ASTRONOMY.

Astronomy (formed of *αστρον, a star,* and *νομος, law,*) is a mixed mathematical science, which treats of the heavenly bodies, their motions, periods, eclipses, magnitudes, &c. and of the causes on which they depend. That part of the science which relates to the motions, magnitudes, and periods of revolutions, is denominated *pure astronomy*; and that which investigates the causes of those motions, and the laws by which they are regulated, is called *physical astronomy.*

§ I. *History of Astronomy.*

General View.

It would be useless, if even the nature of our work would admit of it, to attempt to trace the history of this science from its earliest state of infancy, which is probably nearly coeval with that of society itself; at least if we regard the rude observations of shepherds and herdsmen as exhibiting the first dawn of astronomy. A man must be strangely divested of the curiosity peculiar to his species, who, while exposed to the varying canopy of the heavens, through successive nights and seasons, could suffer such a brilliant spectacle to pass repeatedly before him, without noticing the fixed or variable objects there presented to his view; and his attention, once drawn to a contemplation of the firmament, he would remark the invariable position of the greater number of those bodies with regard to each other; the irregular motion of others; and hence, by some denomination or other, we should have a distinction made between what we now call the *fixed stars* and the *planets;* while the sun and moon are in their appearances sufficiently distinct from the rest of the heavenly bodies, to have called for a farther distinguishing appellation, and to have claimed the particular regard of these rude observers.

Such was probably the origin of astronomy; and in this state, in all likelihood, it might remain for many ages, and in many countries unknown to and unconnected with each other. The length of the year, the duration of a lunar revolution, the particular rising of certain stars at certain seasons, and a few other common and obvious phenomena, might therefore be predicted with a certain degree of accuracy, long before those observations assumed any thing like a scientific form, and long anterior to that time from which we date the origin of astronomy as a science, properly so called.

Claims of the Chaldeans, &c.

The hon[...] being the first inventors of this sublime [...] [...]en attributed to various nations; [...]gyptians, the Chinese, the In[...] [...] their advocates amongst our [...]; and even a certain unknown [...]ed by the enthusiasm of some [...] traces are supposed to have [...] whom all original knowledge [...]een attributed. The more closely, however, we examine the claims of these actual or imaginary people, the more we shall be convinced that their astronomy consisted of little more than we have indicated above; *viz.* a tolerable approximation to certain periods, and to the re-appearance of certain phenomena, that required nothing more than a continued and patient observation of stated occurrences, which as we have observed, could not long remain unnoticed even in the most infant state of society.

Egyptians.

We may judge of the state of Egyptian astronomy from the circumstance of Thales having first taught them how to find the heights of the pyramids from the length of their shadows. It is true that they had some idea of the length of the year, and had, in a certain measure, approximated towards a determination of the obliquity of the ecliptic, or of the path of the sun, which they stated to be 24°. The Chaldeans appear to have made some rude observations on eclipses, but still little scientific knowledge can be attributed to this people; who after observing these phenomena, were contented to explain them by teaching that the two great luminaries of the heavens were only on fire on one side, and that eclipses were occasioned by the accidental turning of their dark sides towards us. And again, that these bodies were carried round the heavens in chariots, close on all sides except one, in which there was a round hole, and that a total or partial eclipse was occasioned by the complete or partial shutting of this aperture. Similar absurd and extravagant notions will be found amongst all the early pretenders to the study of astronomy; but we cannot concede to such knowledge and pretences the term science; they had in fact no science, they had amassed together a number of rude observations, and had been thus enabled to determine certain periods, and to predict some few phenomena; but we have no proof, nor even any reason whatever to imagine, from any facts that have been handed down to us, that these predictions rested upon any other basis than that of simply observing, the repeated returns of these appearances within certain periods.

Astronomy as it was received by the early Greeks.

If to the knowledge above indicated, we add an arbitrary collection of certain clusters or groups of stars into constellations; the division of the zodiac into twelve signs, corresponding to the twelve months of the year; into twenty-seven or twenty-eight hours, answering to the daily motion of the moon; an obscure idea of the revolution of the earth upon its axis, which was afterwards lost; a knowledge of five planets; and some contradictory notions respecting the nature and motion of comets, we shall have a pretty correct picture of the state of astronomy as it was received amongst the Greeks; and from whom it first derived its scientific character. It is therefore

Astronomy. History.

only from this period that we shall commence our historical sketch, and attempt to trace the rise and progress of astronomy.

Successive periods in the history of astronomy.

We shall follow it, from the state above described, when every thing depended upon observation only, unaided either by a calculus or instruments; through that in which the latter began to be employed, and some assistance was derived from the more elementary positions of geometry. We shall next examine it from the period when astronomical science was enriched and extended by the invention of the telescope, but while the principles of computations were still founded on the elements of pure geometry; and lastly, we shall exhibit the science as it now exists, supported by every aid that can be derived from the present high state of practical and theoretical mechanics and optics; when the effect of every celestial motion, and every disturbing force is made to depend upon one universal law; and the amount of each investigated and submitted to computation, by means of the powerful assistance derived from the modern analysis.

The first period above alluded to, comprehends a long series of ages, during which the science of astronomy passed into the hands of different people and nations. First, to the Greeks, then to the Arabs; from which latter it seems probable that it found its way to India and China, about the same time that it was also brought into Spain; whence it afterwards spread throughout all parts of civilized Europe. We shall therefore divide this period into the following minor sections. The astronomy of the Greeks; of the Arabs; of the Indians and Chinese; and of modern Europe; which latter will bring us up to the time of Copernicus and Galileo; including, in all, about twenty-one centuries.

Of the Astronomy of the Greeks.

Thales. B. C. 600.

Thales is generally considered as the founder of astronomy amongst the Greeks. This philosopher, who must have flourished about 600 years before the commencement of the Christian æra, is said to have taught that the stars were fire, or that they shone by means of their own light; the moon received her light from the sun, and that she became invisible in her conjunctions, in consequence of being hidden or absorbed in the solar rays, which it must be acknowledged is but an obscure way of saying that she then turned towards us her unenlightened hemisphere. He taught farther that the earth is spherical, and placed in the centre of the world; he divided the heavens, or rather found them divided into five circles, the equator, the two tropics, and the arctic and antarctic circles. The year he made to consist of 365 days; and determined " the motion of the sun in declination." What is meant by this expression is not very easy to comprehend; if it only means that he discovered such a motion, it can scarcely be considered as correct, as it must have been known prior to his time; viz. to the first observers; and it cannot mean that he laid down rules for computing it, as we have every reason to know that the most simple principles of trigonometry were not propagated till many centuries after his time.

Predict an eclipse.

Thales is also said to have first observed an eclipse, and to have predicted that celebrated one which terminated the war between the Medes and the Lydians; an eclipse on which much has been written, but from which very little satisfactory information has been obtained. Herodotus says, " it happened that the day was changed suddenly into night, a change which Thales the Milesian had announced to the people of Ionia, assigning for the limit of his prediction, the year in which the change actually took place." Thales had therefore neither predicted the day nor the month; and in all probability he had no other principle to proceed upon, than the Chaldean period of eclipses already alluded to in the preceding part of this article.

The pointed declaration of the historian, that the limits assigned by the astronomer for the appearance of this phenomenon, was the year in which it happened, is a pretty obvious proof of the low state of astronomical science at this time, and it would be of little importance whether the eclipse was itself partial or total; but as there is little doubt that such an event actually took place, it becomes a matter of high importance in chronology, to ascertain whether it was such as it is described, viz. a total eclipse; for no partial obscuration of the sun's light would accord with the description of Herodotus, *of the day being suddenly changed into night*; and such a phenomenon in any particular place being an extremely rare occurrence, it would, if correct, enable us to determine not only the year, but the very day and hour at which it happened, and thus furnish at least one indisputable period in chronology and history.

Dates assigned to this eclipse.

Various dates have been assigned to this eclipse. Pliny places it in the fourth year of the forty-eighth Olympiad which answers to the year 585 B. C. (*Hist. Nat.* lib. 2. cap. 12.), a similar opinion has been advanced by Cicero (*De Divinat.* lib. 1. § 49.) and probably by Eudemus (*Clement. Alex. Strom.* lib. 1. p. 354.); by Newton (*Chron. of Anc. Kings amended*); Riccioli (*Chron. Reform,* vol. 1. p. 228.); Desvignoles (*Chronol.* lib. 4. cap. 5. § 7, &c.); and by Brosses (*Mém. de l'Acad. des Belles Lettres,* tom. 21. Mém. p. 33.)

Scaliger, in two of his writings, (*Animad. ad Euseb.* p. 89.) and in (Ολυμ. ἀναγραφη) has adopted also the opinion of Pliny; but in another work (*De Emen. Temp. in Can. Isag.* p. 321.) he fixes the date of this eclipse to the 1st of October, 583, B. C. Calvisius states it in his (*Opus Chron.*) to have taken place in 607 B. C. Petavius says it happened July 9th, 597 B. C. (*De Doct. Temp.* lib. 10. cap. 1.) which date has likewise been adopted by Marsham, Bouhier, Corsini; and by M. Larcher the French translator of Herodotus, (tom. i. p. 335.) Usher is of opinion that it happened 601 B. C.; and Bayer, May 18, 603, B. C.; which latter opinion has been supported by two English astronomers, Costard and Stukeley, (*Phil. Trans.* for 1753.) But Volney attempts to show in his (*Chronologie d'Herodote,*) that it could be no other than the eclipse which happened February 3d, 626 B. C.

Mr. F. Bailly has examined with great care and labour the probability of these several statements, from which it appears, that most of the eclipses above alluded to happened under circumstances which render it absolutely impossible any of them should be that alluded to by Herodotus; most of them were

Astronomy. not even visible in that country, which must necessarily have been the scene of action between the Medes and the Lydians, and none of them was total in those places. He has therefore with great perseverance, by means of the last new astronomical tables of the *Bureau des Longitudes*, computed backward to find whether any eclipse of the sun actually happened within the probable limits of the event recorded by the historian, and the result of his research is, that on the 10th of September, 610 B.C., there was a solar eclipse, which was total in some parts of Asia Minor; and which, he therefore concludes, with great probability, was the identical one referred to by Herodotus. Admitting, therefore, the conclusion, we have one decided point of time to which we are enabled to refer with confidence, and at which time, the state of astronomy is known to have been such as we have described. See *Phil. Trans. for* 1811.

Anaximander, Anaximenes, Anaxagoras. B.C. 530.

The successors of Thales, Anaximander, Anaximanes. and Anaxagoras, contributed considerably to the advancement of astronomy. The first is said to have invented or introduced the gnomon into Greece; to have observed the obliquity of the ecliptic; and taught that the earth was spherical, and the centre of the universe, and that the sun was not less than it. He is also said to have made the first globe, and to have set up a sun-dial at Lacedæmon, which is the first we hear of among the Greeks; though some are of opinion that these pieces of knowledge were brought from Babylon by Pherecydes, a contemporary of Anaximander. Anaxagoras also predicted an eclipse which happened in the fifth year of the Peloponnesian war; and taught that the moon was habitable, consisting of hills, valleys, and waters, like the earth. His contemporary Pythagoras, however, greatly improved not only astronomy and mathematics, but every other branch of philosophy. He taught that the universe was composed of four elements, and that it had the sun in the centre; that the earth was round, that we had antipodes; and that the moon reflected the rays of the sun; that the stars were worlds, containing earth, air, and ether; that the moon was inhabited like the earth; and that the comets were a kind of wandering stars, disappearing in the superior parts of their orbits, and becoming visible only in the lower parts of them. The white colour of the milky-way he ascribed to the brightness of a great number of small stars; and he supposed the distances of the moon and planets from the earth to be in certain harmonic proportion to one another. He is said also to have exhibited the oblique course of the sun in the ecliptic and the tropical circles, by means of an artificial sphere; and he first taught that the planet Venus is both the evening and morning star. This philosopher is said to have been taken prisoner by Cambyses, and thus to have become acquainted with all the mysteries of the Persian magi; after which he settled at Crotona in Italy, and founded the Italian sect.

Pythagoras. B.C. 540.

Philolaus. B.C. 440-432.

About 440 years before the Christian æra, Philolaus, a celebrated Pythagorean, asserted the annual motion of the earth round the sun; and soon after Hicetas, a Syracusan, taught its diurnal motion on its own axis. About this time also flourished Meton and Euctemon at Athens, who took an exact observation of the summer solstice 432 years before Christ; which is the oldest observation of the kind we have, excepting some doubtful ones of the Chinese. Meton is said to have composed a cycle of 19 years, which still bears his name; and he marked the risings and settings of the stars, and what seasons they pointed out: in all of which he was assisted by his companion Euctemon. The science, however, was obscured by Plato and Aristotle, who embraced the system afterwards called the *Ptolemaic*, which places the earth in the centre of the universe. History.

Eudoxus. B.C. 370.

After Philolaus, the next astronomer we meet with of great reputation is Eudoxus, who flourished 370 B.C. He was a contemporary with Aristotle though considerably older, and is greatly celebrated for his skill in this science. He is said to have been the first to apply geometry to astronomy, and is supposed to be the inventor of many of the propositions attributed to Euclid. Having travelled into Egypt in the early part of his life, he obtained a recommendation from Agesilaus to Nectanebus, king of Egypt, and by his means got access to the priests, who were then held to have great knowledge of astronomy; after which he taught in Asia and Italy. Seneca tells us, that he brought the knowledge of astronomy, i. e. of the planetary motions, from Egypt into Greece: and according to Archimedes, his opinion was, that the diameter of the sun was nine times that of the moon. He was also acquainted with the method of drawing a sun dial on a plane.

Callippus. B.C. 330.

Soon after Eudoxus, we meet with Callippus, whose system of the celestial sphere is mentioned by Aristotle; but he is better known for a period of 76 years, containing four corrected Metonic periods, and which had its beginning at the summer solstice, in the year 330 B.C. And it was about this time, or rather earlier, that the Greeks having begun to plant colonies in Italy, Gaul, and Egypt, became acquainted with the Pythagorean system, and the notions of the ancient Druids concerning astronomy.

Autolycus B.C. 300.

Passing over the names of various other astronomers of this period, who appear to have done very little towards the advancement of the science, we come to Autolycus, the most ancient writer whose works have been handed down to our time. He wrote two books, viz. *Of the Sphere which moves*, and the other, *On the Risings and Settings of the Stars*. These works were composed about 300 B.C.

We have now passed over a period of 300 years from the time of Thales, and therefore, by making a few extracts from these works of Autolycus, we shall be enabled to form some idea of the progress of astronomy during this period. In the work on the moveable sphere, we have several propositions, of which the following are the most important.

Earliest Work extant on Astronomy.

1. If a sphere move uniformly about its axis, all the points on its surface which are not in its axis, will describe parallel circles, having for their common poles, those of the sphere itself, and of which all the planes will be perpendicular to the axis.

2. All these points will describe, of their respective circles, similar arcs in equal times.

3. Reciprocally, similar arcs will indicate equal time.

4. If a great fixed circle, perpendicular to the axis, divide the sphere into two hemispheres, the one visible, the other invisible, and that the sphere turns about its axis; those points on the surface that are

hidden, will never rise; and those that are visible will never set. This is what we now denominate a parallel sphere; the great fixed circle corresponding with our equator.

5. If a great circle pass through the poles, all the points of the surface will rise and set alternately. This corresponds to our horizon, and to our right sphere.

6. If the great circle be oblique to the axis, it will touch two equal parallel circles; of which, that adjacent to the one pole will be always apparent, the other always invisible.

The first of these circles was called by the Greeks, (although not by this author,) as we still denominate it, the *arctic* circle, and the other the *antarctic* circle.

7. If the horizon be oblique, the circles, perpendicular to the axis, will always have their points of rising and setting in the same points of the horizon, to which they are all equally inclined.

8. The great circles which touch the arctic and antarctic circle, will, during the complete revolution of the sphere, twice coincide with the horizon.

9. In the oblique sphere, of all the points which rise at the same instant, those which are nearest to the visible pole will set last; and of the points which set at the same instant, those that are nearest the same pole will rise first.

10. In the oblique sphere, every circle which passes through the poles, will be perpendicular to the horizon twice in the course of one complete revolution.

Remarks on this author.

We omit some other propositions of this author, which are of less importance than the above; and even those which we have given, are such as one would imagine could not have escaped the observation of any one who would think of employing an artificial sphere to represent the celestial motions; yet, from the tenor of the work in question, it would seem, that if they were known, they were never before, at least, embodied in the form of a regular treatise.

Here then we may begin to date the first scientific form of astronomy; because in this work, however low and elementary, we have an application of geometry to illustrate the motions of the heavenly bodies; but we shall still find two other centuries pass away, before the same principles were applied to actual computation.

Euclid. B. C. 300.

Contemporary with Autolycus, was Euclid; whose elements of geometry, after so many ages, still maintain their pre-eminence; and in which we find all the propositions that are necessary for establishing every useful theorem in trigonometry; yet it is perfectly evident that no ideas were yet conceived of the latter science. Neither Euclid nor Archimedes, great as were their skill and talents in geometry, had any idea of the method of estimating the measure of any angle by the arc, which, the two lines forming it, intercepted; nor does it appear that they knew of any instrument whatever for taking angles; a very convincing proof of which appears in the process adopted by the latter justly celebrated philosopher, in order to determine the apparent diameter of the sun.

Aristarchus B. C. 264.

Passing over the poet Aratus, who is supposed to have embodied in his poem all the astronomical knowledge of the time in which he wrote, viz. 270 B. C.; but who had not himself made any observations, we come to Aristarchus, who has left us a work, entitled *Of Magnitudes and Distances*; in which he teaches, that the moon receives her light from the sun, and that the earth is only a point in comparison with the sphere of the moon. He likewise added, that when the moon is dichotomized, we are in the plane of the circle which separates the enlightened part from the unenlightened, which is the most curious and original remark of this author: in this state of the moon, he also observes, that the angle subtended by the sun and moon, is one-thirtieth less than a right angle; which, in other words, is saying, that the angle is 87°, whereas we now know that this angle exceeds 89° 50′. In another proposition he asserts, that the breadth of the shadow of the earth is equal to two semi-diameters of the moon, whereas these are to each other as 83 to 64. In his sixth proposition, he states the apparent diameter of the moon to be one-fifteenth part of a sign, or 2°; whereas we know that it is only about half a degree. Again, the distance of the earth from the moon being assumed as unity, its distance from the sun was said to be 17·107, and the distance of the earth from the sun 19·081. Such was the astronomical knowledge in the time of Aristarchus, who lived about 264 years before the Christian æra.

Eratosthenes. B. C. 230.

In order of time we pass now to Eratosthenes, who may, perhaps, with more propriety than Autolycus, be considered as the founder of astronomical science; particularly if it be true that he placed in the portico of Alexandria certain armillary spheres; of which so much use was afterwards made, and which, it is said, he owed to the munificence of Ptolemy Euergetes, who called him to Alexandria, and gave him the charge and direction of his library.

Ancient armillary sphere.

According to the description given of these armillaries by Ptolemy, they were assemblages of different circles; the principal one of which served as a meridian; the equator, the ecliptic, and the two colures, constituted an interior assemblage, which turned on the poles of the equator. There was another circle, which turned on the poles of the ecliptic, and carried an index to point out the division at which it stopped. The instrument of which the above appears to be the general construction, was applied to various uses; amongst others, it served to determine the equinoxes, after the following manner:—The equator of the instrument being pointed with great care in the plane of the celestial equator, the observer ascertained, by watching the moment when neither the upper nor the lower surface was enlightened by the sun; or rather, which was less liable to error, when the shadow of the anterior convex position of the circle completely covered the concave part on which it was projected. This instant of time was evidently that of the equinox. And if this did not happen, although the sun shone, two observations were selected, in which the shadow was projected on the concave part of the circle in opposite directions; and the mean of the interval between these observations was accounted the time of the equinox. At this time we find enumerated five planets, viz. Φαινων, Φαεθων, Πυροειδηρ, which appear to indicate Jupiter, Saturn, and Mars; and to which were added Venus and Mercury.

Determination of the equinoxes.

Magnitude of the earth.

Eratosthenes not only taught the spherical figure of the earth, but attempted to ascertain its actual circumference, by measuring, as exactly as could be done in his time, the length of a certain terrestrial arc,

Astronomy.

and then finding the astronomical arc in degrees intercepted between the zeniths of the two places. The segment of the meridian which he fixed upon for this purpose, was that between Alexandria and Syene; the measured distance of which was found to be 5,000 stadia; and the angle of the shadow upon the scaphia, which was observed at Alexandria, was equal to the fiftieth part of the circle; and at Syene there was no shadow from this gnomon at noonday of the summer solstice; and that this might be the more accurately taken, they dug a deep well, which, being perpendicular, was completely illuminated at the bottom when the sun was on the meridian. The exact quantity which this philosopher assigned to the circumference of the earth is not known; at least, different opinions have been advanced: some state it at 250,000, and others at 252,000 stadia; the length of this unit of measure is also somewhat uncertain. It is, however, of small importance, as we may be pretty well convinced, that by such means as he employed, no very accurate conclusion could be expected; it is sufficient that he attempted the solution of the problem in a very rational manner, to entitle him to the honour of being one of the most celebrated of the Grecian astronomers.

Obliquity of the ecliptic.

Eratosthenes also observed the obliquity of the ecliptic, and made it to consist of $\frac{11}{166}$th of a circumference, which answers to about 23° 51′ 19·5″. This observation is commonly stated to have been made in the year 230 B. C.

Archimedes. B. C. 227.

Archimedes, the justly celebrated geometer of Syracuse, was contemporary with Eratosthenes; and although most conspicuous as a mechanic and geometrician, the great impulse which he gave to the sciences generally, will not admit of our passing him over in silence in this history. All that we have of this author with reference to astronomy, is found in his *Arenarius*, a work which has been translated into most modern languages; where he undertakes to prove, that the numerical denominations which he has indicated in his books to Zeuxippes, are more than sufficient to express the grains of sand, that would compose a globe, not only as large as our earth, but as the whole universe. He supposes that the circumference of the earth is not more than three million stadia; that the diameter of the earth is greater than that of the moon, and less than that of the sun; that the diameter of the sun is 300 times greater than that of the moon, and moreover, that the diameter of the sun is greater than the side of the inscribed chiliagon, that is greater than $\frac{360}{1000}$, or 21′ 36″.

Archimedes ascertains the apparent diameter of the sun.

The manner in which he arrives at his conclusion is very interesting, as showing the state of the sciences at this time, even in the hands of this great master:—"I have used," says he, "every effort to determine by means of instruments, the angle which comprehends the sun, and has its summit at the eye of the observer; but this is not easy; for neither our eyes nor our hands, nor any of the means which it is possible for us to employ, have the requisite precision to obtain this measure. This, however, is not the place to enlarge upon such a subject. It will suffice to demonstrate that which I have advanced, to measure an angle which is not greater than that which includes the sun's apparent diameter, and has its summit in our eyes; and then to take another angle which is not less than that of the sun, and which equally has its summit in our eyes. Having, therefore, directed a long ruler on a horizontal plane towards the point of the horizon where the sun ought to rise, I place a small cylinder perpendicularly on this ruler. When the sun is on the horizon, and we look at it without injury, I direct the ruler towards the sun, the eye being at one of its extremities, and the cylinder is placed between the sun and the eye in such a manner, that it entirely conceals the sun from view. I then remove the cylinder farther from the eye, until the sun begins to be perceived by a thin stream of light on each side of the cylinder. Now, if the eye perceived the sun from a single point, it would suffice to draw from that point tangential lines to the two sides of the cylinder. The angle included between these lines would be a little less than the apparent diameter of the sun; because there is a ray of light on each side. But as our eyes are not a single point, I have taken another round body, not less than the interval between the two pupils; and placing this body at the point of sight at the end of the ruler, and drawing tangents to the two bodies, of which one is cylindric, I obtained the angle subtended by the sun's (apparent) diameter. Now the body, which is not less than the preceding distance (between the pupils), I determine thus: I take two equal cylinders, one white the other black, and place them before me; the white further off, the other near, so near indeed as to touch my face. If these two cylinders are less than the distance between the eyes, the nearer cylinder will not entirely cover the one that is more remote, and there will appear on both sides some white part of that remote cylinder. By different trials, we may find cylinders of such magnitude, that the one shall completely conceal the other: we then have the measure of our view (the distance between the pupils), and an angle, which is not smaller than that in which the sun appears. Now, having applied these angles successively to a quarter of a circle, I have found that one of them has less than its 164th part, and the other greater than its 200th part. It is therefore evident, that the angle which includes the sun, and has its summit at our eye, is greater than the 164th part of a right angle, and less than the 200th part of a right angle."

History.

By this process, Archimedes found the sun's apparent diameter to be between 27′ and 32′ 56″.

Singular accuracy in the result.

It is not a little remarkable, considering the obvious inaccuracy of the method, that the maximum limit thus obtained, differs only $\frac{1}{3}$ of a minute from 32′ 35″·6, which is the largest angle actually subtended by the sun's diameter, and which is observed about the time of the winter solstice, when the sun is nearest to the earth. But this quotation from the *Arenarius* is extremely curious also on other accounts. We may learn from it, first, that Archimedes, with all his fecundity of genius, and with all the variety of his inventions, had no means of diminishing the effect of the sun's rays upon his eyes, and therefore performed this interesting experiment when the sun was in the horizon, that the optic organ might sustain its light without inconvenience. It also proves to us, that there was not then any instrument known to Archimedes, which he thought capable of giving the diameter of the sun, to within four or six minutes;

Astronomy.

since he found it necessary to devise means at which he stopped, after an attempt not very satisfactory. We see, further, that he carried his angles, or their chords, over a quarter of a circle; but he does not say expressly that his arc had been divided; to render his language accurately, it is simply requisite to say, having carried one of the chords 200 times over upon the arc, he found it exhausted; and that the other chords could only be applied 164 times upon the quadrant.

We see, also, that Archimedes had not the means of computing the angle at the vertex of an isosceles triangle, of which he knew the base and the two equal sides. He was obliged to recur to a graphical operation as uncertain as the observation itself. Thus he was entirely ignorant even of rectilinear trigonometry, and he had not any notion of computing the chords of circular arcs.

Hipparchus B. C. 135.

We come now to the great father of true astronomy, Hipparchus; but our limits will not admit of our entering very deeply into his discoveries and improvements. One of his first cares was to rectify the length of the year, which before his time we have seen had been made to consist of 365 days and 6 hours. By comparing one of his own observations at the summer solstice with a similar observation made 145 years before by Aristarchus, he shortened the year about 7 minutes; making it to consist of 365 days, 5 hours, 53 minutes; which, however, was not sufficient: but the cause of the mistake is said to have rested principally with Aristarchus, and not with Hipparchus; for the observations of the latter, compared with those of modern times, give 365 days, 5 hours, 48 min. 49½ sec. for the duration of the year; a result, which exceeds the truth very little more than a second. It is to be observed, however, that this is no very exact criterion, unless the same be compared with the observation of the more ancient observer; for supposing all the error on the side of Hipparchus, it is more divided by comparing it with others at the distance of 19 or 20 centuries, than in comparing it with one, where the distance of time is only 145 years.

Finds the length of the year.

Introduction of trigonometry by chords.

One of the greatest benefits, which astronomy derived from this philosopher was his enunciation and demonstration of the method of computing triangles, whether plane or spherical. He constructed a table of chords, which he applied nearly in the same manner as we now do our tables of sines. As an observer, however, he rendered great service to the doctrine of astronomy, having made much more numerous observations than any of his predecessors, and upon far more accurate principles. He established the theory of the sun's motion in such a manner, that Ptolemy 130 years afterwards, found no essential alteration requisite; he determined also the first lunar inequality, and gave to the motions of the moon those of the apogee and of its nodes, which Ptolemy afterwards very slightly modified. Hipparchus also prepared the way for the discovery of the second lunar inequality, and from his observation it was, that the fact of the precession of the equinoxes was first inferred. He employed the transit of the stars over the meridian to find the hour of the night, and invented the planisphere, or the means of representing the concave sphere of the stars, on a plane, and thence deduced the solution of problems in spherical astronomy, with considerable exactness and facility. To him also we owe the happy idea of making the position of towns and cities, as we do those of the stars, by circles drawn through the poles perpendicularly to the equator; that is, by latitudes and by circles parallel to the equator, corresponding to our longitudes. From his projection it is, that our maps and nautical charts are now principally constructed; and his rules for the computation of eclipses were long the only ones employed for determining the differences of meridian.

Establishes the theory of the sun's motion, and the first lunar inequality.

Hour of the night found by the stars.

History.

Catalogue of the stars.

Another most important work of Hipparchus, was his formation of a catalogue of the stars. The appearance of a new star in his time, caused him to form the grand project of enabling future astronomers to ascertain, whether the general picture of the heavens were always the same. This he aimed to effect, by attempting the actual enumeration of the stars. The magnitude and difficulty of the undertaking did not deter this indefatigable astronomer; he prepared and arranged an extensive catalogue of the fixed stars, which subsequently served as the basis of that of Ptolemy. So great, indeed, is the merit of this prince of Grecian astronomy, that the enthusiastic language in which Pliny speaks of him in his Hist. Nat. (lib. ii. cap. 26.) may rather be admired than censured.

Ptolemy. A. D. 120.

After Hipparchus, we meet with no astronomer of eminence amongst the Greeks till the time of Ptolemy, who flourished between the years 125 and 140 of the Christian æra; which, therefore, includes a space of nearly three hundred years. There were, however, some astronomical writers, both Greeks and Romans, in the course of this time, whom it may not be amiss to enumerate, although the little progress that the science made in their hands will exempt us from the necessity of entering minutely into an analysis of their several works: these were, Geminus, who lived about 70 years B. C., whose book is entitled *Introduction to the Phenomena*; Achilles Tatius, of about the same period; Cleomedes, who lived in the time of Augustus; Theodosius, Menalaus, and Hypsicles, who are supposed to have written about the year 50 B. C.; Manilius, Strabo, Posidonius, and Cicero, who were about half a century later; after which, we meet with no one to whom it is at all necessary even to refer, till we come to Ptolemy, who was born in the year of Christ 70; and who made, as we have stated above, most of his observations between the years 125 and 140 of our æra.

Various labours of Ptolemy.

Ptolemy has rendered all succeeding astronomers indebted to him, both for his own observations, which were very numerous, and his construction of various tables; but most of all for the important collection which he made of all astronomical knowledge prior to his time, and which he entitled, or the Arabs after him, the *Almagest, or Great Collection*. Of his own labours, we may mention his theory and calculation of tables of the planets, and his determination, with a precision little to be expected in his time, of the ratio of their epicycles to their mean distances; that is to say in other terms, the ratio of their mean distances to the distance of the earth from the sun. This theory, imperfect as it was, was adopted and generally admitted, for the space of fourteen centuries; during which time, it was transmitted to the Arabs, the Persians, and the Indians, and with whom it is still held sacred. The equi-distant centres of the earth, from the excentric and the equant, an hypothesis of Ptolemy, led, in all probability, Kepler to the idea of an ellipse and its foci. Ptolemy thus preparing the way for Kepler, as the laws of the latter may be considered as the precursors of the theory of Newton.

Sines introduced into trigonometry.

To this celebrated Grecian we also owe the substitution of the sines of arcs instead of their chords; as also the first enumeration of some important theorems in trigonometry.

Ptolemy's arguments to prove the immobility of the earth.

Ptolemy was the author of that system of astronomy which still bears his name; or if he did not entirely invent it, (as there is great reason to suppose he did not,) he enforced it by such arguments as led to its establishment; and it was afterwards rendered sacred through the stupid bigotry and intolerance of the Romish church. He endeavours to prove the absolute immobility of the earth, by observing, "If the earth had a motion of translation common to other heavy bodies, it would, in consequence of its superior mass, precede them in space, and pass even beyond the bounds of the heavens, leaving all the animals and other bodies without any support but air; which are consequences to the last degree ridiculous and absurd." In the same place he adds, "Some persons pretend, that there is nothing to prevent us from supposing that the heavens remain immovable, while the earth turns on its own axis from west to east, making this revolution in a day nearly; or, that if the heavens and the earth both turn, it is in a ratio corresponding with the relations we have observed between them. It is true, that as to the stars themselves, and considering only their phenomena, there is nothing to prevent us, for the sake of simplicity, from making such a supposition. But these people are not aware how ridiculous their opinion is, when considered with reference to events which take place about us; for if we concede to them that the lightest bodies, consisting of parts the most subtle, are not possessed of levity, (which is contrary to nature,) or that they move not differently from bodies of a contrary kind, (although we daily witness the reverse); or, if we concede to them that the most compact and heaviest bodies possess a rapid and constant motion of their own (while, it is well known, that they yield only with difficulty to the impulses we give to them), still, they would be obliged to acknowledge, that the earth, by its revolution, would have a motion more rapid than any of those bodies which encompass it, in consequence of the great circuit through which it must pass in so short a period; wherefore such bodies as are not supported on it, would always appear to possess a motion contrary to itself; and neither clouds, nor any projected bodies, nor birds in flight, would ever appear to move towards the east; since the earth, always preceding them in this direction, would anticipate them in their motion; and every thing, except the earth itself, would constantly appear to be retiring towards the west."

If we did not feel convinced that, in certain cases, even the errors and false reasoning of such a man as Ptolemy, possess a greater interest than the more correct and refined arguments of minor philosophers, we should certainly not have laid before our readers this extract from the introduction to the *Almagest*; but considering it as the defence of an hypothesis, which maintained its ascendancy for fourteen centuries amongst all nations, and which is still held sacred throughout every part of Asia, it is impossible to divest it of its interest and importance.

Analysis of the *Almagest*.

The other part of this great work is more worthy of the talents of its author, and is more deserving of our attention; but the limits of this article will not admit of our giving more than a very concise abstract of its contents. The first book, beside what we have hitherto mentioned, exhibits a highly interesting specimen of the ancient trigonometry; and the method of computing the chords of arcs, which, in fact, involves our fundamental theorems of trigonometry, though expressed in a manner totally different.

Theorems in trigonometry.

Ptolemy first shows, how to find the sides of a pentagon, decagon, hexagon, square and equilateral triangle, inscribed in a circle, which he exhibits in parts of the diameter, this being supposed divided into 120. He next demonstrates a theorem equivalent to our expression $\sin(a-b) = \sin a \cos b - \sin b \cos a$; by means of which he finds the chords of the difference of any two arcs, whose chords are known. He then finds the chord of any half arc, that of the whole arc being given, and then demonstrates what is equivalent to our formula for the sine of two arcs; that is, $\sin.(a+b) = \sin a \cos b + \sin b \cos a$; and by means of this he computes the chord to every half degree of the semicircle. These theorems it may be said belong rather to the history of trigonometry than to that of astronomy; but we trust that the obvious dependence of the latter science upon the former, will be found to justify us in introducing them to the reader in this place.

Climates.

We are next presented with a table of climates nearly equivalent to our nonagesimal tables, and it is not a little singular, that amongst them, we find none appertaining to the latitude of Alexandria; because, without such an auxiliary, Ptolemy must have contented himself with interpolations, which were not only difficult to make, but attended at the same time with great inaccuracy; a circumstance from which it has been concluded, that Ptolemy himself made few observations, or that he was not very particular concerning the accuracy of his calculations. The examination of this question would carry us too far out of our track to admit of our entering upon it in this place; but the reader may see it developed in all requisite detail, in the learned *History of Astronomy*, lately published by Delambre.

Length of the year, &c.

Having passed over the above preliminary details, the author treats of the length of the year, the motion of the sun, the mean and apparent anomaly, &c. &c. The length of the year, according to the sexagesimal notation, he makes 365d. $14' 48''$, which answers to 365d. 5h. $55' 12''$; the diurnal motion of the sun is stated to be $0^\circ 59' 8'' 7''' 13^{iv} 12^{v} 31^{vi}$, and the horary motion $2' 27'' 50''' 43^{iv} 3^{v} 1^{vi}$. To this is also added two tables, one of the mean motion of the sun, and the other of the solar anomaly. The fourth book of the *Almagest* is employed in treating of the motion of the moon, being prefaced by a few remarks respecting the observations which are most useful for that purpose: he then gives an abstract of all the lunar motions, with a table of them; in the first of which the motion is exhibited for periods of eighteen years;

in the second for years and hours; and in the third for Egyptian months and days. Four other columns of the same table present the number of degrees which belong to each of the times indicated in the first column; viz. the second, the longitude; the third, the anomaly; the fourth, the latitude; and the fifth, the elongation.

Lunar motion.

The author next treats of various subjects connected with the lunar motion; as, for instance, its general anomaly; its eccentricity; the lunar parallax; the construction of instruments for observing the parallax; the distance of the moon from the earth, which he states at $38{\cdot}4\frac{1}{3}$ terrestrial radii, when in the quadratures; the apparent diameters of the sun and moon; the distance of the sun from the earth, which is stated at 1210 radii of the latter; and the relative magnitudes of the sun, moon, and earth. The diameters of these are stated to be to each other, as the numbers $18{\cdot}8$, 1, and $3\frac{2}{5}$; also their masses as $6644\frac{1}{2}$, 1 and $39\frac{1}{4}$.

The next book is entirely occupied with the doctrine of eclipses of the sun and the moon; the determination of their limits and durations; tables of conjunctions; and methods of computation and construction, &c.

Particular deductions of Ptolemy.

We cannot extend the analysis of this important work to a greater length; but must content ourselves with a few remarks relative to some of the deductions to which we have referred. We have seen that Ptolemy made the length of the year to be more than 365 days, 5h. 55m., which is about 6 minutes longer than it really is; but considering that the observations before his time, with the exception of those of Hipparchus, were very imperfect; and that the distance of time between these two celebrated astronomers, was not sufficient to determine such a question, with the means they possessed, to the greatest nicety, we may rather admire the near approximation to the truth, than be astonished at the difference between his result and that deduced from numerous and long continued observations.

The evection discovered.

His researches on the theory of the sun and moon were however attended with better success. Hipparchus had shown that these two bodies were not placed in the centre of their orbits; and Ptolemy demonstrated the same truths by new observations. He moreover made another important discovery, which belongs exclusively to him, except so far as relates to the observations of Hipparchus, by a comparison of which with his own, his conclusion was deduced,—we allude here to the second lunar inequality, at present distinguished by the term *evection.* It is known, generally, that the velocity of the moon in its orbit, is not always the same, and that it augments or diminishes, as the diameter of this satellite appears to increase or decrease; we know, also, that it is greatest and least at the extremities of the line of the apsides of the lunar orbit. Ptolemy observed that from one revolution to another; the absolute quantities of these two extreme velocities varied, and that the more distant the sun was from the line of the apsides of the moon, the more the difference between these two velocities augmented; whence he concluded that the first inequality of the moon, which depends on the eccentricity of its orbit, is itself subject to an annual inequality, depending on the position of the line of the apsides of the lunar orbit with regard to the sun.

When we consider Ptolemy's system of astronomy, as founded upon a false hypothesis, the complication of his various epicycles, in order to account for the several phenomena of the heavenly bodies; and the rude state of the ancient astronomy, it is impossible to withhold our admiration of the persevering industry and penetrating genius of this justly celebrated philosopher; who, with such means, was enabled to discover an irregularity which would seem to require the most delicate and refined aid of modern mechanics to be rendered perceptible.

The work of this author to which we have hitherto confined our remarks, is the *Almagest*;* but Ptolemy also composed other treatises; which, if not equal to the above in importance, are still such as to be highly honourable to his memory and talents, particularly his geography.

Ptolemy's geography.

This work, although imperfect as to its detail, is notwithstanding founded upon correct principles; the places being marked by their latitude and longitude agreeably to the method of Hipparchus. As to the inaccuracies of their position, although they cannot be denied, they will readily be pardoned, when we consider that he had for the determination of the situation of cities and places of which he speaks, only a small number of observations subject to considerable errors; and the mere report of travellers, whose observations we may readily grant were still more erroneous than those of his own. It requires many years to give great perfection to geography: even in the present time, when observations with accurate instruments have been made in every part of the globe, we are still finding corrections necessary; a remarkable instance of which seems to have occurred lately (1818) to Captain Ross, in his voyage into Baffin's Bay, where he is said to have found some parts of the land laid down nearly a degree and a half out of their proper places. Many other minor pieces on astronomy and optics are also attributed to this author; but we have already extended our accounts of his works to a greater length than we had intended, and must now therefore pass on to his successors.

Greeks posterior to Ptolemy.

After the time of Ptolemy we find no Greek authors of eminence, although we have some few writers on this subject. The science of astronomy had now obviously passed its zenith, and began rapidly to decline. The Alexandrian school, it is true, still subsisted; but during the long period of 500 years, all that can be said is, that the taste for, and the tradition of the science was preserved, by various commentators on Hipparchus and Ptolemy; of whom the most distinguished were Theon and the unfortunate Hypatia, his daughter. The latter is said to have herself computed certain astronomical tables, which are lost.

Destruction of the Alexandrian school.

We now arrive at that period, so fatal to the Grecian sciences. These had for a long time taken refuge in the school of Alexandria; where, destitute of support

* The first printed edition of this celebrated performance, was a Latin translation from the Arabic version of Cremoneus; which, however, abounds so much in the idiom of that language, as to render it nearly unintelligible, without a constant reference to the Greek text. This was published at Venice in 1515; and in 1538 the collection appeared in its original language, under the superintendence of Simon Grynaeus, at Basil, together with the eleven books of the Commentaries of Theon. The Greek text was again republished at the same place, with a Latin version, in 1541; and again, with all the works of Ptolemy, in 1551; and lastly, a splendid French edition with the Greek text, by M. Halma, in three beautiful volumes, royal quarto, Paris, 1813.

Astronomy. and encouragement, they could not fail to degenerate. Still, however, they preserved, as we have said above, at least by tradition or imitation, some resemblance of the original; but about the middle of the seventh century, a tremendous storm arose which threatened their total destruction. Filled with all the enthusiasm a military government is calculated to inspire, the successor of Mahomet ravaged that vast extent of country, which stretches from the east to the southern confines of Europe. All the cultivators of the arts and sciences who had from every nation assembled at Alexandria, were driven away with ignominy: some fell beneath the swords of their conquerors, while others fled into remote countries, to drag out the remainder of their lives in obscurity and distress. The places and the instruments which had been so useful in making an immense number of astronomical observations, were involved with the records of them, in one common ruin. The entire library, containing the works of so many eminent authors, which was the general depository of all human knowledge, was devoted to the devouring flames, by the Arabs; the caliph Omar observing, "that if they agreed with the Koran, they were useless; and if they did not they ought to be destroyed:" a sentiment worthy of such a leader, and of the cause in which he was engaged. In the midst of this conflagration, the sun of Grecian science, which had long been declining from its meridian, finally set; never perhaps again to rise in those regions once so celebrated for the cultivation of every art and science that does honour to the human mind.

Astronomy of the Chinese and Indians.

Astronomy of the Chinese. If we were to adopt the opinions of some authors who have written on the subject of the Chinese astronomical knowledge, we should have now to commence at a much earlier period than we did in giving our account of this science amongst the Greeks; as it is stated that the former possess records of eclipses and other celestial phenomena, so far back as the year 2159 B.C., and that even in the year 2857 B.C. the study of astronomy and the desire of propagating a knowledge of that science amongst his people, were objects of great moment with the emperor Ion Hi; such at least is the doctrine supported in the *Histoire Générale de la Chine, ou Annales de cet Empire*, translated into French from Tong-Kien-Kang-Mou, by the Pere De-Mailla, a French Jesuit, one of the missionaries to Pekin.

We cannot attempt to enter here upon a refutation of the ideas supported in this work, relative to the antiquity of the Chinese astronomy; it will perhaps be sufficient to observe, that of 460 eclipses reported to have been predicted and recorded in the Chinese annals, the first is dated 2159 B.C., and the second, 776 B.C., leaving a great blank of 1383 years, during which no such phenomenon is noticed. That from the latter date to the year 1699 of our æra, they run on in pretty regular succession; but that of this number, the Pere Gaubil, who was at the labour of computing them, found only twelve which answered to the year, month, and day stated in the annals; and that of these twelve, only one of them was anterior to the time of Ptolemy, and even this one is doubtful.

The antiquity of it inadmissible. It appears then that very little credit is to be given to the assumed antiquity of Chinese science; but perhaps the following abstract, which Delambre has made from the annals above referred to, will give the reader a better idea of the state of astronomy amongst this singular people, and place their pretensions in a more tangible form than any thing we can advance respecting the improbability of the notions advanced by Mailla.

History. In the year 687 B.C. we find recorded a night without stars, and without clouds; and that towards midnight, there fell a shower of stars, which vanished before they approached the earth.

141 B.C. The sun and moon appeared of a deep red colour, which produced great alarm among the people.

74 B.C. There appeared a star as big as the moon, followed by many other stars of the ordinary magnitude.

38 B.C. A shower of stones as big as nuts.

88 A.D. Another shower of stars.

321. Spots in the sun visible to the naked eye.

522. The astronomy of Hiuen-Chi-Ly was replaced by the astronomy of Tching-Kouange-Ly. (We do not understand these to be the names of astronomers but of systems.) In the year 892, another change is recorded; and again in 956.

In 949, in the fourth moon, there appeared a star in the mid-day, which was regarded as such a dreadful omen, that the people were forbidden to look at it; and many were put to death for disregarding the injunction.

These facts alone, independent of the superstitious fears and ceremonies, which even to this day are observed during the time of an eclipse, would alone give us a sufficient contempt for the astronomy of the Chinese; and lead us to reject as mere fables their pretensions to ancient observations. All that we can concede to them, and to the Indians, whose claims rest upon nearly the same grounds, is what we have already attributed to the Chaldeans, viz. that they had become, very early, observers of the motions and phenomena of the heavenly bodies; that they registered certain events, and thence were able to discover periods at which these phenomena would return again; sufficiently approximate to admit of their predicting an eclipse or occultation, within certain limits; but frequently, their prediction has been belied by the non-appearance of the expected phenomenon, while others have happened that were not foretold; which omissions, as well as a false prediction, have cost some few unfortunate astronomers the forfeiture of their heads: of which a particular instance is recorded in the case of Hi and Ho.

Upon the whole, therefore, we may conclude, that however ancient may be the rude observations of the Chinese and Indians, they possessed no science, properly so called, but what they obtained from the Greeks, through the medium of the Arabs; which people, after deriving it from the former source, carried it to Persia, whence it was transmitted to India and China. Such at least is the conclusion drawn by M. Delambre from a dispassionate examination of all the claims of these nations. We are aware that this notion is totally at variance with that of Bailly, who has also examined the case in point with great labour and attention, but certainly not without great enthusiasm and prejudice.

Astronomy. History.

Astronomy of the Arabs.

Astronomy of the Arabs. In our last mention of this people, we saw them acting the part of the most savage barbarians, burning and destroying every thing, the most distantly connected with scientific research; we have now to exhibit them in a more honourable and dignified point of view. We stated, in the passage referred to, that some of the philosophers of Alexandria, escaped the vengeance of their barbarous conquerors, and these of course carried with them some remnant of that general learning, for which the school was so deservedly celebrated. Still, however, destitute of books, of instruments, and probably also of the means of subsistence without manual labour, very little farther knowledge could be accumulated, and still less propagated; so that in a few years, every species of knowledge connected with astronomy and mathematics, must have become extinct, had not the Arabians themselves within less than two centuries of the dreadful conflagration of the Alexandrian library, become the admirers and supporters of those very sciences they had before so nearly annihilated. They studied the works of the Greek authors which had escaped the general wreck, with great assiduity; and if they added little to the stock of knowledge these works contained, they became sufficient masters of many of the subjects to enable them to comment upon them, and to set a due estimation upon these valuable relics of ancient science.

The destruction of the Alexandrian school occurred in the year 640; and it is about a century after, before we find any author worthy of particular notice amongst the Arabs; this, therefore, brings us to the middle of the eighth century; and from Ptolemy to this date there had passed away at least six hundred years, during which time the science had rather retrograded than advanced. The caliph Almansor, A. D. 754, is the first to claim our attention; but rather, perhaps, for the impulse, which, as a philosophical prince, he gave to the science amongst his people, than for any actual improvements which he had personally made. This impulse was, however, so great, that most of his successors seem to have thought it their duty to support and to study the different sciences, particularly astronomy. Haroun, the grandson of Almansor, is particularly noticed as following in the steps of his great predecessor; and one of his sons, Almamon, pursued the same path with still greater enthusiasm. He caused to be translated all the Greek works that he could procure; and in particular the *Almagest* of Ptolemy. He is even said to have made the delivery of certain manuscripts deposited at Constantinople, one of the conditions of the peace which he concluded with the Greek emperor Michael III. He himself made numerous observations, and employed and instructed others to supply his place, when public business prevented him from his favourite pursuit. He ordered the obliquity of the ecliptic to be observed at Bagdad and at Dumas; whence it was found to be 23° 35′, which is less than some preceding observations had indicated.

Almansor. A.D. 754.

Haroun. A.D 786

Almamon. A.D. 813.

Translate the Almagest.

Measures a degree of latitude.

Another important operation performed under the orders of Almamon, was the measure of a degree of the terrestrial meridian; but the unit of the Arabic measure in which it is expressed, is too uncertain for us to attempt to form from the reported result any decisive conclusion; nor are we to expect that any very great accuracy could be expected, seeing the great discrepancy between some modern measures made with the assistance of the most perfect instruments. Almamon also directed certain of his philosophers to compose a work for the purpose of facilitating the study of astronomy amongst his people, entitled, according to the Latin translation, *Astronomia elaborata a compluribus D. D. jussu regis Maimon.* which is still preserved in many libraries.

Alfraganus. A.D. 850.

In the reign of this prince, there were many other celebrated Arabian astronomers, particularly Alfraganus, Thebit-Ibn-Chora, and Albatenius. The former composed a work, many editions of which have been made since the invention of printing, besides some other works more or less connected with this science.

Thebit. A.D. 860.

Thebit was an annalyst, a geometer, and astronomer. He observed the obliquity of the ecliptic and reduced it to 23° 33′ 30″. He also determined the length of the year, very nearly the same as it is now established by modern observations.

Albatenius. A.D. 879.

Albatenius was one of the greatest promoters of Arabian astronomy. His numerous observations and important knowledge in all the sciences of his time, were the cause of his being surnamed the Ptolemy of the Arabs; an honour by no means ill merited. By a comparison of many of his own observations with those of Ptolemy and others, he corrected the determination of the latter respecting the motion of the stars in longitude, stating it to be one degree in 70 years instead of 100 years: modern observations make it one degree in 72 years. He determined very exactly the eccentricity of the ecliptic, and corrected the length of the year, making it consist of 365 days 5 hours 46 minutes 24 seconds, which is about 2 minutes too short, but 4 minutes nearer the truth than had been given by Ptolemy. He also discovered the motion of the apogee; and rectified the theories of Ptolemy respecting the motion of the planets, and formed new tables of them.

The works of this author have been collected, and published in two volumes 4to., under the title of *De scientia stellarum*, of which there are two editions, one in 1537, and the other in 1646.

Montucla, in his valuable history of mathematics, enumerates a long list of Arabian astronomers which followed Albatenius; but we meet with none deserving of particular notice till we arrive at Ebn Iounis, who wrote in the year 1004, and even he is rather celebrated for his having collected and embodied the knowledge of his time, than for his discoveries, although he made numerous observations. The work of this author is still extant, a concise notice of which Delambre has given in the *Mém. de l'Institut*, vol. 2. p. 5, where we learn, that it contains 28 observations of eclipses of the sun and moon, made between the years 829 and 1004; seven observations on the equinoxes; one on the obliquity of the ecliptic: and some others highly important in the determination of certain data, particularly as regards the acceleration of the moon.

Let us now turn our attention to Spain, where the Arabs, who had long been masters of that country, pursued the sciences with the same ardour as in the east. The most distinguished, however, of the astro-

Astronomy. Arsachel. A.D. 1020.

nomers in this country were Arsachel and Alhazen; the former of whom is celebrated for having added greater accuracy to the theory of the sun, by employing a principle different to that of Ptolemy and Hipparchus, and susceptible of more accuracy. He made some fortunate changes in the dimensions of the solar orbit, and discovered certain inequalities in the sun's motion, which have since been confirmed by the Newtonian theory of gravitation.

Alhazen.

Alhazen is also esteemed a philosopher and astronomer of high reputation; he is said to have first discovered the laws of refraction, and the effect of it in astronomical observations. He explained the phenomenon of the horizontal moon, and indicated the true cause of the crepuscula in the morning and evening; beside various other minor discoveries highly honourable to his memory.

From this time the science of the Arabians seems to have begun to decline; we meet with very little after this period deserving of particular notice; a general shade appears to have been cast over every species of human knowledge, and nearly four hundred years are again lost in darkness and obscurity. We then find the Greeks making some feeble efforts to re-establish astronomy in its original empire, where some faint glimmerings of the genius which animated Archimedes, Hipparchus, and Ptolemy, once more began to discover itself; but which, alas! like the lustre of a passing meteor, was soon extinguished, and darkness and barbarity once more assumed their reign.

Astronomy of modern Europe.

Copernicus died 1543.

We may without impropriety refer the revival of modern astronomy to the time of Copernicus, although he was preceded by some others, who prevented all traces of the Grecian and Arabic science from being lost and forgotten, by their reading and studying such works as had been preserved, during what are commonly denominated the dark ages. Copernicus was born at Thorn, in Poland, in the year 1473, but he did not commence his studies till about the year 1507; when after having well "fathomed every depth and shoal" of the ancient doctrine of astronomy; made numerous observations, and various comparisons; he became at first a convert, and afterwards the most strenuous advocate of a system of astronomy, commonly attributed to Pythagoras, and which we have seen Ptolemy using so many ingenious but false arguments to refute.

System of Copernicus

According to Copernicus, the sun is placed in the centre of the planetary world, about which the several bodies of our system revolve from west to east; viz. 1st, Mercury, 2d, Venus, 3d, the Earth, 4th, Mars, 5th, Jupiter, and 6th, Saturn; the moon revolves about the earth in the same direction, while the latter body itself is carried in its orbit round the sun. In the next place, he taught that the earth turns on its own axis from west to east, in a little less than 24 hours, and that this axis is always preserved parallel to itself, making an angle of about $23\frac{1}{2}°$ with the ecliptic.

The orbits of the several planets he supposed to be circular; but he did not make the sun their common centre. We have seen that what was anciently called the solar orbit had been long known to be eccentric, as well as those of the planets; and to account for the phenomena produced by these eccentricities, Copernicus rendered them still greater, by giving to each a different centre, and to the sun such a position with regard to them all, as with the addition of certain epicycles, best agreed with the appearances already observed. This part of the Copernican system is not often well illustrated in our elementary works on astronomy, where it is generally asserted, that the sun was placed in the common centre, and the several planetary orbits were circles concentric with it; by which means, the discoveries of Kepler are rendered more astonishing than they actually are; for he had at least this much to proceed upon; too little, it is true, for any man possessed of less genius and perseverance than himself. This is by no means the only service which Copernicus bestowed upon astronomy and trigonometry, but it is the principal one, and that which has added most to the celebrity of his name; we shall therefore not stop to report his other discoveries and improvements, as we shall find numerous important points to refer to before we conclude this sketch, already considerably extended. The work containing his new doctrine was composed in the year 1530; but it was not printed till the year 1543; the author having, it is said, received the last sheet a few hours before he expired.

History.

Copernicus was followed by a great number of excellent astronomers, some of whom were firm supporters of his system; others endeavoured to refute it, as Ptolemy had formerly done a similar doctrine; while some few who saw its beauties and advantages, but who by giving too literal a signification to some scriptural passages, wished to modify it so as to retain as many as possible of its advantages, while the system should still be such as to correspond with the passages in question.

Tycho Brache died 1601.

Of this number was Tycho Brache, a noble Dane, one of the greatest observers perhaps, if we except Kepler, that ever lived. His system consisted in depriving the earth of its orbicular and diurnal motion; he places the earth in the centre, and made the moon and sun revolve about it, agreeably to the doctrine of Ptolemy; but he made the sun the centre of the other planets, which, therefore, he supposed to revolve with the former about the earth. By this means, the different motions and phases of the planets may be reconciled, the latter of which could not be by the Ptolemaic system; and he was not obliged to retain the epicycles, in order to account for their retrograde and stationary appearances. This theory was extremely complicated, and did not long survive its author. Many of his other labours were attended with much more important consequences; particularly his discovery of the variation and annual equation of the moon; the greater and less inclination of the lunar orbit; the correction for refraction in astronomical observations; his astronomy of comets; his account of the appearance and disappearance of a great star which happened in his time: his reformation of the calendar, and some other subjects which might be enumerated: these have contributed most to establish his name as a great astronomer, and will not fail to hand it down to the latest posterity.

His system.

Other discoveries of Tycho

Kepler died 1631.

Kepler was about twenty years younger than Tycho, but by no means inferior to him in genius and perseverance; the latter quality is proved by the numerous

Astronomy.

His observations on Mars.

observations that he made on all the planets, particularly on Mars; and the former from the memorable consequences which he drew from them. Kepler first determined the line of the apsides by a method independent of the form of the orbit of Mars, and ascertained the ratio of the aphelion and perihelion distances; the former he found to be to the latter, as 166,780 to 138,500; or calling the distance of the earth from the sun 100,000; the above numbers will express the actual distances. Hence the mean distance of Mars was 152,640, and the eccentricity of its orbit 14,140. He then determined, in like manner, three other distances, and found them to be 147,750; 163,100; 166,255. He next computed the same three distances upon a supposition that the orbit was a circle, and found them to be 148,539; 163,883; 166,605; the errors, therefore, of a circular hypothesis, were 789, 783, 350. But he had too good an opinion of Tycho's observations, (upon which he founded all these calculations), to suppose, that the differences arose from their inaccuracies; and as the distance between the aphelion and perihelion was too great according to the hypothesis in question, he conceived that the orbit must be an oval. And, as of all ovals the ellipse is the most simple, he naturally made trial of this figure, placing the sun in one of its foci; and upon making the requisite calculation, he found the agreement complete. He did the same for other points of the orbit, and still found the same accurate agreement; and hence he pronounced the orbit of Mars to be an ellipse, and that the sun occupied one of its foci. Having established this important point for the orbit of Mars, he conjectured the same to have place in the other planets; and upon trial he found his conjecture fully verified: and, hence, he concluded, that the six primary planets revolve about the sun in elliptic orbits, that body occupying one of the foci.

Kepler's first law.

Kepler's second law.

Having thus discovered the relative mean distances of the planets from the sun, and knowing their periodic times, he next endeavoured to find, whether there were any relations between them; and having naturally a strong turn for numerical analogies, he began by comparing the powers of those quantities with each other; and even in the first instance (March 8th, 1616,) he assumed the correct law; viz. *that the squares of the times of revolution are proportional to the cubes of the mean distances;* but, in consequence of some error in his calculation, his comparison did not appear to be complete. Nor did he discover it till the following May. He then found his first conjecture to be correct, and thus established this celebrated law, which Newton afterwards demonstrated to be the necessary consequence of a body revolving in an orbit about a central attracting point coinciding with one of its foci. *Prin. Phil.* lib. i. sec. ii. pr. 15.

Kepler's third law.

Kepler also discovered from observation, that the velocities of the planets, when in their apsides, are inversely as their distances from the sun; whence it followed, that they described at these points equal areas in equal times; and although he could not prove the same for every point of their orbits, he had still no doubt that it was so. He therefore applied this principle to determine the equation of the orbit; and finding that his calculations agreed with observation, he concluded that it was true in general; "that the planets describe about the sun equal areas in equal times." This discovery was perhaps the foundation of the *Principia*, probably suggesting to Newton the idea, that all the planets of our system were governed by one general law! and that the sun was the general focus of action; a proposition which he afterwards succeeded in demonstrating, and thus founded the basis of physical astronomy.

History.

Kepler also speaks of *gravity* as a power which is mutual between all bodies; and tells us that the earth and moon would move towards each other, and meet at a point so much nearer the earth than the moon, as the former is greater than the latter, if these motions did not prevent it. And he farther adds, that the tides rise from the gravity of the waters towards the moon.

Invention of the telescope.

The beginning of the 17th century was distinguished by two of the most important events for astronomy that we have yet recorded; viz. the invention of the telescope and logarithms; by means of the one, we are enabled to penetrate into the remotest parts of space, and to bring under our immediate view phenomena, which the most sanguine minds could never, if they had suspected their existence, have hoped to bring within the limits of human observation: by means of the other, all the laborious calculations of former astronomers were reduced to mere operations in addition and subtraction; and what was before the labour of a month, became, as it were, the amusement of an hour. With two such powerful auxiliaries, the progress of astronomy could not but be extremely rapid; but still the extent to which it has been since carried, must certainly far have exceeded what the boldest of the astronomers and philosophers of that day could have dared to hope for. It never could have been contemplated that this science, apparently so far beyond the reach of all human power, would become the most perfect of all the physical sciences; that every disturbing force, and every celestial phenomenon of our system, should be submitted, with the greatest precision, to one general simple principle, at that time wholly unknown; and that an analysis would be discovered, to enable us to investigate and compute the motion of bodies of immense magnitudes, and at almost immeasurable distances, with greater accuracy than we can calculate the motion of a projectile from a piece of ordnance. Yet such is actually the case. We know, to a greater nicety, the moment when a planet will arrive at a certain point in the heavens, than we can tell the time that a cannon ball will employ in passing from the gun to the extremity of its destined range, the moment of explosion being given.

Origin of physical astronomy.

As far as we have hitherto traced the progress of astronomy, all our knowledge has been supposed to consist in observations on the motions of the heavenly bodies; and every species of computation relating to it, rested on no other foundation. We are now arrived nearly at that period, when our illustrious Newton, by his discovery of the law of universal gravitation, added an entire new branch to this important science, commonly denominated physical astronomy. From this time a greater range was given to celestial researches; our knowledge was not confined to the mere observation of phenomena, and the return of certain bodies to certain parts of space: the causes of these phenomena, and of these motions, were now to be

Astronomy. investigated, and the amount of them computed from the pure principles of celestial mechanics. Every minute inequality was henceforward to be traced to the same source—the action of gravitation: while, on the other hand, every minute variation indicated by this theory, ought to be observable in the heavens. And nothing, perhaps, is better calculated to demonstrate the generality of this law, and the beauty and profundity of the modern analysis, than the fact, that certain small inequalities were indicated by the theory, before the accuracy of instruments, and the delicacy of observation, were sufficiently attained to render them appreciable; but the existence of which, the present high state of practical mechanics, and a corresponding improvement in optics, have confirmed in the most satisfactory manner. We shall, therefore, now divide the remaining part of this historical sketch under two distinct heads; viz. practical and physical astronomy; and slightly glance at the successive steps that have been made in each; but a simple indication of them is all that can be expected, and indeed it is all that is requisite, since we must necessarily meet the same subjects again in the course of the following treatise; where we shall enter upon them as much at length, as is consistent with the limits allotted to this department of our work.

Discoveries depending upon the telescope.

Our business here, is neither to give the history of the invention of the telescope nor of logarithms; the former has already been treated of in our treatise of optics, and the other will be found in its appropriate place in this work. We have seen in the article above referred to, that the telescope is perhaps an earlier invention than it is commonly supposed to be; and that Harriot, in a very early part of the 17th century, viz. between the years 1610 and 1613, had actually observed the spots on the sun's disc with telescopes of various magnifying powers; a fact, which has been but lately ascertained by Baron Zach, of Saxe Gotha, who, in a visit to this country, had access to certain MSS. of this English author, in the possession of the Earl of Egremont, a descendant of the Earl of Northumberland, who was Harriot's patron. This, after Galileo's, is one of the earliest well attested facts of the application of this instrument to astronomical purposes.

Galileo discovers the satellites of Jupiter. A.D. 1610.

Galileo, as we have seen in our history of optics, was informed of the accidental discovery of Jansen's children, in the year 1609; and on the 8th of January, 1610, he perceived, by directing his new instrument to the heavens, three small stars near the planet Jupiter; two on the one side, and the third on the opposite side; which he took at first for fixed bodies. But having continued to observe them on the next and the following nights, he found that they changed their places and positions; and that they performed their revolutions about that planet, and were, in fact, satellites to it, as the moon is to the earth; and some days after he discovered a fourth. These four satellites he named, in honour of the house of Medici, the Medicean stars; but the appellation was soon lost in the more general and appropriate name of Jupiter's satellites. This discovery was published in the month of March following, in a writing, entitled *Nuncius Sydereus;* he undertook even to investigate the theory of their motion; and, in the year 1613, to predict their configurations for two consecutive months.

History.

Irregularity in the form of Saturn.

Directing his telescope to Saturn, a new surprize, and new pleasure presented itself. After a few imperfect observations on this planet, he was led to believe that Saturn was not a simple globe like the other bodies of our system, but was compounded of three stars touching each other, immovable with regard to themselves, and so disposed, that the largest occupied the centre, with a smaller one on each side of it; but it was not long before he discovered this idea to be erroneous, so far at least as regarded their immobility, or invariable appearance: he found that the figure was changeable, but his telescope had not sufficient power for him fully to unravel the mystery; he found that Saturn appeared irregularly formed, and supposed the extreme parts, of what was afterwards found to be a ring encompassing the body of the planet, were attached to it, forming *ansæ*, or handles; and it was some years after, that Huygens discovered the actual conformation of this beautiful telescopic object.

Phases of Venus, and spots on the sun.

Galileo also first observed the phases of Venus, predicted by Copernicus; as he did also the spots on the sun's disc, unless indeed our countryman Harriot had the advantage of him, in point of time, in this observation. There are others who contest with Galileo for the honour of the important discovery of Jupiter's satellites, as Simon Marius, of Brandebourg, who asserts, that he observed them in 1609; but the truth of this seems to rest on no better foundation than the mere declaration of the author, in his *Nuncius Jovialis, anno* 1609, *detectus,* &c. published in 1614.

Other claims for these discoveries.

The spots on the sun were also observed by Fabricius and Scheiner; and it is perhaps doubtful by whom they were first seen. The telescope soon became well known in all parts of Europe; the sun was an object which would naturally claim the regard of astronomers, and it is not unlikely these spots might be observed by many at nearly the same time.

Galileo defends the Copernican system.

The system of Copernicus, even before these discoveries, had made considerable progress; and what had now occurred, had rendered the truth of it still more certain. Galileo, therefore, in 1615, had the courage openly to support it; but it caused him a reprimand from the *Holy Inquisition;* he was imprisoned, but afterwards liberated upon certain conditions of silence. Twenty years afterwards, greater advances being made in the progress of this science, he again ventured to assert his opinion, but still with the same effect: he convinced all unprejudiced people; but the holy fathers were not in this class, and he was once more obliged to abjure upon his knees his heretical doctrine. Such are the fruits of ignorance blended with bigotry and superstition.

Huygens discovers Saturn's 6th satellite. A.D. 1635.

We shall deviate a little in the order of time, in order to bring together those discoveries which have added to the number of the bodies of the solar system. Huygens, therefore, according to this arrangement, is the first to claim our attention. This eminent philosopher, having in 1635 constructed for his own use two excellent telescopes, the one of twelve, and the other of twenty-four feet, discovered a satellite to Saturn, which is now the sixth in the order of distances, and determined the dimensions of its orbit, the duration of its revolution, &c. with an astonishing degree of exactness; but falling into a metaphysical error, concerning the number of the bodies of our system, he sought to find no more, considering that

Astronomy. Discovers Saturn's ring.

this one was all that was wanting to render the whole planetary scheme complete. He was drawn, however, to a minute examination of Saturn itself, and was the first to announce the actual conformation of it. He stated, that this planet was encircled by a flat broad circular ring, detached from it on all sides, but which, according to the position under which it was observed from the earth, would assume the several appearances of a circle, an ellipse, and a right line; he never, however, observed it under its latter form. He discovered that the diameter of the exterior edge of the ring, was to the diameter of the planet as 9 to 4; and that its breadth was equal to the space contained between the globe and its interior circumference; results which, with some slight modifications, have been confirmed by more modern observations.

Cassini discovers four other satellites, A.D. 1684. Herschel two others.

Four other satellites to this planet were discovered by Dominique Cassini: which, in the order of distances, are now the 3d, 4th, 5th, and 7th; the 5th and 7th in 1671, and the 4th and 5th in 1684; two others have since been added by Sir W. Herschel in 1789; making in all seven attendant luminaries to this remote and otherwise solitary planet.

Herschel discovers the Georgium Sidus.

Eight years prior to the above discovery, viz. in 1781, Sir W. Herschel had observed a small star, which after a little attention, he found changed its place; having well ascertained this fact, he communicated a knowledge of the circumstance to M. Laxel, a celebrated astronomer of the academy of St. Petersburgh, who was then in London; the same information was also transmitted to other eminent astronomers, who observed it with great care, and soon afterwards it was announced as a new planet, the most remote in our system, circulating about the sun at the astonishing distance of nearly *eighteen hundred million miles*, and performing its orbicular revolution in about 80 of our years. This new planet was first named by foreign astronomers after its observer, *the Herschel*; but Herschel himself, in imitation of Galileo, dedicated it to his late Majesty, under the name of the *Georgium Sidus*; both these appellations are, however, now nearly become extinct, that of *Uranus* being almost universally adopted. The same indefatigable observer has since discovered six satellites to this planet, which revolve about him under very peculiar circumstances; but the particulars must be reserved for the proper place in the subsequent treatise.

Discovers six of its satellites.

In order to complete this part of our historical sketch, we must now pass to the beginning of the 19th century; the *first day* of which is remarkable for the discovery of another new planet, between the orbits of Mars and Jupiter; this we owe to the observation of Piazzi. This planet has received the name of Ceres. Another new planet was discovered by Dr. Olbers, on the 28th of March, in the same year, 1801, which is called *Pallas*; its distance and periodic revolution being nearly the same as that of the former. A third, having likewise nearly the same mean distance, was discovered by M. Harding, at Lilienthal, near Bremen, September the 4th, 1804; and a fourth by Dr. Olbers, on the 19th of March, 1807, being the second we owe to this indefatigable observer. Of these two, the former has been named *Juno*, and the latter *Vesta*.

Four other newplanets. Ceres, by Piazzi. Pallas, by Olbers. Juno, by Harding. Vesta, by Olbers.

For the elements, and other particulars respecting these new planets, we must refer the reader to the respective articles in the following treatise.

History.

Uniting together these several discoveries, it will appear, that in less than two centuries, there have been added to the known bodies of our system, no less than five planets, and seventeen satellites, about three times as many as were known at the time of the promulgation of our present system by its venerable author Copernicus.

In the paragraphs immediately preceding, we have in some degree disregarded the order of time, for the purpose of bringing under one point of view those discoveries which have increased the number of the bodies in the solar system; we must now, therefore, retrace our steps, in order to glance at some other facts connected with the history of this science; but the bare enumeration of them is all that our limits will allow us to attempt.

Bayer's catalogue of the stars, A.D. 1603.

In 1603, Bayer formed a catalogue of the stars, which he published under the title of *Uranometria*, a highly important and useful work.

Transit of Mercury. A.D. 1632.

Nov. 7, 1631, Gassendi observed the passage of Mercury over the solar disc, agreeably to the prediction of Kepler; and published his account of it in 1632, in a work entitled *Mercurius in Sole visus*, &c.

Transit of Venus. A.D. 1638.

1638. The transit of Venus was observed over the sun by Horrox, a young English astronomer, which is described in his *Venus in Sole visus*, &c.

These are the two first observations of this kind that had ever been made, and much importance was in consequence attached to them, although their great utility in establishing certain astronomical data was not then foreseen.

Hevelius died. A.D. 1638.

In 1638, the sciences had to deplore the loss of Hevelius, a celebrated astronomer and senator of Dantzic, to whose indefatigable labours we owe many valuable observations; but our limits will not permit of our entering into particulars.

Royal Society and Academy of Sciences.

We pass now to that period when the Royal Society of London, and the Academy of Sciences at Paris, were first instituted, and the observatories of Paris and Greenwich were erected. The advantage of these institutions to the science of astronomy, is unbounded; instruments of the best kind that could be obtained, were immediately constructed: the members of the two academies communicated with each other; various experiments were proposed and executed; and an impulse was thus given to science in general, and to astronomy in particular, which it would have been in vain to have expected from individual perseverance and talent, however conspicuous.

Royal Observatories of London and Paris.

The charter of the Royal Society was granted by Charles II. in 1660; that of the Academy of Sciences by Louis XIV. in 1666. In 1667, the observatory of Paris was erected; and the first stone of the observatory of Greenwich, was laid by Flamstead (who was appointed astronomer royal) on the 10th of August, 1675, at the recommendation of Sir Jonas Moore, to whose influence we are indebted even for the institution itself. Flamstead continued to fill this situation in a manner equally honourable to himself and his country, for 43 years; he was succeeded by Dr. Halley, who continued in it for 23 years; Dr. Bradley followed Dr. Halley, and held the office for twenty years: Mr. Bliss only remained two years, and was followed by Dr. Maskelyne, who died in 1811, after holding it for 46 years.

English astronomers royal.

Never has any appointment been more honourably

Astronomy. History.

filled than that of the astronomer royal of England. Of the names which we have above enumerated, four of them will be handed down to the latest posterity; but we shall only have occasion to refer to Halley and Bradley, because we can only notice the more prominent features of the history of this science; and the observations and results of the first and last of those great men, although of infinite advantage to the attainment of accurate knowledge, and would therefore form an important part in a complete history of astronomy, are still not of that striking nature to claim more than a general notice in this place.

Dr. Halley.

Dr. Halley, who was equally celebrated as an analyst, a geometer, and astronomer, very early distinguished himself by a geometrical method of determining the apsides, the eccentricities and dimensions of the principal planetary orbits; he afterwards undertook a voyage to the island of St. Helena, for the purpose of forming a catalogue of the stars in the southern hemisphere; he projected the method which was afterwards put in practice for observing the transit of Venus in 1761 and 1769, for determining the parallax of the sun; and predicted the return of a comet in 1759, fixing the period of its revolution at 75 years; this comet, which is the only one whose orbit is known, and which is expected to reappear in 1834, bears his name, as an honourable testimony of the truth of his prediction, and the profundity of his knowledge. Many other valuable works of this author we must necessarily pass over.

Predicts the return of a comet.

In 1667, a highly important discovery was made by Roemer, a Danish mathematician, at that time resident at Paris. He had long been engaged in making very accurate observations on the motion and eclipses of Jupiter's satellites; in the course of which he noticed that, at certain times, these bodies emerged from the shadow of the planet some minutes later, and at others, as much before the time given by the most accurate tables. By comparing these variations with each other, it appeared, that the satellite emerged too late from the shadow, when the distance between the earth and Jupiter was the greatest, and too soon when that distance was the least; whence, after some conjecture, he hit upon the happy idea, that as the apparent emersion happened too late or too soon, according as the planet was more remote or nearer to us, it must proceed from the time that light employed in passing over the difference in the two distances; in fact, that the propagation of light is not instantaneous; but that it requires a certain time to pass from one point to another; and submitting his ideas to calculation, it appeared, that a luminous ray employs about eleven minutes in describing a distance equal to that of the earth from the sun. This bold idea he communicated to the Academy of Sciences on the 16th of November, 1667: it has since been confirmed with some modification, reducing the time to 7½ minutes, and has immortalized the name of Roemer.

Roemer. A.D. 1667.

Determines the velocity of light.

Aberration of the fixed stars.

Although the system of Copernicus had now gained a complete ascendancy, yet many persons were inclined to imagine, that if it were true, some parallax ought to be observed in the fixed stars: that these bodies should be at such an immense distance, that a base of nearly *two hundred millions of miles*, the diameter of the earth's orbit, should not sensibly alter their positions, was, it must be allowed, a doctrine sufficient to startle those who could not expand their minds to a contemplation of the infinitude of celestial space; and, accordingly, various observations were made with a view of ascertaining whether or not such a parallax had place; and instruments had now arrived at that degree of perfection, that it was thought by many eminent astronomers such an effect ought to be rendered appreciable.

The minuteness and accuracy of these observations, did, indeed, show a small change in the relative position of some stars, but it was, generally speaking, directly contrary to that which ought to result from a parallax. This motion the cause of which was unknown, was denominated *aberration*; and it was this which Dr. Bradley undertook to examine, and endeavoured to reduce to a general law. In the prosecution of this design, he found that certain stars appeared to have, in the course of a year, a sort of vibration, in longitude, without changing their latitude; some varied only in latitude, while others, and that the greater number, appeared to describe in the heavens, in the course of the year, a small ellipse, more or less elongated. This period of a year to which these variations answered, although so different from each other, was a certain indication that they were connected with the annual motion of the earth in its orbit about the sun; and after a time, he fortunately perceived the cause of all the irregularities he had observed. He attributed the apparent aberration of the fixed stars to the combined motion of the earth, and that with which light is propagated.

Dr. Bradley explains the cause of it. A.D. 1736.

Roemer had shown that the velocity of light is about 10,000 times greater than that of the earth in its orbit, therefore, a ray of light issuing from a star, will not carry the impression of this star to the eye, till after the earth has sensibly changed its place; and consequently, when the eye receives the impression, it ought necessarily to refer the object to a different point in the heavens to that in which it is actually placed, or to that in which it would appear, if the earth were at rest. This explanation satisfied every doubt on the subject, and amounted to nearly a mathematical demonstration of the truth of the Copernican system.

Nutation of the earth's axis.

We owe to this celebrated astronomer, another discovery no less important than the above, viz. the nutation of the earth's axis. The celestial mechanics of our illustrious Newton had shown, that the unequal attractions of the sun and moon on the different parts of the terrestrial spheroid, ought to produce a variation in the position of its axis as referred to the plane of the ecliptic. Bradley undertook to examine the effect of this motion by means of a long series of delicate observations, made in those positions of the sun and moon most proper to render them manifest. The result of his researches were,—1. That the axis of the earth has a conical motion, by which its extremities describe about the pole of the ecliptic, and contrary to the order of the signs, an entire circle in 25,900 years, or an arc of about 50″ annually. This explains the cause of the precession of the equinoxes. 2dly, That this axis has, with reference to the plane of the ecliptic, a libration, by which it inclines about 18″ in the course of one revolution of the nodes, and is also made contrary to the order of the signs; after which it returns to its first position, inclines again,

Astronomy.

and so on, in each successive period of nineteen years. Such are the discoveries which render the name of Bradley immortal in the history of this science. We are also highly indebted to him for numerous other important results, to which we shall have occasion to refer in the course of the following treatise.

There still remains one department of practical astronomy, which requires to be briefly noticed in this sketch, viz. the measurement of the terrestrial circumference; which is important, because the terrestrial radius, in all our actual determination, is the only unit of measure to which we can have recourse, in order to reduce the several distances of the heavenly bodies to known measures: but as this is also an important datum in the physical branch of astronomy, we shall defer the consideration of those measurements, till we have to consider it under the latter point of view.

Progress of physical astronomy.

Dr. Hook's ideas of gravitation. A.D. 1668.

We have seen in our account of Kepler, that he had formed some general ideas of the nature of universal gravity; but it was too vague to become the foundation of any mechanical principle. Dr. Hook, an English philosopher of a most extraordinary genius, had made a much nearer approximation to a development of this great law of the universe. In one of his communications to the Royal Society, May 3d, 1668, he expressed himself as follows:—"I will explain a a system of the world very different from any yet received. It is founded on the three following positions:

"1. That all the heavenly bodies have not only a gravitation of their parts to their own proper centres, but that they also mutually attract each other within their spheres of action.

"2. That all bodies having a simple motion will continue to move in a straight line, unless continually deflected from it by some extraneous force, causing them to describe a circle, an ellipse, or some other curve.

"3. That this extraction is so much the greater as the bodies are nearer. As to the proportion in which those forces diminish by an increase of distance, I own I have not yet discovered it, although I have made some experiments to this purpose. I leave this to others who have time and knowledge sufficient for the task."

This is a very precise enunciation of a proper philosophical theory. The phenomenon of the change of motion, is considered as the mark and measure of a change of force, and his audience is referred to experience for the nature of this force: he having before exhibited to the society a very neat experiment, contrived to shew the nature of it. A ball, suspended by a long thread from the ceiling, was made to swing round another ball, laid on a table immediately below the point of suspension. When the impulse given to the pendulum was nicely adjusted to its deviation from the perpendicular, it described a perfect circle round the ball on the table; but when the impulse was very great, or very small, it described an ellipse, having the other ball in its centre.

Hook shewed that this was the operation of a deflecting force proportioned to the distance from the other ball; and he added, that although this illustrated the planetary motions in some degree, yet it was not suitable to their case; for the planets describe ellipses, having the sun, not in their centre, but in their focus. Therefore, they are not retained by a force proportional to the distance from the sun.

History.

The exalted genius of Newton can suffer no diminution by the enumeration of the above opinions; for though the idea of such a principle as gravitation was not suggested first by Newton, yet so very obscure were the notions of even the most enlightened philosophers on this subject, that it had never been successfully applied to the explanation of a single astronomical phenomenon.

Newton's discoveries. A.D. 1666.

The important discovery of the law of universal gravitation is so intimately connected with the history of philosophical science, that every circumstance relating to it has been recorded with the greatest care. Dr. Pemberton relates, that Newton, in the year 1666, having retired from Cambridge to the country on account of the plague, was led to meditate on the probable cause of the planetary motions, and upon the nature of that central force which retained them in their orbits; when it occurred to him, that the same force, or some modification of the same force, which caused a heavy body with us to descend to the earth, might likewise retain the moon in her orbit, by causing a constant deflection from her rectilinear path. But before this could be submitted to the test of computation, it was necessary that some hypothesis should be formed relative to the modification of its action with respect to distance; and probably, that which has actually place, namely, that it is reciprocally as the square of the distance, almost immediately suggested itself to his mind, as being observed in all kinds of emanations with which we are acquainted.

When Newton first attempted to verify this conjecture, the requisite data with regard to the distance of the moon in terrestrial radii, and the measure of the radius itself were but imperfectly known; the result therefore which he obtained, though nearly, did not accurately agree with observed phenomena; and he in consequence abandoned his theory as untenable: a remarkable instance of the cool and dispassionate frame of mind which this great philosopher observed, even at the moment when he flattered himself with the idea of having discovered one of the most important secrets of nature.

Sometime afterwards, however, he was induced to renew his calculations; as, in the interval, more correct data had been obtained by the measurement of Picard, in France. This attempt succeeded; and he is stated to have been extremely agitated towards the conclusion of his calculation. Whether this were the case or not we cannot attempt to settle: at all events, a moment of greater interest will never be recorded in the annals of science.

Principles of calculation.

Let us briefly illustrate the nature of the calculation to which we have referred. Our author having established as a datum, that the distance of the moon from the earth, was about 60 semi-diameters of the latter, that is, 60 times as far from the earth's centre as a heavy body placed at its surface; and assuming the power of gravity to decrease inversely as the square of the distance, it would follow, that at the moon, this force would only be one 3600th part of what it is at the surface of the earth; and, consequently, the space passed over by a body at this distance, when submitted to the terrestrial gravitation, would be only $\frac{1}{3600}$ of that which is actually described here; or since the spaces are as the squares of the times, and the force inversely as the square of the distance; the moon ought to fall towards the earth through the same space in a minute, as a heavy body here describes in a second, viz. about $16\frac{1}{12}$ feet.

Hence it follows, supposing the moon to be retained in her orbit by this force, that her deflection from the tangent to her orbit at any point, ought to be 16·083 feet in one minute, or $\frac{16\cdot083}{3600}$ in one second. Now the distance of the moon from the earth in semi-diameters being known, and the radius of the earth itself being supposed determined by actual measurement of terrestrial arcs, as also the exact time of one lunar revolution, it would be very easy to find the circumference of the lunar orbit, and the measure of the arc which the moon describes in one second, whence again the versed sine of that arc, or the difference between the secant and radius, which is nearly the same thing, may also readily be obtained in fcet; and this, as above observed, ought to be $\frac{16\cdot083}{3600}$. This measure Newton verified by his calculation, and thus established this most important law with regard to the earth and moon, which was afterwards readily extended to the whole planetary system.

Huygens's central forces.

Several years before this discovery of Newton's, Huygens had given in thirteen propositions the properties of centrifugal and centripetal forces in a circle; but he did not think of applying his theory to the motion of the earth on its axis, and to the moon about the earth; had he done this, it is highly probable he would have arrived at the same conclusion as Newton; this grand step in his process was, however, wanting, and the entire honour of the discovery justly devolved upon our illustrious countryman.

Law of the planetary motions.

In order to extend the same principles to the other planetary motions, Newton was led to consider, that when two bodies act on each other by attraction, the action is reciprocal; moreover, that the attraction of each body is equal to the sum of the attraction of all its parts, and therefore proportional to the mass; consequently, some other data were requisite for demonstrating that the whole system was regulated by this one general principle;—the motion of the moon appertaining only to the earth, while the earth and all the other planets revolved about the sun.

Newton demonstrated, generally, that if a body projected into space be continually turned from its direction, by any force which urges it towards a fixed centre, and which causes it to describe a curve, the areas of thc sectors comprised between the arc of the curve and the right lines joining the body and fixed centre, are proportional to the times of description, and *vice versa*, if the areas are proportional to the times the revolving body is necessarily urged towards a fixed centre. Now the primary planets observe this law in revolving about the sun; therefore regarding this body as fixed, it follows, that each planet is continually attracted towards the sun as a centre. But from this condition alone he would have been able to determine nothing respecting the nature of this force. Kepler, however, furnished the law nccessary for this determination; for if to the condition of equal areas being described in equal times, we add the other condition, that the orbit is an ellipse, then it will follow, as demonstrated by Newton, that the force must vary reciprocally as the square of the distance. Still, however, this only proved that the law obtained for each planet individually in different parts of its orbit, and it still remained to be shown that the same had place for every planet in the system; viz. that each was attracted by a force which was reciprocally as the square of its respective distance. Here again another datum of Kepler's was of the highest importance; he had shown that the squares of the periodic times were as the cubes of the distances; and this was sufficient for demonstrating the universality of the law in question; namely, that every material particle in nature attracts with a force proportional to its mass, and reciprocally as the square of its distance from the body on which it acts. It is this law which regulates all the planetary motions, and to which we must have recourse for explaining any irregularities observable in them.

Masses of certain planets determined.

Our planetary world is compounded of different systems: thus the sun and the primary planets may be considered as one system, the earth and moon as another, Jupiter and his satellites as a third, and so on; and it will bc necessary to distinguish betwcen these, when we are comparing the motions of bodies in one of those systems with those in another; thc mass of the central body being a necessary datum in determining the actual motion of the circulating body; and conversely, thc motion of two or more revolving bodies being known, the masses of the attracting bodies may be determined. It was thus Newton found, that, denoting the mass of the sun by unity, that of Jupiter would be expressed by the fraction $\frac{1}{1067}$, Saturn by $\frac{1}{3021}$, and the earth by $\frac{1}{169282}$.

We cannot follow Newton through the numerous inferences, calculations, and deductions, which resulted from this universal law; it will be sufficient to observe, that it accounted for the flux and reflux of the tides, the nutation of the earth's axis, the precession of the equinoxes, the spheroidal figure of the earth, and various irregular motions in the planetary system; but some of these have been illustrated and submitted to calculations within the last few years, in a manner more exact and conclusive than in the time of our author. Analysis has taken a much greater range, and every advance that it has made, has furnished some further confirmation of the truth and generality of the great law of universal gravitation.

Difficulty of the general problem.

If there were but two bodies in our system, as, for example,—the sun and earth; or but three,—the sun, earth, and moon, the determination of their actual and disturbing forces on each other would be comparatively a problem of easy solution; but when we consider the influence of several bodies, whose position with respect to each other is perpetually changing, and then endeavour to estimate their effects on any one in particular, we shall find the question involved in the greatest perplexity, and apparently far beyond the reach of human intellect to comprehend; yet such has been the progress of analysis within the last half century in the hands of D'Alembert, Euler, Lagrange, La Place, and some others, that not a single phenomenon now remains, nor the smallest irregularity in the motions of the principal bodies of our system, that is not accounted for; its amount computed, and its origin traced to that law which it is the glory of Newton to have first discovered.

As these subjects must necessarily come before us in our treatise on physical astronomy, we shall not detain the reader with the nature of the particular solutions and investigations of the authors above alluded to; but shall pass on to the only remaining topic—the figure and magnitude of the earth.

Magnitude and figure of the earth.

We have seen that this problem had been attempted by Eratosthenes, and again by the Arabian mathematician Almamon, but we know very little of their determinations; we may however easily imagine, knowing how great the difficulty of the operation is, even with the most perfect instruments, that their approximations must necessarily have been very remote from the truth; and probably the early attempts of this kind in Europe were not attended with much better success. We shall briefly notice those which seem to be of the greatest importance.

Different measures.

In 1525, M. Fernelius measured a degree of the meridian northward from Paris, which he made 68·7634 English miles. Snellius, professor of mathematics at Leyden, and our countryman, Norwood, measured each of them a meridional degree; the former in Holland in 1620, and the latter in England between London and York, in 1635. Snellius' measure, when reduced to English miles, is 66·91; and that of Norwood 69·545 miles. In 1644, Riccioli also undertook a similar task, and performed it according to three different measures, between mount Parderno and the tower of Modena in Italy, and obtained a mean length of 75·066 English miles to a degree.

Figure of the earth.

These results differed too much from each other to inspire the least confidence in any of them, and consequently nothing could thence be deduced respecting the figure of the earth; nor was this indeed, at that time, suspected to be any other than a perfect sphere, abstracting from the irregularities on its surface. But after the construction of the telescope had received considerable improvement, and when observations had been reduced to greater nicety, it was found that the planet Jupiter was considerably flattened at his poles; and the pendulum experiment of Richer, 1671, had shown that there was a difference in the action of gravity at the equator and in the latitude of Paris; and these two circumstances probably first suggested to Huygens that the earth was not spherical; and its rotatory motion about its axis naturally led him to conclude that it was flattened at its poles; from the combination of its centrifugal force with that of gravity, he calculated that its polar axes were to its equatorial diameter as 578 to 579. But as he regarded all the power of terrestrial attraction to be collected in the centre of the earth only, his solution was obviously defective; and Newton soon after undertook the same determination, on more correct principles, by supposing every particle in the whole mass to have a reciprocal attraction towards all the others; from which he found the figure to be an ellipsoid, having its polar and equatorial diameters to each other at 229 to 230.

Determined by Newton and Huygens.

Determined by practical measurements.

In this state the question remained for many years, till M. Picard undertook the measurement of a degree in France, in 1669, which was afterwards revised by Cassini in 1718; the result of which tended to show, that the earth was not an oblate but a prolate spheroid. Picard obtained for the length of a degree 68·945 English miles, and Cassini 69·119; whereas the latter, being the most southern, ought to have been the shortest. A circumstance so unexpected as this, naturally produced a great curiosity, and a considerable degree of inquiry and controversy between the astronomers and mathematicians of that period; and the French government, at the recommendation of the Academy of Sciences, in 1735, sent out two companies of mathematicians to determine the point in question, by measuring two degrees, one at the equator, and the other in as high a northern latitude as possible. Accordingly, MM. Godin, Bouguer, and Condamine, from France, with Dons Juan and Ulloa, from Spain, proceeded to Peru; while Maupertuis, Clairaut, Camus, La Monnier, &c. accompanied by Celsus, a Swedish astronomer, proceeded to Lapland.

After experiencing many unforeseen difficulties and delays, both parties accomplished the object of their missions, and returned to France; those from Lapland in 1737, and the other division in 1744. The former made the length of their degree, the middle point of which was in latitude 66° 20′, equal to 69·403 English miles; while at the equator the length, as determined by the other party, was found to be 68·724 English miles, taking the mean of three different results, deduced from the same operation. In the interval between the outfitting of the above expeditions, and the publication of their results, the degrees of Picard and Cassini were examined, recomputed, and found now to be, the first, whose middle point was in latitude 49° 22′, 69·121 English miles, and other 69·092 miles, its middle point being 45°.

Oblate figure established.

The results therefore of all these measures confirmed the earth to be an oblate spheroid; but as they were compared together in pairs, they gave very different degrees of ellipticity; nor has all the accuracy that has since been introduced into these operations been sufficient to establish this point. Colonel Mudge in England, has measured an arc extending from the southernmost point in the kingdom to the northernmost of the Shetland islands; and Messrs. Delambre, Mechain, Biot, Arago, &c. have carried another from Dunkirk to Fomentara, one of the Balearic isles. It is perhaps never to be expected that greater accuracy can be introduced into any operations, nor to see greater talents employed in conducting them, than in the cases to which we have last referred; and yet these two measures, compared with each other, in different portions, give very different ellipticities; different parts of the same arcs give very discordant results; even some of those of the English survey, are such as to lead again to the idea of the prolate figure of the earth. What are we then to conclude from the whole of these deductions, but that the irregularities of local attraction, or some other causes which we cannot discover, produce a certain influence or disturbing power, which renders useless the comparisons of small arcs, and that those results only can be depended upon that are drawn from measurements that extend through several degrees of latitude. Adopting this principle, we may state, that the mean length of a degree in the latitude 45° is 68·769 English miles, and that the ellipticity of the earth is between $\frac{1}{307}$ and $\frac{1}{314}$.

Ellipticity still uncertain.

Medium results.

One other point still requires to be alluded to before we conclude our history. It was observed by Bouguer, in his operations in Peru, that the high mountains in some parts of that country, very sensibly disturbed the verticality of his plumb-line; that is, the lateral attraction drew the line out of its perpendicular direction, a certain quantity which he determined. This suggested to the Royal Society the idea of employing this deflection, in order to ascertain the actual density of the earth as compared with water, or any other known substance. In order to this, it was necessary to select some isolated mountain, whose density and magnitude might be ascertained; when by observing how much a pendulum or plumb-line was deflected on opposite sides of it, at given distances, the proportional forces between the earth and mountain would become known, and hence from the established laws of attractions, the relative masses of the two bodies would be determined, and hence the density of the earth, its magnitude being supposed already ascertained.

Density of the earth.

The mountain selected for this purpose was Schehallian, in Scotland, and Dr. Maskelyne, in 1742, was requested to direct the operations. The mean height of Schehallian above the surrounding valley is about 2000 feet, and its direction is nearly east and west. Two stations were chosen for observation, one on the north, and the other on the south side of the mountain. Every circumstance that could contribute to the accuracy of the experiment, was attended to with particular care; and from the observations on ten stars near the zenith, Dr. Maskelyne found the apparent difference of the latitudes of the stations to be 54′ 6″: and from a measurement by triangles from the two bases, on different sides, he found the actual difference of their parallels to be 4364 feet, which, in the latitude of Schehallian, 56° 40′, answers to an arc of the meridian of 43′, which is less, by 11′ 6″, than that found by the sector. These data being submitted to the calculation indicated above, a task which was performed by Dr. Hutton, it appeared that the density of the whole mass of the earth as compared with water, was nearly in the ratio of 4½ to 1. *Phil. Trans.* vol. 65, and vol. 68; but from more accurate observations on the geology of this mountain by Playfair, it appears that there was, in the first instance, some error in assuming its density; and the corresponding reduction being made in Dr. Hutton's determination, gives the density of the earth 4·95, that of water being 1; which more nearly agrees with the determination of Mr. Cavendish, who on other principles, which are explained in the *Phil. Trans.* for 1798, makes it 5·48.*

We have now given, it is presumed, a sketch of all the more prominent points connected with the history of astronomy; much highly important matter is, we are aware, also either wholly passed over, or very slightly alluded to; but to have noticed all the circumstances worthy of record would have carried us far beyond our proposed limits; moreover, many of the minutiæ we shall have occasion to refer to in our illustration of the various topics in the following treatise; where we shall also take the opportunity of introducing the titles of some of the more important and useful works on this subject, and to which we would refer our readers for the attainment of a perfect knowledge of the ancient and present state of astronomical science.

* In the last volume of the Phil. Trans. Dr. Hutton has given a paper on this subject, pointing out certain errors of calculation in Mr. Cavendish's Memoir; and which, when corrected, make the result more approximative to the last determination of this venerable mathematician.

PLANE ASTRONOMY.

§ II. *Introduction.—Containing a popular view and illustration of the most remarkable phenomena of the heavens.*

1. *General remarks.*

Introduction.

1. Astronomy differs very essentially from all other mathematical sciences, with regard to the connection of its propositions, and the force of its demonstrations. In geometry, for example, after the requisite definitions have been laid down, and certain axioms and postulates granted, the reader finds no farther claims made upon him for the admission of this or that hypothesis; he judges for himself at every step; and every step furnishes him with some new truth, as certain and incontestable as his own existence. So also, in theoretical mechanics: our investigations, if they do not always preserve the same concatenation, as in geometry, furnish an equal conviction to the mind of the reader, because he is never required to give his assent to a proposition, till the means are prepared to rest its demonstration upon something previously established.

In astronomy, on the contrary, a student is called upon for the admission of an hypothesis which is contrary to the evidence of his senses. While he and every thing about him are in a state of apparent permanent rest, he must admit that they are moving with an inconceivable velocity; and on the other hand, that those bodies, which, judging from his senses only, he supposes to be in rapid motion, are actually at rest; moreover, he must wait till the whole chain of reasoning is established before he will be able to judge, and to be convinced of the truth of the hypothesis which he has previously admitted. He will then at least feel this conviction, viz. if the law of gravitation, and the constitution of the solar system were such as he has assumed, that every phenomenon, the most important as well as the most minute, would happen exactly in the same manner as they actually do; and hence, he will be able to judge of the high degree of probability, that the supposition he has made is strictly and positively correct.

Facts on which the truth of the modern astronomy is founded.

To mention a few of the most prominent facts, we may observe, that the law of gravitation, and the hypothesis of the earth's rotation on its axis, render it highly probable, that the terrestrial globe is not a perfect sphere, but an oblate spheroid; and the various geodetic operations that have been carried on in different countries, leave no longer any doubt that such is its actual figure. Again, the same hypotheses indicate, that the intensity of gravity ought to be different in different latitudes; not merely in consequence of the figure of the earth, but on account of the difference in the centrifugal force in different latitudes; and therefore, that the seconds pendulum ought to be longer at or near the poles, than at the equator; and that such is truly the case, has been demonstrated by various experiments. The aberration of the stars, the nutation of the earth's axis; the precession of the equinoxes: phenomena, whose existence have been ascertained by observation, are inexplicable upon any other supposition; and to this we may add, that of all the minute inequalities which the accuracy of modern instruments, and a corresponding

Astronomy. delicacy of observation have rendered appreciable in the motion of the planetary bodies, there is not one of them but what is immediately shown to be the necessary consequence of the law of universal gravitation, and the assumed constitution of the solar system.

These facts, and many others which we might have advanced, if they do not amount to actual demonstration, necessarily involve in them such a high degree of probability as falls very little short of it; leaving on the mind a conviction little inferior to that which we derive from absolute certainty.

This conviction, however, as we have before observed, cannot be felt till the entire chain of reasoning has been established; till this is effected, some doubt may be allowed to remain on the mind of the student; he will, it is true, as he advances, find greater and more substantial reasons for admitting the truth of his hypothesis, but he will not feel complete satisfaction till he is able to compare and weigh the whole.

Sketch of the arrangement of the several subjects.

2. In the following sketch, therefore, which we have given of the constitution of the solar system, we do not require the implicit assent of the reader; we wish him only to consider what we have stated, as the enunciation of a proposition, or of a chain of propositions, of which we shall endeavour, step by step, to demonstrate the truth; nor shall we attempt, in the first instance, to give the particulars with the utmost nicety; because this is unnecessary, where we wish simply to indicate a general view of the planetary motions; but in the subsequent part of our treatise, we shall exhibit only the most modern and authentic results. We shall enter upon our subject, on the supposition that our reader is wholly unacquainted with even the first principles of the science, and therefore that he requires an explanation of every phenomenon. This explanation we propose to render in the first instance merely popular, viz. without attempting any but the most simple computations; for example, we shall first illustrate the most remarkable celestial phenomena, the changes of season, the alternation of night and day, the phases of the moon and planets, the general principles of eclipses, &c. &c. Having thus indicated to the student, the first principles of the science, and given him a concise view of what we propose to establish; we shall then describe to him the circles of the sphere, the construction of some of the most indispensable astronomical instruments, introduce him to the observatory, and point out to him the nature of the observations that are necessary as the groundwork of his computations. These observations, however, will stand in need of certain corrections, the cause and quantity of which he must therefore be next instructed in; after which, he will find no difficulty in following us through the remaining part of this division of our subject.

Having said thus much with reference to the plan we propose to follow in this treatise, we shall begin by exhibiting a brief description of the stars, and of the constellations into which they have been divided.

2. *Of the fixed stars.*

Of the fixed stars.

3. It is impossible to conceive a more magnificent spectacle, nor one more worthy of the contemplation of an intelligent being, than that which is presented to our view by the starry firmament in an unclouded sky; a few hours after the setting of the sun. We then observe innumerable brilliant points spread in every direction over the azure canopy of the heavens; various in magnitude and lustre; differently disposed with regard to each other, and arranged in groups to which the imagination will readily attribute numerous ideal forms and characters.

Plane Astronomy.

If this scene be attentively observed for some successive nights, it will be perceived that the greater number of those bodies preserve constantly the same relative position with regard to each other; to these therefore has been given the name of *fixed stars*. Others of them are perpetually changing their places, varying in their lustre, and apparently traversing different circles of the celestial space; these are, therefore denominated, *wandering stars* or *planets*; while a third class, much more rare in their appearance, and totally distinct from all others in their figure and motion, are sometimes observed, surrounded by a faint sphere of light, and followed by a like luminous train, which at times extend over a considerable arc of the celestial vault. This sphere of light, from its supposed resemblance to hair, has given to these celestial visitants the name of *comets*. These three classes of bodies will form the subject of distinct chapters in the course of this treatise; at present our business is to take a popular view of the fixed stars, in order to illustrate the important uses to which they are applied, both in practical and physical astronomy.

Divisions of the stars into constellations.

4. One of the first objects of a student in this science should be to make himself acquainted with the names and situations of the most conspicuous stars and constellations; the latter being a term used to denote those groups of stars which are supposed to have some resemblance in their form to the animal or other objects, whose names they bear. The division of the stars into constellations is of very remote antiquity; and though it may be useless, and sometimes even inconvenient, for the purpose of minute observation; yet for a general recollection of the great features of the heavens, these arbitrary names and associations cannot but greatly assist the memory. It is also usual to describe particular stars by their situation with respect to the imaginary figure, to which they belong, as *Spica Virginis, Cor Hydræ*, &c. or more commonly at present, by the letters of the Greek alphabet, which were first applied by Bayer in 1603, and in addition to these by the Roman letters, and by the numbers of particular catalogues.

Apparent magnitudes.

5. The stars are again divided into different classes or magnitudes, according to the degrees of their apparent brightness. The largest or most vivid of which, are said to be of the first magnitude; the next in order of brightness of the second, and so on to the sixth, which latter class includes all those that are barely perceptible to the naked eye; all of a smaller kind, are generally called *telescopic stars*, being invisible without the assistance of the telescope.

Method of acquiring a knowledge of the stars.

In order to become familiar with the names and positions of the principal stars and constellations, the student must place himself in an open situation, where he can observe the whole of the celestial hemisphere; he will then perceive, after an attentive observation of their motion, that certain of the stars exposed to his view, will, after attaining their greatest altitude, decline and sink below the horizon; others will attain to the meridian, descend again, but will not sink

Astronomy.

below the horizon; and by directing his view with attention towards that part of the heavens where this occurs, he will perceive some few stars, and one in particular, which have no perceptible motion: this latter is called the pole star, because it is about an axis passing very near to this, that the apparent motion of the heavens takes place. The place of this star referred to the horizon is called the *north*, the opposite point the *south*, that to the right hand the *east*, and to the left the *west*.

Different methods of representing the stars.

6. There are two principal modes of representing the stars; the one by delineating them on a globe, where each star occupies the spot in which it would appear to an eye placed in the centre of the globe, and where the situations are reversed when we look down upon them; the other mode is by a chart, where the stars are generally so arranged as to represent them in positions similar to their natural ones, or as they would appear on the internal concave surface of the globe.

In acquiring a knowledge of the particular stars, it will be most convenient to begin with such as never set in our climates; after which, we may readily refer the situations of others to their position with respect to these.

Different constellations.

7. The *Great Bear* is the most conspicuous of the constellations, which never set in our latitude; it consists of seven principal stars, placed like the four wheels of a waggon, and its three horses, except that the horses are fixed to one of the wheels.

Their connections, &c.

The hind wheels are called the pointers, because they direct us to the pole star, in the extremity of the tail of the Little Bear; and further on to the constellation Cassiopeia, which is situated in the milky way (See Plate I. Astronomy,) which consists of several stars nearly in the form of the letter W; or we may easily imagine them to represent a *chair*; whence these few in particular form what is more commonly called Cassiopeia's Chair.

The two northernmost wheels of the Great Bear, or Wain, point at the bright star Capella, in Auriga. Descending along the milky way from Cassiopeia, if we go towards Capella, we come to Algenib, in Perseus, and a little further from the pole Algol, or Medusa's Head; but if we take the opposite direction, we arrive at Cygnus, the Swan, and beyond it, a little out of the milky way, is the bright star Lyra. The Dragon consists of a chain of stars, partly surrounding the Little Bear; and between Cassiopeia and the Swan, is the constellation Cepheus.

Near Algenib, and pointing directly towards it, are two stars of Andromeda, and a third is a little beyond them. A line drawn through the Great Bear and Capella, passes to the Pleiades, and then turning at a right angle towards the milky way, reaches Aldebaran, or the bull's eye, and the shoulder of Orion, who is known by his belt, consisting of three stars placed in the middle of a quadrangle. Aldebaran, the Pleiades, and Algol, make the upper end, Menkar, or the whale's jaw, with Aries, the lower point of a W. In Aries we observe two principal stars, one of them with a smaller attendant.

Plane Astronomy.

A line from the pole, midway between the Great Bear and Capella, passes to Gemini, the twins, and to Procyon; and then in order to reach Sirius, it must bend across the milky way. Algol and the Twins point at Regulus, the lion's heart, which is situated at one end of an arch, with Denabola at the other end.

The pole star, and the middle horse of the wain, direct us to Spica Virginis, considerably distant: the pole and the first horse, nearly to Arcturus in the waggoner, or Bootes. Much further southward, and towards the milky way, is Antares in the Scorpion; forming, with Arcturus and Spica, a triangle, within which are the two stars of Libra. The northern crown is nearly in a line between Lyra and Arcturus; and the heads of Hercules and Serpentarius are between Lyra and Scorpio. In the milky way, below the part nearest to Lyra, and on a line drawn from Arcturus, through the head of Hercules, is Aquila, making, with Lyra and Cygnus, a conspicuous triangle. The last of the three principal stars in Andromeda, make, with three of Pegasus, a square, of which one of the sides points to Fomalhaut, situated at a considerable distance in the southern fish, and in the vicinity of the whale, which has been already mentioned. By means of these supposititious lines, all the principal stars that are ever visible in our climates may be easily recognised. Of those which never rise above the horizon, there are several of the first magnitude; Canopus in the ship Argo, and Achernar in the river Eridanus, are the most brilliant of them; the feet of the Centaur, and the Crosier are the next; and, according to Humboldt's observations, some others perhaps may require to be admitted into the same class. See Plate II.

The constellations to which we have above referred, are divided into three principal classes; those on the northern side of the equator are called northern constellations, and those on the opposite, southern constellations; while those which are situated in that part of the concave hemisphere where the principal planets and other bodies of our system are observed to move, forming a zone, which crosses the equator in two points, are called zodiacal constellations. The names of these, with the number of stars belonging to each, in four of the principal catalogues; the names of the principal stars; and the characters of the zodiacal constellations, are as follow:—

Astronomy. Plane Astronomy.

Table of the Constellations.

Zodiacal.

Names and Characters of the Constellations.			Ptolemy.	Tycho.	Hevelius.	Flamstead.	Principal Stars.	Mag.
Aries	♈	the Ram	18	21	27	66		
Taurus	♉	the Bull	44	43	51	141	Aldebaran	1
Gemini	♊	the Twins	25	25	38	85	Castor and Pollux	1—2
Cancer	♋	the Crab	13	15	29	83		
Leo	♌	the Lion, with Coma Berenices	35	30	49	95	Regulus	1
Virgo	♍	the Virgin	32	33	50	110	Spica Virginis	1
Libra	♎	the Scales	17	10	20	51	Zubenich Meli	2
Scorpio	♏	the Scorpion	24	10	20	44	Antares	1
Sagittarius	♐	the Archer	31	14	22	69		
Capricornus	♑	the Goat	28	28	29	51		
Aquarius	♒	the Water Bearer	45	41	47	108	Scheat	3
Pisces	♓	the Fishes	38	36	39	113		

Northern Constellations.

Names of the Constellations.	Ptolemy.	Tycho.	Hevelius.	Flamstead.	Principal Stars.	Mag.
Ursa Minor, the Little Bear	8	7	12	24	Pole-star	2
Ursa Major, the Great Bear	35	29	73	87	Dubhe	1
Perseus	29	29	46	59	Algenib	2
Auriga, the Waggoner	14	9	40	66	Capella	1
Bootes	23	18	52	54	Arcturus	1
Draco, the Dragon	31	32	40	80	Rastaber	3
Cepheus	13	4	51	35	Alderamin	3
* Canes Venatici, viz. Asterion et Chara, the Greyhounds	..	..	23	25		
* Cor Caroli	..	..	..	3		
Triangulum, the Triangle	4	4	12	16		
* Triangulum Minus	..	..	..	10		
* Musca	..	..	..	6		
* Lynx	..	..	19	44		
* Leo Minor, the Little Lion	..	..	..	53		
* Coma Berenices, Berenice's Hair	..	14	21	43		
* Camelopardalus	..	..	32	58		
* Mons Menelaus	..	..	..	11		
Corona Borealis, the Northern Crown	8	8	8	21		
Serpens, the Serpent	13	13	22	64		
* Scutum Sobieski, Sobieski's Shield	..	..	7	8		
Hercules, cum Ramo et Cerbero						
Hercules, since called Engonasia	29	28	45	113	Ras Algiatha	3
Serpentarius, sive Ophiuchus	29	15	40	74	Ras Alhagus	3
* Taurus Poniatowski	..	..	..	7		
Lyra, the Harp	10	11	17	22	Vega	1
*Vulpeculus et Anser, the Fox and Goose	..	..	27	37		
Sagitta, the Arrow	5	5	5	18		
Aquila, the Eagle, with Antinous	..	12—3	23—19	71	Altair	1
Delphinus, the Dolphin	10	10	14	18		
Cygnus, the Swan	19	18	47	81	Deneb Adige	1
Cassiopeia, the Lady in her Chair	13	26	37	55		
Equulus, the Horse's Head	4	4	6	10		
* Lacerta, the Lizard	..	..	..	16		
Pegasus, the Flying Horse	20	19	38	89	Markab	2
Andromeda	23	23	47	66	Almaac	2

Those constellations distinguished * are new constellations.

Southern Constellations.

Names of the Constellations.	Number of Stars in different Catalogues.				Principal Stars.	
	Ptolemy.	Tycho.	Hevelius.	Flamstead.		Mag.
* Phœnix	..	..	..	13		
* Officina Sculptoria	..	..	..	12		
Eridanus	34	10	27	84	Achernar	1
* Hydrus, the Water Snake	..	..	..	10		
Cetus, the Whale	22	21	45	97	Menkar	2
* Fornax Chemica	..	..	..	14		
* Horologium	..	..	..	12		
* Reticulus Rhomboidalis	..	..	..	10		
* Xiphias, Dorado, the Sword Fish	..	..	..	7		
* Celapraxitellis	..	..	..	16		
Lepus, the Hare	12	13	16	19		
* Columba Noachi, Noah's Dove	..	..	..	10	Betelgense	1
Orion	38	42	62	78	Canopus	1
Argo Navis, the Ship	45	3	4	64		
Canis Major, the Great Dog	29	13	21	31	Sirius	1
* Equuleus Pictorius	..	..	..	8		
* Monoceros, the Unicorn	..	..	19	31		
Canis Minor, the Little Dog	2	2	13	14	Procyon	1
* Chamæleon	..	..	..	10		
* Pyxis Nautica	..	..	..	4		
* Piscis Volans, the Flying Fish	..	..	..	8	Cor Hydræ	1
Hydra	27	19	31	60		
* Sextans	..	..	11	41		
* Robur Carolinum, the Royal Oak	..	..	..	12		
* Machina Pneumatica	..	..	..	3		
Crater, the Cup	7	3	10	31	Alkes	3
Corvus, the Crow	7	4	..	9	Algorab	3
* Crosiers, et Cruzero	..	..	..	6		
* Apis Musca, the Bee or Fly	..	..	..	4		
*Apus, or Avis Indica, the Bird of Paradise	..	..	..	11		
* Circinus, the Compass	..	..	..	4		
Centaurus, the Centaur	37	..	..	35		
Lupus, the Wolf	19	..	..	24		
* Quadra Euclidis	..	..	..	12		
* Triangulum Australe, the Southern Triangle	..	..	..	5		
Ara, the Altar	7	..	..	9		
* Telescopium	..	..	..	9		
Corona Australis, the Southern Crown	13	..	..	12		
* Pavo, the Peacock	..	..	..	14		
* Indus, the Indian	..	..	..	12		
* Microscopium	..	..	..	10		
* Octans Hadleianus	..	..	..	43		
* Grus, the Crane	..	..	..	14		
Toucan, the American Goose	..	..	..	9		
Piscis Australis, the Southern Fish	18	..	..	24	Fomalhaut	1

Total Number of Constellations.		Number of Stars in each Magnitude, British Catalogue.						Total No.
		1st Mag.	2d Mag.	3d Mag.	4th Mag.	5th Mag.	6th Mag.	
Zodiacal	12	5	16	44	120	183	646	1014
Northern	34	6	24	95	200	291	635	1251
Southern	45	9	36	84	190	221	323	863
	91	20	76	223	510	695	1604	3128

Astronomy.

Number of stars visible to the naked eye.

8. It appears from the preceding summary of the stars in the British catalogue, that the number of them in the entire celestial sphere, including all those of the sixth magnitude, does not much exceed 3000; and it is generally stated, that not more than 1000 are ever visible at one time to the naked eye: but when a telescope is employed, their number appears to increase without any other limit than that of the perfection of the instrument. Sir W. Herschel has observed in the *milky way* above 10,000 stars in the space of one square degree. This luminous track

Milky way.

> "Which nightly, as a circling zone, thou seest
> Powder'd with stars,"

encompasses the heavens, and forms nearly a great circle of the celestial sphere. It traverses the constellations Cassiopeia, Perseus, Auriga, Orion, Gemini, Canis Major, and the Ship, where it appears most brilliant; it then passes through the feet of the Centaur, the Cross, the southern Triangle, and returns towards the north by the Altar, the tail of the Scorpion, and the arc of Sagittarius, where it divides into two branches, passing through Aquila, Sagitta, the Swan, Serpentarius, the head of Cepheus, and returns to Cassiopeia. The ancients had many singular ideas respecting the cause of this phenomenon, but modern astronomers have long attributed it to an immense assemblage of stars too feeble to make distinct impressions; and Herschel has shown, by the observation above referred to, that these conjectures were well founded.

Nebulæ.

Besides the milky way, there are numerous other parts of the heavens, which exhibit an appearance of very nearly the same kind, which are called *Nebulæ*; the most considerable of which is that midway between the two stars in the blade of Orion's sword. This was first observed by Huygens; it contains only seven stars; the other part appearing like a luminous spot upon a dark ground, or like a bright opening into regions beyond.

Proper motion of some stars.

9. We have hitherto spoken of the fixed stars, as preserving actually the same relative situations with regard to each other; this, however, is not strictly true. The delicacy of modern observations has shown that some of these bodies have a progressive motion: Arcturus, for example, has a proper motion, amounting to about two seconds annually; and Dr. Maskelyne found, that out of 36 stars, of which he ascertained the places with great accuracy, 35 of them had a progressive motion.

Mr. Michell and Sir W. Herschel have conjectured that some of the stars revolve round others which are apparently situated near them; and perhaps even all the stars may in reality change their places more or less, although their relative situations, and the direction of their paths may render their motions imperceptible.

Variable stars.

That some of the stars have a periodical change of brightness has been well ascertained from repeated observations; and different hypotheses have been advanced to account for these singular phenomena. New stars have also appeared at certain times, remained stationary like the others, and have afterwards disappeared. Such a temporary star was observed by Hipparchus; and it was this circumstance which suggested to him the idea of forming a catalogue of them, and of delineating their situations. A new star was also discovered in Cassiopeia in 1572, which was so bright as to be seen in the day-time, but it gradually disappeared in 16 months. Another was observed by Kepler in 1604, more brilliant than any other star or planet, and changing perpetually into all the colours of the rainbow, except when it was near the horizon. This star remained visible about a year; and several other cases of the like kind are recorded.

Plane Astronomy.

Distance and magnitude unknown.

As to the actual magnitude and distance of the stars, we may be said to be almost wholly unacquainted with either; all that we are able to state with certainty is, that their distance is immense. We shall show, in a subsequent chapter, that the distance of the sun from the earth is nearly 100 million miles; and, consequently, the diameter of our orbit is nearly 200 million miles; we therefore view the stars, or any one of them in particular, from two points at different times of the year, which are distant from each other 200 million miles, and yet we are not able to detect any difference in their apparent places. If a change of place, amounting only to one second, actually obtained, there is no doubt that it would be detected by the accuracy of modern observations;* we know, therefore, that an isosceles triangle having the diameter of the earth's orbit for its base, and having its vertex in the nearest fixed star, does not subtend an angle of one second; the nearest star must therefore be distant from us more than 20 *billions of miles*. How much their distance may exceed this, it is impossible for us to say; and much less are we able to offer even the most distant conjecture as to their actual magnitude; judging from analogy, we can only suppose them to be bodies resembling our sun, some of greater, and some of less magnitude; that they shine like the sun by their own light, each forming the centre of its particular system, dispensing light, heat, and animation to thousands of worlds, and to myriads of beings. What a wonderful idea do these reflections give us of the immensity of the universe, and of the power and omniscience of the great Creator and Director of so stupendous a machine!

3. *Of the Solar System.*

The solar system.

10. The system of Copernicus has been already referred to in our historical chapter. We have seen, that, according to this astronomer, all the principal planets revolve about the sun as a general, but not as a common centre; the several planets being supposed by him to describe circular orbits, each however having its particular point of circulation. The Copernican and the modern system are not therefore the same, although these terms are frequently employed synonimously. We shall here confine our description to the modern or true system, without, however, deducing any facts in proof of its accuracy: we shall be content

* It is proper to observe, that since this was written, Dr. Brinkley, of Trinity College, Dublin, has been led to think, that he has detected a sensible parallax in several stars; but as Mr. Pond, the Astronomer Royal, has not yet been able to verify Dr. Brinkley's observations, we do not feel ourselves justified in stating the existence of a parallax; although, from the confidence we feel in the talents and accuracy of the observer, we have no doubt that his deductions will be hereafter verified by other astronomers.

Astronomy. that the reader receives it at present merely as an hypothesis, or rather as the enunciation of a proposition; and that he suspends his decision, till, in the course of the following chapters we advance such proofs as can leave him no longer in any doubt respecting the constitution of the solar system, nor of the mechanical principles on which its motions depend.

The sun is the central body. The sun has probably, as well as every body in the universe, a progressive motion in space; but if such a motion has place, he is accompanied in it with the whole system of which he forms the general centre, as well as the source of its light, heat, and motion; we may therefore consider this body at rest, as far as regards the relative motion of the other constituents of our system.

The sun, then, according to the modern hypothesis occupies a fixed centre, about which the several principal planets revolve, in elliptic orbits, one foci of each of which is found in the same point, coinciding very nearly with the solar centre. Several of these planets are again accompanied with attendant luminaries, which observe the same laws in their revolutions about their primaries, as these primaries do with respect to the sun.

Inclination of the orbits of the planets. 11. The orbits of the several planets are not all situated in the same plane, but are variously inclined to each other, and to a fixed plane passing through the sun, called the plane of the ecliptic. Even this plane is not actually fixed, but is subject to a certain annual motion, to which, however, we shall not refer in this description. The several planets of our system, with the exception of the four small ones lately discovered, have their inclinations very small; the particular measures of which will be stated below; and their motions are all made in one direction, viz. from west to east. Several of these bodies are also known to have a rotatory motion on their respective axes, but these axes are differently inclined with respect to the plane of the ecliptic, each, however, always preserving its own direction parallel to itself. This rotatory motion is likewise performed from west to east, at least in all those bodies in which it has been hitherto observed; but some, either from their immense distance, their inconsiderable magnitude, or from their proximity to the sun, have not yet had their diurnal revolution decidedly ascertained.

Motion of the planets. Inclination of their axes.

Eccentricities of the orbits. 12. The orbits of the planets we have seen are all elliptical, but these ellipses have very different degrees of eccentricity; that is, the ratio of the major and minor axes vary very considerably in the orbits of the different planets. The orbits of the satellites have also various eccentricities, and are differently inclined to the ecliptic, and to each other; in some of the secondaries this inclination is extremely great, while in all the principal bodies of our system it is very inconsiderable.

These general remarks being premised, we shall proceed concisely to describe the particular circumstances attending the motion of each planet, beginning with the sun, and proceeding orderly with the other bodies, according to their respective distances from him.

Of the sun. 13. The sun, which is known to be 882,270 English miles in diameter, performs its diurnal revolution in 25 days 14h. 8′, about an axis, which is inclined 82° 30′ from the plane of the ecliptic.

Plane Astronomy. Mercury. 14. Mercury is the nearest planet to the sun; its mean distance is about 36 millions English miles; the time of its sideral revolution 87 days 23h.; its diameter 3123 miles; the inclination of its orbit to the ecliptic, 7° 0′ 9″; and the ratio of the eccentricity of its orbit to that of its semi major axis ·2055149. The inclination of its axis is unknown, and the time of its diurnal revolution doubtful; it has been stated at 24h. 5′.

Venus. 15. Venus is the second planet in order from the sun; her mean distance being 68 millions of miles, and the time of her sideral revolution 224 days 16 hours. The diameter of this planet is 7702 English miles; the inclination of its orbit 3° 23′; and the ratio of its eccentricity to the semi major axis of its orbit ·0068529. Its diurnal revolution is performed in 23h. 21′, about an axis, which, like that of Mercury, is of unknown inclination.

The Earth. 16. The Earth is the third planet in the system; its distance from the sun is 93 million miles, and the time of its sideral revolution 365 days 6h. 9′; which must not be confounded with the length of the solar year, this being only 365 days 5h. 48′ 48″; the diameter of this planet is 7916 miles. The orbit of the earth being that to which the plane of the other planetary orbits are referred, its inclination is nothing; the eccentricity of its orbit is ·0168531; its diurnal revolution is performed in 23h. 56m. 4″, (which is called a sideral day), about an axis which inclines to the ecliptic in an angle of 66° 32′ 3″, the complement of which, viz. 23 27′ 57″, is the obliquity of the ecliptic as referred to the equator.

Mars. 17. The next primary planet in our system is Mars; its mean distance from the sun is 142 millions of miles, and its sideral revolution is performed in 686 days 23½h.; its diameter is little more than half that of the earth, being only 4398 miles.

Its orbit is inclined to the ecliptic in an angle of 1° 51′ 3″; the eccentricity of its orbit is ·0931340, and its diurnal revolution is performed in 24h. 39m., about an axis, which is inclined to the plane of the ecliptic at an angle of 59° 41′ 49″.

Vesta, Juno, Ceres, Pallas. 18. The next planets in order from the sun, are the four new ones; Vesta, Juno, Ceres, Pallas, of which little is known, except their times of revolution, and the inclination of their orbits; and these may even hereafter be found to stand in need of some corrections: they are at present stated as follows:—

Vesta; mean distance, 225 million miles; time of sideral revolution, 1335 days 4h.; inclination of orbit, 7° 8¾′; eccentricity, ·0932200.

Juno; mean distance, 253 million miles; period of sideral revolution, 1590 days 23h.; inclination of orbit, 21°; eccentricity, ·254944.

Ceres; mean distance, 263 million miles; period of revolution, 1681 days 12h.; inclination of orbit, 10° 37′; eccentricity, ·0783486.

Pallas; mean distance, 265 million miles; period of revolution, 1681 days 17h.; inclination of orbit, 34° 50′; eccentricity, ·245384.

The diameters of these planets (which must, however, be considered as doubtful,) have been given as follow:—Vesta, 238 miles; Juno, 1425 miles; Ceres, 1024 miles; and Pallas, 2099 miles.

Jupiter. 19. Jupiter, the largest of all the planets of our system, is the next after Pallas; its mean distance from the sun is 485 million miles, and its diameter is 91522 miles, more than eleven times that of the earth : the inclination of his orbit to the ecliptic is 1° 18′ 51″, and his sideral revolution 4,332 days 14 hours, the ratio of its eccentricity to the semi major axis ·0481784. The diurnal revolution of Jupiter is performed in 9 hours 55′ 49″, about an axis which is inclined to the ecliptic at an angle of 86° 54½′.

Saturn.

20. Saturn is next to Jupiter, as well in magnitude as in distance ; the latter being 890 million miles, and its diameter 76,068 miles ; his sideral revolution is performed in 10,758 days, or about thirty of our years, and the eccentricity of his orbit ·0561683. The diurnal revolution of Saturn is made in 10 hours 16 19″, about an axis which is inclined to the ecliptic at an angle of 58° 41′.

Uranus, or Georgium Sidus.

21. Last in the solar system is the Georgium Sidus, or Uranus ; whose mean distance is more than double that of Saturn, being no less than 1800 million miles ; its diameter is 35,112 miles, and the eccentricity of its orbit ·0446703. Its sideral revolution is performed in about eighty of our years : but its diurnal revolution, and the inclination of its axis, are not at present determined.

22. We have endeavoured to represent, the planetary distances, eccentricities, and proportional magnitude in Plate III., with the exception of the sun, which is there represented nearly as a point, whereas, in comparison with the magnitude we have given to the planets, its diameter ought to have exceeded even that of the orbit of Saturn. The several orbits in the plate are all represented as if situated in one plane ; it is necessary therefore for the reader to bear in mind the measures of the several inclinations above indicated.

Characters of the planets.

23. The following are the characters or symbols employed by astronomers for denoting the several planets :—

The Sun	☉	Ceres	⚳
Mercury	☿	Pallas	⚴
Venus	♀	Jupiter	♃
The Earth	⊕	Saturn	♄
Mars	♂	Uranus	♅
Vesta	⚶	The Moon	☽
Juno	⚵		

24. At present we have spoken only of the primary planets, it now remains for us to say a few words with reference to the satellites by which some of them are attended.

The moon.

The first, and that which in this place claims our particular attention, is the moon, which revolves about the earth, as the earth itself does about the sun. Her mean distance from the terrestrial centre is 237,000 miles ; her diameter is 2,160 miles, and the eccentricity of her orbit is ·0548553. The period of her orbicular and diurnal revolutions, are exactly alike, being completed in 27 days 7 hours 43′ 4″·7. The inclination of her orbit is 5° 9′, and of her axis 88° 29′ 49″.

Satellites of Jupiter, Saturn, &c.

It is unnecessary in this place to enter into the same minutiæ with respect to the other satellites ; it will be sufficient to observe, that Jupiter has four, Saturn seven, and Uranus six, as represented in the plate above referred to.

Saturn's ring.

Besides the seven satellites which accompany Saturn in his dreary path, he is also encompassed by a double ring, by which he is distinguished from all the other planets of our system. This ring, which is very thin, not exceeding 4,500 miles, is inclined to the plane of the ecliptic at an angle of 31° 19′ 12″, and revolves from west to east in 10 hours 29′ 16″·8 ; this rotation is performed about an axis perpendicular to the plane of the ring, passing through the centre of the planet.

We shall not enter more particularly in this place into a description of this singularly beautiful telescopic object ; as we shall of course have to treat of it at length in a subsequent chapter, in which we shall state its several dimensions as determined from the best observations ; it will be sufficient here to observe, that its outside diameter is 204,883 miles, and its inside 146,345, consequently, its mean breadth is about 30,000 miles.

Comets.

25. We come now to the third class of bodies, which are only visible to us for a short time. The planets perform their revolutions about the sun within the limits of our observation, and at distances from him which vary comparatively very little in the different parts of their orbits ; but those to which we now refer, if they all actually revolve about the sun, are, for a very considerable time far beyond the known limits of the solar system. These are called comets : they generally appear attended with a nebulous light, either surrounding them as a coma, or stretched out to a considerable length as a tail, and sometimes they appear to consist of such light only. Their orbits are so eccentric that in the remoter parts of them they are invisible to us, although at other times they approach much nearer to the sun than any of the planets : the comet of 1680, for example, when nearest the sun, was at the distance of only one sixth of the sun's diameter from its surface. Their tails are frequently of great extent, appearing as a faint light directed towards a point always opposite to the sun. It is quite uncertain of what matter they consist, and it is difficult to say which of the conjectures concerning them is the least improbable. Nearly 500 comets are recorded as having been seen at different times, and certain particulars relative to the orbits of about a hundred of them, have been accurately ascertained : but commonly, we have no opportunity of observing a sufficient portion of the cometary path, to determine with accuracy the entire dimensions of the ellipse or other conic section to which the observed part appertains ; on which account little can be known of the periods and other circumstances of these wandering bodies. Only one comet has been recognized in its return to our system, which is that of 1759. Dr. Halley, by comparing together the elements of the several comets that had been observed up to his time, conjectured that those recorded to have appeared in 1531, in 1607, and in 1682, were, in fact, one and the same comet, and, consequently, that its return might be expected about the year 1758 or 1759, and its actual appearance in the last of those years, verified the conjecture. Another comet, which appeared in 1770, was suspected to move in an elliptic orbit ; and if so, its period ought, by Mr. Lexel's computation (which has been since remade by Burckhardt), to be about 5 years and 7 months ; it has never, however, been since observed ; but this circumstance must not lead us to discredit either the observations made on it, or the calculations founded on them, for it has

Astronomy.

been satisfactorily shown, that supposing all the data correct, this comet must have passed so near to Jupiter, that its orbit would be deranged, and the body rendered in future invisible to us.

We shall not enter further on the subject of comets in this place, except to observe, that whether their orbits be all ellipses, or some of them parabolas or hyperbolas, a very small portion of them fall within the limits of our system; if they are all ellipses, they are of very great eccentricity, and are only for a short period visible in these regions of celestial space. The orbits of two of these bodies are shown in our Plate III.

4. *Phenomena in the heavens, due to the motion of the earth and planets.*

Diurnal motion of the earth.

26. We have already in our second section given a general view of the celestial sphere, with an enumeration of the several constellations; we propose now to illustrate a few particular phenomena, and to describe certain circles which astronomers have imagined for the better comprehension of the celestial motions.

We have seen that a person being situated in an open plane, in a star light evening, and watching attentively the motion of the fixed stars, will perceive them rise or emerge above the earth, continue to ascend, till they have attained a certain height, and then descend and disappear in the opposite side of the heavens to that in which they first rose. He will perceive, that accordingly as these stars are nearer the northern or southern points of the heaven, so they will appear visible to him for a greater or less time; and that certain stars very near the north point, never either rise or set, but would be always visible were not their light rendered imperceptible by the more refulgent rays of the sun.

One of the stars in this quarter of the heavens, to which we have already alluded, has scarcely any sensible motion, but, as far as the naked eye can distinguish, retains constantly the same situation; this is called the *pole star*, and those to which we have referred above, that appear to be constantly circulating about it, are, for that reason, called, *circumpolar stars.*

Apparent motion of the heavens.

The first impression that this apparent motion of the stars would make on the mind of an uninformed observer would be, that the entire celestial vault was uniformly revolving from east to west, about an imaginary axis, which passes through or near the pole star, in a direction perpendicular to the planes of the circles described by the several stars which are alternately observed to rise and set; and consequently, that this line produced would again meet the celestial sphere in an opposite point, which may thence be denominated the *south pole.* This, as we have observed, would doubtless be the first impression made on an uninformed observer; but at the same time, if he possess the requisite intelligence, it would not be difficult to convince him that the very same appearances would be produced by supposing the earth to be of a globular form, and that it performed a motion of rotation about an axis corresponding with the supposed axis of the heavens, but in an opposite direction, that is, from west to east; numerous instances might be recalled to his recollection in which he had appeared at rest, when he was actually in rapid motion, and when objects were apparently seen to move with great velocity, although they were in an absolute state of rest. This phenomenon must have presented itself to every one who has travelled in a close carriage in a narrow road, when at times it is difficult to be persuaded but that the trees, gates, &c. which we pass are not moving in an opposite direction to that of the vehicle; and the same appearances are observed still more strikingly in the cabin of a ship when sailing with a moderate gale near the land. In fact, every intelligent observer, whether he admits the actual motion of the earth or not, will not for a moment deny, that if it had such a motion as we have supposed, the appearances would be exactly such as he observes in the heavens. This then will be one step towards his conviction, and various others will afterwards suggest themselves to his mind; which, however, we shall not at present insist upon, because from what has been stated, it is obvious, that we may at any rate be allowed to advance such an hypothesis as the diurnal revolution of the earth, without in any respect changing the appearance of any observed phenomenon. We shall therefore proceed upon the supposition of the earth being a sphere or a spheroid of small ellipticity, and that it performs a motion of rotation from west to east in about 24 hours; or rather, as we shall see in a subsequent chapter, in 23 hours $56' \, 4''\cdot 1$.

Plane Astronomy.

Apparent motion of the sun.

27. By observing attentively the stars which first appear visible in that part of the heavens where the sun sets, and continuing to observe them for several successive nights, we shall soon perceive that those stars which in the first instance we had observed to set immediately after the sun, are no longer to be seen, but that their places are supplied by certain others, which in their turn will also be lost in the solar beams. If now we observe the heavens in the morning before the rising of the sun, we shall find that those stars which in the first instance we had observed to set just after the sun, and which in the course of a few nights were absorbed by his rays, are now rising before him; the sun therefore has made an apparent motion in the heavens contrary to the general motion of the stars, that is, from west to east; and by following his progress in this manner during the course of a year, we shall find that he has described a complete circle of the heavens, and now rises and sets with the same star as we had observed exactly a year before. This circle which the sun thus appears to describe in the heavens is called the *ecliptic*, it is not directly east and west, but deviates nearly 24° from these points of the heaven, as shown in Plates I. and II., and to his obliquity it is, that we owe the variations in the seasons, and various other phenomena, as we shall show in a future chapter.

This apparent motion of the sun is due to the proper motion of the earth. Fig. 1.

28. This progressive motion of the sun in the heavens, which is only apparent, is due to the actual proper motion of the earth in its orbit about the sun. For let AB (Plate IV., fig. 1) represent two positions of the earth in its orbit ABC about the sun S, and let ♈, ♉, ♊, ♋, &c. represent the ecliptic or the apparent path of the sun. Then when the earth is at A, a spectator will refer that body to that part of the heavens marked ♈; but when the earth is arrived at B, he will then see it in ♊; and being in the mean time insensible of his own motion, the sun will appear to him to have described the arc ♈ ♊,

Astronomy.

just the same as if it had actually passed over the arc SS′, and the earth had, during that time, remained quiescent in its first position A. This fact will explain why certain remarkable stars and constellations are seen in the south in different seasons of the year, and at different hours of the night. For the hour depends wholly on the sun; it is noon when the sun is south, and midnight when it is north; the stars directly opposed to him will therefore, by the rotation, appear in the south about midnight; and as the sun, from one day to another, shifts its place in the heavens, so, of necessity, will different stars be opposed to him, and become south at midnight at different seasons of the year.

Proper motion of the planets.

29. Similar phenomena may be observed with regard to the planets; by tracing their motions in the heavens, and comparing them with the stars near to which they appear, they will also be seen to have a motion of their own, but it will be more irregular than that of the sun; for we shall sometimes observe them moving like that body from west to east, then become stationary, maintaining the same position for several nights; then moving in a contrary direction, or from east to west; again become stationary, and again assume their direct motion.

These phenomena, which the ancients spent so much labour and ingenuity to account for, by epicycle upon epicycle, are perfectly consistent with the system as we have described it in our third plate, being due to the proper motions of the earth and planets in their respective orbits. In order to illustrate this, let $v\ v'\ v''$ represent the orbit of the planet Venus; and suppose her to be in the point v when the earth is at A; then it is obvious that a spectator will refer her place in the heavens to ♈, and as her motion is from v towards v', while the earth is moving from A to B, her apparent motion will be direct; or from ♈ towards ♉: if, on the other hand, Venus had been at v'' while the earth was at A, as their motions now are made in the same way, we may suppose Venus to have arrived at v''', while the earth had passed from A to A′; and during this time, it is manifest her place in the heavens will appear to retrograde, or go backward, contrary to the order of the signs: in like manner it will appear, that for a very short time before and after the Earth and Venus attained their positions A and v'', their motions would so agree with each other, that the planet, during this period, would appear stationary. We shall enter more at length upon these phenomena in a subsequent chapter; we have merely referred to the subject here, to show that these appearances are strictly conformable with the constitution which we have supposed of the solar system.

Notwithstanding these irregularities in the apparent motion of the planets, they each, respectively, are observed to describe great circles of the sphere, but more or less inclined to the plane of the ecliptic; they deviate, however, but little from it; all their motions, with the exception of the new planets, being performed in a zone, whose breadth does not exceed 16 degrees, and which we have already spoken of and described in our first and second plates, under the denomination of the zodiac.

Similar observations show that the moon also describes her particular path amongst the stars; but we shall reserve this subject for a subsequent chapter.

Plane Astronomy.

5. *Of the Seasons.*

Of the seasons.

30. It will appear sufficiently obvious from what has been shown in the preceding sections, that the alternations of day and night are attributable generally to the rotation of the earth on its axis; but the difference in the length of the days in different seasons of the year still remains to be illustrated.

Equator defined. Fig. 2, 3.

We have seen that the two extremities of the terrestrial axis about which the diurnal rotation is performed, are called its poles, as N S, (fig. 2 and 3); and if at a quadrant distance from these we conceive a circle QEQ′ to be described, dividing the earth into two equal hemispheres, that circle is called the equator; the hemisphere towards the north pole N is called the northern hemisphere; and that towards the south pole, the southern. Now if the path of the earth in its orbit, or, which is the same, the apparent motion of the sun in the heavens coincided with the plane of the terrestrial equator, then it is obvious, that, at all times during this revolution, we should have the same equal alternations of day and night, each 12 hours in duration. For example: let NQSQ′ represent the earth; NS, its axis; and QEQ′, its equator: and let the plane of this circle produced pass through the sun S; then it is obvious, that, if we suppose the earth to revolve round the sun at the extremity of the line SE as a radius; and that during this revolution, it performed uniformly its rotatory motion about its axis NS: that line, NS, or the circle of which it is the projection, would terminate the limits of day and night; and the rotatory motion being uniform, every point of the globe, except the two poles, would have an equal succession of light and darkness during the entire revolution. We should then have no spring, no summer, no winter; these changes so pleasing in themselves, and so necessary for the production and re-production of the fruits of the earth would be wanting, and nature would thus be divested of a great portion of its charms.

Consequences that would follow if the equator and ecliptic coincided.

Consequences of the obliquity of the ecliptic.

This, however, is not the case; we have seen that the sun appears to describe an oblique motion in the heavens; it rises higher in the summer than in the winter, and by thus darting upon us more perpendicularly its refulgent beams, produces a greater portion of heat, and describes a larger circuit in the heavens, lengthens out the days, and thus gives time for that heat to become more effective.

The actual motion of the earth is therefore as represented in (fig. 3,) where we still suppose it to describe its annual motion at the extremity of the radius S′E; but such, notwithstanding, that the axis NS preserves its inclination and parallelism, whereby it is always directed to the same point in the heavens. The axis NS being now inclined to the plane of the earth's motion, it is obvious, that, as it revolves on its axis, some parts of the earth will experience perpetual day for a certain portion of the year, while other parts will have to contend with an equal duration of night. In the position of the earth, as shown in the figure, the parts about the north pole will be in continued darkness, and those near the southern pole in perpetual light; while, from the nature of the annual motion, it is clear that it will be exactly the reverse when the earth shall have attained the opposite point of its orbit; the regions of the south pole will then be involved in darkness, and those of the northern will enjoy their return of light, as will appear obvious by supposing the sun to be transferred to the other side of the earth, which is exactly equivalent to the change of place in the earth.

31. Dr. Long, in his astronomy, gives us a pleasing illustration of this change of the seasons, with the variable lengths of the days and nights, by means of the following experiment:—

Illustrated by experiment. Fig. 4.

Take about seven feet of a strong wire, and bend it into a circular form, as *b c d*, (fig. 4,) which being viewed obliquely, appears elliptical, as in the figure. Place a lighted candle on a table, and having fixed one end of a thread K to the north pole of a small terrestrial globe H, about three inches in diameter; cause another person to hold the wire circle, or fix it so that it may be parallel to the table, and as high as the flame of the candle I, which should be in or near the centre; then having twisted the thread as towards the left hand, it will, by untwisting, turn the globe round eastward, or contrary to the way which the hands of a watch move; hang the globe by the thread within the circle, nearly contiguous to it; and, as the thread untwists, the globe, which is enlightened half round by the candle as the earth is by the sun, will turn round its axis, and the different places upon it will be carried through the light and dark hemisphere, and have the appearance of a regular succession of days and nights, as our earth has in reality by such a motion. As the globe turns, move the hand slowly, so as to carry the miniature earth round the candle according to the orders of the letters *a*, *b*, *c*, *d*, &c., keeping its centre even with the wire circle, and it will be perceived, that the candle, being still perpendicular to the equator, or rather in the plane of the equator produced, it will enlighten the globe from pole to pole, as we have shown in fig. 2, and that during the whole of its orbicular revolution; consequently every place on the globe goes equally through the light and dark, as it turns round by the untwisting of the thread. The motion of the globe turning in this way represents the earth revolving on its axis; and the motion of the same in the circle of wire, its revolution round the sun; and shows, that if the orbit of the earth had no inclination to its equator, all the days and nights in every part of the globe would be of equal duration throughout the year, as described in the first of the preceding figures.

Now desire the person who holds the wire to incline it obliquely, in the position ABCD, raising the side ♋ just as much as he depresses the side ♑, that the flame may be still in the plane of the circle; and twisting the thread as before, that the globe may turn round its axis the same way as you carry it round the candle; that is, from west to east. Let the globe down into the lowermost part of the circle ♑; and if the latter be properly inclined, the candle will shine perpendicularly on the globe at *a*, and the region about the north pole will be all in the light, as shown in the figure, and will still keep in the light while the globe revolves on its axis.

From the equator to the north polar circle, all the places have longer days and shorter nights; but from the equator to the north pole just the reverse takes place—the nights being longer than the days. The sun does not set to any part of the northern frigid zone, as is shown by the candle constantly shining upon it while the globe is in this position: and on the same principles, it will appear that the southern arctic regions are, during this time, involved in constant darkness; the revolution of the globe never bringing any part of it within the illuminated hemisphere; and, therefore, if the earth remained constantly in this part of its orbit, the sun would never set in the northern frigid zone, nor rise in the southern. In fact, we should thus have perpetual summer in the former, and perpetual winter in the latter; and the same would be the case with the hemispheres to which these zones appertain.

But as the globe moves round its axis, move your hand slowly forward, so as to carry it from H towards E, and the boundary of light and darkness will approach towards the north pole, and recede from the south; the northern places will pass through less and less light, while the southern will be more and more involved in it; whence is shewn the decrease in the length of the days in the northern hemisphere, and the increase of the same in the southern. When the globe is at E, it is in a mean state between the highest and lowest points of its orbit; the candle is directly over the equator, the boundary of light and darkness just reaches both the poles, and all places on the globe pass equally through the light and dark hemispheres, thereby showing that the days and nights are then equal in all parts of the earth excepting only the poles, the sun there appearing as setting to the northern and rising to the southern.

The globe being still moved forward, as it passes towards A, the north pole is more and more involved in the dark hemisphere, and the south pole advances more into the enlightened part; till, when it arrives at F, the candle is directly over the circle *b b*, and the days are then the shortest and the nights the longest throughout the northern hemisphere: and the reverse in the southern, the days there being the longest and the nights the shortest. The southern polar zone is now in perpetual light, and the northern in continual darkness.

If the motion of the globe be still continued, as it moves through the quarter B, the north pole advances towards the light, and the south pole recedes from it; the days lengthen in the northern hemisphere and shorten in the southern; till, when the globe arrives at G, the candle will be again over the equator (as when it was at E); the days and nights will be again equal in all parts of the earth; the north pole will be just emerging out of darkness, and the southern pole beginning to be involved in it, as the northern pole was in the former instance when the earth was at E.

Hence is shown the reason of the days lengthening and shortening from the equator to the polar circle every year, and why there is sometimes no day or no night for several revolutions of the earth within the two frigid zones; and why there is but one day and one night in a year at the poles themselves. We see also that the days and nights at the equator are equal all the year round, this being always equally bisected by the circle bounding the light and darkness.

A similar representation of these phenomena is exhibited in fig. 5; and in a subsequent chapter we shall Fig. 5.

Astronomy.

enter more particularly on the subject; showing the method of computing the length of the day in any given latitude; and the time of the rising, setting, and southing of the sun on any proposed day, and in any given place.

6. *Of the Phases of the Moon.*

Phases of the moon.

32. The moon, of all the celestial bodies, is that which perhaps attracts most the attention, both of illiterate and scientific observers. The former class are drawn to their observations by the remarkable beauty and serenity of her light, her numerous changes or phases, the inconstancy of her illumination, the enjoyment of her light at certain seasons of the year, when the sun sinks early below the horizon, and the want of it when both luminaries rise and set at nearly the same hour.

The philosophic observer examines her varying phases through his telescope; sees the shadows of the hills on her surface projected to a considerable distance through plains and vallies, rendered by the power of optics nearly as distinct as those of a terrestrial plain at the distance of only a few miles; he examines what he considers to be lunar volcanoes, and marks the progress of the lava from the crater to the vallies below; he looks to determine some indication of eruptions in present action; endeavours to distinguish her seas and continents, to measure the heights of her mountains, and to ascertain the existence or non-existence of a lunar atmosphere.

Again, the practical astronomer is watching carefully her deviating course in the heavens, and is endeavouring to submit her oscillating motion to the general principles of celestial mechanics; and to construct formulæ and tables, for the purpose of predicting her place at any appointed day and hour.

These subjects will each engage our attention in the course of the following chapters of this treatise; but at present we only propose to present to the reader an explanation of her most obvious phenomena, and to show their complete accordance with the motions and constitution of the solar system as described generally in the third chapter of this introduction.

Phases of the moon.

33. The first lunar phenomena to which we shall call the readers' attention, is the continual change of figure, or the phases which she exhibits to a terrestrial spectator. At one time perfectly full or globular, at others half or a quarter illuminated, and at others again exhibiting only a fine arched line, barely perceptible to the naked eye. These appearances are doubtless due to the revolution of the moon in her orbit; and the reflection of her light (which she receives from the sun) towards the earth. The lunar globe is necessarily always one half illuminated as we have shown the earth to be in the last section; and therefore, to a spectator placed in a line between the moon and sun, she would always present a full illuminated circle or hemisphere; but out of that line a greater or less part of the enlightened surface will be perceived, and which will obviously entirely vanish in certain positions. This may be illustrated by means of an ivory ball, as in the experiment described in the last section; for the ball being held before a candle in various positions, will present a greater or less portion of the illuminated hemisphere to the view of the observer, according to his situation with regard to the illuminated axis.

Plane Astronomy.

Phases illustrated. Fig. 6.

34. The same may be otherwise exhibited by means of our figure 6; where T is the earth, S the sun, and A, B, C, &c. the moon in different parts of her orbit. When the moon is at A, in conjunction with the sun S, her dark hemisphere being entirely turned towards the earth, she will disappear as at *a*, there being no light on that side to render her visible. When she comes to her first octant at B, or has gone an eighth part of her orbit from her conjunction, a quarter of her enlightened side is towards the earth, and she appears horned, as at *b*. When she has gone a quarter of her orbit from between the earth and sun to C, she shows us one half of her enlightened side, as at *c*, and we say she is a quarter old. At D, she is in her second octant; and by showing us more of her enlightened side she appears gibbous, as at *d*. At E, her whole enlightened side is towards the earth; and therefore she appears round, as at *e*; when we say it is full moon. In her third octant at F, part of her dark side being towards the earth, she again appears gibbous, and is on the decrease, as at *f*. At G, we see just one half of her enlightened side; and she appears half decreased, or in her third quarter, as at *g*. At H, we only see a quarter of her enlightened side, being in her fourth octant; where she appears horned, as at *h*. And at A, having completed her course from the sun to the sun again, she disappears; and we say it is new moon. Thus, in going from A to E, the moon seems continually to increase; and in going from E to A, to decrease in the same proportion; having like phases at equal distances from A to E. But as seen from the sun S she is always full.

The moon appears not perfectly round when she is full in the highest or lowest part of her orbit, because we have not a full view of her enlightened side at that time. When full in the highest point of her orbit, a small deficiency appears at her lower edge, and the contrary when full in the lower point of her orbit.

The earth is a moon to the moon.

35. The moon, as we have seen, shines by her reflected light; in the same manner, the earth, by throwing back the light it receives from the sun, becomes in its turn a moon to the moon; being full to the inhabitants of the lunar sphere when our moon changes, and *vice versa*. For, when the moon is at A, new to the earth, the whole enlightened side of the earth is turned towards the moon; and when the moon is at *d*, full to the earth, the dark side of the latter is turned towards the former. Hence a new moon answers to a full earth, and a full moon to a new earth. The quarters are also reversed with respect to each other.

Position of the cusps of the moon.

36. The position of the moon's cusps, or a right line touching the points of her horns, is very differently inclined to the horizon, at different hours of the same day of age. Sometimes she stands as it were upright on her lower horn, which is then necessarily perpendicular to the horizon; when this happens, she is said to be in her *nonagesimal degree*, which is the highest point of the ecliptic above the horizon; the ecliptic at that time being 90° from each side of the horizon, reckoning from the point where it is then cut by the former circle. But this never happens when the moon is on the meridian, except when she is in the beginning of Cancer or Capricorn.

Astronomy.

The moon always presents the same face to the earth.

37. The moon turns on an axis which is nearly perpendicular to the plane of the ecliptic, and in such a way as to make one complete diurnal rotation during one lunation, and therefore always presents to us the same face or hemisphere, as is demonstrable by observations made on her by the telescope; both these revolutions are performed in 27 days 7 hours 43′, viz. from any star to the same star again, but from one lunation to another is about $29\frac{1}{2}$ days. In consequence of this remarkable coincidence, which is still unaccounted for on physical principles, the earth must appear to a spectator on the moon to be permanently at rest. It will never quit any position in which it is once found, at whatever height it may be above the lunar horizon, or in whatever quarter it may appear; it will go through all its changes, but retain its position; consequently to one half of the moon the earth is always invisible, while the other half enjoys a constant illumination from its reflected rays; but each side has an equal participation of the solar light.

Phases of the planets.

38. The phases of the inferior planets, Mercury and Venus, strongly resemble those of the moon, but they are invisible except by means of the telescope. Copernicus, after having laid down his system of celestial motions, predicted that future astronomers would find that Venus underwent the same changes, and exhibited similar phases to the moon; which prediction was first fulfilled by Galileo, who directing his telescope to this planet, observed the phases foretold by the father of modern astronomy; he observed her to be sometimes full, sometimes horned, and sometimes gibbous.

Mercury also presents similar appearances. All the difference being, that when these are full, the sun is between them and us, whereas, when the moon is full, we are between her and the sun. Mars appears sometimes gibbous, but never horned, its orbit being exterior to that of the earth.

We shall have again to recur to this subject, in a subsequent chapter, our purpose here being merely a popular illustration of the most striking lunar phenomena.

7. *Of lunar and solar eclipses.*

Of eclipses.

39. The phenomena of eclipses are amongst the number of those which have most engaged the attention of mankind; the learned and the unlearned have found an equal interest in them; the one to observe their appearances, to discover the cause of the deprivation of light which we then encounter, to predict their return, &c.; and we have seen, in our historical sketch, the various absurd ideas that have been entertained on this subject by many of the most eminent philosophers of antiquity. Amongst the common people, the interest excited by such phenomena, arose from the fear which they inspired; they were considered by them as alarming deviations from the regular course of nature, and as the forerunners of some portentous event; hence, actuated either by curiosity or timidity, the subject of eclipses has probably from the earliest times engaged the serious attention of mankind, and they are still amongst the most interesting of the celestial phenomena. In illustrating the cause of eclipses, we shall follow still the method we have hitherto pursued in this introduction; that is, we shall render our explanation as popular and simple as possible, leaving all the minute particulars for another place, where we shall enter more fully upon the subject, and illustrate the principles of those calculations on which the prediction of eclipses depends.

Plane Astronomy.

Eclipse of the moon.

40. The earth being an opaque body enlightened by the sun, it necessarily projects a shadow into the regions of space in a contrary direction; and when it so happens that the moon in the course of her revolution about the earth, falls into this shadow, she loses the sun's light by which alone she is visible, and appears to us eclipsed. Let us suppose two straight lines drawn from the opposite parts of the solar disc tangents to the surface of the earth as AB, *a b*, (fig. 7) these lines will represent the limits of the shadow, and as the sun is much larger than the earth, these lines will meet at a point, and cross each other behind the earth; and the shadow will thus assume the figure of a right cone. When the moon enters this shadow, and a part of her disc is still enlightened by the sun, this part is not terminated by a straight line, but has the form of a luminous crescent, the concave part being turned towards the shade. The same circumstance happens again, when the moon begins to quit the shadow.

Fig. 7.

When the moon approaches the terrestrial shadow, she does not lose her light suddenly, but it gradually becomes more and more faint till the obscurity arrives at its greatest intensity. In order the better to comprehend this phenomenon, we have only to attend to the figure, and observe, that an opaque body may be so placed between an object and the sun as only to intercept a part of its light; let us suppose this object to be M, it will then be less illuminated than if it received the whole of the light, but more of it than if it were placed at *m* in total obscurity.

Penumbra.

41. This intermediate state comprehended between the angular space EBC on one side of the umbra or shadow, and FBC on the other, is called the *penumbra*; and it is the entrance of the moon into this partial shade, which produces the faint obscurity observed immediately before the eclipse commences, and after it is over.

The limits of this penumbra may be found by drawing two lines as A *e*, *a* E touching the surface of the sun and the earth, so as to cross at a point C between them. The angles EBC, *e b c* will determine the space occupied by the penumbra; for at a point situated beyond this space, the whole disc of the sun will be visible, and the visible portion will diminish from the line EB to CB, when it will entirely disappear, and consequently the penumbra will gradually increase in its intensity from its first limit EB, to its second BC, where it ceases, or is confounded with the shadow itself.

Faint light observed in a total lunar eclipse.

42. When the moon enters completely into the shadow of the earth, we still do not entirely lose sight of it; its surface is still faintly illuminated with a reddish light, something similar to that reflected by the clouds after the setting of the sun. This effect arises from the solar rays that have been refracted by our atmosphere, and afterwards inflected behind the earth; for those rays which are not enough refracted to reach the surface of the earth, continue their course through the atmosphere, and if not entirely absorbed by it, are inflected towards a focus or point in the same manner as in a convex lens.

The light thus refracted behind the earth is very

Astronomy. considerable: regarding only one luminous point of the solar disc, it can only project one ray to every point of the surrounding space, but through the medium of the terrestrial atmosphere, a course of luminous points is collected behind the earth, an object placed in the focus or vertex of this cone, would be more strongly illuminated than by the direct light of the sun; every point of the sun producing a similar effect, the length and extension of the terrestrial shadow are much diminished; and if the atmosphere did not absorb a very great portion of the solar rays, the light reflected from the disc of the moon would be very great; it is, however, so much modified by the circumstances alluded to, as to exhibit only that faint red light above described.

It may be proper to observe, that this faint illumination of the lunar disc at the time of a total eclipse, has been accounted for upon different principles, but the above appears to us the most satisfactory.

Solar eclipse.

43. An eclipse of the sun is an occultation of the sun's body, occasioned by the interposition of the moon between the earth and sun. On this account it is by some considered rather as an eclipse of the earth, because the light of the sun is hidden from the earth by the moon, whose shadow involves a part of the terrestrial surface. The cause of a solar eclipse, and the circumstances attending it are represented in fig. 8, where S is the sun, *m* the moon, and CD the earth, *rmso* the moon's conical shadow traversing a part of the earth CoD, and thus producing an eclipse to all the inhabitants residing in that track, but no where else; excepting that for a large space around it, there is a fainter shade included within all the space *r*CD*s*, which, as in the lunar eclipse, is called the penumbra.

Fig. 8.

Hence, solar eclipses happen when the moon and sun are in conjunction, whereas the lunar eclipses only take place when they are in opposition; that is, the former happen at the time of the new moon, and the latter at the full.

Different kind.

44. Notwithstanding the moon is very considerably less than the sun, yet from its proximity to us, it so happens, that its apparent diameter differs very little from that of the latter body, and even sometimes exceeds it. Suppose an observer situated in a right line which joins the continuation of the sun and moon, he will see the former of these bodies eclipsed, as we have above stated. If the apparent diameter surpasses that of the sun, the eclipse will be total, and the observer will be entirely immerged in the conical shadow which is projected behind the moon: if the diameters are equal, the point of the cone will terminate at the earth's surface, and there will be a momentary total eclipse. If the diameter of the moon be less than that of the sun, the observer will see a zone of the sun surrounding the moon like a ring, and the eclipse will be central and annular. And lastly, if the observer be not exactly in the line joining the centres, the eclipse may be partial; that is, a part of the solar disc may be hid while the remaining part continues perfectly visible. Total eclipses, which are very rare occurrences in any particular place, are remarkable for the darkness which accompanies them, and which they spread over different parts of the surface of the earth, in the same manner as the shadow of a dense cloud, carried along by the wind, sweeps over the mountains and the plains, depriving them for some instants of the light of the sun. This total darkness under the most favourable circumstances may last about five minutes. The smallest apparent diameter of the sun is 31′ 30″; the diameter of the moon at its mean distance 31′ 25″, that is less than that of the sun; consequently there cannot be a total eclipse when the moon is beyond its mean distance. Eclipses of the sun are also modified as to quantity by the height of the moon above the horizon which increases her diameter; other circumstances also contribute to produce certain changes which must be considered in the computations relative to these phenomena, but which it would be useless to detail in this place. From what has been already stated we may draw the following general conclusions.

Total. Annular. Partial.

Plane Astronomy.

General deductions.

1. That no solar eclipse is universal; that is, none can be visible to the whole hemisphere to which the sun is risen: the moon's disc being too small and too near the earth to hide the sun from a whole terrestrial hemisphere. Commonly, the moon's dark shadow covers only a spot on the earth's surface, about 180 miles broad, when the sun's distance is greatest, and the moon's least. But her partial shadow or penumbra, may then cover a circular space of 4,900 miles in diameter, within which the sun is more or less eclipsed, as the places are nearer to or farther from the centre of the penumbra. In this case, the axis of the shade passes through the centre of the earth, or the new moon happens exactly in the *node*, and then it is evident that the section of the shadow is circular; but in every other case the conical shadow is cut obliquely by the surface of the earth, and the section will be oval, and very nearly a true ellipsis.

2. Nor does the eclipse appear the same in all parts of the earth, where it is seen; but when in one place it is total, in another it is only partial. Moreover, when the apparent diameter of the moon is less than that of the sun, as happens when the former is in apogee and the latter in perigee, the lunar shadow is then too short to reach the earth's surface; in which case, although the conjunction be central, yet the sun will be to no place totally eclipsed, but to certain observers, a bright rim of light will be seen surrounding the moon while the latter is on the solar disc; and the eclipse is then said to be annular.

3. A solar eclipse does not happen at the same time in all places where it is seen; but appears earlier to the western parts, and later to the eastern; as the motion of the moon, and consequently of her shadow, is from east to west.

4. In most solar eclipses, the moon's disc is covered with a faint light; which is attributed to the reflection of the rays from the illuminated part of the earth.

Lastly. In total eclipses of the sun, the moon's limb is seen surrounded by a pale circle of light, which has been considered as indicative of a lunar atmosphere; others, however, doubt this explanation, and offer different conjectures as to the cause of the phenomenon; but this is not the place for discussing this question.

Having thus given a brief description of the constitution of the solar system, with an illustration of the most remarkable phenomena which it presents to naked vision, as far as they can be illustrated inde-

Astronomy. pendent of astronomical computation, we shall now conclude our introduction by describing certain astronomical machines, constructed for the purpose of exhibiting, in a simple and popular manner, all the most remarkable celestial movements and phenomena.

8. *Description of astronomical machines.*

Astronomical machines.

45. By astronomical machines is here to be understood any pieces of mechanism constructed for exhibiting the motions and phenomena of the heavenly bodies, being thus distinguished from astronomical instruments, which include all such constructions as are employed for the purpose of measuring altitudes, angles, &c. necessary for astronomical computation. The one, in fact, are merely employed for explanation; the other, for the purpose of research and calculation. Various improvements have been made by different artists in the construction of planetary machines, and that which is now exhibited in the lectures of the Royal Institution, is perhaps the most perfect of its kind; but it would carry us too far to describe this instrument, with all its apparatus, wheel work, &c. Beside, we cannot help observing, that much time, ingenuity, and expence are frequently wasted in these kind of constructions; because, after all, they are only, as we have before observed, explanatory; for which purpose the same degree of accuracy is not required as in instruments employed in astronomical observation. That student who stands in need of the assistance of such machines, will never become a great proficient in astronomy; to pursue this study with effect, a beginner must acquire the habit of constructing his own planetarium in his mind's eye, and of soaring with it into the regions of celestial space; he ought to conceive the orbits of the heavenly bodies in a free non-resisting medium, their nodes, their inclinations, and eccentricities unimpeded by the intervention of brass rings or ebony frames; which have always the effect of giving a stiffness and unnatural representation extremely offensive to the eye of the professed astronomer. We must, however, acknowledge, that to children or mere novices, these machines may be of some assistance, and shall therefore describe one or two of the most simple of them in this place.

Planetarium.

Planetarium, by Jones. Fig. 9.

46. The machine exhibited in fig. 9, is a planetarium, constructed by the late Mr. Jones, of Holborn. It represents, in a general manner, by various parts of it, all the principal motions and phenomena of the heavenly bodies.

The sun occupies the centre, with the planets Mercury, Venus, the Earth with its moon, Mars, Jupiter, with his four satellites, Saturn with his seven, and an occasional long arm may be attached for exhibiting the Herschel or Uranus, with his several attendants. To the earth and moon is applied a frame, CD, containing only four wheels and two pinions, which serve to preserve the earth's axis in its proper parallelism in its motion round the sun, and to give the moon her due revolution round the earth at the same time. These wheels are connected with the wheel work in the round box below, and the whole is set in motion by the winch H. The arm M that carries round the moon, points out on the plate C her age and phases for any situation in her orbit, which are engraved upon it. In the same manner, the arm points out her place in the ecliptic B, in signs and degrees, called her geocentric place; that is, as seen from the earth. The moon's orbit is represented by the flat rim A; this orbit is made to incline to any desired angle. The earth of this instrument is usually made of a 3-inch, or $1\frac{1}{2}$-inch globe, papered, &c. for the purpose; and by means of the terminating wire, that goes over it, points out the changes of the seasons, and the different length of days and nights. It may also be made to represent the Ptolemaic system, which places the earth in the centre, and the planets and sun revolving about it. This is done by an auxiliary small sun and earth, which change their places in the instrument; but at the same time it affords a most manifest confutation of it. For it is obviously perceived in this construction, first, that the planets Mercury and Venus being both within the orbit of the sun, cannot at any time be seen to go behind it, whereas, in nature, we see them as often go behind as before the sun in the heavens. It shows that as the planets move in circular orbits about the central earth, they ought at all times to be of the same apparent magnitude; whereas, on the contrary, we observe their apparent magnitude in the heavens to be very variable; and so far different, that Mars, for instance, will sometimes appear nearly as large as Jupiter; while, at others, he will scarcely be distinguishable from a fixed star.

Plane Astronomy.

Refutation of Ptolemaic system.

The planetarium, when thus adjusted, shows also that the motions of the planets ought always to be regular and uniform; that they ought always to move in the same direction; whereas, we find them, sometimes direct, at others stationary, and even retrograde; which plainly shows the fallacy of the Ptolemaic hypothesis, at the same time that the modern system is thus clearly represented. Let us, for example, take the earth from the centre, and replacing it by the ball representing the sun, also restoring the earth to its proper situation amongst the planets, and every phenomena will then correspond and agree exactly with celestial observations. For turning the handle H, we shall see the planets Mercury and Venus go both before and behind the sun, or have two conjunctions; we shall perceive also that Mercury can never have more than a certain angular distance 21° from that body, nor Venus a greater than 47°. It will likewise be seen, that the superior planets, particularly Mars, will sometimes be much nearer to the earth than at others; and, consequently, must vary considerably in their apparent magnitude; we shall see that these planets cannot appear from the earth to move with equal velocities; but that this will appear greater when they are nearest, and less as they are more remote; that their apparent motions ought sometimes to be direct, sometimes retrograde, while in particular positions they will seem to be stationary; all which are consistent with the actually observed phenomena.

These particulars are shown somewhat more minutely in fig. 10, where a hollow wire, with a slit at top, is placed over the arm of the planet Mercury or Venus, at E. The arm DG represents a ray of light proceeding from the planet at D to the earth, and is put over the centre which carries the earth at F. The machine being then put in motion, the planet D, as Fig. 10.

Astronomy. seen in the heavens from the earth at F, will undergo the several changes of position as above described; and a similar application may be made to the superior planets.

This apparatus serves also to illustrate the diurnal rotation of the earth on its axis; the cause of the different seasons, the difference in the lengths of the days and nights, &c. For as the earth is placed on an axis inclined to the plane of the ecliptic at an angle of $23\frac{1}{2}°$, we shall have, when the machine is in motion, the most satisfactory illustration of the different inclination of the sun's rays upon the earth. The different quantities which fall on a given space, the unequal quantities of the atmosphere they pass through, and the unequal duration of the sun above the horizon at the same place at different times of the year; which circumstances constitute the primary causes of all that change of seasons and variable lengths of days and nights which we experience.

The globe representing the earth being moveable about an axis, if we draw upon it a circle to denote our own horizon, we may, by means of the terminating wire going over it, very naturally exhibit the cause of the different lengths of the days and nights in our particular latitude, by simply turning the artificial earth with the hand to imitate its diurnal rotation; but in some of the more modern instruments of this kind, this rotatory motion is communicated to the globe by the wheel work of the machine itself.

The eclipses of the sun and moon are still more perfectly shown by this machine than the phenomena to which we have above alluded; for by placing a light in the centre instead of the brass ball, denoting

Fig. 11. the sun (fig. 11), and turning the handle till the moon comes into a right line, between the centres of the light, or sun, and the earth, the shadow of the moon will fall upon the latter, and all the inhabitants of those parts over which the shadow passes, will see more or less of the eclipse; and on the other side the moon passes through the shadow of the earth, and is by that means eclipsed to the inhabitants of those parts to which the lunar disc is at that time visible.

All the phenomena of the satellites of Jupiter, Saturn, &c., might also, with equal facilities, be exhibited by this machine, and are actually so exhibited in some of the larger apparatus, denominated orreries; in the machine we are at present describing, these satellites are only moveable by hand.

Orreries.

Of the Orrery. 47. The term Orrery, to denote such a machine as that we are about to describe, appears rather singular, and is one of those derivations, which, if the history were lost, would involve future etymologists in inexplicable difficulties. The first machine of this kind appears to have been made by the celebrated instrument maker, Graham, by whom it was probably considered only as an improved planetarium: but Rowley, an artist of reputation in his time, copied Graham's machine, and the first of his construction was made for the Earl of Orrery; whence Sir R. Steel, who knew nothing of Graham's original claim, called the instrument after the name of the supposed first purchaser an *Orrery*, which designation it still bears. One of the most usual constructions of this kind is shown in

Fig. 12. (fig. 12,) which may be briefly described as follows:—

Plane Astronomy. The frame of it, which contains the wheel work, &c. and regulates the whole machine, is made of ebony, and about four feet in diameter. Above the frame is a broad ring supported by twelve pillars, which ring represents the plane of the ecliptic. Upon it are two circles divided into degrees with the names and characters of the twelve signs of the zodiac. Near the outside is a circle of months and days, exactly corresponding to the sun's place at noon each day throughout the year. Above the ecliptic stand some of the principal circles of the sphere corresponding with their respective situations in the heavens; viz. *aa* are the two colures, divided into degrees and half degrees; *b* is one half of the equinoctial circle, making an angle of $23\frac{1}{2}$ degrees with the ecliptic. The tropic of Cancer and the arctic circle are each fixed parallel at their proper distances from the equinoctial. On the northern half of the ecliptic is a brass semicircle moveable upon two fixed points in ♈ and ♎.

This semicircle serves as a moveable horizon, to be put to any degree of latitude on the north part of the meridian, and the whole machine may be set to any latitude, without disturbing any of the internal motions, by means of two strong hinges fixed to the bottom frame upon which the instrument moves, and a strong brass arch, having holes at every degree, through which a pin may be passed at any required elevation. These hinges, with the arch, support the whole machine when set to the proposed latitude.

When the Orrery is thus adjusted, set the moveable horizon to any degree upon the meridian, whence may be formed a pretty correct idea of the respective altitude or depressions of the several planets, both primary and secondary. The sun S stands in the centre of the system on a wire making an angle with the ecliptic of about 82°; next in their order follow the planets Mercury, Venus, and the Earth, the axis of the latter being inclined to the plane of the ecliptic, at an angle of $66\frac{1}{2}°$, which is the measure of the inclination of the earth's axis.

Near the bottom of this axis is a dial plate, having an index pointing to the hours of the day, as the Earth revolves; and about the latter is a small ring, supported by two small pillars, representing the orbit of the moon, with divisions answering to the moon's latitude. The motion of this ring represents the motion of the lunar orbit according to that of the nodes; and within it is a small ball with a black cup, or case, by which are exhibited all the phases of this celestial body.

Beyond the orbit of the earth are those of Mars, Jupiter, and Saturn, and in some instruments the Georgium Sidus, or Uranus. Jupiter is attended by his four satellites, and Saturn by his seven satellites and ring.

The machine is put in motion by turning a handle, or winch; and by pushing in, and pulling out a small pin above the handle. When it is in, all the planets, both primary and secondary, will move according to their respective periods. When it is out, the motion of the satellites of Jupiter and Saturn are stopped, while all the rest move without interruption. There is also a brass lamp, having two convex glasses to put in the place of the sun; and also a smaller earth and moon made somewhat in proportion to their distance from each other, and which may be put on or

Astronomy. removed at pleasure. The lamp turns round at the same time with the earth, and the glasses of it cast a strong light upon her. When it is intended to use the machine, the planets must be first placed each in its respective position by means of an astronomical ephemeris, and a black patch or wafer may be placed on the middle of the sun; against the first degree of ♈ (Aries); patches may also be placed upon Venus, Mars, and Jupiter. Now turn the handle, one revolution of which corresponds to one diurnal revolution of the earth about its axis, and consequently answers to 24 hours upon the dial plate, placed at the foot of the wire on which the ball is fixed.

Again, when the index has moved over the space of 10 hours, Jupiter will have made one revolution on his axis, and so of the rest according to their respective periods of diurnal rotation. By these means the revolution of the planets, and their motion round their axis, will be represented to the eye, if not exactly, yet in nearly their due intervals of time.

48. We might have entered into the description of orreries on other and more correct principles than the above, but the explanation must have been proportionally longer; and we have already observed, that such machines are, in our opinion, rather calculated to show the ingenuity of their constructors, than to offer any advantages to the student. It is true, that they may convey to the uninformed reader some ideas of the planetary motions, but we think it is extremely probable, that the idea thus given, if not actually false, may, in many cases, be rather injurious than useful; and as instruments for computation, the most perfect of them are wholly incompetent, we shall therefore make no apology for not having extended our description of orreries to a greater length. What has been said, and a reference to the plate, will be quite sufficient for showing the general principle of their construction and operation, which, we conceive, is all that is requisite to be introduced in this place.

Cometarium.

Cometarium. 49. This machine, which must also be considered rather as an object of curiosity than utility, shows the motion of a comet, or very eccentric body, moving round the sun, and describing equal areas in equal times; and may be so adjusted, as to show such a motion for any degree of eccentricity. The first projection of it we owe to Desaguliers.

Fig. 13, 14. The dark elliptical groove, *a b c d* &c. (fig. 13,) is the orbit of the comet Y; which is carried round in this groove according to the order of those letters, by the wire W fixed to the sun S, and slides on the wire as it approaches nearer or recedes further from, the sun; being nearest, or in its perihelion in *a*, and most distant in the aphelion *g*. The areas *a* S *b*, *b* S *c*, *c* S *d*, &c. or the contents of these several trilaterals, are all equal; and in every turn of the winch, or handle, N, the comet Y is carried over one of these spaces; consequently, in the same time as it moves from *f* to *g*, or from *g* to *h*, it will also move from *m* to *a*, or from *a* to *b*, and so of the rest, its motion being quickest at *a*, and slowest at *g*. Thus the comet's velocity in its orbit continually decreases from the perihelion to the aphelion, and increases in the same proportion from *g* to *a*.

The ecliptic orbit is divided into 12 equal parts or signs, with their respective degrees, as is also the circle *n o p q*, &c., which represents a great circle in the heavens, and to which the comet's motion is referred by a small knob on the point of the wire W. While the comet moves from *f* to *g* in its orbit, it appears to move only about five degrees in this circle, as is shown by the small knob on the end of the wire W; but in as short a time as the comet moves from *m* to *a*, or from *a* to *b*, it appears to describe the large space in the heavens *t n*, or *n o*, either of which spaces contains 120°, or four signs. If the eccentricity of the orbit were greater, the greater also would be the difference in the cometary motion. Plane Astronomy.

The circular orbit ABC, &c. is for showing the equable motion of a body about the sun S, describing equal areas in equal times, with those of a body Y in its elliptic orbit above referred to, but with this difference, that the circular areas ASB, BSC, &c. or the equal arcs AB, BC, &c. are described in the same times as the unequal elliptic arcs *a b*, *b c*, &c.

If we conceive the two bodies Y and R, to move from the points *a*, A, at the same moment of time, and each to go round its respective orbit, and to arrive at the same points again at the same instant, the body Y will be more forward in its orbit than the body R all the way from *a* to *g*, and from A to G: but R will be forwarder than Y through all the other half of the orbit; and the difference is equal to the equation of the body Y in its orbit. At the points *a* A, and *g* G, that is, in the perihelion and aphelion they will be equal; and then the equation vanishes. This shows why the equation of a body moving in an elliptic orbit, is added to the mean or supposed circular motion from the perihelion to the aphelion, and subtracted from the aphelion to the perihelion, in bodies moving round the sun, or from the perigee to the apogee, and from the apogee to the perigee in the moon's motion round the earth.

This motion is performed in the following manner by the machine, (fig. 15.) ABC is a wooden bar (in Fig. 15. the box containing the wheel-work), above which are the wheels D and E, and below it the elliptic plates FF and GG; each plate being fixed on an axis in one of its foci, at E and K; and the wheel E is fixed on the same axis with the plate FF. These plates have grooves round their edges precisely of equal diameters to one another, and in these grooves is the cat-gut string *g g*, *g g* crossing between the plates at *h*. On H, the axis of the handle or winch N in fig. 13, is an endless screw in fig. 15, working in the wheels D and E, whose numbers of teeth being equal, and equal to the number of lines *a* S, *b* S, *c* S, &c. in fig. 14, they turn round their axis in equal times to one another, as do likewise the elliptic plates. For, the wheels D and E having equal numbers of teeth, the plate FF being fixed on the same axis with the wheel E, and turning the plate GG of equal size by a cat-gut string round them both, they must all go round their axis in as many turns of the handle N as either of the wheels has teeth.

It is easy to see, that the end *h* of the elliptical plate FF being farther from its axis E than the opposite end I is, must describe a circle so much the larger in proportion, and must therefore move through so much more space in the same time; and for that

Astronomy. reason the end h moves so much faster than the end I, although it goes no sooner round the centre E: at the same time the quick-moving end h of the plate FF leads about the short end h K of the plate GG with the same velocity; and the slow-moving end I of the plate FF coming half round as to B, must then lead the long end k of the plate GG about, with a corresponding slow motion: so that the elliptical plate FF and its axis E move uniformly and equally quick in every part of its revolution; but the elliptical plate GG, together with its axis K, must move very unequally in different parts of its revolution; the difference being always inversely as the distance of any point of the circumference of GG from its axis at K; or in other words, if the distance K k, be four, five, or six times as great as the distance K h, the point h will move in that position, four, five, or six times as fast as the point k does when the plate GG has gone half round; and so on for any other eccentricity or difference of the distances K k, K h. The I on the plate EF, falls in between the two teeth at k on the plate GG; by which means the revolution of the latter is adjusted to that of the former, so that they can never vary the one from the other.

On the top of the axis of the equally moving wheel D (fig. 15) is the sun S (fig. 14) which by means of the wire attached to it, carries the ball R round the circle, ABC, &c. with an equable motion, according to the order of the letters; and on the top of the axis K of the unequally moving ellipse GG in (fig. 15) is the sun S (fig. 14) carrying the ball Y unequally round the elliptic groove $a\,b\,c\,d$, &c. which elliptic groove must be exactly equal and similar to the verge of the plate GG, which again is also equal to that of EF.

Eclipsarean.

Eclipsarean

50. The eclipsarean is an instrnment invented by Mr. Ferguson for exhibiting the time, quantity, duration, and progress of solar eclipses, in all parts of the earth. This machine consists of a terrestrial globe A, (fig.

Fig. 16, 17. 17) turned by a winch M, round its axis B, inclining $23\frac{1}{2}°$, and carrying an index round the hour circle D; a circular plate E, on which the months and days of the year are inserted, and which supports the globe in such a manner, that when the given day of the month is turned to the annual index G, the axis has the same position with the earth's axis at that time; a crooked wire F, which points to the middle of the earth's enlightened disc, and shows to what place of the earth the sun is vertical at any given time; a penumbra or thin circular plate of brass I, divided into twelve digits by twelve concentric circles and so proportioned to the size of the globe, that its shadow, formed by the sun, or a candle, placed at a convenient distance, with its rays transmitted through a convex lens, to make them fall parallel on the globe, may cover those parts of the globe which the shadow and penumbra of the moon cover on the earth; an upright frame HHHH, on the sides of which are scales of the moon's latitude, with two sliders K and K, fitted to them, by means of which the centre of the penumbra may be always adjusted to the moon's latitude; a solar horizon C, dividing the enlightened from the darkened hemisphere, and showing the places where the general eclipse begins and ends with the rising or setting sun, and a handle M, which turns the globe round its axis by the wheel work, and moves the penumbra across the frames by threads over the pulleys LLL, with a velocity duly proportioned to that of the moon's shadow over the earth as the earth turns round its axis.

Plane Astronomy.

Rectification.

51. If the moon's latitude at any conjunction exceeds the number of divisions on the scales, there can be no eclipse; if not, the sun will be eclipsed to some parts of the earth; the appearance of which may be represented by the machine, either with the light of the sun, or of a candle. For this purpose, let the indexes of the sliders KK, point to the moon's latitude, and let the plate E be turned till the day of the given new moon comes to G, and the penumbra be moved till its centre comes to the perpendicular thread in the middle of the frame, which thread represents the axis of the ecliptic; then turn the handle till the meridian of London on the globe comes under the point of the wire F, and turn the hour circle D till 12 at noon comes to its index; also turn the handle till the hour index points to the time of new moon in the circle D, and then screw fast the collar N. Lastly, elevate the machine till the sun shines through the sight holes in the small upright plates OO, on the pedestal, or place a candle before the machine, at the distance of about four yards, so that the shadow of the intersection of the cross thread in the middle of the frame, may fall precisely on that part of the globe to which the wire F points; with a pair of compasses take the distance between the centre of the penumbra and the intersection of the threads, and set the candle higher or lower, according to that distance; and place a large convex lens between the machine and candle, so that the candle may be in the focus of the lens; and thus the machine is rectified for use.

Phenomena shown by the machine.

52. Let the candle be turned backward till the penumbra almost touches the side, HF, of the frame, and then turning it forward, the following phenomena may be observed.

1. Where the eastern edge of the shadow of the penumbral plate I, first touches the globe at the solar horizon, those who inhabit the corresponding part of the earth, see the eclipse begin on the uppermost edge of the sun, just at the time of its rising.

2. In that place where the penumbra's centre first touches the globe, the inhabitants have the sun rising upon them centrally eclipsed.

3. When the whole penumbra just falls upon the globe, its western edge at the solar horizon touches, and leaves the place where the eclipse ends at sunrise on his lowermost edge.

4. By continued turning, the cross lines in the centre of the penumbra will go over all those places on the globe where the sun is centrally eclipsed.

5. When the eastern edge of the shadow touches any place of the globe, the eclipse begins there; when the vertical line in the penumbra comes to any place, then is the greatest obscuration at that place; and when the western edge of the penumbra leaves the place, the eclipse ends there, and the times are shown on the hour circle; and from the beginning to the end, the shadows of the concentric penumbral circles show the number of digits eclipsed at all the intermediate times.

6. When the eastern edge of the penumbra leaves

Astronomy.

the globe at the solar horizon C, the inhabitants see the sun beginning to be eclipsed on its lowermost edge, at its setting.

7. Where the centre of the penumbra leaves the globe, the inhabitants see the sun centrally eclipsed; and lastly, where the penumbra is wholly departing from the globe, the inhabitants see the eclipse ending on the uppermost part of the sun's edge, at the time of its disappearing in the horizon.

This instrument will likewise serve for exhibiting the time of sun rising and setting; and of morning and evening twilight, as well as the places to which the sun is vertical on any day, by setting the day on the plate E to the index G, turning the handle till the meridian of the place comes under the point of the crooked wire F, and bringing XII on the hour circle D to the index: then if the globe be turned till the place touches the eastern edge of the horizon C, the index shows the time of sun setting; and when the place comes out from below the other edge of C, the index shows the time when evening twilight ends; morning twilight and sun rising are shown in the same manner on the other side of the globe. And the places under the point of the wire F are those to which the sun passes vertically on that day. *Ferguson's Astronomy*, by Brewster, or *Phil. Trans.* vol. xlviii.

53. The celestial and terrestrial globes may also be considered as astronomical machines of the kind we have been describing; but it would too much interrupt the order of our treatise to enter upon a description of these instruments in this place; they will, therefore be described under the proper head in our alphabetical arrangement; and we shall now proceed, having given the foregoing succinct view of the more popular celestial phenomena, to treat the subject under a more scientific point of view, in the following sections.

PART II.

PLANE ASTRONOMY.

§ III. *Containing the principles of astronomical computation.*

1. *Definitions.*

Of the sphere definitions.

54. Previous to our entering upon this subject, it will be requisite for the reader to render himself familiar with the following definitions. Some of them have been already given, but the convenience of having one decided place of reference will compensate for the few repetitions that occur.

A great circle. Fig. 18.

1. A *great circle* of a sphere is any circle QRST (fig. 18) whose plane passes through the centre of the sphere; and a *small circle* is any circle, BHK, whose plane does not pass through the centre. All great circles bisect each other.

A diameter

2. The *diameter* of a sphere is any line, PE, passing through the centre and terminated on both sides by the circumference; this diameter is said to be the *axis* of that great circle to which it is perpendicular; and the extremity of the axis PE are called the poles of that circle.

Hence it follows, that the pole of a great circle is 90° distant from every point of it upon the sphere; and that the arcs subtending any angles at the centre of a sphere are those of great circles. Consequently, all the triangles formed on the surface of a sphere for the solution of spherical problems must be formed by the arcs of great circles.

Plane Astronomy.

Secondaries.

3. *Secondaries* to a great circle are great circles, as PQE, PRE, which pass through its poles, and whose planes are therefore perpendicular to the plane of the latter. Hence every secondary bisects its great circle; a secondary also bisects every small circle that is parallel to the great circle to which it is secondary. Since every secondary passes through the pole of its great circle, and is perpendicular to it; it follows, that if a secondary passes through the poles of two great circles, it is perpendicular to each of them. And conversely, if one circle be perpendicular to two others, it must pass through their poles.

Poles of the celestial and terrestrial sphere.

55. The above definitions belong wholly to the sphere considered abstractedly as a geometrical solid; the following appertain to the sphere considered with reference to astronomy.

Fig. 19.

1. Let pep' of (fig. 19) represent the earth which at present we shall consider as a perfect sphere, and let pp' be the line about which it performs its diurnal rotation; then pp' are called its *poles*, and the line pp' its *axis*. And if we assume the circle PEP′Q to denote the circle of the celestial sphere, and conceive pp' to be produced to the heavens meeting them in PP′, these will be the *poles* of the celestial sphere.

2. The *terrestrial equator* is a great circle $erqs$, of the earth perpendicular to its axis; and if we conceive the plane of this circle to be produced to the sphere of the fixed stars, it will mark out the great circle ERQS, which is called the *celestial equator*.

Hence it follows, that the poles of the terrestrial and celestial spheres are the same as the poles of the respective equators.

Equator.

The equator divides either spheres into two equal portions, called the *northern* and *southern hemispheres*, and the corresponding poles are in like manner denominated the *north* and *south* poles. The northern hemisphere is the part of the earth which lies on the side of the equator which we inhabit, and which in the figure we may assume to be epq.

Latitude terrestrial.

2. The *latitude* of a place on the earth's surface is its angular distance from the equator, measured upon a secondary to it: thus the arc eb, measures the latitude of the point b. Any circle on which we measure the latitude of a place is called a *terrestrial meridian*; and when produced to the heavens a *celestial meridian*.

Meridian.

3. The small circles parallel to the terrestrial equator are called *parallels of latitude*.

4. The secondaries to the celestial equator are called circles of *declination*; and the small circles parallel to the equator on the earth's surface parallels of *declination*. Declination, therefore, in the celestial sphere, corresponds to latitude on the terrestrial sphere, thus the arc eb which measures the latitude of a place on the earth, corresponds to EZ, the declination in the heavens.

Longitude terrestrial

5. The longitude of a place on the earth's surface is an arc of the equator, intercepted between the meridian passing through the place, and another called a first meridian, passing through that place from which you begin to measure; which latter is different in different countries. Most nations account their first

meridian that passing over their capital. The English, for example, take the meridian of London, or rather that of the Royal Observatory, Greenwich; the French, that of Paris, &c.

Horizon.

6. If b be supposed to denote any place on the earth, and a tangent plane be supposed to be drawn to that place, and produced to the heavens, meeting them in the points a c, the circle a b c, which is here projected into a right line, is called the *sensible horizon*.

And the great circle HOR, which is drawn parallel to it, passing through the centre of the earth O, is called the *rational horizon*. It is in the former of those circles, which all the heavenly bodies are observed to rise and set.

Small circles parallel to the horizon are called *almucantars*.

Zenith and nadir.

7. If the radius O b of the earth at the place b of a spectator be produced both ways to the heavens, that point Z vertical to him is called the *zenith*, and the opposite point N the *nadir*. Consequently, the zenith and nadir are the poles of the rational horizon.

Vertical circles.

8. *Vertical circles* are those secondaries which are perpendicular to the horizon, and which therefore pass through the zenith: it is in these circles the altitude of the heavenly bodies are taken. The celestial meridian of a place is therefore a vertical circle passing through the pole and zenith of that place; as PEHP′ in the figure above referred to.

The two points in the horizon R H, which are cut by the meridian of any place, are called the *north* and *south* points, according as they are towards the north or south poles.

Prime vertical.

9. That vertical circle which cuts the meridian of any place at right angles, dividing it into two equal hemispheres, and which cuts the meridian in the east and west points, is called the *prime vertical*, as ZN, which is projected into the right line ZN.

Azimuth and amplitude.

10. When a body is referred to the horizon by a vertical circle, the distance of that point of the horizon from the north or south points, is called the *azimuth*, and its distance from the east or west points its amplitude.

Ecliptic.

11. The *ecliptic* is that great circle of the heavens which the sun appears to describe in the course of the year.

Obliquity of the ecliptic.

12. The angle which the ecliptic forms with the celestial equator, is called the *obliquity of the ecliptic*; and the two points in which these circles intersect and bisect each other, are called the *equinoctial points*. The times when the sun comes to these points are called the *equinoxes*.

For the signs, order, and characters of the twelve signs of the ecliptic, or zodiac, see our table of constellations, p. 506.

Of these signs, the first six which lie on the northern side of the equator, are called northern signs; viz. Aries, Taurus, Gemini, Cancer, Leo, Virgo: and the other six, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces, the southern signs.

The equinoctial points correspond to the first points of Aries and Libra.

The six signs, Capricorn, Aquarius, Pisces, Aries, Taurus, and Gemini, are called ascending signs, the sun approaching our's, or the north pole, while it passes through them; the others, Cancer, Leo, &c., are, for a corresponding reason, called descending signs; the sun, while passing through them, being receding from our pole.

Zodiac.

14. The *zodiac* is a zone of the celestial sphere, extending 8° on each side of the ecliptic, within which the motion of all the principal planets are performed.

The signs of the ecliptic and of the zodiac are the same.

Direct and retrograde.

15. When any of the heavenly bodies appear to move according to the order of the signs, viz. through Aries, Taurus, &c. their motion is said to be *direct*, or *in consequentia*; when contrary to that order, *retrograde*, or *in antecedentia*. (See art. 29)

The real motion of all the planets is according to the order of the signs.

Right ascension.

16. We have seen, that the declination of any heavenly body is measured by the arc of a declination circle, or secondary to the equator, and the distance of that secondary on the equator from the first point of Aries, estimated according to the order of the signs, is called its right ascension.

Hence the right ascension of a heavenly body corresponds with the longitude of a terrestrial one, except as to the point whence it is measured.

Latitude and longitude in the heavens. Fig. 20.

17. The *latitude* of a heavenly body is measured by a secondary to the ecliptic passing through that body; that is, by the angular distance between it and the ecliptic, as by the arc m s, fig. 20: and the *longitude* is measured by the arc of the ecliptic, intercepted between the first point of Aries and the point where the secondary meets the ecliptic, estimated according to the order of the signs, as o m, considering o as the first point of Aries.

Hence the latitude and longitude of a heavenly body are the same with reference to the ecliptic and its secondaries; as those of a terrestrial body or place with reference to the equator and meridians.

Oblique ascension.

18. The *oblique ascension* is an arc of the equator, intercepted between the first point of Aries, and that point of the equator which rises with any body; and the difference between the right and oblique ascension is called the *ascensional difference*.

Tropics.

19. The tropics are two celestial circles parallel to the equator, and touching the ecliptic; the one at the beginning of Cancer, called the tropic of Cancer; the other at the beginning of Capricorn, called the tropic of Capricorn. The two points where the tropics touch the ecliptic, are called the *solstitial* points.

Arctic and antarctic circles.

20. The *arctic* and *antarctic* circles are two parallels of declination; the former about the north, and the latter about the south pole: the distance of each from the pole is equal to the distance of the tropics from the equator.

Zones.

These two circles, and those of the tropics, when referred to the earth, divides it into five zones; two called the *frigid zones*, which are those towards the two poles; the temperate zones being those between each tropic and its corresponding polar circle; and one torrid zone, including all the space between the two tropics, extending therefore about 23½° on each side of the equator.

Nodes.

21. The *nodes* are the points where the orbit of a planet cuts the plane of the ecliptic; and the nodes of a satellite, are the points where its orbit cuts the plane of the orbit of its primary, or that about which it revolves.

Astronomy.

The *ascending* node is that where the body passes from the south to the north side of the ecliptic, and the other is called the *descending* node.

22. The *aphelion* is that point of the orbit of a planet which is farthest from the sun; and the *perihelion* that point where it is nearest.

The *apogee* and *perigee* have the same signification with reference to the earth. The moon, for instance, is said to be in apogee when farthest from us; and in perigee, when nearest.

The above definitions will be sufficient for our present purpose; others, which require a previous knowledge of certain subjects not yet discussed, are reserved for those places in the course of the subsequent sections in which they naturally occur.

2. *Illustration of certain celestial phenomena.*

General remarks on the celestial sphere.

56. Let us now proceed to show the application of the doctrine of the sphere, to the illustration of certain celestial phenomena, as the rising, setting, southing, &c. of the heavenly bodies.

57. In art. 55. we have defined the sensible and rational horizon; but with reference to the sphere of the fixed stars, these may be considered as coinciding, the angle which the arc H a (fig. 19) subtends at the earth, becoming then insensible in consequence of the immense distance of these bodies. Now, if we suppose, as we have hitherto done, the earth to revolve daily about its axis, all the heavenly bodies must successively appear to rise and set, or revolve about the pole, in circles, whose planes are perpendicular to the earth's axis, and consequently parallel to each other; and will, to every appearance, be the same as if the spectator were at rest in the centre of a concave sphere, which revolved uniformly about him; or that the stars each revolved in parallel circles on such a sphere. We may therefore consider the earth but as a point with reference to the radius of the sphere of the fixed stars, and leave it out of the consideration in our farther inquiries upon this subject, and only employ the zenith, equator, poles, horizon, &c. of the celestial sphere, and such circles of declinations, as corresponding with the motion of the given bodies.

Oblique sphere constructions. Fig. 21.

58. Let then fig. 21. represent the position of the heavens to an observer, whose zenith is Z in north latitude; EQ the equator, PP′ the poles, HOR the rational horizon, PZHP′R the meridian of the spectator.

Draw the great circle ZON perpendicular to the meridian, and passing through the zenith Z, which from our definition will be the prime vertical; and being in the plane of the eye, this being supposed to be perpendicular over the pole of the meridian, will be projected into a right line ZN, as will be shown in our treatise on Projection and Perspective. The same will also be the case with the equator EQ, the horizon HR, and the great circle POP′, supposed to be drawn perpendicular to the meridian; the common intersection of all these circles being in the point O, the pole of the meridian.

Draw the small circles, or parallels of declinations, w H, $m\,t$, $a\,e$, Rv, $y\,x$, which will represent the circles described by any of the heavenly bodies; and as the great circle POP′ bisects the equator, it will bisect all the small circles parallel to it, consequently $m\,t$, ae, are bisected in r and c; and we shall have $a\,c = c\,e$, and $m\,t = r\,t$, each equal to a quadrant, or 90°.

Plane Astronomy.

Now if we conceive the figure referred to as exhibiting the eastern hemisphere, the several arcs QE, $a\ e$, $t\ m$, &c. will represent the paths of bodies placed at those distances from the pole, as they ascend from the meridian under the horizon to the meridian above; and the points b, O, s, will be the places where they rise, or begin to appear above the horizon; and t E e, the points where they attain their greatest or meridian altitude; as $a\,e$, QE, $m\,t$, are bisected in c, O, r; $e\,b$ must be greater than $b\ a$; QO, equal to OE, and $t\,s$ less than $s\,m$.

Whence it follows, that a body on the same side of the equator as the spectator, will be longer above the horizon than below it; when the body is in the equator it will be as long below as above the horizon; and when it is on the contrary side of the equator to the spectator, it will be longer below the equator than above it; for the arc $e\ b$ is greater than $b\ a$, EO=OQ, and $t\,s$ less than $s\,m$, and the motion with which these arcs are described are uniform.

The bodies describing $a\,e$, $m\,t$, rise at b and s; and as O is the east point of the horizon, and H and R the north and south points; a body on the same side of the equator as the spectator rises between the east and the north; and a body on the contrary side, between the east and the south, the spectator being supposed in north latitude; and a body in the equator rises in the east at O.

59. When bodies come to d and r, they are in the prime vertical, or in the east; hence, a body on the same side of the equator as the spectator comes to the east after it has risen; a body in the equator rises in the east, and one on the contrary side of the equator has passed the east before it rises. The body which describes the circle Rv, or any one nearer to P, never sets; and such circles are called circles of *perpetual apparition*, and the stars which describe them *circumpolar stars*. The body which describes the circle w H, just become visible at H, and then instantly descends below the horizon; but those bodies which describe the circles nearer to P′ are never visible. Circles of perpetual apparition.

Such is the apparent diurnal motion of the heavenly bodies, when the spectator is situated any where between the equator and either poles; and this is called an *oblique* sphere; because all bodies rise and set obliquely to the horizon.

In the above deductions we have supposed the figure to represent the eastern hemisphere, and the bodies to ascend through their respective arcs; but it may be equally supposed to denote the western hemisphere, only in this case, these arcs will represent the paths of the body as they descend from their greatest altitude above the horizon to their meridian below the horizon. And hence, it is obvious, that supposing a body not to change its declination, it will be at equal altitudes at equal times before and after it has attained its meridian altitude.

60. In the preceding article we have supposed the spectator to be in north latitude, or, which is the same, the zenith of the spectator to be between the equator and north pole; and it is obvious, that what we have said would apply equally to a spectator similarly situated in south latitude; but when he is situated either in the equator or in one of the poles, Right sphere.

Astronomy.

the considerations become less complicated; in the former case we call it a *right sphere*, and in the latter a parallel sphere. If the spectator be at the equator, then E coincides with Z, and Q with N, as in (fig. 22); consequently also PP′ coincides with HR; and the declination circles *ea*, *tm*, which are always parallel to the equator, are in this case perpendicular to the horizon; and as these circles are always bisected by PP′, they must now be bisected by the horizon HR; hence in this position of the spectator all the heavenly bodies, which change not their declination, will be an equal time above and below the horizon, and will rise perpendicularly to it; whence the denomination of the right sphere.

Fig. 22.

Parallel sphere. Fig. 23.

On the other hand, if the spectator be situated at the pole, then the sphere will be as represented in fig. 23, that is, we shall have P coincide with Z, and EQ, with HR, or the equator will coincide with the horizon; and all the parallels of declination, *ae*, *mt*, described by the heavenly bodies, will be therefore parallel to the horizon; any body, therefore, which is above the horizon, and which changes not its declination, will remain constantly above the horizon, and at the same altitude; and those which are below the horizon will continue constantly below. Consequently, a spectator at the pole would never see the heavenly bodies rise and set, but would observe them to describe circles in the heavens parallel to the horizon; whence the denomination *parallel sphere*.

Of the change of declination.

61. We have had two or three times occasion to use the words, "those heavenly bodies which change not their declination;" it may be here proper to explain to the reader, that by this we mean the fixed stars only, these being the only celestial body that are not subject to a change of declination; and some of these even are liable to such a change, it is however too inconsiderable to be attended to in this place. But the sun, moon, and planets are constantly changing their declination, in consequence of the proper motion of the earth and themselves; let us, therefore, now bestow a few words in explanation of these cases, particularly as regards the sun.

Apparent motion of the sun.

We have already stated in our introduction (art. 27) that by attentively observing the stars which set and rise with the sun during the course of the year, that he appears to have described a great circle of the celestial sphere, forming with the ecliptic an angle of about $23\frac{1}{2}°$, or more exactly 23° 28′. This circle is called the ecliptic, and is denoted by the line LC, into which it is projected in the three last figures; which circle cuts the equator, as we have seen, in two points called the equinoctional points. The sun, therefore, during one part of the year, is on one side of the equator, and in the other, on the contrary side; and by this means his rising and setting is subject to all that variety which we have noticed in the stars in the two hemispheres, and hence the cause of the different lengths of the days at different times of the year, the succession of seasons, and the several phenomena attending them, as we have already endeavoured to explain in a popular manner in the preceding introduction.

Change of seasons.

Let us now endeavour to illustrate the same a little more particularly by referring again to our fig. 21. Here since in the course of the year, the sun appears to de cribe the circle of which LC is the projection, it is obvious that he will be sometimes to the north of the equator, as in *q*, at others to the south, as in *p*, and at others in the equator, as at O. In the former case, that is, when he is in *q*, P denoting the north pole, it is obvious, as we have already remarked respecting any body describing the declination circle *ae*, that he will rise between the north and east, attain to the prime vertical, after he has risen, and will be longer above the horizon than below it, which is the case in our latitudes, from about the 21st of March to the 22d of September, these being nearly the times when the sun crosses the equator. On those two days he rises in the east, and is an equal time above and below the horizon to every part of the globe except the two poles; and the days and nights being then equal, these points are called the *equinoctial* points, and the sun itself is said to be in the *equinoxes*. The former of these is called the *vernal*, and the latter the *autumnal* equinox. When the sun is on the south side of the equator, as at *p*, then the same remarks apply as we have already made with respect to any body describing the declination circle, *mt*, that is, he will rise between the south and the east, and will be longer below than above the horizon, and our days will be shorter than our nights, as is the case from the autumnal to the vernal equinox, that is, from about the 22d of September to the 21st of March.

Plane Astronomy.

When the spectator is in the equator, then the sphere being right (see fig. 23), the sun will be always an equal time above and below the horizon; and when the sun is also in the equator, he will rise east and describe a great circle corresponding with the prime vertical, and will be vertical over the head of the spectator in the middle of his course: at other times he will rise between the north and the east or the south and the east, according as his declination is north or south. There is, therefore, even in these regions, a change of seasons; but as the sun will dart his vertical beams upon every point of the equator, twice in the course of one revolution, the inhabitants may be said to have two summers and two winters in the course of a year. When the spectator is at the pole, the sphere will be parallel (see fig. 23), and the sun from the vernal to the autumnal equinox, will be constantly visible to the north pole, and perpetually hidden below the horizon during the other half of the year; and the contrary for the south pole. That is, in the former, he will be visible from the time he passes from O to L, and from L to O, and be invisible while he is describing the other half of the ecliptic. At each of the poles, therefore, the days and nights are each half a year in length. It must not, however, be understood here, that the length of the days and nights are each exactly equal to half a year, for we shall see hereafter, that the sun is not so long on the northern as on the southern side of the equator, the cause of which will likewise be illustrated in a subsequent chapter. At present, it will be sufficient to observe, that such is known to be the case from observation.

Zones described.

62. We have seen in our definitions that the earth is considered as divided into five zones, by referring to the earth the two polar circles, and the two tropics. Now from what has been above stated, it is obvious, that to an observer in either hemisphere, in the latitude of 23° 28′, his zenith will coincide with the sun's

place at noon, on that day when it has attained its greatest north declination, if the observer be in north latitude; or south, if he be situated in south latitude; consequently, in either situation, on one day in a year the sun will be vertical to the inhabitants of either tropic, and of course to all places situated between them, except those on the equator, who, as we have seen, have the sun vertical twice.

Beyond the tropics, either to north or south, the sun is never vertical, the zenith of all such places being farther from the equator than the extreme declination of the sun or the obliquity of the ecliptic.

Those who inhabit the two polar circles will have one day and one night of twenty-four hours, or there will be one day in each of those circles when the sun will not set, another on which he will not rise, as will be immediately obvious by referring to the preceding figures.

From these circles to the poles themselves the sun will be for a greater or less time above and below the horizon, till in the actual poles, the nights and days will be half a year each, as we have already stated.

Synopsis of spherical trigonometry.

3. *Synopsis of spherical trigonometry.*

62. As in the course of the following articles, we shall have frequent occasion to refer to the several cases of spherical trigonometry, we conceive that it will be very convenient for the reader to have a general synoptic table of all the principal results and theorems belonging to this doctrine, the investigations of which will be given in our Treatise on Trigonometry, Part I. We collect them here merely for the convenience of reference, and have chosen those only which are most general in their application. They are sufficient for the solution of any spherical problem, although, in certain cases, they may not offer the most expeditious mode of solution; we shall not therefore uniformly adopt the formulæ given in the table, but when a more expeditious form presents itself, we shall avail ourselves of it; in the greater number of cases, however, our solutions will be deduced from the tabula formulæ.

TABLE I. *For the solution of all the cases of right angled spherical triangles.*

Given.	Required.	Value of the Terms required.	Cases in which the terms required are less than 90°.
I. Hypothenuse and one leg.	Angle opposite the given leg.	Its sin $= \dfrac{\text{sin given leg}}{\text{sin hypoth.}}$	If the given leg be less than 90°.
	Angle adjacent to the given leg.	Its cos $= \dfrac{\text{tan given leg}}{\text{tan hypoth}}$	If the things given be of the same affection*.
	Other leg.	Its cos $= \dfrac{\text{cos hypoth.}}{\text{cos given leg}}$	Idem.
II. One leg and its opposite angle.	Hypothenuse.	Its sin $= \dfrac{\text{sin given leg}}{\text{sin given ang.}}$	Ambiguous.
	Other leg.	Its sin $= \dfrac{\text{tan given leg}}{\text{tan given ang.}}$	Idem.
	Other angle.	Its sin $= \dfrac{\text{cos given ang.}}{\text{cos given leg}}$	Idem.
III. One leg, and the adjacent angle.	Hypothenuse.	Its tan $= \dfrac{\text{tan given leg}}{\text{cos given ang.}}$	If the things given be of like affection.
	Other angle.	Its cos $=$ cos giv. leg $\times$ sin giv. ang.	If the given leg be less than 90°.
	Other leg.	Its tan $=$ sin giv. leg tan giv. ang.	If the given angle be less than 90°.
IV. Hypothenuse and one angle.	Adjacent leg.	Its tan $=$ tan hyp. $\times$ cos giv. ang.	If the things given be of like affection.
	Leg opposite to the given angle.	Its sin $=$ sin hyp. $\times$ sin giv. ang.	If the given angle be acute.
	Other angle.	Its tan $= \dfrac{\text{cot giv. angle}}{\text{cos hypothen}}$	If the things given be of like affection.
V. The two legs.	Hypothenuse.	Its cos $=$ rectan. cos given legs	If the given legs be of like affection.
	Either of the angles.	Its tan $= \dfrac{\text{tan opposite leg}}{\text{sin adjacent leg}}$	If the opposite leg be less than 90°.
VI. The two angles.	Hypothenuse.	Its cos $=$ rect. cot given angles	If the angles be of like affection.
	Either of the legs.	Its cos $= \dfrac{\text{cos opposite angles}}{\text{sin adjacent angle}}$	If the opposite be acute.

* Angles or sides are of the same affection when they are both greater or both less than a right angle; and in the third column of our Table we have stated when the result is of the same affection with the things given, and when it is ambiguous.

In working by logarithms the reader must observe, that when the resulting logarithm is the log. of a quotient, 10 must be added to the index; and when it is the log. of a product, 10 must be subtracted from the index. Thus, when the two angles are given,

log. cos hypothenuse = log. cos one angle + log. cos other angle — 10.

log. cos either leg = log. cos opp. angle — log. sin adjac. angle + 10.

Quadrantal triangles.

In a quadrantal triangle if the quadrantal side be called radius, the supplement to the angle opposite to that side be called hypothenuse, the other sides be called angles, and their opposite angles be called legs; then the solution of all the cases will be as in the above table for right angled spherical triangles.

Napier's *analogies for right angled spherical triangles.*

Napier's circular parts are, the complements of the two angles that are not right angles, the complement of the hypothenuse and the other two sides; that is, denoting the sides by a, b, c, and the angles by A, B, C, A being the right angle, the parts are $90°-a$, $90°-$B, $90°-$C, and b, c; of these, any one may be the middle part, and the two parts next adjacent, one to either hand (not including the right angle) the adjacent parts; and the other two the opposite parts; then the analogies are

rad × sin middle part = rectangle of the tangents of adjacent parts.

rad × sin middle part = rectangle of the cosines of the opposite parts.

And by making the transformations above explained, these will also apply to quadrantal spherical triangles.

Table II. *For the solution of oblique angled spherical triangles.*

Given.	Required.	Values of the quantities required.	
I. Two angles and a side opposite to one of them.	Side opposite to other angle.	By common analogy	Sines of angles, are as sines of opp. sides.
	Third side.	Let fall a perpendicular upon the side contained between the given angles.	Tan 1 seg. of this side = cos adj. angle × tan given side. Sin 2 seg. $= \frac{\text{sin 1 seg.} \times \text{tan ang. adj. given side.}}{\text{tan ang. opp. given side.}}$
	Third angle.	Let fall a per. as before	Cot 1 seg. of this ang. = cos giv. side × tan adj. ang. Sin 2. seg. $\times \frac{\text{sin 1 seg.} \times \text{cos ang. opp. given side.}}{\text{cos ang. adj. given side.}}$
II. Two sides and an angle opposite to one of them.	The angle opposite to the other side.	By the common analogy.	Sines of sides are as sines of their opposite angles.
	Angle included between the given sides.	Let fall a perpendicular from the included angle.	Cot 1 seg. ang. req. = tan giv. ang. × cos adj. side. Cos 2 seg. $= \frac{\text{cos 1 seg.} \times \text{tan giv. side adj. giv. ang.}}{\text{tan side opp. given angle.}}$
	Third side.	Let fall a perpendicular as before.	Tan 1 seg. side req. = cos giv. ang. × tan adj. side. Cos 2 seg. $= \frac{\text{cos 1 seg.} \times \text{cos side opp. giv. ang.}}{\text{cos side adj. given angle.}}$
III. Two sides and the included angle.	An angle opposite to one of the given sides.	Let fall a perpendicular from the third angle.	Tan 1 seg. of div. side = cos giv. ang. × tan side opp. ang. sought. Tan ang. sought $= \frac{\text{tan giv. ang.} \times \text{sin 1 seg.}}{\text{sin 2 seg. of div. side.}}$
	Third side.	Let fall a perpendicular on one of the given sides.	Tan 1 seg. of div. side = cos giv. ang. × tan other given side. Cos side sought $= \frac{\text{cos side not div.} \times \text{cos 2 seg.}}{\text{cos 1 seg. of side divided.}}$
IV. A side and the two adjacent angles.	A side opposite to one of the given angles.	Let fall a perpendicular on the third side.	Cot 1 seg. of div. ang. = cos giv. side × tan ang. opp. side sought. Tan side sought $= \frac{\text{tan giv. side} \times \text{cos 1 seg. div. ang.}}{\text{cos 2 seg. of divided angle.}}$
	Third angle.	Let fall a perpendicular from one of the given angles.	Cot 1 seg. div. ang. = cos given side × tan other giv. angle. Cos angle sought $= \frac{\text{cos ang. not div.} \times \text{sin 2 seg.}}{\text{sin 1 seg. div. angle.}}$

Table II.—*Continued.*

Given.	Required.	Value of the quantities required.
V. The three sides.	An angle by the sine or cosine of its half.	Let a, b, c, be the three sides; A, B, C, the angles; b and c, including the angles sought, and $s = a + b + c$. Then, $\sin \frac{1}{2} A = \sqrt{\frac{\sin. (\frac{1}{2} s - b) \sin (\frac{1}{2} s - c)}{\sin b \sin c.}}$ $\cos \frac{1}{2} A = \sqrt{\frac{\sin \frac{1}{2} s. \sin (\frac{1}{2} s - a)}{\sin b. \sin c.}}$
VI. The three angles.	A side by the sine or cosine of its half.	Let S be the sum of the angles A, B, C, and B and C be adjacent to a, the side required; then, $\sin \frac{1}{2} A = \sqrt{- \frac{\cos \frac{1}{2} S \cos (\frac{1}{2} S - A)}{\sin B. \sin C.}}$ $\cos \frac{1}{2} A = \sqrt{- \frac{\sin (\frac{1}{2} S - B) \sin (\frac{1}{2} S - A)}{\sin B. \sin C.}}$

Table III.

For the solution of all the cases of oblique angled spherical triangles, by the analogies of Napier.

Given.	Required.	Value of the terms required.
I. Two angles and the sides opposite to one of them.	Side opposite to the other given angle Third side. Third angle.	By the common analogy; viz. The sines of the angles are as the sines of the opposite sides tan of its half $\begin{cases} = \frac{\tan \frac{1}{2} \text{dif. giv. sides} \times \sin \frac{1}{2} \text{sum opp. ang.}}{\sin \frac{1}{2} \text{dif. of those angles.}} \\ = \frac{\tan \frac{1}{2} \text{sum giv. sides} \times \cos \frac{1}{2} \text{sum opp. ang.}}{\cos \frac{1}{2} \text{dif. those angles.}} \end{cases}$ By the common analogy.
II. Two sides and an opposite angle.	Angle opposite to the other known side. Third angle. Third side.	By the common analogy. cot of its half $\begin{cases} = \frac{\tan \frac{1}{2} \text{dif. other two ang.} \times \sin \frac{1}{2} \text{sum giv. sides}}{\sin \frac{1}{2} \text{dif. those sides.}} \\ = \frac{\tan \frac{1}{2} \text{sum other two ang.} \times \cos \frac{1}{2} \text{sum giv. sides}}{\cos \frac{1}{2} \text{dif. those sides.}} \end{cases}$ By common analogy.
III. Two sides and the included angle.	The two other angles. Third side.	$\begin{cases} \tan \frac{1}{2} \text{dif.} = \frac{\cot \frac{1}{2} \text{giv. angle} \times \sin \frac{1}{2} \text{dif. given sides}}{\sin \frac{1}{2} \text{sum of those sides.}} \\ \tan \frac{1}{2} \text{sum.} = \frac{\cot \frac{1}{2} \text{giv. angle} \times \cos \frac{1}{2} \text{dif. given sides}}{\cos \frac{1}{2} \text{sum of those sides.}} \end{cases}$ By the common analogy.
IV. Two angles and the included side.	The other two sides. Third angle.	$\begin{cases} \tan \frac{1}{2} \text{dif.} = \frac{\tan \frac{1}{2} \text{giv. side} \times \sin \frac{1}{2} \text{dif. giv. angle}}{\sin \frac{1}{2} \text{sum of those angles}} \\ \tan \frac{1}{2} \text{sum} = \frac{\tan \frac{1}{2} \text{giv. side} \times \cos \frac{1}{2} \text{dif. giv. angle}}{\cos \frac{1}{2} \text{sum of those angles.}} \end{cases}$ By common analogy.
V. The three sides.	Either of the angles.	Let fall a perpendicular on the side adjacent to the angle sought; then, $\left\{ \tan \frac{1}{2} \text{sum or } \frac{1}{2} \text{dif. of the segments of the base} \right\} = \frac{\tan \frac{1}{2} \text{sum} \times \tan \frac{1}{2} \text{dif. of the sides}}{\tan \frac{1}{2} \text{base.}}$ Cos angle sought = tan adj. seg. $\times$ cot adj. sides.
VI. The three angles.	Either of the sides.	These will be determined, by finding the corresponding angle, by the last case, of a triangle, which has all its parts supplemental to those of the triangles, whose three sides are given.

Astronomy.

4. *Problems relative to the determination of the position of the heavenly bodies.*

Rising and setting of the heavenly bodies.

63. It is obvious from what we have stated relative to the position of the different circles of the sphere, that one of the most important data in the solution of astronomical problems is the latitude of the place of the observer, from which the zenith Z in the preceding figures is determined. This may be found as follow.

PROBLEM I.

To find the latitude of any place on the earth's surface.

To determine the latitude of the place.

1. Observe the altitude of the pole above the horizon of any place, and that altitude will be equal to the latitude.

The latitude of any place on the earth is measured by the arc EZ, that is, by the arc subtended between the equator and zenith. But $EZ + ZP = 90^\circ$ and $ZP + PR = 90^\circ$; whence

$$EZ + ZP = ZP + PR$$

Consequently $EZ = PR$

That is, the latitude of any place is equal to the elevation of the pole above the horizon of that place.

By observations on the circumpolar stars.

The elevation of the pole above the horizon, may be practically determined by observing the greatest and least altitude of any of the circumpolar stars, and taking half the sum of the two altitudes; the proper corrections being made for refraction, parallax, &c. according to the principles explained in a subsequent chapter.

For let xy, fig. 21, represent the circle of declination described by any circumpolar star, then Rx will be its greatest meridian altitude, and Ry its least, and it is obvious that

$$RP = \tfrac{1}{2}(Ry + Rx)$$

By observations on the sun.

2. The latitude may also be found by observing the altitudes of the sun when he has attained his greatest north and greatest south declination. Half the sum will be the elevation of the equator above the horizon, and the complement of that angle the latitude of the place of the observer.

Referring to the same figure, let ea be the declination circle described by the sun when he has the greatest north declination, then eH will be his greatest altitude on that day; let st in like manner be the declination circle described on the day when he has the greatest south declination; then He, will be its meridian altitude on that day; and since $Ee = Es$, it is obvious that

$$HE = \tfrac{1}{2}(He + Hs)$$

And $90^\circ - HE = EZ$ the latitude.

64. It is obvious, also, from what is stated above, and referring to our definition of the *obliquity* of the ecliptic, that this angle is measured by half the arc se, that

$$\tfrac{1}{2}(He - Hs) = \text{the obliquity of the ecliptic}$$

PROBLEM II.

To find the time of the rising, setting, &c. of the heavenly bodies.

To find the time of rising, &c

65. Let the proposed body be the sun, and let us suppose that its declination remains constant during its passage from one meridian to the other, and that a clock is adjusted to go 24 hours during this one apparent revolution of the sun, and moreover, that the clock shows 12 exactly, when the sun is on the meridian; *to find the time of its rising, and its azimuth at that time; the latitude of the place and the declination of the sun being given.*

Plane Astronomy.

Fig. 24.

Referring to fig. 24, and comparing it with what has been stated with reference to fig. 21 (art. 59,) it appears that the sun rises when it comes to b; that it is twelve o'clock when the sun is upon the meridian at e, and that the whole circle aea is described in 24 hours; that is uniformly at the rate of $\frac{360}{24}^\circ = 15^\circ$ per hour; to find the time of rising, therefore, we have only to compute the angle ZPb, and to convert it into time at the rate of 15° to an hour in time; and to find its azimuth from the north we must compute the angle RPb, for which computations we have the following data:

$bZ = 90^\circ$, Ee = declination; $eP = Pb$ co-declination
EZ = latitude; ZP = co-latitude

Hence in the triangle ZPb, we have the three sides given and one of them $Zb = 90^\circ$, to find the angle ZPb. This case may therefore be solved by our fifth form in the preceding Table II. for oblique angled triangles, but one of the sides being 90°, it will be more readily solved by means of Napier's analogy; viz

$$\text{rad} : \cot bP :: \cot ZP : \cos ZPb = \text{hour angle}$$
$$\text{or rad} : \tan.\text{dec} :: \tan.\text{lat} : \cos ZPb = \text{hour angle}$$

EXAM. 1. Let us, for example, suppose the latitude of the place to be 52° 13′ north; the declination 23° 28′, to find the time of sun's rising.

By the above analogy,

rad........	10·0000000
tan 23° 28′..	9·6376106
tan 52° 13′..	10·1105786
Cos 124° 2′	9·7481892

(taking the supplement of the tabular angle 55° 58′, the angle being obviously greater than 90°.)

According to either solution, therefore, we have the hour angle = 124° 2′,

which converted into time gives

15° : 124° 2′ :: 1h. : 8h. : $19\frac{1}{2}'$ time from noon

consequently,

h.	m.	″.	
12	0	0	
8	$19\frac{1}{2}$	0	
3	$40\frac{1}{2}$	0	the time of rising

That is in the latitude 52° 13′, on the longest day, or when the sun has 23° 28′ north declination, he will rise at 3h. $40\frac{1}{2}$m.

To find the azimuth from the north, we have in the same triangle the same data to find the angle PZb, which is the measure of the azimuth sought. This may therefore be determined by means of our form 5, oblique spherical triangles; but more concisely by the following analogy:—

$$\sin ZP : \text{rad.} :: \cos bP : \sin PZb = \text{azimuth.}$$

Hence	ar. com. cos 52° 13′........	0·2127683
	rad.	10·0000000
	sin 23 28	9·6001181
	cos 49° 32′ azim...........	9·8128864

EXAM. 2. Let it be required to determine the time of sun rising at the same place, lat. 52° 13′ on the 25th of February, 1818.

The declination on this day by nautical almanack is 9° 11½′ south. Consequently, referring to the same fig. 24, b' = 99° 11½′. Hence, then, in the triangle ZPb', we have as before, ZP = co. lat. = 37° 47′, P b' codec. 99° 11½′, Z b' = 90° Whence, as in the former case,

rad.	10·0000000
tan dec. 9° 11½′........	9·2090197
tan 52° 13′	10·1105786
cos 77° 57′ hour angle..	9·3195983

This converted into time gives 5h. 12m. from noon; which, taken from 12 hours, gives 6h. 48m. for the time sought.

PROBLEM III.

To find the sun's altitude at 6 o'clock, azimuth, &c.

Altitude at 6 o'clock. Fig. 25.

66. Here, since PP′ (fig. 25) bisects *e a*, it is clear that the sun will be at *c*, at 6 o'clock; and we have therefore, in this case, the hour angle ZP*c* = 90° given, and the two sides ZP, P*c*, to find Z*c*, the co-altitude; that is, two sides and the contained angle are given to find the third side. Whence, by our fifth form for right-angled spherical triangles, we have

$$\cos Zc = \frac{\cos ZP \times \cos Pc}{\text{rad.}}$$

Or, which is the same, (adopting the data of the first of the preceding examples,)

rad.	10·0000000
sin lat. 52° 13′	9·8978103
sin dec. 23° 28′	9·6001181
sin. alt. 18° 21′	9·4979284

Time of easting.

67. To find at what time in the day the sun will be east and west; that is, in the prime vertical, and its altitude at that time; taking the latitude of the place 52° 12′ 35″, and the sun's declination 23° 28′.

Here, taking ZP to denote the co-latitude = 37° 47′ 25″ P *b* the co-declination; and the angle *b* ZP being 90°, we have the two sides ZP, P *b*, of the right-angled spherical triangle *b* ZP, and the angle *b* ZP, a right angle, to find the hour angle ZP *b*, and the co-altitude Z *b*.

By preceding Table 1, form 1,

$$\cos Zb = \frac{\cos Pb}{\cos ZP} = \frac{\sin \text{dec.}}{\sin \text{lat.}} = \sin \text{alt.}$$

$$\cos ZPb = \frac{\tan ZP}{\tan Pb} = \frac{\cos \text{lat.}}{\cos \text{dec.}} = \text{hour angle.}$$

sin dec. 23° 28′	= 9·6001181
sin lat. 52° 12′ 35″	= 9·8977695
sin alt. = 30° 15′ 31″	9·7023486

Again,

cos lat. 52° 12′ 35″ =	9.8995301
cos dec. 23 28	= 10·3623894
cos hour angle = 70° 19′ 44″....	9·5371407

Which, converted into time, gives 4h. 41′ 19″ from apparent noon.

Azimuth and hour.

68. Given the latitude of the place, the sun's declination; and altitude to find the hour and azimuth.

Here, referring to the same figure, we have the co-latitude ZP, the co-altitude Z *b*, and the co-declination P *b* to find the hour angle ZP *b*, and the azimuth PZ *b*. That is, in a spherical triangle, we have three sides to find the angles.

Whence, calling the sum of the three sides *s*, we have, by form 5, of our Table II.,

$$\tfrac{1}{2} \cos ZPb = \sqrt{\left\{\frac{\sin \tfrac{1}{2}\, s \ \sin\left(\tfrac{1}{2}\, s - Z\, b\right)}{\sin ZP \,.\, \sin P\, b}\right\}}$$

Let the latitude be 34° 55′, the declination 22° 22′ 57″ N, the altitude 36° 59′ 39″. Then ZP = 55° 5′, Z *b* = 53° 0′ 21″, P *b* = 67° 37′ 3″. Hence the operation,

P *b* =	67°	37′	3″	sin arith. comp.	0·0340191
ZP =	55	5	0	sin arith. comp.	0·0861939
Z *b* =	53	0	21		
sum	175	42	24		
½ *s*	87	51	12	sin =	9·9996942
Z *b*	53	0	21	sin =	9·7569320
½ *s*—Z*b*=	34	50	51		2)19·8768392

½ cos ZP *b* = 29° 47′ 44″ = 9·9384196

Hence, ZP *b* = 59° 35′ 28″, which, reduced to time, gives 3h. 58′ 22″ the time from apparent noon. In the same manner

$$\tfrac{1}{2} \cos PZ\, b = \sqrt{\left\{\frac{\sin \tfrac{1}{2}\, s \,.\, \sin\left(\tfrac{1}{2}\, s - P\, b\right)}{\sin Z\, b \,.\, \sin ZP}\right\}}$$

The numerical solution of which, we leave as an exercise for the reader.

69. By means of the azimuth determined as above, it will not be difficult to draw a meridian line, by simply determining its position, so as to make the requisite angle with the direction of the sun at that time. This, however, of course supposes that the altitude of the sun has been properly corrected for refraction and parallax, which being subjects we have not yet touched upon, we merely indicate the principle of the determination, and shall leave our further discussion on it to a subsequent chapter. Meridian line.

70. In the preceding examples relative to those questions in which the altitude is supposed to be determined from observation, it is obvious that we not only suppose the requisite corrections to have been applied, but also that the observation is made without any appreciable error; but as such error is easily made, particularly if the instrument be not of the most perfect construction, it may not be amiss to ascertain what effect any such error will produce in the computed time, and the azimuth the sun ought to have, so that an error in altitude shall produce the least error in time; we propose therefore the following problem. Error in altitude.

PROBLEM IV.

Given the error in altitude to find the error in time.

71. Let EQ, fig. 26, represent the equator, P the pole, *a e* the parallel of declination, in which the sun, or Error in time. Fig. 26.

Astronomy.

any other heavenly body is on the day of observation; and let r be real, and s the apparent place of the body, or $r\,s$ the error in altitude. Draw $m\,s$ parallel to the horizon; from Z, the zenith, draw Z m, Z r; and from P the pole, the meridians P $m\,p$, P $r\,q$, to pass through $m\,r$, and to cut the equator in the points $p\,q$, then m and r will be the real and apparent places of the body, on the parallel of declination; and the arc $m\,r$ on that circle, or the arc $p\,q$ on the equator, will measure the angle m P r, the corresponding error in time. Now the triangle $m\,s\,r$ being of course exceedingly small, it may be regarded as a rectilinear right-angled triangle, right angled at s;
and hence we have

$$s\,r : r\,m :: \sin s\,m\,r : \text{rad.}$$

by spherics $m\,r : p\,q :: \cos q\,r : \text{rad.}$

Multiplying these together, rejecting the like factors, we have

$$s\,r : p\,q :: \sin s\,m\,r \cos q\,r : \text{rad.}^2$$

Whence

$$p\,q = s\,r \times \frac{\text{rad.}^2}{\sin s\,m\,r \cdot \cos q\,r}$$

But Z r P $= s\,m\,r$, $s\,r\,m$ being the complement of each, and $\sin \text{Z}\,r\text{P} = (\sin s\,m\,r) : \sin \text{PZ} :: \sin r\text{ZP} : \sin \text{P}r$

Consequently,

$$\sin s\,m\,r \,.\, \sin \text{P}\,r = \sin s\,m\,r \,.\, \cos q\,r = \sin \text{PZ} \,.\, \sin r\text{ZP}$$

Whence, substituting for the denominator in the above expression, its equivalent in the last, we have,

$$p\,q = r\,s \times \frac{\text{rad.}^2}{\sin \text{PZ} \cdot \sin r\,\text{ZP}} = \frac{r\,s}{\cos \text{ lat. } \sin r\,\text{ZP}}$$

taking radius $= 1$.

Hence, since all the quantities in this expression are supposed to be given, except $\sin r\,\text{ZP}$, it is obvious, that $p\,q$ will *cæteris paribus* be the least, when $\sin r\,\text{ZP}$ is the greatest, or when the azimuth is the greatest; that is, when the body, whose altitude is observed, is in the prime vertical: it is best, therefore, to deduce the time from an observation, when the body is in or near that circle.

As an example in numbers, suppose the latitude to be 52° 12′ 35″; the declination 15° 24′ 25″; and the altitude as corrected, 40°; required the error in time, supposing an error of 1″ in the altitude.
This gives

$$1' \times \frac{1}{\cos \text{ lat. } \sin \text{ azim.}} = 1'.882$$

which answers to 7.528 seconds in time.

Again, supposing the azimuth to be 46° 22′, the latitude 52° 12′, and the error in altitude 1′, we have

$$1' \times \frac{1}{.612 \times .690} = 2'.334 \text{ of a degree} = 9''.336 \text{ in}$$

time.

Time of the sun ascending through an arc equal to its diameter.

72. By means of this solution, we may ascertain the time in which the sun will pass either the horizontal or vertical wire of a telescope: we have seen, for instance, that the time during which the sun will ascend through any small altitude $= r\,s$, or the arc that he will describe when referred to the equator during that time, is

$$p\,q = \frac{r\,s}{\cos \text{ lat. } \sin r\,\text{ZP}}$$

Plane Astronomy.

in which, substituting d'' the apparent diameter of the sun in seconds for $r\,s$, this becomes

$$\frac{d''}{\cos \text{ lat. } \sin \text{ . azimuth}}$$

consequently,

$$\frac{d''}{15 \text{ . } \cos \text{ lat. } \sin \text{ . azim.}} = \text{time in seconds.}$$

Time of its passing a vertical wire.

73. To find the time in which the sun will pass over the vertical wire, we may take $m\,r$ to denote the apparent diameter of the sun; then, as the seconds in $m\,r$, considered as a small circle, must be increased in proportion as the radius is diminished; (because, when the arc is given, the angle is inversely as the radius,) we have

$\sin \text{P}\,r$, or cos . dec. : rad :: d'' the seconds in $m\,r$. considered as a great circle, to the seconds in the same considered as a small circle, which are the seconds in $p\,q$; whence

$$p\,q = \frac{\text{rad. } d''}{\cos \text{ dec.}} = d''\text{. sec. dec.}$$

and, consequently,

$$\text{the time} = \frac{d'' \times \text{sec. dec.}}{15''}$$

Hence $p\,q = d''$ sec. dec. expresses the sun's diameter in right ascension. If therefore we assume his apparent diameter at 32′ = 1920″, and its declination 20°, its right ascension is

$$1920'' \times \text{sec. } 20° = 34'\ 2''\ .\ 88$$

which, divided by 15, will give the same in time.

In the Nautical Almanack a column is given for indicating the time in which the semi-diameter of the sun passes over a vertical wire, by means of which a single observation on either limb, may be referred to the centre.

Problem V.

To find the time when the apparent diurnal motion of a fixed star is perpendicular to the horizon.

74. Let $x\,y$ (fig. 26) be the parallel of declination described by the star; draw the vertical Z h, touching it at o; then, when the star arrives at o, its apparent motion will be perpendicular to the horizon; the two circles having in the point o, a common tangent. And Z o P is a right angle, we have

rad : tan o P :: cos PZ : cos ZP o

that is, rad : cos dec. :: tan lat. : cos ZP o

which, converted into time, gives the time from the star's being on the meridian; the latter therefore being supposed known, the former is readily determined.

Problem VI.

Given the right ascension and declination of a heavenly body, and the obliquity of the ecliptic, to find its latitude and longitude.

To determine the latitude and longitude. Fig. 27.

75. Let s (fig. 27) be the body, VC the ecliptic, VQ the equator, the angle QVC denoting the given angle of the obliquity of the ecliptic; let s V be joined by the arc of a great circle s V, and let fall the perpendiculars $s\,p$, $s\,r$; then it is clear, from our definitions, that V p will be the right ascension of the body s, $s\,p$

Astronomy.

its declination, Vr its longitude, and sr its latitude: we have therefore given the Vp, sp, and the angle pVr to find Vr, and rs.

Now, first in the right-angled spherical triangle Vps, we have, by form 5, Table I.

$$\tan sVp = \frac{\tan sp}{\sin Vp}$$

whence the angle sVp becomes known, and consequently also sVr, because

$$sVr = sVp + pVr$$

Again,

$$\cos Vs = \cos Vp \,.\, \cos ps$$

whence Vs also becomes known.

Therefore, in the right-angled triangle rVs, we have the hypothenuse Vs, and the angle rVs to find the two sides Vr, rs:

which, by our form 4, Table I. are determined as follows:

$$\tan Vr = \tan Vs \times \cos sVr = \tan \text{long.}$$

$$\sin sr = \sin Vs \times \sin sVr = \sin \text{lat.}$$

In like manner, the right ascension and declination of any body may be found, when its latitude and longitude are given; but generally the problem is to determine the latter from the former, these being first ascertained from actual observation: it is thus the tables of the latitudes and longitudes have been computed.

But as both the ecliptic and the equator are subject to a change in their positions, the right ascension, declination, latitude, and longitude of all the fixed stars are constantly varying, and therefore those tables formed for any particular epoch, will not answer correctly after a certain time; the annual variations, however, being computed, their right ascensions, &c. may be determined for any proposed time, as will be hereafter explained.

5. *Of the crepusculum, or twilight.*

Twilight.

76. The crepusculum, or twilight, is that faint light which is perceived before the rising of the sun, and after its setting. It is occasioned by the terrestrial atmosphere, refracting the rays of the sun, and reflecting them amongst its particles.

The depression of the sun below the horizon, at the beginning of the morning and at the end of the evening twilight, has been variously stated at different seasons, and by different authors; for example,

Alhazen observed it to be		19°
Tycho		17°
Rothman		24°
Stevinus		18°
Cassini		15°
Riccioli, at the equinox		16°
	summer solstice	21°
	winter solstice	17¼°

but we more commonly now in our latitudes assume it 18°; the same both for morning and evening and for all seasons of the year.

Problem I.

Given the latitude of the place and the sun's declination, to find the time when twilight begins.

Time when it commences. Fig. 28.

77. Here S denoting the place of the sun when twilight begins, we have in the triangle ZPS, (fig. 28) $ZS = 90° + 18° = 108°$, the co-latitude ZP, and the co-declination PS, to find the hour angle ZPS from apparent noon.

Plane Astronomy.

Having thus the three sides, the angle required may be found by our 5th form, Table II.; that is, assuming $ZP + ZS + PS = s$

$$\tfrac{1}{2} \cos ZPS = \sqrt{\frac{\sin \frac{1}{2} s \,.\, \sin(\frac{1}{2} s - ZS)}{\sin ZP \sin PS}}$$

Whence the angle ZPS may be determined, and consequently the time from apparent noon.

Problem II.

Given the latitude of the place to find the time and duration of the shortest twilight.

Time of shortest twilight.

78. Let P fig. 28, denote the pole, Z the zenith of the observer, S the sun, 18° below the horizon $ZS = 90° + 18° = 108°$, or more generally, $= 90° + 2a$, where we suppose the twilight to begin when the sun is $2a$ degrees below the horizon; or that the observer whose zenith is Z, will then see the commencement of the morning twilight.

Now in consequence of the diurnal rotation, the declination circle PS, turning about the axis will bring the sun from S to S′, in the horizon, or which is the same, the zenith Z will approach nearer to S, by describing about P the little circle $ZmbQ$, such that the distances Zm, Pm, Pb, PQ are all equal.

When, therefore, the zenith shall have descended from Z to any point m, such that $mS = 90°$, the sun will appear at 90° from the zenith, the day will commence and the twilight terminate. And the arc Zm of the small circle will be the measure of the angle ZPm, and consequently, of the duration of the twilight.

In order to determine this angle, draw the arc ZBm of a great circle, and to its middle B the perpendicular arc PB, then will

$$\sin \tfrac{1}{2} ZPm = \sin ZPB = \frac{\sin \frac{1}{2} Zm}{\sin PZ} = \frac{\sin \frac{1}{2} Zm}{\cos \text{lat.}}$$

now in the spherical triangle ZmS, we have

$$Sm + mZ > ZS, \text{ or}$$
$$90° + mZ > 90° + 2a$$

consequently, $mZ > 2a$ and $\frac{1}{2} mZ > a$

Let $\quad \frac{1}{2} mZ = a + x$

then,

$$\sin \tfrac{1}{2} ZPm = \frac{\sin(a + x)}{\cos \text{lat.}} = \frac{\sin a \cos x + \sin x \cos a}{\cos \text{lat.}}$$

$$= \frac{\sin a + \cos a \sin x - 2 \sin a \sin^2 \frac{1}{2} x}{\cos \text{lat.}}$$

because, $1 - \cos x = 2 \sin^2 \frac{1}{2} x$ (See Trigonometry)

Now the last expression is equivalent to

$$\sin \tfrac{1}{2} ZPm = \frac{\sin a}{\cos \text{lat.}} + \frac{2 \sin \frac{1}{2} x \,(\cos a + \frac{1}{2} x)}{\cos \text{lat.}}$$

And here $\frac{1}{2} x$ is essentially positive, and a and $\frac{1}{2} x$ small angles, such that $a + \frac{1}{2} x < 90°$. Whence the latter part of the expression will be positive, and we shall have

$$\sin \tfrac{1}{2} ZPm > \frac{\sin a}{\cos \text{lat.}}$$

It is further evident that the twilight will be longer as x is greater, and shorter as x is less, and that it

Astronomy.

will be the shortest possible when x is zero, because in that case, the second member of the second side of the equation will vanish.

But this is what occurs when the triangle ZmS is reduced to the arc ZS, that is, when the point m falls on b, or when the distance PS is such, that the part Zb, of the vertical ZS, lying within the small circle ZmQ is equal to $2a$, and the exterior part $bS = 90°$. It is also manifest, that if PS increases, the opposite angle PZS will in like manner increase, and that on the contrary that angle will diminish as PS diminishes. In these variations, the point m will approach to or recede from Z, the intercepted part bZ will vary between the limits 0 and ZQ=2 PZ. Thus the intercepted part may have all values from 0 to 2 (90° — lat.) = 180° — 2 lat. and consequently may have the value $2\,a$: hence in the case where $Zb = ra$, and $bS = 90°$, the shortest twilight will obtain; and the semi duration in degrees will be found by means of the equation

$$\sin ZPB = \frac{\sin a}{\cos \text{lat.}} = \sin a \sec \text{lat.}$$

Whence the duration in time is readily determined.

79. Other theorems are deducible from the same construction and investigations; for on the arc $Zb = 2a$, and with the complement of ZP of the latitude constitute the isosceles triangle ZPb, (fig. 29) and let fall the perpendicular Pm; then ZPb will be the angle which measures the duration. Prolong Zb, till $bS = 90°$, and draw the arc PS which will be the sun's co-declination for the day of the shortest twilight.

Fig. 29.

Sun's azimuth.

Now,

$$\cos mP = \frac{\cos PZ}{\cos Zm} = \frac{\cos PS}{\cos mS}$$

Wherefore,

$$\cos Zm : \cos mS :: \cos PZ : \cos PS$$

or, $\cos a : \cos (90° + a) : \sin \text{lat.} : \sin \text{dec.}$

or, $\cos a : - \sin a :: \sin \text{lat.} : \sin \text{dec.}$

whence, $\sin \text{dec.} = \frac{- \sin a}{\cos a} \sin \text{lat.} = - \tan a . \sin \text{lat.}$

Again, in the right angled triangle ZPm, we have

$$\tan Zm = \cos Z \tan PZ$$

and $\cos Z = \tan Zm \cot PZ = \tan a . \tan \text{lat.}$

Now the angle Z or PZm is the sun's azimuth at the commencement of the twilight, and

$$PZb = PbZ = 180° - PbS$$

But PbS is the sun's azimuth at the instant, when his centre is upon the rational horizon; wherefore *the sun's azimuth at the beginning and ending of the twilight are supplements to each other on the day of the shortest twilight,*

Hence, since $\cos PZS = \tan a . \tan \text{lat.}$

we have $\cos PbS = - \tan a . \tan \text{lat.}$

Hour angle.

80. In like manner ZbS and bPS are the hour angles, at the beginning and end of the twilight; let the former be denoted by P′ and the latter by P; then

$$ZPS - bPS = P' - P = ZPb$$

which is the angle that measures the duration of twilight. Hence we have from what has been done above

$$\sin \tfrac{1}{2} (P' - P) = \frac{\sin a}{\sin \text{lat.}}$$

Also,

$mPS = bPS + mPb = P + \frac{1}{2} (P' - P) = \frac{1}{2} (P' + P)$

and

$$\sin mPS = \frac{\sin Sm}{\sin PS} = \frac{\sin (90° + a)}{\cos \text{dec.}} = \frac{\cos a}{\cos \text{dec.}}$$

Plane Astronomy.

that is, $\sin \frac{1}{2} (P' + P) = \frac{\cos a}{\cos \text{dec.}}$

Whence from the two equations,

$$\sin \tfrac{1}{2} (P' - P) = \frac{\cos a}{\cos \text{dec.}}$$

$$\sin \tfrac{1}{2} (P' + P) = \frac{\sin a}{\cos \text{lat.}}$$

the hour angles P′ and P are easily determined, the declination itself being given by the equation

$$\sin \text{dec} = - \tan a . \sin \text{lat.}$$

We have also,

$$\cos PSZ = \frac{\sin \text{lat.}}{\cos \text{dec.}}$$

Lastly, let $ST = 2a = Zb$; and draw PT, then ZT = 90°, and T is a point in the horizon for the moment when the zenith was in Z. Whence the triangle PZT gives

$$\cos PT = \cos Z \sin PZ \sin ZT + \cos PZ \cos ZT$$
$$= \cos Z \sin PZ = \tan a . \tan \text{lat.} \cos \text{lat.}$$
$$= \tan a . \sin \text{lat.} = \sin \text{dec.}$$

Therefore, 90° — PT = dec., or PT = 90° — dec.

But PS = 90° + dec.

therefore, PT + PS = 180°

which is another remarkable property of the shortest twilight.

81. In all the above deductions, the latitude has been left general, or indeterminate, as has also the quantity $2\,a$, which denotes the number of degrees that the sun is below the horizon when the twilight begins. The latitude may therefore be introduced for any given place, as may also $2\,a$; but commonly, we assume $2a = 18°$, that is, the twilight is supposed to commence when the sun is 18° below the horizon.

Example.

Example. Required the day on which the twilight is the shortest at Woolwich, the latitude of which is 51° 28′ 40″, in the year 1820, with its duration, and the time of its beginning and end.

First, for the declination we have

$$\sin \text{dec.} = - \tan a \sin \text{lat.}$$

Now	log tan a = 9° =	9·1997125
	log sin lat. or 51° 28′⅔ =	9·8934104
	log sin dec. = — 7° 7′ 5″	9·0931229

The declination, therefore, is 7° 7′ 5″ south, which answers to March 2 and October 11.

Again, for the duration, we have

$$\sin \tfrac{1}{2} (P' - P) = \frac{\sin a}{\cos \text{lat.}}$$

from log sin 9° =	9·1943324
take log cos 51° 28′⅔ =	9·7943613
log sin 14° 32′ 41″	9·3999711

Whence P′ — P = 29° 5′ 22″

Also $\sin \frac{1}{2} (P' + P) = \frac{\cos a}{\cos \text{dec.}}$

Hence from cos 9° =	9.9946199
take cos 7° 7′ 5″ =	9.9966399
sin 95° 31′ 19″	9.9979800

Astronomy.

whence $P' + P = 191° 2' 38''$
$P' - P = 29\ 5\ 22$

2)220 8

half sum = ... 110 — 4 = angle P′
half dif. = ... 80 58 — 38 = angle P

	h	m	s
The former in time answers to	7	20	$16\frac{1}{4}$
and the latter to	5	23	$54\frac{3}{4}$

The former showing the time when the evening twilight ends, and the latter the time of the sun's setting, or the time of its beginning; consequently their difference, 1h. 56m. $21\frac{1}{2}$s., is the duration.

The above numbers, taken respectively from 12, will leave the time of the beginning and end of the morning twilight.

Distinction between mean and apparent time.

82. In all the preceding investigations, we have considered only apparent time; that is, we have supposed it to be 12 o'clock when the sun is on the meridian, and that it is exactly 24 hours in passing from one meridian to the same again; but if a clock be adjusted to go thus for one day, that is, if it show exactly 24 hours between the time of the sun being twice successively in the same place, it will not continue to show 12 o'clock every day when the sun comes to the meridian, because the intervals of time from the sun's leaving a meridian to his return to it again, are not always equal. This difference between the sun and a well regulated clock is called the *equation of time*, which will be treated of in a subsequent chapter; at present, we shall not enter farther upon the subject, it is sufficient to apprize the reader that the time as determined in all the preceding problems, is what is called *apparent time*, or the time shown by the sun, and not *mean or true time*, which is that shown by a well regulated clock.

Corrections for parallax, &c.

83. Beside this correction for the time, there are also other corrections which must be introduced into the data of the preceding examples, in order to render the results perfectly conformable with observation. Thus we have all along supposed the body to rise as soon as it is found in the rational horizon; but all bodies in the heavens, when in or near the horizon, are elevated 33′ by refraction above their true places; this, therefore, would make them appear when they are actually 33′ below the horizon, or when they are 90° 33′ from the zenith; and all the way from the horizon to the zenith refraction has the effect of elevating the apparent places of the heavenly bodies, but in a less degree as the altitude is greater; till it vanishes in the zenith. The altitudes, therefore, as we have given them, are supposed to have been subjected to these corrections, the method of making which will be explained hereafter. This is one of the principal corrections for the fixed stars; but for the sun or any other of the bodies of our system, a different correction becomes necessary, these being all depressed below their true places by the effect of parallax, as will also be explained in a subsequent chapter; that is, we have confounded the sensible and rational horizon, which is admissible as far as relates to the fixed stars, in consequence of their immense distance; but the angle subtended by them or by the earth's radius, at any of the bodies of our system, is a sensible quantity that must not be neglected in any computations relative to such bodies. The parallax has therefore a tendency to increase the apparent zenith distance of any body in our system, while the refraction tends to diminish it; therefore the actual zenith distance of a body when it first becomes visible to a spectator on the earth, is equal to 90° — hor. parallax + refraction.

Plane Astronomy.

84. What has been hitherto done, has been merely to indicate the nature of the calculations after certain observations have been made and corrected; we now propose to explain the instruments used for making these observations, the method of employing them for these purposes, and the nature and quantity of the corrections that are requisite for reducing the results thus obtained, so as to render them proper for the purposes of astronomical computation.

§ IV. *Description and use of the most indispensable astronomical instruments.*

Astronomical instruments.

85. We shall not attempt in this place a general description of astronomical instruments, this will be given under a distinct head in another part of this work; our purpose here is only to describe those whose use is indispensable in the pursuit of this science, and with the principle of whose construction and operation, it is essential that the student be well informed, if he wish to proceed upon any other data than those furnished by the report of other observers.

1. *Of the astronomical telescope.*

Astronomical telescope.

86. We have already in our treatise on Optics described and illustrated the principles of this instrument, we have therefore in this place merely to speak of its application to astronomical observations.

Fig. 30.

Let, then, AB (fig. 30) represent the diameter of any heavenly body, as the sun, moon, or planet; *m* its middle point, will send out a luminous pencil of rays, which by means of the refraction of the lenses will be united in the focus of the telescope. The same will be the case with all the points of the disc, and an image of the object GH will be formed, as we have already explained in our treatise on Optics; H being the image of the point B, and G of the point A; that is, the image will be reversed with respect to its actual position. The motion of the object is also reversed, so that if it moves from left to right in the heavens, it will move from right to left in the telescope; but the effect being the same for all bodies, there will result from it no practical inconvenience; it is sufficient that we are apprized of it.

Let us farther observe, that the angle $HeG = ACB$, and if the focal distance eF is to the breadth HG in such a ratio that

$$\frac{\frac{1}{2}HG}{eF} = \tan \tfrac{1}{2} ACB$$

the object will be shown entire in the telescope; but if the focal distance be $ef > CF$, the image hg, will be larger than the field of the telescope, and consequently, we shall not then see it entire. On the contrary, if the focal distance be CQ, less than CF, the image $h'Qg'$ will not fill the telescope.

Generally, the field of the telescope is found by the

Astronomy.

formula

$$\tan \tfrac{1}{2}\,\text{field} = \frac{\tfrac{1}{2}\text{ diam. of the tube}}{\text{focal length}}$$

$$\tan \tfrac{1}{2}\,\text{field} = \frac{\tfrac{1}{2}\text{ diam. of the tube}}{\text{radius of curvature of the object glass}}$$

Or in case of a diaphragm being placed in the tube, as is commonly practised to prevent the reflection of the oblique rays, then

$$\tan \tfrac{1}{2}\ \text{field} = \frac{\tfrac{1}{2}\text{ diam. diaphragm}}{\text{focal length}}$$

These results, which are immediately deducible from what has been done in the preceding treatise, are stated here merely for the sake of avoiding frequent references to those articles; with the same view we may state the following relative to the magnifying power of such an instrument as we are here speaking of, viz.

$$\text{magnifying power} = \frac{\text{focal distance obj. glass}}{\text{focal distance eye glass}}$$

By which is to be understood, that the angle under which we view the image is equal to that under which we should see the object if it were brought so many times nearer as is indicated by the above fraction.

Having said thus much with regard to the telescope more commonly employed in astronomical observations, let us offer a few remarks relative to the apparatus attached to it, for rendering those observations more precise than they could be obtained with this instrument in the simple state in which it has been described above.

Use of the cross wires Fig. 31.

87. *Of the reticule or cross wires.*—Let ABDF (fig.31) represent a section of the telescopic tube or of the diaphragm with which it is furnished. On the edges of this ring or circle, are attached with two screws a metallic thread or fine wire DF; this is in a transit instrument called the horizontal wire. Perpendicular to this thread are placed five others; the centre one AB bisecting DF in C, and the other four at equal distances, two on each side of AB.

A star or any other heavenly body passes the field of view of a telescope in different times according to the diameter of the instrument, and the polar distance of the body; therefore, if the telescope were unprovided with some such an apparatus as here described, it would be difficult or even impossible to say precisely the moment when it was in the axis or in the centre of the instrument, and the accuracy of modern science requires this determination to the utmost accuracy: let us see what precision is attainable by means of the cross wires above indicated.

The point C or G being supposed either in the centre of DF, or in the vertical line bisecting DF, at the moment a star passes it, if the diameter of the thread be equal to that of the star, it will be entirely hidden by it, and that moment will be the time of its passage; but more commonly the thread is sufficiently fine, that at the instant of passage, it will bisect the star, an equal portion of the latter being observable on each side of the former; and thus the time of passage might be found to within one-fourth or one-fifth of a second. But it is obvious that we may observe the same with respect to all the other four vertical wires, which being at equal distances, the times of passing each in succession from the first will form an arithmetical progression; and by taking the mean of the five, we shall have the time of the star passing the centre wire still more exactly, and by this means, we may generally depend upon our observation to within one-tenth of a second.

Plane Astronomy.

Our figure and the description of it, applies to the case of an instrument fixed in the plane of the meridian, in which case the motion of a heavenly body will be apparently horizontal. In any other case, the star ascends or descends obliquely, and then it is necessary to give to the wire DF, a similar inclination, so that the motion of the star may be parallel to it, as in the line ES, as shown in fig. 32, the proper apparatus being supplied for this purpose. The only difficulty of observation is when the night is very dark, and when we are unable to see the threads except for the moment when the star is bisected by them; which being almost instantaneous, we are not sufficiently prepared for noting the time. In order to obviate this difficulty, the interior of the tube is illuminated by the following apparatus. Fig. 32.

88. In the side of the tube of the telescope, and commonly in the axis on which it turns, is made a small hole, directly opposite to which is placed, in the tube, a small mirror, inclined to the axis or sides of the telescope, at an angle of 45°. The light of a small lamp falls on the mirror, and forming with it an angle of 45°, it is reflected at the same angle, and therefore passes in a line parallel to the axis of the instrument, and thus renders the wires sufficiently visible. If the star on which the observation is made be of the 9th or 10th magnitude, we must however be careful to modify the intensity of the illumination, as otherwise, the artificial light will render the natural light of the star imperceptible, and it will be in danger of passing unobserved. Method of enlightening the tube.

89. At present we have spoken only of the transit of a fixed star, if it be the sun or moon that we are observing, we must ascertain the time when their respective centres pass the axis of the instrument. For this purpose we note very accurately, the instant when the eastern or western limb comes in contact with each of the five wires in succession, and the sum of the times divided by 5, will be the instant when that limb passed the centre wire. These five observations being made, the other limb of the sun or moon will be just about leaving the first wire, we therefore, in like manner, note these other five instants, the mean of which will give the time when the last limb passed the centre wire, and the mean of the two will be the time of the transit of the centre. Observations on the sun or moon.

90. Having said thus much with reference to the telescope and the apparatus with which it is supplied, it remains for us to describe the instruments to which it is attached, and the nature of their adjustment; for the accuracy of observations indicated above with reference to the object traversing the axis of the telescope would be to little purpose, unless we could be equally precise in the determination of the direction of that axis, both as regards the horizontal and vertical position of the instrument.

By far the greater number of astronomical observations are made in the plane of the meridian; but some are made at different azimuths, and different instruments are best employed for these purposes; at the same time some are so constructed as to answer in both cases; at present we shall confine our explanation to only one of the most perfect of each sort, but in a future part we shall enter upon the subject more at length, and endeavour to illustrate the advantages and defects of different constructions.

Of astronomical quadrants.

91. These may be either portable or fixed; in the former case they are commonly mounted on a tripod, and may be used for taking altitudes in any azimuth, or be made to follow the body observed in its apparent path; but in the latter case, the instrument is fixed in the plane of the meridian against a substantial wall, and is hence denominated a *mural quadrant.*

2. *Portable astronomical quadrant.*

Portable astronomical quadrant.

92. The great variety of forms under which this instrument has appeared, the numerous methods proposed by different artists for adjusting it to its verticality, &c. would be sufficient of themselves, were we to enter at length upon the subject, to form a considerable volume; we shall, therefore, select one of those esteemed the most perfect, and limit ourselves to the description of that only; which will be amply sufficient for our present purpose.

Fig. 33.

Fig. 33 represents a portable quadrant, constructed by Ramsden for the observatory of Christ's College, Cambridge. The tripod on which it is mounted has screws of adjustment to set the stem, on which the horizontal motion is performed, perpendicular, which is proved to be so in all directions when the plumb-line bisects both the superior and inferior dots during the whole revolution in a horizontal circle. The visible stem is a brass tube, and through it ascends a solid steel vertical axis, which fitting closely at the upper and lower extremity, has not the least shake, and preserves the position once given to it, so long as the feet screws are unmolested. The telescope is of the achromatic construction, and has the usual apparatus for a slow motion. The telescope lies on a bar that carries the counterpoise, and in which is the centre of its motion. It has a system of wires in the focus of the eye glass, which are adjustable by screws both upwards and sideways, as well as in a circular direction, so that the adjustments for collimation, and for zero in the altitude of the circle, may be thereby effected. The point of suspension of the plumb-line is also adjustable by a proper screw apparatus. At the top of the vertical tube or stem, is a small horizontal circle with a clamping apparatus for slow horizontal motion, by means of which the whole quadrant with its attached telescope turns gradually round in azimuth. When the observation is made in or near the zenith, the plumb-line of this instrument falls in the way of the telescope, and is obliged to be removed. This, however, is supplied by the addition of a spirit level, suspended from an adjustable horizontal brass rod, under the uppermost radical bar of the quadrant, which level not only supplies the place of the plumb-line when taken off, but at all times serves as a check on its adjustment, and when furnished with a graduated scale, may very well be made its substitute. We shall not here attempt to describe the nature and division of the vernier scale, it will be sufficient to observe, that the angles may be ascertained to the 10ths of seconds. We proceed next to the adjustments of the instrument.

To adjust the axis of the pedestal vertical.

Adjustment of the instrument to verticality.

93. This adjustment may be performed either by the plumb-line or by the level. When the plumb-line is used, turn the quadrant in azimuth till its plane, or which is the same thing, till the telescope lies parallel to a line joining any two of its three feet, and turn one of the two screws of the feet till the wire bisects the lower dot, and with the proper screw bring the upper dot to the same wire; then reverse the telescope by turning 180° in azimuth, and if both dots are again bisected, the axis is vertical in the direction that the telescope has pointed; in the next place, turn the telescope the space of a quadrant till it points in the same direction as the third foot of the tripod, and make the wire bisect the lower dot by the screw of this foot, and it will be found to bisect the upper dot also, if the first adjustment was properly made, but if not, repeat the operation till both dots are bisected in all the reversed situations of the telescope, and then the axis will be vertical in every direction.

In making this adjustment by the level alone, the process must be thus; first, the level must be made parallel to the rod which it hangs on, and secondly, this rod must be put perfectly horizontal, and the level will be horizontal also, with the bubble in the middle. In order to make the level parallel to the rod, place it parallel to a line joining two of the feet screws, and bring the bubble to the middle by one of the feet screws in question; then take off and reverse the position of the level, and if the bubble is found in the middle now, the parallelism is perfect; if not, one half of the error must be rectified by the same foot screw, and the other half by the adjusting screws at the end of the rod, by releasing one and screwing up the other. A repetition or two of this process will make the bubble stand in the middle in both of the reversed situations. In the next place, with the level thus parallel to the rod of suspension, turn the quadrant round its axis an entire semicircle as nearly as can be estimated, and if the bubble will now rest in the middle, the rod is level, and being at right angles with the axis of the quadrant's motion, proves that this axis is vertical in every direction; but if the bubble be found to run to one end of the tube, bring it one half way back by the adjusting screws of the rod, releasing one and fixing the other, as the case may require, and the other half by the proper foot screw. A repetition of this process will soon settle the bubble in the middle during a whole revolution in azimuth, and then the adjustment of the axis is perfect, as well as of the rod and level.

Adjustment for collimation.

94. The second adjustment is that by which the line of collimation of the telescope, is made parallel to the horizontal line that passes to the centre of the quadrant to zero on the limb, or quadrantal arc, at the same time that zero on the vernier coincides with zero on the limb. This important adjustment may be made in several ways, some of which are tedious and otherwise objectionable; but we shall confine ourselves to two, which apply one to the vertical, and the other to the horizontal line of the quadrant, which two methods, when duly effected, will not only check each other, but detect the error of

the total arc, if there be any at the same time; which is an acquisition of the utmost importance. First then to adjust by the vertical line, let the axis of the quadrant be made truly perpendicular in all directions by the adjustment we have already described, and fix on a star within a few degrees of the zenith when exactly on the meridian, and measure its altitude by the cross wire in the field of view in the usual way, and note down the result; if these readings prove to be at equal distances from the point 90°, one on the quadrant arc, and the other on the arc of excess beyond 90°, the horizontal wire is truly placed in the eye piece, but if not, half of the difference of the readings must be corrected by the proper screw for raising or depressing the said wire. This may be done by directing the telescope to a distant mark till the cross wire bisects it, then by moving the screw of slow motion of the vernier the half quantity required, and by bringing back the cross wire thus displaced to its original mark again. This operation repeated will place the cross wire in such situation, that zero on the vernier will be in its proper place with respect to the point 90°; or the half difference thus ascertained may remain, without altering the cross wire, as an error of adjustment to be constantly applied with the sign + or —, as the case may be, in all subsequent observations. Again, to adjust by the horizontal line passing through the zero of the quadrantal arc, it will be necessary to have a second telescope turning on pivots in adjustable *y*'s attached to the back of the quadrant, on the same level with the said horizontal line of the quadrant. This telescope may be called the adjusting telescope, and may be also used to watch a distant mark, before and after an altitude is taken, in order to detect any deviation in the position of the vertical axis that may happen during the operation of measuring. Let the adjusting telescope bisect a fine distant mark with its cross wire, and turn the tube of the telescope round one half way on its pivots, as it lies in a horizontal position, and if the wire now bisects the same mark it is truly fixed, if not, look out for a new mark a little higher or lower, as the case may require, and make it cut that in the reversed positions of the cross wire, by means of the proper screw for the purpose; now this adjusting telescope will be adjusted for collimation. In the next place, put zero on the vernier to zero on the limb, and direct the telescope of observation to the distant mark, by which the adjusting telescope had its wire adjusted, and let this mark be bisected by both telescopes, the level and plumb-line at the same time showing that the vertical axis is perpendicular; now turn the quadrant half round its azimuth, and reverse the adjusting telescope so as to view the same distant mark again, and if it be found to bisect it as before, the horizontal line of the quadrant is right, and all the quadrantal arc without error, supposing the telescope of observation to have its adjustment for collimation as fixed by the point 90°, above described; but if this adjustment of the point zero on the limb be first made, half the apparent error must be rectified by the screw at the eye piece by means of reversed positions and marks; and then afterwards the adjustment by a new star near the zenith will detect the error of the whole arc. If, however, no error in the total arc exists, then the adjustment for collimation may be made either from the horizontal or from the vertical measurement, as may be most convenient; one of which is more practicable by day, and the other by night. When this delicate and most essential adjustment is finally settled, the object glass of the telescope should not be disturbed, and therefore it would be adviseable to have its interior surface well cleaned previously. It was taken for granted that the cross wire was perfectly horizontal during the time the preceding adjustment was made, or, which is the same thing, that the parallel wires were perpendicular to the horizon. This is proved in a simple manner thus; direct the telescope to a fine small distant mark, or make the adjustment for vision, if necessary, then if one of the vertical wires will continue to bisect the said mark through the whole field of view while the telescope is elevated or depressed, the wires are right, but if not, they must be made so by the proper screws for that purpose, near the focus of the eye-glas. This preparation ought to precede the last adjustment, and when once made, seldom requires altering, except in case of accidental injury. It has also been assumed in the preceding adjustment, that the maker of the instrument placed the plane of the quadrant parallel to the axis of its motion, and also its line of collimation of the telescope parallel to the said plane. The former may be known to be true thus: if, when the plumb-line is adjusted at its centre of suspension, just to escape touching the limb (which should always be the case) the motion of the quadrant in azimuth will not alter it in this respect, the plane is truly fixed; but if not, the screws that fix the quadrant to its axis must be resorted to for its alteration, which is best done by the maker. When there is no plumb-line, a small spirit level fixed at right angles to the plane of the quadrant will answer the same purpose; for the resting of the bubble during the quadrant's revolution in azimuth, will be a proof that the plane to which it is at right angles is vertical. With respect to the parallel position of the telescope, as this is guided by the vernier sliding on the limb, it is the business of the maker to adjust it properly, which he will best do by a comparison with a good transit instrument of the passages of a high and of a low star in each of the two instruments, but a small deviation of the telescope with respect to parallelism, though to be avoided, if practicable, will not sensibly affect the measurement of altitudes, which is the sole business of this instrument. If, however, this deviation be considerable, the eye end of the telescope must be set nearer to or farther from the limb, as the case may require, by the maker himself. We have been the more minute in describing these adjustments, not only because they are indispensably necessary in making good observations, but because they will apply, one or other of them, by means of the plumb-line or of the spirit level, to all other astronomical quadrants that have a motion in azimuth.

3. *Of the transit instrument.*

Transit instrument.

95. Transit instruments, as they are now constructed, may be considered either as fixed or portable; the former of which was the original construction, and is still that commonly made use of in permanent observatories, for the purpose of determining in conjunction with a good astronomical clock, the right ascensions of the heavenly bodies; but the latter

may be used in any place for ascertaining the rate of a clock or chronometer, and when nicely brought into the meridian, for determining also the right ascension with considerable accuracy. It is one of the latter only that we proposed to describe in this place.

Fig. 34. The best construction of a portable transit instrument, is that represented in our fig. 34, where are shown all the parts that are necessary to be described under this article. This instrument is one of the numerous inventions of Troughton, and by no means one of the least useful.

The telescope of this transit is 20 inches in length, and magnifies from 20 to 35 times, according to the eye pieces that are employed; two of which are usually of the prismatic or diagonal kind, to be used at considerable altitudes: the aperture is $1\frac{5}{8}$ inches, and the power is sufficient to enable us to see the pole star by clear day light. The base of the instrument is a thick ring or rim of brass, which receives three equidistant screws for feet, besides the four screws that fix the two vertical frames to the base, and which constitute the supports of the axis; one of these is shown complete in the first of our two figures. These supports are kept perpendicular by the interior bracing bars, of which two are seen in the second figure; they are attached by thumb screws at both ends to the base and upright frames respectively. The circular figure of the base is not only firm, but preserves its shape in all degrees of temperature; and when the parts are detached, by loosening the thumb screws, the whole pack into a box, which is of convenient size for carriage; the diameter of the circular base, and the consequent length of the axis, is a foot within, and the height of the supports thirteen inches.

The graduated circle being of six inches diameter, admits of reading by each of the two opposite verniers to minutes, which is sufficient for finding the meridian altitude of any celestial body of which the declination is known, when the latitude is given; or for determining the latitude when unknown, to the accuracy of the nearest minute.

If the circle were made a little larger, and three verniers substituted for the two, a longer level might be used, and the readings made accurate to 20″ or 30″; but as the intention of the inventor was not to make it an altitude instrument, he has limited himself only to such conditions as were requisite for constituting an useful transit instrument in a portable form. The level of this transit is wholly detached, is equal in length to the axis itself; and is intended to be placed over the same, by resting upon it, having notches on its end pieces, which become tangents to the cylindrical parts of the axis or pivot; so that the reversion is performed without any inconvenience: but it is necessary to remove the level when the altitudes are great, in order to avoid its being displaced and broken by any alteration in the elevation of the telescope. There are usually three studs of brass included, with the darkening glasses, lantern, and other appendages; two of which studs have conical holes to receive the points of the screws or feet of the circular base; and for this purpose all the studs must be made fast to the slab or pillar which supports the instrument, by plaster of Paris or putty, inserted into as many holes in the plane of the marble or stone, care being taken that the line which joins the two conical points be in the direction of the meridian, or so nearly so, that the adjusting screws of one of the Y's will bring it into that situation.

Mr. Jones, of Charing-cross, has made several 30 and 42 inch transit instruments of the portable sort, supported by oblong frames of cast iron, which look neat, and answer the purpose very well. These have all the advantages of the instrument last described, and at the same time of course have greater powers in the telescope, and are cheaper in proportion to their size. He has also made some with telescopes of only 20 inches, for the sake of being more portable.

Of the adjustments.

1. *Of the level.*

When the level hangs on, or is made fast to the axis, put the telescope in its place, and observe to which end of the level the bubble runs, which will always be the more elevated end; bring it back to the middle by the Y screw for vertical motion, or by the foot screw under the end of the axis, and then invert the axis end for end; then, if the bubble is again found in the middle, the level is already parallel to the axis; but if not, adjust one half of the error by the adjusting screw of the level, and the other half by the Y screw, or that at the foot of the support, and let the operation of reversing and adjusting by halves be repeated until the bubble will remain stationary in either position of the axis, in which case the level will be right. When the detached level is used, that notch must be made a little deeper where the bubble is, by scraping it with a penknife, instead of using an adjusting screw, with which it is not commonly provided. And when the notches which rest on the pivots are once made right, they will seldom require a second rectification.

2. *To place the axis of the telescope horizontal.*

If the spirit-level be made use of, the same operation which we have just described, will put the axis level, at the same time that it brings the level parallel to the axis; for unless both these conditions be fulfilled, the adjustment of the level will be deranged by reversion; and when this is not the case, it is a proof that both the level and the axis are truly horizontal. Hence, when the level is previously adjusted, it will be sufficient to bring the bubble to the middle of the level, by the Y screw, or the foot screw alone, as the construction may require.

It is not necessary for our present purpose to describe the adjustments of the larger instruments, but we may just observe, that when in these the plumb line is employed, it is applied to a frame suspended by the pivots of the axis, that will reverse in position according to Ramsden's method, or hanging on the tube of the telescope parallel to the line of collimation; in either case a dot is bisected by the plumb-line near the point of suspension, and another near the lower end of the line, in both the reversed positions of the axis, when the adjustment is truly made by the proper screws as above directed.

3. *To adjust the telescope.*

That is, to place an eye-glass and object-glass at such a distance from each other, that their respective foci may coincide: after which, the wires are to be brought into their common focus. To effect this, some telescopes have the eye-glass and cell, which carries the wires, moveable, while the object-glass is fixed: others have the wires fixed, and the two glasses moveable. In the former case, by pushing in or drawing out the eye piece, adjust the telescope so that the sun or a planet appears perfectly distinct through it; then move the wires nearer to, or farther from, the eye-glass, as may be required, until they also appear perfectly distinct, and the telescope will be adjusted ready for use. In the latter construction, push in, or draw out, the eye piece, till the wires appear perfectly distinct; then alter the object-glass until the sun, or a planet, appears perfectly distinct also, and the telescope will be adjusted ready for use.

As it is of importance to have the telescope adjusted very exactly in this respect, the following method of trying whether it be so or not, may be practised.

The telescope being adjusted to distinct vision for distant objects, when a fixed star is on the meridian, bring the horizontal wire to bisect it very exactly, and the star will run along the wire through the whole extent of the field of the telescope. While the star is thus running along the wire, move your eye a little upward or downward; and if the wires be not exactly in the common focus of the two glasses, the star will appear to quit the wire when the eye is moved. If this be the case, the wires or glasses must be altered until the star will not quit the wire by the motion of the eye; the objects appearing perfectly distinct at the same time.

To bring a transit instrument into the plane of the meridian.

Take the altitude of the sun, noting the times by the watch or clock, and thence find the apparent time, the latitude of the place, and the sun's declination being known. The difference between this time and the mean of the times shown by the watch when the observations were made, will be what the watch is too fast or too slow, for apparent time.

If the watch is too fast, add the difference to 12 hours: but if it be too slow, subtract it from 12 hours, and you will have the time by the watch when the sun will be on the meridian, as near as the going of the watch can be depended upon. Take the time which the sun's semidiameter is in passing the meridian from the Nautical Almanack, and add it to, and subtract it from the time by the watch, when the sun will be on the meridian, and you will have the times when the sun's eastern and western limbs will be on the meridian. A few minutes before the time when the western limb will be on the meridian, let your assistant count the seconds as they pass, by the watch; but instead of calling the 60th second, let him name the minute the watch is then at. While he is doing this, you must bring the sun into the telescope by elevating it to the proper altitude; and turning the whole instrument round on the screw pin U. Having by this means brought the middle wire apparently to the eastward of what appears to be the eastern limb of the sun, (because the sun will appear to move that way in the telescope) tighten the screw U by turning the nut; and when the sun's limb arrives at the middle wire, keep it on it by turning the screw *g*, at the rate the sun moves, till your assistant calls the second by the watch at which you had computed the western limb of the sun would be on the meridian; and the instrument will be nearly in the meridian. Let your assistant count on till the watch arrives at the second, when, according to your calculation, the eastern limb of the sun should be on the meridian; and, if it is not exactly on it, you will have another opportunity of rectifying the instrument by turning the screw *g*.

Having thus brought the instrument into, or very near the meridian, its real situation with respect to the meridian may be verified several ways; of which we shall point out two. If the latitude of the place be considerable, that is, 30 degrees or upward, there are a variety of stars in both hemispheres sufficiently bright, which never set: and consequently, they may be observed with the instrument both above and below the pole.

Let the transits of such a star over the meridian be observed above and below the pole; and it is manifest, that if the time of the first transit above the pole be subtracted from the time of the second transit above the pole (adding 24 hours if necessary), the remainder will be the time by the watch, in which the earth (or the star apparently) makes one diurnal revolution. It is also evident, that if the two intervals between the time of the transit below the pole be equal, the instrument must be exactly in the meridian. If the interval between the first transit above the pole, and the transit below the pole be greater than the interval between the transit below the pole and the second transit above it, the object end of the telescope, when directed toward the elevated pole, lies to the east of the true meridian; but if the latter interval be greatest, the object end of the telescope, when directed towards the elevated pole, lies west of the true meridian.

To correct the error, bring the instrument into the meridian; add 24 hours to the time of the latter transit above the pole, subtract the time of the former from it, and take half the remainder. Take the difference between this and the interval between the transits above and below the pole, and take half this difference. Then, as the time by the watch of an entire revolution is to 24 hours, so is this half difference to the half difference in sideral time. Add to the logarithm of this half difference, the logarithmic tangent of the star's polar distance; and the logarithmic secant of the latitude of the place, the sum, rejecting 20 from the index, will be the logarithm of the number of seconds in time, which expresses the angle made by the instrument and the meridian.

Consider what part this angle makes of the interval, between the wires which are in the focus of the telescope; and turn the instrument on its axis till the telescope points at the horizon. Look out for some tolerably distant object which is cut by one of the wires; and by turning the screw *g*, remove the wire to the east or west of this object (as may be required), such a part of the space between that wire and the next to it, as the angular error which the instrument makes of that interval. You must then proceed to examine the position of the instrument again, either by the same, or some other circumpolar star, and to correct it, if it requires correction, until you get it exactly into the plane of the meridian; and when you have, a mark must be set up in the meridian at as great a distance from the instrument as may be convenient, or as it can be seen distinctly; and the telescope must be carefully adjusted to this mark before every observation. See also vol. 1 of the Transactions of the Astronomical Society.

4. *To adjust a clock or watch by observation.*

96. In the preceding article we have supposed the watch to be already regulated, and have shown how from this the transit may be adjusted; we shall now suppose the instrument correctly fixed, and show the method of adjusting the clock or watch.

Regulation of the clock.

The instrument then being thus adjusted, observe the hour, minute, second, &c. when any particular star traverses over the centre of the field of the telescope, by taking the mean of the times when it passes each wire as described (art. 87). Observe the same the next day, and for several days in succession, as also with different stars; and if the clock be properly adjusted to sideral time, each star ought to transit the meridian at the same instant every day that it is observed. If it marks different times on the different days, it will be easy to determine from the mean of these observations, how much it gains or loses per day; and if this error be considerable, the pendulum must be lengthened or shortened by the proper adjusting screw according as it gains or loses; but if the error be not more than a fraction of a second, or even a second or two, and uniform for every equal interval of time, the pendulum may be allowed to remain, and the proper correction applied whenever any observation is made.

We here suppose the clock to be adjusted to sideral time, or to register exactly 24 hours from one transit of any fixed star to another; if the clock be adjusted to mean solar time, like those employed for common purposes, it ought to show only 23h. 56′ 41″.1, which is the length of a sideral day in mean solar time. But for all the purposes of an observatory, sideral time is to be preferred. The clock may also be regulated by the transit of the sun; but before we can employ it for this purpose, it will be necessary to enter at some length into an explanation of what is termed the equation of time. We shall therefore only further observe, that the best way of observing the transit of any heavenly body, is to watch it first into the telescope; then the clock being supposed close at hand, note the hour, minute, and second of its entrance; and with your eye then applied to the telescope, count the beats of the pendulum till the body passes the first wire, and note down the exact time; between the time of its passing the first and second wire observe again the time by the clock, and proceed again as before, and so on with all the wires; and the mean of the several results will be the true time of its passing the centre wire as already explained. (art. 87.)

Estimation of right ascension in time.

97. The right ascension of any heavenly body is, as we have already observed, the arc of the equator intercepted between the first point of aries, or that point where the equator is cut by the ecliptic, and the point where a secondary to the equator, passing through the body, meets the latter circle. But as the motion of the earth, or the apparent motion of the heavens, is uniform, we may also denote any arc of right ascension, by the interval which elapses between the time when the first point of Aries passes the meridian of any place, and that of the transit of the proposed body. The right ascension may be therefore estimated in measure or time; but the latter is the most common: in the Nautical Almanack, for example, we always find the sun's right ascension noted in sideral hours, minutes, seconds, &c. The right ascension of any heavenly body is then readily obtained by means of our clock and transit instrument; for the former being set to 0 hours at the moment when the first point of Aries passes the meridian of any place; and being supposed correctly adjusted to sideral time by means of the preceding observations, the right ascension of any body will be shown in time by the clock, and this, when requisite, may be immediately reduced to angular measure by saying, as 24 hours : to the time shown by the clock :: 15° : to the measure sought; or the reduction may be made by means of a table computed for the purpose. If the clock has any rate, that is, if it does not show correct sideral time, the proper correction must be applied, as indicated above; and if the clock be not so adjusted as to show 0h. 0m. 0s. when the first point of Aries passes the meridian, then it is obvious that the difference of the two times will be the right ascension sought.

Certain requisite corrections.

It is proper, however, to make here one important remark, which is, that what we have called above the first point of Aries, is not a fixed point, but that it changes its place by a slow retrograde motion from year to year, called *the precession of the equinoxes;* therefore, the right ascension of the stars is also variable, and stand in need of constant corrections; we shall not, however, enter upon this subject at present, our purpose here was only to show how the right ascension of a heavenly body might be determined, that of any particular body being given.

Most of the principal stars have had their right ascensions ascertained with the utmost precision, particularly 36 of them, by Dr. Maskelyne, as well as their annual variations; these, therefore, are now commonly employed by most astronomers for the purpose of regulating their clocks, and then by means of the clock, the right ascension of the sun, moon, and planets, at any time, is ascertained; as well as that of any fixed star, (which is supposed to be not correctly established,) may also be determined according to the principles above explained.

Corrections for observed altitudes

98. We have seen in our illustration of the circles of the sphere, that the altitude of any body when on the meridian will be sufficient for determining its declination; for the altitude of the equator or point E in our fig. 22, is equal to the co-latitude; and the difference therefore, between the meridian altitude and the co-latitude of the place of observation is equal to the declination, which in our latitude will be north or south, according as the former of those quantities is greater or less than the latter; and having the right ascension and declination, the latitude and longitude of the body may be computed as explained in art. 75; or, on the contrary, the latitude and longitude being given, the right ascension and declination may be computed, the obliquity of the ecliptic being supposed given. But as we have before observed, the altitude of a body, as determined from observation, stands in need of certain corrections for parallax, refraction, &c. which we have not yet investigated, consequently, the student cannot yet proceed to apply

such observations to actual astronomical determinations.

§ V. *Of sideral and mean solar days, years, &c.*

99. We have seen that the earth performs its revolution on its axis with a motion perfectly uniform; and that the interval between the return of any fixed star to the same meridian, is what is called a sideral day; observing that by the same meridian is here to be understood, the star appearing at the same meridian altitude; for the circumpolar stars, as we have seen, may be observed twice on the same meridian in the course of 24 hours, once in their inferior and once in their superior passage; but they are only once at the same meridian altitude. This interval, then, is the length of the sideral day; but it remains for us now to explain what is to be understood by a mean solar day, or that day which is employed in the common concerns of life.

1. *Of the mean solar day.*

Mean solar day.

100. It will be seen in the following articles, that the interval time between the sun's leaving the first point of Aries to its return again to the same, which is what is denominated a solar year, is performed in about $365\frac{1}{4}$ solar days; or the sun will have appeared in that interval 365 times on the meridian, and will besides have performed nearly one-fourth of his 366th revolution: hence, if all the solar days were equal, that is, if the sun returned to the meridian of an observer always after the same interval, the increase of his right ascension every day, or the additional angle which the earth (having performed a complete revolution) would have to move through to bring the sun again upon the meridian of the observer, would be equal to $\frac{360^\circ}{365\frac{1}{4}} = 59' \, 8''{\cdot}2$; if, therefore, to the sideral day we add the time which the earth employs in describing $59' \, 8''{\cdot}2$, we shall have the length of the mean solar day; that is, the sideral day is to the mean solar day as $360^\circ : 360^\circ \, 59' \, 8''{\cdot}2$.

Consequently, if we call the mean solar day 24 hours, according to common reckoning of time, we shall have

$$360^\circ \, 59' \, 8'' : 360^\circ :: 24\text{h.} : 23\text{h. } 56\text{m. } 4{\cdot}1''$$

which is the length of a sideral day in mean solar hours; and by reversing the first two terms of the proportion, we shall have the mean solar day expressed in sideral hours.

2. *Sideral year.*

Sideral year.

101. It appears from what is stated above, and we have before made the same remark in a general way, that the sun has a continual motion in the heavens from east to west; and that after a certain period, he will again obtain with respect to the same star, the same relative situation; so that if he were in conjunction with it in the first instance, he will return again to conjunction after this interval, which is therefore called a *sideral year*.

Determined by observation.

In order to determine the duration of this period from observation, take on any day the difference between the sun's right ascension and that of the star, and when the sun returns to the same part of the heavens the next year, compare its right ascension with that of the same star for two days, one when their difference of right ascension is less, and the other when greater than the difference before observed; then it will be obvious that at some instant between these two observations, the right ascension will be exactly the same as it was when first observed.

Now in order to find the precise instant when this happens, let us suppose D to be the difference in right ascension as observed on the two consecutive days; and at the difference between the differences of the sun's and stars' right ascension on the first of these two days, and on the day when the observation was made the day before; and t be the exact time between the intervals of the two transits of the sun over the meridian on the two days; then assuming the motion of the sun to be equal in right ascension during this interval, we shall have

$$D : d :: t : \frac{d\,t}{D}$$

the time from the passage of the sun over the meridian on the first day, to the instant when it had the same right ascension, compared with the star, which it had the year before; and the interval between these two times when the difference of right ascension was the same, is the length of the sideral year.

Or if instead of supposing the second observation to have been repeated on the second year, there is an interval of several years between the two observations, and the observed interval of time be divided by the number of years, the length of the year will be had more exactly, any error in the observations being thus rendered less important by being divided into the great number of parts. The best time for these observations is, when the sun is in or near one of the equinoxes or one of the solstices, his motion in right ascension being then exactly or very nearly uniform.

As an example of this kind, we may state the following:—

By MM. Picard and La Caille.

102. April 1, 1669, at 0h. 3m. 47s. of mean solar time, M. Picard observed the difference between the longitude of the sun and the star Procyon to be 3s. 8° 59′ 36″. And M. La Caille found the difference of longitude between the sun and star to be the same, on April 2, 1745, at 11h. 10′ 45″. The sun, therefore, made 76 complete revolutions with regard to the same fixed star in 76 years 1 day 11h. 6m. 58s.; or in 27,759 days 11h. 6m. 58s.; we have, therefore, by dividing this interval by 76, 365 days 6h 8′ 47″ for the length of the sideral year; more recent observations, however, give 365 days 6h. 9m. 11·5s. for this interval.

3. *Of the tropical year.*

The length of the tropical year.

103. The length of the tropical year is the interval between the sun leaving either equinoctial point to its return again to the same; which it does in a less time than it passes from any fixed star to the same again, the latter we have seen is what is termed the sideral year; and the former being that on which the change of seasons depend is called the *solar* or *tropical* year. To determine the length of this year, we may proceed as follows:

Determined from observation.

Observe the meridian altitude of the sun on the day nearest the equinox, and the next year take its meridian altitude again on two successive days, on the one when its altitude is greater, and on the other when it is less, than in the first observation; then it is obvious, that at some intermediate time between the two last observations, the sun must have had the same altitude or declination as in the first instance. Now, to find the precise instant, let D be the difference of altitude in the two last observations, t the interval between them, which may be here taken as 24 hours; also d the difference between the altitude as observed on the first of the two latter days, and that taken the year before; then, assuming the declination to be uniform, as it actually is at this time, say, as $D : d :: 24 \text{ hours} : \frac{24\,d}{D}$ the time from the first of the two latter observations, to the instant when the declination was the same as in the preceding year. This time, therefore, being added to the number of days between the two first observations, will give the true length of the solar year.

Here again, as in last case, if the two observations be repeated after an interval of several years, we may look for a result more nearly approximating to the truth.

By the Cassinis.

104. The two following observations were made by Cassini and his son, after an interval of 44 years.

March 20, 1672, Meridian alt. sun's upper limb		41° 43′ 0″
March 20, 1722	ditto	41° 27′ 10″
March 21	ditto	41° 51′ 0″

41 51 0   41 43 0
41 27 10   41 27 10

23 50 : 15 50 :: 24 : 15h. 56m. 39s.

Whence on the 20th of March, at 15h. 56m. 39s., the sun's declination was the same as on the 20th of March, 1672, at noon.

Now the interval between the two first of these observations was 44 years, 34 of these were common years of 365 days each, and 10 of 366 days each, making in all an interval of 16,070 days, and therefore the interval between the two periods when the sun's declination was the same was, 16,070 days 15h. 56m. 39s. and this interval embraces 44 tropical years, whence $\frac{16{,}070\text{d. } 15\text{h. } 56\text{m. } 39\text{s.}}{44}$ 365d. 5h. 49′ 0″ 53‴ the length of the tropical year, as determined from these results; more recent observations give for the true length of the tropical year, 365 days 5 hours 48′ 48″.

4. *Precession of the equinoxes.*

Precession of the equinoxes.

105. It appears from what is stated above, that the sun returns to the equinoxes every year, before it returns again to the same fixed star, or to the same point in the heavens; the equinoctial points must, therefore, have a retrograde motion with respect to that of the earth, the cause of which it is not for us at present to explain; we shall, however, hereafter, see that it is due to a regular mechanical effect, viz. the attraction of the sun and moon upon the earth in consequence of its spheroidal figure. The effect of this is, that the longitude of the stars, which are always estimated from the intersection of the equator and ecliptic, or from the equinoctial point, or first point of Aries, must constantly increase, and by comparing the longitude of the same stars at different times, the mean

motion of the equinoctial points, or the precession of the equinoxes may be determined.

We have observations of this kind from the time of Timocharis and Hipparchus; but we may be allowed to entertain considerable doubt as to accuracy; it will be sufficient to observe, that from a comparison of the best observation, the secular precession, or that which takes place in 100 years, amounts to 1° 23′ 45″ or to 50·34″ annually.

5. *Anomalistic year.*

Anomalistic year.

106. At a certain time of the year, the sun's diameter, if measured instrumentally, would be found to be the least; at which time it is obvious he would be the most distant from the earth or in his apogee, that is, the earth will then be at one extremity of the transverse axis of its orbit, and at that extremity which is furthest from the sun. Now, if at the end of a certain interval, a year from the first observation, a second were made, and the sun were found in precisely the same relative situation with regard to certain fixed stars, when his diameter was least, then it would be obvious that the sun had always the least apparent diameter after the completion of a sideral year; but the astronomical fact is not so; the sun does not return to a point in the heavens where his diameter is least in a sideral year; but in an interval a little exceeding it, and this interval is what astronomers have called the *anomalistic year*, the apogee has therefore a progressive motion, as we have seen the equinoxes have a precession; the quantity of the former, like that of the latter, being found from observation. According to the most recent determination, the increase in longitude of the sun's apogee in 100 years, is 1° 42′ 28″, or 1′ 2·2″ annually; but since the precession, which is a regressive motion, is 50·34″ annually, the annual sideral progression is 62·2″ — 50·34″ = 11·86″. Now, the time of describing 11·86″ added to the length of the sideral year, will compose an anomalistic year; and since the sun near its apogee moves in longitude about 58′ in 24 hours; the time will be about 4m. 50s.; hence the length of the anomalistic year, is equal to

365d. 6h. 9m. 11·4s. + 4m. 50s. = 365d. 6h. 14m. 1·4s.

6. *Obliquity of the ecliptic.*

Obliquity of the ecliptic.

107. The angle contained between the plane of the equator and the ecliptic is what is denominated the obliquity of the ecliptic; which is shown from repeated observations to be variable, like the other quantities we have been just examining. We have already, in the preceding section, indicated the method of determining the measure of this angle by means of the greatest and least meridian altitudes of the sun; it will therefore be sufficient in this place to show the result of a long succession of such observations by different astronomers, which are as follows :—

	°	′	″
Eratosthenes, 230 years B.C.	23	51	20
Hipparchus, 140 years B.C.	23	51	20
Ptolemy, A.D. 140	23	51	10
Pappus, A.D. 390	23	30	0
Albatenius, in 880	23	35	40
Arzachel, in 1070	23	34	0
Prophatius, in 1300	23	32	0
Regiomontanus, in 1460	23	30	0

Determined by different astronomers.

Astronomy.

	°	′	″
Waltherus, in 1490	23	29	47
Copernicus, in 1500	23	28	24
Tycho, in 1587	23	29	30
Cassini (the father), in 1656	23	29	2
Cassini (the son), in 1672	23	28	54
Flamstead, in 1690	23	28	48
De la Caille, in 1750	23	28	19
Dr. Bradley, in 1750	23	28	18
Mayer, in 1750	23	28	18
Dr. Maskelyne, in 1769	23	28	8,5
M. de Lalande, in 1768	23	28	0
M. Pond, Ast. Roy. 1816	23	27	50

The observations of Albatenius, an Arabian, are here corrected for refraction. Those of Waltherus, M. De la Caille computed. The obliquity by Tycho is put down as correctly computed from his observations; also the obliquity, as determined by Flamstead, is corrected for the nutation of the earth's axis; these corrections M. de Lalande applied. It is manifest from the above observations, that the obliquity of the ecliptic continually decreases; and the irregularity which here appears in the diminution, we may ascribe to the inaccuracy of the ancient observations, as we know that they are subject to greater errors than the irregularity of this variation. If we compare the first and last observations, they give a diminution of 70″ in 100 years. If we compare the observation of Lalande with that of Tycho, it gives 45″. The same compared with that of Flamstead gives 50″. If we compare that of Dr. Maskelyne with Dr. Bradley's and Mayer's, it gives 50″. The comparisons of Dr. Maskelyne's determination, with that of M. de Lalande, which he took as the mean of several results, gives 50″, as determined from the most accurate observations. This result agrees very well with that deduced from theory; but the observation of Mr. Pond, as compared with those of Bradley, gives 66″ for the variation in the obliquity in 100 years, or 0·40″ annually.

§ VI. *Of the corrections for refraction, parallax, &c.*

1. *Of refraction.*

Refraction. 108. When a ray of light passes out of a vacuum into any medium, or out of a medium into any other of a greater density, it is found to deviate from its regular course, towards a perpendicular to the surface of the medium into which it enters. (See Optics, p. 427) Hence light passing out of a vacuum into the atmosphere, will, where it enters, be bent or deflected towards a radius drawn to the earth's centre, the extreme surface of the atmosphere being supposed spherical and concentric with the centre of the earth; and as in approaching the earth's surface the density of the atmosphere continually increases, the rays of light are constantly entering into a denser medium, and therefore the course of the rays will continually deviate from a right line, and describe a curve; whence, at the surface of the earth, the rays of light enter the eye of a spectator, in a different direction from that in which they would have entered it, if there had been no atmosphere. Consequently, the apparent place of a body from which the light comes must be different from the true place, as shown in fig. 35. Fig. 35.

Observed by the ancients. 109. Although we here state the fact of the refraction of the atmosphere as a necessary consequence of established laws in Optics, it must not be understood to have been introduced into astronomy as such; for it was observed by the ancients long before they were able to trace its cause to optical principles. Nothing is indeed more easy to be detected in astronomical observation; for by taking the greatest and least altitudes of the circumpolar stars, it will be seen that their apparent north polar distances will be different accordingly as it is taken at the time of their superior or inferior passage; and this variation is observed to be very nearly constant for the same place and the same star, and for all stars that have the same declination; but to vary according to a certain law, in stars that pass at different altitudes; it is also determinable from observation, that refraction does not alter the azimuth of bodies, and the same may also be demonstrated on physical principles, as we have shown in our treatise on Optics. Refraction, therefore, has a tendency only to increase the apparent altitude of a heavenly body, its entire effect being produced in a vertical circle, and its effect is less and less from the horizon to the zenith, where it vanishes.

Plane Astronomy.

Both Ptolemy and Alhazen were acquainted with this irregularity, and attributed it to its true cause, but neither of them undertook to determine the quantity of it. Tycho perceiving that the altitude of the equator, as deduced from the two solstices, was not the exact complement of the height of the pole, endeavoured to determine the quantity of refraction due to each zenith distance; he made the horizontal refraction 34″, and supposed it to become insensible at 45° of altitude; in the former, he was not far from the truth, but the latter conclusion was wholly erroneous; it is, in fact, to Dominic Cassini that we are indebted for the first regular hypothesis on the subject of astronomical refraction; but as his solution leads to an expression in which the refraction is made proportional to the tangent of the zenith distance, diminished by a quantity which itself depends upon the refraction sought; we shall not insist upon it in this place, but proceed to illustrate the formulæ given by Bradley. Tycho endeavoured to estimate its quantity.

Bradley's formulæ.

110. Various tables of refraction more or less correct had been already formed, when the above celebrated astronomer commenced his observations for the purpose of deducing more exact formulæ than were at that time in existence, and of these tables he availed himself in settling the law of this important astronomical correction. Bradley's formulæ.

By means of numerous observations on Polaris and other circumpolar stars, Bradley deduced the apparent zenith distance P of the pole. By observations also on the sun at the equinoxes, when this body had the same zenith distance but opposite right ascensions, he deduced the height of the equator. In this instance, as in the former, it was only the apparent altitude that was obtained on account of the refraction, and therefore greater than the true height, and consequently, the apparent zenith distance was less than the true; whence the sum of the two zenith distances of the poles and the equator (which if true ought to be = 90°) will be less than 90°, by the sum of the two refractions due respectively to the zenith distances

Astronomy.

P and Q. Conceive, for instance, the two refractions to be p and q, then,

$$P + Q = 90^\circ - (p + q)$$

and $$p + q = 90^\circ - (P + Q)$$

consequently, the sum of the two refractions $p + q$ is given, but the object is to determine them separately; but, by referring to the best tables extant on this subject, it was found, that the difference $q - p$ was about equal to 2 seconds, and hence the two equations

$$p + q = 90^\circ - (P + Q)$$
$$p - q = 2''$$

gave

$$q = 45^\circ\, 0'\, 1'' - \tfrac{1}{2}(P + Q)$$
$$p = 44^\circ\, 59'\, 59'' - \tfrac{1}{2}(P + Q)$$

Now, according to Bradley's observation, he found

$$P = 38^\circ\, 30'\, 35''$$
$$Q = 51^\circ\, 27'\, 28''$$

whence, $p = 57''{\cdot}5$ and $q = 59''{\cdot}5$

but this being only an approximation, in order to obtain a more correct result; the author separated the sum $p + q = 1'\, 57''$ into two parts $p' + q'$ which should be to one another as the tangents of the observed zenith distances; whence he obtained

$$p' = (p' + q') \times \frac{\tan 38^\circ\, 31'\, 32''{\cdot}5}{\tan 38^\circ\, 31'\, 32''{\cdot}5 + \tan 51^\circ\, 28'\, 27\tfrac{1}{2}''}$$

$$q' = (p' + q') \times \frac{\tan 51^\circ\, 28'\, 27\tfrac{1}{2}''}{\tan 38^\circ\, 31'\, 32\tfrac{1}{2}'' + \tan 51^\circ\, 28'\, 27\tfrac{1}{2}''}$$

and by means of the new operation, he found

$p' = 45''{\cdot}5$ at the co-altitude $38^\circ\, 31'\, 20\tfrac{1}{2}''$
$q' = 1'\, 11''{\cdot}5$ ditto $51^\circ\, 28'\, 39\tfrac{1}{2}''$

111. We have stated above that Bradley assumed the refraction to vary as the tangent of the zenith distance; the principle on which this assumption is founded, may be illustrated as follows:—

Fig. 36.

Let C An, fig. 36. be the angle of incidence, C Am the angle of refraction; and, consequently, mAn the quantity of refraction; let CT be the tangent of the arc Cm, mr its sine, nw the sine of Cm, and draw rm parallel to vw; then as the refraction of the arc is very small, we may consider mrn as a rectilinear triangle; and hence by similar triangles,

$$Av : Am :: rn : mn = \frac{Am \times rn}{Av}$$

but Am is constant, and as the ratio of mv to mw is also constant by the laws of refraction; their difference rn must vary as mv; hence mn varies as $\frac{mv}{Av}$; but $CT = \frac{Am \times mv}{Av}$; which varies as $\frac{mv}{Av}$ because Am is constant: consequently, the refraction mn varies as CT, the tangent of the apparent zenith distance of the star; for the angle of refraction CAm, is the angle between the refracted ray and the perpendicular to the surface of the medium, which perpendicular is directed to the zenith; therefore, while the refraction is very small, so that rmn may be considered as rectilinear, this rule may be considered as furnishing a good approximation.

Formation of tables.

112. Assuming, therefore, the preceding quantities (art. 110) as the true or actual refractions, at the altitude of the pole and equator; and adopting the above analogy with reference to the tangents of the zenith distances, Bradley deduced the refractions for other altitudes exceeding that of the equator, and for less ones or greater zenith distances, he employed the circumpolar stars. That is, supposing xy, (fig. 37) to be the true places of a circumpolar star at its least and greatest altitude, then Zx, by correcting the observed distance is known, consequently ZP is known, and Px is so likewise. Again, the apparent zenith distance of the star at y, is observed, and subtracting from it ZP, the apparent distance from P is known, which is less than the true distance Py, by a certain quantity which is determinable because Px = Py, and the former has been determined, and this difference is obviously the correction due to the zenith distance Zy.

Plane Astronomy.

Fig. 37.

As an example, it was found by observation on Cassiopea at its greatest altitude, that the apparent zenith distance was

	°	′	″
	13	48	12½
Correct for refraction	0	0	14
True zenith dist.	13	48	26½
Zenith dist. of pole	38	31	20½
North polar dist.	24	42	54

Again, by observation, the star being at its least altitude above the horizon, the zenith distance was found to be

	°	′	″
	63	13	21·8
Subtract	38	31	20·5
App. N. P. dist.	24	42	1·3
True ditto	24	42	54
Refraction at 63° 13′ 21″·8 Z. D.	0	0	52·7

By means of similar observations, Bradley determined the refractions for other altitudes; and after tabulating the results and a due examination of them, he found that the law of the refraction instead of being simply proportional to the tangent of the zenith distance, was of the form

$$r = \frac{m}{n} \tan (z - 3\,r)$$

And deducing from observation the values of m and n under different temperatures and different barometrical pressures, he obtained the formula

$$\text{refrac.} = \frac{a}{29{\cdot}6} \times \tan (Z - 3\,r) \times 57'' \times \frac{400}{350 + h}$$

In which a is the altitude of the barometer in inches
Z the zenith distance,
$r = 57'' \times \tan Z$,
p = height of Fahrenheit's thermometer,
29·6 = mean height of barometer.

Groombridge's formulæ.

113. This formula is found to apply with considerable accuracy for all altitudes greater than 10°, but for less altitudes it is very erroneous; and different formulæ and tables have accordingly been computed by more recent astronomers which approach nearer to the truth: of these the results published by Mr. Groombridge in the *Phil. Tran. for* 1814, are perhaps the most valuable; his formula, under the medium temperature and pressure, is as follows:

$$\text{refrac.} = \tan (z - 3{\cdot}6342956\, r) \times 58''{\cdot}132967$$

which answers for all altitudes above 3°; for less altitudes a farther correction becomes necessary; viz.

for every minute below 3° of altitude; or for every minute more than 87° of zenith distance, the result found as above must be reduced ·00462. By means of these formulæ, Mr. Groombridge has computed a very extensive table of refractions, with the requisite tables of corrections for the different states of barometrical pressure, as well as for thermometrical temperatures, both for the outside and inside of the observatory. We cannot allow ourselves to transcribe this table in the extended form given to it by its author, but the following abridgment of it will, it is presumed, be found highly acceptable to our readers. It will be understood that the correction for the barometer and thermometer will be the sum of the two factors in Table II. and Table III. multiplied into the mean refraction, and the product added or subtracted therefrom, according as the sum of the factors is plus or minus.

Table of mean refractions, computed from the preceding formula.

Zen. Dist.	Refrac.	Zen. Dist.	Refrac.	Zen. Dist.	Refrac.	Zen. Dist.	Refrac.	Zen. Dist.	Refrac.
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	0 0·00	22 0	0 23·46	44 0	0 56·03	66 0	2 9·77	87 8	14 46·37
0 30	0 0·51	22 30	0 24·05	44 30	0 57·01	66 30	2 12·85	87 16	15 14·40
1 0	0 1·01	23 0	0 24·65	45 0	0 58·01	67 0	2 16·05	87 24	15 44·00
1 30	0 1·52	23 30	0 25·25	45 30	0 59·03	67 30	2 19·38	87 32	16 15·27
2 0	0 2·03	24 0	0 25·85	46 0	1 0·07	68 0	2 22·85	87 40	16 48·35
2 30	0 2·53	24 30	0 26·46	46 30	1 1·13	68 30	2 26·47	87 48	17 23·37
3 0	0 3·04	25 0	0 27·07	47 0	1 2·20	69 0	2 30·25	87 56	18 0·46
3 30	0 3·55	25 30	0 27·69	47 30	1 3·30	69 30	2 34·21	88 0	18 19·83
4 0	0 4·06	26 0	0 28·32	48 0	1 4·41	70 0	2 38·34	88 6	18 49·98
4 30	0 4·57	26 30	0 28·95	48 30	1 5·55	70 30	2 42·68	88 12	19 21·51
5 0	0 5·08	27 0	0 29·58	49 0	1 6·72	71 0	2 47·23	88 18	19 54·47
5 30	0 5·59	27 30	0 29·80	49 30	1 7·90	71 30	2 52·01	88 24	20 28·89
6 0	0 6·10	28 0	0 30·87	50 0	1 9·11	72 0	2 57·03	88 30	21 5·08
6 30	0 6·62	28 30	0 31·52	50 30	1 10·34	72 30	3 2·33	88 36	21 42·88
7 0	0 7·13	29 0	0 32·18	51 0	1 11·60	73 0	3 7·92	88 42	22 22·47
7 30	0 7·64	29 30	0 32·85	51 30	1 12·89	73 30	3 13·82	88 48	23 3·95
8 0	0 8·16	30 0	0 33·52	52 0	1 14·21	74 0	3 20·07	88 54	23 47·40
8 30	0 8·68	30 30	0 34·20	52 30	1 15·55	74 30	3 26·69	89 0	24 32·94
9 0	0 9·20	31 0	0 34·88	53 0	1 16·93	75 0	3 33·73	89 4	25 4·51
9 30	0 9·72	31 30	0 35·57	53 30	1 18·33	75 30	3 41·22	89 8	25 37·09
10 0	0 10·24	32 0	0 36·27	54 0	1 19·78	76 0	3 49·21	89 12	26 10·71
10 30	0 10·76	32 30	0 36·98	54 30	1 21·25	76 30	3 57·75	89 16	26 45·40
11 0	0 11·29	33 0	0 37·70	55 0	1 22·77	77 0	4 6·89	89 20	27 21·20
11 30	0 11·81	33 30	0 38·42	55 30	1 24·31	77 30	4 16·72	89 24	27 58·14
12 0	0 12·34	34 0	0 39·15	56 0	1 25·91	78 0	4 27·30	89 28	28 36·26
12 30	0 12·87	34 30	0 39·89	56 30	1 27·54	78 30	4 37·72	89 32	29 15·60
13 0	0 13·40	35 0	0 40·64	57 0	1 29·21	79 0	4 51·09	89 36	29 56·19
13 30	0 13·94	35 30	0 41·40	57 30	1 30·93	79 30	5 4·53	89 40	30 38·07
14 0	0 14·48	36 0	0 42·17	58 0	1 32·69	80 0	5 19·18	89 44	31 21·28
14 30	0 15·02	36 30	0 42·95	58 30	1 34·51	80 30	5 35·21	89 48	32 5·85
15 0	0 15·56	37 0	0 43·74	59 0	1 36·38	81 0	5 52·83	89 52	32 51·82
15 30	0 16·10	37 30	0 44·53	59 30	1 38·30	81 30	6 12·26	89 56	33 39·84
16 0	0 16·65	38 0	0 45·34	60 0	1 40·28	82 0	6 33·79	90 0	34 28·13
16 30	0 17·20	38 30	0 46·16	60 30	1 42·32	82 30	6 57·78	90 2	34 53·15
17 0	0 17·75	39 0	0 47·00	61 0	1 44·42	83 0	7 24·63	90 4	35 18·55
17 30	0 18·31	39 30	0 47·84	61 30	1 46·59	83 30	7 54·87	90 6	35 44·34
18 0	0 18·86	40 0	0 48·69	62 0	1 48·83	84 0	8 29·13	90 8	36 10·52
18 30	0 19·43	40 30	0 49·56	62 30	1 51·14	84 30	9 8·18	90 10	36 37·10
19 0	0 19·99	41 0	0 50·44	63 0	1 53·53	85 0	9 53·03	90 12	37 4·08
19 30	0 20·56	41 30	0 51·34	63 30	1 56·00	85 30	10 44·88	90 14	37 31·47
20 0	0 21·13	42 0	0 52·25	64 0	1 58·56	86 0	11 45·57	90 16	37 59·28
20 30	0 21·71	42 30	0 53·17	64 30	2 1·21	86 24	12 41·02	90 18	38 27·50
21 0	0 22·29	43 0	0 54·11	65 0	2 3·96	87 48	13 44·62		
21 30	0 22·87	43 30	0 55·06	65 30	2 6·81	87 0	14 19·80		

Correction to preceding table of refraction.

BAROMETER.				FAHRENHEIT'S THERMOMETER.					
Inches.	Correc. −	Inches.	Correc. +	Degree.	Within. +	Without. +	Degree.	Within. −	Without.
28·60	·0350	29·60	·0000	24·0	·0575	·0420	49·0	·0000	·0080
62	·0342	62	·0007	24·5	·0563	·0410	49·5	·0011	·0090
64	·0335	64	·0014	25·0	·0552	·0400	50·0	·0022	·0100
66	·0328	66	·0020	25·5	·0540	·0390	50·5	·0033	·0110
68	·0321	68	·0027	26·0	·0529	·0380	51·0	·0044	·0120
28·70	·0314	29·70	·0034	26·5	·0517	·0370	51·5	·0055	·0130
72	·0306	72	·0041	27·0	·0506	·0360	52·0	·0066	·0140
74	·0299	74	·0047	27·5	·0494	·0350	52·5	·0077	·0150
76	·0292	76	·0054	28·0	·0483	·0340	53·0	·0088	·0160
78	·0285	78	·0061	28·5	·0471	·0330	53·5	·0099	·0170
28·80	·0278	29·80	·0068	29·0	·0460	·0320	54·0	·0110	·0180
82	·0271	82	·0074	29·5	·0448	·0310	54·5	·0121	·0190
84	·0264	84	·0081	30·0	·0437	·0300	55·0	·0132	·0200
86	·0256	86	·0088	30·5	·0425	·0290	55·5	·0143	·0210
88	·0249	88	·0095	31·0	·0414	·0280	56·0	·0154	·0220
28·90	·0242	29·90	·0101	31·5	·0402	·0270	56·5	·0165	·0230
92	·0235	92	·0108	32·0	·0391	·0260	57·0	·0176	·0240
94	·0228	94	·0115	32·5	·0379	·0250	57·5	·0187	·0250
96	·0221	96	·0122	33·0	·0368	·0240	58·0	·0198	·0260
98	·0214	98	·0128	33·5	·0356	·0230	58·5	·0209	·0270
29·00	·0207	30·00	·0135	34·0	·0345	·0220	59·0	·0220	·0280
02	·0200	02	·0142	34·5	·0333	·0210	59·5	·0231	·0290
04	·0193	04	·0149	35·0	·0322	·0200	60·0	·0242	·0300
06	·0186	06	·0155	35·5	·0310	·0190	60·5	·0253	·0310
08	·0179	08	·0162	36·0	·0299	·0180	61·0	·0264	·0320
29·10	·0172	30·10	·0169	36·5	·0287	·0170	61·5	·0275	·0330
12	·0165	12	·0176	37·0	·0276	·0160	62·0	·0286	·0340
14	·0158	14	·0182	37·5	·0264	·0150	62·5	·0297	·0350
16	·0151	16	·0189	38·0	·0253	·0140	63·0	·0308	·0360
18	·0144	18	·0196	38·5	·0241	·0130	63·5	·0319	·0370
29·20	·0137	30·20	·0203	39·0	·0230	·0120	64·0	·0330	·0380
22	·0130	22	·0210	39·5	·0218	·0110	64·5	·0341	·0390
24	·0123	24	·0216	40·0	·0207	·0100	65·0	·0352	·0400
26	·0116	26	·0223	40·5	·0195	·0090	65·5	·0363	·0410
28	·0109	28	·0230	41·0	·0184	·0080	66·0	·0374	·0420
29·30	·0102	30·30	·0237	41·5	·0172	·0070	66·5	·0385	·0430
32	·0096	32	·0243	42·0	·0161	·0060	67·0	·0396	·0440
34	·0089	34	·0250	42·5	·0149	·0050	67·5	·0407	·0450
36	·0082	36	·0257	43·0	·0138	·0040	68·0	·0418	·0460
38	·0075	38	·0264	43·5	·0126	·0030	68·5	·0429	·0470
29·40	·0068	30·40	·0270	44·0	·0115	·0020	69·0	·0440	·0480
42	·0061	42	·0277	44·5	·0103	·0010	69·5	·0451	·0490
44	·0054	44	·0284	45·0	·0092	—	70·0	·0462	·0500
46	·0048	46	·0291	45·5	·0080	·0010	70·5	·0473	·0510
48	·0041	48	·0297	46·0	·0069	·0020	71·0	·0484	·0520
29·50	·0034	30·50	·0304	46·5	·0057	·0030	71·5	·0495	·0530
52	·0027	52	·0311	47·0	·0046	·0040	72·0	·0506	·0540
54	·0020	54	·0318	47·5	·0034	·0050	72·5	·0517	·0550
56	·0014	56	·0324	48·0	·0023	·0060	73·0	·0528	·0560
58	·0007	58	·0331	48·5	·0011	·0070	73·5	·0539	·0570

Astronomy.

Other effects of refraction.

114. What we have hitherto stated, relates principally to corrections requisite to be made in astronomical observations in consequence of the effect of refraction in elevating all the heavenly bodies, but the same principles will also explain some other astronomical phenomena, as for instance, the morning and evening twilight, the oval appearance of the sun in the horizon, the horizontal moon, &c.

Cause of the twilight

115. With respect to the twilight, it arises both from the refraction and reflection of the sun's rays by the atmosphere. It is probable, that the reflection arises principally from the exhalations of different kinds with which the lower beds of the atmosphere are charged; for the twilight lasts till the sun is farther below the horizon in the evening than it is in the morning when it begins, and it is longer in summer than in winter. Now in the former case, the heat of the day has raised the vapours and exhalations; and in the latter, they will be more elevated from the heat of the season; therefore, supposing the reflection to be made by them, the twilight ought to be longer in the evening than in the morning, and longer in the summer than in the winter.

Commonly, it is assumed, that the twilight begins when the sun is 18° below the horizon; but in our investigations relative to the time of shortest twilight (art. 78), we have left the solution general for any number of degrees, and which may therefore be supplied at pleasure.

Oval figure of the sun and moon in the horizon.

116. Another effect of refraction, as we have above observed, is that of giving the sun and moon an oval appearance in consequence of the refraction of the lower limb being greater than the upper; whereby the vertical diameter is diminished. For assuming the diameter of the sun to be 32′, and the lower limb to touch the horizon, then the mean refraction at that limb is 33′; but the altitude of the upper limb being 32′, its refraction is only 28′ 6″, the difference of which is 4′ 54″, the quantity by which the vertical diameter appears shorter than that parallel to the horizon. This, however, will only happen when the body is in or very near the horizon, for when the altitude is any considerable quantity, the refraction of both limbs being then very nearly the same, the apparent disc will not differ sensibly from a circle.

2. *Of parallaxes.*

Paralla .

117. Parallax is a term used by astronomers to denote an arc of the heavens intercepted between the true and apparent place of a star, or other heavenly body, or its place as viewed from the centre and the surface of the earth.

Fig. 37.

Let s (fig. 37) represent a star, C the centre of the earth, Z the zenith of a spectator, then the observed zenith distance of s is the angle Z A s, but its actual zenith distance, as viewed from the centre C, is ZCs, and the difference between the angles ZCs and ZAs is is AsC, which is called the angle of parallax.

Now we know from the principles of trigonometry that

$$\sin \mathrm{C}s\mathrm{A} = \sin \mathrm{CA}s \times \frac{\mathrm{CA}}{\mathrm{C}s} = \sin \mathrm{ZA}s \times \frac{\mathrm{CA}}{\mathrm{C}s}$$

whence if CA, Cs remain the same, the sine of CsA, that is, the sine of the parallax varies as the sine of the star's zenith distance. Consequently, the parallax must be greater, the greater the zenith distance; it must therefore be greatest when the body is seen in the horizon, or when the zenith distance is 90°. Let p represent the parallax at any zenith distance Z, P the greatest or horizontal parallax, then we shall have

Plane Astronomy.

$$\sin p = \frac{\mathrm{CA}}{\mathrm{C}s} \times \sin \mathrm{Z}, \text{ and } \sin \mathrm{P} = \frac{\mathrm{CA}}{\mathrm{C}s} \times \sin 90° = \frac{\mathrm{CA}}{\mathrm{C}s}$$

Consequently, $\sin p = \sin \mathrm{P} \sin \mathrm{Z}$

If therefore the parallax be known for any one zenith distance, it may be determined for any other; and moreover, if CA the earth's radius, and Cs the distance of the body, were given, the parallax P would become known; and conversely, if the parallax were given, the distance Cs might also be determined, the radius of the earth being supposed known from geodetic operations. It is, in fact, from knowing the parallax of one or more of the heavenly bodies, that their distances have been determined.

The correction for parallax is one of the most important in practical astronomy, and accordingly, various methods have been proposed by different astronomers for determining it; but of these we shall only specify one or two of the most obvious.

Parallax of the moon.

118. First, to find the parallax of the moon. Take the meridian altitudes of the moon when it has its greatest north and south latitudes, and correct them for refraction, as explained in the preceding chapter. Then, if there were no parallax, or if the parallax were the same at both altitudes, the difference of the altitudes thus corrected would be equal to the sum of the latitudes, and consequently, what those quantities want of equality will be equal to the difference of the parallax. We have, therefore, by means of these observations, the difference of the parallaxes at these altitudes, and it is required to find them separately. For this purpose, let us denote the two zenith distances, by Z, z, the parallaxes by P, p: then from what has been stated above we have

$$\sin \mathrm{Z} : \sin z :: \mathrm{P} : p$$

or $$\sin \mathrm{Z} - \sin z : \sin . z :: \mathrm{P} - p : p$$

whence $$p = \frac{\sin z \; (\mathrm{P} - p)}{\sin \mathrm{Z} - \sin z}$$

the parallax at the greatest altitude.

We here suppose the moon to be at the same distance from the earth at both observations; when this is not the case, one of the observations must be reduced to what it would have been had the distance been the same.

Determined by observations. Fig. 38.

119. Let a body P (fig. 38) be observed from two places, A, B, in the same meridian, then the whole angle APB is the sum of the two parallaxes at those two places. Now we have seen in article 117, that the

$$\text{parallax APC or } p = \mathrm{P} \times \sin \mathrm{PAL}$$
$$\text{parallax PBC or } p' = \mathrm{P} \times \sin \mathrm{PBM}$$

Hence $\mathrm{APB} = p + p' = \mathrm{P} \times (\sin \mathrm{PAL} + \sin \mathrm{PBM})$

Consequently

$$\text{hor. parallax (P)} = \frac{p + p'}{\sin \mathrm{PAL} + \sin \mathrm{PBM}}$$

In order to illustrate this by actual observation, we may state the following example:

October 5, 1751, M. De la Caille, at the Cape of Good Hope, observed Mars to be 1′ 25″·8 below the parallel of λ in Aquarius, and at 25° distance from the zenith. On the same day, at Stockholm, Mars was

Astronomy. observed to be $1' \, 57''{\cdot}7$ below the parallel of the same star, and at $68° \, 14'$ zenith distance. Hence

$$\begin{array}{ll} \text{from} & 1' \; 57''{\cdot}7 \\ \text{take} & 1 \;\; 25{\cdot}8 \\ \hline \end{array}$$

$$\text{angle APB} = p + p' = 0 \;\; 31{\cdot}9$$

$$\text{Whence } P = \frac{31{\cdot}9''}{\sin 25° + \sin 68° \, 14'} = 23{\cdot}6$$

The horizontal parallax of Mars being thus determined, we may hence find that for the sun; for we have seen that, generally,

$$P = \frac{CA}{Cs} \quad \text{(see fig. 36)}$$

Consequently, since CA, the radius of the earth, is constant, the parallax will vary reciprocally as the distance of the body; knowing, therefore, the proportional distance of the sun and Mars from the earth at the time of observation; the parallax of the former may be determined when that of the latter is given. This method, however, of determining the solar parallax is not sufficiently accurate for the purposes of modern astronomy.

120. Method of determining the parallax in right ascension and declination.

Parallax in right ascension and declination. Fig. 39.

Let EQ (fig. 39) be the equator, P its pole, Z the zenith, v the true place of the body, and r the apparent place, as depressed by parallax in the vertical circle Zk, and draw the secondaries Pva, Prb, then ab is the parallax in right ascension, and rs in declination.

Now $\quad vr : vs :: \text{rad} : \sin vrs$ or ZvP

and $\quad vs : ab :: \cos va : \text{rad}$

Hence by multiplication, and rejecting the like factors,

$$vr : ab : \cos va : \sin ZvP$$

$$\text{therefore} \quad ab = \frac{vr \sin ZvP}{\cos va}$$

but $\quad vr = \text{hor. par. (P)} \times vZ$

and $\quad \sin vZ : \sin ZP :: \sin ZPv : \sin ZvP$

$$\text{hence} \quad \sin ZvP = \frac{\sin ZP \times \sin ZPv}{\sin vZ}$$

whence by substitution

$$ab = \frac{P \times \sin ZP \times \sin ZPv}{\cos va}$$

Hence it follows, that for the same star, where the *hor. par.* or (P) is given, the parallax in right ascension varies as the sine of the hour angle.

Also,

$$\textit{hor. par.} = \frac{ab \cos va}{\sin ZP \times \sin ZPv}$$

In the eastern hemisphere, the apparent place b lies on the equator to the east of a its true place, and therefore the right ascension is diminished by parallax; but in the western hemisphere b lies to the west of a, and therefore the right ascension is increased. Hence, if the right ascension be taken before and after the meridian, the whole change of parallax in right ascension between the two observations, is the sum (s) of the two parts before and after the meridian, we have therefore

$$s = \frac{vr}{\cos va} \times S$$

S denoting the sum of the sines of the two hour angles, and

$$\textit{hor. par.} \text{ (P)} = \frac{s \cos va}{\sin ZP \times S}$$

Plane Astronomy.

On the meridian there is no parallax in right ascension.

Illustration of this method.

In order to apply this rule, observe the right ascension of the planet when it passes the meridian, compared with that of a fixed star, at which time there is no parallax in right ascension; about six hours after, take the difference of their right ascensions again, and observe how much the difference (d) between the apparent right ascension of the planet and fixed star has changed in that time. Next observe the right ascension of the planet for three or four days when it passes the meridian in order to get its true motion in right ascension. Then if its motion in right ascension in the above interval of time, between the taking of the right ascensions of the fixed star and planet on and off the meridian, be equal to d, the planet has no parallax in right ascension; but if it be not equal to d, the difference is the parallax in right ascension, and hence, on the above principles, the horizontal parallax will be known. Or one observation may be made before the planet comes to the meridian and another after, by which a greater difference will be obtained.

By example.

121. In order to illustrate this method by an example, let the following be taken:

On August 15, 1719, Mars was very near a star of the 5th magnitude in the eastern shoulder in Aquarius: and at 9h. 18m. in the evening, Mars followed the star in $10' \, 17''$; and on the 16th, at 4h. 21m. in the morning, it followed it in $10' \, 1''$, therefore in that interval, the apparent right ascension of Mars had increased $16''$ in time.

But according to observations made in the meridian for several days after, it appeared that Mars approached the star only $14''$ in that time, from its proper motion; therefore $2''$ in time or $30''$ in motion was the effect of parallax in the interval of the observations.

Now the declination of Mars was 15°

$$\begin{array}{ll} \text{the co-latitude} & 41° \; 10' \\ \text{the two hour angles} & \begin{cases} 49° \; 15' \\ 56° \; 39' \end{cases} \end{array}$$

Consequently, the horizontal parallax

$$P = \frac{30'' \times \cos 15°}{\sin 41° \, 10' \, (\sin 49° \, 15' + \sin 56° \, 39')} = 27\tfrac{1}{2}$$

At the time of these observations, the distance of the earth from Mars was to its distance from the sun as $37 : 100$; whence the sun's horizontal parallax is found to be $10{\cdot}17''$.

Effect of parallax in latitude and longitude

122. Besides the effect of parallax in right ascension and declination, it is manifest that the latitude and longitude of the moon and planets must also be effected by it, and as the determination of this, in respect to the moon, is, in many cases, particularly in solar eclipses, of great importance, we shall proceed to show how it may be computed, supposing the latitude of the place, the time, and consequently the sun's right ascension, the moon's true latitude and longitude, and her horizontal parallax to be given.

Fig. 40.

Let HZR (fig. 40) be the meridian, ♈EQ the equator, p its pole; ♈C the ecliptic, P its pole; ♈ the first point of Aries, HQR the horizon, Z the zenith, ZL a secondary to the horizon, passing through the true place r, and apparent place t, of the moon; draw Pt, Pr, which produce to s, drawing the small

Astronomy.

circle ts parallel to ov, and rs is the parallax in latitude, and ov the parallax in longitude. Draw the great circles ♈P, PZAB, Ppde, and ZW perpendicular to pe; then as ♈P = 90°, ♈p = 90°, ♈ must be the pole of Ppde, and therefore d♈ = 90°; consequently, d is one of the solstitial points; viz. either ♋ or ♑, draw also Zx perpendicular to Pr, and join Z♈, p♈. Now ♈E, or the angle ♈pE, or Zp♈ is the right ascension of the mid heaven which is known; PZ = AB, (because A Z the complement to both) the altitude of the highest point A of the ecliptic above the horizon, called the nonagesimal degree, and ♈A, or the angle ♈PA, is its longitude.

Now in the right angled triangle ZpW, we have Zp the co-latitude, and the angle ZpW, the difference between the right ascension of the mid heaven ♈pE and ♈e, to find pW; hence PW = pW ∓ pP, where the upper sign is to be taken when the right ascension of the mid heaven is less than 180°, and the under when greater.

Again, in the triangle WZp, WZP, we have sin Wp : sin WP :: cot WpZ : cot WPZ, or tan AP♈ and as we know ♈o, or ♈Po, the true longitude of the moon, we know APo, or ZPx: also cos WPZ, or sin ♈PZ : rad :: WP : tan ZP.

Hence in the triangle ZPr, we know ZP, Pr, and the angle P, from which the angles ZrP, or trs, and Zr may be found; for in the right angled triangle ZPx we know ZP, and the angle P to find Px, therefore we know rx, and hence, as the sines of the segments of the base of any triangle are inversely as the tangents of the angles at the base adjacent to which they lie, we may find the angle Zrx, with which and rx, we may find Zr, the *true* zenith distance, to which, as if it were the *apparent* zenith distance find the parallax by (art. 117) and add it to the true zenith distance, and we shall have very nearly the apparent zenith distance, corresponding to which find the parallax rt; then in the right angled triangle rst, which may be considered as plane, we know rt and the angle r to find rs the parallax in *latitude*; find ts, which multiplied by the secant of tv, the apparent latitude gives the arc ov the parallax in longitude.

Example.

Example.

123. On January 1, 1771, at 9 hours apparent time, in latitude 53° north, the moon's true longitude was 3s. 18° 27′ 35″, and latitude 4° 5′ 30″ S; also its horizontal parallax was 61′ 9″, to find its parallax in latitude and longitude.

The sun's right ascension was by the tables 282° 22′ 2″, and its distance from the meridian 135°, also the right ascension ♈E of the mid heaven was 57° 22′ 2″, hence the whole operation for the solution of the triangles may stand thus :

Triangle ZpW

	° ′ ″		
ZpW	= 32 37 58	cos	9·9253864
Zp	= 37 0 0	tan	9·8871144
pW	= 32 23 57	tan	9·8025008
Pp	= 23 28 0		
PW	= 55 51 57		

Triangle WpZ

pW	= 32 23 57	AC sin	0·2709855
PW	= 55 51 57	sin	9·9178865
ZpW	= 32 37 53	cot	10·1935941
AP♈	= 67 29 8	tan	10·3824661
oP♈	= 108 27 35		
oPA	= 40 58 27		

Triangle WPZ

APZ	= 67 29 8	sin	9·9655700
WP	= 55 51 57	tan + 10 −	20·1688210
ZP	= 57 56 36	tan	10·2032510

Triangle ZPx

ZP	= 57 56 36	tan	10·2032555
ZPx	= 40 58 27	cos	9·8779500
Px	= 50 19 33	tan	10·0812055
Pr	= 94 5 30		

Triangles ZPx, Zrx

rx	= 43 45 57	AC sin	0.1600743
Px	= 50 19 33	sin	9·8863144
ZPx	= 40 58 27	tan	9·9387676
Zrx	= 44 1 16	tan	9·9851563
Zrx	= 44 1 16	cos + 10	19·8567795
rx	= 43 45 57	tan	9·9812846
Zr	= 53 6 10	cot	9·8754949
Zr	= 53 6 10	sin	9·9029362
hor. par.	= 61′ 9″ = 3669″	log	3·5645477
rt, uncorrected 2934″ = 48′ 54″		log	3·4674839
App. Z. Dist. Zt 53° 55′ 4″ nearly		sin	9·9075042
hor. par 3669″		log	3·5645477
par. rt corrected = 2965″ = 49′ 25″		log	3·4720519

Triangles trs

par. rt corr. 2965 = 49′ 25″	log	3·4720519
trs = 44° 1′ 16″	cos	9·8567795
rs par. in lat. = 2132″ 35′ 32″	log	3·3288314
rt corrected = 2965″	log	3·4720519
trs = 44° 1′ 16.	sin	9·8419369
ts = 2061″ = 34′ 21″	log	3·3139888
true lat. ro = 4° 5′ 30″		
app. lat. tv = ro − rs = 4° 41′ 2″	sec	10·0014528
ov par in long. = 2067″ = 34′ 27	log	3·3154416

The value of tv is equal to ro ∓ rs, according as the moon has north or south latitude.

124. The above operation supposes the moon's horizontal parallax to have been determined. According to the tables of Mayer, the greatest parallax of the moon, or when she is in her perigee and in opposition, is 61′ 32″; the least parallax (or when she Remarks.

Plane Astronomy.

Astronomy. is in her apogee and in conjunction) is 53′ 52″ in the latitude of Paris: now as the parallax varies inversely as the distance, the parallax at the mean distance of the moon is 57′ 24″ viz. an harmonical mean between the two former.

But Delambre recalculated the parallax from the same observations from which Mayer calculated it, and found a slight disagreement in the two results. He made the equatorial parallax 57′ 11″·4; Lalande made it 57′ 5″ at the equator, 56′ 53″·2 at the pole, and 57′ 1″ for the mean radius of the earth; upon a supposition that the ratio of the equatorial and polar axes was as 300 : 299.

Assuming then the mean parallax to be 57′ 1″, we have, referring to fig. 36,

AC : mean radius r :: D, dist. of moon : sin 57′ 1″

hence D = 60·3 rad = 60·3 × 3964 = 239029 miles the mean distance of the moon from the earth.—*Vince's Astronomy.*

The preceding methods by which the parallaxes of the moon and of Mars have been determined are not sufficiently exact for us to employ them in determining that of the sun. And since this is in astronomy a most important element, and requires the most exact determination, it has, as we have before remarked, engaged the most anxious attention of philosophers, and no one has rendered in this respect a more essential service to the science than Dr. Maskelyne, our late worthy astronomer royal; but the method which he employed, and which was first pointed out by the celebrated astronomer Dr. Halley, cannot be with propriety illustrated in this place, because it supposes the planetary motions to be determined to the utmost accuracy: in a subsequent chapter we shall, however, enter at some length upon the explanation of this method, at present it will be sufficient to state that it depends upon the transit of either of the two inferior planets, but particularly that of Venus over the sun's disc; and that it has been thus found to be 8″·75 according to Maskelyne, but 8″·81 according to Laplace; whereas we have seen (art. 121) that as deduced from observations on Mars, it was found to be 10″·17.

From what has been stated it appears, that the parallaxes of the planets answer two important purposes, for we hence may determine their actual distances from the earth, and moreover, without a knowledge of the quantity of this important datum, we should be unable to correct our observations, and much uncertainty would consequently attend all our deductions. The parallax of the sun being very small, its mean horizontal parallax may be considered as constant, viz. 8″·75, and consequently, the parallax for any altitude is readily determined by means of the formula

$$p = \mathrm{P} \times \text{sin zen. dist., or}$$
$$p = 8''{\cdot}75 \times \text{sin zen. dist.}$$

But for the moon, the parallax being considerable, we ought, in delicate observations, to compute it for her actual distance, and accordingly, in the *Nautical Almanack*, we find the lunar parallax, as well as her semi diameter, stated for 12 o'clock, both at noon and midnight, for every day in the year.

125. Let us now propose an example to show the method of introducing the corrections for parallax and refraction, agreeably to what has been taught in the preceding articles. Plane Astronomy.

Observation.

	°	′	″	
Alt. ☉ upper limb....	62	44	11	
Lower limb....	62	15	41	
2)	124	59	52	
App. alt. ☉ centre ..	62	29	56	
App. zen. dist.	27	30	4	sin = 4617
+ Refraction........	0	0	29	
	27	30	33	
(8″¾ × ·4617) parallax	0	0	4	
Corr. alt. ☉'s centre..	27	30	29	
Latitude of the place..	48	50	14	
☉'s declination......	21	29	45	

3. *Of the correction for aberration.*

126. In our historical chapter, we have given some account of this important astronomical discovery; it remains for us in this place to enter a little more at length into an illustration of the principles, and to describe the method of computing and applying the requisite corrections. Aberration.

The situation of any object in the heavens is determined by the position of the axis of the telescope, attached to the instrument with which we measure; for such a position is given to the telescope, that the rays of light from the object may descend down the axis, and in that situation, the index shows the angular distance required; if, therefore, the observer were in a state of absolute rest, while he was making his observation, the direction of his telescope would coincide with that of the object, as it would also, although he were in motion, if light was instantaneously transmitted from the luminous body to the eye. But if, as is actually the case, the motion of light is progressive, while the observer is also carried forward in space, then, except in the particular instance in which both motions take place in the same line, a different direction must necessarily be given to the axis of the telescope; and consequently, the place measured in the heavens will be different from the true place.

127. This may be illustrated in a general manner as follows. Let S′ (fig. 41) be a fixed star, VF the direction of the earth's motion, S′F the direction of a particle of light, entering the axis ac of the telescope, at a, and moving through aF, while the earth moves from c to F; and if the telescope be kept parallel to itself, the light will descend in the axis. For let the axis nm, Fw, continue parallel to ac; and if each motion be considered as uniform, that of the spectator occasioned by the earth's rotation being disregarded on account of its being too small to produce any sensible effect, the spaces described in the same time will preserve the same proportion; but cF and Fa being described in the same time, and as we have Illustrated. Fig. 41.

$$c\mathrm{F} : \mathrm{F}a :: cn : av:$$

it follows, that cn and av will be described in the same time; therefore, when the telescope is in the situation nm, the particle of light will be at v in the telescope; and the case being the same at every

Astronomy.

moment of its descent, the place measured by the telescope at F is s', and the angle $S'F's'$, is the aberration or the difference between the true place of the star and the place measured by the instrument.

Hence it appears, that if we take

FS : Ft :: vel. of light : vel. of the earth,

join St and complete the parallelogram Ft Ss, the aberration will be equal to FSt; S will be the true place of the star, and s the place measured by the instrument, and this latter is the same with the apparent place of the object as it would be seen by the naked eye. This will appear as follows.

The place shown by the telescope the same as shown to the naked eye.

128. If a ray of light fall upon the eye in motion, its relative motion with respect to the eye, will be the same as if equal motions were impressed in the same direction upon each, at the instant of contact; for equal motions in the same direction impressed upon two bodies will not effect their relative motions, and therefore the effect one upon the other will not be altered. Let then VF be a tangent to the earth's orbit at F, and therefore represent the direction of the earth's motion at that instant, and S′ a star: join S′F and produce it to G; take

FG : Fn :: vel. of light : vel. of the earth,

and complete the parallelogram FGHn; join also FH: then since FG, and Fn represent the motions of light and of the earth, we shall have on the principle of the composition of motion explained (art. 24) Mechanics, FH, for the corresponding resulting motion; that is, the object will appear in the direction of the diagonal FH, and GFH or its equal $S'Fs'$ or FSt will be the aberration; consequently, the apparent place to the naked eye is the same as that determined by the telescope.

Ratio of vel. of light to the vel. of earth.

129. Now we have by trigonometry,

sin FSt : sin FtS :: Ft : FS :: vel. of earth : the vel. of light : whence

$$\sin FSt = \sin FtS \times \frac{Ft}{Fs}$$

or $$\sin \text{of aberration} = \sin FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$$

therefore considering the ratio of the velocity of light and of the earth as constant, the sine of aberration or the aberration itself, will vary as the sine of FtS, and is therefore greatest when that angle is a right angle, and it will be zero when the angle FtS vanishes, that is, when the motions of the earth and of light are made in the same right line.

By observations it has been determined, that the greatest effect of aberration is 20″, and as this corresponds to the case of FtS = 90°, or sin FtS = 1 we have

$$\sin 20'' = 1 \times \frac{\text{vel. of earth}}{\text{vel. of light}}$$

vel. of earth : vel. of light as sin 20″ : rad. :: 1 : 10314

Compared with other observations.

130. This result is obtained from observation, and is independent of any deductions drawn from the actual velocity of the luminous rays; it only shows that if light moved with the velocity indicated, that such phenomena ought to have place, and that it may therefore be employed as an illustration of them. But the actual velocity of light has been otherwise determined; let us see, therefore, how nearly the results in the two cases agree with each other.

For this purpose, call the radius of the earth's orbit r, and we shall have $3{\cdot}1416 \times 2r$ for the circumference which is described by the earth in 365¼ days, or 31557600 seconds; consequently, $\frac{3{\cdot}1416 \times 2r}{31557600}$ will be the velocity per second, and by the above determination,

$$\frac{10314 \times 3{\cdot}1416 \times 2r}{31557600} = \text{the velocity of light per second.}$$

Hence as $\frac{10314 \times 3{\cdot}1416 \times 2r}{31557600} : 2r : 1$ sec.

$$\frac{31557600}{10314 \times 31416} = 97''4 \text{ or } 16\tfrac{1}{4} \text{ minutes, the time}$$

light employs in traversing a space equal to the diameter of the earth's orbit, which agrees very nearly with the results previously deduced by astronomers from observations on the eclipses of Jupiter's satellites.

Plane Astronomy.

Considered as a proof of the earth's revolution.

The coincidence of these deductions, founded upon observations wholly independent of each other is generally considered as furnishing one of the most satisfactory proofs of the truths of the Copernican or modern system of astronomy.

Direction of aberration.

131. The aberration $S's'$ lies from the true place of a star in a direction parallel to that of the earth's motion, and towards the same part; and its effect is therefore produced at one time wholly in declination, at another wholly in right ascension, and at others, it will effect both these quantities, but in a greater or less degree, according to circumstances.

Fig. 42.

In order to illustrate this, let E, E′, E″, E‴, (fig. 42) represent four positions of the earth when the sun is in the signs ♈, ♑, ♎, and ♋, and let tT, t'T′, t''T″, &c. be tangents to the earth's orbit at those points; s, s' the same star seen on the meridian of any place. Now in the position E, corresponding to the vernal equinox, the sun is in the equator; therefore, if Pmp be a meridian passing through the poles of the equator and ecliptic, Pmp will coincide with the solstitial colure, and a line drawn from S to E will be perpendicular to the plane of the meridian Pmpn: therefore, if the plane of the ecliptic EE′E″, be conceived to lie in the plane of the paper, that of Pmpn must be perpendicular to it; and SE will be also perpendicular to the tangent tT, which tangent, therefore, is in the plane of the meridian Pmpn.

Now the earth being supposed to rotate on its axis in the direction nEm, and SE being perpendicular to the plane mPpn, the position as shown in the figure corresponds to that of 6 o'clock in the evening; and supposing the star s, to be on the meridian at this instant, it is obvious that it will be situated in a plane corresponding with the line of the earth's motion; the aberration will therefore produce its effect wholly in that plane, and will tend to elevate the star s, from s to s', that is, it will increase its declination, or diminish its zenith distance; and it is obvious that a directly contrary effect will be produced in the opposite position E″ corresponding to the autumnal equinox; for the direction of light being then in the line SE″, while that of the earth is from E″ towards t'', and in the plane of sE″t'' the aberration will be produced wholly in the same plane, and will tend to depress the star; that is, to diminish its declination; or to increase its zenith distance.

...mer solstice, ...the meridian ...meridian at 12 ...$E't'$ will be per- ...eridian; therefore, ...ce in a plane passing ...ill take place in a plane ...f the meridian $P'm'p'$, and ...o the right or west of the ..., the declination of the star, ... such plane, will not be at all ...ht ascension only, which will be ...the opposite position E''' corres- ...inter solstice, the effect will still be ...pt that as the motion of the earth is ...osite to that at E', the effect of the aber- ...be to increase the right ascension of the star, ...f diminishing it as in the former instance.

...e in the particular instances referred to it appears, ...That a star which comes to the meridian at 6 ...lock in the evening, on the 20th March, will be ...fected by aberration only in its declination which will be increased.

2. That a star which comes to the meridian at 12 o'clock at noon, will be affected only in right ascension, which will be diminished.

3. The star which passes the meridian at 6 o'clock in the morning, September 23d, will like that which passes on March 20 have its declination only affected, which in this case will be diminished.

4. The star which comes upon the meridian at 12 o'clock at night, Dec. 23, will experience the effect of aberration only in right ascension, which will be increased.

Directly the reverse of all this will happen with respect to a star whose right ascension is twelve hours less than that we have supposed; for then the declination will be diminished in the first case and increased in the third; and the right ascension increased in the second and diminished in the fourth, as is obvious from what is above stated.

The star γ Draconis on which Bradley first made his observations, corresponds very nearly in position with the supposition in the first case above.

This particular instance has been selected for the purpose of showing, that in particular cases the effect of aberration may be wholly in right ascension or declination; but generally both these quantities will require correction, and consequently also the latitude and longitude, and therefore we shall now proceed to show how these effects may be separated from each other and computed; in which we shall follow the method laid down by professor Vince in his Treatise on Astronomy.

To estimate the effect of aberration in latitude and longitude.

Aberration in latitude and longitude. Fig. 43.

132. Let ABCD (fig. 43) be the earth's orbit supposed to be a circle, with the sun in the centre at x; let Px be a line drawn from x perpendicular to ABCD, P representing the pole of the ecliptic: let S be the true place of the star, and conceive $apcq$ to be the circle of aberration parallel to the ecliptic, and $abcd$ the ellipse into which it is projected; let γT represent an arc of the ecliptic, and draw to it a secondary PSG, which will coincide with the minor axis bd, into which the diameter pq is projected. Draw ... and it is parallel to pq; and BxD perpendicul... AC, must be parallel to the major axis ac; th... when the earth is at A, the star is in conjunction, and in opposition when at C. Now as the place of the star in the circle of aberration is always 90° before the earth in its orbit, when the earth is at A, B, C, D, the apparent place of the star in the circle will be at a, p, c, q, or in the ellipse at a, b, c, d: hence to find the place of the star in the circle when the earth is at any point E, take the angle $pSs = ExB$, and s will be the corresponding place of the star in the circle. To find its projected place in the ellipse, draw sv perpendicular to Sc, and vt perpendicular to the same; in the plane of the ellipse join st, and it will be perpendicular to vt, because the projection of the circle into the ellipse is in lines perpendicular to the latter. Draw the secondary $PvtK$, which will, as to sense, coincide with vt; unless the star be very near the pole of the ecliptic; and the rules here given will be sufficiently accurate except in that case. Now as cvS is parallel to the ecliptic, S and v must have the same latitude, hence vt is the aberration in latitude; and as G is the true and K the apparent place of the star in the ecliptic, GK is the aberration in longitude.

In order to find these quantities, we may observe, that the angle sSc or CxE is the angle of the earth's distance from syzygies, and as the angle svt = complement of stars latitude, vst will be the latitude itself. Now putting Sa, or $Ss = r = 20''$ the greatest effect of aberration; we have in the right angled triangle Ssv.

$$\text{rad} = 1 : \sin sSc :: r : vs = r \sin sSc$$

and in the triangle vts

$$1 : \sin vst :: vs : tv = r \sin sSc \sin vst$$

Therefore, $r \sin sSc \sin vst =$ *aberration in latitude*

Again, in the triangle Ssv, we have

$$1 : \cos sSc : r : vs = r \cos sSc$$

But $\cos vst : 1 :: Sv : \text{GK}$

or $\cos vst : 1 :: r \cos sSc : \dfrac{r \cos sSc}{\cos vst} = \text{GK}$

the *aberration in longitude*.

133. When the earth is in syzygies, $\sin sSc = o$, therefore, there is then no aberration in latitude, and as $\cos sSc$ is then greatest, the aberration in longitude is at its maximum, as we have already explained in a particular case with reference to declination and right ascension.

If the earth be at A, or the star in conjunction, the apparent place of the star is at a, and reduced to the ecliptic at H, therefore GH is the aberration which diminishes the longitude of the star, the order of the signs being γ, G, T; but when the earth is at C, or the star in opposition, the apparent place c reduced to the ecliptic is at F, and the aberration GF increases the longitude, hence the longitude is the greatest when the star is in opposition, and least when in conjunction; corresponding with what has been already shown in a particular case (art. 131) with respect to the right ascension.

When the earth is in quadratures at D or B, then $\cos sSc = o$, and $\sin sSc$ is greatest; there is, therefore, then no aberration in longitude, while that in latitude is the greatest. When the earth is at D, the apparent place of the star is at d, and the latitude is increased, but when at B, the apparent place being at b, the latitude is diminished. Hence the latitude is least in quadrature before opposition, and greatest in quadrature after. From the mean of a great number of observations, Dr. Bradley determined the value of the greatest aberration to be 20″ as we have already stated (art. 129.).

Deductions. 134. It follows from what has been shown above,

1. That the greatest *aberration in latitude* is equal to 20″ multiplied by the sine of the star's latitude, and that the aberration in latitude *for any time* is equal to 20″ multiplied by the star's latitude and by the sine of the elongation found for the same time; and that it is subtractive before opposition and additive after it.

2. The *greatest aberration in longitude* is equal to 20″ divided by the cosine of the latitude; and the aberration for *any time* equal to 20″ multiplied by the cosine of the elongation, and divided by the cosine of the latitude; it will be subtractive in the first and last quadrant of the argument, or of the difference between the longitude of the sun and star, and additive in the second and third.

Examples.

1. Find the greatest aberration of γ *Ursa Minoris*, whose latitude is 75° 13′.

Here sin 75° 13′ is ·9669
Mult. by 20″

19″·34 the *greatest aberration in lat.*

$$\text{And } \frac{20''}{\cos 75^\circ\ 13'} = \frac{20''}{\cdot 2551} = 78''\cdot 4 \text{ the } \textit{greatest}$$

aberration in longitude.

2. Required the aberration in latitude and longitude of the same star when the earth is 30° from syzygies.

sin 30° = m = ·5; hence

20″ × (sin 75° 13′) + ·5 = 9″·67 aberration in latitude; and since cos 30° = ·866

$$\frac{20'' + \cdot 866}{\cdot 2551} = 67''89 \text{ the aberration in longitude.}$$

In the case of the sun, we have always sin $s\,S\,c$ = o and cos = 1, also cos lat. = 1; and, consequently, 20″ × sin $s\,S\,c$ = o, or there is no aberration in latitude, and the aberration in longitude is constant and equal to 20″. This quantity 20″ of aberration of the sun answers to its mean motion in 8′ 7″½; and is therefore the time in which light moves from the sun to the earth, at its mean distance, agreeably to what we have already stated (art. 130.) Hence the sun always appears 20″ behind its true place.

The following table is intended to expedite the calculation in the preceding cases.

The argument for the longitude is *long sun — long star*. The argument for the latitude is *long sun — long star — 3 signs.*

Degree.	0 VI. − +	I VII. − +	II. VIII. − +	Degree.
	″	″	″	
0	20·00	17·32	10· 0	30
1	20·00	17·14	9·70	29
2	19·99	16·96	9·39	28
3	19·97	16·77	9· 8	27
4	19·95	16·58	8·77	26
5	19·92	16·58	8·45	25

Table.—*continued.*

Degree.	0 VI. − +	I. VII. − +	II. VIII. − +	Degree.
6	19·89	16·18	8·13	24
7	19·85	15·97	7·81	23
8	19·81	15·76	7·49	22
9	19·75	15·54	7·17	21
10	19·70	15·32	6·84	20
11	19·63	15· 9	6·51	19
12	19·56	14·86	6·18	18
13	19·49	14·63	5·85	17
14	19·41	14·39	5·51	16
15	19·32	14·14	5·18	15
16	19·23	13·89	4·84	14
17	19·13	13·64	4·50	13
18	19· 2	13·38	4·16	12
19	18·91	13·12	3·81	11
20	18·80	12·86	3·47	10
21	18·67	12·59	3·12	9
22	18·54	12·21	2·78	8
23	18·41	12· 4	2·44	7
24	18·27	11·76	2· 9	6
25	18·13	11·47	1·74	5
26	17·98	11·18	1·40	4
27	17·82	10·89	1·50	3
28	17·66	10·60	0·70	2
29	17·49	10·30	0·35	1
30	17·32	10· 0	0· 0	0

135. For the aberration *in longitude,* multiply the corresponding quantities in the table, by the secant of the star's latitude. Application of the table.

For the aberration *in latitude* multiply the quantities taken from the table by the sine of the star's latitude.

Example.

1. Let the longitude of the sun be 7s 5° 18′, the longitude of the star 5s. 11° 14′, and its latitude 31° 10′; to find its aberration in latitude and longitude.

long. ☉ 7s. 5° 18′
long. ✳ 5 18 14′

1 17 4 correspond in tab. to 13″·62
see 31° 10′........ 1″·69

aberration in long. — 15″·92 product

For the latitude.

	s	°	′
long.☉—long. ✳ =	1	17	4
— 3 signs	3	0	0

10 17 4 cor. t a .—14″·65
sin 31° 10 0·5175

aberration in latitude — 7·58 prod.

To find the aberration in declination and right ascension.

136. Let AEL (fig. 44) represent the equator, p its pole, ACL the ecliptic, P its pole, S the true place of a star, s the apparent place in the ellipse; draw the great circles PSa, Psb, pSw, pSv, and St, Sv per- Aberration in right ascension and declination.

Astronomy. pendicular to pv, Pb. Now vs, and vS, being found as in the preceding articles, we shall have,

Fig. 44.

$$\frac{v\mathrm{S}}{vs} = \tan \mathrm{S}sv$$

which angle hence becomes known.

Again in the triangle Psp, the three sides being given, compute the angle of position Psp, and hence find

$$\mathrm{S}sp = \mathrm{S}sv \mp \mathrm{P}sp$$

Then again,

$$\cos \mathrm{S}sv : \cos \mathrm{S}sp :: sv : st = \frac{\cos \mathrm{S}sp.\, sv}{\cos \mathrm{S}sv} =$$

aberration in declination.

and $\sin \mathrm{S}sv : \sin \mathrm{S}sp :: \mathrm{S}v : \mathrm{S}t = \dfrac{\sin \mathrm{S}sp.\mathrm{S}v}{\sin \mathrm{S}sv}$ Plane Astronomy.

Consequently,

$$vw = \frac{\mathrm{S}t}{\cos \mathrm{dec.}} = \frac{\sin \mathrm{S}sp.\ \mathrm{S}v}{\sin \mathrm{S}sv.\cos \mathrm{dec.}} = \text{the } aberration$$

in right ascension.

137. Notwithstanding the process of solution is rendered very obvious by the preceding investigation, it still involves considerable numerical computation, to facilitate which, different tables and formulæ have been contrived, of which the following, due to Delambre, is deserving of preference.

General TABLES *for the aberration of the fixed stars.*

Note.—A = right ascension, D the declination of the star, and S = the longitude of the sun.

TABLE I. Arg. A – S.				
Signs.	0 VI.	I.—VII.	II. VIII.	S.
Deg.	— +	— +	— +	Deg.
	″	″	″	
0	19·17	16·60	9·59	30
1	19·17	16·43	9·30	29
2	19·16	16·26	9·00	28
3	19·15	16·08	8·70	27
4	19·13	15·89	8·40	26
5	19·10	15·71	8·10	25
6	19·07	15·51	7·80	24
7	19·53	15·31	7·49	23
8	18·99	15·11	7·19	22
9	18·94	14·90	6·87	21
10	18·88	14·69	6·56	20
11	18·82	14·47	6·24	19
12	18·75	14·25	5·93	18
13	18·68	14·02	5·61	17
14	18·60	13·79	5·28	16
15	18·52	13·56	4·96	15
16	18·43	13·32	4·64	14
17	18·33	13·08	4·31	13
18	18·23	12·83	3·99	12
19	18·13	12·56	3·66	11
20	18·02	12·32	3·33	10
21	17·90	12·07	3·00	9
22	17·78	11·80	2·67	8
23	17·65	11·54	2·34	7
24	17·52	11·27	2·00	6
25	17·38	11·00	1·67	5
26	17·23	10·72	1·34	4
27	17·08	10·44	1·00	3
28	16·93	10·16	0·67	2
29	16·77	9·87	0·33	1
30	16·60	9·59	0·00	0
Deg.	— +	— +	— +	Deg.
Signs.	XI. V.	X. IV.	IX. III.	S.

TABLE II. Arg. A + S.				
S.	0 VI.	I. VII.	II. VIII.	S.
Deg.	+ —	+ —	+ —	Deg.
	″	″	″	
0	0·83	0·72	0·41	30
1	0·83	0·71	0·40	29
2	0·82	0·70	0·39	28
3	0·82	0·69	0·38	27
4	0·82	0·68	0·37	26
5	0·82	0·67	0·35	25
6	0·82	0·67	0·33	24
7	0·82	0·66	0·32	23
8	0·82	0·65	0·30	22
9	0·82	0·64	0·29	21
10	0·82	0·63	0·28	20
11	0·82	0·62	0·27	19
12	0·82	0·61	0·25	18
13	0·81	0·61	0·24	17
14	0·81	0·60	0·23	16
15	0·80	0·58	0·22	15
16	0·80	0·57	0·20	14
17	0·80	0·56	0·19	13
18	0·79	0·55	0·17	12
19	0·78	0·54	0·15	11
20	0·78	0·53	0·14	10
21	0·77	0·52	0·12	9
22	0·76	0·51	0·11	8
23	0·76	0·50	0·10	7
24	0·75	0·49	0·09	6
25	0·75	0·47	0·07	5
26	0·75	0·46	0·06	4
27	0·74	0·45	0·05	3
28	0·73	0·44	0·03	2
29	0·72	0·43	0·02	1
30	0·72	0·41	0·00	0
Deg.	+ —	+ —	+ —	Deg.
S.	XI. V.	X. IV.	IX. III.	S.

TABLE III. Arg. S + D & S – D.				
S.	0 VI.	I. VII.	II. VIII.	S.
Deg.	— +	— +	— +	Deg.
	″	″	″	
0	3·98	3·45	1·99	30
1	3·98	3·42	1·93	29
2	3 98	3·38	1·87	28
3	3·98	3·34	1·81	27
4	3·97	3·30	1·75	26
5	3·97	3·26	1·68	25
6	3·96	3·22	1·62	24
7	3·95	3·18	1·56	23
8	3·94	3·14	1·49	22
9	3·93	3·10	1·43	21
10	3·92	3·05	1·36	20
11	3·91	3·10	1·30	19
12	3·90	2·97	1·23	18
13	3·89	2·92	1·17	17
14	3·87	2·87	1·10	16
15	3·85	2·82	1·03	15
16	3·83	2·77	0·97	14
17	3·81	2·72	0·90	13
18	3·79	2·67	0·83	12
19	3·77	2·62	0·66	11
20	3·74	2·56	0·69	10
21	3·72	2·51	0·63	9
22	3·70	2·46	0·56	8
23	3·67	2·40	0·49	7
24	3·64	2·34	0·42	6
25	3·61	2·28	0·35	5
26	3·58	2·23	0·28	4
27	3·55	2·17	0·22	3
28	3·52	2·11	0·14	2
29	3·49	2·05	0·07	1
20	3·45	1·99	0·00	0
Deg.	— +	— +	— +	
S.	XI. V.	X. IV.	IX. III.	

Astronomy.

Use of the Tables

For the aberration in right ascension.

Use of the tables.

138. Enter Table I, with the argument A — S, and Table II. with A + S. Then the sum of the two corresponding numbers, multiplied by the sec. of D will be the aberration in *right ascension.*

For the aberration in declination.

Enter Table I. with the argument A — S + 3 signs, and Table II. with A + S + 3 signs, and the sum of the two corresponding numbers multiplied by the sine of D, will be the *first part* of the aberration in declination.

Enter Table III., with the arguments S + D and S — D, by which will be found the other two parts of the aberration in declination, and the sum of the three will give the whole of the *aberration in declination.*

If the declination of the star be south, add 6 signs to the arguments S + D and S — D.

Example.

Illustrated by an example.

Required the aberration of *a* Aquila, Feb. 15, 1819, at 12 o'clock in the evening.

Here by the tables, A = 9s 25° 29′ D = 8° 24′ 25″
S = 10 26 2

A — S = 10s 29° 27′	Tab. I. . . —16″·5
A + S = 8 21 31	Tab. II. . — 0·15
	— 16·6
Sec. dec. 8° 24′ 25″ =	1·0108
Aberration in *right ascen.*	16·66 Product
A — S + 3 signs 1s 29° 27′	Tab. I. —16″·68
A + S + 3 signs 11 21 31	Tab. III. + 0.82
	— 15·86
Sin dec. 8° 24′ 25″	·1461
S = 10 26° 2′ 9″	— 2·317
S + D = 11 4° 26′ 34″	Tab. III. — 3·595
S — D = 10 17 37 44	Tab. III. — 2·945
Aberration in declination	— 8·857

If the declination had been south, the two latter arguments would have been S + D + 6 signs, and S — D + 6 signs, as stated above.

4. *Of nutation.*

Nutation.

139. We have in the preceding articles endeavoured to illustrate the principles of the most important astronomical corrections, and have shown the method of computation; there however still remains for explanation the doctrine of nutation, but as the complete developement of the principles upon which this theory rests, involves considerations of a physical nature which we have not hitherto examined, we must in this place content ourselves with a very general view of the subject, leaving the more minute particulars for our treatise on physical astronomy. By nutation is to be understood a kind of trepidation or tremulous motion of the earth's axis, whereby its inclination to the plane of the ecliptic is not always the same; but vibrates within certain limits, never, however, exceeding a few seconds; the period of variation is also limited to a certain number of revol[illegible] orbit. This inequality in the terrestrial motion was, like that described in the foregoing chapter, discovered by Dr. Bradley, to whom we owe likewise a just explanation of the cause of it, and a near approximation of its effects.

Astronomy.

140. We have observed above, that it is impossible in this place to give more than a very general explanation of this doctrine; it will be perhaps sufficient to observe here, that the first cause of nutation is due to the mutual gravitation of matter, and to the laws which it is known to observe; viz. that it is directly as the mass, and inversely as the square of the distance. If the orbit of the earth were a circle, and the terrestrial globe a perfect sphere, the attraction of the sun would have no other effect than to keep it in its orbit, and would cause no irregularity in the position of its axis; but neither of these conditions takes place, the earth is not a perfect sphere, nor is its orbit a circle: when the position of the earth is such, that the plane of its equator passes through the centre of the sun, the attractive power of the latter body will still have no other tendency than that of drawing the earth towards it, and the parallelism of its axis will not be disturbed: this happens in the equinoxes. But as the earth recedes from these points, the sun deviates so much the more from the plane of the equator, and the latter, in consequence of its protuberance is more powerfully attracted than the rest of the globe, which causes some alteration in its position, that is, in the inclination of its axis to the plane of the ecliptic; and at that part of the orbit, which is described between the vernal and autumnal equinox, is less than that passed over between the latter and the former; it follows that the irregularity caused by the sun, during its passage through the northern signs, is not entirely compensated by that which takes place during the other part of the revolution; and consequently, that the parallelism of the terrestrial axis, and its inclination to the ecliptic will be a little changed. Physical cause of.

141. Again, the same effect which the sun produces upon the earth by its attraction, or at least an analogous effect, is also produced by the moon, which is more powerful in proportion as it is more distant from the equator; and therefore when its nodes concur with the equinoctial points, the power which causes the irregularity in the position of the terrestrial axis acts with the greatest force; and the revolution of the moon's nodes being performed in about eighteen years, the nodes will twice in this period concur with the equinoctial points; and consequently, twice in the same period, or once every nine years, the earth's axis will be more influenced than at any other time; and during this interval, the pole of the earth will describe an ellipse in the heavens, whose transverse axis is 19″·2, and conjugate axis 15″, which correspond with the ratio between the cosine of the obliquity and the cosine of twice the obliquity of the ecliptic; or to the ratio of cos 23° 28′ and cos 46° 56′. Effect of the moon.

142. Let π (fig. 45) be the pole of the ecliptic, and P the mean place of the pole of the equator, ADC a circle whose radius is equal to the semi transverse axis of the ellipse C*d* A described by the pole as above stated, A the true pole of the equator when the ascending node of the moon's orbit is at ♈, and let A be supposed to move contrary to the order of the signs. Take Computed. Fig. 45.