THE BOOK WAS DRENCHED

ELEMENTARY TREATISE

ON

ASTRONOMY.

BY

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PREFACE.

It may be necessary briefly to state the arrangement of the present Treatise.

In the first Chapters, I have explained, in a general way, certain of the obvious Phenomena of the Heavens: and then, with a view of affording the Student the means of distinctly apprehending the methods, by which, those Phenomena are observed, and their quantities and laws ascertained, I have described, although not minutely, some of the principal instruments of an Observatory. By an attentive consideration of the means, by which, in practice, right ascensions and latitudes are estimated and computed, a more precise notion of those quantities may, perhaps, be obtained, than either from the terms of a definition, or from their representation in a geometrical diagram.

But, an observation expressed by the graduations of a quadrant, or the seconds of a sidereal clock, cannot be immediately used for Astronomical purposes. It must previously be *reduced* or *corrected*. To the theories, then, of the necessary *corrections*, I have very soon called the attention of the Student: since, without a knowledge of them, he would be unable to understand the common process of regulating a sidereal clock, or that, by which, the difference of the latitudes of two places is usually determined.

The corrections are five; Refraction, Parallax, Aberration, Precession, and Nutation. The two latter, although they may be investigated on the principles of Physical Astronomy, are yet, in the ordinary processes of Plane Astronomy, equally necessary with the preceding.

To the Theory of the fixed Stars, which includes, as subordinate ones, the theories of the corrections that have been enumerated, succeed, the Solar, Planetary, and Lunar Theories. Of these, the last is, by many degrees, the most difficult. And, since, in its present improved state, it is not made to rest solely on observation, I have been compelled, in endeavouring to elucidate it, slightly to trespass on the province of Physical Astronomy.

The *Equation of Time*, which, essentially, depends on the Sun's motion, is placed immediately after its Theory.

On the same principle of arrangement, Eclipses are made to succeed the Solar and Lunar Theories. The method of computing them is that, which M. Biot has, in the last Edition of his Physical Astronomy, adopted, probably, from a memoir of Delambre's* on the passage of Mercury

* Mem. *Inst.* tom. III. p. 392. (1802.)

over the Sun's disk. The traces of this method, may be discerned in a Posthumous work*, of the celebrated Tobias Mayer, on Solar Eclipses.

The method just noticed is as extensive as it is simple. For, it equally applies to Eclipses, Occultations of fixed Stars by the Moon, and the Transits of inferior Planets over the Sun's disk. And this circumstance has determined the places of the two latter subjects, which are immediately after that of the former.

In the last Chapters are discussed, the methods of computing Time, Geographical Latitude and Longitude, and the Calendar.

Such is the arrangement of the present Treatise. And, since it could not be entirely regulated by the necessary connexion of the subjects, it has, occasionally, been so, by certain views, of what seemed, their proper and natural sequence. It so happens, therefore, that the more difficult investigations are not invariably preceded by the more easy. The methods, for instance, of computing the Time, Geographical Latitude and Longitude, follow the Lunar Inequalities, Eclipses, Occultations, and Transits; but, since they do not follow by strict consequence, the latter, if it so suits the convenience of the Student, may, in a first perusal, be omitted.

I have been solicitous to supply every part of the Treatise with suitable Examples. For, they are found to

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* Mayer, *Opera Inedita.* vol. I. p. 23.

be in Astronomy, more than in any other science, the means of explanation.

They become the means of explanation for reasons different from those which operate in other cases. For, Astronomical Examples are not always the mere translations of a rule, or an algebraical formula, or a geometrical construction, into arithmetical results. But, frequently, they are of a different description, and require the aid of certain subsidiary departments of Astronomical Science not then the subjects of consideration.

For instance, the difference of the latitudes of two places is equal to the sum or the difference of the zenith distances of the same Star. This rule cannot be applied according to its strict letter; for, when we descend into its detail, we may be obliged to reduce the observed zenith distances by four corrections. Consequently, we ought either to have previously established, or we must proceed to investigate, the theories of those corrections. This instance will also serve to shew, what frequently happens, that a rule shall possess a seeming facility in its general enunciation, which vanishes when we become minute and are in quest of actual results.

There is, in fact, scarcely any thing in Astronomical science single, or produced, at first, perfect by its processes. No series of propositions, as in Geometry, originating from a simple principle and terminating in exactness of result. But, every thing is in connexion; when first disengaged, imperfect, and advanced towards accuracy only by successive approximations.

Consider, for instance, the Sun's Parallax. That essential element is determined by no simple process, but is, as it were, extricated by laborious calculations from a phenomenon in which, at first sight, it does not seem involved. Again, the common method of determining the Longitude at Sea rests on whatever is most refined in theory and exact in practice: on Newton's system in its most improved state, and on the most accurate of Maskelyne's observations.

The preceding remarks, besides their proper purpose, may perhaps serve to shew that an Astronomical Treatise, with any pretensions to utility, cannot be contained within a small compass. It ought to teach the Principles of Astronomy ; but, it cannot well do that, except by detailing and explaining its best methods: that is, by explaining methods such as are practised, and as they are practised. Now, the methods of Astronomy are very numerous, and the details of several of them very tedious.

There are methods merely speculative; such as cannot be practised, although founded precisely on the same principle as other methods that are practised. For instance, the separation of the Sun from a Star, in a given time, is equally certain and of the same kind, as the separation of the Moon from a Star, but since, in practice, it is not so *ascertainable*, it cannot be made the basis, as the latter is, of a method of finding the Longitude.

The exclusion then of methods merely curious, and of no practical utility, has been one mean of contracting the bulk of this Treatise. Another I have found, in omitting to explain the systems of Ptolemy and of Tycho Brahe. These do not now, as formerly, require confutation. The spirit of defending them is extinct. They are not only exploded but forgotten. And, were they not, it would be right to divert the attention of the Student, from what is foreign, fanciful, and antiquated, to real inventions and discoveries of more modern date, and purely of English origin.

The present Treatise is not intended to explain Physical Astronomy and the system of Newton. But, the discoveries and inventions of Bradley and Halley are within its scope. Their numerous and accurate observations and their various Astronomical methods, would alone place them in the first rank of illustrious Astronomers. But, they have an higher title to pre-eminence. In point of genius, they are, after Newton, unrivalled. The first, for his two Theories of Aberration and Nutation: the last, for his invention of the methods of determining the Sun's Parallax from the Transit of Venus, and the Longitude from the Lunar motions.

This Lunar method of determining the Longitude was not reduced to practice by its author. That it has been since, is owing to Hadley and Maskelyne. The first, by his Quadrant, furnishing the instrumental, the latter, by the Nautical Almanack, the mathematical means.

This last-mentioned Astronomical Work, for such it is, and the most useful one ever published, is alone a sufficient basis for the fame of its author. Besides its results, it contains many valuable remarks and precepts. It is a collection of most convenient Astronomical Tables, and should be in the hands of every Student who is desirous of learning Astronomy; and who, for that end, must be conversant with Examples and Tables.

But, mere precepts and instances will not effect every thing. In order to remove the imperfection necessarily attached to knowledge, acquired solely in the closet, instruments must be used and observations made. The means of doing this, however, are not easily had; and, it is to be regretted, they are not afforded to the Students of this University. An Observatory is still wanting to its utility and splendor.

AN

ELEMENTARY TREATISE

ON

ASTRONOMY.

CHAP. I.

Certain Phenomena of the Heavens explained by the Rotation of the Earth.

In an Elementary Treatise on Plane Astronomy, two objects are required to be accomplished: 1st, The description and general explanation of the heavenly phenomena. 2dly, The establishment of methods for exactly ascertaining and computing such phenomena. Our attention will be first directed to the former of these two objects.

If, on a clear night, we observe the Heavens*, they will appear to undergo a continual change. Some stars will be seen ascending from a quarter called the East, *or rising;* others descending towards the opposite quarter the West, *or setting.* In some intermediate point, between the East and West, each star will reach its greatest height, or, will *culminate:* The greatest heights of the several stars will be different, but they will all appear to be attained towards the same part of the Heavens; which part is called the South.

If we now turn our backs to the South and observe the North, the opposite quarter, new phenomena will present them-

* *Exposition du Système du Monde,* p. 2.

selves. Some stars will appear as before, rising, reaching their greatest heights, and setting ; but besides, other stars will be seen that never set, moving with different degrees of velocity ; and some, to appearance, nearly stationary. About one of these stationary stars, the other stars that never set appear to revolve, or describe circles : that stationary star is called the *Polar* Star : the stars revolving round it, *Circumpolar*.

The *Polar* Star, that which is to be seen in the Heavens, is not, when nicely observed, stationary ; it is not the place of the *Pole*, which is an imaginary point, but in which a star, if situated, would appear perfectly quiescent.

Almost all the stars in the Heavens retain towards each other the same relative position ; there is no mutual approach or recess : and accordingly they are called *fixed* stars. There are, however, certain stars, called *Planets*, not under the above conditions, but, which continually change their places. The Sun and Moon also, the two celestial objects of the greatest interest, are from day to day varying their place in the Heavens.

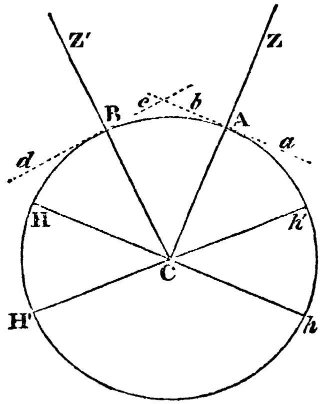
The figure of the Earth is nearly that of a spheroid of small excentricity, that is, not much differing from a sphere. If at the place of a spectator we conceive a tangent plane to the spheroid indefinitely extended on all sides, such plane is called the spectator's *Horizon ;* and an imaginary line drawn from the spectator perpendicularly to the plane, will tend upwards to a point in the Heavens called the *Zenith* : The opposite point in the line's direction continued downwards is called the *Nadir*.

This plane, called the Horizon, will bound the spectator's view: stars are said to rise when they first appear above it ; and to set, when they sink beneath it.

This is nearly, but not exactly true : for, since the spectator by his stature is elevated a little above the Earth's surface, he will be able to see an object a little beneath the horizontal plane, that is extended at his feet. In other words, a line drawn from his eye and a tangent to the Earth's surface, falls beneath the plane of the horizon. And, if his horizon were defined to be that in which such lines should continually be found, it would be a conical superficies, the vertex being in the eye of the spectator.

The tangent plane, in which *aAb* [see Fig. in opposite page] lies, has been called by Astronomers the *Sensible Horizon :* but, parallel to this, they imagine, for the purposes of calculation, another plane, such as *HCh*, passing through the center of the Earth: to which, they have given the name of *Rational Horizon.*

It is plain, that both the sensible and rational horizon are merely relative, that is, will change with the change of the spectator's place. To a spectator at *A*, *ab* perpendicular to *CAZ*



is the *sensible*, and *Hh* parallel to *ab* the *rational* horizon; and *Z* is his zenith. To a spectator at *B*, *ed* perpendicular to *CBZ'* is the sensible, and *H'h'* parallel to *ed* the rational horizon: and *Z'* is his zenith.

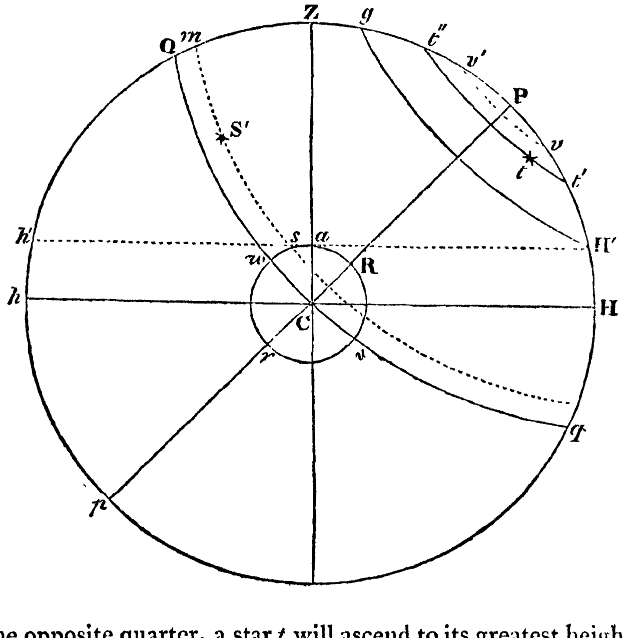
It has been said that stars reach their greatest heights towards the same part of the Heavens. In fact, if we conceive the places of the greatest altitudes of any two stars to be joined by the arc of a circle, in that circle will the greatest altitudes of all stars happen. It must pass through the zenith, which is the point of the greatest altitude of a star that passes directly over our heads: and through the pole which is the place of a quiescent star. This circle is called the *Meridian.*

The meridian cuts the plane of the horizon in two points South and North. The bounding line of the horizon is feigned to be a circle, which being divided into 360 parts, the North and South points are distant from each other 180°, and exactly intermediate to these are the East and West points.

The phenomena of the Heavens will now admit of an adequate, if not a just explanation. Let *aRr* represent the

Earth, *a* a spectator, *H′h′* the *sensible* and *Hh* the *rational* horizon, *Z* the zenith, *P* the pole: Let through *P* and *C*, the center of the Earth, a line *PCp* be drawn.

Now if the Earth be quiescent, and the sphere *PHpZ* revolve, in 24 hours, in the direction *qCQ*, round the imaginary axis *Pp*, the appearances described in the preceding pages will take place; a star will rise at *s* *, ascend through *s S′m* to *m* its greatest height.



In the opposite quarter, a star *t* will ascend to its greatest height *t″*, then descend to its lowest point *t′*, and again ascend: another star *v*, will, like the preceding, never set, but being nearer to *P* will move more slowly; and a star situated at *P* would be quiescent.

A star at *h′* will just appear above the horizon: another at *H′* will be 24 hours above the horizon: a third at *C*, will be as long (12 hours) above as below the horizon.

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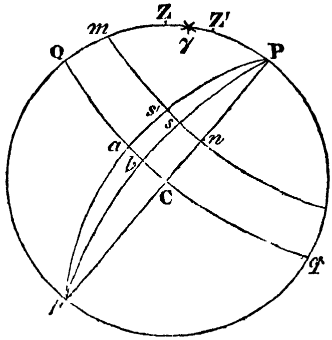
* *s* is meant to be the projection of a point as far distant from *a* as *h′* is. *Hh* and *H′h′* are represented as separate; but relatively to all measurement and calculation of angular distance, they must be considered as coincident. In fact to a spectator at *a*, *HH′* subtends no angle.

The circles $t't''$, $sS'm$, in which the stars appear to move being parallel to CQ (the equator) are called *Parallels.*

Since the sphere is supposed to revolve uniformly, the point b will be transferred to a in a time proportional to the magnitude of ab: for instance, if $ab = 1°$, CQ containing $90°$, the time from b to a will be $\frac{1}{90}$th of the time from C to Q, and therefore will be $\frac{1}{90}$th of 6 hours: in the same time s will be transferred to s', ss' being



less than ba, as the sine of Ps is less than radius; and, since ab measures the angle aPb, or $s'Ps$, the time through ss' is proportional to the angle aPb: and similarly the times through aQ, bQ, Cb are proportional to the angles aPQ, bPQ, bPC, which on that account are called *Hour Angles.*

Noon is determined by the Sun being on the meridian; therefore since CPQ is $90°$, it will be 6 hours before noon, that is, six in the morning, when the Sun is at C, or at n.

This happens if the sphere $PHpZ$ be supposed to revolve round Pp in the direction CQ, from the east towards the west, the Earth remaining quiescent; but, the same, with regard to the appearances, will also happen, if the sphere remain at rest, and the Earth Rra revolve round an axis Rr, in an opposite direction, in the direction wC, [see Fig. p. 4.] from the west towards the east. The first supposition is the antient one, and the revolution of the sphere was denominated that of the *primum mobile.* Either of these two hypotheses will account for the phenomena that have been described: but, the latter, that of the rotation of the Earth, and of the quiescence of the Heavens, is assumed to be the true one, as being the more simple: for if, instead of the Earth, the Heavens be carried round in 24 hours, the stars which are immensely distant must move with a most prodigious velocity. But, besides this reason, which is not indeed conclusive, there are others which will be subsequently explained, that render probable the assumed hypothesis.

In the figure which has been drawn for illustrating the phenomena, $PmQp$ is the meridian: and on a plane passing through this, that is, on the plane of the meridian, the sphere and its lines are supposed to be projected: Hh, for instance, is drawn a line, and is intended for the projection of the circular boundary of the horizontal plane: QCq, likewise, is meant to represent a circle, the plane of which passes through C, and to which a right line PCp, is perpendicular. The plane of the meridian $PmQq$ divides the sphere into two parts one called the Eastern, the other the Western hemisphere, and the present diagram is intended as the representation of lines, &c. drawn on the former. This mode of representation, imperfect as it is, is adopted for convenience, and to avoid the confusion of lines which an attempt to represent solids as they ought to appear, might introduce.

PCp is perpendicular to the plane of the circle QCq, and consequently PQ, Pq are quadrants containing 90 degrees, and also, if Pap [see Fig. p. 5.] be a circle of which the diameter is Pp, Pn, pn are quadrants. The circle QCq is called the *Equator*, and any great circle such as Pap is called a *secondary* to the equator: and the general definition of a secondary to a great circle is, that it is also a great circle passing through the poles of the former. Accordingly, a great circle passing through Z and any point in Hh would be a secondary to the horizon, which secondaries however are further distinguished by the name of *vertical* circles. The altitude of an heavenly body is its angular distance from the horizon, measured in a vertical circle passing through the body. The complement of the altitude is a *zenith distance*.

In the diagram PQp (p. 4,) C is the center of the Earth, and P, p, are the two poles in the Heavens: no determinate distance, however, is intended to be represented by CP: for, the astronomical computations involving angles merely, do not depend on it. Still, it is supposed to be very great; so great, that the Earth's radius bears no proportion to it, and that the sensible and rational horizon may be considered as coincident.

Hitherto the poles and the imaginary circles that have been spoken of have been referred to the Heavens: but the Earth also is made to have its poles and equator. In the Figure, [p. 4.] the former are *R*, *r* and the latter is *Cw*, the plane of which is perpendicular to *RCr*, and consequently in the plane of the celestial equator. The Earth is divided into two hemispheres, the northern *Raw Cu*, the Southern *ruCw*, by the equator *w Cu*.

The more distant a place on the Earth's surface is from its equator, the greater latitude it is said to have. And *latitude* defined is the angular distance of a place from the equator: hence of a place *a*, the latitude is *aCw*, measured by *aw*, or, in degrees, by *ZQ**: that is, it is the angle between its zenith and the celestial equator.

Since *PQ* is a quadrant, *PZ* is the complement of *ZQ*: hence *ZP* may be called the *co-latitude* of a place. If a person were to proceed northwards, that is, towards *R*, he would perceive the polar star situated near *P* to be more and more elevated above the horizon; in other words, *P* would approach his zenith *Z*, or the co-latitude *ZP* would diminish. If he possessed the means, therefore, of measuring an angular distance, he would be able to determine the successive latitudes of the places he arrived at: for, the zenith point *Z* is determined by the direction of a plumb-line; and the polar star is very near the pole. If it be assumed for the pole, the co-latitude will be determined nearly: and, by a method which will be hereafter described, we shall find that by the aid of the polar star and certain appropriate tables, the latitude may be determined with the greatest exactness.

On principles the same as the preceding, the angular distance between any known star on the meridian and the zenith will serve to determine the difference of the latitudes of places. For, the angular distance of a fixed star from the pole remaining the same, but the distance of the zenith from the pole, varying with the change of latitude, the latter must vary as the zenith distance of the star varies. For instance, the star called γ *Draconis* is $2' \ 24''$. *north* of the zenith of Greenwich Observatory:

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* *ZQ* measures the angle *ZaQ* or *ZCQ*: for the difference is the angle *CQa* which may be neglected, since *Ca* is incomparably less than *CQ*. See p. 4. Note.

but 19′ 23″.3, *South* of the zenith of Blenheim Observatory; therefore the difference of the latitudes of the two observatories is 21′ 47″.3; and since the latitude of Greenwich is 51° 28′ 40″, the latitude of Blenheim is 51° 50′ 27″.3

In the diagram, (p. 5,) Z, Z' are the zeniths of the two observatories, and $ZZ' = Z\gamma + Z'\gamma$, γ being the star between the two zeniths.

The distance of a place from the terrestrial equator, we have seen, is called its latitude: but, the angular distance of a star from the celestial equator is not called its latitude but its *Declination*; in the diagram, if m be the star or sun, on the meridian, it is mQ: and since $Pm + mQ =$ a quadrant, Pm is its *co-declination*, or, which is now the more usual term, its *North polar distance*. Hence, as in the case of the latitude, if we possessed the means of measuring angular distances, we could, since we have two objects, the star and the polar star, determine the north polar distance, and consequently the star's declination.

The means of measuring angular distances will soon claim our attention; as soon as, by the aid of two or three short chapters of general explanation, we shall have sufficiently enabled the Student to proceed with us in our course.

Several lines have been already described, and terms defined, as the process of explanation required; others, during the course of the Work, will be similarly so, when the necessity of their introduction arises.

CHAP. II.

On the proper Motions of the Earth, Moon, and Planets.

In the preceding Chapter several of the common phenomena have been accounted for on the simple hypothesis of the rotation of the Earth round its axis. There are, however, other phenomena not so explicable: such are the continual changes of place which the Sun, the Moon, and the Planets undergo.

There is no need of instruments to ascertain these changes. They are obvious to every spectator of the Heavens. If we observe, on any particular day, a star setting soon after the Sun, then on each successive day, its setting will more nearly follow that of the Sun, till its proximity to the Sun will become so close, that the effulgence of the latter will overpower the feeble light of the star and prevent it from being seen. Some days after this has happened, if we direct our view towards the rising Sun, we shall perceive the star first, as it were, emerging from the Sun, and on succeeding mornings, still preceding, in its risings, the Sun, by greater and greater intervals.

The phenomenon of the star just rising before the Sun is called its *heliacal* rising: and it was by such observations that the rude nations of antiquity recognised the seasons, and regulated the labours of the year*.

The mere hypothesis of the rotation of the Earth, or of that of the Heavens, will not explain this phenomenon. The Sun has evidently a proper and peculiar motion. He moves towards stars that set after him, and from stars that rise before him. In other words, amongst the fixed stars he moves from the west towards the east: that is, to a spectator, in our hemisphere, facing the south, from the right hand to the left.

This fact of a motion of the Sun from the west to the east will adequately explain why certain remarkable stars and groups of stars, called *Constellations*, are seen in the south at different hours of the night during the year. For the hour depends solely on the Sun: it is noon, when he is in the south; stars

* The Egyptians looked for the Inundation of the Nile at the time of the heliacal rising of Sirius, or, as they called it, of *Thoth* the Watch-dog.

directly opposite to him are therefore, by the rotation of the Earth, brought on the meridian at midnight: but the stars on the meridian at 12 one night, cannot again be there, at the same hour, on the succeeding night: for the Sun having shifted his place a little to the east, the stars before opposite to him are now opposite a part of the Heavens to the west of the Sun: that is, they must come on the meridian a little before midnight: and on succeeding nights more and more before midnight; so that, in the course of the year, they are in the south, though, by reason of the Sun's brightness, not always seen, at all hours of the four and twenty.

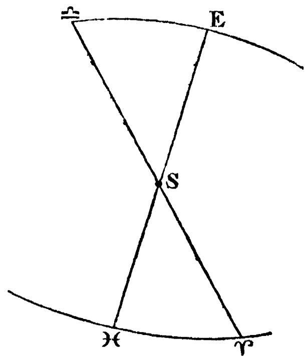
By observations like those that have been described, imperfect indeed, but sufficiently exact to ascertain the fact, the Moon is found to move, amongst the stars, from the west towards the east; and, more rapidly, that is, by greater changes of place, than the Earth.

The planets, if they be observed on successive nights, will be perceived also to have proper motions, and to change their places amongst the stars. Viewed from the Earth, indeed, they will not always appear to move towards the east; but sometimes towards the west, and at other times, for several nights together, they will appear nearly stationary.

It will be seen hereafter that the planets, as to the direction of their motion, form no exception to the Earth and Moon. Viewed from the Sun, their motion is from the spectator's west to his east, never in the contrary direction, or retrograde, and, never stationary. The two latter phenomena are merely optical; and, in a certain sense illusory, arising from the combination of the Earth's motion with that of the planet.

The motions from west to east that we have spoken of, take place, and must be combined with, the diurnal motion from east to west that arises from the rotation of the Earth. This latter motion is so great, that, as it were, it overpowers the former, and, with an inattentive spectator, prevents it from being observed. Even the Moon, which of all the planets has the swiftest proper motion towards the east, shifts her place in the course of a day by not more than 13°; whilst, by the rotation of the Earth, she is seemingly carried in the same time though 360°; yet, there are conjunctures when we cannot but recognise her proper motion; when, for instance, the Moon is near a star previously to an occultation: for moving over a space equal to her diameter in an hour she then visibly approaches the star.

The Earth's motion and the Sun's have both been spoken of; but only one, that of the former, really takes place. The Sun is at rest and the Earth moves round him. Yet we may consider the reverse to be the true case: for, if a spectator at *E* sees the



Sun *S* opposite to a star or a point in the heavens ♓; moving on to ♎, he sees the Sun at ♈. But the same appearances would be observed if the spectator should be quiescent at *E*, and the Sun should move from ♓ to ♈.

According to the solar system, as now established, and called from its inventor the *Copernican*, the Sun is in the center. Round it, in their order, revolve Planets called, Mercury, Venus, the Earth, Mars, Juno, Ceres, Pallas, Jupiter, Saturn, the Georgium Sidus, or, as the French call it, Uranus; and Astronomers are accustomed to designate these, as well as the Sun and Moon, by appropriate symbols:

The Sun	☉	Ceres	⚳
Mercury	☿	Pallas	⚴
Venus	♀	Jupiter	♃
The Earth	⊕	Saturn	♄
Mars	♂	Georgium Sidus	♅
Juno	⚵	The Moon	☽

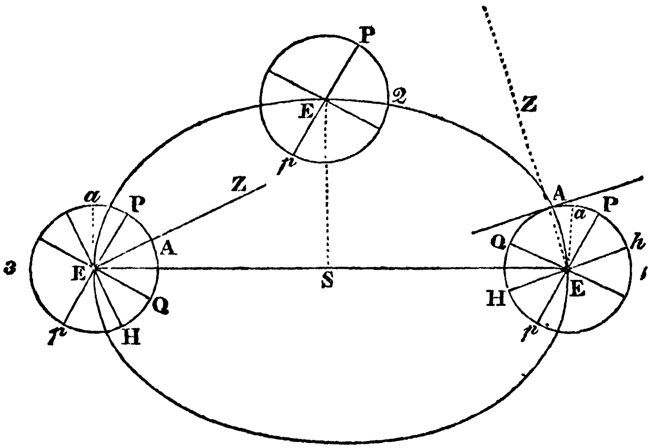
These Planets, considering the Earth as one, have proper motions of their own round the Sun and in the same direction. Such motions will account for their changes of place; as the diurnal rotation of the Earth accounts for the more obvious phenomena of the risings and settings of the Sun, Stars, and Planets.

CHAP. III.

On the Vicissitude of Seasons, and of Day and Night.

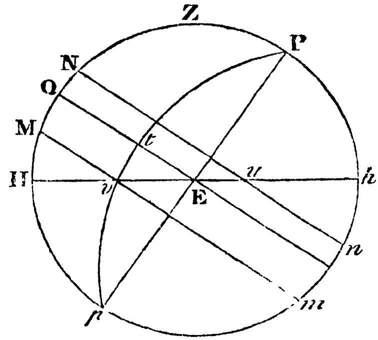
In the preceding Chapter it has been stated, that the Earth revolves in an orbit round the Sun; and the latter being always seen opposite to the Earth, seems in the course of a year to describe a circuit in the Heavens. This orbit, or circuit, is called, the *Ecliptic*. To the plane of the ecliptic the Earth's axis is not perpendicular, but inclined, (so it is determined by observation), at an angle of 23° 28′. From these two facts or circumstances of the Earth's revolution in an orbit round the Sun, and the inclination of the axis of diurnal rotation, at an invariable angle, to the plane of the annual orbit, the vicissitude of seasons may be explained.

Let *S* be the Sun, *E* the Earth in three positions 1, 2, 3, of her orbit; let also *Pp* be the Earth's axis, *EQ* the equator,



and *PAQp* must be conceived to be a section of the Earth perpendicular to the plane passing through the orbit *EEE*; so that *PAQp* will be opposite to the Sun, and to a spectator at *A* will be a meridian*. The axes *Pp* are drawn parallel to each other in the three positions.

Let us first consider the position 1, *A* [see the Figure in the preceding page] is the place of the spectator. *Z* is his zenith, and *Hh*, parallel to a tangent at *A*, his rational horizon [see pp. 3, 4]. The Sun (as it appears by inspection) is below the equator *EQ*: its zenith distance is *SEZ*, and its altitude above the horizon is *SEH*. In the subjoined diagram, which, like that in p. 4, is intended to represent the phenomena of diurnal rotation, let *M* be the Sun's place on the meridian (the angle *MEH*



being = *SEH* in Fig. [p. 12.]) then *Mm* is the parallel described by the Sun in 12 hours; whilst he describes 2 *Mv* he is above the horizon, and whilst he describes 2 *vm*, below; hence, the day, in the common acceptation, is shorter than the night: and

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* Diagrams in Astronomy are not only imperfect representations, since solids are to be represented *in plano*, but with regard to proportion, preposterous representations. The first is a real evil, the latter none; for, the demonstration in the text is equally clear whether *E*♌ be the half or the double of what it is in Figure, p. 12. it is in fact, independent of the represented relative proportion of *E*♌ to *SE*; yet, the former is to the latter, in fact, as 1 to 22984, and not, as in the Figure, as 1 to about 3 or 4. The first evil, however, if we do not recur to schemes of *solid* representation, admits of no remedy, except from the student's attention. The orbit 1, 2, 3, must be conceived as viewed obliquely, and then *PAHp* to be perpendicular to it. Or, if the orbit be conceived coincident with the plane of the paper on which it is drawn, then the plane passing through *PAHp* is perpendicular to the paper.

half the difference is $2. \angle vPE$, or $2Et$ *: and will equal (in latitude $51° 52'$), $4^h 28^m 34^s$, if the position (1) is meant to represent the Earth in Winter, when the Sun is most below the equator: as will appear by the following computation:

By Naper †, $r \times \sin. Et = \tan. tv \times \tan.$ lat.

log. tan. tv [23° 28′] - - - - -	9.63761
log. tan. lat. [51° 52′] - - - - -	10.10510
10 + log. sin. Et - - - - - - -	19.74271

$\therefore$ log. sin. $Et = 9.74271$; $\therefore Et = 33° \ 34' \ 20''$, or, in time, $= 2^h \ 14^m \ 17^s$; $\therefore 2 \ Et = 4^h \ 28^m \ 34^s$.

In this position it is plain the Sun, rising at v, rises between the east point E and the south point H of the horizon; and, in the above instance, rises at 40° 9′ 25″ from the east point: which is thus shewn:

By Naper†, $r \times \sin. tv = \cos.$ lat. $\times \sin. Ev$

log. r + log. sin. 23° 28′ - - - -	19.60011
log. cos. 51° 52′ - - - -	9.79063
log. sin. Ev = - - - - - - -	9.80948

$\therefore Ev = 40° \ 9' \ 25''$.

In the position (3), as before, Z is the zenith: but now the Sun is above the equator EQ: its zenith distance is SEZ, and its height above the horizon SEH: transferring therefore, as before, the place of the Sun to N in the scheme of diurnal rotation, p. 13, Nn is the parallel which he describes in 12 hours, and he is above the horizon, or it is *day* whilst describing 2 Nu: below, or it is night, whilst describing 2 nu: hence the day is longer than the night, and half the excess is $4^h \ 28^m \ 34^s$, if the position (3) be opposite to that of (1); or if it represents the Earth in Summer.

The Sun rises at u between h the north and E the east point; and 40° 9′ 25″ from the east.

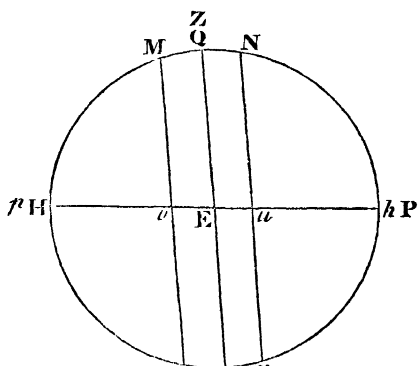
The instances taken have been those, when the Sun is most below and most above the horizon: but the scheme will serve for other positions of the Earth: and, the computations for the

* See p. 5. † *Trigonometry*, p. 105.

lengths of day and night, and for the distance from the east will be similar: since, instead of 23° 28′ we have only to substitute some other number of degrees, representing the declination.

In the position (2), the Sun is neither above nor below the equator, but in its plane produced. Transferring its place to the diagram, p. 13, *Q* would be it: and the parallel described in 12 hours would be *Qq*, and *EQ* being = *Eq*,* the days and nights would be equal. The position (2) represents the Earth in Spring.

In what has hitherto preceded, the spectator was placed at *A*, between the pole and the equator. If we suppose him placed in the latter, then his zenith will be *EQ* produced, and, perpendicular to *EQ* must be his rational horizon; that is, his horizon must be *Pp*: hence the diagram, p. 13, for representing the daily phenomena, must be slightly altered: we must make *Q* and *Z*, *P* and *h* coincident: it must be as it is here represented; in



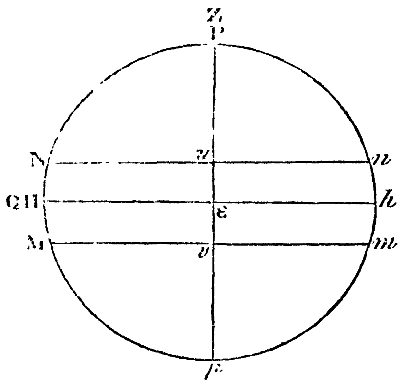
which Figure, it is evident that the parallels *Mm*, *Nn*, are all bisected by *Pp* or *Hh*: and consequently, the days and nights must be always equal wherever the Sun be; whether in the equator, or above it, or below it: in other words, to a spectator in the equator the days and the nights are equal throughout the year.

If we place the spectator at *P*, then his zenith is in *EP* produced†, and his rational horizon will be the equator. If the Sun is above the equator at *N*, then *Nn*‡ is his parallel, which

* *q* is omitted (Fig. p. 13.) in the point where *QE* produced cuts the circle.

† Fig. p. 12. ‡ Fig. p. 16.

being parallel also to the horizon, the Sun during the 24 hours is always at the same distance above the horizon: that is, the 24 hours consist of day alone and no night. If the Sun be at *M*, then the 24 hours consist of night alone. What is true for *N* and *M* is true also for all points between *N* and *Q*, and for all between *M* and *Q*: consequently during half the Earth's revolution, or half the year, the Sun is constantly above, and in the other half, constantly below the horizon.



If in the position (1) which is intended to represent the Sun at its greatest declination below the equator, or in its greatest southern declination, we draw *Ea* perpendicular to *SE* and the plane of the ecliptic, *a* will be the extreme point illuminated by the Sun, and *Ea* the section of the bounding circle of light and darkness. Since, at the greatest declination, $SEQ = 23° 28'$, PEa, or $Pa = 23° 28'$; for $\angle PEa + \angle aEQ$, $[= PEQ$ a right angle$]$, $[= SEa$ a right angle$]$, $= \angle SEQ + \angle aEQ$; and $\therefore \angle PEa = \angle SEQ$.

If from *a*, a small circle, parallel to the equator *QE*, be described, such a parallel is called the *Arctic Circle*, *P* being supposed to be the north pole. A similar small circle about *p* the south pole is called the *Antarctic Circle*.

The vicissitudes of seasons, inasmuch as they are shewn by the varying lengths of day and night, have been made apparent by combining the annual revolution of the Earth with its diurnal rotation. With regard to the variation of heat, or of temperature, at the different seasons, that is accounted for by the greater or less obliquity of the Sun's rays, combined indeed with the duration of day. In the position (1) the Sun is distant from the zenith of the spectator *A*, by the angle *SEZ*; in the position (3), by an angle less than the former, by twice the angle *SEQ*, or $2 \times (23^{\circ}\ 28')$; hence his rays in the first position fall much more obliquely than in the third: consequently, for the same portion of time they would in position (1) less warm the regions of the Earth near the spectator *A*: and besides, the duration of their action, as we have seen [p. 13.] is less.

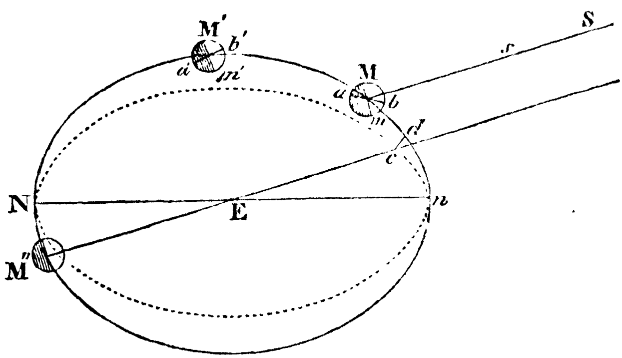
CHAP. IV.

On the Phases and Eclipses of the Moon.

If phenomena were arranged according as they were the more obvious, and excited greater curiosity, the Moon would have claimed our attention before the proper motions of the Sun and the Planets; these latter are not immediately nor very readily detected. The variations of the appearances of the Moon, on the contrary, continually force themselves on our notice.

According to what is stated in p. 11, the Earth moves round the Sun; and the Moon moves round the Earth; but the two orbits or paths in which they move lie not in the same plane. If we add to this, that the Sun illuminates the Moon, and that spectators at the Earth perceive the effects of that illumination, we shall have the means of explaining, why sometimes the whole face or disk of the Moon is luminous, and why at other times only portions of her disk: in other words, we shall be able to explain the *Phases* of the Moon.

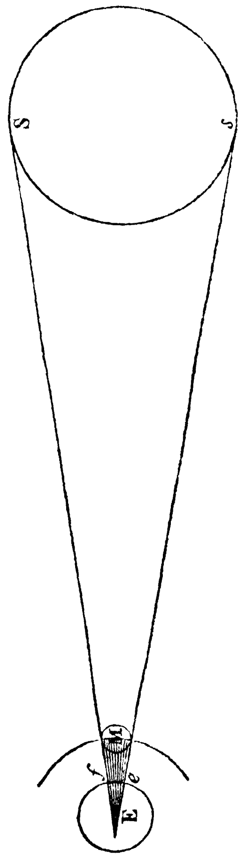
Thus, let *M*, *M'*, *M''*, &c. be different positions of the Moon in her orbit, which is not in the plane represented by the dotted line, and in which the Earth's orbit, or the ecliptic, is supposed to lie. *E* is the Earth, *Ss* the direction of rays of light coming from the Sun very distantly situated. Now a plane perpendicular to *Ss*, and passing through the center of the Moon, will divide her into two hemispheres, one illuminated, the other dark: let this plane pass through *Mm*. A spectator at *E* will see an



hemisphere of the Moon, to be determined by drawing through the Moon's center a plane perpendicular to a line drawn from *E* to the center of the Moon: hence, it is plain, in the position *M*, a spectator at *E* will see only a small portion of the Moon's illuminated disk: and if the Moon were at *c*, that is, in a line joining *E* and *S*, the dark part would be entirely turned towards him; but, in the position *M'*, he will see nearly half the Moon's illuminated disk: and at *M''*, he will see the whole.

The Moon at *c* is said to be a *new* Moon, at *M''* a *full* Moon, at *M'*, if half her disk should be illuminated, she would be said to be *dichotomized;* hence from the Moon's departure from *c* to her return to the same place, in a period of about 29 days, her disk exhibits to a spectator at *E* all her *Phases;* the narrowest crescent near *d*, and a full orb at *M''*: thence she becomes *deficient*, or *wanes*, till reaching a line joining the Earth and Sun she turns her dark side entirely to the spectator.

When the Moon is new at *c*, she comes on the meridian with the Sun, or at noon: when advanced to the position *M*, the Earth revolving from east to west, she comes on the meridian after the Sun, and it is her western limb which the spectator sees illuminated. At *M''*, at her full, she comes on the meridian at midnight: and when past *M''*, and beginning to wane, she becomes deficient on her western side. The Moon's orbit is elevated above the plane of the ecliptic, and is supposed to intersect it in the line *Nn*: now the line *Nn* shifts its position *: it may come into the position *M″Ec*. What then would happen if the Moon arrived at the extremities of this line?



In this case, the Moon, Earth, and Sun are in the same plane: and to ascertain what happens, we must no longer put points for these bodies, but must, in some sort, represent their magnitudes: let *Ss* then represent the Sun. *M* the Moon at *n*. *Eef* the Earth: now, it is plain, that a portion of the Sun's light will be stopped by the interposition of the body *M*; in other words, the Sun will be *eclipsed* by the Moon; or, a Solar eclipse will take place.

Let now the Moon be at the other extremity *N*, or be at her full: then the Earth's shadow will fall on the Moon, and eclipse her; or a Lunar eclipse will take place.

This does not, as we have said, always happen, since it depends on the position of the line *Nn*: in general, the Earth's shadow falls above or below the Moon. Yet that an eclipse may happen, it is not requisite that *Nn* should lie exactly in the direction of a line joining the Earth and Sun: if nearly in that direction, an eclipse may happen, as will be more fully explained in a subsequent Chapter.

* To be hereafter shewn.

CHAP. V.

On the Earth; its Figure and Dimensions.

THE enquiry into the figure and dimensions of the Earth is of considerable Astronomical use, and, if conducted to exactness, of very great difficulty. Here, however, it is intended to shew merely on popular grounds and for probable reasons, the roundness of the Earth, and then, its magnitude, supposing it to be spherical.

The Earth is probably round from the phenomena which we may observe at sea. A ship first comes in sight by shewing us the tops of the masts. Then, as it approaches, we see more and more of the masts, and at last, the hull. And this phenomenon is also so discernible, whatever be the quarter it appears in; whether it be north, south, east, or west.

The Earth also is probably round from the circumstance of Navigators, who by constantly leaving the port they departed from more and more behind them have at last arrived at it. They must therefore have surrounded, or *girded* the Earth.

We may infer also the roundness of the Earth from the seemingly circular boundary of its shadow on the face of the Moon during a lunar eclipse: for, if the Earth be a sphere, its shadow will be conical, and a section perpendicular to the axis will be a circle.

These arguments tend to shew that the Earth is round: it certainly cannot be flat like a plane, nor concave like the inside of a bowl. But if round, why not spherical? this it was at first supposed to be, since of round bodies, the sphere is the most simple. Observation, however, has proved this supposition to be erroneous. And, which is worthy of notice, the same body, the Moon, that has been employed to shew the roundness of the Earth, has been employed to establish its *non*-sphericity. This will be subsequently shewn.

The Earth, however, although not exactly, is very nearly, a sphere. And if we assume it to be such, its dimensions may be computed by the following method. By Chap. I. p. 7, by observations on the height of the polar star, or of the zenith distances of the same star, the latitudes of places may be determined. Suppose the difference of the latitudes of two places on the same meridian* to be 1°: let the linear and actual distance of those places be measured: which will be found to be about 69½ miles: suppose it exactly such: then since the Earth's circumference, supposed to be circular, contains 360°, it will be equal to 360 × 69. 5, that is, 25020 miles; and its diameter will be about 7960.

From this very method of determining the Earth's magnitude, its defect from perfect sphericity may be ascertained. If the Earth were a sphere, then between two places on the same meridian, and differing in their latitude by 1°, the same linear distance of 69½ miles ought always to be found, at whatever distance from the equator the places were situated. This, however, is found not to be the case; between two places differing in latitude by 1°, in latitude about 66° the linear distance is 122457 yards. Between two places near the equator, the linear distance is 121027 yards: the former distance being 69½ miles + 137 yards, the latter, 69½ miles − 1293 yards. And similar measurements establish as a general fact, that degrees, that is, their linear values, increase as we move from the equator towards the pole.

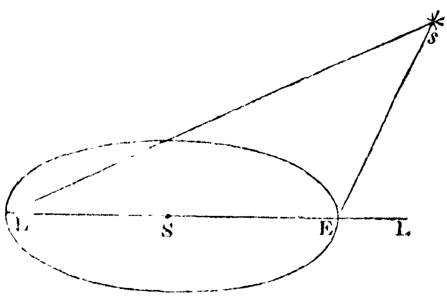
But, if not spherical, what is the Earth's form? It probably, does not differ considerably from that of a spheroid. If we suppose it such, and, from 2 degrees, the one measured at the equator, the other at the pole, determine the eccentricity of the ellipse that would generate it, it will be found to be nearly $\frac{1}{335}$, and the polar and equatorial diameters will be to one another as 335 to 336.

If the Earth be not a sphere, the direction of gravity, which is no other than the direction of a plumb-line, will not generally, that is, in all latitudes, tend towards the Earth's center. If we measure a degree at the pole, the two plumb-lines that are inclined to each other at 1°, will meet in a point of the polar diameter beyond the center. If, at the equator, in a point of the equatorial diameter between the center and the part of the equator where the measurement is made. In other situations, the directions of the plumb-lines will not meet in a diameter drawn to the point where the arc is measured.

* The method of determining when two places are on the same meridian, or have the same *longitude*, will be given in a subsequent Chapter.

The Earth's radius, stated at nearly 4000 miles, seems considerable; yet, compared with other quantities with which Astronomy is conversant, it may from its relative smallness be neglected. The distance of a fixed star, for instance, is infinitely greater: it is so great, that no proportion can be assigned between it and the Earth's radius. The method of ascertaining this may be here described.

Let *s* be a star: when the Earth is at *E*, let the angle *sEL* be determined by observation and computation: and half a year

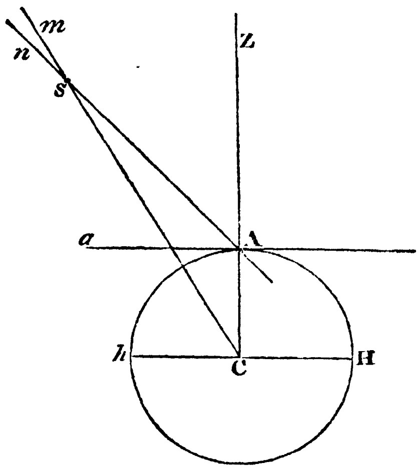


after, let *sE'L* be also determined. Now, it is found, that the angle *sEL* = angle *sE'L*: consequently the angle *EsE'* must be said to be nothing: *s* therefore is so immensely distant, that *EE'* the diameter of the Earth's orbit, bears no proportion whatever to *Es* or *E's*: *a fortiori*, therefore, the Earth's radius which is to *EE'* as 1 to 45968, bears no proportion to *Es* *.

The inference hence intended to be drawn is this: The sensible and rational horizon, may, when we treat of the angular distances of stars from the equator or ecliptic, be considered as coincident, and in the first Chapter, p. 4, it was so asserted. The zenith distance of a star *s* is to an observer at *A*, the angle *sAZ*, equal to the angle *sCZ* (see Fig. p. 23.): since, by what has preceded, the angle *CsA* is too small to be ascertained by observation. The same holds for angular distances from the equator and ecliptic.

* If the star *s* were near the pole of the ecliptic, and *Es* to *ES* as 200000 to 1: the angle *EsE'* would be 2″. But, since no such angle or difference of latitude can be detected, *Es* must be to the Earth's radius in a greater proportion than that of 4569800000 to 1.

Hence in the diagram, p. 4, we may reduce the radius *Ca* to a point at *C*, and compute accordingly. And here is a great



distinction between the computations for fixed stars and Planets. With the latter, the angle *CsA* is of assignable magnitude, and then the sensible and rational horizons cannot be esteemed coincident.

What has preceded relates merely to the general explanation of phenomena. Angular distances, (and such are latitudes and declinations,) have been spoken of; but, no methods given of exactly ascertaining and computing them. Yet, on such methods, Astronomy, as a theory that will account for every phenomenon, essentially rests. Our attention therefore, must chiefly be directed towards them. From popular explanation we must proceed to exact methods: first, to the extrication and determination of Astronomical elements, and then, to their combination in a system.

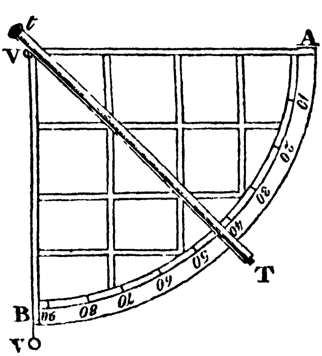
Of such methods, the first in order and of essential importance, are the *Instrumental* methods; and, it fortunately happens that the instruments by which the most important and necessary observations are made, are few in number and easy to be understood. This advantage arises from the wonderful simplicity of modern observations. Formerly, indeed, Astronomers thought, that the instruments most proper for observation were those that imitated the celestial sphere: that were formed in *cœli* *effigiem:* hence their astrolabes and armillary spheres. Now, Astronomers do not follow a star from the east to the west, but wait for it in the south. Their instruments, therefore, are fixed in the plane of the meridian. By these are determined merely the height of a star in that plane, and its passage or *transit* over it. From such simple observations they deduce all the Elements of the Solar System.

The observations spoken of are those which are essential and important; such as are practised in the observatories of Europe. To these, at first, the attention of the Student should be confined: it may afterwards be directed to other observations made out of the plane of the meridian; and for which appropriate instruments have been constructed.

CHAP. VI.

Description and Uses of the Astronomical Quadrant: Of the Circle: Of the Transit Instrument.

The first instrument to be described is the Astronomical Quadrant. *AB* is a quadrantal arc of brass equally divided



into 90 degrees: each degree into smaller equal divisions: the number of divisions plainly depending on the size of the instrument. *Tt* is a Telescope moveable about a center at *V*: *Vv* is a plumb-line, that is, a chord or fine wire with a weight attached to it, and hanging in the direction of gravity, or perpendicularly to the Earth's surface.

Suppose, the plane of the instrument, by proper adjustments, to be made coincident with the imaginary plane of the meridian, and that the plumb-line is brought exactly over the division marking 90°; or, technically speaking, that it *bisects* such division: then, it is plain, if the telescope *Tt* be directed towards any star in the plane of the meridian, the number of degrees between *A* and *T*, will mark the star's height or altitude in the meridian: and the number of degrees between *T* and *B*, will mark its *zenith* distance; the imaginary quadrant of the meridian being supposed to be similarly divided to the instrumental quadrant, and between the horizon and zenith to contain 90 degrees. If the star be in the horizon, the telescope will be parallel to, or coincident with *AV*; if in the zenith, with *Vv*: and in the Figure, the telescope is directed towards a star having an altitude of about 42°.

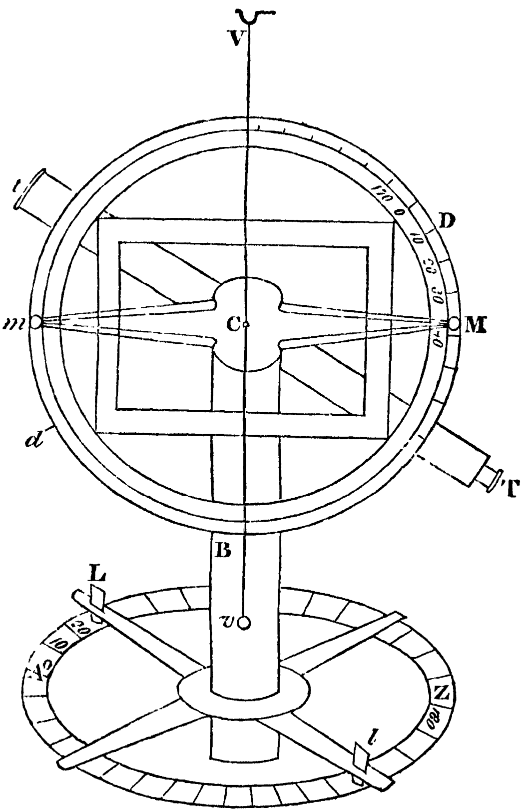
The quadrant may be placed out of the meridian, in the plane of any other vertical circle, and then [see p. 6.] it will measure altitudes in such vertical circle.

When the quadrant is fixed to the side of a vertical wall in the plane of the meridian, it is called a *mural* quadrant. Such are the quadrants in the Observatory at Greenwich.

The above explanation is not intended to describe accurately the nice construction of the instrument, but merely the principle of its operation; and it is perhaps sufficient for the purpose of this Treatise. The improved state of this instrument, however, may be briefly described; and the improvement principally consists in putting together four quadrants, and in forming a *circular instrument*.

Let the graduated Figure [p. 26.] represent the *circle*, with its telescope *Tt*, not as before, moveable on the limb of the circle, but attached to that limb, and moving only when the circle itself moves. The motion of the circle and telescope is round an horizontal axis, not represented in the Figure, but which may be imagined to lie in the direction of a line drawn through *C* perpendicularly to the plane of the instrument. *M*, *m*, are two microscopes, directly opposite each other, detached from the limb of the circle, and not moving with it, but having a proper motion of their own; *Vv* is the plumb-line.

Suppose, as before, by means of proper adjustments, the plane of the instrument to be in the plane of a vertical circle, and besides, *Vv* to bisect 0, when the telescope is horizontal; then, if the microscopes were in the horizontal position such as the Figure represents them to be in, *M* would be opposite 90°, and *m* 90° (for the rim is graduated into two 180 degrees). Let



now, the circle and telescope be moved till the latter is opposite a star, then the division *bisected* by *M* denotes the star's zenith distance: so does that bisected by *m*.

One use, then, of the two microscopes, is, that instead of one we have two *readings off*, as they technically are called: the mean of the two, then, if they happen to differ, is more likely to give a true result, than one would furnish. But, the microscopes need not occupy their horizontal position: they have a proper motion of their own: by means of it, suppose them to be in the position *D*, *d*, these points being the extremities of a diameter; then, the plumb-line as before being made to bisect 0, *M* and *m* will not be opposite to 90°, but to 65°; and, when the telescope is again directed to the star, *M* and *m* will be opposite 10° and 10°, and the star's altitude will be 65° − 10°, or 55°, and its zenith distance 35°; and, similarly, if we put the microscopes into any other position.

The use of this last change of the microscope's place consists in this, that, should the instrument be unequally divided, which must take place to a certain extent, the errors of division will, probably, less affect the result. Suppose in the two positions we have described, the *readings off* to have been

35° 0′ 3″, 35° 0′ 2″, 35° 0′ 4″, 35° 0′ 2″.5.

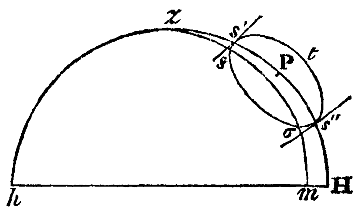
the mean altitude would be 35° 0′ 2″.875.

The primary position of the circle previous to an observation, may be determined either by a plumb-line, or spirit level: But, for ascertaining mere differences of altitudes, the microscope alone is sufficient. For instance, suppose, when the telescope is opposite the star *s*, the microscope to be opposite to the division 33° 14′ 25″, and afterwards to be opposite to the division 85° 31′ 7″, the telescope being directed towards another star *s*′: then, the difference of the altitudes of the two stars is

$$(85° \; 31' \; 7'') - (33° \; 14' \; 25'') = 52° \; 16' \; 42''$$

and, upon this principle, it has been proposed to find the north polar distances of stars without the intervention of the plumb-line.

Since the instrument will measure any, it will measure, meridian altitudes. But, to put the plane of the instrument in the plane of the meridian, a new adjustment is requisite. This is thus effected: we have seen [p. 2], that there are certain stars called circumpolar stars that never set, but apparently describe circles round the Pole; and consequently, in one point of their circuit are nearest to the horizon, and in another, the opposite point, are most remote from it. Now, this phenomenon has been accounted for, [p. 4.] on the hypothesis of the Earth's rotation round its axis: if we add to that hypothesis, the circumstance or condition of an equable motion of rotation, it will follow, that a circumpolar star will appear to move from the lowest point of its circuit to the highest, in the same time exactly as it descends from the highest to the lowest: but the highest and lowest points are in the plane of the meridian: for Zs' is $< Zs$, and Zs'' is $> Z\sigma$. Hence, when the instrument is so placed, that a star being observed at s'' and s', the time of describing $s''ts'$,



shall be found equal to the time of describing $s'ss''$, we may be certain that the telescope moves in the plane of the meridian: for, by adjustments already made, it moves in a vertical circle, and if it moved in one such as $Zs\sigma m$, it is plain, that $\sigma s''ts$ is greater than the remainder $s\sigma$ of the circuit, and consequently, the interval of time between the star observed at σ and s cannot be half the time of the star's apparent revolution *.

This adjustment is a most important one: for, almost all the calculations in Astronomy are founded on observations made in the meridian.

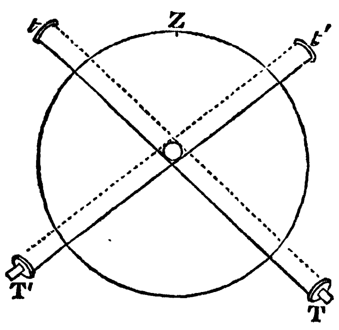
When the circular instrument does not exceed a certain magnitude, it is so contrived, that its horizontal base, is made to revolve in an horizontal plane. The whole instrument also moves, but the vertical pillar still remains vertical, and the axis round which the circle moves, remains horizontal. This horizontal motion of the instrument is called a motion in *azimuth* †.

* In Wollaston's *Fasciculus*, Appendix, p. 74, there is a formula for correcting the error of a meridian telescope by the observation of any circumpolar star above and below the Pole.

† The complement to azimuth, or the distance from the east or the west point of the horizon is *amplitude*, a term unnecessarily introduced.

The term azimuth formally defined is, the angular distance of a star or object from the north or south point of the horizon; and if the star, or other object, be not in the horizon, it must be referred to it by a vertical circle: thus, [see Fig. above] *hm* is the azimuth of a point *m*; it is also the azimuth both of *s* and *σ*.

The graduated rim *AZ* in Fig. p. 26, is intended to measure the quantity of the azimuth motion. Thus, if the microscope or index *L* has been translated from 0 to the division 25° over which it is represented, the instrument has been moved through 25° of azimuth. A great advantage is derived from the azimuth motion of the instrument; for, it enables the observer to deter-



mine the zenith distance of stars without the spirit level, or plumb-line. Suppose, in the above Figure, when the telescope is in the position *Tt*, directed to a star *s*, that the division on the circle opposite to the microscope is 83° 41′ 15″. Now, if the telescope were directed to a star in the south, the face of the instrument which we may be supposed, in the Figure, to look on, will be towards the east. But, if the whole instrument by means of the horizontal motion in the plate *AZ*, (which is graduated like the limb of the circle) be turned through 180° of azimuth; the face before opposite the east, will now be opposite the west: and the position of the telescope will be *T′t′*: consequently in order to be directed towards *s*, that is, in order to assume a position parallel to its former one *Tt*, *t′* must move through an arc *t′t*, which is plainly equal to 2*t′Z*, or 2*tZ*, that is, twice the zenith distance: consequently, if in the second position we should read off by the microscope 7° 11′ 23″, then, we should have the zenith distance half 76° 29′ 52″, or 38° 14′ 56″.

By these means also, a certain instrumental error called the error of the line of *Collimation*, is avoided: Now, this line is an imaginary one, extending from the center of the object glass to the focus, where the middle of the cross wire is: when the index of the instrument points to 0, it ought to be horizontal, or directed towards an object in the horizon: suppose it not to be so, but slightly slanting upwards at a small angle, 34″ for instance: then, a star at that altitude 34″, would, by the instrument, seem to be horizontal: another star at an altitude of 50° 21′ 52″, would, by the instrument, seem to be at 50° 21′ 18″. Turn now the instrument half round in azimuth, by which, the face originally towards the east is turned towards the west; then the telescope *Tt* will occupy the position *T′t′*: and the upper side of the telescope denoted by the dotted line, will still remain so: but, when the telescope is turned from the position *T′t′*, and again directed to the star, the dotted line will become the lower line, and accordingly if the index pointed to 0, the line of collimation would be slightly slanted downwards at an angle of 34″: and the former star at the altitude of 50° 21′ 52″, would, by the instrument, seem to be at 50° 22′ 26″: half the sum therefore of the two altitudes, taken in the manner above described will be the true altitude: for if ϵ be the error of the line of collimation, and A the true altitude,

$$A = \frac{1}{2}[A - \epsilon + A + \epsilon]$$

and half the difference of the two altitudes will be the error of the line of collimation, for

$$\epsilon = \frac{1}{2}[(A + \epsilon) - (A - \epsilon)].$$

We will illustrate this by two examples:

		Altitudes.		
6th Sept. Star *Rigel*, position E.* - -		30°	21′	36″.25
	position W. - -	30	20	22 . 05
	sum =	60	41	58.30
	true altitude =	30	20	59. 15
	difference =	0	1	14. 20
	error of collimation =			37 10
Again,	δ *Sagittarii* W. - -	8°	56′	45″.8
	E. - -	8	58	7 . 1
	sum - -	17	54	52 . 9
	true altitude - -	8	57	26 . 45
	difference - -		1	.21 . 3
	error of collimation - -			40 . 65

If great accuracy be required, the above operations are repeated with several stars, and the *mean* of the whole taken for the error of collimation: thus,

	Error of Collimation.
Rigel	37″. 05
Sirius	40 . 05
δ Sagittarii	40 . 06
x	42 . 0
α Capellæ	39 . 45
θ	37 . 12
γ	37 . 35
δ	37 . 87
	8) 310 . 95
Mean error of collimation	38 . 87

By the same principle, may the error of the line of collimation be corrected in circular instruments, which are too large to admit of an apparatus for the azimuth motion. For, the instrument may

* Position E, position W, denote respectively the graduated side of the circle turned towards the east and west. Rigel, Sirius, α θ, γ, &c. Capellæ, are the names of certain known stars.

be lifted out of the angles of *bearing*, and again placed with the extremities of its axis reversed.

The instrument which has been described is adapted to measure any, but principally meridian altitudes, and when fixed in the plane of the meridian it is called a Declination Circle; for, the declinations of stars are known, from their meridian and greatest altitudes, and the latitude of the place of observation. But, the instrument may be made to serve other Astronomical purposes: it may be used as a *Transit* instrument: that is, the presence of a star on the meridian may be ascertained by it with sufficient accuracy. Whether, however, it is expedient to use the same instrument for two purposes, declinations and transits, it is not our business now to consider: general explanation is, at present, only aimed at; and it sufficiently appears, that the instrument which has been described, will shew the object when on the meridian, and also, its elevation on the same meridian *.

The accurate description of the several parts of the instrument must be learnt from other treatises: to the preceding general and imperfect one, it may here be added, that several fine wires are placed in the field of view: one horizontal, and running through the center and line of collimation, when the instrument is to be used as a declination circle: several (in general five) vertical, when it is to be used as a *transit* instrument: and when the star covers either the horizontal or one of the vertical wires, it is technically said to be *bisected.*

To determine the altitude of a star, the star must be *bisected* by the horizontal wire: but, this operation cannot take place with bodies that have disks; the Sun and Moon, for instance. To determine their altitudes, the horizontal wire must be brought into contact either with their upper or their lower limb, and then to the number of degrees denoted by the instrument, the semidiameter, if known, must be either subtracted or added: and the

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* The common and usual transit instrument, is, to appearance, a very simple one. It is a telescope attached to an horizontal axis, with some simple contrivance for elevating the telescope tolerably near to any proposed altitude. The nice operation with this instrument is, the making the telescope to move in the plane of the meridian. For this purpose, several adjustments are necessary, and contrivances prepared.

result is the height of the center. Or, if the altitudes both of the lower and upper limb are observed, and added together, half their sum will be the altitude of the center: for instance,

Nov. 19.	Altitude of the Sun's upper limb..........	18°	39′	39″
	Altitude of the Sun's lower limb	18	7	13
	Sum..........	36	46	52
	[Difference .		32	26]
∴ half Sum, or altitude of the Sun's center		18	23	26

By the same observation we have the Sun's diameter. It is the difference of the heights of the upper and lower limb, and accordingly equals 0° 32′ 26″.

Having now explained the instruments by which observations can be made in the plane of the meridian, we will proceed to shew, how, by means of them, the Sun's motion may be detected, and certain inferences drawn relatively to its nature and law.

CHAP. VII.

Sun's Motion.—Path.—Ecliptic.—Obliquity of Ecliptic.

If by the instrument and method described in the preceding Chapter, the altitude of the Sun's center be observed from day to day, it will be found continually to vary. For instance, in the four first days of the year 1810 the altitudes were,

	Altitudes.	Differences.
1810, Jan. 1	14° 44′ 40″	
		5′ 4″
2	14 49 44	
		5 31
3	14 55 15	
		5 58
4	15 1 13	

Hence the Sun during these four days was ascending in the meridian, but not by equal increases of altitudes, as appears by the column of differences. Again, the altitudes of the Sun on four successive days in March and June, were

	Altitudes.	Diff.		Altitudes.	Diff.
Mar. 19	37° 5′ 46″		June 20	61° 14′ 32″	
		23′ 41″			29″
20	37 29 27		21	61 15 1	
		23 41			4
21	37 53 8		22	61 15 5	
		23 39			−21
22	38 16 47		23	61 14 44	

Hence during the four days in March, the Sun was continually ascending, and by increments of ascent very nearly equal: in June it was still higher, and on the twenty-second at its greatest altitude; for, on the succeeding day, its altitude was diminished by twenty-one seconds. Its increments of altitude, as appears by the column of differences, are, like those in January, unequal.

Thus far it appears then; the phenomenon of the Sun's continually varying altitude cannot be accounted for, by supposing the Sun to have an equable motion in the meridian; ascending for half the year from December 1809 to June 22, 1810.

and then descending: let us next consider whether an adequate explanation of the phenomenon is to be expected from attributing to the Sun an *unequal* motion in the meridian *.

If the Sun had a motion merely in the meridian, then, since the Earth's rotation is supposed to be equable, the intervals of successive transits over the meridian would always be equal, one with another, and besides, would be equal to the intervals of the transits of a fixed star; that is, of a star which, by its definition, has neither a motion in the meridian nor transversely to it: now neither of these conditions take place; for on Aug. 21, 1810, *α Reguli* was on the meridian 1 minute 20 seconds before the Sun was: but on the succeeding day 5 minutes 2 seconds: on the next, or twenty-third, 8 minutes 44 seconds: so that it is plain the Sun must have shifted away from the meridian, and moved transversely towards the east of it. Hence, to account for the phenomena, two motions must be attributed to the Sun; one in the meridian, the other transverse to it: or what amounts to the same thing, by the doctrine of the composition of motions, an oblique single motion.

From the preceding instance it appears, that the Sun moves to the east of the meridian, and of a fixed star, through an angle which, in time, is equal to 3 minutes 42 seconds: but this angle, or its value the time is not constant: if, for instance, one of the stars of *Sagittarius* was, with the Sun, on the meridian January 2, 1810, the next day the Sun would come later, than the star, to the meridian, by 4 minutes 24 seconds; on January 4th, later by 8 minutes 48 seconds; on the 5th, by 13 minutes 12 seconds, &c. Hence, the Sun's motion perpendicular to the meridian is not equable, neither, as it has appeared, is his motion in the meridian. These are the two resolved parts of the Sun's oblique motion.

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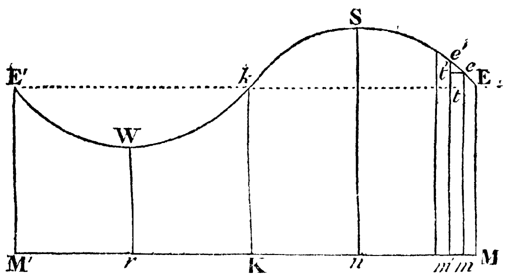
* In proving the Sun's motion in the meridian not uniform, we have supposed, what is not strictly true, the intervals between his successive transits over the meridian to be equal. But the result will be the same, that is, his motion will be found to be unequal, if we correct the supposition, and allow for the inequalities between successive transits

The following Table exhibits the Sun's meridian heights on the 22d days of the several months of the year 1810.

January.	*February.*	*March.*	*April.*	
18° 2′ 7″	23° 55′ 19″	38° 16′ 47″	39° 17′ 28″	
May.	*June.*	*July.*	*August.*	
41° 32′ 27″	61° 15′ 5″	58° 10′ 54″	49° 44′ 19″	
September.	*October.*	*November.*	*December.*	*January*, 1811
38° 16′ 23″	26° 51′ 49″	17° 42′ 42″	14° 19′ 42″	17° 58′ 47″

From this Table it is easy to determine, in a general way, the form of the curve in which the Sun may be supposed to move. For, if MM' be taken to represent the whole space from March 1810 to March 1811, and perpendiculars be erected respectively equal to the Sun's meridian altitudes, the curve drawn through their extremities will be $ESWE'$. If E be the Sun's



place on March 20, e, e' on the two successive days, then ME, me, $m'e'$ must be taken respectively proportional to 37° 29′ 27″; 37° 53′ 8″, 38° 16′ 47″ [see p. 34]. The intervals Mm, mm', &c. are not exactly equal, since they are the spaces through which the Sun retires each day, from his place on the meridian the preceding day [see Note, p. 35.]: and in the present case they are respectively equal, in degrees, &c. to 54′ 33″, 54′ 31″.5, 54′ 31″.

The spaces te, $t'e'$, &c., or the increments of the Sun's altitude in the meridian, are respectively equal to

$23' \ 41''$, $23' \ 39''$;

and the motions, in these directions, combined with the transverse motions in the directions Et, et', compound, as it has been before remarked, [p. 35,] the oblique motions Ee, ee', &c.

In the Figure ESE', there are two altitudes nS, rW, one the greatest, the other the least, which for the year 1810, [see p. 34], would happen on June 22d, and December 22d, and the mean of these two altitudes is

$$\frac{1}{2}\left[(61^\circ \ 15' \ 5'') + (14^\circ \ 19' \ 42'')\right] \doteq 37^\circ \ 57' \ 23''.5,$$

which is, very nearly, the Sun's altitude (ME) on March 21, or Kk his altitude, Sept. 22*.

Now, it is to be remarked, that these mean altitudes ME, Kk are equal to the altitude of the imaginary circle called the equator [see p. 6,] whatever be the place of observation. In the preceding instance the place has been supposed to be Cambridge, of which the latitude is $52^\circ \ 12' \ 36''$: but, the latitude [see p. 7,] is the distance of the zenith from the equator: the distance of the zenith from the horizon is a quadrant, or 90°: consequently, the meridian height of the equator above the horizon is the complement of the latitude, or, as it may be called the *co-latitude*.

The method of determining the latitude has already [p. 7,] been pointed out. By the instrument described in Chap. VI. the height of the polar star (the α *Polaris* of Astronomical Catalogues) is to be observed: then, from such observed height, and by certain appropriate tables, the latitude may be computed. This relates to the best practical method of determining the latitude: but in a general way, the latitude may be said to be determined by the mean between the greatest and least altitudes of *Polaris*, or of any other circumpolar star; for *Polaris* is one, since its north polar distance in 1810 was $1^\circ \ 42' \ 19''.64$. Thus, Fig. p. 4; if v, v' are the two places of a circumpolar star on the meridian, since $Pv = Pv'$

$$PH = \frac{1}{2}[Hv + Pv + Hv' - Pv'] = \frac{1}{2}[Hv + Hv'].$$

$$\text{But } PH + PZ = HZ = \text{a quadrant} = PZ + ZQ;$$

* The greatest and least altitudes (nS, rW) are supposed to happen on the *noons* of June, and of Dec. 22; which is not exactly true. See next page, line 14.

$\therefore PH = ZQ =$ the latitude, accordingly

the latitude $= \frac{1}{2}[Hv + Hv']$;

and the co-latitude equals half the sum of the greatest and least zenith distances. Thus, by observations made at Blackheath,

corr^d. least zen. dist. *Ursa min.* Bod. 4 - - -	37°	35′	55″
corr^d. greatest zen. dist. - - - - - - - -	39	27	57
	$\frac{1}{2}$)77	3	52
co-latitude of Observatory - - - -	38	31	56

Again,

corr^d. least zen. dist. *ο Cephei* - - - - - -	15°	35′	21″
corr^d. greatest zen. dist. - - - - - - - -	61	28	31
	$\frac{1}{2}$)77	3	52
co-latitude - - - -	38	31	56

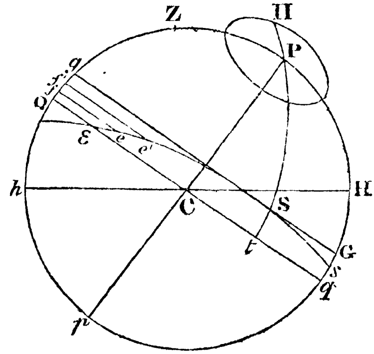
By these means we should be able to recognise that twice in a year, in March and September, the Sun was in the equator. But, if the latitude were determined accurately, we should find that no meridian altitude of the Sun was exactly equal to the co-latitude: for instance, in the former cases, the place of observation was Cambridge; its latitude by observations on the polar star, is 52° 12′ 36″; its co-latitude, 37° 47′ 24″: now amongst the altitudes stated in p. 34, there is no one exactly equal to 37° 47′ 24″: the altitude on March 20th is too small, that on the 21st too large: the reason of this is, that when the Sun was exactly in the equator, he was not on the meridian of the observer's station. There is some place to the east of Cambridge, at which the Sun was on the meridian when in the equator: and this place may easily be determined.

We may now pass from the Sun's tabulated place, obtained by daily observations of his meridian altitudes, to the explanation of the changes of places, as originating from his oblique motion.

The line MM' [see Fig. p. 36,] is intended to represent the aggregate of the angular distances through which the Sun recedes each day from a fixed star, that was with him on the meridian when he was at E. This aggregate is 360°*, and $MK = KM'$, more-

* This part being intended for general explanation only, the *precession of the equinoxes* is not taken account of.

over $ME = Kk = M'E'$ is the height of the equator, and a line EkE' containing 360° may represent it when extended on a plane.



Reversely, the lines EkE', $ESkE'$ may be conceived wound round a sphere, the line EkE', coinciding with QEq, &c *., $ESkE'$ with ESs, &c., and the points E, e, e' &c. in Fig. p. 36, with the points in Fig. p. 39, denoted by the same letters. Suppose now the Sun in the equator at E, then by the revolution of the sphere, the point E and the Sun, would be transferred to the meridian at the point Q, and $hQ = ME$ will be the height: next let the Sun recede through the space Ee; the point e and the Sun will be on the meridian at f, and $fh = me$ [Fig. p. 36,] will be the meridian altitude: on the succeeding day let the Sun, having still farther receded through the space ee', be at e'; then his place on the meridian will be f', and his meridian altitude $f'h = m'e'$ [Fig. p. 36]: and similar circumstances will take place till the Sun has receded through the space ES (ES = a quadrant) when his place on the meridian will be at g, and his meridian altitude $gh = nS$ [Fig. p. 36] then the greatest: after this the meridian altitudes will decrease.

By supposing therefore the Sun to move in the curve ES, &c. from E towards S, whilst the sphere revolves in the opposite direction, from E towards Q, all the phenomena indicated by observation admit of an adequate explanation. And, as the diurnal phenomena were shewn [p. 5,] equally explicable either by supposing the whole celestial sphere to revolve, the Earth being quiescent, or, the Earth to revolve in a contrary

* The ε in the diagram ought to have been E.

direction, the Heavens being at rest; so, these latter phenomena may be accounted for, either by supposing the Sun to move in an orbit such as *ESs*, &c., and the Earth to be at rest, or the Earth to move, but in a reverse direction, in an orbit similar to *ES* whilst the Sun remains at rest.

The above explanation does not depend, on the *real form* of the orbit *ESs*, which may be either circular or elliptical, or of any figure, provided it lies in the same plane. For the Sun is continually seen in the direction of a line drawn from him to the Earth; but, whatever be his place in that line, he will always be transferred to the imaginary concave spherical surface of the Heavens.

This imaginary path of the Sun in the Heavens is called the *Ecliptic*: the points *E*, *E'*, [Fig. p. 36,] where it intersects the equator, are called the *Equinoctial* points: they are the *nodes* of the equator; the points *S*, *W*, those of the greatest and least elevations above the horizon, or, where the Sun is respectively at his greatest northern and southern declinations, are called the *Solstitial* points.

The points of intersection of the equator and the ecliptic have been called the nodes of the former; they can be so called, only by likening the equator to the orbit of a revolving body; for, generally, *nodes* are defined to be the intersections of the orbit of a planet or other revolving body, with the plane of the ecliptic.

The planes in which the ecliptic and equator lie, are inclined to each other: and the angle of their inclination is, for distinction, called the *Obliquity of the Ecliptic*: the angle of the inclination of the planes is the same as the angle made by two tangents, at the point *E*, to the arcs *Ee*, *Eq* *. [see Fig. p. 39.]

If from *S* a solstitial point, a great circle *PS* be drawn perpendicular to the ecliptic, and ΠS be taken equal to a quadrant, then Π is the pole of the ecliptic †.

The circle *Gg*, a tangent to the ecliptic at the solstitial point *S*, and consequently parallel to the equator (and therefore a parallel of declination) is called a *Tropic*. A similar one touches the ecliptic at the other solstitial point.

The small circle described round *P* in the circumference

* Woodhouse's *Trigonometry*, p. 89.

† Ibid. P. 89. l. 2. from bottom. This pole is situated in the *Dragon* between the stars δ and ζ, but nearer the latter.

of which the pole of the ecliptic is always found, is called a *Polar* Circle: sometimes the *Arctic Circle* [p. 16]; and a similar one about the Earth's opposite pole is called the *Antarctic Circle*.

A secondary [see p. 6], to the equator, passing through *E*, the equinoctial point, is called the *Equinoctial Colure;* one passing through *S*, the *Solstitial Colure*.

Astronomers have divided the ecliptic into twelve equal parts called *Signs:* consequently each sign contains thirty degrees. Their names and characteristic symbols are,

Northern.		Southern.	
Aries - - - - - -	♈	Libra - - - - - - -	♎
Taurus - - - - -	♉	Scorpio - - - - - -	♏
Gemini - - - - -	♊	Sagittarius - - - - -	♐
Cancer - - - - - -	♋	Capricornus - - - - -	♑
Leo - - - - - - - -	♌	Aquarius - - - - - -	♒
Virgo - - - - - -	♍	Pisces - - - - - - -	♓

These signs are situated within an imaginary belt called the *Zodiac*, extending eight degrees on each side of the ecliptic. To each of the signs, certain clusters, or groups of stars, called [see p. 9.] *Constellations* *, are appropriated. But the signs, astronomically, serve merely to denote a certain number of degrees: thus, in the Nautical Almanack, the Sun's longitude for July 1, 1810, is stated at 3 signs, 8 degrees, 54 minutes, 19 seconds; equivalent to 98 degrees, 54 minutes, 19 seconds.

The longitude is also sometimes expressed by means of the symbols of the constellations of the Zodiac. Thus, in Flamsteed's catalogue of the fixed stars, the longitude of γ *Draconis* is expressed by:

$$♐\ 23^{\circ}\ 42'\ 48''$$

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* These groups of stars, or *constellations*, are by fancy imagined to form the outlines of the figures of animals and instruments, and are designated by their names. Thus, one group forms the figure of a Bear, another of a Lion, a third of a Dragon, a fourth of a Lyre. So there are stars in the tail of the Bear, the head of the Dragon, the heart of the Lion: which are farther distinguished by Greek characters. In the Catalogues we find, η *Ursæ majoris*, γ *Draconis*, α *Lyræ*, α *Leonis*, &c.

which, since *Sagittarius*, represented by ♐ *, is the 9th sign whose first point from that of *Aries* is accordingly distant by 8 × 30°, or 240°, denotes the longitude of γ *Draconis* to be

263° 42′ 48″.

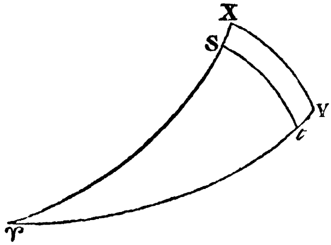
The term *Longitude*, which has been introduced, means angular distance measured or computed along the ecliptic, and from one of the intersections of the equator and ecliptic: which intersection is called the *First Point of Aries*.

After having passed through the 30° of *Aries*, the Sun enters *Taurus*, then *Gemini*, and successively the signs according to the order in which they were enumerated [p. 41]. The motion of the Sun according to this order is said to be *direct*, or *in consequentia;* any motion in the reverse direction is said to be *retrograde*, or in *antecedentia*.

What longitude is with respect to the ecliptic, *right ascension* is with respect to the equator. It is angular distance, from the first point of *Aries*, [see l. 8,] measured along the equator. And what declination is relatively to the equator, *latitude* is to the ecliptic: it is, angular distance from the ecliptic, measured by that arc, of a secondary to the ecliptic passing through the star, which lies between the star and the ecliptic. Thus if ♈ be the first point of *Aries*, or denote the intersection of the equator and ecliptic, and *S t* be perpendicular to the part ♈*t* of a great circle: *St*, ♈*t* are respectively the latitude and longitude *S*, if ♈*t* be part of the ecliptic: or, they are respectively the declination and right ascension of *S*, if ♈*t* be part of the equator. The Sun being always in the ecliptic has no latitude: at the first point of *Aries*,

* The particular stars of a constellation also are usually symbolically represented: thus α ♉ means the first or principal star in *Taurus* or the Bull; λ ♒, one of the inferior stars in *Aquarius*; β ♍, a star of the second magnitude in *Virgo;* γ ♎, a star of the third magnitude in *Libra*.

his declination, longitude, and right ascension, are nothing : at the



solstitial points, his declination is the greatest, and his longitude and right ascension 90°, or 270°.

The longitude of the Sun varying, in the year, from 0 to 360°, becomes successively during that period, equal to the several longitudes of the stars. The longitude of *α Arietis* being in 1809, $1^s\ 4^\circ\ 59'\ 31''$, that of the Sun is equal to it on April 25th. The longitude of *Regulus* being $4^s\ 27^\circ\ 10'\ 27''$, that of the Sun is equal to it on August 20th. When this happens, that is, when the Sun has the same longitude as the star, he is said to be in *conjunction* with the star. And, for conciseness of expression, Astronomers have invented another term called *Opposition*, which happens, when the longitude of the Sun differs from that of the star by 180°, or 6 signs. The symbol for conjunction is ☌, for opposition ☍. Both the preceding terms are comprehended under a third called *Syzygy*. Thus, the Sun having on Oct. 28th, a longitude of $7^s\ 4^\circ\ 39'\ 54''$, he is during that day in *opposition* to *α Arietis*. On April 25th then, he is in *conjunction* with *α Arietis*, on Oct. 28th, in *opposition*, and on both days in *Syzygy* with that star.

The Sun was stated to be in conjunction with *α Arietis* on April 25th. But, the exact time was not specified ; that however, may be found by a formula given in the Appendix : or very nearly by a simple proportion. Thus,

☉ long^e. Apr. 25 -	= $1^s\ 4^\circ\ 49'\ 58''$	- - -		$1^s\ 4^\circ\ 49'\ 58''$
Apr. 26 -	= 1 5 48 15	long. of α♈		1 4 59 21
Inc. of long. in 24^h - - -	58 17	diff. of long. - -		9 33

$$\therefore\ 58'\ 17'' : 9'\ 33'' :: 24^h : 3^h\ 55^m\ 57^s;$$

consequently the conjunction was April 25th, $3^h\ 55^m\ 57^s$ without estimating *the precession of the equinoxes* by which the star's longitude was increased.

The Sun is said to be in *quadrature* with a star, or planet, when the difference of their longitudes is 90°, or 270°; that is, 3^s, or 9^s: for instance, the Sun is in quadratures with α *Arietis* when his longitude is either $4^s\ 4°\ 59'\ 31''$, or $10^s\ 4°\ 59'\ 31''$: that is, either on July 28th, or January 24th. In quadratures with *Regulus*, when his longitude is either $7^s\ 27°\ 10'\ 27''$, or $1^s\ 27°\ 10'\ 27''$: that is, either on Nov. 19th, or May 18th. The symbol for quadratures is □; Thus ☉ □ α *Aquilæ* denotes the Sun to be in quadratures with the first star in the *Eagle*.

In the preceding Figure, if *S* should be the Sun, ♈*S*, the ecliptic and ♈*t* the equator, we could by the solution of a right-angled spherical triangle determine the longitude ♈*S*, if the right ascension ♈*t*, and the obliquity were known. But hitherto, no methods of observation or computation have been given, for determining these latter quantities. We will first shew how the obliquity may be determined. By p. 34, l. 14, the meridian altitudes of the Sun at Cambridge, on four successive days, were

$$61°\ 14'\ 32'',\quad 61°\ 15'\ 1'',\quad 61°\ 15'\ 5'',\quad 61°\ 14'\ 44'',$$

and since the co-latitude of Cambridge is 37° 47′ 24″ the corresponding declinations of the Sun, were

$$23°\ 27'\ 8'',\quad 23°\ 27'\ 37'',\quad 23°\ 27'\ 41'',\quad 23°\ 27'\ 20''.$$

If the greatest of these, that is, 23° 27′ 41″, represented the Sun's greatest declination, it would measure the obliquity: for when ♈*S*, ♈*t* are each equal to a quadrant, *St* is the measure of the spherical angle at ♈*. But it plainly does not represent the greatest declination, since, if it did, the two adjacent declinations would be equal, which they are not: the greatest declination then must have happened sometime between the noons of June 21st, and June 22d, but nearer to the noon of the latter day. It is a quantity somewhat greater than 23° 27′ 41″, certainly not differing from it by four seconds. For, assume it to be the greatest declination, then, in fact, we assume the Sun's longitude to be (what it is at the Solstice) 3 signs or 90°: now, this latter assump-

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* Woodhouse's *Trigonometry*, p. 90.

tion cannot err from the truth 30′, for the change in the Sun's longitude for 12 hours is not quite equal to that quantity: but, suppose it to be 30′, that is, in the Figure, let X be the true place of the solstice, and $SX = 30'$, or $\Upsilon S = 89° 30'$, then by Naper's rule *,

$$\text{rad.} \times \sin. St = \sin. \Upsilon \times \sin. S\Upsilon$$

and $$\text{rad.} \times \sin. Xy = \sin. \Upsilon \times \sin. X\Upsilon;$$

consequently, eliminating sin. Υ, there results [since $\sin. X\Upsilon = 1$]

$$\sin. Xy = \frac{\sin. St}{\sin. S\Upsilon} = \frac{\sin. St}{\cos. SX}$$

$$\therefore \log. \sin. Xy = 10 + \log. \sin. 23° 27' 41'' - \log. \cos. 30',$$

but, 10 + log. sin. 23° 27′ 41″	- - - =	19 . 6000260
log. cos. 30′	- - - - - - - =	9 . 9999835
		9 . 6000425

$$\therefore Xy = 23° 27' 44''.5.$$

But since in the case we have taken, the error in longitude must be less than 30′, the real obliquity must be some quantity between 23° 27′ 41″, and 23° 27′ 44″. And if the error in longitude instead of being 30′ were only 3′, the error in declination instead of being 3″.5 would be only $3''.5 . \frac{1}{(10)^2}$, or .035″ †.

In the present instance the former error is about 20′, and therefore the latter is 1″.5 nearly, and consequently the obliquity differs very little from 23° 27′ 42″.5.

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* Woodhouse's *Trigonometry*, p. 105.

† For the variations in declination near the solstice, are nearly, as the square of the variation in longitude: for, in the former Figure,

$$r \times \sin. p = \sin. \Upsilon . \sin. l \qquad [l = S\Upsilon,\ p = St]$$

$$\therefore r . dp . \cos. p = dl . \sin. \Upsilon . \cos. l \quad \text{[taking the differentials.]}$$

$$\therefore dp = \frac{dl}{r} . \frac{\sin. \Upsilon}{\cos. p} \cos. l = \frac{dl}{r} . \tan. \Upsilon . \cos. l \ \text{[since at sols., } p = \Upsilon \text{ nearly,]}$$

$$\therefore dp = \frac{dl}{r} \tan. \Upsilon . \sin. (90 - l) = \frac{dl}{r} \tan. \Upsilon . \sin. dl = \frac{(dl)^2}{r} \tan. \Upsilon,$$

since at the solstice $l = 90 - dl$ nearly.

In the preceding illustration, the instance taken has been that of the Summer solstice: the declination of the Sun at the Winter solstice may be similarly found; and then the mean of the two declinations will give, more exactly, the obliquity of the ecliptic.

Thus, by observations made at Blackheath, 1807,

Winter solstice zenith distance - - - -	74°	55′	56″.02
Summer - - - - - - - - - - - - - - -	28	0	8.68
2)	46	55	47.34
Mean obliquity of ecliptic - - -	23	27	53.67

On the footing of mere theoretical explanation, it is sufficiently exact to say, that the obliquity of the ecliptic is equal to half the difference of the greatest and least meridian altitudes or zenith distances of the Sun. But, as it is very unlikely to happen, that the Sun should be on the meridian of the place of observation, when at his greatest declinations, or since there is only one longitude or terrestrial meridian at which the greatest declinations can be observed, the practical difficulty is to infer and compute the really greatest declination from the greatest observed one. One method of effecting this has been already given: but, there is perhaps a better that requires the previous determination of the *time* at which the Sun comes to the solstice: this latter method, however, requires the Sun's longitude to be known; which can indeed be, from the declination and right ascension, by the solution of a right-angled spherical triangle [see Fig. p. 43]. But, since as yet no method of ascertaining the latter of these quantities has been given, the second way of determining the obliquity must be postponed.

There is some calculation necessary, so it has appeared, to determine the Sun's greatest declination, but the common declination is very readily determined, when the latitude is known, for instance,

1810, 20th June, ⊙. U. L* - - - -	61°	29′	16″
L. L. - - - - - -	61	0	46
2)	122	30	2
Altitude of Sun's center - - - - -	61	15	1
Co-latitude of Cambridge - - -	37	47	24
Sun's declination - - - - - - - -	23	27	37

In this Chapter, has been shewn the use that may be made of the meridional altitudes of heavenly bodies, observed either by the quadrant or circle. The height of the equator, the equinoctial points, the obliquity of the ecliptic, have been determined. But, the methods of using observations and of determining these points, must, for the present, be considered rather as roughly sketched than as accurately described. No mention has been made of *corrections* to the observations Hitherto the observer has been supposed to be, in an atmosphere not refracting, at the center of the Earth, and without motion. Yet, these suppositions, although erroneous, will be continued in the two succeeding Chapters; for, the principle of things therein to be determined will not depend on the theory of corrections, although the detail and process may.

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* ⊙ *U. L.* ⊙ *L. L,* denote the Sun's upper limb and lower limb.

CHAP. VIII.

Transit of Stars over the Meridian.—Right Ascension.—Sidereal Day.—Mean Solar Day.—Year.—Longitude of the Sun.—Latitude and Longitude of the fixed Stars.—Angle of Position.

ALMOST all Astronomical computation is founded, as it has been already observed, on observations made on the meridian. The method of making one class of these observations, that of meridian altitudes and zenith distances, has been already explained. But, the place of a star in the Heavens depends not solely on his height in the meridian, but also on its lateral angular distance from some point, by convention, made a fixed point; if this latter point be to the east of the star, it will come to the meridian of the observer after the star, by a certain interval of time. The measure of this interval depends on the value of the angle contained between two secondaries to the equator, one passing through the star, the other through the fixed point. For instance, if the angle be 30°, the interval will be $\frac{1}{12}$th of the whole interval between two successive transits of a star over the meridian: if the whole interval were called 24 hours, it would be 2 hours. Hence, it appears, that the lateral angular distance of a star from a fixed point, or from another star, may be expressed in time: and hence, arises the necessity of marking the exact time of the *transit* of stars over the meridian. On this second class of observations in the meridian, depends the *right ascension* of a star. That term, formally defined, is the angular distance, of a star referred to the equator by a secondary passing through the Star, from the intersection of the equator and ecliptic.

For the purpose of marking the transits of stars, there are, as it has been mentioned, in the field of view of the telescope, several equidistant vertical wires, in number, usually five. The instant at which the star is on the middle wire, is the time of transit;

when on either of the two wires to the left of the middle one, the time precedes the time of transit exactly by as much as it is after when the star is on the corresponding wire that is to the right of the middle one: hence, if the several times at which the star is successively on the 5 wires be added together and the mean taken, it will be the time of transit.

Thus, by the Greenwich observations,

Stars.	1st Wire	2d Wire	mid. Wire	4th Wire	5th Wire
	h. m. s.	h. m s.	h. m. s,	h. m. s.	h. m. s.
Regulus.........	9 55 51 .5	9 56 29	9 57 6.5	9 57 44.3	9 58 21.7
Aldebaran......	4 22 31.4	4 23 9.5	4 23 47.6	4 24 25.7	4 25 3.7

Here the mean of the transits of

Regulus is $\frac{1}{5}$ [49^h 45^m 33^s] $= 9^h\ 57^m\ 6^s.6$,

and of *Aldebaran* $\frac{1}{5}$ [21 58 57 .9] $=$ 4 23 47 . 58:

the first differing from the transit at the middle wire by $\frac{1}{10}$ of a second, the latter by $\frac{1}{50}$th *.

The time of the transit is to be marked by a clock or chronometer †. But how is the clock to be adjusted and regulated? If we say its index or hand ought to perform an exact circuit in the course of *a day*, we may farther enquire what portion of time that term is meant to denote? The natural and obvious portion seems to be, that which is contained between, two successive transits of the Sun over the meridian, or, two successive noons; and accordingly, clocks were originally constructed to measure it, and were supposed to go right if they agreed with the Sun. But, by the

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* In Fig. p. 5, *ab*, and *ss'* are passed over in equal times, but *ss'* is less than *ab* in the proportion of cosine of *sb* to radius; therefore the time of describing a space = *ab* on the parallel *asm* is to the time of describing the same space on C♎ the equator as radius to cosine of *sb*, or as radius to cos. declination. Now, analogous to the equal spaces *ab*, is the constant interval of the vertical wires; therefore, the times of passing that interval by different stars are inversely as their cosines of declination: hence, from the times we may determine the declination, and hence the cos. declination of *Regulus* is to the cos. declination of *Aldebaran* as 38″.1 to 37″.5 : which by the above *Obs*. line 9, 10. are the times of the passages of those stars cross the intervals of the wires.

† This is the third essential instrument of an observatory. The transit instrument, the quadrant, the clock, are, as Robison well expresses it, the capital furniture of an observatory.

improvements in machinery, these *time-keepers* were soon made so nicely and accurately, as to indicate an irregularity in the Sun's diurnal motion: they seemed to shew that *Solar days* were unequal: for the more nicely constructed the clock, the more plainly did it indicate the inequality. Solar days then being variable in their duration, it became necessary to seek for some other means of regulating clocks: such were found in the intervals of the transits of fixed stars: these intervals, in other words, *Sidereal days*, are equal, on the supposition of the Earth's uniform motion of rotation; which supposition is, at least probable, since neither observation nor theory indicate any thing to the contrary.

Sidereal days, then, are equal; and, a clock regulated by the transit of fixed stars, or adapted to sidereal time, would plainly indicate real solar days to be unequal. A *mean* solar day, however, is invariable: for that, as its name imports, is divested of irregularity. It is a portion of time not marked by any phenomenon, but merely fictitious: exceeding a sidereal day by a constant quantity.

A *Year* is the time elapsed between the departure of the Sun, from, a certain part of the Heavens, or, a fictitious point in the Heavens, and his return to the same. If the elapsed time be between the Sun's leaving a star and his return to it, the year is called *Sidereal*: if between his entering the *equinoctial point* or first point of *Aries*, and his return to the same point, (which is not fixed,) an *Astronomical* year: let this latter year be supposed to be divided into as many portions as there are days, and one of the divisions is a mean solar day.

By observation it appears that in the time between the Sun's leaving the first point of *Aries* and his return to it, 365 days and about $\frac{1}{4}$th of a day elapse; in other words, the Sun has passed the meridian of the observer 365 times, and, if he was on the meridian when he first left the equinoctial point, had, besides, passed the meridian nearly 6 hours when he returned to it: hence, if all the solar days were equal, the increase [see p. 35.] of the Sun's right ascension every day, or the additional angle which the Earth, having performed a complete rotation, must move through, again to bring the Sun, moving equably, on the meridian of the observer, is $= \frac{360^\circ}{365.25} = 59' \, 8''.2$.

A clock or chronometer then, the index of which performs an exact circuit whilst the Earth, or, what is the same thing, the meridian of the observer, moves through an angle equal to $360° 59' 8''.2$, is said to be adjusted to mean solar time.

A sidereal day is the interval between two successive transits of a star over the meridian, and is completed when the meridian of the observer has moved through 360°: it is less than a mean solar day in the proportion of 360° to $360° 59' 8''.2$; and consequently, expressed in the hours, minutes, seconds, of mean solar time, is equal to $\frac{360°}{360°.59' 8''.2} \times 24^{\text{hours}} = 23^{h}\ 56^{m}\ 4^{s}.098$.

The transits of fixed stars are used for regulating clocks, and, since we now know the value of a sidereal day [$23^{h}\ 56^{m}\ 4^{s}.098$], they may be used for regulating clocks adapted to mean solar time. But, in practice, the clocks of observatories are adapted to sidereal time; in such time, the right ascensions of stars are expressed, and certain tables are calculated, from which, with little difficulty, sidereal time may be converted into mean solar time. The right ascension of the Sun, is always expressed, in the Nautical Almanack, in sidereal time.

If the index of a sidereal clock were at 0 when the first point of *Aries* [see pp.42, 47.] were on the meridian, then, the times indicated by the clock, when other stars were on the meridian, would express the right ascensions of such stars: for instance, a clock set, as we have described, would in the year 1810, thus indicate the times of passage over the meridian,

δ *Andromedæ* - - - - -	0^{h}	28^{m}	$39'.02$
Aldebaran - - - - - -	4	24	51 . 50
Spica Virginis - - - -	13	15	11 . 48

which times are the right ascensions.

In this method we must know the time, when the imaginary intersection of the equator and ecliptic, or the first point of *Aries*, is on the meridian: now, there being no star in that point, we can only determine it, by ascertaining when the Sun enters the equator; that is, by ascertaining the time at which, if on the meridian, his height above the horizon would equal the co-latitude of the place of observation *: and here the

* See pp. 37, 38.

practical difficulty is, of the same nature, as that which occurs in finding the solstitial declination [see p. 44]. For instance, the co-latitude of Cambridge is 37° 47′ 24″: but [see p. 34,] on March 20, 1810, the Sun's altitude was 37° 29′ 27″, on March 21, 37° 53′ 8″: the one too small, the other too great: at some intermediate time then, the Sun was at the altitude 37° 47′ 24″, or in the equator. In order to find it, we must use this formula*:

$$y = a + dx + d' \frac{x(x-1)}{2} + \&c.$$

in which d, d', &c. are the 1st, 2d, differences of a, b, c, &c. which represent the declinations on the 19th, 20th, 21st, 22d of March: that is,

	1st Difference	2d Difference
$a = -41' \, 38''$		
$b = - \;\; 17 \;\; 57$	$-$ 23 41	0
$c = + \;\; 5 \;\; 44$	23 41	2″
$e = + \; 29 \;\; 23$	23 39	

Hence, putting $y = 0$, (the declination of the Sun in the equator) we have nearly, rejecting the third term,

$$0 = -41' \, 38'' + 23' \, 41''. x;$$

$$\therefore \quad x = \frac{41 \times 60 + 38}{23 \times 60 + 41} = 1^d \; 18^h \; 11^m \; 23^s +$$

Hence, March 20th, at 18 hours 11 minutes 23 seconds, mean solar time, the first point of *Aries* was on the meridian.

If therefore, at this time, a sidereal clock be adjusted to $0^h \; 0^m \; 0^s$, it will shew the right ascensions of stars that pass the meridian at subsequent hours. But the clock may not move equably: its rate of going must be ascertained by the transits of fixed stars: their right ascensions therefore must be known; that is, right ascensions must be supposed to be known to regulate that, which was described as determining right ascensions. This seems to be something like arguing in a vicious circle: and in fact, there is no independent and original method of determining the right ascensions of stars: they are ascertained only by methods tentative and approximate, and by successive corrections: and can be supposed to be known, only after a long series of accurate observations. The Astronomers of Greenwich, from Flamsteed to

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* See *Trigonometry*, p. 193.

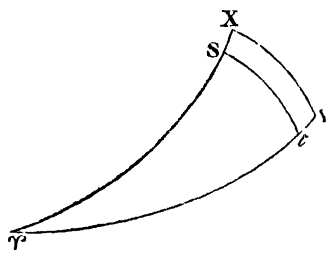
Maskelyne, have been employed in fixing, with still increasing degrees of precision, those important elements of Astronomy, the right ascension of stars. [See *Appendix*.]

The right ascensions of thirty-six principal stars are now ascertained with considerable precision, and have been registered in a table by Dr. Maskelyne. These are used to regulate the sidereal clock: and the latter for ascertaining the right ascensions of the Sun and Moon, of planets, and of any star the place of which may be considered as not settled with sufficient accuracy.

It was said [p. 51,] that the index-hand of the sidereal clock should be 0 when the first point of *Aries* is on the meridian. But it will easily be seen, that this condition is not necessary, and was introduced only for simplicity of explanation. When one of the principal Stars is used for regulating the clock, the latter may indicate any time. For instance, suppose, on Nov. 26, 1810, the sidereal clock to indicate $3^h\ 12^m\ 15^s$, when α *Arietis*, whose right ascension is $1^h\ 56^m\ 32^s.75$ was on the meridian, then the right ascension of a Star on the meridian, when the clock indicated $14^h\ 29^m\ 55^s.65$, would be $13^h\ 14^m\ 13^s.4$: since the clock by the first comparison is too fast by $1^h\ 15^m\ 42^s.25$. It is not necessary, then, that the sidereal clock should immediately indicate the right ascension of stars: neither is it necessary, that it should move at the same rate as the Stars; that is, the seconds which it *beats* may be either less or greater than the real seconds of sidereal time: the sole important requisite is, that it should gain or lose *equably*. That circumstance taking place, the corrections for its variation from true sidereal time are easily made: thus, suppose by comparison with certain of the 36 fixed Stars previously mentioned, it appeared at $6^h\ 40^m$, that the clock was $3^s.123$ too fast, and 16 hours afterwards by comparison with other fixed stars $3^s.095$ too fast; then, in 16 hours the clock had lost $3^s\ 123$—$3^s.095$ or $.028^s$: therefore, if it lost equably, in 8 hours it would lose $.014^s$ in 4 hours $.007^s$, &c.; or, at $10^h\ 40^m$ the clock would be too fast 3 .123—.007, or $3^s.116$; at $8^h\ 40^m$ too fast $3^s.123$—$.0035^s$, or $3^s.1195$.

When the right ascensions of stars are known, it is easy to determine at what time of the day they will be on the meridian. For instance, at the vernal equinox the first point of *Aries* and the Sun are on the meridian together: or it is noon when a star whose right ascension is 0 is on the meridian. A star situated in the solstitial colure has a right ascension either equal to 90°, or to 270°, therefore at the time of the equinox comes on the meridian, either at six in the evening, or at six in the morning, γ *Draconis* is nearly on the solstitial colure, its right ascension being [in 1800] 267° 59′ 40″: and accordingly in March comes on the meridian at six in the morning: in June, the opposite part of the solstitial colure being on the meridian at noon, γ *Draconis* comes on the meridian about midnight. But, in December the Sun and the Star are on the meridian together *.

We may now be supposed to understand the methods of determining, from observation, two of the most important elements in Astronomy, the declinations and right ascensions of stars: on these the places of celestial objects depend: longitudes and latitudes are not observed, but, from the former, computed. The Sun's longitude is computed either from his observed declination and right ascension; or, from his declination and the obliquity of the ecliptic; or, from his right ascension and the obliquity: thus, let ♈ *S* be part of the ecliptic, and ♈ *t* part of the



equator, and let *St* be part of a circle of declination: and let the Sun's longitude Nov. 28, 1810, be required, his declination being 21° 16′ 4″, and right ascension 16h 14m 58s.4, or in space, 243° 44′ 36″.

By Naper's rule, $r \times \cos.$ ♈ $S = \cos.$ ♈ $t \times \cos. St$;

∴ log. cos. ♈ *t* or log. cos. 243° 44′ 36″.......... = 9 .6458083
log. cos. *St*, or log. cos. 21 16 4........... = 9 .9693672

10 + log. cos. ♈ *S*........ 19.6151755

* See the Appendix for a simple Table for determining the time of a given Star crossing the meridian.

$\therefore \Upsilon S = 245° \; 39' \; 10''$ the longitude required;
or $= \; 8^s \; 5° \; 39' \; 10''$.

2dly, Required the Sun's longitude Nov. 29, from his declination $= 21° \; 26' \; 35''$, and obliquity $= 23° \; 27' \; 41''.3$.

By Naper, $r \times \sin, st = \sin. \Upsilon S \times \sin. S \Upsilon t$;

$\therefore \log. r + \log. \sin. 21° \; 26' \; 35'' \ldots\ldots = 19.5629781$
$\log. \sin. 23 \;\; 27 \;\; 41.3 \ldots\ldots = 9.6000276$

$\log. \sin. \Upsilon S \ldots\ldots = 9.9629505$

$\therefore$ longitude $= 246° \; 40' \; 6''$, or $8^s \; 6° \; 40' \; 6''$.

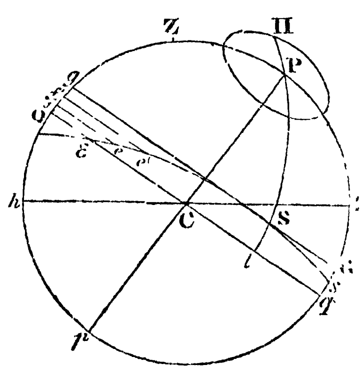
3dly, Required the Sun's longitude Nov. 30, from his R. A. $= 16^h \; 23^m \; 34^s$, and the obliquity of the ecliptic $= 23° \; 27' \; 42''.3$.

By Naper $r \times \cos. S \Upsilon t = \text{cotan}. S \Upsilon \times \tan. \Upsilon t$;

$\therefore \log. r + \log. \cos. 23° \; 27' \; 42''.3 = 19.9625237$
$\log. \tan. 16^h \; 23^m \; 34^s.1 = 10.3492191$

$\log. \text{cotan}. \Upsilon S \ldots\ldots = 9.6133046$
$\therefore$ longitude$\ldots\ldots = 247° \; 40' \; 56''$,
or $= 8^s \; 7° \; 40' \; 56''$.

The longitude in these examples is computed from the right ascension and declination, conditions given by observation. But, in the construction of the Nautical Almanack, the reverse operation takes place. The *Solar Tables* give the Sun's longitude: thence, and from the obliquity of the ecliptic, the right ascension and declination are computed, by trigonometrical operations, similar to the preceding.



The latitudes and longitudes of stars also are computed from their observed declinations and right ascensions and the obliquity of the ecliptic. In Fig. p. 55, if ES, et are quadrants, St is the measure of the angle at E*. or the measure of the obliquity: but, Pt, πS, being quadrants, are equal; $\therefore P\pi = St$; $\therefore P\pi$ is the measure of the obliquity; hence, if we conceive an oblique angled spherical triangle $sP\pi$, s being the star, P and π the poles of the equator and ecliptic, we have given Ps the complement of declination, or north polar distance, $P\pi$ the obliquity, $sP\pi$ the complement of the right ascension, in order to determine $s\pi$ the complement of the latitude. The determination of the latitude of a star s then, is reduced to the solution of this trigonometrical problem: given the two sides $P\pi$, sP and the included angle $sP\pi$, it is required to determine $s\pi$.

$$\text{Let } s\pi = c,\ Ps = a,\ P\pi = b,\ \angle sP\pi = C,$$

then assuming a subsidiary angle θ, such, that

$$[\tan.\ \theta]^2 = \frac{\sin.\ a \sin.\ b . \text{ver. sin. } C}{\text{ver. sin. } [a - b]} \dagger,$$

there results

$$2\,[\sin.\ \tfrac{c}{2}]^2 = \text{ver. sin. } [a - b]\,[\sec.\ \theta]^2$$

for the determination of c, the complement of the Star's latitude.

For the determination of the angle $s\pi P$, the complement of the Star's longitude, we have (c being found)

$$\sin.\ s\pi P, \text{ or cos. of the Star's long}^e, = \frac{\sin.\ a . \sin C}{\sin.\ c},$$

or, independently of the latitude, the longitude may be determined by means of Naper's analogies ‡.

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* *Trigonometry*, p. 91. † Ibid. p. 129. ‡ Ibid. 126.

EXAMPLE

EXAMPLE I.

Required the latitude of γ *Draconis*, its right ascension being [1726] 267° 33′ 44″: its north polar distance [*a*] 38° 28′ 11″, and the obliquity [*b*] being 23° 28′ 18″.

$a =$	38°	28′	11″	- log. sin.......	9.79386
$b =$	23	28	18	- log. sin.......	9.60020
[270°—267° 33′ 44″] $C =$	2	26	16	- log. ver. sin.	6.65556 *
					26.04962
$a - b =$	14	59	53	- log. ver. sin	8.23132 [1]
				2 log. tan. θ - - - -	17.81830
				log. tan. θ - - - -	8.90915
$\therefore 2\theta =$	4°	38′	16″;	log. sec. - - - -	20.00285
				[1.] - - - - - -	8.23132
					28.23417

$\therefore$ log. 2 + log. sin. $\frac{c}{2}$, or, log. ver. sin. $c = 8.23418$

$\therefore$ c, or the complement of Star's latitude = 15° 2′ 55″.

In order to find the longitude, we have from the logarithmic expression of the preceding formula, p. 56,

log. sin Ps - - - - - -	9.79386
log. sin. C - - - - - -	8.62874
	18.42260
log. sin. πs - - - - - -	9.41436
log. sin. $s\pi P$ - - - -	9.00824 = log. sin. [180°— (5° 50′ 58″)]

$\therefore$ longitude = 90° + 174° 9′ 2″ = 264° 9′ 2″.

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* The numbers in this Example, as in that p. 59. l. 18, were taken from M. Mendoza's Tables: in which the log. ver. sines are diminished by log. 2. [see *Trig.* p. 32.]

EXAMPLE II.

Required the latitude and longitude of η *Ursæ majoris.*

[1725] right ascension =	204°	10′	8″			
North polar distance	[a] 39	18	5	- - -	sin.	9.8016778
Obliquity of the ecliptic	[b] 23	28	18	- - -	sin.	9.6002054
270°—204° 10′ 8″ = - -	65	49	52	- -	ver. sin.	9.7712730
						29.1731562
[a—b] - - - -	15	49	47	- -	ver. sin.	8.5788920
						2)20.5942642

log. tan. θ - - - - - - - -	10.2971321
∴ 2 log. sec. θ = - - - - - - -	20.6927580
log. ver sin. [$a-b$] - - - - - -	8.5788920
	29.2716500
log. r + log. 2 - - - - - - - - - -	10.3010300
	2)18.9706200
log. sin. $\frac{c}{2}$ - - - - - - - - - - - -	9.4853100

∴ c, the complement of latitude, is equal to 35° 36′ 7″, and the latitude accordingly is 54° 23′ 53″. For the longitude,

sin. 39°	18′	5″	- - - -	9.8016778
sin. 65	49	52	- - - -	9.9601579
				19.7618357
sin. 35	36	7	- - - -	9.7650353
				9.9968004

∴ the longitude is 90° + 83° 3′ 13″, or 173° 3′ 13″.

When through a Star great circles are drawn respectively from the poles of the equator and ecliptic, they form at the Star an angle called the *Angle of Position*: P and π, being the poles, it is the angle $Ps\pi$: which may be computed from the obliquity [$P\pi$] and from the star's complements of latitude and declination πs, Ps; or, from the obliquity $P\pi$, north polar distance [Ps] and right ascension [$sP\pi$]. The value of the angle will evidently vary according to the position of the star. If s be situated in $P\pi$ produced, it will lie in the solstitial colure [see p. 41.], and then $Ps\pi = 0$, as indeed the formula for the value of the cosine of $PS\pi$ shews: for since in this case

$$\pi s = P\pi + Ps,$$

$$\cos. Ps\pi = \frac{\cos. P\pi - \cos. Ps \cos. \pi s}{\sin. Ps \sin. \pi s} \text{ [see } Trig. \text{ p. 99.]}$$

$$= \frac{\cos. [\pi s - Ps] - \cos. Ps . \cos. \pi s}{\sin. Ps . \sin. \pi s}$$

$$= \frac{\sin. \pi s . \sin. Ps}{\sin. Ps . \sin. \pi s} = 1;$$

consequently, $Ps\pi = 0$.

This angle of position is employed in several Astronomical computations: we shall perceive its use in the theory of the aberration of light: it there performs the office of an involved expression, or of an undeveloped function of the obliquity, right ascension and declination.

If we wish to calculate the angle of position of γ *Draconis*, we have, sin. angle position = sin. obliquity $\times \frac{\sin. C}{\sin. Ps}$:

∴ by logarithmic computation,

log. sin.......	23°	28′	18″	............9.60020
log. sin.......	2	26	16	............8.62874
				18.22894
log. sin.......	15	2	55	............9.41436
angle of position =	3	44	28	............8.81458

Angle of position of η *Ursæ majoris* [see p. 58]

log. sin.......	23°	28′	18″	............9.6002054
log. sin.......	65	49	52	.............9.9601579
				19.5603633
log. sin.......	35	36	7	.... 9.7650353
log. sin. angle of position				............9.7953280

∴ angle of position is 38° 37′ 26″.

By such methods, then, may the angles of position, the latitudes and longitudes of stars, be computed. But, these latter are of much less use, to the practical Astronomer, than the right ascensions and declinations of Stars; and accordingly they are rarely inserted in the catalogues of the fixed Stars. Flamsteed's contains them, and Lalande's, but not Bradley's, nor Lacaille's, nor Mayer's Still however, the latitudes and longitudes of Stars and the angles of position * are useful results: for instance, in the theory of the aberration of light, in which they enter into the composition of the formulæ. And, in the ensuing Chapter, we shall have an additional instance of their use, in the computation of the precession of the equinoxes.

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* M. Lalande, *Astron.* 2d Edit. vol. I. p. 488, has given a table of the angles of position of stars, with their *variations* (from the effect of precession). Thus,

	Angles of Pos. for 1750.	Variation for 10 following years.
β Draconis - -	13° 54′ 17″	— 5′ 25″
γ Draconis - -	3 31 53	— 5 22
η Ursæ majoris -	38 32 0	— 2 10

In the *Connoissance des Tems* for 1804, there is also inserted a catalogue of the latitudes, longitudes, and angles of position of stars.

CHAP. IX.

Precession of the Equinoxes.—Solar or Tropical Year.—Its Length.—Sidereal and Anomalistic Years.—Their Lengths, and the Methods of computing them.

In the former Chapter, methods have been given for computing the time of the Sun's coming to the equator in the equinoctial point; the interval between the Sun leaving that imaginary point, usually called the *first point of Aries*, and his return to the same: that is, the interval between the meridian altitudes, equal to one another, and to the complement of the latitude of the place of observation, is the length of the Astronomical year: now, in this interval, the Sun has returned to the meridian 365 times, and besides, has gone through, nearly, one fourth of his diurnal revolution: that is, the Astronomical year is not exactly equal to 365 days, but to $365^d\ 5^h\ 49^m\ 0^s.53$.

From this imaginary intersection of the equator and ecliptic, the longitudes of Stars are measured: if the point of intersection remain fixed, that is, if this year and the next, it be at the same distances from two fixed Stars, the longitudes of stars will remain unaltered. But it may change; that is, the Sun, after the interval of a year, may return to the equator at a point different from that in which he quitted it. When his declination is nothing, his distances from two fixed Stars, may not be the same as they were, at the beginning of the year. In such case, the longitudes of stars will be changed. If this latter be the fact, by what means, can it be ascertained?

We have already seen how to compute the time of the Sun's entering the equinox, even if his declination should not be nothing, when he is on the meridian [see p. 52.] A sidereal clock will note that time, and on the same day will note also the right ascension of a certain fixed Star; its declination may be observed: and thence (the obliquity of the ecliptic being known) its longitude may be computed. The same process may be repeated at the end of the year: then, if the longitude last determined does not differ from the former, the equinoctial point, or the first point of *Aries*, (the imaginary intersec tionof the equator and ecliptic,) has not changed. But, the fact is otherwise; the two resulting longitudes, are different: that at the end of the year is found to be the greater: consequently, since the star is supposed to be fixed, or not to have varied its place, the point of intersection, or the first point of *Aries*, must: and its place, to account for an increase of longitude, must have shifted to the westward, or in a direction contrary to the order of the signs. It is this westerly motion of the imaginary intersection of the ecliptic and equator, that is called the *precession of the equinoxes*. An instance may illustrate this curious Astronomical fact: by computation from the meridian heights of the Sun in 1809, on March 19, 20, 21, the Sun entered the equinox March 20th, at 12^h 14^m: suppose the sidereal clock at that time to have been set to 0^h: then, the Star *Aldebaran* being on the meridian, it indicated 4^h 24^m 58^s; this was *Aldebaran's* right ascension: his declination observed was 16° 6′ 2″: therefore taking the obliquity of the ecliptic = 23° 27′ 44″.8, his longitude by calculation [see p. 56.] will be found to be 2^s 7° 6′ 59 5″: again, by a similar method, *Aldebaran's* right ascension March 20, 1810, was shewn by the clock to be 4^h 25^m $1^s.5$, and his observed declination was 16° 6′ 10″; consequently by calculation, his longitude is 2^s 7° 7′ 49″.8 the difference of these two quantities shews that the equinoctial point has moved to the right of *Aldebaran*, or to the west, about 50 seconds

This is the method of ascertaining the precession as an Astronomical fact: and every part of such method, whether of observation, or of computation, has been already explained.

The intersection of the equator and ecliptic (always called the *first point of Aries*,) continually varying, the right ascensions and longitudes of stars which are reckoned from that point must also continually vary. Hence, a catalogue of fixed stars, if it merely expressed their right ascensions would be useless, except on the year for which it was constructed: but, it expresses besides, the yearly change, or *annual precession* in right ascension: by means of which, the right ascension and declination of a star from the year for which it is tabulated, may be *reduced* to a preceding, or *brought up* to a succeeding year. Celestial Atlases also, or maps of the Heavens, although exact in their representation for one epoch only, since allowance can be made for the changes introduced by precession, do not cease to be commodious to the Astronomer.

In order to ascertain the quantity of the *precession* to a greater exactness, we ought to compare the longitude of a star, not, at the interval, of a year, but, of several years: since any error committed in the observation will, in that case, less affect the result: for instance, if two longitudes of the same star are compared together, after an interval of 100 years, and the error of observation and computation be $5''$, the corresponding error, in the quantity of the annual precession, will be only $\frac{5''}{100}$, or $.05''$. It is, in fact, diffused over the number of years. But, Astronomers have taken a greater interval than 100 years: M. Lalande in his Astronomy has compared an observation of Hipparchus made 128 years before Christ with an observation made in 1750: thus

128. A. C. longitude of *Spica Virginis* =	5^s	24°	$0'$
In 1750.................................... =	6	20	21
Augmentation of longitude......... =	0	26	21

$$\therefore \text{ the mean annual precession} = \frac{26^\circ\ 21'}{1878} = 50''\ 30''', \&c.$$

The same author, by a great number of like comparisons, found the *secular* precession, that is, the accumulated precessions of 100 years to be $1^\circ\ 23'\ 54''$ *: and consequently the *mean* annual precession, that is, the precession, supposing its quantity to be the same every year of the hundred, to be $50''.34$. In the new French Solar Tables, however, the precession is stated at $50''.1$. The secular precession of $1^\circ\ 23'\ 54''$, gives nearly 1° for the precession in $71\frac{1}{2}$ years: and about 25745 years, for the period in which the precession moves through 360 degrees: or 25869 years, if we take the precession at $50''.1$.

The precession may be geometrically exhibited: let P be the

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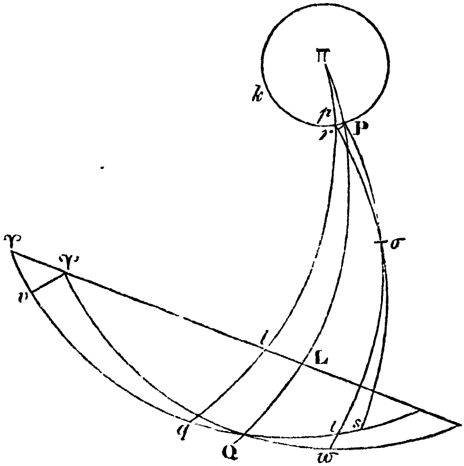
* 2d Edition, p. 392

pole of the equator ♈Q, π the pole of the ecliptic ♈L; πP, the distance of the two poles, being the measure of the obliquity of the ecliptic. Now if ♈, the intersection, shifts by a retrograde motion to ♈′, Q the solstitial point, distant from ♈ 90 degrees, shifts to q: consequently, the pole P is transferred to a point p in a circle Ppk described round π as a pole: for although



the intersection of the planes of the equator and ecliptic varies, the inclination does not, and therefore $\pi P = \pi p$.

If ♈♈′$=50''.1$, Pp, in seconds of the circle Ppk,$=50''.1$: but, in seconds of a great circle,$=50.1 \times \sin. 23^{\circ} 28'$ [$\pi P = 23^{\circ}.28$] $=$ $19''.9$. Since Pp is described in one year, the whole circumference of Ppk is described by P in 25869 years.

This principle of finding the annual error, by taking the accumulated error of several years, and then dividing by the number of years, has been applied to the determination of the length of the solar year: thus, by observations made at Paris (where the latitude is 48° $50'$ $14''$ and the co-latitude accordingly 41° $9'$ $46''$.)

March 20, 1672. meridian altitudes ☉. U. L. was	41°	43′	0″
March 20, 1716 - - - - - - - - - - - - -	41	27	10
March 21, 1716 - - - - - - - - - - - - -	41	51	

Now the difference of declination, between the two first observations, is $15'$ $50''$: between the two last, $23'$ $50''$: and since 24 hours is due to this latter change of declination, we have the time, due to the change of 15′ 50″, equal to $\frac{15' \ 50''}{23' \ 50''} \times 24^h$, or $15^h \ 56^m \ 39^s$: consequently, March 20, 1716, $15^h \ 56^m \ 39^s$ the Sun's declination, was the same as on March 20, 1672: now, the number of intervening years is 44, that is, [$365 \times 34 + 366 \times 10$ *] days, or 16070 days: hence, the whole interval of time between the equal declinations is $16070^d \ 15^h \ 56^m \ 39^s$. But, by the definition of an Astronomical, or solar year, an exact number of years must have elapsed between these two observations; which number is 44: consequently, the mean length of each year must be $\frac{16070^d \ 15^h \ 56^m \ 39^s}{44}$, or $365^d \ 5^h \ 49^m \ 0^s \ 53'''$.

This is the length of the solar year according to the observations that have been recorded, and, deduced from such observations, it is called the *apparent* solar year, in order to distinguish it from the *mean* solar year, which M. Lalande †, by the comparison of the most exact observations, has determined to be $365^d \ 5^h \ 48^m \ 48^s$. The reason of the distinction between the *apparent* and *mean* solar year will be hereafter shewn.

In the precept given, p. 61, for finding the length of the year, the interval elapsed between two appearances of the Sun in the equator, that is, between two meridian altitudes, each equal to the co-latitude of the place of observation, was directed to be computed; but, in the preceding instance, the interval elapsed was between two equal declinations near the equinoctial point: the principle of this latter computation, however, is plainly the same as that of the precept. The length of the year may also be similarly computed, from the interval of time elapsed between two appearances of the Sun in the solstices, or, from two equal declinations near the solstices. And, M. Cassini, p. 233 of his *Elemens d'Astronomie*, states that there are some peculiar advantages belonging to the latter method. It does not require the latitude to be known, nor the actual altitude of the Sun to be

* Ten leap years amongst the forty-four.

† *Abregé d' Astron.* pp. 35, 120. Also, *Mem. sur la veritable dureé de l' année solaire.*

exactly assigned by the instrument. It is sufficient to observe the Sun when it is lowest or highest: and this operation, to a certain extent, is independent both of parallax and refraction.

The year thus computed, from observations of the Sun near the solstices, and, consequently [see p. 40.] near the tropics, is called the *Tropical* Year.

Since the imaginary intersection of the equator and ecliptic may be conceived to have a westerly motion, and seemingly to move, in order to meet the Sun before he has completed an entire circuit of the Heavens, the solar or tropical year must be less than the interval of time due to such an entire circuit; that is, less than the interval elapsed between the Sun's quitting a star and his return to the same star; or, less than the interval between two successive situations of the Sun and a star, when the respective differences of their longitudes are equal. This latter interval Astronomers have denominated a *Sidereal* year.

The length of the sidereal year is easily found: for the solar year is $365^d\ 5^h\ 48^m\ 48^s$: in that time, the Sun describes 360° minus the precession ($50''.34$): in a sidereal year he describes 360°: hence,

$360^\circ - 50''.34 : 360^\circ :: 365^d\ 5^h\ 48^m\ 48^s : 365^d\ 6^h\ 9^m\ 11^s$;
the fourth term $365^d\ 6^h\ 9^m\ 11^s$, is the length of a sidereal year.

The length of a sidereal year (an element of little or no importance in Astronomy) has been determined from the ascertained quantity of precession: but, it may be determined independently, and on principles the same as what have been already stated: for instance,

1669, April 1, at $0^h\ 3^m\ 47^s$: difference of the long. of ☉ and *Procyon* $3^s\ 8^\circ\ 59'\ 36''$.

1745, April 2, at $11^h\ 10^m\ 45^s$ the same difference: hence, in this interval of $76^y\ 19^d\ 11^h\ 6^m\ 58^s$, or $27759^d\ 11^h\ 6^m\ 58^s$, by the definition of a sidereal year, an exact number of such years had elapsed: but, that number can be only 76: consequently, the

$$\text{length of one year} = \frac{27759^d\ 11^h\ 6^m\ 58^s}{76} = 365^d\ 6^h\ 8^m\ 47^s.$$

The common *civil* year consists of 365 days: every fourth year, however, it consists of 366 days: and there are other regulations prescribed by the *Calendar* which will be explained hereafter. But, it may be now observed that the Calendar is constructed solely with reference to the solar or tropical year. On that the seasons depend. For, *ceteris paribus*, at the same place, the temperature, or the degree of heat, will depend on the Sun's declination. When the declination is greatest and northern, it will be, to an inhabitant of the northern Hemisphere, the height of Summer; it will be the time of the *Summer solstice:* when the greatest and southern, the depth of Winter, or the time of the *Winter solstice*, [see Chap. III. p. 16.]

As it does not require very exact instruments *, nor very nice observation, to ascertain the two days of the year, when the meridian altitudes of the Sun are respectively the greatest and the least, the Antients easily discovered that the year consisted of days, the number of which was neither less nor greater than 365. And, conjecturing the simplest theory to be the true theory, they expected that this number of days was the exact time of the year: or, in other words, that the Sun would return to the equator, after he had passed the meridian an exact number of times. Many years, however, passed away, before observations, sufficiently accurate, proved this not to be the case: and still more accurate observations were requisite, to determine the exact excess of time above 365 days, that was necessary for the return of the Sun to the equator, in that part of the Heavens which he had quitted at the beginning of the year.

The length of the year depends on a mere Astronomical fact; the return of the Sun, after an interval of about 365 days, to the same meridian altitude: for, it is meridian altitude which is observed. If we take the altitude at noon on March 20, 1809, then, on March 20, 1810, the altitude will be less, on March 21, greater: consequently, if the observer would *see* the second meridian altitude that is equal to the first, he must, on the same parallel of latitude, travel towards the East through 89° 30′ of longitude, and March 21, near the Albany River in America, would observe the Sun at the same meridian height above the horizon, as at Greenwich the preceding year. The necessity of this second observation, is, as it has been explained [p. 52.] superseded by an

* The shadow of a *stile* would be almost sufficient for this purpose.

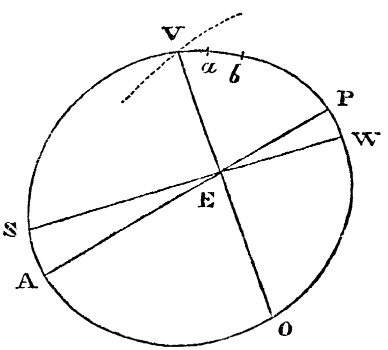
easy process, by which the difference of time (about $5^h\ 58^m$) is computed for the Sun's defect of declination, on the noon of March 20, 1810, from his declination on the noon of March 20, 1809.

Previously to observation, the bias of the mind is towards a belief in the simplicity of theories. We feel inclined to suppose the simplest hypothesis to be the true one. Accordingly, the return of the Sun to the equinoxes after a complete circuit of the Heavens, and, after an exact number of days, are circumstances, which antecedently to experience, we should conceive as *likely* to happen. But, observation shews that this nice adjustment of circumstances is not preserved. The Sun, at the end of the tropical year, has still a little farther to move before he has completed his revolution amongst the stars: and has passed the meridian of the observer nearly 6 hours, if he were on the meridian at the beginning of the year. Perceiving, therefore, this departure from simplicity, the Student might be led to enquire, whether the westerly motion of the equinoctial points, were uniform, or whether the *Precession*, as it is technically called, were each year the same: he might also enquire whether each year were exactly the same length: for, anomalies introduced into one part of the theory, may affect the whole.

It must be obvious, that the only satisfactory answer to these enquiries must be drawn from observation: and that proves the precession, each year, not to be invariably the same: and besides, the length of each year to be subject to an inequality or variation. Both these points are established by taking at different intervals, by the methods given in pp. 63, 65, the quantity of the precession, and the length of the year. This latter point, the length of the year, if followed, would lead into discussions of a nice and intricate nature: mere observation, indeed, seems hardly able to solve their difficulties. If we estimate the interval of time elapsed between the Sun entering the equator on March 20, 1809, and his return to the same on March 21, 1810, by the definition, that interval is a real solar year. If we estimate the interval between two equal declinations, near the equinox, that happened in 1672, and 1716, by the method given in p. 64, then we get the length of a solar year, supposing each of the 44 years to be equal: such is the mean length of one of those years. But, which may seem strange, such length is not the length of a *mean* solar year. Astronomers intend something different by the term *Mean* solar year:

and they have agreed to call the former *Apparent* solar year. And the reason for the distinction is this; the real path of the Sun is an ellipse, in which he moves with a variable velocity: now it happens, that the part of the curve in which he is moving when he comes to the equinox, is not the same as that in which he will be moving at the end of the year; consequently, since his velocity is continually varying, he must then move with a different velocity.

In the Figure, supposing *V* to be the intersection, of the solar



ellipse *PVAO*, and of the equator, for the year 1809; then, the next year, the point *V*, the vernal equinox, will have shifted to *a*: in other words, the Sun will be at *a* when he enters the equator: but, from the properties of elliptical motion, the velocity at *a* is greater than the velocity at *V*: the year following, 1811, the intersection will again have shifted to *b*, and at *b*, the Sun will move with greater velocity than he did at *a*. This is the statement of the fact, and its farther developement and explanation is reserved for a subsequent part. But, we may use what has been just shewn, to explain the length of a mean solar year. At the beginning of an æra, suppose the equinoctial point to be at *V*; then, after this point has moved through *a*, *b*, *P*, *O*, to *V* again, that is, when the line *PEA*, called the axis major, shall have, round *E* as a center, moved through 360°, the æra will be finished, and all the variations of velocities will have taken place, and will again begin to recur, similarly through an equal æra: hence, if we could estimate the interval of time due to such an æra, and divide it by the number of years, we should obtain the length of a *mean* solar year: this, theoretically, is the length: but since there can be no observations so far distant * as to give the interval, Astronomers, by the aid of theory, have endeavoured to supply the deficiency of real observations.

Two kinds of years, the Solar or Tropical, and the Sidereal, have been defined and determined. There is, besides, another, like the latter, of no essential use, called by Astronomers, the *Anomalistic* year. At a certain time of the year the Sun's diameter, if measured instrumentally, would be found to be the least; at that same time, the Sun would be seen in a part of his orbit called his *Apogee*; for, since his disk is smallest, he must then be most distant from the spectator; this point in the Figure [p. 69,] is *A*: it is one of the extremities of the major axis of the solar ellipse. Now, if at the end of an interval, nearly a year, from a first observation of the Sun's least diameter, a second were made, and the Sun were seen towards the same part of the Heavens, (at the same distance, for instance, from two known stars,) then would the Sun have moved from *A* through *OPV* in the time of a *sidereal* year. But, the Astronomical fact is different. The Sun, at the end of the interval, if apparently of the least diameter, will not be seen towards *A*, the place of the first observation, but, at a point (*A'*) † between *A* and *O*: consequently, the interval, the *anomalistic* year, must be greater than a sidereal year, and greater by as much time, as is requisite to describe the space *AA'*, the *progression* of the *apogee A*: or, greater than a solar year, by as much time as is requisite to move through the *precession*, together with the *progression* (with regard to the stars) of the point *A*: or, which is still the same thing, greater than a solar year, by as much time, as is requisite to describe the increase of the longitude of the apogee; for longitude is ever measured from the intersection of the equator and ecliptic.

The *progression* of the apogee is found as the precession of the equinoxes was found [p. 63]: Thus,

According to Flamsteed in	1690. Long. ⊙ apogee	97°	34′	59″
According to Delambre	1780 - - - - - -	99	8	19
	Increase in 90 years - - -	1	33	20

* The period would be more than 25740 years.

† The point *A'*, in Fig. p. 69, must be conceived to be, between *A* and *O*, and near to *A*.

or the annual increase in longitude is equal to $1' \, 2''.2$: but since the precession, which is a *regressive* motion, is $50''.34$, the annual sidereal progression is $1' \, 2''.2 - 50''.34 = 11''.86$.

The time of describing $11''.86$ added to the length of a sidereal year, will compose an anomalistic year, and, since the Sun near his apogee, moves in longitude about $58'$ in 24 hours *, the time will be about $4^m \, 50^s$: hence the length of the anomalistic year $= 365^d \, 6^h \, 9^m \, 11^s + 4^m \, 50^s = 365^d \, 6^h \, 14^m \, 1^s$.

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* By computation [see p. 54.] Sun's longitude

June 21	2^s	$29°$	$8'$	$32''$
June 22	3	0	5	48
Increase in 24 hours	-	-	$57'$	$16''$

CHAP. X.

On the Corrections necessary to be made to the observed Right Ascensions and Declinations of Stars. — Refraction. — Parallax.— Aberration.—Precession.—Inequality of Precession. —Nutation.

In the preceding pages have been given the methods of finding the most important of all Astronomical elements, the right ascensions [p. 48,] and declinations [p. 32,] of stars; which have been determined from observations made on the meridian. From these, by calculations, have resulted the latitudes [p. 56, 57.] and longitudes of Stars. By observations made also on the meridian, with the aid of certain simple processes of computation, the obliquity of the ecliptic [p. 44,] and the precession of the equinoxes [p. 62,] have been ascertained. But, what has hitherto been done, has been, on the simplest suppositions. The declinations and right ascensions have been stated as if they could be immediately used in Astronomical computation; in short, as if they wanted no *corrections*: as if, for instance, we could assign for the declination of a Star merely the difference between his observed, or instrumental meridian altitude and the co-latitude of the place [p.32]: for the right ascension, merely the time as noted by the sidereal clock [p. 51], and then proceed to compute the latitude and longitude [p. 56]. But, corrections exist, not only many in number, but relatively to their value, difficult to be ascertained, and besides, connected with certain most curious and interesting theories. These corrections, and their connected theories, form no inconsiderable part of Astronomical science. Or, it may be said, the science is not complete and exact without them.

It is intended, in the present Chapter, to enumerate the several corrections which must be made to the observed declinations and right ascensions of stars, in order to reduce them, to the conditions of true Astronomical elements. Some slight and general the explanation of their causes will be added; but in the sequel, a separate Chapter will be appropriated to the fuller explanation of each cause.

A star, or other heavenly body, is seen in that direction in which a ray of light enters the spectator's eye. But, such direction may not coincide with a straight line drawn between the star and the spectator: the ray of light in its passage from the star may, from physical causes, be deflected from its original direction; and, if deflected, we should wrongly assign the true place of the star, if we assigned it to be in the direction of its light. Hence would arise a source of error: an inequality affecting the observation, and the necessity of a correction to compensate the inequality.

In the second place, at the time of an observation, to all appearance, we are at rest. But, if the solar or Copernican system be right, the Earth and the observer are really in motion. If so, may not the latter, by moving transversely to the line of the light's progress, refer the star to a point in the Heavens different from the true place? And, if so, a second inequality would arise, affecting the observation at the very time it is made, and depending on the direction of the observer's motion: and hence also, as before, the necessity of a second correction, to compensate this new inequality.

The above corrections arise from inequalities that may be said to be apparent and illusory; and when applied to observations, would enable the observer to refer the heavenly body to its true place: true, however, solely with respect to the observer, situated on the surface of the Earth, and in a particular part of that surface: but, if the place is to be rendered true to all observers, wherever situated, then, some common point, to which all are similarly related, must be selected; and, such will be the center of the Earth. Observations, therefore, seen at the surface, must be *reduced* to the center: that is, the star's place must be assigned, such as it would be seen in, if we could put a spectator in the Earth's center. The *reduction* of the star's place seen from the surface, to the center, or the requisite *correction*, cannot be said to arise from any inequality as a cause, but is a correction invented for the sake of simplicity and the convenience of Astronomical computations.

What precedes, relates to the correction of observations at the time of making them. But, many Astronomical processes depend on the comparison of observations distant from each by considerable intervals. In such intervals, may not the pole of the Earth have changed its situation; or, which is the same thing, may not certain stars be more or less distant from the zenith of the spectator? And if so, in order to institute a comparison, we must so *correct* the observations that they be reduced to the same point of time. Hence, new corrections would arise of a different class from the preceding.

We have attempted to indicate the causes of inequalities which are now known to exist, and distinguished by the titles of *Refraction*, *Aberration*, *Parallax*, *Precession*, *Nutation*. But, if these inequalities exist, how can they be ascertained and made apparent? We will begin with the first correction, arising from a cause called *Refraction*, by which a star, to appearance, is elevated above its true place; the elevation taking place in a vertical circle passing through a star: the effect of this, may be easily ascertained; thus, take at Cambridge, (the latitude of which is 52° 13′ 24″) the distance of γ *Andromedæ* and *Polaris* (which is about 47 degrees) first, when the former is nearest the horizon; next, when most remote and near the zenith: and the difference of these two distances will be about 10 minutes, the excess of the latter over the first distance. If the same observations were made at Paris, where the latitude is 48° 50′ 14″, the difference instead of being 10, would be 30 minutes. And this deviation is clearly connected with the different heights of the stars above the respective horizons of Cambridge and Paris.

The effect of refraction may be shewn also by the following instance:

If at Cambridge, 1810, the meridian altitude of *Spica Virginis* be observed, it would by the instrument appear to be 27° 39′ 17″; and, if the latitude were sought from the height of the pole star, it would appear to be 52° 13′ 20″; and consequently the apparent co-latitude would be 37° 46′ 40″: whence by the rule [p. 8,] the apparent declination of *Spica Virginis*, or the difference of these quantities, would be 10° 7′ 23″. If, however, at a place 10 degrees north of Cambridge the same operations were made, we should have

Co-latitude - - - - - - - - - - -	27° 46′ 54″
Meridian altitude of *Spica Virginis*	17 40 27
Apparent declination of star - - -	10 6 27

Now, since the declination remains the same, the two results representing it ought to be the same: but they differ nearly by one minute; and this difference can be accounted for, if we suppose a star to be the more elevated by refraction, the greater its zenith distance, but, in a higher proportion than the increase of that zenith distance; in fact, to be elevated proportionally to the tangent of its zenith distance: thus, if the elevations, from refraction, were according to the following Table:

Apparent Altitudes.		Elevations by Refraction.		True Altitudes.
27° 39′ 17″	- - - -	1′ 47″	- - - - - - -	27° 37′ 30″
52 13 20	- - - -	44	- - - - - - -	52 12 36
17 40 27	- - - -	2 57	- - - - - - -	17 37 30
62 13 6	- - - -	30	- - - - - - -	62 12 36

then, since the true co-latitudes would be 90° − (52° 12′ 36″) 90° − (62° 12′ 36″); or, 37° 47′ 24″, 27° 47′ 24″, respectively, we should have, according to the rule for the declination, either 37° 47′ 24″ − 27° 37′ 30″, or 27° 47′ 24″ − 17° 37′ 30″, which are evidently the same.

The effect of refraction is also made apparent by observing the distance between two stars, at different times: for, their distance when they are near to the horizon, becomes less at a greater elevation: the effect of refraction being, as we shall hereafter see, to elevate bodies in a vertical circle.

The effect of refraction, is also to be recognised in determining the latitude from the greatest and least heights of circumpolar stars: for, half the sum of those heights, which, if they were the true heights, ought to be the latitude, is not always the same quantity, but varies with different circumpolar stars.

This correction may be said to arise from a physical cause; it takes place in a vertical circle, and consequently will affect the determination of the declinations, but not of the right ascensions of heavenly bodies, when such determination is made from observations taken in the plane of the meridian.

We will now turn our attention to the correction arising from a cause called *Parallax* *. Observations are made on the surface of the Earth; but, Astronomers wish to fix the position of stars, as

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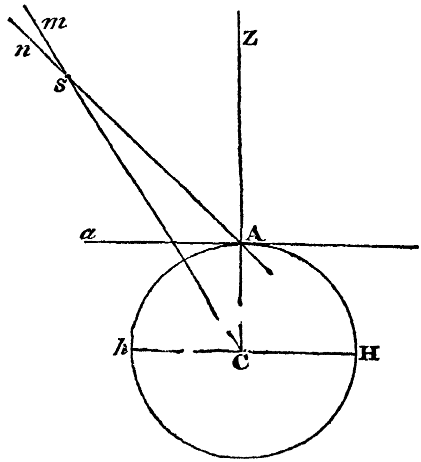
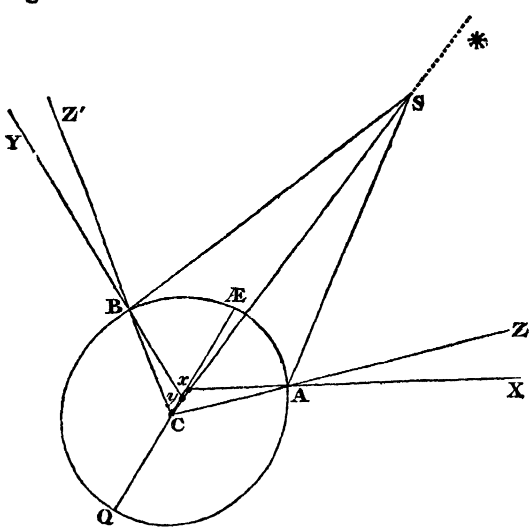
* This is now taken in the second place, which in the former enumeration [p. 73,] was taken in the third.

if they were seen from its center. A star *s*, therefore, seen by a spectator at *A*, whose zenith is *Z*, in the direction *Asn*, at an angular distance *ZAs* from the zenith, is, if we suppose a spectator



at *C*, seen in the direction *Csm*: the difference of the two places, *m n*, to which the star is referred, or the angle *msn* is called the *Parallax*.

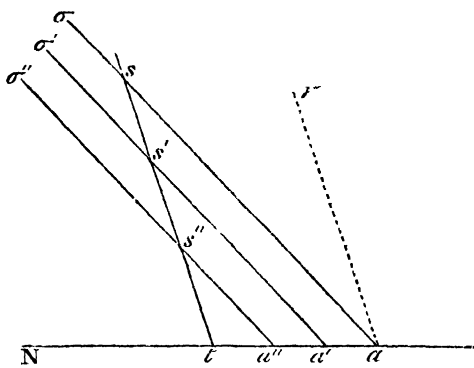
Although this correction has been said to be one merely of



computation, yet, there are certain circumstances which strongly suggest its necessity; and the same circumstances enable us to recognise its existence. If *B* and *A* are two places on the Earth's surface, and the meridian altitudes of a fixed star (λ♒) be respectively observed, and from such altitudes and the latitudes of the places, its declination [see p. 8,] be computed, it will appear to be the same both from the observation at *B*, and from that at *A*. The same would happen with *S*, if *S* were a fixed star: and accordingly the difference of the declinations of λ♒ and *S*, or what is the same thing, the difference of their zenith distances, would be the same, whether observed at *A* or at *B*. If this condition took place, whatever were the celestial body that was observed, there would be no occasion for introducing parallax. But, with certain celestial bodies, the Sun, the Moon, and the planets, the condition does not take place. If *S*, for instance, were Mars, the difference of the zenith distances of *S* and λ, seen from *A*, would not equal the difference seen from *B*. Here then, is a material circumstance of distinction; since the apparent declination of a body like Mars would depend, which it ought not to do, on the place of the observer: it becomes necessary then, to feign an observer in the center of the Earth, whence all declinations would be seen as they ought to be, and to reduce to him, observations made on the surface. To effect this, is the object of the theory of parallax.

The third inequality arising from the *Aberration* of light, cannot, by the test of any simple observations, be easily ascertained and detected. The method of detecting it will be hereafter shewn. At present, we must be content with expatiating a little farther on its probable cause.

The two preceding corrections are totally independent either of the rest, or the motion of the spectator. But, if light should not instantaneously come from the star to the eye; in other words, if it should be *propagated;* and if the spectator should be moving transversely or obliquely to the light's direction, may not the light seem to come from a place different from the star's true place? And if so, from these causes, that is, the motion of the spectator and the propagation of light, there must arise a *correction* to be applied, in addition to the preceding corrections, in order to determine from the apparent instrumental place, the true place: which correction also, with the same star, must vary with the change in the direction of the spectator's motion: or, in other words, will not be the same, whether the spectator move directly towards the star, or transversely to a line between him and the star. Now, it can be shewn, independently of this phenomenon of aberration, that light does not come instantaneously to the eye from a star, but is propagated: suppose then, a ray of light to be descending in the direction $ss's''$; when at s, a spectator at a, would see it in the direction $as\sigma$, or would refer it to a point σ; when at s', the spectator at a' would see it in the direction $a's'\sigma'$; when at



s'', the spectator translated to a'', in the direction $a''s''\sigma''$, &c.; consequently, with reference to the line aN, he constantly sees it at the angle saN, for as, $a's'$, &c. are parallel: but, if stationary at t, or at a, he would see the star at an angle stN, or raN: the difference, of these two angles, then, $stN - saN$, or ast is the *Aberration.*

Of the corrections enumerated, that of parallax, which is arbitrary and scientific, depends with a given star, solely on the star's altitude, if the Earth be supposed spherical; if, (as it is,) spheroidical, then conjointly on the star's altitude and the spectator's latitude. Refraction also depends, with a given state of the atmosphere, solely on the altitude of the body; and, those two conditions remaining the same, is the same, at whatever part of the day (the 24 hours) the observation is made. Aberration, on the contrary, depends neither on the state of the atmosphere, nor on altitude, merely as altitude, but, only inasmuch as it is connected with declination; nor on the spectator's latitude; and consequently with the same star will be the same, whether the spectator be at 50, or 60 degrees of latitude.

But, it will vary according to the time, or hour of observation, supposing, as is always the case, such observation to be made on the meridian.

The preceding corrections enable us to reduce the apparent place of a star to its true and Astronomical place at the time of the observation. We come now to those which, as we stated in p. 74, form a separate class: which enable us to connect distant observations; and, from a star's place computed for one point of time, to assign it at any other, previous or subsequent. Corrections of this kind occur in various Astronomical processes; and, we shall, in a subsequent page, exemplify them and shew their use, in determining the latitudes of places, and in regulating the Astronomical clock. The causes of the corrections or the theories on which they depend, will be treated of in the following order, *Precession*, *Inequality of Precession*, *Nutation*.

The first of these, *Precession*, has been already explained, but explained merely as an Astronomical fact, that of a difference of about 50 seconds between two successive places of the Sun in the equator after the interval of a year, [see p. 62]. And this view of the precession would be sufficient, if we wanted only to know the alterations of precession in the right ascensions and declinations of stars, that took place in *complete* years: for instance, the precession being 50″, the right ascension of *Sirius* would in one year be changed 40″.18; in 2 years 2′ 20″.36; in 10 years 6′ 41″.8, &c. the right ascension of *Regulus* would in one year be changed 48″.39; in 3 years 2′ 25″.17; in 10 years 8′ 3″.9, &c. But, it may be necessary to know the right ascensions for intervals of time, which are not expressed by any number of complete years: in such case, we must understand by the precession of the equinoxes something more than the mere Astronomical fact.

We have already said [pp. 40, 42.] that the equinoctial point may be conceived to be the intersection of the equator and ecliptic: and, if we so conceive it, we may also conceive the point of intersection to move gradually, day by day, through the whole arc (50″) of the precession. In such case, if the motion were uniform, the equinoctial point would have moved to the west, or have been regressive, in half a year, about 25″; in three quarters of a year about 37″, &c.; so that, for 2 years and an half the precession in the right ascension of *Sirius* would be (1′ 20″.36) + 20″.09, or 1′ 40″.45.

This would be the case if the *precession* of the equinoxes during the year were uniform; but it is not, being subject to an inequality, called the *inequality* of the precession. This must be briefly explained.

We have already passed from the consideration of the precession as an Astronomical fact, to that of the intersection of the ecliptic and equator endowed with a regressive motion, and describing gradually the whole arc of the precession. We may now go a step farther, and consider the cause of this regressive motion: if the cause be variable, the effect will; or, the regressive motion, though it may be gradual, cannot be equable: and this is the case; and the cause of it is the action of the Sun and Moon on the bulging equatorial parts of the terrestrial spheroid: which action varies, from the varying situation of each of the two bodies, the Sun and Moon: and accordingly, an *inequality* (requiring an equation,) will arise due to each; one, varying with the Sun's declination, called the *Solar inequality of Precession*; the other, due entirely to the variable action of the Moon, as it depends on the inclination of her orbit, to the plane of the ecliptic, and called the *Nutation*.

Not only from the two preceding causes, an inequality will arise in the precession of the equinoxes, * but the obliquity of the ecliptic will be also affected.

Hitherto has been given a general explanation of the causes of the several corrections. Their importance, in practice and in theory, claims for each a separate Chapter. In practical Astronomy, a knowledge of the quantities or values of all the corrections is essential. Without it, the business of an observatory would be at a stand. From physical Astronomy are derived, the laws of the variation of the latter corrections. Then, from the expressed laws, or formulæ, Tables are formed. The registered results of these Tables are at once applied to observations and confirmed by them. Thus, by a reciprocal operation practice is made exact, and theory is elucidated.

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* Dr. Maskelyne, in his Tables, separates the *Nutation* into two *equations*: one called the *Equation of the Equinoxes*; the other *Deviation* in right ascension, and *Deviation in North Polar distance*.

Series of Chapters, from XI. *to* XVI. *inclusive;*

ON

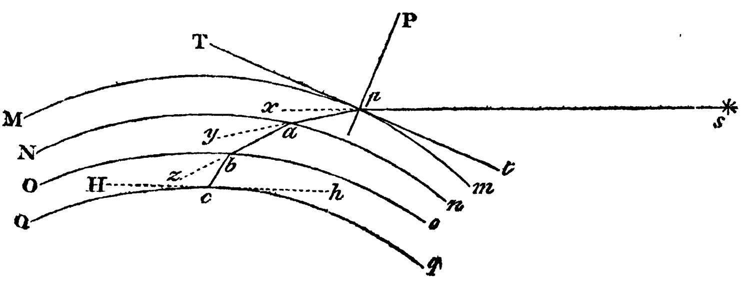
CORRECTIONS,

THEIR THEORIES AND USES.

CHAP. XI.

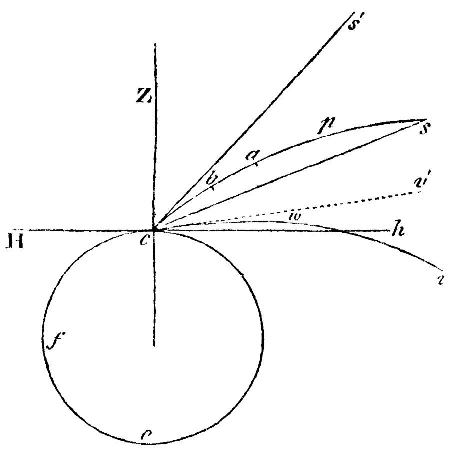
Refraction.—Bradley's Formula.—Application of it, as a Correction of Observations.

It is a law of Optics, established by experiment, that a ray of light passing, from a rarer, into a denser medium, is refracted towards the denser. For instance, if *Mpm*, should be the boundary between two media, then a ray of light *sp*, instead of pursuing its direction *spx*, is deflected in the direction *pa*, that is, towards a perpendicular *Pp* drawn at *p*, to *Tt* a tangent at *p*. Similarly, if *Nn* should be a similar boundary, that is, should separate the rarer



medium contained between *Mm*, *Nn*, from the denser medium contained between *Nn*, *Oo*, the ray of light instead of pursuing its new direction *pay*, is again deflected into the direction *ab*: and like circumstances must take place, if more media and their boundaries are added: hence, the course of the ray instead of being rectilinear and continued, is broken into portions *pa*, *ab*, *bc*, &c. inclined to each other at the angles *pab*, *abc*, &c: and if, according to the principles of the Infinitesimal Calculus, we suppose the number of media to be indefinitely increased, and their boundaries indefinitely to approach, the portions *pa*, *ab*, &c. will become curvilinear elements, and the course of the ray instead of being polygonal will be a curve concave towards the denser medium ; and the representation of the media, and of the course of the ray, will be more adequately exhibited in the subjoined Figure.

By this last step, the medium is no longer parcelled out into different strata, or shells, of variable density, but becomes a medium of a density continually varying. Such an one is the Earth's atmosphere, most dense at its surface, and rarer, the higher the region. A ray of light then, will, in its passage through the atmosphere, be deflected into a curve, concave towards the Earth's surface, exactly as we have seen that it must be, in the preceding case: for the medium, there imagined, and the atmosphere, are alike.



If *c* be the place of the spectator, and *s*, of the star, and if we suppose a plane, perpendicular to the Earth's surface, to pass through *c* and *s*, the refraction can only happen in such a plane, that is, in the plane of a vertical circle: for, at the same distance from the Earth's center, on each side of that plane, the atmosphere is supposed to be of equal density, and probably is; therefore, there is no reason to be assigned, why the ray should be deflected towards one side of the plane, rather than towards the other; which amounts to this, that it will be deflected towards neither.

The ray of light entering the spectator's eye, in the direction of the tangent to *cb*, the star must appear to be in that direction;

and consequently to be at s': and if Hch be horizontal, or a tangent at the point c, to the circle cef, a section of the Earth, the apparent elevation of s above the horizon, that is, the altitude of s will be $s'ch$: whereas the true altitude is sch; and the refraction, accordingly, is the angle scs'.

Hence, by refraction, a star is elevated; and, as we have just seen, in the plane of a vertical circle: now the plane of the meridian is vertical; consequently, the declination of a body, as determined by its meridian altitude, will be affected by the whole quantity of refraction: but its right ascension, determined by its transit over the meridian, will not be at all affected.

If a star were at Z, the zenith of the observer, its light would suffer no refraction; the parts to the right and left of the perpendicular line cZ, at equal distances from the surface, being equally dense: at h, or in the horizon, it will suffer the greatest refraction.

Between the zenith and the horizon, refraction takes place through all its degrees. The greater the star's zenith distance, the greater the refraction; but it is not simply proportional to the zenith distance. It *depends*, however, on the zenith distance, and it becomes an object of mathematical investigation, to *express* the refraction in terms of the zenith distance.

The refraction then depends on the zenith distance: but this supposes the medium or atmosphere to remain the same: if the density be changed, the refraction will: but if the density of the air be changed, its weight or pressure is: and the common barometer indicates, by its variations, the changes of weight: hence, the refraction depends on the star's altitude, and on the state of the atmosphere with regard to its density: or, it may be said to depend on the star's altitude, and on the height of the barometer.

The quantity of refraction, so it appears by experiment, in a medium of a given density, will vary with a change of temperature: hence, the refraction depends, on the star's altitude, barometer, and the air's temperature: or, since the thermometer indicates changes of temperature, the refraction may be said to depend, on the star's altitude, and the heights of the barometer and thermometer.

A formula involving these three conditions, would in specific instances, assign the quantity of refraction: or, would be immediately subservient to the construction of a table of refractions; and, the skill and labour of Mathematicians have succeeded in assigning the following:

$$\text{Refraction} = \frac{a}{29.6} \times \tan.\,(z - 3r) \times 57'' \times \frac{400}{350 + h}\ ^{*}.$$

in which a = altitude of barometer in inches,
z = zenith distance,
$r = 57''.\tan.\, z$,
h = height of Fahrenheit's thermometer,
29.6 is the mean standard height of the barometer.

This formula, undoubtedly of great elegance, probably was not derived by a direct mathematical process: but, was rather the result of many trials, conjectures, and experiments. The refraction, varying, in the space of a quadrant, through all its degrees, from nothing to its maximum, the simplest hypothesis suggested to the mind, would be, the diminution of the quantity of refraction with that of the zenith distance. Observation having overthrown this hypothesis, and shewn the refraction to be diminished in some higher ratio, the next, in point of simplicity, would be the variation of the refraction according to the tangent of the zenith distance. But, nature does not accommodate herself to mathematical simplicity: and this latter hypothesis, although nearer the truth than the former, was found not to give results exactly conformable with observation: at length, Bradley inferred the law, such as is expressed in the preceding formula.

In settling the law, Bradley availed himself of the existing tables of refraction; of results obtained on grounds purely mathematical; and of his own observations, and, nearly, according to the following description.

By numerous observations on *Polaris* and other circumpolar stars, he deduced the zenith distance [P] of the pole. This was the apparent distance, on account of refraction, and therefore less than the true zenith distance. By observations also of the Sun at the equinoxes, when the Sun had the same zenith distance,

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* See Dr. Maskelyne's Explanation and Use of the Tables, p. 5. Dr. Brinkley, *Phil. Trans.* 1810, Part II. p. 204, suggests this formula; $\frac{a}{29.6} \times \tan.\,[z - 3.2r]\ 56.9 \times \frac{500}{450 + h}$.

but opposite right ascensions, he deduced the height of the equator. This also was the apparent height on account of refraction, and therefore greater than the true height; and consequently, the apparent zenith distance [Q], was less than the true: hence, the sum of the two zenith distances of the pole and equator, which, if the true ought to equal 90°, would be less than 90° by the sum of two refractions, due respectively to the zenith distances P and Q: let the two refractions be p and q, then

$$P + Q = 90^\circ - [p + q],$$
$$\text{and } [p + q] = 90^\circ - [P + Q],$$

and consequently, since P and Q were given by observation, $p + q$ would be known. Again, by the best Tables of refraction then extant, it appeared, that $q - p$ was equal to about two seconds: hence, from

$$q + p = 90^\circ - [P + Q],$$
$$\text{and } q - p = 2''$$
$$q = 45^\circ\ 0'\ 1'' - \tfrac{1}{2}[P + Q]$$
$$p = 44^\circ\ 59'\ 59'' - \tfrac{1}{2}[P + Q].$$

According to Bradley's observations, P was 38° 30′ 35″, and Q 51° 27′ 28″: in which case $q = 59''.5$, and $p = 57''.5$: but this being only an approximation, in order to obtain a more exact result, Bradley separated the sum of p and q, that is, 1′ 57″, into two parts, p' and q', which should be to one another as the tangents of the zenith distances: accordingly,

$$p' : q' :: \tan.\ 38^\circ\ 31'\ 32''.5 : \tan.\ 51^\circ\ 28'\ 27''.5,$$

$$\text{whence, } p' = [p' + q'] \frac{\tan.\ 38^\circ\ 31'\ 32''.5}{\tan.\ 38^\circ\ 31'\ 32''.5 + \tan.\ 51^\circ\ 28'\ 27''.5};$$

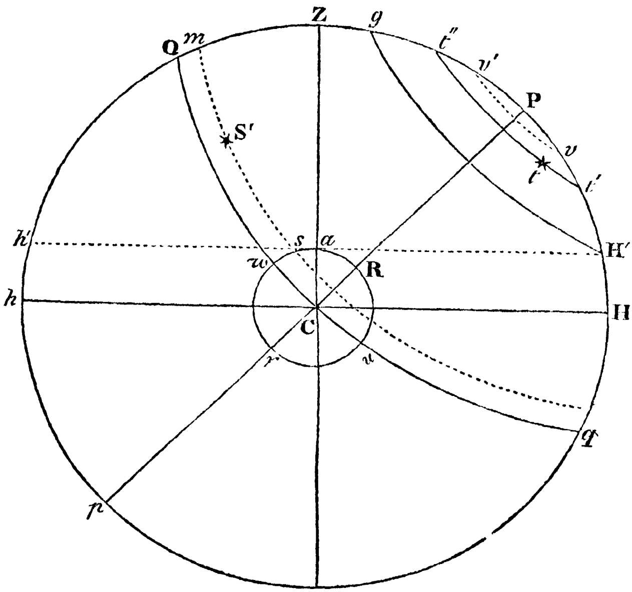
$$q' = [p' + q'] \frac{\tan.\ 51^\circ\ 28'\ 27''.5}{\tan.\ 38^\circ\ 31'\ 32''.5 + \tan.\ 51^\circ\ 28'\ 27''.5};$$

and, by this new operation, (since $p' + q' = p + q = 1'\ 57''$) there results

$p' = 45''.5$ refraction due to the zenith distance 38° 31′ 20″.5,
$q = 1'\ 11''.5$ * - - - to - - - - - - - 51° 28′ 39″.5.

* The observation being made at Greenwich, and $\mathfrak{Q}$ being = 51° 27′ 28″, $\mathfrak{Q} + q' = 51^\circ\ 28'\ 39''.5$, the latitude of the observatory.

Taking these, as the true quantities of refraction, at the zenith distances of the equator and pole, Bradley deduced the refractions for zenith distances less than that of the equator, by assuming them to vary as the tangents of the zenith distances; and he deduced the



quantities of refraction for greater zenith distances, or lower altitudes, by means of circumpolar stars. Thus, suppose v, v' to be the true places of a circumpolar star at its least and greatest altitude, then, since Zv', by correcting the observed distance, is known, and ZP also is known, Pv' is: next, the apparent zenith distance of the star at v is observed, and, subtracting ZP, the apparent distance from P is known: this is less than the true distance Pv, which is known, since it equals Pv': consequently, the difference between the true and apparent distances from P is the refraction due to the zenith distance Zv. For instance, by observation, the zenith distance of *α Cassiopeiæ* when at the greatest altitude was 13° 48′ 12″.5: but the refraction being 14″, the true zenith distance was 13° 48′ 26″.5: and since the zenith distance of the pole was 38° 31′ 20″.5, the north polar distance of the star [Pv', or Pv] was 24° 42′ 54″. Again, by observation, the star being at its least altitude above the horizon, the zenith distance was found to be 63° 13′ 21″.8 ; therefore, the apparent distance from

P was 63° 13′ 21″.8 − 38° 31′ 20″.5, or, 24° 42′ 1″.3 ;

consequently, the refraction was 24° 42′ 54″ −24° 42′ 1″.3, or, 52″.7.

By like operations, Bradley determined the refractions for other altitudes ; and when he had tabulated the results, by examining them, he found that the law of the refraction [r] instead of being represented by

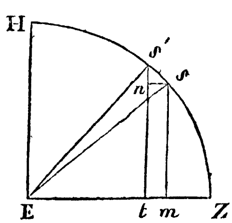
$$r = \frac{m}{n} \tan. z,$$

could be more exactly represented by

$$r = \frac{m}{n} \tan. [z - 3r] ;$$

and deducing from observations the values of m and n, he obtained the formula which was given in p. 84, and from which the Table, in the Appendix, was computed.

It has not yet been explained, why Bradley assumed the refraction to vary as the tangent of the zenith distance, which indeed, is nearly the law of its variation. Let the angle of incidence of a



ray of light be ZEs', the angle of refraction ZEs ; then since the arc or angle sEs' is small (being only 57″ when $ZEs = 45°$) the side ss' may be considered to be rectilinear : consequently,

$$ss' = Es \times \frac{s'n}{Em} \propto \frac{s'n}{Em} ; \text{ but,}$$

since the sines of incidence and refraction are in a given ratio, or

since $\frac{s't}{sm} = \frac{m}{n}$ (m and n given quantities,) $\frac{s'n}{sm} = \frac{m-n}{n}$, or, $s'n \propto sm$

consequently, $ss' \propto \frac{sm}{Em} \propto \frac{\sin. ZEs}{\cos. ZEs} \propto$ tangent $ZEs \propto$ tangent of the zenith distance. This part of Bradley's method is purely mathematical.

Suppose now, that it was required to compute, from the preceding formula, for the refraction at the altitude of 30° above the horizon. Here $z = 60$; $\therefore r = 57''.\tan. 60° = 57'' \times 1.7320 =$ $1'\ 38''.7$ for a first approximation; $\therefore 3r = 4'\ 56''$, and corrected value of $r = 57''.\tan.\ [59°\ 55'\ 4''] = 57'' \times 1.7263 = 1'\ 38''.4$: and if greater accuracy were required, the last value might be substituted for r in $57''\ \tan.\ [z - 3r]$, and a still nearer value obtained.

By a similar process, for the altitudes 27° 39′ 17″, 62° 13′ 6″ (quoted in page 61), the refractions

1st Approx. $r = 57'' \times \tan.\ 62°\ 20'\ 43''$	$r = 57''\ \tan.\ 27°\ 46'\ 54''$
$= 57'' \times 1.908$	$= 57'' \times .5268$
$= 108''.756$	$= 30''.027$
$= 1'\ 48''.75$	
2d Approx. $r' = 57''\ \tan.\ [62°\ 15']$	$r' = 57'' \times \tan.\ 27°\ 45'\ 24''$
$= 57'' \times 1.9006$	$= 57'' \times .5262$
$= 108''.33$	$= 29''.99$
$= 1'\ 48''.3$	

The quantity 1′ 38″.4, is deduced on the supposition that the barometer is at 29.6 inches, and the thermometer at 50 of its degrees: for, then, $\frac{a}{29.6} \times \frac{400}{350 + h} = 1$, and the refraction, in this case, is called the *mean* refraction. The Table is computed on the same supposition.

If the refraction is required for other states of the thermometer and barometer, then, the *mean* refraction must be multiplied by $\frac{a}{29.6} \times \frac{400}{350 + h}$: thus, suppose the refraction to be required for an altitude of 30°, the barometer being 29.85, and the thermometer, at 65 degrees of temperature: here a being 29.85, and h, 65°, the multiplier is $\frac{29.85}{29.6} \times \frac{400}{415}$, or, .9716, and the refraction therefore is $1'\ 38''.4 \times .9716 = 1'\ 35'.6$.

Having now obtained a formula and Table of refractions, that enable us to make the first of the corrections enumerated in page 74, the process for finding the Sun's declination, or a star's, instead of being as in page 47, will be after the following manner:

EXAMPLE.

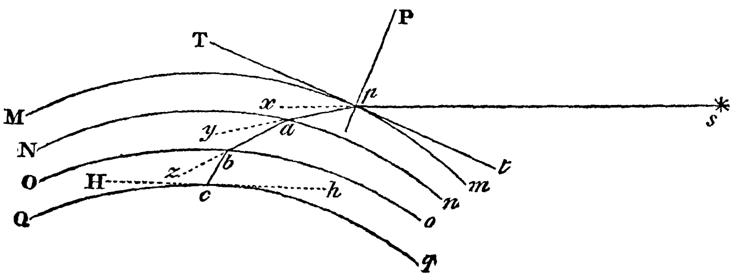
Alt. ☉'s. Upper limb - - - - - - - - - - - -	61°	29′	16″
Lower limb - - - - - - - - - - - -	61	0	46
	2)122	30	2
Apparent alt. of the ☉'s center - - - - - - -	61	15	1
Refraction - - - - - - - - - - - - - - - - -	0	0	31.28
True alt. of the ☉'s center - - - - - - -	61	14	29.72
Co-latitude of Cambridge - - - - - - - -	37	47	24
Declination of the ☉ - - - - - - - - - -	23	27	5.72

In the next Example, a correction for the error of collimation [see pp. 29, 30, &c.], as well as one for refraction is introduced,

☉'s Upper limb - - - - - - - - - - - -	62°	30′	30″.5
Error of collimation - - - - - - - - - -	0	0	34.5
	62	29	56
Apparent zenith distance - - - - - - - -	27	30	4
Refraction - - - - - - - - - - - - - - -			29
	27	30	33
Semi-diameter of the ☉ - - - - - - - -		15	46
	27	46	19
Latitude of place of observation (Paris) -	48	50	14
Declination of the ☉ - - - - - - - - - -	21	3	55

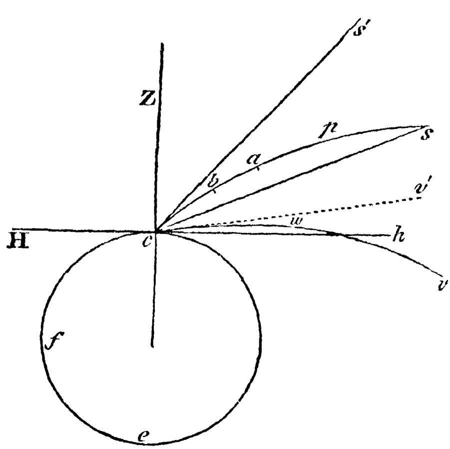
The refraction in this last Example is *added* to the zenith distance; an operation, equivalent to that of subtracting it from the altitude.

The formula and table of refractions are found to answer very well, for all altitudes, greater than ten degrees. At less altitudes, the refraction is very inconstant. The light from a star *s* near the horizon, is obliged, as is plain from the Figure, to pass slantingly through a considerable portion of the lower strata or laminæ of the atmosphere: and the lower strata, in parts remote from the observer, are subject to great variations, which are not



expressed by the thermometer and barometer at the place of observation.

For the mere purposes of Astronomical observation, the Table of refractions is sufficient. But, the principle of refraction may be employed in successfully explaining certain celestial phenomena. For instance, the refraction varying very rapidly near the horizon, the lower limb of the Sun when setting, will be much



more elevated than the upper: and accordingly, the figure of the Sun, instead of being circular will be oval; the *flattening* taking place in the direction of a vertical circle. Again, a star *v* below the horizon *Hh*, the course of its light being by refraction bent into the curve vwc, will appear at v' above the horizon: and accordingly, it is possible to see both the Sun and Moon above the horizon, at the time of a central lunar eclipse: for, if the Sun be just elevated above the plane of the horizon, the Moon diametrically opposite will be beneath, but so little beneath, that the refraction will make her appear above. This phenomenon was observed at Paris on July 19, 1750 *.

Since the quantity of refraction at 10 degrees of altitude amounts to upwards of 5 minutes, it is an object of very considerable moment, in practical Astronomy, to possess an exact formula of refraction: that is, exact in the variable part that expresses the law, and in the constant part that expresses the numerical value of the coefficients. For, the declinations of stars are deduced from meridian altitudes, and, consequently, would partake of their errors: and so it might happen, that certain smaller corrections, belonging, either to inequalities that arise from peculiar motions in the stars themselves; or, to some change in the position of the poles of the Earth; or, to causes merely visual and optical, would not be discerned, but be lost, and absorbed amongst the uncertain errors of refraction.

Bradley, as we have partly seen, paid particular attention to this subject, and in his researches, united observation with theory. The formula which he deduced, is found to be very exact: some, although no great, alteration has, since his time, been made in it. It is, however, most exact for altitudes above 10 degrees, at which the refraction seems to depend, almost entirely, on the pressure of the atmosphere, and on its temperature at the place of observation. For altitudes below 10 degrees, the formula is less to be relied on; but, fortunately for Astronomy, none of the more nice observations, from which its elements are computed, require to be made at such low elevations.

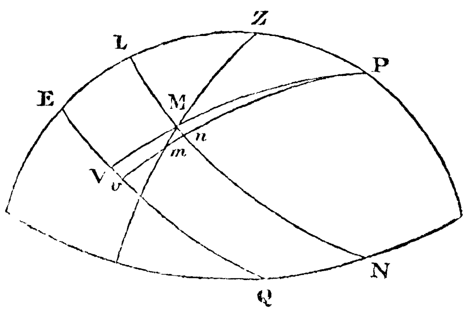
In an observatory, almost all observations that are important, (as it has been more than once said,) are made on the meridian. On such the methods hitherto explained are founded, and, as it has appeared, the calculation of the quantity and law of refraction by Bradley. But, it ought not to be unnoticed, that the refraction may, very conveniently, be computed from observations

* Pliny records a similar fact, Book II. Chap. 13.

made out of the plane of the meridian by the instrument described in Chap. IV. p. 26.

That instrument has an azimuth motion; and by direct observation of a star, its azimuth distance may be determined. Now, refraction takes place entirely in a vertical circle: and such circle is perpendicular to the horizon, along which azimuth is reckoned; consequently, the azimuth of a star is not at all affected by refraction; and therefore, the instrument which, by one operation, determines the altitude and azimuth of a star, determines the latter truly, although not the former, which is greater from refraction than it ought to be: hence, if the altitude of a known star be observed, we have

PM [b] the north polar distance,
PZ [a] the co-latitude,
* PZM [B] the azimuth;



hence, if $\angle PMZ = A$, and $ZM = c$,

we have $\sin. A = \sin. B \times \frac{\sin. a}{\sin. b}$;

and by Naper's Analogies, [see *Trigonometry*, p. 137.]

$$\tan. \frac{c}{2} = \tan. \tfrac{1}{2}[b + a] \frac{\cos. \frac{1}{2}[B + A]}{\cos. \frac{1}{2}[B - A]}.$$

For instance, in latitude 51° 31′ north, the observed altitude and azimuth of a known star (declination = 23° 28′) were observed to be respectively 18° 13′ 5″, and 74° 53′ 30″.

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* *PZ, ZM, PM* are the only lines in the Figure that are wanted for the demonstration.

To find A.

sin. 74° 53′ 30″ - - - - - - - - - - -	9 .9847229
sin. 38 29 - - - - - - - - - - - - - -	9 .7939907
	19 .7787136
sin. 66 32 - - - - - - - - - - - - - -	9 .9625076
sin. A - - - - - - - - - - - - - - - -	9.8162060

$\therefore A$ - - - - - - - - =	40°	54′	56″
since, B - - - - - - - =	74	53	30
$\frac{1}{2}(B+A)$ - - - - - =	57	54	13
$\frac{1}{2}(B-A)$ - - - - - =	16	59	17

To find $\frac{c}{2}$.

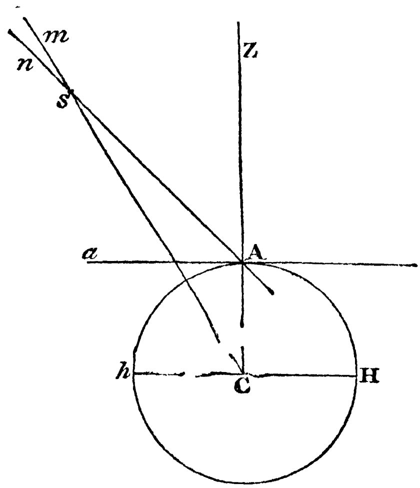
tan. 52° 30′ 30″ - - - - - - - - - - -	10 . 1151503
cos. 57 54 13 - - - - - - - - - - -	9 .7253768
	19 . 8405271
cos. 16 59 17 - - - - - - - - - - -	9.9806240
tan. $\frac{c}{2}$ - - - - - - - - - - - - - - - - -	9.8599031

$\therefore \frac{c}{2}$ = - - - - - - - - - - - - - - -	35°	54′	53″.5
c = - - - - - - - - - - - - - - - - - -	71	49	47
∴. true altitude - - - - - - - - - - -	18	10	13
but observed altitude - - - - - - - =	18	13	5
∴ diff. or the refrac. - - - - - - - =	0	2	52

CHAP. XII.

On Parallax.

If s be a star, C the center of the Earth, Z the zenith of the spectator, then the observed zenith distance of s is the $\angle ZAs$: and the difference of this angle and the angle ZCs, is



$\angle AsC$, called the angle of parallax, [see p. 76].

By *Trigonometry* *,

$$\sin. CsA = \sin. CAs \times \frac{CA}{Cs} = \sin. ZAs \times \frac{CA}{Cs};$$

hence, if CA, Cs, remain unaltered, the sine of CsA, that is, the sine of parallax, varies as the sine of the star's zenith distance.

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* P. 16.

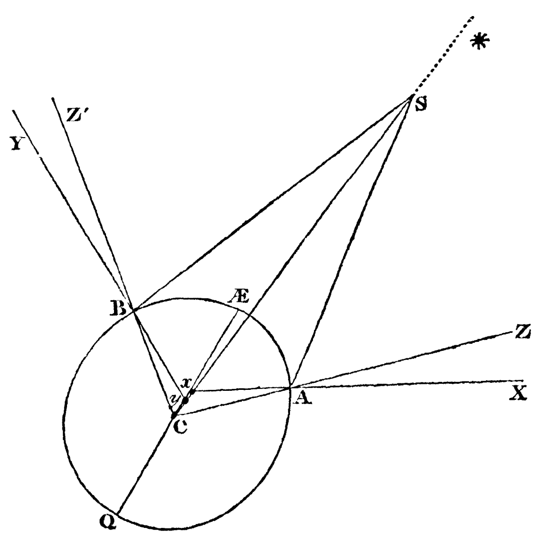
Hence, the parallax must be greater, the greater the zenith distance; it must therefore be the greatest, when the zenith distance is 90°, that is, when the star is in the horizon: let p represent the common parallax, P the greatest, called the *Horizontal* parallax, and let z be the zenith distance; then,

$$\sin. p = \frac{CA}{Cs} \times \sin. z, \text{ and } \sin. P = \frac{CA}{Cs} \times \sin. 90^\circ = \frac{CA}{Cs};$$

and consequently, $\sin. p = \sin. P \times \sin. Z.$

Since the zenith distance can be observed, p and P would be known if the radius of the Earth [CA] and the star's distance [Cs] were known, or their proportion. But, hitherto no method has appeared of finding these quantities: we must therefore either investigate such method, or seek some other means of determining the actual parallax.

Let A, B, be two places on the same meridian of the Earth's surface, that is, which contemporaneously have the same noon: suppose by the methods described in p. 38, their latitudes to be exactly determined. When S is on the meridian,



let its zenith distances ZAS [z], $Z'BS$ [z'] be respectively observed; then since ACB, the sum or difference of the latitudes (in the diagram, the sum) is known, we have

$$\angle ASB = 360^\circ - [180^\circ - z + 180^\circ - z' + ACB]$$
$$= z + z' - ACB;$$

hence the angle ASB, (sometimes called the parallax, being the angle which a chord AB subtends at S,) is known: call this angle A, and the angles CSB, CSA, p', p, respectively.

A is not the angle [see the former Figure] we are seeking: it is, either the angle CSB (p') or the angle CSA (p). Now $\sin. p' = \sin. z' . \frac{CB}{CS}$, and $\sin. p = \sin. z . \frac{CA}{CS} = \sin. z \frac{CB}{CS}$:

hence, $\sin. p'$ or, $\sin. (A - p) = \sin. p . \frac{\sin. z'}{\sin. z}$, and expanding

$$\sin. A . \cos. p - \cos. A . \sin. p = \sin. p \frac{\sin. z'}{\sin. z}:$$

whence dividing by $\sin. A . \sin. p$, and transposing,

$$\cot. p = \cot. A + \frac{\sin. z'}{\sin. z . \sin. A}.$$

This formula may be thus adapted to logarithmic computation:

$$\cot. p = \cot. A \left[1 + \frac{\sin. z'}{\sin. z . \cos. A};\right]$$

let $\frac{\sin. z'}{\sin. z . \cos. A} = [\tan. \theta]^2$; $\therefore \cot. p = \cot. A . [\sec. \theta]^2$;

and consequently,

$$\log. \cot. p = \log. \cot. A + 2 \log. \sec. \theta - 20;$$

θ being determined from

$$\log. \tan. = \tfrac{1}{2} [30 + \log. \sin. z' - \log. \sin. z - \log. \cos. A].$$

From this formula may p be computed; but since, in point of fact, the parallax of all heavenly bodies that are observed is very small, a much simpler formula, and accurate enough for computation, may be exhibited: thus, A, p, p', being very small, are therefore nearly equal their sines; hence, instead of

$$\sin. [A - p] = \sin. p . \frac{\sin. z'}{\sin. z}, \text{ we may write}$$

$$A - p = p . \frac{\sin. z'}{\sin. z}; \text{ whence}$$

$$p = \frac{A \sin. z}{\sin. z + \sin. z'};$$

or, if we wish to express the *horizontal* parallax, since

$$\text{sin. } p = \text{sin. } P \,.\, \text{sin. } z, \quad \text{or} \quad p = P \,.\text{sin. } z,$$

$$P = \frac{A}{\text{sin. } z + \text{sin. } z'};$$

and, if we restore the value of A, making $\angle\, ACB = L \pm L'$

$$P = \frac{z + z' - [L \pm L']}{\text{sin. } z + \text{sin. } z'}.$$

As an example to this formula, we may take the observations of Lacaille, at the Cape of Good Hope, and of Wargentin, at Stockholm:

1751, Oct. 6.

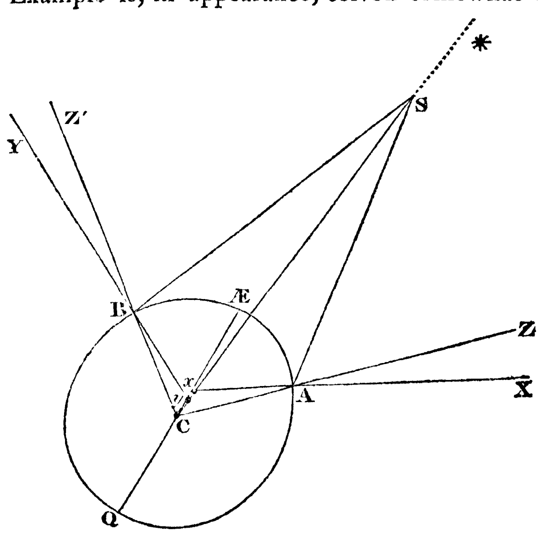
At the Cape, zen. dist. [z] of ♂	25°	2′	0″	- - - sin. z = .4231
At Stockholm, zen. dist. [z']	68	41	6	- - - sin. z' = .9287
$z+z'$ -	93	16	6	sin. z + sin. z' = 1.3518,

Lat. [L] of the Cape (South) - - -	33°	55′	5″
Lat. [L'] of Stockholm - - - - - -	59	20	30
$L + L'$ -	93	15	35

$$\therefore\ z + z' - [L + L'] = 31''$$

$$\therefore\ P, \text{ the horizontal parallax, } = \frac{31''}{1.3518} = 22''.9$$

This Example is, in appearance, solved somewhat differently



by Lacaille: instead of computing the latitudes, he immediately computes the angle *A*: thus, if a star λ♒, were on the meridian with *Mars* [*S*], *Mars* would appear *below* λ♒ to an observer at *B*, or Stockholm; below, in this case by 1′ 26″: it would also appear, to an observer *A* at the Cape, below λ♒, and by 1′ 57″; and the difference of 1′ 57″ and 1′ 26″ is 31″ the angle *A*.

λ♒, whose declination in 1751 was about 8° 50′, in fact, was not on the meridian with *Mars*; therefore, Lacaille says, "*Mars* was below the *parallel* of λ♒": now, where this parallel crossed the meridian, he could easily ascertain by observing the declination of λ; it was simply the place of λ on the meridian.

The two places of observation are the Cape of Good Hope and Stockholm: now, the longitudes of these two places are, respectively 18° 23′ 7″ E., 18° 3′ 51″ E.; consequently, they are not under the same meridian; therefore, a condition of the method [see p. 95. l. 14.] is not preserved: and indeed it is not essentially necessary to preserve it; for, the difference of longitude 19′ 16″, in time, answers to $1^m\ 17^s$: accordingly, *Mars* would be on the meridian of the Cape $1^m\ 17^s$, before he had been on that of Stockholm. If, in that interval, his *declination* had not altered, no correction would be necessary: but, if in 24 hours his declination should have altered 1 minute, then the change of declination due to $1^m\ 17^s$ would be $\frac{60''}{24 \times 60 \times 60} \times 77$, or $\frac{77''}{24 \times 60}$ or .0534″; that is, if *Mars* had been on the meridian at the Cape when observed at Stockholm, the zenith distance instead of being 25° 2′ 0″ would have been 25° 2′ 0″ ± .0534″: hence it appears that it is of no use, in an example like the preceding, to notice the very small correction arising from a difference of longitudes: it also appears that the method itself is applicable, even if the difference of longitudes should be greater than in the example.

By the result of the computation [p. 97. l. 18,] the parallax of *Mars* was found to be about 23 seconds. For planets more distant than *Mars*, the parallax must, it is plain, be less. Hence, for such planets, the above method, although in theory very exact, can practically be of little use. It cannot be relied on: for, when the parallax does not exceed 10 or 12 seconds, the probable errors of observation will bear so large a proportion to it, as materially to affect the certainty of the result. Hence, the method cannot be successfully applied to the Sun, whose parallax is less than 9 seconds : neither to *Jupiter*, *Saturn*, nor the *Georgian* Planet.

The Moon, however, whose parallaxes are considerable, the greatest being 61′ 32″, the least 53′ 52″, and the mean, (or rather the parallax at the mean distance,) 57′ 11″.4, is a proper instance for the method. Yet, with the Moon, the method requires some modification. We must take into consideration, the spheroidical figure of the Earth.

Suppose the meridian *AÆB* not to be circular; then, the produced radii *CA*, *CB*, are not necessarily perpendicular to it, and consequently, *Z*, *Z′* are not the zeniths of the observers at *A* and *B* : but, if *XAx*, *YBy*, be perpendicular to the meridian, or vertical, or in the direction of a plumb-line, then *X*, *Y* are the true zeniths, and the angles *SAX*, *SBY*, are the observed zenith distances : now

$$\sin. ASC, \text{ or, } \sin. p = \frac{CA}{CS} \times \sin. CAS =$$

$$\frac{CA}{CS} \times \sin. \; [SAX - ZAX;]$$

∴ if *z* still represents the angle *SAZ*, it will equal the difference of the zenith distance and the angle contained between the radius and vertical. Hence,

$$\sin. \; p = \frac{CA}{CS} . \sin. \; z, \text{ similarly } \sin. \; p' = \frac{CB}{CS} . \sin. \; z';$$

and hence, if we take, instead of sin. *p*, sin. *p′*, *p* and *p′*,

$$p + p', \text{ or } A = \frac{CA \sin. z + CB . \sin. z'}{CS};$$

and since *P*, the horizontal parallax $= \frac{\text{rad.} \oplus}{CS}$, [p. 95.]

$$P = \frac{\text{rad.} \oplus \times A}{CA . \sin. z + CB . \sin. z'}.$$

Let us take, as an example to this method, the observations of Lacaille and Wargentin, see *Mem. Acad. des Sciences.* Paris 1761 :

1751, Nov. 5.

		Correct.
At the Cape, zen. dist. ☽'s north. limb	56° 39′ 40″	13′ 54″
Parallel of ζ♉ more north than ☽ -	1 46 32.8	
At Stockholm zen. dist. ☽'s limb - -	38 4 52	14 14
Parallel of ζ♉ more north than ☽ -	0 18 37.2	

Hence [see p. 98,] the difference of the quantities in the 2d and 4th line being 1° 27′ 55″.6,

$$A = 1^\circ\ 27'\ 55''.6.$$

Now to find z, z', we must from the zenith distances subtract the corrections 13′ 54″, 11′ 14″, which are the angles between the vertical and the radius. Accordingly,

$$z = 56^\circ\ 25'\ 46'' \text{ - - - - - - } \sin. z = .8332$$
$$z' = 37\ \ 50\ \ 38 \text{ - - - - - - } \sin. z' = .6135$$

$$\sin. z + \sin. z' = 1.4467$$

Hence if we suppose CA, CB equal, we shall have [p. 99. l. 25] the horizontal parallax $= \dfrac{1^\circ\ 27'\ 55''}{1.4467} = 1^\circ\ 0'\ 46''$: the only difference, between this and the preceding method, consisting in the reduction of the zenith distances.

The reduction, or the value of the angle of the vertical, is taken from one of Lalande's Tables, computed for an *Ellipticity* $\dfrac{1}{230}$, and is, in fact, too large.

The expression or formula, from which the table just alluded to is computed, may be easily deduced. It is only requisite to investigate the angle contained between the normal and radius vector, in an ellipse of small eccentricity.

In a sphere, the horizontal parallax $P = \dfrac{CA}{CS}$, and consequently the distance CS remaining the same, the horizontal parallax, whatever be the place of observation, would be the same. In a spheroid,

$$P = \frac{A . \times \text{rad.} \oplus}{CA \sin. z + CB \sin. z'},$$

consequently, the horizontal parallax observed at different places would be different. And with the Moon this is found to be the case : so that, (and there is something curious in the circumstance), this planet which, by her eclipses, shews, in a general way, the Earth to be *round*, by her parallaxes, proves the Earth not to be *spherical* [see p. 20].

The preceding method, by which the parallaxes of *Mars* and the Moon have been determined, is not sufficiently accurate in practice, to determine the Sun's. But that, since it is in Astronomy a most important element, requires the most exact determination: and this it has received from the labour and skill of Dr. Maskelyne, by means of the transit of *Venus*; a method of determination, not immediate and direct, but which infers the quantity required, on the supposition that the planetary motions are known to a very considerable degree of exactness*. The method, then, although very refined and important, would not, in this part of the Treatise, find a proper place.

It is the distance of an heavenly body, as it is clear from pages 22, 95, that causes its parallax to be small: and the Sun's tance is so great, that its parallax, equal to 8″.75, [8″ 81, according to Laplace] cannot accurately be determined by the preceding method [p. 95]. The same method therefore, will not apply to bodies more distant from us than the Sun; neither to *Jupiter*, nor *Saturn*, nor the *Georgian* Planet.

Observation, indeed, shews these bodies to have parallax, but, what are called, fixed Stars to be destitute of it; or, which is the same thing, shews their distance to be infinitely great, [see p. 22.]

The smaller the parallax of a body, the greater is its distance: and if we take, which we may do by reason of its smallness, the parallax for its sine, the mathematical relation between the parallax and distance (d), is

$$d = \frac{\text{rad. } \oplus}{P}.$$

This last expression is not, as it stands, fit for computation. It was deduced from sin. $P = \frac{\text{rad. } \oplus}{d}$, in which the radius is supposed 1. But to a tabular radius r, [see *Trig.* p: 11,] $\frac{\text{sin. } P}{r} = \frac{\text{rad. } \oplus}{d}$: hence,

---

* It is with this, as with many other parts in Astronomy, described in the following passage by the Abbe Lacaille: "Dans l'Astronomie on ne parvient à donner une certaine précision a quelque théorie qu'en revenant incessament sur ces pas et en remaniant tous les Calculs, a mesure que l'on decouvre quelque nouvel element, qui y devoit entrer, ou que l'on perfectionne quelqu'un de ceux qui se compliquent avec les autres." *Mem. de l'Acad.* 1757, p. 108.

$$d = \frac{r}{\sin. P} \times \text{rad.} \oplus, \text{ or } = \frac{r}{P} \times \text{rad.} \oplus.$$

Now, if we wish to compute from this latter expression, since P is to be expressed in degrees, minutes, seconds, &c. we must express the radius r also, in degrees, minutes, &c: and since to a radius 1, the circumference = 2 [3.14159], we have

$$2\,[3.14159] : 360 :: 1 : r = \frac{180^\circ}{3.14159} = 57^\circ.2957795.$$

Hence, the last of the two expressions for d, becomes

$$d = \frac{57^\circ.2957795}{P} \times \text{rad.} \oplus:$$

and from this or the former, $d = \frac{r}{\sin. P} \times \text{rad.} \oplus$, may the distances of heavenly bodies be computed.

If we express the radius r, in degrees, minutes, &c. of French measure (*Trig*. p. 2.), we shall have

$$d = \frac{63^\circ.6619}{P} \times \text{rad.} \oplus.$$

Hence, for the Sun, if P 8″.81, or, in French measure, = 27″.2,

$$d = \frac{57^\circ.2957795}{8''.81} \times \text{rad.} \oplus, \text{ or} = \frac{63\ .6619}{27''.2} \times \text{rad.} \oplus = 23405 \text{ rad.} \oplus$$

For *Mars*, P = 24″.624, or in French measure, = 76″,

$$d = \frac{57^\circ.2957795}{24''.624} \text{ rad.} \oplus, \text{ or} = \frac{63^\circ.6619}{0^\circ.0076} \text{ rad.} \oplus = 8376 \text{ . rad.} \oplus$$

the distance of *Mars* from the Earth at the time of observation.

For the Moon, P=57′ 11″.4, or in French measure, = 1°.059,

$$d = \frac{57^\circ.2957795}{57'\ 11''.4} \text{ rad.} \oplus, \text{ or } = \frac{63^\circ.6619}{1^\circ.059} \text{ rad.} \oplus = 60.1 \text{ . rad.} \oplus.$$

Hence the mean distance of the Moon is about 60 radii of the Earth.

Here, the greatest and least distances are, respectively

$$\frac{63^\circ.6619}{0.99567} \text{ rad.} \oplus, \text{ and } \frac{63^\circ.6619}{1.13714} \text{ rad.} \oplus, \text{ or,}$$

63 . 94145 × rad. ⊕, and 55.98725 rad. ⊕ *.

The general use of parallax, is then, to determine the distances of heavenly bodies: but the special object for which it has been here introduced, is the reduction or correction, which must be made, by means of it, to the observed place of a body; to *prepare*, for instance, an observed altitude of the Moon, for the deducing its declination. Now since, by the principle of the reduction, we imagine a spectator in the center of the Earth, it is plain, from the inspection of the Figure, p. 94, that the place of a planet seen from the surface, must be lower, that is, nearer to the horizon than its place seen from the center: but, this last is assumed to be the true place, or, it is made the place in Astronomical computations: and accordingly, a body seen from the surface must be said to be below its true place, or to be *depressed by parallax*.

This depression takes place in a plane passing through, the center of the Earth, the spectator, and the observed heavenly body; it takes place, therefore, in the plane of a vertical circle. Now, the meridian is a vertical circle; the declination of an heavenly body then, as determined by its meridian altitude [see p. 32,] will be affected by the whole quantity of parallax; but its right ascension, as determined by the time of transit over the meridian, will not be at all affected.

The Example of page 89, may now be still farther corrected, in which, since the zenith distance is employed, the parallax must be *subtracted*: in the second Example, it is *added* to the altitude.

---

* It is plain from the above instances, that it is shorter to compute by the French than by the English expression: for, in the former, we may immediately divide the numerator (63°.6619) by the denominator; which we cannot do in the latter.

EXAMPLE I. [see p. 89.]

Altitude of Sun's upper limb - - -	62°	30′	30″.5	
Error of collimation - - - - - - -	0	0	34.5	
	62	29	56	
Apparent zenith distance - - - - -	27	30	4	sin. .4617
Refraction - - - - - - - - - - - - -	0	0	29	
	27	30	33	
[8″$\frac{3}{4}$ × .4617] *Parallax* - - - - - -	0	0	4	
	27	30	29	
Semi-diameter of the Sun - - - - - -	0	15	46	
	27	46	15	
Latitude of place of observation - -	48	50	14	
Declination of the Sun - - - - - - -	21	3	59	

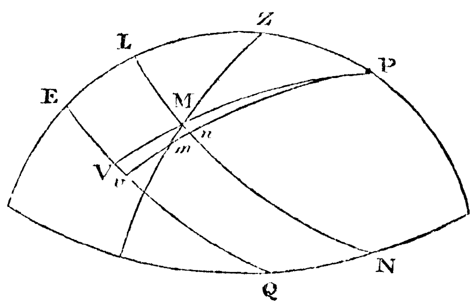
EXAMPLE II.

Altitude of ☽'s upper limb - - -	51°	11′	24″
Refraction - - - - - - - - - - - - -	0	0	45
	51	10	39
[55′ 24″ × .6246] *Parallax* - - - -	0	34	36.2
	51	45	15.2
Semi-diameter - - - - - - - - - - - -	0	15	8.8
Altitude of ☽'s center - - - - - - -	51	30	6.4
Co-latitude of Greenwich - - - - - -	38	31	20
Declination of the Moon - - - - - -	12	58	46.4

In this case, the horizontal parallax for Greenwich is taken = 55′ 24″; and the multiplier .6246 is the natural sine of 38° 39′ 18, which is the zenith distance 38° 49′ 21″ diminished by 10′ 3″, the value of the vertical angle [see p. 99.]

55′ 24″ represents the horizontal parallax for Greenwich, being the parallax on a spheroid at the latitude 51° 28′ 40″, deduced from, what is called, the *Equatoreal* parallax; which is the difference of the Moon's place in the Heavens seen from the equator and the Earth's center: the Moon being in the horizon of the spectator. But this equatoreal parallax is deduced from the equatoreal parallax at the *mean* distance of the Moon *, which according to Mayer, is 57′ 11″.4. There is, therefore, the equatoreal parallax at the mean distance; the horizontal equatoreal at any distance; the horizontal for any latitude, and the common parallax for any altitude: and, in observations of the Moon and in calculations from them, all these circumstances must be attended to.

The quantity of parallax has been computed [see p. 95.] by means of observations made in the meridian; but it may also be computed, as refraction was [p. 92,] by observations out of the plane of the meridian; for in these latter, parallax causing to vary the right ascension, its quantity may be computed from that of the variation. Thus, let M be a planet in its true place,



m in its apparent place, Mm lying in a vertical circle ZMm [see p. 103.] Now, m being the place instead of M, the time from the passage over the meridian, will be represented by the angle ZPm, instead of the angle ZPM: the change therefore, in the time, or in the apparent right ascension of the planet, caused by parallax, is represented by the angle VPv; and this change may be thus estimated: If M were a fixed star, Mm would be nothing, and there would be no parallax affecting the time, or the right ascension: two fixed stars then, that crossed the vertical wire of a telescope in the plane of the meridian, after an interval of t seconds, would also cross the vertical wire of the telescope in a plane, not that of meridian, after the same interval t: but if, instead of one of the fixed

* The Moon's greatest parallax is 61′ 32″; her least 53′ 52″.

stars, we take a planet having parallax, then if the above-mentioned interval were t seconds on the meridian (where parallax does not affect the right ascension,) it could not be t seconds out of the meridian, but, as the figure shews, something more; for instance, $t+\epsilon$ seconds: now ϵ is reckoned, or known, by means of a chronometer; and thence, the horizontal parallax [P] may be computed from this formula

$$P = \frac{15 \times \epsilon \times \text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}},$$

which may be thus proved:

$$\begin{aligned} Vv &= Mn \,.\, \text{sec. } VM = Mm \,.\, \text{sin. } ZMP \,.\, \text{sec. } VM \\ &= P \,.\, \text{sin. } ZM \,.\, \text{sin. } ZMP \,.\, \text{sec. } VM \\ &= P \,.\, \text{sin. } ZP \,.\, \text{sin. } ZPM. \text{ sec. } VM \end{aligned}$$

[for sin. ZM . sin. ZMP = sin. ZP . sin. ZPM *Trig*. p. 102.]

$$\text{Hence,} \qquad P = \frac{Vv}{\text{sin. } ZP \,.\, \text{sin. } ZPM \,.\, \text{sec. } VM},$$

or, since $360° : 24^h :: Vv : \epsilon$; and since sin. ZP = cos. latitude, sin. ZPM = sin. hour angle (h) sec. $VM = \frac{1}{\text{cos. } VM} = \frac{1}{\text{cos. dec.}}$

$$P = \frac{15 \,.\, \epsilon \,.\, \text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}}.$$

This expression applies to the case when the planet and star are observed first on the meridian, and afterwards when they have passed it: if they are observed before they are on the meridian, then a similar expression would obtain for a line $V'v'$ analogous to Vv; and we should have

$$V'v' = \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}}.$$

Hence if the difference ϵ belongs to two observations of the star and planet, the one made to the east, the other to the west of the meridian, we have

$$Vv + V'v', \text{ or } \epsilon \times 15 = \frac{P \text{ cos. lat. sin. } h}{\text{cos. dec.}} + \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}},$$

and accordingly,

$$\begin{aligned} P &= \frac{\epsilon \times 15 \text{ cos. dec.}}{\text{cos. lat.} \times [\text{sin. } h + \text{sin. } h']}, \\ &= \frac{\epsilon \times 15 \times \text{cos. dec.}}{2 \text{ cos. lat.} \left(\text{sin. } \frac{h+h'}{2}\right) \text{cos. } \left(\frac{h-h'}{2}\right)}, \text{ [\textit{Trig}. p. 19.]} \end{aligned}$$

In the preceding investigation it has been supposed, that ε arises solely from parallax: but since, during the observations, the planet will have moved either from, or towards, the star, the noted difference of time, or excess above t seconds, will be compounded of the effect of parallax, and of the time due to the planet's motion, during the interval of the observations.

EXAMPLE.

Aug. 15, 1719. Paris. By the observations of M. Maraldi at 9^h 18^m, *Mars* passed the vertical wire 10^m 17^s after a small star in *Aquarius;* and, 7 hours being elapsed, 10^m 1^s after.

But in this interval (7 hours) *Mars* had approached the star by 14 seconds; that is, had there been no parallax, the former difference of passage, which was 10^m 17^s, would have been reduced to 10^m 17^s — 14^s, or, 10^m 3^s: but by the second observation, the difference of passage is only 10^m 1^s, consequently, the effect of parallax is $(10^m\ 3^s) - (10^m\ 1^s)$, or 2^s: and this is the value to be substituted for ε in the preceding expression: and since, by observations at the time, it appeared that

Declination =	15° 0′ 0″	log. cos. - -	9.9849438
h =	56 39	[log. 15 - -	1.1760913]
h' =	49 15		
$\frac{h+h'}{2}$ =	52 57	log. sin. - -	9.9020628
$\frac{h-h'}{2}$ =	3 42	log. cos. - -	9.9990938
Latitude of Paris =	48 50 12	log. cos. - -	9.8183630

We have, from the logarithmic formula of p. 106,

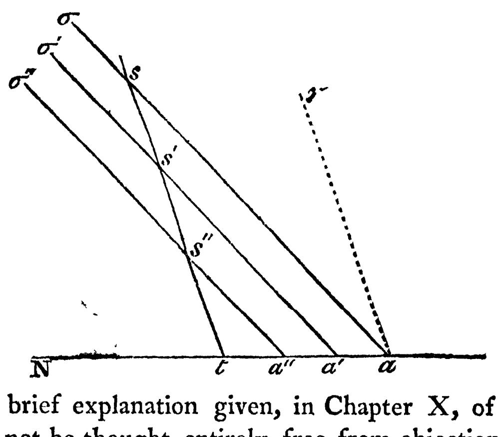
$$\log. P = \log. 15 + 20 + \log. \cos. 15^\circ$$
$$- [\log. \cos. 48^\circ\ 50'\ 12'' + \log. \sin. 52^\circ\ 57' + \log. \cos. 3^\circ\ 42']$$
$$= 1.4415155;$$

∴ P, the horizontal parallax of *Mars*, is 27″.638 [See *Mem. de l'Acad.* 1722; and Lalande's *Astron.* tom. II. p. 356].

CHAP. XIII.

On Aberration.

In Chapter X, p. 78, some explanation was given of this inequality, which was there shewn to arise from the motion of the Earth and the propagation of light. A ray of light descending in the direction $ss's''$, whilst the spectator moved from a through aa', $a'a''$, &c. would, to such spectator, appear in a direction parallel to $a\sigma$.

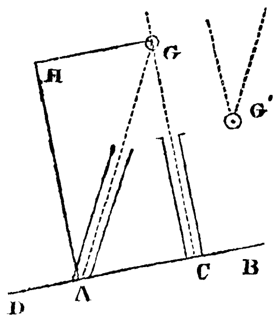


As the brief explanation given, in Chapter X, of the principle, may not be thought entirely free from objection: and, as, indeed, the principle, however established, is somewhat of a refined nature, and remote from common apprehension, it may be proper to elucidate it by one or two illustrations.

That of Clairaut's *, is ingenious and satisfactory. Suppose, G to be one of many drops of rain falling rapidly in the direction

* *Mem. de l'Acad.* 1737, p. 207.

GA. How ought a tube to be held, by a person walking from



C towards *A* and *D*, so that the drops shall descend down its axis?

It cannot be held in the direction *AG*, for then, whilst it was moved from *C* to *A*, the drop would come in contact with the hinder side of the tube. That side of the tube, therefore, must be withdrawn from the direction of the falling drop; and, the quantity through which it must be withdrawn, or its change of inclination from that of *AG*, is to be determined by drawing *GH* parallel and equal to *CA*, and by completing the parallelogram *CGHA*; then, *CG* is the direction in which the tube must be held.

The spectator is supposed to move from *C* to *A*, whilst the drop at *G* falls through *GA*. *CA*, *GA*, therefore, are the relative velocities of the spectator and the drop. Now, if two equal and contrary motions be conceived to be communicated to *G*, the drop *G* will not be affected by them. But, conceive them communicated separately, their quantities being respectively + *GH*, and − *GH*: then, if the latter − *GH*, that is, an impulse from *H* towards *G* be communicated, this combined with the motion *GA*, will cause the drop to take the direction *GC*. By the remaining part + *GH*, the drop will be translated from *G* through *GH*; but the spectator is transferred through an equal space *CA*, in the same time. Since the parts of the system, then, are affected with the same motion and towards the same parts, the drop *G* will fall as if the spectator and tube were at rest; but then it falls, as it has been shewn, in the direction *GC*.

The explanation of the phenomenon has also been attempted, by analogy, from the law of the composition of forces. Light is likened to matter, and conceived to impinge on the eye, and its direction to be determined by the resulting one of impact.

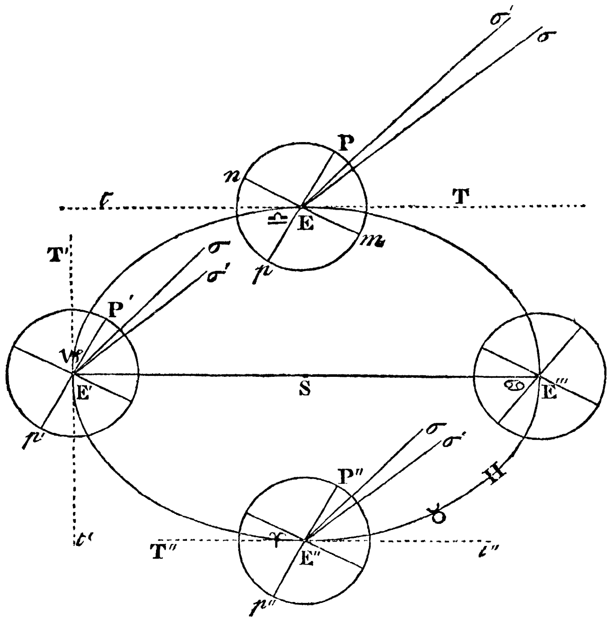
If therefore, the eye at *A*, suffers impact, from the light's force represented by *GA*, and the force of its own motion represented by *AC*, and in that direction, the direction of the resulting force or impact will be *AH*, or *CG*; according to which direction light will be judged to come.

If the principle then be assumed as established, this must follow. The light of a star, from *r*, (Fig. p. 108.) instead of the direction *ra*, will appear, to a spectator moving from *a* towards *N*, to have proceeded in the direction *sa* and if a plane be conceived to be drawn through *ra*, *aN*, the *Aberration* will take place in such plane, and the angle, its measure, will be *ras*. Hence, if *aN* meant as a tangent to the Earth's orbit, be called the Earth's way, then, if the spectator move from *a* to *N*, or towards the star, the direction of the star's light, will, by reason of aberration, appear to form with the Earth's way, an angle *saN* less than *raN*; or the star may be said to be *depressed* towards *aN*: but, the contrary will happen, or the star will seem to be elevated from the line *aN*, the Earth's way, if the spectator move from the star, or from *N* towards *a*.

But, since a star's place is determined from its right ascension and declination, the main object of explanation is, to shew, how the preceding circumstances will affect and modify observations made in the plane of the spectator's meridian. With this object in view, we will begin with the illustration of a simple instance.

Let *S* be the Sun, *E*, *E'*, *E''*, *E'''*, four positions of the Earth, when the Sun is in the signs, ♈, ♑, ♎ ♋; that is, positions at the vernal and autumnal equinoxes, and the two solstices. Let σ be a star, and suppose the Earth to revolve round the Sun according to the order *E E' E'' E'''*. Now, in the position *E*, (the Vernal equinox,) the Sun is in the equator [see p. 15.]; therefore if *Pmp* be a meridian passing through the pole of the equator and the pole of the ecliptic, *Pmp* will coincide with the solstitial colure, and a line drawn from *S* to *E* will be perpendicular to the plane passing through *Pmpn*: therefore, if the plane of the ecliptic

$EE'E''$ be conceived to lie in the plane of the paper, that of $Pmpn$ must be perpendicular to it. SE [conceived to be drawn



between $S\sigma$ and E,] is perpendicular to the tangent Tt (the Earth's orbit being supposed to be circular); and consequently, the line Tt lies in the plane of the meridian $Pmpn$.

The Earth being supposed to turn round its axis in the direction nEm, and SE being perpendicular to the plane $Pmpn$, the meridian $Pmpn$ was opposite to the Sun six hours before it occupied the position represented in the Figure: in other words, $Pmpn$ is the position at six in the evening. A star σ therefore, situated in the solstitial colure, with a right ascension = 18 hours, comes on the meridian at six in the evening: $E\sigma$, since it lies in the solstitial colure, lies, in this case, in the plane of the meridian; so does, Tet the Earth's way. But, by what has preceded [p. 110,] the aberration lies, in a plane passing through $E\sigma$, Et, therefore, in the plane of the meridian; and, since the Earth moves *from* the star σ, σ' the apparent place will be above σ the true place: that is, will be elevated towards P; and if P be the north pole, will to a spectator appear to the *north* of its true place. Moreover, since the aberration takes place in the plane of the meridian, in which plane declinations are estimated, the whole effect of aberration will be to increase the declination of the star, or to lessen its north polar distance.

In the opposite position E'', at the Autumnal equinox, the aberration, as in the preceding case, will take place entirely in declination; but, since the Earth and spectator are moving *towards* the star σ, the star will appear depressed towards $T''E''t''$, or from the north pole P''; in other words, the star will be seen to pass the meridian more southerly: its declination will be lessened, or its north polar distance increased. Since $P''m''p''$ * must revolve through 90° before it is brought opposite the Sun, that is, before noon, the star will be on the meridian at six in the morning.

Since the star in this last position was depressed, and in the former (E) elevated, the star will be more *northerly* in the position E, than in the position E'', by the sum of the two aberrations, at E and E''.

In the second position E', at the Summer solstice, the star will come with the Sun on the meridian $P'm'p'$, that is, will be on the meridian at 12 o'clock. In this position, $T'E't'$ will be perpendicular to the plane of the meridian; therefore, since the aberration takes place in a plane passing through $E'\sigma$ and $T'E't'$, it will take place in a plane perpendicular to that of the meridian $P'm'p'$, and towards $E't'$, that is, to the right or the west of the meridian. But, the aberration being in a plane perpendicular to the meridian, the declination of the star, which is estimated in such plane, will not be at all affected, but solely the right ascension, which will be lessened.

In the opposite position at E''', at the Winter solstice, the aberration, as in the preceding case, will take place entirely in right ascension, which will be increased. In this last position, the star comes on the meridian at midnight.

Hence, recapitulating, a star situated in the solstitial colure with a right ascension equal to 270°, or 18 hours, passes, March 20, the meridian about six in the evening; and, the aberration is wholly in declination, which is increased.

June 21, it passes the meridian about noon, and the aberration is wholly in right ascension, which will be lessened.

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* m', m'' (not expressed in the Figure,) like the point m, are intersections of the small circle, representing the section of the Earth, and of diameters perpendicular to the axes $P'p'$, $P''p''$.

September 23. The star passes the meridian about six in the morning, and the aberration is wholly in declination, which is diminished.

December 23. The star passes the meridian about midnight, and the aberration is wholly in right ascension, which is increased.

A star situated very nearly, as the imagined one, in the preceding illustration, is γ *Draconis*, whose right ascension in 1800 was $17^h\ 52^m$.

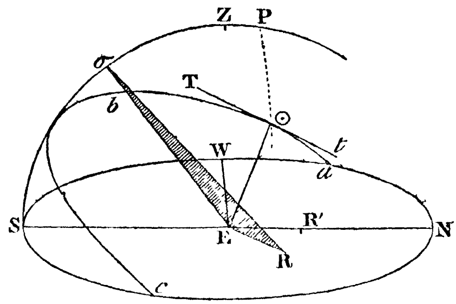
In the preceding illustrations, for the sake of simplicity, a particular star, and particular positions of the Earth, were assumed: and, in such, the aberration was either entirely in declination, or entirely in right ascension. But, in positions intermediate between those that have been assumed, and with the same star, the aberration will not take place in a plane, either coinciding with that of the meridian, or perpendicular to it. In other words, the aberration will be partly in declination and partly in right ascension; or, if we take a star not situated, like the one quoted in the preceding illustration, in the solstitial colure, then, when such a star comes to the meridian, either at six in the morning, or at six in the evening, the aberration is not entirely in declination; and, when it comes, at noon or midnight, the aberration is not entirely in right ascension. Although this may easily be inferred from what will be proved in a general way, in a subsequent part of this Chapter, yet it may aid the Student in his conception of the theory, to establish, by a separate proof, the truth of one of the preceding assertions.

With a view to this end, we will now shew, that the effect of aberration on a star *not* situated in the solstitial colure, at six o'clock, either evening or morning, is partly in declination and partly in right ascension.

Conceive $cSWa$ to be the horizon *, and $cb\odot a$ the ecliptic elevated above it; also S to be the south, W the west, P the pole of the equator, and $\odot$ the Sun at six in the evening, above the horizon, and consequently to the north of the point W. Draw $\odot T$ a tangent to the ecliptic, and ER representing the

* See Figure in p. 114.

Earth's way parallel to it, and in the plane of the ecliptic *cb*☉*a*:



then if σ be the star, the aberration [see p. 110,] will take place in a plane passing through *E*σ, *ER*.

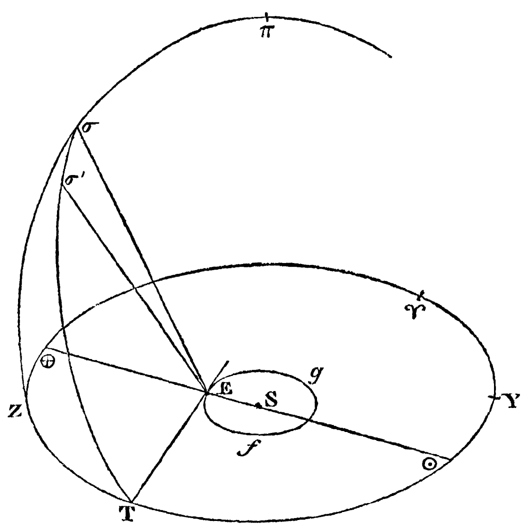
Now, if the star were in the solstitial colure, then, when on the meridian at six o'clock, the Sun would be in the horizon, and the ecliptic, instead of being as in the Figure, would pass through *W*: in that case also *WE* would be perpendicular to a line *ER′*, and since it is perpendicular to *E*σ, it would be so also to a plane passing through *E*σ, *ER′*: but, *EW* is perpendicular to the plane of the meridian; consequently in this case, the plane of the meridian, would coincide with that passing through *E*σ, *ER′*, in which the aberration takes place, and accordingly as it has been before shewn [p. 111. l. 19.] the aberration would take place wholly in the meridian. If, however, ☉ be to the north of *W*, *E*☉ will not be perpendicular to the plane of the meridian, and the plane passing through *E*σ, *ER*, instead of coinciding with the plane of the meridian passing through *E*σ, *ER′*, will be withdrawn from it towards the east. But, the aberration takes place in such plane, and any line representing its effect, may be resolved into two others, one perpendicular to the plane of *SE*σ, representing the aberration in right ascension, the other in that plane and representing the aberration in declination.

The aberration therefore of a star, not in the solstitial colure, which passes the meridian at six o'clock, is not wholly in declination.

There is, however, during the Earth's diurnal rotation, a point of time, at which, the aberration of a star in declination, although not wholly in that direction, would, if on the meridian, be a maximum: another, at which it would equal nothing. There is also, whatever be the star, a day during the Earth's annual revolution, at which the star's aberration in declination (although it may not be wholly in that direction) is a maximum, and another, at which it is equal to nothing. And the like may be affirmed of the aberration in right ascension.

We will now proceed to demonstrate these latter points, and to establish general formulæ, from which the quantity of aberration may be computed.

Let *S* be the Sun, *E* the Earth; *Efg* its orbit; *ZT*♈ that orbit extended to the fixed stars, and in which the signs are



supposed to lie; *ET* a tangent to the Earth's orbit at *E*; ☉ the place of *S* amongst the fixed stars, or in the ecliptic as seen from *E* the Earth; ⊕ the place of *E* the Earth in the ecliptic, as seen from the Sun *S*; σ a fixed star; σT the arc of a circle, (of which the center is *E*) passing through σ and *T*: then, by what has preceded [p. 110] the aberration of a star σ takes place in a plane σET, passing through σE and *ET*; and the Earth moving according to the order *Efg*, and towards *ET*, the aberration may be represented by $\sigma E \sigma'$.

The circle σT, in the Figure, is not a great circle; it would be one, if *E* coincided with *S*. Now this latter condition may be conceived to take place: for, [see p. 22.] the annual parallax of the

Earth's orbit is insensible; in other words, the radius SE of its orbit, with regard to SZ, or ST, (the radius of the imaginary concave in which the stars are conceived to be placed) may, by reason of its smallness, be neglected.

If E then be considered as coincident with S, the arc σT measures the angle σET: hence, since

sin. $\sigma E\sigma'$: sin. σET :: vel^y. of the Earth : vel^y. of light;

and, since the velocities of the Earth and of light may be considered as constant;

sin. $\sigma E\sigma'$, or $\sigma E\sigma'$ [$\sigma E\sigma'$ being very small] $\propto$ sin. σT,

or, the aberration $\propto$ sin. σT: consequently, the aberration is the greatest, when sin. σT is, that is, when σT equals a quadrant, or when σ is in π the pole of the ecliptic.

By observation, the greatest effect of aberration is about $20''$. Hence generally,

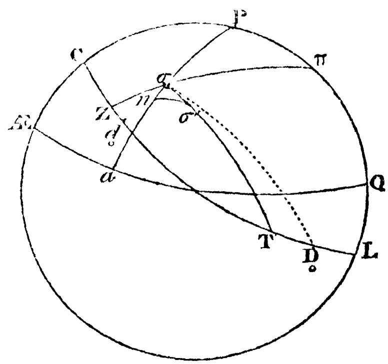
The aberration $= 20''$ sin. σT.

The Earth's orbit being nearly circular, SE is nearly perpendicular to ET: and $\oplus T$ is a quadrant, or T is 90° degrees before the Earth's place seen from the Sun: and if ♈ represents the first point of *Aries*, the longitude of T is ♈ T; and the longitude of the Sun, which, by a spectator on the Earth's surface, is referred to $\odot$, is ♈ $\odot$ = ♈ $T + 90^\circ$.

The aberration $\sigma\sigma'$ is made in the circular arc σT: but, except in particular instances (such as were stated in pp 110, 111,) observations are not made in the plane σET, nor in one perpendicular to it. An observer, who notes the heights of stars in the plane of the meridian, and their transits over it, discerns only the resolved parts of the aberration $\sigma\sigma'$: one part, resolved in the plane of the meridian, and, the other, in a direction perpendicular to it. These are the parts that affect the declinations and right ascensions of stars; and consequently, the enquiry is naturally directed to their investigation.

Let, as before, σ be the star, π the pole of the ecliptic CTL, P the pole of the equator, $ÆQ$, and σT the arc of the circle, in the plane of which, aberration takes place. Then, if σT coincide with πZ, or with Pa, there is, respectively, no aberration perpendicular to πZ, or none perpendicular to Pa: in other words, there is no aberration in longitude, or none in right ascension. If

σT be perpendicular to πZ or to Pa, there is no aberration in the plane of πZ, or none in that of Pa; in other words, there is



either no aberration in latitude, or none in declination. And the determination, on these principles, of the several positions of the point T when the respective aberrations are equal to nothing, is preparatory to the investigation of the formulæ that generally expound the laws of the aberration.

Investigation of the Position of the Point T when the Aberration in Declination = 0.

Let $D_0\sigma$ be perpendicular to $P\sigma a$: then D_0 is the place of T when the aberration in declination is 0. Now T is always 90° before the Earth [p. 116]: let the place of the Earth corresponding to D_0, be d_0; then $D_0 d_0 = 90°$: now, $D_0\sigma Z$ (as the Figure is drawn) is $> 90°$; $\therefore D_0 Z$ is $> 90°$ [see *Trig.* p. 91]; $\therefore d_0$ is between Z and D_0: and in order to compute $d_0 Z$, (since the angle at Z is a right one) we have, by Naper's rule,

$$r \times \sin. \sigma Z = \cot. D_0\sigma Z \times \tan. D_0 Z$$

$$= \cot. [90° + P\sigma\pi] \times \tan. [90° + d_0 Z]$$

$$= \frac{-\tan. P\sigma\pi}{-\tan. d_0 Z} \times r^2. \qquad \text{[see } \textit{Trig.} \text{ p. 12, 13]}$$

Now the angle $P\sigma\pi$ is called [see p. 58,] the angle of position [P] and since σZ is the star's latitude, we have, for computing $d_0 Z$, the Earth's distance from syzygy, this expression

$$* \ \tan. d_0 Z = r . \frac{\tan. P}{\sin. \text{star's latitude}}, \qquad [1]$$

Formula for the Aberration in Declination. (Fig. p. 117.)

Draw $\sigma' n$ perpendicular to $P \sigma a$, then σn is the aberration $[A]$, and we have

$$\begin{aligned} \sigma n &= \sigma \sigma' . \cos. n \sigma \sigma' \\ &= \sigma \sigma' . \sin. D_0 \sigma T \\ &= 20''. \sin. \sigma T . \sin. D_0 \sigma T \ [\text{p. } 116.] \\ &= 20'' . \sin. D_0 T . \sin. T D_0 \sigma \ [\textit{Trig. } \text{p. } 102.] \end{aligned}$$

But, $D_0 \sigma Z$ is a right-angled triangle; ∴ by Naper's rule,

$$\begin{aligned} 1 \times \cos. D_0 \sigma Z, \text{ or, } 1 \times \sin. P \sigma \pi &= \sin. T D_0 \sigma \times \cos. D_0 Z \\ &= \sin. T D_0 \sigma \times \sin. d_0 Z \end{aligned}$$

consequently, σn, or $A = 20'' . \sin. D_0 T \times \dfrac{\sin. P}{\sin. d_0 Z}$.

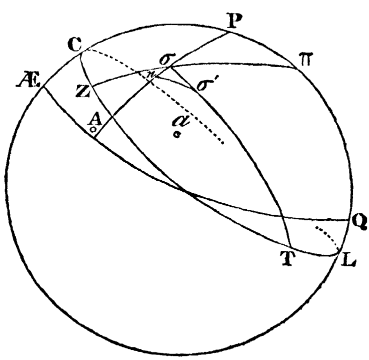
Hence, A is a maximum $[M]$ when $D_0 T = 90°$;

$$\therefore M = 20'' . \frac{\sin. P}{\sin. d_0 Z}, \qquad [2]$$

and $\dagger \ A = M . \sin. D_0 T. \qquad [3]$

Investigation of the Position of the Point T, when the Aberration in Right Ascension = 0.

If σT coincide with $P\sigma$, then A_0 is the corresponding position



of the point T: and a_0 $[A_0 a_0$ being $= 90°]$ that of the Earth. Now, by Naper,

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* This is Bradley's Rule. See *Phil. Trans.* No. 406, p. 650. See also Clairaut *Mem de l'Acad.* 1737, p. 213, and T. Simpson's *Essays*, p. 14.

† See Bradley, *Phil. Trans.* No. 406. p. 650. Clairaut, *Mem. Acad.* 1737, p. 213, and T. Simpson's *Essays*, p. 16.

$$1 \times \sin.\ \sigma Z = \tan.\ A_0 Z \times \cot.\ P\sigma\pi$$

$$= \frac{\cot.\ P}{\cot.\ A_0 Z} = \frac{\cot.\ P}{\tan.\ a_0 Z}.$$

Hence, $$\tan.\ a_0 Z = \frac{\cot.\ P}{\sin.\ \text{star's latitude}}. \qquad [4]$$

Formula for the Aberration in Right Ascension.

Draw $\sigma' n$ perpendicular to $P\sigma A_0$: then the aberration in right ascension [a] equals the angle $n P\sigma'$; therefore

$$a = \frac{n\sigma'}{\sin.\ P\sigma} = \frac{\sigma\sigma'.\sin.\ n\sigma\sigma'}{\cos.\ \text{dec.}}$$

$$= 20''.\sin.\ \sigma T\ .\ \frac{\sin.\ n\sigma\sigma'}{\cos.\ \text{dec.}}: \quad \text{but, [}Trig.\text{ p. 102.]}$$

$$\sin.\ \sigma T \times \sin.\ n\sigma\sigma' = \sin.\ A_0 T \times \sin.\ TA_0\sigma$$

and by Naper,

$$1 \times \cos.\ P = \cos.\ A_0 Z \times \sin.\ ZA_0\sigma$$

$$= \cos.\ A_0 Z \times \sin.\ TA_0\sigma.$$

Hence, $$a = 20''\ \frac{\sin.\ A_0 T}{\cos.\ \text{dec.}} \times \frac{\cos.\ P}{\cos.\ A_0 Z},$$

consequently, a is a maximum (m) when $A_0 T = 90°$;

$$\therefore\ m = 20''.\ \frac{\cos.\ P}{\cos.\ \text{dec.} \times \cos.\ A_0 Z} * \qquad [5]$$

and † $$a = m\ .\ \sin.\ A_0 T. \qquad [6]$$

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* The method used for finding the expressions for the aberrations in declination and right ascension, very nearly resembles one given by M. Lalande, *Astron.* tom. III, 2d Edit. pp. 199, &c. His formulæ too are similar: instead of [5] he deduces [p. 206.]

$$m = 20''.\frac{\cos.\ 23°\ 28'}{\cos.\ \text{dec.} \times \cos.\ \text{dec.}\ A_0};$$

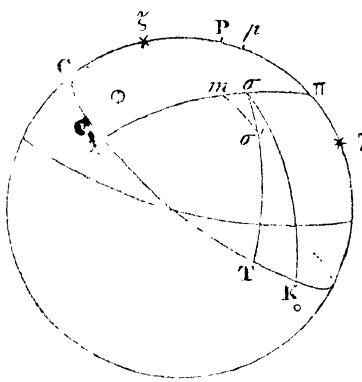
but which is immediately transmutable into [5],

$$\text{since}\ \frac{\cos.\ P}{\cos.\ A_0 Z} = \frac{\cos.\ 23°\ 28'}{\cos.\ \text{dec.}\ A_0}.$$

† See Clairaut, *Mem. de l'Acad.* 1737, p. 225: also T. Simpson's *Essays*, pp. 17, 18, 19.

Investigation of the Position of the Point T when the Aberration in Latitude = 0.

Draw σK_0 perpendicular to $\pi\sigma$, a secondary to the ecliptic;



then σK_0 is the position of σT, and K_0 of T, when the aberration in latitude is = 0.

Now $K_0 Z$ is perpendicular to πZ; and since $K_0\sigma$ is drawn so, K_0 [see *Trig.* p. 90] is the pole of the circle πZ; ∴ $K_0 Z$ is a quadrant; ∴ since K_0 is 90° before the corresponding place of the Earth, the Earth is at Z, or is in syzygy with the star.

Formula for the Aberration in Latitude.

Draw $\sigma' m$ perpendicular to πZ; then σm [= K] is the aberration in latitude,

$$\begin{aligned} \text{and } \sigma m, \text{ or } K &= \sigma\sigma' . \cos. m\sigma\sigma' \\ &= 20'' . \sin. \sigma T . \sin. T\sigma K_0 \\ &= 20'' . \sin. K_0 T . \sin. TK_0\sigma. \end{aligned}$$

But, since K_0 is the pole of πZ, the angle $TK_0\sigma$ is measured by σZ, the star's latitude. Hence,

$$K = 20'' . \sin. K_0 T \times \sin. \text{ star's latitude.}$$

Hence, K, the aberration, is a maximum (N) when $K_0 T$ is equal 90°; that is, when T is in Z or 180° distant from it; or when the

Earth is in *quadratures* [see p. 44,] with the star: the formulæ become then

$$N = 20''. \text{sin. star's latitude} \qquad [7]$$
$$K = N. \text{sin. } K_0 T \qquad [8]$$

Investigation of the Position of the Point T when the Aberration in Longitude = 0. [See Fig. in p. 120.]

This must happen, when σT coincides with σZ: or, when T falls in Z: that is, since T is 90° before the corresponding place [⊕] of the Earth, when the Earth is in *quadratures* with the star.

Formula for the Aberration in Longitude.

$$\text{The aberration } (k) = \angle\, m\pi\sigma' = \frac{m\sigma'}{\text{sin. } \pi\sigma} = \frac{\sigma\sigma' . \text{sin. } Z\sigma T}{\text{cos. } Z\sigma}$$

$$= 20'' . \frac{\text{sin. } \sigma T . \text{sin. } Z\sigma T}{\text{cos. } Z\sigma} .$$

But, since $Z\sigma T$ is a right-angled spherical triangle, by Naper's rule, we have

$$1 \times \text{sin. } ZT = \text{sin. } \sigma T \times \text{sin. } Z\sigma T$$

$$\therefore k = 20'' . \frac{\text{sin. } ZT}{\text{cos. star's latitude}} = 20'' . \frac{\text{cos. } \oplus Z}{\text{cos. star's latitude}} .$$

Hence k is a maximum (n) when cos. $\oplus Z$ is the greatest, that is when $\oplus Z$ either = 0, or 180°: in other words, when the Earth, or Sun, is in syzygy with the star:

$$\text{hence the maximum, or } n = \frac{20''}{\text{cos. star's latitude}}, \qquad [9]$$

$$\text{and } k = n . \text{cos. } \oplus Z. \qquad [10]$$

The preceding formulæ are the foundation of those rules which Clairaut has given at the end of his Memoir in the *Mem. de l'Acad. des Sciences* for 1737; which same rules also, are inserted in Thomas Simpson's *Essays*, as communicated by Dr. Bevis*.

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* Simpson's Essays were published subsequently to Clairaut's Memoir, which Simpson himself (Preface, p. 6,) acknowledges. Clairaut, therefore, must be held as the author of the Rules. Mr. Vince, in the first Volume of his *Astronomy*, has, without their proof, inserted these Rules, and erroneously attributed them to Dr. Maskelyne.

The preceding formulæ also, numerically expounded for particular instances, contain the complete explanation of those phenomena of aberration which Bradley (*Phil. Trans.* No. 406,) has recorded. This, after the solution of certain examples, will be made manifest.

EXAMPLE I.

It is required to find the time when the Star γ Draconis *had no Aberration in Declination.* [For the Longitude, Latitude, and Angle of Position of this Star, see pp. 57. 59.]

The formula of computation is [1] p. 118, and, logarithmically expressed, is

$$\log.\tan.\ d_0 Z = 10 + \log.\tan.\ P - \log.\sin.\ \text{star's latitude}$$

10 + log. tan. P, or 10 + log. tan. 3° 44′ 28″	- -	18.815495
log. sin. star's latitude, or log. sin. 74 57 5	- -	9 984844
log. tan. $d_0 Z$	- - -	8.830651

$$\therefore\ d_0 Z = 3^{\circ}\ 52'\ 25''.$$

But the star's longitude is 264° 9′ 22″; ∴ the longitude of d_0 is 268° 1′ 47″, or 8ˢ 28° 1′ 47″: consequently, when the Sun's longitude is either 8ˢ 28° 1′ 47″, or 2ˢ 28° 1′ 47″, there is no aberration in declination. These two longitudes of the Sun answer to Dec. 20th, and to June 20th.

By p. 118, the maximum (M) of the aberration in declination happens when $D_0 T = 90°$, or $= 3^s$; that is, when the Earth is at D_0; or its longitude is 11ˢ 28° 1′ 47″, or 5ˢ 28° 1′ 47″: or, adding six signs, when the Sun's longitude is 5ˢ 28° 1′ 47″, or 11ˢ 28° 1′ 47″: which longitudes correspond to Sept. 21, and March 19.

EXAMPLE II.

Let it be required to find the Quantity of the Maximum of Aberration in Declination of the same Star.

By formula [2] p. 118,

$$M = 20''\,.\,\frac{\sin.\ P}{\sin.\ d_0 Z},$$

log. 20″ - - - - - - - - - - - - -	1.3010300
log sin. 3° 44′ 28″ - - - - - - - -	8.8145693
	10.1155993
log. sin. $d_0 Z$, or log. sin. 3° 52′ 25″ - -	8.8296625
∴ log. M =	1.2859368

$$\therefore M = 19''.31.$$

Example III.

It is required to find the Aberration (A) of the same Star at any proposed time; Dec. 3, for instance.

By formula [3] p. 118, $A = M . \sin. D_0 T$. Now, Dec. 3, the Sun's long^e. was 8^s 10° 28′; ∴ that of T was 5^s 10° 28′; ∴ $D_0 T$ = 11^s 28° 1′ − [5^s 10° 28′] = 6^s 17° 33′: and the natural sine of 6^s 17° 33′ is .3015 nearly; consequently,

$$A = 19''.31 \times .3015 = 5''.8 \text{ nearly.}$$

Example IV.

It is required to find the Time at which the Aberration of γ Draconis *in Right Ascension is* = 0.

By formula [4] p. 119, $\tan a_0 Z = r \times \dfrac{\cot. P}{\sin. \text{ star's latitude}}$,

log. r + log. cot. P, or 10 + log. cot. 3° 44′ 28″ * -	21.184504
log. sin. star's latitude, or log. sin. 74 57 5 - -	9.984844
log. tan. $a_0 Z$ - - -	11.199660

$$\therefore a_0 Z = 86° \ 23' \ 13''.$$

But, the star's longitude is 264° 9′ 2″, ∴ the longitude of the point a_0 is 177° 45′ 49″, or 5^s 27° 45′ 49″; ∴ when the Sun's longitude is either 11^s 27° 45′ 49″, or 5^s 27° 45′ 49″, that is, either on March 18, or Sept. 21, the aberration of γ *Draconis* in right ascension is = 0.

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* The angles of position of γ *Draconis* and of η *Ursæ majoris* have [p. 59,] been computed for 1725, the time of Bradley's observations, and purposely with the view of explaining and illustrating them.

By p. 119, the maximum (m) happens when $A_0T = 90°$, or 3^s, that is, when the Earth is in A_0, the longitude of which point is $8^s\ 27°\ 45'\ 49''$; ∴ when the Sun's longitude is either $2^s\ 27°\ 45'\ 49''$, or $8^s\ 27°\ 45'\ 49''$; that is, either on June 19, or Dec. 19, the aberration in right ascension of γ *Draconis* is a maximum.

Example V.

It is required to find the Maximum (m) of the above Star.

By formula [5] p. 119,

$$m = 20'' . \frac{r . \cos. P}{\cos. \text{dec.} \times \cos. A_0Z}, \quad \text{[supplying the radius]}$$

log. r + log. 20 - - - - - - - - - -	11.3010300	
log. cos. P, or log. cos. 3° 44′ 28″ -	9.9990736	
	21.3001036	[a]
log. cos. decⁿ., or log. cos. 51° 31′ 49″ -	9.7938609	
log. cos. A_0Z, or log. sin. 86 23 13 -	9.9991359	
	19 7929968	[b]
∴ log. m =	1.5071068	[a]−[b]

∴ $m = 32''.145$, or in time $= 2^s.14$, nearly.

Example VI.

It is required to find the Aberration in Right Ascension of the same Star.

By formula [6] p. 119, $a = m . \sin.\ A_0T$

Longitude of the Sun, Dec. 3, is - - -	8ˢ	11°	30′	
∴ longitude of the point T - - - - - -	5	11	30	
But (see above) longitude of A_0 - - - -	8	27	45	
∴ A_0T - - -	8	13	45	natˡ. sin = .96

∴ $a = 32''.145 \times .96 = 30''.8$ nearly, or in time $= 2^s.05$: which, as it tends to lessen the right ascension, or to bring the star on the meridian before it would come, if there were no aberration, is subtractive and to be noted by —

EXAMPLE VII.

It is required to find the Time when η Ursæ majoris *has no Aberration in Declination.* [For the Latitude, Longitude, and Angle of Position of this Star, see p. 58, 59.]

By formula [1] p. 118, $\tan. d_0 Z = r \frac{\tan. P}{\sin. \text{star's latitude}}$.

log. r + log. tan. P, or 10 + log. tan. 38° 37′ 26″ - 19.9025318
log. sin. star's latitude, or log. sin. 54 23 53 - 9.9101338

$\therefore$ log. tan. $d_0 Z$ = - 9.9923980

$\therefore d_0 Z = 44° \ 29' \ 55''$

but, the star's longitude = 173 3 15

$\therefore$ the longitude of d_0 = 217 33 10, or $7^s \ 7° \ 33' \ 10''$;

$\therefore$ when the Sun's longitude was either $1^s \ 7° \ 33' \ 10''$, or $7^s \ 7° \ 33' \ 10''$; that is, either on April 28, or Oct. 31, the aberration in declination was = 0.

Since the aberration in declination is a maximum when $D_0 T$ is 90°, or 3^s: that fact will happen, either when the Sun's longitude is $4^s \ 7° \ 33' \ 10''$, or $10^s \ 7° \ 33' \ 10''$; that is, either on July 31, or Jan. 28: on the latter day, it will be most southerly; on the former, most northerly.

EXAMPLE VIII.

It is required to find the Maximum (M) of Aberration of Declination in η Ursæ majoris.

By formula [2] $M = 20''. \frac{\sin. P}{\sin. d_0 Z}$,

log. 20 - - - - - - - - - - - - - - - - 1.3010300
log. sin. 38° 37′ 26″ - - - - - - - - 9.7953275

11.0963575
log. sin. 44 29 55 - - - - - - - - 9.8456511

$\therefore$ log. M = 1.2507064

$\therefore M = 17''.81$ nearly.

Example IX.

It is required to find when η Ursæ majoris *has no Aberration in Right Ascension.*

By formula [4]

$$\tan. a_0 Z = r \frac{\cot. P}{\sin \text{ star's latitude}}, \quad [\text{supplying the radius } r]$$

log. r + log. cot. 38° 37′ 26″	- - -	20.0974682
log. sin. 54 23 53	- - -	9.9101338
∴ log. tan. $a_0 Z$ =		10.1873344

∴ $a_0 Z = 57°$ nearly;

but the star's longitude = 173° 3′ 15″;

∴ the longitude of a_0 = 116 3 15, or 3^s 26° 3′ 15″;

∴ when the Sun's longitude is either 9^s 26° 3′ 15″, or 3^s 26° 3′ 15″, that is, on Jan. 26, or July 19, there will be no aberration in right ascension.

These numerical results, with the previous theoretical illustrations, completely explain the phenomena observed by Bradley. According to the first example [p. 122,] γ *Draconis* had no aberration in declination, that is, with respect to its height in the meridian, was stationary, on Dec. 20. Bradley says (*Phil. Trans.* No. 406, p. 639,) that on the 5th, 11th, and 12th, there appeared 'no material difference in the place of the star.' But this is according to the old stile; and the days therefore would now answer to the 16th, 22^d, and 23^d. Again, pp. 122, 123, the maximum of aberration happened in March 19, and its quantity was 19″.368. Bradley says (p. 640,) "about the beginning of March (old stile) the star was found to be more southerly than at the time of the first observation. It now indeed seemed to have arrived *at its utmost limit* southward." Again, by p. 122, the second time at which the star has no aberration is June 20. And, Bradley (p. 640,) says, 'about the beginning of June (old stile) 'it passed at the same distance from the zenith as it had done in December.'

Again, with regard to η *Ursæ majoris*. By Exam. VII, p. 125, it appears that the maximum of declination happens on Jan. 28: on that day the star is most to the south of its true place.

Bradley, p. 658, says it was 'farthest south about the 17th of January; that is, according to the new stile, the 28th of January.

It has been shewn (p. 111, 112), and without the aid of the formulæ [1], [2], &c. that a star situated in the solstitial colure, is farthest north and south, or is at its maxima of declination, when the Sun is in the equinoxes. At that time of the year, such star passes at six o'clock. And this result would follow from the formulæ; for, in

$$\tan.\, d_0 Z = \frac{\tan.\, P}{\sin.\ \text{star's latitude}}$$

$P = 0$, $\therefore\ d_0 Z = 0$, or $= 180°$. But the maximum happens when $D_{11} T = 90°$; that is, when the Sun or Earth is in quadratures with the star. The star therefore being in the solstitial colure, the Sun must be in the equinoxes.

When a star is situated in the solstitial colure, the aberration in right ascension is nothing, when that in declination is a maximum. This *nearly* happens with γ *Draconis*: for its aberration in right ascension is $= 0$ on March 18, (see Example IV, p. 123,) and its aberration in declination is a maximum on March 19. The reason of this is, the proximity of γ *Draconis* to the solstitial colure*. But, for a star, such as η *Ursæ majoris*, not situated near the solstitial colure, the aberration in right ascension is $= 0$ on July 19, [see Example IX, p. 126,] and the aberration in declination a maximum on July 31, [see Example VII, p. 125.]

As far as instances prove then, the aberration in declination is not necessarily a maximum, when that in right ascension is nothing. And, it is no difficult matter to prove the same thing generally.

A star situated in the solstitial colure is farthest north, when it passes the meridian at six in the evening, and farthest south when it passes at six in the morning. γ *Draconis* is nearly in this predicament. But, other stars are not; η *Ursæ majoris*, for instance, has a right ascension $= 204°\ 10'\ 8''$: subtract $90°$; therefore, (equal to 6 hours,) and $114°\ 10'\ 8''$ is the Sun's right

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* In 1725, when Bradley made his observations, it was distant from the solstitial colure about $2°\ 26'$.

ascension, when the star passes at six in the evening: add 90°, and 294 10′ 8″ is the Sun's right ascension when the star passes at six in the morning: these right ascensions of the Sun answer to July 15, and Jan. 12, respectively; but, by Example VII, [p. 125,] the star was farthest north, July 31, and farthest south, Jan. 28: that is, was not farthest north and south when it passed at six in the evening and six in the morning. This is agreeable to what Bradley says, p. 644. 'I have since discovered, that the maxima in most of these stars do not happen exactly when they come to my instrument at those hours.'

The time, at which a star is on the meridian when its aberration in declination is a maximum, is easily determined: thus, with η *Ursæ majoris* the maximum happens July 31, and Jan. 28. On those days, the Sun's right ascension was $8^h\ 38^m\ 53^s$, and $20^h\ 40^m\ 40^s$; but the star's right ascension is 204° 10′ 8″, or $13^h\ 36^m\ 40^s$;

$$\therefore \text{ since } 13^h\ 36^m\ 40^s - (8^h\ 38^m\ 53^s) = 4^h\ 57^m\ 47^s,$$
$$\text{and } 20\ \ 40\ \ 40 - (13\ \ 36\ \ 40) = 7\ \ \ 4\,*$$

The star η *Ursæ majoris* passed farthest north July 31, in the evening at $4^h\ 57^m\ 47^s$: and Jan. 28, farthest south, in the morning about $4^h\ 57^m$ before noon, or about 3 minutes after 7 o'clock.

No Examples have been given to the formulæ [7], [8], [9], [10], of the aberrations in latitude and longitude: and indeed the formulæ themselves might have been omitted. They have been inserted rather because they are usually given in Astronomical Treatises, than from any perception of their utility. The practical Astronomer, who wants corrections for his observations, has little need of them; for the latitudes and longitudes of stars are not *observed* †, but computed (see p. 56,) from right ascensions and declinations. The Tables of aberration are accordingly adapted to correct these latter quantities.

The *Arguments* of these Tables (the quantities with which you must enter the Tables to find the numbers sought) are the Sun's right ascension (or the day of the year) the star's declination and right ascension. Now, the formulæ [1], [2], &c. do not ex-

* These results are not exact; but sufficiently so, for the purpose for which they are intended.

† Except such stars be in the solstitial colure.

pound the aberrations by those quantities; therefore, they are not so proper for the construction of tables, as the formulæ of Delambre and Cagnoli.

These latter formulæ express the aberration in terms of the right ascension and declination. But, to balance this advantage, they are rather difficult of investigation, and not neatly expressed. Those which we have used [p. 118,] are indebted for their compactness, principally, to the angle of position P; which, analytically speaking, is an *involved* expression, or an undeveloped function of the right ascension and declination. But, as we have seen, in order to compute the quantity of aberration, we must, as a previous and preparatory step, compute P: the same cause, therefore, that gives to the expressions analytical neatness, impedes their numerical exhibition.

In the formula, the numerical coefficient 20″ has been used, and [p. 116,] that was said to be the value of the greatest aberration, or the aberration of a star situated in π the pole of the ecliptic. In fact, however, Bradley did not determine such quantity of aberration, by actual observation of a star situated in the pole of the ecliptic, but, otherwise, thus: In formula [2],

$$M = 20''.\frac{\sin. P}{\sin. d_0 Z},$$

instead of 20″, substitute x an unknown quantity, and to be determined; then,

$$x = M.\frac{\sin. d_0 Z}{\sin. P}.$$

Now M is known by actual observation, and $\sin. d_0 Z$, $\sin. P$ may be computed [see pp. 59, 117]; ∴ x may be determined; for instance, by observation, the greatest aberration of γ *Draconis*, from the most northward to the most southward point, was found to be 39″: and since this is double the aberration from the true place,

$$2x = 39'' \times \frac{\sin. 3^\circ\ 52'\ 25''}{\sin. 3^\circ\ 44'\ 28''}, \qquad \text{[see p. 122.]}$$

Hence, since

log. 39 - - - -	1.5910646
log. sin. 3° 52′ 25″ - - - -	8.8296625
	10.4207271
log. sin. 3° 44′ 28″ - - - -	8.8145693
∴ log. $2x$ =	1.6061578

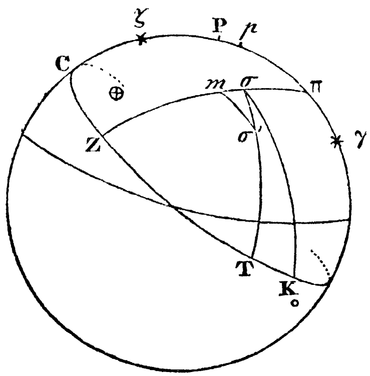
$$\therefore\ 2x = 40''.3, \text{ and } x = 20''.15.$$

And, after this manner, from the observed maxima of the aberrations in declination of other stars, Bradley found the mean value of x to be nearly 20″: as appears from the following Table:

Stars.	Values of M.	Values of $2x$.
α *Persei*	25″	40″.2
α *Cassiopeæ*	34	40 . 8
β *Draconis.*	39	40 . 2
Capella.	16	40
η *Ursæ majoris.*	36	40 . 4

The first and principal star in Bradley's observations was γ *Draconis*; and, the reason of its selection was its proximity, when on the meridian, to the zenith of the observatory. A star observed, as it was, within 2 or 3 minutes of the zenith, would be scarcely at all (by a small fraction of a second) affected by refraction: which, in zenith distances beyond 36°, would exceed the whole quantity of the aberration, and consequently, would, by being mixed and confounded with it, have considerably impeded the disentangling the quantity and law of the latter inequality.

The history of this discovery, one of the most curious and interesting in Astronomical science, resembles the histories of many other discoveries. It was not soon found out, nor immediately suggested. Many fruitless trials and erroneous conjectures preceded it. Bradley devised several hypotheses for the explanation of the phenomenon he had discovered. A *Nutation* of the Earth's axis, or an inclination of its position, naturally suggested itself.



In September *γ Draconis* was more northerly, that is, nearer to the north pole, than it had been in the preceding June: might not then the pole *P* have shifted its place from *P* to *p*? if it had so shifted, then this must happen: the north polar distance of a star *ζ*, situated also in the solstitial colure, but in an opposite part of it, that is, differing from it in its right ascension by 180°, would, instead of being *Pζ*, be increased to *pζ*, and precisely by the quantity *Pp*. Now what was the fact? The north polar distance of *ζ*, or *Pζ*, was found to be increased, *but not by the quantity Pp*, that, by which the north polar distance of *γ* had been diminished, but, by about half that quantity. This, therefore, was quite decisive against the hypothesis of a nutation of the axis, or of a shifting of the pole from *P* to *p*.

But, on Bradley's last hypothesis, that which has been propounded as the true one, is the phenomenon, just mentioned, explicable? The star *ζ* was one in the constellation of *Camelopardalus*, with a north polar distance equal to that of *γ Draconis*; its co-latitude, therefore was equal to the obliquity of the ecliptic + north polar distance, that is, it was about 62°, and its latitude accordingly, would be 28°. Therefore since the latitude of *γ Draconis* [see p. 57.], is 74°: and the maximum $(N) = 20'' \times \sin.$ star's latitude: hence,

N (γ *Draconis*) : *N* (ζ *Camelopardali*) :: sin. 74° : sin. 28°
:: 9612 : 4694
:: 2.04, &c. : 1

which result agrees with the observed phenomenon; and accordingly, Bradley's theory explains it.

It seems scarcely necessary to mention that the principle of aberration will affect the apparent places of the Sun and planets as well as those of the fixed stars. The Sun, for instance, must always appear behind his true place by the same quantity of aberration, which is 20″*: hence, that the computed place may agree with the observed, we must compute the former not for the instant of observation, but for that which precedes it, by the time elapsed during the passage of light, from the Sun to the Earth.

The Sun has been said to be constantly behind his true place by the same quantity 20″. This is not strictly true, except the Earth's velocity be constant, which it is not. A small correction therefore is due to the variation of the velocity, or, (as it is expressed in Astronomical Tables) to the eccentricity of the Earth's orbit; for, if there were no eccentricity, the velocity would be constant.

The principle of aberration having now been explained, and the law of its variation expounded by formulæ, we proceed to shew its use and application as a *correction* to Astronomical observations.

Certain stars, it has been observed, [p. 53.] (those, which situated near the equator, move most quickly) are used to correct and regulate Astronomical clocks and chronometers. When a star, *α Serpentis*, for instance, is on the meridian, the clock denotes a certain time. If the clock denoted time truly, it would express the right ascension of *α Serpentis*. By previous observations and computations founded on them, this latter quantity is known. Tables are constructed, which, after the application of certain cor-

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* For n, formula [9] p. 121 $= \frac{20''}{\text{cos. ⊙'s lat.}} = 20''$,

and k, formula [10] p. 112, $= n \times \text{cos. } 180° = n = -20''$.

rections, assign the star's mean right ascension. But, the star observed on the meridian is in its *apparent* place. It is affected by aberration. In consequence of that, it may seem to be on the meridian before it really is, or after it has passed it. The time of the year regulates that circumstance. Hence, to compare the observed right ascension, which the clock denotes, with the computed, we must make, either, both to be mean, or both to be apparent. We must either *add* the aberration to the *mean* tabulated right ascension, or *subtract* it from the *observed* and *apparent*. The former is the usual mode, and Tables are constructed accordingly: that is, besides the corrections (such as precession, nutation, &c.) due to the real change of the star's place, they express the correction due to the aberration, which is an optical inequality: so that, in fact, the star's right ascension computed from the Tables is the *apparent* right ascension at the time of observation.

The following instance will illustrate the preceding explanation:

April 30, 1810,				
α *Serpentis*, by obser. on the merid. at - - - -	15^h	35^m	55^s	[1]
right ascension of α *Serpentis*, by Tables - - -	15^h	34^m	$55^s.45$	
Aberration in right ascension on April 30 - -	0	0	1.25	
∴ apparent right ascension on April 30 - -	15	34	56.7	[2]
[1] − [2] - - - - - - - -	0	0	58.3	

consequently, the error of the clock is $58^s.3$.

As the preceding instance was intended solely to illustrate the use of the aberration in right ascension as a correction, other corrections, such as precession, nutation, &c. are not specified in it.

We will now explain the use of the aberration in declination in correcting observations.

The difference of the distances of the same star from the zeniths of two places, is the difference of the latitudes of those places, if the star be either north or south of both zeniths [see p. 7.]. If north of one, and south of the other, then the sum of the distances is the difference of the latitudes. If the star be observed on the same day by two observers, then, since the aberration would equally affect each observation, no corrrection, beyond that of refraction, would be necessary. The zenith distances might be immediately added or subtracted. But, which generally is the case, if we make an observation in one place, and avail ourselves of an observation made previously in another, then this latter will need correction. In the interval between the two observations, or, in the interval between the actual observation, and the epoch at which the star's place is registered in Tables, the star with respect to the pole, and consequently to the zenith, will have changed its mean place: it must therefore, by the means of Tables be *brought up* from its tabulated place, to its mean place at the time of observation. But, at that time, from the effect of aberration, the observed star is either seen to the north or the south of its true place. The quantity of deviation therefore, or the aberration in declination, must be either added to or subtracted from the place of the observed star; or, subtracted from and added to the place of the tabulated star. The latter is the usual mode, by which, accordingly, the *apparent* and not the *mean* zenith distances of stars are compared. The following instance will illustrate the preceding explanation:

May 10, 1802, Blenheim Observ. apparent zenith distance (north) of γ *Draconis* - - - - - - -	0° 19′ 44″.59
1802. Greenwich *mean* zen. dist. (south) - - -	0 2 16.65
Aberration to May 10 - - - - - - - - - - - - -	0 0 12.58
May 10, 1802, *Apparent* zenith distance of γ *Draconis* at Greenwich - - - - - - - - - - - - - - - - - -	0 2 4.07
∴ sum of zen. dist. or difference of latitudes * -	0° 21′ 48″.66
and since latitude of Greenwich Observatory - - -	51 28 40
Latitude of Blenheim - - - - - - - - - - - -	51 50 28.66

In a similar way, may the difference of the latitudes of places be determined, if, instead of a recorded observation and one actually made, we use two recorded observations. Thus, we may determine the difference of the latitudes of Cambridge and Greenwich, by means of a zenith distance of γ *Draconis* made, in the former place, June, 3, 1790, and of a zenith distance of the same star made in the latter, Jan. 5, 1797. The two observations, by applying, with other corrections, that of aberration

* The *aberration* is additive to the north polar distance; ∴ since γ *Draconis* is north of the zenith, *subtractive* of such zenith distance.

may be *reduced* either to June 3, 1790, or to Jan. 1797, or both may be reduced to some other; for instance Jan. 1, 1790, or Jan. 1, 1800.

In the two preceding instances, the selection ofproper stars is regulated by different principles. In the first, where the error of the clock was to be detected, α *Serpentis* was chosen; because that star is situated near the equator, and moves with considerable velocity: in the second, where the latitude was to be determined, γ *Draconis* was chosen, because that star, when on the meridian, is near the zenith, and consequently, little affected by refraction.

In the preceding explanation and deduction of formulæ no reference has been made to the form of the apparent path, which a star may, by aberration, seem to describe. Whatever that form be then, it is no *condition* in the investigation. The law of the variation of aberration, whether in right ascension, or in declination, is not founded on it: which law in a certain sense, therefore, is independent of the form: that is, in order to be expressed or expounded, it does not require the form to be either circular, or elliptical. Yet, connected with the law, is the form: the former being established, the latter cannot be varied at pleasure: still, the enquiry into the form, since the theory is complete without it, is one of pure curiosity and speculation: but, which, on those grounds, will lead us to one or two theorems of singular geometrical elegance and beauty.

By p. 110, the aberration always takes place in a plane passing through σE, ET*. But, T in the course of a year is carried through the circle ZTY; therefore, if we conceive $E\sigma$ to remain parallel to itself, (which it may be conceived to do, by reason of the relative smallness of ES) $E\sigma'$, will in a year generate round $E\sigma$ a conical surface.

Draw $r\sigma$ parallel to ET; then, by p. 116,

$$r\sigma : E\sigma :: \text{velocity of the Earth} : \text{velocity of light}.$$

Now, the latter is assumed to be constant, and if the first be, then $r\sigma$ is so also; that is, during the revolution $r\sigma$ will describe a

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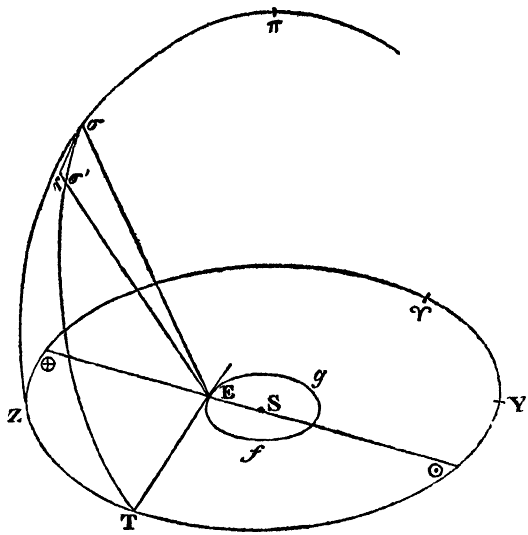
* See Fig. in next page.

circle, parallel to the plane in which ET is, or parallel to the



plane of the ecliptic. This circle may be considered as the base of the conical surface described by Er.

Since $E\sigma$ is not necessarily perpendicular to the plane of the ecliptic, and consequently not so to the plane of the circle described by $r\sigma$, the generated surface belongs to that species of cone which is called oblique.

This is merely geometrical: the spectator sees no circle. The star always appears to him in the direction of $E\sigma'$, and he constantly refers σ' to the imaginary concave surface of the heavens to which $E\sigma$ is perpendicular: consequently, since the intersection of the oblique cone by the concave surface, or by a tangent plane at σ, is an ellipse *, the star, during the year, will constantly appear to be in the circumference of such curve.

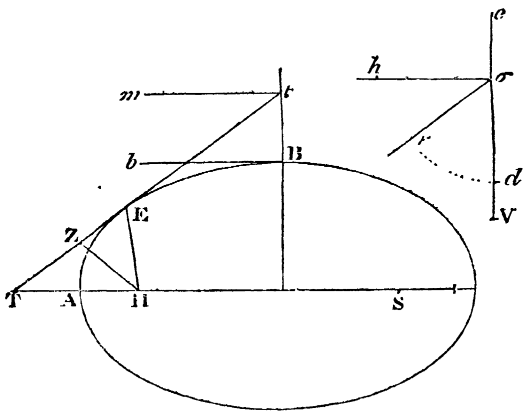
In one case, when σ is situated in π the pole of the ecliptic, the star's apparent path will be circular, for, then, $E\sigma$ will be perpendicular to the plane of the ecliptic, and the conical surface

* The intersection of an oblique cone by a plane not parallel to the circular base of the cone, and not a *sub-contrary* section, is an ellipse.

generated by $E\sigma'$, will belong to a *right* cone, or a cone of revolution.

This is sufficiently plain, if σr be constant, or if the Earth's velocity be constant. But, if we suppose, which is the case in nature, the Earth's velocity to vary, what then will be the imaginary curve which σr describes, or, what will appear to be the curve of aberration of a star situated in π the pole of the ecliptic? It is a curious result, that, in this, as well as in the preceding simple case, the curve is a circle.

Let E be the Earth, in her elliptical orbit; S the Sun in one focus, and let H be the other focus, HZ a perpendicular to



TEt, a tangent at E. Draw from σ the star, σh parallel to Bb, and σr to TEt; and take σr proportional to the Earth's velocity at E.

Since the $\angle h\sigma r = \angle mtT = \angle ZTH$; $\therefore$ the complement of $h\sigma r$, or $\angle r\sigma V = \angle THZ$, the complement of $\angle ZTH$, in other terms, σr, HZ make equal angles with σV, HT. Moreover, the Earth's velocity varies inversely as a perpendicular from S on the tangent TEt, or, by Conics, directly as HZ: but, σr varies, as the Earth's velocity, and therefore as HZ. Hence, σr varying as HZ, and revolving towards σV with the same angular velocity as that with which HZ revolves towards HT, r and Z must describe similar curves: but [Vince's *Conics*, p. 17. Edit. 1781.] Z describes a circle, consequently r does.

At the point A, HZ is the least, and the angle $THZ = 0$; therefore in the line σV, the aberration σd is the least, and consequently there perpendicular to the circle. In the opposite part of the line σe is the greatest and also perpendicular to the circle.

Hence the center of the circle is in the line de, and its distance from σ is equal to $\frac{\sigma e - \sigma d}{2}$.

These latter propositions, in some treatises, are first established, and then become the foundation of the formulæ of aberration. According to the view which we have taken of the theory, they are not essential to it, of no use to the practical Astronomer, and are merely speculative and mathematical. Their excellence, however, as such, has been the cause of their present introduction.

CHAP. XIV.

Precession of the Equinoxes.

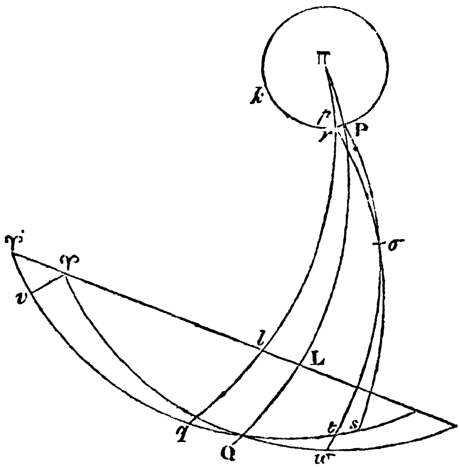
According to the order observed in the preceding pages, the precession of the equinoxes was first considered as an Astronomical fact, and afterwards as an effect arising from a physical cause.

In this latter view, the intersection of the equator and ecliptic is continually regressive; or, from day to day, is moving towards the west, that is, contrary to the order of the signs. This mode of conceiving the precession is essential to Astronomical purposes; since, without it, observations of the declination and right ascensions of stars would not receive their proper corrections. Thus, let π be the pole of the ecliptic ΥlL, P the pole of the equator ΥQw, and $\Upsilon\Upsilon'$ the precession for any time *; then πPQ, which passes through the poles of the equator and ecliptic, is the solstitial colure: but when, in consequence of the precession, $\Upsilon' q$ becomes the position of the equator, then p is its pole, and πpq the solstitial colure, the pole P having moved through an arc Pp, subtending the angle $P\pi p$ the measure of which is $\Upsilon\Upsilon'$. Now, π remaining the same, the latitude of a star σ is not altered by the precession: its longitude is increased by $\Upsilon\Upsilon'$; its right ascension also is increased, since, instead of ΥQw it is $\Upsilon' q s$; the north polar distance, instead of $P\sigma$ becomes $p\sigma$, and according to the construction of the diagram, is increased.

Hence, it is plain, that if, as in the Examples of pp. 133, 134,

* See Fig. in following page.

we employed the tabulated right ascensions and declinations of



stars, without correcting them for the effect of precession, we should have erroneous conclusions.

We must now, from the quantity of precession, and an invariable obliquity, or inclination of the planes of the equator and ecliptic, compute the variations in right ascension and declination.

Precession in North Polar Distance.

$$\begin{aligned} pr\ [= p\sigma - P\sigma \text{ nearly}] &= Pp \,.\cos.\, Ppr \\ &= Pp \,.\sin.\, qps \\ &= \angle\, P\pi p \,.\sin.\, \pi P \,.\sin.\, qps \\ &= \Upsilon\Upsilon' \,.\sin.\, \pi P \,.\sin.\, qps. \end{aligned}$$

Now, the measure of the angle qps, is $qs = \Upsilon' qs - \Upsilon' q =$ ✳'s right ascension $- 90°$, and πp [see p. 64,] is the measure of the obliquity $[I]$. Hence, if $\Upsilon\Upsilon' = 50''.34$, we have

the annual precession in north polar distance
$= 50''.34 \sin.\ I \times -\cos.$ ✳'s right ascension;

and if we suppose the precession to be equably generated, or to be proportional to the time $[t]$, we have, putting for $\Upsilon\Upsilon'$, $50''\ 34\ t$,

the precession $= 50''.34\ t \times \sin.\ I \times -\cos.$ ✳'s R. A.

In this expression, t is a fraction: for instance, in finding the precession from January 1 to April 30, $t = \frac{120}{365}$.

In the Figure, as constructed, the star's right ascension is $> 90°$, and then cos. right ascension is negative; therefore $-$ cos. right ascension is positive; consequently pr, the quantity by which the north polar distance is increased, is positive. The same happens if the star's right ascension greater than $90°$, or 6^h, be less than $270°$, or 18^h.

Hence, in the form of Rules,

When the right ascension (R. A.) is between $90°$, and $270°$, or expressed in time, between 6 and 18 hours, the north polar distance (N.P.D.) is increased by the effect of precession.

When the right ascension of ✱ is between $0°$ and $90°$, or between $270°$ and $360°$; that is, expressed in time, between 0^h and 6^h, or between 18^h and 24^h, the north polar distance is diminished.

$$\text{Since } pr = 50''.34 \times t \times \sin. I . \cos. \text{R. A.} \quad [d]$$

the precession in N.P.D. is greatest, (and $= 20''$) when cos. right ascension is; that is, when R. A. $= 0$, or $180°$; that is, when the star is situated in the equinoctial colure: and least, when R. A. $= 90°$, or $270°$: that is, when the star is situated in the solstitial colure.

Precession in Right Ascension.

By the effect of the precession $\gamma\gamma'$, the right ascension instead of being $\gamma Q w$ becomes $\gamma' q s$: now

$$\gamma' q s = \gamma' v + vt + ts$$
$$= \gamma' v + \gamma Q w + ts;$$

consequently, the change, or variation in right ascension, is

$$= \gamma' v + ts.$$

Now $\gamma' v = \gamma\gamma' \cos. I = 50''.34\, t \times \cos. I = 50''.34\, t \times \cos. 23° \; 28'$,

$$\text{and } ts = Pr \times \frac{\sin. s\sigma}{\sin. P\sigma} = Pr \times \frac{\cos. P\sigma}{\sin. P\sigma} = \frac{Pr}{\tan. P\sigma}$$

$$= \frac{Pp . \sin. Ppr}{\tan. P\sigma} = \gamma\gamma' \times \sin. \pi P \times \frac{\sin. Ppr}{\tan. P\sigma}$$

$$= 50''.34\, t \times \sin. I \times \frac{\sin. ✱\text{'s . R. A.}}{\tan. ✱\text{'s N. P. D.}}.$$

Hence, the whole precession in right ascension, is

$$= 50''.34\, t \left[\cos. I + \sin. I \times \frac{\sin. *\text{'s R. A.}}{\tan. *\text{'s N. P. D.}}\right],$$

or $\left[\text{since tan. N. P. D.} = \text{cotan. dec}^{n}. = \frac{r^2}{\tan.\text{dec}^{n}.}\right]$

$$= 50''.34\, t\, [\cos. I + \sin. I . \sin. *\text{'s R. A.} \times \tan. *\text{'s dec.}]$$

In which expression, the first part, $50''.34\, t \cos. I$ (the value of $\Upsilon' v$) is common to all stars.

If $t = 1$, the formula represents the *annual* precession in right ascension.

If the star be situated in the equator, the declination and its tangent = 0; consequently the annual precession in right ascension equal to

$$50''.34 . \cos. 23^\circ\, 28', \text{ that is, } 46''.17$$

is reduced to that part which is said to be common to all stars.

The preceding Figure is so constructed, that the star σ is nearer to the pole of the equator than to that of the ecliptic; consequently it is situated within the first 180°, or 12^h, of right ascension. For a star situated in the last 180°, or 12^h, the construction will be similar, but ts will be subtractive: the second term of the preceding formula, then, must be subtractive, or affected with a negative sign.

When the second term is negative, it may exceed the first, and then the annual precession in right ascension will be negative. This cannot happen with any of the 36 principal stars in Dr Maskelyne's Catalogue; for, amongst the last 20 stars whose right ascensions exceed 12^h, the star of the greatest declination is *α Lyræ*. Its declination is 38° 36′ 25″, and consequently, the tan. decl^n. $<$ rad. < 1 : therefore since $\cos. I > \sin. I$,

$$50''.34\, [\cos. I - \sin. I . \sin. *\text{'s right ascension tan. decl}^n.]$$

cannot be negative, even if we were to put an extreme case, and make sin. *'s right ascension = 1. The sine of *α Lyræ's* right ascension differs very little from 1 (radius), being the sine of $18^h\ 30^m\ 13^s$; and this circumstance, joined with the magnitude of its declination, is the cause why its precession in right ascension is less than that of any of the other 35 stars. [See Catalogue in Appendix.]

In Wollaston's Catalogue of Circumpolar Stars, there are abundant instances of stars, whose annual precessions in right ascension are negative.

But, if there are some stars with positive precessions in right ascension, and others with negative, there must be certain stars either so exactly, or nearly so, situated, as not to be affected in their right ascensions by the precession of the equinoxes. With such stars then cos. I – sin. $I \times$ sin. R. A. $\times$ cot. N. P. D. must $= 0$, or sin. right ascension $=$ tan. north polar distance $\times$ cot. I, and consequently, the angle of position must equal a right angle. For let, in a spherical triangle $P\sigma\pi$ (where P, π are the poles of the equator and ecliptic, and σ the star), the angle of position, $P\sigma\pi = 90^\circ$; then, by Naper's rule,

$$r \times \cos.\ \sigma P\pi = \tan.\ \sigma P \times \cot.\ \pi P;$$

that is, $r \times$ sin. R. A. $=$ tan. N. P. D. $\times$ cot. obliquity.

Hence, it is easy to assign the relative longitudes and latitudes of such stars, as are not affected in their right ascensions, by the precession of the equinoxes. For, by Naper,

$$r \,.\, \cos.\ P\sigma\pi = \cot.\ P\pi \times \tan.\ \sigma\pi,$$

or, r . sin. longitude $=$ cot. $23^\circ.28'$ $\times$ cot. latitude.

For instance, take the longitude $= 10^\circ$; then,

$$\begin{array}{rl} \log. r + \log. \sin.\ 10^\circ\ 0' \text{ - - - } = & 19.2396702 \\ \log. \cot.\ 23\ \ 28 \text{ - - - - } & 10.3623894 \\ \hline \log. \cot. \text{ lat}^{e}. = & 8.8772808 \end{array}$$

∴ latitude $= 85^\circ\ 41'$; and by a similar computation the following Table of the relative longitudes and latitudes is formed:

Long^e.	0,	10° 0′,	20 0′,	30° 0′,	50° 0′,	70° 0′,	90 0′
Lat^e.	90°	85 41,	81 34,	77 44,	71 36,	67 47,	66 32.

The precession in right ascension (as we have just seen) is nothing, when the angle of position is a right angle; it is also positive, when that angle is acute, and negative, when obtuse. For, by the relation subsisting between the sides a, c, and the angles A, B, of a spherical triangle, we have (see *Trig.* p. 116.),

$$\cot.\ A = \frac{\sin.\ c}{\sin.\ B}\left[\cot.\ a - \frac{\cos.\ B}{\tan.\ c},\right]$$

and making $A = \angle P\sigma\pi$, $a = P\pi\ (I)$, $B = 90° -$ R. A. $c =$ N. P. D.

$$\text{cot.}\ \angle\ P\sigma\pi = \frac{\text{sin. N P. D.}}{\text{cos. R. A.}}\left[\text{cot.}\ I - \frac{\text{sin. R. A.}}{\text{tan. N. P. D.}};\right]$$

hence, if cot. $\angle\ P\sigma\pi$ be negative, that is, if $P\sigma\pi$ be obtuse,

cot. I is $< \frac{\text{sin. R. A.}}{\text{tan. N. P. D.}}$; ∴ (see p. 142, l. 20,) the precession is negative: and the contrary takes place, if $P\sigma\pi$ be acute.

The introduction of the angle of position enables us concisely to state a rule relative to the sign affecting the precession: but, since the angle of position is known only by actual computation, we gain by such rule, nothing more than we can ascertain by considerations like those stated in p. 142.

We will now give Examples to the formulæ, [p. 141, &c.] for the precessions in right ascension, and north polar distance.

Required the Annual Precession of the Right Ascension of γ Pegasi; *its R. A.* (*in* 1800) = $0^h\ 2^m\ 56^s.79$, *and N. P. D.* = 75° 55′ 44″.

Here, since the precession for a year is required, $t = 1$.

Computation of 50″.34 cos. I

log. 50.34 - - - - - - - - - - - - - -	1.7019132
log. cos. 23° 28′ - - - - - - - - - - - -	9.9625076
	11.6644208

∴ (taking away) 10, the log. rad. 1.6644208 = log. 46″.177;

∴ the precession = 46″.177, or in time = $3^s.07$.

Computation of 50″.34 sin. $I \times$ sin. right ascension tan. × decl^n.

log. 50.34 - - - - - - - - - - - - -	1.7019132
log. sin. 23° 28′ - - - - - - - - - - - -	9.6001181
log. sin. $2^m\ 56^s$ - - - - - - - - - - - -	8.1071669
log cot. 75° 55′ 44″ - - - - - - - - - -	9.3990620
	28.8082602

∴ (taking away 30 the log. r^3,) $\bar{2}$.8082602 = log. 0643″.

Hence, adding the two parts together, we have the annual precession

= 46″.241, or in time, = 3″.99

If the precession, instead of 50″.34, be taken equal to 50″.1, the result will be 46″.02, or in time 3ˢ.07.

EXAMPLE II.

Required the Annual Precession in North Polar Distance of the same Star.

log. 50″.34	- - - - - - - - - - - - - -	1.7019132
log. sin. 23° 28′	- - - - - - - - - - - - - -	9.6001181
log. cos. 2^m 56^s	- - - - - - - - - - - - -	9.9999644
		21.3019957

take away 20, and 1.3019957 = log. 20″.044; ∴ (since the right ascension is > 0, and < 90°) − 20″.044, is the precession in N. P. D.

EXAMPLE III.

Required the Annual Precessions in Right Ascension and North Polar Distance of α Serpentis, *its Right Ascension* (*in* 1800) = 15^h 34^m $25^s.2$, *and North Polar Distance* = 82° 56′ 9″.2.

Computation of 50″.34 sin. I × sin. right ascension × tan. dec.

log. 50″.34	- - - - - - - - - - - -	1.7019132
log. sin. 23° 28′	- - - - - - - - - - - -	9.6001181
log. sin. 15^h 34^m 25^s	- - - - - - - - - -	9.9057696
log. cot. 82 56 9	- - - - - - - - - -	9.0933020
		30.3011029

take away 30 (3 log. rad.) and .3011029 = log. 2″.0004.

Now, since the right ascension is > 12^h, this number 2″ 0004 must be taken from the common part (50″.34 × cos. I) before computed, that is, from 46″. 177; consequently, the precession = 44″.176, or in time, = 2ˢ.93

If instead of 50″.34, the precession be taken equal to 50″.1, the result will be 43″.96 in space, or in time, 2ˢ.92.

Computation of 50″.34 sin. I × cos. right ascension.

log. 50″.34	- - - - - - - - - - - - -	1.7019132
log. sin. 23° 28′	- - - - - - - - - - - -	9.6001181
log. cos. 15^h 34^m 25^s	- - - - - - - - - -	9.7733128
		21.0753441

take away 20 (log. r), and 1.0753441 = log. 11″.89; therefore the annual precession in north polar distance of α *Serpentis*, is 11″.89 and +, since its right ascension is > 90° and < 270°.

If instead of 50″.34 the precession be taken equal to 50″.1, the result will be 11″.837.

Example IV.

It is required to find the Annual Precession in North Polar Distance of γ Draconis, *its Right Ascension being, in* 1800, = 267° 59′ 51″.

log. 50″.34 - - - - - - - - - - - - -	1.7019132
log. sin. 23° 28′ - - - - - - - - - - - -	9.6001181
log. cos. 267° 59′ 51″ - - - - - - - - - -	8.5433595
	19.8453908

take away 20 (log. r^2), and $\bar{1}$.8453908 = log. .7004;

therefore the annual precession of γ *Draconis* may be put down at .7 with sufficient exactness, and + .7, since the right ascension > 90°, and < 270°.

In 1727, the time of Bradley's observations, when the right ascension of this star was only 267° 50′, the annual precession in north polar distance was .85. In the year 2146, when it will be on the solstitial colure, its precession in north polar distance will be nothing.

These annual precessions in right ascension and declination are inserted in the Tables of the fixed stars, and, by the side of the right ascensions and declinations: thus,

	Right Ascen. in 1800.	Annual Precession.	North Polar Distance.	Annual Precession
γ *Pegasi*	0h 2m 56s.79	+ 3s. 09	75° 55′ 44″	− 20″.04
α *Serpentis*	15 34 25	+ 2.93	82 56 92	+ 11.89
γ *Draconis*	17 51 52	+ 1.389	38 28 53	− 0.7

There are also certain general Tables, from which, the annual precessions, and the precessions to any day of the year, may very easily be computed.

We may now be able to perceive the use and application of *Precession*, as a correction in Astronomical processes. In the Example, p. 133, the right ascension of *α Serpentis* up to April 30, 1810, is put down at $15^h\ 34^m\ 55^s.4$. Now, this right ascension was obtained from a Table (of which the above is a specimen) in which the right ascension of the star was registered for Jan. 1800: and the process of computation was as follows:

Jan. 1800.	Right ascension of *α Serpentis* - -	15^h	34^m	$25^s.2$
	Precession for 10 years - - - - - -	0	0	29.3
Jan. 1810.	Right ascension - - - - - - - - -	15	34	54.5
	Precession from Jan. to April 30 - -	0	0	.94
∴	right ascension on April 30, 1810 - - -	15^h	34^m	$55^s.44$.

This is very nearly the right ascension given in p. 133 : but not exactly so, since the last quantity added, viz. $.94 = \frac{120}{365} \times 2^s.93$ was obtained, by supposing the precession proportional to the time; or, in other words, to be uniformly generated: which it is not; but the reasons of this, and the mode of allowing for the *inequality of the precession*, must be reserved to the ensuing Chapter.

We have a similar illustration of the use of the precession in north polar distance in the second Example of the preceding Chapter, p. 134, l. 19. In that, the zenith distance of *γ Draconis* for Jan. 1802, at Greenwich, is, expressed by $2'\ 16''.6$; but was thus obtained from Tables:

Jan. 1800.	North polar distance *γ Draconis* - -	38°	28′	53″
	Precession for 2 years - (2 × .7) - -	0	0	1.4
	Precession to May 10 - - - - - - -	0	0	0.234
Nuta^n. to May 10, (a cor^n. to be hereafter explained)		0	0	9.42
∴ May 10, 1802.	North polar distance *γ Draconis*	38	29	4.054
	Co-latitude of Greenwich Observatory - - -	38	31	20
		0	2	15.946

In this process, the precession to May 10 has been stated at .234 Now the number of days elapsed from Jan. to May 10, is 130; ∴ if the precession were proportional to the time, that is, were equably generated, it would equal $\frac{120}{365} \times .7$ to [see p. 140,] or .252: hence, some correction and subtractive, has been applied to this mean quantity [.252] the explanation of its cause must also be deferred to the ensuing Chapter.

CHAP. XV.

On the Solar Inequality of Precession.

The Earth is not a perfect sphere, but an oblate spheroid, protuberant in the parts about the equator. The action, or the attractive force, of the Sun and Moon, on such protuberance, causes the precession of the equinoxes; that is, a continual regression of the intersection of the equator and ecliptic, or of the *Nodes* of the equator [see p. 40.] whilst the inclination of its plane to that of the ecliptic remains the same. Now, the precession depends on the distance of the Sun and Moon from the equator; there would be none, if each luminary continued in its plane. Its mean quantity, which has been stated at $50''.34$, is that which is produced in a whole year. But, it cannot have been equably produced. For, the Sun is sometimes in the equator, when its force in causing precession is nothing; at other times, more than 23 degrees distant, when its force is greatest. Hence (not at present regarding the Moon's action) the Sun's action in producing precession must continually vary, from the equinox in March to the solstice in June: in other words, there must be an *inequality of precession* dependent on the Sun's position, or on the day of the year.

The precession, therefore, for any number of days is not proportional to that number: for 61 days, it is not necessarily equal to $\frac{61}{365} \times 50''.34$, or $8''.56$: it may, or it may not be: it is nearly equal to that quantity from Jan. 1, to March 3: but, from

March 1, to May 1, it is equal only to $6''.5$, and from May 1, to July 1, it becomes equal to $10''$. Hence to the expression $50''.34 \times t$ for any time t, or, to the precession in north polar distance for the same time, which precession [see p. 140,] is =

$$50''.34 \times t \times \sin. 23^\circ\ 28'. \cos. \ast\text{'s right ascension}$$

we must apply a *correction* due to the *solar inequality*, or in other words, an *Equation**, according to the day of the year, either additive or subtractive, to the mean quantity of the precession. The equation spoken of, arising solely from the Sun, is called by Astronomers the *Semi-Annual Solar Equation.* †

The law of this *inequality*, or the formula for the computing the *equation* must be sought for on mechanical principles, in other words, it belongs to *physical* Astronomy to assign its expression. That department of science, however, is beyond the purpose of the present Tract. Yet, it may be observed, that if the inequality of the precession were known, that of declination and right ascension might be computed from the preceding formulæ: since ♈♈′ may as well represent the error, or the inequality of the precession, as the precession itself.

The existence of the *solar inequality* is the reason why, in the former Example [p. 147,] we expressed the precession in north polar distance of γ *Draconis* up to May 10, by .234, and not by the mean quantity .252. This latter, which is the mean, is made equal to the true by means of the *equation* − .018.

On the principles of physical Astronomy a formula has been investigated, from which, any equation such as − .018 may, (a specific case being proposed) be computed. The formula numerically expounded for all cases becomes, *a Table for finding the semi-annual solar equation in north polar distance.* [See Wollaston's *Fasciculus*, Appendix, p. 54.]

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* *Equation,* Astronomically used, is something to be added to, or subtracted from, a mean quantity, to make it *equal* to a true quantity: it is the correction of an inequality.

† *Semi-Annual,* because the equation is the same in corresponding points of the Sun's *half year's* passage from *Aries* to *Libra*, as it is from *Libra* to *Aries*.

If we separated the mean precession in north polar distance, from its equation, in the last Example, p. 147, it would stand thus:

Jan. 1800. N. P. D. of γ *Draconis* - -	38°	28′	53″
Precession for 2 years - - - - - -	0	0	1.4
Mean precession to May 10 - - - -	0	0	0.252
Semi-annual solar equation to May 10 -	−0	0	0.018
Nutation [see p. 147,] - - - - - -	0	0	9.42
N. P. D. of γ *Draconis* - - - - - -	38	29	4.054

For stars situated like γ *Draconis*, we must find the precession after the preceding process. But, of the 36 principal stars which are constantly used by Astronomers in correcting their clocks, &c. the corrections both in north polar distance and in right ascension are inserted in special Tables for every tenth day of the year. The corrections include those of precession, solar inequality and aberration. Thus, in the Tables alluded to, the corrections in north polar distance for α *Serpentis*, and *Capella* are stated for April 30, to be, respectively 10″.4, and − 0″.5: which are to be separated into their component parts, as follows:

α *Serpentis*.		*Capella*.	
Precession [11″.89 × .32]	3″.8	Precession [−5″.11 × 32]	− 1″.63
Solar inequality - -	0.16	Solar inequality - -	.002
Aberration - - -	6.71	Aberration - - - - -	− 3.41
	10 35		− 5.04

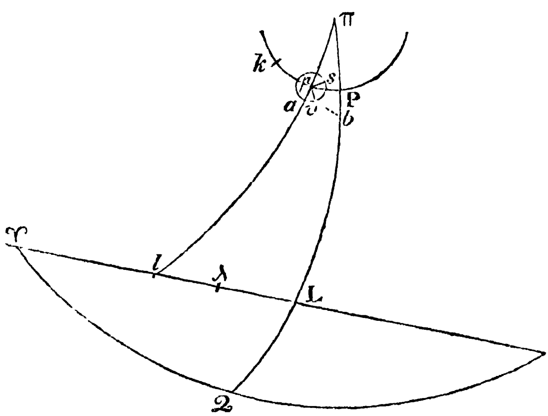
The precession in right ascension will, like that in north polar distance, be affected by the solar inequality, and will require a correction: but, however, is not effected by a Table similar to the one for [p. 150,] the precession in north polar distance: but more simply, by a change in the multiplier of the annual precession. Thus, for α *Serpentis*, the annual precession in right ascension is 2ˢ.935: the proportional *mean* quantity to April 30, would be $\frac{120}{365}$ × 2ˢ.935, or .32 × 2ˢ.935 (.32 being the multiplier) now this is too large: the mean precession, by reason of the *solar inequality*, is unequal to the true: but, instead of correcting the *inequality* by an *equation* (as in the former instance of γ *Draconis*) the multiplier .32 is reduced to .30: and accordingly, the precession in right ascension, inclusive of the semi-annual equation of precession, is .8805. [See Wollaston's *Fasciculus*, Appendix, p. 42.]

This relates to stars not included amongst the 36 principal ones previously mentioned. For those latter stars, as we have said, Tables are calculated which at once for every tenth day express the result of the several corrections of aberration, precession and solar inequality.

The solar inequality affecting all stars, would affect all stars situated in the ecliptic: it must affect then the Sun's place: and for this reason, amongst the new French Tables, there is one entitled *Solar Nutation*.

The solar inequality may be represented by the following construction:

Let π, P, as before, be the poles of the ecliptic and equator, then, by reason of the precession, P will move in a circle Ppk,



and in that order, which is contrary to the order of the signs; and, in a year, it will move through a space such as $Pk =$ (in seconds of that circle) 50″.34. Now, on account of the solar inequality, the true pole will be sometimes, nearer to π, sometimes farther, from π, than the mean; sometimes to the right, and at other times, to the left.

Let now p be the mean place of the pole after the lapse of any time t. Round p, as a center, with a radius $=0''.4345$ describe a a small circle *. Take the $\angle apv = 2.$☉'s longitude, then v is the true place of the pole, the pole having been at b [$Pb = pa$] when the Sun was at ♈.

For instance, on April 30, the mean regressive motion of the pole being $= \frac{40}{365} \times 50''.34 \times \sin. 23^\circ\ 28'$, set off Pp equal to that quantity: and the Sun's longitude being $1^s\ 8^\circ\ 11'$, the angle apv must be taken equal to $2^s\ 16^\circ\ 22'$, that is, $76^\circ\ 22'$.

About May 5, when the Sun's longitude is $1^s\ 15^\circ$, take the angle $apv = 3^s$, and the point v, the place of the true pole, will fall in the circle Ppk, and will be at the same distance from π, that the mean pole p is.

Again, June 19, take $Pp = .25 \times 50''.34 \times \sin. 23^\circ\ 28'$, and the angle apv' [the ☉'s longitude being $2^s\ 27^\circ\ 28'$] $= 5^s\ 24^\circ\ 56' = 174^\circ\ 56'$: then the true place of the pole is at v', very near to the upper point of intersection of the small circle, and the secondary (πpa) to the ecliptic.

On Sept. 22, the ☉'s longitude being 6^s, the $\angle apv$ will $= 12^s$, and the true place of the pole will be at the lower intersection of the small circle, and of the secondary πpa, but not in the same place in which it was in March when the Sun was in ♈, and his longitude nothing, since, in the interval [half a year], the mean place p of the pole has regressed through $\frac{1}{2}\ 53''.34 \times \sin. 23^\circ$.

Since the point a, the true pole, is carried in the direction av, that is, according to the order of the signs, whilst the center p,

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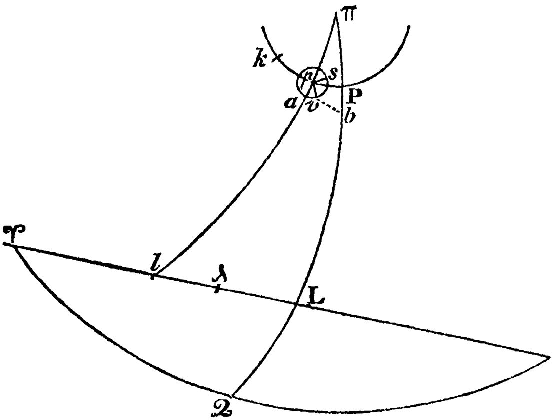
* In mathematical demonstration, it is of no consequence (as we have before remarked, p. 13,) that the geometrical signs should bear, as to magnitude, any proportion to the things signified: if it were, the preceding results would be entirely vitiated; since the proportion as geometrically represented is preposterous; for, the real proportion between pa, and π ♌, is as 1 : 648000.

the place of the mean pole, is carried from P to p, contrary to



that order, the true pole will move in an epicycloid, lying between the points b and v.

The *equation of precession*, due to the solar inequality, may easily be computed by the aid of the preceding construction. Thus, if the precession had been uniformly generated, P would have been transferred to p, and the solstice from L to a point l, where a secondary $\pi p a$ continued would cut the ecliptic. But, owing to the inequality of precession, the true pole, instead of being at p, is at v, and the solstice, instead of being at l, is at some point λ, where a secondary passing through π and v cuts the ecliptic: the defect from the mean place then, or *the equation*, is measured by $l\lambda$; and

$$l\lambda = pv \times \sin. apv \times \frac{1}{\sin. \pi p}$$

$$= \frac{0''.4345 \times \sin. 2\ \odot\text{'s long}^{e}.}{\sin. 23^{\circ}\ 28'}$$

$$= 1''.1 \times \sin. 2\ \odot\text{'s longitude, nearly.}$$

EXAMPLES.

	2 Sun's Longitude.	Natural Sines.	Equation of Precession.
April 5....	....30° 5′	... .50126...	.........0″.55.......
May 11 ...	.... 100.........	... .98486...	.........1 . 08.......
Feb. 19 ...	....660..........	... .86617...	.........0 . 95.......

By means of these equations, adding or subtracting them, we can always assign the quantity of the true precession for any day of the year *: and accordingly, by substituting such quantity in the two expressions pp. 141, 142, at once compute the precessions in north polar distance and right ascension.

If we call πp the mean obliquity, then, the true pole being at v, the difference between the true and mean obliquity, in other words, the *correction* of the obliquity, is the difference between the arc of a great circle that will lie between π and v, and the arc πp.

Now this difference, or equation, or *correction*

$$= pv \times \cos. apv = 0''.4345 \times \cos. 2 \odot\text{'s longitude}$$

EXAMPLES.

	2 Sun's Longitude.	Natural Cosines.	Corrections of Obliquity.
April 5...	......30° 5′...	.. .86530....	..... .37.........
May 11...	......100......	—.17363....	..... .04.........
Feb. 19...	......660......	.. .49975....	..... .21.........

* In the Table of the Precession for every day of the year, the Solar equation is included. See Vince's *Astronomy*, vol. II. p. 23.

The changes in the place of the solstice and in that of the pole, constitute two equations that arise from the solar inequality, and might, with propriety, have been termed, Equations of the *Solar Nutation*. But, under this latter title, there is a table in the new Solar Tables of Delambre (see *Tables du Soleil*, tab. XIII, and Vince, vol. III. pp. 41, 88.) containing one column under the head *Longitude*, for the equation of precession due to the inequality of the Sun's action, where the results agree with those deduced from the formula, p. 154, l. 14: but, the second column, under the head *Obliquity*, does not contain equations arising from the inequality of the Sun's action, but of the Moon's. The equations due to the former inequality are given (combined with the secular diminution of the obliquity) in a Table inserted in the Introduction. (See *Tables du Soleil*, and Vince, vol. III. p. 7.)

The radius $pa = 0''.4345$, is the greatest equation of the obliquity arising from the solar inequality. And, since in the space of a quarter of a year, the pole will move from *a* to the opposite end of the diameter of the small circle, the whole change in that time, of its place, and, consequently, of the obliquity, is $2 \times 0''.4345$, or $0''.869$.

The correction of the obliquity is, very nearly, the correction for the precession in north polar distance of a star situated in the solstitial colure. Thus, for the star ζ *Canis majoris*, whose right ascension in 1800 was $6^h\ 13^m$ the semi-annual solar equations in north polar distance, are as follow:

	Equation: 0″.4345 × cos. 2 Sun's longitude.		
	2 Sun's Longitude.	Natural Cosines.	Corrections.
April 30	....76° 22′....	..... .2357	..... .1 ....
May 10	....96 4	....−.1056........	...− .05.......
Jan. 0......	...199 48......	....− .9408.......	... − .4........

If, instead of 0″.4345 the radius *pa* of the small circle had been assumed equal to 0″.5 (which it used to be before the publication of Delambre's Tables), the equations would have been respectively 0″.11, – 0″.05, — 0″.47: which, however, as well as those in p. 156, l. 28, &c., since they are corrections of north polar distances, must be written with contrary signs.

CHAP. XVI.

On Nutation.—Bradley's Explanation of it.—Formulæ for the Effects of Nutation in North Polar Distance, and in Right Ascension.—Obliquity of the Ecliptic affected by Nutation.—Use of Nutation as a Correction to Observations.

THE precession of the equinoxes arises from the action of the Sun and Moon on the equatoreal parts of the Earth; and for its production, it is necessary that these bodies should be out of the plane of the equator. In the preceding Chapter, we have considered the inequality of the precession arising from the varying distance of the Sun from the equator. The Sun's action is always producing precession, but not equably; and the mean precession for a year, is the aggregate of the unequal daily parts. But, since the Sun's force is greater, the more distant it is from the equator, if, on the latter of two successive years, the Sun were constantly more distant from the equator, each day, than on the corresponding day of the former year, it would constantly produce greater parcels of precession: and their sum, which is the *annual* precession, would be greater. This circumstance of a greater distance from the equator would happen, if the inclination of the plane of the ecliptic to that of the equator were increased.

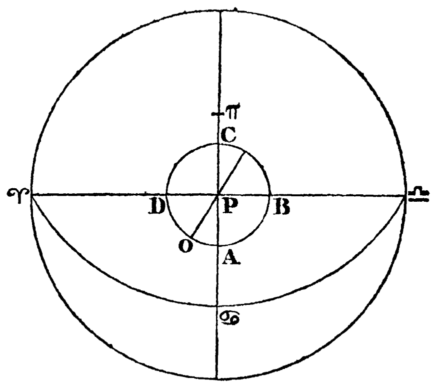
Now, the variation of the obliquity of the ecliptic is not, within short periods, sufficient to cause any sensible effect in the quantity of precession: but, the Moon is in like predicament with the Sun; her action in causing precession varies with her change of distance from the equator: and the annual mean effect would be increased, if the inclination of her orbit to the equator were increased.

This latter circumstance really takes place, and in a sensible degree: consequently, solely from the Moon's action, the precession of one year must differ from that of the preceding. Or, in other words, the mean precession of the equinoxes arising from the action of the Sun and Moon, will constantly be *unequal* to the true precession: and must therefore be made *equal* to the latter, by the application of an *equation*: called, for distinction, the *Equation of the Equinoxes*: and which equation must, it is clear, depend on, or have for its *argument*, the inclination of the Moon's orbit to the equator, or, some quantity involving, or significant of, the inclination.

Of the solar inequality, as we have seen, an equation in the precession is not the sole effect: besides that, there is a variation produced in the distance between the poles of the equator and ecliptic: in other words, the obliquity of the ecliptic is made to vary. The variation is explained in p. 155, and its expression is $0''.4345 \times$ cos. 2 Sun's longitude. This variation is technically called the *Correction of the Obliquity of the Ecliptic* for days of the year, in order to distinguish it from the *equation* of the obliquity arising from nutation. Similar effects take place from the inequality of the Lunar action. Not only the motion of the precession becomes unequable, but the inclination of the equator to the ecliptic is varied. The change of place in the pole of the equator, is called, for distinction, *Nutation*.

The pole of the Earth then, from the inequality of the Moon's action in causing precession, is affected with a motion in the circumference of the circle, and besides, with a motion to, or, from π the pole of the ecliptic. What will be the compound effect of these two motions? or, can any curve be, either analytically expressed, or geometrically described, in which, for any assigned time, the *true* pole of the equator shall be found? Bradley, the discoverer of *Nutation*, in the first instance, affirmed the curve, (the *locus* of the pole) to be a circle.

Let π be the pole of the ecliptic, and P the mean place of the pole of the equator. Describe round P a small circle, with a radius $PA = 9''.6$. Let A be the true pole of the equator, when the ascending node [see p. 40,] of the Moon's orbit is at ♈. Let *A* move, contrary to the order of the signs, and, by an equable



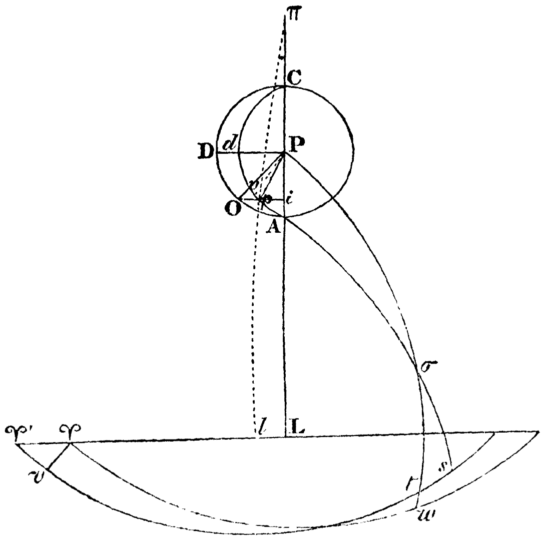
retrograde motion, let it describe the circumference of the small circle, in a period equal to that of the Moon's nodes, that is, of 18 years 7 months: then if the angle *APO* be taken equal to the distance of the Moon's node from ♈ the first point of *Aries*, *O* will be the *true* place of the pole of the equator.

This is the mode of geometrically representing the law of the nutation that was first given by Bradley *. But, towards the end of his Memoir, he suggests that a more exact mode might be obtained, by substituting, instead of the circle *ABCD*, an ellipsis, the transverse axis *AC* and the conjugate *DB* being nearly 18″, and 16″ respectively. Not perfectly satisfied, however, with this alteration, he refers to theory, for the more accurate determination of the *locus* of the *true* pole.

Theory has verified Bradley's suggestions, and shewn the locus of the pole to be an ellipse, with some slight alterations, however, in its proportions. Instead of *AC* being equal to 18″, it must be made equal to 19″.2; and the minor axis of the ellipse must be made equal to 15″. The axes are to be to one another in the proportion of the cosine of the obliquity (23° 28′) to the

* *Phil. Trans.* No. 485. p. 35.

cosine of twice the obliquity. And, APO being taken, as in the former case, equal to the distance of the ascending node of the Moon's orbit from ♈, a line (Oi) drawn from O perpendicular



to PA, will cut the ellipse in a point which is the true place of the pole; that is, in the Figure, p will be the true place, the true place having been at A when the node of the Moon's orbit was at ♈.

This construction being admitted, we may easily calculate the nutations, or the *effects of nutation* in right ascension and north polar distances, and by a method very nearly the same as that which was employed in calculating the effect of precession.

Effect of Nutation in North Polar Distance.

Conceive a perpendicular to be let fall from P on σp (produced, or not, according to the position of p): in the Figure, let u be the point in which the perpendicular cuts σp, then the effect of nutation in N.P.D. is $= \pm P\sigma - p\sigma = \pm pu$, nearly.

Now, $pu = Pp \,.\, \text{sin.}\, pPu = Pp.\, \text{sin}\,.(APu - APp)$

$$= Pp\,.\,\text{sin.}\,(\sigma Pu - \sigma PA - APp)$$
$$= Pp\,.\,\text{sin.}\,(90^\circ - \sigma PA - APp)$$
$$= Pp.\text{sin.}\,[90^\circ - (\ast\text{'s R. A.} - 90^\circ) - APp]$$
$$= Pp.\text{sin.}\,[180^\circ - (\ast\text{'s R. A.} + APp)]$$
$$= Pp\,.\,\text{sin.}\,(\ast\text{'s R. A.} + APp)\,;$$

$\therefore$ effect of nutation in N. P. D. $= \pm\, Pp\,.\,\text{sin.}\,(\ast\text{'s R. A.} + APp.)$

Effect of Nutation in Right Ascension.

The right ascension of the star σ is changed, by the effect of nutation, from Υw into $\Upsilon' ts$. And $\Upsilon' ts = \Upsilon' v + vs = \Upsilon' v + \Upsilon w + ts$; in which expression, as in the case of precession, [see p. 142,] the part $\Upsilon' v$ is common to all stars. Now,

$$\Upsilon' v = \Upsilon\Upsilon' \times \text{cos.}\, I = Ll \times \text{cos.}\, I = pi \times \frac{\text{cos.}\ I}{\text{sin.}\ I} = Pp.\,\text{sin.}\, APp\ \text{cot.}\, I\,;$$

$$\text{and, } ts = Pu\,\frac{\text{sin.}\ \sigma s}{\text{sin.}\ \sigma p} = Pu \times \text{tan. dec.} = Pp\,.\,\text{cos.}\, pPu \times \text{tan. dec.}$$

$$= Pp\,.\,\text{cos.}\,[180^\circ - (\ast\text{'s R. A.} + APp)]\ \text{tan. dec.}$$
$$= -Pp\,.\,\text{cos.}\,(\ast\text{'s R. A.} + APp)\ \text{tan. dec.}$$

In order to exhibit the preceding expressions for the effects of nutation in north polar distance and in right ascension, under a more commodious form, we must, from the properties of the ellipse, investigate the values of Pp, and of the angle APp.

$$\frac{Pp}{PO} = \frac{\text{sec.}\ APp}{\text{sec.}\ APO} = [\textit{Trig.}\ \text{p. 10,}]\ \frac{\text{cos.}\ APO}{\text{cos.}\ APp}\ \text{: but, [see p. 161,]}$$

$APO = 12^s -$ longitude of Moon's ascending node, and accordingly,

$$Pp = PO \times \frac{\text{cos. long. ☽'s ☊}}{\text{cos.}\ APp} = 9''.6 \times \frac{\text{cos. long}^e\text{. ☽'s ☊}}{\text{cos.}\ APp}.$$

Again,

$$\frac{\text{tan.}\ APp}{\text{tan.}\ APO} = \frac{pi}{Oi} = [\text{by property of the ellipse}]\,\frac{Pd}{PD} = \frac{15''}{19''.2}\,;$$

$$\therefore \text{tan.}\ APp = -\frac{15''}{19''.2} \times \text{tan. long. ☽'s ☊} = -\frac{25}{32} \times \text{tan. long. ☽'s ☊}$$

Substituting in the two former expressions, we have the nutation in N. P. D. that is, Pp . sin. (✳'s R. A. + APp) =

$$\frac{9''.6}{\cos. APp} . \cos. \text{long. ☽'s ☊} [\sin. \text{✳'s R. A.} \cos. APp$$

$$+ \cos. \text{✳'s R. A.} \sin. APp] =$$

$$9''.6 \cos. \text{long. ☽'s ☊} . \sin. \text{✳'s R. A.}$$
$$+ 9''.6 \cos. \text{long. ☽'s ☊} \times \cos. \text{✳'s R. A.} \tan. APp.$$

The last term, substituting for tan. APp, becomes

$$- 7''.5 \sin. \text{long. ☽'s ☊} . \cos. \text{✳'s R. A.}$$

substituting now for the products of the sine and cosine, their equivalent expressions [see *Trig*. p. 18.] we have

nutation in N. P. D. =

$$4''.8 \times \sin. [\text{long. ☽'s ☊} + \text{✳'s R. A.}]$$
$$- 4''.8 \times \sin. [\text{long. ☽'s ☊} - \text{✳'s R. A.}]$$
$$- 3.7 \times \sin. [\text{long. ☽'s ☊} + \text{✳'s R. A.}]$$
$$- 3.7 \times \sin. [\text{long. ☽'s ☊} - \text{✳'s R. A.}]$$

$$= 1''.1 \times \sin. [\text{long. ☽'s ☊} + \text{✳'s R. A.}]$$
$$- 8''.5 \times \sin. [\text{long. ☽'s ☊} - \text{✳'s R. A.}]$$

$$\text{or,} = 1''.1 . \sin. [\text{✳'s R. A.} + \text{long. ☽'s ☊}]$$
$$+ 8''.5 . \sin. [\text{✳'s R. A.} - \text{long. ☽'s ☊}.]$$

If in this last formula, we substitute, instead of ✳'s R. A. 180° + ✳'s R A. the expression for the nutation in N. P. D. of a star, with an opposite R. A. to the former, will be

$$1''.1 \sin. [180° + \text{✳'s R. A.} + \text{long. ☽'s ☊}]$$
$$+ 8''.5 \sin. [180° + \text{✳'s R. A.} - \text{long. ☽'s ☊}]$$
$$= - 1''.1 \sin. [\text{✳'s R. A.} + \text{long. ☽'s ☊}]$$
$$- 8''.5 \sin. [\text{✳'s R. A.} - \text{long. ☽'s ☊}]$$

that is, equal in quantity to the former, but in a different direction (see *Phil. Trans*. No. 485, pp. 12, 13.) The preceding formula, numerically expounded, furnishes corrections to be applied to the *apparent*, in order to deduce the *mean* N. P. D.: if we wish to deduce the apparent from the mean, we must employ the formula, and its numerical results, with contrary signs.

When the star is in the equinoctial colure, that is, when its R. A. is either = 0, or = 180°,

Nutation in N. P. D. = ∓ 7″.5 sin. long. ☽'s ☊,
where the upper sign is to be used in deducing the mean N. P. D. from the apparent; the lower, in the reverse process.

When the ☽'s ☊ is either in ♋, or in ♑,
the nutation in N. P. D. = 7″.5.

When the star is in the solstitial colure, and its R. A. = 90°,
nutation in N. P.D. = ± 9″.6 cos. long ☽'s ☊

When its R. A = 270°,
nutation in N. P. D. = ∓ 9″.6 cos. long. ☽'s ☊
in which expressions, the former rule, with regard to the signs, is to be observed.

When the ☽'s ☊ is at ♈, the true pole (see Fig. p. 161) is at *A*, and then the correction to the north polar distance is + 9″.6.

When the Moon's node is in the first point of *Aries*, that is, when long. ☽'s ☊ = 0, the nutation in N. P. D. of a star situated in the solstitial colure is the greatest (= + 9″.6); but, the nutation in N. P. D. of a star in the equinoctial colure is nothing.

In the regress of the Moon's node from ♈ to ♑, the mean north polar distance of a star in the solstitial colure would change, by the effect of nutation, from

9″.6 . cos. 0° to 9″.6 . cos. 270°; that is,
from 9″. 6 . - - - - to - - 0.

In the same period of regress, the mean N. P. D. of a star in the equinoctial colure would change, (but in a contrary direction) by the effect of nutation, from

7″.5 . sin. 0° to 7″. 5 sin. 270°, that is,
from 0 - - - - to 7″. 5.

Hence, from 1727 to May 1732, during Bradley's observations, and in the above stated regress of the Moon's node, the N. P. D. of γ *Draconis* (which is near the solstitial colure) would have changed, from precession (see p. 146.), and nutation

$4.5 \times .8'' - 9''.6$ nearly,

the N. P. D. of γ *Ursæ majoris*, $18''.24 \times 4.5 + 7''.5$ nearly; that is, according to Bradley's words, "Some of the stars near the solstitial colure, had changed their declinations 9″ or 10″ *less* than a precession of 50 would have produced, whilst others near the equinoctial colure had altered their's about the same quantity more than a like precession would have occasioned". [*Phil. Trans.* No. 48, p. 11.]

Effect of Nutation in R. A.; *the Part* (see p. 162.)

$$Pp . \cos. [*\text{'s R. A.} + APp] \tan. \text{dec.} =$$

$$\frac{9''.6}{\cos. APp} . \cos. \text{long. ☽'s ☊} [\cos. *\text{'s R. A.} \cos. APp - \sin. *\text{'s R. A.} \sin. APp] \tan. \text{dec.}$$

the latter term, substituting for the tangent of *APp*, becomes

$$9''.6 \times \frac{25}{32} \times \sin. \text{long. ☽'s ☊} . \sin. *\text{'s R. A.} \tan. \text{dec.}$$

Hence, the variable part of the nutation in R. A. =

$$9''.6 \tan. \text{dec.} \cos. \text{long. ☽'s ☊} \times \cos. *\text{'s R. A.}$$
$$+ 7''.5 \tan. \text{dec.} \sin. \text{long. ☽'s ☊} \times \sin. *\text{'s R. A.}$$

= [*Trigonometry*, p. 18.]

$$\tfrac{1}{2} 9''.6. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} - *\text{'s R. A.}]$$
$$+ \tfrac{1}{2} 9''.6. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} + *\text{'s R. A.}]$$
$$+ \tfrac{1}{2} 7''.5. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} - *\text{'s R. A.}]$$
$$- \tfrac{1}{2} 7''.5. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} + *\text{'s R. A.}]$$
$$= 8''.5. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} - *\text{'s R. A.}]$$
$$+ 1''.1. \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} + *\text{'s R. A.}]$$

The part ♈'*v* (common to all stars) = *Pp* sin. *APp* . cot. *I* =

$$9''.6 . \cos. \text{long. ☽'s ☊} \tan. APp . \cot. 23^{\circ} 28' =$$

$$9''.6 \times -\frac{25}{32} \times 2.3035. \sin. \text{long. ☽'s ☊} = - 17''.2 . \sin. \text{long. ☽'s ☊};$$

∴ the whole effect of nutation in R. A. = ♈'*v* + *ts*

$$= \pm 8''.5 \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} - *\text{'s . R. A.}]$$
$$+ 1''.1 \tan. \text{dec.} \cos. [\text{long. ☽'s ☊} + *\text{'s R. A.}]$$
$$+ 17''.2 . \sin. \text{long. ☽'s ☊}.$$

It is plain, from inspection of the Figure, that this part ♈'*v* is common to all stars, or, is the sole effect of nutation on a star situated in the equator: the same also appears from the preceding analytical expression of the nutation: for, that expression, when the declination, and consequently its tangent, is nothing, is reduced to

$$17''.2 . \sin. \text{long. ☽'s ☊}.$$

The above expression for the effect of nutation in R. A. of a star situated in the equator, is technically distinguished by the title of the *Equation of the Equinoxes in Right Ascension* *.

It is this equation of the equinoxes that is used in correcting the mean longitude of a fictitious Sun moving in the equator as the regulator of mean solar time. (See Chap. *On the Equation of Time.*)

Examples to the preceding Formulæ in pp. 163, 165.

EXAMPLE I.

Required the quantity of Nutation in N. P. D. of γ Draconis *on May* 10, 1802. (See p. 151.)

Long. ☽'s ☊ - - - - - - - - 11ˢ 17° 43′

✳'s R. A. - - - - - - - - - 8 28.

Long. ☽'s ☊ + ✳'s R. A. - - - 8 15 43 − sin. = −.96909,

Long. ☽'s ☊ − ✳'s R. A. - - - 2 19 43 − sin. = .98394;

∴ [see form[a]. p. 163], nutation in N.P.D. = − 1″.66 − 8″.36 = − 9″.42.

EXAMPLE II.

Required the Nutation in R. A. of α Serpentis *on April* 30, 1810.

✳'s dec. - - - - - - - - - - - - - 0ˢ 7° 4′ - - - tan. = .12397,

Long. ☽'s ☊ - - - - - - - - - - 6 13 31 - - - sin. = .23373,

✳'s R. A. − long. ☽'s ☊ - - - 1 10 7 - - - cos. = .76473,

✳'s R. A. + long. ☽'s ☊ - - - 2 7 7 - - - cos. = .38886;

∴ by formulæ, p. 165, nutation in R. A. =

17″.2 × .23373 − [8″.5 × .76473 + 1″.1 × .38886] × .12397

= 3″.16, or, in time, = 0ˢ.2, nearly.

The results of computations, like the preceding, registered and arranged, form tables for the effects of nutation in N. P. D. and in R. A. The arguments of the tables must be (as it is plain

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* See Maskelyne's Tables, p. 2. and Explanation of the Tables, p. 3.; also Vince, vol. II. p. 368, and Wollaston's *Fasciculus*, Appendix, p. 49.

from the formulæ, p. 163,) the right ascension of the star, and the longitude of the Moon's ascending node. To the thirty-six principal stars, however, special tables are assigned: and in these, one argument, the longitude of the Moon's node, is sufficient; the other, the right ascension, is, in fact, included in the particularization of each star.

In the general tables of nutation for N. P. D., the numbers are taken out, by means of the two arguments, from the same table. In those for the nutation in R. A., the numbers must be taken out from two separate tables, one appropriated to the analytical expression for ts, the other, to that for $\Upsilon' v$: the numbers corresponding to the values for ts, must, besides, be multiplied by the tangent of the star's declination.

The small ellipse, which, during a revolution of the Moon's nodes, the true pole [p] of the equator describes, represents, at once, the effects of nutation and of the inequality of precession *. The meaning of which is this: from the varying action of the Moon, as from that of the Sun, an inequality of precession would arise. Such inequality alone, would cause to vary both the right ascensions and declinations of stars, even if the pole of the equator deviated neither from, nor towards, the ecliptic. But, the pole does deviate, by reason of a motion, properly speaking, of *nutation*. The right ascensions and declinations of stars, then, from this latter, as well as from the former cause, will be made to vary. But, both variations, relatively to their combined law and quantity, are represented by the small ellipse AdC, of which the dimensions have been assigned.

We have already seen how to compute from the small ellipse, the formulæ of nutation in N. P. D. and in R. A.; and we may conveniently avail ourselves of the same curve, and the connected construction, to explain, in a general way, the effects of nutation.

The change in the obliquity of the ecliptic is one of the chief effects of nutation. When the ascending node of the Moon's orbit is in *Aries* (at Υ) the pole is at A and the obliquity is equal to πA: when the node has regressed to *Capricorn*, or

* See Figure in p. 161.

Cancer, to the 10th, or to the 4th sign, the pole of the equator will be at *d*, or in the opposite point in which *dP* cuts the ellipse: and in this case, the obliquity, instead of πA, will be equal to πd, or, to πP, nearly: and, πP being the mean, the increase at *A*, or, as it is technically called, the *Equation of the Obliquity of the Ecliptic*, [see Maskelyne's Tables, XXXII.] will be equal to *PA*, equal, accordingly, to 9″.6. This equation of the obliquity is applied to the mean place (*P*) of the pole; consequently, it will have a different sign, when the node of the Moon's orbit is in *Libra*, and the pole at *C*, from what it had when the node was in *Aries*, and the pole at *A*. The sum of the two equations, or the total difference of the obliquity corresponding to the two positions of the pole at *A* and *C*, is $AP+PC = 19''.2$. This change in the obliquity happens after an interval of time equal to half the period of the revolution of the Moon's nodes; which period is equal to about 18 years 7 months.

The equation of the obliquity in other positions of the Moon's nodes, is, generally, $\pm$ 9″.6 cos. long. ☽'s ☊, and, in practice, is found by the aid of an appropriate Table. [See Maskelyne, Table XXXII.]

When the Moon's node is in ♈, and the pole in *A*, the declinations of all stars situated in the solstitial colure will be affected nearly by the whole quantity of nutation: but, the declinations of stars situated in the equinoctial colure will not be altered. When, however, the node of the Moon's orbit approaches ♑, or ♋, the pole will be in *d*, or in the opposite corresponding point; and consequently, will approach the stars that are situated in the equinoctial colure; and, the declinations of such stars will be changed by a quantity nearly equal to 7″.5 [see p. 164.]

The stars in the equinoctial colure are most affected, in their declinations, by precession [see p. 141]: the annual change = 50″.1 × sin. 23° 28′ amounting to about 20″; hence, since the change in 4 years and an half, (in which time nearly the node of the Moon's orbit regresses from ♈ to ♑) would amount to 90″, the change of 7″.5 might, with regard to the numerical result, have been accounted for, by substituting, for the precession, instead of 50″.1, some larger quantity, 55″, for instance: but then, such larger precession, instead of accounting for the changes in the north polar distances of stars near the solstitial colure, would afford results more widely differing from observation, than the common and less quantity of precession: since these stars, in Bradley's words, 'appeared to move, during the same time, in a manner contrary to what they ought to have done by an increase of precession.' [See *Phil. Trans.* No. 485, p. 12.]

Since the nutation does not at all influence the ecliptic, it cannot change the distances of stars from that circle: in other words, the latitudes of stars are not affected by nutation.

Since, however, the change in the place of the pole of the equator alters the position of the solstitial colure, and, consequently, the two places of the intersections of the equator and ecliptic, from one of which (the first point of *Aries*) the longitudes of stars are reckoned; the longitudes of stars will, by nutation, be changed, and, by the quantity ♈ ♈′, [see Fig. p. 168,] which quantity is denominated the *Equation of the Equinoxes in Longitude*. [See Maskelyne's Tables, Tab. XLIII.]

This equation ♈♈′, or Ll [see Fig. p. 168,] is 0 when the pole is in A, or in C; and consequently, when the Moon's node is either in ♈, or in ♎; it is greatest when p is at d, or in the opposite corresponding point; and its value then, is equal to $\frac{Pd}{\sin. 23^\circ\ 28'}$.

The line Ll (the general representative of the equation of the equinoxes in longitude,) is =

$$\frac{pt}{\sin. 23^\circ\ 28'} = \frac{Pp . \sin. APp}{\sin. 23^\circ\ 28'} = - \frac{Pp . \sin. \text{long. ☽'s ☊}}{\sin. 23^\circ\ 28'}$$

(nearly.)

We may now, which is a main object of the Treatise, shew the uses of the theory, formulæ, and Tables of nutation, in completing the preceding Examples of pages 147, 151.

EXAMPLE. [See p. 147.]

It is required to find γ Draconis's *North Polar Distance on May* 10, 1802, *from its tabulated North Polar Distance on Jan.* 1800.

Jan. 1800. N. P. D. γ *Draconis* - - - - - -	38°	28′	53″
Precession for 2 years (2 × .7) - - - - -	0	0	1.4
Precession to May 10, inclusive of the solar inequa^l.	0	0	.234
Nutation to May 10, [see p. 166,] - - - - -	0	0	9.42
	38	29	4.054

In this Example, the mean place of the star is *brought up* from the beginning of 1800 to May 10, 1802. In the following we will *reduce* or carry back the observed zenith distance of γ *Draconis* on May 10, 1802, to its mean place on the beginning of the same year:

May 10, 1802. At Blenheim Observatory*, zen. distance γ *Draconis* - -	0°	53′	30″.1	north
Aberration - - - - - - - - - - - - - -	0	0	12.58	
Precession - - - - - - - - - - - - - -	0	0	0.252	
Semi-annual solar equation - - - - - -	−0	0	.018	
Nutation - - - - - - - - - - - - - - -	0	0	9.42	
Mean place of γ *Drac.* at the begin. of 1802	0	53	52.334	

Here the corrections of aberration, precession, nutation, applied to a *preceding* point of time, are, relatively to the N. P. D. *subtractive*, but relatively to the zenith distance (since the star is north of the zenith) *additive*.

The uses of the zenith distances of such stars as γ *Draconis* have been already [p. 134,] shewn in finding the latitudes of places.

To exemplify the use of nutation in right ascension, we will complete the instance of the observation of α *Serpentis* made for the purpose of correcting the clock:

* This instance is taken from the *Phil. Trans.* 1803; the observation is quoted in p. 416; and the place *reduced* from the observation in p. 470: the results differ by about half a second, owing probably to the nutation in the text being taken larger than Colonel Mudge took it.

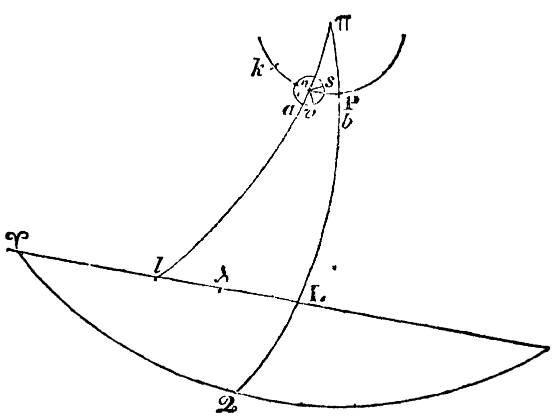
April 30, 1810. *α Serpentis*, by observations on the meridian at - - - - - - - - - - - - - -	15^h	35^m	55^s
R. A. of *α Serpentis*, by Tables - - - - - - -	15	34	25.51
April 30. Aberⁿ., precessⁿ., inequal. of precessⁿ.	0	0	2.16
Nutation -	0	0	0.2
	15	34	27.87

∴ difference of the R. A. shewn by the clock and the tabulated R. A. is 27ˢ.13, or the clock is too fast by that quantity.

α Serpentis is one of the 36 principal stars, and its aberration, precession, and semi-annual equation, are not taken separately from the general tables appropriated to these corrections, but their sum or result is at once set down in special Tables: so is its nutation, the sole *argument* of the nutation being the longitude of the Moon's ascending node.

We have now explained the cause of nutation; deduced formulæ expounding its quantity and law; applied those formulæ to instances; and shewn the practical use of the numerical results from the formulæ, in the correction of Astronomical observations. It may be proper, however, before we dismiss the subject, to make a few observations on the different *loci*, which the pole of the equator has been supposed to describe, by reason of the three causes of, precession, the inequality of precession, and nutation.

In consequence of the first of these, *precession*, the mean



pole *P* describes round *π* the pole of the ecliptic, a small circle

Ppk, &c. in a period of about 25869 years; and, annually, an arc equal to $19''.9$ in a direction contrary to the order of the signs. By reason of this motion, or translation of the pole *P*, the longitudes, right ascensions, and declinations, but not the latitudes, of stars, are changed.

In consequence of the second, the *inequality of precession*, the true place of the pole differs from the mean place; and describes round the latter as a center which we may suppose to be *p*, a small circle *avv'*, according to the order of the signs, and, in a period of half a year. Now, since the true pole is supposed to move round *p*, whilst *p* itself moves, it is plain, that the true pole must describe some curve arising from these two compound motions: and if *b* were the pole's place when *p* coincided with *P*, then, after the description of *Pp*, the curve traced out by the true pole will be an epicycloid lying between *b* and *v*. By reason of this inequality in the translation of the pole along the circle *Ppk*, the precessions of stars in longitude, right ascension and declination, are, at different times of the year, slightly and variously affected.

In consequence of the third inequality, *nutation*, the true pole would describe round the mean pole, as a center, a small ellipse, contrary to the order of the signs, and in a period equal to that of the revolution of the Moon's nodes [$18^y\ 7^m$]; the major axis of the small ellipse coincides with the solstitial colure. This imaginary description of the ellipse, is supposed to proceed whilst the mean pole, the center of the ellipse, is, from the effect of precession, regressing: consequently, as in the case of the solar inequality, the true pole, by the combination of the above circular and elliptical motions, will describe an epicycloidal curve.

The epicycloidal curve will be described, if we abstract the consideration of the small circle described in consequence of the solar inequality of precession: but, in nature, the three causes operate together; hence, the real path of the true pole will be a curve described, by virtue of three motions simultaneously existing, one elliptical, the other two circular.

Since then, the small ellipse of nutation can be supposed to be described, only by abstracting the perturbation of the pole's place arising from the solar inequality of precession, are not the formulæ of nutation [pp. 163, 165,] deduced from such ellipse, erroneous? In theoretical strictness, they are erroneous; but, in practice and numerical exhibition, erroneous to so small a degree, that, without any danger of vitiating Astronomical computation, they may be considered as exact. For the radius of the small circle, described in consequence of the solar inequality, is only 0″.4345.

The investigation then of the nature of the curve described, by the pole of the equator, in consequence of the real motions impressed on it, is of no essential practical use, but rather one of curiosity. If indeed the curve were mathematically determined, then, from such curve, we might deduce expressions for the corrections of the right ascensions and north polar distances of stars; which corrections, must, as it is plain, include the three corrections of precession, the inequality of precession, and the nutation: but nothing, in effect, would be gained by this; for the above mentioned expressions would not consist of a single term, but would be separated, or expanded, into several; each of which would be equivalent to a separate correction, and numerically expounded, require a separate Table.

We have already advanced through a tolerably long Chapter on the subject of a small correction, amounting to a few seconds only, and which the instruments in Flamsteed's time were unable to detect. Still, the smallness of the correction is no test of its want of importance. And, that of nutation is peculiarly interesting and important, by reason of the elucidation and confirmation it bestows on Newton's system of Gravitation. On this account, we shall still continue the subject, in the ensuing Chapter, and there speak of the means used, by Bradley, in detecting the inequality of nutation, and in disengaging it, from the other inequalities, with which, under the form of an observation, it was combined.

CHAP. XVII.

On the means by which Bradley separated Nutation from the Inequalities of Precession and Aberration.—On the successive Corrections applied to the Apparent Place of a Star.—On the Secular Diminution of the Obliquity.

In treating of the several inequalities of precession, aberration, and nutation, it is necessary, in order to avoid being perplexed by the mere words of a theory, to recur to the simple facts of observation. Now, the observations of Bradley were on the Declinations of Stars, or, what amounts to the same thing at a given place, on their zenith distances *: and, the *phenomena* of his observations, were *changes* in the observed zenith distances of the same stars; happening sometimes, at different parts of the same year, and at other times, at corresponding seasons of different years.

The star γ *Draconis*, passing the meridian very near his zenith, and being, consequently, little affected by refraction, was the chief star of his observations. This star (see p. 126.) in March passed more to the south of the zenith by about 39″ than it did in September: that is, whatever was its mean place, the difference of its two zenith distances, or of its declinations, was, in half a year, observed to be about 39″. Other stars, also, changed their

* Bradley's observations were made with a *Zenith Sector*, which, strictly speaking, is a *declination* instrument: adapted, however, to measure the small portions, or minute differences of declination, and of zenith distances, near the zenith.

declinations. The changes of declination of a small star in *Camelopardalus*, with an opposite right ascension to that of γ *Draconis*, were observed at the same times as those of the latter star: then, Bradley argued, if these phenomena (changes of declination) arise from a real nutation of the Earth's axis, the pole must move as much towards γ *Draconis*, as from the star in *Camelopardalus*, but, (see p. 131.) this not being found to be the case, the hypothesis of a nutation of the Earth's axis would not account for the observed phenomenon: more strictly speaking, it would not completely account for it, for, in fact, some part of the observed changes of declination was due to the effect of nutation.

Bradley, as we have seen, (p. 132.) solved the above phenomena by the theory of aberration. Now, if such theory, with the known one of precession, would account for all observed changes of zenith distances, or, of north polar distances, then, there could be no changes but what arose from precession and aberration. Hence, since [p. 122.] the aberration is the same, at the same season of the year, the distance of γ *Draconis*, in September 1728, ought to have differed from his distance, in September 1727, only by the annual precession in N. P. D.: the distance, in September 1729, from the distance, in September 1727, by twice the annual precession in N. P. D.; and so on. Now, this was not the observed fact. In 1728, after the effect of precession had been allowed for, γ *Draconis* was nearer the north by about 0″.8 than in 1727. In 1729, nearer than in 1727, by 1″.5. In 1730, by 4″.5. In 1731, by nearly 8″. Here then was a new phenomenon, a change of north polar distance, indicating an inequality not yet discovered.

Bradley observed other stars besides γ *Draconis*; amongst others, the small star above-mentioned [p. 131.] of *Camelopardalus*: and, it is not a little worthy of notice, this same star, which, in the case of the former inequality, (that of aberration) directed him to reject the hypothesis of a nutation of the Earth's axis, here determined him to adopt it. For, within the same periods, the changes in north polar distance of γ *Draconis* and of the star in *Camelopardalus*, were equal and in contrary directions: that is, whilst the former, through the years 1728, 1729, 1730, 1731, was approaching the zenith, and consequently, the pole, the latter was, by equal steps, receding from the zenith, and consequently from the pole. These phenomena then of the changes in north polar distances, which (not like those of aberration that take place in different parts of the same year, and recur in the corresponding parts of different years) were observed, through a term of years, could adequately be explained by supposing, during that term, a nutation in the Earth's axis, *towards* γ *Draconis*, and, *from* the small star in *Camelopardalus*.

After 1731, Bradley observed contrary effects to happen; that is, γ *Draconis* receded from the zenith and north pole, and the star in *Camelopardalus*, by equal steps, approached those points; and this access and recess continued till 1741: (a period of more than nine years) after which, the former star again began to approach the zenith, and the latter to recede from it. These phenomena, then, that took place between 1731 and 1741, could be adequately explained by supposing, during that term, a nutation in the Earth's axis, *from* γ *Draconis* and *towards* the small star in *Camelopardalus*.

The mere hypothesis of a nutation, or vibratory motion in the Earth's axis, would have found little reception amongst men of science, if no arguments had been adduced to render such nutation probable: that is, if some physical cause, likely to produce it, had not been suggested. Previously, however, to the suggestion of the real and immediate physical cause, Bradley enquired, whether this seeming nutation of the Earth's axis was connected with any concomitant circumstance, or phenomenon: and such circumstance he found to be the position of the nodes of the Moon's orbit.

The star γ *Draconis* was (after the effects of precession had been allowed for) most remote from the pole when the Moon's node was in *Aries*, and least, when in *Libra*: and after a complete revolution of the Moon's nodes, the distances of all the observed stars, at the end, differed from the distances at the beginning, by the effect of precession only. Hence, the phenomenon of a nutation, and the longitude of the Moon's node were connected. But, the inclination of the Moon's orbit varies with the longitude of the node: the former is greatest, when the latter is equal to nothing; least, when the latter is six signs. Hence, the nutation and inclination were connected together. But, the Moon's action, on the bulging equatoreal parts of the Earth, is greater the more distant the Moon is from the equator; and her mean action greater, the greater the inclination of her orbit, and besides, her mean action varying with a variation of the inclination. Hence, the phenomenon of the nutation was connected, with the variable action of the Moon in causing precession; and this last connexion made nutation the effect, and the variation of the Moon's action the cause. And this was the physical cause which seemed to Bradley to afford an adequate solution of the phenomenon he observed: and subsequent researches have confirmed the sagacity of his conjectures.

The real distance of any star (γ *Draconis* for instance) from the north pole of the equator, is increased continually and constantly, by the effect of precession only. The variations in that distance from aberration and nutation are periodical, and recur, the former in the space of a year, the latter in the time of a revolution of the Moon's nodes. Hence, although, as it was asserted in p. 175, in any phenomenon of a change in the north polar distance of a star, the effects of several causes may be blended together and compounded; yet the method is plain, by which we may disengage and separate them. For instance, since the revolution of the Moon's nodes is completed in about 18 years, and since the aberration and the solar inequality are the same, at the same time of the year, the north polar distance of γ *Draconis* in 1745, ought to differ from its north polar distance in 1727 almost solely by the effect of precession: that is, since the latter N. P. D. was 38° 28′ 10″.2, [see p. 57.] and the precession [see p. 146.] 0″.8, the N. P. D. in 1745 ought to have been 38° 27′ 56″: and this difference was, by Bradley's observations, [see *Phil. Trans.* No. 485, p. 27.] found very nearly to exist.

Again, between September 6, 1728, and September 6, 1730, the aberration and solar inequality being the same, the respective north polar distances of γ *Draconis* at those periods ought to differ, by twice the annual precession in N. P. D, and by the effect of nutation: and hence the effect of nutation in an interval of two years, between two known positions of the Moon's ascending node, would be known.

Again, between September 6, 1728, and March 6, 1729, the solar inequality being the same, the respective north polar distances of γ *Draconis* ought to differ by the precession in N. P. D.

due to half a year, by the nutation for the same time, and [see p. 122.] nearly by the sum of the greatest aberrations in N. P. D.; and the whole difference would consist almost entirely of aberration, since the precession and nutation together would not amount to a second.

Again, the Moon's ascending node being, March 28, 1727, in *Aries*, and July 17, 1736, in *Libra*; the respective N. P. D. of γ *Draconis* would differ by the precession due to 9 years 3 months, by the solar inequality of precession, by aberration, and by the sum of the two maximum effects of nutation. But, between March 28, 1727, and March 28, 1736, (since then the solar inequality and the aberration would be the same) the north polar distances would differ by the effect of precession, (a known quantity) and nearly the sum of the two maximum effects in nutation. Hence, it would be easy to disengage, and numerically exhibit, a material element, the maximum effect of nutation.

By examining various and numerous observations and by discriminating from amongst them, those that happened at particular conjunctures, Bradley found abundant confirmation of the truth of his two theories, aberration and nutation. During a period of more than twenty years, he accounted for the phenomena of observation, that is, the changes in the declinations of various stars, by making those changes or variations consist of three parts; the first due to precession; the second to aberration; and, the third to nutation: the quantities and laws of the two latter, being assigned on the principles and by the formulæ of his theories.

We cannot sufficiently admire the patience, the sagacity, and the genius of this Astronomer, who, from a previously unobserved variation not amounting to more than 40 seconds, extricated, and reduced to form and regularity, two curious and beautiful theories.

The following Table exhibits the coincidence of his theories with observations. [See *Phil. Trans.* No. 485, p. 27.]

γ Draconis.		South of 38° 25′.	Precession.	Aberration.	Nutation.	Mean Distance.
1727	Sept. 3	70″.5	− 0″.4	+ 19″.2	− 8″.9	80″.4
1728	Mar. 18	108.7	− 0.8	− 19	− 8.6	80.3
	Sept. 6	70.2	− 1.2	+ 19.3	− 8.1	80.2
1729	Mar. 6	108.3	− 1.6	− 19.3	− 7.4	80.0
	Sept. 8	69.4	− 2.1	+ 19.3	− 6.9	80.2
1730	Sept. 8	68.0	− 2.9	+ 19.3	− 3.4	80.5
1731	Sept. 8	66.0	− 3.8	+ 19.3	− 1.0	80.5
1732	Sept. 6	64.3	− 4.6	+ 19.3	+ 2.0	81.0
1733	Aug. 29	60.8	− 5.4	+ 19.0	+ 4.8	79.2
1734	Aug. 11	62.3	− 6.2	+ 16.9	+ 6.9	79.9
1735	Sept. 10	60.0	− 7.1	+ 19.3	+ 7.9	80.1
1736	Sept. 9	59.3	− 8.0	+ 19.3	+ 9.0	79.6
1737	Sept. 6	60.8	− 8.8	+ 19.3	+ 8.5	79.8
1738	Sept 13	62	− 9.6	+ 19.3	+ 7.0	78.7
1739	Sept. 2	66.6	− 10.5	+ 19.2	+ 4.7	80.0
1740	Sept. 5	70.8	− 11 3	+ 19.3	+ 1.9	80.7
1741	Sept. 2	75.4	− 12.1	+ 19.2	− 1.1	81.4
1742	Sept. 5	76.7	− 12.9	+ 19.3	− 4.0	79.1
1743	Sept. 2	81.6	− 13.7	+ 19.1	− 6.4	80.6
1745	Sept. 5	86.3	− 15.4	+ 19.2	− 8 9	81.2
1746	Sept. 17	86.5	− 16.2	+ 19.2	− 8.7	80.8
1747	Sept. 2	86.1	− 17.0	+ 19.2	− 7.6	80.7

A brief explanation will suffice for this Table. The *apparent* place of a star is deduced from the *mean*, by applying to the latter the several corrections: or, the mean is deduced from the apparent, by applying the same corrections with contrary signs.

If therefore γ *Draconis* were, at the beginning of any period, a certain number of seconds, south of the zenith, or south of any particular division in the zenith sector; it would, at the end of the period, be *really* farther from the zenith by precession; *really* farther or nearer, by nutation; and *apparently* nearer or farther by aberration. By the *mean distance* of the star from the division 38° 25′ of the zenith sector (see last column in preceding Table), Bradley means the distance on March 27, 1727, which would have been, had there been neither nutation, nor aberration. But in that year, the nutation, (the node of the Moon's orbit being in *Aries*) was the greatest. Hence, in September 1727, (see the first horizontal row of the preceding Table) the *observed* or *apparent* distance of γ *Draconis* would differ from the mean, by the effect of precession [$\frac{1}{2} \times .8$] in half a year, by the maximum effect of aberration (see p. 122), and by nearly the greatest effect of nutation. The *apparent* distance then of the star being 70″.5, the *mean* (according to Bradley) would be

$$70''.5 - 0''.4 + 19''.2 - 8''.9 = 80''.4.$$

Again, reversing the process. If 80″ were the mean distance, then on March 6, 1729, the star would appear by aberration farther distant about 19″.3 : would really be more distant by the effect of two years precession in N. P. D. ($2 \times .8$;) and, would really be more distant than it would be if the Moon's orbit were at its mean inclination (the ☊ being either in ♋ or in ♑) by the effect of nutation (7″.4). The apparent * distance therefore would be

$$80'' + 1''.6 + 19''.3 + 7''.4 = 108''.3$$

The mean distances deduced according to the preceding explanation, by means of corrections, from Bradley's two theories of aberration and nutation, and from the known effect of precession, ought, if the theories be true, to be invariably the same: and their very near equality (see last column in Table, p. 179,) establishes, almost beyond doubt, the truth of those theories.

The division in the zenith sector, from which, as a fixed point, Bradley measured the distances of γ *Draconis*, and the north polar distance of which he calls 38° 25′, is not the division corresponding to the zenith of the observatory at Wansted,

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* There is some violation of the propriety of language in calling that *apparent*, which depends on real causes, viz. the changes of the place of the pole from precession and nutation. In strictness, apparent should have been confined to *aberration*, refraction,

and consequently, not the co-latitude of the place. If it had been, the apparent north polar distances of γ *Draconis* on Sept. 3, 1727, and on March 6, 1729, would have been respectively, 38° 26′ 10″.5, and 38° 26′ 48″.3.

The formulæ for the computation of the variations of the north polar distances of stars in precession, aberration and nutation, are determined by theory, and by processes purely mathematical; the numerical coefficients of the formulæ from numerous and accurate observations.

More exact instruments and multiplied observations, may, perhaps, indicate new inequalities in the north polar distances, and right ascensions of stars. If such should exist, they may be detected by the method pursued by Bradley in the case of the inequalities of aberration and nutation. Suppose, for instance, that the star γ *Draconis*, should, from the year 1810, during 20 years, be observed; then, if the several observed north polar distances reduced, by the corrections of precession, aberration, and nutation [see p. 179,] to the beginning of 1810, always gave, as a result, the same *mean* north polar distance, the laws and quantities of the several corrections would seem to be right, and, there would apear no inequality, requiring an explanation. But, if the mean north polar distance should not result the same from every observation, then, either the laws and quantities of the common corrections would require some alteration: or, if these were held to be exact, the errors of the resulting mean distances, would appear as new inequalities, as phenomena without an explanation, as effects with unassigned causes.

The comparison of numerous observations indicate, in several stars, inequalities of the latter description, which cannot be accounted for, by altering, either the laws, that is, the analytical expressions, or the numerical coefficients of the formulæ of the corrections. An alteration adapted to one star would not suit others. The above inequalities, then, following no general law, are attached and appropriated, as peculiar errors, to the several stars; and, in the Tables, are classed, as *proper motions*. (See Maskelyne's Table of the principal Stars for 1802; also Wollaston's *Fasciculus*, Appendix, p 25.)

These unaccounted for deviations in the places of stars may arise from *proper* motions, truly such, or, as Dr. Herschell supposes, from some translation of the solar system. They cannot be accounted for, by supposing the observations to be inaccurate, the instruments deranged, or, the formulæ of corrections erroneous. For, these causes of irregularity, if they existed, would equally affect two contiguous stars. Now, the relative position of *Arcturus* and of a small contiguous star marked *b* (in the Celestial Maps) was during the last century considerably changed.

The proper motion of *Arcturus* is, relatively to other stars, very considerable both in right ascension and in north polar distance. That of *Antares*, and of *α Herculis* in right ascension nothing. (See Maskelyne's Tables, Tab. X.)

The obliquity of the ecliptic, can be observed from year to year (see p. 46,) like any other Astronomical phenomenon; for instance, the north polar distance of a star. Now, the obliquity is subject to two periodical inequalities: one (see p. 155,) arising from the solar inequality of precession, and passing through all its degrees of variation, in the space of half a year; the other (see p. 167,) arising from nutation, and passing through all its degrees of variation, in the space of half a period of the revolution of the Moon's nodes. The observed or apparent obliquity, then, must, by means of the corrections due to the preceding inequalities, be reduced to the mean. Now, by an examination of the several successive *mean* obliquities, resulting from this method, it appears that the mean obliquity of the ecliptic does not remain invariable. During a great number of years, it appears to have been slightly and gradually diminishing. The diminution in 100 years, which is called the *secular*, is stated to be 52″. And, on the supposition that it proceeds equably and gradually, the diminution, from any proposed epoch is proportional to the numbers of years elapsed. Thus, the mean obliquity of the ecliptic being, for the beginning of the year 1800, 23° 27′ 57″, for the beginning of 1810 it would be 23° 27′ 57″, $-10 \times 0''.52$, that is, 23° 27′ 51″.8: for the year 1794, $23° 27' 57'' + 6 \times 0''52$, that is, 23° 28′ 0″.12.

If, however, the *apparent* obliquity were required for April 1, 1810, t hen, besides the proportional part for the secular diminution, we must correct for the solar inequality of precession and for the nutation; and the operation would stand thus:

1800. Mean obliquity - - - - - - - - - - - -	23°	27′	57″
Secular diminution for 10 years - - - - - - -	0	0	−5.2
Proportional secular diminution to April 1 - -	0	0	− .13
* Solar inequality due to April 1 - - - - - -	0	0	+ .4
Nutation - - - - - - - - - - - - - - - - - - -	0	0	−8.76
1810, April 1, Apparent obliquity - - - - - -	23	27	43.3.

This secular diminution of the obliquity of the ecliptic is not, like the inequalities called the proper motions of stars, without an assignable cause. It arises, (so it is proved on the principles and by the processes of physical Astronomy) from the attractions of the planets; which attractions cause not the equator, but the ecliptic to vary: that is, they cause the apparent path described by the Earth, amongst the fixed stars, to vary. But, the investigations that prove the diminution of the obliquity, assign also its limits: which being attained, the obliquity would begin to be increased. [See Euler, *Mem. de l'Acad.* Berlin, 1754, p. 296.]

This change in the obliquity, arising from a change in the ecliptic itself, must affect the latitudes of stars, which (as we have seen in pp. 139, 169,) remain unaltered by the inequalities of precession and nutation. The latitude of *Regulus*, for instance, in 100 years would be increased by $20''.5$: and, accordingly, its *annual variation of latitude* would be $+ 0''.205$. In the Catalogue of the 9 principal stars, which Dr. Maskelyne used to insert at the end of the Nautical Almanack, the last column expresses the variations of latitudes arising from the diminution of the obliquity of the ecliptic.

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* In the new French Tables, and in Mr. Vince's, vol. III, p. 7, Introduction, the proportional diminution, and the solar inequality are included under a single Table. The analytical expression for the obliquity, is

$$E - \frac{0''.52 \times n}{365} + 0''.4345 \times \cos. 2 \odot + 9''.63 \times \cos. N,$$

E being the obliquity at the beginning of the year, and N the supplement of the node.

The same cause, the attraction of planets on the Earth, that produces a change in the obliquity, will produce one in the precession of the equinoxes. Hence, besides the variations in the longitudes of stars already enumerated, one will arise from this cause.

If the regressive motion of the equinoctial points be subject to any secular equation, the length of the tropical year will. This motion is now about 0″.386 *faster* than it was at the beginning of the Christian Æra; and, on that account, the *secular* diminution of the length of the tropical year has been about half a second. And the above is one of the causes that render difficult, as we stated in p. 68, the consideration of the length of the *mean* solar year.

In the instances given in pages 55, 57, where exemplification of method and illustration, and not extreme accuracy, were required, the obliquity was stated in degrees, minutes, &c., without any great attention to its exact values at those times. If, however, great accuracy should be required, (as in deducing the right ascension of the Sun from his longitude resulting from solar tables) then, the obliquity must be determined by the method given in p. 183, l. 3.

The knowledge of the places of the fixed stars has been properly said to be the foundation of all Astronomy; and, the places cannot be determined independently of the preceding corrections. These, as we have seen, are naturally separated into two distinct classes: one, consisting of refraction, parallax, and aberration, applied to an observation at the instant at which it is made, and entirely independent of physical Astronomy; the other, consisting of precession, the solar inequality of precession, and of nutation: corrections not intended to divest an observation of any optical or illusory inequality, but to allow for the real change in the place of the pole of the equator that has happened between two observations, either a present and a past recorded observation; or, two preceding recorded observations. These corrections are dependent on physical Astronomy for a knowledge of their causes, and for the expressions of their laws.

We may perceive then in what way or manner the fixed stars, although beyond the influence of the disturbing forces of the solar system, require, for the determination of their positions, the aid of physical Astronomy. Their place, indeed, is not changed, but that of the observer is.

If we turn our attention from *fixed* to *wandering* stars or planets, still more, and for a two-fold cause, will our researches require the aid of physical Astronomy. For, in the interval between observations, both the place of the observer and that of the observed body will be changed. Here then a new field of enquiry is opened. We may use the settled places of the fixed stars, for the purpose of determining those of a planet, at certain epochs. But the places thus determined will be only certain elements in the planet's theory. The return of the planet after any period, to the same, or nearly the same place; the times of moving from one position to another; these, and other points, must depend on the real curve described by the planet, and on the laws of its curvilinear motion. And, although observation will be the main support of the enquiry, still alone it will not be sufficient. We must guide it by aid derived from the discoveries of Kepler, and the inventions of Newton.

In this new class of investigations, the form and elements of the Earth's orbit, and the laws of its motion, first claim our attention.

CHAP. XVIII.

ON THE SOLAR THEORY.

On the Orbit described by the Earth.—The Law of its Motion.—Kepler's Problem.—Its Use.

By computations founded on the observed right ascensions and declinations of the Sun, its longitude may be determined [see p. 55]. Conversely, if by the knowledge of his real motion his longitude were known, then, by the simple rules of spherical Trigonometry, and from the obliquity of the ecliptic, we could compute his right ascension and declination; and afterwards, by direct observation, ascertain the accuracy of the computation. According to this latter method, the Sun's place is said to be determined by *Solar Tables*.

If the Earth really moved uniformly in a circle, the daily increments of the Sun's longitude would be equal; and, the merely registered observations of his longitude, each day on any one year, would themselves constitute Solar Tables: since, on all succeeding years, at the same distance of time from the equinox, the longitude would be always the same, as also would the right ascension, if the obliquity of the ecliptic remained invariable. In this case, therefore, the sole enquiry, of any difficulty, would be concerning the exact time of the equinoxes.

But, the Earth certainly does not move uniformly in a circle, the Sun being in its center. Observations, of the simplest kind, are sufficient to shew this. For instance, by computations founded on the Sun's observed right ascension and declination, the following results were obtained:

1807. July 1, ☉'s longitude - - - - - - - -	3ˢ	8°	38′	44″
- - - - - 2. - - - - - - - - - - - - - - - -	3	9	35	57
Increase of longitude, or daily motion - - -	0	0	57′	13″
Again,				
Jan. 1, ☉'s longitude - - - - - - - - - - -	9ˢ	10°	18′	56″
- - - 2. - - - - - - - - - - - - - - - - - - -	9	11	15	7
Increase of longitude, or daily motion - - -	0	1°	1′	11″

Here the daily motions, instead of being equal, are to one another as 3671 to 3433.

But, may not the Earth move uniformly in a circle, the Sun not being, indeed, in the center, but situated elsewhere within the circumference? For, the increases of longitudes, or daily motions, are merely angular spaces, and may at different distances correspond to equal real spaces. If such be the case, that is, if the Earth move uniformly, then, since an angle varies as its subtending arc directly, and as its radius inversely, the above daily motions ought to vary inversely as the respective distances of the Sun from the Earth on July 1, and January 1 : since, the subtending arcs, the described parts of the Earth's orbit, are supposed to be equal. Now, it happens, that we possess easy means of ascertaining the ratio of the distances of the Sun from the Earth: for, his apparent diameter varies as the real diameter directly, and the distance inversely; therefore, since the real diameter is invariable, as the distance inversely. By instrumental measurements, the apparent diameters of the Sun in July and January were respectively 31′ 31″, and 32′ 35″.6; in the ratio, therefore, of 1 to 1.0339. But the daily motions have been shewn to be as 3671 to 3433, that is, nearly as 1.0693 : 1; consequently, the real arcs described by the Earth are not equal, or the Earth does not move uniformly in a circle, whether the Sun be supposed to be situated in or out of the center.

The above is not the exact description of Kepler's reasonings. But, by methods, not very dissimilar from those which have been stated, that great Mathematician first examined and then rejected his original hypothesis of the Earth's motion in a circle, the Sun not being in the center; and afterwards examined and established the last and true one, that *of the Earth's motion in an ellipse, the Sun being situated in one of the foci.*

This important truth was not soon found out: there was no analogy to suggest it: and no direct process could lead to it; for, physical Astronomy was unknown. It was the fruit of conjecture, of trial, and of unwearied research.

In point of history, it was preceded by another truth, nearly of equal importance; which is, that *the areas or spaces which may be conceived to be described by a line joining the Sun and Earth, are proportional to the times of description.*

Kepler did not establish this generally, but for those portions of the ellipse that lie contiguous to the extremities of the axis major: the *aphelion* and *perihelion* of the orbit, the Earth revolving; or, the *apogee* and *perigee* of the solar ellipse, the Sun being supposed, for Astronomical convenience, to revolve round the Earth. [See p. 11.]

If we take two arcs (A, a) at the greatest and least distances (D, d) of the Earth from the Sun, then, by Kepler's discovery of the equable description of areas,

$$\frac{A \times D}{2} = \frac{a \times d}{2}, \text{ consequently,}$$

$$\frac{A}{D} : \frac{a}{d} :: d^2 : D^2.$$

Hence $\frac{A}{D}$, $\frac{a}{d}$, which are the measures of small angles described about the Sun; or, the exponents of the angular velocities; or, as they were called in p. 187, the daily motions and increments of longitudes, are not inversely as the distances, (which they would be, if their inequality arose from merely an optical cause,) but, inversely as the squares of the distances.

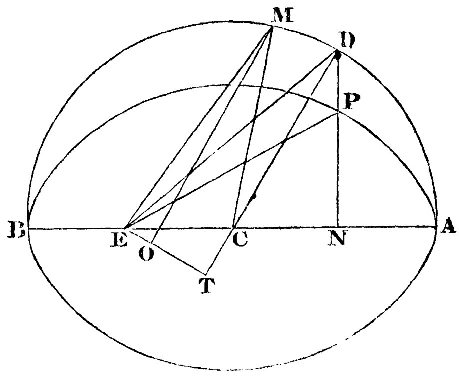
In p. 187, l. 28, the proportion of the daily motions, (or angular velocities,) was stated to be as 1 : 1.0693; that of the distances, to be as 1 : 1.0339. Hence, according to the result in the preceding paragraph, this proportion ought to be true:

$$1^2 : (1.0339)^2 :: 1 : 1.0693;$$

and, on trial, it will be found to be nearly true.

By observation, Kepler's law of the equable description of areas is found to be true; not only at the extremities of the major axis, but at all points of the ellipse. Newton, on mechanical principles, shewed that it must be true. [See *Principia*, Prop. I. Sect. 2.]

These two discoveries, of the elliptical orbit of the Earth, and of the equable description of areas, enabled Kepler to make a grand step towards the assigning the Sun's place at any proposed time; or, towards the construction of Solar Tables. If the Sun be in the apogee on the first day of July, where will be his place, at the end of the month? This question would be resolved by setting of the area AEP proportional to the time, or to 30 days: that is, since the whole ellipse is described in $365\frac{1}{4}$ days, by so drawing EP, that the area $AEP =$ whole area of ellipse $\times \frac{30}{365\frac{1}{4}}$.



The cutting off an elliptical area proportional to the time became then, the main object of enquiry: and the problem in which this is effected, from the importance of its investigation, and the celebrity of its proposer, has, for distinction, been called *Kepler's Problem*.

The difficulties of the problem are not inconsiderable. Various means and mathematical artifices have been resorted to, in order to lessen them. Hence have resulted many excellent solutions; the best indeed, not established by obvious and direct processes, but, by processes found, after many trials, to be the most commodious. Since it would be wide of our present purpose, historically to trace the problem through its several stages of successive solution, we shall proceed, first, to deduce the equations that contain the exact solution of the problem, and next, to lay down a method, approximate indeed, but leading, by easy and certain steps, to results sufficiently exact for all practical purposes.

If we suppose a fictitious body to revolve uniformly in the circle *ADMB*, completing its revolution in the same period in which *P*, representing the real body, and following the law of its elliptical motion, describes the entire ellipse: then, if after any assigned time (t) the former body is at *M*, and the latter at *P*, the angle *ACM* is the *mean*, and the angle *AEP* the *true anomaly*.

The problem will be as effectually solved by finding the angle *AEP* proportional to the time, as by cutting off the area *AEP* proportional to the time: and since *ACM*, (*AM* being described uniformly,) is proportional to the time; the problem announced, in formal terms, is this: *It is required to find the true anomaly in terms of the mean.*

The two bodies are supposed to set off from the *apside*, or with reference to the Sun, the *aphelion A*; consequently, the place *M* of the body in the circle must be before that of *P*: for, since (p. 188, l. 19,) the angular velocity varies inversely as the square of the distance, and the distance *EA* is the greatest, the angular velocity, of the body in the ellipse at *A*, is the least, less certainly therefore than the mean angular velocity, with which the body in the circle constantly revolves.

Besides the mean and true, another anomaly called the *Eccentric*, represented by $\angle DCA$, has been invented, and, specially, for the purpose of facilitating the solution of the problem. It is made a mean term between the true and mean anomaly: one equation connects the true and eccentric: a second, the eccentric and mean anomaly. In order to deduce the second, that is, to express the eccentric anomaly in terms of the mean:

Let t = time of describing *AP*,

P = periodic time in the ellipse,

$a = CA$,

$ae = EC$,

$v = \angle PEA$,

$u = \angle DCA$; ($\therefore ET$, perpendicular to $DT = EC \times \sin. u$

$\varrho = PE$,

$\pi = 3.14159$, &c.;

then, by Kepler's law of the equable description of areas,

$$t = P \times \frac{\text{area } PEA}{\text{area of ellip.}} =^{*} P \times \frac{\text{area } DEA}{\text{area } \odot} = \frac{P}{\pi a^2}[DEC + DCA]$$

$$= \frac{P}{\pi a^2}\left[\frac{ET . DC}{2} + \frac{AD . DC}{2}\right] = \frac{Pa}{2\pi a^2}[EC . \sin. u + DC . u]$$

$= \frac{P}{2\pi}[e \sin. u + u]$: hence, if we put $\frac{P}{2\pi} = \frac{1}{n}$,

we have

$$nt = e . \sin. u + u \qquad [a],$$

an equation connecting the mean anomaly nt, and the eccentric u.

In order to find the other equation, that subsists between the true and eccentric anomaly, we must investigate, and equate, two values of the radius vector ϱ, or EP.

First value of ϱ, in terms of v the true anomaly;

$$\varrho = \frac{a . (1 - e^2)}{1 - e . \cos. v} \dagger \qquad [1]$$

second, in terms of u the eccentric anomaly

$$\varrho = a(1 + e . \cos. u) \qquad [2]$$

For, $\varrho^2 = EN^2 + PN^2$
$= EN^2 + DN^2 \times (1 - e^2)$
$= (ae + a . \cos. u)^2 + a^2 (\sin. u)^2 . (1 - e^2)$
$= a^2 [e^2 + 2e . \cos. u + (\cos. u)^2] + a^2 . (1 - e^2)(\sin. u)^2$
$= a^2 [1 + 2e . \cos. u + e^2 (\cos. u)^2]$

Hence, extracting the square root,

$$\varrho = a(1 + e . \cos. u.)$$

Equating the expressions [1], [2], we have

$$(1 - e^2) = (1 - e . \cos. v) . (1 + e \cos. u), \text{ whence,}$$

$\cos v. = \frac{e + \cos. u}{1 + e . \cos. u}$, an expression for v in terms of u; but, in order to obtain a formula fitted to logarithmic computation, we must find an expression for $\tan. \frac{v}{2}$: now, [see *Trig.* p. 20,]

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* Vince's *Conics*, p. 15. 4th Ed. † Ibid. p. 23. Bridge, p. 93.

$$[b]\ \tan. \frac{v}{2} = \sqrt{\left[\frac{1 - \cos. v}{1 + \cos. v}\right]} = \sqrt{\left[\frac{(1-e)(1-\cos. u)}{(1+e)(1+\cos. u)}\right]}$$

$$= \sqrt{\left[\frac{1-e}{1+e}\right]} \tan. \frac{u}{2}.$$

These two expressions [*a*] and [*b*], that is,

$$nt = e . \sin. u + u$$

$$\tan. \frac{v}{2} = \sqrt{\left[\frac{1-e}{1+e}\right]} . \tan. \frac{u}{2},$$

analytically resolve the problem, and, from such expressions, by certain formulæ belonging to the higher branches of analysis may v be expressed in the terms of a series involving nt *.

Instead, however, of this exact but operose and abstruse method of solution, we shall now give, as we proposed (p. 190,) an approximate method of expressing the true anomaly in terms of the mean.

MO is drawn parallel to *DC*. (1.) Find the half sum of the angles at the base of the triangle *ECM*, from this expression,

$$\tan. \tfrac{1}{2} [CEM - CME] = \tan. \tfrac{1}{2} [CEM + CME] \times \frac{1-e}{1+e},$$

[see *Trig*. p. 27,] in which, $CEM + CME = ACM$, the mean anomaly.

(2.) Find CEM by adding $\frac{1}{2}[CEM + CME]$ and $\frac{1}{2}[CEM - CME]$ and use this angle as an approximate value to the eccentric anomaly *DCA*, from which, however, it really differs by $\angle EMO$.

---

* The following is the series for v in terms of nt;

$$v = nt -$$

$$\left[2e - \frac{1}{4}e^3 + \frac{5}{96}e^5\right] . \sin. nt + \left[\frac{5}{4}e^2 - \frac{11}{24}.e^4 + \frac{17}{192}.e^6\right] \sin. 2nt.$$

$$- \left[\frac{13}{12}.e^3 - \frac{43}{64}.e^5\right] . \sin. 3nt + \left[\frac{103}{96}.e^4 - \frac{451}{480}.e^6\right] . \sin. 4nt$$

$- \left[\frac{1097}{960}e^5 . \sin. 5nt + \frac{1223}{960}e^6\right] \sin. 6nt$, in which the approximation is carried to quantities of the order e^6.

(3) Use this approximate value of $\angle DCA = \angle ECT$ in computing ET which equals the arc DM: for, since [see p. 188,]

$t = \frac{P}{\text{area } \odot} \times DEA$, and (the body being supposed to revolve in the circle ADM) $= \frac{P}{\text{area } \odot} \times ACM$; $\therefore$ area AED = area ACM,

or, the area DEC + area ACD = area DCM + area ACD;

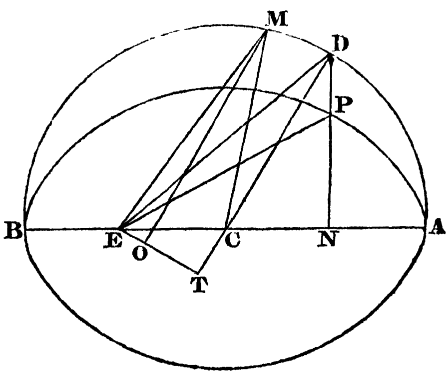
consequently, the area DEC = the area DCM,

and, expressing their values,

$$\frac{ET \times DC}{2} = \frac{DM \times DC}{2} \text{ and } \therefore ET = DM.$$

Having then computed $ET = DM$, find the sine of the resulting arc DM, which sine $= OT$: the difference then, of the arc and sine $(ET - OT)$ gives EO.

(4.) Use EO in computing the angle EMO, the real difference, between the eccentric anomaly DCA, and the $\angle MEC$: add



the computed $\angle EMO$ to $\angle MEC$, in order to obtain $\angle DCA$. The result, however, is not the exact value of $\angle DCA$, since $\angle EMO$, has been computed only approximately; that is, by a process which commenced by assuming $\angle MEC$, for the value of the $\angle DCA$.

For the purpose of finding the eccentric anomaly, this is the entire description of the process, which, if greater accuracy be required, must be repeated; that is, from the last found value of $\angle DCA = \angle ECT$, ET, EO, and $\angle EMO$ must be again computed.

(5.) A sufficiently correct value of the eccentric anomaly (u)

being found, investigate the true (v), from the formula [b] p. 192, that is,

$$\tan.\ \frac{v}{2} = \sqrt{\left[\frac{1-e}{1+e}\right]} \cdot \tan.\ \frac{u}{2}.$$

EXAMPLE I.

The Eccentricity of the Earth's Orbit being .01691, *and the Mean Anomaly* = 30°, *it is required to find the Eccentric and the true Anomalies.*

(1) Log. tan. 15 - - - - - - - 9.4280525

Log. $(1-e)$, or log. .98309 - - $\bar{1}$.9925933

Arith. Comp. $\overline{1+e}$, or of 1.01691 $\bar{1}$.9927218

Log. tan. $\frac{1}{2}$ $(CEM-CME)$ - - 9.4133676 = log. tan. 14° 31′ 22″.

(2.) $\frac{1}{2}$ $(CEM-CME) = 14°\ 31'\ 22''$

$\frac{1}{2}$ $(CEM+CME) = 15$

CEM = 29 31 22. 1st approx^{e}. value of CDA.

(3.) Log. sin. 29° 31′ 22″ - - - 9.6926438

Log. .01691 - - - - - - - - $\bar{2}$.2281436

+ Log. (arc = rad^{s}.) - - - - - 5.3144251

Log. DM in seconds - - - 3.2352125 = log. 1718.7.

DM = 28′ 38″.7, and its sine expressed in seconds differs from the arc DM by less than half a second.

(4.) The operation prescribed in this number [see p. 193, l. 12.] is, in this case, needless, since the correction for the angle EMC is so small, that the first approximate value of the eccentric anomaly may be stated at 29° 31′ 22″.

(5.) Log. tan. $\frac{u}{2}$, or log. tan. 14° 45′ 41″ - - - - 9.4207651

$\frac{1}{2}$ log. $(1-e)$, or $\frac{1}{2}$ log. .98309 - - - - - - - - 4.9962966

$\frac{1}{2}$ log. $(1+e)$, or $\frac{1}{2}$ log. 1.01691 - - - - - - - 4.9963608

Log. tan. $\frac{v}{2}$ - - - - - - - - - - - - - - - - 9.4134225

$= \log.\ \tan.\ 14^\circ\ 31'\ 28''$;

$\therefore$ the true anomaly $= 29^\circ\ 2'\ 56''$.

The difference of the mean and true anomalies, or, as it is called, the *Equation of the Center*, equals $57'\ 4''$.

If the eccentricity had been assumed $= .016813$, or $.016791$, the equation of the center would have resulted $= 56'\ 46''.4$, or $= 56'\ 41''.4$, respectively.

EXAMPLE II.

Instead of .01691, *suppose the Eccentricity of the Earth's Orbit be taken at* .016803 *, *and the Mean Anomaly, reckoning from Perigee, according to the Plan in the new Solar Tables, be* $10^s\ 12^\circ\ 22'\ 12''.4$.

Taking out 6 signs, we have the mean angular distance from apogee $= 4^s\ 12^\circ\ 22'\ 12''.4$.

(1.) Log. tan. $66^\circ\ 11'\ 6''.2$ 10.3552029
Log. .983197 - - - - $\bar{1}.9926406$
Arith. comp. 1.016803 $\bar{1}.9927645$

10.3406080 = log. tan. $65^\circ\ 27'\ 56''.4$.

(2.) $\frac{1}{2}(CEM - CME)$ $65^\circ\ 27'\ 56''.4$
$\frac{1}{2}(CEM + CME)$ 66 11 6.2

131 39 2.6 approx^e. value of CDA (u)

(5.) Log. tan. $\frac{u}{2}$, or log. tan. $65^\circ\ 49'\ 31''.3$ - - - 10.3478640
$\frac{1}{2}$ log. .983197 - - - - - - - - - - - - - 4.9963203
$\frac{1}{2}$ arith. comp. 1.06803 - - - - - - - - - - 4.9963816

Log. tan. $\frac{v}{2}$ - - - - - - - - - - - - - - 10.3405659;

$\therefore\ \frac{v}{2} = 65^\circ\ 27'\ 49''.2$, and $v = 4^s\ 10^\circ\ 55'\ 38''.4$;

$\therefore$ the true anomaly, reckoning from perigee, $= 10^s\ 10^\circ\ 55'\ 38''.4$, and difference of the mean and true anomaly $= 1^\circ\ 26'\ 34''$.

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* In 1750, the eccentricity was 0.016814, and, the secular variation being .000045572, in 1800, ti was 0.016791.

This difference, or *Equation of the Center*, is stated, for 1800, in Lalande's Tables, vol. I. *Astron.* ed. 3. p. 23, at 1° 26′ 38″.6; but, in the new Tables, Vince, vol. III. p. 38, at 11^s 28° 32′ 44″.4. Now the difference of this, and of 12 signs, is 1° 27′ 15.6, which is still greater than Lalande's result by 45″. But, it is purposely made greater; for these 45″ are the sum of the maxima of several very small equations. [See the explanation in Delambre's Introduction, and in Vince's, p. 6.]

In the two preceding Examples, it appears that, by reason of the small eccentricity of the Earth's orbit, the true anomaly and equation of the center are found by an easy and short process; no second approximation being found necessary. It appears also, by the results, that a small change in the eccentricity makes a variation of several seconds in the equation of the center. Thus arranging the results in the preceding Examples:

Mean Anomaly.	Eccentricity.	Equation of Center.
30° 0′ 0″	.016910	0° 57′ 4
30 0 0	.016813	0 56 46.4
30 0 0	.016791	0 56 41.4

Now, by observation and theory, it appears, that the eccentricity of the Earth's orbit is diminishing. Hence, Tables of the equation of the Earth's orbit, computed for one epoch, will not immediately suit another: but, they may be made to suit, by appropriating a column to the *secular variation* of the equation of the center. Thus, in Lalande's Tables, tom. I. ed. 3. p. 18, the equation of the center is stated to be 56′ 41″.2, and in a column by the side, the corresponding secular diminution to be 9″.36. Now Lalande's Tables were computed for 1800*: (when the eccentricity of the Earth's orbit was .016791) consequently, for the *preceding* epochs of 1750, 1550, the equations of the center would be 56′ 41″.2 + 4″.68, and 56′ 41″.2 + 23″.44, that is, 56′ 45″.9, and 57′ 4″.6 respectively. These are nearly the results previously obtained in p. 194, which they ought to be,

* Delambre states, that Lalande's Tables answer better to the epoch of 1809, or 1810, than to 1800. See Introduction to his new Tables.

since, the secular diminution of the eccentricity being .000045572, the eccentricities corresponding to 1750 and 1560 will be, nearly, .016813 and .016910.

By this mode we may also reconcile the two results in Example 2, in p. 195; for the equation of the orbit in Lalande's Tables is 1° 26′ 30″, (that is, for an eccentricity, .016791) therefore, for 1760, when the eccentricity was .016803, the equation will be, the secular diminution being 13″.9, equal to

$$1° \; 26' \; 30''.6 + 3''.4, \text{ that is, } 1° \; 26' \; 34''.$$

EXAMPLE III.

The Eccentricity of the Orbit (that of Pallas) *being* 0.259, *the Mean Anomaly*=45°: *it is required to find the Eccentric and true Anomalies.*

(1.) Log. tan. 22° 30′ - - 9.6172243
Log. tan. .741 - - - $\bar{1}$.8698182
Arith. comp. 1.259 - - 9.8999743

Log. tan. $\frac{1}{2}(CEM - CME)$ 9.3870168 = log. tan. 13° 42′ 3″.3.

(2.) $\frac{1}{2}(CEM - CME) = 13° \; 42' \; 3''.3$
$\frac{1}{2}(CEM + CME) = 22 \; 30$

$\therefore \; CEM = 36 \; 12 \; 3.3$ = 1st approx^e. value of $\angle CDA$,
and $CME = 8 \; 47 \; 56.7$.

(3.) Log. sin. 36° 12′ 3″.3 - - - 9.7713071
Log. .259 - - - - - - - - $\bar{1}$.4132998
Log. (arc = radius) - - - - 5.3144251

Log. DM in seconds - - - 4.4990320 = log. 31552.4;
$\therefore DM = 31552''.4 = 8° \; 45' \; 52''.4$;
∴ log. sin. - - - - - - - - - 9.1829067
Log. (arc = rad.) - - - - 5.3144251

4.4973318 = log. 31429;

∴ since $DM = 31552.4$
and sin. $DM = 31429$

$EO = 123.4$.

(4) [a] Log. .259 - - - - - - - - - - - - -	$\bar{1}$.4132998
Log. sin. 45° - - - - - - - - - - - - -	9.8494850
	9.2627848
Log. sin. 8° 47′ 56″.7 - - - - - - - - -	9.1845968
	.0781880
	5.3144251
	5.3926131
Log. *r* - - - - - - - - - - - - - - - -	10
Log. 123.4 - - - - - - - - - - - - - -	2.0913152
	12.0913152
[a] Log. (arc = radius) + log. *EM* - - -	5.3926131
Log. sin. *EMO* - - - - - - - - - - -	6.6987021

$\therefore$ *EMO* = 1′ 43″.1.

Hence, since	*CDA* =	36°	12′	3″.3
and	*EMO* =	0	1	43.1
corrected value of	*CDA* =	36	13	46.4, the eccentric anomaly.

Log. tan. 18° 6′ 53″.2 - -	9.5147282
$\frac{1}{2}$ log. .741 - - - - - - -	4.9349091
$\frac{1}{2}$ arith. comp. 1.259 - - -	4.9499871
Log. tan. $\frac{v}{2}$ - - - - - -	9.3996244 = log. tan. 14° 5′ 19″;

$\therefore$ the true anomaly is 28° 10′ 38″.

The eccentric and true anomalies being determined, the radius vector ρ may be computed from either of the two expressions, [1] [2] p. 191, but most conveniently from the latter.

Example IV.

Required the Earth's Distance from the Sun, the Mean Anomaly (reckoning from Aphelion) being 4^s 12° 22′ 12″.4, *and the Eccentricity* = .016803. See Ex. 2. p. 194.

$$\rho = 1 + e \,.\, \cos. u, \text{ if } a = 1,$$

and $u = 131° \; 39' \; 2''.6$.

Log. cos. 131° 39′ 2″.6 - - - - - - - - - -	9.8225523
Log .016803 - - - - - - - - - - - - - -	$\bar{2}$.2253868
Log. .011167 - - - - - - - - - - - - -	8.0479391

(since cos. is —), ρ = 1—.011167 = .988833.

Example V.

Required the Distance of Pallas *from the Sun, in the conditions of* Ex. III.

Log. cos. 36° 13′ 46″.4 - - - - - - - - - -	9.9066881
Log. 0.259 - - - - - - - - - - - - - -	$\bar{1}$.4132998
Log. .20892 - - - - - - - - - - - - -	9.3199879

∴ distance = 1.20892.

The knowledge of these distances is useful*, as we shall hereafter see, in computations of the heliocentric longitudes and latitudes of planets. But, in such computations, the *logarithms* of the distances are required. Those can, indeed, be immediately found from the computed distances, by means of the common Tables; with more brevity and facility of computation, however, by taking out, during the process of finding the true anomaly, when the log. sine is taken out, the log. cosine of the eccentric anomaly.

Assume then, $e \,.\cos. u = \cos. \theta$, or, log. cos. θ = log e + log. cos. u; thence θ is known: and, lastly,

$$\log. \rho = \log.(1 + e.\cos. u) - 10 = \log.(1 + \cos.\theta) - 10$$

$$= \log. 2.\left(\cos. \frac{\theta}{2}\right)^2 - 20$$

$$= \log. 2 + 2 \log. \cos. \frac{\theta}{2} - 20 = 2 \log. \cos. \frac{\theta}{2} - 19.6989700.$$

The sole object of this latter method, is compendium of calculation.

By means of the preceding rule, [see pp. 192, 193,] the true anomaly (as in the Examples) may always be computed from the mean, which is known, by a single proportion from the time.

* The Nautical Almanack expresses the logarithm of the Sun's distance for every 6th day of the year.

The difference of the true and mean anomalies, is the equation of the center, or the equation of the orbit. And, the Solar Tables assign to the mean anomaly, as the *argument*, this latter quantity, instead of the true anomaly. It serves then as a *correction* or *equation* to the mean anomaly, by means of which the inequality between the *mean* and *true* places of a planet, at any assigned time, may be compensated. It is additive or subtractive, according as the mean is less or greater than the true anomaly: subtractive, therefore, whilst the body *P* moves, from *A* the aphelion to *B* the perihelion, or, through the first 6 signs of mean anomaly, (reckoning anomaly from the aphelion) and additive, whilst *P* moves, from *B* to *A*, or, through the last 6 signs of mean anomaly.

These circumstances, Lalande's Tables (ed. 3.) used to express, in the common way, by the algebraical signs – and +. But the new Solar Tables, [see Delambre's Tables, and Vince's *Astronomy*, vol. III.] adapted to the operation of addition only, when the mean anomaly exceeds the true, express not the *equation of the center*, but its *supplement to* 12 *signs* (360°). The 12 signs, therefore, must be subsequently struck out of the result. This is not the sole difference in the construction of the Tables. In Delambre's last*, the mean anomaly is reckoned from the perihelion, and the equations of the center are increased by 45′, the sum of several small inequalities: an arrangement made for the same purpose as the former, l. 18; that of avoiding the operation of subtraction.

The *greatest equation of the center*, it is plain, can mean nothing else than the greatest difference between the true and mean anomalies; and that must happen when the body *P* moves with its mean angular velocity. For, if we conceive a body to move uniformly in a circle round *E* as a center, with an angular velocity, the mean between the least of *P* at *A*, and its greatest at *B*, and such, that departing with *P* from *AB* the line of the apsides, it shall, in the same time, attain to it, together with *P*; then, it is plain, at the commencement of the motion, the first day, for instance, *P* moving with its least angular velocity, describes round *E* a less angle than the fictitious body does: the next day,

* Both Tables were constructed by Delambre.

a greater angle than on the first, but still less than the angle described by the fictitious body: similarly for the third, fourth day, &c.: so that, at the end of any assigned time, the two angular distances of the two bodies from the aphelion, will differ by the accumulation of the daily excesses, of the angular velocity of the fictitious body, above that of the body *P*. And this accumulation must continue, till *P*, always moving till it reaches *B* with an increasing angular velocity, attain its mean angular velocity, or, that velocity with which the body moves in the circle; then, this latter body can, in its daily rate, no longer gain on *P*; and past this term, it must lose: exactly at that term, then, the difference of its angular distance from *A*, or from the line of the apsides, must be the greatest.

The point in the elliptic orbit is easily determined; it must be the intersection of the circle, in which the body moves with the mean angular velocity, and the ellipse: let x be the radius, then, since the circle and ellipse are described in equal times, by Kepler's law, (see p. 188,) the areas must be equal: hence

$$\pi x^2 = \pi \times \text{semi-axis major} \times \text{semi-axis minor},$$

consequently, $x = \sqrt{(\text{semi-axis major} \times \text{semi-axis minor})}$

$$= a\,(1 - e^2)^{\frac{1}{4}}$$

From the above value of the radius vector, may the true and eccentric anomalies, at the time of the greatest equation, be computed; and by the expressions [1], [2], p. 191, viz,

$$\varrho = \frac{a\,.\,(1-e^2)}{1 - e\,.\cos.\,v},\quad \varrho = a(1 + e\,.\cos.\,u)$$

Hence, the mean anomaly (nt) is known by the expression

$$nt = u + e\,.\sin.\,u$$

and finally there results the greatest equation of the center =

$$\pm\,(v - nt.)$$

EXAMPLE.

In the Earth's orbit, e being very small (= .016814)

$$\text{since } (1 - e^2)^{\frac{1}{4}} = 1 + e\,.\cos\;u$$

$$1 - \frac{e^2}{4} = 1 + e\,.\cos.\,u\,;\quad \therefore \cos.\,u = -\frac{e}{4},$$

$$\text{and } 1 - \frac{e^2}{4} = (1 - e^2)\,(1 + e\cos.\,v)\,;\ \therefore \cos.\,v = \frac{3}{4}e\,;$$

∴ by the series for the arc in terms of the cosine, and by neglecting the powers of e,

$$nt = \text{quadrant} + \frac{e}{4} + e$$

$$v = \text{quadrant} - \frac{3}{4}e;$$

∴ $nt - v$, (the greatest equation) $= \frac{8e}{4} = 2e$, and consequently, in the Earth's orbit, the eccentricity $= \frac{1}{2}$ the greatest equation.

This is the method of computing the greatest equation, but it is usually determined from observations. For that purpose we must observe the longitude of the body when its angular velocity is equal to its mean angular velocity; thus, according to Lacaille,

1751. Oct. 7, ☉'s longitude - - - -	6^s	13°	47′	13″.7
1752. Mar. 28, - - - - - - - - - -	0	8	9	25.5
Difference of the two longitudes - - - -	5	24	22	11.8
The mean motion proportional to the interval of time was - - - - - - -	5	20	31	43.2
The difference, double of the greatest equation	0	3	50	28.6

Hence, the greatest equation of the center in the Earth's orbit is 1° 55′ 14″.3: more nearly, by correcting the above calculation 1° 55′ 33″.

The difference of the longitudes of the two points in the orbit, at which the real motion nearly equals the mean, is equal to 5^s 24° 22′ 11″, or 174° 22′ 11″. This is a very obtuse angle formed by two lines drawn from the two points to the focus of the solar ellipse. The two points then are not very remote from the extremities of the axis minor; they would be exactly there, if the angle were 180°. Hence, the greatest equation happens when the body is nearly at its mean distance.

In the Example that preceded, the Sun's longitude was taken on October 7, and March 28; because, at those times, his daily motions or increases of longitudes were equal to his mean motion. That circumstance was ascertained by taking his longitudes on two successive days, and then the difference, which is his angular motion. The mean angular motion is nearly 59′ 8″: the greatest, about the beginning of January, being 1° 1′ 10″; the least, about the beginning of July, being 57′ 11″.

We shall perceive the use of this greatest equation of the center, when we treat of the equation of time. Astronomers have also used it in determining the eccentricity of the orbit*. If E be the greatest equation, and $\frac{E}{57^{\circ}.2957795}$ be put $= K$, then the eccentricity, or

$$e = \frac{K}{2} - \frac{11\,K^3}{3.2^6} - \frac{587\,K^5}{3.5.2^{10}} - \&c. \dagger$$

Hence in the case of the Earth's orbit, the eccentricity of which is very small, we have, retaining only the first term of the series, and taking $E = 1^{\circ}\ 55'\ 33''$,

$$e = \frac{K}{2} = \frac{1^{\circ}\ 55'\ 33''}{2 \times 57^{\circ}.2957795} = .016807.$$

If E be taken $= 1^{\circ}\ 55'\ 36''.5$, the greatest equation in 1750,

$$e = .016814.$$

If E be taken $= 1^{\circ}\ 55'\ 26''.8$, the greatest equation in 1800,

$$e = .016791.$$

From these two Examples, the diminution of the greatest equation for 50 years appears to be 9″.7: and, consequently the

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* See Lacaille, *Mem. Acad.* 1757, p. 123.

† This series was invented by Lambert. The reverse series for the greatest equation is

$$2e + \frac{11}{48}e^3 + \frac{599}{5120}e^5 + \&c.$$

In a Note to page 192, we gave the series expressing the true anomaly in terms of the mean and the eccentricity. The following is Delambre's expression for the equation of the center, for the year 1810, in terms of the greatest equation and of the mean anomaly z reckoned from the perigee:

$$1^{\circ}\ 55''\ 26'.352 \text{ sin. } z + 1'\ 12''.679 \text{ sin. } 2z + 1''.0575 \text{ sin. } 3z$$
$$+ 0''.018 \text{ sin. } 4z.$$

secular diminution to be 19″.4. Lalande, in his Tables, states it to be 18″.8.

In the case of the orbit of *Saturn*, $E = 6° \ 26' \ 42''$

$$= 6°.445; \therefore K = \frac{6.445}{57.2957795} = .112486,$$

$$\text{and } e = .056243 - .000031 = .056212.$$

The whole of this Problem of Kepler * whether we regard the analytical solution, or the approximate method, is extremely artificial. On account of its importance in the theory of the Sun and the planets, Mathematicians have taken great pains with it. The results of their labours have been, formulæ of great analytical compactness, and very commodious rules of solution. But these, not being obtained by obvious and direct processes, from the conditions of the Problem, are apt, in some degree, to perplex the Student.

But, it may be asked, may not the preceding formulæ and rules of solution be superseded by merely registering, from observation, each day, the Sun's longitude? Will not at the same distance of time from the equinox, the Sun's longitude be the same in 1810 as it was in 1800? This would be the case, if the longitude of the aphelion, from which the true anomaly is reckoned, continued invariable. But, the contrary is the fact; and, each year, the place of the aphelion is changed. Hence, the intersection of the equator and ecliptic continually corresponds to a different point in the solar ellipse: at that point, then, the Sun is moving, year after year, with a continually varying velocity; and if his velocity at the equinox in 1800, were less than at the equinox in 1810, then, one month after the equinox of

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* The reverse Problem, that of finding the mean from the true anomaly, being of little use, has not been introduced into the text. Its solution is very easy; find u from v by means of the expression

$$\tan. \frac{u}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)} \cdot \tan. \frac{v}{2},$$

and then the mean anomaly (nt) from

$$nt = e \sin. u + u.$$

the former period, his longitude would be less, than at the same distance of time from the equinox of 1810.

Although Kepler's Problem, then, is of essential importance, still it cannot be applied to the determination of the Sun's place, except we previously know the situation of the apsides of the solar orbit. The enquiry therefore, is naturally directed to the determination of their longitude.

CHAP XIX.

On the Place and Motion of the Aphelion of an Orbit.—Duration of Seasons. —Application of Kepler's Problem to the determination of the Sun's Place.

It follows from what was remarked in p. 187, that, the Sun in his perigee being at his least distance, and in his apogee, at his greatest, his apparent diameter in those positions would be respectively the greatest and least. If therefore, we could, by means of instruments, measure the Sun's apparent diameter with sufficient nicety, so as to determine when it were the least, the Sun's longitude computed for that time, would be the longitude of the apogee.*

Or if, computing day by day, from the observed right ascension and declination, the Sun's longitude, we could determine when the increments of longitude were the least, the Sun's longitude, computed for that time, would be that of the apogee: for, the Sun's angular motion in that point is the least (see p. 187.)

The difference of two longitudes thus observed, after an interval of time (t,) would be the angle described by the apogee in that interval. And, if the longitudes were not accurately those of the apogee, still, if they belonged to observations, distant from each other by a considerable interval of time, the motion of the

* *Apogee*, if the Sun be supposed to revolve, *Aphelion*, if the Earth; and, although, in reality, it is the latter body which revolves, yet, since it affects not the *mathematical* theory, we speak sometimes of one revolving, and sometimes of the other; and, with a like disregard of precision, we use the terms apogee and aphelion.

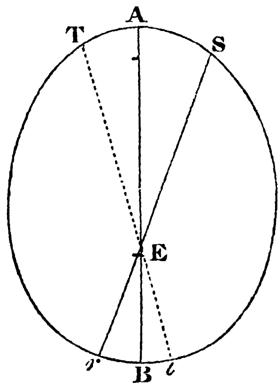
apogee would result with tolerable exactness; since the error would be diffused over a great number of years.

Thus, by the observations of Waltherus,

1496. Longitude of the apogee - - -	3^s	3°	$57'$	$57''$
In 1749, (by Lacaille) - - - - - - - - -	3	8	39	0
∴ progressive motion in 253 years - - -	0	4	41	3

whence the mean annual *progression* * results $1'\ 6''$: differing, however, from the result of better observations and methods, by $4''$. Lalande states the annual progression to be $1'\ 2''$.

The more accurate method, however, of determining the progression of the apogee rests upon a very simple principle. Let SEr be a right line, and draw TEt making with the



axis major AB of the ellipse, an angle $TEA = SEA$: now, the time through $rBtS$ is less than the time through the remaining arc $SATr$: for, the equal and similar areas SEt, TEr, are described in equal times, but the area rEt is < area SET; ∴ by Kepler's law (p. 188,) it is described in less time; ∴ $rEt + SEt$, that is, the area $SErtS$, is described in less time than $SET + TEr$, which compose the area $SErTS$; ∴ the body moves through the arc $rBtS$ in less time than through $STrB$. And this property belongs to every line drawn through E, except the line AEB, the major axis, or, the line of

* *Progression* is here meant to be used technically: a motion *in consequentia*, or, according to the order of the signs.

the apsides, that which joins the aphelion and perihelion of the orbit.

Hence it follows, if, on comparing two observations of the Sun at S and at r, that is, when the difference of the longitudes is 6 signs or 180 degrees, it appear that the time elapsed is not half a year, we may be sure, that the Sun has not been observed in his perigee and apogee. If the interval should be exactly, or nearly half a year, then we may as certainly conclude, that the Sun was at the times of observation, exactly, or very nearly, in the line of the apsides.

If the interval of time be nearly half a year, (which is the case that will occur in practice,) then we must find the true position of the apogee by a slight computation, which shall be first algebraically stated, and then exemplified.

The time from r to S = the time from r to B + the time from B to A − the time from S to A;

$$\therefore \text{ time from } B \text{ to } A - \text{time from } r \text{ to } S = \ldots\ldots [a]$$
$$\text{time from } S \text{ to } A - \text{time from } r \text{ to } B.$$

Now the first difference is known, being the difference between half an anomalistic year [see p. 70,] and the observed interval of observation: and of the second difference, the second term may be expressed by means of the first: thus, let the first term = t: then by Kepler's law, [see p. 188,]

$$\text{time from } r \text{ to } B = t \times \frac{\text{area } rEB}{\text{area } SEA}$$

$$= t \times \frac{rB \times EB}{SA \times EA} \quad [r \text{ and } S \text{ being near the apsides}]$$

$$= t \times \frac{rB}{EB} \times \frac{EA}{SA} \times \frac{EB^2}{EA^2}$$

$$= t \times \frac{EB^2}{EA^2} \left[\text{since } \frac{rB}{EB} = \angle rEB = \angle SBA = \frac{SA}{EA}\right]$$

$$= t \times \frac{\text{angular velocity at } A}{\text{angular velocity at } B} \quad [\text{see p. 188, l 20.}]$$

Now, the angular velocities at A and B, or the increments of the Sun's longitudes at the apogee and perigee, being known from observation [see p. 187,] and the time from r to B being expressed in terms of those velocities and of t, the quantity t is the only unknown quantity in the equation [a] l. 17, and accordingly may be determined from it. Having obtained t then, we can determine the exact time when the Sun (S) is at the apogee A: and his longitude, computed for that time, is the longitude of the apogee.

EXAMPLE.

1743. Dec. 30, $0^h\ 3^m\ 7^s$ ☉'s longitude	-	9^s	8°	$29'$	$12''.5$
1744. June 30, 0 3 0 - - - - - - - -	-	3	8	51	1.5
∴ difference of 2d and 1st longitudes	- -	6	0	21	49

∴ at the 2d observation June 30th, the Sun was past S. In order to find when he was exactly at S, that is, when the difference of the longitudes was exactly 6^s; or (supposing the perigee to have been progressive through $31''$), when the difference of longitudes was $6^s\ 0^\circ\ 0'\ 31''$, we must find the time of describing the difference of $21'\ 49''$, and $31''$, that is, $21'\ 18''$: now this time, since on June 30, the Sun's daily motion in longitude was $57'\ 12''$, equals $\frac{21'\ 18''}{57'\ 12''} \times 24^h$, equals $8^h\ 56^m\ 13^s$: take this then from the time [June 30, $0^h\ 3^m$] of the second observation, and there results, June 29, $15^h\ 6^m\ 47^s$, for the time when the difference of the longitudes of the Sun at r and near S was $180^\circ\ 0'\ 31''$.

The interval between this last time, and Dec. 30, $0^h\ 3^m\ 7^s$, the time of the first observation, is $182^d\ 15^h\ 3^m\ 40^s$, nearly the time from r to S: but, this time is less than half an anomalistic year, which is $182^d\ 15^h\ 7^m\ 1^s$ *: and see [a] p. 208, l. 17,

$$t - \text{time from } r \text{ to } B = 3^m\ 21^s.$$

* In this method which is to determine accurately the given place of the apogee, the motion of the latter, and the length of the anomalistic year are supposed to be known to some degree of accuracy. The one is stated to be $62''$; the other, $365^d\ 6^h\ 14^m\ 2^s$. But, if both be supposed unknown, if we take the difference of the longitudes of r and S to be simply 6^s, and the elapsed time to be half the *tropical* year, still the method will give the place of the apogee very nearly, which may serve as a first approximation to the true place.

But, see the same page, l. 24,

the time from r to $B = t \times \frac{57'\ 12''}{61'\ 12''}$;

$$\therefore, \text{ substituting, } t \times \frac{4'}{61'\ 12''} = 3^m\ 21^s, \text{ and consequently,}$$

$$t = 47^m\ 54^s.$$

Add this to the time, June 29, $15^h\ 6^m\ 47^s$, when the Sun was at S, and we have, June 29, $15^h\ 54^m\ 41^s$ for the time when the Sun was in the apogee.

The Sun's longitude at that time must be less than his longitude ($3^s\ 8^\circ\ 51'\ 1''.5$) on June 30, $0^h\ 3^m$ by the difference due to the difference of the times, which is $8^h\ 8^m\ 19^s$, equal, (since the increase of longitude in 24 hours was $57'\ 12''$)

$$\frac{8^h\ 8^m\ 19^s}{24^h} \times 57'\ 12'' = 19'\ 21'';$$

hence the longitude of the apogee $= 3^s\ 8^\circ\ 51'\ 1''.5 - 19'\ 21'' = 3^s\ 8^\circ\ 31'\ 40''.5$, or $98^\circ\ 31'\ 40''.5$, or $8^\circ\ 31'\ 40''.5$, past the summer solstice.

By the preceding method* the longitude of the apogee is found, and, in the Example, for the year 1744. Find similarly the longitude for another year, 1810 for instance, and the difference of the longitudes, divided by the time intervening, will give the progression of the apogee [see p. 207.]

The above method † of determining the place of the apogee is due to Lacaille. That author, on the grounds of simplicity

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* On most occasions, the nearly circular forms of the orbits of the Earth and the planets are, with respect to facility of calculation, very advantageous circumstances; on other occasions, when, for instance, the eccentricities and aphelia are to be determined, disadvantageous. For the orbits being circular, the observed body moves nearly as in a circle, in which there is neither apside nor eccentricity. The difficulty hence arising, does not consist in the prescribing an exact geometrical method, but in executing a tolerably exact practical one.

† The method is explained, with singular clearness, by Dalembert, in the historical part (L'Histoire) of the *Memoirs of the Academy of Sciences* for 1742.

and uniformity, suggested the propriety of reckoning the anomalies from the perihelia of orbits, since, in the case of Comets, they are necessarily reckoned from those points. In the new Solar Tables of Delambre this suggestion is adopted, [see Introduction: also Vince's *Astronomy*, vol. III. Introduction, p. 2.]

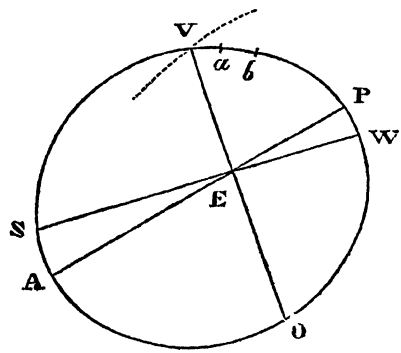
In these new Tables the progression of the perigee, and consequently that of the apogee, is made to be about $61''.9$; and the mean longitudes of the perigee for 1750, 1800, 1810, are respectively stated at $9^s\ 8^\circ\ 37'\ 28''$; $9^s\ 9^\circ\ 29'\ 3''$; $9^s\ 9^\circ\ 39'\ 22''$.

The longitude of the winter solstice is 9^s; therefore in 1810 the perigee was $9^\circ\ 39'\ 22''$ beyond it; at this time, the daily motion of the Sun was $61'\ 11''$; therefore, the solstice happening on Dec. 22, the Sun would be in his perigee about nine days after, or about Dec. 31.

From the longitude for any given epoch, and its annual progression, the position of the apogee and of the axis of the solar ellipse, may, by simple proportions, be found for any other epoch. Suppose, for instance, it were enquired when the axis of the solar ellipse was perpendicular to the line of the equinoxes? This, in other words, would be to enquire, when the longitude of the perigee was 270°, or 9^s. Now, its longitude, in 1750, was $9^s\ 8^\circ\ 37'\ 28''$: the number of years therefore requisite to describe the difference, or $8^\circ\ 37'\ 28''$, taking the annual progression at $62''$, equals $\frac{8^\circ\ 37'\ 28''}{62''}$, or about 500 years; that is, the major axis was perpendicular to the line of the equinoxes in the year 1250.

The epoch when the axis coincided with the line of the equinoxes is a remarkable one: this, since the longitude of the perigee was 180°, or 6^s, and since $\frac{3^s\ 8^\circ\ 37'\ 28''}{62''}$ is nearly 5720 years, happened about 4000 years before the Christian æra, at which time, Chronologists have fixed the beginning of the world.

The knowledge of the place of the perigee is necessary to determine the durations of seasons; which are perpetually varying from its progression. If *W*, *S*, in the Figure, represent the winter and summer soltices, *V* and *O* the vernal and autumnal equinoxes, *PEA* the axis of the solar ellipse; then, in the year 1250, *P* coincided with *W*; and, on that account, the time from the autumnal equinox *O* to the summer solstice *W* was equal to the time from *W* to the vernal equinox *V*. Past that year, *P*, by reason of its progressive motion, began to separate from *W*; and in 1800, the separation, measured by the angle *PEW*, was 9° 29′ 3″. By means of this separation, those parts of the elliptical orbit in which the Earth's real motion is the quickest, being thrown



nearer to *V* and away from *O*, the time from the autumnal equinox *O* to the solstice *W*, became gradually greater than the time from *W* to the vernal equinox: and the time from *V* to *S* became less than the time from *S* to *O*. In 1800, the following were nearly the lengths of the seasons:

V to *S* - - - - - - - - - - - -	92^d	21^h	44^m	28^s
S to *O* - - - - - - - - - - - -	93	13	34	47
O to *W* - - - - - - - - - - - -	89	16	47	20
W to *V* - - - - - - - - - - - -	89	1	42	23

This motion of the perigee also, as will be shewn in a subsequent Chapter, continually causes to vary the equation of time.

What has been said concerning the duration, and change of duration, of the Seasons, is, in some degree, digressive; the main object of the Chapter being to explain the method of finding the place, that is, the longitude of the perigee, in order that Kepler's problem might be applied to the determination of the Sun's place.

By Kepler's problem, we are enabled, from the mean anomaly, to assign the true anomaly, or true angular distance, reckoning from perigee *. The mean anomaly of the Sun, is his mean angular distance computed from perigee : in the Figure, if b be the $\odot$'s mean place, it is $\angle PEb$. Now,

$$\angle PEb = \angle PEV - \angle VEb,$$

and, if V be the first point of *Aries*,

$$\angle PEV = 12^s - \text{mean long. perigee},$$

$$\text{and} \quad \angle VEb = 12^s - \text{mean long. } \odot.$$

Hence, the mean anomaly is the difference between the mean longitudes of the Sun and of the perigee. And the Solar Tables assign the mean anomaly by assigning these longitudes. And then, in the same Tables, the mean anomaly is used as an *argument* for finding the equation of the center. The process may be illustrated by specimens from the Tables, and their application to an Example.

From Table I.

Years.	Mean Longitude of the Sun.	Longitude of Sun's Perigee.
1809.	$9^s\ 10^\circ\ 42'\ 49''.8$	$9^s\ 9^\circ\ 38'\ 20''$
1810.	9 10 28 30.2	9 9 39 22
1811.	9 10 14 10.5	9 9 40 24

From Table IV.

Motion for Days. *November*.

Years. Com.	Years. Bissex.	Mean Longitude. of the Sun.	Perigee.
Days.			
12	11	$10^s\ 10^\circ\ 28'\ 44''$	$53''.5$
13	12	10 11 27 52.3	53.6
14	13	10 12 27 0.7	53.8

* The mean anomaly is stated to be reckoned from perigee, since the succeeding extracts are from Delambre's new Solar Tables.

From Table V. Motion of the Sun for Hours, Minutes, Seconds.					
Hours.		Minutes		Seconds.	
H.	Motion of Sun	M.	Motion of Sun.	S.	Motion of Sun.
1	2′ 27″.8	1	2″.5	1	0″ .0
2	4 55.7	2	4.9	2	0 .1
3	7 23.5	3	7.4	3	0 .1

From Table VII. Equation of the Sun's Center for 1810, with the Secular Variation. (S. V.)			
Mean Anomaly.	Equation.	Diff. +	S. V.
10ˢ 12° 0′	11ˢ 28° 32′ 14″.7	13″.5	13″.13
10 12 10	11 28 32 28.2	13.5	13.09
10 12 20	11 28 32 41.7	13.5	13.06
10 12 30	11 28 32 55.2	13.6	13.03

Suppose now the Sun's longitude were required for 1810, Nov. 13, 2^h 3^m 2^s.

Table I. 1st. the mean longitude for the beginning of 1810, is - - - -	9ˢ	10°	28′	30″.2
Table IV. Nov. 13. - - - - - - - -	10	11	27	52 .3
Table V. 2^h - - - - - - - - - - -	0	0	4	55 .7
Table V. 3^m - - - - - - - - - - -	0	0	0	7 .4
Table V. 2^s - - - - - - - - - - -	0	0	0	.1
Rejecting 12ˢ, mean long. at time required [a]	7	22	1	25.7

The longitude of the perigee is had from the same Tables; thus:

Table I. Long. at beginning of 1810 - -	9ˢ	9°	39′	22″
Table IV. Nov. 13. - - - - - - - - - -	0	0	0	53.6
Longitude of perigee at the time required - -	9	9	40	15.6
Subtract this from [a] increased by 12 signs, there results the mean anomaly - - -	10	12	21	10.1

With this mean anomaly enter Table VII, and there results the equation to the center	- - - -	11^s	28°	$32'$	$42''.2$
Add to this the mean longitude [*a*]	- -	7	22	1	25.7
		7	20	34	7.9

This result, $7^s\ 20^\circ\ 34'\ 7''.9$, is (if no other corrections are required to be performed) the true longitude reckoned from the *mean* equinox. But, as it has been shewn [pp. 149, 158.], the place of the equinox varies from the inequalities of the Sun's action, and the Moon's action in causing the precession. Two *equations*, therefore, must be applied to the above longitude, in order to compensate the above inequalities, and so to correct the longitude, that the result shall be the true longitude, reckoned from the *true place of the equinox*. Now, it happens, by mere accident, that, in the above instance, the *lunar* and *solar* nutations are equal to $1''$, but affected with contrary signs. These corrections, therefore, affect not the preceding result. The correction for *aberration* [see p. 132,] has, in fact, been applied; for, since that, in the case of the Sun, must be nearly constant, (and it would be exactly so, if the Sun were always at the same distance from the Earth) the Solar Tables are constructed so as to include, in assigning the mean longitude, the constant aberration ($20''$). The variable part of the aberration (variable on account of the eccentricity of the orbit) is less than the 5th of a second. Let us see then, whether the longitude that has been determined, from a knowledge of the place of the perigee, and from Kepler's problem, expressed by means of Tables, be a true result:

By the Nautical Almanack for 1810; we have

Nov. 13, ☉'s longitude	- - - - - - -	7^s	20°	$29'$	$8''$
Nov. 14,	- - - - - - - - - - - - - -	7	21	29	36
Increase in 24 hours	- - - - - - - - -	0	1	0	28

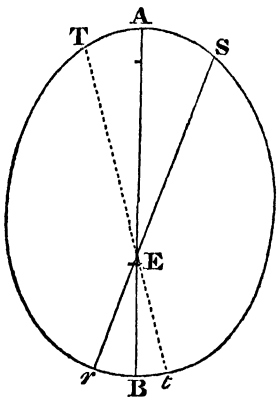
Now the Sun's longitude is expressed in the Nautical Almanack for *apparent* time: and the equation of time being $-15^m\ 33^s$, the mean time is $11^h\ 44^m\ 27^s$. Hence, we must find the increase proportional to $2^h\ 18^m\ 35^s$, which is about $5'\ 47''$; consequently the Sun's longitude, on Nov. 13, $2^h\ 3^m\ 2^s$, (mean time) was $7^s\ 20^\circ\ 34'\ 55''$, which differs from the preceding result, p. 215, l. 4, by about $47''$; consequently, Kepler's problem is not alone sufficient to determine the Sun's place, but some other corrections are requisite to compensate this error of 47 seconds.

Such corrections are to be derived from a new source of inequality; the perturbation of the Earth caused by the attracting force of the Moon and planets; the nature of which will be briefly explained in the ensuing Chapter.

CHAP. XXI.

On the Inequalities of the Earth's Orbit and Motion, caused by the Disturbing Forces of the Moon and the Planets.

It was originally proved by Newton (see *Princ.* Sect. 3.) that a body projected from *A*, perpendicularly to *EA*, a line joining *A* and a body placed at *E*, (the latter body attracting according to the law of the inverse square of the distance,) would describe an ellipse round *E*.



The body placed at *E* is supposed to exert a *centripetal* force, or attraction, proportional, at a given distance, to its mass, or to the number of particles which it contains.

If in *EA* produced, we place, at an equal distance from *A*, another body of equal mass, and, accordingly, of equal attractive force with the body at *E*, and again suppose the body at *A* to be projected; then, since it is equally urged to describe an ellipse round the new mass, as round that originally placed at *E*, it can describe an ellipse round neither, but must proceed to move in a direction perpendicular to *EA*.

In this extreme case, the elliptical orbit, and the law of elliptical motion would be entirely destroyed.

If now we suppose the mass of the new body to be diminished, or its distance from *A* to be increased; or, if we suppose both circumstances to take place, then, the derangement, or *perturbation*, of the body that is to revolve round *E*, will still continue, but in a less degree. An orbit, or curvilinear path, concave towards *E* in the commencement of motion, will be described; but, neither elliptical, nor of any other class and denomination.

In this latter case, the new body, being supposed less than the body placed at *E*, may be called the *disturbing* body; disturbing, indeed, by no other force than that of attraction, with which the body at *E* is supposed to be endowed; but which latter, from a difference of circumstance merely, is denominated a *Centripetal* force. In the first supposition, of an equality of mass and distance in the two bodies, from the similarity of circumstance, either body might be pronounced to be attracting or equally disturbing.

The disturbing body, whatever be its mass and distance, will always derange the laws of the equable description of areas, and of elliptical motion. If its mass be considerable, and its distance not very great, the derangement will be so much as to render the knowledge of those laws useless in determining the real orbit, and law of motion, of the disturbed body. In such case, Kepler's problem would become one of mere curiosity; and the place of the body would be required to be determined by other means.

If, however, the mass of the disturbing body be, with reference to that of the attracting body, inconsiderable, then the derangements, or perturbations, may be so small, that the orbit shall be nearly, though not strictly, elliptical; and the equable description of areas, nearly, though not exactly, true. Under such circumstances, Kepler's problem will not be nugatory. It may be applied to determine the place of the revolving body, supposing it to revolve, which is not the case, but is nearly so, in an ellipse. The erroneous supposition, and consequent erroneous results, being afterwards *corrected* by supplying certain small *equations*, that shall compensate the inequalities arising from the disturbing body.

In the predicaments just described, are the bodies of the solar system. The mass of the Sun, round which the Earth revolves, is amazingly greater than that of the Moon *, which disturbs the Earth's motion: greater also, than the masses of the planets, which, like the Moon, must cause perturbations. The Earth, therefore, describes very nearly an ellipse round the Sun.

As a first approximation then, and a very near one, we may, as in the last Chapter, determine the Sun's, or Earth's place, by means of Kepler's Problem: and subsequently correct such place, by small equations due to the perturbations of the Moon, and of the planets.

But, how are these small corrections to be computed? By finding, for an assigned time, an expression for the place of a body, attracted by one body, and disturbed by another; the masses, distances, and positions, of the bodies being given; that is, by solving what, for distinction, has been called the *Problem of the three bodies*.

The consideration of *three* bodies is sufficient: for suppose, by the solution of the problem, the equation, or correction, for the Sun's longitude, to be expressed, by means of the Sun's and Earth's masses, distances, &c., and of other terms denoting the mass, distance, &c., of a third body; then, substituting, for these latter terms, the numbers that, in a specific instance, belong to the Moon, the result will express the perturbation due to the Moon. Instead of the Moon, let the third body be *Jupiter*: substitute, as before, the proper quantities, and the result expresses the perturbation due to *Jupiter*: and similarly for the other planets. The sum of all these corrections, separately computed, will be the correction of the longitude arising from the action of all the planets.

The above corrections are what are necessary to complete the process of finding the Sun's longitude, in p. 215, of the preceding Chapter; and to supply the deficiency there noted of

* The Sun is 1300000 times greater than the Earth, and the Earth more than 68 times greater than the Moon.

several seconds, from the true longitude. The number of corrections which it is necessary to consider, and which the latest Solar Tables enable us to assign, is five; arising from the perturbations of the *Moon*, *Venus*, *Mars*, *Jupiter*, and *Saturn*. Those of *Mercury*, the *Georgium Sidus*, *Ceres*, *Juno*, and *Pallas*, are disregarded.

No attempt will here be made to compute the perturbations, by solving the problem of the three bodies. That problem presents great and peculiar difficulties: so great, that, instead of a complete and general solution, Mathematicians have been obliged to content themselves with an approximate one: yet, even by what we have seen, the problem is essential. Newton's theory is incomplete without it. The perturbations caused by the planets are as much a consequence of his principle of universal attraction, as their elliptical motion round the Sun; and when, computed according to that principle and its law, by long and intricate processes, they are found to be verified in the exactest manner by observation, they present, although not the most simple, yet, the most irrefragable proof of its truth.

Observation, it is plain, must furnish numerous results, before the formulæ of perturbations can be numerically exhibited, or, what is the same thing, be reduced into Tables. The positions and distances of the planets must be known: for, without any formal proof, we may perceive, that, according to the position of a planet, the effect of its disturbing force may be to draw the Earth either directly from, or towards, the Sun, or, in some oblique and transverse direction. In fact, the *heliocentric* longitudes of the Earth and the planets form the arguments in the Tables of perturbations.

Having thus explained, in a general way, the theory of perturbations, we will complete the Example of p. 214, by adding certain corrections, computed from that theory, to the Sun's longitude.

By p. 215, ☉'s longitude	- - - - - - - -	7^s	20°	$34'$	$8''.2$
Correction due to ☽	- - - - - - - - - -	0	0	0	5.5
to ♀	- - - - - - - - - -	0	0	0	17.49
to ♂	- - - - - - - - - -	0	0	0	4.32
to ♃	- - - - - - - - - -	0	0	0	12.7
to ♄	- - - - - - - - - -	0	0	0	0.65
∴ Nov. 13, 1810. 2^h 3^m 2^s; ☉'s true long^e.	-	7	20	34	48.86*

By computations like these carried on by the aid of Tables (see pp. 213, &c.,) the Sun's longitude is computed for every day in the year, and then registered; in the *Nautical Almanack* of Great Britain, the *Connoissance des Tems* of France, and in the *Ephemerides* of Berlin and other cities. The use of registering the Sun's longitude is explained in the Nautical Almanack, p. 146.

Here terminates the exposition of the Solar Theory; and, that of the Planetary, might, with propriety, immediately commence. But, the subject of the *equation of time*, is too important to be either omitted or postponed; and it could not, without anticipating processes and results, have preceded the method of determining the Sun's longitude. It is now therefore introduced as being strictly dependent on the solar theory, and not as forming any link, or connection, between that and the planetary theory.

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* This determination of the Sun's longitude is less by about 7 seconds than the longitude as stated in the Nautical Almanack. But, this latter was computed, (see Preface to the Nautical Almanack) from Lalande's Tables, inserted in the 3d Edition of his *Astronomy*: which differ by a few seconds from Delambre's last Solar Tables (Vince's, vol. III,) and from which the numbers in the text were taken.

CHAP. XXII.

On the Equation of Time.

THE two preceding Chapters having enabled us to compute the true longitude of the Sun, we can now proceed to the explanation of the *Equation of Time*, and to the mode of computing it.

The term *Equation* has, in this case, its usual Astronomical meaning; it is a correction to be applied to the *mean*, in order to make it *equal* to the *true* time.

The noon of true time is marked by a phenomenon: the actual presence of the Sun's center on the meridian, which can always be ascertained either by a transit telescope, or by a quadrant, or other Astronomical instrument, which determining generally the altitude, determines, consequently, the greatest altitude of an heavenly body.

The interval between two successive noons is a natural day. But, this interval, in different parts of the year, is a variable quantity. The index of a clock moving equably, which, on the second of November, should perform an exact circuit, or mark precisely 24 hours, in the interval between two successive noons, would, between the real noon of the 13th, and that of the 14th, perform more than a circuit. Between the real noons of the 1st and 2d of March, it would perform less.

A clock then constructed to move equably, could never be adjusted so as to agree with the Sun: that is, always to denote twelve hours when his center was on the meridian. Still, however, the Sun being the natural and obvious regulator of the civil day, it is desirable so to adjust a clock, that it shall differ as little as possible from the Sun: and that, agreeing with him at a particular point of time, it should, at least, again agree after the lapse of a year.

For this purpose, Astronomers have feigned an imaginary time, called *Mean Solar Time*, marked indeed by no phenomenon, but the noon of which, we may conceive, to be marked by a fictitious Sun moving in the equator, with the mean motion in right ascension of the real Sun. This mean daily motion in right ascension is 59′ 8″.3. For, the Sun in $365^d\ 5^h\ 48^m\ 48^s$ moves through 360°, and consequently, in one day, if he moved equably, through

$$\frac{1^d}{365^d\ 5^h\ 48^m\ 48^s} \times 360^\circ = \frac{360^\circ}{365.242222, \&c.} = 59'\ 8''.3$$

The difference of time, between the passages of the real and fictitious Sun over the meridian, is called the *Equation of Time.*

This interval may be marked by a sidereal clock; and since that, if duly adjusted, denotes the true right ascensions of heavenly bodies on the meridian, [see p. 49,] the difference of the true right ascensions of the real and fictitious Suns, must represent the *equation of time.*

This, however, is merely a mode of explaining the subject. The presence of the fictitious Sun on the meridian, or its distance from it, cannot be marked, as the very terms import, by any phenomenon. The defect of a phenomenon, therefore, must be supplied by calculation. We must so correct, by means of an equation, the true right ascension of the real Sun, indicated by the phenomenon of his presence on the meridian, that the result shall be the right ascension of the fictitious Sun.

The main reason why we are able to assign the time, at which, the Sun marking mean solar time would be on the meridian, is, that we can determine the true right ascension of the real Sun, both by observation and computation. The true right ascension of the real Sun may be computed from his true longitude by Naper's rules: or, in practice more expeditiously, by the aid of a Table entitled the Reduction of the Ecliptic to the Equator [see Vince, vol. II, p. 352, and Maskelyne's Tables, tab. XXXIII]. The true longitude, as we have seen (p. 169,) is measured not from the *mean* but the *apparent*, or equated place of the equinox: and the true right ascension must be computed from such true longitude, and the *apparent* obliquity [see p. 183.]

The mean equinox differs from the apparent, and the mean obliquity from the apparent, by reason of the effect of nutation

[pp. 168, &c.] We will, however, first abstract the consideration of this cause, and then examine after what manner its introduction modifies the results, in the computation of the equation of time.

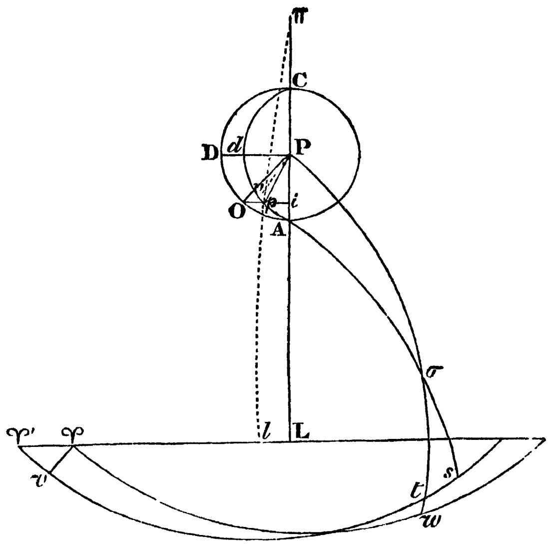
If there were no nutation, the longitude of the real Sun would be measured from the mean place of the equinox, and its right ascension, computed from the longitude and the mean obliquity of the ecliptic, would, consequently, be reckoned from the same mean place of the equinox.

If there were no nutation, the right ascension of the fictitious mean Sun could only be reckoned from the same mean place of the equinox: and the difference in the transits, cross the meridian, of the two Suns, or, in other words, the equation of time, would be the difference of the two right ascensions (reckoned from the same point of the mean equinox,) converted into time, at the rate of 15° for one hour.

The fictitious Sun is supposed to move with the real Sun's mean motion in right ascension. That motion, for 24 hours is 59′ 8″.3 (p. 223). The same quantity also expresses the mean daily increase of the Sun's longitude. Hence, in n days the accumulation of the daily motions in R. A., or the R. A. of the fictitious Sun ($=59' \, 8''.3 \times n$) is equal to the mean longitude of the real Sun. We may therefore vary the preceding expression for the equation of time, and state it to be the difference of the right ascension of the real Sun and of the mean longitude of the real Sun.

Since the equinoctial point (the first point of *Aries*) must, when we abstract nutation, regress equably, the right ascension of the mean fictitious must increase by equal quantities: and the intervals, between its successive transits over the meridian, must be equal portions of absolute time. A clock, then, whose index should always point to 12^h when the center of the Sun was on the meridian of a given place, would be an exact time-keeper.

Let us now consider the effect of nutation. By reason of that, the mean pole is transferred to its true or apparent place, the mean place of the equinoctial point to its apparent place, and the mean equator is changed into the apparent. P is the mean, and p the apparent pole; ♈ the mean, and ♈′ the apparent equinoctial point; ♈ *w* the mean, and ♈′*vt* the true or apparent equator.



The rotation of the Earth is supposed to remain unaltered by nutation. But, by its effect, *P* being transferred to *p*, and the secondary *Pσt* passing through a star *σ*, into the position *pσs*, a star *σ* will, by the effect of nutation, be brought, sooner or later, to the meridian of a given place, by the difference of two angles formed respectively by *σP*, *σp*, with a given meridian: and, this difference in the Figure is *ts*.

Now *ts* (pp. 69, 161,) is equal to $Pp \,.\, \cos. pPu \,.\, \tan. \text{decl}^n.$, consequently, when *σ* is in the equator, where the fictitious mean Sun is supposed to be situated, since $\tan. \text{decl}^n. = 0$ *, the value of *ts* is $= 0$.

Hence it follows, that the absolute time of the mean Sun's return to the meridian is not affected by nutation, and consequently, since the mean Sun in the mean equinoctial, by the interval of its passages over the meridian, marks out exactly equal portions of time, it must continue to do so, when the mean

* By the oscillation of the equator, the mean Sun would acquire declination from the true equator, but so minute, as to give no sensible value to $Pp \,.\, \cos. pPu \,.\, \tan. \text{dec}.$

equator, by the effect of nutation, is changed into the true or apparent.

The absolute time of the real Sun's return to the meridian is affected by nutation; and when most affected, by the maximum value of *ts*, that is, by 9″.6 × tan. 23° 28′; (making $Pp = 9''.6$, and the declination = 23° 28′) which quantity, in time, is about $\frac{1}{4}$ of a second.

But the change in the absolute time of the real Sun's return to the meridian is of no consequence in this enquiry. The *difference* of time between the transits of the real and imaginary Sun over the meridian must always, in a given state of the poles and equator, be proportional to the difference of their right ascensions, reckoned from the same point.

Now, the state of the poles and equator is that which is assumed in consequence of nutation, which is almost always operating. In such disturbed state, the Sun is observed on the meridian, or at a given distance from it. His right ascension, therefore, ought to be reckoned, not from the mean, but the true equinoctial point, or, it ought to be computed from his longitude, reckoned from that same true point, and the *apparent* obliquity.

The right ascension of the fictitious mean Sun must be measured from the same true equinoctial point. It will, therefore, be greater or less than its right ascension, measured from the mean equinoctial point, by the quantity ♈′*v*. [See Fig. in p. 225.]

The distance of the fictitious Sun from the mean equinoctial point, or his right ascension computed from the same point, can only arise from the accumulation of the daily mean motions in R. A. attributed to him. It must, therefore, in *n* days be equal to 59′ 8″.3 × *n*. But, the same quantity numerically expounds the Sun's mean longitude [p. 223]. Hence, using the latter term, the right ascension of the fictitious Sun is equal to the mean longitude of the real Sun, plus or minus the quantity ♈′*v*.

The quantity ♈′*v* is called [see p. 166] *the equation of the equinoxes in right ascension.* In deducing the right ascensions of stars from the Tables, it is used as a *correction*, to compensate the deviation of the equinoctial point from its mean place. It is [see p. 165] the sole correction in nutation, of a star situated in the equator. We may call it then a correction to the right ascension of the mean Sun, and accordingly state the right ascension of the mean fictitious Sun to be equal to the mean longitude of the real Sun, *corrected by the equation of the equinoxes in R. A.*

The equation of time, then, is equal to the *difference of the Sun's true right ascension, and of his mean longitude, corrected by the equation of the equinoxes in right ascension.*

There is a practical convenience in this last mode of expressing the equation of time. For, amongst the collection of Astronomical Tables, there is one for finding the equation of the equinoxes in R. A.; and, in finding the Sun's true longitude, (preparatory to the finding of the true right ascension) the mean longitude, and under that denomination, is one of the results.

If the obliquity be called, - - - - I,

equat. of equinoxes, ν $\left[Ll \text{ in Fig. p. 225} = \frac{-Pp \text{ sin. lon. ☽'s ☊}}{\text{sin. } 23^\circ\ 28'}\right]$

the Sun's true right ascension - - - A

the mean longitude - - - - - - - - M.

Then, since $\Upsilon' v = \nu . \cos. I$,

the equation of time, in space, $= A - M - \nu . \cos. I$

$$\text{and, in time, } = \frac{A - M - \nu . \cos. I}{15}$$

and,

$$\text{(since cos. } I = \cos. 23^\circ\ 28' = \frac{9173}{10000} = \frac{11}{12}\text{, nearly)} = \frac{A - M - \frac{11}{12}\nu}{15}$$

The expression for the equation of time, may be still farther resolved into its component parts. Thus, let

S = Sun's true longitude

E = equation of the center

P = equations due to the planetary perturbations [See p. 219.]

then, [see p. 221], $S = M + E + P + \nu$.

Now, A (as it has been observed in p. 223,) may be deduced from S and the apparent obliquity ($I \pm 9''.6$ cos. long. ☽'s ☊) by the solution of a right-angled spherical triangle, or, which is the usual way, by the application of a correction, called the *Reduction*: let R express that reduction; then,

$$A = S + R = M + E + P + R + \nu.$$

Hence, if T denote the true time, and t the mean, the equation of time, or,

$$T - t = \frac{A - M - \nu . \cos. I}{15}$$

$$= \frac{E + P + R + \nu - \nu \cos. I}{15}$$

$$= \frac{E + P + R + 2\nu . \sin.^2 \frac{I}{2}}{15}.$$

If we do not consider the effect of nutation, the

Sun's longitude $= M + E + P$,

his right ascension $= M + E + P + R'$;

and consequently, since the right ascension of the mean Sun is M,

$$\text{the equation of time} = \frac{E + P + R'}{15}.$$

Now R is the reduction, when the longitude is S, and by supposition, R', when the longitude is $S - \nu$; hence,

$(S < A) \qquad R = A - S$

and $R' = A - \nu \cos. I - (S - \nu)$;

consequently, $R' = R + \nu - \nu \cos. I$,

$$\text{and the equation of time} = \frac{E + P + R + \nu - \nu \cos. I}{15},$$

the same expression, as before l. 5. Hence the equation of time is the same quantity, whether we consider the effect of nutation, or not: or, is the same, whether we compute the right ascensions of the two Suns from the true or mean equinoctial point; and the same conclusion may be obtained independèntly of calculation.

In the preceding reasonings, for the sake of simplicity, we have supposed the noon of mean time to be determined, by the aid of the noon of true or apparent time marked by the phenomenon of the real Sun on the meridian. But, if by means of

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* For the computation of E, P, &c., see *Explication et Usage des Tables du Soleil*, by Delambre.

the Sun's altitude observed out of the meridian, and a knowledge of his declination and of the latitude of the place, or by other means we compute the hour angle measuring the time from apparent noon, we may, as easily as in the preceding case, compute the equation of time for such time, and thence deduce the corresponding mean solar time.

EXAMPLE.

Required the Equation of Time, Nov. 13, 1810, 2^h 3^m 2^s

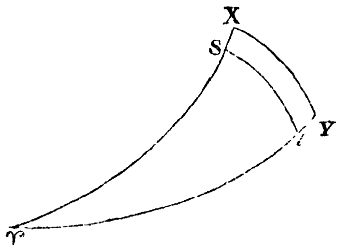
By page 22, the Sun's true longitude or, S (= $M + E + P + \nu$) - - -	7^s	20°	$34'$	$48''.86$
Reduction, (R) - - - - - - - - - -	--	2	26	36.36
True R. A. (A) - - - - - - - - -	7	18	8	12.5
Sun's mean longitude (M) p. 214, - -	7	22	1	26
Equation of equinoxes in R. A. (ν cos. I) -	0	0	0	0.9
Sun's mean longitude corr[d]. ($M + \nu$ cos. I)	7	22	1	26.9

$$\therefore \text{ equation of time, or, } \frac{A - (M + \nu \cos. I)}{15} = -\frac{3^\circ\ 53'\ 14''.4}{15^\circ}$$

$$= -15^m\ 33^s, \text{ nearly.}$$

What has preceded contains the principle and the mode of computing the equation of time; all, therefore, that concerns the practical Astronomer. But if, for the purpose of new and farther illustration, we continue our speculations, we shall find that the equation of time, relatively to its causes, depends on two circumstances; *the obliquity of the ecliptic to the equator*, and *the unequal angular motion of the Sun in its real orbit.*

The Sun moves every day through a certain arc of the ecliptic: which, in other words, is his daily increase of longitude. If we suppose two meridians to pass through the extremities of this arc, they will cut off, in the equator, an arc which is the daily increase of the Sun's right ascension. This latter arc will not remain of the same value, even if the former, that of the ecliptic, be supposed constant. At the solstice it will be larger than at the equinox: the reason is purely a geometrical one: let $S\Upsilon$ be the ecliptic, and Υy the equator, then by Naper's rule, if I be the obli-



quity, l the longitude, A the right ascension, D the declination

$$1 \times \cos. I = \text{cotan.}\ \Upsilon S \times \tan.\ \Upsilon t = \frac{\tan. A}{\tan. l},$$

hence, $\tan. l \times \cos. I = \tan. A$, and, taking the differential,

$$dl \cdot \frac{\cos. I}{(\cos. l)^2} = \frac{dA}{(\cos. A)^2}, \text{ or, since } \cos. l = \cos. A \times \cos. D$$

$$dl.\cos. I = dA\,(\cos. D)^2, \text{ or } dA = dl.\cos. I.(\sec. D)^2.$$

Hence I being the same, dA varies, if dl be given, as $(\sec. D)^2$; ∴ is least at the equinoxes and greatest at the solstices, and its value is easily estimated at the former, for since $D = 0$, $dA = dl.\cos. I$; at the latter, since $\sec. D = \frac{1}{\cos. D} = \frac{1}{\cos. I}$, $(dA) = \frac{dl}{\cos. I}$

$$\therefore dA\ [\text{equinox}] : dA\ [\text{solstice}] :: (\cos. I)^2 : 1$$
$$:: (\cos. 23^\circ\ 28')^2 : 1^2$$
$$:: 8414 : 10000.$$

Hence, even on the hypothesis of the Sun's equable motion in the ecliptic, the true right ascension will not increase equably; but since, by the very definition of the term, the mean longitude does, the equation of time, which is the difference of the true right ascension and the mean longitude (disregarding the equation of the equinoxes) would be a quantity, throughout the year, continually varying, and vanishing at the solstices.

The hypothesis, however, of the Sun's equable motion is contrary to fact; the Sun moves in an ellipse, and consequently, does not move uniformly, or equably in it. If a fictitious Sun, moving with the Sun's mean angular velocity, be supposed to leave, at the same time with the real Sun, the apogee; they will again come together at the perigee: but in the interval, the fictitious Sun would constantly precede the real Sun: the latter therefore, would be first brought on the meridian; true noon therefore, would precede the noon of mean time, supposing now, mean time to be measured by the imaginary Sun moving uniformly in the ecliptic.

If therefore, we hypothetically annul the first cause of the equation of time, by supposing the ecliptic to coincide with the equator, still from the second, (the elliptical motion of the Sun,) there would exist a difference between true and mean time; in other words, an equation of time, continually varying; vanishing, however, at the apogee and perigee.

But, both causes in nature exist; the Sun moves unequably, and not in the equator. From their combination then, the actual equation of time must depend. It cannot be nothing at the solstices, except the solstitial points coincide with those of the apogee and perigee, but, (see p. 210,) in the solar orbit, there is no such coincidence.

At what conjunctures then, will the equation of time be nothing? We have already, for the purposes of explanation, introduced two fictitious Suns, one moving equably in the ecliptic, the other in the equator; let the former be represented by S'', and the latter by S''', and the true Sun, that which moves unequably in the ecliptic, by S'; then, true time depends on S', and mean time on S'''; and consequently, when the meridian passing through one, passes also through the other, then is mean time equal to the true, therefore no equation is requisite, or the equation of time is nothing. Let us suppose the two fictitious Suns S'', S''' to move from the autumnal equinox towards the perigee; S''', in this case, must constantly precede S'', till they arrive at the solstice, where the meridian that passes through one will pass through the other *.

* We shall frequently use the expression of S' *rejoining* S''', or, *coinciding* with it. Nothing, however, farther will be meant by such expression, than that the meridian, which passes through the former in the ecliptic, passes through the latter in the equator; and when S' is said to precede S''', nothing more is meant, than that the point in the equator in which a meridian through S' cuts it, is beyond the place of S''', or, to the eastward of it.

Hence, the real Sun S', which coincided with S'' at the apogee, being constantly behind it [see pp. 190, 200,] till their arrival at the perigee, must certainly be behind it, at and before the solstice, which is previous to the perigee (see p. 210). Hence, before the winter solstice, the order of the Suns is

$$S' \quad S'' \quad S'''.$$

At the solstice $S' \left\{ \begin{matrix} S'' \\ S''' \end{matrix} \right\}$; for S'' then ceases to be preceded by S'''. Immediately after the solstice, S'' takes the lead of S''': therefore, then, the order is

$$S' \quad S''' \quad S''.$$

But, at the perigee, S' must rejoin S'': it cannot effect that, except by previously passing S''': the moment of passing it, is that in which true time is equal to mean time, in which, in other words the equation of time is nothing.

The equation of time then is nothing, between the winter solstice and the time of the Sun's entering the perigee: for the year 1810, (when the longitude of the perigee was $9^s\ 9^\circ\ 39'\ 22''$) between Dec. 21, and Dec. 30. By the Nautical Almanack the exact time was Dec. 24 at midnight: since the equation for the noon of that day is $-$ 15^s, and for the noon of the succeeding day $+$ 15^s.

In the year 1250, when the perigee coincided with the winter solstice [see p. 211,] the equation of time was nothing on the shortest day.

Immediately after the passage of the perigee, S', the true Sun, moving with its greatest angular velocity (see p. 188,) precedes S''; therefore, (since up to the vernal equinox S'' precedes S''',) the order is

$$S''' \quad S'' \quad S';$$

and this order must continue up to the equinox; consequently, S''' and S' cannot come together: and therefore between Dec. 24, (for 1810,) and March 21, the equation of time cannot equal nothing.

After the vernal equinox, S''' precedes S'', and the order is

$$S'' \quad S''' \quad S',$$

S'', and S' are then, (see p. 202,) near the point of their greatest separation, but S'' and S''' begin to separate and reach the point of their greatest separation *, about 46° 14′ from the equinox that is, about the 8th of May. Now, this greatest separation, or, technically, greatest *equation*, is 2° 28′ 20″, or in time 9^m 52^s, whereas the greatest equation of the center, being only 1° 55′ 33″, [p. 202,] the greatest corresponding separation in the equator cannot exceed 2° 6′ †, and that is already past. Hence, before *S″* is at its greatest separation from *S‴*, it is impossible that the order

S″ S‴ S′

should not have been changed. *S′* must have become nearer to *S″* than *S‴* is: consequently, *S‴* must have passed *S′*: but at the moment of passage, mean and true time are equal, that is, the equation of time is nothing : and this must happen between

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* 1 × cos. I = tan. A . cot. l, by Naper, or cos. I × tan. l = tan. A; ∴ $l > A$; ∴ if Y be supposed the place of *S‴*, so that, ♈Y = ♈S, Y is beyond t, and the *separation* is tY (since on that the difference solely depends.)

To find tY, is a common problem, (see Simpson's *Fluxions*, vol. II, p. 551. Vince's *Fluxions*, p. 27.) Since tY = ♈Y − ♈t = $l - A$;

$$\therefore \tan. tY = \frac{\tan. l - \tan. A}{1 + \tan. l . \tan. A} = \frac{\tan. A . (\sec. I - 1)}{1 + (\tan. A)^2 . \sec. I}.$$

Hence, since $d(tY) = d \tan. tY . (\cos. tY)^2$, which must = 0; if we take the differential of the quantity equal to it, make it = 0, and reduce it, there results

$$\tan. A = \sqrt{\cos. I} = \sqrt{(\cos. 23° \; 27' \; 58'')}$$

A = 43° 43′ 50″, and l (from equation, l. 2 of Note) = 46° 14′, and $l - A$ (in its greatest value) = 2° 28′ 20″.

† By p. 230, it appears that the arc of the equator included between two meridians passing through the extremities of a given arc in the ecliptic, is greatest when the latter arc is at the solstice. The arc of the equator measures the *separation* of the Suns *S″*, *S‴*. Hence, putting in the formula of p. 230, dl = 1° 55′ 33″, and $D = I$, which it is at the solstice, we have, very nearly,

$$dA = 1° \; 55' \; 33'' \times \sec. 23° \; 27' \; 58'' = 2° \; 5' \; 55''.$$

The two common problems then of the maximum equation of time, are not merely mathematical problems, exercises for the skill of the Student, or Examples to a fluxionary rule, but of use in the discussion of the real problem of nature.

March 21, and the end of April. In the year 1810, it happened, according to the Nautical Almanack, on April 15, 11^h 12^m.

This second point, at which the equation of time is nothing, being passed, the order of the Suns will become

$$S'' \quad S' \quad S'''.$$

At the solstice, S'' must rejoin S''': but, previously to the solstice, it cannot effect that by passing S': since S'' does not rejoin S' till their arrival at the apogee, which point is more distant than the solstitial: the coincidence of S'' and S''' then can only take place, by S' previously passing S''': but, as before, the moment of passage, is the time when the equation of time is nothing: that circumstance therefore, must happen, before the summer solstice: therefore, between the middle of April and June 22: and, in 1810, according to the Nautical Almanack, it happened on June 15, 14^h.

In the year 1250, the equation of time was nothing on the longest day.

After this third *evanescence* of the equation of time, the order of the Suns will become

$$S'' \quad S''' \quad S'.$$

At the solstice on June 22, S'' will rejoin S''': immediately afterwards, the order becomes

$$S''' \quad S'' \quad S',$$

which will continue to the time of the Sun's entering the apogee: then, S'' rejoins S': and immediately after, S'' moving with greater angular velocity than S' will precede it, and the order becomes

$$S''' \quad S' \quad S''.$$

Now S' cannot rejoin S'' till their arrival at the perigee: but S' will rejoin S''' at the autumnal equinox, consequently previously to that time, it must pass S': but, as before, the moment of passage is, when the equation of time is nothing. It must happen then, between the time of the apogee and the autumnal equinox: between (for 1810) June 30, and September 24; and, by the Nautical Almanack, it happened August 31, 20^h.

It is plain, from the preceding explanation, that the days of the year in which the equation of time is nothing depend on the position, or the longitude of the perigee and apogee: and consequently, since those points are perpetually progressive, the equation of time will not be nothing on the same days of any specified year, as it was, of preceding years: nor, when not nothing, the same in quantity, on the corresponding days of different years.

The preceding statement (beginning at p. 229,) is to be regarded merely as a mode of explaining the subject of the equation of time. It is not essential, and might have been omitted; for, the two causes of inequality are considered and mathematically estimated, in the processes of finding the true longitude and true right ascension. But, it has been inserted, since it serves to illustrate more fully, and, under a different point of view, a subject of considerable difficulty and importance.

With regard to results, very little is effected by the preceding statement. Four points are determined, at which, mean time is equal to apparent: in other words, four particular values (evanescent values) of the equation of time. But, according to the process in p. 229, we are enabled to assign its value for every day in the year: and accordingly, in constructing Tables of the equation of time, the above four particular values would be necessarily included amongst the 365 results.

If the question were, merely to determine when the equation was nothing, it would certainly be an operose method of resolution, to deduce all the values of the equation of time, and then, to select the *evanescent* ones. In such case, it would be better to have recourse to considerations like the foregoing (pp. 230, &c.). But, both these methods would be superseded, if, which is not the case *, the equation of time could be expressed by a simple analytical formula.

The mere inspection of such formula, or some easy deduction from it, would enable us to assign the times when the equation of time vanished.

Instead of a formula, we must use a process consisting of

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* Lagrange, however, although by no direct process, has succeeded in assigning a formula for the equation of time. See *Mem.* Berlin, 1772 So also has M. Schulze, *Mem.* Berlin, 1778, p. 249.

several distinct and unconnected steps, for computing the equation of time. And, in point of fact, the process is quite as convenient as a formula could be; since the concern of the Astronomical Computist is not with special, as such, but with the general values of the equation of time.

If special values are sought after, it must be principally on the grounds of curiosity The method of ascertaining four such values, independently of direct computation, has been already exhibited. And, on like grounds, a similar method might be used in the investigation of other special values: in determining, for instance, when the equation of time is of a mean value; or, when minute, the two causes of inequality counteracting each other; or, when large, the two causes co-operating. We will confine ourselves to two instances.

After the evanescence of the equation of time between the winter solstice and the perigee, the order, as we have seen, (p. 234,) is

$$S''' \quad S' \quad S'',$$

but S' is gaining fast on S'' in order to rejoin it at the perigee, and S'', after parting with S''' at the solstice, is preceding it, by still greater and greater intervals. Consequently, both causes of inequality conspire to make mean time differ from the true, and the equation of time goes on increasing till the Sun is about 40° distant from the vernal equinox, that is, past the point, at which the equation arising from the obliquity is a maximum, (see p. 233,) and before the point at which the equation from the Sun's anomalous motion is a maximum. For the year 1810, the time would be about Feb. 10, and the maximum of the equation is $14^m\ 36^s$.

About the Summer solstice, on the contrary, between that and the apogee, the order is

$$S''' \quad S'' \quad S'.$$

S''' is indeed separating from S'', but S'' is approaching S' to reach him at the apogee: consequently, the two causes of inequality, in some degree, counteract each, and the equation between the two periods at which it is successively nothing, (June 15, and August 31, for 1810,) never attains to the value of seven seconds.

In a similar way, we may form a tolerably just conjecture of the limits of the quantity of the equation of time, for other parts of the year.

The greatest quantity of the real equation of time can never reach the sum of the greatest equation arising from the separate causes. It must therefore be less than

$$2^\circ\ 28'\ 29'' + 2^\circ\ 6', \quad \text{or} \quad 4^\circ\ 34'\ 29'',$$

or in time less than $18^m\ 15^s$ of mean solar time.

The equation of time computed for every day in the year, according to the method given in p. 229, or, by some equivalent method, is inserted in the Nautical Almanack; and, for the purpose of deducing mean solar, from apparent time. In order to regulate its application, the words *additive* and *subtractive* are interposed into the column that contains its several values. And, there will be no ambiguity belonging to that application, if we consider, that the equation is to be applied to a certain time marked by some phenomenon: which phenomenon is the real Sun on the meridian: determined to be so, either by a transit telescope, or by a quadrant, or declination circle that enables us to ascertain, when the Sun is at its greatest altitude. Apparent time, then, is what is instrumentally determined; and to such time, the equation, with its concomitant sign, must be applied, in order to deduce mean time, which, it is plain, is indicated by no phenomenon.

Thus, Dec. 31, 1810, the equation of time in the Nautical Almanack is stated to be $3^m\ 12^s.7$ *additive*; therefore, when the Sun was on the meridian, at its greatest height, on that day the mean solar time was $12^h\ 3^m\ 12^s.7$. Again, Nov. 13, 1810, the equation is stated at $15^m\ 33\ .2$ *subtractive*; therefore, on that day, the Sun was at its greatest height at $12^h - 15^m\ 33^s.2$, that is, $11^h\ 44^m\ 26^s.8$, mean solar time.

Independently of computation, very simple considerations will shew that this procedure is just. In the first instance, the true Sun precedes the mean; that is, is more to the east, or more to the left hand of a spectator facing the south: consequently, by the rotation of the Earth, from west to east, the meridian of the spectator must first pass through the hinder Sun, which, in this instance, is the mean Sun; 12^h therefore of mean time happens before the meridian has reached the true Sun, when it does reach it, then, the time is, in mean time, 12^h + the difference of right ascensions, or $12^h +$ the equation of time. In the second instance, the true Sun is behind the fictitious: therefore the meridian of the spectator first passes through the former: true noon therefore, or 12 hours apparent time, happens before the meridian has reached the fictitious mean Sun; before therefore the noon of mean solar time. The time consequently is not 12 hours, but 12 hours — some quantity, which quantity is the equation of time.

We may now proceed to the consideration of the planetary theory; a subject of the same kind, in essential respects, with the solar, and naturally following it; yet, presenting greater difficulties, and requiring in many points, new considerations and methods.

CHAP. XXIII.

ON THE PLANETARY THEORY.

On the Phases of the Planets: — their Points of Stations, Retrogradations, &c.

The simple hypothesis of the rotation of the Earth accounts for several of the phenomena of the fixed stars: their risings, settings, and the different degrees of velocity in their apparent motions. If we combine with this, the hypothesis of the Earth's revolution round the Sun, we become possessed of the means of explaining all the phenomena of the change of place and variation of motion, which the latter body exhibits. Something more however, is still requisite to account for the phenomena of the planets. Their risings and settings can indeed be explained by the first hypothesis, but, they change their place amongst the fixed stars, and therefore are endowed with a proper motion.

If the motion be one of revolution, what is the point or body round which they revolve? It cannot be the Earth, for then the motion would always take place and seem to take place in one and the same direction. If the Sun, will the combination of the planets' motion round it, with that of the Earth and spectator, explain the anomalous retrogradations and quiescences of the planets? (see p. 10.)

It will be the object of this and the succeeding Chapters to explain the latter and other phenomena of the planets, on the principle of the combination of their motion and of the Earth's round the Sun. But, first we wish, by describing some of the obvious appearances, to shew the probability of the hypothesis, of the planets' revolution round the Sun.

We will begin with the planet *Venus:*

This brilliant star when seen in the west setting soon after the Sun, is known by the name of the *Evening Star.* If observed on several successive nights, it will be found to vary its distance from the Sun; increasing that distance, till it becomes equal to about 45°. Past that term, it approaches the Sun, till its proximity becomes so close, that the effulgence of the latter prevents its appearance. It then ceases to be an evening star. But, some days being elapsed, if we turn our attention to the east, we shall perceive *Venus* rising just before the Sun, and becoming the *Morning Star.* On successive mornings, *Venus* will rise still sooner: will continue separating from the Sun, till having reached an angular distance of about 45°, she will again approach, and finally rejoin the Sun. She then ceases to be the morning star: but soon after, she again becomes the evening star, and the same appearances, in the same order, are renewed.

These appearances prove, not decisively, that *Venus* describes an oval, or a circle about the Sun, but that, at least, she oscillates about the Sun: they prove too, that her orbit can neither be round the Earth, as its center, nor inclusive of the Earth; for, she is never seen in opposition; that is, in the production of a line drawn from the Sun through the Earth.

To the naked sight, or to unassisted vision, the disk of *Venus* appears circular and nearly of the same magnitude. But, the telescope and its micrometer* prove both appearances to be delusive. Viewed through the former, *Venus*, when the evening star, at her greatest separation from the Sun, assumes the form of a crescent, the points or horns being opposite the Sun. As she approaches the Sun, the crescent diminishes. Having passed the Sun, she appears as the morning star, and the crescent is turned the other way. Day after day, the crescent increases, till it is changed into a full orb, just at the time when *Venus* is about to rejoin the Sun.

In this last situation the disk of *Venus*, though most illuminated, is least in magnitude. It is greatest in magnitude, when the disk is least illuminated, and *Venus* is about to rejoin the Sun, ceasing to be the evening star. These latter circumstances, relative to the magnitude of the disk, are determined by the micrometer.

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* An instrument for measuring small angles, and commonly attached to the telescope.

This last-mentioned instrument enables us to determine the greatest and least apparent diameters of *Venus* to be about 2′ 46″, and 26″.

If we now enumerate the circumstances relative to *Venus*, they are as follow:

Venus, whatever be the Sun's place in the ecliptic, always attends on him, and is never separated by a greater angle of elongation, (technically so called) than 45°.

Venus is continually at different distances from the Earth: when at her greatest, that is, when her apparent diameter is the least, she shines with a full orb: when seen at her least distance, that is, when her apparent diameter is the greatest, her crescent is very small; and there are conjunctures, when *Venus* eclipses part of the Sun's disk, and passes over it like a dark spot.

Venus, when the evening star and separating from the Sun, moves from west to east; or according to the order of the signs, or, as the phrase may still be varied, *in consequentia*. Returning towards the Sun, from her greatest elongation, she moves towards the west, that is, *in antecedentia*, contrary to the order of the signs. And, in like manner, she moves, when the morning star, alternately, according and contrary to, the order of the signs.

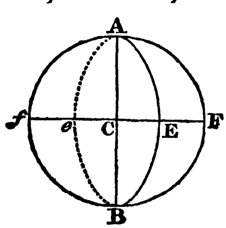
These are the phenomena of observation, that are proposed for explanation, on the grounds of two hypotheses: the first, that *Venus* is an opaque spherical body illuminated by the Sun: the second, that *Venus* revolves in an orbit round the Sun, and interior to the Earth's orbit.

If *Venus* be a sphere, only half of it can be illuminated by the Sun. And the illuminated hemisphere, called, for distinction, the Hemisphere of *Illumination*, is thus to be determined. From the center of the Sun, to that of *Venus*, conceive a right line to be drawn; perpendicular to this line, and passing through the center of *Venus*, conceive also a plane to be drawn; then, such plane will divide the body of the planet into two hemispheres, the one luminous, the other dark.

But, a spectator, whatever be his distance from a sphere, can never see more than half of the same. The hemisphere which he sees, called the Hemisphere *of Vision*, is thus to be determined: Conceive the eye of the spectator and the center of the planet to be joined by a right line; a plane perpendicular to this line, passing through the center of the planet, divides its body into two hemispheres; the one towards the spectator, is that of *vision*.

The two hemispheres, and their boundaries, the circles of illumination and of vision, do not necessarily coincide: indeed, they can coincide only when the Sun, which illuminates the planet, is between it and the spectator on the Earth's surface. In every other situation, part of the planet's illuminated hemisphere is turned away from the spectator; and, when the planet is between the Sun and spectator, wholly turned away: in other words, the planet's disk can either not be seen, or must appear as a dark circle or spot on the Sun's face.

When the spectator, Sun, and *Venus* (for of that planet we are now speaking) lie not in the same right line, the delineation of the illuminated disk, or phase, is reduced to a very simple proposition in orthographic projection. On the plane of projection which is always perpendicular to a line joining the eye of the spectator and the center of the planet, it is required to delineate the ellipse into which the circular boundary of light and darkness will be projected. The minor axis of the ellipse, will, as it is well known, bear that proportion to the major, which the radius bears to the cosine of the inclination of the planes. The inclination is equal to the angle formed by two lines, one drawn from the Sun to the center of *Venus*, the other, from that same center di-



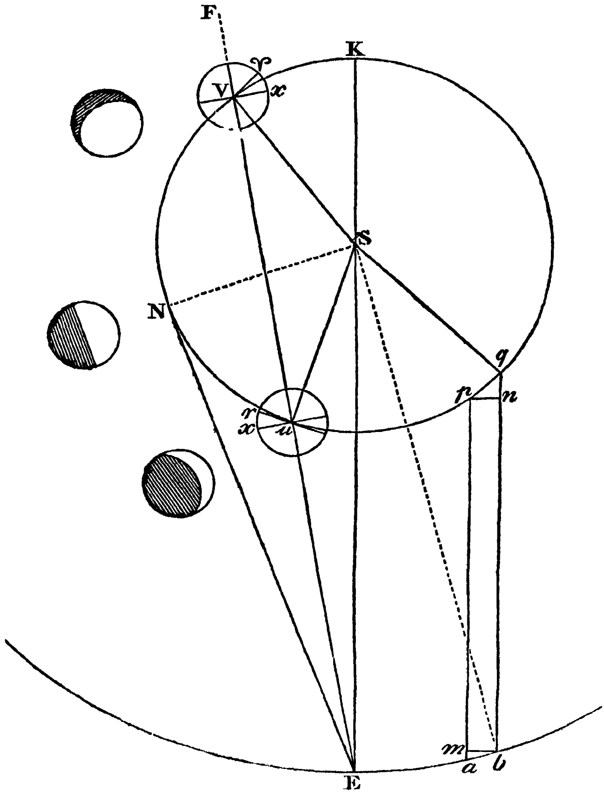
rectly from the spectator*. Hence, if *AFBA* represent the disk, and we take $CF : CE ::$ rad. : cos. planet's inclination, then de-

* *ru* is the half of the projection of the circle of illumination, *xu* of vision, and

$\angle rux = \angle Fux - \angle Fur = 90° - \angle Fur = 90° - [\angle Sur - \angle SuF]$
$= 90° - [90° - SuF] \angle = \angle SuF.$

cribing with the semi axes AC, CE, the semi-ellipse AEB, we shall have the illuminated disk represented by $AFBEA$.

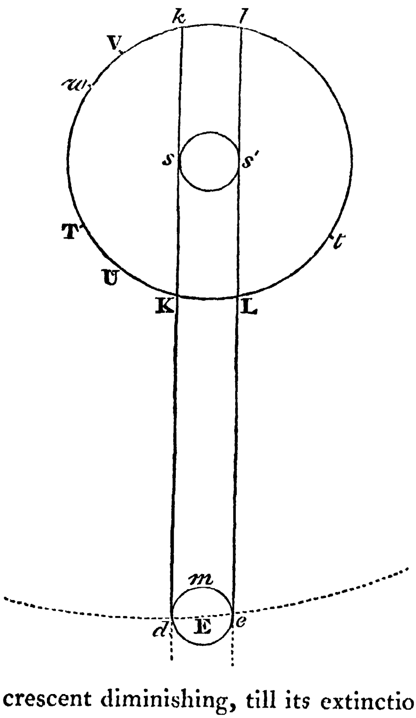
If $KVuL$ be the orbit of *Venus*, S the Sun, E the Earth; then, the angle of inclination of the planes of illumination, and vision at V, is the angle SVF, and at u, the angle $S\,u\,F$. In the latter, the angle is acute, in the former, obtuse; consequently, if CE



in Fig. p. 242, be taken to represent the cosine of the acute angle, to the right of the line AB, Ce must be taken to the left of the same line, in order to represent the cosine of the obtuse angle SVF. At K, when (see p. 43,) the planet is in superior conjunction*, the angle

* An inferior planet is in superior conjunction, when it lies in the direction of a line drawn from the Earth to the Sun, and produced beyond the Sun.

SVF is equal to two right angles; consequently, the cosine (with a negative sign) becomes equal to radius, and the point *E*, falls in *F*; or the whole orb is illuminated. At *L*, when the planet is in inferior conjunction, the angle, such as *SuF*, becomes nothing; therefore the cosine becomes equal to radius, and the point *E* falls in *F*: or the whole orb is dark. From *K* to *L*, in the intermediate points, *Venus* exhibits all her varieties of phases; the full orb, near *K*; the half illuminated orb at *N*, where $SNE = 90^\circ$,



and then the crescent diminishing, till its extinction at *L* *.

These phenomena that would happen if *Venus* an opaque spherical body be illuminated by the Sun, and revolve in an orbit round him, are strictly conformable to the phenomena that are observed, and have been described in pp. 239, &c..

* The phases which *Venus* at *V*, *N*, and *u*, exhibits to a spectator at *E*, are represented by the small circular Figures that are, respectively, to the left of the points *V*, *N*, and *u* [see p. 243.]

Thus far then the hypothesis of *Venus's* revolution round the Sun is probable, and seems to involve no contradiction; it will be still farther confirmed, if we can shew, that it affords an adequate explanation of the other phenomena which the planet exhibits.

Suppose *emd* to be the Earth, and two tangents *dsk*, *es′l*, to the points *d* and *e*, to represent the respective horizons to a spectator at *d* and *e* *, Then, if the Earth's rotation be according to the order *emd*, when the horizon *dsk* of the spectator at *d* shall touch the Sun's disk, the Sun will *set* to that spectator; the moment after, by the rotation of the Earth, the point *k* will be transferred to some point between *l* and *V*, the line *dsk* will no longer touch the Sun's disk, or, the Sun will be below the horizon. But, *Venus*, if at *V*, will be above the line of the horizon, andabove as an evening star, till the Earth, by its farther rotation, shall have so transferred the line *dsk*, that its extremity *k* shall be in some point between *V* and *U*. In the interval between this and the next night, *V* will have moved forward in its orbit to some point *w*; therefore, the line *dsk*, after leaving the Sun's disk, must revolve through a greater angle than it did the preceding evening, before it reaches *V* at *w*. The planet therefore, is now separated from the Sun by a greater angle of elongation: and the elongation on succeeding nights will still continue, till *V* reaches a point *T*, where a line drawn from *E* touches her orbit. Hence from superior conjunction at *k*, to the greatest elongation at *T*, *Venus* is continually separating or *elongating* from the Sun; and if we refer her place to the fixed stars, will seem to move amongst them in a direction *kVwT*, that is, according to the order of the signs.

From *T* to *L* the inferior conjunction, the line *dsk*, after quitting the Sun's disk, will reach the planet after the description of still less and less angles, and the planet will be found approaching the Sun: but, referred to the fixed stars, will be found to change its place amongst them in a direction from *T* towards *L*, contrary to the direction of the former change of place, and contrary to

* In the explanation, intended only to be general, *Venus's* orbit and the Earth's equator *emd*, are supposed to be projected on the plane of the ecliptic, (represented by the plane of the paper,) and, the spectator is supposed to be placed in the equator.

the order of the signs. In other words, the planet is now *retrograde*, (see p. 42.)

Suppose now the planet to have passed the inferior conjunction at *L*. *Day breaks* to a spectator at *e*, when the line *es'l*, representing his horizon, touches the Sun's disk. But, before this has happened, the line *es'l* has passed the planet, or the planet is above the horizon, and has risen as the morning star: on succeeding mornings, the planet having moved forward in its orbit from *L* towards *t*, will rise before the Sun by greater and greater intervals; will continue, to appearance, separating from him, till its arrival at its greatest elongation *t*. From *L* to *t*, the planet will, as from *T* to *L*, still continue retrograde. From *t* to *l*, it will again approach the Sun, and move according to the order of the signs.

These phenomena then, that would happen if *Venus* revolve either in a circular or elliptical orbit round the Sun, are in strict conformity with the phenomena that are observed, and which have been described in p. 239, &c.

In the preceding explanation of the phases and retrogradations of *Venus*, we have, for the sake of simplicity, supposed the Earth to be at rest at *E*. But, there is one phenomenon, that of the seeming quiescence of *Venus* during several successive days, which cannot be explained, except we depart from that supposition, and combine, according to the actual state of things, the motion of the Earth with that of *Venus*.

If *Venus* be at *L*, and the Earth at *e*, and both describe in the same time (24 hours for instance), two small arcs of their orbits, such arcs will be nearly parallel to each other. If they were equal, then during their description, *Venus* would be referred by a spectator on the Earth, to the same point in the heavens. But, *Venus* revolving round the Sun according to the laws of planetary motion (see p. 247, l. 16,) describes a greater arc than the Earth does in the same time. She must, therefore, at the end of the 24 hours be referred by a spectator on the Earth, to a point in the heavens situated to the right of her former place. But, as *Venus* advances from *L* towards *t* in her orbit, the arcs of her orbit (or tangents to them) will become more and more inclined to the arcs of the Earth's orbit. There will then be somewhere between *L* and *t* an arc *pq* (see Fig. p. 243,) such that, its obliquity compensating its greater length, two lines *pa*, *qb*, drawn to the contemporaneously described arc *ab* of the Earth's orbit, shall be parallel; when that happens, *Venus* must appear *stationary*.

We may determine the exact time when that happens, by computing the angle *bSq*, which is, in the same time, the excess of the angular motion of *Venus* above that of the Earth *.

It is plain that *Venus* will be retrograde whilst moving through an arc such as *NLt*, whether the Earth be supposed to be at rest, or to be in motion. The case however, is different with a superior

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* *bSq* may be thus computed: (see Fig. p. 243.)

Draw from *p* and *b*; *pn*, *bm* perpendicular to the parallel lines *qb*, *pa*, then $pn = bm$: call *Sb*, *r*, and *Sq*, *r'*;

$$\text{then } pn = pq \,.\, \sin. pqn = pq \,.\, \cos. Sqb,$$
$$bm = ab \,.\, \cos. mba = ab \,.\, \cos. Sbq;$$

$$\therefore \frac{\cos. Sqb}{\cos. Sbq} = \frac{ab}{pq} = \frac{\text{vel.} \oplus}{\text{vel.} ♀} = \frac{\sqrt{r'}}{\sqrt{r}} \text{(Newton, Sect. II. Prop. 4. Cor. 6;)}$$

$$\therefore \cos.^2 Sbq = \cos.^2 Sqb \times \frac{r}{r'}.$$

$$\text{But, } \sin.^2 Sbq = \sin.^2 Sqb \times \frac{r'^2}{r^2} \quad (\textit{Trigonometry}, \text{ p. 16,})$$

∴ adding these two latter equations, and putting for $\cos.^2 Sqb$, $1 - \sin.^2 Sqb$,

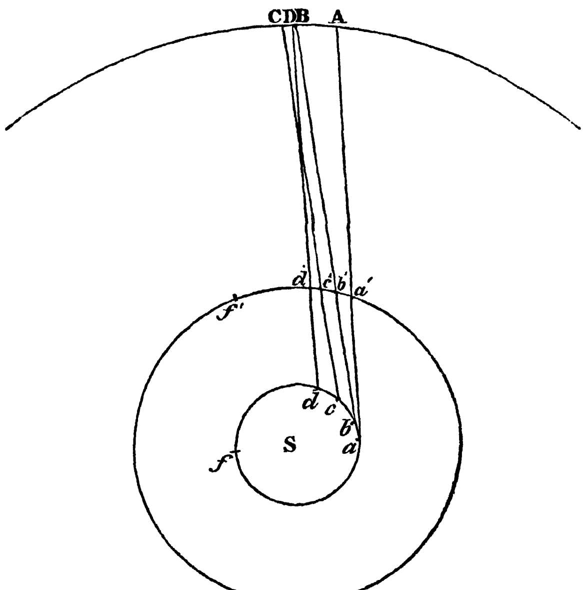
$$1 = \frac{r}{r'}[1 - \sin.^2 Sqb] + \frac{r'^2}{r^2} \sin.^2 Sqb,$$

$$\text{and } \sin. Sqb = \sqrt{\left(\frac{r^3 - r^2 r'}{r^3 - r'^3}\right)} = \frac{r}{\sqrt{(r^2 + rr' + r'^2)}}.$$

$$\text{Hence, } \sin. Sbq = \frac{r'}{\sqrt{(r^2 + rr' + r'^2)}}.$$

The two angles *Sqb*, *Sbq*, being thus determined, $bSq = 180° - (Sqb + Sbq)$ is known; and thence the time from conjunction at *L*. Thus, the mean daily motions of *Venus* and the Earth being 1° 36′ 7″.8, and 59′ 8″.35, the daily excess is 36′ 59″.5: therefore, if the angle *bSq* be 13°, the time from conjunction will be $\frac{13°}{36' \, 59''.5}$, or about 21 days.

planet *, which can only be shewn to be retrograde by combining with its motion, the Earth's. Thus, let *ab*, *bc*, *cd*, be three equal arcs in the Earth's orbit, *a'b'*, *b'c'*, *c'd'*, three equal arcs in *Jupiter's* (for instance,) contemporaneously described, but less (see p. 247, l. 16,) let also *A*, *B*, *C*, *D*, be four points in the imaginary sphere of the fixed stars, to which *a'*, *b'*, *c'*, *d'* are successively referred by a spectator at *a*, *b*, *c*, *d*. Now, if *ABC* be according to the order of the signs, the body in the orbit *a'b'c'd'*, is transferred in that direction or is progressive; whilst the spectator moves from *c* to *d*,



and the planet from *c'* to *d'*, the latter, amongst the stars, is transferred from *C* to *D* towards *B* and *A*, that is, contrary to the order of the signs. During the description then of the intermediate arcs *cb*, *c'b'*, the planet must have been stationary. The retrogradation will continue from *c* through opposition, where it will be the greatest, to a point *f*, situated similarly to *c*; that is, such

* A superior planet includes within its orbit, the Earth's; an inferior planet's orbit is included within that of the Earth's

that the angle made by two lines joining $f'f$, fs shall $=$ the angle $c'cS$. From f through conjunction to c, the planet will move according to the order of the signs.

Here then is a material circumstance of distinction, in this part of their theory, between inferior and superior planets. In the explanation of the quiescences and retrogradations of the former, the Earth's motion is not an essential circumstance; it merely modifies their extent and duration. But, with superior planets, the Earth's motion is an indispensable circumstance. The very nature of the explanation depends on its combination with that of the planets.

In speaking of the stations and retrogradations of the planets, we have been obliged to use a language and phrases by no means descriptive of the observations by which those phenomena are ascertained. But, the Student must be reminded upon this, as upon other occasions, to attend to the simple facts of observations. When a planet is stationary, the fact of observation is, that the right ascension continues the same: when retrograde, that the right ascension diminishes. The right ascension being determined by the hour, minute, &c. at which the observed body comes on the middle vertical wire of a transit telescope.

Jupiter, in treating of his retrogradations, has been assumed to be a superior planet. One proof of his being such, as well as that *Mars*, *Saturn*, and the *Georgium Sidus* are, is to be derived from their phases; which have not as yet been described.

Now, *Mars* exhibits no such variation of phases as *Venus* does; he is seen indeed, sometimes a little *gibbous*, but never in the shape of a crescent, nor as a black spot on the Sun's disk. If we add to these circumstances, that he is seen at all angles of elongation from the Sun, we may presume that *Mars* revolves in an orbit round the Sun inclusive of the Earth's; that he is therefore a superior planet. He certainly cannot revolve round the Earth, for then he would never be stationary, nor retrograde; nor can his orbit pass between the Sun and Earth.

Jupiter, *Saturn*, and the *Georgium Sidus* do not appear gibbous, but shine, almost constantly, with full orbs.

These phenomena can be accounted for, by supposing *Mars*, *Jupiter*, *Saturn*, and the *Georgium Sidus*, to be opaque spherical bodies illuminated by the Sun; and *Mars* to be the least distant: and if not very distant (relatively to the Earth's distance), his illuminated disk may, in some situations, be so much averted from the spectator, as to give him the appearance of being a little gibbous; and, he will be most gibbous in quadratures: where, however, the breadth of the illuminated part will be to that of the whole disk as 175 to 200.

If we were to increase the distance of *Mars*, the above proportion would approach more nearly to one of equality. Hence the reason, why *Jupiter*, *Saturn*, and the *Georgium Sidus*, much more distant from the Sun, than *Mars*, do not appear gibbous, even in quadratures.

From what has preceded, we may draw this conclusion; that, the adequate explanation of the phases, the stations, and the retrogradations of the planets, on the hypothesis of their revolution round the Sun, renders, at least, that hypothesis probable. But, since the explanation has been one, of obvious and general appearances, and not of phenomena precisely ascertained by accurate observations, the mere fact of a *revolution* has alone been rendered probable, without any determination of the nature of the curve of revolution. It may be either circular or elliptical. The system of Copernicus, therefore, is rather proved to be true, than Kepler's laws, or Newton's theory. Their truth, however, is intended to be shewn, and, that the planets revolve round the Sun in orbits very nearly elliptical: the deviations from the exact elliptical forms, being what would result from the mutual disturbances of the planets computed according to the law of gravitation. For this end, phenomena of a different kind from the preceding, must be selected and examined, and explanation, instead of being general, must be particular, and proceed by calculation. The *elements* of the orbits and the motions of the planets must be deduced from observations; arranged in Tables; again compounded according to theory; and, in this last state, as results subjected to the test of the nicest observations.

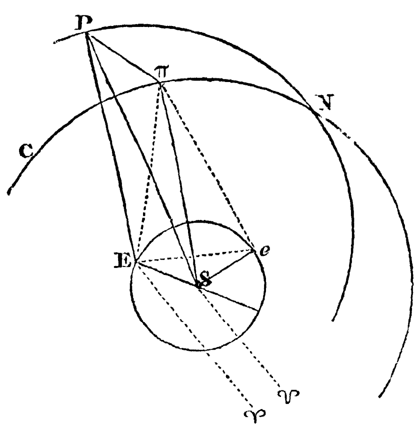
The elements of the orbits of planets depend on certain distances, linear and angular, measured from the Sun. But, the observations, from which these elements are to be deduced, are made at the Earth. The first step then, in the succeeding investigation, must be towards the invention of a method, for transmuting observations made at the Earth, into observations that would be made by a spectator supposed to be placed in the Sun; in technical language, for converting *geocentric* into *heliocentric* angular distances.

This method is necessary for the extrication of the elements. For the examination of the system founded on those elements, the reverse method is required; in other words, we must be possessed of the means of converting *heliocentric* into *geocentric* angular distances.

CHAP. XXIV.

On the Method of reducing Observations made at the Earth to Observations that would, at the same time, be made by a Spectator situated in the Sun.

LET S be the Sun; E, e, two positions of the Earth; P the planet in its orbit, of which NP is part; $P\pi$, part of a circle of latitude, π being a point in the ecliptic, and $N\pi$ part of the



imaginary great circle lying in the plane of the ecliptic; N, a point of intersection of the planet's orbit with the ecliptic, called the node: ♈ the first point of *Aries*.

Now, the points E, e, S, ♈, N, π, lie in the same plane, which is that of the ecliptic; but the point P lies in the orbit NP, the plane of which must be supposed to lie below or above the plane of the ecliptic.

The angle $PE\pi$ is the latitude of the planet to a spectator at

E; it is the geocentric latitude, not immediately observed, but computed (see p. 56,) from the observed right ascension and declination.

The angle contained between πE and a line directed from E towards ♈, is the geocentric longitude.

The angle $PS\pi$ is the heliocentric latitude, and the angle contained between $S\pi$ and S ♈ the heliocentric longitude; and these two latter must be deduced from the two former, which, by observation and computation (p. 56,) are known.

It is required to determine the Heliocentric Longitude of a Planet.

Suppose that, the Earth being at E, observations are made and registered of a planet at P, and, after a lapse of time equal to one, two, or more periods of the planet; that the planet is again observed nearly in the same point of its orbit, the Earth being at e: then, by computing from observation, the geocentric longitudes, and the Sun's longitudes, we can determine the angles $SE\pi$, $Se\pi$; and, from the solar theory and Tables we know, the angle ESe, and the distances ES, eS (pp. 192, 198). Hence, we have

Given.		To be determined.
ES, eS, $\angle ESe$	- - - - - - -	Ee, $\angle SEe$, $\angle SeE$
then,		
$\angle SEe$, $\angle SeE$, $\angle SE\pi$, $\angle Se\pi$	-	$\angle \pi Ee$, $\angle \pi eE$,
next,		
Ee, $\angle \pi Ee$, $\angle \pi eE$	- - - - - -	πE, πe,
lastly,		
SE, $E\pi$, $\angle SE\pi$	- - - - - -	$\angle ES\pi$, and $S\pi$.

Hence, since the angle between ES and S♈ is known, we can determine the angle between $S\pi$ and S♈, and consequently, the heliocentric longitude of the planet.

By the preceding method, besides the $\angle ES\pi$, $S\pi$, called the *Curtate Distance*, (*distance accourcie*) is determined. If we suppose, what is nearly true, that $S\pi$ remains constant, then, for any other time, and consequently, generally, we can determine the heliocentric longitude: for, we can determine an angle such as $ES\pi$, from $SE\pi$, and from the proportion of SE to $S\pi$

The angle formed by two lines drawn from the Earth respectively, to the Sun and the reduced place of the planet in the ecliptic, is called the Angle of *Elongation*. In the Figure, it is the angles $\pi E S$.

If two lines be drawn from E and S to ♈, they may be considered as parallel. Hence,

$$\angle \pi E ♈ - \angle SE♈ = \angle \pi E S,$$

or, geocentric longitude of planet — long. ☉ = ∠ elongation.

The angle $E \pi S$ is called the *Angle of Commutation*. We now proceed to the second Problem, in which

It is required to find the Heliocentric Latitude.

By *Trig*. in the triangle $PE\pi$, we have *, $P\pi = E\pi \times \tan. \angle PE\pi$
in the triangle $PS\pi$, - - - - $P\pi = S\pi \times \tan. \angle PS\pi$

By equating these two expressions for $P\pi$, and reducing,

$$\tan. \angle PS\pi = \tan. \angle PE\pi \times \frac{E\pi}{S\pi} = \tan. \angle PE\pi \times \frac{\sin. \angle ES\pi}{\sin. \angle SE\pi}.$$

From this expression may $\angle PS\pi$, the heliocentric latitude, be computed. For there are known $\angle PE\pi$ the geocentric latitude, determined by computation (p. 56,) from the planet's observed right ascension and declination; $\angle ES\pi$, determined in the preceding problem, p. 255, l. 23, and $\angle SE\pi$, the angle of elongation: see l. 2.

If we wish to exhibit, under the form of a proportion, the preceding expression, (l. 14,) then,

tan. heliocentric latitude : tan. geocentric :: sin. diff. long^{e}. of Earth and planet : sin. planet's elongation.

The heliocentric latitude being determined by the preceding method, and the *curtate* distance $S\pi$, by the method of the former problem, (p. 253, l. 23,) we have the real distance, or, $SP = S\pi \times \sec. \angle PS\pi$. In other terms,

rad. vect. of planet's orbit = *curtate* dist. × sec. helioc. lat.

If from N we take, in the plane of the planet's orbit, an arc A = dist. of N from ♈, then $A + NP$ is called the *Longitude of a Planet on its Orbit*. Now, the inclination of the planet's orbit (the

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* On account of the supposed smallness of the inclination of the planet's orbit $P\pi$, instead of the arc of a great circle, is assumed to be a straight line.

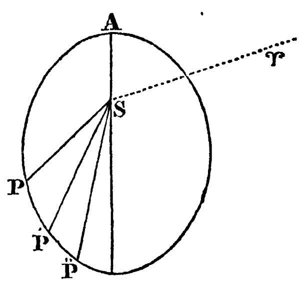
spherical angle at N) being known, and $N\pi$ [$= \Upsilon N\pi - \Upsilon N$], being determined by the preceding methods, NP may be computed by means of Naper's Rules, or, as it usually is, by a table of Reductions (see p. 227,) and then, from NP, and Δ*, $\Delta + NP$ the longitude of the planet on its orbit is known.

Kepler, in his progress towards one of his grand discoveries, the elliptical form of the planetary orbits, followed, nearly, the preceding methods. He investigated three heliocentric longitudes and three distances of the star *Mars*. From these, as data, an ellipse can be described. If the ellipse, belonging to three longitudes and three distances deduced from observation, was the true orbit of *Mars*, then in such ellipse all the other places of the planet ought to be †; on trial, Kepler found this to be the case, and thence concluded that *Mars* revolved in an ellipse, round the Sun placed in one of the foci.

We will proceed to solve the Problem, in which,

It is required, from Three given Distances and Longitudes, to describe an Ellipse.

Let APP', &c. be the ellipse, a the semi axis-major, ae the eccentricity. Let SP, SP', SP'', be represented respectively by



ϱ, ϱ', ϱ'', ASP, PSP', PSP'' - - - - - by x, θ, ϕ,

* If we can find, generally, the longitude of a planet, we can find the longitude of the node, since that is the longitude of a planet when its latitude is equal to nothing.

† What amounts to the same as this, is, the *same* ellipse ought to result, whatever be the three distances and three heliocentric longitudes that are selected.

then, by the property of the ellipse,

$$\varrho = \frac{a.(1-e^2)}{1+e.\cos. x} \qquad [1]$$

$$\varrho' = \frac{a.(1-e^2)}{1+e.\cos.(\theta+x)} \qquad [2]$$

$$\varrho'' = \frac{a.(1-e^2)}{1+e.\cos.(\varphi+x)} \qquad [3]$$

Hence, by the [1] and [2],

$$\varrho + \varrho e.\cos. x = \varrho' + \varrho' e.\cos.(\theta+x)$$

$$\text{and } e = \frac{\varrho'-\varrho}{\varrho \cos. x - \varrho'.\cos.(\theta+x)} \qquad [4]$$

similarly, by [1] and [3],

$$e = \frac{\varrho''-\varrho}{\varrho.\cos. x - \varrho''.\cos.(\varphi+x)} \qquad [5]$$

Hence, equating [4] and [5], and making $\varrho'-\varrho=R'$, $\varrho''-\varrho=R''$,
$R'\varrho.\cos. x - R'\varrho''.\cos.(\varphi+x) = R''\varrho \cos. x - R''.\varrho'.\cos.(\theta+x)$.

Hence, expanding, cos. $(\varphi+x)$, cos. $(\theta+x,)$ and dividing by cos. x,
$R'\varrho - R'\varrho''.\cos.\varphi + R'\varrho'' \sin.\varphi.\tan. x = R''\varrho - R''\varrho'.\cos.\theta + R''\varrho' \sin.\theta.\tan. x$,

$$\text{whence, } \tan. x = \frac{(R'\varrho - R''\varrho) - (R'\varrho'' \cos\varphi - R''\varrho'.\cos.\theta)}{R''\varrho'.\sin.\theta - R'\varrho''.\sin.\varphi}.$$

Now, the three heliocentric longitudes, that is, the three angles ♈SP, ♈SP', ♈SP'' are given; therefore, $\theta = \angle$ ♈$SP' - \angle$ ♈SP, and $\varphi = \angle$ ♈$SP'' - \angle$ ♈SP, are known; and, since ϱ, ϱ', ϱ'', are also known, tan. x, the tangent of the angular distance of the planet at P from perihelion, may be computed. Also e, the ratio of the eccentricity to the semi axis-major may be computed, either from the expression, [4], or [5]; and the semi axis a, either from [1], [2], or [3].

The preceding is an important problem in determining some of the elements of a planet's orbit. Its use, however, in the practice of Astronomy, depends entirely on that other problem, by which geocentric longitudes are converted into heliocentric.

Now, if we examine this last-mentioned problem, we shall find, that although the angle of elongation ($SE\pi$) can be easily determined from one observation of the planet's right ascension and declination, yet the angle $ES\pi$, and the ratio of ES to $S\pi$, cannot, except by two observations, separated from each other by an interval equal to the periodic time of the planet. Hence, if the above were the sole method of determining the ratio of ES to $S\pi$, after one observation of a planet, we should be compelled to abide a lapse of time equal to its period, before we could proceed to compute the elements of its orbit. The *Georgium Sidus*, then, would have still been truly a wandering star; and since its period is 83 years, its discoverer must have died ignorant whether or not, it formed an exception to Kepler's laws.

The fact, however, is that the above planet was discovered in 1781, and we have already Tables of its motions. Its right ascension, declination, &c. are regularly inserted in the Nautical Almanack. And, even so near the time of its discovery as 1782, we find, in the *Memoirs of the Academy of Paris* for that year*, the elements of its orbit computed by M. Lalande. We have a practical proof, then, that some other method than what was given in p. 253, must have been resorted to.

The method used by Lalande was one of trial and conjecture. He was possessed of three geocentric observations of the planet made, respectively, April 25, 1781, July 31, 1781, Dec 12, 1781, and, in order to reduce them to heliocentric, he *assumed* for the radius vector (SP), a number which he conjectured to be nearly its true value. With this assumed value, he computed the angle $ES\pi$ [see Fig. p. 252,] for the times of the first and third observation (April 25, and Dec. 12.) Corresponding to such assumption and computation, there resulted the difference of two heliocentric longitudes, in the interval between April 25, and Dec. 12. This difference, was, in fact, the angle (A) described round the Sun in the same interval of time (T). Hence, supposing the orbit to be circular, the period of the planet's revolution would result from this proportion,

$$A^{\circ} : 360 :: T : \text{period}.$$

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* This volume appeared in 1782, but is said to be for the year 1779. Mr. Robison, in the 1st vol. *Edin. Trans.* has investigated the elements of the orbit.

Having thus obtained a value for the periodic time of the planet, the truth of the original assumption for the distance, could be examined by the test of Kepler's law, which states the squares of the periodic times of planets revolving round the Sun to be as the cubes of their mean distances *. Hence, supposing the Earth to be one planet, and the *Georgium Sidus* the other, there results

$$(\text{Period})^2 = (365^d.256384)^2 \times (\text{assumed rad. ♅'s orbit})^3,$$

[taking the Earth's radius $=1$]. Now, if the period thus obtained had been the same, as the value of the period resulting from the preceding proportion, it would have been proved, that the original assumption of the value of the distance (SP) was a right one. But, on trial, the two values differing from each other, M. Lalande was obliged to amend his first assumption; to assign partly by conjecture, and partly by the guidance of the first trial, a new value for the distance, and then to examine that, as the former, by the preceding process. By repetition of like trials and examinations, a radius vector at length resulted, which agreed, to a sufficient degree of accuracy, with all observations.

This method of M. Lalande's, is akind of sample and exemplar of almost all Astronomical processes. In these, at first, nothing is determined exactly. Approximate quantities are assumed and substituted, the results derived from them, examined and compared, and then other approximations probably nearer the truth, suggested. Astronomy leans for aid on Geometry; but the precision of Geometry does not extend beyond the limits of its theorems. In Astronomy scarcely one element is presented simple and unmixed with others. Its value when first disengaged, must partake of the uncertainty to which the other elements are subject; and can be supposed to be settled to a tolerable degree of correctness,

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* The sidereal revolution of *Jupiter* is $4332^d.60228$; that of *Mercury* $87^d.969255$, and the squares of these numbers are as 2425.7 : 1. In the same proportion, nearly, (as 2427.9 : 1) are the cubes of the numbers 5.202778, 0.3871, which, respectively denote, on the assumption of 1 for the Earth's mean distance, the mean distances of *Jupiter* and *Mercury* from the Sun.

only after multiplied observations, and many revisions. There are no simple theorems for determining at once the parallax of the Sun, the right ascension of a star, or the heliocentric latitude of a planet.

We have already seen in Problem, p. 255, the method of determining some of the elements of a planet's orbit. But, others still remain to be determined, before we can construct Tables of the planet's motion, by means of Kepler's Problem, and subject the accuracy of such Tables to the test of observations. What are usually called the elements of a planet's orbit, are in number seven, of which the following is the enumeration:

The longitude of the ascending node of the orbit.
The inclination of the planet's orbit to the plane of the ecliptic.
The mean motion of the planet round the Sun.
The mean distance of the planet from the Sun.
The eccentricity of the orbit.
The longitude of the aphelion.
The epoch at which the planet is in the aphelion.

We shall proceed in the next Chapter, to determine these elements.

CHAP. XXV.

Determination of the Elements of the Orbits of Planets.

THE latitude of a planet arises from the plane of its orbit being inclined to that of the ecliptic. In consequence of the inclination, the orbit considered to be of an oval form, must intersect the plane of the ecliptic in two points, denominated (see p. 40,) *Nodes*. The imaginary line connecting them, is called the *Line of the Nodes*. The node, which the planet quits when rising from the ecliptic towards the north pole, is called *ascending*, and its symbol is ☊. The other, from which the planet (seeming to descend from our hemisphere) moves towards the south pole *descending*, and its symbol is ☋, the reverse of the former.

Method of determining the Nodes of a Planet's Orbit.

The Astronomical phenomenon indicating the node, is the planet in the plane of the ecliptic: at that time, its latitude is nothing. From amongst the latitudes, then, resulting, by computation, from the observed right ascensions and declinations, select that of which the value is nothing, or nearly so *, and then compute, according to the method of the preceding Chapter, the corresponding heliocentric longitude of the planet. The result will be, the longitude of the planet in the ecliptic, or, the longitude of the node.

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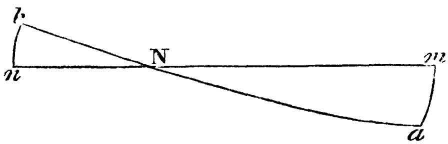
* The geocentric latitude is computed from observations made on the meridian. It will, therefore, probably happen, that the latitude is not exactly nothing, when the planet is on the meridian. In that case the time when the latitude is $=0$, must be computed on the same principles, as the time of the equinox was in p. 52.

econd method of determining the Longitude of the Node.

Let N be the place of the node, nNm a portion of the ecliptic,



aNb a portion of the planet's orbit, am, bn, two heliocentric latitudes reduced from their geocentric. Now, by Naper's Rules,

$$\text{rad.} \times \sin. Nm = \cot. \angle N \times \tan. am,$$
$$\text{rad.} \times \sin. Nn = \cot. \angle N \times \tan. bn;$$

∴ eliminating the cot. ∠ N,

$$\frac{\sin. Nm}{\sin. Nn}, \text{ or, } \frac{\sin. (mn - Nn)}{\sin. Nn} = \frac{\tan. am}{\tan. bn};$$

and, by reduction,

$$\sin. mn \times \cot. Nn - \cos. mn = \frac{\tan. am}{\tan. bn},$$

$$\text{whence, } \tan. Nn = \frac{\sin. mn \times \tan. bn}{\cos. mn \times \tan. bn + \tan. am}.$$

In this expression, bn, am, the two reduced latitudes, are known: also, nm, the difference of the two longitudes in the interval of the observations, and, accordingly, nN is known, or, the difference of the longitudes of n and N; the longitude of n, then, being known, the longitude of the node N is.

Method of determining the Inclination of a Planet's Orbit.

This is deducible from the preceding method: for, in that, nN was determined; and since bn is known, the spherical angle at N, the measure of the inclination, may be obtained from this expression,

$$\tan. \angle N = r \times \frac{\tan. bn}{\sin. Nn}.$$

Second method of determining the Inclination.

From the geocentric longitudes and latitudes deduce the heliocentric latitudes (see pp. 253, 254). Then the latter, (if a period of time sufficiently large has been taken) will form a series of terms increasing for a time, and then decreasing: amongst these, the greatest term, or the greatest latitude, is the measure of the inclination (see *Trigonometry*, p. 90.)

Method of determining the Periodic Time, and mean Motion of a Planet.

Observe (see p. 260,) the planet in its node, and again, when it returns thither; the interval elapsed is nearly the period, but not exactly, by reason (amongst other things) of the retrogradation of the nodes.

Second Method of determining the Periodic Time.*

Observe the planet in opposition, then its place, with regard to longitude, is the same as if the observation were made at the Sun. Amongst succeeding oppositions note that, in which the planet is in the same part of the heavens, as at the time of the first opposition. The interval between the two similar oppositions is nearly the periodic time of the planet.

Since the planet, at the last of the two similar oppositions, will not be exactly in the place in which it was at the time of the first, the *error*, or *deviation*, must be corrected and accounted for, by means of a slight computation, similar, in principle, to several preceding computations, and the nature of which will be sufficiently explained by an Example.

	Sept. 16, 1701, 2^h ♄'s long. in ☍	$353°$	$21'$	$16''$	S. lat.	$2°$	$27'$	$45''$
[2]	Sept. 10, 1730, $12^h\,27^m$ ♄'s long. in ☍	347	53	57	S. lat.	2	19	6
	Interv. $29^y - 5^d\ 13^h\ 33^m$, diff. of long.	5	27	19.				

Hence, it is plain, we must find the time of describing this difference $5°\ 27'\ 19''$: and the means of finding it may be drawn from other observations of the planet made in September 1731.

* The periodic times of planets are important elements, and admit of being very exactly determined; and when determined, become the best means of determining the mean distances, which by parallax, or other methods, are very inaccurately found.

[3] Sept. 23, 1731, $15^h\ 51^m$ ♄'s long. in ☍ 0° 30′ 50″ S. lat. 2° 36′ 55″
Interval betw. [3] and [2] $1^y\ 13^d\ 3^h\ 24^m$, diff. of long. = 12° 36′ 53″

Hence,

12° 36′ 53″ : 5° 27′ 19″ :: $1^y\ 13^d\ 3^h\ 24^m$: time required, which time = $163^d\ 12^h\ 41^m$.

Hence, adding this time to the former interval between opposition and opposition, we have

$$♄\text{'s periodic time} = \left\{ \begin{array}{rrrr} 29^y & 7^d & 0^h & 0^m \\ +\ 163 & & 12 & 41 \\ -\ \ \ 5 & & 13 & 33 \\ \hline 29 & 164 & 23 & 8^m \end{array} \right. \quad [7 \text{ Bissex.}]$$

And consequently, *Saturn's* mean motion for one year, or mean annual motion $= 360° \times \dfrac{1^y}{29^y\ 164^d\ 23^h\ 8^m} =$ 12° 13′ 23″ 50‴.

If the major axis of *Saturn's* orbit be, like that of the Earth's, progressive, then the above determination of the periodic time will not be very exact. And indeed, it ought rather to be regarded as a first approximation, and as the means of obtaining the true value of the periodic time more exactly. Using it therefore as an approximation, we may, by comparing oppositions of the planet, distant from each other by so large an interval of time, that the inequalities of the several revolutions will be mutually balanced and compensated, determine the periodic time to much greater, and indeed, to very great exactness. Thus,

228 *A. C.* March 2, 1^h ♄'s long. in ☍ 98° 23′ 0″ N. lat. 2° 50″
[2] Feb. 26, 1714, $8^h\ 15^m$ ♄'s long. in ☍ 97 56 46 N. lat. 2 3
* Interval $1943^y\ 105^d\ 7^h\ 15^m$, diff. of long. 26 14.

In order to find the time of describing 26′ 14″, as before, l. 1, 2, &c.

[3] March 11, 1715, $16^h\ 55^m$ ♄'s long. in ☍ 111° 3′ 14″ N. lat. 2° 25′
Interval betw. [2] and [3] $378^d\ 8^h\ 40^m$; diff. of long. 13° 6′ 23″

* 11 days are subtracted, in order to reduce it to the stile of the first observation, and 485 days added on account of the Bissextiles.

$\therefore$ time of describing $26' \; 4'' = 378^d \; 8^h \; 40^m \times \frac{26' \; 4''}{13^{\circ} \; 6' \; 28''} = 13^d \; 14^h$.

Adding this to the former interval, we have $1943^y \; 118^d \; 21^h \; 15^m$ for the interval, during which, *Saturn* must have made a complete number of revolutions. Now, if the periodic time ($29^y \; 164^d \; 23^h \; 8^m$) previously determined, had been exactly determined, then, dividing the interval by the periodic time, the result would have been an integer, the exact number of revolutions. But, the period having been only nearly determined, the result of the division (the quotient) will be an integer and some small fraction: still the number of revolutions which can only be denoted by an integer, must be denoted by that same integer. And in the case before us, it will be 66. The number of revolutions then being exactly 66, the exact time of one revolution

$$= \frac{1943^y \; 118^d \; 21^h \; 15^m}{66} = 29^y \; 162^d \; 4^h \; 27^m.$$

Hence, according to this more correct value of the periodic time, the mean annual motion is $12^{\circ} \; 13' \; 35'' \; 14'''$, and the mean daily $2'.0097$.

In the preceding method of determining the periodic time, *Saturn* was reduced to the same *longitude*. And longitude is measured from the first point of *Aries*, which point is continually moving westward $50''.1$ annually, and therefore, in $29^y \; 162^d \; 4^h \; 27^m$ moves through $24' \; 35''$. The period, then, of *Saturn*, which has been determined ($29^y \; 162^d \; 4^h \; 27^m$) belongs to his *tropical* revolution, and is shorter than that of his *sidereal*, by the time requisite to describe $24' \; 35''$, that is, about $12^d \; 7^h$. Hence, *Saturn's* period of sidereal revolution will be $29^y \; 174^d \; 11^h \; 27^m$.

It is equally easy to determine, directly from observations, the period of the sidereal revolution. Since, instead of *reducing Saturn* to the same longitude, we should have so to reduce his place, that it should be at the same distance from a fixed star at the end, as it was at the beginning of the period.

By similar means may the periods of *Mars*, *Jupiter*, and the *Georgium Sidus* be determined. The periodic times of *Venus* and *Mercury* are to be determined by observations of their conjunctions.

Method of determining the Major Axis of a Planet's Orbit.

Having, by the last method, found the periodic time of a planet, its major axis may be determined by Kepler's law (see p. 258,) thus making the Earth's sidereal periodic time $= p$, and *Saturn's* $= P$, and the Earth's mean distance from the Sun (half its major axis) $= 1$, we have

$$♄\text{'s mean distance} = \left[\frac{P}{p}\right]^{\frac{2}{3}} \times 1.$$

Since the sidereal periods can be determined to great exactness, this is the best method of determining the mean distance.

Second method of determining the Major Axis.

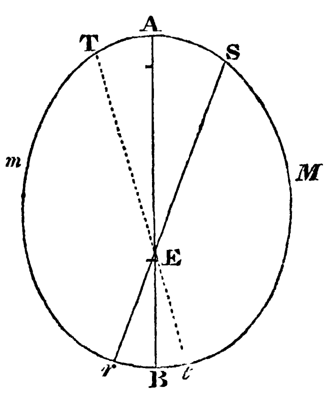
By the expression in p. 256,

$$a = \frac{e\,(1 + e \,.\, \cos.\, x)}{1 - e^2},$$

and, x, e, being determined by the process described in p. 256, a is known.

Method of determining the Eccentricity of a Planet's Orbit.

The eccentricity (e) may be determined from the expression [4], in p. 256, or by means of the greatest equation (see p. 203.) Now, the greatest equation is to be obtained by ascertaining that heliocentric longitude of the planet at which it is moving with



its mean angular velocity. Thus, by the preceding Example, *Saturn's* mean annual motion is about $12° \; 13' \; 35''$, and his mean daily motion about $2'.0097$. From Feb. 26, 1714, to March 11, 1715, (p. 263,) that is, in 378 days he moved through $13° \; 6' \; 28''$; at that time, then, he was moving with, very nearly, his mean motion. At that time, let M represent his place (nearly, see p. 202, the place of the mean distance,) then the greatest equation is equal to the mean anomaly (proportional to the time from M to A) minus the angle AEM; or, if m be, on the opposite side of the orbit, a point situated similarly to M, the angle proportional to the time through MAm minus the angle MEm, is equal to twice the greatest equation.

Method of determining the Aphelion of a Planet's Orbit.

The longitude of the aphelion may be determined from the expression given in p. 256, l. 15; for the quantity x, is its angular distance from one of the observed places of the planet.

Second method of determining the Place of the Aphelion.

This method supposes the greatest equation of the center to be known. Let the planet be observed near A; if it should happen to be there exactly, that circumstance would be known, by the mean angle, proportional to the time from M to A, minus the difference of the observed longitudes at M and A, being equal to the greatest equation. But, suppose it less, and, by the quantity e; then the planet is at some point S: make a second observation, and now let the difference of the true and mean motions be greater than the greatest equation, and, by the quantity e': then, the planet is at T, past the aphelion. Now, the longitudes of S and T are known; consequently, the difference, $\angle SET$; and also, the interval between the two observations: then, the points S and T being supposed to be very near to A,

$$e + e' : \angle SET :: e : \angle SEA.$$

Add therefore, the $\angle SEA$ to $\angle SEM$, and the angle MEA, or the angular distance of the aphelion (A) from M is known.

Method of determining the Epoch at which the Planet is in the Aphelion.

Since the interval (t) between the two observations at S and T are known, and the $\angle SEA$ has been determined; we have

$$e + e' : e :: t : \text{time from } S \text{ to } A;$$

which time, thus determined, added to the time of the observation at S, gives the time at which the planet was in the aphelion of its orbit.

CHAP. XXVI.

On the Formation of Tables of the Planets' Motions, &c.—The Variation of the Elements of the Orbits.—Method of deducing the Place of a Planet from Tables.

TABLES of the Planets are formed, precisely on the same principles as Tables of the Sun, and by means of the same problems; Kepler's, and that of the three bodies [see pp. 213, 221.]

In a theoretical point of view, there is no difference between the Earth revolving in an elliptical orbit round the Sun, and *Jupiter* revolving round the same body in a similar orbit; between the Earth disturbed in her elliptical motion by the action of *Venus*, and *Jupiter* disturbed in his elliptical motion, by the action of *Saturn*.

In order to apply Kepler's Problem, the mean motion, the eccentricity, and the place of the aphelion must be known; the two former are, by the methods in the preceding Chapters. By those methods also are known, the true longitudes of the aphelion and node, and the inclination of the planet's orbit. By the solution, then, of a spherical triangle, or, as it is usually effected, by a Table of Reduction (see Appendix) the *longitude* of the *aphelion in the orbit*, (which is the condition requisite in the application of Kepler's Problem,) may be determined.

In order to apply the problem of the three bodies, the masses and distances, angular as well as linear, of the bodies, must be known. The theory of their motions, therefore, must be known. We seem then to require to know that, which it is our object to investigate. There is, however, in the actual computation, no *arguing in a circle*. For, the corrections due to the perturbations are very small, and result, nearly of the same value, whether or not, in the theory of the planets' motions, account is made of the perturbations. The problem of the three bodies then can be applied, without requiring the motions of the planets to be most exactly known.

The process for computing the planet's heliocentric longitude is similar to that by which (p. 213,) the Sun's longitude was computed. To the mean anomaly, the equation of the center, for distinction called the *first Inequality*, and computed by Kepler's problem, must be applied as a correction, and then, other smaller corrections, due to certain disturbing forces and computed on Newton's *Principle of Gravitation.*

On merely mathematical considerations, *Mercury* and *Venus*, with respect to the theory of perturbations, are precisely under the same predicament, as the *Earth*, *Mars*, *Jupiter*, *Saturn*, and the *Georgium Sidus*. But, in point of fact, their perturbations, when numerically expounded, are so insignificant, that they have not been inserted in Tables. The Tables, then, of *Mercury* and *Venus* are constructed solely by the aid of Kepler's problem, and, by reason of their simplicity, give, very readily, the longitudes of those planets. Thus, suppose it were required to find *Mercury's* longitude on his orbit, June 3, 1793, at 5^h:

	Longitude.				*Aphelion.*			
Epoch for 1793,	2^s	$28°$	$5'$	$16''$	8^s	$14°$	$14'$	$17''$
Mean motion to June 3,	9	0	13	34	0	0	0	24
....................for 5^h	0	0	51	9				
Mean longitude..........	11	29	9	59	8	14	14	41
Equation of center......		− 23	39	58.5	11	29	9	50
Longitude on orbit......	11	5	30	0.5	3	14	55	9
					the mean anomaly.			

Here the mean longitude of the planet minus that of the aphelion, gives the planet's mean anomaly: from the mean anomaly is deduced the equation of the center; the equation of the center is applied to the mean longitude, and the final result is the longitude required.

This simplicity of process does not extend to the other planets. They require, like the Earth, several corrections for perturbations. *Jupiter* and *Saturn* remarkably affect each other. Their mutual perturbations are so considerable, that the expressions of them are separated into several terms, which become, in other language, so many *equations*. Lalande's Tables of *Jupiter* and *Saturn* contain no other corrections than what are due to the mutual perturbations of these planets.

If we examine the preceding process for finding the longitude of *Mercury*, we shall perceive, in the right hand column, the addition of 24″ to the longitude of the aphelion at the beginning of the year. And, in fact, the aphelia of all planets, like the Earth's are perpetually *progressive;* the progression arising from planetary perturbation. In the system of two bodies alone, the Sun and a planet, for instance, an accurate ellipse would be described, and its major axis would be ever quiescent.

The method of finding the *progression* of the aphelion of a planet's orbit, requires merely the repetition of the method for finding the place of the aphelion itself. The difference of the longitudes of the aphelion at two observations, is its progression in the interval between the observations. If the interval be n years, and the difference d, the annual progression is equal to $\frac{d}{n}$ and the secular to $100 \times \frac{d}{n}$.

The progression of the aphelion being known, the planet's anomalistic year may be determined (see p. 70.)

In the planetary system, and almost in all parts of it, every thing is in perpetual change. The nodes of orbits, like the aphelia, are in motion, but, not like the aphelia, according to the order of the signs. They are regressive; and their regression is to be found by a process similar to that which has just been described for finding the progression of an aphelion. Or, both motions may be investigated on the principles of Physical Astronomy.

The eccentricities and inclinations of the orbits of planets are also, from like causes, subject to variations, and which, by similar methods, may be investigated.

We shall conclude this Chapter by an Example, in which the longitude and latitude of *Mars* is found; and which differs from the former (p. 269,) by the introduction of certain small equations arising from the theory of perturbations, and by the *reduction* of the longitude on the orbit, to the true *ecliptical* longitude.

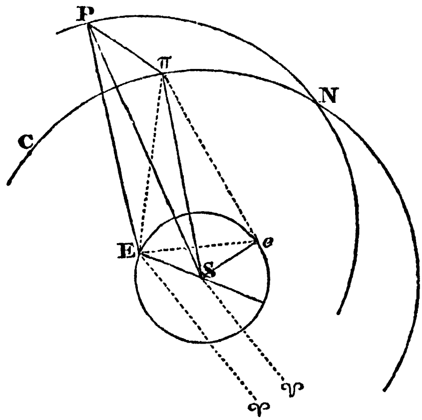
Required the Heliocentric Longitude and Latitude of Mars, Nov. 13, 1800, $11^h\ 8^m\ 20^s$.

	Longitude.				*Aphelion.*				*Node.*			
Epoch for 1800	7^s	22°	$34'$	$21''.8$	5^s	2°	$23'$	$17''$	1^s	18°	$1'$	$1''$
Nov.......	5	9	19	3.4	0	0	0	55.8	0	0	0	22.8
13^d........	0	6	48	46.5	0	0	0	2 4	0	0	0	1
11^h........	0	0	14	24.7								
8^m........	0	0	0	10.5								
20^s.........	0	0	0	0.4								
Mean longitude	1	8	56	47.3	5	2	24	15.2	1	18	1	24.8
(*e*)Sum of equa.	0	10	13	26.9	1	8	56	47.3	1	19	10	14.2
Long. on orbit	1	19	10	14.2	8	6	32	32.1	0	1	8	49.4
Reduction	0	0	0	−2.2	the mean anom.				argument of lat.			
Heliocen. long	1	19	10	12					Heliocen lat. =0° 2′ 13″.4 N.			

In this process, *e*, the sum of the equations, contains, besides the equation of the center (= 10° 13′ 13″.5) three small equations arising from the perturbations of *Venus*, the *Earth*, and *Jupiter*. The sum of these three equations is 13″.4, which added to the equation of the center make *e*.

The reduction − 2″.2, applied to the longitude on the orbit, gives the heliocentric longitude, measured along the ecliptic, and from the mean equinox. If this result be corrected for the effect of nutation, (by applying the equation of the equinoxes) there will be obtained, the longitude measured from the *apparent* equinox.

The longitude (A) of the node, taken away from ($A+NP$) the longitude of the planet on the orbit, gives (NP) the distance



of the planet from the node; this is technically called the *Argument of the Latitude;* since it is the quantity, by means of which the latitude may, either be computed (see p. 254,) or, be taken out of the Tables.

CHAP. XXVII.

On the mode of examining, by Observations, the Tables of the Motions of Planets.—Heliocentric Longitudes and Latitudes reduced to Geocentric.

It has been attempted, in the preceding Chapters, to explain the methods of extricating, from geocentric observations, the elements of the orbits and motions of planets, and then of combining such elements according to the laws of planetary motion. The combinations that result are heliocentric longitudes and latitudes. But these are of no immediate use to the observer, whether he wishes to find the planets in the heavens, or to examine the accuracy of the Tables. They must therefore be transformed into corresponding geocentric longitudes; and such transformation is the object of the ensuing problem.

It is required to determine, from the Heliocentric, the Geocentric Longitude and Latitude of a Planet.

The heliocentric longitude of the planet, and the longitude of the Earth (from the Solar Theory and Tables) being known, that is, the angles formed by πS, ES, with S♈, E♈ being known, the angle $ES\pi$, *the angle of commutation*, is known.

Again, from the heliocentric latitude $\angle PS\pi$, and SP, given by the Planetary Theory, (see p. 198,) the *curtate* distance $S\pi$ may be computed, and from the expression,

$$S\pi = SP \times \cos. PS\pi.$$

But, SE is also known by the Solar Theory (see p. 197,) therefore to determine $\angle SE\pi$, the difference of the heliocentric and geocentric longitudes, we have $\angle ES\pi$, SE and $S\pi$.

The angle $SE\pi$ may be thus determined:

Assume (see *Trig.* p. 28, &c.) an angle θ, such, that

$\tan.\ \theta = r \times \frac{S\pi}{SE} = r \times \frac{SP . \cos.\ PS\pi}{SE}$, then (see *Trig.* p. 29, 30,)

$$r \times \tan.\ \left(\frac{SE\pi - S\pi E}{2}\right) = \tan.\ \frac{ES\pi}{2} \tan.\ (\theta - 45^\circ,)$$

from which formula $SE\pi - S\pi E$ may be computed, and $SE\pi + S\pi E$ being known, the separate angles $SE\pi$, $S\pi E$ may be determined.

The angle $SE\pi$, the angle of elongation, is the difference (see p. 254,) of the geocentric, and of the Sun's longitude. Hence,

geocentric long. planet = longitude of ☉ ± ∠ elongation.

The geocentric latitude may be thus determined,

$$\tan.\ PE\pi = \frac{P\pi}{E\pi} = \frac{S\pi}{E\pi} . \tan.\ PS\pi = \frac{\sin.\ \angle\ SE\pi}{\sin.\ \angle\ ES\pi} \tan.\ \angle\ PS\pi$$

or,

$$\tan.\ \text{geocentric lat.} = \frac{\sin.\ \angle\ \text{elong}^n.}{\sin.\ \angle\ \text{commut}^n.} \times \tan.\ \text{heliocentric lat.}$$

EXAMPLE.

The Heliocentric Longitude and Latitude of Jupiter *being, on July* 11, $5^h\ 48^m\ 39^s$, 1800, $6^s\ 29^\circ\ 9'\ 14''.3$, *and* $1^\circ\ 13'\ 42''$ *respectively, required the corresponding Geocentric Longitude and Latitude.*

Heliocentric long. ♃ - - - - - - - - -	6ˢ	29°	9′	14″.3
(From Solar Tables) long. ☉ - - - - - -	3	19	52	28.3
∠ $ES\pi$ - - - - - - - -	3	9	16	46
∴ $\frac{1}{2}$ $ES\pi$ - - - - - - - -	1	19	38	23

θ computed from tan. $\theta = r \frac{SP . \cos.\ \text{heli}^c.\ \text{lat.}}{SE}$ (l. 3,)

From Tables of the planet. } log. SP - - - - -		.7355821
Log. cos. helio^c. lat. - - -		9.9999001
Arith. comp. SE - - - - -		9.9928989
(log. tan. 79° 24′ 48″) - -		10.7283811 (reject^s. 10)

$\therefore\ \theta = 79°\ 24'\ 48''$

$\theta - 45° =$	34 24 48	- - log. tan.	- -	9.8357262
$\frac{1}{2} ES\pi =$	49 38 23	- - log. tan.	- -	10.0706464
				19.9063726

therefore rejecting 10, 9.9063726 = log. tan. $\dfrac{Se\pi - S\pi E}{2}$;

$$\therefore\ \frac{Se\pi - S\pi E}{2} = 38°\ 52'\ 16''.$$

$$\text{But}\ \frac{Se\pi + S\pi E}{2} = 49\ \ 38\ \ 23\,;$$

$\therefore\ Se\pi = 88\ 30\ 39 = 2^s\ 28°\ 30'\ 39''$

But [p. 274, l. 2,] long. ☉ - - = 3 19 52 28.3

∴ [p. 274, l. 11,] geocen. long. = 6 18 23 7.3

To find the Latitude (from the expression, p. 274, l. 14,)

Log. sin. ∠ elon. ($SE\pi$ = 88° 30′ 39″)	9.99985
Ar. comp. sin. ∠ com. ($ES\pi$ = 99 16 46)	0.00573
Log. tan. heliocentric lat. (= 1 13 42)	8.33126

∴ log. tan. geocentric lat. = 8.33684 (reject. 10)

∴ geocentric latitude = 1° 14′ 39″.

CHAP. XXVIII.

On the Synodical Revolutions of Planets.—On the Method of computing the Returns of Planets to the same Point of their Orbit.—Tables of the Elements of the Orbits of the Planets.

IN the preceding pages, the conjunctions and oppositions of planets have been spoken of, but hitherto no method has been given of computing the times between successive conjunctions, or successive oppositions.

In the method also of determining the mean motions of planets (see p. 262,) directions were given for observing the planet in the same, or nearly the same point of its orbit, but no process or formula given, of computing the time at which such event would take place.

Towards these points then our attention will be now directed: we shall find that they depend on the same principles, and require, in the business of computation, nearly the same formulæ.

The time between conjunction and conjunction, or between opposition and opposition, is denominated, a *Synodical* period. Suppose we assume at a given instant, the *Sun*, *Mercury* and the *Earth* to be in the same right line: then after any elapsed time (a day for instance,) *Mercury* will have described an angle m, and the Earth an angle M, round the Sun. Now, m is greater than M (p. 247,) therefore at the end of a day, the separation of *Mercury* from the Earth (measuring the separation by an angle formed by two lines drawn from *Mercury* and the Earth to the Sun) will be $m - M$: at the end of two days, (the mean daily motions continuing the same,) the angle of separation will be $2(m - M)$; at the end of three days, $3(m - M)$; at the end of s days, $s(m - M)$.

When the angle of separation then amounts to 360°, that is, when $s(m - M) = 360°$, the Sun, *Mercury* and the Earth must be again in the same right line, and, in that case,

$$s = \frac{360°}{m - M} \qquad [1]$$

Where s denotes the time of a synodical revolution, m and M were taken to denote the mean daily motions, and s to denote the number of days; but, as it is plain, m and M may denote any portions, however small, of the mean motions, and s will still be the corresponding time, however reckoned, whether by days, or hours, or seconds.

Let P and p denote the periodic times of the Earth and the planet; then, since $1^d : M° :: P : 360°$,
and $1 : m :: p : 360$,

$$M = \frac{360}{P} \text{ and } m = \frac{360°}{p}; \therefore \text{ substituting}$$

$$s = \frac{360°}{360°\left(\frac{1}{p} - \frac{1}{P}\right)} = \frac{Pp}{P - p} \qquad [2]$$

and from either of these expressions, [1], [2,] the synodical revolution of the planet may be computed.

For instance, let the planet be *Mercury*, then $p = 87^d.969$, and $P = 365^d.256$; ∴ from expression [2]

$$s = \frac{365.256 \times 87.969}{277.287} = 115^d\ 21^h.$$

In the case of the Moon, $m = 13°.1763$, and $M = 59'\ 8''.3$

$$s = \frac{360°}{12°.1906} = 29^d\ 12^h, \text{ nearly.}$$

It is upon this synodical revolution of the Moon, that its phases depend.

$$\text{Since, } s = \frac{Pp}{P - p},\ p = \frac{sP}{s + P};$$

therefore, from the Earth's period (P) known, and the synodic (s) observed, we can determine the periodic time (P) of the planet. This method will not be accurate, if only one synodic period be observed, since that will be affected with all the deviations of the planet's real from its mean motion. The return of the planet then, to a conjunction nearly in the same part of its orbit, where a previous one was observed, must be noted, and then the interval of time divided by the number of synodical revolutions will give the time of a *mean* synodical period. For in this case there will take place, very nearly a mutual compensation of the inequalities arising from the elliptical form of the planet's orbit.

By the above method, the sidereal periods of *Mercury* and *Venus* may be accurately determined.

One reason already assigned for the necessity of knowing those particular conjunctions at which the planet will be nearly in the same part of its orbit, is the mutual compensation that will probably take place of the inequalities (relatively to mean motion) arising from the planet's elliptical motion. Another reason is, that on such conjunctions depend observations of great importance in Astronomy; the transits of *Venus* and *Mercury* over the Sun's disk. This will be manifest, if we consider that *Venus* to be seen on the Sun's disk must not only be in conjunction, but near the node of her orbit: at the next conjunction, after one synodical revolution, she cannot be near her node, and can only be again near, (supposing the motion of the nodes not to be considerable,) when she returns to the same part of her orbit as at the time of the first observation. The importance of knowing these particular conjunctions then is manifest, and we shall be possessed of the means of knowing them, by modifying the formulæ of p. 227, by which the times between successive conjunctions may be computed.

The time (t) of a synodical revolution $= \dfrac{Pp}{P-p}$.

At the times $\dfrac{2Pp}{P-p}$, $\dfrac{3Pp}{P-p}$, $\dfrac{4Pp}{P-p}$ and $\dfrac{nPp}{P-p}$; therefore the planet is still in conjunction: it will, therefore, be for the first time in conjunction, and the Earth and planet will be in the same part of their orbits, when $\dfrac{nPp}{P-p} = P$, or when, $n = \dfrac{P-p}{p}$.

Now, n must be a whole number, but $\frac{P-p}{p}$ may not be a whole number; in such case therefore, after one revolution of the Earth, the planet cannot be in conjunction, or if viewed, about that time, in conjunction, it cannot be in the same part of its orbit.

But, the conditions of the planet in conjunction, and in the same part of its orbit, although they cannot take place in 1 or 2 or 3 years ($P = 1$ year), yet they may take place in m years: and if such conditions take place, then must

$$\frac{nPp}{P-p} = mP,$$

$$\text{and } \frac{m}{n} = \frac{p}{P-p},$$

and the question now is purely a mathematical one, that of determining two integer numbers m and n, such, that $\frac{m}{n} = \frac{p}{P-p}$.

Thus, in the case of *Mercury*, whose tropical revolution is $87^d\ 23^h\ 14^m\ 32^s$ (= 87.968,)

$$\frac{m}{n} = \frac{87.968}{365.256 - 87.968} = \frac{87.968}{277.288};$$

consequently, in 87968 periods of the Earth, in which will happen 277288 synodic revolutions, *Mercury* will be observed in conjunction, and in the same part of his orbit. But, this result is, on account of the length of the period, practically useless: we must find then the lowest terms of the fraction $\frac{87.968}{277.288}$, and if the lowest terms still give periods too large, we must investigate some integer numbers, which are very nearly in the ratio of 87968 to 277288; so that we may know these periods at which the conditions required, will *nearly* take place.

$$\text{Now, } \frac{87968}{277288} = \frac{1}{\frac{277288}{87968}} = \frac{1}{3 + \frac{13384}{87968}},$$

$$= \cfrac{1}{3 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{5726}{7664}}}}$$

and, by continuing the operation, there is at last obtained a remainder equal nothing, and the greatest common measure is 8, and the fraction in its lowest terms is $\frac{10996}{34661}$ *, which result, for obvious reasons, is of no practical use: we must therefore find two near integer numbers; and this we are enabled to do by the preceding operation, which, as we take more and more terms of the continued fraction, affords fractions alternately less and greater than the proposed $\left[\frac{87968}{277288}\right]$ but continually, approximating, nearer and nearer, to its true value. Thus, the first approximation is $\frac{1}{3}$: or, in one year, in which happen 3 synodical periods, the planet will not be very distant from conjunction, nor from those parts of his orbit in which he was first observed. Again, the second approximation is $\cfrac{1}{3 + \cfrac{1}{6}} = \frac{6}{19}$, or in 6 years, in which happen 19 synodical revolutions, the planet will be less distant than he was before, from conjunction, and from those parts of his orbit in which he was in the former instance. The third approximation is $\cfrac{1}{3 + \cfrac{1}{6 + 1}} = \frac{7}{22}$, or, in 7 years, in which happen 22

* The operation in finding the *continued* fraction terminates, and gives a greatest common measure, because, since great accuracy is not requisite, we took $\frac{87968}{277288}$ to represent, which it does nearly, but not exactly, the ratio of the mean motions of *Mercury* and the Earth. If we had taken a fraction more exact to the true value, then the operation would not have happened to terminate.

synodical revolutions, the planet will be nearer to conjunction than he was at either of the two preceding points of time, and so on. This follows from the very nature of the process, by which the successive approximations are formed from the continued fraction [see Euler's *Algebra*, tom. II, p. 410, Ed. 1774]; but it may be useful to exemplify its truth by means of the instance before us. Thus, at the end of 1 year, since the diurnal tropical motion of *Mercury* is $4° \; 5' \; 32''.5 = 4°.092$ nearly, the angle described by that planet is

$$365.25 \times 4°.092 = 1494°.6 \text{ nearly}$$

$= 4 \times 360° + 54°.6$, and consequently, *Mercury* at the end of 1 year, is elongated (reckoning from the Sun) from the line joining the Sun and Earth, and beyond that line, by an angle $= 54°.6$; again, at the end of 6 years, the angle described by the planet is equal to

$(4 \times 360° + 54°.6) \times 6 =$ [rejecting 24 circumferences] $327°.6$; or at the end of 6 years, *Mercury* is elongated from the line joining the Earth and Sun, and not *up* to that line, by an angle $= 32°.4$.

At the end of 7 years, the angle described by *Mercury* is $[4 \times 360 + 54°.6] \times 7 =$ (rejecting 29 circumferences) $22°.2$: or *Mercury* is then *beyond* the line joining the Earth and Sun, by that angle. At the end of 13 years, *Mercury*, (rejecting 54 circumferences,) is separated from the line joining the Earth and Sun, and not *up* to that line, by an angle $= 10°.2$.

The series of fractions, formed as those in p. 280 were formed, is

$$\frac{1}{3}, \; \frac{6}{19}, \; \frac{7}{22}, \; \frac{13}{41}, \; \frac{33}{104}, - \frac{46}{145}, \; \&c.$$

The denominators denote the number of synodical revolutions, corresponding to the number of years denoted by the numerators: the number of periods of the planet must evidently be

$$3 + 1, \quad 6 + 19, \quad 7 + 22, \quad 13 + 41, \quad \&c.$$

that is, 4, 25, 29, 54, &c.

and therefore the series of fractions, in which the denominators are the number of periods of *Mercury*, will be

$$\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \text{\&c}$$

If we form a series of fractions to ascertain the probable transits of *Venus*, assuming $\frac{6501825.37}{4000084.72}$ to represent the ratio of her annual movement to that of the Earth, and making the denominator to represent the number of the periods of *Venus*; then its two first terms will be $\frac{8}{13}$ and $\frac{291}{473}$: which, it is plain, are quite sufficient, since they give the probable transits for three centuries to come.

Since the transit may probably happen (taking the above instance) in 8 years, and more probably in 291, it may happen in 291 − 8, or 283 years; in 291 − 2 × 8, or in 275 years; in 291 − 7 × 8, or in 235 years; and generally in $291 \mp m \times 8$ years, m being an integer.

By the preceding methods, we ascertain the separation of the planet from the line joining the Earth and Sun, when the Earth is in the same point, or nearly so, of its orbit; and, from such ascertained separation, it is easy to determine the exact time of the planet's conjunction: thus, at the end of 13 years, *Mercury* not having reached the line joining the Earth and Sun, by an angle = 10°.2, will, since the respective daily motions of the planet and Earth are 4° 5′ 32″.5, and 59′ 8″.3 arrive at conjunction in about 3 days. At this time, however, *Mercury* will not be in the same point of his orbit as at the time of the first observation.

If at the time of the first observation, there happened a transit of *Mercury* over the Sun's disk; then, at the end of 13 years, and about 3 days, *Mercury* will be nearly in the same part of his orbit, at nearly the same distance from his node, and consequently, another transit may be expected. To determine whether one will happen, compute for the time of conjunction, his geocentric atitude; then, if such latitude be less than the Sun's apparent diameter, a transit will happen.

This is the only method to be pursued; a method evidently of trial and some uncertainty. The numerators of the fractions, (see p. 280,) inform us when certain conjunctions are likely to take place : the exact times of the conjunctions must be computed from the Tables of the planets; and, from the same Tables, the geocentric latitudes. By these means Lalande computed the following Table for the transits of *Venus:*

	Years.	
[at ☊ of *Venus's* orbit] - - - - -	1631 - - -	Dec. 6,
[at ☊, 8 added] - - - - - - - -	1639 - - -	Dec. 4,
[at ☋,] - - - - - - - - - - - -	1761 - - -	June 5,
[at ☋, 8 added] - - - - - - -	1769 - - -	June 3,
[at ☊, 235=291 − 7 × 8 added to 1639]	1874 - - -	Dec. 8,
[at ☊, 235 added to 1769] - - -	2004 - - -	June 7.

We now subjoin Tables of the elements of the orbits of planets, principally taken from Laplace, and reduced from the new French measures which he has adopted.

Sidereal Periods of the Planets.*

Mercury - - - - - - - - - - - - - -	$87^{d}.969258$
Venus - - - - - - - - - - - - - - -	224.700824
The Earth - - - - - - - - - - - - -	365.256384
Mars - - - - - - - - - - - - - - -	686.979619
Vesta - - - - - - - - - - - - - -	1335.205
Juno - - - - - - - - - - - - - -	1590.998
Ceres - - - - - - - - - - - - - -	1681.539
Pallas - - - - - - - - - - - - - -	1681.709
Jupiter - - - - - - - - - - - - -	4332.596308
Saturn - - - - - - - - - - - - -	10758.969840
The Georgian Planet - - - - - - -	30688.712687

* The tropical periods may be deduced from the sidereal, by deducting the times which the several planets require respectively, for the description of an arc of longitude equal to the precession.

Mean Distances, or Semi-Axes of the Orbits.

Mercury	0.387098
Venus	0.723332
The Earth	1.000000 *
Mars	1.523694
Vesta	2.373000
Juno	2.667163
Ceres	2.767406
Pallas	2.767592
Jupiter	5.202791
Saturn	9.538770
The Georgian Planet	19.183305

Ratio of the Eccentricities (ae) to the Semi-Axis at the beginning of 1801: with the Secular Variation of the Ratio, (see p. 196). *The sign — indicates a diminution.*

	Ratio of the Eccentricity.	Secular Variation.
Mercury	0.205514	0.000003867
Venus	0.006853	0.000062711
The Earth	0.016853	0.000041632
Mars	0.093134	0.000090176
Juno	0.254944	not ascertained.
Vesta	0.093220	not ascertained.
Ceres	0.078349	not ascertained.
Pallas	0.245384	not ascertained.
Jupiter	0.048178	0.000159350
Saturn	0.056168	0.000312402
The Georgian Planet	0.046670	0.000025072

* The Earth's distance is here assumed as a standard and = 1: its distance from the Sun, in statute miles, is reckoned to be 93, 726, 900.

Mean Longitudes at the beginning of 1801; *reckoned from the Mean Equinox, at the Epoch of the Mean Noon of January* 1, 1801, *Greenwich.*

Mercury - - - - - - - - - - - - -	166°	0′	48″.2
Venus - - - - - - - - - - - - -	11	33	16.1
The Earth - - - - - - - - - - -	100	39	10
Mars - - - - - - - - - - - - - -	64	22	57.5
Vesta - - - - - - - - - - - - - -	267	31	49
Juno - - - - - - - - - - - - - -	290	37	16
Ceres - - - - - - - - - - - - - -	264	51	34
Pallas - - - - - - - - - - - - -	252	43	32
Jupiter - - - - - - - - - - - - -	112	15	7
Saturn - - - - - - - - - - - - -	135	21	32
The Georgian Planet - - - - - -	177	47	38

Mean Longitudes of the Perihelia, for the same Epoch as the above, with the Sidereal and Secular Variations.

	Long. Perihelion.			Sec. Var.	
Mercury - - - - -	74°	21′	46″	- - 9′	43″.5
Venus - - - - -	128	37	0.8	- −4	28
The Earth - - -	99	30	5	- - 19	39
Mars - - - - - -	332	24	24	- - 26	22
Vesta - - - - - -	249	43	0	not ascertained.	
Juno - - - - - -	53	18	41	not ascertained.	
Ceres - - - - - -	146	39	39	not ascertained.	
Pallas - - - - -	121	14	1	not ascertained.	
Jupiter - - - - -	11	8	35	- - 11	4
Saturn - - - - -	89	8	58	- - 32	17
The Georgian Planet	167	21	42	- 4	

Inclinations of Orbits to the Ecliptic at the beginning of 1801, *with the Secular Variations of the Inclinations to the true Ecliptic.*

	Inclination.			Secular Variation
Mercury	7°	0′	1′	19″.8
Venus	3	23	32	−4.5
The Earth	0	0	0	
Mars	1	51	3.6	−1.5
Vesta	7	8	46	not ascertained.
Juno	13	3	28	not ascertained.
Ceres	10	37	34	not ascertained.
Pallas	34	37	7.6	not ascertained.
Jupiter	1	18	51	−23
Saturn	2	29	34.8	−15.5
The Georgian Planet	0	46	26	3.7

Longitudes of the Ascending Nodes on the Ecliptic, at the beginning of 1801, *with the Sidereal and Secular Motions.*

	Longitude of ☊.			Secular and Sidereal Variation.	
Mercury	45°	57′	31″	13′	2″
Venus	74	52	38.6	−31	10
The Earth	0	0	0		
Mars	48	14	38	−38	48
Juno	103	0	6	not ascertained.	
Vesta	171	6	37	not ascertained.	
Ceres	80	55	2	not ascertained.	
Pallas	172	32	35	not ascertained.	
Jupiter	98	25	34	−26	17
Saturn	111	55	46	−37	54
The Georgian Planet	72	51	14	−59	57

The use of the secular variation of the eccentricity has been already explained (see p. 196.) The secular variations of the longitudes of the perihelia and the nodes are *sidereal*: consequently, they cannot be immediately applied to find a longitude at an epoch, different from that of the Tables; but first, the precession of the equinoxes must be added, and then the result will be a variation relatively to the equinoxes, or tropics. Thus, the secular sidereal variation of the longitude of the perihelion of *Mercury's* orbit is stated to be 9′ 43″.5; therefore, if we take the annual precession at 50″.1, and consequently the secular at 1° 23′ 30″, the secular variation with *regard to the equinoxes*, is 1° 33′ 13″.5; and, accordingly, the longitude of the perihelion of *Mercury's* orbit, for the beginning of 1901, will be

$$74° 21′ 46″ + 1° 33′ 13″.5 = 75° 54′ 59″.5.$$

For the beginning of 1821, will be

$$74° 21′ 46″ + 0° 18′ 38″.7 = 74° 40′ 24″.7.$$

Again, the *sidereal* secular variation of the perihelion of *Venus* is stated to be − 4′ 28″ (− indicating the motion of the perihelion to be contrary to the order of the signs); therefore the variation with regard to the equinoxes, is

$$1° 23′ 30″ − 4′ 28″ = 1° 19′ 2″;$$

and accordingly the longitude of the perihelion for the beginning of 1811, is

$$128° 37′ 0″.8 + 0° 7′ 54″.5 = 128° 44′ 55″.3;$$

and for the beginning of 1781,

$$128° 37′ 0″.8 − 0° 15′ 49″ = 128° 21′ 11″.8.$$

It is easy to see that, both for the nodes and perihelia, a column of the tropical secular variations might be immediately formed from the sidereal by the simple addition of 1° 23′ 30″. The motions of the aphelia and nodes in Lalande's (vol. I. p. 117, &c.) and Mr. Vince's Tables, (vol. III. p. 17, &c.) are motions relative to the equinoxes.

CHAP. XXIX.

On the Satellites of the Planets.—On Saturn's Ring.

The planet *Jupiter* is always seen accompanied by four small stars, which are denominated *Satellites*, and sometimes, *Secondary* planets, *Jupiter* being called the primary.

The satellites of *Jupiter* werediscovered in 1610, by Galileo · they are discernible by the aid of moderate telescopes, and are useful in Practical Astronomy. *Saturn* also, and the *Georgian Planet*, are acompanied by satellites, not however, to be seen except through excellent telescopes, and of no practical use to the observer. The number of *Saturn*'s satellites is seven, and of the *Georgian*'s six.

The satellites are to their primary planet, what the Moon is with respect to the Earth: they revolve round him, cast a shadow on his disk, and disappear on entering his shadow: phenomena perfectly analogous to solar and lunar eclipses, and which render it probable that the primary and their secondary planets are opaque bodies illuminated by the Sun.

That the satellites when they disappear, are eclipsed by passing into the shadow of their primary, is proved by this circumstance: that the same satellite disappears at different distances from the body of the primary, according to the relative positions of the primary, the Sun, and the Earth, but always towards those parts, and on that side of the disk, where the shadow of the primary caused by the Sun ought, by computation, to be. When the planet is near opposition the eclipses happen close to his disk.

There is an additional confirmation of this fact. The third and the fourth of *Jupiter's* satellites disappear and again appear on the same side of the disk; and the durations of the eclipses are found to correspond exactly to the computed times of passing through the shadow.

The motions of *Jupiter's* satellites are according to the order of the signs. The satellites are observed moving sometimes towards the east, and at other times towards the west: but when they move in this latter direction they are never eclipsed; when the eclipses happen, the satellite is always moving eastward; when the transits over the disk, the satellite is always moving westward: the motion therefore towards the east, or, according to the order of the signs, must be the true motion.

By the same proof it is ascertained, that the satellites of *Saturn* perform their motions, round their primary, according to the order of the signs. But the satellites of the *Georgian Planet* may be thought to form an exception, at least, the direction of their motions is ambiguous; for, motions performed in orbits perpendicular to the ecliptic (and such, nearly, are the orbits of the satellites of the *Georgian*) cannot be said to be either direct or retrograde.

The mean motions and periodic times of the satellites are determined by means of their eclipses, and, most accurately, by those eclipses that happen near opposition.

The middle point of time between the satellite entering and emerging from the shadow of the primary, is the time when the satellite is in the direction, or nearly so, of a line joining the centers of the Sun and the primary. If the latter continued stationary, then the interval between this and the succeeding central eclipse would be the periodic time of the satellite. But, the primary planet moving in its orbit, the interval between two successive eclipses is a *synodic period* (see p. 277). This synodic period, however, being observed, and the period of the primary being known, the sidereal period of the satellite may be computed *. Instead of two successive eclipses, two, separated from each other by a large interval, and happening when the Earth, satellite, and primary, are in the same position (in the direction of the same

* Since $\tau = \frac{Pp}{P-p}$, $p = \frac{P\tau}{P+\tau}$.

right line, for instance,) are chosen, and then the interval of time divided by the number of sidereal periods, will give, to greater accuracy, the mean time of one revolution.

The mean motions of the satellites do not differ considerably from their true motions. Hence, the forms of the orbits, must be nearly circular. The orbit, however, of the third satellite of *Jupiter* has a small eccentricity: that of the fourth, a larger.

The distances of the satellites from their primary are ascertained by measuring those distances, by means of a *Micrometer*, at the times of the greatest elongations.

The distance of one satellite being determined, the distances of others, whose periodic times should be known, might be determined by means of Kepler's law, namely, that the squares of the periodic times are as the cubes of the mean distances.

In order to obtain such results, we suppose Kepler's law to be true. But we may adopt a contrary procedure, and by ascertaining the periodic times and distances of all the satellites according to the preceding methods, determine the above-mentioned Law of Kepler to be true. See *Principia Phil. Natur*. lib. 3^{tius} p. 7, &c. Ed. *La Seur*, &c.

The eclipses of *Jupiter's* satellites are used in determining the longitudes of places, and, on account of this, their practical usefulness, have been studied with the greatest attention. Thence has resulted the curious and important discovery of the *Successive Propagation of Light*, which is the basis of the theory of aberration (see pp. 77, 108, &c.) The phenomenon that led to the discovery of the propagation of light, was, that an eclipse of a satellite did not always happen according to the computed time, but later, in proportion as *Jupiter* was farther from the Earth. If, for instance, an eclipse happened, *Jupiter* being in opposition, exactly according to the computed time, then about six months afterwards, when the Earth was more distant from *Jupiter* by a space nearly equal to the diameter of its orbit, an eclipse would happen about 16 minutes later than the computed time. And by similar observations it appeared, that the *retardation* of the time of the eclipse was proportional to the increase of the Earth's distance from *Jupiter*. This fact, the connexion of the retarded eclipse with the Earth's increased distance from *Jupiter*, was first noted by Roemer, a Danish Astronomer, in 1674: who suggested as an hypothesis, and as an adequate cause of the retardation, the successive propagation of light.* Subsequent observations accord so well with this hypothesis, that it is impossible to doubt of its truth: and it receives an additional, although an indirect, confirmation from Bradley's Theory of Aberration which is founded thereon.

The following Table, from Laplace, exhibits the mean distances and sidereal revolutions of the satellites of *Jupiter*, *Saturn*, and the *Georgium Sidus*.

Mean Distances, (the radius of the planet being = 1.)		Sidereal Revolutions.
Jupiter.		Day.
1st. Satellite....	5.81296	1.7691378
2	9.24868	3.5511810
3	14.75240	7.1545528
4	25.94686	16.6887697
Saturn.		
1st Satellite.....	3.080	0.94271
2	3.952	1.37024
3	4.893	1.88780
4	6.268	2.73948
5	8.754	4.51749
6	20.295	15.94530
7	59.154	79.32960
Georgium Sidus.		
1st Satellite......	13.120	5.8926
2	17.022	8.7068
3	19.845	10.9611
4	22.752	13.4559
5	45.507	38.0750
6...	91.008	107.6944

* Light is propagated through a space equal to the diameter of the Earth's orbit in 16^{m} 26^{s}.

On the Ring of Saturn.

Besides his seven satellites, *Saturn* is surrounded by a flat and thin ring of coherent matter. Dr. Herschell has discovered that the ring instead of being entire is divided into two parts, the two parts lying in the same plane.

The ring is luminous, by reason of the reflected light of the Sun; it is visible to us therefore, when the faces illuminated by the Sun are turned towards us: invisible, when the opposite faces; invisible also, when the plane of the ring produced passes through the center of the Earth; since then no light can be reflected to us; invisible also in a third case, when the plane of the ring produced passes through the center of the Sun; since, in that case, it can receive no light from that luminary. The plane of the ring is inclined to that of the ecliptic in an angle of about $31^\circ 24'$, and revolves round an imaginary axis perpendicular to its plane in $10^h\ 29^m\ 16^s$: and, which is worthy of notice, this period is that in which a satellite, having for its orbit the mean circumference of the ring, would revolve according to Kepler's law *.

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* The squares of the periodic times varying as the cubes of the mean distances, is frequently called, the *Third* law of Kepler.

CHAP. XXX.

ON THE LUNAR THEORY.

On the Phases of the Moon.—Its Disk.—Its Librations, in Longitude, in Latitude, and Diurnal.

Of all celestial bodies, the Moon is the most important, by reason of its remarkable and obvious phenomena: the intricacy of the theory of its motions; and, the usefulness of the practical results derived from such theory.

Some of the phenomena admit of an easy explanation, and require no great nicety of computation. Such are the phases of the Moon. Others, with regard to their general cause, admit also of an easy explanation; but, with regard to the exact time of their appearance and recurrence, require the most accurate knowledge of the lunar motions. Of this latter description, are the eclipses of the Moon.

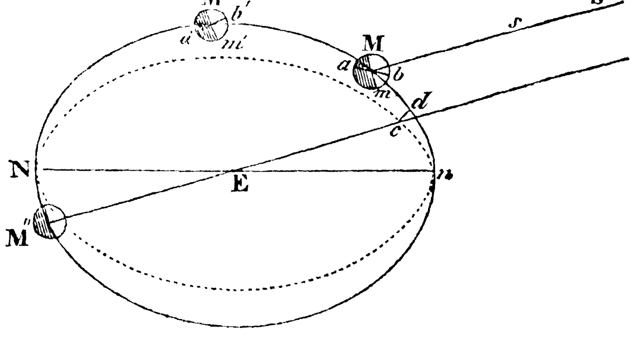
If therefore with a view to simplicity, we arrange the subjects of the ensuing Chapters, we ought first to place the phases of the Moon, next, the elements and form of the orbit, then, the lunar motions and their laws, and lastly, the lunar eclipses.

The explanation of the phases of *Mercury* and *Venus* was founded on the hypotheses, of their being opaque bodies illuminated by the Sun, and, of their revolution round the Sun. A similar explanation, on similar hypotheses, will apply to the Moon. We shall perceive the cause of its phases, if we suppose the Moon to shine by the reflected light of the Sun, and to revolve round the Earth: and, as in the case of the two inferior planets, the explanation does not require a knowledge of the exact curve in which the revolution is performed.

The Moon moves through 12 signs, or 360° degrees of longitude, in about 27 days. This is ascertained by observing each day, on the meridian, her right ascension and declination, and thence deducing, by calculation, (see p. 56,) the corresponding latitude and longitude. Hence, in a period somewhat more than the preceding, the Moon is on the meridian at all hours of the day, and the angle, formed by two lines drawn from the Moon to the Earth and Sun respectively, passes through all degrees of magnitude. The *exterior* angle therefore, (see p. 242,) on the magnitude of which, the visible illuminated disk depends, passes also through all degrees of magnitude: and the Moon accordingly, like *Venus*, must exhibit all variety of phase; the crescent near conjunction; the *half Moon* in quadratures; and the entire orb illuminated, or *full Moon* in opposition.

Venus revolves round the Sun, and the Moon round the Earth: but this difference of circumstance, in no wise affects the principle on which the phases depend: they are regulated by the inclination of the planes of the circles of illumination and vision: and their magnitude depends, as it was shewn in p. 242, on the versed sine of the exterior angle at the planet: that is, in Fig. p. 243, on the versed sine of the angle *SuF.*

The angle, analogous to *SuF* in the annexed Figure, will be



contained between a line *Ss* drawn to the center of the Moon at

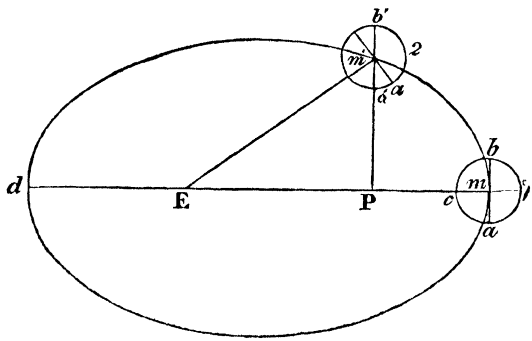
M, and a line drawn from *E* and produced through the same center. This angle, by reason of the parallelism of the lines drawn from *E* to the Sun, will equal the interior angle continued between *cE* and a line drawn from *E* to the center of the Moon; which angle, in other words, is the angle of elongation.

Hence, in delineating the Moon's phases, we may use a simpler expression, and state *the visible enlightened part to vary as the versed sine of the Moon's elongation.*

This is nearly true; not exactly, because the radius of the Moon's orbit subtends some, though a small, angle at the Sun; or, which is the same thing, because lines drawn from the Sun to the several points of the Moon's orbit are not strictly parallel.

The period of the Moon's phases, or the interval of time which must elapse before the phases, having gone through all their variety, begin to recur, must depend upon the return of the Moon to a situation similar to that which it had, at the beginning of the period. If we date then the beginning of the period from the time of conjunction, (the time of new Moon,) the end of the period must be when the longitudes of the Moon and Sun are again the same. Now the longitude of the Sun is continually increasing; when the Moon therefore has made, from its first position, the circuit of the heavens, it will be distant from the Sun, by the angular space through which, during the Moon's sidereal period, the Sun has moved. In order to rejoin the Sun then, and to be again in conjunction, it must move through this space, and a little more; and when it does rejoin the Sun, a *synodic* revolution is completed. And the period therefore of the Moon's phases is a *synodic* period. From the inequality of the Moon's motion, this *synodic* period, or *lunation*, is not always of the same length.

If we conceive a plane passing through the center of the Moon and perpendicular to a line drawn from the Earth to the Moon, then on such plane the Moon's face will be seen projected. This, since the Moon has ever been an object of the attention of Astronomers, has been delineated, and a map made of its seeming Seas, Mountains, and Continents. But, one map of the same hemisphere, has always served to represent the Moon's face: in other words, the same face of the Moon is always turned towards us. This is a curious circumstance, and the immediate inference from it is, that the Moon must revolve round its axis, with an



angular velocity equal to that with which it revolves round the Earth. For *, suppose in the position (1) a to be on the verge of the disk, then, if in the position (2) we still see the point a, in the verge, and in the same position, it must have been transferred, by rotation, through an arc $a'a$: since, in the case of no rotation, $b'a'$, parallel to ba, would have been the position of ba. Now, a being seen on the verge of the Moon's disk, $\angle Em'a =$ a right angle $= \angle Em'a' + \angle a'm'a$. But since EPm' is a right angle $\angle Em'P + \angle PEm'$ is one also: consequently,

$$\angle Em'a + \angle a'm'a = \angle Em'P\ (\angle Em'a) + \angle PEm;$$
$$\therefore\ \angle a'ma = \angle PEm',$$

and the angle $a'ma$ measures the rotation of the Moon round her axis that has taken place since it occupied the position (1),

---

* In the Figure, acb is supposed to represent the Moon's equator, and (which is not strictly true) to lie in the plane of the orbit: the axis of rotation, then, is perpendicular at m to that plane: perpendicular, for instance, to the plane of the paper, if the latter be imagined to represent that of the Moon's orbit.

and the angle PEm', the angular motion of the Moon round E from the same position.

If the angle PEm', the measure of the Moon's true angular distance from one of the apsides of its orbit, increased uniformly, and the Moon's rotation round her axis were uniform, the above result would always take place, that is, the same face of the Moon ought always to be turned to the spectator: and such phenomenon ought constantly to be observed. But since, which is the case, the Moon's true motion differs from the mean, and the angle PEm' does not increase uniformly, the preceding result will not be precisely true, if we suppose, (which is a probable supposition,) the Moon's rotation round her axis to be uniform. If after any time, 3 days for instance, mEm' should measure the Moon's angular distance from the position (1), then, by reason of the Moon's elliptical motion, in 6 days twice the angle mEm' will certainly not measure the Moon's angular distance: but, on the supposition of the Moon's uniform rotation, twice the angle $a'm'a$ would measure the quantity of rotation in 6 days. Hence, if the Moon's angular velocity should be diminishing from the position at (1), at the end of 6 days the point a previously seen on the verge of the Moon's western limb would have disappeared, and some points towards the verge of the Moon's eastern limb would be brought into view; and such, by observation, appears to be the case, and the phenomenon is called the Moon's *Libration in Longitude.*

Since this libration in longitude arises from the unequal angular motion of the Moon in her orbit, it must depend on the difference of the true and mean anomalies, in other words, on the equation of the center, or equation of the orbit; and would be proportional to that equation, and its maximum value would be represented by the greatest equation [6° 18′ 32″] in case the axis of the Moon's rotation were perpendicular to the plane of its orbit.

In the preceding reasonings, we have supposed the section bca, representing the Moon's equator, to be coincident with $mm'd$ the plane of the orbit: in other words, the axis of rotation to be perpendicular to the same plane. Now, the axis is not perpendicular but inclined to the plane at an angle of 5° 8′ 49″; the preceding results therefore will be modified by this circumstance. For, take the extreme case, and suppose the axis of rotation to be parallel to the plane of the orbit, andin the position (1) to be represented by *ce**: then it is plain, we should at position (1) see the pole *c*, and the hemisphere, projected upon a plane passing through *ba* perpendicular to the orbit; and, half a month after, at *d*, we should see the *opposite* pole *e*, and the *opposite* hemisphere, notwithstanding the equality between the Moon's revolution round the Earth, and her rotation round her axis. In intermediate inclinations then of the Moon's axis of rotation, part of this effect must take place, or must modify the preceding results (p. 297). If in the position (1) the Moon's axis being inclined to the plane of her orbit, we perceive, for instance, the Moon's north pole and not her south, we shall in the opposite position at *d*, after the lapse of half a month, perceive the Moon's south, and not her north pole; and, this effect is precisely of the same nature, as that of the north pole being turned towards the Sun at the summer, and of the south pole at the winter solstice, (see p. 12.) The perpendicularity therefore of the axis of rotation to the plane of the orbit is a condition equally essential, with that of the equality of rotation and revolution, in order that the same face of the Moon should be always turned to the spectator.

This second cause, preventing the same face of the Moon from being always seen, is called, with some violation of the propriety of language, the *Libration in Latitude.* For, it is plain, from the preceding explanation, that there are properly and physically no librations, but librations only seemingly such.

There is a third libration, discovered by Galileo, and called the *Diurnal Libration.* If the two former librations did not exist, the same face of the Moon would be turned, not to a spectator on the surface, but, to an imaginary spectator placed in the center of the Earth. Now, two lines drawn respectively from the center and the surface of the Earth to the center of the Moon, (the directions of two visual rays from the two spectators) form, at that center, an angle of some magnitude; and, when the Moon is in the horizon, an angle equal to the Moon's horizontal parallax. Hence, when the Moon rises, parts of her surface, situated towards the boundary of her upper limb, are seen by a spectator, which

* *e*, omitted in the Figure, ought to have been where *cm* produced cuts the circle *cba*.

would not be seen from the Earth's center. As the Moon rises, these parts disappear: but as the Moon, having passed the meridian, declines, other parts, situated near that boundary, which, whilst the Moon was rising, was the lower, are brought into view, and which would not be seen by a spectator placed in the center of the Earth. The greatest effect of this diurnal libration will be perceived, by observing the Moon first at her rising, and then at her setting.

This last libration, like the two preceding, is purely optical.

CHAP. XXXI.

On the Elements of the Lunar Orbit; Nodes; Inclination; Mean Distance; Eccentricity; Mean Motion; Apogee; Mean Longitude at a given Epoch.

In what has hitherto preceded, no attention has been paid to the actual form and position of the Moon's orbit. It has been sufficient for the purposes of general explanation to suppose the Moon describing a great circle situated, like the ecliptic, in the imaginary concave of the heavens.

If, however, we wish to form Tables of the Moon's motions in order to predict, at future periods, its exact positions in the heavens, we must proceed with greater accuracy, and, as in the case of the planets (see p. 259,) we must determine the elements of its orbit and motion.

It is necessary to determine, the longitude of the nodes; the inclination of the plane of the orbit to that of the ecliptic; the major axis of the orbit (supposing it to be elliptical); the eccentricity; the mean motion; the place of the apogee; and the Moon's mean longitude at an assigned epoch.

Position of the Nodes of the Moon's Orbit.

The longitudes of the nodes are determined, as in the case of a planet. From the Moon's observed right ascensions and declinations, the corresponding latitudes and longitudes are computed: when the latitude is equal nothing, the Moon is in the ecliptic; in the intersection therefore of the ecliptic and its orbit: in other words, in its node (see p. 260): the longitude corresponding to such latitude [= 0] is the longitude of the node.

If from two series of observations we deduce two latitudes, each equal to nothing, the difference of the two corresponding longitudes will give the motion of the node. This motion is, as in the case of the planets, a *regressive one;* and, what is remarkable, the regression is so rapid, that it passes through 360° in about 19 years. This is most surely determined by computation. But, there are certain phenomena which very plainly indicate the regression and its quickness. For instance, the star *Regulus* situated nearly in the ecliptic, (its latitude is about 27′ 35″,) was eclipsed by the Moon in 1757: the Moon therefore, must have been nearly in the ecliptic, and consequently, in its node. But, a few years after, the Moon instead of eclipsing *Regulus* passed at the distance of 5 degrees from the star. Again, if the Moon be observed at a certain time in conjunction with a star, and passing very near it, after the interval of a month, it will pass the star at a greater distance; after two months, at a still greater distance; and having reached a certain point, it will, in its conjunctions with the star, again approach it, and, at the end of about 19 years, pass it at the same distance, as at the beginning.

If we take the difference of two longitudes of the same node, we shall have, corresponding to the interval of time, the regression or motion of the node: if the interval be 100 years, the result will be *the secular motion* of the node. But, the mere difference of the two longitudes will not give the whole motion of the node, since the node may have regressed through several entire circuits of the heavens. For instance, in 100 years the mere difference of two longitudes is $4^s\ 14^\circ\ 11'\ 15''$: but, since the revolution of the Moon's nodes is performed in about $18^y\ 7^m$, in 100 years, besides this angle of $4^s\ 14^\circ\ 11'\ 15''$, 5 circumferences must have been described by the node: the proper exponent, therefore, of the secular motion of the node is

$$5 \times 360^\circ + 134^\circ\ 11'\ 15'' = 1934^\circ\ 11'\ 15'',\ [= 1934^\circ.1875.]$$

Hence, the tropical revolution of the node

$$= \frac{36000^\circ}{1934.1875} = 6798^d.54019 = 6798^d\ 12^h\ 57^m\ 50^s.6,$$

And since the equinoctial point in that time has regressed through 15′ 34″, the sidereal period is *less* than the former by nearly five days.

The annual regression of the node has been stated to be 19°.341875. This, as is plain from the mode of deducing it, is the *mean* regression. It will differ from the true annual regression, that which belongs to any particular year, 1810, for instance, by reason of several inequalities to which it is subject. And, as we shall hereafter see, the regression, besides its periodical inequalities, is affected with a secular inequality, by which its *mean motion* is, from century to century, retarded.

Inclination of the Plane of the Moon's Orbit.

Amongst the latitudes computed from the Moon's right ascensions and declinations, the greatest, at the distance of 90° from the node, measures the inclination of the orbit. This, sometimes, is found nearly equal to 5°: at other times, greater than 5°. For instance, the greatest latitude of the new and full Moon, when at 90° from the node, is found equal to 5° nearly: but the greatest latitude when the Moon is in quadrature, and also 90° from the node, is found equal to 5° 18′. Hence the inclination of the Moon's orbit is variable: it is greatest in quadratures and least in syzigies.

Major Axis of the Moon's Orbit.

The Moon's distance is to be determined by her parallax. The method of Lacaille, described in Chap. XII, p. 95, (which is inapplicable, in the case of the Sun, on account of his great distance,) applied to the Moon, affords practical results of great exactness.

The degree of exactness is known by knowing the probable error of observation, and the consequent error in the resulting distance: now, a variation of 1″ in the parallax would cause a difference of about 67 miles in the determination of the distance *: therefore, as the Moon's parallax can certainly be determined within 4″, the greatest error in the resulting distance cannot exceed 280 miles, out of about 240000 miles.

Since, generally, the Moon's distance can be determined, her greatest and least may: and consequently, supposing her orbit to be elliptical, the major axis of the ellipse.

Eccentricity of the Moon's Orbit.

This is known from the greatest and least distances of the Moon, the apogean and perigean. Or, it may be determined from the greatest equation (see p. 203.) Its quantity, according to Lalande, (*Astronomy*, tom II, p. 312,) is 0.055036: which gives for the greatest equation 6° 18′ 32″.076 (see p. 203.) M. Laplace however, states the eccentricity for 1800 to be 0.0548553, which gives the greatest equation of the center, 6° 17′ 54″.492.

The Moon's Mean Motion.

By p. 277, the time (τ) of a synodic revolution equals $\frac{Pp}{P-p}$. Hence, if τ be computed from observation, since P the Earth's period is known, p, the Moon's, may be computed from the expression

$$p = \frac{P\tau}{P+\tau}.$$

* Let p = ☽'s parallax, then, see p. 101, ☽'s dist. $= \frac{\oplus\text{'s rad.}}{p}$. Let e be the error of parallax, then the corresponding error in the Moon's distance $= \frac{\oplus\text{'s rad.}}{p} - \frac{\oplus\text{'s rad.}}{p+e} = \frac{\oplus\text{'s rad.}}{p}\left[1 - \frac{1}{1+\frac{e}{p}}\right]$

$$= \frac{\oplus\text{'s rad.}}{p}\left[1 - 1 + \frac{e}{p}\right] = \frac{\oplus\text{'s rad.}}{p}\left[\frac{e}{p}\right] \text{ nearly,}$$

(rejecting the terms involving e^2, &c.) Hence, if $e = 1''$, and $p = 1°$, and $\frac{\oplus\text{'s rad.}}{p}$, the ☽'s dist. = 240,000 miles, the error $= \frac{1}{60.60} \times$ 240,000 = 67 miles nearly. In the case of *Mars*, an error of 1″ includes in the distance an error of 40,000 miles.

If the Moon and Earth revolved equably in circular orbits, the above method would give accurately the Moon's period; but since the Moon and Earth are subject to all the inequalities of a disturbed elliptical motion, the result obtained, by the above process, from one observed synodic revolution, would differ considerably from the mean period. In order, therefore, to obtain a mean period, we must observe and compute two conjunctions, or two oppositions, separated from each other by a long interval of time; and then, the interval divided by the number of synodic revolutions will give nearly the length of a mean synodic period, and very nearly indeed, if the Moon's apogee at the time of the second conjunction or opposition should be nearly in the same place in which it was, at the time of the first conjunction or opposition. From this *mean value* of the synodic period (τ), the mean period (p) may be computed from the expression in p. 303, l. 21.

Now the phenomena of eclipses are very convenient for ascertaining the times of oppositions at which lunar eclipses must happen, (see Chap. IV, and p. 43.) And great certainty is obtained by their means. For, the recorded time of an eclipse by an antient Astronomer must be nearly the exact time of its happening; whereas, the assigned time of a conjunction or opposition happening long since, might, from the imperfection of instruments and methods, be erroneous, to a very considerable degree.

If we use two oppositions indicated by two eclipses, separated from each other by a short interval, we may deduce, but with no great exactness, (as has been already observed in this page,) the time of a synodic revolution. Thus, according to Cassini, a lunar eclipse happened in Sept. 9, 1718, $8^h\ 4^m$; another eclipse in Aug. 29, 1719, $8^h\ 32^m$. The interval between the two eclipses was $354^d\ 0^h\ 28^m$: and in the interval, 12 synodical revolutions had taken place; consequently, the mean length of one of these 12, is equal to $\frac{354^d\ 0^h\ 28^m}{12}$, equal to $29^d\ 12^h\ 2^m$.

This result cannot be exact: it is affected by the inequalities of the Moon's elliptical motion: for, independently of other causes, the place of the apogee of the Moon's orbit at the time of the second observation is distant from its place at the first by about 40°.

In order to obtain a true mean result we must employ eclipses very distant, in time, from each other. Such are, an eclipse recorded by Ptolemy to have been observed by the Chaldeans in the year 720 before Christ, March 19, $6^h\ 11^m$ (mean time at Paris, according to Lalande,) and an eclipse observed at Paris in 1771, Oct. 23, $4^h\ 28^m$. The interval between the eclipses, is 910044 days minus $1^h\ 43^m$, and expressed in seconds, 78627795420^s. In this interval 30817 synodic revolutions had happened; the mean length of one of these, then,

$$= \frac{78627795420^s}{30817} = 29^d\ 12^h\ 44^m\ 2^s.2.$$ Substituting this value in the expression, p. 303, l. 21, we may obtain the value of p.

The synodic period, if computed from different observations, does not always result of the same magnitude. Its *mean* length therefore is subject to a variation, arising from a cause called the Acceleration of the Moon's Mean Motion, which will be hereafter explained.

According to M. Laplace, the mean length of a synodic revolution of the Moon for the present time, is

$$29^d\ 12^h\ 44^m\ 2^s.8032\ (= 29^d.530588).$$

The periodic revolution of the Moon computed from the expression of p. 303,

$$= \frac{365.242264 \times 29.530588}{365.25 + 29.530588} = 27^d.321582$$

$$= 27^d\ 7^h\ 43^m\ 4^s.6848.$$

This is the *tropical* revolution of the Moon, or the revolution with respect to the equinoxes, since for P was substituted

365.242264 *, which expresses the Earth's tropical revolution.

The diurnal tropical movement of the Moon

$$= \frac{360^\circ}{27.321582} = 13^\circ.17636 = 13^\circ\ 10'\ 34''.896.$$

The *sidereal* revolution of the Moon differs from the tropical, for the same reasons, (see p. 66,) as the sidereal year differs from the tropical: and the difference must be computed on similar principles: thus, the mean precession of the equinoxes being 50″.1 in a year, or about 4″ in a month, the sidereal revolution of the Moon will be longer than the tropical, by the time which the Moon, with a mean diurnal motion of 13°.17636, takes up in describing 4″: which time is nearly 7^s. The exact length of a sidereal revolution is $27^d\ 7^h\ 43^m\ 11^s.510$, [= $27^d.321661$.] †

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* In pages 65, 223, the mean length of the solar year was stated to be $365^d\ 5^h\ 48^m\ 48^s$ (= 365.2422221): which length was adopted by Delambre when he inserted his Tables in Lalande's Astronomy. But, after a new examination, Delambre has found the mean length of the tropical year to be $365^d\ 5^h\ 48^m\ 51^s.6 = 365^d.242264$: which is the number employed in the text.

† We may easily deduce a formula of computation: thus, let p be the Moon's tropical revolution ($=27^d.321582$,) and x the sidereal period to be investigated; then, the arc of the precession described in the time $= \frac{50''.1 \times x}{365.25}$,

and the time of the Moon's describing it $= \frac{p}{365.25} \times \frac{50''.1}{360^\circ} \times x$.

Hence, $x = p + \frac{p}{365.25} \times \frac{50''.1}{360^\circ} \times x$, and thence

$$x = \frac{p}{1 - \frac{p}{365.25} \times \frac{50''.1}{360^\circ}},$$

= (expanding)

$$p\left[1 + \frac{p}{365.25} \times \frac{50''.1}{360^\circ} + \left(\frac{p}{365.25}\right)^2 \times \left(\frac{50''.1}{360^\circ}\right)^2 + \&c.\right]$$

in

Since the equinoctial point (from which longitudes are measured) regresses, the Moon departing from a point, where its longitude is $= 0$, returns to a point at which its longitude is again $= 0$, before it has completed a revolution amongst the fixed stars. In like manner, the node of the Moon's orbit regressing, and faster than the equinoctial point, the Moon quitting a node, will return to the same before completing a revolution amongst the fixed stars, and in a period less than the *tropical.*

This period may be thus found; the diurnal tropical movement of the Moon is $13^\circ\ 10'\ 34''.896$, and that of the node (see p. 301,) $= \frac{19^\circ.341875}{365.242264} = 3'\ 10''.6386$. Hence, the diurnal separation, which is the sum of the above quantities, since the node regresses, $= 13^\circ\ 13'\ 45''.535$ *: and consequently,

$$13^\circ\ 13'\ 45''.535 : 360^\circ :: 1^d : 27^d\ 5^h\ 5^m\ 35^s.6,$$

the revolution of the Moon with respect to its node.

This latter revolution may also be found by the aid of the formula given in the Note to p. 306.

By like processes, from the ascertained quantity of the apogee of the Moon's orbit, we may determine the *anomalistic* revolution

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in which, since $\frac{p}{365.25} \times \frac{50''.1}{360^\circ}$ is a very small quantity, two terms will be sufficient to give a value of x sufficiently near.

The same series may be used for determining the length of the sidereal from the tropical year, by substituting for p, $365^d.25$: in that case, the length of the sidereal year =

$$365.25\left[1 + \frac{50''.1}{360^\circ} + \&c.\right]$$

and a like series would serve to determine the length of an anomalistic year, substituting instead of $50''.1$, the quantity expressing the progression of the apogee.

* The Moon's motion with regard to its node may be found from eclipses; for, when these are of the same magnitude, the Moon is at the same distance from the node. Hipparchus, by comparing the eclipses observed from the time of the Chaldeans to his own, found that in 5458 lunations, the Moon had passed 5923 times through the node of its orbit: thence he deduced the daily motion of the Moon with regard to its node, to be $13^\circ\ 13'\ 45''\ 39'''\frac{2}{5}$. See Lalande, tom. II, p. 189.

of the Moon, M. Lalande (*Astronomie*, tom. II, p. 185,) states it to be $27^d\ 13^h\ 18^m\ 33^s.9499$, but M. Delambre, $27^d\ 13^h\ 18^m\ 37^s.44$ [$= 27^d.5546$.]

There is another revolution, of some consequence in the Lunar Theory, called the *Synodic Revolution of the Node*: this is completed when the Sun departing from the Moon's node first returns to the same. It is to be computed as the preceding periods have been. Thus, since the mean daily increase of the Sun's longitude is $59'\ 8''.33$, and the daily regression of the node is $3'\ 10''.638$, the sum of these quantities, which is the separation of the Sun from the node in a day, is $1^\circ\ 2'\ 18''.96$. Hence, $1^\circ\ 2'\ 18''.96 : 360^\circ :: 1^d : 346^d\ 18^h\ 28^m\ 16^s.032$ ($= 346^d.61963$ *.)

We will now exhibit, under one point of view, the different kinds of lunar periods and motions:

Synodic revolution - - -	29^d	12^h	44^m	$2^s.8032$	$= 29^d.530588$
Tropical - - - - - - -	27	7	43	4.6848......	27.321582
Sidereal - - - - - - -	27	7	43	11.5101......	27.321661
Anomalistic - - - - -	27	13	18	37.44..........	27.5546
Revol^n. in respect of node	27	5	5	35.6...........	27.212217
Tropical revolu^n. of node	6798^d	12^h	57^m	$50^s.416$	6798.54019
Sidereal - - - - - -	6793	10	6	29.952...	6793.42118

☽'s mean tropical daily motion - - - -	13°	$10'$	$34''.896$
☽'s sidereal daily motion - - - - - - -	13	10	35.034
☽'s daily motion in respect to the node -	13	13	45.534

Place of the Apogee.

The Moon's diameter is least at the apogee, and greatest in

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* This and the preceding periods are frequently found on like principles, but by different expressions, from the values of the *secular motions.* Thus, in 100 Julian years, each consisting of $365^d.25$, the *secular motion* of the Sun is $36000^\circ\ 45'\ 45''$ ($36000^\circ.7624998$) and the secular motion of the node (see p. 301,) $1934^\circ.1875$: and the sum of these is $37934^\circ.95$ nearly: thence $37934.95 : 360 :: 100 : \text{period} = \frac{36000}{37934.95}$.

the perigee: and since the diameter can be measured by means of a micrometer, or can be computed from the time it takes up in passing the vertical wires of a transit instrument, the times of the least and greatest diameter, or the times when the Moon is in her apogee and perigee, can be ascertained. Instead of endeavouring to ascertain when the Moon's diameter is the least, Lalande, *Astron.* tom. II, p. 162, says, that it is preferable to observe the diameters towards the Moon's mean distances when the diameter is about 31′ 30″. If two observations can be selected when the diameter is of the same quantity, then we may be sure that, at these two observations, the Moon was at equal distances from the apsides of its orbit. The middle time then between the two observations is that in which the Moon was in her apogee.

By finding the places of the apogee, according to the preceding plan, and comparing them, it appears that the apogee of the Moon's orbit is progressive: completing a sidereal revolution in $3232^d\ 11^h\ 11^m\ 39^s.4$, and a tropical, in $3231^d\ 8^h\ 34^m\ 57^s.1$. Laplace states the sidereal revolution of the apogee to be $3232^d.579$, that is, $3232^d\ 13^h\ 53^m\ 45^s.6$. (See *Exposition du Systeme du Monde*, Edit. 2. p. 20.)

Mean Longitude of the Moon at an assigned Epoch.

By observations on the meridian, the right ascension and declination of the Moon are known; thence may be computed, the Moon's longitude. This resulting longitude is the true longitude, differing from the mean by the effect of all the inequalities, elliptical, as well as those that arise from the perturbations of the Sun and planets. The *mean* longitude therefore, is the difference of the true longitude and of the sum (mathematically speaking) of the *equations* due to the inequalities. In order to be determined, then, the Lunar Theory must be known to some degree of exactness. Any new inequality discovered will affect the previous determination of the mean motion: and accordingly, keeping pace with the continual improvements in the Lunar Theory, repeated alterations have been made in the quantity of the mean longitude. In the last Lunar French Tables, the epoch of the mean longitude for Jan. 1, 1801, midnight at Paris, is $3^s\ 21^\circ\ 36'\ 30''.6$: which for Greenwich, Jan. 1 at noon, is $3^s\ 28^\circ\ 16'\ 56''.1$.

CHAP. XXXII.

On the Secular Equations that affect the Elements of the Lunar Orbit.

THE correction, which is called a *Secular Equation*, is strictly speaking periodical, but requiring a very large period, in order to pass through all its degrees of magnitude before it begins to recur. Its quantity, in general, is very small, and usually expounded by its aggregate in the space of 100 years.

The nodes, the apogee, the eccentricity, the inclination of the Moon's orbit, the Moon's mean motion, are all subject to secular inequalities. And the mode of detecting these inequalities is nearly the same in all.

If we subtract the longitude of the Moon's node now, from what it was 500 years ago, the difference (see p. 301,) is the regression of the node in that interval: the mean annual regression is the difference divided by 500. If we apply a similar process to an observation of the Moon's node, made now, and to one made 1000 years ago, the result must be called, as before, the mean annual regression of the node; and this last result ought, if the regression were always equable, to agree with the former: if not, as is the case in nature, the difference indicates the existence of a *secular inequality*, requiring for its correction, a *secular equation.*

By a similar method the motion of the perigee of the Moon's orbit is found to be not, strictly, a mean motion, but subjectto a secular inequality.

But the most remarkable inequality is that which has been detected in the Moon's mean motion, and which is now known by the title of the *Acceleration of the Mooon's Mean Motion.* The fact of such acceleration was first ascertained by Halley, from the comparison of observations: the cause of the acceleration has been assigned by Laplace*. Although the method of detecting the existence of these inequalities does not differ in principle, from methods just described, yet, on account of its importance, we will endeavour to explain it more fully.

As we have before remarked, eclipses are a species of observations on which we may rely with great certainty; quite distinct from merely registered longitudes which must partake of all the imperfections of methods used at the times of their computation. Now, in the year 721 before Christ, with a specified day and hour, Ptolemy records a lunar eclipse to have happened. The Sun's longitude then being known, the Moon's, which must at the time of the eclipse differ from it by 6 signs, is known also. The Moon's longitude however, computed for the time of the eclipse, and by means of the Lunar Tables, does not agree with the former †. In some part or other, then, the Tables are defective, or without some modification, are not applicable to ages past.

The Moon's place computed from the eclipse is advanced beyond the place computed from the Tables by 1° 26′ 24″; an error too great to be attributed to any inaccuracies in the coefficients of the equations belonging to the *periodic* inequalities, and which would seem rather to be the aggregate, during many years, of a small error in some reputed constant element, such as the Moon's mean motion.

On the hypothesis then of an acceleration in the Moon's motion, that is, if we suppose the Moon now to move more rapidly than it did 2000 years ago, the error of 1° 26′ 24″ can be accounted for. With a mean motion too large, we should throw the Moon too far back in its orbit. And, with the same motion, but for a point of time less remote than the preceding, we ought, if the hypothesis of the acceleration be true, to throw the

* See Laplace, *Exposition du Syst. du Monde*, Edit. 2, pp, 20, 214, &c. also *Mec. Celeste*, pp. 175, &c. Lalande, tom. II, p. 185: Halley, *Phil. Trans.* Nos. 204, and 218, Newton, p. 481, Ed. 2.

† The *true* longitudes are not compared, but the *mean*.

Moon less far back in her orbit: for that would produce an error of the *same kind* as the one already stated, (p. 311, l. 22). Now this is the case, and has been ascertained to be so, by means of an eclipse observed at Cairo by *Ibn Junis*, towards the close of the tenth century.

The acceleration of the Moon's motion therefore, discovered by Halley, may be assumed as established: or, in other words, in the former estimates of the quantity of the Moon's motion, a large secular inequality was included, and which it is now necessary to deduct, in order that what remains may be truly a *mean motion*.

The variation in the mean motion of the Moon, will, it is plain, affect the durations of its synodic, tropical, and sidereal revolutions.

With this secular equation in the Moon's mean motion, the equations in the motions of the nodes and of the apogee are connected. The latter are subtractive, whilst the former is positive; and, according to Laplace, (*Mec. Celeste*, tom. III, p. 236,) the secular motions of the perigee, the nodes, and mean motion, are to each other, as the numbers 3.00052, 0.735452, and 1.

The mean anomaly of the Moon, which is the difference of her mean longitude and the mean longitude of the apogee, must be subject to a secular equation, which is the difference of the secular equations affecting the longitudes of the Moon and of the apogee.

All quantities, in fact, dependent on the Moon's mean motion, the apogee and nodes, must be modified by their secular equations.

The Moon's distance from the Earth, the eccentricity and inclination of her orbit, are, according to M. Laplace, also affected with secular equations connected with that of the mean motion. But, the major axis is not.

CHAP. XXXIII.

On the Parallax and apparent Diameter of the Moon.

In the preceding Chapter, it was asserted that the apogean and perigean lunar distances, and, accordingly, the major axis of the Moon's orbit, might be computed from observations of the Moon's horizontal parallaxes.

The same may be effected by Astronomical measurements of the Moon's apparent diameters. And, there is, at the same place, and at the same time, a constant ratio subsisting between the Moon's horizontal parallax (P) and her apparent diameter (D). For, since the former is the angle which the Earth's radius subtends at the Moon, we have

$$P = \frac{\text{rad. } \oplus}{\text{☽'s dist. from } \oplus},$$

and since, $$D = \frac{\text{☽'s real diameter}}{\text{☽'s dist. from } \oplus},$$

there results $$\frac{P}{D} = \frac{\text{rad. } \oplus}{\text{☽'s real diameter}}.$$

This ratio remains constant, if the Earth be supposed a sphere, for then the radius is invariable; it is also a constant ratio at the same place, whatever be the Earth's figure.

If we suppose $P = 57' \; 4''.16844$, and $\frac{D}{2} = 31' \; 7''.7304$,

then $\frac{P}{D} = \frac{1}{.54586}$, and $\frac{D}{2P} = .27293 =$ nearly (by the method of continued fractions) $\frac{3}{11}$. Hence, from the Moon's apparent semi-diameter, we may deduce the corresponding horizontal parallax, by multiplying the former by $\frac{11}{3}$: and *vice versâ.*

The horizontal parallax of the Moon is the angle subtended by the Earth's radius at the Moon. Hence, the Earth not being spherical, the horizontal parallax is not the same, at the same instant of time, for all places on the Earth's surface. One proof that the Earth is not spherical, is by reversing this inference, namely, that the horizontal parallaxes computed for the same time are found not to be the same. Hence, in speaking of the horizontal parallax it is necessary to specify the place of observation. The Moon's parallax computed for Greenwich is different from the equatoreal parallax. Several corrections therefore, must be applied to an observed parallax, in order to compute, at the time of the observation, the Moon's distance from the center of the Earth. For, that distance, it is plain, ought to result the same, whatever be the latitude of the place of observation.

The greatest and least horizontal parallaxes of the Moon, computed from observations at Paris, are, according to Lalande, (*Astron.* tom. II, p. 197,) 1° 1′ 28″.9992, and 53′ 49″.728, and the corresponding perigean and apogean distances respectively, 63.8419, 55.9164. The corresponding apparent diameters are 33′ 31″, and 29′ 22″.

The mean diameter, that which is the arithmetical mean bebetween the greatest and least, is 31′ 26″.5; but, the diameter at the mean distance is smaller and equal to 31′ 7″.

Whatever be the quantity, which is the subject of their investigation, Astronomers are accustomed to seek for a constant and mean value of it, from which, the true and apparent values are perpetually varying, or, about which, they may be conceived to oscillate. In the subjects of time and motion, they search for *mean time* and *mean motion*, and by applying corrections or *equations* deduce the true. The Moon's parallax not only varies in one revolution, from its perigean to its apogean, but the parallaxes which are the greatest and least in one revolution, remain not of the same value, during successive revolutions: they may not be the greatest and least compared with other perigean and apogean parallaxes. But, all may be conceived to oscillate about one fixed and mean parallax, which has been designated by the title of *Constant* Parallax (*la Constante de la Parallaxe*).

We should obtain no standard of its measure, if we assumed it to be an arithmetical mean between its least and greatest values. For, the eccentricity of the lunar orbit varying, and consequently, the apogean and perigean distances, from the action of the Sun's disturbing force, the greatest parallax, if increased, would not be increased by exactly the quantity of the diminution of the least parallax; the mean of the parallaxes, therefore, would not always be the same constant quantity.

The constant parallax is assumed to be that angle, under which the Earth's radius would be seen by a spectator at the Moon, the Moon being at her mean distance and mean place: such, as would belong to her, abstracting all causes of inequalities. But then, even by this definition, the *constant* parallax would be represented by the same quantity only at the same place; for although the Moon's distance remains the same, the radius of the Earth, supposing it spheroidical, would vary with the change of latitude in the place of observation.

In order therefore, to rescind the occasion of ambiguity which might be attached to the phrase of *constant parallax*, Astronomers, in expressing its quantity, have stated also the place for which it was computed. Thus, the equatoreal diameter being greater than the polar, the *constant parallax under the equator* (as it is termed) is greater than the *constant parallax* under the pole: the former, Lalande, by taking a mean of the results obtained by Mayer and Lacaille, states it to be 57′ 5″, the latter 56′ 53″.2; the same author also, states the constant parallaxes for Paris, and for the radius of a sphere, equal in volume to the Earth, to be respectively 56′ 58″ 3, and 57′ 1″ (see *Astron.* tom. II. p. 315).

M. Laplace, however, proposes to deduce the several constant parallaxes from one alone: and to appropriate the term *constant*, to that parallax, belonging to a latitude, the square of the sine of which, is $\frac{1}{3}$ *. This parallax, by theory, he has determined to be 57′ 4″.16844, the corresponding apparent semi-diameter of the Moon being 31′ 7″.7304, (= 57′ 4″.16844 × .27293.)

This parallax being reckoned the mean parallax, the true parallax is to be deduced from it; if analytically expressed to be so, by a series of terms: if arithmetically computed, by the application of certain *equations*; the terms and equations arising, partly, from mere elliptical inequality, and partly, from the perturbation of the Sun.

The terms due to the first source of inequality are easily computed: for, if we call P the horizontal parallax to the mean distance (a), then since we have any distance (ϱ) in an ellipse expressed (see p. 191,) by this equation,

$$\varrho = \frac{a\,.\,(1-e^2)}{1 \pm e\,.\cos.\,\theta},$$

and since, the parallax $\times\ \varrho = P \times a$, we have the parallax $=$ $P \times \frac{1 + e\,.\cos.\,\theta}{1 - e^2}$, and expanding as far as the terms containing e^3, &c. $= P\,[1 + e\,.\cos.\,\theta + e^2.]$

The terms due to the theory of perturbation are not easily computed. In the extent of mathematical science, there is no computation of equal importance and greater difficulty †.

The formula for the parallax, in which the constant quantity is 57′ 4″.16844, belongs to a latitude, the square of the which is $\frac{1}{3}$. The corresponding formula for any other latitude is to be deduced by multiplying the former by $\frac{r}{r'}$, or by applying a correction

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* Laplace chose this parallel, since the attraction of the Earth on the corresponding points of its surface, is very nearly, as at the distance of the Moon, equal to the mass of the Earth, divided by the square of its distance from the center of gravity. Laplace, *Mec. Cel.* Liv. II, p. 118.

† The difficulty belongs equally to the formulæ for the latitude and longitude. See Lalande, tom. II, pp. 180. 193. 314.

proportional to $r - r'$; r and r' being the radii corresponding to two latitudes, and computed on the supposition that the Earth is a spheroid with an eccentricity $= \frac{1}{300}$. [See Tables XLV, and XLVI; in the collection (1806) of French Tables, and the Introduction. See also Vince, vol. III, p. 50.]

The Moon's horizontal parallax and apparent semi-diameter, for Greenwich, are inserted in the Nautical Almanack, and, for every 12 hours; the former is computed by the formula that has been mentioned (p. 316): the latter, by multiplying the parallax by .27293.

The Moon's distance may, as it has been already noted, be determined from her parallax; her greatest and least distance from her least and greatest parallax; and her mean distance from her mean parallax; and, taking for the value of the latter that determined by Laplace, we shall have

$$☽\text{'s distance} = \frac{57^{\circ}\ 2957795}{57'\ 4''.16844} \times \text{rad.}\ \oplus = \frac{57.2957795}{0.9511579} \times \text{rad.}\ \oplus$$

$= 60.23799 \times \text{rad.}\ \oplus$; therefore, if we assume the Earth's mean radius to be 3964 miles, the Moon's distance will be about 238783 miles.

The distances of the Sun and of the Moon from the Earth are inversely as their parallaxes. Hence, if the parallax of the former be considered equal to $8''.7$, the distances will be to each other, nearly, as 394 : 1.

The comparison of their respective mean parallaxes shews the Sun's distance from the Earth to be much greater than the Moon's: the comparison of the respective ratios of the greatest and least parallaxes will shew the variation of the Moon's distance to be much greater than that of the Sun, or, what amounts to the same, the Moon's orbit to be more eccentric than the Sun's.

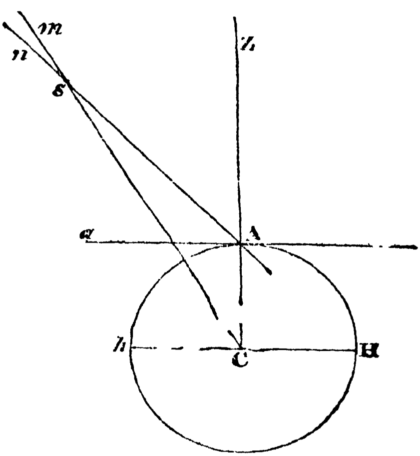
Thus,

$$\frac{☽\text{'s greatest parallax}}{☽\text{'s least parallax}}, \text{ or } \frac{1^{\circ}\ 1'\ 28''.9992}{53'\ 49''.728} = 1.1417;$$

$$\text{but, } \frac{\odot\text{'s greatest parallax}}{\odot\text{'s least parallax}} \text{ (see p. 187,) } = 1.0339.$$

Lacaille's method of determining the distance from the parallax applies successfully to the Moon, on account of her proximity to the Earth. It fails, with regard to the Sun, by reason of his distance. That distance is more than 24090 radii of the Earth: consequently, a radius of the Earth bears a very small proportion to it. The Sun's apparent diameter then seen from the surface of the Earth, is nearly the same, as if it were seen from the center; and his diameter on the meridian cannot be sensibly larger than his horizontal diameter. But, with the Moon, the case is different: since her distance is not much more than 60 radii of the Earth, her apparent diameter at its surface will be one 60th part greater than her diameter viewed from the center: and as she rises from the horizon, and approaches the spectator, her apparent diameter will increase and be greatest on the meridian. It is easy to assign a formula for its augmentation.

Let s be the Moon, p the parallax represented by the angle



msn, D the ☽'s apparent distance from the zenith, Δ the ☽'s diameter viewed from the Earth's center, a the augmentation of the diameter, then

$$☽\text{'s real diameter} = \Delta \times Cs = (\Delta + a) \times As;$$

$$\therefore \frac{\Delta + a}{\Delta} = \frac{Cs}{As} = \frac{\sin. CAs}{\sin. ACs} = \frac{\sin. D}{\sin. (D - p)}.$$

$$\text{Hence, } a = \frac{\Delta . \sin. D - \Delta . \sin. (D - p)}{\sin. (D - p)}$$

$$= \frac{2\ \Delta \left[\sin. \frac{p}{2} . \cos. \left(D - \frac{p}{2}\right)\right]}{\sin. (D - p)}$$

(see *Trig.* p. 18.)

From this formula, in which $p = P \, . \sin. D$, (P the horizontal parallax) a may be computed; but in practice, more easily from a formula, into which, by the known theorems of Trigonometry, the preceding may be expanded. (See Table XLIV, in Delambre's Tables; and the Introduction: also Vince, vol. III, p. 49)

When the Moon is in the horizon, $p = P$, and $D = 90°$;

$$\therefore \; a = \frac{\Delta \, [1 - \cos. P]}{\cos. P} = \Delta \, . [\sec. P - 1].$$

Hence, the ☽'s horizontal diameter is greater than the diameter [Δ] seen from the center, in the proportion of the secant of P to radius, that is, if we assume $P = 1°$, in the proportion of 1.0001523 : 1.

With the preceding value of the parallax (1°) the diameter (Δ) see p. 313, will = 2° × .27293 = 32′ 49″.9 nearly, and accordingly the augmentation = 32′ 49″.9 × (sec. 1° − 1)

$$= 32' \; 49''.9 \times .0001523$$
$$= 0''.3 \text{ nearly.}$$

It is plain, independently of any computation, that the Moon's horizontal diameter must appear larger than it would do, if seen from the center: since the visual ray, in the latter case, is the hypothenuse, in the former, the side of a right-angled triangle. In order to find how much the Moon must be depressed, so that, if it could, it would be seen under the same angle, as when viewed from the Earth's center, draw a line from the bisection of the radius joining the spectator and the Earth's center, perpendicularly towards the Moon's orbit: the intersection with the orbit is the Moon's place, and the depression, below the horizon, is, as it is plain, half the Moon's horizontal parallax.

CHAP. XXXIV.

On the Inequalities affecting the Moon's Orbit.—The Evection.—Variation. — Annual Equation, &c. — The Inequalities of Latitude and Parallax.

By a comparison of the Moon's longitudes, and of her distances deduced from her parallaxes, it appears that the lunar orbit is nearly an ellipse with the Earth in one of the foci. It appears also, that the Moon not only wanders from the ellipse which may be traced out as her mean orbit, and transgresses the laws of elliptical motion, but, that the ellipse itself is subject, in its dimensions, to continual variation: at one time, contracted within its mean state, at another, dilated beyondit.

In strictness of speech, neither the Earth's orbit nor the Moon's are to be called ellipses. If they are considered as such, it is purely on the grounds of convenience. It is *mathematically* commodious, or it may be viewed as an artifice of computation, first, to find the approximate place of each body in an assumed elliptical orbit, and then to compensate the error of the assumptions, and to find a truer place, by means of corrections, or, as they are astronomically called, *Equations*.

In a system of two bodies, when only forces, denominated centripetal, act, an accurate ellipse is described by the revolving round the attracting body; and, in such a system, the apsides, the eccentricities, the mean motions, &c., would remain perpetually unchanged. The introduction of a third, or of more bodies, and the consequent introduction of *disturbing forces*, destroys at once the beautiful simplicity of elliptical motion, and puts every element of the system into a state of continual mutation. Yet, the change and the departure from the laws of elliptical motion, are less in some cases than in others. The Earth's orbit approaches much more nearly to the form of an ellipse than the Moon's: the Sun's longitude, as we have seen in p. 215, computed by Kepler's Problem, did not differ from the true place by more than seven seconds: and that quantity, in those circumstances, represented the perturbations of the planets; and, the equations representing the perturbations were only four. But, in the case of the Moon, one inequality alone will require an equation nearly equal to two degrees, and the number of equations amounts to 28.

The quantity of perturbation, and the difficulty of computing it, depend less on the number than on the proximity of the disturbing bodies. In the case of the Sun, one equation suffices for the perturbation of *Venus*, and another for that of *Jupiter*. But, all the equations compensating the inequalities in the Moon's place, arise from different modifications of the Sun's disturbing force. It is not, however, solely the proximity, but the mass of the disturbing body, that gives rise *to equations*. The strictly *mathematical* solution of the problem of the three bodies, (see Chap. XXI.) is equally difficult, whatever be the mass of the disturbing body. The *practical* difficulty of merely approximating to the true place of the disturbed body, is very considerably lessened by supposing the mass to be small.

If we consider the subject merely in a mathematical point of view, the Moon's place, at any assigned time, results from the compound action of the Earth's centripetal force and the Sun's disturbing force; and the deviation from her place in the exact ellipse, arises entirely from the latter. We are at liberty to call the deviation, or error, one uncompounded effect: yet, since the quantity of the deviation cannot be computed from one single analytical expression, but must be so, by means of several terms, we may separate and resolve the effect into several, (analogous to the above-mentioned terms,) and the causes of some of which we may distinctly perceive and trace in certain simple resolutions and obvious operations of the Sun's disturbing force.

Long before Newton's time, and the rise of Physical Astronomy, this separation, or resolution of the error of the Moon's place from her elliptical place was, in fact, made. And, the error was said to arise from three inequalities, distinguished by the titles of *Evection*, *Variation*, and *Annual Equation*.

These three inequalities were noted because they rose, under certain circumstances, to a conspicuous magnitude; and, were distinguished from each other, because they were found to have an obvious connexion with certain positions of the Sun and Moon and of the elements of their orbits. Although their real physical cause was not discovered, yet the law of their variation was ascertained.

The other Lunar inequalities have not, like the three preceding, been distinguished by titles. This is owing principally to their want of historical celebrity; and they were not detected like the others, by reason of their minuteness, and the imperfection of antient instruments and methods.

Some explanation has already been given, (Chap. XVII, p. 174,) of the principles and mode of detecting and decompounding inequalities. The difference between an observed and computed place, indicates the operation of causes either not taken account of, or, not properly estimated in the previous computation.

Take, for instance, the Moon: her mean place computed from her mean motion, differs from her observed place; and the difference, if we suppose her to move in an elliptical orbit, is the equation of the center, or, of the orbit, called, the *first Lunar Inequality*.

Compute the Moon's place from a knowledge of her mean motion and of the equation of the center, and then compare the computed, with the observed, place. In certain situations, a great difference will be noted between the places, ascending in its greatest value to nearly 1° 18′ 3″. This difference is chiefly owing to the *Evection* discovered by Ptolemy, and named the *second Lunar Inequality*.

In like manner, we may conceive the *third Lunar Inequality* to be discovered. But, we will now proceed to consider more particularly the second inequality; the mode of ascertaining its maximum; its general effect; the formula expressing the law of its variation; and its cause, reckoning as such, some particular modification of the Sun's disturbing force.

Evection.

This inequality has a manifest dependence on the position of the apogee of the Moon's orbit. Let us suppose, the Moon to quit the apogee, the line of the apsides to lie in syzigy, and, that we wish to compute the Moon's place 7 days after her departure from syzigy, that is, when she will be nearly in quadratures. The Moon's place computed by deducting the equation of the center *, (then nearly at its greatest value and $= 6^{\circ}\ 37'\ 54''.492$,) from the mean anomaly (see Chap. XVIII.) will be found *before* the observed place by more than 80 minutes; in other words, the computed longitude of the Moon is so much greater than the observed longitude. But, if we suppose the apsides to lie in quadratures, then the Moon's place, 7 days after quitting her apogee, computed, as before, by subducting the equation of the center from the mean anomaly, will be found *behind* the observed place by more than 80 seconds; in other words, the computed longitude of the Moon is so much less than the observed.

It is an obvious inference, then, from these two instances, that some inequality, besides that of the elliptic anomaly, and, having a marked connexion with the longitude of the lunar apogee, affects the Moon's motion.

What, from the two preceding instances, would be an obvious inference to an Astronomer acquainted solely with the elliptic theory of the Moon? In the first case, the computed place being *before* the observed, it would *seem* that the equation of the center, to be subducted from the mean anomaly, had not been taken of sufficient magnitude; in the latter case, it would *seem* that the equation of the center had been taken too large.

Let us take another case: suppose, instead of comparing the computed with the observed place, that it was intended to deduce the quantity of the equation of the center from an observation of the Moon in syzigy. In that case, the equation of the center, reckoned as the difference of the true and mean longitudes, would result *too small* a quantity. And, this circumstance has really happened. For, the antient Astronomers who determined the elements of the lunar orbit by means of eclipses, when the Moon is in syzigy, have assigned too small a quantity to the equation of the center.

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* The anomaly is here supposed to be reckoned from apogee.

In the preceding instance, when the Moon is in syzigy and the apsides in quadrature, the determination of the equation of the center would be too small by the maximum value of the *Evection* [1° 20′ 29″.5]. But, in other positions of the apsides, the effect of the evection is to lessen, though not by its whole quantity, the equation of the center.

Astronomers, having found that the augmentation and diminution of the equation of the center arose from an inequality, soon ascertained the inequality to be periodical; in other words, that, after passing through all its degrees of magnitude, from 0 to its maximum value, it would recur. Now, of such recurring quantities the cosines and sines of angles are most convenient representations; for instance, $\pm K.\sin. E$ is competent to represent the evection: its maximum value is K, when $E = 90°$: and it is nothing, when E is. If then, the value of K could be assigned and the form for E, the numerical quantity of the evection could be always exhibited. After the comparison of numerous observations, and after many trials, it was found that

$$K = 1° \; 20' \; 29''.5, \text{ and } E = 2(☽ - ☉) - A,$$

A representing the mean anomaly of the Moon, and ☽ — ☉ signifying the angular distance of the Sun and Moon, or, the difference of their mean longitudes viewed from the Earth.

In the *equation* 1° 20′ 29″.5 . sin. [2 (☽ − ☉) — A], 1° 20′ 29″.5 is called the *coefficient*, and 2 (☽ — ☉) − A the *argument*.

If we represent the equation of the center by

$$(6° \; 17' \; 54''.49) \sin. A,$$

in which, the coefficient 6° 17′ 54″.49, is the greatest equation, and A (the mean anomaly) the argument, the Moon's longitude expressed by means of the two equations, that of the center *, and the evection, would stand thus:

$$☽\text{'s longitude} =$$
$$☽\text{'s mean long.} - (6° \; 17' \; 54''.49) \sin. A$$
$$- (1° \; 20' \; 29''.5) \sin. [2 (☽ - ☉) - A];$$

* If A be the mean anomaly, the equation of the center cannot be represented by a single term such as $a \sin. A$, but is, by a series of terms, such as $a \sin. A + b \sin. 2A + c \sin. 3A +$ &c. in which, however, the coefficients b, c, &c. decrease very fast.

now in syzigies ☽ $-$ ☉ $= 0$; ∴ sin. [2 (☽ $-$ ☉) $- A$] $= -$ sin. A; consequently, in this case, the former expression becomes

☽'s longitude =

☽'s mean long. $- (6° \ 17' \ 54''.49)$ sin. $A + (1 \ 20' \ 29''.5)$ sin. A, in which, the argument for the evection assumes that form, which is the general one of the equation of the center; and on this account, the former is sometimes said to *confound* itself with the latter, in syzigies. It also seems to lessen it, since the preceding expression may be put under this form,

☽'s longitude =

☽'s mean long. $- [6° \ 17' \ 54''.49 - 1° \ 20' \ 29''.5]$ sin. A, in which, the coefficient of sin. A would be the difference of the two coefficients $6° \ 17' \ 54''.49$, and $1° \ 20' \ 29''.5$; and, accordingly, A being the argument of the equation of center, that equation would appear to be lessened.

The evection itself, and, very nearly, its exact quantity, were discovered by Ptolemy in the first century after Christ, but the cause of it remained unknown till the time of Newton. That great Philosopher shewed that it arose from *one kind of alteration* which the Moon's centripetal force towards the Earth receives from the Sun's perturbation. Let us see how it may be explained:

When the line of the apsides is in syzygies, the equation of the center (p. 323,) is increased. The equation of the center depends on the eccentricity; (see pp. 196, 303.) an increase therefore in the former would indicate an increase in the latter. Hence, if it can be shewn that the Moon's orbit will, when the line of the apsides is in syzigies, be made more eccentric, by the action of the Sun's disturbing force, an adequate explanation will be afforded of the increase of the equation of the center above its mean value, and which increase is stiled the Evection.

Again, when the line of the apsides is in quadratures, the equation of the center, is lessened: the eccentricity therefore (see expression, p. 203,) is lessened: and now, in order to afford an explanation, it is necessary to shew that, in this position of the line of the apsides, the Sun's disturbing force necessarily renders the orbit less eccentric.

The Sun's disturbing force admits of two resolutions, one in the direction of the radius vector of the Moon's orbit: the other in the direction of a tangent to the orbit. The former sometimes augments, at other times, diminishes the gravity of the Moon towards the Earth, and always (see Newton, Sect XI, Prop. 66,) proportionally to the Moon's distance from the Earth. When the Moon is in syzigy it diminishes; consequently, in the first case, when the line of the apsides is also in syzigy, the perigean gravity, the greatest, (since it varies inversely as the square of the distance) is diminished, and by the least quantity; the apogean gravity, the least, is also diminished, but by the greatest quantity: the disproportion therefore between the two gravities is augmented; the ratio between them becomes greater than that of the inverse square of the distance: the Moon, therefore, if moving towards perigee, is brought to the line of the apsides in a point between its former and mean place, and the Earth: or, if moving towards apogee, reaches the line of the apsides in a point more remote from the Earth than its former and mean place. The orbit then becomes more eccentric; the equation of the center is increased; and, the increase is the *Evection.*

Thus is the first case accounted for: in the second, the Sun's resolved force *increases* the gravity of the Moon towards the Earth, and, as it has been said, proportionally to the distance. The perigean gravity, therefore, the greatest, is increased by the least quantity; the apogean, the least, is also increased, and by the greatest quantity. The disproportion, therefore, between these two gravities is lessened; the ratio between them is less than that of the inverse square of the distance. The Moon, therefore, if moving towards perigee, meets the line of the apsides, in a point more remote from the Earth than the mean place of the perigee: if moving towards the apogee, in a point between the Earth and the mean place of the apogee. The orbit, by these means, becomes less eccentric; the equation of the center is diminished, and, the diminution is the *Evection.*

We will now proceed to consider the third inequality called,

The Variation.

By comparing the Moon's place computed, from her mean motion, the equation of the center, and the evection, with her observed place, Tycho Brahe, in the sixteenth century, discovered that the two places did not always agree. They agreed only in opposition and conjunction, and varied most, when the Moon was half way between quadratures and syzigies, that is, in *Octants*. At those points the new inequality seemed to be at its maximum value [35′ 41″.6].

It appeared clearly from the observations, that this new inequality was connected with the angular distance of the Sun and Moon: and that its *argument* must involve, or, be some function of, that distance. At length, it was found, that the equation due to the inequality, was

$$(35' \; 41''.6) \,.\, \sin.\, 2\,(☽ - ☉)$$

35′ 41″.6 being the *coefficient*, and 2 (☽ − ☉) the *argument*.

According to the above form, the variation is 0 in syzigies and in quadratures, and at its maximum (35′ 41″.6) in octants.

If now, by means of this new equation, we farther correct the expression (p. 324,) for the Moon's place, we shall have

☽'s longitude =

☽'s mean longitude − (6° 17′ 54″.49) sin. A
− (1 20 29.5) . sin. [2 (☽ − ☉) − A]
+ (35′ 41″.6) . sin. 2 (☽ − ☉).

We will now proceed to Newton's explanation of the cause of this inequality.

One effect, from one resolved part of the Sun's disturbing force, we have already perceived in the evection. The variation is occasioned by the other resolved part, that which acts in the direction of a tangent to the Moon's orbit. This latter force will accelerate the Moon's velocity in every point of the quadrant which the Moon describes, in moving from quadrature to conjunction. The force will be greatest in octants and nothing in conjunction; and, when the Moon is past conjunction, the tangential force will change its direction, and retard the Moon's motion. The greatest acceleration therefore, of the Moon's velocity must happen in syzigy: exactly at the termination or cessation of the accelerating force. At that point, therefore, the Moon's velocity must differ most from her mean, or, rather from that velocity which she would have, if the effect of the accelerating tangential force were abstracted. When the Moon moves from that point, her place at the end of any portion of time, a day, for instance, will be beyond her mean place, or beyond the place of an imaginary Moon endowed with a motion from which the effect of *Variation* is abstracted. At the end of the second portion of time, the real Moon will have described a space less, by reason of the retarding force (see p. 327,) than the space described in the first, but still, greater than the space described by the imaginary Moon; so that, at the end of the second portion of time, the two Moons will be distant from each other, by the effect of two separations; and, for succeeding portions of time, the real Moon will still continue describing greater angular spaces than the imaginary Moon, and the separation of the two Moon's, which is the accumulation of the individual excesses, will continue, till the retarding force, by the continuance of its action, and the increase of its quantity, shall have reduced the Moon's velocity to its mean state: at that term which is the octant, the separation will cease to increase, and, be at its greatest. And this greatest separation, 35′ 41″.6, is the maximum effect of the *Variation*: and the separation, previously described, in any point between conjunction and octants, is its common effect.

The preceding reasoning is precisely similar to that which was used in p. 200, on the subject of the greatest equation of the center. At the apogee, the mean velocity differs most from the true, and then the two Suns are together; and, they are most separated, when the real Sun moves with its mean angular velocity.

We will now proceed to a fourth inequality called,

The Annual Equation.

The two former inequalities, of which the periods are short, may be ascertained by observing the Moon during one revolution. But, in order to detect this fourth inequality, it is necessary to compare similar positions of the Moon, computed according to the theory of the three preceding inequalities, in different months of the year. If the computed place agreed with the observed place in January, it would not in March, and it would most differ in July. The inequality was soon found to have a connexion with the Earth's distance from the Sun, and its *equation* was at length found to be

$$11' \; 11''.97 \times \sin. \odot\text{'s mean anom.}$$

11′ 11″.97 being the coefficient, and ⊙'s mean anomaly the argument.

According to the preceding form, the maximum (11′ 11″.97) of the annual equation happens when the Sun's mean anomaly is = 90°, or 270°. The equation is nothing, either when the Earth is in the aphelion or perihelion.

If now, by means of this new equation, we farther correct the expression for the Moon's longitude, we shall have

$$
\begin{aligned}
&☽\text{'s longitude} = \\
☽\text{'s mean longitude} &- (6^\circ \; 17' \; 54''.49) \sin. A \\
&- (1^\circ \; 20' \; 29''.5) \sin. [2(☽ - \odot) - A] \\
&+ \quad (35' \; 41''.6) \sin. 2(☽ - \odot) \\
&+ \quad (11' \; 11''.97) \sin. \odot\text{'s mean anom.}
\end{aligned}
$$

We will now proceed to an explanation of the cause of this inequality.

The variation has been explained from the effect of that resolved part of the Sun's disturbing force that acts in the direction of the tangent; the evection, from the effect of the resolved part in the direction of the radius vector, and which effect alters the ratio of the perigean and apogean gravities from that of the inverse square of the distance. The present inequality depends not, on any immediate effect, either of the one, or of the other resolved part; but on *an alteration in the mean effect* of the disturbing force in the direction of radius; and, which mean effect lessens the gravity of the Moon towards the Earth.

By the mean effect, that is intended which is the result of the disturbing forces in the direction of the radius in one revolution. The disturbing force does not always diminish the Moon's gravity to the Earth; it does in opposition and conjunction, but it augments the gravity in quadratures (see Newton Sect. XI; prop. 66.) The augmentation however, is only half the diminution (Newton Prop. 66, Cor. 7). In the course therefore of a synodic revolution, there results, what may be called a mean force tending to diminish the Moon's gravity to the Earth, the measure of the mean force being equal to (see Newton, Prop. 66.)

$$\frac{\odot\text{'s mass} \times \text{rad. } ☽\text{'s orbit}}{\text{cube } \oplus\text{'s dist. from } \odot}.$$

By reason of this diminution, the Moon is enabled to preserve a greater distance from the Earth, than it could do, by the influence of gravity alone. But, since the disturbing force acts in the direction of the radius, the equal description of areas is not altered (see Newton, Prop. 66). The area however varying as the product of the radius vector and the arc (the measure of the real velocity) and the former (see l. 2.) being increased, the real velocity must be diminished: so also must the angular, which varies inversely as the square of the distance.

These results are derived from that effect of the disturbing force of the Sun, which is a mean effect diminishing the Moon's gravity. If this mean effect of diminution be increased, similar results will follow, but in an enlarged degree; the Moon's angular velocity will be still more diminished and her distance from the Earth increased: now the measure of the mean effect is

$$\frac{\odot\text{'s mass} \times \text{rad. ☽'s orbit}}{\text{cube ♄'s distance from } \odot},$$

which will be increased, by diminishing the denominator: and is, therefore, in nature, increased when the Earth approaches the Sun. That happens in winter. In winter, therefore, the Moon's gravity to the Earth is more diminished, by the Sun's disturbing force, than in summer. Her angular velocity therefore is more diminished. A greater time is requisite to the description of a complete revolution round the Earth: in other words, a periodic month is longer in winter than in summer. Now, as the Earth approaches the Sun, its velocity increases. An acceleration therefore of the Earth's motion, is attended, by reason of this new inequality, with a retardation of the Moon's, and reversely. On this account it is, that the *annual equation* is said to resemble the equation of the Sun's center. For, supposing the Sun to be approaching his perigee, then his place (reckoning from apogee and neglecting the perturbations of the planets) is equal to the mean anomaly — the equation of the center (E,) E *decreasing* as the Sun approaches the perigee; if m be the Moon's place independently of the *annual equation* (e) then her place, correcting by that is $m + e$, e increasing (since it varies as sin. ⊙'s mean anomaly,) and affected with a contrary sign.

When the annual equation is $+$ (11′ 11″.976) sin. ⊙'s mear anomaly, the corresponding equation of the center, for the Sun is (1° 55′ 26″.3748) sin. ⊙'s mean anomaly.

We have now gone through the explanation of the three principal lunar inequalities, which were discovered before the time of Newton and the rise of Physical Astronomy. These inequalities were, by reason of their magnitude, *fished out*, (as a late writer has significantly expressed it) from the rest. The discovery of the rest, in number 25 *, is entirely due to Physical Astronomy. Without the aid of this latter science, it would have been, perhaps, impossible, from mere observation and conjecture, to have assigned the forms of the arguments. These latter being ascertained, it is the proper business of observation to assign the numerical value of their coefficients

The three equations that have been explained are, with regard to magnitude, eminent above the rest; but, it must not be forgotten, that the other equations, on the footing of theory, are of equal importance, and in practice, considering the use that is now made of the Lunar Tables, of very essential importance.

The three equations, with all the others, are derived from theory by the same process. And, as we have seen, the causes of the former may, independently of any formal calculation, be discerned in certain modifications of the Sun's disturbing force. The causes of the other equations are not so easily discernible: yet, the source of some of them may be pointed out in certain changes, which the conditions or circumstances belonging to the three principal equations must necessarily undergo.

For instance, suppose the Moon and the line of the nodes to be in syzigies; then, the Sun's disturbing force, represented by part of a line joining the Sun and Moon, lies entirely in the plane of the Moon's orbit; and two resolutions of it, one in the direction of the radius, the other of the tangent, are sufficient. But, the nodes are regressive; in a subsequent position of them, then, the line representing the Sun's disturbing force, will be inclined to the plane of the Moon's orbit: consequently, a threefold resolution of the force is requisite, the third being in a direction perpendicular to the plane of the Moon's orbit; consequently,

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* Strictly speaking there are more than 25. But Astronomers have *confined* themselves to this number, since other equations, that analytically present themselves, never rise to a numerical value worth considering.

if the line representing the absolute quantity of the disturbing force, be supposed to be the same, the resolved parts in the directions of the radius and of the tangent, must be less than they were before. The inequalities caused by them must therefore be less, and less, according to the position of the nodes. Hence, if the equation of the evection

$$1° \; 20' \; 29''.5 \times 2 \sin. [(☽ - ☉) - A]$$

were adapted to the first position of the nodes, it could not suit the second, since the longitude of the nodes forms no part of the argument [2 (☽ − ☉) − A]. For this reason, therefore, a correction would be wanting for the evection, that is a *new equation*, the argument of which should depend on the position of the nodes *. The same cause, the change in the Sun's disturbing force from being more or less inclined to the Moon's orbit, must introduce new corrections, that is, *new equations*, belonging to the variation and annual equation.

Again, the annual equation arises from the change in that mean effect of the Sun's disturbing force by which the Moon's gravity is diminished. In adjusting therefore the value of the coefficient of the annual equation, the Moon's gravity must be supposed to be of a certain value: consequently, the Moon must be assumed to be at a certain distance from the Earth. When therefore the Moon is at a different distance, the equation, if adjusted for the previous distance, cannot suit this: a small correction, therefore, or a *new* equation will be necessary, the argument of which must involve or contain, in its expression, the Moon's distance, or her mean anomaly, or some term connected with these quantities †.

Again, the argument for the variation involves simply the angular distance of the Sun and Moon; and its coefficient must be supposed to be settled for certain values of the Moon's gravity and the Sun's disturbing force; and consequently, when the Sun and Moon are at certain distances from the Earth. The changes therefore in those distances which are continually happening,

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* The equation in Lalande, p. 180, is

$$60''.4 \times \sin. \; 2 \text{ dist. } ☽\text{'s } ☊ \text{ from } ☉.$$

† The supplementary equation, according to Mayer, is

$$42'' \sin. [☽\text{'s mean anom.} - ☉\text{'s mean anom.}]$$

which however is not the sole correcting equation due to this cause. See Lalande, *Astron.* tom. II, p. 178.

must render necessary two corrections, or two *new equations:* one for the approach of the Sun to the Earth, the other for the elongation of the Moon from the Earth. Generally, any equation furnished with its numerical coefficient on the supposition of the Sun and Moon revolving round the Earth in circular orbits, will require new supplemental or subsidiary equations due to the real and elliptical forms of the orbits*.

Again, the inclination of the Moon's orbit is variable; therefore any equations adjusted to a mean state of inclination will require subsidiary equations, to correct the errors consequent on changes in that state.

From considerations like the preceding, the existence of the *smaller inequalities* is established: and by an attentive consideration of the circumstances that occasion them, the forms of their arguments may be detected; with much less certainty however, than by the direct investigation of the disturbed place of the Moon.

It is one thing to prove the existence of an *inequality*, and another to establish the necessity of its corresponding equation. Whether it is expedient to introduce the latter, is a matter of mere numerical consideration. The correction of a correction, the subsidiary equation to a principal equation, is, in the lunar theory, very minute: and some equations, arising from the causes that have been enumerated, are so minute, as to be disregarded by the Practical Astronomer.

We have at present considered only the inequalities that affect the Moon's longitude: but the Sun's disturbing force causes also *inequalities* in the Moon's *latitude* and in her *parallax*.

The inequalities of the latitude and of the parallax have nothing peculiar in them, nor distinct, (whether we regard their physical cause or the mode of ascertaining the laws of their variation,) from the inequalities of longitude. It is not necessary therefore to dwell on them, since the latter have been explained. We

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* The evection, for instance, is variable from the variation of the distances of the Sun from the Moon and Earth: and for the purpose of correcting the evection, there are 4 subsidiary, or, as Lalande calls them, accessory equations, which in his Tables are the 5th, 6th, 7th, and 9th. See *Astron.* tom. II, p. 177.

will only mention, that the principal inequality in latitude, and its law, were discovered by Tycho Brahé, and by the comparison of observations of the greatest latitudes of the Moon, at different epochs, and when that planet was differently situated, relatively to the nodes of its orbit. The equation is

(8′ 47″.15) . cos. 2 ☉'s dist. from ☽'s ☊.

[See Lalande, tom II, p. 193. Mayer, *Theoria Lunæ*, p. 57. Laplace, *Méc. Cél.* Liv. VII, p. 283, &c. French Tables, Introduction.]

If the Moon's orbit coincided with the plane of the ecliptic, the Sun's disturbing force resolved into the directions of a tangent to the Moon's orbit, and of a radius vector, could only, by the first resolution, alter the law of elliptical angular motion, and, by the second, the length of the radius vector (such as it would be in an ellipse); in other words, it could only produce inequalities in longitude and in parallax, for the parallax varies inversely as the radius vector. But, the Moon's orbit being inclined to the ecliptic, the Sun's disturbing force (represented by a line drawn from the Moon towards the Sun) cannot be entirely resolved into the two former directions: a third resolved part will remain perpendicular to the plane of the Moon's orbit, which will cause the Moon to deviate from that plane; in other words, will cause inequalities in the Moon's latitude.

In order to correct these inequalities in the Moon's latitude, eleven equations are necessary, according to Lalande, (see *Astron.* tom. II, p. 193.) In the New French Tables an additional one is added.

The formula * for the parallax in Lalande (see tom. II, p. 314,) consists of 19 terms, but most of which have very small coefficients: and, in practice, three tables suffice to contain the necessary equations, (see Tables 40, 41, 42, in the New French Tables.)

The nature of the present Work obliges us here to terminate the explanation of the Lunar theory; a subject of great extent,

* ☽'s parallax = 57′ 11″.4—3′ 7″.7 . cos. ☽'s anom.+10″. cos. 2 ☽'s anom. — 37″.3 cos. ang. evection + 26″. cos. 2 (☽ — ☉) — &c. none of the coefficients of the other equations exceed 2″.

difficulty, curiosity, and practical utility. The lunar motions being ascertained, eclipses may be computed, and, for assigned times, the distances of the Moon from the Sun and certain fixed stars. From such computations, either the one or the other, the longitudes of places may be determined. The latter, however, are most useful, since they are subservient to a commodious and sufficiently exact method of computing the longitude of a ship at sea. The determination of the longitudes of places on the Earth's surface is a matter rather of curiosity than of essential use.

Eclipses of the Moon are of no great use in determining the longitudes of places; and this happens, not from any defect of computation, but from the difficulty of marking the exact times when the *phases** of an eclipse commence and terminate. Yet, since lunar eclipses are phenomena of great interest, of celebrity in the history of Astronomy, and of importance in settling certain of the lunar elements †, it is incumbent on us to state some of their principal circumstances, and to explain, generally at least, the modes of computing their magnitudes and epochs.

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* *Phase* of an eclipse, or appearance of the Moon's disk eclipsed, varying with the quantity eclipsed.

† The uncertainty of the time of an eclipse, to the amount of one minute, would render the determination of the longitude of places by means of the eclipse little to be relied on; but, the same error of one minute would be of little consequence, when eclipses distant from each by several centuries, are employed in determining an element in the lunar theory, such as, for instance, the Moon's mean motion.

CHAP. XXXV.

On Eclipses of the Moon.

In Chapter IV, an eclipse of the Moon was shewn to arise from the interposition of the Earth between the Moon and Sun, and from the falling of part of the shadow of the former, on part, or on the whole, of the Moon's disk.

An opaque body interposed, between the Sun and Moon, at a given distance, does not necessarily cause an eclipse: if its diameter should be below a certain magnitude, its shadow could not reach the Moon.

The existence of eclipses then must depend on the relative magnitudes of the diameters of the Sun and Earth, supposing the mutual distances of the Sun, Earth and Moon, to be assigned.

When the Moon is in opposition and at her mean distance, the apparent diameters of the Sun and Earth seen from the Moon's center are $31' \ 59''.08$, and $1^{\circ} \ 55' \ 8''$. Now, at the extremity, or conical point of the Earth's shadow, the apparent diameters of the Sun and Moon are the same. The Moon therefore must be considerably removed towards the Earth, from the extremity of the shadow; or, what amounts to the same, the length of the shadow must be greater than the Moon's distance from the Earth; and, by computation, it is found to be four times as great.

If the preceding result were established relatively to any distance of the Moon from the Earth, either the mean or the least, since the eccentricity of her orbit is only 0.0548553, it might be inferred almost certainly, without calculation, that the Earth's shadow would, in all cases, extend far beyond the Moon. With the aid of calculation, however, the following results are obtained.

		Length of Axes of Shadow.
☉	in perigee - - - - - -	212.896 rad. ⊕
	at mean distance - - -	216.531
	in apogee - - - - -	220.238.

Hence, the least length of the shadow is more than 212 radii of the Earth, whereas the Moon's distance from the Earth never exceeds 64 radii.

Hence it appears, that a lunar eclipse must always happen whenever the Earth is *interposed* between the Sun and Moon; understanding, by such expression, the Earth's center to lie in a line joining the centers of the Sun and Moon and between those centers. In this latter situation of the three bodies, the Moon is in opposition. In such *kind* of opposition then, an eclipse must always happen, and there would be only that kind, if the plane of the Moon's orbit coincided with that of the ecliptic.

The Moon's orbit being inclined to the ecliptic, and, opposition (see p. 43) meaning nothing more, than the difference, in longitude, of a semi-circle, or of 180°, the Moon may be in opposition, and still either directly above or below the right line joining the centers of the Sun and Earth; and, consequently, may either be above or below the conical shadow, the axis of which lies in the direction of the above-mentioned line.

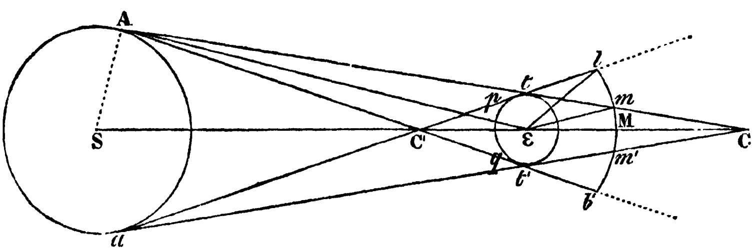
Since the inclination of the Moon's orbit, (see p. 302,) is about 5° 9′, if the Moon in opposition should be either in its greatest northern or southern latitude, that is, 5° 9′ above or below the ecliptic, no eclipse can take place, since the greatest section of the Earth's shadow at the Moon never exceeds 64′. But, in the next succeeding opposition, after the lapse of a synodic period, the Moon cannot be again in her greatest latitude, since (see p. 302,) the nodes of her orbit would have regressed through about 1° 35′, and during succeeding oppositions, the nodes still regressing, the Moon in opposition would approach nearer and nearer to the ecliptic, till at length an opposition would occur, in which the Moon would be either, exactly, or very nearly, in its node: and if in its node, then it would be in the ecliptic, and in such case, (see l. 14,) an eclipse must happen.

An eclipse may happen, if the Moon be *near* the node of her orbit; and, the least degrees of proximity, are called the *Lunar Ecliptic Limits.*

These limits are easily determined from the inclination of the Moon's orbit, the Moon's apparent diameter, and the apparent diameter of a section of the Earth's shadow at the Moon. The two former conditions may be supposed to be known by previous methods, (see pp. 302, &c.) and it is the latter only that now requires to be investigated.

Apparent Diameter of a Section of the Earth's Shadow at the Moon.

Let S represent the Sun's center, E the Earth's, and let the circles described round the centers S, E represent sections of those bodies. Draw AtC, $at'C$, tangents to the circular



sections of the Sun and Earth, then the triangular space included within tC, $t'C$, will represent the section of the conical shadow of the Earth. Let $m\,M\,m'$ be part of the Moon's orbit, then the section of the Earth's shadow at the Moon is $m\,M\,m'$, and its apparent semi-diameter at the Earth, which we have to estimate, is the angle mEM *.

$$\angle\ mEM = \angle\ Emt - \angle\ ECm,$$
$$= \angle\ Emt - [\angle\ AES - \angle\ EAt]$$

* We have, more than once, adverted to the necessary defect which diagrams in Astronomy are subject to, in representing distances and magnitudes according to their true proportion in nature. The Figure in the page is an instance of it. The Earth's radius is there made not less than one third of the Sun's, whereas it is about $\frac{1}{110}$th part. But, if it had been so drawn, then we should have had a most inconvenient diagram, in which it would have been difficult to discern the lines and angles, which are the subjects of investigation.

Now, $\angle\, Emt$, the angle subtended by the Earth's radius at the Moon, is the Moon's horizontal parallax (P),

$\angle\, AES$ is the Sun's apparent semi-diameter $\left(\frac{D}{2}\right)$

$\angle\, EAt$, the angle subtended by the Earth's radius at the Sun, is the Sun's horizontal parallax (p).

Hence,

The apparent semi-diameter of ⊕'s shadow $= p + P - \frac{D}{2}$.

Hence, the distance of the centers of the Moon and of the Earth's shadow, when the Moon's disk just touches the shadow, will be the preceding expression plus the Moon's apparent semi-diameter $\left[\frac{d}{2}\right]$, that is,

$$p + P - \frac{D}{2} + \frac{d}{2}.$$

If we take $P = 57'\ 1''$, $p = 8''.8$, and $\frac{D}{2} = 16'\ 1''.3$ we shall have

The mean apparent semi-diameter of ⊕'s shadow $= 41'\ 8''.5$, which is nearly three apparent semi-diameters of the Moon. Hence, since the Moon in the space of an hour moves over a space nearly equal to its diameter, the Moon may be entirely within the shadow, or a total eclipse may endure, about two hours.

In order to find the greatest value of the preceding expression, we must take the greatest parallax of the Moon, and the least of the Sun: for since there is a constant ratio between the Sun's horizontal parallax and his apparent semi-diameter, when the former is the least, the latter will be: and although in the expression the parallax is additive, yet its diminution below its mean or even its greatest quantity is trifling, relatively to that of its apparent diameter.

Hence, since the ☽'s greatest horizontal parallax is 1° 1′ 29″
and the ☉'s least semi-diameter - - - - - - 15 45.48
the corresponding parallax of the ☉ - - - - - - 0 8.6

We have

the greatest semi-diameter of the ⊕'s shadow $= 45'\ 52''$, nearly, and the diameter 1° 31′ 44″.

Precisely after this manner, and by the same formula, $\left[p + P - \frac{D}{2}\right]$ may the apparent diameters of the Earth's shadow be computed, for other distances of the Sun and the Moon. Thus,

		Apparent Diameter of ⊕'s Shadow.		
☉ in perigee.	☽ in apogee - - - - - - -	1°	15′	24″.3036
	at mean distance - - -	1	23	2.31
	in perigee - - - - - - -	1	30	40.3164
☉ at mean distance.	☽ in apogee - - - - - - -	1	15	56.8656
	at mean distance - - -	1	23	34.872
	in perigee - - - - - - -	1	31	12.8784
☉ in apogee.	☽ in apogee - - - - - - -	1	16	28.2936
	at mean distance - - -	1	24	6.3
	in perigee - - - - - - -	1	31	44.3064

In p. 337, the length of the Earth's shadow was expressed in terms of the Earth's radius: this was obtained from the value of the angle ECt, which is (see p. 339, l. 5,) $\frac{D}{2} - p$.

For then,

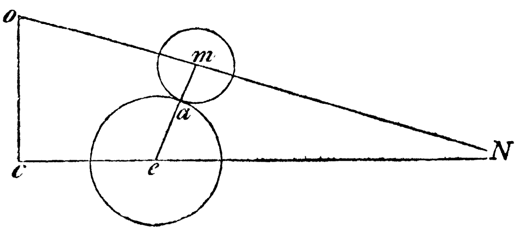
$$EC = \frac{Et}{\sin. \angle ECt} = \frac{\text{rad. } \oplus}{\sin. \left(\frac{D}{2} - p\right)}.$$

Since there is a constant ratio (see p. 313,) between the Sun's semi-diameter and horizontal parallax, which ratio is that of the radius of the Sun to the radius of the Earth, and in numbers, as 110 : 1 nearly, the denominator of the preceding fraction may be expressed either, in terms of the semi-diameter, or of the parallax; thus,

$$\text{Length of shadow} = \frac{\text{rad. } \oplus}{\sin. (109\, p)},$$

$$\text{or} = \frac{\text{rad. } \oplus}{\sin. \frac{109\, D}{220}}.$$

But to return to the investigation of the extreme cases in which eclipses can happen. To the greatest apparent semi-diameter of the Earth's shadow (see p. 338,) add the greatest apparent semi-diameter of the Moon, and the result will be the greatest apparent distance of the Moon's center from the ecliptic, at which an



eclipse can happen. Thus, in the Figure, if Ne be part of the ecliptic, Nm part of the Moon's orbit, e the center of a section of the Earth's shadow; if we take (see p. 339,) ea, in its greatest value, equal to 45′ 52″, and ma, the greatest apparent semi-diameter of the Moon, = 16′ 45″.5, then me = 62′ 37″.5, is the greatest distance of the Moon at which an eclipse can happen. If the distance be greater, there can be no eclipse, if less, and less within certain limits, there may or may not be an eclipse; its happening depending on the relative proximities of the Earth to the Sun and Moon.

The *ecliptic limit* Ne, corresponding to the greatest value of me, may be thus computed:

By Naper's Rules,

$$\text{rad.} \times \sin. me = \sin. Ne \times \sin. \angle eNm;$$

∴ taking me = 62′ 38″, and the inclination of the Moon's orbit, what it generally is, in these circumstances, equal to 5° 17′, we have

10 + log. sin. 62′ 38″ - - - - - - - - - -	18.2605076
log. sin. 5° 17′ - - - - - - - - - - -	8.9641697
∴ log. sin. Ne - - - - - - - - - - - - -	9.2963379

∴ Ne = - - - - - - - - - - - 11° 25′ 40″ nearly.

The species of eclipse represented in the above Figure, where the two circular sections of the Moon and shadow are in contact, is called an *Appulse*.

The opposition of the Moon must have happened soon before this appulse, if the direction of the Moon's motion be supposed from *m* towards *N*. For, the Moon moving more quickly * than the Sun, and consequently, than the center (*e*) of the shadow, cannot long have quitted a point *o*, such that the corresponding position of the center of the shadow would be at *c*. And in these positions of the Moon and shadow, the former (see pp. 43, 337,) is in opposition.

In the computation of eclipses there are several expedients employed for abridging its labour. Eclipses are to be expected when the Moon is near her node, and in opposition. But the labour of a direct and formal computation may frequently be spared, by roughly ascertaining certain limits, beyond which, it is useless to expect an eclipse. Thus, as we have seen in the preceding page, if Ne be greater than 11° 26′, no eclipse can happen. But Ne is the difference of the true longitudes of the center of the ⊕'s shadow and of the ☽'s ☊ at the time of the appulse; the time of appulse differs a little from the time of true opposition, and therefore, for two causes, from the time of mean opposition. The mean longitude of the center of the Earth's shadow differs from the true longitude, by reason of the equation of the center, and other small equations. If therefore, we compute the *mean* longitude of the Earth's shadow at the time of *mean* opposition, it will differ from the longitude of *e*, (see Fig. p. 341,) at the time of appulse for three causes; the difference, of the times of appulse and of true opposition, of the times of mean and true opposition, and of the mean and true longitudes. But, notwithstanding these sources of inequality, the consequent error in the value of Ne computed, from the mean longitude of the Earth, and for the time of mean opposition, is within certain limits; and accordingly M. Delambre states that if Ne be $>$ 12° 36′, there cannot be an eclipse, if $<$ 9°, there must be one. Between 9°, and 12° 36′, the happening of the eclipse is doubtful, and the doubt must be removed by a more exact cal-

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* The diurnal motions of the Moon and Sun are respectively 13° 10′ 35″.027, and 59′ 8″.33.

culation. The time of mean opposition may be computed from the Tables of the Sun and Moon. But, the computation is facilitated by means of a Table of *Epacts*. The *Epact for a year*, meaning the Moon's age at the beginning of the year, the age commencing from the last *mean* conjunction; and the *Epact for any month*, meaning the Moon's age at the beginning of the month, supposing the age to have begun from the beginning of the year. Delambre in his Astronomical Tables has given a new method of computing the probable times of the happening of eclipses. (See Vince, vol. III. Introduction, p. 56.)

In the preceding explanations we have supposed an eclipse to begin when the Moon enters the Earth's shadow at m'. A spectator at the Moon in any point within m' and m, (see Fig. p. 338.) would, by reason of the intervention of the Earth, be unable to see any part of the Sun's disk. But, before and after this eclipse, properly so called, the Moon's light would be obscured; or, what amounts to the same thing, the spectator, on the Moon's surface, before being entirely deprived of the Sun's light, would lose sight of portions of his disk. In order to determine, when this obscuration first begins, and when it ends, draw two tangents $AC'ql$, $aC'pl$, to the Sun and Moon; then, the moment the Moon enters $l'l$, part of the Sun's light is stopped; or, a spectator at the Moon situated any where between $l'm'$ sees part only of the Sun's disk. Entering $m'm$, the spectator loses sight of the Sun entirely; emerging from $m'm$, he recovers, in his progress through ml, the sight of successively greater portions of the disk, and finally, emerging from ml, he again sees the full orb of the Sun.

The space included within the lines pl, ql', is the section of what is, properly enough, denominated the *Penumbra*; and its angle is $lC'l'$.

Angle of the Penumbra.

$$\angle AC'S = \angle AES + \angle EAC',$$

$$= \odot\text{'s apparent semi-diameter} + \odot\text{'s hor. parallax},$$

$$= \frac{D}{2} + p.$$

Hence, may be deduced,

The Apparent Semi-diameter of a Section of the Penumbra at the Moon's Orbit.

For, $\angle lEC = \angle ElC' + \angle EC'l,$

$$= ☽\text{'s hor. par}^{x}. + \frac{D}{2} + p \quad \text{(p. 343, l. 34,)}$$

$$= P + p + \frac{D}{2}.$$

From this formula, as in the case of the umbra (p. 340, l. 4,) the several values of the apparent semi-diameter of the penumbra, corresponding to certain positions of the Sun and Moon, may be computed.

Since the apparent semi-diameter of the Moon's penumbra is

$$P + p + \frac{D}{2},$$

the distance of the Moon's center and of the center of the shadow, when the Moon first enters the penumbra, is

$$P + p + \frac{D}{2} + \frac{d}{2};$$

d representing the Moon's apparent diameter.

In the preceding investigations we have supposed, the cones of the umbra and penumbra to be formed by lines drawn from the Sun and touching the Earth's surface. This, probably, is not the exact case in nature; for, the apparent diameter of the Earth's shadow is found, by observation, to be somewhat greater than what would result from the preceding formula. This circumstance is, with great appearance of probability, accounted for, by supposing those solar rays, that, from their direction, would glance by and rase the Earth's surface, to be stopped and absorbed by the lower strata of the atmosphere. In such a case, the conical boundary of the Earth's shadow would be formed by certain exterior rays, and be larger.

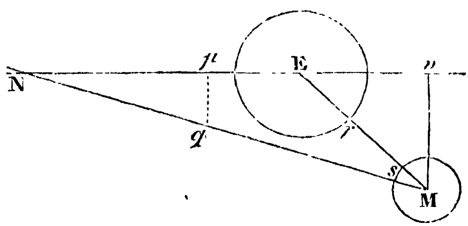
This is not the sole effect of the atmosphere in eclipses; but, another, totally of a different nature, results from it. Certain of the Sun's rays, instead of being stopped and absorbed, are bent from their rectilinear course, by the refracting power of the atmosphere; and, so form a cone of faint light interior to that cone which has been mathematically described as the

Earth's shadow. The effect of this, or the phenomenon of which the preceding statement is presumed to be the explanation, is a reddish light visible on the Moon's disk, during an eclipse.

We may now proceed to shew how the time, duration and magnitude, of a lunar eclipse, may be computed.

Let NqM represent part of the Moon's orbit, vEN the ecliptic, N the node.

Suppose the Moon to have been in opposition when at q, p being the corresponding place of the center of the Earth's



shadow, and the latter to have described pE, whilst the Moon's center was describing qM. Let also

m = ☽'s motion in longitude,
n = ☽'s motion in latitude,
s = ☉'s (or, the shadow's center's,) motion in longitude,
λ = ☽'s latitude when in opposition at q,
t = time from q to M,
c = distance of M from E (ME);

then, in the time t, the ☽'s motion in longitude $= mt$ $[vp]$,
in latitude $= nt$ $[Mv-pq]$
the ☉'s motion in longitude $= st$ $[Ep]$;
consequently, $Mv = pq + nt = \lambda + nt$, and $Ev = pv - Ep = mt - st$;

$$\therefore c^2 \ (ME^2) = Mv^2 + Ev^2 = (\lambda + nt)^2 + (mt - st)^2,$$

which expression expanded produces a quadratic equation, of which t is the quantity to be determined, and the value of which will depend on that of c; or, if we assign to c such values as belong to the different *phases* (see p. 335,) of an eclipse, the results will be intervals of time between the happening of such phases, and the time of opposition, which latter time may be computed from the Tables of the Sun and Moon.

If we expand the preceding expression for t^2, and substitute in it, instead of $\frac{n}{m-s}$, tan. θ, there will result,

$$n^2 t^2 + 2\lambda n \sin.^2 \theta \times t = (c^2 - \lambda^2) \sin.^2 \theta,$$

and if from this, by the Rule for the solution of a quadratic equation, we deduce the value of t,

$$t = \frac{1}{n} [- \lambda \sin.^2 \theta \pm \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}],$$

from which expression, as it has been stated, may be deduced values of the time corresponding to any assigned values of c.

For instance, if we wish to determine the time, at which the Moon first enters the Earth's penumbra, we must assume (see p. 344,)

$$c = P + p + \frac{D}{2} + \frac{d}{2};$$

t has two values, and the second value will denote the time at which the Moon quits the penumbra If we wish to determine the time at which the Moon enters the umbra, we must assume, (see p. 339,)

$$c = P + p + \frac{d}{2} - \frac{D}{2}.$$

If we wish to determine the time when the whole disk has just entered the shadow, we must subduct d from the preceding value, and make

$$c = P + p - \frac{d}{2} - \frac{D}{2},$$

and similarly for other phases.

The two values (t', t'') of t are

$$t' = \frac{1}{n} [-\lambda \sin.^2 \theta + \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}]$$

$$t'' = \frac{1}{n} [-\lambda \sin.^2 \theta - \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}],$$

which values can never equal each other, except the quantity under the radical sign, or, $\lambda^2 - c^2 \cos.^2 \theta = 0$;

in which case, the value of t which is equal to $-\lambda\frac{\sin.^2\theta}{n}$, represents the *middle* of the eclipse, when the distance of the centers (c) is $=\lambda\cos.\theta$.

This value ($\lambda\cos.\theta$) of c that corresponds to the middle of the eclipse, is the least distance, or, the nearest approach of the centers of the Moon and shadow. For, if by the rules for finding the maxima and minima of quantities, we deduce from the expression, p. 346, l. 6, the value of t, it will be found equal to $-\frac{\lambda\sin.^2\theta}{n}$, which (see l. 1,) is the value of t, when $c=\lambda\cos.\theta$.

The nearest approach of the centers being known, the magnitude of the eclipse is easily ascertained. Thus, on the supposition that $\lambda\cos.\theta$, is less than the distance $\left(P+p+\frac{d}{2}-\frac{D}{2}\right)$ at which the Moon's limb just touches the shadow, some part of the Moon's disk is eclipsed; and the portion of the diameter of the eclipsed part is

$$P+p+\frac{d}{2}-\frac{D}{2}-\lambda\cos.\theta.$$

The portion of the diameter of the non-eclipsed part, is the Moon's apparent diameter (d) minus the preceding expression, and therefore is

$$\lambda\cos.\theta+\frac{d}{2}+\frac{D}{2}-P-p.$$

If this expression should be equal nothing, the eclipse would be *just* a total one. If the expression should be negative, the eclipse may be said to be *more than* a total one, since the upper boundary of the Moon's disk would be below the upper boundary of the section of the shadow: and the distance of the two boundaries would be the preceding expression.

The preceding formulæ for the parts eclipsed, which are parts of the Moon's diameter, are usually expressed in twelvths of that diameter; which twelvths are, with no great propriety of language, called *Digits*. Thus, if the part eclipsed should be 24′ 52″, the Moon's diameter being 33′ 18″; then, the part eclipsed

$$=\frac{24'\ 52''}{33'\ 18''}\times\overset{\text{Digits.}}{12}=\overset{\text{Digits.}}{8.96}.$$

By p. 346, the second root of the quadratic, or

$$t'' = -\frac{1}{n}[\lambda \sin.^2 \theta + \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}]$$

which is negative with respect to the other value t'; that is, if the first be previous to opposition, the latter is subsequent to it: hence the whole duration of that part of the eclipse which takes place between equal values of the distance of the centers is the sum of the two times, and therefore =

$$t' + (-t'') = \frac{2}{n} \sin. \theta \sqrt{[c^2 - \lambda^2 \cos.^2 \theta.]}$$

If in this expression we substitute that value of c, which is $P + p + \frac{d}{2} - \frac{D}{2}$, (see p. 346,) the expression $\frac{2}{n} \sin. \theta \sqrt{[c^2 - \lambda^2 \cos.^2 \theta]}$, denotes the time from the Moon's first entering, to her finally quitting the shadow or *umbra*. And, if we substitute for c, $P + p + \frac{d}{2} + \frac{D}{2}$, (see p. 346,) the resulting expression denotes the whole time of an eclipse, from the Moon's first entering, till her finally quitting the *penumbra*.

Example.

Of the Eclipse, which happened on March 17, 1764, *it is required to calculate the beginning, middle, and the end: also the number of Digits eclipsed.*

By the Lunar and Solar Tables it appears that the epoch, or the time of true opposition, happened on the 18th of March 1764, at $0^h\ 6^m\ 12^s$, mean solar time at Paris (reckoned from midnight).

By the above-mentioned Tables the following numerical results were obtained.

☽'s lat. at the time of opposition -	$\lambda =$	38′ 42″	N.
☽'s horary motion in latitude - -	$n =$	—3 26	(lat. decreasing)
☽'s horary motion in longitude -	$m =$	37 23	
☉'s horary motion in longitude -	$s =$	2 29	
☽'s apparent diameter - - - - -	$d =$	33 18	
☽'s corresponding hor^l. parallax -	$P =$	61 0	
☉'s apparent diameter - - - - -	$D =$	32 10	
☉'s corresponding hor^l parallax -	$p =$	0 9.	

Hence, (see p. 346,)

$$\tan.\,\theta = \frac{n}{m-s} = -\frac{3'\ 26''}{34'\ 54''} = -\frac{206}{2094}$$

$$\therefore\ \theta = -5^\circ\ 37'\ 6''.5.$$

Hence, (see p. 347, l. 1,) the middle of the eclipse, or,

$$-\frac{\lambda \sin.^2 \theta}{n} = \frac{2322}{206} \times \sin.^2 [5^\circ\ 37'\ 6''.5] = 6^m\ 29^s.$$

Now this is the time reckoned from the epoch of opposition, which is March 18, 0^h 6^m 12^s, consequently, the middle of the eclipse was March 18, 0^h 12^m 41^s. Now, in order to find the times when the Moon first entered and when it finally quitted the shadow, we must first compute, (see p. 348,) the corresponding values of c, and accordingly

$$c = \frac{d}{2} - \frac{D}{2} + p + P = 61'\ 43'',$$

or, adding (see p. 344, l. 2,) 1′ 40″ for the effect of the Earth's atmosphere,

$$c = 63'\ 23'',$$

which value being substituted in

$$-\frac{\lambda \sin.^2 \theta \pm \sin.\,\theta \sqrt{[c^2 - \lambda^2 \cos.^2 \theta]}}{n}$$

there result for the two values of t,

(end of eclipse) $t'' = 6^m\ 29^s + 1^h\ 26^m\ 8^s = 1^h\ 32^m\ 37^s$

(beginning) $t' = 6\ \ 29 - 1\ \ 26\ \ 8 = -1\ \ 19\ \ 39$

and consequently, the duration of the eclipse - 2^h 52^m 16^s.

Since $t' = -1^h\ 19^m\ 39^s$ is negative, the commencement of the eclipse happened before the time of opposition, therefore at Paris, 1^h 19^m 39^s before March 18, 0^h 6^m 12^s, that is, on March 17, 22^h 46^m 33^s, and the eclipse terminated 1^h 32^m 37^s after the time of opposition March 18, 0^h 6^m 12^s, that is, on March 18, 1^h 38^m 49^s.

Since the preceding times are computed, as is the usage of French Astronomers, from midnight, and since, at the time of opposition the Moon was nearly on the meridian, it is plain that the whole of this eclipse must have been seen at Paris, and could not have been seen on the hemisphere opposite to that, on which Paris is situated.

The distance of the centers corresponding to the middle of the eclipse, and to the greatest *phase*, that is, to the greatest quantity of eclipsed disk, or

$$\lambda \cos. \theta = - \quad - \quad - \quad - \quad - \quad - \quad - \quad 38' \; 31''.$$

The eclipsed part, or

$$\frac{d}{2} - \frac{D}{2} + p + P - \lambda \cos. \theta = \quad - \; 23' \; 12'',$$

or (see p. 344,) accounting for the effect of atmosphere - 24′ 52,″

$$\text{and expressed in digits} = \overset{\text{Digits.}}{12} \times \frac{24' \; 52''}{33' \; 18''} = \overset{\text{Digits.}}{8.96}.$$

In deducing the equation that involves the time (t) we supposed the Moon to describe the space $M q$, whilst the center of the shadow described $E p$: and, expressed by means of the horary motions, the line $p v$ was $= m t$*, and the line, which is the difference of $M v$ and $p q$, was $= n t$. According to this notation, therefore, the tangent of the inclination of the Moon's orbit $\left(\text{which} = \frac{M v}{N v}\right) = \frac{n t}{m t} = \frac{n}{m}$. Now the Moon approaches the shadow for two reasons, the motion in latitude ($n t$) and the *excess* ($m t - s t$) of its motion in longitude above that of the shadow. Hence, its approach to the shadow would evidently be the same, if we suppose the center of the shadow to be *quiescent*, the Moon to move with its proper motion in latitude ($n t$), and besides with imaginary proper motion, in longitude, equal to the relative one, $m t - s t$; with such an hypothesis the equation (see p. 345,)

$$c^2 = (\lambda + n t)^2 + (m - s)^2 t^2,$$

would equally result, and the same conclusions relative to t, &c. would also equally result. In this case, since we suppose the shadow to be at rest, and the two motions of the Moon to be $n t$, and $(m - s)\, t$, the Moon must move towards the shadow along an imaginary orbit, the tangent of whose inclination would be $\frac{nt}{(m - s)t}$, or $\frac{n}{m - s}$, an inclination greater therefore than that of the real orbit.

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* The Reader must observe that $m t$, $n t$, &c. are not lines like $p q$, &c. but the products of two algebraical symbols, m, t and n, t.

This imaginary orbit, (which originates by a species of translation of the equation involving t,) has, for the purpose of graphically representing the phases of an eclipse, been invented by Astronomers, and been termed the Moon's *relative* Orbit. If we prolong the line pq, below NqM, till it equals $pq + n \times t$, or $\lambda + nt$, and then, from the extremity of the prolonged line, draw a line parallel to pv, towards M, and equal to $(m - s)t$, and lastly, join p and the extremity of the line parallel to pv; then, the joining line will represent a portion of the relative orbit, and be equal to ME (c).

The *relative* orbit is a mere mathematical fiction, convenient enough for representing the phases of an eclipse, but not essential to their computation, as the very fact of the preceding computations, made without reference to it, sufficiently proves. If, however, by independent reasonings, it be established and laid down as the basis of investigation, then may all the preceding results relative to the duration and quantity of an eclipse be obtained. It may not be improper to note, that the artifice of computation which substitutes tan. θ instead of $\frac{n}{m - s}$, when geometrically exhibited, introduces the *relative* orbit.

In the preceding computations of the duration, &c. of a lunar eclipse, we have supposed the motion of the Sun in longitude, and the motions of the Moon in longitude and latitude to be uniform. This, during the short continuance of an eclipse, is nearly, but not exactly, true. The error of the supposition, however, may be corrected by means of the Lunar and Solar Tables, which give the true motions of the Sun and Moon for every instant of time, and then the eclipse may be computed to the greatest exactness.

Since the computation of eclipses, and especially, of solar, is attended with considerable difficulties, it is natural to search for expedients that may lessen them. Now, an eclipse depends on two circumstances, the syzigy of the Moon, and, the proximity to the node of her orbit. The first circumstance, whether it be an opposition or a conjunction, recurs after a synodic period, or, 29^d. But, at the end of this period, the proximity of the Moon to the node of her orbit cannot be the same, in degree, as it was at the beginning. It must, according as the

Moon is approaching or receding from the node, be less or greater. This arises from the regression of the nodes. But, the nodes still regressing, before they have performed a circuit of the heavens, an opposition or conjunction must happen, in which the Moon would be either exactly, or very nearly, at the same distance from the node, as it was at the beginning of the period. If, for the sake of illustration, we suppose the synodic period to be 30 days, and the Sun after quitting the node of the Moon's orbit, to return to the same after 330 days, then at the end of this latter period, and after eleven lunations, if the Sun and Moon were in conjunction, or opposition, at the beginning, they would be again so, and besides the Moon would be in the same degree of proximity to the node. But, if the return of the Sun to the node should not be performed exactly in 330 days, but in 330 days 12 hours, then at the end of 661 days, after two revolutions with respect to the node and 60 lunations, the Moon would be in syzigy with the Sun, and at the same distance from the node, as it was at the beginning. Now, if the Moon, at different periods, be in syzigy with the Sun, and at the same distance from the node, the same phases of an eclipse must be always seen at those periods (supposing the mutual distances of the Moon, Sun, and Earth, not to alter). Hence, an eclipse computed for one period would serve for other periods, and, eclipses could be predicted; since, after the lapse of a certain number of days, they would recur.

A lunation, and the Sun's period with regard to the node of the Moon's orbit, are not of the values, which, in the preceding illustration, we have supposed them to be. The former is $29^d\ 12^h\ 44^m\ 2^s.8$, (29.530588) the latter $346^d\ 14^h\ 52^m\ 16^s.032$ (346.61963). But, with these true values, the period of the recurrence of the Moon to the same position, relatively to the Sun and the node of its orbit, is to be determined on the same principles, which indeed, are those which have been previously used on the occasion of the transits of *Venus* and *Mercury* over the Sun's disk, (see p. 278). We must find two numbers in the proportion of 29.530588 to 346.61963: if not exactly, nearly so, employing the method of continued fractions, (see p. 280). Now two numbers, nearly so, are 19 and 223; the Moon's node, therefore, after 223 lunations has, relatively to the Sun, returned 19 times to the same position. And accordingly at the end of 223 lunations, that is, of 18 years 11 days *, there are the same conditions requisite for an eclipse, as at the beginning; after such interval, then eclipses, solar as well as lunar, will recur, and in the same order. If we know, therefore, previous, we can predict subsequent eclipses.

This simple method of predicting eclipses was known to the antient Astronomers. It, however, is not exact, since 19 to 223, is only an approximate ratio: even were it exact, still the lunar inequalities, the periodical and secular, would prevent the Moon from being at the end of $18^{y}\ 11^{d}$, or of $36^{y}\ 22^{d}$, &c. precisely at the same distance from the node, as at the beginning.

The method, however, may, with advantage, be used for ascertaining, very nearly, the happening of eclipses; after which, the exact times may be calculated by means of the Astronomical Tables.

By means of the period of 223 lunations, called by the Chaldean Astronomers, the *Saros*, eclipses may be predicted; but, independently of this, there is, for finding directly those syzigies at which eclipses may happen, the method of *Astronomical Epacts*, (see p. 343).

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* More exactly, $18^{y}\ 10^{d}\ 7^{h}\ 43^{m}$, or $18^{y}\ 11^{d}\ 7^{h}\ 43^{m}$, accordingly as four or five years happen in the interval of 223 lunations.

CHAP. XXXVI.

On Solar Eclipses.

An eclipse of the Sun, (see p. 19,) is caused by the interposition of the Moon between the Sun and Earth; by reason of which, the whole, or part of the Sun's light is prevented from falling on certain parts of the Earth's surface.

A spectator deprived of the whole of the Sun's light is involved in the Moon's shadow; deprived of part, in the penumbra.

There is one material circumstance of distinction, between lunar and solar eclipses: the former are seen, at the same time, by every spectator above whose horizon the Moon is. The latter may be seen by different spectators at different times; or may be seen by one spectator and not by another. The passage of the Moon's shadow across the Earth's surface, during a solar eclipse has been properly likened to that of the shadow of a cloud.

In the case of the Moon, it was shewn, that, if that body were within certain limits of distance from the node of her orbit, an eclipse must happen in opposition; because, (see p.337,) the shadow of the Earth, in all distances of the Moon and Sun, extends far beyond the lunar orbit. The length of the Moon's shadow must be determined as that of the Earth's has, on the same principles, and, by similar formulæ. But, the result, in certain respects, will be different. The Moon's shadow will never extend far beyond the Earth, and sometimes will fall short of it. Hence, the happening of a solar eclipse, will depend not solely on the ecliptic limits, but also on the relative distances of the Sun, Moon, and Earth.

Since, (see p. 317,) the Moon is nearly $\frac{1}{60}$th nearer a place when on its zenith than when on the horizon, and since a solar eclipse depends so much on the *condition of distance*, a spectator may, or may not see it, according to the altitude of the Sun above his horizon.

In order to determine the length of the Moon's shadow, we may use the Figure of page 338.

Now, by p. 340, l. 18, $CE = \frac{Et}{\sin. \angle ECt}$,

$$= \frac{Et}{\sin. (\angle AES - \angle EAt)}.$$

Here then E must represent the Moon, and accordingly $\angle AES$, is the apparent semi-diameter of the Sun seen from *the Moon*, equal, therefore, to

$$\text{apparent semi-diameter} \odot \text{ seen from } \oplus \times \frac{\text{dist. } \odot \text{ from } \oplus}{\text{dist. } \odot \text{ from } ☽},$$

and, the angle EAt is the Sun's horizontal parallax for the *Moon*, equal therefore, to

$$\odot\text{'s horizontal parallax for } \oplus \times \frac{☽\text{'s rad.}}{\oplus\text{'s rad.}} \times \frac{\text{dist. } \odot \text{ from } \oplus}{\text{dist. } \odot \text{ from } ☽}.$$

Hence, calling the radii of the Moon and Earth, r, R, and the distances of the Sun from the Moon, and Earth, k, K respectively, there results

$$\text{length of Moon's shadow} = \frac{r}{\sin. \left[\frac{D}{2} \times \frac{K}{k} - p\frac{rK}{Rk}\right]}$$

$$= \frac{r}{\sin. \left[\left(\frac{D}{2} - p\frac{r}{R}\right)\frac{K}{k}\right]}$$

$$= \frac{r}{\sin. \left[\left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p}\right]}$$

For, since $p = \frac{R}{K}$, and $P = \frac{R}{K-k}$, $\frac{K}{k} = \frac{P}{P-p}$.

By means of this formula, we have

	Length of Shadow.	☽'s Dist.
☉ in apogee, ☽ in perigee - - - -	59.730	55.902
☉ in perigee, ☽ in apogee - - - -	57.760	63.862

And this latter case is one of those mentioned in p. 354, and in which the Moon's shadow never reaches the Earth.

The formula for the length of the Earth's shadow has been adapted, so as to express the length of the Moon's shadow. Similar alterations may be applied to the other formulæ. For instance, (see p. 338,)

the appa^t. semi-diameter of ⊕'s shadow $= \angle Emt - (\angle AES - \angle EAt)$

Now we have already shewn (p. 355, l. 22,) that

$$\angle AES - \angle EAt = \left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p},$$

and $\angle Emt$, (the Moon being at E, and the Earth at M,) equals the ☽'s apparent semi-diameter $\left(\frac{d}{2}\right)$.

Hence,

$$\text{the appa}^{t}\text{. semi-diam}^{r}\text{. of ☽'s shadow} = \frac{d}{2} - \left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p}$$

$$\left(\text{since see p. 311, } \frac{d}{2P} = \frac{r}{R}\right) = \frac{d-D}{2} \times \frac{P}{P-p}.$$

Hence, when the Moon's apparent diameter (d) equals the Sun's (D), the apparent semi-diameter of the Moon's shadow is equal nothing; or, the vertex of the conical shadow just reaches the Earth.

When the Moon's apparent diameter (d) is less than the Sun's (D), the expression for the apparent diameter of a section of the Moon's shadow is negative; in other words, the shadow never reaches the Earth.

In a similar manner may the formulæ for the penumbra of the Earth be transformed, and adapted to the case of the Moon.

In order to find the distance of the centers of the Moon's shadow and of the Earth, when the Earth's disk just touches the section of the Moon's shadow, we must add to the expresssion, l. 17, the apparent semi-diameter of the Earth, seen from the

Moon, which, in other words, is the Moon's horizontal parallax (P). Hence (see p. 339,)

$$\text{distance} = P + \frac{d - D}{2} \times \frac{P}{P - p}.$$

From this expression the solar ecliptic limits may be computed, precisely as the lunar were (see p. 341,) and they will be found equal to 17° 21′ 27″.

The same diagram and formulæ, as we have seen, apply equally to solar as to lunar eclipses; and, to a spectator placed in the Moon, our solar eclipses must appear, precisely, as lunar eclipses appear to us; the fictitious spectator too, might compute the duration, and magnitude, of an eclipse caused by the shadow of the globe on which he insists, by processes like those which have already been used, (p. 345,) in the case of lunar eclipses. The forms of the resulting equations, and the steps of the process, would be the same in each case. It would be only necessary to make such slight alterations as we have already made. And, under this point of view there is no difference between lunar and solar eclipses. The computation of the one is as easy as that of the other. But, still the fact is, that the subject of solar, is much more difficult than that of lunar eclipses. There is then some material circumstance of difference between them, which it is now necessary to point out.

In the preceding computations relative to lunar eclipses, no consideration was had of any particular parts of the Moon's disk which might either be covered by, or approach within a certain distance of, the Earth's shadow. In the ingress, for instance, merely the time of contact was determined, and nothing said concerning the position of the point of contact relatively to any fixed point in the Moon's equator. The lunar latitude and longitude of the point of contact is a matter of indifference to the observer on the Earth's surface. But, to an observer at the Moon the case is quite different: to such an one, the eclipse does not begin when the Earth's shadow comes in contact with the Moon's disk, but when it begins to obscure his station. Now, in the predicament of this fictitious observer at the Moon, during what to us is a lunar eclipse, is an observer at the Earth during a solar eclipse. It is necessary for him to know when, and how long, the shadow of the Moon will obscure a station of an assigned longitude and latitude.

Solar eclipses then are more difficult of computation because more is required to be done in them, than in lunar eclipses. If in the investigation of the latter, there had been solved a problem, in which it was required to determine the time when a particular point on the Moon's surface was eclipsed, then from such solution we should possess the means of determining, what it is essential to determine, in solar eclipses.

The method, however, of computing lunar eclipses (given in pp. 345, &c.) may be adapted to solar; and, in such a manner as to determine the times of the happening of the latter at an assigned place. This we will endeavour to explain.

First, that method may (making such substitutions as have already been made in pp. 355, &c.) be employed in computing the time and duration of a solar eclipse with reference to the *whole disk* of the Earth; that is, the eclipse being supposed to begin, at the first contact between the Moon's shadow and any part of the Earth, and to end at the last contact.

At any time (t) included within the duration (T) of such an eclipse, we are able to compute the apparent distance of the centers of the Sun and Moon, supposing the spectator to be placed in the center of the Earth. The problem is precisely the same as the one in p. 345, relative to a lunar eclipse. Corresponding to the time t, the solar and lunar Tables, will furnish us with the longitude of the Sun, the longitude and latitude of the Moon, &c.; such quantities in fact, as λ, m, ϱ, &c.; and, involving these quantities precisely as they were in pp. 345, &c., an equation exactly similar to the one of p. 346, would result: and from its solution, since t is supposed to be given, c would result; but, if c be assigned, then is t the resulting quantity.

If, instead of a spectator in the Earth's center, we suppose one on the surface, in what respects and degree ought the conditions of the preceding problem to be changed? The latitudes and longitudes (l, λ) computed for the former spectator, cannot belong to the latter, because angular distances (and such are latitudes and longitudes) seen from the center are not the same as when seen from the surface. But, they differ solely by *parallax*. If therefore the true longitudes and latitudes at any time be diminished by parallax, the resulting longitudes and latitudes (l', λ') will belong to a spectator on the Earth's surface, for the same time. These latter being substituted as in page 345, the equation

$$n^2 t^2 + 2\,\lambda' n \sin.^2 \theta \times t = (c^2 - \lambda'^2) \sin.^2 \theta,$$

will express the relation between t and c.

In finding therefore the time, at which, the apparent distance of the centers of the Sun and Moon should be of an assigned magnitude, or in finding the magnitude for an assigned time, the chief thing that is required to be done, is to diminish the angular distances, which the Astronomical Tables furnish us with, by the effects of parallax in the directions of those angular distances.

The angular distances, as we have seen (p. 358,) are measured along the circles of latitude and longitude. What we require then, are formulæ for computing the *parallaxes in longitude and latitude.* The investigation of such formulæ is the chief object of the ensuing Chapter.

That Chapter is on the *Occultation of fixed Stars by the Moon.* A subject which, equally with solar eclipses, requires the aid of formulæ for computing the parallax in longitude and latitude. The investigation of those formulæ might have been introduced into the present Chapter, but it was judged right to defer it to the next, because its subject may *mathematically* be viewed in the light of the simplest case of a solar eclipse For, if from this last we make abstraction of all the ordinary phenomena, the two cases are similar. In the one, we have to find the apparent distance of the centers of the Sun and Moon, in the other, the apparent distance of the center of the Moon and a fixed Star. In each we must take the latitudes and longitudes from the Tables, and then correct such for parallax; but, the latter case is somewhat the more simple, because it is necessary to compute the parallax in latitude and longitude for one body only, the Moon; the other, the fixed Star, having no parallax.

There is a third phenomenon, *The Transit of an inferior Planet over the Sun's Disk,* which is nearly similar to an occultation and a solar eclipse in its general circumstances, and exactly so in its mathematical conditions. In the two latter phenomena, the Moon by its interposition obscures the light of the Sun, or suddenly extinguishes that of the Star: in the former, the planet successively darkens parts of the Sun's disk; this effect then, like an occultation, is a species of eclipse. But, without any forced analogies or violation of the proprieties of language, it is a sufficient reason for classing these phenomena together, that it is *mathematically convenient* so to do. To each, the same equations and formulæ apply; and, as we shall hereafter perceive, they may all be employed in attaining the same object, the longitudes of places.

The next Chapter will put us in possession of the means of computing the apparent distance of the centers of the Sun and Moon. If that distance be the sum of the semi-diameters of those bodies, their disks will be just in contact, and the corresponding time will be that of the beginning or the end of an eclipse. Such, considering the practical use of solar eclipses in determining the longitudes of places, is the essential problem; and to that we shall restrict ourselves: still, it must not be forgotten that it is only one out of many that may be proposed on the same subject.

The times of the beginnings of solar eclipses can be exactly noted: it is that circumstance which gives them utility and distinguishes them from lunar. In order therefore that the observer may be prepared to note the times of the phases of an eclipse, he ought to know them approximately at least, by previous computation. This he may do, by computing for the several times included within the whole duration of the eclipse, the apparent distances of the centers of the Sun and Moon: and then, from such results he may determine nearly (which is all he wants) the time when the distance shall be equal the sum of the semi-diameters of those bodies.

CHAP. XXXVII.

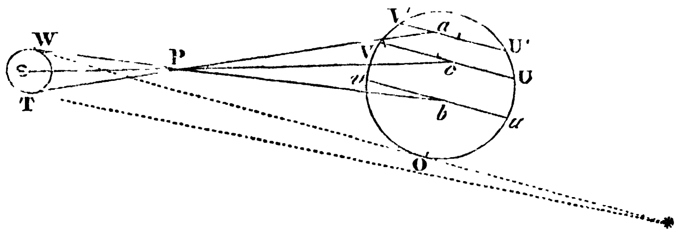
On the Occultation of fixed Stars by the Moon.

PARALLAX enters as a condition into almost all Astronomical calculations; because, we agree to reckon from the center of the Earth, observations which we must make on its surface. The parallax in its greatest value (the horizontal,) being the greatest angle under which the Earth's radius can be seen at an heavenly body, is less, the more distant the body. Fixed Stars are so distant that they have none. Hence, if the Moon were conceived to be equally distant, her center, or any point of her disk, would be seen at the same angular distance from a fixed Star, whether viewed from the Earth's center or surface. If her disk therefore were in contact with a fixed Star, the contact would be seen, at the same instant of time, by an imaginary spectator in the Earth's center, and by all spectators (to whom the Moon should be visible) on its surface. The same instant of time, however, would be differently reckoned by different spectators, according to the situation of their meridians. If 3^h at Greenwich, it might be 7^h at a place to its east, or noon at a place to its west. And, in this case, the mere differences of the reckoned times of the happening of the phenomenon would be the angular distances of the several meridians, or the differences of *the longitudes* of the stations of the several observers.

The Moon, by reason of her great relative proximity, is more affected by parallax than any other heavenly body. Suppose in the Figure (which is intended subsequently to illustrate the transit of *Venus*) *V'VOU* &c. to be the Moon's disk, *W t T* the Earth *, then

* *P* and the lines *V U*, *V' U'*, &c. are of no use in the present illustration.

a spectator at *W* would see a Star * in apparent contact with the point *O* in the Moon's disk, and (if the Moon's center be supposed moving towards *WO*) in the instant immediately previous to an



occultation. A spectator at *T* would see the Star * separated from the Moon's disk; so would a spectator in ε, the Earth's center, but separated by a less angle. To these latter spectators the instant of contact, immediately preceding an occultation, would not have arrived. Hence, it is plain, that the *absolute time* of an occultation would be different to different observers; and, accordingly, the mere difference of the reckoned times of the happening of the phenomenon, would not, in all cases, give the difference of the longitudes of the places of observation. Account must be made of that difference in the absolute time, which would be nothing, were it not for the effects of parallax.

The effects of parallax in longitude and latitude cannot be computed except by a process of considerable length, involving several subordinate ones. These latter, so many distinct steps in the investigation, may be proposed as independent problems. And, on such occasions, authors have been accustomed so to treat a complicated process. They resolve it into its parts, and propose such for solution under the form of problems, and towards the beginnings of their treatises. The object in view, in this arrangement, is the accommodation of the student, who, it is intended, should thus separately subdue the parts of a formidable calculation. But then he must be content to learn the solutions of problems, without discerning the objects of their application. He must take them on trust, and consider that, although, not of independent and immediate, they may be of subsidiary and future use.

It is now intended to resolve the process for computing the parallax in longitude and latitude into its several parts; previously to propose such parts as problems for solution; and then immediately to proceed to their use and application. On this plan, therefore, we are required to find

The right ascension of the mid-heaven, *or of the Medium Cœli.*

The altitude of the *Nonagesimal.*

The longitude of the Nonagesimal.

1st. *The Right Ascension of the Mid-Heaven.*

The right ascension of the mid-heaven at any time is the right ascension of a point of the equator on the meridian at that same time: if at the time assigned or required, a Star were on the meridian, its right ascension would be that of the mid-heaven. If the Sun, either the true, or the imaginary mean, Sun, then the true right ascension of the former, or the mean longitude of the latter, would be the right ascension of the mid-heaven. Suppose, the Star, or the Sun, to have passed the meridian and to be to the west of it, then the right ascension of the mid-heaven must be the right ascension of the Star, or of the Sun, plus the angular distance of the Star or Sun from the meridian, that is, plus the *hour* or *horary angle* (see p. 5) of the Star or Sun. If the true Sun be used, the right ascension of the mid-heaven will be the

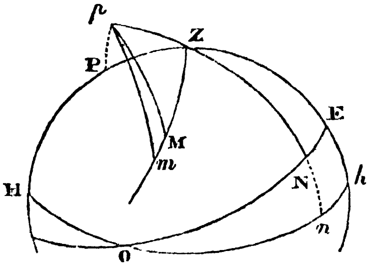
$\odot$'s true right ascension + true time from meridian [*A*]

If the mean Sun, then the right ascension required is

$\odot$'s mean longitude + mean time.

The Altitude of the Nonagesimal.

The *Nonagesimal* is that point of the ecliptic, which, at any assigned time, is highest above the horizon. If *Hh* be the horizon,



ONE a portion of the ecliptic, then, if *ON* be taken = 90°, the point N is the nonagesimal, and its height is Nn; Nn being the continuation of a vertical circle passing through N and the zenith Z.

Nn the height of the nonagesimal is (see *Trig.* p. 91) the measure of the spherical angle EOH, the inclination of the ecliptic to the horizon.

If we continue NZ to p, the pole of the ecliptic, then, since pN and Zn are each equal to a quadrant, by taking away the common part ZN, we have

$$pZ\ (= Nn) \text{ the height of the nonagesimal,}$$

which is accordingly the measure of the inclination of the ecliptic to the horizon.

In order to find pZ, take P the pole of the equator; then in the triangle PpZ, we have

PZ the co-latitude of the place,
Pp the obliquity of the ecliptic,
$\angle pPZ = 270° -$ right ascension of the mid-heaven.

For the right ascension of E is clearly the same as the right ascension of the mid-heaven.

This case then is that of oblique spherical triangles, in which, from two sides and an included angle, it is required to find the third side; a problem of the same kind as that of the latitude of a Star to be determined from its right ascension and north polar distance (see p. 52) and which we shall similarly solve by the aid of a subsidiary angle (θ) (see *Trig.* p. 129).

Assume then θ such, that

$$\tan.^2\theta = \frac{\text{sin. obl}^{y}.\times\text{cos. lat.}\times\text{ver. sin. }(90°+\text{R. A. of mid-heaven})}{\text{ver. sin. (co-latitude} - \text{obliquity)}},$$

$$\text{then, ver. sin. } pZ = \text{ver. sin. (co-lat.} - \text{obliquity)}\times\text{sec.}^2\,\theta\,*$$

$$\text{or, sin. }\frac{pZ}{2} = \text{sin. }\frac{1}{2}\text{ (co-lat.} - \text{obliquity)}\times\text{sec. }\theta$$

* Examples to these several methods will be given under that belonging to the general problem of 'the distance of two bodies.'

and in logarithms,

$$\log.\sin.\frac{pZ}{2}=10+\log.\sin.\frac{1}{2}(\text{co-lat}-\text{obliquity})-\log.\cos.\theta$$

The complement of the altitude (pZ) of the nonagesimal is ZN, sometimes called the *Latitude of the Zenith.*

Longitude of the Nonagesimal.

This longitude is the longitude of the pole P (which is 90°) plus the angle PpZ. It is necessary therefore to find this latter angle, which may be found either from

$$\sin.\ PpZ=\frac{\sin.pPZ\times\cos.\text{lat.}}{\sin.\ \text{height of nonagesimal}},$$

or from this expression, (see *Trig.* p. 118,)

$$\cos.^2\frac{1}{2}\ PpZ=\frac{1}{\sin.pZ.\sin.pP}\times$$

$$\left[\sin.\frac{1}{2}(pP+pZ+PZ).\sin.\frac{1}{2}(pP+pZ-PZ\right].$$

And, in certain cases, it will be expedient to compute the angle PpZ by this latter expression, (for the reasons of which, see *Trig.* pp. 38, 122.)

From the right ascension of the mid-heaven have been found the height and longitude of the nonagesimal; and from these latter we may proceed to, what are the objects of search, the parallaxes in longitude and latitude.

Parallax in Longitude.

Let M be the true place of an heavenly body, m the apparent place depressed, in a vertical circle ZMm, by parallax, (see Chap. XII,) then the parallax in longitude is the angle Mpm, the measure of which, since Mm is small, is very nearly the fluxion, or the differential of the angle ZpM: and such we shall assume it to be. Now, let

L, l, be the latitudes of M, m, $(= 90^\circ - pM,\ 90^\circ - pm,)$
K, k the angles ZpM, Zpm,
h, (pZ) the height of the nonagesimal,
p, the common parallax, P $(= p\,.\,\text{sec. alt.})$ the horizontal,
α, the parallax in longitude ; δ the parallax in latitude,
Z, z, the zenith distances ZM, Zm.

Now, by *Trigonometry*, p. 116, l. 9, we have

$$\cot. z\,.\,\sin. h = \cot. k\,.\,\sin. \angle pZm + \cos. h\,.\,\cos. \angle pZm.$$

Of this equation take the *differential* or fluxion, then, since $\angle pZm$ is constant, and dk or $\dot{k} = \alpha$, and dz, or $\dot{z} = p$, there results

$$p\,.\,\frac{\sin. h}{\sin.^2 z} = \alpha\,.\,\frac{\sin. \angle pZm}{\sin.^2 k}\,.$$

But, (*Trigonometry*, p. 102.)

$$\sin. pZm = \sin. k \times \frac{\sin. pm}{\sin. Zm} = \sin. k \times \frac{\cos. l}{\sin. z};$$

$$\therefore\ \alpha, \text{ the parallax in longitude, } = \frac{p}{\sin. z} \times \frac{\sin. h\,.\,\sin. k}{\cos. l},$$

$$= P\,\frac{\sin. h\,.\,\sin k}{\cos. L} \text{ (very nearly).}$$

In this expression $k = K + dk = K + \alpha$; $\therefore$ α the quantity sought, is contained in the formula that is meant to express its value. That we may not argue therefore in a circle, we must approximate to the value of α, by supposing, in the first case, k to equal K: thus, first find a value (e) of α from this expression

$$\alpha\,(e) = P\,\frac{\sin. h\,.\,\sin. K}{\cos. L},$$

then investigate a nearer value of α, from

$$\alpha = P.\frac{\sin. h \sin. (K + e)}{\cos. L},$$

and if this last value is not sufficiently accurate, the above process must be repeated.

Parallax in Latitude.

By a formula similar to that which we have just used, and which differs from it only, in the angle k being used for pZm, l for z, &c., we have

in $\triangle$ Zpm, $\tan. l \sin. h = \cot. pZm . \sin. k + \cos. h . \cos. k$,
in $\triangle$ ZpM, $\tan. L \sin. h = \cot. pZm \sin. K + \cos. h . \cos. K$,

eliminate, from these two equations, $\cot. pZm$, and there results

$$\sin. h(\tan. L.\sin. k - \tan. l.\sin. K) = \cos. h(\sin. k \cos. K - \cos. k.\sin. K)$$
$$= \cos. h \times \sin. (k - K.)$$

Now, $k - K = \alpha$, and $\sin. (k - K) = \sin. \alpha = \alpha$ (nearly) $= P\frac{\sin. h . \sin. k}{\cos. L}$: substituting ∴ and dividing by $\sin. h \times \sin. k$,

$$\tan. L - \tan. l\frac{\sin. K}{\sin. k} = P\frac{\cos. h}{\cos. L};$$

$$\therefore \tan. L - \tan. l = P\frac{\cos. h}{\cos. L} - \tan. l\left[1 - \frac{\sin. K}{\sin. k}\right],$$

$$= P\frac{\cos. h}{\cos. L} - \frac{\tan. l}{\sin. k}[\sin. k - \sin. K].$$

Now, (*Trig*. p. 22, bottom line) $\tan. L - \tan. l = \frac{\sin. (L - l)}{\cos. L . \cos. l}$,

and (p. 19,) $\sin. k - \sin. K = 2 . \cos. \left(\frac{k + K}{2}\right) \sin. \left(\frac{k - K}{2}\right)$,

and since, $k - K = \alpha$, $\frac{k + K}{2} = K + \frac{\alpha}{2}$: substituting therefore,

$$\frac{\sin. (L - l)}{\cos. L.\cos. l} = P\frac{\cos. h}{\cos. L} - \frac{2 \tan. l}{\sin. k}\left[\cos. \left(K + \frac{\alpha}{2}\right) \sin. \frac{\alpha}{2}\right].$$

But $\sin. (L - l) = \sin. dl = \sin. \delta = \delta$, nearly, and $\sin. \frac{\alpha}{2} = \frac{\alpha}{2}$
$= \frac{P \sin. h \sin. k}{2 \cos. L}$.

$\therefore \delta$, the par. in lat., $= P \cos. h.\cos. l - P.\sin. h \sin. l \times \cos. \left(K + \frac{\alpha}{2}\right)$ *

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* See *Mem.* Gottingen, tom. II, p. 168; where Mayer has given, very nearly, the same expressions; also Lalande, tom. II, p. 305. Edit. 3.

This expression, since $l = L - \delta$, is under the same predicament as the former one, (p. 366,) and must be treated in the same manner; that is, we must find a value of δ by supposing $l = L$, and then a nearer value. Since the Moon's latitude is never very large, and at the time of an eclipse (for computing which the above expressions are useful) is always very small, (and consequently sin. l is very small) we may assume, as a first step in the approximation,

$$\delta = P \cos. h \cos. L \ (= f \text{ suppose,})$$

and then the second step may be made by computing δ, from

$$\delta = P \cos. h \cos. (L - f) - P \sin. h \sin. (L - f) . \cos. \left(K + \frac{\alpha}{2}\right)$$

and the investigation continued, will give more exact values of δ, the parallax in latitude *.

The formulæ for computing the parallaxes in longitude and latitude, have been deduced by, what has technically been called, *the Method of the Nonagesimal.* This method, of no recent invention, naturally suggested itself, as Lalande observes, to the mind of Kepler. For, since the parallax takes place in the direction of a vertical circle, if the heavenly body were situated in a vertical circle, such as ($pZNn$) passing through N, the nonagesimal point, then in such circle, parallax taking place, its effect would be entirely in latitude and be nothing in longitude: since ON, the ecliptic, is perpendicular to pZN. Again, if the Moon, always near the ecliptic at the time of an eclipse, should also be near the nonagesimal, then the greater its altitude the less would be the parallax in latitude, (see Lalande, tom. II, p. 291.)

Distance of the Moon and a Star at the time of an Occultation.

Computing by the preceding formulæ the parallaxes, we must apply them, with their proper signs, to the true longitudes and latitudes furnished by the Tables, or by observation, and the results will be the apparent longitudes and latitudes of the center of the Moon and of the star. Suppose these to be l, l', k, k', respectively;

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* The expressions for the parallaxes in right ascension and declination may easily be deduced from the preceding processes. We must then consider p to be the pole of the equator.

then, in order to find the distance (D), we have (in a triangle such as Mpm, Fig. p. 363), the two sides $90^\circ - l$, $90^\circ - l'$ (analogous to Mp, mp), and the included angle, $k-k'$ (analogous to Mpm); and D is the side opposite to the angle $k - k'$: therefore, (*Trig.* pp. 100, 129, 131),

$$\cos. D = \cos. l . \cos. l' \cos. (k - k') + \sin. l . \sin. l',$$

and substituting for cos. D, &c. $1 - 2 \sin.^2 \frac{D}{2}$, &c. there results

$$\sin.^2 \frac{D}{2} = \sin.^2 \left(\frac{l - l'}{2}\right) + \cos. l . \cos. l' . \sin.^2 \left(\frac{k - k'}{2}\right),$$

whence D may be deduced, and most conveniently, by means of a subsidiary angle, (see the pages just referred to.)

The preceding method is not confined to the case of an occultation, but is equally applicable to the finding of the distances of the Sun and Moon during a solar eclipse, and of the Sun and an inferior planet during a transit. And, in all the cases, since the distances are small, a more simple formula for computing D may be introduced. For, D may be considered as the hypothenuse of a right-angled triangle, the sides of which are $l-l'$, and $(k - k') \cos. l$ *, and then

$$D^2 = (l - l')^2 + (k - k')^2 . \cos.^2 l$$

$$= (l - l')^2 \left[1 + \left(\frac{k - k'}{l - l'}\right)^2 \cos.^2 l\right];$$

$$\therefore D = (l - l') . \sec. \theta,$$

$$\text{making } \tan. \theta = \frac{k - k'}{l - l'} . \cos. l.$$

The latter expression (l. 22,) for the value of D is easily deducible from the former, by substituting in it, $\frac{D}{2}$, $\frac{l - l'}{2}$, &c. instead of their sines, which may be done with inconsiderable error, by reason of the smallness of those angles, during the contiguity of the Moon and star, &c.

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* For $k - k'$ is the arc on the great circle, $(k - k') . \cos. l$, on the parallel; for instance, in Fig. p. 5, if $ab = \angle aPb$ $(k - k')$ $ss' = ab . \cos. sb = (k - k') \cos. sb$.

The first term of the expression for $\sin.^2 \frac{D}{2}$, (see p. 369,) is $\sin.^2 \left(\frac{l-l'}{2}\right)$. Now l, l', are the apparent latitudes, therefore if δ, δ', were the parallaxes, we should have

$l - l' = \Delta + \delta - \delta'$ (Δ = difference of the true latitudes.)

Suppose one of the bodies (that to which the latitude l' belongs) to have no parallax in latitude, but the other to have a parallax equal to $\delta - \delta'$, then, still as before,

$$l - l' = \Delta + (\delta - \delta'),$$

and a similar result will hold good with regard to $\sin.^2 \frac{k - k'}{2}$; therefore, if the coefficient of this latter term, instead of being $\cos. l . \cos. l'$, were a constant quantity a, for instance, (or involved merely the *difference* of the parallaxes), from

$$\sin.^2 \frac{D}{2} = \sin.^2 \frac{l - l'}{2} + a \cdot \sin.^2 \frac{k - k'}{2}.$$

the distance D would result precisely of the same value, if instead of assigning to each body its proper parallax, we supposed one to be entirely without, and *attributed* to the other an imaginary parallax in latitude and longitude, equal to the difference of the real parallaxes. And in this case, the rule given by Astronomers, (see Lalande, 434, tom. II, and *Cagnoli*, p. 463,) would be proved to be true. Since however, the coefficient $\cos. l . \cos. l'$, is not a constant quantity such as a, but [since it equals $\frac{1}{2}$ [$\cos. (l - l') + \cos. (l + l')$], involves, besides the difference, the *sum* of the parallaxes, the rule is not perfectly exact. It, however, is nearly so, since $\sin.^2 \frac{k - k'}{2}$, which is multiplied into $\cos. l . \cos. l'$, is a very small quantity.

We have spoken of the general case of the Problem, when the distance of the centers of two heavenly bodies is to be found. But, if we speak of each particular case, then the rule is slightly inaccurate in a solar eclipse and in a transit, but exact in an occultation, since one of the bodies, the fixed star, is devoid of parallax.

The *Distance of the Centers* is the last step in the mathematical process belonging to the subject of the occultation of a fixed star by the Moon; and since the process is somewhat complicated, we will endeavour to illustrate it, and its subordinate methods, by an Example.

Required the Apparent Distance of Antares *from the Center of the Moon at the instant of Immersion, which was observed at Paris in April* 6, 1749, $13^h\ 1^m\ 20^s$, *Apparent Time.* *

(1.) *Right Ascension of the Mid-Heaven.*

Converting the time into degrees, and taking from the Tables the Sun's longitude, we have (see p. 363,)

R. A. of Mid-heaven $(A) = 15°\ 58' + 195°\ 20'$
$= 211°\ 18'$

Since, $15°\ 58' =$ ☉'s R. A.
and $195\ \ 20 = 13^h\ 1^m\ 20^s$.

(2.) *Altitude of the Nonagesimal,* (*see* 1st *Form p.* 364,)

log. sin.	23° 28′ 22″	(obliquity) - - -	9.60022 †	
cos.	48 38 50	(lat. cor. see p. 99,)	9.82000	
ver. sin.	301 18 0	(90° + *A*) - -	9.68167	
			29.10189	
ver. sin.	17 52 48	(co-lat.—obliquity)	8.68395	- - [*a*]
			20.41794	= 2 log. tan. θ
2. sec.	58 16 54	(θ) - - - - - -	20.55845	
		[*a*] - - - - - -	8.68395	
		20 + log. ver. sin. *pZ* =	29.24240	

∴ *pZ* (*h*) the altitude of the nonagesimal is 34° 23′ 9″.

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* Lalande, tom. II, pp. 437, &c.

† Five decimals are sufficient: more, such is the nature of the process, would not add to the accuracy of the result.

(3.) *Longitude of the Nonagesimal,* (*see Form p.* 365,) *

$p Z$ (h)	34°	23′	9″	log. sin. =	9.75186
$P p$	23	28	22	sin.	9.60022
$P Z$	41	21	10	(b)	18.35208
Sum =	99	12	41		
$\frac{1}{2}$ Sum	49	36	20.5	log. sin.	9.88172
$\frac{1}{2}$ Sum − $P Z$	41	21	10	sin.	9.15697
				(20 added)	39.03869
				(b)	18.35208
				2 log. cos. $P p Z$ =	19.68661

∴ $P p Z$ = 91° 36′ 30″, and consequently, (see p. 365,) the longitude of the nonagesimal = 181° 36′ 30″.

Hence, since by the Lunar Tables the longitude of the Moon was 245° 31′ 42″.4, K, or the Moon's distance from the nonagesimal, (see Fig. p. 363,)

is 245° 31′ 42″.4 — 181° 36′ 30″ = 63° 55′ 12″.

(4.) *Parallax in Longitude,* (see p. 366,)

log. 0° 57′ 16″.2 (P, from Tables)	3.53608	} sum = 13.28890	
log. sin. 34 23 9 (h)	9.75186		
Ar.com. cos. 3 47 58.7 (L ☽'s true lat.)	0.00096		
sin. 64 10 † ($K + a$)	9.95427		
(rejecting 10)	3.24317	= log. 29′ 10″	

---

* The angle $P p Z$ being nearly 90°, is the reason, why it is expedient to use the second, (see p. 365,) of the formulæ, which, in the first instance, gives only half the angle $P p Z$. For a more full explanation of this point consult *Trig.* p. 38, &c.

† K (see l. 16.) = 63° 55′ 12″, and, since a is some small quantity, it is *conjecturally* taken, in the first trial, equal to 14′ 48″, which added to K, makes $K + a$ = 64° 10′.

$\therefore\ \epsilon$, or the first approximate value of α, is 29′ 10″, and

$$K' + \epsilon = 64^\circ\ 24'\ 22'',$$

log. sin. 64° 24′ 20″ $(K + \epsilon)$	- -	9.95515
Sum (see p. 372, l. 19,) rejecting 10		3.28890
	(rejecting 10)	3.24405 = log. 29′ 14″.1 ;

$\therefore\ \alpha$, the parallax in longitude, is 29′ 14″.1.

(5.) *Parallax in Latitude*, (*see p.* 367,)

Computation of the first part of the expression,

log. P - - - - - - - - -	3.53608	} sum = 13.45267
log. cos. 34° 23′ 9″ (h) - -	9.91659	
cos. 3 47 58.7 (L) - -	9.99903	
(rejecting 20)	3.45170 = log. 47′ 9″ = first	

approximate value of δ.

Again,

log. cos. 4° 35′ 8″.7 $(L + \delta)$	9.99861
log. P + log. cos. h - -	3.45267
(rejecting 10)	3.45128 = log. 47′ 6″.7, 2d value of δ

Computation of the second part of the expression, (p. 367,)

log. $P \times$ sin. h (see p. 372, l. 19, 20,) -	3.28794
log. cos. 64° 9′ 47″ $\left(K + \frac{\alpha}{2}\right)$ - - - - -	9.63929
sin. 4 35 8.7 (☽'s latitude) - - -	8.90283
(rejecting 20) - - - -	1.83006 = log. 1′ 9″

Since the Moon's latitude was south, this last part (1′ 9″) of the parallax in latitude must be added; consequently, the whole parallax in latitude $(\delta) = 47'\ 6''.7 + 1'\ 9'' = 48'\ 15''$ nearly. Hence, applying the parallaxes thus found, to the true longitude and latitude,

☽'s apparent long. = 245° 31′ 42″.4 + 29′ 14″.1 = 246° 0 56″.5
☽'s apparent lat. = 3 47 58.7 + 48 15 = 4 36 13.7.

(6.) *Apparent Distance of the Moon and Antares*, (*see p.* 369.)

Long. of *Antares* (k') -	246° 16′ 19″.2	- lat. (l')	4° 32′ 10″.5
☽'s longitude (k) -	246 0 56.5	- lat. (l)	4 36 13.7
$k' - k$ -	0 15 22.7	- $l - l'$	0 4 3.2

$\therefore$ log. cos. 4° 34′ 12″ $\left(\frac{l + l'}{2}\right)$ - - 9.9986171

log.	- 0	15	22.7	- - - - - -	2.9650605
Ar.comp.log.	0	4	3.2	- - - -	7.6140364
					10.5777140 = log. tan. θ
log. sec.	75	11	21 (θ)	- - - - -	10.5923906
Ar.comp.log.	0	4	3.2	- - - - - -	7.6140364
				log. 951″.38 =	2.9783542

therefore the distance required is 15′ 51″.38.

Thus has been found in an occultation, the distance of a fixed star and the Moon's center. And a like process will give the distance of the Sun and Moon in a solar eclipse, and of the Sun and an inferior planet in a transit. And the only differance between these two latter cases and the former, is pointed out in the rule (p. 370,) which directs us to suppose one body to be without parallax, and the other to be endowed with a parallax equal the difference of the parallaxes of the two bodies.

The parallaxes in longitude and latitude being known, it is evident we may deduce the true longitudes and latitudes from the apparent, as we have the apparent from the true; and thence, similarly, deduce the true distance of the star and Moon. And according to some methods, the true distance is made subservient to the same end, which we shall shew the apparent to be, namely, the determination of the longitude of the place of observation. (See Vince, vol. I, pp. 534.)

CHAP. XXXVIII.

On the Transits of Venus and Mercury over the Sun's Disk.

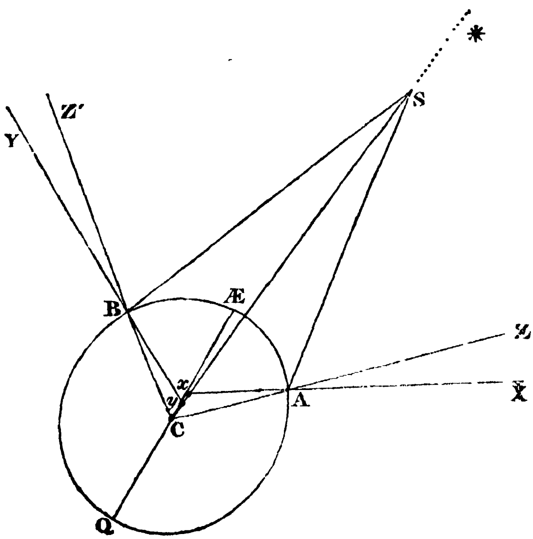
WE have already stated in p. 359, that the phenomena of eclipses, occultations, and transits are very nearly alike in their general circumstances, and exactly alike in their mathematical theories. In these theories, the essential problem which is to be solved, is the apparent angular distance of two heavenly bodies, in apparent proximity to each other, to a spectator on an assigned station on the Earth's surface.

In an eclipse and occultation, the Sun's parallax is supposed to be known. were it supposed to be known in a transit, then there would be an additional circumstance of similarity between its theory and those of the former phenomena: for, they would have same object, and would equally serve to the determination of the longitudes of places. And, in point of fact, this is the present state of the case. One transit of *Venus* has already answered a special purpose, that of determining the parallax of the Sun, and future transits may be either used, in confirming the accuracy of that determination, or for the general purposes which *eclipses*, in their extended signification, (see p. 359,) are made subservient to.

It is the object of the present Chapter to explain the use that *has been made* of the transit of *Venus*; to shew the special use of that phenomenon in determining the important element of the Sun's parallax.

The Sun's parallax is the angle subtended at the Sun by the Earth's radius; that angle can be found, if another subtended by a chord, lying between two known places, can. And to find this latter angle is the object of the method given in Chap. XII, p. 95.

The angle *ASB*, is the object of investigation. Now, in the instrumental measurement of that angle an error of three or four seconds may be committed ; which, in the case of the Moon, whose



parallax is about 1°, is of little consequence. but a probable error of that magnitude in the case of the Sun, whose parallax is less than nine seconds, would render the result of the method so uncertain, as entirely to vitiate it.

Preserving the principle of the method, Astronomers have sought to correct its error, by *computing* instead of instrumentally measuring an angle such as *ASB*, or an angle from which it may be immediately deduced.

Suppose, for the sake of illustration, *S* to be a point in Venus's disk, and *BS* continued to be a tangent to the Sun's disk: then the direction of a line *AS* would be to the left of the Sun's disk. In other words, the moment of contact or ingress would be present to a spectator at *B*, but to a spectator at *A* would not have arrived. It would arrive some minutes after, when by the retrograde motion (see p. 246) of *Venus*, the line *AS*, always a tangent to the disk of *Venus*, should become one to that of the Sun. Suppose *AS* in this latter direction (to the right of its present position) to intersect BS' produced in some point S' situated in the Sun's disk: then, the angle SAS' is proportional to the time elapsed between the contacts at B and A: that time is known from observation and the ascertained difference of longitudes of the places B and A: suppose it t, and let h be the horary approach of *Venus* to the Sun (about $240''$); then,

$$1 : t :: h : ht, \text{ the angle } SAS',$$

which is by these means *computed*.

SAS' being known, $SS'A$, or $AS'B$, may be determined from the known ratio between SA and SS'. (See Chap. XXIV.)

The preceding is a very imperfect description of the method that was actually used in the problem of the transit of *Venus*. But it shews the principle of the method and the reason of its superior accuracy: for, since the time of contact can be observed as nearly as two seconds, or since the limit of the error in time is two seconds, and since the excess of the horary motion of *Venus* above the Sun's is $240''$, that is, $4''$ in 1^m, or $\frac{1''}{15}$ in 1^s, an error of 4^s (2^s at each place of observation) would only cause an error of $\frac{4''}{15}$ in the estimation of the angle SAS', and an error in the estimation of $SS'A$, (on which the parallax depends) less in the proportion of SA to SS', that is, in the case of *Venus*, of one to two and a half nearly.

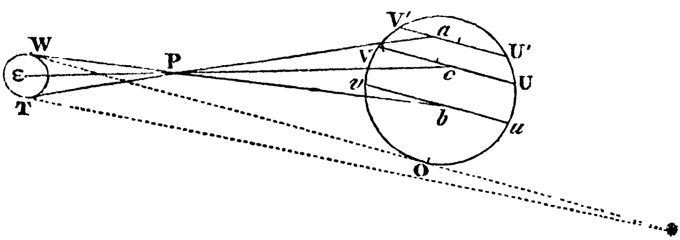
The imperfection of the method, as it has been described, consists in this; that it requires to be known, what it is very difficult to determine, the difference of the longitudes of the places A and B. For, t (see p. 361,) is the difference of *actual* or *absolute time*, which depends on the reckoned time at each place of observation, and the difference of the longitudes of those places. If the contact was observed at Greenwich at $3^h\ 40^m$, and at a place $15°$ east of Greenwich, at $4^h\ 41^m$, the difference in absolute time would be only 1^m; since 1^h, in the reckoned, is entirely due to the difference of the meridians.

The longitude of the Cape of Good Hope, which had been long the station of an European colony, and where the transit of 1761 was observed, was known to a considerable degree of accuracy. That of Otaheite, where it was expedient to observe the transit of

1769, was not known. And, from the difficulty of ascertaining with sufficient precision this nice condition of the longitude, Astronomers, by modifying their process of calculation, have got rid of it entirely. Instead of observing the mere ingress, they observe the duration of the transit, and from the difference of durations, at different places, deduce the quantity of the Sun's parallax.

The difference in the durations of transits does not amount to many minutes. To make it as large as possible, it is expedient so to select the places of observation, that, at one, the duration should be accelerated, at another, retarded beyond the *true time* of duration, which is supposed to be that which would be observed at the Earth's center.

If P were *Venus*, ε the Earth, W a place towards the north pole (Wardhus for instance) and T (Otaheite) towards the south, and $V'V$, &c. the Sun's disk, then the *true* line of transit, seen from the center ε would be VU: from W, vu would be the line; from T, $V'U'$. If T should be the *true duration* of the transit,

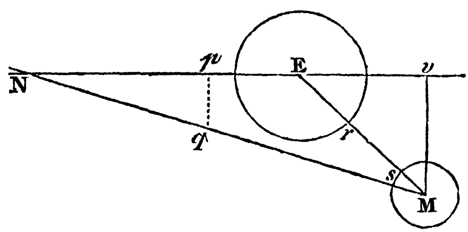


or the time of describing VU, then the time of describing vu nearer to the Sun's center than VU, would be $T + t$: of describing $V'U'$ more remote than VU from the Sun's center, $T - t'$: and accordingly the difference of the durations of the transits seen from T and W, would be $T + t - (T - t') = t + t'$. This, as it is plain, is entirely the effect of parallax.

We now purpose to enter farther into the mathematical process of the subject; and to shew, how from T, t, and t' computed, the Sun's parallax may be determined.

In this process we shall have a proof of what we have more than once asserted, namely, the similarity of the mathematical theories of eclipses, occultations, and transits. For, T, $T + t$, $T - t'$ will be computed by means of the formula employed in Chap. XXXV, p. 345. And the only difference in the computation of T and of $T+t$ consists in assuming in the former, the angular distances seen from the Earth's center and given by the Astronomical Tables, and in the latter, those angular distances *corrected* for the effects of parallax in longitude and latitude.

In the above-mentioned formula, the time and the apparent angular distance of two heavenly bodies were involved. And the diagram employed on that occasion will suit the present*. Instead of E and M representing the centers of the Earth's shadow and



the Moon, let them represent the centers of the Sun and *Venus*; then, EM will represent the distance of their centers previous to, or after a transit: and, the Tables of the Sun and of the planets, will, as in an eclipse (see p. 345,) furnish us with quantities analogous to λ, m, n, &c. Suppose then, at the time of conjunction,

♀'s lat. - - - λ - - - horary motion in lat. - - n;
♀'s long. - - - l - - - horary motion in long. - - m
☉'s horary motion in long. - - s

then, forming an equation, precisely as the one in p. 345, was formed, we have

$$n^2 t^2 + 2\lambda n t \, . \sin.^2 \theta = (c^2 - \lambda^2) \, . \sin.^2 \theta$$

$$\text{whence, } t = \frac{1}{n} [- \lambda \sin.^2 \theta \pm \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}]$$

---

* The same diagram will serve for an occultation, M being the Moon, and E the star.

t being the time from conjunction, and c the distance of the centers.

Substitute in this equation, instead of c, the sum of the apparent semi-diameters of the Sun and *Venus*, and the resulting time will be that of the first or last *exterior contact*: substitute the difference, and the resulting time will be that of the first or last *interior contact*. The duration of a transit is the difference between the times of the last and first exterior contacts, and is to be found exactly as the duration of an eclipse was in pp. 348, &c.

The times which we have mentioned as resulting from the preceding equation, would be noted by a spectator in the Earth's center: they belong to the points V, U, and the line VU. But to a spectator at T, for instance, the contact instead of V would appear to take place at V'; and, it would appear to happen at a time, different from T' the computed time of its happening at V; at $T' + t'$, for instance, t' being a small quantity and entirely the effect of parallax.

The latitude and longitudes of *Venus* and the Sun perpetually altering, those quantities at the time $T' + t'$ from conjunction would be different from what they were at the time T': their change would be proportional to t'. The time T' being computed from the preceding equation, the latitudes and longitudes may be taken from the Tables, or easily computed from their values at the time of conjunction. At this latter time, we have supposed the latitude of *Venus* to be λ. It is convenient for us to use that symbol (λ) to denote the latitude at the time T' of contact; let also the corresponding longitudes of *Venus* and the Sun be l, l'; and the horary motions m, n, s: then (see p. 345,) at the time t' from contact,

$$♀\text{'s long.} - - - l + mt' - - ♀\text{'s lat.} - - - \lambda + nt,$$
$$☉\text{'s long.} - - - l' + st'.$$

And accordingly, the distance of the centers (such as EM) would be the hypothenuse of a right-angled triangle, of which the sides respectively, are $(l + mt') - (l' + st')$, and $\lambda + nt'$.

These angular distances belong to the center of the Earth; but when they are diminished, as in the case of an occultation, (see p. 368,) by the parallaxes in longitude and latitude, they will belong to a spectator on the Earth's surface. Let the parallaxes in longitude be α, α'; in latitude δ, δ'; then, the sides of the right-angled triangle are

$$(l + m t' - \alpha) - (l' + s t' - \alpha'), \text{ and } \lambda + n t' - \delta + \delta',$$
$$\text{or } l - l' + (m - s)t' - (\alpha - \alpha'), \text{ and } \lambda + n t' - (\delta - \delta').$$

The hypothenuse is the distance of the centers. But, the time is that at which a contact of the limbs of the Sun and *Venus* is seen; if the contact therefore be an *internal* one, (when the whole of *Venus*'s disk is just within the Sun's,) the distance will be the difference of the semi-diameters of *Venus* and the Sun: let it equal Δ, then,

$$\Delta^2 = [l - l' + (m - s)t' + \alpha' - \alpha]^2 + [\lambda + n t' - (\delta - \delta')]^2.$$

In which expression, $\alpha - \alpha'$, $\delta - \delta'$, and t' are very small quantities; rejecting therefore their squares and products in the expression expanded;

$$\Delta^2 = (l - l')^2 + 2(l - l') \times (m - s)t' - 2(l - l') \times (\alpha - \alpha')$$
$$[a] \qquad + \lambda^2 + 2\lambda n t' - 2\lambda(\delta - \delta').$$

But, since by hypothesis, (see p. 380, l. 27,) l, l', &c. are the longitudes, &c. at the time of contact seen from the center, we have

$$\Delta^2 = (l - l')^2 + \lambda^2,$$

thence deducing t' from $[a]$,

$$t' = \frac{(l - l')(\alpha - \alpha') + \lambda(\delta - \delta')}{(l - l')(m - s) + \lambda n}.$$

In this expression, l, l', λ, m, s, n, are to be computed from the Tables, and the parallaxes in longitude and latitude, (α, α', δ, δ') from the expressions in pages 366, 367, that is, if P, P' represent the horizontal parallaxes of *Venus* and the Sun,

$$\alpha = \frac{P \,.\, \sin. h \,.\, \sin. k}{\cos. \text{lat. } ♀}, \quad \alpha' = \frac{P' \,.\, \sin. h \,.\, \sin. k'}{1},$$

$$\delta = P \cos. h \,.\, \cos. ♀\text{'s app. lat.} - P \sin. h \,.\, \sin. ♀\text{'s app. lat.} \times \cos. \left(\frac{K + \delta}{2}\right),$$

$$\delta' = P' \cos. h \quad (\text{since } ☉\text{'s apparent latitude is nearly} = 0.)$$

At the time of a transit, *Venus*'s latitude is very small, and her longitude nearly equal to that of the Sun's, the coefficients of P, P', therefore, in the expressions for α, α', and for δ, δ', must be nearly equal. Let these coefficients be a, a', b, b' respectively, then

$$t' = \frac{(l - l')\ (aP - a'P') + \lambda\ (bP - b'P')}{(l - l')(m - s) + \lambda n};$$

or, since $aP - a'P' = a'(P - P') + (a - a')P$, and $(a - a')P$, as well as $(b - b')P$, are very small quantities and may be therefore neglected, we have,

$$t' = \frac{a'l - a'l' + \lambda b'}{(l - l')(m - s) + \lambda n} \times (P - P').$$

From this equation, if t' should be known from observation, $P - P'$, the excess of the parallax of *Venus* above that of the Sun, (which is the object of investigation,) could be determined. We must consider therefore, by what means t' may be ascertained.

The Astronomical Tables, from which the quantities, l, l', &c. are supposed to be taken, are computed for Greenwich. For such a place, let the time of the conjunction of *Venus* and the Sun be T; then at any place to the west of Greenwich and by a longitude $= M$ (expressed in time) the reckoned time, at which the conjunction would be seen from the center of the Earth would be $T - M$; the time of internal contact, seen also from the center, would be $T - M + T'$; and the time, at which the contact would be seen from the place of observation (whose longitude is M) would be

$$T - M + T' + t'.$$

Now, the observer by means of his regulated clock is able to note this time; suppose it H', then

$$t' = H' - T + M + T', \text{ and consequently,}$$

$$H' - T + M + T' = \frac{a'l - a'l' + \lambda b'}{(l - l')(m - s) + \lambda n} \times (P - P')$$

$= f(P - P')$, f representing the coefficient of $P - P'$.

From this equation $P - P'$ could be determined, if M, the longitude of the place, were known. But, for the reasons alledged in p. 378, we must seek to dispense with that condition. This is simply effected by observing the *last* interior contact, that is, the one immediately preceding the egress of *Venus*'s disk from the Sun. Let the quantities analogous to T', H' belonging to this last contact be T'', H'', and the coefficient of $P - P'$ (analogous to f) be f'; then,

$$H' - T + M - T' = f(P - P')$$
$$H'' - T + M - T'' = f'(P - P')$$

consequently,

$$(H' - H'') - (T' - T'') = (f - f')(P - P) \quad [h]$$

$$\text{and } P - P' = \frac{H' - H'' - (T' - T'')}{f - f'} .$$

This expression is deduced by observing at the same place the times of ingress and egress. If we take a second place of observation, then there will result an equation similar to (h), that is,

$$H_{,} - H_{,,} = T' - T'' = (f_{,} - f_{,,})\, P - P',$$

and subtracting this from the former (h),

$$(h')\ (H' - H'') - (H_{,} - H_{,,}) = [(f - f') - (f_{,} - f_{,,})] \times (P - P') \text{*}$$

whence results $P - P'$: a value obtained from the difference of the *durations* of the transit.

The parallax is inversely as the distance; but, by observation and the Planetary Theory, (see Chap. XXIV,) the ratio of the distances of the Earth from *Venus* and the Sun, is known, and therefore the ratio of P to P'; let it be as $g : 1$, and let the coefficient of $P - P'$ in (h') be q, the right hand side being $= A$; then

$$(g - 1)\, q\, P' = A,$$

$$\text{and } P = \frac{A}{q\,(g - 1)} .$$

This is the value of P' when the Sun is at some distance ϱ from the Earth. At the mean distance (1)

☉'s horizontal parallax (nearly his mean) $= \varrho\, P'$.

The preceding formula, applied to the transit of *Venus* which happened in 1769, would give

$$P - P' = \frac{14'\ 16''}{65.72962} \times 1'' = 21''.5428.$$

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* This last operation, although unnecessary in the preceding simple statement, is not so in practice: since, by means of it, the errors of the Tables introduced into the calculation as arbitrary quantities are got rid of.

And the Astronomical Tables, at the epoch of the observations, gave

♁'s distance from ☉ (ϱ) - - - - 1.01515
♀'s distance from ☉ - - - - - - .72619

and therefore $g - 1 = \frac{72619}{28896}$, and

P' the Sun's parallax $= 21''.5428 \times \frac{28896}{72619} = 8''.5721$

and (see p. 383, l. 20,) the ☉'s hor. par. $= 8''.5721 \times 1.01513 = 8''.7017$

In the fraction $\frac{1416}{65.72962}$ $(= P - P')$ the numerator is obtained from observations on the times of contact. If that numerator had been $1416 - 65.72962$, then the quotient, instead of being $21''.5428$, would have been $20''.5428$. In other words, a difference of $65^s.72962$, made in noting the times of the transit, would have produced an error of one second only in the difference of the parallaxes, and consequently, an error in the Sun's parallax less in the ratio of 28896 to 72619, or (of 2 to 5 nearly). Or, what amounts to the same thing, it would require an error in time equal to $164^s \left(= 65.7 \times \frac{5}{2}\right)$ to have produced an error of $1''$ in the value of the Sun's parallax.

The special Astronomical use of the transit of *Venus* is, as it has been observed, the determination of the Sun's horizontal parallax. But, that important element being once determined, the transit of an inferior planet, even with regard to its use and object, may be made to enter the class of eclipses and occultations, and like them be made subservient to the determination of the longitudes of places.

That a transit may be adapted to this latter purpose, is evident from the equation of p. 382, namely,

$$H' - T + M + T' = f \cdot (P - P'),$$

for in that, if $P - P'$ be supposed to be known, M, the longitude of the place of observation, is the only unknown quantity.

Transits, however, are phenomena of such rare occurrence, that their use, in this latter respect, is very inconsiderable.

The fixed stars, the Sun, the planets, and the Moon, with their peculiar and connected theories, have already been treated of. There is another class of heavenly bodies, called *Comets*, which ought not to be passed over. Yet their strictly mathematical theory is so difficult, that, instead of attempting to put the Student in possession of it, we shall content ourselves with acquainting him with some of its general circumstances, and with referring him to ampler sources of information.

CHAP. XXXIX.

On Comets.

COMETS are bodies that occasionally are seen in the heavens, with ill-defined and faint disks, and usually accompanied with a *coma* or stream of faint light in the direction of a line drawn from the Sun through the Comet.

Comets resemble the Moon and planets in their changes of place amongst the fixed Stars: but, they differ from them in never having been observed to perform an entire circuit of the heavens. There are also, other points of difference; the inclinations of the planes of their orbits observe not the limits of the Zodiac, as the planes of the orbits of the Moon and planets do; and, the motions of some of them are not according to the order of the signs.

Comets, like planets, move in ellipses, but, of such great eccentricity, that thence has arisen a ground of distinction, and Comets are said to *differ* from planets, because they move in orbits so eccentric. The eccentricities of those that have been observed have been found so great, that, it has been found, parabolas would nearly represent them.

What are called the elements of a Comet's orbit are less in number than those of a planet's; they are only five. It is impossible from the observations made, during one appearance of a Comet, to compute the major axis of its orbit and its period, and consequently the area described by it in a given time: what Astronomers seek to compute, and what they with difficulty compute, are the perihelion distance; its place, or longitude; the epoch of that longitude; the longitude of the ascending node, and the inclination of the orbit.

The elements of the orbits of planets are capable of being determined by observations made on the meridian: by longitudes and latitudes computed from right ascensions and declinations. But, Comets require observations of a different kind: by the rotation of the Earth they are brought on the meridian, but, (from their proximity to the Sun whilst they are visible,) not during the night, when alone the faintness of their light does not prevent them from being discerned. They must therefore be observed out of the meridian; and, in that position, the differences between their right ascensions and declinations and those of a known contiguous Star must be determined.

It is difficult to make these latter observations with accuracy by reason of the doubtful and ill-defined disk of the Comet; and a small error in the observations, will materially affect the elements of the orbit.

The rigorous solution of the problem of the elements of a Comet's orbit requires three observations only. But then, the solution is attended with so many difficulties, that in this, as in other like cases, Astronomers have sought, by the indirect methods of trial and conjecture, to avoid them. If, what always happens, more than three observations are obtained, the redundant ones are employed in correcting and confirming previous results.

The periodic time, as we have observed, cannot be determined from observations during one appearance of a Comet. If known, it can only be so, by recognising the Comet during its second appearance. And the only mode of recognising a Comet, is by the identity of the elements of its orbit with those of the orbit of a Comet already observed. If the perihelion distance, the position of the perihelion and of the nodes, the inclination of the orbit, are the same or nearly so, we may presume, with considerable probability, that the Comet we are observing, has been previously in the vicinity of the Sun; and that, after moving round by the aphelion of its oval orbit, it has again returned towards its perihelion distances.

Comets not having been formerly observed with great accuracy, it so happens, that the period of one alone, that of the Comet observed in 1682, 1607, and 1531, is known to any degree of certainty. Its period is presumed to be about 76 years. Assuming the Earth's mean distance to be unity, the perihelion distance of the Comet was 0.58, and the major axis of the orbit 35.9. The inequalities which are noted in its period are supposed to arise from the influence of some disturbing forces *.

The chief business of the present Treatise, hitherto, has been with calculations founded on observations made on the meridian. But, there are many important processes dependent on angular distances observed *out* of the meridian : such, for instance, as those for ascertaining the latitude and longitude of a ship at sea. The nature of the observations, in these cases, require a peculiar instrument; which, besides being adapted to the measuring of angular distances out of the meridian, may be held in the hand of the observer, and used by him, even when made unsteady by the motion of the vessel. The description and use of such an instrument will be explained in the ensuing Chapter.

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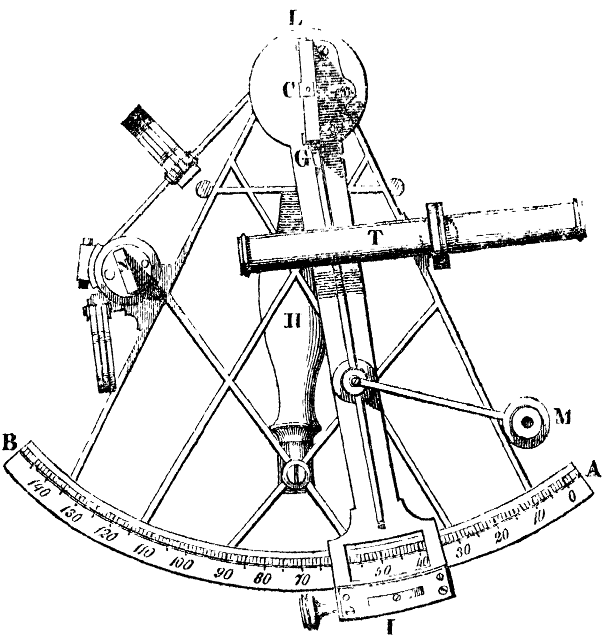
* On the subject of Comets, see Laplace, *Mec. Celeste*, Liv. II, p. 20, &c. Biot, tom. III, Add. p. 186, Englefield : Cagnoli, p. 429, Newton, *Arith. Univ*. Sect. 4, Chap. II, Prob. xxx.

CHAP. XL.

On Hadley's Quadrant and the Sextant.

THE larger figure is intended to represent a *Sextant*, as it is usually fitted up, with its handle *H*, the telescope *T*, the micro-



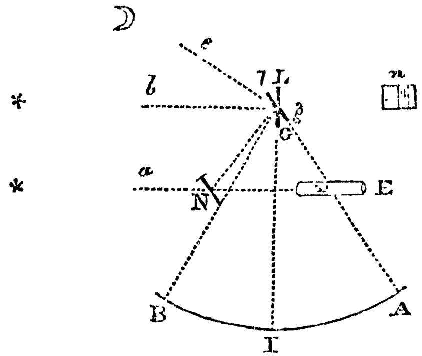
scope *M* moveable about a center, and capable of being adjusted so as to *read off* the divisions on the graduated limb *AB*. The less Figure is intended as a sketch of the larger and for the purpose of explaining its properties.

LCG and *N* (in the large Figure) must be supposed to represent the edges of two plane reflectors; the planes of which are perpendicular to the plane of the instrument in which the graduated limb and the connecting bars lie. The upper part of the reflector *N*, which is fixed, and called *the Horizon glass*, is transparent and free from quicksilver, as in *n* (in the small Figure) which is represented as *N* appears when viewed through the tube of the telescope *T*; the other reflector *LCG* (*the index glass*) is attached to the limb and index *I*, and with them moveable round a center placed near *C*. Now, the instrument is so constructed that, when the index *I* is moved up to *A*, and points to *o* on the graduated arc, the planes of the two reflectors *LCG* and *N* are exactly parallel to each other. In the small Figure, *lg* represents this position of *LG*.

In this position of the index *I* and the reflector *LG*, if the eye at *E* (small Figure) look through the upper part of the horizon glass at *N*, and perceive a distant object such as a Star (✱), it will also perceive the image of the same Star reflected from the under and silvered part of *N*. For, by hypothesis, the reflectors are parallel: and since the Star is extremely distant, two rays from it (*a N*, *b g*) falling on *N* and *LG* must be parallel; therefore the latter ray, after two reflections, the first at *LG*, the second at *N*, must proceed towards the eye in the direction of *a N* produced.

Suppose now, the eye still looking through the telescope at the same object (the ✱), the index *I*, the limb *GI*, and with them the reflector *LCG*, to be moved from *A* towards *B* (*LGI* is their position in the small Figure); then, the Star ✱ can no longer be seen by two reflexions, but some other object such as the ☽ may: and if so, two objects, the ✱ and ☽, would be seen nearly in contact; the former in the upper part of the horizon glass *N*, the latter on the lower silvered part.

In consequence then of this translation of the index *I* from *A* where it was opposite *o*, to another position between *A* and *B*; two objects (✱ and ☽) inclined to each other at a certain angle (*b g c* in small Figure) are brought into contact. If, therefore, the arc moved through (*AI* in the small Figure) bore any relation to the angular distance of the two objects, and we could ascertain such relation, we should then be able by measuring *AI*, or by reading off its graduations, to determine the angular distances of the two observed objects. This relation we will proceed to investigate.



In the first position (LG) both the direct and the reflected rays from ✳ are seen in the direction of the telescope (T); the direct ray from ✳ is always seen in the same direction: but, in a new position, the reflected ray (in order that ☽ may be seen) must also be seen in that direction; therefore, the ray must come from the under part of N in the same direction: and therefore, since N is fixed, the ray must always be incident on N in the same direction, and consequently be *always reflected* from LCG in the same direction. What we have to determine then, is reduced to this. *To find the inclination of two incident rays, such, that the position of the reflector (from* LG *to* lg *for instance,) being changed, each shall be reflected into the same direction.*

Let the first incident ray (and consequently the reflected ray) be inclined to the reflector at an angle $= A$: let the reflector be moved through an angle $= \theta$, and towards the reflected ray: (for instance, from the position $g\,l$ to GL in small Figure), then the angle between the reflected ray and the plane in its new position $= A - \theta$ between the first incident ray and the plane - - $= A + \theta$. But, by the laws of reflection, the second incident ray must form with the reflector, an angle equal to that which the reflected ray does; an angle, therefore, $= A - \theta$. Now, the difference between the angles which the incident rays form with the same position of the plane, is no other than the inclination of the incident rays, equal therefore,

$$(A + \theta) - (A - \theta), \text{ or, } 2\,\theta.$$

This is the important principle in the construction of the instrument. For, suppose the arc *AB* to be one sixth part of a circle, and the index *I*, when the two objects are seen in contact, be one third of the way between *AB*; then, the inclination of the two reflectors (for the reflector *N* is always parallel the first position *lg*) would be one third of one sixth of 360° or 20°: and accordingly the angular distances of the two objects would be 40°. Instead of dividing *AB* into a number of degrees proportional to its magnitude (60° for instance, if $AB = \frac{1}{6}$th circumference), it is usual to divide it into *twice* that number; then, the number of degrees, minutes, &c. intercepted between *o* and the index will at once determine the angular distance of the two objects.

The objects must be brought into contact: in the case of a Star and the Moon, the former must be made just to touch the limb of the latter: in the case of the Sun and Moon, their two limbs must be made to touch.

For the sake of illustration, we have supposed the two objects to be a Star and the Moon: and in practice, those are frequently the observed bodies: but, the instrument is capable of measuring the angular distance of any two objects and lying in any plane: the Sun and Moon, for instance, and in such cases there are certain darkened glasses, near *N*, and between *N* and *L* (see Fig.) contrived for the purpose of reducing the Sun's light to that of the Moon's, or the Moon's to that of a Star's.

The upper and lowest points in the disks of the Sun, or of the Moon, may be considered as two objects; therefore, their distances which are the diameters of the Sun and the Moon, may be measured by the described instrument. Instead of the points in the direction of a vertical circle, we may observe two opposite points in an horizontal direction; and accordingly, we can measure the horizontal diameters of the Sun and Moon.

If we make a Star, or the upper or the lower limb of the Sun or Moon, one object, and the point in the horizon directly beneath the other, we can measure their angular distance, which, in these cases, is either the *altitude* of the Star, or the altitude either of the upper or the lower limb of the Sun and Moon. In this observation, the horizon is viewed through the upper part of the reflector *N*, which is the reason why that is called the *horizon*-glass. At sea, where the horizon is usually defined with sufficient accuracy, the altitude of the Sun or of a Star may be taken, by the above method; but at land, the inequalities of the Earth's surface oblige us to have recourse to a new expedient, in the contrivance of what is called a a *False Horizon.* This, in its simplest state, is a bason either of water, or of quicksilver: to the image of the Sun or other object seen therein we must direct the telescope *T*, and view it directly through the upper part of *N*, and then move, backwards, or forwards the limb and index, till by the double reflexion, the upper or the under limb of the reflected Sun is brought into contact, or exactly made to touch the under or the upper limb of the image of the Sun seen in the *False Horizon.* The angle shewn by the instrument is double either of the altitude of the Sun's upper or under limb: subtract or add the Sun's diameter, divide by two, and the result is the altitude of the Sun's center: all other proper corrections, instrumental as well as theoretical, being supposed to be made.

It is evident from the preceding description, that the plane of the instrument must be held in the plane of the two bodies whose angular distance is required: in a vertical plane, therefore, when altitudes are measured; in an horizontal, when, for instance, the horizontal diameters of the Sun and Moon are to be taken. In the management of the instrument, this adjustment of its plane, or the holding it in the plane of the two bodies, is the most difficult part.

The instrument is to be held by the handle *H*, and generally is, in the left hand of the observer: his right being employed in moving and adjusting the index, its connected limb, and the reflector *LCG*. Its great and eminent advantage is, that it does not require to be fixed, nor that the observer using it should himself be steady. It is the chief instrument in Nautical Astronomy: since by its means alone, the position of a vessel at sea may be determined.

The instrument represented and described in this Chapter is, the *sextant*: which is an improvement on the *quadrant*, called, from its inventor, *Hadley's Quadrant*. Besides these, on the same principle, but of better contrivance, there is the *reflecting circle* *:

* Invented by Mr. Troughton: for a description of it, see Rees' *Encyclopedia*, new edit. Art. *Circle*.

also, Borda's *reflecting repeating* circle, on the principle of Mayer's. (See *Mem. Gottingen*, tom. II, also *Tabulæ Motuum*, &c. 1770).

We subjoin two instances:

Angular Distance of the Sun's Center, and of the Horizon (at Sea,) or (see p. 392.) Altitude of the Sun's Center.

Alt. ☉'s lower limb	49°	10′	0″
[a] ☉'s semi-diameter	0	15	51
	49	25	51
* Refrac. (Chap. XI.)	0	0	43
true alt. ☉'s center	49	25	8

Distance of Eastern and West. limbs, or ☉'s horizon'. semi-diam.	31	42″
[a] ☉'s semi-diameter	15	51

Altitude of the ☉'s Center, by means of the false Horizon, (see p 393,)

By inst. ☉'s upper limb - - - - - - - -	100°	2′	47″
Apparent altitude - - - - - - - - - - -	50	1	23.5
[b] ☉'s semi-diameter - - - - - - - -	0	15	50
	49	45	33.5
Refraction - - - - - - - - - - - - - -	0	0	43
True alt. ☉'s center - - - - - - - - -	49	44	50.5

☉'s horizontal diameter - - - - - - - - -	31′	40″
[b] ☉'s semi-diameter - - - - - - - - - -	15	50

* The Nautical Tables of Refraction include within their results the correction for the Sun's parallax.

CHAP. XLI.

On the Mode of computing Time and the Hour of the Day; by the Sun; by the Transit of Stars; by equal Altitudes; by the Altitude of the Sun or of a Star.

We will begin with those methods which depend on observations made on the meridian.

Transit of the Sun over the Meridian.

When the Sun's center is on the meridian, it is true or apparent noon. It is determined to be there, by means of a transit instrument, (see Chap. XV.) With this, observing the contacts of the Sun's western and eastern limbs with the middle vertical wire (see p. 48,) note, by means of the clock, the interval of time, and half that interval added to the time of the contact of the western, or subtracted from that of the eastern, will give the time at which the Sun's center is on the meridian. For greater accuracy, the times of contact of the Sun's limbs with the vertical wires to the right and left of the middle one may be noted.

The time thus determined is *apparent* noon; in order to deduce the *mean* time, apply the *equation of time*. (see Chap. XXII.) For instance, the *equation* on Nov. 8, 1808, is stated in the Nautical Almanack to be $-$ 16^m $3^s.7$, therefore when the Sun's center on that day was on the meridian, the mean solar time was $12^h - 16^m\ 3^s.7$, or $11^h\ 43^m\ 56^s.3$; 12^h being supposed to denote the time when the center of the mean Sun is on the meridian.

Transit of a fixed Star; of the Moon; of a Planet over the Meridian.

The mean Sun leaves a meridian and returns to the same in 24^h, describing $360° \ 59' \ 8''.3$; $59' \ 8''.3$ being the increase of his mean right ascension in that time. Since the mean Sun, by its definition, moves equably, the time from mean noon must be always proportional to the Sun's distance from the meridian. If a star then, were on the meridian, the time would be proportional to the Sun's angular distance from the star; it would be proportional therefore, to the difference of the right ascensions of the star and the Sun, at the time when the star is on the meridian.

The Sun's right ascension in the Nautical Almanack is expressed solely for noon, that is, when his center was on the meridian of Greenwich; and since that right ascension is continually increasing, it will be greater when the star comes on the meridian, and the Sun is more to the west, than it was at noon. In the interval between the transits of the Sun and Star, the former will have moved to the east, and towards the latter, by an increase of right ascension proportional to the interval. The angular distance therefore of the Star and Sun, or the difference of their right ascensions, when the former is on the meridian, is

$$\ast\text{'s R. A.} - \odot\text{'s R. A. } \textit{at preceding noon} - \text{increase of } \odot\text{'s R. A.}$$

and, to this angular distance is the time proportional.

The time from noon is nearly proportional to the $\ast$'s right ascension − $\odot$'s right ascension at noon; therefore the increase of $\odot$'s right ascension is nearly proportional to that angle. If a therefore denote the increase of the Sun's right ascension in 24^h we have, the time =

$$\ast\text{'s R. A.} - \odot\text{'s R. A.} - \frac{D}{24} \times a.$$

$$[\text{making } D = \ast\text{'s R. A.} - \odot\text{'s R. A.}]$$

EXAMPLE.

A Star in Capricorn *whose R. A.* = $20^h\ 30^m\ 7^s$ *was on the Meridian at Greenwich, Nov. 8, 1808. Required the time.*

✱'s R. A. - - - - - - - - - - -	20^h	30^m	7^s
By Naut. Alm. ☉'s R. A. (noon of Nov. 8.)	14	53	52 *
✱'s R. A. — ☉'s R. A. (*D*) - - - - - - - -	5	36	15
☉'s R. A. Nov. 9. - - - - - -	14	57	53.5
8. - - - - - -	14	53	52
$a =$ - - - -		4	1.5

* The Sun's right ascension is expressed in time, the Moon's in degrees, and to be expressed in the hours, minutes, &c. of *sidereal time*, must be converted into such at the rate of 15° for 1^h; for $\frac{24}{360} = \frac{1}{15}$.

For facilitating this operation and its reverse, appropriate Tables are provided; but, it may be, nearly with as much ease, effected by dividing and multiplying by 4. Thus, to convert $7^h\ 21^m\ 56^s.21 = 7^h\ 21^m\ 56^s\ 12'''$ into degrees, &c. begin with the minutes, and take the fourth of them, then, of the seconds, &c. reckoning the minutes of the quotient as degrees, the seconds as minutes, &c. thus:

	4) 21^m	56^s	$12'''$
	5°	29′	3″
But 7^h =	105		
	110	29	3

For the reverse operation, multiply by 4, reckoning the seconds of the product as thirds, the minutes as seconds, &c.

Thus - -	36°	8′	34″	30‴		- [36° = 30 + 6 = 2^h + 6°]
				4		
	2^h	24^m	34^s	18‴	0	

or dividing 18‴ by 6 to reduce it to a decimal, the product is $2^h\ 24^m\ 34^s.3$.

The reasons of the two operations are these; in the first we ought to multiply by 15, or, which is the same thing, by $\frac{60}{4}$; therefore we may divide by 4 and dispense with the multiplication by 60, by merely *raising the denomination* of the quotient; for $60 \times 1'' = 1'$. In the second case, we ought to *divide* by 15, or which is the same thing,

∴ the apparent time =

$$5^h\ 36^m\ 15^s - \frac{5^h\ 36^m\ 15^s}{24^h} \times 4^m\ 1^s.5 = 5^h\ 35^m\ 19^s.3,$$

and the mean time =

$5^h\ 35^m\ 19^s.3 - 16^m\ 2^s$ (the equation of time) $= 5^h\ 19^m\ 17^s.3$

Since the increase of the Sun's mean R. A. is 59′ 8″.3 in 24 hours, a meridian of the Earth describes, in that time, 360° 59′ 8″.3; therefore it describes 360° in, $24^h \times \frac{360°}{360°\ 59'\ 8''.3}$, or in $23^h\ 56^m\ 4''.09$; this is the time of the Earth's rotation, or the length of a sidereal day, expressed in mean solar time. If the Sun, therefore, and a Star were together on the meridian on a certain day, on the succeeding one, the Star would return sooner, or more quickly, to the meridian by $3^m\ 55^s.9$ of mean solar time; on this account, stars are said to be *accelerated*. The *acceleration* on mean solar time therefore, when the Star and Sun are distant by 360°, or by 24^h *of sidereal time*, is $3^m\ 55^s.909$; when distant by 180°, or by 12^h *of sidereal time*, it is $1^m\ 57^s.955$; when distant by 60°, or 4^h, it is $39^s.388$, and generally the *acceleration* is

$$\frac{\text{✱'s R. A.} - \text{☉'s R. A.}}{24^h} \times 3^m\ 55^s.909.$$

This is only another mode of expressing the rule given in p. 396; instead of the increase of the Sun's mean R. A., in 24 hours of mean solar time, we took then the real increase between two apparent noons.

There are Tables constructed for the *acceleration of stars on mean solar time*, which render the computation of the hour, by means of the transit of a fixed star, very easy; the rule is, the time =

✱'s R. A. − ☉'s R. A. − acceleration.

Thus, in the former instance,

		h	m	s
	✱'s R. A. - - - - - -	20	30	7″
Nov. 8.	☉'s mean R. A. - - -	15	9	57.3
		5	20	9.7
	Acceleration - - - -			52.3
	Mean time - - - - -	5	19	17.4

---

thing, we may *multiply* by $\frac{1}{15}$ or $\frac{4}{60}$; therefore, we may multiply solely by 4, and dispense with the division by 60 by merely *lowering the denomination* of the product; for $\frac{1'}{60} = 1''$.

The right ascensions of the Sun and of the stars, are always expressed in sidereal time; and, care must be taken to distinguish the hours, minutes, &c. of that time, from the hours, minutes, &c. of mean solar time. If we subtract, from an angle expressed in the symbols of sidereal time, the *acceleration*, the remainder is expressed in mean solar time. Thus,

A star is to the east of the meridian 30° 30′, or	2^h	2′	0″
The acceleration, or the Sun's motion in 2^h 2′ -			19.99
	2	1	40.01

therefore in 2^h 1^m $40^s.01$ of mean solar time, the star will be on the meridian.

The time is proportional to a *less* angle than the difference of the right ascensions of the Star and the Sun; or, stars are *accelerated*, because the Sun, in the interval between his transit and that of the Star, moves towards the latter. In the case of the Moon then, the time must be proportional to a *greater* angle than the difference of the Sun's right ascension on the preceding noon, and the Moon's; or, the Moon must be *retarded*; because, in the interval between the transit of the Sun and that of the Moon, the latter by her greater motion in right ascension, has increased her angular distance from the former. It would be easy, as in the former case, to compute the hour from the Moon's transit over the meridian, (or what is the same thing, to find the hour of the Moon's transit), but instead of it, we will give a formula applicable to all cases:

Let the increment of ☉'s R. A. in 24^h be - - - a
of a ✱, or of the ☽, or of a planet - - - - A

Let also the difference between the R. A. of the heavenly body and that of the Sun at *the preceding noon*, expressed in sidereal time, be } - - t

then, if $a = A$, the hour of transit will be proportional to t
if $a > A$, - - - - - - to some less angle - $t - \tau$
if $a <$ - - - - - - - to some greater - $t + \tau$

Hence in the first case, which can only happen with a planet, the time of transit is proportional to t; that is, if the Sun's right ascension when on the meridian be 30° 30′, or 2^h 2^m less than that of the planet, the latter will be on the meridian at 2^h 2^m of solar time.

In the second case, $a > A$

$$24 : a - A :: t - \tau : \tau ; \therefore \tau = t \times \frac{a - A}{24 + a - A}$$

In the third case $a < A$

$$24 : A - a :: t + \tau : \tau ; \therefore \tau = t \times \frac{A - a}{24 + a - A}$$

Hence, in the second case, the time of transit $= t - t \times \dfrac{a - A}{24 + a - A}$

in the third, $t + t \times \dfrac{A - a}{24 + a - A}$, or, $t - t \times \dfrac{a - A}{24 + a - A}$

therefore, in both cases,

the time of transit $= t \left[1 - \dfrac{a - A}{24 + a - A}\right]$

(expanding) $= t \left[1 - \dfrac{a - A}{24} + \left(\dfrac{a - A}{24}\right)^2 - \left(\dfrac{a - A}{24}\right)^3 + \text{\&c.}\right]$

Hence in the case of a fixed star, when $A = 0$, the time of ✳'s transit $= t - \dfrac{at}{24} + \left(\dfrac{a}{24}\right)^2 t - \text{\&c.}$

in which the two first terms (which are sufficient) give the rule of computation used in p. 396, l. 28.

In the case of the Moon, a is $< A$; therefore all the terms are additive, and

the time of ☽'s transit $= t + \dfrac{A - a}{24} t + \left(\dfrac{A - a}{24}\right)^2 t + \text{\&c.}$

In the case of a planet, a may be less or greater than A; if equal, then the time of transit $= t$, as before, p. 399, l. 34.

There is one case which has not been mentioned, that in which a planet is *retrograde* (see Chap. XXIII). In this case, the approach of the Sun and planet is greater than that of the Sun and a star, and the same, as if, instead of the Sun having a motion in R. A. equal to a, we suppose him endowed with a motion equal to $a + A$; substituting therefore in form, p. 400, l. 9, $a + A$ instead of a

time of the planet's transit $= t - \dfrac{a + A}{24} . t + \left(\dfrac{a + A}{24}\right)^2 . t - \text{\&c.}$

When the planet is *stationary*, its hour of passage is evidently the same as that of a fixed star which has the same right ascension.

EXAMPLE.

Let it be required to find the time of the Moon's passing the Meridian of Greenwich, June 13, 1791.

June 14, ☽'s R.A. .	15^h	43^m	32^s	☉'s R.A. .	5^h	30^m	38^s	
13, ditto	14	42	32	ditto	5	26	29.1	
	1	1	0 = A			4	8.9 = a	
June 13, ☽'s R.A. .	14	42	32	A . . .	1	1	0	
☉'s R.A. .	5	26	29.1	a . . .	0	4	8 9	
	9	16	2.9 = t			56	51.1	$A-a$

∴ t	9^h	16^m	$2^s.9$ =	approx. time of ☽'s transit
$t.\frac{A-a}{24}$, or $\frac{9^h\ 16^m\ 2^s.9}{24} \times 56^m\ 51^s.1$		21	57	
	9	37	59.9 =	more correct time.
$t\left(\frac{A-a}{24}\right)^2$	0	0	49.8	
	9	38	49.7 =	still more corr^t. time.

This last result (in apparent time) is sufficiently exact for Astronomical purposes.

The second additional term $21^m\ 54^s.7 = \frac{9^h\ 16^m\ 29^s}{24^h} \times 56^m\ 51^s.1$, is evidently the *proportional part* * of $56^m\ 51^s.1$, corresponding to $9^h\ 16^m\ 29^s$; the third additional term, $49^s.8$, =

$$\left(\frac{A-a}{24}\right)^2.t = \frac{A-a}{24} \times \frac{A-a}{24}.t = \frac{21^m\ 54^s.7}{24^h} \times A-a$$

$= \frac{21^m\ 54^s 7}{24^h} \times 56^m\ 51^s.1$ is evidently the *proportional part* of

* Tables are computed for facilitating these operations.

56^m $51^s.1$, corresponding to the time 21^m $54^s.7$. This is the explanation of the rule, as it is sometimes given by Astronomers, which directs us to find a first, and a second proportional, and to add them to the approximate time of the Moon's transit, in order to find a more correct time. (See Nautical Almanack, 1811, pp. 154, 155. Also Wollaston's *Fasciculus*, Appendix, p. 76)

The hour, or the mean solar time, may be determined or computed from the transit of a fixed star; and, much more exactly, than from the transit of the Moon or of a planet. With regard therefore to these two latter, the object of the preceding methods is to determine from Astronomical Tables, the times of their transits, or passages over the meridian, rather than the hour of the day from the transits.

Time determined by the Sidereal Clock.

If we can determine the time from the transit of a fixed star, it is an immediate inference that we can determine it from the sidereal clock. For, the clock is regulated by the observed transits of stars, and when regulated, we may suppose it always to indicate the right ascension of some imaginary star: Thus,

July 1, 1790, time by sidereal clock - - - -	13^h	20^m	15^s
☉'s mean longitude (by Tables) - - - -	6	54	35.86
	6	25	39.14
Acceleration (Maskelyne, Tab. XXI.) - - -		1	3.1
Mean solar time - - - -	6	24	36.04

The preceding computations of transits, &c. have been made for Greenwich, for which place our Astronomical Tables, and the Nautical Almanack are constructed. For any other place, we must account for the difference of longitude. Thus, to find, on July 9, 1808, the Sun's R. A. at noon, at a place 35^o (2^h 20^m) east of Greenwich, we have only to find the Sun's R. A. 2^h 20^m previous to noon time at Greenwich: which is easily done by subtracting from the R. A. at noon the proportional increase of R. A. in 2^h 20^m: thus,

* The *Acceleration* is the Sun's mean motion in R. A., and by this latter title it is called by Maskelyne in the Table referred to. See Wollaston's *Fasciculus*, Appendix, p. 69.

July 10, - - - ☉'s R. A. - - - - - - - -	7^h 17^m $48^s.5$	
9, - - - ditto - - - - - - - - - - - -	7 13 43.2	
Increase of R. A. in 24^h - - - - - -	4 5.3	
Proportional increase in 2^h 20^m = -	0 33	

∴ Sun's R. A., at noon, at the required place, $=7^h\ 17^m\ 15^s.5$.

A similar method must be used to find the Moon's right ascension, or longitude, &c. at noon, at any given place, with this difference, however, that the change of R. A. will not be simply proportional to the time, but must be computed more exactly by the differential method and series $\left(a + xd' + x.\frac{x-1}{2}d'' + \&c.\right)$ See *Trigonometry*, p. 193.

We now proceed to the methods of determining the time, by observations made out of the meridian.

Method of equal Altitudes.

By the instrument described in pp. 66. &c., or by the Sextant in Chap. XXIX, take an altitude of the Sun before noon, and wait till the sun descends to an equal altitude in the afternoon; then, half the interval of time elapsed is nearly the time of noon, from the first or last observation.

If the Sun did not alter his declination, the time of noon would be determined exactly by this method: it must therefore be determined very nearly at the solstices, and least exactly, at the equinoxes. It is plain the change of declination must affect the accuracy of the method; for suppose the north declination to be increasing, then the Sun, after passing the meridian, will be longer in descending to the proper altitude in the west, than it was in rising to the meridian from the equal altitude in the east: half the interval therefore would throw the meridian too much to the west. Since, however, the method is a good practical one, it has been made exact by means of corrections computed from a formula which we will now investigate.

In a triangle ZPS, where Z is the zenith, P the pole, S the Sun, the angle ZPS measures the time $\left(\frac{t}{2}\right)$ from noon, and by *Trigonometry*, p. 100,

$$\cos. \frac{t}{2} \times \sin. ZP \times \sin. PS = \cos. ZS - \cos. ZP \times \cos. PS;$$

Now, $\frac{t}{2}$ being the exact time from noon, if PS remain constant, we have to ascertain the variation in $\frac{t}{2}$, from the variation in PS: for that purpose, it will be sufficient to deduce the proportion between the *differentials* or *fluxions* of these quantities; accordingly, taking the differential of the above equation,

$$-\frac{dt}{2} . \sin. \frac{t}{2} . \sin. ZP \sin. PS + d(PS) \cos. PS \cos \frac{t}{2} . \sin. ZP =$$
$$d . (PS) . \sin. PS \cos. ZP,$$

or putting $\frac{dt}{2} = \epsilon, d(PS) = \delta$, and reducing,

$$\epsilon = \delta \left(\tan. \text{decl}^{n}. \times \cot. \frac{t}{2} - \tan. \text{lat.} \times \text{cosec.} \frac{t}{2}\right).$$

Time determined from an observed Altitude of the Sun.

The altitude of the Sun is to be observed and corrected as it was in page 394; then, we have to find the angle ZPS (h), from ZS ($90^\circ - A$) thus determined, from the Sun's north polar distance (p) given by the Tables, and from the latitude (L) of the place, known or previously determined by observation. Then, by *Trig.* pp. 99. 118. $\cos. ZPS$ ($\cos. h$)

$$= \frac{\cos. ZS - \cos. ZP \times \cos. PS}{\sin. ZP. \sin. PS} = \frac{\sin. A - \sin. L. \cos. p}{\cos. L. \sin. p};$$

$$\therefore 2 . \sin.^2 \frac{h}{2} = 1 - \cos. h = \frac{\cos L. \sin. p + \sin L \cos. p - \sin. A}{\cos. L. \sin. p.}$$

$$= \frac{\sin. (p+L) - \sin. A}{\cos. L. \sin. p}$$

(*Trig.* p. 19.) $$= \frac{2}{\cos. L. \sin. p} [\cos. \tfrac{1}{2}(p+L+A) \sin. \tfrac{1}{2}(p+L-A)]$$

and in logarithms, $2 \log. \sin. \frac{h}{2} = 20 +$

$$\log \cos. \tfrac{1}{2}(p+L+A) + \log. \sin. \tfrac{1}{2}(p+L-A) - \log. \cos. L - \log. \sin. p.$$

EXAMPLE.

The Sun's Altitude being 39° 5′ 28″; *his North Polar Distance, from Nautical Almanack,* 74° 51′ 50″, *and the Latitude of Place,* 52° 12′ 42″; *it is required to deduce the Time.*

$L=$	52°	12′	42″	---- cos. =	9.7872806	
$p=$	74	51	50	---- sin. =	9.9846660	
$A=$	39	5	28		19.7719466	[a]
sum	166	10	20			
½ sum	83	5		----- cos. =	9.0807189	
½ sum $-A$	43	59	32	-- sin. =	9.8417102	
					38.9224291	
				[a]	19.7719466	
				2)	19.1504825	

$$\text{log. sin. } \frac{h}{2} = 9.5752412 = \text{log. sin. } 22^{\circ}\ 5'\ 20''\tfrac{1}{3}$$

$$\therefore h = 44^{\circ}\ 10'\ 40''\tfrac{2}{3} = \text{(in time,) } 2^{h}\ 56^{m}\ 43^{s} \text{ nearly.}$$

This is the time for Greenwich; for any other place, we must correct p, taken from the Nautical Almanack, by adding to it, or subtracting from it, the change in the Sun's north polar distance, proportional to the difference of longitude between Greenwich, and the place of the observed altitude. The process is like that given in p. 403,

Time determined from an observed Altitude of a fixed Star.

The altitude is to be observed as in the former instance: the latitude is supposed to be known from previous observation, and, the Star's north polar distance from his *mean* north polar distance (contained in Tables) corrected for the several inequalities of precession, aberration, and nutation, (see Chapters XIII, XIV, &c.) Then, the computation of the angle ZPS, or of h, will be exactly the same as in the preceding case. That angle will be the Star's angular distance from the meridian; therefore, since the Star's right ascension is known, the right ascension of a point of an imaginary Star, then on the meridian, is known. But, the right ascension of a Star on the meridian being known, the hour of the day is (see p. 397).

All stars on the meridian at the same time have the same right ascension; therefore, we may place the imaginary Star on the equator, and then (see p. 363.) its right ascension will be that of the *Mid-Heaven;* consequently we may give the rule for finding the time under the following form:

$*$'s R. A. $\pm h$ = R. A. of mid-heaven,

R. A. of mid-heaven — ☉'s R. A. — *acceleration* = time (see p. 398.)

EXAMPLE.

April 14, 1780. *In Latitude* 48° 56′, *Longitude* W = 66° (4^h 24^m) *the Altitude of* Aldebaran *in the West, was observed* = 22° 20′ 8″. *Required the Time.*

L = 48° 56′ 0″	- - - - - - -	cos.	=	9.8175235
p = 73 56 59	- - - - - - -	sin.		9.9827322
A = 22 17 50	(refrac. = 2′ 18″)			19.8002557
2)145 10 49				20
$\frac{1}{2}$ sum = 72 35 24	- - - - - - -	cos.		9.4759722
$\frac{1}{2}$ sum — A = 50 17 34	- - - - - - -	sin.		9.8861065
				39.3620787
				19.8002557
				2)19.5618230
		log. sin. $\frac{h}{2}$	=	9.7809115

[= l sin. 37° 8′ 39″.75 ;

∴ h = 74° 17′ 19″.5

$*$'s R. A. = 65 49 49 5 (by Tables)

$*$'s R. A. + h = 140 7 9 = R. A. of mid-heaven.

But, April 14, ☉'s R. A = 1^h 31^m 1^s

April 15 - - - = 1 34 42

Increase in 24^h - - - = 0 3 41 ∴ prop^l. inc^e. in 4^h 24^m = 40^s.

Hence, R. A. of mid-heaven (140° 7′ 9″) = 9^h 20^m 28^s.6

☉'s R. A. (= 1^h 31^m 1^s + 40^s) - - - = 1 31 41

7 48 47.6

Acceleration (see pp. 398. 402.) - - - - - - 0 1 16 8

∴ apparent time = 7 47 30.8

This method, as a practical one, is inferior to the former ; partly, by reason of the greater length of the computations, but chiefly from the difficulty of exactly ascertaining the altitude of a star by the sextant. The errors of the Solar Tables affect both methods. Those of observation, however, are lessened by the plan of observing several successive altitudes, at nearly equal small intervals of time, and by taking the mean of these as the true observed altitude. This operation, with the sextant, is attended with some difficulty, from the necessity of *reading off* the divisions at the end of each observation. But, with Borda's *reflecting Circle of Repetition*, there is no such necessity. Since with this, we may take successively ten altitudes (for instance,) and at the end, read off the sum of all. One tenth of these, if made at equal intervals, or if not, reduced to equal intervals, is to be assumed as the true altitude.

In an observatory, where the instruments are fixed in the plane of the meridian, the time of apparent noon is easily determined. It may be also ascertained by a sextant, which (see p. 392,) is adapted to measure altitudes: for when the Sun is in the meridian, it is also at its greatest altitude. But, the altitude of the Sun, when near the meridian, varying very little, it is difficult to ascertain by a sextant the precise time of the greatest altitude, and consequently, that of apparent noon. Out of the meridian, the variations of altitude are quicker: where they are most quick then, there an error in the altitude (and such there will always be in an observation with a sextant) must be of the least consequence, since it least affects the time; which would be truly comcomputed, by the preceding method, if the altitude were rightly observed.

Since the altitude changes most slowly, when the star is near the meridian, either towards the south or the north, it seems probable, that it would change most rapidly, half way between the north and south; and this is the case, as we shall prove in the solution of a problem, which is usually thus announced:

Given the Error in Altitude; it is required to find where the corresponding Error in Time will be the least.

By p. 404,

$$\cos. h = \frac{\sin. A - \sin. L.\cos p}{\cos. L.\sin. p},$$

Take the differential or fluxion of this equation, and put $dh = \epsilon$, $dA = \alpha$, then

$$-\epsilon \sin. h = \alpha \frac{\cos. A}{\cos. L . \sin. p},$$

but *Trig.* p. 102, $\sin. h \times \sin. p = \sin. PZS \times \cos. A$;

$$\therefore \epsilon = \frac{\alpha}{\sin. PZS \times \cos. L};$$

consequently, if L and α, the error in altitude, be given, ϵ is least, when sin. PZS is the greatest, that is, when $PZS = 90°$, or the azimuth, (see p. 28,) is 90°, or the body is on the *Prime Vertical*: which is that vertical circle which passes through the east and west points.

The above is the reason of the precept given by Dr Maskelyne at p. 152, Nautical Almanack, in which he directs the altitude to be observed near the west and east points. To this precept may be added another; that those stars should be selected for observation, which move most quickly; those, therefore, which are situated near the equator.

Besides the error of altitude, there may be an error in the assumed latitude. For between the observation which determines the latter from the Sun's meridian altitude, and the observation of the altitude, the observer, if on board a ship, may have changed his place, and, if so, has probably changed his latitude. The relation between its error and that of the time may be determined exactly as the relation between ϵ and δ was in p. 404. Instead of making PS to vary, we must make ZS, $(90 - L)$; let λ be the variation of L, then,

$$\epsilon = \lambda \left[\tan. \text{dec.} \times \text{cos. sec.} \frac{t}{2} - \tan. \text{lat.} \times \cot. \frac{t}{2}\right].$$

By means of an observed altitude, the time from noon is determined. But, this is not its sole use. The watch or chronometer is always adjusted and regulated by it. One observation will determine the real error, in time, of the chronometer; two, its rate, that is, whether it gains or loses: three or more will determine whether it gains or loses *equably*. With this mode of ascertaining, and allowing for, its degree of inaccuracy, the chronometer is relied on, during short intervals of time, for determining the distance of the ship from the last observations.

There are several methods and instruments used to ascertain, in the interval between observations, the situation of the ship. Dating from a latitude and longitude astronomically determined, navigators *carry* on a latitude and longitude by *account*. This they are enabled to do, by the chronometer, by the *Log* (by which instrument they ascertain the ship's velocity,) and by the *Magnetic Compass*, which shews the direction in which the ship is proceeding.

The *Needle* of the magnetic compass, is a thin bar of steel, moveable about a center, in a plane nearly horizontal, and in different parts of the Earth pointing to different parts of the horizon. In scarcely any place, is its direction true north and south. The *Magnetic North*, almost always, differs from the true. And the difference is, technically, called the *Variation* of the compass, differing in degree at different places, and not remaining the same at the same place. Navigators are provided with charts of this *Variation*; therefore, if they know, nearly, the situation of their ship, they know also nearly the *Variation*. And, since they are enabled, independently of the *Charts*, to ascertain the variation, they are also enabled to examine their accuracy, and to correct them if wrong.

The magnetic north is always known from the direction of the magnetic needle. The true north may be computed from the Sun's azimuth, at the time of his rising, or from his observed altitude at any other time. The azimuth is the angle PZS, the computation of which is exactly similar to that of the hour angle ZPS (h) in p. 404.

Let the declination and zenith distance of the Sun be d, z, then,

$$\cos. PZS = \frac{\cos. PS - \cos. ZP . \cos. ZS}{\sin. ZP . \sin. ZS} = \frac{\sin. d - \sin. L \cos. z}{\cos. L . \sin. z}$$

when the Sun rises, or is on the horizon, $z = 90°$;

$$\therefore \cos. z = 0, \quad \text{and} \quad \sin. z = 1,$$

and cos. PZS, or sin. *amplitude* * $= \frac{\sin. d}{\cos. L}$.

In other situations, deducing 2 log. sin. $\frac{PZS}{2}$, exactly as 2 log. sin. $\frac{h}{2}$ was, in p. 404, we have

$$2 \log. \sin. \text{azimuth} = 20 + \log. \cos. \tfrac{1}{2}(L + z + d) + \log. \sin. \tfrac{1}{2}(L + z - d) - \log. \cos. L - \log \sin. z.$$

Example to the First Method.

In Lat. 51° 52′ *N. the Sun's Declination being* 23° 28′ *N. Required the Amplitude, in the Morning.*

$d = 23°\ 28'$	- - - - - -	sin.	9.6001181
$L = 51\ \ 52$	- - - - - -	cos.	9.7906325
			9.8094856 = log. sin. 40° 9′ 26′

∴ the Sun's distance from the east point = 40° 9′ 26′′.

Example to the Second Method.

In Lat. 51° 32′, *the Sun's Declination being* 23° 28′, *and his Altitude corrected for Refraction* 46° 20′. *Required the Azimuth.*

$L = 51°\ 32'$	- - -	cos. =	9.7938317
$z = 43\ \ 40$	- - -	sin. =	9.8391396
$d = 23\ \ 28$			19.6329713 [a]
Sum = 118 40			20
½ Sum = 59 20	- - -	cos. =	9.7076064
½ Sum − d = 35 52	- - -	sin.	9.7678242
			39.4754306
		[a]	19.6329713
		2)	19.8424593
			9.9212296 = log. sin. 56° 31′ 28′′

* The amplitude is frequently appropriated to signify the complement of the azimuth, when the star *rises* or *sets*.

$\therefore$ the Sun's azimuth $= 56^\circ\ 31'\ 28''$.

The first of these methods is evidently the simplest for determining the point of the true north, since the mariner can easily ascertain the angular distance between the point in the horizon to which the magnetic needle points, and that in which the Sun rises.

In Nautical Astronomy, the determination of the time is an essential operation, that of the azimuth, an occasionally useful one. For this reason the methods of finding them have been inserted: but, for a like reason, other methods, and the solutions of many curious problems, but of doubtful utility, are excluded. One or two however, are subjoined, chiefly for the purpose of shewing the great extent of the application of that formula which expresses the cosine of a spherical angle, in terms of the sines and cosines of its sides. Thus, (see p. 404,) the time at any altitude of the Sun, is to be obtained from this expression,

$$\cos. h = \frac{\cos. z - \sin. L \cos. p}{\cos. L . \sin. p},$$

when the Sun rises or sets, $z = 90^\circ$; $\therefore \cos. z = 0$;

$$\therefore \cos. h = -\frac{\sin. L . \cos. p}{\cos. L . \sin. p}$$

$$= - \tan. L . \text{co-tan}. p,$$

the negative sign indicating (if p be < 90) that h is $> 90^\circ$.

Twilight is the light of the Sun, when below the horizon, faintly reflected by the atmosphere; and, by computation, it is found to be just sensible when the Sun is within 18° of the horizon; or, when $z = 118^\circ$; and we may find the time, therefore, of twilight's beginning or ending, by substituting in the preceding expression, or in that which is immediately deduced from it, (see p. 404,) instead of $A\ (= 90^\circ - z)$, $- 18^\circ$.

The *duration* of twilight, is the interval of time due to the Sun's ascending or descending through 18°, it is, therefore, equal to the difference of the last expression, and that, l. 20, which expresses the time of the Sun's rising or setting.

The boundary of twilight, a small circle parallel to the horizon, and 18° from it, is called the *Almacanter*.

The length of a day, in its common acceptation, is the interval of time between the rising and setting of the Sun; therefore, equal to twice the angle h, estimated from that expression of cos. h, in which $A = 0$, that is, it is equal to $2 . \tan. L . \text{co-tan}. p$.

$$\text{At the equinoxes, } p \text{ the } \odot\text{'s N.P.D.} = 90°;$$

$$\therefore \cot. p = 0; \therefore \cos. h = 0; \therefore h = 90° = (\text{in time})\ 6^h;$$

$$\therefore \text{the length of the day} = 12^h.$$

At the solstices, p, either, $= 90°—23°\ 28'$, or $90°+23°\ 28'$; therefore, the lengths of the longest and shortest day at Greenwich are to be computed from this expression,

$$\cos. h = \mp 2 \tan. 51°\ 28'\ 39''.5 \times \tan. 23°\ 28',$$

the upper sign — for the longest day, denoting h to be $> 90°$, and the lower sign + for the shortest, h being taken $< 90°$, and equal to the supplement of the former.

If we wish to investigate the latitude in which the Sun's center, in its greatest depression, just reaches, but does not descend below, the horizon, then h will equal 180°,

$$\text{and } \cos. 180° = -1 = -\tan. L . \cot. p = -\frac{\tan. L}{\tan. p};$$

$$\therefore \tan. L = \tan. p, \text{ and } L = p, \text{ or } = 90° - \text{declination},$$

or, the co-latitude of the place equals the Sun's declination.

In a similar way, and still using the expression for cos. h, we may express the relation between the latitude and the Sun's declination, when there is *just* twilight all night; thus, z being the zenith distance, since

$$\cos. h = \frac{\cos. z - \sin. L . \cos. p}{\cos. L . \sin. p},$$

$$\cos. 180° = \frac{\cos. 118° - \sin. L . \cos. p}{\cos. L . \sin. p};$$

$$\therefore \sin. L \cos. p—\cos. L \sin. p, \text{ or } \sin. (L—p), = \cos. 118° = -\sin. 18°;$$

$$\therefore L - p = -18°, \text{ or } L - (90° - \odot\text{'s dec.}) = -18°;$$

$\therefore$ ☉'s declination $= (90° - L) - 18°$.

If L therefore be given, search in the Nautical Almanack for that declination, which equals the difference of the co-latitude and 18°.

Since, $L = p - 18°$, and the least value of p, is 66° 32′; therefore the *least* value of L is 48° 32′; or in latitudes less than 48° 32′, there never can be twilight all night.

CHAP. XLII.

On Geographical Latitude.

LATITUDE of places at land, (see p. 7.)

1st. *Method by the Altitudes of Circumpolar Stars.*

This method has been already described in pp. 37, 38. Another instance of it is subjoined, in which, the circumpolar star is that particular one, which, for distinction, is called the *Pole Star*, (the *α Polaris* of Astronomical Catalogues, see p. 37).

By means of an Astronomical Circle, (see Chap. IV,) the following zenith distances (*Z.D*) were observed at Dublin Observatory on Aug. 23, 1808:

Greatest Z. D. - - - - - - - - - - - -	38°	18′	59″.1
Refraction (barom. 29. 97, thermom. 67,) -	0	0	44.01
Corrected Z. D. - - - - - - - -	38	19	43.11
Least Z. D. of *α Polaris* - - - - - - -	34°	53′	10″.1
Refraction, (barom. 29, 99, thermom. 58,) -	0	0	39.45
Corrected Z. D. - - - - - - - - - -	34	53	49.55
	38	19	43.11
	2)73	13	32.66
∴ co-latitude of Observatory, see p. 38, -	36	36	46.33

∴ latitude is 53° 23′ 13″.67.

2dly, *Method by the Zenith Distances of Stars near the Zenith.*

This method determines merely the difference of latitude by means of an instrument, (the zenith sector) capable of measuring small zenith distances with great exactness. We have had already (pp. 7. 117.) specimens of it, and we here subjoin another.

EXAMPLE.

By observation, at the College of Mazarin, (*Mem. Acad.* 1755.)

Z. D. of γ *Draconis reduced* (see p. 134.) to Jan. 1750,	2°	40′	15′
At Greenwich Z. D. reduced to the same epoch -	0	3	4.5
(The star is to the north of both zeniths) diff. lat. -	2	37	10.5
Hence, if the latitude of Greenwich be - - - -	51	28	39.5
Latitude * of observatory, at College of Mazarin -	48	51	29

It is essential, as it has been fully explained in pp. 134, 135. 170, that, for finding the difference of latitudes, by this operation, the zenith distances of the star observed at different epochs, should be reduced to the same. If, however, we should be possessed of two observations of the same star, made on the same day, of the same year, then, since the corrections of aberration, precession, and nutation, (see Chap. XIV, XV, XVI, XVII.) would be the same in each observation, it would be necessary merely to apply the corrections for refraction, before we subtracted or added (see p. 7.) the zenith distances.

This method of determining the latitude, and capable of great accuracy, was employed in the Trigonometrical Survey of England. See *Phil. Trans.* 1803, pp. 483, &c.

3dly, *Method by Observations of Altitudes made near the Meridian and reduced to the Meridian.*

This operation, as it is plain, cannot be made by the Astronomical Quadrant or Circle, if fixed in the plane of the meridian: but it may be, if they are endowed with an azimuth motion according to the contrivance described in p. 28.; still, however, inconveniently, from the necessity (see p. 407,) of *reading off* the altitude at the end of each observation. In Borda's *Circle of*

* See *Phil. Trans.* 1787, pp. 168, &c.

Repetition *, the necessity of these successive and intermediate *readings off* is superseded. The last result alone of the instrument is examined. If, after n observations, the *index*, having made m revolutions, points to d degrees, the mean of the observations is

$$\frac{m \times 360^\circ + d^\circ}{n}.$$

In a series of altitudes successively observed at very small intervals of time, the mean of them would, very nearly, be the mean altitude. For, the increment of the altitude would vary nearly as that of the time. But, near the meridian this law does not obtain. The decrements or increments (according as the passage of the Sun or star is a superior or an inferior one, that is, above or below the pole) vary in a higher ratio than that of the times from the passage of the meridian; and to the investigation of this higher ratio we must now turn our attention.

Taking, as usual, the triangle ZPS, and calling h, h', the horary angles, z, z', the zenith distances, d the declination, and L the latitude, we have, (see p. 404,)

$$\cos. h = \frac{\cos. z - \sin. L . \sin. d}{\cos. L . \cos. d}, \quad \cos. h' = \frac{\cos. z' - \sin. L \sin. d}{\cos. L . \cos. d};$$

$$\therefore \cos. h - \cos. h' = \frac{\cos. z - \cos. z'}{\cos. L . \cos. d};$$

and, (*Trig.* p. 19.) $\sin. \frac{1}{2}(h' + h) . \sin. \frac{1}{2}(h' - h) =$

$$\frac{1}{\cos. L . \cos. d}[\sin. \tfrac{1}{2}(z' + z) . \sin. \tfrac{1}{2}(z' - z)],$$

let $h = 0$, in other words, let its corresponding zenith distance z, be taken on the meridian; then,

$$\sin. \frac{h'}{2} . \sin. \frac{h'}{2} = \frac{1}{\cos. L . \cos. d}[\sin. \tfrac{1}{2}(z' + z) . \sin. \tfrac{1}{2}(z' - z)]$$

if $z' - z = \delta$,

$$\text{then, } \sin. \tfrac{1}{2}(z' - z) = \sin. \frac{\delta}{2} = \frac{\delta}{2}, \text{ nearly}, = \frac{\delta}{2} \times \frac{\sin. 1''}{1''},$$

and $\sin. \frac{1}{2}(z' + z) = \sin. z$, nearly; therefore,

$$\delta \text{ (the reduction to the meridian)} = \frac{2}{\sin. 1''} \times \sin.^2 \frac{h'}{2} \times \frac{\cos. d \cos. L}{\sin. z}$$

* The Circle of Repetition differs from the Astronomical Circle (p. 26.) in the principle of repetition; and, the *Reflecting* Circle of repetition, from the Sextant, or Troughton's Reflecting Circle, by the same principle.

Hence, if z', z'', &c. $z_{\prime}$, $z_{\prime\prime}$, &c.
be n zenith distances to the east and west of the meridian, and if
h', h'', &c. $h_{\prime}$, $h_{\prime\prime}$, &c.
δ', δ'', &c. $\delta_{\prime}$, $\delta_{\prime\prime}$, &c.
be the corresponding times and calculated corrections, the true or corrected meridional zenith distances will be (if the passage of the Star be above the pole,)

$$z' - \delta', \quad z'' - \delta'', \quad \&c. \quad z_{\prime} - \delta_{\prime}, \quad z_{\prime\prime} - \delta_{\prime\prime}, \quad \&c.$$

and, the true mean meridional zenith distance will be

$$= \frac{z' - \delta' + z'' - \delta'' + \&c. + z_{\prime} - \delta_{\prime} + z_{\prime\prime} - \delta_{\prime\prime} + \&c.}{n}$$

$$= \frac{z' + z'' + \&c. + z_{\prime} + z_{\prime\prime} + \&c.}{n} - \frac{\delta' + \delta'' + \&c. + \delta_{\prime} + \delta_{\prime\prime} + \&c.}{n}$$

and the computation, in instances, is made according to this last form.

In the formula, $\delta = \frac{2}{\sin. 1''} \times \sin.^2 \frac{h'}{2} \cdot \frac{\cos. d. \cos. L}{\sin. z}$, d is known from the Tables; h' must be determined by means of a time-keeper; for L, the approximate value of the latitude may be taken, and for z, the meridional zenith distance. The latitude L is to be determined to the greatest accuracy, by means of the correction δ, when computed; but, in computing δ, which will always be a very small quantity, we may, in assigning to L its value, in the factor $\frac{\cos. d. \cos. L}{\sin. z}$, introduce an error of several seconds, without any danger of vitiating the *practical* accuracy of the computation. Now, L may be determined within a few seconds, by taking half of the greatest and least zenith distances corrected for refraction. L, therefore, if not known by other means, may be taken of that approximate value so determined, and be employed in computing the correction δ.

For like reasons, in computing the several corrections δ', δ'', &c., the same value of z, that of the meridional zenith distance, may always be used: for the values of z', z'', &c. cannot differ much from it; since, the observations are separated from each other by a very small interval of time, generally, (though this circumstance must vary with the skill of the observer,) by about half a minute.

EXAMPLE, (*from* Biot's *Astronomy*.)

By observations, at Dunkirk, Dec. 19, 1802.

Half the sum of greatest and least Z. D. of *α Polaris*,	38° 57′ 55″
By Tables of the fixed Stars, N. P. D. of the ✱, -	1 42 18 .5
$z =$	37 15 36.5

$\therefore \log. \left(\frac{\cos. L. \cos d}{\sin. z}\right) = \log. \sin. 38° \ 57' \ 55'' + \log. \sin. 1° \ 42' \ 18''.5$ $- \log \sin. 37° \ 15' \ 36''.5 = 8.4900862$; which is the logarithm of the constant factor in the formula for δ. In the other part of the formula, the several values of $\frac{h'}{2}$ were determined by the clock: thus, the passage of the Star over the meridian being $0^h \ 24^m \ 44^s$: when the clock denoted $23^h \ 57^m \ 2^s$, the horary angle (h') was $0^h \ 24^m \ 44^s - 23^h \ 57^m \ 2^s = 27^m \ 42^s$; when $23^h \ 58^m \ 18^s$, the corresponding value of the horary angle was $0^h \ 24^m \ 44^s - 23^h \ 58^m \ 18^s = 26^m \ 26^s$, and so on. These values being substituted for h', h'', &c., the corresponding values of $\frac{2}{\sin. 1''} . \sin.^2 \frac{h'}{2}$ were computed, and arranged in a Table according to the subjoined specimen.

Times by the Clock.	Values of h'.	Values of $\frac{2}{\sin. 1''} . \sin.^2 \frac{h'}{2}$.
$23^h \ 57^m \ 2^s$	$27^m \ 42^s$	1504″.7
58 18	26 26	1370.4
59 6	25 38	1288.8
&c.	&c.	&c.

The number (n) of observations was 26: the sum of the 26 numbers in the last column was 24811.8: hence,

$\log. \left(\frac{\cos \ d. \cos. L}{\sin. z}\right)$ - - - - - -	8.4900862
correction for retardation of pendulum	.0006986
$\log. \left(\frac{24811 \ 8}{26}\right) = \log. 24811.8 - \log 26,$	2.9796885
	1.4704733 = log. 29″.55

Hence, since the mean of the zenith distances, increased by refraction, or $\dfrac{z'+z''+\&c.+z_{\prime}+z_{\prime\prime}+\&c.}{26}+46''.41$ - - $= 37^\circ\ 16'\ 7''.3$

and mean of *reductions*, or

$\dfrac{\delta'+\delta''+\&c.+\delta_{\prime}+\delta_{\prime\prime}+\&c.}{26}$ - - - - - - - - 0 0 29.55

Meridional Z. D. of *a Polaris* - - - - -	37	15	37.75
but, N. P. D. (from Tables,) - - - - - -	1	42	18.59
Co-latitude - - - - - - - - - - - - -	38	57	56.34

and latitude is, $51^\circ\ 2'\ 3''.66$.

In this Example, the latitude is determined by means of the north polar distance of *a Polaris*, taken from the Tables; but, if we find the meridional *greatest* zenith distance of the star as we have found the least, then the co-latitude will equal half the sum of the zenith distances, and, the north polar distance of *a Polaris* will be half the difference.

This method of determining the latitude was used by the French Astronomers, in their last great operation of measuring an *Arc of the Meridian*; and, since they consider it as convenient and capable of great accuracy, they have computed Tables to facilitate its application.

None of the preceding methods can be practised at sea, where the motion of the vessel renders useless the level and plumb-line. The instrument that must be resorted to is the sextant; and by that, and the aid of the Solar and other Tables, the latitude may be determined to within 30 seconds, or about half a mile: an accuracy quite sufficient for practical purposes, but very inferior to what may be obtained by the last method, which is capable of determining the latitude to within the fraction of a second *.

* The preceding method is useful in determining the meridional altitudes of the Sun when near the solstices: then, by a similar method, the meridional altitudes observed near the solstice may be *reduced* to the solstice. And by these means, the obliquity of the ecliptic may be determined to great exactness. (See pp. 45, &c.)

LATITUDE OF A VESSEL AT SEA.

Method by the Meridional Altitude of the Sun.

If the latitude and the declination be of the same denomination, that is, either both north, or both south, then, the latitude = Z. D.☉ + decl. ☉

or = decl. ☉ − Z. D. ☉, if decl. > lat.

If the latitude and declination be of different denominations then, the latitude = Z. D. ☉ − decl. ☉.

EXAMPLE.

July 24, 1783. *Longitude* 54° (3^h 36^m) *West of Greenwich, the Altitude of the Sun's Lower Limb was observed by the Sextant to be* 59° 16′. *Required the Latitude.*

Altitude of Sun's lower limb - - - -	59°	16′	0″
Refraction [Chap. XI.] - - - - - -	0		— 34
Parallax [Chap. XII.] - - - - - - - -	0		+ 4
Sun's semi-diameter - - - - - - - -	0	15	48
True alt. of Sun's center - - - - -	59	31	18
∴ Z. D. - - - - - - - - - - - - -	30	28	42
Sun's decl. (found as in p. 403,) - - -	19	51	0
∴ latitude (*N*) - - -	50	19	42

By the Meridional Altitude of a fixed Star.

March 29, 1783. *South Latitude, the Meridional Altitude of* Procyon *was* 77° 27′ 15″: *the Height of the Observer's Eye,* 22 *Feet above the Surface of the Sea. Required the Latitude.*

Meridional alt. of *Procyon* - - - -	77°	27′	15′
Refraction - - - - - - - - - - - -	0	0—	13
Dip of the horizon - - - - - - - -	0	— 4	28
True alt. of ✱ - - - - - - - - - - -	77	22	34
∴ true zen. dist. - - - - - - - - -	12	37	26.S
Decl. of *Procyon* (from Tables) - - -	5	46	17 N
∴ (see l. 8.) latitude - - - - - - -	6	51	9 S

In this Example, a correction called the *Dip*, and not before mentioned, is made. That correction arises from the increase of the apparent altitude occasioned by the elevation of the observer above the surface of the sea.

By the Meridional Altitude of the Moon.

March 26, 1810. *Longitude* 40° 47′ *West of Greenwich, the Altitude of the Moon's Upper Limb was observed to be* 46° 14′ 19″. *Required the Latitude.*

Alt. of Moon's upper limb - - - - - - -	46°	14	19″
Horiz. ½ diam. - - - - - - 16′ 6″ } Augmentation (see p. 318.) 0 12 } semi-diam.	0	16	18
Apparent alt. of Moon's center - - - - - -	45	58	1
[Horiz. Parallax 59′ 7″] Parallax (p. 95.) - -	0	41	0
Refraction - - - - - - - - - - - - - -	0		—55
True alt. Moon's center - - - - - - - - -	46	38	6
Declination S. - - - - - - - - - - - - -	17	42	0
Alt. of equator, or co-latitude - - - - - - -	64	20	6
∴ latitude - - - -	25	39	54. N

The difference of the parallax and refraction is given as *one result* in Astronomical Tables, (See Tab. VIII. of the Requisite Tables: also Tab. VIII. of Mr. Mendoza's.)

Of these three methods, the first, in which the altitude of the Sun is observed, is most commonly used: the second, very rarely, by reason of the difficulty of observing the Star's altitude with a sextant: the third, as it is plain, can only be used in certain parts of the month; and, since in all, the observed body must be on the meridian, clouds may prevent any of the three from being used. A subsidiary method, therefore, is provided, in which the latitude may be computed from two observed altitudes of the Sun, and the interval of time between the observations.

Method of finding the Latitude by two Altitudes of the Sun and the Time between.

We have already used a triangle ZPS, and we will now introduce another, ZPs, exactly similar to it: in which s is a position of the Sun, separated from that of S, by the angle SPs, and, in time, by the interval t. Conceive the places S, s (S being nearest to the meridian) to be joined by the arc Ss of a great circle; then we have given

ZS, Zs, (a, a') the observed altitudes,

PS, Ps $(p, p,)$ equal N. P. D. of the Sun,

and $\angle SPs$ (t) measuring the interval between the observations. Now the investigation will consist of several steps, which all tend to the finding of the angle ZsP; for, that being found, we have given Zs, Ps, and the included angle ZsP, to find ZP the co-latitude. The steps for finding ZsP are according to the following order. First,

Ss is found; then $\angle PsS$; next $\angle ZsS$, and last,

$$\angle ZsP = \angle PsS - \angle ZsS$$

S s *found.*

$$\text{Cos. } Ss = \cos. SPs . \sin. SP . \sin. sP + \cos. SP . \cos. sP$$

(*Trigonometry*, p. 100.)

$$\therefore\ 1 - \cos. Ss, \text{ or, } 2\sin.^2 \frac{Ss}{2} = 1 - \cos.^2 p - \cos. t \sin.^2 p$$

$$= \sin.^2 p \cdot 2 . \sin.^2 \frac{t}{2}\text{; and in log}^s.$$

$$\log. \sin. \frac{Ss}{2} = \log. \sin. p + \log. \sin. \frac{t}{2} - 10.$$

Angle S s P *found.*

$$\text{Sin. } SsP = \frac{\sin. p . \sin. t}{\sin. Ss}, \quad *$$

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* The angle might be deduced from this expression; but the last in practice, is more convenient, since, by taking out the log. sin. $\frac{t}{2}$, we can, without turning over the leaves, take out the log. cot. $\frac{t}{2}$.

$$\cos. S s P = \frac{\cos. p\,(1 - \cos. S s)}{\sin. p\,.\,\sin. S s};$$

$$\therefore \tan. S s P = \frac{\sin. t\,.\,\sin.^2 p}{\cos. p\,(1 - \cos. S s)} = \frac{\sin. t\,.\,\sin.^2 p}{2\,.\cos. p\,.\,\sin.^2 p\,.\,\sin.^2 \frac{t}{2}},$$

$$= \frac{\cot. \frac{t}{2}}{\cos. p}.$$

In logarithms,

$$\log. \tan. S s P = 10 + \log. \cot. \frac{t}{2} - \log. \cos p.$$

Angle Z s S *found.*

$$\text{Cos. } Z s S = \frac{\cos. ZS - \cos. S s \cos. Zs}{\sin. S s\,.\,\sin. Z s} = \frac{\sin. a - \sin. a' \cos. S s}{\cos. a'\,.\,\sin. S s}$$

a form exactly similar to the one in p. 404;

$$\therefore \sin.^2 \tfrac{1}{2}\,.\,Z s S =$$

$$\frac{1}{\sin. S s\,.\cos. a}\left[\cos. \tfrac{1}{2}(S s + a' + a)\,.\,\sin. \tfrac{1}{2}(S s + a' - a)\right],$$

and in logarithms, $2 \log. \sin. \frac{1}{2} Z s S =$

$$20 + \log. \cos. \tfrac{1}{2}(S s + a' + a) + \log. \sin. \tfrac{1}{2}(S s + a' - a)$$

$$- \log. \sin. S s - \log. \cos. a;$$

hence, $\angle Z s P = \angle S s P - \angle Z s S$, is known.

ZP, *the Co-latitude, found.*

This Problem, in which from two sides and the included angle the third side is required, is exactly like three that have been already solved in pages 56. 364. 369. Assume, therefore, θ such, that

$$\tan.^2 \theta = \frac{\cos. a'\,.\,\sin. p\,.\,\text{ver. sin. } Z s P}{\text{ver. sin. } (90^\circ - a' - p)},$$

$$\text{and } \sin. \frac{ZP}{2} = \sin. \frac{90^\circ - a' - p}{2} \times \sec. \theta;$$

and in logarithms,

$$\log. \sin. \frac{ZP}{2} = 10 + \log. \sin. \tfrac{1}{2}(90^\circ - a' - p) - \log. \cos \theta.$$

ZP may be also found by a different subsidiary angle (See *Trig.* p. 131).

This direct method, requiring considerable computation, may happen to be beyond the skill of the mariner; and it is, in fact, seldom resorted to. Instead of it, Dr. Brinkley and Mr. Mendoza have given approximate methods and facilitated their application by means of appropriate Tables. (See Nautical Almanack, 1797, 1798, 1799, 1800, and Mendoza's Tables on Nautical Astronomy.)

It is evident, the preceding methods (pp. 420, &c.) which are the only ones that can be practised at sea, may be practised at land,) when the sextant is used with an artificial horizon, (see p. 393). But then, they are to be used only when no great accuracy is required, and in default of better instruments. The errors of observation with the sextant, and those of the Solar Tables, must always be presumed to be of some magnitude; and, of both of these errors, the above-mentioned methods necessarily partake.

CHAP. XLIII.

On Geographical Longitude.

THE Earth revolves round its axis in $23^h\ 56^m\ 4^s.091$ of mean solar time; but, a meridian passing through the Sun returns to it after the lapse of a greater time, viz. 24^h, and consequently, after describing a greater angle than 360°. This arises from the increase of the Sun's right ascension in the time of the Earth's rotation; the mean value of which increase is 59′ 8″.3: and consequently, the angle, through which a meridian revolves in a mean solar day of 24 hours, is 360° 59′ 8″.3.

If we suppose a number of meridians to be drawn at equal intervals, that is, forming successively with each other, equal angles at the poles, then in the course of 24 hours, each of these meridians (supposing their planes produced) will pass through the Sun: and, since both the Earth's rotation, and the Sun's mean motion in right ascension, are supposed to be uniform, at equal intervals of time. If the meridian of a given place passed through the Sun at the beginning of the 24 hours, it would again pass through it at the end; 24 hours then of mean solar time would correspond to 360 degrees of longitude; for, the whole scale of longitude must be comprehended between the eastern and western sides of the meridian of the same place. At places situated on the meridian opposite that on which the Sun was at 0^h, or, in civil reckoning, at 12 at noon, the time would be 12^h, or 12 at night; and 12^h would correspond to 180 degrees of longitude. At places situated on the meridian, at right angles to the former, the time would be 6^h or 18^h; or, in civil reckoning, 6 in the morning, or 6 in the evening; and accordingly, 6 and 18 hours of mean solar time, would correspond to 90°, or 270° of longitude; and similarly for intermediate meridians.

The selection of a meridian, from which the longitudes of all other places are to be reckoned, is entirely arbitrary. The English have selected that which passes through the Royal Observatory at Greenwich: it is called the *First Meridian*, and its longitude is called 0^h. The French use a different one: their *Premier Meridien* passes through the observatory at Paris, and is $9^m\ 21^s$ east of the former.

If then at Greenwich, (and consequently at all places through which its meridian passes) the Sun were 7° 30′ to the west of the meridian, or the time were $0^h\ 30^m$, at places, the meridians of which should be 15°, 30°, 45°, &c. distant from that of Greenwich and to the east, or which should have, respectively, 15°, 30° 45°, &c. of *east longitude*, the times, or the reckoned hours of the day, would be, respectively, $1^h\ 30^m$, $2^h\ 30^m$, $3^h\ 30^m$, &c. At places, 10°, 20°, 30°, &c. of *west longitude*, the times would be respectively, $23^h\ 50^m$, $23^h\ 10^m$, $22^h\ 30^m$, &c. or in civil reckoning, $11^h\ 50^m$, $11^h\ 10^m$, $10^h\ 30^m$, &c. in the morning. Now, some of the methods of determining the longitude, depend solely on the reverse of this; that is, they find the differences between the reckoned time at a given place and at Greenwich, and thence deduce the difference of longitude, or, (since that of Greenwich is 0), the real longitude, converting the time into degrees at the rate of 15 for each hour.

The methods that depend solely on the difference of the reckoned times, are those which are connected with phenomena that happen and are observed at the same point of *absolute time*. Such phenomena are the eclipses of the Moon and of the satellites of *Jupiter*. There are other methods, however, which depend partly on the difference of the reckoned, and partly on that of the absolute times. Such are founded on the phenomena of solar eclipses, of occultations, and of transits, which are not observed, at the same point of absolute time, at all parts of the Earth's surface. (See p. 361.)

This may be illustrated by an instance. Berlin is $44^m\ 10^s$ east of Paris; therefore, if an eclipse of one of *Jupiter's* satellites were observed to happen at the latter place at $13^h\ 1^m\ 20^s$, it would be reckoned to happen at the former at $13^h\ 45^m\ 30^s$: for, since the phenomenon takes place by the actual falling of the shadow on the satellite, the observer at Berlin must see it at the same point of absolute time, as the observer at Paris. But, the occultation of *Antares* by the Moon, (see p. 371,) was observed at Paris at $13^h\ 1^m\ 20^s$, and at Berlin, at $14^h\ 6^m\ 19^s$. The difference ($1^h\ 4^m\ 59^s$) of the *reckoned* times, then, is not entirely due to the *difference of meridians* ($44^m\ 10^s$), but partly to that, and partly to the difference in the absolute times of the observations of the phenomena: which latter difference, equal to $20^m\ 49^s$, is entirely the effect of parallax. In the former case, the satellite was *obscured* by the shadow of *Jupiter*, in this latter, the star is *concealed* by the interposition of the Moon.

The methods of finding the longitude, then, naturally arrange themselves into two classes: one belonging to phenomena of the first description, the other, to phenomena of the second. The methods of the former being very simple in their application, but not very accurate in their results; the latter requiring tedious computations, but capable of great exactness. We will, however, first shew how to determine

The Longitude, by a Chronometer or Time-keeper.

Suppose a chronometer to be adjusted to mean solar time at Greenwich; then, if its motion were equable and of the proper rate, we should always know, whatever the place, the time at Greenwich. By one of the methods given in pp. 396, &c., we could compute the apparent, and by means of the equation of time, the mean time, at the place of observation. The difference between this latter time, and that shewn by the chronometer, would be the longitude, east or west of Greenwich.

A chronometer, however nicely constructed, must always be subject to some irregularities; even if in given instances, it should happen to be entirely free from them, still, if we had no means of ascertaining its accuracy, we could never rely and act on it. On this account it is not entirely relied on. It is trusted to for short intervals of time; and the degree of trust can always be appreciated, since it is subjected to continual examination and correction by the strictly Astronomical methods of estimating the time.

Longitude by an Eclipse of the Moon.

By means of a perfect chronometer we could always, and in all places, determine the longitude. By Lunar eclipses which are rare, we can determine the longitude, only occasionally and at particular conjunctures; but, when such occur, by the following method. The times at which eclipses happen, at the place of observation, are to be computed, by one of them ethods given in pp. 396, &c., or, which is commonly the case, may be known by a chronometer previously regulated by observation. The times at Greenwich, previously computed, are inserted in the Nautical Almanack, or may be computed by the observer from the Lunar Tables. The difference of these times is the longitude.

Since the Lunar Tables are not exact, the comparison of the same eclipse, actually observed at two different places, will give the difference of their longitudes much more accurately than the comparison of the eclipse observed at one place, and *computed for another.*

EXAMPLE.

1729, *Aug.* 28. *By observations of Cassini at Paris* (Mem. Acad. 1779). *and of Mr. Stevenson at Barbados* (Phil. Trans. N° 416. p. 441.)

At Paris, Imm. ☽ - - -	12^h	19^m	13^s	Emer. ☽ - -	13^h	59^m
At Barbados, Imm. - -	8	11		Emer. - - -	9	51
	4	8	13		4	8

By the mean of the two, the difference of longitude is, 4^h 8^m $6^s.5$ or 62° 1′ 30″: that is, Barbados is 62° 1′ 30″ west of Paris.

This method of determining the longitude is rarely used, since, by reason of the penumbra, it is difficult to ascertain the exact time of contact of the Earth's shadow with the Moon's limb. The time is uncertain, to the extent of 2^m, or 30′. It has been proposed to amend the method, by observing the contact of the Earth's shadow with some remarkable spots in the Moon's disk. (See *Phil. Trans.* 1786. pp. 415, &c.)

Longitude by the Eclipses of Jupiter's Satellites.

This method is practically better than the preceding, for two reasons; first, the eclipses of the Satellites are of frequent recurrence; and, secondly, the times of immersion and emersion can be noted with much greater precision, than those of the contacts of the Earth's shadow with the Moon's limb.

EXAMPLE.

At the Cape of Good Hope, May 9, 1769,

Emer. 1st Satellite - - - - - - - - - -	10h	46m	45s
At Greenwich, by computation (Naut. Alm.) - -	9	33	12
Difference of meridians,	1	13	33

or the Cape is 18° 23′ 15″ to the east of Greenwich. The remark which was applied to the former case, applies to this. If we use the emersion *observed* at Greenwich, instead of the emersion *computed* for Greenwich, we shall avoid the errors of the Tables of Jupiter's Satellites, and obtain a more exact value of the longitude.

The errors of the Tables are not the only things to be guarded against. The observer must take care that his telescope be of proper power: for, otherwise, he will perceive an emersion later, and an immersion sooner than he ought to do. If there are two observers at different places, then, in order to determine the difference of their longitudes, with as much exactness as the method is capable of, they ought to be provided with telescopes of the same power and goodness. They ought also to observe both the immersion and emersion and to take the mean of their results. (See Nautical Almanack, p. 151.)

We now proceed to the methods of determining the longitude by means of phenomena of the second class; those, which are not seen by all spectators at the same point of absolute time.

The Longitude determined by an occultation of a fixed Star by the Moon.

In pp. 371, &c. the apparent distance of *Antares*, from the Moon was computed, for the instant previous to its occultation, and found equal to 15′ 51″. 38. The place of observation was

Paris: the hour or apparent time $13^h\ 1^m\ 20^s$ (the mean time $13^h\ 3^m\ 32^s.8$): and the formula for the computation of the distance, was

$$D^2 = (l-l')^2 + (k-k')^2 . \cos.^2 l \qquad (a)$$

In this formula, l, k, are the *apparent* latitude and longitude of the Moon, obtained, by adding to the true, (see p. 373,) the computed parallaxes in longitude and latitude.

The true longitude and latitude of the Moon were taken, from Lunar Tables *computed for the meridian of Paris*, and for $13^h\ 3^m\ 32^s.8$ *mean solar time at Paris*: and were found, respectively, equal to $9^s\ 5^\circ\ 31'\ 42''.4$ and $3^\circ\ 47'\ 58''.7$. (See p. 373.)

If then the Lunar Tables be correct, D would result from the preceding formula (a) exactly of it's proper value, such as the Tables would assign, or (since D is, in this case, the Moon's semi-diameter) such as might easily be ascertained by observation. But, if D computed from the formula (a) should differ from the value of the Moon's semi-diameter assigned by the Tables, then that circumstance would be a proof of the existence of errors in the Tables. And, the difference between the two values of D, would enable us to deduce an Equation between the corresponding errors in the Moon's latitude and longitude. In this case, an occultation would serve to *correct the errors of the Lunar Tables.*

But, there is another method of correcting the Lunar Tables. On the day of observation, let the Moon's declination and right ascension be observed, and thence, let her latitude and longitude be computed. The respective differences between these, and her latitude and longitude computed from the Lunar Tables, will give, for that day, their errors.

Since we have the means then of ascertaining the errors, we will *suppose* the Lunar Tables to be perfectly correct. Let us now see, by what means, D is to be computed, in a place of observation, for the *Meridian of which, there are no Tables constructed.*

In such a place, the observer must use Tables computed for another meridian: either, for the meridian of Greenwich, or for that of Paris: either the Nautical Almanack, or the *Connois-*

sance des Temps *. By these, he must compute *l*, and *k*, and accordingly, previously must compute the Moon's true latitude and longitude, that is, the latitude and longitude that belong to the center of the Earth. The values of these latter depend on the time for which they are computed, and, *on the time as it is reckoned either at Greenwich or Paris*. Now, although (see pp. 396, &c.) the time, at the place of observation, can be exactly known, that, *at the place for which the Tables are computed*, cannot, except by a knowledge of the *longitude* of the former place.

This is easily illustrated: The occultation of *Antares* was observed at Berlin at $14^h\ 7^m\ 31^s$, mean solar time. The observer at that place in order to compute, by the *French Tables*, the Moon's true longitude, must know the corresponding time at Paris. If he *assume* Berlin to be 44^m east of Paris, the corresponding mean time, at the latter place would be, $13^h\ 23^m\ 31^s$: and the Moon's true longitude computed for $13^h\ 23^m\ 31^s$, would be $8^s\ 5^\circ\ 43'\ 16''$. But, if he *assume* the difference of longitude to be $39^m\ 49^s$, the corresponding time at Paris will be $14^h\ 27^m\ 42^s$: and the Moon's true longitude computed for $14^h\ 27^m\ 42^s$, will be $8^s\ 5^\circ\ 45'\ 35''$. The computations for the Moon's true latitude will be similiarly affected by a change in the hypothesis of the longitude of Berlin.

A small error in that hypothesis will very little affect the computation† of the parallaxes in longitude and latitude: those depend chiefly on the hour angle; consequently, since the apparent differ from the true longitudes and latitudes, solely by the parallaxes, the change, or error in the hypothesis of the difference of meridians, will produce the same difference in the apparent, as in the true longitudes and latitudes of the Moon.

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* These Ephemerides may be considered a species of lunar and solar Tables, in which certain results, most commonly wanted in practice, and computed from the *general* Tables, are inserted. Such results are the Moon's right ascension, declination, longitude, latitude, parallax, and semi-diameter, for noon and midnight.

† If we examine the formulæ of computation, (1), (2), (3), &c. in pp. 371, &c. we shall perceive that the parallaxes depend principally on the hour-angle which is not changed by altering the hypothesis of the longitude.

Hence it follows, that an error in the assumed longitude of Berlin (that being still the place used for illustration) will produce errors in the computation of l, k; and consequently, in the computation of D from,

$$D^2 = (l - l')^2 + (k - k')^2 \cos.^2 l \qquad (a)$$

there must be an error in the resulting value of D.

Now, the principle of finding the longitude of Berlin, consists in *correcting* the assumed longitude, by means of the error in D. The correction is thus made.

The Moon's latitude and longitude (l, k,) being supposed to be erroneous, let their true value be $l + nt$, $k + mt$: n, m being the Moon's horary motions in latitude and longitude, and t, as an an unknown quantity, representing the time, or the error of the hypothesis of the *difference of the meridians*; then, if Δ be the Moon's true semi-diameter, we have

$$\Delta^2 = (l + nt - l')^2 + (k + mt - k')^2. \cos.^2 l \qquad (b)$$

and from this and the preceding equation (a), t is to determined.

If we suppose, what will always be the case in practice, the longitude of the place of observation to be *nearly* known, and consequently, the hypothesis of its value to differ but little from the true value, t will be a small quantity; and, if we neglect its square in the expansion of (b), we shall have

$$\Delta^2 = (l - l')^2 + 2nt. (l - l') + [(k - k')^2 + 2mt (k - k')] \cos.^2 l$$

Subtracting (a) from this,

$$\Delta^2 - D^2 = 2t \, [n (l - l') + m. (k - k') \cos^2 l]$$

and consequently,

$$t = \frac{\Delta^2 - D^2}{2 \, [n (l - l') + m (k - k'). \cos.^2 l]} \qquad (c)$$

This value of t, (an approximate one) is the correction to the assumed longitude: suppose, the longitude $= T$, then its corrected value is $T \pm t$; and, if a still more correct value be required, compute again by means of this *corrected hypothesis* of the difference of the meridians ($T \pm t$), the true latitudes and longitudes of the Moon; thence deduce correcter values of l, k, and find a new approximation (t') from the expression (c). The longitude, after this second correction, will be $T \pm t \pm t'$.

This method, from an assumed approximate value, is capable of determining the true value of the longitude, to the greatest exactness. And, we need not be solicitous concerning the *nearness* of the first approximation to the truth. An eclipse of one of *Jupiter's* satellites, which is easily observed, will afford us a first value of the longitude, we might almost say, more than sufficiently near. For, we may even take as a first value, the difference of the *reckoned times* of the occultation at the two places which in the preceding illustration was 1^h 5^m, and which (see pp. 427, &c.) is considerably different from the true value.

We have already illustrated the method, by supposing the occultation to have been observed at Berlin, and the Moon's longitude and latitude to have been computed by Paris Tables. We will now attempt to exemplify the mode of computing the *correction* (*t*), by supposing the occultation to have been *observed at Paris*, and the Moon's longitude and latitude to be computed by Tables adapted to the *Meridian of Greenwich*.

The immersion (see p. 371,) was observed at Paris at 13 1^m 20^s. In order to find the corresponding time at Greenwich, suppose the latter to be 9^m west of the former; then, the reckoned time would be 13^h 1^m $20^s - 9^m$, or 12 52^m 20^s; for this time, compute the Moon's longitude; the simplest mode of effecting which, now, would be, to take from the Nautical Almanack the Moon's longitudes on April 6th at midnight, and April 7th at noon; to find their difference, and then to add to the former that part of the difference which is proportional to 52^m 20^s. The result would be the Moon's true longitude at 12^h 52^m 20^s. (See pp. 403, &c.) Compute in the same way the Moon's latitude: suppose them to be exactly of those values which are assigned to them in the Example of pp. 371, &c.; then, the parallaxes, &c., being computed exactly as in that Example, the Moon's semi-diameter will be found (see p. 374.) equal to 15′ 51″.3. If the Tables be perfectly correct, and the longitude be rightly assumed, that computed value of the semi-diameter ought to be equal to the semi-diameter assigned by the same Tables. But, the latter is found to be 15′ 37″.7. The difference or error 13″.6, *assuming* the Tables to be correct, must arise then solely from an error in the hypothesis of the longitude: computing that error from

$$t = \frac{\Delta^2 - D^2}{2\,[n\,(l - l') + m\,(k - k')\cos.^2 l]}$$

in which $\Delta = 15'\ 37''.7$, - - - - $l - l' = 4'\ 3''.2$

$D = 15\ 51\ .3$, - - - - $k - k' = 15\ 22\ .7$

$l = 4^\circ\ 36'$, and n and m are the hourly motions * ; t will be found nearly $= 2.5^s$. The corrected longitude of Paris then is $9^m\ 25^s$, and a repetition of the process will give a value still more correct.

Since the illustration of the method of correcting the assumed longitude was our chief object, we have supposed, the Lunar Tables to be correct. But, in practice, their errors, which are frequently considerable, must be always attended to.

If the occultation be observed under a known meridian, such as that of Greenwich or of Paris, then, it may be made subservient to the correction of the Lunar Tables. For such an end, Mayer has employed the immersion and emersion of *Aldebaran* †. And, it is easy to see, since the errors in the computation of the Moon's distance from the Star, can be only three ‡ (those of the Lunar longitude and latitude and of the assumed longitude of the place of observation,) that three observations, of an immersion, at a place of an ascertained longitude, and of an immersion and an emersion at a place whose longitude is required, will furnish three equations sufficient to correct the three errors above-mentioned. (See Cagnoli, *Trig.* pp. 470, &c.)

In page 374, allusion was made to a method, of deducing the longitude from an occultation, in some respects the reverse of the preceding. In the method alluded to, the true latitude and longitude of the *point* of occultation are deduced by correcting

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* To obtain n, m, the hourly motions, compute the Moon's apparent latitudes and longitudes, for $12^h\ 51^m\ 40^s$, and for $13^h\ 51^m\ 40^s$: and the respective differences of these quantities will be the hourly motions in latitude and longitude. In the computation they were assumed to be $1'\ 54''$ and $36'\ 31''$; which are not, however, their exact values.

† Mayer's Lunar Tables, 1770, pp. 39, 40.

‡ The Moon's semi-diameter, on the day of the occultation, may be measured or computed by means of an observation, and accordingly, any error, in it's value assigned by the Tables, corrected.

the apparent latitude and longitude of the Star from the effects of parallax. The true latitude of the Moon is taken from the Nautical Almanack. The true distance D, or the semi-diameter of the Moon may be taken from the same source, or may be determined by observation: and thence, may the Moon's longitude be determined; for, supposing in the equation (p. 369.)

$$D^2 = (l - l')^2 + (k - k')^2. \cos.^2 l,$$

that, l, k, &c. represent the true latitudes and longitudes: if D, l, l', are known, $k-k'$ may be determined; and, since k', or the true longitude of the point of occultation is known, k the longitude of the Moon's center is.

Suppose, then, that by these means, and separate calculations, we obtained, from an occultation, at two different places, the following results:

Greenwich, long. ☽'s center - -	67° 22′ 26″.1 - -	hour = $8^h\ 37^m\ 36^s.8$
Dublin - - - - - - - -	67 18 43 .3	8 4 51.5
	3 42 .8	30 45.3

then, 3′ 42″.8, is the difference between the Moon's true longitudes at the *absolute* times of the observed occultation: and if the Moon's horary motion be 30′ 9″.2, the difference would correspond to $7^m\ 23^s.3$, in time. The occultation therefore at Greenwich *really* happened later than the occultation at Dublin by $7^m\ 23^s.3$: but, it is *reckoned* to happen later at the former by $32^m\ 45^s.3$: part of this then, or that part which remains after $7^m\ 23^s.3$ is subducted, is solely due to the difference of the longitudes of the two places: Dublin therefore is *east* of Greenwich, $25^m\ 22^s$.

The Longitude determined by means of a Solar Eclipse.

This method, in all its parts, is like the preceding. The distance (D) which is to be computed, instead of being the Moon's semi-diameter, will be the sum of the semi-diameters of the Sun and Moon. The immersion of the Star will correspond to the first exterior contact of the limbs of the Sun and Moon, the emersion to the last. Thence will result, two equations for correcting, if the Lunar and Solar Tables be correct, the hypothesis (see p. 434.) of the assumed longitude. But, since we can also observe other *Phases* of the eclipse, that, for instance, of the nearest approach of the centers (see pp. 347. 356), we may deduce equations sufficient to correct both the errors of the Tables and the error of the assumed longitude of the place of observation.

We will now proceed to the description of an excellent method for finding the longitude, which cannot be ranged under either of the two preceding classes.

Method of determining the Longitude by means of the Passage of the Moon over the Meridian.

Let us suppose the meridian of a given place produced to the heavens to pass through the Moon, the Sun, and a fixed Star. In the next instant, the Sun by its motion in R. A. will separate itself from the Star; the Moon by her greater motion in R. A. both from the Star and Sun, and the meridian by the rotation of the Earth, from the Star, Sun and Moon. In other words, in the instant of time (whatever be its magnitude) after that on which the three bodies were on the meridian, the Star will be most to the west of the meridian, the Moon least, and the Sun will be in an intermediate position.

The meridian after quitting these bodies, will approach towards them with different degrees of velocity, and will reach them after different intervals of time. It will again pass through the Star, after describing 360°, in 23^h 56^m $4^s.09$; through the Sun, after describing 360° 59′ 8″.3, in 24^h; and, through the Moon, after describing an angle equal the sum of 360°, and the increase of the Moon's right ascension in 24^h, and in a time equal to the sum of 24 hours, and of the Moon's *retardation* (see p. 399,) in 24 hours.

This takes place in the interval between two successive transits of the Moon over the same meridian. A spectator on a *different* meridian must note similar effects; but less in degree, and less proportionally to the distance of his, from the first, meridian. He will note an increase in the Sun's right ascension, (or a separation of the Sun from the fixed Star) but less than 59′ 8″.3: an increase in the Moon's right ascension (or a separation of the Moon from the Star), but less than its increase between two successive transits; and consequently, an excess of the increase of the Moon's right ascension above that of the Sun's, but less than the excess that takes place between two successive transits of the Moon over the first meridian.

Hence, if the spectator, on this second meridian, knows, or is able to compute, the respective increases in right ascension of the Moon and Sun, that take place between two successive passages of the Moon over the first meridian, then, since he is able, by actual observation, to ascertain at the times of their passages, the right ascension of the Sun and Moon, he may, by simple proportion, determine his longitude; and in fact, he has three ways of effecting it: either with the Sun and Star; or with the Moon and Star; or with the Moon and Sun. Since, however, the first method by reason of the slow motion of the Sun, is not convenient and *practically* useful, we shall not notice it, but consider only the two latter.

Let E be the increase of the ☽'s right ascension during two successive transits at the first meridian, e the difference between ☽'s right ascension at the Moon's first passage at the first meridian, and her right ascension at the passage over the second meridian, then,

$$E : e :: 360^\circ : 360 \times \frac{e}{E} = \text{difference of the meridians.}$$

This is the case with the Moon and Star: and, with the Moon and Sun, there is this only difference, that E (E) must denote the *excess* of the increase of the Moon's right ascension above that of the Sun between two successive transits of the Moon; and e (e') the difference between the hours of Moon's passages over the second and first meridian: for the hour of the Moon's passage is proportional to the angular distance which then exists between the Sun and Moon.

We must now endeavour to render the above formula more convenient for computation, so that (which ought in practical Astronomy to be our constant aim) we may avail ourselves of the facilities of the Nautical Almanack.

E is the increase of right ascension between two successive transits of the Moon over the first meridian; it is, therefore, equal to the increase of right ascension in twenty-four hours, plus the increase of right ascension due or proportional to, the Moon's *retardation* (see p. 399.) in twenty-four hours. We have therefore this rule in the case of the Moon and Star:

Find from the Nautical Almanack, (see p. 403,) the increase of the Moon's right ascension in twenty-four hours.

Compute also by the rule in p. 155, of the Nautical Almanack, (or from the expression in this Treatise, p. 400,) the Moon's retardation in twenty-four hours.

To the increase (A) of the the ☽'s right ascension in 24^h add the increase proportional to the retardation: call the sum E.

Then, substituting in p. 437, l. 20, 24^h instead of $360°$, we have

$$\log.\ \text{longitude} = \log.\ 24 + \log.\ e - \log.\ E.$$

In the case of the Moon and Sun, the rule is somewhat more simple. for E' converted into time in the case of the Moon, is the Moon's retardation, and e' is the proportional retardation between the transits at the first and second meridian. The third step, therefore, in the preceding rule, in this case, need not be made.

The above rule is adapted to the Nautical Almanack. But, it is easy to substitute, instead of it, a general formula of computation expressed in symbols. Thus, let A, a, be the respective increases of the right ascensions of the Moon and Sun in twenty-four hours; then, since the interval between two successive passages of the Moon over the meridian is

$$24^h + 24 \times \frac{A-a}{24} + 24\left(\frac{A-a}{24}\right)^2 + 24\left(\frac{A-a}{24}\right)^3 + \&c.$$

(since in this case $t = 24^h$, see p. 400, l. 16.) the *retardation* in 24^h must equal

$$A - a + \frac{(A-a)^2}{24} + \frac{(A-a)^3}{(24)^2} + \&c.$$

and the increase of A due to the retardation must equal

$$\frac{A}{24}\left[A - a + \frac{(A-a)^2}{24} + \frac{(A-a)^3}{(24)^2} + \&c.\right]$$

and consequently, (see p. 437,)

$$E = A + A\left[\frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right]$$

and the longitude =

$$\frac{24 \times e}{A\left[1 + \frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right]} \quad [1.]$$

In the case of the Sun,

$$E' = A - a + \frac{(A-a)^2}{24} + \frac{(A-a)^3}{24^2} \quad \&c.$$

and $e' = e - \epsilon$, where ϵ expresses the Star's *acceleration*, (see p. 398,) proportional to the time corresponding to the difference of meridians. Hence, the longitude =

$$\frac{24 \times (e - \epsilon)}{(A-a)\left[1 + \frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right]} \quad [2.]$$

Since $e - \epsilon : A - a :: e : A$, it is plain, the two expressions are, as they ought to be, equal.

The Moon's right ascension is expressed in the Nautical Almanack for every 12^h. Instead therefore of the difference of the increases of right ascension $(A - a)$ in 24 hours, we may employ the difference $\left(\frac{A-a}{2}\right)$ in 12 hours: and accordingly in the Rule, (p. 438, l. 1, &c.) and in the two expressions [1], [2], we must use 12^h instead of 24^h.

The denominators of the expressions, [1], [2], are, strictly speaking, infinite series; but, in practice it will be sufficiently accurate to take the sums of three of their terms.

In the application of the rule (p. 434,) to Examples, there would be occasion scarcely for any computation, if the passages of the Moon over the meridian at Greenwich, were more accurately expressed in the Nautical Almanack. For then, the *retardation* in 24^h would be immediately obtained by merely taking the difference between two successive passages. But, the times of the passages are expressed only as far as minutes. For this reason, it becomes necessary to compute them as far as seconds, from the Moon's right ascension, either by the expression of p. 400; or, by the rule given in the Nautical Almanack, p. 155. 1812, and after the Example of p. 401.

This, however, is not the description of the whole of the process of computation. For, since the Moon's right ascension is expressed in the Nautical Almanack, only as far as minutes of space, it becomes necessary to *compute* it, from the declination, latitude and longitude: which two latter quantities are expressed in degrees, minutes, and *seconds*. This, however, it is requisite to do, only when great accuracy is required.

EXAMPLE.

April 8, 1800. By observation at Greenwich,

Right ascension of Moon's center - - -	12^h	36^m	$26^s.6$
On a meridian to the west, - - - - - - -	12	47	56.7
$e=$	0	11	30.1

By computation from Naut. Almanack (see p. 403, l. 5, &c.)

Increase of ☽'s right ascension in 24^h, or A - - -	52^m	6^s
of ☉'s, - - - - - - - or a - - -	3	39.3
$A-a=$	48	26.7
Moon's retardation in 24^h, or time proportional to the description of $A-a$ (see p. 400, also Nautical Almanack, p. 155.) - -	50	7.8
Proportional increase of $52^s.6$, in 50^m $7^s.8$ - - -	1	48.8
$\therefore$ E ($= 52^m$ $6^s + 1^m$ $48^s.8$) - - - - - - - -	53	54.8

Hence, by the rule, p. 438,

log. 24 - - - - - -	1.3802112
log. 11^m $30^s.1$ - - -	2.8389120
	4.2191232
log. 53 54.8 - - -	3.5098474
	0.7092758 = log. 5. 12007;

therefore the longitude $= 5^h.12007 = 5^h$ 7^m $12^s.25$.

We will now solve the same Example, by the second method, which is founded on the difference between the hours of the Moon's passages over the meridian, instead of the difference of her right ascensions at those passages. We will also use 12 instead of 24 hours (see p 439, l. 10.)

EXAMPLE.

Moon's passage at Greenwich - - - -	11^h	26^m	$47^s.82$	
—— at the place of observation - - -	11	37	29 . 5	
e' or, $e - \varepsilon =$	0	10	41 . 68	
Moon's retardation, or E - - - - - -		25	3 . 9	

Hence, log. 12 - - - - - - 1.0791812
log. 10^m $41^s.68$ - - 2.8073185

3.8864997
log. 25 3 . 9 - - 3.1772190

.7092807 = log. 5.1201

$\therefore$ longitude $= 5^h.1201 = 5^h\ 7^m\ 12^s.36$.

The results are expressed as far as decimals of a second, merely for *arithmetical* exactness, and with no view of signifying that, in practice, any such exactness is attainable. The method is an excellent one, if it will determine the longitude within 10 seconds: and its original author Mr. Pigott, does not think it capable of a greater degree of accuracy. (See *Phil. Trans.* 1786, p. 419.)

The method, indeed, in a point of view strictly theoretical, cannot be minutely accurate. For the Moon's motion is continually variable, and the increase of its right ascension in 24 hours, will not be 24 times the increase in one hour. But if, from the strict laws of the Lunar motions, we corrected the method, we should probably obtain an exactness of no practical value; since, we might only get rid of errors much less than the almost unavoidable errors of observation.

Any means, however, of rendering the method more accurate and simple, are not to be neglected. And, on the ground of accuracy, we shall probably gain something, by employing, instead of the Sidereal clock, one of the Stars that regulate it: and, *that Star*, which shall happen to be nearest the Moon in right ascension and declination. Let both observers note the right ascensions of this Star and of the Moon, at the times of their transits over their meridians; then since, in a short interval, the clocks will not err much, the *difference of the differences in right ascension*, on which the method depends, will be given with sufficient accuracy for its successful application.

Again, the method will be rendered more simple, if instead of computing the transit of the Moon's center, we are content to note merely the *transit of one of her limbs*. This we may do, with little error, if the required longitude be not great. For, the error, if there be any, can arise, solely from a change in the Moon's semi-diameter during the interval between the transits over the two meridians.

EXAMPLE (See Vince's *Astronomy*, p. 533.)

June 13, 1791. At Greenwich, difference of R. A. of ☽'s first limb, and of *α Serpentis* -	28^m	$31^s.18$
Difference, at Dublin - - - - - - - - - -	27	24 .74
	1	6.44 = *e*
By Nautical Almanack, $\frac{A}{2}$ - - - - - - - -	30	30
$\frac{a}{2}$ - - - - - - - -	2	4.4
$\frac{A-a}{2}$ - - - - - - -	28	25.6
Retardation, (see p. 440.) - - - - - - -	29	35.2
Increase of $\frac{A}{2}$ proportional to retardation - - -	1	15.2
∴ E $(=30^m\ 30^s + 1^m\ 15^s.2)$ - - - - - - -	31	45.2

Hence, log. 12 - - - - 1.0791812

log. $1^m\ 6^s.44$ - - 1.8224296

2.9016108

log. $31^m\ 45^s.2$ - - 2.2799406

0.6216702 = log. .418475

∴ the longitude $= 25^m\ 6^s.5.$*

The methods of finding the longitude by an occultation and the eclipses of the Sun and Moon, would, even if they could be practised, be of no use at Sea, by reason of the rare occurrence of the phenomena on which they depend. A voyage might be completed before any eclipse happened. The mariner, who continually changes his place, requires a constant method of determining the change of longitude; a method, accordingly, depending on phenomena, continually occurring. Now, such phenomena the passages of the Moon over the meridian, and the eclipses of *Jupiter's* Satellites, must be reckoned. But, of neither of these can he avail himself: for the method founded on the former requires a nice observation with a telescope adjusted to move in the plane of the meridian: which is an operation evidently impracticable on board a ship. And the other method, on trial, has been found to be equally impracticable. Yet all that is wanted, for its success is, a contrivance that shall enable the observer to direct, with steadiness, a telescope of sufficient power, towards *Jupiter*. (See Naut. Alm. p. 151.)

From the defect, however, of the preceding methods, has arisen one of singular simplicity and ingenuity, in which the sole instrument employed is the *Sextant*. This we shall now proceed to describe and illustrate.

Method of determining the Longitude by the Distance of the Moon from a fixed Star, or from the Sun.

1. By means of the sextant (see Chap. XL.) observe the distance between a Star and one of the limbs of the Moon; or between

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* The principle of the preceding method is to be found, in a letter from Mr. Pigott, to Dr. Maskelyne, inserted in the *Philosophical Transactions* for 1786, pp. 417, &c.; and the method was used by the former in determining the longitude of York. The rule, however, p. 417, given by its author, is inaccurate: immaterially so, with regard to a place of so small a longitude as York, but to the extent, nearly, of 3 degrees, if we should seek to determine, by it, the longitude of a place that exceeds 5^h. This inaccuracy, as well as those of other authors, (see Vince's *Practical Astronomy*, p. 91. Wollaston's *Fasciculus*, Appendix, p. 76) who have adopted Mr. Pigott's method, were first pointed out in the *Phil. Mag.* Vol. XV.

the limbs of the Sun and Moon; then, by adding or subtracting in the former case, the Moon's semi-diameter, and in the latter, the sum of the semi-diameters of the Sun and Moon, there will result either the distance between the Moon's center and the Star, or between the centers of the Sun and Moon.

2. If there be two observers besides the one, who takes the above distance, let them, at the instant that distance be taken, observe the altitudes of the Moon and Star, or of the Moon and Sun. If there be only one observer, he must take the altitudes immediately before and after the observation of the distance, and endeavour to allow for the changes of altitude, that may have taken place in the intervals between their observations and that of the distance.

3. These observations being made, the true altitudes must be deduced from the apparent and observed, by correcting the latter for parallax and refraction, (see Chap. XI, XII.). This in practice, is to be effected by means of Tables.

4. The observed distance is also an apparent one: it must be reduced to a true distance, or, (as it is technically expressed,) must be *cleared* of the effects of parallax and refraction. This must be effected in every case, by a distinct computation from a proper formula.

5. The true distance being obtained, find the hour, minute, &c. of *Greenwich time* corresponding to it. This is to be effected by appropriate Tables, previously computed and inserted in the Nautical Almanack. In these Tables the Moon's distances from certain Stars are inserted for every 3^h: and thence, by an easy calculation, the time corresponding to an intermediate and distance may be found.

6. Compute the time at the place of observation from the corrected altitude of the Sun or Star, the Sun's or Star's north polar distance (furnished by Tables), and the latitude.

7. The difference between this latter time and the time at Greenwich, is the *longitude*.

The first thing in the preceding statement that requires our attention, is the

Formula for deducing the True from the observed Distance.

Conceive S, M to be the true places of the Star and Moon in two vertical circles SZ, MZ, forming at the zenith Z, the angle MZS; then, since (see Chap. XI, XII.) both parallax and refraction take place entirely in the directions of vertical circles, some point s above S, in the circle ZS, will be the apparent place of the Star, and m below M, (since, in the case of the Moon, the *depression* by parallax is greater than the *elevation* by refraction) will be the apparent place of the Moon: let

D (SM) be the true, d (sm) the apparent distance,
A, a ($90° - ZM$, $90° - ZS$) the true altitudes,
H, h ($90° - Zm$, $90° - Zs$) the apparent altitudes;

then, see *Trig*. pp. 99, &c.

$$\text{in } \triangle\ SZM,\ \cos. SZM = \frac{\cos. D - \sin. A . \sin. a}{\cos. A . \sin. a},$$

$$\text{in } \triangle\ sZm,\ \cos. sZm\ (= SZM) = \frac{\cos. d - \sin. H . \sin. h}{\cos. H . \cos. h},$$

and D is to be deduced by equating these two expressions.

Hence,

$$\cos. D = (\cos. d - \sin. H . \sin. h)\frac{\cos. A . \cos. a}{\cos. H . \cos. h} + \sin. A . \sin a,$$

(*Trigonometry*, p. 18.)

$$= [\cos. d + \cos. (H+h) - \cos. H . \cos. h]\frac{\cos. A . \cos. a}{\cos. H . \cos. h} + \sin. A . \sin. a$$

$$= 2 . \cos. \tfrac{1}{2}(H + h + d) . \cos. \tfrac{1}{2}(H + h - d)^{*}\ \frac{\cos. A . \cos. a}{\cos. H . \cos. h},$$

$$-\ (\cos. A \cos. a - \sin. A \overset{*}{} \sin. a.)$$

But the last term $= \cos. (A + a)$; subtract both sides of the equation from 1; then, since

$$1 - \cos. D = 2 . \sin.^2 \frac{D}{2},\ \text{and } 1 + \cos. (A + a) = 2 . \cos.^2 \frac{A + a}{2},$$

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* Cos. $\tfrac{1}{2}(d - H - h)$ if d be $> H + h$.

we have, dividing by 2, and making F to represent $\frac{\cos. A . \cos. a}{\cos. H . \cos. h}$,

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2} (A+a) - [\cos. \tfrac{1}{2}(H+h+d) \cos. \tfrac{1}{2}(H+h-d)] \times F$$

$$= \cos.^2 \tfrac{1}{2}(A + a) \left[1 - \frac{\cos. \tfrac{1}{2}(H+h+d).\cos. \tfrac{1}{2}(H+h-d)}{\cos.^2 \tfrac{1}{2}(A+a)} \times F\right]$$

and if we make the fraction, on the right-hand side of the equation, $= \sin.^2 \theta$, we shall have

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A + a) . \cos.^2 \theta,$$

$$\text{and } \sin. \frac{D}{2} = \cos. \tfrac{1}{2}(A + a) . \cos. \theta.$$

Hence, in logarithms, the rule of computation is

1st, $2 . \log. \sin. \theta = \log. \cos. \tfrac{1}{2}(H+h+d) + \log. \cos. \tfrac{1}{2}(H+h-d)$
$+ \log. \cos. A + \log. \cos. a + \text{ar. com. } \log. \cos H$
$+ \text{ar. com. } \log. \cos. h - 2 \log. \cos. \tfrac{1}{2}(A + a)$,

and 2ndly, $\log. \sin. \frac{D}{2} = \log. \cos. \tfrac{1}{2}(A + a) + \log. \cos. \theta - 10$*

The other parts (1,) (2), &c. p. 444, of the statement † have

* This formula of computation is Borda's. If in p. 445, l. 20, instead of substituting for $\sin. H \sin. h$, $\cos. H . \cos. h - \cos. (H + h)$, we substitute $\cos. (H - h) - \cos. H . \cos. h$, we may deduce the formula, which is the basis of Dr. Maskelyne's Rule inserted in the Introduction to Taylor's *Logarithms*, pp. 60, &c.

† The distance (see p. 444, l. 26.) between the Moon and a fixed Star is easily computed from their latitudes and the difference of their longitudes, the proper formula is

$$\sin.^2 \frac{D}{2} = \sin.^2 \left(\frac{l - l'}{2}\right) + \cos. l . \cos. l' . \sin.^2 \frac{k - k'}{2},$$

(see p. 366: also *Trig.* pp. 129. 131.) l, l', k, k', representing, in this case, the true latitudes and longitudes.

The Moon's latitude and longitude being computed and inserted in the Nautical Almanack, for noon and midnight, the Moon's distances from have either already received explanation, in the preceding pages of this Treatise, or are so plain as to need none. We proceed therefore to an Example.

EXAMPLE.

June 5, 1793, about an hour and an half after noon, in 10° 46′ 40″ south latitude, and 149° longitude, *by account* (see p. 409), by the mean of several observations, it appeared, that

Distance of nearest limbs of ☉ and ☽ - -	83°	26′	46″
Altitude of lowest limb of ☉ - - - - -	48	16	10
Altitude of upper limb of ☽ - - - - -	27	53	30

Here see (1) p. 443, we must add to the distance, the semi-diameters of the Sun and Moon, taking them from the Nautical Almanack.

The apparent distance of limbs of ☽ and ☉	83°	26′	46″
semi-diameter of ☉ - - -	0	15	46
of ☽ - - -	0	14	54
Augmentation propor[l]. to altitude, (see p. 318,)	0	0	7
Apparent distance (d) of centers - - - -	83	57	33

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from certain stars are computed, by the above formula, for those times; and, the distances for the intermediate times, at 3^h, 6^h, &c. are determined by *interpolation*, or by the aid of the formula in p. 52, l. 8. The latitudes and longitudes of the stars, are either to be computed, (see p. 56.) from their right ascensions and declinations, or to be immediately taken from certain Tables. (See Lalande's Tables, Nautical Almanack, 1773, *Connois. des Tems,* an. 12). This method is preferable to the computation of the distance from the declinations and difference of right ascensions of the Moon and Star, because the right ascension of the Moon is not computed, in the Nautical Almanack, as far as *seconds,* which the longitude and latitude are.

Reduction of the Apparent to the True Altitude. (See [3] p. 444.)

Altitude of Sun's lower limb - - - -	48°	16′	10″
Dip (see p. 421.) - - - - - - - -	0 –	4	24
	48	11	46
Semi-diameter - - - - - - - - - - -	0	15	46
Apparent altitude of Sun's center (h) - -	48	27	32
Refr. – Par. – correct. for Therm. see p. 88.	0	0 –	43
True alt. of Sun's center (a) - - - -	48	26	49
Altitude of Moon's upper limb - - - -	27°	53′	30″
Dip - - - - - - - - - - - - - - -	0 –	4	24
	27	49	6
Semi-diameter - - - - - - - - - -	0	15	1
Apparent altitude of Moon's center (H) -	27	34	5
Par. – Refr. + corr. for Therm. - - - -	0	46	43
True altitude of Moon's center (A) - - -	28	20	48

Reduction of the Apparent to the True Distance.
(See [5] p. 444, and *Formula*, p 446.)

d	83°	57′	33″		
h	48	27	32	ar. co. cos. =	.1783835
H	27	34	5	ar. co. cos. =	.0523390
Sum	159	59	10		
½ Sum	79	59	35	- cos. =	9.2399686
$d - \frac{1}{2}$ Sum	3	57	58	- cos. =	9.9989587
a	48	26	49	- cos. =	9.8217187
A	28	20	48	- cos. =	9.9445275
$A+a=$	76	47	37		39.2358960
$\frac{1}{2}(A+a)=$	38	23	48	2 log. cos.	19.7883324
				2)	19.4475636

log. sin. θ=9.7237818=log. sin. 31° 57′ 53″

Hence, log. cos. 31 57 53 9.9285875
log. cos. 38 23 48 9.8941662
(10 taken away) 9.8227537=log. sin. 41° 40′ 27″$\frac{3}{4}$

$$\therefore \frac{D}{2} = 41° \; 40' \; 27''\tfrac{2}{3}$$

and, $D = 83 \;\; 20 \;\; 55$, nearly

Time at Greenwich computed. (See [5] p. 444.)

By Nautical Almanack, (p. 70.)

Dist. ☽ from ☉ at 15h	83°	6′	1″	$D =$	83°	20′	55″
at 18	84	28	26	at 15h	83	6	1
Increase of dist. in 3h =	1	22	25		0	14	54

Hence,

1° 22′ 25″ : 14′ 54″ :: 3h : time corresponding to the increase 14′ 54″

* Hence, log. 3 = .4771213
log. 894″ = 2.9513375

3.4284588
log. 4945″ = 3.6941663

$\bar{1}$.7342925 = log. 0h.5425 = log. 32m 33s

Hence, the time at Greenwich = 15h 32m 33s.

Time at the Place of Observation computed. [See (6) p. 444. *also*, pp. 404, 405.]

L (Lat.)	10°	16′	40″	cos.	9.9929749
p	113	22	48	sin.	9.9627922
a	48	26	49		19.9557671
Sum	172	6	17		
½ Sum	86	3	8.5	cos.	8.8378712
½ Sum − *a*	37	36	19.5	sin.	9.7854864
				(20 added)	38.6233576
					19.9557671
					2)18.6675905
					9.3337901 = log. sin. 12° 27′ 17″½

* As this is a frequent operation in Nautical Astronomy, it is facilitated by means of approximate Tables of *Proportional Logarithms*, in which the log. 3h = 1. See Requisite Tables, Tab. XV. also Mendoza's Tables, Tab. XIV.

$\therefore$ hour angle (see p. 404.) = 24° 54′ 35″

(and in time, by Rule, p. 397.) = 1^h 39^m $38^s.3$

Hence, see (7) p. 444,

	h	m	s
Time at Greenwich, - - - - - - -	15	32	33
at place of observation - - - -	1	39	38.3
Long. from Greenwich *reckoning by the west*	13	52	54.7

$\therefore$ longitude *east* of Greenwich 10^h 7^m $4^s.3$.

The process for finding the longitude from the distance of the Moon from a *Star*, will be similar to the preceding: in deducing the true from the observed altitude, somewhat more simple; but, more tedious in the computation of the time from the altitude.

This latter computation, it is desirable to supersede, by reason, (see p. 407.) of the probable errors that will be made in observing the Star's altitude. And it may be superseded, by finding the time and regulating the chronometer by a previous or a subsequent observation of the Sun's altitude: by allowing for the change in longitude (see p. 409.) during the two observations; and then by *computing* the Star's altitude, from its north polar distance, the latitude, and the *estimated* time.

The proper formula of computation for this occasion is one that has repeatedly occurred, (see pp. 52. 364. 369.) If L be the latitude, p the north polar distance, h the estimated hour angle (see. p. 404.) and a the altitude, then,

$$\sin. a = \sin. L. \cos. p + \cos. L \sin. p . \cos. h,$$

whence, a may be computed by means of a subsidiary angle. (See *Trig*. pp. 129, 130, 131.)

Hence, the process for finding the longitude, (see p. 443.) although it does not essentially require the chronometer, is rendered more easy and accurate by its aid.

This is not the sole use of the chronometer. It enables the observer to use the *mean* of several observed distances of the Moon from a Star, or the Sun, instead of a single one. For, he cannot, without error, take the *mean*, except he know the several intervals of time that separate the successive observations. The chronometer enables him to ascertain these intervals.

Since the finding of the longitude is the most important and most difficult operation, in Nautical Astronomy, several expedients have been devised for facilitating it. *The distance has been cleared*, (see p. 444.) by a formula different, from that which has been given in (p. 445.) although derived from the same fundamental expression. Instead of a logarithmic computation, one proceeding solely by addition, and furnished with appropriate Tables, has been substituted. But, for a satisfactory explanation of the means and artifices, by which, on this occasion, the labour of computation is abridged and expedited, we must refer to the treatises that contain them. (See *Requisite Tables:* their explanation and use. Mendoza's *Treatise on Nautical Astronomy:* Brinkley; *Irish Transactions*, 1808: *Connoissance des Tems* for 1808, and for years 12 and 14: Mackay, *On the Longitude.*)

If we wish to reduce, to one of the classes (see p. 427), the preceding method of finding the longitude, we shall find that it belongs to the second. The principle on which it rests, is, indeed, precisely the same as that which forms the basis of the second method (see p. 435.) of finding the longitude from an occultation; For,

Analogous to the distance *D*	83°	20′	55″,	at 1^h 39^m 38^s
is the ☽'s longitude at Dublin,	67	18	43.3,	at 8 4 51.5
Analogous to the distance -	84	28	26,	at 18 (*Greenwich*)
is the Moon's longitude - -	67	22	26.1,	at 8 37 36.8

(for the Moon's longitude is a species of distance, being the distance of her place referred to the ecliptic from ♈). And the reduction of 84° 28′ 26″ to 83° 20′ 55″ by taking away 1° 7′ 31″, corresponding to 2^h 27^m 27^s, is analogous to the reduction of 67° 22′ 26″.1 to 67° 18′ 43″.3, by taking away 3′ 42″.8, corresponding to 7^m $23^s.3$; 1° 22′ 25″, being, in the former case, the change of the Moon's distance in 3^h and 30′ 9″.2, in the latter, the change of the Moon's longitude in 1^h: that is, in other words, the Moon's *horary motion in longitude.*

The problems then of deducing the longitude from an occultation, and from the distance of the Moon from a Star, are the same in principle; but the former is more difficult in its process, because, in *clearing* the observation of parallax, it is necessary to compute its resolved parts in the directions of longitude and latitude; whereas, in the latter, the entire effects of parallax, which take place in altitude, are alone considered.

The former, as a practical method of determining the longitude, is exceedingly more accurate than the latter*; because, we are enabled to mark the distance, which is the Moon's semi-diameter, and the corresponding time, which is that either of the immersion or emersion, with much greater precision, than we can measure the distance by means of a sextant, and compute the time from an observed altitude. But, as it has been observed in p. 443, the degree of accuracy does not alone determine the adoption of a method; we are obliged, in finding the longitude at sea, by the exigencies of the case, to rely solely on, what is called technically, the *Lunar Method*.

In finding the longitudes of places at land, circumstances also must determine which of the preceding methods must be adopted. Several have been proposed, not as if they might be indifferently used, but that observers may select from them, what are suited to their several wants, means, and opportunities. If the observer, furnished with a telescope and chronometer, wishes readily and soon to determine the longitude of the place where he is, he may use the method of the eclipses of *Jupiter's* Satellites, (see p. 429,) and obtain a result probably within 30 or 40 seconds of the truth. If he has the means of adjusting a telescope to move nearly in the plane of the meridian, the method of the transits of the Moon and of a fixed Star, (see p. 436,) will afford a more accurate result, and with an error, perhaps, not exceeding 10 seconds. But, if great accuracy be required, and expedition be not, then the observer

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* "For the present, I infer, we may take the difference of meridians (Greenwich and Paris) $9^m\ 20^s$, as being within a few seconds of the truth, till some *occultations* of fixed stars by the Moon, already observed, or hereafter to be observed, in favourable circumstances, and carefully calculated, shall enable us to establish it with the *last exactness*." Maskelyne, *On the Latitude and Longitude of Greenwich*, &c. *Phil. Trans.* 1787, p. 186. See also *Phil. Trans.* 1790, p. 230.

must wait for the opportunity of a Solar eclipse, or, what is better, of an occultation *, and thence compute the longitude †.

The several methods have their peculiar advantages and disadvantages: the last, which is the most accurate, requires computations of considerable length and nicety; the first probably inaccurate to the extent of $\frac{1}{5}$th of a degree, requires scarcely any. The second is more accurate, and may constantly be used, and therefore, on the whole, it is perhaps the readiest and best practical method.

The *Lunar method*, which is the least exact, is yet founded on the most refined theory, and the most complicated calculations. It depends, for its accuracy, entirely on previous computations. We cannot, in applying it, compare, as in the case of an occultation, (pp. 429, &c.) *actual* observations of the same phenomenon, or give accuracy to the result, by *correcting* (see p. 434,) the errors of the Tables. But, the mariner must be guided by the result, such as it comes out at the time of the observation, and which, a few hours after, will have lost all its utility.

In page 437, it was mentioned, that, in a merely theoretical point of view, the longitude ought to be afforded as a result, from the separation, during a given interval, of the Sun from a Star; but that the slow motion of the former, deprived the method of all

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* An occultation affords a more exact practical result than a Solar eclipse, because, in the former, the instant of immersion can be marked with greater precision, than the instant of contact in the latter.

The *recurrence* of occultations may be found as those of eclipses were, p. 352. We must find two numbers in the proportion, or nearly so, of $27^d.321661$ (the Moon's sidereal period) to $6793^d.42118$ (the sidereal revolution of the nodes): which numbers are 4227, and 17: and the period of recurrence is $316^y\ 72^d.1$ ($= 4227 \times 27.321661$).

† In speaking of the errors in the determination of the longitude, we have supposed the *mean*, of several observations accurately made with excellent instruments, to be taken. The errors of *single* observations will be much greater than what have been assigned to them. With the first satellite of *Jupiter* it may amount to $3^m\ 44^s$ according to Mr. Short. (See his Paper in the *Phil. Trans.* 1763, p. 167, for determining the difference of longitude between Greenwich and Paris, from the transits of *Mercury* over the Sun's disk).

practical utility. Now, the material circumstance that confers, what accuracy it possesses, on the *Lunar method*, is the Moon's quick change of place. Were the change greater, the method would be more accurate. For instance, the Moon now moves through 1° in about 2 hours, and therefore, an error of 1′, in observing and computing her distance, causes an error of 2 minutes of time, or of 30′ of longitude. But, if she moved through the same space (1°) in ½ hour, then the error of 1′ would cause only an error of 30ˢ of time, and of 7′½ of longitude.

Hence it follows, that the first satellite which moves round *Jupiter* in less than two days, (see p. 291,) must enable the inhabitants of that planet to determine very exactly the longitude: as exactly, as we can determine the latitude.

CHAP. XLIV.

On the Calendar.

THE Sun naturally regulates the beginnings, ends, and durations of the seasons (pp. 12, 212); and, the calendar is constructed to distribute and arrange the smaller portions of the year.

The calendar divides the year into 12 months, containing 365 days; and, it is desirable that it should always denote the same parts of the same season by the same days of the same months, that, for instance, the Summer and Winter solstices, if once happening on the 21st of June and 21st of December, should, ever after, be reckoned to happen on the same days; that, the date of the Sun's entering the equinox, the natural commencement of Spring, should, if once, be always on the 20th of March. For thus, the labours of agriculture, which really depend on the situation of the Sun in the heavens, would be simply and truly regulated by the calendar.

This would happen, if the civil year of 365 days were equal to the astronomical; but, (see p. 65,) the latter is greater; therefore, if the calendar should invariably distribute the year into 365 days, it would fall into this kind of confusion; that, in progress of time, and successively, the vernal equinox would happen on every day of the civil year. Let us examine this more nearly.

Suppose the excess of the astronomical year above the civil to be exactly 6 hours, and, on a certain year, on the noon of March 20th, the Sun to be in the equinoctial point; then, after the lapse of a civil year of 365 days, the Sun would be on the meridian, but not in the equinoctial point; it would be to the west of that point; and would have to move 6 hours in order to reach it, and to complete (see pp. 65, &c.) the astronomical or tropical year.

At the completions of a second, and a third civil year, the Sun would be still more and more remote from the equinoctial point: and, would be obliged to move, respectively, for 12 and 18 hours, before he could rejoin it, and complete the astronomical year.

At the completion of a fourth civil year, the Sun would be more distant, than on the two preceding ones, from the equinoctial point. In order to rejoin it, and to complete the astronomical year, he must move for 24 hours, that is, for *one whole day*. In other words, the astronomical year would not be completed till the beginning of the next astronomical day; till, in civil reckoning, the *noon of March* 21st.

At the end of four more common civil years, the Sun would be in the equinox on the noon of March 22. At the ends of 8 and 64 years, on March 23, and April 6, respectively: at the end of 736 years, the Sun would be in the vernal equinox on September 20. And, in a period of about 1508 years, the Sun would have been in every sign of the Zodiac on the same day of the calendar, and in the same sign on every day.

If the excess of the astronomical above the civil year, were really, what we have supposed it to be, 6 hours, this confusion of the calendar might be, most easily, avoided. It would be necessary merely to make every fourth civil year to consist of 366 days; and, for that purpose, to interpose, or to *intercalate* a day in a month previous to March. By this *intercalation* what would have been March 21st is called March 20th; and accordingly, the Sun would be still in the equinox on the same day of the month.

This mode of correcting the calendar was adopted by Julius Cæsar. The fourth year into which the intercalary day is introduced was called *Bissextile*: it is now frequently called the *Leap* year. The correction is called the *Julian correction*, and the length of a mean Julian year is equal to $365^d.25$.

The astronomical year (see p. 306.) is equal to $365^d.242264$, and, accordingly, is less than the mean Julian by $0^d.007736$. The *Julian correction*, therefore, itself needs a correction. The calendar, regulated by it, would, in progress of time, become erroneous, and would require *reformation.*

The intercalation of the Julian correction being too great, its effect would be to *antedate* the happening of the equinox. Thus, to return to the old illustration, the Sun at the completion of the fourth civil year, now the Bissextile, would have passed the equinoctial point, by a time equal to four times $0^d.007736$: at the end of the next Bissextile, by eight times $0^d.007736$: at the end of 129 years, nearly by one day. In other words, the Sun would have been in the equinoctial point 24 *hours previously,* or on the *noon of March* 19th.

In the lapse of ages, this error would continue and be increased. Its accumulation in 1292 years would amount, nearly, to 10 days, and then, the vernal equinox would be reckoned to happen on March 10th.

The error into which the calendar had fallen, and would continue to fall, was noticed by Pope Gregory in 1582. At his time, the length of the year was known to greater precision, than at the time of Julius Cæsar. It was supposed equal to $365^d\ 5^h\ 49^m\ 16^s.23$. Gregory, desirous that the vernal equinox should be reckoned on or near March 21st, (on which day it happened in the year 325, when the Council of Nice was held,) ordered that the day succeeding the 4th October 1582, instead of being called the 5th, should be called the 15th; thus, suppressing 10 days, which, in the interval between the years 325 and 1582, represented, nearly, the accumulation of error arising from the *excessive intercalation of the Julian correction.*

This act *reformed* the calendar: in order to *correct* it in future ages, it was prescribed, that at certain convenient periods, the intercalary day of the Julian correction should be omitted. Thus, the centenary years, 1700, 1800, 1900, are (as every year divisible by 4 is) according to the Julian correction, Bissextiles, but on these it was ordered that the intercalary day *should not be inserted:* inserted again in 2000, but not inserted in 2100, 2200, 2300; and so on for succeeding centuries.

This is a most simple mode of regulating the calendar. It corrects the insufficiency of the Julian correction by omitting, in the space of 400 years, 3 intercalary days. And, it is easy to estimate the degree of its accuracy. For, the real error of the Julian correction is 0 .007736 in 1 year, consequently, $4 \times .7736$, or $3^d.0944$ in 400 years. Consequently, $0^d.0944$, or, $2^h\ 15^m\ 56^s.16$ in 400 years, or 1 day in 4237 years is the measure of the degree of inaccuracy in the Gregorian correction. Against such, it plainly is not worth the while to make any formal provision in the mode of regulating the calendar.

The calendar may be thus examined and regulated, without the aid of mathematical processes and formulæ. Yet, on this subject, the method of *continued Fractions** is frequently employed. This, however, is to use an instrument too *fine* for the occasion. The results have a degree of exactness, beyond what we require, or can practically avail ourselves of. The only thing, in the correction of the calendar, that requires a high degree of mathematical science, is the determination of the length of the astronomical year. Had this been known, to a greater exactness, by the Astronomers of the time of Julius Cæsar, the Julian correction would, probably, have superseded the necessity of the Gregorian.

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* Since the excess of the tropical year above the civil is $0^d.242264$, the exact intercalation is that of 242264 days, in 1000000 years. But, since this intercalation would be of no practical use, we must find numbers nearly in the ratio of 242264 to 1000000: which may be effected by the method of continued fractions, as in pages 279, 280, &c. See on this subject, Euler's *Algebra*. Addition, pp. 426, &c. edit. 1774.

TABLES.

TABLE I. The mean Right Ascensions of 36 principal Fixed Stars, to the beginning of 1802; with their Annual Precessions, and Annual Proper Motions, from the latest Observations.

NAMES OF STARS.	Right Ascension in Signs, Degrees, &c. S. D. M. S.	Annual Precession. S.	Annual proper Motion. S.
γ Pegasi	0 0 45 46.8	46.13	—0.09
α Arietis	0 29 0 34.1	50.08	+0.10
α Ceti	1 12 59 6.3	46.84	—0.12
Aldebaran	2 6 8 34.5	51.34	+0.03
Capella	2 15 31 14.4	66.01	+0.21
Rigel	2 16 15 24 1	43.17	—0.03
β Tauri	2 18 26 45.8	56.70	+0.01
α Orion	2 26 6 48.9	48.65	+0.01
Sirius	3 9 6 21.8	40.21	—0.42
Castor	3 20 29 7.7	57.95	—0 15
Procyon	3 22 13 54.0	47.93	—0 80
Pollux	3 23 17 40.2	56.05	—0 74
α Hydræ	4 19 27 49.5	44.28	—0.09
Regulus	4 29 27 12.0	48.41	—0.22
β Leonis	5 24 44 13.1	46.58	—0.57
β Virginis	5 25 5 41.6	46.14	+0.74
Spica Virginis	6 18 41 40 5	47.22	—0 02
Arcturus	7 1 39 28.8	42.18	—1.26
1 α Libræ	7 9 56 22.2	49.54	—0.11
2 α Libræ	7 9 59 12.0	49 56	—0.11
α Cor. Bor.	7 21 34 36 8	37.92	+0.26
α Serpentis	7 23 37 50.8	44.05	+0.11
Antares	8 4 19 21.5	54.87	0.00
α Herculis	8 16 24 21.1	40.97	0 00
α Ophiuchi	8 21 26 12.1	41.58	+0 06
α Lyræ	9 7 33 29.5	30 20	+0.23
γ Aquilæ	9 24 12 38.0	42.80	—0.11
α Aquilæ	9 25 16 46.9	43.40	+0.48
β Aquilæ	9 26 23 46.1	44.21	—0.03
1 α Capricorni	10 1 39 55.3	50.04	0.00
2 α Capricorni	10 1 45 52.2	50.04	+0 05
α Cygni	10 8 40 13.7	30 63	—0.08
α Aquarii	10 28 54 6.4	46.29	—0.08
Fomalhaut	11 11 40 12.1	49.80	+0.35
α Pegasi	11 13 43 33.5	44.64	—0 06
α Andromedæ	11 29 32 39.1	45.97	+0.03

TABLE I. *continued.* The North Polar Distances of 36 principal Stars, to the beginning of 1802; with their Annual Precessions, and Annual Proper Motions, from the latest Observations.

NAMES OF STARS.	Distance from the North Pole. D.	M.	S.	Annual Precession. s.	Annual Proper Motion. s.
γ Pegasi	75	54	55 9	−20.05	−0.15 N
α Arietis	67	28	45.2	−17.54	+0 07 S
α Ceti	86	41	36.6	−14 67	−0 08 N
Aldebaran	73	54	0.6	− 8.11	+0.12 S
Capella	44	13	12.4	− 5.01	+0 44 S
Rigel	98	26	19.0	− 4.76	−0.16 N
β Tauri	61	34	23.1	− 4.02	+0.10 S
α Orion	82	38	27.8	− 1.36	−0.13 N
Sirius	106	27	4.7	+ 3.17	+1.04 S
Castor	57	41	28.0	+ 7.02	+0.04 S
Procyon	84	16	34.4	+ 7.59	+0.95 S
Pollux	61	30	25.7	+ 7.93	0.00
α Hydræ	97	48	19.3	+15.24	−0.14 N
Regulus	77	4	9.5	+17.27	−0.08 N
β Leonis	74	19	14.6	+19.97	+0.07 S
β Virginis	87	7	6.7	+19.98	+0.24 S
Spica Virginis	100	7	14.6	+18 99	−0.19N
Arcturus	69	46	45.4	+17.07	+1.72 S
1 α Libræ	105	9	41.9	+15.37	−0.18 N
2 α Libræ	105	12	26.0	+15.36	−0.15 N
α Cor. Bor.	62	36	35.5	+12 46	+0.03 S
α Serpentis	82	56	24.6	+11.89	−0.19 N
Antares	115	58	31.2	+ 8.69	−0.26 N
α Herculis	75	22	15.6	+ 4.71	−0.23 N
α Ophiuchi	77	17	0.0	+ 2.99	+0.05 S
α Lyræ	51	23	35.3	− 2.64	−0.27 N
γ Aquilæ	79	51	26.8	− 8.22	−0.16 N
α Aquilæ	81	38	33.8	− 8.56	−0.54 N
β Aquilæ	84	4	33.0	− 8.91	+0.35 S
1 α Capricorni	103	6	22.9	−10.53	−0.28 N
2 α Capricorni	103	8	41.6	−10.56	−0.26 N
α Cygni	45	25	16.4	−12.53	−0.03 N
α Aquarii	91	16	25.1	−17.17	−0.19 N
Fomalhaut	120	39	52.7	−19.04	−0.06 N
α Pegasi	75	51	18.2	−19.25	−0.18 N
α Andromedæ	62	0	5.8	−20.05	+0.06 S

TABLE II.

The mean Astronomical Refractions.
Barom. 29.6. Thermom. 50. See Chap. IX.

Apparent Zen. Dist.	Refraction.	Apparent Zen. Dist.	Refraction.	Apparent Zen. Dist.	Refraction.
D.	M. S.	D.	M. S.	D.	M. S.
1	0 1.0	31	0 34.2	61	1 42.5
2	0 2.0	32	0 35.6	62	1 46.8
3	0 3.0	33	0 37.0	63	1 51 5
4	0 4.0	34	0 38.4	64	1 56 4
5	0 5.0	35	0 39.9	65	2 1.7
6	0 6.0	36	0 41.4	66	2 7.4
7	0 7.0	37	0 42.9	67	2 13.6
8	0 8.0	38	0 44.5	68	2 20.3
9	0 9.0	39	0 46.1	69	2 27.5
10	0 10.0	40	0 47.8	70	2 35.5
11	0 11.1	41	0 49.5	71	2 44.3
12	0 12.1	42	0 51.3	72	2 53.9
13	0 13.1	43	0 53.[illegible]	73	3 4.7
14	0 14.2	44	0 55.0	74	3 16.7
15	0 15.3	45	0 56 9	75	3 30 2
16	0 16.3	46	0 58.9	76	3 45.5
17	0 17.4	47	1 1.0	77	4 3.0
18	0 18.5	48	1 3.2	78	4 23.2
19	0 19.6	49	1 5.4	79	4 46.8
20	0 20.7	50	1 7.8	80	5 14.8
21	0 21.9	51	1 10.2	81	5 48.4
22	0 23.0	52	1 12.8	82	6 29.5
23	0 24.2	53	1 15.5	83	7 20.9
24	0 25.4	54	1 18.3	84	8 26.4
				85	9 52.5
25	0 26.6	55	1 21.2		
26	0 27.8	56	1 24.3		
27	0 29.0	57	1 27.5		
28	0 30.3	58	1 31.0		
29	0 31.6	59	1 34.6		
30	0 32.9	60	1 38.4		

TAB. III.

The ☉'s Paral^x. in Altitude.

Zen. Dist	Sun's Parallax.
O.	S.
0	0.0
3	0.4
6	0 9
9	1 4
12	1.8
15	2.3
18	2.7
21	3.1
24	3.6
27	4.0
30	4.4
33	4.8
36	5.2
39	5.5
42	5.9
45	6.2
48	6.5
51	6.8
54	7.1
57	7.4
60	7.6
63	7.8
66	8.0
69	8.2
72	8.4
75	8.5
78	8.6
81	8.7
84	8.7
87	8.8
90	8.8

TABLE IV.

Table for determining, nearly, the Times when the Principal Stars are on the Meridian. (See pp. 54. 396, &c.)

Star's Mean R. A. in 1811.

	Mean R. A.	
γ Pegasi -	0^h	3^m
Polaris -	0	55
α Arietis -	1	56
α Ceti - -	2	52
Aldebaran	4	25
Capella -	5	3
Rigel - -	5	5
β Tauri - -	5	14
α Orionis -	5	45
Sirius - -	6	37
Castor -	7	22
Procyon -	7	29
Pollux -	7	34
α Hydræ -	9	18
Regulus -	9	58
β Leonis - -	11	39
β Virginis -	11	41
Spica Virginis	11	15
Arcturus	14	7
α Libræ -	14	40
α Coronæ Bor.	15	27
α Serpentis	15	35
Antares -	16	18
α Herculis -	17	6
α Ophiuchi	17	26
α Lyræ - -	18	30
α Aquilæ -	19	41
α Capricorni	20	7
α Cygni -	20	35
α Aquarii -	21	56
Fomalhaut	22	47
α Pegasi -	22	55
α Andromedæ	23	59

Sun's Mean R. A. in 1811.

	Hours.	Days.	M.	S.
Jan. 6,	19	1	3	56
21,	20	2	7	53
Feb. 5,	21	3	11	49
20,	22	4	15	46
Mar. 7,	23	5	19	43
22,	0	6	23	39
Apr. 7,	1	7	27	36
22,	2	8	31	32
May 7,	3	9	35	29
22,	4	10	39	28
June 7,	5	11	43	22
22,	6	12	47	18
July 7,	7	13	51	15
22,	8	14	55	12
Aug. 7,	9	15	59	8
22,	10			
Sept. 6,	11			
21,	12			
Oct. 6,	13			
21,	14			
Nov. 6,	15			
21,	16			
Dec. 6,	17			
21.	18			

Rule, see p. 396,
✱'s R. A. – ☉'s R. A. = meridional passage ✱, nearly.

EXAMPLE.

Required the Meridional Passage of α Capricorni. Nov. 8, 1811.

☉'s R. A. Nov. 6	15^h	0^m	0^s
Motion for two days	0	7	53
☉'s R. A. Nov. 8 -	15	7	53
✱'s R. A. - - - -	20	7	0
App. time of passage	4	59	7

TABLE V.

Epochs of the mean Longitude of the Moon's ascending Node.

Years.	S.	D.	M.	Years.	S.	D.	M.	Years.	S.	D.	M.	Years.	S.	D.	M.
1752	8	1	37	1785	10	23	20	1817	2	4	27	1849	5	15	31
1753	7	12	17	1786	10	4	1	1818	1	15	7	1850	4	26	11
1754	6	22	57	1787	9	14	41	1819	0	25	48	1851	4	6	51
1755	6	3	37	1788	8	25	18	1820	0	6	25	1852	3	17	28
1756	5	14	14	1789	8	5	58	1821	11	17	5	1853	2	28	8
1757	4	24	55	1790	7	16	39	1822	10	27	45	1854	2	8	48
1758	4	5	35	1791	6	27	19	1823	10	8	26	1855	1	19	29
1759	3	16	15	1792	6	7	56	1824	9	19	3	1856	1	0	6
1760	2	26	52	1793	5	18	36	1825	8	29	43	1857	0	10	46
1761	2	7	33	1794	4	29	17	1826	8	10	23	1858	11	21	26
1762	1	18	13	1795	4	9	57	1827	7	21	4	1859	11	2	7
1763	0	28	53	1796	3	20	34	1828	7	1	41	1860	10	12	44
1764	0	9	30	1797	3	1	14	1829	6	12	21	1861	9	23	24
1765	11	20	11	1798	2	11	55	1830	6	23	1	1862	9	4	4
1766	11	0	51	1799	1	22	35	1831	5	3	42	1863	8	14	45
1767	10	11	31	1800	1	3	16	1832	4	14	19	1864	7	25	22
1768	9	22	8	1801	0	13	55	1833	3	24	59	1865	7	6	2
1769	9	2	49	1802	11	24	36	1834	3	5	39	1866	6	16	42
1770	8	13	29	1803	11	5	16	1835	2	16	20	1867	5	27	23
1771	7	24	9	1804	10	15	53	1836	1	26	57	1868	5	8	0
1772	7	4	46	1805	9	26	33	1837	1	7	37	1869	4	18	40
1773	6	15	27	1806	9	7	14	1838	0	18	17	1870	3	29	20
1774	5	26	7	1807	8	17	54	1839	11	28	58	1871	3	10	1
1775	5	6	47	1808	7	28	31	1840	11	9	35	1872	2	20	38
1776	4	17	24	1809	7	9	11	1841	10	20	15	1873	2	1	18
1777	3	28	5	1810	6	19	52	1842	10	0	55	1874	1	11	58
1778	3	8	45	1811	6	0	32	1843	9	11	35	1875	0	22	39
1779	2	19	25	1812	5	11	9	1844	8	22	13	1876	0	3	16
1780	2	0	2	1813	4	21	49	1845	8	2	53	1877	11	13	56
1781	1	10	42	1814	4	2	30	1846	7	13	33	1878	10	24	36
1782	0	21	23	1815	3	13	10	1847	6	24	13	1879	10	5	17
1783	0	2	3	1816	2	23	47	1848	6	4	51	1880	9	15	54
1784	11	12	40												

TABLE VI.

Retrograde Motion of the Moon's Node to every Day in the Year. Subtract from the Longitude of the Epoch.

Days.	Jan.		Feb.		Mar.		April		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.	
	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M
1	0	3	1	42	3	11	4	49	6	24	8	3	9	38	11	17	12	55	14	31	16	9	17	44
2	0	6	1	45	3	14	4	52	6	28	8	6	9	41	11	20	12	58	14	34	16	12	14	48
3	0	10	1	48	3	17	4	55	6	31	8	9	9	45	11	23	13	2	14	37	16	15	17	51
4	0	13	1	51	3	20	4	59	6	34	8	12	9	48	11	26	13	5	14	40	16	19	17	54
5	0	16	1	54	3	23	5	2	6	37	8	16	9	51	11	29	13	8	14	43	16	22	17	57
6	0	19	1	58	3	27	5	5	6	40	8	19	9	54	11	33	13	11	14	46	16	25	18	0
7	0	22	2	1	3	30	5	8	6	44	8	22	9	57	11	36	13	14	14	50	16	28	18	3
8	0	25	2	4	3	33	5	11	6	47	8	25	10	1	11	30	13	17	14	53	16	31	18	7
9	0	29	2	7	3	36	5	15	6	50	8	28	10	4	11	42	13	21	14	56	16	34	18	10
10	0	32	2	10	3	39	5	18	6	53	8	32	10	7	11	45	13	24	14	59	16	38	18	13
11	0	35	2	13	3	42	5	21	6	56	8	35	10	10	11	49	13	27	15	2	16	41	18	16
12	0	38	2	17	3	46	5	24	6	59	8	38	10	13	11	52	13	30	15	6	16	44	18	19
13	0	41	2	20	3	49	5	27	7	3	8	41	10	16	11	55	13	33	15	9	16	47	18	23
14	0	44	2	23	3	52	5	30	7	6	8	44	10	20	11	58	13	37	15	12	16	50	18	26
15	0	48	2	26	3	55	5	34	7	9	8	47	10	23	12	1	13	40	15	15	16	54	18	29
16	0	51	2	29	3	58	5	37	7	12	8	51	10	26	12	4	13	43	15	18	16	57	18	32
17	0	54	2	33	4	1	5	40	7	15	8	54	10	29	12	8	13	46	15	21	17	0	18	35
18	0	57	2	36	4	5	5	43	7	18	8	57	10	32	12	11	13	49	15	25	17	3	18	38
19	1	0	2	39	4	8	5	46	7	22	9	0	10	36	12	14	13	52	15	28	17	6	18	42
20	1	4	2	42	4	11	5	50	7	25	9	3	10	39	12	17	13	56	15	31	17	9	18	45
21	1	7	2	45	4	14	5	53	7	28	9	7	10	42	12	20	13	59	15	34	17	13	18	48
22	1	10	2	48	4	17	5	56	7	31	9	10	10	45	12	23	14	2	15	37	17	16	18	51
23	1	13	2	52	4	21	5	59	7	34	9	13	10	48	12	27	14	5	15	40	17	19	18	54
24	1	16	2	55	4	24	6	2	7	38	9	16	10	51	12	30	14	8	11	44	17	22	18	57
25	1	19	2	58	4	27	6	5	7	41	9	19	10	55	12	33	14	11	15	47	17	25	19	1
26	1	23	3	1	4	30	6	9	7	44	9	22	10	58	12	36	14	15	15	50	17	28	19	4
27	1	26	3	4	4	33	6	12	7	47	9	26	11	1	12	39	14	18	15	53	17	32	19	7
28	1	29	3	7	4	36	6	15	7	50	9	29	11	4	12	43	14	21	15	56	17	35	19	10
29	1	32			4	40	6	18	7	53	9	32	11	7	12	46	14	24	16	0	17	38	19	13
30	1	35			4	43	6	21	7	57	9	35	11	10	12	49	14	27	16	3	17	41	19	17
31	1	38			4	46			8	0			11	14	12	52			16	6			19	20

In the Months Jan. and Feb. in Leap-Year, take out for the Day preceding the given Day.

GENERAL TABLES OF ABERRATION.

TABLE VII.

Argument, the Sun's true Longitude.

Degrees.	0s	6s	1s	7s	2s	8s	
	Log. a−	x +	Log. a−	x +	Log. a −	x +	
0	1,2690	0° 0′	1,2790	2° 11′	1,2977	2° 6′	30
1	1,2690	0 5	1,2796	2 14	1.2983	2 3	29
2	1,2691	0 11	1,2802	2 16	1,2988	2 0	28
3	1,2692	0 16	1,2808	2 18	1,2993	1 57	27
4	1,2692	0 22	1,2815	2 20	1,2998	1 54	26
5	1,2693	0 27	1,2821	2 21	1,3003	1 51	25
6	1,2695	0 32	1,2827	2 23	1,3008	1 47	24
7	1,2696	0 37	1,2834	2 24	1,3012	1 44	23
8	1,2698	0 43	1,2840	2 25	1,3017	1 40	22
9	1,2700	0 48	1,2847	2 26	1,3021	1 36	21
10	1,2703	0 53	1,2853	2 27	1,3025	1 32	20
11	1,2705	0 58	1,2860	2 28	1,3028	1 28	19
12	1,2708	1 3	1,2866	2 28	1,3032	1 24	18
13	1,2711	1 8	1,2873	2 28	1,3036	1 20	17
14	1,2714	1 12	1,2879	2 28	1,3039	1 16	16
15	1,2718	1 17	1,2886	2 28	1,3042	1 11	15
16	1,2721	1 22	1,2892	2 28	1,3045	1 7	14
17	1,2725	1 26	1,2899	2 27	1,3048	1 3	13
18	1,2729	1 30	1,2905	2 27	1,3050	0 58	12
19	1,2733	1 34	1,2912	2 26	1,3053	0 53	11
20	1,2738	1 39	1,2918	2 25	1,3055	0 49	10
21	1,2742	1 42	1,2924	2 24	1,3057	0 44	9
22	1,2747	1 46	1,2931	2 22	1,3059	0 39	8
23	1,2752	1 50	1,2938	2 21	1,3060	0 34	7
24	1,2757	1 53	1,2944	2 19	1,3061	0 30	6
25	1,2762	1 57	1,2949	2 17	1,3063	0 25	5
26	1,2768	2 0	1,2956	2 15	1,3064	0 20	4
27	1,2773	2 3	1,2961	2 13	1,3064	0 15	3
28	1,2779	2 6	1,2966	2 11	1,3065	0 10	2
29	1,2785	2 9	1,2972	2 8	1,3065	0 5	1
30	1,2790	2 11	1,2977	2 6	1,3065	0 0	0
	Log. a	x −	Log. a	x −	Log. a	x −	Degrees.
	5s	11s	4s	10s	3s	9s	

TABLE VIII.

Argument, the Sun's Longitude plus or minus the Declination.

Degrees.	0s 6s − +	1s 7s − +	2s 8s − +	
0	4″,03	3″,49	2″,02	30
1	4 ,03	3 ,46	1 ,95	29
2	4 ,03	3 ,42	1 ,89	28
3	4 ,03	3 ,38	1 ,85	27
4	4 ,02	3 ,34	1 ,77	26
5	4 ,02	3 ,30	1 ,70	25
6	4 ,01	3 ,26	1 ,64	24
7	4 ,00	3 ,22	1 ,58	23
8	3 ,99	3 ,18	1 ,51	22
9	3 ,98	3 ,13	1 ,45	21
10	3 ,97	3 ,09	1 ,38	20
11	3 ,96	3 ,04	1 ,31	19
12	3 ,95	3 ,00	1 ,25	18
13	3 ,93	2 ,95	1 ,18	17
14	3 ,91	2 ,90	1 ,11	16
15	3 ,90	2 ,85	1 ,04	15
16	3 ,88	2 ,80	0 ,98	14
17	3 ,86	2 ,75	0 ,91	13
18	3 ,84	2 ,70	0 ,84	12
19	3 ,81	2 ,65	0 ,77	11
20	3 ,79	2 ,59	0 ,70	10
21	3 ,77	2 ,54	0 ,63	9
22	3 ,74	2 ,48	0 ,56	8
23	3 ,71	2 ,43	0 ,49	7
24	3 ,68	2 ,37	0 ,42	6
25	3 ,66	2 ,31	0 ,35	5
26	3 ,63	2 ,26	0 ,28	4
27	3 ,59	2 ,20	0 ,21	3
28	3 ,56	2 ,14	0 ,14	2
29	3 ,53	2 ,08	0 ,07	1
30	3 ,49	2 ,02	0 ,00	0
	+ − 5s 11s	+ − 4s 10s	+ − 3s 9s	Degrees.

The Logarithms of *a* Tab. VII. and of *b* Tab. IX. have the sign – prefixed, to indicate that they belong to negative coefficients.

When the sines and cosines are negative, affix – to their logarithms. Their product will be negative if the number of – signs is odd.

Affix – to the declination, if it be south,

GENERAL TABLE OF NUTATION.

TABLE IX.

Argument, the mean Longitude of the Node.

Degrees	0°		6s	1s		7s	2s		8s	
	Log. *b* −	*B* −	*c* −+	Log. *b* −	*B* −	*c* −+	Log. *b* −	*B* −	*c* −+	
0	0,9944	0° 0′	0″00	0,9588	6° 45′	8″27	0,8960	7° 48′	14″35	30
1	0,9944	0,15	0,29	0,9571	6 54	8,52	0,8939	7 40	14,47	29
2	0,9943	0,31	0,58	0,9554	7 3	8,77	0,8917	7 32	14,61	28
3	0,9942	0,46	0,87	0,9536	7 12	9,01	0,8896	7 23	14,74	27
4	0,9940	1, 1	1,15	0,9518	7 20	9,25	0,8875	7 14	14,87	26
5	0,9937	1,16	1,44	0,9500	7 28	9,49	0,8854	7 4	14,99	25
6	0,9834	1,32	1,75	0,9481	7 36	9,72	0,8834	6 53	15,11	24
7	0,9830	1,47	2,02	0,9462	7 43	9,96	0,8814	6 42	15,23	23
8	0,9825	2, 2	2,03	0,9442	7 49	10,19	0 8795	6 29	15,34	22
9	0,9821	2,17	2,59	0,9422	7 55	10,41	0,8776	6 17	15,45	21
10	0,9815	2,31	2,87	0,9402	8 1	10,63	0,8758	6 3	15,55	20
11	0,9809	2,46	3,16	0,9382	8 6	10,85	0,8740	5 49	15,64	19
12	0,9802	3, 1	3,44	0,9361	8 10	11,07	0,8723	5 35	15,73	18
13	0,9795	3,15	3,72	0,9340	8 14	11,28	0,8707	5 20	15,82	17.
14	0,9787	3,29	4,00	0,9318	8 17	11,49	0,8692	5 4	15,90	16
15	0,9779	3,45	4,28	0,9297	8 20	11,70	0,8677	4 48	15,98	19
16	0,9770	3,57	4,56	0,9275	8 23	11,90	0,8663	4 31	16,05	14
17	0,9760	4,11	4,84	0,9253	8 24	12,10	0,8659	4 14	16,12	13
18	0,9750	4,24	5,11	0,9231	8 25	12,30	0,8637	3 56	16,18	12
19	0,9739	4,37	5,39	0,9208	8 25	12,49	0,8625	3 38	16,24	11
20	0,9728	4,50	5,66	0,9186	8 25	12,67	0,8615	3 20	16,29	10
21	0,9716	5, 3	5,93	0,9163	8 24	12,86	0,8605	3 1	16,34	9
22	0,9704	5,16	6,20	0,9140	8 23	13,04	0,8596	2 41	16,38	8
23	0,9691	5,28	6,46	0,9118	8 21	13,21	0,8588	2 22	16,42	7
24	0,9678	5,40	6,73	0,9095	8 18	13,38	0,8582	2 2	16,45	6
25	0,9664	5,51	6,99	0,9072	8 15	13,55	0,8576	1 42	16,48	5
26	0,9650	6, 3	7,25	0,9050	8 11	13,72	0,8571	1 22	16,50	4
27	0,9635	6,14	7,51	0,9027	8 6	13,83	0,8568	1 2	16,52	3
28	0,9620	6,24	7,77	0,9005	8 1	14,03	0,8565	0 41	16,53	2
29	0,9604	6,35	8,02	0,8983	7 55	14,18	0,8563	0 21	16,54	1
30	0,9588	6,45	8,27	0,8960	7 48	14,35	0,8563	0 0	16,54	0
	Log. *b*	+ *B*	−+ *c*	Log. *b*	+ *B*	−+ *c*	Log. *b*	+ *B*	−+ *c*	Degrees.
	5s		11s	4s		10s	3s		9s	

USE OF THE TABLES.

TAB. I. *Required the Right Ascension and North Polar Distance of* α Cygni, *on Dec.* 17, 1807.

R. A. in 1802,	10ˢ	8°	40′	13″.7	- Precession	-	30″.63
Preces. for 6 years	0	0	3	3.78			6
∴ R. A. required	10	8	43	17.48		3	3.78

N. P. D. in 1802,	45°	25′	16″.4	- Precession		− 12″.53
Precess. for 6 years		−1	15.18			6
N. P. D. required	45	24	1.22		−1	15.18
∴ Declination =	44	35	58.78			

The interval between Jan. 1, 1802, and Dec. 17, 1807, is nearly 6 years: if instead of Dec. 17, the time had been June 17, 1807, then, the annual precessions instead of being multiplied by 6, would have been multiplied by 5.5: and, like alterations must be made for other cases.

TAB. V. VI. *Required the Longitude* (☊) *of the Moon's ascending Node on Dec.* 17, 1807.

Tab. V. Epoch - - - - - - - - - - -	8ˢ	17°	54′
Tab. VI. Dec. 17. - - - - - - - - - -	0	18	35
∴ ☊ =	7	29	19

These Tables are introduced as being subsidiary to the Table of Nutation.

TAB. VII. VIII. *These Tables are constructed by M. Gauss, and inserted in the Connoissance des Tems for* 1810. *Instead of* 20″ (see p. 129,) *M. Gauss used* 20″.255 *to express the major axis of the Circle of Aberration*; *and, in this he agrees with M. Delambre.*

The expressions from which these Tables are constructed, are

Aberration in R. A. $(dA) = -\frac{\alpha \cos. \omega . \cos. \odot}{\cos. (\odot + x)} \times \frac{\cos. (\odot + x - A)}{\cos. D}$,

$$\text{Aber. in dec. } (dD) = -\frac{\alpha \cdot \cos.\omega \cdot \cos.\odot}{\cos.(\odot + x)} \cdot \sin. D . \sin. (\odot + x - A) \, [m]$$

$$-\frac{\alpha}{2} \cdot \sin.\omega \cdot \cos.(\odot + D) - \frac{\alpha}{2} \sin.\omega \cos.(\odot - D),$$

in which, $\alpha = 20''.255$, ω = obliquity, $\odot$ = Sun's longitude, A = ✱'s R. A. D = ✱'s dec. and x is to be determined from tan. $(\odot + x)$ = tan. $\odot$. sec. ω : a, in the Tables, represents the coefficient $\frac{\alpha \cdot \cos.\omega \cdot \cos.\odot}{\cos.(\odot + x)}$.

EXAMPLE. *Required the Aberrations in Right Ascension and Declination of* α Cygni, *on Dec.* 17, 1807.

(From the Nautical Almanack) - - - -	$\odot$ =	8^s	25° 9′
Tab. VII. - - - - - - - - - - - - - -	x =		+24
	$\odot + x$ =	8	25 33
(see p. 469, l. 6.) A		10	8 43
	$\odot + x - A$ =	10	16 50
(See p. 469, l. 10.)	D =	1	14 35.58

Tab. VII. log. a - -	1.3063	log. a - - - - - -	1.3063
log. cos. $(\odot + x - A)$	9.8629	log. sin. $(\odot + x - A)$	9.8351
Ar. com. log. cos. D	0.1475	log. sin. D - - -	9.8464
∴ log. (dA) - - -	1.3167	∴ log. m - - - -	0.9878+
∴ $dA = -20''.735$			∴ $m = +9.72$

Again, $\odot + D$ =	10^s 9° 45′	Tab. VIII. - - - -	− 2.58
$\odot - D$ =	7 10 33	Tab. VIII. - - -	+ 3.06
		∴ $(d\,D)$ =	+10.2

TAB. IX. *This Table also is constructed by M. Gauss. The expressions, from which the Tables are constructed, are*

$$\text{Nutation in R. A.} = -\frac{a \cdot \cos.☊}{\cos.(☊ - B)} \cdot \cos.(☊ - B - A) \tan. D,$$

$$\text{Nutation in declination} = -\frac{a. \cos.☊}{\cos.(☊ - B)} \cdot \sin.(☊ - B - A).$$

EXAMPLE.

Required the Nutation in Right Ascension and Declination of α Cygni, *on Dec.* 17, 1807.

See p. 469, l. 20. - - - - - - - - -	☊ =	7^s	29°	19′
Tab. IX. - - - - - - - - - - - - -	B =		− 7	53
	☊ − B =	7	21	26
(see p. 469, l. 6.) - - - - - - - -	A =	10	8	43
	☊ − B − A =	9	12	43

Tab. IX, log. b, -	0.8976 −	log. b - - - - - -	0.8976
log. cos. (☊ − B − A)	9.3426 +	log. sin. (☊ − B − A)	9 9892
log. tan. D - -	9.9939	∴ log. 7″.705 =	0.8868
log. − 1.71 =	0.2341 −	Nut. in dec. = + 7″.705	
Tab. IX. c =	+ 14.23		
Nut. in R A.	+ 12.52		

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PRINCIPALLY OF THE TERMS.

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