APPLIED BALLISTICS
THE SCIENCE OF ACCURACY

Aerodynamic Drag Modeling for Ballistics

By Bryan Litz

Part 1: Aerodynamic Drag 101

Aerodynamic drag is an important consideration for accurate long range trajectory prediction. The data and methods used to account for aerodynamic drag can make or break a long range shot. This article will describe how aerodynamic drag affects modern small arms trajectory predictions, and how drag modeling has evolved from its meager beginnings to its current level of refinement.

The Physics of Aerodynamic Drag

Please don't let the title scare you! This is the *shooter-speak* version of the physics; distilled down to the practical elements. There is a technical appendix to this article which goes into greater depth on some of the math and explanations, if you're so inclined.

Some of you may have read about G1 and G7 standard projectiles and standard drag models. Figure 1 below shows these two standard projectile shapes and the associated drag curves that go with them. Now, a lot has been made of these so called *drag curves,* and how well they represent the *drag curves* of modern bullets. But what IS a *drag curve?* What physical significance does it have? Why does it appear the drag goes down as velocity (Mach number) goes up? Shouldn't drag increase with speed? Certainly feels like it when I put my head out the car window and hit the gas…

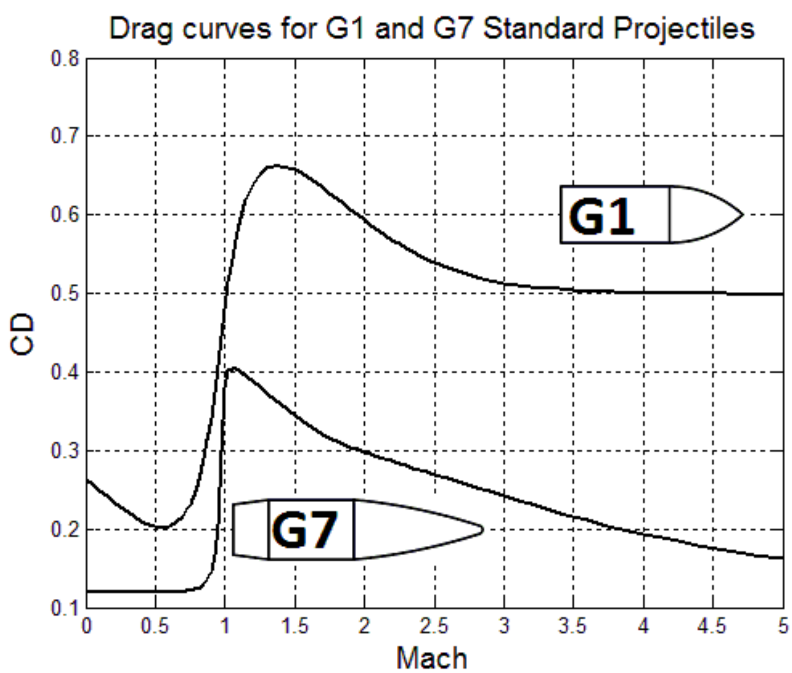

Figure 1. G1 and G7 standard projectile models and their associated drag curves.

Here's what's going on in Figure 1. The Coefficient of Drag (CD) is plotted against the Mach number. The Mach number is how fast you're going in relation to the speed of sound. For example, Mach 2 is 2 times the speed of sound, Mach 3 is 3 times the speed of sound, etc.

It's clear to see that the drag coefficient peaks at or near the speed of sound (Mach 1), then tapers down as Mach number (speed) increases.

Before we talk anymore about the drag curves, we have to address the elephant in the room: ***Why does the drag coefficient (CD) go down as speed increases!?!?***

It's a good question, and one that needs a clear answer if this article is to make any sense at all.

The key is understanding that CD is a *coefficient.* It doesn't represent the force of aerodynamic drag in pounds or any other units. The *coefficient* of drag is just a number that says how much drag a certain shape will have at any given speed. More streamlined shapes have lower drag coefficients, and blunter shapes have higher drag coefficients. But how does the drag coefficient relate to *actual drag in pounds?* It's probably best if we start from the top on this.

All of external ballistics is based on how much velocity the bullet loses as it flies thru the air. The amount of: drop, wind drift, time of flight, and every other aspect of a bullets trajectory are all determined by the bullets velocity, and the rate it's slowing down. In physics, there's a name for the rate in change of velocity: it's called *acceleration.* When something is slowing down, it's tempting to say it's *decelerating,* but the correct terminology is *negative acceleration.* Remembering that this is the *shooter speak* explanation, I'll shamelessly refer to bullets as *decelerating* throughout this article.

In order to know the exact amount of deceleration the bullet at all points in its flight, we need to know the force that's acting on it. Newtons second law of motion tells us clearly that an objects acceleration is equal to the force applied to it, divided by its mass. Since the mass of a bullet is easy to know, it all comes down to the force that's applied; *the aerodynamic drag force*.

In words, the aerodynamic drag force is equal to the ***dynamic pressure,*** times the bullets ***frontal area,*** times its ***drag coefficient***. These 3 terms bear some discussion.

Dynamic Pressure

Dynamic pressure is basically the pressure of the oncoming air flow. One of the important factors in determining the dynamic pressure is the air density. Every shooter knows that ballistics programs need to know the air temperature, pressure and humidity in order to calculate a long range trajectory. Well, this is exactly where those things come into play. The air temperature, pressure and humidity determine what the air density will be, and this directly affects the dynamic pressure on the bullet, which affects the aerodynamic drag, and hence the bullets deceleration. To imagine the difference that air density has on drag, imagine moving your hand as fast as you can thru air, then under water in a swimming pool.

The resistance you feel is greater in water because the *density* of the fluid is greater. The same thing applies with air that's more or less dense; higher density air creates more drag, which results in greater drag, which decelerates the bullet faster…

One interesting property of dynamic pressure is that it *increases with the square of velocity*. In shooter speak, that means that if you double velocity, the dynamic pressure is 4 times greater. If you triple velocity, dynamic pressure is 8 times greater, etc. It's not linear. The units of dynamic pressure are pounds per square foot. If you're really interested in knowing the equation for dynamic pressure, you can skip to the technical appendix. Just for an example, a projectile moving along at Mach 3 which is 3348 fps in standard conditions would experience 13,310 pounds per square foot of dynamic pressure. At Mach 2 (2232 fps) the bullet feels 5,916 pounds per square foot of pressure and by Mach 1 (1116 fps) the bullet feels a mere 1,479 pounds per square foot of pressure. Of course if the air density is higher or lower than standard, the dynamic pressure would be more or less accordingly.

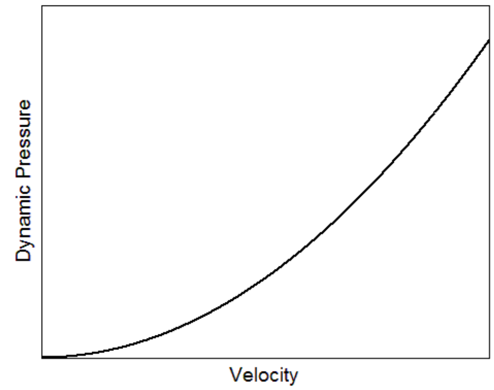

Figure 2. Dynamic Pressure increases more at higher speeds.

Frontal Area

Dynamic pressure gives us the *pounds per square foot,* so in order to know the *actual force of aerodynamic drag in pounds*; we need to know the *area* on which the dynamic pressure is applied. This is simply the frontal area of the bullet in square feet.

Frontal area is the most straightforward and least exciting aspect of calculating aerodynamic drag. The equation for frontal area is given in the technical appendix so you can calculate it for any bullet. As an example, a .308 caliber bullet has a frontal area of 0.000517 square feet. To calculate a drag force for Mach 1, 2, and 3, we simply multiply the dynamic pressure at each of these speeds by the projectiles frontal area giving: 6.9 pounds at Mach 3, 3.1 pounds at Mach 2, and 0.8 pounds at Mach 1 for a .308 caliber projectile in standard conditions.

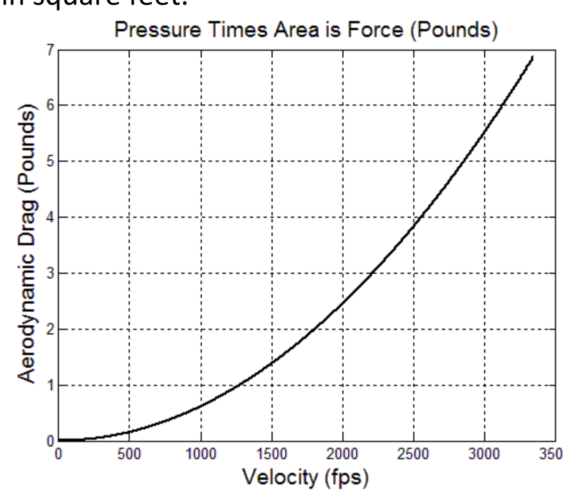

Figure 3. Multiplying the dynamic pressure times frontal area gives force of drag in pounds.

Now that we know the pressure and the area it acts on, we can show the actual aerodynamic drag force over a range of velocities. Figure 3 is basically the same as Figure 2 with the exception it shows velocity units in feet per second and the force of drag in pounds.

At this point we can say that dynamic pressure acting on the bullets frontal area is what makes aerodynamic drag. But what about the *shape* of the bullet; surely the shape has an effect on drag?

Drag Coefficient

And now we're finally getting to the point. As we stated in the beginning, *the coefficient of drag (CD) is a number that scales the basic drag calculation for the shape of the projectile*. A bullet like a wadcutter which is purely blunt in front and back will have a drag coefficient close to 1 because the frontal area is taking the full brunt of the dynamic pressure. It's experiencing all the drag possible. But if you give the projectile an ogival nose and maybe a boat-tail, *it will experience less drag at the same speed*. The drag coefficient is the number that describes how much. Going back to Figure 1, you can see that at Mach 3 (3348 fps), the G1 projectile has a drag coefficient of 0.51, while the G7 projectile has a drag coefficient of only 0.24. In the previous section we learned that at Mach 3, a .308 caliber bullet has 6.9 pounds of drag applied to it at this speed (dynamic pressure times bullet frontal area). However, this is the maximum potential drag that could be experienced by something; the wadcutter shape. In reality, a modern projectile shaped similar to the G7 standard will only experience about 24% of that 6.9 pounds due to its shape (CD is 0.24).

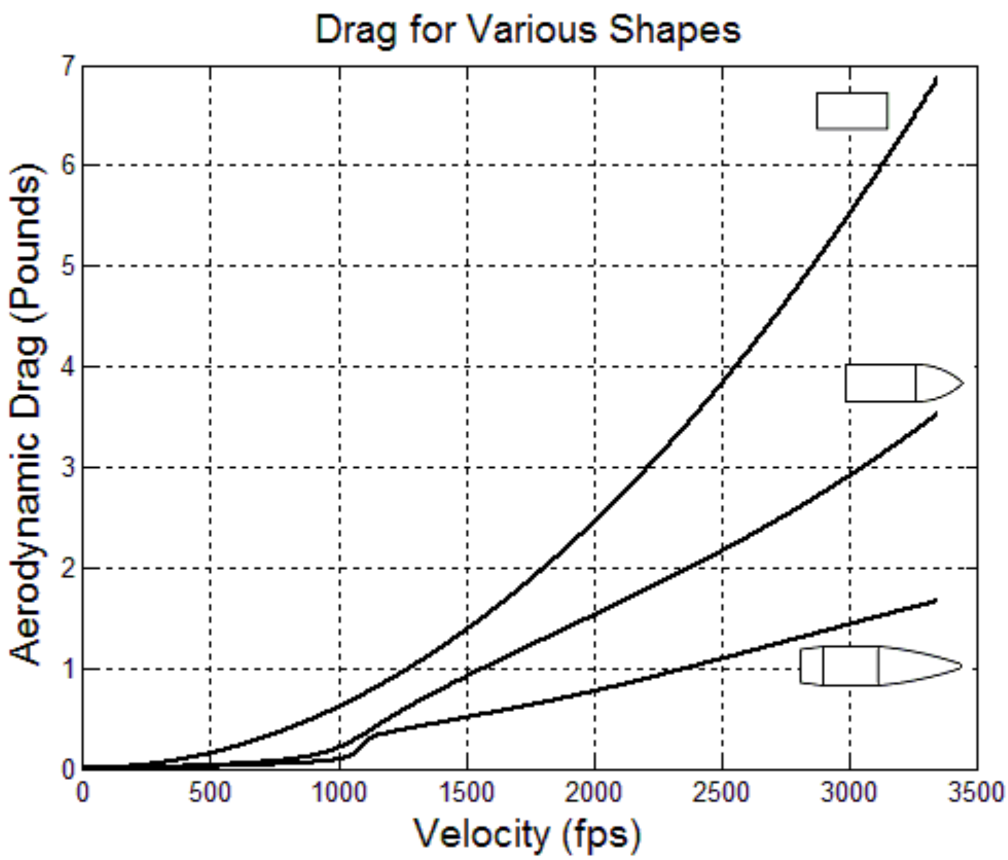

Figure 4. Actual drag experienced by various projectile shapes from zero to Mach 3.

There is some minor simplification going on here for the sake of clarity and remaining at the *shooter speak* level, but the main ideas are all here. Figure 4 above shows the culmination of aerodynamic drag including: dynamic pressure, bullet frontal area, and drag curve to account for projectile shape. If you look closely, you can see where the drag curve plot affects the force of drag around Mach 1. The steep ramp at this speed is what is referred to as the *sound barrier;* the sharp rise in drag as you approach the speed of sound. Most flight vehicles such as aircraft and rockets approach the sound barrier from the left side of Figure 4, as they accelerate to higher speeds. Bullets are an exception here, as they are high supersonic as soon as they exit the muzzle (right side of Figure 4) and spend all their time slowing down to the sound barrier at Mach 1.

Hopefully this background has shown you how the drag coefficient plots like those in Figure 1 actually relate to something physical. The following summary will highlight the important insights you should move forward with:

Summary

- The force of aerodynamic drag is made up of the *dynamic air pressure* applied to the bullets *frontal area*, times a *drag coefficient.*
- The *drag coefficient (CD)* scales the drag at each speed based on the shape of the bullet.
- The *drag curve* is just the drag coefficient for *all speeds*.
- The drag curve of a bullet is determined by measuring its drag at multiple flight speeds; measure enough points at different speeds and connect the dots to make a drag curve.

It's important to know what the drag curve *is not:*

- A drag curve *is not* a trajectory path for a bullet.
- A drag curve *is not* a series of 3 or 4 banded BC's. To be effective, a CDM is comprised of dozens of points which define a bullets actual drag at all speeds.
- A drag curve *is not* a mathematical equation.
- A drag curve *is not* a predictive algorithm

Part 2: Custom Drag Models and Ballistic Coefficients

You may recall from other sources that all projectile shapes have a unique drag curve based on their shape. Furthermore, bullets within a given class can all be represented with a *Ballistic Coefficient (BC)* referenced to a standard curve such as G1 or G7. For more background on this, refer to Chapter 2 of *Applied Ballistics for Long Range Shooting.* The basic idea is that it's much easier to represent the drag of a class of bullets by referencing all bullets to a common standard. This is where you get G1 BC's, G7 BC's, etc.

The simplicity of the standard curve approach is offset by the compromise that *the actual unique projectile drag is not accurately being modeled for each and every bullet shape*. In Figure 4 you can see that the G7 standard may be a close representation of any given modern bullet, but in reality they are different in shape. Those differences in shape mean that the drag curves are different.

In most cases, the drag shapes are similar enough that simply scaling the drag curve with a form factor results in trajectory predictions that are accurate enough. However for the ultimate in accurate drag modeling, nothing beats the use of *Custom Drag Models (CDM's).* CDM's dispense with the compromise of matching 'G' standards and basically makes every bullet its own standard by modeling its unique drag. The benefit of CDM's over BC's is maximized at extended range near transonic speeds (near Mach 1). This is where the bullets drag curve is most unique; each one being like a fingerprint describing how a particular bullet shape makes its way from supersonic to subsonic speed.

Figure 4. G7 standard (left), modern LR bullet (right).

This is the perfect place for a historical footnote.

The use of standard projectiles and Ballistic Coefficients was established prior to the advent of the modern computer. At that time, firing tables for small arms were computed by hand. It was very tedious work that sometimes took months to calculate a single trajectory **[REF 2]**. During that time, the military (let alone the sporting arms industry) couldn't make use of custom drag models due to computational constraints. This is why the standard 'G' projectiles and drag curves were created. By creating tables for only a small number of standard projectiles, then referencing each bullet to its closest matching standard, reasonably accurate tables could be produced efficiently. This practice remained common until about the 1950's when modern computers enabled the use of custom trajectory calculations in the field. The use of BC referenced to G standards has continued in the sporting arms industry and much of the military's small arms ballistics calculators. Only recently has the modern standard migrated from the G1 standard to the G7 which is a much better match for modern small arms ballistics.

Why, you might ask, did it take so long for the modern standard to move from G1 to G7? Furthermore, you might ask, why haven't we done away with BC's in favor of CDM's now that computational power is no longer a constraint? The answer is two-fold. First, you have the natural reluctance of people to change and adopt a new paradigm. But even if people were all gung-ho about changing to the G7 standard, what good would it do if there weren't an accurate and extensive library of G7 BC DATA?

Without accurate data, G7 BC's would just be a good idea with no way to implement.

Recognizing this impediment to progress, the sporting arms industry slowly migrated to providing BC's referenced to the better fitting G7 standard. The creation of accurate data, combined with capable computers and software have enabled the shooting world to take advantage of this *better matching* G7 BC.

Even as the world embraced the *better matching* BC, one couldn't help but wonder why not go straight to the CDM's for each bullet rather than accepting another approximation albeit an improved approximation. The hold up with widespread use of CDM's was again, availability of DATA. It's one thing to generate a G7 BC based on some limited measurements of downrange velocity or time of flight. But to map out the entire drag curve for each bullet takes a lot more work! Slowly but surely, the sporting arms industry is catching up with the state of the art and beginning to provide some CDM's for modern long range bullets. When properly measured, CDM's are the most accurate and complete means to model drag for modern bullets. Below are a few examples of the test firing showing the Applied Ballistics Custom Drag Model compared to G1 and G7 approximations of drag.

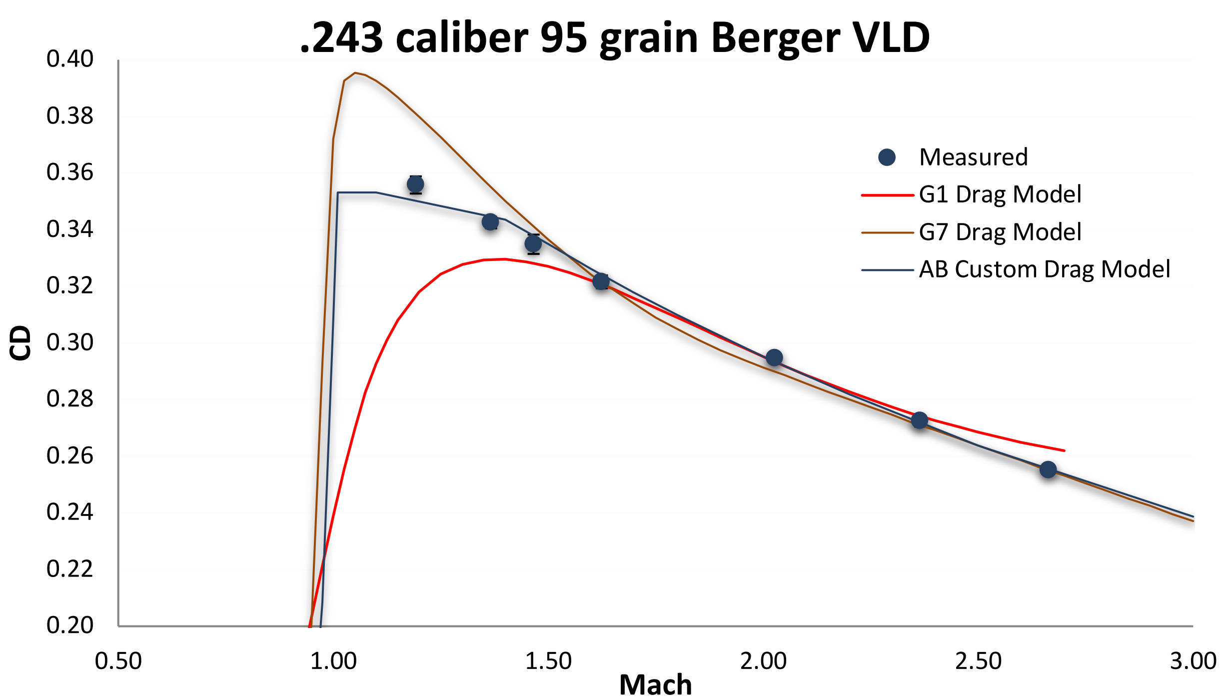

Figure 5. Custom Drag Model compared to the G1 and G7 standard curves.

The first plot is for the .243 caliber 95 grain Berger VLD. Each of the blue data points is an average of multiple shots fired at that velocity. The CDM is determined by measuring discrete points of drag at various speeds. The error bounds are shown on the measured data points which represent +/- 2 standard errors. In the case of this bullet, the actual drag is somewhere between the G1 and G7 curves. In other words, neither a G1 or a G7 BC would accurately model this bullets drag at all speeds, only the CDM can do this.

If you've been paying attention, you'll recall that in Figure 1 the G1 drag curve was much higher than the G7, and here they're shown as nearly equal in supersonic speeds. This is because the drag curves are scaled to the projectile drag measurements via a form factor. This is explained in great detail in Chapter 2 of *Applied Ballistics for Long Range Shooting* **[REF 1]**.

Below is another example of carefully collected live fire test data, this time on the Berger .308 caliber 155.5 grain FULLBORE bullet. Note how the drag curve is very similar to the G7 standard but not quite the same. These subtle differences in drag modeling between the G standards and the actual drag are the last frontier in eliminating error from modern drag modeling. With CDM's you don't have to settle for the *best fitting* representation of your bullet, you can actually model the drag of your specific bullet.

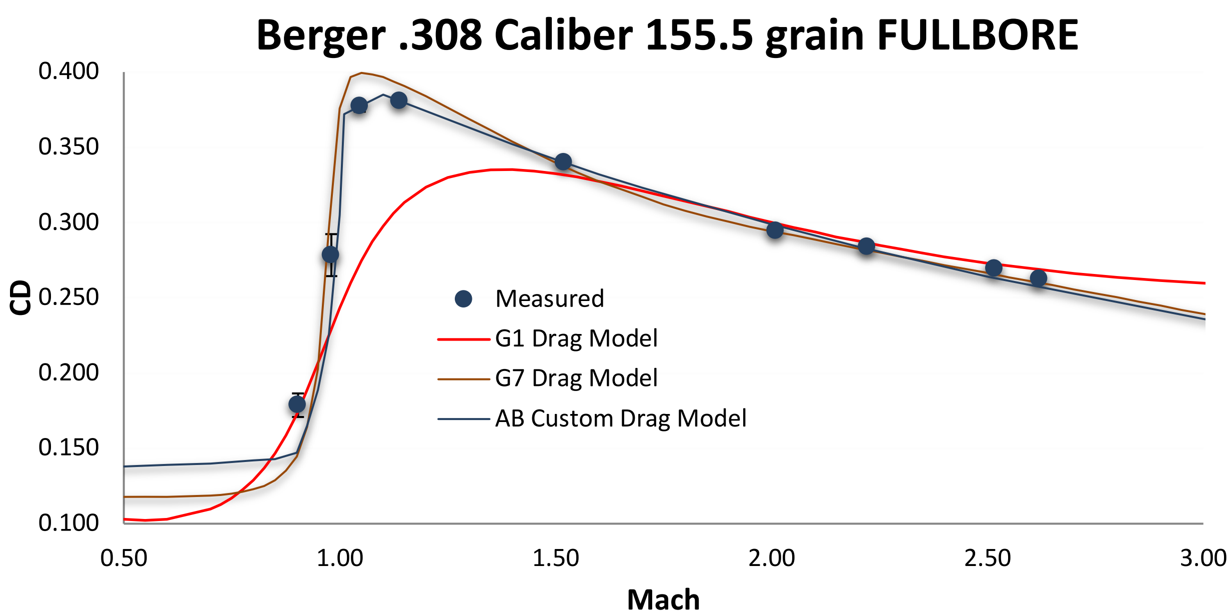

Figure 6. Custom Drag Model compared to the G1 and G7 standard curves.

To get an idea of the experimental nature of these live fire tests, consider the following plot which shows each single data point from the test; each data point representing a single shot. In the plot below you can see that the data points measured in the live fire test are quite repeatable and rarely stray far from the average. This plot shows the dense collection of data points around transonic and down thru Mach 1. High confidence data like this is the best way to support the most accurate long range trajectory predictions.

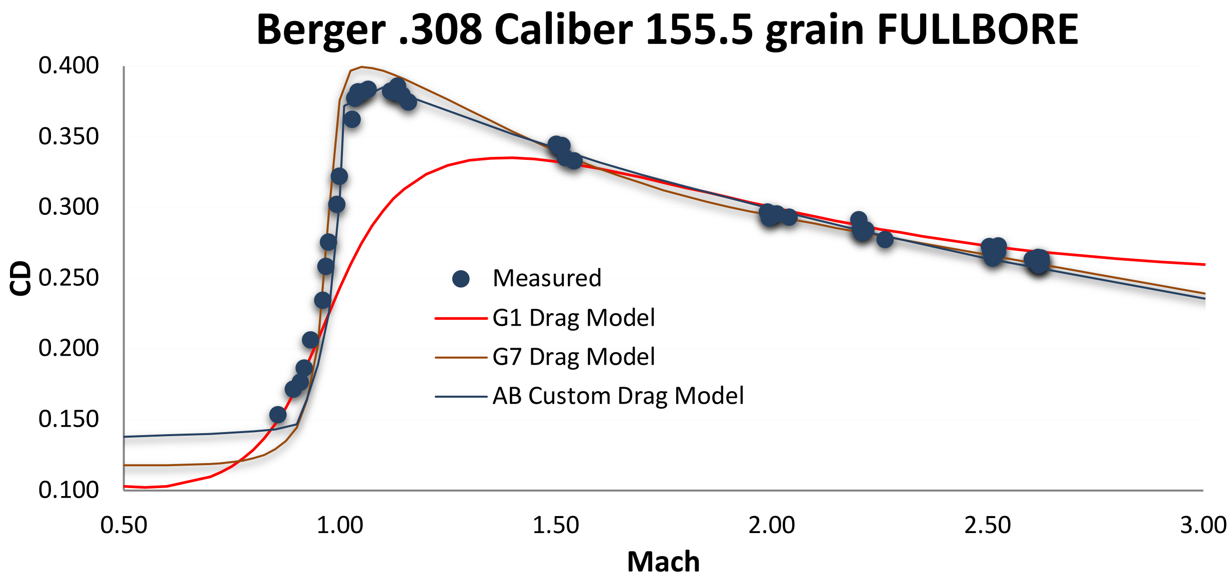

Figure 6. Custom Drag Model compared to the G1 and G7 standard curves.

The technical appendix has more details on how the data points are collected thru live fire to create a Custom Drag Model. The big idea is that the test firings need to be conducted under very controlled conditions. The slightest error in measurement will result in great uncertainty in the measured data. It's not possible to replicate a custom drag model by simply observing drop over long range. Measuring drop at long range brings an entire host of other variables into play such as wind, scopes, shooter skill, etc. which skew the perception of fundamental drag.

Long range shooters who are familiar with ballistics programs are very familiar with the following phrase: *Garbage in, Garbage out.* The phrase is referring to the users ability to supply accurate inputs such as muzzle velocity, range, BC, wind, etc. Although internal to the ballistic solver, the drag model is sort of like an input. If you input a G1 or G7 BC, the properly written ballistic solver is scaling and applying the G1 or G7 standard drag curve inside the solver according to your BC input. Any mismatch between your bullets drag curve and the G1 and G7 curves will manifest as subtle error in trajectory prediction at extended range. However if you're using a carefully measured custom drag model to represent the drag curve for your bullet, then that's the best you can do.

The combination of: modern computers, ballistics software, and an extensive library of custom drag models based on live fire have enabled an unprecedented level of accuracy in long range trajectory prediction.

Part 3: Application of Custom Drag Models

A concept that many long range shooters are familiar with is *truing* or *calibrating* the ballistic solver. This is basically the process of firing shots at long range, telling the ballistic computer where you hit, so it can self-correct itself. This process is necessary in some applications where shooters may not have good information on their bullets muzzle velocity or BC. Muzzle Velocity (MV) uncertainty will always be an issue in such applications and so robust calibration process are important, and it's possible to determine MV with a good deal of certainty based on observed drop. However shooting to determine BC or drag is a very different thing which is much more difficult to do accurately. See the technical appendix for more details on the uncertainties involved in measuring drag with drop observations vs. measuring velocity decay or time of flight.

As a user of ballistic software, it's important to understand the distinction in the various types of ballistic solvers. Just like G7 BC's and CDM's are only useful if the data exists, those things also require compatible software to properly use that data in a bullet fly-out simulation. The Point Mass (PM) class of ballistic solvers has been the modern standard for trajectory computation since the 1950's when computers became powerful enough to crunch the numbers **[REF 2]**. ***Only point mass solvers are capable of modeling the CDM's that have been measured for various bullets***. There are different classes of ballistic solvers (non-Point Mass) which solve the math in ways that prevent them from working with a live fire derived CDM's. For example, all solvers based on the Pejsa method and similar approaches use mathematical functions to approximate the shape of drag curves. Using these mathematical functions, it's not possible to model the true drag of the bullet as it was measured and represented in the CDM. A few modern solvers use these methods because they're easier to program, but there is no live fire database of BC's or CDM's that is technically compatible with non-Point Mass solvers.

So how accurately can a ballistic solver using CDM's predict trajectories at extended ranges? The following tables summarize some carefully collected data that was fired at extended range, deep into transonic where trajectory predictions typically fall apart. A short barreled 308 Winchester firing 175 grain bullets was used to engage targets out to 1323 yards, which is deep into the transonic range for that bullet. Table 1 shows the observed drop compared to the drop predicted by a PM Solver using a CDM. Note that all of the predicted data in Table 1 is *un-calibrated/un-trued* meaning the MV was taken from a chronograph prior to the test, and not adjusted afterwards to match up with observed points.

Sierra .30 caliber 175 MatchKing					
		Actual	Custom Curve - No Truing		
Range	Vel/Mach	Drop MILS	Prediction	Error MILS	Error Inches
300	2074/1.88	-1.1	-1.09	-0.01	-0.1
600	1627/1.47	-4.3	-4.36	0.06	1.3
1000	1135/1.03	-11.2	-11.36	0.16	5.8
1101	1058/0.96	-13.7	-13.80	0.10	4.0
1166	1025/0.92	-15.5	-15.38	-0.12	-5.0
1200	1010/0.92	-16.6	-16.55	-0.05	-2.2
1323	967/0.87	-19.9	-20.08	0.18	8.6

Table 1. Actual vs. Predicted drop for the 175 grain Sierra MatchKing thru transonic speed.

Table 1 shows the actual vs. predicted drop for the .30 caliber 175 grain Sierra MatchKing, fired at an average muzzle velocity of 2570 fps. The observed drop is based on what was required to center the group on a steel target, so there is some minor uncertainty in the *observed* data, maybe +/- 1 click (0.1 MIL). Note that the velocities and Mach numbers shown in red are indicating transonic range, where the bullet has slowed below Mach 1.2, or about 1340 fps. This is the range that's most difficult to predict drop due to the mismatch in drag curves between the standard G1/G7 and the projectiles actual CDM. Using the CDM to model the bullets actual flight path results in predictions that are within +/- 9" all the way to 1323 yards which is Mach 0.87 for this bullet.

Berger .30 caliber 175 OTM Tactical					
		Actual	Custom Curve - No Truing		
Range	Vel/Mach	Drop MILS	Prediction	Error MILS	Error Inches
300	2088/1.89	-1.1	-0.94	-0.16	-1.7
600	1660/1.50	-4.0	-4.08	0.08	1.7
700	1531/1.39	-5.6	-5.47	-0.13	-3.3
1000	1182/1.07	-10.7	-10.66	-0.04	-1.4
1101	1088/0.99	-12.8	-12.93	0.13	5.2
1166	1059/0.95	-14.2	-14.43	0.23	9.7
1200	1032/0.94	-15.4	-15.45	0.05	2.2
1323	993/0.90	-18.6	-18.76	0.16	7.6

Table 2. Actual vs. Predicted drop for the 175 grain Berger OTM Tactical thru transonic speed.

Table 2 shows the same data for the Berger .30 caliber 175 grain OTM Tactical bullet. Again you can see the CDM prediction matches the observed drop within +/- 10" for the full trajectory which includes deep transonic flight.

Remember that the tables above are showing the UN-TRUED raw predictions from the Point Mass solver and CDM's. In other words; nothing was tweaked to bring these predictions into alignment with the observed drop. This is a live fire demonstration of the first shot accuracy that's possible with the Point Mass solver and CDM's.

You can read about another demonstration of the raw accuracy of the Applied Ballistics solver and CDM's here: http://www.longrangeonly.com/forum/showthread.php?529-Field-Test-of-Ballistic-Apps-Cold-Bore-1-0-Applied-Ballastics-amp-others&highlight=applied+ballistics

In this carefully done test, Jeff Brozovich presents his live fire results which are summarized in Table 3 below. As with the previous test cases, *all of this shooting was done with an un-trued solution*, meaning the raw, first shot accuracy of the Applied Ballistics solver and CDM's.

Range Yards	AB – Custom Curve Prediction	Error
800	11.4 MOA	-0.1"
1200	21.9 MOA	+4.0"
1600	35.1 MOA	-1.8"
1773	42.2 MOA	-1.8"

Table 3. Long range test showing excellent agreement between predicted and actual drop out to a mile.

The rifle was a .338 caliber wildcat that fires 300 grain Berger Hybrids at 3198 fps! So even at 1773 yards, the bullet was still supersonic under the test conditions so this demonstration was fully supersonic yet the error was kept to within 4" to beyond a mile. It simply doesn't get much better than that for first round, predictive accuracy.

The preceding examples of a modern Point Mass solver and CDM's are typical of what you can expect when using these tools in the field. The hardest part is getting good field data into the solver such as MV, range and making sure your scope is dialing accurately.

Conclusion

The barriers to truly accurate ballistic modeling have been lack of data, and the hardware/software to run it. These barriers have been overcome by conducting the careful live fire testing necessary to establish the CDM's for over 500 bullets commonly used for long range shooting. Furthermore, there are numerous software tools and apps available now which give you access to the state of the art in drag modeling and trajectory prediction. If you're the average long range shooter who stays within supersonic ranges, then accurate G7 BC's are enough to keep you on target. If you routinely shoot into transonic ranges and need your first shots on target, look for a Point Mass solver capable of running the highly accurate CDM's that are measured with live fire.

Technical Appendix

Dynamic pressure

The equation for dynamic pressure (pounds per square foot) is:

$$q = 1/2\rho V^2$$

Where: ρ (greek letter rho) is the air density (slugs per cubic foot; standard value is 0.002377 sl/ft^3)

V is the velocity of the bullet (feet per second)

Bullet Frontal Area

The equation for a bullets frontal area (square feet) is:

$$S = \pi\left(\frac{cal}{24}\right)^2$$

Where: cal is the bullet caliber (inches)

Aerodynamic Drag

The equation for aerodynamic drag on a bullet (pounds) is:

$$drag = qSCd$$

Where: q is the dynamic pressure (pounds per square foot)

S is the bullets frontal area (square feet)

Cd is the bullets drag coefficient (unitless)

Drag Coefficient

The drag coefficient of a bullet can be approximated within +/- 10% using predictive methods, but *can only be known accurately if measured by live fire*. By shooting many shots at various speeds (Mach numbers), a *drag curve* is established. This drag curve is used to determine the aerodynamic drag on a bullet at any speed, which is used to determine the deceleration of the bullet, it's time of flight, drop and every other metric of a ballistic trajectory. Drag coefficients and drag curves are difficult to measure and require carefully instrumented live fire testing to determine with accuracy.

As stated by NASA **[REF 5]:** "The drag coefficient then expresses the ratio of the drag force to the force produced by the dynamic pressure times the area."

Live Fire Testing to determine Custom Drag Models (CDM's)

The collection of data to determine Mach vs. CD points is pretty basic in principle, but the devil, as they say, is in the details.

The basic task is to measure the velocity of a bullet at two points to determine how much velocity was lost over a carefully measured distance. The loss in velocity, along with the atmospheric density and the bullets mass, caliber, and atmospheric density combine to determine the drag coefficient according to the math given at the end of this section.

Measuring Mach-CD points is all about measurement uncertainty. How accurately are your measurements of: distance, start and stop velocity, and atmospherics. At the Applied Ballistics lab, atmospherics are measured with a NIST traceable weather station, verified by numerous other NIST traceable weather meters (Kestrels). Velocity is measured using Oehler Chronographs that are tested prior to each test firing to insure their reading the same speed for a number of shots fired thru both chronographs. The chronographs are verified to match to within +/- 1 fps. When set up in a test, the spacing between the chronograph placements was determined by survey equipment that is accurate to within 0.1" over the entire range.

Given the uncertainty of all the experimental measurements, determination of Mach-CD points is accurate to within +/- 1% for any given shot, and much less than that for the average of a cluster of shots. Furthermore, results are repeatable within +/- 1% on any given day.

The other method used to determine Mach-CD points is to measure muzzle velocity and time of flight to distant ranges. Both the time of flight and velocity decay methods produce the same results within +/- 1% which adds to the confidence that both methods are accurate.

In addition to managing experimental uncertainty, there's also the issue of matching stability conditions for the low speed shots. Firing a bullet at reduced charges to measure Mach-CD points over short range can produce different results from loading the rounds to full speed and letting if fly all the way out to transonic ranges *unless stability conditions are matches.* When conducting live fire testing to measure Mach-CD points, it's important to fire the low velocity shots in a barrel twist that's faster than a conventional twist barrel to match the stability conditions of a real long range shot. Extensive live fire testing was completed and some of it published in *"Modern Advancements in Long Range Shooting – Volume 1"* **[REF 4]** to determine the spin rate decay of bullets on long range trajectories. These realities of spin rate decay are accounted for in the measurement of Mach-CD points by selecting a barrel with the proper twist based on the Mach number being tested. Failure to match stability conditions for each Mach number would result in measurements that are not representative of actual bullet flight.

On the subject of measuring custom drag models, some shooters think this is what they're doing when they shoot their bullets for drop and segment a BC in their ballistics program.

Shooing for drop and defining a piecewise BC is not the same thing as measuring Mach-CD points to create a custom drag model.

The biggest problem with this approach is uncertainty. Observations of drop are influenced by so many other things that it's very difficult to resolve actual drag with any kind of practical accuracy. Scope tracking, wind, shot dispersion and variables related to the shooter and gun handling make up a long list of error sources that are not an issue when measuring raw velocity decay or time of flight. For example, suppose you fire a .30 caliber 175 grain bullet at 2650 fps to the extent of its supersonic range (about 800 yards) and measure the drop to be 7.6 MILS. Such an experiment easily has +/- 0.2 MILS (2 clicks) of error based on the conditions and uncertainty budget. In this example, the 2 clicks of error translates to at least +/- 5% of error in the average drag from the muzzle to transonic range, and this is assuming perfectly accurate MV and range information which usually isn't available in most drop tests. If the shooter were to then fire at another target at 1000 or 1200 yards to get a second point of drop, the accuracy suffers even more because the separation between the points is much less than the first target, and any error from the first point is compounded when a second point is fired. In this example, you could theoretically calculate 2 Mach CD points that would have way more than 5% error. By contrast, a properly conducted velocity decay or time of flight test produces dozens of points all having less than 1% error. For these reasons, drop testing to determine Mach-CD points and custom curves is unrealistic.

Drop tests conducted over shorter ranges are even more highly subject to error due to the magnitude of drop being less. In general, if you have a good CDM or BC for your bullet, you can accurately determine MV by shooting to transonic range and observing drop if you're careful. However, the idea of measuring many points of drag by observing drop in supersonic range is highly subject to error, and is not advisable anywhere that accuracy is a concern.

Once the raw velocity decay or time of flight data is collected, the calculation of Mach-CD points is pretty straightforward. The following match shows how to compute Mach-CD points from live fire velocity decay data.

You begin by calculating:

$$K = \frac{\ln(v_1/v_2)}{x}$$

Where:

v_1= muzzle velocity (feet per second)
v_2= downrange velocity (feet per second)
x= distance between the two velocity measurements (feet)

Furthermore:

$$c = \frac{m}{\pi r^2}$$

Where:

m= bullet mass (slugs)
r = radius of bullet (feet)

And finally:

$$Cd = \frac{2Kc}{\rho}$$

Where:

ρ (greek letter rho) is the air density (slugs per cubic foot; standard value is 0.002377 sl/ft^3)

This gives you the drag coefficient, now to get the Mach number for this point, you simply calculate the average velocity of the shot (start velocity plus end velocity divided by 2). Then divide the average velocity by the speed of sound to get the Mach number. You have now calculated a drag coefficient for a given Mach number. All such points over a range of Mach numbers combine to form a *drag curve*. A drag curve created in such a way is accurate to within +/- 1% which is enough to predict a bullets trajectory to within 1 click all the way thru transonic ranges. Of course, other uncertainties of the shooting environment such as muzzle velocity and scope tracking often prevent the cumulative accuracy from being this good, but a good CDM is good enough to support this level of accuracy if other variables are known with equally high degrees of certainty.

References

[REF 1] Bryan Litz, "Applied Ballistics for Long Range Shooting – 3rd Edition", 2015 Applied Ballistics, LLC

[REF 2] Robert L. McCoy, "Modern Exterior Ballistics", 1999 Schiffer Publishing, LTD

[REF 3] Jochem Peelen, "A Look at Drag Models in Old Small Arms Firing Tables", 2006 Jochem Peelen, München

[REF 4] Bryan Litz, "Modern Advancements in Long Range Shooting – Volume 1", 2014 Applied Ballistics, LLC

[REF 5] Glenn Research Center, National Aeronautics and Space Administration (NASA), https://www.grc.nasa.gov/www/k-12/airplane/dragco.html