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1868

The Cambridge Course of Physics.

ELEMENTS

OF

ASTRONOMY.

BY

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TEACHERS IN THE HIGH SCHOOL, CAMBRIDGE, MASS.

SECOND EDITION,
REVISED AND ENLARGED.

BOSTON:
CROSBY AND AINSWORTH.
NEW YORK: O. S. FELT.
1868.

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UNIVERSITY PRESS: WELCH, BIGELOW, & Co.,
CAMBRIDGE.

PREFACE

TO THE SECOND EDITION.

THE method of treatment in this ASTRONOMY agrees with that adopted in other parts of the Course, so far as the nature of the subject will admit. The aim throughout has been to show the scholar from what facts of observation, and by what processes of reasoning, astronomers have reached their present knowledge of the structure of the universe.

The authors believe that the principles which lie at the bottom of the explanation of most astronomical phenomena are really simple, and, if rightly presented, capable of being understood by high-school scholars of ordinary ability. They do not assume, however, that the explanations given in this book are in all cases full enough to enable the teacher to dispense with oral instruction.

The first part of the book treats of the motions and distances of the heavenly bodies; the second, of their physical features; and the third, of gravity, or the force by which they act upon one another. In this edition a fourth part, treating of the origin, transmutation, and conservation of energy, has been added, since it forms a fitting conclusion to the Astronomy, and also to the whole Course.

In no portion of the book is there assumed, on the part of the pupil, any knowledge of mathematics beyond that of the elements of plane geometry, and an ability to prove that in plane triangles the sines of the angles are proportional to their opposite sides. That it may not be necessary for scholars to study trigonometry before taking up this book, this last proposition is demonstrated in an Appendix.

In the Appendix, the principal constellations have been described, and illustrated by seventeen star-maps. These maps have been reduced by photography from the excellent charts in Argelander's *Uranometria Nova*. In order, however, that the maps in this reduced form might not be too crowded, all stars below the fourth magnitude have been omitted, as well as the circles of right ascension and declination. The dotted lines have been added by the authors to assist in tracing the leading stars in each constellation.

The Appendix also contains an outline of the history and mythology of the constellations; an account of the metric system and the calendar; and various astronomical tables.

In the preparation of the first and third parts of this book the authors have made free use of Airy's "Popular Astronomy" (London, 1866). In many instances material has been taken from this source with little alteration, except that it has been condensed and the language simplified. Yet the method of treatment which we have employed is quite different from that adopted by Airy.

The material of the second part of the volume has been drawn largely from the English translation of Guillemin's "Heavens" (London, 1866), Hind's "Solar System" (London, 1851), and Hind's "Astronomy" (London, 1863.)

The division of labor in preparing the book has been the same as was explained in the Preface to Part First.

CAMBRIDGE, March 5, 1868.

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I.

MOTIONS AND DISTANCES OF THE HEAVENLY BODIES.

MOTIONS AND DISTANCES OF THE HEAVENLY BODIES.

1. At the beginning of our study of Physics we came to the conclusion that matter is made up of insensible masses called *molecules*, and that these molecules are separated by spaces, which, though probably thousands of times greater than their own bulk, are yet insensibly small. We have also learned that many, and probably all, of these molecules are made up of yet smaller parts, called *atoms*.

Hitherto we have been mainly occupied with the consideration of the forces which act upon these atoms and molecules through insensible distances.

We now pass to the study of the earth and the heavenly bodies, and the forces which act upon them through the spaces by which they are separated.

THE SHAPE OF THE HEAVENLY BODIES.

2. *The Shape of the Earth.* — For several thousand years men supposed that the earth was a large platform, and that, if one went far enough, he would everywhere come to the edge, as one does at the sea-shore. As soon, however, as they began to make long voyages at sea, it was seen that the sea is not flat, but rounded like a low hill; for wherever we go at sea, we always see the masts of ships a long way off before we can see the hull or body of the ship, though, so far as size goes, the latter would be

much easier to be seen. The sea cuts off the view just like a hill rising between the two ships. It was also found that the distance at which ships of the same height begin to be seen is everywhere the same, and as light is known to come in straight lines, the hill of sea between the two ships must be everywhere the same. This can be so only on a globe, that is, on a body whose surface is rounded equally in every direction.

We see, then, that the surface of the ocean is spherical, and when we remember that about three fourths of the surface of the earth is covered with water, it seems probable that the whole surface of the earth is spherical.

Again, in an eclipse of the moon, we always see a shadow with a round edge moving across its disc. This shadow has never any other shape, whether the eclipse be great or small, and whatever part of the earth be facing the moon at the time. Now this shadow is known to be the shadow of the earth, and a body which casts a round shadow in every position must be a sphere.

The earth, therefore, must be a globe, or sphere.

3. *The Sun, Moon, and Planets are Globes.* — The disc of the sun is always circular. The same is true of that of the moon, though at times we see only a part of its disc. The same is true of all the planets, which bodies always present sensible discs when viewed with the telescope. Now it is well known that these bodies do not always present the same side to the earth; and, as a sphere is the only body that presents a circular outline from whatever position it is looked at, we see that the sun, moon, and planets are also globes.

The fixed stars present no sensible disc when viewed with the most powerful telescope, therefore we know nothing about the shape of these bodies.

THE APPARENT MOTIONS OF THE STARS.

4. If, on a clear night, we watch the eastern horizon through its whole extent from north to south, we see stars continually rising; and if we watch the western horizon through its whole extent from north to south, we see stars continually setting. We see, also, that the stars do not rise perpendicularly, but obliquely. Those which rise near to the north or near to the south rise very slantingly indeed. Those nearest to the east rise less obliquely. The same is true of their setting. Those near to the north or to the south set very obliquely; those which set nearest to the west set with a sharp incline. If we trace the whole path of any one of these stars, we find that it rises somewhere in the east in the sloping direction already described; that it continues to rise with a path becoming more and more horizontal, till it reaches a certain height in the south, when its course is exactly horizontal; and that it then declines by similar degrees, and sets at a place in the west just as far from the north point as the place where it rose in the east.

If we select a star that has risen near to the north, it takes it a long time to rise to its greatest height, which is very high in the south, and then an equally long time to set. Lastly, if we look to the north and observe those stars which are fairly above the horizon, we find them going around the Polar Star and describing a complete circle. These stars are called *circumpolar.*

The Polar Star to an ordinary observer does not appear to change its place during the whole night. Whenever he looks out, he finds it in the same place. Careful observation, however, shows that it does change its place and moves in a small circle. The stars of the Great Bear and of Cassiopeia turn in a circle considerably larger than the

Polar Star, but they go completely round in it without descending below the horizon. Capella and Vega describe still longer circles, of which the Pole Star is the centre. These stars pass below the horizon in the north, and pass nearly overhead when farthest to the south.

Thus, if we fix a straight rod in a certain standard direction, pointing nearly, but not exactly, to the Polar Star, we find that the stars which are close in the direction of this rod, as seen by viewing along it, describe a very small circle; the stars farther from it describe a larger circle; others just touch the northern horizon; whilst, in regard to others, if they do describe a whole circle at all, part of that circle is below the horizon; they are seen to come up in the east, to pass the south, and to go down in the west, and they are lost below the horizon from that place till they rise again in the east.

5. *Are the Movements of the Stars such that they appear to describe accurate Circles about a Point of the Sky near the Polar Star as a Centre?* — To answer this question we must use an instrument called the Equatorial. One form of this instrument is represented in Figure 1. It turns round an axis *A B*, which is placed in the direction which leads to the point of the sky around which the stars appear to turn, and which is not far from the Polar Star. This axis carries the telescope *C D*. As the instrument turns on its axis the telescope retains the same inclination to this axis unless another motion is given it at the same time. The telescope is, however, so arranged that another motion may be given to it, so as to place it in different positions, as *C′ D′*, *C″ D″*. It can thus be directed to stars in different parts of the heavens. If now the telescope is directed to any one star, it is found, by turning the instrument on its axis, that the telescope, without any alteration of its inclination to the axis, will follow that star from its rising to its setting. It is the same wherever the star may

Fig. 1.

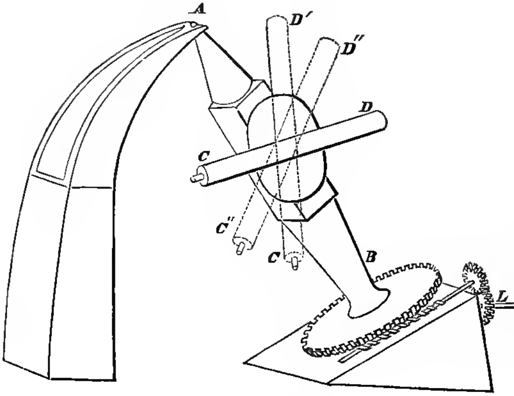
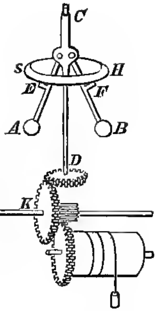


Fig. 2.



be, whether near the Polar Star or far from it. The telescope will follow the star by merely turning the instrument on its axis.

The movement of the stars, then, is of such a kind that they appear to describe accurate circles about a point of the sky near the Pole Star as a centre; for it is evident that the telescope, when the instrument is turned on its axis, describes such a circle, and, as seen, the telescope always points to the star to which it was directed.

6. *The Stars move at a uniform Rate, and all describe their Circles in the same Time.* — The best equatorials are furnished with a toothed wheel attached to the axis, in which works an endless screw or worm, as seen in Figure 1. By turning this worm the whole instrument is made to revolve. The worm is turned by an apparatus constructed especially for producing uniform movement. The one usually adopted, with some modification, is represented in Figure 2. The lower drum is turned by a falling weight, and its motion is regulated by the centrifugal balls *A B*, similar to those which are used to regulate the mo-

tion of a steam-engine. It is well known that whirling the balls, by the rotation of the axis to which they are attached, causes them to spread out, and the more rapidly they are whirled the more they spread.

When the speed has reached a certain limit, the spreading out of the balls causes their arms to rub against the fixed part, *S H.* This friction prevents further acceleration, and thus a uniform speed is produced with very great nicety. The spindle *K L* from this apparatus is attached to the screw which carries the equatorial. It causes the telescope of the equatorial to revolve around its axis uniformly, and thus gives us the means of ascertaining with the utmost exactness whether or not the stars move with uniform speed. When the machinery is in operation the telescope is pointed to a star. Whether this star be near the pole or at a distance from the pole, it is found that it is constantly seen in the field of view of the telescope; that is, the telescope turns just as fast as the star moves. Now as the telescope is moving with a uniform speed, it follows that all the stars are describing their orbits in the same time, and that each star moves with the same speed in every part of its orbit. The stars then move as though they were attached to a shell, which rotates at a uniform rate from east to west about an axis passing from a point of the sky near the Polar Star through the centre of the earth. This axis is therefore called the *celestial axis*, and the point of the sky near the Polar Star is called the north *celestial pole.*

7. *Refraction.* — When a telescope of considerable power is attached to the equatorial, so that we can see a small departure from the centre of the telescope in the position of the star we are looking at, and when we trace the course of the star down to the horizon, we find it to be a universal fact that the star is not quite so near the horizon as we should be led to expect. This is due to refraction.

We have already learned that a ray of light in passing from a rarer into a denser medium is always bent toward a perpendicular to the surface of this medium. The earth is surrounded by an atmosphere, and we will suppose, at first, that this atmosphere has a definite boundary and a uniform density. Let Figure 3 represent a part of the earth covered by the atmosphere. Suppose a beam of light is coming from a star in the direction *A B*, and that it meets the atmosphere at *B*. In passing into this denser medium it is bent toward a line perpendicular to the surface of the atmosphere, and takes the direction *B C*. Consequently the star would be seen in the direction *C B*. This would be the case if the atmosphere had a definite boundary and a uniform density. But if the atmosphere has not a definite boundary and varies in density from stratum to stratum, becoming more and more dense as we near the earth, the ray of light would be bent as described above in passing from one stratum to the next. This condition of things is shown in Figure 4. The star would be seen in the position *I*, instead of its real position *A*. The effect of refraction is, then, to cause all the stars to appear nearer the zenith (the point directly overhead) than they really are. If a star were exactly in the zenith, its position would not be changed by refraction, since the ray of light coming from the star

Fig. 3.

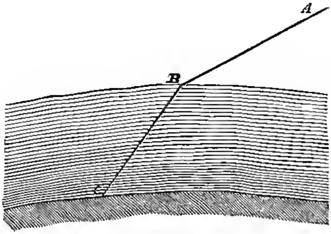
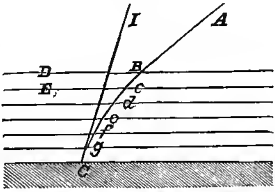


Fig. 4.



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would meet each stratum of air perpendicularly. The farther a star is from the zenith, the more is it displaced by refraction, since the light coming from it would meet each stratum more obliquely, and the more obliquely the ray of light meets the surface of the denser medium, the more is its direction bent.

8. *Does the Celestial Sphere really rotate about the Earth from East to West?* — It was for a long time supposed that the starry heavens turned round the earth daily, and that the apparent motion of the stars was real. But when it was discovered that the earth is round, it was at once seen that the visible motion of the heavenly bodies could be explained as well by supposing that the earth rotates from west to east about an axis which has the same direction as the axis about which the celestial sphere appears to rotate from east to west, as by supposing that this sphere does really rotate. The supposition that the earth rotates instead of the heavens is the simpler of the two, and the fact that the earth does thus rotate is capable of the following direct proof. It is found that when a heavy ball is suspended by a long and flexible string, at the equator, and made to vibrate, its vibrations appear to take place always in the same direction; that is, if it is set vibrating in a north and south direction, it will continue to vibrate in that direction; while if a ball similarly suspended is set to vibrating anywhere north of the equator, the direction in which it vibrates appears to be continually changing. Now we know that a body when once put in motion tends always to move in the same absolute direction, and it is reasonable to suppose that the heavy ball, when once set swinging, will always vibrate in the same absolute direction, provided that nothing interferes with its motion. If the point from which the ball is suspended is twirled while the ball is swinging, its direction of vibration does not change; and if the point is moved forward

in a straight line, or in the circumference of a circle, its directions of vibration are always parallel to one another. Suppose now that the earth be rotating from west to east, and that a ball be suspended at the equator; the motion of the earth would carry the point of suspension around in a circle, and the same would be true were the ball suspended north of the equator. Suppose now that a ball at the equator, and one some way north of the equator, were both set swinging in a north and south direction; what would be the apparent direction of the vibration of the ball in each case, on the supposition that the earth rotates? In each case, as we have seen, the vibrations of the ball would be parallel to one another. On the supposition that the earth rotates on its axis from west to east, are the directions which at different times we call north and south parallel to one another, or not? We must first see what we mean by north and south.

Suppose that two straight rods are fastened to a terrestrial globe, one at the equator and the other some way north of the equator, close by the brass meridian and parallel to it. These rods, of course, would point north and south to observers at these two points on the globe. Now rotate the globe, and the rod fastened at the equator is seen to remain parallel to the brass meridian, while the rod north of the equator begins to deviate at once from a direction parallel to the meridian, and deviates more and more till the globe has rotated through 90°, when it begins again to approach this direction, and when the globe has rotated through 180° it is again parallel to the brass meridian. At the equator, then, on the supposition that the earth is rotating, the directions which an observer at a given place would at different times call north and south are parallel with one another; while at a place north or south of the equator the directions which an observer at different times calls north and south are continually changing, although they appear to be always the same.

If, then, the vibrations of the ball suspended as described above are always in the same absolute direction, and the ball be set swinging from north to south, its vibrations at the equator ought to appear to be always in the same direction; while at a point north of the equator its plane of vibration ought to appear continually shifting in a direction contrary to that in which the north and south direction is really shifting. This is just what is found to be true on trial. The experiment was first made by Foucault at Paris in 1851, and has been often repeated with the same results. It was tried in this country some years ago, in the Bunker Hill Monument.

The supposition, then, that the earth rotates on its axis from west to east must be true, and the starry heavens are really at rest. That the earth appears to be at rest and the heavens rotating, is not surprising when we remember that, on looking out at the window of a railway car while the train is in rapid motion, we seem to be at rest, and the fences and trees to be shooting past us in the opposite direction.

9. *The Direction of the Circles described by the Apparent Motion of the Stars is different in different Parts of the Earth.* — The circles described by the stars in their apparent daily motion are always perpendicular to the axis of the earth. In our latitude they are oblique to the horizon, sloping from the south. They are oblique to the horizon because the horizon here is inclined to the earth's axis.* At the equator the horizon is parallel to this axis, and the circles described by the stars are consequently perpendicular to the horizon. South of the equator the horizon is inclined to the axis in a direction opposite to that in which it is inclined north of the equator, and the circles described by the stars are again oblique to the horizon, but sloping from the north. At the poles the horizon is perpendicular to the earth's axis, and the circles described

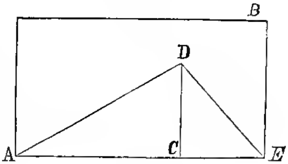
* See Appendix, II.

by the stars are consequently parallel to the horizon. Those stars which are nearer to the elevated pole than the horizon is, are always in view; while those nearer the depressed pole than the horizon is, are always out of sight. At any point north of the equator, the north pole of the heavens is elevated above the horizon, and the south pole depressed below it. Hence north of the equator the north pole is the elevated pole, and the south pole the depressed pole. South of the equator, of course, it is just the reverse.

10. *The Fixed Stars appear in the same Position in the Sky, from whatever Part of the Earth they are observed.* — It is necessary to find some way of describing the position of a star, as seen in the sky, in order to ascertain whether it is seen in the same position from different parts of the earth.

I wish to describe the position of a speck, *D* (see Figure 5), on a wall, so that any person could mark the position of the speck on a similar wall elsewhere. I could describe the position of the speck by giving the horizontal distance, *A C*, from one end of the wall, and the vertical distance, *C D*, from the floor, or by giving the measure of the distance *A D* from the corner *A*, and of the distance *E D* from the corner *E*. I might also give the measure of the distance *A C*, and of the inclination of the line *A D* to the horizon.

Fig. 5.



It is thus seen that there are several ways of describing the position of the speck, but that in every case two measures are necessary. The two measures which are necessary for defining the position of any point on a surface are called the *co-ordinates* of the point. Thus the distances *A C* and *C D* are *co-ordinates* of the point *D*. So also the distances $A D$ and $E D$, and the distance $A C$ and the angle $D A C$ are *co-ordinates* of the point D.

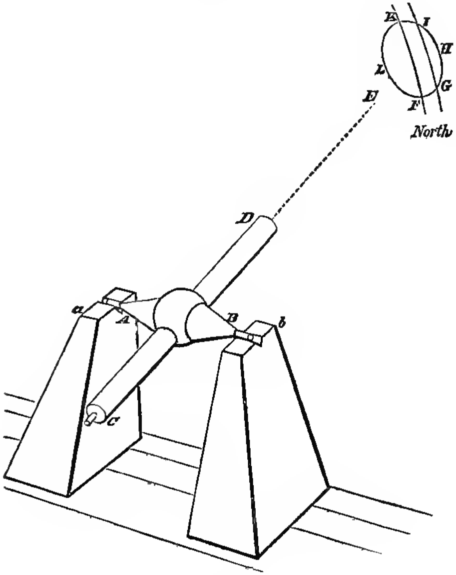
Suppose we wish to define the position of a point on a celestial globe. We could do it most conveniently by giving its distance from the pole of the globe measured in degrees, and the distance that the globe must rotate from a certain point before this point comes under the brass meridian. These two distances, both measured in degrees, would be the co-ordinates of the point.

Suppose, for instance, there is a fixed point marked on every celestial globe, and I wish to tell some one at a distance the position of a second mark, which I have found on my globe. I find that I must rotate the globe 23° to the westward from the fixed point to bring the mark under the brass meridian, and that the mark is 36° from the north pole of the globe. I send these co-ordinates to the distant person, and he sets his globe so that the fixed point is under the brass meridian, and rotates it 23° westward. He now knows that the point is under the meridian. He next measures along the meridian 36° from the given pole, and he knows the exact position of the point. If the globe were stationary and the brass meridian movable, we could describe the position of the point equally well by giving its angular distance from one of the poles of the globe, and the angular distance through which the meridian must be turned from a fixed point, in order to bring the point under it.

If now we suppose an imaginary arc of a great circle passing directly over our heads and through the celestial poles, and fixed to the earth in such a manner as to be carried around with it in its rotation from west to east, we can evidently describe the position of a star in the sky by ascertaining how far this imaginary meridian must sweep from a fixed point to bring the star under it, and by ascertaining its angular distance from one of the celestial poles when under this meridian. The angular distance of a star from a celestial pole, and the angular distance that an imaginary meridian must sweep over from a fixed point in order to bring the star under it, are the most convenient co-ordinates of a star.

11. *The Measurement of the Angular Distance that the Meridian must sweep over in order to bring the Star under it.* — The measurement of the angular space over which the meridian must sweep from a fixed point to bring the star under it is effected by means of a *Transit Instrument.* This instrument is represented in Figure 6. It consists

Fig. 6.



of a telescope mounted on an axis, *A B*, in such a way that when it turns around on this axis the line *C D E* prolonged to the sky will describe the imaginary meridian just spoken of. This curve must be perpendicular to the horizon, must divide the visible heavens into two equal parts, and must pass through the poles of the heavens.

To meet the first condition, it is necessary that the axis *A B* be exactly horizontal. To meet the second condition, it is necessary that the telescope *C D* be exactly square with its axis *A B*. The astronomer ascertains whether the telescope be exactly square with its axis or not, by looking at a distant mark, first, with the pivots *A* and *B* of the instrument resting on the piers *a* and *b*, and then with the axis turned over so that the pivots *A* and *B* rest on the piers *b* and *a*. If the telescope points equally well to the mark in both positions of the axis, it is exactly square with its axis.

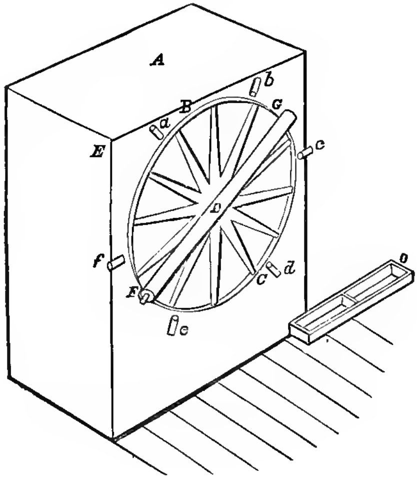
The astronomer ascertains whether the instrument is so adjusted that the curve described by the line *C D E* prolonged shall pass through the celestial pole, by means of the Polar Star. This star, as has already been stated, describes a small circle about the celestial pole as a centre. Let *F G H I K L* represent this circle. Suppose that in turning the transit instrument about its axis, the line *C D E* prolonged, traces the line *G I* or *F K*, as the case may be. The Polar Star in its revolution passes that line twice; and if the line passes through the celestial pole, which is the centre of the circle, the arcs *F H K* and *K L F* must be equal, and as the motion of the star is uniform, it will take it just as long to pass from *F* to *K*, as from *K* to *F*. If these arcs are described in equal times, the instrument is properly adjusted.

By means of the transit instrument objects can be viewed only when they come under the meridian. Some bright star, as Altair, is selected, and the time when it comes under the meridian accurately observed by means of the transit instrument and the clock. The time when it next comes under the meridian is also carefully observed, and the interval between these two *transits* of the star across the meridian is called a *sidereal day.* This day is divided into twenty-four equal parts called sidereal hours. The clock used in the observatory is called a sidereal clock, and is so constructed that it would describe just twenty-four hours between two successive transits of the same star, if its movements were perfectly accurate. In practice it is found impossible to construct a perfectly accurate clock. The rate at which the clock gains or loses time is ascertained by observing the successive transits of the same star, and, this being known, it becomes easy to reduce every observation to true sidereal time. The sidereal clock should beat seconds, and the beats should be very distinctly given.

To facilitate the determination of the exact time when a star or planet comes under the meridian, a number of cobweb threads are strung across the focus of the telescope at equal distances, parallel to one another and to the motion of the telescope as it is turned on its axis. These are called cross-wires. The observer notices a star approaching the meridian. He directs the telescope so as to observe the star when it actually crosses the meridian, and looks into the telescope. Just before the star begins to cross the wires, he looks at the face of the clock for the hours and minutes; he then listens to the beats of the clock, and thus finds the hour, minute, second, and fraction of a second when the star crosses each wire; then, by taking the mean of these times, he finds the time at which the star passes the meridian. The same observations are made with star after star, as they approach the meridian, and when these observations have been corrected for the error of the clock, the interval of time which elapses between the transit of a given star and the other stars observed becomes known. And as the apparent rotation of the celestial sphere is uniform, we thus find one of the co-ordinates by which the position of a star is defined, —that is, the distance that the imaginary meridian must turn from a given point in order to bring the star under it. Suppose the bright star Vega to be taken as the starting-point, and suppose we find that a given star is under the meridian two hours afterward. The meridian must then turn through 30° from Vega in order to bring the star under it.

12. *The Mural Circle.*—The next thing is to ascertain the angular distance of the star from the pole of the heavens when it is under the meridian. This is ascertained by means of the *Mural Circle.* One form of this

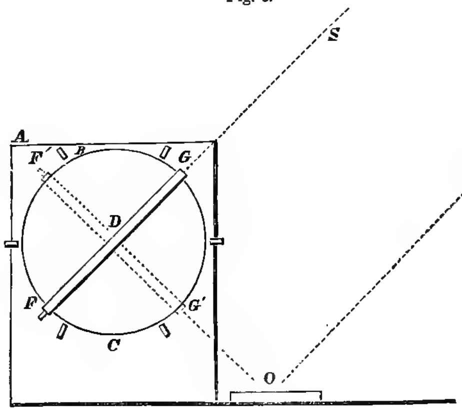
Fig. 7.



instrument is represented in Figure 7. *A* is a stone pier which supports the axis of the instrument, and to which microscopes, *a*, *b*, *c*, *d*, *e*, and *f*, are attached. The face of the pier which carries the microscopes fronts either the east or the west. The axis carries the circle *B C* and the telescope FG. The telescope is fastened to the circle, so that both must move together. This circle is graduated on its outside into degrees, minutes, and other subdivisions. The microscopes serve as pointers for observing the exact position of the circle, and by their aid the space between the divisions can be subdivided with great exactness.

We wish to know in any observation how far the telescope points above the horizon. This can be easily ascertained, if we know what is the reading of the circle when the telescope points horizontally. For example, if the reading of the circle is $5^\circ\ 15'$ when the telescope points horizontally, and $27^\circ\ 16'\ 25''$ when the telescope is pointing to the star, the telescope must point $27^\circ\ 16'\ 25'' - 5^\circ\ 15' = 22^\circ\ 1'\ 25''$ above the horizon. The reading of the circle when the telescope points horizontally is ascertained as follows. It is well known that a star seen by reflection from the surface of water or quicksilver appears just as far below the horizon as it is above it. The trough o is filled with quicksilver, and the telescope first directed to a star, S (see Figure 8), on the me-

Fig. 8.

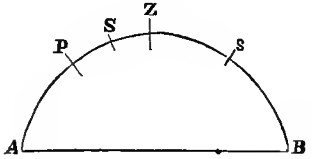


ridian, and the reading of the circle observed; the telescope is then turned so as to observe the star as reflected by the quicksilver, and the reading of the circle again observed. The horizontal reading of the circle is evidently midway between these two readings.

The elevation of the north celestial pole must next be ascertained. This is done by observing the Pole Star. This star, as has already been stated, describes a small circle about the celestial pole as its centre. With the mural circle the angular elevation of this star above the horizon is observed at its highest and lowest points. These observations are corrected for refraction, and their mean gives the angular elevation of the pole above the horizon. The angular elevation of any body above the horizon is called its *altitude*, and its altitude when on the meridian is called its *meridian altitude.*

13. *Polar Distance.* — By observing now the altitude of any star when under the meridian, we can easily ascertain its angular distance from the pole. This angular distance is called the *polar distance.* If the star be north of the zenith, its polar distance is equal to the difference between its meridian altitude and the altitude of the pole. If the star is south of the zenith, its polar distance will be 180° *minus* the sum of the altitude of the pole and of the meridian altitude of the star. This is at once seen by a reference to Figure 9. Let $A\ Z\ B$ represent the arc of the meridian above the horizon. It contains, of course, 180°. Let P represent the position of the pole whose altitude we will suppose to be 42°. Let S be a star north of the zenith, whose meridian altitude is 75°: its polar distance $PS =$

Fig. 9.



$75^\circ - 42^\circ = 33^\circ$. Let s be a star south of the zenith, whose meridian altitude Bs is 45°: its polar distance $Ps = 180^\circ - (42^\circ + 45^\circ) = 93^\circ$.

It is often found more convenient to refer the position of the pole and of the star to the zenith rather than to the horizon. The angular distance of the pole or of a star from the zenith is evidently equal to 90° *minus* the altitude of the pole or star.

14. *Summary.* — By means, then, of the transit instrument and the mural circle, we can determine the two co-ordinates necessary for describing the position of a star or planet on the surface of the sky. These co-ordinates for the fixed stars are found to be precisely the same, whether they are measured at Washington, Greenwich, or the Cape of Good Hope. This shows that the stars are seen in exactly the same position, from whatever part of the earth they are observed.

We have now learned that the starry heavens appear to rotate, all in a piece, from west to east, once in twenty-four hours, about an axis which passes through the centre of the earth to a point near the Pole Star; and that the fixed stars, whether viewed from one part of the earth or another, always appear in the same parts of the heavens.

THE APPARENT MOTIONS OF THE SUN.

15. *The Sidereal and the Solar Day and Year.* — We have already defined a sidereal day as the interval of time between two successive transits of the same star across the meridian. The *solar* or ordinary day is the interval between two successive transits of the sun across the meridian. This interval is ascertained by means of the transit instrument, in the same way as the interval between the successive transits of a given star. This interval is found to be a little longer than a sidereal day.

If a bright star which can be seen with the telescope of the transit instrument at noonday comes under the meridian at precisely the same instant as the sun, when this star comes under the meridian the next day, the sun will be about a degree eastward of the meridian; and the next day, when the star comes under the meridian, the sun will be about two degrees eastward; and so on. In about 360 days after both came under the meridian together, they will come under the meridian together a second time. When the sun and a given star come under the meridian together, they are said to come into *conjunction*. The interval between two successive conjunctions of the sun and a given fixed star is called a *sidereal year*. This year is about $20\frac{1}{3}$ minutes longer than the ordinary year. The sun, then, appears to move entirely around the heavens from west to east in a period of about one year. The fact that the sun is apparently moving eastward among the stars is evident without the use of the transit instrument. If we notice the position of the constellations above the western horizon night after night at the same time after sunset, we shall find that they come nearer and nearer the horizon, until many of them have disappeared below it, and we shall find, in about a year from the first observation, that these constellations occupy the same position in the heavens as at first.

One of the co-ordinates of the sun's position in the sky is daily changing. This co-ordinate is the angular space over which the meridian must sweep, from a fixed point, in order to bring the sun on the meridian.

16. *The Solar Days are of Unequal Length.* — Careful observation with the transit instrument shows that the sun is not only apparently moving around the heavens eastward, but that it is moving at unequal rates from day to day. The interval, therefore, between two successive passages of the sun across the meridian varies in length.

The solar days are therefore of unequal length. The solar day, like the sidereal, is divided into twenty-four equal parts called hours. The solar hours are a little longer than sidereal hours, and of unequal length. An hour of ordinary clock time is the average length of all the solar hours; and the ordinary civil day, consisting of twenty-four hours of clock time, is the average length of the solar days. Hence ordinary clock time is called *mean* solar time.

17. *The Polar Distance of the Sun is continually changing.* — In midwinter the sun appears low in the south, and from that time till midsummer its meridian altitude gradually increases. It then begins to diminish, and goes on diminishing till midwinter again. Now, since at a given place on the earth's surface the altitude of the celestial pole remains the same, we must conclude that the second co-ordinate of the sun's position in the heavens, its polar distance, is continually changing. By means of the mural circle the least polar distance of the sun, which occurs in midsummer, is found to be about 66½°, and its greatest polar distance in midwinter is 113½° The circle described by the sun in its apparent journey round the earth, then, is not perpendicular to the earth's axis, but is inclined to that axis at an angle of about 66½°. The plane of the sun's orbit passes through the centre of the earth, and the circle formed by the intersection of this plane with the celestial sphere is called the *ecliptic.*

When the polar distance of the sun is 90°, it is directly overhead at midday at the equator of the earth When its polar distance is 66½°, it is directly overhead at midday at any place 23½° north of the equator; and when its polar distance is 113½°, it will be directly overhead at midday at any place 23½° south of the equator. At every place, then, situated within about 23½° of the equator either north or south, — that is, between the tropic of Cancer and the tropic of Capricorn, — the sun comes directly overhead at least once a year. This belt of the earth is called the *Torrid Zone.*

18. *The Relative Length of Day and Night, and the Changes of the Seasons.* — The sun, of course, always illumines just one half of the earth; and when the sun is just over the equator, the illumined part just reaches the north and the south poles. When the sun is directly over the tropic of Cancer, of course the illumined part of the earth reaches $23\frac{1}{2}°$ beyond the north pole, and only to within $23\frac{1}{2}°$ of the south pole. Hence an observer situated within $23\frac{1}{2}°$ of the north pole would see the sun during the entire rotation of the earth, while an observer within $23\frac{1}{2}°$ of the south pole would not see the sun at all. When the sun is directly over the tropic of Capricorn, the illumined part of the earth reaches $23\frac{1}{2}°$ beyond the south pole, and only to within $23\frac{1}{2}°$ of the north pole. Hence a person situated anywhere within $23\frac{1}{2}°$ of the north pole would not see the sun at all during twenty-four hours, while any person situated anywhere within $23\frac{1}{2}°$ of the south pole would see the sun the whole 24 hours. At every place, then, which is situated within $23\frac{1}{2}°$ of the north or south pole, that is, north of the arctic or south of the antarctic circle, there is at least one day in the year in which the sun does not come above the horizon at all, and one day in which the sun does not sink below the horizon at all. The belts of the earth betwen these circles and the poles are called the *Frigid Zones.* Just at the arctic and antarctic circles there is only one day in the year in which the sun does not rise above or sink below the horizon. But as you go nearer the poles, the number of days when the sun does not rise above or sink below the horizon increases. Between the arctic and antarctic circles and the tropics there is no place where the sun comes directly overhead, or where it does

not rise and set every day. These belts of the earth are called the *Temperate Zones.*

A moment's reflection will make it clear that, when the sun is directly over the equator, every place on the surface of the earth is illumined twelve hours and is in darkness twelve hours; and as the sun is directly over the equator twice a year, the days and nights are equal in every part of the earth twice a year. When the sun is directly overhead at any place north of the equator, every part of the earth south of the equator is in the sunshine a shorter time than it is out of it, while every place north of the equator is in the sunshine longer than it is out; that is, in this position of the sun the days south of the equator are shorter than the nights, while north of the equator the days are longer than the nights. The farther a place is south of the equator, the shorter the day and the longer the night; while the farther north of the equator a place is, the longer the day and the shorter the night. When the sun is directly over any place south of the equator, the relative length of the day and night is the reverse of that just described; that is, the day north of the equator is shorter than the night, while south of the equator it is longer than the night.

When the sun's north polar distance is less than 90°, it is summer in the northern hemisphere and winter in the southern, since the northern hemisphere then receives more heat from the sun than the southern. When the north polar distance of the sun is more than 90°, it is winter in the northern hemisphere and summer in the southern, since the latter then receives more heat from the sun than the former.

The variation of the sun's polar distance, then, gives rise to the change of seasons and the varying length of day and night. If the axis of the earth were perpendicular to the plane of the sun's path among the stars, there would be no change of seasons and no variation in the relative length of day and night.*

19. *Declination and Right Ascension.* — It is often convenient to refer the position of the sun and stars to the celestial equator rather than to the celestial poles. The celestial equator is an imaginary circle perpendicular to the earth's axis, and dividing the celestial sphere into two equal parts. The angular distance of the sun or other heavenly body from the equator, is the difference between 90° and its polar distance. This angular distance is called *declination;* north declination when it is north of the equator, and south declination when it is south of the equator. When the polar distance of the sun is 66½°, its declination is 23½° north; when its polar distance is 113½°, its declination is 23½° south.

The plane of the sun's apparent orbit is evidently inclined about 23½° to the equator, which it bisects. Twice a year the sun's declination is 0°. The points at which the plane of the sun's orbit cuts the equator are called *equinoctial* points, since, as we have already seen, the days and nights are equal in every part of the earth when the sun is at these points. The sun is at one of these points on the 21st of March, and at the other on the 21st of September. The first point is called the *spring* or *vernal equinox*, and the second the *autumnal equinox.* The Spring equinox is not far from Algenib, a bright star in the constellation Pegasus; and the autumnal equinox is not far from Denebola, the bright star in the tail of Leo. The Spring equinox is usually taken as the fixed point on the celestial sphere from which the imaginary meridian is supposed to start in sweeping over the heavens, so as to bring a given star under it; and the angular distance which

* The change of seasons and of the relative length of day and night can be more clearly illustrated by means of one of the little globes made for that purpose than by any description or figures.

the meridian must sweep over from this point to bring a star or planet under it is called *right ascension.* Right ascension is, therefore, always measured eastward.

20. *The Precession of the Equinoxes.* — It is found that the path of the sun does not always cross the equator at exactly the same place from year to year. The equinoctial point shifts along the equator westward 50.1″ yearly. This yearly shifting of the equinoctial point is called the *precession of the equinoxes.*

21. *The Tropical Year.* — The interval between two successive conjunctions of the sun with the same fixed star is called, as we have seen, a *sidereal* year. The interval between two successive appearances of the sun at the same equinoctial point is evidently a little shorter than the sidereal year, since this point is continually shifting westward. This interval is called a *tropical* year. It is 20⅓ minutes shorter than the sidereal year. The seasons are evidently completed in a tropical year, since they depend on the declination of the sun. Hence the tropical year is the year of common life, which is regulated by the change of the seasons.

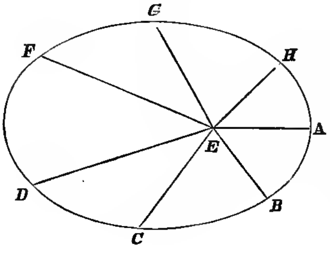
22. *Solstitial Points.* — When the sun has reached his greatest northern or southern declination, his declination scarcely changes for two or three days. He seems to halt a little in his journey toward the poles before he turns back toward the equator. These two points in his path are called *solstices* (*sun-stands*). The one north of the equator, where the sun appears in midsummer, is called the *summer* solstice; and the one south of the equator, where the sun appears at midwinter, is called the *winter* solstice.

23. *The Variation of the Sun's apparent Diameter.* — The angular diameter of the sun is about 32′, but this diameter is continually changing. From a certain point it goes on gradually increasing for six months. It then begins to

diminish, and continues to diminish for the next six months, when it becomes the same as at first. We must, therefore, conclude that the sun's distance from the earth is continually changing. It is obviously at the greatest distance from the earth when its diameter is least, and nearest to the earth when its diameter is greatest. It is a well-known fact that the angle subtended by any object diminishes in the same proportion as its distance increases. If its distance is doubled, the angle which it subtends is diminished one half. Hence, by measuring the angular diameter of the sun from time to time we find out the relative distance of the sun from the earth at those times.

24. *The Form of the Sun's apparent Path among the Stars.* — If we draw a straight line, *E A*, to represent the direction and distance of the sun at any time, and observe the sun's angular diameter and his right ascension at this time, we can, by observing his right ascension and angular diameter at any other time, find the length and direction of another line, *E B*, which shall represent the direction and distance of the sun at the time of the second observation. For the length of the line *E B* will be to the length of the line *E A* in the inverse ratio of the observed angular diameters of the sun, and the angle which the line *E B* makes with *E A* will evidently be the difference of the observed right ascensions. In the same way the length and direction of the lines *E C*, *E D*, *E F*, *E G*, and *E H*, are determined at different times in the course of a year. Now if a curve be drawn through the ends of these lines, it will evidently represent the form of the

Fig. 10.



sun's path among the stars during a year. This curve is found to be, not a circle, but an ellipse. The earth is situated at one of the *foci* of this ellipse. The nearer the sun is to the earth, the more rapid is his motion in right ascension.

25. *Summary.* — We find, then, that the sun is continually changing his position with reference to the fixed stars; that he is travelling eastward among them at the rate of about a degree a day, thus making an entire circuit of the heavens in a year. We find that the axis of the earth is not perpendicular to this path, but inclined to it at an angle of about $66\frac{1}{2}°$; and that the sun travels at unequal rates in different parts of his path.

The plane of the sun's path passes through the earth's centre, and its intersection with the celestial sphere is called the *ecliptic*.

The inclination of the earth's axis to the plane of the ecliptic gives rise to the change of seasons and the change in the relative length of day and night.

TWILIGHT.

26. Darkness does not come on at once after sunset. Full daylight gradually fades away into the darkness of night, and in the morning the darkness gradually melts away into full daylight again. This gradual transition from daylight to darkness in the evening, and from darkness to full daylight in the morning, is called *twilight*.

27. *Cause of Twilight.* — After the sun sinks below the horizon, it still shines upon the particles of air above the earth, and these reflect the light to the earth again. At first there is a large number of these illumined particles above the horizon, but as the sun sinks lower and lower they become fewer and fewer, and the light which they reflect to the earth feebler and feebler, until it becomes im-

perceptible when the sun has sunk 18° below the horizon. And again in the morning, when the sun has come within 18° of the horizon, it begins to shine on the particles of air above the horizon, and they begin to reflect a feeble light to the earth. As the sun comes nearer and nearer to the horizon, more and more particles above the horizon become illumined, and the light which they reflect to the earth becomes more and more intense, till full daylight bursts forth at the rising of the sun.

28. *Duration of Twilight.* — Twilight, then, lasts at night till the sun is 18° below the horizon, and begins in the morning when the sun has come within 18° of the horizon. Is twilight of the same length in different parts of the earth, and in the same parts of the earth at different seasons of the year? This is equivalent to inquiring whether the sun gets 18° below the horizon in the same time in different parts of the earth, or in the same part of the earth at different seasons. When we say that the sun is 18° below the horizon, we mean 18° measured on a *vertical* circle, that is, a circle perpendicular to the horizon and passing through the zenith. We will suppose the time to be the 21st of March, and the place to be the equator of the earth. Here the circle described by the sun in his apparent daily motion is perpendicular to the horizon and passes through the zenith, that is, it is a vertical circle; and when the earth has rotated through 18° after the sun has reached the horizon, he will be 18° below it, and twilight will end. We will next suppose the time to be the same, but the observer to be either north or south of the equator. The circle described by the sun in his apparent daily motion is still a great circle, but it is inclined to the horizon, and so is not a vertical circle. The sun here will evidently not be 18° below the horizon when the earth has rotated 18° after the sun has reached the horizon. For the sun has gone down obliquely below the horizon, and

he must descend more than 18° degrees in this oblique path in order to get 18° below the horizon; just as one in walking obliquely from a wall must walk more than ten feet to get ten feet from the wall. Hence when the sun is on the equator, twilight is shorter at the equator than at places north or south of the equator. The farther north or south of the equator, the more obliquely does the sun rise and set, and of course the longer the twilight.

We will next suppose that the sun is at the summer or winter solstice, and that the observer is at the equator. The diurnal path of the sun is still perpendicular to the horizon, but does not pass through the zenith. It is not a great circle, but a small circle. When, therefore, the earth has rotated 18° after the sun has reached the horizon, the sun is not 18° below the horizon, since an arc of 18° on a small circle is shorter than an arc of 18° on a vertical circle, which is a great circle. Hence twilight at the equator grows longer as the sun passes north or south of the equator.

Suppose now that an observer is north of the equator, and the sun is at one of the solstices. The plane of the horizon is now inclined to the axis of the earth's rotation, and is carried around that axis with a wabbling motion. This motion may be illustrated by passing a wire through the centre of a piece of cardboard, and fastening the card so that it will be inclined to the wire, and then rotating the wire.

Let us first see what would be true of twilight, provided the horizon were rotating on an axis which coincided with its own plane. When the sun is either north or south of the equator, its daily motion is in a small circle, and as these small circles are all equally inclined to the plane of the horizon, twilight would grow longer as these circles become smaller; that is, as the sun moves north and south from the equator. But by referring to the illustration of the wabbling motion of the plane of the horizon in consequence of its being inclined to the axis of the earth's rotation, it will be seen that to a person north of the equator the portion of the heavens south of the east and west points is carried, as it rotates, bodily away from the stars as they sink below it; while the portion north of those points is carried bodily towards the stars as they sink below it. Thus this wabbling motion of the horizon tends to increase the length of twilight in summer, and to shorten it in winter; while the fact that the sun is moving in small circles at each of these times tends to increase the length of twilight as well in winter as in summer. The tendency of this wabbling motion of the horizon to diminish the length of twilight in winter, in our latitude, almost exactly balances the effect of the sun's moving in a small circle. The length of twilight varies but a few minutes from the autumnal to the vernal equinox; while in midsummer in our latitude, twilight is half an hour longer than at the equinoxes. At the poles the sun never sinks more than $23\frac{1}{2}°$ below the horizon, hence the twilight there lasts about two thirds of the long winter night.

29. *Summary.* — The twilight at the equator is shortest when the sun is on the equator, and longest when the sun is farthest north or south of the equator. The shortest twilight at any time is at the equator.

As you go from the equator either north or south, the twilight lengthens, and it is longer in summer than in winter. The shortest twilight at the equator is about one hour and twelve minutes; in our latitude it is about an hour and a half.

THE APPARENT MOTION OF THE MOON.

30. *The Lunar Month.* — The new moon is always seen near the western horizon soon after sunset. It is farther

and farther from the horizon at sunset, night after night, until about a fortnight from new moon, when the moon becomes full and rises just as the sun sets. It then lags farther and farther behind the sun until it does not rise till just before sunrise.

We see, then, that the moon is also moving eastward among the stars, and that it is moving more rapidly than the sun. When the moon and sun both rise or set together, they are said to be in *conjunction*, and when the moon rises just as the sun sets, it is said to be in *opposition*. The moon passes from conjunction to conjunction, or from opposition to opposition, in 29½ days. This period is the ordinary *lunar month*.

The moon, like the sun, in her eastward journey among the stars, changes not only her right ascension from day to day, but also her declination. The points at which the moon's path cuts the ecliptic are called the *nodes* (knots). The point where its path cuts it from south to north is called the *ascending* node, while the other is called the *descending* node.

31. *The Lunar Day.* — The interval between two successive passages of the moon across the meridian is sometimes called a *lunar day*. This interval is nearly an hour longer than the solar day. The lunar days are found to be even more unequal in length than the solar days.

32. *The Moon's Orbit is an Ellipse.* — By a method similar to that used in the case of the sun, the orbit of the moon is found to be an ellipse, which has the earth at one of its foci. When the moon is in that part of her orbit which is nearest to the earth, she is said to be in *perigee* (near the earth), and when in that part of her orbit farthest from the earth, she is said to be in *apogee* (away from the earth). The line joining these points of the moon's orbit is the major axis of an ellipse, and is called the line of *apsides*. The moon moves faster at perigee than at apogee.

THE APPARENT MOTIONS OF THE PLANETS.

33. *Venus.* — There is a conspicuous star sometimes seen in the west in the early evening, and sometimes in the east before sunrise. This star is familiarly known as the morning and evening star. If it be watched for some time it will be seen, after it has ceased to be a morning star, close to the horizon in the west soon after sunset. Then night after night it will be seen to have moved farther and farther to the eastward from the sun, until its angular distance from him is about 47°. Venus, at this point, is said to be at its greatest eastern *elongation* from the sun. It then begins to approach the sun, and finally passes him and appears again on his western side as a morning star. It separates farther and farther from him to the westward, until its angular distance from him is again about 47°, when it is said to be at its greatest western elongation from the sun. It then approaches the sun again, and passes by him to the eastward. The interval between two successive appearances of Venus at its greatest eastern or its greatest western elongation is about nineteen months. We see, then, that Venus is apparently carried on with the sun in his eastward journey among the stars, sometimes falling behind him an angular distance of 47°, and again overtaking and passing beyond him a like distance.

34. *Mercury.* — The movements of Mercury are similar to those of Venus, but his greatest elongation from the sun never exceeds 29°, so that he sinks below the horizon too soon after sunset, and rises too short a time before the sun, to be often visible to the naked eye.

Stars, like Mercury and Venus, which are thus continually changing their position in the sky, are called *planets* (wanderers), to distinguish them from the fixed stars, which we have seen do not sensibly change their position in the sky.

35. *Movements of the other Planets among the Stars.*— Besides the two planets, Mercury and Venus, there are three other planets, Mars, Jupiter, and Saturn, which are conspicuous to the naked eye; and two large planets, Uranus and Neptune, which are so distant that they cannot be seen except with the telescope; and a large number of smaller ones, called *asteroids*, which, though nearer, can be seen only with a telescope.

If one of the most conspicuous of these planets, as Jupiter, be watched for a series of years, it will be found to work its way gradually to the eastward among the stars, and to complete the circuit of the heavens in a longer or shorter time. This time is found to be always the same for the same planet; but different for different planets. But this eastward motion of the planet is not regular and uniform. The planet advances quite rapidly at times, then halts and remains stationary, then actually goes backward or retrogrades, then halts and remains stationary again, and again advances.

Fig. 11.

Figure 11 represents the apparent motion of Jupiter among the stars during the year 1866.

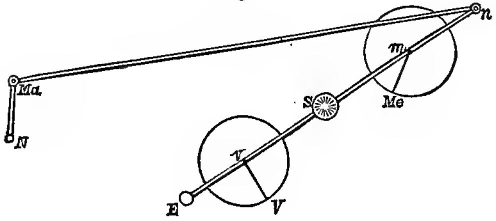
THE PTOLEMAIC SYSTEM.

36. We have now seen that the sun appears to revolve about the earth from west to east once in a year in an orbit whose exact form is an ellipse; that the moon appears to revolve about the earth in the same direction and in an orbit of the same form once in a month; that

Mercury and Venus appear to swing backward and forward across the sun, while they are at the same time carried onward with him in his eastward motion; and that the other planets also appear to move about the earth in longer or shorter periods and in very irregular paths. It seems improbable that any planet should really move in so irregular a path as Jupiter appears to move in.

The ancient astronomers assumed that the planets all moved in circular orbits, and attempted to account for their apparent irregular motions by the combination of various circular motions. They supposed that the earth is fixed, and that the sun moves. They supposed that a bar, or something equivalent, was connected at one end with the earth, and that on some part it carried the sun; and as they saw that the planet Venus is apparently sometimes on one side of the sun and sometimes on the other, they said that the planet Venus moves in a circle whose centre is on the same bar. Then if we suppose that Venus is revolving around this centre at the same time that the bar is moving about the earth, we get a perfect representation of the apparent motion of Venus and the sun as seen from the earth. This is illustrated by Figure 12. Sup-

Fig. 12.



pose *E* to be the fixed earth; *E v S m n*, a bar turning in a circle, having one end fixed at *E*; *S* the sun carried by it; *v*, the centre of the orbit in which Venus revolves;

V, the planet Venus, connected with *v* by a bar (real or imaginary), and thus describing a circle round *v*, while *v* itself is carried on the bar round the earth.

It was supposed that Mercury revolved in another circle, whose centre was also on the same bar, but perhaps beyond the sun, as at *m*. They did not pretend to say exactly where these centres were: all that they were certain about was this; that the centre of motion of each planet was on the same bar that supported the sun. Now it is easy to be seen, on these suppositions, that both Mercury and Venus would appear, when viewed from the earth, at one time on the right and again on the left of the sun, and at the same time they would appear to be carried around the earth with him.

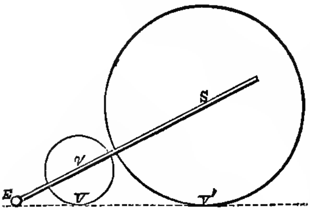
With regard to Mars, they found out that its motion could be represented pretty well by supposing that this same bar carried another centre at *n*, around which Mars revolved as at *Ma*, carried by an arm long enough to project beyond the earth, so that its orbit completely surrounded the earth as well as the sun. In the same way the apparent motions of Jupiter and Saturn were accounted for.

The motion of the planet Mars, however, still presented some discordances, and there were some smaller discordances with regard to all the other planets. These led to the invention of those things known as epicycles, deferents, etc., the nature of which may be thus explained. By the contrivance which we have already described, they found that the movement of the point *Ma* at the end of the rod *n Ma* would nearly, but not exactly, represent the motion of Mars. To make it represent the motion more exactly, they supposed that another small rod, *Ma N*, was carried by the longer rod, jointed at *Ma*, and turning around in a different time. To make it still more exact, they supposed another shorter rod carried at *N*, and that its extremity carried the planet Mars. The same complications were necessary for all the other planets. It will be thus seen that a combination of no less than five circular motions was necessary to account for the apparent irregularities in the motion of a single planet. Thus the joint *n* moved in a circle about *E*; the joint *Ma* in a circle about *n*; the joint *N* about *Ma*; and the planet Mars about *N*. Of all the complicated systems that man ever devised, there never was one like this Ptolemaic system. The celebrated king of Castile, Alfonso, the greatest patron of astronomy in his age, alluding to this theory of epicycles, said that "if he had been consulted at the creation, he could have done the thing better."

THE SYSTEM OF TYCHO DE BRAHE.

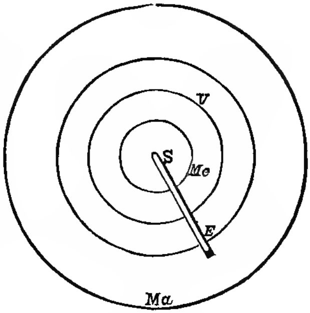
37. If we suppose the earth fixed as at *E* (Figure 13), and Venus to be revolving around a centre situated somewhere in the line *E S*, we may remove that centre as far from the earth as we please, and yet get the same appearance, provided we enlarge the dimensions of the orbit of Venus in the same proportion. For instance, suppose *E* to be the earth, the smaller circle to be the orbit of Venus, and the sun to be at *S*; then, in revolving in her orbit, Venus will appear to go to a certain distance to the right and to the left of the sun. But we may take any other point on the bar, even the point *S* itself, as the centre of the orbit of Venus, provided we give Venus a larger circle to revolve in. If the larger circle in the figure represents the orbit

Fig. 13.



of Venus, she will appear to move just as far to the right and to the left of the sun as when she moved in the small orbit. We may then fix the centre of the orbit of Venus where we please; and so with the centres of the orbits of Mercury, Mars, and of each of the other planets, provided we give proper dimensions to their orbits. By having all their centres at the centre of the sun, we have all the planets revolving about the sun, while the sun revolves about the earth, as shown in Figure 14. This system is much less complex than the Ptolemaic system, though the theory of epicycles and deferents is still retained. This modification of the Ptolemaic system was adopted by the great Danish astronomer, Tycho de Brahe.

Fig. 14.



THE COPERNICAN SYSTEM.

38. Now, instead of supposing the sun to be travelling, and by some imaginary power causing the planets to revolve about himself as their travelling centre, suppose we say that the earth revolves about the sun, and that the sun is a fixed, or nearly fixed, body, and that all the planets, including the earth, go around the sun; that is, in Figure 14, instead of supposing *S* with the whole system of orbits to be travelling around *E*, suppose *Me*, *V*, *E*, and *Ma* to travel in separate orbits about *S*, and the appearances of the planets, as viewed from the earth, will be represented exactly as well as before.

This great step of assuming the sun to be the centre of motion of all the planets, including the earth, was taken by Copernicus. But he could not get rid of the epicycles to account for the apparent irregularities in the motion of the planets.

THE SYSTEM OF KEPLER.

39. Tycho de Brahe had employed a good part of his life in observing and recording the position of the heavenly bodies. His pupil, Kepler, by examining carefully the observations which Brahe had made of the planets, and especially of the planet Mars, and comparing them with his own, ascertained that the whole could be represented with the utmost accuracy by supposing that Mars moves in an ellipse, one of whose foci is occupied by the sun. It is difficult to explain in a few words how Kepler came to this conclusion; generally speaking, it was by the method of trial and error. The number of suppositions he made to account for the motion of the planets is beyond belief: that the planets turned round centres at a little distance from the sun; that their epicycles and deferents turned on points at a little distance from the ends of the bar to which they were jointed; and the like. After trying every device he could think of with epicycles, eccentrics, and deferents, and computing the apparent place of Mars from these different assumptions, and comparing them with the places really observed by Brahe, he found that he could not bring them nearer to Brahe's observations than eight minutes of a degree. He then said boldly that so good an observer as Brahe could not be wrong by eight minutes, and added, "Out of these eight minutes we will construct a new theory that will explain the motion of all the planets." The theory thus constructed was, that all the planets move in ellipses

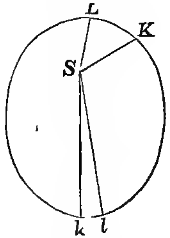
which have the sun at one of their foci. This theory has been found to explain accurately all the seeming irregularities in the motion of the planets.

The planets appear to advance and retrograde because they are seen from the earth, which is itself revolving about the sun. If they were seen from the sun, their advance would be steady and regular.

To construct an ellipse, stick two pins into a board a little way apart; fasten to the pins the ends of a string somewhat longer than the distance between the pins; then, keeping the string stretched by the point of a pencil, carry the pencil round. The curve described will be an ellipse, and the points where the pins are stuck into the board will be the *foci* of the ellipse.

If the ellipse in Figure 15 be the orbit of a planet, *S* will be the place of the sun. The sun is at the focus of the ellipse described by every planet. Every planet describes a different ellipse. The degree of flatness of the ellipse is different for every planet, and the direction of the long diameter of the ellipse is different for every planet. There is, in fact, the greatest variety among the ellipses described by the different planets.

Fig. 15.



40. *Kepler's First Law.*—The first great fact, then, that Kepler made out with regard to the motion of the planets is, that *they all move in ellipses, which have the sun at one focus.* This fact is usually called *Kepler's first law.*

41. *Kepler's Second Law.*—The second fact made out by this astronomer is, that the planets move at unequal rates in different parts of their orbits. He found that each planet, when in its *perihelion*, that is, the part of its orbit which is nearest the sun, travels quickly, and when

in its *aphelion*, or the part of its orbit which is farthest from the sun, travels slowly.

Kepler expressed this law of motion in this way: if in one part of the planet's orbit the lines *S K* and *S L* (see Figure 15) enclose a certain portion of the area of the ellipse, and in another part the two lines *s k* and *s l* enclose a space equal to that enclosed by *S K* and *S L*, the planet will be just as long in moving over the short arc *k l* as over the large arc *K L*; that is, *the planets describe equal areas in equal times.*

42. *Kepler's Third Law.* — Kepler made out another very important fact with regard to the motions of the planets compared with their distances from the sun; namely, that *the squares of the periodic times of the planets are to each other as the cubes of their mean distances from the sun.*

SUMMARY.

The earth, sun, moon, and planets are *globes*. The shape of the fixed stars is unknown. (2, 3.)

The starry heavens appear to rotate in a piece from east to west about an axis which passes through the centre of the earth and a point near the Polar Star. This rotation is completed in twenty-four hours. (6.)

The earth rotates from west to east once in twenty-four hours about an axis which passes through the centre of the earth and a point near the Polar Star. This rotation of the earth causes the heavens to appear to rotate in the opposite direction. (8.)

The stars describe accurate circles, though their paths are somewhat disturbed by *refraction*. (5, 7.)

The circles described by the stars are differently inclined to the plane of the horizon in different parts of the earth. (9.)

The co-ordinates of a heavenly body are the angular distance of the body from the celestial pole, and the angular distance that the meridian must sweep over from a fixed point to bring the body under it. (10.)

The latter of these co-ordinates is measured by means of the *Transit Instrument;* the former, by means of the *Mural Circle.* (11, 12.)

The co-ordinates of the fixed stars are always the same wherever they are measured. (14.)

The co-ordinates of the sun are found to change daily. He travels eastward among the stars and completes the circuit of the heavens once a year.

The plane of his orbit passes through the centre of the earth, and is inclined to the earth's axis at an angle of $66\frac{1}{2}°$. (15, 17.)

The sun in his eastward journey describes an *ellipse* with the earth at one of its foci. (24.)

The moon travels eastward among the stars more rapidly than the sun. She completes a circuit of the heavens in a month, and describes an ellipse with the earth at its focus. (30, 32.)

Venus and Mercury are seen to vibrate to and fro across the sun. At their greatest elongations, Venus is 47°, and Mercury 29°, from the sun. (33, 34.)

The other planets describe very circuitous paths among the stars. They advance to the eastward for a time, then halt, and then even go backward. (35.)

The ancient astronomers assumed that the planets all moved in circular orbits, and attempted to account for their apparent irregular motion by the combination of various circular motions. They constructed a system of cycles, epicycles, and deferents. (36.)

Tycho de Brahe simplified the Ptolemaic system by placing the centres of all the cycles at the centre of the sun. (37.)

Copernicus simplified it further by making the earth revolve about the sun. (38.)

Kepler was the first who did away with the complex system of cycles and epicycles by showing that all the planets move in *ellipses, all of which have the sun at one focus.* (39, 40.)

He further showed that *the planets describe equal areas in equal times,* and that the *squares of the periodic times of the planets are to each other as the cubes of their mean distances from the sun.* (41, 42.)

A *sidereal* day is the interval between two successive transits of a star across the meridian.

The *solar* day is the interval between two successive transits of the sun across the meridian. (15.)

The solar days are of *unequal* length. The ordinary civil day is the average length of these. (16.)

The variation of the sun's polar distance gives rise to the change of seasons and the varying length of day and night. (18.)

The circle formed by the intersection of the plane of the sun's orbit with the celestial sphere is called the *ecliptic.*

The *celestial equator* is a circle which is perpendicular to the earth's axis and which divides the celestial sphere into two equal parts.

The points where the sun's orbit cuts the celestial equator are called the *equinoxes.*

Angular distance measured north or south from the celestial equator is called *declination.*

Angular distance measured from the vernal equinox eastward is called *right ascension.* (19.)

The equinoxes slowly shift along the equator to the westward. This shifting is called the *precession of the equinoxes.* (20.)

The interval between two successive conjunctions of the sun with the same fixed star is a *sidereal* year.

The interval between two successive appearances of the sun at the same equinox is a *tropical* year. (21.)

The points in the sun's path at which he gains his greatest northern or southern declination are called the *solstices*. (22.)

Twilight is caused by the reflection of the sun-light from the clouds and the particles of air.

It continues while the sun is within 18° of the horizon. It is shortest at the equator and longest at the poles. In our latitude it is longer in the summer than in the winter. (27.)

HOW TO FIND THE PERIODIC TIMES OF THE PLANETS.

43. *The Periodic time of the Earth determined by direct Observation.* — We have seen that the real motion of the earth about the sun causes the sun to appear to revolve about the earth. It is evident that the time that it takes the earth to revolve about the sun is the same as that which it takes the sun to make an apparent revolution around the earth. The periodic time of the earth is found by observing the interval between two successive appearances of the sun at the same equinox, or two successive conjunctions with the same star.

44. *Synodic Period of a Planet determined by direct Observation.* — The planets Venus and Mercury, as we have already seen, never appear in the part of the heavens opposite to the sun. Hence the orbits of these planets must lie wholly inside the orbit of the earth. When these planets come between the earth and the sun they are said to be in *inferior conjunction*, and when the sun is between them and the earth they are said to be in *superior conjunction*. The planets whose orbits lie wholly

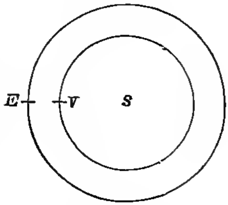
within the earth's orbit are called *inferior* planets. Those whose orbits lie wholly without the earth's orbit are called *superior* planets. When a superior planet appears in the same part of the heavens as the sun, that is, when the sun is between the earth and planet, it is said to be in *conjunction*. When the planet appears in the opposite part of the heavens to that of the sun, that is, when the earth is between the planet and the sun, it is said to be in *opposition*.

The interval between two successive oppositions of a planet, or between two successive conjunctions of the same kind, is called the *synodic* revolution of the planet. This interval is determined by direct observation.

45. *How to find the Sidereal Period of an inferior Planet.* — The sidereal period of a planet is the time it takes to make a complete revolution about the sun. This time can be easily computed when we know the sidereal period of the earth and the synodic period of the planet.

Let P be the sidereal period of the earth, S the synodic period of Venus, and p the sidereal period of Venus. P and S are known by direct observation, and p is required.

Fig. 16.



We will suppose that Venus is at inferior conjunction; then E, V, and S will represent the respective places of the earth, Venus, and the sun. If the earth were stationary as well as the sun, then Venus would come again into conjunction when it had just completed a revolution about the sun; but the earth is moving in the same direction as Venus, hence Venus must make a complete revolution and then overtake the earth before it comes into inferior conjunction again. If two persons start together at some point on the circumference of a circle, and the first walks faster than the other, he must gain the whole length of the circumference before he comes up to the second again. In the same way, after Venus has come into inferior conjunction, it must gain 360° upon the earth before it can come into inferior conjunction again.

$\frac{360°}{P}$ = the angular space passed over by the earth in one day. $\frac{360°}{p}$ = angular space passed over by Venus in one day.

$\frac{360°}{p} - \frac{360°}{P}$ = the angular gain of Venus upon the earth in one day. But Venus gains 360° in S days, hence $\frac{360°}{S}$ = daily gain of Venus.

Hence $\frac{360°}{p} - \frac{360°}{P} = \frac{360°}{S}$.

Divide by 360, and we have

$$\frac{1}{p} - \frac{1}{P} = \frac{1}{S}.$$

$$PS - pS = Pp.$$

$$p(S + P) = PS.$$

$$p = \frac{PS}{S + P}.$$

46. *To find the Sidereal Period of a Superior Planet.*— The sidereal period of a superior planet can be found by a similar method. In this case the earth gains upon the planet.

Let p and S represent the sidereal and synodical period of a superior planet. Then $\frac{360°}{P} - \frac{360°}{p}$ = daily gain of the earth upon the planet:

$$\frac{360°}{P} - \frac{360°}{p} = \frac{360°}{S}.$$

$$p = \frac{PS}{S - P}.$$

47. *Synodical and Sidereal Periods of the Planets.* — The following table gives the synodical and sidereal periods of the principal planets: —

	Synodical Period.	Sidereal Period.			
Mercury	115.877 days	87.969 days	or	3	months
Venus	583.921 "	224.701	"	7½	"
Earth		365.256	"	1	year
Mars	779.936 "	686.980	"	2	years
Jupiter	398.884 "	4332.585	"	12	"
Saturn	378.092 "	10759.220	"	29	"
Uranus	369.656 "	30686.821	"	84	"
Neptune	367.489 "	60126.722	"	164	"

SUMMARY.

The *sidereal period* of the earth is found by observing the interval between two successive appearances of the sun at the same equinox, or two successive conjunctions of the sun with the same star. (43.)

An *inferior* planet is one whose orbit lies wholly within the earth's orbit.

A *superior* planet is one whose orbit lies wholly without the earth's orbit.

A planet is in *inferior conjunction* when it lies in the same part of the heavens as the sun, and is between the earth and sun.

A planet is in *superior conjunction* when it lies in the same part of the heavens as the sun, and is beyond the sun.

A planet is in *opposition* when it lies in the opposite part of the heavens from the sun.

The superior planets are never in *inferior conjunction*, and the inferior planets are never *in opposition.*

The *synodical period* of a planet is the interval between two successive oppositions of the planet, or between two successive conjunctions of the same kind.

The synodical period is ascertained by *direct observation.* (44.)

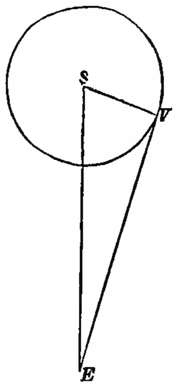
The *sidereal period* of a planet is the time it takes the planet to make a complete revolution about the sun.

The sidereal period of a planet can be computed when the sidereal period of the earth and the synodical period of the planet are known. (45, 46.)

HOW TO FIND THE DISTANCE OF THE PLANETS FROM THE SUN.

48. *To find the relative Distances of the Inferior Planets from the Sun.*—We have now seen how to find the periodic times of the planets, which must have been known to Kepler before he could discover the simple relation which the periodic times of the planets bear to their mean distances from the sun. We must next see how we can find the relative distances of the planets from the sun. We will begin with the inferior planets.

Fig. 17.



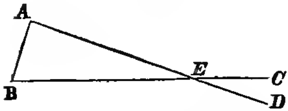
Let *V*, in Figure 17, represent the position of Venus at its greatest elongation from the sun; *S*, the position of the sun; and *E* that of the earth. The line *E V* will evidently be tangent to a circle described about the sun with a radius equal to the distance of Venus from the sun at the time of this greatest elongation. Draw the radius *S V* and the line *S E*. Since *S V* is a radius, the angle at *V* is a right angle. The angle at *E* is known by measurement, and the angle at $S = 90° -$ the angle *E*. In the right-angled triangle

$E\ V S$, we then know the three angles, and we wish to find the ratio of the side $S\ V$ to the side $S\ E$.

The ratio of these two sides may be found by construction as follows:—

Draw any line, as $A\ B$ (see Figure 18), and from the point A draw the line $A\ D$ at right angles to the line $A\ B$. From the point B draw the line $B\ C$, making with the line $B\ A$ an angle equal to the angle at S in Figure 17. These lines will intersect at some point, as E, and $E\ A\ B$ will be a right-angled triangle similar to $E\ V S$, and the side $A\ B$ will have the same ratio to $B\ E$ as $V S$ has to $S\ E$. Measure now the two lines $A\ B$ and $B\ E$ by means of the scale and dividers, and the ratio of $A\ B$ to $B\ E$, and consequently of $V S$ to $E\ S$, becomes known.

Fig. 18.



The ratio of these lines may be found with greater accuracy by trigonometrical computation, as follows:—

$$V S : E\ S = \sin S\ E\ V : (\sin S\ V E = 1).^*$$

Substitute the value of the sine of $S\ E\ V$, and we have

$$V S : E\ S = .723 : 1.$$

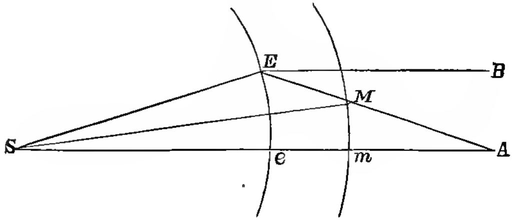
Hence the relative distances of Venus and of the earth from the sun are .723 and 1.

As Venus moves in an ellipse, and its greatest elongation takes place in different parts of its orbit, the angle $S\ E\ V$ will not always be the same. In order to get the mean distance of Venus from the sun, we must know the average value of its greatest elongation from the sun. This is obtained by observing a large number of such elongations.

* See Appendix, I. 1 and 3.

49. *To find the relative Distances of the Superior Planets from the Sun.* — Let *S*, *e*, and *m*, in Figure 19, represent the relative positions of the sun, the earth, and

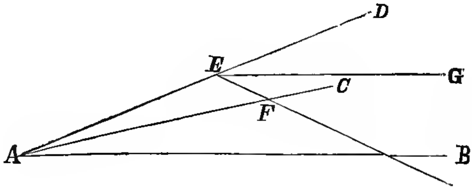
Fig. 19.



Mars, when the latter planet is in opposition. Let *E* and *M* represent the relative positions of the earth and Mars the day after opposition. At the first observation Mars will be seen in the direction *e m A*, and at the second observation, in the direction *E M A*.

But the fixed stars are so distant that, if a line, *e A*, were drawn to a fixed star at the first observation, and a line, *E B*, drawn from the earth to the same fixed star at the second observation, these two lines would be sensibly parallel; that is, the fixed star would be seen in the direction of the line *e A* at the first observation, and in the direction of the line *E B*, parallel to *e A*, at the second observation. But if Mars were seen in the direction of the fixed star at the first observation, it would appear back, or west, of that star at the second observation by the angular distance *B E A*; that is, the planet would have retrograded that angular distance. Now this retrogression of Mars during one day at the time of opposition can be measured directly by observation. This measurement gives us the value of the angle *B E A*. But we know the rate at which both the earth and Mars are moving in their orbits, and from this we can easily find the angular distance passed over by each in one day. This gives us the angles $E S A$ and $M S A$. We can now find the relative length of the lines $M S$ and $E S$ (which represent the distance of Mars and of the earth from the sun) both by construction and by trigonometrical computation. The relative length of these lines can be found by construction, as follows. Draw any line,

Fig. 20.



$A B$, then through the point A draw two lines, $A C$ and $A D$, making with $A B$ angles respectively equal to the angles $M S A$ and $E S A$, as found above. Through any point on the line $A D$ draw the line $E G$, parallel to the line $A B$; and draw $E F$, making with $E G$ an angle equal to the angle $B E A$, as found by observation. The triangle $A E F$ will evidently be similar to the triangle $E S M$, and the side $F A$ will bear to $E A$ the same ratio as $M S$ bears to $E S$. This ratio can be found by measurement of the two lines $A F$ and $A E$.

This ratio can be determined with much greater accuracy by the following trigonometrical calculation.

Since $E B$ and $e A$ are parallel, the angle $E A S$ is equal to $B E A$.

$$S E A = 180° - (E S A + E A S).$$
$$E S M = E S A - M S A.$$
$$E M S = 180 - (S E A + E S M).$$

We have then

$$M S : E S = \sin S E A : \sin E M S.$$

Substituting the values of the sines, and reducing the ratio to its lowest terms, we have

$$MS : ES = 1.524 : 1.$$

Thus we find that the relative distances of Mars and the earth from the sun are 1.524 and 1. By the simple observation of its greatest elongation we are able to determine the relative distance of an inferior planet and the earth from the sun; and by the equally simple observation of the daily retrogression of a superior planet we can find the relative distance of such a planet and the earth from the sun.

50. *The Relative Distances of the Planets from the Sun.* — In this way the relative distances of the principal planets have been found to be as follows:—

Mercury	0.387
Venus	0.723
Earth	1.00
Mars	1.524
Jupiter	5.203
Saturn	9.539
Uranus	19.183
Neptune	30.037

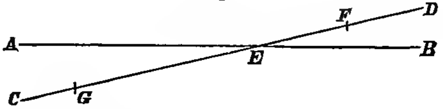
Knowing the periodic times of the planets and their relative distances from the sun, Kepler found the ratio which these bear to each other by the method of trial and error, which he had previously used in ascertaining the form of the orbits of the planets.

51. *To find the Distance of the Earth from the Sun.* — We have now found the relative distances of all the planets, including the earth, from the sun. If now we can find the distance of the earth from the sun in miles, we can easily find the distances of all the planets from the sun in miles.

Now it is evident that, when two straight lines cross each other, the distances between these two lines at any two points are proportionate to the distance of these points from the intersection of the two lines.

Thus, suppose the two straight lines *A B* and *C D* (Figure 21) cross each other at *E*, and suppose the point

Fig. 21.

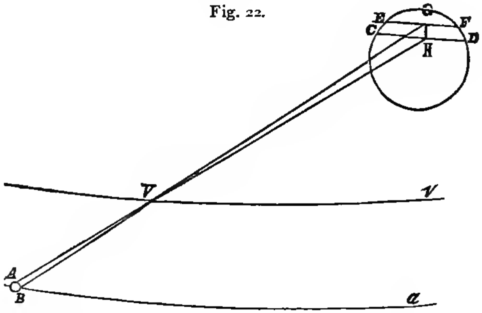


G be twice as far from *E* as *F* is from the same point; the distance between the two lines will be twice as great at *G* as at *F*.

Now it occasionally happens that Venus, at inferior conjunction, passes directly across the disc of the sun.

In Figure 22 let *V* represent the position of Venus at

Fig. 22.



inferior conjunction, *A B* the position of the earth, and *C D* that of the sun.

An observer at *A* would see Venus crossing the sun in the line *C D*, and an observer at *B* would see it cross the sun in the line *E F*. These two chords will be parallel to each other, and the distance between them will be equal to the distance between the two lines *A H* and *B G* at the distance *H* from their intersection at *V*. This distance bears to the distance between the two lines at *A* the same ratio that the distance *V H* bears to the distance *V A*.

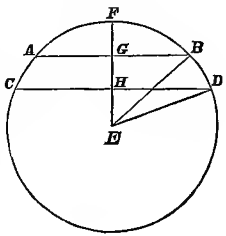
But we have already found that the distance *A H* bears to the distance *V H* the ratio of 1 to .723. But $VA = AH - VH$. Hence *V A* bears to *V H* the ratio 277 to 723. Then $GH : AB = 723 : 277$.

Hence, if we know the distance between the two observers at *A* and *B* in a straight line in miles, we can find the distance *G H* in miles.

Since Venus revolves about the sun from west to east, it will appear to us to be moving westward when it crosses the sun, while the sun is apparently moving eastward. We however know the rate at which both the sun and Venus are moving. Hence, if we know the time that it takes Venus to cross the sun's disc, we can find what angular distance both have passed over in this time. And as they are moving in opposite directions, the sum of these distances will give the angular measure of the chord described by Venus across the sun's disc.

Each observer, then, thus notices carefully the time that it takes Venus to cross the sun's disc. From this observed time each is able to find the angular value of the chord described by the planet.

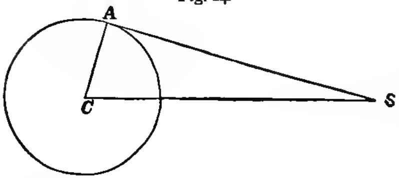
Fig. 23.



Let *A B* and *C D* (Figure 23) represent the chords described on the sun's disc by the passage of Venus. Draw the radii *E F*, *E B*, and *E D*. The radius *E F* is drawn perpendicular to the two chords, and therefore bisects them. The angular diameter of the sun can be found directly by observation. Hence the angular value of each of the radii $E B$ and $E D$ is known; also the angular values of $G B$ and $H D$, the halves of $A B$ and $C D$. Hence in each of the right-angled triangles $E G B$ and $E H D$ the angular values of the hypothenuse and of one side are known, and the angular value of the other side can be easily found. For $\overline{E B}^2 - \overline{G B}^2 = \overline{GE}^2$; and $\overline{E D}^2 - \overline{H D}^2 = \overline{H E}^2$. But $G H = G E - H E$.

The angular value of $G H$ is then known, and also its linear value. Dividing the linear value by the number of seconds in its angular value, we find how long a line will subtend an angle of 1″ at the distance of the sun. Knowing this, we know how large an angle will be subtended by the earth's radius at the distance of the sun.

Fig. 24.



In Figure 24, let S represent the place of the sun, and C the centre of the earth. Draw the line $S A$ tangent to the surface of the earth, and draw the radius $A C$. The triangle $S A C$ will be right-angled at A. The comparative length of the lines $A C$ and $C S$ can now be determined either by construction, as in section 45, or by the following computation: —

$$C S : C A = 1 : \sin C S A.$$

Substituting the value of sin $C S A$, we find

$$C S : C A = 23{,}750 \text{ (in round numbers)} : 1.$$

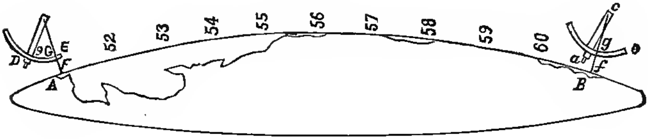
Hence the distance of the sun from the earth is, in round numbers, 23,750 times the length of the earth's radius in miles.

Now if we can find the length of the earth's radius in miles, and the distance in a straight line between the points *A* and *B* (Figure 22), we can find the distance of the earth and of each of the planets from the sun in miles.

52. *To find the Length of the Radius of the Earth in Miles.* — We know that the circumference of a circle is 3.1416 times as long as its diameter. Now, if the earth is an exact sphere, every meridian of the earth will evidently be an exact circle; and if we can measure any known fraction of one of these meridians, we can easily find its whole length; and from this the length of the diameter of the earth, which is of course the diameter of every great circle of the earth.

53. *How to measure a known Fraction of a Meridian.* — Suppose two places, *A* and *B* (Figure 25), to be situ-

Fig. 25.



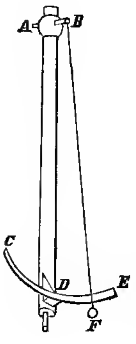
ated, the one, for instance, at Shanklin Down, in the Isle of Wight, and the other on the little island of Balta, in the Shetland Isles. We wish to know, first, what fraction of a whole meridian is the arc included between these two places; and second, how long this arc is in miles.

Any line which is perpendicular to a tangent to a circle at the point of contact is said to be vertical to the circle; and, if two verticals be prolonged within the circumference of the circle, they will meet at the centre of the circle. The fraction of the circumference included between the two verticals depends upon their inclination to each other. If they are inclined to each other at an angle of 1°, the arc included between them is $\frac{1}{360}$ of the whole circumference, and, if they are inclined to each other at an angle of 12°, the arc included between them is $\frac{1}{30}$ of the whole circumference.

But a plumb-line is always vertical to a great circle of the earth; hence, if we can find the inclination of a plumb line at *A* to one at *B*, we know what fraction of the whole meridian the arc included between *A* and *B* is. The angle which the directions of the plumb-lines at the two stations make with each other can be ascertained by means of the *Zenith Sector.*

This instrument is shown in Figure 26. It consists of a telescope swinging upon pivots *A B*, and having attached to it an arc *C D E*, graduated into degrees and minutes. There is a plumb-line, *B F*, connected with the upper end of the telescope, or with one of the pivots. This plumb-line is a very fine silver wire supporting a weight, and kept steady by hanging in water. It gives us the direction of the vertical.

Fig. 26.

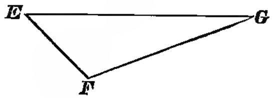


The zenith sector is taken to one of the stations, and the telescope directed to a bright star upon the meridian near the zenith. The number of degrees and minutes which the plumb-line hangs from the zero point at the centre of the telescope on the graduated arc is observed. The sector is now taken to the other station, and the same star is again observed when upon the meridian, and the number of degrees and minutes that the plumb-line hangs from the zero point on the arc is again observed. Now, since the directions of the telescope at the two stations are sensibly parallel, the difference of the degrees and minutes that the plumb-line at the two stations hangs away from the telescope must be the amount by which the verticals at the two places are inclined to each other. Suppose that this difference amounts to 12°, then the arc *A B* is equal to $\frac{1}{30}$ of the whole circumference of the meridian.

As these two stations are 700 or 800 miles apart, we cannot of course measure the distance between them by using only a yard-stick, though the distance between these stations must be calculated by means of a distance first measured by a yard-stick.

54. *Triangulation.* — Such measurements are effected by means of a system of triangulation. Suppose that we have three places, *E*, *F*, and *G* (Figure 27); the two nearest, *E* and *F*, on a plain, and perhaps six or eight miles apart; a third, *G*, at a considerable distance, perhaps inaccessible from *E* and *F*, at least in a straight line. The distance of *G* from either *E* or *F*, which cannot well be measured, can be readily found by measuring the line *E F*, and the angle which this line makes with lines drawn from *E* and *F* to the point *G*. For we may draw any line, *E F*, and from its extremities draw the lines *E G* and *F G*, making with the first line the angle determined by measurement, and by means of the scale and dividers the lines *E G* and *F G* can be measured. We can also find the length of the line by trigonometrical computation.*

Fig. 27.

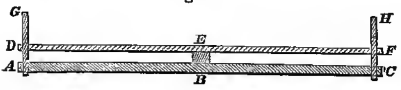


55. *The Measurement of a Base Line.* — For every system of triangulation one line must be measured. This line is called a *base line.* In extensive systems of triangulation, where great accuracy is required, the measurement of the base line is a very troublesome operation. It seems at first thought extremely easy to measure a straight line,

* See Appendix, I. 5.

but, in fact, there is nothing more difficult. In the first place what are we to measure it with? If we use bars of metal, they, as we have seen, expand when warmed and contract when cooled, and consequently are not always of the same length. The line must be measured by the yard. But the yard is a certain definite length, and cannot therefore be the length of any rod whose length changes with the temperature to which it is exposed. A rod can be a yard long only at a particular temperature. Many base lines have been measured with rods and chains of iron or brass, but every precaution has been used in every part of the operation to screen them from changes of temperature, by covering them with tents; putting perhaps half a dozen bars at a time in a row, with several thicknesses of tent over them, so as to protect them effectually from the sun and wind. Having taken this precaution to shield them from the effects of changes of temperature, thermometers are placed by the side of the bars, and their temperature thus ascertained. Knowing, then, their length at a given temperature, and how much they expand for a given rise of temperature or contract for a given fall of temperature, the length represented by the bars when used can be ascertained. Figure 28 represents another contrivance which has been used with great success. It consists of a combination of two bars;

Fig. 28.

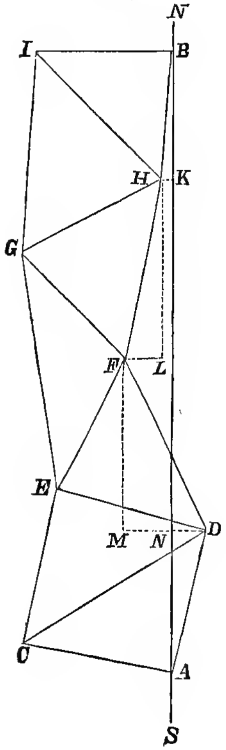


one, *A B C*, of brass, and the other, *D E F*, of iron. These bars are connected at the middle, *E B*, and they have projecting tongues, *A D G* and *C F H*. These tongues turn easily on pivots at *A*, *D*, *C*, and F. The length *G D*, is just $\frac{3}{5}$ of *G A*, and *H F* just $\frac{3}{5}$ of *H C*. Now brass expands more than iron for the same rise of temperature. Suppose the rod *A B C* to remain of the same length, and the rod *D E F* to expand; the points *G* and *H* would evidently be carried farther apart. But suppose the rod *D E F* to remain of the same length, and the rod *A B C* to expand; the points *G* and *H* will come nearer together. Suppose now that both rods expand together, but that the rod *A B C* expands just as much more rapidly than the other rod as the distance *A G* is greater than the distance *D G;* the points *H* and *G* will evidently remain at the same distance from each other. Now we know that brass expands $1\frac{2}{3}$ times as fast as iron, and the distance *A G* is $1\frac{2}{3}$ times the distance *D G.*

A number of combined bars like these are placed one after another, with a small interval between each two; and then the question is, how is the interval between them to be measured? It will not do to make one bar touch the other, because expansions may be going on in one of the series of bars, and it would jostle the others throughout the whole extent. This small distance is sometimes measured by means of microscopes mounted on the same principle as the bars, so that the measure which they give is not affected by temperature. In some cases, glass wedges have been dropped between the successive bars, in others sliding tongues have been used. The result of all this has been, that a distance of eight or ten miles has been measured to within a very small fraction of an inch.

We have been thus minute in our account of the measurement of a base line, to give an illustration of the extreme care that must be taken in measuring lines and angles which are to be used in the computation of celestial distances. And we see the necessity of this great carefulness when we remember that this base line, which does not exceed eight or ten miles, is to be used, first, in the computation of the radius of the earth, which is some 4,000 miles long; then in computing the distance of the sun, which is some 24,000 times the length of the earth's radius; then also in computing the distance of Neptune from the sun, which is some thirty times the distance of the earth from the sun; and finally, as we shall see, in computing the distance of the fixed stars, which are thousands of times more remote than Neptune from the sun. It is like measuring a length of a hundredth of an inch on the wall in hair-breadths as a basis for computing the dimensions of a house in hair-breadths. A slight mistake in the measure of the first length would lead to an enormous error in the final result.

Fig. 29.



56. But the point *B* (Figure 25) is too distant to be seen from the extremities of our base line, one end of which may be at *A*. We must, therefore, approach it step by step with our triangles.

The method by which this is accomplished, and the distance between the points finally determined, is illustrated by Figure 29. Suppose we are to find the distance between the points *A* and *B*. First measure the base line *A C*. The angle which this line makes with a north and south line is then ascertained. To do this, a transit instrument is set up at *A*, and its telescope so adjusted that it will point exactly to the north celestial pole. It is then turned down to the horizon, and a mark is set up at a distance in the direction of the north as thus found. A *theodolite* is then set up at *A*. This instrument consists of a telescope attached to a graduated circle which is arranged so as to turn horizontally. When, therefore, the telescope of the instrument is turned horizontally, the graduated circle carried around with it shows how many degrees it has been turned. The telescope is first directed to the mark, and then turned around till it points to the station *C*. The number of degrees that the telescope has been turned, as indicated by the graduated circle, shows the angle which the line *A C* makes with the north and south line. A third station, *D*, is then chosen, which can be distinctly seen from both *A* and *C*, and a signal erected at this point. The telescope of the theodolite is again turned to *C* and then to *D*, and the number of degrees that the telescope is turned in passing from *C* to *D* gives the angle *C A D*. The line *A D* is then drawn so as to make this angle with *A C*. The theodolite is next carried to *C* and pointed to *A*, and then turned till it points to *D*. The number of degrees it has to be turned gives the angle *D C A*. The line *D C* is then drawn, making this angle with *C A*.

A fourth station, *E*, is now selected, which can be readily seen at *C* and *D*. Then *E C D* and *E D C* are measured by means of the theodolite, as before, and the lines *C E* and *D E* are drawn, making the angles at *C* and *D* equal to those measured. Thus we go on step by step, measuring the angles and drawing the triangles, till we reach the point *B*. The distance *B A* can then be measured by means of the scale and dividers, and compared with the length of the base line *A C*. The distance of the stations *D*, *E*, *F*, etc. from the preceding station, and the distance of *B* from *A*, can be more accurately determined by trigonometrical computation. In the first triangle, the side CA and the angles at C and A are known by measurement; hence the other parts of the triangle can be readily computed. Then, in the second triangle, the side CD and the angles at C and D become known, and the other part of this triangle can be computed; and so on to the end. Then, by drawing the dotted lines, a series of right-angled triangles is formed, in each of which the hypothenuse is known, and one of the acute angles can be readily found. For in the right-angled triangle AND the angle $NAD = DAC - NAC$. In the right-angled triangle FMD, $MFD = 180° - (DAN + CDA + CDE + EDF)$. This may be shown by drawing a line through D parallel to NS. The sum of all the angles at the point D will then be 180°. But the angle formed by FD with the line supposed to be drawn will be equal to MFD, and the angle formed by AD with the same line will be equal to DAN. In a similar manner the angles FHL and HBK can be found. Hence the parts of these right-angled triangles can be computed, and the lengths of the sides AN, MF, LK, and KB can be found. But the sum of these sides is evidently equal to the length of the line AB.

57. In this way it has been ascertained, that, if the two plumb-lines at A and B (Figure 25) are inclined to each other at an angle of 12°, the length of the arc between them in miles is about 830 miles. From this we conclude that we must pass over an arc $69\frac{1}{6}$ miles long in order to find the distance of two places whose verticals are inclined one degree.

$69\frac{1}{6} \times 360$, then, gives the circumference of the earth, and this divided by 3.1416 gives the diameter of the earth, one half of which is the radius, which is thus found to be about 4,000 miles long.

58. *The Earth not a perfect Sphere.* By measuring arcs of meridian in different parts of the earth, we find that the arcs included between two places whose verticals are inclined to each other one degree are not always of the same length in miles. It has been found that towards the poles two places must be farther apart than near the equator, to have their verticals inclined to each other one degree. The earth, then, cannot be an exact sphere. Since the arc between two places whose verticals are inclined one degree is longer near the poles than at the equator, the curvature of the earth at the poles must be that of a larger circle than the curvature at the equator, and since the larger the circle the less rapidly does it curve, we see that the earth is slightly flattened at the poles. The polar diameter of the earth has been found to bear to the equatorial the ratio of 299 to 300.

59. Knowing the exact size and form of the earth, the distance between the two stations *A* and *B* (Figure 22) in a straight line can be computed.

The transits of Venus, by which the sun's distance from the earth can be determined, occur in pairs at intervals of eight years separated by more than one hundred years. The last pair occurred in 1761 and 1769, and the next will be in 1874 and 1882.

It is now supposed by many that there was an error in one of the observations of the last transit of Venus, so that the distance of the sun as computed from these observations is some 3,000,000 of miles greater than it really is. Hence the next transits of Venus are looked forward to with great interest.

60. *The Mean Distances of the Planets.* — The following is a table of the mean distances of the most important planets from the sun, as computed from the last transit of Venus: —

Mercury	37,000,000	miles
Venus	69,000,000	"
Earth	95,000,000	"
Mars	145,000,000	"
Jupiter	436,000,000	"
Saturn	909,000,000	"
Uranus	1,828,000,000	"
Neptune	2,862,000,000	"

SUMMARY.

The distance of a planet from the sun compared with the earth's distance from the same is called its *relative distance* from the sun.

To find the relative distance of an *inferior planet* from the sun, its greatest elongation must be measured; and the right-angled triangle which the earth, the planet, and the sun then form must be computed. (48.)

To find the relative distance of a *superior planet* from the sun, the retrogression of the planet during one day, when it is in opposition, must be measured; and the triangle which the earth, the planet, and the sun then form must be computed. (49.)

The distance of the earth from the sun is found by means of the *transits of Venus.*

To find this distance, we must determine the angle subtended by the diameter of the sun, and by two chords which Venus, as seen by two observers, one north and the other south of the equator, describes across the sun's disc; the relative distance of Venus and of the earth from the sun; and the size and shape of the earth.

The angle subtended by the diameter of the sun is ascertained by *direct measurement.*

The angle subtended by the chords described by Venus is ascertained by *observing the transit of Venus.* (51.)

The size and shape of the earth are found by *measuring known arcs of meridians on different parts of the earth.* (53–56.)

The *actual distance* of any planet from the sun can be found by multiplying its relative distance by the distance of the earth from the sun.

HOW TO FIND THE DISTANCE OF THE MOON.

61. We have already ascertained the actual distance of the earth, as well as of all the other planets, from the sun, but as yet we do not know the distance of the moon from the earth. We have also seen that all the planets, though they appear to revolve about the earth, really revolve about the sun, but the moon's motion among the stars can be explained only on the supposition that she revolves about the earth.

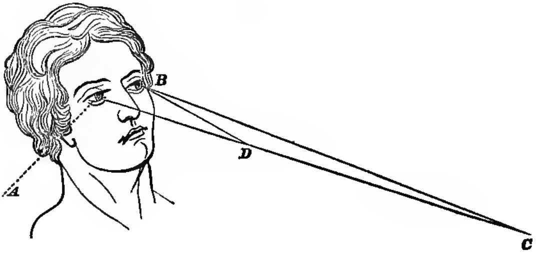
The ancients ascertained the distance of the moon with considerable accuracy by the observation of her eclipses. They knew that these phenomena were caused by the passage of the moon through the earth's shadow. Now the earth's shadow is not very much narrower at the distance of the moon than at the surface of the earth, and the average length of time the moon takes to pass through that shadow is about four hours. The moon passes over a part of her orbit equal to the diameter of the earth in about four hours, and consequently she passes over a length of her orbit equal to six diameters of the earth in a day, and, as she completes a revolution in about thirty days, her whole orbit must be about one hundred and eighty times as long as the diameter of the earth. Consequently the diameter of the moon's orbit is about sixty times the diameter of the earth, and the distance of the moon about thirty times the diameter of the earth.

62. *Parallax.* — The distance of the moon can now be measured by means of *parallax.*

The following illustration shows that this is really the method by which we commonly estimate distance. If you place your head in a corner of a room or against a high-backed chair, and close one eye, and allow another person to put a lighted candle on a table before you, and if you then try to snuff the candle, with one eye still shut, you will find that you cannot do it: you will probably fail nine times out of ten. But if you open the other eye, the charm is broken; or if, without opening the other eye, you move your head sensibly, you are enabled to judge of the distance.

In Figure 30 let *A* and *B* be the two eyes, and *C* an object which is first viewed with the eye *A* alone. This

Fig. 30.



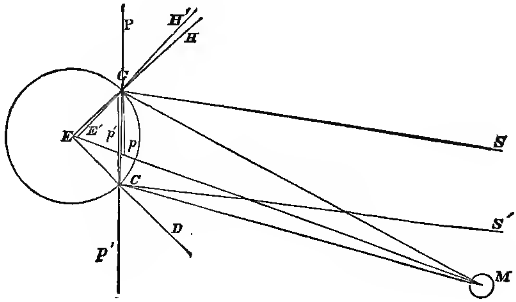
eye alone has no means of judging of the distance of *C*. All that it can tell is that this object is in the direction of *A C*, but there is nothing by which it can judge of its distance in that line. Suppose now the other eye, *B*, is opened and turned to *C*, then there is a circumstance introduced which is affected by the distance, namely, the difference of direction of the two eyes. While the object is at *C*, the two eyes are turned inward but very little to see it; but if the object is brought quite close, as at *D*, then the two eyes have to be turned inward considerably to see it; and from this effort of turning the eyes we acquire some notion of the distance. We cannot lay down any accurate rule for the estimation of the distance; but we see clearly enough in this explanation, and we feel distinctly enough when we make the experiment, that the estimation of distance does depend upon this difference of direction of the eyes. Now, this difference of direction of the two eyes is a veritable parallax; and this is what we mean by *parallax*, that it is the difference of direction of an object as seen in two different places. The two places in the above experiment are the two eyes in the head. The distance of the moon is found by precisely the same method. The two eyes in the head will be two telescopes, one in the observatory at Greenwich, and the other in the observatory at the Cape of Good Hope; and the difference of direction of the eyes becomes the difference of direction of these two telescopes when pointed at the moon. When this difference of direction becomes known the distance of the moon is easily computed.

63. *How to find the Difference of the Direction of the two Telescopes when turned to the Moon.*—The difference of direction of the two telescopes when pointed at the moon is found by means of the mural circle. By means of this the zenith distances of the north celestial pole and of the moon are observed at Greenwich, and the sum of these distances gives the north polar distance of the moon as seen at Greenwich.

By means of this same instrument the zenith distances of the south celestial pole and of the moon are also observed at the Cape of Good Hope at the same time that the observations were made at Greenwich. The sum of these zenith distances gives the south polar distance of the moon. It is found, for instance, that the north polar distance of the moon as obtained at Greenwich is 108°, and that the south polar distance as observed at the Cape of Good Hope is $73\frac{1}{2}°$. The sum of these two polar distances is then $181\frac{1}{2}°$.

Suppose now that *G* and *C* in Figure 31 represent the position of the observatories at Greenwich and at the

Fig. 31.



Cape of Good Hope, and that *G P* and *C P'* represent the direction of the north and south poles of the heavens respectively from each of these stations. The line *G P* has of course a direction just opposite to that of the line *C P'*. The line *G M* represents the direction of the telescope at Greenwich when turned toward the moon, and the line *C M* represents the direction of the telescope at the Cape of Good Hope when turned toward the moon. Suppose now that the telescope at each observatory be turned toward the same fixed star. *G S* will be the direction of the telescope at Greenwich, and *C S'* the direction of the telescope at the Cape of Good Hope. Now when the north polar distance of any fixed star is measured at Greenwich, and the south polar distance of the same star is measured at the Cape of Good Hope, the sum of these polar distances always equals 180°; that is, the angle $S\ G\ P + S'\ C\ P' = 180°$. Hence $S\ G$ and $S'\ C$ must be parallel. For $S\ G\ P + S\ G\ P' = 180°$. Therefore $S'\ C\ P' = S\ G\ P'$; that is, $S\ G$ and $S'\ C$ are parallel.

We have found that $M\ G\ P + M\ C\ P' = 181\frac{1}{2}$. The two lines $M\ G$ and $M\ C$ must then be inclined to each other at an angle of $1\frac{1}{2}°$, for if these two lines were parallel the two angles $M\ G\ P + M\ C\ P'$ would be equal to 180°.

We have now found the difference of direction of the two telescopes when turned toward the moon. Now since the exact size and form of the earth are known, the length of the line $G\ C$ and the angle $M\ G\ C$ can be computed, and the distance $M\ G$ can then be found by construction or by computation.

64. *The Moon's Parallax.* — This gives the distance of the moon from each of the observatories G and C. It is convenient, however, to make all our calculations of the moon's place with reference to the centre of the earth. By reference to the above figure it will be seen that the direction of the moon is not the same when seen from the centre of the earth E as when seen from its surface at G or C. The difference of the directions of the moon as seen at Greenwich and as seen from the centre of the earth is called the moon's parallax at Greenwich. Thus the angle $G\ M\ E$ is the moon's parallax at Greenwich, and the angle $C\ M\ E$ is her parallax at the Cape of Good Hope.

The sum of the moon's parallaxes at Greenwich and at the Cape of Good Hope is evidently equal to the angle $G\ M\ C$. The method by which the moon's distance is actually found is as follows. From a knowledge of the earth's dimensions, the length of $E\ G$ is known with considerable accuracy. And though the plumb-line at *G* is not directed actually to the earth's centre, *E*, but in a slightly different direction, *H' G E'*, yet from knowing the form of the earth, we can calculate accurately how much it is inclined to the line *H E*, which is directed to the earth's centre. Then we know the angle *H' G H*, and we have observed the angle *H' G M* with the mural circle, and their difference is the angle *H G M*, which is therefore known. The difference between 180° and the angle *H G M* gives the angle *E G M*. Then we assume, for trial, a value of the distance *E M*. With the length *E M*, the length *G E*, and the angle *E G M*, it is easy to compute the angle *G M E*.* The same process is used to calculate the angle *C M E*. We then add these two calculated angles together, and find whether their sum is equal to the angle *G M C*, which we have found from observation. If their sum is not equal to this angle found from observation, we must try another assumption for the length of *E M*, and go through the calculation again; and so on till the numbers agree.

We supposed at first that the observations were made at the same instant at Greenwich and at the Cape of Good Hope. This is not strictly correct; but the difference of time is known, and the moon's motion is well enough known to enable us to compute how much the angle *P' C M* changes in that time; and thus we know what would have been the direction of the line *C M*, if the observations had been made at exactly the same instant as the observation at *G*.

When now we wish to know the position of the moon at any observation as seen from the centre of the earth, its parallax must be computed and applied. By correcting the observed place of the moon for parallax, it has been found that the plane of the moon's orbit passes through the centre of the earth in the same way as the

* See Appendix, I. 6.

orbits of the earth and of the other planets pass through the centre of the sun.

65. *Another Method of finding the Difference in the Direction of the two Telescopes pointed at the Moon.* — The method given above for ascertaining the difference of the direction of the two telescopes when turned towards the moon, is liable to only one error. Since the inclination of these lines is determined by observations made with the mural circle, it is necessary that every observation should be corrected for refraction, which, as we have seen, causes objects in every part of the heavens to appear higher than they really are. Now this correction is very troublesome, since the amount of refraction changes with the altitude of the object and with the different conditions of the atmosphere. Prof. Airy, the Astronomer Royal of England, says that refraction is the very abomination of astronomers. It changes with the condition of the atmosphere in so irregular a manner that every correction made for it is liable to a slight error.

There is another way of ascertaining the difference of direction of the lines *G M* and *C M*, which is liable to error from refraction to a much less extent.

We have already seen that, if the telescopes at the two observatories are pointed to the same fixed star, their direction will be precisely the same. If, then, a fixed star, *S*, be chosen, which is quite near the moon, and which comes upon the meridian at nearly the same time, and the telescope of the mural circle at each station be first directed to this star, and then turned to the moon, we shall get the inclination of each of the directions *G M* and *C M* to the direction *G S*. The difference of their inclinations to this direction will evidently be their inclination to each other. Suppose, for instance, that at *G* the moon is seen two degrees below the star, and at

C half a degree below it; the two lines *G M* and *C M* will evidently be inclined to each other $1\frac{1}{2}°$. As the moon and star must be observed at very nearly the same altitude, and under almost precisely the same conditions of the atmosphere, refraction can make no appreciable difference in their angular distance from each other.

SUMMARY.

The ancients ascertained the distance of the moon with considerable accuracy by the observation of her eclipses. (61.)

The distance of the moon is now found by means of *parallax*. (62.)

Parallax is the difference of direction of any body as seen from two different places.

We really determine the distance of ordinary bodies by means of parallax.

When a telescope at Greenwich and another at the Cape of Good Hope are pointed at the moon, their difference of direction can be ascertained, either by measuring the polar distance of the moon at both observatories (63), or by measuring at both observatories the angular distance of the moon from a fixed star near her. (65.)

The parallax of the moon is the difference of her direction as seen from the centre and from the surface of the earth.

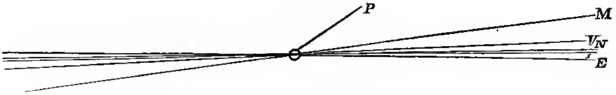
When the difference of direction of two telescopes which have been pointed at the moon from the observatories at Greenwich and the Cape of Good Hope has been ascertained, the parallax of the moon is found by the method of *trial and error*. (64.)

A GENERAL SURVEY OF THE ORBITS OF THE PLANETS.

66. *The Inclination of the Orbits of the Planets to the Plane of the Ecliptic.* — We have now learned that the earth and all the planets revolve about the sun from west to east; that each describes a curve called an ellipse, one of whose foci is occupied by the sun; and that the plane of this orbit always passes through the centre of the sun. Also that each planet moves at such a rate in different parts of its orbit that a line joining the planet with the sun always sweeps over equal areas in equal times; consequently the planets all move faster in their orbits at perihelion than at aphelion. We have found, also, that the different planets move at such rates that the squares of their periodic times bear the same ratio as the cubes of their mean distances from the sun. We have learned, too, that these ellipses differ in form, some being flatter than others; also in the direction of their major axes, and in their inclination to the ecliptic.

The orbits of the larger planets are, however, but slightly inclined to the ecliptic, as Figure 32 shows. *E*

Fig. 32.



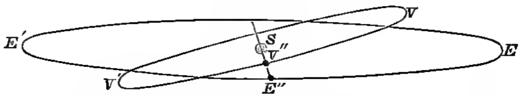
represents the plane of the ecliptic; *J*, the plane of Jupiter's orbit; *N*, Neptune's; *V*, Venus's; *M*, Mercury's. *P* is the plane of the orbit of Pallas, one of the *minor* planets.

67. *Nodes.* — As the orbits of all the planets are some-

what inclined to the ecliptic, they must all intersect it. The points at which they intersect the ecliptic are called the *nodes* of their orbits.

68. *Transits of Venus.* — The considerable inclination of the orbit of Venus to the ecliptic explains why the transits of Venus occur so seldom. This will be made clear by Figure 33. *S* represents the position of the sun;

Fig. 33.



V V′ V″, the orbit of Venus; and *E E′ E″*, the orbit of the earth, that is, the plane of the ecliptic. Now it is evident from inspection of the figure that Venus will not be seen from the earth to cross the disc of the sun, unless the earth be very near one of the nodes of Venus's orbit, as at *E″*, at the time of inferior conjunction.

HOW TO FIND THE DISTANCE OF THE FIXED STARS.

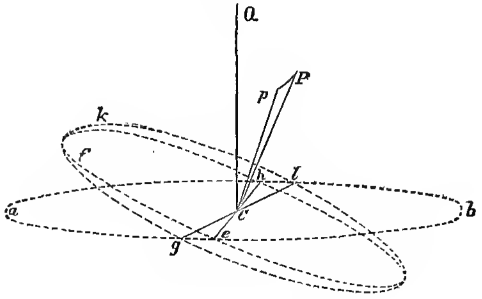
69. We have already seen that two telescopes so far apart as those at the observatories at Greenwich and at the Cape of Good Hope do not sensibly differ in direction when each is turned to the same fixed star; so that we cannot estimate the distance of the stars by the method employed in finding the distance of the moon.

It will be remembered that, in our first illustration of parallax (62), we were enabled to estimate the distance of the candle, either by turning both of the eyes upon it at the same time and so becoming aware of their difference of direction, or by using only one eye and moving the head, and so becoming aware of the amount by which the eye must change its direction for a given movement of the head, to be still turned toward the candle. The first method of estimating the distance of the candle was imitated in ascertaining the distance of the moon. Can we ascertain the distance of the fixed stars by imitating the second? In sweeping round the sun the earth, as we have seen, describes an ellipse, whose mean diameter is some 190,000,000 of miles. Can we, by using the telescope of the observatory at Greenwich as a single eye, estimate the distance of a fixed star by observing how much the direction of the telescope must be changed in order always to point to that star when on the meridian throughout an entire revolution of the earth? Suppose that the north polar distance of a fixed star be measured by means of the mural circle at Greenwich, and six months from this time, when that observatory has moved away from its former position 190,000,000 miles, its north polar distance be measured again. If the polar distances of the star thus measured differ, their difference must be the difference of direction of the telescope at the two observations.

70. *Does the Earth's Axis always point in the same Direction?* — But since these observations are made at long intervals, it becomes necessary to know whether the pole of the heavens from which the angular distance of the star is measured remains unchanged during the year. It must be borne in mind that the pole of the heavens is that part of the heavens to which the axis of the earth points. Now we must ascertain whether the axis of the earth always points to exactly the same part of the heavens, or not; that is, whether the axis of the earth always points in exactly the same direction. That the axis of the earth always points in very nearly the same direction is evident from the fact that the pole of the heavens does not sensibly change its position from year to year. But

have already noticed the fact (20) that the points ere the equator cuts the ecliptic are slowly shifting along the ecliptic to the westward at the rate of 50″ annually. It is also found by observation that the inclination of the celestial equator to the ecliptic does not change. Let us suppose that a top, with its upper surface perfectly flat, be spinning upon a perfectly level surface. After a time, the end of the handle of the top will be seen to describe a small circle, and the upper surface of the top will then be seen to be inclined at a certain angle to the floor. Suppose now that the top keep on spinning for a time, and that the inclination of the upper surface of the top to the floor remain the same. Conceive a plane parallel with the floor passing through the point of contact of the handle with the top. This plane may represent the ecliptic; the upper surface of the top may represent the plane of the earth's equator; and the points where the circumference of the upper surface of the top cuts this plane may be considered as the equinoctial points. Now as the top goes on spinning, the direction of its inclination to the floor is constantly shifting, though its amount remain unchanged. It is evident, then, that the points where the circumference of this surface cuts our imaginary ecliptic are continually shifting, and that they will pass entirely around while the end of the top handle is describing a circle. In this experiment the top handle represents the axis of the earth. In Figure 34 let *a b* represent the ecliptic, and *e f h* represent the position of the equator at one time; then *g k l* must represent its position after the equinoxes have shifted from *h* to *l* and from *e* to *g*. The shifting of the equinoctial points, then, seems to be due to a shifting of the direction of the inclination of the earth's equator similar to that of the upper surface of the top in the above experiment. If this is really so, the end of the axis of the earth, CP, ought to describe in the mean time an arc, Pp, and eventually to describe

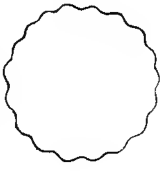
Fig. 34.



a complete circle like the end of the top handle. Now observation reveals the fact that the pole of the heavens is actually describing a circle in the heavens whose radius is an arc of $23\frac{1}{2}°$, and that it is describing this circle at the rate of $50''$ annually. It therefore describes a complete circle in about 26,000 years. The earth, then, as it spins on its axis in its journey around the sun, wabbles like a spinning top, not several times a minute, but once in 26,000 years.

Careful observation has also shown that the earth's axis, while describing this circle in 26,000 years, has a slight tremulous movement, swinging back and forth through a space of $18''$ in nineteen years. The effect of both these movements is to cause the pole of the heavens to describe in 26,000 years such a curve as is represented in Figure 35. The effect of the first movement of the axis is called *precession;* that of the second, *nutation.*

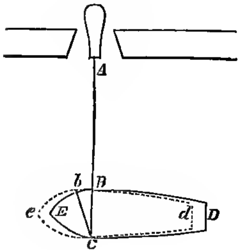
Fig. 35.



The polar distance of a star is not in any case changed more than $21''$ by precession and nutation. But these are quantities so large that we must be perfectly acquainted with their laws and magnitude when we are dealing with changes in the places of the stars not exceeding one or two seconds.

71. *The Aberration of Light.* — In observing, then, the polar distance of our fixed star at an interval of six months, we must allow for precession and nutation. This can, however, be done with the greatest accuracy. Is there anything else that would make the star appear in a different direction at the end of the six months, except the change of position of the observer? On a rainy day, when the drops are large and there is no wind, if one goes out and stands still, he will see the drops of rain falling directly downwards. If he then walks forward, he will see the drops fall towards him; and if he walks backward, he will see them fall away from him. Again, in Figure 36, let *A* be a gun of a battery, from which a shot is fired at a ship, *D E*, that is passing. Let *A B C* be the course of the shot. The shot enters the ship's side at *B*, and passes out at the other side at *C*. But in the mean time the ship has moved from *E* to *e*, and the part *B* where the shot entered has been carried to *b*. If a person on board the ship could see the ball as it crossed the ship, he would see it cross in the diagonal line *b C*. And he would at once say that the cannon was in the direction of *C b*. If the ship were moving in the opposite direction, he would say that the cannon was just as far the other side of its true position.

Fig. 36.



Now we see a star in the direction in which the light coming from the star appears to be moving. When we examine a star with a telescope we are in the same condition as the person who on shipboard saw the cannon ball cross the ship. The telescope is carried along by the earth at the rate of eighteen miles a second, hence the light will appear to pass through the tube in a slightly different direction from that in which it is really moving; just as the cannon ball appears to pass through the ship in a different direction from that in which it is really moving. As light moves with enormous velocity, it passes through the tube so quickly that it is apparently changed from its true direction only by a very slight angle, but it is sufficient to displace the star. This apparent change in the direction of light caused by the motion of the earth is called *aberration of light*. Now as it is at once seen that the earth is moving in opposite directions at the beginning and the end of six months, it is clear that the observations must be corrected for aberration as well as for precession and nutation. This correction can, however, be made with great accuracy, since we can compute the exact effect of this disturbance. There however remains the correction for refraction, which in this case is more troublesome than usual, since stars which are on the meridian at twelve at night are, six months from that time, on the meridian during the day, and the atmospheric conditions which affect refraction are widely different by day and by night.

72. By this method the inclination of the directions of a telescope, when turned to a bright star in the constellation of the Centaur (Alpha Centauri) in the Southern Hemisphere, has been found to be an angle of about two seconds (Airy). This inclination would make the distance of this star some 100,000 times the radius of the earth's orbit, which, as we know, is about 95,000,000 miles.

An angle of two seconds is that which a circle of $\frac{6}{10}$ of an inch in diameter would subtend at the distance of a mile. Yet this is the greatest parallax that any star is found to have, when seen at opposite parts of the earth's orbit. The parallax of Vega, in the constellation Lyra, is not more than $\frac{1}{4}$ of a second; and there are comparatively few of the stars whose parallax is above $\frac{1}{10}$ of a second. But considering the uncertainty of refraction, Airy, the Astronomer Royal of England, says that the determination of a parallax of $\frac{2}{10}$ of a second by the above method is more than he can undertake to answer for.

73. *Another Method of finding the Parallax of a Fixed Star.* — In consequence of this uncertainty another method has been devised, admitting of far greater accuracy; namely, by comparing two stars whose declinations are very nearly the same. This method is very similar to that which is used for measuring the distance of the moon.

Suppose that two stars have very nearly the same declination, and that from observation by the first method we have reason to suppose that one of the stars is at such an enormous distance that it will have no sensible parallax, while from the same observation we have reason to believe that the other is very much nearer. If now we measure the angular distance between the two stars at opposite points of the earth's orbit, the difference of these angular measurements will be the parallax of the nearer of the two stars.

By this method, since we compare stars which are seen very nearly in the same direction, we get rid of the uncertainty of refraction, and also of precession, nutation, and aberration; because these produce the same effect on both stars. This is the method which the celebrated Bessel, of Königsberg, used for determining the distance of the small star known as 61 Cygni. He thus found the parallax of this star as seen from opposite parts of the earth's orbit to be $\frac{6}{10}$ of a second; and this corresponds to a distance of 660,000 times the radius of the earth's orbit, or 63,000,000,000,000 miles. "Enormous as this distance is," says Airy, "I state it as my deliberate opinion, founded upon a careful examination of the whole process of observation and calculation, that it is ascertained with what may be called in such a problem considerable accuracy." The distance of those stars whose parallax is estimated at $\frac{1}{10}$ of a second is about 2,000,000 times the distance of the earth from the sun.

74. *The Immensity of these Distances.*—We have no appreciation of distances so vast. The moon has been found to be distant from the earth about 240,000 miles. If a locomotive were travelling at the rate of 1000 miles a day it would take it eight months to reach the moon; travelling at the same rate it would reach the sun in 260 years, and the star 61 Cygni in 171,600,000 years, a period of which we can form no conception whatever.

Light, it will be remembered, travels at the rate of about 190,000 miles a second. It would accordingly take light about $7\frac{1}{4}$ years to reach us from 61 Cygni. If this star were suddenly annihilated, it would be over seven years before we should become aware of it. Yet this is one of the nearer fixed stars. It has been estimated from tolerably accurate measurements that it would take light fourteen years to reach us from Sirius, twenty years from Vega, and twenty-five years from Arcturus; and Sir John Herschel has estimated that light requires at least 2000 years to reach us from the smaller stars with which the Milky Way is crowded.

75. *Nebulæ.*—Nor is this all. Near the belt of Andromeda there may be seen with the naked eye a faint glimmer of light on a very clear night. Figure 37 shows this object as it appeared in Sir John Herschel's tele-

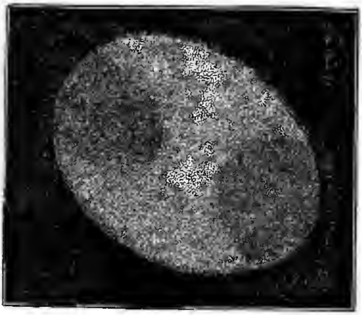
Fig. 37.

scope, and Figure 38 as it appears in the powerful refractor of the Observatory at Cambridge, Mass. Such an object is called a *nebula*. Nebulæ are really very nu-

Fig. 38.

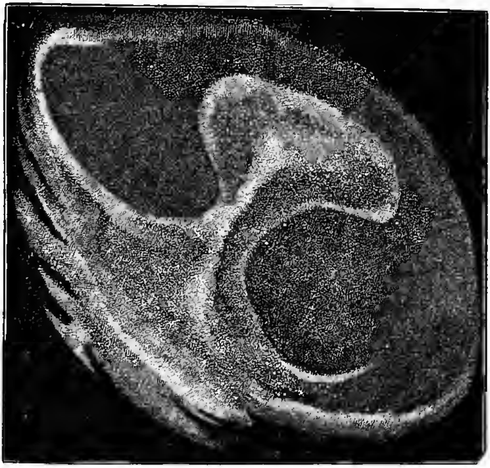
merous, though very few of them are bright enough to be seen with the unaided eye.

Fig 39.



In Figure 39 is represented the celebrated nebula in the constellation of Vulpecula, as it appeared to Sir

Fig. 40.



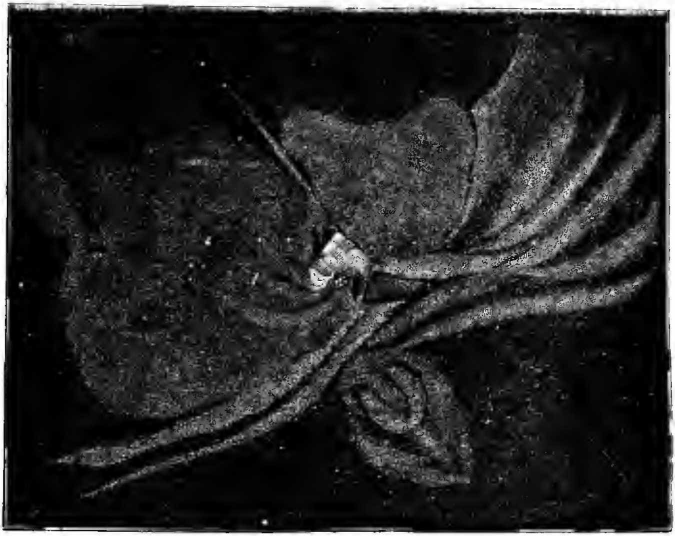
John Herschel. Figure 40 shows the same nebula as it appears in the great reflecting telescope of Lord Rosse.

Fig. 41.

Figure 41 represents the Crab nebula in Taurus, as delineated by Lord Rosse; and Figure 42, the great nebula of Orion, as figured by Professor G. P. Bond of the Cambridge Observatory. Another celebrated nebula, known as the Ring nebula in Cygnus, is shown in Figure 43.

About five thousand nebulæ have been observed. It is seen from the above figures that they differ greatly in form. It will be noticed in the case of the Dumb-bell nebula (Figure 39), and also in the Crab nebula, that these nebulæ often have the appearance of clusters of minute stars when viewed with very powerful telescopes. This has led astronomers generally to believe that many of the nebulæ are vast systems of stars like our own, seen at immense distances. Figure 44 is a representation of what Sir John Herschel conceived would be the appearance

Fig. 42.

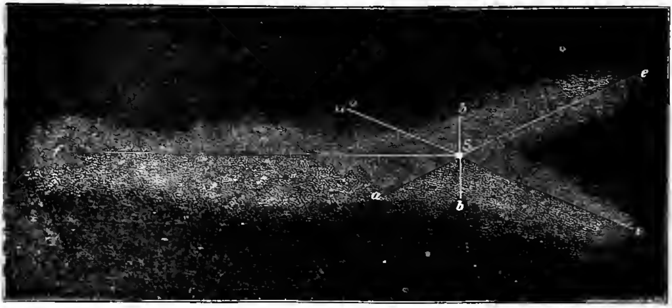


of our system of stars including the Milky Way, when seen at the distance of one of the nebulæ. According to this theory, our system of stars, which is to us so great that we can have no appreciation of its vast dimensions, is really but one of countless systems which are scattered throughout space at such distances, that, when viewed from one another with the most powerful telescopes, they appear as small faintly luminous spots.

Fig. 43.

We now begin to see that the supposition which we made a long time ago (Part First, Section 4) of beings small enough to dwell on a molecule of a stone as we dwell upon the earth, is after all not so very

Fig. 44.



unreasonable. For as we gain some faint conception of the dimensions of the universe as a whole, we feel that we are such minute beings, dwelling, not upon a molecule, but upon one of the smallest atoms of which a single molecule of the universe is made up. But the greatest wonder of all is that we, after measuring but a small fraction of the distance around our atom, and making a few simple observations of the position of the heavenly bodies, can then, by simple geometric and trigonometric computation, measure the dimensions of the universe.

76. *The Nebulæ are not all Systems of Stars.* — It is not at all clear as yet that the nebulæ are all systems of stars. While it seems quite certain that some of them are such systems at immense distances, there are strong reasons for believing that others are but thin clouds of luminous gases which may be much nearer the earth than the nearest fixed star.

SUMMARY.

A telescope at the Cape of Good Hope, when pointed at a fixed star, does not differ appreciably in direction from a telescope at Greenwich pointed at the same star.

If a telescope is turned to the same fixed star at opposite points of the earth's orbit, the difference of its direction, if appreciable, can be found by measuring the star's *polar distance.* (69.)

The earth's axis has a slight *wabbling motion*, which causes the celestial pole to describe a circle with a radius of $23\frac{1}{2}°$ in about 26,000 years.

This movement of the pole is called *precession.*

In addition to its regular wabble, the earth's axis has a *tremulous motion.* The effect of this motion is called *nutation.*

In an exact measurement of the polar distances of a star, a correction must be made for the effect of *precession* and *nutation.* (70.)

Aberration of light is the apparent change in the direction of light caused by the motion of the earth in its orbit. The observation of a star's polar distance must also be corrected for aberration. (71.)

The parallax of a fixed star may often be best determined by measuring, at opposite points of the earth's orbit, its angular distance from another star near it. (73.)

It takes light about *seven years* to come to us from the nearest fixed stars of our system, and probably *several thousand* to come from the most remote. (74.)

Many of the *nebulæ* are supposed to be outlying systems of stars, so distant that they appear no larger than a man's fist, yet so large that it takes light several thousand years to cross them. (75.)

SYSTEMS OF SATELLITES AND SUNS.

77. *Satellites.* — We have already seen that the earth in its revolution about the sun is attended by a moon, which revolves about it as a centre. Such an attendant is called a *satellite.*

The telescope shows that Jupiter is accompanied by a system of four satellites revolving about him as a centre, and that Saturn is accompanied by a system of eight satellites, Uranus by a system of four (perhaps six), and Neptune by one satellite.

In our solar system, then, we have satellites revolving about planets, and these planets with their systems of satellites revolving about the sun. Is there now any evidence that the sun, with this complex system of planets and satellites, revolves about some other sun?

78. *The Motion of our Solar System through Space.*— We have heretofore spoken of the stars as *fixed.* But upon comparing the places of stars as we observe them in different years, and applying the corrections for precession, nutation, and aberration, so as to reduce every observation of every star to what it ought to exhibit on the first day in the year, agreeably to the common practice of astronomers, we find that the vast majority of the stars which have been well observed seem to have a motion of their own. This motion is known as the *proper motion* of the stars. In all good catalogues of stars, the direction in which the stars appear to be moving, and the amount of their motion in a year, are given. This proper motion has been discovered only after many years' observations; it is in every case a small quantity; yet in most instances this quantity has been correctly ascertained. The proper motions of Sirius and Arcturus are pretty large; but the largest observed are those of two small stars known to astronomers as 61 *Cygni* and *Groombridge* 1830. The motion of the former is about five seconds a year, and of the latter nearly seven seconds. The proper motions of many of the stars are very irregular in direction and magnitude, but with regard to others there is a certain approach to regularity.

If you are walking through a forest, and keep your attention on the objects directly in front of you, they do not appear to change their place; but if you look at the objects to the right or to the left, they appear to be spreading away to the right or to the left. Even if you did not know that you were moving yourself, yet by seeing these objects spreading away you could infer with tolerable certainty that you were moving in a certain direction. Now if it should appear that, taking the stars generally, we can fix on any direction and see that the stars in that direction do not appear to be moving, but that the stars right and left appear to be moving away from that point, then there is good reason to infer that we are travelling toward that point. This speculation was first started by Sir William Herschel. He found a point in the heavens, in the constellation Hercules, such that the great majority of the stars about the point have no sensible proper motion, while the stars to the right and left of it have apparently a motion to the right and left respectively. He inferred from this that the solar system is travelling in a body towards this point. Every astronomer who has examined this subject carefully has come to a conclusion very nearly the same as that reached by Sir William Herschel; namely, that the whole solar system is moving toward that point in the constellation Hercules.

It is probable, then, that our sun, with his complex system of planets and satellites, is really revolving about some other sun, whose position is not yet ascertained.

79. *Double Stars.*—There is in the vicinity of Vega, the brightest star of the constellation Lyra, a small star called *Epsilon Lyræ*, which appears elongated to some persons of very keen eyesight, and this appearance suggests that it may really be composed of two luminous points. It is only necessary to examine it with an opera-glass to see that it really consists of two stars, separated

by an interval equal to about a ninth part of the apparent diameter of the moon.

Here, then, we have an example of an easily divided double star, which a keen eye or an opera-glass of small magnifying power is sufficient to separate into its components. If now we examine each of these components with an instrument of considerable optical power, we find that each consists of two stars so near together that the intervals separating them are not more than the seventieth part of the total distance of the couples themselves: so that we have here a *double-double* star, or a *quadruple* star. A star which appears single to the naked eye becomes quadruple when examined with a powerful telescope.

A century ago only about twenty double stars were known; now, however, we have catalogues of more than six thousand.

80. *This Union of two Stars is not in all cases Accidental.* —The first impression is that this proximity of two or more stars is purely accidental, being due to an effect of perspective; the stars themselves, though differing widely in their distance from us, lying in the same line of sight.

When, however, these double stars are carefully scrutinized, it is found that in some cases their components have the same proper motion, while in other cases they do not. Again, those whose components have the same proper motion are often found to revolve about each other in periods of greater or less length, while nothing of the kind is observed in the other class. Hence double stars have come to be classified as *optical* and *physical pairs*. Those whose components do not have the same proper motion, and do not revolve about one another, are regarded as optical pairs, or *optically double stars*. Their apparent proximity is regarded as purely accidental, owing to the fact that, though they are at very different distances from us, they happen to be in very nearly the same direction.

Those whose components have the same proper motion, or have been observed to revolve about one another, are supposed to be about the same distance from the earth and to be physically connected, so as to form systems like that of the sun and the planets. These are called *physical* pairs, or *physically double stars*.

81. *Theta Orionis.* — There is a remarkable system of stars in the constellation of Orion, near the centre of the great nebula already mentioned. The unaided sight distinguishes this system only as a luminous point. With the help of a good telescope, however, this point is divided into four stars, which are seen in the form of a *trapezium*. When viewed with a telescope of five or six inch aperture, two of the stars of the trapezium are seen to be accompanied by two other very small stars, forming altogether a group of six stars, as shown in Figure 45.

Fig. 45.

This sextuple star, known as Theta Orionis, or more commonly as the trapezium of Orion, probably constitutes a real system, since the five smaller stars have the same proper motion as the principal one. Mr. Lassell has discovered a seventh star in this remarkable system, so that Theta Orionis is a septuple star

82. *Xi Ursæ Majoris.* — In the constellation of the Great Bear, or Ursa Major, there is a star, designated in the catalogues by the Greek letter ξ, or Xi, which has been known as a double star since 1782. One of the components of this system is of the fourth, the other of the fifth, magnitude. The movement of revolution of the

second around the first having been detected, a French astronomer, Savary, determined by calculation the elements of the orbit. The period of revolution is sixty-one years. Since the discovery of the system, then, one revolution has been completed, and more than a third of another.

The elliptical or oval form of the orbit of this binary system is very decided. It is much more elongated than that of any of the planets of the solar system. But among the double stars there are some whose orbits are even more elongated. Such is that of Alpha Centauri, whose period of revolution exceeds seventy-eight years. The orbits of all the binary stars, so far as known, are ellipses, like those of the planets.

83. *Other Binary Stars and their Periods.* — The following are a few of the double stars whose periods have been determined: —

Zeta Herculis	36	years
Zeta Cancri	59	"
Mu Coronæ Borealis	66	"
70 Ophiuchi	93	"
Gamma Virginis	150	"
61 Cygni	450	"

There is, as seen by this table, great variety in the periods of double stars. But there are others whose periods probably differ even more widely. In Berenice's Hair, and in the Lion, there are two pairs, the first of which seems to have a period of twelve years, while the second seems to have a period of twelve centuries.

84. *The Dimensions of the Orbits of Binary Stars.* — When we know the distance of the binary stars, we can calculate, not only the form of their orbits and their periods of revolution, but also the dimensions of their orbits. It has been calculated that the mean distance between

the components of Alpha Centauri is not less than 1 319,000,000 miles. The mean distance of the companion of 61 Cygni is about forty-five times the length of the radius of the earth's orbit.

Of the six thousand double stars six hundred and fifty have been demonstrated to be physically connected systems.

85. *The Sun as a Fixed Star.* — If now we imagine the sun to be plunged into space to the distance of the nearest fixed star, and calculate, according to the laws of optics, what will be the diminution of its light, we find that it would become of the brightness of a star of the second magnitude, that is, it would shine with the brilliancy of the Pole Star, or of the principal stars in the constellation of the Great Bear.

86. *The Heavenly Bodies are all in Motion.* — The stars, then, are manifestly suns, and the double, triple, sextuple, and other multiple stars are systems of revolving suns. Each of these suns is probably attended by systems of planets like our own, which of course could not be seen at their immense distance with the most powerful telescope. It would seem that all the heavenly bodies are in motion, satellites about planets, planets about suns, and suns about suns, and systems of suns about systems of suns; and that the reason why we do not detect this motion in all cases is that the stars are situated at such enormous distances that their motion cannot be detected in the brief space of two or three thousand years which has elapsed since recorded observations first began.

We have already seen that the proper motion of the stars amounts at most to about seven seconds a year, and these are probably the very nearest fixed stars. Yet from their known distance and proper motion it has been computed that the following stars must move at the rates given below: —

Arcturus	54 miles a second.
61 Cygni	40 " "
Capella	30 " "
Sirius	14 " "
Alpha Centauri	13 " "
Vega	13 " "

It will be remembered that the earth's velocity in its orbit is eighteen miles a second. Notwithstanding this enormous velocity, it has required years of patient observation to establish the fact of the proper motion of the stars. We need not be surprised, then, that the more remote stars appear fixed, though they may be moving at the enormous rate of fifty miles a second.

SUMMARY.

Many of the planets are accompanied by systems of satellites.

Our solar system is a complex system of planets and satellites. (77.)

Our sun is travelling through space.

Many of the so-called fixed stars have *proper motions.* (78.)

Stars which appear close together and have the *same* proper motion constitute a *physical system.* (80.)

Theta Orionis is a remarkable system of this kind. (81.)

The components of these systems are in many cases known to revolve about a common centre in *elliptical* orbits. (82 – 84.)

II.

PHYSICAL FEATURES OF THE HEAVENLY BODIES.

PHYSICAL FEATURES OF THE HEAVENLY BODIES.

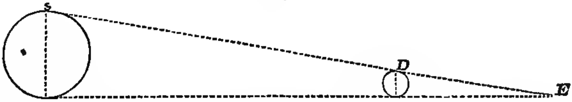
THE SUN.

We have seen that the universe is made up mainly of satellites, planets, and suns, and we have learned something of the motions and distances of these, and have thus got a general notion of the structure of the universe as a whole. We now turn our attention to these bodies to inquire what is known of each individually. We naturally begin at the centre of the solar system.

87. *The Size of the Sun.*—We have already learned that the sun is distant from the earth about 95,000,000 miles. Its disc is well known to be circular, and careful measurement shows that it is an exact circle.

Knowing the distance of the sun, it is an easy problem to find its size. Suppose a pasteboard disc, say about four inches in diameter, be held before the sun, it will be found that it must be held about twelve yards from the eye in order exactly to cover the sun's disc. Now since the distance between the lines in Figure 46 at any two

Fig. 46.



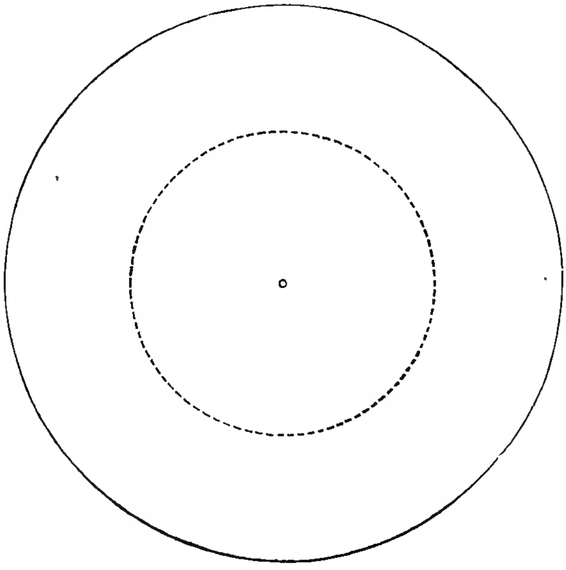
points is evidently proportioned to their distance from the vertex of the angle which the lines make, it follows that twelve yards is to 95,000,000 miles as four inches is to the diameter of the sun.

We thus learn that the diameter of the sun is about 112 times the diameter of the earth, or 887,076 miles. The circumference of the sun, therefore, exceeds 2,785,400 miles.

We have already seen that the moon's mean distance from the earth is about thirty times the earth's diameter. If, then, we imagine the centre of the sun to be at the centre of the earth, his circumference would extend beyond the orbit of the moon by a distance equal to twenty-six diameters of the earth.

In Figure 47, the inner circle represents the size of the earth, the middle one the moon's orbit, and the outer one the size of the sun, all drawn to the same scale. The

Fig. 47.

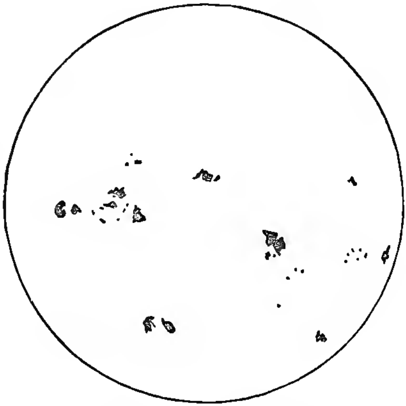


volume of the sun is about a million and a half times that of the earth.

88. *The Sun-Spots.*—When the sun is examined with a telescope of moderate magnifying power, the eye being properly protected, his disc usually appears sprinkled with irregularly grouped dark points. These dark spots are the sun-spots.

Figure 48 gives an idea of the manner in which the spots are distributed, and their grouping at any one time.

Fig. 48.



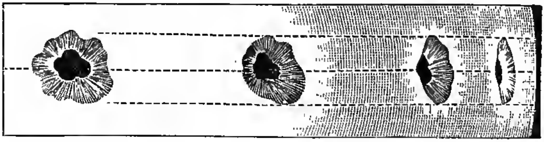
The number of the spots, their relative positions, and their forms are found to vary continually. Sometimes, though rarely, the solar disc is free from them; and sometimes as many as eighty spots have been visible at once. From a series of observations continuing through a period of some forty years, it appears that the spots occur with greater frequency at regular intervals. They diminish in number during five or six years, and then increase again through about the same length of time. Their greatest number thus occurs at intervals of ten or twelve years. The last maximum was in 1860.

89. *The Motion of the Spots.* — When observed with care during several consecutive days, the spots are seen to vary in both form and position. But amidst all their variations, a common movement of the whole in the same direction can be distinguished.

Let us suppose a spot to appear on the eastern edge or *limb* of the sun. From day to day it will be seen to progress with increasing rapidity until it occupies a central position on the disc. It still continues to advance, but now with decreasing rapidity, until it finally disappears on the western border. The same is true of all the spots, which at first appear scattered over the sun's disc. They all describe, in the same direction, with nearly equal velocities, either straight lines, or curved ones whose convexity always lies in the same direction for all the spots observed at the same time.

Let us suppose that the particular spot that we have observed is of an oval form, its greatest length being at right angles to the direction of its motion across the sun's disc at the moment when it appeared on the eastern limb. As it approaches the centre the spot widens, until at the centre it becomes nearly circular; then, having passed the centre, its form becomes more and more oval again until its disappearance, its apparent size in one direction meanwhile not having sensibly changed. Figure 49 shows these changes of form during the first and last half of the period of the visibility of the spot.

Fig. 49.



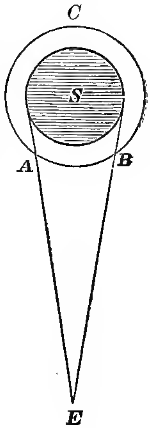
About fourteen days is the time during which the spots remain visible, and this time is nearly the same for all, though they do not all traverse arcs of the same length on the sun's surface.

It is also fourteen days after the disappearance of a spot on the western border before it appears again on the eastern, often changed in form, yet generally recognizable.

Precise measurements have proved both the general uniformity and the parallelism of these movements.

90. *The Spots are not Planets.* — It was at first thought that these spots might be caused by small planets revolving about the sun and presenting to us their unillumined faces. But since the time of their disappearance is equal to that of their visibility, they cannot be such planets. For it is evident from Figure 50 that, were they such planets, they would remain invisible longer than they are visible. For such a planet would be visible only while describing the arc *A B* across the sun's disc, while it would be invisible while describing the much longer arc *A C B*.

Fig. 50.



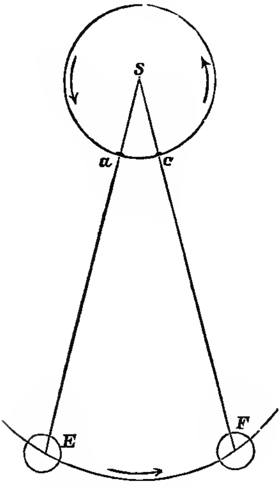
It was also thought possible to explain the movements of the spots from the eastern to the western border, by an actual translation of them across the surface of the sun, the surface itself being immovable. This supposition is, however, inconsistent with their uniformity of movement.

91. *The Sun's Rotation on his Axis.* — The movements of the spots, then, show beyond a doubt that the sun is rotating on an axis from west to east. The change of form of the spots illustrated in Figure 49, and their unequal rate of motion on different parts of the sun's disc, also prove beyond a doubt that the sun's surface is spherical. These

changes of form and of speed are the effect of perspective. The sun, then, like the earth, is not only moving through space, but whirls on its axis as it moves.

We have already stated that the interval between two successive appearances of the same spot on the same edge of the sun is twenty-eight days. The period of rotation of the sun is less than this. This will be evident by a reference to Figure 51.

Fig. 51.



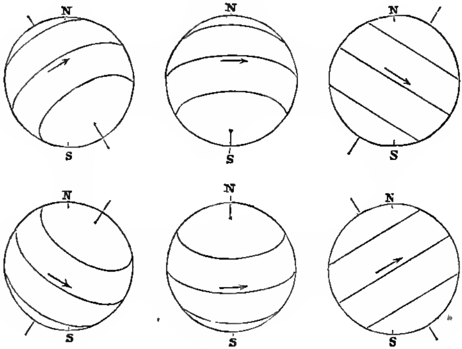
Let *S* be the centre of the sun; *a* the position of a spot; and *E* the position of the earth. The spot will then appear on the centre of the sun. The sun will evidently have made a complete rotation when the spot comes upon the line *S E* again, but it will not appear in the centre of the sun till it comes on the line *S F*, since the earth has in the mean time passed from *E* to *F*.

Between the two successive appearances of the same spot upon the centre of the sun, the sun will have more than completed a rotation. Since we know the angular velocity of the earth in its orbit, we can find how much more than a rotation the sun has completed. For he will evidently have completed a rotation *plus* the angular space that the earth has passed over in twenty-eight days. It is thus found that the sun rotates on its axis once in about twenty-five days.

92. *The Axis of the Sun is not perpendicular to the Plane of the Ecliptic.*—If the axis of the sun were perpendicular

to the plane of the ecliptic, the spots would evidently appear always to describe straight lines across the sun's disc. Observation shows that twice in the year they describe straight lines across the sun, and that for half of the remaining time they describe curves which are convex towards the upper limb of the sun, and the other half curves which are convex towards the lower limb of the sun, as shown in Figure 52. It has thus been found that the sun's axis is inclined 82° 45' to the plane of the ecliptic.

Fig. 52.

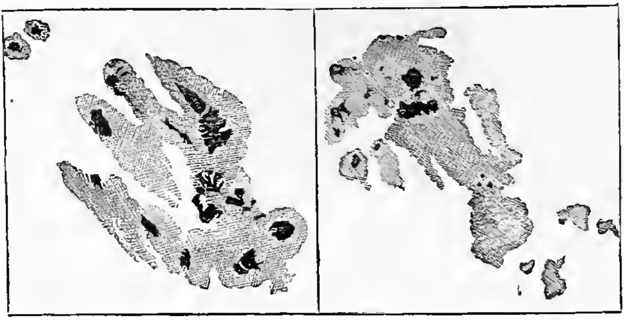


The sun-spots are confined in the main to two zones, situated on each side of the equator. They are seldom observed elsewhere on the sun's disc.

93. *The Appearances of the Spots.*—We have already learned from the spots on the sun the direction and duration of his rotation, and the inclination of his axis to the plane of the ecliptic. We will now study the appearance of these curious phenomena more minutely. Figure 53 represents a series of sun-spots. It will be seen that the spots consist almost invariably of one or more dark portions called *umbræ*, which seem black when com-

pared with the luminous parts of the disc. Around these a gray tint forms what is named, improperly, the *penumbra.* The majority of spots are composed of one or several umbræ, enclosed in one penumbra. But sometimes spots appear without the grayish envelope, as also occasionally a penumbra without an umbra.

Fig. 53.

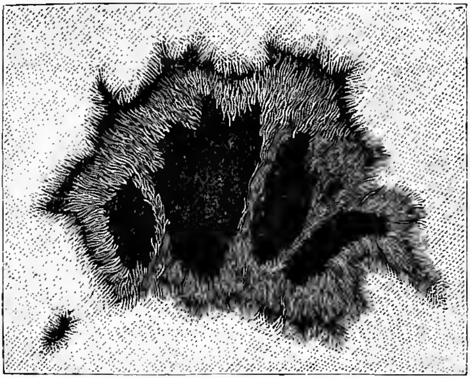


The forms of the spots, as shown by the drawings, are very varied. The penumbra most frequently reproduces the principal contour of the umbra, and often presents a great variety of shades when examined with a considerable magnifying power. On the exterior edges of the penumbra, the gray tint seems generally the deepest; either by the effect of contrast with the brilliant portions that surround it, or because in reality there is at these points a more decided tint. Figure 54 affords a striking example of this aspect of the penumbra.

This spot presents the peculiarity, not at all unfrequent, of the division of the dark umbra into several fragments by luminous bridges spanning it, as it were, from one side of the penumbra to the other.

The umbra itself is far from offering a uniform black tint. In reality it always presents a variety of shades, as if the penumbra and umbra were mingled and their tints mixed up in various proportions.

Fig. 54.



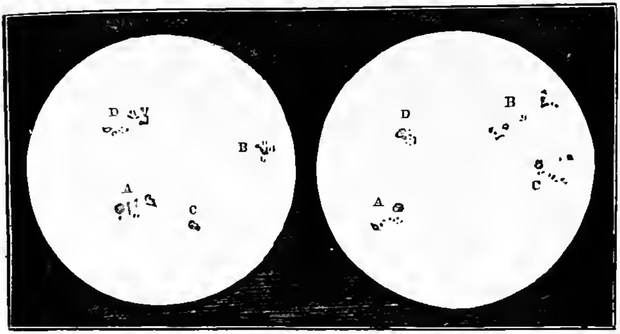
Under the best conditions of air and of instrument, the umbra seems to be pierced, and to afford a view of a much darker portion underneath. This darker portion has been called the *nucleus.* It appears to be of the most intense blackness, but it must be remembered that the word *black* as applied to the sun is only comparative. Sir J. Herschel has shown that a ball of ignited quick-lime, in the oxyhydrogen flame, though it seems to give out a light approaching that of the sun, appears, when projected on the sun, *as a black spot.*

94. *Dimensions of the Spots.* — The spots sometimes cover enormous areas. It is not uncommon to see one with a surface larger than that of the earth. Schröter measured one whose extent was four times the whole surface of the globe. Its diameters were more than 29,000 miles. Sir W. Herschel measured a spot consisting of *two* parts, the diameter of which was not less than 50,000 miles. The most extensive spot measured

was not less than 186,000 miles in its greatest length, and its surface embraced about 25,000,000,000 square miles.

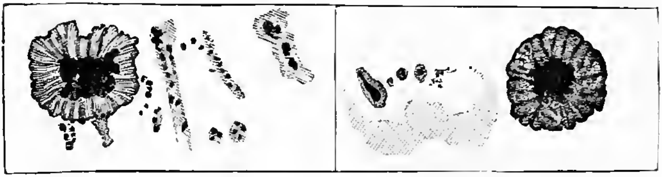
If the spots are deep rents in the sun's envelope, the larger ones must form gulfs, at the bottom of which the earth might lie like a boulder in the crater of a volcano.

Fig. 55, *a*.



95. *The Form and Size of the Spots are continually changing.* — Not only are the sun-spots not permanent, rarely lasting for many successive rotations, but their forms and dimensions differ from one rotation to another; sometimes even within a single day.

Fig. 55, *b*. (An enlarged view of the group *A*, in Fig. 55, *a*.)



The modifications which spots undergo in the course of a rotation are illustrated by Figure 55. These different groups, though easily recognized again, are considerably changed in general outline, and still more in detail.

These changes indicate two phenomena going on simultaneously, which may be best studied separately. First, there is indicated a proper motion of the spots, more or less rapid, and distinct from the apparent movement produced by rotation.

The proper motion of the spots was investigated in the most thorough manner by Mr. Carrington, who observed the sun every fine day for more than eight years, — from 1853 to 1861. After his discoveries there need be no wonder that different observers have varied so greatly in the time they have assigned to the sun's rotation. He shows that all sun-spots have a movement of their own, and that the rapidity of this movement varies regularly with their distance from the solar equator. The spot near the equator travels faster than those farther from it, so that if we take an equatorial spot, we shall say that the sun rotates in about twenty-five days; while, if we take one situated half way between the equator and the poles, in either hemisphere, we shall say that it rotates in about twenty-eight days. These facts show that we are ignorant of the exact period of the sun's rotation.

In the second place, the changes in the *form* of the spots are no less remarkable. Sometimes a spot divides into several separate nuclei; sometimes many distinct nuclei reunite into one. Arago quotes a curious instance of a spot which seemed to break upon the surface of the sun, in the same manner as a block of ice thrown upon the frozen surface of a sheet of water divides into several pieces which slide in all directions.

Diligent observation, moreover, of the umbra and penumbra with a powerful instrument, shows that change is going on incessantly in the region of the spots. Sometimes after the lapse of an hour, many changes in detail are noticed: here, a portion of the penumbra setting sail across the umbra; there, a portion of the umbra melting from sight; in another place, an evident change of position and direction in masses which retain their form.

96. *Faculæ.*—These spots are not the only exception to the uniform brightness of the sun's surface. Near the edge of the solar disc, and especially about spots approaching the edge, it is quite easy, even with a small telescope, to discern certain very bright streaks of diversified form, quite distinct in outline, and either entirely separate or coalescing in various ways into ridges and net-work. These appearances, which have been called *faculæ*, are the most brilliant part of the sun. Where near the limb the spots become invisible, undulated shining ridges still indicate their place. Faculæ vary much in magnitude. Professor Phillips has observed them from barely discernible, faintly-gleaming, narrow tracts, 1,000 miles long, to continuous, complicated, and heavy ridges, 40,000 miles and more in length, and 1,000 to 4,000 miles broad.

Ridges of this kind often surround a spot, and here appear more conspicuous; but sometimes there appears a very broad platform round the spot, and from this the white crumpled ridges extend in various directions.

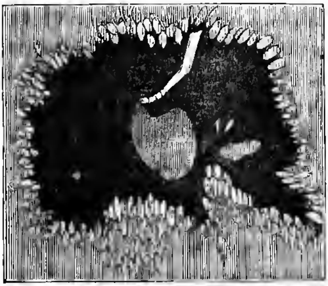
There would appear to be a close connection between spots and faculæ. An eminent French observer holds that spots are distributed for the most part in groups, with their greatest length parallel to the sun's equator, and that the first spot of the group is the blackest, the most regular, and lasts the longest. As the spots in the wake of the first disappear, they give place to faculæ, which cover the region where the spots showed themselves; then the original spot appears followed by a train of faculæ.

97. "*Pores*," "*Willow-Leaves*," *or* "*Granules*."—The

whole surface of the sun, except those portions occupied by the spots, is *coarsely mottled.* When examined with a large instrument it is seen that the surface is made up principally of luminous masses imperfectly separated from each other by rows of minute dark dots called *pores.* Mr. Nasmyth has recently announced his discovery that these pores are the "polygonal interstices between certain luminous objects of an exceedingly definite shape and general uniformity of size, which is that of the oblong leaf of the willow-tree." According to other observers, however, these luminous masses present almost every variety of irregular form; they are "rice grains," granules or granulations, "untidy circular masses," "things twice as long as broad," and so on.

Mr. Dawes asserts, indeed, that he has seen some nearly in contact differ so greatly in size that one was four or five times as large as another; and while, in a remarkably bright mass, one somewhat resembled a blunt and ill-shaped arrow-head, another, very much smaller, and within 5″ of it, was an irregular trapezium with rounded corners.

Fig. 56.



The occasional "willow leaf" appearance of the penumbra is shown in Figure 56.

98. *Appearances about the Sun during a Total Eclipse.* — Some minutes before and after, but especially during the totality of an eclipse of the sun, a luminous appearance in the form of a halo surrounds his disc, and throws in every direction rays of light separated by dark spaces. In many total eclipses, independently of the regular *corona*

(as this halo is called), other light portions have been noticed irregularly situated on the sun's contour. The color of the corona which immediately surrounds the dark disc is sometimes of a pearly or silvery white, sometimes yellowish, and even red.

The explanation generally given of this corona is, that it indicates the presence of a solar atmosphere, enveloping the radiant body to an enormous distance.

During the total eclipse of 1842 prominences of various forms and of a reddish color were visible throughout the contour of the moon's limb, during the period of totality. Some took the form of mountain peaks; others rose vertically from the sun's disc, and then turned at right angles; others, again, appeared completely detached, like floating clouds. Their tint was sometimes of a bright red, sometimes rosy, here and there variedly greenish-blue. These rose-colored flames or prominences had been noticed as early as 1733, but special attention was not given to them till 1842. Since then they have been observed with great care.

It has now been proved beyond all question that these protuberances belong to the sun. In the observations of the total eclipse of July, 1860, it was seen that as soon as the last delicate *line* of light disappeared behind the eastern edge of the moon, the rose-colored prominences were seen on those borders where the solar crescent had just disappeared. On the opposite side they were not yet entirely visible; their tops extended beyond the obscure disc only at its upper and lower parts. The moon's advance hid by degrees the prominences first observed, and exposed to view, at the opposite side, those previously covered.

In July, 1860, the heliograph of the Kew Observatory was carried to Spain, and photographs of the sun were taken at intervals during the eclipse. These photographs showed that the form of these protuberances does not change during the moon's passage across the sun's disc. Similar photographs were taken at Rome by Father Secchi, and a comparison of these with those taken in Spain shows that the prominences did not sensibly change their outline during the interval between the occurrence of the eclipse at the two places.

These facts show that the prominences do not belong to the lunar disc, and are not optical effects produced by its presence, but that they are absolutely parts of the sun itself.

They were first supposed to be enormous mountains on the surface of the sun. But the forms of many of them, and their occasional complete separation from the solar disc, contradict this hypothesis. All the observed facts lead to the conclusion that these immense appendages, rising 25,000 and even 50,000 miles in height and length, are clouds floating in the solar atmosphere, whose presence is indicated by the corona.

THE NATURE OF SUN-SPOTS.

99. *Wilson's Observations.*—In 1769 Professor Wilson of Glasgow watched a large spot as it passed across the sun's disc. He first saw it as it was passing towards the western limb. At first the penumbra was seen entirely to surround the umbra. As the spot approached the limb, the penumbra on the side nearest the sun's centre became narrower and narrower, until it finally disappeared, and the umbra also began to disappear on this side. On the reappearance of the spot on the eastern limb, the penumbra reappeared on the eastern side of the umbra, but had vanished on the western. As the spot approached the sun's centre, the penumbra again became visible on the western side, as a narrow line

which grew broader and broader till the spot reached the centre of the disc, when the breadth of the penumbra was equal on all sides of the umbra. These appearances led Professor Wilson to suppose that the body of the sun is surrounded by a luminous envelope of the consistency of a very dense fog, from which emanate all the solar light and heat, and that the spots are vast cavities or rents in this luminous envelope, through the bottom of which the dark body of the sun becomes visible, the shelving sides of the cavity giving rise to the penumbra. When a spot is near the centre of the sun the shelving sides of the cavity should appear on all sides of the darker bottom; that is, the umbra should be surrounded on all sides equally by the penumbra. When the spot is near either limb, the shelving side of the cavity should disappear on the side towards the observer, that is, on the side next the sun's centre, and there should be a penumbra only on the side of the umbra next the sun's limb. (Compare Figure 49.)

100. *Herschel's Theory.* — Herschel supposed that the sun is surrounded by an outer or luminous envelope, and an inner non-luminous envelope, somewhat like the stratum of clouds that surrounds our earth. The body of the sun itself he supposed to be dark in comparison with the luminous envelope, and the non-luminous envelope to be capable of reflecting the light of the luminous envelope about it. A spot he supposed to be caused by a rent in one or both of these envelopes. If only the outer envelope was rent, then the spot would be wholly penumbra without any nucleus, since the non-luminous envelope would reflect the light which fell upon it from the outer envelope. If both envelopes were rent, then the spot would have a nucleus, since the dark body of the sun would be revealed. If the opening in the non-luminous envelope were smaller than that in the lu-

minous envelope, as is usually the case, the spot would appear with a black nucleus surrounded by a penumbra. On this theory, as on Wilson's, the spots are cavities in the sun's luminous atmosphere.

101. *Recent Investigations on the Nature of Sun-spots.*— A very important and elaborate series of researches on the nature of the sun-spots has been recently begun by Warren De La Rue, President of the Royal Astronomical Society, and Balfour Stewart and Benjamin Loewy of the Kew Observatory.

These investigators have in their hands all the original drawings of sun-spots executed by Carrington between the years 1853 and 1861, the collection of drawings of the sun made by Schwabe during a course of about forty years, and the photographs of the sun taken by the Kew heliograph. A few pictures were taken by this instrument as early as 1858, and since February, 1862, it has been used in continuous observations.

These investigators have already arrived at several important conclusions.

An elaborate examination of the drawings and photographs of the sun's spots has sustained Wilson's conclusion that the umbra of a sun-spot is at a lower level than the penumbra.

The faculæ are regarded as portions of the sun's photosphere raised above the general surface. The fact that the faculæ are more conspicuous near the edge of the sun's disc than at the centre supports this conclusion. For there is good reason to suppose that the sun is surrounded by an atmosphere, and the absorption of light by this atmosphere would be much greater near the edge than at the centre; since the light reaching us has to travel through a much greater extent of atmosphere. If, then, the photosphere near the borders should be thrown up to a great elevation, the light coming from it will escape much of this atmosphere. On the other hand, very little will be gained in this way, when the elevation is near the centre of the disc, where the atmospheric absorption is small. The stereoscopic pictures of the sun that have been obtained also show some of the faculæ as ridges.

It is found, also, that the faculæ often retain the same appearance for many days together, as if the matter of which they are composed were capable of remaining suspended for some time. This permanence of form would seem to show that the faculæ cannot be elevations of the nature of waves in a liquid ocean resting on the surface of the sun, but that they are rather of the nature of a cloud; that is, of solid or liquid matter formed from the condensation of vapor, either slowly sinking or suspended in equilibrium in a gaseous medium. Kirchoff has shown by means of the spectroscope that there exist in the atmosphere of the sun vapors of such substances as iron, which are condensed into liquids or solids at a comparatively high temperature. It would be natural, then, that such vapors should be condensed and should float in the sun's atmosphere, as aqueous vapor is condensed and floats in the form of clouds in our atmosphere. The portion of the sun, then, which appears luminous to us, is probably a condensed stratum of those vapors which exist in the sun's atmosphere, and which are capable of condensation at a high temperature.

A comparison of a large number of spots shows that faculæ are, on an average, to the left of their accompanying spots. From the way in which the spots often break up, and from the fact that detached portions of luminous matter often appear to float across the spot without producing any permanent alteration, these observers are disposed to think that the umbra and penumbra of a spot both lie beneath the sun's photosphere.

They conclude that the spots are produced by the cooling of the sun's photosphere, and they believe that this cooling is occasioned by downward currents in the solar atmosphere which bring the cooler atmosphere of the higher regions down into that of the photosphere. The supposition that the spots are produced by the cooling effects of downward currents is supported by the proper motion of the spots. It will be remembered that the spots have all a proper motion in the direction of the sun's rotation, and of greater rapidity in proportion as they are nearer the equator. Now if they are produced by downward currents, the air in the upper regions must have a greater velocity than that in the lower regions, and it must consequently tend to increase the velocity of this lower air, which would tend to give the spots a proper motion in the direction of the sun's rotation. This velocity in the upper regions would be greatest at the equator, hence the proper motion of the spots should be greatest there.

But these downward currents should be accompanied with upward currents. There should then be faculæ in the neighborhood of the spots. And as the air in the lower regions has less velocity than that in the upper regions, the faculæ should fall behind the spots. This is exactly confirmed by observation.

This theory of the nature and cause of the sun-spots is certainly simpler and more satisfactory than that framed by Herschel.

The same observers have measured the area of the sun-spots on Carrington's drawings and on the photographs of the sun. They have thus discovered that the maximum area of visible spots is always on the part of the sun's disc which is opposite the planet Venus. When Venus is at superior conjunction, the spots attain the greatest area where they cross a line drawn through the centre of the sun's disc perpendicular to the plane of the ecliptic. As Venus passes on from superior conjunction, the spots attain their greatest area to the right of this central line.

They have found, too, that when Venus is at or near the plane of the sun's equator, the belts of spots approach the sun's equator; and that when Venus is most distant from that plane, they recede from the equator. The planet Venus, then, has a remarkable influence on the size and position of the sun's spots. They have also found that the area of the spots on crossing the central line on the sun's disc is much greater when Venus and Jupiter are both on the opposite side of the sun from the earth, than when Venus is on the opposite side and Jupiter on the same side as the earth. This shows that the planet Jupiter has also considerable influence on the sun's spots.

SUMMARY.

Place the centre of the sun at the centre of the earth, and his circumference would extend far beyond the moon's orbit. (87.)

The sun's disc is seldom free from dark patches. These patches are called *sun-spots*.

Sun-spots appear in greatest numbers at intervals of about twelve years. (88.)

These spots belong to the disc of the sun. (90.)

The rotation of the sun on his axis, and the inclination of his axis to the plane of the ecliptic, have been ascertained by means of the sun-spots. (91, 92.)

The spots have usually an *umbra* and a *penumbra*. The umbra is not *uniformly* dark. (93.)

The spots sometimes cover enormous areas. (94.)

The spots have a proper motion in the direction of the sun's rotation. Those near the equator move much faster than those away from it. They are also constantly changing their form. (95.)

Faculæ are bright streaks on the sun's disc. They seem to be connected with the spots. They usually appear in the rear of these. (96.)

The general disc of the sun is *coarsely mottled.* "*Willow leaves*" are sometimes seen about the spots. (97.)

During a total eclipse the sun is surrounded by a *corona*, and by *rose-colored clouds.* (98.)

Professor Wilson supposed that the spots are vast rents in the sun's luminous atmosphere, through which the dark body of the sun is visible. (99.)

According to Herschel's theory the sun is surrounded by two envelopes, the outer one luminous, and the inner one non-luminous. When both envelopes are rent, and the opening in the outer envelope is larger than that in the inner, the spot has both an umbra and a penumbra. (100.)

An important investigation of the nature of the sun-spots has been recently begun by De La Rue, Stewart, and Loewy.

They regard the sun's photosphere as of the nature of a cloud, and the faculæ as portions of this cloud raised above the general surface. They find that the faculæ are more often to the left of the spots than elsewhere. They have decided that both the umbra and penumbra are beneath the sun's photosphere.

They think the spots are produced by the cooling of the sun's photosphere by downward currents. They have shown that Venus and Jupiter have a marked influence on the sun-spots. (101.)

MERCURY.

102. *Its Distance from the Sun, Period of Revolution, etc.* — Mercury, as we have seen, is the planet nearest the sun. Its greatest elongation from the sun is about 29°; hence it is never seen at a great distance from the horizon, or long after sunset or before sunrise. And as the atmosphere near the horizon is usually charged with vapors, Mercury is seldom a very conspicuous object.

The mean distance of this planet from the sun is 37,000,000 miles, and its period of revolution is about three months. Its orbit is a very flat ellipse, so that its distance from the sun varies greatly. At perihelion it is less than 30,000,000 miles from the sun, while at aphelion it is more than 44,000,000 miles from him. Its distance from the sun, then, varies by about 15,000,000 miles. Its orbit is inclined to the ecliptic about 7°, which is more than that of any other of the larger planets.

If its inferior conjunction occurs when it is near one of its nodes (67), it will be seen to cross the sun. It then appears as a black circular disc projected upon the sun. Accurate measurements taken at a large number of observations of these transits of Mercury show that the disc is always an exact circle. Hence we conclude that Mercury is an exact sphere.

103. *Its Distance from the Earth, its Diameter, etc.* — Since the orbit of Mercury lies inside of the orbit of the earth, its distance from our planet must vary greatly. At inferior conjunction it is nearer the earth by the whole diameter of its orbit than it is at superior conjunction. Hence it appears much larger at the former time than at the latter. Its angular diameter varies from 5″ to 12″. Knowing this angular diameter, and the distance of the

planet from the earth, we can readily compute its diameter in miles by the method already used in the case of the sun (87). Its diameter is thus found to be about 3,000 miles.

When examined with a telescope this planet presents phases exactly like those of the moon. Figure 57 shows

Fig. 57.

these phases, and also the comparative size of the planet as seen in different parts of its orbit. It appears at first as a luminous disc, nearly circular, which by degrees is reduced on the side towards the east, until not more than half a circle is visible at the time of its greatest elongation from the sun; it then becomes a crescent, which grows narrower and narrower, until just before inferior conjunction it is visible only as a fine luminous thread. It repeats these phases in the opposite order when it reappears on the other side of the sun.

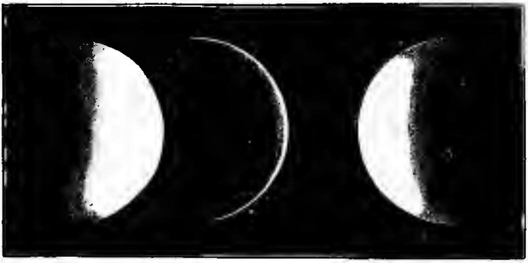
104. *The Explanation of these Phases.* — These phases prove that Mercury is not self-luminous, but shines by reflected sunlight. One half of the planet is of course always illuminated by the sun, and it is evident that at superior conjunction this illuminated half is turned towards the earth, so that its whole disc then appears luminous. At inferior conjunction it is equally evident that the illumined half is turned away from the earth; since it will always be turned toward the sun. At its

greatest elongation half of the illumined hemisphere will be turned towards the earth, and the illumined disc will then appear as a semicircle. Between the greatest elongation and superior conjunction more than half of the illumined hemisphere is turned toward the earth; while between the elongation and inferior conjunction less than half of the illumined hemisphere is turned towards us.

105. *Mercury's Period of Rotation.* — The great proximity of Mercury to the sun renders the observation of this planet somewhat difficult, so that very little is known of its surface. The German astronomer Schröter, who observed Mercury very carefully in the latter part of the last century, considered that he had decided evidence of the existence of high mountains on the planet; and by watching these he came to the conclusion that Mercury rotates on its axis in a little over twenty-four hours.

Schröter observed that during the crescent phase of the planet the line which separated the illumined from the dark portion of the disc appeared somewhat jagged, as shown in Figure 58. These markings evidently indi-

Fig. 58.



cate the existence of high mountains, which intercept the sunlight, and of valleys plunged in the shade, which lie near the parts of the surface of the planet then illumined. These markings are not always visible, but appear and disappear at intervals; and it was from these intervals that Schröter determined the rotation of the planet. But as he is the only astronomer who has been able to make out these irregularities on the disc of Mercury, the period of rotation must be considered as still very uncertain.

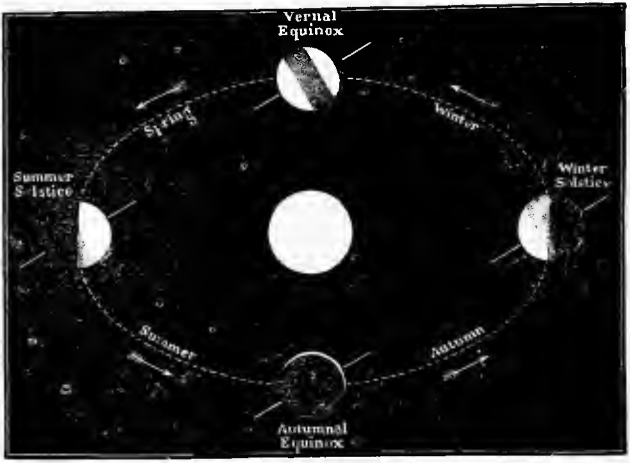
106. *The Inclination of its Orbit, and its Seasons.* — Schröter observed some dark bands on the disc of Mercury, which he considered as an equatorial zone. It was from the direction of these bands that he deduced the inclination of the axis of rotation to the plane of the planet's orbit. This he estimated at about 20°, which would make the equator of Mercury inclined to the plane of the orbit at an angle of about 70°. It will be seen that this inclination is about three times as great as that of the earth's equator to the plane of the earth's orbit, or the ecliptic. This inclination, together with the great variation of Mercury's distance from the sun, would give a remarkable variety of seasons, with great extremes of heat and cold.

Figure 59 shows, according to Schröter, the angle at which Mercury presents himself to the sun at the commencement of each season. It will be seen that very extensive zones about the poles enjoy at one season, during the summer, continuous day; while at another, during their winter, they are plunged in profound darkness. It is only during a short period, and near the planet's equinoxes, that these zones see light and darkness succeed one another in the same day.

These polar zones are of course much broader than the corresponding zones on the earth.

Since the light and heat which a planet receives from the sun diminish as the square of the distance increases, it follows that the intensity of the sunlight at Mercury is about seven times as great as at the earth.

Fig. 59.



It will be noticed in Figure 58 that the illumined part of the disc shades off gradually into the dark part. This gradual shading off indicates the presence of an atmosphere about Mercury, the darkish part being the zone of twilight which separates the full light of day from the darkness of night. This zone of twilight, as at the earth, must be caused by an atmosphere.

VENUS.

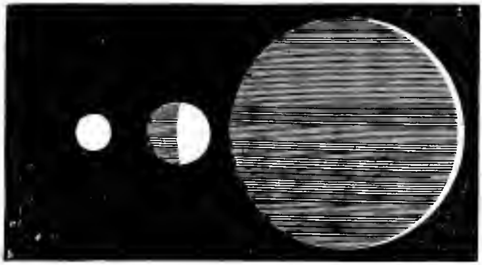
107. *Its Distance from the Sun, Time of Revolution, Diameter, etc.* — The second planet from the sun is Venus. Her mean distance (48) is 69,000,000 miles, and her period of revolution about $7\frac{1}{2}$ months.

While the orbit of Mercury differs more from a circle than that of any other planet, the orbit of Venus is found to be nearer a circle than that of any other planet. Hence she is but little nearer the sun at perihelion than at aphelion. Her orbit is inclined to the ecliptic at an angle of 3° 23′. When examined with a telescope she presents phases like those of Mercury; proving that she is not self-luminous, but shines with light borrowed from the sun.

The distance of Venus from the earth varies more than that of Mercury, since the diameter of her orbit is greater than his. At inferior conjunction Venus is of course about 138,000,000 miles nearer the earth than at superior conjunction. Hence the apparent diameter of the planet varies greatly, ranging from 70″ to less than 10″. The actual diameter of Venus is about 7,800 miles, or a little less than that of the earth.

Figure 60 shows the apparent size of Venus at its greatest, its mean, and its least distance from the earth.

Fig. 60.



Venus appears as an evening star for a period of about $9\frac{1}{2}$ months, and is then a morning star for the same length of time.

108. *Time of Rotation, Inclination of Axis, etc.* — Venus is the most conspicuous of all the planets. Her light is often brilliant enough to cast a shadow, and she is sometimes visible at midday. Her greatest brilliancy is not near her inferior conjunction, since the illumined portion of her disc is then reduced to a mere thread of light. It occurs at an elongation of about 40°, and her phase is

then about the same as that of the moon three days from new moon.

The brilliancy of Venus is so great as to render accurate observation of her disc almost impossible. We do not know whether she is flattened at the poles, like the earth, or not.

The *terminator* of Venus (the line dividing the illumined from the dark part of the disc), as seen at times by Schröter, presented considerable irregularity of outline. This irregularity, as in the case of Mercury (105), indicates the presence of mountains and valleys. By observing the interval between the disappearance and reappearance of certain of these irregularities, this astronomer came to the conclusion that Venus rotates on her axis in a little less than twenty-four hours.

Schröter made the inclination of the planet's axis to the plane of its orbit about 20°, and several other observers have arrived at very nearly the same result. The change of seasons and of the length of day and night would therefore be about the same on Venus as we have supposed them to be on Mercury.

The gradual shading off of the illumined portion of the disc of Venus indicates the existence of twilight on this planet, and consequently of an atmosphere (106).

109. *Has Venus a Satellite?* — According to Cassini and several observers of the last century, Venus is attended by a satellite. But recent astronomers have not been able to detect it, though they are provided with much better instruments for observation. It is, therefore, now generally believed that this planet has no satellite.

110. *Transits of Venus.* — Venus, as we have already seen, crosses the sun's disc whenever her inferior conjunction happens near one of her nodes (67).

THE ZODIACAL LIGHT.

111. *Its Appearance.* — In the evenings about the time of the vernal equinox, when in our latitude the twilight is of short duration, if we examine the horizon towards the west soon after sunset, we may see a faint light that rises in a triangular form among the constellations.

This appearance is known as the *Zodiacal Light.* Those not familiar with it might confound this glimmering with the milky way, or with the ordinary twilight, or even with an aurora. But with a little attention it is impossible to mistake it. Its triangular shape, its elevation, and its inclination to the horizon, all serve to distinguish it.

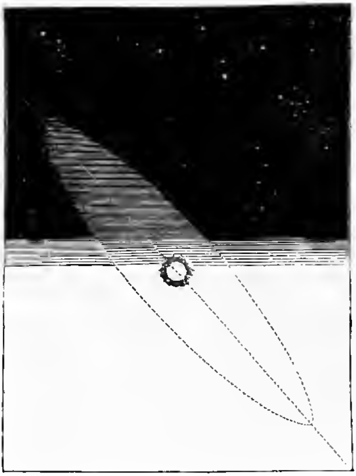
As the days lengthen, and with them the duration of twilight, the zodiacal light becomes invisible, at least in our latitude. But it may be again seen before sunrise in the east, about the time of the autumnal equinox, in September and October, when the morning twilight is short. At places favorably situated, even in the temperate zone, the zodiacal light can be seen at almost every season of the year.

In our climate, the light of the zodiacal light is rather more intense than that of the Milky Way, and much more uniform.

If now we pass from the temperate zone of either hemisphere towards the tropical zone, the zodiacal light increases in intensity and height; and it can be observed throughout the year.

112. *Its Cause.* — The most probable of the many explanations that have been given of the zodiacal light is that it is a flattened nebulous body surrounding the sun at some distance. The direction of the axis of the cone of light, if prolonged below the horizon, always passes through the sun, as shown in Figure 61.

Fig. 61.



It was believed at first that this direction coincided with the solar equator, but it has been found to coincide more nearly with the plane of the earth's orbit, or the ecliptic.

The distance from the summit of the cone to the middle of its base at the horizon varies with the time of observation. This ring sometimes extends as far as the earth's orbit, and even beyond it; at other times it is enclosed within this orbit. This may be explained by supposing either that the form of the ring is oval, or that it is circular and that the sun does not lie in its centre.

Some suppose that the zodiacal light is formed of myriads of solid particles analogous to aerolites, having a common general movement, but travelling separately about the sun as a centre. The light of the ring would thus be produced by the accumulation of this multitude of brilliant points, reflecting towards us the light borrowed by each of them from the sun. Others regard it as a ring of thin nebulous matter much like the train of comets; others as a vaporous ring which surrounds the earth at some distance.

THE EARTH.

113. The third planet in the order of distance from the sun is the earth. We have already found its mean distance from the sun to be 95,000,000 miles; that it revolves about the sun in an ellipse, in a period of one year; that it rotates on its axis in a little less than twenty-four hours; that this axis is inclined to the plane of its orbit by an angle of $66\frac{1}{2}°$; that this inclination gives rise to the change of seasons and the varying length of day and night; that the mean diameter of our planet is about 8,000 miles; and that it is not a perfect sphere, but is flattened somewhat at the poles.

The earth has also an atmosphere of known composition, which gives rise to the phenomena of twilight. The land surface of the earth is also jagged with mountains, as we have seen there is reason to believe is also the case with the surface of Mercury and Venus.

The earth is the first planet in order from the sun that is known to be accompanied by a satellite. The earth, like Mercury and Venus, shines by reflected light, as will be proved hereafter.

THE MOON.

114. *Her Distance, Diameter, Periodic Time, etc.* — The mean distance of the moon, as we have seen, is about thirty times the diameter of the earth, and her mean angular diameter is about 31′. From this her real diameter is found to be somewhat more than 2,000 miles. The moon revolves around the earth in an ellipse whose plane

is inclined to the plane of the earth's orbit by an angle of about 5°. She performs a revolution in about $27\frac{1}{2}$ days. Owing, however, to the motion of the earth in its orbit, the synodical revolution of the moon, or the interval between two successive new moons, is about $29\frac{1}{2}$ days.

115. *The Phases of the Moon.* — The *phases* of the moon depend on the position of the moon with respect to the sun, or, what amounts to the same thing, her distance from *conjunction*, which is termed in astronomical language the *age* of the moon. Being an opaque spherical body reflecting the sun's light, she can appear fully illuminated only when opposite the sun, and in all other positions her illuminated disc appears less than a circle. Soon after conjunction with the sun she may be seen as a very narrow crescent, a little above the western horizon at sunset; for, being then between the earth and sun, her illuminated surface is in a great measure turned from us. As she advances in her orbit, the dark part gradually diminishes until the moon is 90° from conjunction, which is called the *first quarter*, and then the illumined and unillumined parts are equal. After this point, the illumined surface increases till the moon is in opposition, when she is said to be *full*, and presents to us her whole enlightened disc. The bright part then begins to diminish, and again forms one half of her surface when the moon is 90° from conjunction, or at the *last quarter.* It then becomes narrower as she approaches conjunction, till a thin crescent above the eastern horizon shortly before sunrise is all that remains. These *phases* repeat themselves after the interval of a *synodical* revolution, and depend upon the position of the *visible*, with reference to the *enlightened*, hemisphere of the moon.

116. *Libration.* — The most casual observer of the moon can hardly have failed to remark that she always presents

very nearly the same face towards us, and a little reflection will convince him that the cause of this must lie in the very near equality of her periods of axial rotation and synodical revolution round the earth. If these periods were exactly equal, and the moon's motions exactly uniform, we should have the same hemisphere turned towards us without the slightest variation. But the motion in her orbit is subject to small irregularities, while that on her axis is perfectly uniform, and for this reason a phenomenon termed *libration* takes place, whereby we occasionally see a little more of one edge of the moon than usual, either on the eastern or western side of her equatorial region. Suppose, for instance, that conjunction occurs when the moon is at *perigee*, or nearest the earth, and that the moon rotates on its axis at a uniform rate and once during a sidereal revolution. But the moon moves in her orbit faster at perigee than elsewhere. She would accordingly perform the first quarter of her revolution before she had performed a quarter of a rotation. We should therefore see a little more of her western edge than we should if she had performed a quarter of her rotation. Again, suppose that conjunction occurs when the moon is at *apogee*, or farthest from the earth. Then while she is performing a quarter of a revolution, she will perform more than a quarter of a rotation, and we should see more of her eastern edge than we should if she had performed just a quarter rotation.

This libration, which is due to the moon's unequal rate of motion in her orbit, is usually called *libration in longitude*.

The moon, then, rotates on her axis in the mean period of a sidereal revolution, and it has been found by observation that the axis of this rotation is not quite perpendicular to the plane of her orbit. Now since the axis always maintains the same direction, it follows that we are enabled at times to see a little more of the polar regions than at others. This phenomenon is called *libration in latitude.*

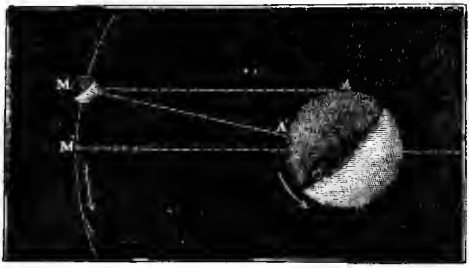
Since we are situated four thousand miles above the centre of the earth, we see at the rising of the moon a little more of the western edge than we should if we were viewing the moon from the centre of the earth. For the same reason, we see at the setting of the moon a little more of the eastern edge of the moon than we should if we were situated at the centre of the earth. This phenomenon is called *parallactic libration.*

117. *The Earth's Phases as seen from the Moon.* — It is a well-known fact that at the time of new moon the dark part of the moon's surface is partially illumined, so that it becomes visible to the naked eye. This must be due to the light reflected to the moon from the earth. Since at new moon the moon is between the earth and sun, it follows that when it is new moon at the earth, it must be *full earth* at the moon. Hence while the bright crescent is enjoying full sunlight, the dark part of its surface is enjoying the light of the full *earth.*

118. *The Apparent Size of the Moon.* — The apparent magnitude of the lunar disc not only varies with the distance of the moon from the earth, but even on the surface of our globe, and at the same instant, the disc does not appear of equal magnitude to all observers. It appears larger to an observer who sees the moon rising or setting, than to him who sees it at the zenith.

In Figure 62 the distance *A M* of the moon from an observer at *A* is seen to be nearly equal to its real distance from the centre of the earth, while at the same time its distance *A′ M* from an observer at *A′* is 4,000 miles less than its distance from the centre of the earth. Hence the moon is nearer an observer at *A′* by about $\frac{1}{60}$ of its distance than to an observer at *A*.

Fig. 62.



Notwithstanding it is much nearer when at the zenith than at the horizon, it seems to us much larger at the horizon.

This is a pure illusion, as we become convinced when we measure the disc with accurate instruments, so as to make the result independent of our ordinary way of judging. When the moon is near the horizon, it seems placed beyond all the objects on the surface of the earth in that direction, and therefore farther off than at the zenith, where no intervening objects enable us to judge of its distance. In any case, an object which keeps the same apparent magnitude seems to us, through the instinctive habits of the eye, the larger in proportion as we judge it to be more distant.

A recent discovery of very great interest shows us that in the case of the moon the word *apparent* means much more than it does in the case of other celestial bodies. Indeed, its brightness causes our eyes to play us false. As is well known, the crescent of the new moon seems part of a much larger sphere than that which it has been said, time out of mind, to "hold in its arms." We now learn that the bright portion of the moon, as seen with our measuring instruments, as well as when seen with the naked eye, covers a larger space in the field of the telescope than it would if it were not so bright. This has been recently proved by measuring the *dark moon* by means of the *occultation* of a star. In this way the Astronomer Royal of England has shown that the diameter of the moon hitherto received is too large by 2″.

119. *The Path of the Moon through Space.* — Since the earth is moving in its orbit at the same time that the moon is revolving about it, it follows that the path described by the moon through space is much the same as that described by a point on the circumference of a wheel which is rolled over another wheel. If we place a circular disc against the wall, and carefully roll along its edge another circular disc to which a piece of lead pencil has been fastened so as to mark upon the wall, the curve described will somewhat resemble that described by the moon. This curve is called an *epicycloid*, and it will be seen that at every point it is concave towards the centre of the larger disc. In the same way the moon's orbit is at every point concave towards the sun.

120. *Harvest Moon.* — The full moon which occurs nearest to the autumnal equinox has long been called the *Harvest Moon*, from the fact that the difference between the times of the moon's rising on two successive nights is then at a minimum, and the long duration of moonlight, thus afforded soon after sunset, is very advantageous to the farmer at this busy season. This near coincidence in the times of several successive risings occurs every lunar month, when the moon is in the signs Pisces and Aries, but it attracts attention only when the moon is at the full in these signs, and this can happen only in August or September. The least possible difference between two successive risings in the latitude of Boston is about twenty-three minutes. When the moon is in Libra, and at the same time near the descending node of her orbit, the difference between the times of rising on two evenings is the greatest possible, amounting to about one hour and seventeen minutes.

121. *The Surface of the Moon.* — The moon is much the nearest to us of all the heavenly bodies, and we are consequently best acquainted with its surface. The naked eye readily discerns that the disc of the full moon is not uniformly bright: light and dark regions diversify it, giving the idea of continents and seas like those on our own globe. In fact, the earlier selenographers considered the dull, grayish spots to be water, and termed them the lunar seas, bays, and lakes. They are so called on lunar maps to the present day, though we have strong evidence to show that if water exists at all on the moon, it must be in very small quantity.

On examining the moon with suitable magnifying powers, we perceive on every part of the surface, even in the midst of the so-called oceans and seas, ring-like spots, evidently of volcanic character, with extensive chains of mountains and steep isolated rocks, presenting altogether a very rugged and desolate appearance. If we choose for observation the first or last quarter of the moon, the portions near the edge of the illuminated part appear eaten into cavities surrounded by circular walls, which cast shadows away from the sun, at one side towards the interior, and on the other towards the exterior of the cavity. Along the whole line which separates the light and dark parts of the moon, called the *terminator*, the interior of the ring-like cavities seems quite black, while here and there luminous points show themselves detached from the illuminated portion of the moon.

These spots indicate mountain tops or ranges, which, according as we observe them at the first or last quarter, are receiving the rays of the moon's rising or setting sun while the lowlands are in the shade.

Small spots of annular form, which are regarded as craters, are exceedingly numerous, and are seen to cover the whole visible surface of the moon. In some places they are thickly crowded together, small volcanoes having formed on the sides of the large one: in other regions they are comparatively isolated. Their dimensions are far greater than those of the largest volcanoes on the earth, the breadth of the chasm occasionally exceeding one hundred miles, while the sides of the mountains attain a very considerable elevation. The best time for viewing a crater is when it is just clear of the dark part of the moon, or when the sun is just above its horizon. We can then trace the shadows thrown by the side of the mountain upon its interior and exterior surface, and, by measuring these shadows, we may approximate to the true altitude of the mountain. Some of the steep isolated rocks throw their shadows for many miles across the plains surrounding them.

Of course the angle subtended by the shadow can be directly measured, and since we know the angle subtended by the diameter of the moon, and the length of this diameter in miles, we can readily determine the length of the shadow in miles. It will evidently be the same fraction of the diameter in miles, as the angle which it subtends is of the angle subtended by the diameter of the moon. Knowing, then, the length of the shadow in miles, and the height of the sun above the horizon, we can easily ascertain the height of the mountain which casts the shadow.

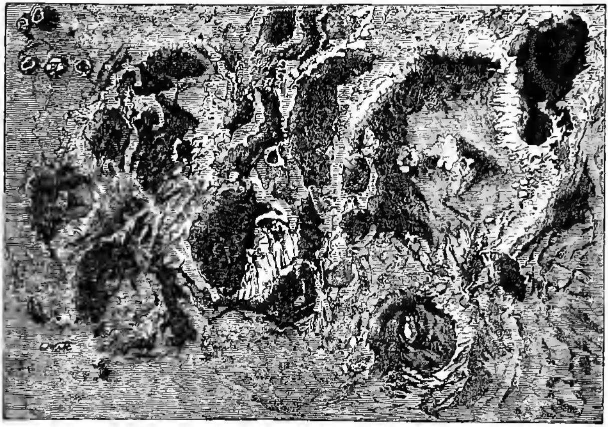
We have only to ascertain the length of the shadow cast by a mountain of known height on the earth when the sun is the same distance above the horizon. The height of the lunar mountain will be just as many times greater as its shadow is longer.

122. *Tycho.* — One of the most remarkable of the lunar spots is that called *Tycho*, which is readily distinguished in the southern part of the full moon by the number of luminous rays, or streaks of light, which di-

verge from it in a northeasterly direction. Tycho is an annular mountain or crater, no less than fifty-four miles in diameter. The height of the western wall above the interior level is, according to Mädler, 17,100 feet, and of the eastern borders somewhat more than 16,000 feet. A mountain nearly a mile high marks the centre of the crater. Tycho is surrounded by a great number of craters, peaks, and ridges of mountains, lying so close together that in some directions it is impossible to find the smallest level place.

Figure 63 gives a view of the region to the southeast of Tycho.

Fig. 63.



This mountain, as we have already said, is the centre of a number of luminous streaks or rays, which extend therefrom over fully one fourth of the moon's disc. The brightest one branches off in a northeasterly direction, and there are others very conspicuous on the western side of the crater. These rays become visible as soon as the sun has risen from 20° to 25° above their horizon. Their color is perhaps a little whiter or more silvery than the general lunar surface. Many opinions have been advanced by Cassini, Schröter, Herschel, and others, respecting the nature of these appearances, and they have been variously styled mountains, streams of lava, and even roads. There is nothing on the surface of the earth bearing the slightest analogy to them. Perhaps the most plausible theory is that first started by Mr. Nasmyth, that they have been caused by a general volcanic upheaval of the moon's crust in former ages, which has produced an appearance on the lunar surface similar to that of a pane of glass broken by a sharp-pointed instrument. The mere fact of their divergence from the great crater Tycho proves that it was the focus of this volcanic outbreak, whenever it may have occurred.

123. *Copernicus.*—Another very beautiful annular mountain is that known as *Copernicus*, shown in Figure 64.

The diameter of the crater is somewhat larger than that of Tycho, being rather more than fifty-five miles. The highest point is about 11,250 feet above the surrounding plains. It is readily discernible on the full moon, but is most favorably viewed when the sun's rays have just reached its eastern side, about the time of quadrature, or first quarter. The shadows of the western side of the crater are then thrown on the interior level, that of the central peak on the same level towards the eastern side, while the shadow of this side of the mountain darkens for some distance the exterior plain on the rugged edge of the moon. Generally speaking, these shadows are extremely well defined. The divergent streaks of light from this mountain are best seen near the time of full moon. They vary in breadth from three to ten miles, the principal one branching off towards the northeast.

Fig. 64.

124. *Kepler.* — This is also a conspicuous ring-mountain, the focus of similar rays of light. The crater is about twenty-two miles in diameter, and the altitude of the eastern edge above the level of the interior is about 10,000 feet.

Tycho, Copernicus, and Kepler are the principal craters which form the radiating points of the luminous streaks which are so remarkable upon the surface of the full moon.

125. *Eratosthenes.* — This is a very beautiful annular mountain situated at the extremity of the long range called the Apennines, which cover a surface of more than 16,000 square miles. The crater is not less than

thirty-seven miles in diameter, and in its centre a steep rock rises 15,800 feet above the level surface of the interior. The outside of the circular mountain is about 3,300 feet high on the western border, while on the eastern side its height is more than twice as great.

The volcanic character of the lunar mountains is unmistakable. All the crust of our satellite is pierced with craters which indicate an innumerable series of volcanic eruptions, some limited to a small space, others embracing an immense area.

The darkish portions of the moon are supposed to be large plains.

126. *Are there Active Volcanoes on the Moon?* — In 1787 Sir William Herschel announced that he had observed three volcanoes in a state of eruption in different parts of the moon; and modern astronomers have repeatedly noticed luminous spots in the dark portion of the lunar disc, some of which were so distinct and striking that they might readily be taken for active volcanoes. The prevailing opinion among astronomers, however, is, that these appearances are due to the reflection of the "earth-light" (117) from certain mountain tops, which from their nature or their position have a greater reflective power than other parts of the moon.

Recent observations, however, show that changes of some kind are still going on in lunar craters. In October and November, 1866, Schmidt, the Director of the Observatory of Athens, noticed that the deep crater *Linné*, whose diameter is 5.6 miles, had completely disappeared, and in its place there was only "a little whitish luminous cloud." He at once called the attention of other European astronomers to the facts, and in December the locality of the lost crater was carefully examined by many of them. All agreed with Schmidt, that Linné could not be seen at the time when it was most favorably situated for observation, and when smaller craters in its immediate neighborhood were very distinct *with the shadows within them.*

The obscuration, whatever may have been its cause, appears to have ceased in the latter part of December, when the crater was distinctly seen by Dr. Tietjen at Berlin.

It is said that one of Schröter's maps gives a dark spot in the place of Linné, and that the crater is not to be found on Russell's globe or maps of the year 1797; from which it may be inferred that the crater has previously been obscured.*

127. *The Moon has little or no Atmosphere.* — If there is a lunar atmosphere, it must be one of great rarity and of no great extent; otherwise it would give rise to phenomena which could not fail to attract the attention of astronomers.

The two main reasons for thinking that there is no atmosphere of any considerable density at the moon are, (1.) the sharpness of the line which separates the bright and dark portions of her disc, and (2.) the absence of refraction, as shown in the occultation of stars.

(1.) It is well known that there is no gradual shading off from the illuminated parts of the moon's disc, as there appears to be in the case of Mercury and Venus, and as there is known to be in the case of the earth. There is, therefore, no perceptible twilight on the moon, and consequently no atmosphere, unless of great rarity.

(2.) According to the well-known laws of refraction, if there were an atmosphere about the moon, the rays of light would be so bent that a star, on passing behind the moon, would be seen even after it was really behind the disc, just as the sun is visible after it is really below the horizon; and so that it would become visible before it had really emerged from behind the disc, just

* See Appendix, V.

as the sun is visible before it is really above the horizon. It would then follow that a star would appear to be behind the moon a shorter time than that computed from the known rate of motion and angular diameter of the moon. Now it is found that the observed time of the occultation of any star by the moon is very slightly, if at all, shorter than the computed time. Airy has shown that, if the discrepancy of 2″ which he has found (118) between the angular diameter of the moon, as determined by observation, and as computed from the rate of the moon's motion and the time of the occultation of a star, is wholly due to refraction caused by a lunar atmosphere, the refractive power of that atmosphere is only $\frac{1}{2000}$ part of that of the earth's atmosphere; hence it must be of extreme tenuity.

If there is no atmosphere at the moon, there can be no water on her surface; for the heat of the sun would cause that water to evaporate, and thus an atmosphere of aqueous vapor would be formed.

Furthermore, there has never been discovered any positive evidence of the existence of *clouds* at the moon, which would be the necessary result of the existence of water there.

Some astronomers have supposed that the side of the moon turned towards us may be a huge mountain, and that there may really be both air and water on the farther side, though not enough to rise above this mountain. Both Adams and Le Verrier have, however, shown that such a hypothesis is, to say the least, extremely improbable.

ECLIPSES.

128. *The Shadows of the Earth and Moon.* — The earth and moon are two spherical and opaque bodies, and the halves of both are constantly illuminated by the rays of

the sun, while the other halves are in the shade. The illuminating body is itself a sphere of much greater size. Not only, therefore, have the earth and the moon always one of their hemispheres dark, but each of these bodies throws behind it, in a direction opposite from the sun, a shadow of conical form, the length and diameter of which depend upon the distance and diameter of the illuminating body, and the diameter of the illuminated body.

This cone of shade encloses all those parts of space where, by reason of the interposition of the opaque body, no rays of light from the sun can be received. Beyond the apex of this cone of pure shade, which is called the *umbra*, and in the direction of its axis, are situated those portions of space from which a part of the sun is seen in the form of a luminous ring bordering the obscure disc of the opaque body. Lastly, these two regions are themselves surrounded by what is called the *penumbra;* or those portions of space which receive light only from a part of the sun, one side of whose luminous disc is obscured by the disc of the opaque body. The darkness of the penumbra at any point is more intense, in proportion as the point is nearer the umbra.

The moon and the earth in their movements carry with them their cones of umbra and penumbra, and it is by projecting these total or partial shadows upon each other that they produce the phenomena of *eclipses.*

These various cones are represented in Figure 65.

129. *When Eclipses may Occur.* — On examining this figure, it will be seen at once why an eclipse of the sun can happen only at the time of new moon, and why, on the other hand, an eclipse of the moon is possible only at full moon.

In all other positions of the moon, her cone of shade is projected into space away from the earth, and the

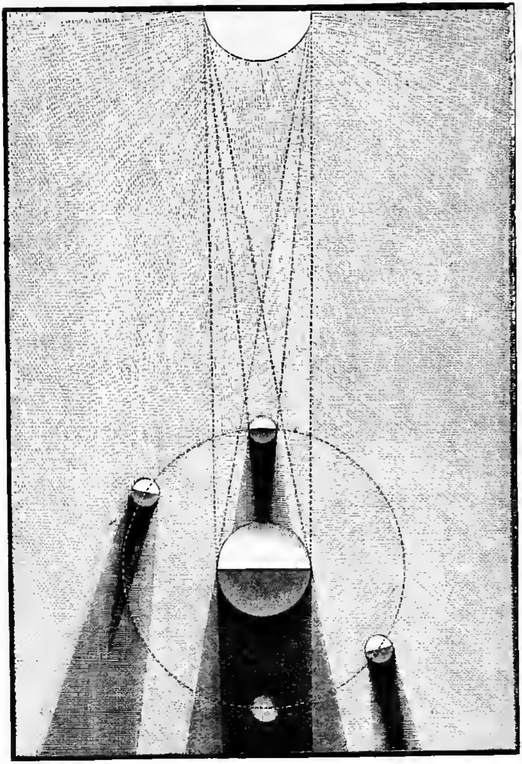


Fig. 65.

errestrial cone of shade does not meet the moon. It does not follow, however, that there is an eclipse of the sun at every new moon, or of the moon at every full moon. This would be true if the orbits of the earth round the sun and of the moon round the earth were described in the same plane. Then at each opposition or conjunction the centres of the three bodies would necessarily lie in a straight line.

But the orbit of the moon is inclined to the ecliptic at an angle of about 5°, so that it often happens at the time of new moon that our satellite throws its cone of shadow above or below the earth. In like manner, at the time of opposition, the moon, in consequence of her being out of the plane of the ecliptic, passes sometimes above and sometimes below the cone of the earth's shadow. In such cases there can be no eclipse.

In order, then, that there should be an eclipse of the sun, new moon must occur when the moon is at or near one of the nodes (67) of her orbit; and in order that there should be an eclipse of the moon, full moon must occur when the moon is at or near one of her nodes.

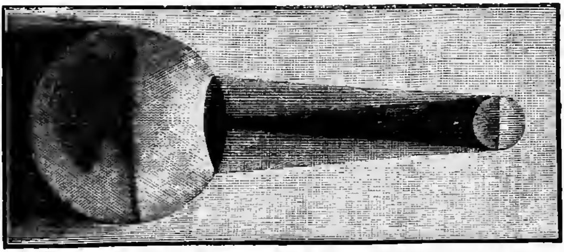
130. *Eclipses of the Sun.* — Solar eclipses are of three kinds. When the dark disc of the moon entirely covers the sun, the eclipse is *total;* when only a portion, large or small, of one side of the sun is covered by the moon, the eclipse is *partial;* and when the disc of the moon is not large enough to cover the whole disc of the sun, and thus leaves a luminous ring visible around its own body, the eclipse is *annular.*

As the moon is much smaller than the sun, it will be understood that it is her small relative distance that causes her disc to appear equal to or greater than that of the sun. This distance varies by reason of the elliptical form of the moon's orbit, and hence the lunar disc is sometimes larger, sometimes smaller than, and sometimes equal to, that of the sun.

This is the same as saying that the cone of the moon's real shadow, or umbra, sometimes reaches the earth and sometimes does not. If it reaches the earth there is a total eclipse of the sun to all parts of the earth within it, and a partial eclipse to all parts within the penumbra. This will be readily seen from Figure 66.

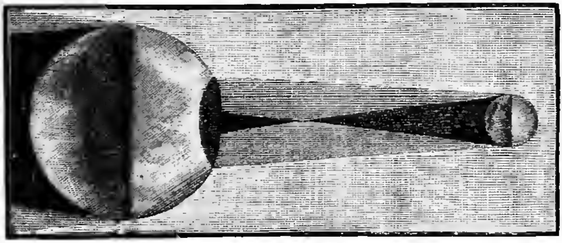
If the cone of the lunar shadow does not reach the earth, there will be an annular eclipse in those places

Fig. 66.



which are in the direction of the axis of the cone, and a partial eclipse to those which are only within the penumbra. This case is represented in Figure 67.

Fig. 67.



It will be seen, then, that the conditions under which a total eclipse of the sun is possible are the following:— (1.) the moon must be in conjunction, or *new*; (2.) she must at the same time be at or near a node; (3.) her distance from the earth must be less than the length of her shadow.

The same conditions, except the last, are necessary for an annular eclipse.

The breadth of the cone of the moon's umbra at the distance of the earth seldom equals 160 miles. Hence a total eclipse is seen only over a very narrow tract; but, owing to the rotation of the earth, this tract has considerable length.

A total eclipse of the sun is a rare occurrence at best, and a total eclipse at any given place is rarer still. It will be seen from Figure 67 that even the penumbra of the moon's shadow traverses but a small part of the earth; so that a partial eclipse of the sun is by no means visible to the whole earth.

Hind thus describes the appearances during the total eclipse of the sun, July 28th, 1851, in Sweden:—

"The aspect of nature during a total eclipse was grand beyond description. A diminution of light over the earth was perceptible a quarter of an hour after the beginning of the eclipse, and about ten minutes before the extinction of the sun the gloom increased very perceptibly. The distant hills looked dull and misty, and the sea assumed a dusky appearance, like that it presents during rain. The daylight that remained had a yellowish tinge, and the azure blue of the sky deepened to a purplish-violet hue, particularly towards the north. But, notwithstanding these gradual changes, the observer could hardly be prepared for the wonderful spectacle that presented itself when he withdrew his eye from the telescope, after the totality had come on, to gaze around him for a few seconds. The southern heavens were then of a uniform purple-gray color, the only indication of the sun's position being the luminous corona, the light of which contrasted strikingly with that of the surrounding sky. In the zenith, and north of it, the heavens were of a purplish-violet, and appeared very near, while in the northwest and northeast, broad bands of yellowish-crimson light, intensely bright, produced an effect which no person who witnessed it can ever forget. The crimson appeared to run over large portions of the sky in these directions irrespective of the clouds. At higher altitudes the predominant color was purple. All nature seemed to be overshadowed by an unnatural gloom, the distant hills were hardly visible; the sea turned lurid red, and persons standing near the observer had a pale, livid look, calculated to produce the most painful sensations. The darkness, if it can be so termed, had no resemblance to that of night. At various places within the shadow, the planets Venus, Mercury, and Mars, and the brighter stars of the first magnitude, were plainly seen during the total eclipse. Venus was distinctly visible at Copenhagen, though the eclipse was only partial in that city; and at Dantzic she continued in view ten minutes after the sun had reappeared. Animals were frequently much affected. At Engelholm, a calf which commenced lowing violently as the gloom deepened, and lay down before the totality had commenced, went on feeding quietly enough very soon after the return of daylight. Cocks crowed at Helsingborg, though the sun was there hidden only thirty seconds, and the birds sought their resting-places as if night had come on."

131. *Eclipses of the Moon.* — Like the eclipses of the sun, those of the moon may be either *total* or *partial*, but they are never *annular*, since the breadth of the cone of the earth's shadow at the distance of the moon is always much greater than the diameter of the moon's disc.

The fundamental difference between the two phenomena is, that an eclipse of the sun is visible to only a part of the hemisphere which has him above the horizon, while an eclipse of the moon is visible from every part of the earth from which the moon herself is visible; and an eclipse of the sun is seen at different stations successively, as the umbra and penumbra of the moon's shadow traverse the earth, while an eclipse of the moon everywhere begins and ends at the same instant. The reason of this difference is that the sun's disc is not really darkened, but only hidden by the obscure disc of the moon, so that the interposition is an effect of perspective, varying according to the respective position of the observer, of the moon, and of the sun. The lunar eclipse is, on the contrary, produced by the real fading out of the moon's light, and the darkness consequent upon it is observed at the same instant wherever the moon is in view.

When the moon passes through the centre of the earth's shadow the eclipse is *total* and *central*. The earth's shadow at the moon's distance is, however, so broad that an eclipse may be total without being also central.

The *magnitude* of an eclipse, if partial, and the continuance of the obscuration, if total, depend upon the direction of the moon's passage through the earth's shadow, which is sufficiently broad to allow of her being hidden by it one hour and fifty minutes, when she passes through its centre.

It is not possible to ascertain, with any degree of accuracy, the time when the moon first enters the penumbra, for the darkening effect upon her disc is so slight that some minutes must elapse before sufficient shade is produced to attract attention. Neither does the time of contact with the umbra admit of exact observation, since the penumbra shades off into the umbra by imperceptible degrees.

When the moon is totally immersed in the dark shadow, she does not, except on rare occasions, become invisible, but assumes a dull reddish hue, somewhat like that of tarnished copper. This arises from the refraction of the sun's rays in passing through the earth's atmosphere.

In a total lunar eclipse in 1848, the spots on the moon's surface were distinctly seen by many observers, and the general color of the moon was a full glowing red. Her appearance was so singular that many persons doubted of her being eclipsed at all.

Once in about eighteen years the earth, sun, and moon occupy the same relative positions. This is a fact which the ancients established by observation long before the theory of the celestial movements had demonstrated its near approach to the truth. If, then, we start from the epoch of an eclipse of the sun or moon, that is to say, from a lunar conjunction or opposition coinciding with one of the moon's nodes, after 18 years the three bodies will be found in situations nearly identical. Hence the eclipses which succeeded one another in the first period follow again in the same order during the second period. This is the starting point in the calculation of eclipses, but the approximation is too rough for the exactness of modern astronomy. Now-a-days the time of eclipses is foretold to a second several years in advance of their occurrence.

132. *Occultations.* — The moon in traversing her orbit round the earth produces another kind of eclipse, to which the name of *occultation* has been given. A star or planet is said to be *occulted* when it passes behind the lunar disc. These phenomena have already been mentioned with reference to the question of the existence of an atmosphere on the surface of the moon (127).

The occultations of the stars are calculated with the same precision as the eclipses, and as they are of frequent occurrence they are of great use to navigators in determining their longitude. As the moon is very near the earth, compared with the distance of the stars and even of the planets, it follows that two observers at different points on the earth do not see it projected at the same instant on the same part of the heavens. The occultation of a star does not, therefore, take place to both of them at the same instant of time.

By correcting these observations for refraction and parallax, the exact time is found at which an occultation would take place to an observer at the centre of the earth. Now the times at which the occultation of stars would occur to an observer situated at the centre of the earth are computed in Greenwich time and published in the Nautical Almanac. An observer at sea, then, finds by observation the time of occultation as seen from the earth's centre in his own local time, and he can then compare his own local time with Greenwich time, and find the difference between the two. He can then readily determine the longitude of his place.

As we have already seen, the meridian of a place sweeps over the whole heavens, from the sun around to it again, in twenty-four hours. Hence it will sweep over 15° in one hour, and 1° in four minutes; and when the sun is on the meridian of a given place, it will be 15° east of the meridian of a place 15° to the west of it. The sun will then come upon the meridian an hour later at the second place, and it will be one o'clock at the more easterly place when it is twelve o'clock at the more westerly. Hence local time becomes an hour earlier as we travel westward 15°.

If, then, we know that Greenwich time is three hours later than our time, we know that we are 45° west from Greenwich; and if Greenwich time is two hours earlier than our time, we know that we are 30° east of that place, or that we are in longitude 30° east.

Not only is the time at which the occultation of the star would occur to an observer at the centre of the earth computed in Greenwich time and published in the Nautical Almanac, but also the time when the moon passes all the principal stars near her path is computed and published in the same manner, as well as the distance of the moon from these principal stars for every day during the year.

Thus the heavens become a universal dial over which the moon sweeps as a minute-hand, marking, as she passes the fixed stars, Greenwich time to every part of the earth.

But this hand moves with considerable unsteadiness, now faster and now slower, according as the moon is at perigee or apogee, and subject to various other fluctuations; while by reason of parallax and refraction her real position in front of the dial is seldom what it appears to be, so that it has required the patient observation and study of years to learn to read this time aright.

The learning to read time accurately by this clock of nature has been one of the greatest triumphs of astronomy, and is a good illustration of the practical bearing of such scientific studies.

SUMMARY OF THE MOON AND ECLIPSES.

The *mean distance* of the moon is about thirty diameters of the earth; her *mean angular diameter* about 31′; the *inclination of her orbit* to the ecliptic about 5°; her *sidereal period* about $27\frac{1}{2}$ days; and her *synodical period* about $29\frac{1}{2}$ days. (114.)

The *phases* of the moon depend upon the position of her *visible* with reference to her *enlightened* hemisphere. They repeat themselves after the interval of a *synodical* revolution. (115.)

The moon completes a rotation on her axis in the same time that she completes a revolution about the earth. Her rotation on her axis is performed at a *uniform* rate, while the rate of her revolution about the earth *varies*. This gives rise to *libration in longitude.* The axis of the moon is not quite perpendicular to the plane of her orbit. This gives rise to *libration in latitude.* We see the moon from a point about 4,000 miles above the centre of the earth. This gives rise to *parallactic libration.* (116.)

The earth presents to the moon *phases* similar to those which the moon presents to us. When it is *new moon* to us, it is *full earth* to the moon. (117.)

The moon is *nearest* to us when she is in the zenith, but she *appears largest* when she is near the horizon. Owing to her *brightness*, the moon appears larger than she really is. (118.)

The moon describes an *epicycloidal* path, every part of which is concave toward the sun. (119.)

The least difference between two successive risings of the moon in our latitude is about twenty-three minutes. When this least difference occurs at the time of full moon, we have what is called the *Harvest Moon.* (120.)

The surface of the moon is covered with steep isolated rocks, volcanic craters, and extensive mountain chains. The existence of these rocks and mountains is indicated by the shadows which they cast. By means of these shadows we can estimate the height of the objects which cast them. (121.)

The four most remarkable lunar mountains are *Tycho*, *Copernicus*, *Kepler*, and *Eratosthenes.* (122 – 125.)

There is no evidence of active volcanoes on the moon, though changes of some kind are still going on within the lunar craters. (126.)

That the moon has little or no atmosphere is shown by the sharpness of the line which separates the bright and dark portions of her disc, and by the absence of refraction. (127.)

The earth and the moon cast *conical shadows* behind them. (128.)

When the earth passes through the shadow of the moon, there is an *eclipse of the sun*, which may be *partial*, *annular*, or *total.* This can happen only at the time of *new moon*, and when the moon is near her node. As the moon's shadow barely reaches the earth, total eclipses of the sun are of rare occurrence and of short duration. (129, 130.)

When the moon passes through the shadow of the earth, there is an *eclipse of the moon*, which may be either *partial* or *total*. This can happen only at full moon, and when the moon is near one of her nodes. In an eclipse of the moon her light is extinguished, while in an eclipse of the sun his light is only hidden by the moon. (129, 131.)

The planets and stars are *occulted* by the moon, and by their occultation *longitude* at sea is determined. The heavens are a universal dial, upon which the moon points Greenwich time to every part of the earth. (132.)

METEORIC RINGS.

133. *Shooting stars* are those evanescent meteors which dart across the sky at night in all directions, and generally leave behind them luminous trains visible some seconds after the extinction of the brighter part. The number of the shooting stars varies greatly with the time of the year ; hence the distinction between *sporadic meteors* and the *showers* of shooting stars which appear in the sky in large numbers and generally periodically. During ordinary nights, the mean number of shooting stars observed in the interval of an hour is from four to five, according to some observers, and as high as eight, according to others.

But at two periods of the year, about the 10th of August and the 12th of November, these phenomena are much more numerous, and the number of shooting stars observed in the interval of an hour is often more than tenfold that seen on ordinary nights. The August showers used to be popularly known as "St. Lawrence's tears"; the luminous trains being nothing else to the untutored people of Ireland than the burning tears of that martyr, whose feast fell on the 10th of August. The November shower is usually more brilliant than the August, and at intervals of about thirty-three years it is of extraordinary splendor.

On the 12th of November, 1799, Humboldt, who was then at Cumana, relates that, between the hours of two and five in the morning, the sky was covered with innumerable luminous trains, which incessantly traversed the celestial vault from north to south, presenting the appearance of fire-works let off at an enormous height; large meteors, having sometimes an apparent diameter of one and a half times that of the moon, blending their trains with the long, luminous, and phosphorescent paths of the shooting stars. In Brazil, Labrador, Greenland, Germany, and French Guiana, the same phenomena were observed.

The shower of November 12th, 1833, was no less extraordinary. The meteors were observed along the eastern coast of America, from the Gulf of Mexico as far as Halifax, from nine o'clock in the evening till sunrise, and in some places, even in full daylight, at eight o'clock in the morning. They were so numerous, and visible in so many parts of the sky at once, that in trying to count them one could only hope to arrive at a very rough approximation. At Boston, Prof. Olmsted compared the shower, at the moment of maximum, to half the number of flakes which one sees in the air during an ordinary snow-storm. When the brilliancy of the display was considerably reduced he counted six hundred and fifty in fifteen minutes, though he confined his observations to a zone which was not a tithe of the visible horizon. He estimated that the number he counted was not more than two thirds of the number which fell; making the whole number 866 in the zone observed, and some 8,660 in all the visible heavens.

Now the phenomena lasted more than seven hours, and as the above estimate would give an average of 34,640 an hour, the number seen at Boston exceeded 240,000; and yet it must be remembered that the basis of this calculation was obtained at a moment when the display was notably on the decline.

Again, on the morning of the 14th of November, 1866, an extraordinary display of meteors was seen in England. The display was very brilliant, but those who saw both, pronounced it much less splendid than the show of 1833.

134. *The Probable Cause of these Phenomena.* — The great majority of the meteors of the November shower radiate in all directions from a point in Leo, called from this fact the *radiant point;* while the radiant point of the August shower is in Perseus. These points are precisely those toward which the earth is moving at the time.

Astronomers have therefore concluded that the appearance of shooting stars is caused by the passage of the earth through rings composed of myriads of these bodies, which circulate, like the larger planets, round the sun, and whose parallel movements, seen from the earth, seem to radiate from that part of the heavens which the earth is approaching. The appearance required by this theory is exactly that presented to us.

Professor Newton, of New Haven, an astronomer who has given much attention to this subject, finds that the average number of meteors which traverse the atmosphere daily, and which are large enough to be visible to the naked eye on a dark, clear night, is no less than 7,500,000; and applying the same reasoning to telescopic meteors, the number will have to be increased to 400,000,000.

It is now generally held, that these little bodies are not scattered uniformly throughout space, or collected into either one or two rings, but that they are collected into several rings round the sun; and that, when the earth in its orbit breaks through one of these rings, or passes near it, her attraction overpowers that of the sun, and causes them to impinge on our atmosphere, where, their motion being arrested and converted into heat and light, they become visible to us as meteors, fire-balls, or shooting stars, according to their size.

It has been suggested, not without some probability, that the earth's attraction may sometimes retain these meteors as permanent satellites. A French astronomer believes he has detected one of these bodies that revolves around our globe in a period of three hours and twenty minutes. The distance of this singular companion of the moon is 5,000 miles from the surface of the earth. Occasionally these meteors are drawn to the earth by its superior attraction, and fall to the ground as *meteoric stones.*

MARS.

135. *His Distance, Period of Revolution, etc.* — The next planet in the order of distance from the sun is Mars. He is consequently the first of those planets whose orbit encloses that of the earth, and which have therefore been called *exterior* or *superior planets.* Mars appears to the naked eye as the reddest star in the heavens.

His mean distance from the sun is 145,000,000 miles, but, in consequence of the eccentricity or flattened form of his orbit, he is about 27,000,000 miles nearer the sun at perihelion than at aphelion. His sidereal period is somewhat less than two years, and his synodical period somewhat more than two years.

The plane of Mars's orbit is inclined to the ecliptic at an angle of less than 2°. The apparent diameter of the planet varies considerably, since his distance from the earth varies considerably. He must be the diameter of the earth's orbit nearer to us at opposition than at conjunction. When nearest the earth his diameter subtends an angle of more than 30″, while at his greatest distance the diameter subtends an angle of only 4″. The real diameter of this planet is about 4,500 miles. When examined with a telescope of sufficient power, the disc of Mars appears perfectly round at opposition and conjunction, while in every other part of his orbit the disc is more or less gibbous, according to the distance of the planet from quadrature, when the illuminated disc differs most from a circle. These phases prove that Mars, like Mercury and Venus, shines with light borrowed from the sun. At opposition and conjunction he turns the same face toward us and toward the sun, and hence we see the whole of his illuminated hemisphere; while in other parts of his orbit he does not turn quite the same face to us and to the sun, hence he appears more or less gibbous.

This planet is most favorably situated for observation at perihelion and opposition. Opposition occurs, as already stated, once in a little over two years; and opposition and perihelion occur together once in about eight years.

136. *The Physical Characteristics of Mars.* — When viewed under proper optical powers, the surface of this planet presents outlines of seas and continents similar to those on our globe, and usually white spots are discernible near the poles, which, from their alternate diminution and increase, according as one pole is turned to or from the sun, are conjectured to be masses of snow. The color of the continents is a dull red; that of the seas greenish, as by contrast with the land it should be. It is this prevailing color of the land which gives the planet that ruddy light by which it is at all times readily distinguished from the other planets and from the fixed stars. By observing the spots on the surface the time of the

axial rotation of Mars has been determined to be about $24\frac{1}{2}$ hours. His axis is inclined to the plane of his orbit at an angle of about 61°.

Consequently Mars experiences about the same changes of seasons as the earth, though each season is about twice as long.

Mars, like the earth, is not perfectly spherical; it is somewhat flattened at the poles, though the amount of the flattening is not yet accurately ascertained.

It is quite certain that Mars has an atmosphere of considerable density, since small stars are obscured as they

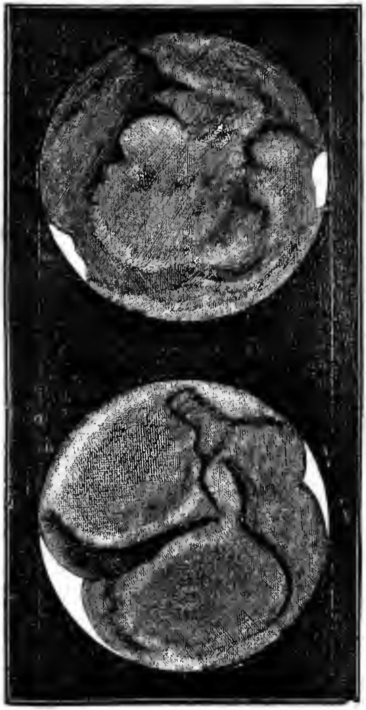


Fig. 68.

approach its disc. The existence of snow near the poles proves that there must be aqueous vapor in the atmosphere of Mars; and the existence of the aqueous vapor goes to prove that there are seas on the surface of the planet, as is also indicated by the greenish spots.

Figure 68 shows the white spots, supposed to be masses of snow, and also the markings on the disc. It will be seen that the spots are not exactly at the poles.

137. *The Inner Group of Planets.* — We now see that the four planets, Mercury, Venus, the Earth, and Mars, form a group of planets with certain resemblances. They all, so far as known, have an axial rotation of about twenty-four hours; all have atmospheres; they differ comparatively little in size, and are all small in comparison with another group with which we shall soon become acquainted; and only one of them, the earth, has a satellite.

SUMMARY OF THE INNER GROUP OF PLANETS.

The first planet of this group is *Mercury*. He revolves about the sun in about three months, and rotates on his axis in about twenty-four hours. (102.)

The orbit of Mercury lies wholly within that of the earth. His diameter is somewhat less than one half that of the earth. He presents *phases* like the moon. (103.)

These phases are owing to the fact that Mercury shines by reflected light. (104.)

Schröter thought he detected spots on the disc of Mercury which indicate the presence of mountains. By observing these he decided that this planet rotates on its axis in about twenty-four hours. (105.)

The existence of twilight shows that Mercury has an atmosphere. (106.)

The second planet of this group is *Venus*. Her diameter is a little less than that of the earth. She completes a revolution round the sun in about $7\frac{1}{2}$ months, and presents *phases* like Mercury. (107.)

Schröter detected slight irregularities in the terminator of Venus, which indicate the existence of mountains. By observing certain spots he concluded that Venus rotates on her axis in about twenty-four hours. Venus also has an atmosphere. (108.)

She is believed to have no satellite. (109.)

The next and largest planet of this group is the *earth.* This planet is attended by one satellite. (113.)

The last planet of the inner group is *Mars.* His orbit lies wholly without that of the earth. He revolves about the sun in a period of somewhat less than two years. The *diameter* of Mars is a little more than half that of the earth. Mars often appears somewhat *gibbous.* (135.)

His disc presents outlines of seas and continents similar to those which exist on the earth, and white spots near the poles, which are thought to be patches of snow. Mars resembles the earth in his change of seasons. He also has an atmosphere of considerable density. (136.)

The four planets, Mercury, Venus, the Earth, and Mars, have well-marked resemblances. They constitute what may be called the *Inner Group of Planets.* (137.)

The *Zodiacal Light* (111, 112) and *Meteoric Rings* (133, 134) lie within the limits of this group.

THE MINOR PLANETS.

138. *Bode's Law.* — Between the orbit of Mars and that of Jupiter, the next of the planets known to the ancients, there is an interval of 350,000,000 miles, in which no planet was known to exist before the beginning of the present century.

Three hundred years ago, Kepler had pointed out something like a regular progression in the distances of the planets as far as Mars, which was broken in the case of Jupiter, and he is said to have suspected the existence of another planet in the great space separating these two bodies. The question attracted little further attention until Uranus was discovered by Sir William Herschel in 1781, when several German astronomers revived the opinion held by Kepler, and, guided by a law of planetary distances published by Professor Bode of Berlin, even approximated to the period of the supposed latent body. According to this law, the distance of a planet is about double that of the next interior one, and half that of the next exterior one, and, roughly speaking, this rate of progression of the planetary distances is found to hold good with this exception. Mars is situated at a distance about twice that of the earth, but very much less than half that of Jupiter; and again, Jupiter revolves at half the distance of the next exterior planet, Saturn, but considerably more than twice that of Mars. If, therefore, another planet existed between Mars and Jupiter, the progression of Bode's law, instead of being interrupted at this point, might perhaps be found to hold good as far as Uranus. For this reason, an association of astronomers was formed, and a regular plan of search was devised with a view to the discovery of the suspected planet.

139. *The Discovery of Ceres.* — Professor Piazzi, Director of the Observatory at Palermo, repeatedly sought for a star numbered in Wollaston's Catalogue, but finding none in the position there assigned, he observed all the stars of similar brightness in the vicinity. On the 1st of January, 1801, or about the time the search for the supposed body was begun, he determined the place of an object shining as a star of the eighth magnitude not far from the position of the missing one. On the fol-

lowing night the place of this star was sensibly altered. Piazzi regarded this object as a comet, and announced its discovery as such on the 24th of January. On the publication of the whole series of positions observed at Palermo, Professor Gauss of Göttingen undertook the determination of the orbit of Piazzi's star, and announced that it revolved round the sun at a mean distance of 2.7 times the distance of the earth. This distance agreeing so closely with that indicated by Bode's law for the planet supposed to exist between Mars and Jupiter, astronomers were very soon led to regard Piazzi's comet as in reality a new planet, fulfilling, in a remarkable manner, the condition, in respect to distance from the sun, which had been found to hold good for the other members of the planetary system. This new planet was named *Ceres*. Its minuteness has prevented any exact determination of its diameter. Sir William Herschel's measurement makes it one hundred and sixty-three miles in diameter. Observers have remarked a haziness surrounding the planet, which is attributed to the density and extent of its atmosphere. Ceres is generally just beyond the range of unaided vision, though it has been seen without the telescope.

140. *The Discovery of Pallas.* — In order to find Ceres more readily, Dr. Olbers examined minutely the configuration of the small stars lying near her path. On the 28th of March, 1802, after observing the planet, he swept with his telescope over the north wing of Virgo, and was astonished to find a star of the seventh magnitude, where he was certain no star was visible in January and February preceding. In the course of three hours he found that the right ascension and declination of the star had changed. On the following evening he found the star had moved considerably, and he became convinced that it was a planet. He named this new body *Pallas*. Its orbit was

soon determined by Professor Gauss, who found that its most remarkable peculiarity consisted in the great inclination of its plane to the ecliptic. This inclination is 34°, while that of Mercury, which is the greatest among the larger planets, is only 7°. Its mean distance from the sun was found to be nearly the same as that of Ceres.

Dr. Olbers showed that the orbits of the newly discovered planets approached very near each other at the ascending node of Pallas, a circumstance which led him to make his remarkable conjecture as to the common origin of these bodies. He thought that a much larger planet had, in remote ages, existed near the mean distance of Ceres and Pallas; that, by some tremendous catastrophe, this body had been shattered; and that the two small planets were among the fragments. This hypothesis seemed to be materially strengthened by subsequent discoveries, though it is now generally admitted to be without foundation. The diameter of Pallas seems to be about 600 miles.

141. *The Discovery of Juno.* — In 1804 Professor Harding, of Lilienthal, while preparing a chart of the small stars lying near the paths of Ceres and Pallas, with a view to assist the identification of these minute bodies, discovered a small object which he recognized as a planet by its movement. This planet he named *Juno.*

142. *The Discovery of Vesta.* — Dr. Olbers, following up his idea respecting the origin of this zone of planets, considered, from the fact that the orbits of the three already found intersect one another in Virgo and Cetus, that the explosion must have taken place in one or the other of those regions, and consequently that all the fragments should pass through them. Provided with an ordinary night glass, he examined every month the small stars of Virgo and of Cetus, according as the one or the other of these constellations was the more favorably situated

for observation. In 1807 he discovered a small star in Virgo, where there had been none on a previous examination, and he soon satisfied himself that the star was really in motion, and thus recognized it as a planet. This planet was named *Vesta.* Its diameter is about 300 miles.

Dr. Olbers continued his systematic examination of the small stars of Virgo and Cetus between the years 1808 and 1816, and was so closely on the watch for a moving body, that he considered it highly improbable that a planet could have passed through either of these regions in the interval without detection. No further discovery being made, the plan was relinquished in 1816.

143. *More Recent Discoveries of Minor Planets.* — After Dr. Olbers discontinued his search for planets, the subject appears to have attracted little attention until Hencke, an amateur astronomer, took up the search with a zeal and diligence which could hardly fail in producing some important result, and he was rewarded in 1845 by the discovery of a fifth planet, which was named *Astræa,* and in 1847 by the discovery of *Hebe.* Since 1845 the discovery of these bodies has been very frequent, and their number has now reached 95.

144. *The Group of Minor Planets.* — There is, then, between Mars and Jupiter, a group of minute planets, spread through a zone some 50,000,000 miles in diameter. Their orbits are generally more flattened than those of the larger planets, and their planes are more inclined to the ecliptic; though they differ greatly in this respect, since the orbits of some of them nearly coincide with the ecliptic. There is good reason to suppose that many more will be discovered.

These planets are often called *asteroids* (*star-like* bodies).

SUMMARY OF THE MINOR PLANETS.

Kepler suspected the existence of an unknown planet between Mars and Jupiter. The discovery of Uranus in 1781 led the German astronomers to undertake a systematic search for this suspected body. They were guided by Bode's law of planetary distances. (138.)

In 1801 Piazzi discovered *Ceres*, but supposed it to be a comet. Professor Gauss showed it to be a planet, and that it is at about the distance from the sun required by Bode's law. (139.)

In 1802 Dr. Olbers discovered *Pallas*. The marked peculiarity of this planet is the *great inclination of its orbit* to the plane of the ecliptic. Since near the ascending node of Pallas the orbit of this planet and of Ceres very nearly coincide, Dr. Olbers was led to believe that Ceres and Pallas were the *fragments of a broken planet*. (140.)

In 1804 *Juno* was discovered by Harding. (141.)

Dr. Olbers now began a systematic search for other fragments of his broken planet. In 1807 he discovered *Vesta*. He continued his search till 1816 without further fruit. (142.)

No more of these minute planets were discovered till 1845, when Hencke discovered *Astræa*. In 1847 he discovered *Hebe*. Since that time these bodies have been discovered with considerable frequency. Their number now reaches 95. Astronomers do not now believe that these bodies are fragments of a broken planet. (143.)

The Minor Planets form a well-marked group. (144.)

JUPITER.

145. *Its Distance, Period, Size, etc.* — From the regions of space where we have just seen the smallest members of our system moving in their orbits, we pass abruptly to a group of planets of a very different order. The first planet of this group is *Jupiter.*

To the naked eye, Jupiter appears as a star of the first magnitude, the brightness of which is sometimes sufficient to cast a shadow. It is the most brilliant of the planets except Venus.

The mean distance of this planet from the sun is 496,000,000 miles. He moves in an orbit which differs considerably from a circle, so that his distance from the sun at aphelion is about 48,000,000 miles greater than at perihelion. Of course the difference of his distance from the earth at conjunction and opposition is still greater. He completes a revolution in about twelve years.

The plane of Jupiter's orbit very nearly coincides with that of the ecliptic, being inclined to it by an angle of a little over 1°.

The diameter of Jupiter is about 89,000 miles, or about eleven times the diameter of the earth. The bulk, therefore, of this immense planet is about 1400 times that of our globe. If seen at the distance of our satellite, his disc would cover a space in the sky 1200 times that occupied by the disc of the full moon. Yet this mass is travelling through space with a velocity about eighty times that of a cannon ball.

Jupiter, like the earth, is not a perfect sphere, but an ellipsoid flattened at the poles. This flattening is much greater than in the case of the earth.

As has already been stated (77), Jupiter is accompanied by four satellites. These appear in the telescope as so many points of light oscillating in short periods across the planet.

146. *Its Physical Characteristics.* — On examining the surface of Jupiter with a telescope, we see no appearance of regular continents or seas, as on the surface of Mars; but dark streaks, or *belts*, are found to cross his disc, presenting some of the modifications of clouds in our own atmosphere. Occasionally these belts retain nearly the same form and position for months together, while at other times they undergo great and sudden changes, and, in one or two instances, have been observed to break up and spread themselves over the whole disc of the planet. Generally there are two belts much more strongly marked than the rest, and more nearly permanent in their character, one situated a little north and the other a little south of the planet's equator. The prevailing opinion among astronomers is that these phenomena are produced by disturbances in the planet's atmosphere, which occasionally render its dark body visible; and as the belts are found to traverse the disc in lines uniformly parallel to Jupiter's equator, (see Figure 69,) we are naturally led to the conclusion that these disturbances are connected with the rotation of the planet on its axis, in the same way that the trade winds on the earth are connected with its rotation on its axis.

In July, 1665, Cassini, of Paris, remarked a black spot of considerable apparent magnitude on the upper edge of the southern belt of Jupiter, which remained visible two years. This spot, or one supposed to be identical with it, has repeatedly appeared since that time, but at very irregular intervals.

In 1834, a remarkable spot was discovered on the northern belt. It was black and well defined; about two thirds of its breadth was above the belt, and one third upon it.

Fig. 69.

Shortly after this spot was first noticed a second distinct spot was discovered on the same belt. These spots remained wholly unchanged for nearly a year. Cassini noticed that the spot of 1665 appeared to traverse the disc of the planet from east to west. It was very conspicuous near the centre of the disc, but gradually faded away as it approached the western limb. The motion seemed quickest when the spot was near the centre, and became slower towards the edge of the planet. Hence he inferred that the spot adhered to the surface of the planet, and was carried across the disc by the rotation of Jupiter upon his axis. This hypothesis would account fully for the appearances observed. By closely watching the movements of the spot, Cassini ascertained that the time of rotation of the planet was 9 hours 56 minutes.

By careful observations of the spots of 1834 the time of rotation was found to be about half a minute less. This enormous globe, whose diameter is eleven times greater than that of the earth, is therefore whirled upon its axis in less than ten hours. The axis of rotation is very nearly perpendicular to the plane of the orbit.

147. *The Satellites of Jupiter.* — These bodies were discovered by Galileo in 1610. They shine with the brilliancy of stars between the sixth and seventh magnitude; but owing to their proximity to the planet they are invisible to the naked eye.

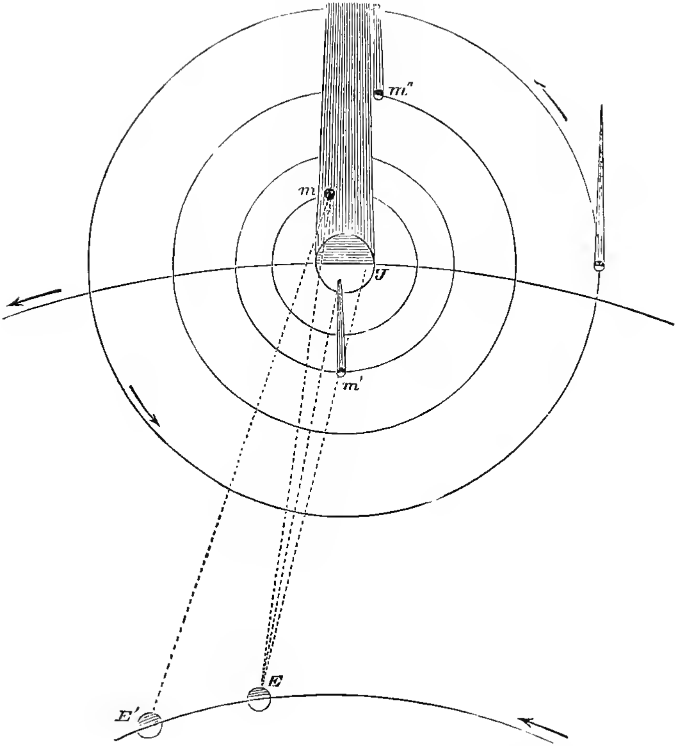
Their configuration is continually changing. Sometimes they are all situated on one side of the planet, though more often one at least is found on each side. In very rare instances all four have been invisible, as on the 21st of August, 1867, when the planet appeared thus unattended for one hour and three quarters. It is not a rare occurrence to find only one satellite visible.

Sir William Herschel, from a long series of observations on the satellites, concluded that they rotate on their axis in the time of one synodical revolution around Jupiter, thus presenting an analogy to our own satellite. He was led to this conclusion on remarking the great changes in the relative brightness of the satellites in different positions, which were found to follow such a law as could be reconciled only with this hypothesis.

148. *The Eclipses of these Satellites.* — If Jupiter were a self-luminous body, the satellites would disappear only when they pass behind him. But they disappear at other times and in such a way as to show that the planet must be an opaque, non-luminous body. This will be seen by reference to Figure 70.

If Jupiter be non-luminous, he will cast a shadow directly away from the sun, as shown in the figure. The satellites will then disappear, not only when they pass behind the planet (as in the case of the satellites *m* and *m''* when the earth is at *E*), but also when they pass through the shadow of the planet (as in the case of *m* when the earth is at *E'*); and this they are found to do.

Fig. 70.



When they disappear behind the planet they are *occulted;* when they pass through the shadow they are *eclipsed.* The entrance of the satellite into the shadow is called its *immersion*, and its exit from the shadow its *emersion.* The shadow is sometimes so projected that both the immersion and emersion of the satellite can be seen, and at other times so that only one of them can be seen. Three of the satellites are totally eclipsed at every revolution, while the fourth is often only partially eclipsed, or not eclipsed at all, since its orbit is inclined to the orbit of Jupiter by a greater angle than that of the others.

When a satellite passes between us and Jupiter it makes a *transit* across his disc. As seen by the figure the shadow of the satellite, as *m'*, is often projected on the disc at a different place from the satellite itself. The shadow always appears as a round black spot upon the disc, while the satellite usually appears as a bright spot, often brighter than the general disc of Jupiter. They have, however, been observed as dark spots, a phenomenon which can be accounted for only by supposing that such spots really exist on the satellites themselves, for their illuminated face must be turned towards us at the time.

SATURN.

149. *Distance, Period, Size, etc.* — The next planet in the order of distance from the sun is *Saturn*, who performs his revolution in about 29½ years, at a mean distance of 909,000,000 miles from the sun, in an orbit considerably flattened, and inclined to the ecliptic at an angle of about 2½ degrees.

The diameter of Saturn is found to be about 73,000 miles, or about nine times that of the earth. The bulk of Saturn is consequently about eight hundred times that of the earth.

Though belts are frequently observed with good telescopes on the surface of Saturn, they are much less distinct than those of Jupiter. Spots are of rare occurrence. One was seen by Sir William Herschel in 1780 for several days, and another quite distinct was seen in 1847.

In 1793, Sir W. Herschel saw a quintuple belt. By very frequent and careful examination of the appearance of this belt he ascertained that the time of rotation of Saturn is a little over ten hours.

The axis of rotation is inclined to the plane of the planet's orbit at an angle of about 63°. His seasons are therefore more diversified than those of Jupiter.

150. *The Satellites and the Rings of Saturn.* — Though Saturn is smaller than Jupiter, his system is far more complicated. He is attended by eight satellites. While the satellites of Jupiter are known respectively as the *first*, *second*, *third*, and *fourth*, those of Saturn have mythological names. Their names in the order of distance are Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion, and Japetus. Of these Titan is the largest. But the most interesting feature of the Saturnian system is his *rings*.

When Galileo turned his newly constructed telescope upon Saturn, he saw that the figure of the planet was not round as in the case of Jupiter. At first he thought it to be oblong, but on further examination he concluded that the planet consisted of a large globe, with a smaller one on each side of it. Continuing his observation he remarked that this appearance was not constantly the same, the appendages on each side of the central globe gradually diminishing until they vanished entirely, and left the planet nearly round, without anything extraordinary about it. He therefore concluded that he had been mocked by an optical illusion.

Huyghens, who possessed telescopes of greater power than those of the Italian astronomer, was the first who gave a correct explanation of these varied appearances, and detected a luminous ring surrounding the globe of Saturn. In 1675, Cassini discovered a division separating the ring into two concentric rings. This division had been detected ten years previously by two English amateurs.

The ring of Saturn may be described as broad and flat, situated exactly in the plane of the planet's equator, and consequently inclined to the ecliptic at an angle of about 28°. It keeps this same inclination throughout the revolution of Saturn. The plane of the ring therefore intersects the ecliptic. It is owing to this inclination that the ring is sometimes observed as a broad ellipse,

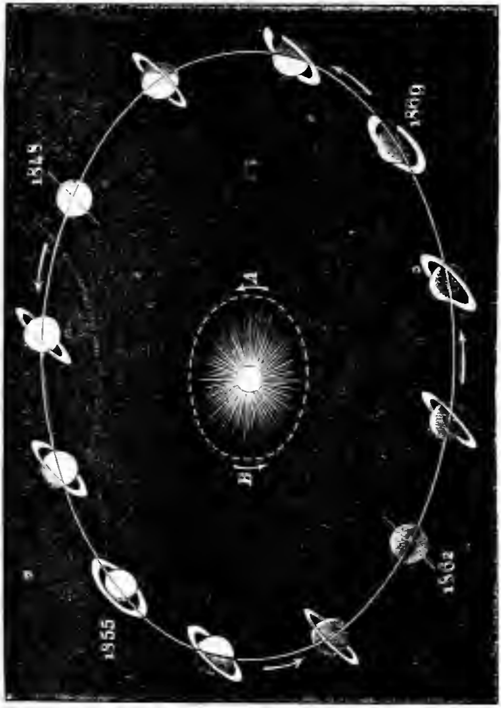


Fig. 71.

at other times as a straight line barely discernible with the most powerful telescopes, and that at other times the ring entirely disappears. These phases of the ring will be understood by reference to Figure 71.

In two positions of the planet the plane of the ring is seen to pass through the centre of the sun, and of course only its edge is illumined. Now the ring is estimated to be about one hundred miles thick. This thickness, at the distance of Saturn, would subtend an angle just about equal to that of a good-sized pin at the distance of two miles. Hence in these two positions the ring will appear as a line of light discernible only with the most powerful telescopes. Suppose that some time before the ring came into the position marked 1848 the earth was at *A*: then the plane of the ring would pass between the earth and the sun, and the unillumined side of the ring would be turned towards us, and the ring would of course disappear. So, too, if some time after the ring came into this position the earth were at *B*, the plane of the ring would again pass between the earth and the sun, and the ring would again disappear. At the positions marked 1855 and 1869, the ring would appear as a broad ellipse.

More recent observations go to show that the two divisions of Saturn's ring are further subdivided, so as to constitute a *multiple* ring.

The most recent, and at the same time one of the most remarkable discoveries with reference to the rings of Saturn, is that of a dusky or obscure ring, nearer to the planet than the intensely bright one. This dusky ring seems to have been first noticed by Dr. Galle of Berlin, but this notice attracted but little attention till 1850, when the phenomenon was remarked by Prof. Bond, of Cambridge, Mass., and by Mr. Dawes, of England. The latter detected a division of the obscure ring.

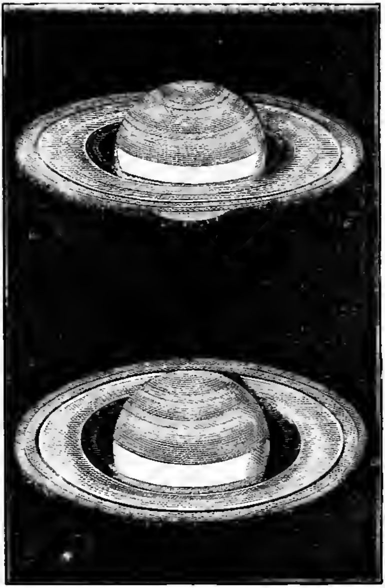
This newly discovered ring is quite transparent, allowing the disc of the planet to be seen through it.

Fig. 72 represents Saturn with his system of rings.

When the ring last appeared, as a mere line, in 1861 and 1862, singular appendages, like clouds of less intense light, were noticed lying on each side of the ring.

The ring of Saturn has been supposed to be solid, and

Fig. 72.



to be liquid; and able mathematicians have in turn demonstrated that it can be neither. It is now held by some that it is composed of innumerable little satellites revolving about the planet, in the plane of his equator.

The system of rings seems to be increasing in breadth at the rate of about twenty-nine miles a year.

URANUS.

151. *Its Discovery.*—Previous to the year 1781, the only planets known, beside the one we inhabit, were Mercury, Venus, Mars, Jupiter, and Saturn. All of these are more or less conspicuous to the naked eye, and were recognized

from the earliest antiquity as wandering bodies. Saturn was supposed to be the most distant member of the solar system, and very little suspicion of the existence of an exterior planet was entertained. The close examination of the heavens, begun by Sir William Herschel in 1781, led to a discovery which more than doubled the area of our system.

On the 13th of March, 1781, while exploring with his telescope the constellation of the Twins, he observed a star, the disc of which attracted his attention. Perceiving, after a few nights of observation, that the new body moved, he at first mistook it for a comet; but it soon became evident that it was a new planet, outside the orbit of Saturn.

152. *Its Period, Distance, Size, etc.*—This planet may just be discerned by a person of very good eyesight, without a telescope. Uranus revolves about the sun in a period of about eighty-four years, at a mean distance of 1,828,000,000 miles, in an elliptical orbit, whose plane is inclined to the ecliptic at an angle of less than 1°. Its diameter is about 36,000 miles.

No telescopes hitherto constructed have succeeded in showing any spots or belts upon this planet, owing to its enormous distance and the consequent minuteness of its disc. The time of its rotation, and the position of its axis with respect to the plane of its orbit, are, therefore, unknown, and are likely to remain so.

153. *Its Satellites.*— Sir William Herschel thought he had detected six satellites of this planet, but it is now pretty well established that there are but four. It is a curious fact that the satellites of Uranus, unlike those of the earth, Jupiter, and Saturn, revolve in a *retrograde* direction,—that is, from east to west,—and that their orbits are inclined at a very large angle to that of the planet.

NEPTUNE.

154. *Its Period, Distance, Size, etc.* — The next planet in the order of distance from the sun, and, so far as we know, the last of the solar system, is Neptune. He revolves around the sun in a period of 164 years, at a mean distance of 2,862,000,000 miles, in an orbit inclined to the ecliptic at an angle of a little less than 2°. His diameter is about 35,000 miles. No spots can be detected on his disc, and consequently nothing is known about his time of rotation or the inclination of his axis. He is certainly attended by one satellite, which, like those of Uranus, moves in a retrograde direction.

155. *Its Discovery.* — Though we know so little of this most distant member of our system, yet the circumstances of its discovery give it an enduring interest.

It will be shown, further on, that the planets, by their mutual action, disturb one another's orbits to a slight extent, so that none of them describe exact ellipses. It had been noticed for many years that the motion of Uranus was not exactly what it was calculated it should be, after making allowance for all known causes of disturbance. Two young mathematicians, M. Le Verrier, of France, and Mr. Adams, of England, were led, unknown to each other, to inquire into the cause of this apparent anomaly, and both soon came to the conclusion that a planet of considerable magnitude must exist outside the orbit of Uranus. Their next object was to ascertain the position of the planet amongst the stars, with a view to its actual discovery in the telescope; but the problem to be solved was one of excessive difficulty, — so much so, in fact, that several of the most eminent astronomers had declared their conviction that the place of the latent planet could never be discovered by calculation. M. Le Verrier and Mr.

Adams were of a different opinion, and finally succeeded in their researches, which assigned nearly the same position to the body whose influence had been so visibly exercised on the movements of Uranus. Mr. Adams, however, did not make his conclusions public through the press, and much of the first glory of this great discovery was consequently given to the French astronomer, who had announced the position of the new planet to the Academy of Sciences at Paris in the summer of 1846. On the 23d of September, of the same year, Dr. Galle, of the Royal Observatory, Berlin, acting upon the urgent representations of M. Le Verrier, contained in a letter which reached Berlin at this date, turned the large telescope of the observatory to that part of the heavens in which M. Le Verrier had informed him he would find the disturbing planet. Hardly was this done when a pretty bright telescopic star appeared in the field of view, at a point where no such object was marked in a carefully prepared map of that part of the heavens. This proved to be the predicted planet, and the name *Neptune* was given to it by the common consent of M. Le Verrier, Mr. Adams, and the chief astronomers of Europe.

In calculating the position of this planet, they had assumed, according to Bode's law, that it would be about twice as distant as Uranus. But the mean distance of Neptune is found to be considerably less than double that of Uranus; hence this law, which led to the discovery of minor planets, and helped in the discovery of Neptune, has singularly enough been overthrown by these discoveries.

156. *The Outer Group of Planets.* — The third outer group of planets comprises the large bodies outside the ring of telescopic planets. Jupiter, Saturn, Uranus, and Neptune belong to this group. So far as known, they rotate in periods of about ten hours, and three of them at least are attended by a number of satellites. They have also a very slight density.

SUMMARY OF THE OUTER GROUP OF PLANETS.

The first and largest planet of this group is *Jupiter*. His *diameter* is about eleven times that of the earth. He completes *a revolution* in about twelve years, and is attended by *four satellites*. (144.)

The disc of Jupiter is crossed by a number of parallel *belts*, which sometimes resemble clouds. The most strongly marked of these are situated near the equator of the planet. They are probably due to a disturbance in Jupiter's atmosphere analogous to our trade winds. By observation of certain well-marked spots Jupiter has been found to *rotate on his axis* in a little less than ten hours. (145.)

The satellites of Jupiter were first discovered in 1610. They are *occulted* when they pass behind the planet; *eclipsed* when they pass into its shadow; and make *transits* across the planet's disc when they pass between it and the sun. At rare intervals Jupiter is seen without satellites. (146, 147.)

The next planet of this group is *Saturn*. He is not so large as Jupiter, his *diameter* being but nine times that of the earth. He completes a revolution in about 29½ years. He has *belts*, but they are less marked and less permanent than those of Jupiter. He is found to *rotate* on his axis in about ten hours. (148.)

Saturn is attended by *eight satellites*, and by a most remarkable *system of rings*. These rings are parallel to the plane of the planet's equator, and inclined to that of its orbit at an angle of about 28°. It is owing to this inclination that the rings sometimes appear as broad ellipses, and at other times as mere straight lines. The rings occasionally disappear entirely. Little is known of the physical constitution of these rings. They are certainly several in number, and appear to be slowly increasing in breadth. (149.)

Mercury, Venus, Mars, Jupiter, and Saturn are the only planets visible to the naked eye, and up to the year 1781, they were the only ones known to exist. In that year Herschel announced the discovery of a new planet, since named Uranus. (150.)

Uranus is about *twice as distant* as Jupiter, and his *diameter* is about five times that of the earth. He completes a *revolution* in about 84 years, but nothing is known about his *rotation* on his axis. (151.)

He is attended by at least *four satellites*, all of which have a *retrograde* motion. (152.)

The last member of this group, and, so far as we know, of our planetary system, is *Neptune*. His *disc* is about the same as that of Uranus. He completes a *revolution* in about 164 years, while nothing is known about his *rotation*. Neptune is attended by *one satellite*, which has a *retrograde* motion. (153.)

A peculiar interest attaches itself to this remote body, owing to the circumstances of its discovery. (154.)

The planets, so far as known, all rotate on their axes from *west to east*. They also revolve about the sun from west to east, while the satellites, with the exception of those of Uranus and Neptune, revolve about their primaries from west to east. The sun and moon also rotate from west to east.

COMETS.

157. There is another group of well-known bodies called *Comets*, which differ in so many respects from the planets that they seem hardly to belong to the same system.

These bodies are observed only in those parts of their orbits which are nearest to the sun. They are not confined, like the larger planets, to the zodiac, but appear in every quarter of the heavens, and move in every possible direction. They usually continue visible a few weeks or months, and very rarely so long as a year. Their appearance, with some few exceptions, is nebulous or cloud-like, whence it is inferred that they consist of masses of vapor, though in a highly attenuated state, since very small stars are often seen *through* them.

The more conspicuous comets are accompanied by a *tail*, or train of light, which sometimes stretches over an arc of the heavens of 50° or upwards, but more frequently is of much less extent.

The same comet may assume very different appearances during its visibility, according to its position with respect to the earth and sun. When first perceptible, a comet resembles a little spot of faint light upon the dark ground of the sky; as it approaches the sun its brightness increases, and the tail begins to show itself. Generally the comet is brightest when it arrives near its perihelion, and gradually fades away as it recedes from the sun, until it cannot be seen with the best telescopes we possess.

Some few have become so intensely brilliant as to be seen in *full daylight*. A remarkable instance of this kind occurred in 1843, when a comet was discovered within a few degrees of the sun himself; and there are one or two similar cases on record.

The brighter or more condensed part of a comet, from which the tail proceeds, is called the *nucleus;* and the nebulous matter surrounding the nucleus is termed the *coma;* frequently the nucleus and coma are included under the general term *head*. Some comets have no nuclei, their light being nearly uniform.

The tail of a comet is merely a prolongation of the nebulous envelope surrounding the nucleus, and it almost always extends in a direction opposite to that of the sun at the time. In some cases it is long and straight; in others, curved near the extremity, or divided into two or more branches. A few comets have exhibited two distinct tails. The real length of this train has sometimes exceeded 100 or 150 millions of miles; that of the great comet of 1843 is said to have been 200 millions of miles long.

Comparatively few of the many comets that visit our system are visible to the naked eye. Most of them are faint filmy masses, without tails, which can be seen only with the telescope.

It is supposed that the general form of the orbits of these bodies is a highly elongated ellipse.

158. *Halley's and other famous Comets.* — Astronomers have ascertained with great precision the periods which certain comets require to perform their revolutions round the sun, and are able to predict the times of their reappearance, and their paths among the stars. This was first done by Dr. Halley, in the case of the comet observed in 1682, which he discovered to be the same that had appeared in 1456, 1531, and 1607, and hence concluded that its revolution is accomplished in about seventy-five years. He foretold its reappearance in 1759, which actually took place after a retardation of between one and two years. The same body appeared again in 1835, and will again visit our solar system about the year 1911. It may be traced in history as far back as the year 11 B. C.

A comet called *Encke's* has a period of $3\frac{1}{2}$ years; another, *Biela's*, of $6\frac{3}{4}$ years; and several others perform their revolutions in from five to eight years.

There are a few comets, besides the ones above mentioned, which complete their journey round the sun in from sixty to eighty years; but it is certain that by far the greater number require hundreds or even thousands of years to perform their revolutions. When this is the case, it becomes almost impossible to assign their exact periods.

Remarkable comets appeared in 1680 and 1843, both of which approached so near to the sun as almost to *graze his surface.* The comet of 1811 has acquired great celebrity. It remained visible to the naked eye several months, shining with the lustre of the brighter stars, and attended by a beautiful fan-shaped tail. This body is supposed to require upwards of 3000 years to complete its excursion through space.

The splendid comet of 1858, generally known as *Donati's*, will long be remembered for the remarkable physical appearances it presented in the telescope, as well as on account of its imposing aspect to the naked eye. It is presumed to have a period of revolution of about 2100 years.

Hardly less famous in future times will be the grand comet which appeared in 1861. This comet had a tail 100 degrees in length. Its period of revolution would appear to be much shorter than that of Donati's comet, probably not exceeding 450 years.

It is probable that there are many thousands of comets belonging to the solar system, of which a large proportion never come sufficiently near the sun to be seen from the Earth.

SUMMARY.

The comets are a group of bodies quite unlike the planets. They are visible only when near the sun. They appear in every quarter of the heavens, and move in every possible direction.

The largest comets consist of a *nucleus*, a *coma*, and a *tail*. Their trains are often of great length. A comet usually changes its appearance considerably during its visibility.

Comparatively few comets are visible to the naked eye. (157.)

Several of the comets are known to return to the sun at intervals of greater or less length.

Some of the most famous comets are *Halley's*, which has a period of about seventy-five years; *Encke's*, whose period is about three and a half years; those of 1680 and 1843, which approached very near the sun; that of 1811, which remained visible several months, and was attended by a beautiful fan-shaped tail; *Donati's*, which is thought to have a period of about 2100 years; and that of 1861, noted for the splendor of its train. (158.)

THE FIXED STARS.

159. We have already seen (78) that the stars are not absolutely fixed: many of them are known to be moving at the rate of several miles a second. It is only owing to their immense distance from us and from one another that their configurations do not appear to change. The most noticeable feature of the fixed stars is their scintillation or *twinkling*, which contrasts so strongly with the steady light of the principal planets. This twinkling is an optical phenomenon, supposed to be due to what is termed the *interference* of light. Humboldt states that in the pure air of Cumana, in South America, the stars do not twinkle after they attain an elevation, on the average, of 15° above the horizon. Hence we must conclude that the twinkling of the stars is due to atmospheric conditions.

160. *Their Number.* — The actual number of stars visible to the naked eye, at the same time, on a clear, dark

night, is between two and three thousand, though a person forming an estimate of their number from casual observation is almost certain to make it very much larger. It is a well-ascertained fact that, *in the whole heavens*, the stars which can be distinctly seen without the telescope, by any one gifted with good sight, do not exceed six thousand.

The telescopic stars are innumerable. It has been conjectured that more than *twenty millions* might be seen with one of Herschel's twenty-feet reflectors; and if we could greatly increase the power of our telescopes, there is no doubt that the number actually discernible would be vastly augmented.

161. *Magnitudes.* — The stars are divided, according to their degrees of brightness, into separate classes, called *magnitudes.* The most conspicuous are termed stars of the *first* magnitude: there are about twenty of these. The next in order of intensity of light are stars of the *second* magnitude, which are about fifty or sixty in number. Of the *third* magnitude there are two hundred or upwards, and many more of the *fourth*, *fifth*, and *sixth*. These six classes comprise all the stars that can be well seen with the naked eye on a clear night. Telescopes in common use will show fainter stars to the *tenth* magnitude inclusive, while the powerful instruments in observatories reveal an almost infinite multitude of others, even down to the *eighteenth* or *twentieth* magnitudes.

This classification of the stars is, to a great degree, arbitrary, so that it is not unusual to find astronomers differing greatly in their estimates of brightness.

162. *Constellations.** — For the sake of more readily distinguishing the stars, and referring to any particular quarter of the heavens, they have been arranged into groups called *constellations*, each named from some object to which the configuration of its stars may be supposed to

* See Appendix, VI.

bear a resemblance. Many are figures or heroes, birds, and animals, connected with the fables of classical mythology. This mode of grouping the stars is of very ancient date.

There are twelve constellations in the *zodiac* (a belt of the heavens extending 8° on each side of the ecliptic, and including the paths of all the larger planets), and hence called the *zodiacal constellations*, viz. Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces. These are also the names of the twelve *signs*, or divisions of 30° each, into which the zodiac was formerly divided; but the effect of precession (20), which throws back the place of the equinox among the stars from year to year, prevents a constant agreement between these constellations and the corresponding signs.

163. *Names of the Stars.* — Many of the brighter stars had proper names assigned to them at a very early date, as *Sirius*, *Arcturus*, *Rigel*, *Aldebaran*, etc., and by these names they are still commonly distinguished.

The stars are now usually designated by the letters of the Greek alphabet, to which the genitive of the Latin name of the constellation is added. The brightest star is called α (alpha), the next β (beta), and so on. Thus, Aldebaran is termed α Tauri; Rigel, β Orionis; Sirius, α Canis Majoris.

164. *The Color of the Stars.* — Many of the stars shine with colored light, as red, blue, green, or yellow.

These colors are exhibited in striking contrast in many of the double stars. Combinations of blue and yellow, or green and yellow, are not uncommon, while in fewer cases we find one star white and the other purple, or one white and the other red. In several instances each star has a rosy light.

The following are a few of the most interesting colored double stars: —

Name of star.	Color of larger one.	Color of smaller one.
γ (gamma) Andromedæ,	Orange,	Sea-green.
α Piscium,	Pale green,	Blue.
β Cygni,	Yellow,	Sapphire-blue.
η (eta) Cassiopeiæ,	Yellow,	Purple.
σ (sigma) Cassiopeiæ,	Greenish,	Fine blue.
ζ (zeta) Coronæ,	White,	Light purple.
A star in Argo,	Pale rose,	Greenish blue.
A star in Centaurus,	Scarlet,	Scarlet.

Single stars of a fiery red or deep orange color are common enough. Of the first color may be mentioned Aldebaran, Antares, and Betelgeuse. Arcturus is a good example of an orange star. Isolated stars of a deep blue or green color are very rarely found.

It is now a well-established fact that the stars change their color. Sirius was described as a fiery red star by the ancients. Some years ago it was pure white, while it is now becoming of a decided green color. Capella was also called a red star by the ancients; it was afterwards described as a yellow star; and is now bluish. Many other instances of change of color, though less decided, have been detected.

165. *Variable Stars.* — Besides those stars which are known to undergo changes of color, there are many stars, not only among those visible to the naked eye, but also belonging to telescopic classes, which exhibit periodical changes of brilliancy. These are called *variable* stars.

Algol, or β (beta), in the constellation Perseus, is one of the most interesting of the variable stars. For about $2^d\ 13^h$ it shines as an ordinary star of the second magnitude, and is therefore conspicuously visible to the naked eye. In somewhat less than four hours it diminishes to the fourth magnitude, and thus remains about twenty minutes; it then as rapidly increases to the second, and continues so for another period of $2^d\ 13^h$, after which similar changes recur. The exact period in which all these variations are performed is $2^d\ 20^h\ 48^m\ 55^s$.

Another remarkable star of this kind is *o* (omicron) in Cetus, often termed *Mira*, or the *wonderful* star. It goes through all its changes in 334 days, but exhibits some curious irregularities. When brightest it usually shines as a star of the second magnitude, yet on certain occasions has not appeared brighter than the fourth. Between five and six months afterwards it disappears altogether. Sometimes it will shine without perceptible change of brightness for a whole month; at others there is a very sensible alteration in a few days. Its variability was discovered in the seventeenth century.

The list of variable stars visible to the naked eye is pretty numerous. Among these are the following:—

δ (delta) Cephei,	goes through	its changes in	5 d.	9 h.
η Aquilæ,	"	"	7	4
α Herculis,	"	"	66	days.
A star in Aquila,	"	"	72	"
A star in Corona Borealis,		"	323	"
A star near χ (chi) Cygni,		"	406	"
30 Hydræ,	"	"	442	"

In some cases the periods extend to many years. 34 Cygni, a star whose fluctuations were noticed as long since as 1600, is supposed to complete its cycle of changes in about eighteen years.

The bright star Capella, in the constellation Auriga, is believed to have increased in lustre during the present century, while within the same period one of the seven bright stars (delta) in Ursa Major, forming *the Dipper*, has diminished. Many instances of a similar kind might be mentioned.

Telescopic variable stars are very numerous, and have lately excited much attention.

166. *Irregular or Temporary Stars.* — In the present state of our knowledge, it appears necessary to distinguish between the variable stars, properly so called, which go through their changes with some degree of regularity, and are either always visible or seen at short intervals, and those wonderful objects that have occasionally burst forth in the heavens with a brilliancy in some instances far surpassing the light of stars of the first magnitude, or even the lustre of Jupiter and Venus, remaining thus for a short time, and then gradually fading away. This latter class are called *irregular* or *temporary* stars.

The most celebrated star of this kind recorded in history is one which made its appearance in 1572, and attracted the attention of Tycho de Brahe, the Danish astronomer, who has left us a particular description of the various changes it underwent while it continued within view. It was situated in Cassiopeia, one of the circumpolar constellations, was first seen early in the autumn of 1572, and afterwards dwindled down, until it became so faint, in March, 1574, that Tycho could no longer perceive it. During the early part of its apparition it far surpassed Sirius, and even Jupiter, in brilliancy, and could only be compared to the planet Venus when she is in her most favorable position with respect to the earth. Persons with keen sight could see the star at noon-day; and at night it was discernible through clouds that obscured every other object. It twinkled more than the ordinary fixed stars; was first white, then yellow, and finally very red.

Another temporary star became suddenly visible in Ophiuchus, in 1604, and was observed by Kepler. Though somewhat inferior to Venus, it exceeded Jupiter and Saturn in splendor. Like Tycho's star, it twinkled far more than its neighbors, but was not characterized by successive changes of color; when clear from the vapors prevalent about the horizon, it was always white. This object remained visible till March, 1606, and then disappeared.

Other stars, evidently of the same class, are mentioned by historians in remote times. One of a less conspicuous character was discovered by Anthelme, in 1670, not far from β Cygni; and another in April, 1848, in the constellation Ophiuchus, which rose to the fourth magnitude, and has now faded away to the twelfth, so that it cannot be seen without a good telescope.

In May, 1866, a remarkable temporary star appeared in the constellation of Corona Borealis. At its greatest brilliancy it was somewhat above the second magnitude. It rapidly faded away and early in June was not above the ninth magnitude. It may be that these temporary stars are merely variable stars of long period.

167. *The Via Lactea, or Milky Way.* — The Via Lactea, Galaxy, or Milky Way, as it is variously termed, is that whitish luminous band of irregular form which is seen on a dark night stretching across the heavens from one side of the horizon to the other.

To the naked eye it presents merely a diffused milky light, stronger in some parts than in others; but when examined with a powerful telescope it is found to consist of myriads of stars, — of millions upon millions of suns, so crowded together that only their united light reaches the unassisted eye.

The general course of the Milky Way is in a great circle, inclined about 63° to the celestial equator, and intersecting it in the constellations Cetus and Virgo.

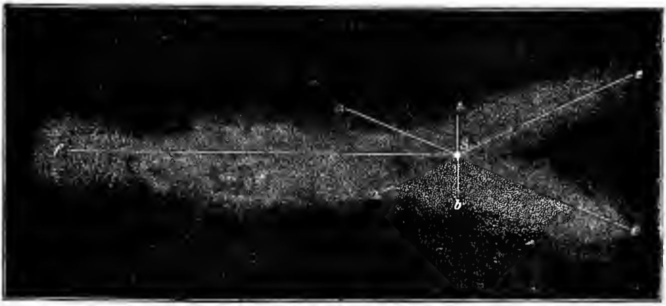
The distribution of the telescopic stars within its limits is far from uniform. In some regions several thousands (or as many as are seen by the naked eye on a clear night over the whole firmament) are crowded together within the space of a square degree: in others a few glittering points only are scattered on the black ground of the heavens. It presents in some parts a bright glow of light to the naked eye, from the closeness of the constituent stars; in others there are dark spaces with scarcely a single star upon them. A remarkable instance of the kind occurs in the broad stream of the Via Lactea, near the Southern Cross, where its luminosity is very considerable; but there exists in the midst of it a dark oval or pear-shaped vacancy, distinguished by the early navigators under the name of the *Coal-Sack.* Similar vacancies occur in the constellations Scorpio and Ophiuchus.

From the results of a numerical estimate of the stars at various distances from the circle of the Via Lactea, it has been proved that the stars are fewest in number near the poles of that circle, and increase — slowly at first, afterwards more rapidly — until we arrive at the Milky Way itself, where their number is greatest.

Hence it is inferred that the stars which cover our heavens are not uniformly distributed throughout space, but, as described by Sir John Herschel, "form a stratum, of which the thickness is small in comparison with its length and breadth." The solar system would appear to be placed somewhat to the northern side of the middle of its thickness, since the density of the stars is rather greater to the south than to the north of the plane of the Via Lactea. From these and other considerations, Sir William Herschel was led to regard our starry firmament as possessing in reality a form of which Figure 73 will convey some idea, one portion being subdivided into two branches slightly inclined to each other.

The earth being placed at *S* (not far from the point of divergence of the two streams), the stars in the direction of *b* and *b* would appear comparatively few in number,

Fig. 73.



but would increase rapidly as the line of vision approached *e*, *e*, or *f*, in which directions we should see them most densely crowded, the rate of transition from the poorer regions to those richest in stars being such as we have alluded to above.

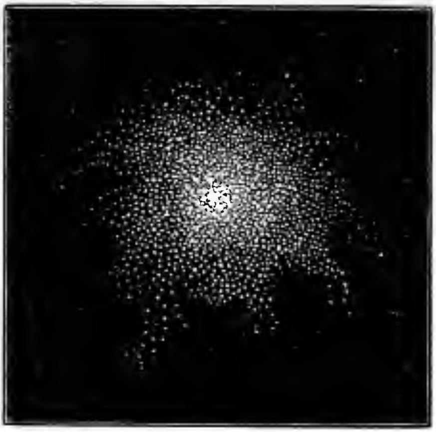
Near the intersection of the streams *e* and *e*, few stars would present themselves, and there would consequently be a dark space, or rather a space thinly covered with stars, included within the two branches of the Milky Way. This exactly represents the actual appearance of the heavens: the luminosity of the Milky Way does separate into two distinct streams of light, which remain thus over an arc of about 150°, and then unite again.

168. *Clusters of Stars and Nebulæ.* — On casting our eyes over the heavens on a clear dark evening, we at once perceive that in some directions the stars are clustered together, and in a few instances so compressed that the unaided eye cannot separate the members of the group, which assumes a hazy, undefined, or cloud-like appearance.

Among these close assemblages of stars may be mentioned the Pleiades in Taurus, Præsepe (popularly termed the Beehive) in Cancer, and a remarkable group in the sword-handle of Perseus, in which the stars are readily seen with a common night-glass, though the whole has a blurred aspect to the naked eye.

One of the most magnificent globular clusters in the northern hemisphere occurs in the constellation Hercules, between the stars *Eta* and *Zeta*. It is visible to the naked eye on dark nights as a hazy-looking object; and the stars composing it are readily seen with a telescope of moderate power. When examined with a powerful instrument, its aspect is grand beyond conception: the stars, which are coarsely scattered at the borders, come up to a perfect blaze in the centre. It is shown in Figure 74.

Fig. 74.



There is another cluster in the constellation Centaurus, of which Figure 75 is a representation. To the naked eye it appears like a nebulous or hazy star of the fourth magnitude; while in the telescope it is found to cover a space two thirds of the apparent diameter of the Moon, over which the stars are congregated in countless numbers.

Fig. 75.

169. *Nebulous Stars.* — The *nebulæ* have been already described (75). In some instances, a faint nebulosity, usually of a circular figure and several minutes in diameter, envelopes a star which is placed in or very near the centre. Such stars are called *nebulous* stars. There is nothing in their appearance to distinguish them from others entirely destitute of such appendages; nor can the nebulosity in which they are situated be resolved into stars with any telescopes hitherto constructed. As instances of nebulous stars, may be mentioned one of the fifth magnitude, numbered 55 in Andromeda, and one of the eighth magnitude on the borders of Perseus and Taurus, particularly pointed out by Sir William Herschel as a remarkakble object of this class.

170. *Variable Nebulæ.* — Nebulæ, as well as stars, are known sometimes to change in brilliancy. A remarkable case of this kind is that of a nebula situated near Epsilon

Tauri. At the time of its discovery in 1852 it was easily seen with a good telescope, whereas in 1861 and 1862 it was invisible with instruments of far greater power, thus proving that its light must have undergone very considerable diminution in the course of a few years. A small star close to this nebula likewise faded within the same lapse of time. No probable cause has yet been assigned for this variation in the brightness of a nebula.

171. *The Nubeculæ, or Magellanic Clouds.*—In the southern hemisphere, not far from the pole of the equator, are two nebulous clouds of unequal extent, the greater covering an area about four times that of the lesser one. They were termed *Magellanic clouds*, after Magellan the navigator, a name still in very common use, but on astronomical maps they are usually called the *nubeculæ*, *major* and *minor*.

Both these cloud-like masses are distinctly visible to the naked eye when the moon is absent; the smaller one, however, disappears in strong moonlight. Their light is white and diffused, resembling that of the Milky Way. Sir John Herschel examined these remarkable objects with his powerful instrument at the Cape of Good Hope, and describes them as consisting of swarms of stars, globular clusters, and nebulæ of various kinds, some portions being quite irresolvable, and presenting the same milky appearance in the telescope that the nubeculæ themselves do to the naked eye.

It is believed that the nubeculæ are independent of the Via Lactea, since they offer combinations of nebulous forms which rarely occur in that zone.

SUMMARY.

The stars are not absolutely *fixed*. Their *twinkling* is due to *atmospheric* conditions, and is caused by *interference*. (159.)

The number of stars in the whole heavens, visible to the naked eye, is about 6,000. The *telescopic* stars are *innumerable*. (160.)

There are about 20 stars of the *first magnitude;* about 50 or 60 of the *second;* and some 200 of the *third*. The faintest stars that can be seen without the aid of the telescope are of the *sixth magnitude*. (161.)

The stars are arranged in groups called *constellations*. Twelve of these are called *zodiacal constellations*. (162.)

Many of the brightest stars have *proper names*, but stars are usually designated by the letters of the Greek alphabet. (163.)

Many stars shine with *colored light*, and in double stars the colors of the components are often *contrasted*. A few stars are known to change their color. (164.)

Stars often undergo changes of brightness. Sometimes the variations are periodic, like those of *Algol*, *Mira*, and some others. (165.) In other instances the changes are temporary, as is the case with *Tycho's* and *Kepler's* stars. But the changes of the temporary stars may be periodic, recurring at very long intervals. This seems to be the case with the temporary star of 1866. (166.)

The stars are most numerous in the vicinity of the circle of the *Milky Way*, and fewest in number near the poles of that circle. (167.)

The stars are also often collected into *clusters*, some of which are visible to the unaided eye, as in the case of the Pleiades, Præsepe, and the faint cluster in the sword handle of Perseus; others, like the cluster in Hercules, and that in Centaurus, can be made out only with a telescope. (168.)

Stars which are enveloped in a faint nebulosity are called *nebulous* stars. (169.)

The nebulæ are sometimes variable. (170.)

Near the south celestial pole are two nebulous masses called the *Magellanic clouds,* or *Nubeculæ.* (171.)

III.

GRAVITY,

OR THE FORCE BY WHICH THE HEAVENLY BODIES ACT UPON ONE ANOTHER.

GRAVITY.

THE LAWS OF MOTION.

172. *First Law of Motion.*—When a body is put in motion, it will keep on moving in a straight line with unvarying velocity, unless some forces act upon it. This fact is called the *first law of motion.* It is impossible to establish this fact by direct experiment, since we cannot put a body in motion where no forces shall act upon it; and since we cannot observe its motion through an infinite distance and time. If a wheel be made to rotate or a pendulum made to vibrate in the air, they soon come to rest. But the friction of the turning point, and the resistance of the air, are constantly offering hindrance to their motion. It is found that if the friction be diminished by more delicate adjustment of their points of support, they will keep in motion a proportionally longer time. Again, the resistance of the air can be got rid of by putting the wheel or pendulum under the exhausted receiver of an air-pump. When the resistance of the air is thus removed, and the friction reduced to the least possible amount, the bodies are found to keep up their motion a very long time. Hence we must conclude that, if we could get rid of all external hindrances, the wheel, when once started, would keep on rotating with an unvarying velocity, and also that the pendulum would keep on vibrating at the same rate.

Neither of these is an example of motion in a straight line, but from these circular motions we conclude that a body when once put in motion, and not acted upon by any force, would move in a straight line with a uniform velocity. For mathematical investigation has shown that if the parts of a body when unconstrained would move in a straight line with uniform velocity, then, when they are so constrained by their connection with one another that they are obliged to move in circles, the body will rotate with a uniform velocity; and that if the parts of a body, when unconstrained, would not move in straight lines with an unvarying velocity, then they would not rotate with a uniform velocity. Mathematical investigation has also shown that the only way to account for the uniform vibration of a free pendulum is to suppose that the pendulum-ball would move in a straight line with a uniform rate, if left to itself after it has been once started.

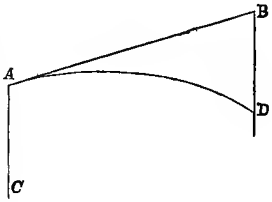
173. *The Planets and Satellites are acted on by some Force.* — We have seen that all the planets and satellites move in curved lines; hence they must be acted on by some force, and this force must be constant in its action, since they move in straight lines in no part of their orbits.

174. *Second Law of Motion.* — It is well known that when a body is held in the hand it presses upon it, and when it is left unsupported it falls to the ground. This shows that there is a force acting between the earth and the body, which draws the two together. It is this force which gives bodies weight; that is, which causes them to exert pressure. It is therefore called *gravity.*

The second law of motion is, that when a moving body is acted upon by gravity alone, it will, at a given time, be just as far from the point which it would have reached had it been left to itself, as it would have been had it been at rest at that point in the first place, and been acted upon by gravity alone during the same time.

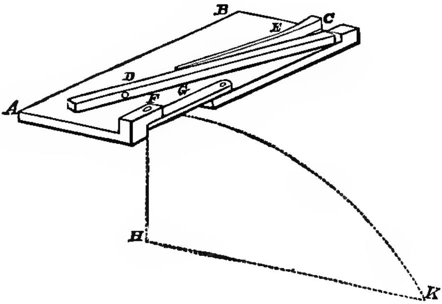
Suppose that a body, for instance, is moving in the direction of *A B* (Figure 77) with a velocity which would carry it from *A* to *B* in one second of time, and suppose that the force of gravity would draw it in the same time from a position of rest at *A* to *C;* then at the end of the second the body will be found at *D*, having been pulled away from the place *B*, which it would have reached with the original direction and velocity, just as much as if pulled away from the state of rest. That is, *BD* just equals *AC*.

Fig. 77.



This law may be illustrated by experiments in the following manner: *AB* (Figure 78) is a board; *CD* an arm

Fig. 78.



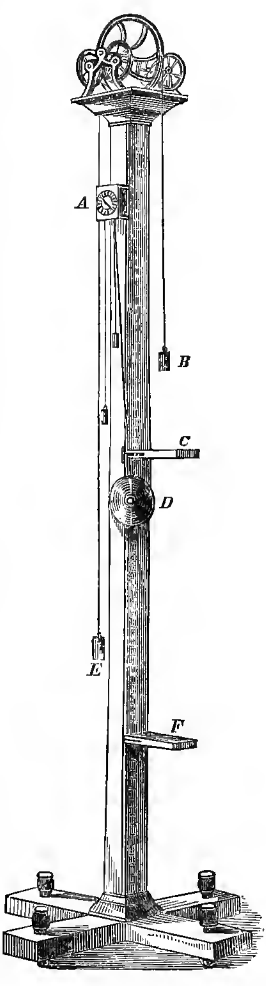
moving upon it, turning on a hinge at *C*, and driven by a spring *E;* at the end *D* of the arm is a hollow, with its opening in the side of the arm large enough to contain a small ball, so that when the arm is driven by the spring *E*, the ball will be thrown horizontally from the hollow at *D;* at *F* is another chamber opening downwards, its lower opening being stopped by a board *G*, which will be knocked away by a blow of the arm *CD;* then it is plain that if one ball be put in *D* and another in *E*, the very same movement which throws one ball forward causes the other ball to drop at the same instant; and, if the second law of motion be true, one of them will fall down vertically to the floor at *H* at the same instant at which the other, which is projected forward, reaches the floor at *K*. And the two balls do reach the floor at the same instant; proving that if a ball is thrown horizontally, it falls from that horizontal line down to the ground in the same time as a ball which dropped from a state of rest.

This experiment is equally applicable to an inclined throw, if the floor upon which the balls fall be inclined exactly in the same degree.

175. *Atwood's Machine.* — The same is found to be true whatever be the direction in which the ball is projected; though the apparatus used in the above illustration is not suited to show this when the ball is thrown either vertically upward or downward. The case in which the body is projected vertically downward can be best illustrated by an instrument called Atwood's machine, shown in Figure 79. It consists of an upright column, with a pulley at the top arranged to run with the least possible friction. Over this pulley passes a cord, to which are attached the equal weights *B* and *E*. *C* and *F* are movable shelves, the former of which has a circular hole in the centre large enough to let the weight *B* pass through it. *A* is a clock beating seconds, and carried by the pendulum *D*. When we wish to make the weight *B* fall, we place upon it a small horizontal bar of iron, which is too long to pass through the hole in the shelf *C*. When, therefore, the weight drops through this hole, the bar will be caught off and will remain upon the shelf.

If the weight *B* with the bar upon it be allowed to fall, it will be found that the force of gravity will pull it down one inch during one second. Now adjust the shelf *C* so that the bar shall be removed at the end of the first second; it will then be found that the weight will fall two inches the next second. At the end of the first second, then, the weight is projected vertically downward with a velocity of two inches a second. If now the bar is left on during both seconds, the weight will be found to have fallen three inches during the second second. Hence the force of gravity pulls the weight down the second second an inch farther than its velocity at the beginning of this second would have carried it; that is, just as far as gravity would have pulled it from a state of rest.

Fig. 79.

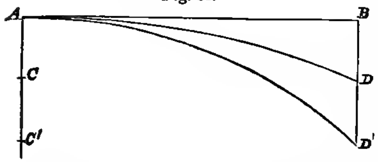


By means of this same machine the case of a body projected vertically upward can be illustrated. While one of the weights is falling the other weight is rising. Suppose that one bar be placed upon the ascending weight, and two on the descending weight; the second a little heavier than the first, so that it shall bear the same ratio to the whole weight now as the one bar used at first. We have already seen that this bar, acting during one second, will give one of the weights a velocity downward of two inches a second, and the other weight the same velocity upward. Suppose now that at the end of the first second both bars be caught off the descending weight, the other weight will rise not two inches but only one during the next second. Had it not been for the action of gravity upon the bar resting on it, it would have risen two inches. But we have already seen that gravity acting upon this bar will cause the weight to fall one inch from a state of rest; hence it is pulled just as far from the place it would have reached as it would have been pulled from a state of rest.

This action of gravity upon a moving body causes it to move in a curve in every case except when the body is moving vertically upward or downward. In the first of these cases it merely retards the motion of the body at a uniform rate, and in the second case it accelerates it in the same manner.

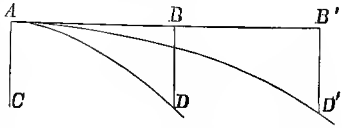
176. *Curvilinear Motion.* — The form of the curve described by a moving ball depends upon the velocity with which it is moving and the strength of the force of gravity. The velocity of the body being the same, the greater the force of gravity, the more rapidly does the path described by the body curve. This is shown in Figure 80. If the ball is moving with a velocity which would carry it from *A* to *B* in a second, and the force of gravity would carry it from a state of rest at *A* to *C* in the same time, then *A D* will be the curvature of its path, *BD* being equal to *A C*. If gravity could carry it from *A* to *C'* in a second, then *A D'* would represent the curvature of its path, *B D'* being equal to *A C'*.

Fig. 80.



A D′ obviously curves more rapidly than *A D*. Again, the force of gravity being the same, the less the velocity with which the ball is moving, the more rapidly does its path curve. Thus, in Figure 81, if the force of gravity would carry the body from a state of rest at *A* to *C* in a second, and the velocity of the body be such that it would move from *A* to *B* in the same time, then *A D* will represent its path. But if its velocity would carry it from *A* to *B′* in a second, then *A D′* would represent its path. But *A D* obviously curves more rapidly than *A D′*. Everybody knows that when a stone is thrown from the hand, its path is much curved, and it reaches the ground before it has gone far. But if you watch the motion of a cannon-ball, which you may do if you stand behind a cannon when it is fired, as you can then see the ball from the time it leaves the cannon's mouth to a distance of half a mile or more, you will perceive that its path is curved, but very much less curved than the path of the stone: in fact it is nearly straight. The ball drops downward through the same space as the stone in one second of time, but it moves much farther in a horizontal direction in the same time.

Fig. 81.

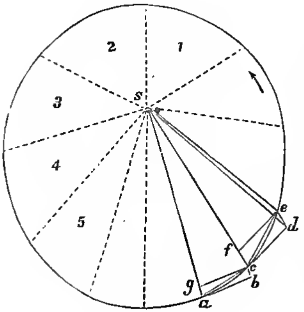


177. *The Force which curves the Paths of the Planets is always directed towards the Sun.* — Since the planets are carried along with the sun in his journey among the stars, it would seem that there must be some force which holds them together. Is this the same force as that which curves their orbit, and as that which draws a stone to the earth? Since the path of a planet curves round in the same direction throughout its whole extent, the force which curves this path must be constantly pulling the planet towards some point inside its orbit.

Long before the theory of gravitation was established by Newton, Kepler had discovered that if a line be drawn from the sun to a planet, this line sweeps over equal areas in equal times in every part of the planet's orbit.

In Figure 82, suppose *S* to be the position of the sun, and the curved line to be the orbit of the planet. Suppose the planet to be at *a*, moving in the direction *a b* with a velocity which would carry it to *b* in a unit of time, and that a force pulling it toward the sun acts upon it for only an instant, giving it a velocity which would carry it to *g* in the same time. Draw the line *b c* parallel to *a g* and take *b c* equal to *a g;* the point *c* will be the position of the body at the end of the unit of time, and the straight line *a c* will be the path passed over by the body. If the body were left to itself the next unit of time, it would pass on in a straight line to *d*, a distance equal to *a c*. Suppose now it be acted upon by another instantaneous impulse drawing it towards the sun, and sufficient to carry it from a state of rest at *c* to *f* in the same time; its position at the end of the time will be at *e*, found by drawing *d e* equal and parallel to *c f*, and its path will be the straight line *c e*. Since *d e* is parallel to *c f*, the two triangles *S c e* and *S c d* have the same base and equal altitudes, and are consequently equal.

Fig. 82.



But *a c* and *c d* are also equal, and the two triangles *a S c* and *c S d* have equal bases and the same altitude, since their vertices are at the same point; hence these triangles are equal, and therefore *S a c* is equal to *S c e*. So long then as the planet is subjected to a succession of instantaneous impulses at equal intervals which draw it towards the sun, the areas passed over by an imaginary line joining the planet and the sun during each of these intervals will be equal. This will be true whatever be the duration of the intervals and the strength of the instantaneous impulses, whether these impulses are equal or not. It will therefore be true when the interval between the successive impulses, *equal or not*, is infinitely small; that is, when the force drawing the planet toward the sun acts continuously. In this case the path becomes a curve, that is, a line bent at every point. Not only are these triangles equal when the impulses are directed toward the sun, but they will not be equal when the impulses are directed toward any other point. The force, then, that curves the paths of the planets must be directed toward the sun. The same is true of the moon's path and the earth.

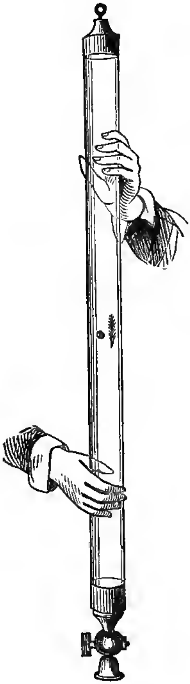
178. *The Force that deflects the Moon's Path is the same as that which draws a Stone to the Earth.* — Having thus determined from Kepler's second law that the force which deflects the moon's path is directed towards the earth, Newton inquired whether this force was the same as that which drew a stone to the earth, on the supposition that this force, like light, diminishes in intensity as the square of the distance increases; that is, becomes four and nine times less when the distance becomes twice or thrice as great.

In order to understand how Newton pursued this inquiry, we must see how gravity causes bodies to fall at the surface of the earth.

179. *Gravity would cause all Bodies to fall at the same Rate, were it not for the Resistance offered by the Air.* — As we observe bodies light and heavy falling through the air, we come to think that the force of gravity causes heavy bodies to fall more rapidly than light ones; but if

we place a coin and a feather in a long glass tube and exhaust the air completely, on inverting the tube (Figure 83) the two bodies will fall through it in the same time. It is therefore the resistance of the air which causes a light body to fall more slowly through the atmosphere than a heavy one does.

Fig. 83.



When therefore the force of gravity is unimpeded in its action, it will cause every body, whatever may be its size, shape, or density, to fall with exactly the same speed. We next inquire how far gravity will cause a body to fall in a second of time. This is determined by means of the pendulum.

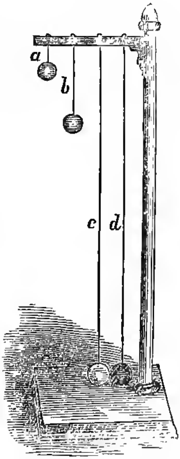
THE PENDULUM.

180. A pendulum is a heavy body suspended from a fixed point by means of a cord or rod. When the centre of gravity of the body is directly under the point of support, the body remains at rest; but if the body be drawn out of this position, it will, on being released, fall towards a vertical line passing through the point of support, and when it has reached this line it will, owing to its inertia, pass beyond it. On coming to rest, it again falls toward this vertical line and again passes beyond, and thus continues to oscillate from side to side.

In studying the movements of the pendulum, mathematicians have considered two kinds of pendulum, which they have called the *simple pendulum* and the *compound pendulum*.

181. *The Simple Pendulum.* — A *simple* pendulum consists of a material point, suspended to a fixed point by means of a thread without weight, perfectly flexible, and incapable of stretching. Such a pendulum has of course no real existence; but we can approach sufficiently near to it, for purposes of illustration, by suspending a small lead bullet to a fixed point by means of a fine silk thread.

Fig. 84.



182. *First Law of the Oscillation of the Pendulum.* — Suppose *d*, in Figure 84, to be a leaden ball hanging by a fine silk thread. Pull it to one side so that it shall vibrate through an arc of some 3°, and count the number of its oscillations in a minute. Now bring it to rest again, and draw it to one side so that it shall oscillate through an arc of 2°, and again count its oscillations in a minute. Again bring the ball to rest, then cause it to oscillate through an arc of 1°, and count the oscillations in a minute. In all three cases the number of oscillations in a minute will be equal.

By an *oscillation* is meant the whole of the pendulum's movement in one direction. The arc through which the pendulum oscillates is called the *amplitude* of its oscillation.

The above experiment shows that *when the length of the pendulum remains the same, and the amplitude of the oscillation does not exceed* 3°, *the pendulum always oscillates in the same time, whatever be the amplitude of the oscillation.*

This singular property of the pendulum is called its *isochronism*, from two Greek words signifying *equal times*, and the oscillations of the pendulum are said to be *isochronous.*

183. *Second Law of the Oscillation of the Pendulum.* — Let *d* and *c*, in figure 84, be two pendulums exactly alike, except that the ball of one is lead and of the other ivory. Let each oscillate through a small arc, and count its oscillations in a minute. It will be found that, making allowance for the resistance of the air, each performs the same number of oscillations in the same time. This gives the second law of the oscillation of the pendulum, namely: *for pendulums of the same length the duration of the oscillation is the same, whatever be the substance of which the pendulum is formed.*

184. *Third Law of the Oscillation of the Pendulum.* — Let *b*, in Figure 84, be a pendulum one fourth the length of *c*, and *a* another, one ninth the length of *c*. Set each oscillating through a small arc, and count the oscillations of each in a minute. It will be found that *b* oscillates twice as fast as *c*, and *a* three times as fast as *c*. This shows that, *for pendulums of unequal length, the duration of the oscillation is proportional to the square root of the length*; that is, the lengths of the pendulum being made 4, 9, and 16 times greater, the duration of the oscillation of the pendulum will be only 2, 3, and 4 times longer. This is the third law of the oscillation of the pendulum.

185. *Fourth Law of the Oscillation of the Pendulum.* — It is found that when a pendulum of a given length is placed on different parts of the earth's surface, the duration of the oscillations is not always the same. Towards the poles it is found to oscillate more rapidly than at the equator. Mathematicians have shown that this is because the force of gravity is stronger at the poles. They have shown that, *in different parts of the earth the duration of the oscillations for pendulums of the same length is in the inverse ratio of the square root of the intensity of gravity*; that is, if the intensity of gravity were four times as great in one place as in another, the duration of the oscillations of a

pendulum of the same length would be half as great, and so on.

186. *The Formula of the Pendulum.* — Let t represent the duration of an oscillation; l, the length of the pendulum; g, the intensity of gravity, that is, the velocity acquired at the end of a second by a body which falls in a vacuum; and $\pi = 3.1416$: and mathematicians have found that the above laws can be summed up in the following expression: —

$$t = \pi \sqrt{\frac{l}{g}}.$$

This expression is called the *formula of the pendulum.*

187. *The Compound Pendulum.* — The simple pendulum, as has been stated, can have no real existence. Every pendulum actually used is a *compound* pendulum, consisting of a heavy weight suspended from a fixed point by means of a rod of wood or metal. The particles of such a pendulum must of course be at different distances from the point of suspension, and must therefore tend to oscillate in different times. Hence the time of oscillation of the whole pendulum will not be the same as that of a simple pendulum of the same length.

The compound pendulum may be regarded as consisting of as many simple pendulums as it contains particles. If these were free to move, they would oscillate in times depending upon their distances from the point of suspension; but since they are united in one body, they are all compelled to oscillate in the same time. Consequently, the oscillations of the particles near the point of suspension are retarded by the slower oscillations of the particles below them; and, on the other hand, the oscillations of the particles near the lower end of the pendulum are accelerated by the more rapid oscillations of those above them. At some point between these there must be a particle whose oscillation is neither retarded nor accelerated, — all the particles above having just the same tendency to oscillate faster that those below have to oscillate slower. This point is called the *centre of oscillation*, and it is obvious that the time of oscillation of a compound pendulum is the same as that of a simple pendulum whose length is equal to the distance of the centre of oscillation from the point of suspension. This distance is the *virtual length* of the pendulum, and the formula given above (186) will apply to compound pendulums by making $l =$ their *virtual length.* By the length of a pendulum, whatever may be its form, we are always to understand the virtual length, unless the reverse is expressly stated.

When the form of the pendulum is given, the position of the centre of oscillation can be calculated by methods belonging to the higher mathematics. It can also be found experimentally by making use of a remarkable property of the compound pendulum, by virtue of which, if such a pendulum be inverted and suspended by its centre of oscillation, its former point of suspension will become its new centre of oscillation, and the time of vibration will be the same as before; or, as it is usually expressed, *the centres of oscillation and suspension are interchangeable.*

To find the centre of oscillation, then, we have only to reverse a pendulum, and by trial find the point at which it must be suspended in order to oscillate in the same time as it did before it was reversed. A pendulum constructed for this purpose is called a *reversible pendulum.*

188. *The Use of the Pendulum for measuring the Force of Gravity.* — By transposing, we get from the equation above (185) $g = l \frac{\pi^2}{T^2}$; from which, when we know the length of a pendulum which oscillates in a given time, T, we can easily calculate the value of g for the place of experiment. If, in the last equation, we make $T = 1$, then l denotes the length of a pendulum beating seconds, and we find that

$g = l\,\pi^2$. In order, then, to measure the intensity of gravity at any place, we have only to oscillate a pendulum whose virtual length is known, and observe the length of a single oscillation. This is readily done by counting a large number of oscillations, and observing the time occupied by the whole number. This time, divided by the number of oscillations, gives the time of a single oscillation very accurately, because any error which may have been made in observing the time is thus greatly divided.

Now it has been found that a pendulum beating seconds at London must be 39.13929 inches in length. Substituting this value, and also that of π^2 in the equation, $g = l\,\pi^2$, we get $g = 386$ inches.

Since the velocity of a ball is zero at starting, and at the end of a second it attains the velocity of 386 inches, its mean velocity would be 193 inches, and this is evidently the distance the body would be drawn towards the earth by gravity in a second of time.

We have thus found the answer to the question asked in section 179, and have proved that *gravity causes a body to fall sixteen feet in a second of time.*

SUMMARY OF THE PENDULUM.

A *pendulum* is a heavy body suspended from a fixed point by means of a cord or rod. (180.)

The laws of the oscillation of the pendulum are best investigated by means of a *simple pendulum.* (181.)

These laws are four in number.

1st. *When the length of the pendulum remains the same, and the amplitude of the oscillations does not exceed 3°, the pendulum always oscillates in the same time.* (182.)

2d. *For pendulums of the same length, the duration of the oscillations is the same, whatever be the substance of which the pendulum is formed.* (183.)

3d. *For pendulums of different lengths, at the same place, the duration of the oscillation is proportional to the square root of the lengths.* (184.)

4th. *In different parts of the earth the duration of the oscillation for pendulums of the same length is in the inverse ratio of the square root of the intensity of gravity.* (185.)

These laws can be summed up in the following expression, called the *formula of the pendulum*:—

$$t = \pi \sqrt{\frac{l}{g}} \quad (186.)$$

The pendulum in actual use is a *compound pendulum.* (187.)

The pendulum is used for measuring *the force of gravity.* (188.)

GRAVITY ACTS BETWEEN THE EARTH AND THE MOON.

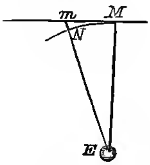
189. *The Intensity of Gravity varies directly as the Mass of the Body acted upon by this Force.*—It has been shown, by the falling of bodies in a vacuum (179), and the pendulum (188), that gravity acting upon a body at rest will cause it to fall one hundred and ninety-three inches in a second, whatever may be the amount of matter that the body contains. But if one body contains twice the amount of matter that another does, it will clearly take twice the force to pull it through the same distance. Hence the intensity of gravity acting upon bodies near the earth must vary directly as the amount of matter which they contain, or as the *mass* of the bodies.

190. *The Moon's Path is curved by the Force of Gravity.*—We now return to the consideration of the Moon's path, and the force which deflects it. (178).

We have found that gravity acts between the earth and a body near its surface with a force whose intensity varies directly as the mass of the body, and is sufficient to draw it from a state of rest 193 inches in a second.

If, then, the earth's attraction at the distance of 3,959 miles from its centre draws a body 193 inches in a second, how far in a second ought it to draw a body distant 238,800 miles from this centre, supposing that the force diminishes in proportion as the square of the distance increases? This distance will, of course, diminish with the intensity of gravity, and can easily be ascertained by squaring these two numbers, finding how many times the less of these squares is contained in the greater, and dividing 193 inches by the quotient. This distance is thus found to be .05305 of an inch. We must now find how much the moon's path is really deflected in one second of time. Let *E*, in Figure 85, represent the position of the earth; *M*, the position of the moon; and *M m*, the direction in which the moon is moving at the instant. Let *M m* be the distance the moon would pass over in one second of time if nothing interfered with her motion. Then *m N* will be the distance the moon is drawn towards the earth in this time.

Fig. 85.



Now, considering the moon as moving in a circle whose semidiameter is her mean distance, *E M*, is 238,800; the whole circumference is 1,500,450 miles; and her periodic time is 27 days, 7 hours, 43 minutes. Dividing the length of the whole orbit by the number of seconds in her periodic time, we find her velocity in any part of her orbit, as at *M*, is .6356 of a mile. This is the distance, *M m*, that she would have travelled, had no force interfered with her motion. But *E M m* is a right-angled triangle, and we know the length of the sides *E M* and *M m*. By squaring these and extracting the square root of their sum, we find the length of the hypothenuse *E m* to be

238,800.0000008459 miles. But $m\ N$ is equal to $m\ E$ minus $E\ N$. Therefore $m\ N$ is equal to .0000008459 of a mile, or .0536 of an inch. This is found to be almost exactly the same distance that the moon ought to be drawn to the earth in a second of time, provided she is drawn downward by the same force which draws a stone to the earth, the intensity of the force having diminished as the square of the distance has increased. This slight difference is exactly accounted for by disturbing causes which are known to exist. It is therefore certain that the attraction of the earth which causes the stone to fall, and the attraction of the earth which bends the moon's path from a straight line to a circle, are really the same attraction, only diminished for the moon in the inverse proportion of the square of her distance.

If we had taken the interval of an hour instead of a second, we should have found that the moon was drawn to the earth 10.963 miles; and we should have found that a stone at the distance of the moon would have fallen through a corresponding space in the same time.

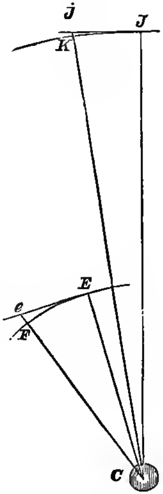
GRAVITY ACTS BETWEEN THE SUN AND PLANETS, AND BETWEEN PLANETS AND THEIR MOONS.

191. *The Paths of the Earth and of Jupiter are curved by the Force of Gravity.* — We will proceed to see whether the paths of the planets are curved by gravity.

Let us first compare the spaces through which the sun draws the planets in one hour; and, as an instance, we will take the earth and Jupiter. In Fig. 86 let $E\ F$ be the path described by the earth in one hour, and $E\ e$ the path in a straight line which the earth would have described in one hour if nothing had disturbed it; and let $J\ K$ be the path described by Jupiter in one hour; and

Jj the path Jupiter would have described in one hour if nothing had disturbed it. Then *e F* is the space through which the sun's attraction has drawn the earth in one hour, and *j K* the space through which the sun's attraction has drawn Jupiter in one hour. We wish to find the ratio of *e F* to *j K*. Taking *C E* as 95,000,000 miles, the circumference of the earth's orbit is 596,900,000 miles, which the earth describes in 365.26 days; and therefore the line *E e*, which is the earth's motion in one hour, is 68,091 miles. Adding the square of *C E* to the square of *E e*, and extracting the square root of the sum, we find that *C e* is 95,000,024.402 miles; and therefore *e F*, the space through which the sun draws the earth in an hour, is 24.402 miles. For Jupiter, *C J* is 494,000,000 miles; the circumference of its orbit is therefore 3,104,000,000 miles; which is described in 4,332.62 days; therefore *J j*, the motion in one hour, is 29,850 miles; and the length of *C j*, found in the same manner, is 494,000,000.9019 miles; and *j K*, the space through which the sun draws Jupiter in one hour, is 0.9019 miles.

Fig. 86.



The distances through which these planets are drawn towards the sun are therefore in the ratio of 24.402 to 0.9019. But if we compute, from the rule of the inverse square of the distances, what would be the proportion of the force of the sun on the earth to the force of the sun on Jupiter, we find that it is the proportion of 24.402 to 0.9024. These proportions may be regarded as exactly the same, the trifling difference between them arising mainly from the circumstance that we have used only round numbers for the distances of the two planets from the sun. It is true, then, for these two planets that the strength of the sun's attraction is inversely proportional to the square of the distance of the attracted body from the sun.

192. *The Paths of all the Planets and their Satellites are curved by Gravity.* — If we should compare any two planets in the same way, we should arrive at the same conclusion.

In fact it has been demonstrated that whenever the rule known as Kepler's third law (42) holds, — namely, that "the squares of the periodic times of several bodies moving round a central body are proportional to the cubes of the distances of the several bodies from that central body," — then it will be found, by a process exactly similar to that which we have gone through, that the effects of the central body's attraction at the different distances are inversely as the squares of the distances. Now, this law was discovered by Kepler, long before the theory of gravitation was invented, to hold in regard to the times and distances of the planets in their revolutions round the sun. Moreover, in regard to the four satellites of Jupiter, the same law holds. For we are able without difficulty to observe their periodic times; we are able also to ascertain their angular distance from Jupiter; and from this, knowing the distance of Jupiter from the earth in miles, we can compute the distance of each satellite from Jupiter in miles. We thus find that the squares of their times are proportional to the cubes of their distances. Consequently the attraction of Jupiter upon his several satellites is inversely proportional to the squares of their distances from him. In like manner it is found that the attraction of Saturn upon his eight satellites is inversely proportional to the squares of their distances from him; and, so far as we can examine, the same law holds with regard to the attraction of Uranus and Neptune on their satellites.

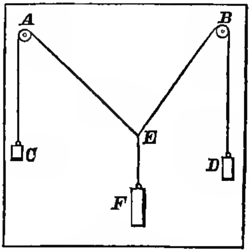
Thus, for every body which we know around which other bodies revolve, the force of attraction of the central body on the different bodies that revolve round it is inversely proportional to the squares of their distances.

193. *Gravity causes the Planets and their Moons to move in Ellipses.* — Kepler also discovered that the planets do not move in circles but ellipses. Hence their distance from the sun varies. We have now found it true that the attraction of the central body upon the bodies revolving about it follows the law of the inverse squares of the distances; does this law hold in the case of each planet as its distance from the sun varies?

We have already seen that when a planet moves in an ellipse, the deflecting force must be directed to the focus of the ellipse, in order that a line drawn from that focus to the planet may describe equal areas in equal times. Since the planets are thus constantly pulled towards the sun, and since they at times really approach him, it would seem that they would be unable to recede from him again.

194. *The Resolution of Forces.* — Before we show how it is that a planet can thus recede from the sun, we must explain what is called the *resolution of forces.* This may be illustrated by the apparatus represented in Figure 87.

Fig. 87.



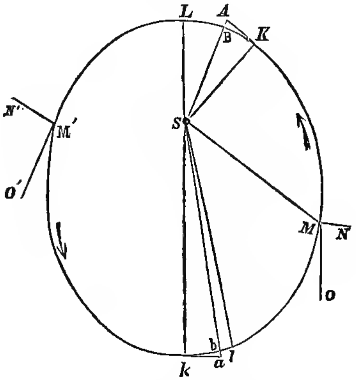
Suppose *A* and *B* to be two pulleys, fixed upon an upright frame, and suppose two cords to pass over them, carrying the two weights, *C* and *D*, at their ends; and where they meet at *E* let a third cord be attached, carrying the weight *F*; then the tension or pull produced by this one weight *F*, acting at the point *E*, supports two tensions in different directions acting at the same point, namely, the tension produced by the weight *C* acting in the direction *E A*, and the tension produced by the weight *D* acting in the direction *E B*. Thus we may say, that one pull in the direction *E F* exerts two pulls in different directions, *A E* and *B E*, for it keeps the two cords strained so as to support the two other weights. We may say, on the other hand, that these two outside weights, *C* and *D*, support the middle one. The three pulls of the cords keep the point *E* in equilibrium; but they will support it only in one position, according to the amount of weight which is hung to each cord. If we put another weight upon *C*, the position of the point *E* and the direction of the cords will immediately change. Regarding the action of the two tensions in the directions *E A*, *E B*, as supporting the one tension in the direction *E F*, this may be considered as an instance of the *combination* of forces; and regarding the one tension in the direction *E F*, as supporting two in the directions *A E*, *E B*, this may be considered as an instance of the *resolution* of forces, the one force in the direction *E F* being *resolved* into two forces in the directions *A E*, *B E*, and producing in all respects the same effects as two forces in the directions *A E*, *B E*.

Having, then, a force in any one direction, we may resolve it into two forces acting in any two directions suited to the nature of the case, and we may use those two forces instead of the one original force.

195. We can now see how it is that when a planet has once begun to approach to the sun, it can recede again from it. Suppose, in Figure 88, a planet is moving from *l*,

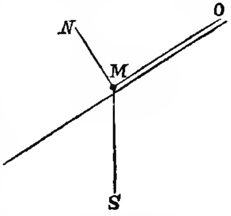
through *M*, towards *L*. The attraction of the sun pulls it in the direction of the line *M S*. Upon the principle of the resolution of forces, we may consider the force in the direction of *M S* to be resolved into two, one of which is in

Fig. 88.



the direction of *N M*, perpendicular to the orbit, and the other is in the direction of *O M*, parallel to that part of the orbit. Now that part of the force which is in the direction *N M*, perpendicular to the orbit, makes the orbit curved. But that part which acts in the direction of *O M*, parallel to the orbit, produces a different effect; it accelerates the planet's motion in its orbit. Thus, in going from *l* towards *L*, the planet is made to go quicker and quicker. If the diagram (Figure 88) be turned in such a manner that *M S* is vertical, *S* being downwards, the planet is under the same circumstances as a ball rolling down a hill. If a ball is going down a hill, as at *M*, Figure 89, the force of gravity, which is in the direction *M S*, may be resolved

Fig. 89.



into two parts, one of which acts in the direction *N M*, perpendicular to the hillside, and merely presses the ball towards the hill; the other acts in the direction *O M*, making it go the faster down the hill. In this manner, as long as the planet goes from *k* through *M* towards *L*, it is going quicker and quicker. It has been explained above (176), that the curvature of a planet's orbit, or the curvature of the path of a cannon-ball, depends upon two circumstances; one is the velocity with which it is going, and the other is the force which acts to bend its path. The greater its speed, the less its path is curved. Consequently, as the planet is going so fast in the neighborhood of *K*, its orbit may be very little curved there, even though the sun is there pulling it with a very great force. The effect of this is, that the planet passes the sun and begins to recede from it. But it does not recede perpetually. Suppose, for instance, that it has reached the point *M′*; the force of the sun in the direction *M′S* may be resolved into two, in the directions *N′M′*, *O′M′*, of which the former only curves the orbit, while the latter retards the planet in its movement in the orbit. Therefore, as the planet recedes from the sun, it goes more and more slowly, till at last its velocity may be diminished so much, that the power of the sun, reduced as it is there, is enabled to bring it back again. Thus the planet goes on in its orbit, alternately approaching to, and receding from, the sun.

196. In Figure 88, let *K A* and *k a* be the path in a straight line which the planet would describe in an hour in those parts of its orbit: then will *A B* and *a b* be the distance which the planet is drawn towards the sun in an hour in each of those positions. Now, by a somewhat difficult mathematical investigation, it can be shown that the lines *A B* and *a b* stand in the inverse ratio of the squares of the distances *S K* and *S k*, and this is found to be true of the planet at any two positions in its orbit. We therefore con-

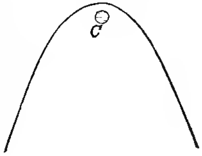
clude, that when any body moves in an elliptical orbit round a certain body situated in the focus of this ellipse, the deflecting force, exerted by the central body, varies in the inverse proportion of the squares of the distances between the two bodies.

GRAVITY ACTS BETWEEN THE SUN AND COMETS.

197. *The Parabolic Motion of Comets.* — There is, however, another remarkable class of bodies of which we have already spoken; namely, the comets. Can the curved form of their paths be accounted for on the supposition that they are drawn toward the sun by the same force as the planets? A few of the comets, as has been stated (157), are now known to move in very long ellipses, and to return periodically to our sight. Of course their motion can be accounted for in the same way as that of the planets; but, in Newton's time, the idea of a periodical comet was wholly unknown; and it is now certain that many of the comets, after visiting the sun, never return to him again.

But since curvature of the path of a planet depends both upon the force which draws it towards the sun and upon its velocity, we can readily see that the velocity of a planet might be so great that its path would never be bent around enough to bring it back to the sun. Newton showed, by an investigation similar to that made in the case of an elliptical orbit, that a body subject to the attraction of a central body (as the sun) might, if the force varied inversely as the square of the distance, describe the curve called the *parabola;* but no other law of force would account for the description of such a curve. The form of the parabola is represented in Figure 90, *C* be-

Fig. 90.



ing the sun; and this curve, it is evident, possesses two of the peculiarities which distinguish the motions of comets; it comes very near to the sun at one part, and it goes off to an indefinitely great distance at other parts.

Now, when Newton had found out that the same laws of gravitation which were established from the consideration of elliptical motion would account for motion in a parabola, he began to try whether the parabola would not represent the motion of a comet. It was found, that by taking a parabola of certain dimensions, and in a certain position, the motions of the comet which had been observed most accurately could be represented with the utmost precision. Since that time, the same investigation has been repeated for hundreds of comets; and it has been found, in every instance, that the comet's movements could be exactly represented by supposing it to move in a parabola of proper dimensions and in the proper position, the sun being always situated at a certain point called the focus of the parabola. This investigation tends most powerfully to confirm the law of gravitation; showing that the same moving body, which at one time is very near to the sun, and at another time is inconceivably distant from it, is subject to an attraction of the sun varying inversely as the square of the distance.

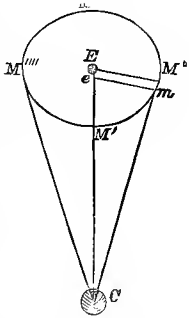
GRAVITY ACTS AMONG ALL THE HEAVENLY BODIES.

198. *The Moon's Perturbations.* — We have seen that the sun attracts the earth. Does it also attract the moon? If so, as these bodies are always either at different distances from the sun, or lie in different directions from the sun, they will be differently attracted by him; and hence their relative motions will be disturbed. We find that these motions are thus disturbed, giving rise to what are called *perturbations*. These perturbations were discovered from

observation long before the theory of gravitation was invented. One of the first triumphs of the theory was their complete explanation. We shall attempt here to explain only the one which is called the Moon's Variation.

In Figure 91, suppose E to be the earth, M' M'' M''' M'''' the moon's orbit, and C the sun. The sun, by the law of gravitation, attracts bodies which are near with greater force than those which are far distant from it. Therefore, when the moon is at M' the sun attracts the moon more than the earth, and tends to pull the moon away from the earth. When the moon is at M''' the sun attracts the earth more than the moon, and therefore tends to pull the earth from the moon, producing the same effect as at M' or tending to separate them. When the moon is at M'' the force of the sun on the moon is nearly the same as the force of the sun upon the earth, but it is in a different direction. If the sun pulls the earth through the space $E e$, and if it also pulls the moon through the same space $M''m$, these attractions tend to bring the earth and the moon nearer together, because the two bodies are moved as it were along the sides of a wedge which grows narrower and narrower. Thus, at M' and M''' the action of the sun tends to separate the earth and the moon, and at M'' and M'''' the action tends to bring the earth and the moon together.

Fig. 91.

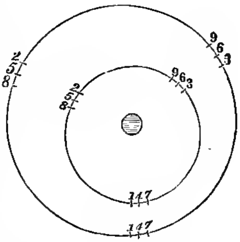


One might perhaps infer from this that the moon's orbit is elongated in the direction M' M'''; but the effect is exactly the opposite. The fact really is, that the moon's orbit is elongated in the direction M'' M''''. And it can easily be shown that it must be so. The moon, we will suppose, is travelling from M'''' to M'. All this time the sun is attracting her more than the earth, and therefore increasing her velocity till she reaches M'. When she is passing from M' to M'' the sun is pulling her back, and her velocity is diminished till she reaches M''. From this point her velocity increases again till she reaches M''', and then diminishes again till she reaches M''''. Therefore, when the moon is nearest to the sun, and farthest from the sun, she is moving with the greatest velocity; when she is at those parts of her orbit at which her distance from the sun is equal to the earth's distance from the sun, she is moving with the least velocity. We have learned that the curvature of the orbit depends on two considerations. One is the velocity; and the greater the velocity is, the less the orbit will be curved. The other is the force; and the less the force is, the less the orbit will be curved. Since, then, the velocity is greatest at M' and M''', and the force directed to the earth is least (because the sun's disturbing force there diminishes the earth's attraction), the orbit must be the least curved there. At M'' and M'''' the velocity has been considerably diminished; the force which draws the moon towards the earth is greatest there (because the sun's disturbing force there increases the earth's attraction), and therefore the orbit must be most curved there. The only way of reconciling these conclusions is by saying that the orbit is lengthened in the direction M'' M''''; a conclusion opposite to what we should have supposed if we had not investigated closely this remarkable phenomenon. It will easily be understood that the amount of this effect is modified in some degree by the change which the earth's attraction undergoes in consequence of the change of the moon's distance, (the earth's attractive force varying inversely as the square of the moon's distance,) but still the reasoning applies with perfect accuracy to the kind of alteration which is produced in the moon's orbit.

This particular inequality was discovered by Tycho de Brahe before gravitation was known; and it was explained by Newton as a result of gravitation. There are other perturbations, even more important than this, which were discovered before gravitation was known, and which were most fully and accurately explained by gravitation.

199. *The Mutual Perturbations of the Planets.* — Again, it is found that there are disturbances in the motions of the planets generally; and these disturbances can be explained only by supposing that every planet attracts every other planet, and that therefore the motions of the planets are not exactly the same as if only the sun attracted them. These disturbances are exceedingly complicated. There is one kind, however, of which possibly some notion may be given. They are the most remarkable in Jupiter and Saturn. There are many books, written as late as the beginning of the present century, in which the motions of Jupiter and Saturn are spoken of as irreconcilable with the theory of gravitation. It was one of the grand discoveries of La Place that the great disturbances of those two planets are caused by what is called the "inequality of long period," requiring some hundreds of years to go through all its changes.

Fig. 92.



Let Figure 92 represent the orbits of Jupiter and Saturn. They are both ellipses, and the positions of their axes do not correspond. Now, the thing which La Place pointed out as affecting the perturbations of these planets is one which applies more or less to several other planets; namely, that the periodic times of Jupiter and Saturn are very nearly in the proportion of two small numbers, 2 to 5.

Inasmuch as these periodic times are in the proportion of 2 to 5, it follows that, while Saturn is describing two thirds of a revolution in its orbit, Jupiter is describing almost exactly five thirds of a revolution in its orbit. If, therefore, the two planets have been in conjunction, then about twenty years afterwards Saturn has described two thirds of a revolution, and Jupiter a whole revolution and two thirds, and the planets will be in conjunction again, but not in the same parts of their orbits as before, but in parts farther on by two thirds of a revolution. Thus, in Figure 92, suppose 1 1 to be the place of the first conjunction of which we are speaking. Saturn describes two thirds of his orbit as far as the figure 2. Jupiter goes on describing a whole revolution and two thirds of a revolution, and arrives at the same time at the figure 2 in his orbit, and the planets are in conjunction at 2 2. Saturn goes on describing two thirds of the orbit again, and comes to figure 3. Jupiter goes on describing a whole revolution and two thirds of another, and he comes to figure 3, and they are in conjunction there. The next time they are in conjunction at figure 4, the next at figure 5, and the next at figure 6, and so on. These conjunctions occur in this manner from the circumstance that the periodic times are nearly in the proportion of 2 to 5; there are three points of the orbit at nearly equal distances, at which the conjunctions occur.

But we will suppose that they occurred exactly at three equidistant points, and that time after time they happened exactly at the same points. It is plain that in that case there would be a remarkable effect of the disturbances, particularly at those parts of the orbit 1 1, 2 2, 3 3, etc., where Jupiter and Saturn are nearer to each other than at other times. They are very large planets; each of them larger than all the rest of the solar system, except the sun. They exercise very great attractive force each upon the other; and therefore they would disturb each other in a very great degree, if their conjunctions occurred exactly at the same place.

Now, these conjunctions do not occur exactly at the same place. The periodic times are nearly in the proportion of 2 to 5, but not exactly in that proportion. Consequently their places of conjunction travel on, until after a certain time the points of conjunction of the series 1, 4, 7, etc., would have travelled on until they met the series 3, 6, 9, etc. A period of not less than nine hundred years is required for this change.

Now, so long as three conjunctions take place at any definite set of points, the effect on the orbits is of one kind. As they travel on, the effect is of another description (because, from the eccentricity of their orbits, the distance between the planets at conjunction is not the same), and so they go on changing slowly until the points of the series 1, 4, 7, etc., are extended so far as to join the series 3, 6, 9, etc.; and then the conjunctions of the two planets occur at the same points of their orbits as at first, and the effect of each planet in disturbing the other is the same as at first; and thus we have the same thing recurring over and over again for ages. During one half of each period of nine hundred years, the effect that one planet has upon the other is, that its orbit has been slowly changing; and then, during the other half, it comes back to what it was before. Suppose that, during half the nine hundred years, one planet has been causing the other to move a little quicker, and that, during the other half of that nine hundred years, it has been causing it to move a little slower; although that change may be extremely small as regards the velocity of the planets, yet, as that velocity has four hundred and fifty years to produce its effect in one way, and an equal time to produce its effect in the opposite way, it does produce a considerable irregularity. If the place of Saturn be calculated on the supposition that its periodic time is always the same, then at one time its real place will be behind its computed place by about one degree, and four hundred and fifty years later its real place will be before its computed place by about one degree, so that in four hundred and fifty years it will seem to have gained two degrees. The corresponding disturbances of Jupiter are not quite so large.

These are the most remarkable of all the planetary disturbances, their magnitude being greater than any other, on account of the magnitude of the planets, and the eccentricity of their orbits. There are, however, others of the same kind. One of these depends upon the circumstance, that eight times the periodic time of the earth is very nearly equal to thirteen times the periodic time of Venus.

We have attempted to explain only one limited class of perturbations. There are some which may be described as a slow increase and decrease of the eccentricities of the orbits, and a slow change in the direction of the longer axes of the orbits; but there are others of which no intelligible account can be given in an elementary book.

200. *The Calculation of the Amount of these Perturbations.* — In order, however, to bring these theories into actual calculation, it is necessary to know not only the general tendency of the disturbances, but also their actual magnitude. In the perturbations produced by the earth, by Jupiter, and by Saturn, there is no difficulty in doing this. We have shown (191) that we can calculate the number of miles through which the earth's attraction draws the moon in one hour. We know also (179) that the earth's attraction draws every body at the earth's surface through the same space in the same time; so that a ball of lead and a feather will fall to the ground with equal speed, if the resistance of the air is removed. We say, therefore, that the earth's attraction would draw a planet through the same space as the moon, provided the planet were at the

moon's distance; and, for the greater distance of the planet, we must, by the law of gravitation, diminish that space in the inverse proportion of the square of the distance. We have already learned (192) how to compute the space through which the sun draws a planet in one hour; and therefore the problem now is, to compute the motion of a planet, knowing exactly how far, and in what direction, the sun will draw it in one hour, and also knowing exactly how far, and in what direction, the earth will draw it in one hour.

In like manner we can, from observations of Jupiter's satellites, compute how far Jupiter draws one of his satellites in one hour, and therefore how far Jupiter would draw a planet at the same distance in one hour: and then by the law of gravitation we can compute (by the proportion of inverse squares of the distances) how far Jupiter will draw a planet at any distance in one hour; and this is to be combined, in computation, with the space through which the sun will draw the planet in one hour. In like manner, by similar observations of Saturn's satellites, and similar reasoning, we can find how far Saturn will draw any planet in one hour, and we can combine this with the space through which the sun would draw it in one hour. Thus we are enabled to compute completely the perturbations which these three planets produce in any other planets.

201. *Do the Planets' Motions, as computed with these Disturbances, agree with what we see in actual Observation?* — They do agree most perfectly. Perhaps the best proof which can be given of the care with which astronomers have looked to this matter, is the following. The measures of the distances of Jupiter's moons in use till within the last sixteen years, had not been made with due accuracy; and, in consequence, the perturbations produced by Jupiter had all been computed too small by about one fiftieth part. So great a discordance manifested itself

between the computed and the observed motions of some of the planets, that many of the German astronomers expressed themselves doubtful of the truth of the law of gravitation. Airy, the Astronomer Royal of England, was led to make a new set of observations of Jupiter's satellites, and discovered that these bodies were farther from Jupiter than was supposed, that the space through which Jupiter drew them in an hour was greater than was supposed, and that the perturbations ought to be increased by about one fiftieth part. On using the corrected perturbations, the computed and the observed places of the planets agreed perfectly.

The motions of our moon are sensibly disturbed by the planet Venus. An irregularity, which had been discovered by observation, and had puzzled all astronomers for fifty years, was explained a short time ago by Professor Hansen, of Gotha, on the theory of gravitation, as a very curious effect of the attraction of Venus.

202. *The Theory of Gravitation holds good throughout the known Universe.*—We thus see that the theory of gravitation holds good as far as the solar system extends. We have learned, moreover, that the binary stars which have been observed to revolve about one another all move in elliptical orbits. Hence, they must act upon one another with a force which varies inversely as the square of the distance. Hence, the law of gravitation seems to hold, as far as we are able to extend our observations into space.

SUMMARY.

From the rotating of a wheel, and the vibrating of a free pendulum in a vacuum, we infer that *a moving body will continue to move in a straight line and with a uniform velocity until it is acted upon by some force.* (172.)

By means of a projecting machine and Atwood's machine, we find that *a moving body, when acted upon by gravity alone, will, at any given time, be just as far from the point which it would have reached had it been left to itself, as it would have been had it been at rest at that point in the first place and been acted upon by gravity alone during the same time.* (174, 175.)

By the falling of bodies in a vacuum, and by the pendulum, we find that gravity acts upon a body near the earth with a force whose intensity varies directly as the mass of the body, and is sufficient to draw it from a state of rest one hundred and ninety-three inches in a second. (179, 188, 189.)

The planets and their moons all *describe curved paths.* Hence they must be acted upon by some force. (173.)

They all *describe equal areas in equal time.* Hence the force which curves their paths must, in the case of the planets, be directed toward the sun, and, in the case of the moons, toward the planet about which they revolve. (177.)

At the distance of the moon, gravity, if its intensity diminishes as the square of the distance increases, will cause a body to fall towards the earth .05305 of an inch. It is found by computation that, when allowance is made for disturbing causes which are known to exist, the moon is drawn toward the earth exactly this distance each second. Hence we conclude that *gravity acts between the earth and the moon* with a force whose intensity varies directly as the mass of the body acted upon, and inversely as the square of its distance. (190.)

It is found by computation that, during a given time, all the planets are drawn toward the sun, a distance which varies inversely as the square of their distance from him. Hence they must be acted upon by a force which varies directly as their mass and inversely as the square of their distance from the sun. It is also found that the planets must be acted upon by such a force, in order that they may move in ellipses. From this we conclude that gravity acts between the sun and the planets. For a like reason we conclude that gravity acts between the planets and their moons. (191 – 196.)

The comets describe *elliptical* and *parabolic* orbits. Hence they must be acted upon by *gravity*. (197.)

The perturbations of the moon show that gravity must act between the sun and the moon, as well as between the sun and the earth, and the earth and the moon. (198.)

The perturbations of the planets can all be exactly accounted for by supposing that each acts upon every other with a force varying directly as the mass and inversely as the square of the distance of these bodies. Hence we conclude that gravity acts among all the bodies of the solar system. (199 – 201.)

The binary and multiple stars revolve about one another in *elliptical* orbits. Hence we conclude that gravity acts between stars and stars as well as among the members of our solar system. (202.)

The planets not only describe ellipses which differ from one another in their eccentricity and their inclination to the ecliptic, but the path described by each planet is constantly undergoing changes in each of these particulars. These changes go on increasing in one direction for a certain length of time, when they again begin to diminish. It has been shown by mathematical investigations that these changes can in no case reach such a point as to break up the present order of our system, provided that system be acted upon by no external force, but that they all repeat themselves in cycles which are in some cases of enormous length.

These changes of very long period are often called *secular* changes.

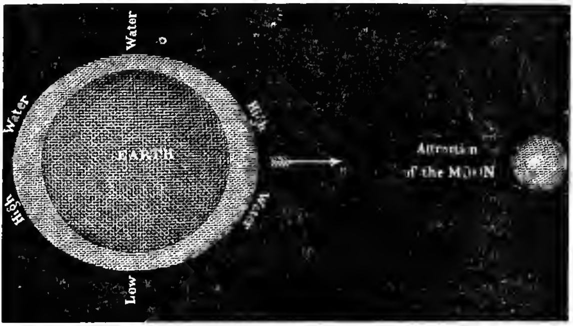
GRAVITY ACTS UPON THE PARTICLES OF MATTER.

203. We have now learned that gravity acts upon all the heavenly bodies with a force varying directly as their masses, and inversely as the squares of their distances from one another. Does gravity act upon these bodies as *wholes*, or upon the *particles* of which they are composed?

204. *The Tides.* — If the force acting between the sun and the earth, and between the moon and the earth, acts upon the particles of which the earth is made up, those particles which are nearest the sun or moon ought to be pulled more strongly than those which are farther away, and, if they were free to move, they ought to be drawn away from those behind them. But the particles of water have great freedom of motion among themselves. Are the waters of the ocean heaped up under the sun and moon, as they should be if gravity acts, not upon the earth as a whole, but upon the particles of which it is composed?

Now it is well known that *tides* exist in the ocean, and that the tidal wave follows the moon in her daily round. The particles of water nearest the moon are then drawn

Fig. 93.



away from those behind them, thus giving rise to the tides. It is only the mutual attraction among the particles of which the earth is composed, that keeps these particles from being drawn entirely away from the earth.

On the side of the earth opposite the moon, the particles of the water are drawn less strongly than those of the solid

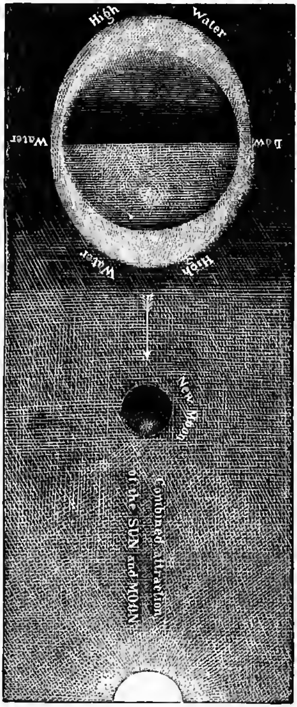


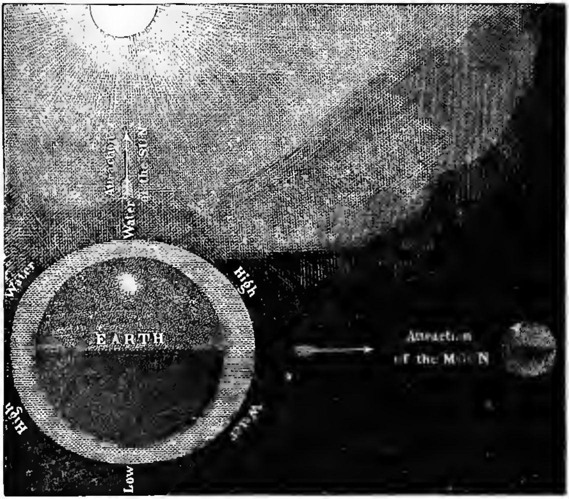
Fig. 94.

earth which are nearer the moon. For this reason the solid part of the earth is here drawn away from the water. This gives rise to a tidal wave also on the part of the ocean opposite the moon.

The tides follow the moon generally, but not entirely.

They do not follow the time of the moon's meridian passage by the same interval at all times; and they are much larger shortly after new moon and full moon than at other times. From a careful examination of all the phenomena of tides, it appears that they may be most accurately represented by the combination of two independent tides, the larger produced by the moon (as shown in Figure 93), and

Fig. 95.



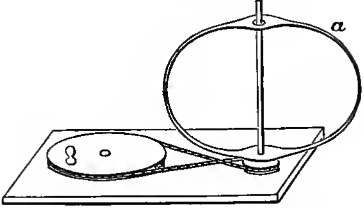
the smaller produced by the sun. When these two tides are added together, they make a very large tide (as shown in Figure 94), which is called a *spring-tide;* but when the high water produced by the sun is combined with the low water produced by the moon, and the low water produced by the sun is combined with the high water produced by the moon (see Figure 95), a small tide is produced, which is called a *neap-tide.* We conclude then that the force of gravity which acts between the sun and the earth, and between the moon and the earth, does not act upon the earth as a whole, but upon each particle of which the earth is composed.

205. *The Spheroidal Form of the Earth.*—Another important fact bearing upon the theory of gravitation is the spheroidal form of the earth.

We have seen in Part First of this work (138) that the materials of the earth are evidently the result of burning. The heat developed by this combustion must have been sufficient to reduce the whole mass to the liquid state. It then slowly solidified at the surface, forming the rocks and soil. There is the best of evidence that the interior of the earth is still a molten mass. Portions of this molten matter are often poured forth from the craters of volcanoes, which act as so many safety-valves to relieve the pent-up forces within. Again, in deep mines the temperature, as we descend, is found to increase at a rate which shows that a few miles below the surface the heat must be sufficient to fuse all known rocks.

The earth was, then, without doubt, once a liquid mass; and, obeying the tendency of all liquids, would have shaped itself into a perfect sphere, had nothing interfered with this tendency. But we have already seen that the earth is not a perfect sphere, but is flattened at the poles, and protrudes at the equator. How can this form be explained?

Fig. 96.



When the hoop, in Figure 96, is made to rotate rapidly on its axis, it flattens in the direction of the axis, and bulges out in the direction at right angles to this axis. To understand why the hoop bulges out in this way we must remember the first law of motion, namely, that when a body is once put in motion it will move on a straight line if it can. No matter in what way the part of the hoop *a* is put in motion, it has no tendency to move in a circle, but will move in a horizontal straight line, if this is possible. By motion in a straight line, this part *a* would go farther and farther from the central bar. In order to keep it at the same distance from the central bar, a restraining force is necessary. The term *centrifugal force* has been used to express the tendency of the various parts of the hoop to get farther off from the central bar. This term is not a good one, since there is in reality no force. Centrifugal *tendency* would be less objectionable.

The form that the hoop assumes depends upon this centrifugal tendency, and upon the restraining force of the hoop which keeps the parts from moving in a straight line. The less this restraining force, and the greater the centrifugal tendency, the more flattened the hoop becomes.

In the same way, the flattened form of the earth must be accounted for by the centrifugal tendency of its parts when in the liquid state, developed by its rotation on its axis, together with the restraining force which binds the particles of the earth together.

Now, it has been found by careful investigation, that the present form of the earth can be accounted for by the action of the centrifugal tendency, on the supposition that every particle of the earth attracts every other particle with a force varying in the inverse ratio of the square of the distance. We have, then, an evidence here, that every particle of matter in a body attracts every other particle in the inverse ratio of the square of the distance; and, in the case of tides, an evidence that the sun and moon attract every particle of the earth in the inverse ratio of the square of the distance.

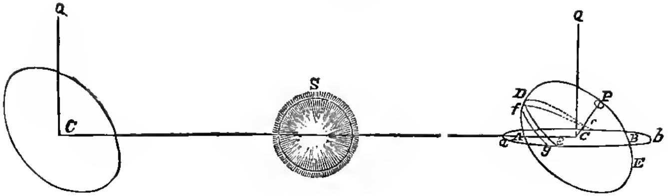
6. *The Precession of the Equinoxes is caused by Gravity.* 'he *precession of the equinoxes* has already been described (70). According to the theory of gravitation, the sun's attraction is stronger at *A* (Figure 97) than at *C*, since *A* is nearer than *C*. Hence the sun is always acting on *A*, the part nearest to it, as if it were pulling it away from the earth's centre. If it pulled the centre and the surface of the earth equally, it would not tend to separate them; but since it pulls the former at *A* more than the latter, it tends to draw it away from the centre towards *S*.

In like manner, as the sun pulls the centre more powerfully than it pulls *B*, it tends to separate them, not by pulling the opposite side *B* from the centre, but by pulling the centre from the opposite side *B*. The general effect of the sun's attraction, therefore, as tending to affect the different parts of the earth, is this: that it tends to pull the nearest parts towards the sun, and to push the most distant parts from the sun.

If the earth were a perfect sphere, this would be a matter of no consequence: it would produce tides of the sea, but it would not affect the motion of the solid parts. But the earth, as we have seen, is not a sphere; it is flattened like a turnip, or has the form of a spheroid. Moreover, the axis of the earth is not perpendicular to the ecliptic; the earth's equator, at all times except the equinoxes, is inclined to the line joining the earth's centre with the sun.

Let us now consider the position of the earth at the winter solstice, shown in Figure 97. The North Pole is away from the sun; the South Pole is turned towards the sun. This spheroidal earth, at this time, has its protuberance, not turned exactly towards the sun, but raised above it. As we have seen, the attraction of the sun is pulling the part *D* of the earth more strongly than it pulls the centre. The immediate tendency of that action is to bring the part *D* towards *a*, supposing *a* to be in the plane

Fig. 97.



passing through the centre of the sun and the centre of the earth. This is because the force pulling *D* towards the sun is equivalent to two other forces: one in the direction of *C D*, tending to pull *D* from *C*; and the other acting at right angles to this direction, tending to carry *D* towards *a*. In like manner, as the sun attracts the centre of the earth more than it attracts the protuberance *E*, which amounts to the same thing as pushing the protuberance *E* away from the sun, there is a tendency to bring *E* towards *b*. The immediate tendency of the sun's pulling, therefore, is so to change the position of the earth that its axis will become more nearly perpendicular to the plane of the ecliptic; but this tendency to change the inclination of the axis is entirely modified by the rotation of the earth. Undoubtedly, if the earth were not revolving, and if the earth were of a spheroidal shape, the attraction of the sun would tend to pull it into such a position that the axis of the earth would become perpendicular to the line *S C*; or (if in the position of the winter solstice) it would become perpendicular to the plane of the ecliptic; but, in consequence of the rotation of the earth, the attraction produces a wholly different effect. Let us consider the motion of a mountain in the earth's protuberance, which, passing through the point *c* on the distant side of the earth, would, in the semi-revolution of half a day, describe the arc *c D e*, if the sun did not act on it (*c* and *e* being the points at which this circle *c D e* intersects the plane of the ecliptic, or the plane of the circle *a b* that passes through the sun). While the protuberant mountain is describing the path *c D e* it is constantly nearer to the sun than the earth's centre is; the difference of the sun's action therefore tends to pull that mountain towards *S*, and therefore (as it cannot be separated from the earth) to pull it downwards, giving to the earth a tilting movement; it will, therefore, through the mountain's whole course, from *c*, make it describe a lower curve than it would otherwise have described, and will make it describe the curve *c f g* instead of the curve *c D e*. The result of this is, that, as it mounts from *c* to *f*, the sun's downward pull draws it towards the ecliptic, and consequently renders its path less steep than it would otherwise be. At *f* it will be a very little lower than it would otherwise have been at *D*; but as the sun's downward pull still acts upon it till it comes to *g*, the steepness of its path between *f* and *g* is increased more than belongs naturally to its elevation at *f*, and becomes in fact the same as it was at *c*, or very nearly so; so that the inclination of the path to the plane of the circle *a b* is the same as at first. But, instead of crossing the circle *a b* at *e*, it will cross at *g*: in other words, it will, in consequence of the sun's action, come to the crossing place earlier than it would have come had the sun not acted. In the remaining half of its rotation, from *g* towards *c* again, it is farther from the sun than the earth's centre is; therefore the sun's action does, in fact, tend to push it away (as already explained); and as it cannot be separated from the earth, this force tends to push the mountain upwards towards the circle *b*, tilting the earth in the same direction as before; the mountain, therefore, in this part, will move in a path higher than it would have moved in without the sun's action, and therefore it will come to its intersection with the circle *a b* sooner than it would if not subject to that action. The inclination of its path (just as in the former half of its rotation) will not be altered. Thus the effect produced by this action of the sun, in both halves of the rotation of this mountain is, that it comes to the place of intersection with the plane of the circle *a b*, or with the plane of the ecliptic, sooner than it otherwise would. And whatever number of points or mountains in the protuberant part of the earth we consider, we shall find the same effect for every one; and therefore, the effect of the sun's action upon the entire protuberance will be the same; that is, its inclination to the circle *a b*, or the plane of the ecliptic, will not be altered; but the places in which it crosses that plane will be perpetually altering in such a direction as to meet the direction of rotation of the earth.

We have spoken as if the protuberance were the only part to be considered. In reality, this protuberance is attached to the remaining spherical part of the earth; but the action of the sun on the different masses of that spherical part balances exactly; so that we need not consider the sun's action upon it at all. The only effect, therefore, of that spherical part will be to impede the motion which the protuberant part would otherwise have: not to destroy it, but to diminish it.

On the whole, therefore, the effect of the sun's action on the spheroidal earth will be, that the points at which the earth's equator intersects the plane of the ecliptic move very slowly in the direction opposite to that in which the earth revolves; but the inclination is not altered.

All that I have said here applies to the position of the earth at the winter solstice. But if we consider the earth at the summer solstice, on the opposite side of the sun, we shall find that the effect of the sun's action is exactly the same. At the equinoxes, the plane of the earth's equator passes through the sun, and then the sun's action does not tend to tilt the earth at all, and consequently does not tend to alter the position of its equator; but at all other times the sun's action produces a motion, greater or less, of the intersection of the earth's equator with the plane of the ecliptic, in a direction opposite to that of the earth's rotation. And this is the motion called the *precession of the equinoxes.*

It is to be observed that the principal part of this precession is produced, not by the sun, but by the moon. The moon's mass is not a twenty millionth part of the sun's, but she is four hundred times as near as the sun. Still she does not pull the earth as a mass with more than a hundred and twentieth part of the sun's force. But since the difference of the distances of the different parts of the earth from the moon bears a greater proportion to the whole distance than in the case of the sun, the effect of the moon in pulling the nearer parts of the earth from the earth's centre, and in pushing the more distant parts of the earth from the earth's centre, is about treble the effect of the sun.

We now see that the precession of the equinoxes is perfectly explained by the supposition that the sun attracts every particle of the earth, and that it cannot be explained without making this supposition.

SUMMARY.

Every particle of matter in our solar system acts upon every other particle with a force varying inversely as the square of their distance from one another.

This supposition is necessary to explain the *tides* (204), the *spheroidal form of the earth* (205), and the *precession of the equinoxes.* (206.)

The *tides* result from the struggle between the attraction of gravity among the particles of the earth, and that between the particles of the earth and those of the sun and moon. (204.)

The *spheroidal form of the earth* resulted from the struggle between the attraction of gravity among the particles of the earth, and the centrifugal tendency of the particles of the earth in the neighborhood of the equator, when the earth was in the liquid state. (205).

The precession of the equinoxes is the result of the struggle between the attraction of the sun and moon upon the earth's equatorial protuberance, and the tendency of this ring always to rotate in the same plane. (206).

HOW TO FIND THE WEIGHT OF THE HEAVENLY BODIES.

207. *The Weight of the Sun and Planets.* — Having thus established the law of gravitation, we next inquire after the strength of this force. Of course we know nothing about the absolute strength of this or any force. We only know their relative strength, and we must compare the force exerted between two bodies with the force exerted between the earth and a pound of water. We want, first, to find the weight of the earth; that is, to find how the force acting between any given body and a pound of water compares with the force acting between this same body and the earth.

208. *The Schehallien Experiment.* — The first experiment for ascertaining this was made at the Schehallien mountain, in Scotland. It was argued that if the theory of gravitation were true, — that is to say, if attraction were produced, not by a tendency to the centre of the earth, or to any special point, but to every particle of the earth's structure, — then, by the fundamental law of gravitation, the attraction of a mountain would be a sensible thing; for a mountain is a part of the earth, with this difference only, that though the mountain is small in comparison with the earth, yet you get so close to the mountain, that its effect

may be very sensible as compared with the effect produced by the rest of the earth. Some parts of the earth are eight thousand miles from us, and their attraction will be comparatively small. It was therefore thought worth while to ascertain whether the attraction of a mountain, would be sensibly felt.

The Schehallien mountain ranges east and west. It was possible to make astronomical observations on the north and south sides; and it was also possible to connect the two places of observation by triangulation (54). Suppose

Fig. 95.

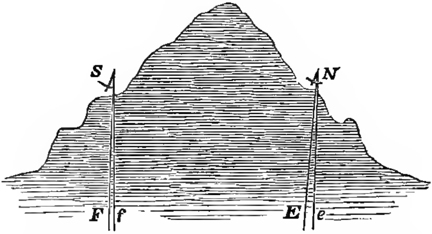


Figure 95 to represent a section of the mountain, north and south; and *N* the northern, *S* the southern observing station. Observations were made at *N* and *S* upon stars with the Zenith Sector (53). By the use of this instrument the difference of the directions of gravity at these two stations was found, exactly in the same manner as the difference of the directions of gravity in the two stations in Figure 25.

The direction of gravity at each station is the result of the gravity of the whole earth (as considered for a moment independently of the mountain), combined with the attraction of the mountain. Supposing that at *N*, if there were no mountain, the direction of gravity would be *Ne;* then introducing the supposition of the mountain, the attraction of the mountain would pull the plumb-line sideways towards the centre of the mountain, and the direction of the gravity would be *N E*. And in like manner, supposing that if there were no mountain, the direction of the gravity at *S* would be *S F*; then, introducing the mountain, the effect of its attraction is to pull the plumb-line towards the centre of the mountain, and the direction of gravity would be *S f*. We see, then, the effect of the mountain. At *N* the direction of gravity is *N E* instead of *N e*, and at *S* the direction of gravity is *S f* instead of *S F*, — that is to say, the two directions which are taken by the plumb-line of the Zenith Sector, make a greater angle than they would if the mountain were not there.

Now we know the general dimensions of the earth; we know what the inclination of the plumb-line at *N* and *S* would be if there were no mountain in the case. If, then, we can find the distance from our observing station at *N* to that at *S*, then we can tell from that distance how much the directions of the plumb-line at *N* and *S* would be inclined if there were no mountain; and we can compare that inclination with the inclination observed by means of the Zenith Sector.

Accordingly, the observations were made in exactly the same manner as those made for determining the figure of the earth (53 – 56). The Zenith Sector was carried to *N*, and certain stars were observed; the Zenith Sector was then carried to *S*, and the same stars were observed at that place. By means of these observations of the stars, the actual inclinations of the plumb-line at the two places were found. The next thing done was to carry a survey by triangulation across the mountain. This was done in the most careful way in which the best surveyors of the time could accomplish the task. The result was, that the distance between the stations was found to be such, that, supposing there were no mountain in the case, the inclination of the two plumb-lines ought to be 41 seconds.

It was found, from the observations by the Zenith Sector, that the inclination of the plumb-lines actually was 53 seconds.

The difference between the two was the effect of the mountain. The mountain had pulled the plumb-line at one station in one direction, and at the other station in the opposite direction, to such a degree, that the two plumb-lines, instead of making an angle of 41 seconds, made an angle of 53 seconds; or, in other words, the sum of the effects of the two attractions of the mountain, on opposite sides, was twelve seconds.

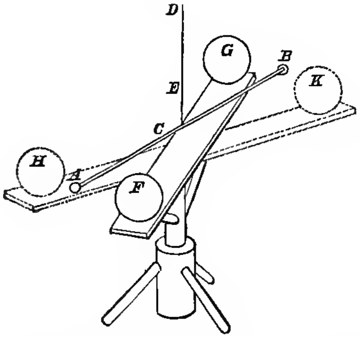
The next thing was, to draw from this observation a determination of the mean density of the earth. The mountain was surveyed, mapped, levelled, and measured, in every way, so completely that a model of it might have been made; it was then (for the sake of calculation) conceived to be divided into prisms of various forms: the attraction of every one of these was computed, on the supposition that the mountain had the same density as the mean density of the earth; and by means of this, the attraction of the whole mountain was found on the same supposition.

Thus it was found, that, if the density of the mountain had been the same as the mean density of the earth, the sum of the effects of the attractions of the mountain at *N* and *S* would have been about $\frac{1}{9933}$ part of gravity. But the observed sum of effects was 12 seconds, which corresponds to $\frac{1}{17804}$ part of gravity. Hence the density of the mountain is only about $\frac{5}{9}$ of the earth's mean density; or the earth's mean density is nearly double the mountain's density. The nature of the rocks composing the mountain was carefully examined, and their density as compared with that of water was ascertained; and thus the mean density of the earth was found to be something less than five times the density of water.

209. *The Cavendish Experiment.* — After this another set of experiments was made; first by Mr. Henry Cavendish, a rich man, much attached to science (from whom the experiment of which we are speaking received the name of the Cavendish Experiment); afterwards by a Dr. Reich; and finally, in a very much more complete way, by Mr. Francis Baily, as the active member of a committee of the Astronomical Society of London, to whom funds were supplied by the British Government.

The shape in which the apparatus is represented, in Figure 96, is that in which it was used by Mr. Baily. There are two small balls, *A*, *B*, (generally about two

Fig. 96.



inches in diameter,) carried on a rod *A C B*, suspended by a single wire *D E*, or by two wires at a small distance from each other. By means of a telescope, the positions of these balls were observed from a distance. It was of the utmost consequence that the observer should not go near, not only to prevent his shaking the apparatus, but also because the warmth of the body would create currents of air that would disturb everything very much, even though the balls were enclosed in double boxes, lined with gilt paper, to prevent as much as possible the influence of such currents. When the position of the small balls had been observed, large balls of lead, *F*, *G*, about twelve inches in diameter, which moved upon a turning frame, were brought near to them; but still they were separated from each other by half a dozen thicknesses of wooden boxes, so that no effect could be produced except by the attraction of the large balls. Observations were then made to see how much these smaller balls were attracted out of their places by the large ones. By another movement of the turning frame, the larger balls could be brought to the position *H K*. In every case, the motion of the small balls produced by the attraction of the larger ones was undeniably apparent. The small balls were always put into a state of vibration by this attraction; then by observing the extreme distances to which they swing both ways, and taking the middle place between those extreme distances, we find the place at which the attraction of the large balls would hold them steady.

Suppose, now, the attraction of the large balls was found to pull the small balls an inch away from their former place of rest, what amount of *dead pull* does that show? In order to ascertain this we must compare the vibrations of the balls with those of an ordinary pendulum.

We have seen that when a pendulum-ball is pulled aside and let go, it begins to vibrate. The force of gravity acting upon the pendulum-ball is resolved into two parts, one of which acts on the ball in the direction of the pendulum-rod, and the other sideways. The former of course does not affect the movement of the pendulum at all; it is the latter which causes the pendulum to vibrate. This latter force increases with the distance the pendulum is pulled aside, and always bears the same ratio to the weight of the pendulum-ball as the distance the ball is drawn aside bears to the length of the pendulum. As a pendulum beating seconds is 39.139 inches long, the force which will pull it one inch sideways will then be $\frac{1}{39.139}$ of its weight.

In the case of the lead balls suspended by a wire, when they are pulled aside by the large balls, they begin to vibrate. This vibration is caused by the *torsion*, or twist, of the wire. This torsion increases with the distance the balls are pulled aside, precisely as the force which causes the pendulum to vibrate increases with the distance it is pulled aside. Hence the balls will vibrate exactly like a pendulum. If, then, the balls vibrate in one second, and are pulled aside one inch, the force which pulls them aside must bear the same proportion to their weight that the force which pulls a seconds pendulum one inch aside bears to the weight of the pendulum-ball; that is, it must be $\frac{1}{39.139}$ of their weight.

Then it is known, as a general theorem regarding vibrations, that, to make the vibrations twice as slow, we must have forces (for the same distances of displacement) four times as small; and so in proportion to the inverse square of the times of vibration. Thus if balls or anything else vibrate once in ten seconds, the dead pull sideways corresponding to an inch of displacement is $\frac{1}{3913.9}$ of their weight. So that, in fact, all that we now want for our calculation, is the time of vibration of the suspended balls. This is very easily observed; and then, on the principles already explained, there is no difficulty in computing the dead pull sideways corresponding to a sideways displacement of one inch; and then (by altering this in the proportion of the observed displacement, whatever it may be) the sideways dead pull or attraction corresponding to any observed displacement is readily found. The delicacy of this method of observing and computing the attraction of the large balls may be judged from the fact that the whole attraction amounted to only about $\frac{1}{20,000,000}$ part of the weight of the small balls, and that the uncertainty in the measure of this very small quantity did not amount probably to $\frac{1}{40}$ or $\frac{1}{50}$ of the whole.

Then the next step was this: knowing the size of the large balls and their distances from the small balls in the experiment, and knowing also the size of the earth, and the distance of the small balls from the centre of the earth, we can calculate what would be the proportion of the attraction of the large balls on the small balls to the attraction of the earth on the small balls (that is, the weight of the small balls), if the leaden balls had the same density as the mean density of the earth. It was found that this would produce a smaller attraction than that computed from the observations. Consequently, the mean density of the earth is less than the density of lead in the same proportion; and thus the mean density of the earth is found to be 5.67 times the density of water.

210. *Another Method of finding the Weight of the Earth.* — It has been found that if the law of universal gravitation be true, the attraction of the whole earth, considered as a sphere, on a body at its surface, is the same as if the whole matter of the earth were collected at its centre. It has also been found that the attraction of the earth on a body within its surface is the same as if the spherical shell situated between the body and the earth's surface were removed; or is the same as if all the matter situated nearer to the earth's centre than the body were collected at the centre, and all the matter situated at a greater distance were removed.

If the earth were of uniform density throughout, it would follow from these propositions that the force of gravity at the bottom of a mine would be less than the force at the top. To show this, suppose that the mine reached halfway to the centre of the earth. Then (since the volumes of spheres vary as the cubes of their diameters) the quantity of matter nearer to the earth's centre than the bottom of the mine would be only one eighth of the whole quantity of matter in the earth. But the attraction of a quantity of matter at the earth's centre would be more powerful on a body at the bottom of a mine than on one at the top, in the inverse ratio of the squares of the distances of the bodies from the earth's centre; that is, in the present case, in the ratio of four to one. Hence the attraction on a body at the bottom of a mine would be, on the whole, less than the attraction on a body at the top, in the ratio of one to two.

If, however, the earth be not of uniform density, but its density increase towards the centre, then, though the attracting mass which acts on a body at the bottom of the mine be smaller, yet the diminution in the force of gravity so occasioned may be more than compensated by the comparative nearness of the attracted body to the denser parts of the earth. From the two laws of the attraction of spheres, which have been stated above, it is possible to calculate the ratio which the force of gravity at the bottom of the mine would bear to that at the top, on any supposition we choose to make as to the ratio between the mean density of the earth and the density at the surface; so that, if we know one ratio we can immediately infer the other. Now, pendulum observations afford us the means of determining the force of gravity at any place; and therefore, if the times of vibration of a pendulum at the top and bottom of a mine be found, the ratio of the force of gravity at the top to that at the bottom may be calculated, and thence the ratio of the mean density of the earth to that of its surface.

The pendulum is made of metal; it turns with a hard steel prism, having a very fine edge, upon plates of agate, or some very hard stone. It swings like the pendulum of a clock. But it must be observed that a clock pendulum will not do for this purpose, because there are other forces besides gravity acting upon it; that is to say, the clock weights acting through the train of the clock wheels. It is necessary to have a detached pendulum. Now we wish to know how many vibrations that pendulum would make in a day.

In modern experiments of this kind, the vibrations of the detached pendulum have been compared with the vibrations of a clock pendulum. The mode usually adopted is this: a detached pendulum is placed in front of a clock; a person is watching with a telescope; he watches when the two pendulums are going the same way; he remarks whether the vibrations of the detached pendulum recur faster or slower than those of the clock pendulum; he sees that the vibrations separate more and more, till the two pendulums actually move in opposite ways; after this, they begin to move more nearly in the same way, and at length move exactly in the same way. Perhaps the number of vibrations between these two agreements of motion may be 500. If we can determine the time when the two pendulums swing the same way, we find how long it is before one pendulum gains two vibrations upon the other. Then suppose that the detached pendulum is going slower than the clock pendulum; and suppose that $7\frac{1}{2}$ minutes elapse between two agreements of motion of the pendulum: this shows that while the clock has gone $7\frac{1}{2}$ minutes, or while its pendulum has made 450 vibrations, the detached pendulum has made only 448 vibrations. Now, the clock is going day and night, and by means of observation with the transit instrument, we can find how many hours, minutes, and seconds the clock hands pass over in one day, or how many vibrations the clock pendulum makes in one day. Then, as the detached pendulum makes 448 vibrations for every 450 made by the clock pendulum, we find at once how many vibrations the detached pendulum makes in twenty-four hours.

Some corrections for the effect of temperature in altering the length of the pendulum, and for other circumstances, are necessary. The method just described is exceedingly delicate. There is no difficulty in ascertaining by it the number of vibrations which the detached pendulum will make in a day, with no greater error than one tenth of a vibration, or with an error not exceeding one eight hundred thousandth part of the whole.

This mode of determining the weight of the earth was put in practice by Airy, the Astronomer Royal of England, at the Harton Coal Pit, in the year 1854. The mean density deduced from his observations is 6.565 times that of water: a value considerably exceeding that found from the Schehallien and Cavendish experiments.

Each cubit foot of the earth is thus found to weigh, on an average, about six times as much as a cubic foot of water. In the earth there are 259,800,000,000 cubic miles, and in each cubic mile 147,200,000,000 cubic feet. From these data the whole weight of the earth may be readily found.

211. *The Weight of the Sun.* — We have already seen that the motion of a body in a second, when drawn by gravity, is directly proportional to the force acting upon it. Remembering this, we can easily find the weight of the sun as compared with that of the earth.

We have found, in Figure 85, that the earth draws the moon through 10.963 miles in one hour, the moon being at the distance of 238,800 miles from the earth; and in Figure 86, that the sun draws the earth through 24.402 miles in one hour, the earth being at the distance of 95,000,000 miles from the sun. In order to compare these attractions, we must reduce them both to the same distance. If the earth draws the moon through 10.963 miles in an hour when at the distance of 238,800 miles, how far would it draw the moon in an hour if it were at the distance of 95,000,000 miles? Diminishing 10.963 in the proportion of the inverse squares of the distances, we find that the earth would draw the moon through 0.00006927 mile or 4.389 inches in an hour, if it were at the distance of 95,000,000 miles. Comparing this with 24.402 miles, through which the sun draws the earth or moon when at the same distance, we find that the sun's attraction is 352,280 times as great as the earth's, and therefore, that the sun's mass is 352,280 times as great as the earth's.

We have learned (87) that the sun's bulk is some 1,400,000 times as great as the earth's bulk. Therefore the sun's mean density is only about $\frac{1}{4}$ of the earth's mean density, or about 1.4 times the density of water.

212. *The Weight of the Planets.* — The method which has been used above for comparing the mass of the earth with that of the sun, is also used for comparing the mass of Jupiter, Saturn, Uranus, or Neptune, with that of the sun; and in all cases where the moons can be easily observed, it can be applied with very great accuracy. For those planets which have no moons there is considerable uncertainty. The only way in which their weight can be determined is by their disturbance of other planets. For instance, in certain positions, the earth is disturbed by Mars a few seconds, say six or eight. We compute what would be the amount of perturbation if the planet Mars were as big, or half as big, as the earth, and we alter the supposition till we find a mass which will produce perturbations equal to those which we observe. This is the process of trial and error, which has already been described. In this manner the masses of Mars and Venus are determined. That of Mars is not very certain; that of Venus is more certain, — both because it produces larger perturbations of the earth, and because its attraction tends to produce a continual change in the plane of the ecliptic,

which in many years amounts to a very sensible quantity. The mass of Mercury is still very uncertain. Lately attempts have been made to deduce it from the perturbations which Mercury produces in the motions of one of the comets.

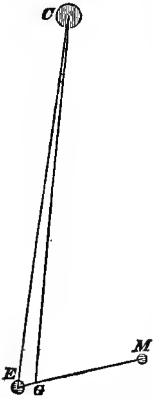
213. *The Weight of the Moon obtained from the Tides.* — There is, however, one mass which is more important than the others, and that is the mass of the moon. There are several methods by which this mass is determined.

One method is by comparing the tides at different times. By comparing the spring-tides with the neap-tides, we can find the proportion of the effect produced by the moon to that produced by the sun. Now, the tides are produced, not by the whole attraction of the moon and the sun upon the water, but by the difference between their attraction upon the water and their attraction upon the mass of the earth, by which difference the moon (and also the sun) draws the water nearest to it away from the earth, and draws the earth away from the water which is farthest from the moon.

With proper investigation it is possible to find, from the tidal effects of the sun and moon, the proportion of their differences of attraction. And knowing this proportion, and knowing the distances of the sun and moon, we can find the proportion of their masses.

214. *Another Method of finding the Moon's Weight.* — In Figure 100, suppose C to be the sun, E the earth, M the moon. We have spoken continually of the sun's attraction upon the earth, and of the earth's revolution round the sun, as if the sun were the only body whose attraction influenced in a material degree the earth's movement. But in reality the moon also acts in a very sensible degree upon the earth. And the immediate effect upon the motion of the earth can be found. Draw a line from E to M, and in this line take the point G, so

Fig. 100.



that the proportion of *E G* to *G M* is the same as the proportion of the weight of *M* to the weight of *E;* or so that if *E* and *M* were like two balls fastened upon the ends of a rod, they would balance at *G*. This point would be their common *centre of gravity*. Then investigation shows that the motion of the earth may be almost exactly represented by saying that the point *G* travels round the sun in an ellipse, describing areas proportional to the times (according to Kepler's laws), and that the earth *E* revolves round the point *G* in a month, being always on the side opposite to the moon.

Consequently, the direction in which the earth would be seen from the sun (and therefore the direction in which the sun is seen from the earth) depends in a certain degree on the distance *E G*. And, therefore, if we observe the sun regularly, and if we compute where we ought to see the sun, according to Kepler's laws, the difference between these two directions will be the angle *E C G;* and knowing the distance *C G*, we can then compute the length of *E G*, and the proportion which it bears to *G M;* and this proportion is the same as the proportion of the mass of the moon to the mass of the earth.

These different methods agree very well in giving the result that the mass of the moon is about $\frac{1}{80}$ of the earth's mass.

SUMMARY.

The earth weighs about six times as much as a globe of water of the same size would weigh.

The weight of the earth can be found in three ways:—

(1.) By comparing the attraction exerted by the whole earth with that exerted by a mountain, and then ascertaining the density of the mountain; as in *the Schehallien experiment.* (208.)

(2.) By comparing the force exerted by the earth upon two small balls of lead with that exerted upon the same by two large balls of lead; as in *the Cavendish experiment.* (209.)

(3.) By observing the vibration of a pendulum at the earth's surface, and at the bottom of a deep mine; as in *the Harton Coal Pit experiment.* (210.)

Knowing the weight of the earth, *the weight of the sun* can be found by comparing the distance the moon is drawn by the earth in an hour with the distance the sun draws the earth in the same time. (211.)

Having found the weight of the sun, we can find *the weight of any planet which has moons* by comparing the distance they draw their moons in an hour with the distance they themselves are drawn by the sun in the same time. The *weight of those planets which have no moons* can be found by the perturbations they cause in the motions of other planets. (212.)

The *weight of the moon* can be found by comparing her effect with that of the sun in producing the *tides* (213), and by means of the apparent displacement of the sun, occasioned by the action of the moon upon the earth. (214.)

The earth not only draws the moon towards itself, but is also itself drawn towards the moon; and the moon does not really revolve about the earth, but both these bodies revolve about their common centre of gravity. This centre of gravity is very much nearer the earth than the moon, because the mass of the earth is much greater than that of the moon. (214.)

The same thing is true of the sun and the planets. He not only draws the planets towards himself, but is himself drawn towards them; and they do not really revolve about him as a centre, but both they and he revolve about their common centre of gravity. Since the mass of the sun very much exceeds that of all the planets, the common centre of gravity of the whole solar system lies within the surface of the sun.

So, too, in the binary and multiple stars, there is no one star about which the others revolve, but each revolves about the common centre of gravity of the system. If both components of a binary system have equal masses, their common centre of gravity would be midway between them.

GENERAL SUMMARY.

When a free pendulum is vibrating in any part of the earth except at the equator, its direction of vibration appears constantly to change; and from this we know that the earth must rotate on its axis from west to east once in twenty-four hours.

The planets follow such irregular paths among the stars that their motion can be explained only by supposing that they all describe ellipses which have the sun at one focus; and the motion of the moons can be explained only on the supposition that they describe ellipses about their planets.

The time it takes the earth to complete a revolution about the sun is found by direct observation; and then, by observing their synodical revolutions, the sidereal periods of the planets can be computed.

The relative distances of the earth and the planets from the sun are found by observing the greatest elongation of the inferior planets, and the daily retrogression of the superior planets at the time of their opposition.

The real distance from the earth to the sun is found by observation of the transits of Venus, and by measuring an arc of the earth's meridian.

Telescopes which are pointed at the moon from Greenwich and the Cape of Good Hope, differ in direction; and by measuring this difference of direction, the distance of the moon from the earth is ascertained.

When a telescope is pointed to a fixed star, from opposite points of the earth's orbit, there is often an appreciable difference in its direction. This difference can be measured, and, by means of it, we can find the distance of the stars.

All the stars in the neighborhood of the constellation Hercules appear to be spreading away from a point in that constellation; and from this we know that our sun is travelling through space towards that point.

The stars are slowly changing their configurations, and must therefore be in motion.

The sun is about a million and a half times as large as the earth; he rotates on his axis in about twenty-five days; he is surrounded by an atmosphere several thousand miles in depth; and his photosphere is probably a stratum of clouds suspended in this atmosphere. The spots on his disc are probably caused by the cooling of portions of these clouds by downward currents; while the faculae are caused by the raising of other portions into ridges by upward currents.

The moon presents a very rough and rugged surface; and there are indications of similar inequalities in the case of Mercury, Venus, and Mars.

The four inner planets are all of moderate size. They all have atmospheres, and all appear to rotate on their axes in about twenty-four hours; and of these, the earth alone has a moon. The four outer planets form another group, and differ strikingly from these in their enormous size, their rapid rotation on their axes, and in their complex systems of satellites. Between this inner and outer group of planets there is a large number of telescopic bodies, forming the well-marked group of the Minor Planets.

Many of the stars change their color and their brightness. Some undergo these changes in short and regular periods, and others at long and perhaps irregular intervals.

The vibration of a free pendulum in a vacuum can be explained only on the supposition that a moving body always tends to move in a straight line and with an unvarying velocity.

Gravity has the same effect upon a moving body as upon a body at rest.

Gravity acting upon a body near the earth causes it to fall from a state of rest 193 inches in a second.

Since the planets and moons describe equal areas in equal times, the force which deflects their paths must be directed in the case of the planets towards the sun, and in the case of the moons towards their planets.

On the supposition that a stone at the distance of the moon would be still acted upon by gravity, but with a force which diminishes as the square of the distance from the earth's centre increases, it would fall just as far in a unit of time as the moon is drawn towards the earth in the same time. We therefore conclude that gravity extends to the moon.

The distances the planets are drawn towards the sun in a given time are exactly in the inverse ratio of the squares of their distances from him; and the force which causes a planet to describe an ellipse must vary inversely as the square of its distance from the sun. Hence we conclude that gravity acts between the sun and the planets.

The perturbations of the planets can be explained only by the supposition that each attracts the others with a force which varies directly as the mass of the bodies acted upon, and inversely as the square of their distances from one another. We therefore conclude that all the members of the solar system are acted upon by gravity.

Many of the stars move in elliptical orbits. Hence we conclude that gravity acts among all the heavenly bodies.

The tides, the spheroidal form of the earth, and the precession of the equinoxes can be explained only on the supposition that gravity acts among the particles of the earth, and between these and the sun and moon. Gravity therefore acts upon the heavenly bodies, not as wholes, but upon the particles of which they are composed.

By comparing the pull exerted by one piece of lead upon another with the pull exerted by the earth upon the sun, we are able to express the pull of the earth in pounds. The distance a body is pulled from a state of rest in a second is directly proportional to the effective pull of gravity acting upon it. The earth's gravity can then be compared with that of the sun by comparing the distance the moon is drawn through by the earth in a second with the distance the earth is drawn through in a second by the sun. The gravity of a planet which has a moon can be compared with the gravity of the sun by comparing the distance the planet draws its moon in a given time with the distance the sun draws the planet in the same time. The weight of a planet without a moon can be found by comparing its disturbance of another planet's motion with the disturbance of the same planet's motion by a planet whose weight is known.

CONCLUSION.

We have now become somewhat acquainted with the heavenly bodies, and with the force by which they act upon one another.

Our attention was first called to the motions of these bodies, and we find that they are all describing accurate circles about the earth from east to west once in twenty-four hours. An examination of the vibration of a free pendulum taught us, however, that the earth is rotating from west to east, and that the motion we had first observed is only apparent. We soon, however, discovered that certain of these bodies are moving eastward among the stars with considerable rapidity, and usually in very irregular paths. We noticed the rude and complex systems by which the ancients attempted to account for these motions, and were taught by Kepler that these bodies, together with the earth, form a group by themselves, and that they all, except the moon, revolve in ellipses about the sun. These bodies are the planets.

We next inquired how long it takes each of these bodies to complete a revolution about the sun, and the simple observation of the interval between two successive conjunctions of the sun with the same fixed star, and with each of the planets, enabled us to answer this inquiry.

We next sought the relative distances of the planets from the sun, and were enabled to find these distances by simply observing the greatest elongation of the inferior planets and the retrogression of the superior planets during one day when they are in opposition. We then found the actual distance of the earth from the sun simply by observing a transit of Venus, and by measuring a short distance upon the earth.

We next directed our attention to the moon, and saw that her motion could be explained only on the supposition that she revolves about the earth in an ellipse; and that her distance from the earth's centre could be found by simply pointing a telescope at her from the observatories at Greenwich and the Cape of Good Hope, and measuring the inclination of these telescopes to each other.

Having learned so much about the motions and distances of the moon and the planets, we next inquired the distance between our earth and the stars. We found that in some cases the telescope does not have quite the same direction when pointed at a fixed star from opposite points in the earth's orbit, and that we can measure this difference of direction, and thus ascertain the distance of the stars from the earth. This inquiry led us to the unexpected conclusion that our sidereal system is so vast, that, were all the stars which compose it suddenly destroyed, it would be some four years before light, travelling with the velocity of 190,000 miles a second, could inform us of the destruction of the nearest, and several thousand years before it could inform us of the destruction of the most remote. And at the same time we caught sight of outlying sidereal systems, which probably are not inferior in magnitude to our own, but which even with the aid of a telescope appear only as minute patches of hazy light.

Since our sun at the distance of the fixed stars would shine only as a star of the second magnitude, we saw that all the stars are suns; and a more careful examination of these bodies taught us that they appear fixed only because they are separated from us by such immense distances. We found that our sun is moving toward a point in the constellation Hercules; that many of the stars have proper motions; and that Arcturus and other stars are really moving through space with a velocity about six hundred times that of a cannon-ball.

We thus learned that our sun, moons, and planets probably form but one of millions of such groups, and that while in each of these groups the moons are revolving about the planets and the planets about the suns, the suns themselves are revolving about one another in systems more or less complicated.

After having thus learned the motions and distances of the members of our solar system, and the motions and distances of a few of the stars, we next inquired what is known of the physical features of each of these bodies. On this point we found our knowledge to be extremely limited. The telescope has revealed to us that there are rocks, mountains, and volcanic craters upon the surface of the moon, and we think that Mercury, Venus, and Mars resemble the earth and moon in this respect. We know that the earth and Mars rotate on their axes in about twenty-four hours, and we think that Mercury and Venus rotate in the same time. We know that each one of these planets has an atmosphere. We know nothing of the physical constitution of the minor planets. And of the outer planets, we know only that Jupiter and Saturn rotate on their axes in about ten hours, and that Jupiter is attended by four moons, and Saturn by eight moons and a complex system of rings. Of Uranus and Neptune we know only that they are attended by a few moons, whose motions are retrograde. We know that the sun rotates on his axis in about twenty-five days, that he has an atmosphere several thousand miles deep, and that there are dark spots on his disc.

We think that in the atmosphere of the sun the vapors of such substances as iron, which pass from the gaseous to the liquid state at a temperature above a white heat, condense into a cloudy stratum, giving rise to the solar photosphere, and that these clouds are sometimes cooled by downward and sometimes lifted into ridges by upward currents, and that this is the cause of the spots and faculæ.

We next learned that a moving body when left to itself will always move in a straight line and with uniform velocity.

A further study of the paths of bodies falling to the earth, and of those of the heavenly bodies, taught us that these bodies are all acted upon by a force which tends to draw them together, and whose intensity varies directly as the mass of the bodies acted upon and inversely as their distance from one another; and that this force is the same as that which draws a stone to the earth.

We have found that the action of this force is so well understood that we can explain by it not only the general form of the paths described by the heavenly bodies, but the many and complicated disturbances which the planets cause in one another's motions; that we are able to predict disturbances whose existence had not been revealed by observation, and by the study of perturbations already observed to point out the position of planets before unknown.

We have found, too, that the knowledge of the force which acts among the heavenly bodies and through planetary distances is of no less practical value than the knowledge of the forces of affinity and electricity which act among the atoms and molecules of matter and through atomic and molecular spaces. For it is only by a correct knowledge of the action of this force that astronomers become acquainted with all the irregularities of the moon's motion, and are enabled to construct tables by which navigators at sea can read longitude from the heavens with accuracy.

We have thus seen that the groupings and movements of the heavenly bodies result from a single force acting among them; and that, by a study of these groupings and movements, we have arrived at a complete knowledge of the law by which this force acts.

In another part of this Course we have seen that bodies are made up of atoms which are grouped into molecules and into masses. Are these atoms also in motion, and are their groupings and movements regulated by a single force acting among them; and are electricity, light, and heat only disturbances of these atomic and molecular movements? The achievements of astronomy lead us to hope that we may at a future day be able to measure the minute distances between the atoms, and to discover in what paths they move and the laws by which the force acting among them is governed.

IV.

ORIGIN, TRANSMUTATION, AND CONSERVATION OF ENERGY.

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215. *Actual and Potential Energy.* — A weight at rest on the ground cannot move, or do work of any kind. It is therefore said to have no *energy*, or ability to do work. If the same weight is raised from the ground and suspended, it still manifests no energy; that is, so long as it remains suspended it can do no work. The moment it is released, however, it begins to fall, and in falling can drive a clock, or do other work. While, then, a weight rests on the earth, it has no energy, and it is not possible for it to manifest any. When the weight is raised from the earth, it may not manifest any energy, but it is always possible for it to do so. In this case it is said to have a *possible* or *potential* energy. When the weight is falling it has an *actual* or *dynamical* energy. Every moving body, then, has a *dynamical* energy; and every body which is so situated that it can be moved by gravity has a *potential* energy.

216. *Mechanical, Molecular, and Muscular Energy.* — When, as in the case of the falling weight, the motion of a body is visible, its energy is called *mechanical*. But we have seen that heat can separate and rearrange the molecules of a body, and, in so doing, it does work. Again, light can effect chemical changes; and electricity in motion can turn a magnetic needle, or make a magnet. The motions which manifest themselves as heat, light, and electricity are invisible. They are not movements of a body

as a whole, but of its molecules among themselves. The energy of these movements is called *molecular*.

The movements of bodies which manifest themselves as sound are either visible, or can be easily made so. The energy of these movements, then, is to be regarded as mechanical rather than as molecular.

The energy manifested in the bodies of animals is called *muscular* energy.

There are, then, three kinds of dynamical energy, — that manifested in ordinary motion and in a sounding body, called *mechanical* energy; that manifested in the molecular movements of light, heat, and electricity, called *molecular* energy; and that manifested in the bodies of animals, called *muscular* energy.

It must be understood, however, that these kinds of energy do not differ in themselves, but only in the way in which they are manifested.

217. *Affinity, Cohesion, and Gravity are the Forces which tend to convert Potential into Dynamical Energy.* — We have seen that a weight raised from the earth has a potential energy, which gravity tends to convert into dynamical energy. When the molecules of a body are separated by melting and boiling, cohesion tends to draw them together again, and thus to convert their potential into dynamical energy, which appears as heat. Again, when the elements of a compound are separated, they have a potential energy, since it is possible for them to unite again. The force which tends to draw them together, and thus to convert their potential into dynamical energy, is affinity.

When, therefore, the *atoms* of different elements are separated, they have a potential energy, which *affinity* tends to convert into dynamical energy. When the *molecules* of a body are separated, they have a potential energy, which *cohesion* tends to convert into dynamical energy. When *bodies* are separated, they have a potential energy, which *gravity* tends to convert into dynamical energy. The first two forces act through *insensible* distances, and give rise to *molecular* energy; the last acts through *all* distances, and gives rise to *mechanical* energy.*

218. *Mechanical Energy may be converted into Heat.* — We have a familiar illustration of this in the lighting of a friction match. A portion of the energy employed in rubbing the match is converted by the friction into heat, which ignites the phosphorus. Here there is a double transfer of energy. The muscular energy of the arm is converted into mechanical energy in the moving match, and a part of this into heat by the friction.

Before matches were invented, the flint and steel were used for the same purpose. The steel was struck against the flint, and the spark obtained was caught in tinder. A part of the mechanical energy of the steel appeared as heat in the spark.

Indians are said to obtain fire by vigorously rubbing together two pieces of dry wood. In this case, too, the heat is nothing but mechanical energy appearing in a new form.

Iron can be heated red-hot by hammering it. And, generally, heat is developed by friction and percussion.

219. *Count Rumford's Experiment.* — Until recently, heat has been regarded as a substance, or a form of matter. It was supposed that this substance was taken up by bodies much as water is taken up by a sponge, and in this state it was said to be *latent;* but when bodies were rubbed or struck together, a portion of the heat was squeezed out of them, and thus became *sensible.* The more heat a body could take up and render latent, the greater was said to be its *capacity* for heat.

This theory was first attacked by Count Rumford, in

* Magnetism, like gravity, tends to draw bodies together; but it does not, like gravity, act upon all bodies.

1798. While superintending the boring of cannon in the workshops of the Military Arsenal at Munich, he was struck by the high temperature which the cannon acquired in the process, and the still more intense heat of the metallic chips separated from it by the borer. He was thus led to inquire into the source of this heat. Was it squeezed out of the chips? If so, their capacity for heat must be reduced sufficiently to account for all the heat rendered sensible. He found, however, that the chips had exactly the same capacity for heat as slices of the same metal cut off by a fine saw where heating was avoided. It was evident, then, that the heat could not have been furnished at the expense of the metallic chips.

Count Rumford then constructed a machine for the express purpose of generating heat by friction. It consisted of a metallic cylinder, which was turned on its axis by horse-power while a blunt borer was forced against its solid bottom. To measure the heat, a small hole was bored into the cylinder, in which was placed a thermometer. At the beginning of the experiment the temperature of the cylinder was 60°. At the end of 30 minutes, or after the cylinder had made 960 revolutions, the temperature had risen to 130°. He now removed the dust which the borer had detached from the bottom of the cylinder, and found it to weigh only 837 grains, or less than two ounces. "Is it possible," he exclaimed, "that the very considerable quantity of heat produced in this experiment — a quantity which actually raised the temperature of 113 pounds of gun-metal at least 70° Fahrenheit — could have been furnished by so inconsiderable a quantity of metallic dust, and this merely in consequence of a change in its capacity for heat?" But he found by careful experiment that the capacity of the metal for heat was changed very slightly, if at all, by the boring. He concluded, therefore, that heat could not be a material substance, but was merely a mode of *motion*. The mechanical motion of the borer had been converted into molecular motion, and appeared as heat.

220. *Sir Humphrey Davy's Experiment.* — Sir Humphrey Davy proved that heat is not material, by an experiment even more conclusive than Count Rumford's.

We have already seen that a large amount of heat is rendered latent in converting ice into water. A pound of water, therefore, contains considerably more heat than a pound of ice; and, of course, it would be impossible to melt ice by the heat squeezed out of it. But Sir Humphrey Davy found that he could melt ice by rubbing two pieces of it together. If, then, heat is material, the friction must have induced some change in the pieces of ice which enabled them to attract heat from the bodies with which they were in contact. He next, by means of clock-work, caused two pieces of ice to rub together in an exhausted receiver, and they melted as before, showing that the heat could not have been taken from the air. It might, however, have been conducted to the ice through the pump-plate and the clock-work. He next placed a piece of ice upon the pump-plate, and set the clock-work upon that; and the ice was again melted by the friction. If the heat had been drawn up through the ice, the ice would have been colder at the top than at the bottom. But he had cut a groove in the upper face of the ice, and had filled it with water. As the temperature of the ice was 32°, this water would have frozen had the temperature of this surface fallen; but the water did not freeze. It was evident, then, that the heat could not have come up through the pump-plate. It might still be objected that the heat could have come by radiation from the receiver; but the receiver was kept colder than the ice, and hence must have radiated less heat to the ice than the ice radiated back again.

If, then, heat is matter, we are driven to the conclusion that this matter must have been created by the friction. Now, we know that in the friction mechanical energy disappeared, while heat was produced. The only rational inference, then, seems to be, that the mechanical energy was transformed into heat; in other words, that the mechanical motion of the ice was converted into the molecular motion of heat.

221. *All Mechanical Energy is ultimately converted into Heat.* — When a falling body strikes the earth, it becomes heated. In this case the whole energy of the body is converted into heat. When bodies are rubbed together, their energy, as we have seen, is converted into heat.

The energy of a running stream is gradually converted into heat by the friction against its banks and bed and among its particles. If it is made to turn the wheels of a factory on its way, the rubbing of the parts of the machinery against each other and against the air, together with the various kinds of work done by the machinery, converts the mechanical energy of the water-wheel into heat.

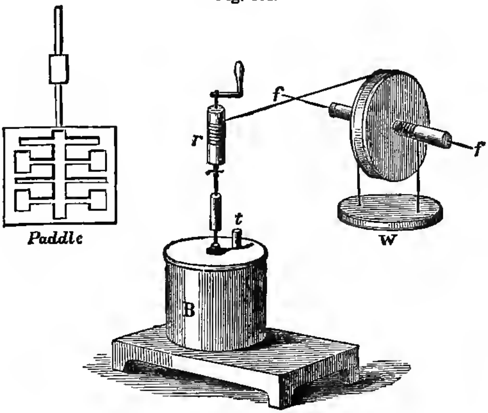
A railway train is really stopped by the conversion of its motion into heat. When this has to be done quickly, the change is hastened by increasing the friction by means of the brakes. On the other hand, in order to prevent the loss of energy while the train is in motion, the axles of the wheels are kept carefully oiled, that they may turn with as little friction as possible.

When unlike substances are rubbed together, a part of the energy is first converted into electricity, but ultimately into heat.

222. *When Mechanical Energy is converted into Heat, the same Amount of Energy always gives rise to the same Amount of Heat.* — This was first shown by Joule, who began his experiments in 1843 and continued them till

1849. He converted mechanical energy into heat by means of friction. He first examined cases of the friction of solids against liquids. The apparatus used for this purpose is shown in Figure 101. *B* is a cylindrical box hold-

Fig. 101.



ing the liquid. In the centre of the box is an upright axis, to which are attached eight paddles like the one shown in the figure. These revolve between four stationary vanes, which prevent the liquid from being carried round. The paddles are turned by means of the cord *r* and the weight *W*. The size of the weight is such that it descends without acquiring any velocity, and hence all its energy is expended in the friction of the paddles. The degree to which the liquid became heated by the friction was shown by a thermometer at *t*. Knowing the weight of the liquid, its specific heat, and the rise of temperature during the experiment, the amount of heat generated could be readily calculated.

With this machine Joule found that, whatever the liquid he used, a weight of one pound falling through 772 feet, or 772 pounds falling one foot, generated heat enough to raise one pound of water one degree Fahrenheit in temperature, or one *unit of heat*, as it is called.

He also found that, when solids were rubbed together by the action of a falling weight, one pound falling through 772 feet generated a unit of heat. In this experiment iron discs were made to rotate together, one against the other, in a vessel of mercury.

If a metallic disc be put into rapid rotation and then brought between the poles of a powerful electro-magnet, it soon comes to rest. It will now be found very difficult to turn it, and that it becomes heated as it rotates. Joule found in this case, as in the others, that, if the disc were turned by a falling weight, one pound descending 772 feet generated a unit of heat.

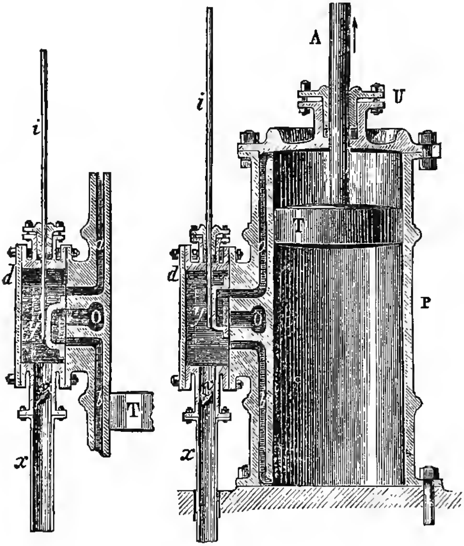
The force necessary to raise one pound one foot is called a *foot-pound;* and this is the same force which a pound acquires in falling one foot from a state of rest.

We see, then, that when mechanical energy is converted into heat, the same amount of energy always gives rise to the same amount of heat, and that 772 foot-pounds of mechanical force are equivalent to one unit of heat. For this reason, we call 772 foot-pounds the *mechanical equivalent of heat.*

223. *Heat may be converted into Mechanical Energy.* — The steam-engine is a contrivance for converting heat into mechanical energy. The heat converts the water into steam, and gives to this steam an expansive force; and this expansive force is made to move a piston by means of the arrangement shown in Figure 102. The steam coming from the boiler by the tube x passes into the box d. From this box run two pipes, a and b, for carrying the steam, one above and the other below the piston. A sliding-valve y is so arranged that it always closes one of these pipes. In the right-hand figure, the lower pipe b is open,

and the steam can pass in under the piston and force it up. At the same time, the steam which has done its work on the other side of the piston passes out through the pipes *a*

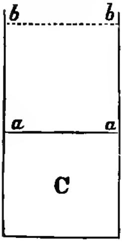
Fig. 102.



and *O*. The sliding-valve is connected by means of the rod *i* with the crank of the engine, so that it moves up and down as the piston moves down and up. As soon, then, as the piston has reached the top of the cylinder, the sliding-valve is brought into the position shown in the left-hand figure. The steam now passes into the cylinder above the piston by the pipe *a* and forces the piston down, while the steam on the other side which has done its work goes out through *b* and *O*. The sliding-valve is now in the position shown in the right-hand figure, and the piston is driven up again as before; and thus it keeps on moving up and down, or in and out. By means of a crank it can be made to drive machinery.

224. *The same Amount of Heat always gives rise to the same Amount of Mechanical Energy.* — In Figure 103, *C* is a box a foot square. Suppose *a a* to be a partition one foot from the bottom, so as to shut in a cubic foot of air. Suppose this partition to be immovable, and the air beneath to be heated. Its elastic force will be increased, but it cannot expand. We will next suppose that *a a* is movable, but without weight, and that the air beneath is heated as before. On raising its temperature 490° its volume will be doubled, and *a a* will of course be raised one foot to *b b*. In raising *a a* one foot it has had to raise the air above it. Now, this air presses 15 pounds upon every square inch, and $15 \times 144 = 2,160$ pounds upon the whole surface. From the specific heat of air, we know that to raise a cubic foot of air 490°, when it is free to expand, 9.5 units of heat are required.

Fig. 103.



But we have seen that a part of the heat which enters a body is used in expanding it, and a part in raising its temperature. In the above experiment, how much heat is used in raising the temperature? This is equivalent to asking how much heat is required to raise the cubic foot of air 490° when it is not allowed to expand. We have learned that the computed velocity of sound in air is less than its observed velocity, and that this is owing to the heat developed in the compressed portion of the sound-wave. From the ratio which exists between the observed and the computed velocity, it is found that the specific heat of air when free to expand must be 1.42 of its heat when not allowed to expand. Hence the heat required to raise the temperature of the cubic foot of air 490°, when it is not allowed to expand, is found by the following proportion to be 6.7 units: —

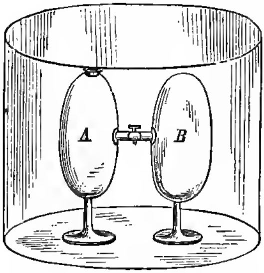
$$1.42 : 1 = 9.5 : 6.7.$$

The amount of heat, then, used in expanding the air — that is, in raising 2,160 pounds one foot high — is 2.8 units. Dividing 2,160 by 2.8, we get 772, nearly.

Since there is no cohesion among the particles of air, the whole expansive force is used in raising the weight.

There is a prevalent impression that the expanding of air is in itself a cooling process (that is, consumes heat); but this is not the case, unless the air in expanding performs work. This was proved by Joule in the following manner. In Figure 104 we have two strong vessels, of which *A* contains air compressed under a pressure of some 20 atmospheres, while in *B* is a vacuum. The two are connected by a tube with a stop-cock. The whole apparatus is placed in a vessel of water. After the temperature of the water has been very carefully ascertained, open the stop-cock and allow the air to expand. The temperature of the water remains unchanged. As there is no resistance to the expansion of the air, no heat is consumed in the expansion.

Fig. 104.



We see, then, that 772 foot-pounds of mechanical force are equivalent to a unit of heat, and that a unit of heat is equivalent to 772 foot-pounds of mechanical force.

225. *The Molecular Energy produced by Cohesion is converted into Heat.* — We have seen that the heat which enters a body is employed in three ways, — in raising the

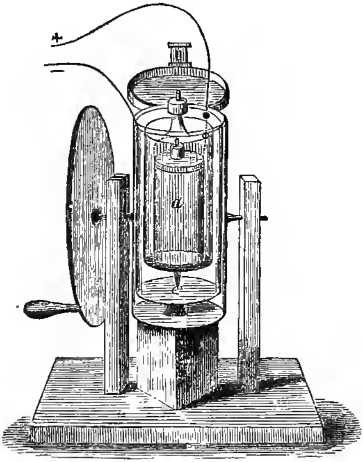
temperature of the body, in expanding it, and in changing its state. A body is expanded by the separation of its molecules; and its state is changed either by the separation or the rearrangement of its molecules. A part of the heat, then, which enters a body performs work; and, as this work is done within the body, it is called *interior* work. We have already noticed that, when it does this interior work, it gives to the molecules potential energy; and that, when these molecules are brought together again by cohesion, the potential energy appears as heat. The enormous strength of the cohesive force is shown by the amount of heat which it requires to melt ice, or to convert water into steam. We have seen that merely to *melt* a pound of ice at a temperature of 32° Fahrenheit requires 143 units of heat, which is equivalent to the force required to lift 10,396 pounds, or about 55 tons, a foot high. And to convert a pound of boiling water into steam requires 967 units of heat, equivalent to the force required to lift 746,524 pounds, or about 373 tons, a foot high. The force of gravity is almost as nothing compared with this molecular force.

226. *The Molecular Energy of Affinity is sometimes converted into Electricity.* — We have seen that, in all the forms of voltaic battery, electricity is generated by chemical action. Here the energy of affinity is converted into electricity. The same amount of chemical action always gives rise to the same amount of electricity.

227. *The Energy of Affinity always gives rise to Heat.* — Every form of chemical combination develops heat, which is nothing but the energy of affinity reappearing in a new form. The strength of affinity is shown by the amount of heat developed when oxygen combines with hydrogen. The heat thus generated can be found by the apparatus shown in Figure 105. Two measures of hydrogen and one of oxygen are put into the strong copper vessel *a*, and

this vessel is put into a larger one filled with water. This in turn is suspended in a cylinder with a movable cover at each end, and the whole is enclosed in a fourth cylindrical vessel, which may be rotated on a horizontal axis. The

Fig. 105.



apparatus is first rotated for some time, in order to bring all its parts to the same temperature, which is measured by a very delicate thermometer. The electric current is sent through the vessel *a*, where it heats a fine platinum wire red-hot, and explodes the mixed gases. The apparatus is again rotated about half a minute to make the temperature uniform throughout, and the temperature is again measured by the thermometer. The rise of temperature shows the amount of heat generated by the combination of the gases. It is found in this way that, when the oxygen combines with one pound of hydrogen, 61,000 units of heat are generated. Hence the force which has combined the two gases is equal to $61{,}000 \times 772 = 47{,}092{,}000$ foot-pounds, or the force necessary to raise 23,546 tons a foot high, or to throw one ton to a height of more than four miles.

We see, then, that the force even of cohesion is insignificant compared with that of affinity.

By a modification of the apparatus described above, it is found that a pound of carbon, in combining with oxygen, gives out about 14,500 units of heat, equivalent to 11,194,000 foot-pounds.

228. *The Energy of Affinity sometimes reappears as Muscular Force.* — The body is continually wasting away. The waste is supplied by the food we eat. The waste products of the body are burned-up food, — that is, the elements of the food combined with oxygen. The energy of this combination reappears as heat and muscular force. The materials of the body, as we have seen, are either *ternary* (compounds of oxygen, hydrogen, and carbon) or *quaternary* (compounds of oxygen, hydrogen, carbon, and nitrogen). It has been found that the muscular energy is mainly due to the combustion of the former.

We have now seen that the energy of affinity may be converted into electricity, heat, and muscular force. Through any one of these it may be converted into mechanical force. The electricity, for instance, may be made to develop magnetism, and thus to drive an electro-magnetic engine; the heat may be made to work a steam-engine; and the muscular energy may be employed in moving the body, or in any form of manual labor.

It is mainly the affinity of *oxygen* which develops these different forms of energy. In ordinary combustion and respiration, the substance with which the oxygen unites is mainly *carbon.*

229. *Energy may be transmuted, but not destroyed.* — We have now seen that the mechanical energy of motion, as well as that of affinity, may be converted into heat, light, and electricity. Heat and light are only different mani-

festations of the same agent; and heat and electricity are mutually convertible. The energy of affinity may also be converted into muscular energy. Heat is the form of energy into which all the other forms of energy seem destined ultimately to be converted. In all these transformations no energy is lost. The heat which finally results from them may either be radiated into space (where it may become insensible on account of its extreme diffusion), or else it may be made to separate atoms, molecules, and masses, and thus to confer potential energy upon them. When these are drawn together again by affinity, cohesion, and gravity, they develop the same amount of energy which was consumed in separating them.

Energy, like matter, may assume a great variety of forms; but, like matter, it is wholly indestructible.

230. *Source of Energy.* — If left to itself, affinity would soon bring all dissimilar atoms together, and lock them up in compounds; cohesion would bring all the molecules of these compounds together, and lock them up in solids; and gravity would bring all these solids together, and hold them in its iron grasp; while the heat developed by these forces would be radiated into space, and our earth become one dreary waste, void of all signs of life and activity. What, then, is the source of the energy which is thus manifesting itself in Protean forms?

Let us consider, first, the energy developed by gravity. This energy is seen in the winds, the falling rain, and running streams. The atmosphere on each side of the equator is an immense wheel. The side of this wheel next the equator is continually expanded, and thus made lighter by the heat of the sun. Hence gravity pulls down the colder and heavier side in the polar regions, and thus the wheel is made to turn round and round. Were it not for the sun's heat, it would soon come to rest.

Again, the heat of the sun evaporates the waters of the ocean, and in their gaseous state they are swept round with the atmospheric wheel till they come to colder regions, where they are condensed, and fall to the earth as rain, and flow to the ocean in rivers. It is due, then, to the heat which comes to the earth in the sunbeam, that gravity can thus unceasingly manifest its energy.

The energy of chemical affinity which is manifested in heat, light, and muscular force is, as we have seen, developed by its action between oxygen and carbon. How are these elements separated from carbonic acid, so that they may be reunited by affinity?

Place a leafy plant in a glass vessel, and let a current of carbonic acid stream over it in the dark, and no change takes place. Let the same gas stream over the plant in the sunshine, and a part of it will disappear, and be replaced by oxygen. When acted upon by the sunbeams, leaves of plants remove carbonic acid from the air, separate its carbon and oxygen, retain the former, and give the latter back to the air. When plants are consumed by combustion in our furnaces, and by respiration in our bodies, this oxygen combines with carbon and develops energy, which appears as mechanical force in our engines, and as muscular force in our bodies.

In the summer, when more sunshine than we need is poured upon the earth, a part of it is absorbed by the leaves of plants, and used to decompose carbonic acid, to build up the varied forms of vegetable life. In this way, the forests and the fields become vast storehouses of force which has been gathered from the sunbeam. When, therefore, we burn fuel in our stoves and food in our bodies, the light, heat, and muscular force developed are only the reappearance in another form of the sunbeams stored up in plants.

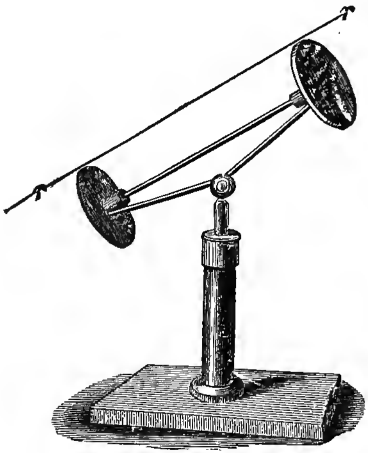
But this process of gathering force from the sunlight has been going on for ages; and when we burn anthracite or bituminous coal, we are merely releasing the sunbeams imprisoned in plants which grew upon the earth before it became the dwelling-place of man.

The energy of affinity, then, like that of gravity, is nothing but transmuted sunshine.

The only form of energy known to us which does not come to the earth in the sunbeam is that developed by the ebb and flow of the tidal wave. This wave is dragged round the earth mainly by the attraction of the moon; and it acts as a brake upon the earth's rotation, since it is drawn from east to west while the earth is turning from west to east. The energy of this wave, then, is developed at the expense of the earth's motion on its axis; and it must tend to retard this motion, though to so slight a degree that the observations of thousands of years have not served to make it appreciable.

231. *The Amount of Heat given out by the Sun.* — In Figure 106 we have an apparatus for measuring the heat

Fig. 106.



radiated to the earth by the sun. At one end of the instrument there is a shallow iron box, the cover of which is blackened. The box is filled with mercury, into which a delicate thermometer is introduced. At the other end of the instrument is a disc of the same diameter as the box. If the instrument be set in such a way that the shadow of the box just covers the disc, it is evident that the sun's rays will fall perpendicularly upon the cover. In the first place, the instrument, sheltered from the sun, is allowed to radiate its heat into the clear sky for five minutes, and its loss of heat is noted. It is then turned to the sun for the same length of time, and its gain of heat noted. It is then turned again toward the sky, and after five minutes its loss of heat is again noted. The mean between this loss and its loss during the first five minutes will be its loss during the five minutes it was turned towards the sun. This loss added to the gain already observed will be the whole heat which it received from the sun in five minutes.

Now, as one half of the earth is always turned towards the sun, the amount of heat received by the earth in five minutes will be as many times greater than that received by the box as the surface of a great circle of the earth is greater than that of the cover of the box. Making allowance for the heat absorbed by the atmosphere, it has been calculated that the amount received by the earth during a year would be sufficient to melt a layer of ice 100 feet thick and covering the whole earth. But the sun radiates heat into space in every other direction as well as towards the earth; and if we conceive a hollow sphere to surround the sun at the distance of the earth, our planet would cover only $\frac{1}{2,300,000,000}$ of its surface. Hence the sun radiates into space 2,300,000,000 times as much heat as the earth receives. Sir John Herschel has calculated that if a cylinder of ice 45 miles thick were darted into the sun with the velocity of light (190,000 miles a second), it might be melted by the heat radiated by the sun, without lowering the temperature of the sun itself.

232. *Source of the Sun's Heat.* — What, then, is the source of this enormous amount of heat?

It has been supposed by some that the materials of the sun are undergoing combustion, and that this combustion develops the light and heat which it sends forth. There are, however, no substances known to us whose burning would produce so much heat for so long a time as we know the sun has been shining. Carbon is one of the most combustible substances with which we are acquainted; but if the sun, large as he is, were a mass of pure carbon, and were burning at a rate sufficient to produce the light and heat that he is giving out, he would be utterly consumed in 5,000 years. It seems hardly possible, then, that the solar light and heat can be generated by ordinary combustion.

One of the most satisfactory theories of the origin of the solar heat is that recently devised by a German physician, Mayer, and known as the *meteoric* theory.

We have seen that a pound-weight which has fallen through 772 feet will, when its motion is arrested, generate a unit of heat. Now, we know that a body falling that distance will acquire a velocity of about 223 feet a second. Hence a pound ball moving with a velocity of 223 feet a second will generate a unit of heat when its motion is arrested. We know, too, that the velocity with which a falling body strikes the ground is in proportion to the square root of the height from which it falls; that is, in order to double or treble its velocity, a body must fall from four or nine times the height. A pound ball, then, moving with a velocity of twice 223 feet a second will be able to generate 4 units of heat; one moving with thrice this velocity, 9 units of heat; and so on. When, therefore, we know the weight of a body and the speed with which it is moving, we can easily calculate how much heat will be generated on stopping it.

Were the earth's motion arrested, its elements would melt with fervent heat, and most of them would be converted into vapor. Were the earth to fall into the sun, the heat generated by the shock would be sufficient to keep up the solar light and heat for 95 years. We know that countless swarms of meteoric bodies are revolving in rings about the sun, and the well-known retardation of Encke's comet shows that these bodies must be moving in a resisting medium. If so, they must eventually be drawn into the sun, and, from the velocity with which they must strike, it has been shown that they could fall in sufficient numbers to generate all the light and heat of the sun, without increasing his magnitude enough to be detected, since accurate measures of his diameter were first made. This theory of the sun's heat was first published by Mayer in 1848, and was further developed by Thomson in 1854.

The following account of the meteoric theory, as developed by Thomson, is taken from Tyndall: —

"'In conclusion, then,' writes Professor Thomson, 'the source of energy from which solar heat is derived is undoubtedly meteoric. . . . . The principal source — perhaps the sole appreciable efficient source — is in bodies circulating round the sun, at present inside the earth's orbit, in the sunlight by us called "zodiacal light." The store of energy for future sunlight is at present partly dynamical, — that of the motions of these bodies round the sun; and partly potential, — that of their gravitation towards the sun. This latter is gradually being spent, half against the resisting medium, and half in causing a continuous increase of the former. Each meteor thus goes on moving faster and faster, and getting nearer and nearer the centre, until some time, very suddenly, it gets so much entangled in the solar atmosphere as to begin to lose velocity. In a few seconds more it is at rest on the sun's surface, and the energy given up is vibrated across the district where it was gathered during so many ages, ultimately to penetrate, as light, the remotest regions of space.'

"From the tables published by Professor Thomson I extract the following interesting data, firstly with reference to the amount of heat equivalent to the rotation of the sun and planets round their axes, — the amount, that is, which would be generated supposing a brake applied at the surfaces of the sun and planets until the motion of rotation was entirely stopped; secondly, with reference to the amount of heat due to the sun's gravitation, — the heat, that is, which would be developed by each of the planets in falling into the sun. The quantity of heat is expressed in terms of the time during which it would cover the solar emission.

	Heat of Gravitation, equal to Solar Emission for a Period of	Heat of Rotation, equal to Solar Emission for a Period of
Sun	. . . .	116 years 6 days.
Mercury	6 years 214 days	15 "
Venus	83 " 227 "	99 "
Earth	94 " 303 "	81 "
Mars	12 " 252 "	7 "
Jupiter	32,240 "	14 " 144 "
Saturn	9,650 "	2 " 127 "
Uranus	1,610 "	71 "
Neptune	1,890 "	

"Thus, if the planet Mercury were to strike the sun, the quantity of heat generated would cover the solar emission for nearly seven years; while the shock of Jupiter would cover the loss of 32,240 years. The heat of rotation of the sun and planets, taken together, would cover the solar emission for 134 years; while the total heat of gravitation (that produced by the planets falling into the sun) would cover the emission for 45,589 years.

"Whatever be the ultimate fate of the theory here sketched, it is a great thing to be able to state the conditions which certainly would produce a sun, — to be able to discern in the force of gravity, acting upon dark matter, the source from which the starry heavens *may* have been derived; for, whether the sun be produced and his emission maintained by the collision of cosmical masses or not, there cannot be a doubt as to the competence of the cause assigned to produce the effects ascribed to it. Solar light and solar heat lie latent in the force which pulls an apple to the ground. The potential energy of gravitation was the original form of all the energy in the universe. As surely as the weights of a clock run down to their lowest position, from which they can never rise again unless fresh energy is communicated to them from some source not yet exhausted, so surely must planet after planet creep in, age by age, towards the sun. When each comes within a few hundred thousand miles of his surface, if he is still incandescent, it must be melted and driven into vapor by radiant heat. Nor, if he be crusted over and become dark and cool externally, can the doomed planet escape its fiery end. If it does not become incandescent, like a shooting-star, by friction in its passage through his atmosphere, its first graze on his surface must produce a stupendous flash of light and heat. It may be at once, or it may be after two or three bounds like a cannon-shot ricochetting on a surface of earth or water, the whole mass must be crushed, melted, and evaporated by a crash, generating in a moment some thousands of times as much heat as a coal of the same size would produce by burning."

233. *The Nebular Hypothesis.* — According to Laplace, the material of our solar system was once a nebulous mass of extreme tenuity, and the sun, moon, and planets were formed by its gradual condensation. Let us suppose such a nebulous mass slowly rotating, and gradually cooling by

radiation into space. As it cools, it must begin to contract; and as it contracts, its rotation must be quickened, since the matter at the surface must be moving faster than nearer the centre. It thus goes on contracting and rotating faster and faster, until the centrifugal tendency becomes so great that cohesion and gravity can no longer hold it together. A ring is then detached from the circumference, which continues to rotate by itself. The central mass goes on contracting and rotating with ever-increasing velocity, until a second ring is thrown off. In this way, ring after ring is detached, and all these rings continue to rotate round the central mass in the same direction. But the rings themselves would go on condensing, and at last they would be likely to break up, each forming one or several globular masses. These would, of course, all revolve about the central mass in the same direction, and their condensation would cause them to rotate on their axes; and it has been proved that, with the exception of one or two of the outer ones, they must all rotate on their axes in the same direction in which they revolve in their orbits.

But as these masses condensed, their rotation would be accelerated, and they would be very likely to throw off rings, which would either remain as rings, or be condensed into secondary masses revolving about their primaries.

The central mass, of course, forms the sun; the rings which it throws off, the planets; and the rings thrown off by the planets, the moons. In the case of Saturn, a part of the rings still remain uncondensed, while a part appear as moons.

The rings thrown off by the central mass usually condensed into one body, but, in the case of the minor planets and the meteoric rings, into many.

234. *Helmholtz's Theory of Solar Heat.* — The nebular hypothesis not only accounts for the motion of the planets, but it explains the internal heat of the earth and the solar

heat; for, as the molecules of the nebulous mass were drawn nearer and nearer together, their potential energy must have been converted into heat.* Helmholtz has made this the basis of his theory of solar heat, an account of which is given by Tyndall as follows: —

"He starts from the nebular hypothesis of Laplace, and, assuming the nebulous matter in the first instance to have been of extreme tenuity, he determines the amount of heat generated by its condensation to the present solar system. Supposing the specific heat of the condensing mass to be the same as that of water, then the heat of condensation would be sufficient to raise their temperature 28,000,000° Centigrade. By far the greater part of this heat was wasted ages ago in space. The most intense terrestrial combustion that we can command is that of oxygen and hydrogen, and the temperature of the pure oxyhydrogen flame is 8,061° C. The temperature of a hydrogen flame, burning in air, is 3,259° C.; while that of the lime-light, which shines with such sunlike brilliancy, is estimated at 2,000° C. What conception, then, can we form of a temperature more than thirteen thousand times that of the Drummond light? If our system were composed of pure coal and burnt up, the heat produced by its combustion would only amount to $\frac{1}{3500}$ of that generated by the condensation of the nebulous matter to form our solar system. Helmholtz supposes this condensation to continue; that a virtual falling down of the superficial portions of the sun towards the centre still takes place, a continual development of heat being the result. However this may be, he shows by calculation that the shrinking of the sun's diameter by $\frac{1}{10000}$ of its present length would generate an

* It would seem at first that this heat would prevent further condensation, but it is gradually radiated off into space, and as the molecules come nearer together the force of gravity increases more rapidly than the repulsive force of the remaining heat.

amount of heat competent to cover the solar emission for 2,000 years; while the shrinking of the sun from its present mean density to that of the earth would have its equivalent in an amount of heat competent to cover the present solar emission for 17,000,000 of years.

"'But,' continues Helmholtz, 'though the store of our planetary system is so immense that it has not been sensibly diminished by the incessant emission which has gone on during the period of man's history, and though the time which must elapse before a sensible change in the condition of our planetary system can occur is totally beyond our comprehension, the inexorable laws of mechanics show that this store, which can only suffer loss and not gain, must finally be exhausted. Shall we terrify ourselves by this thought? We are in the habit of measuring the greatness of the universe, and the wisdom displayed in it, by the duration and the profit which it promises to our own race; but the past history of the earth shows the insignificance of the interval during which man has had his dwelling here. What the museums of Europe show us of the remains of Egypt and Assyria we gaze upon with silent wonder, in despair of being able to carry back our thoughts to a period so remote. Still, the human race must have existed and multiplied for ages before the Pyramids could have been erected. We estimate the duration of human history at 6,000 years; but, vast as this time may appear to us, what is it in comparison with the period during which the earth bore successive series of rank plants and mighty animals, but no men? — periods during which, in our own neighborhood (Königsberg), the amber-tree bloomed, and dropped its costly gum on the earth and in the sea; when in Europe and North America groves of tropical palms flourished, in which gigantic lizards, and, after them, elephants, whose mighty remains are still buried in the earth, found a home. Different geologists, proceed-ing from different premises, have sought to estimate the length of the above period, and they set it down from one to nine millions of years. The time during which the earth has generated organic beings is again small compared with the ages during which the world was a mass of molten rocks. The experiments of Bischof upon basalt show that our globe would require 350 millions of years to cool down from 2,000° to 200° Centigrade. And with regard to the period during which the first nebulous masses condensed, to form our planetary system, conjecture must entirely cease. The history of man, therefore, is but a minute ripple in the infinite ocean of time. For a much longer period than that during which he has already occupied this world, the existence of a state of inorganic nature, favorable to man's continuance here, seems to be secured; so that for ourselves, and for long generations after us, we have nothing to fear. But the same forces of air and water, and of the volcanic interior, which produced former geologic revolutions, burying one series of living forms after another, still act upon the earth's crust. They, rather than those distant cosmical changes of which we have spoken, will put an end to the human race, and perhaps compel us to make way for new and more complete forms of life, as the lizard and the mammoth have given way to us and our contemporaries.' "

It will be noticed that Mayer's theory is not inconsistent with that of Helmholtz, but supplementary to it. The former merely assumes that the meteors and planets, which were thrown off from the nebulous mass as it condensed, are slowly falling into it again. When these shall all have fallen into it and the condensation shall have ceased, our sun will cease to shine, like many other stars which have disappeared from the heavens.

SUMMARY.

When bodies or their molecules are in motion, they are said to have an *actual* energy; and when they are so situated that they can be moved by gravity or by the molecular forces, they are said to have a *potential* energy. (215.)

Energy may be *mechanical*, *molecular*, or *muscular*. (216.)

Affinity, cohesion, and gravity are the forces which are constantly tending to convert potential into actual energy. (217.)

Heat is not a substance, but only a *mode of motion*, as was shown by Count Rumford and by Sir Humphrey Davy. (219, 220.)

Mechanical energy may be converted into *heat*, into which, in fact, all mechanical energy is ultimately converted. (218, 221.)

The same amount of mechanical energy, on conversion into heat, always gives rise to the same amount of heat. (222.)

Heat may be again converted into mechanical energy; and the same amount of heat always gives rise to the same amount of mechanical energy. (223, 224.)

The energy of affinity is converted partially into electricity, partially into heat, and partially into muscular energy. (225 – 228.)

The strength of affinity is shown by the great amount of heat generated in the burning of hydrogen and carbon. (227.)

However energy may be transmuted, it can never be destroyed. (229.)

All the energy manifested at the surface of the earth, except that of the tides, is drawn from the sunbeams. (230.)

The sun radiates sufficient heat to melt a cylinder of ice 45 miles thick, at the rate of 190,000 miles a second. (231.)

According to Mayer, the solar heat is developed by meteoric showers; according to Helmholtz, by the gradual condensation of the sun. (232 – 234.)

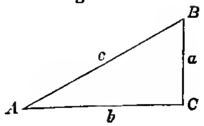
APPENDIX.

APPENDIX.

I.

1. A *right triangle* is one which contains a right angle. Thus $A\,C\,B$, Figure 1, is a right triangle.

Fig. 1.

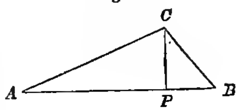


2. An *oblique triangle* is one which does not contain a right angle.

Every oblique triangle can be resolved into two right triangles by dropping a perpendicular from one of the angles upon the opposite side.

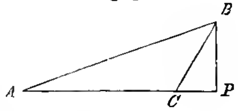
Thus in Figure 2, by dropping the perpendicular $C\,P$, two right triangles, $A\,P\,C$ and $B\,P\,C$, are formed.

Fig. 2.



In Figure 3, by dropping the perpendicular $B\,P$ upon $A\,C$ produced, we get two right triangles, $A\,P\,B$ and $C\,P\,B$.

Fig. 3.



2. *The sine of an angle is the quotient of the opposite side divided by the hypothenuse.* — Thus, by designating the angles by the large letters and the sides opposite them by the corresponding small ones, we have in Figure 1,

$$\sin A = \frac{a}{c}, \quad \sin B = \frac{b}{c}, \quad \text{and } \sin C = \frac{c}{c} = 1.$$

The sine of a right angle equals unity.

In Figures 2 and 3, we have

$$\sin A = \frac{CP}{b},\ \sin B = \frac{CP}{a},\ \text{and}\ \sin C = \frac{BP}{a}.$$

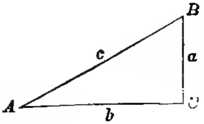
$$BCA + BCP = 180°.$$

When the sum of two angles is 180°, the angles are said to be *supplements* of each other.

It will be seen that, in Figure 3, $\frac{BP}{a}$ is the sine of both BCA and BCP.

An angle and its supplement have the same sine.

3. *The sides of a plane triangle are proportional to the sines of their opposite angles.*



In Figure 1, we have

$$\sin A = \frac{a}{c},\ a = c \sin A,\ c = \frac{a}{\sin A}$$

$$\sin B = \frac{b}{c},\ b = c \sin B,\ c = \frac{b}{\sin B}$$

$$\sin C = \frac{c}{c},\ c = c \sin C,\ c = \frac{c}{\sin C}$$

whence

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

which, converted into proportions, give

$$a : \sin A = b : \sin B$$

$$a : \sin A = c : \sin C$$

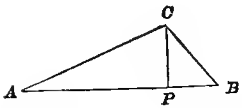
$$b : \sin B = c : \sin C$$

and these, by transposing the means, give

$$a : b = \sin A : \sin B$$

$$a : c = \sin A : \sin C$$

$$b : c = \sin B : \sin C$$



In Figure 2, we have

$$\sin A = \frac{CP}{b},\ CP = b \sin A$$

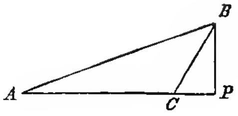
$$\sin B = \frac{CP}{a},\ CP = a \sin B$$

whence $$b \sin A = a \sin B$$

which, converted into a proportion, gives

$$a : b = \sin A : \sin B.$$

In Figure 3, we have



$$\sin A = \frac{B\,P}{c},\quad B\,P = c \sin A$$

$$\sin C = \frac{B\,P}{a},\quad B\,P = a \sin C$$

whence $$a \sin C = c \sin A$$

which, converted into a proportion, gives

$$a : c = \sin A : \sin C.$$

By dropping a perpendicular from A upon $B\,C$ produced, it can be found in the same way that

$$b : c = \sin B : \sin C.$$

In both right and oblique triangles, then, the sides are proportional to the sines of their opposite angles.

4. It is shown in Geometry that two right triangles which have an acute angle of one equal to an acute angle of the other, have their corresponding sides proportional. Hence whatever the length of the sides b and c in the right triangle, Figure 1, $\frac{a}{c}$ and $\frac{b}{c}$ will have the same values as long as the angles A and B remain the same. The value of the sine, then, depends wholly on the size of the angle.

The value of the sines have been computed for every angle between 0° and 90°, and these values have been arranged in tables called "Tables of Natural Sines."

5. By means of (3), the other parts of a plane triangle can be found when one side and two angles are given.

To find the third angle, subtract the sum of the given angles from 180°.

To find the other sides, form a proportion as follows: As the sine of the angle opposite the given side is to the sine of the angle opposite the required side, so is the given side to the required side.

Thus in the triangle $A\,B\,C$, Figure 2, given the side a, and the angles B and C.

To find the angle A.

$$180° - (B + C) = A.$$

To find the side b.

$$\sin A : \sin B = a : b.$$

Find the side c.

Since one of the angles of a right triangle is always a right angle, the parts of such a triangle can be computed when one side and one acute angle are given.

In the right triangle $A\ B\ C$, Figure 1, given the side c and the angle A. Find the other parts.

6. By means of (3), the other parts of a plane triangle can be computed when two sides and an angle opposite one of them are given.

To find a second angle, form the following proportion: As the side opposite the given angle is to the side opposite the required angle. so is the sine of the given angle to the sine of the required angle.

After the second angle is found the case becomes the same as (5).

In the triangle $A\ B\ C$, Figure 2, given a and c and the angle A. Find B, C, and b.

II.

The horizon is the plane which at any point on the earth would separate the visible from the invisible part of the heavens, if the earth were everywhere level like the surface of the sea. It divides the celestial sphere into two equal parts, and its intersection with it forms the circumference of a great circle. Every part of this circumference is 90° from the zenith.

At the equator the celestial poles are just 90° from the zenith, hence the horizon will pass through these, and its plane will coincide in direction with the earth's axis. As we go north from the equator, the zenith passes northward, and the horizon passes below the north pole and becomes more and more inclined to the earth's axis till we reach the pole, when the inclination becomes 90°. As we go southward from the equator, a corresponding change takes place.

III.

THE CALENDAR.

OLD AND NEW STYLE.

THE *solar year*, or the interval between two successive passages of the same equinox by the sun, is 365 days, 5 hours, 48 minutes, 48 seconds. If then we reckon only 365 days to a common or *civil year*, the sun will come to the equinox 5 hours, 48 minutes, 48 seconds, or nearly a quarter of a day, later each year; so that, if the sun entered Aries on the 20th of March one year, he would enter it on the 21st four years after, on the 22d eight years after, and so on. Thus in a comparatively short time the spring months would come in the winter, and the summer months in the spring.

Among different ancient nations different methods of computing the year were in use. Some reckoned it by the revolutions of the moon; some by that of the sun: but none, so far as we know, made proper allowances for deficiencies and excesses. Twelve moons fell short of the true year; thirteen exceeded it: 365 days were not enough; 366 were too many. To prevent the confusion resulting from these errors, Julius Cæsar reformed the calendar by making the year consist of 365 days, 6 hours (which is hence called a *Julian* year), and made every fourth year consist of 366 days. This method of reckoning is called *Old Style.*

But as this made the year somewhat too long, and the error in 1582 amounted to ten days, Pope Gregory XIII., in order to bring the vernal equinox back to the 21st of March again, ordered ten days to be struck out of that year; calling the next day after the 4th of October the 15th. And to prevent similar confusion in the future he decreed that three leap-years should be omitted in the course of every 400 years. This way of reckoning time is called *New Style.* It was immediately adopted by most of the European nations, but was not accepted by the English until the year 1752. The error then amounted to 11 days, which were taken from the month of September, by calling the 3d of that month the 14th.

According to the Gregorian calendar, *every year whose number is divisible by* 4 is a *leap-year;* except that *in the case of the years whose numbers are exact hundreds, those only are leap-years which are divisible by* 4 *after cutting off the last two figures.* Thus, the years 1600, 2000, 2400, etc., are leap-years; 1700, 1800, 1900, 2100, 2200, etc., are not. Under this mode of reckoning, the error will not amount to a day in 5,000 years.

THE DOMINICAL LETTER.

The *Dominical Letter* for any year is that which we often see placed against Sunday in the almanacs, and is always one of the first seven in the alphabet. Since a common year consists of 365 days, if this number be divided by 7, the number of days in a week, there will be a remainder of one. Hence a year commonly begins one day later in the week than the preceding one did. If a year of 365 days begins on Sunday, the next will begin on Monday; if it begins on Thursday, the next will begin on Friday; and so on. If Sunday falls on the 1st of January, the *first* letter of the alphabet, or *A*, is the *Dominical Letter.* If Sunday falls on the 7th of January (as it will the next year, unless the first be leap-year) the *seventh* letter, *G*, is the Dominical Letter. If Sunday falls on the 6th of January (as it will the third year, unless the first or second be leap-year) the *sixth* letter, *F*, will be the Dominical Letter. Thus, if there were no leap-years, the Dominical Letters would regularly follow a retrograde order, *G*, *F*, *E*, *D*, *C*, *B*, *A*.

But *leap* years have 366 days; which, divided by 7, leaves 2 remainder. Hence the years following leap-years will begin two days later in the week than the leap-years did. To prevent the interruption which would hence occur in the order of the Dominical Letters, leap-years have *two* Dominical Letters; one indicating Sunday till the 29th of February, and the other for the rest of the year.

By *Table I.* below, the Dominical Letter for any year (New Style) for 4,000 years from the beginning of the Christian

Era may be found; and it will be readily seen how the Table could be extended indefinitely.

To find the Dominical Letter by this Table, *look for the hundreds of years at the top, and for the years below a hundred at the left hand.*

Table I.

Years less than One Hundred.				Centuries.			
				100	200	300	400
				500	600	700	800
				900	1000	1100	1200
				1300	1400	1500	1600
				1700	1800	1900	2000
				2100	2200	2300	2400
				2500	2600	2700	2800
				2900	3000	3100	3200
				3300	3400	3500	3600
				3700	3800	3900	4000
				C	E	G	BA
1	29	57	85	B	D	F	G
2	30	58	86	A	C	E	F
3	31	59	87	G	B	D	E
4	32	60	88	FE	AG	CB	DC
5	33	61	89	D	F	A	B
6	34	62	90	C	E	G	A
7	35	63	91	B	D	F	G
8	36	64	92	AG	CB	ED	FE
9	37	65	93	F	A	C	D
10	38	66	94	E	G	B	C
11	39	67	95	D	F	A	B
12	40	68	96	CB	ED	GF	AG
13	41	69	97	A	C	E	F
14	42	70	98	G	B	D	E
15	43	71	99	F	A	C	D
16	44	72	..	ED	GF	BA	CB
17	45	73	..	C	E	G	A
18	46	74	..	B	D	F	G
19	47	75	..	A	C	E	F
20	48	76	..	GF	BA	DC	ED
21	49	77	..	E	G	B	C
22	50	78	..	D	F	A	B
23	51	79	..	C	E	G	A
24	52	80	..	BA	DC	FE	GF
25	53	81	..	G	B	D	E
26	54	82	..	F	A	C	D
27	55	83	..	E	G	B	C
28	56	84	..	DC	FE	AG	BA

Table II.

	A	B	C	D	E	F	G
Jan. 31. Oct. 31.	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31	..	..	..	..
Feb. 28–29. March 31. Nov. 30.	..	..	..	1	2	3	4
	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30	31	..
April 30. July 31	..	..	..	..	..	..	1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31	..	..	..	..	..
Aug. 31.	..	..	1	2	3	4	5
	6	7	8	9	10	11	12
	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31	..	..
Sept. 30. Dec. 31.	..	..	..	..	..	1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
	31	..	..	..	..	..	..
May 31.	..	1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31	..	..	..
June 30.	..	..	..	..	1	2	3
	4	5	6	7	8	9	10
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	..

Thus, the letter for 1867 will be opposite the number 67, and in the column having 1800 at the top; that is, it will be *F*. In the same way, the letters for 1868, which is a leap-year, will be found to be *ED*.

Having the Dominical Letter of any year, *Table II.* shows what days of every month of the year will be *Sundays*.

To find the Sundays of any month in the year by this Table, *look in the column under the Dominical Letter, opposite the name of the month given at the left.*

From the Sundays the date of any other day of the week can be readily found.

Thus if we wish to know on what day of the week Christmas will fall in 1867, we look opposite December under the letter *F*, (which we have found to be the Dominical Letter for the year,) and find that the 22d of the month is a Sunday. The 25th, or Christmas, will then be Wednesday.

In the same way we may find the day of the week corresponding to any date (New Style) in history. For instance, the 17th of June, 1775, the day of the fight at Bunker Hill, is found to have been a *Saturday*.

These two Tables then serve as a *perpetual almanac.*

THE GOLDEN NUMBER.

It has been found that after a period of 19 years the sun, earth, and moon occupy nearly the same relative positions. Starting then with new moon or full moon, it is evident that after the expiration of this period it will be new moon or full moon again. Hence the phases of the moon which succeeded one another in the first period will repeat themselves in the same order in the second.

This period is called the *Cycle of the Moon*, and the number 19 is known as the *Golden Number*.

To find the Golden Number for any year, *add* 1 *to the number of that year, divide by* 19, *and the remainder is the Golden Number. If nothing remains, the Golden Number is* 19.

In *Table III.* the Golden Numbers under the months stand against the days of new moon in the left hand column. It is adapted chiefly to the second year after leap year, and will indicate *the time of new moon* (within one day) till the year 1900. A perfectly correct table of this kind cannot be easily made.

To show the use of the Table, suppose we wish to know nearly the time of new moon in September, 1867. By the rule given above, we find the Golden Number for the year to be 6. Looking in the Table under September, we find the number 6 opposite the 27th day of the month.

The error can in no case exceed one day.

TABLE III.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	9	..	9	17	17	..	..	..	..	11	..	19
2	..	17	..	..	6	14	14	3	11	..	19	..
3	17	6	17	6	..	..	3	11	..	19	8	8
4	6	..	6	14	14	3	..	..	19	8	..	16
5	..	14	..	..	3	11	11	19	8	..	16	..
6	14	3	14	3	..	..	19	..	..	16	5	5
7	3	..	3	11	11	19	..	8	16	..	..	13
8	..	11	..	..	19	8	8	16	5	5	13	..
9	11	19	11	19	..	..	..	..	..	13	..	2
10	..	..	19	8	8	16	16	5	13	..	2	10
11	19	8	..	..	..	..	5	13	2	2	10	..
12	8	16	8	16	16	5	..	..	..	10	..	18
13	..	..	..	..	5	13	13	2	10	..	18	7
14	16	5	16	5	..	..	2	10	18	18	7	..
15	5	..	5	13	13	2	..	..	..	7	..	15
16	..	13	..	..	2	10	10	18	7	..	15	..
17	13	2	13	2	..	..	18	7	..	15	4	4
18	2	..	2	10	10	18	..	..	15	..	..	12
19	..	10	..	..	18	7	7	15	4	4	12	..
20	10	18	10	18	..	..	15	..	..	12	1	1
21	18	..	18	7	7	15	..	4	12	..	..	9
22	..	7	..	..	15	4	4	12	1	1	9	..
23	7	15	7	15	..	..	12	..	..	9	17	17
24	..	..	15	4	4	12	..	1	9	..	..	6
25	15	4	..	..	12	..	1	9	17	17	6	..
26	4	..	4	12	..	1	..	..	..	6	..	14
27	..	12	..	1	1	9	9	17	6	..	14	..
28	12	1	12	..	9	..	17	6	14	14	3	3
29	1	..	1	9	..	17	..	..	..	3	..	11
30	..	..	..	..	17	6	6	14	3	..	11	..
31	9	..	9	..	..	..	14	3	..	11	..	19

EPACT.

A *solar* year, as we have seen, is about 365 days, 6 hours; a *lunar* year, of 12 lunar months, is about 354 days, 9 hours. The difference of nearly 11 days between the two is the *Annual Epact.* Since the epact of one year is 11 days, that of 2 years will be 22 days; of 3 years 33 days, or rather 3 days, being 3 days over a lunar month. Thus, by yearly adding 11, and casting out the 30's, it will be found that on every 19th year 29 remains; which is reckoned a complete lunar month, and the epact is 0. Thus the cycle of epacts expires with the lunar cycle, or that of the Golden Numbers; and on every 19th year the solar and lunar years begin together.

By the epact of any year *the moon's age* (115) *for the 1st of January* is shown.

Table IV. gives the Golden Numbers with the corresponding Epacts till the year 1900.

TABLE IV.

Golden No.	Epact.	Golden No.	Epact.	Golden No.	Epact.	Golden No.	Epact.	Golden No.	Epact.
1	0	5	14	9	28	13	12	17	26
2	11	6	25	10	9	14	23	18	7
3	22	7	6	11	20	15	4	19	18
4	3	8	17	12	1	16	15		

IV.

THE METRIC SYSTEM.

SINCE the measurement of length is required for almost every purpose of construction, as well as for every intelligible statement of the size of material objects and their distance from one another, it is indispensable that every community should fix upon some common standard, some well known *unit*, by whose repetition and subdivision length, distance, and size, whether great or small, can be expressed in words and numbers.

The standards which almost all communities have taken for their unit of length, have been either some portion of the human body, such as the length of the arm, of the fore-arm (the *ell* or *cubit*), of the foot, of the breadth of the hand (*span* or *palm*), or the length of the ordinary step (*pace*); and for measuring smaller lengths, the length of certain cereal grains, such as those of rice or barley (the *barley-corn*).

Thus an old English statute defined an inch as the length of three barleycorns.

But no part of the body of a full grown man has invariably the same length; hence it became necessary for each community to take the length of a particular fore-arm or foot for their unit. In this way the units fixed upon by different communities did not agree with one another. Thus we find the length of the Roman foot equivalent to 11.6 of our inches; the English to 12; the Grecian to 12.1; the French to 12.8; and the Egyptian to 13.1.

The English unit is the *yard*, which is said to have been introduced by King Henry the First, who ordered that the *ulna*, or ancient *ell*, which corresponds to the modern yard, should be made the exact length of his own arm, and that the other measures of length should be based upon it.

In 1790, at the time of the French Revolution, that people undertook to establish a new metrical system, and sought for some natural object of invariable length upon which to base their unit of length. They chose the length of a meridian of the globe, and they called the ten-millionth part of a quadrant of the meridian a *mètre*. They then set about measuring an arc of a meridian so as to compute exactly the length of its quadrant. A mistake was made in this computation, so that the French *mètre* does not correctly represent the ten-millionth part of the quadrant of a meridian.

As nations come into closer relations with one another, it becomes more and more desirable that they should have a common metrical system.

Accordingly many governments, and among them our own, have enacted that the French mètre shall be regarded as a legal unit of measure.

There are, however, objections to this unit. It is true it is based upon the quadrant of a meridian, which is of invariable length, but it does not represent a ten-millionth of that quadrant accurately.

We have already seen that the radius or diameter of the earth is the natural unit with which we begin the measurement of the distance of the heavenly bodies. Hence it would be much more convenient that the unit of linear measurement should be based upon the diameter of the earth than upon the length of a meridian, and it would be a great objection to the French mètre that it is not based upon the length of the earth's diameter, even if it represented accurately what it professes to do.

Now, Sir John Herschel has called attention to the fact that if the English inch were made .001 longer than it now is, then 50 such inches would represent almost exactly a ten-millionth part of the polar diameter of the earth. He consequently proposes 50 such inches as the unit of measure, and would call it a *module*. This seems on the whole the most satisfactory unit that has been proposed.

It has been proposed that the length of a pendulum beating seconds be taken as the unit of length. This unit would differ but little from the mètre.

It has this advantage; that it is much easier to measure the length of such a pendulum accurately than to measure the arc of a meridian. But the length of the pendulum beating seconds as a standard of length has the same disadvantage that the human foot has for the same purpose. It has already been seen that pendulums which beat seconds in different parts of the earth are not all of the same length. If then the length of a seconds pendulum is to be taken as a standard, it must be stated at what particular place the pendulum is to beat seconds.

Having decided upon a unit of length it is necessary to decide according to what scale this unit shall be multiplied or divided. The French have multiplied and divided their mètre on the decimal scale, as the following table shows.

TABLE OF LINEAR MEASURE.

10 millimètres	= 1 centimètre
10 centimètres *	= 1 décimètre
10 décimètres	= 1 *mètre*
10 mètres	= 1 décamètre
10 decamètres	= 1 hectomètre
10 hectomètres	= 1 kilomètre.

The names of the *higher* orders of units, or the *multiples* of the *mètre*, are formed from the word *mètre* by means of prefixes taken from the *Greek* numerals: namely, *deca*-(10), *hecto*-(100), *kilo*-(1,000).

The names of the *lower* orders of units, or the *subdivisions* of the *mètre*, are formed in a similar manner by means of prefixes taken from the *Latin* numerals: namely, *déci*-(10), *centi*-(100), *milli*-(1,000).

This is certainly a great improvement upon our clumsy multiplication and subdivision of the yard and foot.

It is very desirable that the units of length, of capacity, and of weight, should be connected in some natural manner. This the French do very neatly. The *cubic décimètre* is their unit of capacity, and is called a *litre*. This litre is multiplied and divided decimally in the same way as the mètre, as is seen in the following table.

TABLE OF MEASURES OF CAPACITY.

10 millilitres	= 1 centilitre
10 centilitres	= 1 décilitre
10 décilitres	= 1 *litre*
10 litres	= 1 décalitre
10 décalitres	= 1 hectolitre
10 hectolitres	= 1 kilolitre.

The names of the higher and lower orders of units are formed from the name of the *litre*, in the way explained under the preceding table.

They have taken the weight of a cubic centimètre of water,

* The new five-cent piece (1866) is 2 centimetres in diameter, and weighs 5 grammes.

at a temperature of 4° Centigrade in a vacuum, as their unit of weight, and have called it a *gramme*. This also they multiply and divide decimally, according to the following table.

TABLE OF WEIGHTS.

10 milligrammes	=	1 centigramme
10 centigrammes	=	1 décigramme
10 décigrammes	=	1 *gramme*
10 grammes	=	1 décagramme
10 décagrammes	=	1 hectogramme
10 hectogrammes	=	1 kilogramme.

The names are formed from the word *gramme* by means of prefixes, in the same manner as those in the other tables.

It would probably be impossible to improve upon the method in which the French have multiplied and subdivided the unit of measure, and in which they have connected the units of length, capacity, and weight.

Our unit of weight is the *grain*. This is defined as the weight of such an amount of distilled water that, at a temperature of 62° Fahrenheit, 252.46 such grains shall fill a cubic inch. A pound *avoirdupois* contains 7,000 of these grains: and an imperial gallon of distilled water at a temperature of 62° F. 70,000 of them. This gallon is our unit of capacity. An ounce contains 437½ such grains. According to this system a cubic foot of distilled water at a temperature of 62° F. contains 997.145 ounces, falling short of 1,000 ounces by nearly three ounces. It is evident that nothing could be more clumsy than the way in which we connect our units of weight, of capacity, and of length.

But, Sir John Herschel has called attention to the fact that if our inch, and consequently our foot, were increased .001 in length, then a cubic foot of water at the temperature of 62° F. would contain almost exactly 1,000 ounces, and that it would require only the slightest change in the value of the ounce to make it contain exactly 1,000 ounces. Thus by taking the ounce thus slightly modified as our unit of weight, it would be connected decimally with our unit of length. Also our half-pint, which he proposes to call a *beaker*, would be just .01 of a cubic foot in capacity. And by taking this as our unit of capacity, our units of length, of weight, and of capacity would all be connected decimally. "And thus," as Sir John Herschel says, "the change which would place our system of linear measure on a perfectly faultless basis, would at the same time rescue our weights and our measures of capacity from their present utter confusion, and secure that other advantage, second only in importance to the former, of connecting them decimally with that system on a regular, intelligible, and easily remembered principle, and that, too, by an alteration practically imperceptible in both cases, and interfering with no one of our usages and denominations."

These units once established, they could of course be divided and multiplied decimally in the same way as the French units are. If these decimal divisions and multiples were found more convenient than the old ones, they would soon displace them, as the decimal system in our currency has displaced the old system of pounds, shillings, and pence.

V.

THE latest observations upon *Linné* (according to Silliman's Journal, July, 1867), appear to show in the centre of the bright spot covering the former crater a minute black spot indicating a crater of about six hundred yards diameter. The original crater appears to have been a deep one, and about ten thousand yards in diameter. This small crater was so plainly visible as to have been noticed (independently as it would seem) by Dr. Schmidt at Athens, by Father Secchi at Rome, and by Professor Lyman at New Haven. It was detected at New Haven three days after the sun had risen over the horizon of *Linné*, and when the sun was therefore 30° or 35° high upon it. These observations show that any change which has taken place is not in the nature of a development of a cloud, but imply rather that the old crater has been filled up by an eruption from the small one now visible.

TABLES.

THE SUN AND PRINCIPAL PLANETS.

Name.	Symbol.	Time of Axial Rotation.	Inclination of Orbit to Ecliptic.	Mean Diameter in Miles.	Volume.	Mass.	Density.
		h. m. s.	° ′ ″				
The Sun	☉	600		887,000	1416000	354936	0.25
Mercury	☿	24 5 28	7 0 8	2,950	0.059	0.118	2.01
Venus	♀	23 21 21	3 23 31	7,800	0.912	0.883	0.97
The Earth	⊕	23 56 4		7,912	1.000	1.000	1.00
Mars	♂	24 37 22	1 51 5	4,500	0.183	0.132	0.72
Jupiter	♃	9 55 26	1 18 40	88,000	1412.000	338.034	0.24
Saturn	♄	10 29 17	2 29 28	73.000	770.000	101.064	0.13
Uranus	♅		0 46 30	36.000	95.900	14.789	0.15
Neptune	♆		1 46 59	35,000	89.500	24.648	0.27

THE PLANETS (*continued*).

Name.	Sidereal Period in Days.	Relative Distance from Sun.	Mean Distance from Sun in Miles.	Relative Light and Heat received from Sun.
Mercury	87.969	0.387	37,000,000	6.67
Venus	224.701	0.723	69,000,000	1.91
Earth	365.256	1.000	95,000,000	1.00
Mars	686.980	1.524	145,000,000	0.43
Jupiter	4,332.585	5.203	436,000,000	0.037
Saturn	10,759.220	9.539	909,000,000	0.011
Uranus	30,686.821	19.183	1,828,000,000	0.003
Neptune	60,126.722	30.037	2,862,000,000	0.001

THE MOON.

Mean Distance from the Earth .... 238,900 miles.
Sidereal Period of Revolution.............. $27^d\ 7^h\ 43^m\ 11.46^s$.
Synodical Period of Revolution .... $29^d\ 12^h\ 44^m\ 2.87^s$.
Diameter .... 2160 miles.
Inclination of the Orbit .... $5^\circ\ 8'\ 48''$.
Density (the Earth = 1) .... .05657.
Mass (the Earth = 1) .... $\frac{1}{81}$.

THE MINOR PLANETS.

No.	Name.	Date of Discovery.	Discoverer.	Sidereal Rev. in Days.
1	Ceres	1801, Jan. 1	Piazzi	1680
2	Pallas	1802, March 28	Olbers	1682
3	Juno	1804, Sept. 1	Harding	1596
4	Vesta	1807, March 29	Olbers	1326
5	Astræa	1845, Dec. 8	Hencke	1512
6	Hebe	1847, July 1	Hencke	1379
7	Iris	1847, Aug. 13	Hind	1346
8	Flora	1847, Oct. 18	Hind	1193
9	Metis	1848, April 25	Graham	1346
10	Hygieia	1849, April 12	Gasparis	2043
11	Parthenope	1850, May 11	Luther	1403
12	Victoria	1850, Sept. 13	Hind	1303
13	Egeria	1850, Nov. 2	Gasparis	1511
14	Irene	1851, May 19	Hind	1519
15	Eunomia	1851, July 29	Gasparis	1570
16	Psyche	1852, March 17	Gasparis	1828
17	Thetis	1852, April 17	Luther	1421
18	Melpomene	1852, June 24	Hind	1271
19	Fortuna	1852, Aug. 22	Hind	1393
20	Massilia	1852, Sept. 19	Gasparis	1365
21	Lutetia	1852, Nov. 15	Goldschmidt	1388
22	Calliope	1852, Nov. 16	Hind	1813
23	Thalia	1852, Dec. 15	Hind	1556
24	Themis	1853, April 5	Gasparis	2036
25	Phocæa	1853, April 7	Chacornac	1358
26	Proserpine	1853, May 5	Luther	1580
27	Euterpe	1853, Nov. 8	Hind	1313
28	Bellona	1854, March 1	Luther	1692
29	Amphitrite	1854, March 1	Marth	1492
30	Urania	1854, July 22	Hind	1329
31	Euphrosyne	1854, Sept. 1	Ferguson	2048
32	Pomona	1854, Oct. 26	Goldschmidt	1521
33	Polyhymnia	1854, Oct. 28	Chacornac	1778
34	Circe	1855, April 6	Chacornac	1609
35	Leucothea	1855, April 19	Luther	1903
36	Atalanta	1855, Oct. 5	Goldschmidt	1664
37	Fides	1855, Oct. 5	Luther	1569
38	Leda	1856, Jan. 12	Chacornac	1657
39	Lætitia	1856, Feb. 8	Chacornac	1684
40	Harmonia	1856, March 31	Goldschmidt	1247
41	Daphne	1856, May 22	Goldschmidt	1681
42	Isis	1856, May 23	Pogson	1392
43	Ariadne	1857, April 15	Pogson	1195
44	Nysa	1857, May 27	Goldschmidt	1379
45	Eugenia	1857, June 27	Goldschmidt	1638
46	Hestia	1857, Aug. 16	Pogson	1470
47	Melete *	1857, Sept. 9	Goldschmidt	1529
48	Aglaia	1857, Sept. 15	Luther	1788

* Goldschmidt at first believed it to be *Daphne* (41), but, finding its period different, called it *Pseudo-Daphne*. It was not seen from 1857 to 1861, when Schubert rediscovered it and named it *Melete*.

No.	Name.	Date of Discovery.	Discoverer.	Sidereal Rev. in Days.
49	Doris.........	1857, Sept. 19	Goldschmidt	2003
50	Pales.........	1857, Sept. 19	Goldschmidt	1975
51	Virginia	1857, Oct. 4	Ferguson	1576
52	Nemausa	1858, Jan. 22	Laurent..........	1338
53	Europa	1858, Feb. 6	Goldschmidt	1993
54	Calypso	1858, April 4	Luther...........	1548
55	Alexandra	1858, Sept. 10	Goldschmidt	1634
56	Pandora	1858, Sept. 10	Searle	1674
57	Mnemosyne ...	1859, Sept. 22	Luther...........	2049
58	Concordia	1860, March 24	Luther...........	1615
59	Danaë........	1860, Sept. 9	Goldschmidt	1902
60	Olympia (Elpis)	1860, Sept. 12	Chacornac	1634
61	Erato	1860, Sept. 14	Förster	2023
62	Echo.........	1860, Sept. 15	Ferguson	1352
63	Ausonia	1861, Feb. 10	Gasparis	1355
64	Angelina	1861, March 4	Tempel..........	1601
65	Cybele	1861, March 8	Tempel..........	2311
66	Maia	1861, April 9	Tuttle	1588
67	Asia..........	1861, April 17	Pogson	1375
68	Hesperia......	1861, April 29	Schiaparelli	1893
69	Leto	1861, April 29	Luther...........	1695
70	Panopea	1861, May 5	Goldschmidt	1542
71	Feronia.......	1861, May 29	Peters and Safford.	1245
72	Niobe	1861, Aug. 13	Luther...........	1671
73	Clytie	1862, April 7	Tuttle	1590
74	Galatea.......	1862, Aug. 29	Tempel	1691
75	Eurydice......	1862, Sept. 22	Peters	1594
76	Freia.........	1862, Oct. 21	d'Arrest	2080
77	Frigga........	1862, Nov. 12	Peters	1596
78	Diana	1863, March 15	Luther...........	1554
79	Eurynome	1863, Sept. 14	Watson..........	1399
80	Sappho.......	1864, May 2	Pogson	1270
81	Terpsichore ...	1864, Sept. 30	Tempel	1693
82	Alcmene......	1864, Nov. 27	Luther...........	1659
83	Beatrix	1865, April 26	Gasparis.........	1381
84	Clio..........	1865, Aug. 26	Luther...........	1330
85	Io............	1865, Sept. 19	Peters	1583
86	Semele	1866, Jan. 4	Tietjen	1983
87	Sylvia	1866, May 16	Pogson	2384
88	Thisbe	1866, June 15	Peters	1675
89	Julia	1866, Aug. 6	Stephan	1472
90	Antiope	1866, Oct. 11	Luther...........	2031
91	———......	1866, Nov. 4	Stephan	1495
92	Undina.......	1867, July 7	Peters	2086
93	———......	1867, Aug. 24	Watson..........	1669
94	———......	1867, Sept. 6	Watson..........	—
95	Arethusa......	1867, Nov. 23	Luther...........	—
96				

☞ The numerical order of the minor planets differs somewhat in the lists of English and French astronomers.

Of the first eighty-nine of these planets, the nearest to the sun is *Flora*, whose mean distance is 201,274,000 miles.

The farthest from the sun is *Sylvia*, with a mean distance of about 319,500,000 miles.

The least eccentric orbit is that of *Europa*, which is even nearer to an exact circle than the orbit of Venus. The most eccentric orbit is that of *Polyhymnia*, whose aphelion distance is rather more than double its perihelion distance.

The orbit of *Massilia* has the least inclination to the ecliptic, or 0° 41′; that of *Pallas* the greatest inclination, or 34° 42′.

The brightest planet is *Vesta*, which appears at times as a star of the sixth magnitude. The faintest is *Atalanta*, which, under the most favorable circumstances, is scarcely above the thirteenth magnitude.

The largest planet, according to some authorities, is *Pallas*. Lamont makes its diameter 670 miles, but Galle only 172 miles. According to others, *Vesta* is the largest, with a diameter of 228 miles. The more recently discovered planets are all so small that it is impossible to tell which is smallest. The diameters of those numbered 5 – 39, as given by Chambers, range from 12 miles (*Eunomia*) up to 111 miles (*Hygieia*).

Of these planets (up to the ninety-fourth inclusive), fifteen have been discovered in the United States, — *Euphrosyne*, *Virginia*, *Pandora*, *Echo*, *Maia*, *Feronia*, *Clytie*, *Eurydice*, *Frigga*, *Eurynome*, *Io*, *Thisbe*, and *Undina*, with the 93d and 94th, which are not yet (January, 1868) named.

In several cases minor planets have been discovered independently by two or more observers, each knowing nothing of what the other had done. Thus, *Irene* was discovered by Hind, May 19, 1851, and by Gasparis, May 23; *Massilia*, by Gasparis, September 19, 1852, and by Chacornac, September 20; *Amphitrite*, by Marth, March 1, 1854, by Pogson, March 2, and by Chacornac, March 3.

MOONS OF JUPITER.

Moon.	Sidereal Period of Revolution. d. h. m.	Distance in Radii of Jupiter.	Mean Distance in Miles.
I.	1 18 28	6.049	278,542
II.	3 13 15	9.623	442,904
III.	7 3 43	15.350	706,714
IV.	16 16 32	26.998	1,200,000

MOONS OF SATURN.

Moon.	Sidereal Period of Revolution. d. h. m.	Distance in Radii of Saturn.	Mean Distance in Miles.
I.	0 22 36	3.361	118,000
II.	1 8 58	4.313	152,000
III.	1 21 18	5.340	188,000
IV.	2 17 41	6.840	240,000
V.	4 12 25	9.553	336,000
VI.	15 22 41	22.145	778,000
VII.	21 12 0	28.000	940,000
VIII.	79 7 55	64.359	2,268,000

MOONS OF URANUS.

Moon.	Sidereal Period of Revolution. d. h. m.	Distance in Radii of Uranus.	Mean Distance in Miles.
I.	2 12 17	6.940	119,994
II.	4 3 28	9.720	170,863
III.	8 16 56	15.890	288,600
IV.	13 11 7	21.270	380,000

MOON OF NEPTUNE.

Sidereal Period of Revolution .... $5^d\ 20^h\ 50^m\ 45^s$.
Mean Distance from Neptune .... 236,000 miles.

THE CONSTELLATIONS.

1. All the stars in the heavens have been divided into groups called *constellations* (162). Many of these were recognized and named at a very early period; and some of them, as Orion, are mentioned in the Old Testament.

The method of naming the stars in each constellation has been explained above (163). The characters and names of the Greek alphabet are as follows: —

α,	Alpha.	ν,	Nu.
β,	Beta.	ξ,	Xi.
γ,	Gamma.	ο,	Omicron.
δ,	Delta.	π,	Pi.
ε,	Epsilon.	ρ,	Rho.
ζ,	Zeta.	σ,	Sigma.
η,	Eta.	τ,	Tau.
θ,	Theta.	υ,	Upsilon.
ι,	Iota.	φ,	Phi.
κ,	Kappa.	χ,	Chi.
λ,	Lambda.	ψ,	Psi.
μ,	Mu.	ω,	Omega.

If a constellation has more stars than can be named from the Greek alphabet, the Roman alphabet is used in the same way; and when both alphabets are exhausted, numbers are used.

2. *Circumpolar Constellations.* — One of the most important constellations, and one easily recognized, is the *Great Bear*, or *Ursa Major*. It is represented in Plate I. at the end of this volume. It may be known by the seven stars forming "the Dipper," or "Charles's Wain," as it is sometimes called. These stars are designated by the first seven letters of the Greek alphabet; and the name and position of each should be carefully fixed in the mind, as we shall have frequent occasion to refer to them. The Bear's feet are marked by three pairs of stars. These and the star in the nose can be readily found by means of the lines drawn on the chart. It may be remarked here, that in all cases the stars thus connected by lines are the leading stars of the constellation, and should be thoroughly

learned. The stars α and β are called the *Pointers.* If a line be drawn from β to α, and prolonged about five times the distance between them, it will pass near an isolated star of the second magnitude known as the *Pole Star*, or *Polaris.* This is the brightest star in the *Little Bear*, or *Ursa Minor* (Plate II.). It is in the end of the handle of a second and smaller "dipper." The stars β and γ of this constellation are quite bright, and are nearly parallel with ϵ and ζ of the other Bear.

On the opposite side of the Pole Star from the Great Bear, and at about the same distance, is another conspicuous constellation, called *Cassiopeia.* Its five brightest stars form an irregular *W*, opening towards the Pole Star (Plate II.).

About half-way between the two Dippers three stars of the third magnitude will be seen, — the only stars at all prominent in that neighborhood. These belong to *Draco*, or the *Dragon.* The chart will show that the other stars in the body of the monster form an irregular curve around the Little Bear, while the head is marked by four stars arranged in a trapezium. Two of these stars, β and γ, are quite bright. A little less than half-way from Cassiopeia to the head of the Dragon is the constellation *Cepheus*, five stars of which form an irregular *K*.

These five constellations never set in our latitude, and are called *circumpolar* constellations (page 5).

3. *Constellations visible in September.* — We will now study the remaining constellations visible in our latitude, beginning with those which are above the horizon at eight o'clock in the evening, about the middle of September. At this time the Great Bear will be low down in the northwest, and the Dragon's head nearly in the zenith. If we draw a line from ζ to η of the Great Bear and prolong it, we shall find that it will pass near a reddish star of the first magnitude. This star is called *Arcturus*, or α *Boötis*, since it is the brightest star in the constellation *Boötes.* Of its other conspicuous stars, four form a cross. These and the remaining stars of the constellation can be readily traced with the aid of Plate III.

Near the Dragon's head (Plate IV.) may be seen a very bright star of the first magnitude, shining with a pure white light. This star is *Vega*, or α *Lyræ.* Of this and some of the other stars in the Lyre we shall have occasion to speak hereafter.

If we draw a line from Arcturus to Vega (Plate III.), it will pass through two constellations, — the *Crown*, or *Corona Borealis*, and *Hercules*. The former is about one third of the way from Arcturus to Vega, and consists of a semicircle of six stars, the brightest of which is called *Alphecca*, or *Gemma Coronæ*, — the *gem* of the Crown.

Hercules is about half-way between the Crown and Vega. This constellation is marked by a trapezoid of stars of the third magnitude. A star in one foot is near the Dragon's head; there is also a star in each shoulder, and one in the face.

Just across the Milky Way from Vega (Plate V.) is a star of the first magnitude, called *Altair* or *α Aquilæ*. This star marks the constellation *Aquila*, or the *Eagle*, and may be recognized by a small star on each side of it. These are the only important stars in this constellation.

In the Milky Way, between Altair and Cassiopeia (Plate IV.), there is a large constellation called *Cygnus*, or the *Swan*. Six of its stars form a large cross, by which it will be readily known. *α* Cygni is often called *Deneb*. It forms a large isosceles triangle with Altair and Vega.

Low down in the south, on the edge of the Milky Way (Plate VI.), is a constellation called *Sagittarius*, or the *Archer*. It may be known by five stars forming an inverted dipper, often called "the Milk-dipper." The head is marked by a small triangle. The other stars, as seen by the map, may be grouped so as to represent a bow and an arrow.

Low in the southwest is a bright red star called *Antares*, or *α Scorpionis*. This constellation is described below (12).

The space between Sagittarius and Hercules and Scorpio is occupied by the *Serpent* (*Serpens*) and the *Serpent-bearer*, or *Ophiuchus* (Plates VI. and VII.). The head of the Serpent is near the Crown, and marked by a small triangle. The head of Ophiuchus is close to the head of Hercules, and may be known by a star of the second magnitude. Each shoulder is marked by a pair of stars. His feet are near the Scorpion. The Serpent can be best traced with the aid of the map.

Nearly on a line with Arcturus and γ Ursæ Majoris (Plate I.), and rather nearer the latter, is an isolated star of the third magnitude, called *Cor Caroli*, or *Charles's Heart*. This is the only prominent star in the constellation of *Canes Venatici*, or the *Hunting Dogs*.

Cassiopeia is almost due east of the Pole Star. A line drawn from the latter through β Cassiopeiæ, and prolonged, passes through two stars of the second and third magnitude. These, with two others farther to the south, form a large square, called the *Square of Pegasus*. Three of these, as seen by the map (Plate V.), belong to the constellation *Pegasus*, or the *Winged Horse*. α Pegasi is called *Markab*, and β is called *Algenib*. The bright stars in the neck and nose can be found by the map.

The fourth star in the Square of Pegasus belongs (Plate VIII.) to the constellation *Andromeda*. Nearly in a line with α Pegasi and this star are two other bright stars belonging to Andromeda. The stars in her *belt* may be found by the map.

Following the direction of the line of stars in Andromeda just mentioned, and bending a little towards the east, we come to *Algol*, or β *Persei*, a remarkable *variable star* (165). This star may be readily recognized from the fact that, together with β and γ Andromedæ and the four stars in the Square of Pegasus, it forms a figure similar in outline to the Dipper in Ursa Major, but much larger. If the handle of this great Dipper is made straight instead of being bent, the star in the end of it is α Persei, of the second magnitude. This star has one of the third magnitude on each side of it. The other stars in *Perseus* may be found by the chart.

Just below θ in the head of Pegasus (Plate IX.) are three stars of the third and fourth magnitudes, forming a small arc. These mark the urn of *Aquarius*, the *Water-bearer*. His body consists of a trapezium of four stars of the third and fourth magnitudes. Small clusters of stars show the course of the water flowing from his urn.

This stream enters the mouth of the *Southern Fish*, or *Piscis Australis*. The only bright star in this constellation is *Fomalhaut*, which is of the first magnitude, and at this time will be low down in the southeast.

To the south of Aquarius is *Capricornus*, or the *Goat*. He is marked by three pairs of stars arranged in a triangle. One pair is in his head, another in his tail, and the third in his knees.

Near Altair (Plate V.), and a little higher up, is a small diamond of stars forming the *Dolphin*, or *Delphinus*.

A little to the west of the Dolphin, in the Milky Way, are four stars of the fourth magnitude, which form the constellation *Sagitta*, or the *Arrow*.

4. *Constellations visible in October.* — If we look at the heavens at eight o'clock on the 15th of October, we shall see that all the constellations described above have shifted somewhat towards the west. Arcturus and Antares have set. In the east, below Andromeda (Plate X.), we see a pair of bright stars, which are the only conspicuous ones in *Aries*, or the *Ram*.

About half-way between Aries and γ Andromedæ are three stars which form a small triangle. This constellation is called *Triangulum*, or the *Triangle*.

Between Aries and Pegasus is the constellation *Pisces*, or the *Fishes*. The southernmost Fish may be recognized by a pentagon of small stars lying below the back of Pegasus. There are no conspicuous stars in the other Fish, which is directly below Andromeda. The stars in the band connecting the Fishes may be traced with the help of the map.

5. *Constellations visible in November.* — At eight o'clock in the evening on the 15th of November, we see at a glance that the constellations with which we have become acquainted have moved yet farther to the westward. Boötes, the Crown, Ophiuchus, and the Archer have set; Pegasus, Cassiopeia, and Andromeda are overhead; while new constellations appear in the east.

We notice at once (Plate XI.) a very bright star in the northeast, directly below Perseus. This is *Capella*, or α *Aurigæ*. There are five other conspicuous stars in *Auriga*, or the *Charioteer;* and with Capella they form an irregular pentagon.

Somewhat to the eastward (Plate XII.), and a little lower down, is a very bright red star. This is *Aldebaran*, or α *Tauri*. It is familiarly known as *the Bull's eye*. It will be noticed by the map that it is at one end of a *V* which forms the face of the Bull. This group is known as the *Hyades*. Somewhat above the Hyades is a smaller group, called the *Pleiades*, — more commonly known as the *Seven Stars*, though few persons can distinguish more than six. The bright star on the northern horn, or β Tauri, is also in the foot of Auriga, and counts as γ of that constellation.

All the space between Taurus and the Southern Fish, and below Aries and Pisces (Plate XIII.), is occupied by *Cetus*, the *Whale*. The head is marked by a triangle of rather conspicuous stars below Aries; the tail, by a bright star of the second magnitude, which is now just about as far above the horizon as Fomalhaut. On the body are five stars, forming a sort of sickle. About half-way between this sickle and the triangle, in the head, is *o Ceti*, also called *Mira*, or the *wonderful* star (165).

6. *Constellations visible in December.* — At eight o'clock in the evening in the middle of December, we shall find that Hercules, Aquila, and Capricornus have sunk below the horizon; while Vega and the Swan are on the point of setting. The Great Bear is climbing up in the northeast. In the east we behold by far the most brilliant group of constellations we have yet seen. Capella and Aldebaran are now high up; and below the former (Plate XII.) is the splendid constellation of *Orion*. His *belt*, made up of three stars in a straight line, will be recognized at once. Above this, on one shoulder, is a star of the first magnitude, called *Betelgeuse*, or *a Orionis*. About as far from the belt, on the other side, is another star of the first magnitude, called *Rigel*. There are two other fainter stars which form a large trapezium with Betelgeuse and Rigel. The three small stars below the belt are upon the sword.

Below Orion (Plate XIV.) is a small trapezium of stars which are in the constellation of *Lepus*, or the *Hare*. The head is marked by a small triangle, as seen on the map.

To the north of Orion, and a little lower down (Plate XII.), are two bright stars near together, one of the first and the other of the second magnitude. The latter is called *Castor*, and the former *Pollux*. They are in the constellation *Gemini*, or the *Twins*. A line of three smaller stars just in the edge of the Milky Way marks the feet, and another line of three the knees. Pollux forms a large triangle with Capella and Betelgeuse.

7. *Constellations visible in January.* — At eight in the evening on the 15th of January, Vega, Altair, the Dolphin, Aquarius and Fomalhaut have disappeared in the west; Deneb and the Square of Pegasus are near the horizon; while Capella and

Aldebaran are nearly overhead. Two stars of exceeding brilliancy have come up in the west. The one farthest to the south (Plate XIV.) is the brightest star in the whole heavens. It is called *Sirius*, or the *Dog-star;* and is in the constellation of *Canis Major*, or the *Great Dog*, which can be readily traced by the lines on the map.

The other bright star is between Sirius and Pollux (Plate XII.), and is called *Procyon.* It is in *Canis Minor*, or the *Little Dog.* The only other prominent star in this constellation is one of the third magnitude near Procyon.

Procyon, Sirius, and Betelgeuse form a large equilateral triangle.

Orion and the group of constellations about it constitute by far the most brilliant portion of the heavens, as seen in our latitude. There are, in all, but about twenty stars of the first magnitude, and seven of these are in this immediate vicinity.

8. *Constellations visible in February.* — If we look at the heavens at the same time in the evening about the middle of February, we shall miss Cygnus and Pegasus from the west. Auriga and Orion are nearly overhead.

Southeast of the Great Bear (Plate XV.) is a red star of the first magnitude, called *Regulus*, in the constellation of *Leo*, or the *Lion.* There are five stars near Regulus, which together with it form a group often called the *Sickle.* The star in the tail is *Denebola*, which makes a right-angled triangle with two others near it.

Between Leo and Gemini is the constellation *Cancer*, or the *Crab.* It contains no bright stars, but a remarkable cluster of small stars called *Præsepe*, or sometimes the *Beehive.*

Below Regulus (Plate XIV.) is a bright red star of the second magnitude, called *Cor Hydræ*, or the *Hydra's Heart.* The head of Hydra is marked by five small stars. The coils of the monster can be traced by the map. A portion of the constellation is on Plate XVI.

9. *Constellations visible in March.* — At the middle of March, the heavens will have shifted round somewhat towards the west; but all the conspicuous constellations of the preceding month are still visible, while no new ones at all brilliant have come into view.

If we draw a line from the end of the Great Bear's tail to Denebola, it will pass through two constellations, — *Canes Venatici*, mentioned above; and *Coma Berenices*, or *Berenice's Hair*, a large cluster of faint stars (Plate XV.).

10. *Constellations visible in April.* — At the middle of April, Aries and Andromeda have set; Taurus, Orion, and Canis Major are sinking towards the west; the Great Bear and the Lion are overhead; Arcturus has risen in the northeast (Plate XVI.); and some way to the south of this is seen a star of the first magnitude, which forms a large triangle with Arcturus and Denebola. It is called *Spica Virginis*, and is the chief star in the constellation *Virgo*, or the *Virgin*. The stars on the breast and wings can be found with the aid of the map.

South of Virgo is a trapezium of four stars, which are in the constellation of *Corvus*, or the *Crow*.

11. *Constellations visible in May.* — At the middle of May, Taurus, Orion, and Canis Major have set; Vega has just come up in the northeast; and between Vega and Arcturus we again see Hercules and Corona. Below Spica are two stars of the second magnitude, belonging to the constellation *Libra*, or the *Balance*. A star of the fourth magnitude forms a triangle with these, and marks one pan of the balance (Plate VII.).

12. *Constellations visible in June.*—In June we shall find that Canis Minor, Perseus, Auriga, and Gemini have either set, or are on the point of setting; Arcturus is overhead; Cygnus and Aquila are just rising. Ophiuchus is well up; and low in the southeast we see again the red star Antares, in the constellation *Scorpio*, or the *Scorpion* (Plate VI.). There is a star of the third magnitude on each side of Antares, and several stars of the third and fourth magnitudes in the head and claws. The configuration of these stars is much like a boy's kite with a long tail. Scorpio is a very brilliant constellation, and is seen to better advantage in July and August.

13. *Constellations visible in July and August.* — We have now described all the important constellations visible in our latitude. Those which are seen in July and August are mainly those described under the last two or three months, and under September.

14. *Southern Circumpolar Constellations.* — There are a

number of constellations near the South Pole of the heavens which never rise in our latitude, just as there are some near the North Pole which never set. These are called the *southern circumpolar constellations*, and are shown in Plate XVII.

CONSTELLATIONS VISIBLE EACH MONTH.

☞ THE following table gives the constellations visible at 8 o'clock in the evening, about the middle of each month. The stars opposite the names of the constellations indicate those visible in the month designated at the top.

Name of Constellation.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	April.	May.	June.	July	Aug.
Boötes	*							*	*	*	*	*
Corona Borealis	*	*							*	*	*	*
Ophiuchus	*									*	*	*
Sagittarius	*										*	*
Hercules	*	*							*	*	*	*
Lyra	*	*	*							*	*	*
Aquila	*	*	*								*	*
Delphinus	*	*	*								*	*
Capricornus	*	*	*									*
Cygnus	*	*	*	*						*	*	*
Sagitta	*	*	*								*	*
Aquarius	*	*	*	*								
Piscis Australis	*	*	*	*								
Pegasus	*	*	*	*	*							
Andromeda	*	*	*	*	*	*	*					
Perseus	*	*	*	*	*	*	*	*				
Aries		*	*	*	*	*	*					
Pisces		*	*	*	*							
Cetus			*	*	*	*						
Triangulum			*	*	*	*	*					
Auriga			*	*	*	*	*	*	*			
Taurus			*	*	*	*	*	*				
Lepus				*	*	*	*					
Orion				*	*	*	*	*				
Gemini				*	*	*	*	*	*			
Canis Major					*	*	*	*				
" Minor					*	*	*	*	*			
Cancer						*	*	*	*	*		
Hydra						*	*	*	*	*		
Leo						*	*	*	*	*	*	
Coma Berenices							*	*	*	*	*	*
Canes Venatici							*	*	*	*	*	*
Virgo								*	*	*	*	*
Corvus								*	*	*	*	
Libra									*	*	*	*
Scorpio										*	*	*

The following are the *circumpolar* constellations which are visible all the year round: Ursa Major, Ursa Minor, Draco, Cassiopeia, and Cepheus.

STARS OF THE FIRST MAGNITUDE.

The following is a list of the stars of the first magnitude, in the order of their brightness: —

1. Sirius, or α Canis Majoris.
2. η Argus (*variable*).
3. Canopus, or α Argus.
4. α Centauri.
5. Arcturus, or α Boötis.
6. Rigel, or β Orionis.
7. Capella, or α Aurigæ.
8. Vega, or α Lyræ.
9. Procyon, or α Canis Minoris.
10. Betelgeuse, or α Orionis.
11. Achernar, or α Eridani.
12. Aldebaran, or α Tauri.
13. β Centauri.
14. α Crucis.
15. Antares, or α Scorpionis.
16. Altair, or α Aquilæ.
17. Spica, or α Virginis.
18. Fomalhaut, or α Piscis Australis.
19. β Crucis.
20. Pollux, or β Geminorum.

Some astronomers admit into this class only the first seventeen of the above list. Others add to the list Regulus, or α Leonis; others, α Ursæ Majoris and α Andromedæ.

THE HISTORY OF THE CONSTELLATIONS.

The question with respect to the time when the stars were first grouped into constellations has been much discussed, but cannot be said to have been settled. Some writers believe that the earliest division of the starry sphere into such figures dates back to fourteen hundred years before the Christian era; but the most ancient reference to them in literature is in Homer and Hesiod, some seven hundred and fifty or eight hundred and fifty years before Christ. These poets mention only a few of the more marked stars and asterisms, as Arcturus, Sirius, the Pleiades, the Hyades, Orion, and the Bear. There are references to certain stars or groups of stars in the Bible, Job ix. 9, xxvi. 13, xxxviii. 31, 32; Amos v. 8; but they are probably not more ancient than those in the Greek poets, and the translation of the passages is a matter of dispute.

It is pretty certain that nearly four hundred years before Christ all the leading constellations had been formed; for about that time Eudoxus, of Cnidus, wrote an account of them, which would appear to have become quite a popular work. It has not come down to our day, but we know that it was the basis of the famous "Phænomena" of Aratus, written about 270 B. C. This was the first attempt, so far as we know, to describe in verse the groups and motions of the stars, and, though it was by no means free from mistakes, it was received with the highest favor, and has been famous even down to our own day. It was translated and praised by Ovid, by Cicero, and by Germanicus. Manilius drew from it in the preparation of his *Astronomica*, and Virgil himself borrowed from it in his Georgics.

Ptolemy, about 140 A. D., enumerates forty-eight constellations, — twenty-one northern, twelve zodiacal, and fifteen southern. We have described all the northern, except *Equuleus*, the *Little Horse*, which contains only ten stars, all below the third magnitude, between the head of Pegasus and the Dolphin.

The twelve zodiacal constellations are *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Pisces*. They are situated along the line of the zodiac (page 187), and have all been described above.

Of the fifteen south of the zodiac, we have mentioned eight. The others are *Eridanus*, *Argo Navis*, *Crater*, *Centaurus*, *Lupus*, *Ara*, and *Corona Australis*.

Eridanus, or the *River Po*, winds in an irregular stream through some 130° of the heavens. The portion of it which is visible in the latitude of Boston lies between Orion and Cetus.

Argo Navis, or the *Ship Argo*, is one of the largest and most brilliant of the southern constellations. It contains two stars of the first magnitude, four of the second, and nine of the third. Only a small part of it, containing none of the brightest stars, rises above the horizon in our latitude.

Crater, or the *Cup*, is on the back of Hydra, south of the hind feet of Leo. It is made up of a few stars, only one of which rises to the third magnitude.

Centaurus, or the *Centaur*, is a large and conspicuous constellation, but none of its brighter stars are visible in our latitude.

Ara, or the *Altar*, does not rise above our horizon.

Lupus, or the *Wolf*, is directly south of Scorpio. It contains no conspicuous stars.

Corona Australis, or the *Southern Crown*, is a small group of stars, only one of which is equal to the fourth magnitude, between the fore legs of Sagittarius and the Milky Way.

Of the constellations described by us, two are modern, — *Coma Berenices*, added by Tycho de Brahe, about 1603, and *Canes Venatici*, by Hevelius, in 1690. Fifty or sixty more have been added from time to time, some of which have been rejected by modern uranographers.

The ancient constellations include all the brighter stars in the heavens, except in a small region about the south pole. The later ones are mainly made up of the small stars not included in these early asterisms, and, with very few exceptions, are not worth tracing.

THE MYTHOLOGY OF THE CONSTELLATIONS.

To the Greeks the starry heavens were an illustrated mythological poem. Every constellation was a picture, connected with some old fable of gods or heroes. We shall give a brief sketch of the more important of these myths, with a few out of the many allusions to them in ancient and modern poetry.

The two *Bears* have one story. Callisto was a nymph beloved by Jupiter, who changed her into a she-bear to save her from the jealous wrath of Juno. But Juno learned the truth, and induced Diana to kill the bear in the chase. Jupiter then placed her among the stars as Ursa Major, and her son Arcas afterwards became Ursa Minor. Juno, indignant at the honor thus shown the objects of her hatred, persuaded Tethys and Oceanus to forbid the Bears to descend, like the other stars, into the sea. Hence, Virgil speaks of the Bears as "Oceani metuentes aequore tingi"; and Ovid, as "liquidique immunia ponti."

According to Ovid, Juno changed Callisto into a bear; and when Arcas, in hunting, was about to kill his mother, Jupiter placed both among the stars.

Ursa Minor was also called *Phœnice*, because the Phœnicians made it their guide in navigation, while the Greeks preferred the Great Bear for that purpose. It was also known as *Cynosura* (*dog's tail*) from its resemblance to the upturned curl of a dog's tail. The Great Bear was sometimes called *Helice* (*winding*), either from its shape or its curved path. Ovid says, as Aratus had said before him, —

> "Esse duas Arctos; quarum Cynosura petatur
> Sidoniis, Helicen Graia carina notet."

Boötes (the *Herdsman*) was also called *Arctophylax* and *Arcturus*, both of which names mean the *guard* or *keeper of the bear*. According to some of the stories, Boötes was Arcas; according to others, he was Icarus, the unfortunate son of Dædalus. The name *Arcturus* was afterwards given to the chief star of the constellation.

Cepheus, *Cassiopeia*, *Andromeda*, *Perseus*, and *Pegasus* are a group of star-pictures illustrating a single story.

Cepheus and Cassiopeia were the king and queen of Ethiopia, and had a very beautiful daughter, Andromeda. Her mother boasted that the maiden was fairer than the Nereids, who in their anger persuaded Neptune to send a sea-monster to ravage the shores of Ethiopia. To appease the offended deities Andromeda, by the command of an oracle, was exposed to this monster. The hero Perseus rescued her and married her.

Milton, in *Il Penseroso*, alludes to Cassiopeia as

> "That starred Ethiop queen that strove
> To set her beauty's praise above
> The Sea-Nymphs, and their powers offended."

According to one form of the story, it was her own beauty, and not her daughter's, of which the "Ethiop queen" boasted.

Pegasus, the winged horse, sprang from the blood of the frightful Gorgon, Medusa, whom Perseus had slain not long before he rescued Andromeda from the sea-monster. According to the most ancient account, Pegasus became the horse of Jupiter, for whom he carried the thunder and lightning;

but he afterward came to be considered the horse of Aurora, and finally of the Muses. Modern poets rarely speak of him except as connected with the Muses.

The *Dragon*, according to some of the poets, was the one that guarded the golden apples of the Hesperides; according to others, the monster sacred to Mars which Cadmus killed in Bœotia.

Virgil describes the dragon thus (G. i. 244): —

> "Maximus hic flexu sinuoso elabitur Anguis
> Circum perque duas, in morem fluminis, Arctos."

The *Lyre* is said to be the one which Apollo gave to Orpheus. After the death of Orpheus, Jupiter placed it among the stars at the intercession of Apollo and the Muses.

The *Crown* was the bridal gift of Bacchus to Ariadne, transferred to the heavens after her death. Virgil speaks of it (G. i. 222) as "Gnosia stella Coronae," referring to the Cretan birth of Ariadne. Ovid also calls it (Fasti, iii. 457) "Coronam Gnosida." Spenser refers to it as follows: —

> "Look how the crown which Ariadne wore
> Upon her ivory forehead that same day
> That Theseus her unto his bridal bore,
> When the bold Centaurs made that bloody fray
> With the fierce Lapiths which did them dismay,
> Being now placed in the firmament,
> Through the bright heaven doth her beams display,
> And is unto the stars an ornament,
> Which round about her move in order excellent."

Hercules is spoken of by Aratus as —

> "An *Image* none knows certainly to name,
> Nor what he labors for";

and again, in another part of the poem, as "the inexplicable Image." Ptolemy refers to it in somewhat the same way. Manilius calls it "ignota facies." When the name *Hercules* was given to it would appear to be uncertain.

Aquila is probably the eagle into which Merops was changed. It was placed among the stars by Juno. Some, however, make it the Eagle of Jupiter.

Cygnus or *Cycnus*, according to Ovid, was a relative of Phaëthon. While lamenting the unhappy fate of his kinsman on the banks of the Eridanus, he was changed by Apollo into a swan, and placed among the stars.

Sagittarius was said by the Greeks to be the Centaur Cheiron, the instructor of Peleus, Achilles, and Diomed. It is pretty certain, however, that all the zodiacal constellations are of Egyptian origin, and represent twelve Egyptian deities who presided over the months of the year. Thus Aries was Jupiter Ammon; Taurus, the bull Apis; Gemini, the inseparable gods Horus and Harpocrates; and so on. The Greeks adopted the figures, and invented stories of their own to explain them.

Scorpio, in the Egyptian zodiac, represented the monster Typhon. Originally this constellation extended also over the space now filled by Libra. Thus Ovid (Met. ii. 195) says: —

> "Est locus, in geminos ubi brachia concavat arcus
> Scorpios, et cauda flexisque utrimque lacertis
> *Porrigit in spatium signorum membra duorum*."

Virgil also (G. i. 33) suggests that the deified Augustus may find a place among the stars, —

> "Qua locus Erigonen inter Chelasque sequentes
> Panditur."

Erigone is Virgo, and *Chelæ* are the *claws* of the Scorpion. The poet goes on to picture the Scorpion as drawing himself into narrower space, to make room for the new-comer: —

> "Ipse tibi jam brachia contrahit ardens
> Scorpios, et caeli justa plus parte reliquit."

Ophiuchus represents Æsculapius, the god of medicine. Serpents were sacred to him, "probably because they were a symbol of prudence and renovation, and were believed to have the power of discovering herbs of wondrous powers."

Milton (P. L. ii. 709) speaks of

> "the length of Ophiuchus huge
> In the arctic sky."

Aquarius, in Greek fable, was Ganymede, the Phrygian boy who became the cup-bearer of the gods in place of Hebe.

Capricornus, the god Mendes in the Egyptian zodiac, is the subject of several Greek fables not worth recounting.

There are various stories also with regard to *Auriga*. The star Capella takes its name from the goat which he bears on his shoulder. Aratus says (Dr. Frothingham's translation): —

> "On his left shoulder rests
> The sacred Goat, — said to have suckled Jove;
> Olenian Goat of Jove the priests have named her."

So Ovid (Fasti v. 112): —

> "Nascitur Oleniae signum pluviale Capellae."

Taurus, as has been stated above, was the Egyptian Apis. The Greeks made it the bull which carried off Europa. The *Pleiades* are usually called the daughters of Atlas, whence their name *Atlantides*. Milton (P. L. x. 673) speaks of them as "the seven Atlantic Sisters." The idea that only six of the seven can be seen is very ancient. Aratus says: —

> "As seven their fame is on the tongues of men,
> Though six alone are beaming on the eye."

And Ovid (Fasti iv. 167): —

> "Quae septem *dici*, sex tamen *esse* solent."

According to one legend the seventh was Sterope, who became invisible because she had loved a mortal; according to another, her name was Electra, and she left her place that she might not witness the downfall of Troy, which was founded by her son, Dardanus. Tennyson, in *Locksley Hall*, alludes to the Pleiades: —

> "Many a night I saw the Pleiads, rising through the mellow shade,
> Glitter like a swarm of fire-flies tangled in a silver braid."

The *Hyades*, according to one of several stories, were sisters of the Pleiades. The name probably means *the Rainy*, since their heliacal rising announced wet weather. Hence Virgil speaks of them as *pluviae*, and Horace as *tristes*.

Cetus is said by most writers to be the sea-monster from which Perseus rescued Andromeda.

Orion was a famous giant and hunter, who loved the daughter of Œnopion, King of Chios. As her father was slow to consent to her marriage, Orion attempted to carry off the maiden; whereupon Œnopion, with the help of Bacchus, put out his eyes. But the hero, in obedience to an oracle, exposed his eye-balls to the rays of the rising sun, and thus regained his sight. The accounts of his subsequent life, and of his death, are various and conflicting. According to some, Aurora loved him and carried him off; but, as the gods were angry at this, Diana killed him with an arrow. Others say that Diana loved him, and that Apollo, indignant at his sister's affection for the hero, once pointed out a distant object on the surface of the sea, and challenged her to hit it. It was the head of Orion swimming, and the unerring shot of the goddess pierced it with a fatal wound. Another fable asserts that Orion boasted that he would conquer every animal; but the earth sent forth a scorpion which destroyed him.

Aratus alludes to the brilliancy of this constellation: —

> "What eye can pass him over,
> Spreading aloft in the clear night? Him first
> Whoever scans the heavens is sure to trace."

And again he speaks of him as

> "In nothing mean, glittering in belt and shoulders,
> And trusting in the might of his good sword."

Ovid calls him "ensiger Orion," and Virgil describes him as "armatum auro." We have a vivid picture of him in Longfellow's "Occultation of Orion": —

> "Begirt with many a blazing star,
> Stood the great giant Algebar,
> Orion, hunter of the beast!
> His sword hung gleaming by his side,
> And, on his arm, the lion's hide
> Scattered across the midnight air
> The golden radiance of its hair."

Canis Major and *Minor* are the dogs of Orion, and are pursuing the *Hare*.

The *Twins*, Castor and Pollux, the sons of Jupiter and Leda, are the theme of many a fable. They were especially worshipped as the protectors of those who sailed the seas, for Neptune had rewarded their brotherly love by giving them power over winds and waves, that they might assist the shipwrecked.

Leo, according to the Greek story, was the famous Nemean lion slain by Hercules. Jupiter placed it in the heavens in honor of the exploit.

The *Hydra* also commemorates one of the twelve labors of Hercules, — the destruction of the hundred-headed monster of the Lernæan lake.

Virgo represents Astræa, the goddess of innocence and purity, or, as some say, of justice. She was the last of the gods to withdraw from earth at the close of "the golden age." Aratus thus speaks of her: —

> "Once on earth
> She made abode, and deigned to dwell with mortals.
> In those old times, never of men or dames
> She shunned the converse; but sat with the rest,
> Immortal as she was. They called her Justice.
> Gathering the elders in the public forum,
> Or in the open highway, earnestly
> She chanted forth laws for the general weal."

But when the age became degenerate,

> "Justice then, hating that generation,
> Flew heavenward, and inhabited that spot
> Where now at night may still be seen the Virgin."

Libra, or the *Balance*, is the emblem of justice, and is usually associated with the fable of Astræa.

Argo Navis is the famous ship in which Jason and his companions sailed to find the Golden Fleece.

This slight sketch of the leading fables connected with the constellations will serve to show how completely the Greeks "nationalized the heavens." There have been various attempts to change into Christian titles the whole nomenclature of the skies. Julius Schiller, in 1627, urged such a revolution in his *Coelum Stellatum Christianum*, as Bartsch and others had done before him. According to these reformers of the heavens, the Great Bear becomes the skiff of St. Peter; Cassiopeia, Mary Magdalene; and Perseus with Medusa's head, David with the head of Goliath. The cross in the Swan is the

Holy Cross; the Virgin is Mary; and the Water-bearer, John the Baptist.

In the seventeenth century Weigel, a professor in the University of Jena, proposed the formation of a collection of *heraldic* constellations. In the zodiac he wished to place the escutcheons of the twelve most illustrious houses of Europe; and Orion, Auriga, and other leading asterisms were metamorphosed in the same way.

Sir John Herschel says: "The constellations seem to have been almost purposely named and delineated to cause as much confusion and inconvenience as possible. Innumerable snakes twine through long and contorted areas of the heavens, where no memory can follow them. Bears, lions, and fishes, small and large, northern and southern, confuse all nomenclature. A better system of constellations might have been a material help as an artificial memory."

But the habits of four thousand years are not easily changed. Men will still

> "Hold to the fair illusions of old time, —
> Illusions that shed brightness over life,
> And glory over nature";

and the starry heavens will continue to be for ages to come, as they have been for ages gone by, a picture-book of Greek fable.

QUESTIONS FOR REVIEW AND EXAMINATION.

MOTIONS AND DISTANCES OF THE HEAVENLY BODIES.

1. WHAT was the earth once thought to be? 2. What is the shape of the earth now known to be? 3. How is this shown by observation of ships at sea? 4. How by the observation of the eclipses of the moon? 5. How do we know that the sun, moon, and planets are all globes? 6. What is known of the shape of the stars? 7. How do the stars rise and set? 8. What are circumpolar stars? 9. What is true of the motion of the Polar Star? 10. What is true of the motion of the stars as we go from the Polar Star? 11. How do we know that the stars describe accurate circles about the Polar Star as a centre? 12. How do we know that the stars move at a uniform rate, and all describe their circles in the same time? 13. How do we detect the existence of atmospheric refraction? 14. What is the effect of refraction upon all the heavenly bodies? 15. Explain this effect? 16. Prove that the earth rotates from west to east in 24 hours? 17. Do the heavens really rotate about the earth from east to west? 18. How do we know this? 19. State the direction of the circles described by the stars, as compared with that of the horizon in different parts of the earth? 20. Do the stars as seen in different parts of the earth describe circles which really have different directions? 21. Show by an illustration that two co-ordinates are sufficient to define the position of a point on a plane surface? 22. What two co-ordinates serve to define the position of a point on the surface of a globe? 23. Show that these co-ordinates will serve to define the position of such a point? 24. What are the most convenient co-ordinates of a star? 25. Which of these

co-ordinates is measured by means of the transit instrument? 26. Explain the adjustment of this instrument. 27. Explain how one of the co-ordinates of a star is measured with it. 28. What is a sidereal day? 29. Describe the mural circle. 30. Explain how the horizontal reading of the circle is found. 31. Explain how the altitude of the celestial pole is found. 32. Which co-ordinate of a heavenly body is found by the mural circle? 33. Explain how. 34. Do the fixed stars appear in exactly the same position in the heavens, from whatever part of the earth they are observed? 35. How do we know? 36. What is the solar day? 37. How is its length found, and how does it compare with that of the sidereal day? 38. What does the difference of length of these two days show as to one of the co-ordinates of the sun? 39. By what other observation is this same thing shown? 40. What is a sidereal year? 41. How do the solar days compare with one another in length? 42. Why is ordinary clock time called mean time? 43. What is the ecliptic? 44. What is the inclination of the ecliptic to the earth's axis? 45. What belt of the earth is called the torrid zone? 46. What belts are called the frigid zones? 47. What belts are called the temperate zones? 48. What is true of the sun's coming directly overhead, and of its rising and setting in each of these zones? 49. Explain the changes in the relative lengths of day and night. 50. Explain the change of seasons. 51. What is the celestial equator? 52. Explain declination and right ascension. 53. What are the equinoxes? 54. What is the precession of the equinoxes? 55. What is the tropical year? 56. How does it compare with the sidereal year? 57. Which is the year of common life? 58. What are the solstices? 59. What does the variation of the sun's apparent diameter prove? 60. How can the form of the path described by the sun among the stars be found? 61. What kind of a curve is it? 62. To what does the inclination of the earth's axis to the ecliptic give rise? 63. What is twilight? 64. What causes it? 65. It continues while the sun is within what distance of the horizon? 66. When and where is twilight shortest? 67. Explain why this is so. 68. In the latitude of Boston how does the twilight in the summer compare with that in the winter?

69. Explain why it is so. 70. Where is the new moon always seen? 71. What is her motion? 72. When is the moon in conjunction? 73. When in opposition? 74. What is true of the moon's declination? 75. What are the moon's nodes? 76. What is a lunar day, and how does it compare with the solar day? 77. How do lunar days compare with one another? 78. What is the moon's orbit, and how can it be found? 79. What is meant by the moon's being in perigee? 80. What by her being in apogee? 81. What is the line of apsides? 82. Describe the apparent motion of Venus. 83. The apparent motion of Mercury. 84. Why are these bodies called planets? 85. What is meant by the greatest elongation of these planets? 86. Describe the apparent motion of the other planets. 87. How did the ancients attempt to explain the apparent irregular motion of the planets? 88. Give an account of the Ptolemaic system. 89. Explain what is meant by cycles, epicycles, and deferents. 90. What change did Tycho de Brahe introduce into this system? 91. How was this system further modified by Copernicus? 92. Did he dispense with epicycles and deferents in his system? 93. What three facts did Kepler discover about the planetary motions? 94. Give an account of the method by which he discovered these facts. 95. Whose observations did he make use of? 96. What is the sidereal period of a planet? 97. What is the synodical period of a planet? 98. How is the sidereal period of the earth found? 99. How is the synodical period of a planet found? 100. What planets can be in inferior and superior conjunction? 101. What planets have conjunctions and oppositions? 102. What must be known in order to compute the sidereal period of a planet? 103. Find the sidereal period of an inferior planet. 104. Find the sidereal period of a superior planet. 105. What must be observed in order to find the relative distance of an inferior planet from the sun? 106. Find the relative distance of an inferior planet from the sun. 107. What must be observed in order to find the relative distance of a superior planet from the sun? 108. Find the relative distance of a superior planet from the sun. 109. When the relative distances of the planets from the sun are known, what must be found in order to ascertain their real distances

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P.

THE END.

Cambridge: Electrotyped and Printed by Welch, Bigelow, & Co.