A TEXT-BOOK

OF

GEODETIC ASTRONOMY.

BY

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PREFACE.

(*To be read by the student as well as the teacher.*)

THE purpose of this book is to furnish a text which is sufficiently short and easy to be mastered by the student of civil engineering in a single college term, but which shall give him a sufficiently exact and extensive knowledge of geodetic astronomy to serve as a basis for practice in that line after graduation. Though the book has been prepared primarily for students, the author has endeavored to insert such subject-matter, tables, and convenient formulæ as would make it of value as a manual for the engineer making astronomical observations. In order to make the book sufficiently short it has been necessary to omit all mathematical processes except those actually necessary for developing the working formulæ. And as the object of the work is to teach a certain limited branch of astronomy, rather than to teach mathematics, the simpler and special means of deriving the working formulæ have been chosen, in every case in which there was a chance for choice, instead of the more difficult and general derivation that would naturally be chosen by the mathematician.

The occasion for the book is the fact that in the course of study prescribed for students of civil engineering at Cornell University but five hours per week for one term can be devoted to the text-book work and lectures on astronomy. Under these conditions it is out of the question to use Chauvenet's standard work. Even Doolittle's Practical

Astronomy contains more mathematics than a student can be expected to master thoroughly in that period. Of various other text-books available none seem to fit the special conditions.

In the wording of the book it is tacitly assumed that the observer is in the northern hemisphere. To make the wording general would require too many circumlocutions.

It is assumed that the student has a knowledge of least squares. If, however, he has not such knowledge, it will not debar him from following nearly every part of the text except §§ 107–113, dealing with the treatment of transit time observations by least squares, and §§ 154–157, giving the process of combining the results for latitude with a zenith telescope by that method. If he reads carefully §§ 283–285, stating the technical meaning of the phrase "probable error," the statement of the uncertainty of a given observation in terms of the probable error, or the statement of the errors to be expected from certain sources in such terms, should convey to him a definite meaning.

Considerable space has been devoted in the text to a discussion of the various sources of error in each kind of observation treated. Two separate considerations seem to the author to justify this. One is that the special value of geodetic astronomy as a part of the course of training of an engineer depends largely upon the fact that in studying it he is brought face to face with the idea that instruments are fallible, and that therefore their indications must be carefully scrutinized and interpreted; and that if the best results are to be secured from them, the sources of the various minute errors which combined constitute the errors of observation must be carefully studied. The other consideration is that an observer's success in securing accurate results with moderate effort depends to a considerable extent upon his power to estimate rightly the *relative* importance of the various errors affecting his final result.

The accuracy of a man's thoughts, as well as of his speech, when dealing with a given subject depends largely upon the precision of his understanding of the special vocabulary of that subject. With that idea in view the finder list of definitions given in § 312 has been prepared. The student who is not sure of the exact meaning of a word may turn to this list and so find the exact definition quickly. In reading definitions the context should also be read. When a word is defined in the text it is printed in italics.

The effort has been made to select the formulæ which have been found in practice to lead to accurate and rapid computations. They have been gathered at the end of the volume for convenient reference, and adjacent to each formula will be found references to the corresponding portion of the text, so that for those who may use the book as a manual the list of formulæ with these references may serve as an index or finder for the text.

In the five principal chapters the instrument has first been described, and the adjustments given, as well as directions for observing, and an example of the record. The derivation of the formulæ, the computation, etc., follow. If the text-book work and the practical work of the observatory are carried on together during the same term one naturally wishes the students to become familiar with the instruments and their manipulation as soon as possible. In that case it is recommended that the first portions only of certain chapters be taken and the later portions omitted temporarily. The following order may then be used: §§ 1–27, 37, 51–63, 83–91, 134–146, 177–187, 201–203, 205 to middle of 210, 273–276, 28–50, 64–82, 92–133, 147–176, 188–272, 277 to the end.

During the preparation of this volume the text-books on astronomy written by Chauvenet, Doolittle and Loomis have been freely consulted, as well as various reports of the Coast and Geodetic Survey, of the Northern Boundary Survey, of the U. S. Lake Survey, and the report of the Mexican Boundary Survey of 1892–93. Appendix No. 14 to the Coast Survey Report for 1880, which is written by Assistant C. A. Schott and is used as a manual by the officers of that survey, has been extensively drawn upon as the best exposition of good field methods known to the author. Several tables have been taken from that source, notably the table of factors for the reduction of transit time observations given in § 299. The Superintendent of the Coast Survey has very kindly furnished certain data, and photographs of instruments.

JOHN F. HAYFORD.

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GEODETIC ASTRONOMY.

CHAPTER I.

INTRODUCTORY.

1. THIS book is limited to the treatment of astronomy as applied to surveying, or to what might be called geodetic astronomy. Only such matters are treated as are pertinent to this particular limited branch of the subject. Moreover, the subject as thus limited is treated from the point of view of the engineer who wishes to obtain definite results, rather than from that of a mathematician more interested in the processes concerned than in their final outcome.

2. The bodies considered by the engineer in geodetic astronomy are the stars; the Sun; the planets, including the Earth; the Moon, the Earth's satellite; and to a very limited extent some of the satellites of the other planets. The engineer from his standpoint upon the surface of the Earth sees these different bodies moving about within the range of his vision,—aided by a telescope if necessary. Their apparent motions in the sky as seen by him are quite complicated. His success in locating and orienting himself upon the Earth

by observations upon these heavenly bodies—for that is his particular purpose in observing them—depends first of all upon his having a clear and accurate conception of their apparent motions, and then upon his possession of, and ability to use efficiently, the instruments with which the observations are made. Much of the complexity in the apparent movements of these heavenly bodies is due to the fact that the observer sees them not from a fixed station in space, but from a standpoint upon one of the planets,—the Earth, which is moving rapidly through space with a motion which is in itself quite complicated. He sees then in the *apparent* motion of each heavenly body upon which he gazes not only the actual motion of that body, but also, reflected back upon him, so to speak, he sees the actual motion of the seemingly solid and immovable earth upon which he stands. He is like a passenger upon a train at night who looks out upon the many lights of a town. He sees the lights all apparently in motion. In one case the apparent motion of the particular light may be entirely due to his own motion with the train upon which he is riding, the light itself being at rest. In another case the light may be upon another moving train and its apparent motion will then be due to the actual motion of each of the trains. If the darkness is sufficient to conceal the landscape, he may be at a loss to determine what portions of the apparent motions of the lights are due to his own change of position and what to the motions of the lights themselves. He is then in the position of a man when he first begins to study the apparent movements of the heavenly bodies.

Let us first form concrete conceptions as to the *actual* motion of each of the bodies under consideration, including the Earth itself. We will then be in a position to understand the *apparent* motions.

3. Conceive the Sun to be a very large self-luminous mass of matter. For the present let it be supposed to be fixed in space. Around this central Sun revolve eight planets, namely, in order of their distance from the Sun, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. All these planets move nearly, but not exactly, in the same plane passing through the Sun. The orbit, or path, of any one of them in its own orbital plane is very nearly a perfect ellipse with one focus at the Sun, and the velocity with which the planet moves varies at different parts of its orbit in such a way that the line joining the planet and Sun describes equal areas in equal times. This orbit and law of velocity result from the fact that each planet is pursuing its path in obedience to a single force, gravity, continually directed toward a fixed center, the Sun.

4. The Earth may be taken as a representative planet. It is the most important of the planets *for our present purpose.* It moves about the Sun in an elliptical orbit at a mean distance from the Sun, in round numbers, of 92 800 000 miles.* Though the orbit is an ellipse, its major and minor axes are so nearly equal that if it were plotted to scale the unaided eye could not distinguish it from a circle. The greatest distance of the Earth from the Sun exceeds the least distance by but little more than 3%. The Sun is in that focus of the ellipse to which the Earth is nearest during the winter (of the northern hemisphere). The eccentricity of the ellipse, and therefore the difference of the two axes, is very slowly decreasing. The plane of the Earth's orbit is not absolutely fixed in direction in space. It changes with exceeding slowness—so slowly, in fact, that it is used as one of the astronomical reference planes. Moreover, the position of the

* See "The Solar Parallax and its Related Constants," Harkness, p. 140.

elliptical orbit in the plane is slowly changing; that is, one focus necessarily remains at the Sun, but the direction of the major axis of the ellipse gradually changes.

Roughly speaking, the Earth makes one complete circuit of its orbit in the period of time which is ordinarily called one year. At different portions of the orbit its linear velocity varies according to the law (common to all the planets) that the line joining it and the Sun describes equal areas in equal times. Each portion of the nearly circular path being almost perpendicular to the line joining the Earth and Sun at that instant, the linear velocity is nearly inversely proportional to the distance of the Earth from the Sun. Evidently the angular velocity varies still more largely than the linear, since the greatest linear velocity comes at the same time as the least distance from the Sun, and *vice versa*.

At the same time that the Earth, as a whole, is swinging along in its orbit it is rotating uniformly about one of its own diameters as an axis.* This rotation is so nearly uniform in rate that it is assumed to be exactly uniform and is used to furnish our standard of time. Roughly speaking, the interval of time required for one rotation of the Earth on its axis is what is called one day. The more exact statement will be made later.

5. The axis of rotation of the Earth points at present nearly to the star called Polaris, or North Star, and makes an angle of about $66\frac{1}{2}°$ with the plane of the Earth's orbit, or $23\frac{1}{2}°$ with the perpendicular to that plane. The direction of this axis of rotation is not fixed in space, but changes just as

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* The diameter about which the rotation takes place is, however, not strictly fixed with respect to the Earth,—is not, in other words, always the same diameter,—but varies through a range of a few feet only on the surface of the Earth. See §§ 286–7. For present purposes, however, it will be considered as fixed.

the axis of a rapidly spinning top is seen to wabble about. This change is quite slow, but extends through a large range of motion. It is compounded of two motions called respectively *precession* and *nutation.* By virtue of the motion called *precession* the axis of the earth tends to remain at an angle of about $23\frac{1}{2}°$ with the perpendicular to the plane of the Earth's orbit (usually known as the *plane of the ecliptic*), and to revolve completely around it, describing a cone of two nappes with an angle of about 47° (twice $23\frac{1}{2}°$) between opposite elements. The time required to make one such complete revolution is, at the present rate, about 26 000 years.* The motion of the Earth's axis called *nutation* is compounded of several periodic motions, the principal one of which is such as to cause the axis to describe a cone of which the right section is an ellipse, and of which the greatest angle between opposite elements is about eighteen seconds of arc and the least about fourteen.†

* The change of seasons is caused by the inclination of the Earth's axis to the plane of its orbit. At present the northern end of the axis is inclined directly away from the Sun at about Dec. 21st; the Sun then appears to be farther south than at any other time, and it is winter in the northern and summer in the southern hemisphere. At about June 20th the reverse is true, namely, it is summer in the northern and winter in the southern hemisphere. On account of the precession the winter of the northern hemisphere will occur in June, July, and August about 13000 years hence.

† All the various motions of the planets and their satellites—the peculiar mathematical properties of their orbits, the variability of the planes of the orbits and of the orbits in the planes, etc.—are by celestial mechanics shown to be due simply to the action of gravitation. Or, stating the matter from the converse point of view, given these various bodies in their actual positions and having their actual motions at a given instant, and given the law that gravitation acts between each pair of them with an intensity inversely proportional to the square of their distance apart and directly proportional to the product of the two masses, the position and motion of each one of them, their orbits, etc., at any other stated time may be computed from the principles of celestial mechanics alone. Even the precession and nutation are caused by gravitation and are thoroughly

6. The Earth was taken as representative of the planets. Most or all of the various phenomena which have been indicated in the motion of the Earth are repeated in each of the other planets. Each has an orbital plane of its own which is slightly variable and does not in any case at present make an angle of more than about 7° with the plane of the Earth's orbit. Each moves in that plane in an ellipse of which the eccentricity and position are slowly changing. Each has a rotation about its own axis,—with which, however, the engineer is not concerned.

The Moon is a satellite of the Earth, revolving about it under the action of gravity just as the Earth revolves about the Sun, and in its orbit are again found the same peculiarities as in the orbit of the Earth itself. The orbit of the Moon is an ellipse of variable eccentricity and position, with the Earth in one focus, and lying in a variable plane making an angle of about 5° with the plane of the Earth's orbit. The several variations mentioned are much greater in the case of the Moon than of the Earth, and the motion, moreover, is subject to other perturbations. Its motion is therefore a most difficult one to compute.

Each one of the other planets, except Mercury and

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accounted for by principles of celestial mechanics derived from the above law of gravitation. The luni-solar precession is due to the fact that the Earth is not a sphere, but a spheroid, having an excess of matter in the equatorial regions. One component of the attraction of the Moon and Sun acting upon this equatorial excess tends continually to shift the position of the equator in one direction without changing its angle with the ecliptic. The action of the planets upon the Earth as a whole tends to draw it out of the plane of its orbit, or rather to change the orbital plane. This change is called the planetary precession. The luni-solar and planetary precessions together constitute what is often called simply the precession. Nutation is made up of periodic motions which are due to regular periodic fluctuations in the forces which produce precession. Nutation might be described as the periodic part of precession

Venus, has one or more satellites bearing the same relations to it that the Moon does to the Earth.

The comparatively erratic motions of the numerous asteroids, or small planets, moving in orbits between that of Mars and Jupiter, and of the comets and meteors which occasionally visit the solar system, prevent their use by the engineer.*

The Sun, the eight planets and their satellites, and the asteroids together constitute the solar system.

7. The stars are self-luminous bodies at great distances from the solar system. Their remoteness is to a certain extent indicated by the fact that with the best telescopes and with the highest magnifying powers at present available the image of a star cannot be magnified. It remains with all powers and telescopes a point of light of which the apparent size is merely a measure of the imperfection of the telescope and eye. But the best evidence of the immense distance even to the nearest of the fixed stars is the fact that even though the diameter of the Earth's orbit, 186 million miles, be taken as the base of a triangle of which the vertex is at the star, it is only with the greatest difficulty, if at all, that the angle at the star can be detected even though the instruments used be of the highest order of accuracy and a long series of observations are used. In the few cases in which this angle at the star has been successfully measured it has been found to be not greater than one second of arc. For the purposes of the engineer, then, it may be assumed that each and every star is at so great a distance from the Earth that the true direction in space of the straight line from the Earth to the star is the same at all times of the year notwithstanding the widely separated positions the Earth may occupy in its orbit.

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* For an interesting treatment of comets, meteors, and asteroids, see Young's General Astronomy, and Chamber's Astronomy, pp. 104–109, 278–430, 780–816.

If the stars had no motion relative to each other or to the solar system as a whole, the true direction of the line from the Earth to any one star would not vary from year to year. As a matter of observation, however, it is known that in general the true direction of such a line does change, although the change is exceedingly slow in every case. This change will be treated more in detail in a later chapter.

The *apparent* motion of any particular heavenly body as seen by an observer upon the Earth is the compound result of the motion of the Earth and of that body.

8. In the case of a star, the object observed is for most purposes at what may be considered an infinite distance. The line joining the observer and star preserves, therefore, a sensibly constant direction in spite of the motion through space of the observer upon the Earth. The apparent motion of the star is caused by the rotation of the Earth about its axis and the change in the direction of that axis in space. The rotation of the Earth causes the line of sight to a star to seem to describe at a uniform rate a right circular cone, of which the axis is the line joining the observer with a point in the sky at an infinite distance in the axis of the Earth produced. In other words, the axis of the cone is a line from the observer parallel to the axis of the Earth. Such a line, for any point in the northern hemisphere, pierces the sky in a point not far from the North Star, Polaris. The angle between any element of the cone and its axis is the angle between the line joining observer to star, and the axis of the Earth. This angle is called the *polar distance* of the star,—*north polar distance* if measured from the north end of the Earth's axis. So long as these two lines are fixed in direction in space, the line of sight to the star continues to describe the same right circular cone once for every turn which the Earth makes on its axis. For example, the line of sight to

Polaris makes an angle of about $1\frac{1}{4}°$ with the axis of the Earth, and describes a corresponding right circular cone. Or it may be said that Polaris seems to describe a circle in the sky of which the radius subtends an angle at the eye of $1\frac{1}{4}°$. With a good telescope Polaris may be followed completely around the circle, all of which would be above the horizon for any point in the United States. With the naked eye only that portion of the apparent motion which occurs during the hours of darkness could be observed. For an observer at Ithaca, in latitude $42\frac{1}{2}°$, the cone for a star having a north polar distance less than $42\frac{1}{2}°$ is entirely above the horizon. Given one view of the star and an idea of the position of the vanishing point of the Earth's axis in the sky, an observer is able to trace out the whole apparent path of the star. For Ithaca, a star of north polar distance of $42\frac{1}{2}°$ has its cone tangent to the horizon; and if greater than that value, a part of the cone must be below the horizon, and the star is necessarily invisible on that portion. If the north polar distance is 90°, the cone becomes a plane. Stars still farther south describe a right circular cone about the southern portion of the Earth's axis produced, the angle of the cone being the south polar distance of the star.

In every case the diurnal rotation of the Earth causes the line of sight to a star to describe a right circular cone. But, as has already been stated (§ 5), the direction of the Earth's axis is continually changing slowly, and hence the north polar distance or angle between the Earth's axis and the line joining the observer and star is continually changing. The cone of revolution therefore slowly changes from day to day.

If the object observed is not a star at a practically infinite distance, but a planet, the Moon, or the Sun, at a finite distance, the line joining observer to object describes a surface any small portion of which may be considered to be a portion of the surface of a right circular cone. But the north polar distance of the object now continually changes, not only on account of the change in the direction of the Earth's axis, but still more largely on account of the change in the true direction of the line joining observer and object,—two points which are at a finite distance from each other and both in motion. This last cause also makes the *rate* at which the surface is described variable.

9. The two principal reference planes of astronomy are the plane of the equator and the plane of the Earth's orbit, or, as it is generally called, the *plane of ecliptic.* The *plane of the equator* is a plane passing through the center of the Earth and perpendicular to its axis of rotation. Neither of these two planes, from what has already been written, are fixed in space, nor fixed relatively to each other. Their changes of position are, however, very slow.

10. To avoid the necessity of using cumbersome expressions and circumlocutions, it is convenient to make use of the *celestial sphere* as an arbitrary conception. The *celestial sphere* is a sphere of infinite radius, the eye of the observer being supposed to be at its center. Any celestial object is considered to be projected along the line of sight to the surface of this sphere and is referred to as occupying that position upon the sphere. Then for convenience one may speak of arcs, angles, and triangles upon the celestial sphere instead of using the complicated expressions necessary in speaking always of the actual lines and planes which are under consideration. The sphere is assumed to be of infinite radius so that lines which are parallel and at a finite distance apart will intersect the sphere in the same point, or at least what is sensibly one point, since two points at a finite distance apart must appear as one when seen from an infinite distance. So also parallel planes which are at a finite distance apart intersect the celestial sphere in the same arc. For example, the axis of the Earth and a line parallel to it through the eye of the observer both intersect the celestial sphere in the same pair of points called the *poles of the equator*, or more briefly the *poles*,—north and south respectively. Also the plane of the equator, and a plane parallel to it through the eye of the observer, intersect the celestial sphere in the same great circle which is called the *equator of the celestial sphere*, or more frequently simply *the equator*.

11. The *equator*, the *ecliptic*, *hour-circles*, and the *horizon* are all great circles of the celestial sphere formed by the intersection of various planes with that sphere.

The *ecliptic* is the intersection of the plane of the ecliptic, or, in other words, the plane of the Earth's orbit, with the celestial sphere. The Sun, therefore, is always seen projected on some point of the ecliptic.

An *hour-circle* is the intersection of a plane passing through the Earth's axis with the celestial sphere. All hour-circles are then great circles passing through the poles.

The *horizon* is the intersection with the celestial sphere of a plane passed through the eye of the observer perpendicular to the plumb-line, or line of action of gravity, at the observer. All horizontal lines at a given point on the Earth's surface pierce the celestial sphere in the horizon of that point.

In each of these cases it is evident that the great circle on the celestial sphere would not be changed if the intersecting plane were moved parallel to itself a finite distance,—for instance, to pass through any other point in or upon the surface of the Earth. For example, the horizon may be considered to be the intersection with the celestial sphere of a plane passing through the *center of the Earth* and perpendicular to the observer's gravity line, instead of that given above.

12. The angle between a line joining the center of the

Earth to a star (or other celestial object) and the plane of the equator is called the *declination* of that object. It is measured upon the celestial sphere by that portion of the hour-circle passing through the star which is between the star and the equator. The declination is considered positive when measured north from the equator. It follows from the definition of polar distance (given in § 8) that the declination and polar distance are complements of each other.

The equator and the ecliptic intersect each other at an angle of about 23° 27′. Their two points of intersection on the celestial sphere are called the *equinoxes*. That one at which the Sun is found in the spring is called the *vernal equinox*, and that at which it is found in the fall the *autumnal equinox*. As both the equator and ecliptic move slowly in space the equinoctial points slowly shift in position upon the celestial sphere.

The *right ascension* of a star, or other celestial object, is the angle, measured along the equator, between the two hour-circles which pass through the star and the vernal equinox respectively. In other words, the right ascension is the angle between two planes, one passing through the Earth's axis and the star and the other through the Earth's axis and the vernal equinox. It is reckoned in degrees from 0 to 360, in the direction that would appear counter-clockwise if one looked toward the equator from the north pole,—from west to east. Right ascensions are still more frequently expressed in time, 24 hours being equivalent to 360 degrees.

The *zenith* is the point in which the action-line of gravity produced upward intersects the celestial sphere. The opposite point on the celestial sphere is called the *nadir*.

The intersection with the celestial sphere of a plane passed through its center, the zenith, and the pole is called the *meridian*, and the plane itself is called the *meridian plane*.

The intersection of the meridian plane with the plane of the horizon is called the *meridian line.* It connects the north and south points of the horizon. The intersection with the celestial sphere of a plane through the zenith perpendicular to the meridian plane is called the *prime vertical.* The east and west points of the horizon are in the prime vertical.

13. The angle, measured along the equator, between the meridian and the hour-circle passing through a star (or other celestial object) is the *hour-angle* of the star. In other words, the hour-angle is the angle between the meridian plane and a plane passing through the Earth's axis and the star. Hour-angles are reckoned like right ascensions, either in degrees, minutes, and seconds of arc or in hours, minutes, and seconds of time. In this book hour-angles will be measured for 180° each way from the upper branch of the meridian and will always be considered positive.

The student should distinguish carefully between an hour-angle and a right ascension. Each is an angle between two planes. In each case one of the two planes is defined by the Earth's axis and the star, and therefore changes direction but slowly in space. The second plane concerned in the case of a right ascension is defined by the Earth's axis and the vernal equinox. This plane changes its direction very slowly. So the right ascension of a star is an angle which is *slowly* changing,—at a rate of less than one minute of arc per year for nearly all the stars. The second plane concerned in the measurement of an *hour-angle* is the plane of the meridian. This accompanies the Earth in its diurnal rotation. Hence the hour-angle of a celestial object varies rapidly,—360° for each rotation of the Earth on its axis. The right ascension and declination are spherical co-ordinates locating a celestial object with reference to the hour-circle through the vernal equinox and the equator. The hour-angle and declination are two spherical co-ordinates locating a celestial object with reference to the meridian and equator.

14. It is convenient for some purposes to refer the position of a heavenly body by spherical co-ordinates to the planes of the meridian and horizon,—the two co-ordinates in this case being the *altitude* and *azimuth.* The *altitude* of a heavenly body is its angular distance above the horizon, or the angle between the line joining the observer to the star, and the horizontal plane. Any great circle of the celestial sphere passing through the zenith is called a *vertical circle.* The altitude of a star is measured by that portion of the vertical circle passing through the star which is included between the star and the horizon. The *azimuth* of a star, or other celestial body, is the angle between the plane of the meridian and the vertical plane passing through the star. The same definition applies to a line joining two terrestrial points. The azimuth at station *A* on the Earth's surface, of the line joining stations *A* and *B*, is the angle between the vertical plane at *A* passing through the line *AB* and the meridian plane of *A*. The azimuth of a star is measured on the celestial sphere by that portion of the horizon included between the star's vertical circle and the meridian line. In general the altitude and azimuth of a celestial object are both changing rapidly because of the Earth's rotation. The *zenith distance* of a star is its angular distance from the zenith,—measured, of course, along a vertical circle. The zenith distance and altitude are complements of each other.

15. The *astronomical latitude* of a station on the surface of the Earth is the angle between the line of action of gravity at that station and the plane of the equator. It is measured on the celestial sphere along the meridian from the equator to the zenith.

The *astronomical longitude* of a station on the surface of the Earth is the angle between the meridian plane of that station and some arbitrarily chosen initial meridian plane. Usually the meridian of Greenwich, Eng., is taken as the initial meridian, but sometimes that of Paris or of Berlin, or in the case of detached surveys some arbitrary meridian plane to which all points of the survey may be conveniently referred. Unless otherwise stated astronomical latitude or astronomical longitude is meant when the word latitude or longitude is used in this book.

The student should distinguish *astronomical* latitude and longitude from *geodetic* latitude and longitude, and should be careful not to confuse either one of these with *celestial* latitude and longitude. The *geodetic latitudes and longitudes* differ from the astronomical in that, instead of being referred to the actual action-line of gravity at the station, they are referred to a gravity line which has been corrected for local deflection, or station error.* *Celestial latitudes* and *longitudes* form a

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* In the operations of geodesy the action-line of gravity has been found to be nearly perpendicular at all stations to the surface of an imaginary ellipsoid of revolution generated by the revolution of an ellipse about its minor axis, the minor axis coinciding with the axis of rotation of the Earth. This is the form which a rotating liquid mass necessarily assumes under the action of no other forces than the action of gravitation between its component parts. Values for the polar and equatorial diameters, respectively, of this ellipsoid having been determined such that its surface is as nearly as possible perpendicular at all points to the action-lines of gravity, the outstanding difference of direction between the normal to the surface of the ellipsoid at any point and the actual action-line of gravity at that point is called the *station error*, or local deflection of the vertical at that point. The station error is supposed to be due to variations of density in the interior of the Earth near the station, and to the local irregularities of the surface.

The operation of determining the station error at a given place is as follows: The astronomical latitude and longitude of each of a number of stations are determined. The stations are connected by an accurate geodetic survey. All the latitudes and longitudes are then reduced to one of the stations by use of the known elements of the ellipsoid. The mean system of spherical co-ordinates,—frequently used by the astronomer but seldom by the engineer. In this the ecliptic and vernal equinox play the same part as do the equator and vernal equinox in the case of declinations and right ascensions.

16. In general, when the engineer observes a heavenly body he has one of four objects in view, namely, to determine his astronomical latitude, the azimuth of a line joining his station with some other terrestrial point, the true local time at the instant of observation, or the longitude of his station. The determination of longitude always involves a determination of the true local time together with additional operations which are in some cases quite complicated. The instrument used in any case for the determination of time, latitude, or azimuth indicates the position of the horizon, and consequently of the zenith, by means of attached spirit-levels, or

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of the various values of the latitude of this single station as thus obtained is called its *geodetic latitude.* The corresponding statement applies to the longitude. It is evident that the greater the number of stations and the more widely scattered they are the nearer will the vertical as given by the geodetic latitude and longitude coincide with the normal to the ellipsoid. The difference between the astronomical and geodetic latitude at a given point is therefore usually called the *station error in latitude.* A similar statement defines *station error in longitude.* For further information on this subject see Clark's Geodesy, pp. 287–288, Merriman's Geodetic Surveying, pp. 79–88, or any extended treatise on geodesy.

Station errors in longitude, or deflections of the vertical at right angles to the meridian, change the plane of the meridian from the position it would otherwise occupy and so change all azimuths from the values they would otherwise have. Hence there arises the same distinction between the *astronomical azimuth* of a line and its *geodetic azimuth* as is drawn above between the astronomical and geodetic latitudes and longitudes. On account of station error the line of gravity at a station and the axis of the Earth do not, in general, intersect. Hence to be exact the meridian plane must be said to be defined, not by the line of gravity and the Earth's axis of rotation, but by the line of gravity and the point in which the axis of rotation produced intersects the celestial sphere.

by a basin of mercury having a free horizontal surface. The star, or other celestial object, is usually observed with a telescope. The two points on the celestial sphere always observed are, therefore, the zenith and the object. The right ascension and declination of the object observed become known, independently of the observations, by the methods indicated in the next chapter. The process most frequently used is to acquire, by instrumental observation and by the means indicated in the next chapter, a knowledge of three of the elements, arcs and angles, of some triangle on the celestial sphere of which one of the unknown elements, now capable of computation, is the quantity sought, or is one from which the required quantity can be readily derived.

For example, suppose that the latitude of the station of observation is known, and that the zenith distance of a certain star is accurately observed. Let the true local sidereal time (see § 18) at the instant of observation be required. In the triangle on the celestial sphere defined by the pole, the zenith, and the instantaneous position of the star, the arc from the zenith to the pole is known, being the complement of the latitude of the station. The arc from the star to the pole becomes known by the methods indicated in the next chapter, since it is the complement of the declination of the star at the instant of observation. The arc from the zenith to the star, the zenith distance, was directly observed. Hence in the spherical triangle pole-zenith-star all three arcs are known and any of the angles may be computed. The angle at the pole of that triangle is the hour-angle of the star at the instant of observation. This being computed by the methods of spherical trigonometry, a mere addition to or subtraction from the right ascension of the star (which becomes known by the methods of the following chapter) gives the true local sidereal time (as will be shown later).

17. On account of the rapid apparent motion of most celestial objects, time enters as an important element into almost every astronomical problem with which the engineer has to deal. Three kinds of time are in use in astronomy: sidereal time, apparent solar time, and mean solar time.

The passage of a star or other celestial object across the meridian is called its *transit* or *culmination*.

The meridian (a great circle of the celestial sphere) is divided into two half-circles by the poles. If the whole of the meridian be considered, a star has two transits for each complete rotation of the Earth on its axis: one over that half of the meridian stretching from pole to pole which includes the zenith, and the other over that half which passes through the nadir. The first of these is called the *upper transit* or *upper culmination*, and the second the lower transit or lower culmination. The word transit or culmination unmodified usually means the upper transit. The expression "the passage of a star across the meridian" refers, of course, to the apparent motion of the star. It would be more accurate to say that the meridian passes the star. But to refer directly to the apparent motion as if it were real saves circumlocution, is more clear in many cases, and is not misleading if one keeps in mind that this is merely a mode of speech.

18. A *sidereal day* is the interval between two successive transits of the vernal equinox across the same meridian. Its hours are numbered from 0 to 24. The sidereal time is $0^h\ 00^m\ 00^s$ at the instant when the vernal equinox transits across the meridian. The *sidereal time* at a given station and instant is the right ascension of the meridian, or is the same as the hour-angle of the vernal equinox, counted in the direction of the apparent motion of the stars, at that station and instant.

Right ascensions being reckoned from west to east, opposite to the apparent motion of the stars, it follows from the above definition that the sidereal time at the instant of transit of a star is the same as the right ascension of that star.

The sidereal day is substantially the interval of time required for one rotation of the earth on its axis, and the uniformity of the rotation of the earth is depended upon to furnish the ultimate measure of time. Because of the motion of the vernal equinox on the celestial sphere, about 50″ per year, the sidereal day and the time of one rotation of the earth on its axis differ by about one one-hundredth of a second.

19. The interval between two successive transits of the Sun across the meridian is called an *apparent solar day*. The *apparent solar time* for any instant and station is the hour-angle of the Sun, at that instant, from that meridian. "But the intervals between successive returns of the Sun to the same meridian are not exactly equal, owing to the varying motion of the Earth around the Sun, and to the obliquity of the ecliptic."

Let Fig. 1 represent a section of the universe on the plane of the Earth's orbit as seen from some position in space on the side on which the north pole is situated. The Earth is seen moving around its orbit in a counter-clockwise direction, while at the same time its rotation about its own axis appears to be counter-clockwise. The figure is not to scale, but is merely a diagram in which certain dimensions are exaggerated for the sake of clearness. Suppose that *A* is the position of the Earth at a certain time, about March 21, when the Sun is seen projected against the celestial sphere upon the vernal equinox. Let *B* be the position of the Earth one sidereal day later. Then *Aa* and *Ba* are parallel lines, the vernal equinox being at an infinite distance (on the celestial sphere). The Earth has made one complete rotation on its axis between the two positions, and the vernal equinox has returned to the same meridian. The Earth having moved a distance AB along its orbit, the Sun is now seen projected against the celestial sphere at b instead of a. Before the Sun will return to the meridian of position A again the Earth must rotate through the additional angle represented by *aBb reduced to the plane of the Earth's equator.* (The figure represents a section in the plane of the *ecliptic.*) The apparent solar day will then be longer than the sidereal day by the time required for the Earth to rotate through this angle,—on an average a little less than four minutes.

Let the angle governing the excess of the apparent solar over the sidereal day be examined further. As the Earth proceeds forward along its orbit the Sun will apparently move backward on the celestial sphere along the ecliptic to points b, c, d, etc. One of the laws of gravitation governing the motion of the Earth in its orbit is that the line joining the Earth to the Sun sweeps over equal areas in equal times. The linear velocity then varies nearly inversely as the distance to the Sun, and the angular velocity varies still more than the linear. The angular velocity is about 7% greater during the winter (of the northern hemisphere) than during the summer. The various arcs ab, bc, cd, etc., along the ecliptic, each corresponding to one sidereal day, will vary in value through that range. But the excess of the apparent solar over the sidereal day depends upon these arcs projected upon the equator along hour-circles, the rotation of the Earth being uniform when measured along the equator. When such a small arc as ab near either equinox is projected upon the equator, it will be considerably reduced, being at an angle of $23\frac{1}{2}°$ to the equator,—the angle between the equator and ecliptic at the equinoxes.

On the other hand, when a portion of the ecliptic about midway between the equinoxes is projected along its limiting hour-circles upon the equator, the projected length will be greater than the original. In short, the difference between the sidereal and apparent solar day varies by a rather complicated law from about $4^m\ 26^s$ to $3^m\ 35^s$, being on an average $3^m\ 56^s.555$ (in sidereal time).

20. Apparent solar time is a natural and direct measure of duration, inasmuch as it is indicated directly by the hour-angle of the Sun, the most conspicuous of all the heavenly bodies. But a clock or chronometer cannot be regulated to keep this kind of time accurately, since the different days are of unequal length. To avoid the difficulties thus arising from the direct use of the Sun as a measure of time, a fictitious *mean Sun* is used. The *mean Sun* is supposed to move in the equator with a uniform angular velocity, and to keep as near the real Sun as is consistent with perfect uniformity of motion. This mean Sun makes one complete circuit around the equator at a uniform rate while the Earth is making a complete circuit around its orbit, at a variable rate. It is sometimes as much as 16 minutes ahead of the real Sun, and sometimes behind it by that amount. A *mean solar day* is the interval between successive transits of the mean Sun over the same meridian. The *mean solar time* for any instant and station is the hour-angle of the mean Sun at that instant from that meridian. For brevity mean solar time is often called simply *mean time*. The mean solar day is about $3^m\ 56^s$ longer than the sidereal day,—that being the amount by which the apparent solar day exceeds the sidereal day on an average. Stated more exactly, 24 hours of mean solar time is the same interval as $24^h\ 03^m\ 56^s.555$ of sidereal time.

The sidereal and mean solar time coincide for an instant about March 21 each year. The former gains 24 hours on the latter in a year.

The *equation of time* is the correction to be applied to apparent time to reduce it to mean time. It is the interval of time by which the mean Sun precedes or follows, or is fast or slow of, the real Sun at a given instant. Its limiting values are about $+ 16^m$ and $- 16^m$. It is given in the American Ephemeris and Nautical Almanac for every noon at Washington (and Greenwich). It can be obtained for any intermediate instant with an error not greater than $0^s.1$, usually much less, by a simple straight-line interpolation.

21. The *civil day*, according to the customs of society, commences and ends at midnight. The hours from midnight to noon are counted from 0 to 12 and are marked A.M. The remaining hours from noon to midnight are again numbered from 0 to 12 and marked P.M.

The *astronomical day* commences at noon on the civil day of the same date. Its hours are numbered from 0 to 24, from noon of one day to noon of the next. The astronomical time as well as the civil time may be either apparent solar or mean solar. The convenience of the astronomical day for the astronomer arises from the fact that he does not have to change the date on his record of observations in the midst of a night's work as he would be obliged to if he used civil dates.

The zeros of sidereal, apparent solar, and mean solar time are, by definition, the instants of transit, across the meridian, of the vernal equinox, the Sun, and the mean Sun, respectively. The time, therefore, (of any of the three kinds,) will be the same for two stations at a given instant only in case those stations are on the same meridian. If the stations are not on the same meridian the difference of their times (of any of the three kinds) is a difference of two hour-angles measured from the respective meridians to the same object, and is therefore the angle between the meridians or the difference of longitude of the two stations. A difference of longitude is then a difference of time.

22. In the United States, excluding Alaska, for every mile of distance, east or west along a parallel of latitude, the longitude changes by about four or five seconds of time. If each city and town used its own local mean solar time, the traveller would find himself at considerable inconvenience, on the modern railroads which transport him from 500 to 1000 miles per day, to keep his watch regulated to the time of his various stopping points. Even when the railroads and the general public used one particular time for considerable areas, —that time being usually that of some large city or important railroad division terminus,—as was the case a few years ago, there was still confusion and annoyance arising from the fact that each kind of time was changed to the next by the addition or subtraction of some irregular number of minutes, which was apt to be forgotten when most needed. These, and other reasons, have led to the general adoption in this country of what is called *standard time.* The *standard time* for each particular locality is the mean solar time of the nearest meridian which is an exact whole number of hours, four, five, six, seven, etc., west of Greenwich. The standard meridians for this country are thus:

75° or 5^h west of Greenwich, running near Utica, N. Y., Philadelphia, Pa., and off Cape Hatteras.

90° or 6^h west of Greenwich, running near St. Louis, Memphis, and New Orleans.

105° or 7^h west of Greenwich, running near Denver, Colorado.

120° or 8^h west of Greenwich, running along the east line of the northern part of California and near Santa Barbara, Cal.

135° or 9^h west of Greenwich, running near Sitka, Alaska.

To reduce the local mean solar time to standard time it is merely necessary in each case to apply as a correction the difference of longitude of the station and the standard meridian. Only the astronomer or engineer, however, is obliged to use this process. The traveller has occasion simply to change from one kind of standard time to another which differs from it by exactly one hour,—an interval which is easy to remember.

To Convert Mean Solar to Sidereal Time.

23. To convert mean solar to sidereal time or *vice versa*, it is necessary to take account logically of two facts, that the zeros of the two kinds of day differ by a certain interval, to be derived from the Ephemeris, and that the two kinds of hours bear a fixed ratio to each other which is nearly, but not quite, unity.

The local mean solar time at St. Louis, Mo., $52^m\ 37^s.07$ west of Washington, is $9^h\ 21^m\ 23^s.35$ A.M., July 29, 1892. What is the local sidereal time ?

Local mean solar time	=		$9^h\ 21^m\ 23^s.35$
Time of mean noon	=		12 00 00 .00
Mean solar interval to nearest mean noon	=		2 38 36 .65
Reduction to sidereal interval (see § 290)	= +		0 00 26 .06
Sidereal interval to nearest mean noon			2 39 02 .71
Sidereal time of mean noon, July 29, 1892, at Washington	=	$8^h\ 31^m\ 14^s.23$	
Correction due to longitude to reduce to St. Louis (see below)	= +	0 00 08 .64	
Sidereal time of mean noon, July 29, 1892, at St. Louis	=		8 31 22 .87
Required sidereal time at St. Louis	=		5 52 20 .16

The first step is to obtain the mean solar interval between the given time and the nearest mean noon, and to reduce it to an equivalent sidereal interval by use of the tables in § 290 (reprinted from the back part of the Ephemeris). The derivation of these tables from the equation given at the end of each is sufficiently obvious.

The next step is to derive from the American Ephemeris and Nautical Almanac the sidereal time of that mean noon, or, in other words, the difference of the zero points of the two kinds of time at noon of that day. The Ephemeris, in the part headed "Solar Ephemeris" (pp. 377–384 in the volume for 1892), gives directly the sidereal time of every Washington mean noon for the year. What is required is the sidereal time of St. Louis mean noon. The vernal equinox, marking the zero of sidereal time, shifts $3^m\ 56^s.555$ per mean solar day with respect to the mean Sun, marking the zero of mean solar time. The sidereal time of mean noon for a given point then increases $3^m\ 56^s.555$ per day. St. Louis being $52^m\ 37^s.07$ west of Washington, its mean noon occurs at that interval of mean solar time later than the mean noon of Washington. Its sidereal time of mean noon is evidently that of Washington increased by the motion of the vernal equinox relative to the mean Sun in $52^m\ 37^s.07$, or

$$[(52^m\ 37^s.07) \div (24^h)][3^m\ 56^s.555].$$

This proportional part is precisely that given by the table, § 290, for the reduction of mean solar to sidereal time, and hence the correction is taken directly from that table.

Having now the sidereal interval to the nearest local mean noon, and the local sidereal time of that mean noon, the required sidereal time is obtained by a simple subtraction (or addition, as the case may call for).

Note that the longitude of the station is used only in reducing the sidereal time of mean noon at Washington to the local sidereal time of mean noon. An error of 4^s in the longitude produces an error of only $0^s.01$ in this reduction.

Example of the Reduction from Sidereal to Mean Time.

24. At a certain instant in the evening of May 21, 1892, at Harvard Observatory, it was found by an observation upon a star that the sidereal time was $13^h\ 41^m\ 27^s.34$. What was the mean time at that instant? Harvard Observatory is $23^m\ 41^s$ east of Washington.

Given sidereal time	=		$13^h\ 41^m\ 27^s.34$
Sidereal time of mean noon, May 21, 1892, at Washington	=	$3^h\ 59^m\ 11^s.73$	
Correction, due to longitude, to reduce to Harvard Observatory (§ 290)............ ...	=	— 0 00 03 .89	
Sidereal time of mean noon, May 21, 1892, at Harvard Observatory.........................	=		3 59 07 .84
Sidereal interval after mean noon................	=		9 42 19 .50
Reduction to mean time interval (§ 291)	=		— 0 01 35 .40
Required mean time at Harvard Observatory.....	=		9 40 44 .10 P.M.

The Ephemeris.

25. The American Ephemeris and Nautical Almanac referred to in the above computation is an annual publication of the United States Government. It can be obtained at any time by sending one dollar to the Nautical Almanac Office, Washington, D. C. It, or its equivalent, is a necessity to an engineer making astronomical determinations, as will be seen by the many references to it in the following chapters. As it forms a part of the outfit of the astronomical observer and computer, the student should become familiar with its general arrangement, should acquire a general understanding of all parts of it, and should obtain a thorough grasp of those particular portions to which he finds especial reference in the text of this book. To gain familiarity with the most frequently used portions of the Ephemeris, it is especially desirable that the following pages of the text at the back of the Ephemeris headed "On the Arrangement and Use of the American Ephemeris" be read; viz., the first four pages of the explanation of Part I (pp. 493–496 in the volume for 1892), and the first three pages of the explanation of Part II (pp. 501–503 of the volume for 1892). The Governments of Germany, France, and England, and some others, issue similar publications.

QUESTIONS AND EXAMPLES.

26. 1. The position of the Sun projected upon the celestial sphere is always at some point of the ecliptic. Explain why this statement is not true in regard to a planet.

2. What is the relation between the latitude of a station and the altitude of the pole at that station?

3. Given the latitude of a station and the declination of a star, how may the zenith distance of the star at the instant of upper culmination be determined?

4. In the case of a circumpolar star how may the zenith distance at lower culmination be determined, the declination and latitude being given?

A *circumpolar star* is one comparatively near the pole, say within ten degrees.

5. How would you determine the zenith distance at upper culmination, and also at lower, for a circumpolar star of which the polar distance is given? The latitude of the station is supposed to be known.

6. The hour-angle of the star Vega, east of the meridian, at a certain instant on the evening of June 30, 1892, at the Cornell Observatory was $2^h\ 11^m\ 14^s$. The right ascension of Vega at that instant was $18^h\ 33^m\ 19^s$. What was the local sidereal time? Also, what was the Washington sidereal time, —Cornell being $2^m\ 16^s$ east of Washington?

7. At a certain instant the hour-angle and zenith distance of a star are observed. The declination of the star is known. In the spherical triangle star-zenith-pole what parts are known and how may the latitude of the station be computed?

8. What was the hour-angle of the Sun on September 29, 1892, at a station $4^h\ 19^m\ 46^s.3$ west of Washington, when a clock which was $31^s.9$ fast of local mean time indicated $2^h\ 41^m\ 18^s.9$ P.M.?

The equation of time for apparent noon at Washington on September 29 was $-9^m\ 57^s.71$ and for the 30th, $-10^m\ 17^s.05$. *Ans.* $2^h\ 50^m\ 50^s.5$ west of the meridian.

9. The mean time was $5^h\ 16^m\ 21^s.34$ P.M., August 10, 1892, at a station $2^h\ 19^m\ 31^s$ west of Washington. What was the sidereal time? The sidereal time of mean noon at Washington on that day was $9^h\ 18^m\ 32^s.91$.

Ans. $14^h\ 36^m\ 09^s.14$.

10. The sidereal time was $23^h\ 49^m\ 59^s.92$, the astronomical date August 21, 1892, and the station $1^h\ 29^m\ 21^s$ west of Washington. What was the mean time and the civil date? The sidereal time of mean noon at Washington on the 21st (civil date) was $10^h\ 01^m\ 55^s.02$, and on the 22d, $10^h\ 05^m\ 51^s.57$. *Ans.* Mean time $= 1^h\ 45^m\ 34^s.60$ A.M. Civil date, August 22.

11. The apparent solar time at a station $1^h\ 46^m\ 18^s.2$ west of Washington was at a certain instant on April 17, 1892, $10^h\ 33^m\ 14^s.3$ A.M. What was the mean time? The equation of time at Washington apparent noon on that date was $-38^s.95$, and on the 18th was $-52^s.49$.

Ans. $10^h\ 32^m\ 35^s.2$ A.M.

12. The hour-angle of the Sun as observed at a certain instant, at a station $2^h\ 14^m\ 34^s$ east of Washington, on the forenoon of May 21, 1892, was found to be $2^h\ 48^m\ 19^s.3$. What was the sidereal time? The equation of time for apparent noon at Washington was $-3^m\ 37^s.96$ on May 20,

and $- 3^m\ 33^s.96$ on the 21st. The sidereal time of mean noon at Washington on the 21st was $3^h\ 59^m\ 11^s.73$.

Ans. $1^h\ 06^m\ 27^s.3$.

13. Suppose you are carrying a watch which is 20^s fast of standard (75th meridian) time, and that you wish to start a sidereal clock on correct time to within 1^s at Cornell ($2^m\ 16^s$ east of Washington or $5^h\ 05^m\ 56^s$ west of Greenwich). Suppose the date to be Sept. 30, 1892, and the sidereal time of mean noon for Washington on that date to be $12^h\ 39^m\ 37^s.2$. What time should the sidereal clock indicate when your watch reads $7^h\ 00^m\ 00^s$? *Ans.* $19^h\ 34^m\ 29^s$.

14. Explain why the "sidereal time of mean noon" as given in the last column of the "Solar Ephemeris" (pp. 377–384 in the volume for 1892), in the American Ephemeris and Nautical Almanac is not the same as the "apparent right ascension" at "mean noon" as given in the second column.

15. Why is not the "equation of time for apparent noon" as given in the eighth column, the same as the difference of the two columns mentioned in the preceding example?

16. What is the relation between the right ascension of the Sun at mean noon, the equation of time at mean noon, and the sidereal time of mean noon?

17. Look up the sidereal time of mean noon for to-day in the Ephemeris. Then, knowing the time of day and your latitude, hold two sheets of paper parallel respectively to the plane of the equator and the plane of the ecliptic.

CHAPTER II.

COMPUTATION OF RIGHT ASCENSION AND DECLINATION.

27. In the astronomical practice of the engineer the right ascension and declination of the object observed are usually known quantities determined from sources external to his own observations. The object of this chapter is to show how the right ascension and declination for the instant of observation are obtained from the available sources of information.

The various heavenly bodies which the engineer is called upon to observe have all been observed frequently at the various fixed observatories with large instruments and at many different times extending over a long period of years. From these observations the positions, that is, right ascensions and declinations, at various stated times are determined, and the motions are carefully computed. This makes it possible to compute the position of each of these bodies at any stated future time with an accuracy depending on the precision of the observations and the remoteness of the future time. The results of such computations of positions made in advance, and also the data for such computations, are given in the ephemerides issued by various governments: the American Ephemeris, Berliner Jahrbuch, Connaissance du Temps (Paris), British Nautical Almanac, etc. Various other occasional publications also give the data for such computations. The engineer uses these computations of position made in advance, and the published data for such computations, to obtain the right ascension and declination at the instant of his observation.

When references are given in the following text to data in the American Ephemeris, it should be understood that substantially the same data may also be obtained from the other national ephemerides.

Position of the Sun and Planets.

28. The right ascension and declination of the Sun are given for Washington mean and apparent noon in the American Ephemeris (pp. 377–384 of the volume for 1892) for every day of the year, together with some other data that are frequently needed for computation purposes. The corresponding data are also given for Greenwich in first part of the Ephemeris. The right ascension and declination are also given in the American Ephemeris for each planet for every day of the year when its transit is visible at Washington (on pp. 393–411 of volume for 1892). The corresponding data are given in more complete form for Greenwich in the first part of the Ephemeris (pp. 218–249 of the volume for 1892). For the methods by which the right ascension and declination of the Sun, or a planet, at *any* given intermediate time, are to be derived from the values stated in the Ephemeris, see the following sections, Nos. 29–34.

Interpolation.

29. By *interpolation* is meant the process by which, having given a series of numerical values of a function corresponding each to a stated value of the independent variable, the value of the function for any other intermediate value of the variable is found independently of a knowledge of the analytical form of the function. The independent variable is often called the *argument*. For example, the right ascension of the Sun is a known function of time as the independent variable. It is given in the Ephemeris for certain stated times. When it is required for any other time, instead of computing it directly from the known function, it is much more convenient and rapid to deduce it by interpolation from the stated numerical values.

Interpolation always leads to approximate results which may be made more exact as the process of interpolation is made more complicated and laborious. The *error of interpolation* is the difference between an interpolated value and the value which would be found if one resorted to direct computation from the known function. Of the multitude of methods of interpolation, with widely varying degrees of convenience, rapidity, and accuracy, three methods will be found sufficient for the ground covered by this book. These three may be described briefly as *interpolation along a chord*, *interpolation along a tangent*, and *interpolation along a parabola*.

Interpolation along a Chord.

30. In *interpolation along a chord* the rate of change of the function, between the two stated values of the variable which are adjacent to the value for which the interpolation is to be made, is assumed to be constant and equal to the total change of the function between those points divided by the interval between the stated values of the variable. If the actual values of the function were represented graphically, all interpolated values would lie along chords of the function curve, connecting points on the curve corresponding to stated values of the variable. For example, the right ascension of Jupiter at $12^h\ 35^m.5$ mean time at Washington, on Oct. 1, 1892, was $1^h\ 21^m\ 07^s.81$ and at $12^h\ 31^m.1$ on Oct. 2, was $1^h\ 20^m\ 38^s.80$ (Ephemeris, p. 405). Required its right ascension at $15^h\ 14^m.2$ Washington mean time, on Oct. 1? The interval between stated values of the variable is $23^h\ 55^m.6 = 23^h.93$. The change in the value of the function is $-\ 29^s.01$. The rate of change is then $-\ 29^s.01 \div 23^h.93 = -\ 1^s.212$ per hour. The interval over which the interpolation is carried from the nearest given value is $15^h\ 14^m.2 - 12^h\ 35^m.5 = 2^h\ 38^m.7 = 2^h.64$. The change during that interval is $(2.64)(-1.212) = -3^s.20$. The required right ascension is $1^h\ 21^m\ 07^s.81 - 3^s.20 = 1^h\ 21^m\ 04^s.61$. The result would have been identical with this had the interpolation been made from the other adjacent value, namely, that at $12^h\ 31^m.1$ on Oct. 2d. In algebraic form this interpolation may be expressed thus:

$$F_I = F_1 + (F_2 - F_1)\frac{V_I - V_1}{V_2 - V_1}$$

$$\text{or} \qquad F_I = F_2 - (F_2 - F_1)\frac{V_2 - V_I}{V_2 - V_1}; \quad . \quad . \quad . \quad . \quad (1)$$

the first form being used when the interpolation is made forward from the value F_1, and the second when it is made backward from F_2.

F_I is the required interpolated value corresponding to the value V_I of the independent variable. V_1 and V_2 are the adjacent stated values of the argument to which correspond the given values F_1 and F_2 of the function.

Interpolation along a Tangent.

31. *Interpolation along a tangent* is, in general, more accurate than interpolation along a chord, but can only be used conveniently when the rates of change, or first differential coefficients of the function, are given at the stated values of the variable, in addition to the values of the function itself. In this interpolation the rate of change, for the interval from the *nearest* stated value of the variable to the value for which the interpolation is to be made, is assumed to be constant and equal to the given rate of change at the stated value of the variable.

The interpolated points represented graphically would lie on a tangent, at the nearest stated value of the variable, to the curve representing the function. For example, let it be required to find the declination of the Sun on Sept. 5, 1892, at $9^h\ 30^m$ A.M., Washington mean time. On page 382 of the Ephemeris for that year, the nearest time for which the declination is given, is Washington mean noon of that day. For that instant the declination is $+ 6° 28' 18''.6$. Its rate of change for that instant is stated to be $- 55''.92$ per hour. The interval over which the interpolation is to extend is $2^h.5$ *backward* from noon. Then by interpolation along the tangent to the curve (representing declinations) at noon of Sept. 5th, there is obtained as the declination at $9^h\ 30^m$ A.M., $6° 28' 18''.6 + (2.5)(55''.92) = 6° 30' 38''.4$. In this method of interpolation the shorter the tangent the smaller the error of interpolation, and therefore care should be taken to interpolate from the *nearest* stated value of the variable. The formula for this interpolation is

$$F_I = F_1 + \left(\frac{dF}{dV}\right)_1 [V_I - V_1]. \quad . \quad . \quad . \quad . \qquad (2)$$

F_I and V_I are the required interpolated value and the corresponding given argument, V_1 and F_1 are the nearest tabular value of the argument and the corresponding value of the function, and $\left(\frac{dF}{dV}\right)_1$ is the given first differential coefficient corresponding to V_1.

Interpolation along a Parabola.

32. In *interpolation along a parabola* it is assumed that the *second* differential coefficient of the function is constant between adjacent stated values of the independent variable, or, in other words, that the rate of change of slope of the function curve is constant between those points. This assumption places the interpolated points along a parabola, with axis vertical, passing through two points of the function curve,—the uniform rate of change of slope being a property of such a parabola. There are two cases arising under this method, depending upon whether the first differential is, or is not, given for the stated values of the variable.

33. For an example of the first case take the problem proposed in the preceding section, in which it is required to find the declination of the Sun at $9^h\ 30^m$ A.M., Washington mean time, Sept. 5, 1892. The data given in the Ephemeris for 1892 for Washington mean noon Sept. 4 are declination $= +6° 50' 37''.5$, and the first differential coefficient $= -55''.66$; and for Sept. 5, declination $= 6° 28' 18''.6$, and first differential coefficient $= -55''.92$. It is proposed to place the interpolated value on a parabola (with axis vertical) coinciding with the curve of declinations at the two given points, and also having a common tangent at each of these points. To make the interpolation, the principle will be used that a chord of such a parabola is parallel to the tangent at a point of which the abscissa is the mean of the abscissæ of the two ends of the chord. The slope of the chord (of the parabola) corresponding to the interval $9^h\ 30^m$ to 12^h, on Sept. 5, is then the same as the slope of the tangent at the middle of that interval, $10^h\ 45^m$. The slope of the tangent changes by $(-55''.92) - (-55''.66) = -0''.26$ in 24 hours, or $-0''.0108$ per hour. The slope at $10^h\ 45^m = -55''.92$

$+ (0''.0108)(1.25) = - 55''.91$. The interpolated value at $9^h\ 30^m$ is $6°\ 28'\ 18''.6 + (55''.91)(2.5) = 6°\ 30'\ 38''.4$.

This method of interpolation, though most easily remembered, perhaps, in the geometrical form, may be put in convenient algebraic form as follows:

$$F_I = F_1 + [V_I - V_1]\left[\left(\frac{dF}{dV}\right)_1 + \left\{\left(\frac{dF}{dV}\right)_2 - \left(\frac{dF}{dV}\right)_1\right\}\left\{\frac{\frac{1}{2}(V_I - V_1)}{V_2 - V_1}\right\}\right];^* \quad (3)$$

or

$$F_I = F_2 + [V_I - V_2]\left[\left(\frac{dF}{dV}\right)_2 + \left\{\left(\frac{dF}{dV}\right)_2 - \left(\frac{dF}{dV}\right)_1\right\}\left\{\frac{\frac{1}{2}(V_I - V_2)}{V_2 - V_1}\right\}\right]; \quad (3a)$$

according to whether the interpolation is made forward from V_1 or backward from V_2. The notation is the same as in the preceding paragraphs. The two results are identical, but the arithmetical work will be shorter if the interpolation is made from whichever of the given points happens to be the nearer.

34. The second case of interpolation along a parabola occurs when the first differential coefficients are not given. The assumptions involved are just as before. As an example, take the problem proposed a few paragraphs back, of finding the right ascension of Jupiter, at $15^h\ 14^m.2$, Washington mean time, on Oct. 1, 1892. The Ephemeris gives the right ascension

$= 1^h\ 21^m\ 36^s.59$ at $12^h\ 39^m.9$ on Sept. 30;
$= 1^h\ 21^m\ 07^s.81$ at $12^h\ 35^m.5$ on Oct. 1;
$= 1^h\ 20^m\ 38^s.80$ at $12^h\ 31^m.1$ on Oct. 2.

It is proposed to interpolate the required point on a parabola, with axis vertical, passing through these three given

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* If the second derivative is constant, then $\left[\left(\frac{dF}{dV}\right)_2 - \left(\frac{dF}{dV}\right)_1\right] \div (V_2 - V_1)$ is really $\frac{d^2F}{dV^2}$. Call $V_I - V_1$, ΔV. Then (3) put in the calculus notation becomes $F_I = F_1 + \frac{dF}{dV} . \Delta V + \frac{d^2F}{dV^2}\frac{\overline{\Delta V}^2}{2}$, in which $\frac{dF}{dV}$ and $\frac{d^2F}{dV^2}$ are values corresponding to the point F_1. V_1.

points. Again, using the principle that in such a parabola, a chord, and the tangent at a point of which the abscissa is the mean between the abscissæ of the two ends of the chord, are parallel, the slope of the parabola at any point may be computed. The slope of the parabola at the middle of the first interval, at $0^h\ 37^m.7 = 0^h.63$ on Oct. 1, is $(07^s.81 - 36^s.59) \div 23.93 = 1^s.203$ per hour. At the middle of the second interval, at $0^h\ 33^m.3 = 0^h.56$ on Oct. 2, it is $(38^s.80 - 67^s.81) \div 23.93 = -1^s.212$ per hour. The interval over which the interpolation is made, from the nearest given value, is $12^h\ 35^m.5$ to $15^h\ 14^m.2$ on Oct. 1, or $2^h\ 38^m.7 = 2^h.64$. The slope of the chord for this interval is that of the tangent at its middle, $13^h\ 54^m.8 = 13^h.91$. This slope is, assuming the rate of change of the slope constant, $-1^s.203 + \dfrac{13^h.91 - 0^h.63}{24^h.56 - 0^h.63}[(-1^s.212) - (-1^s.203)] = -1^s.208$ per hour. The right ascension at $15^h\ 14^m.2$ is $1^h\ 21^m\ 07^s.81 - (1^s.208)(2.64) = 1^h\ 21^m\ 04^s.62$. This sample interpolation is made in the present form simply for the purpose of illustrating the principles involved. The numerical work of interpolation should ordinarily be done as indicated in formula (4) of the following section.

Putting this method in the algebraic language it takes the following form: Let F_1, F_2, and F_3 be three successive given values of the function corresponding to the values V_1, V_2, and V_3 of the independent variable; and let V_2 be the stated value of the variable nearest to which lies the value for which the interpolation is to be made. Let F_I be the required value of the function corresponding to V_I. Then

$$F_I = F_2 + \left[\frac{F_2 - F_1}{V_2 - V_1} + \left\{ \frac{F_3 - F_2}{V_3 - V_2} - \frac{F_2 - F_1}{V_2 - V_1} \right\} \frac{\dfrac{V_I + V_2}{2} - \dfrac{V_2 + V_1}{2}}{\dfrac{V_3 + V_2}{2} - \dfrac{V_2 + V_1}{2}} \right] [V_I - V_2];$$

or, in simplified form,

$$F_I = F_2 + \left[\frac{F_2 - F_1}{V_2 - V_1} + \left\{\frac{F_3 - F_2}{V_3 - V_2} - \frac{F_2 - F_1}{V_2 - V_1}\right\}\left\{\frac{V_I - V_1}{V_3 - V_1}\right\}\right][V_I - V_2]. \quad (4)$$

If, as is usually the case, the successive differences between V_1, V_2, and V_3 are all the same and equal to D, this may be further simplified to the form

$$F_I = F_2 + \left[\frac{F_2 - F_1}{D} + \frac{d_2}{2D^2}(V_I - V_2 + D)\right]\{V_I - V_2\}; \quad (4a)$$

in which d_2 is the second difference, or $(F_3 - F_2) - (F_2 - F_1)$. The second term in the square bracket will usually be comparatively small, and therefore easy to compute.

For a more complete discussion of interpolation, giving other more complex and accurate formulæ, see Chauvenet's Spherical and Practical Astronomy, vol. I. pp. 79–91; Doolittle's Practical Astronomy, pp. 69–98; and Loomis' Practical Astronomy, pp. 202–212.

Accuracy of Interpolation of Position of Sun and Planets.

35. An interpolation along a tangent,—the first differential coefficients or hourly changes being given,—from the values given for noon of each day in the Ephemeris (pp. 377–384 of the volume for 1892), will give the right ascension of the Sun at any time with an error of interpolation not exceeding $0^s.6$, and the declination with an error of interpolation not exceeding $1''.8$. For nearly all cases the error of interpolation will be much less than these extreme limits. Approximately, the extreme error of interpolation along a tangent is one-eighth of the second difference at that point,*—meaning

* The interpolation along a tangent will evidently give the greatest error when the interpolated point is midway between the tabulated values,

by a second difference the difference between successive first differences. If greater accuracy is desirable,—which will often be true of declinations, but seldom of the right ascensions,—an interpolation along a parabola will always give all needful accuracy. In dealing with the planets an interpolation along a tangent, or along a chord in those cases in which the first differential coefficients are not given, will in many cases give a sufficient degree of accuracy, and interpolation along a parabola will give all needful precision in every case.

Position of the Moon.

36. In the first part of the Ephemeris, in which the standard meridian is that of Greenwich (pp. 2–217 of the volume for 1892), the Moon's right acension and declination are given for every hour during the year, together with the corresponding first differential coefficients. An interpolation along a tangent, from the nearest hour, will give the Moon's right ascension at any time with an error of interpolation not exceeding $0^s.05$. The corresponding limit for declination interpolated along a tangent is $1''$. This will usually be a sufficient degree of accuracy. But if for some special reason a greater precision is required, an interpolation along a parabola will give the results far within the limits of error of the tabular values themselves.

Positions of Stars.

37. The American Ephemeris gives the right ascension and declination of four close circumpolar stars for every upper

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the tangent then used, corresponding to one-half of a tabular interval, being longer than is necessary in any other case. If for this case the interpolation along a parabola be used, the interpolated value will differ from that found by using the tangent by one-eighth the second difference, —as may be seen by inspection of the formulæ (2), § 31, and (3), § 33. If, then, the second interpolation be assumed to be exact, this value is the error of the first interpolation.

transit at Washington (pp. 302–313 of 1892); of every tenth transit for about 200 stars; and the right ascension only for every tenth transit visible at Washington of about 200 more. Other national Ephemerides contain similar lists, which often comprise about the same stars. This list is made up of stars whose positions are well determined by many observations at various observatories. They are also chosen with especial reference to the needs of the engineer and navigator as regards brightness and distribution on the celestial sphere. An idea of the care with which their positions have been determined may be gained from the mere statement of the fact that in computing many of these declinations fifty catalogues of recorded observations, at many different observatories, made at various times during a total interval of a century and a quarter, were consulted, and the various observations upon any one star combined in each case in a single least-square computation.*

The positions of the close circumpolars at any time may be obtained with all needful accuracy by interpolation along a chord from the values given in the Ephemeris. For the other stars given in the Ephemeris (at 10-day intervals) an interpolation along a parabola will usually be necessary.

When other stars must be observed than these Ephemeris stars of which the places are given at frequent intervals, a complicated procedure is necessary to obtain the position of the star at the time of the observation. This process forms the subject of the remainder of this chapter.

The position or place of a star is usually given in one of

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* See "Survey of the Northern Boundary from the Lake of the Woods to the Rocky Mountains" (Washington, 1878), pp. 409–615, for a complete report on the computation of star places for that survey by Lewis Boss (pp. 421–424 give catalogues consulted). Many of the star places given in the Ephemeris are from this computation.

three ways, which should be carefully distinguished. Either its *apparent* place, *true* place, or *mean* place is given. The right ascension and declination, as defined in § 12, indicate the *true* place of a star or other celestial object. But the apparent direction of a star, even aside from the refraction of the line of sight by the terrestrial atmosphere, is affected by aberration. The *apparent* place of a star is its true place modified by the aberration of light. An observer sees a star in a position which differs from what is technically called its *apparent* place by the effect of refraction only.* It should be carefully noted that the word "apparent" is not here used in the ordinary sense, but in the special technical sense which it must be understood to have hereafter throughout this book.

Aberration.

38. *Aberration* is an apparent displacement of a star resulting from the fact that the velocity of light is not infinite as compared with the velocity of motion through space of the observer, stationed at a point on the Earth's surface.

If one is standing in a rain which is falling in vertical lines, the umbrella must be held directly overhead. If, however, one is riding rapidly through such a rain-storm, the umbrella must be inclined forward. In the first case a drop of rain entering at the centre of one end of a straight open tube held with its axis vertical would pass along the axis of the tube to the other end without touching the tube. In the second case, however, if it is desired that drops which enter the tube at the upper end shall continue down the tube without touching the sides, it will be necessary to incline the tube forward from the vertical to a certain angle which is dependent on the relative velocity of the horizontal motion of the tube and the vertical motion of the rain. So when a

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* For a detailed consideration of refraction see §§ 67–69.

telescope is to receive along its axis the light undulations from a star, it must be inclined forward in the direction of the actual motion of the telescope in space so as to make a slight angle—the aberration—with the actual line joining telescope to star. This small angle, the aberration, amounting at most to about 20″, is evidently dependent upon the relative velocity of light and of the telescope, and the angle between those two velocities. (The student may easily draw a diagram for himself showing the geometrical relations concerned.) The motion of the telescope is compounded of that due to the diurnal rotation of the Earth on its axis and the annual revolution of the Earth about the Sun. These give rise to the *diurnal aberration* and *annual aberration*, respectively. The diurnal aberration evidently affects right ascensions directly, but has no effect upon declinations. The annual aberration in general affects both. For an example of the way in which diurnal aberration is taken into account in computations, see § 96. The effect of annual aberration is included in the apparent place computation treated later in this chapter.

The velocity of light is, according to the best determinations, about 186300 miles per mean solar second.* It requires about eight minutes for light to travel from the Sun to the Earth. An observer, then, does not see a celestial object in its true position at the instant when the light enters the eye, but in the position which it occupied when that light left the object—an appreciable interval earlier, for all celestial objects. This phenomenon is called *planetary aberration*. With this form of aberration the engineer is not concerned.

39. The *mean place* of a star is its position referred to the mean equator and mean ecliptic, as distinguished from its

* See "The Solar Parallax and its Related Constants," Wm. Harkness, Washington, 1891, pp. 142 and 29–32.

position as referred to the actual or true equator and ecliptic. The equator and ecliptic as they would be if unaffected by periodic variations, in other words by nutation, are called the *mean equator* and *mean ecliptic.*

The mean place of a star, then, at a given instant, differs from the true place by the effect of nutation at that instant, and from the apparent place by the effects of both nutation and aberration.

To avoid inconveniences arising in the course of computations of star places, if any other form of year is employed in reckoning time, the astronomer uses what is called the *Besselian fictitious year.* The beginning of the *fictitious year* is the instant at which the celestial longitude of the mean Sun is 280°, or, in other words, when the mean Sun is 280° from the vernal equinox measured along the ecliptic.* The beginning of the fictitious year differs from the beginning of the ordinary year by a fraction of a day, which varies for different years.

The places given in the Ephemeris, referred to in § 37, for every day or every ten days, are apparent places, and are so marked. When the engineer is obliged to have recourse to stars which are not so given in the Ephemeris, he consults one or more of the various available star catalogues or star lists.† These catalogues and lists give the mean places of the stars at the beginning of some stated fictitious year, together with other data relative to each star. The problem which then confronts the engineer is to derive, from that given mean place, the apparent place at the time at which his observation was made. This is done in two steps. Firstly, the mean place of the star is reduced from the epoch of the catalogue to the beginning of the fictitious year at some

* See definition of celestial longitude, § 15.

† For references to a few of such catalogues and lists see § 141.

part of which its apparent place is desired. Secondly, the apparent place of the star at the time of observation is deduced from the mean place at the beginning of the fictitious year.

Reduction of Mean Places from Year to Year.

40. To serve as a concrete example, let it be supposed that the star μ Hercules was observed at its transit across the meridian at St. Louis, Mo., on July 16, 1892; and the authority depended upon for its position is Boss's Catalogue of 500 Stars for 1875.0.* This star is No. 312 in that catalogue and its mean place as there given for the beginning of the fictitious year 1875 is

$$\alpha_{1875.0} = 17^h\ 41^m\ 34^s.0 = \text{mean right ascension};$$
$$\delta_{1875.0} = +\ 27°\ 47'\ 42''.17 = \text{mean declination.}$$

(Throughout this book α and δ will be used to indicate the apparent right ascension and declination, respectively, at the time of the observation under consideration. The same letters with the subscript $_m$, thus, α_m, δ_m, will be used to indicate the mean place. With a year as a subscript as above, they will be understood to indicate the mean place at the beginning of that fictitious year.)

41. The reduction from the mean place at 1875.0 to that at 1892.0 involves simply the change in the mean equator and mean ecliptic during that time. The determination of the laws of change of these two fundamental reference circles, and the method of computing the effect of those changes upon right ascensions and declinations, belong rather to the province of the astronomer than to that of the engineer. It

* Survey of the Northern Boundary from the Lake of the Woods to the Rocky Mountains (Washington, 1878), pp. 592–615.

suffices for the engineer to accept the results of the investigations of the astronomer in the following form:

$$\frac{d\alpha_m}{dt} = m + n \,.\, \sin \alpha_m \tan \delta_m + \mu; \quad . \quad . \quad . \quad (5)$$

$$\frac{d\delta_m}{dt} = n \,.\, \cos \alpha_m - \mu'; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{d^2\delta_m}{dt^2} = \frac{dn}{dt} \cos \alpha_m - n \,.\, \sin \alpha_m \,.\, \frac{d\alpha}{dt}; \quad . \quad . \quad . \quad (7)$$

in which $m = 46''.0623 + 0''.0002849(t - 1800)$ (t being expressed in years), and $n = 20''.0607 - 0''.0000863(t - 1800)$. The numerical values for m and n as here given are those most extensively used, and are the result of exhaustive investigations by the astronomers Peters and Struve. μ and μ' are *proper motions* per year in right ascension and declination respectively, for an account of which see §§ 44, 45. For the present these proper motions may be considered simply as changes at a uniform rate in each of the two co-ordinates, without any reference to their meaning or method of derivation. $\frac{d\alpha_m}{dt}$ and $\frac{d\delta_m}{dt}$ are rates of change per year.

The formulæ given above are neither complete nor exact, many terms of the exact formulæ having been dropped, and those which are retained having been somewhat modified. But they furnish the complete basis for a reduction, with sufficient accuracy for the purposes of the engineer, from the mean place given in a catalogue to the mean place at the beginning of any other fictitious year within thirty or perhaps fifty years. For the formulæ in complete form adapted to the use of the astronomer to bridge over long intervals of time,—sometimes more than a century,—see "Survey of the Northern Boundary from the Lake of the Woods to the

Rocky Mountains," pp. 416–420; and for a detailed discussion of them see Doolittle's Practical Astronomy, pp. 560–578 and 583–589.

42. The engineer is, however, relieved of the necessity for performing the numerical operations indicated by formulæ (5), (6), and (7). For star catalogues and lists give in addition to α_m and δ_m the values of $\frac{d\alpha_m}{dt}$, $\frac{d\delta_m}{dt}$, and the term $\frac{d^2\delta_m}{dt^2}$ is tabulated,* in § 292 of this book, for the arguments α_m and $\frac{d\alpha_m}{dt}$, of which it is evidently a function.

Students will find slight differences between different authorities in regard to the nomenclature of this part of the

* This table is, so far as the author knows, a new one. It was computed from formula (7) above, for the date 1900, to six places of decimals and afterward reduced to five. It is hoped that all the tabular values are, in so far as the computation is concerned, within 0.6 of a unit in the fifth place. The formula used for $\frac{d^2\delta_m}{dt^2}$ is, however, approximate in itself in having omitted the effect of proper motion. Theory indicates that this omission should produce an error so small as to be negligible for our present purpose. To test that conclusion, as well as the accuracy of computation of the table, $\frac{d^2\delta_m}{dt^2}$ for fifty stars (every tenth) of Boss' List, Northern Boundary Report, was derived from the table and compared with that given by Boss in the list. Boss' values were computed from the exact formulæ. The greatest difference found was $0''.00003$. This would cause an error of only $0''.01$ in a reduction extending over 30 years, and only $0''.04$ in 50 years. It is believed, therefore, that the table is abundantly accurate within the limits over which its arguments extend. It should not, however, be assumed to hold good beyond those limits. The table does not cover the comparatively rare cases in which $\frac{d\alpha}{dt}$ is negative (for stars near the pole). For these cases the formula (7) must be used. The table is computed for the year 1900. The same computation made for any date between 1700 and 2100 would give values differing from those of the table by not more than one unit in the last decimal place given in the table.

subject. $\frac{d\alpha_m}{dt}$ is usually known as the "annual variation in right ascension." $\frac{d\delta_m}{dt}$ is sometimes given under the heading "annual variation in declination." Sometimes the terms $n \cos \alpha$ and μ' are given separately as "annual precession" and "proper motion," respectively. $\frac{d^2\delta_m}{dt^2}$ is sometimes given in the form of a "change per 100 years" in the annual precession.

Having given the mean place of a star, α_m and δ_m for a date t_m at the beginning of some fictitious year, the place at time t_0 at the beginning of any other fictitious year is

$$\alpha_0 = \alpha_m + (t_0 - t_m)\left(\frac{d\alpha_m}{dt}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\delta_0 = \delta_m + (t_0 - t_m)\left(\frac{d\delta_m}{dt}\right) + \frac{1}{2}(t_0 - t_m)^2 \frac{d^2\delta_m}{dt^2} . \quad . \quad (9)$$

43. To return to the numerical case in hand, the following data are also given in Boss' list for the star μ Hercules:

$\frac{d\alpha_m}{dt}$ = annual variation in right ascension $= + 2^s.345$;

$\frac{d\delta_m}{dt}$ = annual variation in declination, including proper motion $= - 2''.3701$;

$\frac{d^2\delta_m}{dt^2} = + 0''.003380$;

—all for the date 1875.

The mean place for 1892 is then, by (8) and (9),

$$\alpha_{1892} = 17^h\ 41^m\ 34^s.0 + (17)(2^s.345) = 17^h\ 42^m\ 13^s.86;$$

$$\delta_{1892} = 27°\ 47'\ 42''.17 + (17)(-2''.3701) + \tfrac{1}{2}(17)^2(0''.003380)$$
$$= 27°\ 47'\ 42''.17 - 40''.29 + 0''.49 = 27°\ 47'\ 02''.37.$$

If $\frac{d^2\delta_m}{dt^2}$ had not been given in the star list, as frequently it is not, it could have been obtained by entering the table § 292 with the arguments $\alpha_m = 17^h\ 41^m.6$ and $\frac{d\alpha_m}{dt} = + 2^s.345$. The value as then found from the table would have been $= 0''.00341$, and the final value for δ_{1892} would have been identical with that given above.

Proper Motion.

44. When the co-ordinates of a star, as observed directly at widely separated times, are reduced to the same epoch, it is usually found that, aside from discrepancies arising from accidental errors of observation, there are systematic differences in the various values indicating a steady movement of the star in some one direction with the lapse of time. Observations on another star indicate usually that it has also such a motion peculiar to itself, which is without any apparent relation to the motion of the first star. So each star is, in general, found to have an unexplained motion peculiar to itself, called its *proper motion*. This proper motion is always exceedingly small, and is assumed to take place along an arc of a great circle of the celestial sphere, and at a uniform rate in each case. Probably neither of these assumptions are strictly true; but the accumulated proper motion for several centuries even would be so small that observations of the highest degree of accuracy now obtainable would not be sufficient to prove the path of star to be curved, or its motion to be other than uniform.

When the mean position of a star for a given date is to be derived from the results of many observations at various times in the past by the process indicated briefly in § 37, an unknown annual proper motion in declination, and another in right ascension, are introduced into the least square adjustment. The annual proper motions in declination and in right ascension as thus derived are then used in deducing the place of the star at any future date in the manner indicated in formulæ (8), (9), (5), and (6), §§ 41, 42. For a full discussion of the treatment of proper motion, from the astronomer's point of view, with the refinements necessary when reductions are to be made covering very long periods of time, see Doolittle's Practical Astronomy, pp. 578–583, and Chauvenet's Astronomy, vol. I. pp. 620–623.

A concrete idea of the magnitude of the proper motion usually found may be gained from the fact that in the Boss Catalogue of 500 Stars for the epoch 1875.0 there are only 8 stars out of the 500 for which the annual proper motion in declination exceeds $0''.50$. In 367 cases it is less than $0''.10$. Proper motions in right ascension are of the same order of magnitude,—keeping in mind, of course, that 1^s of right ascension represents, for a star near either pole, a much smaller displacement upon the celestial sphere than 1^s for a star near the equator.

45. That the so-called proper motion is not really due to an erroneous determination of the precession, is put in evidence by the fact that the various proper motions for different stars do not show the systematic relation which they must necessarily have if due to a shifting of the reference circles. Precession does not change the *relative* positions of the stars. Proper motions do. To what are the proper motions due? 1st. If they are due to a motion of the solar system as a whole through space, the stars which are ahead in the direction of motion must seem to be separating in all directions from the point toward which we are moving, must seem to be going backward at the sides, and apparently closing together behind us,—just as points of the landscape seem to a traveller to move. 2d. If the proper motions are due to actual motions of the stars themselves, acting as entirely independent bodies, the proper motions should seem to be without any relation to each other. 3d. If, on the other hand, they are due to actual motions of the stars, which are not, however, independent, one would expect to find laws connecting the proper motions,—laws, however, which would differ from those called for by the last supposition above. A close study of the proper motions seems to indicate that there is some truth in each of the three suppositions.

Computations based upon hundreds of observed proper motions, made by different astronomers at various times, have all agreed, in a general way, in indicating that there is a slow motion of the solar system as a whole through space toward a point in the neighborhood of $\alpha = 17^h$, $\delta = +35°$. For details in regard to the computations, see Chauvenet's Astronomy, vol. I. pp. 703–708. In regard to the third supposition, it may be noted that in a few rare cases of double stars, two stars apparently very near to each other, the observed proper motions indicate that the two revolve about some common centre—are linked together by gravitation. But though some laws connecting the various proper motions have been thus discovered, the salient fact to keep in mind is that the second supposition is very largely true,—that the discovered laws only account for an extremely small fraction of the actually observed proper motions.

Reduction from Mean to Apparent Place.

46. To reduce from the mean place at the beginning of the year to the apparent place at a given date, it is necessary to reduce the mean place up to date, and then apply to that result the effect of nutation and aberration at that date.

This computation, if made directly from the known laws of nutation and aberration, is very laborious.*

But such a direct computation is not necessary. This is again one of the cases in which it is advisable for the engineer simply to accept the results of the astronomer's investigations in the convenient form in which they are given in the Ephemeris, without going through all the details of the derivation of those results.

47. Suffice it to say that this reduction has been put in the following convenient form:

$$\alpha = \alpha_0 + f + \tau\mu + \tfrac{1}{15}g \sin(G + \alpha_0) \tan \delta_0 + \tfrac{1}{15}h \sin(H + \alpha_0) \sec \delta_0 \quad . \; . \; . \text{ (in time)}; \qquad (10)$$

$$\delta = \delta_0 + \tau\mu' + g \cos(G + \alpha_0) + h \cos(H + \alpha_0) \sin \delta_0 + i \cos \delta_0 \quad . \text{ (in arc)}; \qquad (11)$$

in which α and δ are the required apparent right ascension and declination at some stated time; α_0 and δ_0 are the mean right ascension and declination at the beginning of that fictitious year; τ is the elapsed portion of the fictitious year expressed in units of one year; μ and μ' are the annual proper motions in right ascension and declination; and f, G, H, g, h, and i are quantities called *independent star-numbers*, which are functions of the time only, and are given in the Ephemeris for every Washington mean midnight during the year (pp. 285–292 of the volume for 1892). τ expressed in units of a year is also given in the Ephemeris on the same pages. The values of these constants may be derived for the exact instant at which they are required, with sufficient accuracy, by interpolations along chords.

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* For an exhibit of the formulæ for the computation if made thus, see Doolittle's Practical Astronomy, p. 610.

48. The computation of the apparent place of μ Hercules at its transit at St. Louis, Mo., July 16, 1892, as proposed in § 40 and partially carried out in § 43, may now be continued, as follows. From § 43:

$$\alpha_0 = 17^h\ 42^m\ 13^s.9 = 265^\circ\ 31' \text{ (to nearest minute).}$$

$$\delta_0 = 27^\circ\ 47'\ 02''.37.$$

St. Louis is west of Washington....................................	53^m
St. Louis sidereal time of transit (same as α)....................	17^h 42
Washington sidereal time (to nearest minute)....................	18 35
Sidereal time of mean midnight (at end of the civil day, July 16) by interpolation between sidereal times of mean noon as given in Ephemeris, p. 381, for July 16 and 17 (to nearest minute)....	19 42
Hence the sidereal interval before Washington midnight for the stated time is..	1 07

This interval is, with sufficient accuracy for the purpose of interpolation of the star-numbers, $\frac{1^h\ 07^m}{24^h} = 0.05$ day.

The Ephemeris, p. 289, gives directly the following values:

July 15, Washington mean midnight:

τ	f	G	H	log g	log h	log i
0.54	+ $1^s.00$	315° 26′	157° 43′	+ 0.9607	+ 1.3048	+ 0.5212

July 16, Washington mean midnight:

0.54	+ $1^s.00$	315° 41′	156° 49′	+ 0.9620	+ 1.3043	+ 0.5317

The signs attached to log g, log h, log i in the Ephemeris are the signs of g, h, and i, and not signs applying to their logarithms, as might naturally be supposed from the way in which they are printed.

For the stated time, 0.05 day before Washington mean midnight of July 16th, following the order indicated by formula (10),

$\alpha_0 =$	$17^h\ 42^m\ 13^s.86$
$f =$	$+ 1.00$
(μ not being given) $\tau\mu =$	0.00

$$\log \tfrac{1}{15} = 8.8239$$

$$\log g \ = 0.9619$$

$G = 315° \ 40'$, $(G + \alpha_0) = 221° \ 13'$, $\log \sin (G + \alpha_0) = 9.8185_n$

$$\log \tan \delta_0 = 9.7217$$

$$\log \tfrac{1}{15} g \sin (G + \alpha_0) \tan \delta_0 = 9.3263_n$$

$$\tfrac{1}{15} g \sin (G + \alpha_0) \tan \delta_0 = -0.21$$

$$\log \tfrac{1}{15} = 8.8239$$

$$\log h = 1.3043$$

$H = 156° \ 52'$, $(H + \alpha_0) = 62° \ 25'$, $\log \sin (H + \alpha_0) = 9.9475$

$$\log \sec \delta_0 = 0.0532$$

$$\log \tfrac{1}{15} h \sin (H + \alpha_0) \sec \delta_0 = 0.1290$$

$$\tfrac{1}{15} h \sin (H + \alpha_0) \sec \delta_0 = +1.35$$

α, at $16^h \ 10^m \ 14^s.1$. St. Louis Sidereal Time, July 16, 1892 $= 17^h \ 42^m \ 16^s.00$

The computation for δ, following the order of (11) is

$$\delta_0 = 27° \ 47' \ 02''.37$$

$\tau\mu' = (0.54)(-0''.76)$, [$\mu'$ is given $= -0''.76$ in Boss' list] $= -0 \ .41$

$$\log g = 0.9619$$

$$\log \cos (G + \alpha_0) = 9.8766_n$$

$$\log g \cos (G + \alpha_0) = 0.8382$$

$$g \cos (G + \alpha_0) = -6 \ .89$$

$$\log h = 1.3043$$

$$\log \cos (H + \alpha_0) = 9.6661$$

$$\log \sin \delta_0 = 9.6685$$

$$\log h \cos (H + \alpha_0) \sin \delta_0 = 0 \ 6384$$

$$h \cos (H + \alpha_0) \sin \delta_0 = +4 \ .35$$

$$\log i = 0.5363$$

$$\log \cos \delta_0 = 9.9468$$

$$\log i \cos \delta_0 = 0.4831$$

$$i \cos \delta_0 = +3 \ .04$$

δ, at $16^h \ 10^m \ 14^s.1$. St. Louis Sidereal Time, July 16, 1892 $= 27° \ 47' \ 02''.46$

The above example shows how far out the computation needs to be carried. Where many star places are to be computed, the computation is materially shortened by using printed blank forms so arranged as to facilitate the work.

Especially convenient forms of that nature are in use in the Coast and Geodetic Survey.

49. In computing a number of stars on a single night—which will usually be the case in dealing with latitudes observed with a zenith telescope—considerable time will be saved at an exceedingly small sacrifice of accuracy by the following procedure. First interpolate the values of the independent star-numbers for every whole hour from Washington mean midnight for the period over which the obervation extends. Then for each star use the interpolated value of each star-number for the nearest hour as interpolated, instead of making a special interpolation for each.

For an account of the method of computation of the independent star-numbers, and the method of computing star places by the use of the Besselian star-numbers, see Doolittle's Practical Astronomy, pp. 609–617; Chauvenet's Astronomy, vol. I. pp. 645–651; and the Ephemeris, pp. 280–284 (of the volume for 1892). The Besselian star-numbers are not ordinarily so convenient for the engineer as the independent star-numbers.

If one has a great number of star places to compute, under certain conditions, the work may be abridged somewhat by using differential and graphic methods. For the details of a differential method which reduces the labor of computation about one-half in case the place of each star is to be computed on three or more nights, see Coast and Geodetic Survey Report, 1888, pp. 465–470. A somewhat similar method to be used when the places are to be computed for a few stars on many nights will be found in the Coast and Geodetic Survey Report for 1892, Part II, pp. 73–75. For a graphic method of reducing from the mean to the apparent place in declination, see Coast and Geodetic Survey Report, 1895, pp. 371–380.

QUESTIONS AND EXAMPLES.

50. 1. At a certain instant in the forenoon of May 27, 1892, the observed hour-angle of the Sun at Cornell Observatory was $2^h\ 30^m\ 41^s$. What was its apparent right ascension and declination at that instant? Cornell is $2^m\ 16^s$ east of Washington. The Ephemeris for 1892 (p. 380) gives for Washington apparent noon May 26th $\alpha = 4^h\ 15^m\ 47^s.93$, $\delta = 21°\ 17'\ 45''.3$, hourly motion in right ascension $= + 10^s.141$, hourly motion in declination $= + 25''.16$; and for Washington apparent noon May 27th $\alpha = 4^h\ 19^m\ 51^s.57$, $\delta = 21°\ 27'\ 38''.3$, hourly motion in right ascension $= + 16^s.160$, in declination $= + 24''.23$.

Ans. By interpolation along a tangent $\alpha = 4^h\ 19^m\ 25^s.67$, $\delta = 21°\ 26'\ 36''.5$.

By interpolation along a parabola $\alpha = 4^h\ 19^m\ 25^s.67$, $\delta = 21°\ 26'\ 36''.4$.

2. What was the apparent declination of the Sun at $7^h\ 41^m\ 32^s$ A.M. Greenwich mean time on Dec. 21, 1892? The Ephemeris for 1892 (p. 201) gives the apparent declination of the Sun at Greenwich mean noon Dec. 21st $= - 23°\ 27'\ 18''.6$, and its hourly motion $= + 0''.18$; and for Dec. 20th $\delta = - 23°\ 27'\ 08''.7$, with an hourly motion $= - 1''.00$. *Ans.* $- 23°\ 27'\ 18''.9$.

3. Work the preceding problem, as a check, from the following data from the Ephemeris (p. 384): Declination of Sun at Washington mean noon Dec. 21st $= - 23°\ 27'\ 17''.0$, hourly motion $= + 0''.43$. Hourly motion for Washington mean noon Dec. 20th $= - 0''.75$. Washington is $5^h\ 8^m\ 12^s$ west of Greenwich. *Ans.* $- 23°\ 27'\ 18''.9$.

4. At a station $2^h\ 58^m$ west of Washington the south zenith distance of Jupiter, at the instant of its meridian transit on July 16, 1892, was observed to be $29°\ 22'\ 17''.4$.

What was the latitude of the station? The apparent declination of Jupiter at its meridian transit at Washington is given in the Ephemeris (p. 404) as follows: July 15th $+7^\circ$ $56' 11''.8$, July 16th $+7^\circ 57' 53''.8$, and July 17th $+7^\circ 59'$ $32''.0$. *Ans.* $37^\circ 21' 23''.5$.

5. What was the right ascension and declination of the Moon at $8^h 30^m 09^s$ P.M. local mean time at Cornell April 8, 1892? Cornell is $5^h 5^m 56^s$ west of Greenwich. In the Ephemeris (p. 62) the position of the Moon is given for 2^h A.M. Greenwich mean time April 9th, $\alpha = 11^h 33^m 43^s.91$, $\delta = +7^\circ 34' 02''.2$; "difference for 1 minute" in right ascension $= +1^s.7980$; "difference for 1 minute" in declination $= -13''.192$. The differences for 1 minute in right ascension and declination respectively at 1^h A.M. are $+1^s.8003$ and $-13''.167$.

Ans. By interpolation along a tangent $\alpha = 11^h 33^m 00^s.91$, $\delta = 7^\circ 39' 17''.7$.

By interpolation along a parabola $\alpha = 11^h 33^m 00^s.90$, $\delta = 7^\circ 39' 17''.6$.

6. What was the apparent right ascension of the star λ Aquarii at transit at Mount Hamilton, Cal., Sept. 1, 1892? For upper transit at Washington (Ephemeris, p. 362) on August 27th $\alpha = 22^h 46^m 61^s.51$, and $\frac{d\alpha}{dt} = +0^s.10$ per ten days. Also for Sept. 6th $\alpha = 22^h 46^m 61^s.66$, and $\frac{d\alpha}{dt} = +$ $0^s.05$ per ten days. The longitude of Mount Hamilton is $2^h 58^m 22^s$ west of Washington.

Ans. By interpolation along a chord $\alpha = 22^h 47^m 01^s.63$.

By interpolation along a tangent from Sept. 6th $\alpha = 22^h$ $47^m 01^s.64$.

By interpolation along a parabola $\alpha = 22^h 47^m 01^s.63$.

7. For star *BAC* 5706 $\alpha_{1875} = 16^h 50^m 43^s.9$, $\delta_{1875} = 46^\circ$

44′ 31″.22. Its annual variation in right ascension for that date $= + 1^s.721$, and in declination (including proper motion) $= - 6''.0105$. What was its mean declination for 1895.0?

Ans. $\delta_{1895} = 46° 42' 31''.49$.

8. For the star η Geminorum $\alpha_{1875} = 6^h 07^m 20^s.0$, $\frac{d\alpha_m}{dt} = + 3^s.622$ per year, $\delta_{1875} = 22° 32' 27''.18$, annual precession in declination $= - 0''.6415$, annual proper motion in declination $= - 0''.0161$. What is its mean declination for 1892.0? *Ans.* $\delta_{1892} = 22° 32' 15''.24$.

9. For the star *BAC* 7440, $\alpha_{1892} = 21^h 19^m 39^s$, $\delta_{1892} = - 4° 01' 11''.30$, and annual proper motion in declination $= - 0''.068$. For the star *BAC* 7482 $\alpha_{1892} = 21^h 25^m 41^s$, $\delta_{1892} = 66° 20' 16''.00$, and annual proper motion $= - 0''.042$. What was the apparent declination of each of these stars at transit on August 9th and 16th at San Bernardino Ranch, Arizona, $2^h 09^m$ west of Washington? The Ephemeris (pp. 289, 290) gives the following data for Washington mean midnight:

	Aug. 9.	Aug. 10.	Aug. 16.	Aug. 17.
τ..........	0.61	0.61	0.63	0.63
G..........	319° 33′	319° 29′	320° 38′	320° 54′
H..........	134° 38′	133° 30′	127° 38′	126° 39′
log g.......	1.0309	1.0328	1.0418	1.0449
log h......	1.2907	1.2900	1.2863	1.2857
log i.......	0.7815	0.7879	0.8226	0.8276

The sidereal time of mean midnight at Washington on Aug. 9th was $21^h 17^m$, and on Aug. 16th, $21^h 44^m$.

Ans. BAC 7440, Aug. 9th, $\delta = - 4° 01' 03''.43$.
Aug. 16th, $\delta = - 4° 01' 02''.75$.
BAC 7482, Aug. 9th, $\delta = 66° 20' 18''.63$.
Aug. 16th, $\delta = 66° 20' 21''.26$.

10. Justify the half square in the last term of formula (9), § 42. That is, show that (9) is exact if $\frac{d^3\delta_m}{dt^3} = 0$.

11. Show from formula (5), § 41, that $\frac{d\alpha_m}{dt}$ cannot be negative for any star unless its declination is quite large (near 90°). What other condition must also be fulfilled ?

12. Draw and explain the diagram called for in the parenthesis in § 38, showing the geometrical relation between the aberration, velocity of light, and velocity and direction of motion of the observer.

13. Look in the Ephemeris and see whether τ as given with the independent star-numbers for Jan. 0 of the current year is zero. If not, why not ? At what time during this year was it exactly zero ?

CHAPTER III.

THE SEXTANT.

51. The sextant is an instrument for measuring angles; especially useful at sea and on exploratory surveys because of its lightness and portability, and because it requires no fixed support. It also commends itself in certain other cases because results of a sufficient degree of accuracy can be obtained with it more conveniently than with larger instruments with fixed supports which might otherwise be employed. It is principally used in astronomy for the determination of local time and of latitude at sea, and on explorations by land. It is also used extensively in hydrographic surveying for the measurement of horizontal angles serving to locate soundings. With it an angle may be measured from the deck of a rolling vessel where an engineer's transit or a theodolite would be unavailable.

Description of the Sextant.

52. A view of a sextant is shown in Fig. 3. The main frame *ABC* carries a graduated arc, *DE*, of which the center is at *F*; it carries a bearing at *F* which receives the axis (perpendicular to the plane of the frame) about which the arm *GF* swings; a ring secured to the frame at *H* into which the telescope *I* is screwed and held in a fixed position relatively to the frame; a plane mirror at *J*, called the *horizon-glass*, which is fixed to the frame in a plane perpendicular to it; and certain colored glasses at *K* and *L*, which may be used to absorb some of the light when observing the Sun (or the Moon with a star), so that the images seen in the telescope may not be too bright for comfort. *N* is the handle by which the sextant is held in the observer's right hand. The arc *DE* is graduated to five-minute spaces, but has the graduation marked upon it *as if each space were* TEN *minutes*. The arm *GF* carries a vernier at *G*, which reads against the arc *DE* to five seconds (real). It is marked, however, as if it read to *ten seconds*. Any reading on the arc *DE* made by means of the vernier indicates, therefore, twice the angle between that position of the arm *GF* and the position corresponding to the zero reading. *M* is a small glass used in reading the vernier.

The arm *GF* also carries at *F* a plane mirror, called the *index-glass*, which is perpendicular to the plane of the frame for any position of the arm. The horizon-glass *J* has only that half of its surface which is nearest the sextant frame silvered. The other half is merely a plane clear glass, or is cut away entirely. The telescope *I* may be adjusted to such a distance from the sextant frame that the edge of the silvering of mirror *J* is in the axis of the telescope produced. The observer thus sees at the same time both the images reflected from the silvered surface and whatever may be in the line of sight of the telescope beyond the mirror.

The Principle of the Sextant.

53. The principle underlying sextant observations is indicated by Figs. 4 and 5. Suppose the sextant to be in perfect adjustment. Let *OP*, Fig. 4, be a ray of light, from a distant object, which passes through the unsilvered portion of mirror *J*, without change of direction, into the telescope *I*, parallel to its axis. Let *QR* be a ray of light, parallel to *OP*, which strikes the index-glass *F*. The positions of *F* relative to the arm *FG*, and of *J* relative to the telescope, are such that if the reading of the arc taken from vernier *G* is zero the ray *QR* will be reflected from *F* along the line *FJ*, and reflected again from the silvered portion of *J* along a line parallel to *OP* into the telescope. The image of the object from which *QR* came will therefore be seen in the telescope in coincidence with the image of the object from which *OP* came. To secure the above result mirrors *F* and *J* must, for this zero position of the arm, be parallel, and the perpendiculars to the mirrors, *FS* and *JT*, must bisect the angles *QFJ* and *FJP*. Note that the arc reading is the same as the angle (zero in this case) between the two rays of light *QR* and *OP* which eventually enter the telescope as parallel rays.

In Fig. 5 let *OP* be as before; but let *Q'R'* be a ray of light at an angle β with the ray *OP* (or with the ray *QR* of the preceding paragraph) and striking the index-glass. Evidently *Q'R'* will not be reflected to *J* from *F* unless *F* is first rotated through the angle $\frac{\beta}{2}$ by moving the arm *FG* to the position *FG'*. In this position of the mirror *F* the perpendicular *FS'* will bisect the angle *Q'FJ*. The angle *GFG'* will be $\frac{\beta}{2}$, but on account of the peculiar graduation of the arc as indicated in § 52 the reading of the arc will be β. The ray *FJ* will evidently be reflected into the telescope, as before, along a line parallel to *OP*, and the object from which *Q'R'* came will be seen in the telescope apparently in coincidence with the object from which *OP* came. Note that here, as before, the reading of the arc is the angle between the two rays of light *Q'R'* and *OP*.

So for any case, if the sextant is in perfect adjustment, the reading of the arc is the angle between two rays of light, one coming to the index-glass and the other through the unsilvered portion of the horizon-glass, which finally reach the telescope as parallel rays and produce coincident images. When the images of two celestial objects, or any two objects at a great distance from the observer, are made to apparently coincide in the telescope the arc reading is the angle at the eye between the two objects, measured in the plane defined by the two objects and the eye. This plane may happen to be in a horizontal, a vertical, or an oblique position. If the two objects observed are at a comparatively short distance it may be necessary to take account of the fact that the angle indicated by the arc is the angle between the two objects *from the point U* in which $Q'R'$ produced intersects OP, and not the angle at the eye. The difference between these two angles is called the *sextant parallax*.

Adjustments of the Sextant.

54. *To make the index-glass perpendicular to the plane of the sextant.**—Place the vernier near the middle of the arc. Hold the instrument with the arc away from you, and look obliquely into the index-glass in such a way as to see a portion of the arc both directly and by reflection at the same time. If the direct and reflected portions appear to form one continuous arc the adjustment is perfect. If not, the inclination of the glass to the plane of the sextant must be changed by whatever means have been provided on that particular instrument. This adjustment once carefully made will not require frequent attention; for this reason some makers do not provide a convenient means of making it.

55. *To make the horizon-glass perpendicular to the plane of the sextant.*—Having first adjusted the index-glass, point the

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* By "*plane of the sextant*" is meant the plane of the graduated arc, to which the axis about which the arm rotates is necessarily perpendicular.

telescope to any well-defined object and move the vernier slowly back and forth past the zero. The reflected image will be seen moving back and forth past the direct image. If in passing it coincides with the direct image, the adjustment is perfect. If not, the correction must be made by use of the screws provided for that purpose at the back of the horizon-glass. With most sextants this adjustment must be inspected frequently.

A star is the best object for this purpose. The Sun may also be employed. The accuracy will be increased by making the two images of the Sun appear of different colors by use of the colored glass shades. The sea horizon may be used by holding the plane of the sextant horizontal and keeping the arc reading nearly zero.

56. *To make the axis of the telescope parallel to the plane of the sextant.*—Rotate the eye end of the telescope until two of the four dark lines seen in the telescope are parallel to the plane of the sextant.* Point the telescope to one of two objects at an angle of 90° or more apart. Bring the reflected image of the second object into contact with the image of the first at that one of the parallel lines which is apparently nearest the sextant frame. This may be done by rotating the sextant about the telescope as an axis until it is in the plane of the two objects and the eye, and then bringing the vernier to the proper reading by trial. Now move the instrument, without changing the vernier reading, so that the two images are upon the other of the two parallel lines. If the contact is still perfect no adjustment is required. Otherwise the ring into which the telescope is screwed must be adjusted to change the inclination of the telescope to the sextant plane until the above test fails to detect any error.

* These lines are placed by the instrument-maker in symmetrical positions on each side of the middle of the field of the telescope. All observations are to be made at about the middle of the space defined by them.

This adjustment may also be made as follows: Place the sextant face upward on a table or other firm horizontal support. Sight across in the plane of the graduation, or a parallel plane, and mark a point at that height and in line with the telescope, upon a wall distant fifteen feet or more. Measure upward from this point a distance equal to the measured distance at the sextant from the sight plane just used to the axis of the telescope, and mark this second point. The ring carrying the telescope must now be moved, if necessary, in such a way as to change the inclination of the telescope until this second point is exactly in the centre of the field of the telescope.

It should be noted that when this adjustment has been accurately made a contact made upon one of the side lines will not necessarily be perfect when shifted to the middle of the field. The reading of the arc is *slightly* less for a contact made at the middle of the field than for one made on *either* side when all adjustments are perfect.*

This adjustment will usually remain sensibly perfect for a long period.

57. *To make the index error zero.*—The reading of the arc when the direct and reflected images of the same point† are made to coincide is called the *index error* of the sextant. The negative of the index error is the *index correction,* which evidently must be applied to every reading. To make the index error zero the horizon-glass may be rotated about a line perpendicular to the plane of the sextant. Screws for producing this rotation are often, though not always, provided

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* For a detailed statement of the theory of the errors arising from non-parallelism of telescope to the plane of the sextant, see Chauvenet's Astronomy, vol. II, pp. 112–114.

† Provided, of course, that the point is so distant that the sextant parallax (§ 53) may be neglected.

by the sextant-maker. Since this adjustment cannot ordinarily be depended upon to remain perfect even for a day, it is advisable to determine the index error at the time of each series of observations, and apply the derived correction, instead of trying to keep the value of the error down to zero by adjustment. When this procedure is adopted it is only necessary to make the adjustment at rare intervals when the index error has become inconveniently large. The method of determining the index error will be found in § 62.

Directions for Observing the Sun's Altitude with a Sextant to Determine the Local Time.

58. The altitude of the Sun is a known function of the latitude of the station of observation, of the declination of the Sun, and of the local apparent solar time. Hence if the latitude and declination are known, and the altitude is measured, at a given instant, the local apparent solar time may be computed. From this the mean solar time may be derived.

In determining the altitude of the Sun at a station on land the artificial horizon must be used. The *artificial horizon* is a shallow rectangular basin filled with mercury, molasses, or oil, protected from the wind by a roof consisting of two pieces of plate glass held in a suitable mounting. These glass plates each have faces which are as nearly as possible plane and parallel, so that rays of light may pass through them without unequal change of direction.

Let *MN*, Fig. 6, represent the surface of the mercury in the artificial horizon. *MN* is necessarily a horizontal surface, that is, a surface which is perpendicular at every point to the action line of gravity at that point. A ray of light *SB* from the Sun will be reflected along a line *BA* in the same vertical plane with *SB*, and such that the angle *NBA* is equal to the angle *MBS*. An observer at *A* will see the reflected image along the line AS''. He may also see the Sun directly along the line AS'. The distance AB being very small as compared with the distance to the Sun, $S'A$ and SB are sensibly parallel, and the angle $S'AS''$, which is to be measured with the sextant, is evidently the double altitude of the Sun.

Before commencing the observations, the adjustments should be examined and corrected if necessary, the telescope should be *carefully* focused to give well-defined images of the Sun, and such colored glasses interposed in the path of the light that the two images of the Sun will be of about the same brightness, and dim enough so that continued gazing at them will not fatigue the eye.

To begin observations, place the eye in such a position that an image of the Sun can be seen reflected from the artificial horizon. Without moving the eye, bring the telescope of the sextant up to it, and point upon this image. Being careful to hold the plane of the sextant vertical, swing the vernier slowly back and forth along the arc. If this is done with sufficient care, a second image of the Sun, formed by light coming to the telescope by way of the index-glass, will be seen in the telescope when the vernier is near the reading of the arc corresponding to the double altitude of the Sun.

For convenience let the two images of the Sun be called the horizon image and the index-glass image, respectively.

If the observer has not had sufficient experience to handle the sextant with facility, it will be well for him at this point to familiarize himself with the following motions and their effects. Rotate the sextant about the telescope as an axis: the horizon image will appear to remain fixed while the index-glass image will appear to move sidewise horizontally. Move the vernier slowly along the arc, keeping the sextant frame and telescope fixed: the index-glass image will appear to move vertically, while the horizon image apparently remains fixed. Rotate the sextant about a line in its plane perpendicular to the telescope at the eye end: the images will appear to move sidewise without change of relative position. Rotate the sextant about a line perpendicular to its plane at the eye: and the images will appear to move vertically without change of relative position. Move the eye horizontally in the vertical plane passing through the artificial horizon: and the horizon image will appear to be cut off by a straight line on the upper edge, or lower, as the eye is moved forward or backward. Similarly, if the eye is moved sidewise, the horizon image will be seen partially cut off by the side of the artificial horizon on one side or the other, as the case may be. The effects of these various movements of the sextant have been commented upon because the ease and rapidity with which one can use the sextant depends largely upon having accurate conceptions of these effects, as well as upon manual skill. To secure steadiness of the sextant, it is well, in addition to holding it by the handle in the right hand, to rest the lower edge of the arc upon the fingers and thumb of the left hand. Care must be taken, however, not to touch the graduations at any time; nor to touch any part of the vernier arm, or of the clamp and screws attached to it, at the instant when an observation is made.

59. The observer having secured control of the images, let them be placed so as to be near together, one above the other, and approaching each other, let us say. For the images will, in general, be moving relatively to each other, since the altitude of the Sun is continually changing. Clamp the vernier in this position. Pick up the beat of the chronometer.* Then watch the approaching images, keeping their adjacent portions about in the middle of the field of the

* See § 60.

telescope, and carefully keeping one image vertically above the other. Note the exact time by the chronometer when the two images are first tangent to each other. Observe and record the corresponding reading of the vernier. Unclamp the vernier and place the images in about the same relative position as before, and repeat the process until six readings (say) of time and the corresponding angle have been made. Then repeat the whole process, with the difference that now the images are slightly overlapped at first and allowed to separate, the instant when the tangency of the images takes place being noted. To complete the observation of the Sun's altitude it now remains to determine the index error (see § 62).

When a tangency is observed with images approaching, the noted time is too late unless the observer has accurately kept the images in the same vertical plane. The reverse is true of an observation made upon separating images. To guard against an error in locating the vertical plane, it is well to continually rotate the sextant very slightly back and forth around the telescope as an axis so as to be certain to secure the first, or last, tangency, as the case may be.

60. To pick up the beat of the chronometer, first look at some second-mark two seconds or more ahead of the seconds hand. Fix the name of that second in mind as the seconds hand approaches it. Name it exactly with the tick at which the seconds hand reaches it, keeping the rhythm of the chronometer beat. Count it either aloud, in a whisper, or mentally. In counting it will be found easier to keep the rhythm if the names of the numerals are elided in such a way as to leave but a single staccato syllable in each. The half-second beat should be marked by the word "half" thus: one, half; two, half; three . . .; *twenty*, half; twenty-*one*, half; twenty-*two*, . . .; and so on. With practice, an observer can carry the count of the beat for an indefinite period without looking at the chronometer face, provided he can hear the tick. If he becomes expert, he will even be able to carry the count for a half-minute or more during which he has not even heard the tick. When an observation is made of the time of tangency of two images, or of any other visual event, the eye observes the event and the chronometer is read by ear at the same instant. It is conducive to accuracy for the observer to acquire the habit of deciding definitely, at once, without hesitation, upon the second and fraction as soon as he has seen the event. He who hesitates is inaccurate.

The observation of time may be made by the observer at the sextant calling "tip," at the instant of tangency, to an assistant who reads the face of the chronometer by eye. This is an easier process, but is also a much less accurate process than the one described above. The nerve times (or intervals of time required for the nerves concerned to perform their functions), and errors of judgment, of *two* men instead of one, enter into the result. Moreover, the assistant at the chronometer is observing an event which comes upon him suddenly instead of one of which he sees the gradual approach. If, however, an ordinary watch is used instead of a chronometer, it is necessary to let an assistant read the time, both on account of the faintness of the tick, and because it is difficult to carry by ear a beat of five ticks per second.

61. The image of the Sun seen in the surface of the mercury is reversed by the reflection in such a way that the apparent upper edge, or *limb*,* of the image is really the image of the lower limb of the Sun. The image of the Sun received by way of the index-glass and horizon-glass is reversed at each of the two reflecting surfaces, and is finally

* The word *limb* is here used in the technical sense, in which it means the edge of the visible disk—of the Sun, Moon, or other heavenly body.

seen as if no reversal had taken place. If, then, one makes the lower limb of the index-glass image tangent to the upper limb of the mercury image, there are really at the point of tangency two coincident images of the *same point* of the Sun, namely, the lower limb. If an inverting telescope is used *both* images are reversed again in addition to the reversals stated above. The record of observations must be made to show which limb of the Sun is used in each case. The object of making each complete set of observations include pointings upon each of the two limbs is the elimination of certain errors, which will be commented upon later (§ 74).

62. To determine the index error, point the telescope at the Sun, with the vernier set near zero, and with the sextant in such a position that a line in the plane of the sextant perpendicular to the telescope is horizontal. Make the direct and reflected images of the Sun tangent to each other, with the zero of the vernier on the positive part of the graduated arc, and read the vernier. Make the two images tangent to each other in the reverse position, with the zero of the vernier on the negative portion of the graduated arc, and read the vernier again. Repeat the process two or three times for greater accuracy. Any reading on the positive portion of the arc evidently gives a measure of the Sun's apparent diameter. So also does any reading on the negative arc. But these two measures are affected equally and in opposite directions by the index error. Hence both the index error and the Sun's diameter become known.

The Sun's *horizontal* diameter is measured in order that the results may not be affected by the refraction * in the vertical plane. In making readings on the negative arc care must be taken to mentally reverse the numbering of the graduations on the vernier.

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* See §§ 67–69.

EXAMPLE OF RECORD.

63. Determination of Time by Sextant Observations upon the Sun.

Astronomical Station No. 10.

Latitude 31° 19′ 35″ — Longitude $2^h 12^m$ west of Washington.

Date—October 14, 1892, A.M. — Observer—J. F. H.

Chronometer—Dent No. 2186. — Sextant—Greenough No. 2083.

Thermometer Reading, 17° C. — Barometer Reading, 25.56 in. (Aneroid).

Reading of Arc. On Sun's Upper Limb.	Chronometer Time.	Reading of Arc. On Sun's Lower Limb.	Chronometer Time.
64° 40′ 00″	$11^h\ 01^m\ 35^s.0$	65° 40′ 00″	$11^h\ 07^m\ 37^s.5$
65 00 00	02 32 .0	66 00 00	08 36 .0
65 20 00	03 31 .0	66 20 00	09 35 .0
65 40 00	04 29 .5	66 40 00	10 33 .5
66 00 00	05 28 .5	67 00 00	11 33 .0
66 20 00	06 26 .0	67 20 00	12 33 .5

DETERMINATION OF INDEX ERROR.

By Measurement of Sun's Horizontal Diameter.

Readings on Arc.	Readings on Negative Arc.
32′ 40″	31′ 40″
32 30	31 40

Each half of the above set of observations was computed separately. The computation for the first half of the set, on Sun's upper limb, is given below. The explanation of the computation follows (§§ 65–70).

64. Computation.

Mean reading of arc,		65° 30′ 00″
Index error $= I$	=	−28
Eccentricity (not determined)	=	00
$2A_u$	=	65 29 32
Approx. altitude $= A_u$	=	32 44 46
Sun's semi-diameter	=	−16 05
Parallax $= p$	=	+ 07
Refraction $= R$	=	− 1 16

Index Error.

Mean reading on arc	=	32′ 35″
Mean reading on negative arc	=	31′ 40″
Diff.	=	00′ 55″
½ Diff. = Index error $= I$	=	−00′ 28″

Altitude $= A$	$= 32\ 27\ 32$	
Zenith distance $= \zeta$	$= 57\ 32\ 28$	
Latitude $= \phi$	$= 31\ 19\ 35$	$\log \cos \phi = 9.9315695$
Declination $= \delta$	$= -8\ 29\ 33$	$\log \cos \delta = 9.9952117$
$(\phi - \delta)$	$= 39\ 49\ 08$	9.9267812
$\frac{1}{2}[\zeta + (\phi - \delta)]$	$= 48\ 40\ 48$	$\log \sin \frac{1}{2}[\zeta + (\phi - \delta)] = 9.8756596$
$\frac{1}{2}[\zeta - (\phi - \delta)]$	$= 8\ 51\ 40$	$\log \sin \frac{1}{2}[\zeta - (\phi - \delta)] = 9.1876329$
		9.0632925
		$\log \sin^2 \frac{1}{2}t = 9.1365113$
$\frac{1}{2}t$	$= 21^\circ\ 43'\ 06''$	$\log \sin \frac{1}{2}t = 9.5682556$
t = hour-angle	$= 43\ 26\ 12$	
T_A = apparent solar time	$= 9^h\ 06^m\ 15^s.2$	
E = Equation of time	$= -14\ 08.6$	
T_M = Mean solar time	$= 8\ 52\ 06.6$	
T_c Time by chronometer = Mean of six given readings	$= 11\ 04\ 00.3$	
ΔT_c = Chronometer correction	$= -2\ 11\ 53.7$	

Explanation of Record and Computation.

65. The above observations were made at uniform intervals of 20′ on the sextant arc by setting the vernier *before* each observation to that exact reading, instead of taking the readings on the arc after each random pointing. A rough method of detecting any single wild observation was furnished by the fact that the time intervals between successive readings must be nearly the same throughout.

The mean of the arc readings is assumed to correspond to the mean of the observed chronometer times. This would be strictly true if the rate of change of the Sun's altitude, as affected by refraction, were constant during the period covered by each half set of observations. The rate of change varies so little during this short interval that the error introduced is negligible.

The method of computing the index error has already been indicated (§ 62).

The eccentricity of this sextant had not been determined, but was known to be small. For the method of determining eccentricity, see § 76.

Half the corrected reading of the sextant is the *approximate* altitude of the Sun, to which must be applied the corrections for the Sun's semi-diameter, for parallax, and for refraction, as indicated in the following sections.

The pointings were made upon the Sun's upper limb. But the position of the Sun as given in the Ephemeris is for the Sun's center. Hence the angle subtended at the observer by the Sun's semi-diameter must be subtracted to reduce the altitude to the value which would have been obtained had the observations been made upon the center. This angular semi-diameter of the Sun is given in the Ephemeris (pp. 377–384 of the volume for 1892) for every day at Washington apparent noon. It can be obtained for any other time with all needful accuracy by an interpolation along a chord.

Parallax.

66. Moreover, since the right ascension and declination of the Sun define its position on the celestial sphere as seen from the Earth's *center*, it is necessary to reduce the observed altitude to what it would have been had the observer placed himself at the center of the Earth and had used the same horizon as before.

In Fig. 7, let S represent the position of the Sun's center, and let the circle BFG represent a section of the Earth made by a plane passing through the observer, at B, and the centers of the Sun and Earth at S and C, respectively. Let BD represent the plane of the observer's horizon. Let CE be parallel to BD. Then DBS is the altitude of the Sun as seen by an observer at B. ECS is the altitude that would be observed by him if he were at the Earth's center and used a horizon plane parallel to the one he used at B. The difference between these two angles, which is evidently equal to the angle BSC, is the reduction required.

In general, the *parallax* of an object is its apparent displacement due to a change in the position of the observer. The *parallax* of any celestial object is the difference of direction of two straight lines drawn to it from two different points of view. It is, then, the angle *at the object* between the two straight lines drawn to it from the two points from which it is supposed to be viewed. The word *parallax*, unmodified, will be used in this book to indicate the difference of direction of a celestial object as seen from the center of the Earth and from a station on the surface. The *horizontal parallax* is the parallax for an object which is *in the horizon of the observer*. The *equatorial horizontal parallax* is the parallax of a celestial object seen in the horizon by an observer at a station *on the Earth's equator*.

In Fig. 7, if S' represents a position of the Sun in the horizon of the observer at B, the angle $BS'C$ is the horizontal parallax of the Sun. It is the angle subtended at the Sun by the radius BC of the Earth. If p_h is the horizontal parallax of the Sun in seconds of arc, r is the radius of the Earth, and d_s is the distance between the centers of the Earth and Sun, then

$$p_h = \frac{r}{d \sin 1''} = \text{about } 9''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The exact value of the equatorial horizontal parallax of the Sun is given in the Ephemeris at intervals of ten days (p. 278 of the volume for 1892). The different radii of the Earth are so nearly equal that the Sun's horizontal parallax for a station anywhere on the surface will not differ from that for an equatorial station by more than 0″.03,—a quantity which may be disregarded for our present purpose.

Returning to the figure, let *BH* be drawn perpendicular to *CS*. If p is the parallax of the Sun at any position, *S*, above the horizon, then, keeping in mind that p and p_h are very small angles,

$$p : p_h = \overline{BH} : \overline{BC},$$

or

$$p = p_h \cos A. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The table in § 293, abridged from Connaissances des Temps, serves to give the parallax of the Sun for any date and altitude. The distance of the Sun is so nearly the same for the same date in different years that the table may be used for any year for several centuries to come.

Refraction.

67. The path of a ray of light from any celestial object to an observer upon the Earth's surface is, to the best of our knowledge, a straight line until the ray enters the Earth's atmosphere. From that point onward the ray encounters at each successive element of its path a stratum of air which is more dense than the stratum left behind, since the density of the air continually increases from the top downward with the increasing pressure due to weight of the superincumbent strata. The ray is, therefore, continually being refracted, or bent, out of the straight line, and this portion of its path is a curve.

The two general laws of refraction are: That when a ray passes from a rarer to a denser medium it is refracted toward the normal to the separating surface by an amount which is a function of the angle between the ray and the normal, and of the densities of the two media; and that a plane containing the normal and the original ray also contains the refracted ray. The refraction is reversed in passing from a denser to a rarer medium.

Let Fig. 8 represent a portion of a section of the Earth and its atmosphere made by a plane passing through the center of the Earth and the straight portion of a ray of light from the celestial object O to the point of incidence, a, of the ray with the Earth's atmosphere. At a the ray is refracted out of the straight line Oa to a new direction ab, nearer to the normal aC, and still in the plane OaC. At the point b at which the ray passes to a denser stratum the ray is again bent, toward the normal bC, to the new direction bc. The ray is thus refracted at the successive points a, b, c, d, etc., remaining always in the plane OaC until it finally reaches the observer at A. In reality the path is a continuous curve, since the increase of density is continuous. An observer at A sees the object in the direction AO' along the tangent at A to the path of the ray. The angle between the original direction of the ray, Oa, and its final direction, $O'A$, is called the *astronomical refraction*, or, for convenience, simply the *refraction*. It should be noted that the refraction as described above affects altitudes directly, always making the observed altitude too great, but has no effect on azimuth.

In the above treatment it is assumed that the various strata of air are horizontal at every point. For a statement of the extent to which the azimuth is affected by refraction because of the error of the above assumption, see § 219.

Even if the law of variation of density of the air with the height were a simple one, the computation of the refraction would be a complicated process. But the laws governing the variation of density are themselves complicated, and not thoroughly known. Hence the theory of refraction is long and difficult. The treatment of refraction in this book will therefore be limited to an explanation of the tables given in §§ 294–297, which contain the results of the astronomer's investigations in convenient form for the engineer.

68. § 294 gives the *mean refraction*,* or refraction under the mean conditions, at the station of observation, stated at the head of the table, viz., pressure 760 mm. (= 29.9 in.) and temperature 10° C. (= 50° F.). The mean refraction is a function of the altitude, since the refraction of a ray of light in passing from one medium into another is a function of the angle between the ray and the normal to the dividing surface.

§ 295 gives the factor, C_B, by which the mean refraction must be multiplied if the reading of the barometer is not exactly 760 mm. The argument of this table is the barometer reading uncorrected for temperature, but corrected if necessary for its index error when its temperature is 10° C. If a mercurial barometer is used with a brass reading-scale it is necessary to apply a correction to the reading to take account of the difference of expansion of the brass scale and the mercury. This correction is usually applied directly as a correction to the barometer reading. But for convenience, in dealing with refractions, it has been expressed as a correction to the mean refraction, and is given in § 297 in terms of the reading of the thermometer which is attached to the

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*This table was obtained by combining the table of mean refractions given in Doolittle's Practical Astronomy, p. 628, with that given in the *Connaissances des Temps* for 1897, p. 658. The values of Prof. Doolittle's table were first reduced to the same basis as those of the French table, and then the indiscriminate mean of corresponding values taken. Prof. Doolittle's table is said to be based upon Bessel's tables, which in turn were based upon certain long series of observations as reduced by Bessel, using the theory of refraction elaborated by him. The French tables, on the other hand, depend upon other observations and upon a different theory—that of Laplace.

barometer. In case the barometer used is not of the kind referred to above,—if, for example, it is a mercurial barometer with an ivory or steel scale, or if it is an aneroid,—the proper corrections (including the temperature correction) must be applied to its readings to reduce to millimeters of mercury at 10° C., and then the table of § 295 must be used, but that of § 297 ignored.

§ 296 shows the factor, C , by which the mean refraction must be multiplied to take into account the temperature of the open air at the station of observation.

To sum up, the refraction R, as computed from these tables, is

$$R = R_M(C_B)(C_D)(C_A); \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

in which R_M is the mean refraction as given in § 294, and C_B, C_D, and C_A are the factors given in §§ 295–297.

The density of the air along the line of sight, and therefore the refraction, is dependent upon the pressure and temperature at all points along that line.* But observations of temperature and pressure can be made at the station of observation only. The refraction is expressed, as above, in terms of the pressure and temperature at the station, and the temperature and pressure are assumed to vary with the height according to certain laws which depend to a considerable extent upon theory only.

69. It is in order here to inquire what errors may be expected in the refractions as thus computed. The values for R_M as derived from a long series of observations, extending over several years, at one observatory when compared with the corresponding values derived from a similar series at

* It is also dependent to a very small extent upon the humidity of the air,—to so small an extent, however, that no attempt is ordinarily made to take the humidity into account.

another observatory are found to differ in an extreme case by as much as 0.5%,* and differences of half that amount are not infrequent. No table of refractions can, therefore, be depended upon to give even the *average refraction for a term of years* at an arbitrarily chosen station within $\frac{1}{400}$ part of its true value. The error of the refraction for *any one* observation, as derived from *any* table, must be uncertain to a much greater extent. It is probable that the refraction corresponding to any single observation as computed by the use of *any* available tables or formulæ will often be in error by more than 1%.† This amounts to $0''.0$ to $0''.5$ for altitudes from 90° to 50°, $0''.5$ to $1''.5$ for altitudes from 50° to 20°, $1''.5$ to $3''.0$ for altitudes from 20° to 10°, and increases rapidly for smaller altitudes. From considerations which may not be entered into here it seems probable that the refractions above 50° of altitude are subject to greater uncertainty than that indicated above.

Theories of astronomical refraction all depend upon the assumption that surfaces of equal density in the atmosphere are *everywhere horizontal*, and that the density varies with the height according to some *fixed* law. If one reflects upon the unceasing changes of pressure in the air as indicated by the winds and by the fluctuating barometer, upon the large and irregular changes in temperature near the Earth's surface, and upon the continual changes in humidity indicated by the

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* Astronomical Papers, American Ephemeris and Nautical Almanac, vol. II. Part VI.

† This is the reason why the more cumbersome and more accurate method of computing the refraction by Bessel's factors has been omitted in this book. With the limited number of observations which the engineer usually makes in determining any one quantity, the actual accuracy of the final result attained by the use of the tables given will not differ sensibly from that which would be obtained with a greater expenditure of time from other tables or formulæ.

evanescent clouds, the large margin of uncertainty in the refraction stated above seems not only reasonable, but inevitable. If, however, more direct evidence is needed to convince one, it will usually be found in making any long series of astronomical observations in our climate. For, on some nights, in attempting to make an accurate pointing upon a star with a telescope it may be observed to be apparently oscillating irregularly through a range of three or four seconds (say) about its mean position on account of momentary changes in refraction.

Derivation of Formula.

70. The three corrections for Sun's semi-diameter, parallax, and refraction being applied to the approximate altitude A_u, the result is the measured altitude, A, of the Sun's center. The complement of this altitude, or the zenith distance, ζ, of the Sun's center, is a side of the spherical triangle (Fig. 9) Sun-zenith-pole, upon the celestial sphere. The arc zenith to pole of that triangle is the complement of the latitude, which is supposed to be known. The declination of the Sun at the instant of observation may be obtained from the Ephemeris by interpolation along a tangent as indicated in § 35. The necessary interpolation extends over the interval from the nearest Washington mean noon. To obtain this interval one may assume an error for the chronometer (to be checked later). For this purpose it is only necessary to know the error within one minute. For example, in the computation in hand it was known from previous observations that the error of the chronometer on Washington mean time was about 0^m. The Washington mean time of observation was then $11^h\ 04^m$, and the interpolation interval 56^m. The complement of the declination is the third side, Sun to pole, of the spherical triangle Sun-zenith-pole. The three arcs of this spherical triangle being known, the angle at the pole, which is the hour-angle of the Sun, may be computed by spherical trigonometry.

Let A_t, B_t, and C_t be the angles of any spherical triangle, and a_t, b_t, and c_t the sides opposite, respectively. From spherical trigonometry

$$\sin \tfrac{1}{2}A_t = \sqrt{\frac{\sin (s - b_t) \sin (s - c_t)}{\sin b_t \sin c_t}}, \quad . \quad . \quad . \quad (15)$$

in which $$s = \frac{a_t + b_t + c_t}{2}.$$

In the triangle Sun-zenith-pole let A_t be the angle at the pole, namely, the hour-angle of the Sun, t. Let b_t be the side zenith to pole ($= 90° - \phi$), and c_t be the side Sun to pole ($= 90° - \delta$). a_t must be ζ, the zenith distance of the Sun. Making the substitutions indicated, squaring both members of the equation, and simplifying, there is obtained

$$\sin^2 \tfrac{1}{2}t = \frac{\sin \tfrac{1}{2}[\zeta + (\phi - \delta)] \sin \tfrac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}, \quad (16)$$

by the use of which the hour-angle may be computed as indicated in § 64.

The hour-angle subtracted from 12^h, the Sun being east of the meridian, is the local apparent solar time, T_A. The equation of time, E (see § 20), may be obtained from the Ephemeris with sufficient accuracy by an interpolation along a chord from the nearest Washington *apparent* noon. $T_A + E$ is the local mean solar time, T_M. ΔT_C, the correction to the chronometer to give local mean time, is evidently the difference between T_C, the reading of the chronometer, and T_M. It should be kept definitely in mind that ΔT_C is strictly the correction to the reading of the chronometer only at the one instant when the reading of the chronometer is T_C. At any other instant the chronometer will have a different correction, depending upon its rate.

If the chronometer used in the sextant observations keeps mean time, and it is proposed to obtain from it the error of a sidereal chronometer, the two chronometers should be compared by the method indicated in § 250.

Discussion of Errors.

71. The various errors which affect the final result in any astronomical observation may be grouped in three classes: 1st, *external errors*, or errors arising from conditions outside the instrument and observer; 2d, *instrumental errors*, or errors due to the instrument, arising from lack of perfect adjustment, from imperfect construction, from instability of the relative positions of different parts, etc.; 3d, *observer's errors*, or errors due directly to the inaccuracies of the observer, arising from his unavoidable errors in judgment as to what he sees and hears, and from the fact that his nerves and brain do not act instantaneously. By the phrase *errors of observation* is meant the errors arising from all these sources combined.

External Errors.

72. Following the order indicated above, let us first consider the errors arising from conditions outside the instrument and observer.

The accuracy of a determination of time from observations upon the Sun depends largely upon the part of the day at which the observations are made. Near apparent noon the altitude of the Sun is changing quite slowly. A few hours later or earlier the change of altitude is comparatively rapid.

Evidently, the effect of a given error in the measured altitude upon the computed time will be less the greater is the rate of change of the altitude. It may be shown from the differential formulæ * applicable to the spherical triangle used in the preceding computation (§ 70), that this rate of change is greatest when the Sun is in the prime vertical, or when it is nearest the prime vertical in case it does not reach it while above the horizon. This condition by itself would fix the most favorable time for observations at sunrise or sunset during the months when the Sun is south of the equator, and from three to six hours from the meridian during the remainder of the year, for nearly all points in the United States.

Another condition, however, must also be considered in determining the most favorable time for observing. As indicated in § 69, the uncertainty in the computed refraction increases rapidly as the altitude diminishes. In so far, then, as the refraction is concerned, the nearer to apparent noon the observations are made the better. Taking both conditions into account the most favorable time for observing is from two to four hours from the meridian. Within these limits and for stations in the United States (excluding Alaska) the rate of change of altitude is from 5 to 14 seconds of arc per second of time, and the error in the derived chronometer correction arising from the uncertainty of the computed refraction, adopting the estimate of that uncertainty as given in § 69, may be from $0^s.02$ under the most favorable conditions (latitude 24° 30′, midsummer) to $0^s.5$ for an observation at two hours from the meridian in latitude 49° in midwinter, or even 2^s if this last observation is made near sunrise or sunset.

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* See Chauvenet's Astronomy, vol. I. pp. 213, 214; or Doolittle's Practical Astronomy, p. 223.

The position of the Sun is so well determined that there is no sensible error in the result from that cause.

Instrumental Errors.

73. If the telescope is not perfectly focused upon the Sun, or if the colored glasses introduced make the images of the Sun very dim, or leave them too bright to be gazed at with comfort, there is a tendency to see the images either larger or smaller than they really are, and so to misjudge the position of tangency. This is not eliminated by the determination of index error, as described in § 62, for the effect would be to increase (or decrease) both plus and minus readings by the same amount, and so leave the computed index error unchanged. But it is eliminated by taking half of the observations on the upper limb of the Sun and half on the lower limb, as shown in the set of observations given in § 63.

The inclination of the index-glass to the perpendicular to the sextant plane, and the inclination of the axis of the telescope to that plane, combine to produce an error which varies as the tangent of one-quarter of the measured angle.* The error of adjustment of the index-glass and of the telescope may each be made less than 5′ by the methods given in §§ 54, 56. If each is 5′, the maximum error introduced into a measured angle of 120° is 4″.0, and for other angles in the ratio indicated above. This error is therefore small provided the adjustments are carefully made and frequently verified, but it is sensibly a constant affecting the mean of a set of observations made at nearly the same reading of the arc. If the telescope is parallel to the plane of the sextant, but, in observing, the contacts are made with the images out of the center of the field, the sight line is inclined to the plane of the

* Chauvenet's Astronomy, vol. II. p. 116.

sextant, and the effect on the measured angle is the same as if the telescope were so inclined. It is important, therefore, that every observation should be made nearly in the middle of the field of the telescope.

If the horizon-glass is not perpendicular to the plane of the sextant, the error introduced is greater the smaller the angle observed, and will ordinarily be appreciable only in the determination of the index error. In determining the index error by observing the Sun's semi-diameter, the error in any one reading from this cause will be less than 1″ even if the horizon-glass is inclined as much as 30″ to its normal position (which is about the maximum error of this adjustment made as indicated in § 55). This error is eliminated from the derived index correction, for both positive and negative readings are numerically too small by the same amount.

If the center about which the index-arm swings does not coincide with the center about which the graduated arc is described, an error due to this eccentricity will be introduced into every reading. The magnitude of this error will evidently depend upon the size of the angle measured as well as upon other conditions. See § 76 for the method of determining, and correcting for, eccentricity.

The errors treated in the last three paragraphs are functions of the angle measured, but are constant for a given reading of the sextant so long as the condition of the instrument remains unchanged. Their effect may therefore be eliminated almost wholly from the final result in determining time by measured altitudes of the Sun, by making observations both in the *forenoon and afternoon at about the same altitude*. The computed altitude will be too great, or too small, by the same amount in both cases, if the two altitudes are equal, and one computed time will be as much too late as the other is too early. This procedure will also eliminate the error in the computed time arising from an error in the assumed latitude.

The errors arising from changes in the relative position of different parts of the sextant due to stresses or to changes of temperature are probably small in comparison with the other errors considered under the next heading. So also are the errors of graduation of the sextant arc.

The error which may arise from either or both glasses of the horizon roof being prismatic instead of plane, may be eliminated by reversing the roof when half the observations have been taken.

To avoid errors arising from the prismatic form of the shades, some instrument-makers provide a contrivance by which the colored shades may be rotated 180° from their original position; but it is better to use colored shades between the eyepiece and the eye instead of the colored shades in front of the index and horizon glasses. A shade in this position may be of a prismatic form without vitiating the observed results.

Observer's Errors.

74. The errors which are classed as instrumental depend to a considerable extent upon the care and judgment with which the sextant is manipulated. But aside from the manipulation, which is an important as well as a difficult portion of the observer's duty, the final result also depends upon his estimates of the positions of contact of the two images and of the chronometer times of those contacts.

His estimate of the position of contact is subject to both an *accidental* and a *constant* error.* The accidental error

* A *constant error* is one which has the same effect upon all the observations of the series, or portion of a series, under consideration. *Accidental errors* are not constant from observation to observation; they are as apt to

depends mainly upon the personality and experience of the observer and the care with which he observes, but also to a certain extent upon the steadiness of the refraction, the power of the telescope, the brightness and definition of the images, and the physical conditions affecting the observer's comfort. A probable error of $\pm 14''$ seems from various recorded series of observations to be a fair estimate of the accidental error in a single measurement of the Sun's double altitude by an experienced observer with an ordinary sextant under average conditions. The accidental error in the mean of twelve observations constituting a set is on this basis $\pm 14'' \div \sqrt{12} = 4''$. This corresponds, for observations taken in the United States when the Sun is observed from two to four hours from the meridian, to $\pm 0^s.15$ to $\pm 0^s.40$. The constant error of the observer's estimate of the position of contact is eliminated from the mean for a set if half the observations are taken upon the Sun's upper limb and half upon the lower.

The observer's estimate of the *time* of contact is also subject to both an accidental and a constant error. The accidental error, judging from time observations made with a transit instrument, is about $\pm 0^s.1$ for a single observation, or $\pm 0^s.03$ for a mean of twelve observations constituting a set,—a small error as compared with that arising from the uncertainty of the *position* of contact. The constant error made in estimating the time, or personal equation * of the observer, may be as great as $0^s.5$ for some men. It affects all the observations of a set alike.

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be minus as plus, and they presumably follow the law of error which is the basis of the theory of least squares. It is then the effect of accidental errors upon the final result, which may be diminished by continued repetition of the observations and by the least square methods of computation, whereas the effect of constant errors must be eliminated by other processes.

* See § 125.

Error of the Computed Time.

75. The mean result from a set of observations such as that given in § 63 is subject, then, to an accidental error of about $\pm 0^s.25$, on an average, arising almost entirely from the observer's accidental errors. It is also subject to an error which is constant for the set, arising from uneliminated instrumental errors and the error of the computed refraction (neglecting for the time being the personal equation of the observer). A fair estimate of this constant error under average field conditions seems to be $\pm 0^s.25$. This makes the probable error of the result from the set about $\sqrt{\overline{0.25}^2 + \overline{0.25}^2} = \pm 0^s.35$, aside from personal equation. It is evident from the above estimate that increasing the number of observations in a set, or number of sets taken under the same circumstances, diminishes the final error but little (only one term under the radical above being reduced). The constant instrumental error may, however, be almost entirely eliminated by making observations at about the same altitude in both forenoon and afternoon as indicated in § 73. There is no feasible way of eliminating the personal equation error in the field.

The above estimate of the errors from various sources is believed to be a fair one for average conditions. A special investigation for a particular observer and set of conditions may show errors either somewhat smaller or much larger than those indicated.

Correction for Eccentricity.

76. Unless the center about which the index-arm swings coincides exactly with the center of the graduation, every sextant reading will be in error by the effect of this eccentricity (as noted in § 73), which effect is different for readings taken on different parts of the arc. To eliminate the effect of eccentricity upon the sextant readings one may proceed as follows:

First. The values of angles as measured with the sextant may be compared with their true values determined in some other way.

For example, the angles between certain terrestrial objects may be measured with the sextant and then with a good theodolite. In making this comparison it must be kept in mind that a theodolite as ordinarily used measures horizontal and vertical angles, while the sextant measures directly the angle between the two objects in the plane (horizontal, oblique, or vertical) passing through the two objects and the sextant. Also, in this case, the sextant parallax, § 53, must be taken into consideration unless the objects are very distant.

The angular distance between two known stars may be observed and compared with its value as computed from the known right ascensions and declinations of the stars, corrected for the effect of refraction at the time of observation. This computation will be found, unfortunately, to be rather laborious.

Or, the altitude of a known star (or of the Sun) may be measured at a known time at a station of which the latitude is known. The true altitude of the star may be computed, and becomes comparable, after correction for refraction, with that measured with the sextant.

Second. For each sextant observation which is compared with a known angle an observation equation of the form

$$Jx + Ky + v = D_A \quad . \quad . \quad . \quad . \quad . \quad (17)$$

is formed, in which x, y, and v are unknowns to be determined, and $D_A = \theta_t - \theta_m$ is the difference between the true value of the angle θ_t and the measured value θ_m. θ_m is the reading of the sextant corrected for index error.

$$J = \sin\frac{\theta_m}{4}\cos\frac{\theta_m}{4} \quad \text{and} \quad K = \sin^2\frac{\theta_m}{4}. \quad . \quad . \quad (18)$$

Third. The most probable values of x, y, and v are determined from these observation equations by the method of least squares.

Fourth. These values of x, y, and v may now be substituted in equation (17) and a table of corrections computed by substituting 0°, 10°, 20°, . . . successively for θ_m, the corresponding computed values of D_A being evidently the corrections for eccentricity which must be applied to measured angles.*

So many and such accurate observations are required for a satisfactory determination of the eccentricity of a sextant that it will usually be found more convenient to eliminate the effect of eccentricity upon time observations by observing both in the forenoon and afternoon with the Sun at about the same altitude, as indicated in § 73. But in sextant observations for latitude a special determination of the eccentricity is necessary if the highest attainable degree of accuracy is desired.

Other Uses of the Sextant.

77. In determining time with a sextant by the preceding method the latitude is supposed to be known. If, conversely, the time of observation of the altitude of the Sun (or a star)

* This method of determining the corrections to be applied for eccentricity, which is here given in condensed form, and without the derivation of the formulæ, will be found treated in full in Doolittle's Practical Astronomy, pp. 196–206. Certain refinements there given, which add much to the labor of computation and little to the accuracy of the computed result, have here been omitted.

is known, the latitude of the station may be computed by the method of § 171, or by that of § 172, if the observation is made near the meridian. The most favorable time for thus determining the latitude by observations upon the Sun is about apparent noon, for then the altitude is changing very slowly, and hence an error in time will have but little influence on the computed result. The refraction is also a minimum at apparent noon.

78. At sea, observations with the sextant for time are usually made in the middle of the forenoon and of the afternoon, and for latitude at apparent noon. To make this latitude observation the Sun is watched with a sextant for a few minutes before apparent noon, as its altitude increases slowly at a diminishing rate. The observation is made when the altitude stops increasing and is at its maximum. With sufficient accuracy it may be assumed that the Sun is then on the meridian, and that therefore the latitude is the declination of the Sun plus its south zenith distance. In observing at sea the natural horizon is used, and an allowance must be made for the dip of the horizon, or downward inclination of the line of sight to the apparent horizon, due to the height of the sextant above the surface of the sea (see table, § 298 *). The observation of latitude and of local time serves to locate the observer at sea, provided he also knows the Greenwich time of the observation. This last he obtains from the known rate of his chronometer and its known error on Greenwich time at some previous date.

79. An observation at sea of the altitude, at a known instant of Greenwich time, of any celestial object (Sun, Moon, planet, or star) serves to locate the observer upon an

* This table is reproduced, with a slight extension, from Chauvenet's Astronomy. It is computed for a mean state of the atmosphere.

arc of a small circle on the Earth's surface, of which the pole is at a point in the line joining the object and the Earth's center, and of which the polar distance is equal to the observed zenith distance of the object. This small circle, or such a portion of it as is necessary, may be plotted on a sphere or chart. A second such observation on an object in some other azimuth serves to locate the observer on another small circle intersecting the first in two points. These two points of intersection are usually so far apart that there is no difficulty in discriminating between them, and the observer's position becomes definitely known. This process of determining a position at sea is known as Sumner's method. For a more complete statement of this method see Chauvenet's Astronomy, vol. I. pp. 424–428.

80. If, for the purpose of determining local time, an observation is taken upon a star east of the meridian, and the observation is repeated west of the meridian at the same reading of the sextant, the computation of time may be made independently of any knowledge of the index error, eccentricity, or other errors of the sextant, and independently of any computation of the refraction, upon the assumption that these quantities retain the same values at the second observation which they had at the first, and that therefore the two observations are made at the *same* (unknown) altitude. During such an interval of a few hours, the declination of any star is for the present purpose sensibly constant. Upon these assumptions it may be shown that the mean of the two observed chronometer times is the chronometer time corresponding to the transit of the star across the meridian. (Let the student prove this.) If the object observed is a planet, the Moon, or the Sun, the same method of computation may be used, but it will be necessary to apply a correction for the change of declination during the interval between the two observations.*

The advantages of this method are the ease and simplicity of the computation. Its disadvantage is the liability of losing the second observation on account of clouds or other hindrances. If observations are taken within one hour (say) of the same hour-angle east and west of the meridian respectively, are computed as indicated in § 64, and the mean taken, the elimination of errors is almost as complete, advantage may be taken of temporary breaks in the clouds, and the observer is not put to the inconvenience of being ready at some particular moment. The computation will consume a little more time.

81. The Covarrubias method of observing, developed by the Mexican astronomer of that name, serves to eliminate the instrumental errors, and accomplishes that purpose without the necessity of the long wait between observations which is required in the method stated above. Two stars are selected which are several hours apart in right ascension, and have declinations not very different. At a certain time each night, which is first estimated roughly by the observer, these two stars will for an instant be at the same altitude, one east and the other west of the meridian. A few minutes before this time he observes one of the stars, noting the chronometer time and the sextant reading. He then turns to the second star, which he finds approaching the same altitude, and observes the chronometer time at which the sextant reading, and therefore the altitude, is the same for this second star as that before observed upon the first star. From this observation of the two chronometer times at which the two stars reach

* For the computation of this correction, see Doolittle's Practical Astronomy, pp. 230, 231; Chauvenet's Astronomy, vol. I. pp. 198-201; or Loomis' Practical Astronomy, pp. 126–130.

the same altitude the error of the chronometer may be computed independently of any knowledge of the exact absolute value of that altitude. This method will sometimes be found desirable, especially in case the only available sextant is an inferior one, or has been damaged to such an extent that its indications are unreliable. It is not developed in detail here for lack of space. For a complete statement of the method see "Nuevos Metodos Astronomicos para determinas la hora, el azimut, la latitude y la longitude"; F. D. Covarrubias, Mexico, 1867.

QUESTIONS AND EXAMPLES.

82. 1. Prove, using figures if necessary, that the test as given in § 54 for determining whether the index-glass is perpendicular to the plane of the sextant is valid.

2. Explain why two images of the same object cannot be made to coincide in the sextant telescope if the index-glass is in perfect adjustment but the horizon-glass is inclined to the plane of the sextant (see § 55). Explain also how it is possible that such coincidence may be secured if *both* the index and horizon glasses are inclined. Why is it advisable to make certain of the index-glass adjustment before adjusting the horizon-glass?

3. Suppose that the index correction of a certain sextant is found to be $-15''$. Through what angle and in what direction must the horizon-glass be rotated to make the correction zero?

4. Show that the errors due to the inclination to the plane of the sextant of the sight line and of the index-glass are not eliminated by the process of eliminating the index error indicated in § 62.

5. The radius of the graduated circle of a certain sextant is 4 in. What linear movement of the vernier corresponds

to a change of 10″ in its reading? Explain the need of the caution in the last sentence of § 58.

6. Explain, by diagrams if necessary, why the images behave as stated in § 58 when the sextant is moved in the various ways there described.

7. Show that the limb of an image of the Sun seen in a sextant telescope, which is preceding with respect to the apparent motion of the image, corresponds necessarily to the preceding limb of the Sun, regardless of the number of reversals to which said image may have been subject in its progress to and through the telescope. By the use of this principle show that when observations are made with approaching images it is the upper limb of the Sun which is being observed if it is forenoon and the lower limb if it is afternoon.

8. What is the error of the chronometer on local mean time from the last half of the set of observations given in § 63? *Ans.* $-2^h\ 11^m\ 53^s.5$.*

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* In this example the student may find that his computation gives a result differing by as much as $0^s.2$ from the one here given on account of calling 0″.5 a whole second, where it has in this computation been called zero, or *vice versa*. The fact that such a difference may exist may be used as an argument for carrying the computation to one more decimal place. A careful investigation indicates, however, that such a procedure would add so much to the labor of computation, especially in making the various interpolations, that it is not considered advisable. If the computation were carried one decimal place farther, the computed result from a complete set of observations would seldom be changed by more than $0^s.1$, whereas the probable error of that result is $\pm\ 0^s.3$ or $\pm\ 0^s.4$. For a further discussion of the question of the number of decimal places to which a computation should be carried, see § 277.

CHAPTER IV.

THE ASTRONOMICAL TRANSIT.

83. THE astronomical transit is designed primarily to be used for the determination of time with its telescope in the plane of the meridian. Its essential parts are a telescope, an axis of revolution fixed at right angles to the telescope, a suitable support for said axis, such that it shall be stable in azimuth and inclination, and a striding level with which the inclination of the axis may be determined.

Fig. 10 shows an astronomical transit which is now and has been for several years past in use in the U. S. Coast and Geodetic Survey for time determinations of the highest order of accuracy. The focal length (distance from the lines of the eyepiece diaphragm to the optical center of the objective) of the telescope AB is 94 cm. (37 in.). The clear aperture of the object-glass is 7.6 cm. (3 in.), and the magnifying power with the diagonal eyepiece, A, ordinarily used is 104 diameters. In the focus of the eyepiece is a thin glass diaphragm upon which are ruled lines which serve the same purpose as the spider lines or cross wires more commonly placed in that position in a telescope. The system of lines consists of two horizontal lines near the middle of the field, and thirteen vertical lines. The milled head shown at C controls, by means of a rack and pinion, the distance of the diaphragm from the object-glass, and serves therefore to focus the object-glass,—or, in other words, to bring the image formed by the object-glass into coincidence with the diaphragm. The diaphragm, and the corresponding image formed by the object-glass, are much larger than the field of view of the eyepiece. To enable the observer to see various parts of the diaphragm, and the corresponding portions of the image, the whole eyepiece proper is mounted upon a horizontal slide controlled by the milled head shown at *D*.

The lines of the diaphragm are seen black against a light field. The illumination of the field at night is obtained from one of the lamps shown at *E*. The light from the lamp passes in through the perforated end of the horizontal axis, and is reflected down to the eyepiece by a small mirror in the interior of the telescope, fastened to a spindle of which the milled head *G* is the outer end. The perforated disk shown at *F* carries plain, ground, and colored glasses, to be used by the observer to temper the illumination.

The horizontal axis *FF* is 51.5 cm. (20¼ in.) long, ending in pivots of bell metal. *HH* is the striding level, in position, resting upon the pivots of the horizontal axis.

The iron sub-base, a portion of which shows at *I*, is cemented firmly to the pier. The transit base is carried by three foot-screws resting upon this sub-base. The device shown at *J* serves to give the instrument a slow motion in azimuth.

The lever *K* actuates a cam to raise the cross-piece *L*, and with it the columns *MM*. The horizontal axis is then raised sufficiently upon the forks at the upper end of *M* and *M* to clear the Ys. The cross-piece *L* is then free to turn (180°) until arrested by the fixed stops, and thus to reverse the horizontal axis *FF* in the Ys.

The setting circles *NN* are 10 cm. (4 in.) in diameter, are graduated to 20′ spaces, and are read to single minutes by verniers. They are set to read zenith distances.

Fig. 11 shows another type of transit in use in the U. S. C. & G. S. Its peculiarities are the folding frame, the graduated scale at Q to facilitate putting the telescope in the meridian, and the fact that the eyepiece is furnished with a movable line carried by a micrometer screw, while one of the setting circles carries a *sensitive* level so that the instrument may be used both as a zenith telescope (see Chapter V) and a transit. The screw T moves the upper base SS in azimuth with respect to the lower base RR.

The Theory of the Transit.

84. If a transit were in perfect adjustment the *line of collimation* * of the telescope as defined by the mean line of the reticle would be at right angles to the transverse axis upon which it revolves, and that transverse axis would be in the prime vertical, and horizontal. Under these circumstances the line of collimation would always lie in the meridian plane

* The *line of collimation* of a telescope is that line of sight to which all observations are referred. In an engineer's transit the line of collimation is the line of sight on which all observations are made, and is defined by a vertical line in the middle of the field of view of the telescope. In an astronomical transit the observations are made on the several lines of sight defined by the several lines of the reticle. These various observations are all referred to an imaginary line of sight, or line of collimation, which is defined, however, by the mean of all the lines, and not by the middle line. The mean line is, of course, near to the middle line, the spacing of the lines in the reticle being made as nearly uniform as possible.

Imagine a plane passed through any line of the reticle of a telescope and through the center of the object-glass. Imagine the plane produced indefinitely beyond the object-glass. Evidently every point of which the image is seen in the telescope in apparent coincidence with this line of the reticle must lie in this plane in space. The *line* of the reticle may be said to define this *plane* in space, or a *point* of the reticle line may be said to define a *line* in space. For convenience a line of the reticle is ordinarily spoken of as defining a *line* of sight rather than a *plane* of sight, it being tacitly understood that one *point* only of the reticle line is referred to—ordinarily the middle point.

and the local sidereal time at which any star might be seen in the line of collimation of the telescope would necessarily be the same as the right ascension of that star. In observing meridian transits for the determination of time, these conditions are, by careful adjustment of the instrument, fulfilled as nearly as possible. The time observations themselves, and certain auxiliary observations, are then made to furnish determinations of the errors of adjustment; and the observed times of transit are corrected as nearly as may be to what they would have been if the observations had been made with a perfectly adjusted instrument. The observed time of transit of any star, as thus corrected, minus the right ascension of the star, is the error (on local sidereal time) of the chronometer with which the observation was made.

Adjustments of the Transit.

85. Let it be supposed that observations are about to be commenced at a new station at which the pier and shelter for the transit have been prepared. By daylight make the following preparations for the work of the night.

By whatever means are at your disposal determine the direction of the meridian, mark it upon the top of the pier, and put the foot-plates of the transit in such positions that the transit telescope will swing nearly in the meridian (true, not magnetic). A compass needle will serve for this purpose if no other more accurate and equally convenient means is at hand. An *accurate* determination of the meridian is not yet needed. To give the foot-plates a good bearing upon the pier and to fix them rigidly in position, it is well to cement them in position with plaster of Paris.

Set up the transit and inspect it. Focus the telescope carefully if it is not already in good focus. The eyepiece may be first focused upon the reticle with the telescope turned up to the sky. The focus for most distinct vision of the reticle lines is what is required. Now direct the telescope to some distant object (at least a mile away, and preferably much farther) and focus the object-glass, by changing its distance from the reticle, so that when the eye is shifted about in front of the eyepiece there is no apparent change of relative position (or parallax) of the lines of the reticle and of the image of the object. If the eyepiece itself has been properly focused, this position of the object-glass will also be the position of most distinct vision. The focus of the object-glass will need to be inspected again at night, and corrected if necessary, using a star as the object. None but the brightest stars will be seen at all, unless the focus is nearly right.

Bisect some well-defined distant object, using the apparent upper part of the middle vertical line of the reticle. Rotate the telescope slightly about its horizontal axis until the object is seen upon the apparent lower part of this same line. If the bisection is still perfect, no adjustment is needed. If, however, the bisection is no longer perfect, the reticle must be rotated about the axis of figure of the telescope until the line is in such a position that this test fails to discover any error.

86. Now bisect the distant object with the middle line of the reticle. Reverse the telescope axis in its Ys. If the bisection still remains perfect, the line of sight defined by the *middle* line of the reticle is at right angles to the horizontal axis and the *mean* line may be assumed to be sufficiently near to that position. If necessary, however, make the adjustment by moving the reticle sidewise, so as to make the *error of collimation* small. By *error of collimation* is meant the angle between the line of sight defined by the mean line of the reticle and a plane perpendicular to the horizontal axis of the telescope.

Level the horizontal axis of the telescope. Adjust the level so that when it is reversed its reading will change but little.

Test the finder circles to see that they have no index error. Point upon an object and read one of the finder circles. Reverse the telescope, point upon the object again, and read the same finder circle as before. The mean of these two readings is evidently the zenith distance of the object (if the circle is graduated to read zenith distances) and their half difference is the index error of the circle. This index error may be made zero by raising or lowering one end or the other of the level attached to the vernier of the finder circle. This same process will evidently serve if the circle reads elevations instead of zenith distances. If the circle is to be made to read declinations directly, the same process is still applicable. For if the circle be made to read zenith distances with an index error equal to the latitude of the station, its readings will be declinations for one position of the telescope (though not for the other, after reversal). The other circle may be made to read declinations for the other position of the telescope.

A "finder list" of stars, showing for each star to be observed its name, magnitude, setting of finder circle, and the right ascension or the chronometer time of transit to the nearest minute, will be found to be a convenience in the night work. In making out a finder list the refraction may be neglected, not being sufficient to throw a star out of the field of the telescope. The zenith distance of a star is then $\phi - \delta$, south zenith distances being reckoned as positive.

The Azimuth Adjustment.

87. In the evening, before the regular observations are commenced, it will be necessary to put the telescope more accurately in the meridian. Having estimated the error of the chronometer in any available way, within say five minutes, and having carefully levelled up the axis, set the telescope for some bright star which is about to transit within 10° (say) of the zenith. Observe the chronometer time of transit of the star. This star at transit being nearly in the zenith, its time of transit will be but little affected by the azimuth error of the instrument. The collimation error and level error have been made small by adjustment. Therefore the difference between the right ascension of the star and its chronometer time of transit will be a close approximation to the error of the chronometer. Now set the telescope for some slow-moving star which will transit well to the northward of the zenith (let us say, of declination greater than 60° and a north zenith distance of more than 20°). Compute its chronometer time of transit, using the approximate chronometer error just obtained. As that time approaches bisect the star with the middle line of the reticle, and keep it bisected, following the motion of the star in azimuth by the use of whatever means have been furnished on that particular transit for that purpose. Keep the bisection perfect till the chronometer indicates that the star is on the meridian. The telescope is now approximately in the meridian.

The adjustment may be tested by repeating the process, i.e., by obtaining a closer approximation to the chronometer error by observing another star near the zenith, and then comparing the computed chronometer time of transit of a slow moving northern star with the observed chronometer time of its transit. If the star transits, apparently, too late, the object-glass is too far west (for a star above the pole), and *vice versa*. The slow-motion azimuth-screw may then be used to reduce the azimuth error. This process of reducing the azimuth error will be much more rapid and certain, if instead of simply guessing at the amount of movement which must be given to the azimuth-screw, one computes roughly what fraction of a turn must be given to it. This may be done by computing the azimuth error of the instrument roughly by the method indicated in § 102, having previously determined the value of one turn of the screw. An experienced observer will usually be able on the second or third trial to reduce the azimuth error to less than 1^s (= 15″).

The table given in § 310 will be found convenient in making the first approximation to the meridian.

Directions for Observing.

88. The instrument being completely adjusted and the axis levelled, set the telescope for the first star. It is not advisable to use the horizontal axis clamp during observations, for its action may have a slight tendency to raise one end or the other of the axis. See to it, loading one end if necessary, that the centre of gravity of the telescope is at its horizontal axis, and then depend upon the friction at the pivots to keep the telescope in whatever position it is placed. When the star enters the field, bring it between the horizontal lines of the reticle, if it is not already there, by rapping the telescope lightly. Center the eyepiece so that the vertical line nearest the star is in the apparent middle of the field of view. As the star approaches the line pick up the beat of the chronometer.* Observe the chronometer time of transit across the line, estimating to tenths of seconds. Then center the eyepiece on the second line and observe the transit there, and so on, until observations have been made upon all the lines, taking care always to keep the eyepiece centered upon the line which is in use.

The directions and suggestions given in § 60 for observing time apply here with equal force. In order to estimate

* The eye and ear method of observing without a chronograph is here referred to. For a description of the chronograph and the method of using it in observing, see § 89.

tenths of seconds it is necessary to divide the half-second interval given directly by the chronometer into fifths by some mental process. To secure accuracy and ease in making this estimate it is advisable to transform it into a process of estimating relative distances. The apparent motion of the star image is nearly uniform (not quite so on account of disturbance by irregular refraction). Let us suppose that at the instant when the chronometer tick which indicates the time to be $1^s.5$ is heard the star is seen at A (Fig. 12). Let the observer retain a mental picture of this relative position of the star and the line. When the chronometer tick for $2^s.0$ is heard, he sees the star at B. If he has retained (for $0^s.5$ only) the mental picture referred to above, he has before his mind's eye exactly what is indicated in the figure. He estimates the ratio of the *distances* from A to the line and from A to B, and concludes that the ratio is nearer to $\frac{2}{5}$ than to $\frac{1}{5}$, or $\frac{3}{5}$, and calls the time of transit of the star across line I, $1^s.7$. Though this mental process may seem awkward at first, it will ultimately be found to be both easier and more accurate than the direct process, for all cases in which the star has an apparent motion which is sufficiently rapid to make the distance AB appreciable in a half-second. An experienced observer, using this process, is able to estimate the time of transit of a star's image across each line of the reticle with a probable error of about $\pm 0^s.1$.

It is well here to bear in mind the suggestion given in § 60, that he who hesitates is inaccurate. The successful observer decides promptly, but without hurry, upon the second and tenth at which the transit occurred.

At convenient intervals between stars the striding level should be read in each of its positions upon the horizontal axis. At about the middle of the observations which are to constitute a set the telescope should be reversed, so that the effect of the error of collimation (and inequality of pivots) upon the apparent time of transit may be reversed in sign. Each half-set should contain one slow-moving star (of large declination) to furnish a good determination of the azimuth error of the instrument.

The telegraphic longitude parties of the Coast and Geodetic Survey make the best time determinations that are made with portable instruments at present in this country. In their practice ten stars are observed in each set, five before and five after reversal of the telescope. From two to four readings of the level are taken in each of its positions in each half-set. Care is taken to have the telescope at different inclinations during the different readings of the level, inclined sometimes to the south and sometimes to the north, so that the level may rest in turn upon various parts of the pivots. This is done to eliminate, in part at least, the effect of irregularity in the figure of the pivots upon the determination of the inclination of the axis.

The Chronograph.

89. The preceding directions for observing were given on the supposition that the eye and ear method of observing the times of transit is to be used. If, instead, the time observation proper is made with a chronograph, the method is changed in that one particular only.

A common form of the chronograph is shown in Fig. 13. The train of gear-wheels partially visible through the back glass of the case at *F* is driven by a falling weight, and drives the speed governor at *AECCDD*, the screw *I*, and the cylinder *H*. As the speed of rotation of the governor increases, the weights *CC* move farther from the axis until a small projection on one of them strikes the hook at *E* and carries it along. This hook carries with it in its rotation the small weight *A*. The result of the impact and of the added friction at the base of *A* is to cause the speed of the governor to decrease until the hook E is released. The speed then increases until the hook is engaged, decreases again until it is released, and so on. The total range of variation in the speed is, however, surprisingly small,—so small that in interpreting the record of the chronograph the speed is assumed to be uniform during the intervals between clock breaks. By moving the adjusting nuts DD upon their screws, the critical speed at which the hook is engaged may be adjusted. The carriage M is moved parallel to the axis of the cylinder H by the screw I. The pen G, carried by an arm projecting from the carriage M, tends to trace a helix at a uniform rate upon the paper, or "chronograph sheet," stretched upon the cylinder. The magnets KK are in an electric circuit (through the wires L), with a break-circuit clock or chronometer. Whenever the circuit is broken, the armature N is released and the back portion of the arm carrying the pen is drawn back, by a spring, to contact with the stop at J. The pen then makes a small offset from the helix. It returns to the helix as soon as the current is renewed. As a result the equal intervals of time between the instants at which the chronometer breaks the electric circuit are indicated by equal linear intervals between offsets on the line drawn by the pen, the speed being kept constant by the governor. The chronometer is usually arranged to break the circuit every second or every alternate second, and to indicate the beginning of each minute by omitting one break. The hours and minutes may be identified by recording at some point upon the sheet the corresponding reading of the face of the chronometer.

The electric circuit passing through the magnets KK, the chronometer, and battery also passes through a break-circuit key in the hand of the observer. To record the exact time of occurrence of any phenomenon he presses the key at that instant, breaks the circuit, and produces an additional offset in the helix, of which the *position* indicates accurately the *time* at which it was made. In observing a star, to determine a chronometer error the instant of transit of the star image across each line of the reticle is so recorded. To read the fractions of seconds from the chronograph sheet it is convenient to use a scale divided into intervals corresponding to tenths of seconds. The process of reading is also facilitated by writing the number of the second at the head of each column, and of the minute on each line, of the chronograph sheet before beginning to read.

The experienced observer gains very little in accuracy by substituting the chronographic method of observing time for the eye and ear method. He gains somewhat in the convenience and rapidity with which his night's record is made. These small gains are not usually sufficient to justify the use of the chronograph in the field, except in connection with telegraphic determinations of longitude. In that case the chronographic method has special advantages (see §§ 234–242).

Example of Record and Computation.

90. The following time set was observed as a part of the telegraphic longitude work of the Coast and Geodetic Survey in May, 1896. The observations were made with a chronograph. The explanation of each separate portion of the computation is given in detail under the appropriate heading in the following sections.

The constants of the instrument used in this example are here inserted for convenient reference.

One division of the striding level = $1''.674$.

Pivot inequality = $-0^s.010$ with band west.

Equatorial intervals of lines with band west:

Line 1.	$-15^s.20$	Line 5.	$-2^s.52$	Line 9.	$+10^s.09$
" 2.	$-12^s.69$	" 6.	$+0^s.09$	" 10.	$+12^s.65$
" 3.	$-10^s.15$	" 7.	$+2^s.52$	" 11.	$+15^s.15$
" 4.	$-5^s.06$	" 8.	$+5^s.11$		

Time of Transit Across Mean Line.

92. If the transit of the star across every line of the reticle is observed, the time of transit across the mean line, or line of collimation, is evidently obtained by taking the mean of the several observed times. In obtaining the sum of the several times for this purpose an error of a whole second in any one observed time, which might otherwise remain unnoticed, will be detected by the use of the auxiliary sums shown in the little column just after the observed times, namely, the sum of the first and last times, of the second and last but one, third and last but two, etc. These auxiliary sums should be nearly the same and nearly equal to double the time on the middle line. The unexpressed minute for each is the same as that for the middle line. The sum of these auxiliary sums and of the middle time is the total sum required in computing the mean.

It will frequently happen, especially on partially cloudy or hazy nights, that the transits of a star across several lines of the reticle will be successfully observed, and yet the observer may fail utterly to secure the transits across the remaining lines. It then becomes necessary to reduce the mean of the observed times of transit across certain of the lines to the mean of *all* of the lines.

Let t_1, t_2, t_3, . . . be the observed times of transit across the successive lines, and let t_m be their mean, or the time of transit across the mean line.

Let i_1, i_2, i_3, . . . be the *equatorial intervals* of the successive lines from the mean line, or the intervals of time which elapse for an equatorial star (star of zero declination) between transits across the separate lines and the transit across the mean line.

Then for an equatorial star $i_1 = t_1 - t_m$, $i_2 = t_2 - t_m$, $i_3 = t_3 - t_m$, . . .

To determine the relation for any other star between i_n, the equatorial interval for any line, and $t_n - t_m$ (in which t_n is the time of transit over that line), deal with the spherical triangle defined by the pole, the star at the instant when it is on any line, and the star at the instant when it is on the mean line. In Fig. 14, let *P*, *A*, and *B* represent these points respectively. The sides *PB* and *PA* are the polar distance of the star ($= 90° - \delta$). The angle at *P* expressed in seconds of arc is $15(t_n - t_m)$, $(t_n - t_m)$ being expressed in seconds of time. The side *AB*, expressed in seconds of arc, may be taken equal to $15i_n$, i_n being expressed in seconds of time. Using the law that the sines of the sides of any spherical triangle are proportional to the sines of the opposite angles, there is obtained

$$\sin 15i_n : \sin (90° - \delta) = \sin 15(t_n - t_m) : \sin A. \qquad (19)$$

The arc *AB* corresponding to the equatorial interval for a line will seldom exceed 15′ in any transit, and is usually much less. For such an isosceles spherical triangle as this, *AB* being short, angles *A* and *B* are necessarily nearly equal to 90°. Assuming as an approximation that $A = 90°$, (19) may be written $\sin 15i_n = \sin 15(t_n - t_m) \cos \delta$. Again, assuming that the small angles $15i_n$ and $15(t_n - t_m)$ are proportional to their sines, and dividing both members by 15, we obtain

$$i_n = (t_n - t_m) \cos \delta. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In the derivation of (20) besides the approximations mentioned, there is another in the assumption that *AB* corresponds to the perpendicular distance between the two lines of the reticle concerned, whereas none but images of equatorial stars will pursue a path across the reticle which is perpendicular to all the lines. Every other star image follows an apparent path which is curved (the apparent radius of curvature being less, the greater the declination), and therefore not perpendicular to more than one line of the reticle. So *AB* corresponds, except for an equatorial star, to an oblique distance between lines of the reticle, the obliquity being exceedingly small. In spite of all these approximations it has been found by comparing (20) with the formula expressing the exact relation between i_n and $(t_n - t_m)$ that for an extreme value of $i_n = 60^s$ the computed value of $(t_n - t_m)$ will be in error by less than $0^s.01$ for any star of declination less than 70°, and that for a star of declination 85° the error is only $0^s.3$.*

93. Let us deal now with such a case as that of star 17 H. Can. Ven. in the preceding computation, in which the star image was observed to transit across the first ten of the eleven lines of the reticle and the transit across the eleventh line was missed. Suppose that the equatorial intervals i_1, i_2, i_3, . . . have previously been determined by special observations as indicated in § 114.

From (20) we may write

$$\left.\begin{aligned} t_m &= t_1 - i_1 \sec\delta; \\ t_m &= t_2 - i_2 \sec\delta; \\ t_m &= t_3 - i_3 \sec\delta; \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ t_m &= t_{10} - i_{10} \sec\delta; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (21)$$

whence

$$t_m = \frac{t_1 + t_2 + t_3 + \ . \ . \ . + t_{10}}{10} - \frac{(i_1 + i_2 + i_3 + \ . \ . \ . + i_{10}) \sec\delta}{10}, \quad (22)$$

* For the exact treatment of this problem, see Chauvenet's Astronomy, vol. II. pp. 146–149; or Doolittle's Astronomy, pp. 291–293.

in which all quantities are known. By the use of (22) the time of transit t_m across the mean line may be computed even though the transits across certain of the lines are missed.

Adding the corresponding terms of the separate equations of (21), together with the equation $t_m = t_{11} - i_{11} \sec \delta$ (which is true though t_{11} is unknown), and dividing by 11, there is obtained

$$t_m = \frac{t_1 + t_2 + t_3 + \ldots + t_{11}}{11} - \frac{(i_1 + i_2 + i_3 + \ldots + i_{11}) \sec \delta}{11}. \quad (23)$$

But t_m is by definition $\frac{t_1 + t_2 + t_3 + \ldots + t_{11}}{11}$. Therefore (23) becomes

$$0 = -(i_1 + i_2 + i_3 + \ldots + i_{11}) \sec \delta,$$

and

$$-(i_1 + i_2 + i_3 + \ldots + i_{10}) \sec \delta = (i_{11}) \sec \delta:$$

(22) may now also be written in the form

$$t_m = \frac{t_1 + t_2 + t_3 + \ldots + t_{10}}{10} + \frac{(i_{11}) \sec \delta}{10}. \quad . \quad (24)$$

For the particular case in hand of the imperfect transit of 17 H. Can. Ven., $\sec \delta = C = 1.26$, $i_{11} = +15^s.15$, and (24) becomes

$$t_m = 13^h\ 30^m\ 14^s.20 + \frac{(15^s.15)(1.26)}{10} = 13^h\ 30^m\ 14^s.21 + 1^s.91$$
$$= 13^h\ 30^m\ 16^s.12.$$

For the general case (22) and (24) may be written, respectively,

$$t_m = \text{mean of observed times} - \frac{\text{(sum of equatorial intervals of observed lines) (sec } \delta)}{\text{number of observed lines}}; \quad . \quad (25)$$

$$t_m = \text{mean of observed times} + \frac{\text{(sum of equatorial intervals of missed lines) (sec } \delta)}{\text{number of observed lines}}. \quad . \quad (26)$$

The first or second of these two formulæ is to be used, respectively, according to whether less than half or more than half of the lines were observed.

Inclination Correction.

94. In the following treatment it will be assumed that the two pivots upon which the telescope turns are circular cylinders having nearly, but not exactly, equal radii, and that the two inverted Ys forming a part of the striding level, and the two Ys in which the telescope pivots rest, all have equal angles.

If Fig. 15 represent a cross-section through one of the pivots perpendicular to its axis, it is assumed that the curve *GEBD* is a perfect circle, of which the center is at *C*, and that the angles *GFE* and *DAB* are equal to each other and to the corresponding angles at the other pivot. With these assumptions the distances *FC* and *AC*, from the vertex of the level Y and from the vertex of the supporting Y to the center of the pivot, are equal.

In Fig. 16 let *CC'* be the line joining the centers of the two pivots, the axis about which the telescope rotates, of which the inclination is required. Let *F* and *F'* be the vertices of the level Ys, and *A* and *A'* the vertices of the supporting Ys. Suppose the radius of the pivot of which the centre is *C'* is greater than that of the pivot at *C*. Then the distance $F'C'(= A'C')$ is greater than the distance $FC(= AC)$. Draw CG parallel to FF' and CH parallel to AA'. The inclination of the line FF' (equal to the inclination of the line CG) is directly measured by the level readings. The angle between FF' and CC' is the angle $GCC'\left(= \frac{GCH}{2}\right)$. This is the correction to be applied to the inclination as given directly by the level readings to obtain the inclination of the axis. If the telescope axis were reversed in the Ys the line AA' joining the vertices of the supporting Ys would remain unchanged, and the line FF' would assume a new position $F''F'''$ such that the angle between FF' and $F''F'''$ is *four times* $GCC'[= 2(GCH)]$. Let β_W and β_E designate the inclination, as given by the level readings, for lamp west and lamp east, respectively (using the position of the lamp which illuminates the interior of the telescope as a convenient means of designating the position of the telescope), and b_w and b_e the corresponding inclinations of the axis CC'. Let the angle GCC' be called p_i, or *pivot inequality*. Let all inclinations be considered positive when the west end is higher than the east. Then

$$* \left\{ \begin{aligned} b_w &= \beta_W + p_i; \\ b_e &= \beta_E - p_i; \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and $$p_i = \frac{\beta_E - \beta_W}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Let w and e be the readings of the west and east ends, respectively, of the bubble of the striding level for a given

* These formulæ are exact only in case the angle of the level Ys is the same as the angle of the supporting Ys. For ordinary cases, however, in which p_i is small and the Ys have angles which do not differ greatly, they are sufficiently exact. For the full treatment of this problem for the general case, see Chauvenet's Astronomy, vol. II. pp. 153–158.

position of the telescope axis. Let w' and e' be the corresponding west and east readings after the level is reversed, the telescope axis remaining as it was. Let d be the value of a division of the level in seconds of arc. Then for β, the apparent inclination of the telescope axis, expressed in seconds of time, we may write, if the level divisions are numbered in both directions from the middle,

$$\beta = \tfrac{1}{4}\{(w + w') - (e + e')\}\frac{d}{15};$$

or in more convenient form for numerical work,

$$\beta = \{(w + w') - (e + e')\}\frac{d}{60}, \quad . \quad . \quad . \quad (29)$$

in which $\frac{d}{60}$ is a constant for the level.

If the level divisions are numbered continuously from one end of the level to the other, (29) takes the form

$$\beta = \{(w + e) - (w' + e')\}\frac{d}{60}; \quad . \quad . \quad . \quad (30)$$

in which the primed letters refer to that position of level in which the division marked zero is at the western end.

95. It still remains to derive the relation between b, the inclination of the rotation axis of the telescope, and the correction to the observed time of transit of a star to reduce it to what it would be if the rotation axis were truly horizontal.

If the error of collimation were zero and the rotation axis of the telescope horizontal and in the prime vertical, the line of collimation would describe the meridian upon the celestial sphere when the telescope was rotated upon its axis. If now the other errors, of azimuth and collimation, be assumed to remain zero but the rotation axis is moved slightly out of the horizontal by an angle b, the line of collimation will describe a great circle intersecting the meridian at the south and north points of the horizon and making an angle b with it at those points. In Fig. 17 let B be the position occupied by a star at the instant when it would be observed to cross the mean line of a transit in perfect adjustment. Let B' be its position when it would be observed to cross the mean line of a transit having an axis inclination b, but otherwise in perfect adjustment. Let S be the south point of the horizon. In the spherical triangle $BB'S$ the angle S in seconds of arc $= 15b$, SB is the altitude of the star when it is on the meridian, $= 90° - (\phi - \delta)$; and the angle $BB'S$ is almost exactly a right angle. From the law of proportionality of sines, $\sin 15b : \sin BB' = \sin BB'S : \sin \{90° - (\phi - \delta)\}$.

Whence, the small angles $15b$ and BB' being assumed proportional to their sines,

$$BB' = 15b \cos \zeta. \qquad (31)$$

Treating now the spherical triangle defined by B, B', and the pole, just as the spherical triangle in figure was treated, it may be shown that the hour-angle subtended at the pole by BB' is $BB' \sec \delta$. Substituting this in (31) and reducing to time, there is obtained as the hour-angle, or elapsed interval of time (in seconds) between the position B and the position B',

$$b \cos \zeta \sec \delta. \qquad (32)$$

This is the required correction to the observed time of transit. For convenience this may be written

$$\text{Inclination correction} = Bb, \qquad (33)$$

in which $B = \cos \zeta \sec \delta$, and may be found tabulated for the arguments ζ and δ in § 299.

The approximations in the derivation of (32) are of the same order as those made in deriving (20). If 4^s be allowed as a maximum value for b, the error of the formula will be much less than $0^s.01$ for any star of declination less than 80°. Under ordinary circumstances b will seldom be as great as 1^s.

In deriving β from the level readings it is sometimes assumed that the inclination is variable, and that each set of level readings gives the inclination at that particular time. Under good conditions, however, the variation of the inclination during any half set is probably less than the error of any one determination of the inclination. It is advisable, therefore, to assume the inclination constant during a half set. The method of computing this mean inclination from the level readings is sufficiently shown in the example in § 91. A distinction is made between level readings with objective north and readings with objective south on account of the possibility that the level readings may be affected by irregularities in the shape of the pivots.

Correction for Diurnal Aberration.

96. The effect of the annual aberration, due to the motion of the Earth in its orbit (§ 46), is taken into account in computing the apparent star place. But the effect of the diurnal aberration, due to the rotation of the Earth on its axis, must be dealt with in the present computation. In round numbers the velocity of light is 186 000 miles per second, and the linear velocity of a point on the Earth's equator due to the diurnal rotation is 0.288 mile per second. The linear velocity of any point on the Earth in latitude ϕ is then 0.288 cos ϕ mile per second. The apparent displacement of a star on the meridian is

$$k' = \tan^{-1} \frac{0.288 \cos \phi}{186\,000}, \quad . \quad . \quad . \quad . \quad . \quad (34)$$

the motion of the observer being at right angles to the line of sight. The hour-angle k, corresponding to the displacement k', is found by applying the method of the latter part of § 92 to the spherical triangle defined by the true position of the star, its displaced position, and the pole. It is thus found that

$$k = k' \sec \delta = \tan^{-1} \frac{0.288 \cos \phi}{186\,000} \sec \delta.$$

Keeping in mind that the angles k' and k are very small, this may be written

$$k = \frac{0.288}{186\,000 \tan 1''} \cos \phi \sec \delta = 0''.319 \cos \phi \sec \delta$$

$$= 0^s.021 \cos \phi \sec \delta. \quad . \quad (35)$$

For convenience k is tabulated in terms of ϕ and δ in § 301.

As the aberration causes the star to appear too far east, the observed time of transit is too late, and k is negative when applied as a correction to the observed times (except for subpolars).

Azimuth Correction.

97. If the transit is otherwise in perfect adjustment but has a small error in azimuth, the line of collimation will describe a vertical circle, i.e., a great circle passing through the zenith, at an angle with the meridian (measured at the zenith) which we will call a.

In Fig. 18, let z be the zenith, B be the position of a star when it is on the meridian, and B' its position when observed crossing the line of collimation of a transit which has an azimuth error a. By applying the process of the latter part of § 92 to this spherical triangle it may be shown that $BB' = a \sin \zeta$.

Applying the same process again to the spherical triangle defined by B, B', and the pole, it may be shown that the corresponding hour-angle is $BB' \sec \delta$.

Let the angle a be expressed in seconds of time, and be called positive when the object-glass is too far east with the telescope pointing southward. Then the required correction to the observed times, equal to the time elapsed between position B' and B of the star, may be written

$$\text{Azimuth correction} = a \sin \zeta \sec \delta = Aa, \quad . \quad (36)$$

in which A is written for $\sin \zeta \sec \delta$, and is tabulated in terms of ζ and δ in § 299.

The methods of deriving a from the time observations will be treated later (§§ 100–110).

Collimation Correction.

98. If the instrument is otherwise in perfect adjustment but has a small error of collimation (§ 86), the mean line describes a small circle parallel to the meridian, at an angular distance c, the error of collimation, from it, when the telescope is rotated about its horizontal axis. By the same line of reasoning that was used in § 92 in dealing with the intervals of the various lines from the mean line, it may be shown that if c be expressed in time, then the

$$\text{Collimation correction}^* = c \sec \delta = Cc, \quad . \quad . \quad (37)$$

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* Objection may be made to the methods used in deriving the formulæ of §§ 92–98 because of the many approximations involved in them. For, in addition to the stated approximations that have been made in the derivations, the fact that the formulæ for inclination, azimuth, and collimation are not independent has been neglected, and each treated as if entirely independent of the other.

Is such objection valid? Our present purpose is to furnish the engineer with such mathematical formulæ (together with an intelligent in which C is written for sec δ and is tabulated in terms of δ in § 299. The collimation error necessarily changes sign when the telescope axis is reversed in its Ys. Let c be the error of collimation with lamp or band east, and let it be called positive when the star (above the pole) is observed too soon. To take account of the change in sign of the collimation error with lamp west, let the sign of C be reversed (so that $C = -$ sec δ) whenever the lamp is west. With this convention as to the sign of C the algebraic sign of the product Cc in (37) will always be correct.

The methods of deriving c from the time observations will be treated later (§§ 100–112).

Correction for Rate.

99. If the rate of the chronometer is known to be large, it may be necessary to apply a correction to each observed time to reduce it to the mean epoch of the set. The correction required is the change in the error of the chronometer in the interval between the observation and the mean epoch of the set. If the rate of the chronometer is less than 1^s per day and the interval in question is not more than 30^m, the greatest correction for rate will be $0^s.02$. In such a case, if the correction for rate is ignored, the computed correction to the

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understanding of them) as will serve him most efficiently in making certain astronomical determinations with portable instruments. The formulæ furnished are sufficiently accurate for his purpose. The degree of accuracy is roughly indicated to give him a basis for confidence. The alternative procedure is to derive the exact formulæ, at a large expenditure of time and mental energy; to find that said formulæ are too complicated for actual use in computation; to simplify them by dropping terms and making transformations that are approximate; and to arrive finally, when ready for actual numerical computation, *at the same simple formulæ as are here derived directly* (or their equivalents in simplicity and inaccuracy). This procedure furnishes more mathematical training than that adopted in the text. But mathematical training is not the primary object of this treatise.

chronometer will be sensibly exact, but the computed probable errors will be slightly too large.

If the rate of the chronometer is very large, it may even be necessary to apply a rate correction to the reduction to the mean line, in case of an incomplete transit, as derived in § 92.

Computation of Azimuth, Collimation, and Chronometer Corrections, without the Use of Least Squares.

100. Having corrected each observed time of transit for inclination and aberration, the azimuth error a and the collimation error c, as well as the required chronometer correction, may be derived from the observations by writing an observation equation of the following form for each star observed,

$$\Delta T_c + aA + cC - (\alpha - T_c') = 0, \quad . \quad . \quad . \quad (38)$$

forming the corresponding normal equations, and solving for the required quantities, ΔT_c the chronometer correction, a, and c. In (38) α is the apparent right ascension of the star (reduced to mean time if a mean-time chronometer is used), and T_c' is the observed chronometer time of transit of the star *corrected for diurnal aberration, inclination, and rate of chronometer.*

This least square process is rather laborious, and a shorter method is desirable for obtaining approximate results. Such a short method * without least squares will now be treated. It is a method of successive approximations to the required results.

101. The exact form of the computation is shown below in a numerical example dealing with the observations shown in § 91.

* This method, which has been in continual use in the field on the longitude parties of the Coast and Geodetic Survey for many years, was devised in the '70's by Mr. Edwin Smith, then an aid on that Survey.

STATION: Washington, D. C. DATE: May 17, 1896.

Star.	Position.	$\alpha - T'c$	C	A	Cc	Aa	$\Delta Tc = \alpha - T'c - Cc - Aa$	v	
17 H. Can. Ven......	W	$-4^s.07$	+1.26	+0.02	$+0^s.04$	$+0^s.01$	$-4^s.12$	$-0^s.10$	
η Urs. Maj.........	W	−4 .09	+1.56	−0.30	+0 .05	−0 .17	−3 .97	+0 .05	
η Bootis...........	W	−3 .69	+1.06	+0.36	+0 .03	+0 .20	−3 .92	+0 .10	
11 Bootis...........	W	−3 .89	+1.13	+0.22	+0 .04	+0 .12	−4 .05	−0 .03	At $14^h\ 02^m.0$
α Draconis.........	W	−4. 52	+2.36	−1.03	+0 .08	−0 .58	−4 .02	0 .00	$\Delta T_c = -04^s.024$
d Bootis............	E	−3 .94	−1.11	+0.25	−0 .04	+0 .13	−4 .03	−0 .01	
α Bootis............	E	−3 .81	−1.06	+0.35	−0 .03	+0 .18	−3 .96	+0 .06	
λ Bootis............	E	−4 .23	−1.46	−0.20	−0 .05	−0 .10	−4 .08	−0 .06	
θ Bootis............	E	−4 .29	−1.64	−0.38	−0 .05	−0 .19	−4 .05	−0 .03	
5 Urs. Min.........	E	−5 .44	−4.18	−2.53	−0 .13	−1 .28	−4 .03	−0 .01	
		$\alpha - T'c$	C	A	Cc	$\alpha - T'c - Cc$	Aa	$\alpha - T'c - Cc - Aa$	
1st Approx. { Mean of time stars..........	W	−3 .94	+1.25	+0.08	+0 .06	−4.00	+0.05	−4.05	$c = +0^s.051$
1st Approx. { Azimuth star...	W	−4 .52	+2.36	−1.03	+0 .12	−4.64	−0.59	−4.05	$a_W = +0.577$
1st Approx. { Mean of time stars..........	E	−4 .07	−1.32	0.00	−0 .07	−4.00	0.00	−4.00	$a_E = +0.486$
1st Approx. { Azimuth star...	E	−5 .44	−4.18	−2.53	−0 .21	−5.23	−1.23	−4.00	
2d Approx. { Mean of time stars.........	W				+0 .04	−3.98	+0.04	−4.02	$c = +0.032$
2d Approx. { Azimuth star...	W				+0 .08	−4.60	−0.58	−4.02	$a_W = +0.559$
2d Approx. { Mean of time stars..........	E				−0 .04	−4.03	0.00	−4.03	$a_E = +0.506$
2d Approx. { Azimuth star...	E				−0 .13	−5.31	−1.28	−4.03	

102. The first five columns of the main portion of the computation are compiled from § 91, and from the table of § 299 (factors A, B, C). The remaining columns are filled out after the computation of a and c, shown in the lower part of the tabular form, is completed.

103. It should be noted that the five stars of each group, observed in one position of the instrument, have been so selected that one is a slowly-moving northern star at a considerable distance from the zenith; while the other four are all comparatively near the zenith, some transiting to the northward of it and some to the southward, and so placed that their mean azimuth factor (A) is nearly zero. These four stars of each group are for convenience called *time stars*, since the determination of the time falls mainly upon them, while the slowly-moving star serves to determine the azimuth error of the instrument and is called the *azimuth star*.

104. In the computation * to derive c and a, the four time stars in each position of the instrument are combined and treated as one star, by taking the means of their $(\alpha - T_c')$'s and of their factors C and A, respectively, the means being written below the separate stars in the computation form, together with the azimuth stars. On the assumption that the means of the time stars in the two positions of the instrument are equally affected by the azimuth error, the first approximation to c is found by dividing the difference between the two mean values of $\alpha - T_c'$ by the difference between the two mean C's. Or

$$c = \frac{(\alpha - T_c')_W - (\alpha - T_c')_E}{C_W - C_E} . \quad . \quad . \quad . \quad (39)$$

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* This example of the method of computing a and c without least squares, and much of the explanation of it, is taken with little modification from Appendix No. 9 of the Coast and Geodetic Survey Report for 1896, by Asst. G. R. Putnam.

In the example in hand

$$c = \frac{-3.94 - (-4.07)}{+1.25 - (-1.32)} = \frac{+0.13}{+2.57} = +0^s.051. \quad (40)$$

Using this approximation to c, the correction Cc is then subtracted from the $\alpha - T_c'$ of the means of the time stars and of the azimuth stars, and the values of $\alpha - T_c' - Cc$ obtained.

Separate values for the azimuth error of the instrument are then derived for each position of the instrument as follows, upon the assumption that the difference between the $(\alpha - T_c' - Cc)$ for the mean of the time stars and for the azimuth star of a group is due entirely to azimuth error. Upon this assumption, for each position of the instrument

$$a = \frac{(\alpha - T_c' - Cc)_{\text{time stars}} - (\alpha - T_c' - Cc)_{\text{azimuth stars}}}{A_{\text{time stars}} - A_{\text{azimuth stars}}}. \quad (41)$$

Numerically, in the present case

$$a_W = \frac{-4.00 - (-4.64)}{+0.08 - (-1.03)} = \frac{+0.64}{+1.11} = +0^s.577,$$

and

$$a_E = \frac{-4.00 - (-5.23)}{0.00 - (-2.53)} = \frac{+1.23}{+2.53} = +0^s.486. \quad (42)$$

With these approximate values of a_W and a_E the corrections Aa are applied, giving the values $\alpha - T_c' - Cc - Aa$ in the last column in the lower part of the computation form.

105. If these do not agree for the two positions of the instrument, it indicates that the mean values of $\alpha - T_c'$ used in (40) in deriving c were not equally affected by the azimuth error, so that their difference was not entirely due to c, as was assumed in using (39). A second approximation to the true value of c may now be obtained by considering the differences in the last column to be due to error in the first approximate value of c; substituting from that column in formula (39); and thus obtaining a correction to the first approximate c. Thus in the present case the second member of (39) becomes

$$\frac{-4.05-(-4.00)}{+1.25-(-1.32)}=\frac{-0.05}{+2.57}=-0^s.019. \quad . \quad (43)$$

The second approximation to the true value of c is then

$$+0^s.051-0^s.019=+0^s.032.$$

Proceeding as before, improved values for a_W and a_E are found by the use of formula (41). Thus in the present case there are obtained as second approximations

$$\left.\begin{aligned} a_W&=\frac{-3.98-(-4.60)}{+0.08-(-1.03)}=\frac{+0.62}{+1.11}=+0^s.559, \\ \text{and} \\ a_E&=\frac{-4.03-(-5.31)}{0.00-(-2.53)}=\frac{+1.28}{+2.53}=+0^s.506. \end{aligned}\right\} \quad (44)$$

This process of making successive approximations to the values of c and a may be continued until the values $(\alpha - T_c' - Cc - Aa)$ show a sufficiently good agreement. In general, with a well-chosen time set, the final value for ΔT_c will not be changed by as much as $0^s.01$ by any number of approximations made after the above agreement has been brought within the limit $0^s.05$.

When satisfactory values for c, a_W, and a_E have been obtained, the corrections Cc and Aa are applied separately to each star, as shown in the sixth, seventh, and eighth columns of the upper part of the computation form, and the values of the chronometer correction (ΔT_c) derived separately from each star. The residuals furnish a check on the computation.

Any large error in observation or computation will be indicated by the residuals, and may often be located by a careful study of them. The mean value of ΔT_c is the required chronometer correction at the epoch of the mean of the observed chronometer times.

106. A study of the above process of successive approximation to the values of c, a_W, and a_E shows that the rapidity with which the true values are approached depends upon three conditions. The mean A for the time stars for each position of the instrument should be as nearly zero as possible. In each position of the instrument the A for the azimuth star should differ as much as possible from the A for the mean time star, while corresponding C's should differ as little as possible. The last two conditions are difficult to satisfy simultaneously, but the fact that both must be considered leads one to avoid observing sub-polars. The conditions here stated show why the stars for the above time set were chosen as indicated in § 103. It is not advisable to spend time in observing more than one azimuth star in each half set.

It should be noted that the choice of stars indicated above also insures the maximum degree of accuracy in the determination of ΔT_c for a given expenditure of time, regardless of the method of computation.

The two things which especially commend this approximate method of computing time to those observers who have used it much in the field are the *rapidity* with which the computation may be made (especially when Crelle's multiplication-tables are used), and the *accuracy* which results from the fact that the derived values of a and c depend upon all the observations, and not upon observations upon a few stars only, as is frequently the case with other approximate metnods.

Computation of the Azimuth, Collimation, and Chronometer Corrections by Least Squares.

107. We start with the observation equations* indicated in (38). For lamp west each of these equations are of the form

$$\Delta T_c + A_W a_W + Cc - (\alpha - T_c') = 0, \quad . \quad . \quad (45)$$

and for lamp east, of the form

$$\Delta T_c + A_E a_E + Cc - (\alpha - T_c') = 0. \quad . \quad . \quad (46)$$

The subscripts added to A discriminate between factors applying to stars observed with lamp east and those observed with lamp west. This is done for the purpose of avoiding confusion in the normal equations. The parenthesis $(\alpha - T_c')$ is an approximate value for the clock correction after taking account of inclination, rate, and aberration. It is the observed quantity. The coefficients A and C may be obtained from the table in § 299.

Treating the observation equations all together as a single group, as many equations as stars, the four derived normal equations are of the form

$$\left.\begin{aligned}
\Sigma\Delta T_c + \Sigma A_W a_W + \Sigma A_E a_E + \Sigma Cc - \Sigma(\alpha - T_c') &= 0; \\
\Sigma A_W \Delta T_c + \Sigma A^2_W a_W + \Sigma A_W Cc - \Sigma A_W(\alpha - T_c') &= 0; \\
\Sigma A_E \Delta T_c + \Sigma A^2_E a_E + \Sigma A_E Cc - \Sigma A_E(\alpha - T_c') &= 0; \\
\Sigma C\Delta T_c + \Sigma CA_W a_W + \Sigma CA_E a_E + \Sigma C^2 c - \Sigma C(\alpha - T_c') &= 0.
\end{aligned}\right\} \quad (47)$$

The solution of these equations gives the required quantities ΔT_c, a_W, a_E, and c.

To obtain the probable error of a single observation sub-

* Observation equations are also called conditional equations by some authors.

stitute these values back in the observation equations, (45) and (46), and obtain residuals v_1, v_2, v_3, . . . , one for each equation. The probable error of a single observation is

$$\epsilon = 0.674\sqrt{\frac{\Sigma v^2}{n_0 - n_v}}, \quad . \quad . \quad . \quad . \quad . \quad (48)$$

in which n_0 is the number of observations, and n_v is the number of normal equations (and of unknowns).

To obtain the probable error, ϵ_0, of the computed ΔT_c, proceed as follows: Rewrite the normal equations (47), putting Q in the place of ΔT_c, -1 in the place of $-\Sigma(\alpha - T_c')$, and 0 in the place of the other absolute terms. Solve the resulting equations for Q. Notice that since all the coefficients in these new equations are just as before, it is only that part of the computation which deals with the absolute terms that is changed in the solution of the normal equations.

$$\epsilon_0 = \epsilon\sqrt{Q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

The form of the normal equations, and method of computing the probable error, are here stated for convenience of reference. For the corresponding reasoning the student must depend upon his knowledge of least squares, the methods here given being the ordinary least square methods for dealing with a set of observation equations in case there are no rigid conditions to be satisfied.*

108. As a concrete illustration of this least square adjustment for determining ΔT_c we may take the set of observations given in § 91, from which ΔT_c has been computed without

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* See Wright's Adjustment of Observations (Van Nostrand, New York), or Merriman's Least Squares (John Wiley & Sons, New York).

the use of least squares in § 106. The observation equations are

	v	v^2
$\Delta T_c + 0.02a_W \qquad + 1.26c + 0^s.07 = 0$	$-0^s.10$	0.0100
$\Delta T_c - 0.30a_W \qquad + 1.56c + 0\ .09 = 0$	$+0\ .05$	0.0025
$\Delta T_c + 0.36a_W \qquad + 1.06c - 0\ .31 = 0$	$+0\ .09$	0.0081
$\Delta T_c + 0.22a_W \qquad + 1.13c - 0\ .11 = 0$	$-0\ .03$	0.0009
$\Delta T_c - 1.03a_W \qquad + 2.36c + 0\ .52 = 0$	$0\ .00$	0.0000
$\Delta T_c \qquad + 0.25a_E - 1.11c - 0\ .06 = 0$	$-0\ .01$	0.0001
$\Delta T_c \qquad + 0.35a_E - 1.06c - 0\ .19 = 0$	$+0\ .07$	0.0049
$\Delta T_c \qquad - 0.20a_E - 1.46c + 0\ .23 = 0$	$-0\ .06$	0.0036
$\Delta T_c \qquad - 0.38a_E - 1.64c + 0\ .29 = 0$	$-0\ .02$	0.0004
$\Delta T_c \qquad - 2.53a_E - 4.18c + 1\ .44 = 0$	$+0\ .01$	0.0001
	Sum =	$0.0306 = \Sigma v^2$

From the absolute term in each equation $4^s.00$ has been dropped, as is frequently the case in least square computations, for the purpose of shortening the numerical work. The true value of $-(\alpha - T_c')$ is then, in each case, that written above, $+ 4^s.00$.

The four normal equations formed from the above observation equations in the usual way are

$$+10.00\Delta T_c - 0.73a_W - 2.51a_E - 2.08c + 1.97 = 0;$$
$$-0.73\Delta T_c + 1.33a_W - 2.24c - 0.70 = 0;$$
$$-2.51\Delta T_c + 6.77a_E + 10.84c - 3.88 = 0;$$
$$-2.08\Delta T_c - 2.24a_W + 10.84a_E + 36.64c - 5.56 = 0.$$

The solution of these equations for the unknowns gives $a_W = +0^s.568$, $a_E = +0^s.511$, $c = +0^s.034$, and $\Delta T_c = -0^s.020$, which combined with the $4^s.00$ which was dropped to ease the numerical work gives $\Delta T_c = -4^s.020$.

If these values are now substituted in the observation equations, § 108, the residuals (v) there shown are obtained.

From these the probable error of a single observation, see formula (48), is

$$\epsilon = 0.674\sqrt{\frac{0.0306}{10-4}} = \pm 0^s.048.$$

The modified normal equations being solved for Q as indicated in § 107, its value is found to be 0.1158.

Hence the probable error of the result (ΔT_c) is, see formula (49),

$$\epsilon_0 = \pm 0.048\sqrt{0.1158} = \pm 0^s.016.$$

109. If, instead of computing a separate value for the azimuth error, a, for each of the positions of the telescope axis, before and after reversal, the azimuth error is assumed to be the same throughout the whole set, the principles involved in the computation are the same as before; the distinction between a_W and a_E is dropped; there are but three unknowns and three normal equations instead of four; and the work of solving the normal equations is correspondingly shortened. The loss of accuracy in the computed result depends upon the magnitude of the actual change in the azimuth error at reversal. If no more than six stars are observed in a set, it may be advisable to use this process so as to reduce the number of unknowns.

110. Experience shows that the process outlined in §§ 105–106 gives such an accurate value for c that the value subsequently derived from a least square adjustment is found to be substantially identical with it. When such a preliminary computation has been made, the least square adjustment is shortened considerably, with little loss of accuracy, by accepting this preliminary value of c, applying the collimation corrections (as well as the inclination, rate, and aberration corrections) before the least square adjustment, and treating the clock correction and the two azimuth errors as the only unknowns. It is well in this case to treat each half set separately. The discrepancy between the two values for the clock correction thus derived, when reduced for clock rate to the same epoch, indicates the amount of error in the assumed value for c.

To illustrate this method we may use the same set of observations as in §§ 91, 101. Let it be assumed that the preliminary computation shown in § 101 has been made, and let the value $+ 0^s.032$ for c given there be accepted as a basis for this computation. The observation equations now become

$$\left.\begin{array}{l} \Delta T_c + 0.02a_W + 0^s.11 = 0 \\ \Delta T_c - 0.30a_W + 0\ .14 = 0 \\ \Delta T_c + 0.36a_W - 0\ .28 = 0 \\ \Delta T_c + 0.22a_W - 0\ .07 = 0 \\ \Delta T_c - 1.03a_W + 0\ .60 = 0 \end{array}\right\} \text{For the first half of set.}$$

$$\left.\begin{array}{l} \Delta T_c + 0.25a_E - 0\ .10 = 0 \\ \Delta T_c + 0.35a_E - 0\ .22 = 0 \\ \Delta T_c - 0.20a_E + 0\ .18 = 0 \\ \Delta T_c - 0.38a_E + 0\ .24 = 0 \\ \Delta T_c - 2.53a_E + 1\ .31 = 0 \end{array}\right\} \text{For the second half of set.}$$

The normal equations for the first half of the set are

$$+ 5.00\Delta T_c - 0.73a_W + 0.50 = 0;$$
$$- 0.73\Delta T_c + 1.33a_W - 0.75 = 0;$$

and for the second half of the set,

$$+ 5.00\Delta T_c - 2.51a_E + 1.41 = 0;$$
$$- 2.51\Delta T_c + 6.77a_E - 3.54 = 0.$$

The solution gives for the first half-set $\Delta T_c = - 0^s.019$, $a_W = + 0^s.553$, and $Q = 0.217$, and for the second half-set $\Delta T_c = - 0^s.024$, $a_E = + 0^s.514$, and $Q = 0.246$.

The probable error of a single observation derived from the first half set is $\pm 0^s.058$, and from the second $\pm 0^s.032$. The probable error of ΔT_c from the first half set is $\pm 0^s.027$, and from the second $\pm 0^s.016$. The final result from the complete set is, by this method of computation, $\Delta T_c = - 4^s.022 \pm 0^s.016$.

The difference between the two values for ΔT_c derived from the two halves of the set serves to indicate the degree of accuracy of the assumed value of c.

Introduction of Unequal Weights.

111. In the preceding treatment it has been tacitly assumed that all observations are of equal weight. But incomplete transits should be given less weight than complete transits. For if only a few lines of the reticle are observed upon, evidently the accidental errors made in estimating the times of transit across the separate lines will not be eliminated to as great an extent as if all the lines were observed. Then, too, the image of a star of large declination moves much more slowly across the reticle than does the image of an equatorial star, and it is therefore more difficult to estimate the exact time of its transit across each line. If it is found by the investigation of many records for such slow-moving stars that the error of observation is larger for such stars than for equatorial stars, it is proper to give them less weight in the computation of time. An extended discussion of this matter of weights may be found in the Annual Report of the Coast and Geodetic Survey for 1880, pp. 213, 235–237. It suffices for our purpose here to give, in slightly abridged form, the tables of relative weights which were derived from that discussion (see §§ 302, 303). The weights as given in these tables were deduced for transit instruments having a clear aperture of object-glass from $1\frac{3}{4}$ to $2\frac{3}{4}$ inches, and a magnifying power from 70 to 100 diameters. It may be extended with little error to instruments of the same nature which are considerably larger or smaller.

112. To introduce the unequal relative weights, w, into the least-square adjustment, it is necessary to multiply each observation equation by $\sqrt{w}$, and to make the usual subsequent modifications in the least square computation. These modifications are indicated in the following example,—the same problem as that treated in § 108, without the use of unequal weights. If an incomplete observation is made upon a slow star, so that both the tables of § 302 and of § 303 must be used, first multiply the two relative weights w together, and then take the square root of that product as the multiplier for the observation equations.

The square roots of the weights given to the ten stars, in order of observation, are respectively 0.9, 0.8, 1.0, 0.9, 0.6, 0.9, 1.0, 0.8, 0.8, and 0.3. The observation equations shown in § 108 are multiplied by these factors respectively. The normal equations resulting from the weighted observation equations so obtained are

$$
\begin{aligned}
+ 6.80\Delta T_c - 0.01a_W - 0.04a_E + 0.53c + 0^s.14 &= 0;\\
- 0.01\Delta T_c + 0.61a_W \qquad\qquad - 0.57c - 0^s.34 &= 0;\\
- 0.04\Delta T_c \qquad\qquad + 0.87a_E + 0.93c - 0^s.50 &= 0;\\
+ 0.53\Delta T_c - 0.57a_W + 0.93a_E + 13.79c - 0^s.61 &= 0.
\end{aligned}
$$

The solution of these equations gives $\Delta T_c = - 0^s.019$, $a_W = + 0^s.583$, $a_E = + 0^s.544$, and $c = + 0^s.033$.

The probable error of an observation of weight unity is

$$\epsilon = 0.674\sqrt{\frac{\Sigma w\, v^2}{n_0 - n_v}} = 0.674\sqrt{\frac{0.0223}{10-4}} = \pm\, 0^s.041.$$

The probable error of

$$\Delta T_c = \epsilon_0 = \epsilon\sqrt{Q} = \pm\, 0^s.041\sqrt{0.147} = \pm\, 0^s.016.$$

113. Unless an extreme degree of accuracy is required, the assumption that all observations are of equal weight is sufficiently exact. The introduction of unequal weights adds so little to the accuracy of the computation that economic considerations will often indicate that the least square adjustment should be made on the basis of equal weights.*

Auxiliary Observations.

114. Aside from the observations and computations which have been treated in detail, certain others are necessary for the determination of the instrumental constants which have been assumed in the preceding treatment to be known.

The equatorial intervals of the lines of the reticle may be determined from any series of complete transits, i.e., observations in which the transit of each star was observed across every line. A special series of observations is not required, for the complete transits of the particular series of time observations under treatment may be utilized for this purpose in addition to using them to determine the clock correction. For every complete transit every term in equation (20) (see § 92), namely, $i_n = (t_n - t_m) \cos \delta$, is known except i_n. Every

* The computation of a series of time observations taken by the author on the shore of Chilkat Inlet, Alaska (in latitude 59° 10′), in 1894, was made in the field by least squares, giving all stars equal weight, regardless of their declinations and of the number of missed lines. In the final computation subsequently made at the Coast and Geodetic Survey Office in Washington unequal weights were assigned. In the series there were 46 sets, each consisting, generally speaking, of observations upon 10 stars. The average difference, without regard to sign, between the chronometer corrections as computed in the two ways from the same set of observations was $0^s.04$. This is about equal to the probable error of the clock correction computed from a set. But it must be remembered that the conditions were extreme. On account of the high latitude of the station many of the stars were slow-moving stars (even those observed in the zenith). There was so much interference by clouds that complete observations on all the stars were secured on only 10 nights out of the 46, and observations on a single line only of the reticle were not infrequent.

complete transit observed furnishes, then, a determination of the equatorial interval of every line. The transit of a slow-moving star gives a more accurate determination of the equatorial intervals than the transit of a star of small declination, for the errors in observing t_n do not increase so rapidly, with increase of declination, as cos δ decreases. For this reason some observers prefer to make a special series of observations for equatorial intervals using stars of large declination only. In computing and using the equatorial intervals it must be borne in mind that when the telescope axis is reversed in its Ys the order in which the star transits across the lines is reversed, and also the algebraic sign of the equatorial interval of each line.

115. The portion of a set of observations given below will serve to show how the pivot inequality, p_i (see § 94), is determined by a series of readings of the striding level, upon the telescope axis placed alternately in each of its two possible positions with clamp west and clamp east (the clamp instead of the lamp being here used to indicate the position of the axis).

OBSERVATIONS FOR INEQUALITY OF PIVOTS OF TRANSIT NO. 4.

STATION: Seaton, Washington.—G W. D., observer.—June 19, 1867.

Altitude.	Time.	Temperature, Fahr.	Clamp West.			Clamp East.			
			Object-glass S.		$\frac{1}{4}(\Sigma w - \Sigma e)$	Object-glass N.		$\frac{1}{4}(\Sigma w - \Sigma e)$	$\frac{b_e - b_w}{4} = p_i$
			Level.		b_w	Level.		b_e	
			W. end.	E. end.		W. end.	E. end.		
°	h. m.	°	d.	d.	d.	d.	d.	d.	d.
55	10.30 A.M.	73	60 0	64.0	+0.600	59.0	65.2	−0.425	−0.256
			65.2	58.8		64.0	59.5		
50	45 A.M.	72	65.0	59.0	+0.950	64 0	59.5	−0.250	−0.300
			60.8	63.0		59.0	64.5		
45	50 A.M.	72.5	60.8	63.0	+1.450	59.5	64 0	−0.125	−0.394
			66.0	58.0		64.0	60.0		
40	11.00 A.M.	72.8	65.0	58.8	+1.050	64.0	60.0	−0.175	−0.306
			61.0	63.0		59 3	64.0		
35	05 A.M.	73	60.5	63 0	+1.200	59.2	64 0	−0.575	−0 444
			65.5	58 2		63.0	60.5		

The value of one division of this striding level was known to be $1''.05$. The whole set, of which this is a part, gave for a mean value of p_i — 0.337 divisions of the level = $0''.354$ = $0^s.024$.

116. The most accurate way of determining the value of one division of a level is by means of what is known as a level-trier or level-tester. The level-trier is essentially a bar supported at one end upon two pivots so that it is free to rotate about that end in a vertical plane, and carried at the other end by a good micrometer screw with its axis vertical. The length of the bar between supports and the pitch of the screw being known, the change of inclination of the bar corresponding to one turn of the screw is known. To determine the value of a division of a level it is placed upon the bar, and movements of the bubble corresponding to successive small movements of the micrometer screw are observed. Both ends of the bubble must of course be read, as its length is apt to change rapidly with changes of temperature. By comparing successive movements of the bubble corresponding to *equal* successive movements of the screw the uniformity of the value of a level division, or, in other words, the constancy of the radius of curvature of the upper inner surface of the level tube,* may be inferred. Observations of the length of time required for the bubble to come to rest in a new position also give an indication of the value of the level as an instru-

* The longitudinal section of the upper inner surface of a level tube is made as nearly a perfect circle as possible. If the student will consider how great is this radius of curvature in a sensitive striding level. he will appreciate to a certain extent the wonderful accuracy with which this surface must be ground. He will also understand why small deformations of the level tube by unequal changes of temperature have such a marked effect upon the movement of the bubble. The radius of curvature for a level of which each division is two millimeters long and is equivalent to one and a quarter seconds of arc,—a common type of level,—is more than three hundred meters (about a thousand feet).

ment of precision. This time is greater the smaller is the value of a division (of a given length) expressed in arc, but for levels of the same division value, is less the more perfect is the inner upper surface. If the level tube is so held in its metallic mounting that there is any possibility that it may be put under stress by a change of temperature, it is advisable to determine the value of a division *with the tube in its mounting* at two or more widely different temperatures. It may be well also to determine whether changing the length of the bubble, by changing the amount of liquid in the chamber at the end of the level tube, changes the apparent value of one division.

117. If an observer is forced to determine the value of a level division in the field, remote from a level-trier, after some accident let us say, which leads to replacing an old, well-known (but broken) level by another of which the value is unknown, his ingenuity will lead him to devise a method of utilizing whatever apparatus is at his disposal. The three methods given below will be found suggestive.

118. If a telescope having an eyepiece micrometer similar to that of a zenith telescope (§ 135), measuring altitudes or zenith distances, is available, the unknown angular value of a division of the level may be found by comparison with the known angular value of a division of the micrometer. Place the level in an extemporized mounting fixed to the telescope. Point with the micrometer upon some distant well-defined fixed object and read the micrometer and level. Change the micrometer reading by an integral number of divisions, point to the same object again by a movement of the telescope as a whole, and note the new reading of the level. Every repetition of this routine gives a determination of the value of a level division.

119. If in his instrumental outfit he has another well-determined level of sufficient sensibility the observer may use it as a standard with which to compare the unknown level. Put the unknown level in an extemporized mounting fastened to that of the known level. Adjust so that both bubbles are near the middle at once. Compare corresponding movements of the two bubbles for small changes of inclination common to both levels.

120. The following method * gives fully as great precision as either of the other two outlined above, and is especially valuable because the required means are apt to be at hand in the field even when the apparatus required for the other two methods is wanting.

For this method the only instrument required is a theodolite, or an engineer's transit, or any other instrument having both horizontal and vertical circles (not necessarily with a fine graduation) and a good vertical axis. Mount the level on the plate of the instrument parallel to the plane of the telescope and adjust it as if it were a plate level. Make the vertical axis truly vertical in the usual way. Measure the zenith distance of some well-defined stationary object, taking readings with (vertical) circle right and circle left to eliminate index error. Now incline the vertical axis directly toward or from the object, from 1° to 3°, by use of the foot-screws. The direction of this inclination may be assured by use of the plate level which is at right angles to the plane of the telescope. Measure the apparent zenith distance of the object again. The apparent change in the zenith distance is evidently the inclination of the axis to the vertical, which we will call γ. If, now, the instrument is revolved completely

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* Described in full by Prof. G. C. Comstock in the Bulletin of the University of Wisconsin, Science Series, vol. I, No. 3, pp. 68–74, and said by him to be due originally to Braun. Those desiring further details are referred to that article, from which this statement is condensed.

around its vertical axis, two positions will be found at which the bubble of the level is in the middle of the tube. For positions near these two the bubble is within such limits that it may be read. It is from readings of the bubble in such positions, in connection with readings of the horizontal circle and the above outlined determination of γ, that the value of a division of the level is derived.

121. Let Fig. 19 "represent a portion of the celestial sphere adjacent to the zenith, Z, and let V and S be the points in which the axis of the theodolite, and the line drawn from the center of curvature of the level tube through the middle of the bubble, respectively, intersect the sphere." "Since the bubble always stands at the highest part of the tube, its position, S, and the corresponding value of q are found by letting fall a perpendicular from the zenith upon the arc VS, and in the right-angled spherical triangle thus formed we have the relation"

$$\tan q = \tan \gamma \cos \beta. \quad . \quad . \quad . \quad . \quad . \quad (50)$$

"Since the level tube turns with the theodolite when the latter is revolved in azimuth, while the positions of the points V and Z remain unchanged, it appears that the angle β must vary directly with the readings of the azimuth circle." "If we represent by A_0 the reading of the circle when the arc VS is made to coincide with VZ, we shall have corresponding to any other reading A'"

$$\tan q = \tan \gamma \cos (A_0 - A'). \quad . \quad . \quad . \quad (51)$$

The value of A_0 may be obtained by taking the mean of any two readings of the circle for which the bubble stands at the same part of the tube.

"If A' and A'' denote slightly different readings of the azimuth circle, b' and b'' the corresponding readings of the middle of the bubble on the level scale, we may write two equations similar" to (51), "and taking their difference obtain"

$$\frac{\sin(q' - q'')}{\cos q' \cos q''} = 2 \sin \frac{A' - A''}{2} \sin \left(A_0 - \frac{A' + A''}{2}\right) \tan \gamma. \quad (52)$$

"Since $q' - q''$ is the distance moved over by the bubble, we may write $q' - q'' = (b' - b'')d$, where d is the value of a division of the level, and transform" (52) into

$$d = \frac{2 \tan \gamma \cos^2 q \sin \frac{1}{2}(A' - A'')}{\sin 1''} \frac{\sin [A_0 - \frac{1}{2}(A' + A'')]}{b' - b''}. \quad (53)$$

In this equation $\cos^2 q$ may usually be placed equal to unity. For greater accuracy the average value of q from equation (51) may be used. A_0 may be determined as indicated just below equation (51). Only an approximate value for it is required. All other quantities in the second member of (53) are known. Hence d may be computed. It simplifies the computation to take the readings at equidistant points on the circle. Sin $\frac{1}{2}(A' - A'')$ will then be constant.

It would seem at first sight that a value of d derived in this way by use of vertical and horizontal circles reading to half-minutes only (say) must necessarily be crude. If, however, the inclination of the vertical axis is made $3°$, γ and its tangent, and therefore d, may be determined within one four-hundredth part. The readings of the horizontal circle do not need to be very refined, because for the positions used a comparatively large change in the circle reading is necessary to produce an appreciable change in the position of the bubble. This method, then, serves to determine the level value with an accuracy which bears little relation to the fineness of the circle graduations.

Discussion of Errors.

122. Following the same general plan as in discussing the errors of sextant observations, the external errors, instrumental errors, and observer's errors will be discussed separately, and then their combined effect will be considered.

The two principal *external errors* are the error in the assumed right ascension of the star, and the lateral refraction of the light from the star.

If only such stars as are given in the various national ephemerides are observed for time, the probable errors in the right ascensions will usually be on an average $\pm 0^s.04$ or $\pm 0^s.05$, and no appreciable constant errors need be apprehended from this source.

From considerations which need not be stated in detail here, one is led to the conclusion that the effect of lateral refraction upon transit time observations must be quite small in comparison with the other errors; but it is difficult to estimate, because it is always masked by other errors following about the same law of distribution. For further consideration of this matter see § 219.

123. Among the instrumental errors may be mentioned those arising from change in azimuth, collimation, and inclination, from non-verticality of the lines of the reticle, from poor focusing and poor centering of the eyepiece, from irregularity of pivots, and from variations in the clock rate.

The errors of azimuth and collimation being determined from the observations themselves are quite thoroughly cancelled out from the final result, *provided* they remain constant during the period over which the observations extend, and provided also that the stars observed are so distributed in declination as to furnish a good determination of these constants. Their *changes*, however, during that interval, arising from changes of temperature, shocks to the instrument, or other causes, produce errors in the final result. It is in this connection that the stability of the pier is of especial importance. Such changes will evidently be smaller the more rapidly the observations are made and the more carefully the instrument is handled. In general they are probably small but not inappreciable.

To a considerable extent the same remarks also apply to the inclination error. The *changes* in inclination during each half-set evidently produce errors directly. Hence again the desirability of rapid manipulation. But the mean value of the inclination is determined from readings of the striding level, not from the time observations, and the level may give an erroneous determination of the mean inclination. Different observers seem to differ radically as to the probable magnitude of errors from this source, but the best observers are prone to use the striding level with great care. However small this error may be under the best conditions and most skilful manipulation, there can be no doubt that careless handling and slow reading* of the striding level, or a little heedlessness about bringing a warm reading lamp too near to it, may easily make this error one of the largest affecting the result. An error of 0.0002 inch in the determination of the difference of elevation of the two pivots of such an instrument as that described in § 83 produces an error of $0^s.1$ or more in the deduced time of transit of a zenith star.

If the lines of the reticle are not carefully adjusted so as to define vertical planes (§ 85), stars will be observed too early or too late if observed above or below the middle of the reticle. Such errors may be made very small by careful

* It is here assumed that before attempting to read the level it has been in position long enough for the bubble to come to rest in the position of equilibrium.

adjustment and by always observing within the narrow limits given by the two horizontal lines of the reticle.

Poor focusing of either the object-glass or the eyepiece leads to increased accidental errors because of poor definition of the star image. But poor focusing of the object-glass is especially objectionable, because it puts the reticle and the star image in different planes, and so produces parallax. The parallax error may largely be avoided by centering the eyepiece each time over the line of the reticle upon which the star is next to be observed. This repeated centering should never be omitted even though the observer may be confident that the focusing is perfect. It also serves in a measure to avoid errors which might otherwise be produced by the imperfections of the eyepiece.

If the inequality of the two pivots has been carefully determined as indicated in § 115, the errors arising from defects in their shapes may ordinarily be depended upon to be negligible.

Changes in the rate of the timepiece during a set of observations evidently produce errors in the deduced clock correction at the mean epoch of the set. Under ordinary circumstances such errors must be exceedingly small. If, however, an observer is forced to use a very poor timepiece, or if clouds interfere so as to extend the interval required for a set of observations over several hours, this error may become appreciable. It is less the more rapidly the observations are made.

The errors introduced by irregularity in the action of a chronograph of the form described in § 89 are too small to be considered, especially if its speed is assumed to be constant simply during the interval between successive clock breaks, and the chronograph sheet is read accordingly.

124. The *observer's errors* are by far the most serious in transit time observations. He is subject to both accidental and constant errors in his estimate of the time of transit .

From computations based upon thousands of observed transits it is known that an experienced observer is subject to an *accidental* error of from $\pm 0^s.06$ to $\pm 0^s.15$ in estimating the time of transit of a star of declination less than 60° across a single line of the reticle of such instruments as those described in § 83. For slower stars his error, expressed in time, is of course still greater. If the observations of the transits of a given star across the different lines of the reticle were not subject to any error common to all the lines, the probable error of the deduced time of transit across the mean line would vary inversely as the square of the number of lines in the reticle. But experience, as above, indicates that there is an error common to all the lines of $0^s.05$ to $0^s.12$. This error, sometimes called the culmination error, is an observer's error, which is constant for the interval during which the star is transiting across the reticle, but which may change before the next star is observed. From the method by which this value ($0^s.05$ to $0^s.12$) was deduced it also necessarily includes the small errors due to lateral refraction and irregularities in clock rate, as well as some small outstanding instrumental errors. The probable error, r, of the time of transit of a star (of declination less than 60°) across the mean line of a reticle is, therefore, given by an equation of the form

$$r^2 = (0^s.05 \text{ to } 0^s.12)^2 + \frac{(0^s.06)^2 \text{ to } (0^s.15)^2}{n},$$

in which n is the number of lines in the reticle. (Compare §§ 111, 302, 303.) For a more extended discussion see Coast and Geodetic Survey Report for 1880, Appendix No. 14, pp. 235, 236, or Doolittle's Practical Astronomy, pp. 318–322

125. In addition, still, to these errors there is another which is constant for all the observations of a set. Every experienced observer, though doing his best to record the time of transit accurately, in reality forms a fixed habit of observing too late, or too early, by a constant interval. This interval between the time when the star image actually transits across a line of the reticle and the recorded time of transit is called the *absolute personal equation* of the observer. The difference between the absolute personal equations of two observers is called their *relative personal equation*. The relative personal equation of two experienced observers has been known to be as great as $1^s.2$, and values greater than $0^s.25$ are common. For a more detailed discussion of personal equation, see §§ 243, 244.

126. To sum up, it may be stated that the *accidental* errors in the determination of a clock correction from observations with a portable astronomical transit upon ten stars may be reduced within the limits indicated by the probable error $\pm 0^s.02$ to $\pm 0^s.10$, but that the result is subject to a large *constant* error, the observer's absolute personal equation, which may be ten times as great as this probable error.

Miscellaneous.

127. In the field it is often necessary to use other instruments as transits for the determination of time. A theodolite when so used is apt to give results of a higher degree of accuracy than would be expected from an instrument of its size as compared with the astronomical transits whose performance has just been discussed,—unless, indeed, one has it firmly fixed in mind that the principal errors in a transit time determination are those due directly to the observer. On the other hand, a zenith telescope of the common form in which the telescope is eccentric with respect to the vertical axis has been found to give rather disappointing results,—perhaps because of the asymmetry of the instrument and of the fact that there can be no reversal of the horizontal axis in its bearings, but only of the instrument as a whole.

128. The mathematical theory for the determination of time by the use of the transit in any position out of the meridian has been thoroughly developed. That practice has been advocated. But the additional difficulty of making the computation, over that for a transit nearly in the meridian, and other incidental inconveniences, much more than offset the fact that the adjustment for putting the transit in the meridian is unnecessary. The transit is generally used in the meridian for time, at least in this country.

129. The use of the transit for time in the vertical plane passing through Polaris at the time of observation has also been advocated and has been used to a considerable extent in Europe. "The obvious advantage which this mode of observing possesses lies in the shorter period of time during which the observer depends upon the stability of his instrumental constants. For meridian observations this period is rarely much less than half an hour, while by the method suggested"—in which the whole time set consists of a pointing upon Polaris immediately followed by an observation of the transit of a zenith or southern star across that vertical plane —"it need never exceed five minutes."* This method is open, to a less extent, to the same objections as that of the preceding paragraph. This, in connection with the fact that it is rarely used in this country, makes its extended discussion inadvisable here.

130. If the transit is turned at right angles to the plane of the meridian, in other words, is put in the prime vertical,

* See Bulletin of the University of Wisconsin, Science Series, vol. I., No. 3. pp. 81–93.

an observation of the time of transit of a star across the mean line of its reticle furnishes a good determination of the latitude of the station if the clock correction is known. Or, if both transits, east and west of the zenith, are observed, the latitude may be computed without a knowledge of the clock correction. Formerly this method * was often used for the determination of latitude. Now it is almost entirely superseded by the use of the zenith telescope for latitude.

131. The Sun or a planet may sometimes be observed for time. In the case of the Sun the transit of both the preceding and the following limb may be observed, and the mean taken as the time of transit of the center. Both limbs of a planet may possibly be observed if a chronograph is used. Otherwise the preceding and following limbs may be observed alternately on successive lines of the reticle, taking care that the number of observations on each limb is the same, and the mean of all taken as the transit of the center across the mean line.

132. It is not advisable to observe the Moon for time, for its place is not well determined. Usually but one limb can be observed, the other being either obscure or invisible; and the observation of the limb on a side line of the reticle is affected by the rapid change in the Moon's right ascension and by a parallax due to its comparative nearness to the Earth.

QUESTIONS AND EXAMPLES.

133. I. An observer who is trying to get his transit into the meridian to begin observations for time finds that an observation upon η Draconis ($\delta = 61° 45'$) indicates that his chronometer is $4^s.2$ fast of local sidereal time, while an observation upon β Herculis ($\delta = 21° 43'$) indicates that his

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* For the detail of this method see Doolittle's Practical Astronomy, pp. 348–377, or Chauvenet's Astronomy, vol. II., pp. 238–271.

chronometer is $1^s.3$ slow. Assuming that the instrument is in perfect adjustment with respect to collimation and inclination, how much must he turn the slow-motion screw which shifts his instrument in azimuth, if one turn produces a change of $200''$ in azimuth? The latitude of the station is $39° 58'$. Is the object-glass too far east, or too far west, when the telescope is pointing northward?

Ans. 0.37 turn. Too far west.

2. The star 5 Ursæ Minoris ($\delta = 76° 09'$) was observed to transit as follows: Line I, $9^h 48^m 41^s.03$; II, $49^m 38^s.52$; III, $50^m 35^s.12$; IV, $51^m 31^s.70$; V, $52^m 28^s.79$; VI, $53^m 26^s.40$; VII, $54^m 22^s.58$. Derive the equatorial intervals of the various lines from the mean line.

Ans. I, $-40^s.93$; II, $-27^s.17$; III, $-13^s.62$; IV, $-0^s.08$; V, $+13^s.59$; VI, $+27^s.38$; VII, $+40^s.83$.

3. While observing a transit of the star 24 Comæ ($\delta = 18° 57'$) clouds interfered so that observations upon the first and second lines of the reticle were missed. The observed times of transit across the remaining lines were as follows: III, $8^h 13^m 20^s.40$; IV, $13^m 34^s.93$; V, $13^m 49^s.28$; VI, $14^m 03^s.88$; VII, $14^m 18^s.10$. The known equatorial interval of the first line is $-40^s.86$, and of the second $-27^s.31$. Deduce the time of transit across the mean of the seven lines.

Ans. $8^h 13^m 34^s.90$.

4. Draw two diagrams illustrating the geometric relations from which formulæ (34) and (35) of § 96 are derived.

5. The following ten stars were observed for time with a Troughton and Simms transit at Cornell University on May 23, 1896. Given the partially reduced results as indicated below, compute the correction to the Howard clock (keeping mean time) with which the observations were made.

Star.	δ.		Position of Lamp.	Corrected Transit Across Mean Line.*			Right Ascension Reduced to Mean Time.		
ε Virginis	11°	31′	W	8h	56m	18s.84	8h	48m	22s.66
43 Comæ...........	28	24	W	9	06	18 .93	8	58	22 .48
20 Can. Ven.......	41	07	W	9	12	09 .90	9	04	13 .35
ζ Ursæ Maj........	55	28	W	9	19	00 .84	9	11	04 .30
Gr. 2001...........	72	56	W	9	22	47 .60	9	14	50 .19
17 H. Can. Ven....	37	43	E	9	29	23 .87	9	21	26 .84
η Ursæ Maj........	49	50	E	9	42	39 .58	9	34	42 .52
η Bootis...........	18	55	E	9	48	54 .94	9	40	58 .23
11 Bootis..........	27	53	E	9	55	37 .63	9	47	40 .83
α Drac.............	64	52	E	10	00	45 .68	9	52	48 .27

Ans. By the method of § 104, $\Delta T_c = -7^m\ 56^s.73$, $c = +0^s.17$, $a_W = +0^s.73$, and $a_E = +0^s.30$.

By the method of § 107, $\Delta T_c = -7^m\ 56^s.74, \pm 0.02$, $a_W = +0^s.69$, $a_E = +0^s.34$, $c = +0^s.17$.

6. The following ten stars were observed for time at Washington, D. C. ($\phi = 38°\ 54'$), on June 22, 1896, with a sidereal chronometer. Given the following data, compute the chronometer correction on local sidereal time:

Star.	δ.		Position of Lamp.	Corrected Transit Across Mean Line.*			Right Ascension Reduced to Mean Time.		
3 Serpentis........	5°	19′	E	15h	10m	03s.96	15h	10m	04s.14
1 H. Urs. Min.....	67	46	E	15	13	32 .01	15	13	30 .38
μ Bootis...........	37	44	E	15	20	37 .07	15	20	36 .65
i Draconis.........	59	20	E	15	22	41 .21	15	22	40 .29
γ′ Bootis	41	11	E	15	27	15 .11	15	27	14 .70
ζ Cor. Bor. seq....	36	58	W	15	35	30 .18	15	35	30 .68
κ Serpentis	18	28	W	15	44	05 .86	15	44	06 .54
ζ Urs. Min.........	78	07	W	15	47	52 .19	15	47	51 .32
ε Cor. Bor........	27	11	W	15	53	19 .42	15	53	19 .92
θ Draconis.........	58	51	W	15	59	59 .56	15	59	59 .92

Ans. By the method of § 104, $\Delta T_c = +0^s.08$, $c = +0^s.32$, $a_E = +0^s.69$, and $a_W = +0^s.82$.

By the method of § 107, $\Delta T_c = +0^s.08 \pm 0^s.03$, $c = +0^s.32$, $a_E = +0^s.66$, and $a_W = +0^s.80$.

* Transit corrected for diurnal aberration, pivot inequality, and inclination.

7. Suppose that a striding level carries a continuous graduation of one hundred divisions each one-twentieth of an inch long, and that each division represents one second of arc. By about how much does the arc which is the longitudinal section of the upper inner surface of the level tube depart from the chord of that arc joining the end graduations ?

Ans 0.00030 inch.

CHAPTER V.

THE ZENITH TELESCOPE AND THE DETERMINATION OF LATITUDE.

The Principle of the Zenith Telescope.

134. The zenith distance of a star when on the meridian is the difference between the latitude of the station of observation and the declination of the star. Hence a measurement of the meridional zenith distance of a known star furnishes a determination of the latitude. In the zenith telescope, or Horrebow-Talcott, method of determining the latitude there is substituted for this measurement of the absolute zenith distance of a star the measurement of the small *difference* of zenith distances of two stars culminating* at about the same time on opposite sides of the zenith. The effect of this substitution is the attainment of a much higher degree of precision, arising from the increased accuracy of a differential measurement, in general, over the corresponding absolute measurement; from the elimination of the use of a graduated circle † in the measurement; and from the fact that the computed result is affected, not by the error in estimating the absolute value of the astronomical refraction, but simply by the error in estimating the very small difference of refraction of two stars at nearly the same altitude.

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* A star is said to *culminate* at the instant when it crosses the meridian.

† The zenith telescope carries a graduated circle, but it is used simply as a finder or setting circle, and its readings do not enter the computed result.

One may form a concrete conception of the relation between the latitude and the measured difference of zenith distance as follows: Suppose an observer, *A*, measures the difference of the meridional zenith distances of two stars and finds one to be 1° farther south of his zenith than the other is north of it. Suppose that another observer, *B*, is stationed just 1′ due north of *A*, and measures the difference of zenith distances of those same stars at the same times. For *B* the southern star will evidently be 1′ farther from the zenith than for *A*, and the northern star 1′ nearer the zenith. Hence *B* will find the difference of the zenith distances to be 1° 02′. Or, a given change in the position of the observer, along a meridian, produces double that change in the difference of zenith distances of two stars which culminate on opposite sides of the zenith. (Let the student draw a figure, in the plane of the meridian, to illustrate this paragraph.)

Description of the Zenith Telescope.

135. Fig. 20 shows a zenith telescope which is the property of the Coast and Geodetic Survey.

The arm *A* turns with the superstructure of the instrument and may be clamped to the horizontal circle, which is fixed to the base. At *B* and *B* are two stops which may be clamped to the circle in such positions that the telescope will be in the meridian when the arm *A* is in contact with either of them. One end of the horizontal axis is shown at *C*. The striding level is shown at *D*. It is counterweighted so as to make it balance on the horizontal axis. By means of the vernier and tangent screw at *E*, the levels *FF* (called latitude levels) can be set at any required angle with the telescope. These levels each carry a 2-mm. graduation of 50 divisions, numbered continuously from one end. The value of one division is about 1.5 seconds. One level would serve the purpose, but two were placed upon this instrument so that increased accuracy might be secured by reading both. By means of the clamp at *G* and the tangent screw at *H*, operating upon the sector *I*, the telescope may be brought to any desired inclination.

The object-glass has a clear aperture 7.6 cm. (= 3.0 in.) in diameter, and its focal length is 116.6 cm. (= 45.9 in.). The eyepiece has a magnifying power of 100 diameters. The focal plane of the object-glass lies in the rectangular brass box shown at *J*. The micrometer screw, of which the graduated head is shown at *K*, controls a rectangular brass frame sliding in parallel guides within this box. The movable line with which the star bisections are made is stretched across the sliding frame. While in use the object-glass is so focused as to make the focal plane coincide with the plane in which this line moves.

To facilitate counting the whole turns of the micrometer screw a small brass strip is placed in one side of the field of view of the eyepiece nearly in the plane of the micrometer line. The edge of the strip is filed into notches 0.01 in. apart. The pitch of the screw being 0.01 in., the micrometer line appears to move one notch along this comb for each complete turn of the screw. The whole turns are thus read from the comb, and the fractions are read from the head of the screw, which is graduated into one hundred equal divisions.

In Fig. 21, drawn in a vertical plane through the center of the telescope, let *O* be the optical center * of the object-glass. Let *S* be the position of a star. The star image is formed at the focus *T*, which is necessarily in the line *SO* produced. If the star is to appear bisected, the micrometer line must be placed at *T*. If another star later occupies the

* The *optical center* of a lense is that point through which all incident rays pass without permanent change of direction.

position S', its image will be formed at T', in $S'O$ produced, and to make a bisection the micrometer screw must be turned until the micrometer line is at T'. The recorded number of turns of the micrometer screw required to move the line from T to T' gives a measurement of the linear distance TT'. For the small angles concerned this linear distance is proportional to the angle TOT', the equal of SOS'. Hence the observed movement of the micrometer screw gives a measurement of the difference of zenith distances, SOS', of the two stars. In this particular instrument, the pitch of the screw being about 0.01 in. and the focal length OT about 45.9 in., one turn of the screw measures an angle * of about $\sin^{-1} \frac{0.01}{45.9}$, or about 40″.

The more common form of zenith telescope differs from the one here shown in having the telescope mounted eccentrically on one side of the vertical axis instead of in front of it, as in this case; in having a clamp which acts directly upon the horizontal axis in the place of the clamp at G acting on the sector I; and in having only one latitude level instead of two.

Adjustments.

136. The vertical axis must be made truly vertical. In adjusting and using the instrument it will be found convenient to have two of the three foot-screws in an east and west direction. The vertical axis may be made approximately vertical by use of the plate level, if there is one on the instrument, and the final adjustment made by using the latitude

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* The value of one turn cannot be determined with sufficient accuracy by such linear measurements. They are given here merely to illustrate the principle involved. The indirect process by which the value is ordinarily determined will be found described in §§ 158–164.

level. The process in each case is precisely the same as that of using the unadjusted plate levels of an engineer's transit to adjust its vertical axis.

The horizontal axis must be perpendicular to the vertical axis. This may be tested, after the vertical axis has been adjusted, by reading the striding level in both of its positions. If the horizontal axis is inclined, it must be made horizontal by using the screws which change the angle between the horizontal and vertical axes.

137. The line of collimation must be perpendicular to the horizontal axis. If the instrument is of the form shown in Fig. 20, this adjustment may be made as for an astronomical transit (§ 86) by reversing the horizontal axis in the Ys. If the instrument is of the form in which the telescope is eccentric with respect to the vertical axis, the method of making the test must be modified accordingly. It may be made as for an engineer's transit, but using *two* fore and *two* back points, the distance apart of each pair of points being made double the distance from the vertical axis to the axis of the telescope. Or, a single pair of points at that distance apart may be used and the horizontal circle trusted to determine when the instrument has been turned 180° in azimuth. If one considers the allowable limit of error in this adjustment (see § 167), it becomes evident that a telegraph pole or small tree, if *sufficiently distant* from the instrument, may be assumed to be of a diameter equal to the required distance between the two points. Or, a single point at a known distance may be used and a computed allowance made on the horizontal circle for the parallax of the point when the telescope is changed from one of its positions to the other.

138. During daylight the object-glass should be carefully focused on the most distant well-defined object available, to insure that stars may be seen at night. A neglect to do this may cause the observer, especially if inexperienced, much annoyance while he is trying to find out why stars for which the settings are properly made do not appear in the telescope. At the first opportunity the focus should be tested upon a star. When once the focus has been satisfactorily adjusted at a station, so that there is no parallax, it should not again be changed at that station. For any change in the object-glass focus changes the angular value of a division of the micrometer. It is well to clamp the slide so as to make an accidental change of focus impossible.

The stops on the horizontal circle must be set so that when the abutting piece is in contact with either of them the line of collimation is in the meridian. For this purpose, and throughout the observations, the chronometer correction must be known roughly, within one second, say. Set the telescope for an Ephemeris star which culminates well to the northward of the zenith, and look up the apparent right ascension for the date. Follow the star with the middle vertical line of the reticle, at first with the horizontal motion free, and afterward using the tangent screw on the horizontal circle, until the chronometer, corrected for its error, indicates that the star is on the meridian. Then clamp a stop in place against the abutting piece. Repeat for the other stop, using a star which culminates far to the southward of the zenith. It is well to test the setting of each stop again by an observation of another star before commencing latitude observations.

139. The movable line, attached to the micrometer, with which pointings are to be made must be truly horizontal. This adjustment may be made, at least approximately, in daylight after the other adjustments. Point, with the movable line, upon a distant well-defined object, with the image of that object near the apparent right-hand side of the field of the eyepiece. Shift the image to the apparent left-hand side of the field by turning the instrument about its vertical axis. If the bisection is not still perfect, half the correction should be made with the micrometer and half with the slow-motion screws which rotate the whole eyepiece and reticle about the axis of figure of the telescope. The adjustment should be carefully tested at night after setting the stops, by taking a series of pointings upon a slow-moving star as it crosses the field with the telescope in the meridian. If the adjustment is perfect the mean reading of the micrometer before the star reaches the middle of the field should agree with its mean reading after passing the middle, except for the accidental errors of pointing. It is especially important to make this adjustment carefully, for the tendency of any inclination is to introduce a *constant* error into the computed values of the latitude.

The Observing List.

140. Before commencing the observations at a station, an observing list should be prepared, showing, for each star to be observed, its catalogue number or its name, its magnitude, mean * right ascension and declination (at the beginning of the year), zenith distance, whether it culminates north or south of the zenith; and for each pair the setting of the vertical circle (the mean of the two zenith distances), the difference of the zenith distances with its algebraic sign as given by formula (54); and finally the micrometer comb setting for each star. For the purposes of the observing list the right ascensions to within one second of time, and the declinations and derived quantities within one minute of arc, are sufficiently accurate. If the micrometer comb reading is one minute per notch, and the middle notch is called 20, the comb setting for

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* For the definition of the mean place of a star see §§ 37, 39.

one star is 20 + the half-difference of zenith distances for one star, and 20 — that half-difference for the other star of a pair.

The requisites for a pair of stars for this list are that their right ascensions shall not differ by more than 20^m, to avoid too great errors from instability in the relative positions of different parts of the instrument; nor by less than 1^m, that interval being required to take the readings upon the first star and prepare for the second star of a pair; that their difference of zenith distances shall not exceed half the length of the micrometer comb, 20′ for the usual type of instrument; that each star shall be bright enough to be seen distinctly—not fainter than the seventh magnitude for the instruments here described; and that no zenith distance shall exceed 35°, to guard against too great an uncertainty in the refraction. The selection of a series of such pairs from the stars of a catalogue requires much time and patience.

The total range of the list in right ascension is governed by the hours of darkness on the proposed dates of observation, and by the convenience of the observer. The third of the above conditions may perhaps be used more conveniently in this form: the sum of the two declinations must not differ from twice the latitude by more than 20′. To prepare the list the latitude of the station should be known within a minute. It may possibly be secured from a map; if not, then from a sextant observation of the Sun, or from an observation of the meridional zenith distance of a star with the finder circle of the zenith telescope.

141. The stars selected should be such that their computed mean places may be made to depend in each individual case upon observations at several different observatories. The declination of a star as derived from observations at a single observatory will not in general be sufficiently accurate for the purpose in hand.

If the observer knows that in making the computation an ample collection of catalogues * of original observations at each of various observatories will be available, he may select all the suitable pairs he can find in any *extensive* list of stars, say the British Association Catalogue, or any of the Greenwich Catalogues, and trust to finding afterward in the various catalogues a sufficient number of observations upon each star at various observatories to give an accurate determination of its place. This is the usual procedure in the Coast and Geodetic Survey. A computer in the office at Washington calculates the declinations of the stars which have been observed for latitude by bringing together in a least square adjustment all the observations upon each star that he finds in his large collection of star catalogues.

If the necessary collection of catalogues of original observations is not known to be available, the observing list had best be made up from star lists in which the declinations given are the result of the compilation and computation of original observations at various observatories as outlined above. Among such available lists of mean places are those in the various national ephemerides; Preston's Sandwich Island List in the Coast and Geodetic Survey Report for 1888, Appendix No. 14, pp. 511–523; the list given in Appendix No. 7, pp. 83–129, C. & G. S. Report for 1876; and Boss' list in the report of the Survey of the Northern Boundary from the Lake of the Woods to the Rocky Mountains, pp. 592–615. These lists are given in about the order of the accuracy of their star places when reduced to the present time, the more recently computed places being more accurate, if other conditions are about the same. In the report of the Mexican Boundary Survey of 1892–93, which is about to be published, there will

* An indication of what is meant by "an ample collection of catalogues" may be gained by reading § 37.

be given an unusually accurate list of declinations prepared for that survey by Prof. T. H. Safford.

142. If an observer finds difficulty in securing a sufficient number of pairs from the available lists of accurately computed places, he may extend his list in two ways. Firstly, and preferably, if he has a good instrument, by extending the limits given in § 140. The limit of difference of right ascensions may not safely be extended much; the limit of difference of zenith distances may be extended to the full length of the micrometer comb, say 40′; and zenith distances as great as 45° may be allowed. Secondly, he may have recourse in his extremity to a catalogue of original observations at the Greenwich observatory for enough pairs to complete his list. The number of pairs required may be estimated by the considerations dealt with in § 169.

In the list of pairs resulting directly from the search in the star catalogues there will be many pairs which overlap in time. A feasible observing list may be formed by omitting such pairs that among the remainder the shortest interval between the last star of one pair and the first star of the next shall not be less than 2^m. In that interval a rapid observer can finish the readings upon one pair, set and be ready for the next, under favorable circumstances. The omitted pairs may be included in a list prepared for the second or third night of observation if one uses the second plan outlined in § 169. Also, it will frequently be found that the same star occurs in two or more different pairs. Such pairs may be treated like those which overlap in time, or the three or more stars forming what might be called a compound pair may all be observed at one setting of the telescope and then treated in the computation as two or more separate, but not independent, pairs.

It is desirable to so select the pairs that the algebraic sum of all the differences of zenith distances for a station shall be nearly zero, so as to make the computed latitude for the station nearly free from any effect of error in the mean value of the micrometer screw.

Directions for Observing.

143. The instrument being adjusted, set the vertical circle to read the mean zenith distance, or "circle setting" as marked in the observing list, of the first pair. Direct the telescope to that side of the zenith on which the first star of the pair will culminate. Put the bubble of the latitude level nearly in the middle of the tube by using the tangent screw which changes the inclination of the telescope. Place the micrometer thread at that part of the comb at which the star is expected, as shown by the observing list. Watch the chronometer to keep posted as to when the star should appear. When the star enters the field place the micrometer thread approximately upon it, and center the eyepiece over the thread. As soon as the star comes within the limits indicated by the vertical lines of the reticle bisect it carefully. As the star moves along watch the bisection and correct it if any error can be detected. Because of momentary changes in the refraction, the star will usually be seen to move along the thread with an irregular motion, now partly above it, now partly below. The mean position of the star is to be covered by the line. An attempt is being made to secure a result which is to be in error by much less than the apparent width of the thread, hence too much care cannot be bestowed upon the bisection. It is possible, but not advisable, to make several bisections of the star while it is passing across the field. As soon as the star reaches the middle vertical line of the reticle read off promptly from the comb the whole turns of the micrometer, read the level, and then the fraction of a micrometer turn, in divisions, from the micrometer head. Set promptly for the next star, even though it is not expected soon. In setting for the second star of a pair all that is necessary is to reverse the instrument in azimuth and set the micrometer thread to a new position.

144. The instrument must be manipulated as carefully as possible. Especial care should be taken in handling the micrometer screw, as any longitudinal force applied to it produces a flexure of the telescope which tends to enter the result directly as an error. The last motion of the micrometer head in making a bisection should always be in the same direction (preferably that in which the screw acts positively against its opposing spring), to insure that any lost motion is always taken up in one direction. The bubble should be read promptly, so as to give it as little time as possible to change its position after the bisection. The desired reading is that at which it stood at the instant of bisection. Avoid carefully any heating of the level by putting the reading lamp, warm breath, or face any nearer to it than necessary. During the observation of a pair the tangent screw of the setting circle must not be touched, for the angle between the level and telescope must be kept constant. If it is necessary to relevel, to keep the bubble within reading limits, use the tangent screw which changes the inclination of the telescope. Even this may introduce an error, due to a change in the flexure of the telescope, and should be avoided if possible.

145. For first-class observing it is desirable to have a recorder. He may count seconds from the face of the chronometer for a minute before culmination in such a way as to indicate when the star is to culminate according to the right ascension given on the observing list, taking the known chronometer correction into account. Such counting aloud serves a double purpose. It is a warning to be ready and indicates where to look for the star if it is faint and difficult to find. It also gives for each star a rough check upon the position of the azimuth stops and warns the observer when they need readjustment. It is only a rough check, because the observing list gives mean right ascensions (for the beginning of the year) instead of apparent right ascensions for the date. But in view of § 167 it is sufficiently accurate. The observer can easily make allowance for the fact that all stars will appear to be fast or slow according to the observing list by about the same interval, 0^s to 5^s (the difference between the mean and the apparent place). If a star cannot be observed upon the middle line, on account of temporary interference by clouds or tardiness in preparing for the observation, observe it anywhere within the safe limits of the field as indicated by the vertical lines of the reticle and record the chronometer time of observation.

EXAMPLE OF RECORD.

146. Station—No. 8, near San Bernardino Ranch, Arizona.
Instrument—Wurdemann Zenith Telescope No. 20.
Observer—J. F. H.
Date—August 9, 1892.

No. of Pair.	Star No. B. A. C.	N. or S.	Micrometer.		Level.		Remarks.
			Turns.	Divisions.	N.	S.	
88	7528	S.	22	82.0	19.9	54.9	Sky perfectly clear.
	7544	N.	16	98.9	56.0	20.9	Chronometer 21^s fast.
91	7566	N.	24	71.2	52.9	17.9	
	7586	S.	13	68.9	17.9	52.8	
93	7631	N.	30	29.0	52.5	17.6	
	7662	S.	9	13.0	16.4	51.7	

Derivation of Formula.

147. Let ζ and ζ' be the *true* meridional zenith distance, and δ and δ' the declination, of the south and north star of a pair, respectively. Then the latitude of the station is

$$\phi = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(\zeta - \zeta'). \quad . \quad . \quad . \quad . \quad (54)$$

Let the student draw the figure and prove this formula.

Let $(z - z')$ be the *observed* difference of zenith distances of the two stars, the primed letter referring to the north star. $(z - z')$ is in terms of the observed micrometer readings $= (M - M')r$, in which M and M' are the micrometer reading upon the south and north star, respectively, expressed in turns, and r is the angular value of one turn. Before this observed difference of zenith distances may be used in (54) it must be corrected for the inclination of the vertical axis as given by the level readings, for refraction, and for reduction to the meridian if either star is observed off the meridian.

148. Let d be the value of one division of the latitude level. Let n and s be the north and south reading, respectively, of the level for the south star, and n' and s' the same for the north star. Then, if the level tube carries a graduation of which the numbering increases each way from the middle, the inclination of the vertical axis, considered positive if the upper end is too far south, is

$$\frac{d}{4}\{(n + n') - (s + s')\}. \quad . \quad . \quad . \quad . \quad (55)$$

If the level tube carries a graduation which is numbered continuously from one end to the other, with the zero nearest the eyepiece, the inclination of the vertical axis is

$$\frac{d}{4}\{(n' + s') - (n + s)\}. \quad . \quad . \quad . \quad . \qquad (56)$$

If the zero is nearest the object-glass the algebraic sign must be changed from that given above.

The inclination of the vertical axis makes the south zenith distance too small by the amount indicated by (55) or (56) and the north too large by the same amount. Hence the correction to $\frac{1}{2}(z - z')$ is $\frac{d}{4}\{(n + n') - (s + s')\}$, or the corresponding expression (56).

149. The refraction makes each apparent zenith distance too small. If R and R' represent the refraction for the south and north star, respectively, the correction to $(z - z')$ is $(R - R')$, and to $\frac{1}{2}(z - z')$ is $\frac{1}{2}(R - R')$.

Let m be the correction to the apparent zenith distance of a south star observed slightly off the meridian to reduce it to what it was when on the meridian, and m' the corresponding reduction for a north star observed off the meridian. The correction to $\frac{1}{2}(z - z')$ will then be $\frac{1}{2}(m - m')$. m and m' are of course zero in the normal case, when the observation is made in the meridian.

Formula (54) may now be written, for an instrument with the level graduated both ways from the middle,

$$\phi = \frac{1}{2}(\delta + \delta') + (M - M')\frac{r}{2} + \frac{d}{4}\{(n + n') - (s + s')\}$$
$$+ \frac{1}{2}(R - R') + \frac{m}{2} - \frac{m'}{2}. \quad . \qquad (57)$$

This is the working formula for the computation, but the values of the last two terms may be conveniently tabulated.

150. The difference $R - R'$ being very small, the variation of the state of the atmosphere at the time of observation from its mean state (see refraction tables, §§ 294–297) may be neglected, except for stations at high altitudes. It has been shown, by the investigations of the laws of refraction which have been referred to in §§ 67–69, that this differential refraction, for the mean state of the atmosphere, is, with sufficient accuracy for the present purpose,

$$R - R' = 57''.7 \sin (z - z') \sec^2 z. \quad . \quad . \quad (58)$$

By computation from this formula the value of the term $\frac{1}{2}(R - R')$ of formula (57) has been tabulated, in § 304, for the arguments $\frac{1}{2}(z - z')$ as directly observed with the micrometer, and the zenith distance.

If the station is so far above sea-level that the mean barometric pressure is less than $\frac{9}{10}$, say, of 760 mm. (§ 294), the mean pressure at sea-level, it is necessary to take this fact into account by diminishing the values of the differential refraction given in § 304 in the ratio of the mean pressures. That is, if the mean pressure is 10% less than at sea-level diminish the values of § 304 by 10%; if 20% less subtract 20%, and so on. Inspection of the table shows that this allowance need only be made roughly, since the tabular values are small.

151. The value of $\frac{m}{2}$, and of its equal $\frac{m'}{2}$, is tabulated in § 305. The table gives directly the correction to the latitude for any case of a star observed off the meridian, but within one minute of it. If both stars of a pair are observed off the meridian two such corrections must be applied, one for each star. For the difficult derivation of the formula from which this table is computed see Chauvenet's Astronomy, vol. II. pp. 346, 347; or Doolittle's Practical Astronomy, pp. 505, 506.

EXAMPLE OF COMPUTATION.

152. Station—No. 8, near San Bernardino Ranch, Arizona.
Instrument—Wurdemann Zenith Telescope No. 20.
Observer—J. F. H.
Date—August 9, 1892.

[*Left-hand page of Computation.*]

Number of Pair.	Star Number B. A. C.	N. or S.	Micrometer. 1 turn = 100 div. = 62''.099. Reading.		Diff. Z. D.		Level. 1 div. = 1''.28. N.	S.	Level Corr. in Div.	Meridian Distance.	Declination.			
			t.	*d.*	*t.*	*d.*								
88	7528	S.	22	82.0			19.9	54.9			19°	46'	48''	.62
	7544	N.	16	98.9	+ 5	83.1	56.0	20.9	+0.52	——	42	47	05	.83
91	7566	N.	24	71.2			52.9	17.9			37	47	26	.64
	7586	S.	13	68.9	−11	02.3	17.9	52.8	+0.03	——	25	03	55	.38
93	7631	N.	30	29.0			52.5	17.6			55	17	23	.93
	7662	S.	9	13.0	−21	16.0	16.4	51.7	+0.50	——	7	44	26	.71

[*Right-hand page of Computation.*]

Sum and Mean of Declinations.	Corrections. Micrometer.	Level.	Refraction.	Meridian.	Latitude.	Remarks.
62° 33' 54''.45						
31 16 57 .22	+ 3' 01''.05	+0''.67	+0''.04	——	31° 19' 58''.98	
62 51 22 .02						
31 25 41 .01	− 5 42 .26	+0 .04	− 0 .08	——	58 .71	
63 01 50 .64						
31 30 55 .32	−10 57 .01	+0 .64	− 0 .18	——	58 .77	

Explanation of Computation.

153. The first seven columns of the computation need no explanation. The eighth column gives the values of $\frac{1}{4}\{(n' + s') - (n + s)\}$, § 148, the level tube of this instrument being graduated continuously from end to end with the zero nearest the eyepiece. The ninth column gives the meridian distance of such stars as were not observed upon the meridian. It is the hour-angle of the star expressed in seconds of time. To obtain the apparent right ascension within one second, which is sufficiently accurate for the determination of the required hour-angle for the present purpose, proceed as follows: Select a star from the mean place list of the Ephemeris which has nearly the same right ascension and declination as the star in hand. Compare its mean right ascension with its apparent right ascension for the date and assume that the corresponding change for the star in hand is the same.

The declinations given in column ten are those resulting from the apparent place computation made as indicated in §§ 46–49, or from the Ephemeris by interpolation in case the star is one of which the apparent place is there given.

The computation of the values in the second column of the right-hand page of the computation is facilitated, if there are many observations, by first constructing a table giving 10, 20, 30, etc., turns of the micrometer reduced to arc by multiplying by $\frac{r}{2}$: then of 1, 2, 3, 4, 5, 6, 7, 8, 9 turns reduced to arc; of 10, 20, 30, 40, 50, 60, 70, 80, 90 divisions thus reduced to arc; of 1, 2, 3, . . . etc.; and of 0.1, 0.2, 0.3, . . . etc. Such a table reduces the multiplication process otherwise required to a process of adding five tabular quantities.

The corrections for refraction were obtained by subtracting 20% from the values of § 304, the barometric pressure being only about four-fifths as great at San Bernardino as at sea-level.

The latitude is obtained from each pair by adding the various corrections algebraically to the mean declination, as indicated in formula (57).

With sufficient accuracy for some purposes the indiscriminate mean of all the individual values may be taken as the final value of the latitude. If the best, or most probable, value is desired the procedure outlined below must be followed.

Combination of Individual Results by Least Squares.

154. Let us first deal with the simplest case. Namely, let it be supposed that p separate pairs have been observed on each of n' nights at a station, each pair being observed on every night. For this case it will be found that the indiscriminate mean is, after all, the most probable value of the latitude, but the principles developed will be found useful in dealing with other more difficult cases in which this is not true.

The differences Δ obtained by subtracting the mean result for any one pair from the result on each separate night for that pair are evidently independent of errors of declination. We may compute from these differences, or residuals, the probable error of a single observation e. This error of observation includes the observer's errors, instrumental errors, and all external errors except the errors of the assumed declinations. Then by least squares

$$e = \sqrt{\frac{(0.455)[\Delta\Delta]}{n'p - p}},\text{*} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

* This square bracket [ ] is here used to indicate summation, as it frequently is in text-books on least squares.

in which $[\Delta\Delta]$ stands for the sum of the squares of all the residuals Δ obtained from all the pairs.

The probable error e_p of the mean result from any one pair may also be computed from the observations by the formula

$$e_p = \sqrt{\frac{(0.455)[vv]}{p - 1}}, \quad . \quad . \quad . \quad . \quad . \quad (60)$$

in which v is the residual obtained by subtracting the mean result for the station from the mean result for each pair. There are p such residuals. $[vv]$ stands for the sum of the squares of these residuals.

Let e_δ be the probable error of the mean of the two declinations. e_p evidently includes the declination errors of the two stars of a pair. From the ordinary law of transmission of accidental errors

$$e_p = \sqrt{e_\delta^2 + \frac{e^2}{n'}}. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Whence

$$e_\delta = \sqrt{e_p^2 - \frac{e^2}{n'}}. \quad . \quad . \quad . \quad . \quad . \quad (62)$$

e_δ may thus be obtained, from the observations for latitude, by substituting the values e and e_p computed by (59) and (60) in (62).

n' being the same for all pairs, it is evident from (61) that the means from the various pairs have equal weight. The most probable value for the latitude is then the indiscriminate mean of the results from the separate pairs, or, what is numerically the same in this case, the indiscriminate mean of all the individual results for latitude.

The probable error of the final result for latitude is

$$e_\phi = \frac{e_p}{\sqrt{p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

155. The simple case just treated seldom occurs in practice. Observations upon certain pairs are missed on some of the nights by accident or by cloud interference; work may be entirely stopped by clouds after half the observations of an evening have been made; or on the later evenings of a series the observer may purposely, with a view to more effectual elimination of declination errors, include in his observing list certain pairs which have not before been observed at that station, in the place of pairs which have already been observed once or more.

In the usual case, then, a total of p pairs are observed, pair No. 1 being observed n_1 times (i.e., on n_1 nights), pair No. 2 n_2 times, . . . , and the total number of observations is $n = n_1 + n_2 + n_3 \ldots$

By the same reasoning as before we have, by the ordinary least square formula, the probable error of a single observation

$$e = \sqrt{\frac{(0.455)[\Delta\Delta]}{n - p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

To obtain the probable error e_p of the mean result from any one pair with rigid exactness it is necessary to take into account the fact that different pairs must now be given different weights, since some are observed more times than others. To do this would make the computation considerably longer than is otherwise necessary. Fortunately, investigation of the numerical values concerned shows that the results are abundantly accurate if formula (60) is here used and the fact that the pairs are of unequal weight neglected in deriving e_p. With sufficient accuracy, then,

$$e_p = \sqrt{\frac{(0.455)[vv]}{p - 1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

According to the laws of accidental errors the probable errors of the mean results from the separate pairs are

$$e_{p\cdot 1} = \sqrt{e_\delta^2 + \frac{e^2}{n_1}}, \quad e_{p\cdot 2} = \sqrt{e_\delta^2 + \frac{e^2}{n_2}}, \text{ etc.} \quad . \quad (66)$$

The values $e_{p\cdot 1}$, $e_{p\cdot 2}$, . . . differ from each other because of the various values of n_1, n_2, n_3, . . .

For use in deriving e_δ an average value of the second term under the radical must be obtained. Again neglecting the unequal weights of the pairs, this average value, which will be called ϵ^2, is

$$\epsilon^2 = \frac{\left(\frac{e^2}{n_1} + \frac{e^2}{n_2} + \frac{e^2}{n_3} \ldots\right)}{p} = \frac{e^2}{p}\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \ldots\right). \quad (67)$$

Corresponding to (62), there is now obtained

$$e_\delta = \sqrt{e_p^2 - \epsilon^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

from which e_δ may be computed from the latitude observations.

156. The proper weights for the mean results from the separate pairs are inversely proportional to the squares of their probable errors. Hence these weights w_1, w_2, w_3, . . . , are proportional to

$$\frac{1}{e_\delta^2 + \frac{e^2}{n_1}}, \quad \frac{1}{e_\delta^2 + \frac{e^2}{n_2}}, \quad \frac{1}{e_\delta^2 + \frac{e^2}{n_3}}, \text{ etc.,} \quad . \quad . \quad (69)$$

and may now be computed from the known values of e_δ and e.

The most probable value ϕ_0 for the latitude of the station is the weighted mean of the mean results from the various pairs, or

$$\phi_0 = \frac{w_1\phi_1 + w_2\phi_2 + w_3\phi_3 \ldots}{w_1 + w_2 + w_3 \ldots} = \frac{[w\phi]}{[w]}, \quad . \quad . \quad (70)$$

in which ϕ_1 is the mean result from the first pair, ϕ_2 from the second pair, and so on.

Also, the probable error of this result is

$$e_\phi = \sqrt{\frac{(0.455)[wv'^2]}{(p - 1)[w]}}, \quad . \quad . \quad . \quad . \quad . \quad (71)$$

in which $[wv'^2]$ stands for the sum of the products of the weight for each pair into the square of the residual obtained by subtracting ϕ_0 from the mean result for that pair, and $[w]$ is the sum of the weights.

In case two north stars are observed in connection with the same south star, or *vice versa*, and the computation is made as if two independent pairs had been observed, the weight of each of these pairs as given by (69) should be multiplied by $\frac{2}{3}$ to take account of the fact that they are but partially independent. Similarly if three north stars have been observed in connection with the same south star the weights from (69) for each of the three resulting pairs should be multiplied by $\frac{1}{2}$.*

If, however, a given north star is observed in connection with a certain south star on a certain night or nights, and

* Coast and Geodetic Survey Report, 1880, p. 255; or Professional Papers of the Corps of Engineers, No. 24 (Lake Survey Triangulation), p. 625.

on a certain *other* night or nights is observed in connection with some *other* south star, the case is different, and the computation is sufficiently accurate, though not exact, if each of such pairs is given the full weight resulting from (69).

If very few pairs are observed more than once at a station the determination of e_s from the latitude observations obviously fails, and it must be estimated in some other way—from the star catalogues, for example.

157. As an example of the application of formulæ (64) to (71), the process of combining the various values for the latitude of station No. 8 on the Mexican Boundary Survey may be given. At this station 100 observations were made on 75 pairs, 25 of the pairs being observed twice each, and the other 50 once each. The observations extended over four nights.

The sum of the squares of the fifty residuals, Δ, obtained by subtracting the mean for each pair which was observed twice, from each of the two values from that pair, was 2.50 square seconds. The probable error of a single observation was then, from (64),

$$e = \sqrt{\frac{(0.455)(2.50)}{100 - 75}} = \sqrt{0.0455} = \pm 0''.213.$$

The indiscriminate mean of the 75 results, one from each pair, was found to be 31° 19′ 59″.02. By subtracting this value from each of the 75 separate values, squaring, and adding, it was found that $[vv] = 11.62$ square seconds. From (65) it followed that

$$e_p = \sqrt{\frac{(0.455)(11.62)}{75 - 1}} = \sqrt{0.0714} = \pm 0''.267.$$

The term $\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \ldots\right)$ of formula (67) is here $(25)(0.5) + 50 = 62.5$, and

$$\epsilon^2 = \frac{(0.0455)}{75}(62.5) = 0.0379.$$

Whence from (68)

$$e_\delta^2 = 0.0714 - 0.0379 = 0.0335 \quad \text{or} \quad e_\delta = \pm\, 0''.183.$$

Substituting the values of e_δ^2 and ϵ^2 in (69), the weights for the pairs observed twice was found to be 17.8, and for those upon which one observation only was made, 12.7. Since it is the *relative* weights only which affect the final result these weights were for convenience written 1.0 and 0.7, respectively.

The resulting weighted mean as indicated in (70) was found to be 31° 19′ 59″.01.

From the residuals corresponding to this value it was found that $[wv'^2] = 9.26$ square seconds. Hence

$$e_\phi = \sqrt{\frac{(0.455)(9.26)}{(74)(60.0)}} = \sqrt{0.000949} = \pm\, 0''.031.$$

Determination of Micrometer and Level Values.

158. The most advantageous method of determining the screw value is to observe the time required for a close circumpolar star near elongation to pass over the angular interval measured by the screw. Near elongation the apparent motion of the star is very nearly vertical and uniform.

That one of the four close circumpolars given in the Ephemeris, namely, α, δ, and λ Ursæ Minoris, and 51 Cephei, may be selected which reaches an elongation at the most convenient hour. In selecting the star it may be assumed that the elongations for such stars occur when the hour-angle is six hours, on either side of the meridian. Having selected the star, it is necessary both for planning the observations and for the computation to compute the time of elongation more accurately. The spherical triangle defined by the pole, the star, and the zenith is necessarily, for a star at elongation, right-angled at the star. Hence it may be shown that

$$\cos t_E = \tan \phi \cot \delta, \quad . \quad . \quad . \quad . \quad (72)$$

in which t_E is the hour-angle of the star at elongation (always less than 6^h). This hour-angle added to or subtracted from the right ascension of the star, for a western or eastern elongation respectively, gives the sidereal time of elongation, whence the chronometer time of elongation becomes known by applying the chronometer error. It is advisable to have the middle of the observations about at elongation. The observer should obtain an approximate estimate of the rate at which the star moves along his micrometer by a rough observation or from previous record, and time the beginning of his observations accordingly.

Everything being ready for the observations, the star is brought into the field of the telescope, the telescope clamped, the star image so placed as to be approaching the micrometer line with the micrometer reading some exact integral number of turns, and the level bubble brought near to the middle of the tube. The chronometer time of transit of the star across the line is observed, and the level read. Then the micrometer line is moved one whole turn in the direction of the motion of the star, the time of transit is again observed and the level read, and the process repeated until as much of the middle portion of the screw has been covered by the observations as is considered desirable. If desired, an observation may be made at every half-turn, or even every quarter-turn by allowing an assistant to read the level.

159. In Fig. 22, let P be the pole, S the position of the star when observed transiting across the micrometer line, and S_E its position at elongation. The small circle SS_E is a portion of the apparent path of the star. Let SK be a portion of the vertical circle through S limited by the great arc PS_E. Let the length (in seconds of arc) of SK, measuring the change in zenith distance of the star in passing from the position S to elongation, be called z. Let the sidereal interval of time, in seconds, from position S to elongation be called τ. In the spherical triangle SKP the angle at K is 90°, and that at P is, in seconds of arc, 15τ. Therefore

$$\sin z = \cos \delta \sin (15\tau). \quad . \quad . \quad . \quad . \quad (73)$$

The various values of z corresponding to observed values of τ might be directly computed from (73). But the computation becomes much easier and shorter, though based upon a more difficult conception, if one proceeds as follows:

(73) may be written

$$z = \cos \delta \frac{\sin (15\tau)}{\sin 1''}. \quad . \quad . \quad . \quad . \quad . \quad (74)$$

From the known expansion of the sine in terms of the arc there is obtained

$$\sin (15\tau) = (15\tau) \sin 1'' - \tfrac{1}{6}(15\tau \sin 1'')^3 + \tfrac{1}{120}(15\tau \sin 1'')^5. \quad (75)$$

By substitution from (75) in (74)

$$z = 15 \cos \delta \{\tau - \tfrac{1}{6}(15 \sin 1'')^2\tau^3 + \tfrac{1}{120}(15 \sin 1'')^4\tau^5\}. \quad (76)$$

By inspection of (76) it is evident that 15 cos δ is the rate of change of z (in seconds of arc per second of time) *at elongation.* If the rate of change of z were always constant at this value, z for any position of the star would be $(15 \cos \delta)\tau$. Compare this with (76), and it becomes evident that if the observed value of τ is corrected by applying to it the small correction $-\frac{1}{6}(15 \sin 1'')^2\tau^3 + \frac{1}{120}(15 \sin 1'')^4\tau^5$, the resulting value is that at which the value of z would be the same as the actual value if instead of the actual star there were substituted one whose motion is *vertical at the constant rate 15 cos* δ. These corrections $-\frac{1}{6}(15 \sin 1'')^2\tau^3 + \frac{1}{120}(15 \sin 1'')^4\tau^5$ are tabulated in § 306 for the argument τ. By using the signs there indicated they may be applied directly to the observed chronometer times. The corrected times correspond to uniform motion in a vertical great circle, in the place of the actual motion in a small circle.

160. Suppose the reading of the level has not remained constant during the observations. The change of reading indicates that the inclination of the telescope as a whole has changed. Let n_0, s_0 be the readings of the north and south ends of the bubble, respectively, at some selected time during the observations. Let it be proposed to reduce the observed times to what they would have been if the level readings had been n_0, s_0 throughout the observations. If n and s are the north and south readings, respectively, at a given observation, the correction to the inclination of the telescope to reduce it to the state n_0, s_0 of the level is evidently, in seconds of arc, for a graduation numbered both ways from the middle,

$$\tfrac{1}{2}\{(n - s) - (n_0 - s_0)\}d,$$

in which d is the value of one division in seconds of arc. From (76), noting the smallness of the tabular values of § 306, it is evident that with sufficient accuracy for the present purpose the rate of change of z is 15 cos δ (in seconds of arc per second of time) at any instant during the observations. Hence the correction, in seconds, to the observed chronometer time is

$$\pm\tfrac{1}{2}\{(n-s)-(n_0-s_0)\}\frac{d}{15\cos\delta}=\pm\{(n-s)-(n_0-s_0)\}\frac{d}{30\cos\delta}. \quad (77)$$

The upper sign is to be used for western elongation and the lower for eastern. The corresponding formula for a level with a graduation numbered continuously from one end to the other, with the zero nearest the eyepiece, is

$$\pm\{(n+s)-(n_0+s_0)\}\frac{d}{30\cos\delta}. \quad . \quad . \quad . \quad (78)$$

161. Having applied the corrections of § 306, and of formula (77) or (78), to the observed chronometer times, the results are the times which would have been obtained if the star had moved uniformly in a vertical circle and the telescope had remained fixed in position. We may now subtract the first corrected time from the middle one of the series, the second from the next following the middle, and so on, thus obtaining a series of values of the time interval corresponding to a known number of turns of the micrometer. The mean of these values may be taken, and from it the time interval t, in seconds, corresponding to one turn may be derived. The value of one turn, expressed in seconds of arc, is (15 cos δ)t.

To this must still be applied corrections for differential refraction and rate of chronometer. The refraction correction to the derived value of one turn is the change of refraction, or differential refraction, corresponding to one turn at the star's altitude. This is most conveniently derived by multiplying the value of one turn in seconds of arc by one-sixtieth of the change of refraction for one minute at the star's altitude as given in § 294. The refraction correction may be derived more exactly from formula (58), § 150. The correction is always negative, as the effect of refraction is always to make a heavenly body appear to move slower than it actually does.

The correction for rate of chronometer may be computed from the proportion:

$$\frac{\text{rate of chronometer in seconds per day}}{86\,400} = \frac{\text{required correction to value of one turn}}{\text{value of one turn}}; \qquad (79)$$

(86 400 seconds = 1 day.) The correction is negative if the chronometer runs too fast, otherwise positive. If the value of one turn is approximately one minute, this correction is less than $0''.001$ if the rate of the chronometer is less than $1^s.4$ per day.

162. At Astronomical Station No. 10, on the Mexican Boundary Survey, the value of the micrometer of the zenith telescope was determined by observations upon Polaris near eastern elongation on October 4, 1892. The computed chronometer time of elongation was $21^h\ 34^m\ 30^s.0$. The rate of the chronometer on sidereal time was $0^s.7$ per day, gaining. The declination of Polaris was $88°\ 44'\ 06''.9$. The latitude of the station was $31°\ 20'$. The star was observed at every half-turn from the reading 30 turns to the reading 10.5 turns. The level numbering was continuous from one end to the other with the zero nearest the eyepiece. $\frac{d}{30 \cos \delta} = 1^s.94$. Below is a portion of the record and computation.

Micrometer Reading. Turns.	Chronometer Time.	Level Readings.		Time from Elongation=τ.	Correction for Curvature.	Level Correction to State. 50.6 N. 17.2 S.	Level Correction.	Corrected Times.	Interval for Ten Turns.
		N.	S.			d			
22.5	21^h 20^m 51^s.0	50.4	17.2	13^m 39^s.0	+0^s.5	+0.20	+0^s.4	21^h 20^m 51^s.9	
22	22 23 .0	50.5	17.2	12 07 .0	+0 .3	+0.10	+0 .2	22 23 .5	
21.5	23 57 .5	50.4	17.1	10 32 .5	+0 .2	+0.30	+0 .6	23 58 .3	
21	25 32 .5	50.6	17.1	8 57 .5	+0 .1	+0.10	+0 .2	25 32 .8	
20.5	27 05 .0	50.6	17.1	7 25 .0	+0 .1	+0.10	+0 .2	27 05 .3	
...	....	...	...	.....	....	....	...	....	
12.5	52 08 .5	51.1	17.1	17 38 .5	−1 .0	−0.40	−0 .8	52 06 .7	31^m 14^s.8
12	53 44 .0	51.2	17.1	19 14 .0	−1 .3	−0.50	−1 .0	53 41 .7	18 .2
11.5	55 16 .0	51.3	17.1	20 46 .0	−1 .7	−0.60	−1 .2	55 13 .1	14 .8
11	56 50 .0	51.2	17.1	22 20 .0	−2 .1	−0.50	−1 .0	56 46 .9	14 .1
10.5	58 25 .0	51.2	17.1	23 55 .0	−2 .6	−0.50	−1 .0	58 21 .4	16 .1

Mean for 10 turns (from all the observations) = 31^m 14^s.65 ± 0^s.33.
Mean time for 1 turn = 187^s.465.

$$
\begin{aligned}
\text{Log } 187.465 &= 2.2729202 \\
\text{Log } \cos\delta &= 8.3438467 \\
\text{Log } 15 &= 1.1760913 \\
\hline
\text{Log } 62''.067 &= 1.7928582
\end{aligned}
$$

One turn	=	62″.067
Correction for refraction *	=	− 0 .051
" " rate	=	0 .000
Final value	=	62 .016 ± 0″.011

163. If the values of both the level and the micrometer are unknown, one may observe for micrometer value as outlined above, and also derive the value of the level in terms of the micrometer as indicated in § 118. We may first compute the micrometer value, omitting the level corrections; then derive the value of the level division from this approximate value of micrometer. The level corrections being now introduced into the micrometer computation will be found to modify it so slightly that a second approximation for the level value will not ordinarily be required.

* Refraction at this station was only ⅗ of that at sea-level.

164. If no special observations for micrometer value have been made, or if such observations have proved defective, the micrometer value may be derived directly from the latitude observations. Let ϕ_P be the mean latitude, as deduced with an approximate micrometer value, from all pairs for which the micrometer difference (taken $S - N$) was positive, ϕ_N the mean latitude from pairs with minus micrometer differences, D_P the mean of the positive micrometer differences, and D_N the mean of the negative differences. Then the correction to the approximate value of one turn is *

$$\frac{2(\phi_N - \phi_P)}{D_P - D_N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

For methods of determining the level value alone, see §§ 116–121.

Discussion of Errors.

165. The *external errors* affecting a zenith telescope observation are those due to defective declinations and those due to abnormal refraction.

The declinations used in the computation have probable errors which are sufficiently large to furnish much, often more than one-half, of the error in the final computed result. This arises from the fact that a good zenith telescope gives results but little inferior in accuracy to those obtained with the large instruments of the fixed observatories which are used in determining the declinations.

The following three examples will serve to indicate the

* This formula is not exact, from the least square point of view, that is, it does not give the most probable value of the required correction. But it gives so nearly the same numerical results as the exact least square treatment, and leads to so short and simple a computation, that its use is advisable.

improvement in the available declinations during the last few years, and the magnitude of the declination errors to be expected. The probable error of the mean of two declinations, e_δ, was found* to be $\pm 0''.55$ for the list of stars furnished to the U. S. Lake Survey by Prof. T. H. Safford in 1872. Similarly, for the list of stars furnished from the Coast and Geodetic Survey Office for use in determining the variation of latitude at the Hawaiian Islands in 1891–92,† $e_\delta = \pm 0''.18$; for the list furnished to the Mexican Boundary Survey by Prof. Safford in 1892–93, $e_\delta = \pm 0''.18$. Such a high degree of precision as that of the last two examples is only attainable by an up-to-date computation from many catalogues of many observatories. By the time such lists are available in print their accuracy has ordinarily diminished considerably with the lapse of time.

The errors in the computed differential refractions are probably very small, and it is not likely that they increase much with an increase of the mean zenith distance of a pair, up to the limit, 45°. If there were a sensible tendency, as has been claimed, for *all* stars to be seen too far north, or south, on some nights,—because of the existence of a barometric gradient for example,—it should be detected by a comparison of the mean results for different nights at the same station. Many such comparisons made by the writer indicate that in zenith telescope latitudes there is no error peculiar to the night. The variation in the mean results from night to night was found in all the cases examined to be about what should be expected from the known probable errors of observation and declination.

166. The *observer's errors* are those made in bisecting the star, and in reading the level and micrometer. Here also may

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* Professional Papers of the Corps of Engineers, No. 24, pp. 622–638.

† Coast and Geodetic Survey Report, 1892, Part 2, p. 158.

perhaps be classed the errors due to unnecessary longitudinal pressure on the head of the micrometer.

Indirect evidence indicates that the error of bisection of the stars is one of the largest errors concerned in the measurement. It probably constitutes the major part of the computed error of observation, and the bisection should be made with corresponding care.

With care in estimating tenths of divisions on the micrometer head and on the level tube, each of these readings may be made with a probable error of ± 0.1 division. For the ordinary case of a micrometer screw of which one turn represents about 60″, and of a level of which the value is about 1″ per division, such reading would produce probable errors of ± 0″.04 and ± 0″.05, respectively, in the latitude from a single observation. These errors are small, but by no means insignificant when it is considered that for first-class observing the whole probable error of a single observation, arising from all sources except declination, is less than ± 0″.30, and sometimes even less than ± 0″.20.

While reading the level the observer should keep in mind that a very slight unequal or unnecessary heating of the level tube may cause errors several times as large as the mere reading error indicated above; and that if the level bubble is found to be moving, a reading taken after allowing it to come to rest deliberately may not be pertinent to the purpose for which it was taken. The level readings are intended to fix the position of the telescope *at the instant when the star was bisected.*

It requires great care in turning the micrometer head to insure that so little longitudinal force is applied to the screw that the bisection of the star is not affected by it. A displacement of $\frac{1}{4000}$ part of an inch in the position of the micrometer line relative to the object-glass produces in the telescope of Fig. 20 a change of more than 1″ in the apparent position of the star. The whole instrument being elastic, the force required for even such a displacement is small. An experienced observer has found that in a series of his latitude observations, during which the level was read both before and after the star bisections, the former readings continually differed from the latter, from $0''.1$ to $0''.9$, always in one direction.*

167. Among the *instrumental errors* may be mentioned those due: 1st, to an inclination of the micrometer line to the horizontal; 2d, to an erroneous level value; 3d, to inclination of the horizontal axis; 4th, to erroneous placing of the azimuth stops; 5th, to error of collimation; 6th, to irregularity of micrometer screw; 7th, to an erroneous mean value of the micrometer screw; 8th, to the instability of the relative positions of different parts of the instrument.

The first-mentioned source of error must be carefully guarded against, as indicated in § 139, as it tends to introduce a *constant* error. The observer, even if he attempts to make the bisection in the middle of the field (horizontally), is apt to make it on one side or the other according to a fixed habit. If the line is inclined his micrometer readings are too great on all north stars and too small on all south stars, or *vice versa*.

The error from using an erroneous level value is smaller the smaller are the level corrections and the more nearly the plus and minus corrections in a series balance each other. To insure that it shall be negligible it is necessary to relevel every time the correction becomes more than two seconds, at most.

The errors from the third, fourth, and fifth sources may easily be kept negligible. An inclination of one minute of arc in the horizontal axis, or an error of that amount in either collimation or azimuth, produces only about $0''.01$ error in the

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* Coast and Geodetic Survey Report, 1892, Part 2, p. 58.

latitudes. All three of these adjustments may easily be kept far within this limit.

Most micrometer screws are so regular that the uneliminated error in the mean result for a station from the sixth cause is usually very small. But it should not be taken for granted that a given screw is regular. Large irregularities may be detected by inspection of the computation of the micrometer value. Errors with a period of one turn may be detected by making the observations for micrometer value at every quarter-turn, and then deriving the value of each quarter, 0 to 25 divisions, 25 to 50 divisions, etc., of the head separately. The four mean values thus derived should agree within the limits indicated by their probable errors.

168. To guard against error from the seventh source the pairs must be so selected as to make the plus and minus micrometer differences at a station balance as nearly as possible.* For example, at the fifteen astronomical stations occupied on the Mexican Boundary Survey of 1892–93 the mean micrometer difference, taken with regard to sign, never exceeded 0.36 turn at any station, and was less than 0.10 turn at nine of the stations. If the plus and minus micrometer differences balance exactly at a station, an erroneous micrometer value does not affect the computed latitude, but merely increases the computed probable errors.

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* It seems an easy matter to make an accurate determination of micrometer value. But experience shows that such determinations are subject to unexpectedly large and unexplained errors. For example, in the Hawaiian Island series of observations, mentioned above, the micrometer value was carefully determined twelve times. The results show a range of nearly $\frac{1}{300}$ of the total value. This corresponds to a range of about one-sixth of an inch in the focus of the object-glass. In the San Francisco series, and in general wherever the micrometer value has been repeatedly measured, the same large discrepancies have been encountered. Hence the need of carrying out the suggestions of the above paragraph.

The errors from the eighth source may be small on an average, but they undoubtedly produce at times some of the largest residuals. They may be guarded against by protecting the instrument from sudden temperature changes, and from shocks and careless handling, and by avoiding long waits between the two stars of a pair. The closer the agreement in temperature between the instrument room and the outer air the more secure is the instrument against sudden and unequal changes of temperature.

The computed probable error of a single observation, e, including all errors except those of declination, was found to be as follows in three recent first-class latitude series: In the observations for variation of latitude at San Francisco * in 1891–92, 1277 observations (in two series) gave $e = \pm 0''.19$ and $e = \pm 0''.28$; in a similar series at the Hawaiian Islands † for the same purpose in 1891–92, 2434 observations gave $e = \pm 0''.16$; from 1362 observations at fifteen stations on the Mexican Boundary in 1892–93, $e = \pm 0''.19$ to $\pm 0''.38$.‡

169. When an observer begins planning a series of observations to determine the latitude of a given point two questions at once arise. How many observations shall be made? How many separate pairs shall be observed? Increasing the number of observations increases the cost of both field work and computation. An increase in the total number of separate pairs adds proportionally to the work of computing the mean places, but otherwise has little effect on the total cost. The economics of the problem demand that the ratio of observations to pairs shall be such as to give the maximum accuracy for a given expenditure. Two extremes

* Coast and Geodetic Survey Report, 1893, Part 2, p. 494.

† Coast and Geodetic Survey Report, 1892, Part 2, pp. 54, 158.

‡ Transactions of the Association of Civil Engineers of Cornell University, 1894, p. 58.

of practice are to take 210 observations on 30 pairs, each pair being observed on 7 nights; and to take 100 observations on 100 pairs, each pair being observed but once. The first is the old practice of the Coast Survey.* The recent practice of that Survey is intermediate between these extremes. The latter extreme was approached, but not quite reached, on the Mexican Boundary Survey of 1892–93.

Let it be supposed that $e = \pm 0''.21$ and $e_\delta = \pm 0''.18$, as in the example given in § 157. Then for the former extreme method the effect of the errors of observation on the result would be reduced to $\pm 0''.21 \div \sqrt{210} = \pm 0''.014$, and the effect of the declination errors to $\pm 0''.18 \div \sqrt{30} = \pm 0''.033$, giving for e_ϕ, the probable error of the result, $\sqrt{(0.014)^2 + (0.033)^2} = \pm 0''.036$. In the latter extreme case the error of observation would be reduced to $\pm 0''.21 \div \sqrt{100} = \pm 0''.021$, the declination error to $\pm 0''.18 \div \sqrt{100} = \pm 0''.018$, and the probable error of the result to $\sqrt{(0.021)^2 + (0.018)^2} = \pm 0''.028$. To look at the matter in another light: if with the above data as to e and e_δ the weight for a pair observed once is called unity, that for a pair observed twice is 1.40, by formula (69), § 156; observed seven times is 1.98; and for a pair observed an infinite number of times 2.36. Little is gained in accuracy from the second observation on a pair, and less from each succeeding observation.

170. The zenith telescope furnishes a latitude determination which is so far superior to that given by any other portable instrument that it should always be used where great accuracy is desired. A theodolite, or an astronomical transit, may be used as a zenith telescope if furnished with a suitable eyepiece micrometer, and with a sufficiently sensitive level parallel

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* C. & G. S. Report, 1893, Part 2, p. 301.

to the plane in which the telescope rotates upon its horizontal axis. For the convenience, however, of those who may desire to determine the latitude with a sextant on explorations or at sea, and of those who may be forced by circumstances to determine the latitude by a measurement of the altitude of the Sun, or a star, with a theodolite or an altazimuth, the following formulæ are here collected:

To compute the latitude from an observed altitude of a star, or the Sun, in any position, the time being known.

171. The requisite formulæ are

$$\tan D = \tan \delta \sec t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

$$\cos (\phi - D) = \sin A \sin D \operatorname{cosec} \delta; \quad . \quad . \quad (82)$$

in which δ is the declination and t the hour-angle of the star (or Sun) at the instant of observation; D is an auxiliary angle introduced merely to simplify the computation; A is the altitude resulting from the measurement after applying all instrumental corrections, the correction for refraction, and, if the Sun is observed, the corrections for parallax and semidiameter (see §§ 65, 66). D is to be taken less than 90°, and $+$ or $-$ according to the algebraic sign of the tangent. Formula (82) is ambiguous in that $\phi - D$, determined from the cosine, may be either positive or negative. But the latitude of the station is always known beforehand with sufficient accuracy to decide between these two values. These formulæ are exact, no approximations having been made in deriving them.*

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* For this derivation see Doolittle's Practical Astronomy, pp. 236, 237; or Chauvenet's Astronomy, vol. I. pp. 229, 230.

To compute the latitude from zenith distances of a star, or the Sun, observed near the meridian, the time being known.

172. The rate of change of zenith distance (or of altitude) of a given star is smaller the nearer the star is to the meridian. Hence the effect of a small error in the time, which is assumed to be known, is less the nearer the observation is made to the meridian, and is zero for an observation made precisely on the meridian. Only a single pointing can be made when the star is on the meridian, whereas it is desirable to take several pointings so as to decrease the effect of errors of observation. Hence the desirability of a rapid method for computing the latitude from circummeridian observations.

Let ϕ be the required latitude of the station; δ the declination of the star at observation; ζ_1, ζ_2, ζ_3, . . . successive observed values of the zenith distance of the star corresponding to the hour-angles t_1, t_2, t_3, . . . ; and ζ_0 the meridional zenith distance of the star. Then

$$A = \frac{\cos\phi\cos\delta}{\sin\zeta_0}, \quad B = A^2\cot\zeta_0, \quad C = A^3\tfrac{2}{3}(1 + 3\cot^2\zeta_0); \qquad (83)$$

$$m = \frac{2\sin^2\frac{1}{2}t}{\sin 1''}, \quad n = \frac{2\sin^4\frac{1}{2}t}{\sin 1''}, \quad o = \frac{2\sin^6\frac{1}{2}t}{\sin 1''}. \quad . \qquad (84)$$

A, B, and C are evidently constant for a series of observations made near a given meridional passage of the star. Let m_1, m_2, m_3, . . . , n_1, n_2, n_3, . . . , o_1, o_2, o_3, . . . , be values of m, n, and o corresponding, respectively, to t_1, t_2, t_3, ... Then for a star upon which u observations are made near upper culmination

$$\phi = \delta \pm \frac{\zeta_1+\zeta_2+\zeta_3\ldots\zeta_u}{u} \mp A\frac{m_1+m_2+m_3\ldots m_u}{u} \pm B\frac{n_1+n_2+n_3\ldots n_u}{u} \mp C\frac{o_1+o_2+o_3\ldots o_u}{u}, \qquad (85)$$

in which the upper signs are to be used if the star crosses the meridian south of the zenith, and the lower signs if it crosses north of the zenith.

Similarly, for a star upon which u observations are made near lower culmination

$$\phi = 180° - \delta - \frac{\zeta_1 + \zeta_2 + \zeta_3 \ldots \zeta_u}{u} - A\frac{m_1 + m_2 + m_3 \ldots m_u}{u} - B\frac{n_1 + n_2 + n_3 \ldots n_u}{u} - C\frac{o_1 + o_2 + o_3 \ldots o_u}{u}, \quad (86)$$

t being now reckoned from lower culmination.

These formulæ * are not exact. They are derived by an expansion into a converging series, of which only the first three terms are retained. The errors of the formulæ are greater the greater the hour-angle, and the smaller the zenith distance of the star. Their accuracy is sufficient for the reduction of sextant observations if the hour-angle is limited to 30^m, and is also not allowed to exceed (in minutes of time) the zenith distance of the star (in degrees). To be certain of sufficient accuracy for the reduction of observations made with a theodolite or altazimuth the hour-angle must be limited to 30^m, and to one-half (in minutes of time) the zenith distance of the star (in degrees).

It will be noted that ϕ and ζ_0 are required at the start for the reduction since they occur in the second member of (85) and (86). Only approximate values are required, however, as the terms involving A, B, and C are always small. Such approximate values may be derived from previous computations; from a preliminary approximate reduction of a single observation; or, if the observations extend on both sides of the meridian, the smallest observed zenith distance may be assumed for this purpose to have been made upon the meridian.

The values of m, n, and o will be found tabulated in terms of t in § 307, for values of t up to 30^m.

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* For their derivation see Doolittle's Practical Astronomy, pp. 238–244; or Chauvenet's Astronomy, vol. I. pp. 238–240.

If the Sun is observed, δ is not a constant during the period of observation. But formula (85) may still be used for the reduction of sextant observations if for δ is substituted the mean of the declinations corresponding to each instant of observation.

To compute the latitude from observations of the altitude of Polaris at any hour-angle, the time being known.

173. The apparent motion of Polaris is so slow that it may be observed *at any hour-angle* with the assurance that the effect of an error in the assumed chronometer correction upon the computed latitude will be small. It is so bright as to be quickly found and brought into the field of a telescope. For stations in the United States it is always in a convenient position for observation. These considerations often lead to the adoption of Polaris as the object when the latitude is to be determined by direct observations of altitude with a theodolite or smaller instrument.

If A is the altitude of Polaris at the hour-angle t, and P is its polar distance ($= 90° - \delta$) expressed in seconds of arc, then *

$$\begin{aligned}\phi = A - P \cos t + \tfrac{1}{2}P^2 \sin 1'' \sin^2 t \tan A \\ - \tfrac{1}{3}P^3 \sin^2 1'' \cos t \sin^2 t + \tfrac{1}{8}P^4 \sin^3 1'' \sin^4 t \tan^3 A \\ - \tfrac{1}{24}P^4 \sin^3 1'' (4 - 9 \sin^2 t) \sin^2 t \tan A, \quad . \quad . \quad . \quad . \quad (87)\end{aligned}$$

in which all terms in the second member except the first are in seconds of arc.

The last term will never exceed 0''.01 at any station of which the latitude is not greater than 82°. The fifth term will never exceed 0''.01 for any station below latitude 48°, nor 0''.10 at any station below 67°. The maximum value of

* For the derivation of this formula see Doolittle's Practical Astronomy, pp. 256–259; or Chauvenet's Astronomy, vol. I. pp. 253–255.

the fourth term is about $0''.33$. The preceding statements are based upon $1^\circ 20'$ as the value of P, and the terms are correspondingly reduced when P is below that limit.

174. Formulæ (81), (82), and (87) furnish the means of computing the latitude from a *single* observation of altitude. How must one proceed if a series of observations of the altitude have been made? If the chosen formula is applied to each observation separately, each computation will be exact; the computations will be so nearly alike as to furnish convenient rough checks by differences; but if there are many observations the computation of the series becomes tediously long. If one takes the mean of all the measured altitudes, assumes that it corresponds to the mean of the hour-angles, and applies either formula to this mean altitude and mean hour-angle, the result is approximate. For, the mean of the altitudes does not correspond to the mean of the hour-angles, in general, because of the curvature of the apparent path of the star and the corresponding variation in the rate of change of the altitude. The error involved in the assumption that the two means correspond, evidently decreases as the interval covered by the series of observations decreases. If, then, one breaks up the series into sufficiently short groups, the means for each group may be treated by a single application of the formula without sensible error. For sextant observations it suffices when observing upon Polaris to break up the series into groups not more than fifteen minutes long. For more accurate observations, made upon Polaris with a theodolite or altazimuth, one cannot be certain of sufficient accuracy unless each group is limited to about five minutes. In using (81) and (82) with the Sun, or with other stars than circumpolars, it will usually be necessary to make the groups very much shorter than indicated above.

175. The process of making a direct measurement of altitude with an instrument having a vertical circle (theodolite or altazimuth) is so simple that a detailed statement of it is hardly necessary here. Suffice it to say that the effect of the index error of the vertical circle must be eliminated from each observation or group of observations before applying the formula for the latitude computation; that the chronometer time of each pointing upon the star must be noted with a chronometer of which the error is known (to fix the hour-angle); and that the inclination of the vertical axis must be determined during the observations by readings of a level having its tube parallel to the plane of the telescope. The effect of index error may be eliminated either by making special observations to determine the index error, by making half of the observations of each group with the telescope in a reversed position, or by a combination of both these processes. The accuracy with which the chronometer error must be known may be inferred from the observed rate of change of the altitude. The correction to the measured altitude due to the inclination of the vertical axis may be computed by formula (55) or (56), § 148, using the algebraic signs there given if the star is north of the zenith, and reversing them if south of the zenith.

The method of observing the altitude of the Sun, or a star, with a sextant, has already been given in Chapter III, in connection with the determination of time.

If for a given purpose it is only required to determine the latitude within 30″ the table given in § 308 may be utilized as there indicated.

QUESTIONS AND EXAMPLES.

176. 1. State in detail how you would make the computed allowance for parallax upon the horizontal circle if a single point at a known distance is used in making the collimation adjustment of a zenith telescope mounted on one side of its vertical axis. (See § 137.)

2. Prove formulæ (55) and (56), for the level corrections to be applied to observations with a zenith telescope.

3. At the same station and with the same instrument as given in § 152, the following latitude observations were also made. Compute the results.

Date, 1892.	B. A. C. No. of Star.	N. or S.	Micrometer Reading. t	d	Level Readings. N.	S.	Declinations.
August 9	7380	S	10	70.9	18.1	52.8	4° 48′ 12″.76
	7417	N	28	36.0	50.8	16.0	58 10 03 .10
August 9	7440	S	29	06.4	16.9	51.7	−4 01 03 .43
	7482	N	9	09.0	53.1	18.1	66 20 18 .63
August 16	7440	S	30	21.1	12.9	56.5	−4 01 02 .75
	7482	N	10	25.9	57.6	13.8	66 20 21 .26

Diminish the differential refractions of § 304 by 20% (see § 150). *Ans.* $\phi = 31° \ 19' \ 58''.41$, $58''.83$, and $59''.62$.

4. What is the correction to the computed latitude due to observing the star off the meridian, with a zenith telescope, in each of the following cases? The declinations of the four stars were, respectively, 23° 19′, − 5° 42′, 46° 06′, 81° 27′, and they were observed too late by the following intervals, respectively, 12^s, 21^s, 18^s, 53^s.

Ans. $+ 0''.01$, $- 0''.01$, $+ 0''.05$, and $+ 0''.11$.

5. At Astronomical Station No. 5, on the Mexican Boundary, 99 observations with a zenith telescope were taken on 50 pairs, each pair excepting one being observed twice. It was found that $[\Delta\Delta] = 5.17$ square seconds, and $[vv] =$

7.52 square seconds. Compute e and e_δ. For notation and formulæ, see §§ 154, 155.

Ans. $e = \pm 0''.22$; $e_\delta = \pm 0''.21$.

6. Prove formula (72), for the computation of the hour-angle of a star at elongation. If δ is less than ϕ, this formula leads to an absurdity, namely, a cosine greater than unity. To what does this absurdity correspond in nature?

7. What was the sidereal time of western elongation of δ Ursæ Minoris at Anchorage Point, Chilkat Inlet, Alaska ($\phi = 59° 10' 19''.4$) on October 13, 1892? Its declination was then $86° 36' 54''.1$, and its right ascension $18^h 06^m 49^s.6$.

Ans. $23^h 44^m 04^s.5$.

8. State the reason for the double signs in formulæ (77) and (78), for the level corrections in a computation of micrometer value, and prove that the signs as given are correct.

9. What is the mean value of one turn of the micrometer deduced from the following observations, forming a portion of the series from which the observations given in § 162 were also extracted?

Micrometer Reading. t	Chronometer Time.	Level Readings. N. d	Level Readings. S. d
27.0	21^h 06^m $43^s.5$	50.1	17.4
26.5	08 18.5	50.3	17.7
26.0	09 50.5	50.2	17.2
25.5	11 27.0	50.1	17.1
...	....	...	...
17.0	21 38 03.0	50.9	17.1
16.5	39 35.5	50.9	17.1
16.0	41 12.0	51.0	17.2
15.5	42 41.5	51.0	17.1

Ans. $61''.987$.

10. The following altitudes, uncorrected for refraction, of the star Altair ($\alpha = 19^h 45^m 32^s.5$, $\delta = 8° 35' 08''.7$) were observed with a theodolite on October 13, 1892: $51° 53' 28''$, $51° 35' 49''$, $51° 06' 40''$, and $50° 48' 09''$. The times of the

separate observations as read from the face of a chronometer known to be $13^s.1$ fast of local sidereal time were, respectively, $21^h\ 32^m\ 16^s.1$, $21^h\ 34^m\ 19^s.6$, $21^h\ 37^m\ 42^s.5$, and $21^h\ 39^m\ 50^s.1$. Compute the latitude of the station. Carry your computation to tenths of seconds of arc, and treat each observation separately.

Ans. Mean value of $\phi = 38°\ 14'\ 11''$.

11. The following observations of the altitude of the Sun were made with a theodolite on February 15, 1892. The times are given as read from the chronometer, which was known to be 11^s fast of local mean solar time, and the altitudes are given after all corrections have been applied. The following data in regard to the Sun were derived from the Ephemeris:

Times.	Corrected Altitudes.
$11^h\ 42^m\ 19^s$	$42°\ 05'\ 11''$
43 12	05 16
44 03	05 27
44 50	05 29
45 41	05 35
46 32	05 30
47 26	05 36
48 13	05 28
49 08	05 16
50 02	04 55

$\delta = -12°\ 40'\ 39''$.
Equation of time $= +14^m\ 20^s$.
These values correspond to the mean of the observed times.
What was the latitude of the station?

Ans. $\phi = 35°\ 13'\ 46''$.

12. Four observations of the altitude of Polaris with an altazimuth gave for its mean altitude, corrected for refraction, $40°\ 19'\ 48''.9$. The sidereal times of the observations were, respectively, $22^h\ 19^m\ 16^s$, $22^h\ 20^m\ 31^s$, $22^h\ 22^m\ 51^s$, and $22^h\ 24^m\ 10^s$. At the time of observation the right ascension of Polaris was $1^h\ 20^m\ 05^s.1$, and its declination $88°\ 44'\ 05''.9$. What was the latitude of the station?

Ans. $39°\ 26'\ 06''.8$.

CHAPTER VI.

AZIMUTH.

177. The instrumental process of determining the azimuth of a terrestrial line astronomically consists of a measurement of the angle between two vertical planes—one defined by the azimuth mark and the vertical line through the instrument, and the other by the observed star and the vertical at the instrument. Since the angle between these two planes is continually changing, the exact time at which each pointing is made upon the star must be noted upon a chronometer of which the error is known. From this recorded time the hour-angle of the star and its azimuth as seen from the station may be computed. The computed azimuth of the star combined by addition or subtraction (as the case may be) with the measured horizontal angle at the station between the star and the azimuth mark gives the azimuth of the mark from the station.

Description of Instrument.

178. Any one of the many theodolites used for the measurement of the horizontal angles of a triangulation may be used for azimuth determinations,—provided the telescope can be inclined enough to point to the star,—each instrument giving a degree of accuracy dependent upon its size, power, and workmanship. The larger instruments often called altazimuths, designed primarily for astronomical work, do not differ in principle from the smaller theodolites. The instrument shown in Fig. 24 is a 20-in.* theodolite which belongs to the U. S. Coast and Geodetic Survey.

The Troughton and Simms altazimuth now in use in the College of Civil Engineering of Cornell University may be taken as a type of the larger altazimuths. Its horizontal circle is 36 cm. (= 14 in.) in diameter, and is graduated to 5′ spaces. It may be quickly unclamped from the fixed center of the instrument, and its zero shifted to a new position. The index microscope carries in its field of view (as the comb is carried in the micrometer of the zenith telescope; see § 135) a small pointer which is seen projected against a portion of the horizontal circle, and gives the degrees and the nearest preceding five minutes of the reading. The remaining minutes and the seconds of the reading are obtained from three reading microscopes. Each of these microscopes is furnished with an eyepiece micrometer similar to that of the zenith telescope described in § 135. A line of the circle graduation, the object-glass, and the micrometer line bear the same relation to each other here as the star, the object-glass, and the micrometer line bear to each other in the zenith telescope (see Fig. 21, and the corresponding text, § 135). Each microscope is so adjusted that five turns of the micrometer screw correspond as nearly as may be to one space on the circle. Each turn represents therefore approximately one minute, and each of the sixty equal divisions of the micrometer head one second. The reading of the micrometer *increases* as the micrometer line apparently moves in the direction of *decreasing* graduations. The reading given directly by the comb and head is the distance, measured in the backward direction along the circle graduation, from the zero of the micrometer (middle of the comb) as seen projected upon the circle, to the nearest graduation representing *too small* a

* That is, a theodolite with a horizontal circle twenty inches in diameter.

reading of the circle. This reading added to that of the index microscope gives the complete reading of the circle. The three reading microscopes give nominally identical readings, and the mean is taken to secure increased accuracy. The use of two or more microscopes or verniers on a circle also serves of course to eliminate, wholly or in part, the errors which would otherwise enter the measurements from the effect of eccentricity and of periodic errors in the graduation of the circle. Instead of a single micrometer line with which to bisect the graduations, these reading microscopes are usually provided with two parallel lines at such a distance apart that when they are placed symmetrically on the two sides of a graduation a narrow strip of light is seen between each line and the edge of the dark graduation. More accurate pointings upon a graduation can be made with such a double line than with a single line.

The telescope has a focal length of 24 in., its object-glass has a clear diameter of 2½ in., and its eyepiece a magnifying power of about 45 diameters. The value of one division of the striding level is $1''.8$. The vertical circle is graduated to $5'$ spaces, and is read to seconds by two micrometer microscopes. The telescope is furnished, in addition to a fixed reticle, with an eyepiece micrometer similar to that of the zenith telescope, but turned 90° in its position so that it measures small angles in the plane defined by the telescope and its horizontal axis instead of differences of zenith distances. (For the use of this micrometer see §§ 205–214.) The other features of the instrument are sufficiently shown by the figure.

179. The three reading microscopes are often, especially on smaller instruments, replaced by two microscopes or by two verniers. A hanging level may be used instead of a striding level. The eyepiece micrometer of the telescope, the separate index microscope, and the vertical circle are often omitted. A reflecting prism is sometimes placed at the intersection of the telescope with the horizontal axis to turn the light rays at right angles, and the eyepiece is then placed at the end of the horizontal axis. The proportions of the various parts and their absolute size may be varied greatly. The graduated circle may be furnished with a clamp and tangent screw, similar in principle and purpose to that of the lower motion of an engineer's transit. These various modifications produce great changes in the outward appearance of the instrument, may introduce certain obvious limitations to their use, but the principles involved are changed only in minor details.

Adjustments.

180. The vertical axis must be made truly vertical by the same process that is used in levelling up an engineer's transit. Whatever levels are to be used should for convenience be adjusted so that each will reverse with but little change in the position of the bubble.

The adjustments of the focus of the telescope, of the collimation, and for bringing the middle line of the reticle into a vertical plane, should be made precisely as for the astronomical transit (see §§ 85, 86).

The reading microscopes must be kept in adjustment. Ordinarily the only adjustment that will be found necessary is to fit the microscope to the eye by drawing out or pushing in the eyepiece until the most distinct vision of the micrometer lines and of the graduation is obtained. Sometimes it may be found that the micrometer lines are apparently not parallel to the graduation upon which the pointing is to be made. This may be remedied by rotating the micrometer box about the axis of figure of the microscope. If to do this it is necessary to loosen the microscope in its supporting clamp, great caution is necessary to insure that the distance of the objective from the circle graduation is not changed.

If one turn of the micrometer is found to differ very much from its nominal value, in terms of the circle graduation (one turn = one minute for the instrument described in § 178), it may be restored to its nominal value by changing the distance of the objective from the circle graduation. An inspection of Fig. 21, and the corresponding text, § 135, will show that for a given microscope the nearer the objective is to the graduation the smaller is the value of one turn, and *vice versa.* A change in this distance also necessitates a change in the distance from the objective to the micrometer lines—these lines and the graduation being necessarily at conjugate foci of the objective. This adjustment of the micrometer value is a difficult one to make, and so should not be attempted unless it is certainly necessary. Once well made, it usually remains sufficiently good for a long period.

Directions for Observing.

181. The azimuth mark and the instrument fix the two ends of a line of which the azimuth is to be determined. The azimuth mark must be placed so far away from the instrument that no change from the sidereal focus of the telescope will be required to give a well-defined image of it. One mile will usually be found sufficient. A convenient mark is a bull's-eye lantern shining through a small hole in a box which serves to protect it from the wind. An ordinary tubular lantern mounted in that way has been found satisfactory on distances from one to three miles. The size of the hole must be suited to the distance and telescope. Too large a hole gives a blur of light instead of a well-defined point. Too small a hole makes the light appear too faint. A diameter of one inch has been preferred by the writer at distances of one to three miles for observations with a telescope having an objective 45 mm. in diameter and an eyepiece magnifying 30 diameters. Most observers prefer a much smaller hole. If the distance to the mark exceeds five miles, a stronger light, with a parabolic reflector behind it or a lens in front of it, may be required. Black and white stripes on the front of the box, or a pole accurately in line, may serve as a target for the measurements in daylight of the horizontal angles necessary to connect with a triangulation. A ground mark (stake, bolt, or stone monument) must always be provided to hold the point in case the box and light are accidentally disturbed. If the line of sight passes near the ground, the light usually appears more unsteady than if the line is high above the ground. A long cut through woods along the line of sight, or the presence of objects very near to the line on either side, tends to make the light appear unsteady, and introduces a liability to a small constant error in the observed azimuth. The direction to the azimuth mark is immaterial, except when a micrometer is to be used as indicated in § 205.

182. Occasionally it is difficult to find a satisfactory location for an azimuth mark. For example, the astronomical station may be one of the stations of a triangulation located on the flat top of a mountain in such a position that none but very near or very distant points are visible from it. In such a case one may resort to a collimator for an azimuth mark. The collimator is an auxiliary telescope rigidly mounted so as to face the instrument, and adjusted to sidereal focus. The instrument telescope being also at sidereal focus and pointed upon the collimator, the two object-glasses then being toward each other, the lines of the reticle of the collimator may be seen as if they were at an infinite distance, for rays of light proceeding from them are parallel rays in the space between the two object-glasses. The middle line of the reticle then defines a fixed direction from the instrument and serves the same purpose as the ordinary distant azimuth mark, *provided* the collimator remains fixed in direction. Great care is necessary in mounting a collimator and in protecting it to insure that this last condition is sufficiently well satisfied.

183. In accurate azimuth work, to insure that errors of time and latitude have but little effect upon the computed azimuths, only close circumpolar stars should be observed. It will be found advisable to use only the four close circumpolars of which the apparent places are given in the Ephemeris, namely, α, λ, and δ Ursæ Minoris and 51 Cephei. Fig. 23, showing their relative positions, will be found a convenient aid in finding and identifying them. The figure also shows roughly their right ascensions, declinations, and magnitudes. The position of the figure on the page corresponds to the position of the stars in the sky at 0^h sidereal time. The figure in the sky rotates once around in a counter-clockwise direction every twenty-four sidereal hours. The arrow at the pole indicates by its length and direction the apparent motion of the pole among the stars in a century.

184. Three methods of observing will be treated. First, that in which a close circumpolar star is observed at any hour-angle, and the instrument is used as a direction instrument; second, a similar method in which the instrument is used as a repeater; and third, a method of observing upon a close circumpolar near elongation with an eyepiece micrometer only, without using the horizontal circle.

185. For the first method the following program of observing may be used: Point upon the mark and read the horizontal circle, twice each; point approximately upon the star and place the striding level in position; perfect the pointing upon the star and note* the chronometer time of the bisection; read the horizontal circle; point again upon the star, noting the chronometer time; read the circle; read the striding level and reverse it; point upon the star and note the chronometer time; read the circle; read and remove the striding level; point upon the mark and read the circle, twice each. This completes a half-set. Reverse the telescope in altitude, and the instrument in azimuth, and repeat the same routine for a similar half-set. The whole set will thus be made up of six pointings upon the star and eight pointings upon the mark, with the corresponding circle readings, and four readings of each end of the bubble of the striding level. It will be found a convenience when making the computation if the altitude of the star is read to the nearest minute from the vertical circle once for each half-set. The special considerations that lead to the recommendation of the above routine are: that the effect of twisting of the instrument in azimuth should be eliminated from the result for each half-set as far as possible; that the level bubble should have time to settle without delaying the observer for that purpose; and that the observations of a set should be completed as quickly as possible to avoid the effects of instability of instrument. More pointings in a set would serve to decrease the effect of errors of observation, but tend to increase the errors due to instability.

To secure accurate results, the pointings upon the star, mark, and graduations must be carefully made; all heating of the instrument above the temperature of the outside air, and especially all unequal heating of its parts, must be avoided as far as possible; the manipulation must be made with as little applied force as possible, especially at the instants when

* The allowable error in time being comparatively large in azimuth work, the observer may simply call "Tip" at bisection and let an assistant read the chronometer.

bisections are being made. Other conditions being unchanged, the more rapidly the observations are made the greater the accuracy, because the errors due to instability are smaller.

186. Before commencing the next set of observations, the graduated circle should be shifted to another position so that each microscope will come over a different part of the graduation. To insure as complete an elimination of periodic errors of graduation as possible, it is advisable to shift the circle between the successive sets of a series so that the various positions of a given microscope, when pointing upon the mark in a given position of the telescope, shall divide the interval between successive microscopes (120° or 180°, usually) into as many equal parts as there are sets in the series.

187. EXAMPLE OF RECORD.

Station—West Base.
$\phi = 41° 29' 03''.5$
Date—Sept. 13 1877.
Star—δ Ursæ Minoris.
$\alpha = 18^h 11^m 47^s.5$
$\delta = 86° 36' 41''.0$

Observer—A. F. Y.
Instrument—T. & S. Altazimuth No. 72.
1 div. of striding level = 2''.12.
Chronometer—Negus 1431 (Sidereal).
Chronometer correction = + 4^s.5.

Object.	Position of Tel.	Level Readings.		Chronometer Times.	Horizontal Circle.							
		W.	E.		Index.	Mic. A.			Mic. B.		Mic. C.	
						t	*d*	*d*	*d*	*d*	*d*	*d*
Mark	D				142° 25′	3	19.7	19.0	20.5	20.0	21.7	20.5
"	"				142 25	3*	21.0	19.7	20.0	21.0	20.0	18.3
Star	"	*d*	*d*	0^h 08^m $00^s.0$	158 20	0	24.0	23.0	25.0	26.5	24.5	24.3
"	"	60.3	47.1	09 01 .0	158 20	0	26.0	25.5	27.5	28.2	27.1	26.7
"	"	41.5	65.6	09 50 .5	158 20	0	26.8	26.8	28.0	28.8	28.0	28.2
Mark	"				142 25	3	23.5	22.0	22.0	23.0	22.4	20.2
"	"				142 25	3	22.2	21.4	20.7	22.0	21.4	18.9
"	R				322 25	3	09.0	08.1	07.3	08.8	08.4	06.2
"	"				322 25	3	07.2	06.6	05.3	07.2	06.3	04.7
Star	"			0 20 03 .5	338 20	0	56.2	55.2	57.5	58.2	56.1	55.7
"	"	46.4	62.4	21 33 .5	338 20	1	07.2	06.6	08.3	09.2	07.3	07.4
"	"	62.8	46.0	22 46 .0	338 20	1	19.5	18.9	20.7	20.9	19.0	19.0
Mark	"				322 25	3	07.8	07.4	06.4	07.9	07.6	05.5
"	"				322 25	3	07.9	07.4	06.2	08.2	07.6	06.5

The mean zenith distance of the star during the observations, from two approximate readings of the vertical circle, was found to be 40° 52′.

* The reading of the whole turns, for the other micrometers, is not repeated in the record, but in making the readings it is called out to the recorder. If he finds it the same as for micrometer A, the record is as above. If it falls a unit below that for A, he indicates it by writing a minus sign *over* the recorded reading of the head; and if it is a unit above, he calls attention to it and records it as 60 + the given reading.

The Circle Reading.

188. In the record above, two readings are given for each position of each reading microscope. When commencing to read microscope *A* for the first time, for example, in the above set of observations, the field of view looked as shown in Fig. 25.* The reading was evidently 25′ plus the angle represented by the interval from the zero of the microscope (the position in which the micrometer lines are shown) to the 25′ graduation. This plus quantity is read directly from the micrometer when the 25′ graduation is bisected (called the forward reading), namely, 3′ 19″.7, *provided* there is no error of pointing, and *provided* each turn of the micrometer represents exactly 1′. But neither of these conditions are realized in practice, and a more reliable result may be secured if a reading is also made on the 30′ graduation (called the backward reading). With perfect pointing and perfect adjustment of the micrometer, the screw would necessarily be turned exactly five revolutions backward to pass from a pointing on the 25 line to a pointing on the 30 line, and the reading of the head of the micrometer would be the same in both cases. It is actually 0.7 less. Neglect for a moment all consideration of possible errors in pointing and reading. These two readings would then indicate that one turn of the micrometer represents† $\frac{5'}{5.012} = 59''.856$. Hence the measured interval of 3′ 19″.7 from the zero of the micrometer to the 25′ line represents $\left(3\frac{19.7}{60}\right)(59''.856) = 3'\ 19''.2$. This procedure does

* The field of view is here shown as it actually appears to the observer. The microscopes invert, and therefore the graduation really increases in the opposite direction from that here shown.

† 0.7 division = 0.012 turn. (60^d = 1 turn.)

not involve any assumption as to the exact value of one turn, but in fact derives the circle reading from each pair of micrometer readings upon the assumption merely that the graduated interval on the circle is exact. This process of making the correction for the run * of the micrometer may be put in convenient form for rapid computation for the above-described instrument as follows. The above figure and explanation will serve as a sufficient proof of the formulæ given.

189. Let F' be the forward reading of the micrometer, both comb and head, expressed in turns, this reading being taken on the line of the graduation which is adjacent to the zero of the micrometer in the direction of *increasing readings of the micrometer;* and let F be the corresponding reading of the micrometer head, expressed in turns. Let B be the backward reading of the micrometer head, expressed in turns, taken on that line of the graduation which is adjacent to the zero of the micrometer in the opposite direction. Let the true reading of the circle to be derived be called T. Let the interval between lines of the graduation be called I (5′ in the preceding illustration). Then one turn of micrometer $= \frac{I}{5 + F - B} = \frac{5'}{5 + F - B}$. Strictly, the required value of T is then $T = \frac{5'}{5 + F - B}(F')$. But remembering that $F - B$ is ordinarily only one or two sixtieths of a turn, and is therefore small as compared with the complete interval (5 turns), we may write $T = \frac{5' + B - F}{5}(F') = \left(1' + \frac{B - F}{5}\right)(F')$, in which the difference $B - F$ is now taken in divisions of

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* The *run* of a micrometer is the amount by which one turn exceeds, or falls short of, its nominal value,—0.7 in the above example. The *error of runs* is the error in the result which is introduced by neglecting the run of the micrometer.

the head, and considered to represent seconds. The nominal reading of the micrometer being (1′) (F'), the correction, C_r, to this nominal reading, or correction for run to be applied to the forward reading, is

$$C_r = \frac{B-F}{5}(F') = \frac{F'}{5}(B-F). \quad . \quad . \quad . \quad . \quad (88)$$

The values of C_r will be found tabulated in § 309 for the arguments F', the nominal forward reading, and $B-F$, expressed in divisions, or nominally in seconds. This table applies of course only to a reading microscope of the above type in which five turns are nominally equal to 5′, one space of the circle graduation, and each division of the head is nominally 1″. The table may be used to correct each forward reading of a series, or the mean value of C_r taken out from the table for each reading may be applied to the mean of the forward readings. A similar table may be constructed on the same principle for any other micrometer.

190. In developing the preceding method of computing the true reading of the circle, the accidental errors of pointing upon the graduation have been entirely ignored, it being tacitly assumed that they are small as compared with the error of runs. Let us now make the converse supposition—that the error of runs is small as compared with the error of pointing. On this supposition the forward and back readings of the head differ simply because of errors of pointing. Hence they are equally good determinations of the seconds of the reading, and their mean is to be taken. On this supposition the true reading, so far as the seconds are concerned, is

$$T = \frac{F+B}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

191. The use of (89) instead of (88), or the corresponding tables, leads to quite a saving of time. Two other considerations also point to such use as advisable. Firstly, the true result sought is in reality *between* the results given by these two methods, since errors of run and errors of pointing both exist, and in general neither are insensible as compared with the other. Secondly, $\frac{F'}{5}$ in (88) is as apt to be greater than $\frac{1}{2}$ as it is to be less than $\frac{1}{2}$. If $\frac{F'}{5}$ is $\frac{1}{2}$, the use of (88) gives numerically the same result as (89). Hence the results from the use of (88) are as apt to be greater as to be less than those from (89), and the greater the number of observations treated the nearer the results from the two formulæ agree. Hence, in general, there is not a sufficient gain in accuracy over the procedure indicated in (89) to justify the time required to correct for errors of run.*

The Level Correction.

192. Any inclination of the horizontal axis affects the circle reading corresponding to the pointing upon the star, and necessitates a correction which is to be determined from the readings of the striding level.

In Fig. 26 let *NESW* represent the horizon. Let *s* be the star, and *Z* the zenith. If the instrument is in perfect

* Sometimes the mean value for the run of a given micrometer is derived from a special series of observations for that purpose: the run is assumed to be constant; and a correction based upon this mean value is applied to the mean results computed by (89). This procedure shortens the work of applying the correction for run, *after* the mean value of the run has been computed. The validity of the assumption that the run is a constant is so doubtful, however, that it seems that if the correction for run is to be applied at all, it should be based upon a value for the run derived from the very readings that are to be corrected.

adjustment, when the telescope is pointed upon the star the plane in which the telescope is free to swing about its horizontal axis is defined by the arc ZsP, in which P is the pole of the great circle passing through Z, and A, the point in which the horizontal axis produced pierces the celestial sphere. If, now, the horizontal axis be given an inclination b, the west end being placed too high, A will move to a point A', at a distance b along Az. The zenith of the instrument will virtually be shifted to Z' (such that the arc $AZZ' = 90°$, and $ZZ' = b$), that being the nearest point to the true zenith to which the telescope can be pointed. The telescope is now free to swing in the arc $Z'P$. But this arc does not pass through s. To bisect the star it is necessary to turn the instrument about its vertical axis, which now passes through Z', until the telescope swings in the arc $Z's$. A' will then be in such a position as A''. The change in the circle reading, due to the inclination of the axis, is evidently measured by the angle $PZ's$. The circle, if graduated clockwise, now reads too small by that amount, which will be called C_L. Consider the spherical triangle $Z'Ps$. In this triangle the angle at Z' is the required C_L, the angle at P is b, the side Ps is the altitude of the star A, and the side $Z's$ is the zenith distance of the star as measured with the displaced instrument. But, the displacement being small, $Z's$ may be for the present purpose considered equal to Zs, the zenith distance, ζ, of the star. From the proportionality of the sines of angles and opposite sides in the triangle $Z'Ps$, we may write $\dfrac{\sin C_L}{\sin A} = \dfrac{\sin b}{\sin \zeta}$. Replacing $\sin C_L$ and $\sin b$ by C_L and b, those angles being small, and solving for C_L, there is obtained

$$C_L = b\frac{\sin A}{\sin \zeta} = b \tan A \quad . \quad . \quad . \quad . \quad (90)$$

Expressing the inclination b in terms of the readings of the striding level, there is obtained the complete formula for the level correction,

$$C_L = \{(w + w') - (e + e')\}\frac{d}{4} \tan A, \quad . \quad . \quad (91)$$

for a level having its divisions numbered both ways from the middle.

$$C_L = \{(w + e) - (w' + e')\}\frac{d}{4} \tan A \quad . \quad . \quad . \quad (92)$$

for a level numbered continuously in one direction, the primed letters referring to the readings taken in the position in which the numbering increases toward the east. C_L as given by these formulæ is the correction to the circle reading on the supposition that the numbers on the circle graduation increase in a clockwise direction.

Similar corrections to the circle readings upon the mark, derived from corresponding readings of the striding level, are necessary if the line of sight to the mark is much inclined. Ordinarily the line of sight to the mark is so nearly horizontal that such corrections are negligible, and the corresponding level readings may be dispensed with, provided that care is taken to keep the instrument well levelled up.

Azimuth of the Star.

193. The preceding formulæ suffice for the computation of the horizontal angle between the star and mark. It remains to compute the azimuth of the star.

The detail of the process of computing the hour-angle of the star from the chronometer reading need not be stated here. The hour-angle t and the declination δ of the star being known, as well as the latitude of the station ϕ, the azimuth of the star may be computed from the spherical triangle defined by the star, the zenith, and the pole. Certain sides and angles of this spherical triangle have the values indicated in Fig. 9, in terms of the angles, z the azimuth of the star reckoned from the north, A its altitude, and t, ϕ, and δ. From the principle that the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides, we may write

$$\frac{\sin z}{\cos \delta} = \frac{\sin t}{\cos A}. \quad \ldots \ldots \quad (93)$$

Also, from the principle that in any spherical triangle the cosine of any side is equal to the product of the cosines of the other two sides plus the product of their sines into the cosine of the opposite angle, we may write the two formulæ,

$$\sin \delta = \sin \phi \sin A + \cos \phi \cos A \cos z; \quad . \quad (94)$$

$$\sin A = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \quad . \quad . \quad (95)$$

By substituting sin A from (95) in the first term of the second member of (94), and solving the resulting equation for cos Z, there is obtained

$$\cos z = \frac{(1 - \sin^2 \phi) \sin \delta - \sin \phi \cos \phi \cos \delta \cos t}{\cos \phi \cos A}. \quad (96)$$

By substituting $\cos^2 \phi$ for $1 - \sin^2 \phi$, and dividing both numerator and denominator by cos ϕ, (96) reduces to

$$\cos z = \frac{\cos \phi \sin \delta - \sin \phi \cos \delta \cos t}{\cos A}. \quad . \quad . \quad (97)$$

If (93) is now divided by (97), and both denominators of the resulting equation are divided by cos δ, there results

$$\tan z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}, \quad . \quad . \quad (98)$$

from which z may be computed from the known values of ϕ, δ, and t. In (98) z is the azimuth of the star east or west, respectively, of north, as the hour-angle, is reckoned from upper culmination to the eastward or westward, respectively.

The Curvature Correction.

194. To apply this formula to the star for each pointing made upon the star during a set of observations would be too laborious a process. If for the t of the formula is taken the mean of the hour-angles of the set, the computed azimuth is that corresponding *to the mean hour-angle*, but is not the required *mean of the azimuths corresponding to the separate hour-angles*, since the rate of change of the azimuth is continually varying. The difference between the two quantities indicated by the italics is small, though not usually negligible, for the interval of time covered by a set of observations. It is proposed to derive a sufficiently precise expression for this difference, and to apply it as a correction to the result obtained by using a mean value of t in formula (98).

195. Let t_1, t_2, t_3, . . . t_n be the observed hour-angles, and z_1, z_2, z_3, . . . z_n the respective corresponding azimuths. Let t_0 be the mean of the observed hour-angles, $= \frac{t_1 + t_2 + t_3 \ldots t_n}{n}$, and let z_0 be the azimuth corresponding to t_0. Let $\Delta t_1 = t_1 - t_0$, $\Delta t_2 = t_2 - t_0$, . . . $\Delta t_n = t_n - t_0$. Then

$$\Delta t_1 + \Delta t_2 + \Delta t_3 \ldots \Delta t_n = 0. \quad . \quad . \quad . \quad (99)$$

If the third and following derivatives of z with respect to t be neglected, we may write

$$\left.\begin{aligned} z_1 &= z_0 + \frac{dz}{dt}\Delta t_1 + \frac{d^2z}{dt^2}\tfrac{1}{2}\overline{\Delta t_1}^2; \\ z_2 &= z_0 + \frac{dz}{dt}\Delta t_2 + \frac{d^2z}{dt^2}\tfrac{1}{2}\overline{\Delta t_2}^2; \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ z_n &= z_0 + \frac{dz}{dt}\Delta t_n + \frac{d^2z}{dt^2}\tfrac{1}{2}\overline{\Delta t_n}^2; \end{aligned}\right\} \quad . \quad . \quad . \quad (100)$$

in which $\frac{dz}{dt}$ and $\frac{d^2z}{dt^2}$ are the first and second derivatives corresponding to z_0 and t_0. The mean value of z is

$$\frac{z_1 + z_2 + z_3 \ldots z_n}{n} = z_0 + \frac{d^2z}{dt^2}\frac{\tfrac{1}{2}\overline{\Delta t_1}^2 + \tfrac{1}{2}\overline{\Delta t_2}^2 + \tfrac{1}{2}\overline{\Delta t_3}^2 \ldots \tfrac{1}{2}\overline{\Delta t_n}^2}{n}. \quad (101)$$

The terms of the form $\frac{dz}{dt}\Delta t$ disappear by virtue of the relation expressed by (99).

196. An approximate value of $\frac{d^2z}{dt^2}$ may be derived from equation (98) as follows: Since only close circumpolar stars are being considered, z is always small, and as an approximation we may write z (in arc measure) for $\tan z$. Also, since for the stars under consideration δ is nearly 90°, $\cos\phi \tan\delta$ is much larger than $\sin\phi \cos t$, and the quantity $\frac{1}{\cos\phi\tan\delta - \sin\phi\cos t}$ may be assumed constant for the interval of a few minutes over which the observations of a set extend, and will be called C. Equation (98) becomes after these substitutions $z = C \sin t$, whence $\frac{dz}{dt} = C \cos t$, and

$$\frac{d^2z}{dt^2} = -C \sin t = -\frac{\sin t}{\cos\phi \tan\delta - \sin\phi \cos t} = -\tan z.$$

For substitution in (101) this must be written $\frac{d^2z}{dt^2} = -\tan z_0$, according to the statement following equations (100). Equation (101) then becomes

$$\frac{z_1 + z_2 + z_3 \ldots z_n}{n} = z_0 - \tan z_0 \frac{\tfrac{1}{2}\overline{\Delta t_1}^2 + \tfrac{1}{2}\overline{\Delta t_2}^2 + \tfrac{1}{2}\overline{\Delta t_3}^2 \ldots \tfrac{1}{2}\overline{\Delta t_n}^2}{n}. \quad (102)$$

197. In deriving the expression $-\tan z_0$ for $\frac{d^2z}{dt^2}$, z was considered to be in arc measure. Therefore, putting Δt_1, Δt_2, Δt_3, . . . Δt_n also in arc in that term, and then dividing the whole term by sin 1″ to reduce to seconds, it becomes

$$-\tan z_0 \frac{\tfrac{1}{2}\overline{\Delta t_1}^2 + \tfrac{1}{2}\overline{\Delta t_2}^2 + \tfrac{1}{2}\overline{\Delta t_3}^2 \ldots \tfrac{1}{2}\overline{\Delta t_n}^2}{n \sin 1''}.$$

Finally, to facilitate the computation by enabling the computer to use the table given in § 307, we may write Δt (in arc) $= 2 \sin \tfrac{1}{2}\Delta t$, or $\tfrac{1}{2}\overline{\Delta t}^2 = 2 \sin^2 \tfrac{1}{2}\Delta t$.

Equation (102) may then be written

$$\frac{z_1+z_2+z_3 \ldots z_n}{n} = z_0 - \tan z_0 \frac{1}{n}\left(\frac{2\sin^2\tfrac{1}{2}\Delta t_1}{\sin 1''} + \frac{2\sin^2\tfrac{1}{2}\Delta t_2}{\sin 1''} + \ldots \frac{2\sin^2\tfrac{1}{2}\Delta t_n}{\sin 1''}\right). \quad (103)$$

in which the values of the terms $\frac{2 \sin^2 \frac{1}{2}\Delta t_1}{\sin 1''}$, etc., are known from the table given in § 307. Certain approximations have been made in deriving the last term in (103),* but the total value of that term is so small that the errors due to these approximations are negligible.

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* For a slightly different method of deriving the same formula, see Doolittle's Practical Astronomy, pp. 537, 538.

Correction for Diurnal Aberration.

198. To the results as computed by the above formulæ there must still be applied a small correction for the effect of diurnal aberration. Because of the rapid motion of the observer due to the rotation of the Earth on its axis, the star is seen slightly displaced from its real position, and the apparent azimuth of the star is correspondingly affected.

Suppose that at the instant of observation the station from which the observation is made is moving in the direction AB of Fig. 27. AB necessarily passes through the east point, on the celestial sphere, of the observer's horizon at that instant. Let SA be the true direction of the ray of light from the star. The figure is drawn, then, in the plane defined by the star, the observer, and the east point of the observer's horizon. In consequence of the aberration the star will be seen in the direction AS'. Let CA and AD be drawn proportional to the distance V traversed by a ray of light in one second, and the distance v traversed by the station of observation in one second, respectively. Complete the parallelogram $CADG$. Call the angle SAB β, and the angle $CAG = SAS'$ $d\beta$. $d\beta$ is the apparent displacement of the star measured in the plane of the figure. From the triangle CAG, $\frac{\sin d\beta}{\sin \beta} = \frac{v}{V}$, or

$d\beta = \sin^{-1} \frac{v}{V} \sin \beta.$

Substituting for v, 0.288 cos ϕ, and for V 186 000 (see § 96), there is obtained

$$d\beta = 0''.319 \cos \phi \sin \beta. \quad . \quad . \quad . \quad . \quad . \qquad (104)$$

199. It remains to determine the effect of this displacement upon the star's azimuth. In Fig. 28 let $NESW$ be the horizon of the observer, Z his zenith, and s the star. Prolong the great arc Zs to F. In the spherical triangle sFE the angle at F is 90°; the side sF is the altitude of the star A, the side FE is $90° - z$ (z is the azimuth of the star), and the side sE is the β of the preceding paragraph. Call the angle sEF v. Then from Napier's rules for a right spherical triangle we may write

$$\cos A \sin FsE = \cos v; \quad . \quad . \quad . \quad . \quad (105)$$

$$\sin FsE \sin \beta = \cos z; \quad . \quad . \quad . \quad . \quad (106)$$

$$\cos A \sin z = \cos \beta. \quad . \quad . \quad . \quad . \quad . \quad (107)$$

Substituting the value of sin FsE from (106) in (105), there is obtained

$$\cos A \cos z = \sin \beta \cos v. \quad . \quad . \quad . \quad . \quad (108)$$

We require the effect upon z of a small change in β. This may be found by differentiating (107) and (108) with respect to A, z, and β (but not with respect to v, since that is not changed by the change in β). Thus we find

$$\cos A \cos z dz - \sin A \sin z dA = - \sin \beta d\beta; \quad (109)$$

$$- \cos A \sin z dz - \sin A \cos z dA = \cos \beta \cos v d\beta. \quad (110)$$

Multiply (109) by cos z and (110) by sin z, and subtract the second of the resulting equations from the first, and there is obtained

$$\cos A dz = - \cos z \sin \beta d\beta - \sin z \cos \beta \cos v d\beta. \quad (111)$$

Solve for dz, multiply both numerator and denominator of the resulting fraction by sin β, and eliminate cos v by substituting its value from (108), and the result is

$$dz = - \frac{\cos z}{\sin \beta \cos A} d\beta. \quad . \quad . \quad . \quad . \quad (112)$$

Substituting the value of $d\beta$ from (104), we have, finally,

$$dz = -0''.319 \frac{\cos\phi \cos z}{\cos A}. \quad . \quad . \quad . \quad (113)$$

For a close circumpolar $\frac{\cos\phi}{\cos A}$ will always be nearly unity, and so will be $\cos z$, except for stations very near the pole. Hence for such stars it is nearly exact to use

$$dz = -0''.32. \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

The greatest variation from this value for the four circumpolars mentioned in § 183, and for stations below latitude 50°, is 0''.02. Hence for most purposes it suffices to use (114), that is, to dispense with the computation of the factor $\frac{\cos\phi \cos z}{\cos A}$ of (113). That will be done in the examples of this book.

The sign in (113) and (114) is for the correction when the azimuth of the star is expressed as an angle west of north. If the azimuth of the star is expressed as an angle east of north, the sign must be changed to +.

Example of Computation.

200. The computation of the set of observations given in § 187 is as follows. No correction was applied for run of micrometers, the true reading of the circle being derived by (89).

For First Half-set.

Mean circle reading on star	=	158°	20′	26″.38
Level correction = (101.8 − 112.7)(0.530)(0.865)	=			− 5 .00
Corrected mean reading on star	=	158	20	21 .38
Mean reading on mark	=	142	28	20 .88
Mark west of star	=	15	52	00 .50

For Second Half-set.

Mean circle reading on star	$= 338° \ 21' \ 07''.94$
Level correction = (109.2 − 108.4)(0.530)(0.865)	$= \quad + 0 \ .36$
Corrected mean reading	$= 338 \ 21 \ 08 \ .30$
Mean reading on mark	$= 322 \ 28 \ 07 \ .15$
Mark west of star	$= 15 \ 53 \ 01 \ .15$
Mark west of star from whole set (mean)	$= 15 \ 52 \ 30 \ .82$
Mean of observed chronometer times	$= 0^h \ 15^m \ 12^s.4$
Chronometer correction	$= \quad + 4 .5$
Mean of sidereal times	$= 0 \ 15 \ 16 .9$
α	$= 18 \ 11 \ 47 .5$
Mean hour-angle, t_0, west from upper culmination	$= \begin{cases} 6^h \ 03^m \ 29^s.4 \\ 90° \ 52' \ 21''.0 \end{cases}$

$\log \cos \phi = 9.8745614$ $\quad \log \sin \phi = 9.8211300$ $\quad \log \sin t = 9.9999497$

$\log \tan \delta = 1.2275941$ $\quad \log \cos t_0 = 8.1826260n$ $\quad \log 12.66198 = 1.1025016$

$$\begin{cases} 1.1021555 \\ + 12.65189 \\ + 0.01009 \end{cases} \qquad \begin{cases} 8.0037560n \\ - 0.01009 \end{cases} \qquad \log \tan z = 8.8974481$$

$$12.66198 \qquad\qquad z = 4° \ 30' \ 54''.47$$

$$- \tan z_0 \frac{1}{n} \left(\frac{2 \sin^2 \frac{1}{2} \Delta t_1}{\sin 1''} + \ldots \right) = \quad - 6 \ .20$$

Correction for diurnal aberration	$= \quad - 0 \ .32$
Azimuth of star, west of north	$= 4 \ 30 \ 47 \ .95$
Mark west of star	$= 15 \ 52 \ 30 \ .82$
Azimuth of mark, west of north	$= 20 \ 23 \ 18 \ .77$

Δt	$\frac{2 \sin^2 \frac{1}{2} \Delta t}{\sin 1''}$
$7^m \ 12^s.4$	$101''.97$
6 11 .4	75 .22
5 21 .9	56 .51
4 51 .1	46 .21
6 21 .1	79 .20
7 33 .6	112 .21
	6)471 .32

Mean = $78''.55$

$\log 78.55 = 1.89515$

$\log \tan z = 8.89745$

Curvature correction $\begin{cases} 0.79260 \\ 6''.20 \end{cases}$

Program of Observing for the Method of Repetitions.

201. To measure a horizontal angle by repetitions one must use an instrument having a clamp and tangent screw to control the motion of the lower or graduated circle, in addition to a similar clamp and tangent screw to control the relation between the upper circle carrying the verniers and the lower graduated circle. At the beginning of the measurement the circle is read. By a suitable manipulation of the two motions, upper and lower, the angle to be measured is multiplied mechanically three, five, or more times. The circle being read again gives the measured value of the multiple angle, from which the required angle is readily derived. This process serves to greatly decrease the errors arising from erroneous readings of the verniers, and errors of graduation. But any lost motion, or false motion, in clamps and tangent screws, affects the measured angle directly. To eliminate this error as far as possible one may measure both the required angle and its explement,* always revolving the instrument in a clockwise direction with *either* motion loose, and making all pointings with either tangent screw so that the last motion of that screw is in the direction in which the opposing spring is being compressed. With this procedure, unless the action of the clamps and tangent screws is variable, the derived values of both the angle and its explement will be too large or too small by the same amount. The mean of the measured angle and 360° minus the measured explement will be the correct value of the angle unaffected by the constant errors of the clamps and tangent screws.

202. The following is a convenient program for the measurement of an azimuth by repetitions. After all adjustments have been made and the instrument carefully levelled,

* 360° minus a given angle is called the *explement* of that angle.

clamp the upper circle to the lower in any arbitrary position; point approximately upon the star; place the striding level in position and read it; reverse it, read it again, and remove it; point accurately upon the star, noting the chronometer time, and using the lower clamp and tangent only; read the horizontal circle; unclamp the upper motion and point upon the mark using the upper clamp and tangent screw; unclamp the lower motion and point upon the star, using the lower clamp and tangent screw and taking care to note the chronometer time of bisection; loosen the upper motion and point again upon the mark, using the upper clamp and tangent screws; take another pointing upon the star with the lower motion, noting the time; point again upon the mark, using the upper motion; read the horizontal circle. This completes the observations of a half-set if three repetitions are to be made. In passing from the star to the mark, and *vice versa*, the instrument should always be rotated in a clockwise direction, and the precaution stated in the preceding paragraph as to the use of the tangent screws must be kept in mind. Before commencing the second half-set the lower motion should be unclamped, the telescope reversed in altitude, and the instrument reversed 180° in azimuth. The program will be as for the first half-set, except that now the first pointing is to be upon the mark; *all* pointings on the mark are to be made with the *lower* clamp and tangent screw, and upon the star with the *upper* clamp and tangent screw; and the striding level is to be read just after the *last* pointing upon the star. The direction of motion of the instrument must always be clockwise as before, and the tangent screws must be used as before.

203. EXAMPLE OF RECORD; METHOD OF REPETITIONS.

Station—Dollar Point, Texas.
$\phi = 29^\circ\ 26'\ 02''.6$.
Date—April 5, 1848.
Star—Polaris.
$\alpha = 1^h\ 04^m\ 04^s.7$.
$\delta = 88^\circ\ 29'\ 57''.82$.

Observer—A. F. Y.
Instrument—Gambey Theodolite.
1 div. of striding level = 3''.68.
Chronometer—Hardy No. 50 (Sidereal).
Chronometer correction = $-1^s.8$.

Object.	Pos. of Tel.	Level Readings.		No. of Repetitions.	Chronometer Times.	Circle Readings.	
		W.	E.			Vernier A.	Vernier B.
Star.	D.	129.0 81.0	71.5 119.0	3	$9^h\ 03^m\ 33^s.5$ 04 47 .5 06 07 .5	91° 10′ 30″	271° 10′ 40″
Mark.	D.					128 14 50	308 14 50
Mark.	R.			3		128 14 50	308 14 50
Star.	R.	121.5 80.0	79.0 120.0		9 08 06 .5 09 24 .5 10 23 .5	91 13 40	271 13 50

204. No reading of the altitude was taken. The altitude may be derived with sufficient accuracy for use in computing the level corrections from the table in § 310. The level correction must here be applied to the angle between the star and mark, not directly to the circle reading. Formulæ (91) and (92) will not give the sign of the level correction; that must be derived from the consideration that the star appears to be farther west than it really is if the west end of the horizontal axis is too high, and *vice versa*.

The angle between the star and mark, computed from the first half-set, is

$$(128^\circ\ 14'\ 50'' - 91^\circ\ 10'\ 35'')\tfrac{1}{3} = 12^\circ\ 21'\ 25''.0,$$

and from the second half-set is

$$(128^\circ\ 14'\ 50'' - 91^\circ\ 13'\ 45'')\tfrac{1}{3} = 12^\circ\ 20'\ 21''.7.*$$

* Evidently this method of computing the second value of the angle necessarily always gives the same numerical result as first computing the explement and then subtracting from 360°.

The remainder of the computation may be made as indicated in § 200.

Directions for Observing Azimuth with a Micrometer.

205. If the instrument is provided with a good eyepiece micrometer measuring angles in the plane defined by the telescope and its horizontal axis, the most accurate as well as the most rapid way of determining azimuth with it is to place the azimuth mark nearly in the vertical plane of a close circumpolar star at elongation, and then to measure the horizontal angle between the star and mark with the micrometer, independently of the graduated horizontal circle of the instrument.

206. To place the azimuth mark with sufficient accuracy in the required position one may take a single pointing upon Polaris on the first night after the station is ready for observations, noting the sidereal time and the reading of the horizontal circle. The instrument may then be left standing, *with the lower motion clamped*, until the next day. During the next day the instrument may be set to such a reading of the horizontal circle computed roughly from the observations of the night before, by the table of § 310, or by formula (98), as would place the telescope in the vertical plane of the star about 30^m before or after the elongation at which the observations are to be made. An assistant may then, by previously arranged signals, be aligned at the proposed site of the azimuth mark so as to place it in the direction defined by the telescope. The "alignment" of the mark may be made at night, as soon as the pointing is made upon Polaris, instead of waiting until the next day, if necessary; but it is usually easier to pick out a good location for the mark and to transmit signals from the station to the mark in daylight than at night. The mark may be placed either to the northward or to the southward of the station.

207. The adjustments of the vertical axis, of the levels, of focus (see end of § 216), and for bringing the movable micrometer line into a vertical plane, must be made as indicated in § 180.

208. The following is a good program for the observations. Place the micrometer line at such a reading that it is nearly in the line of collimation of the telescope. If this reading is not already known, it may be determined by taking the mean of two readings upon the mark with the micrometer, the instrument being rigidly clamped in azimuth, and the horizontal axis of the telescope reversed in its Ys between the two readings. Clamp the lower circle, and point upon the mark by use of the upper clamp and tangent-screw. Then, with the instrument clamped rigidly in azimuth, take five pointings with the micrometer upon the mark, direct the telescope to the star; place the striding level in position; take three pointings upon the star with the micrometer, noting the chronometer time of each; read and reverse the striding level; take two more pointings upon the star, noting the times; read the striding level. This completes a half-set. Reverse the horizontal axis of the telescope in its Ys; point approximately to the star; place striding level in position; take three pointings upon the star, noting the chronometer times; read and reverse the striding level; take two more pointings upon the star, noting the times; read the striding level; and finally, make five pointings upon the mark.

Such a set of observations may be made very quickly; the effect of a uniform twisting of the instrument in azimuth is eliminated from the result; and the bubble of the striding level has plenty of time to settle without delaying the observer for that purpose.

209. With the instrument used for the following observations, increased readings of the micrometer correspond to a movement of the line of sight toward the east when the vertical circle is to the east, and toward the west if the vertical circle is to the west.

210. EXAMPLE OF RECORD AND COMPUTATION.

Station No. 10.
$\phi = 31° 19' 35''.0$.
Date—October 13, 1892.
Star—Polaris, near eastern elongation.
One turn* of micrometer = 123″.73.

Observer—J. F. H.
Instrument—Fauth Theodolite No. 725.
One division of striding level = 3″.68.
Chronometer—Negus No. 1716 (Sidereal).
Chronometer corr. = $-2^h 11^m 28^s.2$.

Cir. E. or W.	Level Readings.		Chronometer Time.	Δt	$\frac{2 \sin^2 \frac{1}{2}\Delta t}{\sin 1''}$	Micrometer Readings.		
	W.	E.				On Star.	On Mark.	
	d.	*d.*				*t.*	*t.*	
E.	8.0	9.9	$9^h 06^m 38^s.0$	$3^m 58^s.6$	31.05	18.379	18.310	Longitude
	10.0	7.3	07 32 .0	3 04 .6	18.59	.388	.315	$2^h 12^m$ west
			08 05 .5	2 31 .1	12.45	.400	.315	of Washington.
	+ 18.0	− 17.2	09 13 .0	1 23 .6	3.82	.424	.311	
E.	+ 0.8		09 48 .0	0 48 .6	1.29	.430	.316	
						18.4042	18.3134	Means.
W.	9.0	9.0	9 12 01 .8	1 25 .2	3.96	18.100	18.290	
	7.0	10.9	12 24 .7	1 48 .1	6.37	.100	.275	
			12 48 .3	2 11 .7	9.46	.090	.279	
	+ 16.0	− 19.9	13 36 .3	2 59 .7	17.61	.086	.281	
W.	− 3.9		13 58 .1	3 21 .5	22.14	.080	.279	
	Mean =	− 1.55	9 10 36 .6		12.67	18.0912	18.2808	Means.

α of Polaris = $1^h 20^m 07^s.4$ }
δ of Polaris = $88° 44' 10''.4$ } at the time of observation.

Altitude of star at the mean epoch of the observations, per section 308, = 31° 13′
Altitude of star at the middle of the first half-set, per section 308, = 31 12
Altitude of star at the middle of the second half-set, per section 308, = 31 14
Collimation reads, $\frac{1}{2}(18.3134 + 18.2808) = 18^t.2971$

---

* The head of this micrometer was graduated to 100 equal parts. For the meaning of an increased reading see § 209.

Mark east of collimation, 18.3134 − 18.2971	= 0 .0163	=	02″.02
Circle E., star E. of collimation, (18.4042 − 18.2971) ÷ 0.8554 *	= 0 .1252		
Circle W., star E. of collimation, (18.2971 − 18.0912) ÷ 0.8551 †	= 0 .2408		
Mean, star E. of collimation	= 0 .1835	=	22 .70
Mark west of star		=	20 .68
Level correction, (1.55)(0.92)(0.606)		=	− 0 .86
Mark west of star, corrected		=	19 .82

Mean chronometer time of observation	=	21^h	10^m	$36^s.6$
Chronometer correction	= −	2	11	28 .2
Mean sidereal time of observation	=	18	59	08 .4
α	=	1	20	07 .4
Hour-angle (= t) east of upper culmination = 95° 14′ 45″.0	=	6	20	59 .0

$\log\cos t = 9.9315695$ $\quad\log\sin\phi = 9.71593$ $\quad\log\sin t = 9.9981771$

$\log\tan\delta = 1.6563815$ $\quad\log\cos t = 8.96108n$ $\quad\log 38.76893 = 1.5884838$

$\begin{cases} 1.5879510 \\ 38.72140 \end{cases}$ $\quad\begin{cases} 8.67701n \\ -0.04753 \end{cases}$ $\quad\log\tan z_0 = 8.4096933$

$+ 0.04753$ $\quad z_0 = 1°\ 28'\ 16''.92$

38.76893

$$-\tan z_0\,\frac{1}{n}\left(\frac{2\sin^2\frac{1}{2}\Delta t_1}{\sin 1''} + \text{etc.}\right) = -0''.33$$

Correction for diurnal aberration = $+0''.32$

Star east of north = z = 1° 28′ 16″.91

Mark west of star from above = 19 .82

Mark east of north = 1 27 57 .09

$$\log\frac{1}{n}\left(\frac{2\sin^2\frac{1}{2}\Delta t_1}{\sin 1''} + \text{etc.}\right)$$

$= \log 12.67 \doteq 1.10278$

$\log\tan z_0 = 8.40969$

$\begin{cases} 9.51247 \\ 0''.33 \end{cases}$

211. Here again the sign of the level correction as applied to the angle between the star and mark must be derived directly from the fact that the star appears to be farther west

* 0.8554 = cosine 31° 12′ (natural).

† 0.8551 = cosine 31° 14′ (natural).

than it really is if the west end of the axis is too high, and *vice versa*.

212. The micrometer measures angles *in the plane defined by the telescope and its horizontal axis*. In Fig. 29 let Z be the zenith, s the star at the instant when a pointing is made upon it with the micrometer, and Zn the vertical circle described by the line of collimation of the telescope. Then sm, a great circle through s perpendicular to Zn, is the arc measured directly with the micrometer. In the right spherical triangle smZ the side Zs is $90° - A$, the complement of the altitude of the star, and from Napier's rules

$$\sin sm = \sin mZs \cos A. \quad . \quad . \quad . \quad . \quad (115)$$

Or, writing the angles for the sines of sm and mZs, and solving for mZs,

$$mZs = sm \sec A. \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

mZs, the angle at the zenith, is the required angle between the vertical plane through the star and the vertical plane described by the line of collimation.

The computation form shows how this factor, sec A, is most conveniently applied. To be absolutely exact, this factor should be applied to every pointing upon the star. But the computation as given is abundantly accurate, the factor being applied to the mean angle between the line of collimation and the star for each half-set. In fact, the computation will often be sufficiently exact, if the factor is applied to the mean value of this angle for the set.

213. The use of the table given in § 308 is the most convenient way of securing the required values of the altitude of the star, unless they are read approximately from the vertical circle during the observations. First compute the mean hour-angle of the star and take out the corresponding altitude for use in deriving the level correction, then the other two angles may be derived by interpolating over the interval to the middle of each half-set with the rate of change of altitude taken from § 308. The altitude need only be known within one minute, ordinarily.

For any other star than Polaris, the table of § 308 not being available, one must either read the required altitude from the vertical circle of the instrument, if it has one, or else resort to the computation upon which the table of § 308 is founded.

The use of the factor sec A is not necessary with the pointings upon the mark, both because the line of collimation was purposely placed nearly upon the mark, and because sec A is very nearly unity for the small altitude of the mark.

214. Inspection will show that there is nothing in this method of observing or computing which limits its use to the time near elongation. The micrometer may be used in this way and the azimuth computed as above with the star at any hour-angle, even at culmination. But if the star is not near elongation, its motion in azimuth is more rapid, it remains near the vertical plane of the mark a shorter time, and larger angles must be measured with the micrometer or else the series of observations made shorter. Errors in the time also have less effect the nearer the star is to elongation.

If the azimuth mark is placed to the *southward* of the station, the program of observing and the computation are not materially modified.

Micrometer Value.

215. To determine the value of one turn of the micrometer the observer may use a process similar to that used in determining the value of the zenith telescope micrometer.

That is, one may observe the times of transit of a close circumpolar star near culmination across the micrometer line set at successive positions one turn apart (or one-half a turn), the instrument being rigidly clamped in azimuth. The corrections for curvature may be made by use of the same table, § 306, as for the zenith telescope, but now using in the place of τ the hour-angle of the star reckoned from the nearest culmination, and making the corrections to the observed times positive before culmination and negative after. The striding level may be read during the observations and a corresponding correction applied.

The correction in seconds of time to be applied to each observed time to reduce it to what it would have been with the axis level is

$$\{(w + w') - (e + e')\}\frac{d}{4} \,.\, \sin A \,.\, \frac{\sec \delta}{15}$$
$$= \{(w + w)' - (e + e')\}\frac{d \,.\, \sin A \,.\, \sec \delta}{60}, \quad (117)$$

for a level with a graduation numbered both ways from the middle. The observer must depend upon his instrument to remain fixed in azimuth,—unless, fortunately, he has an azimuth mark so nearly in the meridian that he can occasionally take a pointing upon it, without unclamping the horizontal circle, during the progress of the observations, and so determine the twist of the instrument.

216. Another convenient way of determining the micrometer value, without doing any work at night, is to measure a small horizontal angle at the instrument between two terrestrial objects, both with the horizontal circle and the micrometer. If the two objects pointed upon are much above or below the instrument, the measured angle between them may be reduced to the horizon for comparison with the circle measurement by use of the factor sec A, as indicated in § 212.

As the micrometer value is depended upon to remain constant for the station the focus must be left undisturbed if possible after the micrometer value has been determined.

Discussion of Errors.

217. The *external errors* are those due to errors in the right ascension and declination of the star observed, to lateral refraction of the rays of light from the star or mark to the instrument, and to error in the assumed latitude of the station of observation.

218. Errors of declination enter the computed azimuth with full value when the star is observed at elongation, and errors of right ascension enter with a maximum effect when it is observed at culmination. At intermediate positions both errors enter the computed result with partial values. The errors arising from this source are usually small as compared with the errors of observation, but are nearly constant if all the observations at a station are taken with the star at about the same position in its diurnal path, say near eastern elongation. They may be eliminated to a considerable extent by observing the same star at various positions of its diurnal path, or by observing upon two or more different stars.

219. When the computed results of a long series of accurate azimuth observations at a station are inspected it is usually found that they tend to group themselves by nights. That is, the results for any one night agree better with each other than do the results on different nights. They thus appear to indicate that some source of error exists which is constant during each night's observations, but changes from night to night. For example, from 144 sets of micrometric observations of azimuth, made on 36 different nights, at 15 stations on the Mexican Boundary in 1892–93, it was found that the error peculiar to each night was represented by the probable error $\pm 0''.38$, and the probable error of the result from a single set exclusive of this error was $\pm 0''.54$. In other words, in this series of observations, the error peculiar to each night, which could not have been eliminated by increasing the number of observations, was two-thirds as large on an average as the error of observation in the result from a single set.

The most plausible explanation seems to be that there is lateral refraction between the mark and the instrument, and that this lateral refraction is dependent on the peculiar atmospheric conditions of each night. But whether that explanation be true or not, the fact remains that an increase of accuracy in an azimuth determination at a given station may be attained much more readily by increasing the number of nights of observation than by increasing the number of sets on each night.

220. The accuracy with which the latitude must be known when observing upon Polaris may be inferred from an inspection of the table of § 310. It must be known with greater accuracy when a star farther from the pole is used.

221. The *observer's errors* are his errors of pointing upon the mark and star, errors of pointing upon the circle graduation if reading microscopes are used, errors of vernier reading if verniers are used, errors of reading the micrometer heads, errors in reading the striding level, and errors in estimating the times of bisection.

There is such a large range of difference in the designs of the various instruments used for azimuth work that little can be stated in regard to the relative and absolute magnitude of these different errors that will be of general application. Each observer may investigate these various errors for himself with his own instrument. In designing instruments the attempt is often made to so fix the relative power of the telescope and means of reading the horizontal circle that the errors arising from telescope pointings and circle readings shall be of the same order of magnitude.

The effect of errors in time may be estimated by noting the rate at which the azimuth of the star was changing at the time it was being observed. The table of § 310 will serve this purpose for Polaris. Such errors are usually small, but not insensible except near elongation.

222. Of the relative magnitude of the *instrumental errors* arising from imperfect adjustment and imperfect construction little of general application can be said, because of the great variety of instruments used. With the more powerful instruments, however, it may be stated that the errors due to instability of the instrument become relatively great, and must be guarded against by careful manipulation and rapid observing.

The errors due to the striding level become more serious the farther north is the station (see formula (91)). If the level is not a good one, it may be advisable to take more level readings than have been suggested in the preceding programs of observation. With a very poor level, or at a station in a high latitude, it may be well to avoid placing any dependence upon the level by taking half of the observations upon the star's image reflected from the free surface of mercury (an artificial horizon). The effect of inclination of the axis upon the circle reading will be the negative for the reflected star of what it is for the star seen directly. Considerable care will be necessary to protect the mercury from wind and from tremors transmitted to it through its support.

223. The micrometric method treated in §§ 205–214 gives a higher degree of accuracy than the other methods described, if a good micrometer is available. It avoids several sources of error, and the observations may be made so rapidly that the conditions are quite favorable for the elimination of errors due to instability. The error, in the final result for a station, due to an error in the value of the micrometer screw may be made as small as desired by so placing the azimuth mark and so timing the observations that the sum of the angles measured eastward from the mark to the star shall be nearly equal to the sum of the angles measured westward from the mark to the star.

Other Instruments and Methods.

224. An astronomical transit furnished with an eyepiece micrometer is especially well adapted to give results of a high degree of accuracy in determining azimuths by the micrometric method.

225. If the transit has no micrometer, a secondary azimuth may be determined incidentally to time observations with little extra expenditure of time. Put an azimuth mark as nearly as possible in the meridian of the transit. At the beginning of each half-set of the time observations point upon the mark with the middle line of the reticle. If the mark is nearly in the horizon of the instrument, the collimation and azimuth errors of the transit as derived from each half-set, reduced to arc and combined by addition and subtraction with each other and with the equatorial interval of the middle line, give the azimuth of the mark. The azimuth of a certain mark was so determined from the time observations required for a determination of the longitude of a station at Anchorage Point, Chilkat Inlet, Alaska, in 1894. The computed azimuth of the mark from 38 nights of observation varied through a range of $12''.8$. The probable error of a single determination was $\pm 2''.1$.

226. The transit may also be made to furnish a good determination of azimuth by observations in the vertical of Polaris by the method already referred to in § 129.

227. If the allowable error of a given azimuth determination exceeds 2″, a convenient method is to observe upon Polaris at any hour-angle and use the table given in § 310 to compute its azimuth at the time of each observation. The tabulated values * were computed by formula (98). The only correction to be applied to the value as taken from the table is that due to the difference between the apparent declination of Polaris at the time of observation and the value 88° 46′ with which the table was computed. This may be computed by use of the columns, given at the right-hand side of the table, headed "Correction for 1′ increase in declination of Polaris," by assuming that the correction is proportional to the increase, and must be changed in sign if the declination is less than 88° 46′. The table may also be used as a convenient rough check on computations made by formula (98).

If a star which is not a close circumpolar, or the Sun, is observed for azimuth at a known hour-angle, its azimuth may be computed by formula (98) for each observation, or the observations may be treated in groups covering short intervals of time. But formula (103) will not apply, since certain approximations were made in its derivation which are only allowable when δ is nearly 90°.

228. For rough determinations of azimuth in daylight, say within 30″, when the time is only approximately known, the Sun may be observed with a small theodolite, or with an engineer's transit, as follows: Point upon the mark and read the horizontal circle; point upon the Sun, making the horizontal line of the transit tangent to the upper limb and the

* See Coast and Geodetic Survey Report, 1895, Appendix No. 10, for the original of this and the following table.

vertical line tangent to the western limb; note the time, and read both horizontal and vertical circles; repeat this pointing upon the Sun twice more, noting the times and reading the circles; reverse the instrument 180° in azimuth and the telescope in altitude; again take three readings upon the Sun, but now make the horizontal line tangent to the lower limb and the vertical line tangent to the eastern limb; finally, point upon the mark again and read.

To compute the azimuth of the Sun one may use the formula

$$\tan^2 \tfrac{1}{2}z = \frac{\sin (s - \phi) \sin (s - A)}{\cos s \cos (s - P)}, \quad . \quad . \quad (118)$$

in which z is the azimuth counted from the north, P is the Sun's north polar distance ($= 90° - \delta$), and $s = \frac{1}{2}(\phi + A + P)$.

If desired, the hour-angle of the Sun, and thence the chronometer error, may also be computed from the observations by the formula

$$\tan^2 \tfrac{1}{2}t = \frac{\cos s \sin (s - A)}{\sin (s - \phi) \cos (s - P)}. \quad . \quad . \quad (119)$$

These formulæ may readily be derived from the ordinary formulæ of spherical trigonometry as applied to the triangle defined by the Sun, the zenith, and the pole.

229. *Observations of Sun for Azimuth.*

Niantilik, Cumberland Sound, British America, Sept. 18, 1896, P.M.
Instrument—Theodolite Magnetometer No. 19. $\phi = 64° 53'.5$.
Chronometer correction on Greenwich Mean Time $+ 2^m 09^s.8$.

Object.	Position of Instrument.	Time. Chronometer, 1842.	Horizontal Circle.			Vertical Circle.		
			A.	B.	Mean.	A.	B.	Mean.
Azimuth mark............	D.		53° 58′	59′	58′.5			
	R.		233 55	55	55 .0			
Means..................			53		56 .8			
Sun's first and upper limb..	R.	7^h 39^m 20^s	229 46	44	45 .0	17° 17′	18′	17′.5
		40 40	230 05	03	04 .0	11	13	12 .0
		41 34	19	16	17 .5	06	09	07 .5
Sun's second and lower limb	D.	42 58	51 16	17	16 .5	73 33	33	33 .0
		44 04	33	33	33 .0	39	39	39 .0
		45 33	54	55	54 .5	46	47	46 .5
Means..................		7 42 21 .5	50		48 .4	16		46 .4
Sun's second and lower limb	D.	7 46 34	52 09	09	09 .0	73 50	51	50 .5
		47 33	23	24	23 .5	55	57	56 .0
		48 31	38	38	38 .0	74 02	03	02 .5
Sun's first and upper limb...	R.	51 04	232 38	36	37 .0	16 16	17	16 .5
		52 05	52	50	51 .0	11	13	12 .0
		53 21	233 12	09	10 .5	03	07	05 .0
Means..................		7 49 51 .3	52		38 .2	16		07 .4
Azimuth mark.....	R.		233 56	55	55 .5			
	D.		53 58	58	58 .0			
Means..................			53		56 .8			

Computation.

	First Half.	Second Half.
Chronometer time *	$7^h\ 42^m\ 21^s.5$	$7^h\ 49^m\ 51^s.3$
Chronometer corr. on Greenwich Mean Time	+ 02 09 .8	+ 02 09 .8
Greenwich Mean Time	7 44 31 .3	7 52 01 .1
Sun's Apparent Declination, δ, interpolated from Ephemeris	1° 26'.9	1° 26'.8
Observed Altitude	16 46 .4	16 07 .4
Correction for parallax	+ 0 .1	+ 0 .1
Correction for refraction	− 3 .1	− 3 .3
Corrected Altitude, A	16 43 .4	16 04 .2
$P(= 90° - \delta)$	88 33 .1	88 33 .2
Latitude, ϕ	64 53 .5	64 53 .5
$\frac{1}{2}(\phi + A + P) = s$	85 05 .0	84 45 .4
$s - \phi$	20 11 .5	19 51 .9
$s - A$	68 21 .6	68 41 .2
$s - P$	− 3 28 .1	− 3 47 .8
Log sin $(s - \phi)$	9.53802	9.53124
Log sin $(s - A)$	9.96826	9.96923
Log numerator	9.50628	9.50047
Log cos s	8.93301	8.96090
Log cos $(s - P)$	9.99920	9.99904
Log denominator	8.93221	8.95994
Log $\tan^2 \frac{1}{2}z$	0.57407	0.54053
Log tan $\frac{1}{2}z$	0.28704	0.27026
$\frac{1}{2}z$	62° 41'.4	61° 46'.6
z	125 22 .8	123 33 .2
Horizontal circle reads	50 48 .4	52 38 .2
True meridian reads	176 11 .2	176 11 .4
Azimuth mark reads	53 56 .8	53 56 .8
Mark west of north	122 14 .4	122 14 .6

230. The derived value of the azimuth is more exact the farther the Sun is from the meridian, and becomes unreliable when the Sun is very near the meridian. These are the same conditions that limit the use of the solar transit. If, however, the error of the timepiece has been determined earlier in the day by formula (119), or is determined later in the day,

* It is important to notice that the only way in which this observed chronometer time enters this computation is as a means of interpolating the declination from the Ephemeris.

or both, an observation at noon will give a good determination of azimuth by computing the hour-angle from the observed times and then computing the azimuth from the formula

$$\cot \tfrac{1}{2}z = \tan \tfrac{1}{2}t \frac{\cos (s + \delta - 90^\circ)}{\sin (s - A)}, \quad . \quad . \quad . \quad (120)$$

which may be derived readily from (118) and (119).

QUESTIONS AND EXAMPLES.

231. 1. In the azimuth computation of § 200 a correction is applied for diurnal aberration. Why is not a correction also applied for the aberration due to the motion of the Earth in its orbit?

2. Differentiate equation (98) with respect to ϕ, and show in a general way the relative errors introduced, by a given error in ϕ, into the computed azimuth from observations taken upon stars of various declinations observed at various hour-angles and at stations in various latitudes. Check your conclusions by inspection of the table in § 310. Also do the same with respect to errors of time.

3. To what indeterminate form does formula (98) reduce for a star in the zenith? Is that case in nature indeterminate?

4. What difficulties would you expect to encounter in determining the azimuth accurately at a station of which the latitude is nearly 90°, aside from those arising from the climate? In considering this question remember that in most of the methods for determining the azimuth the determination of the error of a chronometer on local time is one of the necessary auxiliary observations.

5. What is the azimuth of the mark from the record given in § 203 ?

6. What is the azimuth of the mark from the following record ?

Station No. 14. February 11, 1893.

Observations for azimuth of mark on Polaris near western elongation.

Chronometer error = $+ 2^h\ 29^m\ 47^s.2$. One division of level = $3''.68$.

One turn of micrometer = $123''.73$. $\phi = 32^\circ\ 29'\ 01''.12$.

Cir. E. or W.	Level Readings. W.	Level Readings. E.	Chronometer Time.	Micrometer Readings. On Star.	Micrometer Readings. On Mark.	
	d	*d*		*t*	*t*	
E.	7.2	6.3	$8^h\ 47^m\ 48^s.5$	19.219	18.400	Longitude
	7.2	6.4	48 31 .0	.189	.391	$2^h\ 31^m$ west
			49 01 .5	.170	.387	of Wash-
	+14.4	−12.7	49 47 .0	.146	.399	ington.
E.			50 20 .0	.124	.393	
				19.1696	18.3940	Means.
W.	6.0	8.0	8 52 09 .0	17.790	18.470	
	6.1	7.8	52 36 .5	.800	.470	
			53 04 .5	.820	.469	
	+12.1	−15.8	53 57 .5	.852	.475	
W.	−2.0	= Sum	54 28 .0	.871	.462	
				17.8266	18.4692	Means.

α of Polaris = $1^h\ 18^m\ 48^s.0$. δ of Polaris = $88^\circ\ 44'\ 33''.4$

7. Prove formula (117) for the level correction to be made when determining the value of an eyepiece micrometer measuring angles in the plane of the telescope and its horizontal axis.

8. Prove the statement of § 225, that under certain conditions the azimuth error, collimation error, and equatorial interval of the middle line of a transit when combined by simple addition and subtraction give the azimuth of the mark.

9. What is the circle reading corresponding to the true north from the following record ?

Station—Capitol, East Park, Washington, D. C.
Sun near prime vertical, August 15 A.M., 1856. Observer—C. A. S.
Instrument—5-in. Magnetic Theodolite. Sidereal Chronometer.

Chronometer Time.	Horizontal Circle.		Vertical Circle.		
	A	B	A	B	
	Sun's upper and first limb. Telescope D.				
$5^h\ 20^m\ 44.0^s$	28° 25′ 00″	208° 25′ 00″	58° 29′ 00″	58° 29′ 30″	
22 01.5	28 37 45	208 38 15	58 14 45	58 14 30	
25 26.5	29 13 30	209 14 00	57 36 00	57 35 45	
	Sun's lower and second limb. Telescope R.				
$5^h\ 27^m\ 32.5^s$	209° 01′ 30″	29° 00′ 30″	57° 48′ 00″	57° 47′ 30″	
28 39.5	209 12 45	29 12 15	57 34 30	57 34 15	
30 01.0	209 27 00	29 26 30	57 19 15	57 18 30	78° Fahr.

$\phi = 38° 53' 18''$ $\lambda = 5^h\ 08^m\ 01^s.0$ west of Greenwich.
δ (at mean of the times) $= 13° 55' 16''$. (Interpolated from Ephemeris.)

10. If both are available, which should be used in formula (98)—the geodetic or the astronomical latitude?

11. Prove formulæ (118), (119), and (120).

CHAPTER VII.

LONGITUDE.

232. To determine the longitude of a station on the Earth's surface, referred to the meridian of Greenwich, is to determine the angle between the two meridian planes passing through the station and Greenwich respectively. (See § 15.) This angle between the two meridian planes is the same as the difference of the local times* of the two stations, considering 24^h to represent $360°$. (See § 21.) Hence to determine the longitude of a station is to determine the difference between the local time of that station and the local time of Greenwich. In general the longitude of an unknown station is not referred to Greenwich directly, but to some station of which the longitude is already known. The astronomical determination of the longitude of a station consists, then, in a determination of the local time at each of two stations, the longitude of one which is known and of the other is to be determined, and the comparison of these two times. Their difference is the difference of longitude expressed in time. This may be reduced to arc by the relations $24^h = 360°$, $1^h = 15°$, $1^m = 15'$, and $1^s = 15''$.

233. The principal methods of determining differences of

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* The times may be *either* sidereal or mean solar. The *vernal equinox* apparently makes one complete revolution about the earth in 24 *sidereal* hours, and the *mean Sun* apparently makes one complete revolution in 24 *mean solar hours*.

longitude are by the use of the telegraph, by transportation of chronometers, by observations of the Moon's place, and by observations of eclipses of Jupiter's satellites. The methods of making the necessary determinations of the local time in each of these methods need not be considered here, as they have already been exploited in Chapters III and IV. We need here consider only the methods by which some signal is transmitted between the stations to serve for the comparison of the times.

The Observing Program and Apparatus of the Telegraphic Method.

234. The telegraphic method has been used very extensively in this country by the Coast and Geodetic Survey, and during the fifty years of its use has been gradually modified. The method and apparatus at present used will be here described.

The nightly program at each station is to observe two sets of ten stars each for time with a transit of the type shown in Fig. 10. Each half-set consists in general of four stars having a mean azimuth factor A (see § 299) nearly equal to 0, and one slow star (of large declination) observed above the pole. Two such half-sets, with a reversal of the telescope in the Ys between them, give a strong determination of the time. The same sets of stars are by previous agreement observed at each station. Between the two time sets, or rather at about the middle of the night's observations, certain arbitrary signals are exchanged by telegraph between the two stations, which serve to compare the two chronometers, and therefore to compare the two local times which have been determined from the star observations.*

235. Fig. 30 shows the arrangement of the electrical apparatus at each station during the intervals when no arbitrary signals are being sent or received, and each observer is busy taking his time observations. In the local circuit, which is now entirely independent of the Western Union lines, are placed the break-circuit chronometer* (or clock), battery, chronograph, and the break-circuit observing keys. All the time observations are recorded on the chronograph. Meanwhile the telegraph operator has at his disposal the usual telegrapher's apparatus upon the main line connecting the two stations, namely, his key, and the sounder relay which controls the sounder in a second local circuit. The operator, a few minutes before the time for exchange of signals, secures a clear line between stations, ascertains whether the observations at the other station are proceeding successfully, and finally an agreement is telegraphed between the two observers as to the exact epoch at which the exchange of signals will be made.

236. When that epoch arrives, time observations are stopped at each station, and by suitable switches the electrical apparatus at each station is arranged as shown in Fig. 31. The only change is that now a relay, called a signal relay, is used to connect each local chronograph circuit with the main line in such a way that the local circuit will be broken every time the main circuit is broken, in addition to the regular breaks made in it by the local chronometer. The observer at station *A* now takes the telegrapher's key (in the main circuit), and sends a series of arbitrary break-circuit signals over the main line by holding the key down *except* when a dot is sent by releasing the key for an instant—the reverse of the ordinary usage of the telegrapher. He listens to his own chronograph, and sends a signal once in each two-second interval at such an instant as will not conflict with his own

* Or the chronometer may be placed in a separate local circuit, breaking this one through a relay.

chronograph record. Each signal is transmitted by the signal relay at station *A* to that local circuit, and its time of receipt recorded automatically by the chronograph. At the same instant, except for the time required for the electrical wave to be transmitted over the main line between stations, the signal relay at station *B* transmits the signal to the local circuit and chronograph there. If these signals coincide with the clock breaks on the chronograph at *B* at any time, the observer at *B* breaks into the main circuit with his telegrapher's key, and produces a rattle at *A*'s sounder which informs him that he must change his signals a fraction of a second to another part of the intervals given him by his chronograph beat. Thirty signals are sent from station *A* at intervals of about two seconds. The observer at *A* then closes his key and the observer at *B* proceeds to send thirty similar signals from *B* to *A*.* The Western Union line is then released, the apparatus at each station is again arranged as shown in Fig. 30, and each observer proceeds to finish his time observations.

For a first-class determination this program is carried out for five nights at each station; the observers then change places (to eliminate the effect of personal equation), the instrumental equipment of each station being left undisturbed; and the same program is again followed for five nights.

Example of Computation.

237. A determination of the difference of longitude of Cambridge, Mass., and of Ithaca, N. Y., was made May 16–June 3, 1896. The following is a portion of the field computation:

* Thirty signals at two-second intervals keep each chronometer in use timing signals for just one revolution (1^m) of the toothed wheel which breaks the circuit in the chronometer, and thus any errors in the spacing on that wheel are eliminated from the final result.

Arbitrary Signals, May 27, 1896.

From Ithaca to Cambridge.		From Cambridge to Ithaca.	
Cambridge Record.	Ithaca Record.	Cambridge Record.	Ithaca Record.
14^h 16^m $46^s.34$	9^h 38^m $19^s.52$	14^h 17^m $56^s.55$	9^h 39^m $29^s.63$
48 .39	21 .54	58 .51	31 .61
50 .32	23 .50	00 .56	33 .63
52 .48	25 .63	02 .50	35 .57
....	....	....	....
....	....	....	....
....	....	....	....
42 .41	15 .45	50 .37	23 .31
44 .31	17 .30	52 .40	25 .36
46 .35	19 .36	54 .44	27 .39
48 .52	21 .52	56 .45	29 .39
14^h 17^m $17^s.441$	9^h 38^m $50^s.532$	14^h 18^m $26^s.426$	9^h 39^m $59^s.440$
— 25 .702	— 07 57 .107	— 25 .700	— 07 57 .109
	9 30 53 .425		9 32 02 .331
	1 33 .783		1 33 .971
	4 22 59 .370		4 22 59 .370
14 16 51 .739	13 55 26 .578	14 18 00 .726	13 56 35 .672
Difference 21^m $25^s.161$		Difference 21^m $25^s.054$	

The heading shows which way the signals were sent over the main line, and the four columns give the times of the signals as read directly from the chronograph sheets at the stations indicated. There were 31 or 32 signals in each series, of which only a portion are here printed. The means are given for the whole series in each case. A mean-time clock was used in the chronograph circuit at Ithaca, and a sidereal chronometer at Cambridge.

238. The first time set of the evening at Cambridge gave for the chronometer correction, on local sidereal time, at the mean epoch of the set, when the chronometer read 13^h $30^m.2$, $-25^s.787$. The second set gave the chronometer correction $= -25^s.677$ at the epoch when the chronometer read 14^h $31^m.3$. By taking the means of the epochs and corrections, on the assumption that the chronometer rate was constant during this interval, it was found that the correction was $-25^s.732$ at the chronometer reading $14^h\ 00^m.7$. Also from the differences of epochs and corrections it was found that the rate of the chronometer during this interval was $0^s.00180$ per minute. Applying this rate for the interval $(14^h\ 17^m.3 - 14^h\ 00^m.7)$ to the value $-25^s.732$ of the correction, there is obtained for the chronometer correction at the mean epoch of the signals sent from Ithaca to Cambridge $-25^s.702$. Similarly, from the time observations at Ithaca it was found that when the clock read $9^h\ 38^m.8$ its correction was $-7^m\ 57^s.107$ (on local mean solar time). The computation * shows how the mean epoch of the signals was derived in Cambridge sidereal time $(14^h\ 16^m\ 51^s.739)$, and in Ithaca sidereal time $(13^h\ 55^m\ 26^s.578)$†. The difference of these two, $21^m\ 25^s.161$ is the difference of longitude of the stations, affected by the transmission time of the electric wave, and by the relative personal equation of the two observers.

239. The longitude difference as computed from the other set of signals shown in the computation is evidently affected in the reverse way by the transmission time. Hence the mean of the two derived values, namely, $\frac{1}{2}(21^m\ 25^s.161 + 21^m\ 25^s.054) = 21^m\ 25^s.108$, is the longitude difference unaffected by transmission time, provided such time remained constant during the two minutes of the exchange. Also, the transmission time ‡ itself is $\frac{1}{2}(21^m\ 25^s.161 - 21^m\ 25^s.054) = 0^s.054$.

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* This computation would be simplified in an obvious manner if sidereal timepieces had been used at both stations.

† $4^h\ 22^m\ 59^s.370$ is the sidereal time of mean moon at Ithaca May 27, 1896.

‡ The mean value of the transmission time on nine nights over this line was 0.070, and the separate values varied from $0^s.054$ to $0^s.084$. The telegraph line from Ithaca to Cambridge, by way of Syracuse, New York, and Boston, was 592 miles long, and passed through one repeater (at New York).

An inspection of Fig. 31 will show that this is merely the transmission time *between* the two signal relays, and does not include the transmission time *through* the relays and the chronograph circuit, as this part of the transmission is always in one direction, no matter where the signal starts from in the main circuit.

240. In the regular program of observation * five values of the longitude would thus be obtained, and then five more similar results after the observers exchanged places. From these two means the effect of relative personal equation would then have been eliminated by computation, as shown in the portion of a field computation given below.

DIFFERENCE OF LONGITUDE.

Albany, N. Y., west of Montreal.

Date, 1896.	Observer at A	Observer at M	Longitude Difference. A Signals.	Longitude Difference. M Signals.	A – M	Mean of A and M Signals.	Personal Equation.	Difference of Longitude.
Sept. 16 ...	F	S	$0^m\ 41^s.050$	$0^m\ 41^s.006$	0.044	$0^m\ 41^s.028$	+ 0.268	$0^m\ 41^s.296$
" 20 ..	F	S	41 .086	41 .047	0.039	41 .066		41 .334
" 24....	F	S	41 .036	40 .998	0.038	41 .017		41 .285
" 28 ...	F	S	41 .022	40 .987	0.035	41 .005		41 .273
Oct. 9......	F	S	41 .052	41 .016	0.036	41 .034		41 .302
				Means =	0.038	0 41 .030		
Oct. 10. ...	S	F	0 41 .525	0 41 .491	0.034	0 41 .508	– 0.268	41 .240
" 15....	S	F	41 .569	41 .538	0.031	41 .553		41 .285
" 19.....	S	F	41 .617	41 .580	0.037	41 .598		41 .330
" 21.....	S	F	41 .639	41 .599	0.040	41 .619		41 .351
" 26.....	S	F	41 .578	41 .526	0.052	41 .552		41 .284
				Means =	0.039	0 41 .566		0 41 .298

Transmission time = $\frac{1}{2}(0^s.038) = 0^s.019$.
Relative personal equation, S – F = $\frac{1}{2}(41.566 - 41.030) = + 0^s.268$.
Difference of longitude, A – M = $0^h\ 00^m\ 41^s.298$.

* The regular program was not carried out at this station,—hence the following illustration is taken from another source.

Discussion of Errors.

241. From the final computed result there has thus been eliminated the average relative personal equation during the series of observations, and the average value of the transmission time during the short interval covered by the exchange of signals on each evening. The errors of the adopted right ascensions are also eliminated from the result, because the *same* stars have been observed at *both* stations.*

242. The final computed result is subject to the following errors: 1st, that arising from the accidental errors of observations of 200 stars at each station, which must be quite small after the elimination due to 400 repetitions; 2d, that arising from the variation of the relative personal equation of the two observers from night to night, of which the magnitude may be estimated from the following paragraphs; 3d, that due to lateral refraction, to which reference will be made in § 245; 4th, that due to variations in the rates of the chronometers during the period covered by the observations, which must usually be quite small, as the chronometers are not disturbed in any way during the observations and are protected as far as possible against changes of temperature; 5th, that arising from the variation of the transmission time, between the two halves of the exchange of signals, on each night, which is probably insensible, as this interval is usually only a minute; 6th, the difference of transmission time through the two signal relays, since this difference always enters with the same sign, as may be seen by an inspection of Fig. 31. This last error is made very small by using specially designed relays which act very quickly, by adjusting the two relays to be as

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* Where the difference of longitude is very large, the observers may be forced to use different star lists to avoid depending upon their chronometer rates for too long an interval.

nearly alike as possible, by controlling the strength of the current passing through the relay so that it shall always be nearly the same, and by exchanging relays when the observers change places, or by a combination of these methods.*

Personal Equation.

243. The extent to which the relative personal equation may be expected to vary may be estimated from the following statement of the experience of two observers who have made the major portion of the primary longitude determinations of the Coast and Geodetic Survey during the period indicated. The plus sign indicates that Mr. Sinclair observes later than Mr. Putnam.

PERSONAL EQUATION BETWEEN C. H. SINCLAIR AND G. R. PUTNAM, ASSISTANTS C. AND G. SURVEY, RESULTING FROM OR CONNECTED WITH THE TELEGRAPHIC LONGITUDE WORK OF THE SURVEY.†

By direct comparison at Washington, D. C., 1890, Sept. 17, 18, 19	$S - P = + 0^s.266$
By direct comparison at St. Louis, Mo., 1890, Nov. 4, 15	$+ 0\ .278$
From interchange of observers during longitude determinations, after *one-half* of the work was completed, generally from 4 or 5 days' results:	
Cape May, N. J., and Albany, N. Y., 1891, May and June	$S - P = + 0^s.184 \pm 0^s.011$
Detroit, Mich., and Albany, N. Y., 1891, June and July	$+ 0\ .140 \pm 0\ .008$
Chicago, Ill., and Detroit, Mich., 1891, July.	$+ 0\ .172 \pm 0\ .006$
Minneapolis, Minn., and Chicago, Ill., 1891, Aug.	$+ 0\ .161 \pm 0\ .010$
Omaha, Neb., and Minneapolis, Minn., 1891, Aug. and Sept	$+ 0\ .176 \pm 0\ .011$
Los Angeles, Cal., and San Diego, Cal., 1892, Feb. and Mar.	$+ 0\ .160 \pm 0\ .006$

* See Coast and Geodetic Survey Report, 1880, p. 241.

† For these data the author is indebted to the Superintendent of the Coast and Geodetic Survey.

San Diego, Cal., and Yuma, Ariz., 1892, March	+ 0 .192 ± 0 .004
Los Angeles, Cal., and Yuma, Ariz., 1892, Mar. and April	+ 0 .140 ± 0 .002
Yuma, Ariz., and Nogales, Ariz., 1892, April	+ 0 .150 ± 0 .005
Nogales, Ariz., and El Paso, Tex., 1892, April and May	+ 0 .126 ± 0 .004
Helena, Mont., and Yellow Stone Lake, Wyo., 1892, June and July	+ 0 .109 ± 0 .010
El Paso, Tex., and Little Rock, Ark., 1893, Feb. and March	+ 0 .082 ± 0 .010

The following values depend on unrevised field computation:

Key West, Fla., and Charleston, S. C., 1896, Feb. and March	$S - P =$ + 0ˢ.147
Atlanta, Ga., and Key West, Fla., 1896, March	+ 0 .121
Little Rock, Ark., and Atlanta, Ga., 1896, April	+ 0 .130
Charleston, S. C., and Washington, D. C., 1896, April and May	+ 0 .183
Washington, D. C., and Cambridge, Mass., 1896, May and June	+ 0 .142 ± 0ˢ.013
Washington, D. C., Naval Observatory and Washington, D. C., Coast and Geodetic Survey Office, 1896, June and July	+ 0 .117 ± 0 .008

Note that the period covered by this record is nearly six years, and that the localities show that the observers were surely submitted to a great variety of climatic conditions. Yet if the first two determinations, made when Mr. Putnam was comparatively new to the work, be omitted, the total range of the results is only 0ˢ.110. It must be remembered, however, that each of these results, except the first two, depend upon from eight to ten nights of observation, four or five nights each before and after the interchange of observers. It is quite probable that the actual variation of the relative personal equation from night to night is somewhat greater than that shown above.

244. The absolute personal equation is the time interval required for the nerves and portions of the brain concerned in an observation to perform their offices. Although the personal equation has been studied by many, little more can be confidently said in regard to the laws which govern its magnitude than that it is a function of the observer's personality, that it tends to become constant with experience, and that probably whatever affects the observer's physical and mental condition affects its value. But so little is known in regard to it, that no observer will predict, before the observations of a night have been computed, that his personal equation was large or small on that particular night.

Discussion of Errors.

245. Returning to a consideration of the errors of the telegraphic longitudes, it may be said that the ten results for a station, after eliminating the personal equation, still show a range, ordinarily, in primary work, of from $0^s.10$ or less, to $0^s.20$. This range is larger than is to be accounted for by the accidental errors of observation, or by any of the other errors enumerated in § 242, except perhaps those of the second and third classes. Those most familiar with the observations are apt to account for the large range as arising either from variation in the personal equation or from lateral refraction. One observer of long experience is inclined to suspect the striding level of giving errors which tend to be constant for the night. To whatever these errors may be due, they seem to be fairly well eliminated from the mean for the station. For in the great network of longitude determinations, made by the Coast and Geodetic Survey, covering the whole United States, the discrepancies arising in closing the various "longitude triangles" are always less than $0^s.10$.*

* For a good example of a check showing the degree of accuracy of this network see Coast and Geodetic Survey Report, 1894, p. 85.

Personal Equation.

246. If, in making a longitude determination circumstances prevent the interchange of observers, the effect of the relative personal equation upon the computed longitude may still be eliminated, in part at least, by a special determination of the equation by joint observations at a common station. The two observers may place their instruments side by side in the same observatory, observe the same stars, and record their observations upon the same chronograph. The difference of the two chronometer corrections computed by them, corrected for the minute longitude difference corresponding to the measured distance between their instruments, is then their relative personal equation. Or, they may observe with the same transit as follows: On the first star A observes the transits over the lines of the first half of the reticle, and then quickly gives place to B, who observes the transits across the remaining lines. On the second star B observes on the first half of the reticle, and A follows. After observing a series of stars thus, each leading alternately, each observer computes for each star, from the known equatorial intervals of the lines and from his own observations, the time of transit of the star across the mean line of the whole reticle. The difference of the two deduced times of transit across the mean line is the relative personal equation. If each has led the same number of times in observing, the mean result is independent of any error in the assumed equatorial intervals of the lines. No readings of the striding level need be taken, and the result is less affected by the instability of the instrument than in the other method.

247. In certain cases in which it is not feasible to use a telegraph line for a longitude determination, the same principles may be used with the substitution of a flash of light between stations in the place of the electric wave. For example, one might so determine the longitudes of the Aleutian Islands of Alaska, the successive islands being in general intervisible.

Longitude by Chronometers—Equipment.

248. If a telegraph line is not available between the two stations, the next method in order of accuracy, aside from the flash method alluded to above, is that of transporting chronometers back and forth between them. The transported chronometers then perform the same duty as the telegraph, namely, that of comparing the local times of the two stations.

The chronometric method may perhaps be best explained by giving a concrete example. The longitude of a station at Anchorage Point, Chilkat Inlet, Alaska, was determined in 1894, by transportation of chronometers between that station and Sitka, Alaska, of which the longitude was known. At Anchorage Point observations were taken on every possible night from May 15th to August 12th, namely, in 53 nights, by the eye and ear method, with a transit of the type shown in Fig. 11, using as a hack for the observations chronometer Bond 380 (sidereal). At the station there were also four other chronometers, two sidereal and two mean. These four were never removed during the season from the padded double-walled box in which they were kept for protection against sudden changes of temperature, and in which the hack chronometer was also kept when not in use. The instrumental equipment at Sitka was similar. A sidereal chronometer was used as an observing hack, and two other chronometers, one sidereal and one mean, were used in addition. Nine chronometers, eight keeping mean time and one sidereal time, were carried back and forth between the stations on the steamer Hassler.

Longitude by Chronometers—Observations.

249. Aside from the time observations the procedure was as follows: Just before beginning the time observations at Anchorage Point and again as soon as they were finished, on each night, the hack chronometer No. 380 (sidereal) was compared with the two mean time chronometers by the method of coincidence of beats, to be described later (§ 250). These two were then each compared with each of the two remaining (sidereal) chronometers at the station. These comparisons, together with the transit observations, served to determine the error of each chronometer on local time at the epoch of the transit observations.* Whenever the steamer first arrived at the station, and again when it was about to leave, the hack chronometer No. 380 was compared with the other station chronometers as indicated above, was carried on board the steamer and compared with the nine steamer chronometers, and then immediately returned to the station and again compared with the four stationary station chronometers. As an extra precaution both the station observer and the observer in charge of the steamer chronometer made each of these comparisons. In the comparisons on the steamer, the hack (380) was compared by coincidence of beats with each of the eight mean time chronometers, and the remaining (sidereal) chronometer was then compared with some of the eight. The comparisons on shore before and after the trip to the steamer served to determine the error of the hack (380) at the epoch of the steamer comparisons. The steamer comparisons determined the errors of each of the

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* The station chronometers were also intercompared on days when no observations were made. But this was merely done to ascertain their performance, and these comparisons were not used in computing the longitude.

steamer chronometers on Anchorage Point time. Similar observations were made at Sitka to determine the errors of the nine steamer chronometers on Sitka time as soon as they arrived, and again just before they departed from Sitka. During the season the steamer, which was also on other duty, made seven and a half round trips between the stations. The distance travelled was about 400 statute miles for each round trip.

250. The process of comparing a sidereal and a mean time chronometer is analogous to that of reading a vernier. The sidereal chronometer gains gradually on the mean time chronometer, and once in about three minutes the two chronometers tick exactly together (one beat = $0^s.5$). Just as one looks along a vernier to find a coincidence, so here one listens to this audible vernier and waits for a coincidence. As in reading a vernier one should also look at lines on each side of the supposed coincidence to check, and perhaps correct, the reading by observing the symmetry of adjacent lines, so here one listens for an approaching coincidence, hears the ticks nearly together, apparently hears them exactly together for a few seconds, and then hears them begin to separate, and notes the real coincidence as being at the instant of symmetry. The time of the coincidence is noted by the face of one of the chronometers. Just before or just after the observation of the coincidence the difference of the seconds readings of the two chronometers is noted to the nearest half-second (either mentally or on paper). This difference serves to give the seconds reading of the second chronometer. The hours and minutes are observed directly. When a number of chronometers are to be intercompared, the experienced observer is able to pick out from among them two that are about to coincide; he compares those; selects two more that are about to coincide and compares them, and so on; and thus to a certain extent avoids the waits—of a minute and a half on an average—which would otherwise be necessary to secure an observation on a pair of chronometers selected arbitrarily.

Computation of a Longitude by Chronometers.

251. The following example (taken from another set of observations) will show how the chronometer comparisons are computed. A certain Dent mean time chronometer was compared with a certain Negus sidereal chronometer on Oct. 14, 1892, at a station $2^h\ 12^m$ west of Washington. It was found that $11^h\ 20^m\ 23^s.0$ A.M. Dent = $12^h\ 54^m\ 41^s.0$ Negus. The correction to the Dent on local mean time was known to be $-\ 2^h\ 11^m\ 53^s.41$, and the correction of the Negus to local sidereal time was required.

Time by Dent chronometer	23^h	20^m	$23^s.00$
Correction to Dent	− 2	11	53.41
Local mean solar time	21	08	29.59
Reduction to sidereal interval (§ 290)		+ 03	28.38
Sidereal interval from preceding mean noon	21	11	57.97
" time of preceding mean noon (Oct. 13)	13	31	14.05
Local sidereal time	10	43	12.02
Time by Negus chronometer	12	54	41.00
Correction to Negus chronometer	− 2	11	28.98

The computation is modified in an obvious manner if it is the error of the sidereal chronometer that is known.

252. This process of comparing chronometers is so accurate, that it was found that the two values of the error of either of the station sidereal chronometers, as derived from the comparisons described above in two different ways from the hack chronometer, seldom differed by more than $0^s.03$. This corresponds to an error of 11^s in noting the time of coincidence of beats, on the supposition that all the error was made in one of the four comparisons concerned. If two chronometers of the same kind, both sidereal or both mean time, are compared directly it requires very careful observing to secure their difference within $0^s.10$ of the truth.

253. The comparisons of the other four station chronometers with the hack chronometer immediately before and after transit observations gave the errors of each of those four chronometers. To compute the errors of the steamer chronometers at the time of the comparisons made on the steamer, it is first necessary to secure as good a determination as possible of the error of the hack chronometer at the epoch of those comparisons. One value for that error was obtained in an obvious manner by assuming that the hack chronometer ran at a uniform rate between the last preceding and the next following transit observations. Four other determinations of the error of the hack at that epoch were obtained, by making that same assumption for each of the other four station chronometers, and deriving the error of the hack from the comparisons made with that chronometer at the station before and after the steamer comparisons. The weighted mean of these five values of the error of the hack was used. For the method of deriving the relative weights which were assigned to these five results see § 260. At Anchorage Point 13 comparisons were made with the steamer chronometers. In six cases out of the thirteen the range of the five derived values of the error of the hack was less than $0^s.2$.

254. Having now the errors of the steamer chronometers on the local time of each station at the time of arrival at and departure from each station, the difference of longitude was computed in the manner indicated by the following illustration. Suppose chronometer No. 231 to have been found to have the following errors on a certain round trip:

Anchorage Point, at departure,	May 15,	9^h 00^m A.M.	40^s fast	of A. P. time.
Sitka, on arrival..............	May 16,	9 00 A.M.	11 "	" Sitka "
Sitka, at departure............	May 22,	9 00 A.M.	4 "	" Sitka "
Anchorage Point..............	May 23,	9 00 A.M.	32 "	" A. P. "

From the two Anchorage Point observations it appears that the chronometer has lost 8^s in the eight days it was gone from there. From the two Sitka observations it appears that 7^s were lost while at Sitka. Hence the chronometer lost 1^s only while travelling both ways between the stations, or its travelling rate was $0^s.5$ per day, losing. Applying this rate to the errors as determined at Anchorage Point, we find that the errors of the chronometer on *Anchorage Point* time at the epochs of the Sitka steamer comparisons were $39^s.5$ fast and $32^s.5$ fast, and that the difference of longitude required is $39^s.5 - 11^s.0 = 32^s.5 - 4^s.0 = 28^s.5$, A. P. west of S. We have thus derived the longitude difference on the supposition that the steamer chronometers have a travelling rate which is constant during the round trip, and without any assumptions as to the rates while in port. The assumptions as to the station chronometers have been simply that each preserves a constant rate between successive transit observations.

255. The longitude was thus computed from each round trip starting from Anchorage Point, and the mean taken. If the chronometers had continually accelerated (or retarded) rates, this mean was subject to an error arising from that fact. To eliminate such a possible error, and to serve as a check upon the computation, a second computation was made from each round trip starting from Sitka, and the mean taken. The error from acceleration (or retardation) of rates was necessarily of opposite sign in this mean. The mean of these two results is then subject only to accidental errors, in so far as the chronometers are concerned.

256. The following table shows the separate results obtained and the manner of combining them:

DIFFERENCE OF LONGITUDE, IN SECONDS, BETWEEN SITKA AND ANCHORAGE POINT, CHILKAT INLET, ALASKA.

SUMMARY OF RESULTS FROM SEVEN ROUND TRIPS, STARTING FROM ANCHORAGE POINT, CHILKAT INLET.

Chronometers, M. T. or Sid.		1st	2d	3d	4th	5th	6th	7th	Means. Δλ	Weights.
M. T.	231	28.03	26.36	28.36	28.19	28.45	28.19	28.18	27.97	3
	1507	28.44	29.06	29.18	28.26	28.27	28.20	28.54	28.56	4
	1510	28.57	29.25	29.00	28.52	28.63	28.06	28.58	28.66	7
	196	28.59	29.09	29.54	28.59	28.43	28.51	28.92	28.81	3
	1542	28.11	28.11	28.66	28.23	28.47	28.38	28.37	28.33	22
	1728	28.66	28.94	29.16	28.63	28.58	28.43	28.59	28.71	6
	208	27.95	27.40	28.21	28.19	28.42	28.42	28.09	28.10	6
	2167	28.21	28.56	28.90	28.55	28.68	28.27	28.64	28.54	17
Sid.	387	28.20	28.44	28.91	27.93	28.41	27.93	28.59	28.34	6
Mean		28.31	28.36	28.88	28.34	28.48	28.27	28.50	28.45	
Weighted mean		28.25	28.38	28.82	28.35	28.52	28.28	28.49	28.44	
Weight.		3	1	2	2	2	1	2		

Weighted mean 0^h 0^m $28^s.44 \pm 0^s.05$

SUMMARY OF RESULTS FROM SEVEN ROUND TRIPS, STARTING FROM SITKA.

Chronometers, M. T. or Sid.		1st	2d	3d	4th	5th	6th	7th	Means. Δλ	Weights.
M. T.	231	28.87	28.78	28.74	28.39	28.37	28.71	28.11	28.57	3
	1507	27.69	29.08	29.11	27.76	28.78	27.93	28.64	28.43	4
	1510	28.37	28.88	28.82	27.91	28.83	28.10	28.58	28.50	7
	196	28.59	29.07	28.95	27.66	28.03	29.56	29.20	28.72	3
	1542	28.93	28.57	28.59	28.22	28.50	28.50	28.32	28.52	22
	1728	27.59	28.90	28.75	27.99	29.01	28.09	28.75	28.44	6
	208	27.71	28.03	28.52	28.58	27.88	28.76	27.65	28.16	6
	2167	28.24	28.71	28.80	28.27	28.77	28.31	28.49	28.51	17
Sid.	387	28.68	28.80	28.43	27.69	28.97	27.98	28.73	28.47	6
Mean		28.30	28.76	28.75	28.05	28.57	28.44	28.50	28.48	
Weighted mean		28.41	28.69	28.70	28.13	28.61	28.38	28.44	28.48	
Weight		1	2	2	2	2	2	2		

Weighted mean 0^h 0^m $28^s.48 \pm 0^s.05$

Final mean Δλ		$= + 0^h\ 00^m\ 28^s.46 \pm 0^s.05$
Longitude of Sitka,		$9\quad 01\quad 21\ .48 \pm 0\ .13$
Longitude of Anchorage Point		$9\quad 01\quad 49\ .94 \pm 0\ .14$
	or	$135°\ 27'\ 29''.10 \pm 2''.10$

257. The steamer started from Anchorage Point at the beginning of the season, and finished at Sitka at the season's end, after 7½ round trips. The last half-trip was omitted in the first part of the above computation, and the first half-trip omitted in the second part. If there had been simply seven round trips starting from Anchorage Point the procedure would have been to deal regularly with all trips in the first half of the computation; and in the last half in addition to the six regular round trips starting from Sitka, the last half-trip (S. to A. P.) and the first half-trip (A. P. to S.) would have been used together as a seventh round trip from Sitka.

258. Let N be the number of days during which the chronometers were depended upon to carry the time during each round trip, reckoned as follows: Add together the two intervals between comparisons of the steamer chronometers with the shore chronometer at the beginning and at the end of each half-trip, and increase this by adding the interval from each comparison of the observing chronometer and steamer chronometers to the *nearest* transit observations made at that station. The weight assigned to each trip in the above computation is proportional to $1/N$.

259. What relative weights shall be assigned to the results from the different chronometers? Some evidently run at a more nearly constant rate than others. Let l_1, l_2, l_3, . . . l_n be the separate values of the longitude as given by any one chronometer, and l_m their mean, and let n be the number of such values, or the number of trips. Then by least squares the probable error of any one value is

$$\pm \sqrt{\frac{(0.455)[(l_1 - l_m)^2 + (l_2 - l_m)^2 \ldots (l_n - l_m)^2]}{n - 1}}.$$

By the rule that the weight of a result is inversely proportional to the square of its probable error, the relative weights to be assigned to the chronometers are proportional to

$$\frac{n - 1}{[(l_1 - l_m)^2 + (l_2 - l_m)^2 \ldots (l_n - l_m)^2]} \quad . \quad . \quad (121)$$

The factor 0.455 is dropped for simplicity since we are dealing with *relative* weights only. In the above computation the sum $[(l_1 - l_m)^2 + (l_2 - l_m)^2 \ldots (l_7 - l_m)^2]$ was determined from each half of the computation, and the mean used in the denominator of (121). The remainder of the computation needs no explanation.

260. The relative weights assigned to the station chronometers as indicated in § 253 may be determined by an analogous process. Let o be the error of a chronometer at the epoch of the transit time observations as determined from those observations. Let I be its error at that same instant interpolated between its errors as determined at the last preceding and first following transit time observations on the assumption that its rate during that interval is constant. Then $I - \text{o}$ is a measure of the behavior of the chronometer. It is the amount by which the chronometer has gone wrong on the supposition that the transit observations may be considered exact. The chronometer apparently indicates that the station at the middle observation was at a distance $I - \text{o}$ in longitude from its position at the preceding and following observations. For a group of chronometers whose errors are all determined a number of times in succession by the *same* transit observations, the relative weights are evidently proportional to the quantities

$$\frac{1}{\Sigma(I - \text{o})^2}.$$

261. The above example serves to illustrate the principles involved in the computation of a longitude by chronometers. The accuracy of the derived longitude is greater, the greater the number of chronometers used, the greater the number of trips, the smaller the average value of N (§ 258), and of course depends intimately upon the quality of the chronometers and the care with which they are protected from jars and from sudden changes of temperature. Unless the round trips are quite short the errors of the transit time observations will be small as compared with the other errors of the process. If considered necessary the relative personal equation of the observers may be eliminated from the result by the same methods that are used in connection with telegraphic determinations of longitude.

262. If the trips are very long, it may possibly be advisable to determine, by a special series of observations, the temperature coefficient of each chronometer and also a coefficient expressing its acceleration (or retardation) of rate, and to apply corresponding computed corrections to the travelling rates.* The chronometers are compensated for temperature as far as possible by the maker, of course, but such compensation cannot be perfect. The thickening of the oil in the bearings tends to increase the friction with lapse of time, and by diminishing the arc of vibration of the balance-wheel to increase the rate of running. Attempts to use rate corrections depending upon the computed coefficients of a chronometer have usually been rather unsatisfactory, and should not be made except in extreme cases.

Longitude Determined by Observing the Moon.

263. If none of the preceding methods are available, one is forced to use those methods which depend upon the motion of the Moon, or perhaps to observe upon Jupiter's satellites.

The place of the Moon has been observed many times at the fixed observatories. From these observations its orbit and the various perturbations to which it is subject have been computed. In the American Ephemeris and similar publica-

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* For details of this process see Doolittle's Practical Astronomy, pp. 383–388.

tions, tables will be found giving the Moon's right ascension and declination for every hour, and also other tables giving its place as defined in other ways. Suppose now that an observer at a station of which the longitude is required determines the position of the Moon and notes the local time at which his observation was made. He may then consult the Ephemeris and find at what instant of Greenwich time the Moon was actually in the position in which he observed it. The difference between this time and the local time of his observation is his longitude reckoned from Greenwich.

Among the processes by which the position of the Moon may be determined for this purpose are the following.

264. The local sidereal time of transit of the Moon across the meridian of the station may be observed with a transit, and a chronometer of which the error is determined in the usual way by observations upon the stars. Or, what is in principle the same thing, the right ascension of the Moon may be derived by comparing its time of transit with that of four stars of about the same declination as the Moon, two transiting shortly before it, and two soon after it. In either case the right ascension of the Moon at the instant of its transit may be computed, and from the Ephemeris the Greenwich time at which the Moon had that right ascension becomes known.

265. The *lunar distance* of a heavenly body is the angle between two lines drawn from the center of the Earth—one to the center of the Moon and the other to the center of the body considered. Or, in other words, it is the angle between the two objects as seen from the Earth's center. The Ephemeris gives the lunar distances of the Sun, the four larger planets, and of certain stars, at intervals of three hours, Greenwich mean time. An observer anywhere may measure the angular distance from the Moon to any one of these objects, with a sextant or other suitable instrument. His measurement *reduced to the Earth's center* gives the lunar distance; from which with the use of the Ephemeris the Greenwich time of the observation becomes known; and also his longitude if he noted the local time of the observation.

266. A star is said to be occulted during the time it is out of sight behind the Moon. The beginning and end of the occultation, that is, the instants of disappearance and reappearance of the star, called its *immersion* and *emersion*, are phenomena capable of being observed with considerable accuracy. The Ephemeris gives the necessary elements for computing the *Washington* times of occultation of various stars as seen from any point upon the surface of the Earth. The local time of the occultation being observed, either of the immersion or emersion, and the Washington time being computed, the longitude becomes known. An observation of the local times of the phenomena of an eclipse of the Sun or Moon furnishes a similar determination of longitude.

267. The computations required in the last two methods are quite long and complicated, and the theories involved require much study for their mastery. The method of culminations gives rise also to rather difficult computations, though not so difficult as those just mentioned. However, the time and labor expended would be fully rewarded if accurate results were obtained. But any of these methods give rise to results which are crude in comparison with those given by the telegraphic method, or by transportation of chronometers. The method of occultations requires the greatest amount of computing, but also gives the greatest accuracy, of the methods named.

268. Three conditions stand in the way of the attainment of accuracy by any method involving the Moon. Firstly, the Moon requires about $27\frac{1}{3}$ days to make one complete circuit in its orbit about the Earth. The apparent motion of the Moon among the stars is then about one-twenty-seventh as fast as the apparent motion of the stars relative to an observer's meridian, which furnishes his measure of time. Any error in determining the position of the Moon is then multiplied by at least twenty-seven when it is converted into time in the progress of the computation. If then the time of transit of the Moon, for example, could be observed as accurately as that of a star, one would expect the errors in a longitude computed from Moon culminations to be twenty-seven times as great as the errors of the local time derived from the same number of star observations.

269. Secondly, the motion of the Moon is so difficult to compute that its positions at various times as given in the Ephemeris, and also of course the data there given in regard to lunar distances and occultations, are in error by amounts which become whole seconds when multiplied by the factor twenty-seven. This source of error is often avoided in the method of Moon's transits (culminations) by using in the computation for each night the Moon's right ascension as corrected at Greenwich, or some other station of known longitude, by direct observation on that same night.

Thirdly, the limb, or edge of the visible disk of the Moon, is necessarily the object really observed, and this is a "ragged edge" rather than a perfect arc, for purposes of accurate measurement.

270. The determination of the points at which the boundary between Alaska and British America (141st meridian) crosses the Yukon and Porcupine rivers was one of the comparatively few instances in late years in which it was necessary to resort to observations upon the Moon to determine an important longitude. To determine the longitude by transportation of chronometers would have been exceedingly difficult and costly, for there is more than a thousand miles of slow river navigation between the mouth of the Yukon River and either station. At a station near the point where the Yukon crosses the boundary, Moon culminations were observed on 23 nights. Four of the results were rejected as worthless. The other 19 gave results ranging from $9^h\ 22^m\ 30^s.0$ to $48^s.9$, with a weighted mean of $38^s.5$. Fourteen of these computed results depend upon the Moon's place as corrected by corresponding observations at Greenwich or San Francisco. At the same station two observed occultations gave for the seconds of the longitude $35^s.5$ and $37^s.2$, and a solar eclipse gave $32^s.2$. At a station near the point where the Porcupine River crosses the boundary, 13 observed Moon culminations computed by the use of corresponding observations on the same nights at San Francisco, Washington, or Greenwich, gave longitudes varying from $9^h\ 23^m\ 45^s.5$ to $63^s.8$, with a weighted mean of $55^s.4$. One observed occultation, both immersion and emersion, gave for the seconds of the longitude $63^s.6$. These examples * will serve to indicate roughly the possibilities of the lunar methods of determining longitude. Experienced observers took the observations at both stations. It should be noted, however, that in such high latitudes (the stations were near the Arctic Circle), the trigonometric conditions are unfavorable to accurate time determinations, and the climatic conditions were such as to make observing difficult.

271. Those wishing to study these lunar methods of determining the longitude are referred for details to Doolittle's Practical Astronomy; to Chauvenet's Astronomy, vol. I.; and in the American Ephemeris (aside from the tables)

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* For a more complete account of these observations see Coast and Geodetic Survey Report, 1895, pp. 331–336.

especially to the pages, in the back of the volume, headed "Use of Tables."

272. The Ephemeris gives, for each night of the year when Jupiter is not too near the Sun to be observed, the Washington mean time of the occultations and eclipses of Jupiter's satellites by that planet, and also the transit of the satellites and their shadows across the face of the planet. An eclipse may be observed at a station of which the longitude is required. By comparison of the computed Washington mean time of the eclipse as given in the Ephemeris, and the observed mean time, the required longitude may be derived. The times of the other phenomena mentioned are given to the nearest minute only. They may be observed simultaneously by two observers using the Ephemeris merely to indicate when to be on the alert. The difference in the local times of observation of the same phenomena is the difference of longitude of the observers, the transit or occultation serving merely as a signal that may be seen at the same instant by both. The difficulty of accurately observing these phenomena (including the eclipses) makes the derived longitudes only rough approximations. The time of a satellite may, for example, be observed a whole minute sooner than it actually occurs, if a low-power telescope is used. Such errors may be partially eliminated by observing the reappearance as well as the disappearance.

CHAPTER VIII.

MISCELLANEOUS.

Suggestions about Observing.

273. Among the characteristics of a good observer, that is, of an observer who will secure the maximum accuracy with a given expenditure of time and money, in making such astronomical determinations as are treated in this book, may be mentioned the following:

He is without bias as to the results to be obtained, his prime motive being always to come as near as possible to the truth. He has that kind of self-control which makes it possible for him to prevent the knowledge that the result he is securing is too small (or too large) to check with other determinations, from having the slightest effect upon his observations. For example, he may know that his observations, in making a telegraphic determination of the longitude of a station, are placing that station $0^s.5$ farther west than it has been fixed by a primary triangulation, and yet have no tendency to observe stars earlier or later than usual. Or, when in reading a micrometer upon an azimuth mark several times in quick succession he secures three or four readings which agree almost exactly, and then one which differs from them by two seconds (say), thus making a bad looking break in his record, he will not suppress or "spring" this reading, though it may serve to make him more careful with following readings.

274. He is well aware of the minuteness of the allowable errors. A student, when warned that he must not apply any longitudinal force to the head of the micrometer of a zenith telescope, will perhaps experiment for himself, by purposely applying a little pressure while making a bisection, and not being able to see any appreciable motion, will become incredulous as to the necessity of the warning. A good observer, on the other hand, knows that he can secure observations such that the combination of errors from *all* sources produce an error in each result which is as apt to be less than $0''.3$ as greater than that value ($e = \pm 0''.30$), although $0''.3$ is a fraction of the apparent width of the line with which he makes the bisection. In other words, he knows that he can make pointings under good conditions, of which the errors are so small as to be invisible in the telescope. He knows that he can make pointings with a probable error of $\pm 0''.5$, say, with a telescope with which it would be hopeless to try to see a rod one-sixth of an inch in diameter placed one mile away ($\frac{1}{6}$ inch subtends $0''.5$ at one mile).

275. He is conscious that the most delicate manipulation is required. He knows that his instrument is built of elastic material, and that unless he is exceedingly careful to apply only such forces as are necessary he may readily produce deformations in his instrument, which though strictly in accordance with the modulus of elasticity of the material composing it, are yet as large as the largest allowable errors of observation. One may sometimes secure striking ocular evidence of this by watching a bisection, in a reading microscope on a horizontal circle (or in the telescope), while a poor observer makes his pointings with another of the reading microscopes on the instrument.

276. A good observer does not consider his instrument to be of fixed dimensions or shape, even when no external forces are applied to it. He knows that it is constantly undergoing changes of shape due to changes of temperature; that these changes even under the best conditions that he can secure may produce errors of the same order of magnitude as the observer's errors; and under adverse conditions may produce errors which are larger than all the others concerned in the measurement.

With respect to movements under stress and under thermal changes, the support of the instrument (tripod, block, or pier) should be considered as a part of the instrument.

Suggestions about Computing.

277. Almost the first question that arises on commencing a given kind of computation for the first time is "To how many decimal places must each part of the computation be carried?" If too few figures are used the errors from the cast away decimal places become larger than is allowable. If too many places are used the computation becomes slower than is necessary,—the work required for interpolations in whatever tables are used being especially liable to increase rapidly with an increase of decimal places. A good general guide in this matter is to carry each part of every computation to as many decimal places as correspond to two doubtful figures in the final result. That is, when the computation is finished, and the probable error is computed, there should in general be two significant figures in the probable error. Or, in other words, the probable error should be between 10 and 100 units in the last place. It may be allowable to drop one more figure than above indicated if to do so decreases the work of computation very much, as in computing sextant observations for time (see foot-note to Example 8, at the end of Chapter III). It is important, after deciding upon the number of places to use in a computation, to adhere to that number strictly. To carry some numbers one place farther than others, in a column to be added, is useless; and worse than useless, for it leads to mistakes such as adding tenths and hundredths, for instance, as if they were in the same column.

If a number ends in a five and the last figure is to be cast away, shall the five be called ten or zero? Both are equally near the truth. A good rule is, in such cases, to make the last retained figure even (not odd). This will mean calling the five a ten about half of the time, and avoids the constant tendency to make the result too large (or too small) that would exist if the five were always called ten (or zero).

278. If many astronomical computations are to be made, Barlow's tables of squares, etc., Crelle's four-place multiplication tables, and a machine for multiplying will be found convenient aids,—the last two for checks, especially. For example, apparent star places may be computed by logarithms in the usual way, and then checked by a separate computation by natural numbers and the use of Crelle's tables (or a computing-machine). The check will be much more efficient than repeating the logarithmic work because an entirely different set of figures are used, and it will take about the same amount of time.

279. The difference between a good computer and a poor one lies largely in the industry and ingenuity with which a good computer applies such checks to his work to find whatever mistakes he makes. Rough checks should not be despised, such as comparing two computations which are nearly alike (computations of two successive sets of observations for example), or such as checking an exact computation by formula (98), § 193, by the use of the table in § 310.

Means should always be checked by residuals. If residuals be obtained by subtracting a mean from each of the separate values, the sums of the positive and of the negative residuals so obtained must not differ by more than $\frac{n}{2}$ units corresponding to the last place of the mean, where n is the number of the separate values.

280. In converting angles into time, or *vice versa*, it is about as rapid to use the relations $360° = 24^h$, $15° = 1^h$, $1° = 4^m$, $1' = 4^s$, $15'' = 1^s$, as it is to use the tables given for that purpose on page 560 of Vega's Logarithmic Tables, and elsewhere. The tables may be used as a check.

281. When several computations of the same kind are to be made, it usually saves time to carry along corresponding portions together. For example, in computing apparent places all the star numbers may be taken out at one time, later all the values of log cos $(G + \alpha_0)$, and so on.

The use of a fixed form for a computation saves time and mistakes. The form should represent a logical order of work, and should involve as little repetition of figures as possible. All scribbling, multiplying, dividing, interpolating, etc., should be done on separate sheets of paper from the regular computation.

Probable Errors.

282. The reader who does not understand the principles of least squares cannot hope to understand the logic of the formulæ given in Chapters IV and V for certain least-square computations. But after a careful perusal of §§ 283–285 a statement of the uncertainty in a certain value in terms of the so-called probable error should not be unintelligible to him.

283. In the expression "probable error" the word "probable" is not used in its ordinary sense, but in a special technical sense. To assert that the probable error of a certain stated value is $\pm e$, is to assert the chances are equal for and against the truth of the proposition that the stated value does not differ from the truth by more than e. Thus, to assert that the azimuth of a certain line west of north as derived from a certain series of observations is $59''.0 \pm 0''.5$, is to assert that it is as likely that the true value of that azimuth is between $58''.5$ and $59''.5$ W. of N. as that it is some value outside of these limits. To assert that the probable error of a single observation in a series $= \pm 0''.5$, is to assert that it is an even chance that any particular observation is within $0''.5$ in either direction of the truth. Or, what is the same thing, it is to assert that if a long series of such observations were made, the chances are that one-half of the observations would give results within $0''.5$ of the truth, and one-half would give results differing from the truth by more than $0''.5$.

More accurately, perhaps, the probable error should be regarded as referring to accidental* errors only, without reference to possible constant errors. Thus the above statements should be modified to read as follows: To assert that the azimuth of a certain line west of north, as derived from a certain series of observations, is $59''.0 \pm 0''.5$, is to assert that if an infinite number of such observations were taken, under the same average conditions, their mean would be as likely to lie between $58''.5$ and $59''.5$ W. of N. as to fall outside those limits. And to assert that the probable error of a single observation $= \pm 0''.5$, is to assert that it is an even chance that that particular observation is within $0''.5$ in either direction of the mean which would result from an infinite number of such observations, made under the same average condi-

* For the distinction between accidental and constant errors see foot-note to § 74.

tions. The second form of the statement is non-committal as to possible constant errors affecting all the series alike, which would not be eliminated by increasing the number of observations. Such a constant error would be introduced into an observed azimuth by placing the azimuth light, unknowingly, a little to one side of the monument which it is supposed to indicate.

284. There seems to be some confusion between these two conceptions of the probable error. It is a common mistake among those who use least squares to derive a probable error by methods which correspond to the second form of statement above, and then to assume that the first form of statement is true. Hence one is always on the safe side to assume the second form of statement to give the true meaning of the probable error, and to form an estimate of the possibility of a constant error from other sources of information. If there is no possibility of a constant error in the observations, the two forms of statement are identical.

285. The relation between the probable error of a single observation, and the total range between the largest and smallest values given by such observations, is as follows: If the probable error of a single observation is $\pm e$, one is to expect that if a large number of such observations were made, only about one per cent will fall outside a total range of $7\frac{1}{2}$ times e. Or, if the probable error of a single observation is $\pm 0''.5$, only about one observation in one hundred would be expected to fall outside a total range of $3''.8$.

The Latitude Variation.

286. Until a few years ago it was supposed that the latitude of a given station was invariable. During the last few years a vigorous investigation of that assumption has been made, both by means of new series of observations of the highest degree of accuracy planned especially for the purpose, and by the re-examination of various old series of observations at the fixed observatories. The result of these investigations may be briefly stated as follows: The axis of rotation of the Earth does not coincide exactly with its axis of figure. By axis of figure is meant that line about which its moment of inertia is a maximum. Roughly speaking, the axis of figure describes a cone, with its vertex at the centre of the Earth, about the axis of rotation once in 428 days. The motion of the pole of figure about the pole of rotation during that interval is roughly an ellipse with a major diameter of about 60 feet, described in the direction of decreasing west longitudes, that is, in a counter-clockwise direction, as seen from above at the north pole. This motion is combined also with one of a period of one year, and is variable as to the diameter and position of the ellipse, and otherwise, so that the above statement serves simply as an approximate description of the motion. The general law governing the motion is not yet known, and all formulæ as yet derived for predicting the future motion are empirical.

The direction of gravity at a given station is sensibly constant as referred to the *axis of figure*, that is, as referred to the solid Earth. But the latitude as measured is referred to the *axis of rotation*—the plane perpendicular to that line, the equator, being the plane to which the declinations of the stars are referred. Hence the latitude of every station on the Earth varies through a range equal to twice the angle between the two axes, a range of about $0''.6$. It changes from its maximum to its minimum value, and back again to the maximum, once, roughly speaking, every 428 days. This motion can be traced in the past, but not as yet predicted for the distant future.

287. Three examples of the long series of latitude observations made with zenith telescopes for the special purpose of determining the latitude variation may be found in Coast and Geodetic Survey Reports for 1892, part 2, pp. 1–159, and for 1893, part 2, pp. 440–508. The principal investigations of the variation by means of latitude observations not specially planned for the purpose have been made by Prof. S. C. Chandler. Indeed, his investigations first proved satisfactorily that such variations are a fact. His results will be found published in various numbers of the Astronomical Journal for several years past. A general statement of the "Mechanical interpretation of the variations of latitudes," by Prof. R. S. Woodward, will be found in the Astronomical Journal No. 345, May 21, 1895.

Station Errors and the Economics of Observing.

288. The author cannot close this book without calling attention briefly to one phase of geodetic astronomy to which little attention has apparently been paid, but which is of great importance in planning the astronomical work in connection with a geodetic survey, namely, the relation between the economics of observing and station errors.

Broadly stated, the purpose of the astronomical observations made in connection with a geodetic survey is to determine the relation between the actual figure of the Earth as defined by the lines of gravity and the assumed mean figure upon which the geodetic computations are based.* This is the purpose, whether the astronomical observations be used simply as a check upon the geodetic operations, or whether they be used as a means of determining the mean figure of the Earth. In determining the relation between the actual figure and the assumed mean figure three classes of errors are

* See foot-note to § 15.

encountered: the errors of the geodetic observations; the errors of the astronomical observations; and the errors due to the fact that only a few scattered stations can be occupied on the large area to be covered, and that the station errors as derived for these few points must be assumed to represent the facts for the whole *area*.

Neglect the first class of errors, as being in the province of geodesy rather than astronomy. The duty of the engineer when planning the astronomical work of a survey is to so fix the number and character of the observations at each station, and the number and position of the stations, as to make the combined errors of the second and third classes a minimum for a given expenditure. By increasing the number of observations at a station the errors of the second class may be diminished, the relation between the number of observations and the error of the result being that said error is inversely proportional to the square of the number of observations in the most favorable case (of no tendency to constant errors in the series of observations). If there are any constant errors affecting the series, then the increase in accuracy with increase in the number of observations is slower than that stated above. The third class of errors may be reduced by increasing the number of stations, and distributing them as uniformly as possible, so as to diminish the area to which the result from each station is assumed to apply.

289. To illustrate, suppose the latitude observations for a geodetic survey of a State are being planned. Let us suppose that the engineer knows that with the available zenith telescopes and star places an observer can secure a latitude with a probable error of about $\pm$ 0''.10 from observations on a single evening, and that he can reduce this to $\pm$ 0''.06 by observing on four evenings. Let us assume that he estimates that it will cost the same, on an average, to observe on four nights at one station, as to observe at three stations on three different nights.* Should he plan to observe at ten different stations distributed uniformly over the State on four nights at each station, or at thirty stations uniformly distributed for one night only at each? Obviously the answer depends mainly on the magnitude of the station errors to be expected. If he estimates the station error by consulting the results obtained on the U. S. Lake Survey, he will expect an average station error of nearly $4''$, with a maximum exceeding $10''$.† If he consults the report ‡ of the "Survey of the Northern Boundary from the Lake of the Woods to the Rocky Mountains" he finds that the average station error there was $2''$, with a maximum of $8''$, and that in one case six successive stations on a total distance of 100 miles along the line showed a nearly uniform change of about $0''.14$ per mile in one direction. If he consults the published results of still other surveys his estimate of the station errors to be expected will not be materially altered. Does it not seem evident that under such circumstances the thirty stations should be occupied on one night each? Yet the usual practice of geodetic surveys in this country corresponds rather to the plan of observing at 10 stations on 4 nights each, even though the observation error in the result from a single night is upon an average only one-twentieth, say, of the station error.

With longitudes and azimuths it will be found that the ratio of the errors of the astronomical observations to the station errors is somewhat larger, but not enough larger to materially modify the above economic problem.

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* It being expected that the observations are to be taken at triangulation stations by the same observers who measure the horizontal angles of the triangulation.

† See Professional Papers of the Corps of Engineers No. 24 (Lake Survey Report), p. 814.

‡ Plate opposite page 267 of that report.

290. CONVERSION OF MEAN SOLAR TIME INTO SIDEREAL.

Correction to be added to a mean solar interval to obtain the corresponding sidereal interval.
(See § 23.)

Mean Solar.	0^h	1^h	2^h	3^h	4^h	5^h	For Seconds.	
0^m	0^m $00^s.000$	0^m $09^s.856$	0^m $19^s.713$	0^m $29^s.569$	0^m $39^s.426$	0^m $49^s.282$	0^s	$0^s.000$
1	0 00 .164	0 10 .021	0 19 .877	0 29 .734	0 39 .590	0 49 .447	1	0 .003
2	0 00 .329	0 10 .185	0 20 .041	0 29 .898	0 39 .754	0 49 .611	2	0 .005
3	0 00 .493	0 10 .349	0 20 .206	0 30 .062	0 39 .919	0 49 .775	3	0 .008
4	0 00 .657	0 10 .514	0 20 .370	0 30 .227	0 40 .083	0 49 .939	4	0 .011
5	0 00 .821	0 10 .678	0 20 .534	0 30 .391	0 40 .247	0 50 .104	5	0 .014
6	0 00 .986	0 10 .842	0 20 .699	0 30 .555	0 40 .412	0 50 .268	6	0 .016
7	0 01 .150	0 11 .006	0 20 .863	0 30 .719	0 40 .576	0 50 .432	7	0 .019
8	0 01 .314	0 11 .171	0 21 .027	0 30 .884	0 40 .740	0 50 .597	8	0 .022
9	0 01 .478	0 11 .335	0 21 .191	0 31 .048	0 40 .904	0 50 .761	9	0 .025
10	0 01 .643	0 11 .499	0 21 .356	0 31 .212	0 41 .069	0 50 .925	10	0 .027
11	0 01 .807	0 11 .663	0 21 .520	0 31 .376	0 41 .233	0 51 .089	11	0 .030
12	0 01 .971	0 11 .828	0 21 .684	0 31 .541	0 41 .397	0 51 .254	12	0 .033
13	0 02 .136	0 11 .992	0 21 .849	0 31 .705	0 41 .561	0 51 .418	13	0 .036
14	0 02 .300	0 12 .156	0 22 .013	0 31 .869	0 41 .726	0 51 .582	14	0 .038
15	0 02 .464	0 12 .321	0 22 .177	0 32 .034	0 41 .890	0 51 .746	15	0 .041
16	0 02 .628	0 12 .485	0 22 .341	0 32 .198	0 42 .054	0 51 .911	16	0 .044
17	0 02 .793	0 12 .649	0 22 .506	0 32 .362	0 42 .219	0 52 .075	17	0 .047
18	0 02 .957	0 12 .813	0 22 .670	0 32 .526	0 42 .383	0 52 .239	18	0 .049
19	0 03 .121	0 12 .978	0 22 .834	0 32 .691	0 42 .547	0 52 .404	19	0 .052
20	0 03 .285	0 13 .142	0 22 .998	0 32 .855	0 42 .711	0 52 .568	20	0 .055
21	0 03 .450	0 13 .306	0 23 .163	0 33 .019	0 42 .876	0 52 .732	21	0 .057
22	0 03 .614	0 13 .471	0 23 .327	0 33 .183	0 43 .040	0 52 .896	22	0 .060
23	0 03 .778	0 13 .635	0 23 .491	0 33 .348	0 43 .204	0 53 .061	23	0 .063
24	0 03 .943	0 13 .799	0 23 .656	0 33 .512	0 43 .368	0 53 .225	24	0 .066
25	0 04 .107	0 13 .963	0 23 .820	0 33 .676	0 43 .533	0 53 .389	25	0 .068
26	0 04 .271	0 14 .128	0 23 .984	0 33 .841	0 43 .697	0 53 .554	26	0 .071
27	0 04 .435	0 14 .292	0 24 .148	0 34 .005	0 43 .861	0 53 .718	27	0 .074
28	0 04 .600	0 14 .456	0 24 .313	0 34 .169	0 44 .026	0 53 .882	28	0 .077
29	0 04 .764	0 14 .620	0 24 .477	0 34 .333	0 44 .190	0 54 .046	29	0 .079
30	0 04 .928	0 14 .785	0 24 .641	0 34 .498	0 44 .354	0 54 .211	30	0 .082
31	0 05 .093	0 14 .949	0 24 .805	0 34 .662	0 44 .518	0 54 .375	31	0 .085
32	0 05 .257	0 15 .113	0 24 .970	0 34 .826	0 44 .683	0 54 .539	32	0 .088
33	0 05 .421	0 15 .278	0 25 .134	0 34 .990	0 44 .847	0 54 .703	33	0 .090
34	0 05 .587	0 15 .442	0 25 .298	0 35 .155	0 45 .011	0 54 .868	34	0 .093
35	0 05 .750	0 15 .606	0 25 .463	0 35 .319	0 45 .176	0 55 .032	35	0 .096
36	0 05 .914	0 15 .770	0 25 .627	0 35 .483	0 45 .340	0 55 .196	36	0 .099
37	0 06 .078	0 15 .935	0 25 .791	0 35 .648	0 45 .504	0 55 .361	37	0 .101
38	0 06 .242	0 16 .099	0 25 .955	0 35 .812	0 45 .668	0 55 .525	38	0 .104
39	0 06 .407	0 16 .263	0 26 .120	0 35 .976	0 45 .833	0 55 .689	39	0 .107
40	0 06 .571	0 16 .427	0 26 .284	0 36 .140	0 45 .997	0 55 .853	40	0 .110
41	0 06 .735	0 16 .592	0 26 .448	0 36 .305	0 46 .161	0 56 .018	41	0 .112
42	0 06 .900	0 16 .756	0 26 .612	0 36 .469	0 46 .325	0 56 .182	42	0 .115
43	0 07 .064	0 16 .920	0 26 .777	0 36 .633	0 46 .490	0 56 .346	43	0 .118
44	0 07 .228	0 17 .085	0 26 .941	0 36 .798	0 46 .654	0 56 .510	44	0 .120
45	0 07 .392	0 17 .249	0 27 .105	0 36 .962	0 46 .818	0 56 .675	45	0 .123
46	0 07 .557	0 17 .413	0 27 .270	0 37 .126	0 46 .983	0 56 .839	46	0 .126
47	0 07 .721	0 17 .577	0 27 .434	0 37 .290	0 47 .147	0 57 .003	47	0 .129
48	0 07 .885	0 17 .742	0 27 .598	0 37 .455	0 47 .311	0 57 .168	48	0 .131
49	0 08 .049	0 17 .906	0 27 .762	0 37 .619	0 47 .475	0 57 .332	49	0 .134
50	0 08 .214	0 18 .070	0 27 .927	0 37 .783	0 47 .640	0 57 .496	50	0 .137
51	0 08 .378	0 18 .234	0 28 .091	0 37 .947	0 47 .804	0 57 .660	51	0 .140
52	0 08 .542	0 18 .399	0 28 .255	0 38 .112	0 47 .968	0 57 .825	52	0 .142
53	0 08 .707	9 18 .563	0 28 .420	0 38 .276	0 48 .132	0 57 .989	53	0 .145
54	0 08 .871	0 18 .727	0 28 .584	0 38 .440	0 48 .297	0 58 .153	54	0 .148
55	0 09 .035	0 18 .892	0 28 .748	0 38 .605	0 48 .461	0 58 .317	55	0 .151
56	0 09 .199	0 19 .056	0 28 .912	0 38 .769	0 48 .625	0 58 .482	56	0 .153
57	0 09 .364	0 19 .220	0 29 .077	0 38 .933	0 48 .790	0 58 .646	57	0 .156
58	0 09 .528	0 19 .384	0 29 .241	0 39 .097	0 48 .954	0 58 .810	58	0 .159
59	0 09 .692	0 19 .549	0 29 .405	0 39 .262	0 49 .118	0 58 .975	59	0 .162
Mean Solar.	0^h	1^h	2^h	3^h	4^h	5^h	For Seconds.	

CONVERSION OF MEAN SOLAR TIME INTO SIDEREAL.

Correction to be added to a mean solar interval to obtain the corresponding sidereal interval.

Mean Solar.	6^h	7^h	8^h	9^h	10^h	11^h	For Seconds.	
0^m	0^m $59^s.139$	1^m $08^s.995$	1^m $18^s.852$	1^m $28^s.708$	1^m $38^s.565$	1^m $48^s.421$	0^s	$0^s.000$
1	0 59 .303	1 09 .160	1 19 .016	1 28 .873	1 38 .729	1 48 .585	1	0 .003
2	0 59 .467	1 09 .324	1 19 .186	1 29 .037	1 38 .893	1 48 .750	2	0 .005
3	0 59 .632	1 09 .488	1 19 .345	1 29 .201	1 39 .058	1 48 .914	3	0 .008
4	0 59 .796	1 09 .652	1 19 .509	1 29 .365	1 39 .222	1 49 .078	4	0 .011
5	0 59 .960	1 09 .817	1 19 .673	1 29 .530	1 39 .386	1 49 .243	5	0 .014
6	1 00 .124	1 09 .981	1 19 .837	1 29 .694	1 39 .550	1 49 .407	6	0 .016
7	1 00 .289	1 10 .145	1 20 .002	1 29 .858	1 39 .715	1 49 .571	7	0 .019
8	1 00 .453	1 10 .310	1 20 .166	1 30 .022	1 39 .879	1 49 .735	8	0 .022
9	1 00 .617	1 10 .474	1 20 .330	1 30 .187	1 40 .043	1 49 .900	9	0 .025
10	1 00 .782	1 10 .638	1 20 .495	1 30 .351	1 40 .207	1 50 .064	10	0 .027
11	1 00 .946	1 10 .802	1 20 .659	1 30 .515	1 40 .372	1 50 .228	11	0 .030
12	1 01 .110	1 10 .967	1 20 .823	1 30 .680	1 40 .536	1 50 .393	12	0 .033
13	1 01 .274	1 11 .131	1 20 .987	1 30 .844	1 40 .700	1 50 .557	13	0 .036
14	1 01 .439	1 11 .295	1 21 .152	1 31 .008	1 40 .865	1 50 .721	14	0 .038
15	1 01 .603	1 11 .459	1 21 .316	1 31 .172	1 41 .029	1 50 .885	15	0 .041
16	1 01 .767	1 11 .624	1 21 .480	1 31 .337	1 41 .193	1 51 .050	16	0 .044
17	1 01 .932	1 11 .788	1 21 .644	1 31 .501	1 41 .357	1 51 .214	17	0 .047
18	1 02 .096	1 11 .952	1 21 .809	1 31 .665	1 41 .522	1 51 .378	18	0 .049
19	1 02 .260	1 12 .117	1 21 .973	1 31 .829	1 41 .686	1 51 .542	19	0 .052
20	1 02 .424	1 12 .281	1 22 .137	1 31 .994	1 41 .850	1 51 .707	20	0 .055
21	1 02 .589	1 12 .445	1 22 .302	1 32 .158	1 42 .015	1 51 .871	21	0 .057
22	1 02 .753	1 12 .609	1 22 .466	1 32 .332	1 42 .179	1 52 .035	22	0 .060
23	1 02 .917	1 12 .774	1 22 .630	1 32 .487	1 42 .343	1 52 .200	23	0 .063
24	1 03 .081	1 12 .938	1 22 .794	1 32 .651	1 42 .507	1 52 .364	24	0 .066
25	1 03 .246	1 13 .102	1 22 .959	1 32 .815	1 42 .672	1 52 .528	25	0 .068
26	1 03 .410	1 13 .266	1 23 .123	1 32 .979	1 42 .836	1 52 .692	26	0 .071
27	1 03 .574	1 13 .431	1 23 .287	1 33 .144	1 43 .000	1 52 .857	27	0 .074
28	1 03 .739	1 13 .595	1 23 .451	1 33 .308	1 43 .164	1 53 .021	28	0 .077
29	1 03 .903	1 13 .759	1 23 .616	1 33 .472	1 43 .329	1 53 .185	29	0 .079
30	1 04 .067	1 13 .924	1 23 .780	1 33 .637	1 43 .493	1 53 .349	30	0 .082
31	1 04 .231	1 14 .088	1 23 .944	1 33 .801	1 43 .657	1 53 .514	31	0 .085
32	1 04 .396	1 14 .252	1 24 .109	1 33 .965	1 43 .822	1 53 .678	32	0 .088
33	1 04 .560	1 14 .416	1 24 .273	1 34 .129	1 43 .986	1 53 .842	33	0 .090
34	1 04 .724	1 14 .581	1 24 .437	1 34 .294	1 44 .150	1 54 .007	34	0 .093
35	1 04 .888	1 14 .745	1 24 .601	1 34 .458	1 44 .314	1 54 .171	35	0 .096
36	1 05 .053	1 14 .909	1 24 .766	1 34 .622	1 44 .479	1 54 .335	36	0 .099
37	1 05 .217	1 15 .073	1 24 .930	1 34 .786	1 44 .643	1 54 .499	37	0 .101
38	1 05 .381	1 15 .238	1 25 .094	1 34 .951	1 44 .807	1 54 .664	38	0 .104
39	1 05 .546	1 15 .402	1 25 .259	1 35 .115	1 44 .971	1 54 .828	39	0 .107
40	1 05 .710	1 15 .566	1 25 .423	1 35 .279	1 45 .136	1 54 .992	40	0 .110
41	1 05 .874	1 15 .731	1 25 .587	1 35 .444	1 45 .300	1 55 .156	41	0 .112
42	1 06 .038	1 15 .895	1 25 .751	1 35 .608	1 45 .464	1 55 .321	42	0 .115
43	1 06 .203	1 16 .059	1 25 .916	1 35 .772	1 45 .629	1 55 .485	43	0 .118
44	1 06 .367	1 16 .223	1 26 .080	1 35 .936	1 45 .793	1 55 .649	44	0 .120
45	1 06 .531	1 16 .388	1 26 .244	1 36 .101	1 45 .957	1 55 .814	45	0 .123
46	1 06 .695	1 16 .552	1 26 .408	1 36 .265	1 46 .121	1 55 .978	46	0 .126
47	1 06 .860	1 16 .716	1 26 .573	1 36 .429	1 46 .286	1 56 .142	47	0 .129
48	1 07 .024	1 16 .881	1 26 .737	1 36 .593	1 46 .450	1 56 .306	48	0 .131
49	1 07 .188	1 17 .045	1 26 .901	1 36 .758	1 46 .614	1 56 .471	49	0 .134
50	1 07 .353	1 17 .209	1 27 .066	1 36 .922	1 46 .778	1 56 .635	50	0 .137
51	1 07 .517	1 17 .373	1 27 .230	1 37 .086	1 46 .943	1 56 .799	51	0 .140
52	1 07 .681	1 17 .538	1 27 .394	1 37 .251	1 47 .107	1 56 .964	52	0 .142
53	1 07 .845	1 17 .702	1 27 .558	1 37 .415	1 47 .271	1 57 .128	53	0 .145
54	1 08 .010	1 17 .866	1 27 .723	1 37 .579	1 47 .436	1 57 .292	54	0 .148
55	1 08 .174	1 18 .030	1 27 .887	1 37 .743	1 47 .600	1 57 .456	55	0 .151
56	1 08 .338	1 18 .195	1 28 .051	1 37 .908	1 47 .764	1 57 .621	56	0 .153
57	1 08 .502	1 18 .359	1 28 .215	1 38 .072	1 47 .928	1 57 .785	57	0 .156
58	1 08 .667	1 18 .523	1 28 .380	1 38 .236	1 48 .093	1 57 .949	58	0 .159
59	1 08 .831	1 18 .688	1 28 .544	1 38 .400	1 48 .257	1 58 .113	59	0 .162
Mean Solar.	6^h	7^h	8^h	9^h	10^h	11^h	For Seconds.	

CONVERSION OF MEAN SOLAR TIME INTO SIDEREAL.

Correction to be added to a mean solar interval to obtain the corresponding sidereal interval.

Mean Solar.	12^h	13^h	14^h	15^h	16^h	17^h	For Seconds.	
0^m	1^m 58^s.278	2^m 08^s.134	2^m 17^s.991	2^m 27^s.847	2^m 37^s.704	2^m 47^s.560	0^s	0^s.000
1	1 58 .442	2 08 .298	2 18 .155	2 28 .011	2 37 .868	2 47 .724	1	0 .003
2	1 58 .606	2 08 .463	2 18 .319	2 28 .176	2 38 .032	2 47 .889	2	0 .005
3	1 58 .771	2 08 .627	2 18 .483	2 28 .340	2 38 .196	2 48 .053	3	0 .008
4	1 58 .935	2 08 .791	2 18 .648	2 28 .504	2 38 .361	2 48 .217	4	0 .011
5	1 59 .099	2 08 .956	2 18 .812	2 28 .668	2 38 .525	2 48 .381	5	0 .014
6	1 59 .263	2 09 .120	2 18 .976	2 28 .833	2 38 .689	2 48 .546	6	0 .016
7	1 59 .428	2 09 .284	2 19 .141	2 28 .997	2 38 .854	2 48 .710	7	0 .019
8	1 59 .592	2 09 .448	2 19 .305	2 29 .161	2 39 .018	2 48 .874	8	0 .022
9	1 59 .756	2 09 .613	2 19 .469	2 29 .326	2 39 .182	2 49 .039	9	0 .025
10	1 59 .920	2 09 .777	2 19 .633	2 29 .490	2 39 .346	2 49 .203	10	0 .027
11	2 00 .085	2 09 .941	2 19 .798	2 29 .654	2 39 .511	2 49 .367	11	0 .030
12	2 00 .249	2 10 .105	2 19 .962	2 29 .818	2 39 .675	2 49 .531	12	0 .033
13	2 00 .413	2 10 .270	2 20 .126	2 29 .983	2 39 .839	2 49 .696	13	0 .036
14	2 00 .578	2 10 .434	2 20 .290	2 30 .147	2 40 .003	2 49 .860	14	0 .038
15	2 00 .742	2 10 .598	2 20 .455	2 30 .311	2 40 .168	2 50 .024	15	0 .041
16	2 00 .906	2 10 .763	2 20 .619	2 30 .476	2 40 .332	2 50 .188	16	0 .044
17	2 01 .070	2 10 .927	2 20 .783	2 30 .640	2 40 .496	2 50 .353	17	0 .047
18	2 01 .235	2 11 .091	2 20 .948	2 30 .804	2 40 .661	2 50 .517	18	0 .049
19	2 01 .399	2 11 .255	2 21 .112	2 30 .968	2 40 .825	2 50 .681	19	0 .052
20	2 01 .563	2 11 .420	2 21 .276	2 31 .133	2 40 .989	2 50 .846	20	0 .055
21	2 01 .727	2 11 .584	2 21 .440	2 31 .297	2 41 .153	2 51 .010	21	0 .057
22	2 01 .892	2 11 .748	2 21 .605	2 31 .461	2 41 .318	2 51 .174	22	0 .060
23	2 02 .056	2 11 .912	2 21 .769	2 31 .625	2 41 .482	2 51 .338	23	0 .063
24	2 02 .220	2 12 .077	2 21 .933	2 31 .790	2 41 .646	2 51 .503	24	0 .066
25	2 02 .385	2 12 .241	2 22 .098	2 31 .954	2 41 .810	2 51 .667	25	0 .068
26	2 02 .549	2 12 .405	2 22 .262	2 32 .118	2 41 .975	2 51 .831	26	0 .071
27	2 02 .713	2 12 .570	2 22 .426	2 32 .283	2 42 .139	2 51 .995	27	0 .074
28	2 02 .877	2 12 .734	2 22 .590	2 32 .447	2 42 .303	2 52 .160	28	0 .077
29	2 03 .042	2 12 .898	2 22 .755	2 32 .611	2 42 .468	2 52 .324	29	0 .079
30	2 03 .206	2 13 .062	2 22 .919	2 32 .775	2 42 .632	2 52 .488	30	0 .082
31	2 03 .370	2 13 .227	2 23 .083	2 32 .940	2 42 .796	2 52 .653	31	0 .085
32	2 03 .534	2 13 .391	2 23 .247	2 33 .104	2 42 .960	2 52 .817	32	0 .088
33	2 03 .699	2 13 .555	2 23 .412	2 33 .268	2 43 .125	2 52 .981	33	0 .090
34	2 03 .863	2 13 .720	2 23 .576	2 33 .432	2 43 .289	2 53 .145	34	0 .093
35	2 04 .027	2 13 .884	2 23 .740	2 33 .597	2 43 .453	2 53 .310	35	0 .096
36	2 04 .192	2 14 .048	2 23 .905	2 33 .761	2 43 .617	2 53 .474	36	0 .099
37	2 04 .356	2 14 .212	2 24 .069	2 33 .925	2 43 .782	2 53 .638	37	0 .101
38	2 04 .520	2 14 .377	2 24 .233	2 34 .090	1 43 .946	2 53 .803	38	0 .104
39	2 04 .684	2 14 .541	2 24 .397	2 34 .254	2 44 .110	2 53 .967	39	0 .107
40	2 04 .849	2 14 .705	2 24 .562	2 34 .418	2 44 .275	2 54 .131	40	0 .110
41	2 05 .013	2 14 .869	2 24 .726	2 34 .582	2 44 .439	2 54 .295	41	0 .112
42	2 05 .177	2 15 .034	2 24 .890	2 34 .747	2 44 .603	2 54 .460	42	0 .115
43	2 05 .342	2 15 .198	2 25 .054	2 34 .911	2 44 .767	2 54 .624	43	0 .118
44	2 05 .506	2 15 .362	2 25 .219	2 35 .075	2 44 .932	2 54 .788	44	0 .120
45	2 05 .670	2 15 .527	2 25 .383	2 35 .239	2 45 .096	2 54 .952	45	0 .123
46	2 05 .834	2 15 .691	2 25 .547	2 35 .404	2 45 .260	2 55 .117	46	0 .126
47	2 05 .999	2 15 .855	2 25 .712	2 35 .568	2 45 .425	2 55 .281	47	0 .129
48	2 06 .163	2 16 .019	2 25 .876	2 35 .732	2 45 .589	2 55 .445	48	0 .131
49	2 06 .327	2 16 .184	2 26 .040	2 35 .897	2 45 .753	2 55 .610	49	0 .134
50	2 06 .491	2 16 .348	2 26 .204	2 36 .061	2 45 .917	2 55 .774	50	0 .137
51	2 06 .656	2 16 .512	2 26 .369	2 36 .225	2 46 .082	2 55 .938	51	0 .140
52	2 06 .820	2 16 .676	2 26 .533	2 36 .389	2 46 .246	2 56 .102	52	0 .142
53	2 06 .984	2 16 .841	2 26 .697	2 36 .554	2 46 .410	2 56 .267	53	0 .145
54	2 07 .149	2 17 .005	2 26 .861	2 36 .718	2 46 .574	2 56 .431	54	0 .148
55	2 07 .313	2 17 .169	2 27 .026	2 36 .882	2 46 .739	2 56 .595	55	0 .151
56	2 07 .477	2 17 .334	2 27 .190	2 37 .047	2 46 .903	2 56 .759	56	0 .153
57	2 07 .641	2 17 .498	2 27 .354	2 37 .211	2 47 .067	2 56 .924	57	0 .156
58	2 07 .806	2 17 .662	2 27 .519	2 37 .375	2 47 .232	2 57 .088	58	0 .159
59	2 07 .970	2 17 .826	2 27 .683	2 37 .539	2 47 .396	2 57 .252	59	0 .162
Mean Solar.	12^h	13^h	14^h	15^h	16^h	17^h	For Seconds.	

CONVERSION OF MEAN SOLAR TIME INTO SIDEREAL.

Correction to be added to a mean solar interval to obtain the corresponding sidereal interval.

Mean Solar.	18^h	19^h	20^h	21^h	22^h	23^h	For Seconds.
0^m	$2^m\ 57^s.417$	$3^m\ 07^s.273$	$3^m\ 17^s.129$	$3^m\ 26^s.986$	$3^m\ 36^s.842$	$3^m\ 46^s.699$	0^s $0^s.000$
1	2 57 .581	3 07 .437	3 17 .294	3 27 .150	3 37 .007	3 46 .863	1 0 .003
2	2 57 .745	3 07 .602	3 17 .458	3 27 .315	3 37 .171	3 47 .027	2 0 .005
3	2 57 .909	3 07 .766	3 17 .622	3 27 .479	3 37 .335	3 47 .192	3 0 .008
4	2 58 .074	3 07 .930	3 17 .787	3 27 .643	3 37 .500	3 47 .356	4 0 .011
5	2 58 .238	3 08 .094	3 17 .951	3 27 .807	3 37 .664	3 47 .520	5 0 .014
6	2 58 .402	3 08 .259	3 18 .115	3 27 .972	3 37 .828	3 47 .685	6 0 .016
7	2 58 .566	3 08 .423	3 18 .279	3 28 .136	3 37 .992	3 47 .849	7 0 .019
8	2 58 .731	3 08 .587	3 18 .444	3 28 .300	3 38 .157	3 48 .013	8 0 .022
9	2 58 .895	3 08 .751	3. 18 .608	3 28 .464	3 38 .321	3 48 .177	9 0 .025
10	2 59 .059	3 08 .916	3 18 .772	3 28 .629	3 38 .485	3 48 .342	10 0 .027
11	2 59 .224	3 09 .080	2 18 .937	3 28 .793	3 38 .649	3 48 .506	11 0 .030
12	2 59 .388	3 09 .244	3 19 .101	3 28 .957	3 38 .814	3 48 .670	12 0 .033
13	2 59 .552	3 09 .409	3 19 .265	3 29 .122	3 38 .978	3 48 .834	13 0 .036
14	2 59 .716	3 09 .573	3 19 .429	3 29 .286	3 39 .142	3 48 .999	14 0 .038
15	2 59 .881	3 09 .737	3 19 .594	3 29 .450	3 39 .307	3 49 .163	15 0 .041
16	3 00 .045	3 09 .901	3 19 .758	3 29 .614	3 39 .471	3 49 .327	16 0 .044
17	3 00 .209	3 10 .066	3 19 .922	3 29 .779	3 39 .635	3 49 .492	17 0 .047
18	3 00 .373	3 10 .230	3 20 .086	3 29 .943	3 39 .799	3 49 .656	18 0 .049
19	3 00 .538	3 10 .394	3 20 .251	3 30 .107	3 39 .964	3 49 .820	19 0 .052
20	3 00 .702	3 10 .559	3 20 .415	3 30 .271	3 40 .128	3 49 .984	20 0 .055
21	3 00 .866	3 10 .723	3 20 .579	3 30 .436	3 40 .292	3 50 .149	21 0 .057
22	3 01 .031	3 10 .887	3 20 .744	3 30 .600	3 40 .456	3 50 .313	22 0 .060
23	3 01 .195	3 11 .051	3 20 .908	3 30 .764	3 40 .621	3 50 .477	23 0 .063
24	3 01 .359	3 11 .216	3 21 .072	3 30 .929	3 40 .785	3 50 .642	24 0 .066
25	3 01 .523	3 11 .380	3 21 .236	3 31 .093	3 40 .949	3 50 .806	25 0 .068
26	3 01 .688	3 11 .544	3 21 .401	3 31 .257	3 41 .114	3 50 .970	26 0 .071
27	3 01 .852	3 11 .708	3 21 .565	3 31 .421	3 41 .278	3 51 .134	27 0 .074
28	3 02 .016	3 11 .873	3 21 .729	3 31 .586	3 41 .442	3 51 .299	28 0 .077
29	3 02 .181	3 12 .037	3 21 .893	3 31 .750	3 41 .606	3 51 .463	29 0 .079
30	3 02 .345	3 12 .201	3 22 .058	3 31 .914	3 41 .771	3 51 .627	30 0 .082
31	3 02 .509	3 12 .366	3 22 .222	3 32 .078	3 41 .935	3 51 .791	31 0 .085
32	3 02 .673	3 12 .530	3 22 .386	3 32 .243	3 42 .099	3 51 .956	32 0 .088
33	3 02 .838	3 12 .694	3 22 .551	3 32 .407	3 42 .264	3 52 .120	33 0 .090
34	3 03 .002	3 12 .858	3 22 .715	3 32 .571	3 42 .428	3 52 .284	34 0 .093
35	3 03 .166	3 13 .023	3 22 .879	3 32 .736	3 42 .592	3 52 .449	35 0 .096
36	3 03 .330	3 13 .187	3 23 .043	3 32 .900	3 42 .756	3 52 .613	36 0 .099
37	3 03 .495	3 13 .351	3 23 .208	3 33 .064	3 42 .921	3 52 .777	37 0 .101
38	3 03 .659	3 13 .515	3 23 .372	3 33 .228	3 43 .085	3 52 .941	38 0 .104
39	3 03 .823	3 13 .680	3 23 .536	3 33 .393	3 43 .249	3 53 .106	39 0 .107
40	3 03 .988	3 13 .844	3 23 .700	3 33 .557	3 43 .413	3 53 .270	40 0 .110
41	3 04 .152	3 14 .008	3 23 .865	3 33 .721	3 43 .578	3 53 .434	41 0 .112
42	3 04 .316	3 14 .173	3 24 .029	3 33 .886	3 43 .742	3 53 .598	42 0 .115
43	3 04 .480	3 14 .337	3 24 .193	3 34 .050	3 43 .906	3 53 .763	43 0 .118
44	3 04 .645	3 14 .501	3 24 .358	3 34 .214	3 44 .071	3 53 .927	44 0 .120
45	3 04 .809	3 14 .665	3 24 .522	3 34 .378	3 44 .235	3 54 .091	45 0 .123
46	3 04 .973	3 14 .830	3 24 .686	3 34 .543	3 44 .399	3 54 .256	46 0 .126
47	3 05 .137	3 14 .994	3 24 .850	3 34 .707	3 44 .563	3 54 .420	47 0 .129
48	3 05 .302	3 15 .158	3 25 .015	3 34 .871	3 44 .728	3 54 .584	48 0 .131
49	3 05 .466	3 15 .322	3 25 .179	3 35 .035	3 44 .892	3 54 .748	49 0 .134
50	3 05 .630	3 15 .487	3 25 .343	3 35 .200	3 45 .056	3 54 .913	50 0 .137
51	3 05 .795	3 15 .651	3 25 .508	3 35 .364	3 45 .220	3 55 .077	51 0 .140
52	3 05 .959	3 15 .815	3 25 .672	3 35 .528	3 45 .385	3 55 .241	52 0 .142
53	3 06 .123	3 15 .980	3 25 .836	3 35 .693	3 45 .549	3 55 .405	53 0 .145
54	3 06 .287	3 16 .144	3 26 .000	3 35 .857	3 45 .713	3 55 .570	54 0 .148
55	3 06 .452	3 16 .308	3 26 .165	3 36 .021	3 45 .878	3 55 .734	55 0 .151
56	3 06 .616	3 16 .472	3 26 .329	3 36 .185	3 46 .042	3 55 .898	56 0 .153
57	3 06 .780	3 16 .637	3 26 .493	3 36 .350	3 46 .206	3 56 .063	57 0 .156
58	3 06 .944	3 16 .801	3 26 .657	3 36 .514	3 46 .370	3 56 .227	58 0 .159
59	3 07 .109	3 16 .965	3 26 .822	3 36 .678	3 46 .535	3 56 .391	59 0 .162
Mean Solar.	18^h	19^h	20^h	21^h	22^h	23^h	For Seconds.

24 mean solar hours = $24^h\ 03^m\ 56^s.555$ of sidereal time.

291. CONVERSION OF SIDEREAL TIME INTO MEAN SOLAR.

Correction to be subtracted from a sidereal interval to obtain the corresponding mean time interval. (See § 23.)

Sid.	0^h	1^h	2^h	3^h	4^h	5^h	For Seconds.	
0^m	$0^m\ 00^s.000$	$0^m\ 09^s.830$	$0^m\ 19^s.659$	$0^m\ 29^s.489$	$0^m\ 39^s.318$	$0^m\ 49^s.148$	0^s	$0^s.000$
1	0 00 .164	0 09 .993	0 19 .823	0 29 .653	0 39 .482	0 49 .312	1	0 .003
2	0 00 .328	0 10 .157	0 19 .987	0 29 .816	0 39 .646	0 49 .475	2	0 .005
3	0 00 .491	0 10 .321	0 20 .151	0 29 .980	0 39 .810	0 49 .639	3	0 .008
4	0 00 .655	0 10 .485	0 20 .314	0 30 .144	0 39 .974	0 49 .803	4	0 .011
5	0 00 .819	0 10 .649	0 20 .478	0 30 .308	0 40 .137	0 49 .967	5	0 .014
6	0 00 .983	0 10 .813	0 20 .642	0 30 .472	0 40 .301	0 50 .131	6	0 .016
7	0 01 .147	0 10 .976	0 20 .806	0 30 .635	0 40 .465	0 50 .295	7	0 .019
8	0 01 .311	0 11 .140	0 20 .970	0 30 .799	0 40 .629	0 50 .458	8	0 .022
9	0 01 .474	0 11 .304	0 21 .134	0 30 .963	0 40 .793	0 50 .622	9	0 .025
10	0 01 .638	0 11 .468	0 21 .297	0 31 .127	0 40 .956	0 50 .786	10	0 .027
11	0 01 .802	0 11 .632	0 21 .461	0 31 .291	0 41 .120	0 50 .950	11	0 .030
12	0 01 .966	0 11 .795	0 21 .625	0 31 .455	0 41 .284	0 51 .114	12	0 .033
13	0 02 .130	0 11 .959	0 21 .789	0 31 .618	0 41 .448	0 51 .278	13	0 .035
14	0 02 .294	0 12 .123	0 21 .953	0 31 .782	0 41 .612	0 51 .441	14	0 .038
15	0 02 .457	0 12 .287	0 22 .117	0 31 .946	0 41 .776	0 51 .605	15	0 .041
16	0 02 .621	0 12 .451	0 22 .280	0 32 .110	0 41 .939	0 51 .769	16	0 .044
17	0 02 .785	0 12 .615	0 22 .444	0 32 .274	0 42 .103	0 51 .933	17	0 .046
18	0 02 .949	0 12 .778	0 22 .608	0 32 .438	0 42 .267	0 52 .097	18	0 .049
19	0 03 .113	0 12 .942	0 22 .772	0 32 .601	0 42 .431	0 52 .260	19	0 .052
20	0 03 .277	0 13 .106	0 22 .936	0 32 .765	0 42 .595	0 52 .424	20	0 .055
21	0 03 .440	0 13 .270	0 23 .099	0 32 .929	0 42 .759	0 52 588	21	0 .057
22	0 03 .604	0 13 .434	0 23 .263	0 33 .093	0 42 .922	0 52 .752	22	0 .060
23	0 03 .768	0 13 .598	0 23 .427	0 33 .257	0 43 .086	0 52 .916	23	0 .063
24	0 03 .932	0 13 .761	0 23 .591	0 33 .420	0 43 .250	0 53 .080	24	0 .066
25	0 04 .096	0 13 .925	0 23 .755	0 33 .584	0 43 .414	0 53 .243	25	0 .068
26	0 04 .259	0 14 .089	0 23 .919	0 33 .748	0 43 .578	0 53 .407	26	0 .071
27	0 04 .423	0 14 .253	0 24 .082	0 33 .912	0 43 .742	0 53 .571	27	0 .074
28	0 04 .587	0 14 .417	0 24 .246	0 34 .076	0 43 .905	0 53 .735	28	0 .076
29	0 04 .751	0 14 .581	0 24 .410	0 34 .240	0 44 .069	0 53 .899	29	0 .079
30	0 04 .915	0 14 .744	0 24 .574	0 34 .403	0 44 .233	0 54 .063	30	0 082
31	0 05 .079	0 14 .908	0 24 .738	0 34 .567	0 44 .397	0 54 .226	31	0 .085
32	0 05 .242	0 15 .072	0 24 .902	0 34 .731	0 44 .561	0 54 .390	32	0 .087
33	0 05 .406	0 15 .236	0 25 .065	0 34 .895	0 44 .724	0 54 .554	33	0 .090
34	0 05 .570	0 15 .400	0 25 .229	0 35 .059	0 44 .888	0 54 .718	34	0 .093
35	0 05 .734	0 15 .563	0 25 .393	0 35 .223	0 45 .052	0 54 .882	35	0 .096
36	0 05 .898	0 15 .727	0 25 .557	0 35 .386	0 45 .216	0 55 .046	36	0 .098
37	0 06 .062	0 15 .891	0 25 .721	0 35 .550	0 45 .380	0 55 .209	37	0 .101
38	0 06 .225	0 16 .055	0 25 .885	0 35 .714	0 45 .544	0 55 .373	38	0 .104
39	0 06 .389	0 16 .219	0 26 .048	0 35 .878	0 45 .707	0 55 .537	39	0 .106
40	0 06 .553	0 16 .383	0 26 .212	0 36 .042	0 45 .871	0 55 .701	40	0 .109
41	0 06 .717	0 16 .546	0 26 .376	0 36 .206	0 46 .035	0 55 .865	41	0 .112
42	0 06 .881	0 16 .710	0 26 .540	0 36 .369	0 46 .199	0 56 .028	42	0 .115
43	0 07 .045	0 16 .874	0 26 .704	0 36 .533	0 46 .363	0 56 .192	43	0 .117
44	0 07 .208	0 17 .038	0 26 .867	0 36 .697	0 46 .527	0 56 .356	44	0 .120
45	0 07 .372	0 17 .202	0 27 .031	0 36 .861	0 46 .690	0 56 .520	45	0 .123
46	0 07 .536	0 17 .366	0 27 .195	0 37 .025	0 46 .854	0 56 .684	46	0 .126
47	0 07 .700	0 17 .529	0 27 .359	0 37 .188	0 47 .018	0 56 .848	47	0 .128
48	0 07 .864	0 17 .693	0 27 .523	0 37 .352	0 47 .182	0 57 .011	48	0 .131
49	0 08 .027	0 17 .857	0 27 .687	0 37 .516	0 47 .346	0 57 .175	49	0 .134
50	0 08 .191	0 18 .021	0 27 .850	0 37 .680	0 47 .510	0 57 .339	50	0 .137
51	0 08 .355	0 18 .185	0 28 .014	0 37 .844	0 47 .673	0 57 .503	51	0 .139
52	0 08 .519	0 18 .349	0 28 .178	0 38 .008	0 47 .837	0 57 .667	52	0 .142
53	0 08 .683	0 18 .512	0 28 .342	0 38 .171	0 48 .001	0 57 .831	53	0 .145
54	0 08 .847	0 18 .676	0 28 .506	0 38 .335	0 48 .165	0 57 .994	54	0 .147
55	0 09 .010	0 18 .840	0 28 .670	0 38 .499	0 48 .329	0 58 .158	55	0 .150
56	0 09 .174	0 19 .004	0 28 .833	0 38 .663	0 48 .492	0 58 .322	56	0 .153
57	0 09 .338	0 19 .168	0 28 .997	0 38 .827	0 48 .656	0 58 .486	57	0 .156
58	0 09 .502	0 19 .331	0 29 .161	0 38 .991	0 48 .820	0 58 .650	58	0 .158
59	0 09 .666	0 19 .495	0 29 .325	0 39 .154	0 48 .984	0 58 .814	59	0 .161
Sid.	0^h	1^h	2^h	3^h	4^h	5^h	For Seconds.	

CONVERSION OF SIDEREAL TIME INTO MEAN SOLAR.

Correction to be subtracted from a sidereal interval to obtain the corresponding mean time interval.

Sid.	6^h	7^h	8^h	9^h	10^h	11^h	For Seconds.	
0^m	0^m $58^s.977$	1^m $08^s.807$	1^m $18^s.636$	1^m $28^s.466$	1^m $38^s.296$	1^m $48^s.125$	0^s	$0^s.000$
1	0 59 .141	1 08 .971	1 18 .800	1 28 .630	1 38 .459	1 48 .289	1	0 .003
2	0 59 .305	1 09 .135	1 18 .964	1 28 .794	1 38 .623	1 48 .453	2	0 .005
3	0 59 .469	1 09 .298	1 19 .128	1 28 .958	1 38 .787	1 48 .617	3	0 .008
4	0 59 .633	1 09 .462	1 19 .292	1 29 .121	1 38 .951	1 48 .780	4	0 .011
5	0 59 .796	1 09 .626	1 19 .456	1 29 .285	1 39 .115	1 48 .944	5	0 .014
6	0 59 .960	1 09 .790	1 19 .619	1 29 .449	1 39 .279	1 49 .108	6	0 .016
7	1 00 .124	1 09 .954	1 19 .783	1 29 .613	1 39 .442	1 49 .272	7	0 .019
8	1 00 .288	1 10 .118	1 19 .947	1 29 .777	1 39 .606	1 49 .436	8	0 .022
9	1 00 .452	1 10 .281	1 20 .111	1 29 .940	1 39 .770	1 49 .600	9	0 .025
10	1 00 .616	1 10 .445	1 20 .275	1 30 .104	1 39 .934	1 49 .763	10	0 .027
11	1 00 .779	1 10 .609	1 20 .439	1 30 .268	1 40 .098	1 49 .927	11	0 .030
12	1 00 .943	1 10 .773	1 20 .602	1 30 .432	1 40 .261	1 50 .091	12	0 .033
13	1 01 .107	1 10 937	1 20 .766	1 30 .596	1 40 .425	1 50 .255	13	0 .035
14	1 01 .271	1 11 .100	1 20 .930	1 30 .760	1 40 .589	1 50 .419	14	0 .038
15	1 01 .435	1 11 .264	1 21 .094	1 30 .923	1 40 .753	1 50 .483	15	0 .041
16	1 01 .599	1 11 .428	1 21 .258	1 31 .087	1 40 .917	1 50 .746	16	0 .044
17	1 01 .762	1 11 .569	1 21 .422	1 31 .251	1 41 .081	1 50 .910	17	0 .046
18	1 01 .926	1 11 .756	1 21 .585	1 31 .415	1 41 .244	1 51 .074	18	0 .049
19	1 02 .090	1 11 .920	1 21 .749	1 31 .579	1 41 .408	1 51 .238	19	0 .052
20	1 02 .254	1 12 .083	1 21 .913	1 31 .743	1 41 .572	1 51 .402	20	0 .055
21	1 02 .418	1 12 .247	1 22 .077	1 31 .906	1 41 .736	1 51 .565	21	0 .057
22	1 02 .582	1 12 .411	1 22 .241	1 32 .070	1 41 .900	1 51 .729	22	0 .060
23	1 02 .745	1 12 .575	1 22 .404	1 32 .234	1 42 .064	1 51 .893	23	0 .063
24	1 02 .909	1 12 .739	1 22 .568	1 32 .398	1 42 .227	1 52 .057	24	0 .066
25	1 03 .073	1 12 .903	1 22 .732	1 32 .562	1 42 .391	1 52 .221	25	0 .068
26	1 03 .237	1 13 .066	1 22 .896	1 32 .726	1 42 .555	1 52 .385	26	0 .071
27	1 03 .401	1 13 .230	1 23 .060	1 32 .889	1 42 .719	1 52 .548	27	0 .074
28	1 03 .564	1 13 .394	1 23 224	1 33 .053	1 42 .883	1 52 .712	28	0 .076
29	1 03 .728	1 13 588	1 23 .387	1 33 .217	1 43 .047	1 52 .876	29	0 .079
30	1 03 .892	1 13 .722	1 23 .551	1 33 .381	1 43 .210	1 53 .040	30	0 .082
31	1 04 .056	1 13 .886	1 23 .715	1 33 .545	1 43 .374	1 53 .204	31	0 .085
32	1 04 .220	1 14 .049	1 23 .879	1 33 .708	1 43 .538	1 53 .368	32	0 .087
33	1 04 .384	1 14 .213	1 24 .043	1 33 .872	1 43 .702	1 53 .531	33	0 .090
34	1 04 .547	1 14 .377	1 24 .207	1 34 .036	1 43 .866	1 53 .695	34	0 093
35	1 04 .711	1 14 .541	1 24 .370	1 34 .200	1 44 .029	1 53 .859	35	0 .096
36	1 04 .875	1 14 .705	1 24 .534	1 34 .364	1 44 .193	1 54 .023	36	0 .098
37	1 05 .039	1 14 .868	1 24 .698	1 34 .528	1 44 .357	1 54 .187	37	0 .101
38	1 05 .203	1 15 .032	1 24 .862	1 34 .691	1 44 .521	1 54 .351	38	0 .104
39	1 05 .367	1 15 .196	1 25 .026	1 34 .855	1 44 .685	1 54 .514	39	0 .106
40	1 05 .530	1 15 .360	1 25 .190	1 35 .019	1 44 .849	1 54 .678	40	0 .109
41	1 05 .694	1 15 .524	1 25 .353	1 35 .183	1 45 .012	1 54 .842	41	0 .112
42	1 05 .858	1 15 .688	1 25 .517	1 35 .347	1 45 .176	1 55 .006	42	0 .115
43	1 06 .022	1 15 .851	1 25 .681	1 35 .511	1 45 .340	1 55 .170	43	0 .117
44	1 06 .186	1 16 .015	1 25 .845	1 35 .674	1 45 .504	1 55 .333	44	0 .120
45	1 06 .350	1 16 .179	1 26 .009	1 35 .838	1 45 .668	1 55 .497	45	0 .123
46	1 06 .513	1 16 .343	1 26 .172	1 36 .002	1 45 .832	1 55 .661	46	0 .126
47	1 06 .677	1 16 .507	1 26 .336	1 36 .166	1 45 .995	1 55 .825	47	0 .128
48	1 06 .841	1 16 .671	1 26 .500	1 36 .330	1 46 .159	1 55 .989	48	0 .131
49	1 07 .005	1 16 .834	1 26 .664	1 36 .493	1 46 .323	1 56 .153	49	0 .134
50	1 07 .169	1 16 .998	1 26 .828	1 36 .657	1 46 .487	1 56 .316	50	0 .137
51	1 07 .332	1 17 .162	1 26 .992	1 36 .821	1 46 .651	1 56 .480	51	0 .139
52	1 07 .496	1 17 .326	1 27 .155	1 36 .985	1 46 .815	1 56 .644	52	0 .142
53	1 07 .660	1 17 .490	1 27 .319	1 37 .149	1 46 .978	1 56 .808	53	0 .145
54	1 07 .824	1 17 .654	1 27 .483	1 37 .313	1 47 .142	1 56 .972	54	0 .147
55	1 07 .988	1 17 .817	1 27 647	1 37 .476	1 47 .306	1 57 .136	55	0 .150
56	1 08 .152	1 17 .981	1 27 .811	1 37 .640	1 47 .470	1 57 .299	56	0 .153
57	1 08 .315	1 18 .145	1 27 .975	1 37 .804	1 47 .634	1 57 .463	57	0 .156
58	1 08 .479	1 18 .309	1 28 .138	1 37 .968	1 47 .797	1 57 .627	58	0 .158
59	1 08 .643	1 18 .473	1 28 .302	1 38 .132	1 47 .961	1 57 .791	59	0 .161
Sid.	6^h	7^h	8^h	9^h	10^h	11^h	For Seconds.	

CONVERSION OF SIDEREAL TIME INTO MEAN SOLAR.

Correction to be subtracted from a sidereal interval to obtain the corresponding mean time interval.

Sid.	12^h	13^h	14^h	15^h	16^h	17^h	For Seconds.	
0^m	1^m 57^s.955	2^m 07^s.784	2^m 17^s.614	2^m 27^s.443	2^m 37^s.273	2^m 47^s.102	0^s	0^s.000
1	1 58 .119	2 07 .948	2 17 .778	2 27 .607	2 37 .437	2 47 .266	1	0 .003
2	1 58 .282	2 08 .112	2 17 .941	2 27 .771	2 37 .601	2 47 .400	2	0 .005
3	1 58 .446	2 08 .276	2 18 .105	2 27 .935	2 37 .764	2 47 .594	3	0 .008
4	1 58 .610	2 08 .440	2 18 .269	2 28 .099	2 37 .928	2 47 .758	4	0 .011
5	1 58 .774	2 08 .603	2 18 .433	2 28 .263	2 38 .092	2 47 .922	5	0 .014
6	1 58 .938	2 08 .767	2 18 .597	2 28 .426	2 38 .256	2 48 .085	6	0 .016
7	1 59 .101	2 08 .931	2 18 .761	2 28 .590	2 38 .420	2 48 .249	7	0 .019
8	1 59 .265	2 09 .095	2 18 .924	2 28 .754	2 38 .584	2 48 .413	8	0 .022
9	1 59 .429	2 09 .259	2 19 .088	2 28 .918	2 38 .747	2 48 .577	9	0 .025
10	1 59 .593	2 09 .423	2 19 .252	2 29 .082	2 38 .911	2 48 .741	10	0 .027
11	1 59 .757	2 09 .586	2 19 .416	2 29 .245	2 39 .075	2 48 .905	11	0 .030
12	1 59 .921	2 09 .750	2 19 .580	2 29 .409	2 39 .239	2 49 .068	12	0 .033
13	2 00 .084	2 09 .914	2 19 .744	2 29 .573	2 39 .403	2 49 .232	13	0 .035
14	2 00 .248	2 10 .078	2 19 .907	2 29 .737	2 39 .566	2 49 .396	14	0 .038
15	2 00 .412	2 10 .242	2 20 .071	2 29 .901	2 39 .730	2 49 .560	15	0 .041
16	2 00 .576	2 10 .405	2 20 .235	2 30 .065	2 39 .894	2 49 .724	16	0 .044
17	2 00 .740	2 10 .569	2 20 .399	2 30 .228	2 40 .058	2 49 .888	17	0 .046
18	2 00 .904	2 10 .733	2 20 .563	2 30 .392	2 40 .222	2 50 .051	18	0 .049
19	2 01 .067	2 10 .897	2 20 .727	2 30 .556	2 40 .386	2 50 .215	19	0 .052
20	2 01 .231	2 11 .061	2 20 .890	2 30 .720	2 40 .549	2 50 .379	20	0 .055
21	2 01 .395	2 11 .225	2 21 .054	2 30 .884	2 40 .713	2 50 .543	21	0 .057
22	2 01 .559	2 11 .388	2 21 .218	2 31 .048	2 40 .877	2 50 .707	22	0 .060
23	2 01 .723	2 11 .552	2 21 .382	2 31 .211	2 41 .041	2 50 .870	23	0 .063
24	2 01 .887	2 11 .716	2 21 .546	2 31 .375	2 41 .205	2 51 .034	24	0 .066
25	2 02 .050	2 11 .880	2 21 .709	2 31 .539	2 41 .369	2 51 .198	25	0 .068
26	2 02 .214	2 12 .044	2 21 .873	2 31 .703	2 41 .532	2 51 .362	26	0 .071
27	2 02 .378	2 12 .208	2 22 .037	2 31 .867	2 41 .696	2 51 .526	27	0 .074
28	2 02 .542	2 12 .371	2 22 .201	2 32 .031	2 41 .860	2 51 .690	28	0 .076
29	2 02 .706	2 12 .535	2 22 .365	2 32 .194	2 42 .024	2 51 .853	29	0 .079
30	2 02 .869	2 12 .699	2 22 .529	2 32 .358	2 42 .188	2 52 .017	30	0 .082
31	2 03 .033	2 12 .863	2 22 .692	2 32 .522	2 42 .352	2 52 .181	31	0 .085
32	2 03 .197	2 13 .027	2 22 .856	2 32 .686	2 42 .515	2 52 .345	32	0 .087
33	2 03 .361	2 13 .191	2 23 .020	2 32 .850	2 42 .679	2 52 .509	33	0 .090
34	2 03 .525	2 13 .354	2 23 .184	2 33 .013	2 42 .843	2 52 .673	34	0 .093
35	2 03 .689	2 13 .518	2 23 .348	2 33 .177	2 43 .007	2 52 .836	35	0 .096
36	2 03 .852	2 13 .682	2 23 .512	2 33 .341	2 43 .171	2 53 .000	36	0 .098
37	2 04 .016	2 13 .846	2 23 .675	2 33 .505	2 43 .334	2 53 .164	37	0 .101
38	2 04 .180	2 14 .010	2 23 .839	2 33 .669	1 43 .498	2 53 .328	38	0 .104
39	2 04 .344	2 14 .173	2 24 .003	2 33 .833	2 43 .662	2 53 .492	39	0 .106
40	2 04 .508	2 14 .337	2 24 .167	2 33 .966	2 43 .826	2 53 .656	40	0 .109
41	2 04 .672	2 14 .501	2 24 .331	2 34 .160	2 43 .990	2 53 .819	41	0 .112
42	2 04 .835	2 14 .665	2 24 .495	2 34 .324	2 44 .154	2 53 .983	42	0 .115
43	2 04 .999	2 14 .829	2 24 .658	2 34 .488	2 44 .317	2 54 .147	43	0 .117
44	2 05 .163	2 14 .993	2 24 .822	2 34 .652	2 44 .481	2 54 .311	44	0 .120
45	2 05 .327	2 15 .156	2 24 .986	2 34 .816	2 44 .645	2 54 .475	45	0 .123
46	2 05 .491	2 15 .320	2 25 .150	2 34 .979	2 44 .809	2 54 .638	46	0 .126
47	2 05 .655	2 15 .484	2 25 .314	2 35 .143	2 44 .973	2 54 .802	47	0 .128
48	2 05 .818	2 15 .648	2 25 .477	2 35 .307	2 45 .137	2 54 .966	48	0 .131
49	2 05 .982	2 15 .812	2 25 .641	2 35 .471	2 45 .300	2 55 .130	49	0 .134
50	2 06 .146	2 15 .976	2 25 .805	2 35 .635	2 45 .464	2 55 .294	50	0 .137
51	2 06 .310	2 16 .139	2 25 .969	2 35 .798	2 45 .628	2 55 .458	51	0 .139
52	2 06 .474	2 16 .303	2 26 .133	2 35 .962	2 45 .792	2 55 .621	52	0 .142
53	2 06 .637	2 16 .467	2 26 .297	2 36 .126	2 45 .956	2 55 .785	53	0 .145
54	2 06 .801	2 16 .631	2 26 .460	2 36 .290	2 46 .120	2 55 .949	54	0 .147
55	2 06 .965	2 16 .795	2 26 .624	2 36 .454	2 46 .283	2 56 .113	55	0 .150
56	2 07 .129	2 16 .959	2 26 .788	2 36 .618	2 46 .447	2 56 .277	56	0 .153
57	2 07 .293	2 17 .122	2 26 .952	2 36 .781	2 46 .611	2 56 .441	57	0 .156
58	2 07 .457	2 17 .286	2 27 .116	2 36 .945	2 46 .775	2 56 .604	58	0 .158
59	2 07 .620	2 17 .450	2 27 .280	2 37 .109	2 46 .939	2 56 .768	59	0 .161
Sid.	12^h	13^h	14^h	15^h	16^h	17^h	For Seconds.	

CONVERSION OF SIDEREAL TIME INTO MEAN SOLAR.

Correction to be subtracted from a sidereal interval to obtain the corresponding mean time interval.

Sid.	18^h	19^h	20^h	21^h	22^h	23^h	For Seconds.	
0^m	$2^m\ 56^s.932$	$3^m\ 06^s.762$	$3^m\ 16^s.591$	$3^m\ 26^s.421$	$3^m\ 36^s.250$	$3^m\ 46^s.080$	0^s	$0^s.000$
1	2 57 .096	3 06 .925	3 16 .755	3 26 .585	3 36 .414	3 46 .244	1	0 .003
2	2 57 .260	3 07 .089	3 16 .919	3 26 .748	3 36 .578	3 46 .407	2	0 .005
3	2 57 .424	3 07 .253	3 17 .083	3 26 .912	3 36 .742	3 46 .571	3	0 .008
4	2 57 .587	3 07 .417	3 17 .246	3 27 .076	3 36 .906	3 46 .735	4	0 .011
5	2 57 .751	3 07 .581	3 17 .410	3 27 .240	3 37 .069	3 46 .899	5	0 .014
6	2 57 .915	3 07 .745	3 17 .574	3 27 .404	3 37 .233	3 47 .063	6	0 .016
7	2 58 .079	3 07 .908	3 17 .738	3 27 .568	3 37 .397	3 47 .227	7	0 .019
8	2 58 .243	3 08 .072	3 17 .902	3 27 .731	3 37 .561	3 47 .390	8	0 .022
9	2 58 .406	3 08 .236	3 18 .066	3 27 .895	3 37 .725	3 47 .554	9	0 .025
10	2 58 .570	3 08 .400	3 18 .229	3 28 .059	3 37 .889	3 47 .718	10	0 .027
11	2 58 .734	3 08 .564	3 18 .393	3 28 .223	3 38 .052	3 47 .882	11	0 .030
12	2 58 .898	3 08 .728	3 18 .557	3 28 .387	3 38 .216	3 48 .046	12	0 .033
13	2 59 .062	3 08 .891	3 18 .721	3 28 .550	3 38 .380	3 48 .210	13	0 .035
14	2 59 .226	3 09 .055	3 18 .885	3 28 .714	3 38 .544	3 48 .373	14	0 .038
15	2 59 .389	3 09 .219	3 19 .049	3 28 .878	3 38 .708	3 48 .537	15	0 .041
16	2 59 .553	3 09 .383	3 19 .212	3 29 .042	3 38 .871	3 48 .701	16	0 .044
17	2 59 .717	3 09 .547	3 19 .376	3 29 .206	3 39 .035	3 48 .865	17	0 .046
18	2 59 .881	3 09 .710	3 19 .540	3 29 .370	3 39 .199	3 49 .029	18	0 .049
19	3 00 .045	3 09 .874	3 19 .704	3 29 .533	3 39 .363	3 49 .193	19	0 .052
20	3 00 .209	3 10 .038	3 19 .868	3 29 .697	3 39 .527	3 49 .356	20	0 .055
21	3 00 .372	3 10 .202	3 20 .032	3 29 .861	3 39 .691	3 49 .520	21	0 .057
22	3 00 .536	3 10 .366	3 20 .195	3 30 .025	3 39 .854	3 49 .684	22	0 .060
23	3 00 .700	3 10 .530	3 20 .359	3 30 .189	3 40 .018	3 49 .848	23	0 .063
24	3 00 .864	3 10 .693	3 20 .523	3 30 .353	3 40 .182	3 50 .012	24	0 .066
25	3 01 .028	3 10 .857	3 20 .687	3 30 .516	3 40 .346	3 50 .175	25	0 .068
26	3 01 .192	3 11 .021	3 20 .851	3 30 .680	3 40 .510	3 50 .339	26	0 .071
27	3 01 .355	3 11 .185	3 21 .014	3 30 .844	3 40 .674	3 50 .503	27	0 .074
28	3 01 .519	3 11 .349	3 21 .178	3 31 .008	3 40 .837	3 50 .667	28	0 .076
29	3 01 .683	3 11 .513	3 21 .342	3 31 .172	3 41 .001	3 50 .831	29	0 .079
30	3 01 .847	3 11 .676	3 21 .506	3 31 .336	3 41 .165	3 50 .995	30	0 .082
31	3 02 .011	3 11 .840	3 21 .670	3 31 .499	3 41 .329	3 51 .158	31	0 .085
32	3 02 .174	3 12 .004	3 21 .834	3 31 .663	3 41 .493	3 51 .322	32	0 .087
33	3 02 .338	3 12 .168	3 21 .997	3 31 .827	3 41 .657	3 51 .486	33	0 .090
34	3 02 .502	3 12 .332	3 22 .161	3 31 .991	3 41 .820	3 51 .650	34	0 .093
35	3 02 .666	3 12 .496	3 22 .325	3 32 .155	3 41 .984	3 51 .814	35	0 .096
36	3 02 .830	3 12 .659	3 22 .489	3 32 .318	3 42 .148	3 51 .978	36	0 .098
37	3 02 .994	3 12 .823	3 22 .653	3 32 .482	3 42 .312	3 52 .141	37	0 .101
38	3 03 .157	3 12 .987	3 22 .817	3 32 .646	3 42 .476	3 52 .305	38	0 .104
39	3 03 .321	3 13 .151	3 22 .980	3 32 .810	3 42 .639	3 52 .469	39	0 .106
40	3 03 .485	3 13 .315	3 23 .144	3 32 .974	3 42 .803	3 52 .633	40	0 .109
41	3 03 .649	3 13 .478	3 23 .308	3 33 .138	3 42 .967	3 52 .797	41	0 .112
42	3 03 .813	3 13 .642	3 23 .472	3 33 .301	3 43 .131	3 52 .961	42	0 .115
43	3 03 .977	3 13 .806	3 23 .636	3 33 .465	3 43 .295	3 53 .124	43	0 .117
44	3 04 .140	3 13 .970	3 23 .800	3 33 .629	3 43 .459	3 53 .288	44	0 .120
45	3 04 .304	3 14 .134	3 23 .963	3 33 .793	3 43 .622	3 53 .452	45	0 .123
46	3 04 .468	3 14 .298	3 24 .127	3 33 .957	3 43 .786	3 53 .616	46	0 .126
47	3 04 .632	3 14 .461	3 24 .291	3 34 .121	3 43 .950	3 53 .780	47	0 .128
48	3 04 .796	3 14 .625	3 24 .455	3 34 .284	3 44 .114	3 53 .943	48	0 .131
49	3 04 .960	3 14 .789	3 24 .619	3 34 .448	3 44 .278	3 54 .107	49	0 .134
50	3 05 .123	3 14 .953	3 24 .782	3 34 .612	3 44 .442	3 54 .271	50	0 .137
51	3 05 .287	3 15 .117	3 24 .946	3 34 .776	3 44 .605	3 54 .435	51	0 .139
52	3 05 .451	3 15 .281	3 25 .110	3 34 .940	3 44 .769	3 54 .599	52	0 .142
53	3 05 .615	3 15 .444	3 25 .274	3 35 .104	3 44 .933	3 54 .763	53	0 .145
54	3 05 .779	3 15 .608	3 25 .438	3 35 .267	3 45 .097	3 54 .926	54	0 .147
55	3 05 .942	3 15 .772	3 25 .602	3 35 .431	3 45 .261	3 55 .090	55	0 .150
56	3 06 .106	3 15 .936	3 25 .765	3 35 .595	3 45 .425	3 55 .254	56	0 .153
57	3 06 .270	3 16 .100	3 25 .929	3 35 .759	3 45 .588	3 55 .418	57	0 .156
58	3 06 .434	3 16 .264	3 26 .093	3 35 .923	3 45 .752	3 55 .582	58	0 .158
59	3 06 .598	3 16 .427	3 26 .257	3 36 .086	3 45 .916	3 55 .746	59	0 .161
Sid.	18^h	19^h	20^h	21^h	22^h	23^h	For Seconds.	

24 sidereal hours = $24^h - [3^m\ 55^s.909]$ of mean solar time.

292. CHANGE PER YEAR IN THE ANNUAL PRECESSION IN DECLINATION.

In units of the fifth decimal place (unit = $0''.00001$).

The side argument $\left(\frac{d\alpha_m}{dt}\right)$ is the annual variation in right ascension. (See § 42.)

$\frac{d\alpha_m}{dt}$	Right Ascension (α_m).								
	$0^h\ 00^m$	$0^h\ 20^m$	$0^h\ 40^m$	$1^h\ 00^m$	$1^h\ 20^m$	$1^h\ 40^m$	$2^h\ 00^m$	$2^h\ 20^m$	$2^h\ 40^m$
$0^s.0$	− 9	− 9	− 8	− 8	− 8	− 8	− 8	− 7	− 7
1 .0	9	21	34	46	58	69	80	91	100
1 .1	9	23	36	50	63	76	88	99	110
1 .2	9	24	38	53	68	82	95	107	119
1 .3	9	25	41	57	73	88	102	116	128
1 .4	9	26	44	61	78	94	110	124	138
1 .5	9	28	46	65	83	100	117	132	147
1 .6	9	29	49	69	88	106	124	141	156
1 .7	9	30	51	72	93	112	131	149	166
1 .8	9	31	54	76	98	119	139	158	175
1 .9	9	33	56	80	103	125	146	166	185
2 .0	9	34	59	84	108	131	153	174	194
2 .1	9	35	62	88	113	137	161	183	203
2 .2	9	36	64	91	118	143	168	191	213
2 .3	9	38	67	95	123	150	175	199	222
2 .4	9	39	69	99	128	156	182	208	232
2 .5	9	40	72	103	133	162	190	216	241
2 .6	9	42	74	106	138	168	197	224	250
2 .7	9	43	77	110	143	174	204	233	260
2 .8	9	44	79	114	148	180	212	241	269
2 .9	9	46	82	118	153	187	219	250	278
3 .0	9	47	84	121	158	193	226	258	288
3 .1	9	48	87	125	163	199	234	266	297
3 .2	9	49	89	129	168	205	241	275	306
3 .3	9	51	92	133	173	211	248	283	316
3 .4	9	52	94	136	178	218	255	291	325
3 .5	9	53	97	140	183	224	263	300	335
3 .6	9	54	99	144	188	230	270	308	344
3 .7	9	56	102	148	193	236	277	316	353
3 .8	9	57	104	151	197	242	284	325	363
3 .9	9	58	107	155	202	248	292	333	372
4 .0	9	60	110	159	207	254	299	342	381
5 .0	9	72	135	197	257	316	372	425	475
6 .0	9	85	160	234	307	377	445	509	569
7 .0	9	98	186	272	357	439	518	593	663
8 .0	− 9	− 110	− 211	− 310	− 407	− 501	− 591	− 676	− 756
	$12^h\ 00^m$	$12^h\ 20^m$	$12^h\ 40^m$	$13^h\ 00^m$	$13^h\ 20^m$	$13^h\ 40^m$	$14^h\ 00^m$	$14^h\ 20^m$	$14^h\ 40^m$
	Right Ascension (α_m).								

Change all signs when using this lower argument.

CHANGE PER YEAR IN THE ANNUAL PRECESSION IN DECLINATION.

In units of the fifth decimal place (unit = 0″.00001).

The side argument $\left(\frac{d\alpha_m}{dt}\right)$ is the annual variation in right ascension.

$\frac{d\alpha_m}{dt}$	Right Ascension (α_m).								
	3^h 00m	3^h 20m	3^h 40m	4^h 00m	4^h 20m	4^h 40m	5^h 00m	5^h 20m	5^h 40m
0^s.0	− 6	− 6	− 5	− 4	− 4	− 3	− 2	− 2	− 1
1 .0	109	117	124	131	136	140	143	145	146
1 .1	120	128	136	143	149	154	157	160	161
1 .2	130	140	148	156	162	167	171	174	175
1 .3	140	151	160	168	176	181	186	188	190
1 .4	150	162	172	181	189	195	200	203	204
1 .5	161	173	184	194	202	208	214	217	219
1 .6	171	184	196	206	215	222	228	231	233
1 .7	181	195	208	219	228	236	242	246	248
1 .8	192	206	220	232	242	250	256	260	262
1 .9	202	218	232	244	255	263	270	274	277
2 .0	212	229	244	257	268	277	284	289	291
2 .1	222	240	256	270	281	291	298	303	306
2 .2	233	251	268	282	294	304	312	317	320
2 .3	243	262	280	295	308	318	326	332	335
2 .4	254	274	292	307	321	332	340	346	349
2 .5	264	285	304	320	334	346	355	361	364
2 .6	274	296	316	333	347	359	369	375	379
2 .7	284	307	327	345	361	373	383	389	393
2 .8	295	318	339	358	374	387	397	404	408
2 .9	305	330	351	370	387	400	411	418	422
3 .0	315	340	363	383	400	414	425	433	437
3 .1	326	352	375	396	414	428	439	447	451
3 .2	336	363	387	408	426	441	453	461	466
3 .3	346	374	399	421	440	455	467	475	480
3 .4	357	385	411	434	453	469	481	490	495
3 .5	367	396	423	446	466	482	495	504	509
3 .6	377	408	435	459	480	496	509	518	524
3 .7	388	419	447	472	493	510	524	533	538
3 .8	398	430	459	484	506	524	538	547	553
3 .9	408	441	471	497	519	537	552	562	567
4 .0	418	452	483	510	532	551	566	576	582
5 .0	522	564	602	636	665	688	707	720	727
6 .0	625	676	722	762	797	825	847	863	872
7 .0	728	787	841	888	929	962	989	1007	1018
8 .0	− 831	− 899	− 961	− 1015	− 1061	− 1099	− 1129	− 1150	− 1163
	15^h 00m	15^h 20m	15^h 40m	16^h 00m	16^h 20m	16^h 40m	17^h 00m	17^h 20m	17^h 40m
	Right Ascension (α_m.)								

Change all signs when using this lower argument.

CHANGE PER YEAR IN THE ANNUAL PRECESSION IN DECLINATION.

In units of the fifth decimal place (unit = 0″.00001).

The side argument $\left(\frac{d\alpha_m}{dt}\right)$ is the annual variation in right ascension.

$\frac{d\alpha_m}{dt}$	Right Ascension (α_m).								
	$6^h\ 0^m$	$6^h\ 20^m$	$6^h\ 40^m$	$7^h\ 00^m$	$7^h\ 20^m$	$7^h\ 40^m$	$8^h\ 00^m$	$8^h\ 20^m$	$8^h\ 40^m$
$0^s.0$	0	+ 1	+ 2	+ 2	+ 3	+ 4	+ 4	+ 5	+ 6
1 .0	− 146	− 144	− 142	− 139	− 134	− 128	− 122	− 115	− 106
1 .1	160	159	156	153	148	142	135	127	117
1 .2	175	174	171	167	161	155	147	139	129
1 .3	190	188	185	181	175	168	160	150	140
1 .4	204	203	200	195	189	181	172	162	151
1 .5	219	217	214	209	202	195	185	174	162
1 .6	233	232	228	223	216	208	198	186	173
1 .7	248	246	243	237	230	221	210	198	184
1 .8	262	261	257	251	244	234	223	210	196
1 .9	277	275	271	265	257	247	236	222	207
2 .0	292	290	286	280	271	261	248	234	218
2 .1	306	304	300	294	285	274	261	246	229
2 .2	321	319	315	308	298	287	274	258	240
2 .3	335	333	329	322	312	300	286	270	251
2 .4	350	348	343	336	326	314	299	282	263
2 .5	364	362	358	350	340	327	311	294	274
2 .6	379	377	372	364	353	340	324	306	285
2 .7	394	391	386	378	367	353	337	318	296
2 .8	408	406	401	392	381	366	349	330	308
2 .9	423	420	415	406	394	380	362	342	319
3 .0	437	435	429	420	408	393	375	354	330
3 .1	452	450	444	434	422	406	387	366	341
3 .2	467	464	458	449	435	419	400	378	352
3 .3	481	478	472	463	449	432	412	389	363
3 .4	496	493	487	477	463	446	425	401	374
3 .5	510	508	501	491	477	459	438	413	385
3 .6	525	522	515	505	490	472	450	425	397
3 .7	540	537	530	519	504	485	463	437	408
3 .8	554	551	544	533	518	499	476	449	419
3 .9	569	566	559	547	531	512	488	461	430
4 .0	583	580	573	561	545	525	501	473	441
5 .0	729	726	717	702	682	657	627	593	553
6 .0	875	871	860	843	819	790	754	712	665
7 .0	1021	1016	1004	984	956	922	880	831	776
8 .0	− 1166	− 1161	− 1148	− 1125	− 1093	− 1054	− 1006	− 951	− 888
	$18^h\ 00^m$	$18^h\ 20^m$	$18^h\ 40^m$	$19^h\ 00^m$	$19^h\ 20^m$	$19^h\ 40^m$	$20^h\ 00^m$	$20^h\ 20^m$	$20^h\ 40^m$
	Right Ascension (α_m).								

Change all signs when using this lower argument.

CHANGE PER YEAR IN THE ANNUAL PRECESSION IN DECLINATION.

In units of the fifth decimal place (unit = 0″.00001).

The side argument $\left(\frac{da_m}{dt}\right)$ is the annual variation in right ascension.

$\frac{da_m}{dt}$	Right Ascension (a_m).								
	$9^h\ 00^m$	$9^h\ 20^m$	$9^h\ 40^m$	$10^h\ 00^m$	$10^h\ 20^m$	$10^h\ 40^m$	$11^h\ 00^m$	$11^h\ 20^m$	$11^h\ 40^m$
$0^s.0$	+6	+7	+7	+8	+8	+8	+8	+8	+9
1 .0	−97	−87	−76	−65	−54	−42	−29	−17	−4
1 .1	107	96	85	73	60	47	33	19	5
1 .2	118	106	93	80	66	52	37	22	7
1 .3	128	115	102	87	72	57	41	24	8
1 .4	138	125	110	95	79	62	45	27	9
1 .5	149	134	118	102	85	67	48	29	10
1 .6	159	143	127	109	91	72	52	32	12
1 .7	169	153	135	116	97	77	56	34	13
1 .8	180	162	143	124	103	82	60	37	14
1 .9	190	172	152	131	109	87	63	39	16
2 .0	200	181	160	138	115	92	67	42	17
2 .1	210	190	168	146	122	97	71	45	18
2 .2	221	200	177	153	128	102	75	47	19
2 .3	231	209	185	160	134	107	78	50	21
2 .4	241	218	193	167	140	112	82	52	22
2 .5	252	228	202	175	146	117	86	55	23
2 .6	262	237	210	182	152	122	90	57	24
2 .7	272	246	219	189	159	127	94	60	26
2 .8	283	256	227	197	165	132	97	62	27
2 .9	293	265	235	204	171	137	101	65	28
3 .0	303	274	244	211	177	142	105	67	30
3 .1	314	284	252	218	183	147	109	70	31
3 .2	324	293	260	226	190	152	112	72	32
3 .3	334	303	269	233	196	157	116	75	33
3 .4	345	312	277	240	202	162	120	77	35
3 .5	355	322	286	248	208	167	124	80	36
3 .6	365	331	294	255	214	171	127	83	37
3 .7	375	340	302	262	220	176	131	85	38
3 .8	386	350	311	269	226	181	135	88	40
3 .9	396	359	319	277	233	187	139	90	41
4 .0	406	368	327	284	239	191	142	93	42
5 .0	509	462	411	357	300	241	180	118	55
6 .0	612	555	494	430	362	291	218	143	68
7 .0	715	649	578	503	424	341	256	169	80
8 .0	−819	−743	−662	−576	−485	−391	−293	−194	−93
	$21^h\ 00^m$	$21^h\ 20^m$	$21^h\ 40^m$	$22^h\ 00^m$	$22^h\ 20^m$	$22^h\ 40^m$	$23^h\ 00^m$	$23^h\ 20^m$	$23^h\ 40^m$
	Right Ascension (a_m).								

Change all signs when using this lower argument.

293. THE PARALLAX OF THE SUN (p) FOR THE FIRST DAY OF EACH MONTH. (See § 66.)

Altitude.	Jan. 1st.	Feb. 1st. Dec. 1st.	Mar. 1st. Nov. 1st.	April 1st. Oct. 1st.	May 1st. Sept. 1st.	June 1st. Aug. 1st.	July 1st.	Zenith Distance.
0°	9″.0	9″.0	8″.9	8″.9	8″.8	8″.7	8″.7	90°
3	9 .0	9 .0	8 .9	8 .8	8 .8	8 .7	8 .7	87
6	9 .0	8 .9	8 .9	8 .8	8 .7	8 .7	8 .7	84
9	8 .9	8 .9	8 .8	8 .8	8 .7	8 .6	8 .6	81
12	8 .8	8 .8	8 .7	8 .7	8 .6	8 .5	8 .5	78
15	8 .7	8 .7	8 .6	8 .6	8 .5	8 .4	8 .4	75
18	8 .6	8 .6	8 .5	8 .4	8 .4	8 .3	8 .3	72
21	8 .4	8 .4	8 .3	8 .3	8 .2	8 .2	8 .1	69
24	8 .2	8 .2	8 .2	8 .1	8 .0	8 .0	8 .0	66
27	8 .0	8 .0	8 .0	7 .9	7 .8	7 .8	7 .8	63
30	7 .8	7 .8	7 .7	7 .7	7 .6	7 .6	7 .6	60
33	7 .6	7 .5	7 .5	7 .4	7 .4	7 .3	7 .3	57
36	7 .3	7 .3	7 .2	7 .2	7 .1	7 .1	7 .0	54
39	7 .0	7 .0	6 .9	6 .9	6 .8	6 .8	6 .8	51
42	6 .7	6 .7	6 .6	6 .6	6 .5	6 .5	6 .5	48
44	6 .5	6 .5	6 .4	6 .4	6 .3	6 .3	6 .3	46
46	6 .3	6 .2	6 .2	6 .2	6 .1	6 .1	6 .0	44
48	6 .0	6 .0	6 .0	5 .9	5 .9	5 .8	5 .8	42
50	5 .8	5 .8	5 .7	5 .7	5 .6	5 .6	5 .6	40
52	5 .6	5 .5	5 .5	5 .4	5 .4	5 .4	5 .4	38
54	5 .3	5 .3	5 .2	5 .2	5 .2	5 .1	5 .1	36
56	5 .0	5 .0	5 .0	5 .0	4 .9	4 .9	4 .9	34
58	4 .8	4 .8	4 .7	4 .7	4 .7	4 .6	4 .6	32
60	4 .5	4 .5	4 .5	4 .4	4 .4	4 .4	4 .4	30
62	4 .2	4 .2	4 .2	4 .2	4 .1	4 .1	4 .1	28
64	4 .0	3 .9	3 .9	3 .9	3 .8	3 .8	3 .8	26
66	3 .7	3 .7	3 .6	3 .6	3 .6	3 .6	3 .5	24
68	3 .4	3 .4	3 .4	3 .3	3 .3	3 .3	3 .3	22
70	3 .1	3 .1	3 .1	3 .0	3 .0	3 .0	3 .0	20
72	2 .8	2 .8	2 .8	2 .7	2 .7	2 .7	2 .7	18
74	2 .5	2 .5	2 .5	2 .4	2 .4	2 .4	2 .4	16
76	2 .2	2 .2	2 .2	2 .1	2 .1	2 .1	2 .1	14
78	1 .9	1 .9	1 .9	1 .8	1 .8	1 .8	1 .8	12
80	1 .6	1 .6	1 .6	1 .5	1 .5	1 .5	1 .5	10
82	1 .2	1 .2	1 .2	1 .2	1 .2	1 .2	1 .2	8
84	0 .9	0 .9	0 .9	0 .9	0 .9	0 .9	0 .9	6
86	0 .6	0 .6	0 .6	0 .6	0 .6	0 .6	0 .6	4
88	0 .3	0 .3	0 .3	0 .3	0 .3	0 .3	0 .3	2
90	0 .0	0 .0	0 .0	0 .0	0 .0	0 .0	0 .0	0

294. MEAN REFRACTION (R_M) BAROMETER 760 MILLIMETERS = 29.9 INCHES.

Temperature 10° C. = 50° F. (See § 68.)

Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.
0° 00'	34' 08".6	11".66	7° 00'	7' 24".2	0".95	19° 00'	2' 47".6	0".16	33° 00'	1' 29".4	0".06
10	32 15 .9	10 .88	10	7 14 .9	0 .91	20	2 44 .6	0 .15	20	1 28 .2	0 .06
20	30 31 .1	10 .10	20	7 06 .0	0 .88	40	2 41 .6	0 .15	40	1 27 .1	0 .05
30	28 53 .9	9 .64	30	6 57 .4	0 .84	20 00	2 38 .7	0 .14	34 00	1 26 .1	0 .05
40	27 18 .2	9 .20	40	6 49 .1	0 .81	20	2 35 .9	0 .14	20	1 25 .0	0 .05
50	25 49 .8	8 .50	50	6 41 .2	0 .78	40	2 33 .2	0 .13	40	1 24 .0	0 .05
1 00	24 28 .3	7 .82	8 00	6 33 .5	0 .76	21 00	2 30 .6	0 .13	35 00	1 23 .0	0 .05
10	23 13 .5	7 .17	10	6 26 .0	0 .73	20	2 28 .1	0 .13	20	1 22 .0	0 .05
20	22 04 .9	6 .58	20	6 18 .9	0 .70	40	2 25 .6	0 .12	40	1 21 .0	0 .05
30	21 01 .8	6 .06	30	6 12 .0	0 .68	22 00	2 23 .2	0 .12	36 00	1 20 .0	0 .05
40	20 03 .7	5 .60	40	6 05 .3	0 .66	20	2 20 .9	0 .12	30	1 18 .5	0 .05
50	19 09 .8	5 .20	50	5 58 .9	0 .63	40	2 18 .6	0 .11	37 00	1 17 .1	0 .04
2 00	18 19 .7	4 .84	9 00	5 52 .7	0 .61	23 00	2 16 .4	0 .11	30	1 15 .7	0 .04
10	17 33 .1	4 .50	20	5 40 .8	0 .58	20	2 14 .2	0 .11	38 00	1 14 .4	0 .04
20	16 49 .7	4 .18	40	5 29 .7	0 .54	40	2 12 .1	0 .10	30	1 13 .1	0 .04
30	16 09 .5	3 .88	10 00	5 19 .2	0 .51	24 00	2 10 .1	0 .10	39 00	1 11 .8	0 .04
40	15 32 .1	3 .62	20	5 09 .4	0 .48	20	2 08 .1	0 .10	30	1 10 .5	0 .04
50	14 57 .1	3 .39	40	5 00 .1	0 .46	40	2 06 .1	0 .10	40 00	1 09 .3	0 .04
3 00	14 24 .3	3 .18	11 00	4 51 .2	0 .43	25 00	2 04 .2	0 .09	30	1 08 .1	0 .04
10	13 53 .6	2 .98	20	4 42 .8	0 .40	20	2 02 .4	0 .09	41 00	1 06 .9	0 .04
20	13 24 .8	2 .79	40	4 35 .0	0 .38	40	2 00 .6	0 .09	30	1 05 .7	0 .04
30	12 57 .8	2 .61	12 00	4 27 .5	0 .37	26 00	1 58 .8	0 .09	42 00	1 04 .6	0 .04
40	12 32 .5	2 .46	20	4 20 .3	0 .35	20	1 57 1	0 .09	30	1 03 .5	0 .04
50	12 08 .7	2 .33	40	4 13 .5	0 .33	40	1 55 .4	0 .08	43 00	1 02 .4	0 .04
4 00	11 46 .0	2 .20	13 00	4 07 .1	0 .32	27 00	1 53 .8	0 .08	30	1 01 .3	0 .04
10	11 24 .6	2 .09	20	4 00 .9	0 .30	20	1 52 .2	0 .08	44 00	1 00 .2	0 .03
20	11 04 .2	1 .98	40	3 55 .1	0 .28	40	1 50 .6	0 .08	30	1 59 .2	0 .03
30	10 44 .9	1 .88	14 00	3 49 .5	0 .27	28 00	1 49 .1	0 .08	45 00	0 58 .2	0 .03
40	10 26 .5	1 .79	20	3 44 .2	0 .26	20	1 47 .6	0 .07	30	0 57 .2	0 .03
50	10 09 .1	1 .70	40	3 39 .1	0 .25	40	1 46 .1	0 .07	46 00	0 56 .2	0 .03
5 00	9 52 .6	1 .61	15 00	3 34 .1	0 .24	29 00	1 44 .6	0 .07	30	0 55 .2	0 .03
10	9 36 .9	1 .54	20	3 29 .4	0 .23	20	1 43 .2	0 .07	47 00	0 54 .2	0 .03
20	9 21 .9	1 .46	40	3 24 .8	0 .23	40	1 41 .8	0 .07	30	0 53 .3	0 .03
30	9 07 .6	1 .40	16 00	3 20 .4	0 .22	30 00	1 40 .5	0 .07	48 00	0 52 .5	0 .03
40	8 54 .0	1 .33	20	3 16 .1	0 .21	20	1 39 .1	0 .07	30	0 51 .6	0 .03
50	8 41 .0	1 .27	40	3 12 .0	0 .20	40	1 37 .8	0 .06	49 00	0 50 .7	0 .03
6 00	8 28 .6	1 .22	17 00	3 08 .2	0 .19	31 00	1 36 .6	0 .06	30	0 49 .8	0 .03
10	8 16 .7	1 .16	20	3 04 .5	0 .19	20	1 35 .3	0 .06	50 00	0 48 .9	0 .03
20	8 05 .3	1 .12	40	3 00 .9	0 .18	40	1 34 .1	0 06	30	0 48 .0	0 .03
30	7 54 .3	1 .07	18 00	2 57 .4	0 .17	32 00	1 32 .9	0 .06	51 00	0 47 .2	0 .03
40	7 43 .9	1 .02	20	2 54 .0	0 .17	20	1 31 .8	0 .06	30	0 46 .3	0 .03
50	7 33 .9	0 .98	40	2 50 .7	0 .16	40	1 30 .6	0 .06	52 00	0 45 .5	0 .03

MEAN REFRACTION (R_M).—Con'd.

Altitude.	Mean Refraction.	Change per Minute.
52° 30′	0′ 44″.7	0″.03
53 00	0 43 .9	0 .03
30	0 43 .1	0 .03
54 00	0 42 .3	0 .03
30	0 41 .6	0 .03
55 00	0 40 .8	0 .03
30	0 40 .0	0 .03
56 00	0 39 .3	0 .025
57 00	0 37 .8	0 .024
58 00	0 36 .4	0 .023
59 00	0 35 .0	0 .023
60 00	0 33 .6	0 .022
61 00	0 32 .3	0 .022
62 00	0 31 .0	0 .022
63 00	0 29 .7	0 .022
64 00	0 28 .4	0 .021
65 00	0 27 .2	0 .021
66 00	0 25 .9	0 .021
67 00	0 24 .7	0 .020
68 00	0 23 .6	0 .020
69 00	0 22 .4	0 .020
70 00	0 21 .2	0 .019
71 00	0 20 .1	0 .019
72 00	0 18 .9	0 .019
73 00	0 17 .8	0 .018
74 00	0 16 .7	0 .018
75 00	0 15 .6	0 .018
76 00	0 14 .5	0 .018
77 00	0 13 .5	0 .018
78 00	0 12 .4	0 .018
79 00	0 11 .3	0 .018
80 00	0 10 .3	0 .018
81 00	0 09 .2	0 .018
82 00	0 08 .2	0 .018
83 00	0 07 .2	0 .018
84 00	0 06 .1	0 .018
85 00	0 05 .1	0 .018
86 00	0 04 .1	0 .017
87 00	0 03 .1	0 .017
88 00	0 02 .0	0 .017
89 00	0 01 .0	0 .017
90 00	0 00 .0	0 .017

295. CORRECTION TO MEAN REFRACTION AS GIVEN IN § 294, DEPENDING UPON THE READING OF THE BAROMETER.

$R = (R_M)(C_B)(C_D)(C_A)$. (See § 68.)

Barometer, Inches.	Barometer. Millimeters.	C_B	Barometer. Inches.	Barometer. Millimeters.	C_B	Barometer. Inches.	Barometer. Millimeters.	C_B
20.0	508	0.670	24.2	615	0.809	28.4	721	0.949
20.1	511	0.673	24.3	617	0.813	28.5	724	0.953
20.2	513	0.676	24.4	620	0.816	28.6	726	0.956
20.3	516	0.679	24.5	622	0.820	28.7	729	0.959
20.4	518	0.682	24.6	625	0.823	28.8	732	0.963
20.5	521	0.685	24.7	627	0.826	28.9	734	0.966
20.6	523	0.688	24.8	630	0.829	29.0	737	0.970
20.7	526	0.692	24.9	632	0.832	29.1	739	0.973
20.8	528	0.696	25.0	635	0.835	29.2	742	0.976
20.9	531	0.699	25.1	637	0.838	29.3	744	0.979
21.0	533	0.703	25.2	640	0.842	29.4	747	0.983
21.1	536	0.706	25.3	643	0.846	29.5	749	0.986
21.2	538	0.709	25.4	645	0.849	29.6	752	0.989
21.3	541	0.712	25.5	648	0.853	29.7	754	0.992
21.4	544	0.716	25.6	650	0.856	29.8	757	0.996
21.5	546	0.719	25.7	653	0.859	29.9	759	0.999
21.6	549	0.722	25.8	655	0.862	30.0	762	1.003
21.7	551	0.725	25.9	658	0.866	30.1	765	1.007
21.8	554	0.729	26.0	660	0.869	30.2	767	1.010
21.9	556	0.732	26.1	663	0.872	30.3	770	1.013
22.0	559	0.735	26.2	665	0.875	30.4	772	1.016
22.1	561	0.739	26.3	668	0.879	30.5	775	1.020
22.2	564	0.742	26.4	671	0.882	30.6	777	1.023
22.3	566	0.746	26.5	673	0.885	30.7	780	1.026
22.4	569	0.749	26.6	676	0.889	30.8	782	1.029
22.5	572	0.752	26.7	678	0.892	30.9	785	1.033
22.6	574	0.755	26.8	681	0.896	31.0	787	1.036
22.7	576	0.759	26.9	683	0.899			
22.8	579	0.762	27.0	686	0.902			
22.9	582	0.766	27.1	688	0.905			
23.0	584	0.770	27.2	691	0.909			
23.1	587	0.773	27.3	693	0.912			
23.2	589	0.776	27.4	696	0.916			
23.3	592	0.779	27.5	699	0.920			
23.4	594	0.783	27.6	701	0.923			
23.5	597	0.786	27.7	704	0.926			
23.6	599	0.789	27.8	706	0.929			
23.7	602	0.792	27.9	709	0.933			
23.8	605	0.796	28.0	711	0.936			
23.9	607	0.799	28.1	714	0.939			
24.0	610	0.803	28.2	716	0.942			
24.1	612	0.806	28.3	719	0.946			

296. CORRECTION TO MEAN REFRACTION AS GIVEN IN § 294. DEPENDING UPON THE READING OF THE DETACHED THERMOMETER.

$R = R_M(C_B)(C_D)(C_A)$. (See § 68.)

Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D
−25°	−31°.7	1.172	20°	−6°.7	1.062	65°	18°.3	0.972	110°	43°.3	0.895
−24	−31 .1	1.169	21	−6 .1	1.060	66	18 .9	0.970	111	43 .9	0.894
−23	−30 .6	1.166	22	−5 .6	1.058	67	19 .4	0.968	112	44 .4	0.892
−22	−30 .0	1.164	23	−5 .0	1.056	68	20 .0	0.966	113	45 .0	0.891
−21	−29 .4	1.161	24	−4 .4	1.054	69	20 .6	0.964	114	45 .6	0.890
−20	−28 .9	1.158	25	−3 .9	1.051	70	21 .1	0.962	115	46 .1	0.888
−19	−28 .3	1.156	26	−3 .3	1.049	71	21 .7	0.961	116	46 .7	0.886
−18	−27 .8	1.153	27	−2 .8	1.047	72	22 .2	0.959	117	47 .2	0.885
−17	−27 .2	1.151	28	−2 .2	1.045	73	22 .8	0.957	118	47 .8	0.884
−16	−26 .7	1.148	29	−1 .7	1.043	74	23 .3	0.955	119	48 .3	0.882
−15	−26 .1	1.145	30	−1 .1	1.041	75	23 .9	0.953	120	48 .9	0.881
−14	−25 .6	1.143	31	−0 .6	1.039	76	24 .4	0.952	121	49 .4	0.880
−13	−25 .0	1 140	32	0 .0	1.036	77	25 .0	0.950	122	50 .0	0.878
−12	−24 .4	1.138	33	+0 .6	1.034	78	25 .6	0.948	123	50 .6	0.877
−11	−23 .9	1.135	34	1 .1	1.032	79	26 .1	0.946	124	51 .1	0.876
−10	−23 .3	1.133	35	1 .7	1.030	80	26 .7	0.945	125	51 .7	0.874
− 9	−22 .8	1.130	36	2 .2	1.028	81	27 .2	0.943	126	52 .2	0.873
− 8	−22 .2	1.128	37	2 .8	1.026	82	27 .8	0.941	127	52 .8	0.871
− 7	−21 .7	1.125	38	3 .3	1.024	83	28 .3	0.939	128	53 .3	0.870
− 6	−21 .1	1.123	39	3 .9	1.022	84	28 .9	0.938	129	53 .9	0.868
− 5	−20 .6	1.120	40	4 .4	1.020	85	29 .4	0.936	130	54 .4	0.867
− 4	−20 .0	1.118	41	5 .0	1.018	86	30 .0	0.934			
− 3	−19 .4	1.115	42	5 .6	1.016	87	30 .6	0.933			
− 2	−18 .9	1.113	43	6 .1	1.014	88	31 .1	0.931			
− 1	−18 .3	1.111	44	6 .7	1.012	89	31 .7	0.929			
0	−17 .8	1.108	45	7 .2	1.010	90	32 .2	0.928			
+ 1	−17 .2	1.106	46	7 .8	1.008	91	32 .8	0.926			
2	−16 .7	1.103	47	8 .3	1 006	92	33 .3	0.924			
3	−16 .1	1.101	48	8 .9	1.004	93	33 .9	0.923			
4	−15 .6	1.099	49	9 .4	1.002	94	34 .4	0.921			
5	−15 .0	1.096	50	10 .0	1.000	95	35 .0	0.919			
6	−14 .4	1.094	51	10 .6	0.998	96	35 .6	0.917			
7	−13 .9	1.092	52	11 .1	0.996	97	36 .1	0.916			
8	−13 .3	1.089	53	11 .7	0.994	98	36 .7	0.914			
9	−12 .8	1.087	54	12 .2	0.992	99	37 .2	0.912			
10	−12 .2	1.085	55	12 .8	0.990	100	37 .8	0.911			
11	−11 .7	1.082	56	13 .3	0.988	101	38 .3	0.909			
12	−11 .1	1.080	57	13 .9	0.986	102	38 .9	0.908			
13	−10 .6	1.078	58	14 .4	0.985	103	39 .4	0.906			
14	−10 .0	1.076	59	15 .0	0.983	104	40 .0	0.905			
15	− 9 .4	1.073	60	15 .6	0.981	105	40 .6	0.903			
16	− 8 .9	1.071	61	16 .1	0.979	106	41 .1	0.902			
17	− 8 .3	1.069	62	16 .7	0.977	107	41 .7	0.900			
18	− 7 .8	1.067	63	17 .2	0.975	108	42 .2	0.899			
19	− 7 .2	1.064	64	17 .8	0.973	109	42 .8	0.897			

297. Correction to Mean Refraction of § 294, depending upon attached Thermometer.

$R = (R_M)(C_B)(C_D)(C_A)$.

See § 68.

Temp. Fahr.	Temp. Cent.	C_A
−30°	−34°.4	1.007
−20	−28 .9	1.006
−10	−23 .3	1.005
0	−17 .8	1.005
+10	−12 .2	1.004
20	− 6 .7	1.003
30	− 1 .1	1.002
40	+ 4 .4	1.001
50	10 .0	1.000
60	15 .6	0.999
70	21 .1	0.998
80	26 .7	0.997
90	32 .2	0 996
100	37 .8	0.996
110	43 .3	0.995
120	48 .9	0.994
130	54 .4	0.993

298. DIP OF THE SEA HORIZON.

(See § 78.)

Height of the Eye. Feet.	Dip.	Height of the Eye. Feet.	Dip.	Height of the Eye. Feet.	Dip.
0	0′ 00″	25	4′ 54″	50	6′ 56″
1	0 59	26	5 00	51	7 00
2	1 23	27	5 06	52	7 04
3	1 42	28	5 11	53	7 08
4	1 58	29	5 17	54	7 12
5	2 11	30	5 22	55	7 16
6	2 24	31	5 27	56	7 20
7	2 36	32	5 33	57	7 24
8	2 46	33	5 38	58	7 28
9	2 56	34	5 43	59	7 31
10	3 06	35	5 48	60	7 35
11	3 15	36	5 53	65	7 54
12	3 24	37	5 58	70	8 12
13	3 32	38	6 03	75	8 29
14	3 40	39	6 07	80	8 46
15	3 48	40	6 12	85	9 02
16	3 55	41	6 17	90	9 18
17	4 02	42	6 21	95	9 33
18	4 09	43	6 25	100	9 48
19	4 16	44	6 30		
20	4 23	45	6 34		
21	4 29	46	6 39		
22	4 36	47	6 43		
23	4 42	48	6 47		
24	4 48	49	6 52		

299. FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

The sign of A is + except for stars between the zenith and the pole, for B it is + except for sub-polars, and for C it is + except for sub-polars with lamp WEST, and − except for sub-polars with lamp EAST. (See §§ 95, 97, 98.)

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	ζ
1°	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	89°
2	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	88
3	.05	.05	.05	.06	.06	.06	.06	.06	.06	.06	.06	.06	87
4	.07	.07	.07	.07	.08	.08	.08	.08	.08	.08	.08	.09	86
5	.09	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.11	85
6	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.13	.13	84
7	.12	.12	.13	.13	.13	.13	.14	.14	.14	.14	.15	.15	83
8	.14	.14	.14	.15	.15	.15	.16	.16	.16	.16	.17	.17	82
9	.16	.16	.16	.17	.17	.17	.17	.18	.18	.18	.19	.19	81
10	.17	.18	.18	.19	.19	.19	.19	.20	.20	.21	.21	.21	80
11	.19	.19	.20	.20	.21	.21	.21	.22	.22	.23	.23	.24	79
12	.21	.21	.22	.22	.22	.23	.23	.24	.24	.25	.25	.26	78
13	.22	.23	.23	.24	.24	.25	.25	.26	.26	.27	.27	.28	77
14	.24	.25	.25	.26	.26	.27	.27	.27	.28	.29	.29	.30	76
15	.26	.26	.27	.28	.28	.28	.29	.29	.30	.31	.31	.32	75
16	.28	.28	.29	.29	.30	.30	.31	.31	.32	.33	.33	.34	74
17	.29	.30	.30	.31	.31	.32	.33	.33	.34	.34	.35	.36	73
18	.31	.31	.32	.33	.33	.34	.34	.35	.36	.36	.37	.38	72
19	.33	.33	.34	.35	.35	.36	.36	.37	.38	.38	.39	.40	71
20	.34	.35	.35	.36	.37	.37	.38	.39	.40	.40	.41	.42	70
21	.36	.36	.37	.38	.39	.39	.40	.41	.41	.42	.43	.44	69
22	.37	.38	.39	.40	.40	.41	.42	.42	.43	.44	.45	.46	68
23	.39	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.48	67
24	.41	.41	.42	.43	.44	.45	.45	.46	.47	.48	.49	.50	66
25	.42	.43	.44	.45	.46	.46	.47	.48	.49	.50	.51	.52	65
26	.44	.45	.45	.47	.47	.48	.49	.50	.51	.52	.53	.54	64
27	.45	.46	.47	.48	.49	.50	.51	.51	.52	.54	.55	.56	63
28	.47	.48	.49	.50	.51	.51	.52	.53	.54	.55	.57	.58	62
29	.48	.49	.50	.52	.52	.53	.54	.55	.56	.57	.58	.60	61
30	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.62	60
31	.52	.52	.53	.55	.56	.56	.57	.58	.59	.61	.62	.64	59
32	.53	.54	.55	.56	.57	.58	.59	.60	.61	.63	.64	.65	58
33	.55	.55	.56	.58	.59	.60	.61	.62	.63	.64	.66	.67	57
34	.56	.57	.58	.59	.60	.61	.62	.63	.65	.66	.67	.69	56
35	.57	.58	.59	.61	.62	.63	.64	.65	.66	.68	.69	.71	55
36	.59	.60	.61	.63	.63	.64	.65	.67	.68	.69	.71	.73	54
37	.60	.61	.62	.64	.65	.66	.67	.68	.70	.71	.73	.74	53
38	.62	.63	.64	.66	.66	.67	.69	.70	.71	.73	.74	.76	52
39	.63	.64	.65	.67	.68	.69	.70	.71	.73	.74	.76	.78	51
40	.64	.65	.67	.68	.69	.70	.72	.73	.74	.76	.77	.79	50
41	.66	.67	.68	.70	.71	.72	.73	.74	.76	.77	.79	.81	49
42	.67	.68	.69	.71	.72	.73	.74	.76	.77	.79	.81	.83	48
43	.68	.69	.71	.73	.74	.75	.76	.77	.79	.80	.82	.84	47
44	.69	.71	.72	.74	.75	.76	.77	.79	.80	.82	.84	.86	46
45	.71	.72	.73	.75	.76	.77	.79	.80	.82	.83	.85	.87	45

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	ζ
46°	.72	.73	.74	.77	.78	.79	.80	.82	.83	.85	.87	.89	44°
47	.73	.74	.76	.78	.79	.80	.81	.83	.84	.86	.88	.90	43
48	.74	.76	.77	.79	.80	.81	.83	.84	.86	.88	.90	.92	42
49	.75	.77	.78	.80	.81	.83	.84	.86	.87	.89	.91	.93	41
50	.77	.78	.79	.82	.83	.84	.85	.87	.89	.90	.92	.95	40
51	.78	.79	.80	.83	.84	.85	.87	.88	.90	.92	.94	.96	39
52	.79	.80	.82	.84	.85	.86	.88	.89	.91	.93	.95	.97	38
53	.80	.81	.83	.85	.86	.87	.89	.91	.92	.94	.96	.99	37
54	.81	.82	.84	.86	.87	.89	.90	.92	.93	.95	.98	1.00	36
55	.82	.83	.85	.87	.88	.90	.91	.93	.95	.97	.99	1.01	35
56	.83	.84	.86	.88	.89	.91	.92	.94	.96	.98	1.00	1.02	34
57	.84	.85	.87	.89	.90	.92	.93	.95	.97	.99	1.01	1.04	33
58	.85	.86	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.05	32
59	.86	.87	.89	.91	.92	.94	.95	.97	.99	1.01	1.03	1.06	31
60	.87	.88	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.07	30
61	.87	.89	.91	.93	.94	.96	.97	.99	1.01	1.03	1.06	1.08	29
62	.88	.90	.91	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.09	28
63	.89	.91	.92	.95	.96	.98	.99	1.01	1.03	1.05	1.07	1.10	27
64	.90	.91	.93	.96	.97	.98	1.00	1.02	1.04	1.06	1.08	1.11	26
65	.91	.92	.94	.96	.98	.99	1.01	1.03	1.05	1.07	1.09	1.12	25
66	.91	.93	.95	.97	.99	1.00	1.02	1.04	1.06	1.08	1.10	1.13	24
67	.92	.94	.95	.98	.99	1.01	1.02	1.04	1.06	1.09	1.11	1.14	23
68	.93	.94	.96	.99	1.00	1.02	1.03	1.05	1.07	1.09	1.12	1.15	22
69	.93	.95	.97	.99	1.01	1.02	1.04	1.06	1.08	1.10	1.13	1.15	21
70	.94	.95	.97	1.00	1.01	1.03	1.05	1.06	1.09	1.11	1.13	1.16	20
71	.95	.96	.98	1.01	1.02	1.04	1.05	1.07	1.09	1.12	1.14	1.17	19
72	.95	.97	.98	1.01	1.03	1.04	1.06	1.08	1.10	1.12	1.15	1.17	18
73	.96	.97	.99	1.02	1.03	1.05	1.06	1.08	1.10	1.13	1.15	1.18	17
74	.96	.98	1.00	1.02	1.04	1.05	1.07	1.09	1.11	1.13	1.16	1.19	16
75	.97	.98	1.00	1.03	1.04	1.06	1.08	1.09	1.12	1.14	1.16	1.19	15
76	.97	.99	1.00	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.20	14
77	.97	.99	1.01	1.04	1.05	1.07	1.08	1.10	1.13	1.15	1.17	1.20	13
78	.98	.99	1.01	1.04	1.05	1.07	1.09	1.11	1.13	1.15	1.18	1.21	12
79	.98	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.16	1.18	1.21	11
80	.98	1.00	1.02	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.19	1.22	10
81	.99	1.00	1.02	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	9
82	.99	1.01	1.03	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	8
83	.99	1.01	1.03	1.06	1.07	1.09	1.10	1.12	1.15	1.17	1.20	1.23	7
84	.99	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	6
85	1.00	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	5
86	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	4
87	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	3
88	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	2
89	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	1
90	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	0

The bottom line on this page is the collimation factor C (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination $\pm \delta$.

ζ	38°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	ζ
1°	.02	.02	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03	89°
2	.04	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	88
3	.07	.07	.07	.07	.07	.07	.07	.07	.08	.08	.08	.08	87
4	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.11	.11	86
5	.11	.11	.11	.12	.12	.12	.12	.13	.13	.13	.13	.13	85
6	.13	.14	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	84
7	.15	.16	.16	.16	.17	.17	.17	.18	.18	.18	.19	.19	83
8	.18	.18	.18	.19	.19	.19	.20	.20	.20	.21	.21	.22	82
9	.20	.20	.21	.21	.21	.22	.22	.22	.23	.23	.24	.24	81
10	.22	.23	.23	.23	.24	.24	.25	.25	.25	.26	.26	.27	80
11	.24	.25	.25	.26	.26	.27	.27	.28	.28	.28	.29	.30	79
12	.26	.27	.27	.28	.28	.29	.29	.30	.30	.31	.32	.32	78
13	.29	.29	.30	.30	.31	.31	.32	.32	.33	.34	.34	.35	77
14	.31	.32	.32	.33	.33	.34	.34	.35	.35	.36	.37	.38	76
15	.33	.34	.34	.35	.35	.36	.37	.37	.38	.39	.39	.40	75
16	.35	.36	.37	.37	.38	.38	.39	.40	.40	.41	.42	.43	74
17	.37	.38	.39	.39	.40	.41	.41	.42	.43	.44	.45	.45	73
18	.39	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.48	72
19	.41	.42	.43	.44	.45	.45	.46	.47	.48	.49	.50	.51	71
20	.43	.45	.45	.46	.47	.48	.48	.49	.50	.51	.52	.53	70
21	.45	.47	.47	.48	.49	.50	.51	.52	.52	.54	.55	.56	69
22	.48	.49	.50	.50	.51	.52	.53	.54	.55	.56	.57	.58	68
23	.50	.51	.52	.53	.53	.54	.55	.56	.57	.58	.60	.61	67
24	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	66
25	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	.64	.66	65
26	.56	.57	.58	.59	.60	.61	.62	.63	.64	.65	.67	.68	64
27	.58	.59	.60	.61	.62	.63	.64	.65	.67	.68	.69	.71	63
28	.60	.61	.62	.63	.64	.65	.66	.68	.69	.70	.72	.73	62
29	.61	.63	.64	.65	.66	.67	.69	.70	.71	.72	.74	.75	61
30	.63	.65	.66	.67	.68	.69	.71	.72	.73	.75	.76	.78	60
31	.65	.67	.68	.69	.70	.72	.73	.74	.75	.77	.78	.80	59
32	.67	.69	.70	.71	.72	.74	.75	.76	.78	.79	.81	.82	58
33	.69	.71	.72	.73	.74	.76	.77	.78	.80	.81	.83	.85	57
34	.71	.73	.74	.75	.76	.78	.79	.80	.82	.84	.85	.87	56
35	.73	.75	.76	.77	.78	.80	.81	.83	.84	.86	.87	.89	55
36	.75	.77	.78	.79	.80	.82	.83	.85	.86	.88	.90	.91	54
37	.76	.79	.80	.81	.82	.84	.85	.87	.88	.90	.92	.94	53
38	.78	.80	.82	83	.84	.86	.87	.89	.90	.92	.94	.96	52
39	.80	.82	.83	.85	.86	.87	.89	.91	.92	.94	.96	.98	51
40	.82	.84	.85	.86	.88	.89	.91	.93	.94	.96	.98	1.00	50
41	.83	.86	.87	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	49
42	.85	.87	.89	.90	.91	.93	.95	.96	.98	1.00	1.02	1.04	48
43	.86	.89	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	47
44	.88	.91	.92	.93	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	46
45	.90	.92	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	45

Use this left-side argument for azimuth factor A (= $\sin \zeta \sec \delta$).

Use this right-side argument for inclination factor B (= $\cos \zeta \sec \delta$).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

ζ	38°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	ζ
46°	.91	.94	.95	.97	.98	1.00	1.02	1.04	1.05	1.07	1.10	1.12	44°
47	.93	.95	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.14	43
48	.94	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.16	42
49	.96	.99	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	41
50	.97	1.00	1.01	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.19	40
51	.99	1.01	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.21	39
52	1.00	1.03	1.04	1.06	1.08	1.10	1.11	1.13	1.15	1.18	1.20	1.23	38
53	1.01	1.04	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.22	1.24	37
54	1.03	1 06	1.07	1.09	1.11	1.12	1.14	1.16	1.19	1.21	1.23	1.26	36
55	1.04	1.07	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.25	1.27	35
56	1.05	1.08	1.10	1.12	1.13	1.15	1.17	1.19	1.22	1.24	1.26	1.29	34
57	1.06	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.31	33
58	1.08	1.11	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.27	1.29	1.32	32
59	1.09	1.12	1.14	1.15	1.17	1.19	1.21	1.23	1.26	1.28	1.31	1.33	31
60	1.10	1.13	1.15	1.17	1.18	1.20	1.22	1.25	1.27	1.29	1.32	1.35	30
61	1.11	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	29
62	1.12	1.15	1.17	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.35	1.37	28
63	1.13	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	1.39	27
64	1.14	1.17	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.37	1.40	26
65	1.15	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.41	25
66	1.16	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.37	1.39	1.42	24
67	1.17	1.20	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.40	1.43	23
68	1.18	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.36	1.39	1.41	1.44	22
69	1.18	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.37	1.40	1.42	1.45	21
70	1.19	1.23	1.25	1.26	1.28	1.31	1.33	1.35	1.38	1.40	1.43	1.46	20
71	1.20	1.23	1.25	1.27	1.29	1.31	1.34	1.36	1.39	1.41	1.44	1.47	19
72	1.21	1.24	1.26	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.45	1.48	18
73	1.21	1.25	1.27	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.49	17
74	1.22	1.25	1.27	1.29	1.31	1.34	1.36	1.38	1.41	1.44	1.46	1.49	16
75	1.23	1.26	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.44	1.47	1.50	15
76	1.23	1.27	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	14
77	1.24	1.27	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.48	1.52	13
78	1.24	1.28	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	12
79	1.25	1.28	1.30	1.32	1.34	1.36	1.39	1.41	1.44	1.47	1.50	1.53	11
80	1.25	1.29	1.30	1.33	1.35	1.37	1.39	1.42	1.44	1.47	1.50	1.53	10
81	1.25	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	9
82	1.26	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.45	1.48	1.51	1.54	8
83	1.26	1.30	1.32	1.34	1.36	1.38	1.40	1.43	1.46	1.48	1.51	1.54	7
84	1.26	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	6
85	1.26	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	5
86	1.27	1.30	1.32	1.34	1.36	1.39	1.41	1.44	1.46	1.49	1.52	1.55	4
87	1.27	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	3
88	1.27	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	2
89	1.27	1.31	1.33	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	1
90	1.27	1.31	1.33	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	0

Use this right-side argument for inclination factor B (= cos ζ sec δ).

The bottom line of this page is the collimation factor C (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ °	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	60½°	61°	ζ °
1°	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.04	.04	89
2	.06	.06	.06	.06	.06	.06	.06	.07	.07	.07	.07	.07	88
3	.08	.08	.09	.09	.09	.09	.10	.10	.10	.10	.11	.11	87
4	.11	.11	.12	.12	.12	.12	.13	.13	.14	.14	.14	.14	86
5	.14	.14	.14	.15	.15	.16	.16	.16	.17	.17	.18	.18	85
6	.17	.17	.17	.18	.18	.19	.19	.20	.20	.21	.21	.22	84
7	.19	.20	.20	.21	.21	.22	.22	.23	.24	.24	.25	.25	83
8	.22	.23	.23	.24	.24	.25	.26	.26	.27	.28	.28	.29	82
9	.25	.25	.26	.26	.27	.28	.29	.29	.30	.31	.32	.32	81
10	.28	.28	.29	.30	.30	.31	.32	.33	.34	.35	.35	.36	80
11	.30	.31	.32	.32	.33	.34	.35	.36	.37	.38	.39	.39	79
12	.33	.34	.35	.35	.36	.37	.38	.39	.40	.42	.42	.43	78
13	.36	.36	.37	.38	.39	.40	.41	.42	.44	.45	.46	.46	77
14	.38	.39	.40	.41	.42	.43	.44	.46	.47	.48	.49	.50	76
15	.41	.42	.43	.44	.45	.46	.48	.49	.50	.52	.53	.53	75
16	.44	.45	.46	.47	.48	.49	.51	.52	.54	.55	.56	.57	74
17	.46	.47	.49	.50	.51	.52	.54	.55	.57	.58	.59	.60	73
18	.49	.50	.51	.53	.54	.55	.57	.58	.60	.62	.63	.64	72
19	.52	.53	.54	.55	.57	.58	.60	.61	.63	.65	.66	.67	71
20	.54	.56	.57	.58	.60	.61	.63	.64	.66	.68	.69	.70	70
21	.57	.58	.59	.61	.62	.64	.66	.68	.70	.72	.73	.74	69
22	.60	.61	.62	.64	.65	.67	.69	.71	.73	.75	.76	.77	68
23	.62	.63	.65	.66	.68	.70	.72	.74	.76	.78	.79	.81	67
24	.65	.66	.68	.69	.71	.73	.75	.77	.79	.81	.83	.84	66
25	.67	.69	.70	.72	.74	.76	.78	.80	.82	.85	.86	.87	65
26	.70	.71	.73	.75	.76	.78	.80	.83	.85	.88	.89	.90	64
27	.72	.74	.75	.77	.79	.81	.83	.86	.88	.91	.92	.94	63
28	.75	.76	.78	.80	.82	.84	.86	.89	.91	.94	.95	.97	62
29	.77	.79	.81	.82	.84	.87	.89	.91	.94	.97	.98	1.00	61
30	.79	.81	.83	.85	.87	.89	.92	.94	.97	1.00	1.01	1.03	60
31	.82	.84	.86	.88	.90	.92	.95	.97	1.00	1.03	1.05	1.06	59
32	.84	.86	.88	.90	.92	.95	.97	1.00	1.03	1.06	1.08	1.09	58
33	.87	.88	.91	.93	.95	.97	1.00	1.03	1.06	1.09	1.11	1.12	57
34	.89	91	.93	.95	.97	1.00	1.03	1.05	1.09	1.12	1.14	1.15	56
35	.91	.93	.95	.98	1.00	1.03	1.05	1.08	1.11	1.15	1.16	1.18	55
36	.93	.95	.98	1.00	1.03	1.05	1.08	1.11	1.14	1.18	1.19	1.21	54
37	.96	.98	1.00	1.02	1.05	1.08	1.10	1.14	1.17	1.20	1.22	1.24	53
38	.98	1.00	1.02	1.05	1.07	1.10	1.13	1.16	1.20	1.23	1.25	1.27	52
39	1.00	1.02	1.05	1.07	1.10	1.12	1.15	1.19	1.22	1.26	1.28	1.30	51
40	1.02	1.04	1.07	1.09	1.12	1.15	1.18	1.21	1.25	1.29	1.31	1.33	50
41	1.04	1.07	1.09	1.12	1.14	1.17	1.20	1.24	1.27	1.31	1.33	1.35	49
42	1.06	1.09	1.11	1.14	1.17	1.20	1.23	1.26	1.30	1.34	1.36	1.38	48
43	1.08	1.11	1.13	1.16	1.19	1.22	1.25	1.29	1.32	1.36	1.39	1.41	47
44	1.10	1.13	1.15	1.18	1.21	1.24	1.28	1.31	1.35	1.39	1.41	1.43	46
45	1.12	1.15	1.17	1.20	1.23	1.26	1.30	1.33	1.37	1.41	1.44	1.46	45

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is for the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	60½°	61°	ζ
46°	1.14	1.17	1.19	1.22	1.25	1.29	1.32	1.36	1.40	1.44	1.46	1.48	44°
47	1.16	1.19	1.21	1.24	1.27	1.31	1.34	1.38	1.42	1.46	1.49	1.51	43
48	1.18	1.21	1.23	1.26	1.30	1.33	1.36	1.40	1.44	1.48	1.50	1.53	42
49	1.20	1.23	1.25	1.28	1.32	1.35	1.39	1.42	1.47	1.51	1.53	1.56	41
50	1.22	1.24	1.27	1.30	1.34	1.37	1.41	1.44	1.49	1.53	1.56	1.58	40
51	1.23	1.26	1.29	1.32	1.35	1.39	1.43	1.47	1.51	1.55	1.58	1.60	39
52	1.25	1.28	1.31	1.34	1.37	1.41	1.45	1.49	1.53	1.58	1.60	1.63	38
53	1.27	1.30	1.33	1.36	1.39	1.43	1.47	1.51	1.55	1.60	1.62	1.65	37
54	1.29	1.31	1.34	1.38	1.41	1.45	1.49	1.53	1.57	1.62	1.64	1.67	36
55	1.30	1.33	1.36	1.39	1.43	1.46	1.50	1.55	1.59	1.64	1.66	1.69	35
56	1.32	1.35	1.38	1.41	1.45	1.48	1.52	1.56	1.61	1.66	1.68	1.71	34
57	1.33	1.36	1.39	1.43	1.46	1.50	1.54	1.58	1.63	1.68	1.70	1.73	33
58	1.35	1.38	1.41	1.44	1.48	1.52	1.56	1.60	1.65	1.70	1.72	1.75	32
59	1.36	1.39	1.42	1.46	1.49	1.53	1.57	1.62	1.66	1.71	1.74	1.77	31
60	1.38	1.41	1.44	1.47	1.51	1.55	1.59	1.63	1.68	1.73	1.76	1.79	30
61	1.39	1.42	1.45	1.49	1.53	1.56	1.61	1.65	1.70	1.75	1.78	1.80	29
62	1.40	1.43	1.47	1.50	1.54	1.58	1.62	1.67	1.71	1.77	1.79	1.82	28
63	1.42	1.45	1.48	1.52	1.55	1.59	1.64	1.68	1.73	1.78	1.81	1.84	27
64	1.43	1.46	1.49	1.53	1.57	1.61	1.65	1.70	1.75	1.80	1.83	1.85	26
65	1.44	1.47	1.51	1.54	1.58	1.62	1.66	1.71	1.76	1.81	1.84	1.87	25
66	1.45	1.48	1.52	1.55	1.59	1.63	1.68	1.72	1.77	1.83	1.85	1.88	24
67	1.46	1.50	1.53	1.57	1.60	1.65	1.69	1.74	1.79	1.84	1.87	1.90	23
68	1.47	1.51	1.54	1.58	1.62	1.66	1.70	1.75	1.80	1.85	1.88	1.91	22
69	1.48	1.52	1.55	1.59	1.63	1.67	1.71	1.76	1.81	1.87	1.90	1.93	21
70	1.49	1.53	1.56	1.60	1.64	1.68	1.73	1.77	1.82	1.88	1.91	1.94	20
71	1.50	1.54	1.57	1.61	1.65	1.69	1.74	1.78	1.84	1.89	1.92	1.95	19
72	1.51	1.54	1.58	1.62	1.66	1.70	1.75	1.80	1.85	1.90	1.93	1.96	18
73	1.52	1.55	1.59	1.63	1.67	1.71	1.76	1.80	1.86	1.91	1.94	1.97	17
74	1.53	1.56	1.60	1.63	1.68	1.72	1.76	1.81	1.87	1.92	1.95	1.98	16
75	1.53	1.57	1.60	1.64	1.68	1.73	1.77	1.82	1.88	1.93	1.96	1.99	15
76	1.54	1.58	1.61	1.65	1.69	1.73	1.78	1.83	1.88	1.94	1.97	2.00	14
77	1.55	1.58	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.95	1.98	2.01	13
78	1.55	1.59	1.62	1.66	1.70	1.75	1.80	1.85	1.90	1.96	1.99	2.02	12
79	1.56	1.59	1.63	1.67	1.71	1.76	1.80	1.85	1.91	1.96	1.99	2.02	11
80	1.56	1.60	1.64	1.67	1.72	1.76	1.81	1.86	1.91	1.97	2.00	2.03	10
81	1.57	1.60	1.64	1.68	1.72	1.77	1.81	1.86	1.92	1.98	2.01	2.04	9
82	1.57	1.61	1.64	1.68	1.73	1.77	1.82	1.87	1.92	1.98	2.01	2.04	8
83	1.58	1.61	1.65	1.69	1.73	1.77	1.82	1.87	1.93	1.99	2.02	2.05	7
84	1.58	1.62	1.65	1.69	1.73	1.78	1.83	1.88	1.93	1.99	2.02	2.05	6
85	1.58	1.62	1.65	1.69	1.74	1.78	1.83	1.88	1.93	1.99	2.02	2.05	5
86	1.59	1.62	1.66	1.70	1.74	1.78	1.83	1.88	1.94	2.00	2.03	2.06	4
87	1.59	1.62	1.66	1.70	1.74	1.79	1.83	1.88	1.94	2.00	2.03	2.06	3
88	1.59	1.62	1.66	1.70	1.74	1.79	1.83	1.89	1.94	2.00	2.03	2.06	2
89	1.59	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.94	2.00	2.03	2.06	1
90	1.59	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.94	2.00	2.03	2.06	0

The bottom line on this page is the collimation factor C (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	61½°	62°	62½°	63°	63½°	64°	64½°	65°	65½°	66°	66½°	67°	ζ°
1°	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	89
2	.07	.07	.08	.08	.08	.08	.08	.08	.08	.09	.09	.09	88
3	.11	.11	.11	.12	.12	.12	.12	.12	.13	.13	.13	.13	87
4	.15	.15	.15	.15	.16	.16	.16	.17	.17	.17	.18	.18	86
5	.18	.19	.19	.19	.19	.20	.20	.21	.21	.21	.22	.22	85
6	.22	.22	.23	.23	.23	.24	.24	.25	.25	.26	.26	.27	84
7	.26	.26	.26	.27	.27	.28	.28	.29	.29	.30	.31	.31	83
8	.29	.30	.30	.31	.31	.32	.32	.33	.34	.34	.35	.36	82
9	.33	.33	.34	.35	.35	.36	.36	.37	.38	.39	.39	.40	81
10	.36	.37	.38	.38	.39	.40	.40	.41	.42	.43	.43	.44	80
11	.40	.41	.41	.42	.43	.44	.44	.45	.46	.47	.48	.49	79
12	.44	.44	.45	.46	.47	.47	.48	.49	.50	.51	.52	.53	78
13	.47	.48	.49	.50	.50	.51	.52	.53	.54	.55	.56	.58	77
14	.51	.52	.52	.53	.54	.55	.56	.57	.58	.59	.61	.62	76
15	.54	.55	.56	.57	.58	.59	.60	.61	.62	.64	.65	.66	75
16	.58	.59	.60	.61	.62	.63	.64	.65	.66	.68	.69	.71	74
17	.61	.62	.63	.64	.66	.67	.68	.69	.70	.72	.73	.75	73
18	.65	.66	.67	.68	.69	.70	.72	.73	.74	.76	.77	.79	72
19	.68	.69	.70	.72	.73	.74	.76	.77	.78	.80	.82	.83	71
20	.72	.73	.74	.75	.77	.79	.79	.81	.83	.84	.86	.88	70
21	.75	.76	.78	.79	.80	.82	.83	.85	.86	.88	.90	.92	69
22	.78	.80	.81	.82	.84	.85	.87	.89	.90	.92	.94	.96	68
23	.82	.83	.85	.86	.88	.89	.91	.92	.94	.96	.98	1.00	67
24	.85	.87	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.04	66
25	.89	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	1.08	65
26	.92	.93	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	1.12	64
27	.95	.97	.98	1.00	1.02	1.04	1.05	1.07	1.09	1.12	1.14	1.16	63
28	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.18	1.20	62
29	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.22	1.24	61
30	1.05	1.07	1.08	1.10	1.12	1.14	1.16	1.18	1.21	1.23	1.25	1.28	60
31	1.08	1.10	1.11	1.13	1.15	1.17	1.20	1.22	1.24	1.27	1.29	1.32	59
32	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.33	1.36	58
33	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.29	1.31	1.34	1.37	1.39	57
34	1.17	1.19	1.21	1.23	1.25	1.27	1.30	1.32	1.35	1.37	1.40	1.43	56
35	1.20	1.22	1.24	1.26	1.29	1.31	1.33	1.36	1.38	1.41	1.44	1.47	55
36	1.23	1.25	1.27	1.30	1.32	1.34	1.37	1.39	1.42	1.45	1.47	1.51	54
37	1.26	1.28	1.30	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	53
38	1.29	1.31	1.33	1.36	1.38	1.40	1.43	1.46	1.48	1.51	1.54	1.58	52
39	1.32	1.34	1.36	1.39	1.41	1.43	1.46	1.49	1.52	1.55	1.58	1.61	51
40	1.35	1.37	1.39	1.42	1.44	1.47	1.49	1.52	1.55	1.58	1.61	1.65	50
41	1.37	1.40	1.42	1.45	1.47	1.50	1.53	1.55	1.58	1.61	1.64	1.68	49
42	1.40	1.42	1.45	1.47	1.50	1.53	1.55	1.58	1.61	1.64	1.68	1.71	48
43	1.43	1.45	1.48	1.50	1.53	1.56	1.58	1.61	1.64	1.68	1.71	1.75	47
44	1.46	1.48	1.50	1.53	1.56	1.58	1.61	1.64	1.67	1.71	1.74	1.78	46
45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.74	1.77	1.81	45

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

ζ	61½°	62°	62½°	63°	63½°	64°	64½°	65°	65½°	66°	66½°	67°	ζ
46°	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.74	1.77	1.80	1.84	44°
47	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.73	1.76	1.80	1.83	1.87	43
48	1.55	1.58	1.60	1.63	1.66	1.69	1.72	1.75	1.79	1.82	1.86	1.90	42
49	1.58	1.61	1.63	1.66	1.69	1.72	1.75	1.79	1.82	1.86	1.89	1.93	41
50	1.60	1.63	1.66	1.69	1.72	1.75	1.78	1.81	1.85	1.88	1.92	1.96	40
51	1.63	1.66	1.68	1.71	1.74	1.77	1.80	1.84	1.87	1.91	1.95	1.99	39
52	1.65	1.68	1.71	1.74	1.77	1.80	1.83	1.86	1.90	1.94	1.98	2.02	38
53	1.67	1.70	1.73	1.76	1.79	1.82	1.85	1.89	1.93	1.96	2.00	2.04	37
54	1.69	1.72	1.75	1.78	1.81	1.85	1.88	1.91	1.95	1.99	2.03	2.07	36
55	1.72	1.74	1.77	1.80	1.84	1.87	1.90	1.94	1.98	2.01	2.05	2.10	35
56	1.74	1.77	1.80	1.83	1.86	1.89	1.93	1.96	2.00	2.04	2.08	2.12	34
57	1.76	1.79	1.82	1.85	1.88	1.91	1.95	1.98	2.02	2.06	2.10	2.15	33
58	1.78	1.81	1.84	1.87	1.90	1.93	1.97	2.01	2.05	2.08	2.13	2.17	32
59	1.80	1.83	1.86	1.89	1.92	1.95	1.99	2.03	2.07	2.11	2.15	2.19	31
60	1.81	1.84	1.88	1.91	1.94	1.97	2.01	2.05	2.09	2.13	2.17	2.22	30
61	1.83	1.86	1.89	1.93	1.96	2.00	2.03	2.07	2.11	2.15	2.19	2.24	29
62	1.85	1.88	1.91	1.94	1.98	2.01	2.05	2.09	2.13	2.17	2.21	2.26	28
63	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.11	2.15	2.19	2.23	2.28	27
64	1.88	1.91	1.95	1.98	2.02	2.05	2.09	2.13	2.17	2.21	2.25	2.30	26
65	1.90	1.93	1.96	2.00	2.03	2.07	2.11	2.14	2.19	2.23	2.27	2.32	25
66	1.91	1.95	1.98	2.01	2.05	2.08	2.12	2.16	2.20	2.25	2.29	2.34	24
67	1.93	1.96	1.99	2.03	2.06	2.10	2.14	2.18	2.22	2.26	2.31	2.36	23
68	1.94	1.97	2.01	2.04	2.08	2.11	2.15	2.19	2.24	2.28	2.32	2.37	22
69	1.96	1.99	2.02	2.06	2.09	2.13	2.17	2.21	2.25	2.30	2.34	2.39	21
70	1.97	2.00	2.03	2.07	2.11	2.14	2.18	2.22	2.27	2.31	2.36	2.40	20
71	1.98	2.01	2.05	2.08	2.12	2.16	2.20	2.24	2.28	2.32	2.37	2.42	19
72	1.99	2.03	2.06	2.09	2.13	2.17	2.21	2.25	2.29	2.34	2.38	2.43	18
73	2.00	2.04	2.07	2.11	2.14	2.18	2.22	2.26	2.31	2.35	2.40	2.45	17
74	2.01	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.32	2.36	2.41	2.46	16
75	2.02	2.06	2.09	2.13	2.16	2.20	2.24	2.29	2.33	2.37	2.42	2.47	15
76	2.03	2.07	2.10	2.14	2.17	2.21	2.25	2.30	2.34	2.39	2.43	2.48	14
77	2.04	2.07	2.11	2.15	2.18	2.22	2.26	2.31	2.35	2.40	2.44	2.49	13
78	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.36	2.40	2.45	2.50	12
79	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	11
80	2.06	2.10	2.13	2.17	2.21	2.25	2.29	2.33	2.38	2.42	2.47	2.52	10
81	2.07	2.10	2.14	2.18	2.21	2.25	2.29	2.34	2.38	2.43	2.48	2.53	9
82	2.08	2.11	2.15	2.18	2.22	2.26	2.30	2.34	2.39	2.43	2.48	2.53	8
83	2.08	2.12	2.15	2.19	2.22	2.26	2.31	2.35	2.39	2.44	2.49	2.54	7
84	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.35	2.40	2.45	2.49	2.55	6
85	2.09	2.12	2.16	2.19	2.23	2.27	2.31	2.36	2.40	2.45	2.50	2.55	5
86	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.45	2.50	2.55	4
87	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.46	2.50	2.56	3
88	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.46	2.51	2.56	2
89	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	2.56	1
90	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	2.56	0

Use this left-side argument for azimuth factor *A* (= sin ζ sec δ).

Use this right-side argument for inclination factor *B* (= cos ζ sec δ).

The bottom line on this page is the collimation factor *C* (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	67½°	68°	68½°	69°	69½°	70°	70¼°	70½°	70¾°	71°	71¼°	71½°	ζ
1°	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	89°
2	.09	.09	.10	.10	.10	.10	.10	.10	.11	.11	.11	.11	88
3	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	.16	.16	87
4	.18	.19	.19	.20	.20	.20	.21	.21	.21	.21	.22	.22	86
5	.23	.23	.24	.24	.25	.25	.26	.26	.26	.27	.27	.27	85
6	.27	.28	.28	.29	.30	.31	.31	.31	.32	.32	.33	.33	84
7	.32	.33	.33	.34	.35	.36	.36	.37	.37	.37	.38	.38	83
8	.36	.37	.38	.39	.40	.41	.41	.42	.42	.43	.43	.44	82
9	.41	.42	.43	.44	.45	.46	.46	.47	.47	.48	.49	.49	81
10	.45	.46	.47	.49	.50	.51	.51	.52	.53	.53	.54	.55	80
11	.50	.51	.52	.53	.54	.56	.56	.57	.58	.59	.59	.60	79
12	.54	.56	.57	.58	.59	.61	.62	.62	.63	.64	.65	.66	78
13	.59	.60	.61	.63	.64	.66	.67	.67	.68	.69	.70	.71	77
14	.63	.65	.66	.68	.69	.71	.72	.72	.73	.74	.75	.76	76
15	.68	.69	.71	.72	.74	.76	.77	.78	.78	.79	.80	.81	75
16	.72	.74	.75	.77	.79	.81	.82	.83	.84	.85	.86	.87	74
17	.76	.78	.80	.81	.83	.85	.86	.88	.89	.90	.91	.92	73
18	.81	.83	.84	.86	.88	.90	.91	.93	.94	.95	.96	.97	72
19	.85	.87	.89	.91	.93	.95	.96	.98	.99	1.00	1.01	1.03	71
20	.89	.91	.93	.95	.98	1.00	1.01	1.02	1.04	1.05	1.06	1.08	70
21	.94	.96	.98	1.00	1.02	1.05	1.06	1.07	1.09	1.10	1.11	1.13	69
22	.98	1.00	1.02	1.05	1.07	1.09	1.11	1.12	1.14	1.15	1.17	1.18	68
23	1.02	1.04	1.07	1.09	1.12	1.14	1.16	1.17	1.19	1.20	1.21	1.23	67
24	1.06	1.09	1.11	1.14	1.16	1.19	1.20	1.22	1.23	1.25	1.27	1.28	66
25	1.10	1.13	1.15	1.18	1.21	1.24	1.25	1.27	1.28	1.30	1.31	1.33	65
26	1.15	1.17	1.20	1.22	1.25	1.28	1.30	1.31	1.33	1.35	1.36	1.38	64
27	1.19	1.21	1.24	1.27	1.30	1.33	1.34	1.36	1.38	1.39	1.41	1.43	63
28	1.23	1.25	1.28	1.31	1.34	1.37	1.39	1.41	1.42	1.44	1.46	1.48	62
29	1.27	1.29	1.32	1.35	1.38	1.42	1.43	1.45	1.47	1.49	1.51	1.53	61
30	1.31	1.33	1.36	1.39	1.43	1.46	1.48	1.50	1.52	1.54	1.56	1.58	60
31	1.35	1.38	1.40	1.44	1.47	1.51	1.52	1.54	1.56	1.58	1.60	1.62	59
32	1.39	1.42	1.45	1.48	1.51	1.55	1.57	1.59	1.61	1.63	1.65	1.67	58
33	1.42	1.45	1.49	1.52	1.55	1.59	1.61	1.63	1.65	1.67	1.69	1.72	57
34	1.46	1.49	1.53	1.56	1.60	1.63	1.65	1.68	1.70	1.72	1.74	1.76	56
35	1.50	1.53	1.56	1.60	1.64	1.68	1.70	1.72	1.74	1.76	1.78	1.81	55
36	1.54	1.57	1.60	1.64	1.68	1.72	1.74	1.76	1.78	1.80	1.83	1.85	54
37	1.57	1.61	1.64	1.68	1.72	1.76	1.78	1.80	1.83	1.85	1.87	1.90	53
38	1.61	1.64	1.68	1.72	1.76	1.80	1.82	1.84	1.87	1.89	1.91	1.94	52
39	1.65	1.68	1.72	1.75	1.80	1.84	1.86	1.88	1.91	1.93	1.96	1.98	51
40	1.68	1.72	1.75	1.79	1.84	1.88	1.90	1.93	1.95	1.97	2.00	2.03	50
41	1.71	1.75	1.79	1.83	1.87	1.92	1.94	1.96	1.99	2.01	2.04	2.07	49
42	1.75	1.79	1.83	1.87	1.91	1.96	1.98	2.00	2.03	2.05	2.08	2.11	48
43	1.78	1.82	1.86	1.90	1.95	1.99	2.02	2.04	2.07	2.09	2.12	2.15	47
44	1.82	1.85	1.90	1.94	1.98	2.03	2.06	2.08	2.11	2.13	2.16	2.19	46
45	1.85	1.89	1.93	1.97	2.02	2.07	2.09	2.12	2.14	2.17	2.20	2.23	45

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

Use this right-side argument for inclination factor B (= cos ζ sec δ).

ζ	67½°	68°	68½°	69°	69½°	70°	70¼°	70½°	70¾°	71°	71¼°	71½°	ζ
46°	1.88	1.92	1.96	2.01	2.05	2.10	2.13	2.15	2.18	2.21	2.24	2.27	44°
47	1.91	1.95	2.00	2.04	2.09	2.14	2.16	2.19	2.22	2.25	2.27	2.30	43
48	1.94	1.98	2.02	2.07	2.12	2.17	2.19	2.22	2.25	2.28	2.31	2.34	42
49	1.97	2.01	2.06	2.11	2.16	2.21	2.23	2.26	2.29	2.32	2.35	2.38	41
50	2.00	2.04	2.09	2.14	2.19	2.24	2.27	2.29	2.32	2.35	2.38	2.41	40
51	2.03	2.07	2.12	2.17	2.22	2.27	2.30	2.33	2.36	2.39	2.42	2.45	39
52	2.06	2.10	2.15	2.20	2.25	2.30	2.33	2.36	2.39	2.42	2.45	2.48	38
53	2.09	2.13	2.18	2.23	2.28	2.33	2.36	2.39	2.42	2.45	2.48	2.52	37
54	2.11	2.16	2.21	2.26	2.31	2.37	2.39	2 42	2.45	2.48	2.52	2.55	36
55	2.14	2.19	2.23	2.29	2.34	2.40	2.42	2.45	2.48	2.52	2.55	2.58	35
56	2.17	2.21	2.26	2.31	2.37	2.42	2.45	2.48	2.51	2.55	2.58	2.61	34
57	2.19	2.24	2.29	2.34	2.39	2.45	2.48	2.51	2.54	2.58	2.61	2.64	33
58	2.22	2.26	2.31	2.37	2.42	2.48	2.51	2.54	2.57	2.61	2.64	2.67	32
59	2.24	2.29	2.34	2.39	2.45	2.51	2.54	2.57	2.60	2.63	2.67	2.70	31
60	2.26	2.31	2.36	2.42	2.47	2.53	2.56	2.59	2.63	2.66	2.69	2.73	30
61	2.29	2.33	2.39	2.44	2.50	2.56	2.59	2.62	2.65	2.69	2.72	2.76	29
62	2.31	2.36	2.41	2.46	2.52	2.58	2.61	2.64	2.68	2.71	2.75	2.78	28
63	2.33	2.38	2.43	2.49	2.54	2.60	2.64	2.67	2.70	2.74	2.77	2.81	27
64	2.35	2.40	2.45	2.51	2.57	2.63	2.66	2.69	2.73	2.76	2.80	2.83	26
65	2.37	2.42	2.47	2.53	2.59	2.65	2.68	2.71	2.75	2.78	2.82	2.86	25
66	2.39	2.44	2.49	2.55	2.61	2.67	2.70	2.74	2.77	2.81	2.84	2.88	24
67	2.41	2.46	2.51	2.57	2.63	2.69	2.72	2.76	2.79	2.83	2.86	2.90	23
68	2.42	2.47	2.53	2.59	2.65	2.71	2.74	2.78	2.81	2.85	2.88	2.92	22
69	2.44	2.49	2.55	2.61	2.67	2.73	2.76	2.80	2.83	2.87	2.90	2.94	21
70	2.46	2.51	2.56	2.62	2.68	2.75	2.78	2.81	2.85	2.89	2.92	2.96	20
71	2.47	2.52	2.58	2.64	2.70	2.77	2.80	2.83	2.87	2.90	2.94	2.98	19
72	2.49	2.54	2.59	2.65	2.72	2.78	2.81	2.85	2.88	2.92	2.96	3.00	18
73	2.50	2.55	2.61	2.67	2.73	2.80	2.83	2.86	2.90	2.94	2.97	3.01	17
74	2.51	2.57	2.62	2.68	2.74	2.81	2.84	2.88	2.92	2.95	2.99	3.03	16
75	2.52	2.58	2.64	2.70	2.76	2.82	2.86	2.89	2.93	2.97	3.00	3.04	15
76	2.54	2.59	2.65	2.71	2.77	2.84	2.87	2.91	2.95	2.99	3.02	3.06	14
77	2.55	2.60	2.66	2.72	2.78	2.85	2.88	2.92	2.95	2.99	3.03	3.07	13
78	2.56	2.61	2.67	2.73	2 79	2.86	2.89	2.93	2.97	3.00	3.04	3.08	12
79	2.57	2.62	2.68	2.74	2.80	2.87	2.91	2.94	2.98	3.02	3.05	3.09	11
80	2.57	2.63	2.69	2.75	2.81	2.88	2.91	2.95	2.99	3.02	3.06	3.10	10
81	2.58	2.64	2.69	2.76	2.82	2.89	2.92	2.96	3.00	3.03	3.07	3.11	9
82	2.59	2.64	2.70	2.76	2.83	2.90	2.93	2.97	3.00	3.04	3.08	3.12	8
83	2.59	2.65	2.71	2.77	2.83	2.90	2.94	2.97	3.01	3.05	3.09	3.13	7
84	2.60	2.66	2.71	2.78	2.84	2.91	2.94	2.98	3.02	3.06	3.09	3.13	6
85	2.60	2.66	2.72	2.78	2.84	2.91	2.95	2.98	3.02	3.06	3.10	3.14	5
86	2.61	2.66	2.72	2.78	2.85	2.92	2.95	2.99	3.03	3.06	3.10	3.14	4
87	2.61	2.67	2.72	2.79	2.85	2.92	2.95	2.99	3.03	3.07	3.11	3.15	3
88	2.61	2.67	2.73	2.79	2.85	2.92	2.96	2.99	3.03	3.07	3.11	3.15	2
89	2.61	2.67	2.73	2.79	2.86	2.92	2.96	3.00	3.03	3.07	3 11	3.15	1
90	2.61	2.67	2.73	2.79	2.86	2.92	2.96	3.00	3.03	3.07	3.11	3.15	0

The bottom line on this page is the collimation factor C (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

ζ	71¾°	72°	72¼°	72½°	72¾°	73°	73¼°	73½°	73¾°	74°	74¼°	ζ
1°	.05	.06	.06	.06	.06	.06	.06	.06	.06	.06	.06	89°
2	.11	.11	.11	.12	.12	.12	.12	.12	.12	.13	.13	88
3	.17	.17	.17	.17	.18	.18	.18	.18	.19	.19	.19	87
4	.22	.23	.23	.23	.23	.24	.24	.24	.25	.25	.26	86
5	.28	.28	.29	.29	.29	.30	.30	.31	.31	.32	.32	85
6	.33	.34	.34	.35	.35	.36	.36	.37	.37	.38	.39	84
7	.39	.39	.40	.41	.41	.42	.42	.43	.44	.44	.45	83
8	.44	.45	.46	.46	.47	.48	.48	.49	.50	.50	.51	82
9	.50	.51	.51	.52	.53	.53	.54	.55	.56	.57	.58	81
10	.55	.56	.57	.58	.59	.60	.60	.61	.62	.63	.64	80
11	.61	.62	.63	.63	.64	.65	.66	.67	.68	.69	.70	79
12	.66	.67	.68	.69	.70	.71	.72	.73	.74	.75	.77	78
13	.72	.73	.74	.75	.76	.77	.78	.79	.80	.82	.83	77
14	.77	.78	.79	.80	.82	.83	.84	.85	.87	.88	.89	76
15	.83	.84	.85	.86	.87	.89	.90	.91	.93	.94	.95	75
16	.88	.89	.91	.92	.93	.94	.96	.97	.99	1.00	1.02	74
17	.93	.95	.96	.97	.99	1.00	1.01	1.03	1.05	1.06	1.08	73
18	.99	1.00	1.01	1.03	1.04	1.06	1.07	1.09	1.10	1.12	1.14	72
19	1.04	1.05	1.07	1.08	1.10	1.11	1.13	1.15	1.16	1.18	1.20	71
20	1.09	1.11	1.12	1.14	1.15	1.17	1.19	1.20	1.22	1.24	1.26	70
21	1.14	1.16	1.17	1.19	1.21	1.22	1.24	1.26	1.28	1.30	1.32	69
22	1.20	1.21	1.23	1.25	1.26	1.28	1.30	1.32	1.34	1.36	1.38	68
23	1.25	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.44	67
24	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.48	1.50	66
25	1.35	1.37	1.39	1.41	1.42	1.45	1.47	1.49	1.51	1.53	1.56	65
26	1.40	1.42	1.44	1.46	1.48	1.50	1.52	1.54	1.57	1.59	1.61	64
27	1.45	1.47	1.49	1.51	1.53	1.55	1.58	1.60	1.62	1.65	1.67	63
28	1.50	1.52	1.54	1.56	1.58	1.60	1.63	1.65	1.68	1.70	1.73	62
29	1.55	1.57	1.59	1.61	1.63	1.66	1.68	1.71	1.73	1.76	1.79	61
30	1.60	1.62	1.64	1.66	1.69	1.71	1.73	1.76	1.79	1.81	1.84	60
31	1.64	1.67	1.69	1.71	1.74	1.76	1.79	1.81	1.84	1.87	1.90	59
32	1.69	1.71	1.74	1.76	1.79	1.81	1.84	1.87	1.89	1.92	1.95	58
33	1.74	1.76	1.79	1.81	1.84	1.86	1.89	1.92	1.95	1.98	2.01	57
34	1.79	1.81	1.83	1.86	1.89	1.91	1.94	1.97	2.00	2.03	2.06	56
35	1.83	1.86	1.88	1.91	1.93	1.96	1.99	2.02	2.05	2.08	2.11	55
36	1.88	1.90	1.93	1.95	1.98	2.01	2.04	2.07	2.10	2.13	2.16	54
37	1.92	1.95	1.97	2.00	2.03	2.06	2.09	2.12	2.15	2.18	2.22	53
38	1.97	1.99	2.02	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.27	52
39	2.01	2.04	2.06	2.09	2.12	2.15	2.18	2.22	2.25	2.28	2.32	51
40	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.26	2.30	2.33	2.37	50
41	2.09	2.12	2.15	2.18	2.21	2.24	2.28	2.31	2.34	2.38	2.42	49
42	2.14	2.16	2.19	2.22	2.26	2.29	2.32	2.36	2.39	2.43	2.46	48
43	1.18	2.21	2.24	2.27	2.30	2.33	2.37	2.40	2.44	2.47	2.51	47
44	2.22	2.25	2.28	2.31	2.34	2.38	2.41	2.45	2.48	2.52	2.56	46
45	2.26	2.29	2.32	2.35	2.38	2.42	2.45	2.49	2.53	2.56	2.60	45

Use this right-side argument for inclination factor B (= cos ζ sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

ζ	71¾°	72°	72¼°	72½°	72¾°	73°	73¼°	73½°	73¾°	74°	74¼°	ζ
46°	2.30	2.33	2.36	2.39	2.42	2.46	2.49	2.53	2.57	2.61	2.65	44°
47	2.33	2.37	2.40	2.43	2.47	2.50	2.54	2.57	2.61	2.65	2.69	43
48	2.37	2.40	2.44	2.47	2.51	2.54	2.58	2.62	2.66	2.70	2.74	42
49	2.41	2.44	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	41
50	2.45	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.82	40
51	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.82	2.86	39
52	2.52	2.55	2.58	2.62	2.66	2.69	2.73	2.77	2.82	2.86	2.90	38
53	2.55	2.58	2.62	2.66	2.69	2.73	2.77	2.81	2.85	2.90	2.94	37
54	2.58	2.62	2.65	2.69	2.73	2.77	2.81	2.85	2.89	2.94	2.98	36
55	2.62	2.65	2.69	2.72	2.76	2.80	2.84	2.88	2.93	2.97	3.02	35
56	2.65	2.68	2.72	2.76	2.80	2.84	2.88	2.92	2.96	3.01	3.05	34
57	2.68	2.71	2.75	2.79	2.83	2.87	2.91	2.95	3.00	3.04	3.09	33
58	2.71	2.74	2.78	2.82	2.86	2.90	2.94	2.99	3.03	3.08	3.12	32
59	2.74	2.77	2.81	2.85	2.89	2.93	2.97	3.02	3.06	3.11	3.16	31
60	2.76	2.80	2.84	2.88	2.92	2.96	3.01	3.05	3.09	3.14	3.19	30
61	2.79	2.83	2.87	2.91	2.95	2.99	3.04	3.08	3.13	3.17	3.22	29
62	2.82	2.86	2.90	2.94	2.98	3.02	3.06	3.11	3.16	3.20	3.25	28
63	2.84	2.88	2.92	2.96	3.00	3.05	3.09	3.14	3.18	3.23	3.28	27
64	2.87	2.91	2.95	2.99	3.03	3.07	3.12	3.16	3.21	3.26	3.31	26
65	2.89	2.93	2.97	3.01	3.06	3.10	3.14	3.19	3.24	3.29	3.34	25
66	2.92	2.96	3.00	3.04	3.08	3.13	3.17	3.22	3.27	3.31	3.37	24
67	2.94	2.98	3.02	3.06	3.10	3.15	3.20	3.24	3.29	3.34	3.39	23
68	2.96	3.00	3.04	3.08	3.13	3.17	3.22	3.26	3.31	3.36	3.42	22
69	2.98	3.02	3.06	3.10	3.15	3.19	3.24	3.29	3.34	3.39	3.44	21
70	3.00	3.04	3.08	3.12	3.17	3.21	3.25	3.31	3.36	3.41	3.46	20
71	3.02	3.06	3.10	3.14	3.19	3.24	3.28	3.33	3.38	3.43	3.48	19
72	3.04	3.08	3.12	3.16	3.21	3.25	3.30	3.35	3.40	3.45	3.50	18
73	3.05	3.09	3.14	3.18	3.22	3.27	3.32	3.37	3.42	3.47	3.52	17
74	3.07	3.11	3.15	3.20	3.24	3.29	3.33	3.38	3.44	3.49	3.54	16
75	3.08	3.13	3.17	3.21	3.26	3.30	3.35	3.40	3.45	3.50	3.56	15
76	3.10	3.14	3.18	3.23	3.28	3.32	3.37	3.42	3.47	3.53	3.58	14
77	3.11	3.15	3.19	3.24	3.29	3.33	3.38	3.43	3.48	3.54	3.59	13
78	3.12	3.16	3.21	3.25	3.30	3.34	3.39	3.44	3.49	3.55	3.60	12
79	3.13	3.18	3.22	3.26	3.31	3.36	3.41	3.46	3.51	3.56	3.62	11
80	3.14	3.19	3.23	3.27	3.32	3.37	3.42	3.47	3.52	3.57	3.63	10
81	3.15	3.20	3.24	3.28	3.33	3.38	3.43	3.48	3.53	3.58	3.64	9
82	3.16	3.20	3.25	3.29	3.34	3.39	3.44	3.49	3.54	3.59	3.65	8
83	3.17	3.21	3.26	3.30	3.35	3.40	3.45	3.49	3.55	3.60	3.66	7
84	3.18	3.22	3.26	3.31	3.35	3.40	3.45	3.50	3.55	3.61	3.66	6
85	3.18	3.22	3.27	3.31	3.36	3.41	3.46	3.51	3.56	3.61	3.67	5
86	3.19	3.23	3.27	3.32	3.36	3.41	3.46	3.51	3.57	3.62	3.68	4
87	3.19	3.23	3.28	3.32	3.37	3.42	3.47	3.52	3.57	3.62	3.68	3
88	3.19	3.23	3.28	3.32	3.37	3.42	3.47	3.52	3.57	3.62	3.68	2
89	3.19	3.24	3.28	3.33	3.37	3.42	3.47	3.52	3.57	3.63	3.68	1
90	3.19	3.24	3.28	3.33	3.37	3.42	3.47	3.52	3.57	3.63	3.68	0

Use this right-side argument for inclination factor B (= cos ζ sec δ).

The bottom line on this page is the collimation factor C (= sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

ζ	74½°	74¾°	75°	75¼°	75½°	75¾°	76°	76¼°	76½°	76¾°	77°	77¼°	ζ
1°	.06	.07	.07	.07	.07	.07	.07	.07	.07	.08	.08	.08	89°
2	.13	.13	.13	.14	.14	.14	.14	.15	.15	.15	.16	.16	88
3	.20	.20	.20	.21	.21	.21	.22	.22	.22	.23	.23	.24	87
4	.26	.27	.27	.27	.28	.28	.29	.29	.30	.30	.31	.32	86
5	.33	.33	.34	.34	.35	.35	.36	.37	.37	.38	.39	.40	85
6	.39	.40	.40	.41	.42	.42	.43	.44	.45	.46	.46	.47	84
7	.46	.46	.47	.48	.49	.50	.50	.51	.52	.53	.54	.55	83
8	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	82
9	.58	.59	.60	.61	.62	.64	.65	.66	.67	.68	.70	.71	81
10	.65	.66	.67	.68	.69	.71	.72	.73	.74	.76	.77	.79	80
11	.71	.73	.74	.75	.76	.77	.79	.80	.82	.83	.85	.86	79
12	.78	.79	.80	.82	.83	.85	.86	.88	.89	.91	.92	.94	78
13	.84	.86	.87	.88	.90	.91	.93	.95	.96	.98	1.00	1.02	77
14	.91	.92	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	76
15	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	75
16	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.23	1.25	74
17	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.32	73
18	1.16	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.32	1.35	1.37	1.40	72
19	1.22	1.24	1.26	1.28	1.30	1.32	1.35	1.37	1.39	1.42	1.45	1.47	71
20	1.28	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	70
21	1.34	1.36	1.38	1.41	1.43	1.46	1.48	1.51	1.54	1.56	1.59	1.62	69
22	1.40	1.42	1.45	1.47	1.50	1.52	1.55	1.58	1.60	1.63	1.66	1.70	68
23	1.46	1.49	1.51	1.54	1.56	1.59	1.62	1.64	1.67	1.70	1.74	1.77	67
24	1.52	1.55	1.57	1.60	1.63	1.65	1.68	1.71	1.74	1.77	1.81	1.84	66
25	1.58	1.61	1.63	1.66	1.69	1.72	1.75	1.78	1.81	1.84	1.88	1.91	65
26	1.64	1.67	1.69	1.72	1.75	1.78	1.81	1.84	1.88	1.91	1.95	1.99	64
27	1.70	1.73	1.75	1.78	1.81	1.85	1.88	1.91	1.95	1.98	2.02	2.06	63
28	1.76	1.78	1.81	1.84	1.87	1.91	1.94	1.97	2.01	2.05	2.09	2.13	62
29	1.81	1.84	1.87	1.90	1.94	1.97	2.00	2.04	2.08	2.11	2.15	2.20	61
30	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.10	2.14	2.18	2.22	2.27	60
31	1.93	1.96	1.99	2.02	2.06	2.09	2.13	2.17	2.21	2.25	2.29	2.33	59
32	1.98	2.01	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.36	2.40	58
33	2.04	2.07	2.10	2.14	2.18	2.21	2.25	2.29	2.33	2.38	2.42	2.47	57
34	2.09	2.13	2.16	2.20	2.23	2.27	2.31	2.35	2.40	2.44	2.49	2.53	56
35	2.15	2.18	2.22	2.25	2.29	2.33	2.37	2.41	2.46	2.50	2.55	2.60	55
36	2.20	2.24	2.27	2.31	2.35	2.39	2.43	2.47	2.52	2.56	2.61	2.66	54
37	2.25	2.29	2.33	2.36	2.40	2.44	2.49	2.53	2.58	2.63	2.67	2.73	53
38	2.30	2.34	2.38	2.42	2.46	2.50	2.55	2.59	2.64	2.69	2.74	2.79	52
39	2.35	2.39	2.43	2.47	2.51	2.56	2.60	2.65	2.70	2.75	2.80	2.85	51
40	2.40	2.44	2.48	2.52	2.57	2.61	2.66	2.70	2.75	2.80	2.86	2.91	50
41	2.45	2.49	2.53	2.58	2.62	2.66	2.71	2.76	2.81	2.86	2.92	2.97	49
42	2.50	2.54	2.58	2.63	2.67	2.72	2.77	2.81	2.87	2.92	2.97	3.03	48
43	2.55	2.59	2.63	2.68	2.72	2.77	2.82	2.87	2.92	2.98	3.03	3.09	47
44	2.60	2.64	2.68	2.73	2.77	2.82	2.87	2.92	2.98	3.03	3.09	3.15	46
45	2.65	2.69	2.73	2.78	2.82	2.87	2.92	2.97	3.03	3.08	3.14	3.20	45

Use this right-side argument for inclination factor B (= cos ζ sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination $\pm \delta$.

Use this left-side argument for azimuth factor A (= $\sin \zeta \sec \delta$).

Use this right-side argument for inclination factor B (= $\cos \zeta \sec \delta$).

ζ	74½°	74¾°	75°	75¼°	75½°	75¾°	76°	76¼°	76½°	76¾°	77°	77¼°	ζ
46°	2.69	2.73	2.78	2.82	2.87	2.92	2.97	3.03	3.08	3.14	3.20	3.26	44°
47	2.74	2.78	2.83	2.87	2.92	2.97	3.02	3.08	3.13	3.19	3.25	3.31	43
48	2.78	2.82	2.87	2.92	2.97	3.02	3.07	3.13	3.18	3.24	3.30	3.37	42
49	2.82	2.87	2.92	2.96	3.01	3.07	3.12	3.18	3.23	3.29	3.35	3.42	41
50	2.87	2.91	2.96	3.01	3.06	3.11	3.17	3.22	3.28	3.34	3.41	3.47	40
51	2.91	2.95	3.00	3.05	3.10	3.16	3.21	3.27	3.33	3.39	3.45	3.52	39
52	2.95	3.00	3.04	3.09	3.15	3.20	3.26	3.31	3.38	3.44	3.50	3.57	38
53	2.99	3.04	3.09	3.14	3.19	3.24	3.30	3.36	3.42	3.48	3.55	3.62	37
54	3.03	3.08	3.13	3.18	3.23	3.29	3.34	3.40	3.47	3.53	3.60	3.67	36
55	3.07	3.11	3.16	3.22	3.27	3.33	3.39	3.45	3.51	3.57	3.64	3.71	35
56	3.10	3.15	3.20	3.26	3.31	3.37	3.43	3.49	3.55	3.62	3.68	3.76	34
57	3.14	3.19	3.24	3.29	3.35	3.41	3.47	3.53	3.59	3.66	3.73	3.80	33
58	3.17	3.22	3.28	3.33	3.39	3.45	3.51	3.57	3.63	3.70	3.77	3.84	32
59	3.21	3.26	3.31	3.37	3.42	3.48	3.54	3.61	3.67	3.74	3.81	3.88	31
60	3.24	3.29	3.35	3.40	3.46	3.52	3.58	3.64	3.71	3.78	3.85	3.92	30
61	3.27	3.33	3.38	3.44	3.49	3.55	3.62	3.68	3.75	3.82	3.89	3.96	29
62	3.30	3.36	3.41	3.47	3.53	3.59	3.65	3.72	3.78	3.85	3.92	4.00	28
63	3.33	3.39	3.44	3.50	3.56	3.62	3.68	3.75	3.82	3.89	3.96	4.04	27
64	3.36	3.42	3.47	3.53	3.59	3.65	3.72	3.78	3.85	3.92	4.00	4.07	26
65	3.39	3.45	3.50	3.56	3.62	3.68	3.75	3.81	3.88	3.95	4.03	4.11	25
66	3.42	3.47	3.53	3.59	3.65	3.71	3.78	3.84	3.91	3.99	4.06	4.14	24
67	3.44	3.50	3.56	3.62	3.68	3.74	3.81	3.87	3.94	4.02	4.09	4.17	23
68	3.47	3.53	3.58	3.64	3.70	3.77	3.83	3.90	3.97	4.05	4.12	4.20	22
69	3.49	3.55	3.61	3.67	3.73	3.79	3.86	3.93	4.00	4.07	4.15	4.23	21
70	3.52	3.57	3.63	3.69	3.75	3.82	3.89	3.95	4.03	4.10	4.18	4.25	20
71	3.54	3.60	3.65	3.71	3.78	3.84	3.91	3.98	4.05	4.13	4.20	4.28	19
72	3.56	3.62	3.67	3.74	3.80	3.86	3.93	4.00	4.07	4.15	4.23	4.31	18
73	3.58	3.64	3.69	3.76	3.82	3.89	3.95	4.02	4.10	4.17	4.25	4.33	17
74	3.60	3.65	3.71	3.78	3.84	3.91	3.97	4.04	4.12	4.19	4.27	4.36	16
75	3.61	3.67	3.73	3.79	3.86	3.92	3.99	4.06	4.14	4.21	4.29	4.38	15
76	3.64	3.69	3.75	3.82	3.88	3.94	4.01	4.08	4.16	4.23	4.31	4.40	14
77	3.65	3.70	3.76	3.83	3.89	3.96	4.03	4.10	4.17	4.25	4.33	4.41	13
78	3.66	3.72	3.78	3.84	3.91	3.97	4.04	4.11	4.19	4.27	4.35	4.43	12
79	3.67	3.73	3.79	3.86	3.92	3.99	4.06	4.13	4.21	4.28	4.36	4.45	11
80	3.68	3.74	3.81	3.87	3.93	4.00	4.07	4.14	4.22	4.30	4.38	4.46	10
81	3.70	3.75	3.82	3.88	3.94	4.01	4.08	4.16	4.23	4.31	4.39	4.48	9
82	3.71	3.76	3.83	3.89	3.96	4.02	4.09	4.17	4.24	4.32	4.40	4.49	8
83	3.72	3.77	3.84	3.90	3.96	4.03	4.10	4.18	4.25	4.33	4.41	4.50	7
84	3.72	3.78	3.84	3.91	3.97	4.04	4.11	4.18	4.26	4.34	4.42	4.51	6
85	3.73	3.79	3.85	3.91	3.98	4.05	4.12	4.19	4.27	4.35	4.43	4.51	5
86	3.73	3.79	3.85	3.92	3.98	4.05	4.12	4.20	4.27	4.35	4.43	4.52	4
87	3.74	3.79	3.86	3.92	3.99	4.06	4.13	4.20	4.28	4.36	4.44	4.52	3
88	3.74	3.80	3.86	3.92	3.99	4.06	4.13	4.20	4.28	4.36	4.44	4.53	2
89	3.74	3.80	3.86	3.93	3.99	4.06	4.13	4.21	4.28	4.36	4.44	4.53	1
90	3.74	3.80	3.86	3.93	3.99	4.06	4.13	4.21	4.28	4.36	4.44	4.53	0

The bottom line on this page is the collimation factor C (= $\sec \delta$).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor A (= sin ζ sec δ).

ζ	77½°	77¾°	78°	78¼°	78½°	78¾°	79°	79¼°	79½°	79¾°	80°	ζ
1°	.08	.08	.08	.09	.09	.09	.09	.09	.10	.10	.10	89°
2	.16	.16	.17	.17	.18	.18	.18	.19	.19	.20	.20	88
3	.24	.25	.25	.26	.26	.27	.27	.28	29	.29	.30	87
4	.32	.33	.34	.34	.35	.36	.37	.37	.38	.39	.40	86
5	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50	85
6	.49	.49	.51	.51	.52	.54	.55	.56	.57	.59	.60	84
7	.56	.57	.59	.60	.61	.62	.64	.65	.67	.69	.70	83
8	.64	.66	.67	.68	.70	.71	.73	.75	.76	.78	.80	82
9	.72	.74	.75	.77	.78	.80	.82	.84	.86	.88	.90	81
10	.80	.82	.84	.85	.87	.89	.91	.93	.95	.98	1.00	80
11	.88	.90	.92	.94	.96	.98	1.00	1.02	1.05	1.07	1.10	79
12	.96	.98	1.00	1.02	1.04	1.07	1.09	1.11	1.14	1.17	1.20	78
13	1.04	1.06	1.08	1.10	1.13	1.15	1.18	1.21	1.23	1.26	1.30	77
14	1.12	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	76
15	1.20	1.22	1.25	1.27	1.30	1.33	1.36	1.39	1.42	1.46	1.49	75
16	1.28	1.30	1.33	1.35	1.38	1.41	1.44	1.48	1.51	1.55	1.59	74
17	1.35	1.38	1.40	1.44	1.47	1.50	1.53	1.57	1.60	1.64	1.68	73
18	1.43	1.46	1.49	1.52	1.55	1.58	1.62	1.66	1.70	1.74	1.78	72
19	1.51	1.53	1.57	1.60	1.63	1.67	1.71	1.75	1.79	1.83	1.87	71
20	1.58	1.61	1.65	1.68	1.72	1.75	1.79	1.83	1.88	1.92	1.97	70
21	1.65	1.69	1.72	1.76	1.80	1.84	1.88	1.92	1.97	2.01	2.06	69
22	1.73	1.77	1.80	1.84	1.88	1.92	1.96	2.01	2.06	2.11	2.16	68
23	1.81	1.84	1.88	1.92	1.96	2.00	2.05	2.09	2.14	2.20	2.25	67
24	1.88	1.92	1.96	2.00	2.04	2.08	2.13	2.18	2.23	2.29	2.34	66
25	1.95	1.99	2.03	2.07	2.12	2.17	2.22	2.27	2.32	2.38	2.43	65
26	2.02	2.07	2.11	2.15	2.20	2.25	2.30	2.35	2.41	2.46	2.52	64
27	2.10	2.14	2.18	2.23	2.28	2.33	2.38	2.43	2.49	2.55	2.61	63
28	2.17	2.21	2.26	2.31	2.36	2.41	2.46	2.52	2.58	2.64	2.70	62
29	2.24	2.28	2.33	2.38	2.43	2.48	2.54	2.60	2.66	2.73	2.79	61
30	2.31	2.36	2.40	2.46	2.51	2.56	2.62	2.68	2.74	2.81	2.88	60
31	2.38	2.43	2.48	2.53	2.58	2.64	2.70	2.76	2.83	2.89	2.97	59
32	2.45	2.50	2.55	2.60	2.66	2.72	2.78	2.84	2.91	2.98	3.05	58
33	2.52	2.57	2.62	2.67	2.73	2.79	2.85	2.92	2.99	3.06	3.14	57
34	2.58	2.64	2.69	2.75	2.80	2.87	2.93	3.00	3.07	3.14	3.22	56
35	2.65	2.70	2.76	2.82	2.88	2.94	3.01	3.08	3.15	3.23	3.30	55
36	2.72	2.77	2.83	2.89	2.95	3.01	3.08	3.15	3.23	3.30	3.38	54
37	2.78	2.84	2.90	2.95	3.02	3.08	3.15	3.23	3.30	3.38	3.47	53
38	2.85	2.90	2.96	3.02	3.09	3.16	3.23	3.30	3.38	3.46	3.55	52
39	2.91	2.97	3.03	3.09	3.16	3.23	3.30	3.37	3.45	3.53	3.62	51
40	2.97	3.03	3.09	3.16	3.22	3.29	3.37	3.45	3.53	3.61	3.70	50
41	3.03	3.09	3.16	3.22	3.29	3.36	3.44	3.52	3.60	3.69	3.78	49
42	3.09	3.15	3.22	3.29	3.36	3.43	3.51	3.59	3.67	3.76	3.85	48
43	3.15	3.21	3.28	3.35	3.42	3.50	3.57	3.66	3.74	3.83	3.93	47
44	3.21	3.27	3.34	3.41	3.48	3.56	3.64	3.72	3.81	3.91	4.00	46
45	3.27	3.33	3.40	3.47	3.55	3.62	3.71	3.79	3.88	3.97	4.07	45

Use this right-side argument for inclination factor B (= cos ζ sec δ).

FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS.

This top argument is the star's declination ± δ.

Use this left-side argument for azimuth factor *A* (= sin ζ sec δ).

ζ	77½°	77¾°	78°	78¼°	78½°	78¾°	79°	79¼°	79½°	79¾°	80°	ζ
46°	3.32	3.39	3.46	3.53	3.61	3.69	3.77	3.86	3.95	4.04	4.14	44°
47	3.38	3.45	3.52	3.59	3.67	3.75	3.83	3.92	4.01	4.11	4.21	43
48	3.43	3.50	3.57	3.65	3.73	3.81	3.89	3.98	4.08	4.18	4.28	42
49	3.49	3.56	3.63	3.71	3.79	3.87	3.96	4.05	4.14	4.24	4.35	41
50	3.54	3.61	3.68	3.76	3.84	3.93	4.02	4.11	4.20	4.30	4.41	40
51	3.59	3.66	3.74	3.82	3.90	3.98	4.07	4.17	4.26	4.37	4.48	39
52	3.64	3.71	3.79	3.87	3.95	4.04	4.13	4.22	4.32	4.43	4.54	38
53	3.69	3.77	3.84	3.92	4.01	4.09	4.19	4.28	4.38	4.49	4.60	37
54	3.74	3.81	3.89	3.97	4.06	4.15	4.24	4.34	4.44	4.55	4.66	36
55	3.78	3.86	3.94	4.02	4.11	4.20	4.29	4.39	4.50	4.60	4.72	35
56	3.83	3.91	3.99	4.07	4.16	4.25	4.34	4.44	4.55	4.66	4.77	34
57	3.88	3.95	4.04	4.12	4.21	4.30	4.39	4.50	4.60	4.72	4.83	33
58	3.92	4.00	4.08	4.16	4.25	4.35	4.44	4.55	4.65	4.77	4.88	32
59	3.96	4.04	4.12	4.21	4.30	4.39	4.49	4.60	4.70	4.82	4.94	31
60	4.00	4.08	4.17	4.25	4.34	4.44	4.54	4.64	4.75	4.87	4.99	30
61	4.04	4.12	4.21	4.29	4.39	4.48	4.58	4.69	4.80	4.92	5.04	29
62	4.08	4.16	4.25	4.34	4.43	4.53	4.63	4.73	4.85	4.96	5.08	28
63	4.12	4.20	4.29	4.38	4.47	4.57	4.67	4.78	4.89	5.01	5.13	27
64	4.15	4.24	4.32	4.41	4.51	4.61	4.71	4.82	4.93	5.05	5.18	26
65	4.19	4.27	4.36	4.45	4.55	4.65	4.75	4.86	4.97	5.09	5.22	25
66	4.22	4.31	4.40	4.49	4.58	4.68	4.79	4.90	5.01	5.14	5.26	24
67	4.26	4.34	4.43	4.52	4.62	4.72	4.82	4.94	5.05	5.18	5.30	23
68	4.28	4.37	4.46	4.55	4.65	4.75	4.86	4.97	5.09	5.21	5.34	22
69	4.32	4.40	4.49	4.58	4.68	4.79	4.89	5.00	5.12	5.25	5.38	21
70	4.34	4.43	4.52	4.61	4.71	4.82	4.93	5.04	5.16	5.28	5.41	20
71	4.37	4.46	4.55	4.64	4.74	4.85	4.96	5.07	5.19	5.32	5.45	19
72	4.39	4.48	4.57	4.67	4.77	4.88	4.98	5.10	5.22	5.34	5.48	18
73	4.42	4.51	4.60	4.70	4.80	4.90	5.01	5.13	5.25	5.37	5.51	17
74	4.44	4.53	4.62	4.72	4.82	4.93	5.04	5.15	5.27	5.40	5.53	16
75	4.46	4.55	4.65	4.74	4.84	4.95	5.06	5.18	5.30	5.43	5.56	15
76	4.48	4.57	4.67	4.76	4.87	4.97	5.09	5.20	5.32	5.45	5.59	14
77	4.50	4.59	4.68	4.78	4.89	4.99	5.11	5.22	5.35	5.47	5.61	13
78	4.52	4.61	4.70	4.80	4.91	5.01	5.13	5.24	5.37	5.50	5.63	12
79	4.54	4.63	4.72	4.82	4.92	5.03	5.14	5.26	5.39	5.52	5.65	11
80	4.55	4.64	4.74	4.84	4.94	5.05	5.16	5.28	5.40	5.54	5.67	10
81	4.56	4.65	4.75	4.85	4.95	5.06	5.18	5.30	5.42	5.55	5.69	9
82	4.57	4.67	4.76	4.86	4.97	5.08	5.19	5.31	5.43	5.56	5.70	8
83	4.59	4.68	4.78	4.87	4.98	5.09	5.20	5.32	5.45	5.58	5.72	7
84	4.60	4.69	4.79	4.88	4.99	5.10	5.21	5.33	5.46	5.59	5.73	6
85	4.60	4.69	4.79	4.89	5.00	5.11	5.22	5.34	5.47	5.60	5.74	5
86	4.61	4.70	4.80	4.90	5.00	5.11	5.23	5.35	5.47	5.61	5.74	4
87	4.62	4.71	4.81	4.90	5.01	5.12	5.23	5.35	5.48	5.61	5.75	3
88	4.62	4.71	4.81	4.91	5.01	5.12	5.24	5.36	5.48	5.61	5.75	2
89	4.62	4.71	4.81	4.91	5.01	5.12	5.24	5.36	5.49	5.62	5.76	1
90	4.62	4.71	4.81	4.91	5.02	5.13	5.24	5.36	5.49	5.62	5.76	0

Use this right-side argument for inclination factor *B* (= cos ζ sec δ).

The bottom line on this page is the collimation factor *C* (= sec δ).

300. FACTORS FOR THE REDUCTION OF TRANSIT TIME OBSERVATIONS AT CORNELL UNIVERSITY.

$\phi = 42° 27'$.

δ	A	B All +	C All + for lamp west, and — for lamp east.	δ	A	B All +	C All + for lamp west, and — for lamp east.
— 40°	+ 1.29	0.17	1.31	+ 62½°	— 0.74	2.03	2.17
— 35	+ 1.19	0.26	1.22	63	— 0.77	2.06	2.20
— 30	+ 1.10	0.35	1.15	63½	— 0.80	2.09	2.24
— 25	+ 1.02	0.42	1.10	64	— 0.84	2.12	2.28
— 20	+ 0.94	0.49	1.06	64½	— 0.87	2.15	2.32
— 15	+ 0.87	0.56	1.04	65	— 0.91	2.19	2.37
— 10	+ 0.80	0.62	1.02	65½	— 0.94	2.22	2.41
— 8	+ 0.78	0.64	1.01	66	— 0.98	2.25	2.46
— 6	+ 0.75	0.67	1.01	66½	— 1.02	2.29	2.51
— 4	+ 0.73	0.69	1.00	67	— 1.06	2.33	2.56
— 2	+ 0.70	0.71	1.00	67½	— 1.10	2.37	2.61
0	+ 0.67	0.74	1.00	68	— 1.15	2.41	2.67
+ 2	+ 0.65	0.76	1.00	68½	— 1.20	2.45	2.73
4	+ 0.62	0.79	1.00	69	— 1.25	2.50	2.79
6	+ 0.60	0.81	1.01	69½	— 1.30	2.54	2.86
8	+ 0.57	0.83	1.01	70	— 1.35	2.59	2.92
0	+ 0.54	0.86	1.02	70¼	— 1.38	2.62	2.96
12	+ 0.52	0.88	1.02	70½	— 1.41	2.64	3.00
14	+ 0.49	0.91	1.03	70¾	— 1.44	2.67	3.03
16	+ 0.46	0.93	1.04	71	— 1.47	2.70	3.07
18	+ 0.43	0.96	1.05	71¼	— 1.50	2.73	3.11
20	+ 0.41	0.98	1.06	71½	— 1.53	2.76	3.15
22	+ 0.38	1.01	1.08	71¾	— 1.56	2.79	3.19
24	+ 0.35	1.04	1.09	72	— 1.60	2.82	3.24
26	+ 0.32	1.07	1.11	72¼	— 1.63	2.85	3.28
28	+ 0.28	1.10	1.13	72½	— 1.66	2.88	3.33
30	+ 0.25	1.13	1.15	72¾	— 1.70	2.91	3.37
32	+ 0.22	1.16	1.18	73	— 1.74	2.95	3.42
34	+ 0.18	1.19	1.21	73¼	— 1.78	2.98	3.47
36	+ 0.14	1.23	1.24	73½	— 1.82	3.02	3.52
38	+ 0.10	1.27	1.27	73¾	— 1.86	3.05	3.57
40	+ 0.06	1.30	1.31	74	— 1.90	3.09	3.63
41	+ 0.03	1.32	1.33	74¼	— 1.94	3.13	3.68
42	+ 0.01	1.35	1.35	74½	— 1.98	3.17	3.74
43	— 0.01	1.37	1.37	74¾	— 2.03	3.21	3.80
44	— 0.04	1.39	1.39	75	— 2.08	3.26	3.86
45	— 0.06	1.41	1.41	75¼	— 2.13	3.30	3.93
46	— 0.09	1.44	1.44	75½	— 2.18	3.35	3.99
47	— 0.12	1.46	1.47	75¾	— 2.23	3.40	4.06
48	— 0.14	1.49	1.49	76	— 2.28	3.45	4.13
49	— 0.17	1.51	1.52	76¼	— 2.34	3.50	4.21
50	— 0.20	1.54	1.56	76½	— 2.40	3.55	4.28
51	— 0.24	1.57	1.59	76¾	— 2.46	3.60	4.36
52	— 0.27	1.60	1.62	77	— 2.52	3.66	4.44
53	— 0.30	1.63	1.66	77¼	— 2.59	3.72	4.53
54	— 0.34	1.67	1.70	77½	— 2.65	3.78	4.62
55	— 0.38	1.70	1.74	77¾	— 2.72	3.85	4.71
56	— 0.42	1.74	1.79	78	— 2.80	3.91	4.81
57	— 0.46	1.78	1.84	78¼	— 2.87	3.98	4.91
58	— 0.51	1.82	1.89	78½	— 2.95	4.06	5.02
59	— 0.55	1.86	1.94	78¾	— 3.03	4.13	5.13
60	— 0.60	1.91	2.00	79	— 3.12	4.21	5.24
60½	— 0.63	1.93	2.03	79¼	— 3.21	4.29	5.36
61	— 0.66	1.96	2.06	79½	— 3.31	4.38	5.49
61½	— 0.68	1.98	2.10	79¾	— 3.41	4.47	5.62
+ 62	— 0.71	2.01	2.13	+ 80	— 3.51	4.57	5.76

This table is computed for latitude 42° 27′. It may be used, however, for any station whose latitude does not differ from that value by more than 7′. For a station in latitude 42° 20′, or in latitude 42° 34′, the maximum error in the table is two units in the last place.

301. CORRECTION TO TRANSIT OBSERVATIONS FOR DIURNAL ABERRATION.

The correction is negative when applied to observed times, except for sub-polars. (See § 96.)

Latitude $= \phi$.	Declination $= \delta$.								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
0°	$0^s.02$	$0^s.02$	$0^s.02$	$0^s.02$	$0^s.03$	$0^s.03$	$0^s.04$	$0^s.06$	$0^s.12$
10	0 .02	0 .02	0 .02	0 .02	0 .03	0 .03	0 .04	0 .06	0 .12
20	0 .02	0 .02	0 .02	0 .02	0 .03	0 .03	0 .04	0 .06	0 .12
30	0 .02	0 .02	0 .02	0 .02	0 .02	0 .03	0 .04	0 .05	0 .11
40	0 .02	0 .02	0 .02	0 .02	0 .02	0 .03	0 .03	0 .05	0 .09
50	0 .01	0 .01	0 .01	0 .02	0 .02	0 .02	0 .03	0 .04	0 .08
60	0 .01	0 .01	0 .01	0 .01	0 .01	0 .02	0 .02	0 .03	0 .06
70	0 .01	0 .01	0 .01	0 .01	0 .01	0 .01	0 .01	0 .02	0 .04
80	0 .00	0 .00	0 .00	0 .00	0 .00	0 .01	0 .01	0 .01	0 .02

302. RELATIVE WEIGHTS FOR TRANSIT OBSERVATIONS DEPENDING ON THE STAR'S DECLINATION. (See § 111.)

δ	w	$\sqrt{w}$	δ	w	$\sqrt{w}$	δ	w	$\sqrt{w}$
0	1.00	1.00	45	0.69	0.83	70	0.24	0.49
10	0.99	0.99	50	0.61	0.78	75	0.14	0.37
20	0.95	0.97	55	0.52	0.72	80	0.07	0.26
30	0.87	0.93	60	0.42	0.65	85	0.02	0.14
40	0.76	0.87	65	0.33	0.57			

In the application of the multiplier $\sqrt{w}$ it generally suffices to employ but one significant figure.

303. RELATIVE WEIGHTS FOR INCOMPLETE TRANSITS.

(See § 111.)

No. of Lines Obs.	For eye and ear observations.				For observations with a chronograph.					
	5 Lines in Reticle.		7 Lines in Reticle.		9 Lines in Reticle.		11 Lines in Reticle.		13 Lines in Reticle.	
	w	$\sqrt{w}$	w	$\sqrt{w}$	w	$\sqrt{w}$	w	$\sqrt{w}$	w	$\sqrt{w}$
1	0.40	0.63	0.36	0.60	0.42	0.65	0.42	0.65	0.41	0.64
2	0.64	0.80	0.57	0.75	0.62	0.79	0.62	0.79	0.60	0.77
3	0.80	0.89	0.71	0.84	0.73	0.85	0.73	0.85	0.71	0.84
4	0.92	0.96	0.82	0.91	0.82	0.91	0.81	0.90	0.79	0.89
5	1.00	1.00	0.90	0.95	0.87	0.93	0.86	0.93	0.84	0.92
6			0.95	0.97	0.92	0.96	0.90	0.95	0.88	0.94
7			1.00	1.00	0.95	0.97	0.93	0.96	0.91	0.95
8					0.98	0.99	0.95	0.97	0.93	0.96
9					1.00	1.00	0.97	0.98	0.95	0.97
10							0.99	0.99	0.97	0.98
11							1.00	1.00	0.98	0.99
12									0.99	0.99
13									1.00	1.00

304. CORRECTION TO LATITUDE FOR DIFFERENTIAL REFRACTION.

The sign of the correction is the same as that of the micrometer difference. (See § 150.)

½ Diff. of Zenith Distances.	Zenith Distance.							
	0°	10°	20°	25°	30°	35°	40°	45°
0′.0	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00
0 .5	0 .01	0 .01	0 .01	0 .01	0 .01	0 .01	0 .01	0 .02
1 .0	0 .02	0 .02	0 .02	0 .02	0 .02	0 .02	0 .03	0 .03
1 .5	0 .03	0 .03	0 .03	0 .03	0 .03	0 .03	0 .04	0 .05
2 .0	0 .03	0 .03	0 .04	0 .04	0 .04	0 .05	0 .06	0 .07
2 .5	0 .04	0 .04	0 .05	0 .05	0 .05	0 .06	0 .07	0 .08
3 .0	0 .05	0 .05	0 .06	0 .06	0 .07	0 .08	0 .09	0 .10
3 .5	0 .06	0 .06	0 .07	0 .07	0 .08	0 .09	0 .10	0 .12
4 .0	0 .07	0 .07	0 .08	0 .08	0 .09	0 .10	0 .11	0 .13
4 .5	0 .08	0 .08	0 .09	0 .09	0 .10	0 .11	0 .13	0 .15
5 .0	0 .08	0 .09	0 .10	0 .10	0 .11	0 .13	0 .14	0 .17
5 .5	0 .09	0 .10	0 .10	0 .11	0 .12	0 .14	0 .16	0 .18
6 .0	0 .10	0 .10	0 .11	0 .12	0 .13	0 .15	0 .17	0 .20
6 .5	0 .11	0 .11	0 .12	0 .13	0 .14	0 .16	0 .19	0 .22
7 .0	0 .12	0 .12	0 .13	0 .14	0 .15	0 .18	0 .20	0 .24
7 .5	0 .13	0 .13	0 .14	0 .15	0 .16	0 .19	0 .21	0 .25
8 .0	0 .13	0 .14	0 .15	0 .16	0 .18	0 .21	0 .23	0 .27
8 .5	0 .14	0 .15	0 .16	0 .17	0 .19	0 .22	0 .24	0 .29
9 .0	0 .15	0 .16	0 .17	0 .18	0 .20	0 .23	0 .26	0 .30
9 .5	0 .16	0 .17	0 .18	0 .20	0 .21	0 .24	0 .27	0 .32
10 .0	0 .17	0 .18	0 .19	0 .21	0 .23	0 .26	0 .29	0 .34
10 .5	0 .18	0 .19	0 .20	0 .22	0 .24	0 .27	0 .30	0 .35
11 .0	0 .18	0 .19	0 .21	0 .23	0 .25	0 .28	0 .31	0 .37
11 .5	0 .19	0 .20	0 .22	0 .24	0 .26	0 .30	0 .33	0 .39
12 .0	0 .20	0 .21	0 .23	0 .25	0 .27	0 .31	0 .34	0 .40
12 .5	0 .21	0 .21	0 .24	0 .26	0 .28	0 .32	0 .36	0 .42
13 .0	0 .22	0 .22	0 .25	0 .27	0 .29	0 .33	0 .37	0 .44
13 .5	0 .23	0 .23	0 .26	0 .28	0 .30	0 .34	0 .39	0 .45
14 .0	0 .23	0 .24	0 .27	0 .29	0 .31	0 .35	0 .40	0 .47
14 .5	0 .24	0 .25	0 .28	0 .30	0 .32	0 .36	0 .41	0 .49
15 .0	0 .25	0 .26	0 .28	0 .31	0 .34	0 .38	0 .43	0 .50
15 .5	0 .26	0 .27	0 .29	0 .32	0 .35	0 .39	0 .44	0 .52
16 .0	0 .27	0 .28	0 .30	0 .33	0 .36	0 .40	0 .46	0 .54
16 .5	0 .28	0 .29	0 .31	0 .34	0 .37	0 .41	0 .47	0 .55
17 .0	0 .28	0 .29	0 .32	0 .35	0 .38	0 .42	0 .49	0 .57
17 .5	0 .29	0 .30	0 .33	0 .36	0 .39	0 .44	0 .50	0 .59
18 .0	0 .30	0 .31	0 .34	0 .37	0 .40	0 .45	0 .52	0 .60
18 .5	0 .31	0 .32	0 .35	0 .38	0 .41	0 .46	0 .53	0 .62
19 .0	0 .32	0 .33	0 .36	0 .39	0 .43	0 .48	0 .54	0 .64
19 .5	0 .33	0 .34	0 .37	0 .40	0 .44	0 .49	0 .56	0 .66
20 .0	0 .34	0 .35	0 .38	0 .41	0 .45	0 .50	0 .57	0 .67

305. CORRECTION TO LATITUDE FOR REDUCTION TO MERIDIAN.

The sign of the correction to the latitude is positive except for stars of negative declination (south of the equator). (See § 151.)

	Hour-angle.											
δ	10^s	15^s	20^s	25^s	30^s	35^s	40^s	45^s	50^s	55^s	60^s	δ
0°	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00	90°
5	0 .00	0 .01	0 .01	0 .01	0 .02	0 .03	0 .04	0 .05	0 .06	0 .07	0 .09	85
10	0 .00	0 .01	0 .02	0 .03	0 .04	0 .06	0 .08	0 .09	0 .12	0 .14	0 .17	80
15	0 .01	0 .02	0 .03	0 .04	0 .06	0 .08	0 .11	0 .14	0 .17	0 .21	0 .24	75
20	0 .01	0 .02	0 .04	0 .05	0 .08	0 .11	0 .14	0 .18	0 .22	0 .27	0 .32	70
25	0 .01	0 .02	0 .04	0 .07	0 .09	0 .13	0 .17	0 .21	0 .26	0 .32	0 .38	65
30	0 .01	0 .03	0 .05	0 .07	0 .11	0 .14	0 .19	0 .24	0 .30	0 .36	0 .42	60
35	0 .01	0 .03	0 .05	0 .08	0 .12	0 .16	0 .21	0 .26	0 .32	0 .39	0 .46	55
40	0 .01	0 .03	0 .05	0 .08	6 .12	0 .16	0 .22	0 .27	0 .34	0 .41	0 .48	50
45	0 .01	0 .03	0 .06	0 .08	0 .12	0 .17	0 .22	0 .28	0 .34	0 .41	0 .49	45

306. CORRECTION FOR CURVATURE OF APPARENT PATH OF STAR, IN MICROMETER VALUE DETERMINATIONS.

The correction as tabulated is $\frac{1}{6}(15 \sin 1'')^2\tau^3 - \frac{1}{120}(15 \sin 1'')^4\tau^5$.

Apply the corrections given in the table directly to the observed chronometer times, adding them before either elongation to the times, and subtracting them after either elongation.

(See § 159.)

τ	Corr.	τ	Corr.	τ	Corr.	τ	Corr.	τ	Corr.
6^m	$0^s.0$	18^m	$1^s.1$	30^m	$5^s.1$	42^m	$14^s.1$	54^m	$29^s.9$
7	0 .1	19	1 .3	31	5 .7	43	15 .1	55	31 .6
8	0 .1	20	1 .5	32	6 .2	44	16 .2	56	33 .3
9	0 .1	21	1 .8	33	6 .8	45	17 .3	57	35 .1
10	0 .2	22	2 .0	34	7 .5	46	18 .5	58	37 .0
11	0 .2	23	2 .3	35	8 .2	47	19 .7	59	39 .0
12	0 .3	24	2 .6	36	8 .9	48	21 .0	60	41 .0
13	0 .4	25	3 .0	37	9 .6	49	22 .3	61	43 .1
14	0 .5	26	3 .3	38	10 .4	50	23 .7	62	45 .2
15	0 .6	27	3 .7	39	11 .3	51	25.2	63	47 .4
16	0 .8	28	4 .2	40	12 .2	52	26 .7	64	49 .7
17	0 .9	29	4 .6	41	13 .1	53	28 .3	65	52 .1

307. $m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}$. (See § 172.)

t	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m
0^s	0″.00	1″.96	7″.85	17″.67	31″.42	49″.09	70″.68	96″.20	125″.65
1	0 .00	2 .03	7 .98	17 .87	31 .68	49 .41	71 .07	96 .66	126 .17
2	0 .00	2 .10	8 .12	18 .07	31 .94	49 .74	71 .47	97 .12	126 .70
3	0 .00	2 .16	8 .25	18 .27	32 .20	50 .07	71 .86	97 .58	127 .22
4	0 .01	2 .23	8 .39	18 .47	32 .47	50 .40	72 .26	98 .04	127 .75
5	0 .01	2 .31	8 .52	18 .67	32 .74	50 .73	72 .66	98 .50	128 .28
6	0 .02	2 .38	8 .66	18 .87	33 .01	51 .07	73 .06	98 .97	128 .81
7	0 .02	2 .45	8 .80	19 .07	33 .27	51 .40	73 .46	99 .43	129 .34
8	0 .03	2 .52	8 .94	19 .28	33 .54	51 .74	73 .86	99 .90	129 .87
9	0 .04	2 .60	9 .08	19 .48	33 .81	52 .07	74 .26	100 .37	130 .40
10	0 .05	2 .67	9 .22	19 .69	34 .09	52 .41	74 .66	100 .84	130 .94
11	0 .06	2 .75	9 .36	19 .90	34 .36	52 .75	75 .06	101 .31	131 .47
12	0 .08	2 .83	9 .50	20 .11	34 .64	53 .09	75 .47	101 .78	132 .01
13	0 .09	2 .91	9 .64	20 .32	34 .91	53 .43	75 .88	102 .25	132 .55
14	0 .11	2 .99	9 .79	20 .53	35 .19	53 .77	76 .29	102 .72	133 .09
15	0 .12	3 .07	9 .94	20 .74	35 .46	54 .11	76 .69	103 .20	133 .63
16	0 .14	3 .15	10 .09	20 .95	35 .74	54 .46	77 .10	103 .67	134 .17
17	0 .16	3 .23	10 .24	21 .16	36 .02	54 .80	77 .51	104 .15	134 .71
18	0 .18	3 .32	10 .39	21 .38	36 .30	55 .15	77 .93	104 .63	135 .25
19	0 .20	3 .40	10 .54	21 .60	36 .58	55 .50	78 .34	105 .10	135 .80
20	0 .22	3 .49	10 .69	21 .82	36 .87	55 .84	78 .75	105 .58	136 .34
21	0 .24	3 .58	10 .84	22 .03	37 .15	56 .19	79 .16	106 .06	136 .88
22	0 .26	3 .67	11 .00	22 .25	37 .44	56 .55	79 .58	106 .55	137 .43
23	0 .28	3 .76	11 .15	22 .47	37 .72	56 .90	80 .00	107 .03	137 .98
24	0 .31	3 .85	11 .31	22 .70	38 .01	57 .25	80 .42	107 .51	138 .53
25	0 .34	3 .94	11 .47	22 .92	38 .30	57 .60	80 .84	107 .99	139 .08
26	0 .37	4 .03	11 .63	23 .14	38 .59	57 .96	81 .26	108 .48	139 .63
27	0 .40	4 .12	11 .79	23 .37	38 .88	58 .32	81 .68	108 .97	140 .18
28	0 .43	4 .22	11 .95	23 .60	39 .17	58 .68	82 .10	109 .46	140 .74
29	0 .46	4 .32	12 .11	23 .82	39 .46	59 .03	82 .52	109 .95	141 .29
30	0 .49	4 .42	12 .27	24 .05	39 .76	59 .40	82 .95	110 .44	141 .85
31	0 .52	4 .52	12 .43	24 .28	40 .05	59 .75	83 .38	110 .93	142 .40
32	0 .56	4 .62	12 .60	24 .51	40 .35	60 .11	83 .81	111 .43	142 .96
33	0 .59	4 .72	12 .76	24 .74	40 .65	60 .47	84 .23	111 .92	143 .52
34	0 .63	4 .82	12 .93	24 .98	40 .95	60 .84	84 .66	112 .41	144 .08
35	0 .67	4 .92	13 .10	25 .21	41 .25	61 .20	85 .09	112 .90	144 .64
36	0 .71	5 .03	13 .27	25 .45	41 .55	61 .57	85 .52	113 .40	145 .20
37	0 .75	5 .13	13 .44	25 .68	41 .85	61 .94	85 .95	113 .90	145 .76
38	0 .79	5 .24	13 .62	25 .92	42 .15	62 .31	86 .39	114 .40	146 .33
39	0 .83	5 .34	13 .79	26 .16	42 .45	62 .68	86 .82	114 .90	146 .89
40	0 .87	5 .45	13 .96	26 .40	42 .76	63 .05	87 .26	115 .40	147 .46
41	0 .91	5 .56	14 .13	26 .64	43 .06	63 .42	87 .70	115 .90	148 .03
42	0 .96	5 .67	14 .31	26 .88	43 .37	63 .79	88 .14	116 .40	148 .60
43	1 .01	5 .78	14 .49	27 .12	43 .68	64 .16	88 .57	116 .90	149 .17
44	1 .06	5 .90	14 .67	27 .37	43 .99	64 .54	89 .01	117 .41	149 .74
45	1 .10	6 .01	14 .85	27 .61	44 .30	64 .91	89 .45	117 .92	150 .31
46	1 .15	6 .13	15 .03	27 .86	44 .61	65 .29	89 .89	118 .43	150 .88
47	1 .20	6 .24	15 .21	28 .10	44 .92	65 .67	90 .33	118 .94	151 .45
48	1 .26	6 .36	15 .39	28 .35	45 .24	66 .05	90 .78	119 .45	152 .03
49	1 .31	6 .48	15 .57	28 .60	45 .55	66 .43	91 .23	119 .96	152 .61
50	1 .36	6 .60	15 .76	28 .85	45 .87	66 .81	91 .68	120 .47	153 .19
51	1 .42	6 .72	15 .95	29 .10	46 .18	67 .19	92 .12	120 .98	153 .77
52	1 .48	6 .84	16 .14	29 .36	46 .50	67 .58	92 .57	121 .49	154 .35
53	1 .53	6 .96	16 .32	29 .61	46 .82	67 .96	93 .02	122 .01	154 .93
54	1 .59	7 .09	16 .51	29 .86	47 .14	68 .35	93 .47	122 .53	155 .51
55	1 .65	7 .21	16 .70	30 .12	47 .46	68 .73	93 .92	123 .05	156 .09
56	1 .71	7 .34	16 .89	30 .38	47 .79	69 .12	94 .38	123 .57	156 .67
57	1 .77	7 .46	17 .08	30 .64	48 .11	69 .51	94 .83	124 .09	157 .25
58	1 .83	7 .60	17 .28	30 .90	48 .43	69 .90	95 .29	124 .61	157 .84
59^s	1 .89	7 .72	17 .47	31 .16	48 .76	70 .29	95 .74	125 .13	158 .43

$$m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}.$$

t	9^m	10^m	11^m	12^m	13^m	14^m	15^m	16^m
0^s	159″.02	196″.32	237″.54	282″.68	331″.74	384″.74	441″.63	502″.46
1	159 .61	196 .97	238 .26	283 .47	332 .59	385 .65	442 .62	503 .50
2	160 .20	197 .63	238 .98	284 .26	333 .44	386 .56	443 .60	504 .55
3	160 .80	198 .28	239 .70	285 .04	334 .29	387 .48	444 .58	505 .60
4	161 .39	198 .94	240 .42	285 .83	335 .15	388 .40	445 .56	506 .65
5	161 .98	199 .60	241 .14	286 .62	336 .00	389 .32	446 .55	507 .70
6	162 .58	200 .26	241 .87	287 .41	336 .86	390 .24	447 .54	508 .76
7	163 .17	200 .92	242 .60	288 .20	337 .72	391 .16	448 .53	509 .81
8	163 .77	201 .59	243 .33	289 .00	338 .58	392 .09	449 .51	510 .86
9	164 .37	202 .25	244 .06	289 .79	339 .44	393 .01	450 .50	511 .92
10	164 .97	202 .92	244 .79	290 .58	340 .30	393 .94	451 .50	512 .98
11	165 .57	203 .58	245 .52	291 .38	341 .16	394 .86	452 .49	514 .03
12	166 .17	204 .25	246 .25	292 .18	342 .02	395 .79	453 .48	515 .09
13	166 .77	204 .92	246 .98	292 .98	342 .88	396 .72	454 .48	516 .15
14	167 .37	205 .59	247 .72	293 .78	343 .75	397 .65	455 .47	517 .21
15	167 .97	206 .26	248 .45	294 .58	344 .62	398 .58	456 .47	518 .27
16	168 .58	206 .93	249 .19	295 .38	345 .49	399 .52	457 .47	519 .34
17	169 .19	207 .60	249 .93	296 .18	346 .36	400 .45	458 .47	520 .40
18	169 .80	208 .27	250 .67	296 .99	347 .23	401 .38	459 .47	521 .47
19	170 .41	208 .94	251 .41	297 .79	348 .10	402 .32	460 .47	522 .53
20	171 .02	209 .62	252 .15	298 .60	348 .97	403 .26	461 .47	523 .60
21	171 .63	210 .30	252 .89	299 .40	349 .84	404 .20	462 .48	524 .67
22	172 .24	210 .98	253 .63	300 .21	350 .71	405 .14	463 .48	525 .74
23	172 .85	211 .66	254 .37	301 .02	351 .58	406 .08	464 .48	526 .81
24	173 .47	212 .34	255 .12	301 .83	352 .46	407 .02	465 .49	527 .89
25	174 .08	213 .02	255 .87	302 .64	353 .34	407 .96	466 .50	528 .96
26	174 .70	213 .70	256 .62	303 .46	354 .22	408 .90	467 .51	530 .03
27	175 .32	214 .38	257 .37	304 .27	355 .10	409 .84	468 .52	531 .11
28	175 .94	215 .07	258 .12	305 .09	355 .98	410 .79	469 .53	532 .18
29	176 .56	215 .75	258 .87	305 .90	356 .86	411 .73	470 .54	533 .26
30	177 .18	216 .44	259 .62	306 .72	357 .74	412 .68	471 .55	534 .33
31	177 .80	217 .12	260 .37	307 .54	358 .62	413 .63	472 .57	535 .41
32	178 .43	217 .81	261 .12	308 .36	359 .51	414 .59	473 .58	536 .50
33	179 .05	218 .50	261 .88	309 .18	360 .39	415 .54	474 .60	537 .58
34	179 .68	219 .19	262 .64	310 .00	361 .28	416 .49	475 .62	538 .67
35	180 .30	219 .88	263 .39	310 .82	362 .17	417 .44	476 .64	539 .75
36	180 .93	220 .58	264 .15	311 .65	363 .07	418 .40	477 .65	540 .83
37	181 .56	221 .27	264 .91	312 .47	363 .96	419 .35	478 .67	541 .91
38	182 .19	221 .97	265 .68	313 .30	364 .85	420 .31	479 .70	543 .00
39	182 .82	222 .66	266 .44	314 .12	365 .75	421 .27	480 .72	544 .09
40	183 .46	223 .36	267 .20	314 .95	366 .64	422 .23	481 .74	545 .18
41	184 .09	224 .06	267 .96	315 .78	367 .53	423 .19	482 .77	546 .27
42	184 .72	224 .76	268 .73	316 .61	368 .42	424 .15	483 .79	547 .36
43	185 .35	225 .46	269 .49	317 .44	369 .31	425 .11	484 .82	548 .45
44	185 .99	226 .16	270 .26	318 .27	370 .21	426 .07	485 .85	549 .55
45	186 .63	226 .86	271 .02	319 .10	371 .11	427 .04	486 .88	550 .64
46	187 .27	227 .57	271 .79	319 .94	372 .01	428 .01	487 .91	551 .73
47	187 .91	228 .27	272 .56	320 .78	372 .91	428 .97	488 .94	552 .83
48	188 .55	228 .98	273 .34	321 .62	373 .82	429 .93	489 .97	553 .93
49	189 .19	229 .68	274 .11	322 .45	374 .72	430 .90	491 .01	555 .03
50	189 .83	230 .39	274 .88	323 .29	375 .62	431 .87	492 .05	556 .13
51	190 .47	231 .10	275 .65	324 .13	376 .52	432 .84	493 .08	557 .24
52	191 .12	231 .81	276 .43	324 .97	377 .43	433 .82	494 .12	558 .34
53	191 .76	232 .52	277 .20	325 .81	378 .34	434 .79	495 .15	559 .44
54	192 .41	233 .24	277 .98	326 .66	379 .26	435 .76	496 .19	560 .55
55	193 .06	233 .95	278 .76	327 .50	380 .17	436 .73	497 .23	561 .65
56	193 .71	234 .67	279 .55	328 .35	381 .08	437 .71	498 .28	562 .76
57	194 .36	235 .38	280 .33	329 .19	381 .99	438 .69	499 .32	563 .87
58	195 .01	236 .10	281 .12	330 .04	382 .90	439 .67	500 .37	564 .98
59	195 .66	236 .82	281 .90	330 .89	383 .82	440 .65	501 .41	566 .08

$$m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}.$$

	17^m	18^m	19^m	20^m	21^m	22^m	23^m	24^m	25^m
0^s	567″.2	635″.9	708″.4	784″.9	865″.3	949″.6	1037″.8	1129″.9	1225″.9
1	568 .3	637 .0	709 .7	786 .2	866 .6	951 .0	1039 .3	1131 .4	1227 .5
2	569 .4	638 .2	710 .9	787 .5	868 .0	952 .4	1040 .8	1133 .0	1229 .2
3	570 .5	639 .4	712 .1	788 .8	869 .4	953 .8	1042 .3	1134 .6	1230 .8
4	571 .6	640 .6	713 .4	790 .1	870 .8	955 .3	1043 .8	1136 .2	1232 .5
5	572 .8	641 .7	714 .6	791 .4	872 .1	956 .7	1045 .3	1137 .8	1234 .1
6	573 .9	642 .9	715 .9	792 .7	873 .5	958 .2	1046 .8	1139 .3	1235 .7
7	575 .0	644 .1	717 .1	794 .0	874 .9	959 .6	1048 .3	1140 .9	1237 .3
8	576 .1	645 .3	718 .4	795 .4	876 .3	961 .1	1049 .8	1142 .5	1239 .0
9	577 .2	646 .5	719 .6	796 .7	877 .6	962 .5	1051 .3	1144 .0	1240 .6
10	578 .4	647 .7	720 .9	798 .0	879 .0	963 .9	1052 .8	1145 .6	1242 .3
11	579 .5	648 .9	722 .1	799 .3	880 .4	965 .4	1054 .3	1147 .2	1243 .9
12	580 .6	650 .0	723 .4	800 .7	881 .8	966 .9	1055 .9	1148 .8	1245 .6
13	581 .7	651 .2	724 .6	802 .0	883 .2	968 .3	1057 .4	1150 .4	1247 .2
14	582 .9	652 .4	725 .9	803 .3	884 .6	969 .8	1058 .9	1152 .0	1248 .9
15	584 .0	653 .6	727 .2	804 .6	886 .0	971 .2	1060 .4	1153 .6	1250 .5
16	585 .1	654 .8	728 .4	806 .0	887 .4	972 .7	1062 .0	1155 .2	1252 .2
17	586 .2	656 .0	729 .7	807 .3	888 .8	974 .1	1063 .5	1156 .8	1253 .8
18	587 .4	657 .2	730 .9	808 .6	890 .2	975 .5	1065 .0	1158 .3	1255 .5
19	588 .5	658 .4	732 .2	809 .9	891 .6	977 .0	1066 .5	1159 .9	1257 .1
20	589 .6	659 .6	733 .5	811 .3	893 .0	978 .5	1068 .1	1161 .5	1258 .8
21	590 .8	660 .8	734 .7	812 .6	894 .4	979 .9	1069 .6	1163 .1	1260 .5
22	591 .9	662 .0	736 .0	813 .9	895 .8	981 .4	1071 .1	1164 .7	1262 .2
23	593 .0	663 .2	737 .3	815 .2	897 .2	982 .9	1072 .6	1166 .3	1263 .8
24	594 .2	664 .4	738 .5	816 .6	898 .6	984 .4	1074 .2	1167 .9	1265 .5
25	595 .3	665 .6	739 .8	817 .9	900 .0	985 .8	1075 .7	1169 .5	1267 .1
26	596 .5	666 .8	741 .1	819 .2	901 .4	987 .3	1077 .2	1171 .1	1268 .8
27	597 .6	668 .0	742 .3	820 .5	902 .8	988 .8	1078 .7	1172 .7	1270 .5
28	598 .7	669 .2	743 .6	821 .9	904 .2	990 .3	1080 .3	1174 .3	1272 .1
29	599 .9	670 .4	744 .9	823 .2	905 .6	991 .8	1081 .8	1175 .9	1273 .7
30	601 .0	671 .6	746 .2	824 .6	907 .0	993 .2	1083 .3	1177 .5	1275 .4
31	602 .2	672 .8	747 .4	825 .9	908 .4	994 .7	1084 .8	1179 .1	1277 .1
32	603 .3	674 .1	748 .7	827 .3	909 .8	996 .2	1086 .4	1180 .7	1278 .8
33	604 .5	675 .3	750 .0	828 .6	911 .2	997 .6	1087 .9	1182 .3	1280 .4
34	605 .6	676 .5	751 .3	829 .9	912 .6	999 .1	1089 .5	1183 .9	1282 .1
35	606 .8	677 .7	752 .6	831 .2	914 .0	1000 .6	1091 .0	1185 .5	1283 .8
36	607 .9	678 .9	753 .8	832 .6	915 .5	1002 .1	1092 .6	1187 .1	1285 .5
37	609 .1	680 .1	755 .1	833 .9	916 .9	1003 .5	1094 .1	1188 .7	1287 .1
38	610 .2	681 .3	756 .4	835 .3	918 .3	1005 .0	1095 .7	1190 .3	1288 .8
39	611 .4	682 .6	757 .7	836 .6	919 .7	1006 .5	1097 .2	1191 .9	1290 .5
40	612 .5	683 .8	759 .0	838 .0	921 .1	1008 .0	1098 .8	1193 .5	1292 .2
41	613 .7	685 .0	760 .2	839 .3	922 .5	1009 .4	1100 .3	1195 .1	1293 .8
42	614 .8	686 .2	761 .5	840 .7	923 .9	1010 .9	1101 .9	1196 .7	1295 .5
43	616 .0	687 .4	762 .8	842 .0	925 .3	1012 .4	1103 .4	1198 .3	1297 .2
44	617 .2	688 .7	764 .1	843 .4	926 .8	1013 .9	1105 .0	1199 .9	1298 .9
45	618 .3	689 .9	765 .4	844 .7	928 .2	1015 .4	1106 .5	1201 .5	1300 .5
46	619 .5	691 .1	766 .7	846 .1	929 .6	1016 .9	1108 .1	1203 .1	1302 .2
47	620 .6	692 .4	768 .0	847 .5	931 .0	1018 .4	1109 .6	1204 .7	1303 .9
48	621 .8	693 .6	769 .3	848 .9	932 .4	1019 .9	1111 .2	1206 .4	1305 .6
49	623 .0	694 .8	770 .6	850 .2	933 .8	1021 .4	1112 .7	1208 .0	1307 .3
50	624 .1	696 .0	771 .9	851 .6	935 .2	1022 .8	1114 .3	1209 .6	1309 .0
51	625 .3	697 .3	773 .1	852 .9	936 .6	1024 .3	1115 .8	1211 .2	1310 .7
52	626 .5	698 .5	774 .5	854 .3	938 .1	1025 .8	1117 .4	1212 .9	1312 .4
53	627 .6	699 .7	775 .7	855 .7	939 .5	1027 .3	1118 .9	1214 .5	1314 .1
54	628 .8	701 .0	777 .1	857 .1	940 .9	1028 .8	1120 .5	1216 .1	1315 .7
55	630 .0	702 .2	778 .4	858 .4	942 .3	1030 .3	1122 .0	1217 .7	1317 .4
56	631 .2	703 .5	779 .7	859 .8	943 .8	1031 .8	1123 .6	1219 .4	1319 .1
57	632 .3	704 .7	781 .0	861 .1	945 .2	1033 .3	1125 .1	1221 .0	1320 .8
58	633 .5	705 .9	782 .3	862 .5	946 .6	1034 .8	1126 .7	1222 .6	1322 .5
59	634 .7	707 .1	783 .6	863 .9	948 .1	1036 .3	1128 .3	1224 .2	1324 .2

$$m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}.$$

t	26m	27m	28m	29m
0s	1325″.9	1429″.7	1537″.5	1649″.0
1	1327 .6	1431 .4	1539 .3	1650 .9
2	1329 .3	1433 .2	1541 .1	1652 .8
3	1331 .0	1434 .9	1542 .9	1654 .7
4	1332 .7	1436 .7	1544 .8	1656 .6
5	1334 .4	1438 .5	1546 .6	1658 .5
6	1336 .1	1440 .3	1548 .4	1660 .4
7	1337 .8	1442 .1	1550 .2	1662 .3
8	1339 .5	1443 .9	1552 .1	1664 .2
9	1341 .2	1445 .6	1553 .9	1666 .1
10	1342 .9	1447 .4	1555 .8	1668 .0
11	1344 .6	1449 .2	1557 .6	1669 .9
12	1346 .3	1451 .0	1559 .5	1671 .9
13	1348 .0	1452 .8	1561 .3	1673 .8
14	1349 .7	1454 .5	1563 .2	1675 .7
15	1351 .4	1456 .3	1565 .0	1677 .6
16	1353 .2	1458 .1	1566 .9	1679 .5
17	1354 .9	1459 .9	1568 .7	1681 .4
18	1356 .6	1461 .6	1570 .5	1683 .3
19	1358 .3	1463 .4	1572 .4	1685 .2
20	1360 .1	1465 .2	1574 .3	1687 .2
21	1361 .8	1466 .9	1576 .1	1689 .1
22	1363 .5	1468 .7	1578 .0	1691 .0
23	1365 .2	1470 .5	1579 .8	1692 .9
24	1367 .0	1472 .3	1581 .7	1694 .8
25	1368 .7	1474 .1	1583 .5	1696 .7
26	1370 .4	1475 .9	1585 .3	1698 .6
27	1372 .1	1477 .7	1587 .2	1700 .5
28	1373 .9	1479 .5	1589 .1	1702 .5
29	1375 .6	1481 .3	1590 .9	1704 .4
30	1377 .3	1483 .1	1592 .7	1706 .3
31	1379 .0	1484 .9	1594 .6	1708 .2
32	1380 .8	1486 .7	1596 .5	1710 .2
33	1382 .5	1488 .5	1598 .3	1712 .1
34	1384 .2	1490 .3	1600 .2	1714 .0
35	1385 .9	1492 .1	1602 .1	1715 .9
36	1387 .7	1493 .9	1604 .0	1717 .9
37	1389 .4	1495 .7	1605 .9	1719 .8
38	1391 .2	1497 .5	1607 .7	1721 .7
39	1392 .9	1499 .3	1609 .6	1723 .6
40	1394 .7	1501 .1	1611 .5	1725 .6
41	1396 .4	1502 .9	1613 .3	1727 .5
42	1398 .2	1504 .7	1615 .2	1729 .5
43	1399 .9	1506 .5	1617 .1	1731 .5
44	1401 .7	1508 .4	1619 .0	1733 .4
45	1403 .4	1510 .2	1620 .8	1735 .3
46	1405 .2	1512 .0	1622 .7	1737 .2
47	1406 .9	1513 .8	1624 .6	1739 .2
48	1408 .7	1515 .6	1626 .5	1741 .2
49	1410 .4	1517 .4	1628 .3	1743 .1
50	1412 .2	1519 .2	1630 .2	1745 .1
51	1413 .9	1521 .0	1632 .1	1747 .0
52	1415 .7	1522 .9	1634 .0	1749 .0
53	1417 .4	1524 .7	1635 .9	1750 .9
54	1419 .2	1526 .5	1637 .7	1752 .8
55	1420 .9	1528 .3	1639 .6	1754 .8
56	1422 .7	1530 .2	1641 .5	1756 .8
57	1424 .4	1532 .0	1643 .3	1758 .7
58	1426 .2	1533 .8	1645 .2	1760 .7
59	1427 .9	1535 .6	1647 .1	1762 .6

$$n = \frac{2 \sin^4 \frac{1}{2}t}{\sin 1''}.$$

t	n	t	n
0m 0s	0″.00	20m 0s	1″.49
1 0	0 .00	10	1 .54
2 0	0 .00	20	1 .60
3 0	0 .00	30	1 .65
4 0	0 .00	40	1 .70
5 0	0 .01	50	1 .76
6 0	0 .01	21 0	1 .82
7 0	0 .02	10	1 .87
8 0	0 .04	20	1 .93
9 0	0 .06	30	1 .99
10 0	0 .09	40	2 .06
11 0	0 .14	50	2 .12
12 0	0 .19	22 0	2 .19
10	0 .20	10	2 .25
20	0 .22	20	2 .32
30	0 .23	30	2 .39
40	0 .24	40	2 .46
50	0 .25	50	2 .54
13 0	0 .26	23 0	2 .61
10	0 .28	10	2 .69
20	0 .30	20	2 .77
30	0 .31	30	2 .85
40	0 .33	40	2 .93
50	0 .34	50	3 .01
14 0	0 .36	24 0	3 .10
10	0 .38	10	3 .18
20	0 .39	20	3 .27
30	0 .41	30	3 .36
40	0 .43	40	3 .45
50	0 .45	50	3 .55
15 0	0 .47	25 0	3 .64
10	0 .49	10	3 .74
20	0 .52	20	3 .84
30	0 .54	30	3 .94
40	0 .56	40	4 .05
50	0 .59	50	4 .15
16 0	0 .61	26 0	4 .26
10	0 .64	10	4 .37
20	0 .67	20	4 .48
30	0 .69	30	4 .60
40	0 .72	40	4 .72
50	0 .75	50	4 .83
17 0	0 .78	27 0	4 .96
10	0 .81	10	5 .08
20	0 .84	20	5 .20
30	0 .88	30	5 .33
40	0 .91	40	5 .46
50	0 .95	50	5 .60
18 0	0 .98	28 0	5 .73
10	1 .02	10	5 .87
20	1 .06	20	6 .01
30	1 .09	30	6 .15
40	1 .13	40	6 .30
50	1 .18	50	6 .44
19 0	1 .22	29 0	6 .59
10	1 .26	10	6 .75
20	1 .30	20	6 .90
30	1 .35	30	7 .06
40	1 .40	40	7 .22
50	1 .44	50	7 .38
20 0	1 .49	30 0	7 .55

$$o = \frac{2 \sin^6 \frac{1}{2}t}{\sin 1''}.$$

t	o
14m	0″.000
15	0 .001
16	0 .001
17	0 .001
18	0 .002
19	0 .002
20	0 .003
21	0 .004
22	0 .005
23	0 .007
24	0 .009
25	0 .011
26	0 .014
27	0 .017
28	0 .021
29	0 .026
30	0 .032

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(See § 175.)

Hour-angle before or after upper Culmination.	The Correction to be applied to the Latitude of the station to obtain the apparent altitude of Polaris. Computed for the declination 88° 46′ and the mean refraction. Latitude 30°	Latitude 35°	Latitude 40°	Latitude 45°	Latitude 50°	Latitude 55°	Latitude 60°	Correction for 1′ increase in the declination of Polaris.
0h 00m	+1° 15′.6	+1° 15′.3	+1° 15′.1	+1° 14′.9	+1° 14′.8	+1° 14′.6	+1° 14′.5	−1′.0
0 15	+1 15 .4	+1 15 .2	+1 14 .9	+1 14 .8	+1 14 .6	+1 14 .4	+1 14 .3	−1 .0
0 30	+1 14 .9	+1 14 .7	+1 14 .5	+1 14 .3	+1 14 .2	+1 14 .0	+1 13 .8	−1 .0
0 45	+1 14 .2	+1 13 .9	+1 13 .7	+1 13 .5	+1 13 .3	+1 13 .2	+1 13 .0	−1 .0
1 00	+1 13 .0	+1 12 .8	+1 12 .5	+1 12 .3	+1 12 .2	+1 12 .0	+1 11 .9	−1 .0
1 15	+1 11 .6	+1 11 .3	+1 11 .1	+1 10 .9	+1 10 .8	+1 10 .6	+1 10 .4	−0 .9
1 30	+1 09 .9	+1 09 .6	+1 09 .4	+1 09 .2	+1 09 .0	+1 08 .8	+1 08 .6	−0 .9
1 45	+1 07 .9	+1 07 .6	+1 07 .3	+1 07 .2	+1 07 .0	+1 06 .8	+1 06 .6	−0 .9
2 00	+1 05 .6	+1 05 .3	+1 05 .0	+1 04 .8	+1 04 .6	+1 04 .4	+1 04 .2	−0 .8
2 15	+1 03 .0	+1 02 .7	+1 02 .4	+1 02 .2	+1 02 .0	+1 01 .8	+1 01 .6	−0 .8
2 30	+1 00 .1	+0 59 .8	+0 59 .5	+0 59 .3	+0 59 .1	+0 58 .9	+0 58 .7	−0 .8
2 45	+0 57 .0	+0 56 .7	+0 56 .5	+0 56 .2	+0 56 .0	+0 55 .8	+0 55 .5	−0 .7
3 00	+0 53 .7	+0 53 .4	+0 53 .1	+0 52 .9	+0 52 .6	+0 52 .3	+0 52 .1	−0 .7
3 15	+0 50 .1	+0 49 .8	+0 49 .5	+0 49 .2	+0 49 .0	+0 48 .8	+0 48 .5	−0 .6
3 30	+0 46 .4	+0 46 .0	+0 45 .7	+0 45 .5	+0 45 .2	+0 45 .0	+0 44 .7	−0 .6
3 45	+0 42 .4	+0 42 .1	+0 41 .8	+0 41 .5	+0 41 .3	+0 41 .0	+0 40 .7	−0 .5
4 00	+0 38 .3	+0 38 .0	+0 37 .6	+0 37 .4	+0 37 .1	+0 36 .8	+0 36 .5	−0 .5
4 15	+0 34 .0	+0 33 .6	+0 33 .3	+0 33 .0	+0 32 .8	+0 32 .5	+0 32 .1	−0 .4
4 30	+0 29 .6	+0 29 .2	+0 28 .9	+0 28 .5	+0 28 .3	+0 28 .0	+0 27 .6	−0 .4
4 45	+0 25 .0	+0 24 .6	+0 24 .3	+0 24 .0	+0 23 .7	+0 23 .4	+0 23 .0	−0 .3
5 00	+0 20 .4	+0 20 .0	+0 19 .7	+0 19 .4	+0 19 .1	+0 18 .8	+0 18 .4	−0 .2
5 15	+0 15 .6	+0 15 .3	+0 14 .9	+0 14 .6	+0 14 .3	+0 14 .0	+0 13 .6	−0 .2
5 30	+0 10 .8	+0 10 .4	+0 10 .1	+0 09 .9	+0 09 .6	+0 09 .2	+0 08 .8	−0 .1
5 45	+0 06 .0	+0 05 .6	+0 05 .3	+0 05 .0	+0 04 .7	+0 04 .4	+0 04 .0	0 .0
6 00	+0 01 .2	+0 00 .8	+0 00 .5	+0 00 .2	−0 00 .1	−0 00 .5	−0 00 .9	0 .0
6 15	−0 03 .6	−0 04 .0	−0 04 .4	−0 04 .7	−0 05 .0	−0 05 .3	−0 05 .7	+0 .1
6 30	−0 08 .4	−0 08 .8	−0 09 .2	−0 09 .5	−0 09 .2	−0 10 .1	−0 10 .4	+0 .1
6 45	−0 13 .2	−0 13 .6	−0 14 .0	−0 14 .3	−0 14 .5	−0 14 .9	−0 15 .2	+0 .2
7 00	−0 17 .9	−0 18 .3	−0 18 .6	−0 18 .9	−0 19 .2	−0 19 .6	−0 19 .9	+0 .3
7 15	−0 22 .5	−0 22 .9	−0 23 .2	−0 23 .6	−0 23 .8	−0 24 .2	−0 24 .6	+0 .4
7 30	−0 27 .0	−0 27 .4	−0 27 .7	−0 28 .0	−0 28 .3	−0 28 .6	−0 29 .0	+0 .4
7 45	−0 31 .4	−0 31 .8	−0 32 .1	−0 32 .4	−0 32 .7	−0 33 .0	−0 33 .3	+0 .5
8 00	−0 35 .6	−0 36 .0	−0 36 .3	−0 36 .6	−0 36 .9	−0 37 .2	−0 37 .5	+0 .5
8 15	−0 39 .7	−0 40 .1	−0 40 .4	−0 40 .7	−0 41 .0	−0 41 .2	−0 41 .6	+0 .6
8 30	−0 43 .6	−0 44 .0	−0 44 .3	−0 44 .6	−0 44 .8	−0 45 .1	−0 45 .4	+0 .6
8 45	−0 47 .3	−0 47 .7	−0 48 .0	−0 48 .3	−0 48 .5	−0 48 .8	−0 49 .0	+0 .7
9 00	−0 50 .8	−0 51 .2	−0 51 .5	−0 51 .7	−0 51 .9	−0 52 .1	−0 52 .4	+0 .7
9 15	−0 54 .1	−0 54 .5	−0 54 .7	−0 55 .0	−0 55 .2	−0 55 .5	−0 55 .7	+0 .8
9 30	−0 57 .2	−0 57 .5	−0 57 .8	−0 58 .0	−0 58 .2	−0 58 .5	−0 58 .7	+0 .8
9 45	−1 00 .0	−1 00 .3	−1 00 .6	−1 00 .8	−1 01 .0	−1 01 .2	−1 01 .4	+0 .8
10 00	−1 02 .5	−1 02 .8	−1 03 .1	−1 03 .3	−1 03 .4	−1 03 .6	−1 03 .9	+0 .9
10 15	−1 04 .7	−1 05 .0	−1 05 .3	−1 05 .5	−1 05 .7	−1 05 .9	−1 06 .1	+0 .9
10 30	−1 06 .7	−1 07 .0	−1 07 .2	−1 07 .5	−1 07 .6	−1 07 .9	−1 08 .0	+0 .9
10 45	−1 08 .4	−1 08 .7	−1 08 .9	−1 09 .2	−1 09 .3	−1 09 .5	−1 09 .7	+0 .9
11 00	−1 09 .8	−1 10 .1	−1 10 .3	−1 10 .5	−1 10 .6	−1 10 .9	−1 11 .0	+1 .0
11 15	−1 10 .8	−1 11 .1	−1 11 .4	−1 11 .6	−1 11 .8	−1 12 .0	−1 12 .1	+1 .0
11 30	−1 11 .6	−1 11 .9	−1 12 .2	−1 12 .4	−1 12 .5	−1 12 .8	−1 12 .9	+1 .0
11 45	−1 12 .1	−1 12 .4	−1 12 .6	−1 12 .9	−1 13 .0	−1 13 .2	−1 13 .3	+1 .0
12 00	−1 12 .3	−1 12 .6	−1 12 .8	−1 13 .0	−1 13 .2	−1 13 .3	−1 13 .4	+1 .0

309. CORRECTION FOR ERROR OF RUN OF A MICROMETER

The tabular value is the correction to the forward reading.

The sign of the correction is the same as that of the difference Backward—Forward.

Use this table with such a micrometer as is described in § 189, and no other.

Forw'd Reading.	B. − F.											Forw'd Reading.
	0".2	0".4	0".6	0".8	1".0	1".2	1".4	1".6	1".8	2".0	3".0	
0' 15"			0".0	0".0	0".0	0".1	0".1	0".1	0".1	0".1	0".2	0' 15"
0 30		0".0	0 .1	0 .1	0 .1	0 .1	0 .1	0 .2	0 .2	0 .2	0 .3	0 30
0 45		0 .1	0 .1	0 .1	0 .2	0 .2	0 .2	0 .2	0 .3	0 .3	0 .4	0 45
1 00		0 .1	0 .1	0 .2	0 .2	0 .2	0 .3	0 .3	0 .4	0 .4	0 .6	1 00
1 15	0".0	0 .1	0 .2	0 .2	0 .2	0 .3	0 .4	0 .4	0 .4	0 .5	0 .8	1 15
1 30	0 .1	0 .1	0 .2	0 .2	0 .3	0 .4	0 .4	0 .5	0 .5	0 .6	0 .9	1 30
1 45	0 .1	0 .1	0 .2	0 .3	0 .4	0 .4	0 .5	0 .6	0 .6	0 .7	1 .0	1 45
2 00	0 .1	0 .2	0 .2	0 .3	0 .4	0 .5	0 .6	0 .6	0 .7	0 .8	1 .2	2 00
2 15	0 .1	0 .2	0 .3	0 .4	0 .4	0 .5	0 .6	0 .7	0 .8	0 .9	1 .4	2 15
2 30	0 .1	0 .2	0 .3	0 .4	0 .5	0 .6	0 .7	0 .8	0 .9	1 .0	1 .5	2 30
2 45	0 .1	0 .2	0 .3	0 .4	0 .6	0 .7	0 .8	0 .9	1 .0	1 .1	1 .6	2 45
3 00	0 .1	0 .2	0 .4	0 .5	0 .6	0 .7	0 .8	1 .0	1 .1	1 .2	1 .8	3 00
3 15	0 .1	0 .3	0 .4	0 .5	0 .6	0 .8	0 .9	1 .0	1 .2	1 .3	2 .0	3 15
3 30	0 .1	0 .3	0 .4	0 .6	0 .7	0 .8	1 .0	1 .1	1 .3	1 .4	2 .1	3 30
3 45	0 .2	0 .3	0 .4	0 .6	0 .8	0 .9	1 .0	1 .2	1 .4	1 .5	2 .2	3 45
4 00	0 .2	0 .3	0 .5	0 .6	0 .8	1 .0	1 .1	1 .3	1 .4	1 .6	2 .4	4 00
4 15	0 .2	0 .3	0 .5	0 .7	0 .8	1 .0	1 .2	1 .4	1 .5	1 .7	2 .6	4 15
4 30	0 .2	0 .4	0 .5	0 .7	0 .9	1 .1	1 .3	1 .4	1 .6	1 .8	2 .7	4 30
4 45	0 .2	0 .4	0 .6	0 .8	1 .0	1 .1	1 .3	1 .5	1 .7	1 .9	2 .8	4 45
5 00	0 .2	0 .4	0 .6	0 .8	1 .0	1 .2	1 .4	1 .6	1 .8	2 .0	3 .0	5 00

310.

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 30°.	Lat. 31°.	Lat. 32°.	Lat. 33°.	Lat. 34°.	Lat. 35°.	Lat. 30°.
0h 15m	0° 05′ 40″	0° 05′ 43″	0° 05′ 47″	0° 05′ 51″	0° 05′ 55″	0° 06′ 00″	− 5″
0 30	0 11 18	0 11 25	0 11 33	0 11 41	0 11 49	0 11 58	− 9
0 45	0 16 53	0 17 04	0 17 15	0 17 27	0 17 40	0 17 53	− 14
1 00	0 22 23	0 22 38	0 22 53	0 23 09	0 23 26	0 23 44	− 18
1 15	0 27 48	0 28 06	0 28 25	0 28 45	0 29 06	0 29 28	− 23
1 30	0 33 05	0 33 26	0 33 49	0 34 13	0 34 38	0 35 04	− 27
1 45	0 38 13	0 38 38	0 39 04	0 39 32	0 40 00	0 40 30	− 31
2 00	0 43 12	0 43 40	0 44 09	0 44 40	0 45 12	0 45 46	− 35
2 15	0 47 58	0 48 29	0 49 02	0 49 36	0 50 12	0 50 50	− 39
2 30	0 52 32	0 53 06	0 53 42	0 54 19	0 54 59	0 55 40	− 43
2 45	0 56 52	0 57 29	0 58 07	0 58 48	0 59 30	1 00 15	− 46
3 00	1 00 58	1 01 37	1 02 18	1 03 01	1 03 46	1 04 34	− 50
3 15	1 04 47	1 05 28	1 06 12	1 06 58	1 07 46	1 08 36	− 53
3 30	1 08 19	1 09 02	1 09 48	1 10 36	1 11 27	1 12 20	− 56
3 45	1 11 33	1 12 18	1 13 06	1 13 56	1 14 49	1 15 45	− 58
4 00	1 14 28	1 15 15	1 16 05	1 16 57	1 17 52	1 18 50	− 61
4 15	1 17 04	1 17 52	1 18 44	1 19 37	1 20 34	1 21 34	− 63
4 30	1 19 19	1 20 09	1 21 02	1 21 57	1 22 55	1 23 57	− 64
4 45	1 21 14	1 22 05	1 22 59	1 23 55	1 24 55	1 25 57	− 66
5 00	1 22 48	1 23 40	1 24 35	1 25 32	1 26 32	1 27 36	− 68
5 15	1 24 00	1 24 53	1 25 48	1 26 46	1 27 47	1 28 51	− 69
5 30	1 24 51	1 25 44	1 26 40	1 27 38	1 28 39	1 29 44	− 69
5 45	1 25 20	1 26 13	1 27 09	1 28 07	1 29 09	1 30 14	− 70
6 00	1 25 27	1 26 19	1 27 15	1 28 14	1 29 15	1 30 20	− 70
6 15	1 25 12	1 26 04	1 26 59	1 27 57	1 28 59	1 30 03	− 69
6 30	1 24 34	1 25 27	1 26 21	1 27 19	1 28 19	1 29 23	− 68
6 45	1 23 36	1 24 27	1 25 21	1 26 18	1 27 17	1 28 20	− 67
7 00	1 22 16	1 23 06	1 23 59	1 24 55	1 25 53	1 26 55	− 66
7 15	1 20 35	1 21 25	1 22 16	1 23 10	1 24 08	1 25 08	− 65
7 30	1 18 34	1 19 22	1 20 12	1 21 05	1 22 00	1 22 59	− 64
7 45	1 16 13	1 16 59	1 17 48	1 18 39	1 19 33	1 20 29	− 62
8 00	1 13 33	1 14 17	1 15 04	1 15 53	1 16 45	1 17 39	− 60
8 15	1 10 34	1 11 16	1 12 01	1 12 48	1 13 37	1 14 29	− 57
8 30	1 07 17	1 07 57	1 08 40	1 09 25	1 10 12	1 11 01	− 54
8 45	1 03 43	1 04 22	1 05 02	1 05 44	1 06 29	1 07 15	− 51
9 00	0 59 54	1 00 30	1 01 07	1 01 47	1 02 29	1 03 12	− 48
9 15	0 55 49	0 56 23	0 56 58	0 57 34	0 58 13	0 58 54	− 45
9 30	0 51 31	0 52 01	0 52 34	0 53 08	0 53 43	0 54 21	− 42
9 45	0 46 59	0 47 27	0 47 57	0 48 28	0 49 00	0 49 34	− 38
10 00	0 42 16	0 42 42	0 43 08	0 43 36	0 44 05	0 44 35	− 34
10 15	0 37 23	0 37 45	0 38 08	0 38 33	0 38 59	0 39 26	− 30
10 30	0 32 20	0 32 39	0 32 59	0 33 20	0 33 43	0 34 06	− 26
10 45	0 27 09	0 27 25	0 27 42	0 28 00	0 28 18	0 28 38	− 22
11 00	0 21 51	0 22 04	0 22 18	0 22 32	0 22 47	0 23 03	− 18
11 15	0 16 28	0 16 38	0 16 48	0 16 59	0 17 10	0 17 22	− 13
11 30	0 11 01	0 11 08	0 11 14	0 11 22	0 11 29	0 11 37	− 9
11 45	0 05 31	0 05 34	0 05 38	0 05 42	0 05 45	0 05 49	− 4
Elongation: Azimuth....	1 25 27	1 26 20	1 27 16	1 28 14	1 29 16	1 30 20	− 69
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*s.*
Hour-angle.	5 57 09	5 57 02	5 56 55	5 56 48	5 56 40	5 56 33	+ 2

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 35°.	Lat. 36°.	Lat. 37°.	Lat. 38°.	Lat. 39°.	Lat. 40°.	Lat. 40°.
0h 15m	0° 06′ 00″	0° 06′ 05″	0° 06′ 10″	0° 06′ 15″	0° 06′ 20″	0° 06′ 26″	− 5″
0 30	0 11 58	0 12 08	0 12 18	0 12 28	0 12 39	0 12 50	− 10
0 45	0 17 53	0 18 07	0 18 22	0 18 38	0 18 54	0 19 11	− 16
1 00	0 23 44	0 24 02	0 24 22	0 24 43	0 25 04	0 25 27	− 21
1 15	0 29 28	0 29 51	0 30 15	0 30 41	0 31 08	0 31 36	− 26
1 30	0 35 04	0 35 31	0 36 00	0 36 31	0 37 02	0 37 36	− 31
1 45	0 40 30	0 41 02	0 41 35	0 42 11	0 42 47	0 43 26	− 36
2 00	0 45 46	0 46 22	0 47 00	0 47 39	0 48 21	0 49 04	− 40
2 15	0 50 50	0 51 29	0 52 11	0 52 55	0 53 41	0 54 29	− 45
2 30	0 55 40	0 56 23	0 57 09	0 57 57	0 58 47	0 59 40	− 49
2 45	1 00 15	1 01 02	1 01 51	1 02 43	1 03 37	1 04 34	− 53
3 00	1 04 34	1 05 24	1 06 17	1 07 12	1 08 10	1 09 12	− 57
3 15	1 08 36	1 09 29	1 10 25	1 11 24	1 12 25	1 13 30	− 60
3 30	1 12 20	1 13 16	1 14 14	1 15 16	1 16 21	1 17 29	− 63
3 45	1 15 45	1 16 43	1 17 44	1 18 49	1 19 57	1 21 08	− 66
4 00	1 18 50	1 19 50	1 20 54	1 22 01	1 23 11	1 24 25	− 69
4 15	1 21 34	1 22 36	1 23 42	1 24 51	1 26 03	1 27 20	− 72
4 30	1 23 57	1 25 01	1 26 08	1 27 19	1 28 33	1 29 52	− 74
4 45	1 25 57	1 27 03	1 28 12	1 29 24	1 30 40	1 32 00	− 75
5 00	1 27 36	1 28 42	1 29 52	1 31 06	1 32 23	1 33 44	− 76
5 15	1 28 51	1 29 59	1 31 09	1 32 24	1 33 42	1 35 04	− 77
5 30	1 29 44	1 30 52	1 32 03	1 33 18	1 34 37	1 35 59	− 78
5 45	1 30 14	1 31 21	1 32 33	1 33 48	1 35 07	1 36 30	− 78
6 00	1 30 20	1 31 27	1 32 39	1 33 54	1 35 13	1 36 35	− 78
6 15	1 30 03	1 31 10	1 32 21	1 33 36	1 34 54	1 36 16	− 78
6 30	1 29 23	1 30 30	1 31 40	1 32 54	1 34 11	1 35 32	− 77
6 45	1 28 20	1 29 26	1 30 35	1 31 48	1 33 04	1 34 24	− 76
7 00	1 26 55	1 27 59	1 29 07	1 30 18	1 31 33	1 32 52	− 75
7 15	1 25 08	1 26 11	1 27 17	1 28 26	1 29 39	1 30 56	− 73
7 30	1 22 59	1 24 00	1 25 04	1 26 12	1 27 23	1 28 38	− 72
7 45	1 20 29	1 21 28	1 22 30	1 23 36	1 24 45	1 25 57	− 69
8 00	1 17 39	1 18 36	1 19 36	1 20 39	1 21 45	1 22 54	− 66
8 15	1 14 29	1 15 24	1 16 21	1 17 22	1 18 25	1 19 31	− 64
8 30	1 11 01	1 11 53	1 12 48	1 13 45	1 14 45	1 15 48	− 61
8 45	1 07 15	1 08 04	1 08 56	1 09 50	1 10 47	1 11 47	− 58
9 00	1 03 12	1 03 58	1 04 47	1 05 38	1 06 31	1 07 27	− 54
9 15	0 58 54	0 59 37	1 00 22	1 01 09	1 01 59	1 02 51	− 50
9 30	0 54 21	0 55 00	0 55 42	0 56 25	0 57 11	0 57 59	− 46
9 45	0 49 34	0 50 10	0 50 48	0 51 27	0 52 09	0 52 53	− 42
10 00	0 44 35	0 45 08	0 45 42	0 46 17	0 46 54	0 47 34	− 38
10 15	0 39 26	0 39 54	0 40 24	0 40 55	0 41 28	0 42 03	− 34
10 30	0 34 06	0 34 30	0 34 57	0 35 24	0 35 52	0 36 22	− 29
10 45	0 28 38	0 28 59	0 29 20	0 29 43	0 30 07	0 30 32	− 24
11 00	0 23 03	0 23 19	0 23 37	0 23 55	0 24 14	0 24 35	− 20
11 15	0 17 22	0 17 35	0 17 48	0 18 02	0 18 16	0 18 31	− 15
11 30	0 11 37	0 11 46	0 11 54	0 12 04	0 12 13	0 12 23	− 10
11 45	0 05 49	0 05 53	0 05 58	0 06 02	0 06 07	0 06 12	− 5
Elongation: Azimuth....	1 30 20	1 31 28	1 32 40	1 33 55	1 35 14	1 36 36	− 78
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*s.*
Hour-angle.	5 56 33	5 56 25	5 56 17	5 56 09	5 56 00	5 55 52	+ 3

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 40°.	Lat. 41°.	Lat. 42°.	Lat. 43°.	Lat. 44°.	Lat. 45°.	Lat. 40°.
0h 15m	0° 06′ 26″	0° 06′ 32″	0° 06′ 39″	0° 06′ 45″	0° 06′ 52″	0° 07′ 00″	− 5″
0 30	0 12 50	0 13 03	0 13 15	0 13 29	0 13 43	0 13 58	− 10
0 45	0 19 11	0 19 30	0 19 48	0 20 08	0 20 29	0 20 52	− 16
1 00	0 25 27	0 25 51	0 26 16	0 26 43	0 27 10	0 27 40	− 21
1 15	0 31 36	0 32 05	0 32 36	0 33 09	0 33 44	0 34 21	− 26
1 30	0 37 36	0 38 11	0 38 48	0 39 27	0 40 09	0 40 52	− 31
1 45	0 43 26	0 44 07	0 44 50	0 45 35	0 46 22	0 47 12	− 36
2 00	0 49 04	0 49 50	0 50 39	0 51 29	0 52 23	0 53 19	− 40
2 15	0 54 29	0 55 20	0 56 14	0 57 10	0 58 10	0 59 12	− 45
2 30	0 59 40	1 00 35	1 01 34	1 02 36	1 03 41	1 04 49	− 49
2 45	1 04 34	1 05 34	1 06 38	1 07 44	1 08 54	1 10 08	− 53
3 00	1 09 12	1 10 16	1 11 24	1 12 35	1 13 50	1 15 09	− 57
3 15	1 13 30	1 14 38	1 15 50	1 17 06	1 18 25	1 19 49	− 60
3 30	1 17 29	1 18 41	1 19 57	1 21 16	1 22 39	1 24 08	− 63
3 45	1 21 08	1 22 23	1 23 42	1 25 04	1 26 32	1 28 04	− 66
4 00	1 24 25	1 25 43	1 27 05	1 28 31	1 30 01	1 31 37	− 69
4 15	1 27 20	1 28 40	1 30 04	1 31 33	1 33 07	1 34 45	− 72
4 30	1 29 52	1 31 14	1 32 41	1 34 12	1 35 48	1 37 29	− 74
4 45	1 32 00	1 33 24	1 34 53	1 36 25	1 38 04	1 39 47	− 75
5 00	1 33 44	1 35 10	1 36 40	1 38 14	1 39 54	1 41 38	− 76
5 15	1 35 04	1 36 30	1 38 02	1 39 37	1 41 18	1 43 04	− 77
5 30	1 35 59	1 37 26	1 38 58	1 40 34	1 42 16	1 44 02	− 78
5 45	1 36 30	1 37 57	1 39 29	1 41 05	1 42 47	1 44 34	− 78
6 00	1 36 35	1 38 02	1 39 34	1 41 10	1 42 51	1 44 38	− 78
6 15	1 36 16	1 37 43	1 39 14	1 40 49	1 42 30	1 44 16	− 78
6 30	1 35 32	1 36 58	1 38 28	1 40 03	1 41 42	1 43 27	− 77
6 45	1 34 24	1 35 48	1 37 17	1 38 50	1 40 28	1 42 12	− 76
7 00	1 32 52	1 34 15	1 35 42	1 37 13	1 38 49	1 40 31	− 75
7 15	1 30 56	1 32 17	1 33 42	1 35 11	1 36 45	1 38 24	− 73
7 30	1 28 38	1 29 56	1 31 19	1 32 46	1 34 17	1 35 53	− 72
7 45	1 25 57	1 27 13	1 28 33	1 29 56	1 31 25	1 32 58	− 69
8 00	1 22 54	1 24 07	1 25 24	1 26 45	1 28 10	1 29 40	− 66
8 15	1 19 31	1 20 41	1 21 55	1 23 12	1 24 33	1 25 59	− 64
8 30	1 15 48	1 16 55	1 18 05	1 19 18	1 20 35	1 21 57	− 61
8 45	1 11 47	1 12 49	1 13 55	1 15 05	1 16 18	1 17 35	− 58
9 00	1 07 27	1 08 26	1 09 28	1 10 33	1 11 41	1 12 54	− 54
9 15	1 02 51	1 03 45	1 04 43	1 05 43	1 06 47	1 07 54	− 50
9 30	0 57 59	0 58 49	0 59 42	1 00 38	1 01 37	1 02 38	− 46
9 45	0 52 53	0 53 39	0 54 27	0 55 18	0 56 11	0 57 07	− 42
10 00	0 47 34	0 48 15	0 48 58	0 49 44	0 50 32	0 51 22	− 38
10 15	0 42 03	0 42 39	0 43 18	0 43 58	0 44 40	0 45 25	− 34
10 30	0 36 22	0 36 53	0 37 26	0 38 01	0 38 38	0 39 16	− 29
10 45	0 30 32	0 30 58	0 31 26	0 31 55	0 32 26	0 32 58	− 24
11 00	0 24 35	0 24 56	0 25 18	0 25 42	0 26 06	0 26 32	− 20
11 15	0 18 31	0 18 47	0 19 04	0 19 22	0 19 40	0 20 00	− 15
11 30	0 12 23	0 12 34	0 12 45	0 12 57	0 13 09	0 13 23	− 10
11 45	0 06 12	0 06 18	0 06 23	0 06 29	0 06 36	0 06 42	− 5
Elongation: Azimuth....	1 36 36	1 38 03	1 39 35	1 41 11	1 42 53	1 44 40	− 78
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*s.*
Hour-angle.	5 55 52	5 55 43	5 55 34	5 55 24	5 55 14	5 55 04	+ 3

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 45°.	Lat. 46°.	Lat. 47°.	Lat. 48°.	Lat. 49°.	Lat. 50°.	Lat. 50°.
0h 15m	0° 07′ 00″	0° 07′ 08″	0° 07′ 16″	0° 07′ 25″	0° 07′ 34″	0° 07′ 44″	— 6″
0 30	0 13 58	0 14 13	0 14 30	0 14 48	0 15 06	0 15 25	— 13
0 45	0 20 52	0 21 15	0 21 40	0 22 06	0 22 33	0 23 02	— 19
1 00	0 27 40	0 28 11	0 28 44	0 29 18	0 29 55	0 30 33	— 25
1 15	0 34 21	0 34 59	0 35 40	0 36 23	0 37 08	0 37 56	— 32
1 30	0 40 52	0 41 38	0 42 26	0 43 17	0 44 11	0 45 08	— 38
1 45	0 47 12	0 48 05	0 49 01	0 49 59	0 51 02	0 52 07	— 43
2 00	0 53 19	0 54 19	0 55 22	0 56 28	0 57 38	0 58 52	— 49
2 15	0 59 12	1 00 18	1 01 28	1 02 41	1 03 59	1 05 21	— 54
2 30	1 04 49	1 06 01	1 07 17	1 08 38	1 10 03	1 11 32	— 59
2 45	1 10 08	1 11 26	1 12 48	1 14 15	1 15 47	1 17 24	— 64
3 00	1 15 09	1 16 32	1 18 00	1 19 33	1 21 11	1 22 54	— 68
3 15	1 19 49	1 21 17	1 22 50	1 24 29	1 26 13	1 28 02	— 72
3 30	1 24 08	1 25 40	1 27 18	1 29 02	1 30 51	1 32 46	— 76
3 45	1 28 04	1 29 41	1 31 23	1 33 11	1 35 05	1 37 06	— 80
4 00	1 31 37	1 33 17	1 35 03	1 36 55	1 38 54	1 40 59	— 83
4 15	1 34 45	1 36 29	1 38 18	1 40 14	1 42 16	1 44 25	— 86
4 30	1 37 29	1 39 15	1 41 08	1 43 06	1 45 11	1 47 24	— 88
4 45	1 39 47	1 41 35	1 43 30	1 45 31	1 47 39	1 49 54	— 90
5 00	1 41 38	1 43 29	1 45 25	1 47 28	1 49 38	1 51 55	— 91
5 15	1 43 04	1 44 55	1 46 53	1 48 57	1 51 08	1 53 27	— 92
5 30	1 44 02	1 45 54	1 47 53	1 49 58	1 52 10	1 54 30	— 93
5 45	1 44 34	1 46 26	1 48 25	1 50 30	1 52 43	1 55 03	— 94
6 00	1 44 38	1 46 31	1 48 29	1 50 34	1 52 46	1 55 06	— 93
6 15	1 44 16	1 46 08	1 48 05	1 50 10	1 52 21	1 54 40	— 93
6 30	1 43 27	1 45 18	1 47 14	1 49 17	1 51 27	1 53 44	— 92
6 45	1 42 12	1 44 01	1 45 56	1 47 56	1 50 04	1 52 20	— 91
7 00	1 40 31	1 42 18	1 44 10	1 46 09	1 48 14	1 50 27	— 89
7 15	1 38 24	1 40 09	1 41 59	1 43 54	1 45 57	1 48 06	— 87
7 30	1 35 53	1 37 35	1 39 21	1 41 14	1 43 13	1 45 19	— 85
7 45	1 32 58	1 34 36	1 36 19	1 38 08	1 40 03	1 42 05	— 82
8 00	1 29 40	1 31 14	1 32 53	1 34 38	1 36 29	1 38 26	— 79
8 15	1 25 59	1 27 29	1 29 04	1 30 44	1 32 30	1 34 22	— 76
8 30	1 21 57	1 23 23	1 24 53	1 26 28	1 28 09	1 29 55	— 72
8 45	1 17 35	1 18 56	1 20 21	1 21 51	1 23 26	1 25 07	— 68
9 00	1 12 54	1 14 10	1 15 30	1 16 54	1 18 23	1 19 57	— 64
9 15	1 07 54	1 09 05	1 10 19	1 11 38	1 13 01	1 14 28	— 59
9 30	1 02 38	1 03 44	1 04 52	1 06 04	1 07 21	1 08 41	— 55
9 45	0 57 07	0 58 07	0 59 09	1 00 15	1 01 24	1 02 38	— 50
10 00	0 51 22	0 52 16	0 53 12	0 54 11	0 55 13	0 56 19	— 45
10 15	0 45 25	0 46 12	0 47 01	0 47 53	0 48 49	0 49 47	— 40
10 30	0 39 16	0 39 57	0 40 40	0 41 25	0 42 12	0 43 02	— 34
10 45	0 32 58	0 33 32	0 34 08	0 34 46	0 35 26	0 36 08	— 29
11 00	0 26 32	0 27 00	0 27 28	0 27 59	0 28 31	0 29 05	— 23
11 15	0 20 00	0 20 20	0 20 42	0 21 05	0 21 29	0 21 55	— 18
11 30	0 13 23	0 13 36	0 13 51	0 14 06	0 14 22	0 14 39	— 12
11 45	0 06 42	0 06 49	0 06 56	0 07 04	0 07 12	0 07 21	— 6
Elongation: Azimuth....	1 44 40	1 46 32	1 48 31	1 50 36	1 52 48	1 55 08	— 93
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*s.*
Hour-angle.	5 55 04	5 54 53	5 54 42	5 54 31	5 54 20	5 54 07	+ 5

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 50°.	Lat. 51°.	Lat. 52°.	Lat. 53°.	Lat. 54°.	Lat. 55°.	Lat. 50°.
0h 15m	0° 07′ 44″	0° 07′ 54″	0° 08′ 05″	0° 08′ 17″	0° 08′ 29″	0° 08′ 42″	— 6″
0 30	0 15 25	0 15 46	0 16 08	0 16 31	0 16 56	0 17 22	— 13
0 45	0 23 02	0 23 33	0 24 06	0 24 41	0 25 18	0 25 57	— 19
1 00	0 30 33	0 31 14	0 31 58	0 32 44	0 33 33	0 34 25	— 25
1 15	0 37 56	0 38 47	0 39 40	0 40 38	0 41 38	0 42 43	— 32
1 30	0 45 08	0 46 08	0 47 12	0 48 20	0 49 32	0 50 49	— 38
1 45	0 52 07	0 53 17	0 54 31	0 55 49	0 57 12	0 58 41	— 43
2 00	0 58 52	1 00 11	1 01 34	1 03 03	1 04 37	1 06 16	— 49
2 15	1 05 21	1 06 48	1 08 21	1 09 59	1 11 43	1 13 33	— 54
2 30	1 11 32	1 13 08	1 14 48	1 16 35	1 18 29	1 20 30	— 59
2 45	1 17 24	1 19 07	1 20 55	1 22 51	1 24 54	1 27 04	— 64
3 00	1 22 54	1 24 44	1 26 41	1 28 44	1 30 55	1 33 15	— 68
3 15	1 28 02	1 29 59	1 32 02	1 34 13	1 36 32	1 39 00	— 72
3 30	1 32 46	1 34 49	1 36 58	1 39 16	1 41 42	1 44 18	— 76
3 45	1 37 06	1 39 14	1 41 29	1 43 52	1 46 25	1 49 07	— 80
4 00	1 40 59	1 43 12	1 45 32	1 48 01	1 50 39	1 53 27	— 83
4 15	1 44 25	1 46 42	1 49 07	1 51 40	1 54 23	1 57 16	— 86
4 30	1 47 24	1 49 44	1 52 13	1 54 50	1 57 37	2 00 35	— 88
4 45	1 49 54	1 52 17	1 54 49	1 57 29	2 00 20	2 03 21	— 90
5 00	1 51 55	1 54 21	1 56 54	1 59 37	2 02 31	2 05 35	— 91
5 15	1 53 27	1 55 54	1 58 29	2 01 15	2 04 10	2 07 16	— 92
5 30	1 54 30	1 56 58	1 59 34	2 02 20	2 05 16	2 08 23	— 93
5 45	1 55 03	1 57 31	2 00 08	2 02 53	2 05 50	2 08 58	— 94
6 00	1 55 06	1 57 34	2 00 10	2 02 56	2 05 52	2 08 58	— 93
6 15	1 54 40	1 57 06	1 59 41	2 02 26	2 05 21	2 08 26	— 93
6 30	1 53 44	1 56 09	1 58 43	2 01 25	2 04 18	2 07 22	— 92
6 45	1 52 20	1 54 42	1 57 14	1 59 54	2 02 44	2 05 45	— 91
7 00	1 50 27	1 52 47	1 55 15	1 57 52	2 00 39	2 03 36	— 89
7 15	1 48 06	1 50 23	1 52 48	1 55 21	1 58 04	2 00 57	— 87
7 30	1 45 19	1 47 32	1 49 52	1 52 21	1 54 59	1 57 47	— 85
7 45	1 42 05	1 44 13	1 46 29	1 48 53	1 51 26	1 54 08	— 82
8 00	1 38 26	1 40 29	1 42 40	1 44 58	1 47 25	1 50 01	— 79
8 15	1 34 22	1 36 20	1 38 25	1 40 38	1 42 58	1 45 27	— 76
8 30	1 29 55	1 31 48	1 33 47	1 35 52	1 38 06	1 40 28	— 72
8 45	1 25 07	1 26 53	1 28 45	1 30 44	1 32 50	1 35 04	— 68
9 00	1 19 57	1 21 37	1 23 22	1 25 13	1 27 11	1 29 17	— 64
9 15	1 14 28	1 16 01	1 17 38	1 19 22	1 21 12	1 23 08	— 59
9 30	1 08 41	1 10 06	1 11 36	1 13 12	1 14 53	1 16 40	— 55
9 45	1 02 38	1 03 55	1 05 17	1 06 44	1 08 16	1 09 53	— 50
10 00	0 56 19	0 57 28	0 58 42	1 00 00	1 01 23	1 02 50	— 45
10 15	0 49 47	0 50 48	0 51 53	0 53 02	0 54 15	0 55 32	— 40
10 30	0 43 02	0 43 56	0 44 52	0 45 51	0 46 54	0 48 01	— 34
10 45	0 36 08	0 36 52	0 37 39	0 38 29	0 39 22	0 40 18	— 29
11 00	0 29 05	0 29 41	0 30 18	0 30 58	0 31 41	0 32 26	— 23
11 15	0 21 55	0 22 22	0 22 50	0 23 20	0 23 52	0 24 26	— 18
11 30	0 14 39	0 14 57	0 15 16	0 15 37	0 15 58	0 16 21	— 12
11 45	0 07 21	0 07 30	0 07 39	0 07 49	0 08 00	0 08 11	— 6
Elongation:							
Azimuth....	1 55 08	1 57 36	2 00 13	2 02 59	2 05 55	2 09 02	— 93
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h m. s.*	*h. m. s.*	*s.*
Hour-angle.	5 54 07	5 53 54	5 53 41	5 53 27	5 53 12	5 52 57	+ 5

Hour-angle before or after Upper Culmination.	AZIMUTH OF POLARIS COMPUTED FOR DECLINATION 88° 46′.						Correction for 1′ Increase in Declination of Polaris.
	Lat. 55°.	Lat. 56°.	Lat. 57°.	Lat. 58°.	Lat. 59°.	Lat. 60°.	Lat. 60°.
0h 15m	0° 08′ 42″	0° 08′ 56″	0° 09′ 12″	0° 09′ 28″	0° 09′ 45″	0° 10′ 03″	— 8″
0 30	0 17 22	0 17 50	0 18 20	0 18 53	0 19 27	0 20 04	— 17
0 45	0 25 57	0 26 39	0 27 24	0 28 12	0 29 03	0 29 58	— 25
1 00	0 34 25	0 35 21	0 36 20	0 37 23	0 38 31	0 39 44	— 33
1 15	0 42 43	0 43 52	0 45 06	0 46 24	0 47 48	0 49 19	— 41
1 30	0 50 49	0 52 11	0 53 39	0 55 12	0 56 52	0 58 40	— 49
1 45	0 58 41	1 00 16	1 01 56	1 03 44	1 05 40	1 07 44	— 57
2 00	1 06 16	1 08 03	1 09 57	1 11 58	1 14 08	1 16 28	— 64
2 15	1 13 33	1 15 31	1 17 37	1 19 52	1 22 16	1 24 51	— 71
2 30	1 20 30	1 22 39	1 24 56	1 27 24	1 30 01	1 32 50	— 78
2 45	1 27 04	1 29 23	1 31 52	1 34 31	1 37 21	1 40 23	— 84
3 00	1 33 15	1 35 43	1 38 22	1 41 12	1 44 13	1 47 28	— 89
3 15	1 39 00	1 41 37	1 44 25	1 47 25	1 50 37	1 54 03	— 94
3 30	1 44 18	1 47 03	1 50 00	1 53 08	1 56 30	2 00 07	— 99
3 45	1 49 07	1 52 00	1 55 04	1 58 21	2 01 51	2 05 37	— 104
4 00	1 53 27	1 56 26	1 59 37	2 03 01	2 06 40	2 10 34	— 108
4 15	1 57 16	2 00 21	2 03 38	2 07 09	2 10 54	2 14 55	— 111
4 30	2 00 35	2 03 44	2 07 06	2 10 42	2 14 32	2 18 39	— 114
4 45	2 03 21	2 06 34	2 10 00	2 13 40	2 17 35	2 21 47	— 116
5 00	2 05 35	2 08 51	2 12 20	2 16 03	2 20 02	2 24 17	— 118
5 15	2 07 16	2 10 34	2 14 05	2 17 50	2 21 51	2 26 09	— 119
5 30	2 08 23	2 11 42	2 15 14	2 19 01	2 23 04	2 27 23	— 120
5 45	2 08 58	2 12 17	2 15 50	2 19 36	2 23 39	2 27 58	— 120
6 00	2 08 58	2 12 17	2 15 49	2 19 35	2 23 37	2 27 56	— 120
6 15	2 08 26	2 11 44	2 15 14	2 18 59	2 22 59	2 27 15	— 119
6 30	2 07 22	2 10 37	2 14 05	2 17 47	2 21 44	2 25 57	— 118
6 45	2 05 45	2 08 57	2 12 21	2 16 00	2 19 53	2 24 03	— 116
7 00	2 03 36	2 06 44	2 10 05	2 13 39	2 17 27	2 21 32	— 114
7 15	2 00 57	2 04 00	2 07 16	2 10 45	2 14 27	2 18 26	— 111
7 30	1 57 47	2 00 45	2 03 55	2 07 18	2 10 54	2 14 46	— 108
7 45	1 54 08	1 57 00	2 00 04	2 03 20	2 06 49	2 10 32	— 104
8 00	1 50 01	1 52 47	1 55 43	1 58 52	2 02 12	2 05 47	— 100
8 15	1 45 27	1 48 06	1 50 54	1 53 54	1 57 06	2 00 32	— 96
8 30	1 40 28	1 42 58	1 45 39	1 48 30	1 51 32	1 54 47	— 91
8 45	1 35 04	1 37 26	1 39 57	1 42 39	1 45 31	1 48 35	— 86
9 00	1 29 17	1 31 30	1 33 51	1 36 23	1 39 05	1 41 57	— 80
9 15	1 23 08	1 25 12	1 27 24	1 29 44	1 32 14	1 34 55	— 75
9 30	1 16 40	1 18 34	1 20 36	1 22 45	1 25 03	1 27 30	— 69
9 45	1 09 53	1 11 37	1 13 28	1 15 25	1 17 31	1 19 45	— 63
10 00	1 02 50	1 04 23	1 06 03	1 07 48	1 09 41	1 11 41	— 56
10 15	0 55 32	0 56 54	0 58 22	0 59 55	1 01 34	1 03 20	— 50
10 30	0 48 01	0 49 12	0 50 27	0 51 48	0 53 14	0 54 45	— 43
10 45	0 40 18	0 41 18	0 42 21	0 43 28	0 44 40	0 45 57	— 36
11 00	0 32 26	0 33 14	0 34 05	0 34 59	0 35 57	0 36 59	— 29
11 15	0 24 26	0 25 02	0 25 41	0 26 21	0 27 05	0 27 51	— 22
11 30	0 16 21	0 16 45	0 17 10	0 17 38	0 18 07	0 18 38	— 14
11 45	0 08 11	0 08 23	0 08 36	0 08 50	0 09 04	0 09 20	— 7
Elongation: Azimuth....	2 09 02	2 12 21	2 15 54	2 19 40	2 23 43	2 28 02	— 120
	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	s.
Hour-angle.	5 52 57	5 52 41	5 52 24	5 52 06	5 51 47	5 51 27	+ 7

311. NOTATION AND PRINCIPAL WORKING FORMULÆ.

The following general notation is used throughout the book. The special notation involved in each working formula will be found below each group of formulæ.

GENERAL NOTATION.

α and $\delta \equiv$ the apparent right ascension and declination, respectively, at the time of the observation under consideration.

α_m and $\delta_m \equiv$ the mean right ascension and declination, respectively.

α_{1875} and δ_{1875} (with a year as a subscript) $\equiv$ the mean right ascension and declination, respectively, at the beginning of the fictitious year indicated.

α_0 and $\delta_0 \equiv$ the values of α_m and δ_m at the beginning of the fictitious year during which the observation under consideration was made.

μ and $\mu' \equiv$ proper motions, per year, in right ascension and declination, respectively.

$A \equiv$ altitude.

$\zeta \equiv$ zenith distance.

$\phi \equiv$ astronomical latitude of the station of observation.

$t \equiv$ hour-angle, measured eastward or westward from the upper branch of the meridian as the case may be, but always considered positive, and never exceeding 180° (or 12^h).

$z \equiv$ the azimuth of a star, measured to the eastward or westward from north as the case may be, but always considered positive and never exceeding 180°.

$d \equiv$ value, in arc, of one division of a level.

WORKING FORMULÆ,

WITH THEIR SPECIAL NOTATION, AND WITH REFERENCES TO CORRESPONDING PORTIONS OF THE TEXT AND TO THE TABLES.

To convert mean solar to sidereal time.

See § 23 and the table of § 290.

To convert sidereal to mean time.

See § 24 and the tables of §§ 290, 291.

To interpolate along a chord.

$$F_I = F_1 + (F_2 - F_1)\frac{V_I - V_1}{V_2 - V_1}$$

or

$$F_I = F_2 - (F_2 - F_1)\frac{V_2 - V_I}{V_2 - V_1}; \quad . \quad . \quad . \quad . \quad (1)^*$$

the first form being used when the interpolation is made forward from the value F_1, and the second when it is made backward from the value F_2. F_I is the required interpolated value of the function corresponding to the value V_I of the independent variable. V_1 and V_2 are the adjacent stated values of the independent variable to which correspond the given values F_1 and F_2 of the function. See § 30.

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* The number assigned to each formula corresponds to that used in the body of the text.

To interpolate along a tangent.

$$F_I = F_1 + \left(\frac{dF}{dV}\right)_1 (V_I - V_1). \quad . \quad . \quad . \quad . \quad (2)$$

F_I is the required interpolated value of the function corresponding to the value V_I of the independent variable. V_1 and F_1 are, respectively, the *nearest* given value of the independent variable, and the corresponding value of the function. $\left(\frac{dF}{dV}\right)_1$ is the given first differential coefficient corresponding to V_1. See § 31.

To interpolate along a parabola.

If the first differential coefficients are given,

$$F_I = F_1 + [V_I - V_1]\left[\left(\frac{dF}{dV}\right)_1 + \left\{\left(\frac{dF}{dV}\right)_2 - \left(\frac{dF}{dV}\right)_1\right\}\left\{\frac{\frac{1}{2}(V_I - V_1)}{V_2 - V_1}\right\}\right], \quad (3)$$

or

$$F_I = F_2 + [V_I - V_2]\left[\left(\frac{dF}{dV}\right)_2 + \left\{\left(\frac{dF}{dV}\right)_2 - \left(\frac{dF}{dV}\right)_1\right\}\left\{\frac{\frac{1}{2}(V_I - V_2)}{V_2 - V_1}\right\}\right], \quad (3a)$$

according to whether the interpolation is made forward from V_1 or backward from V_2. F_I is the required interpolated value of the function corresponding to the value V_I of the independent variable. V_1 and V_2 are the adjacent stated values of the independent variable to which correspond the given values F_1 and F_2 of the function, and the given values $\left(\frac{dF}{dV}\right)_1$ and $\left(\frac{dF}{dV}\right)_2$ of the first differential coefficient. See § 33.

If the first differential coefficients are not given,

$$F_I = F_2 + \left[\frac{F_2 - F_1}{V_2 - V_1} + \left\{\frac{F_3 - F_2}{V_3 - V_2} - \frac{F_2 - F_1}{V_2 - V_1}\right\}\left\{\frac{V_I - V_1}{V_3 - V_1}\right\}\right][V_I - V_2], \quad (4)$$

or

$$F_I = F_2 + \left[\frac{F_3 - F_1}{D} + \frac{d_2}{2D^2}(V_I - V_2 + D)\right][V_I - V_2]. \quad (4a)$$

(4) is the general formula which is applicable even when the successive differences between V_1, V_2, V_3, . . . are not all the same, and (4*a*) is the formula for the special case in which those differences are all the same.

F_I is the required interpolated value of the function corresponding to the value V_I of the independent variable. F_1, F_2, and F_3 are three successive given values of the function corresponding, respectively, to the values V_1, V_2, V_3 of the independent variable, V_2 being the stated value of the variable nearest to which lies the value V_I.

In (4*a*) $D = (V_2 - V_1) = (V_3 - V_2) = \ldots$, and d_2 is the second difference, or $(F_3 - F_2) - (F_2 - F_1)$. See § 34.

Having given the mean place, α_m and δ_m, of a star for a date t_m at the beginning of some fictitious year, to compute its mean place, α_0, δ_0, at t_0, the beginning of the fictitious year during which the observations under consideration were made.

$$\alpha_0 = \alpha_m + (t - t_m)\left(\frac{d\alpha_m}{dt}\right); \quad . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

$$\delta_0 = \delta_m + (t_0 - t_m)\left(\frac{d\delta_m}{dt}\right) + \frac{1}{2}(t_0 - t_m)^2 \frac{d^2\delta_m}{dt^2}; \ . \quad (9)$$

in which t_0 and t_m are expressed in years, $\frac{d\alpha_m}{dt}$ is the change in α_m per year, $\frac{d\delta_m}{dt}$ is the change in δ_m per year, and $\frac{d^2\delta_m}{dt^2}$ is the change in $\frac{d\delta_m}{dt}$ per year. Lists of star places usually give the numerical values of $\frac{d\alpha_m}{dt}$ and $\frac{d\delta_m}{dt}$, and sometimes $\frac{d^2\delta_m}{dt^2}$, for each star. $\frac{d^2\delta_m}{dt^2}$ is tabulated in § 292. (N.B. $\frac{d\alpha_m}{dt}$ and $\frac{d\delta_m}{dt}$ include *both* the effect of precession and of proper motion.) See §§ 40–43.

Having given α_0 *and* δ_0, *the mean place of a star at the beginning of the fictitious year during which the observations under consideration were made, to compute the apparent place,* α *and* δ, *at the instant of observation.*

$$\alpha = \alpha_0 + f + \tau\mu + \tfrac{1}{15}g \sin (G + \alpha_0) \tan \delta_0 + \tfrac{1}{15}h \sin (H + \alpha_0) \sec \delta_0 \quad \ldots\ldots \text{(in time)}; \quad (10)$$

$$\delta = \delta_0 + \tau\mu' + g \cos (G + \alpha_0) + h \cos (H + \alpha_0) \sin \delta_0 + i \cos \delta_0 \quad \ldots \text{(in arc)}; \quad (11)$$

in which f, G, H, g, h, and i are quantities called independent star-numbers which are functions of the time only and are given in the Ephemeris for every Washington mean midnight. Their values for the instant of observation may be derived by interpolations along chords between the Ephemeris values. τ is the elapsed portion of the fictitious year expressed in units of one year. It is given in the Ephemeris with the star-numbers. See §§ 46–49.

To compute the correction to a timepiece on mean time from observations of the double altitude of the Sun with a sextant and artificial horizon. See §§ 62–70.

The mean reading of the sextant arc corrected for index error and eccentricity is $2A_u$. A_u is the approximate altitude of the Sun. For the method of correcting for index error see § 62, and for eccentricity see § 76. The altitude

$$A = A_u \pm \text{Sun's semi-diameter} + p - R.$$

The Sun's semi-diameter, as taken from the Ephemeris, is to be added if the Sun's lower limb was observed, and subtracted if the upper limb was observed.

The parallax, p, is given in the table of § 293.

The refraction is $R = R_M(C_B)(C_D)(C_A)$. R_M, C_B, C_D, and C_A are given in the tables of §§ 294–297. The refraction is required for the altitude A_u, not A.

$$\zeta = 90° - A.$$

$$\sin^2 \tfrac{1}{2}t = \frac{\sin \tfrac{1}{2}[\zeta + (\phi - \delta)] \sin \tfrac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}. \qquad (16)$$

See general notation. δ may be obtained from the Ephemeris by interpolation along a tangent.

$12^h \pm t =$ apparent solar time $= T_A$.

$T_A + E =$ mean solar time $= T_M$.

E is the equation of time which is given in the Ephemeris for Washington apparent noon, and may be obtained for the instant of observation by interpolation along a chord.

$T_c =$ mean reading of timepiece.

$T_M - T_c = \Delta T_c =$ required correction to timepiece.

If the observations are taken at sea using the natural horizon, the mean reading of the sextant arc corrected for index error and eccentricity is A_u (not $2A_u$).

$$A = A_u \pm \text{Sun's semi-diameter} + p - R - \text{Dip}.$$

The first four terms in the second member are the same as before. The dip, or downward inclination of the line of sight to the apparent horizon due to the height of the sextant above the surface of the sea, is given in the table in § 298. The remainder of the computation is as given above.

TO COMPUTE THE CORRECTION TO A SIDEREAL TIMEPIECE FROM OBSERVATIONS WITH AN ASTRONOMICAL TRANSIT PLACED IN THE MERIDIAN. §§ 90–99.

If the times of transit of a star across some (but not all) of the lines of the reticle were observed, the time, t_m, of transit across the *mean line* of the reticle may be computed by the formula

$$t_m = \text{mean of observed times} - \frac{\text{(sum of equatorial intervals of observed lines) (sec } \delta)}{\text{number of observed lines}}, \quad . \quad (25)$$

or

$$t_m = \text{mean of observed times} + \frac{\text{(sum of equatorial intervals of missed lines) (sec } \delta)}{\text{number of observed lines}}. \quad . \quad . \quad (26)$$

For the process of finding the equatorial intervals see § 114.

$$T_c' = t_m + Bb + k;$$

in which T_c' is the observed time of transit across the mean line corrected for inclination of the horizontal axis and for diurnal aberration.

$B = \cos \zeta \sec \delta$ is tabulated in § 299.

$b = \beta \pm p_i$, p_i being the pivot inequality derived as indicated in §§ 94, 115.

$$\beta = \{(w + w') - (e + e')\}\frac{d}{60} \quad . \quad . \quad . \quad (29)$$

if the level divisions are numbered from the middle toward each end, w and w' being the west end readings of the bubble before and after reversal of the striding-level, and e and e' the corresponding east end readings.

$$\beta = \{(w + e) - (w' + e')\}\frac{d}{60} \quad . \quad . \quad . \quad (30)$$

if the level divisions are numbered continuously from one end to the other, the primed letters indicating the readings taken with the zero of the level to the westward.

k is tabulated in § 315.

$$T_c = T_c' + Aa + Cc,$$

in which T_c is the reading of the timepiece when a star crosses the meridian, and Aa and Cc are the corrections for azimuth error and collimation error, respectively.

$A = \sin \zeta \sec \delta$ and $C = \sec \delta$ are tabulated in § 299.

To compute the azimuth and collimation errors (a and c) *from the observations*, WITHOUT THE USE OF LEAST SQUARES (§ 101–106), use the formulæ

$$c = \frac{(\alpha - T_c')_W - (\alpha - T_c')_E}{C_W - C_E}, \quad . \quad . \quad . \quad (39)$$

$$a = \frac{(\alpha - T_c' - Cc)_{\text{time stars}} - (\alpha - T_c' - Cc)_{\text{azimuth star}}}{A_{\text{time stars}} - A_{\text{azimuth star}}}, \quad (41)$$

to derive a and c, by successive approximations, as indicated in §§ 101–106.

The clock correction as determined by each observation is $\Delta T_c = \alpha - T$.

To compute the azimuth and collimation errors (a and c), *and* ΔT_c, *from the observations*, BY LEAST SQUARES. (§§ 107–110.)

The observation equations are of the form

$$\Delta T_c + A_W a_W + Cc - (\alpha - T_c') = 0, \quad . \quad . \quad (45)$$

and

$$\Delta T_c + A_E a_E + Cc - (\alpha - T_c') = 0, \quad . \quad . \quad (46)$$

for the observations made with illumination west and east, respectively.

The normal equations are

$$\left.\begin{array}{llllll}
\Sigma \Delta T_c & + \Sigma A_W a_W & + \Sigma A_E a_E & + \Sigma Cc & - \Sigma(\alpha - T_c') & = 0; \\
\Sigma A_W \Delta T_c & + \Sigma A^2_W a_W & & + \Sigma A_W Cc & - \Sigma A_W(\alpha - T_c') & = 0; \\
\Sigma A_E \Delta T_c & & + \Sigma A^2_E a_E & + \Sigma A_E Cc & - \Sigma A_E(\alpha - T_c') & = 0; \\
\Sigma C \Delta T_c & + \Sigma C A_W a_W & + \Sigma C A_E a_E & + \Sigma C^2 c & - \Sigma C(\alpha - T_c') & = 0.
\end{array}\right\} \quad (47)$$

The solution of these four equations gives the values of ΔT_c, a_W, a_E, and c.

The probable error of a single observation is

$$\epsilon = 0.674\sqrt{\frac{\Sigma v^2}{n_0 - n_u}},$$

in which the v's are the residuals of the observation equations, n_0 is the number of observations, and n_u is the number of unknowns (and of normal equations).

The probable error of the computed ΔT_c is $\epsilon_0 = \epsilon\sqrt{Q}$, in which Q is a quantity obtained as follows: In equation (47) write Q in the place of ΔT_c, -1 in the place of $\Sigma(\alpha - T_c')$, and 0 in the place of the other absolute terms, and then solve for Q.

For two modifications of this method of computing, which may be used if considered advisable, see §§ 109, 110.

For the form of computation if unequal weights (depending upon the declination of the star and the number of lines of the reticle observed upon) are assigned to the separate observations, see §§ 111–113.

TO COMPUTE THE LATITUDE FROM OBSERVATIONS MADE WITH A ZENITH TELESCOPE. (§§ 146–157.)

The latitude from a single pair of stars is

$$\phi = \tfrac{1}{2}(\delta + \delta') + (M - M')\frac{r}{2} + \frac{d}{4}\{(n + n') - (s + s')\}$$

$$+ \tfrac{1}{2}(R - R') + \frac{m}{2} - \frac{m'}{2} \quad \ldots \quad (57)$$

In (57) the primed letters correspond to the northern star of the pair; M is the micrometer reading expressed in turns; r is the angular value of one turn; n and s are the north-end and south-end readings, respectively, of the level, for the northern star, and n' and s' for the southern star; R is the refraction, and m the reduction to the meridian of a star observed off the meridian.

The level correction as given above is for a level tube which carries a graduation of which the numbering increases each way from the middle. If the level-tube graduation is numbered continuously from one end to the other the level correction becomes $\frac{d}{4}\{(n' + s') - (n + s)\}$. (See § 148.)

The term $\tfrac{1}{2}(R - R')$ is tabulated in § 304. (See § 150.)

The term $\frac{m}{2}$ is tabulated in § 305. (See § 151.)

To combine the separate values of ϕ and to compute the probable errors. (§§ 154–157.)

The probable error of a single observation is

$$e = \sqrt{\frac{(0.455)[\Delta\Delta]}{n-p}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

in which the Δ's are the differences obtained by subtracting the mean result for each pair from the result on each separate night from that pair; $[\Delta\Delta]$ is the sum of the squares of the Δ's; n is the total number of observations; and p is the total number of pairs observed.

The probable error of the mean result from any one pair is

$$e_p = \sqrt{\frac{(0.455)[vv]}{p-1}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

in which the v's are the residuals obtained by subtracting the indiscriminate mean result for the station from the mean result from each pair; and $[vv]$ is the sum of the squares of the v's.

The probable error of the mean of the two declinations of the stars of a pair is

$$e_\delta = \sqrt{e_p^2 - \epsilon^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

in which

$$\epsilon^2 = \frac{e^2}{p}\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \ldots\right). \quad . \quad . \quad . \quad (67)$$

n_1, n_2, n_3, . . . are the numbers of times that pair No. 1, pair No. 2, pair No. 3, . . . , respectively, are observed.

The proper weights w_1, w_2, w_3, . . . for the mean results from the separate pairs are proportional to

$$\frac{1}{e_\delta^2 + \frac{e^2}{n_1}}, \quad \frac{1}{e_\delta^2 + \frac{e^2}{n_2}}, \quad \frac{1}{e_\delta^2 + \frac{e^2}{n_3}}, \ldots$$

The most probable value, ϕ_0, for the latitude of the station is

$$\phi_0 = \frac{w_1\phi_1 + w_2\phi_2 + w_3\phi_3 \ldots}{w_1 + w_2 + w_3 \ldots} = \frac{[w\phi]}{[w]}, \quad . \quad . \quad (70)$$

in which ϕ_1 is the mean result from the first pair, ϕ_2 from the second pair, and so on.

The probable error of ϕ_0 is

$$e_\phi = \sqrt{\frac{(0.455)[wv'^2]}{(p-1)[w]}}, \quad . \quad . \quad . \quad . \quad . \quad (71)$$

in which $[wv'^2]$ stands for the sum of the products of the weight for each pair into the square of the residual obtained by subtracting ϕ_0 from the mean result for that pair; and $[w]$ is the sum of the weights.

Computation of the Micrometer Value from Observations upon a Circumpolar Star near Elongation. (§§ 158–162.)

For an example of this computation see § 162.

To compute the hour-angle of the star at elongation use the formula

$$\cos t_E = \tan \phi \cot \delta, \quad . \quad . \quad . \quad . \quad (72)$$

t_E added to or subtracted from the right ascension of the star, for a western or eastern elongation respectively, gives the sidereal time of elongation.

The correction for curvature is given in § 306.

The level correction is given by formulæ (77) and (78) of § 160.

If t is the time interval in seconds corresponding to one turn of the micrometer, then the value of one turn expressed in seconds of arc is $(15 \cos \delta)t$. (See § 161.)

To this value must still be applied the corrections for chronometer rate and for refraction as indicated at the end of § 161.

To Compute the Latitude from an Observed Altitude of a Star, or the Sun, in any position, the time being known.

See § 171.

To Compute the Latitude from Zenith Distances of a Star, or the Sun, Observed near the Meridian, the time being known.

See § 172.

To Compute the Latitude from Observations of the Altitude of Polaris at any Hour-angle, the time being known.

See § 173.

To compute azimuth from observations upon a circumpolar star with a direction instrument.

Example of record, § 187.

Example of computation, § 200.

Level correction

$$C_L = \{(w + w') - (e + e')\}\frac{d}{4} \tan A, \quad . \quad . \quad (91)$$

for a level having its divisions numbered both ways from the middle.

$$C_L = \{(w + e) - (w' + e')\}\frac{d}{4} \tan A \quad . \quad . \quad . \quad (92)$$

for a level numbered continuously in one direction, the primed letters referring to the readings taken in the position in which the numbering increases toward the east. C_L as given by these formulæ is the correction to the circle reading for the star upon the supposition that the circle graduation increases in a clockwise direction (§ 192).

$$\tan z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}. \quad . \quad . \quad (98)$$

See § 193.

Curvature correction

$$= - \tan z \frac{1}{n}\left(\frac{2 \sin^2 \frac{1}{2}\Delta t_1}{\sin 1''} + \frac{2 \sin^2 \frac{1}{2}\Delta t_2}{\sin 1''} + \ldots \frac{2 \sin^2 \frac{1}{2}\Delta t_n}{\sin 1''}\right);$$

in which Δt_1, Δt_2, . . . Δt_n are the differences between each hour-angle and the mean of the hour-angles, and the values of the terms $\frac{2 \sin^2 \frac{1}{2}\Delta t_1}{\sin 1''}$, etc., are known from the table in § 307. (See §§ 194–197.)

Correction for diurnal aberration,

$$0''.319 \frac{\cos \phi \cos z}{\cos A}. \quad . \quad . \quad . \quad . \quad . \quad (113)$$

or, with sufficient accuracy for nearly all cases 0″.32. See §§ 198, 199. The sign of the correction is + if applied to the computed azimuth of the star expressed as an angle *east* of north, and — if the angle is measured westward from the north.

To compute azimuth from observations upon a circumpolar star with a repeating instrument.

Example of record, § 203.

The computation is made as indicated above with the exception of certain modifications indicated in § 204.

To compute azimuth from observations upon a circumpolar star with an eyepiece micrometer.

See example of record and computation, §§ 210–214.

312. LIST OF DEFINITIONS.

INDEX.

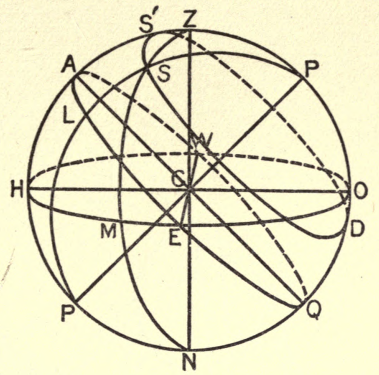


FIG. I

CELESTIAL SPHERE.—C = position of observer; Z = his zenith and N his nadir; P, P' = the north and south poles; $HMEOW$ = horizon; ZM, ZH, ZO = vertical circles; O, E, H, W = north, east, south, and west points, respectively; $ALEQW$ = equator; S = a star; $S'SD$ = small circle defining the diurnal motion of the star; PAP', PSP' = hour-circles; $OPZAH$ = the meridian; ZA = latitude of station; ZS = zenith distance of star; PS = north polar distance of star; LS = declination of star; ZPS = hour-angle of star; MS = altitude of star; OM = azimuth of star.

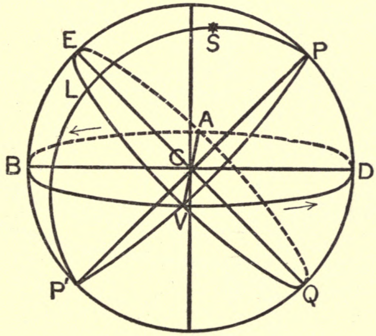


FIG. Ia

CELESTIAL SPHERE.—P, P' = north and south poles; $EVQA$ = equator; $VDAB$ = ecliptic (the arrows indicating the apparent motion of the Sun); V = vernal equinox; A = autumnal equinox; VL = right ascension of the star S.

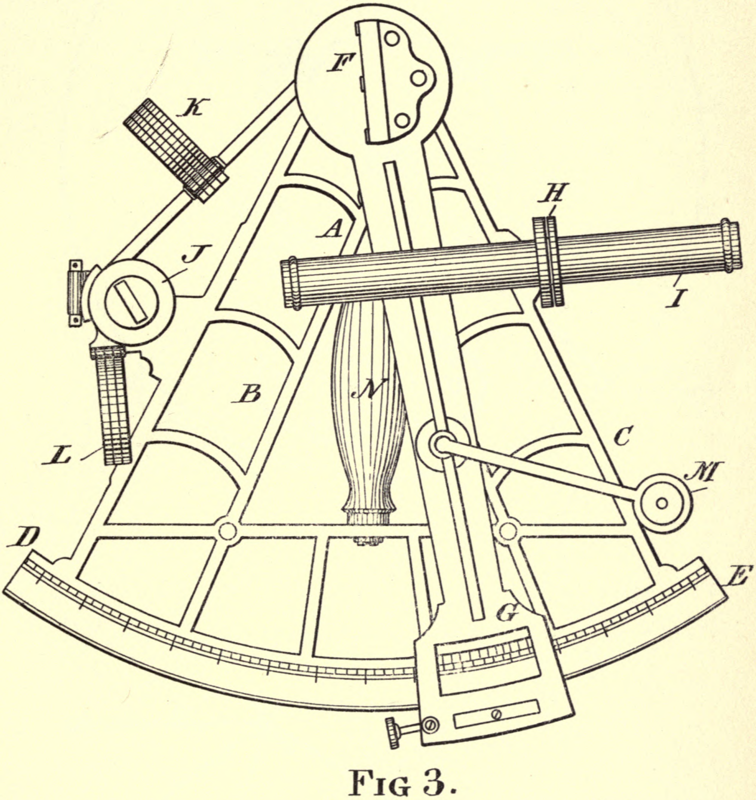


Fig 3.

SEXTANT

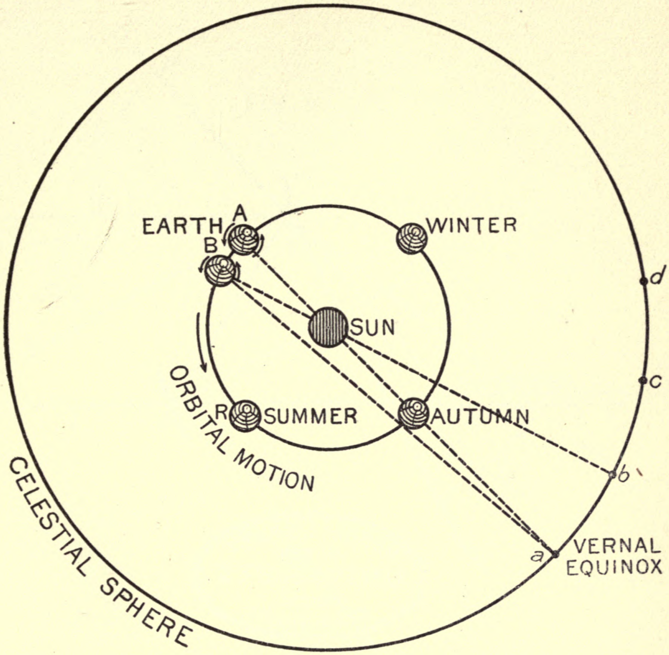

EARTH
A
B
WINTER
SUN
d
c
R
SUMMER
AUTUMN
ORBITAL MOTION
b
a
VERNAL
EQUINOX
CELESTIAL SPHERE

FIG. 2

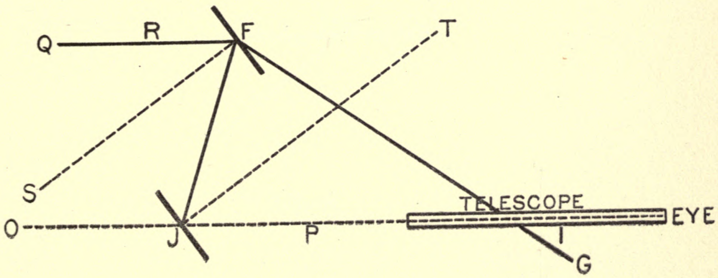

Q
R
F
T
S
TELESCOPE
EYE
O
J
P
I
G

FIG. 4

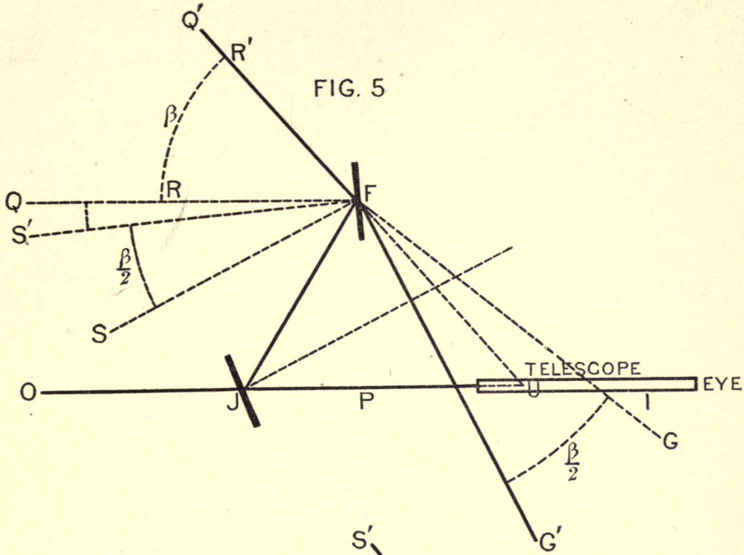


FIG. 5

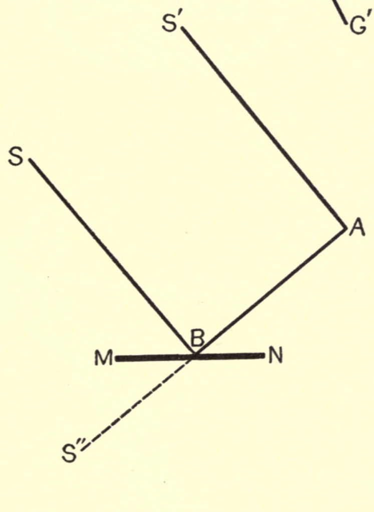
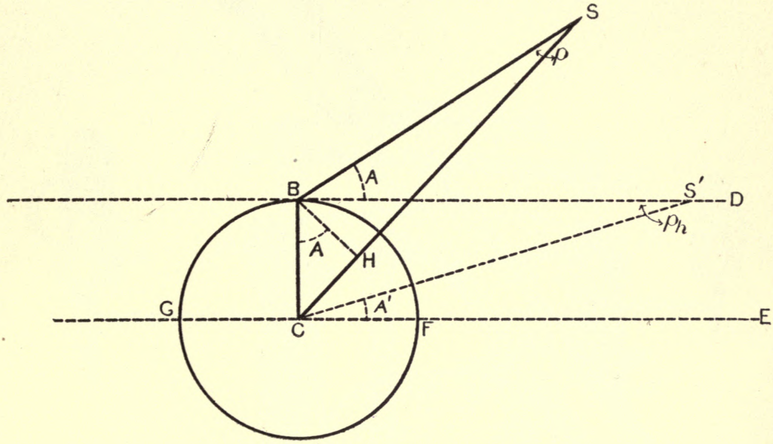


FIG. 6



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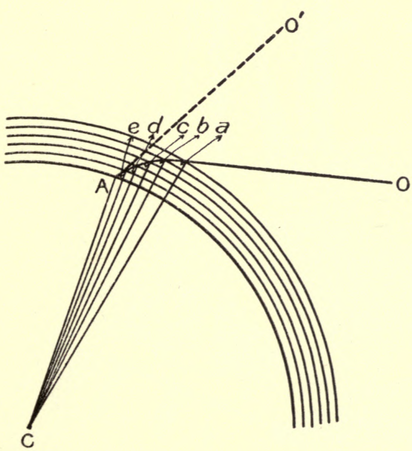
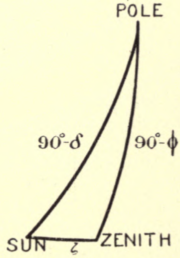


FIG. 8



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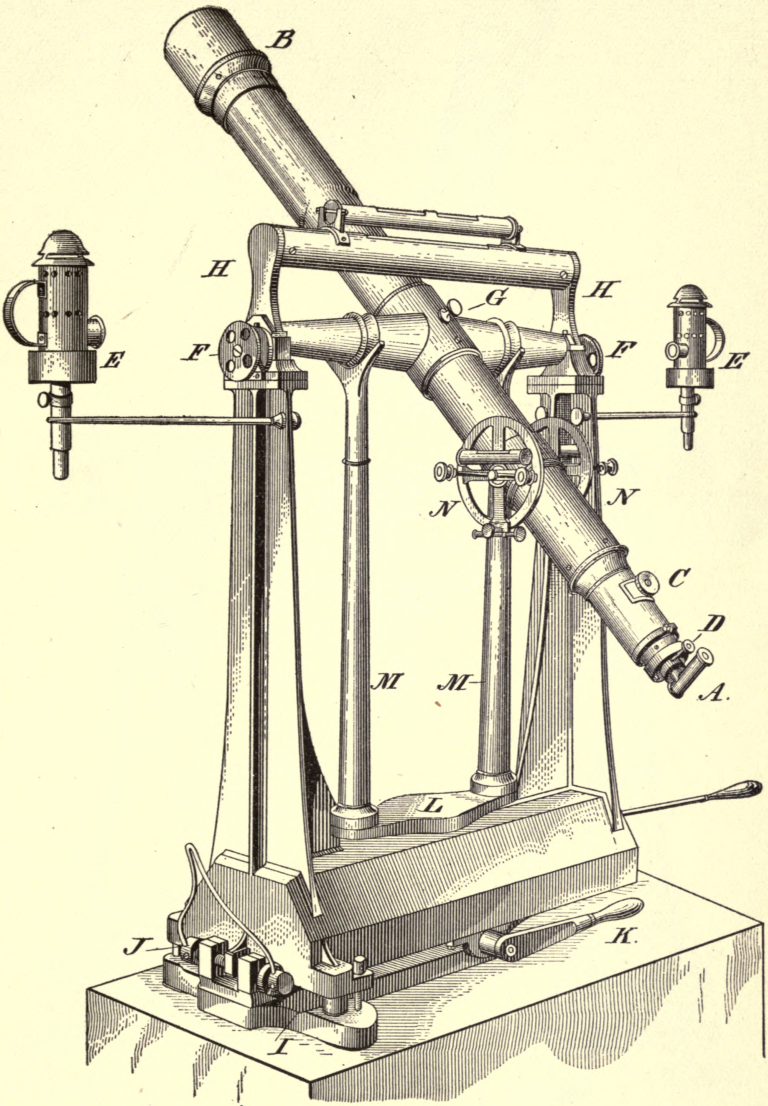


FIG. 10.

ASTRONOMICAL TRANSIT.

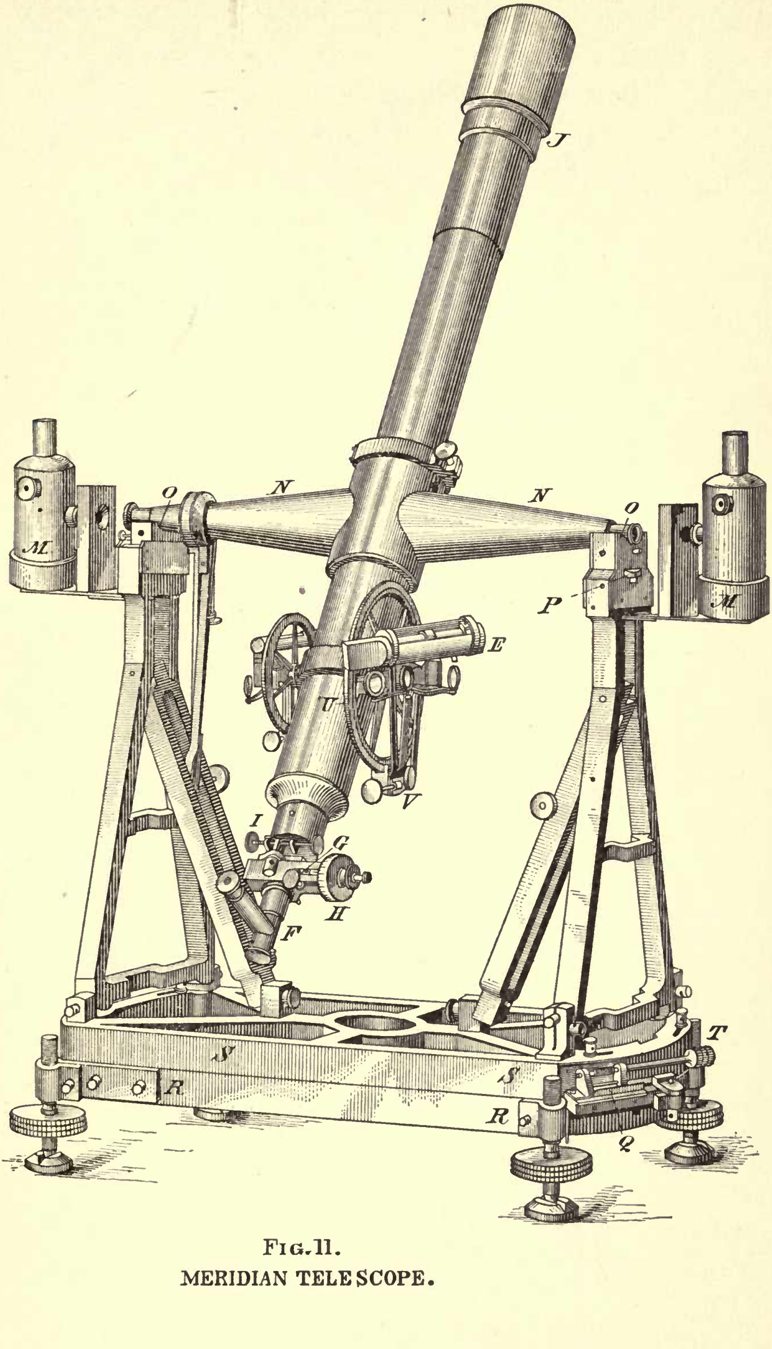


FIG. 11.
MERIDIAN TELESCOPE.

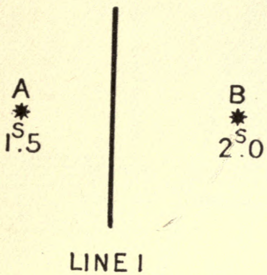
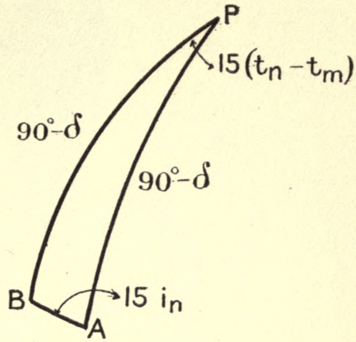


FIG. 12



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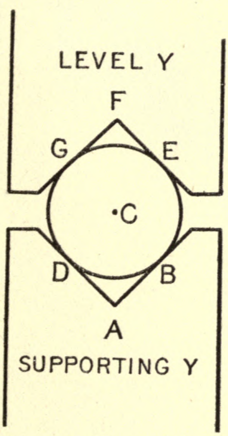

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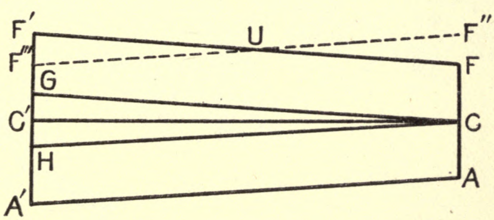


FIG. 16

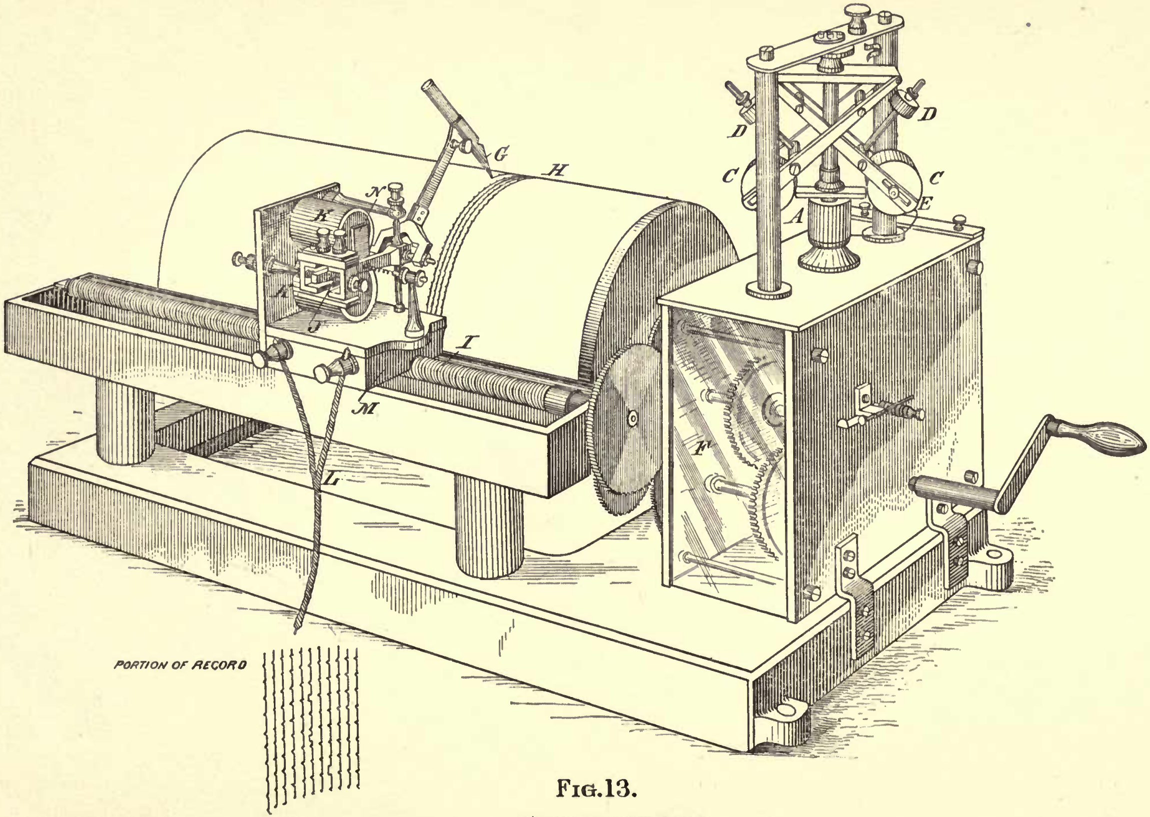


Fig. 13.
CHRONOGRAPH.

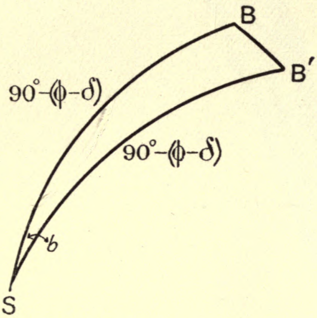


FIG. 17

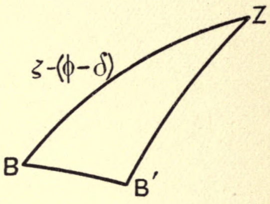
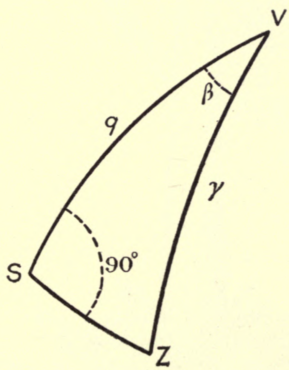


FIG. 18



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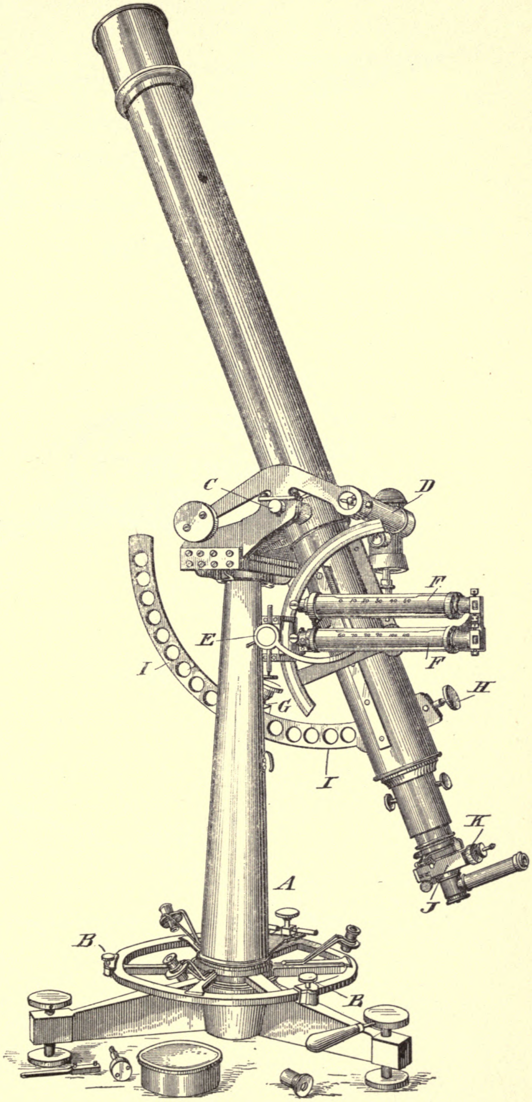
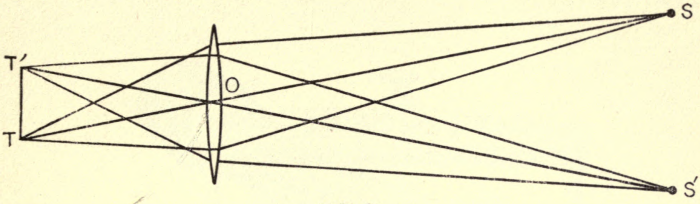


FIG. 20.
ZENITH TELESCOPE.



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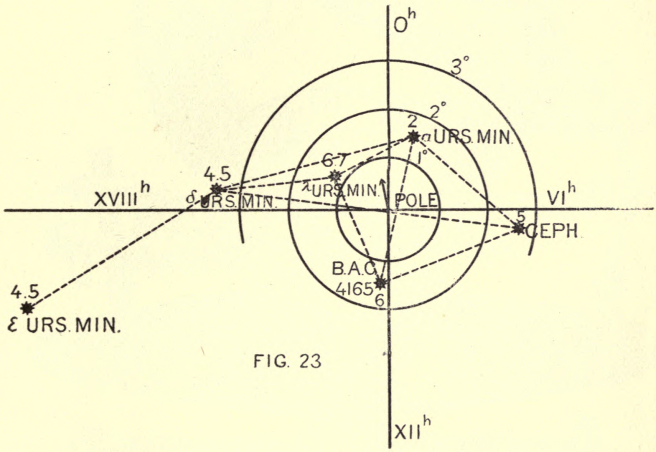

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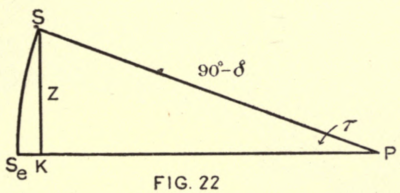

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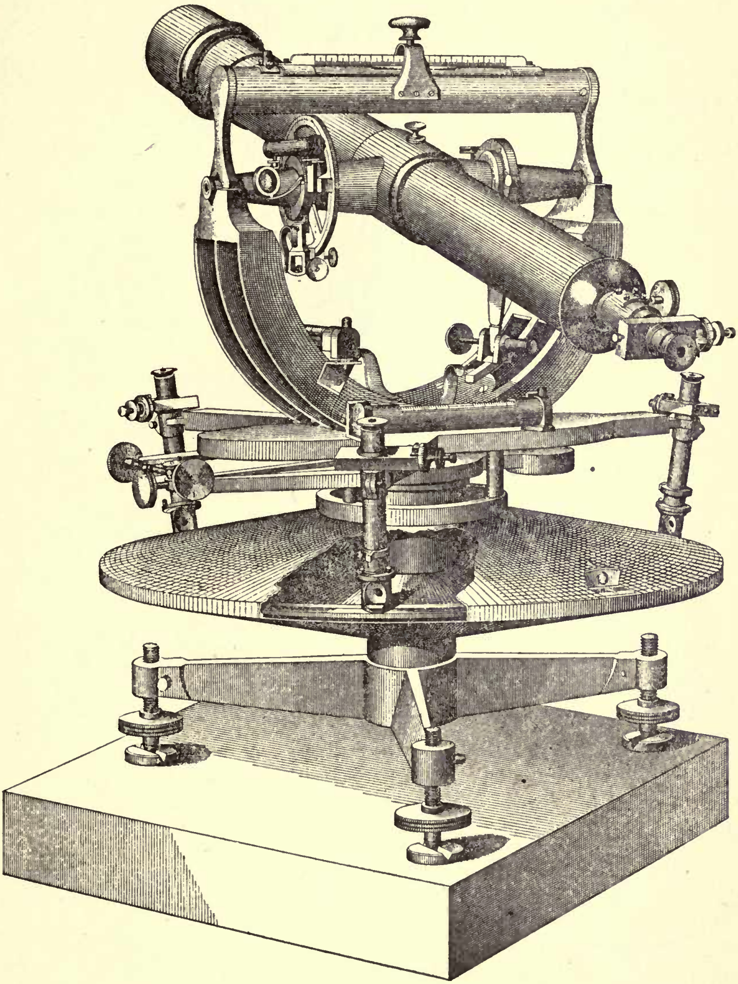


FIG. 24.—ALTAZIMUTH.

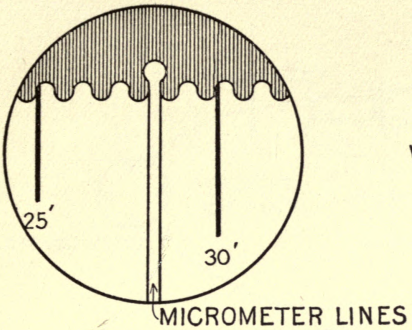
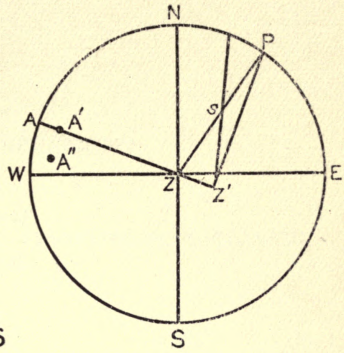


FIG. 25



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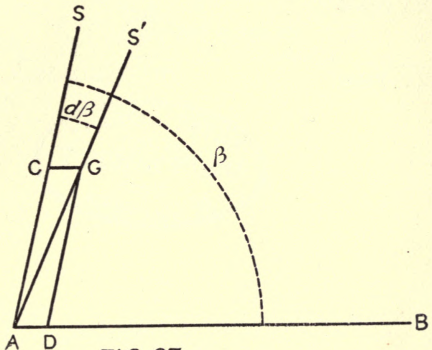
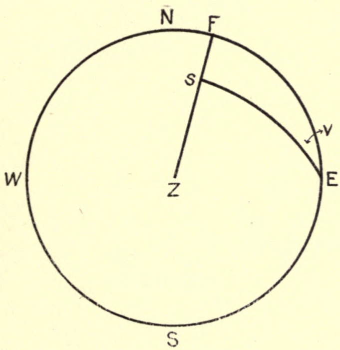


FIG. 27



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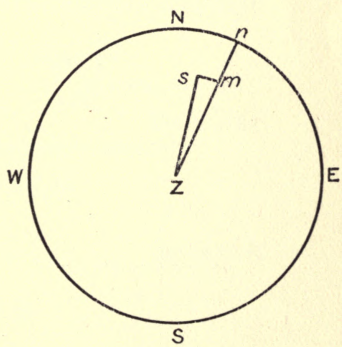
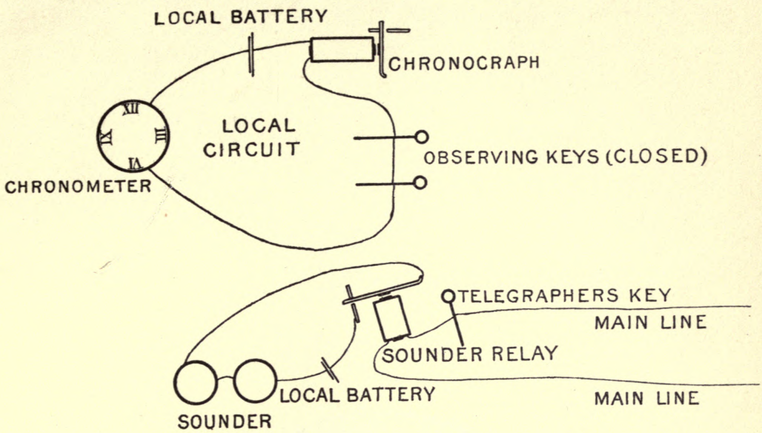


FIG. 29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CHRONOCRAPH
LOCAL
CIRCUIT
OBSERVING KEYS (CLOSED)
CHRONOMETER
TELEGRAPHERS KEY
MAIN LINE
SOUNDER RELAY
LOCAL BATTERY
MAIN LINE
SOUNDER

FIG. 30

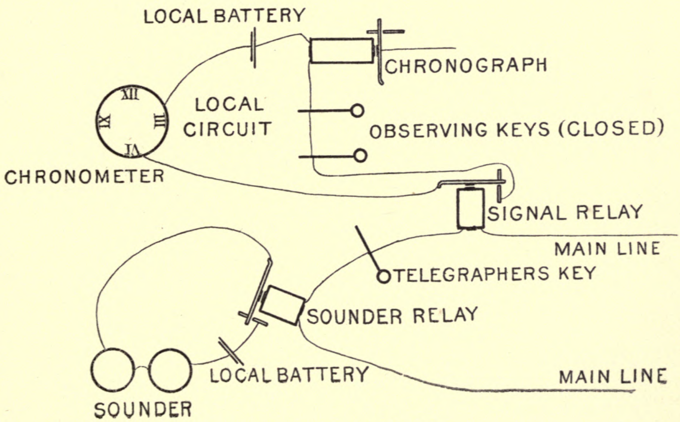
LOCAL BATTERY
CHRONOGRAPH
LOCAL
CIRCUIT
OBSERVING KEYS (CLOSED)
CHRONOMETER
SIGNAL RELAY
MAIN LINE
TELEGRAPHERS KEY
SOUNDER RELAY
LOCAL BATTERY
MAIN LINE
SOUNDER

FIG. 31

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