Analyzing the BrowserID SSO System with Primary Identity Providers Using an Expressive Model of the Web

Daniel Fett, Ralf Küsters, and Guido Schmitz

University of Trier, Germany {fett,kuesters,schmitzg}@uni-trier.de

Abstract. BrowserID is a complex, real-world Single Sign-On (SSO) System for web applications recently developed by Mozilla. This open source system employs new HTML5 features (such as web messaging and web storage) and cryptographic assertions to provide decentralized and federated login, with the intent to respect users' privacy. It can operate in two modes, the primary identity provider mode and the secondary identity provider mode. While in the primary mode BrowserID runs with arbitrary identity providers, in the secondary mode there is one identity provider only, namely Mozilla's default identity provider.

In prior work, we have proposed an expressive general model for the web infrastructure and based on this model, we analyzed the security of the secondary identity provider mode of BrowserID. The analysis revealed several severe vulnerabilities.

In this paper, we complement our prior work by analyzing the even more complex primary identity provider mode of BrowserID. During the proof, we discovered a new practical and interesting identity injection attack, which violates a central security property of SSO systems. This attack cannot be carried out in the secondary mode. We propose a fix and prove that the fixed system satisfies all security requirements we consider.

The security analysis performed in this paper constitutes the most complex formal analysis of a web application based on an expressive model of the web infrastructure. The model that we employ is in fact the most comprehensive such model.

Another contribution of this work, besides the analysis of BrowserID, is that we identify and prove important security properties of generic web features in our model. These properties will facilitate future analysis efforts of web standards and web applications using the model.

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1 Introduction

Single sign-on (SSO) systems have become an important building block for authentication in the web. Over the last years, many different SSO systems have been developed, for example, OpenID, OAuth, and proprietary solutions such as Facebook Connect. These systems usually allow a user to identify herself to a so-called relying party, which provides some service, using an identity that is managed by an identity provider, such as Facebook or Google.

Given their role as brokers between identity providers and relying parties, the security of SSO systems is particularly crucial: Numerous attacks have shown that vulnerabilities in SSO systems usually compromise the security of many services (relying parties) and users at once (see, e.g., [2, 5, 19-22]).

BrowserID [17] is a relatively new complex SSO system which allows users to utilize any of their existing email addresses as an identity. BrowserID, which is also known by its marketing name *Persona*, was initiated by Mozilla and is an actively developed open source project, providing decentralized and federated login, with the intent to respect users' privacy. Several web applications support BrowserID authentication. For example, popular content management systems, such as Drupal and WordPress allow users to log in using BrowserID. Also Mozilla uses this SSO system on critical web sites, e.g., their bug tracker Bugzilla and their developer network MDN.

BrowserID is based solely on native web technologies. It uses many new HTML5 web features, such as web messaging and web storage. For example, BrowserID uses the postMessage mechanism for cross-origin inter-frame communication (i.e., communication within a browser between different windows) and the web storage concept of modern browsers to store user data on the client side.

There are two modes for BrowserID: For the best user experience, email providers can actively support BrowserID (they are then called primary identity providers). For all other email providers that do not support BrowserID yet, the user can register her email address at a default identity provider, namely Mozilla's login.persona.org, the so-called secondary identity provider.

In prior work [10], we have proposed a general and expressive Dolev-Yao style model for the web infrastructure and based on this model, we analyzed the security of the secondary identity provider mode of BrowserID. The analysis revealed several severe vulnerabilities. The web model proposed in [10] is designed to closely mimic standards and specifications for the web. It is the most comprehensive one to date. Among others, HTTP(S) requests and responses, including several headers, such as host, cookie, location, strict-transport-security (STS), and origin headers, are modeled. The model of web browsers captures the concepts of windows, documents, and iframes, including the complex navigation rules, as well as new technologies, such as web storage and cross-document messaging. JavaScript is modeled in an abstract way by so-called scripting processes. These processes can be sent around and, among others, they can create iframes and initiate XMLHTTPRequests (XHRs). Browsers may be dynamically corrupted by the attacker.

Contribution of this Paper. In this paper, we first slightly extend the web model proposed in [10]. We complement our modeling of the web storage concept of modern browsers by including both localStorage and sessionStorage [23]. So far, only localStorage has been supported in the model. These mechanisms provide a convenient way for JavaScript to store user data in the browser. (Both kinds of storage are used by BrowserID in its primary mode.) While data in the localStorage is stored per origin, data in the sessionStorage is stored per origin and (top-level) window. We also extend the model to include a set of user identities (e.g., user names or email addresses) in addition to user secrets.

We then complement the security analysis of BrowserID by analyzing the *primary* identity provider mode of BrowserID. This mode is even more complex than the secondary mode (see also the remarks in Section 4.2). It involves more components (such as an arbitrary set of identity providers, more iframes), a much more complex communication structure, and requires weaker trust assumptions (for example, some identity providers, and hence, the JavaScript they deliver, might be malicious).

While carrying out the security proof for the primary mode of BrowserID, we discovered a new attack, which violates a central security property of SSO systems. The attack is an identity injection attack which allows the adversary to log in the browser of an honest user at a relying party as the attacker. Depending on the service provided by the relying party, this could allow the attacker to track the honest user or to obtain user secrets. We confirmed the attack on the actual implementation and reported it to Mozilla [7]. We note that this attack does not apply to the secondary mode of BrowserID.

We have proposed a fix and proved that the fixed system satisfies central security properties one would expect an SSO system to have.

The analysis we have performed is interesting by itself as it considers a relevant real-world and complex web application, finds a practical attack, and establishes security properties for the fixed system. Moreover, so far there have been only very few efforts for the formal analysis of web application based on a formal model of the web infrastructure, with the work presented here certainly constituting the most complex analysis based on an expressive web model (see Section 6). Our web model is in fact the only one to date that can be applied to BrowserID, given the many and diverse web features and technologies that BrowserID employs.

The security analysis of BrowserID carried out here and the case study done in [10] showed that certain security properties of the web model need to be established in most security proofs for web standards and web applications. As another contribution of this paper, we therefore identify and summarize central security properties of generic web features in our model and formalize them in a general way such that they can be used in and facilitate future analysis efforts of web standards and web applications.

Structure of this Paper In Section 2, we present the basic communication model and the web model, including the extensions mentioned above. We deduce general properties of this model, which are independent of specific web applications, in Section 3. For our security analysis, we first, in Section 4, provide a description of the BrowserID system, focusing on the primary mode. We then, in Section 5, present the analysis of BrowserID in primary mode using our model. Related work is discussed in Section 6. We conclude in Section 7. Full details can be found in the appendix.

2 The Web Model

In this section, we present the model of the web infrastructure as proposed in our previous work [10, 11], along with the extensions mentioned in the introduction. Full details are provided in Appendices A to C. We first present the generic Dolev-Yao style communication model which the model is based on.

2.1 Communication Model

The main entities in the communication model are *atomic processes*, which are used to model web browsers, web servers, DNS servers as well as web and network attackers. Each atomic process has a list of addresses (representing IP addresses) it listens to. A set of atomic processes forms what we call a *system*. The different atomic processes in such a system can communicate via events, which consist of a message as well as a receiver and a sender address. In every step of a run, one event is chosen non-deterministically from the current "pool" of events and is delivered to an atomic process that listens to the receiver address of that event; if different atomic processes can listen to the same address, the atomic process to which the event is delivered is chosen non-deterministically among the possible processes. The (chosen) atomic process can then process the event and output new events, which are added to the pool of events, and so on. More specifically, messages, processes, etc. are defined as outlined in the following.

Terms, Messages and Events. As usual in Dolev-Yao models, messages are expressed as formal terms over a signature. Based on this notion of messages, we introduce events. In our setting, messages may, for instance, represent HTTP(S) requests and responses.

The signature Σ for the terms and messages considered in this work is the union of the following pairwise disjoint sets of function symbols: (1) constants $C = \mathsf{IPs} \cup \mathbb{S} \cup \{\top, \bot, \Diamond\}$ (IPs for (IP) addresses and \mathbb{S} for ASCII strings) where the three sets are pairwise disjoint, (2) function symbols for public keys, asymmetric/symmetric encryption/decryption, and digital signatures: $\mathsf{pub}(\cdot)$, $\mathsf{enc}_a(\cdot, \cdot)$, $\mathsf{dec}_a(\cdot, \cdot)$, $\mathsf{enc}_s(\cdot, \cdot)$, $\mathsf{sig}(\cdot, \cdot)$, $\mathsf{checksig}(\cdot, \cdot)$, $\mathsf{extractmsg}(\cdot)$, (3) *n*-ary sequences $\langle \rangle, \langle \cdot \rangle, \langle \cdot, \cdot \rangle, \langle \cdot, \cdot, \rangle$, etc., and (4) projection symbols $\pi_i(\cdot)$ for all $i \in \mathbb{N}$. The equational theory associated with the signature Σ is defined as usual. For instance, we have that $\mathsf{dec}_a(\mathsf{enc}_a(x,\mathsf{pub}(y)), y) = x$. Ground terms over this signature are terms that do not contain variables. These terms represent messages. By \mathcal{M} we denote the set of messages. An event (over IPs and \mathcal{M}) is of the form (a:f:m), for $a, f \in \mathsf{IPs}$ and $m \in \mathcal{M}$, where a is interpreted to be the receiver address and f is the sender address.

Atomic Processes, Systems and Runs. We now define atomic Dolev-Yao processes, systems, and runs of systems.

A (generic) atomic process is a tuple $p = (I^p, Z^p, R^p, s_0^p)$ where I^p is a set of addresses (the set of address the process listens to), Z^p is a set of states, $s_0^p \in Z^p$ is an initial state, and R^p is a relation that takes an event and a state as input and (non-deterministically) returns a new state and a set of events. This relation models a non-deterministic computation step of the process, which upon receiving an event in a given state non-deterministically moves to a new state and outputs a set of messages (events).

For our web model, we consider specialized atomic processes only, called *atomic Dolev-Yao (DY) processes*. For these processes, we require that the events and states that they output can be computed (more formally, derived in the usual Dolev-Yao style) from the current input event and state. In the rest of this paper, we will consider DY processes only.

We also consider a special *attacker process*, which records all messages it receives and outputs all messages it can possibly derive from its recorded messages. Hence, an attacker process is the maximally powerful DY process. It carries out all attacks any DY process could possibly perform. The attacker process is parametrized by the set of sender addresses it may use.

A system is a (possibly infinite) set of atomic processes. Its state (i.e., the states of all contained atomic processes) together with a multi-set of waiting events is called a *configuration*.

A run of a system for an initial set E_0 of events is a sequence of configurations, where each configuration (except for the first one, which consists of E_0 and the initial states of the atomic processes) is obtained by delivering one of the waiting events of the prior configuration to an atomic process (which listens to the receiver address of the event), say p, and which in turn performs a computation step according to its relation R^p .

2.2 Scripting Processes

For our web model, we also define scripting processes, which model client-side scripting technologies, such as JavaScript.

A scripting process (or simply, a script) is a relation of a certain form. It is called by the browser which provides it with a (fresh, infinite) set N of nonces and state information s. The script then outputs a term s', which represents the new internal state and some command which is interpreted by the browser (see Section 2.5 for details). Again, we require that a script's output s' is derivable from its input (s, N).

Similarly to an attacker process, we define the *attacker script* R^{att} . This script outputs everything that is derivable from the input.

2.3 Web System

In our web model, we formalize the web infrastructure and web applications by what we call a web system. A web system, among others, contains a (possibly infinite) set of DY processes, which model web browsers, web servers, DNS servers, and attackers. As we will see, attackers may corrupt other entities, such as browsers.

The model has been carefully designed to closely follow published (de-facto) standards, for instance, the HTTP/1.1 standard, associated (proposed) standards (mainly RFCs), and the HTML5 W3C candidate recommendation.

Web System. A web system $WS = (W, S, script, E_0)$ is a tuple with its components defined as follows:

The first component, \mathcal{W} , denotes a system (a set of DY processes) and is partitioned into the sets Hon, Web, and Net of honest, web attacker, and network attacker processes, respectively. In this paper, we do not consider web attackers, i.e., Web = \emptyset , and one network attacker only, i.e., Net contains one atomic process, namely the attacker process, which may listen to and spoof all addresses (and may corrupt other entities). Hence, this attacker is the maximally powerful attacker. Honest processes (contained in Hon) can be either web servers, web browsers, or DNS servers. In this paper, DNS servers will be subsumed by the network attacker, and hence, we do not need to model DNS servers. The modeling of a *web server* heavily depends on the specific web application under consideration. Our concrete models for the web servers of the BrowserID system are provided in Sections 4 and following. Below, we present the modeling of web browsers, which is independent of a specific web application (with full details provided in Appendices B and C).

The second component, S, is a finite set of scripts, which include the attacker script $R^{\text{att}} \in S$. In a concrete model of a web application, the set $S \setminus \{R^{\text{att}}\}$ typically describes the set of honest scripts used in the considered application. Malicious scripts are modeled by the "worst-case" malicious script, R^{att} .

The third component, script, is an injective mapping from S to S, i.e., by script every $s \in S$ is assigned its string representation script(s). Finally, E_0 is a multiset of events, containing an infinite number of events of the form (*a:a:TRIGGER*) for every process *a* in the web system. A *run* of WS is a run of W initiated by E_0 .

2.4 HTTP Messages

Before we present the model of web browsers, we model HTTP requests and responses. HTTP requests and responses are represented as ground terms that contain a nonce, a method (e.g., GET or POST), a domain name, a path, URL parameters, request headers (such as Cookie or Origin), and a message body. For example, an HTTP GET request for the URL http://example.com/show?p=1 is modeled as the term $r := \langle \text{HTTPReq}, n_1, \text{GET}, \text{example.com}, /\text{show}, \langle \langle p, 1 \rangle \rangle, \langle \rangle, \langle \rangle \rangle$, where headers and body are empty. An HTTP response contains a nonce (matching the one in the request), a status code, response headers, and a body. A response to r could be $s := \langle \text{HTTPResp}, n_1, 200, \langle \langle \text{Set-Cookie}, \langle \text{SID}, \langle n_2, \bot, \top, \bot \rangle \rangle \rangle$, $\langle \text{script1}, initst \rangle$, where $\langle \text{SID}, \langle n_2, \bot, \top, \bot \rangle \rangle$ is a cookie with the name/value pair SID = n_2 and the attributes httpOnly, secure, session set or not set, and

(script1, initst) is the body of the response, in this case an HTML document that is to be delivered to the browser (an HTML document is modeled by the string representation of a script and its initial state, see below).

For the HTTP request and response in the above examples, the corresponding HTTPS request would be of the form $enc_a(\langle r, k' \rangle, pub(k_{example.com}))$ and the response of the form $enc_s(s, k')$ where k' is a fresh symmetric key (a nonce) which is typically generated by the sender of the request. The responder is supposed to use this key to encrypt the response.

2.5 Web Browsers

We think of an honest browser to be used by one honest user. However, we also allow browsers to be taken over by attackers. The honest user is modeled as part of the web browser model. Actions a user takes are modeled as nondeterministic actions of the web browser. For example, the web browser itself can non-deterministically follow the links provided by a web page. Secrets, such as passwords, typically provided by the user are stored in the initial state of a browser and are given to a web page when needed, similar to the AutoFill function in browsers (see below).

A web browser p is modeled as a DY process $(I^p, Z^p, R^p, s_0^p, N^p)$ where $I^p \subseteq$ IPs is a finite set of addresses p may listen to and $N^p \subseteq \mathcal{N}$ is an infinite set of nonces p may use. The set of states Z^p , the initial state s_0^p , and the relation R^p are defined next.

Browser State: Z^p and s_0^p . The set Z^p of states of a browser consists of terms of the form

(windows, ids, secrets, cookies, localStorage, sessionStorage, keyMapping, sts, DNSaddress, nonces, pendingDNS, pendingRequests, isCorrupted).

Windows and documents. The most important part of the state are windows and documents, both stored in the subterm *windows*. A browser may have several windows open at any time (resembling the tabs in a real browser). Each window contains a list of documents of which one is "active". A document represents a loaded HTML page with the current JavaScript execution state and other information (see below). Being active means that this document is currently presented to the user and is available for interaction, similarly to the definition of active documents in the HTML5 specification [12]. The document list of a window represents the history of visited web pages in that window. A window may be navigated forward and backward (modeling forward and back buttons). This deactivates one document and activates its successor or predecessor. Documents can contain iframes, which are represented as windows (subwindows) nested inside of document terms in our model. This creates a tree of windows and documents. Documents also contain (the string representation of) a script, which is meant to model the static HTML code, including, for example, links and forms, and possibly multiple JavaScript code parts. When called by the browser, a script essentially outputs a command which is then interpreted by the browser, such as following a link, creating an iframe, or issuing an XHR (see below).

Secrets and IDs. This subterm holds the secrets and the identities of the user of the web browser. Secrets (such as passwords) are modeled as nonces and they are indexed by origins (where an origin is a domain name plus the information whether the connection to this domain is via HTTP or HTTPS). Secrets are only released to documents (scripts) with the corresponding origin, similarly to the AutoFill mechanism in browsers. Identities are arbitrary terms that model public information of the user's identity, such as email addresses. Identities are released to any origin. As mentioned in the introduction, identities were not considered in [10].

Cookies, localStorage, and sessionStorage. These subterms contain the cookies (indexed by domains), localStorage data (indexed by origins), and sessionStorage data (indexed by origins and top-level window references) stored in the browser. As mentioned in the introduction, sessionStorage were not modeled in [10].

KeyMapping. This term is our equivalent to a certificate authority (CA) certificate store in the browser. Since, for simplicity, we currently do not formalize CAs in the model, this term simply encodes a mapping assigning domains $d \in \mathsf{Doms}$ to their respective public keys $\mathsf{pub}(k_d)$.

STS. Domains that are listed in this term are contacted by the web browser only over HTTPS. Connection attempts over HTTP are transparently rewritten to HTTPS requests. Web sites can issue the Strict-Transport-Security header to clients in order to add their domain to this list.

DNSaddress. This term contains the address of the DY process that is to be contacted for DNS requests; typically a DNS server.

Nonces, pendingDNS, and pendingRequests. These terms are used for bookkeeping purposes, recording the nonces that have been used by the browser so far, the HTTP requests that await successful DNS resolution, and HTTP requests that await a response, respectively.

Is Corrupted. This term indicates whether the browser is corrupted $(\neq \perp)$ or not $(= \perp)$. A corrupted browser behaves like a web attacker.

Initial state s_0^p of a web browser. In the browser's initial state, keyMapping, DNSAddress, secrets, and ids are defined as needed, isCorrupted is set to \perp , and all other subterms are $\langle \rangle$.

Web Browser Relation R^p . Figure 1 outlines how the web browser processes incoming messages. In the first two lines, special messages that cause the browser to become corrupted are processed. If the browser becomes corrupted, it essentially acts like the attacker process. There are two types of corruption: If the browser gets fully corrupted, the attacker learns the entire current state of the browser. If it gets close-corrupted, first any information about open windows, documents and used nonces (i.e., HTTPS encryption keys) is discarded from the browser's state before it is handed over to the attacker, modeling that Processing Input Message mm = FULLCORRUPT: isCorrupted := FULLCORRUPTm = CLOSECORRUPT: is Corrupted := CLOSECORRUPT $m = \text{TRIGGER: non-det. choose action from } \{1, 2\}$ action = 1: Call script of some active document. Outputs new state and command *cmd*. $cmd = \text{HREF:} \rightarrow Initiate \ request$ $cmd = \text{IFRAME: Create subwindow,} \rightarrow Initiate request$ $cmd = FORM: \rightarrow Initiate \ request$ cmd = SETSCRIPT: Change script in given document. cmd = SETSCRIPTSTATE: Change state of script in given document. $cmd = \texttt{XMLHTTPREQUEST:} \rightarrow Initiate \ request$ cmd = BACK or FORWARD: Navigate given window. cmd = CLOSE: Close given window. cmd = POSTMESSAGE: Send postMessage to specified document. $action = 2: \rightarrow Initiate request to some URL in new$ window m = DNS response: send corresponding HTTP request m = HTTP(S) response: (decrypt,) find reference. reference to window: create document in window reference to document: add response body to document's script input

Fig. 1. The basic structure of the web browser relation R^p with an extract of the most important processing steps, in the case that the browser is not already corrupted.

a user closed the browser, but a malicious user now uses the browser (and all information left in the browser's state).

The browser can receive special trigger messages, which model non-deterministic actions of the browser or user. One of these actions is to select one of the current documents, trigger its JavaScript and evaluate the output of the script. Scripts can change the state of the browser (e.g., by setting cookies) and can trigger specific actions (e.g., following a link), which are modeled as *commands* issued by the script.

Scripts or non-deterministic triggering of browsers can cause new HTTP(S) requests to be issued. In this case, the browser always first sends a DNS request to the configured DNS resolver, asking for the IP address that belongs to the domain name in the HTTP(S) request. As soon as the DNS response arrives, the browser sends the HTTP(S) request to the IP address in the DNS response.

If the HTTP(S) response arrives, its headers are evaluated and the body of the request becomes the script of a newly created document that is then inserted at an appropriate place in the window/document tree. However, if the HTTP(S) response is a response to an XMLHTTPRequest (triggered by a script in a document), the body of the response is added to the corresponding document and can later be processed by the script of that document.

3 General Security Properties

As mentioned in the introduction, as one of the contributions of this work, we have identified central application independent security properties of web features in our model and have formalized them in a general way such that they can be used in and facilitate future analysis efforts of web standards and web applications. In this section, we provide a brief overview of these properties, with precise formulations and proofs presented in Appendix D.

The first set of properties concerns encrypted connections (HTTPS): We show that HTTP requests that were encrypted by an honest browser for an honest receiver cannot be read or altered by the attacker (or any other party). This, in particular, implies correct behavior on the browser's side, i.e., that browsers that are not fully corrupted never leak a symmetric key used for an HTTPS connection to any other party. We also show that honest browsers set the host header in their requests properly (i.e., the header reflects an actual domain name of the receiver) and that only the designated receiver can (successfully) respond to HTTPS requests.

The second set of properties concerns origins and origin headers. Using the properties stated above, we show that browsers cannot be fooled about the origin of an (HTTPS) document in their state: If the origin of a document in the browser's state is a secure origin (HTTPS), then the document was actually sent by that origin. Moreover, for requests which contain an origin header with a secure origin we prove that such requests were actually initated by a script that was sent by that origin to the browser. In other words, in this case, the origin header works as expected.

4 The BrowserID System

BrowserID [18] is a decentralized single sign-on (SSO) system developed by Mozilla for user authentication on web sites. It is a complex full-fledged web application deployed in practice, with currently ~47k LOC (excluding some libraries). It allows web sites to delegate user authentication to email providers, where users use their email addresses as identities. The BrowserID implementation makes use of a broad variety of browser features, such as XHRs, postMessage, local- and sessionStorage, cookies, etc.

Following [10], we first, in Section 4.1, provide a high-level overview of the BrowserID system. A more detailed description of the BrowserID implementation is then given in Section 4.2.

The description of the BrowserID system presented in the following as well as our BrowserID model (see Section 5.1) is extracted mainly from the BrowserID source code [16] and the (very high-level) official BrowserID documentation [18].

4.1 Overview

The BrowserID system knows three distinct parties: the user, who wants to authenticate herself using a browser, the relying party (RP) to which the user



Fig. 2. BrowserID authentication: basic overview

wants to authenticate (log in) with one of her email addresses (say, user@eyedee.me), and the identity/email address provider, the IdP. If the IdP (eyedee.me) supports BrowserID directly, it is called a *primary IdP*. Otherwise, a Mozilla-provided service, the so-called *secondary IdP*, takes the role of the IdP. As mentioned before, here we concentrate on the primary IdP mode. However, we briefly discuss the differences between the two modes at the end of Section 4.2.

A primary IdP provides information about its BrowserID setup in a so-called *support document*, which it provides at a fixed URL derivable from the email domain, e.g., https://eyedee.me/.well-known/browserid.

A user who wants to log in at an RP with an email address for some IdP has to present two signed documents to the RP: A user certificate (UC) and an identity assertion (IA). The UC contains the user's email address and the user's public key. It is signed by the IdP. The IA contains the origin of the RP and is signed with the private key corresponding to the user's public key. Both documents have a limited validity period. A pair consisting of a UC and a matching IA is called a certificate assertion pair (CAP) or a backed identity assertion. Intuitively, the UC in the CAP tells the RP that (the IdP certified that) the owner of the email address is (or at least claims to be) the owner of the public key. By the IA contained in the CAP the RP is ensured that the owner of the given public key (i.e., the one who knows the corresponding private key) wants to log in. Altogether, given a valid CAP, RP would consider the user (with the email address mentioned in the CAP) to be logged in.

The BrowserID authentication process (with a primary IdP) consists of three phases (see Figure 2 for an overview): $\boxed{1}$ provisioning of the UC, $\boxed{1}$ CAP creation, and $\boxed{11}$ verification of the CAP.

In Phase $\boxed{1}$, the (browser of the) user creates a public/private key pair $\boxed{\mathbb{A}}$. She then sends her public key as well as the email address she wants to use to log

in at some RP to the respective IdP \mathbb{B} . The IdP now creates the UC \mathbb{C} , which is then sent to the user \mathbb{D} . The above requires the user to be logged in at IdP.

With the user having received the UC, Phase \square can start. The user wants to authenticate to an RP, so she creates the IA \blacksquare . The UC and the IA are concatenated to a CAP, which is then sent to the RP \blacksquare .

In Phase \square , the RP checks the authenticity of the CAP. For this purpose, the RP could use an external verification service provided by Mozilla or check the CAP itself as follows: First, the RP fetches the public key of the IdP \square , which is contained in the support document. Afterwards, the RP checks the signatures of the UC and the IA \square . If this check is successful, the RP can, as mentioned before, consider the user to be logged in with the given email address and send her some token (e.g., a session ID), which we refer to as an *RP service token*.

4.2 Implementation Details

We now provide a more detailed description of the BrowserID implementation. Since the system is very complex, with many HTTPS requests, XHRs, and postMessages sent between different entities (servers as well as windows and iframes within the browser), we here describe mainly the phases of the login process without explaining every single message exchange done in the implementation. A more detailed step-by-step description can be found in Appendix E. Note that BrowserID's specification of IdPs fixes the interface to BrowserID only, but otherwise does not further detail the specification of IdPs. Therefore, in what follows, we consider a typical IdP, such as the example implementation provided by Mozilla [16].

An overview of the BrowserID implementation is given in Figure 3. For brevity of presentation, several messages and components, such as the CIF (see below), are omitted in the figure.

In addition to the parties mentioned in the overview in Section 4.1, the actual implementation uses another party, Mozilla's login.persona.org (LPO). Among others, LPO provides the HTML and JavaScript files of the implementation core.

Windows and iframes in the Browser. By RP-Doc (see Figure 3) we denote the window containing the document loaded from some RP, at which the user wants to log in with an email address of some IdP. This document typically includes JavaScript from LPO and contains a button "Login with BrowserID". The LPO JavaScript running in *RP-Doc* opens an auxiliary window called the *login dialog* (LD). Its content is provided by LPO and it handles the interaction with the user. During the login process, a temporary invisible iframe called the *provisioning iframe* (PIF) can be created in the LD. The PIF is loaded from IdP. It is used by LD to communicate (cross-origin) with the IdP via postMessages: As the BrowserID implementation mainly runs under the origin of LPO, it cannot directly communicate with the IdP, thus it uses the PIF as a proxy. Temporarily, the LD may navigate itself to a web page at IdP to allow for direct



Fig. 3. Simplified BrowserID implementation overview. Black arrows (open tips) denote HTTPS messages, blue arrows (filled tips) denote XHRs (over HTTPS), red (dashed) arrows are postMessages, snake lines are commands to the browser. Note that the communication iframe for automatic CAP creation is omitted for brevity.

user interaction with the IdP. We then call this window the *authentication dialog* (AD).

Login Process. To describe the login process, for the sake of presentation we assume for now that the user uses a "fresh" browser, i.e., the user has not been logged in before. As mentioned, the process starts by the user visiting a web site of some RP. After the user has clicked on the login button in RP-Doc, the LD is opened and the interactive login flow is started. We can divide this login flow into seven phases: In Phase (i), essentially the LD is initialized and the user is prompted to provide her email address. Also, LD fetches the support document (see Section 4.1) of the IdP via LPO. In Phase (ii), LD creates the PIF from the *provisioning URL* provided in the support document. As (by our assumption) the user is not logged in yet, the PIF notifies LD that the user is not authenticated to the IdP yet. In Phase (iii), LD navigates itself away to the authentication URL which is also provided in the support document and links to the IdP. Usually, this document will show a login form in which the user enters her password to authenticate to the IdP. After the user has been authenticated to IdP (which typically implies that the IdP sets a session cookie in the browser). the window is navigated to LPO again. (This is done by JavaScript loaded from LPO that the IdP document is supposed to include.)

Now, the login flow continues in Phase (iv), which basically repeats Phase (i). However, the user is not prompted for her email address (it has previously been saved in the localStorage under the origin of LPO along with a nonce, where the nonce is stored in the sessionStorage). In Phase (v), which essentially repeats Phase (ii), the PIF detects that the user is now authenticated to the IdP and the provisioning phase is started ((1) in Figure 2): The user's keys are created by LD and stored in the localStorage under the origin of LPO. The PIF forwards the certification request to the IdP, which then creates the UC and sends it back to the PIF. The PIF in turn forwards it to the LD, which stores it in the localStorage under the origin of LPO.

In Phases (i) and (ii), mainly the IA is generated by LD for the origin of RP-Doc and sent (together with the UC) to RP-Doc (II) in Figure 2). In the localStorage, LD stores that the user's email address is logged in at RP. Moreover, the user's email address is recorded at LPO (see the explanation on LPO Sessions below). For this purpose, LD generates an IA for the origin of LPO and sends the UC and IA to LPO.

LPO Session. LPO establishes a session with the browser by setting a cookie browserid_state (Step 5 in Figure 3) on the client-side. LPO considers such a session authenticated after having received a valid CAP (Step 22 in Figure 3). In future runs, the user is presented a list of her email addresses (which is fetched from LPO) in order to choose one address. Then, she is asked if she trusts the computer she is using and is given the option to be logged in for one month or "for this session only" (*ephemeral* session). In order to use any of the email addresses, the user is required to authenticate to the IdP responsible for that

address to get an UC issued. If the localStorage (under the origin LPO) already contains a valid UC, then, however, authentication at the IdP is not necessary.

Automatic CAP Creation. In addition to the interactive login presented above, BrowserID also contains an automatic, non-interactive way for RPs to obtain a freshly generated CAP: During initialization within RP-Doc, an invisible iframe called the *communication iframe* (CIF) is created inside RP-Doc. The CIF's JavaScript is loaded from LPO and behaves similar to LD, but without user interaction. The CIF automatically issues a fresh CAP and sends it to RP-Doc under specific conditions: among others, the email address must be marked as logged in at RP in the localStorage. If necessary, a new key pair is created and a corresponding new UC is requested at the IdP.

Differences to the Secondary IdP Mode. In the secondary IdP mode there are three parties involved only: RP, Browser, and LPO, where LPO also takes the role of the IdP; LPO is the only IdP that is present, rather than an arbitrary set of (external) IdPs. Consequently, in the secondary IdP case the PIF and the AD do not exist, LPO takes care of user authentication and UC generation itself. Creating CAP_{LPO} for LPO is not necessary either. Altogether, the secondary IdP case requires much less communication between parties/components and trust assumptions are simpler: while some external IdPs might be malicious (and hence, also the scripts they deliver for the PIF and the AD), one has to assume LPO to be trusted. The secondary IdP mode is described in detail in [11] (Appendix F, Figure 6). Figure 6 in [11] includes the description of the CIF which, however, in Figure 3 we have omitted for the sake of brevity of presentation. In the primary mode, the behavior of the CIF is even more complex than the one in the secondary mode. For example, in the primary mode, just like the LD, the CIF might contain a PIF (iframe in iframe) and interact with it via postMessages.

5 Analysis of BrowserID

In this section, we present the analysis of the BrowserID system with primary IdPs. As already mentioned, in [10], we merely analyzed the simpler case with a secondary IdP. We first, in Section 5.1, describe our model of BrowserID with primary IdPs, followed in Section 5.2 by a description of two central security properties one would expect from an SSO system. As mentioned in the introduction, during the analysis of BrowserID it turned out that one of the security properties is not satisfied and that in fact there is an attack on BrowserID. We confirmed that this attack works on the actual implementation of BrowserID and reported it to Mozilla. In Section 5.3, the attack is presented along with a fix. (The detailed model of BrowserID presented in Appendix F contains this fix as well as fixes pointed out in [10].) In Section 5.4, we prove that the fixed BrowserID system with primary IdPs satisfies both security properties.

5.1 Modeling of BrowserID with Primary IdPs

We model the BrowserID system with primary IdPs as a web system (in the sense of Section 2). Note that while in Section 4 we give only a brief overview of the BrowserID system, our modeling and analysis considers the complete system with primary IdPs, where we have extracted the model from the BrowserID source code [16].

We call a web system $\mathcal{B}I\mathcal{D} = (\mathcal{W}, \mathcal{S}, \mathsf{script}, E_0)$ a *BrowserID web system* if it is of the form described in what follows.

The system $\mathcal{W} = \mathsf{Hon} \cup \mathsf{Web} \cup \mathsf{Net}$ consists of the (network) attacker process attacker, the web server for LPO, a finite set B of web browsers, a finite set RP of web servers for the relying parties, and a finite set IDP of web servers for the identity providers, with $\mathsf{Hon} := \mathsf{B} \cup \mathsf{RP} \cup \mathsf{IDP} \cup \{\mathsf{LPO}\}, \mathsf{Web} := \emptyset$, and $\mathsf{Net} := \{\mathsf{attacker}\}$. DNS servers are assumed to be dishonest, and hence, are subsumed by attacker. IdPs and RPs can become corrupted (similar to browsers, by a special message); LPO is assumed to be honest.

The set IPs of IP addresses (see Section 2.1) contains for LPO, attacker, every relying party in RP, every identity provider in IDP, and every browser in B one address each. By addr we denote the corresponding assignment from processes to addresses. We define the set $Doms \subseteq S$ to contain one domain for LPO, one for every relying party in RP, a finite set of domains for every identity provider in IDP, and a finite set of domains for attacker. Browsers do not have a domain.

The definition of the processes in \mathcal{W} follows the description in Section 4.2. We define the processes in \mathcal{W} in full detail in Appendix F. In the following, we only highlight important details.

All processes in \mathcal{W} contain in their initial states all public keys and the private keys of their respective domains (if any). For RP, we explicitly follow the security considerations in [18] (Cross-site Request Forgery protection, e.g., by checking origin headers and HTTPS only with STS enabled). When RP receives a valid CAP (see below), RP responds with a fresh *RP service token for ID i* where *i* is the ID (email address) for which the CAP was issued. Intuitively, a client having such a token can use the service of the RP.

Each browser $b \in B$ owns a set of email addresses (identities) of the form $\langle name, d \rangle$ with $name \in S$ and $d \in Doms$ and associated secrets (i.e., nonces), modeling passwords. The domains belong to IdPs. The secrets and identities are contained in the initial states of the browsers.

In our model, a UC uc for a user u with email address $\langle name, d \rangle$ and public key (verification key) $pub(k_u)$, where $d \in dom(y)$ is a domain of the IdP y that issued the UC and k_u is the private (signing) key of u, is modeled to be a message of the form $uc = sig(\langle name, d \rangle, pub(k_u) \rangle$, signkey(y)), with signkey(y) being the signing key of y. An IA *ia* for an origin o is a message of the form $ia = sig(o, k_u)$. Now, a CAP is of the form $\langle uc, ia \rangle$. Note that the time stamps are omitted both from the UC and the IA. This models that both certificates are valid indefinitely. In reality, as explained in Section 4, they are valid for a certain period of time, as indicated by the time stamps. So our modeling is a safe overapproximation. The set S of $\mathcal{B}I\mathcal{D}$ contains six scripts, with their string representations defined by script: the honest scripts running in RP-Doc, CIF, LD, AD, and PIF, respectively, and the malicious script R^{att} . The scripts for CIF and LD (issued by LPO) are defined in a straightforward way following the implementation outlined in Section 4. The script for RP-Doc (issued by RP) also includes the script that is (in reality) loaded from LPO. In particular, this script creates the CIF and the LD iframes/subwindows, whose contents (scripts) are loaded from LPO. The scripts for the AD and PIF are modeled following the example implementation provided by Mozilla [16]. Full formal specifications of all the above mentioned scripts are provided in Appendix F.

The set E_0 contains only the trigger events specified in the definition of a web system (Section 2.3).

5.2 Security Properties of the BrowserID System

While the documentation of BrowserID does not contain explicit security goals, we state two fundamental security properties every SSO system should satisfy. These properties are adapted from [10].

Informally, these properties can be stated as follows: (A) The attacker should not be able to use a service of RP as an honest user. In other words, the attacker should not get hold of (be able to derive from his current knowledge) an RP service token for an ID of an honest user (browser), even if the browser was closed and then later used by a malicious user (i.e., after a CLOSECORRUPT). (B) The attacker should not be able to authenticate an honest browser to an RP with an ID that is not owned by the browser. We refer the reader to Appendix G for the formal definition of these properties.

We call a BrowserID web system $\mathcal{B}I\mathcal{D}$ secure if the above conditions are satisfied in all runs of the system.

5.3 Attack on BrowserID with Primary IdPs

As mentioned, our analysis of BrowserID with primary IdPs w.r.t. the above security properties revealed a serious attack, which does not apply to the case with secondary IdPs. We confirmed the attack on the actual implementation and reported it to Mozilla [7].

During the provisioning phase \boxed{v} (see Figure 3), the IdP issues a UC for the user's identity and public key provided in 16. This UC is finally sent to the LD by the PIF in 20. Unfortunately, LD fails to check whether the UC contains the correct email address and public key, namely, the email address/public key pair that was sent in 16.

If the IdP is malicious, it can issue a UC with different data. In particular, it could replace the email address by a different one, but keep the original public key. This (malicious) UC is then later included in the CAP by LD. The CAP will still be valid, because the public key is unchanged. Now, as the RP determines the user's identity by the UC contained in the CAP, RP issues a service token

for the spoofed email address. As a result, the honest user will use RP's service (and typically will be logged in to RP) under an ID that belongs to the attacker, which, for example, could allow the attacker to track actions of the honest user or obtain user secrets. This violates Condition (**B**).

Proposed fix. To fix this problem, upon receipt of the UC in 20, LD should check whether it contains the correct email address and public key, i.e., the one requested by LD in 16. The same is true for the CIF, which behaves similarly to the LD. The formal model of BrowserID presented in Appendix F contains these fixes.

5.4 Security of the Fixed System

For the fixed BrowserID system with primary IdPs, we have proven the following theorem, which says that a fixed BrowserID web system (i.e., the system where the above described fix is applied) satisfies the security properties (A) and (B).

Theorem 1. Let BID be a fixed BrowserID web system. Then, BID is secure.

We prove Conditions (A) and (B) separately. For both conditions, we assume that they are not satisfied and lead this to a contradiction. In our proofs, we make use of the general security properties of the web model presented in Section 3, which helped a lot in making the proof for the primary IdP model more modular and concise. The complete proof with all details is provided in Appendix H.

6 Related Work

The formal treatment of the security of the web infrastructure and web applications based on this infrastructure is a relatively young discipline. Of the few works in this area even less are based on a general model that incorporates essential mechanisms of the web.

Early work in formal web security analysis includes work by Kerschbaum [14], in which a Cross-Site Request Forgery protection proposal is formally analyzed using a simple model expressed using Alloy, a finite-state model checker [13]. Formal methods were also used by Armando et al. [2] to analyze SAML single sign-on and by Sun et al. [21] to perform security analysis of OpenID. Chari et al. [8] analyze OAuth using the Universal Composability approach. However, all these works are based on very limited models developed specifically for the application under scrutiny.

The first work to consider a general model of the web is the work by Akhawe et al. [1]. Their model is again written in Alloy and they apply it to analyze the Origin header concept, Cross-Origin Resource Sharing, Kerschbaum's Referer Validation (as mentioned above), HTML5 forms, and the WebAuth authentication mechanism. Inspired by this work, Bansal et al. [4,5] built a more expressive model, called WebSpi, in ProVerif [6], a tool for symbolic cryptographic protocol analysis. They use the model to analyze encrypted web storage and website authorization. Recently, Kumar [15] presented a high-level Alloy model and applied it to SAML single sign-on. Compared to our web model, on the one hand, the above mentioned models are formulated in the specification languages of specific analysis tools, and hence, are tailored towards automation (while we perform manual analysis). On the other hand, the models considered in these works are much less expressive and precise. For example, these models do not incorporate a precise handling of windows, documents, or iframes; cross-document messaging (postMessages) or session storage are not included at all. In fact, several general web features and technologies that have been crucial for the analysis of BrowserID are not supported by these models. Moreover, the complexity of the system we analyzed, namely BrowserID, exceeds that of the systems analyzed in these works in terms of the use of web features and technologies and in terms of the complexity of the protocols. For example, BrowserID in primary mode is a protocol consisting of 48 different (network and inter-frame) message types compared to typically about 10–15 in the protocols analyzed in other models.

The BrowserID system in the primary mode has been analyzed before using the AuthScan tool developed by Bai et al. [3]. Their work focusses on the automated extraction of a model from a protocol implementation. This tool-based analysis did not reveal the identity injection attack, though. Dietz and Wallach demonstrated a technique to secure BrowserID when specific flaws in TLS are considered [9].

7 Conclusion

We have presented a detailed analysis of the BrowserID single sign-on system in the so-called primary IdP mode. For this, we first slightly extended the web model proposed in prior work. Our model is the most complete model of the web so far. It contains many security-relevant features and is designed to closely mimic standards and specifications for the web. As such, it constitutes a solid basis for the analysis of a broad range of web standards and applications. During the proof of the security of the primary IdP mode of BrowserID, we found a flaw in BrowserID that leads to an identity injection attack, which violates security Condition (B). We confirmed that the attack can in fact be carried out on the BrowserID implementation and reported the flaw to Mozilla. We proposed a fix and proved that the fixed system fulfills the security properties (A) and (B), which are fundamental security properties of SSO systems. The analysis carried out in this work constitutes the most complex formal analysis of a web application to date. It illustrates that (manual) security analysis of complex web applications in detailed web models, while laborious, is feasible and yields meaningful and relevant results. In addition, we have identified and proved important security properties of general application independent web features in order to facilitate future analysis efforts of web standards and web applications using the model.

As for future work, it is clear that our web model can serve as a basis and a reference for automated approaches, where one could try to extend the existing automated approaches or develop new ones (e.g., based on theorem provers, where higher accuracy is typically paid by more interaction). The main obstacle to overcome during this step will be the complex data structures and algorithms that form essential parts of modern web browsers and are already part of our model. Finally, we also plan to apply our model to other web applications and web standards and to identify and prove further web properties.

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A Communication Model

Extending Section 2.1, we here present details and definitions on the basic concepts of our communication model. For readability, some parts from Section 2.1 are repeated here.

A.1 Terms, Messages and Events

The signature Σ for the terms and messages considered in this work is the union of the following pairwise disjoint sets of function symbols:

- constants $C = |\mathsf{Ps} \cup \mathbb{S} \cup \{\top, \bot, \Diamond\}$ where the three sets are pairwise disjoint, \mathbb{S} is interpreted to be the set of ASCII strings (including the empty string ε), and $|\mathsf{Ps}$ is interpreted to be a set of (IP) addresses,
- function symbols for public keys, asymmetric/symmetric encryption/decryption, and signatures: $pub(\cdot),~enc_a(\cdot,\cdot),~dec_a(\cdot,\cdot),~enc_s(\cdot,\cdot),~dec_s(\cdot,\cdot),~sig(\cdot,\cdot),~checksig(\cdot,\cdot),~extractmsg(\cdot),$
- *n*-ary sequences $\langle \rangle, \langle \cdot \rangle, \langle \cdot, \cdot \rangle, \langle \cdot, \cdot \rangle$, etc., and
- projection symbols $\pi_i(\cdot)$ for all $i \in \mathbb{N}$.

Definition 1. Let $X = \{x_0, x_1, ...\}$ be a set of variables and \mathcal{N} be an infinite set of constants (nonces) such that Σ , X, and \mathcal{N} are pairwise disjoint. For $N \subseteq \mathcal{N}$, we define the set $\mathcal{T}_N(X)$ of terms over $\Sigma \cup N \cup X$ inductively as usual: (1) If $t \in N \cup X$, then t is a term. (2) If $f \in \Sigma$ is an n-ary function symbol in Σ for some $n \ge 0$ and t_1, \ldots, t_n are terms, then $f(t_1, \ldots, t_n)$ is a term.

By $\mathcal{T}_N = \mathcal{T}_N(\emptyset)$, we denote the set of all terms over $\Sigma \cup N$ without variables, called *ground terms*. The set \mathcal{M} of messages (over \mathcal{N}) is defined to be the set of ground terms $\mathcal{T}_{\mathcal{N}}$.

Example 1. For example, $k \in \mathcal{N}$ and $\mathsf{pub}(k)$ are messages, where k typically models a private key and $\mathsf{pub}(k)$ the corresponding public key. For constants a, b, c and the nonce $k \in \mathcal{N}$, the message $\mathsf{enc}_{\mathsf{a}}(\langle a, b, c \rangle, \mathsf{pub}(k))$ is interpreted to be the message $\langle a, b, c \rangle$ (the sequence of constants a, b, c) encrypted by the public key $\mathsf{pub}(k)$.

For strings, i.e., elements in S, we use a specific font. For example, HTTPReq and HTTPResp are strings. We denote by $Doms \subseteq S$ the set of domains, e.g., www.example.com \in Doms. We denote by Methods $\subseteq S$ the set of methods used in HTTP requests, e.g., GET, POST \in Methods.

The equational theory associated with the signature Σ is given in Figure 4.

$$dec_{a}(enc_{a}(x, pub(y)), y) = x$$
(1)

$$\mathsf{dec}_{\mathsf{s}}(\mathsf{enc}_{\mathsf{s}}(x,y),y) = x \tag{2}$$

$$\mathsf{extractmsg}(\mathsf{sig}(x, y)) = x \tag{3}$$

 $\mathsf{checksig}(\mathsf{sig}(x,y),\mathsf{pub}(y)) = \top \tag{4}$

$$\pi_i(\langle x_1, \dots, x_n \rangle) = x_i \text{ if } 1 \le i \le n \tag{5}$$

$$\pi_j(\langle x_1, \dots, x_n \rangle) = 0 \quad \text{if } j \notin \{1, \dots, n\}$$
(6)

$$\pi_j(t) = \Diamond \text{ if } t \text{ is not a sequence}$$
(7)

Fig. 4. Equational theory for Σ .

By \equiv we denote the congruence relation on $\mathcal{T}_{\mathcal{H}}(X)$ induced by this theory. For example, we have that $\pi_1(\mathsf{dec}_a(\mathsf{enc}_a(\langle \mathbf{a}, \mathbf{b} \rangle, \mathsf{pub}(k)), k)) \equiv \mathbf{a}$.

Definition 2. An event (over IPs and \mathcal{M}) is of the form (a:f:m), for $a, f \in IPs$ and $m \in \mathcal{M}$, where a is interpreted to be the receiver address and f is the sender address. We denote by \mathcal{E} the set of all events.

A.2 Atomic Processes, Systems and Runs

We here define atomic processes, systems, and runs of systems.

An atomic process takes its current state and an event as input, and then (non-deterministically) outputs a new state and a set of events. **Definition 3.** A (generic) atomic process is a tuple $p = (I^p, Z^p, R^p, s_0^p)$ where $I^p \subseteq \mathsf{IPs}, Z^p$ is a set of states, $R^p \subseteq (\mathfrak{E} \times Z^p) \times (2^{\mathfrak{E}} \times Z^p)$, and $s_0^p \in Z^p$ is the initial state of p. We write (e, z)R(E, z') instead of $((e, z), (E, z')) \in R$. A system \mathcal{P} is a (possibly infinite) set of atomic processes.

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Definition 4. A configuration of a system \mathcal{P} is a tuple (S, E) where S maps every atomic process $p \in \mathcal{P}$ to its current state $S(p) \in Z^p$ and E is a (possibly infinite) multi-set of events waiting to be delivered.

Definition 5. A processing step of the system \mathcal{P} is of the form

$$(S, E) \xrightarrow[p \to E_{out}]{e \to p} (S', E')$$

such that (1) there exist $e = (a:f:m) \in E$, $E_{out} \subseteq E'$, and $p \in \mathcal{P}$ with $(e, S(p))R^p(E_{out}, S'(p))$ and $a \in I^p$, (2) S'(p') = S(p') for all $p' \neq p$, and (3) $E' = (E \setminus \{e\}) \cup E_{out}$ (multi-set operations). We may omit the superscript and/or subscript of the arrow.

Definition 6. Let \mathcal{P} be a system and E_0 be a multi-set of events. A run ρ of a system \mathcal{P} initiated by E_0 is a finite sequence of configurations $(S_0, E_0), \ldots, (S_n, E_n)$ or an infinite sequence of configurations $(S_0, E_0), \ldots$ such that $S_0(p) = s_0^p$ for all $p \in \mathcal{P}$ and $(S_i, E_i) \to (S_{i+1}, E_{i+1})$ for all $0 \leq i < n$ (finite run) or for all $i \geq 0$ (infinite run).

A.3 Atomic Dolev-Yao Processes

We next define atomic Dolev-Yao processes, for which we require that the messages and states that they output can be computed (more formally, derived) from the current input event and state. For this purpose, we first define what it means to derive a message from given messages.

Definition 7. Let $N \subseteq \mathcal{N}$, $\tau \in \mathcal{T}_N(\{x_1, \ldots, x_n\})$, and $t_1, \ldots, t_n \in \mathcal{T}_N$. Then, by $\tau[t_1/x_1, \ldots, t_n/x_n]$ we denote the (ground) term obtained from τ by replacing all occurrences of x_i in τ by t_i , for all $i \in \{1, \ldots, n\}$.

Definition 8. Let $M \subseteq \mathcal{M}$ be a set of messages. We say that a message m can be derived from M with nonces N if there exist $n \geq 0, m_1, \ldots, m_n \in M$, and $\tau \in \mathcal{T}_N(\{x_1, \ldots, x_n\})$ such that $m \equiv \tau[m_1/x_1, \ldots, m_n/x_n]$. We denote by $d_N(M)$ the set of all messages that can be derived from M with nonces N.

For example, $a \in d_{\{k\}}(\{\mathsf{enc}_{\mathsf{a}}(\langle a, b, c \rangle, \mathsf{pub}(k))\}).$

Definition 9. An atomic Dolev-Yao process (or simply, a DY process) is a tuple $p = (I^p, Z^p, R^p, s_0^p, N^p)$ such that (I^p, Z^p, R^p, s_0^p) is an atomic process and (1) $N^p \subseteq \mathcal{N}$ is an (initial) set of nonces, (2) $Z^p \subseteq \mathcal{T}_{\mathcal{N}}$ (and hence, $s_0^p \in \mathcal{T}_{\mathcal{N}}$), and (3) for all $a, a', f, f' \in \mathsf{IPs}$, $m, m', s, s' \in \mathcal{T}_{\mathcal{N}}$, set of events E with ((a:f:m), s)R(E, s') and $(a':f':m') \in E$ it holds true that $m', s' \in d_N(\{m, s\})$. (Note that $a', f' \in d_N(\{m, s\})$.)

Definition 10. An (atomic) attacker process for a set of sender addresses $A \subseteq$ IPs is an atomic DY process $p = (I, Z, R, s_0, N)$ such that for all $a, f \in$ IPs, $m \in T_{\mathcal{N}}$, and $s \in Z$ we have that ((a:f:m), s)R(E, s') iff $s' = \langle \langle a, f, m \rangle, s \rangle$ and $E = \{(a':f':m') \mid a' \in$ IPs, $f' \in A, m' \in d_N(\{m, s\})\}.$

B Message and Data Formats

We now provide some more details about data and message formats that are needed for the formal treatment of our web model and the analysis of BrowserID presented in the rest of the appendix.

B.1 Notations

Definition 11 (Sequence Notations). For a sequence $t = \langle t_1, \ldots, t_n \rangle$ and a set s we use $t \subset \langle \rangle$ s to say that $t_1, \ldots, t_n \in s$. We define $x \in \langle \rangle t \iff \exists i : t_i = x$. We write $t + \langle \rangle y$ to denote the sequence $\langle t_1, \ldots, t_n, y \rangle$. For a sequence $t = \langle t_1, \ldots, t_n \rangle$ we define |t| = n. If t is not a sequence, we set $|t| = \langle \rangle$. For a finite set M with $M = \{m_1, \ldots, m_n\}$ we use $\langle M \rangle$ to denote the term of the form $\langle m_1, \ldots, m_n \rangle$. (The order of the elements does not matter; one is chosen arbitrarily.)

Definition 12. A dictionary over X and Y is a term of the form

$$\langle \langle k_1, v_1 \rangle, \ldots, \langle k_n, v_n \rangle \rangle$$

where $k_1, \ldots, k_n \in X$, $v_1, \ldots, v_n \in Y$, and the keys k_1, \ldots, k_n are unique, i.e., $\forall i \neq j : k_i \neq k_j$. We call every term $\langle k_i, v_i \rangle$, $i \in \{1, \ldots, n\}$, an element of the dictionary with key k_i and value v_i . We often write $[k_1 : v_1, \ldots, k_i : v_i, \ldots, k_n : v_n]$ instead of $\langle \langle k_1, v_1 \rangle, \ldots, \langle k_n, v_n \rangle \rangle$. We denote the set of all dictionaries over X and Y by $[X \times Y]$.

We note that the empty dictionary is equivalent to the empty sequence, i.e., $[] = \langle \rangle$. Figure 5 shows the short notation for dictionary operations that will be used when describing the browser atomic process. For a dictionary $z = [k_1 : v_1, k_2 : v_2, \ldots, k_n : v_n]$ we write $k \in z$ to say that there exists *i* such that $k = k_i$. We write $z[k_j] := v_j$ to extract elements. If $k \notin z$, we set $z[k] := \langle \rangle$.

$$[k_1 : v_1, \dots, k_i : v_i, \dots, k_n : v_n] [k_i] = v_i$$

$$[k_1 : v_1, \dots, k_{i-1} : v_{i-1}, k_i : v_i, k_{i+1} : v_{i+1}, \dots, k_n : v_n] - k_i =$$
(8)

$$[k_1:v_1,\dots,k_{i-1}:v_{i-1},k_i+1:v_{i+1}\dots,k_n:v_n] = k_i = [k_1:v_1,\dots,k_{i-1}:v_{i-1},k_{i+1}:v_{i+1}\dots,k_n:v_n]$$
(9)

Fig. 5. Dictionary operators with $1 \le i \le n$.

Given a term $t = \langle t_1, \ldots, t_n \rangle$, we can refer to any subterm using a sequence of integers. The subterm is determined by repeated application of the projection π_i for the integers *i* in the sequence. We call such a sequence a *pointer*:

Definition 13. A pointer is a sequence of non-negative integers. We write $\tau.\overline{p}$ for the application of the pointer \overline{p} to the term τ . This operator is applied from left to right. For pointers consisting of a single integer, we may omit the sequence braces for brevity.

Example 2. For the term $\tau = \langle a, b, \langle c, d, \langle e, f \rangle \rangle$ and the pointer $\overline{p} = \langle 3, 1 \rangle$, the subterm of τ at the position \overline{p} is $c = \pi_1(\pi_3(\tau))$. Also, $\tau.3.\langle 3, 1 \rangle = \tau.3.\overline{p} = \tau.3.3.1 = e$.

To improve readability, we try to avoid writing, e.g., o.2 or $\pi_2(o)$ in this document. Instead, we will use the names of the components of a sequence that is of a defined form as pointers that point to the corresponding subterms. E.g., if an *Origin* term is defined as $\langle host, protocol \rangle$ and o is an Origin term, then we can write *o*.protocol instead of $\pi_2(o)$ or o.2. See also Example 3.

In our pseudocode, we will write, for example,

let x, y such that $(Constant, x, y) \equiv t$ if possible; otherwise doSomethingElse

for some variables x, y, a string Constant, and some term t to express that $x := \pi_2(t)$, and $y := \pi_3(t)$ if Constant $\equiv \pi_1(t)$ and if $|\langle \text{Constant}, x, y \rangle| = |t|$, and that otherwise x and y are not set and doSomethingElse is executed.

B.2 URLs

Definition 14. A URL is a term of the form (URL, protocol, host, path, params) with protocol \in {P,S} (for plain (HTTP) and secure (HTTPS)), host \in Doms, path \in S and params \in [S $\times T_N$]. The set of all valid URLs is URLs.

Example 3. For the URL $u = \langle \text{URL}, a, b, c, d \rangle$, u.protocol = a. If, in the algorithm described later, we say u.path := e then $u = \langle \text{URL}, a, b, c, e \rangle$ afterwards.

B.3 Origins

Definition 15. An origin is a term of the form $\langle host, protocol \rangle$ with host \in Doms, $protocol \in \{P, S\}$. We write Origins for the set of all origins. See Example 6 for an example of an origin.

B.4 Cookies

Definition 16. A cookie is a term of the form $\langle name, content \rangle$ where $name \in \mathcal{T}_{\mathcal{N}}$, and content is a term of the form $\langle value, secure, session, httpOnly \rangle$ where $value \in \mathcal{T}_{\mathcal{N}}$, secure, session, $httpOnly \in \{\top, \bot\}$. We write Cookies for the set of all cookies.

If the *secure* attribute of a cookie is set, the browser will not transfer this cookie over unencrypted HTTP connections. If the *session* flag is set, this cookie will be deleted as soon as the browser is closed. The *httpOnly* attribute controls whether JavaScript has access to this cookie.

Note that cookies of the form described here are only contained in HTTP(S) requests. In responses, only the components *name* and *value* are transferred as a pairing of the form $\langle name, value \rangle$.

B.5 HTTP Messages

Definition 17. An HTTP request is a term of the form shown in (10). An HTTP response is a term of the form shown in (11).

 $\langle \text{HTTPReq}, nonce, method, host, path, parameters, headers, body \rangle$ (10) $\langle \text{HTTPResp}, nonce, status, headers, body \rangle$ (11)

The components are defined as follows:

- nonce $\in \mathcal{N}$ serves to map each response to the corresponding request
- $method \in Methods$ is one of the HTTP methods.
- host \in Doms is the host name in the HOST header of HTTP/1.1.
- $path \in S$ is a string indicating the requested resource at the server side
- status ∈ S is the HTTP status code (i.e., a number between 100 and 505, as defined by the HTTP standard)
- parameters $\in [\mathbb{S} \times T_{\mathcal{N}}]$ contains URL parameters
- headers $\in [S \times T_{\mathcal{N}}]$, containing request/response headers. The dictionary elements are terms of one of the following forms:
 - $\langle \texttt{Origin}, o \rangle$ where o is an origin
 - $\langle \texttt{Set-Cookie}, c \rangle$ where c is a sequence of cookies
 - $(\operatorname{Cookie}, c)$ where $c \in [\mathbb{S} \times T_{\mathcal{N}}]$ (note that in this header, only names and values of cookies are transferred)
 - (Location, l) where $l \in \text{URLs}$
 - $\langle \texttt{Strict-Transport-Security}, \top \rangle$
- $body \in T_{\mathcal{N}}$ in requests and responses.

We write HTTPRequests/HTTPResponses for the set of all HTTP requests or responses, respectively.

Example 4 (HTTP Request and Response).

$$\begin{split} r := & \langle \texttt{HTTPReq}, n_1, \texttt{POST}, \texttt{example.com}, /\texttt{show}, \langle \langle \texttt{index}, 1 \rangle \rangle, \\ & [\texttt{Origin} : \langle \texttt{example.com}, \texttt{S} \rangle], \langle \texttt{foo}, \texttt{bar} \rangle \rangle \end{split} \tag{12} \\ s := & \langle \texttt{HTTPResp}, n_1, \texttt{200}, \langle \langle \texttt{Set-Cookie}, \langle \langle \texttt{SID}, \langle n_2, \bot, \bot, \top \rangle \rangle \rangle \rangle, \langle \texttt{somescript}, x \rangle \rangle \\ & (13) \end{split}$$

An HTTP GET request for the URL http://example.com/show?index=1 is shown in (12), with an Origin header and a body that contains (foo,bar). A

possible response is shown in (13), which contains an httpOnly cookie with name SID and value n_2 as well as the string representation somescript of the scripting process script⁻¹(somescript) (which should be an element of S) and its initial state x.

Encrypted HTTP Messages. For HTTPS, requests are encrypted using the public key of the server. Such a request contains an (ephemeral) symmetric key chosen by the client that issued the request. The server is supported to encrypt the response using the symmetric key.

Definition 18. An encrypted HTTP request is of the form $enc_a(\langle m, k' \rangle, k)$, where $k, k' \in \mathcal{N}$ and $m \in \mathsf{HTTPRequests}$. The corresponding encrypted HTTP response would be of the form $enc_s(m', k')$, where $m' \in \mathsf{HTTPResponses}$. We call the sets of all encrypted HTTP requests and responses HTTPSRequests or HTTPSResponses, respectively.

Example 5.

$$enc_{a}(\langle r, k' \rangle, pub(k_{example.com}))$$
 (14)

$$\operatorname{enc}_{\mathsf{s}}(s,k')$$
 (15)

The term (14) shows an encrypted request (with r as in (12)). It is encrypted using the public key $pub(k_{example.com})$. The term (15) is a response (with s as in (13)). It is encrypted symmetrically using the (symmetric) key k' that was sent in the request (14).

B.6 DNS Messages

Definition 19. A DNS request is a term of the form $\langle DNSResolve, domain, n \rangle$ where domain \in Doms, $n \in \mathcal{N}$. We call the set of all DNS requests DNSRequests.

Definition 20. A DNS response is a term of the form $\langle DNSResolved, result, n \rangle$ with result \in IPs, $n \in \mathcal{N}$. We call the set of all DNS responses DNSResponses.

DNS servers are supposed to include the nonce they received in a DNS request in the DNS response that they send back so that the party which issued the request can match it with the request.

C Detailed Description of the Browser Model

Following the informal description of the browser model in Section 2.5, we now present a formal model. We start by introducing some notation and terminology.

C.1 Notation and Terminology (Web Browser State)

Before we can define the state of a web browser, we first have to define windows and documents. Concrete window and document terms are shown in Example 6.

Definition 21. A window is a term of the form $w = \langle nonce, documents, opener \rangle$ with nonce $\in \mathcal{N}$, documents $\subset^{\langle \rangle}$ **Documents** (defined below), opener $\in \mathcal{N} \cup \{\bot\}$ where d.active $= \top$ for exactly one $d \in^{\langle \rangle}$ documents if documents is not empty (we then call d the active document of w). We write Windows for the set of all windows. We write w.activedocument to denote the active document inside window w if it exists and $\langle \rangle$ else.

We will refer to the window nonce as (window) reference.

The documents contained in a window term to the left of the active document are the previously viewed documents (available to the user via the "back" button) and the documents in the window term to the right of the currently active document are documents available via the "forward" button, as will be clear from the description of web browser model (see Section C.2).

A window a may have opened a top-level window b (i.e., a window term which is not a subterm of a document term). In this case, the *opener* part of the term b is the nonce of a, i.e., b.opener = a.nonce.

Definition 22. A document d is a term of the form

(nonce, origin, script, scriptstate, scriptinput, subwindows, active)

where nonce $\in \mathcal{N}$, origin \in Origins, script, scriptstate, scriptinput $\in \mathcal{T}_{\mathcal{N}}$, subwindows $\subset^{\langle\rangle}$ Windows, active $\in \{\top, \bot\}$. A limited document is a term of the form $\langle nonce, subwindows \rangle$ with nonce, subwindows as above. A window $w \in^{\langle\rangle}$ subwindows is called a subwindow (of d). We write Documents for the set of all documents.

We will refer to the document nonce as (document) reference.

Example 6. The following is an example of a window term with reference n_1 , two documents, and an opener (n_4) :

$$\langle n_1, \langle \langle n_2, \langle \texttt{example.com}, \mathsf{P} \rangle, \texttt{script1}, \langle \rangle, \langle \rangle, \langle \rangle, \bot \rangle, \\ \langle n_3, \langle \texttt{example.com}, \mathsf{S} \rangle, \texttt{script2}, \langle \rangle, \langle \rangle, \langle \rangle, \top \rangle \rangle, n_4 \rangle$$

The first document has the reference n_2 . The document was loaded from the origin (example.com, P), which translates into http://example.com. Its scripting process has the string representation script1, the last state and the input history of this process are empty. The document does not have subwindows and is inactive (\perp). The second document has the reference n_3 , its origin corresponds to https://example.com, the scripting process is represented by script2, and the document is active (\top). All other components are empty.

We can now define the set of states of web browsers. Note that we use the dictionary notation that we introduced in Definition 12.

Definition 23. Let $OR := \{ \langle o, r \rangle | o \in \text{Origins}, r \in \mathcal{N} \}$. The set of states Z^p of a web browser atomic process p consists of the terms of the form

(windows, ids, secrets, cookies, localStorage, sessionStorage, keyMapping, sts, DNSaddress, nonces, pendingDNS, pendingRequests, isCorrupted)

where windows $\subset^{\langle\rangle}$ Windows, $ids \subset^{\langle\rangle} T_{\mathcal{H}}$, $secrets \in [\text{Origins} \times \mathcal{N}]$, cookies is a dictionary over Doms and dictionaries of Cookies, $localStorage \in [\text{Origins} \times T_{\mathcal{H}}]$, $sessionStorage \in [OR \times T_{\mathcal{H}}]$, $keyMapping \in [\text{Doms} \times T_{\mathcal{H}}]$, $sts \subset^{\langle\rangle}$ Doms, $DNSaddress \in IPs$, $nonces \subset^{\langle\rangle} \mathcal{N}$, $pendingDNS \in [\mathcal{N} \times T_{\mathcal{H}}]$, $pendingRequests \in T_{\mathcal{N}}$, $isCorrupted \in \{\bot, FULLCORRUPT, CLOSECORRUPT\}$. We call the set of all states of standard HTTP browsers SHBStates.

Definition 24. For two window terms w and w' we write $w \xrightarrow{childof} w'$ if $w \in \langle \rangle$ w'.activedocument.subwindows. We write $\xrightarrow{childof^+}$ for the transitive closure.

In the following description of the web browser relation R^p we will use the helper functions Subwindows, Docs, Clean, CookieMerge and AddCookie.

Given a browser state s, Subwindows(s) denotes the set of all pointers¹ to windows in the window list s.windows, their (active) documents, and the subwindows of these documents (recursively). We exclude subwindows of inactive documents and their subwindows. With Docs(s) we denote the set of pointers to all active documents in the set of windows referenced by Subwindows(s).

Definition 25. For a browser state s we denote by Subwindows(s) the minimal set of pointers that satisfies the following conditions: (1) For all windows $w \in \langle \rangle$ s.windows there is a $\overline{p} \in$ Subwindows(s) such that $s.\overline{p} = w$. (2) For all $\overline{p} \in$ Subwindows(s), the active document d of the window $s.\overline{p}$ and every subwindow w of d there is a pointer $\overline{p'} \in$ Subwindows(s) such that $s.\overline{p'} = w$.

Given a browser state s, the set Docs(s) of pointers to active documents is the minimal set such that for every $\overline{p} \in Subwindows(s)$, there is a pointer $\overline{p'} \in Docs(s)$ with $s.\overline{p'} = s.\overline{p}$.activedocument.

The function Clean will be used to determine which information about windows and documents the script running in the document d has access to.

Definition 26. Let s be a browser state and d a document. By Clean(s, d) we denote the term that equals s.windows but with all inactive documents removed (including their subwindows etc.) and all subterms that represent non-same-origin documents w.r.t. d replaced by a limited document d' with the same nonce and the same subwindow list. Note that non-same-origin documents on all levels are replaced by their corresponding limited document.

¹Recall the definition of a pointer in Definition 13.

The function **CookieMerge** merges two sequences of cookies together: When used in the browser, *oldcookies* is the sequence of existing cookies for some origin, *newcookies* is a sequence of new cookies that was outputted by some script. The sequences are merged into a set of cookies using an algorithm that is based on the *Storage Mechanism* algorithm described in RFC6265.

Definition 27. The set CookieMerge(oldcookies, newcookies) for two sequences oldcookies and newcookies of cookies (where the cookies in oldcookies have pairwise different names) is defined by the following algorithm: From newcookies remove all cookies c that have c.content.httpOnly $\equiv \top$. For any c, c' $\in^{\langle \rangle}$ newcookies, c.name \equiv c'.name, remove the cookie that appears left of the other in newcookies. Let m be the set of cookies that have a name that either appears in oldcookies or in newcookies, but not in both. For all pairs of cookies (c_{old}, c_{new}) with $c_{old} \in^{\langle \rangle}$ oldcookies, $c_{new} \in^{\langle \rangle}$ newcookies, c_{old} .name $\equiv c_{new}$.name, add c_{new} to m if c_{old} .content.httpOnly $\equiv \bot$ and add c_{old} to m otherwise. The result of CookieMerge(oldcookies, newcookies) is m.

The function AddCookie adds a cookie c to the sequence of cookies contained in the sequence *oldcookies*. It is again based on the algorithm described in RFC6265 but simplified for the use in the browser model.

Definition 28. The sequence AddCookie(oldcookies, c), where oldcookies is a sequence of cookies with different names and c is a cookie c, is defined by the following algorithm: Let m := oldcookies. Remove any c' from m that has $c.name \equiv c'.name$. Append c to m and return m.

The function NavigableWindows returns a set of windows that a document is allowed to navigate. We closely follow [12], Section 5.1.4 for this definition.

Definition 29. The set NavigableWindows(\overline{w}, s') is the set $\overline{W} \subseteq$ Subwindows(s') of pointers to windows that the active document in \overline{w} is allowed to navigate. The set \overline{W} is defined to be the minimal set such that for every $\overline{w'} \in$ Subwindows(s') the following is true:

- If $s'.\overline{w}'$.activedocument.origin $\equiv s'.\overline{w}$.activedocument.origin (the active documents in \overline{w} and $\overline{w'}$ are same-origin), then $\overline{w'} \in \overline{W}$, and
- If $s'.\overline{w} \xrightarrow{\text{childof}^*} s'.\overline{w'} \land \nexists \overline{w''} \in \text{Subwindows}(s') \text{ with } s'.\overline{w'} \xrightarrow{\text{childof}^*} s'.\overline{w''} (\overline{w'} \text{ is a top-level window and } \overline{w} \text{ is an ancestor window of } \overline{w'}), \text{ then } \overline{w'} \in \overline{W}, \text{ and } \overline{W'} \in \overline{W} \in \overline{W}, \text{ and } \overline{W'} \in \overline{W} \in \overline{W} \in \overline{W}, \text{ and } \overline{W'} \in \overline{W} \in \overline{W}, \text{ and } \overline{W'} \in \overline{W} \in \overline{$
- If ∃ p̄ ∈ Subwindows(s') such that s'.w' childof⁺ s'.p
 ∧ s'.p̄.activedocument.origin = s'.w.activedocument.origin (w' is not a top-level window but there is an ancestor window p̄ of w' with an active document that has the same origin as the active document in w), then w' ∈ W, and
- If ∃ p̄ ∈ Subwindows(s') such that s'.w'.opener = s'.p.nonce ∧ p̄ ∈ W̄ (w' is a top-level window—it has an opener—and w̄ is allowed to navigate the opener window of w', p̄), then w' ∈ W̄.

C.2Description of the Web Browser Atomic Process

We will now describe the relation R^p of a standard HTTP browser p. For a tuple r = (((a:f:m), s), (M, s')) we define r to belong to R^p iff the non-deterministic algorithm presented in Section 7, when given ((a:f:m), s) as input, terminates with stop M, s', i.e., with output M and s'. Recall that (a:f:m) is an (input) event and s is a (browser) state, M is a set of (output) events, and s' is a new (browser) state.

The notation let $n \leftarrow N$ is used to describe that n is chosen non-deterministically from the set N. We write for each $s \in M$ do to denote that the following commands (until end for) are repeated for every element in M, where the variable s is the current element. The order in which the elements are processed is chosen non-deterministically.

We first define some functions which will be used in the main algorithm presented in Section 7.

Functions. In the description of the following functions we use a, f, m, s and N^p as read-only global input variables. Also, the functions use the set N^p as a read-only set. All other variables are local variables or arguments.

TAKENONCE returns a nonce from the set of unused nonces and modifies the browser state such that the nonce is added to the sequence of used nonces. Note that this function returns two values, the nonce n and the modified state s'.

Algorithm 1 Non-deterministically choose a fresh nonce.

1: function TAKENONCE(s')

- $\begin{array}{l} \textbf{let } n \leftarrow \left\{ x \middle| x \in N^p \land x \not\in^{\langle \rangle} s'. \texttt{nonces} \right\} \\ \textbf{let } s'. \texttt{nonces} := s'. \texttt{nonces} + {}^{\langle \rangle} n \end{array}$ 2:
- 3:
- return n, s'4:
- 5: end function

The following function, GETNAVIGABLEWINDOW, is called by the browser to determine the window that is *actually* navigated when a script in the window $s'.\overline{w}$ provides a window reference for navigation (e.g., for opening a link). When it is given a window reference (nonce) window, GETNAVIGABLEWINDOW returns a pointer to a selected window term in s':

- If window is the string BLANK, a new window is created and a pointer to that window is returned.
- If window is a nonce (reference) and there is a window term with a reference of that value in the windows in s', a pointer w' to that window term is returned, as long as the window is navigable by the current window's document (as defined by NavigableWindows above).

In all other cases, \overline{w} is returned instead (the script navigates its own window).

Algorithm 2 Determine window for navigation.

▷ Open a new window when BLANK is used 2: if $window \equiv$ BLANK then

^{1:} function GETNAVIGABLEWINDOW(\overline{w} , window, s')

```
let n, s' := \mathsf{TAKENONCE}(s')
 3:
 4:
                  let w' := \langle n, \langle \rangle, s'.\overline{w}.\texttt{nonce} \rangle
                  \mathbf{let} \ s'.\mathtt{windows} := s'.\mathtt{windows} \ +^{\langle\rangle} \ w'
 5:
                   \hookrightarrow and let \overline{w}' be a pointer to this new element in s'
 6:
                  return (\overline{w}', s')
 7:
            end if
            let \overline{w}' \leftarrow \mathsf{NavigableWindows}(\overline{w}, s') such that s'.\overline{w}'.\mathsf{nonce} \equiv window
 8:
             \hookrightarrow if possible; otherwise return (\overline{w}, s')
 9:
            return (\overline{w'}, s')
10: end function
```

The following function takes a window reference as input and returns a pointer to a window as above, but it checks only that the active documents in both windows are same-origin. It creates no new windows.

Algorithm 3 Determine same-origin window.

function GETWINDOW(w, window, s')
 let w' ← Subwindows(s') such that s'.w'.nonce ≡ window
 if possible; otherwise return (w, s')
 if s'.w'.activedocument.origin ≡ s'.w.activedocument.origin then
 return (w', s')
 end if
 return (w, s')
 end function

The next function is used to stop any pending requests for a specific window. From the pending requests and pending DNS requests it removes any requests with the given window reference n.

Algorithm 4 Cancel pending requests for given window.

1: function CANCELNAV(n, s')

```
2: remove all \langle n, req, key, f \rangle from s'.pendingRequests for any req, key, f
```

- 3: remove all $\langle x, \langle n, message, protocol \rangle \rangle$ from s'.pendingDNS
 - \hookrightarrow for any x, message, protocol return s'
- 4: return s'5: end function

The following function takes an HTTP request *message* as input, adds cookie and origin headers to the message, creates a DNS request for the hostname given in the request and stores the request in s'.pendingDNS until the DNS resolution finishes. For normal HTTP requests, *reference* is a window reference. For XHRs, *reference* is a value of the form $\langle document, nonce \rangle$ where *document* is a document reference and *nonce* is some nonce that was chosen by the script that initiated the request. *protocol* is either P or S. *origin* is the origin header value that is to be added to the HTTP request.

Algorithm 5 Prepare headers, do DNS resolution, save message.

1: function SEND(reference, message, protocol, origin, s')

```
2: if message.host \in^{\langle\rangle} s'.sts then
```

 $\mathbf{let} \ protocol := \mathtt{S}$ 3: 4: end if let $cookies := \langle \{ \langle c.name, c.content.value \rangle | c \in \rangle s'.cookies [message.host] \rangle$ 5: $\hookrightarrow \land (c.\texttt{content.secure} \implies (protocol = \texttt{S})) \} \rangle$ **let** *message*.headers[Cookie] := *cookies* 6: if $origin \not\equiv \bot$ then 7: 8: **let** *message*.headers[Origin] := *origin* end if 9: let $n, s' := \mathsf{TAKENONCE}(s')$ 10:let s'.pendingDNS[n] := $\langle reference, message, protocol \rangle$ 11: **stop** {(s'.DNSaddress : a : (DNSResolve, host, n)}, s'12:13: end function

The following two functions have informally been described in Section 2.5.

The function RUNSCRIPT performs a script execution step of the script in the document $s'.\overline{d}$ (which is part of the window $s'.\overline{w}$). A new script and document state is chosen according to the relation defined by the script and the new script and document state is saved. Afterwards, the *command* that the script issued is interpreted. Note that **for each** (Line 13) works in a non-deterministic order.

Algorithm 6 Execute a script.

1: function RUNSCRIPT($\overline{w}, \overline{d}, s'$) 2: let $n, s' := \mathsf{TAKENONCE}(s')$ 3: let tree := $Clean(s', s', \overline{d})$ $\texttt{let } cookies := \langle \{ \langle c.\texttt{name}, c.\texttt{content.value} \rangle | c \in {}^{\langle \rangle} s'.\texttt{cookies} [s'.\overline{d}.\texttt{origin.host}] \rangle$ 4: $\hookrightarrow \land c.\texttt{content.httpOnly} = \bot$ $\hookrightarrow \land (c.\texttt{content.secure} \implies (s'.\overline{d}.\texttt{origin.protocol} \equiv \texttt{S}))\}\rangle$ let $tlw \leftarrow s'$.windows such that tlw is the top-level window containing \overline{d} 5: 6: let $sessionStorage := s'.sessionStorage [\langle s'.d.origin, tlw.nonce \rangle]$ let localStorage := s'.localStorage [s'.d.origin]7: let secret := s'.secrets $[s'.\overline{d}.origin]$ 8: let nonces be an infinite subset of $\left\{ x \mid x \in N^p \land x \notin (x) \right\}$ 9: let $R \leftarrow \text{script}^{-1}(s'.\overline{d}.\text{script})$ 10: let $in := \langle tree, s'.\overline{d}.nonce, s'.\overline{d}.scriptstate, s'.\overline{d}.scriptinput, cookies,$ 11: \hookrightarrow localStorage, sessionStorage, s'.ids, secret let $state' \leftarrow T_{\mathcal{N}},$ 12: \hookrightarrow cookies' \leftarrow Cookies, \hookrightarrow localStorage' $\leftarrow T_{\mathcal{N}}$, \hookrightarrow command $\leftarrow T_{\mathcal{N}}$, \hookrightarrow out := $\langle state', cookies', localStorage', sessionStorage', command \rangle$ \hookrightarrow such that $((in, nonces), out) \in R$. for each $n \in d_{\mathcal{N}}(\langle in, out \rangle) \cap N^p$ do 13:let s'.nonces := s'.nonces + $\langle \rangle$ n 14: 15:end for let s'.cookies $[s'.\overline{d}.origin.host]$ 16: $\hookrightarrow := \langle \mathsf{CookieMerge}(s'.\mathsf{cookies}[s'.\overline{d}.\mathsf{origin.host}], cookies') \rangle$ let s'.localStorage $[s'.\overline{d}.origin] := localStorage'$ 17:let s'.sessionStorage [$\langle s'.\overline{d}.origin, tlw.nonce \rangle$] := sessionStorage' 18:

```
let s'.\overline{d}.scriptstate := state'
19:
20:
          switch command do
21:
                case \langle \rangle
22:
                    stop \{\}, s'
                case \langle \text{HREF}, url, hrefwindow \rangle^2
23:
24:
                    let \overline{w}', s' := \mathsf{GETNAVIGABLEWINDOW}(\overline{w}, hrefwindow, s')
25:
                    let req := \langle \texttt{HTTPReq}, n, \texttt{GET}, url.\texttt{host}, url.\texttt{path}, \langle \rangle, url.\texttt{params}, \langle \rangle \rangle
26:
                    let s' := \mathsf{CANCELNAV}(s'.\overline{w}'.\texttt{nonce}, s')
                    \mathsf{SEND}(s'.\overline{w}'.\texttt{nonce}, \ req, \ url.\texttt{protocol}, \ \bot, \ s')
27:
                case \langle IFRAME, url, window \rangle
28:
                    let \overline{w}', s' := \mathsf{GETWINDOW}(\overline{w}, window, s')
29:
30:
                    let req := \langle \texttt{HTTPReq}, n, \texttt{GET}, url.\texttt{host}, url.\texttt{path}, \langle \rangle, url.\texttt{params}, \langle \rangle \rangle
31:
                    let n, s' := \mathsf{TAKENONCE}(s')
32:
                    let w' := \langle n, \langle \rangle, \bot \rangle
                    let s'.\overline{w}'.activedocument.subwindows
33:
                     \hookrightarrow := s'.\overline{w}'.activedocument.subwindows + ^{\langle\rangle}w'
34:
                    SEND(n, req, url.protocol, \perp, s')
                case \langleFORM, url, method, data, hrefwindow\rangle
35:
                    if method \notin {GET, POST} then <sup>3</sup>
36:
37:
                          stop \{\}, s'
                    end if
38:
                    let \overline{w}', s' := \mathsf{GETNAVIGABLEWINDOW}(\overline{w}, hrefwindow, s')
39:
40:
                    if method = GET then
                         let body := \langle \rangle
41:
42:
                          let params := data
43:
                          let origin := \perp
44:
                    else
45:
                          let body := data
                          let params := url.params
46:
                         let origin := s'.\overline{d}.origin
47:
                    end if
48:
                    let req := \langle \texttt{HTTPReq}, n, method, url.\texttt{host}, url.\texttt{path}, \langle \rangle, params, body \rangle
49:
50:
                    let s' := \mathsf{CANCELNAV}(s'.\overline{w}'.\mathsf{nonce}, s')
                    SEND(s'.\overline{w}'.nonce, req, url.protocol, origin, s')
51:
52:
                case (SETSCRIPT, window, script)
                    let \overline{w}', s' := \mathsf{GETWINDOW}(\overline{w}, window, s')
53:
54:
                    let s'.\overline{w}'.activedocument.script := script
                    stop {}, s'
55:
                case (SETSCRIPTSTATE, window, scriptstate)
56:
                    let \overline{w}', s' := \mathsf{GETWINDOW}(\overline{w}, window, s')
57:
                    let s'.\overline{w}'.activedocument.scriptstate := scriptstate
58:
59:
                    stop \{\}, s'
60:
                case (XMLHTTPREQUEST, url, method, data, xhrreference)
                    if method \in \{CONNECT, TRACE, TRACK\} then
61:
```

 2 See the definition of URLs in Appendix B.2.

³The working draft for HTML5 allowed for DELETE and PUT methods in HTML5 forms. However, these have since been removed. See http://www.w3.org/TR/2010/WD-html5-diff-20101019/#changes-2010-06-24.
```
stop {}, s'
62:
63:
                       end if
                      if url.host \neq s'.\overline{d}.origin.host
64:
                        \hookrightarrow \forall url. protocol \neq s'. \overline{d}. origin. protocol then
65:
                            stop \{\}, s'
                       end if
66:
                       if method \in \{GET, HEAD\} then
67:
                            let data := \langle \rangle
68:
                            let origin := \bot
69:
70:
                       else
                            let origin := s'.\overline{d}.origin
71:
72:
                       end if
73:
                      let req := \langle \texttt{HTTPReq}, n, method, url.\texttt{host}, url.\texttt{path}, url.\texttt{params}, data \rangle
                      SEND(\langle s'.\overline{d}.nonce, xhrreference \rangle, req, url.protocol, origin, s')
74:
                 case \langle BACK, window \rangle<sup>4</sup>
75:
                       let \overline{w}', s' := \mathsf{GETNAVIGABLEWINDOW}(\overline{w}, window, s')
76:
                       77:
                            let s'.\overline{w'}.\texttt{documents}.\overline{j}.\texttt{active} := \bot
78:
                            let s'.\overline{w'}.\texttt{documents}.(\overline{j}-1).\texttt{active} := \top
79:
                            let s' := \mathsf{CANCELNAV}(s'.\overline{w}'.\texttt{nonce}, s')
80:
81:
                       end if
82:
                       stop \{\}, s'
                 case \langle FORWARD, window \rangle
83:
                       let \overline{w}', s' := \mathsf{GETNAVIGABLEWINDOW}(\overline{w}, window, s')
84:
                        \mathbf{if} \ \exists \ \overline{j} \in \mathbb{N} \ \mathbf{such} \ \mathbf{that} \ s'. \overline{w'}. \mathtt{documents}. \overline{j}. \mathtt{active} \equiv \top \\
85:
                        \hookrightarrow \land s'.\overline{w'}.\texttt{documents}.(\overline{j}+1) \in \texttt{Documents} then
86:
                            let s'.\overline{w'}.\texttt{documents}.\overline{j}.\texttt{active} := \bot
87:
                            let s'.\overline{w'}.\texttt{documents}.(\overline{j}+1).\texttt{active} := \top
                            let s' := \mathsf{CANCELNAV}(s'.\overline{w}'.\texttt{nonce}, s')
88:
89:
                       end if
90:
                      stop \{\}, s'
                 case (CLOSE, window)
91:
                       let \overline{w}', s' := \mathsf{GETNAVIGABLEWINDOW}(\overline{w}, window, s')
92:
                      remove s'.\overline{w'} from the sequence containing it
93:
94:
                       stop \{\}, s'
95:
                 case (POSTMESSAGE, window, message, origin)
                      let \overline{w}' \leftarrow \mathsf{Subwindows}(s') such that s'.\overline{w}'.\texttt{nonce} \equiv window
96:
                      if \exists \overline{j} \in \mathbb{N} such that s'.\overline{w'}.documents.\overline{j}.active \equiv \top
97:
                        \hookrightarrow \land (\textit{origin} \not\equiv \bot \implies s'.\overline{w'}.\texttt{documents}.\overline{j}.\texttt{origin} \equiv \textit{origin}) \texttt{ then}
98:
                            let s'.\overline{w'}.documents.\overline{j}.scriptinput
                             \hookrightarrow := s'.\overline{w'}.documents.\overline{j}.scriptinput
                             \hookrightarrow +^{\langle\rangle} \langle \text{POSTMESSAGE}, s'. \overline{w}. \text{nonce}, s'. \overline{d}. \text{origin}, message \rangle
99:
                       end if
100: end function
```

⁴Note that navigating a window using the back/forward buttons does not trigger a reload of the affected documents. While real world browser may chose to refresh a document in this case, we assume that the complete state of a previously viewed document is restored.

The function PROCESSRESPONSE is responsible for processing an HTTP response (*response*) that was received as the response to a request (*request*) that was sent earlier. In *reference*, either a window or a document reference is given (see explanation for Algorithm 5 above). Again, *protocol* is either P or S.

The function first saves any cookies that were contained in the response to the browser state, then checks whether a redirection is requested (Location header). If that is not the case, the function creates a new document (for normal requests) or delivers the contents of the response to the respective receiver (for XHR responses).

Algorithm 7 Process an HTTP response.

1:	function PROCESSRESPONSE(response, reference, request, protocol, s')
2:	let $n, s' := TAKENONCE(s')$
3:	${f if}$ Set-Cookie \in $response$ headers then
4:	$\mathbf{for}\;\mathbf{each}\;c\in^{\langle angle}\;response.\mathtt{headers}\;[\mathtt{Set-Cookie}],c\inCookies\;\mathbf{do}$
5:	$\texttt{let} \ s'.\texttt{cookies} \left[request.\texttt{url.host} \right]$
	$\hookrightarrow \hspace{0.1 cm} := AddCookie(s'.\texttt{cookies}\left[\mathit{request.url.host}\right], c)$
6:	end for
7:	end if
8:	if Strict-Transport-Security \in response.headers \land protocol \equiv S then
9:	$\texttt{let} \ s'.\texttt{sts} := s'.\texttt{sts} + \checkmark \ request.\texttt{host}$
10:	end if
11:	$\textbf{if Location} \in \textit{response.headers} \land \textit{response.status} \in \{303, 307\} \textbf{ then }^5$
12:	$let \ url := response.headers [Location]$
13:	let $method' := request.method$ ⁶
14:	$\mathbf{let} \ body' := request.\mathtt{body}''$
15:	$\mathbf{if} \; \texttt{Origin} \in request. \mathtt{headers \; \mathbf{then}}$
16:	$\texttt{let } \textit{origin} := \langle \textit{request.headers[Origin]}, \langle \textit{request.host}, \textit{protocol} \rangle \rangle$
17:	else
18:	let $origin := \bot$
19:	end if
20:	$\mathbf{if} \ response.\mathtt{status} \equiv 303 \land request.\mathtt{method} \not\in \{\mathtt{GET}, \mathtt{HEAD}\} \ \mathbf{then}$
21:	let $method' := GET$
22:	$\mathbf{let} \ body' := \langle \rangle$
23:	end if

⁵The RFC for HTTPbis (currently in draft status), which obsoletes RFC 2616, does not specify whether a POST/DELETE/etc. request that was answered with a status code of 301 or 302 should be rewritten to a GET request or not ("for historic reasons" that are detailed in Section 7.4.). As the specification is clear for the status codes 303 and 307 (and most browsers actually follow the specification in this regard), we focus on modeling these.

⁶While the standard demands that users confirm redirections of non-safe-methods (e.g., POST), we assume that users generally confirm these redirections.

⁷If, for example, a GET request is redirected and the original request contained a body, this body is preserved, as HTTP allows for payloads in messages with all HTTP methods, except for the TRACE method (a detail which we omit). Browsers will usually not send body payloads for methods that do not specify semantics for such data in the first place.

```
if \nexists \overline{w} \in \text{Subwindows}(s') such that s'.\overline{w}.\text{nonce} \equiv reference then
24:
                                                                                                    \triangleright Do not redirect XHRs.
25:
                       stop \{\}, s
26:
                  end if
                 let req := \langle \text{HTTPReq}, n, method', url.host, url.path, \langle \rangle, url.params, body' \rangle
27:
28:
                 SEND(reference, req, url.protocol, origin, s')
29:
            end if
           if \exists \overline{w} \in \text{Subwindows}(s') such that s'.\overline{w}.\texttt{nonce} \equiv reference then
30:
                                                                                                              \triangleright normal response
31:
                 let script := \pi_1(response.body)
32:
                 let scriptstate := \pi_2(response.body)
33:
                 let d := \langle n, \langle request.host, request.protocol \rangle, script, scriptstate, \langle \rangle, \langle \rangle, \top \rangle
34:
                 if s'.\overline{w}.\texttt{documents} \equiv \langle \rangle then
35:
                       let s'.\overline{w}.\texttt{documents} := \langle d \rangle
36:
                  else
                       let \overline{i} \leftarrow \mathbb{N} such that s'.\overline{w}.documents.\overline{i}.active \equiv \top
37:
38:
                       let s'.\overline{w}.\texttt{documents}.\overline{i}.\texttt{active} := \bot
                       remove s'.\overline{w}.documents.(\overline{i}+1) and all following documents
39:
                        \hookrightarrow from s'.\overline{w}.\texttt{documents}
                       let s'.\overline{w}.documents := s'.\overline{w}.documents +\langle \rangle d
40:
                 end if
41:
                 stop \{\}, s'
42:
            else if \exists \overline{w} \in \text{Subwindows}(s'), \overline{d} \text{ such that } s'.\overline{d}.\text{nonce} \equiv \pi_1(reference)
43:
             \hookrightarrow \land s'.\overline{d} = s'.\overline{w}.activedocument then
                                                                                                    \triangleright process XHR response
                 let s'.\overline{d}.\texttt{scriptinput} := s'.\overline{d}.\texttt{scriptinput} + \langle \rangle
44:
                       \langle \mathsf{P}, response.body, \pi_2(reference) \rangle
            end if
45:
46: end function
```

Main Algorithm. This is the main algorithm of the browser relation. It was already presented informally in Section 2.5 and follows the structure presented there. It receives the message m as input, as well as a, f and s as above.

Algorithm 8 Main Algorithm

Input: (a:f:m), s1: let s' := s2: if $s.isCorrupted \equiv FULLCORRUPT$ then 3: let s'.pendingRequests := $\langle m, s.$ pendingRequests \rangle \triangleright Collect incoming messages 4: let $m' \leftarrow d_{N^p}(s')$ let $a' \leftarrow \mathsf{IPs}$ 5: **stop** $\{(a':a:m')\}, s'$ 6: 7: else if $s.isCorrupted \equiv CLOSECORRUPT$ then let s'.pendingRequests := $\langle m, s.$ pendingRequests \rangle 8: \triangleright Collect incoming messages let $N^{\text{clean}} := N^p \setminus \{n | n \in {}^{\langle \rangle} s.\texttt{nonces}\}$ 9: let $m' \leftarrow d_{N^{\text{clean}}}(s')$ 10:let $a' \leftarrow \mathsf{IPs}$ 11: let s'.nonces := s.nonces 12:

```
stop \{(a':a:m')\}, s'
13:
14: end if
15: let n, s' := \mathsf{TAKENONCE}(s')
16: if m \equiv \text{TRIGGER then}
                                                                                     \triangleright A special trigger message.
          let switch \leftarrow \{1, 2\}
17:
          if switch \equiv 1 then
18:
                                                                                                  \triangleright Run some script.
               let \overline{w} \leftarrow \mathsf{Subwindows}(s') such that s'.\overline{w}.\mathsf{documents} \neq \langle \rangle
19:
                \hookrightarrow if possible; otherwise stop {}, s'
               let \overline{d} := \overline{w} + \langle \rangle activedocument
20:
               RUNSCRIPT(\overline{w}, \overline{d}, s')
21:
22:
          else if switch \equiv 2 then
                                                                                     \triangleright Create some new request.
23:
               let w' := \langle n, \langle \rangle, \bot \rangle
               let s'.windows := s'.windows +\langle \rangle w'
24:
25:
               let protocol \leftarrow {P, S}
26:
               let host \leftarrow Doms
27:
               let path \leftarrow \mathbb{S}
               let parameters \leftarrow [\mathbb{S} \times \mathbb{S}]
28:
               let n', s' := \mathsf{TAKENONCE}(s')
29:
               let req := \langle \text{HTTPReq}, n', \text{GET}, host, path, \langle \rangle, parameters, \langle \rangle \rangle
30:
31:
               SEND(n, req, protocol, \perp, s')
          end if
32:
33: else if m \equiv \texttt{FULLCORRUPT} then
                                                                                  \triangleright Request to corrupt browser
          let s'.isCorrupted := FULLCORRUPT
34:
          stop {}, s'
35:
36: else if m \equiv \text{CLOSECORRUPT} then
                                                                                                \triangleright Close the browser
37:
          let s'.secrets := \langle \rangle
          let s'.windows := \langle \rangle
38:
          let s'.pendingDNS := \langle \rangle
39:
          let s'.pendingRequests := \langle \rangle
40:
          let s'.sessionStorage := \langle \rangle
41:
          let s'.cookies \subset^{\langle\rangle} Cookies such that
42:
           \hookrightarrow (c \in {}^{\langle} s'.\texttt{cookies}) \Longleftrightarrow (c \in {}^{\langle} s.\texttt{cookies} \land c.\texttt{content.session} \equiv \bot)
          let s'.isCorrupted := CLOSECORRUPT
43:
          stop \{\}, s'
44:
45: else if \exists \langle reference, request, key, f \rangle \in \langle \rangle s'.pendingRequests
      \hookrightarrow such that \pi_1(\operatorname{dec}_{\mathsf{s}}(m, key)) \equiv \operatorname{HTTPResp} then \triangleright Encrypted HTTP response
          let m' := \operatorname{dec}_{s}(m, key)
46:
47:
          if m'.nonce \neq request.nonce then
48:
               stop \{\}, s
49:
          end if
          remove \langle reference, request, key, f \rangle from s'.pendingRequests
50:
          \mathsf{PROCESSRESPONSE}(m', reference, request, S, s')
51:
52: else if \pi_1(m) \equiv \text{HTTPResp} \land \exists \langle reference, request, \bot, f \rangle \in \langle \rangle s'.\text{pendingRequests}
      \hookrightarrow such that m'.nonce \equiv request.key then
          remove \langle reference, request, \bot, f \rangle from s'.pendingRequests
53:
          \mathsf{PROCESSRESPONSE}(m, reference, request, P, s')
54:
55: else if m \in \mathsf{DNSResponses then}
                                                                                      \triangleright Successful DNS response
          if m.\texttt{nonce} \notin s.\texttt{pendingDNS then}
56:
57:
                stop \{\}, s
58:
          end if
```

```
59:
        let \langle reference, message, protocol \rangle := s.pendingDNS[m.nonce]
60:
        if protocol \equiv S then
             let k, s' := \mathsf{TAKENONCE}(s')
61:
62:
             let s'.pendingRequests := s'.pendingRequests
              \hookrightarrow +^{\langle \rangle} \langle reference, message, k, m.result \rangle
             let message := enc_a(\langle message, k \rangle, s'.keyMapping[message.host])
63:
64:
         else
             let s'.pendingRequests := s'.pendingRequests
65:
              \hookrightarrow +^{\langle\rangle} \langle reference, message, \bot, m.result \rangle
66:
         end if
        let s'.pendingDNS := s'.pendingDNS - m.nonce
67:
68:
        stop {(m.result:a:message)}, s'
69: else
70:
        stop \{\}, s
71: end if
```

D General Security Properties of the Web Model

We now formally state and prove the general application independent security properties of the web which in Section 3 have been sketched only.

Let $\mathcal{Web} = (\mathcal{W}, \mathcal{S}, \mathsf{script}, E_0)$ be a web system. In the following, we write $s_x = (S_x, E_x)$ for the states of a web system.

Definition 30. In what follows, given an atomic process p and a message m, we say that p emits m in a run $\rho = s_0, s_1, \ldots$ if there is a processing step of the form

$$s_{u-1} \xrightarrow[p \to E]{} s_u$$

for some $u \in \mathbb{N}$, a set of events E and some addresses x, y with $(x:y:m) \in E$.

Definition 31. We say that a term t is derivably contained in (a term) t' for (a set of DY processes) P (in a processing step $s_i \rightarrow s_{i+1}$ of a run $\rho = s_0, s_1, \ldots$) if t is derivable from t' with the knowledge available to P, i.e.,

$$t \in d_{\eta}(\{t'\} \cup \varsigma) \text{ with } \eta := \bigcup_{p \in P} N^p \text{ and } \varsigma := \bigcup_{p \in P, j \leq i} S_j(p) \text{ .}$$

Definition 32. We say that a set of processes P leaks a term t (in a processing step $s_i \rightarrow s_{i+1}$) to a set of processes P' if there exists a message m that is emitted (in $s_i \rightarrow s_{i+1}$) by some $p \in P$ and t is derivably contained in m for P' in the processing step $s_i \rightarrow s_{i+1}$. If we omit P', we define $P' := \mathcal{W} \setminus P$. If P is a set with a single element, we omit the set notation.

Definition 33. We say that an DY process p created a message m (at some point) in a run if m is derivably contained in a message emitted by p in some processing step and if there is no earlier processing step where m is derivably contained in a message emitted by some DY process p'.

Definition 34. We say that a browser b accepted a message (as a response to some request) if the browser decrypted the message (if it was an HTTPS message) and called the function PROCESSRESPONSE, passing the message and the request (see Algorithm 7).

Definition 35. We say that an atomic DY process p knows a term t in some state s = (S, E) of a run if it can derive the term from its knowledge, i.e., $t \in d_{N^p}(S(p))$.

Definition 36. We say that a script initiated a request r if a browser triggered the script (in Line 12 of Algorithm 6) and the first component of the command output of the script relation is either HREF, IFRAME, FORM, or XMLHTTPREQUEST such that the browser issues the request r in the same step as a result.

For a run $\rho = s_0, s_1, \ldots$ of any *Web*, we state the following lemmas:

Lemma 1. If in the processing step $s_i \to s_{i+1}$ of a run ρ of Web an honest browser b (I) emits an HTTPS request of the form

$$m = \operatorname{enc}_{\mathsf{a}}(\langle req, k \rangle, \operatorname{pub}(k'))$$

(where req is an HTTP request, k is a nonce (symmetric key), and k' is the private key of some other DY process u), and (II) in the initial state s_0 the private key k' is only known to u, and (III) u never leaks k', then all of the following statements are true:

- 1. There is no state of Web where any party except for u knows k', thus no one except for u can decrypt req.
- 2. If there is a processing step $s_j \to s_{j+1}$ where the browser b leaks k to $\mathcal{W} \setminus \{u, b\}$ there is a processing step $s_h \to s_{h+1}$ with h < j where u leaks the symmetric key k to $\mathcal{W} \setminus \{u, b\}$ or the browser is fully corrupted in s_j .
- 3. The value of the host header in req is the domain that is assigned the public key pub(k') in the browsers' keymapping s_0 .keymapping (in its initial state).
- 4. If b accepts a response (say, m') to m in a processing step $s_j \rightarrow s_{j+1}$ and b is honest in s_j and u did not leak the symmetric key k to $\mathcal{W} \setminus \{u, b\}$ prior to s_j , then u created the HTTPS response m' to the HTTPS request m, i.e., the nonce of the HTTP request req is not known to any atomic process p, except for the atomic process b and u.

Proof. (1). follows immediately from the condition. If k' is initially only known to u and u never leaks k', i.e., even with the knowledge of all nonces (except for those of u), k' can never be derived from any network output of u, k' cannot be known to any other party. Thus, nobody except for u can derive req from m.

(2). We assume that b leaks k to $\mathcal{W} \setminus \{u, b\}$ in the processing step $s_j \to s_{j+1}$ without u prior leaking the key k to anyone except for u and b and that the browser is not fully corrupted in s_j , and lead this to a contradiction.

The browser is honest in s_i . From the definition of the browser b, we see that the key k is always chosen from a fresh set of nonces (Line 61 of Algorithm 7)

that are not used anywhere else. Further, the key is stored in the browser's state in *pendingRequests*. The information from *pendingRequests* is not extracted or used anywhere else (in particular it is not accessible by scripts). If the browser becomes closecorrupted prior to s_j (and after s_i), the key cannot be used anymore (compare Line 9 of Algorithm 8). Hence, b does not leak k to any other party in s_j (except for u and b). This proves (2).

(3). Per the definition of browsers (Algorithm 8), a host header is always contained in HTTP requests by browsers. From Line 63 of Algorithm 8 we can see that the encryption key for the request *req* was chosen using the host header of the message. It is chosen from the *keymapping* in the browser's state, which is never changed during ρ . This proves (3).

(4). An HTTPS response m' that is accepted by b as a response to m has to be encrypted with k. The nonce k is stored by the browser in the *pendingRequests* state information. The browser only stores freshly chosen nonces there (i.e., the nonces are not used twice, or for other purposes than sending one specific request). The information cannot be altered afterwards (only deleted) and cannot be read except when the browser checks incoming messages. The nonce k is only known to u (which did not leak it to any other party prior to s_j) and b (which did not leak it either, as u did not leak it and b is honest, see (2)). The browser b cannot send responses. This proves (4).

Corollary 1. In the situation of Lemma 1, as long as u does not leak the symmetric key k to $\mathcal{W} \setminus \{u, b\}$ and the browser does not become fully corrupted, k is not known to any DY process $p \notin \{b, u\}$ (i.e., $\nexists s' = (S', E') \in \rho$: $k \in d_{N^p}(S'(p))$).

Lemma 2. If for some $s_i \in \rho$ an honest browser b has a document d in its state $S_i(b)$.windows with the origin $\langle dom, \mathbf{S} \rangle$ where $dom \in \mathsf{Domain}$, and $S_i(b)$.keyMapping $[dom] \equiv \mathsf{pub}(k)$ with $k \in \mathcal{N}$ being a private key, and there is only one DY process p that knows the private key k in all s_j , $j \leq i$, then b extracted (in Line 33 in Algorithm 7) the script in that document from an HTTPS response that was created by p.

Proof. The origin of the document d is set only once: In Line 33 of Algorithm 7. The values (domain and protocol) used there stem from the information about the request (say, req) that led to loading of d. These values have been stored in *pendingRequests* between the request and the response actions. The contents of *pendingRequests* are indexed by freshly chosen nonces and can never be altered or overwritten (only deleted when the response to a request arrives). The information about the request req was added to *pendingRequests* in Line 62 (or Line 65 which we can exclude as we will see later) of Algorithm 8. In particular, the request was an HTTPS request iff a (symmetric) key was added to the information in *pendingRequests*. When receiving the response to req, it is checked against that information and accepted only if it is encrypted with the proper key and contains the same nonce as the request (say, n). Only then the protocol part of the origin of the newly created document becomes **S**. The domain part

of the origin (in our case dom) is taken directly from the *pendingRequests* and is thus guaranteed to be unaltered.

From Line 63 of Algorithm 8 we can see that the encryption key for the request req was actually chosen using the host header of the message which will finally be the value of the origin of the document d. Since b therefore selects the public key $S_i(b)$.keyMapping $[dom] = S_0(b)$.keyMapping $[dom] \equiv pub(k)$ for p (the key mapping cannot be altered during a run), we can see that req was encrypted using a public key that matches a private key which is only (if at all) known to p. With Lemma 1 we see that the symmetric encryption key for the response, k, is only known to b and the respective web server. The same holds for the nonce n that was chosen by the browser and included in the request. Thus, no other party than p can encrypt a response that is accepted by the browser b and which finally defines the script of the newly created document.

Lemma 3. If in a processing step $s_i \to s_{i+1}$ of a run ρ of Web an honest browser b issues an HTTP(S) request with the Origin header value $\langle \text{dom}, \mathbf{S} \rangle$ where and $S_i(b).\texttt{keyMapping}[\text{dom}] \equiv \texttt{pub}(k)$ with $k \in \mathcal{N}$ being a private key, and there is only one DY process p that knows the private key k in all s_j , $j \leq i$, then that request was initiated by a script that b extracted (in Line 33 in Algorithm 7) from an HTTPS response that was created by p.

Proof. First, we can see that the request was initiated by a script: As it contains an origin header, it must have been a POST request (see the browser definition in Appendix C.2). POST requests can only be initiated in Lines 51, 74 of Algorithm 6 and Line 28 of Algorithm 7. In the latter instance (Location header redirect), the request contains at least two different origins, therefore it is impossible to create a request with exactly the origin $\langle dom, \mathbf{S} \rangle$ using a redirect. In the other two cases (FORM and XMLHTTPRequest), the request was initiated by a script.

The Origin header of the request is defined by the origin of the script's document. With Lemma 2 we see that the content of the document, in particular the script, was indeed provided by p.

E Step-By-Step Description of BrowserID (Primary IdP)

We now present additional details of the implementation of BrowserID. While the basic steps have been shown in Section 4.2, we will now again refer to Figure 3 and provide a step-by-step description. As above, for brevity of presentation, we focus on the main login flow without the CIF, and we leave out steps for fetching additional resources (like JavaScript files) and some less relevant postMessages and XHRs. Also, we assume that a typical IdP implementation like the example implementation provided by Mozilla is used.

We emphasize, however, that our formal model of BrowserID with primary IdPs (cf. Section F) closely follows the full BrowserID implementation.

We (again) assume that the user uses a "fresh" browser, i.e., the user has not been logged in before. The user has already opened a document of some RP (RP-Doc) in her browser. RP-Doc includes a JavaScript file, which provides the BrowserID API. The user is now about to click on a login button in order to start a BrowserID login.

Phase (i). After the user has clicked on the login button, RP-Doc opens a new browser window, the login dialog (LD) 1. The document of LD is loaded from LPO 2. Now, LD sends a *ready* postMessage 3 to its opener, which is RP-Doc. RP-Doc then responds by sending a *request* postMessage 4. This postMessage may contain additional information like a name or a logo of RP-Doc. LD then fetches the so-called *session context* from LPO using 5. The session context contains information about whether the user is already logged in at LPO, which, by our assumption, is not the case at this point. The session context also contains an XSRF protection token which will be sent in all subsequent POST requests to LPO. Also, an httpOnly cookie called browserid_state is set, which contains an LPO session identifier. Now, the user is prompted to enter her email address (login email address), which she wants to use to log in at RP [6]. LD sends the login email address to LPO via an XHR 7, in order to get information about the IdP the email address belongs to. The information from this so-called *sup*port document may be cached at LPO for further use. LPO extracts the domain part of the login email address and fetches an information document 8 from a fixed path (/.well-known/browserid) at the IdP. This document contains the public key of IdP, and two paths, the provisioning path and the authentication path at IdP. These paths will be used later in the login process by LD. LPO converts these paths into URLs and sends them in its response 9 to the requesting XHR 7.

Phase ii). As there is no record about the login email address in the localStorage under the origin of LPO, the LD now tries to get a UC for this identity. For that to happen, the LD creates a new iframe, the *provisioning iframe* (PIF) 10. The PIF's document is loaded 11 from the provisioning URL LD has just received before in 9. The PIF now interacts with the LD via postMessages 12. As the user is currently not logged in, the PIF tells the LD that the user is not authenticated yet. This also indicates to the LD that the PIF has finished operation. The LD then closes the PIF 13.

Phase (iii). Now, the LD saves the login email address in the localStorage indexed by a fresh nonce. This nonce is stored in the sessionStorage to retrieve the email address later from the localStorage again. Next, the LD navigates itself to the authentication URL it has received in ③. The loaded document now interacts with the user and the IdP [14] in order to establish some authenticated session depending on the actual IdP implementation, which is out of scope of the BrowserID standard. For example, during this authentication procedure, the IdP may issue some session cookie.

Phase (iv). After the authentication to the IdP has been completed, the authentication document navigates the LD to the LD URL again. The LD's document is fetched again from LPO and the login process starts over. The following steps

are similar to Phase (i): The ready and request postMessages are exchanged and the session context is fetched. As the user has not been authenticated to LPO yet, the session context still contains the same information as above in 5. Now, the user is not prompted to enter her email address again. The email address is fetched from the localStorage under the index of the nonce stored in the sessionStorage. Now, the address information is requested again from LPO.

Phase v. As there still is no UC belonging to the login email address in the localStorage, the PIF is created again. As the user now has established an authenticated session with the IdP, the PIF asks the LD to generate a fresh key pair. After the LD has generated the key pair 15, it stores the key pair in the localStorage (under the origin of LPO) and sends the public key to the PIF as a postMessage 16. The following steps 17–19 are not specified in the BrowserID protocol. Typically, the PIF would send the public key to IdP (via an XHR) 17. The IdP would create the UC 18 and send it back to the PIF 19. The PIF then sends the UC to the LD 20, which stores it in the localStorage. Now, the LD closes the PIF.

Phase (vi). The LD is now able to create a CAP, as it has access to a UC and the corresponding private key in its localStorage. First, LD creates an IA for LPO [21]. The IA and the UC is then combined to a CAP, which is then sent to LPO in an XHR POST message [22]. LPO is now able to verify this CAP with the public key of IdP, which LPO has already fetched and cached before in [8]. If the CAP is valid, LPO considers its session with the user's browser to be authenticated for the email address the UC in the CAP is issued for.

Phase (vii). Now, in [23], the LD fetches a list of email addresses, which LPO considers to be owned by the user. If the login email address would not appear in this list, LD would abort the login process. After this, the LD fetches the address information about the login email address again in [24]. Using this information, LD validates if the UC is signed by the correct party (primary/secondary IdP). Now, LD generates an IA for the sender's origin of the request postMessage [4] (which was repeated in Phase (iv)) using the private key from the localStorage [25] (the IA is generated for the login email address). Also, it is recorded in the localStorage that the user is now logged in at RP with this email address. The LD then combines the IA with the UC stored in the localStorage to the CAP, which is then sent to RP-Doc in the *response* postMessage [26].

This concludes the login process that runs in LD. Afterwards, RP-Doc closes LD $^{\boxed{27}}$.

F Model of BrowserID with Primary IdPs

We now present the full details of our formal model of BrowserID with primary IdPs. We consider ephemeral sessions (the default), which are supposed to last until the browser is closed. We note that the model is with the fixed version of BrowserID (see Section 5.3).

We model the BrowserID system as a web system (in the sense of Section 2). We call a web system $\mathcal{B}I\mathcal{D} = (\mathcal{W}, \mathcal{S}, \text{script}, E_0)$ a *BrowserID web system* if it is of the form described in what follows.

F.1 Outline

The system $\mathcal{W} = \text{Hon} \cup \text{Web} \cup \text{Net}$ consists of the (network) attacker process attacker, the web server for LPO, a finite set B of web browsers, a finite set RP of web servers for the relying parties, and a finite set IDP of web servers for the identity providers, with $\text{Hon} := B \cup \text{RP} \cup \text{IDP} \cup \{\text{LPO}\}$, $\text{Web} := \emptyset$, and $\text{Net} := \{\text{attacker}\}$. DNS servers are assumed to be dishonest, and hence, are subsumed by attacker. More details on the processes in \mathcal{W} are provided below. Figure 6 shows the set of scripts \mathcal{S} and their respective string representations that are defined by the mapping script. The set E_0 contains only the trigger events as specified in Section 2.3.

$s \in \mathcal{S}$	script(s)
R^{att}	att_script
$script_rp_index$	<pre>script_rp_index</pre>
$script_lpo_cif$	<pre>script_lpo_cif</pre>
$script_lpo_ld$	<pre>script_lpo_ld</pre>
$script_idp_pif$	<pre>script_idp_pif</pre>
$script_idp_ad$	<pre>script_idp_ad</pre>

Fig. 6. List of scripts in \mathcal{S} and their respective string representations.

This outlines $\mathcal{B}I\mathcal{D}$. We will now define the DY processes and their addresses, domain names, and secrets in more detail.

F.2 Addresses and Domain Names

The set IPs contains for LPO, attacker, every relying party in RP, every identity provider in IDP, and every browser in B one address each. By addr we denote the corresponding assignment from a process to its address. The set Doms contains one domain for LPO, one for every relying party in RP, a finite set of domains for every identity provider in IDP, and a finite set of domains for attacker. Browsers (in B) do not have a domain.

By addr and dom we denote the assignments from atomic processes to sets of IPs and Doms, respectively. If dom or addr returns a set with only one element, we often write dom(x) or addr(x) to refer to the element.

F.3 Keys and Secrets

The set \mathcal{N} of nonces is partitioned into four sets, an infinite set $N^{\mathcal{W}}$, an infinite set K_{SSL} , an infinite set K_{sign} , and a finite set Secrets. We thus have

$$\mathcal{N} = \underbrace{N^{\mathcal{W}} \ \dot{\cup} \ K_{\rm SSL} \ \dot{\cup} \ K_{\rm sign} \ \dot{\cup} \ \underbrace{\mathsf{Secrets}}_{\rm finite} \ \underbrace{\mathsf{inite}}_{\rm finite} \ \underbrace{\mathsf{inite}}_{\rm finite} \ \underbrace{\mathsf{inite}}_{\rm finite}$$

The set $N^{\mathcal{W}}$ contains the nonces that are available for each DY process in \mathcal{W} . It is partitioned into infinite sets of nonces, one set $N^p \subseteq N^{\mathcal{W}}$ for every $p \in \mathcal{W}$.

The set K_{SSL} contains the keys that will be used for SSL encryption. Let sslkey: Doms $\rightarrow K_{\text{SSL}}$ be an injective mapping that assigns a (different) private key to every domain.

The set K_{sign} contains the keys that will be used by IdPs for signing UCs. Let signkey: $IdPs \rightarrow K_{\text{sign}}$ be an injective mapping that assigns a (different) private key to every identity provider.

The set Secrets $\subseteq \mathcal{N}$ is the set of passwords (secrets) the browsers share with the identity providers.

F.4 Identities

Indentites are email addresses, which consist of a user name and a domain part. For our model, this is defined as follows:

Definition 37. An identity (email address) i is a term of the form $\langle name, domain \rangle$ with $name \in S$ and $domain \in Doms$.

Let ID be the finite set of identities. By ID^y we denote the set $\{\langle name, domain \rangle \in ID \mid domain \in dom(y)\}.$

We say that an ID is governed by the DY process to which the domain of the ID belongs. Formally, we define the mapping governor : $ID \to W$, $\langle name, domain \rangle \mapsto dom^{-1}(domain)$.

The governor of an ID will usually be an IdP, but could also be the attacker. Note that we omit delegation of authority over domains.

We further define UCs, IAs and CAPs formally:

Definition 38. A (valid) user certificate (UC) uc for a user u with email address $id = \langle name, d \rangle$ and public key (verification key) $pub(k_u)$, where $d \in dom(y)$ is a domain of the governor y of id and k_u is the private key (signing key) of u, is a message of the form $uc = sig(\langle name, d \rangle, pub(k_u) \rangle, signkey(y))$.

An (valid) identity assertion (IA) ia for an origin o (e.g., $\langle \text{example.com}, S \rangle$) signed with the key k_u is a message of the form $ia = sig(o, k_u)$.

A certificate assertion pair (CAP) is of the form $\langle uc, ia \rangle$, with uc and ia as above.⁸

⁸Note that the time stamps are omitted both from the UC and the IA. This models that both certificates are valid indefinitely. In reality, they are valid for a certain period of time, as indicated by the time stamps. So our modeling is a safe overapproximation.

Each browser $b \in B$ owns a set of secrets (\in Secrets). Each secret is assigned a set S of IDs for a specific IdP y such that $S \subseteq ID^y$. Browsers have disjoint secrets and secrets have disjoint sets of IDs. The IdPs of the secrets of a browser are disjoint. An *ID* i is owned by a browser b if the identity associated with i belongs to b:

Let ownerOfSecret : Secrets \rightarrow B denote the mapping that assigns to each secret the browser that owns this secret. Let secretOfID : ID \rightarrow Secrets denote the mapping that assigns to each identity the associated secret. Now, we define the mapping ownerOfID : ID \rightarrow B, $i \mapsto$ ownerOfSecret(secretOfID(i)), which assigns to each identity the browser that owns this identity (we say that the identity belongs to the browser).

F.5 Corruption

RPs and IdPs can become corrupted: If they receive the message CORRUPT, they start collecting all incoming messages in their state and (upon triggering) send out all messages that are derivable from their state and collected input messages. We say that an RP or IdP is *honest* if the according part of their state (*s.corrupt*) is \bot , and that they are corrupted otherwise.

We are now ready to define the processes in $\mathcal W$ as well as the scripts in $\mathcal S$ in more detail.

F.6 Processes in \mathcal{W} (Overview)

We first provide an overview of the processes in \mathcal{W} . All processes in \mathcal{W} contain in their initial states all public keys and the private keys of their respective domains (if any). We define $I^p = \{\mathsf{addr}(p)\}$ for all $p \in \mathsf{Hon}$.

Attacker. The attacker process is a network attacker (see Section 2.3), who uses all addresses for sending and listening. All parties use the attacker as a DNS server. See Appendix F.7 for details.

Browsers. Each $b \in B$ is a web browser as defined in Section 2.5. The initial state contains all secrets owned by b, stored under the origin of the respective IdP. See Appendix F.8 for details.

LPO. LPO is a web server that serves important scripts (script_lpo_cif and script_lpo_ld) and manages user sessions. See Appendix F.9 for details.

IdPs. Each IdP is a web server. IdPs are modeled following the example implementation provided by Mozilla. As outlined in Section 4, users can authenticate to the IdP with their credentials. IdP tracks the state of the users with sessions. Authenticated users can receive signed UCs from the IdP. When receiving a special message (CORRUPT) IdPs can become corrupted. Similar to the definition of corruption for the browser, IdPs then start sending out all messages that are derivable from their state.

Relying Parties. A relying party $r \in \mathsf{RP}$ is a web server. The definition of R^r follows the description in Section 4 and the security considerations in [18]

(Cross-site Request Forgery protection, e.g., by checking origin headers, and HTTPS only with STS enabled). RP answers any GET request with the script script_rp_index (see below). When receiving an HTTPS POST message, RP checks (among others) if the message contains a valid CAP. For this purpose, all signing keys of the identity providers (see below) are contained in the initial state of all RPs. If successful, RP responds with an *RP service token for ID i* of the form $\langle n, i \rangle$, where $i \in ID$ is the ID for which the CAP was issued and n is a freshly chosen nonce. The RP r keeps a list of such tokens in its state. Intuitively, a client having such a token can use the service of r for ID i. See Appendix F.10 for details. Just like IdPs, RPs can become corrupted.

F.7 Attacker

As mentioned, the attacker attacker is modeled to be a network attacker as specified in Section 2.3. We allow it to listen to/spoof all available IP addresses, and hence, define $I^{\text{attacker}} = \mathsf{IPs}$. His initial state is $s_0^{\text{attacker}} = \langle attdoms, sslkeys, signkeys \rangle$, where attdoms is a sequence of all domains along with the corresponding private keys owned by the attacker, sslkeys is a sequence of all domains and the corresponding public keys, and signkeys is a sequence containing all public signing keys for all IdPs. All other parties use the attacker as a DNS server.

F.8 Browsers

Each $b \in B$ is a web browser as defined in Section 2.5, with $I^b := {addr(b)}$ being its address.

To define the initial state, first let $ID^b := \text{ownerOfID}^{-1}(b)$ be the set of all IDs of b, $ID^{b,d} := \{i \mid \exists x : i = \langle x, d \rangle \in ID^b\}$ be the set of IDs of b for a domain d, and *SecretDomains*^b := $\{d \mid ID^{b,d} \neq \emptyset\}$ be the set of all domains that b owns identities for.

Then, the initial state s_0^b is defined as follows: the key mapping maps every domain to its public (ssl) key, according to the mapping sslkey; the DNS address is addr(attacker); the list of secrets contains an entry $\langle \langle d, \mathbf{S} \rangle, s \rangle$ for each $d \in SecretDomains^b$ and s = secretOfID(i) for some $i \in ID^{b,d}$ (s is the same for all i); ids is $\langle ID^b \rangle$; sts is empty.

F.9 LPO

LPO is a an atomic DY process $(I^{LPO}, Z^{LPO}, R^{LPO}, s_0^{LPO}, N^{LPO})$ with the IP address $I^{LPO} = \{ \mathsf{addr}(\mathsf{LPO}) \}$. The initial state s_0^{LPO} of LPO contains the private key of its domain, and the signing keys of all IdPs (LPO does not need the public ssl keys of other parties, which is why we omit them from LPO's initial state.). The definition of R^{LPO} follows the description of LPO in Section E.

HTTP responses by LPO can contain strings representing scripts, namely the script script_lpo_cif run in the CIF and the script script_lpo_ld run in the LD. These scripts are defined in Appendices 12 and 13, respectively.

Client sessions at LPO. Any party can establish a *session* at LPO. Such a session can either be authenticated or unauthenticated. Roughly speaking, a session becomes authenticated if a client has provided a valid CAP (for the origin of LPO) to LPO during the session. LPO manages groups of IDs, i.e., lists of email addresses. If a user authenticates a session using any ID in the group, she is authenticated for all IDs in the group. An authenticated session can (non-deterministically) *expire*, i.e. the authenticated session can get unauthenticated or it is removed completely. Such an expiration is used to model a user logout or a session expiration caused by a timeout.

More specifically, a session is identified by a nonce, which is issued by LPO. Each session is associated with some xsrfToken, which is also a nonce issued by LPO. LPO stores all information about established sessions in its state as a dictionary indexed by the session identifier. In this dictionary, for every session LPO stores a pair containing the xsrfToken and, in authenticated sessions, the sequence of all IDs associated with the secret provided in the session, or, in unauthenticated sessions, the empty sequence $\langle \rangle$ of IDs. On the receiver side (typically a browser) LPO places, by appropriate headers in its HTTPS responses, a cookie named **browserid_state** whose value is the session identifier (a nonce). This cookie is flagged to be a session, httpOnly, and secure cookie.

Before we provide a detailed formal specification of LPO, we first provide an informal description.

HTTPSRequests to LPO. LPO answers only to certain requests (listed below). In reality, all such requests have to be over HTTPS, and all responses send by LPO contain the Strict-Transport-Security header. We overapproximate safely here in omitting these two requirements from the model.

GET /cif. LPO replies to this request by providing the script script_lpo_cif. GET /ld. LPO replies to this request by providing the script script_lpo_ld.

- GET /ctx. This requests the session context information from LPO. The response body is of the form $\langle loggedIn, xsrfToken \rangle$, where loggedIn is \top or \bot , depending on whether the user is logged in at LPO or not, and xsrfToken is the token that the client is supposed to include into the auth request (see below).
- **POST** /auth. With this request, a client can log into LPO. The client has to provide a sequence of a CAP and an XSRF token. The CAP must be valid and issued for the origin of LPO.

We now define LPO formally as an atomic DY process $(I^{LPO}, Z^{LPO}, R^{LPO}, s_0^{LPO}, N^{LPO})$. As already mentioned, we define $I^{LPO} = \{ addr(LPO) \}$.

In order to define the set Z^{LPO} of states of LPO, we first define the terms describing the session context of a session.

Definition 39. A term of the form $\langle ids, xsrfToken \rangle$ with $ids \subset \langle \rangle$ ID and $xsrfToken \in \mathcal{N}$ is called an LPO session context. We denote the set of all LPO session contexts by LPOSessionCTXs.

Now, we define the set Z^{LPO} of states of LPO as well as the initial state s_0^{LPO} of LPO.

Definition 40. A state $s \in Z^{LPO}$ of LPO is a term of the form (nonces, sslkey, signkeys, sessions) where nonces $\subset^{\langle\rangle} \mathcal{N}$ (used nonces), sslkey = sslkey(dom(LPO)), signkeys is a mapping of domain names to public signing keys of the form signkeys = $\langle \{ \langle d, \mathsf{pub}(\mathsf{signkey}(y)) \rangle | y \in \mathsf{IdPs}, d \in \mathsf{dom}(y) \} \rangle$, and sessions $\in [\mathcal{N} \times \mathsf{LPOSessionCTXs}]$.⁹

The initial state s_0^{LPO} of LPO is a state of LPO with s_0^{LPO} .nonces = $\langle \rangle$ and s_0^{LPO} .sessions = $\langle \rangle$.

Example 7. Let k be a private signing key for some identity provider which owns the domain example.com. A possible state s of LPO may look like this:

 $s = \langle \langle n_1, \dots, n_m \rangle$, sslkey(dom(LPO)), [example.com : pub(k)], sessions \rangle

with

$$sessions = \langle \langle \texttt{sessionid}_1, \langle \langle id'_1, \dots, id'_l \rangle, \texttt{xsrfToken} \rangle \rangle, \dots \rangle$$

We now specify the relation $R^{LPO} \subseteq (\mathcal{E} \times Z^{LPO}) \times (2^{\mathcal{E}} \times Z^{LPO})$ of LPO. Just like in Appendix C.2, we describe this relation by a non-deterministic algorithm.

Algorithm 9 Relation of $LPO R^{LPO}$

Input: (a:f:m), s1: let s' := s2: let $sts := \langle \texttt{Strict-Transport-Security}, \top \rangle$ 3: if $m \equiv \text{TRIGGER then}$ 4: if s'.sessions $\equiv \langle \rangle$ then stop $\{\}, s$ 5: 6: end if let sessionid $\leftarrow \{id \mid id \in \langle \rangle \ s'.sessions\}$ 7: 8: let $choice \leftarrow \{logout, expire\}$ 9: if $choice \equiv logout then$ 10: let s'.sessions[sessionid].ids := $\langle \rangle$ 11: else 12:let s'.sessions := s'.sessions - sessionid 13:end if 14: stop $\{\}, s$ 15: end if 16: let m_{dec} , k such that $\langle m_{\text{dec}}, k \rangle \equiv \text{dec}_a(m, s.\text{sslkey})$ \hookrightarrow if possible; otherwise stop {}, s 17: let n, method, path, params, headers, body such that \hookrightarrow (HTTPReq, n, method, dom(LPO), path, params, headers, body) $\equiv m_{dec}$ \hookrightarrow if possible; otherwise stop {}, s 18: if $method \equiv \text{GET} \land path \equiv /\text{cif then}$ \triangleright Deliver CIF script let scriptinit := (init $, \bot, \bot, \bot, \bot, \bot, \bot, \langle \rangle, \bot, \bot \rangle$ 19:

⁹As mentioned before, the state of LPO does not need to contain public keys.

```
20:
          let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle \text{script lpo } cif, scriptinit \rangle \rangle, k,)
21:
          stop {(f:a:m')}, s'
22: else if method \equiv \text{GET} \land path \equiv /\text{ld then}
                                                                                              \triangleright Deliver LD script
          let scriptinit := \langle \texttt{init}, \bot, \bot, \bot, \bot, \bot, \langle \rangle, \bot, \bot, \bot \rangle
23:
          let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle \text{script lpo ld}, scriptinit \rangle \rangle, k,)
24:
          stop {(f:a:m')}, s'
25:
26: else if method \equiv \text{GET} \land path \equiv /\text{ctx then}
                                                                                           \triangleright Deliver context info.
          let sessionid := headers[Cookie][browserid state]
27:
          if sessionid \not\in^{\langle\rangle} s.sessions then
28:
               let sessionid, s' := \mathsf{TAKENONCE}(s')
29:
               let xsrfToken, s' := TAKENONCE(s')
30:
31:
               let s'.sessions := s'.sessions + \langle \rangle (sessionid, \langle \langle \rangle, xsrfToken \rangle \rangle
          end if
32:
33:
          let context := \langle \bot, s'. sessions[sessionid]. xsrfToken \rangle
34:
          if s'.session[sessionid].ids \neq \langle \rangle then
35:
               let context.1 := \top
36:
          end if
          let setCookie := (Set-Cookie, \langle \langle browserid state, sessionid, \top, \top, \top \rangle \rangle \rangle
37:
38:
          let headers := \langle sts, setCookie \rangle
          let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, headers, context \rangle, k)
39:
          stop {(f:a:m')}, s'
40:
41: else if method \equiv POST \land path \equiv /auth then
42:
          let uc, ia, xsrfToken such that \langle \langle uc, ia \rangle, xsrfToken \rangle \equiv body
           \hookrightarrow if possible; otherwise stop {}, s
43:
          let sessionid := headers[Cookie][browserid state]
44:
          if s'.sessions[sessionid].xsrfToken \neq xsrfToken then
45:
               stop \{\}, s
46:
          end if
47:
          let name, domain, userpubkey such that
           \hookrightarrow \langle \langle name, domain \rangle, userpubkey \rangle \equiv \mathsf{extractmsg}(uc)
           \hookrightarrow if possible; otherwise stop {}, s
48:
          let id := \langle name, domain \rangle
          let origin := extractmsg(ia)
49:
          if checksig(uc, s.signkeys[domain]) \neq \top \lor checksig(ia, userpubkey) \neq \top
50:
           \hookrightarrow \lor origin \not\equiv \langle s. \texttt{domain}, \texttt{S} \rangle then
51:
               stop \{\}, s
52:
          end if
          if s'.sessions[sessionid].ids \equiv \langle \rangle then
53:
               if \not\exists n \in \mathbb{N} such that id \in {}^{\langle \rangle} s'.idgroups.n then
54:
                    let s'.idgroups := s'.idgroups + \langle \rangle \langle id \rangle
55:
               end if
56:
               let n \leftarrow \mathbb{N} such that id \in {}^{\langle \rangle} s'.idgroups.n
57:
58:
          else
               let n \leftarrow \mathbb{N} such that s'.idgroups.n \equiv s'.sessions[sessionid].ids
59:
                \hookrightarrow if possible; otherwise stop {}, s
               if id \notin s'.idgroups.n then
60:
                    let s'.idgroups.n := s'.idgroups.n + \langle \rangle \langle name, domain \rangle
61:
               end if
62:
63:
          end if
          let s'.sessions[sessionid].ids := s'.idgroups.n
64:
```

 $\begin{array}{ll} 65: & \mathbf{let} \ m' := \mathsf{enc}_{\mathsf{s}}(\langle \mathtt{HTTPResp}, n, 200, \langle sts \rangle, \top \rangle, k) \\ 66: & \mathbf{stop} \ \{(f{:}a{:}m')\}, \ s' \\ 67: \ \mathbf{end} \ \mathbf{if} \\ 68: \ \mathbf{stop} \ \{\}, \ s \end{array}$

F.10 Relying Parties

A relying party $r \in \mathsf{RP}$ is a web server modeled as an atomic DY process $(I^r, Z^r, R^r, s_0^r, N^r)$ with the address $I^r := \{\mathsf{addr}(r)\}$. Its initial state s_0^r contains its domain, the private key associated with its domain, the DNS server address, and the signing keys of all IdPs.¹⁰ The full state additionally contains the set of service tokens the RP has issued. The definition of R^r again follows the description in Appendix E. RP accepts only HTTPS requests.

In a typical flow with one client, r will first receive an HTTP GET request. In this case, it returns the script script_rp_index (see below) and sets the Strict-Transport-Security header.

Afterwards, it will receive an HTTPS POST request. Provided that the message contains a CAP, r checks that the UC and IA are valid and matching, and that the IA contains the Origin of r (with HTTPS). If the check is successful, r creates a new RP service token for the identity i, $\langle n, i \rangle$, and sends it to the browser. The RP keeps a list of such tokens in its state. Intuitively, a client in possession of such a token can use the service of r for ID i (e.g., access data of i at r).

We now provide the formal definition of r as an atomic DY process $(I^r, Z^r, R^r, s_0^r, N^r)$. As mentioned, we define $I^r = \{\mathsf{addr}(r)\}$. Next, we define the set Z^r of states of r and the initial state s_0^r of r.

Definition 41. A state $s \in Z^r$ of an RP r is a term of the form (nonces, domain, sslkey, signkeys, serviceTokens, corrupt) where nonces $\subset^{\langle\rangle} \mathcal{N}$ (used nonces), domain = dom(r), sslkey = sslkey(dom(r)), signkeys = $\langle \{ \langle d, \mathsf{pub}(\mathsf{signkey}(y)) \rangle \mid y \in \mathsf{IdPs}, d \in \mathsf{dom}(y) \} \rangle$ (same as for LPO), serviceTokens $\in [\mathcal{N} \times \mathbb{S}]$, corrupt $\in T_{\mathcal{N}}$.

The initial state s_0^r of r is a state of r with s_0^r .nonces = s_0^r .serviceTokens = $\langle \rangle$ and s_0^r .corrupt = \bot .

We now specify the relation $R^r \subseteq (\mathcal{E} \times Z^r) \times (2^{\mathcal{E}} \times Z^r)$ of r. Just like in Appendix C.2, we describe this relation by a non-deterministic algorithm. We note that we use the function TAKENONCE introduced in Section C.2 for this purpose.

Algorithm 10 Relation of a Relying Party R^r

Input: (a:f:m), s

1: let s' := s

2: if s'.corrupt $\not\equiv \bot \lor m \equiv \texttt{CORRUPT}$ then

¹⁰We add the IdP signing keys to the initial status (instead of having RPs retrieve them dynamically) in order to reduce the overall complexity.

3: let s'.corrupt := $\langle \langle a, f, m \rangle, s'.corrupt \rangle$ 4: let $m' \leftarrow d_{N^p}(s')$ let $a' \leftarrow \mathsf{IPs}$ 5: **stop** $\{(a':a:m')\}, s'$ 6: 7: end if 8: let $sts := \langle \texttt{Strict-Transport-Security}, \top \rangle$ 9: let m_{dec} , k such that $\langle m_{\text{dec}}, k \rangle \equiv \text{dec}_a(m, s.\text{sslkey})$ \hookrightarrow if possible; otherwise stop $\{\}, s$ 10: let n, method, path, params, headers, body such that \hookrightarrow (HTTPReq, n, method, s.domain, path, params, headers, body) $\equiv m_{dec}$ \hookrightarrow if possible; otherwise stop {}, s 11: if method \equiv GET then \triangleright Deliver CIF script 12:let scriptinit := $\langle \text{init}, \bot, \bot, \bot, \langle \rangle, \langle \rangle, \bot \rangle$ let $m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle \text{script rp index}, scriptinit \rangle \rangle, k)$ 13:14:**stop** {(f:a:m')}, s' 15: else if $method \equiv POST \land headers \equiv \langle Origin, \langle s.domain, S \rangle \rangle$ then let uc, ia such that $\langle uc, ia \rangle \equiv body$ if possible; otherwise stop {}, s 16:17:let name, domain, userpubkey such that $\hookrightarrow \langle \langle name, domain \rangle, userpubkey \rangle \equiv \mathsf{extractmsg}(uc)$ \hookrightarrow if possible; otherwise stop {}, s 18:let $id := \langle name, domain \rangle$ 19:let origin := extractmsg(ia)if $checksig(uc, s.signkeys[domain]) \not\equiv \top \lor checksig(ia, userpubkey) \not\equiv \top \lor$ 20: \hookrightarrow origin $\not\equiv \langle s. \texttt{domain}, \texttt{S} \rangle$ then 21: stop $\{\}, s$ 22: end if let $n_{\text{token}}, s' := \mathsf{TAKENONCE}(s')$ 23:let s'.serviceTokens := s'.serviceTokens + $\langle \rangle \langle n_{\text{token}}, id \rangle$ 24:let $m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle n_{\text{token}}, id \rangle \rangle, k)$ 25:**stop** {(f:a:m')}, s' 26:27: end if

F.11 Identity Providers

An identity provider $i \in \mathsf{IdPs}$ is a web server modeled as an atomic process $(I^i, Z^i, R^i, s_0^i, N^i)$ with the address $I^i := \{\mathsf{addr}(i)\}$. Its initial state s_0^i contains a list of domains and (private) SSL keys (see below), a list of users and identities (see below), and a private key for signing UCs. Besides this, the full state of *i* further contains a list of used nonces, and information about active sessions.

Sessions are structured as a dictionary: For each session identifier (session ID) the dictionary contains the list of identities for which the session is authenticated.

IdPs, in our model, only accept SSL connections. Thus, after receiving a request, an IdP first decodes the message. It then checks whether a valid session ID is contained in the cookie that was sent with the request. If there is no such ID, a new session with a freshly chosen session ID is created. IdP saves this ID into its list of active sessions, along with the initial session data (an empty list of authenticated identities). A Set-Cookie header is added to IdPs response to the browser in order to add the session cookie to the client's cookie store. The IdP then checks the method and the path of the request and acts as follows:

If the method is GET, IdP serves, depending on the path, the provisioning iframe (script idp pif) or the authentication dialog (script idp ad).

If the method is POST, the IdP can either authenticate the user or sign a UC. In the first case, IdP extracts the identity of the user (an email address) and the user's secret from the request. If the secret and the identity are found in the user database, the session is considered to be logged in for all identities associated with this secret. In the second case (signing UC), the IdP extracts the user's identity and the public key of the user from the request. If the session is considered to be logged in for this identity, the IdP creates a UC and signs it with its signing key before sending it to the user.

Formal description. In the following, we will first define the (initial) state of i formally and afterwards present the definition of the relation R^{i} .

To define the initial state, we will need a term that represents the "user database" of the IdP *i*. We will call this term $userset^i$. This database defines, which secret is valid for which set of identities. It is encoded as a mapping of secrets to lists of identities for which these secrets are valid. For example, if the secret $secret_1$ is valid for the identities id_1 and id_2 and the secret $secret_2$ is valid for the identities id_3 and id_4 , the $userset^i$ looks as follows:

$$userset^i = [secret_1: \langle id_1, id_2 \rangle, secret_2: \langle id_3, id_4 \rangle]$$

To define $userset^i$ (for the identity provider i), we first define the set $Secrets^p = \bigcup_{j \in \mathsf{ID}^p} secretOfID(j)$, the function $\mathsf{IDsofSecret} : Secrets \to \mathsf{ID}, s \mapsto \{j \mid j \in \mathsf{ID}, secretOfID(j) = s\}$, and finally $userset^i = \langle \{\langle s, \langle \mathsf{IDsofSecret}(s) \rangle \rangle \mid s \in \mathsf{Secrets}^i \} \rangle$.

We also need a term that represents a dictionary that maps domains to (private) SSL keys of the IdP *i*. We define $sslkeys^i = \langle \{ \langle d, sslkey(d) \rangle \mid d \in dom(i) \} \rangle$.

Definition 42. A state $s \in Z^i$ of an IdP *i* is a term of the form (nonces, sslkeys, users, signkey, sessions, corrupt) where nonces $\subset^{\langle\rangle} \mathcal{N}$ (used nonces), sslkeys = sslkeys^{*i*}, users = userset^{*i*}, signkey $\in \mathcal{N}$ (the key used by the IdP *i* to sign UCs), sessions $\in [\mathcal{N} \times \mathcal{T}_{\mathcal{N}}]$, corrupt $\in \mathcal{T}_{\mathcal{N}}$.

The initial state s_0^i of *i* is the state $\langle \langle \rangle, sslkeys^i, userset^i, signkey(i), \langle \rangle, \bot \rangle$.

The relation R^i that defines the behavior of the IdP *i* is defined as follows:

Algorithm 11 Relation of IdP R^i

Input: (a:f:m), s1: let s' := s2: if $s'.corrupt \neq \bot \lor m \equiv CORRUPT$ then 3: let $s'.corrupt := \langle \langle a, f, m \rangle, s'.corrupt \rangle$ 4: let $m' \leftarrow d_{N^p}(s')$ 5: let $a' \leftarrow IPs$ 6: stop {(a':a:m')}, s'7: end if

```
8: let sts := \langle \texttt{Strict-Transport-Security}, \top \rangle
 9: let m_{dec}, k, k', inDomain such that
       \hookrightarrow \langle m_{\text{dec}}, k \rangle \equiv \text{dec}_{a}(m, k') \land \langle inDomain, k' \rangle \in s.sslkevs
       \hookrightarrow if possible; otherwise stop {}, s
10: let n, method, path, params, headers, body such that
      \hookrightarrow (HTTPReq, n, method, inDomain, path, params, headers, body) \equiv m_{dec}
       \hookrightarrow if possible; otherwise stop {}, s
11: if method \equiv POST then
12:
          if path \not\equiv /certreq then
                                                                                                           \triangleright User logs in.
13:
                let id, secret such that (id, secret) \equiv body
                 \hookrightarrow if possible; otherwise stop {}, s
                if headers \not\equiv \langle \texttt{Origin}, \langle inDomain, \texttt{S} \rangle \rangle then
14:
15:
                     stop \{\}, s
                end if
16:
17:
                let ids := s.users[secret]
                if ids \equiv \langle \rangle \lor id \equiv \langle \rangle \lor id \notin \langle \rangle ids then
18:
                                                                                              \triangleright Check id/secret pair.
19:
                     stop \{\}, s
                end if
20:
                let sessionid, s' := \mathsf{TAKENONCE}(s')
21:
22:
                let s'.sessions[sessionid] := ids
23:
                let setCookie := (Set-Cookie, ((sessionid, sessionid, \top, \top, \top)))
24:
                let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts, setCookie \rangle, \top \rangle, k)
25:
                stop {(f:a:m')}, s'
26:
           else
                                                                                         \triangleright User wants a certificate.
27:
                let id, pubkey such that \langle id, pubkey \rangle \equiv body
                 \hookrightarrow if possible; otherwise stop {}, s
                let \ sessionid := headers[Cookie][sessionid]
28:
                if id \notin s'.sessions[sessionid] then
                                                                                       \triangleright Check if user is logged in.
29:
30:
                     stop \{\}, s
31:
                end if
                let uc := sig(\langle id, pubkey \rangle, s.signkey)
32:
33:
                let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, uc \rangle, k)
34:
                stop {(f:a:m')}, s'
35:
           end if
36: else
37:
           if path \equiv /pif then
                let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle \text{script\_idp\_pif}, \rangle)
38:
                 \, \hookrightarrow \, \langle \texttt{init}, \langle \rangle, \langle \rangle, \langle \rangle, \bot, \bot, \bot \rangle \rangle \rangle, k)
39:
           else
40:
               let m' := \text{enc}_{s}(\langle \text{HTTPResp}, n, 200, \langle sts \rangle, \langle \text{script idp ad}, \langle \rangle \rangle \rangle, k)
41:
           end if
42:
           stop \{(f:a:m')\}, s'
43: end if
44: stop \{\}, s
```

F.12 BrowserID Scripts

As already mentioned in Section F.1, the set S of the web system $\mathcal{B}I\mathcal{D}_{\text{primary}} = (\mathcal{W}, S, \text{script}, E_0)$ consists of the scripts $R^{\text{att}}, script_rp_index$,

script_lpo_cif, script_lpo_ld, script_idp_pif, and script_idp_ad with their string representations being att_script, script_rp_index, script_lpo_cif, script_lpo_ld, script_idp_pif, and script_idp_ad (defined by script).

The script R^{att} is the attacker script (see Section 2.3). The formal model of the other scripts follows the description in Appendix E. The script $script_rp_index$ defines the script of the RP index page. In reality, this page has its own script(s) and includes a script from LPO. In our model, we combine both scripts into $script_rp_index$. In particular, this script is responsible for creating the CIF and the LD iframes/subwindows, whose contents are loaded from LPO.

In what follows, the scripts $script_rp_index$, $script_lpo_cif$, and $script_lpo_ld$ are defined formally. First, we introduce some notation and helper functions.

Notations and Helper Functions. In the formal description of the scripts we use an abbreviation for URLs at LPO. We write URL_{path}^{LPO} to describe the following URL term: $\langle URL, S, dom(LPO), path, \langle \rangle \rangle$. Also, we call origin_{LPO} the origin of LPO which describes the following origin term: $\langle dom(LPO), S \rangle$.

In order to simplify the description of the scripts, several helper functions are used.

CHOOSEINPUT. As explained in Section 2.5, the state of a document contains a term, say, *scriptinputs*, which records the input this document has obtained so far (via XHRs and postMessages). If the script of the document is activated, it will typically need to pick one input message from *scriptinputs* and record which input it has already processed. For this purpose, the function CHOOSEINPUT(s', *scriptinputs*) is used, where s' denotes the scripts current state. It saves the indexes of already handled messages in the scriptstate s' and chooses a yet unhandled input message from *scriptinputs*. The index of this message is then saved in the scriptstate (which is returned to the script).

Algorithm 12 Choose an unhandled input message for a script

- 1: function CHOOSEINPUT(s', scriptinputs)
- 2: let *iid* such that *iid* $\in \{1, \dots, |scriptinputs|\} \land iid \notin \$ handledInputs \hookrightarrow if possible; otherwise return (\bot, s')
- 3: let $input := \pi_{iid}(scriptinputs)$
- 4: let s'.handledInputs := s'.handledInputs + $\langle \rangle$ iid
- 5: **return** (input, s')
- 6: end function

PARENTWINDOW. To determine the nonce referencing the parent window in the browser, the function PARENTWINDOW(*tree*, *docnonce*) is used. It takes the term *tree*, which is the (partly cleaned) tree of browser windows the script is able to see and the document nonce *docnonce*, which is the nonce referencing the current document the script is running in, as input. It outputs the nonce referencing the window which directly contains in its subwindows the window of the document referenced by *docnonce*. If there is no such window (which is the

case if the script runs in a document of a top-level window), $\mathsf{PARENTWINDOW}$ returns $\bot.$

SUBWINDOWS. This function takes a term *tree* and a document nonce docnonce as input just as the function above. If docnonce is not a reference to a document contained in *tree*, then SUBWINDOWS(*tree*, *docnonce*) returns $\langle \rangle$. Otherwise, let $\langle docnonce, origin, script, scriptstate, scriptinput, subwindows, active \rangle$ denote the subterm of *tree* corresponding to the document referred to by docnonce. Then, SUBWINDOWS(*tree*, *docnonce*) returns subwindows.

AUXWINDOW. This function takes a term *tree* and a document nonce *docnonce* as input as above. From all window terms in *tree* that have the window containing the document identified by *docnonce* as their opener, it selects one non-deterministically and returns its nonce. If there is no such window, it returns the nonce of the window containing *docnonce*.

OPENERWINDOW. This function takes a term *tree* and a document nonce *docnonce* as input as above. It returns the window nonce of the opener window of the window that contains the document identified by *docnonce*. Recall that the nonce identifying the opener of each window is stored inside the window term. If no document with nonce *docnonce* is found in the tree *tree*, \Diamond is returned.

GETWINDOW. This function takes a term *tree* and a document nonce *docnonce* as input as above. It returns the nonce of the window containing *docnonce*.

GETORIGIN. To extract the origin of a document, the function GETORIGIN(tree, docnonce) is used. This function searches for the document with the identifier *docnonce* in the (cleaned) tree *tree* of the browser's windows and documents. It returns the origin *o* of the document. If no document with nonce *docnonce* is found in the tree *tree*, \Diamond is returned.

Web storage under LPO's origin. The web storage under the origin of LPO used by the script *script_lpo_cif* and *script_lpo_ld* (see below) is organized as follows:

The localStorage is a dictionary. There are two types of entries in this dictionary: Under the key siteInfo, a dictionary is stored which has origins as keys and IDs as values. An entry in this dictionary indicates that the user is logged in at the referenced origin with a certain ID. The second type of entry has a nonce as a key. The value is an email address (ID). This models the email address a user entered in the LD before being navigated away to the AD. The nonce is also stored in the sessionStorage (see below).

Example 8.

$$\begin{array}{l} \langle \langle \texttt{siteInfo}, \langle \langle (\texttt{domain}_{\texttt{RP1}}, \texttt{S} \rangle, id_1 \rangle, \\ & \langle \langle \texttt{domain}_{\texttt{RP2}}, \texttt{S} \rangle, id_1 \rangle, \\ & \langle \langle \texttt{domain}_{\texttt{RP3}}, \texttt{S} \rangle, id_2 \rangle \rangle \rangle, \\ \langle n_1, id_1 \rangle, \\ & \langle n_2, id_3 \rangle \rangle \end{array}$$

This example shows a localStorage under the origin of LPO, indicating that the user is logged in at domain_{RP1} and domain_{RP2} with id_1 and at domain_{RP3} with

 id_2 (using HTTPS). Further, the nonces n_1 and n_2 each refer to an email address which the user entered in the LD.

The sessionStorage is also a dictionary. It may only contain one key, idpnonce. Its value is a nonce (like n_1 or n_2 in the example above) which references the latest email address entry in the localStorage (see above).

login.persona.org Communication Iframe Script (script_lpo_cif). As defined in Section 2.3, a script is a relation that takes as input a term and a set of nonces it may use. It outputs a new term. As specified in Section 2.5 (Triggering the Script of a Document (m = TRIGGER, action = 1)) and formally specified in Algorithm 6, the input term is provided by the browser. It contains the current internal state of the script (which we call *scriptstate* in what follows) and additional information containing all browser state information the script has access to, such as the input the script has obtained so far via XHRs and postMessages, information about windows, etc. The browser expects the output term to have a specific form, as also specified in Section 2.5 and Algorithm 6. The output term contains, among other information, the new internal scriptstate.

As for *script_lpo_cif*, this script models the script run in the CIF, as sketched in Appendix E.

We first describe the structure of the internal scriptstate of the script script_lpo_cif.

Definition 43. A scriptstate s of script <u>lpo</u> cif is a term of the form $\langle q, requestOrigin, loggedInUser, pause, context, key, uc, handledInputs, refXHRctx, PIFindex <math>\rangle$ where $q \in \mathbb{S}$, requestOrigin \in Origins $\cup \{\bot\}$, loggedInUser \in ID $\cup \{\langle \rangle, \bot\}$, pause $\in \{\top, \bot\}$, context $\in \mathcal{T}_{\mathcal{N}}$, key $\in \mathcal{N} \cup \{\bot\}$, uc $\in \mathcal{T}_{\mathcal{N}}$, handledInputs $\subset^{\langle \rangle} \mathbb{N}$, refXHRctx $\in \mathcal{N} \cup \{\bot\}$, PIFindex $\in \mathbb{N} \cup \{\bot\}$. The initial scriptstate initState_{cif} of script_lpo_cif is the state $\langle \text{init}, \bot, \bot, \bot, \bot, \bot, \bot, \rangle$.

Before we provide the formal specification of the relation that defines the behavior of $script_lpo_cif$, we present an informal description. The behavior mainly depends on the state q the script is in.

- q = init This is the initial state. Its only transition handles no input and outputs a postMessage cifready to its parent window and transitions to default.
- q = default This is the state to which script_lpo_cif always returns to. This state handles all postMessages the CIF expects to receive from its parent window. If the postMessage received was sent from the parent window of the CIF, it behaves as follows, depending on the first element of the received postMessage:
 - **postMessage loaded** The script records the sender's origin of the received postMessage as the remote origin in the scriptstate if the scriptstate did not contain any information about the remote origin yet. Also, an ID, which represents the assumption of the sender on who it believes to be

logged in, is saved in the scriptstate. If the *pause* flag in the scriptstate is \top it transitions to the state default. Otherwise, it is checked, if the current context in the scriptstate is \bot . If the check is true, the script transitions to the state fetchContext, or to the state checkAndEmit otherwise.

- **postMessage dlgRun** The script sets the *pause* flag in the scriptstate to \top and transitions to default.
- **postMessage dlgCmplt** The script sets the *pause* flag in the scriptstate to \perp . It then transitions to the state fetchContext.
- **postMessage loggedInUser** This message has to contain an ID. This ID is saved in the scriptstate and then the script transitions to **default**.
- **postMessage logout** The script removes the entry for the RP (recorded in the scriptstate) from the localStorage and then transitions to the state **sendLogout**. If no remote origin is set in the script's state, it is now set to the sender's origin of the received postMessage.
- q = fetchContext In this state, the script sends an XHR to LPO with a GET request to the path /ctx and then transitions to the state <code>receiveContext</code>.
- q = receiveContext In this state, the script expects an XHR response as input containing the session context. This context is saved as the current context in the scriptstate. The script transitions to <code>checkAndEmit</code>.
- q = checkAndEmit This state lets the script create the provisioning iframe and transition to startPIF iff (1) some email address is marked as logged in at RP in the localStorage, (2) if an email address is recorded in the current scriptstate, this email address differs from the one recorded in the localStorage, and (3) the user is marked as logged in in the current context. Otherwise, if the email address recorded in the current scriptstate is $\langle \rangle$, the script transitions to default, else it transitions to sendLogout.
- q = startPIF In this state, the script waits for a postMessage from the PIF containing a ping message. If such a message is received and the sender's window and origin match the PIF, the script sends a pong message back to the PIF and transitions to the state <code>runPIF</code>.
- q = runPIF This is the state in which script_lpo_cif interacts with the PIF. This state handles all postMessages the CIF expects to receive from the latest PIF (as recorded in PIFindex in its state). If the postMessage received was sent from the PIF's window and the PIF's origin, it behaves as follows, depending on the first element of the postMessage:
 - **postMessage beginProvisioning** The script responds with a postMessage to the PIF containing the email address of the identity which is to authenticate to the relying party (as recorded in the CIF's state).
 - **postMessage genKeyPair** The script creates a fresh key pair (i.e. the CIF chooses a fresh nonce) and sends the public key contained in a postMessage to the PIF.
 - postMessage registerCertificate The script stores the UC received in this postMessage in the CIF's state and transitions to the state createCAPforRP.

- **postMessage raiseProvisioningFailure** This message indicates that no one is logged in. This is recorded in the CIF's state accordingly. The script transitions to the state sendLogout in which the CIF's parent window will be notified that no one is logged in.
- q = createCAPforRP In this state, the script creates an IA for the request origin (as recorded in the script's state), combines the IA with the UC to a CAP, and sends the CAP in a postMessage to its parent restricting the receiver to the request origin.
- q =sendLogout In this state, the script sends a logout postMessage to the parent document and then transitions to the default state.

We now specify the relation $script_lpo_cif \subseteq (\mathcal{T}_{\mathcal{N}} \times 2^{\mathcal{N}}) \times \mathcal{T}_{\mathcal{N}}$ of the CIF's scripting process formally. Just like in Appendix C.2, we describe this relation by a non-deterministic algorithm.

Just like all scripts, as explained in Section 2.5 (see also Algorithm 6 for the formal specification), the input term this script obtains from the browser contains the cleaned tree of the browser's windows and documents *tree*, the nonce of the current document *docnonce*, its own scriptstate *scriptstate* (as defined in Definition 43), a sequence of all inputs *scriptinput* (also containing already handled inputs), a dictionary *cookies* of all accessible cookies of the document's domain, the localStorage *localStorage* belonging to the document's origin, the secrets *secret* of the document's origin, and a set *nonces* of fresh nonces as input. The script returns a new scriptstate s', a new set of cookies *cookies'*, a new localStorage *localStorage'*, and a term *command* denoting a command to the browser.

Algorithm 13 Relation of script lpo cif

Input: $\langle tree, docnonce, scriptstate, scriptinputs, cookies, localStorage, sessionStorage, <math>\hookrightarrow ids, secret \rangle$, nonces

```
1: let s' := scriptstate
 2: let cookies' := cookies
 3: let localStorage' := localStorage
 4: let sessionStorage' := sessionStorage
 5: switch s'.q do
 6:
        case init
 7:
            let command := \langle POSTMESSAGE, PARENTWINDOW(tree, docnonce),
             \hookrightarrow \langle \texttt{cifready}, \langle \rangle \rangle, \perp \rangle
 8:
            let s'.q := default
 9:
            stop \langle s', cookies', localStorage', sessionStorage', command \rangle
10:
        case default
            let input, s' := CHOOSEINPUT(s', scriptinputs)
11:
12:
            if \pi_1(input) \equiv \text{POSTMESSAGE then}
                let senderWindow := \pi_2(input)
13:
14:
                let senderOrigin := \pi_3(input)
15:
                let m := \pi_4(input)
16:
                if senderWindow \equiv \mathsf{PARENTWINDOW}(tree, docnonce) then
17:
                    switch m do
```

18:	$\mathbf{case}~\langle \texttt{loaded}, id angle$
19:	$\mathbf{if} s'. \texttt{requestOrigin} \equiv \perp \mathbf{then}$
20:	$\texttt{let} \ s'.\texttt{requestOrigin} := senderOrigin$
21:	end if
22:	$\texttt{let} \ s'.\texttt{loggedInUser} := id$
23:	$\mathbf{if} s'. \mathtt{pause} \equiv \top \mathbf{then}$
24:	stop $\langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle$
25:	$\mathbf{else \ if} \ s'.\texttt{context} \equiv \bot \ \mathbf{then}$
26:	${f let}\;s'.{f q}:={f fetchContext}$
27:	$\mathbf{stop} \ \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle$
28:	else
29:	${f let}\; s'.{f q}:={f checkAndEmit}$
30:	$\mathbf{stop} \ \langle s', cookies', localStorage', sessionStorage', \langle angle angle$
31:	end if
32:	$\mathbf{case} \hspace{0.1 cm} \langle \mathtt{dlgRun}, \langle \rangle \rangle$
33:	$\texttt{let} \ s'.\texttt{pause} := \top$
34:	$\mathbf{stop} \langle s', cookies', localStorage', sessionStorage', \langle \rangle angle$
35:	$\mathbf{case} \hspace{0.1 cm} \langle \mathtt{dlgCmplt}, \langle \rangle \rangle$
36:	$\texttt{let} \ s'.\texttt{pause} := \bot$
37:	let s'.q := fetchContext
38:	stop $\langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle$
39:	case $\langle \texttt{loggedInUser}, id \rangle$
40:	let s' .loggedInUser := id
41:	stop $\langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle$
42.	case $(logout \langle \rangle)$
43.	if s' request 0 rigin = 1 then
44·	let s' requestorigin $= \pm \text{ senderOrigin}$
45:	end if
46:	let s'loggedInUser :=
47:	remove the element with key s' .requestOrigin
	\hookrightarrow from the dictionary <i>localStorage</i> '[siteInfo]
48:	let s' .g := sendLogout
<u>/</u> 0·	and if
49. 50.	end if
51.	case fetchContext
52.	let s' refXHBctx \leftarrow nonces
53.	let command := $(XMI HTTPREDIFST IR _{PO} GET () s' refXHRctx)$
50.54	let $s' a :=$ receiveContext
55.	stop (s' cookies' localStorage' sessionStorage' command)
56.	and received ant art
50.	case receive context
57.	if $(\pi_i(input) = \text{VMI UTTDEFOURET}) \land (\pi_i(input) = a' \text{ refYHP et } then$
50.	In $(\pi_1(mpat) = \text{Animitracional}) \land (\pi_3(mpat) = s \text{.ierAncess})$ then let s' context := $\pi_s(inpat)$
60.	let $e' a := check lndEmit$
61.	stop /s' cookies' local Storacs' consistor Storacs' /\\
62.	ond if
62.	case checkIndEmit
64.	let s' email :- localStorage'[siteInfo][s' requestOrigin]
04.	ion a semart normanade [arrennto][a rednearontkin]

```
65:
              if (s'.email \neq \langle \rangle)
               \hookrightarrow \land (s'. \texttt{loggedInUser} \notin \{\langle \rangle, \bot\} \Rightarrow (s'. \texttt{loggedInUser} \not\equiv s'. \texttt{email}))
               \hookrightarrow \land (\pi_1(s'.\texttt{context}) \equiv \top) then
66:
                  let s'.q := \texttt{startPIF}
                  let url := \langle URL, S, \pi_2(s'.email), /pif \rangle
67:
                  let s'.PIFindex := |subwindows| + 1
68:
                                                    \triangleright Index of the next subwindow to be created.
69:
                  let command := \langle IFRAME, url, SELF \rangle
                  stop \langle s', cookies', localStorage', sessionStorage', command \rangle
70:
71:
              else if s'.loggedInUser \equiv \langle \rangle then
                  let s'.q := default
72:
                  stop \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle
73:
74:
              else
                  let s'.q := sendLogout
75:
76:
                  stop \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle
77:
              end if
         case startPIF
78:
              let idpOrigin := \langle \pi_2(s'.email), S \rangle
79:
              let input, s' := CHOOSEINPUT(s', scriptinputs)
80:
              let pifNonce := \pi_{s'.PIFindex}(subwindows).nonce
81:
              if \pi_1(input) \equiv \text{POSTMESSAGE then}
82:
                  let senderWindow := \pi_2(input)
83:
84:
                  let senderOrigin := \pi_3(input)
                  let m := \pi_4(input)
85:
86:
                  if m \equiv ping \land senderWindow \equiv pifNonce
                   \hookrightarrow \land senderOrigin \equiv idpOrigin then
87:
                       let command := \langle POSTMESSAGE, pifNonce, pong, idpOrigin \rangle
88:
                       let s'.q := runPIF
                       stop \langle s', cookies', localStorage', sessionStorage', command \rangle
89:
90:
                  end if
              end if
91:
92:
         case runPIF
93:
              let idpOrigin := \langle \pi_2(s'.\texttt{email}), \mathbf{S} \rangle
              let input, s' := CHOOSEINPUT(s', scriptinputs)
94:
95:
              let pifNonce := \pi_{s'.PIFindex}(subwindows).nonce
96:
              if \pi_1(input) \equiv \text{POSTMESSAGE then}
97:
                  let senderWindow := \pi_2(input)
98:
                  let senderOrigin := \pi_3(input)
99:
                  let m := \pi_4(input)
                   if senderWindow \equiv pifNonce \land senderOrigin \equiv idpOrigin then
100:
101:
                        switch \pi_1(m) do
102:
                             case beginProvisioning
103:
                                 let jschannel nonce := \pi_2(m)
                                 let command := \langle POSTMESSAGE, pifNonce, \rangle
104:
                                \hookrightarrow \langle jschannel nonce, s'.email \rangle, idpOrigin \rangle
                                 stop \langle s', cookies', localStorage', sessionStorage', command \rangle
105:
106:
                             case genKeyPair
107:
                                 let jschannel nonce := \pi_2(m)
108:
                                 let s'.key \leftarrow nonces
```

109:	let command := $\langle POSTMESSAGE, pifNonce, \rangle$
	$\hookrightarrow (jschannel nonce, pub(s'.\mathtt{key})), idpOrigin)$
110:	stop $\langle s', cookies', localStorage', sessionStorage', command \rangle$
111:	case registerCertificate
112:	if $\pi_1(extractmsg(\pi_2(m))) \equiv s'.email \land s'.email \neq \langle \rangle then$
	\triangleright This check is our fix against identity injection.
113:	$\mathbf{let} \ s'. \mathbf{uc} := \pi_2(m)$
114:	$\texttt{let} \ s'.\texttt{q} := \texttt{createCAPforRP}$
115:	end if
116:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
117:	case raiseProvisioningFailure
118:	$\texttt{let} \ s'.\texttt{loggedInUser} := \bot$
119:	$let\ s'.q:=\mathtt{sendLogout}$
120:	$\mathbf{stop}~\langle s', cookies', localStorage', sessionStorage', command \rangle$
121:	end if
122:	end if
123:	case createCAPforRP
124:	$\mathbf{let} \ ia := sig(s'.\texttt{requestOrigin}, s'.\texttt{key})$
125:	$\mathbf{let} \ cap := \langle s'. \mathbf{uc}, ia \rangle$
126:	let command := $\langle POSTMESSAGE, PARENTWINDOW(tree, docnonce), \rangle$
	$\hookrightarrow \ \langle \texttt{response}, \mathit{cap} angle, s'.\texttt{requestOrigin} angle$
127:	$\texttt{let} \ s'.\texttt{q} := \texttt{null}$
128:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
129:	case sendLogout
130:	let command := $\langle POSTMESSAGE, PARENTWINDOW(tree, docnonce), \rangle$
	$\hookrightarrow \ \langle \texttt{logout}, \langle angle angle, \perp angle$
131:	$\texttt{let} \ s'.\texttt{q} := \texttt{default}$
132:	$\textbf{stop} \ \langle s', \textit{cookies'}, \textit{localStorage'}, \textit{sessionStorage'}, \textit{command} \rangle$
133:	stop $\langle scriptstate, cookies, localStorage, sessionStorage, \langle \rangle \rangle$

login.persona.org Login Dialog Script (script_lpo_ld). This script models the LD contents. Its formal specification, presented next, follows the one presented above for script_lpo_cif.

Definition 44. A scriptstate s of script_lpo_ld is a term of the form $\langle q, requestOrigin, context, email, key, uc, handledInputs, refXHRctx, refXHRLPOauth, PIFindex \rangle$ with $q \in \mathbb{S}$, requestOrigin \in Origins $\cup \{\bot\}$, context $\in \mathcal{T}_{\mathcal{N}}$, email \in ID $\cup \{\bot\}$, key $\in \mathcal{N} \cup \{\bot\}$, $uc \in \mathcal{T}_{\mathcal{N}}$, handledInputs $\subset^{\langle \rangle} \mathbb{N}$, refXHRctx, refXHRLPOauth $\in \mathcal{N} \cup \{\bot\}$, PIFindex $\in \mathbb{N} \cup \{\bot\}$. The initial scriptstate initState_{ld} is the state $\langle \text{init}, \bot, \bot, \bot, \bot, \downarrow, \downarrow, \langle \rangle, \bot, \bot, \bot \rangle$.

Before we provide the formal specification of the relation that defines the behavior of $script_lpo_ld$, we present an informal description. The behavior mainly depends on the state q the script is in.

- $q \equiv \text{init}$ This is the initial state. Its only transition takes no input and outputs a postMessage ldready to its parent window and transitions to start.
- $q \equiv \texttt{start}$ In this state, the script expects a <code>request</code> postMessage. The sender's origin of this postMessage is recorded as the requesting origin in the script-

state. An XHR is sent to LPO with a GET request to the path /ctx and then the script transitions to the state receiveContext.

 $q \equiv \texttt{receiveContext}$ In this state, the script expects an XHR response as input containing the session context. This context is saved as the current context in the scriptstate. The script checks if an <code>idpNonce</code> is recorded in the sessionStorage. The presence of this nonce indicates that there was a run of <code>script_lpo_ld</code> in the same window previously. Indexed by this nonce, there can be an email address (identity) recorded in the localStorage which is then copied to the script's state. Otherwise an email address is non-deterministically choosen (and copied to the script's state) out of the email addresses owned by the browser.

The script now always issues the command to create an iframe, the PIF. The URL for the PIF is determined by the domain of the email address now recorded in the state. The script then transitions to the state startPIF.

- q = startPIF In this state, the script waits for a postMessage from the PIF containing a ping message. If such a message is received and the sender's window and origin match the PIF, the script sends a pong message back to the PIF and transitions to the state <code>runPIF</code>.
- q = runPIF This is the state in which script_lpo_ld interacts with the PIF. This state handles all postMessages the LD expects to receive from the latest PIF (as recorded in PIFindex in its state). If the postMessage received was sent from the PIF's window and the PIF's origin, it behaves as follows, depending on the first element of the received postMessage:
 - **postMessage beginProvisioning** The script responds with a postMessage to the PIF containing the email address of the identity which is to authenticate to the relying party (as recorded in the LD's state).
 - **postMessage genKeyPair** The script creates a fresh key pair (i.e. the LD chooses a fresh nonce) and sends the public key contained in an postMessage to the PIF.
 - **postMessage registerCertificate** The script stores the UC received in this postMessage in the LD's state. If the context contained in the script's state indicates that the browser is authenticated to LPO, the script transitions to the state createCAPforRP. Otherwise, the script transitions to the state createCAPforLPO.
 - **postMessage raiseProvisioningFailure** This message indicates that no one is logged in. The script now chooses a fresh nonce, the so-called *idpNonce*, which is stored in the sessionStorage. In the localStorage, this nonce is used as a key under which the email address is stored, the LD is currently trying to get an UC for. The script navigates the window it is running to the authentication path at the identity provider responsible for the email address.
- q = createCAPforLD In this state, the script creates an IA for LPO, combines it with the UC (stored in the script's state) to a CAP and sends the CAP to LPO in an XHR. The nonce identifying the XHR is stored as <code>refXHRLPOauth</code> in the script's state.

- q = receiveLPOauthresponse In this state, the script expects the response to the XHR identified by the nonce <code>refXHRLPOauth</code>. If the response indicates a successful authentication at LPO, the context recorded in the script's state is changed accordingly and the script transitions to the state <code>createCAPforRP</code>.
- q = createCAPforRP In this state, the script creates an IA for the request origin (as recorded in the script's state), combines the IA with the UC to a CAP, and sends the CAP in a postMessage to its parent restricting the receiver to the request origin. The script records in the localStorage that the email address it is currently using is logged in at the request origin. The script then transitions to the state null.
- $q \equiv$ null In this state, the script does nothing.

We now formally specify the relation $script_lpo_ld \subseteq (\mathcal{T}_{\mathcal{N}} \times 2^{\mathcal{N}}) \times \mathcal{T}_{\mathcal{N}}$ of the LD's scripting process. Just like in Appendix C.2, we describe this relation by a non-deterministic algorithm. Like all scripts, the input term given to this script is determined by the browser and the browser expects a term of a specific form (see Algorithm 6)

Algorithm 14 Relation of script_lpo_ld

Input: (tree, docnonce, scriptstate, scriptinputs, cookies, localStorage, sessionStorage
\rightarrow lus, secret/, nonces
1: let $s := scriptstate$
2: let $cookies := cookies$
3: let localStorage := localStorage
4: let sessionStorage := sessionStorage
b: switch s' q do
6: case init
7: let command := (POSTMESSAGE , OPENERWINDOW (tree, docnonce),
$\hookrightarrow \ \ \left< \texttt{ldready}, \left< \right> \right>, \ \ \bot \right>$
8: let $s'.q := start$
9: stop $\langle s', cookies', localStorage', sessionStorage', command \rangle$
10: case start
11: let <i>input</i> , $s' := CHOOSEINPUT(s', scriptinputs)$
12: if $\pi_1(input) \equiv \text{POSTMESSAGE then}$
13: let sender Window := $\pi_2(input)$
14: let senderOrigin := $\pi_3(input)$
15: let $m := \pi_4(input)$
16: if $m \equiv \langle \text{request}, \langle \rangle \rangle$ then
17: let s' .requestOrigin := senderOrigin
18: let s' .refXHRctx \leftarrow nonces
19: let <i>command</i> := $\langle XMLHTTPREQUEST, URL_{(ctr}^{LPO}, GET, \langle \rangle, s'.refXHRctx \rangle$
20: let $s'.q := receiveContext$
21: $stop(s', cookies', localStorage', sessionStorage', command)$
22: end if
23: end if
24: case receiveContext
25: let <i>input</i> , $s' := CHOOSEINPUT(s', scriptinputs)$
26: if $(\pi_1(input) \equiv XMLHTTPREQUEST) \land (\pi_3(input) \equiv s'.refXHRctx)$ then

27:let s'.context := $\pi_2(input)$ 28:let s'.q := startPIF29:**let** *idpnonce* := *sessionStorage*[idpnonce] 30: if $idpnonce \equiv \langle \rangle \lor localStorage[idpnonce] \equiv \langle \rangle$ then let s'.email $\leftarrow ids$ 31: 32: else 33: let s'.email := localStorage[idpnonce] 34: let $sessionStorage[idpnonce] := \langle \rangle$ 35:end if 36: let $url := \langle URL, S, \pi_2(s'.email), /pif \rangle$ 37: let s'.PIFindex := |subwindows| + 1 \triangleright Index of the next subwindow to be created. let $command := \langle IFRAME, url, _SELF \rangle$ 38: 39: **stop** $\langle s', cookies', localStorage', sessionStorage', command \rangle$ 40: end if 41: case startPIF 42: let $idpOrigin := \langle \pi_2(s'.\texttt{email}), \mathsf{S} \rangle$ 43: let input, s' := CHOOSEINPUT(s', scriptinputs)44: let $pifNonce := \pi_{s'.PIFindex}(subwindows).nonce$ if $\pi_1(input) \equiv \text{POSTMESSAGE then}$ 45:46:let $senderWindow := \pi_2(input)$ 47:let senderOrigin := $\pi_3(input)$ 48: let $m := \pi_4(input)$ if $m \equiv ping \land senderWindow \equiv pifNonce$ 49: $\hookrightarrow \land$ senderOrigin \equiv idpOrigin then 50: **let** command := $\langle POSTMESSAGE, pifNonce, pong, idpOrigin \rangle$ 51:let s'.q := runPIF**stop** $\langle s', cookies', localStorage', sessionStorage', command \rangle$ 52:end if 53: 54: end if case runPIF 55: 56: let $idpOrigin := \langle \pi_2(s'.\texttt{email}), \mathbf{S} \rangle$ let input, s' := CHOOSEINPUT(s', scriptinputs)57:58:let $pifNonce := \pi_{s', PIFindex}(subwindows).$ nonce if $\pi_1(input) \equiv \text{POSTMESSAGE then}$ 59:60: let senderWindow := $\pi_2(input)$ let senderOrigin := $\pi_3(input)$ 61: 62: let $m := \pi_4(input)$ 63: $\mathbf{if} \ senderWindow \equiv pifNonce \land senderOrigin \equiv idpOrigin \ \mathbf{then}$ 64:switch $\pi_1(m)$ do 65:case beginProvisioning 66: let *jschannel* nonce := $\pi_2(m)$ let command := $\langle POSTMESSAGE, pifNonce, \rangle$ 67: $\hookrightarrow \langle jschannel nonce, s'.email \rangle, idpOrigin \rangle$ 68: **stop** $\langle s', cookies', localStorage', sessionStorage', command \rangle$ 69: case genKeyPair 70: let $jschannel_nonce := \pi_2(m)$ 71:let s'.key \leftarrow nonces

72:	let $command := \langle POSTMESSAGE, pifNonce, \rangle$
	$\hookrightarrow \langle jschannel \ nonce, pub(s'.\mathtt{key}) angle, \ idpOrigin angle$
73:	stop $\langle s', cookies', localStorage', sessionStorage', command \rangle$
74:	case registerCertificate
75:	if $\pi_1(\text{extractmsg}(\pi_2(m))) \equiv s'.\text{email} \land s'.\text{email} \not\equiv \langle \rangle$ then
	▷ This check is our fix against identity injection.
76:	$let\ s'.uc:=\pi_2(m)$
77:	let $loggedIn := \pi_1(s'.context)$
78:	$\mathbf{if} \ loggedIn \equiv \top \ \mathbf{then}$
79:	$\texttt{let} \ s'.\texttt{q} := \texttt{createCAPforRP}$
80:	end if
81:	$\mathtt{let}\;s'.\mathtt{q}:=\mathtt{createCAPforLPO}$
82:	end if
83:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
84:	${f case}$ raiseProvisioningFailure
85:	$\textbf{let} \ idpnonce \leftarrow nonces$
86:	$let \ localStorage'[idpnonoce] := s'.email$
87:	$let \ sessionStorage'[idpnonce] := idpnonce$
88:	$\mathbf{let} \ command := \langle \mathtt{HREF}, \langle \mathtt{URL}, \mathtt{S}, \pi_2(s'.\mathtt{email}), \langle \rangle \rangle, _\mathtt{SELF} \rangle$
89:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
90:	end if
91:	end if
92:	case createCAPforLPO
93:	$\mathbf{let} \ ia := sig(\langle dom(LPO), \mathtt{S} \rangle, s'.\mathtt{key})$
94:	$\mathbf{let} \ cap := \langle s'. \mathbf{uc}, ia \rangle$
95:	$\texttt{let } body := \langle cap, \pi_2(s'.\texttt{context}) \rangle$
96:	$\texttt{let} \ s'.\texttt{refXHRLPOauth} \leftarrow nonces$
97:	$\mathbf{let} \ command := \langle \texttt{XMLHTTPREQUEST}, URL_{/\mathtt{auth}}^{LPO}, POST, \mathit{body}, s'.\mathtt{refXHRLPOauth} \rangle$
98:	${f let}\; s'.{f q}:={ t receiveLPO}{f authresponse}$
99:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
100:	\mathbf{case} receiveLPOauthresponse
101:	let $input, s' := CHOOSEINPUT(s', scriptinputs)$
102:	$\mathbf{if} \ (\pi_1(input) \equiv \mathtt{XMLHTTPREQUEST}) \land (\pi_3(input) \equiv s'.\mathtt{refXHRLPOauth})$
	$ \hookrightarrow \wedge \pi_2(input) \equiv op {f then}$
103:	$\texttt{let} \ \pi_1(s'.\texttt{context}) := \top$
104:	$\texttt{let} \ s'.\texttt{q} := \texttt{createCAPforRP}$
105:	$\mathbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$
106:	end if
107:	case createCAPforRP
108:	${f let} \; ia := {f sig}(s'.{ t requestOrigin}, s'.{ t key})$
109:	$\mathbf{let} \ cap := \langle s'. \mathtt{uc}, ia \rangle$
110:	$let \ command := \langle POSTMESSAGE, \ OPENERWINDOW(tree, \ docnonce),$
	$\hookrightarrow \ \langle \texttt{response}, \mathit{cap} angle, s'.\texttt{requestOrigin} angle$
111:	let s'.q := null
112:	$\texttt{let} \ \textit{localStorage'}[\texttt{siteInfo}][s'.\texttt{requestOrigin}] := s'.\texttt{email}$
113:	$\textbf{stop} \ \langle s', cookies', localStorage', sessionStorage', command \rangle$

114: **stop** $\langle scriptstate, cookies, localStorage, sessionStorage, \langle \rangle \rangle$

Relying Party Web Page Script (script _rp_index). This script models the default web page at a RP. The user usually triggers the login process on this page. Its formal specification, presented next, follows the one presented for the other scripts above.

Definition 45. A scriptstate *s* of *script*_*rp*_*index is a term of the form* $\langle q, CIFindex, LDindex, dialogRunning, cap, handledInputs, refXHRcap <math>\rangle$ with $q \in \mathbb{S}$, *CIFindex* $\in \mathbb{N} \cup \{\bot\}$, *dialogRunning* $\in \{\top, \bot\}$, *cap* $\in T_{\mathcal{N}}$, *handledInputs* $\subset^{\langle\rangle} \mathbb{N}$, *refXHRcap* $\in \mathcal{N} \cup \{\bot\}$. We call *s* the initial scriptstate of *script*_*rp*_*index iff* $s \equiv \langle \texttt{init}, \bot, \bot, \bot, \langle \rangle, \langle \rangle, \bot \rangle$.

Before we provide the formal specification of the relation that defines the behavior of $script_rp_index$, we present an informal description. The behavior mainly depends on the state q the script is in.

- $q \equiv \text{init}$ This is the initial state. The script creates the CIF iframe and then transitions to receiveCIFReady.
- $q \equiv \texttt{receiveCIFReady}$ In this state, the script expects a <code>cifready</code> postMessage from the CIF iframe with the sender origin of LPO. The script chooses some ID, $\langle \rangle$, or \perp and sends this in a <code>loaded</code> postMessage to the CIF iframe with receiver's origin set to the origin of LPO.¹¹ It then transitions to the state <code>default</code>.
- $q \equiv \text{default}$ In this state, the script chooses non-deterministically between (1) opening the LD subwindow and then transitioning to the same state or (2) handling one of the following postMessages (identified by their first element): postMessage login This message has to be sent from the CIF with ori
 - gin of LPO. Handling this postMessage stores the CAP (contained in the postMessage) in the scriptstate and then transitions to the sendCAP state.
 - **postMessage logout** This message has to be sent from the CIF with origin of LPO. Handling this postMessage has no effect and results in the same state.
 - **postMessage ldready** This message can only be handled after the LD has been opened and before a **response** postMessage has been received. The **ldready** postMessage has to be sent from the origin of LPO. The script sends a **request** postMessage to the LD and stays in the **default** state.
 - **postMessage response** This message can only be handled after the LD has been opened and before another **response** postMessage has been received. The ldready postMessage has to be sent from the origin of LPO. Handling this postMessage stores the CAP (contained in the postMessage) in the scriptstate, closes the LD, and then transitions to the dlgClosed state.

¹¹From the point of view of the real scripts running at RP either some ID is considered to be logged in (e.g. from some former "session"), or no one is considered to be logged in ($\langle \rangle$), or the script *script_rp_index* does not know if it should consider anyone to be logged in (\perp). This is overapproximated here by allowing *script_rp_index* to choose non-deterministically between these cases.

- $q \equiv \texttt{dlgClosed}$ In this state, the script sends a loggedInUser postMessage to the CIF and transitions to the loggedInUser state.
- $q \equiv \texttt{loggedInUser}$ In this state, the script sends a dlgCmplt postMessage to the CIF and transitions to the sendCAP state.
- $q \equiv \texttt{sendCAP}$ In this state, the script sends the CAP to RP as a POST XHR and then transitions to the <code>receiveServiceToken</code> state.
- $q \equiv \texttt{receiveServiceToken}$ In this state, the script receives $\langle n, i \rangle$ from RP, but does not do anything with it. The script then transitions to the default state.

We now formally specify the relation $script_rp_index \subseteq (\mathcal{T}_{\mathcal{H}} \times 2^{\mathcal{H}}) \times \mathcal{T}_{\mathcal{H}}$ of the RP-Doc's scripting process. Just like in Appendix C.2, we describe this relation by a non-deterministic algorithm. Like all scripts, the input term given to this script is determined by the browser and the browser expects a term of a specific form (see Algorithm 6). Following Algorithm 15, we provide some more explanation.

Algorithm 15 Relation of *script* rp index

Input: (tree, docnonce, scriptstate, scriptinputs, cookies, localStorage, sessionStorage,
$ \hookrightarrow \mathit{ids}, \mathit{secret} angle, \mathit{nonces}$
1: let $s' := scriptstate$
2: let $cookies' := cookies$
3: let $localStorage' := localStorage$
4: let $sessionStorage' := sessionStorage$
5: switch s' .q do
6: case init
7: let command := $\langle IFRAME, URL_{cif}^{LPO}, GETWINDOW(tree, docnonce) \rangle$
8: let $s'.q := receiveCIFReady$
9: let $subwindows := SUBWINDOWS(tree, docnonce)$
10: let $s'.CIFindex := subwindows + 1$
\triangleright Index of the next subwindow to be created.
11: stop $\langle s', cookies', localStorage', sessionStorage', command \rangle$
12: case receiveCIFReady
13: let $input, s' := CHOOSEINPUT(s', scriptinputs)$
14: if $\pi_1(input) \equiv \text{POSTMESSAGE then}$
15: let senderWindow := $\pi_2(input)$
16: let senderOrigin := $\pi_3(input)$
17: let $m := \pi_4(input)$
18: let $subwindows := SUBWINDOWS(tree, docnonce)$
19: if $(m \equiv \langle \texttt{cifready}, \langle \rangle \rangle)$
$ \hookrightarrow \wedge (senderOrigin \equiv {\sf origin}_{{\sf LPO}})$
$\hookrightarrow \land (senderWindow \equiv \pi_{s'.\texttt{CIFindex}}(subwindows).\texttt{nonce}) ext{ then}$
20: let $id \leftarrow \{\bot, \langle\rangle\} \cup ID$
21: let command := (POSTMESSAGE, $\pi_{s'.CIFindex}(subwindows)$,
\hookrightarrow $\langle \texttt{loaded}, id angle, origin_{LPO} angle$
22: let $s'.q := default$
23: $stop \langle s', cookies', localStorage', sessionStorage', command \rangle$
24: end if

```
end if
25:
26:
         case default
27:
             if s'.dialogRunning \equiv \bot then
28:
                  let choice \leftarrow \{\texttt{openLD}, \texttt{handlePM}\}
29:
              else
30:
                  let choice := handlePM
31:
              end if
32:
             if choice \equiv \texttt{openLD then}
                  let s'.dialogRunning := \top
33:
                  let command := \langle \text{HREF}, \text{URL}_{/\text{ld}}^{\text{LPO}}, \text{\_BLANK} \rangle
34:
                  let s'.q := default
35:
36:
                  stop \langle s', cookies', localStorage', sessionStorage', command \rangle
37:
             else
                  let input, s' := CHOOSEINPUT(s', scriptinputs)
38:
39:
                  if \pi_1(input) \equiv \text{POSTMESSAGE then}
40:
                      let senderWindow := \pi_2(input)
                      let senderOrigin := \pi_3(input)
41:
42:
                       let m := \pi_4(input)
                       let subwindows := SUBWINDOWS(tree, docnonce)
43:
                       if senderOrigin \equiv \text{origin}_{\text{LPO}} then
44:
                           if senderWindow \equiv \pi_{s',CIFindex}(subwindows).nonce then
45:
46:
                               if \pi_1(m) \equiv \text{login then}
47:
                                   let s'.cap := \pi_2(m)
                                   {\bf let} \ s'.{\bf q} := {\tt sendCAP}
48:
49:
                                   stop \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle
50:
                               else if \pi_1(m) \equiv \text{logout then}
51:
                                   let s'.q := default
                                   stop \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle
52:
53:
                               end if
54:
                           else if s'.dialogRunning \equiv \top then
                               if \pi_1(m) \equiv \texttt{ldready then}
55:
56:
                                   let command := \langle POSTMESSAGE,
                                     \hookrightarrow AUXWINDOW(tree, docnonce), \langle \texttt{request}, \langle \rangle \rangle, origin<sub>LPO</sub> \rangle
57:
                                   let s'.q := default
                                   stop \langle s', cookies', localStorage', sessionStorage', command \rangle
58:
59:
                               else if \pi_1(m) \equiv response then
                                   let s'.dialogRunning := \bot
60:
61:
                                   let s'.cap := \pi_2(m)
62:
                                   let command := \langle CLOSE, AUXWINDOW(tree, docnonce) \rangle
63:
                                   let s'.q := dlgClosed
64:
                                   stop \langle s', cookies', localStorage', sessionStorage', command \rangle
65:
                               end if
66:
                           end if
                      end if
67:
68:
                  end if
             end if
69:
70:
         case dlgClosed
             let subwindows := SUBWINDOWS(tree, docnonce)
71:
                                                                               \triangleright Extract ID from CAP.
72:
             let id := \pi_1(\text{extractmsg}(\pi_1(s'.\text{cap})))
```
```
73:
             let command := \langle \text{POSTMESSAGE}, \pi_{s'.CIFindex}(subwindows).nonce,
               \hookrightarrow (loggedInUser, id), origin<sub>LPO</sub>)
74:
             let s'.q := loggedInUser
              stop \langle s', cookies', localStorage', sessionStorage', command \rangle
75:
76:
         case loggedInUser
             let subwindows := SUBWINDOWS(tree, docnonce)
77:
             let command :=
78:
              \hookrightarrow (POSTMESSAGE, \pi_{s'.CIFindex}(subwindows).nonce, (dlgCmplt, ()), origin<sub>LPO</sub>)
79:
             let s'.q := sendCAP
80:
              stop \langle s', cookies', localStorage', sessionStorage', command \rangle
81:
         case sendCAP
82:
              let s'.refXHRcap \leftarrow nonces
83:
              let host, protocol such that
               \hookrightarrow \langle host, protocol \rangle \equiv \mathsf{GETORIGIN}(tree, docnonce)
               \hookrightarrow if possible; otherwise stop
               \hookrightarrow (scriptstate, cookies, localStorage, sessionStorage, command)
84:
             let command := \langle XMLHTTPREQUEST, \langle URL, protocol, host, /, \langle \rangle \rangle, POST, s'.cap,
               \hookrightarrow s'.\texttt{refXHRcap}
                                                                         \triangleright Relay received CAP to RP.
             let s'.q := receiveServiceToken
85:
86:
             stop \langle s', cookies', localStorage', sessionStorage', command \rangle
87:
         case receiveServiceToken
             let input, s' := CHOOSEINPUT(s', scriptinputs)
88:
             if (\pi_1(input) \equiv \texttt{XMLHTTPREQUEST}) \land (\pi_3(input) \equiv s'.\texttt{refXHRcap}) then
89:
90:
                  let s'.q := default
91:
                  stop \langle s', cookies', localStorage', sessionStorage', \langle \rangle \rangle
92:
              end if
93: stop \langle scriptstate, cookies, localStorage, sessionStorage, \langle \rangle \rangle
```

In Lines 7–11 and 33–36 the script asks the browser to create iframes. To obtain the window reference for these iframes, the script first determines the current number of subwindows and stores it (incremented by 1) in the scriptstate (CIFindex and LDindex, respectively). When the script is invoked the next time, the iframe the script asked to be created will have been added to the sequence of subwindows by the browser directly following the previously existing subwindows. The script can therefore access the iframe by the indexes CIFindex and LDindex, respectively.

Identity Provider Authentication Dialog Script (script_idp_ad). This script runs in the LD after *script_lpo_ld* has navigated the LD window. The purpose of this script is to authenticate the browser to the identity provider.

The script non-deterministically chooses if it sends authentication data to the IdP (i.e. its origin) via an XHR, or if it navigates the window to an URL at LPO which servers *script_lpo_ld*. Note that *script_idp_ad* does not read or change its scriptstate. Hence, we omit the definition of the scriptstate for this script.

Algorithm 16 Relation of script idp ad

Input: $\langle tree, docnonce, scriptstate, scriptinputs, cookies, localStorage, sessionStorage, <math>\hookrightarrow$ ids, secret \rangle , nonces

1: let $action \leftarrow \{authenticate, navigate\}$ 2: if $action \equiv authenticate$ then 3: let $email \leftarrow ids$ let $body := \langle email, secret \rangle$ 4: 5: let *host*, *protocol* such that \leftrightarrow (host, protocol) \equiv GETORIGIN(tree, docnonce) \hookrightarrow if possible; otherwise \hookrightarrow **stop** (*scriptstate*, *cookies*, *localStorage*, *sessionStorage*, $\langle \rangle \rangle$ $\mathbf{let} \ command := \langle \mathtt{XMLHTTPREQUEST}, \langle \mathtt{URL}, protocol, host, /\mathtt{auth}, \langle \rangle \rangle, \mathtt{POST}, body, \bot \rangle$ 6: 7: **stop** (*scriptstate*, *cookies'*, *localStorage'*, *sessionStorage'*, *command*) 8: else let *command* := $\langle \text{HREF}, \langle \text{URL}, \text{S}, \text{dom}(\text{LPO}), /\text{ld}, \langle \rangle \rangle$, SELF \rangle 9: 10: **stop** (*scriptstate*, *cookies'*, *localStorage'*, *sessionStorage'*, *command*) 11: end if

Identity Provider Provisioning Iframe Script (script_idp_pif). This script acts as a proxy between the LD or CIF and the IdP server.

Definition 46. A scriptstate s of $script_idp_pif$ is a term of the form $\langle q, emails, pubkeys, ucs, provisioningnonces, genkeypairnonces, xhrnonces, handledInputs <math>\rangle$ with $q \in \mathbb{S}$, emails, pubkeys, ucs $\in T_{\mathcal{H}}$, provisioningnonces, genkeypairnonces, xhrnonces $\in \mathcal{N} \cup \{\bot\}$, handledInputs $\subset^{\langle \rangle} \mathbb{N}$. We call s the initial scriptstate of $script_idp_pif$ iff $s \equiv \langle \texttt{init}, \langle \rangle, \langle \rangle, \bot, \bot, \bot \rangle$.

Before we provide the formal specification of the relation that defines the behavior of $script_idp_pif$, we present an informal description. The behavior mainly depends on the state q the script is in.

- q =init This is the initial state. Its only transition handles no input and outputs a postMessage ping to its parent window, which has to have the origin of LPO, and transitions to waiting.
- q = waiting In this state, the script expects a postMessage containing either ping or pong, which has to be sent by the parent window from the origin of LPO. If such a postMessage has been received, the script transitions to default.
- q = default In this state, the script chooses an action non-deterministically out of the following:
 - beginprovisioning The script sends a postMessage to the parent window, which has to have the origin of LPO, indicating that the provisioning process of a UC should start. A fresh nonce is chosen, stored in the script's state, and included in this postMessage. The postMessage requests the email address of the user from the receiver. The address is to be sent to the PIF in a postMessage which is identified by the nonce in the request.
 - genkeypair The script sends a postMessage to the parent window, which has to have the origin of LPO, indicating that a new key pair should be generated. This postMessage requests the public key of this fresh key pair. As above, a nonce is included to identify the response corresponding to the request.

- registercert The script sends a postMessage containing a UC to the parent window, which has to have the origin of LPO. This postMessage is only sent if the script has received a UC before.
- raisefailure The script sends a postMessage to the parent window, which has to have the origin of LPO, indicating that the browser is currently not authenticated to the identity provider.
- requestuc The script sends an XHR to the origin of the current document if the scriptstate contains at least one email address and one public key. The message contains a non-deterministically chosen email address and a public key (from the scriptstate). The nonce identifying this XHR is non-deterministically chosen and stored in the scriptstate.
- handleresponse The script chooses non-deterministically a script input and distinguishes if this input is a postMessage or an XHR response.

If the chosen input is a postMessage, it is checked if the postMessage was sent by the parent window and if this window has the origin of LPO. If this check is successful, it is checked if the message contains a nonce, which was previously been recorded in the script's state. If this nonce indicates that this message is a response to a beginProvisioning postMessage, the second part is assumed to contain an email address. This address is then recorded in the script's state. If the nonce indicates that this message is a response to a genKeyPair postMessage, the second part is assumed to contain a public key. This public key is then recorded in the script's state.

If the chosen input is an XHR response, it is checked if the nonce identifying the XHR is recorded in the script's state. If this is the case, the message is assumed to contain an UC. The content of the message is stored in the script's state.

Algorithm 17 Relation of script_idp_pif

Inpu	$\textbf{it:} \ \langle tree, \ docnonce, \ script state, \ script inputs, \ cookies, \ local Storage, \ session Storage$
\hookrightarrow	$ids, secret \rangle, nonces$
1: l e	$\mathbf{et} \ s' := scriptstate$
$2: \mathbf{s}$	witch s' .q do
3:	case init
4:	let command := $\langle POSTMESSAGE, PARENTWINDOW(tree, docnonce), \rangle$
	$ \hookrightarrow \langle \texttt{ping}, \langle \rangle \rangle, \langle \texttt{dom}(LPO), \mathtt{S} \rangle \rangle$
5:	let s'.q := waiting
6:	$\textbf{stop}\;\langle s', cookies, localStorage, sessionStorage, command \rangle$
7:	case waiting
8:	let $input, s' := CHOOSEINPUT(s', scriptinputs)$
9:	let senderWindow := $\pi_2(input)$
10:	let senderOrigin := $\pi_3(input)$
11:	let $m := \pi_4(input)$
12:	if $\pi_1(input) \in \{\texttt{ping}, \texttt{pong}\}$
	$\hookrightarrow \land$ sender Window \equiv PARENTWINDOW(tree, docnonce)

 $\hookrightarrow \land senderOrigin \equiv \langle \mathsf{dom}(\mathsf{LPO}), \mathsf{S} \rangle$ then

13:	let s'.q := default
14:	end if
15:	stop $\langle s', cookies, localStorage, sessionStorage, \langle \rangle \rangle$
16:	case default
17:	let $action \leftarrow \{ beginprovisioning, genkeypair, registercert, \}$
	\hookrightarrow raisefailure, requestuc, handleresponse}
18:	switch action do
19:	case beginprovisioning
20:	$\textbf{let } jschannel_nonce \leftarrow nonces$
21:	let command := $(POSTMESSAGE, PARENTWINDOW(tree, docnonce)),$
	$\hookrightarrow \ \langle \texttt{beginProvisioning}, jschannel_nonce angle, \ \texttt{dom}(LPO) angle$
22:	let s' .provisioningnonces :=
	$\hookrightarrow s'$ provisioningnonces $+^{\langle \rangle} jschannel_nonce$
23:	$\mathbf{stop}\;\langle s', cookies, localStorage, sessionStorage, command angle$
24:	\mathbf{case} genkeypair
25:	$\textbf{let } jschannel_nonce \leftarrow nonces$
26:	let command := $(POSTMESSAGE, PARENTWINDOW(tree, docnonce)),$
	$\hookrightarrow \ \langle \texttt{genKeyPair}, jschannel_nonce angle, \ dom(LPO) angle$
27:	$\mathbf{let} \ s'$.genkeypairnonces :=
	$ \hookrightarrow s'.$ genkeypairnonces $+^{\langle angle} jschannel_nonce$
28:	$\mathbf{stop} \ \langle s', cookies, localStorage, sessionStorage, command angle$
29:	case registercert
30:	if s'.ucs $\neq \langle \rangle$ then
31:	$\mathbf{let} uc \leftarrow s'. \mathbf{ucs}$
32:	$let \ command := (POSTMESSAGE, PARENTWINDOW(tree, docnonce),$
	$\hookrightarrow \ \langle \texttt{registerCertificate}, \mathit{uc} angle, \ dom(LPO) angle$
33:	$\textbf{stop} \ \langle s', cookies, localStorage, sessionStorage, command \rangle$
34:	end if
35:	case raisefailure
36:	$let \ command := \langle POSTMESSAGE, PARENTWINDOW(tree, docnonce), \rangle$
	$\hookrightarrow \ \langle \texttt{raiseProvisioningFailure}, \perp angle, dom(LPO) angle$
37:	$\mathbf{stop} \ \langle s', cookies, localStorage, sessionStorage, command angle$
38:	case requestuc
39:	$\mathbf{if} \ s'.\mathtt{emails} \not\equiv \langle \rangle \land s'.\mathtt{pubkeys} \not\equiv \langle \rangle \ \mathbf{then}$
40:	$\texttt{let} \ email \leftarrow s'.\texttt{emails}$
41:	$\mathbf{let} \ pubkey \leftarrow s'.\mathtt{pubkeys}$
42:	$\mathbf{let} \ body := \langle email, pubkey \rangle$
43:	let $xhrnonce \leftarrow nonces$
44:	let s'.xhrnonces := s'.xhrnonces $+^{\langle \rangle} xhrnonce$
45:	let host, protocol such that
	$\hookrightarrow \langle host, protocol \rangle \equiv GETORIGIN(tree, docnonce)$
	\hookrightarrow if possible; otherwise
	$\hookrightarrow \mathbf{stop} \langle s', cookies, localStorage, sessionStorage, \langle angle angle$
46:	let $command := \langle XMLHTTPREQUEST, \rangle$
	$\hookrightarrow \langle \text{URL}, protocol, host, / certreq}, \langle \rangle \rangle, \text{POST}, body, xhrnonce} \rangle$
47:	$stop \langle s', cookies, localStorage, sessionStorage, command \rangle$
48:	end if
49:	case handleresponse
50:	let $input, s' := CHOOSEINPUT(s', scriptinputs)$

51:	if $\pi_1(input) \equiv \texttt{POSTMESSAGE then}$
52:	let senderWindow := $\pi_2(input)$
53:	let senderOrigin := $\pi_3(input)$
54:	let $m := \pi_4(input)$
55:	if $senderWindow \equiv PARENTWINDOW(tree, docnonce)$
	$\hookrightarrow \land senderOrigin \equiv \langle dom(LPO), \mathtt{S} \rangle \ \mathbf{then}$
56:	if $\pi_1(m) \in s'$.provisioningnonces then
57:	$\texttt{let} \ s'.\texttt{emails} := s'.\texttt{emails} \ +^{\langle\rangle} \ \pi_2(m)$
58:	$\mathbf{else} \; \mathbf{if} \; \pi_1(m) \in s'.$ genkeypairnonces \mathbf{then}
59:	${f let}\; s'.$ pubkeys $:=s'.$ pubkeys $+^{\langle angle}\; \pi_2(m)$
60:	end if
61:	stop $\langle s', cookies, localStorage, sessionStorage, \langle \rangle \rangle$
62:	end if
63:	else if $\pi_1(input) \equiv XMLHTTPREQUEST$
	$\hookrightarrow\wedge\pi_3(input)\in s'.$ xhrnonces then
64:	$\mathbf{let} s'.\mathtt{ucs} := s'.\mathtt{ucs} + {}^{\langle\rangle} \pi_2(input)$
65:	stop $\langle s', cookies, localStorage, sessionStorage, \langle \rangle \rangle$
66:	end if
67:	stop $\langle scriptstate, cookies, localStorage, sessionStorage, \langle \rangle \rangle$

G Formal Security Properties

The security properties for BrowserID, informally introduced in Section 5.2, are formally defined as follows. First note that every RP service token $\langle n, i \rangle$ recorded in RP was created by RP as the result of a unique HTTPS POST request m with a valid CAP for ID i. We refer to m as the request corresponding to $\langle n, i \rangle$.

Definition 47. Let BID be a BrowserID web system. We say that BID is secure if for every run ρ of BID, every state (S_j, E_j) in ρ , every $r \in \mathsf{RP}$ that is honest in S_j , every RP service token of the form $\langle n, i \rangle$ recorded in r in the state $S_j(r)$, the following two conditions are satisfied:

(A) If $\langle n, i \rangle$ is derivable from the attackers knowledge in S_j (i.e., $\langle n, i \rangle \in d_{N^{\text{attacker}}}(S_j(\text{attacker}))$), then it follows that the browser b owning i is fully corrupted in S_j (i.e., the value of isCorrupted is FULLCORRUPT) or governor(i) is not an honest IdP (in S_j).

(B) If the request corresponding to $\langle n, i \rangle$ was sent by some $b \in B$ which is honest in S_i , then b owns i.

H Proof for Primary IdPs

In order to prove Theorem 1, we have to prove Conditions A and B of Definition 47. We prove these conditions separately. First, we provide an overview of the proofs.

H.1 Overview

For Condition (A), we analyze the request to an honest RP r upon which r returned a service token $\langle n, i \rangle$, where i is an ID and n a nonce. We show that

it must contain a valid CAP (for the identity *i*). For this, it must in particular contain a valid UC and a matching IA. We show that the UC must have been created by the IdP that governs the identity *i* (which is honest by assumption). We can then show that only *b* can request a UC at the IdP for the identity *i*, and that *b* does not leak the private key that corresponds to the public key used for this UC, and that this key was chosen from *b*'s set of fresh nonces. Thus, only *b* can know the key that is used in the creation of the UC in the CAP. We show that neither the private key corresponding to the public key in the UC, nor the IA can leak to the attacker. Thus, the attacker cannot have sent the request corresponding to $\langle n, i \rangle$ to the RP *r*. Also, $\langle n, i \rangle$ does not leak to the attacker. The attacker can therefore not know $\langle n, i \rangle$, which contradicts the assumption and proves that Condition (A) is satisfied.

For Condition (**B**), we focus on the request corresponding to $\langle n, i \rangle$ as well. We observe that if the request was sent by b, the script that initiated the request was $script_rp_index$, which again got the CAP that is finally used in the request from either $script_lpo_cif$ or $script_lpo_ld$ (any other sources, including the attacker script, can be ruled out). In both of these scripts, the identity in the CAP is checked against the list of identities of the browser (here, the proposed patch comes into play). This ensures that the request corresponding to $\langle n, i \rangle$ contains a CAP for an identity of the browser, which contradicts the assumption that Condition (**B**) is not satisfied and thus proves the theorem.

H.2 Condition A

We assume that Condition A is not satisfied and prove that this leads to a contradiction. That is, we make the following assumption: There is a run $\rho = s_0, s_1, \ldots$ of $\mathcal{B}I\mathcal{D}$, a state $s_j = (S_j, E_j)$ in ρ , an $r \in \mathsf{RP}$ that is honest in S_j , an RP service token of the form $\langle n, i \rangle$ recorded in r in the state $S_j(r)$ such that $\langle n, i \rangle \in d_{N^{\operatorname{attacker}}}(S_j(\operatorname{attacker}))$ and the browser b owning i is not fully corrupted in S_j and governor(i) is an honest IdP in S_j .

By definition of RPs, for $\langle n, i \rangle$ there exists a corresponding HTTPS request received by r, which we call req_{cap} , and a corresponding response $resp_{cap}$. The request must contain a valid CAP c and must have been sent by some atomic process p to r. The response must contain $\langle n, i \rangle$ and it must be encrypted by some symmetric encryption key k sent in req_{cap} .

In particular, it follows that the request and the response must be of the following form, where $d_r \in \mathsf{dom}(r)$ is the domain of r, $n_{cap}, k \in \mathcal{N}$ are some nonces, *path*, *params* $\in \mathcal{T}_{\mathcal{N}}$, c is some valid CAP, and *sts* is the Strict-Transport-Security header (as in the definition of RP's relation):

$$req_{cap} = enc_{a}(\langle \langle HTTPReq, n_{cap}, POST, d_r, path, params, [Origin : \langle d_r, S \rangle], c \rangle, \\ k \rangle, pub(key(d_r)))$$
(16)
$$resp_{cap} = enc_{s}(\langle HTTPResp, n_{cap}, 200, \langle sts \rangle, \langle n, i \rangle \rangle, k)$$
(17)

Moreover, there must exist a processing step of the following form, where $m \leq j$, $a_r \in \operatorname{addr}(r)$, and x is some address:

$$s_{m-1} \xrightarrow[r \to \{(x:a_r:resp_{cap})\}{} s_m$$

From the assumption and the definition of RPs it follows that c is of the following form:

$$c = \langle uc, ia \rangle$$

= $\langle sig(\langle i, pub(k_u) \rangle, k_{sign}), sig(\langle d_r, S \rangle, k_u) \rangle$

where k_u and k_{sign} are some private keys. When we write $i = \langle i_{\text{name}}, i_{\text{domain}} \rangle$, we have that:

$$c \equiv \langle \operatorname{sig}(\langle i_{\operatorname{name}}, i_{\operatorname{domain}} \rangle, \operatorname{pub}(k_u) \rangle, k_{\operatorname{sign}}), \operatorname{sig}(\langle d_r, S \rangle, k_u) \rangle$$
.

As r accepts the CAP c, we know that $pub(k_{sign}) \equiv S_j(r).signkeys[i_{domain}]$. As the subterm signkeys of r's state is never changed, we have $S_j(r).signkeys = S_0(r).signkeys$. With the definition of the initial state of r (See Definition 41), we have that $pub(k_{sign}) \equiv S_j(r).signkeys[i_{domain}] \equiv pub(signkey(dom^{-1}(i_{domain})))$.

The private key signkey(dom⁻¹(i_{domain})) is initially only known to the DY process $idp := dom^{-1}(i_{domain}) = governor(i)$. From the assumption we know that idp is an honest IdP (and not the attacker, a corrupted IdP, or some other DY process). As we can see in Algorithm 11 (that defines the behavior of IdPs), the *signkey* can only be used in Line 4 and in Line 32. We know that Line 4 cannot be invoked as long as idp is honest, which it is in s_j and ever since s_0 . For Line 32, we see that the key is not sent out to other processes. In s_j , the key can therefore not have been leaked to any other DY processes.

Knowing that in or before s_j , only idp can derive k_{sign} from its knowledge, it is easy to see that only idp can derive $sig(x, k_{sign})$ for any x, and in particular, uc.

Now we want to see exactly how *idp* creates *uc* and which data it uses in this process.

We have already seen that idp creates the uc in Line 32 of Algorithm 11. There may be more than one processing step in ρ where idp outputs uc.

Lemma 4. For all processing steps of the form

$$s_{\beta-1} \xrightarrow{(a_{idp}:x:req_{uc}) \to idp}_{idp \to \{(x:a_{idp}:resp_{uc})\}} s_{\beta} \tag{18}$$

(for some addresses x, a_{idp} with $s_{\beta} < s_j$, where $resp_{uc}$ is an encrypted HTTP response with the body $\langle uc \rangle$) it holds that req_{uc} was emitted by b.

Proof. To reach Line 32 of Algorithm 11, several conditions have to be met for req_{uc} : It must be an encrypted HTTPS POST request with the path /certreq.

The body of req_{uc} must be congruent to $\langle i, \mathsf{pub}(k_u) \rangle$. The request must contain a cookie with the name **sessionid** and some value *sessionid*. This value must be a valid key for the dictionary s'.sessions and

$$i \in {}^{\langle \rangle} s'.$$
sessions[sessionid]. (19)

Initially, s'.sessions is empty. It is only populated in Line 22 of Algorithm 11. This line must have been executed in a previous processing step of the following form:

$$s_{\alpha-1} \xrightarrow[idp \to \{(x:a_{idp}:resp_{auth}) \to idp]{} s_{\alpha}$$

$$(20)$$

(for some addresses x, a_{idp} with $s_{\alpha} < s_{\beta}$). In this step, s'.sessions was populated with a new entry for the session id *sessionid*.

From Algorithm 11 we can see that req_{auth} must meet the following conditions: It must be an HTTPS POST request, must contain a specific Origin header and its body must contain a pair $\langle i_{in}, secret_{in} \rangle$ such that the id/password combination matches a combination stored in $S_{\alpha-1}(idp)$.users. As we have that $S_{\alpha-1}(idp)$.users = $S_0(idp)$.users and with the initial definition

$$S_0(idp).\texttt{users} = \langle \{ \langle s, \langle \mathsf{IDsofSecret}(s) \rangle \rangle | \mathsf{Secrets}^i \} \rangle$$
(21)

we can see that $i_{in} \in \mathsf{IDsofSecret}(secret_{in})$. As the list of authenticated ids in the session is then (in Line 22 of Algorithm 11) populated with $\mathsf{IDsofSecret}(secret_{in})$ and with (19) we have that $i \in \mathsf{IDsofSecret}(secret_{in})$. Now, $\mathsf{IDsofSecret}(secret_{in})$ the IDs to their secrets according to $\mathsf{secretOfID}$, i.e., it must hold that

$$\operatorname{secretOfID}(i) = \operatorname{secret_{in}}$$
. (22)

This secret can be owned by at most one browser, and according to the definitions of the initial knowledge of the DY processes in F, it is initially only known to the owner of the secret ownerOfSecret($secret_{in}$) (see Section F.8) and to one specific IdP (see Section F.11), in this case $i_{domain} \in dom(idp)$ (because otherwise, idp would not accept this ID).

From Algorithm 11 we can see that the IdP never uses this secret to create messages as long as it is honest, which it is by precondition.

With (22) we see that initially, only ownerOfSecret(secretOfID(i)) = ownerOfID(i) knows the secret secret_{in}, which, by assumption, is not fully corrupted in s_j , and thus, with the request order given for (18) and (20) is not fully corrupted in s_{α} . (Once fully corrupted, browsers stay fully corrupted.)

(*): Honest browsers release secrets only to scripts that are loaded from a specific origin. In this case, according to the initial state given in Section F.8, the secret secretOfID(*i*) is only released to scripts from the origin $\langle i_{\text{domain}}, S \rangle$. For any such script (or document), with Lemma 2 and the definition of the browser's key mapping in Section F.8, we can see that any script that has access to the secret was sent by *idp*. This DY process is also the governor of *i*, which is, by assumption, not corrupted. Therefore, *idp* can only deliver either

the script $script_idp_pif$ or the script $script_idp_ad$. We can now check, that both scripts, running in a browser, never send this secret to any other DY process than idp, and trigger only encrypted requests to do so.

In $script_idp_pif$ (Algorithm 17), the subterm secret of the state is not used at all; therefore, the script triggers no outgoing message containing the secret at all.

In script_idp_ad (Algorithm 16), secret is only used as a part of a an HTTP request to the document's own origin (which therefore is the origin for which the secret is stored in the browser's list of secrets, which therefore must be $\langle i_{\text{domain}}, \mathbf{S} \rangle$). The request's data is not stored in the script's state.

We now know that all entities that have access to *secret* (the browser *b* and the IdP *idp*) never leak it. As *idp* never creates any HTTP(S) requests, *b* must have created req_{auth} before the processing step $s_{\alpha-1} \rightarrow s_{\alpha}$.

In this processing step, idp creates a new session id (*sessionid*). This id is sent out only once (in Line 25 of Algorithm 11), which, in our case, is $resp_{auth}$. With Corollary 1 we can see that from this (encrypted) response $resp_{auth}$, only b can derive the contents, especially the contents of the Set-Cookie header. As in b, the cookie is stored as a *secure*, HTTP only cookie, b releases the contents of this cookie only as a Cookie header to the origin $\langle i_{domain}, S \rangle$. Given the keymapping in b's state, requests to this origin are handled by idp, and with Algorithm 11 it is easy to see that the Cookie header is only used for validating the UC request, but is not used anywhere else. All in all, b and idp do not leak the session id *sessionid*.

As *sessionid* is an important part of req_{uc} , we can see that this request must have been emitted by b.

Lemma 5. The secret key k_u was chosen by the browser b from its own nonces, i.e., $k_u \in N^b$.

Proof. First of all, we know that for *idp* to generate uc, there must be a processing step in ρ of the form (described in Lemma 4):

$$s_{\beta-1} \xrightarrow[idp \to \{(x:a_{idp}: resp_{uc})\}{idp} \to \{s_{\beta}$$

$$(23)$$

(for some addresses x, a_{idp} with $s_{\beta} < s_j$, where $resp_{uc}$ is an encrypted HTTP response with the body $\langle uc \rangle$). For the request req_{uc} , the method must be POST and the path component must be /certreq.

With Lemma 4 we know that req_{uc} was emitted by b, which is honest at this point in the run. With the same arguments as in (*) we can see that either $script_idp_pif$ or the script $script_idp_ad$ initiated req_{uc} .

For *script_idp_ad* it is easy to see that this script never sends a POST request to *idp*.

The script <u>script_idp_pif</u> can only send a POST request to /certreq in Line 47 of Algorithm 17. In this case, the public key is chosen from the subterm pubkeys of the script's state. This subterm is only populated in Line 59 of Algorithm 17. It can only be populated by a postMessage *pm* from an immediate parent window and from the origin $\langle \mathsf{dom}(\mathsf{LPO}), \mathbf{S} \rangle$ (given how a browser checks and transmits postMessages, see Line 97f. of Algorithm 6). Further, the message in pm must be of the form $\langle n, \mathsf{pub}(k_u) \rangle$ where n is a nonce that was freshly chosen for a $\langle \mathsf{genKeyPair}, n \rangle$ postMessage in Line 28 of Algorithm 17.

Given that b's keymapping assigns the private key of LPO to the domain of LPO and with Lemma 3 we see that the only scripts that can send such a postMessage are *script* lpo cif and *script* lpo ld.

In the script *script_lpo_cif* (Algorithm 13), postMessages of the form of *pm* can only be sent in Line 110 (the message sent in Line 105 would not carry the correct nonce for a response to a genKeyPair message).

The same holds true for the script *script lpo ld* (Algorithm 13).

Therefore, the key k_u is a nonce that was chosen from the browser's nonces.

Lemma 6. k_u does not leak from b.

Proof. As we have seen above, the key k_u was chosen either in the script $script_lpo_cif$ or in the script $script_lpo_ld$ running in the honest browser b.

In both scripts, any nonce that is chosen from the script's *nonces* will not be given to the script (as part of *nonces*) by the browser again, thus, the nonce was chosen freshly. Further, the nonce is stored in the subterm key of the script's state and (besides the derivation of the public key) is only used to sign IAs.

There are no other scripts running in the origin of $(\mathsf{dom}(\mathsf{LPO}), \mathbf{S})$. The (honest) browser *b* does not leak the script's state. Therefore, k_u does not leak from *b*. \Box

With Lemma 4, 5, and 6, we can see that only *b* knows k_u and the attacker cannot know k_u . Therefore, only *b* can create the $ia = sig(\langle d_r, S \rangle, k_u)$. As k_u is only accessible to scripts with the origin $\langle dom(LPO), S \rangle$, only the script $script_lpo_cif$ or the script $script_lpo_ld$ can create $sig(\langle d_r, S \rangle, k_u)$. In both scripts, after creation, *ia* is sent in postMessage only to scripts that have the origin for which *ia* was created (= $\langle d_r, S \rangle$). With Lemma 3 and the definition of relying parties (see Algorithm 10) we see, that the only potential receiver is $script_rp_index$.

After receiving this **response** postMessage, $script_rp_index$ stores the UC and the IA in the subterm called **cap** of its scriptstate (see Algorithm 15, Line 47). After doing so, this subterm is read only in Line 72 (where only the identity is extracted) and in Line 84. There, the *ia* is sent to r (in the encrypted request req_{cap}).

The RP r, which is not corrupted, and the browser b do not leak ia. After receiving ia, r sends the newly created service token $\langle n, i \rangle$ to b, which ignores it (see Algorithm 15 Line 90f.). Therefore, b and r do not leak $\langle n, i \rangle$.

Therefore, the attacker cannot know $\langle n,i \rangle$ in S_j , i.e., $\langle n,i \rangle \notin d_{N^{\text{attacker}}}(S_j(\text{attacker}))$. This is a contradiction to our assumption.

H.3 Condition B

Similar to before, we assume that Condition B does not hold and lead this to a contradiction. We therefore make the following assumption: There is a run ρ of $\mathcal{B}I\mathcal{D}$, some state $s_j = (S_j, E_j)$ in ρ , some $r \in \mathsf{RP}$ that is honest in S_j , some RP service token of the form $\langle n, i \rangle$ recorded in r in the state $S_j(r)$, the request corresponding to $\langle n, i \rangle$ was sent by some $b \in \mathsf{B}$ which is honest in S_j , and b does not own i.

By definition of RPs, for $\langle n, i \rangle$ there exists a corresponding HTTPS request received by r, which we call req_{cap} , and a corresponding response $resp_{cap}$. The request must contain a valid CAP c and must have been sent by some atomic process p to r. The response must contain $\langle n, i \rangle$ and it must be encrypted by some symmetric encryption key k sent in req_{cap} .

In particular, it follows that the request and the response must be of the following form, where $d_r \in \mathsf{dom}(r)$ is the domain of r, $n_{cap}, k \in \mathcal{N}$ are some nonces, *path*, *params* $\in \mathcal{T}_{\mathcal{N}}$, c is some valid CAP, and *sts* is the Strict-Transport-Security header (as in the definition of RP's relation):

$$req_{cap} = enc_{a}(\langle \langle \text{HTTPReq}, n_{cap}, \text{POST}, d_{r}, path, params, [\text{Origin} : \langle d_{r}, \mathbf{S} \rangle], c \rangle, \\ k \rangle, pub(key(d_{r})))$$
(24)

$$resp_{cap} = enc_{s}(\langle HTTPResp, n_{cap}, 200, \langle sts \rangle, \langle n, i \rangle \rangle, k)$$
(25)

Moreover, there must exist a processing step of the following form, where $m \leq j$, $a_r \in \operatorname{addr}(r)$, and x is some address:

$$s_{m-1} \xrightarrow{(a_r:x:req_{cap}) \to r}{r \to \{(x:a_r:resp_{cap})\}} s_m \quad .$$

From the assumption and the definition of RPs it follows that c is of the following form:

$$c = \langle uc, ia \rangle$$

$$\equiv \langle \operatorname{sig}(\langle i, \operatorname{pub}(k_u) \rangle, k_{\operatorname{sign}}), \operatorname{sig}(\langle d_r, \mathbf{S} \rangle, k_u) \rangle$$

where k_u and k_{sign} are some private keys. When we write $i = \langle i_{name}, i_{domain} \rangle$, we have that:

$$c \equiv \langle \operatorname{sig}(\langle \langle i_{\operatorname{name}}, i_{\operatorname{domain}} \rangle, \operatorname{pub}(k_u) \rangle, k_{\operatorname{sign}}), \operatorname{sig}(\langle d_r, S \rangle, k_u) \rangle$$
.

With Lemma 3 we see that this request was initiated by a script that b extracted from an HTTPS response by r. The only script that r sends in its responses is *script*_*rp_index*.

In this script (Algorithm 14), the only place where a request is initiated is in Line 47. We can see that the cap c is taken from the script's state, i.e., $s'.cap \equiv c$ before the execution of Line 47 must hold. Initially, this term is empty, therefore the value must have been set during the prior execution of the script. This happens in Line 47 and in Line 61 of the algorithm. For both lines to be executed, there must arrive a postMessage at *script_rp_index* (either a login or a *response* postMessage) from the origin of LPO.

With Lemma 2, Lemma 3, and the definition of the web browser, we can see that the message must indeed come from one of LPO's scripts, that is, either $script_lpo_ld$ or $script_lpo_cif$. Before we proceed by showing that both scripts never send a UC for an identity that is not owned by browser b to the script $script_rp_index$ (and later to r), we first proof the following lemma:

Lemma 7. The value of s'.email in script lpo ld is always either one of the browser's identities or empty.

Proof. We show this by induction:

Base case: The value of s'.email is initially empty (see initial scriptstate).

Induction step: The value is set only in Lines 31 and 33. In the first case, the identity is chosen non-deterministically from the browser's identities *ids*, which are the identities that the browser owns (see Section F.8).

In the second case, the value of s'.email is taken from the localStorage, with the help of the key *idpnonce* that is taken from the sessionStorage. We can now show that what is retrieved from the localStorage is either empty or a previous value of s'.email:

First, we show that the value of *idpnonce*, taken from sessionStorage in Line 29, is always a nonce or empty: The browser's sessionStorage is separated by origins (and root windows), and therefore, only scripts under the origin of LPO have read or write access. Thus, the only two scripts that can possibly write the *idpnonce* value are *script_lpo_cif* and *script_lpo_ld*. The script *script_lpo_cif* does not write to sessionStorage. The script *script_lpo_ld* only writes to sessionStorage in Line 87. It only writes a fresh nonce (chosen in Line 85). Therefore, the value of *idpnonce* is always a nonce (or empty).

As we are already in the second case of the if-statement in Line 30 (we know that Line 33 was executed) *idpnonce* cannot be empty and must be a nonce.

Now, we can show that localStorage[idpnonce] is either empty or a previous value of s'.email: The browser's localStorage is separated by origins, and therefore, only scripts under the origin of LPO have read or write access. As above, the only two scripts that can write values to the localStorage are $script_lpo_cif$ and $script_lpo_ld$. The script $script_lpo_cif$ does not write to localStorage (it only removes subterms form localStorage in Line 47). We can thus focus on $script_lpo_ld$.

There are two lines where this script writes to the localStorage: Lines 112 and 86. We can safely ignore the first case, as it does not use a nonce as a key (but the fixed string siteInfo instead). In the latter case, it writes a value of s'.email.

This concludes the induction.

We can now show (for both scripts), that they never send a UC for an identity that is not owned by the browser b:

(I) For *script_lpo_ld* (Algorithm 14), it is easy to see that the UC that is finally used to create a CAP for RP in Line 109 is set in Line 76. There, the identity in the UC is checked against the identity in *s'.email* in the script's state (and it is checked that *s'.email* is not empty).

With Lemma 7 and the observations above we can conclude that in $script_lpo_ld$, it is not possible that a UC for an identity that the browser does not own is accepted. Therefore, the UC that is sent to $script_rp_index$ is issued for an identity of the browser b.

(II) For $script_lpo_cif$ (Algorithm 13), it is easy to see that the UC that is finally used in Line 125 is set in Line 113. There, the identity in the UC is checked against the value of s'.email (and, that s'.email is not empty). Initially, s'.email is empty. It is set only in Line 64. There, it is taken from the localStorage, using the key siteInfo. As we have seen above, the only place where values are stored using this key is in Line 112 of Algorithm 14. There, it is taken from the script's s'.email, which, according to Lemma 7, is either empty or one of the browser's identities. Note that the value of siteInfo is a dictionary. The keys which are used inside of this dictionary are not relevant here, but only the values.

Thus, in $script_lpo_cif$, it is not possible that a UC for an identity that the browser does not own is accepted. Therefore, the UC that is sent to $script \ rp \ index$ is issued for an identity of the browser b.

With (I) and (II), we see that all UCs that are sent to $script_rp_index$ (and later to r) are issued for identities of the browser b. This contradicts the assumption, which proves that Condition B holds true.