}}= \pm 4.0
$$

In the second method there are 24 equations to determine 9 unknown quantities, namely, $6 \delta \phi$ 's and $\delta \lambda$ 's and $3 z$ 's at new points. The probable error of an observed direction is therefore,

$$
0.6745 \sqrt{\frac{734.93}{15}}= \pm 4.7
$$

ADJUSTMENT OF A FIGURE CONTAINING LATITUDE, LONGITUDE, AZIMUTH, AND LENGTH CONDITIONS BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

This example illustrates the fitting of a chain of triangulation in between fixed lines at the ends. The necessary preliminary computations of the assumed positions and directions could have been carried out in much the same way as in the preceding examples. A preliminary figure adjustment was, however, available and the results of it were used in the preliminary computations of the triangles and the geographic positions, pages 140-157. The fixed lines at the ends of the chain are shown in the following list of fixed positions. The list of observed directions is not given. The necessary data may be derived from the observed angles of the triangles, pages 140-143, taken in connection with figure 7, page 157.

Table of fixed positions

| Station | Latitude and longitude | Azimuth | Back azimuth | To station | Logarithm of distance. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fort Morgan. | $\begin{array}{lll} 30 & 13 & 42.242 \\ 88 & 01 & 23.228 \end{array}$ | - , " | - , " |  |  |
| Dauphin Island east base. | $\begin{array}{lll} 30 & 14 & 56.379 \\ 88 & 08 & 14.288 \end{array}$ | 2814217.9 | 1014544.9 | Fort Morgan. | 4. 050203 |
| Dauphin Island west base. | $\begin{array}{lll} 30 & 14 & 21.492 \\ 88 & 14 & 51.034 \end{array}$ | 2641122.1 | 841441.9 | Dauphin Island east base | 4. 027832 |
| Biloxi Lighthouse. | $\begin{array}{lll} 30 & 23 & 39.419 \\ 88 & 54 & 03.820 \end{array}$ |  |  |  |  |
| Ship Island Lighthouse. | $\begin{aligned} & 301245.341 \\ & 885757.464 \end{aligned}$ | 1971219.7 | 171417.6 | Biloxi Lighthouse. | 4. 323998 |

Preliminary computation of triangles

| Station | Observed angle | Correction | Spher- ical angle | Spher- ical excess | Plane angle | Logarithm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - " 1 | " | " | " | - , " |  |
| Fort Morgan-Dauphin Island east base |  |  |  |  |  | 4. 050203 |
| Cedar | 435421.8 | $+0.0$ | 21.8 | 0.1 | 21.7 | 0.158968 |
| Fort Morgan | 423337.7 | $+0.3$ | 38.0 | 0.1 | 37.9 | 9.830183 |
| Dauphin Island east base | 933159.6 | + 0.9 | 60.5 | 0.1 | 3200.4 | 9.999174 |
| Cedar-Dauphin Island east base |  | $+1.2$ |  | 0.3 |  | 4.039354 |
| Cedar-Fort Morgan |  |  |  |  |  | 4. 208345 |
| Dauphin Island east base-Dauphin Island west base |  |  |  |  |  | 4.027832 |
| Cedar | 372633.0 | $-0.9$ | 32.1 | 0.1 | 32.0 | 0.216124 |
| Dauphin Island east base | 1035536.4 | $-0.9$ | 35.5 | 0.1 | 35.4 | 9.987043 |
| Dauphin Island west base | 383752.5 | +0.2 | 52.7 | 0.1 | 52.6 | 9.795398 |
|  |  | $-1.6$ |  | 0.3 |  |  |
| Cedar-Dauphin Island west base |  |  |  |  |  | $\text { 4. } 230999$ |
| Cedar-Dauphin Island east base |  |  |  |  |  | $4.039354$ |
| Cedar-Dauphin Island east base |  |  |  |  |  | 4.039354 |
| Cat | 693032.7 | $+0.8$ | 33.5 | 0.0 | 33.5 | 0.028386 |
| Cedar | 601114.2 | $+1.4$ | 15.6 | 0.1 | 15.5 | 9.938349 |
| Dauphin Island east base | 501811.4 | $-0.3$ | 11.1 | 0.1 | 11.0 | 9.886171 |
| Cat-Dauphin Island east base |  | $+1.9$ |  | 0.2 |  | 4.006089 |
| Cat-Cedar |  |  |  |  |  | 3.953911 |
| Cedar-Dauphin Island west base |  |  |  |  |  | 4. 230999 |
| Cat | 1353154.9 | $+2.7$ | 57.6 | 0.0 | 57.6 | 0.154590 |
| Cedar ${ }^{\text {din }}$ | 224441.2 | $+2.3$ | 43.5 | 0.1 | 43.4 | 9.587303 |
| Dauphin Island west base | 214318.7 | + 0.4 | 19.1 | 0.1 | 19.0 | 9.568322 |
| Cat-Dauphin Island west base |  | $+5.4$ |  | 0.2 |  | 3.972892 |
| Cat-Cedar |  |  |  |  |  | 3.953911 |
| Dauphin Island east base-Dauphin Island west base |  |  |  |  |  | 4. 027832 |
| Cat | 660122.2 | $+1.9$ | 24.1 | 0.1 | 24.0 | 0.039191 |
| Dauphin Island east base | 533725.0 | $-0.6$ | 24.4 | 0.1 | 24.3 | 9.905869 |
| Dauphin Island west base | 602111.2 | +0.6 | 11.8 | 0.1 | 11.7 | 9.939066 |
|  |  | $+1.9$ |  | 0.3 |  |  |
| Cat-Dauphin Island west base Cat-Dauphin Island east base |  |  |  |  |  | 3.972892 <br> 4. 006089 |
|  |  |  |  |  |  |  |
| Cedar-Cat |  |  |  |  |  | 3.953911 |
| Pins | 232240.3 | $-1.8$ | 38.5 | 0.1 | 38.4 | 0. 401445 |
| Cedar | 305461.5 | $-3.5$ | 58.0 | 0.1 | 57.9 | 9.710779 |
| Cat | 18.4 |  | 23.7 | 0.0 | $125 \quad 4223.7$ | 9.909565 |
|  |  |  |  | 0.2 |  |  |
| Pins-Cat |  |  |  |  |  | 4.066135-1 |
| Pins-Cedar |  |  |  |  |  | 4.264921 |
| Cedar-Dauphin Island west base |  |  |  |  |  | 4.230999 |
| Pins | 584526.0 | $-2.8$ | 23.2 | 0.2 | 23.0 | 0.068049 |
| Cedar | 533942.7 | $-1.2$ | 41.5 | 0.2 | 41.3 | 9.906082 |
| Dauphin Island west base | 673457.9 | $-2.0$ | 55.9 | 0.2 | 55.7 | 9.965873 |
|  |  | $-6.0$ |  | 0.6 |  |  |
| Pins-Dauphin Island west base Pins-Cedar |  |  |  |  |  | 4. 205130 4. 264921 |
| Cat-Dauphin Island west base |  |  |  |  |  | 3.972892 |
| Pins | 352245.7 | $-1.0$ | 44.7 | 0.1 | 44.6 | 0.237334 |
| Cat in tsland west base | $45 \quad 3135.3$ |  | 38.7 | 0.0 | 984538.7 | 9.994903 9.855908 |
| Dauphin Island west base | 455139.2 | $-2.4$ | 36.8 | 0.1 | 36.7 | 9.855908 |
| Pins-Dauphin Island west base Pins-Cat |  |  |  | 0.2 |  | 4. $205129^{+1}$ <br> 4. 066134 |

Preliminary computation of triangles-Continued


Preliminary computation of triangles-Continued

| Station | Observed angle | Correction | Spherical angle | Spherical excess | Plane angle | Logarithm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - , 1 | " | " | " | - , " |  |
| Pascagoula-Horn |  |  |  |  |  | 4.137527 |
| Belle | 485849.8 | $-1.8$ | 48. 0 | 0.2 | 47.8 | 0.123352 |
| Pascagoula | 694328.0 | - 3.4 | 24.6 | 0.1 | 24.5 | 9.972217 |
| Horn | 611753.5 | - 5.6 | 47.9 | 0.2 | 47.7 | 9.943058 |
|  |  | $-10.8$ |  | 0.5 |  |  |
| Belle-Horn |  |  |  |  |  | 4.232096 |
| Belle-Pascagoula |  |  |  |  |  | 4.202937 |
| Belle-Pascagoula |  |  |  |  |  | 4.202937 |
| Club | 584043.3 | $+1.1$ | 44.4 | 0.1 | 44.3 | 0.068406 |
| Belle Pascagoula | $\begin{array}{llll}89 & 28 & 55.3 \\ 31 & 50 & 23.4\end{array}$ | +1.6 -2.1 | 54.7 +21.3 | 0.1 0.1 | 54.5 21.2 | 9.999982 9.722253 |
|  |  | $-1.6$ |  | 0.4 |  |  |
| Club-Pascagoula |  |  |  |  |  | 4. 271325 |
| Club-Belle |  |  |  |  |  | 3.993596 |
| Belle-Horn |  |  |  |  |  | 4. 232096 |
| Club | 1054356.9 | $-0.4$ | 56.5 | 0.1 | 56.4 | 0.016582 |
| Belle | 403005.5 | + 1.2 | 06.7 | 0.1 | 06.6 | 9. 812561 |
| Horn | . 334553.7 | $+3.4$ | 57.1 | 0.1 | 57.0 | 9.744918 |
|  |  | $+4.2$ |  | 0.3 |  |  |
| Club-Horn |  |  |  |  |  | 4. 061239 |
|  |  |  |  |  |  |  |
| Pascagoula-Horn |  |  |  |  |  | 4. 137527 |
| Club | 470313.6 | -1.5 | 12.1 | 0.1 | 12.0 | 0.135436 |
| Pascagoula Horn | $\begin{array}{llll}37 & 53 & 04.6 \\ 95 & 03 & 47.2\end{array}$ | 1.3 -2.2 | 03.3 45.0 | 0.1 0.2 | 03.2 44.8 | 9.788216 |
|  | 950347.2 | - 2.2 | 4.0 |  | 44.8 | 9.998302 |
|  |  | $-5.0$ |  | 0.4 |  |  |
| Club-Horn <br> Club-Pascagoula |  |  |  |  |  | 4. 061239 |
| Belle-Club |  |  |  |  |  | 3.983596 |
| Decr | 410210.7 | $+0.7$ | 11.4 | 0.1 | 11.3 | 0.182739 |
| Belle | 1023518.5 | $+1.5$ | 20.0 | 0.0 | 20.0 | 9.989432 |
| Club | 362226.8 | + 2.0 | 28.8 | 0.1 | 28.7 | 9.773101 |
|  |  | $+4.2$ |  | 0.2 |  |  |
| $\begin{aligned} & \text { Deer-Club } \\ & \text { Deer-Belle } \end{aligned}$ |  |  |  |  |  | 4.165767 3.949436 |
| Deer-Belle |  |  |  |  |  | 3.949436 |
| Ship | 331060.2 | $-1.8$ | 58.4 | 0.1 | 58.3 | 0.261764 |
| Deer | 974937.9 | $-1.8$ | 36.1 | 0.1 | 36.0 | 9.995935 |
| Belle | 485922.9 | + 2.9 | 25.8 | 0.1 | 25.7 | 9.877717 |
|  |  | $-0.7$ |  | 0.3 |  |  |
| Ship-Belle Ship-Deer |  |  |  |  |  | $\begin{aligned} & 4.207135^{+1} \\ & 4.088917 \end{aligned}$ |
| Deer-Club |  |  |  |  |  | 4.165767 |
| Ship | 70523.5 .0 | $-0.5$ | 34.5 | 0.2 | 34.3 | 0.024654 |
| Deer | 564727.2 | $-2.5$ | 24.7 | 0.1 | 24.6 | 9.922554 |
| Club | 522003.5 | $-2.3$ | 01.2 | 0.1 | 01.1 | 9.898496 |
|  |  | $-5.3$ |  | 0.4 |  |  |
| $\begin{aligned} & \text { Ship-Club } \\ & \text { Ship-Deer } \end{aligned}$ |  |  |  |  |  | $\begin{aligned} & \text { 4. } 112975 \\ & 4.088917 \end{aligned}$ |
| Belle-Club |  |  |  |  |  | 3.993596 |
| Ship | 374134.8 | + 1.3 | 36.1 | 0.1 | 36.0 | 0.213650 |
| Celle | 53 88 88 45 45 51 35.6 | -1.4 | 54.2 30.0 | 0.1 | 54.1 | 9.9057729 9.999890 |
| Club | 884230.3 | $-0.3$ | 30.0 | 0.1 | 29.9 | 9.999890 |
|  |  | -0.4 |  | 0.3 |  |  |
| Ship-Belle |  |  |  |  |  | 4.207136 |

Preliminary computation of iriangles-Continued

| Station | $\begin{aligned} & \text { Observed } \\ & \text { angle } \end{aligned}$ | $\begin{gathered} \text { Correc- } \\ \text { tion } \end{gathered}$ | Spherical angle | Spherical excess exces | Plane angle | Loga- |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | " | " | " | " | - , " |  |
| $\mathrm{Deer-Ship}_{\text {Biloxi Lighthouse }}$ | 481117.4 | $+3.6$ | 21.0 | 0.1 |  | 4.085917 0.127640 |
| Deer | 963031.2 | +3.6 <br> +2.3 | 33.5 | 0.1 | 33.4 | 9.997191 |
| Ship | 351804.4 | +1.4 | 05.8 | 0.1 | 05.7 | 9.761838 |
|  |  | + 7.3 |  | 0.3 |  |  |
| Biloxi Iighthouse-Ship |  |  |  |  |  | 4. 213748 |
| Biloxi Lighthouse-Deer |  |  |  |  |  | 3.978395 |
| Liloxi Lighthouse-Deer |  |  |  |  |  | 3. 978395 |
| Ship Island Lighthouse | 253002.1 | + 1.4 | 03.5 | 0.2 | 03.3 | 0. 366001 |
| Biloxi Lighthouse | 815536.0 | $-1.7$ | 34.3 | 0.1 | 34.2 | 9.995674 |
| Deer | 723426.4 | $-3.7$ | 22.7 | 0.2 | 22.5 | 9.979593 |
|  |  | -4.0 |  | 0.5 |  |  |
| Ship Island Lighthouse-Biloxi Lighthouse |  |  |  |  |  | 4.340070 4.323989 |
| Biloxi Lighthouse-Ship |  |  |  |  |  | 4.213748 |
| Ship Island Lighthouse | 503141.2 | -0.9 | 40.3 | 0.2 | 40.1 | 0.112420 |
| Biloxi Lighthouse | 334418.6 | - 5.3 | 13.3 | 0.2 | 13.1 | 9.744591 |
| Ship | 954407.0 | -0.1 | 06.9 | 0.1 | 06.8 | 9.997S21 |
|  |  | $-6.3$ |  | 0.5 |  |  |
| Ship Island Lighthouse-Ship <br> Ship Island Lighthouse-Biloxi Lighthouse |  |  |  |  |  | $\begin{aligned} & 4.070759 \\ & 4.323989 \end{aligned}$ |
| Deer-Ship |  |  |  |  |  | 4.088917 |
| Ship Island Lighthouse | 250139.1 | $-2.3$ | 36.8 | 0.1 | 36.7 | 0.373615 |
| Deer | 235604.8 | +6.0 | 10.8 | 0.1 | 10.7 | 9.608227 |
| Ship | 1310211.4 | +1.3 | 12.7 | 0.1 | 12.6 | 9.887537 |
|  |  | $+5.0$ |  | 0.3 |  |  |
| Ship Island Lighthouse-ship Ship Island Lighthouse-Deer |  |  |  |  |  | 4.070759 <br> 4. $340069^{+}$ |

Preliminary position computation,
STATION CAT


STATION CEDAR

secondary triangulation
StATION CAT


STATION CEDAR


Preliminary position computation,
STATION PINS


STATION GRAND

sciondary trinngulation-Continued

## STATION PINS



STATION GRAND


Preliminary position computation,

## STATION PETIT



## STATION HORN


secondary triangulation-Continued
STATION PETIT


STATION HORN


Preliminary position computation,
station pascagoula


STATION BELLE

secondary Iriangulation-C'ontinued
STATION PASCAGOULA


STATION BELLE


Preliminary position computation,

## Station club



STATION DEER

secondary triangulation-Continued
STATION CLUB


STATION DEER


Preliminary position computation,
Station ship

| $\stackrel{\alpha}{\boldsymbol{\alpha}}$ Second angle | Deer to Club Club and Ship |  |  |  |  |  | $\begin{array}{r} 322 \\ +56 \end{array}$ | $\begin{aligned} & 03 \\ & 47 \end{aligned}$ | $\begin{array}{r} \prime \prime \prime \\ 14.8 \\ 24.7 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \alpha \\ \Delta \alpha \end{gathered}$ | Deer to Ship |  |  |  |  |  | - 18 | 50 1 | $\begin{aligned} & 39.5 \\ & 14.8 \end{aligned}$ |
| $\alpha^{\prime}$ | Ship to Deer |  |  | First angle of triangle |  |  | $\begin{array}{r} 180 \\ 198 \\ 70 \end{array}$ | $\begin{aligned} & 00 \\ & 49 \\ & 52 \end{aligned}$ | $\begin{aligned} & 00.00 \\ & 24.7 \\ & 34.5 \end{aligned}$ |
| ${ }_{\Delta}^{\phi}{ }_{\phi}$ | $-30$ | 21 27 <br> 6 17 | 7. 220 7. 200 | Deer |  | $\lambda$ $\Delta \lambda$ | 88 $+\quad 8$ | 48 2 | $\begin{aligned} & \text { 41. } 870 \\ & 28.269 \end{aligned}$ |
| $\phi^{\prime}$ | 30 | 15 10 | 0.020 | Ship |  | $\lambda^{\prime \prime}$ | 88 | 51 | 10.139 |
| $\frac{1}{2}\left(\phi+\phi^{\prime}\right)$ | $\begin{array}{ccc}\circ & \prime \\ 30 & 18 & \prime \prime \\ & \prime \prime \\ & \prime \prime\end{array}$ | $\left\|\begin{array}{c}s \\ \cos \alpha \\ \mathrm{~B}\end{array}\right\|$ | 4. 088917 <br> 9.976075 <br> 8.511549 | $\mathrm{sin}^{8^{2}} \mathrm{Si}^{2} \alpha$ | 8. 1778 9.0184 1.1731 | $\mathrm{h}^{\mathbf{D}}$ | 5. 153 2. 332 |  |  |
| $\left.\begin{array}{c} \text { 1st term } \\ \left.\begin{array}{c} \text { d and } 3 \mathrm{~d} \\ \text { terms } \\ -\Delta \phi \end{array}\right\} \mid, ~ \end{array}\right\}$ | $\begin{array}{r}+377.1734 \\ +\quad 0.0265 \\ \hline\end{array}$ | h | 2. 576541 |  | 8.3693 +0.0234 |  | 7.485 +0.0031 |  |  |
|  | +377. 1999 |  |  |  |  |  |  |  |  |
|  | $\begin{array}{c\|c\|c} s & 4.088917 \\ \sin ^{\prime} \alpha & 9.509200 & \\ A^{\prime} & 8.509353 \\ \sec ^{\prime} \phi^{\prime} & 0.063581 \\ & 2.171051 & \Delta \lambda \\ & \sin \frac{1}{2}\left(\phi+\phi^{\prime}\right) \\ & \prime \prime & \\ \Delta \lambda & +148.2692 & -\Delta \alpha \end{array}$ |  |  |  | $\begin{aligned} & 71051 \\ & 02952 \end{aligned}$ |  |  |  |  |
|  |  |  |  |  | $\begin{aligned} & 74003 \\ & \prime \prime \\ & 74.82 \end{aligned}$ |  |  |  |  |

STATION BILOXI LIGHTHOUSE

secondary triangulation-Continued
Station ship


STATION BILOXI LIGHTHOUSE


Preliminary position computation,
Station ship island lighthouse


Fixed $\alpha$ Biloxi Lighthouse to Ship Island Lighthouse, $17^{\circ} \mathbf{1 4}^{\prime} \mathbf{1 7 . 6 ^ { \prime \prime }}$.
secondary triangulation-Continued
STATION SHIP ISLAND LIGHTHOUSE



Fig. 7.

## FORMATION OF OBSERVATION EQUATIONS

The position computation was carried westward from the fixed lines at the eastern end of the scheme and the observation equations were formed in the same order. The treatment of fixed lines is the same as in the first adjustment of figure 6, pages 105 et seq. No new detail arises until the points Deer and Ship are reached, which have lines connecting them with the fixed points Biloxi Lighthouse and Ship Island Lighthouse. Suppose for the moment that Biloxi Lighthouse were not fixed but that its latitude and longitude were to receive corrections of $\delta \phi_{12}$ and $\delta \lambda_{12}$ respectively. The observation equation for $v_{57}$ would then read,

$$
v_{57}=z_{13}-603 \delta \phi_{11}+248 \delta \lambda_{11}+603 \delta \phi_{1 z}-248 \delta \lambda_{12}+0.5
$$

The latitude and longitude of Biloxi Lighthouse as developed by the preliminary position computation are $30^{\circ} 23^{\prime} 399^{\prime \prime} 419$ and $88^{\circ}$ $54^{\prime} 033^{\prime \prime} 954$, while the fixed values are $30^{\circ} 23^{\prime} 39 .^{\prime \prime} 419$ and $88^{\circ} 54^{\prime}$ $03 .{ }^{\prime \prime} 820$, so that to reduce the preliminary to the fixed valucs on which the adjustment is built corrections of $\delta \phi_{12}=0.000^{*}$ and $\delta \lambda_{12}=-0.134$ are necessary. By substituting these values in the equation for $v_{57}$ there results,

$$
v_{57}=z_{13}-603 \delta \phi_{11}+248 \delta \lambda_{11}+603(0.000)-248(-0.134)+0.5
$$

or,

$$
v_{57}=z_{13}-603 \delta \delta \phi_{11}+248 \delta \lambda_{11}+33.7
$$

as given in the table page 16.
The equation for the reverse direction, $v_{63}$, if Biloxi Lighthouse were not fixed, would be,

$$
v_{63}=z_{15}-603 \delta \phi_{11}+248 \delta \lambda_{11}+603 \delta \phi_{12}-24 \delta \delta \lambda_{12}+0.0
$$

which with the use of the above values of $\delta \phi_{12}$ and $\delta \lambda_{12}$ becomes,

$$
v_{63}=z_{15}-603 \delta \partial \phi_{11}+248 \delta \lambda_{11}+33.2
$$

Similar computations must be made for $v_{56}, v_{58}, v_{59}, v_{64}, v_{67}$, and $v_{68}$.
The known terms in the exprossions for $v_{65}$, and $v_{66}$ may be found in a similar way by allowing for the fixity of both ends of the line. If the corrections needed to the preliminary position of Ship Island Lighthouse in order to reduce it to the fixed position are called $\delta \phi_{13}$ and $\delta \lambda_{13}$ then for the line Biloxi Lighthouse to Ship Island Lighthouse the formula gives,

$$
d \alpha=89\left(\delta \phi_{13}-\delta \phi_{12}\right)+250\left(\delta \lambda_{13}-\delta \lambda_{12}\right)
$$

If the line were free to be turned in azimuth, then by the adjustment,

$$
v_{65}=z_{15}+\delta 9\left(\partial \phi_{13}-\partial \phi_{12}\right)+250\left(\partial \lambda_{13}-\partial \lambda_{12}\right)-1.7
$$

[^13]But to reduce the positions of the preliminary computation to the fixed positions, $\delta \phi_{13}=-0.002, \delta \phi_{12}=0.000, \delta \lambda_{13}=-0.080$, and $\delta \lambda_{12}=-0.134$ (see pp. 155, 157). Substituting these values gives,

$$
v_{65}=z_{15}+11.6
$$

as given in the table. $v_{68}$ is found by a similar process.
The effect of using for the preliminary computation values from a previous adjustment for the figure but not for the positions appears in the constant term of the normal equations until the effect of closure in positions comes in. These constant terms should be zero except for the effect of accumulated errors in the last place of the two computations and a slight difference in the treatment of directions (3) and (3a). They are in fact almost negligible until the effect of closure appears in the equation for $\delta \phi_{8}$, from which point they become quite large.

Fort Morgan-Dauphin Island east base:

$$
\begin{aligned}
& \text { Assumed azimuth, } \begin{array}{lll}
101 & 45 & 44.9
\end{array} \\
& \text { Observed azimuth, } 101 \quad 45 \quad 44.9-z_{1}+v_{1} \\
& \begin{array}{c}
0=0.0+z_{1}-v_{1} \\
v_{1}=z_{1}+0.0
\end{array} \\
& \text { Fort Morgan-Cedar: } \\
& \text { Assumed azimuth, } 144 \quad 19 \quad 22.9+\mathrm{d} \alpha \\
& \text { Observed azimuth, } 144 \quad 19 \quad 22.6-z_{1}+v_{2} \\
& 0=+0.3+z_{1}-v_{2}+229 \dot{\partial} \phi_{1}-277 \delta \lambda_{1} \\
& v_{2}=z_{1}+229 \delta \phi_{1}-277 \delta \lambda_{1}+0.3
\end{aligned}
$$

DAUPHIN ISLAND EAST BASE

| Assumed azimuth | Observed azimuth | Equation | Station observed |
| :---: | :---: | :---: | :---: |
| - , " | - , " |  |  |
| 841441.9 | $841441.9-z_{2}+v_{3}$ | $v_{3}=z_{2}+0.0$ | West base |
| 1375206.3 | $1375206.9-z_{2}+v_{4}$ | $v_{4}=+z_{2}+4208 \delta_{2}-4038 \lambda_{2}-0.6$ | Cat |
| 1881017.4 | $1881018.3-z_{2}+v_{5}$ | $v_{3}=+z_{2}-82 \partial \phi_{1}-498 \partial \lambda_{1}-0.9$ | Cedar |
| 2514217.9 | $2814217.9-z_{2}+v_{3} a$ | $v_{3 a}=z_{2}+0.0$ | Fort Morgan |

(1) CEDAR

| 3241625.0 | $3241625.0-z_{3}+v_{4} a$ | $v_{4}=z_{3}+2298 \phi_{1}-2778 \lambda_{1}+0.0$ | Fort Morgan |
| :---: | :---: | :---: | :---: |
| 81046.8 | $81046.8-z_{3}+v_{5} a$ | $v_{50}=z_{3}-82 \delta \phi_{1}-4988 \lambda_{1}+0.0$ | East base |
| 453718.9 | $453719.8-z_{3}+v_{6}$ | $v_{6}=z_{3}-266 \% \phi_{1}-226 \partial \lambda_{1}-0.9$ | West base |
| 682202.4 | $682201.0-z_{3}+v_{7}$ | $v_{7}=z_{3}-657 \delta \phi_{1}-228 \delta \lambda_{1}+657 \delta \phi_{2}+226 \delta \lambda_{2}+1.4$ | Cat |
| 991700.4 | $991702.5-z_{3}+v_{8}$ | $v_{8}=z_{3}-341 \delta \phi_{1}+48 \partial \lambda_{1}+341 \delta \phi_{3}-48 \delta \lambda_{3}-2.1$ | Pins |

(2) CAT

| 235122.0 | $235122.0-z_{1}+v_{17}$ | $v_{17}=2_{4}-2738 \phi_{2}-5378 \lambda_{2}+0.0$ | West base |
| :---: | :---: | :---: | :---: |
| 1223700.7 2481924.4 | 24819 27.1-zi+v15 |  | Pins |
| 3174957.9 | 248 19 27.1 <br> 317 59.8  <br> 59.8   | $\begin{aligned} & v_{15}=z_{4}-657 \delta \phi_{1}-226 \delta \lambda_{1}+657 \delta \phi_{2}+226 \delta \lambda_{2}-2.7 \\ & v_{16}=z_{1}+420 \delta \phi_{2}-403 \delta \lambda_{2}-1.9 \end{aligned}$ | East base |

DAUPHIN ISLAND WEST BASE

| 2641122.1 | $26411.22 .1-z_{5}+v_{14}$ | $v_{14}=z_{5}+0.0$ | East base |
| :---: | :---: | :---: | :---: |
| 785702.1 | $785701.7-z_{5}+v_{9}$ | $v_{9}=z_{5}+367 \delta^{\prime} \phi_{4}+623 \lambda_{1}+0.4$ | Petit |
| 1180341.5 | $1180340.8-z_{5}+v_{10}$ | $v_{10}=z_{5}+2878 \phi_{5}-1338 \lambda_{5}+0.7$ | Grand |
|  | $1575831.7-z_{5}+v_{11}$ | $v_{11}=z_{5}+1488 \phi_{3}-3188 \lambda_{3}+1.8$ | Pins |
| 2035010.3 | ${ }^{203} 503010.9-z_{5}+v_{13}$ | $v_{12}=z_{5}-273 \delta \phi_{2}-5378 \lambda_{2}-0.6$ |  |
| 2253329.4 | $2253329.6-z_{5}+v_{13}$ | $v_{13}=z_{5}-2668 \phi_{1}-226 \delta \lambda_{1}-0.2$ | Cedar |

(3) PINS

| Assumed azimuth | Observed azimuth | Equation | Station observed |
| :---: | :---: | :---: | :---: |
| - , " | - , " |  |  |
| ${ }^{2} 791116.6$ | $2791116.6-z_{6}+v_{18}$ | $v_{18}=z_{6}-341 \delta \phi_{1}+48 \delta \lambda_{1}+341 \delta \phi_{3}-48 \delta \lambda_{3}+0.0$ | Cedar |
| 302335051 337 56 | ${ }_{337} 302335654.9-z_{6}+v_{19}$ |  | West base |
| ${ }^{30} 224929$ | ${ }_{30} 2252.1-2_{6}+v_{21}$ | $v_{21}=z_{6}-153 \delta^{2} \phi_{3}-228 \delta^{2} \lambda_{3}+153 \delta^{\prime} \phi_{4}+228 \delta \lambda_{4}-2.9$ | Petit |
| 630948.1 | $630949.6-z_{6}+v_{22}$ | $v_{22}=z_{6}-450 \delta \phi_{3}-198 \partial^{\prime} \lambda_{3}+450 \delta \phi_{5}+198 \lambda_{5}-1.5$ | Grand |

## (4) PETIT

| 963130.6 | $963130.6-z_{7}+v_{28}$ |
| :---: | :---: |
| 1390841.2 | $1390840.9-z_{7}+v_{29}$ |
| 1771020.4 | $1771019.5-z_{7}+v_{30}$ |
| 2101928.8 | $2101928.2-z_{7}+v_{31}$ |
| 2585148.7 | $2585147.7-z_{7}+v_{32}$ |


| $v_{28}=z_{7}-358 \delta \phi_{4}+36 \delta \lambda_{4}+358 \delta \phi_{6}-36 \delta \lambda_{6}+0.0$ | Horn |
| :---: | :---: |
| $v_{29}=z_{7}-209 \partial^{2} \phi_{4}+211 \partial^{\prime} \lambda_{4}+209 \partial_{\partial} \phi_{7}-211 \partial_{7}+0.3$ | Pascagoula |
| $v_{30}=z_{7}-25 \delta \phi_{4}+442 \lambda^{2} \lambda_{4}+25 \delta \phi_{5}-442 \delta \lambda_{5}+0.9$ | Grand |
|  |  |
| $v_{32}=2_{7}+367 \delta \phi_{4}+62 \delta \lambda_{4}+1.0$ | West base |

(5) GRAND

| 2430615.8 | $2430615.8-z_{8}+v_{23}$ |
| :---: | :---: |
| 2975816.0 | $2975817.4-z_{8}+v_{24}$ |
| 3571008.8 | $3571009.4-z_{8}+v_{25}$ |
| 581446.3 | $581446.6-z_{8}+v_{26}$ |
| 1012906.4 | $1012905.5-z_{8}+v_{27}$ |


|  |
| :---: |
|  |  |
|  |  |
|  |  |
|  |  |

Pins
Petit
Pernt
Horn
$v_{27}=z_{8}-495 \delta \phi_{5}+87 \delta \lambda_{5}+495 \delta \phi_{7}-87 \delta \lambda_{7}+0.9$
Pascagoula

## (6) HORN

| 1041600.3 | $1041600.3-z_{9}+v_{38}$ | $v_{38}=z_{9}-535 \delta^{\prime} \phi_{6}+118 \delta_{6} \lambda_{6}+535 \delta^{\prime} \phi_{8}-118 \delta^{2} \lambda_{8}+0.0$ | Club |
| :---: | :---: | :---: | :---: |
| 1380157.4 | $1380154.0-z_{9}+v_{39}$ | $v_{39}=z_{9}-249 \delta \phi_{6}+240 \lambda_{6}+249 \delta \phi_{9}-240 \lambda^{2} \lambda_{9}+3.4$ | Belle |
| 1991945.3 | $1991947.5-z_{9}+v_{40}$ | $v_{40}=z_{9}+153{ }_{2} \phi_{6}+379 \delta \lambda_{6}-153 \dot{\partial} \phi_{7}-379 \delta \lambda_{7}-2.2$ | Pascagoula |
| 2380928.1 | $2380926.5-z_{9}+v_{41}$ | $v_{41}=z_{9}-272 \delta^{2} \phi_{5}-146 \delta^{3} \lambda_{5}+272 \delta^{2} \phi_{6}+146 \delta^{2} \lambda_{6}+1.6$ | Grand |
| 2762601.4 | $2762600.7-z_{9}+v_{42}$ | $v_{42}=z_{9}-358 \delta \phi_{4}+36 \hat{\delta}^{\prime} \lambda_{4}+358 \delta \phi_{6}-36 \delta_{6}+0.7$ | Petit |

## (7) PASCAGOULA

2812513.6
3190437.2
192111.1
571414.4
890435.7
$2812513.6-z_{10}+v_{33}$ $3190434.2-z_{10}+v_{34}$ $192107.6-z_{10}+v_{35}$ $571412.2-z_{10}+v_{36}$ $890435.6-z_{10}+v_{37}$

```
v33}=\mp@subsup{z}{10}{}-495\delta\mp@subsup{\phi}{5}{}+87\delta\mp@subsup{\lambda}{5}{}+495\mp@subsup{\delta}{\delta}{}\mp@subsup{\phi}{7}{}-87\delta\mp@subsup{\lambda}{7}{}+0.
v34}=\mp@subsup{z}{10}{}-209\partial\mp@subsup{\phi}{4}{}+211\delta\mp@subsup{\lambda}{4}{}+209\mp@subsup{\rho}{\partial}{}\mp@subsup{\phi}{7}{}-211\mp@subsup{1}{\partial}{}\mp@subsup{\lambda}{7}{}+3.
```



```
v36}=\mp@subsup{z}{10}{}-286\delta\phi7-160\delta\mp@subsup{\lambda}{7}{}+286\delta\phi8+160\delta\mp@subsup{\lambda}{8}{}+2.
v37}=\mp@subsup{z}{10}{}-398\delta\mp@subsup{\phi}{7}{}-6\delta\mp@subsup{\lambda}{7}{}+398\delta\mp@subsup{\phi}{9}{}+6\delta\mp@subsup{\lambda}{9}{}+0.
```


## Grand

Petit
Horn
Club
Belle
(9) BELLE

$v_{43}=z_{11}-398 \delta^{\hat{o}} \phi_{7}-6{ }_{\delta} \lambda_{7}+398 \delta^{2} \phi_{9}+6 \delta \lambda_{9}+0.0$
$v_{44}=z_{11}-249 \partial \phi_{6}+240_{\delta} \lambda_{6}+249{ }_{\partial} \phi_{9}-240 \partial^{2} \lambda_{9}-1.8$ $v_{45}=z_{11}-17 \delta \phi_{8}+559 \delta \lambda_{8}+17 \delta \phi_{9}-559_{\delta} \lambda_{9}-0.6$
$v_{46}=z_{11}-311 \delta \phi_{9}-211 \delta \lambda_{9}+311 \delta \phi_{10}+211 \delta \lambda_{10}-2.0$ $v_{47}=z_{11}-700 \delta \phi_{9}+119 \delta \lambda_{9}+700 \delta \phi_{11}-119 \delta \lambda_{11}+0.9$

## Pascagoula

Horn
Club
Ship
Deer
(8) CLUB

$v_{48}=z_{12}-490 \delta \phi_{8}-2 j \lambda_{8}+490 \delta \phi_{10}+2 \delta \lambda_{10}+0.0$ $v_{49}=z_{12}-266 \delta \phi_{8}+297 \delta \lambda_{8}+266 \delta \phi_{11}-297 \delta \lambda_{11}-2.3$ $v_{50}=z_{12}-17 \delta \phi_{8}+559 \delta \lambda_{8}+17 \delta \phi_{9}-559 \delta \lambda_{9}-0.3$ $v_{51}=z_{12}-286 \delta_{\dot{\prime}} \phi_{7}-160 \delta \lambda_{7}+286 \delta_{\delta} \phi_{8}+160 \hat{\delta} \lambda_{8}+0.8$ $v_{52}=z_{12}-535 \delta \phi_{6}+118 \delta \lambda_{6}+535 \delta \phi_{8}-118 \delta \lambda_{8}-0.7$

## Ship

Deer
Belle
Pascagoula Horn
(11) DEER

| Assumed azimuth | Observed azimuth | Equation | Station observed |
| :---: | :---: | :---: | :---: |
| - , " | - , " |  |  |
| 2810103.4 | $2510103.4-z_{13}+v_{53}$ | $v_{53}=z_{13}-700 \delta^{2} \phi_{9}+119 \lambda^{2} \lambda_{9}+7008 \phi_{11}-1193 \lambda_{11}+0.0$ | Belle |
| 3220314.8 | $3220314.1-z_{13}+v_{54}$ | $v_{54}=z_{13}-266 \partial \phi_{8}+2973 \lambda_{8}+266{ }^{\circ} \phi_{11}-297 \partial \lambda_{11}+0.7$ | Club |
| 185039.5 | $185041.3-z_{13}+v_{55}$ | $v_{55}=z_{13}+1678 \phi_{10}+425 \delta^{\prime} \lambda_{10}-1678 \phi_{11}-4258 \lambda_{11}-1.8$ | Ship |
| 424650.3 | $424646.1-z_{13}+v_{56}$ | $v_{56}=z_{13}-197 \delta \phi_{11}-185 \delta \lambda_{11}-11.0$ | Ship Island Lighthouse. |
| 1152113.0 | $1152112.5-z_{13}+v_{57}$ | $v_{57}=z_{13}-603 \partial^{11}+248 \partial_{11}+33.7$ | Biloxi Lighthouse |

(10) SHIP

| 674712.0 | $674712.0-z_{14}+v_{58}$ | $v_{58}=z_{14}-500 \delta^{\prime} \phi_{10}-1778 \lambda_{10}-15.2$ | Ship Island Lighthouse |
| :---: | :---: | :---: | :---: |
| 1633118.9 | $1633119.0-z_{14}+v_{50}$ | $v_{59}=z_{14}-1103 \phi_{10}+3238 \lambda_{10}+43.2$ | Biloxi Lighthouse |
| 1984924.7 | $1984923.4-z_{14}+v_{60}$ | $v_{60}=z_{14}+167 \delta \phi_{10}+425 \delta \lambda_{10}-167 \delta \phi_{11}-42 \nu^{2} \lambda_{11}+1.3$ | Deer |
| 2320023.1 | $2320023.6-z_{14}+v_{61}$ | $v_{61}=z_{14}-311 \delta \phi_{9}-211 \delta \lambda_{9}+311 \delta \phi_{10}+211 \delta \lambda_{10}-0.5$ | Belle |
| 2694159.2 | $2694158.4-z_{14}+v_{68}$ | $v_{62}=z_{14}-490 \delta \phi_{8}-2 \delta \lambda_{8}+490 \delta \phi_{10}+28 \lambda_{10}+0.8$ | Club |

BILOXI LIGHTHOUSE

| 29.51830 .1 | $2951830.1-z_{15}+v_{63}$ | $v_{63}=z_{15}-603{ }_{3} \phi_{11}+24{ }^{\text {d }} \lambda_{11}+33.2$ | Deer |
| :---: | :---: | :---: | :---: |
| 3432951.1 | $3432947.5-z_{15}+v_{64}$ | $v_{64}=z_{15}-110 \partial \phi_{10}+3238 \lambda_{10}+46.9$ | Ship |
| 171404.4 | $171400^{6} .1-z_{15}+v_{65}$ | $v_{65}=z_{15}+11.6$ | Ship Island |

SHIP ISLAND LIGHTHOUSE

| 197 | 12 | 06.5 | 197 | 12 | $06.5-z_{16}+v_{66}$ | $v_{66}=z_{16}+13.3$ <br> 222 <br> 42 <br> 10.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 222 | 42 | $08.6-z_{16}+v_{67}$ | $v_{67}=z_{16}-197 \delta \phi_{11}-185 \delta \lambda_{11}-13.8$ |  |  |  |
| 247 | 43 | 46.8 | 247 | 43 | $47.7-z_{16}+v_{68}$ | $v_{68}=z_{16}-500 \delta \phi_{10}-177 \delta \lambda_{10}-16.1$ |$\quad$| Biloxi Lighthouse |
| :--- |

In order to get the quantities on a better relative basis, it is best to adopt $100 \delta \phi_{1}, 100 \delta \lambda_{1}$, etc., as unknowns in the equations. The coefficients throughout will then be divided by 100, and from the solution we shall determine one hundred times the corrections in seconds to the various latitudes and longitudes.

162 COAST AND GEODETIC SURVEY SPECIAL PUBLICATION NO. 28.


Table for formation of normals, No. 1-Continued


Table for formation of normals, No. 2 *


* All values in this table except those in the $\eta$ and $\Sigma$ columns have been divided by 100 .

Normal equations

| ๙્ન | 툭유융유웅 <br>  －iópiorio் $+++1+11+$ | 4 | ๗in్ <br>  <br>  $1++++++1++\mid 11++1++++1+$ |
| :---: | :---: | :---: | :---: |
| 7 |  유NNㅜ웅 <br>  $++11117$ | $=$ |  <br>  $0-100^{\circ 0} 0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ}{ }^{\circ}$ $++1+11+1++111++1+++$＋1 $1+$ |
| 9 |  <br>  <br>  $+1+111+1++$ | ๙ | 우눈 <br>  <br>  $1+11+1+1 \mid 11+$ |
| $\infty$ |  <br>  <br>  <br> ++++111 ＋＋ | － |  <br>  <br>  $+1++1+1+117$ |
| $\infty$. |  N్ర్ల్రగ్గ్ల్ －icio riocio $+++11+1+$ | ลิ |  రO－ <br>  $+1+++111++$ |
| － | இN 웅N․ ఝ్రీగ్రంగ్ర <br>  $++++11+$ | $\stackrel{\square}{\square}$ |  N． <br>  $+1++11117$ |
| $\bullet$ | 9To <br>  －－irimo $11+1+7$ | $\stackrel{\sim}{\sim}$ |  <br>  <br>  $1+1\|+\|11+1\|+$ |
| 5 |  | $\stackrel{\sim}{\square}$ | Tioniz <br>  <br>  $+1++1+1+1+7$ |
| ＋ |  －i＂min | $\stackrel{\square}{\square}$ |  <br>  $0000 \times 100009$ $11+1+1+11+$ |
| $\infty$ |  | 19 |  <br>  <br>  $+1++1+11+$ |
| $\sim$ |  | $\pm$ |  －inicimoon in $11+11111++$ |
| $\rightarrow$ | $\begin{aligned} & \text { ion } \\ & 0 . \\ & 8 . \\ & + \\ & + \end{aligned}$ | $\cong$ |  － $++1+1+11+$ |

Solution of normals

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | $\eta$ | z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} +90.0150 \\ \delta \phi_{1} \end{gathered}$ | $\begin{aligned} & +4.1730 \\ & -0.046359 \end{aligned}$ | $\begin{array}{\|c} -57.5385 \\ +\quad 0.639210 \\ \hline \end{array}$ | -41.7538 +0.463854 | $\begin{array}{\|} -12.6766 \\ +\quad 0.14082 s \end{array}$ | $\begin{aligned} & -6.3049 \\ & +0.070043 \end{aligned}$ | $\begin{aligned} & +2.6686 \\ & -0.029646 \end{aligned}$ | $\begin{aligned} & +1.8229 \\ & -0.020251 \end{aligned}$ | $\begin{aligned} & +4.3305 \\ & -0.048109 \end{aligned}$ | $\begin{aligned} & +0.7642 \\ & -0.008490 \end{aligned}$ |  |  |  |  | $\left\lvert\, \begin{aligned} & +0.5960 \\ & -0.006621 \end{aligned}\right.$ | $\begin{aligned} & -13.9036 \\ & +0.154459 \end{aligned}$ |
|  | $\begin{array}{r} +45.4427 \\ -0.1935 \\ +45.2492 \\ \delta \lambda_{1} \\ 1 \\ 2 \end{array}$ | $-\quad 3.5323$ $+\quad 2.6674$ | -17.5325 +1.9357 | +11.5126 +0.5877 | -1.7807 +0.2923 | $\begin{array}{r} +1.2372 \\ -0.1237 \end{array}$ | $\begin{aligned} & +0.0158 \\ & -0.0845 \end{aligned}$ | $\begin{array}{r} +0.6543 \\ -0.2008 \end{array}$ | $\begin{aligned} & -0.6 .37 \\ & -0.0354 \end{aligned}$ |  |  |  |  | +1.0071 -0.0276 | $\begin{aligned} & +40.5135 \\ & +0.6446 \end{aligned}$ |
|  |  | $\left\lvert\, \begin{array}{ll} -\quad 0.8649 \\ + & 0.019114 \end{array}\right.$ | $\begin{array}{\|} -15.5968 \\ +\quad 0.344687 \end{array}$ | $\begin{gathered} +12.1003 \\ -\quad 0.267415 \end{gathered}$ | $\begin{aligned} & -1.4884 \\ & +0.032893 \end{aligned}$ | $\begin{aligned} & +1.1135 \\ & -0.024608 \end{aligned}$ | $\begin{aligned} & -0.0687 \\ & +0.001518 \end{aligned}$ | $\begin{aligned} & +0.4535 \\ & -0.010022 \end{aligned}$ | $\begin{aligned} & -0.7191 \\ & +0.015892 \end{aligned}$ |  |  |  |  | $\left\lvert\, \begin{aligned} & +0.9795 \\ & -0.021647 \end{aligned}\right.$ | $\begin{aligned} & +41.1581 \\ & -0.909557 \end{aligned}$ |
|  |  | $\begin{array}{r} +117.5659 \\ -36.7792 \\ -\quad 0.0165 \\ +80.7702 \\ \delta \phi_{2} \\ \\ 1 \\ 2 \\ 3 \end{array}$ | $\begin{aligned} & +33.7556 \\ & -26.6894 \\ & -0.2981 \end{aligned}$ | $\begin{array}{r} -21.7139 \\ -8.1030 \\ +\quad 0.2313 \end{array}$ | $\begin{aligned} & +1.2849 \\ & -4.0302 \\ & -0.0284 \end{aligned}$ | $\begin{aligned} & +3.0590 \\ & +1.7058 \\ & +0.0213 \end{aligned}$ | $\begin{aligned} & +2.3685 \\ & +1.1652 \\ & -0.0013 \end{aligned}$ | $\begin{array}{r} +5.4192 \\ +2.7681 \\ +0.0087 \end{array}$ | $\begin{array}{r} +1.2251 \\ +0.4885 \\ -0.0137 \end{array}$ |  |  |  |  | $\begin{array}{r} -0.4396 \\ +0.3810 \\ +0.0187 \end{array}$ | $\begin{array}{r} +81.4539 \\ +8.8873 \\ +0.7867 \end{array}$ |
|  |  |  | $\begin{aligned} & +6.7681 \\ & -0.083795 \end{aligned}$ | $\begin{aligned} & -29.5856 \\ & +\quad 0.366294 \end{aligned}$ | $\begin{aligned} & -2.7737 \\ & +0.034341 \end{aligned}$ | $\begin{aligned} & +4.7861 \\ & -0.059256 \end{aligned}$ | $\begin{aligned} & +3.5324 \\ & -0.043734 \end{aligned}$ | $\begin{aligned} & +8.1960 \\ & -0.101473 \end{aligned}$ | $\begin{aligned} & +1.6999 \\ & -0.021046 \end{aligned}$ |  |  |  |  | $\left\lvert\, \begin{aligned} & -0.0399 \\ & +0.000494 \end{aligned}\right.$ | $\begin{aligned} & +73.3535 \\ & -0.908175 \end{aligned}$ |
|  |  |  | $\begin{gathered} +78.7027 \\ -19.3677 \\ -5.3760 \\ -0.5671 \\ +53.3919 \\ \delta \lambda_{9} \end{gathered}$ | $\begin{aligned} & +9.7277 \\ & +5.8801 \\ & +4.1708 \\ & +2.4791 \end{aligned}$ | -3.8022 <br> $-\quad 2.9246$ 0.5130810 .2324 | $\begin{aligned} & +2.5098 \\ & +1.2378 \\ & +0.3838 \\ & -0.4011 \end{aligned}$ | $\begin{aligned} & -0.6153 \\ & +0.8456 \\ & -0.0237 \\ & -0.2960 \end{aligned}$ | $\begin{aligned} & +0.2709 \\ & +2.0087 \\ & +0.1563 \\ & -0.6868 \end{aligned}$ | $\begin{array}{r} -2.1972 \\ +0.3545 \\ -0.2479 \\ -0.1424 \end{array}$ |  |  |  |  | $\begin{array}{r} +0.5294 \\ +0.2765 \\ +0.3376 \\ +0.0033 \end{array}$ | $\begin{aligned} & +59.5951 \\ & +6.4492 \\ & +14.1867 \\ & -6.1467 \end{aligned}$ |
|  |  |  |  | $\left\lvert\, \begin{aligned} & +10.4975 \\ & -0.196612 \end{aligned}\right.$ | $\begin{aligned} & -7.0074 \\ & +0.131245 \end{aligned}$ | $\begin{aligned} & +3.7303 \\ & -0.069866 \end{aligned}$ | $\left\lvert\, \begin{aligned} & -0.0894 \\ & +0.001674 \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & +1.7491 \\ & -0.032760 \end{aligned}\right.$ | $\begin{aligned} & -2.2330 \\ & +0.041823 \end{aligned}$ |  |  |  |  | $\left\lvert\, \begin{aligned} & +1.1468 \\ & -0.021479 \end{aligned}\right.$ | $\begin{aligned} & +61.1858 \\ & -1.145975 \end{aligned}$ |
|  |  |  |  | +84.3424 -1.7852 -3.2358 -10.8370 -2.0639 | $\begin{aligned} & +6.2298 \\ & +0.8879 \\ & +0.3980 \\ & +1.0160 \\ & +1.3777 \end{aligned}$ | $\begin{aligned} & -7.0677 \\ & +0.3758 \\ & -0.2978 \\ & +1.7531 \\ & -0.7334 \end{aligned}$ | $\begin{aligned} & -1.7394 \\ & +0.2567 \\ & +0.0184 \\ & +1.2939 \\ & +0.0176 \end{aligned}$ | $\begin{array}{r} -44.2628 \\ +0.6099 \\ -0.1213 \\ +3.0021 \\ -0.3439 \end{array}$ | $\begin{aligned} & -24.1325 \\ & +0.1076 \\ & +0.1923 \\ & +0.6227 \\ & +\quad 0.4390 \end{aligned}$ | $+3.5402$ | +1.1977 | $+5.0745$ | -1. 4231 | $\begin{aligned} & -0.2811 \\ & +0.0839 \\ & -0.2619 \\ & -0.0146 \\ & -0.2255 \end{aligned}$ | $\begin{aligned} & +8.3278 \\ & -1.9580 \\ & -11.0063 \\ & +26.8689 \\ & -12.0299 \end{aligned}$ |
|  |  |  |  | $\begin{gathered} +66.4205 \\ \delta \phi_{3} \end{gathered}$ | $\begin{aligned} & +6.1016 \\ & -0.091863 \end{aligned}$ | $\begin{aligned} & -5.9700 \\ & +0.089882 \end{aligned}$ | $\begin{aligned} & -0.1528 \\ & +0.002300 \end{aligned}$ | $\begin{aligned} & -41.1160 \\ & +0.619026 \end{aligned}$ | $\begin{aligned} & -22.7709 \\ & +\quad 0.342829 \end{aligned}$ | $\begin{aligned} & +3.5402 \\ & -0.053300 \end{aligned}$ | $\begin{aligned} & +1.1977 \\ & -0.018032 \end{aligned}$ | $\begin{aligned} & +5.0745 \\ & -0.076400 \end{aligned}$ | $\begin{aligned} & -1.4231 \\ & +0.021426 \end{aligned}$ | $\begin{aligned} & -0.6992 \\ & +0.010527 \end{aligned}$ | $\begin{aligned} & +10.2025 \\ & -0.153605 \end{aligned}$ |
|  |  |  |  | 1 2 3 4 5 | $\begin{aligned} & +19.9567 \\ & -0.4416 \\ & -0.0490 \\ & -0.0953 \\ & -0.9197 \\ & -0.5605 \end{aligned}$ | $\begin{aligned} & -2.2687 \\ & +0.1869 \\ & +0.0366 \\ & +0.1644 \\ & +0.4896 \\ & +0.5484 \end{aligned}$ | +0.9316 +0.1277 -0.0023 +0.1213 -0.0117 +0.0140 | $\begin{aligned} & -6.7978 \\ & +0.3033 \\ & +0.0149 \\ & +0.2815 \\ & +0.2296 \\ & +3.7770 \end{aligned}$ | $\begin{aligned} & -8.1327 \\ & +0.0535 \\ & +0.0237 \\ & +0.0584 \\ & +0.2931 \\ & +\quad 2.0918 \end{aligned}$ | +2.7178 -0.3252 | +0.4153 -0.1100 | +2.9155 -0.4662 | $\left\lvert\, \begin{aligned} & -1.3059 \\ & +0.1307\end{aligned}\right.$ | -0.1405 +0.0417 +0.0322 -0.0014 +0.1505 +0.0642 | $\begin{aligned} & +3.9182 \\ & +0.9738 \\ & +1.3538 \\ & +2.5190 \\ & +8.0303 \\ & -0.9372 \end{aligned}$ |
|  |  |  |  |  | $\begin{gathered} +17.8906 \\ \delta \lambda_{3} \end{gathered}$ | $\begin{aligned} & -0.8428 \\ & +0.047109 \end{aligned}$ | $\left\|\begin{array}{l} +1.1806 \\ -0.065990 \end{array}\right\|$ | $\begin{aligned} & -2.1915 \\ & +0.122494 \end{aligned}$ | $\begin{aligned} & -6.2458 \\ & +0.349111 \end{aligned}$ | $\begin{aligned} & +2.3926 \\ & -0.133735 \end{aligned}$ | $\begin{aligned} & +0.3053 \\ & -0.017065 \end{aligned}$ | $\begin{aligned} & +2.4493 \\ & -0.136904 \end{aligned}$ | $\begin{aligned} & -1.1752 \\ & +0.065688 \end{aligned}$ | $\left\|\begin{array}{l} +0.1467 \\ -0.008200 \end{array}\right\|$ | $\begin{aligned} & +13.9098 \\ & -0.777492 \end{aligned}$ |

Solution of normals-Continued


| 4 |  |  |  <br>  <br>  $\|\mid 1+++1+1+1$ |  <br>  <br>  $+++11+++11+1$ |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ |  생겅ํㅇ $00^{\circ} 00^{\circ} 00$ $1+111++1+$ |  |  |  <br>  <br>  |
| तิ | $\begin{array}{ll} \text { i } & \text { it } \\ \text { in } \\ \text { i } & \text { it } \\ \text { it } \end{array}$ |  |  |  |
| त |   <br> $\infty$ 010 <br> 0 00 <br> 0 00 <br> + +1 |  |  |  |
| \％ | $\begin{aligned} & +1.0673 \\ & \\ & +1.0673 \\ & -0.012965 \end{aligned}$ |  |  |  |
| O－1 |  |  |  |  |
| 2 |  |  |  |  |
| に |  |  |  |  |
| $\cdots$ |  |  |  |  |
| 12 |  |  |  |  |
| $\pm$ |  |  |  <br>  <br>  $+++++1+11+1$ |  <br>  <br>  $+111111111+$ |
| $\cong$ |  |  <br>  －o்＂～ióNiNi rio $1\|1++++1\|+$ |  <br>  <br>  $+11111111+$ |  |
| $\stackrel{1}{\square}$ |  |  <br> ற्र०००－ióo $0^{\circ}{ }^{\circ}$ <br> $+1111111+$ | ッやためのロニッ1 | － |
| $=$ | － <br>  <br>  ＋111111＋ | $\operatorname{sor} \infty \infty=$ |  |  |
|  | 20t－moso |  |  |  |

Solution of normals－Continued

| $\omega$ |  §్ర 우NNNN <br>  $+1111+1+++1$ |  |  | $\begin{aligned} & \text { స్సె } \\ & \text { ஸ் } \\ & 1 \text { + } \end{aligned}$ |  |  |  |  우NNN <br>  $1+++1++$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| － |  <br>  $\cdots$ 1000000000 $++1111111+1$ |  |  |  <br> 上0808～000 $\infty$ － $0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ} \boldsymbol{\infty}$ $1111++1+1+1+$ |  |  |  <br>  ฒัற $0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ}$ \＃i $++11+1+1+++$ |  |  |
| N |  |  |  | $\begin{aligned} & \text { F } \\ & \text { N } \\ & 0 \\ & 1 \end{aligned}$ |  ON్స <br> $-\mathbf{- 0}^{\circ} 0^{\circ} 0^{\circ}$ $+++1+$ |  | $\begin{aligned} & \text { 10 } \\ & \text { 기 } \\ & \text { + } \\ & + \end{aligned}$ |  － 0 <br>  ｜｜｜＋＋＋ | $\begin{aligned} & \text { ® } \\ & \text { 8. } \\ & \text { © } \\ & \text { io } \\ & +1 \end{aligned}$ |
| － | $\begin{aligned} & \text { N్ } \\ & \underset{\sim}{1} \\ & \stackrel{10}{1} \end{aligned}$ |  | $\begin{aligned} & \text { N } \\ & \text { Nip } \\ & \text { Noస } \\ & \text { io } \\ & 1+ \end{aligned}$ | $\begin{aligned} & \stackrel{8}{8} \\ & \stackrel{1}{2} \\ & \dot{m} \\ & + \end{aligned}$ |  | $\begin{aligned} & \text { N్ర } \\ & \text { ² } \\ & \text { and } \\ & \text { io } \\ & 1+ \end{aligned}$ | $\begin{aligned} & \text { N } \\ & 0 \\ & \text { N } \\ & \text { N } \\ & \text { oi } \end{aligned}$ | 규궁ㅇㅇ웅 §શionin - ión $^{\circ}{ }^{\circ}$ $++++1+$ |  |
| ค | $\begin{aligned} & \text { 厄్ర } \\ & 0 \\ & \infty \\ & + \\ & + \end{aligned}$ |  |  | $\begin{gathered} \text { R⿸广 } \\ \infty \\ \infty \\ \text { in } \\ 1 \end{gathered}$ | が侯がた <br> 우숙 <br> $0^{\circ} 0^{\circ} 0^{\circ} 0^{\circ}$ <br> ｜｜｜1＋ |  | $\begin{aligned} & \stackrel{0}{0} \\ & \text { N } \\ & 0 \\ & 0 \\ & 1 \\ & 1 \end{aligned}$ | － 9 №웅 <br> 두이구웅 <br> －0－iorir <br> $++++++$ |  |
| 9 | $\begin{aligned} & \text { N } \\ & \stackrel{1}{\circ} \\ & \text { pi } \\ & \text { í } \end{aligned}$ |  |  |  |  |  | ¢ ¢ ¢ a | 毋゙ －1． <br>  $+++1 \overrightarrow{1}+$ |  |
| $\stackrel{\sim}{\sim}$ |  |  | $\begin{array}{r} 8 \\ 108 \\ 80 \\ 00 \\ 1+ \end{array}$ |  |  |  |  | N్న్ర్తํ్ర స్ల్లిగ్లి －0゚べガベザ $++1+1+$ | $\begin{aligned} & 08 \\ & 0.8 \\ & 10 \\ & 10 \\ & 10 \\ & 1+ \end{aligned}$ |
| $\dagger$ |  |  |  |  |  | $\begin{aligned} & \text { N } \\ & \text { m } \\ & \text { N } \\ & \text { N } \\ & \text { No } \\ & \infty \\ & +1 \\ & +1 \end{aligned}$ |  |  |  |
| $\stackrel{\square}{-1}$ |  |  | $\begin{aligned} & \text { ơ } \\ & \text { 융ㅇ } \\ & \text { í } \\ & \text { No } \\ & 1+ \end{aligned}$ | $\begin{aligned} & \text { N12 } \\ & \text { 궁 } \\ & \text { ஜio } \\ & +1 \end{aligned}$ |  |  |  |  |  |
| $\stackrel{10}{1}$ |  |  | $\begin{aligned} & 10 \\ & 0.0 \\ & 00_{0}^{\infty} \\ & + \\ & + \end{aligned}$ |  | ニッバプハ |  |  |  |  |
| － |  |  |  |  |  |  |  |  |  |

Solution of normals-Continued


Back solution

| 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-12.0636$ | $+2.6267$ | $-3.7370$ | -1.2439 | $-0.5567$ | -0.6347 | +0.1657 | -0.5425 |
|  | -3.5565 | $-7.9282$ | -3.7327 | - 7.8525 | +0.3765 | -1.0174 | +0.5788 |
| -12.0636 | -0.9298 | $+0.3489$ | -0.2932 | $+1.0047$ | -0.8785 | -0.0196 | -0.1245 |
|  |  | $-11.3163$ | $+5.3757$ | - 3.1330 | $+0.8443$ | $-1.1210$ | +1.8069 |
|  |  |  | +0.1059 | + 0.100 | +0.0176 | +8.4798 | +0.0565 |
|  |  |  |  | -10.4306 | -1.4968 | $+0.5110$ | -0.5808 |
|  |  |  |  |  |  | $-9.9251$ | -0.4057 |
|  |  |  |  |  |  |  | -0.1469 |
| 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 |
| $+0.0043$ | $+0.0035$ | $+0.0040$ | $+0.0056$ | -0.0056 | -0.0042 | +0.0041 | $+0.0019$ |
| -0.8137 | -0.8324 | +0.5052 | -0.5520 | -0.6012 | -0.2065 | +0.0385 | -0.2996 |
| -0.0747 | +0.1548 | -0.1329 | +0.0714 | -0.0805 | +0.1132 | -0.0799 | $+0.0876$ |
| -0.5552 | +0.4238 | -0.4242 | +0.1467 | -0.7224 | +0.1155 | -0.1733 | -0.0302 |
| -0.0089 | -0.0142 | +0.0095 | -0.0087 | +0.0065 | +0.0125 | -0.0066 | +0.0126 |
| -4.8507 | -2.5109 | -0.7118 | +0.0094 | -2.2577 | -0.1831 | -0.0197 | +0.2271 |
| +1.4117 | $-0.9080$ | +0.2615 | $-0.2450$ | $+1.9853$ | -1.3614 | +0.0507 | $-0.3460$ |
| -1.3794 | +1.4171 | -2.2421 | +2.8702 | -1.4399 | +0.3084 | -0.6273 | +1.2773 |
| -0.1261 | $-0.0650$ | -0.0163 | -0.0969 | +0.2647 | +0.0790 | $+0.0945$ | $+0.1663$ |
| -6.3927 | $\begin{array}{r} +0.1406 \\ -2.1907 \end{array}$ | -0.1365 | $\begin{aligned} & -0.8846 \\ & -0.5086 \end{aligned}$ | $-2.8508$ | -0.2828 | -2.2905 +0.0490 | -0.0681 -0.2829 |
|  |  |  | -0.4000 |  | -1.4094 | $\frac{+0.0490}{-2.9605}$ | $\begin{aligned} & -0.2829 \\ & -0.0309 \end{aligned}$ |
|  |  | -7.2323 | +0.4075 |  |  |  | +0.7151 |
| 6 | 5 | 4 | 3 | 2 | 1 |  |  |
| $\begin{aligned} & -0.0082 \\ & -0.4199 \\ & +0.2999 \\ & +0.1234 \\ & -0.0545 \\ & -0.9952 \\ & -0.1726 \\ & +0.1954 \\ & +0.0337 \end{aligned}$ | $\begin{aligned} & +0.0105 \\ & -0.1370 \\ & +0.1674 \\ & +0.1304 \\ & -0.0217 \\ & -0.9773 \\ & -0.8725 \\ & -0.0068 \\ & +0.0643 \\ & +0.0917 \end{aligned}$ | $\begin{aligned} & -0.0215 \\ & -0.1192 \\ & +0.0462 \\ & -0.0050 \\ & -0.0500 \\ & -0.1310 \\ & +0.3049 \end{aligned}$ | $\begin{aligned} & +0.0005 \\ & +0.0600 \\ & +0.1430 \\ & +0.1295 \\ & -0.0424 \\ & -0.0343 \\ & -0.5681 \\ & -0.0020 \end{aligned}$ | $\begin{aligned} & -0.0216 \\ & -0.0453 \\ & +0.0141 \\ & -0.0045 \\ & -0.0176 \\ & -0.0328 \\ & +0.4148 \\ & +0.0084 \\ & -0.0060 \end{aligned}$ | $\begin{aligned} & -0.0066 \\ & +0.0242 \\ & +0.0678 \\ & +0.0600 \\ & -0.0212 \\ & -0.0699 \\ & -0.2184 \\ & +0.0113 \\ & -0.2006 \\ & -0.0143 \end{aligned}$ |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  | +0.0244 | -0.3138 |  |  |  |  |
| -0.9980 |  |  |  | $+0.3095$ |  |  |  |
|  | -1.5510 |  |  |  | $-0.3677$ |  |  |

Computation of corrections.

| 1 | 2 | $z_{1}$ | 3 | 4 | 5 | $3 a$ | $z_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & +0.700 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & -0.842 \\ & -0.857 \\ & +0.700 \\ & +0.3 \end{aligned}$ | $\begin{aligned} & -0.842 \\ & -0.857 \\ & +0.3 \\ & \hline-1.399 \end{aligned}$ | $\begin{aligned} & +1.039 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -1.318 \\ & -0.098 \\ & +1.039 \\ & -0.6 \end{aligned}$ | $\begin{aligned} & +0.301 \\ & -1.541 \\ & +1.039 \\ & -0.9 \end{aligned}$ | $\begin{aligned} & +1.039 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & +0.301 \\ & -1.541 \\ & -1.318 \\ & -0.098 \\ & -1.5 \end{aligned}$ |
|  | -0.699 -0.7 | $\begin{aligned} & -1.399 \\ & +0.700 \end{aligned}$ |  | $\begin{aligned} & -0.977 \\ & -1.0 \end{aligned}$ | $\begin{aligned} & -1.101 \\ & -1.1 \end{aligned}$ |  | $\begin{array}{r} -4.156 \\ +1.039 \end{array}$ |
| $4 a$ | $5 a$ | 6 | 7 | 8 | $z_{3}$ | 17 | 15 |
| $\begin{array}{r} -0.842 \\ -0.857 \\ +1.592 \end{array}$ | $\begin{aligned} & +0.301 \\ & -1.541 \\ & +1.592 \end{aligned}$ | $\begin{array}{r} +0.978 \\ -0.699 \\ +1.592 \end{array}$ | $\begin{aligned} & +2.416 \\ & -0.699 \\ & -2.062 \end{aligned}$ | $\begin{array}{r} +1.254 \\ +0.149 \\ -5.289 \end{array}$ | $+4.107$ <br> $-3.649$ <br> $-2.062$ | $\begin{aligned} & +0.857 \\ & -0.131 \\ & +1.860 \end{aligned}$ | $\begin{aligned} & +2.416 \\ & -0.699 \\ & -2.062 \end{aligned}$ |
| $\begin{aligned} & -0.107 \\ & -0.1 \end{aligned}$ | $\begin{aligned} & +0.352 \\ & +0.4 \end{aligned}$ | $\begin{aligned} & +0.971 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & +1.592 \\ & +1.4 \end{aligned}$ | $\begin{aligned} & +1.592 \\ & +2.1 \end{aligned}$ | $\begin{array}{r} +5.289 \\ +0.479 \end{array}$ | $\begin{aligned} & +2.586 \\ & +2.6 \end{aligned}$ | $\begin{aligned} & +1.860 \\ & +2.7 \end{aligned}$ |
|  |  |  | $\begin{aligned} & +2.702 \\ & +2.7 \end{aligned}$ | $\begin{aligned} & -3.915 \\ & -3.9 \end{aligned}$ | $\begin{aligned} & -7.959 \\ & +1.592 \end{aligned}$ |  | $\begin{aligned} & -1.130 \\ & -1.1 \end{aligned}$ |
| 16 | $z_{4}$ | 14 | 9 | 10 | 11 | 12 | 13 |
| $\begin{aligned} & -1.318 \\ & -0.098 \\ & +1.860 \\ & -1.9 \end{aligned}$ | $\begin{aligned} & +2.416 \\ & -0.699 \\ & -2.523 \\ & -0.174 \\ & -4.6 \end{aligned}$ | $\begin{aligned} & -0.753 \\ & -0.8 \end{aligned}$ | $\begin{aligned} & +2.624 \\ & -1.836 \\ & -0.753 \\ & +0.4 \end{aligned}$ | $\begin{aligned} & -4.045 \\ & +3.792 \\ & -0.753 \\ & +0.7 \end{aligned}$ | $\begin{aligned} & -2.295 \\ & +3.174 \\ & -0.753 \\ & +1.8 \end{aligned}$ | $\begin{aligned} & +0.857 \\ & -0.131 \\ & -0.753 \\ & -0.6 \end{aligned}$ | $\begin{aligned} & +0.978 \\ & -0.699 \\ & -0.753 \\ & -0.2 \end{aligned}$ |
| $\begin{aligned} & -1.456 \\ & -1.4 \end{aligned}$ | $\begin{aligned} & -5.580 \\ & +1.860 \end{aligned}$ |  | $\begin{aligned} & +0.435 \\ & +0.4 \end{aligned}$ | $\begin{aligned} & -0.306 \\ & -0.3 \end{aligned}$ | $\begin{aligned} & +1.926 \\ & +1.9 \end{aligned}$ | $\begin{aligned} & -0.627 \\ & -0.7 \end{aligned}$ | $\begin{aligned} & -0.674 \\ & -0.7 \end{aligned}$ |
| 25 | 18 | 19 | 20 | 21 | 22 | 26 |  |
| $\begin{aligned} & +0.978 \\ & -0.699 \\ & +0.857 \\ & -0.131 \\ & -2.295 \\ & +3.174 \\ & +2.624 \\ & -1.836 \\ & -4.045 \\ & +3.792 \\ & +2.1 \end{aligned}$ | $\begin{aligned} & +1.254 \\ & +0.149 \\ & -5.289 \\ & +0.479 \\ & +3.728 \end{aligned}$ | $\begin{aligned} & +1.440 \\ & +0.062 \\ & -7.119 \\ & +2.545 \\ & +3.728 \\ & -1.8 \end{aligned}$ | $\begin{aligned} & -2.29 .5 \\ & +3.174 \\ & +3.728 \\ & -2.8 \end{aligned}$ | $\begin{aligned} & +2.373 \\ & +2.275 \\ & +1.094 \\ & -6.750 \\ & +3.728 \\ & -2.9 \end{aligned}$ | $\begin{aligned} & +6.980 \\ & +1.976 \\ & -6.342 \\ & -5.645 \\ & +3.728 \\ & -1.5 \end{aligned}$ | $\begin{aligned} & +1.254 \\ & +0.149 \\ & +1.440 \\ & +0.062 \\ & +5.551 \\ & +10.449 \\ & +1.094 \\ & -6.750 \\ & -6.342 \\ & -5.645 \\ & -9.0 \end{aligned}$ |  |
|  | $\begin{aligned} & +0.321 \\ & +0.3 \end{aligned}$ | $\begin{aligned} & -1.144 \\ & -1.2 \end{aligned}$ | $\begin{aligned} & +1.807 \\ & +1.8 \end{aligned}$ | $\begin{aligned} & -0.180 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & -0.803 \\ & -0.8 \end{aligned}$ |  |  |
| $\begin{array}{r} +4.519 \\ -0.753 \end{array}$ |  |  |  |  |  | $\begin{aligned} & -18.640 \\ & +\quad 3.728 \end{aligned}$ |  |
| 28 | 29 | 30 | 31 | 32 | 27 |  |  |
| $\begin{array}{r} -2.560 \\ -1.066 \\ +1.454 \\ +2.604 \\ -0.633 \\ \hline \end{array}$ | $\begin{aligned} & -1.495 \\ & -6.247 \\ & -4.579 \\ & +13.489 \\ & -0.633 \\ & +0.3 \end{aligned}$ | $\begin{aligned} & -0.179 \\ & -13.085 \\ & -0.352 \\ & +12.601 \\ & -0.633 \\ & +0.9 \end{aligned}$ | $\begin{aligned} & +2.373 \\ & +2.275 \\ & +1.094 \\ & -6.750 \\ & -0.633 \\ & +0.6 \end{aligned}$ | $\begin{aligned} & +2.624 \\ & -1.836 \\ & -0.632 \\ & +1.0 \\ & +1.155 \end{aligned}$ | $\begin{aligned} & +2.373 \\ & +2.275 \\ & -0.515 \\ & -28.983 \\ & -0.352 \\ & +12.601 \end{aligned}$ |  |  |
| $\begin{aligned} & -0.201 \\ & -0.1 \end{aligned}$ | $\begin{aligned} & +0.835 \\ & +0.9 \end{aligned}$ | -0.748 -0.7 | $\begin{aligned} & -1.041 \\ & -1.0 \end{aligned}$ | $\begin{aligned} & +1.155 \\ & +1.2 \end{aligned}$ | $\begin{aligned} & +1.454 \\ & +2.604 \\ & -4.579 \\ & +13.489 \\ & +2.8 \\ & \hline+3.167 \\ & -0.633 \end{aligned}$ |  |  |

Compatation of corrections-Continued

| 23 | 24 | 25 | 26 | 27 | 28 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| +6.980 +1.976 -6.342 -5.645 +1.588 | -4.045 +3.792 +1.588 -1.4 -0.065 | -0.179 -13.085 -0.352 +12.601 +1.588 -0.6 | +3.834 +4.162 +1.108 +10.559 +1.588 -0.3 | +6.977 -2.480 -10.844 +5.562 +1.588 +0.9 | +6.980 +1.976 +0.179 +13.085 +0.070 +12.429 |
| $\begin{aligned} & -1.443 \\ & -1.4 \end{aligned}$ | $\begin{aligned} & -0.065 \\ & -0.0 \end{aligned}$ | $\begin{aligned} & -0.027 \\ & -0.0 \end{aligned}$ | $\begin{aligned} & -0.167 \\ & -0.2 \end{aligned}$ | $\begin{aligned} & +1.703 \\ & +1.7 \end{aligned}$ | $\begin{aligned} & +1.108 \\ & -10.559 \\ & -10.844 \\ & +5.562 \\ & -1.4 \end{aligned}$ |
|  |  |  |  |  | $\begin{array}{r} -7.942 \\ +1.588 \end{array}$ |
| 38 | 39 | 40 | 41 | 42 | $z_{9}$ |
| $-2.180$ | $-1.015$ | + 0.623 | $+3.834$ | $-2.560$ | $-2.560$ |
| $-8.534$ | -17.358 | -27.410 | + 4.162 | $-1.066$ | $-1.066$ |
| $-0.786$ | -3.727 | +3.352 | +1.108 | +1.459 | + 3.834 |
| $+11.712$ | +25.033 | +24.228 | -10.559 | + 2.604 | + 4.162 |
| $-1.284$ | -1.284 | $-1.284$ | -1.284 | $-1.284$ | -0.004 |
| $\begin{aligned} & -1.072 \\ & -1.1 \end{aligned}$ | $+3.4$ | $-2.2$ | $+1.6$ | + 0.7 | -61.258 |
|  | $\begin{aligned} & +5.049 \\ & +5.0 \end{aligned}$ | $\begin{aligned} & -2.691 \\ & -2.7 \end{aligned}$ | $\begin{aligned} & -1.139 \\ & -1.2 \end{aligned}$ | $\begin{aligned} & -0.147 \\ & -0.1 \end{aligned}$ | $\begin{aligned} & -1.230 \\ & +3.352 \\ & +24.228 \\ & +0.786 \\ & +11.712 \\ & +3.727 \\ & +25.033 \\ & +3.5 \end{aligned}$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  | - | +6.420 -1.284 |
| 33 | 34 | 35 | 36 | 37 | $z_{10}$ |
| $+6.977$ | $-1.495$ | $+0.623$ | +6.265 | +8.719 | - 1.495 |
| $-2.480$ | $-6.247$ | -27.410 | +10.228 | +0.384 | $-6.247$ |
| -10.844 | $-4.579$ | + 3.352 | $-0.420$ | $-5.957$ | + 6.977 |
| + 5.562 | +13.489 |  | $-15.880$ | -0.626 | -2.480 |
| - 2.538 | $-2.538$ | $-2.538$ | $-2.538$ | $-2.538$ | + 0.623 |
| $\begin{aligned} & -3.323 \\ & -3.3 \end{aligned}$ | + 3.0 | + 3.5 | + 2.2 | +0.1 | -27.410 |
|  | $\begin{aligned} & +1.630 \\ & +1.7 \end{aligned}$ | $\begin{aligned} & +1.755 \\ & +1.8 \end{aligned}$ | $\begin{aligned} & -0.145 \\ & -0.1 \end{aligned}$ | $\begin{aligned} & +0.082 \\ & +0.1 \end{aligned}$ | +53.890 |
|  |  |  |  |  | -0.420 |
|  |  |  |  |  | $-5.957$ |
|  |  |  |  |  | $\begin{aligned} & -0.626 \\ & +8.8 \end{aligned}$ |
|  |  |  |  |  | +12.689 -2.538 |
| 43 | 44 | 45 | 46 | 47 | $z_{11}$ |
| +8.719 | $-1.015$ | $+0.025$ | $+4.655$ | +10.478 | $-1.015$ |
| +0.384 | -17.358 | $-55.481$ | $+22.009$ | -12.412 | -17.358 |
| $-5.957$ | -3.727 | -0.254 | + 0.329 | -6.509 | +8.719 |
| $-0.626$ | +25.033 | +58.307 | $-23.877$ | +14.356 | + 0.384 |
| -2.716 | -2.716 -1.8 | - 2.716 | -2.716 -2.0 | +2.716 +0.9 | $+0.025$ |
| $\begin{aligned} & -0.196 \\ & -0.2 \end{aligned}$ | -1.8 | - 0.6 | - 2.0 | $+0.9$ | +55.481 +5.194 |
|  | $\begin{aligned} & -1.583 \\ & -1.6 \end{aligned}$ | $\begin{aligned} & -0.719 \\ & -0.7 \end{aligned}$ | $\begin{aligned} & =1.600 \\ & -1.6 \end{aligned}$ | $\begin{aligned} & +4.097 \\ & +4.1 \end{aligned}$ | $\begin{array}{r} 0.134 \\ +92.311 \\ +\quad 0.329 \end{array}$ |
|  |  |  |  |  | $-23.877$ |
|  |  |  |  |  | -6.509 |
|  |  |  |  |  | $\begin{aligned} & +14.356 \\ & -3.5 \end{aligned}$ |
|  |  |  |  |  | $\begin{array}{r} +13.578 \\ -2.716 \end{array}$ |

Computation of corrections-Continued


Final computation of triangles


Final computation of triangles-Continued


Final computation of triangles-Continued


Final computation of triangles-Continued


STATION CEDAR
Final position computation,


STATION CAT

secondary triangulation
STATION CEDAR


STATION CAT


Final position computation,
STATION PINS


STATION GRAND

secondary triangulation-Continued
STATION PINS


STATION GRAND


Final position computation,
Station petit


STATION HORN

secondary triangulation-Continued
STATION PETIT


STATION HORN


Final position computation,
STATION PASCAGOULA


## STATION BELLE


secondary triangulation-Continued
STATION PASCAGOULA


STATION BELLE


Final position computation,

## STATION CLUB



STATION DEER

secondary triangulation-Continued
STATION CLUB


## STATION DEER



Final position computation,

## STATION SHIP



STATION BILOXI LIGHTHOUSE

secondary triangulation-Continued
STATION SHIP


## STATION BILOXI LIGIITHOUSE



Final position computation,
STATION SHIP ISLAND LIGHTHOUSE

secondary triangulation-Continued
STATION SHIP ISLAND LIGHTHOUSE


## ADJUSTMENTS BY THE ANGLE METHOD

If the adjustment be made according to the angle method* the complications due to the presence of the $z$ 's are avoided. An angle is the difference of two directions and the observation equation for an observed angle is the difference of the observation equations of its two sides, and in taking the difference the $z$ drops out. To illustrate this suppose that at station Gunner, Figure 6, page 104, the following angles were observed: Duck to Indian Point, Indian Point to Larrabee, Larrabee to Mam, Mam to Lubec Channel Lighthouse, and Lubec Channel Lighthouse to Lubec Church spire. Call the corrections to the observed angles $u_{1}, u_{2}, u_{3}, u_{4}$, and $u_{5}$ respectively, and suppose the observed and assumed values to be as given on page 115. Then

$$
\begin{aligned}
& u_{1}=v_{2}-v_{1}=-6131 \delta \phi_{1}+1800 \delta \lambda_{1}-2.2 \\
& u_{2}=v_{3}-v_{2}=-1997 \delta \phi_{1}+236 \delta \lambda_{1}+7.0
\end{aligned}
$$

In a similar way,

$$
\begin{aligned}
& u_{3}=-2732 \delta \phi_{1}+85 \delta \lambda_{1}-1.2 \\
& u_{4}=-5565 \delta \phi_{1}-2445 \delta \lambda_{1}+3.5 \\
& u_{5}=+5886 \delta \phi_{1}+5413 \delta \lambda_{1}+18.4
\end{aligned}
$$

These contain no $z$ 's and the normal equations may be formed in the usual way.

Observation equations of this kind would arise when at an unknown point angles are taken on known points, as for example when angles are taken with a sextant from a point off-shore to determine its position, and for such observations the angle method is both easier and more logical than the direction method.

[^14]ADJUSTMENT OF VERTICAL OBSERVATIONS
general statement
When reciprocal vertical observations are made over the lines of a triangulation scheme a computation of the differences of elevations is made by the usual Coast and Geodetic Survey formula. For an account of these observations and of the method of computation, see United States Coast and Geodetic Survey Special Publication No. 19, page 140 et seq. As there are always several lines from each station, rigid conditions are present in the figure. Thus it becomes necessary to make an adjustment of the observed values by the method of least squares. In the following figure the differences of elevations as observed are first computed and then the results are adjusted by the method employed in the United States Coast and Geodetic Survey.

The formula used in the following computations is the one given in Special Publication No. 19, mentioned above. On pages 205 et seq. there is given a new development of the formula that takes into account


Fig. 8. some of the small terms that are needed in computation over longer and higher lines. The final form of the new formula differs slightly from the one used in this computation.

## Computation of elevations from reciprocal observations

| Station 1 | Pollywog | Pollywog | Pack Sad- dle | Pack Saddle | Elk | Elk |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Station 2 | $\begin{aligned} & \text { Pack Sad- } \\ & \text { dle } \end{aligned}$ | Elk | $\underset{\text { vide }}{\text { High }} \mathrm{Di}$ | Elk | Long Ridge | $\underset{\text { vide }}{\text { High }} \text { Di- }$ |
|  | - 1 | - ' " | - ' 1 | - ' 1 | - ' 1 | - ' 1 |
| $\zeta 1$ | 90 | 90 | $90 \quad 3000$ | 915350 | 88 | $\begin{array}{lll}89 & 15 & 25\end{array}$ |
| $\zeta_{2}$ | $\begin{array}{llll}90 & 04 & 59\end{array}$ | $89 \quad 1115$ | $89 \quad 37 \quad 33$ | $88 \quad 1059$ | $\begin{array}{llll}91 & 36 & 54\end{array}$ | $\begin{array}{lll}90 & 51 & 04\end{array}$ |
| $\zeta_{2}-\zeta_{1}$ | + $+\quad 46$ $+\quad 6.0432$ | -1 4616 | - 5227 | $\begin{array}{llll}-3 & 42 & 51\end{array}$ | +30338 | +1 3539 |
| $\frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | + 23 | 5314 | 2614 | $-1 \begin{array}{lll}-1 & 51 & 26\end{array}$ | +13149 | + 4750 |
| $\tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 6.04732 | 8.18994 | 7.88258 | 8.51090 | 8.42675 | 8.14348 |
| $\log 8$ | 4.27444 | 4.29253 | 4.15543 | 3.98141 | 4.31524 | 4.17150 |
| $\log 8 \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 0.32176 | 2.48247 | 2.03801 | 2.49234 | 3.74199 | 2.31498 |
| ${ }^{3} \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | +2.10 | -303. 72 | -109.15 | $-310.70$ | +552.06 | +206.53 |
| Second term | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Third term | 0.00 | 0.03 | 0.01 | 0.03 | 0.06 | 0.02 |
| $\mathrm{h}_{2}-\mathrm{h}_{1}$ | $+2.10$ | -303. 75 | -109.16 | $-310.73$ | +552.12 | +206.55 |
| $2 \log 8$ | 8.549 | 8.585 | 8.311 | 7.963 | 8.630 | 8.343 |
| $\log p=9-2 \log s$ | 0.451 | 0.415 | 0.689 | 1.037 | 0.370 | 0.657 |
| $p$ of $\mathrm{h}_{2}-\mathrm{h}_{1}$ | 2.82 | 2.60 | 4.89 | 10.89 | 2.34 | 4.54 |
| Station 1 | $\begin{aligned} & \text { High Di- } \\ & \text { vide } \end{aligned}$ | $\begin{aligned} & \text { High Di- } \\ & \text { vide } \end{aligned}$ | $\begin{aligned} & \text { High Di- } \\ & \text { vide } \end{aligned}$ | Long Ridge | Long Ridge | Gordon |
| Station 2 | Bald Hill | Gordon | Long Ridge | Gordon | Bald Hill | Bald Hill |
|  | - , " | , | - , " | - , | - , | , |
| $\zeta_{1}$ | $\begin{array}{lll}89 & 29 & 43\end{array}$ | $88 \quad 28 \quad 23$ | 88 | 89 | $\begin{array}{lll}91 & 27 & 22\end{array}$ | $\begin{array}{lll}92 & 45 & 09\end{array}$ |
| $\zeta 2$ | $\begin{array}{lll}89 & 37 & 50\end{array}$ | $\begin{array}{lll}91 & 40 & 22\end{array}$ | $\begin{array}{lll}91 & 53 & 03\end{array}$ | $\begin{array}{lll}90 & 52 & 54\end{array}$ | 88 | $87 \quad 210$ |
| $\zeta_{2}-\zeta_{1}$ | - 5153 | +31159 | +34031 | +13853 | $-24536$ | $\begin{array}{lll}-5 & 23 & 59\end{array}$ |
| $\frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | $25 \quad 56$ | + 13600 | +15016 | + 4926 | - $1 \begin{array}{lll}1 & 22 & 48\end{array}$ | $\begin{array}{llll}-2 & 42 & 00\end{array}$ |
| $\tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 7.87759 | 8.44611 | 8.50632 | 8.15777 | 8.38184 | 8.67357 |
| $\log 8$ | 4.21546 | 4.29195 | 4.03443 | 4. 14252 | 4.29013 | 4.15348 |
| $\log 8 \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 2.09305 | 2.73806 | 2.54075 | 2.30029 | 2.67197 | 2.82705 |
| $8 \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | -123.89 | +547.09 | +347.34 | +199.66 | -469.86 | -671.51 |
| Second term | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Third term | 0.01 | 0.08 | 0.05 | 0.03 | 0.06 | 0.07 |
| $\mathrm{h}_{2}-\mathrm{h}_{1}$ | -123.90 | +547.17 | +347.39 | +199.69 | -469.92 | -671.58 |
| $2 \log 8$ | 8.431 | 8.584 | 8.069 | 8.285 | 8.580 | 8.307 |
| $\log p=9-2 \log s$ | 0.569 | 0.416 | 0.931 | 0.715 | 0.420 | 0.693 |
| $p$ of $\mathrm{h}_{2}-\mathrm{h}_{1}$ | 3.71 | 2.61 | 8.53 | 5.19 | 2.63 | 4.93 |
| Station 1 | Gordon | Gordon | Gordon | Child | Child | Rattle |
| Station 2 | Red Moun- tain | Rattle | Child | Rattle | Red Moun- tain | Red Moun- tain |
|  | - 11 | - 1 11 | - ' 1 | - , " | - , "1 | - 11 |
| $\zeta 1$ | 900328 | 903105 | 915600 | 875600 | 883347 | 890726 |
| $\zeta 2$ | $90 \quad 1044$ | 893848 | 881127 | 920903 | 913613 | 905756 |
| $\zeta_{2}-\zeta_{1}$ | $+\quad 0716$ | - 5217 | - 34433 | + 41303 | +30226 | +15030 |
| $\frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | + 0338 | - 2608.5 | $-15216.5$ | + 20631.5 | + 13113 | + 5515 |
| $\tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 7.02404 | 7.88106 | 8.51416 | 8.56610 | 8.42390 | 8.20610 |
| $\log s$ | 4.48786 | 4.31274 | 4.23223 | 4.03839 | 4.34662 | 4.06606 |
| $\log s \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 1.51190 | 2.19380 | 2.74639 | 2.60449 | 2.77052 | 2.27216 |
| $s \tan \frac{1}{1}\left(\zeta_{2}-\zeta_{1}\right)$ | +32.50 | -156.24 | -557.69 | +402.24 | +589.55 | +187. 14 |
| Second term | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Third term | 0.01 | 0.03 | 0.09 | 0.06 | 0.09 | 0.04 |
| $\mathrm{h}_{2}-\mathrm{h}_{1}$ | +32.51 | -156.27 | -557.78 | +402.30 | +589.64 | +187.18 |
| $2 \log 8$ | 8.976 | 8.625 | 8.464 | 8.077 | 8.693 | 8.132 |
| $\log p=9-2 \log 8$ | 0.024 | 0.375 | 0.536 | 0.923 | 0.307 | 0.868 |
| $p$ of $\mathrm{h}_{2}-\mathrm{h}_{1}$ | 1.06 | 2.37 | 3.43 | 8.38 | 2.03 | 7.38 |

Computation of elevations from nonreciprocal observations


The adjustment of vertical observations as practiced in the United States Coast and Geodetic Survey is made by means of observation equations and differs somewhat from the method of conditions. Of course condition equations could be employed if it were desired, just as triangulation can be adjusted by observation equations. (See the adjustment by the Variation of Geographic Coordinates, p. 91 et seq.)

Elevations for the various stations are assumed somewhat near what the final values will be. To these are added $x$ 's to be determined by the adjustment. (See table of assumed elevations on p. 200.) By means of these, observation equations are formed by the comparison of the assumed $h_{2}-h_{1}$ with that determined by computation.

The method of formation is shown below and a tabulated form of all of the computation on page 201.

Fixed elevations.

| Bosley. | Meters $1037.35$ |
| :---: | :---: |
| Stack. | 1062.69 |
| Craggy | 1368. 31 |
| Redding Rock. | 0. 00 |
| Mean sea level. | O. 00 |

Assumed and adjusted clevations

| Station | Elevation |  |
| :---: | :---: | :---: |
|  | $\begin{aligned} & \text { Assumed } \\ & + \text { correc- } \\ & \text { tion } \end{aligned}$ | Adjusted |
| Pollywog | Mrters <br> $811+x_{1}$ | Mcters 811.06 |
| Elk | $507+x_{2}$ | 504.61 |
| Pack Saddle | $817+x_{3}$ | 815.74 |
| High Divide | $710+x_{4}$ | 708.77 |
| Long Ridge | $1059+x_{5}$ | 1055.96 |
| Bald Hill | $589+x_{6}$ | 585.16 |
| Gordon | $1259+x_{7}$ | 1256.12 |
| Child | $701+x_{8}$ | 698.19 |
| Rattle | $1103+x_{9}$ | 1100.46 |
| Red Mountain | $1290+x_{10}$ | 1287.70 |

## FORMATION OF OBSERVATION FQUATIONS

The observation equations are formed as follows:
(1) Pollywog, assumed elevation* $=811+x_{1}$
(2) Craggy, fixed elevation $=1368.31$
$h_{2}-h_{1}$ (assumed)* $\quad=+557.31-x_{1}$
$h_{2}-h_{1}$ (observed)* $\quad=+555.38+v_{1}$
Observed- assumed* $\quad=-1.93+x_{1}+v_{1}=0$

$$
\begin{aligned}
-v_{1} & =-1.93+x_{1} \\
p & =\frac{1}{3} \text { of } 3.96=1.32
\end{aligned}
$$

(1) Elk, assumed elevation*
$=507+x_{2}$
(2) Pollywog, assumed elevation* $=811+x_{1}$
$h_{2}-h_{1}$ (assumed)* $=+304+x_{1}-x_{2}$
$h_{2}-h_{1}$ (observed)* $\quad=+303.75+v_{5}$
Observed-assumed* $\quad=-0.25-x_{1}+x_{2}+v_{5}=0$

$$
\begin{aligned}
-v_{5} & =-0.25-x_{1}+x_{2} \\
p & =2.60
\end{aligned}
$$

In a similar manner the remaining equations are formed. These are usually formed as in the following table. The constant term is found in the column "Observed minus assumed," and the remainder of the equation in the column "Symbol."
Table of formation of observation equations

| $\stackrel{\sim}{*}$ |  <br>  | \% |
| :---: | :---: | :---: |
| $\begin{aligned} & + \\ & \sum_{i=1}^{n} \\ & E_{i}^{2} \end{aligned}$ |  <br>  $1++1+11++11++1+11+1+1$ | กั๊ |
| B |  <br>  $++1++111+1++11++11+1++1++1+$ |  |
|  |  <br>  |  |
|  |  <br>  <br>  |  |
| $\begin{aligned} & \text { O} \\ & \text { E } \\ & \text { En } \\ & \hline \end{aligned}$ |  |  |
|  |  <br>  |  |
|  |  <br>  $+++++1+11+1\|1++1\|$ \| 1 | $1++$ \| 1 +t |  |
|  | लதద్లా <br>  $+++++1+1\|+\|\|\|++\|\|\|\|\| \|++\overrightarrow{1}++$ |  |
|  |  <br>  |  |
|  |  |  |
|  |  |  |

* Computed from nonreciprocal observations. Weight used $=1 p$ from the computation.
+ This column added for convenience in computing.


## COMPUTATION OF PROBABLE ERROR

Probable error $= \pm 0.6745 \sqrt{\frac{\Sigma p v^{2}}{\text { No. observations -No. unknowns }}}$

$$
= \pm \sqrt{\frac{0.455 \sum p v^{2}}{n_{\mathrm{o}}-n_{\mathrm{u}}}}
$$

$$
\Sigma p v^{2}=146.394, \log =2.16552
$$

No. observations - No. unknowns $=27-10=17, \quad \operatorname{colog}=8.76955$
Constant $=0.455, \quad \log =9.65801$
$\log (\text { probable error })^{2}=0.59308$
$\log$ probable error $=0.29654$
Probable error of unit weight $= \pm 1.98 * \mathrm{~m}$.
$\log (\text { probable error, unit weight })^{2}=0.59308$
Weight coefficient for Long Ridge $=2.843, \log =0.45378$
$\log (\text { probable error })^{2}=0.13930$
$\log$ probable error $=0.06965$
Probable error for Long Ridge $= \pm 1.17 \mathrm{~m}$.

## FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The following equations are formed from the table just given:

$$
\begin{aligned}
& -v_{1}=-1.93+x_{1} \\
& -v_{2}=-0.83+x_{1} \\
& -v_{3}=+1.92+x_{1} \\
& -v_{4}=+0.72+x_{2} \\
& -v_{5}=-0.25-x_{1}+x_{2}
\end{aligned}
$$

and so on for the rest of the $27 v$ 's.
The function $u$ to be made a minimum is $\Sigma p_{n} v_{n}^{2}$, or

$$
\begin{aligned}
u= & +1.32\left(-1.93+x_{1}\right)^{2}+0.34\left(-0.83+x_{1}\right)^{2}+1.51\left(+1.92+x_{1}\right)^{2}+0.72\left(+0.72+x_{2}\right)^{2} \\
& +2.60\left(-0.25-x_{1}+x_{2}\right)^{2}+10.89\left(-0.73-x_{2}+x_{3}\right)^{2}+0.47\left(+6.90+x_{3}\right)^{2}+2.82(+3.90 \\
& \left.-x_{1}+x_{3}\right)^{2}+4.54\left(-3.55-x_{2}+x_{4}\right)^{2}+4.89\left(+2.16-x_{3}+x_{4}\right)^{2}+8.53\left(+1.61-x_{4}+x_{5}\right)^{2} \\
& +2.34\left(-0.12-x_{2}+x_{5}\right)^{2}+1.63\left(+4.02-x_{3}+x_{5}\right)^{2}+3.71\left(+2.90-x_{4}+x_{6}\right)^{2}+2.63(-0.08 \\
& \left.-x_{5}+x_{6}\right)^{2}+4.93\left(-1.58-x_{6}+x_{7}\right)^{2}+2.61\left(+1.83-x_{4}+x_{7}\right)^{2}+5.19\left(+0.31-x_{5}+x_{7}\right)^{2} \\
& +0.36\left(-0.67+x_{10}\right)^{2}+0.42\left(+2.73-x_{6}+x_{10}\right)^{2}+1.06\left(-1.51-x_{7}+x_{10}\right)^{2}+3.43 \\
& \left(-0.22-x_{7}+x_{8}\right)^{2}+2.03\left(+0.64+x_{8}-x_{10}\right)^{2}+0.24\left(-0.11+x_{9}\right)^{2}+8.38\left(-0.30-x_{8}+x_{9}\right)^{2} \\
& +2.37\left(+0.27-x_{7}+x_{9}\right)^{2}+7.38\left(+0.18+x_{9}-x_{10}\right)^{2} .
\end{aligned}
$$

The function will be rendered a minimum by equating to zero the partial differential coefficients with respect to $x_{1}, x_{2}$, etc. By this means the following equations are derived:

$$
\begin{aligned}
& +1.32\left(-1.93+x_{1}\right)+0.34\left(-0.83+x_{1}\right)+1.51\left(+1.92+x_{1}\right)-2.60\left(-0.25-x_{1}+x_{2}\right)-2.82 \\
& \quad\left(+3.90-x_{1}+x_{3}\right)=0 \\
& +0.72\left(+0.72+x_{2}\right)+2.60\left(-0.25-x_{1}+x_{2}\right)-10.89\left(-0.73-x_{2}+x_{3}\right)-4.54\left(-3.55-x_{2}+x_{4}\right) \\
& \quad-2.34\left(-0.12-x_{2}+x_{5}\right)=0
\end{aligned}
$$

[^15]```
\(+10.89\left(-0.73-x_{2}+x_{3}\right)+0.47\left(+6.90+x_{3}\right)+2.82\left(+3.90-x_{1}+x_{3}\right)-4.89\left(+2.16-x_{3}+x_{4}\right)\)
    \(-1.63\left(+4.02-x_{3}+x_{5}\right)=0\)
\(+4.54\left(-3.55-x_{2}+x_{4}\right)+4.89\left(+2.16-x_{3}+x_{4}\right)-8.53\left(+1.61-x_{4}+x_{5}\right)-3.71(+2.90\)
    \(\left.-x_{4}+x_{6}\right)-2.61\left(+1.83-x_{4}+x_{7}\right)=0\)
\(+8.53\left(+1.61-x_{4}+x_{5}\right)+2.34\left(-0.12-x_{2}+x_{5}\right)+1.63\left(+4.02-x_{3}+x_{5}\right)-2.63(-0.08\)
    \(\left.-x_{5}+x_{6}\right)-5.19\left(+0.31-x_{5}+x_{7}\right)=0\)
\(+3.71\left(+2.90-x_{4}+x_{6}\right)+2.63\left(-0.08-x_{5}+x_{6}\right)-4.93\left(-1.58-x_{6}+x_{7}\right)-0.42(+2.73\)
    \(\left.-x_{6}+x_{10}\right)=0\)
\(+4.93\left(-1.58-x_{6}+x_{7}\right)+2.61\left(+1.83-x_{4}+x_{7}\right)+5.19\left(+0.31-x_{5}+x_{7}\right)-1.06(-1.51\)
    \(\left.-x_{7}+x_{10}\right)-3.43\left(-0.22-x_{7}+x_{8}\right)-2.37\left(+0.27-x_{7}+x_{9}\right)=0\)
\(+3.43\left(-0.22-x_{7}+x_{8}\right)+2.03\left(+0.64+x_{8}-x_{10}\right)-8.38\left(-0.30-x_{8}+x_{9}\right)=0\)
\(+0.24\left(-0.11+x_{9}\right)+8.38\left(-0.30-x_{8}+x_{9}\right)+2.37\left(+0.27-x_{7}+x_{9}\right)+7.38\left(+0.18+x_{9}\right.\)
    \(\left.-x_{10}\right)=0\)
\(+0.36\left(-0.67+x_{10}\right)+0.42\left(+2.37-x_{6}+x_{10}\right)+1.06\left(-1.51-x_{7}+x_{10}\right)-2.03(+0.64\)
    \(\left.+x_{8}-x_{10}\right)-7.38\left(+0.18+x_{9}-x_{10}\right)=0\)
```

By multiplying and collecting, we obtain the following normals:

(See the table of normals on p. 204.)
The normals are most conveniently formed from the table given on page 204. The various observation equations are written along the horizontal lines in the columns of their respective $x$ 's. The normals are then formed as in condition equations, except that the constant terms must also be multiplied by each column and the sums taken for the constant terms in the normals, as may be seen from the direct computation of the normals above.

After the $x$ 's are determined from the solution of the normals, they are added to the assumed elevations, giving the adjusted final elevations. The $v$ 's are most easily determined by computing $h_{2}-h_{1}$ from the adjusted values; if the observed $h_{2}-h_{1}$ is subtracted from the adjusted value the respective $v$ results. They could, of course, be computed by substituting the $x$ 's in the observation equations, but this would require more work.

For a check the $\Sigma p v$ at any station should equal zero, with the possible exception of a small amount due to dropping the decimals on the $x$ 's. In the table on page 201, use $p v$ from the first column if the $x$ is positive and from the second column if the $x$ is negative.

Table for formation of normal equations

|  | $p$ | $N$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $p N$ |  | $x$ 's |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.32 | -1.93 | +1 |  |  |  |  |  |  |  |  |  | - 2.5476 |  | +0.0595 |
| 2 | 0.34 | -0.83 | $+1$ |  |  |  |  |  |  |  |  |  | - 0.2822 | 2 | -2.3925 |
| 3 | 1.51 | +1.92 | +1 |  |  |  |  |  |  |  |  |  | +2.8992 | 3 | -1.2577 |
| 4 | 0.72 | +0.72 |  | +1 |  |  |  |  |  |  |  |  | + 0.5184 | 4 | -1.2304 |
| 5 | 2.60 | -0.25 | -1 | +1 |  |  |  |  |  |  |  |  | - 0.6500 | 5 | -3.0402 |
| 6 | 10.89 | -0.73 |  | -1 | $\pm 1$ |  |  |  |  |  |  |  | - 7.9497 | 6 | $-3.8403$ |
| 7 | 0.47 | +6.90 |  |  | +1 |  |  |  |  |  |  |  | +3.2430 | 7 | -2.8757 |
| 8 | 2. 82 | +3.90 | -1 |  | +1 |  |  |  |  |  |  |  | +10.9980 | 8 | -2.8107 |
| 9 | 4.54 | $-3.55$ |  | -1 |  | +1 |  |  |  |  |  |  | -16.1170 | 9 | -2.5441 |
| 10 | 4.89 | +2.16 |  |  | -1 | ${ }_{-1}^{+1}$ |  |  |  |  |  |  | +10.5624 | 10 | -2.2951 |
| 11 | 8. 53 | +1.61 |  |  |  | -1 | +1 +1 |  |  |  |  |  | +13.7333 |  |  |
| 13 | 2.34 | -0.12 |  | -1 | -1 |  | +1 +1 |  |  |  |  |  | + 0.2808 |  |  |
| 14 | 3.71 | +2.90 |  |  |  | -1 |  | +1 |  |  |  |  | +10.7590 |  |  |
| 15 | 2.63 | -0.08 |  |  |  |  | -1 | +1 |  |  |  |  | -0.2104 |  |  |
| 16 | 4.93 | $-1.58$ |  |  |  |  |  | -1 | +1 |  |  |  | - 7.7894 |  |  |
| 17 | 2.61 | +1.83 |  |  |  | -1 |  |  | +1 |  |  |  | + 4.7763 |  |  |
| 18 | 5.19 | +0.31 |  |  |  |  | -1 |  | +1 |  |  |  | + 1.6089 |  |  |
| 19 | 0.36 | $-0.67$ |  |  |  |  |  |  |  |  |  | $+1$ | -0.2412 |  |  |
| 20 | 0.42 | +2.73 |  |  |  |  |  | -1 |  |  |  | +1 | + 1.1466 |  |  |
| 21 | 1.06 | -1.51 |  |  |  |  |  |  | -1 |  |  | +1 | - 1.6006 |  |  |
| 22 | 3.43 <br> 2.03 | -0.22 |  |  |  |  |  |  | -1 |  |  |  | +0.7546 +1.2992 |  |  |
| ${ }_{24}^{23}$ | 2.03 0.24 | +0.64 -0.11 |  |  |  |  |  |  |  | +1 |  | -1 | $\begin{array}{r}\text { + } \\ + \\ +0.2992 \\ \hline\end{array}$ |  |  |
| 25 | 8.38 | $-0.30$ |  |  |  |  |  |  |  | -1 | +1 |  | - 2.5140 |  |  |
| 26 | 2.37 | +0.27 |  |  |  |  |  |  | -1 |  | +1 |  | +0.6399 |  |  |
| 27 | 7.38 | +0.18 |  |  |  |  |  |  |  |  | +1 | -1 | + 1.3284 |  |  |

Normal equations

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 9 \\ 10 \end{array}$ | +8.59 | $\begin{array}{r} -2.60 \\ +21.09 \end{array}$ | $\begin{aligned} & -2.82 \\ & -10.89 \\ & +20.70 \end{aligned}$ | $\begin{aligned} & -4.54 \\ & \mathbf{4 . 8 9} \\ & +24.28 \end{aligned}$ | $\begin{aligned} & -2.34 \\ & =1.63 \\ & -8.53 \\ & +20.32 \end{aligned}$ | $\begin{aligned} & -3.71 \\ & -2.63 \\ & +11.69 \end{aligned}$ | $\begin{array}{r} -2.61 \\ -5.19 \\ -4.93 \\ +19.59 \end{array}$ | $\begin{array}{r}\text { - } \\ +13.43 \\ \hline\end{array}$ | $\begin{array}{\|r\|r\|} \hline & -2.37 \\ 4 & 8.38 \\ +18.37 \end{array}$ | $\left.\begin{array}{\|c\|} \hline-0.42 \\ -1.06 \\ -2.03 \\ -7.38 \\ -11.25 \end{array} \right\rvert\,$ | 10.2786 -24.2159 +10.8237 -34.8232 +18.6066 +17.1914 +0.3111 +3.0586 -0.5721 -3.3228 | 7.1086 +24.9359 -10.3537 -34.8232 +18.6066 +17.1914 +0.3111 +3.0586 -0.3321 -2.9628 |

Solution of normal equations

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+\underset{x_{1}}{8.59}$ | $\begin{aligned} & -2.60 \\ & +0.30268 \end{aligned}$ | $\begin{aligned} & -2.82 \\ & +0.32829 \end{aligned}$ |  |  |  |  | $\begin{aligned} & -10.2786 \\ & +1.19658 \end{aligned}$ | $\begin{aligned} & -7.1086 \\ & +0.82754 \end{aligned}$ |
|  | $\begin{gathered} +21.09 \\ -0.7870 \\ +20.3030 \\ x_{2} \end{gathered}$ | $\begin{aligned} & -10.89 \\ & -0.8536 \\ & -11.7436 \\ & +0.57842 \end{aligned}$ | $\begin{aligned} & -4.54 \\ & \\ & -4.54 \\ & +0.22361 \end{aligned}$ | $\begin{aligned} & -2.34 \\ & \\ & -2.34 \\ & +0.11525 \end{aligned}$ |  |  | $\begin{aligned} & +24.2159 \\ & -3.1111 \\ & +21.1048 \\ & -1.03949 \end{aligned}$ | $\begin{aligned} & +24.9359 \\ & -2.1516 \\ & +22.7842 \\ & -1.12221 \end{aligned}$ |
|  |  | $\begin{aligned} & +20.70 \\ & -0.9258 \\ & -6.7927 \\ & +12.9815 \\ & x_{3} \end{aligned}$ | $\begin{aligned} & -4.89 \\ & -2.6260 \\ & -7.5160 \\ & +0.57898 \end{aligned}$ | $\begin{aligned} & -1.63 \\ & -1.3535 \\ & -2.9835 \\ & +0.22983 \end{aligned}$ |  |  | $\begin{array}{r} -10.8237 \\ -3.3744 \\ +12.2074 \\ -1.9907 \\ +\quad 0.15335 \end{array}$ | $\begin{aligned} & -10.3537 \\ & -2.3337 \\ & +13.1788 \\ & +0.4913 \\ & -0.03785 \end{aligned}$ |
|  |  | $\stackrel{2}{3}$ | $\begin{gathered} +24.28 \\ -1.0152 \\ -4.3516 \\ +18.9132 \\ x_{4} \end{gathered}$ | -8.53 -0.5232 -1.7274 -10.7806 +0.570004 | $\begin{aligned} & -3.71 \\ & \\ & -3.71 \\ & +0.196159 \end{aligned}$ | $\begin{aligned} & -2.61 \\ & \\ & -2.61 \\ & +0.137999 \end{aligned}$ | $\begin{aligned} & \hline-34.8232 \\ & +4.7192 \\ & -1.1526 \\ & -31.2566 \\ & +1.652634 \end{aligned}$ | $\begin{aligned} & -34.8232 \\ & +5.0948 \\ & +0.2845 \\ & -29.4440 \\ & +1.556796 \end{aligned}$ |

Solution of normal equations-Continued

| 9 | 8 | 10 | 7 | 6 | 5 | $\eta$ | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+\underset{x_{9}}{18.37}$ | $\begin{aligned} & -8.38 \\ & +0.45618 \end{aligned}$ | $\begin{aligned} & -7.38 \\ & +0.40174 \end{aligned}$ | $\begin{aligned} & -2.37 \\ & +0.12901^{5} \end{aligned}$ |  |  | $\begin{aligned} & -0.5721 \\ & +0.03114 \end{aligned}$ | $\begin{aligned} & -0.3321 \\ & +0.01808 \end{aligned}$ |
|  | $\begin{gathered} +13.84 \\ -3.8228 \\ +10.0172 \\ x_{8} \end{gathered}$ | $\begin{aligned} & -2.03 \\ & -3.3666 \\ & -5.3966 \\ & +0.53873 \end{aligned}$ | $\begin{aligned} & -3.43 \\ & -1.0811 \\ & -4.5111 \\ & +0.45034 \end{aligned}$ |  |  | $\begin{aligned} & +3.0586 \\ & -0.2610 \\ & +2.716 \\ & -0.27928 \end{aligned}$ | $\begin{array}{r} +3.0586 \\ -0.1515 \\ +2.9071 \\ -0.29021 \end{array}$ |
|  | $\begin{aligned} & 9 \\ & 8 \end{aligned}$ | $\begin{aligned} & +11.25 \\ & -2.9648 \\ & -2.9073 \\ & +5.3779 \\ & x_{10} \end{aligned}$ | $\begin{aligned} & -1.86 \\ & =0.9521 \\ & -2.4303 \\ & -4.4424 \\ & +0.82605 \end{aligned}$ | $\begin{aligned} & -0.42 \\ & -0.42 \\ & +0.07810 \end{aligned}$ |  | $\begin{aligned} & -3.3228 \\ & -0.2298 \\ & +1.5072 \\ & -2.0454 \\ & +0.38033 \end{aligned}$ | $\begin{aligned} & -2.9628 \\ & -0.1334 \\ & +1.5661 \\ & -1.5299 \\ & +0.28448 \end{aligned}$ |
|  |  | $\begin{array}{r} 4 \\ 9 \\ 8 \\ 10 \end{array}$ | $\begin{gathered} +19.59 \\ -0.3602 \\ =0.3058 \\ -2.0315 \\ -3.6696 \\ +13.2229 \\ x_{7} \end{gathered}$ | $\begin{aligned} & -4.93 \\ & -0.5120 \\ & -0.3469 \\ & -5.7889 \\ & +0.43779 \end{aligned}$ | $\begin{aligned} & -5.19 \\ & -1.4877 \end{aligned}$ $\begin{aligned} & -6.6777 \\ & +0.50501 \end{aligned}$ | $\begin{aligned} & +0.3111 \\ & -4.3134 \\ & -0.0738 \\ & +1.2599 \\ & -1.6896 \\ & -4.5058 \\ & +0.34076 \end{aligned}$ | +0.3111 -4.0632 -0.0428 +1.3092 -1.2638 -3.7495 +0.28356 |
|  |  |  | $\begin{array}{r} 4 \\ 10 \\ 7 \end{array}$ | $\begin{gathered} +11.19 \\ =0.7277 \\ =0.0328 \\ -2.5343 \\ +\quad 8.3952 \\ x_{6} \end{gathered}$ | $\begin{aligned} & -2.63 \\ & -2.1148 \\ & -2.9234 \\ & -7.6681 \\ & +0.91339 \end{aligned}$ | $\begin{array}{r} +17.1914 \\ -6.1313 \\ -0.1597 \\ -1.9726 \\ +8.9279 \\ -1.06345 \end{array}$ | +17.1914 -5.7757 -0.1195 -1.6415 +9.6550 -1.15006 |
|  |  |  |  | $\begin{aligned} & 2 \\ & 3 \\ & 4 \\ & 7 \\ & 6 \end{aligned}$ | $\begin{aligned} & +20.32 \\ & -0.2697 \\ & =0.6857 \\ & =6.1450 \\ & =3.3723 \\ & -7.0040 \\ & +2.8433 \\ & x_{5} \end{aligned}$ | $\begin{aligned} & +18.6066 \\ & +2.4323 \\ & -0.4575 \\ & -17.8164 \\ & +2.2755 \\ & +8.1547 \\ & +8.6442 \\ & -3.04020 \end{aligned}$ | $\begin{array}{r} +18.6066 \\ +2.6259 \\ +0.1129 \\ -16.7832 \\ -1.8935 \\ +8.8188 \\ +11.4875 \\ -4.04020 \end{array}$ |

Back solution

| 5 | 6 | 7 | 10 | 8 | 9 | 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.0402 | -1.0634 | +0.3408 | +0.3803 | -0.2793 | +0.0311 | +1.6526 | +0.1534 | -1.0395 | +1.1966 |
| -3.0402 | $-2.7769$ | -1.5353 | -0.2999 | -1.2950 | $-0.3710$ | -1.7329 | $-0.6987$ | -0.3504 | -0.4129 |
|  | $-3.8403$ | -1.6812 | -2.3755 | -1.2364 | $\begin{aligned} & -0.9220 \\ & -1.2822 \end{aligned}$ | $\begin{aligned} & -0.7533 \\ & -0.3968 \end{aligned}$ | -0.7124 | $\begin{aligned} & -0.2751 \\ & -0.7275 \end{aligned}$ | -0.7242 |
|  |  | -2.8757 | -2.2951 | -2.8107 | -2.5441 | -1.2304 | -1.2577 | -2.3925 | +0.0595 |

DEVELOPMENT OF FORMULAS FOR TRIGONOMETRIC LEVELING

## GENERAL STATEMENT

The formulas used on pages 198 and 199 in the computation of vertical observations were found to be lacking in some of the quantities that were appreciable when the lines were very long and high. Accordingly, a new derivation is now given that takes into account some of these quantities. As a result, the formulas derived in this development differ slightly from those used in the computation cited above, but they ought to give practically the same result in computing over lines of such length as occur therein.

The following derivation of the formulas for trigonometric leveling is based on certain approximate assumptions which fall under four general heads:

1. Geometric approximations.-The verticals at the two points ( $P_{1}$ the point occupied and $P_{2}$ the point sighted on) are treated as if they lay in one plane and the intersection of this plane with the ellipsoid that represents the surface of the earth is treated as the arc of a circle whose radius is the mean radius of curvature of a vertical section through $P_{1}$ and $\mathrm{P}_{2}$. Helmert (in his Höhere Geodäsie, Vol. I, p. 520 , and Vol. II, p. 563) investigates the error arising from these assumptions and finds it to be about $1 / 40$ meter at a maximum when the distance $P_{1} P_{2}$ is about 100 kilometers.
2. Geodetic approximations.-The difference between the geodetic zenith and the astronomic zenith, i. e., the deflection of the plumb line, is ignored. If these deflections are known, corrections may be applied to the measured zenith distances (which, of course, are referred to the astronomical zenith) to reduce them to the geodetic zenith. Furthermore, the elevations obtained by trigonometric leveling between two points are referred to an assumed ellipsoid, while spirit leveling gives elevations referred to the geoid, so that the distances between geoid and ellipsoid must be known to make the two kinds of leveling comparable. If trigonometric leveling could be carried out with great precision, its use in connection with spirit leveling would give just this information as to the distance of the ellipsoid from the geoid. The change in the distance from geoid to ellipsoid occurring between $P_{1}$ and $P_{2}$ may be found from the deflections of the vertical at those points, provided it is assumed that the deflections vary uniformly between $P_{1}$ and $P_{2}$, an assumption which may be considerably in error.
3. Optical approximations. - The path of the ray of light between $P_{1}$ and $P_{2}$ is assumed to be the arc of a circle in a vertical plane through $P_{1}$ and $P_{2}$. The angle between the chord $P_{1} P_{2}{ }^{\circ}$ and the tangent to the circle at either point is the refraction in zenith distance and it is evidently implied that this refraction is equal at $P_{1}$ and $P_{2}$. If we call $O$ (see figure 9) the center of the circle referred to in approximation 1, and call the angle $P_{1} O P_{2}=\theta$, the refraction in zenith distance of the angle $T P_{1} P_{2}\left(=\angle T P_{2} P_{1}\right)$ is written as $m \theta$ and $m$ is termed the coefficient of refraction. The course of a ray of light through the atmosphere depends on the variations in pressure, temperature and humidity of the medium through which it passes and may be far from circular. Our lack of knowledge of the conditions which govern the refraction is the greatest obstacle to precision in trigonometric leveling.
4. Algebraic approximations.-After tho approximations mentioned above have been made, there is the further approximation arising from the dropping of small terms after an expansion in series. In the following developments it will be seen that only extremely small terms are dropped, and that in cases arising in practice their effect even on the sixth place of logarithms is unimportant, whilo in fact logarithms of only five places are commonly used for this sort of computation. The accuracy of the developments is confirmed by the numerical agreement between the approximate and the exact formulas in the examples given. (Exact is used in the sense of dispensing with the use of series. The formula is inexact, owing to the first three sets of approximations.) The examples represent rather extrome cases of those arising in practice, and other numerical examples of extreme cases give a similar agreement.

## DEVELOPMENT OF THE FORMUIAS

Figure 9 represents the vertical plane of approximation 1 common to $P_{1}$ and $P_{2}$, being in fact the plane parallel to both verticals (seo Helmert, Höhere 'Geodäsie, Vol. I, p. 519) on which the several points aro projected.

The measured zenith distances are assumed equal to

$$
\begin{aligned}
& \angle V_{1} P_{1} T=\zeta_{1} \\
\text { and } \quad & \angle V_{2} P_{2} T=\zeta_{2} .
\end{aligned}
$$

The measurements are not made exactly in this plane, but the error, which is part of that involved in approximation 1 , is negligible.

The refraction in zenith distance is, according to approximation 3,
$\Delta \zeta=\angle T P_{1} P_{2}=\angle T P_{2} P_{1}=m \theta$.
$S_{1}$ and $S_{2}$ are points on the earth's surfaco in the verticals


Fig. 9. of $P_{1}$ and $P_{2}$, so that the rospective elevations of $P_{1}$ and $P_{2}$ above the surface are

$$
\begin{aligned}
& h_{1}=S_{1} P_{1} \\
& h_{2}=S_{2} P_{2} .
\end{aligned}
$$

and
The mean radius of curvature $\rho$ of approximation 1 is given by

$$
\rho=O S_{1}=O S_{2} .
$$

$$
91865^{\circ}-15-14
$$

If $s$ denotes the distance $P_{1} P_{2}$ measured along the are and if $\theta$ be expressed in radians,
or if $\theta$ be in seconds,

$$
s=\rho \theta
$$

$$
\theta^{\prime \prime}=\frac{s}{\rho \sin 1^{\prime \prime}}
$$

There are two cases to be considered according as to whether both or only one of the zenith distances have been measured.

## $C_{A S E}$ I. Reciprocal zenith distances

In the triangle $P_{1} O P_{2}$
also

$$
\begin{aligned}
& \angle P_{2} P_{1} O=180^{\circ}-\zeta_{1}-\Delta \zeta=180^{\circ}-\zeta_{1}-m \theta \\
& \angle P_{1} P_{2} O=180^{\circ}-\zeta_{2}-\Delta \zeta=180^{\circ}-\zeta_{2}-m \theta
\end{aligned}
$$

and

$$
O P_{1}=\rho+h_{1}
$$

$$
O P_{2}=\rho+h_{2}
$$

Therefore by the law of sines

$$
\frac{\rho+k_{1}}{\rho+h_{2}}=\frac{\sin \left(\zeta_{2}+m \theta\right)}{\sin \left(\zeta_{1}+m \theta\right)} .
$$

Treating this as a proportion and taking by division,

$$
\frac{\left(\rho+h_{2}\right)-\left(\rho+h_{1}\right)}{\rho+h_{1}}=\frac{\sin \left(\zeta_{1}+m \theta\right)-\sin \left(\zeta_{2}+m \theta\right)}{\sin \left(\zeta_{2}+m \theta\right)}
$$

or

$$
\begin{equation*}
h_{2}-h_{1}=\frac{2\left(\rho+h_{1}\right) \sin \left(\frac{\zeta_{1}-\zeta_{2}}{2}\right) \cos \left(\frac{\zeta_{1}+\zeta_{2}}{2}+m \theta\right)}{\sin \left(\zeta_{2}+m \theta\right)} . \tag{A}
\end{equation*}
$$

Since the sum of the angles of a triangle is $180^{\circ}$,

$$
180^{\circ}-\zeta_{1}-m \theta+180^{\circ}-\zeta_{2}-m \theta+\theta=180^{\circ}
$$

which gives

$$
\frac{\zeta_{1}+\zeta_{2}}{2}+m \theta=90^{\circ}+\frac{\theta}{2}
$$

also

$$
\zeta_{2}+m \theta=\frac{\zeta_{2}+\zeta_{1}}{2}+m \theta+\frac{\zeta_{2}-\zeta_{1}}{2}=90^{\circ}+\frac{\theta}{2}+\frac{\zeta_{2}-\zeta_{1}}{2}
$$

whence ( $\Lambda$ ) becomes

$$
\begin{equation*}
h_{2}-h_{1}=\frac{2\left(\rho+h_{1}\right) \sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sin \frac{\theta}{2}}{\cos \left(\frac{\zeta_{2}-\zeta_{1}}{2}+\frac{\theta}{2}\right)} \tag{1}
\end{equation*}
$$

The quantity $2\left(\rho+h_{1}\right) \sin \frac{\theta}{2}$ has a simple geometrical interpretation In the figure make $O L_{2}=O P_{1}$ and draw $O M \perp P_{1} L_{2}$. Then

$$
P_{1} M=L_{2} M=O P_{1} \sin P_{1} O M=\left(\rho+h_{1}\right) \sin \frac{\theta}{2} .
$$

Then $2\left(\rho+h_{1}\right) \sin \frac{\theta}{2}$ is the chord $P_{1} L_{2}$ or the chord $S_{1} S_{2}$ increased to allow for the elevation of $P_{1}$ above the earth's surface. In fact, the relation (1) might have been obtained by applying the law of sines directly to the triangle $P_{1} P_{2} L_{2}$, which makes it evident why $P_{1} L_{2}$ appears.

For convenient computation* (1) may be transformed as follows: By the sine series

$$
\begin{gather*}
2 \sin \frac{\theta}{2}=2\left[\frac{\theta}{2}-\frac{1}{6}\left(\frac{\theta}{2}\right)^{3}+\cdots \cdot\right] \\
2\left(\rho+h_{1}\right) \sin \frac{\theta}{2}=\rho\left(1+\frac{h_{1}}{\rho}\right)\left(0-\frac{\theta^{3}}{24}+\cdots \cdot\right) \\
=\rho \theta\left(1+\frac{h_{1}}{\rho}\right)\left(1-\frac{\theta^{2}}{24}\right)=s\left(1+\frac{h_{1}}{\rho}\right)\left(1-\frac{s^{2}}{24 \rho^{2}}\right) \tag{2}
\end{gather*}
$$

The remaining factors of the right-hand side of (1) may be written,

$$
\begin{align*}
\frac{\sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}{\cos \left(\frac{\zeta_{2}-\zeta_{1}}{2}+\frac{\theta}{2}\right)} & =\frac{\sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}{\cos \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \cos \frac{\theta}{2}-\sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sin \frac{0}{2}} \\
& =\frac{\tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sec \frac{\theta}{2}}{1-\tan \frac{\theta}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)} \\
& =\tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\left(1+\frac{\theta^{2}}{8}\right)\left[1+\frac{\theta}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\right] .
\end{align*}
$$

The last transformation comes by expanding sec $\frac{0}{2}$ in powers of $\theta$ and noting that $\tan \frac{0}{2}=\frac{0}{2}$ nearly, and that the product $\frac{0}{2} \tan \frac{\zeta_{2}-\zeta_{1}}{2}$ is small, so that,

$$
\frac{1}{1-\frac{0}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}=1+\frac{\theta}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \text { very nearly. }
$$

By combining (2) and (3) and using $\theta=\frac{s}{\rho}$, equation (1) becomes

$$
\begin{align*}
h_{2}-h_{1} & =s\left(1+\frac{h_{1}}{\rho}\right) \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\left[1+\frac{s}{2 \rho} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\right]\left[1+\frac{s^{2}}{12 \rho^{2}}\right] \\
\text { or } h_{2}-h_{1} & =s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) A B C \tag{4}
\end{align*}
$$

where $A=1+\frac{h_{1}}{\rho}=$ correction for elevation of station whose elevation
is known,

$$
B=1+\frac{s}{2 \rho} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)=\text { correction for approximate difference }
$$

$C=1+\frac{s^{2}}{12 \rho^{2}}=$ correction for distance.
The logarithms of $A, B$, and $C$ are given in the tables on pages 218 and 219 with the respective arguments $h_{1}, \log \left[s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\right]$, and $\log s$. The tables show the limiting values of the respective arguments for which logarithms of $A, B$, and $C$ become $1,2,3$, etc., units of the fifth place of decimals.

Equation (4) may be compared with the expression more commonly given for $h_{2}-h_{1}$,

$$
\begin{equation*}
h_{2}-h_{1}=s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\left[1+\frac{h_{1}+h_{2}}{2 \rho}+\frac{s^{2}}{12 \rho^{2}}\right] \tag{5}
\end{equation*}
$$

With the tables here given ${ }_{4}(5)$ will probably be found slightly more convenient for logarithmic computation than $s^{5}(\bar{A})$. The two forms are equally accurate.

## Case II. Only one zenith distance ( $\zeta_{1}$ ) observed

Where two zenith distances are known, the formula, either (4) or (5), does not involve the coefficient of refraction ( $m$ ) explicitly. Where only one zenith distance is known, a value of $m$ must be assumed from the best sources of information available.

In the triangle $P_{1} L_{2} P_{2}$

$$
\begin{aligned}
\angle P_{1} L_{2} P_{2} & =90^{\circ}+\frac{\theta}{2}=\angle V_{1} P_{1} L_{2} \\
\angle P_{2} P_{1} L & =\angle V_{1} P_{1} L_{2}-\angle V_{1} P_{1} P_{2} \\
& =90^{\circ}+\frac{\theta}{2}-\left(\zeta_{1}+\Delta \zeta\right)=90^{\circ}-\zeta+\left(\frac{1}{2}-m\right) \theta
\end{aligned}
$$

For the third angle we find, by subtracting the sum of the other angles from $180^{\circ}$

By the law of sines

$$
\angle P_{1} P_{2} L=\zeta_{1}-(1-m) \theta \text {. }
$$

$$
\frac{L_{2} P_{2}}{P_{1} L_{2}}=\frac{\sin P_{2} P_{1} L_{2}}{\sin P_{1} P_{2} L_{2}}
$$

or

$$
\begin{equation*}
h_{2}-h_{1}=P_{1} L_{2} \frac{\cos \left[\zeta_{1}-\left(\frac{1}{2}-m\right) \theta\right]}{\sin \left[\zeta_{1}-(1-m \theta)\right]} . \tag{6}
\end{equation*}
$$

The chord $P_{1} L_{2}=$ chord $S_{1} S_{2} \times \frac{\rho+h_{1}}{\rho}=$ chord $S_{1} S_{2} \times A, A$ having the meaning previously given; chord $S_{1} S_{2}=$ arc $s$ very nearly; or, if greater precision is desired, $P_{1} L_{2}=s A R$, where $R$ is the reduction factor from are to chord.

The logarithm of the reduction factor from arc to sine is given in the Coast and Geodetic Survey Special Publication No. 8 (Formulæ and Tables for the Computation of Geodetic Positions), page 17. The logarithm of the reduction to chord is very nearly one-fourth of the reduction from arc to sine. Granting approximations 1, 2, and 3, equation (6) may be rewritten as the so-called exact formula in the following form:

$$
\begin{equation*}
h_{2}-h_{1}=s A R \frac{\cos \left(\zeta_{1}-\left(\frac{1}{2}-m\right) \frac{s}{\rho \sin 1^{\prime \prime}}\right)}{\sin \left[\zeta_{1}-\left(\frac{1}{2}-m\right) \frac{s}{\rho \sin 1^{\prime \prime}}-\frac{1}{2} \frac{s}{\rho \sin \cdot 1^{\prime \prime}}\right]} \tag{7}
\end{equation*}
$$

$\operatorname{Sin} 1^{\prime \prime}$ is introduced to convert the angle from radians to seconds of arc. $A$ and $R$ have the meanings previously indicated. The quantity $\left(\frac{1}{2}-m\right)$ appears in the computation of the refraction from reciprocal zenith distances on the Coast and Geodetic Survey forms. A mean of the determinations of $\left(\frac{1}{2}-m\right)$ from the reciprocal zenith distances should be used in computing the nonreciprocal observations.
Having found the angle $\zeta_{1}-\left(\frac{1}{2}-m\right) \frac{s}{\rho \sin 1^{\prime \prime}}$ for the numerator, the computer should subtract $\frac{1}{2} \frac{s}{\rho \sin 1^{\prime \prime}}$ from it to get the angle for the denominator. The angle in the denominator need not be carried out very accurately, as it is always near $90^{\circ}$ where the sine varies slowly.

The former Coast and Geodetic Survey formula was

$$
\begin{equation*}
h_{2}-h_{1}=s \cot \zeta_{1}+\frac{\left(\frac{1}{2}-m\right) s^{2}}{\rho}+\frac{(1-m) s^{2} \cot ^{2} \zeta_{1} .}{\rho} . \tag{8}
\end{equation*}
$$

It is obtained from (6) or (7) by expanding in series and dropping certain small quantities. On some of the longer lines the quantities dropped are appreciable in computations with five-place logarithms. The development hereafter given will show that the general form of (8) may be retained by the introduction of correction-factors $D_{1}$ and $D_{2}$, which are nearly unity, and by the further factor $\Lambda$, the correc-tion-factor for elevation of the occupied station. The full formula will then be,

$$
h_{2}-h_{1}=A D_{1} s \cot \zeta_{1}+\frac{\left(\frac{1}{2}-m\right) A D_{2} s^{2}}{\rho}+\frac{(1-m) s^{2} \cot ^{2} \zeta_{1}}{\rho} .
$$

This form may be obtained from (6) as follows:
As before

$$
P_{1} L_{2}=2\left(\rho+h_{1}\right) \sin \frac{\theta}{2}
$$

or expanding by the sine series

$$
\begin{equation*}
P_{1} L_{2}=\rho\left(1+\frac{h_{1}}{\rho}\right)\left[\theta-\frac{\theta^{3}}{24}+\cdots\right]=A_{\rho}\left(\theta-\frac{\theta^{3}}{24}+\cdots\right) . \tag{9}
\end{equation*}
$$

The factor $\cos \left[\zeta_{1}-\left(\frac{1}{2}-m\right) 0\right]$ in (6) may be written

$$
\cos \left[\zeta_{1}-\left(\frac{1}{2}-m\right) 0\right]=\cos \zeta_{1} \cos \left[\left(\frac{1}{2}-m\right) 0\right]+\sin \zeta_{1} \sin \left[\left(\frac{1}{2}-m\right) \theta\right]
$$

Since $\left(\frac{1}{2}-m\right) 0$ is a small quantity, the series forms for its sine and cosine may be used, giving

$$
\left.\begin{array}{rl}
\cos \left[\zeta_{1}-\left(\frac{1}{2}-m\right) \theta\right]= & \cos \zeta_{1}
\end{array}\right]\left[1-\left(\frac{1}{2}-m\right)^{2} \frac{\theta^{2}}{2}+\cdots \cdot\right] \quad \begin{aligned}
& +\sin \zeta_{1}\left[\left(\frac{1}{2}-m\right) \theta-\left(\frac{1}{2}-m\right)^{3} \frac{\theta^{3}}{6}+\cdots \cdot\right]
\end{aligned}
$$

The third factor on the right-hand side of (6), namely,

$$
\frac{1}{\sin \left[\zeta_{1}-(1-m) 0\right]}=\operatorname{cosec}\left[\zeta_{1}-(1-m) 0\right]
$$

may be expanded in powers of $(1-m) \theta$ by Taylor's theorem.

$$
\begin{aligned}
f\left(\zeta_{1}\right) & =\operatorname{cosec} \zeta_{1} \\
f^{\prime}\left(\zeta_{1}\right) & =-\cot \zeta_{1} \operatorname{cosec} \zeta_{1} \\
f^{\prime \prime}\left(\zeta_{1}\right) & =\operatorname{cosec} \zeta_{1}\left(1+2 \cot ^{2} \zeta_{1}\right) \\
f^{\prime \prime \prime}\left(\zeta_{1}\right) & =-6 \operatorname{cosec}^{3} \zeta_{1} \cot \zeta_{1}+\cot \zeta_{1} \operatorname{cosec} \zeta_{1} .
\end{aligned}
$$

This gives,

$$
\begin{align*}
\sin \left[\zeta_{1}-(1-m) 0\right] & =\operatorname{cosec} \zeta_{1}+\operatorname{cosec} \zeta_{1} \cot \zeta_{1}(1-m) \theta \\
& +\operatorname{cosec} \zeta_{i}\left(1+2 \cot ^{2} \zeta_{1}\right)(1-m)^{2} \frac{\theta^{2}}{2}  \tag{11}\\
+ & \operatorname{cosec} \zeta_{1} \cot \zeta_{1}\left(6 \operatorname{cosec}^{2} \zeta_{1}-1\right)(1-m)^{3} \frac{\theta^{3}}{6}+\ldots
\end{align*}
$$

The expressions (9), (10), and (11) for the factors on the right-hand side of (6) are now to be multiplied together.

In cases that actually occur, 0 and $\cot \zeta_{1}$ are small quantities of about the same order of magnitude. If we call $\cot \zeta_{1}$ a quantity of the first order, it is evident that $\operatorname{cosec} \zeta_{1}$ differs from unity by a quantity of the second order. In forming the product from (9), (10), and (11) it is seen that the product is of the second order, and will moreover contain only terms of even order, so that if terms of the fourth order are retained the error will be of the sixth order, or the proportional error (the error as compared with the quantity itself) will be of the fourth order or of the order of $\frac{1}{30^{4}}$ part of the difference of elevation, if we suppose a quantity of the first order may be as large as $\frac{1}{30}$, a liberal allowance. The error, then, of the omitted terms should not affect the fifth place of logarithms and probably not the sixth. It will be seen that the expansions (9), (10), and (11) hare been carried out sufficiently far for the purpose in hand, and if these expressions
be multiplied together, retaining in the product no terms of higher order than the fourth, the result may be written:

$$
\begin{gather*}
h_{2}-h_{1}=\left(\rho+h_{1}\right)\left\{\theta \cot \zeta_{1}\left[1+\frac{6(1-m)^{2}-1}{6} \cdot \theta^{2}\right]\right. \\
\left.+\left(\frac{1}{2}-m\right) \theta^{2}\left[1+\frac{5-10 m+4 m^{2}}{12} \cdot \theta^{2}\right]+(1-m) \theta^{2} \cot ^{2} \zeta_{1}\right\} \tag{12}
\end{gather*}
$$

Since $\theta=\frac{s}{\rho}$, we may write

$$
\begin{equation*}
h_{2}-h_{1}=A D_{1} s \cot \zeta_{1}+\frac{A D_{2}\left(\frac{1}{2}-m\right) s^{2}}{\rho}+\frac{(1-m) s^{2} \cot ^{2} \zeta_{1}}{\rho} \tag{13}
\end{equation*}
$$

where $D_{1}=1+\frac{6(1-m)^{2}-1}{6} \cdot \frac{s^{2}}{\rho^{2}}$

$$
\begin{equation*}
\left.D_{2}=1+\frac{5-10 m+4 m^{2}}{12} \cdot \frac{s^{2}}{\rho^{2}}\right\} \tag{13a}
\end{equation*}
$$

The factor $A$ has been omitted from the last term as being unnecessary, the latter being small and $A$ near unity. $D_{1}$ and $D_{2}$ are also near unity. Their logarithms are tabulated in the same manner as the other quantities, the tables showing the limiting values of the argument between which $\log D_{1}$ or $\log D_{2}$ may be taken as $1,2,3$, etc., units of the fifth decimal.

It may be noted that in some European surveys the torm $(1-m) \frac{s^{2} \cot ^{2} \zeta_{1}}{\rho}$ is dropped and the formula for difference of clevation written as

$$
\begin{equation*}
h_{2}-h_{1}=s \cot \zeta_{1}+\left(\frac{1}{2}-m\right) \frac{s^{2}}{\rho} \tag{14}
\end{equation*}
$$

The dropped terms or factors all represent quantities of the fourth order in our expansion. The term $\frac{(1-m) s^{2} \cot ^{2} \zeta_{1}}{\rho}$ is, however, the largest of such quantities as a rule, and might be noticeable where $D_{1}$ and $D_{2}$ would not be.

Probably for short lines and small differences of elevation the most convenient formula would be

$$
\begin{equation*}
h_{2}-h_{1}=A s \cot \zeta_{1}+A\left(\frac{1}{2}-m\right) \frac{s^{2}}{\rho} \tag{15}
\end{equation*}
$$

and for other lines formula (7).

## EXAMPLES

The data for the following examples, which illustrate the use of the formulas, come from The Transcontinental Triangulation, Special Publication No. 4, page 273, et. seq.

|  | At Snow <br> Mountain <br> West | At Ross <br> Mountain |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 0 | , | $\prime \prime$ | 0 |

$\log s=5.007341$.
For mean $\alpha$ and $\phi$ on the Clarke spheroid of $1866, \log \rho=6.80369$.
Example 1. Difference of elevation for reciprocal zenith distances, assuming Snow Mountain West as the known elevation from formula (5).

Example 2. Same data as Example 1 worked by formula (4).
Example 3. Assuming Ross Mountain as known elevation, solve by (4).

Example 4. With refraction from reciprocal zenith distances, but with only zenith distance at Snow Mountain West appearing explicitly, find difference of elevation by (7).

Example 5. Same data as Example 4, worked by (13).
Example 6. Like example 4, except zenith distance at Ross Mountain is used.

Example 7. Like example 5, except zenith distance at Ross Mountain is used.

The agreement of the differences of elevation as computed by the various combinations of data and formulas will give an idea of the accuracy of the latter.

| Example 1 |  |  | Example 2 | Example 3 |
| :---: | :---: | :---: | :---: | :---: |
| Station 1 | $\left\{\begin{array}{c} \text { Snow } \\ \text { West } \end{array}\right. \text { Mountain }$ | $\begin{aligned} & \log s \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right) \\ & \log A \\ & \log B \\ & \log C \end{aligned}$ | $\begin{array}{r} 3.168203 n \\ +146 \\ +50 \\ +11 \end{array}$ | $\begin{array}{r} 3.168203 \\ +46 \\ +50 \\ +11 \end{array}$ |
| Station 2 | Ross Mountain |  |  |  |
| $\zeta_{1}$ | $\begin{array}{llll}91 & 13 & 39.1\end{array}$ |  |  |  |
| $\zeta_{2}$ | $\begin{array}{r}89 \\ \hline 14 \\ -1394.8 \\ \hline\end{array}$ |  |  |  |
| ${ }_{2}^{2}\left(\zeta_{2}-\zeta_{1}\right)$ in | - 4947.15 |  |  |  |
| $\begin{aligned} & \frac{1}{3}\left(\zeta_{1}\right) \text { in secs. } \\ & \log \text { ditto } \end{aligned}$ | $\begin{array}{r} 2987.15 \\ -\quad 3.47525 i n \end{array}$ |  |  |  |
|  | 4.685605 5.007341 |  |  |  |
| $\log ^{2} 8 \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | 5. $3.168203 n$ |  |  |  |
| $8 \tan \frac{1}{2}\left(\zeta_{2}-\zeta_{1}\right)$ | -1473.00 |  |  |  |
| Second term* | $\overline{-} .03$ |  |  |  |
| Third term* | . 33 |  |  |  |
| $h_{2}-h_{1}$ | -1473.36 | $\log \left(h_{2}-h_{1}\right)$ | 3. $168310 n$ | 3. 168310 |
| $h_{1}$ | 2145.66 |  |  |  |
| $h_{2}$ | , 672.30 |  |  |  |
| $\zeta_{1}+\zeta_{2}-150^{\circ}$ | $47 \quad 43.9$ |  |  |  |
| $\begin{aligned} & \zeta_{1}+\zeta_{2}-180 \text { in secs. } \\ & \log \text { ditto } \end{aligned}$ | $\underset{3.45 c 958}{28.3 .9}$ |  |  |  |
| $\log \rho$ | 6.803690 |  |  |  |
| $\log \frac{1}{8}$ | 4.992 C 59 |  |  |  |
| $\sin 1^{\prime \prime}$ |  |  |  |  |
| $\log \frac{1}{2}=4.38454$ | 4.384545 |  |  |  |
| $\begin{aligned} & \log _{(0.5-m)}(0.5-m) \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { 9.637852 } \\ & 0.43436 \end{aligned}$ |  |  |  |


| Example 4 |  | Example 6 |
| :---: | :---: | :---: |
| $\log s$. $\operatorname{colog} \rho$ colog $\sin 1^{\prime \prime}$ | 5. 007341 <br> 3.196310 <br> 5.314425 | (Samo as example 4) |
| $\begin{aligned} & \log \theta \dagger \\ & \log (0.5-m) \end{aligned}$ | $\begin{aligned} & 3.518076 \\ & 9.637852 \end{aligned}$ |  |
| $\begin{aligned} & \log (0.5-m) \theta \\ & (0.5-m) \theta \end{aligned}$ | $\begin{aligned} & 3.155928 \\ & 1431 .^{\prime \prime} 95 \end{aligned}$ |  |
| $\zeta_{1}$ $(0.5-m) \theta$ | $\begin{array}{cc} 91^{\circ} & 13^{\prime} \\ & 39 .^{\prime \prime} 1 \\ 23 & \text { 51. } \end{array}$ | $\begin{array}{ccc} 89^{\circ} 34^{\prime} & 044^{\prime \prime} 8 \\ & 23 & 51 . \end{array}$ |
| $\begin{aligned} & \zeta_{1}-(0.5-m) \theta \\ & \frac{\theta}{2} \end{aligned}$ | $\begin{array}{llll} \hline 90 & 49 & 47 . & 15 \\ & 27 & 28 & \end{array}$ | $\begin{array}{llll} 89 & 10 & 12.85 \\ & 27 & 28 \end{array}$ |
| $\begin{aligned} & l_{1}-(1-m) \theta \\ & \log g \end{aligned}$ | $\begin{array}{lll} 90 & 22 & 19 \\ & 5.007341 \end{array}$ | $\begin{array}{lll} 88 & 42 & 45 \\ & 5.007341 \end{array}$ |
| $\log A$ | $+\quad 146$ | + 5 |
| $\begin{aligned} & \log \cos \left[\zeta_{1}-(0.5-m) \theta\right] \\ & \operatorname{colog} \sin \left[\Gamma_{1}-(1-m) \theta\right] \end{aligned}$ | $\begin{gathered} 8.160817 n \\ +9 \end{gathered}$ | $\begin{array}{r} 8.160817 \\ +\quad 110 \end{array}$ |
| $\begin{aligned} & \log \left(h_{2}-h_{1}\right) \\ & h_{2}-h_{1} \end{aligned}$ | $\begin{gathered} 3.168308 n \\ -1473.36 \end{gathered}$ | $\begin{aligned} & 3.168309 \\ & +1473.36 \end{aligned}$ |

* Second and third terms in example 1 computed by aid of table in General Instructions for Field Work, Coast and Geodetic Survey, pp. 36-37 (edition of 1908).
$\dagger \theta$ is used for $\frac{8}{\rho \sin 1^{\prime \prime}}$, as in the text.

| Example 5 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\log 8$ $\log A$ $\log D_{1}$ $\cot \zeta_{1}$ | $\begin{gathered} 5.007341 \\ +146 \\ +79 \\ 8.330975 n \end{gathered}$ | $\begin{aligned} & \log (0.5-m) \\ & \log s^{2} \\ & \operatorname{colog} \rho \\ & \log A \\ & \log D_{2} \\ & \log \text { second } \end{aligned}$ | 9.637852 10.014682 3.196310 146 | $\begin{aligned} & \log s^{2} \\ & \operatorname{colog}_{\rho} . \\ & \cot ^{2} \zeta_{1} \\ & \log (1-m) \end{aligned}$ <br> $\log$ third | 10.015 3.196 6.662 9.971 |
| log first | $3.338541 n$ |  | 39 |  | 9.844 |
| First Second Third | $\begin{array}{r} -2180.424 \\ +\quad 706.365 \\ +\quad .698 \end{array}$ |  | 2.849029 |  |  |
| $h_{2}-h_{1}$ | -1473.361 |  |  |  |  |
| Example 7 |  |  |  |  |  |
| $\log s$ <br> $\log A$ <br> $\log D_{1}$ <br> $\cot \zeta_{1}$ | $\begin{array}{r} 5.007341 \\ +46 \\ +79 \\ 7.877369 \end{array}$ | $\begin{aligned} & \log (0.5-m) \\ & \log s^{2} \\ & \operatorname{colog} \rho \\ & \log A \\ & \log D_{2} \\ & \log \text { second } \end{aligned}$ | $\begin{array}{r} 9.637852 \\ 10.014682 \\ 3.196310 \\ +46 \\ +39 \end{array}$ | $\log s^{2}$ <br> $\operatorname{colog} \rho$ <br> $\cot ^{2} \zeta_{1}$ <br> $\log (1-m)$ <br> $\log$ third | $\begin{array}{r} 10.015 \\ 3.196 \\ 5.755 \\ 9.971 \end{array}$ |
| log first | 2.884835 |  |  |  | 8.937 |
| First Second Third | $\begin{array}{r} +767.070 \\ +\quad 706.202 \\ +\quad .087 \end{array}$ |  | 2.818929 |  |  |
| $h_{2}-h_{1}$ | +1473.359 |  |  |  |  |

## recapitulation of formulas

(Numbered as in foregoing discussion)

## Case I. Reciprocal observations

Former Coast and Geodetic Survey form,

$$
\begin{equation*}
h_{2}-h_{1}=s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\left[1+\frac{h_{1}+h_{2}}{2 \rho}+\frac{s^{2}}{12 \rho^{2}}\right] . \tag{5}
\end{equation*}
$$

Reference: Page 210 and General Instructions for Field Work Coast and Geodetic Survey, pages 34-37 (edition of 1908).

Logarithmic form,

$$
\begin{equation*}
h_{2}-h_{1}=s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) A B C . \tag{4}
\end{equation*}
$$

Reference: Page 209 and tables.

## Case II. Nonreciprocal observations

Former Coast and Geodetic Survey form,

$$
\begin{equation*}
h_{2}-h_{1}=s \cot \zeta_{1}+\frac{0.5-m}{\rho} s^{2}+\frac{(1-m) s^{2} \cot ^{2} \zeta_{1}}{\rho} \tag{8}
\end{equation*}
$$

Reference: Page 211 and General Instructions for Field Work Coast-and Geodetic Survey, pages 34-37 (edition of 1908).

Corrected form,

$$
\begin{equation*}
h_{2}-h_{1}=A D_{1} s \cot \zeta_{1}+\frac{0.5-m}{\rho} A D_{2} s^{2}+\frac{(1-m) s^{2} \cot ^{2} \zeta_{1}}{\rho} \tag{13}
\end{equation*}
$$

Reference: Page 213 and tables.
"Exact" form,

$$
\begin{equation*}
h_{2}-h_{1}=s A R \frac{\cos \left[\zeta_{1}-\left(\frac{1}{2}-m\right) \frac{s}{\rho \sin 1^{\prime \prime}}\right]}{\sin \left[\zeta_{1}-\left(\frac{1}{2}-m\right) \frac{s}{\rho \sin 1^{\prime \prime}}-\frac{s}{2 \rho \sin 1^{\prime \prime}}\right]} \tag{7}
\end{equation*}
$$

Reference: Page 211 and tables; also Formule and Tables for Position Computation, Coast and Geodetic Survey Special Publication No. 8, for $R$.
See also additional note, page 220.

## notes on construction and dise of tables

The tables are constructed with mean values of $\rho$ and $m$
$\log \rho=6.80444$ corresponding to mean radius of curvature in latitude $40^{\circ}$ for Clarke's spheroid of 1866 .
$\dot{m}=0.06$. $m$ varies between 0.05 and 0.10 in the great majority of cases. This value near the smaller limit was taken as probably nearer the truth for the high lines, in which the correction terms tabulated are most likely to appear, than an intermediate value of 0.07 or 0.08
$A, B, C, D_{1}$ and $D_{2}$ are all very near unity. To compute their logarithms the approximate expression $\log (1+x)=M x$ was used, $M$ being the modulus of common logarithms $=0.43429$.

Formulas for constructing tables:

$$
A=1+\frac{h}{\rho} \quad \log A=\frac{M h}{\rho} \quad h=\frac{\rho}{I I} \log A=146.7 S \log A
$$

$\log A$ being in units of fifth place.

$$
\begin{gathered}
B=1+\frac{s}{2 \rho} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \quad \log B=\frac{M s}{2 \rho} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \\
\log \left[s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\right]=\log \frac{2 \rho}{M 1}+\log (\log B) \\
= \\
=7.4677+\log (\log B) \\
C=1+\frac{s^{2}}{12 \rho^{2}} \quad \log C=\frac{M s^{2}}{12 \rho^{2}} \quad \log s=\log \left(\rho \sqrt{\frac{12}{M}}\right)+\frac{1}{2} \log (\log C) \\
=7.5251+\frac{1}{2} \log (\log C) \\
\begin{aligned}
& D_{1}=1+\frac{6(1-m)^{2}-1}{6} \frac{s^{2}}{\rho^{2}} \quad \log D_{1}=M \frac{6(1-m)^{2}-1}{6} \frac{s^{2}}{\rho^{2}} \\
& \log s=7.0578+\frac{1}{2} \log \left(\log D_{1}\right)
\end{aligned} \\
\begin{aligned}
D_{2}=1+\frac{10-20 m+8 m^{2}}{24} \frac{s^{2}}{\rho^{2}} \quad \log D_{2}=M\left(\frac{5-10 m+4 m^{2}}{12}\right) \frac{s^{2}}{\rho^{2}} \\
\log s=7.2027+\frac{1}{2} \log \left(\log D_{2}\right)
\end{aligned}
\end{gathered}
$$

The values of $\log A, \log B$, etc., were taken successively at $0.5,1.5$, 2.5 , etc., units of the fifth place, namely, at the point where tho value
of $\log A, \log B$, as rounded off to 5 decimals would change by one in the fifth place. The corresponding values of $h, \log \left[s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\right]$ and $\log s$ were then computed by the formulas above. These values are carried out far enough so that the values of $\log A, \log B$, etc., may be obtained by interpolation to six decimals. In the numerical examples here given the values of $\log A, \log B$, etc., were computed independently for the actual values of $\rho$ and $m$. These results as used in the example all agree within a unit of the sixth decimal place with those found by interpolating in the tables.

The unit of length throughout the tables and formulas is the meter.

Tables

| Elevation <br> of occupied <br> station $h_{1}$ | log A units <br> of fifth place | Elevation <br> of occupied <br> station $h_{1}$ | log $A$ units <br> of fifth place |
| :---: | :---: | :---: | :---: |
|  |  | Meters |  |
| 0 | 0.0 | 3009 | 20.5 |
| 73 | 0.5 | 3156 | 21.5 |
| 220 | 1.5 | 3303 | 22.5 |
| 367 | 2.5 | 3449 | 23.5 |
| 514 | 3.5 | 3596 | 24.5 |
| 661 | 4.5 | 3743 | 25.5 |
| 807 | 5.5 | 3890 | 26.5 |
| 954 | 6.5 | 4036 | 27.5 |
| 1101 | 7.5 | 4183 | 28.5 |
| 1248 | 8.5 | 4330 | 29.5 |
| 1394 | 9.5 | 4477 | 30.5 |
| 1541 | 10.5 | 4624 | 31.5 |
| 1688 | 11.5 | 4770 | 32.5 |
| 1835 | 12.5 | 4917 | 33.5 |
| 1982 | 13.5 | 5064 | 34.5 |
| 2128 | 14.5 | 5211 | 35.5 |
| 2275 | 15.5 | 5357 | 36.5 |
| 2422 | 16.5 | 5504 | 37.5 |
| 2569 | 17.5 | 5651 | 38.5 |
| 2715 | 18.5 | 5798 | 39.5 |
| 2862 | 19.5 |  | 5945 |
|  |  |  | 40.5 |

$\log A$ is positive except in the rare case when $h_{1}$ indicates a depression below mean sea level.
$A$ is used for both reciprocal and nonreciprocal observations.

| For reciprocal observations only (unless formula, p. -, is used) |  |  |  | For nonreciprocal observations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\log B$ units of 5th place | $\log s$ | $\log C$ | $\log s$ | $\log D_{1}$ units of 5 th place | $\log s$ | $\log D_{2}$ units of 5 th place |
|  | 0.0 |  | 0.0 |  | 0.0 |  | 0.0 |
| 2. 167 | 0.5 | 4. 875 | 0.5 | 4.407 | 0.5 | 4. 552 | 0.5 |
| 2. 644 | 1.5 | 5.113 | 1.5 | 4.646 | 1.5 | 4. 781 | 1.5 |
| 2.866 | 2.5 | 5.224 | 2.5 | 4.757 | 2.5 | 4.902 | 2.5 |
| 3.011 | 3.5 | 5.297 | 3.5 | 4.830 | 3.5 | 4.975 | 3.5 |
| 3.121 | 4.5 | 5. 352 | 4.5 | 4.8S4 | 4.5 | 5.029 | 4.5 |
| 3.208 | 5.5 | 5.395 | 5.5 | 4.928 | 5.5 | 5.073 | 5.5 |
| 3.281 | 6.5 | 5.432 | 6.5 | 4.964 | 6.5 | 5. 109 | 6.5 |
| 3.343 | 7.5 | 5.463 | 7.5 | 4.995 | 7.5 | 5. 140 | 7.5 |
| 3.397 | 8.5 |  |  | 5.023 | 8.5 | 5. 107 | 8.5 |
| 3.445 | 9.5 | - |  | 5.047 | 9.5 | 5. 192 | 9.5 |
| 3. 489 | 10.5 |  |  | 5.068 | 10.5 | 5. 213 | 10.5 |
| 3.528 | 11.5 |  |  | 5.088 | 11.5 | 5.233 | 11.5 |
| 3. 5¢5 | 12.5 |  |  | 5. 106 | 12.5 | 5.251 | 12.5 |
| 3.598 | 13.5 |  |  | 5.123 | 13.5 |  |  |
| 3.629 | 14.5 |  |  | 5.138 | 14.5 |  |  |
| 3.658 | 15.5 |  |  | 5.153 | 15.5 |  |  |
| 3. 685 | 16.5 |  |  | 5. 167 | 16.5 |  |  |
| 3. 711 | 17.5 |  |  | 5.179 | 17.5 |  |  |
| 3. 735 | 18.5 |  |  | 5.191 | 18.5 |  |  |
| 3.758 | 19.5 |  |  | 5.203 | 19.5 |  |  |
| 3.779 | 20.5 |  |  | 5.214 | 20.5 |  |  |
| 3.800 | 21.5 |  |  | 5.224 | 21.5 |  |  |
| 3.829 | 22.5 |  |  | 5. 234 | 22.5 |  |  |
| 3.839 3.857 | 23.5 |  |  | 5.243 | 23.5 |  |  |
| 3.857 | 24.5 |  |  | 5.252 | 24.5 |  |  |
| 3.874 | 25.5 |  |  | 5.261 | 25.5 |  |  |
|  |  |  |  |  |  |  |  |
| * Or $\log 8 \cot \left[\zeta_{1}-(0.5-m) \frac{8}{\rho \sin 1^{\prime \prime}}\right]$ for nonreciprocal observations. (See note 2, p. 220.) |  |  |  |  |  |  |  |

$\log B$ has the same sign as the approximate difference of elevation. $\log C$ is always positive.
$\log D_{1}$ and $\log D_{2}$ are always positive.

## NOTES ON THE DEVELOPMENTS

Note 1.-The transformation of (1), page 208, may be conducted rather more simply than is there given.

$$
\begin{equation*}
h_{2}-h_{1}=\frac{2\left(\rho+h_{1}\right) \sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sin \frac{\theta}{2}}{\cos \left[\left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)+\frac{\theta}{2}\right]} \tag{1}
\end{equation*}
$$

or

$$
h_{2}-h_{1}=\frac{2\left(\rho+h_{1}\right) \sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sin \frac{\theta}{2}}{\cos \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \cos \frac{\theta}{2}-\sin \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \sin \frac{\theta}{2}}
$$

Divide numerator and denominator by $\cos \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) \cos \frac{\theta}{2}$,

$$
h_{2}-h_{1}=\frac{2\left(\rho+h_{1}\right) \tan \frac{\theta}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}{1-\tan \frac{\theta}{2} \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}
$$

or expanding $\tan \frac{\theta}{2}$ in series and using $\theta=\frac{s}{\rho}$,

$$
\begin{gathered}
h_{2}-h_{1}=\left(1+\frac{h}{\rho}\right) s\left(1+\frac{s^{2}}{12 \rho^{2}}\right) \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)\left[1+\frac{s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right)}{2 \rho}\right] \\
=s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) A B C
\end{gathered}
$$

which is equation (4).
Note 2.-The formula for nonreciprocal observations may be put in the same form as that for reciprocal observations.

From the equation on page 208

$$
\begin{aligned}
\zeta_{2} & =180^{\circ}-\zeta_{1}-2 m \theta+\theta \\
\frac{\zeta_{2}-\zeta_{1}}{2} & =90-\zeta_{1}+(0.5-m) \theta \\
\tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) & =\cot \left[\zeta_{1}-(0.5-m) \theta\right]=\cot \left[\zeta_{1}-(0.5-m) \frac{s}{\rho \sin 1^{\prime \prime}}\right]
\end{aligned}
$$

Substitute in (4)

$$
h_{2}-h_{1}=s \cot \left[\zeta_{1}-(0.5-m) \frac{s}{\rho \sin 1^{\prime \prime}}\right] A B C
$$

for nonreciprocal observations analogous to

$$
h_{2}-h_{1}=s \tan \left(\frac{\zeta_{2}-\zeta_{1}}{2}\right) A B C
$$

for reciprocal observations. $B$ should be taken from table with argument

$$
\log s \cot \left[\zeta_{1}-(0.5-m) \frac{s}{\rho \sin 1^{\prime \prime}}\right] .
$$

This is the present Coast and Geodetic Survey formula for nonreciprocal observations.


[^0]:    * These ralues result from the computation on p. 13.

[^1]:    *For triangles see p. 23.

[^2]:    * These values result from the computation on p. 19.

[^3]:    * For triangles, see p. 23
    $\dagger$ These values result from the computation on p. 21.

[^4]:    * For the method of solution of triangles see United States Coast and Geodetic Survey Special Publication No. 8, p. 6.

[^5]:    * For an explanation of the forms for computing differences of latitude, longitude, and azimuti see Inited States Coast and Geodetic Survey Special Publication No. 8, pp. 6-11.

[^6]:    * The upper sign being used for a right azimuth angle, the lower sign for a left.

[^7]:    * Upper sign for right azimuth angle, lower for left.

[^8]:    * See position computation, p. 40.

[^9]:    * There is some confusion in usage as to the term equation of condition, or condition equation. In this publication the meaning is restricted to that of an equation expressing some condition which is imposed a priori and independently of anything arising from the observations themselves, and which must be rigorously satisfied by the adopted results. An equation which expresses the results of an observation, and which will, in general, be satisfied only approximately by the adopted results, is not herein termed an equation of condition, but an observation equation.

[^10]:    * For more exact formula to be used with longer lines, see Dr. F. R. Helmert's Höhere Geodäsie, vol. 1, pp. 495 and 496. For such lines some of the approximations made in the derivation here given are no longer permissible.

[^11]:    * See Wright and Hayford, Adjustment of Observations, Chap. VII.
    $\dagger$ This does not necessarily hold good when a line whose direction has already been fixed enters into the set.

[^12]:    * This corresponds to the idea followed out in the second method of solution, but in the preliminary computation this is of no consequence, as is shown in the next sentence.

[^13]:    * This zero discrepancy is merely accidental.

[^14]:    *Wright and Hayford, Adjustment of observations, p. 180.

[^15]:    * This vertical net is not of a high degree of accuracy, it being a small spur of secondary triangulation that was executed in some haste with slight attention to vertical observations. It was selected on account of its small size. The more accurate work is usually in larger nets. See list of probable errors ranging from $\pm 0.23 \mathrm{~m}$. to $\pm 1.83 \mathrm{~m}$. in United States Coast and Geodetic Survey Special Publication No. 13.

