PLANE ASTRONOMY.

Part I.

INCLUDING EXPLANATIONS OF CELESTIAL PHENOMENA AND DESCRIPTIONS OF THE PRINCIPAL ASTRONOMICAL INSTRUMENTS.

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PREFACE.

In putting forth the first part of a Treatise on Plane Astronomy, I feel called on to explain why I have entered on a field already extremely well occupied: I have done so because it appears to me that Students have been deterred from this interesting and important study by the very completeness of the works in which it is brought before them. I mean that the easier parts of the subject, such as explanations of ordinary phenomena, and many simple applications of geometry and analysis, have become so mixed up with the very uninviting processes which exclusively concern the professional observer, that the whole subject is often thrown aside as wearisome and repulsive.

I have endeavoured to throw the elementary explanations into a form which may be considered as either complete in itself, or introductory to the higher branches of the subject, which it is my purpose to treat of in another part.

I have devoted a considerable space to the description of Instruments, because I think that in the present state of Astronomical Science, no one can be said to have a complete knowledge of the subject, who is not well acquainted with the means by which observations are made with the required exactness. And indeed, many of the contrivances for this object are well worthy of attention in themselves, from the mechanical skill and ingenuity displayed in them.

In the explanations I have endeavoured to give as clear solutions as I could of the difficulties which naturally present themselves to a Student entering on the subject, and in this endeavour I have neither aimed at nor avoided novelty.

In case my memory may have led me to insert without special notice anything I have derived from others, I beg here to acknowledge all such assistance, and particularly to express my obligations to Sir J. F. W. Herschel, whose admirable work, recently republished, was in its earlier form one of the first books I read on the subject: and also to W. Hopkins, Esq., to whose excellent training in my undergraduate's course I owe whatever knowledge I have of Astronomical Instruments.

In conclusion, I derive much consolation from the reflection, that those who will be most able to criticise my labours are the very persons who can best appreciate and allow for the difficulties of the undertaking—who, knowing how hard it often is to make a single point clear to a beginner, can form some idea of the pains required to explain a whole subject. If, by assisting their labours as well as presenting the subject in a more inviting form to the beginner, I am in any degree able to promote the study of Astronomy, I shall not regret the time and thought which this work has cost me.

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PLANE ASTRONOMY.

1. From the earliest periods, the motions of the heavenly bodies have attracted the attention and exercised the faculties of mankind; and although they appear arbitrary and anomalous to the superficial observer, it seems always to have been the opinon of philosophical and inquiring minds that they are in themselves orderly and regular, and that the laws to which they are obedient may be discovered by proper skill and application.

The efforts employed on this object have resulted in the science of Astronomy.

There are two ways in which the subject may be treated. We may either start from apparent phenomena, and proceed step by step, according to successive discoveries, until we arrive at the true theory of the universe; or, we may begin with assuming the results of previous inquiries, and deduce from them, one after another, the appearances which present themselves to our observation.

The former method has the advantage of historical order, but the latter is so much the more clear and intelligible that we shall not hesitate to adopt it.

2. The earth on which we live was naturally regarded for a long time as stationary, and all the heavenly bodies as in motion about it; and this supposition, as we shall see hereafter, was a source of great difficulty and confusion in explaining celestial phenomena. It has now been satisfactorily

established, not only that the earth is not the fixed centre of the universe, but that it is one of a system of bodies called planets revolving about the sun. We shall first give some general features common to most of these bodies, and then briefly describe each of them in detail.

(*a*). The planetary form is that of an oblate spheroid, which is geometrically defined as the solid generated by the revolution of a semi-ellipse about its minor axis. In general, the planets differ little from spheres.

(*b*). Each planet has a motion of rotation about its least axis, which axis moves nearly parallel to itself in the orbital motion about the sun.

(*c*). The orbit in each case is an ellipse of small eccentricity, having the sun in one focus. The planes of most of the orbits are inclined at small angles to one another.

(*d*). Several of the planets have satellites, which revolve about them in the same way as themselves about the sun.

(*e*). We may also mention that it has been ascertained by observation that the squares of the periodic times, or times of revolution about the sun, are to one another in the same ratio as the cubes of the mean distances, or semi-major axes of the orbits; and also that the radius vector of each planet, that is, the straight line joining the planet and the sun, passes over equal areas in equal times.

From these properties it may be inferred, according to the reasoning of Newton in the 2nd and 3rd Sections of his Principia, that each planet is attracted to the sun by a force varying inversely as the square of the distance from the sun.

The above statements, however, respecting the forms of the orbits, are only approximately true—though the approximation is very close—and consequently the inference we have made is not quite an accurate statement of the physical theory of the universe. It is sufficient here to observe that from the mutual influence of the planets on one another, their forms, and the phenomena presented by their satellites, Sir

Isaac Newton established the existence in all matter of an attractive power, by which every portion, however small, attracts every other portion, with an accelerating force which varies directly as the mass of the attracting body and inversely as the square of its distance from the attracted body.

3. We shall now proceed to describe the *Solar System.*

(1). The Sun is a self-luminous body whose diameter is 880,000 miles, or about 112 times the diameter of the earth. Consequently, its volume is more than a million times that of the earth. It has a motion of rotation about its axis in about 25 days and a half.

Many conjectures have been formed as to the nature and constitution of the Sun. The most probable seems to be that it consists of a solid body surrounded by a luminous atmosphere. The surface, viewed through a telescope, in general appears marked with dark spots, some of them enormously large, which are supposed to be breaks in the atmosphere, through which the opaque body becomes visible.

(2). The nearest planet to the Sun is *Mercury.* It revolves about its axis in 24 hrs. 5 min. Its diameter is about $\frac{2}{5}$ of that of the Earth, and it describes a comparatively eccentric orbit about the Sun in about 88 days. The ratio of the minor and major axes of the orbit is about 4 : 5. The mean distance is nearly $\frac{2}{5}$ of that of the Earth.

(3). The next planet in order is *Venus,* which is rather less than the Earth, and completes its revolution about the Sun in something less than 225 days. It revolves about its axis in 23 hrs. 21 min. The mean distance is $\frac{7}{10}$ of that of the Earth. The orbit is very nearly circular.

(4) Next to Venus is the *Earth,* whose distance from the Sun is 95 millions of miles, its diameter 7935 miles, and its periodic time 365 days and nearly a quarter. The day of 24 hours is the time occupied by the Earth's rotation about its axis.

(5). The first planet without the Earth's orbit is *Mars*, at a mean distance of about $1\frac{1}{2}$, calling that of the Earth 1. Its diameter is about half that of the Earth. It revolves about its axis in 24 hrs. 37 min., and about the Sun in 1 year and 321 days.

(6). After Mars come four small planets, *Vesta, Juno, Ceres*, and *Pallas*, generally called *Asteroids*. Their diameters are very small. Their mean distances are between 2 and 3 times that of the Earth, and their periodic times between 4 and $5\frac{1}{2}$ years. It has been supposed by some astronomers that they are fragments of some planet destroyed by an internal convulsion.

(7). At nearly twice the distance of the Asteroids from the Sun, or about 5 times that of the Earth, revolves the planet *Jupiter*, the largest in the Solar system, in a period of 11 years and 315 days. Its diameter is 11 times that of the Earth, and its rotation about its axis is performed in the short period of 9 hrs. 55 min. It is accompanied by four satellites.

(8). The next planet, *Saturn*, is also of large dimensions, having a diameter more than nine times that of the Earth. It revolves about its axis in 10 hrs. 29 min., and about the Sun in 29 years and 174 days. It has seven satellites, and is besides encircled by a flat double ring, nearly concentric with the planet, and revolving about its axis with great velocity.

(9). Saturn was long thought the most distant planet of the system, until, in the year 1801, Sir William Herschel discovered *Uranus*, which he named *Georgium Sidus* in compliment to George III. After this it was called *Herschel*, in honour of the discoverer, but has now nearly everywhere received the name of *Uranus*. It has six satellites, and revolves about the sun in 84 years and 27 days. Its diameter is between 4 and 5 times that of the Earth. Its mean distance is about twice that of Saturn, and 19 times that of the Earth.

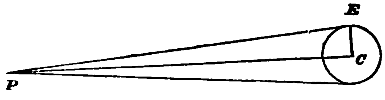
(10). Certain inequalities in the motion of Uranus having long perplexed astronomers, the idea occurred separately to Mr. Adams of St. John's College, and M. Le Verrier in France, that the anomaly was to be accounted for by the attraction of another planet. The search which was on this account set on foot resulted in the discovery of *Neptune*, the most remote planet yet known which obeys the law of gravitation to the Sun.

Besides the planets, the Solar system may be said to include *Comets*, bodies of small density, though sometimes of considerable dimensions, which mostly describe orbits of great eccentricity; periodically approaching very near the Sun, and then receding into space beyond the limits of our observation. They are generally surrounded by a luminous atmosphere, which in some cases extends to a great distance from them in the direction opposite to the Sun, forming a luminous train of very striking and beautiful appearance.

4. The Solar system is surrounded on every side by the Fixed Stars, which appear to pervade all space, and which are mostly at distances too great for our powers of measurement.

In order to give some idea of the space which separates us from them, we may observe that the distances of the nearer heavenly bodies are computed by means of the angles subtended at each of them by the Earth's diameter. This angle being determined in any particular case by observations which we shall afterwards refer to, and the radius of the Earth being known, we have given two parts CE and $\angle CPE$ (half the observed angle) of a right-angled triangle PCE, and may thence find the side PC which gives the distance of the proposed body P from the Earth's centre.

Now, in the case of the fixed stars, not only is this angle inappreciably small, but that subtended by the diameter of the Earth's orbit is imperceptible also, so that two lines



drawn towards a fixed star, from two points, 190 millions of miles distant from one another, do not sensibly converge, but are so nearly parallel, that the angle which one makes with the other is too small to be determined by our instruments. Attempts have been made to calculate the distance of one of these stars, from observations which appear to have determined this angle at a fraction of a second; but the numerical symbols which represent the distance in miles, utterly fail to convey any distinct notion to our minds, but only suggest a vague notion of indefinite magnitude.

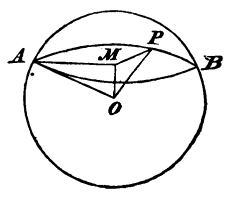
The fixed stars are supposed to be self-luminous, and may with probability be conjectured to be the centres of systems similar to our own, the inferior bodies of which are invisible from their distance. Late discoveries have shewn that several stars which appear single points of light to the naked eye, are really systems of two bodies which revolve about one another. Thus the law of gravitation may be fairly extended to the whole visible universe.

5. Having given a brief account of the different bodies with which the regions of space are peopled, we proceed to explain the appearances which they present to the observer on the Earth's surface,—a process which is purely geometrical, but involving only the simplest investigations of geometry, requires little more than careful attention to follow it completely.

It will be necessary to premise a few propositions relating to the geometry of the *sphere*.

DEF. A sphere is a surface every point of which is equally distant from a point within it, which is called the centre.

(1). Every section of a sphere made by a plane is a circle.



Let APB be the curve in which the sphere is cut by the given plane.

From O the centre of the sphere, draw OM perpendicular to the plane and meeting it in M. Join MA, MP, A, P, being any two points in the curve.

Then, $OP = OA$, by property of sphere.

$\angle OMA = \angle OMP$, each being a right angle,

OM is common to the triangles OMP, OMA,

$\therefore\ MP = MA$,

and in the same way the distances of any two points in the curve from M may be proved to be equal.

Therefore, the curve is a circle, and M its centre.

Def. When the cutting plane passes through the centre of the sphere, the section is called a *great* circle. In all other cases it is called a small circle.

(2). Any two great circles of a sphere bisect each other.

Since the cutting plane in each case passes through the centre, the line of intersection of any two such planes must be a diameter of the sphere, and consequently a diameter of each of the circles, which proves the proposition.

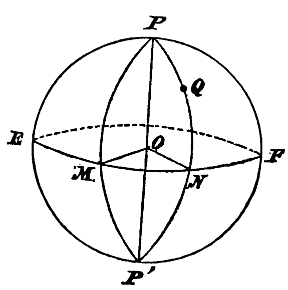
(3). The diameter of the sphere which is perpendicular to the plane of any great circle, cuts the sphere in two points which are called the poles of that circle.

(4). The planes of all great circles passing through the poles will evidently be perpendicular to the plane of the proposed great circle.

Let PMP' be such a circle, cutting the proposed circle in M.

Join PP' passing through O the centre of the sphere.

The angle POM is evidently a right angle, therefore PM, MP', are each quadrants, that is, the proposed great circle bisects the part of any other circle intercepted between its poles.



(5). The angle between any two arcs of great circles which intersect, is the angle between the planes of these two circles at the centre of the sphere.

Hence the angle *PME* is a right angle, and it follows that all great circles passing through the poles of a proposed great circle cut it at right angles.

(6). The angle between any two great circles passing through *P*, as *PMP'*, *PNP'*, is measured by the arc *MN*, on the great circle of which *P*, *P'* are poles.

For *MN* measures the angle *MON* of inclination of the planes of the two great circles, and therefore measures the spherical angle *MPN*.

(7). The position of a point on a sphere is determined by its distance from a fixed point as *P*, measured along a great circle, and by the angle made by that great circle with a fixed great circle.

Thus, if *PMP'* be a fixed great circle, any point *Q* is determined by the distance *PQ* and the angle *MPN*. It is evident we may equally determine it by *NQ* the complement of *PQ*, and the arc *MN*. In terrestrial measurements, the former of these is the latitude, and the latter the longitude. They may be called the spherical coordinates of the point *Q*.

The great circle *PQP'* is called the *meridian* of the place *Q*, and all great circles passing through both poles are called *terrestrial meridians.*

Figure of the Earth.

6. The exact figure of the Earth is that of an oblate spheroid, or the solid generated by the revolution of an ellipse about its minor axis. An orange is an instance of this kind of solid; but to say that the Earth is like an orange, gives an incorrect idea of its form, for it is so nearly spherical, that a perfect representation of it of the dimensions of an orange could not be distinguished by inspection from a perfect sphere. The difference between its major and its minor axis is less than 14 miles, the former being 7935 miles. This is about $\frac{1}{300}$ of the whole diameter; so that, if a ten-inch globe were constructed of the exact form of the Earth, its two axes

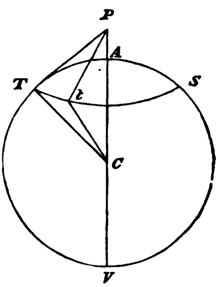
would only differ from one another by $\frac{1}{30}$ of an inch, a quantity too small to be appreciated by the eye.

The irregularities of the Earth's surface prevent us in general from forming any idea of the general form of the whole; but, at sea, we have very distinct evidences of its curved nature. If the surface were one extended plane, as we are apt to suppose at first, an object receding from us would be visible to any distance, as far as the eye could distinguish it, and would become lost to us only in consequence of its remoteness. Nothing would intervene to hide it from us. But this is not the case in observations made at sea. A ship, after receding to a certain distance, appears to dip below the horizon, and gradually to disappear—first the hull, and then the rest by degrees, till the tops of the masts are hidden. This disappearance is not the effect of remoteness, for the most powerful telescope will not restore the vessel to our sight: nor is it the effect of the sudden interposition of an obstacle, for it is slow and gradual. The only explanation we can give is that the convexity of the Earth intervenes between us and the vessel, or, in other words, that the vessel disappears behind the convexity of the Earth. Hence the surface is shewn to be—not *plane*, but *curved*.

Again, the distance at which a receding object disappears behind the convexity is the same in whatever direction it is moving, and whatever may be the position of the observer, so long as his height above the water is the same. A well-defined line, called the *offing*, appears to bound the view on all sides, and every point of this line is found by observation to be equally distant from the observer. Hence it is a *circle*, and its magnitude depends only on the height of the observer, and not on his position on the Earth's surface. It is also observed that a line drawn from the eye to the offing is not exactly at right angles to a vertical line drawn through the eye, but that the angle so formed is slightly obtuse, the obtuseness being greater as the eye is higher above the sea. The angle however is the same in all directions, so long as the observer's height is the same.

Now, if from a point without a sphere we draw tangents to its surface in all directions, the points of contact will all lie on a circle, the magnitude of which will depend on the distance of the point from the surface.

Let P be the point, PT, Pt two tangents, C the centre of the sphere.



Join CT, Ct.

Then, CTP, CtP are right angles, and $CT = Ct$, CP is common,

$$\therefore \; PT = Pt.$$

Therefore the line TtS of points of contact is a circle.

Hence, the appearances described agree with the supposition of the Earth's being a sphere; and, indeed, no other hypothesis would account for them, since a sphere only would give a circular offing of the same dimensions at every part of the surface. These observations are not refined enough to detect the slight variation of the Earth from the spherical form. The determination of the exact figure and the amount of the eccentricity has tasked the ability and energy of the most eminent scientific men.

The line CP in the figure is the vertical direction, or that in which the force of gravity acts. On the supposition of the spherical form, it passes through C, the centre.

The angle CPT is the supplement of the angle formed at the eye between the vertical and the direction of the offing.

If PC be produced to V, cutting the Earth's surface in A, we have $PA.PV = PT^2$.

In general PA is very small compared with AV, therefore we may write AV for PV without sensible error.

Hence
$$PT^2 = AV.PA \text{ nearly,}$$

or the diameter of the circle within which an object may be seen, varies as the square root of the observer's height.

And since the areas of circles are as the squares of their diameters, the actual space commanded varies directly as the height.

It is the result of observation that two objects each 10 feet high just cease to be visible to each other when they are 8 miles apart.

Hence an object 10 feet high will command a circle of 4 miles' radius. We have therefore

$$AV.\tfrac{10}{5280} = 4^2, \quad \text{or} \quad AV = 16 \times 528. = 8448,$$

which is greater than the real diameter of the Earth, because the atmospheric refraction, by elevating distant objects, allows us to see farther than we should be able to see without it.

Motion of the Earth.

The centre of the Earth describes about the Sun an orbit which is of the form of an ellipse, the Sun occupying one focus. The eccentricity is, however, small—indeed the orbit, if accurately represented on a common sheet of paper, would not be distinguishable from a circle.

The minor-axis of the Earth is inclined to the plane of the orbit at an angle of about 66° 32′, and continues parallel to itself throughout the whole rotation—that is to say, if we take any two positions of the axis in any parts of the orbit, they are parallel to one another.

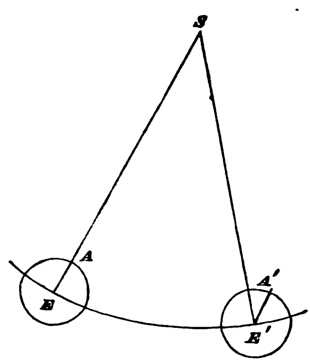
This motion of the centre and axis is not the only motion of the Earth. If it were so, each part of the surface would be once presented to the Sun during each revolution in the orbit, or there would be only one day and one night in the year. The effect would be just the same as if the Earth remained absolutely at rest, and the Sun moved round it in a similar orbit to that in which the Earth actually moves.

But, in fact, if we were to destroy the motion of the centre and the axis, there would still remain a rotation about the axis, none of the parts being permanently moved in space, but all periodically returning to their original positions. This rotation being referred to the axis, may be estimated as if that axis were at rest. The time of one rotation will therefore be determined by the return of the surface to the same position with respect to the axis from which it started. As the axis remains parallel to itself, the rotation will be completed when any radius of the spheroid comes into a position parallel to that in which it was at first.

This, however, is not the exact length of a day and night, or 24 hours, but falls short of that period by about 3 minutes and 56 seconds.

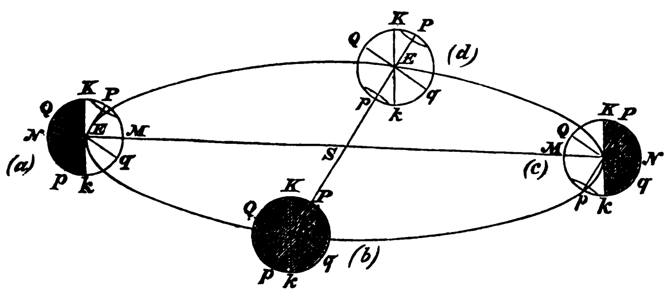
The length of the solar day is the interval from noon to noon at any given place. This is longer than the time of rotation of the Earth upon its axis, on account of the motion round the Sun.

To shew this, let *S* be the Sun, *EE'* part of the Earth's orbit described by the Earth's centre during one complete rotation on the axis. Let the line *SE* cut the Earth's surface in *A*. When the Earth has arrived at *E'*, the line *E'A'*, parallel to *EA* will be, by what has preceded the position of the radius, *EA*. If the Earth had remained at *E*, this radius would have returned to its original position passing through *S*; but as the Earth has moved to *E'*, its direction no longer passes through the Sun, and therefore the solar day is not completed. As in fact the Sun has been left behind, somewhat more than a complete revolution must be performed in order that the place *A* may be again turned towards the Sun,—that is, the interval from noon to noon is greater than the exact time of revolution about the axis. The time of actual rotation is called, for reasons which will be afterwards explained, the *Sidereal* day.



Annual Motion.

7. The Earth's annual orbit is so nearly circular that it may be supposed accurately so in the explanations which follow, the slight modifications which depend on the eccentricity being afterwards separately explained. In the same way the axis may be supposed to continue exactly parallel to itself, although in fact it has slight deviations from this position, which are the causes of minute irregularities, only appreciable by refined instruments, and in no way affecting the general phenomena which we have here to account for.



Let *E* be the Earth's centre, *Pp* its axis, *S* the Sun, *Kk* perpendicular to the plane of the orbit.

Then the angle $PEK = 23° 28'$, which we shall call the 'obliquity', and distinguish by the symbol ω.

Join *SE* cutting the surface in *M* and *N*, and let the axis *Pp* be in the same plain with *SE*, *Kk*, as in (*a*); therefore *PES* is the complement of ω.

It is evident from the position of *S*, that the whole hemisphere of which *M* is the middle point is enlightened by the Sun, and that the hemisphere of which *N* is the middle point is unenlightened. The great circle separating these two hemispheres, and passing through *Kk*, may be called the boundary of light and darkness. The angular distance of *P* from the nearest point of that circle is *PK*, and similarly that of *p* is *pk*.

In the time of each rotation about the axis, the Earth only describes $\frac{1}{365}$ of its annual orbit. Consequently, during one such rotation the position of the axis with respect to the Sun will not sensibly vary.

By the diurnal rotation every point in the surface is carried round in a circle, of which P and p are the poles, all of them being small circles, except that described by a point in the equator.

The points P, p remain stationary, and it is evident from the figure that the former is in the enlightened hemisphere, and the latter in the unenlightened hemisphere. There is therefore perpetual day at P, and perpetual night at p.

Every point in the Earth's equator describes the circle QEq, which is bisected by the great circle KEk, and consequently half of it is in the enlightened, and half in the unenlightened hemisphere. Thus every place in the equator has its days and nights equal.

If we take a point at a distance from P of exactly 23° 28′, that is, a place whose north latitude is 66° 32′, it will evidently describe a circle, which just touches the great circle KEk at the point K. Consequently such a place will have perpetual daylight, as well as all places nearer than it to P. All places between it and the equator will be partly in the unenlightened and partly in the enlightened hemisphere during their daily revolution, but will be longer in the latter than in the former, so that they will have longer days than nights.

In the same way, a place 23° 28′ from p, or in 66° 32′ south latitude, will have no daylight at all; and all places between it and p will be in perpetual night, while all places between it and the equator will have alternations of day and night, but the latter greater than the former.

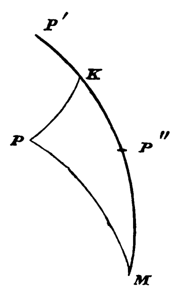
This is the position of the Earth at the time of the summer solstice about June 20.

In the opposite position of the orbit as at (c), the axis retaining its parallelism, the same explanations hold, if we put north for south, and south for north in all cases. Each hemisphere is enlightened precisely in the same way as the other was in the former position; the point *p* with a space 23° 28′ in extent all round it is in perpetual sunshine, and *P* with a space of equal dimensions is in perpetual darkness.

This is the state of things at the winter solstice.

8. In position (*a*) we observe that the angle *SEP* is less than a right angle by 23° 28′. In position (*c*) it exceeds a right angle by the same quantity. These are its least and greatest values respectively. For the angle *SEK* is a right angle, and if *M* be the point in which *SE* cuts the Earth's surface, we have a spherical triangle *PKM*, of which the side *PM* measures the angle *SEP*, the side *MK* is always a quadrant, and the side *PK* equal to 23° 28′.

The angle *PKM* is that which the plane *SEK* makes with the plane *KPE*, which, as the latter plane is always parallel to itself, increases uniformly with the angle described by the line *SE*.



Hence, in the course of a revolution, the angle *PKM* goes through all values from 0° to 360°, and *PM* undergoes corresponding changes.

It is clear that the greatest value of *PM* is *P′M*, and its least value *P″M*, *P′* and *P″* being in the same line with *M* and *K*, *i.e.* the axis *Pp* in the same plane with *SE* and *EK*.

The angle *SEP* is very important, for it measures the angular distance of *P* at any time from the point *M* which is the centre of the enlightened hemisphere. Thus the complement of *SEP* measures the angular distance of *P* from the nearest point of the boundary of light and darkness, and therefore the angular breadth of the space about *P* which, in that position of the Earth, shares with *P* in perpetual day or perpetual night.

9. If we take the position at (*b*) 90° from (*a*), we find the angle *SEP* is a right angle, the plane *PEK* being now perpendicular to *SE*.

Hence P and p are both in the boundary of light and darkness, and every place on the whole surface describes a circle which is bisected by that boundary. Hence days and nights are equal in every latitude. This is the state of things at the *autumnal equinox.*

Nearly the same explanations apply to fig. (d), which represents the position of the Earth at the *vernal equinox.*

Hence we see that the days and nights are equal all over the globe in two opposite positions of the Earth in its orbit; that for half a year the north pole is in perpetual light, and the south pole in perpetual darkness, the days being longer than the nights in the northern hemisphere, and shorter in the southern hemisphere; and that during the remaining half-year these phenomena are reversed, the northern hemisphere being put in the place of the southern, and the southern in the place of the northern.

At the equator there is no variation in the length of the days, which are always eqnal to the nights, because being a *great circle,* it must always be bisected by the great circle which has been called the boundary of light and darkness. The irregularity of the length of day increases as we proceed towards the poles, for the parallel small circles are divided into parts more and more unequal as we recede from the equator on both sides, until we reach the points K or k, after which the circles are not divided at all, but are either wholly within or wholly without the enlightened hemisphere.

The circle which passes through north latitude 66° 32′ is called the arctic circle, and the corresponding southern parallel is called the antarctic circle.

It should be remembered that the points K, M, are not fixed points on the Earth's surface, but that a succession of points continues to occupy those positions from the diurnal rotation. If we suppose a common globe to be fitted with a hemispherical cap which allows it to turn about its axis, the highest point of the cap will be the point K, and the middle point of the uncovered hemisphere will be M. Such globes are often constructed, sometimes with a wire attached to the cap which terminates in a point in the position of M.

10. It is easily seen, that when any point on the Earth's surface is brought by the diurnal circle to the eastern edge of the boundary of light and darkness, the Sun appears to rise to that point, and when it arrives at the western edge, the Sun appears to set. It also appears that the great circle *PM*, which bisects the enlightened hemisphere, will bisect the diurnal path of each such point; or, at midday, the point will be somewhere in *PM*, whatever be its distance from the equator. Thus all points on the same terrestrial meridian have their midday or noon at the same absolute instant of time. The point *M* being that in which the line joining the centres of the Earth and Sun cuts the Earth's surface, will be, as has been said before, the pole of the great circle bounding light and darkness, and the central point of the enlightened hemisphere. At this point the Sun's rays will fall *vertically*, that is, in a line passing through the Earth's centre.

Now the position of *M* in fig. (*a*) is given by the value of *PEM*, for the latitude of *M* is the complement of that angle, *i.e.* 23° 28′ north. At the summer solstice, then, a series of points 23° 28′ distant from the equator on its north side, pass under a vertical Sun during one revolution, and to no other part is the Sun vertical at that time.

In fig. (*c*) the angular distance of *M* from the equator is the same as before, but on the south side; consequently the Sun is vertical to places in 23° 28′ south latitude. At greater distances from the equator the Sun can never be vertical, because the angle *PEM* has its greatest value in fig. (*c*), and its least in fig. (*a*).

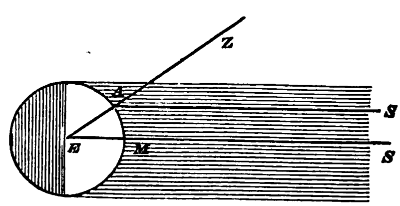
In figs. (*b*) and (*d*), the point *M* will be in the terrestrial equator, since the poles both lie in the great circle bounding light and darkness. Therefore at the equinoxes the Sun is vertical to the equator. The point *M* then recedes from the equator, first towards the north, until it reaches its greatest distance, when it returns, and after crossing the equator proceeds to its greatest distance on the south, when it again returns. Hence, the Sun is vertical twice every year to all places within 23° 28′ of the equator on either side, except at the extreme points, where it is only vertical at the times of the solstices.

The parallels of latitude passing through these extreme points are called *tropics*, from the sun appearing to turn back again after reaching them. The northern tropic is called the tropic of Cancer, and the southern tropic the tropic of Capricorn. The space included between them is called the torrid zone. The spaces between the tropic of Cancer and the arctic circle, and between the tropic of Capricorn and the antarctic circle, are called respectively the Northern and Southern *temperate* zones. The polar regions bounded by the arctic and antarctic circles are called the *frigid* zones.

11. There are two circumstances by which climate is regulated, so far as it depends on astronomical phenomena. One is the length of the day, the other the *inclination of the Sun's rays.*

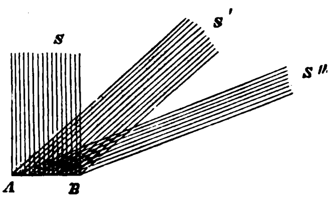
Let us take the case of the summer solstice. The length of the day increases from the equator, where it is 12 hours, to the arctic circle where it is 24 hours; and, the farther a place is from the equator, the longer is its day at that time. The days in the southern hemisphere at the same time decrease from the equator the antarctic circle, beyond which there is no day at all.

If then we take the length of day alone, the climate ought to increase in warmth from the antarctic to the arctic circles, and should be warmest in the north polar regions. But we observe at the same time that the part where the Sun's rays fall most directly is the tropic of Cancer, while everywhere else they fall more obliquely, according to the angular distance from the point *M*.



To shew this, let *A* be any point on the Earth's surface, *E* the centre, *S* the Sun. Join *EA*, and produce it to *Z*, and draw *AS* parallel to *ES*. *AS* will be very approximately the direction of the solar rays at *A*, which, owing to the great distance of the Sun, fall nearly parallel on the Earth. The surface at *A* is perpendicular to *AZ*; and therefore the rays fall obliquely upon it, the obliquity being measured by the angle *SAZ*, or *MEA*, the angular distance of *A* from *M*.

Let *AB* be a small plane: it is evident that if parallel pencils of rays fall on it from different points, the space *AB* will receive a larger pencil if it comes *directly*, as from *s*, than if it comes *obliquely*, as from *s'*, *s''*. Therefore, more heat will be communicated according as the rays are more direct. From this cause, therefore, the heat ought to be greatest at *M*, and least at the boundary of light and darkness.



When these two causes are combined therefore, we find they modify one another. Since the one cause tends to increase and the other to diminish the heat in the northern hemisphere, the variation of climate from the equator to the north pole is at this time of year less than it would be if either cause acted alone; whereas, in the southern hemisphere the two causes strengthen one another and increase the general effect. Accordingly it is found that short periods of considerable heat are experienced within the frigid zones, sometimes greater than in more southern latitudes, because the Sun, although not shining so directly, continues to shine night and day without interruption. Within the tropics, however, the verticality of the Sun at certain periods, and the directness of its rays at all seasons, compensates the effect of shorter days, and perhaps the greatest heat is found in the equatorial regions. It is probable that there is a point of minimum summer heat in some high latitude of the temperate zone, to the north of which the greater length of day more than compensates for the diminished altitude of the Sun.

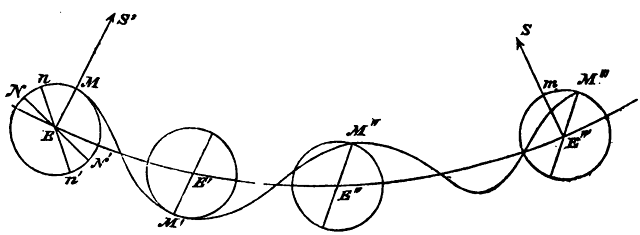
These considerations, however, only apply to one position of the Earth. To determine the general climate of the year, we must take into account the other positions also. At the winter solstice, we find the two causes strengthening one another in the decrease of heat towards the north pole, so that the winter temperature uniformly and rapidly decreases as we leave the equator. At the equinoxes, the days and nights are equal, and the Sun is vertical over the equator; therefore at these epochs the greatest heat falls on the equatorial regions, and the effect is less and less as we go towards the poles. At intermediate times the effects are intermediate between those at the four epochs which we have examined, changing gradually as the Earth goes through each quadrant of its orbit. In general, as far as purely astronomical causes are concerned, the climate is warmest in the torrid zones, and becomes colder from thence to the poles. These considerations are much modified by local circumstances, so that we cannot apply any such uniform rule to the actual estimation of climate. Islands of small size have much less variations of temperature than large continents. The elevation of a place above the sea is an important element in its climate. In the Andes, for instance, may be seen, on the line itself, every variety of climate in the world, from the scorching plain on the sea level, to the polar snows of the mountain summit. Many places in precisely the same latitude are totally different in respect of climate. The winters in parts of America more southerly than any part of Britain are more rigorous than those of the Shetland Islands, and the Canadian summers are very much hotter than our own.

Our rule will however hold good, when we take large intervals. It is colder in London than at Naples; and Scotland, though very far removed from tropical heat, is more genial than Lapland with its days of a month long.

12. Next to the changes of day and night and summer and winter, the most important astronomical phenomena are

presented by the Moon, which revolves about the Earth as the Earth about the Sun. The real orbit of the Moon is an irregular curve, the nature of which will be afterwards described: but we gain a near approximation to its actual motions by supposing the orbit to be a plane circle, having the Earth in its centre, and carried round with the Earth in its annual orbit. The effect may be illustrated by supposing a wheel to move with its centre in the circumference of a much larger wheel, a point at the same time moving uniformly along the circumference of the smaller wheel.

Suppose E to be the Earth's centre, M the Moon. When



the Earth has reached E' in its orbit, let M' be the corresponding position of the Moon. Let E'', M'', E''', M''', be other corresponding positions of the two bodies.

If EM', $E''M''$, $E'''M'''$, be, as in the figure, all parallel to EM, the time from E to E' will be that of half a revolution of the Moon in its orbit, the time from E to E'' will be that of a whole revolution, and from E to E''' that of two whole revolutions. For, whenever the radius EM comes into a position parallel to that from which it started, a revolution in the orbit is completed.

The actual path, it is evident, is the curve $MM'M''M'''$.

The radius EM is only $\frac{1}{400}$ of the radius of the Earth's orbit; consequently the Moon's real path in space coincides much more nearly with the Earth's orbit than would appear by the figure, in which the ratio of EM to SE is much too great.

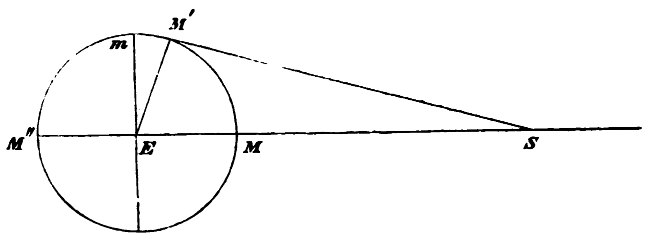
The Moon's orbit, such as we have described, does not lie exactly in the plane of the Earth's orbit, but is inclined to it at an angle of about 5° 35′. The two orbits being planes, and both passing through the Earth, have consequently a straight line of intersection, which also passes through the Earth. At two opposite points, therefore, of the orbit, the Moon crosses the plane of the Earth's orbit. These points are called *nodes,* and the line joining them, the line of nodes. The supposition made respecting the orbit is only approximate. But a slight modification of it will serve to explain the phenomena with accuracy. Instead of supposing the plane of the Moon's orbit to be fixed, or always parallel to a fixed plane, we must suppose that it is continually changing its position by slow degrees. If the plane did not alter its position, but remained always parallel to some fixed plane, the line of nodes would always be parallel to a fixed line. But in fact, as the lunar orbit is carried with the Earth about the Sun, this line is carried not quite parallel to itself, but having a small angular motion in a direction opposite to that of the Moon's motion. This angular motion is irregular, but in the course of a revolution is on an average about $\frac{1}{19}$ of the circumference; so that if NEN' be the line of nodes when the Earth is at E, it will have the position nEn' when the Earth returns to E after a year, the angle NEn being about $\frac{1}{19}$ of 360°. At the end of 18 years 225 days, the line of nodes returns to its original position.

It must be borne in mind that the revolving orbit and transferable line of nodes are only geometrical artifices to simplify the explanations. The nodes of the Moon's orbit are those points at which the Moon actually crosses the plane of the Earth's orbit, which it does twice in every revolution about the Earth. These transits are observed to take place earlier every month than in the preceding month, and this circumstance is represented by the retrogradation of the geometrical line of nodes.

The interval of time occupied by the Moon in passing from M to M'', is called its *Sidereal period;* but as during that period the Earth has moved from E to E'', the Moon has not returned to the same position with respect to the Sun. At M it is said to be in conjunction with the Sun; not that it is necessarily between the Earth and Sun, because it is not generally in the plane of the Earth's orbit, but the angle MES then has its minimum value, or the Moon makes its nearest approach to the Sun as seen from the Earth. At M'' this is evidently not the case, but the Moon has to describe a farther arc in order to return to conjunction with the Sun. The time from conjunction to conjunction is called the *Synodic* period of the Moon, and is what is commonly signified by a lunar month. If the point M had been coincident with one of the nodes, the Moon would have then been in the plane of the Earth's orbit, and therefore directly between the Earth and the Sun. If it had been near one of the nodes, a part of the Earth's surface would have been deprived of the Sun's light; but in general the inclination of the orbit is sufficient to carry the Moon entirely clear of the Sun to all parts of the Earth.

13. The phases of the Moon are too well known to require description. It is not difficult to see that the more nearly the Moon is between us and the Sun, the less we see of the enlightened hemisphere, which is turned the other way; and that the only case in which we can see the whole of that hemisphere is when the Earth is exactly between the Sun and the Moon, supposing the solar light not to be intercepted by that of the Earth. In general the obliquity of the Moon's orbit prevents the three bodies being exactly in a line, when the Moon reaches the part of its orbit opposite to the Sun; but they are so nearly in a line that almost the whole enlightened hemisphere is seen, and that in general without any light being intercepted by the Earth. If, however, the opposition occurs near one of the nodes, the shadow of the Earth is more or less thrown on the Moon, and it suffers an eclipse.

At the time of conjunction the darkened hemisphere is entirely turned to us. Then, as the Moon gets out of the line of the Sun, we see gradually more of the enlightened part, first as a mere strip of light, then in the form of a crescent, which gradually increases to a semicircle. It is then said to be in its first quarter.



When the Moon has reached a point *M'* (fig. 9) such that the angle *SM'E* is a right angle, the face which is presented to us is exactly half enlightened and half dark, for the line *EM'* cuts the Moon's surface exactly in the boundary line of light and darkness. That circle consequently being turned edgeways to us has the appearance of a straight line, and the Moon is technically said to be *dichotomized.*

It is evident that this phenomenon occurs somewhat before the Moon has described a quadrant from *M*, because the angle at *E* is necessarily less than a right angle. The difference between it and a right angle is the angle *ESM'*, which depends on the ratio of *EM'* to *ES*, or the Moon's distance to the Sun's. If we could accurately determine this angle, we should have an excellent means of determining the Sun's distance, for that of the Moon is known with great accuracy.

The only necessary observation would be the exact time at which the Moon appeared dichotomized. Then, the time of New Moon being known, the time of describing the angle *MEM'* would be known; and, supposing the Moon's motion uniform, we should have the proportion, as the whole synodic period to the time of describing *MEM'*, so is the whole circumference of 360° to the angle *MEM'*, the cosine of which is $\frac{EM'}{SE}$, or the ratio of the distance of the Moon to that of the Sun. The practical objection to this method is the difficulty of observing the exact time of dichotomization, which is so great as to make the method entirely useless.

As the Moon proceeds towards opposition, the enlightened hemisphere continues to be more turned towards the Earth until the whole of it is visible, after which the same phenomena take place in an inverse order, until at conjunction the Moon again disappears.

From the point of conjunction to the first quarter, the greater part of the enlightened hemisphere is turned away from us, therefore the Moon appears in the form of a crescent, the points of which are turned away from the Sun. Between the first quarter and the full Moon, the greater part of the enlightened hemisphere is visible, therefore the appearance is that of a bright circle, from which a crescent has been cut off, the circular edge of the bright part being always turned towards the Sun. The technical term for this appearance is *gibbous*.

The reason of these appearances is, that the boundary circle of the enlightened part is seen by us in different views, according to the position of the Moon with respect to the Sun, and appears to us as an ellipse of different degrees of eccentricity, according to its *position*, degenerating into a straight line at the first quarter, and becoming a circle at conjunction and opposition. A familiar illustration of this elliptic appearance of a great circle when seen from different points of view is obtained by looking at the meridians on a globe, which resemble ellipses having the same major axis but of different eccentricities. If we fix on one of these meridians, and bring it into all positions by turning the globe round, we shall get an exact representation of the boundary of the enlightened part of the Moon on the side away from the Sun, the edge next to the Sun being always circular.

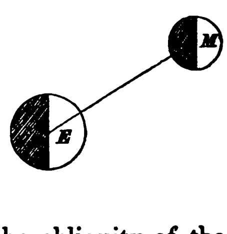
14. It is remarkable that the Moon revolves about an axis perpendicular to the plane of its orbit, in exactly the same time that it revolves about the Earth; so that nearly the same face is always presented to us.

If both the orbital and the rotatory motions were perfectly uniform, and the axis always remained strictly parallel to itself, we could never see more than half the lunar surface:

but owing to some slight irregularities in these respects, some of the parts which are not generally visible occasionally come into view, sometimes on one side, sometimes on the other. By far the greater part, however, of the opposite hemisphere is never visible to us. The slight changes of position by which the Moon appears to oscillate slowly from side to side are called *librations*.

It will tend very much to give clear ideas on the subject of astronomical phenomena, if we consider what would be the appearance of the Earth to a spectator situated in the Moon.

From the remarkable coincidence of motions above described, an inhabitant of this side of the Moon would see the Earth nearly constantly in the same part of the heavens, only appearing to oscillate slowly through very small spaces on account of the librations, while an inhabitant of the opposite side would never see the Earth at all. The apparent diameter of the Earth would be nearly four times that of the Sun, and its surface would present similar phases to those we observe in the Moon, being in conjunction at our full Moon, and in opposition at our new Moon. At intermediate times, the darkened part of the Earth, as seen from the Moon, would be the same portion of the surface as the enlightened part of the Moon as seen from the Earth, and *vice versâ*. This is evident from the figure in which the line *ME* joining the centres, cuts the enlightened hemisphere of the Earth in the same way as the darkened hemisphere of the Moon.



This relation is sometimes expressed by saying that the phases of the Earth and Moon are *complementary* to each other.

15. It has been said that, in general, the obliquity of the lunar orbit is sufficient to prevent the Moon from intercepting the Sun's rays at conjunction, or from being obscured by the Earth's shadow at opposition. When however the Moon, at conjunction or opposition, is near one of the nodes of her orbit, the three bodies are sufficiently nearly in a line for

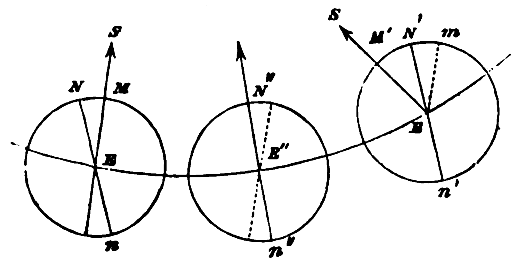
these phenomena to take place; in which cases eclipses of the Sun and Moon are visible to us.

At conjunction, the Moon is in that part of her orbit which at the time lies exactly between the Earth and Sun: and this part is not the same in any two successive conjunctions, since, from the annual motion of the Earth, the Moon has had to describe more than a whole revolution from one conjunction to another. If, for instance, a conjunction had occurred exactly at one of the nodes, the Moon at the time of the next conjunction would have passed the node by a considerable angle.

If the line of nodes were carried exactly parallel to itself, which is approximately the case, it would pass through the Sun twice every year; and if a conjunction or opposition occurred just at the time, there would be an eclipse of the Sun or the Moon. The times at which the line of nodes would have these positions, would be exactly at six months' intervals, and would occur at the same times every year. As it is, however, the line of nodes *retrogrades*, and therefore passes through the Sun earlier than it otherwise would. Hence, in general the intervals are less than six months.

It is found by observation, that when a conjunction occurs within a certain distance of the node, there is always an eclipse of the Sun at some part of the Earth's surface, and that this distance is greater than half that which is intercepted between the places of two successive conjunctions. Consequently there must always be an eclipse of the Sun at one at least of the conjunctions adjacent to each node; and since each node is in conjunction with the Sun at least once a year, it follows that there must be at least two solar eclipses every year.

To make this more clear, let *NEM* be the lunar orbit, *E* the Earth, *M* the Moon in conjunction, *Nn* the line of nodes. At the next conjunction let *E'*, *M'*, *N'n'* be the positions of the Earth, the Moon, and the line of nodes, *N'n'* being very nearly parallel to *Nn*. Draw *E'm* parallel to *EM*. *mM'* is the distance between the places of two successive conjunctions. And the node *N'* lying between *M'* and *m*, the two conjunctions represented are those adjacent to



the node. Now, if *N'M'*, or *N'm* do not exceed a certain quantity, known by observation, there must be an eclipse of the Sun at the former or the latter conjunction; and as this quantity is greater than half *M'm*, it follows that one or other of the distances *N'M'* or *N'm* must be less than this quantity. Hence there *must* be an eclipse of the Sun at one or other of the conjunctions adjacent to the node *N'*. About half a year afterwards, the other node *n* will be nearly in conjunction with the Sun, and it may be shewn in the same way that there must be an eclipse of the Sun about that time. Hence, two is the least number of solar eclipses which can happen in a year. If the distances *M'N'*, *mN'*, are nearly equal, they both fall within the given limit, and then two eclipses of the Sun occur about the node *N*. In like manner two solar eclipses may occur about the node *n*. Thus four solar eclipses may happen in a year. Farther, on account of the retrogradation of the line of nodes, if the first two eclipses occur very early in the year, there may be another eclipse about the first node before the end of the year, thus making five eclipses of the Sun, which is the greatest number that can possibly occur. The angular distance from the node within which, if a conjunction occur there must be an eclipse, and the distance within which there may be an eclipse, are determined by observation, and are called the solar ecliptic limits.

Analogous quantities are also determined for eclipses of the Moon, and called the lunar ecliptic limits. The distance from the node within which, if an opposition occur, there must be an eclipse of the Moon, is less than half the space between the places of the Moon at two successive oppositions. Consequently, both the oppositions adjacent to either node may be too far from the node for an eclipse. Thus a year may pass without any eclipse of the Moon; and there cannot be more than two lunar eclipses in a year. If there are two solar eclipses at each node, there will be necessarily one lunar eclipse, for the two conjunctions being nearly equidistant from the node, the intervening opposition will fall very near the opposite node; since, between the two conjunctions, the line of nodes will pass through the sun, as in the figure at E''. If there is only one solar eclipse at each node, the conjunctions occur near the node, and there may be no lunar eclipses at all.

The greatest number, therefore, of eclipses which can happen in a year is seven—five of the Sun and two of the Moon; and the least number two, which are both of the Sun.

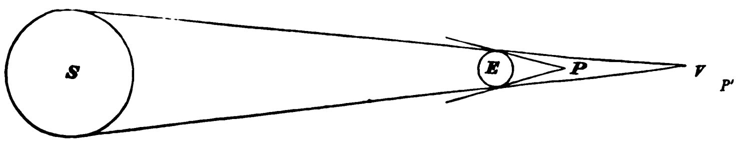
Eclipses of the Moon.

16. These phenomena are visible alike to all parts of the Earth's surface from which the Moon can be seen at the time; because the Earth actually deprives the Moon of the solar light, causing its shadow to fall on the surface, and thus nothing depends on the position of the observer, so long as the Moon is visible to him. In the case of solar eclipses, on the contrary, the light of the Sun being intercepted by a much nearer body, and not actually obscured, the position of the observer is an important consideration, so much so that in one place the Sun may be totally eclipsed, and in another no eclipse at all be visible. This may be familiarly illustrated by holding up a hat between the eye and the Sun. It is sufficient, if held close enough, to shut out the light, but if the observer moves a few inches to the right or the left, it

will no longer be in his way; while any object on which the shadow of the hat falls will appear equally obscured, from whatever point of view it is observed, so long as the hat is kept in the same position.

The Earth's Shadow.

17. The Sun being much larger than the Earth, causes the latter to cast a conical shadow into the opposite parts of space; since if we draw common tangents to the Earth and Sun, on the same side of their centres, they will all converge to a point, and within the space enclosed by these tangents no light can fall from the Sun, it being all intercepted by the opaque body of the Earth.



If *P* be a point within the cone, and we draw tangents from it to the Earth, the angle included by these tangents will be the apparent diameter of the Earth as seen from *P*, which will evidently exceed the apparent diameter of the Sun, and thus the Earth will be sufficient to obscure the whole of the Sun to an observer situated at *P*.

If the observer be supposed placed at *V*, the vertex of the above described cone, the diameters of the Earth and Sun will appear to subtend the same angle, and the Earth to cover the Sun exactly. If the observer be placed beyond *V*, as at *P'*, the angle subtended by the diameter of the Earth will be less than that subtended by the Sun, and the Earth will not be able to obscure the whole of the Sun at the same time. Consequently *V* is the farthest point which can be deprived at once of the whole solar light. If the Moon's distance from the Earth's centre exceeded *EV*, it could never be totally eclipsed; as it is, the distance is considerably less: and if the opposition occur exactly at the node, the Moon passes through a considerable part of the dark cone.

Sphere of Observation.

18. To an observer, at any point of the Earth's surface, the heavens present the appearance of an immense hemispherical vault, the centre of which is occupied by the eye of the observer, and its radius is of vast and indefinite dimensions. If the observer be at sea, the visible horizon or offing will appear to be the boundary of the vault: on land, his view of it will be more or less interrupted by terrestrial objects.

This appearance probably arises from our inability to estimate the distances of the heavenly bodies, or to discern any differences between them. The process of estimating the comparative distances of terrestrial objects is learned by habit and observation, but no such power can be acquired with respect to the stars. Hence, we tacitly regard them as equally remote, and this is equivalent to supposing these bodies to be on the surface of a sphere of unknown diameter of which the eye occupies the centre. This is the same thing as measuring their apparent distances from one another by the angular spaces between them, for the arcs of a sphere are to one another as the angles they subtend at the centre, independently of the magnitude of the radius.

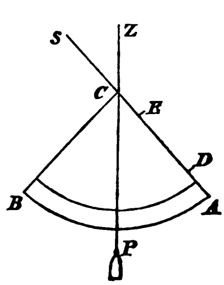
If in looking at a landscape we see two familiar objects such as houses or trees, we estimate their relative positions by various circumstances which we have been accustomed to employ intuitively in the formation of our opinion. They may be nearly in the same line, but, from the difference of their apparent magnitude and relative distinctness, we may judge them to be a mile apart, and their distance from one another may subtend a considerable angle, although it may not be more than a hundred yards. These circumstances we have learned by long practice to take into account, and are generally tolerably correct in our judgments: but if we see two stars near together, there is nothing to shew us whether they are really close, or whether they are only nearly in the same line. We cannot judge by their relative brightness, for we do not know their actual magnitudes. In default therefore of the means of estimation, we compare distances by the angle they subtend at the eye, which is equivalent to measuring them by arcs of an indefinite sphere, of which the eye occupies the centre.

This sphere we shall call the *sphere of observation*. It has been before stated, that not more than half of it is generally visible, although at sea the dip of the horizon enables us to see a very small zone of the lower hemisphere. We shall in general, however, suppose that our view of the sphere is limited by a plane touching the surface of the Earth at the point of observation, and intersecting the sphere of observation in a great circle called the *horizon*. This plane is perpendicular to the Earth's radius at the proposed point, and therefore to the direction of gravity which, on the supposition of the sphericity of the Earth, passes through the centre.

The point in which the Earth's radius produced intersects the sphere of observation is called the *zenith*, and from spherical trigonometry, it is on that account the pole of the horizon.

This property of the direction of gravity enables us to measure the positions of bodies relatively to the horizon by means of the plumb-line and spirit-level. The plumb-line, as is well known, always hangs vertically, and the spirit-level enables us to determine the horizontal direction.

It may be as well to explain one of the simplest methods of observation by which the position of a body in the sphere of observation may be determined. The instrument used is a quadrant of a circle made of any rigid material, and graduated from 0° to 90° along the limb.



Let ACB be such an instrument, to the centre of which a plumb-line is fastened. Let D, E, be two small sights, or pieces of metal with a hole in the centre of each, through which an observer looking at the star S, and bringing them into coincidence with it, brings the line AC into coincidence with the line AS, or the direction in which S is seen from the eye.

The angle ACP, measured by the arc AP, and consequently indicated by the point of the graduation on which the plumb-line rests, is equal to ZCS, the zenith distance of the star. The complement of this angle is called the star's *altitude*, being its perpendicular distance from the horizon.

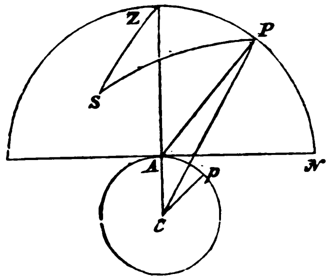
The position of a point on the sphere of observation is determined by its zenith distance, and by the angle which the great circle passing through it and the zenith makes with some fixed great circle, according to the method described in Art. 5 (7). This angle is called the *azimuth*. The zenith distance, therefore, and azimuth, or the altitude and azimuth, completely determine the position of a point in the sphere of observation.

The altitude may be readily determined by the method already described. We shall hereafter describe methods by which much greater accuracy may be attained. The azimuth is measured from an arbitrary position, which we shall explain in a future part of the subject.

19. From what has been said of the sphere of observation, it is obvious that there is a different sphere of this kind for every place, and that every observer refers the heavenly bodies to a sphere of his own. Not only are the centres of these spheres different, but also the directions of their zenith lines, which all converge to the Earth's centre.

Hence it has been found convenient to refer the places of the heavenly bodies to a fixed sphere, namely, that which would be the sphere of observation to the centre of the Earth. We shall call this the "celestial sphere," and we shall shew that although it is impossible to make observations from the position in which the observer is supposed to be placed, yet we may very easily reduce observations on the Earth's surface to those which would be made by a person stationed at the centre.

Let A be a point on the Earth's surface, C the Earth's centre. Join CA, and produce it to Z. Join AP, CP.



Then, if the zenith distance of P be measured from the point A, it will be the angle ZAP. If similar observations could be made at C, the result would be the angle ZCP. The difference between the two observed angles is APC, and if we can find that angle, we may reduce the zenith distance observed at A to that which would be observed at C.

We shall afterwards shew how this angle APC may be found. It evidently depends on the distance of P, and would vanish altogether, if P were so distant as that AC should be indefinitely small compared to CP or AP.

20. In order completely to determine the positions of the heavenly bodies, we must have a fixed point of reference and a fixed great circle in the celestial sphere, from which to measure their distances.

Two fixed points are given by the intersection of the axis of the Earth produced with the celestial sphere. These are called the North and South Poles. The former is generally taken as the point of reference. The great circle, of which it is the pole, is called the *celestial equator*.

Now, supposing an observer to be placed at the Earth's centre, and to partake of the rotatory motion of the Earth, he would see the whole celestial sphere apparently revolving about an axis which is the prolongation of that of the Earth, for he would suppose himself to be at rest, and refer all the motion to the heavenly bodies. The two poles only would appear stationary, every other part of the heavens partaking of the rotatory motion. The equatoreal parts would describe great circles, and all the other parts small circles, decreasing in magnitude as they approached the poles. The effect would be precisely the same as if, the observer being at rest, the celestial sphere actually revolved upon an axis passing through the two poles in the course of a day and night. This would very clearly be the appearance to an observer at the Earth's centre. The question is how that appearance would be modified to a spectator at the surface. In order to investigate this, we must remember that the sphere of observation is of indefinitely great magnitude, as also the celestial sphere: and if we consider the latter as a sphere actually revolving about the poles, those poles are so distant that their direction from the point *A* does not differ perceptibly from their direction as seen from *C*. Consequently the direction in which the pole is seen from *A* is virtually parallel to the Earth's axis. Hence, an observer at *A* sees the sphere of observation apparently revolving about a point *P*, which is so situated that *AP* is parallel to the Earth's axis.

Let *Z* be the zenith, *P* the pole at the place *A*. Join *AZ*, *AP*.

Let *C* be the Earth's centre, *CP* its axis.

Then $ZAP = ACp$, which is the angular distance of the place *A* from the pole of the Earth, and therefore the complement of the latitude.

Hence *PAN* is the latitude of the place, *N* being a point in the horizon; or the altitude of the pole above the horizon is equal to the latitude of the place.

The great circle through *P*, *Z*, is called the celestial meridian of the place *A*, because it bisects all those portions of the diurnal paths of the heavenly bodies which are above the horizon.

It has been said that the whole visible sphere appears to revolve about *P*, that point alone being stationary. When any body is brought by that revolution above the horizon it *rises*, and when it is carried below the horizon it *sets*. When it passes the meridian it *culminates*, being then at its greatest altitude. For, if *S* be a star, *PS* its polar distance, and *PZ* the co-latitude of the place remain constant, while the angle *SPZ* and *ZS* vary. In general *PZ*, *ZS* are together greater than *PS*, or $ZS > PS - PZ$. But when *S* is on the meridian, $ZS = PS - PZ$, and therefore at that point the zenith distance is least, or the altitude greatest.

It is an important question in what manner the diurnal circles described by the heavenly bodies are divided by the horizon, as on that depends the proportion of the whole time of rotation during which they are above the horizon, or visible to the observer.

If we take a circle between *P* and *N*, or at a less distance from *P* than *N*, it is evident that such a circle is wholly above the horizon, so that bodies between *P* and *N* never set at all, but are always visible. Similarly, bodies between the opposite pole *P'* and the opposite part of the horizon to *N*, never rise at all, nor are ever visible to the observer at *A*. The celestial equator being a great circle, will be bisected by the horizon, and consequently all bodies there situated will be equal times above and below the horizon. Between the equator and the parts *N* and *M*, the diurnal circles will be unequally divided by the horizon, but the northern circles will have their greater part above, and the southern circles their greater part below. Consequently, bodies in the former position will be longer above the horizon than below, and bodies in the latter position will be longer below than above.

21. From these results it follows that the whole of the celestial sphere can become visible during one revolution only to an observer who has the poles in his horizon, that is to say, an inhabitant of the equatoreal regions of the Earth; while, in every other part of the Earth's surface, a certain part about the depressed pole is continually invisible, a corresponding part near the elevated pole being always in sight. It follows also that at the equator every one of the heavenly bodies is as long above as below the horizon, because the horizon cutting all the diurnal circles at right angles, bisects them.

If an observer could place himself at either pole, he would see continually one hemisphere which would never vary, the other hemisphere being entirely out of sight.

22. The celestial equator is often taken as the circle of

reference, and the positions of bodies in the celestial sphere determined in the same way as those of places on the Earth's surface. The names, however, of the co-ordinates are different; that which answers to longitude being called *Right Ascension*, and that which answers to latitude *Declination*. The circles which on the terrestrial sphere are called meridians, are called *Declination-circles* in the celestial sphere. The rotatory motion of the Earth and therefore the apparent motion of the heavens being uniform, each declination-circle is carried uniformly round,—its angular distance at any time from the meridian of any place being proportional to the time which has elapsed from its last coincidence with that fixed circle.

23. The above remarks suppose the celestial sphere to be an immoveable whole, appearing to revolve in consequence of the motion of the observer, which he attributes to surrounding objects. Consequently they are strictly applicable to the fixed stars only. The time in which the sphere appears to complete its revolution, or in which any star revolves from the meridian to the meridian again, is a *sidereal day*, which is so called on this account. The sphere of the fixed stars is so vast that the change of position of the Earth in its annual course makes no difference in the phenomena, and the effect is the same as if the Earth were at rest in the centre, and the whole sphere revolving about it.

The only motion therefore observable in the fixed stars is the diurnal motion, they keeping their relative positions, and appearing to revolve *en masse* about the Earth in a sidereal day.

The other heavenly bodies, namely the Sun, Moon, Planets, and Comets, all change their places with reference to the fixed stars, or have other apparent motions besides the diurnal motion, arising either from the change of position of the Earth, or from their own proper motions combined with it.

24. The apparent motion of the Sun arises wholly from the annual rotation of the Earth, which by a common illusion

produces the same appearance as if the Sun described about the Earth an orbit exactly similar to that which the Earth describes about the Sun.

The Sun will therefore appear to move among the fixed stars in the same way as if it described an annual orbit about the Earth. The plane of this orbit will be the same as that of the Earth's actual orbit, and consequently inclined at an angle ω to the plane of the Earth's equator. This plane will therefore intersect the celestial sphere in a great circle inclined to the equator at an angle ω. This circle is the apparent path of the Sun among the fixed stars, and is called the *Ecliptic*.

The Sun's motion would be easily observable, but that the brilliancy of its rays prevents our seeing the stars among which it is moving. Every day we should see a sensible variation in its position, and in a year we should find that it had returned to its first place to describe the same path over again. This motion being in a direction contrary to that in which the whole heavens are apparently carried by the diurnal motion, we should find the Sun come later every day to the meridian than if it remained at rest. Thus if a star and the Sun passed the meridian at the same time on one day, we should find the star pass before the Sun the next day, by between three and four minutes of time. This is exactly the same thing which was shewn in Art. 6, where it appeared that, after the Earth has completed a revolution about its axis, it has still to turn a little further round, in order to return to the same position with respect to the Sun.

Although we cannot trace the Sun's motion among the fixed stars, yet we may very easily satisfy ourselves of the fact that it moves in the manner described. The only necessary observation may be made by any one who possesses a good watch. Let a fixed star be observed when a little to the east of some wall, or other vertical object, not very near the eye of the observer. After a little time, the diurnal motion will cause it to disappear behind the selected object. Let the exact time of its disappearance be noted, and let the same observation be repeated for several successive nights, care being taken to make them from exactly the same place. It will be found that the disappearance occurs between three and four minutes earlier every night. Now the time of disappearance indicates the exact completion of a sidereal day. The time by which the observer's watch is regulated is *solar* time, measured by the length of a *solar* day. The observation shews that the sidereal day is shorter than the solar day by nearly four minutes. The exact time is $3' \ 56''$.

It has been said that the Sun's apparent motion is not in the celestial equator, but in another great circle of the celestial sphere, making with the equator an angle equal to ω. These circles, by the principle of spherical trigonometry, intersect one another at two opposite points. Thus the Sun is for half the year on one side of the equator, and for the remaining half of the year on the other side. The exact position of the Sun is determined by its *right ascension* and *declination*, the former of which is measured from the point in which the ecliptic cuts the equator in passing from south to north.

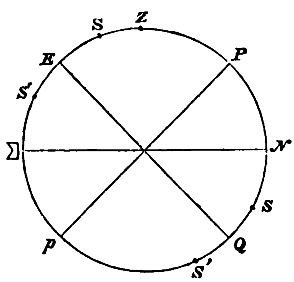
The greatest amount of declination is equal to ω the angle of obliquity, this being the greatest possible angular distance of any point in the ecliptic from a corresponding point in the equator. The declination reaches this its greatest amount when the Sun has passed through a quadrant of the ecliptic from its passage across the equator. After this point it begins to approach the equator again, and after traversing another quadrant, again crosses it. Thus the declination increases from $0°$ to ω in a quarter of a year, in the next quarter it diminishes to $0°$ again, in the third quarter it increases on the other side of the equator till it reaches the maximum value, and in the last quarter it again diminishes to $0°$.

25. The phenomena of the seasons are easily derivable from this apparent annual motion of the Sun. When the Sun is in the equator, the diurnal motion causes it to remain equally long above and below the horizon, consequently the days and nights are equal in all latitudes.

When the Sun is to the north of the equator, or has north declination, the diurnal circle which it describes is cut unequally by the horizon, so that to places in north latitude the days are longer than the nights, and to places in south latitude the nights are longer than the days. If the Sun's north polar distance be less than the latitude, it does not set at all, but after sinking towards the horizon in the north, rises again without disappearing. In order that this may be the case, the latitude must be greater than the complement of ω, or than 66° 32′. Therefore the phenomenon described can only occur between the pole and the arctic circle. When the Sun has south declination, the same thing occurs within the antarctic circle, and the days in the northern hemisphere are shorter than the nights.

26. The meridian altitude of the Sun is at once ascertained when his declination and the latitude of the place are known. For the altitude of the celestial equator above the horizon is the complement of the latitude, and this is also evidently the meridian altitude of the Sun when in the equator. When the Sun is not in the equator we must add its declination, if on the same side of the equator as the latitude, or subtract it if on the opposite side. That is, if the latitude be north, we must add the declination when north, and subtract it when south; and if the latitude be south, we must subtract the declination when north, and add it when south.

Let Pp be the north and south celestial poles, EQ the celestial equator, and let ΣN be the horizon of any place. The arc ΣE is the co-latitude. Let S be the Sun, having the altitude ΣS above the horizon. $\Sigma S = \Sigma E + ES$, or is the sum of the co-latitude and declination. If the declination had been south, as ES', we should have had $\Sigma S' = \Sigma E - ES'$, or we should have had to subtract the declination from the co-latitude in order to get the meridian altitude.



If, reversing the figure, we take ΣpN for the visible half of the celestial sphere, we get the case of a place having south latitude Σp, equal to the north latitude NP of the place under consideration. The meridian altitudes of the Sun as seen from that place will be NS, NS', corresponding to the altitudes ΣS, $\Sigma S'$ in the former place. It is easy to see that the former of these is obtained by subtracting, and the other by adding the declination to the co-latitude, as before stated.

It may be observed that, conversely, if we know the Sun's declination, and can ascertain his meridian altitude, we are immediately in possession of the latitude of the place.

When the sum of the co-latitude and declination is exactly 90°, the Sun is vertical at noon. This can only happen when the co-latitude is greater than the complement of ω, or the latitude itself less than ω. Thus the Sun can only be vertical to places between the tropics.

In this manner, from the Sun's apparent orbit we may obtain all the phenomena before derived from the consideration of the actual course of the Earth.

27. Similar explanations will apply to many of the lunar phenomena. The Moon, as we have seen, does not move in a plane coincident with that of the Earth's orbit; therefore its apparent path in the celestial sphere does not coincide with that of the Sun. If its orbit in space were an immoveable plane, its apparent path in the celestial sphere would be a fixed great circle intersecting the ecliptic in two opposite points. As it is, the orbit, though of an irregular form, may be well represented as a *moveable* plane, as has been before described, and therefore the path in the celestial sphere as a *moveable great circle*, which has small periodical variations in its inclination to the ecliptic, and whose points of intersection with the ecliptic have a constant variation of position.

These points of intersection are called, like those in which the actual lunar orbit in space intersects the plane of the Earth's orbit, the Moon's *nodes*. That by which the Moon passes from the south to the north of the ecliptic is called the *ascending* node, and the other the *descending* node.

These points have a retrograde motion on the ecliptic, which carries them completely round it in 18 years 225 days, because in this period the actual line of nodes of the lunar orbit in space, after describing a complete circle, returns to its original position. The Moon's place in the celestial sphere is often referred, for convenience, to the ecliptic as the great circle of reference. The co-ordinates by which it is determined are called *longitude* and *latitude*, these terms having the same signification as when employed to mark the position of a point on the Earth's surface referred to the terrestrial equator. The perpendicular distance of the Moon from the ecliptic is called the latitude, and the longitude is measured from that point of intersection of the equator with the ecliptic, from which right ascension is also estimated.

Two bodies are said to be in *conjunction* when their longitude is the same, in *opposition* when their longitudes differ by 180°. It follows from spherical trigonometry that the Moon's latitude cannot exceed the angle of inclination of the Moon's orbit, which we shall call ι, and whose average value is 5° 35′. That is, the centres of the Sun and Moon when in conjunction cannot be further apart than the length of the arc ι.

The average apparent diameters of the Sun and Moon are respectively 32 and 31 minutes; so it is evident that where the latitude at all approaches its maximum value, the Moon must be so far from the Sun that there is no possibility of the latter being obscured by it.

In fact, as we have explained before, no obscuration or eclipse of the Sun takes place, unless the conjunction occurs near one of the nodes.

28. During one revolution of the Moon in its orbit, the Sun moves through an arc of the ecliptic which is equal to nearly one-thirteenth of the whole. Consequently when the Moon returns to the place from which it started, the Sun has advanced by a considerable space, and the Moon has to traverse a further arc before it is again in conjunction. Thus each successive conjunction occurs at a different part of the heavens, and similarly each successive opposition.

When the position of the Sun and Moon at conjunction is within a certain distance from that of either of the nodes, the latitude of the Moon is so small, or it approaches so near the Sun, that an eclipse occurs to some part of the Earth; and this interval being greater than half that which is passed over by the Sun in the ecliptic during a lunar month, it follows that there *must* be an eclipse at one of the two conjunctions between whose positions one of the nodes is situated. If one of those conjunctions adjacent to the node occur very near the node, there is no eclipse at the other; but if they are about equidistant from the node, eclipses occur at each of them. Thus there *must* be two eclipses of the Sun in a year, one at each node, and there *may* be four eclipses, two at each node. This however is not the greatest number which can possibly take place, since if the first be very near the beginning of the year, the node near which it occurs may be brought by its retrograde motion into such a position that the last conjunction in the year may be near enough to it for a fifth eclipse.

When the position of the Moon at the time of opposition is within a certain distance of one of the nodes, there is a lunar eclipse. This interval is less than half that between two successive oppositions, and consequently, if the two adjacent oppositions to a node be nearly equidistant from the node, there *may* be no eclipse. And if the same thing happen near the other node, there will be no eclipse of the Moon in the course of the year. There cannot be more than one eclipse at the same node, because if one opposition be within the distance, the other will be necessarily without it.

In the former case, where there is no eclipse, the conjunction between the two oppositions will occur very near the opposite node, and thus there will be only one eclipse of the Sun. In the latter case, if an opposition occur very near the node, the two adjacent conjunctions will be about equidistant from the opposite node, and so there will be two eclipses of the Sun.

Thus there may be only *two* eclipses in a year, both of the Sun, or as many as *seven*, five of the Sun and two of the Moon.

29. The times of rising and setting of the Moon will depend on two circumstances, its age and its declination. At conjunction, being near the Sun, it passes the meridian about noon, and at opposition, being opposite to the Sun, about midnight. Speaking roughly, its meridian passage is about fifty minutes later every day than the day before, thus going through the whole twenty-four hours in its synodic period of twenty-nine days and a half. If the Moon's declination remained unaltered, its rising and setting would each be fifty minutes later every day than the day before, because the same diurnal circle would be always described, and the time above the horizon would be always the same. As it is however, the Moon's declination varies considerably. When its ascending node coincides with the vernal equinoctial point, the inclination of its orbit to the equator is $\omega + \iota$; and when the descending node coincides with the same point, it is $\omega - \iota$, the former being the greatest possible and the latter the least possible inclination. Consequently the inclination is always between $\omega - \iota$ and $\omega + \iota$, and therefore the Moon's declination in the course of each revolution varies at least from $\omega - \iota$ in one side of the equator to $\omega - \iota$ on the other.

The effect of a change in the declination is to vary the time of the Moon's being above the horizon, without altering the time of passing the meridian. And as the Moon passes over a considerable part of its orbit in a day, there may be a considerable change of declination in that interval. Suppose the effect of that change to be the augmentation of the Moon's time above the horizon: the time of rising will be earlier and the time of setting later than if the declination had not varied. Similarly, if the effect of the change in declination be to diminish the time above the horizon, the rising will occur later and the setting earlier than if the declination had not varied. In the first of these cases, the time of rising will be less and the time of setting more than 50 minutes later than the day before. The opposite effect will take place in the second case. If, for instance, the time of the Moon's being above the horizon—or what we may call the lunar day—is increased by an hour in consequence of the change in declination, then the time of rising is half-an-hour earlier, and the time of setting half-an-hour later than if there had been no such change. Hence the time of rising will be 50 - 30 or 20 minutes, and the time of setting 50 + 30 or 80 minutes later than the day before, the meridian passage being 50 minutes later, as we have already shewn.

In places whose latitude is north, the lunar day is greatest when the Moon's northern declination is greatest, and continually increases when the Moon is in that part of its orbit which goes from south to north. If the Moon's orbit coincided with the ecliptic, the lunar day would be on the increase, while the Moon passed from the tropic of Capricorn to the tropic of Cancer. During all that time, therefore, the Moon would rise less than 50 minutes later, and would set more than 50 minutes later every day. And this effect would be greatest just as it crossed the equator, for at that point the declination increases most rapidly. From the tropic of Cancer to that of Capricorn, the declination varying in the opposite direction, the effect on the rising and setting would be opposite. As it is, the lunar orbit being not very far from the ecliptic, the time at which there is least variation in the time of rising, and most in the time of setting, owing to the augmentation of the lunar day, is when the Moon is in that part of its orbit which is near the vernal equinox.

When the Sun is near the opposite equinox, that is about the 22nd of September, the Moon's opposition occurs near the vernal equinox. The consequence is that the nearest full Moon to the autumnal equinox in high northern latitudes rises very nearly at the same time for several days together; a phenomenon which has given to this lunation the name of *Harvest Moon*. It follows from what has been said, that the time of setting is as much more than 50 minutes later each day as the time of rising is less.

In southern latitudes the same phenomenon occurs, as may be easily seen, at our vernal equinox, which, owing to the opposition of the seasons, is the autumnal equinox to the inhabitants of those regions.

The Planets.

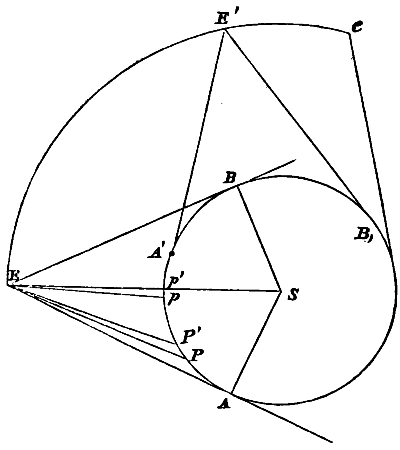
30. These bodies, as we have seen, revolve about the Sun in elliptic orbits of small eccentricity, whose planes are inclined at small angles to the plane of the Earth's orbit. If we could observe the planetary motions from the Sun, they would appear very simple and straightforward, like those of the Moon about the Earth: but as we are limited to looking at them from the Earth, which not only is not the centre of their motion, but has a motion of its own besides, their apparent orbits are by no means so easily traced. The appearances presented by the inferior planets, or those which are between us and the Sun, being very different from those of the superior planets, whose orbits are without that of the Earth, we shall take each case separately.

31. The inferior planets Mercury and Venus, having orbits lying within that of the Earth, can never be in *opposition* to the Sun; and in fact such a planet can never be seen at above a certain angular distance from the Sun, depending on the magnitude of its orbit. This angle is half that subtended by the whole orbit at the Earth, it being manifest that the extremities of the orbit on each side of the Sun, as seen from the Earth, are the farthest points to which the planet can reach. Owing to the small inclination of the orbit to the ecliptic, it is seen nearly edgeways from the Earth, and consequently has nearly the appearance of a straight line, having the Sun in the middle. This effect may be illustrated by observing a candle carried round and round a circular table, at a little distance, on a level with the eye. The curved motion of the candle will be lost in consequence of the circle in which it is carried being seen edgeways, and the appearance

will be much the same as if the candle were carried backwards and forwards in a straight line. Thus an inferior planet appears to oscillate backwards and forwards about the Sun, appearing to turn when it has reached its greatest distance on one side, and proceed in the opposite direction till it has reached its greatest distance on the other side.

Its motion appears most rapid when it is just crossing the Sun, for then its path is exactly perpendicular to the line of vision: and supposing the motion to be uniform and the orbit circular, which is not far from the case, the same arc occupies a greater apparent space than at any other part of the orbit.

To make this more clear, suppose *S* to be the Sun, *P* an



inferior planet, *E* the Earth: and for simplicity we shall first suppose that the Earth is at rest. Join *ES*, and draw the tangents *EA*, *EB*. The angle *AEB* is that subtended at the Earth by the orbit of *P*, and therefore *AES* or *BES* is the greatest angular distance from *S* at which *P* can be seen from *E*.

Suppose *P*, moving in its orbit, to arrive at *A*, and to proceed towards *B*.

Its first direction is towards *E*. When it arrives at *p'* in the line between *S* and *E*, its direction is exactly across *SE*: and if we take equal arcs *pp'*, *PP'*, we find that the angle subtended at *E* by the former is greater than that subtended by the latter, and the more so the nearer *P* is to *A*.

Now it must be remembered that *E* being the centre of the celestial sphere, the apparent motion of *P* is measured by the angle through which it appears to move as seen from *E*. Therefore the apparent motion is quickest at *p'*, and again diminishes as far as *B*. It then changes to the opposite direction, the planet describing the part *BA* of its orbit, and appearing to move through the angle *BEA* in the opposite direction to that in which it was first moving.

It is evident that in its passage from *A* to *B* and back to *A* again, the planet *P* will be twice in conjunction with the Sun. The former of these is called the *inferior* and the latter the *superior* conjunction. At the former, if the planet be sufficiently near the node of its orbit, it passes between the Earth and the Sun. This phenomenon is called a *transit*.

We thus observe that, supposing the Earth to be fixed, and an inferior planet to revolve about the Sun in an orbit edgeways to the Earth, the planet would appear to move backwards and forwards within a certain angular distance, its velocity being greatest about conjunction and least at the extreme points of its orbit. In astronomical language, the motion of an inferior planet is first *direct*, or from west to east; then *retrograde*, or from east to west. Between these two it is for a short time *stationary*.

32. We have here supposed the Earth to be fixed. It remains to investigate the effect of its motion on the phenomena of an inferior planet.

If the Earth be at *E* when the planet is at *A*, let *E'* be the position of the Earth when the planet has arrived at *B*; which, in the former case, was the position of greatest elongation. Draw the tangents *EA'*, *E'B'*. The planet at *B* is evidently not in its position of greatest elongation as seen from E', but would have to move in order to be so through the arc BB'. And as, when it has arrived at B', the Earth will have advanced from E', the planet will not be at its greatest elongation as seen from the Earth, till it has arrived at a farther point b, where be a tangent to its orbit passes through the Earth's place e.

Similarly, if the planet had been at B when the Earth was at E, it would have had to go beyond A before it came into the position of greatest elongation on that side of the Sun.

The effect of the Earth's motion, therefore, is to *prolong* the time from one elongation to another, without interfering with the general phenomena above described. The time from one conjunction to another is similarly prolonged, so that the appearances are exactly the same as if the Earth were at rest, and the period of the planet were increased. The apparent period as seen from the Earth is called the *Synodic* period, the actual time of revolution being the *Sidereal* period.

The one is easily calculated from the other. For, let S be the synodic period, σ the sidereal period, P the sidereal period of the Earth.

Then, as $\frac{1}{\sigma}$ to $\frac{1}{P}$, so is the angular velocity of the planet to that of the Earth.

Therefore in the time S, the arcs described by the planet and the Earth respectively will be $\frac{S}{\sigma}$, $\frac{S}{P}$, the whole circumference being represented by unity. Now the planet has described a whole circumference more than the Earth, and therefore

$$\frac{S}{\sigma} = \frac{S}{P} + 1, \quad \text{or } \sigma = \frac{PS}{P+S}.$$

The inferior planets deriving all their light from the Sun, and therefore being unenlightened in the hemisphere which is turned away from that luminary, present phases like those of the Moon. At the time of inferior conjunction they would be invisible to us, unless when crossing the Sun's disc, even if they were not lost in the Sun's rays, for their darkened part would be turned towards us. At superior conjunction, if we could see them so near the Sun, their enlightened hemispheres would be turned towards us, and we should see them fully illuminated. As it is, they are not visible till they are at some distance from the Sun, owing to the brightness of the solar rays. At the points of greatest elongation on each side they appear *dichotomized*, or like the Moon in its first quarter, the angles *SAE*, *SBE* being right angles. From *A* to *b*, the angle *SPE* is obtuse, and therefore more of the darkened hemisphere is seen than of the other. Consequently the planet appears crescent-shaped. From *b* to the next point of greatest elongation on the other side, the angle *SPE* is acute, and therefore more of the enlightened than of the darkened hemisphere is visible. The planet is then said to be *gibbous*.

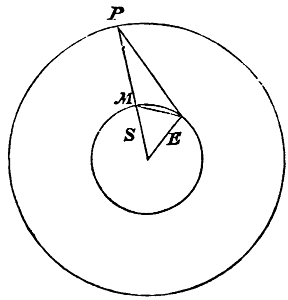
It has been said that the orbits of the inferior planets are inclined to that of the Earth. The consequence is, that in general they are sufficiently far from the ecliptic to pass clear of the Sun at conjunction. Transits of Venus are very rare, and those of Mercury only occur at considerable intervals of time: those of Venus are of important use in Astronomy, for the determination of the distance of the Sun.

The above phenomena are very little affected by the ellipticity of the orbits. The principal difference is that the times of elongation do not occur at exactly regular intervals, and that the distances from the Sun at which the planet begins to turn and move in the opposite direction are not always the same, the elliptic orbits not subtending the same angle at the Earth in all positions, like the supposed circular orbits.

33. The superior planets are those whose orbits are exterior to that of the Earth. Consequently, they can never come between the Earth and the Sun, but there is no limit to their angular distance from the latter. A superior planet is in opposition when the Earth lies between it and the Sun. It is in conjunction when the Sun lies between it and the

Earth. In both these cases the illuminated part of the planet is turned towards the Earth, but in the latter case it is not visible, owing to the strength of the Sun's rays.

A superior planet can never appear *crescent-shaped,* because the angle *SPE* is always less than a right angle, and consequently more than half the enlightened hemisphere is always visible.



If we join *PS,* cutting the Earth's orbit in *M,* it is evident that *SPE* cannot be as great as a right angle, for then *SME,* the exterior angle, must be *obtuse,* which is impossible.

The sidereal period of a superior planet may be found by a similar method to that of an inferior planet.

The notation being the same as before, the Earth will have described a whole circumference more than the planet, and therefore

$$\frac{S}{\sigma} + 1 = \frac{S}{P}, \text{ or } \sigma = \frac{PS}{S - P}.$$

CHAPTER II.

34. We have now explained the principal appearances of the heavenly bodies to a spectator on the Earth's surface, deduced from an assumed theory of their motions. In order, however, to divest our inquiries of all unnecessary complexity, we have made some suppositions which are only approximately true, such approximations being near enough for the purpose of explaining general phenomena. If this were the only object of Astronomy, we need not perhaps go any farther, but as the applications of this science require the utmost attainable accuracy, we cannot rest contented with mere approximations.

We have supposed that the Earth is spherical, whereas really it is a spheroid of very small ellipticity. We have treated the orbits of the Earth, Moon, and Planets as circles, whereas really they are curves nearly resembling ellipses of small eccentricity.

We shall find that the substitution of the true theory in all these cases for the simpler hypothesis which we have adopted, will very little affect the general description of the phenomena; but when we come to measurements and minute observations, we shall find it no longer a matter of indifference which view of things we proceed upon. The modifications of the general phenomena consequent on these modifications of the theory have been at once the principal difficulties, and the causes of the principal discoveries of Astronomy. When increasing accuracy of observation shewed that our first suppositions are not accurately correct, all the ingenuity of the most penetrating minds was employed to determine what the suppositions should be. The first result was that more complex suppositions were made; but as light increased they were abandoned, and it is the triumph of inductive discovery that the whole range of the phenomena is embraced by one grand and simple law.

It was probably with no small regret that men were forced to abandon the simple theory of circular motion and uniform velocity in the heavenly bodies; but it is surely an abundant consolation to us now, that the very irregularities which reduced earlier theorists to despair, are the constant verifications of Newton's great discovery: and of this we need only mention one recent and striking example, that Astronomers of our day, by means of the perturbations of a planet which the ancients never saw, have discovered a yet more distant subject of the empire of gravitation.

35. We shall now endeavour to trace the effects of adopting the true hypothesis in all cases respecting the celestial motions; and as we now pass from popular explanations into the domain of scrupulous accuracy, we shall begin by explaining the means of exact observation. We shall endeavour to shew by what instruments and by what calculations an observer in any given place may determine the positions of bodies in the celestial sphere, and thence find his own position on the Earth's surface.

36. By means of Astronomical instruments we determine the place of a body in the sphere of observation,—either its altitude or its azimuth, or both together. We have already shewn how the former may be roughly measured. Such an instrument, however, as that above described would be almost useless in the present state of astronomical science; for, even supposing it to be mounted on a frame, so that it might be placed in a perfectly firm position, with its plane accurately vertical, the observations made by it would be liable to errors large enough to deprive it of all utility. In the first place, it would be very difficult to graduate the arc to any great nicety. A single degree would occupy so small a space on a circle of moderate size, that it would be next to impossible

to make divisions as near to one another as single minutes. But an angle of a second is of importance in astronomical observations. Besides, if the graduation were made as minute as this, it would be a matter of great difficulty to make out which line exactly the plumb-line coincided with, even were it the finest that could be contrived. In earlier times Astronomers endeavoured to meet this difficulty by making instruments of enormous size, but it was found that the advantage of this was very much counteracted by the liability of such instruments to bend, from the weight of their parts. It is to be remarked too, that instruments in which the observed body is brought to coincide with simple sights by the naked eye are incapable of sufficient accuracy, however good the graduation, because of the inability of the eye to appreciate the very small angular quantities with which modern science deals.

The contrivances, therefore, by which such quantities are measured do not consist only of refinements of graduation and reading off the observed angles, but also of optical aids to the sight. Neither of these means would be of any great use without the other. The adaptation of the telescope to astronomical instruments is justly called an era in the science of practical Astronomy. The telescope usually employed is that which is called, for that reason, the astronomical telescope. In its simplest form it consists of two lenses, the object-glass and the eye-glass, on the same axis, and so placed that their principal foci coincide. When the tube in which the lenses are contained is directed to a heavenly body, pencils of parallel rays from every part of the body fall on the object-glass, and an inverted image is formed at the principal focus. This image being by the construction in the focus of the eye-glass, is made visible by parallel rays to an eye looking into the telescope. In practice the object-glass consists of two lenses in contact, so as to destroy as much as possible the effects of aberration and colour, whence this combination is called *achromatic*. The eye-piece also is generally composed of two lenses, near to one another, although it cannot be made achromatic without sacrificing other advantages. The general effect of the improved telescope is, however, quite the same as that of the simpler instrument. The difference is that the final image is more distinct and free from colour.

The final image is evidently *inverted,* but that is of little consequence; and the usual contrivance for producing an erect image in telescopes for terrestrial objects being composed of four lenses instead of two, the loss of light from absorption by the glass is too great a drawback to make up for any advantage that might be gained. It is perfectly easy in practice to make allowances for the inverted appearance. This telescope enables us to detect differences of position which would be quite inappreciable by the naked eye.

But this is not sufficient for observation. It is obviously necessary that there should be a fixed line of sight into which the observed object must be brought, for merely to bring it into the field of view would not be sufficient. The method in use is to fix two very fine threads, or spider-lines, at right angles to one another in the focus of the eye-glass, intersecting one another nearly in the axis of the tube, just where the image of the observed object is formed by the object-glass. The line joining the intersection of these spider-lines, or *cross-wires* (as they are called from their appearance when magnified by the eye-glass), with the optical centre of the object-glass, is called the *line of collimation* of the telescope; and as the image of every point in the object is in the same straight line with the centre of the object-glass and the point of which it is the image, it is clear that when an object appears to the observer to coincide with the intersection of the cross-lines, it is really in the line of collimation produced.

If therefore we had a quadrant, with such a telescope instead of the line of sight, we might make observations more delicate than those made with the naked eye in proportion to the magnifying power of the telescope. We should in that case, however, be obliged to use corresponding care in so adjusting the telescope that its line of collimation should be exactly perpendicular to the line joining the centre of the arc with the zero line of graduation, so that the reading should be exactly zero when the telescope is horizontal.

With such an apparatus, however, the method of reading the graduation by a plumb-line would be, as was said before, wholly insufficient.

37. There are two principal methods in use for the supply of this defect, the *vernier* and the reading *microscope*.

Before proceeding to explain these contrivances, we may mention that it will not be necessary to have a plumb-line at all, if we have any means of determining in what position of the instrument the line of collimation is accurately horizontal or vertical. If, for instance, we have a fixed index which coincides with the zero point of graduation, when the telescope is horizontal; then, when we have brought the intersection of the cross-wires into coincidence with the body to be observed, the reading of the index will give its altitude. The horizontality of the telescope is often ascertained by means of a spirit-level. It is obvious that if the reading of the index, when the telescope is horizontal, be not zero, but some given number of degrees, minutes, and seconds, we shall come to the right result by subtracting that reading from the reading obtained by directing the telescope to the object which is to be observed; for all we want to do in that particular observation is to measure the angle between the direction of the telescope in its horizontal position and its direction when pointed to the star.

Thus, when we have once either found that the index marks zero when the telescope is horizontal, or observed what it does mark if it is not zero, we are in a condition to make any number of observations without troubling ourselves farther about the horizontal position. The problem in fact reduces itself to finding the angular interval measured on the arc of the instrument, or *limb* as it is technically called, between two given positions of the index, the intercepted arc measuring the angle between the two directions of the telescope. And this is the general problem in all astronomical instruments which are graduated. There is a limb sometimes consisting of a whole circle, sometimes of a part only, according to the object contemplated by the construction. A telescope revolves about an axis differently placed according to circumstances. Sometimes it carries with it an index by means of which the angle it describes is marked on the limb. Sometimes it is fixed to the circle, which revolves with it, and then the angle described is indicated by a fixed index, as in the case above alluded to. The principle in all these cases is the same, and the desideratum is, by delicacy of graduation and other contrivances, to measure with all possible exactness the portion of the limb passed over.

38. The first instrument we shall describe is the *Vernier*.

The most simple kind of index is a pointer or hand carried along the arc—or remaining stationary while the arc is moved, according to the construction of the instrument—and indicating the angular interval by the graduation, like the time by the hand of a clock. The vernier is an improved index by which, with the same graduation, much greater nicety of measurement is obtained. The simple index will not determine intervals much nearer than those of the graduation, although when it does not coincide exactly with one of the lines of graduation, we may guess roughly by inspection at the fraction of the interval represented; just as when the hand of a clock is between two divisions we guess at the fraction of a minute to be added to the former of the two readings in order to get the true time. The vernier, however, accurately subdivides the intervals of the graduation into a great number of equal parts. To continue the illustration of the clock: with a simple index we obtain a reading no more accurate than that of the minute-hand, whereas the vernier may be made to give us the same accuracy as a hand which marks seconds.

In the case of the common index, a single point is carried along the graduated scale by means of an arm revolving about an axis which passes through the centre of graduation. In the vernier index, instead of the single point there is a small portion of the circumference of a circle concentric with the circle of graduation, and marked with divisions which we shall presently describe. This arc by the motion of the index is made to slide along the limb, and from its construction always coincides with a portion of the graduated arc.

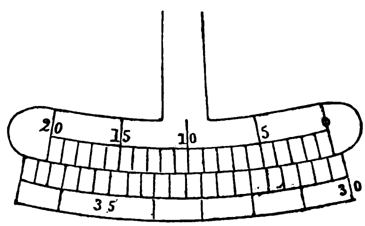
The length of the vernier arc is made such as to include exactly some particular number of divisions of the limb. This length is then divided into a number of equal parts exceeding by one the number of divisions in that part of the limb with which it coincides. It is evident that if the two extremities of the vernier be made to coincide with certain lines of graduation of the limb, none of the intermediate lines of the vernier will coincide with those of the limb.

The first division of the vernier will fall short of the first on the limb, the second will fall short by twice the space, the third by three times the space, and so on till the last, which of course falls short by a whole division.. Hence the space by which the first division of the vernier falls short of that on the limb will be that fraction of a division of the limb whose numerator is unity and whose denominator is the number of divisions in the vernier. If therefore, from the position in which the extremity of the vernier coincides with a line of graduation of the limb, we push forward the vernier through the above space, its next line of graduation will coincide with the next line on the limb. If we push it through such another space, the next line will coincide, and so on; and conversely, if we see any line of the vernier coinciding with one on the limb, we may know, by counting the number of lines from the end, through how many spaces it has been pushed from its first position. And this we may tell by inspection, if we mark the extreme line zero, and the rest one, two, three, &c. Hence, if the zero line, which we may take for the index point, do not exactly coincide with any line of graduation of the limb, we have only to carry on the eye till we see some farther division coincident, and by observing its number we may estimate that fraction of a division of the limb by which the zero point is distant from the line of graduation. It is evident that only one line of the vernier at a time can coincide with one on the limb, excepting the two extreme lines.

In many instruments the graduation of the limb is made to the third part of a degree. Each division therefore contain 20′. The vernier is made to coincide with 19 divisions of the limb, and is divided into 20 parts, the one extremity being marked zero. Thus we are enabled to read off to $\frac{1}{20}$ of each division on the limb, or to measure angles true to a minute of space.

Suppose in taking an observation we see the zero point between two lines of graduation of the limb. We look to see which line of the vernier coincides with one of the limb, and we add that number of minutes to the angle obtained by taking the first line of graduation of the two that are adjacent to the zero point.

We annex a figure of a vernier index with a corresponding part of the limb.



The zero point marks 30°, taking the next line behind it.

The line marked 6 in the vernier coincides with one on the limb, and therefore we add 6′ to the former angle, and the true result of the observation is 30° 6′.

The delicacy of reading by means of a vernier may be considerably increased by the use of a magnifying lens.

39. Another contrivance which is extensively employed for the same purpose as the vernier is the *Reading Microscope.* It consists of an object-glass and an eye-glass placed in a tube, which tube is usually fixed with its axis parallel to the plane of the instrument, and directed towards the graduated limb, the divisions of which are made on the outer circumference.

In the tube, and perpendicular to its axis, is an oblong rectangular frame, at a distance from the eye-glass equal to its focal length. In the position of diagonals to this frame are two spider-lines intersecting one another in its centre. The whole frame is moveable in the direction of its length, backwards or forwards, by means of a screw, and thus the point of intersection of the spider-lines is made to pass across the field of view. When the microscope is properly placed, an image of part of the graduated limb is formed by the object-glass in the focus of the eye-glass, and is seen to coincide with the image of the spider-lines. The adjustments are so made that the point of intersection of the spider-lines is carried from one line of graduation to the next by a certain number of turns of the screw, the head of which is graduated and furnished with a pointer to shew how many turns and parts of a turn it has made.

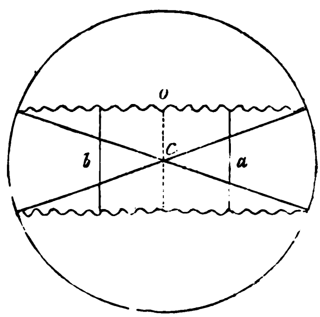
When an observation is made, the graduated limb being attached to the telescope and moveable with it, the microscope acts the part of a fixed index; or to speak more exactly, the intersection of the spider-lines may be taken as the index, when it is in the middle of the field of view, the pointer on the screw-head being so adjusted as to mark zero at the same time. If the observer, on looking into the microscope, sees some line of graduation exactly coincident with the intersection of the lines when so adjusted, he reads off at once from the limb. But if, as is generally the case, the intersection of the lines is between two adjacent lines of graduation, he only gets the reading roughly from the limb, and the object of the microscope is to determine how much must be added to that first reading on account of the index being beyond the line of graduation from which it is taken.

The method is to turn the screw-head in such a way as to move the point of intersection of the wires backwards, till it coincides with the line of graduation from which the reading has been taken, to count the number of whole turns, and to read off on the screw-head the number of parts of a turn which the screw has made. Then, the number of whole turns being known which bring the index from one line of graduation to the next, the space required may be found by proportion. Thus we may subdivide the divisions on the limb into a number of parts, which is the product of the number of turns which carries the index from one line of graduation to the next, and of the number of divisions on the screw-head.

In general, the instruments to which the reading microscope is applied have divisions on the limb to 5′ of space, the screw makes 5 turns between two adjacent lines of graduation, and the screw-head is divided into 60 parts. Hence the reading may be made accurately to single seconds.

In practice, a fixed frame is placed in the field of view edged with teeth, by which the number of whole turns of the screw may be read by inspection, so that it is not necessary to count them.

We subjoin a representation of the appearance of the field of view of the reading microscope; *a*, *b* being two adjacent lines of graduation on the limb, and *c* an imaginary line with which the point of intersection of the spider-lines coincides when the pointer on the screw-head marks zero. The true reading is therefore that given by the line of graduation *a* added to the number of minutes and seconds contained between *a* and *c*.



The exact position of the line *c* is not of great consequence, so long as it is near the middle of the field of view. It is determined by being the nearest position of the index to the middle of the field when the pointer on the screw-head marks zero.

The most important adjustment of the microscope is that by which the images of two adjacent lines of graduation are made to lie just at that distance from one another which is passed over by the index in the given number of turns. This adjustment is made by placing the object-glass at a proper distance from the limb, and at such a distance from the eye-glass that the image may be formed in its focus; for the magnitude of the image depends on the distance of the object from the lens, and its distinctness to the eye on its being in the focus of the eye-glass.

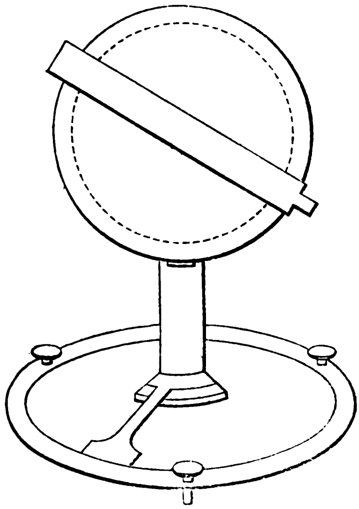
If, for instance, the microscope be placed nearer to the limb, the images of the lines of graduation will be formed farther from the object-glass, and will subtend a greater angle at the centre of the object-glass, since they subtend the same angle at that point as the lines themselves. Therefore the eye-glass must be drawn out in order that the images may be distinctly visible, and the effect of the change will be that the screw must make more turns to carry the index from one to the other. If the microscope, on the other hand, be moved further from the limb, and the eye-glass properly placed, the effect will be just the opposite. Thus the microscope may be accurately adjusted. In practice, however, a perfect adjustment is not attempted, it being found better to determine the error by observing the exact reading given by the screw-head when the index is brought from one line of graduation to the next. This error is called the *error of runs*, and must be allowed for in the result of every observation.

It may be as well here to make the general remark, that though astronomical instruments are usually furnished with the means of accurate adjustment, yet in practice such adjustments are only approximately made, and the small deviations from perfect accuracy are observed and allowed for. This method gives quite as accurate results as if the instruments were perfectly adjusted, and are much more to be relied upon than if perfect adjustment were attempted; for it is impossible to keep an instrument in exact adjustment for any length of time, on account of changes in the weather and the necessary strains on the parts in making observations. Besides, the stability of the instrument, which is a most important element in its usefulness, is very apt to be injured by perpetual changes of the positions of its parts.

The Altitude and Azimuth Instrument.

40. This instrument consists of an astronomical telescope so mounted as to determine by one and the same observation the altitude and azimuth of a heavenly body.

The telescope is firmly fastened to a graduated circle whose plane is parallel to the line of collimation. This circle revolves



about an axis through its centre, which axis itself revolves about another axis at right angles to it. The latter axis is made to pass through the centre of another circle, to the plane of which it is perpendicular.

When the instrument is in adjustment, the plane of the latter circle is horizontal, and therefore that of the former circle vertical. The telescope is therefore capable of motion in a vertical plane, and also the plane in which it moves is capable of motion about a vertical axis. The angles described in each case are indicated by verniers.

The adjustment is usually performed by three screw legs on which the lower circle rests, and the horizontality of the circle is determined by spirit-levels. The upper circle has also a spirit-level to determine the reading when the telescope is exactly horizontal.

When a heavenly body is observed, supposing the instrument to be perfectly adjusted, the reading of the upper circle gives the altitude; and when the direction of the meridian is known, the lower circle gives the azimuth.

The ordinary method of finding the meridian direction is by taking equal altitudes of a star before and after its meridian passage.

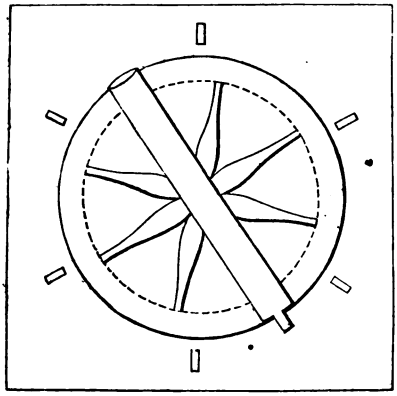
The star is observed some time before passing the meridian, and the reading of the azimuth circle noted. It is observed again when it comes back to the same altitude after passing the meridian, and the reading of the azimuth circle again noted. Now the star's path is exactly symmetrical with respect to the meridian, therefore its azimuth must be the same in the two positions, only on different sides of the meridian. Therefore the reading of the azimuth circle when the telescope points to the meridian must be half-way between the two observed readings. If R_1 be the reading at the first observation, R_2 at the second, $\frac{1}{2}(R_1 + R_2)$ will be the meridian reading; and this being ascertained, the azimuth of any body afterwards observed will be easily found.

By this means bodies may be observed at all points of the sphere of observation, whenever a firm support can be found for the instrument. As however observations of bodies on the meridian are the most important, it is usual in observatories to employ altitude circles which are restricted to moving in the plane of the meridian. The supports of such instruments may be made more firm, and there being only one axis of revolution, the general stability is more perfect.

The axis of revolution is usually of the form of a truncated cone, and works in a socket which is firmly fixed in a wall. Hence the instrument is called the *Mural Circle.* Its importance requires a separate description.

41. The *Mural Circle* consists of an astronomical telescope furnished with vertical and horizontal cross-wires, firmly fixed to a circle graduated from 0° to 360°. It revolves about a horizontal axis very strongly supported. The reading-off is made by means of the microscope above described.

Besides the horizontal and vertical fixed wires in the focus of the eye-piece, there is a moveable horizontal wire, called the *micrometer* wire, which is made to move parallel to itself by means of a screw. This contrivance enables the observer to find the difference of altitude between two points which are



in the field of view at the same moment, as for instance two stars near together: for it may be ascertained, by using a known distance, through what space the wire is moved by each turn of the screw, and the screw-head being graduated, the exact distance of the moveable wire from the fixed horizontal wire may be determined. One means of ascertaining the interval due to each turn of the screw is by measuring in this way the Sun's diameter, which is given for every day in the *Nautical Almanac*. When the Sun enters the field of view, the telescope is so placed as that the Sun's limb appears just to touch the fixed wire, and the micrometer wire is made to touch the other limb. Then the moveable wire is brought into coincidence with the fixed wire, and the number of turns and parts of a turn necessary to bring it into such coincidence is noted. This number corresponding to a known interval, the value of each turn of the screw is easily found by proportion. When the field of view is too small to include the whole disc of the Sun, as is the case in large instruments, two known stars near together, having nearly the same right ascension, may be used instead.

42. In order to observe altitudes with the Mural Circle, it is necessary, as in the case of the instruments described before, to have some means of knowing the reading when the telescope is exactly horizontal or exactly vertical. It is not necessary that the reading should be zero at either of those points, because we can always apply the required correction to the observed angle by simple addition or subtraction. If, for instance, we know the reading when the telescope is horizontal, and then observe a star as it crosses the meridian, the difference between the readings will give the angle through which the telescope has been moved from the horizontal direction to that of the star, and therefore the altitude of the star.

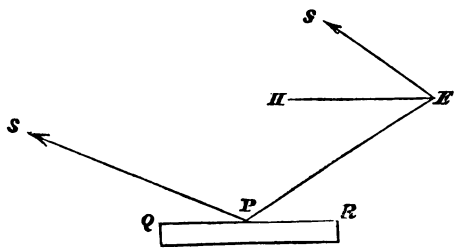
We shall explain only two methods out of several which are in use to find the horizontal or vertical position.

A trough of mercury is placed immediately below the centre of the circle, and the telescope is directed towards it, with the line of collimation pointing nearly vertically downwards. An image of the cross-wires is formed by reflection in the trough of mercury by means of pencils of parallel rays; for the cross-wires are in the focus of the object-glass, and therefore pencils diverging from them emerge parallel from the object-glass upon the mercury. The emergent pencils therefore from the mercury by which the reflected image is formed are parallel, and falling on the object-glass converge to its focus. Thus the image of the cross-wires is formed close to the cross-wires themselves. When the reflected image appears to coincide with the cross-wires, the line of collimation is perpendicular to the surface of the mercury, or accurately vertical; and from the reading of the instrument when in this position, it is easy to find that when the telescope is horizontal.

The trough of mercury above referred to is, however, often employed in a different manner, which, besides giving the horizontal reading, has the advantage of shewing the altitude of the observed star at once.

If an observer be so placed as to see the image of a star reflected from a trough of mercury, the pencil of light by which it becomes visible to him makes with the surface of the mercury the same angle as the incident pencil. Therefore the angle of depression of the image of the star below the horizontal line is equal to the angle of altitude of the star itself from the surface of the mercury. And the altitude of the star from the observer's eye is the same as its altitude from the surface of the mercury; therefore the angle between the directions of the star and its reflected image is twice the altitude of the star.

If *QPR* be the trough of mercury, *E* the eye, *PS* the direction of the star, then the angles *QPS, RPE* are equal to



one another and to the angle *PEH*. Now *ES* parallel to *PS* is the direction of the star as seen from *E*, and by parallel lines the angle *HES* is equal to *QPS*, and therefore to *HEP*. Therefore *PES* is twice the altitude of the star.

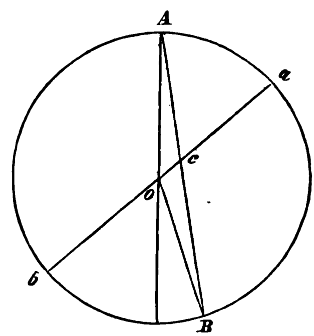
If we could observe the star at the same time by direct vision and by reflection, we could thus immediately determine the altitude and the horizontal reading of the instrument: for if R_1, R_2 were the readings so obtained, $\frac{1}{2}(R_2 - R_1)$ would be the altitude, and $\frac{1}{2}(R_1 + R_2)$ the horizontal reading, being that corresponding to the position of the telescope when half-way between the two observed positions.

Although it is impossible to make the observations simultaneously, yet the same result may be obtained by the following method, which is founded on the principle, that just as the star *culminates*, and changes its motion from ascending to descending, there is very little change of altitude for some few seconds.

First the telescope is directed to the trough of mercury at an angle of depression nearly equal to the altitude of the star, which must be approximately known, so that it may be secured that the star shall pass through the field of view. The reading of the limb in this position is accurately noted down. When the star in its transit has approached near the vertical wire, the micrometer wire is brought into coincidence with it, and immediately the instrument is directed towards the star itself, which is observed in the usual manner just after it has passed the meridian. The reading of the micrometer wire in the first case has to be added to or subtracted from that of the limb, according as the star has passed above or below the horizontal wire; and then this corrected reading, being subtracted from the second reading, will give the double altitude.

43. In observations by the Mural Circle, it is usual to employ six reading-microscopes attached to the wall by which the axis is supported, and at equal angular intervals from one another. One object of this is to get rid of the errors which arise from *imperfect centering*, that is, from the geometrical axis about which the instrument revolves, not passing accurately through the centre of graduation—a source of error which mechanical skill cannot entirely remove. We will briefly explain the principle of this compensation.

Let O be the centre of graduation, C a point in the axis of the instrument, A an index. Let the instrument be turned till the point a coincides with the index. The angle through which the telescope has been moved is ACa, but the angle measured by the graduation is AOa—a less angle than ACa. Suppose B to be another index opposite to A, b the point of the arc which is brought into contact with it when a comes to A.



The angle indicated by the limb in this case is BOb, a greater angle than BCb.

The angles, therefore, shewn by the opposite indexes are affected with opposite errors, and the excess of the one, namely the angle OBC, is equal to the defect of the other, that is OAC, for the triangle OAB is isosceles; therefore if we take the half sum of the two angles, it will be equal to the angle through which the telescope has actually moved.

From this it appears that two microscopes at opposite points would be sufficient to correct the error of false centering, supposing the graduation to be perfect, and the reading off liable to no error. As it is, however, there is a liability to error on both these grounds, to correct which three pairs of opposite microscopes are used, and the mean of all the readings taken as the result of the observation.

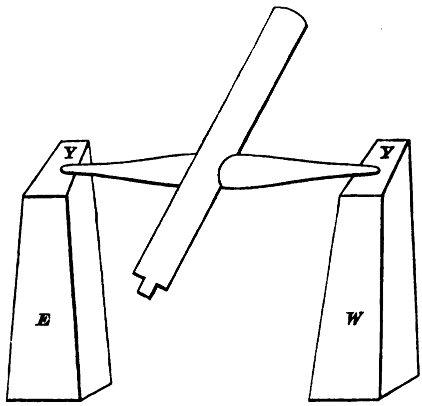
And, as the absolute values of the observed angles are of no consequence, it being only necessary to measure the angle through which the telescope must be turned from one position to another, this method is just as well suited to the purpose as if there were a single index giving the direct result at once.

The student should carefully distinguish between the two kinds of error which are corrected by means of the six microscopes. The error of centering, if any, is entirely and certainly got rid of by having the microscopes in pairs at opposite points, as the investigation above given shews. The errors of graduation and reading off are not of the same definite nature, nor can we in the same way make exact compensation for them. The errors of graduation arise from the limb of the instrument not being accurately divided into equal parts. There is certainly a compensation in this respect: for if one division is too small, there must be some other too large, in order that the circumference may contain the proper number; but we cannot tell where this compensation is to be applied. Errors of reading off are still more purely accidental, being such as arise from the defects of the microscope, or unavoidable mistakes on the part of the observer. The only mode of compensation applicable to these errors is that of having several indexes, it being improbable that all the accidental errors should be in the same direction, and therefore it being to be expected that they will correct one another.

If all the errors were in the same direction, that is all in excess, or all in defect, the mean result would be affected by the mean of the errors; but if some be in defect and some in excess, the gross result will be affected by the excess of the errors in one direction over those in the other, and this divided by the number of observations will be the only error in the mean result. If as many errors were in defect as in excess, the whole error would be very small, and on this principle the result of the mean of a number of observations is more likely to be true than that of a single one. This principle should be carefully recollected, as it is of constant and very important application in Astronomical Observations.

44. The azimuth and altitude instrument, it will be seen, is theoretically sufficient to determine the place of a body in the sphere of observation, supposing it to be accurately adjusted, and the meridian correctly ascertained. It has, however, appeared that the general determination of altitude and azimuth at any time is not of much importance in the greater number of observations, so that an altitude circle limited to the plane of the meridian is sufficient for ordinary purposes, besides possessing the great advantage of stability. The mural circle is obviously a modification of the altitude and azimuth instrument, inasmuch as it determines the azimuth to be zero at the time of the observation. Hence, with the assistance of a clock, such an instrument would give the means of determining the exact time at which a body passed the meridian—a very important observation for many purposes. It has been explained, however, that the change of altitude just at the culminating point is for a short time nearly imperceptible, and it is obvious that as the whole motion of the body is there parallel to the horizon, its motion in azimuth is most rapid; consequently a much more accurate adjustment is required for observing the exact moment of transit across the meridian than for determining the altitude: on this account it is found more convenient to employ a separate instrument for the purpose of observing transits. This we shall now proceed to describe.

45. *The Transit Instrument.* This instrument consists of an astronomical telescope, so moveable about a horizontal axis



that when it is in perfect adjustment, its line of collimation moves in the plane of the meridian.

The axis, which consists of two strong arms terminating in two cylindrical pivots, rests on two equal supports, called Y's, (from their resemblance in form to that letter), which supports are firmly fixed in two strong piers standing east and west. One of the Y's is capable of a horizontal and the other of a vertical motion, given to them by means of screws.

In the focus of the eye-piece of the telescope are five or seven vertical wires, and one horizontal wire. These wires are capable of being illuminated by a lamp placed in one of the arms, which is hollow, so that observations may be made at night-time.

There is also a small altitude circle furnished with a spirit-level, by means of which the meridian altitude of the body to be observed is approximately known, and the instrument may be so directed that the body shall pass through the field of view.

When this arrangement has been made, the observer having the clock beside him watches for the appearance of the star in the field of view. As soon as he sees it, he observes the position of the seconds-hand of the clock, and then returns to the telescope, in which the star is now approaching the first of the wires. He counts the beats of the clock, and remembering the number of that with which he began, he notes the time of transit over the first wire. This he can always do accurately to the nearest second, but an expert observer can obtain greater nicety still.

The star moving through the field of view will not generally coincide exactly with the wire at a particular beat of the clock, but between two adjacent beats. At the former of these beats it has not yet arrived at the wire, at the latter it has passed it by a certain interval. Therefore a fraction of a second has to be added to the time given by the former beat. Now a practised observer compares in his mind the apparent distances from the wire at the two adjacent beats, and thus estimates roughly the proper fraction to be added. In this way transit observations are given to a tenth of a second.

The time of the passage of the first wire being noted down, the successive transits over all the wires are similarly observed in succession, and the mean time of the whole is taken for the actual transit over the meridian.

This is not only a mode of getting rid of errors of observation according to the general principle of compensation, but it gives additional opportunities of observation when the atmosphere is cloudy, there being a greater probability of seeing the transit over one of five or seven wires than over only one wire. It not unfrequently happens that the time of passing one or more of the wires cannot be observed, owing to temporary obscuration of the star. In this case, when the mean time of transit is to be found, allowance must be made for the missing wire or wires.

46. In order that the observed time of transit may be that of the actual meridian passage, it is necessary that the line of collimation should move in the plane of the meridian; or, according to the principle which has been before explained with respect to astronomical instruments in general, that we should have the means of detecting and allowing for any deviation from the right position.

The line of collimation has before been defined as the line joining the optical centre of the object-glass with the point of intersection of the vertical and horizontal wires, in cases where there is only one vertical wire. This definition must be slightly modified in the case of the transit instrument. If the time of passing the middle vertical wire coincided exactly with the mean of the times over all the wires, then the line joining the centre of the object-glass with the intersection of the middle vertical wire with the horizontal wire would be the line of collimation. But as these times do not necessarily coincide, though they are very nearly the same, the true line of collimation, over which the star passes at the observed time of transit, is the line joining the centre of the object-glass with the point where the horizontal wire is intersected by an imaginary wire nearly but not quite coincident with the middle vertical wire. This is sometimes called the *mean of the wires.*

For our present purposes, however, we may consider the mean of the wires to coincide with the middle wire.

The deviations of the line of collimation from its true position are of three kinds.

(1). It may not be perpendicular to the geometrical axis about which the telescope revolves.

(2). The axis of the telescope may not be horizontal.

(3). The axis may not point exactly east and west.

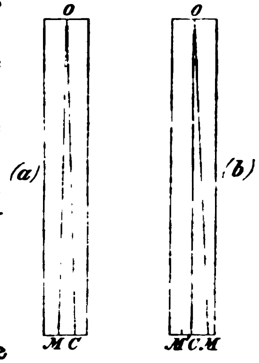
In the first case, the line of collimation describes a conical surface with a very large vertical angle, instead of lying in one plane; and therefore the series of points in the sphere of observation to which the telescope is successively directed lie in a small circle instead of a great circle. In the second case, supposing each error to exist alone, the points to which the telescope is directed lie in a great circle, but in one which is inclined to the horizon. In the third case the great circle is vertical, but does not coincide with the meridian.

In general all these errors coexist, but as they are very small—the instrument being kept nearly in adjustment—they may be investigated and allowed for separately. They are called respectively, the *error of collimation*, the *error of level*, and the error of *deviation from the meridian*. We shall explain here how their existence is detected, but it will be beyond our present province to enter into the calculation of their amount and the corrections to be made for them in the time of observation. As it is necessary that the errors should be small, in order that the corrections may be applied separately for each, the instrument is furnished with the means of adjustment, by which it may be placed approximately in its right position. The line of collimation is adjusted by an apparatus for moving the system of cross-wires horizontally across the field of view. The axis is adjusted by means of the screws attached to the *Y* sockets.

(1). The existence of the collimation error is ascertained by reversing the axis, that is, by lifting the telescope off its bearings, and replacing it with each arm on the opposite pier to that which it before occupied. If the line of collimation is accurately at right angles to the axis, its direction will be the same after this change as before; but if not, whatever angle it made with the axis on one side, it will make the same angle with it on the other side. The test therefore of true adjustment will be that the same distant object will appear to coincide with the intersection of the horizontal and middle vertical wires before and after the reversing of the axis.

Suppose the instrument to be out of adjustment.

Let *O* fig. (*a*) be the centre of the object-glass, *OC* perpendicular to the axis of the instrument, *OM* the line of collimation, *M* being the intersection of the horizontal and middle vertical wires.



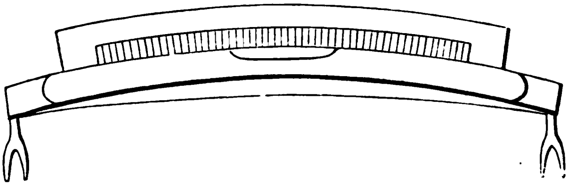
When the telescope is reversed, the direction of *OC* will not be changed, since that of the axis to which it is perpendicular is the same as before; but *OM* will make the same angle with *OC* on the other side of it, as is represented in fig. (*b*). Thus, if a distant object appear to coincide with the intersection of the wires before reversing, it will not appear to coincide with it after reversing, for its direction is in the line MO produced, since its image is formed at M, and therefore in the second figure its image will be formed at M', as far from C on the left, as M is on the right. Therefore it is only when the line of collimation coincides with OC, that the same distant object can coincide with the intersection of the lines before and after reversing.

If the distance MM' can be measured, it gives twice the collimation error, or twice the space through which the cross-wires must be moved to correct the error. It is not absolutely necessary that the distant object should exactly coincide with the point M at the first observation, for the same result is obtained by measuring the difference of apparent distances between it and the intersection of the wires before and after reversing, that difference being the space MM', or twice the error. It should be remembered that the principle of the method is the shifting of the point M to the same distance as before from C on the other side of it, the point C and the position of the image of the distant object not being affected by the reversing of the axis.

The distant object must obviously be terrestrial, as it must not alter its position. It must also be nearly due south of the observer. It is usual in observatories to erect a small well-defined object for this purpose at a considerable distance, and in such a position as to be included in the field of view of the telescope. The meridian mark of the Cambridge Observatory is a small cross of wood on the tower of Grantchester church.

(2). The *error of level*. This error is determined by means of a spirit-level, an instrument which it may be as well here to describe in detail, although its general principle and uses are well known.

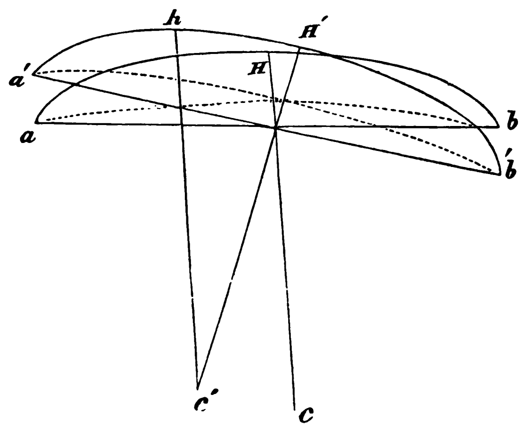
The *Spirit-level* consists of a hollow glass tube of uniform bore, nearly filled with spirits or sulphuric ether, and closed at both ends. The tube is not quite straight, but has a slight uniform curvature, such as would result from its being a small part of a circular ring of very large diameter. When the level is placed in such a position that the convexity of the tube is uppermost, the upper surface of the fluid being necessarily



horizontal, it follows that the air-bubble above the fluid occupies the highest portion of the tube, or to speak more accurately, that the highest point of the tube is half-way between the two extremities of the bubble.

The common use of the level is to determine when the line joining the two lower extremities of the tube is horizontal; and this is done to a sufficient degree of exactness by observing when the bubble stands about the middle of the tube: for if the radius of the circular ring, of which the tube may be supposed to form part, be very large, the position of the bubble will be affected by a very small change of inclination in the tube.

To shew this, let *aHb* be a vertical section of the tube, *ab*



the straight line joining its lower extremities, *C* the centre of the circle of which *ab* is an arc; *H* the highest point.

Let the inclination of the level be changed, and *a'*, *b'*, *H'*, *C'*, the new positions of *a*, *b*, *H*, *C*.

Draw $C'h$ vertical. h is now the highest point, and the arc hH', through which the middle point of the bubble will have shifted is the arc subtended at the centre by the angle through which CH and consequently ab has been turned.

Consequently, the greater the radius, the greater will be the space through which the bubble will move for a given change of inclination in ab, or in technical language, the greater will be the *sensibility* of the instrument.

It follows from this, that if the arc ab be truly circular, the angle through which the base ab is moved may be measured by means of a scale.

The instrument as applied to astronomical purposes is furnished with such a scale accurately graduated, and is calculated to detect variations of level of a very minute order.

The level used for the transit instrument is furnished with two equal and similar legs ending in notches, just so far apart as to rest on the cylindrical extremities of the telescope axis, and of such a length that the tube lies across the telescope without touching it.

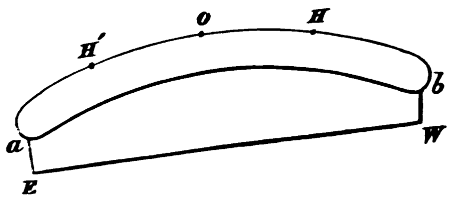
There is a smaller level placed at right angles to the other in order to ensure that the convexity of the larger tube should be exactly uppermost.

The scale is graduated in both directions from the zero point, which ought to be the highest point when the axis on which the legs are placed is horizontal.

Thus, if the adjustment of the scale were perfect, the horizontality of the axis would be denoted by the readings of the two ends of the bubble being the same.

This adjustment however is not necessary, even if it could be made without risk of error; for we may obtain a perfect test of horizontality by reversing the level, or placing it with its legs on the opposite extremities of the axis to those which they occupied before. In this case, supposing the axis perfectly horizontal, and its extremities as well as those of the legs of the level to be perfectly equal and similar, the bubble will evidently assume the same position as before. But if, on reversal, the bubble assumes a different position, the space through which it passes gives twice the angle by which the axis deviates from exact horizontality, the middle position between the two being the position the bubble ought to have when the axis is exactly horizontal.

Let EW be the axis, $aOHb$ the level; H the highest point,



O the point which would be highest if EW were horizontal. Then the arc OH measures the inclination of the axis; and if the level be reversed, the point a being placed at W and b at E, the same inclination will be measured by an arc OH' on the other side of O.

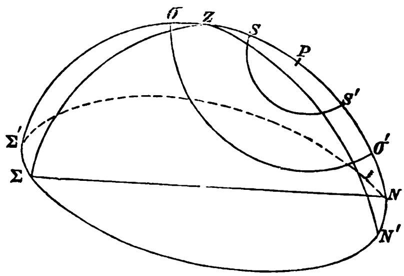
The middle point of the bubble will therefore move from H to H', where $OH' = OH$. The scale gives the means of determining the middle point of the bubble, by taking the mean between the readings of the two ends.

In the figure, the curvature of the level and inclination of the axis are purposely very much exaggerated for facility of explanation.

In order to adjust the axis, that extremity of it which is capable of vertical motion must be raised or lowered, according as the bubble after reversal approaches or recedes from that end of the level which at first coincided with the adjustible extremity of the axis.

(3). *The error of deviation* is determined by observing the upper and lower transits of a circumpolar star, that is, of a star which does not set, but passes the meridian below the pole. If the instrument is in complete adjustment, so that the telescope moves in the meridian plane, these transits occur at exactly equal intervals of time, since the meridian bisects all the diurnal circles described by the stars: but if there be a deviation error, the observed transits of the star being not across the meridian but across another vertical great circle, do not occur at equal intervals, but the time from the upper transit to the lower is either greater or less than the time from the lower to the upper, according to the direction in which the error lies.

Let P be the pole, Z the zenith, ΣZN the meridian,



$\Sigma' ZN'$ another great circle, meeting the horizon in N', Σ'. First, let S be a star whose polar distance PS is less than PZ. It will cross the meridian at S and S'.

If Σ' be on the west side of Σ, as represented in the figure, or the deviation error *west*, the star will come to the great circle $\Sigma' ZN'$ just after it has passed the meridian at the lower transit, and it will pass the same circle again just before it arrives at the meridian at the upper transit: consequently, if the telescope be so placed as to command the great circle $\Sigma' ZN'$ instead of the meridian, the observed time from the lower transit to the upper will be less than it ought to be, and of course the time from the upper to the lower will be greater than it ought to be. Hence, any inequality in the times will detect an error of deviation, which is either to the *east* or to the *west*, according as the time between the upper and lower transit is *less* or *greater* than that between the lower and the upper.

Again, let σ be a star whose polar distance is greater than PZ. It will pass the meridian at σ and σ', the former point being south of the zenith.

As before, it will come to the circle $\Sigma' ZN'$ after passing the meridian at the lower transit; but as the meridian and the circle $\Sigma'ZN'$ intersect in Z, it will not come to the latter circle again till after it has passed the meridian at the upper transit.

Hence both the observed transits will be later than the true.

Since however σZ is a smaller arc than $\sigma' Z$, the one being the sum and the other the difference of the arcs $P\sigma$ and PZ, the space between the meridian and the circle $\Sigma'ZN'$ is less at σ than at σ', and therefore the observed upper transit occurs nearer the true time than the observed lower transit. Thus the time from the lower to the upper transit is less than it should be, and the result is the same as before.

In general it is advisable to select a star for observation which, as in the first case, makes both its transits on the same side of the zenith, for then the inequality of the times is most perceptible. In fact, the nearer the star is to the pole (its diurnal motion being slower), the greater is the effect on its times of transit of a given deviation from the meridian. On this account the pole-star, commonly called *Polaris*, which is about a degree-and-a-half from the north pole, is very often observed for the deviation error.

It is obvious that in or near the equator this method is inapplicable, owing to the pole being nearly in the horizon. In this case the same result may be obtained by observing the transits of two stars, one of which passes the meridian near the zenith, and the other near the horizon. It is necessary, however, that the difference of right ascension of the two stars should be known beforehand. Then the time intervening between the true transits of the two stars will be known, being the same part of 24 sidereal hours that their difference in right ascension is of 360 degrees. If therefore the instrument is in adjustment, the interval between the observed transits will be the time so obtained. But if the telescope command the circle $\Sigma'ZN'$ instead of the meridian, the star passing near the zenith will cross this circle much about the same time as it crosses the meridian, while the star near the horizon will have a much larger space to pass through between the two circles. Thus the time of transit of the first star will be much less affected than that of the second, and therefore the interval between the transits will be altered by the error. Thus the error of deviation may be detected when there is no opportunity of observing a circumpolar star.

47. *The Astronomical Clock.* It is evident from what has preceded, that a clock which can be depended upon is an

essential part of the furniture of an Observatory. The contrivances by which the necessary accuracy is obtained in clocks used for astronomical observations distinguish them so completely from the coarser time-pieces which are sufficient for ordinary purposes, that the *astronomical clock* may be looked upon as a separate instrument, and its description may properly occupy a place in an astronomical treatise.

The moving power of the clock is a weight, which is more suitable than a spring, on account of the uniformity of its action. There is no striking apparatus or superfluous machinery of any kind to interfere with the regularity of the working. The pendulum, which vibrates seconds, is made to move with its extremity in a cycloidal arc, and is fitted with a contrivance by which the effects of change of temperature on the rate of going are obviated.

It will be recollected that the period of vibration of a simple pendulum depends on its length. Such a pendulum obviously can only exist in theory, as it is supposed to consist of a heavy particle at the end of a rod without weight. In opposition to this, all pendulums in use are called compound pendulums, and it may be stated, without entering into farther explanations, that the period of vibration of any compound pendulum depends jointly on its form and on the distance of its centre of gravity from the axis about which it revolves. The ordinary pendulum, as is well known, consists of a rod on which slides a piece of metal, technically called a *bob*, capable of being raised or lowered by means of a screw. The principal weight being in the *bob*, the centre of gravity of the whole is thus varied in position without much change of form, and the period of vibration is increased or diminished at pleasure. Thus, theoretically, a clock might be regulated to any degree of accuracy, the action of the works on the pendulum being uniform. Since, however, all known metals expand with heat and contract with cold, it is found that the time of vibration of a common pendulum changes with the temperature, the length of the rod being greater in warm weather than in cold. To obviate this, instead of the ordinary bob, a cylindrical glass vessel is used, containing a large quantity of mercury. When the rod expands so as to lower the bottom of this vessel, the mercury also expands and rises in the vessel. The centre of gravity of the mercury is therefore farther from the bottom of the vessel than it was before, and this effect compensates that of the lengthening of the rod; so that when the quantity of mercury is properly apportioned to the weight of the pendulum, the position of the centre of gravity of the whole is unchanged, and the time of vibration is unaffected.

By this and other contrivances the astronomical clock has been brought to great accuracy. The grand desideratum is *uniformity* in the rate of going: for when this is secured we may regulate to any required rate, or by observing the difference between the actual rate and the required rate, we may obtain the means of applying corrections to the observed times. The latter is the method commonly employed. The time lost or gained in a day is determined by observation, and is technically termed the *rate*. The *clock error* at any time is the amount to be added to or subtracted from the observed time in order to get the true time. When the clock error and rate are known, the true time at any subsequent observation may be found, by adding to the observed time the error and the amount of the rate multiplied by the number of days and parts of a day intervening since the error was determined. In practice the clock is so regulated as to have a small losing rate, and is set a little too slow. Thus the corrections are additive. The error is never allowed to exceed a small amount.

The *error of rate* is the difference between the rates of two successive days. If the clock were perfect there would be no such error, and in the present state of mechanical skill it is kept within very small limits. An habitual error of rate amounting to a second would entirely condemn a clock for astronomical purposes.

The astronomical clock is usually set to sidereal time, that is, so as to mark 24 hours between two successive transits of a star. Thus it is independent of the variations in the Sun's position. The sidereal day begins when the vernal equinoctial point, called the *first point of Aries* (from its being the point where the Sun enters the zodiacal sign Aries), passes the meridian. This occurs about noon when the Sun is in equinox, and earlier every subsequent day by nearly four minutes till it comes round to noon again. It is not, however, to be supposed that any visible point shews by its transit when the sidereal day begins, the first point of Aries being an imaginary point indicated by no appearance in the heavens, and only geometrically determined by its being at the intersection of the equator and the ecliptic. If such a point existed, the sidereal clock might be set at once by observing its transit. The actual mode of setting it is, however, a matter of much greater difficulty, which we are not yet far advanced enough in the subject to explain.

The *rate* is easily found by observing two successive transits of the same star; and for this purpose it is not necessary that the transit instrument should be in very accurate adjustment, only that it should not be moved between the two observations, for a fixed star occupies exactly 24 sidereal hours in revolving from any given point to the same point again.

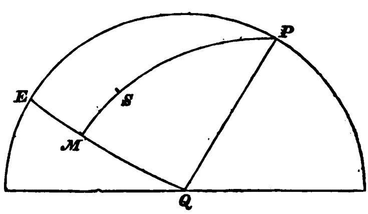
A succession of such observations would detect any error of rate.

48. *The Equatoreal.* This instrument consists of two graduated circles revolving about axes at right angles to one another in the same manner as those of the altitude and azimuth instrument. They are however differently placed, the lower circle having its plane parallel with that of the celestial equator, and consequently inclined to the horizon at an angle equal to the co-latitude. The axis, therefore, which bears the upper circle points to the pole, whereas that of the altitude and azimuth instrument points to the zenith.

The equatoreal, if in exact adjustment, would determine at once the declination of an observed body. For, suppose the telescope to be directed to a star, so that the cross-wires appear to coincide with it; the reading of the upper circle gives the angular distance of the star from the plane of the lower circle, that is, from the celestial equator. On this account the upper circle is called the *declination-circle*. Supposing also the zero point of graduation of the lower circle to be that marked by the index when the telescope is directed to the meridian, the reading of the lower circle will give the angular distance of the star from the meridian at the time of observation, measured along the celestial equator. This angle will be the same as that between the meridian and the declination-circle of the star. It is commonly called the *hour-angle* of the star, and the circle on which it is measured is called the *hour-circle*.

Let P be the pole, EQ the celestial equator, S a star.

The equatoreal gives the arcs SM and EM, the former of which is the declination, and the latter is equal to the angle EPM, included between PE the meridian, and PS the declination-circle of the star.



EPM is called the hour-angle, because whatever part it is of 360°, the same part of 24 hours is the time of S from the meridian, that is, the time S will take in getting to the meridian, or the time by which it has passed the meridian, according as it is on the east or the west of the meridian at the time of observation.

In practice the equatoreal is not much used for accurate determinations of declinations and hour-angles, which may be more certainly made by other methods. Its chief use is in giving approximate positions of bodies observed out of the meridian, and in such observations as occupy any considerable time, as those of double stars by micrometers; for by turning the telescope about the polar axis, which is parallel to that of the diurnal motion of the heavens, a star may be kept continually in the field of view, the space commanded by the telescope in its revolution being in fact the star's diurnal path.

In the case of large instruments the telescope is made to move in this manner by clockwork, properly regulated, so that the observer may devote his whole attention to micrometrical measurements, or whatever other object he may have in view.

49. *Hadley's Sextant.*

From the preceding descriptions it is evident that the instruments which have hitherto occupied our attention depend for their efficiency on the firmness of their supports, without which all modes of adjustment would be vain. Such instruments, therefore, are inapplicable to a very large and important class of observations, namely those made at sea. In fact, the roughest determination of the position of the observer by means of the heavenly bodies, would be perfectly hopeless except in the calmest weather, if we had no instruments to depend on excepting such as have been already described.

The instrument of which a representation is subjoined is found completely to answer all nautical purposes. It is called by the name of its reputed inventor, Hadley, but we have Sir J. Herschel's authority for ascribing it to Newton.*

It consists of a strong frame in the form of a sector of a circle, including, as the name imports, one-sixth of the circumference. The two extreme radii are firmly braced together, as represented in the figure. A moveable radius, having a vernier at its extremity to read off the divisions on the arc, carries a small mirror called the *index-glass*, which stands upon it at right angles to the plane of the instrument, and in such a position as to be bisected by the axis of revolution produced. On the extreme radius, and facing the index-glass, is fixed the *horizon-glass*, which is half silvered, the other half being left so that objects may be seen through it. It is of about the same size as the index-glass, and is fixed in such a manner that it is parallel to the index-glass when the index is at the opposite extremity of the arc. Its plane is therefore perpendicular to that of the instrument, and nearly parallel to the extreme radius on the other side.

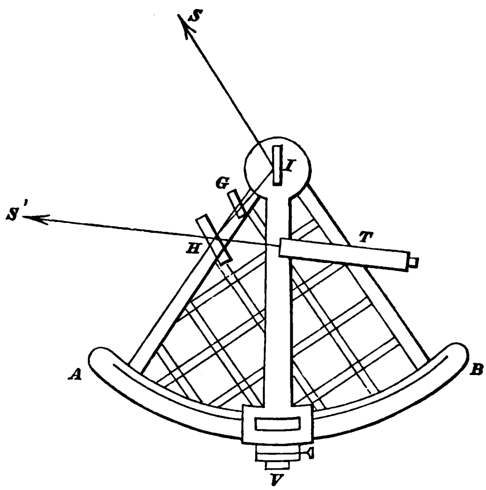
A telescope is so fixed that its line of collimation passes through the centre of the horizon-glass, meeting its surface at the same angle as the line drawn from the same point to the centre of the index-glass. Hence a ray of light reflected from

* Outlines of Astronomy, p. 115.

the centre of the index-glass to that of the horizon-glass is again reflected along the line of collimation of the telescope.

In the accompanying figure, I is the index-glass, H the horizon-glass, T the telescope, and V the vernier-index. The object of the instrument is to measure the angle between two distant objects.

Let S be such an object. If the light from S fall upon the index-glass at I in such a way that the lines SI and HI



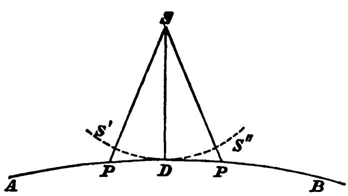
make equal angles with the surface, it will be reflected at I and H, and will emerge from the horizon-glass in the direction of the line of collimation of the telescope, so that an image of S will be seen through the telescope by reflection at the silvered part of the horizon-glass. Since there have been two reflections, the deviation of the course of the light, that is the angle which the line of collimation makes with SI, is double the angle of inclination of the mirrors. And if another object S' be at the same time in the line of collimation produced, the angle between IS and HS', or between the directions of S and S', is twice the angle of inclination of the mirrors. Therefore, if the observer can bring the image of S, seen by reflection at I and H, to coincidence with that of S' seen directly, which may be done just at the common edge of the silvered and unsilvered parts of the horizon-glass, he may conclude that the angular interval between S and S' is twice the angle by which the index-glass is inclined to the horizon-glass.

If then the limb be so graduated that every division of one degree shall count for two, and the zero point be that marked by the index when the index-glass and horizon-glass are parallel, the vernier will give the exact angular interval required; for the arc passed over by the index is the measure of the angle through which the index-glass revolves.

In this way a sextant is capable of measuring an angle of 120°. It is obvious that when the index-glass is parallel to the horizon-glass, there is no deviation, or SI is parallel to the line of collimation. Hence, when the index is at zero, the image of S seen by reflection ought to coincide with that seen directly through the unsilvered part of the horizon-glass, (S being a *distant* object, and therefore its direction from H being the same as from I). This gives an easy test of the right adjustment of the instrument. The horizon-glass is furnished with a screw by which it may be turned on its axis through a small angle, and so may be brought to parallelism with the index-glass when the index marks zero. In practice, however, it is usual to bring the direct and reflected images of some well-defined object to coincidence by moving the index only, and then to observe the reading of the limb, which may be applied as a correction to the angles afterwards observed. This is called the *index-error*, and must be added to or subtracted from the angles afterwards observed, according as the index was behind or before the zero point at the time of coincidence of the images.

In this way the angle between two stars may be observed, or the angle between the moon and a star, the image of the moon's limb as seen by reflection being made to touch the direct image of the star. The latter observation, as will afterwards appear, is of great use in navigation. The principal use, however, of the sextant at sea is in taking altitudes, the reflected image of the observed body being brought into contact with the sea-line or offing seen through the unsilvered part of the object-glass.

The following is the usual mode of finding the altitude of a star. The observer holding the sextant in his right hand, with its plane vertical, directs the telescope to the star, the index being in such a position that the mirrors are parallel. The reflected image therefore coincides with that seen by direct vision. The observer then moves the index gradually forward with his left hand, causing the reflected image to leave the direct image, and, by the principle of the instrument, to descend through twice the angle described by the index. The observer follows the image in its descent by gradually lowering the telescope until at last the sea-line appears in the field of view. If the plane of the instrument were accurately vertical, it would be sufficient to bring the star into coincidence with the middle point of the sea-line in the horizon-glass, and then the observed angle would be the altitude of the star above the visible horizon: but if the plane of the instrument be not vertical the observed angle will be too great, the distance of the star from the point in the offing vertically below it being less than its distance from any other point.



Thus, if S be the star, AB the sea-line, it would not do to observe the distance of S from any point P in the offing, the altitude required being SD, where the arc SD is perpendicular to AB.

There is, however, a very easy practical method of determining the right altitude.

If the plane of the instrument be turned through any angle, without altering the inclination of the index-glass to the direction of the star, or moving the index, the reflected image of the star will still remain visible, and will appear to describe a circular arc, since it is always at the same angular distance from the star itself. This kind of motion is easily communicated to the instrument with the hand; and if the index be moved till the arc described by the reflected image appears just to touch the sea-line, the resulting angle will be the true altitude above the visible horizon.

The altitude of either limb of the sun or moon may be found in the same way by making it appear just to sweep the horizon as the instrument is moved backwards and forwards. In order to take off the glare of the sun, the sextant is furnished with a set of glasses of different degrees of darkness, which may be applied at pleasure between the index-glass and the horizon-glass. There is also generally a shaded glass beyond the horizon-glass, to diminish the light reflected from the sea.

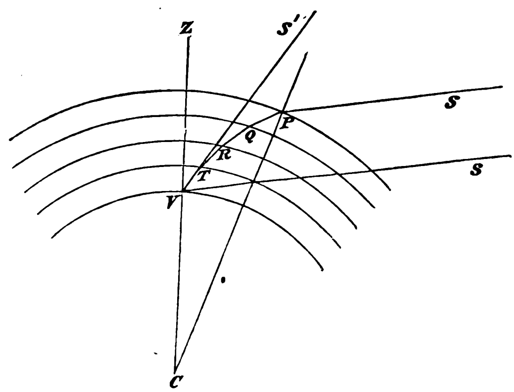
50. The object of all the instruments above described is to give the place of a body at any time, or to determine the time at which a body occupies a place before determined on, in the sphere of observation.

The *true* place of the body on the sphere of observation is that point in which the sphere is intersected by a straight line passing through the eye and the body. The *observed* place is that point in which the sphere of observation is intersected by a straight line drawn from the eye in the *apparent* direction of the body. The observed place and the true place will coincide if light proceeds from the body to the eye in straight lines, for then the apparent and the true direction will be the same. But if the light proceed from a body in a curved instead of a straight line, the apparent direction will be that of the tangent to the curve just as it enters the eye, for the eye is only affected by the light just at the end of its course, and in that case the apparent direction will not necessarily coincide with the true direction.

This is in fact the real state of the case. The Earth's atmosphere deflects the light from the heavenly bodies out of the straight line in which it would proceed if it continued in vacuo, and therefore their apparent directions are not the same as their true directions.

The atmosphere is a refracting medium which surrounds the Earth on all sides, its outer surface being nearly spherical, and its density decreasing with its distance from the Earth. A ray of light falling on such a surface undergoes continual refraction throughout its passage, and describes a curvilinear path. This may be shewn by taking a supposed case of a series of uniform media of finite breadth, each of less density than that which it envelopes, and then passing to the limit when the number of such media is indefinitely increased and their breadth indefinitely diminished.

Let a ray of light from a body S fall upon a series of such media, they being concentric spherical shells of small thickness



whose common centre is C. It will be refracted at each surface and proceed in a straight line through each medium, and therefore it will pursue the polygonal path $PQRTV$. If an eye be situated at V, the direction in which the light will strike it will be TV, giving the impression of a body S' in the direction VT produced. Now if we pass to the limit, the case will be exactly that of light passing through a medium whose density decreases with the distance from the centre, and the path will be the limit of the polygon in the hypothetical case, that is to say, a curve. The direction of the light just as it enters the eye at V will be the tangent to the curve, and in that direction S will appear to be.

The only case in which the *apparent* and the *true* directions coincide will be when the incident light is perpendicular to the surface, in which case it will suffer no deflection. And, *cæteris paribus*, the more oblique the direction of incidence, the greater will be the deflection.

The hypothetical polygon $PQRTV$ lies entirely in one plane; for any two consecutive sides of it, as QR, RT, lie in the same plane with the normal at R, by the law of refraction. Therefore the normals at Q and T, which meet the normal at R in C, both lie in that plane. And thus it may be shewn that the normals at all the points P, Q, R, T, V, lie in the same plane. Hence the whole polygon lies in the same plane with CP, CV, the two extreme radii. In the limit, therefore, the path of the light will be a plane curve, and its tangent at V will lie in the same plane with the radius at V, and the original direction of the light, PS. If we draw VS parallel to PS, it will be the direction in which S, which is very distant, would appear to be if there were no refraction. Hence the apparent direction lies between the true direction and the vertical in the same plane with these lines.

From this we gather that the effect of refraction is to bring a heavenly body nearer to the zenith, or to diminish its zenith distance. And the displacement being directly towards the zenith, the *azimuth* of the body will not be affected.

Hence, all altitudes have to be corrected for refraction, while the observed and the true azimuth are the same. The amount of the correction of the altitude increases with the zenith distance, being zero at the zenith and about 33′ in the horizon. It does not however increase uniformly, but according to a law which we cannot now enter into. Its value is about 57″ at an altitude of 45°.

As the azimuth of a body is not affected by refraction, the observed time of transit over the meridian requires no correction on this account.

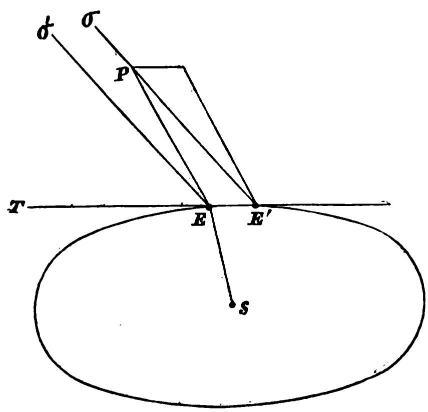
It may be here observed that the length of the day is increased by refraction, the Sun's centre when in the horizon being elevated through a space rather greater than its diameter. In fact, when the Sun appears to us to be just on the horizon, it is really entirely below. Where the diurnal path of the Sun makes a small angle with the horizon, as is the case in high latitudes, the day is by this cause very considerably lengthened.

51. Another effect of the Earth's atmosphere is the phenomenen of *twilight*. If there were no atmosphere, night would begin suddenly and abruptly as soon as the Sun disappeared below the horizon. In fact, there would be no light anywhere except in actual sunshine. The atmosphere however, being itself illuminated by the Sun's rays, reflects them upon the Earth's surface, and thus not only gives light in parts which are not directly enlightened by the Sun, but protracts the day even after the Sun has set. The part of the atmosphere visible to any place is a segment of a sphere cut off by a supposed tangent plane to the Earth's surface at the proposed point. Now as long as any part of this segment is illuminated, it will reflect light upon the Earth; and it is only when the Sun is so far below the horizon that its rays cannot reach any part of the visible atmosphere, that total darkness ensues.

It is found by observation that twilight lasts till the Sun's perpendicular depression below the horizon is 18°. Hence, the more oblique the diurnal path is to the horizon, the longer the twilight lasts. At the equator, where the diurnal path is at right angles to the horizon, the twilight is shortest.

52. The proper correction being determined for each observed altitude, which is done by a process we cannot now enter into, we are enabled to find the direction in which the star would appear if the atmosphere were removed; and if this direction being produced would pass through the star, the place so corrected would be the true place of the body in the sphere of observation. But this is not exactly the case, as we shall endeavour to shew. Another correction is still necessary to reduce the observed to the true place. There is indeed no deflection of the light from its rectilinear course till it reaches the atmosphere. The cause of the displacement to which we now refer is the motion of the Earth in its orbit at a rate which bears a finite though very small ratio to the velocity of light.

If the Earth were stationary, the light of a heavenly body would always proceed in the same straight line, in the direction of which the body would be seen. If the transmission of light were instantaneous, the line joining the body with the eye at any moment would be the apparent direction of the body, whether the Earth were in motion or not. But the case is different when the velocity of light is not indefinitely great compared to that of the Earth; for then the observer, supposing himself at rest, attributes his own velocity in a contrary direction to the light; and this being combined with the actual motion of the light, produces a resultant motion, the direction of which is that in which the body appears. To make this clear, let E be the Earth in its orbit, σ a distant body. Let $E'E$ be a



small arc of the Earth's orbit, nearly coincident with the tangent TE produced backwards. Take P such a point in $E\sigma$ that PE may be to $E'E$ as the velocity of light to that of the Earth. Then if a ray from σ fall on the Earth at E, the Earth will be at E' when the light is at P. The direction in which σ will be seen from E will be that in which the light, considered as a moving body P, strikes E, or the direction of the *relative* motion of the light with respect to E. This relative motion will be unaltered if we suppose the Earth to remain stationary, and its velocity to be communicated to P in the opposite direction, for this is equivalent to impressing the same velocity on both bodies, which does not affect their relative motions. If therefore we suppose the Earth to remain at E', while P has the compound motion which would carry it through PE and EE', at the end of the time P will arrive at E', and its direction will be PE'. Hence, when the Earth arrives at E, the light will appear to strike it not in the direction σE, but in the direction $\sigma' E$ which is parallel to PE', that being the direction of the relative motion.

This effect of the observer's motion may be illustrated by a familiar example. If a man runs at a rapid pace in a shower of rain which falls vertically, he finds that the drops beat into his face exactly in the same way as if he stood still in a driving shower. If instead of running he were carried along unconsciously to himself, he would not be able to distinguish the effect of his own motion from that of the rain. This is quite analogous to the displacement of a heavenly body, the *direction* of the light being changed.

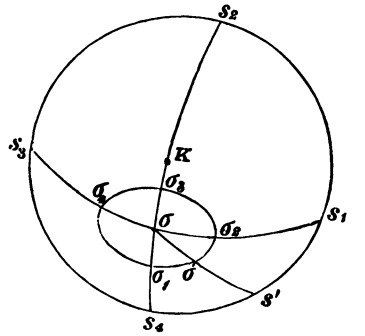
The displacement due to this cause, which is called the *aberration of light*, is measured by the angle $\sigma E \sigma'$, or its equal EPE'; that is, the arc of the sphere of observation which is subtended by the angle $\sigma E \sigma'$ is the correction to be applied to the observed place of the body on this account. On account of the great distance of the observed body, we need make no distinction between the sphere of observation and the celestial sphere. In order to apply the correction, it is necessary we should know its *amount* and its *direction*.

From the triangle PEE', we have, using the circular measure of the small angle EPE' instead of its sine,

$$EPE' = \frac{EE'}{PE} \sin PE'E,$$

where the ratio $\dfrac{EE'}{PE}$ is that of the velocity of the Earth to that of light. The angle $PE'E$ is evidently equal to $\sigma' ET$, that is, the angle between the apparent direction of the body and the tangent to the Earth's orbit. Now if S be the Sun, the line SE is perpendicular to ET, on the supposition of the orbit being circular, and ET lies in the plane of the ecliptic; therefore ET intersects the sphere whose centre is E in a point of the ecliptic 90° behind the Sun's apparent place, and the arc of the sphere which is subtended by the angle $\sigma' ET$ is that joining the star's apparent place with a point in the ecliptic 90° behind the Sun. It appears from the above investigation that the displacement lies in that circle, which is therefore called the *great circle of aberration.*

Let the circle $S_1 S_2 S_3 S_4$ represent the ecliptic, K its pole, σ the place of a star. Draw the great circle $S_2 S_4$ through



K and σ, and also the great circle $S_1 S_3$ through σ and the poles of the former great circle.

Suppose the Earth to be in such a part of its orbit that the Sun's apparent place is S_1. Then S_4 is 90° behind the Sun, and the great circle $S_2 S_4$ will be the great circle of aberration. Therefore the star is displaced in that circle towards the point S_4.

Let $\sigma\sigma_1$ be the displacement. Then, by the above equation,

$$\sigma\sigma_1 = \frac{v}{V} \sin \sigma_1 S_4;$$

but as $\sigma\sigma_1$ is a very small arc never exceeding 20″.1, we may without sensible error write σS_4 for $\sigma_1 S_4$; therefore

$$\sigma\sigma_1 = \frac{v}{V} \sin \sigma S_4.$$

Similarly, if σ_2, σ_3, σ_4 be the apparent places of the star when the Sun occupies the positions S_2, S_3, S_4 respectively, we have

$$\sigma\sigma_2 = \frac{v}{V}\sin\sigma S_1,$$

$$\sigma\sigma_3 = \frac{v}{V}\sin\sigma S_2,$$

$$\sigma\sigma_4 = \frac{v}{V}\sin\sigma S_3.$$

And if S be any other portion of the Sun, S' the point 90° behind it, and σ' the corresponding apparent position of σ, we have

$$\sigma\sigma' = \frac{v}{V}\sin\sigma S'.$$

It is to be observed, that since the sine of an arc is equal to the sine of its supplement, the displacements corresponding to any two opposite positions of the Sun will be equal. Thus $\sigma\sigma_1 = \sigma\sigma_3$, and $\sigma\sigma_2 = \sigma\sigma_4$. Also, since S_1, S_2 are the poles of the great circle through $K\sigma$, σS_1 and σS_3 are each arcs of 90°, therefore $\sigma\sigma_2 = \sigma\sigma_4 = \frac{v}{V}$, and this is the greatest possible amount of displacement. It is found to amount to 20″.1, of which angle $\frac{v}{V}$ is therefore the circular measure.

If we wish to express the correction in seconds, we may substitute 20″.1 for $\frac{v}{V}$, and writing a for the correction, and ϕ for the arc on whose sine it depends, and which is technically called the *Earth's way*, we have

$$a = 20''.1 \sin\phi.$$

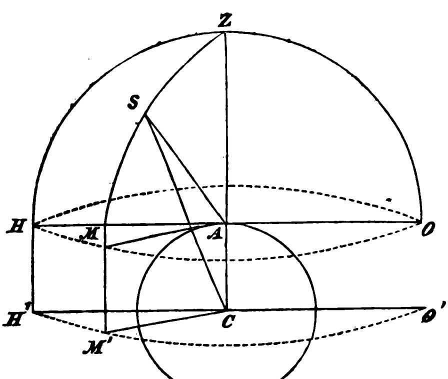
Now σS_2 is the greatest arc, and σS_4 the least which can be drawn from σ to the ecliptic, and σS_2 is equal to the latitude of the star; therefore, calling it λ, we have $20''.1 \sin\lambda$ for the least value of the correction for aberration.

It is evident that the star appears to describe in the course of the year a symmetrical curve about its true place, and it may be shewn by an investigation foreign to our present purpose that the curve is an ellipse.

The major axis of the ellipse is 40″.2, the minor axis that quantity multiplied by the sine of the latitude. Therefore a star situated at the pole of the ecliptic appears to describe a circle, while a star in the ecliptic moves backwards and forwards in the same line.

53. The corrections for refraction and aberration being applied, we arrive at the true place of the observed body in the *sphere of observation.* The next process is to reduce the place in the sphere of observation to that in the *celestial sphere,* which makes the result independent of the place of observation.

Let *A* be the place of observation, *C* the Earth's centre, *Z* the zenith, *HO* the horizon, *S* a heavenly body, whose place



is determined by observation. Draw the great circle *ZSM* cutting the horizon in *M.*

Then *SM* is the altitude, *HM* the azimuth of *S.*

Since *A* is the centre of the sphere of observation, if we join *AS, AM,* the angle *SAM* will measure the altitude, and *HAM* the azimuth.

Through *C* draw a circle *H′M′O′* parallel and equal to *HO.*

In it draw CM' parallel to AM, and complete the rectangle CM, which evidently lies in the same plane with C, A, and S.

The circle $H'O'M'$ is called the *rational* horizon, HOM being called the *sensible* horizon. The former is the horizon of the celestial sphere, the latter the horizon of the sphere of observation.

Hence the angle $H'CM'$ measures the azimuth of S in the celestial sphere, or as seen from the centre of the Earth; and SCM' measures the altitude in the celestial sphere. Therefore SCM', $H'CM'$, are the *reduced* altitude and azimuth of S.

If we join HH', which will be parallel to MM', we see at once that the angle $H'CM'$ is equal to the angle HAM, therefore the reduced azimuth is the same as the observed azimuth.

The reduced altitude is greater than the observed altitude by the angle CSA, which evidently depends on the distance and the altitude of S.

Let CA the Earth's radius be represented by r, CS, the distance of S from C, by R, the observed altitude SAM by a.

Then $$\frac{CA}{CS} = \frac{\sin CSA}{\sin CAS} = \frac{\sin CSA}{\sin ZAS};$$

therefore $$\sin CSA = \frac{r}{R} \cdot \cos a \ldots .$$

In most cases $\frac{r}{R}$ is a very small fraction, and CSA a very small angle, therefore we may write the circular measure of CSA for its sine, and calling it p, we have

$$p = \frac{r}{R} \cos a.$$

when $a = 0$, $p = \frac{r}{R}$ which we may call P, and thus we shall get $p = P \cos a$.

The correction p, which has to be added to the observed altitude in order to obtain the reduced altitude, is called *parallax;* its value when the altitude is 0° is called the *horizontal parallax*.

The above equation expresses therefore the relation between the parallax, the altitude, and the horizontal parallax.

54. We have now given the means commonly employed to determine the altitude and azimuth of a body when observed at a particular place, and to reduce the results of the observation to those of a contemporaneous observation supposed to be made by a fictitious observer at the Earth's centre.

If the position of the place of observation be accurately known, it is easy to translate these results, which are given with reference to the rational horizon of the place, into terms which refer to the fixed circles of the celestial sphere: for the position of the horizon of any place with respect to the celestial equator is always known when the latitude is known.

In the case of meridional observations, we can at once obtain the right ascension and declination from the time of transit and the meridian altitude.

The declination, as has been shewn before, may be found from the altitude and the known latitude by simple addition or subtraction.

Also the time intervening between the transits of any two bodies over the meridian is proportional to their difference of right ascension, for it is the same part of 24 hrs. that the angle at the pole between their declination circles is of 360°. Therefore if the astronomical clock be accurately set, or its error exactly known, which comes to the same thing, the time of transit will be the difference of right ascension between the observed body and the first point of Aries, *i.e.* the actual right ascension of the body.

Thus the place of the body as referred to the celestial equator is completely determined. It may be remarked that right ascension is generally expressed in time, on account of the kind of observations by which it is determined. It may, however, easily be reduced to the ordinary measure of arc, by allowing 15° to an hour, or a degree to every 4 minutes.

It is very important that the student should bear in mind the nature of each of the corrections for refraction, aberration, and parallax. The first two are rendered necessary by the circumstance that the direction of the light which enters the eye from a heavenly body is not the direction which if produced backwards would pass through the body. The third is not required on account of any remaining error in the position of the body as determined at the place of observation, but is applied for the convenience of reducing all observations to such as would be made at the centre of the Earth.

The amount of the first two corrections depends only on the position of the body with reference to the horizon and the ecliptic respectively, and not on its distance from the Earth. The correction for parallax depends on the ratio of the Earth's radius to the distance of the body, that ratio being the sine of the *horizontal* parallax. This circumstance makes a great distinction between parallax and the other two errors, for in the case of the fixed stars the ratio is quite imperceptible; it is small in the case of the Sun and most of the planets. The only body which on account of its nearness is much affected by parallax, is the Moon.

The determination of the horizontal parallax is of great importance, not only for the purpose of reducing observations, but also for the determination of the distance of the proposed body, it being the angle *CPE* in page 5.

It was mentioned in page 5, that not only is the horizontal parallax of a fixed star inappreciable, but also the angle subtended by the radius of the Earth's orbit is in general too small to be determined, giving an overwhelming idea of the vastness of the distances of these bodies. The latter angle, from analogy to that subtended by the radius of the Earth, is called the *annual parallax.*

It was in the endeavour to detect annual parallax by observations of extreme delicacy on the star γ Draconis, that the aberration of light was discovered. Bradley, by whom the observations were made, discovered the annual displacement which we have attempted to describe, and for some time was unable to account for it. He ascertained that it could not be due to the annual parallax which he sought; and as it occurred in other stars, it could not arise from proper motion. At last he conceived the true cause, and acting upon his idea, soon resolved the difficulty. It is a remarkable proof of the accuracy of this theory, and of astronomical observations generally, that the ratio of the velocity of the Earth to that of light given by the phenomena of aberration, is the same as that determined by quite independent observations on the satellites of Jupiter. It was found by Römer, that the eclipses of these bodies occurred sometimes later than the time calculated and sometimes earlier, according as the distance of Jupiter from the Earth was greater or less. He accounted for this seeming irregularity by supposing that light occupied a finite time proportional to the distance, in passing from Jupiter to the Earth. He calculated on this hypothesis the velocity of light, and found it would take about $16\frac{1}{2}$ minutes to traverse the Earth's orbit through the centre. It would thus take about 52 minutes to go round the orbit, supposed circular. The ratio of 52 minutes to 365 days and a quarter would therefore be the ratio of the velocity of the Earth to that of light; and this result, as has been stated, agreed with Bradley's to a great degree of accuracy.

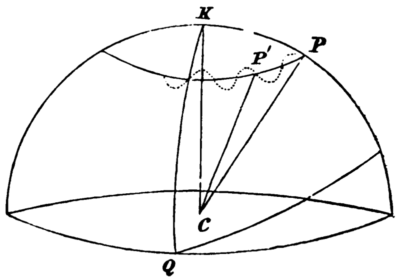
Precession and Nutation.

55. The instruments above described afford the means of making observations, which when properly corrected and reduced give the right ascension and declination of a heavenly body. An observer therefore, who knows his own position and has an astronomical clock rightly set, may form a catalogue of the fixed stars by writing down the right ascension and declination of each. He may also, by an easy process in spherical trigonometry, determine their latitudes and longitudes. And these results being once obtained would be always true, supposing the stars to have no proper motion, if the circles of the celestial sphere always retained the same place. This, however, is not exactly the case. The pole of the heavens is subject to variations of position among the fixed stars, which,

as will be seen, have nothing to do with the stars themselves, but depend solely on the position of the Earth's axis; and of course the celestial equator has consequent variations. Thus the declination is subject to change, not from any motion of the stars, but from that of the circle to which they are referred. The first point of Aries also varies in position, so that it is not strictly accurate to define a sidereal day as the interval between two successive transits of a *star*, although the error in one day is so small as to be scarcely perceptible.

These changes in the position of the pole are caused by a deviation of the axis of the Earth from parallelism, too small to affect the general phenomena of the seasons, and only manifested by delicate and long-continued observations of the fixed stars. The cause of this deviation is the spheroidal form of the Earth, which gives rise to a disturbing action from the Sun and Moon. If the Earth were perfectly spherical, its axis would always keep its parallelism, for the whole attraction of the Sun and Moon on every point of the Earth would produce the same effect as if it acted with the same intensity on the centre alone.

But in the actual case there is an additional action on that part of the Earth which would be cut off by a sphere having the minor axis as its radius. That part is in the form of a shell whose thickness is greatest at the equator and vanishes at the poles. The attraction on this mass produces a twisting effect, which is small, on account of the smallness of the deviation from sphericity, and the consequence is that the axis does not keep exactly parallel to a fixed line, as it would if the Earth were a perfect sphere.



To explain the effect, let C be the centre of a sphere of any radius, CK perpendicular to the Earth's orbit, CP parallel to the Earth's axis at any time. If there were no deviation from parallelism, CP would always remain fixed; but to represent the actual case, *CP* must revolve in a direction contrary to that of the Earth's motion, and maintaining nearly the same inclination to *CK*. Therefore the point *P* must describe nearly a small circle about *K*, the arc *KP* being equal to ω the obliquity of the ecliptic. The motion of the axis is completely represented by supposing *P* to move in the circumference of a very small ellipse whose axes are 18″.5 and 13″.7 respectively, the centre of the ellipse moving uniformly about *K* in a small circle whose radius is ω. Thus *P*'s motion is of an undulating character, as in the figure, though the undulations are much smaller than there represented.

The centre of the ellipse revolves about *K* at a rate which would carry it through the whole circumference of the circle in 25,868 years, the annual arc described being 50″.1. The period of revolution in the small ellipse is about 19 years.

If we suppose *C* to be the centre of the celestial sphere, and *P* its pole; then *CP* being always necessarily in the line of the Earth's axis, the point *P* will describe about *K* the pole of the ecliptic, just such a path as in the supposed sphere. This will continually alter the position of the equator with respect to the ecliptic. The intersection of these two great circles evidently lies in a great circle *KQ* perpendicular to *KP*, and consequently revolving with it. Therefore the equinoctial points go backwards along the ecliptic at a rate which is nearly uniform and averaging 50″.1 in a year, while the inclination of the equator to the ecliptic undergoes fluctuations whose period is about 19 years.

The ecliptic being nearly fixed in space, we may consider these motions of the pole as absolute. The effect is that the fixed stars are subject to continual changes of right ascension and declination. It is to be observed, however, that the latitude and longitude being referred to the ecliptic, do not change except in consequence of the change in the position of the equinoctial point from which longitude is measured. The latitudes therefore are unaltered, while the longitudes increase by an average of 50″.1 a year.

The general effect of the deviation of the axis from parallelism is usually separated into two parts, *precession*, arising from the average retrogradation of the equinoctial points, or the effect which would arise from supposing the pole to move uniformly in the small circle whose radius is KP; and *nutation* arising from the undulations on each side of the circle when we take into account the superadded motion of the pole in the ellipse.

In consequence of precession, the pole and the first point of Aries occupy very different positions now from those in which they were in the first ages of astronomical observation. The pole-star has not always been so near the pole, neither will it keep its present position.

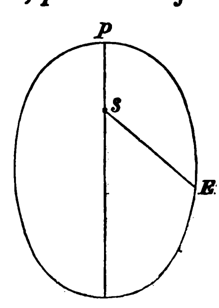
For some very remarkable facts of historical interest on this subject, the reader is referred to Sir J. Herschel's Astronomy.

The obliquity of the ecliptic is subject not only to the above-mentioned fluctuations, but also to a very slow diminution which depends on a change in the plane of the Earth's orbit, and has nothing to do with the position of its axis.

It must be carefully recollected, that though the pole of the heavens and the celestial equator vary in position with respect to the stars, they never change their position with reference to the horizon of a particular place, for it is ascertained beyond a doubt, that terrestrial latitudes are invariable. Therefore at the same place the pole is always at the same altitude, and the equator occupies the same position with respect to the horizon.

56. Effect of the ellipticity of the Earth's orbit.

Let S be the Sun, E the Earth in its orbit, pSa the major-axis of the orbit. Since S is the focus of the orbit, p is the point at which the Earth is nearest to the Sun, and a the point at which it is at the greatest distance. These two points are called the *perihelion* and *aphelion* respectively. From p to a it is evident that the Earth's distance increases, and from a to p it diminishes.



The line pSa is called the line of apses. It is not exactly fixed in space, but has a slow progressive motion of about 11″ in a year. Not that the orbit is really a revolving ellipse, but that this supposition explains the phenomena as in the case of the Moon's orbit. The actual orbit of the Earth is indeed very nearly elliptical, and would be quite so but for the perturbations caused by the other bodies of the solar system. One effect of these perturbations is to retard the times of greatest and least distance from the Sun, so that they occur at intervals of rather more than half-a-year from each other; and this is the same thing as if the orbit were an ellipse, whose line of apses had a corresponding angular motion.

In consequence of the varying distance of the Earth from the Sun, the apparent diameter of the Sun varies continually, being greatest at the beginning of January when the Earth is in perihelion, and least about the beginning of July when the Earth is in aphelion.

It will be remembered that the radius vector, or line joining the Earth and Sun, passes over equal areas in equal times. Consequently it must describe varying angles about S according to the distance of E. For the greater the radius vector, the less will be the angle it must pass over in order to describe the same area. Thus the angular velocity of E about S will be greatest at p and least at a. It will increase from a to p, and decrease from p to a.

Now the apparent motion of the Sun is the same as the actual motion of the Earth. Therefore the Sun will appear to move with varying velocity in the ecliptic, the arc which it describes in any given time being proportional to the angle described by the radius vector. On this account the Sun's motion in longitude will be most rapid in January, and least rapid in July, changing gradually from one of these times to the other.

As the Sun appears to describe about the Earth an orbit exactly similar to that of the Earth about the Sun, it is often convenient to speak of this apparent orbit instead of the true orbit of the Earth. Thus we speak of the Sun being in apogee and perigee, instead of the Earth being in aphelion and perihelion.

57. *On Time.* Solar time, as has been said before, depends on the position of the Sun with reference to the meridian.

If a great circle be drawn joining the pole of any plane with the true position of the Sun, and the angular distance of that circle westward from the meridian be converted into time, allowing 15° to an hour, the result will be the *apparent solar time,* as estimated in astronomy. Consequently the astronomical day begins at noon, and instead of beginning the hours again after 12, we reckon on up to 24. The civil day begins on the midnight before; therefore, in translating ordinary dates into astronomical language, if the time is between midnight and noon, we must refer it to the day preceding, and add twelve to the number of hours. Thus 8 o'clock A.M. on the 10th of January is, in astronomical terms, 20h. on the 9th of January.

The angle between the meridian and the great circle passing through the Sun increases continually from the diurnal motion of the heavens, but not so rapidly as it otherwise would, because the Sun is all the time moving in the opposite direction from west to east. The angle is in fact that through which the heavens have revolved since the Sun was on the meridian, diminished by the Sun's motion in right ascension during the same time.

Now the motion of the heavens is uniform, and therefore if the Sun's motion in right ascension were uniform, the angle which measures apparent time would increase uniformly. But the Sun's motion in right ascension is not uniform, partly on account of the ellipticity of the Earth's orbit, which we have already considered, and partly because of the inclination of the axis, the effect of which we shall presently shew. Consequently apparent time does not increase uniformly. On this account it is impossible to regulate a clock to follow the Sun exactly, as to do this its rate must continually change.

Hence clocks are generally regulated to *mean time,* or the time which would be indicated by a body moving uniformly in right ascension with the mean or average velocity of the Sun.

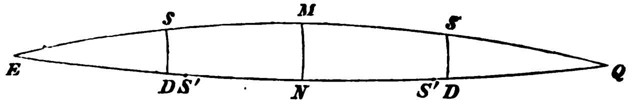
Now as no such body can be observed, but *apparent time* only can be obtained by observation, it is necessary to find what correction must be applied to it in order to obtain mean time. Such a correction is called the equation of time.

The direct method of finding the equation of time is by determining the Sun's true right ascension, and taking the difference between that and his mean right ascension. This difference expressed in time is the correction required. We may, however, explain the general nature of the correction and its comparative amount at different times, without entering into these calculations. There are two causes, as has been observed before, from which it arises—the ellipticity of the orbit, and the inclination of the axis; each of which produces its effect, and by taking these effects separately, we may greatly simplify our explanations. We cannot indeed, in strictness, consider the effect of each as if the other did not exist, for each is modified by the other. The correction due to the obliquity of the ecliptic is not the same as it would be if the orbit were circular, nor that due to the ellipticity of the orbit the same as if the ecliptic coincided with the equator. But the whole amount of the correction is small, and therefore we may without perceptible error form a general estimate of its nature by taking each part independently.

First let us take the part due to the ellipticity of the orbit, and for simplicity we shall suppose the Sun to move in an ellipse about the Earth in one focus. Let a body S move with the Sun's mean angular velocity, starting with the Sun from perigee. It will at first fall behind the Sun, because the Sun's motion is there quicker than the average. But the Sun's velocity is gradually diminishing, and at some point about half-way between perigee and apogee becomes equal to its mean velocity. After this point its velocity will be below the mean, and therefore S will begin to gain on the Sun. The two bodies will arrive at apogee at the same time, because from perigee to apogee is exactly half the ellipse, and the Sun therefore describes it in half-a-year. From apogee to perigee the same thing will occur in inverted order. Therefore from perigee to apogee S is behind the Sun, and from apogee to perigee before it. The distance between S and the Sun is greatest at points about half-way between apogee and perigee, where the Sun's velocity becomes equal to its average value; for then S will begin to gain on the Sun between perigee and apogee, and to lose on it between apogee and perigee.

Taking this cause alone, therefore, and supposing the clock to be regulated by the body S, if the clock and the Sun are together at perigee, they will also be together at apogee. From perigee to apogee, S being behind or to the west of the Sun, will come to the meridian before it. Hence the clock will be *before the Sun*. From apogee to perigee the clock will be *behind the Sun*.

Again, let us consider the effect of the inclination of the Earth's axis, or the inclination of the ecliptic to the equator.



Let EMQ, ENQ be the ecliptic and equator, intersecting in the points E, Q. Each of these arcs therefore is a semicircle. Let EM, EN be quadrants, and join MN. The angles at M, N are both right angles, because E is the pole of MN.

If S be the Sun at any point of its orbit, and SD an arc drawn at right-angles to the equator; ES is the longitude and ED the right ascension. If S be between E and M, we have a right-angled triangle in which the hypothenuse ES is necessarily greater than the side ED. If S be between M and Q, we have in like manner SQ greater than DQ, and therefore ED greater than ES. If S coincide with M, D will coincide with N.

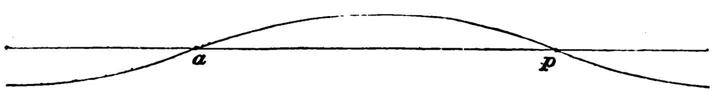
Therefore, from the equinox E to the solstice M, the longitude is greater than the right ascension. At the solstice M the longitude and right ascension are equal. From the solstice to the equinox, the right ascension is greater than the longitude. The same thing may be shewn in the other half of the orbit from Q to E.

Now if a body S' move in the equator at the same rate as S in the ecliptic, we shall have $ES' = ES$; and if we suppose the motion in longitude uniform, ES' will be the mean right ascension, ED being the true right ascension. Hence the mean right ascension will be greater than the true from the solstice to the equinox, and less from the equinox to the solstice.

If then we take the equation of time arising from this cause alone, regulating the clock by S', the clock and the Sun, being together at E, will always agree at the equinoxes and solstices. The clock will be before the Sun from solstice to equinox, and behind it from equinox to solstice. Between each equinox and solstice will be points where the clock and the Sun have their greatest difference, and after that begin to approach one another again.

The whole equation of time is found by adding together, with their proper signs, the corrections due to the two causes separately.

The general effect may be shewn very simply by means of a diagram, as we shall proceed to explain.

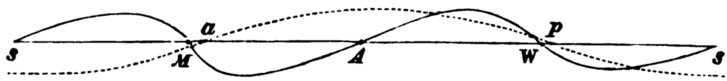


Let ap be a curve* whose ordinate at any point is proportional to the correction due to the ellipticity of the orbit, at the time of year indicated by the abscissa. Let p represent the perigee, a the apogee; therefore at those points the curve will cross the axis. From p to a the Sun is behind the clock, therefore the curve will be below the axis; and similarly, from a to p the curve will be above the axis. The maximum distance of the curve from the axis will be at points intermediate between p and a, a and p.

Similarly, we may represent the correction due to the inclination of the ecliptic by a curve, cutting the axis at the points M, A, W, S, representing the summer solstice, the autumnal

* For this method I am indebted to Prof. Fischer, Fellow of Clare Hall.

equinox, the winter solstice, and the spring equinox, respectively.



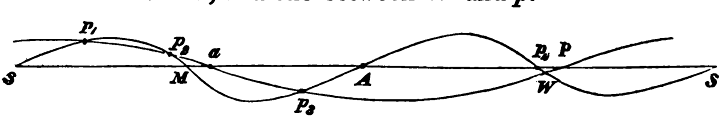
If we describe both curves on the same axis, giving them their proper relative position, and describe a third curve whose ordinate at any point is equal to the sum of the ordinates of the other two, with their proper signs, that third curve will represent the whole equation of time.

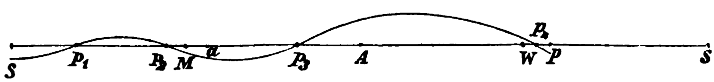
The Sun being in apogee about the beginning of July, and in perigee about the beginning of January, the point *a* will be a little beyond *M*, and *p* a little beyond *W*. The dotted line in the second figure will represent the curve in the first.

The maximum value of the second correction is rather greater than that of the first, therefore the dotted curve must not recede so far as the other from the axis.

The simplest way of drawing the third curve is by describing the other two curves on opposite sides of the same axis; that is, for instance, by drawing the first curve *below* the axis from *a* to *p*, and *above* it from *p* to *a*. Then the distance between two corresponding points of the two curves will be the sum of their ordinates taken with their proper signs, and therefore equal to the corresponding ordinate of the third curve.

Drawing the curves in this way, we observe that they intersect in four points, at which points therefore the third curve cuts the axis. Two of these points are between *S* and *M*, one between *M* and *A*, and one between *W* and *p*.





The curve which actually represents the equation of time will be approximately that in the last figure. The clock will be behind the Sun between p_1 and p_2, p_3 and p_4. It will be before the Sun between p_4 and p_1, p_2 and p_3. The greatest difference will be between p_3 and p_4. Thus it appears that the equation of time vanishes four times a-year at irregular intervals.

58. *The Sidereal Year* is the time occupied by the Earth in making a complete revolution about the Sun. The period which we commonly call a year is, however, somewhat less than this; on account of the motion of the equinoctial points in the opposite direction to that of the Earth. This causes each equinox and solstice to occur earlier than it otherwise would, and therefore the course of the seasons to occupy something less than the time of a sidereal revolution. In astronomy this period is called the *Tropical year.*

The Anomalistic Year is the period of the Earth's revolution from aphelion or perihelion to the same point again: since the line of apses moves in the same direction as the Earth, the anomalistic year is rather longer than the sidereal year. The respective lengths of the sidereal, tropical, and anomalistic years are

365d. 6h. 9m. 10.7s.; 365d. 5h. 48m. 51.6s.; 365d. 6h. 13m. 44.6s.

The tropical year being nearly 365 days and a quarter, the ordinary reckoning of 365 days to a year with the addition of a day every four years is nearly correct: but this addition is rather too great, as the excess over 365 days is 11 minutes less than 6 hours. Consequently the average length of the ordinary year exceeds that of the tropical year. To remedy this we take away 3 days every four centuries. This is generally done by reckoning as an ordinary year the year which completes a century, unless the number of centuries which it indicates is divisible by 4. Thus the year 1800 was not a leap-year, as it completed the 18th century, and 18 is not divisible by 4.

The addition of one day in four years is called the Julian correction, having been introduced by Julius Cæsar. For a long time it was thought sufficient; but it was found that in consequence of the average year being thus made too long, the seasons did not occur at the same times as at first. In fact the vernal equinox, which was at first fixed on the 21st of March, fell several days earlier. The present arrangement was introduced by Pope Gregory in 1582, and bears his name. It was not adopted in this country till the year 1751, when 11 days had to be left out. The former method of reckoning is still called *Old Style*. The difference between it and our present reckoning is now 12 days, another day having been left out in the year 1800. Thus old Christmas-day is the 6th and old New-year's day the 13th of January.

59. *Terrestrial Longitude.*

It has been already explained that the longitude of a place on the Earth's surface is the angle included between the meridian of the place, and some fixed meridian whose position is arbitrary. It is usual in this country to take the meridian passing through the Observatory of Greenwich as the origin of longitudes, and to measure them both eastward and westward up to 180°.

The terrestrial meridian, it will be remembered, is a great circle passing through the poles of the Earth and the proposed place. It is therefore the section of the Earth's surface made by a plane passing through the proposed place and the Earth's axis.

Since this plane passes through the Earth's centre, it will, if produced to the celestial sphere, pass through the zenith of the proposed place; and since it includes the Earth's axis, it will also pass through the pole. Therefore it will intersect the celestial sphere in the celestial meridian of the place. Hence the angle between the celestial meridians of any two places is equal to their difference of terrestrial longitude.

We have seen that the apparent time at any place is measured by the angle included between the great circle joining the Sun's true place with the pole and the celestial meridian. Now the difference between this and a similar angle measured from the meridian of another place must be equal to the angle included between the meridians of the two places, that is, the difference of their terrestrial longitudes. Therefore the difference between the apparent times in two places, at the same absolute instant, is, when converted into arc at the rate of 15° to an hour, the difference of their terrestrial longitudes.

The same thing may be shewn of the difference of their mean times and their sidereal times; for the demonstration would have been exactly the same if instead of the actual position of the Sun, we had taken the place of the fictitious body which regulates mean time, or the first point of Aries which regulates sidereal time.

Since the angle which measures time is estimated westward from the meridian, it will be greater at the more easterly of the two places than at the other, and therefore the time at the former place will be in advance of the time at the latter.

Hence, if a traveller proceed in an easterly direction from Greenwich, carrying with him a watch shewing Greenwich time, he will find his watch too slow by four minutes for every degree of longitude through which he has travelled. In the same way, if he proceed westward, he will find his watch too fast. If he extend his journey either to the east or west through 180° of longitude, he will find his watch just twelve hours different from the time at the place where he has arrived. But if he has travelled eastward, it will be twelve hours too *slow*, and if he has travelled westward, it will be twelve hours too *fast*. Hence, two travellers meeting at such a place would agree as to the time of day, but would differ as to the day itself, for their reckonings would differ by twenty-four hours. Supposing for instance they met at noon, the Greenwich time being midnight, the eastward traveller would reckon it the noon succeeding the Greenwich midnight, and the westward traveller would reckon it the noon preceding the same midnight. The former would be in fact a day in advance of the latter. Supposing the same travellers to continue their progress and to meet at Greenwich after having completed the circuit of the Earth, the one would find his watch twenty-four hours too slow and the other twenty-four hours too fast. They would therefore differ in their reckonings by two whole days. In fact, since the one has gone round the Earth in the direction of its motion, he has made one more revolution than the place from which he started, and so his journey has appeared to occupy one more day than it really did. Similarly, the other having gone round in the contrary direction to the Earth, his journey has appeared to him to occupy one day less than its actual time.

60. Having now described some of the most simple astronomical phenomena, and also the instruments in common use, and the corrections which are to be applied to observations made by them; we shall devote a brief space to the description of some of the most ordinary observations, and the uses to which their results are applied.

We may remark that observations are divided into two great classes: those which are purely astronomical, or intended solely for the purpose of determining the positions of the heavenly bodies; and those which may be called in contradistinction geographical, whose object is to determine the positions of places on the Earth's surface.

Observations of the former kind are made in fixed observatories, with every resource of art and science which can promote accuracy. Those of the latter kind are made chiefly at sea, in unexplored countries, and for the measurement of arcs of the Earth's surface in order to determine its figure and magnitude. It must not, however, be thought that these classes of observations are independent of one another. It is necessary that the geographical position of an observatory should be most accurately known, in order that observations made there may have any value; and again, it is by tables constructed from theory on the data furnished by these observations, that we are enabled to determine geographical positions with readiness and certainty.

We shall begin with observations for finding the latitude and longitude of a proposed place. We shall only give the most simple methods in use, the others being reserved for the second part of this work.

61. *To find the latitude.* This may be done by taking the meridian altitude of any body whose declination is known, as will appear from page 41.

It may also be found by taking the altitudes of a circumpolar star at its upper and lower transit. In this case the declination need not be known, for since the polar distance always remains the same, the half-sum of the observed altitudes will give the elevation of the pole, and therefore the latitude.

All the altitudes must be corrected for refraction. Those of the Sun, Moon, and Planets must also be corrected for parallax: and as the upper or lower limb is the part observed, the semi-diameter must also be added or subtracted to give the altitude of the centre.

The practical method of finding the meridian altitude, when the time of passing the meridian is not exactly known, is to observe the body some little time before its transit, following it with the instrument as long as it continues to rise, and leaving off as soon as it begins to sink. Thus the altitude indicated by the instrument is the greatest which the body has attained, and therefore the meridian altitude required.

At sea this observation is made with the sextant, chiefly on the Sun. The lower limb of the Sun's image being at first made to touch the horizon, as has been described before, appears to rise from it after a while, if the altitude is increasing. The observer brings it down again by moving the index forward, and continues to do so as long as it rises. As soon, however, as the limb appears to dip below the horizon, he knows that the meridian has been passed, and that the reading of his instrument is the altitude required.

On land the observation is best made with an altitude and azimuth instrument. It may, however, be made with a sextant, by taking the angle between the body and its image reflected from a trough of mercury. As however the result of this observation is twice the altitude, and the sextant is not capable of measuring angles much exceeding 120°, this method is very often impracticable.

62. *To find the longitude.*

We cannot find the longitude at once like the latitude, for it is not marked by any peculiarity in astronomical phenomena, and is in fact only an arbitrary quantity depending on the position of the place from which we choose to measure it. It is, however, absolutely necessary to the complete determination of geographical positions, and also to travellers on sea or land in order to make use of the charts on which such positions are laid down.

It will be seen from what has preceded that what is required is to determine the time simultaneously at the place of observation and at the place from which longitudes are measured. Now the time at the place of observation may be easily found, as we shall proceed to shew; but to ascertain the time at Greenwich, or any other place selected as the origin of longitudes, is a matter of no small difficulty, and has long exercised the ingenuity of scientific men.

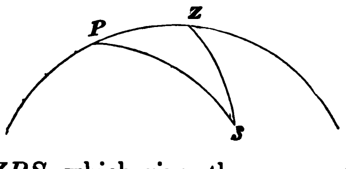
For a long time the progress made in the solution of this problem was very slow, and therefore the longitude could not be determined with any degree of certainty. The importance of greater accuracy to navigation induced the Government to offer large rewards for improvements in this respect; and we are now enabled to arrive at a great degree of exactness, though still inferior to that which is attainable in finding the latitude.

We shall first shew how the time may be found at the place of observation.

The most direct way is by observing the transit of the Sun or any known body across the meridian.

The transit of the Sun occurs of course at apparent noon, and that of any other body at the sidereal time which is equal to its right ascension converted into time.

This method, however, requires a fixed instrument and a knowledge of the meridian. It is therefore useless at sea. The time of transit may however be roughly ascertained by observing the meridian altitude with the sextant; but as the altitude varies very slowly about noon, the result is not to be depended on for great accuracy. If the altitude be observed out of the meridian, and the latitude be known, then three sides of a spherical triangle SZP being known, we may find the angle ZPS, which gives the apparent time.



The difficulty of this method at sea is, that owing to the constant change of position the latitude is not accurately known, except when a meridian observation can be taken. If, however, the Sun be not near the meridian, a very exact knowledge of the latitude is not necessary, and it is sufficient to use the approximate latitude estimated from the distance the ship has run and the course she has steered since the last determination.

By these and other methods, which we cannot now enter into, the apparent or the sidereal time at a given place may be found, and the mean time may of course be derived by the application of the equation of time. We now proceed to explain some of the means employed to determine the time at Greenwich.

If it were possible to construct a chronometer such that its going might be perfectly depended on, it would be sufficient to carry such an instrument regulated to Greenwich time to any place whose longitude was to be ascertained; and a ship provided with such a chronometer would be furnished with the means of determining the longitude whenever an opportunity occurred of observing the time.

It is impossible, however, to construct time-pieces accurate enough to be implicitly relied on, even if we employ several at once, and take the mean of their results, which is much nearer the truth than the time shewn by any one by itself. It is necessary, therefore, to find independent means of determining

Greenwich time. It is obvious that if any phenomenon can be observed of which the Greenwich time is known by previous calculation, it will answer the purpose. Of this nature is the beginning or ending of an eclipse of the Moon, which is seen at the same absolute time from all parts of the Earth to which the Moon is visible, and the time of which at Greenwich may be accurately calculated. As, however, eclipses of the Moon rarely occur, they are not of much use; besides which, owing to the undefined nature of the Earth's shadow, it is difficult to tell exactly when the Moon enters it or leaves it. The eclipses of Jupiter's satellites are more available, for they occur frequently, and the moment of their immersion in the shadow of the planet or emergence from it may be generally observed with considerable precision. As, however, a telescope of considerable power is required, the unsteadiness of a ship renders it impossible to make these observations at sea.

The method which is most used in navigation—indeed the only method generally available at sea—is that of *lunar distances;* or observing with the sextant the angular distance of the Moon from the Sun, or from certain stars selected for the purpose. The Moon's motion among the stars is sufficiently rapid to cause perceptible variations in its distance from any one of them in short intervals of time. The distances from the selected stars are tabulated for every three hours of Greenwich time in the *Nautical Almanac,* being calculated from the lunar tables with all attainable accuracy; and by this means, if any one of these distances be observed at the proposed place, we may find by simple proportion the Greenwich time of the observation, supposing the distance to vary uniformly during the short period of three hours, which will generally be the case. The observed distance will generally be found to lie between two tabulated distances; and by finding in what proportion it divides the difference between them, we can determine the time which must be added to that of the first tabulated distance in order to give the true time of the observation.

The tabulated distances, however, are calculated for the celestial sphere. Therefore the observed distance must first be corrected for refraction and parallax, which by altering the altitudes of the bodies will also change their apparent distance. In order to make these corrections, the altitudes of the bodies must be observed simultaneously with the distance; and this will have the farther advantage of giving the time at the place of observation—the apparent time if one of the bodies be the Sun, the sidereal time if it be a known star: and thus at the same time all the data for the determination of the longitude will be obtained.

63. Such are the usual methods of determining geographical latitude and longitude. In observatories a great many determinations of the geographical position are made, and thus more accurate results are obtained than by any single observations. When the geographical position is perfectly known, the mural circle and transit instrument are employed to determine very accurately the right ascensions and declinations of the Sun, Moon, planets, and fixed stars; and these results are constantly compared with those of calculation from theory, in order to correct and improve the tables by which the motions of these bodies are predicted. The fixed stars having but very small variations in position, catalogues of their places at given epochs are formed, from which, by the application of the corrections for precession, nutation, and aberration, their places at any other times may be found.

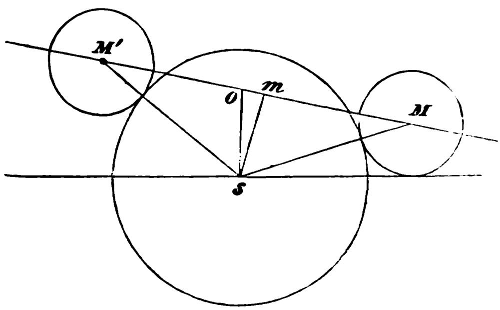
If an observatory has to be erected in a new place, it is necessary to place the transit and mural circle in their right positions, and to set the sidereal clock. The transit may be approximately placed by the method before described for finding the meridian direction by equal altitudes of the Sun. Then its errors of collimation and level being as much as possible got rid of, the error of deviation may be corrected by the transits of a circumpolar star. The mural circle may be adjusted by means of the transit, when that instrument is accurately placed.

To set the sidereal clock, it will be sufficient, if the observer have a catalogue of the stars, to observe the transit of a star whose right ascension is known. This will give the sidereal time at once, and continued observations of the same or other stars will give the rate.

64. We shall conclude this part of the subject with a few general remarks on the subject of eclipses.

It has been already shewn that lunar eclipses present the same appearance to all observers to whom the Moon is visible at the time. It is therefore sufficient to calculate them as they would be seen from the centre of the Earth, which may be done with great exactness from tables which give the positions of the Moon and of the Earth's shadow. It then only remains to find in what parts of the Earth the Moon is above the horizon at the time of the eclipse.

Let *S* be the centre of a section of the Earth's shadow at the distance of the Moon. The point *S* is of course exactly opposite



to the Sun, and therefore moves along the ecliptic at the same rate as the Sun. The Moon's centre, when in opposition, is in conjunction with *S*, or in some point *O* in a great circle perpendicular to the ecliptic at *S*; and, if *O* be near enough to *S*, the Moon's orbital motion, which is much quicker than that of the shadow, will cause it to pass through part of the shadow and to be eclipsed.

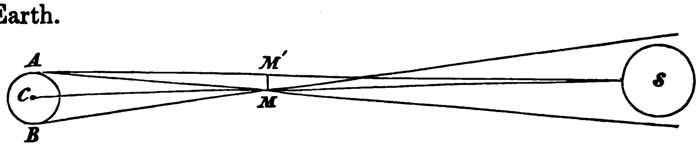
We are able to find from the tables the exact time of opposition, the latitude of the Moon at the time, the apparent diameters of the Moon and the shadow, and the velocities of each; and from these data we can calculate the time when the Moon enters and emerges from the shadow. The shadow, moving in the ecliptic, has of course only a motion in longitude, while the Moon, moving in an inclined path, varies both its longitude and latitude.

The apparent path of the Moon with reference to the shadow is found by applying to each the velocity of the shadow in the opposite direction, that is, by supposing the shadow to remain fixed, and subtracting its velocity in longitude from that of the Moon.

Taking this relative velocity in longitude and the true velocity in latitude, as found from the tables, we may find the Moon's relative path, such as *MM'* represented in the figure. If *SM* and *SM'* be equal to the sum of the radii of the Moon and shadow, *M* and *M'* will be the positions of the Moon's centre for the beginning and ending of the eclipse, and the times of its being in these positions may be calculated from the tables. Thus all the circumstances of lunar eclipses may be calculated.

It is to be observed, that although the Moon at opposition may pass much within the cone of the Earth's shadow, and may therefore be entirely deprived of the direct solar light; yet some light bent out of its course by the Earth's atmosphere finds its way within the shadow, so that in general the Moon is discernible throughout a total eclipse, appearing of a dull red colour.

Solar Eclipses.—Suppose two observers, *A* and *B*, to stand on different points of the Earth's surface about the time of conjunction of the Sun and Moon, as seen from *C* the centre of the Earth.



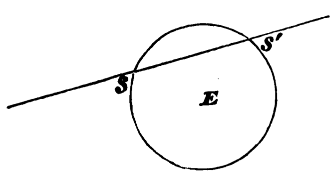
Let *M* be the Moon, *S* the Sun. Join *AM, BM, CM,* and produce them indefinitely. If *CM* produced meet the Sun's surface, *AM* and *BM* may fall without it on different sides, so that the Sun may be eclipsed to *C* while *A* and *B* have an uninterrupted view of it. And if the Moon come to a point *M'* where it passes between *A* and the Sun, there may be no eclipse at *B* or *C*.

Thus it is by no means sufficient to calculate an eclipse of the Sun as seen from the centre of the Earth, for the appearances vary according to the position of the spectator on the surface, and must therefore be separately calculated for any proposed place.

The places on the Earth's surface also to which the eclipse is visible are not at all so easily ascertained as in the case of a lunar eclipse, for they depend on many circumstances besides the Sun's being at the time above the horizon.

It is evident that the Moon, like the Earth, casts a conical shadow in the direction opposite to that of the Sun, and that from all points within the shadow the Sun's light is completely shut out. Now we may calculate the path of the Moon's shadow just like that of the Earth's shadow in a lunar eclipse, and if we can find the parts of the Earth which it successively obscures, we know to what places the Sun will be totally eclipsed. The shadow, however, does not always extend as far as the Earth, for the orbit being elliptic and not circular the distance of the Moon varies, and sometimes the shadow comes to a point without reaching the Earth. When that is the case, the Sun cannot be totally eclipsed, for it will subtend a larger angle than the Moon from any point of the Earth's surface. In that case, when the observer is in the direct line of the axis of the shadow, the Moon will appear surrounded by a bright ring of the Sun's surface. Such eclipses are called *annular*. When an eclipse is central, it is *total* or *annular* according as the apparent diameter of the Moon is greater or less than that of the Sun. The difference between the diameters is never very great, so that neither of these appearances can last very long.

If E be the Earth as seen from the Moon, we may calculate the path SS' of the centre of the shadow, or of the point where the axis of the shadow, produced if necessary, meets the surface. The times corresponding to S and S' will be the times of beginning and ending of the central eclipse upon the Earth. An observer



at S at the first of these times will see the Sun rise centrally eclipsed, and an observer at S' will see it set centrally eclipsed, and all places on the line SS' will have a central eclipse at some part of the day. Of course partial eclipses will be seen over a much larger part of the surface.

If the Earth had no motion of rotation, the path of the central eclipse would not be difficult to find, but the diurnal motion makes the calculation very complicated. The student will best understand the nature of the path by consulting the maps which are given every year in the *Nautical Almanac.*

END OF THE FIRST PART.

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