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Distance-bounding facing both mafia and distance frauds: Technical report ⋆

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Abstract. Contactless technologies such as RFID, NFC, and sensor networks are vulnerable to mafia and distance frauds. Both frauds aim at passing an authentication protocol by cheating on the actual distance between the prover and the verifier. To cope these security issues, distance-bounding protocols have been designed. However, none of the current proposals simultaneously resists to these two frauds without requiring additional memory and computation. The situation is even worse considering that just a few distance-bounding protocols are able to deal with the inherent background noise on the communication channels. This article introduces a noise-resilient distance-bounding protocol that resists to both mafia and distance frauds. The security of the protocol is analyzed with respect to these two frauds in both scenarios, namely noisy and noiseless channels. Analytical expressions for the adversary's success probabilities are provided, and are illustrated by experimental results. The analysis, performed in an already existing framework for fairness reasons, demonstrates the undeniable advantage of the introduced lightweight design over the previous proposals.

Keywords: Authentication, distance-bounding, relay attack, mafia fraud, distance fraud, noise.

1 Introduction

A *mafia fraud* is a man-in-the-middle attack applied against an authentication protocol where the adversary simply relays the exchanges without neither manipulating nor understanding them [1]. The earliest version of this attack was introduced by Conway in 1976 and is known as the *chess grandmaster problem* [5]. In this problem, a little girl is able to compete with two chess grandmasters during a postal chess game, where she transparently relays the moves between the two grandmasters. She eventually wins a game or draws both.

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In modern cryptography, mafia frauds can typically be used against authentication protocols. The adversary relays the messages between the prover and the verifier, who think they communicate together, while there is an adversary in the middle. This so-called mafia fraud was actually suggested by Desmedt, Bengio and Goutier in 1987 [6] to defeat the Fiat-Shamir protocol [8].

One of the most promising field to apply the mafia fraud is the contactless technology, especially Radio Frequency IDentification (RFID) and Near Field Communication (NFC) where the devices answer to any solicitation without explicit agreement of their holder. Some attacks have already been performed against both RFID and NFC systems [10,12]. Nevertheless, mafia fraud is not limited to contactless technologies, it also threats other technologies such as smartcards [7] and e-voting [14].

Two other attacks related to the mafia fraud exist: the *terrorist fraud* and the *distance fraud*. The distance fraud only involves a malicious prover, who cheats on his distance to the verifier. It was first introduced by Brands and Chaum [4], and comes from the distance measuring process used to defeat the mafia fraud. The terrorist fraud is a variant of the mafia fraud where the prover is malicious and actively helps the adversary to succeed the attack [3]. No solution exists yet to avoid this exotic fraud, which is not addressed in this paper. Additional countermeasure must actually be considered to thwart this fraud.

As mentioned above, a distance measuring process can mitigate the mafia and distance frauds. To that aim, Brands and Chaum [4] proposed the *distance-bounding protocols* (DB protocols). The distance estimation relies on the measurement of the Round-Trip-Time (RTT) of single bit exchanges. Considering the physical impossibility to travel faster than the speed of light, RTT bounds the distance between the parties. Several distance-bounding protocols have been proposed [1]. However, none of the current DB protocols are lightweight and resistance to both mafia and distance frauds. Furthermore, just a few of them are able to deal with the inherent background noise of the communication channel.

Contribution. In this paper we introduce a novel DB protocol that significantly reduces the success probability of an adversary capable of mounting both mafia and distance frauds. Our protocol does not rely on computationally expensive primitives, has a very low memory requirement, and is noise-resilient. Therefore, it is efficient and suitable for extremely low resources devices. We provide analytical and experimental results that together show the superiority of our proposal w.r.t. to previous ones.

Organization. Further below Section 2 presents a brief background about DB protocols. Section 3 explains the rationality behind our proposal and Section 4 introduces and details the proposal. Sections 5 and 6 are dedicated to the resistance of the protocol to mafia and distance frauds respectively. Section 7 describes our noise resiliency mechanism. Section 8 provides comparative results with several DB protocols in both scenarios the free-noise case and the noisy case. Finally, Section 9 draws the conclusions.

2 Background on distance-bounding

The first lightweight DB protocol was proposed by Hancke and Kuhn's [11] in 2005. Its simplicity and suitability for resource-constrained devices have promoted the design of other DB protocols based on it [2,13,16]. All these protocols share the same design: (a) there is a slow phase[4] where both prover and verifier generate and exchange nonces, (b) the nonces and a keyed cryptographic hash function are used to compute the answers to be sent (resp. checked) by the prover (resp. verifier). Below, we provide the main characteristics of each of these protocols, especially the technique they use to compute the answers.

Hancke and Kuhn's protocol [11]. The answers are extracted from two n-bit registers such that any of the n 1-bit challenges determines which register should be used to answer.

Avoine and Tchamkerten's protocol [2]. Binary trees are used to compute the prover answers: the verifier challenges define the unique path in the tree, and the prover answers are the vertex value on this path. There are several parameters impacting the memory consumption and the resistance to distance and mafia frauds: l the number of trees and d the depth of these trees. It holds $d \cdot l = n$, where n is the number of rounds in the fast phase. The larger d, the better the frauds resistance and the larger the memory consumption.

Trujillo-Rasua, Martin and Avoine's protocol [16]. This protocol is similar to the previous one, except that it uses particular graphs instead of trees to compute the prover answers.

Kim and Avoine's protocol [13]. This protocol, closer to the Hancke and Kuhn's protocol [11] than [2] and [16], uses two registers to define the prover answers. An important additional feature is that the prover is able to detect a mafia fraud thanks to *predefined challenges*, that is, challenges known by both prover and verifier. The number of predefined challenges impacts the frauds resistance: the larger, the better the mafia fraud resistance, but the lower the resistance to distance fraud.

There exist other DB protocols with different designs and computational complexities (*e.g.,* protocols based on signatures and/or a final extra slow phase [4,17]). However, they are beyond the scope of this article that focuses on lightweight protocols only. The interested reader could refer to [1] for more details.

3 Rationality of our proposal

Being resistant to both mafia and distance frauds is the primary goal of a DB protocol. An important lower-bound for both frauds is $\left(\frac{1}{2}\right)^n$, which is the probability of a naive adversary who answers randomly to the n verifier challenges during the fast phase. However, this resistance is hard to attain for lightweight DB protocols. Therefore, our aim is to design a protocol that is close to this bound for both mafia and distance frauds, without requiring costly operations

[4] In DB protocols, a *fast phase*, which generally consists on n rounds, is a phase where the verifier computes RTTs. Otherwise, we say that it is a *slow phase*.

or an extra final slow phase. We also aim to reach the additional property of being noise-resilient. Below, the intuitions that lead to our design are explained for each of the three considered properties.

Mafia fraud. Among the DB protocols without final slow phase, those achieving the best mafia fraud resistance are round-dependent [2,13,16]. The idea is that the correct answer at the ith round should depend on the ith challenge and also on the $(i-1)$ previous challenges. Our proposal also uses a round-dependent technique, the proposed construction is significantly simpler than those proposed in [2,13,16], though.

Distance fraud. As in mafia fraud, the best protocols in term of distance fraud are round-dependent. However, round-dependency by means of predefined challenges as in the Kim and Avoine's construction [13] fails to properly resist to distance fraud. Intuitively, as more control over the challenges the prover has, the lower the resistance to distance fraud is. For this reason, our proposal allows the verifier to have full and exclusive control over the challenges.

Noise-resiliency. Round-dependent protocols can hardly work in noisy environments. A noise in a given round might affect all the subsequent rounds and thus, these rounds becomes useless from the security point of view. Therefore, in order to deal with noise, round-dependent protocols should be able to detect the noisy-rounds so that the prover responses can be checked considering these noise occurrences. To the best of our knowledge, our protocol is the first round-dependent DB protocol able to detect such a noisy-rounds with a high level of accuracy thanks to the simplicity of its design.

4 Proposal

This section describes the DB protocol introduced in the paper. Initialization, execution, and decision steps are presented below and a general view is provided in Figure 1.

Initialization. The prover (P) and the verifier (V) agree on (a) a security parameter n, (b) a timing bound $\Delta t_{\max}$, (c) a pseudo random function PRF that outputs $3n$ bits, (d) a secret key x.

Execution. The protocol consists of a slow phase and a fast phase.

Slow Phase. P (respectively V) randomly picks a nonce N_P (respectively N_V) and sends it to V (respectively P). Afterwards, P and V compute $PRF\,(x, N_P, N_V)$ and divide the result into three n-bit registers Q, R^0, and R^1. Both P and V create the function $f_Q : \mathcal{S} \rightarrow \{0,1\}$ where $\mathcal{S}$ is the set of all the bit-sequences of size at most n including the empty sequence. The function f_Q is parameterized with the bit-sequence $Q = q_1 \dots q_n$, and it outputs 0 when the input is the empty sequence. For every non-empty bit-sequence $C_i = c_1 \dots c_i$ where $1 \leq i \leq n$, the function is defined as $f_Q(C_i) = \bigoplus_{j=1}^{i}(c_j \wedge q_j)$.

Fast Phase. In each of the n rounds, V picks a random challenge $c_i \in_R \{0,1\}$, starts a timer, and sends c_i to P. Upon reception of c_i, P replies with $r_i = R_i^{c_i} \oplus f_Q(C_i)$ where $C_i = c_1...c_i$. Once V receives r_i, he stops the timer and computes the round-trip-time Δt_i.

Decision. If $\Delta t_i < \Delta t_{\max}$ and $r_i = R_i^{c_i} \oplus f_Q(C_i)\ \forall\ i \in \{1,2,...,n\}$ then the protocol succeeds[5].

Prover		**Verifier**
secret x		secret x
	slow phase	
picks $N_P \in_R \{0,1\}^n$		picks $N_V \in_R \{0,1\}^n$
	$\xrightarrow{N_P}$	
	$\xleftarrow{N_V}$	
$(Q, R^0, R^1) = PRF(x, N_P, N_V)$		$(Q, R^0, R^1) = PRF(x, N_P, N_V)$
	fast phase	
	for $i = 1, \ldots, n$**:**	
		picks $c_i \in_R \{0,1\}$
	$\xleftarrow{c_i}$	start timer
$r_i = R_i^{c_i} \oplus f_Q(C_i)$	$\xrightarrow{r_i}$	stop timer
		computes Δt_i

Fig. 1. Protocol description

5 Resistance to mafia fraud

Analyses of DB protocols usually consider two strategies to evaluate the resistance against a mafia fraud: the pre-ask and the post-ask strategies [1]. Although considering these two strategies only do not provide a formal security proof, this evaluates the resistance of the protocol, at least against these well-known attack strategies, and is the only way known so far to evaluate DB protocols. Providing a formal security proof of DB protocols would be interesting but is clearly out of the scope of this paper.

This section reminds the concept of pre-ask strategy[6], then identifies the adversarial behavior that maximizes the success probability when considering the pre-ask strategy, and the section finally computes this probability.

[5] $\Delta t_{\max}$ is a system parameter that implicitly represents the maximum allowed distance between the prover and the verifier.

[6] Note that the post-ask strategy is not relevant in protocols without an extra final slow phase [1]

5.1 Best behavior

The *pre-ask strategy* consists first, for the adversary, to relays the initial slow phase. Then, she runs the fast phase with the prover. With the answers she obtains, she finally executes the fast phase with the verifier. In our case, we consider that the adversary first sends a sequence of challenges $\tilde{c}_1...\tilde{c}_n$ to the legitimate prover and receives $\tilde{r}_1...\tilde{r}_n$ where $\tilde{r}_i = R_i^{\tilde{c}_i} \oplus f_Q(\tilde{C}_i)$ and $\tilde{C}_i = \tilde{c}_1...\tilde{c}_i$ for every $i \in \{1,...,n\}$. Next, she executes the fast phase with the verifier receiving the challenges $c_1...c_n$. Given $\tilde{r}_1...\tilde{r}_n$, the adversarial behavior that maximizes the success probability is provided in Theorem 1.

Theorem 1. *The adversary's behavior that maximizes her mafia fraud success probability with a pre-ask strategy is: (a) For every round i where $c_i \neq \tilde{c}_i$, answer randomly. (b) For every round i where $c_i = \tilde{c}_i$, guess the value $f_Q(C_i) \oplus f_Q(\tilde{C}_i)$ and answer with the value $f_Q(C_i) \oplus f_Q(\tilde{C}_i) \oplus \tilde{r}_i$ where $C_i = c_1...c_i$ and $\tilde{C}_i = \tilde{c}_1...\tilde{c}_i$.*

Proof. First, let us prove the following lemma.

Lemma 1. *Given that $c_i = \tilde{c}_i$, the adversary's behavior maximizing her success probability at the ith round is equivalent to the best behavior for guessing the value $f_Q(C_i) \oplus f_Q(\tilde{C}_i)$.*

Proof. Given that $c_i = \tilde{c}_i$, then $R_i^{c_i} = R_i^{\tilde{c}_i}$, which means that $\tilde{r}_i \oplus f_Q(\tilde{C}_i) = r_i \oplus f_Q(C_i) \Rightarrow r_i = f_Q(\tilde{C}_i) \oplus f_Q(C_i) \oplus \tilde{r}_i$. Therefore, either the adversary guesses $f_Q(C_i) \oplus f_Q(\tilde{C}_i)$ or the adversary losses this round.

In the case where $c_i \neq \tilde{c}_i$, the prover's response $\tilde{r}_i$ does not help the adversary since $R_i^{c_i}$ and $R_i^{\tilde{c}_i}$ are independent values. Therefore, there does not exist any best behavior, *i.e.,* whatever the adversary behavior, her success probability at this round is $\frac{1}{2}$. This result and Lemma 1 conclude the proof. □

5.2 Adversary's success probability

Given the adversary's behavior provided by Theorem 1, Theorem 2 provides a recursive way to compute her success probability.

Theorem 2. *Let M_i be the event that the adversary has won the first i rounds by following her best behavior with the pre-ask strategy. Let S_i be the event that the adversary guesses $f_Q(C_i) \oplus f_Q(\tilde{C}_i)$ at the ith round. The probability $\Pr(M_i)$ can be recursively computed as follows:*

$$\Pr(M_i) = \left(\tfrac{1}{2}\right)^i + \Pr(M_i | C_i \neq \tilde{C}_i)\left(1 - \left(\tfrac{1}{2}\right)^i\right).$$

$$\Pr(M_i | C_i \neq \tilde{C}_i) = \Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1}) \times \left(\tfrac{1}{2^i - 1} + \Pr(M_{i-1} | C_{i-1} \neq \tilde{C}_{i-1})(1 - \tfrac{1}{2^i - 1})\right).$$

$$\Pr(M_i|C_i \neq \tilde{C}_i, M_{i-1}) = \Pr(S_{i-1}|C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1})\left(1 - \tfrac{2^{i-1}}{2^i-1}\right) + \tfrac{1}{2}\tfrac{2^{i-1}}{2^i-1}.$$

$$\Pr(S_i|C_i \neq \tilde{C}_i, M_i) = \tfrac{1}{2} + \tfrac{1}{2}\frac{\Pr(S_{i-1}|M_{i-1}, C_{i-1} \neq \tilde{C}_{i-1})\Pr(M_{i-1}|C_{i-1} \neq \tilde{C}_{i-1})\left(1-\left(\frac{1}{2}\right)^{i-1}\right)\frac{1}{2}}{\Pr(M_i|C_i \neq \tilde{C}_i)\left(1-\left(\frac{1}{2}\right)^{i}\right)}.$$

Where $\Pr(M_1|C_1 \neq \tilde{C}_1) = 1/2$ *and* $\Pr(S_1|C_1 \neq \tilde{C}_1, M_1) = 1/2$ *are the stopping conditions.*

Proof. If $C_i = \tilde{C}_i$, the adversary knows that $f_Q(C_i) \oplus f_Q(\tilde{C}_i) = 0$ and thus, her success probability until the ith round is 1, which means that $\Pr(M_i|C_i = \tilde{C}_i) = 1$. Considering that $\Pr(C_i = \tilde{C}_i) = (1/2)^i$ and $\Pr(C_i \neq \tilde{C}_i) = 1 - (1/2)^i$, then $\Pr(M_i)$ can be expressed as follows:

$$\Pr(M_i) = \left(\frac{1}{2}\right)^i + \Pr(M_i|C_i \neq \tilde{C}_i)\left(1 - \left(\frac{1}{2}\right)^i\right). \tag{1}$$

Equation 1 states that the computation of $\Pr(M_i)$ requires $\Pr(M_i|C_i \neq \tilde{C}_i)$. Note that M_i holds if M_{i-1} holds, so:

$$\Pr(M_i|C_i \neq \tilde{C}_i) = \Pr(M_i|C_i \neq \tilde{C}_i, M_{i-1})\Pr(M_{i-1}|C_i \neq \tilde{C}_i). \tag{2}$$

Given that M_{i-1} depends on whether $C_{i-1} = \tilde{C}_{i-1}$ or not, and considering that $\Pr(C_{i-1} = \tilde{C}_{i-1}|C_i \neq \tilde{C}_i) = 1/(2^i - 1)$, then $\Pr(M_{i-1}|C_i \neq \tilde{C}_i)$ can be transformed as follows:

$$\begin{aligned}\Pr(M_{i-1}|C_i \neq \tilde{C}_i) \quad &= \Pr(M_{i-1}|C_i \neq \tilde{C}_i, C_{i-1} = \tilde{C}_{i-1})\Pr(C_{i-1} = \tilde{C}_{i-1}|C_i \neq \tilde{C}_i)\\ &+ \Pr(M_{i-1}|C_i \neq \tilde{C}_i, C_{i-1} \neq \tilde{C}_{i-1})\Pr(C_{i-1} \neq \tilde{C}_{i-1}|C_i \neq \tilde{C}_i)\\ &= \frac{1}{2^i - 1} + \Pr(M_{i-1}|C_{i-1} \neq \tilde{C}_{i-1})\frac{2^i - 2}{2^i - 1}. \end{aligned} \tag{3}$$

Assuming that $\Pr(M_i|C_i \neq \tilde{C}_i, M_{i-1})$ can be computed for every i, then according to Equations 2 and 3, $\Pr(M_i|C_i \neq \tilde{C}_i)$ can be recursively computed as follows:

$$\begin{aligned}\Pr(M_i|C_i \neq \tilde{C}_i) = &\Pr(M_i|C_i \neq \tilde{C}_i, M_{i-1})\\ &\times\left(\frac{1}{2^i - 1} + \Pr(M_{i-1}|C_{i-1} \neq \tilde{C}_{i-1})\frac{2^i - 2}{2^i - 1}\right).\end{aligned} \tag{4}$$

Note that, the result $\Pr(M_1|C_1 \neq \tilde{C}_1) = 1/2$ can be used as the stopping condition for the recursion defined in Equation 4. This recursion simplifies the analysis of $\Pr(M_i)$: instead of analyzing the probability to win all the i rounds, only the probability to win the ith round is needed. Since it depends on the adversary's behavior, and the latter depends on whether $c_i = \tilde{c}_i$ or not, we compute $\Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1})$ as follows:

$$\begin{aligned}\Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1}) = \; & \Pr(M_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i) \Pr(c_i = \tilde{c}_i | C_i \neq \tilde{C}_i) \\ & + \Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1}, c_i \neq \tilde{c}_i) \Pr(c_i \neq \tilde{c}_i | C_i \neq \tilde{C}_i).\end{aligned} \tag{5}$$

When $c_i \neq \tilde{c}_i$ the adversary answers randomly and thus $\Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1}, c_i \neq \tilde{c}_i) = 1/2$. Considering this result and that $\Pr(c_i \neq \tilde{c}_i | C_i \neq \tilde{C}_i) = 2^{i-1}/(2^i - 1)$, Equation 5 yields to:

$$\begin{aligned}\Pr(M_i | C_i \neq \tilde{C}_i, M_{i-1}) = & \frac{2^{i-2}}{2^i - 1} \\ & + \Pr(M_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i) \left(1 - \frac{2^{i-1}}{2^i - 1}\right).\end{aligned} \tag{6}$$

From Equation 6, we deduce that computing $\Pr(M_i)$ requires $\Pr(M_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i)$. Theorem 2 states that the adversary's behavior in this case is to guess $f_Q(C_i) \oplus f_Q(\tilde{C}_i)$. Hence:

$$\Pr(M_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i) = \Pr(S_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i) \tag{7}$$

We now aim at computing $\Pr(S_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i)$. Since $c_i = \tilde{c}_i$, then $f_Q(C_i) \oplus f_Q(\tilde{C}_i) = f_Q(C_{i-1}) \oplus f_Q(\tilde{C}_{i-1})$. Therefore, the adversary's strategy maximizing $\Pr(S_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i)$ consists in holding her previous guess for the $(i-1)$th round. So:

$$\Pr(S_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i) = \Pr(S_{i-1} | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}). \tag{8}$$

As pointed out by Equation 8 and Equation 7, computing $\Pr(M_i | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1}, c_i = \tilde{c}_i)$ requires $\Pr(S_{i-1} | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1})$. Since it is indexed by $i-1$, we assume that $\Pr(M_j)$ is already computed for every $j < i$ and as shown by Lemma 2, $\Pr(S_{i-1} | C_{i-1} \neq \tilde{C}_{i-1}, M_{i-1})$ can be recursively computed.

Lemma 2. *Given that* $\Pr(M_j | C_j \neq \tilde{C}_j)$ *can be computed for every* $j \leq i$*, then* $\Pr(S_i | C_i \neq \tilde{C}_i, M_i)$ *can be recursively computed as follows:*

$$\begin{aligned}\Pr(S_i | C_i \neq \tilde{C}_i, M_i) = & \frac{1}{2} + \frac{\Pr(S_{i-1} | M_{i-1}, C_{i-1} \neq \tilde{C}_{i-1})}{\Pr(M_i | C_i \neq \tilde{C}_i) \left(1 - \left(\frac{1}{2}\right)^i\right)} \\ & \times \Pr(M_{i-1} | C_{i-1} \neq \tilde{C}_{i-1}) \left(\frac{1}{2} - \left(\frac{1}{2}\right)^i\right).\end{aligned}$$

where $\Pr(S_1 | C_1 \neq \tilde{C}_1, M_1) = \frac{1}{2}$ *is the stopping condition.*

Proof. By definition of $f_Q(.)$, and because $\Pr(q_i = 0) = \Pr(q_i = 1) = 1/2$, $\Pr(S_i | C_i \neq \tilde{C}_i, M_i, c_i \neq \tilde{c}_i) = 1/2$. Moreover, Theorem 1 states that M_i and S_i are equivalent when $c_i = \tilde{c}_i$, hence $\Pr(S_i | C_i \neq \tilde{C}_i, M_i, c_i = \tilde{c}_i) = 1$. Considering both results we obtain:

$$\Pr(S_i | C_i \neq \tilde{C}_i, M_i) = \frac{1}{2} + \frac{1}{2} \Pr(c_i = \tilde{c}_i | C_i \neq \tilde{C}_i, M_i). \tag{9}$$

Finally, considering that $\Pr(C_i \neq \tilde{C}_i) = 1 - \left(\frac{1}{2}\right)^i$ and $\Pr(c_i = \tilde{c}_i, C_i \neq \tilde{C}_i) = \left(1 - \left(\frac{1}{2}\right)^{i-1}\right) \frac{1}{2}$, the probability $\Pr(c_i = \tilde{c}_i | C_i \neq \tilde{C}_i, M_i)$ can be expressed as follows:

$$\begin{aligned}
\Pr(c_i = \tilde{c}_i | C_i \neq \tilde{C}_i, M_i) =& \frac{\Pr(c_i = \tilde{c}_i, C_i \neq \tilde{C}_i, M_i, M_{i-1})}{\Pr(C_i \neq \tilde{C}_i, M_i)} \\
=& \frac{\Pr(M_i | M_{i-1}, c_i = \tilde{c}_i, C_i \neq \tilde{C}_i)}{\Pr(M_i | C_i \neq \tilde{C}_i) \Pr(C_i \neq \tilde{C}_i)} \\
& \times \Pr(M_{i-1} | c_i = \tilde{c}_i, C_i \neq \tilde{C}_i) \Pr(c_i = \tilde{c}_i, C_i \neq \tilde{C}_i) \\
=& \frac{\Pr(S_{i-1} | M_{i-1}, C_{i-1} \neq \tilde{C}_{i-1})}{\Pr(M_i | C_i \neq \tilde{C}_i) \left(1 - \left(\frac{1}{2}\right)^i\right)} \\
& \times \Pr(M_{i-1} | C_{i-1} \neq \tilde{C}_{i-1}) \left(\frac{1}{2} - \left(\frac{1}{2}\right)^i\right).
\end{aligned} \tag{10}$$

Equations 9 and 10 yield the expected result.

Lemma 2 together with Equations 1, 4, 6, 7, and 8, conclude the proof of this theorem.

6 Resistance to distance fraud

This section analyzes the adversary success probability when mounting a distance fraud. As stated in [1], the common way to analyze the resistance to distance fraud is by considering the early-reply strategy instead of the pre-ask strategy. This strategy consists on sending the responses in advance, *i.e.,* before receiving the challenges. Doing so, the adversary gains some time and might pass the timing constraint. In this section, the behavior that maximizes the success probability using the early-reply strategy is identified, and then a recursive way to compute the resistance w.r.t. a distance fraud is provided.

6.1 Best behavior

With the early-reply strategy, in order to send a response in advance in the ith round with probability of being correct greater that 1/2, the adversary must send either $R_i^0 \oplus f_Q(C_{i-1}||0)$ or $R_i^1 \oplus f_Q(C_{i-1}||1)$ where C_{i-1} is the sequence of challenges sent by the verifier until the $(i-1)$th round. Theorem 3 shows that, guessing the values $f_Q(C_i)$ for every $i \in \{1, ..., n-1\}$ is needed to maximize the adversary success probability.

Theorem 3. *Let C_i be the sequence of challenges $c_1...c_i$ sent by the verifier until the ith round ($i \geq 1$). The adversary's behavior that maximizes her distance fraud success probability is equivalent to the best behavior for guessing the values $f_Q(C_1), f_Q(C_2), ..., f_Q(C_{n-1})$.*

Proof. In order to send a response in advance at the ith round with probability of being correct greater that 1/2, the adversary must send either $R_i^0 \oplus f_Q(C_{i-1}||0)$ or $R_i^1 \oplus f_Q(C_{i-1}||1)$. By definition, $f_Q(C_{i-1}||0) = f(C_{i-1})$ and $f_Q(C_{i-1}||1) = f_Q(C_{i-1}) \oplus q_i$. Therefore, $R_i^0 \oplus f_Q(C_{i-1}||0) = R_i^0 \oplus f_Q(C_{i-1})$ and $R_i^1 \oplus f_Q(C_{i-1}||1) = R_i^1 \oplus f_Q(C_{i-1}) \oplus q_i$. Since the adversary knows the values R_i^0, R_i^1, and q_i, guessing the correct value at this round is equivalent to guessing the correct value of $f_Q(C_{i-1})$.

As stated in Theorem 3, computing the adversary success probability requires the best behavior to guess the outputs sequence $f_Q(C_1), \ldots, f_Q(C_{n-1})$. Theorem 4 solves this problem.

Theorem 4. *The best adversary's behavior to guess $f_Q(C_i)$ is to assume that her previous guess for $f_Q(C_{i-1})$ is correct and to compute $f_Q(C_i)$ as follows: (a) if $q_i = 0$, then consider $f_Q(C_i) = f_Q(C_{i-1})$. (b) if $q_i = 1$, pick a random bit c_i and consider that $f_Q(C_i) = f_Q(C_{i-1}) \oplus c_i$.*

Proof. Assuming that $q_i = 0$, then $f_Q(C_i) = f_Q(C_{i-1})$ and thus, the probability to guess $f_Q(C_i)$ is equal to the probability of guessing $f_Q(C_{i-1})$. In the case of $q_i = 1$, the adversary does not have a better behavior than choosing a random bit of challenge c_i and considering that $f_Q(C_i) = f_Q(C_{i-1}) \oplus c_i$. Given that $f_Q(\epsilon) = 0$ where ϵ is the empty sequence, the proof can be straightforwardly concluded by induction.

6.2 Adversary's success probability

Given the best adversary's behavior provided by Theorems 3 and 4, Theorem 5 shows a recursive way to compute the resistance to distance fraud.

Theorem 5. *Let D_i be the event that the distance fraud adversary successfully passes the protocol until the ith round. Then, $\Pr(D_i)$ can be computed as follows:*

$$\Pr(D_i) = \frac{1}{4}\Pr(D_{i-1}) + \frac{1}{2^i} + \frac{1}{8}\sum_{j=1}^{i-1}\Pr(D_j)\frac{1}{2^{i-j}}.$$

where $\Pr(D_0) = 1$ is the stopping condition.

Proof. Let F_i be the event that the adversary correctly guesses the value of $f_Q(C_i)$. Then, the event D_i depends on the events D_{i-1} and F_{i-1}, which can be expressed as follows:

$$\begin{aligned}\Pr(D_i) = & \Pr(D_i|D_{i-1}, F_{i-1}) \Pr(D_{i-1}, F_{i-1}) \\ & + \Pr(D_i|D_{i-1}, \bar{F}_{i-1}) \Pr(D_{i-1}, \bar{F}_{i-1}).\end{aligned} \tag{11}$$

Two cases occur (a) $R_i^0 = R_i^1$ and (b) $R_i^0 \neq R_i^1$. In the first case, the adversary wins the ith round if and only if she guesses the value $f_Q(C_{i-1})$, so $\Pr(D_i|D_{i-1}, F_{i-1}, R_i^0 = R_i^1) = 1$ and $\Pr(D_i|D_{i-1}, \bar{F}_{i-1}, R_i^0 = R_i^1) = 0$. In the second case, the adversary has no better probability to win than 1/2 and thus, $\Pr(D_i|D_{i-1}, F_{i-1}, R_i^0 \neq R_i^1) = \Pr(D_i|D_{i-1}, \bar{F}_{i-1}, R_i^0 \neq R_i^1) = 1/2$. Therefore, we deduce $\Pr(D_i|D_{i-1}, F_{i-1}) = 3/4$ and $\Pr(D_i|D_{i-1}, \bar{F}_{i-1}) = 1/4$. Using these results and Equation 11 we have:

$$\begin{aligned}\Pr(D_i) &= \frac{3}{4} \Pr(D_{i-1}, F_{i-1}) + \frac{1}{4} \Pr(D_{i-1}, \bar{F}_{i-1}) \\ &= \frac{1}{4} \Pr(D_{i-1}) + \frac{1}{2} \Pr(D_{i-1}, F_{i-1}).\end{aligned} \tag{12}$$

Equation 12 states that $\Pr(D_i)$ can be computed by recursion if we express $\Pr(D_{i-1}, F_{i-1})$ in terms of the events D_j where $1 \leq j < i$. Therefore, in the remaining of this proof we aim at looking for such result. As above, in order to analyze $\Pr(D_i, F_i)$, the events D_{i-1} and F_{i-1} should be considered:

$$\begin{aligned}\Pr(D_i, F_i) = & \Pr(D_i|F_i, D_{i-1}, F_{i-1}) \Pr(F_i|D_{i-1}, F_{i-1}) \Pr(D_{i-1}, F_{i-1}) \\ & + \Pr(D_i|F_i, D_{i-1}, \bar{F}_{i-1}) \Pr(F_i|D_{i-1}, \bar{F}_{i-1}) \Pr(D_{i-1}, \bar{F}_{i-1}).\end{aligned} \tag{13}$$

Four cases should be analyzed depending on the value of q_i and the events F_i and F_{i-1}.

Case 1 ($q_i = 1$, F_i and F_{i-1} hold). This case occurs if the adversary correctly guesses the challenge c_i. Therefore, she provides the correct answer at this round $R_i^{c_i} \oplus f_Q(C_i)$. So, $\Pr(D_i|F_i, D_{i-1}, F_{i-1}, q_i = 1) = 1$.

Case 2 ($q_i = 1$, F_i and $\bar{F}_{i-1}$ hold). Given that $\bar{F}_{i-1}$ and F_i hold, the adversary computed $f_Q(C_i) = f_Q(C_{i-1}) \oplus \tilde{c}_i$ using a challenge different from the verifier's one, *i.e.*, $c_i \neq \tilde{c}_i$. Therefore, $\Pr(D_i|F_i, D_{i-1}, \bar{F}_{i-1}, q_i = 1) = \frac{1}{2}$ because both events F_i and $\bar{F}_{i-1}$ coexist only if $q_i = 1$, then $\Pr(q_i = 1|F_i, D_{i-1}, \bar{F}_{i-1}) = 1$.

Case 3 ($q_i = 0$, F_i and F_{i-1} hold). Given $q_i = 0$, the event F_i has no effect on the event D_i. Thus, $\Pr(D_i|F_i, D_{i-1}, F_{i-1}, q_i = 0) = \Pr(D_i|D_{i-1}, F_{i-1}, q_i = 0) = \frac{3}{4}$ because it depends on whether $R_i^0 = R_i^1$. So, $\Pr(D_i|F_i, D_{i-1}, F_{i-1}, q_i = 0) = \frac{3}{4}$.

Case 4 ($q_i = 0$, F_i and $\bar{F}_{i-1}$ hold). When $q_i = 0$, then $f_Q(C_i) = f_Q(C_{i-1})$, which means that this case cannot occur. Therefore, $\Pr(F_i, D_{i-1}, \bar{F}_{i-1}, q_i = 0) = 0$.

Cases 1 and 3 yield the following result:

$$\begin{aligned}\Pr(D_i|F_i, D_{i-1}, F_{i-1}) =& \Pr(q_i = 1|F_i, D_{i-1}, F_{i-1}) + \frac{3}{4}\Pr(q_i = 0|F_i, D_{i-1}, F_{i-1}) \\ =& \frac{3}{4} - \frac{1}{4}\Pr(q_i = 1|F_i, D_{i-1}, F_{i-1}).\end{aligned} \tag{14}$$

And Cases 2 and 4 yield this other result:

$$\Pr(D_i|F_i, D_{i-1}, \bar{F}_{i-1}) = \frac{1}{2}. \tag{15}$$

Because $\Pr(F_i|F_{i-1}, q_i = 0) = 1$ and $\Pr(F_i|F_{i-1}, q_i = 1) = 1/2$, we have $\Pr(F_i|D_{i-1}, F_{i-1}) = \Pr(F_i|F_{i-1}) = 3/4$. Similarly, $\Pr(F_i|D_{i-1}, \bar{F}_{i-1}) = \Pr(F_i|\bar{F}_{i-1}) = 1/4$ because $\Pr(F_i|\bar{F}_{i-1}, q_i = 0) = 0$ and $\Pr(F_i|\bar{F}_{i-1}, q_i = 1) = 1/2$. Combining these results with Equations 14 and 15, Equation 13 becomes:

$$\begin{aligned}\Pr(D_i, F_i) =& \left(\frac{3}{4} - \frac{1}{4}\Pr(q_i = 1|F_i, D_{i-1}, F_{i-1})\right)\frac{3}{4}\Pr(D_{i-1}, F_{i-1}) \\ &+ \frac{1}{2}\frac{1}{4}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{3}{16}\Pr(q_i = 1|F_i, D_{i-1}, F_{i-1})\Pr(D_{i-1}, F_{i-1}) + \frac{9}{16}\Pr(D_{i-1}, F_{i-1}) \\ &+ \frac{1}{8}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{3}{16}\frac{\Pr(q_i = 1, F_i, D_{i-1}, F_{i-1})}{\Pr(F_i|D_{i-1}, F_{i-1})} + \frac{9}{16}\Pr(D_{i-1}, F_{i-1}) + \frac{1}{8}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{3}{16}\frac{\Pr(F_i|q_i = 1, D_{i-1}, F_{i-1})\Pr(D_{i-1}, F_{i-1})\frac{1}{2}}{\Pr(F_i|D_{i-1}, F_{i-1})} + \frac{9}{16}\Pr(D_{i-1}, F_{i-1}) \\ &+ \frac{1}{8}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{3}{16}\frac{\frac{1}{2}\Pr(D_{i-1}, F_{i-1})\frac{1}{2}}{\frac{3}{4}} + \frac{9}{16}\Pr(D_{i-1}, F_{i-1}) + \frac{1}{8}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{1}{16}\Pr(D_{i-1}, F_{i-1}) + \frac{9}{16}\Pr(D_{i-1}, F_{i-1}) + \frac{1}{8}\Pr(D_{i-1}, \bar{F}_{i-1}) \\ =& \frac{5}{8}\Pr(D_{i-1}, F_{i-1}) + \frac{1}{8}(\Pr(D_i) - \Pr(D_{i-1}, F_{i-1})) \\ =& \frac{1}{2}\Pr(D_{i-1}, F_{i-1}) + \frac{1}{8}\Pr(D_i) \\ =& \frac{1}{2^i} + \frac{1}{8}\sum_{j=1}^{i}\Pr(D_j)\frac{1}{2^{i-j}}.\end{aligned} \tag{16}$$

Considering that $\Pr(D_0) = 1$, Equations 16 and 12 yield the expected result.

7 Noise resilience

Some efforts have been made in order to adapt existing distance-bounding protocols to noisy channels. Most of them rely on using a threshold x representing the maximum number of incorrect responses expected by the verifier [11,13]. Intuitively, the larger x, the lower the false rejection ratio but also the lower the resistance to mafia and distance frauds. Others use an error correction code during an extra slow phase [17]. However, the latter cannot be applied to our protocol given that it does not contain any final slow phase. Consequently, we focus on the threshold technique.

7.1 Understanding the noise effect in our protocol

We consider in the analysis that a 1-bit challenge (on the *forward* channel) can be flipped due to noise with probability p_f and a 1-bit answer (on the *backward* channel) can be flipped with probability p_b. Further, we denote as $\tilde{c}_i$ the bit-challenge received by the prover at the ith round, which might be obviously different to the challenge c_i. Similarly, $\tilde{r}_i$ denotes the response received by the verifier at the ith round. As in previous works [11], the considered forward and backward channels are assumed to be memoryless. Table 1 shows the three different scenarios when considering a noisy communication at the i-th round in our protocol.

Forward Noise	**Backward Noise**	**Forward and Backward Noise**
P receives $\tilde{c}_i = c_i \oplus 1$ P updates $\tilde{C}_i = \tilde{c}_1...\tilde{c}_i$ P sends $r_i = R_i^{\tilde{c}_i} \oplus f_Q(\tilde{C}_i)$ V receives $\tilde{r}_i = r_i$	P receives $\tilde{c}_i = c_i$ P updates $\tilde{C}_i = \tilde{c}_1...\tilde{c}_i$ P sends $R_i^{c_i} \oplus f_Q(\tilde{C}_i)$ V receives $\tilde{r}_i = r_i \oplus 1$	P receives $\tilde{c}_i = c_i \oplus 1$ P updates $\tilde{C}_i = \tilde{c}_1...\tilde{c}_i$ P sends $R_i^{\tilde{c}_i} \oplus f_Q(\tilde{C}_i)$ V receives $\tilde{r}_i = r_i \oplus 1$

Table 1. The three possible scenarios when some noise occurs at the ith round.

According to the protocol, in a noise-free ith round executed with a legitimate prover it holds that $r_i = \tilde{r}_i \Leftrightarrow f_Q(C_i) = f_Q(\tilde{C}_i)$. We thus say that prover and verifier are *synchronized* at the ith round if $f_Q(C_i) = f_Q(\tilde{C}_i)$, otherwise they are said to be *desynchronized*. Intuitively, in a noise-free ith round the answer $\tilde{r}_i$ can be considered correct by the verifier either if $r_i = \tilde{r}_i$ and they are synchronized or if $r_i \neq \tilde{r}_i$ and they are desynchronized.

The challenge is therefore to identify whether the prover and the verifier are synchronized or not. To that aim, we rise the following observation.

Observation 1 *Several consecutive rounds where all, or almost all, the answers hold that $r_i = \tilde{r}_i$ (resp. $r_i \neq \tilde{r}_i$), might indicate that the legitimate prover and the verifier have been synchronized (resp. desynchronized).*

7.2 Our noise resilient mechanism

Based on Observation 1, we propose an heuristic aimed at identifying those rounds where prover and verifier *switch* from being synchronized to desynchronized or vice versa. The heuristic is named *SwitchedRounds* and its pseudocode description is provided by Algorithm 1.

SwitchedRounds creates first the sequence $d_1...d_n$ where $d_i = 0$ if $r_i = \tilde{r}_i$, otherwise $d_i = 1$. Following Observation 1, it searches for the longest subsequence $d_i...d_j$ that matches any of the following patterns[7]: (a) $\wedge$(1+)0 (b) $\wedge$(1+)$ (c) 1(0+)1 (d) 0(1+)0 (e) 1(0+)$ (f) 0(1+)$.

The aim of these patterns is to look for *large* subsequences of either consecutive 0s or 1s in $d_1 \ldots d_n$. Note that, we do not include the patterns $\wedge$(0+)1 and $\wedge$(0+)$ because starting with a sequence of zeros is exactly what the verifier expects. Intuitively, the lower the expected noise the larger the subsequences should be. As an example, let us consider the case where the communication channel is noiseless. Since no noise is expected, the sequence $d_1 \ldots d_n$ should be equal to n consecutive zeros unless an attack is being performed. In Algorithm 1, a threshold Δl defines how large a matching $d_i...d_j$ should be in order to be analyzed. We discuss a computational approach to estimate a practical value for Δl in Section 8.

If a pattern $d_i...d_j$ holds that $j - i \geq \Delta l$, *SwitchedRounds* looks for the closer index r to $i + 1$ such that $q_r = 1$, and assumes that the rth round caused the switch from synchronization to desynchronization or vice versa. To understand this choice, let us note that a pattern $d_i...d_j$ implies that $d_i \neq d_{i+1}$. This could have happened if a switch from synchronization to desynchronization or vice versa occurred in the $(i + 1)$th round. However, due to the probabilistic nature of the noise we cannot precisely determine whether the switch occurred in the $(i+1)$th round or in some (possibly close) round. What we do know is that such a round r must hold that $q_r = 1$, which justifies Step 5 in Algorithm 1.

Once r is found, the pair (r, s) is created where s is 0 if the switch is to synchronization, $s = 1$ otherwise. Finally, *SwitchedRounds* recursively calls itself to analyze the two subsequences lying on the left and right side of $d_i...d_j$. The output is the union of all obtained pairs such that they are in increasing order (according to the round) and every two consecutive pairs have different values (according to the type of switching).

Armed with the *SwitchedRounds* algorithm, the threshold technique can be straightforwardly applied as Algorithm 2 shows. It simply counts the number of errors occurred during the execution of the protocol where an error is defined as either a switched round or a wrong response. Both cases are considered as *error* because, on the one hand, a switched round might be falsely detected during an attack, and on the other hand, there is no distinction between a wrong response due to noise or to an attack. Finally, the protocol is considered to fail if the number of errors is above a threshold x.

[7] The patterns have been written following the POSIX Extended Regular Expressions standard. The symbols $\wedge$ and $ represent the start and the end of the string respectively.

Algorithm 1 SwitchedRounds

Require: The challenges $c_1...c_n$ and the registers R^0, R^1, and Q. The prover's responses $\tilde{r}_1...\tilde{r}_n$. A threshold Δl indicating the minimum matching length.

Let $d_1...d_n$ be a sequence such that $d_i = \tilde{r}_i \oplus R_i^{c_i} \oplus f(C_i)$.
Let $d_i...d_j$ be the longest matching with `∧(1+)0|∧(1+)$|0(1+)0|0(1+)$|1(0+)1|1(0+)$` on $d_1...d_n$.
if $j - i < \Delta l$ or no matching exists **then return** the empty set;
Let s be 0 if the matching is with `1(0+)1|1(0+)$` and 1 otherwise;
Let r be the closest index to $i+1$ such that $q_r = 1$;
Let A be the output of *SwitchedRounds* on $d_1...d_{i-1}$;
Let B be the output of *SwitchedRounds* on $d_{j+1}...d_n$;
Let E be the union of $A \cup \{(r,s)\} \cup B$ such that the indexes are in increasing order and every two consecutive pairs have different boolean values;
return E;

Note that basing the decision on a threshold is a common and easy procedure but not the best one, especially when the channels are not memoryless. Instead, the decision procedure could consist in comparing the vector $d_1...d_n$ with the error distribution on the noisy channels.

Algorithm 2 Authentication in the presence of noise

Require: All the parameters of the protocol; an integer value x representing the noise tolerance; and a threshold Δl.

Let E be the output of *SwitchedRounds* algorithm on input $c_1...c_n$, $\tilde{r}_1...\tilde{r}_n$, R^0, R^1, Q, and Δl;
Let $d_1...d_n$ be a sequence such that $d_i = \tilde{r}_i \oplus R_i^{c_i} \oplus f(C_i)$.
Let s be a boolean variable initialized in 0;
Let $errors$ be a counter initialized in 0;
for all $1 \leq i \leq n$ **do**
if $(i, s') \in E$ **then**
$s \leftarrow s'$;
$errors++$;
else if ($d_i = 0$ and $s = 1$) or ($d_i = 1$ and $s = 0$) **then**
$errors++$;
if $errors > x$ **then return** fail;
else return success;

8 Experiments and comparison

The first part of this section is devoted to compare several DB protocols in term of mafia fraud resistance, distance fraud resistance, and memory consumption. The second part takes noise into account and evaluates our proposal w.r.t. the Hancke and Kuhn's [11] and Kim and Avoine's [13] protocols.

8.1 Noise-free environment

Mafia and distance fraud analyses in a noise-free environment can be found in [11,13,16,2] for KA2, AT, Poulidor, and HK. In the case of AT and Poulidor, only an upper-bound of their resistance to distance fraud have been provided [16,9]. Considering those previous results, Fig. 2(a) and Fig. 2(b) show the resistance to mafia and distance frauds respectively for the five considered protocols in a single chart. For each of them, the configuration that maximizes its security has been chosen: this is particularly important for AT and KA2 because different configurations can be used.

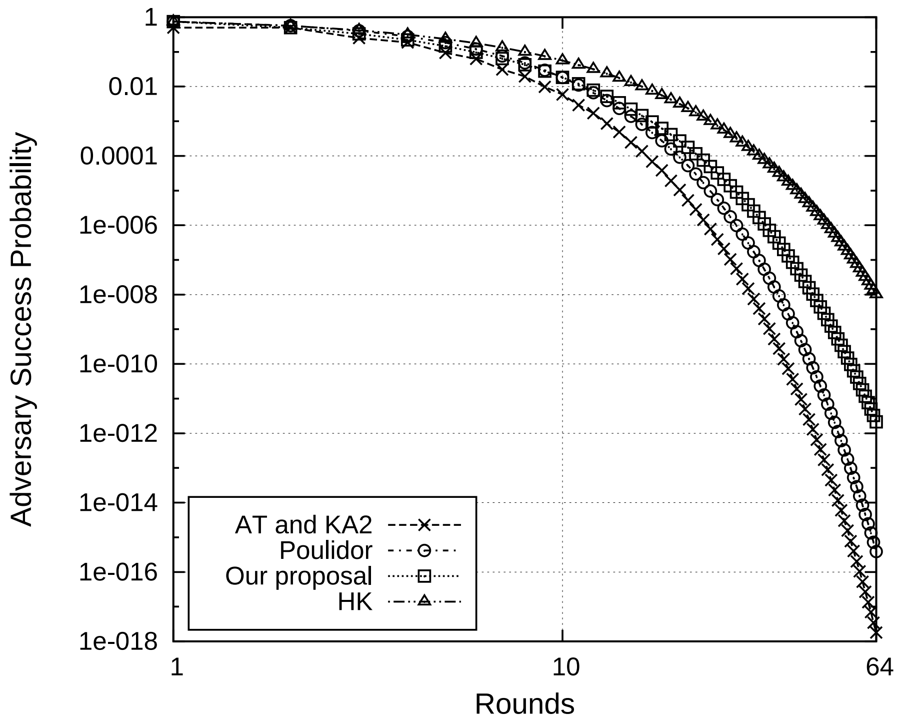
Figures 2(a) and 2(b) show that AT and KA2 are the best protocols in terms of mafia fraud while our proposal is the best in terms of distance fraud. However, it makes sense to consider the two properties together. To do so, we follow the technique used in [16] to seek for a good trade-off. This technique first discretizes the mafia fraud (x) and distance fraud (y) success probabilities. For every pair (x, y), it then evaluates which protocol is the less round-consuming. This protocol is considered as the best for the considered pair. In case of draw between two protocols, the one that is the less memory-consuming is considered as the best protocol. Using this idea, it is possible to draw what we call a *trade-off* chart, which represents for every pair (x, y) the best protocol among the five considered (see Figure 3(a)).

Figure 3(a) shows that our protocol offers a good trade-off between resistance to mafia fraud and resistance to distance fraud, especially when a high security level against distance fraud is expected. In other words, our protocol defeats all the other considered protocols, except when the expected security levels for mafia and distance fraud are unbalanced, which is meaningless in common scenarios.

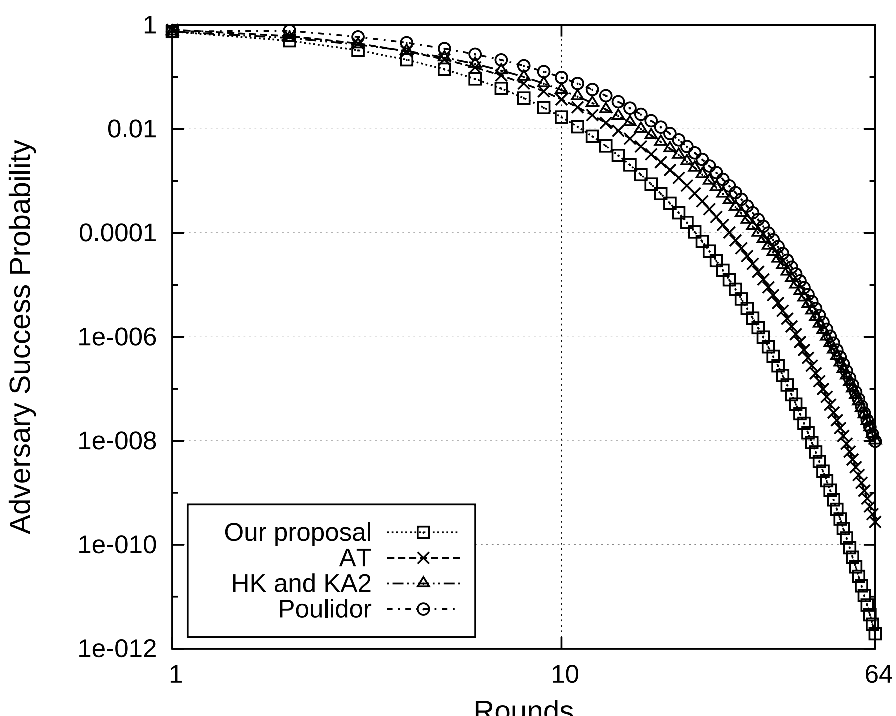
Another interesting comparison could take into consideration the memory consumption of the protocols. Indeed, for n rounds of the fast phase, AT requires $2^{n+1} - 1$ bits of memory, which is prohibitive for most pervasive devices. We can therefore compare protocols that require a linear memory w.r.t. the number of rounds n. For that, we consider a variant of AT [2], denoted AT-3, that uses $n/3$ trees of depth 3 instead of just one tree of depth n. By doing so, the memory consumption of all the considered protocols is below $5n$ bits of memory. The resulting trade-off chart (Figure 3(b)) shows that constraining the memory consumption considerable reduces the area where AT is the best protocol, but it also shows that our protocol is also the best trade-off in this scenario.

8.2 Noisy environment

Among the protocols we are considering, only HK [11] and KA2 [13] claim to be noise resilient. For this reason, we analyze in this section the performance of our proposal in the presence of noise by comparing it with HK and KA2. The comparison is performed by considering two properties: availability and security. Availability is measured in terms of false rejection ratio and security in terms of mafia fraud resistance. It should be remarked that, distance fraud resistance

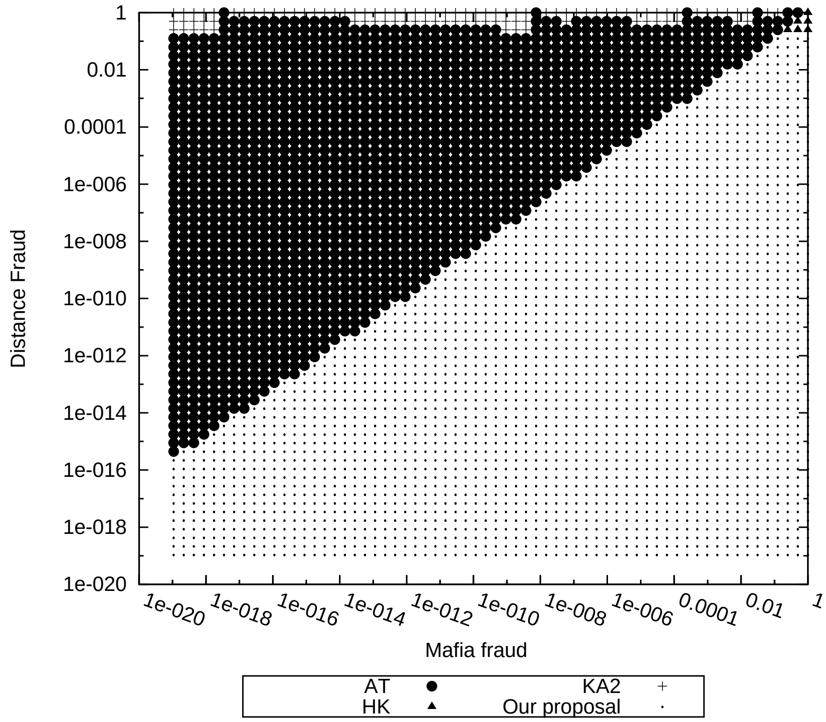


(a) Mafia Fraud

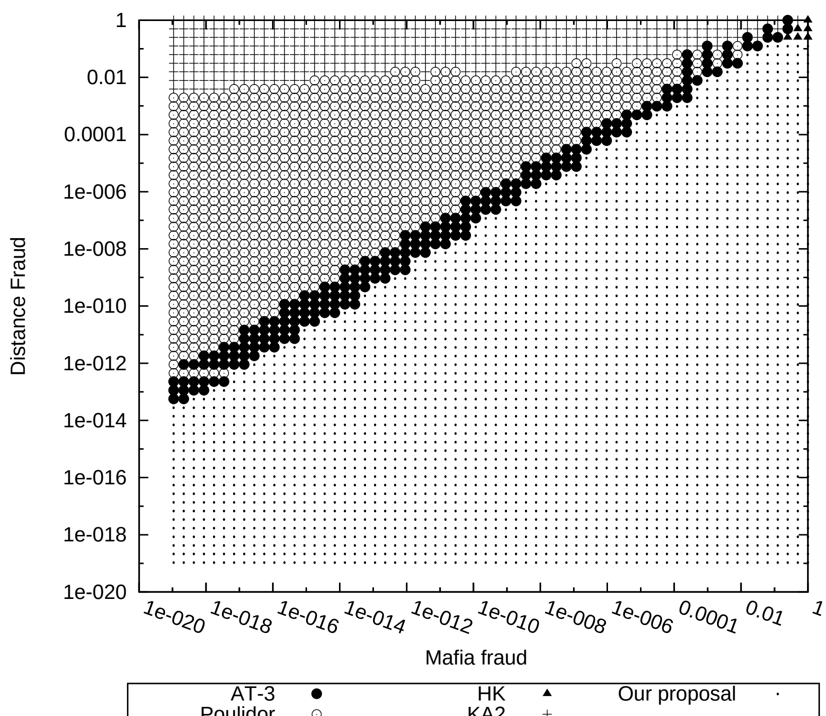


(b) Distance Fraud

Fig. 2. The mafia fraud (Figure 2(a)) and distance fraud (Figure 2(b)) success probabilities considering up to 64 rounds (logarithmic scale). The considered protocols are KA2 [13], AT [2], Poulidor [16], HK [11], and our protocol.



(a) Trade-off without memory constraint



(b) Trade-off with memory constraint

Fig. 3. Two trade-off charts showing the most efficient protocol for each pair of mafia fraud and distance fraud probability values ranging between 1 and $\left(\frac{1}{2}\right)^{64}$. Figure 3(a) considers the protocols KA2 [13], AT [2], Poulidor [16], HK [11], and our proposal, while Figure 3(b) changes AT by its low-resource consuming variant AT-3.

has not been considered for simplicity and because, as shown previously, our proposal outperforms all the other protocols in terms of this fraud.

An important parameter when measuring availability and security for the three protocols is the number of allowed incorrect responses (x). In the case of KA2 and our proposal, other parameters are the minimum size of the pattern (s) (denoted by Δl in Algorithm 1) and the number of predefined challenges p respectively. Theoretical bounds for these parameters in term of the number of rounds and noise probabilities might be provided, however, we left this non-trivial task for future work. Instead, we treat the three parameters $z = (x, s, p)$ as the variables of an optimization problem defined as follows:

Problem 1. Let Π be a distance-bounding protocol, n the number of rounds, p_f the probability of noise in the forward channel, and p_b the probability of noise in the backward channel. Let $\Pi^{security}_{p_f,p_b,n}(z)$ and $\Pi^{availability}_{p_f,p_b,n}(z)$ be the functions that, given a set of parameters $z = (x, s, p)$, compute the adversary success probability when mounting a mafia fraud against Π and the false rejection ration of Π respectively. Given a threshold Δ, the optimization problem consists in:

$$\begin{array}{ll} \text{minimizing} & \Pi^{security}_{p_f,p_b,n}(z) \\ \text{subject to} & \Pi^{availability}_{p_f,p_b,n}(z) \leq \Delta \end{array}$$

To follow notations of Problem 1, we assume that $\Pi \in \{\text{HK, KA2, Our}\}$ where "Our" denotes our proposal. Therefore, algorithms to compute $\Pi^{security}_{p_f,p_b,n}(z)$ and $\Pi^{availability}_{p_f,p_b,n}(z)$ for every $\Pi \in \{\text{HK, KA2, Our}\}$ are required. In case $\Pi =$ HK, $\text{HK}^{security}_{p_f,p_b,n}(z)$ can be computed as shown in [11], while $\text{HK}^{availability}_{p_f,p_b,n}(z)$ is provided by Theorem 6 (see Appendices). For $\Pi = \text{KA2}$, an algorithm to compute $\text{KA2}^{security}_{p_f,p_b,n}(z)$ is given in [13]. Unfortunately, analytical expressions for $\text{KA2}^{availability}_{p_f,p_b,n}(z)$, $\text{Our}^{security}_{p_f,p_b,n}(z)$, and $\text{Our}^{availability}_{p_f,p_b,n}(z)$, seem to be cumbersome to find. Therefore, we address this issue by means of simulation.

A simulation means that, given a protocol Π, all the parties (Verifier, Prover, and Adversary) are simulated. The protocol is then executed 10^6 times and the mean of the results (either security or availability) is taken as the estimation.

For the experiments, we consider 48 rounds and a false rejection ratio lower than 5%. Note that, there does not exist a real consensus on how many rounds should be executed in a distance bounding protocol. For instance, in [13] up to 40 rounds are considered, while others might vary from 20 to 80. We normally choose 64 rounds for our experiments [16]. However, since our optimization problem is solved by means of simulations whose performance decrease with the number of rounds, we drop from 64 to 48 rounds.

Regarding noise we consider two cases: (a) both forward and backward channels introduce noise with the same probability ($p_f = p_b \leq 0.05$), and (b) the noise probabilities for the forward and backward channels are not necessarily equal and are related as follows ($p_f + p_b = 0.05$) [8]. We do not consider an over-

[8] We have performed experiments by considering several other correlations between p_f and p_b. The results are not significantly different to those provided by these two cases, though.

all noise probability higher than 0.1. Actually, a high noise probability makes useless all the distance bounding protocols proposed up-to-date.

Armed with these settings, Figure 4(a) and Figure 4(b) show the maximum resistance to mafia fraud for the three protocols considering the cases (a) and (b) respectively.

Figure 4(a) shows the mafia fraud resistance of the three protocols when $p_f = p_b$. As expected, the higher the noise the lower the provided security of the three protocols. In this scenario, however, our protocol is clearly the best even though KA2 achieves the highest resistance when no noise is considered ($p_f = p_b = 0$).

A different scenario ($p_f + p_b = 0.05$) is shown by Figure 4(b). There, the security of HK improves with p_f, while KA2 and our protocol are clearly sensitive to the increase of p_f. This is an inherent problem of both protocols since a noise in the forward channel could cause a “desynchronization” between the prover and the verifier. Nevertheless, thanks to the noise resilience mechanism proposed in Section 7.2, our protocol deals with noise much better than KA2 and, in general, performs better than both HK and KA2.

9 Conclusions

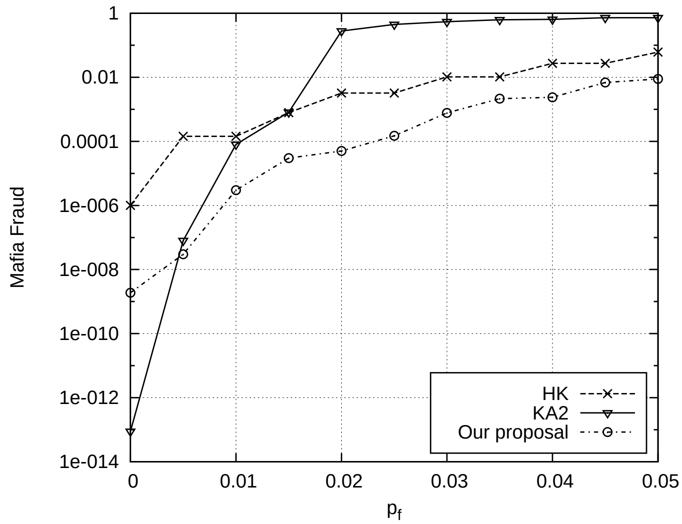
A new lightweight distance-bounding protocol has been introduced in this article. The protocol simultaneously deals with both mafia and distance frauds, without sacrificing memory or requiring additional computation. The analytical expressions and experimental results show that the new protocol outperforms the previous ones. This benefit is obtained through the use of dependent rounds in the fast phase. The protocol also goes a step further by dealing with the inherent background noise on the communication channels. This is a serious advantage compared to the other existing protocols.

Appendix

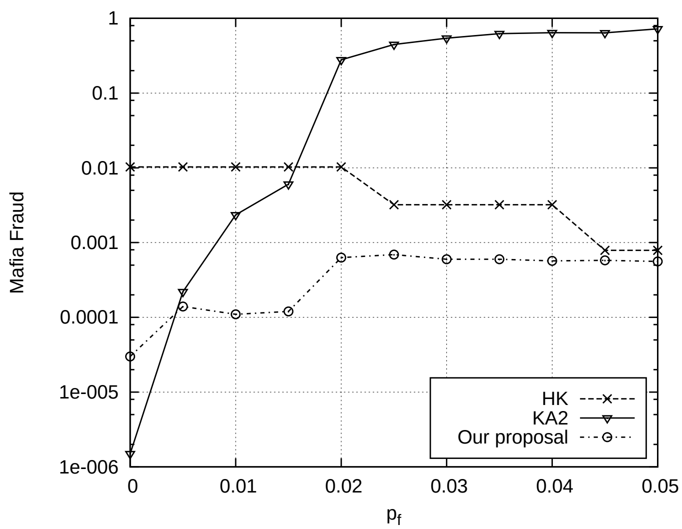
Theorem 6. *Let x be the maximum number of errors allowed by the verifier in the HK protocol. The false rejection ratio $HK^{availability}_{p_f,p_b,n}(x)$ can be computed as follows:*

$$HK^{availability}_{p_f,p_b,n}(x) = \sum_{i=n-x}^{n} \left(\binom{n}{i} \times \left(1 - \frac{p_f}{2} - p_b + p_f p_b\right)^i \left(\frac{p_f}{2} + p_b - p_f p_b\right)^{n-i} \right).$$

Proof. Let W be the event that a legitimate prover's bit-answer is correct for the verifier. The false rejection ratio $HK^{availability}_{p_f,p_b,n}(x)$ can be expressed in terms



(a) $p_f = p_b$



(b) $p_f + p_b = 0.05$

Fig. 4. The maximum resistance to mafia fraud of HK, KA2, and our proposal, considering 48 rounds, a false rejection ratio lower than 5%, and different values p_f and p_b: in Figure 4(a) $p_f = p_b \in \{0, 0.005, ..., 0.045, 0.05\}$, and in Figure 4(b) $p_f + p_b = 0.05$ where $p_f \in \{0, 0.005, ..., 0.045, 0.05\}$.

of W as follows:

$$HK^{availability}_{p_f,p_b,n}(x) = \sum_{i=n-x}^{n} \binom{n}{i} \Pr(W)^i (1-\Pr(W))^{n-i}. \quad (17)$$

It should be noted that if a noise occurs on the forward channel then $\Pr(W) = \frac{1}{2}$, which happens with probability $p_f(1-p_b) + p_f p_b$. Thus:

$$\begin{aligned} \Pr(W) &= \frac{1}{2}(p_f(1-p_b) + p_f p_b) + (1-p_f)(1-p_b) \\ &= 1 - \frac{p_f}{2} - p_b + p_f p_b. \end{aligned} \quad (18)$$

Equations 17 and 18 yield the expected result.

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