

MOLECULAR COULOMB FIELD COLLAPSE--BLACKLIGHT PROCESS

BELOW "GROUND" STATE TRANSITIONS OF HYDROGEN-TYPE MOLECULES AND MOLECULAR IONS

Excited states of orbitalspheres are discussed in the Excited States of the One Electron Atom (Quantization) Section. In the case of ellipsoidal M. O. 's, excited electronic states are created when photons of discrete frequencies are trapped in the ellipsoidal resonator cavity of the M. O. The photon changes the effective charge at the M. O. surface where the central field is ellipsoidal and arises from the protons and the effective charge of the "trapped photon" at the foci of the M. O. Force balance is achieved at a series of ellipsoidal equipotential two dimensional surfaces confocal with the ground state ellipsoid. The "trapped photons" are solutions of the Laplacian in ellipsoidal coordinates, Eq. (12.27).

As is the case with the orbitalsphere, higher and lower energy states are equally valid. The photon standing wave in both cases is a solution of the Laplacian in ellipsoidal coordinates. For an ellipsoidal resonator cavity, the relationship between an allowed circumference, $4aE$, and the photon standing wavelength, λ , is

$$4aE = n\lambda \quad (13.1)$$

where n is an integer and where

$$k = \frac{\sqrt{a^2 - b^2}}{a} \quad (13.2)$$

is used in the elliptic integral E of Eq. (13.1). Applying Eqs. (13.1) and (13.2), the relationship between an allowed angular frequency given by Eq. (12.24) and the photon standing wave angular frequency, ω , is:

$$\frac{\pi\hbar}{m_e A} = \frac{\hbar}{m_e n a_1 n b_1} = \frac{\hbar}{m_e a_n b_n} = \frac{1}{n^2} \omega_1 = \omega_n \quad (13.3)$$

where $n = 1, 2, 3, 4, \dots$

$$n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

ω_1 is the allowed angular frequency for $n = 1$

a_1 and b_1 are the allowed semimajor and semiminor axes for $n = 1$

ENERGY HOLES

From Eq. (13.3), the magnitude of the elliptic field corresponding to a below "ground state" transition of the hydrogen molecule is an integer. The potential energy equations of hydrogen-type molecules are

$$V_e = \frac{-p2e^2}{8\pi\epsilon_o \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (13.4)$$

$$V_p = \frac{pe^2}{8\pi\epsilon_o \sqrt{a^2 - b^2}} \quad (13.5)$$

where

$$a = \frac{a_o}{p} \quad (13.6)$$

$$b = \frac{1}{p\sqrt{2}} a_o \quad (13.7)$$

$$c' = \sqrt{a^2 - b^2} = \frac{\sqrt{2}a_o}{2p} \quad (13.8)$$

and where p is an integer. (These energies are approximate in that they do not include the energy component corresponding to zero order vibration. The exact energies are given by Eqs. (13.4-13.5) where the parameters a and b are those given by Eqs. (13.6-13.7) with the correction for zero order vibration as given in the Vibration Section). From energy conservation, the resonance energy hole of a hydrogen-type molecule which causes the transition

$$H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{p} \quad H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{p+m} \quad (13.9)$$

is

$$mp^2 \times 48.6 \text{ eV} \quad (13.10)$$

where m and p are integers. During the transition, the elliptic field is increased from magnitude p to magnitude $p+m$. The corresponding potential energy change equals the energy absorbed by the energy hole.

$$\text{Energy hole} = -V_e - V_p = mp^2 \times 48.6 \text{ eV} \quad (13.11)$$

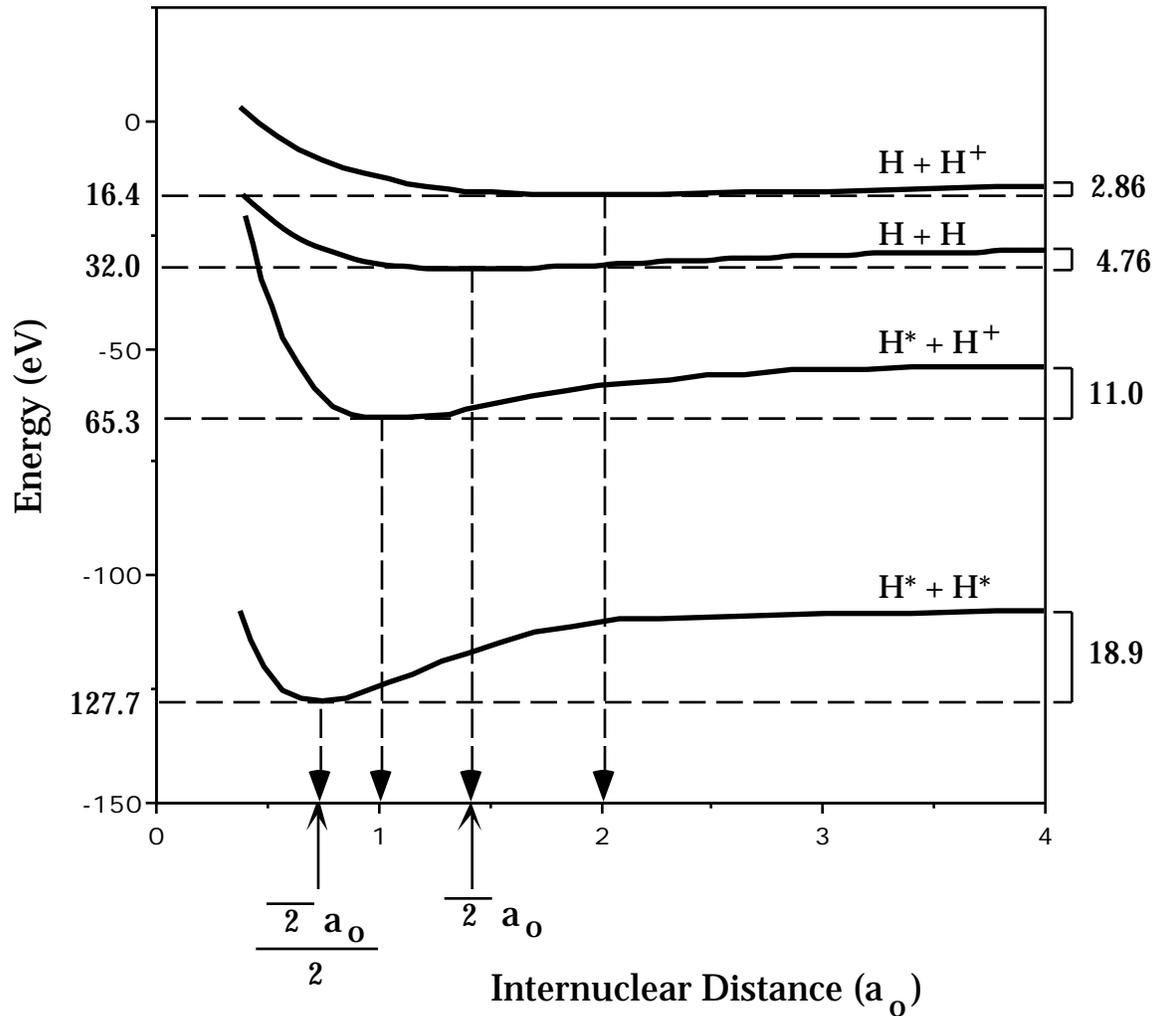
Further energy is released by the hydrogen-type molecule as the internuclear distance "shrinks". The total energy, E_T , released during the transition is

$$E_T = -13.6 \text{ eV} \quad 2(m+p)^2\sqrt{2} - (m+p)^2\sqrt{2} + \frac{(m+p)^2\sqrt{2}}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - (m+p)^2\sqrt{2} \\ + 13.6 \text{ eV} \quad 2p^2\sqrt{2} - p^2\sqrt{2} + \frac{p^2\sqrt{2}}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - p^2\sqrt{2} \quad (13.12)$$

(This energy is approximate in that it does not include the energy component corresponding to zero order vibration. The exact energy is given by Eq. (13.12) with the correction for zero order vibration as given in the Vibration Section).

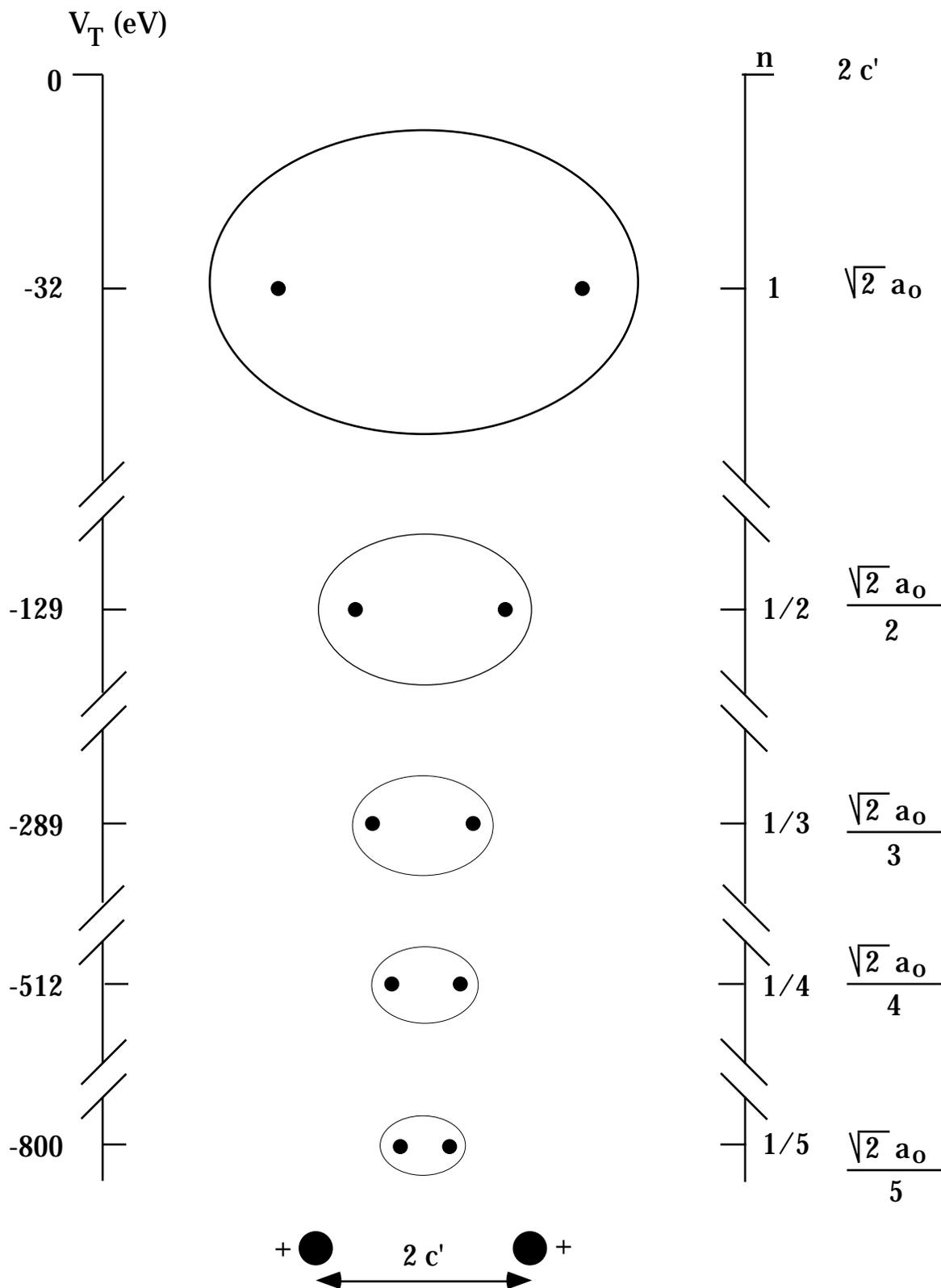
A schematic drawing of the total energy well of hydrogen-type molecules and molecular ions is given in Figure 13.1. The exothermic reaction involving transitions from one potential energy level to a lower level is also hereafter referred to as the Molecular BlackLight Process.

Figure 13.1. The total energy well of hydrogen-type molecules and molecular ions.



A hydrogen-type molecule with its electrons in a lower than "ground state" energy level corresponding to a fractional quantum number is hereafter referred to as a dihydrino molecule. The designation for a dihydrino molecule of internuclear distance, $2c' = \frac{\sqrt{2}a_0}{p}$ where p is an integer, is $H_2^* 2c' = \frac{\sqrt{2}a_0}{p}$. A schematic drawing of the size of hydrogen-type molecules as a function of total energy is given in Figure 13.2.

Figure 13.2. The size of hydrogen-type molecules as a function of total energy.



The magnitude of the elliptic field corresponding to the first below "ground state" transition of the hydrogen molecule is 2. From energy conservation, the resonance energy hole of a hydrogen molecule which excites the transition of the hydrogen molecule with internuclear distance $2c' = \sqrt{2}a_o$ to the first below "ground state" with internuclear distance $2c' = \frac{1}{\sqrt{2}}a_o$ is given by Eqs. (13.4-13.8) where the elliptic field is increased from magnitude one to magnitude two:

$$V_e = \frac{-2e^2}{8\pi\epsilon_o\sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -67.813 \text{ eV} \quad (13.13)$$

$$V_p = \frac{e^2}{8\pi\epsilon_o\sqrt{a^2 - b^2}} = 19.23 \text{ eV} \quad (13.14)$$

$$\text{Energy hole} = -V_e - V_p = 48.6 \text{ eV} \quad (13.15)$$

In other words, the elliptic "ground state" field of the hydrogen molecule can be considered as the superposition of Fourier components. The removal of negative Fourier components of energy

$$m \times 48.6 \text{ eV} \quad (13.16)$$

where m is an integer, increases the positive electric field inside the ellipsoidal shell by m times the charge of a proton at each focus. The resultant electric field is a time harmonic solution of the Laplacian in ellipsoidal coordinates. The corresponding potential energy change equals the energy absorbed by the energy hole.

$$\text{Energy hole} = -V_e - V_p = m \times 48.6 \text{ eV} \quad (13.17)$$

Further energy is released by the hydrogen molecule as the internuclear distance "shrinks". The hydrogen molecule with internuclear distance $2c' = \sqrt{2}a_o$ is caused to undergo a transition to the below "ground state" level, and the internuclear distance for which force balance and

nonradiation are achieved is $2c' = \frac{\sqrt{2}a_o}{1+m}$. In decaying to this internuclear distance from the "ground state", a total energy of

$$\begin{aligned} -13.6 \text{ eV} & \quad 2(1+m)^2\sqrt{2} - (1+m)^2\sqrt{2} + \frac{(1+m)^2\sqrt{2}}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - (1+m)^2\sqrt{2} \\ & \quad + 13.6 \text{ eV} \quad 2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \end{aligned} \quad (13.18)$$

is released.

CATALYTIC ENERGY HOLES FOR HYDROGEN-TYPE MOLECULES

An efficient catalytic system that hinges on the coupling of three resonator cavities involves iron and lithium. For example, the fourth

ionization energy of iron is 54.8 eV. This energy hole is obviously too high for resonant absorption. However, Li^+ releases 5.392 eV when it is reduced to Li . The combination of Fe^{3+} to Fe^{4+} and Li^+ to Li , then, has a net energy change of 49.4 eV.

$$49.4 \text{ eV} + Fe^{3+} + Li^+ + H_2[2c' = \sqrt{2}a_o] \quad Fe^{4+} + Li + H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.19)$$

$$Li + Fe^{4+} \quad Li^+ + Fe^{3+} + 49.4 \text{ eV} \quad (13.20)$$

And, the overall reaction is

$$H_2[2c' = \sqrt{2}a_o] \quad H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.21)$$

Note that the energy given off as the molecule shrinks is much greater than the energy lost to the energy hole. And, the energy released is large compared to conventional chemical reactions.

An efficient catalytic system that hinges on the coupling of three resonator cavities involves scandium. For example, the fourth ionization energy of scandium is 73.47 eV. This energy hole is obviously too high for resonant absorption. However, Sc^{3+} releases 24.76 eV when it is reduced to Sc^{2+} . The combination of Sc^{3+} to Sc^{4+} and Sc^{3+} to Sc^{2+} , then, has a net energy change of 48.7 eV.

$$48.7 \text{ eV} + Sc^{3+} + Sc^{3+} + H_2[2c' = \sqrt{2}a_o] \quad Sc^{4+} + Sc^{2+} + H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.22)$$

$$Sc^{2+} + Sc^{4+} \quad Sc^{3+} + Sc^{3+} + 48.7 \text{ eV} \quad (13.23)$$

And, the overall reaction is

$$H_2[2c' = \sqrt{2}a_o] \quad H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.24)$$

An efficient catalytic system that hinges on the coupling of three resonator cavities involves yttrium. For example, the fourth ionization energy of gallium is 64.00 eV. This energy hole is obviously too high for resonant absorption. However, Pb^{2+} releases 15.03 eV when it is reduced to Pb^+ . The combination of Ga^{3+} to Ga^{4+} and Pb^{2+} to Pb^+ , then, has a net energy change of 48.97 eV.

$$48.97 \text{ eV} + Ga^{3+} + Pb^{2+} + H_2[2c' = \sqrt{2}a_o] \quad Ga^{4+} + Pb^+ + H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.25)$$

$$Ga^{4+} + Pb^+ \quad Ga^{3+} + Pb^{2+} + 48.97 \text{ eV} \quad (13.26)$$

And, the overall reaction is

$$H_2[2c' = \sqrt{2}a_o] \quad H_2^* \quad 2c' = \frac{\sqrt{2}a_o}{2} + 95.7 \text{ eV} \quad (13.27)$$

The rates of electronic transitions of molecules is a function of the change in internuclear distance during the transition. Transitions between electronic states that have equivalent internuclear distances at

some point during their vibrational cycles have much greater rates than transitions that require the energy level of the electrons to change as well as the internuclear distance to change simultaneously. As shown in Figure 13.1, the transition from the $n = 1$ state to the $n = 1/2$ state of molecular hydrogen is not favored for this reason. A more likely transition pathway is a vibrational excitation of molecular hydrogen ($n = 1$) that breaks the bond, followed by a transition reaction of each of the hydrogen atoms via a 27.2 eV energy hole catalyst as given in the Atomic Coulomb Field Collapse-Hydrino Theory-BlackLight Process Section, followed by reaction of the two hydrino atoms ($n = 1/2$) to form dihydrino molecule ($n = 1/2$).

DIATOMIC MOLECULAR ENERGY STATES

EXCITED ELECTRONIC STATES OF ELLIPSOIDAL M.O.'S

Excited states of orbitalspheres are discussed in the Excited States of the One Electron Atom (Quantization) Section. In the case of ellipsoidal M. O. 's, excited electronic states are created when photons of discrete frequencies are trapped in the ellipsoidal resonator cavity of the M. O. The photon changes the effective charge at the M. O. surface where the central field is ellipsoidal and arises from the protons at the foci of the M. O. Force balance is achieved at a series of ellipsoidal equipotential two dimensional surfaces confocal with the ground state ellipsoid. The "trapped photons" are solutions of the Laplacian in ellipsoidal coordinates and are given by Eq. (12.27).

MAGNETIC MOMENT OF AN ELLIPSOIDAL M.O.

The magnetic dipole moment, μ , of a current loop is

$$\mu = iA \quad (14.1)$$

The area of an ellipse is given by Eq. (12.25). For any elliptic orbital due to a central field, the frequency, f , is

$$f = \frac{\frac{L}{2}}{ab} \quad (14.2)$$

where L is the angular momentum. The current, i , is

$$i = ef = \frac{\frac{eL}{2}}{ab} \quad (14.3)$$

where e is the charge. Substitution of Eqs. (14.3) and (12.25) into Eq. (14.1) where L is the angular momentum of the electron, \hbar , gives

$$\mu = \frac{e\hbar}{2m_e} \quad (14.4)$$

which is the Bohr magneton.

MAGNETIC FIELD OF AN ELLIPSOIDAL M.O.

The magnetic field can be solved as a magnetostatic boundary value problem which is equivalent to that of a uniformly magnetized ellipsoid.[1] The magnetic scalar potential inside the ellipsoidal M. O., ϕ^- , is

$$\phi^- = \frac{e\hbar}{2m_e} x \frac{ds}{(s+a^2)R_s} \quad (14.5)$$

The magnetic scalar potential outside of the M. O., ϕ^+ , is

$$\phi^+ = \frac{e\hbar}{2m_e} x \frac{ds}{(s+a^2)R_s} \quad (14.6)$$

The magnetic field inside the ellipsoidal M. O., \mathbf{H}_x^- , is

$$\mathbf{H}_x^- = -\frac{\delta\phi^-}{\delta x} = \frac{-e\hbar}{2m_e} \frac{ds}{(s+a^2)R_s} \quad (14.7)$$

The magnetic field inside the ellipsoidal M. O. is uniform and parallel to the minor axis.

DIATOMIC MOLECULAR VIBRATION

It can be shown that a perturbation of the orbit determined by an inverse square force results in simple harmonic oscillatory motion of the orbit. For the case of a circular orbit of radius a , an approximation of the angular frequency of this oscillation is

$$\omega = \sqrt{\frac{-\frac{3}{a} f(a) + f'(a)}{m}} = \sqrt{\frac{k}{m}} \quad (14.8)$$

Oscillating charges radiate. However, molecules and molecular ions including the hydrogen molecule, the hydrogen molecular ion, dihydrino molecules, and dihydrino molecular ions demonstrate nonradiative zero order vibration which is time harmonic oscillation of the position of the protons along the principal axis. The protons are located at the foci, and nonradiation is due to the geometry of the ellipse where the electron M. O. is ellipsoidal. A fundamental property of an ellipse is that a light ray emitted from a focus in any direction is reflected off of the ellipse to the other focus, and the sum of the lengths of the ray paths is constant, $2a$.

An oscillating charge $r_o(t) = d \sin \omega_o t$ has a Fourier spectrum

$$\mathbf{J}(\mathbf{k}, \omega) = \frac{q\omega_o d}{2} J_m(k \cos \theta d) \{ \delta[\omega - (m+1)\omega_o] + \delta[\omega - (m-1)\omega_o] \} \quad (14.9)$$

where J_m 's are Bessel functions of order m . These Fourier components can, and do, acquire phase velocities that are equal to the velocity of light. Consider two oscillating charges at the foci of an ellipsoidal resonator cavity, an ellipsoidal M. O. A nonradiative standing electromagnetic wave can be excited which has higher order harmonics in addition to the fundamental frequency as given in Eq. (14.9). This nonradiative standing wave gives rise to zero order vibration of the molecule. The zero order mode is a standing wave with destructive interference of all harmonics of the fundamental frequency, ω_o . A ray undergoes a 180° phase shift upon reflection, and the protons oscillate in opposite relative directions. Thus, mutual destructive interference occurs when x , the distance from one focus to the other for a reflected

ray is equal to a wavelength, λ , where λ is

$$\lambda = \frac{h}{mv} \quad (14.10)$$

It follows that

$$v = \frac{h}{m\lambda} = \frac{h}{mx} \quad (14.11)$$

For time harmonic motion,

$$v = v_{average} = \frac{v_{maximum}}{\sqrt{2}} \quad (14.12)$$

The kinetic energy, T , is given by

$$T = \frac{1}{2} mv^2 \quad (14.13)$$

The vibrational energy of the protons, E_{Pvib} , is equal to the maximum vibrational kinetic energy of the protons. Substitution of Eqs. (14.11) and (14.12) into Eq. (14.13) and multiplication by two corresponding to the two protons is

$$T = T_{max} = 2 \frac{1}{2} m \frac{h^2}{m^2 x^2} (\sqrt{2})^2 = 2 \frac{h^2}{mx^2} \quad (14.14)$$

The vibrational energy is the sum of the vibrational energy of the electron M. O. and that of the protons which are equal.

$$E_{vib} = \frac{4h^2}{mx^2} \quad (14.15)$$

where m is the sum of the masses of the protons, each of mass m_p .

$$m = m_p \quad (14.16)$$

And, X is $2a$. Thus, the vibrational energy is

$$E_{vib} = \frac{h^2}{m_p a^2} \quad (14.17)$$

For a in units of a_o ,

$$E_{vib} = \frac{0.59}{a^2} eV \quad (14.18)$$

The time average internuclear distance is increased by the zero order vibration because the total energy versus internuclear distance function is asymmetrical with a lower slope for internuclear distances greater than the internuclear distance at which the total energy is a minimum. Elongation occurs along the principal axis, and shifts the total energy versus internuclear distance function to a new function which includes the contribution due to vibration. The perturbation of E_T , the total energy of the M. O. given by Eq. (12.72) with a fractional increase in the semimajor axis, a , and the reciprocal decrease in the semiminor axis, b is calculated by reiteration. The angular frequency of the M.O. given by Eq. (12.24) is unchanged when a and b are changed by reciprocal fractions. The corrected a and b are obtained when the change in E_T is

equal to the vibrational energy. The vibrational energy is the sum of two equal components, the vibrational energy of the protons and the vibrational energy of the electron M. O. Vibration causes a redistribution of energy within the molecule. The M.O. potential and kinetic energy terms given by Eqs. (12.59), (12.61), and (12.72) add π radians out of phase with the potential and kinetic energies of vibration; thus, the energy of the molecule will decrease by this amount which is equal to one half the vibrational energy. An $x\%$ increase in the semimajor axis and the reciprocal decrease in the semiminor axis decreases E_T by the vibrational energy and releases energy equal to one half vibrational energy. The vibrational energies and bond distances for hydrogen-type molecules and molecular ions are given in the Nature of the Chemical Bond Section.

Zero order vibration arises because the state is nonradiative and is an energy minimum. Furthermore, electromagnetic radiation of discrete energies given by Eq. (14.18) can be trapped in the resonator cavity where constructive interference occurs at the foci. These standing waves change the electric field at the ellipse surface as described in the Excited Electronic States of Ellipsoidal M. O.'s Section; thus, the major and minor axes increase and the total energy of the molecule given by Eqs. (12.73), (12.74), (12.75), and (12.76) increases. The photons of these standing waves drive the vibration of the molecule at a higher frequency than the zero order frequency, but are reradiated. The energy of a vibrational transition is given by the difference of the sum of the energies of the modes excited before and after the transition. The modes are quantized, and from Eq. (14.18), the energy spacing of the modes is closer together as the total vibrational energy increases.

DIATOMIC MOLECULAR ROTATION

A molecule with a permanent dipole moment can resonantly absorb a photon which excites a rotational mode about the center of mass of the molecule. Nonradiative rotational states require that the space-time Fourier Transform of the rotational current-density function not possess Fourier components synchronous with waves traveling at the speed of light. As demonstrated previously in the Spacetime Fourier Transform of the Electron Function Section, the product of a radial Dirac delta function, two spherically harmonic angular functions, and a time harmonic function is nonradiative and is a solution of the wave equation (Eq. (1.11)). Furthermore, momentum must be conserved with excitation of a rotational mode. The photon carries \hbar of angular momentum; thus, the rotational angular momentum of the molecule changes by \hbar . And, the rotational charge-density function is equivalent to the rigid rotor problem considered in the Rotational Parameters of the

Electron (Angular Momentum, Rotational Energy, Moment of Inertia)
Section with the exception that for a diatomic molecule having atoms of masses m_1 and m_2 , the moment of inertia is

$$I = \mu r^2 \quad (14.19)$$

where μ is the reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (14.20)$$

and where r is the distance between the centers of the atoms, the internuclear distance. The rotational energy levels follow from Eq. (1.95)

$$E_{\text{rotationalorbital}} = \frac{\hbar^2}{2I} J(J+1) \quad (14.21)$$

where J is an integer. For Eq. (14.21), $J=0$ corresponds to rotation about the z-axis where the internuclear axis is along the y-axis, and $J=1$ corresponds to a linear combination of rotations about the z and x-axis.

As given in the Selection Rules Section, the radiation of a multipole of order (l, m) carries $m\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$. Thus, the z component of the angular momentum of the corresponding excited rotational state is

$$L_z = m\hbar \quad (14.22)$$

Thus, the selection rule for rotational transitions is

$$J = \pm 1 \quad (14.23)$$

In addition, the molecule must possess a permanent dipole moment. In the case of absorption of electromagnetic radiation, the molecule goes from a state with a quantum number J to one with a quantum number of $J+1$. Using Eq. (14.21), the energy difference is

$$E = E_{J+1} - E_J = \frac{\hbar^2}{I} [J+1] \quad (14.24)$$

DIATOMIC MOLECULAR ROTATION OF HYDROGEN-TYPE MOLECULES

The reduced mass of hydrogen-type molecules, μ_{H_2} , having two protons is given by Eq. (14.20) where $m_1 = m_2 = m_p$, and m_p is the mass of the proton.

$$\mu_{H_2} = \frac{m_p m_p}{m_p + m_p} = \frac{1}{2} m_p \quad (14.25)$$

The moment of inertia of hydrogen-type molecules is given by substitution of the reduced mass, Eq. (14.25), for μ of Eq. (14.19) and substitution of the internuclear distance, two times Eq. (12.95), for r of Eq. (14.19).

$$I = m_p \frac{a_o^2}{n^2} \quad (14.26)$$

where n is an integer which corresponds to $\frac{1}{n}$, the fractional quantum number of the hydrogen-type molecule. Using Eqs. (14.24) and (14.26), the rotational energy absorbed by a hydrogen-type molecule with the transition from the state with the rotational quantum number J to one with the rotational quantum number $J + 1$ is

$$E = E_{J+1} - E_J = \frac{n^2 \hbar^2}{m_p a_o^2} [J + 1] = n^2 [J + 1] 2.37 \times 10^{-21} J \quad (14.27)$$

The energy can be expressed in terms of wavelength in angstroms (\AA) using the Planck relationship, Eq. (2.65).

$$\lambda = 10^{10} \frac{hc}{E} \text{\AA} = \frac{8.38 \times 10^5}{n^2 [J + 1]} \text{\AA} \quad (14.28)$$

Vibration increases the internuclear distance, r of Eq. (14.19), which decreases the rotational energy. The rotational wavelength including vibration given in the Vibration of Hydrogen -Type Molecules Section is

$$\lambda = \frac{8.43 \times 10^5}{n^2 [J + 1]} \text{\AA} \quad (14.29)$$

The calculated wavelength for the $J = 0$ to $J = 1$ transition of the hydrogen molecule $H_2 (n = 1)$ including vibration is $8.43 \times 10^5 \text{\AA}$. The experimental value is $8.43 \times 10^5 \text{\AA}$. The wavelength calculated from Eq. (14.29) for the $J = 0$ to $J = 1$ transition of the hydrogen-type molecule

$H_2^* 2c' = \frac{\sqrt{2} a_o}{p}$ including vibration is given in Table 14.1.

The rotational wavelength for $p = 8$ is given in Table 14.1 as $13,175 \text{\AA}$. Recently, an interstellar band has been discovered for which no satisfactory assignment exists [2]. The experimentally measured wavelength which matches the predicted wavelength is $13,175 \text{\AA}$.

Table 14.1. The wavelength calculated from Eq. (14.29) for the $J = 0$ to $J = 1$ transition of the hydrogen-type molecule H_2^* $2c' = \frac{\sqrt{2}a_0}{p}$ including vibration.

p of $H_2 \frac{1}{p}$	Lambda (Å)
1	8.431×10^5
2	2.108×10^5
3	9.368×10^4
4	5.269×10^4
5	3.372×10^4
6	2.342×10^4
7	1.721×10^4
8	1.317×10^4
9	1.041×10^4
10	8.431×10^3
11	6.968×10^3
12	5.855×10^3
13	4.989×10^3
14	4.302×10^3
15	3.747×10^3
16	3.293×10^3
17	2.917×10^3
18	2.602×10^3

DIATOMIC MOLECULAR ROTATION OF HYDROGEN-TYPE MOLECULAR IONS

The moment of inertia of hydrogen-type molecular ions is given by substitution of the reduced mass, Eq. (14.25), for μ of Eq. (14.19) and substitution of the internuclear distance, two times Eq. (12.67), for r of Eq. (14.19).

$$I = m_p \frac{2a_o^2}{n^2} \quad (14.30)$$

where n is an integer which corresponds to $\frac{1}{n}$, the fractional quantum number of the hydrogen-type molecular ion. Using Eqs. (14.24) and (14.26), the rotational energy absorbed by a hydrogen-type molecular ion with the transition from the state with the rotational quantum number J to one with the rotational quantum number $J + 1$ is

$$E = E_{J+1} - E_J = \frac{n^2 \hbar^2}{m_p 2a_o^2} [J + 1] = n^2 [J + 1] 1.89 \times 10^{-21} J \quad (14.31)$$

The energy can be expressed in terms of wavelength in microns (μm) using the Planck relationship, Eq. (2.65).

$$\lambda = 10^6 \frac{hc}{E} \mu m = \frac{168}{n^2 [J + 1]} \mu m \quad (14.32)$$

Vibration increases the internuclear distance, r of Eq. (14.19), which decreases the rotational energy. The rotational wavelength including vibration given in the Vibration of Hydrogen -Type Molecular Ions Section is

$$\lambda = \frac{169}{n^2 [J + 1]} \mu m \quad (14.33)$$

The calculated wavelength for the $J = 0$ to $J = 1$ transition of the hydrogen molecular ion $H_2 [2c' = 2a_o]^+$ including vibration is $169 \mu m$. The experimental value is $169 \mu m$. The wavelength calculated from Eq. (14.33) for the $J = 0$ to $J = 1$ transition of the hydrogen-type molecular ion $H_2^* 2c' = \frac{2a_o}{p}^+$ including vibration is given in Table 14.2.

The rotational wavelength for $p = 6$ given in Table 14.2 is $4.7 \mu m$. A broad $4.7 \mu m$ solar chromospheric absorption line is observed which was previously assigned to cool carbon monoxide clouds; however, the temperature of the chromosphere, $> 6000 K$, is higher than that at which carbon monoxide completely decomposes into carbon and oxygen, $< 4000 K$ [3]. The assignment of the $4.7 \mu m$ absorption line to the $J = 0$ to $J = 1$ transition rotational transition of $H_2^* 2c' = \frac{a_o}{3}^+$ provides a resolution of the problem of cool carbon monoxide clouds.

Table 14.2. The wavelength calculated from Eq. (14.33) for the $J = 0$ to $J = 1$ transition of the hydrogen-type molecular ion H_2^+ $2c' = \frac{2a_0}{p}$ including vibration.

p of H_2^+ $\frac{1}{p}$	Lambda (μm)
1	1.69×10^2
2	42.2
3	18.7
4	10.5
5	6.74
6	4.68
7	3.44
8	2.63
9	2.08
10	1.69
11	1.39
12	1.17
13	0.998
14	0.860
15	0.749
16	0.659
17	0.583
18	0.520

References

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2. C. Joblin, J. P. Maillard, L. d'Hendecourt, and A. Leger, *Nature*, 346, (1990), pp. 729-732.
3. Phillips, J. H., Guide to the Sun, Cambridge University Press, Cambridge, Great Britain, (1992), pp. 126-127; 360.

SECTION II

Collective Phenomena

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STATISTICAL MECHANICS

The distribution functions of statistical mechanics according to the Mills theory are as usual [1]. However, the nature of each distribution is related to the present classical quantum mechanical model. The underlying physics is deterministic which can be modeled with chaos theory.

Bose-Einstein- indistinguishable photons having \hbar of angular momentum excite quantized energy levels of electron resonator cavities where superposition and conservation of angular momentum are obeyed.

Fermi-Dirac- identical, indistinguishable electrons occupy the lowest energy configuration as given in the Two Electron Atom Section. The Pauli Exclusion Principle arises as a minimum of energy for interacting electrons each having a Bohr magneton of magnetic moment.

Maxwell-Boltzmann- identical, discrete particles such as molecules which are separated such that the predominant interaction is scattering possess a continuum of momenta.

References

1. Beiser, A., Concepts of Modern Physics, Fourth Edition, McGraw-Hill Book Company, New York, (1978), pp. 312-354.

SUPERCONDUCTIVITY

In the case of a superconductor, an applied voltage gives rise to a transient constant electric field in the z direction

$$\mathbf{E}_z = r \cos \theta \mathbf{i}_z \quad (16.1)$$

$$\mathbf{E}_z = E_0 \mathbf{i}_z \quad (16.2)$$

where \mathbf{i}_z is the unit vector along the z-axis.

The applied field polarizes the material into a superconducting current comprised of current dipoles-magnetic dipoles. In Cartesian coordinates, the magnetic field, \mathbf{H} , at the point (x, y, z) due to a magnetic dipole having a magnetic dipole moment of a Bohr magneton, μ_B , at the position (x_0, y_0, z_0) is

$$\mathbf{H} = \frac{\mu_B \left(2(z - z_0)^2 - (x - x_0)^2 - (y - y_0)^2 \right)}{\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right]^{5/2}} \mathbf{i}_z \quad (16.3)$$

$$\mathbf{H} = \frac{(2z^2 - x^2 - y^2)}{[x^2 + y^2 + z^2]^{5/2}} \mu_B \delta(x - x_0, y - y_0, z - z_0) \mathbf{i}_z \quad (16.4)$$

The field is the convolution of the system function, $h(x, y, z)$ or $h(\rho, \phi, z)$, (the left-handed part of Eq. (16.4)) with the delta function (the right-hand part of Eq. (16.4)) at the position (x_0, y_0, z_0) . A very important theorem of Fourier analysis states that the Fourier Transform of a convolution is the product of the individual Fourier Transforms [1]. The Fourier Transform of the system function, $h(x, y, z)$ or $h(\rho, \phi, z)$, is given in Box 16.1.

BOX 16.1. FOURIER TRANSFORM OF THE SYSTEM FUNCTION

The system function, $h(\rho, \phi, z)$, in cylindrical coordinates is

$$h(\rho, \phi, z) = \frac{2z^2 - x^2 - y^2}{[x^2 + y^2 + z^2]^{5/2}} = \frac{2z^2 - \rho^2}{[\rho^2 + z^2]^{5/2}} \quad (1)$$

The spacetime Fourier transform in three dimensions in cylindrical coordinates, $H(k_\rho, \phi, k_z)$, is given [1] as follows:

$$H(k_\rho, \phi, k_z) = \int_0^{2\pi} \int_0^\infty \int_0^\infty h(\rho, \phi, z) \exp\left(-i2\pi\left[k_\rho \rho \cos(\phi - \phi) + k_z z\right]\right) \rho d\rho d\phi dz \quad (2)$$

With circular symmetry [1]

$$H(k_\rho, k_z) = 2\pi \int_0^\infty h(\rho, z) J_0(k_\rho \rho) e^{-i2\pi k_z z} \rho d\rho dz \quad (3)$$

The Fourier transform of the system function is given by the substitution of Eq. (1) into Eq. (3).

$$H = -2 \int_0^{\infty} \frac{2z^2 - \rho^2}{[\rho^2 + z^2]^{5/2}} J_0[k_\rho \rho] \rho d\rho e^{-jk_z z} dz \quad (4)$$

Consider the integral of Eq. (4) with respect to $d\rho$ only. Factorization of $h(\rho, \phi, z)$ gives

$$2 \int_0^{\infty} \frac{2z^2 \rho}{[\rho^2 + z^2]^{5/2}} - \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} J_0[k_\rho \rho] d\rho \quad (5)$$

Consider the definite integral

$$\int_0^{\infty} \frac{t^{v+1} J_v[at] dt}{[t^2 + z^2]^{u+1}} = \frac{a^u z^{v-u} K_{v-u}[az]}{2^u [u+1]} \quad (6)$$

and the modified Bessel function of the third kind relationship,

$$K_{-v}[x] = K_v[x] \quad (7)$$

The first factor of Eq. (5) is the same form as Eq. (6) with $v = 0; u = \frac{3}{2}$,

thus,

$$2z^2(2) \int_0^{\infty} \frac{\rho}{[\rho^2 + z^2]^{5/2}} J_0[k_\rho \rho] d\rho = \frac{2z^2(2) k_\rho^{3/2} z^{-3/2}}{2^{3/2} [5/2]} K_{-3/2}[k_\rho z] = \frac{[2^{1/2}] z^{1/2} k_\rho^{3/2}}{[5/2]} K_{3/2}[k_\rho z] \quad (8)$$

where $K_{-3/2}[k_\rho z] = K_{3/2}[k_\rho z]$ (Eq. (7)). The second factor of Eq. (5) can be made into the same form as Eq. (6) using the Bessel function of the first kind recurrence relationship

$$J_{v-1}[x] + J_{v+1}[x] = \frac{2v}{x} J_v[x] \quad (9)$$

Consider the second factor of the integral of Eq. (5).

$$-2 \int_0^{\infty} \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} J_0[k_\rho \rho] d\rho \quad (10)$$

Eq. (9) with $v = 1$ is

$$J_0[x] + J_2[x] = \frac{2}{x} J_1[x] \quad (11)$$

$$J_0[x] = \frac{2}{x} J_1[x] - J_2[x] \quad (12)$$

Let

$$x = k_\rho \rho \quad (13)$$

Substitution of Eq. (13) into Eq. (12) is

$$J_0[k_\rho \rho] = \frac{2}{k_\rho \rho} J_1[k_\rho \rho] - J_2[k_\rho \rho] \quad (14)$$

Substitution of Eq. (10) into Eq. (14) is

$$\begin{aligned} -2 \int_0^{\infty} \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} J_0[k_\rho \rho] d\rho &= -2 \int_0^{\infty} \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} \frac{2}{k_\rho \rho} J_1[k_\rho \rho] - J_2[k_\rho \rho] d\rho \\ &= -2 \int_0^{\infty} \frac{2\rho^2}{k_\rho [\rho^2 + z^2]^{5/2}} J_1[k_\rho \rho] d\rho + 2 \int_0^{\infty} \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} J_2[k_\rho \rho] d\rho \end{aligned} \quad (15)$$

The first factor of the right hand side of Eq. (15) is the same form as Eq.

(6) with $v = 1; u = \frac{3}{2}$, thus,

$$-2 \int_0^{\infty} \frac{2\rho^2}{k_p[\rho^2 + z^2]^{5/2}} J_1[k_p\rho] d\rho = \frac{-(4)k_p^{3/2}z^{-1/2}}{k_p 2^{3/2} [5/2]} K_{-1/2}[k_p z] = -\frac{[2^{1/2}] z^{-1/2} k_p^{1/2}}{[5/2]} K_{1/2}[k_p z] \quad (16)$$

where $K_{-1/2}[k_p z] = K_{1/2}[k_p z]$ (Eq.(7)). The second factor of the right hand side of Eq. (15) is the same form as Eq. (6) with $v = 2; u = \frac{3}{2}$, thus,

$$2 \int_0^{\infty} \frac{\rho^3}{[\rho^2 + z^2]^{5/2}} J_2[k_p\rho] d\rho = \frac{(2)k_p^{3/2}z^{1/2}}{2^{3/2} [5/2]} K_{1/2}[k_p z] = \frac{z^{1/2} k_p^{3/2}}{[2^{1/2}] [5/2]} K_{1/2}[k_p z] \quad (17)$$

Combining the parts of the integration with respect to $d\rho$ of Eq. (4) by adding Eq. (8), Eq. (16), and Eq. (17) gives

$$- \frac{[2^{1/2}] z^{1/2} k_p^{1/2}}{[5/2]} K_{3/2}[k_p z] - \frac{[2^{1/2}] z^{1/2} k_p^{1/2}}{[5/2]} K_{1/2}[k_p z] + \frac{z^{1/2} k_p^{3/2}}{[2^{1/2}] [5/2]} K_{1/2}[k_p z] e^{-jk_z z} dz \quad (18)$$

The modified Bessel function of the third kind formulae is

$$K_{n+1/2}[x] = \frac{1}{2x} e^{-x} \sum_{m=0}^n [2x]^{-m} \frac{[n+m+1]}{m! [n+1-m]} \quad (19)$$

Substitution of Eq. (13) into Eq. (19) with $v = 1$ is

$$K_{3/2}[k_p z] = \frac{1}{2k_p z} e^{-k_p z} \left[1 + \frac{1}{2k_p z} \right] [3] \quad (20)$$

Substitution of Eq. (13) into Eq. (19) with $v = 0$ is

$$K_{1/2}[k_p z] = \frac{1}{2k_p z} e^{-k_p z} \quad (21)$$

Substitution of Eq. (20) and Eq. (21) into Eq. (18) is

$$- \frac{(2^{1/2})\pi z^{1/2} k_p^{3/2}}{[5/2]} \left[1 + \frac{1}{2k_p z} \right] [3] - \frac{(2^{1/2})\pi z^{1/2} k_p^{1/2}}{[5/2]} + \frac{\pi z^{1/2} k_p^{3/2}}{(2^{1/2}) [5/2]} \frac{\pi}{2k_p z} e^{-k_p z} e^{-jk_z z} dz \quad (22)$$

$$- \frac{\pi^{3/2}}{[5/2]} k_p e^{-[jk_z + k_p]z} + \frac{z^{-1} \pi^{3/2} [3]}{[5/2]2} e^{-[jk_z + k_p]z} - \frac{z^{-1} \pi^{3/2}}{[5/2]} e^{-[jk_z + k_p]z} + \frac{\pi^{3/2}}{[5/2]2} k_p e^{-[jk_z + k_p]z} dz \quad (23)$$

Collecting terms gives

$$- \frac{\pi^{3/2}}{[5/2]} k_p [1 + 1/2] + \frac{[3]}{2} - 1 z^{-1} e^{-[jk_z + k_p]z} dz \quad (24)$$

With $[3] = 2$ and $[5/2] = 3/4\pi^{1/2}$, Eq. (24) is

$$- \frac{\pi^{3/2}}{[5/2]} \left\{ k_p [3/2] + [1-1]z^{-1} \right\} e^{-k_p z} e^{-jk_z z} dz \quad (25)$$

$$- \frac{\pi^{3/2}}{3/4\pi^{1/2}} 3/2 k_p e^{-k_p z} e^{-jk_z z} dz \quad (26)$$

$$2\pi k_{\rho} \int_0^{\infty} e^{-k_{\rho} z} e^{-jk_z z} dz \quad (27)$$

$$4\pi k_{\rho} \int_0^{\infty} e^{-k_{\rho} z} e^{-jk_z z} dz \quad (28)$$

$$4\pi k_{\rho} \int_0^{\infty} e^{-[jk_z + k_{\rho}]z} dz \quad (29)$$

Integration of Eq. (29) with respect to dz gives

$$4\pi k_{\rho} \left. \frac{-1}{jk_z + k_{\rho}} e^{-[jk_z + k_{\rho}]z} \right|_0 \quad (30)$$

$$4\pi k_{\rho} \frac{1}{jk_z + k_{\rho}} \quad (31)$$

Multiplication of Eq. (31) by

$$1 = \frac{-jk_z + k_{\rho}}{-jk_z + k_{\rho}} \quad (32)$$

gives

$$4\pi k_{\rho} \frac{-jk_z + k_{\rho}}{k_z^2 + k_{\rho}^2} \quad (33)$$

The system function (Eq. (1)) is an even function; thus, the spacetime Fourier transform in three dimensions in cylindrical coordinates, $H(k_{\rho}, k_z)$, is given by taking the real part of Eq. (33) [2].

$$H[k_{\rho}, k_z] = \frac{4\pi k_{\rho}^2}{k_z^2 + k_{\rho}^2} \quad (34)$$

The spacetime Fourier transform in three dimensions in Cartesian coordinates, $H(k_x, k_y, k_z)$, is

$$H[k_x, k_y, k_z] = \frac{4\pi[k_x^2 + k_y^2]}{[k_x^2 + k_y^2 + k_z^2]} \quad (35)$$

where the relationship between the wavenumbers and the spatial Cartesian coordinates is as follows:

$$k_x = \frac{2\pi}{\lambda_x} = \frac{1}{x} \quad (36)$$

$$k_y = \frac{2\pi}{\lambda_y} = \frac{1}{y} \quad (37)$$

$$k_z = \frac{2\pi}{\lambda_z} = \frac{1}{z} \quad (38)$$

References

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2. Siebert, W. McC., Circuits, Signals, and Systems, The MIT Press, Cambridge, Massachusetts, (1986), p. 399.

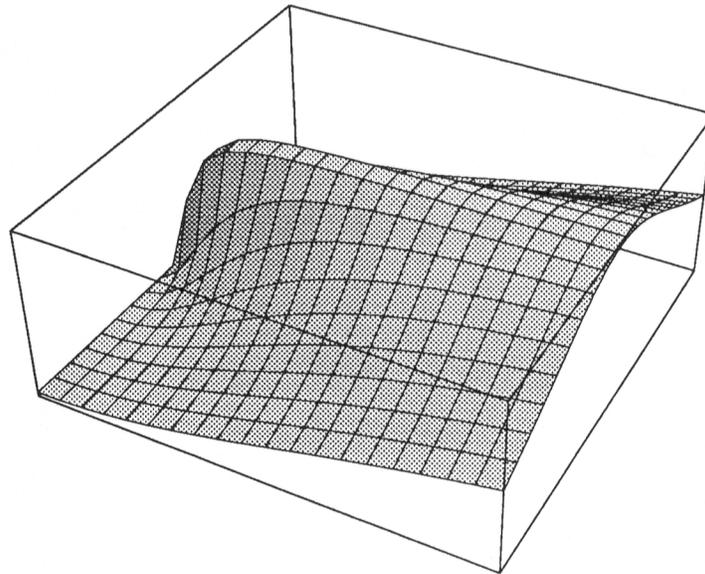
BAND-PASS FILTER

The z component of a magnetic dipole oriented in the z direction has the system function, $h(x, y, z)$, which has the Fourier Transform, $H[k_x, k_y, k_z]$, which is shown in Figure 16.1.

$$H[k_x, k_y, k_z] = \frac{4\pi[k_x^2 + k_y^2]}{[k_x^2 + k_y^2 + k_z^2]} \quad (16.5)$$

$$= H[k_\rho, k_z] = \frac{4\pi k_\rho^2}{k_z^2 + k_\rho^2} = \frac{4\pi}{1 + \frac{k_z^2}{k_\rho^2}} \quad (16.6)$$

Figure 16.1. The Fourier Transform $H[k_x, k_y, k_z]$ of the system function $h(x, y, z)$ corresponding to the z component of a magnetic dipole oriented in the z direction.



As shown in the Electron Scattering by Helium Section, in the far field, the amplitude of the scattered electromagnetic radiation or scattered electron flux density is the Fourier Transform of the aperture function. In the case of a superconductor, the electric field is zero-no voltage drop occurs; however, a magnetic field is present. The relationship between the amplitude of the scattered energy and the Fourier Transform of the aperture function can be applied to the present case of the scattering of magnetic energy by the lattice of the potential superconductor. The spatial aperture function is the convolution of the

array pattern with the elemental pattern. The elemental pattern is the system function, $h(x, y, z)$, -the geometric transfer function for the z component of a z oriented magnetic dipole. And, the array pattern is a periodic array of delta functions each at the position of a magnetic dipole corresponding to a current carrying electron.

$$\frac{(2z^2 - x^2 - y^2)}{[x^2 + y^2 + z^2]^{5/2}} \mu_B \delta(x - nx_0, y - ny_0, z - nz_0) \quad (16.7)$$

The Fourier Transform of a periodic array of delta functions (the right-hand side of Eq. (16.7)) is also a periodic array of delta functions in k-space

$$\frac{1}{x_0 y_0 z_0} \mu_B \delta(k_x - \frac{n}{x_0}, k_y - \frac{n}{y_0}, k_z - \frac{n}{z_0}) \quad (16.8)$$

By the Fourier Theorem, the Fourier Transform of the spatial aperture function, Eq. (16.7), is the product of the Fourier Transform of the elemental function, system function given by Eq. (16.6), and the Fourier Transform of the array function given by Eq. (16.8).

$$\frac{4\pi}{1 + \frac{k_z^2}{k_p^2}} \frac{1}{x_0 y_0 z_0} \mu_B \delta(k_x - \frac{n}{x_0}, k_y - \frac{n}{y_0}, k_z - \frac{n}{z_0}) \quad (16.9)$$

The space-time aperture function corresponding to the current-density function is given by multiplying the spatial aperture function (Eq. (16.7)) by a time harmonic function

$$\exp(-i\omega t) \quad (16.10)$$

Thus, the space-time aperture function is

$$\frac{(2z^2 - x^2 - y^2)}{[x^2 + y^2 + z^2]^{5/2}} \mu_B \delta(x - nx_0, y - ny_0, z - nz_0) \exp(-i\omega t) \quad (16.11)$$

The Fourier Transform of the time harmonic function (Eq. (16.10)) is

$$\frac{[\delta(\omega - \omega_z) + \delta(\omega + \omega_z)]}{2} \quad (16.12)$$

A very important theorem of Fourier analysis states that the Fourier Transform of a product is the convolution of the individual Fourier Transforms. Thus, the Fourier Transform of Eq. (16.11) is the convolution of Eqs. (16.9) and (16.12)

$$\frac{4\pi}{1 + \frac{k_z^2}{k_p^2}} \frac{1}{x_0 y_0 z_0} \mu_B \delta(k_x - \frac{n}{x_0}, k_y - \frac{n}{y_0}, k_z - \frac{n}{z_0}) \frac{[\delta(\omega - \omega_z) + \delta(\omega + \omega_z)]}{2} \quad (16.13)$$

In the special case that

$$k_p = k_z \quad (16.14)$$

the Fourier Transform of the system function (the left-hand side of Eq. (16.13)) is given by

$$H = 4\pi \quad (16.15)$$

Thus, the Fourier Transform of the system function band-passes the Fourier Transform of the time dependent array function. Both the space-time aperture function, Eq. (16.11) and its Fourier Transform, Eq. (16.13), are a periodic array of delta functions. No frequencies of the Fourier Transform of the space-time aperture function are attenuated; thus, no energy is lost in this special case where Eq. (16.14) holds. (This result is also central to a powerful new medical imaging technology- Resonant Magnetic Susceptibility Imaging (ReMSI) [2-3]). No energy loss corresponds to a superconducting state. And the relationship between k-space and real space is

$$\begin{aligned} k_x &= \frac{2\pi}{\lambda_x} = \frac{1}{x} \\ k_y &= \frac{2\pi}{\lambda_y} = \frac{1}{y} \\ k_z &= \frac{2\pi}{\lambda_z} = \frac{1}{z} \end{aligned} \quad (16.16)$$

From Eqs. (16.14) and (16.16), it follows that a cubic array ($x_0 = y_0 = z_0$) of magnetic dipoles centered on the nuclei of the lattice is a superconductor when the temperature is less than the critical temperature such that the superconducting electrons can propagate. Propagating electrons which carry the superconducting current and comprise magnetic dipoles form standing waves centered on the nuclear centers of the cubic lattice. Fermi-Dirac statistic apply to electrons as given in the Statistical Mechanics Section. It follows from Eqs. (16.14) and (16.16) that the Fermi energy is calculated for a cubical cavity L on a side. The number standing waves in a cubical cavity L on a side is given by Eq. (9.33) of Beiser [4]

$$g(j) dj = \pi j^2 dj \quad (16.17)$$

where

$$j = \frac{2L}{\lambda} \quad (16.18)$$

The de Broglie wavelength of an electron is

$$\lambda = \frac{h}{p} \quad (16.19)$$

Electrons in superconductors have non relativistic velocities; so,

$$p = \sqrt{2m_e \varepsilon} \quad (16.20)$$

and

$$j = \frac{2L}{\lambda} = \frac{2Lp}{h} = \frac{2L\sqrt{2m_e \varepsilon}}{h} \quad (16.21)$$

$$dj = \frac{L}{h} \sqrt{\frac{2m_e}{\varepsilon}} d\varepsilon \quad (16.22)$$

Using these expressions for j and dj in Eq. (16.17) gives

$$g(\varepsilon)d\varepsilon = \frac{8\sqrt{2}\pi L^3 m_e^{3/2}}{h^3} \sqrt{\varepsilon} d\varepsilon \quad (16.23)$$

Substitution of V for L^3 gives the number of electron states, $g(\varepsilon)$

$$g(\varepsilon)d\varepsilon = \frac{8\sqrt{2}\pi V m_e^{3/2}}{h^3} \sqrt{\varepsilon} d\varepsilon \quad (16.24)$$

The Fermi energy, E_F , is calculated by equating the number of free electrons, N , to the integral over the electron states of energy ε from zero to the highest energy, the Fermi energy, $E = E_F$.

$$N = \int_0^{E_F} g(\varepsilon)d\varepsilon = \frac{8\sqrt{2}\pi V m_e^{3/2}}{h^3} \int_0^{E_F} \sqrt{\varepsilon} d\varepsilon \quad (16.25)$$

$$= \frac{16\sqrt{2}\pi V m_e^{3/2}}{3h^3} E_F^{3/2} \quad (16.26)$$

and the Fermi energy is

$$E_F = \frac{h^2}{2m_e} \left(\frac{3N}{8\pi V} \right)^{2/3} = \frac{h^2}{2m_e} \left(\frac{3}{8\pi} \right)^{2/3} n^{2/3} \quad (16.27)$$

The quantity $N/V = n$ is the density of free electrons.

In the case of superconducting electrons, comprising an array of magnetic dipoles (each dipole in the xy -plane and oriented along the z -axis), the dimensions of Eq. (9.33) of Beiser [4] is reduced to 2 from 3.

$$2 \frac{1}{4} 2\pi j = g(j) \quad (16.28)$$

For $g(j) = 1$ with the substitution of Eq. (16.18),

$$2\pi L = \lambda \quad (16.29)$$

As the temperature of a superconducting material rises from a temperature below the critical temperature, T_c , the number density, n_s , of superconducting electrons decreases. At the transition temperature, the superconducting electrons condense into a nondissipative electron current ensemble which obeys the statistics of a Bose gas (each electron is identical and indistinguishable as indicated in Eq. (16.8) with the constraint of Eq. (16.14)), and Eqs. (16.28) and (16.29) apply

$$\frac{2\pi}{\lambda} = \frac{1}{L} = n_s \quad (16.30)$$

where

$$n_s E_F = n k_B T_c \quad (16.31)$$

n_s is the number density of superconducting electrons within $k_B T_c$ of the Fermi energy and n is the number density of free electrons. The current carried by each superconducting electron corresponds to a translational

or kinetic energy. The relationship between the electron de Broglie wavelength (Eqs. (16.19) and (16.20)) and the average electron energy, $\bar{\epsilon}$, per degree of freedom, f , given by Beiser [5]

$$\bar{\epsilon} = \frac{f}{2} k_B T_c = \quad f = 3, 2, \text{ or } 1 \quad (16.32)$$

is

$$\lambda = \frac{h}{2m_e \frac{1}{2} f k_B T_c}^{\frac{1}{2}} = \frac{h}{(m_e f k_B T_c)^{\frac{1}{2}}} \quad (16.33)$$

where in the present case of an inverse squared central field, the binding energy or energy gap, Δ , of the superconducting state is one half the negative of the potential energy and equal to the kinetic energy [6]. Consider the case wherein the Fermi energy is that of a three dimensional system, but the motion of superconducting electrons is restricted to 3, 2, or 1 directions corresponding to $f = 3, 2, \text{ or } 1$, respectively. Combining Eqs. (16.30-16.33) gives the transition temperature

$$T_c = \frac{8}{(2\pi)^6} \frac{8\pi^2}{3} \frac{E_F}{k_B f^3} \quad (16.34)$$

where the Fermi energy, E_F , is given by Eq. (16.27). An isotope effect can be manifest indirectly by changing the rms. position of atoms which effects the condition of Eq. (16.14) or the Fermi energy by changing the bond and vibrational energies.

CRITICAL TEMPERATURE, T_c

T_c for Conventional Three Dimensional Metallic Superconductors

In the case of conventional three dimensional metallic superconductors, the number density of conduction electrons is comparable to the number density of atoms-approximately $10^{29} / m^3$. Thus, the calculated transition temperature (Eq. (16.34)) is

$$T_c = 30.8 K$$

As a comparison, the material of this class with the highest known transition of 23.2 K is Nb₃Ge [7].

T_c for One, Two, or Three Dimensional Ceramic Oxide Superconductors

In the case of ceramic oxide superconductors, one, two, and three dimensional conduction mechanisms are possible. The number density of conduction electrons is less than that of metallic superconductors-

approximately $10^{28} / m^3$.

For the three dimensional case, the calculated transition temperature (Eq. (16.34)) is

$$T_c = 7 K$$

As a comparison, a possible material of this class, Li_2TiO_3 has a transition temperature of $13.7 K$ [8].

For the two dimensional case,

$$T_c = 22 K$$

As a comparison, a possible material of this class, the original Bednorz and Muller $Ba - La - Cu - O$ material has a transition temperature of $35 K$ [7].

For the one dimensional case,

$$T_c = 180 K$$

As a comparison, a possible material of this class, $Tl - Ca - Ba - Cu - O$ has a transition temperature of $120 - 125 K$ [9]. The existence of superconductivity confined to stripes has been observed experimentally by neutron scattering [10].

Transition temperatures which are intermediate of each of these limiting cases are possible where combinations of conduction mechanism are present.

JOSEPHSON JUNCTION, WEAK LINK

As shown in the Electron g Factor Section, the electron links flux in units of the magnetic flux quantum. Thus, the magnetic flux that links a superconducting loop with a weak link is the magnetic flux quantum, ϕ_0 .

$$\phi_0 = \frac{h}{2e} \quad (16.35)$$

The factor of $2e$ in the denominator has been erroneously interpreted [11] as evidence that Cooper pairs are the superconducting current carriers which is central to the BCS theory of superconductors. This theory fails to explain so called High Temperature Superconductors. These materials have a transition temperature which corresponds to an internal electron energy that is well above the energy limits at which the BCS theory permits conduction electron pairing. According to the present theory, Cooper pairs do not exist, and the present theory is consistent with the existence of High Temperature Superconductors as well as the experimental result that the magnetic flux that links a superconducting loop with a weak link is the magnetic flux quantum, ϕ_0 .

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QUANTUM HALL EFFECT

GENERAL CONSIDERATIONS

When confined to two dimensions and subjected to a magnetic field, electrons exhibit a range of extraordinary behavior, most notably the Quantum Hall Effect (QHE). Two distinct versions of this phenomenon are observed, the Integral Quantum Hall Effect (IQHE) and the Fractional Quantum Hall Effect (FQHE). The former involves the condition for re-establishment of a superconducting state of one well in the presence of a magnetic field; whereas, the latter involves the condition for re-establishment of a superconducting state of two magnetically linked wells in the presence of a magnetic field.

Consider a conductor in a uniform magnetic field and assume that it carries a current driven by an electric field perpendicular to the magnetic field. The current in this case is not parallel to the electric field, but is deflected at an angle to it by the magnetic field. This is the Hall Effect, and it occurs in most conductors.

In the Quantum Hall Effect, the applied magnetic field quantizes the Hall conductance. The current is then precisely perpendicular to the magnetic field, so that no dissipation (that is no ohmic loss) occurs. This is seen in two-dimensional systems, at cryogenic temperatures, in quite high magnetic fields. Furthermore, the ratio of the total electric potential drop to the total current, the Hall resistance, R_H , is precisely equal to

$$R_H = \frac{h}{ne^2} \quad (17.1)$$

The factor n is an integer in the case of the Integral Quantum Hall Effect, and n is a small rational fraction in the case of the Fractional Quantum Hall Effect. In an experimental plot [1] as the function of the magnetic field, the Hall resistance exhibits flat steps precisely at these quantized resistance values; whereas, the regular resistance vanishes (or is very small) at these Hall steps. Thus, the quantized Hall resistance steps occur for a transverse superconducting state.

As shown in the Superconductivity Section, superconductivity arises for an array of current carrying magnetic dipoles when

$$k_p = k_z \quad (17.2)$$

Thus, the Fourier Transform of the system function band-passes the Fourier Transform of the time dependent array function. Both the space-time aperture function and its Fourier Transform are a periodic array of delta functions. No frequencies of the Fourier Transform of the space-time aperture function are attenuated; thus, no energy is lost in this special case where Eq. (17.2) holds. Consider the case that an external magnetic field is applied along the x-axis to a two dimensional

superconductor in the yz-plane which exhibits the Integral Quantum Hall Effect. (See Figure 17.1.) The magnetic field is expelled from the bulk of the superconductor by the supercurrent (Meissner Effect). The supercurrent-density function is a minimum energy surface; thus, the magnetic flux decays exponentially at the surface as given by the London Equation [2]. The Meissner current increases as a function of the applied flux. The energy of the superconducting electrons increases with flux. This energy increase is equivalent to lowering the critical temperature in Eq. (16.31) of the Superconductivity Section which is given by

$$n_s E_F = nkT_c \quad (17.3)$$

where n_s is the number density of superconducting electrons within kT_c of the Fermi energy and n is the number density of free electrons. At the critical current, the material loses superconductivity and becomes normal at a temperature below that of the critical temperature in the absence of an applied field. Conduction electrons align with the applied field in the x direction as the field permeates the material. The normal current carrying electrons experience a Lorentzian force, F_L , due to the magnetic flux. The y directed Lorentzian force on an electron having a velocity v in the z direction by an x directed applied flux, B , is

$$F_L = ev \times B \quad (17.4)$$

The electron motion is a cycloid where the center of mass experiences an $E \times B$ drift [3]. Consequently, the normal Hall Effect occurs. Conduction electron energy states are altered by the applied field and by the electric field corresponding to the Hall Effect. The electric force, F_H , due to the Hall electric field, E_y , is

$$F_H = eE_y \quad (17.5)$$

When these two forces are equal and opposite, conduction electrons propagate in the z direction alone. For this special case, it is demonstrated in Jackson [3] that the ratio of the corresponding Hall electric field and the applied magnetic flux is

$$E/B = v \quad (17.6)$$

where v is the electron velocity. At a temperature below T_c , given by Eq. (17.3) where E_F is the Fermi energy, Eq. (17.6) is satisfied. The further conditions for superconductivity are

$$n\omega_p = \omega_z \quad (17.7)$$

$$nk_p = k_z \quad (17.8)$$

And, it is demonstrated in the Integral Quantum Hall Effect Section that the Hall resistance, R_H , in the superconducting state is given by

$$R_H = \frac{h}{ne^2} \quad (17.9)$$

where n of Eqs. (17.7), (17.8), and (17.9) is the same integer for the case of a single superconducting well. It is demonstrated in the Fractional Quantum Hall Effect Section that electrons in different superconducting wells can interact when the two wells are separated by a distance comparable to the magnetic length, ℓ_0 .

$$\ell_0 = \frac{\hbar c}{eB}^{1/2} \quad (17.10)$$

In this case, it is further demonstrated that the Hall resistance, R_H , in the superconductivity state is given by Eq. (17.9) where n is a fraction.

INTEGRAL QUANTUM HALL EFFECT

A superconducting current-density function is nonradiative and does not dissipate energy as was the case for single electron current-density functions described previously in the One Electron Atom Section, the Two Electron Atom Section, the Three Electron Atom Section, the Electron in Free Space Section, and the Nature of the Chemical Bond Section. Furthermore, a superconducting current-density function is the superposition of single electron current-density functions-which are spatially two dimensional in nature. Thus, a superconducting current-density function is an electric and magnetic equipotential energy surface.

From Eq. (1.55), the angular frequency in spherical coordinates which satisfies the boundary condition for nonradiation is

$$\omega = \frac{\hbar}{m_e r^2} = \frac{(2r)^2 \hbar}{m_e (2r)^2} \quad (17.11)$$

The relationship between the electron wavelength and the radius which satisfies the nonradiative boundary condition in spherical coordinates is given by Eq. (1.43)

$$2r = \lambda \quad (17.12)$$

Substitution of Eq. (17.12) into Eq. (17.11) gives

$$\omega = \frac{\hbar}{m_e} k^2 \quad (17.13)$$

where

$$k = \frac{2}{\lambda} \quad (17.14)$$

It follows from Eq. (1.47) that

$$v = \frac{\hbar}{m_e r} = \frac{\hbar}{m_e} k \quad (17.15)$$

In a solid lattice, the coordinates are Cartesian rather than spherical. The relationship between the wavelength of a standing wave of a superconducting electron and the length, x , of a cubical unit cell follows from Eqs. (16.28) and (16.29) of the Superconductivity Section

$$\lambda = 2\pi x \quad (17.16)$$

The de Broglie wavelength, λ , is given by

$$\lambda = \frac{h}{m_e v} \quad (17.17)$$

It follows from Eqs. (17.14), (17.16), and (17.17) that the angular velocity, ω , and linear velocity, v , for an electron held in force balance by a periodic array of nuclei comprising a cubical unit cell with internuclear spacing x are given by Eqs. (17.13) and (17.15) where

$$k = \frac{2\pi}{\lambda} = \frac{1}{x} \quad (17.18)$$

In general, the Cartesian coordinate wavenumber, k , given by Eq. (17.18) replaces $\frac{1}{r}$ of spherical coordinates.

In the case of an exact balance between the Lorentzian force (Eq. (17.4)) and the electric force corresponding to the Hall voltage (Eq. (17.5)), each superconducting electron propagates along the z-axis where

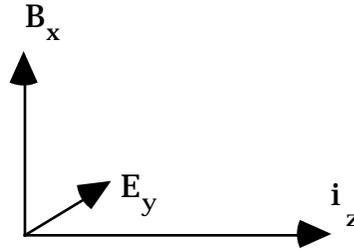
$$E/B = v \quad (17.19)$$

where v is given by Eq. (17.15). Substitution of Eqs. (17.15) and (17.18) into Eq. (17.19) gives

$$E/B = \frac{\hbar}{m_e} k = \frac{\hbar}{m_e x} \quad (17.20)$$

Eq. (17.20) is the condition for superconductivity in the presence of crossed electric and magnetic fields. The Hall resistance for this superconducting state is derived as follows using the coordinate system shown in Figure 17.1.

Figure 17.1. Coordinate system of crossed electric field, E_y , corresponding to the Hall voltage, magnetic flux, B_x , due to applied field, and superconducting current i_z .



The current is perpendicular to E_y , thus there is no dissipation.

This occurs when

$$e\mathbf{E} = e\mathbf{v} \times \mathbf{B} \quad (17.21)$$

or

$$E/B = v \quad (17.22)$$

The magnetic flux, B , is quantized in terms of the Bohr magneton because an electron, and therefore a superconductor, links flux in units of the magnetic flux quantum,

$$\phi_0 = \frac{h}{2e} \quad (17.23)$$

The electric field, E_y , corresponding to the Hall voltage, V_H , is quantized in units of e because this electric field arises from conduction electrons—each of charge e . The energy, E_H , corresponding to the Hall voltage is calculated using the Poynting Power Theorem. The Hall energy of an integer number of electrons, Z , each in the presence of a magnetic dipole and an electric field of magnitude Ze due to the Z electrons follows for Eqs. (7.30) and (7.47) of the Two Electron Atom Section

$$E_H = ZE_{mag} = \frac{Z \mu_0 e^2 \hbar^2 k^3}{Zm_e^2} \quad (17.24)$$

where k is given by Eq. (17.13) and where the electric energy of Eq. (7.47) is zero because each electron is a conduction electron. In the limit to a superconducting state, the trajectory of each electron is a cycloid where ω_p is the frequency in the xy -plane and ω_z is the frequency along the z -axis. In this case, the dipole array function given in the Superconductivity Section is multiplied by a time harmonic function with argument ω_p

$$\frac{(2z^2 - x^2 - y^2)}{[x^2 + y^2 + z^2]^{5/2}} \mu_B \delta(x - nx_0, y - ny_0, z - nz_0) \exp(-i\omega t) \quad (17.25)$$

where

$$\omega = \omega_p + \omega_z \quad (17.26)$$

The Fourier Transform of the convolved functions of Eq. (17.25) is given in the Superconductivity Section as

$$\frac{4\pi}{1 + \frac{k_z^2}{k_p^2}} \frac{1}{x_0 y_0 z_0} \mu_B \delta\left(k_x - \frac{n}{x_0}, k_y - \frac{n}{y_0}, k_z - \frac{n}{z_0}\right) \quad (17.27)$$

The Fourier Transform of the time harmonic function is

$$\frac{[\delta(\omega - (\omega_p + \omega_z)) + \delta(\omega + (\omega_p + \omega_z))]}{2} \quad (17.28)$$

A very important theorem of Fourier analysis states that the Fourier Transform of a product is the convolution of the individual Fourier Transforms. Thus, the Fourier Transform of Eq. (17.25) is the convolution of Eqs. (17.27) and (17.28)

$$\frac{4\pi}{1 + \frac{k_z^2}{k_p^2}} \frac{1}{x_0 y_0 z_0} \mu_B \delta\left(k_x - \frac{n}{x_0}, k_y - \frac{n}{y_0}, k_z - \frac{n}{z_0}\right) \frac{[\delta(\omega - (\omega_p + \omega_z)) + \delta(\omega + (\omega_p + \omega_z))]}{2} \quad (17.29)$$

Eq. (17.29) is a band-pass when

$$nk_p = k_z \quad (17.30)$$

and when

$$\frac{\omega_z}{\omega_p} = n \quad (17.31)$$

where n is an integer. The cyclotron frequency, ω_p , is derived as follows:

The force balance between the Lorentzian force and the centrifugal force is

$$\frac{m_e v^2}{r} = e \mathbf{v} \times \mathbf{B} \quad (17.32)$$

The magnetic flux, \mathbf{B} , from a magnetic moment of a Bohr magneton is

$$B = \frac{\mu_0 e \hbar}{2m_e} k^3 \quad (17.33)$$

Cancellation of \mathbf{v} on both sides of Eq. (17.32) gives

$$m_e \omega = e \times \mathbf{B} \quad (17.34)$$

$$\omega_p = \frac{eB}{m_e} \quad (17.35)$$

Substitution of Eq. (17.33) into Eq. (17.35) gives

$$\omega_\rho = \frac{\mu_0 e^2 \hbar}{2m_e^2} k^3 \quad (17.36)$$

Substitution of Eq. (17.31) into Eq. (17.36) gives

$$\omega_z = \frac{n\mu_0 e^2 \hbar}{2m_e^2} k^3 \quad (17.37)$$

The current, i_z , along the z-axis is given as the product of the charge, e , and ω_z , the frequency along the z-axis

$$i_z = e\omega_z = \frac{n\mu_0 e^3 \hbar}{2m_e^2} k^3 \quad (17.38)$$

The Hall voltage is given as the energy per coulomb:

$$V_H = \frac{E_{mag}}{e} = \frac{\mu_0 e \hbar^2 k^3}{m_e^2} \quad (17.39)$$

Thus, the Hall resistance, R_H , is given as the ratio of the Hall voltage (Eq. (17.39)) and the current, i_z , (Eq. (17.38))

$$R_H = \frac{V_H}{i_z} = \frac{\frac{\mu_0 e \hbar^2 k^3}{m_e^2}}{\frac{n\mu_0 e^3 \hbar k^3}{2m_e^2}} = \frac{h}{ne^2} \quad (17.40)$$

The velocity of each superconducting electron according to Eq. (17.22)

is
$$\frac{E}{B} = v \quad (17.41)$$

which is derived as follows:

The Hall electric field, E_y , is given by the ratio of the Hall voltage and the distance of the cyclotron orbit, $2\pi x$, where the unit cell distance, x , wavenumber, k , by Eq. (17.16)

$$E_y = V_H \frac{k}{2\pi}$$

where V_H is given by Eq. (17.39)

$$E_y = \frac{\mu_0 e \hbar^2 k^4}{2\pi m_e^2} \quad (17.42)$$

The magnetic field, B , is given by Eq. (17.33); thus the velocity v is given as

$$v = \frac{\frac{\mu_0 e \hbar^2 k^4}{2\pi m_e^2}}{\frac{\mu_0 e \hbar k^3}{2m_e}} \quad (17.43)$$

$$= \frac{\hbar}{m_e} k \quad (17.44)$$

Eq. (17.44) is equivalent to the velocity for nonradiation given by Eq. (1.47), where

$$\frac{2}{2r} = \frac{2}{\lambda} = k \quad (17.45)$$

This superconducting phenomenon whereby the Hall resistance occurs as inverse integer multiples of

$$\frac{h}{e^2} \quad (17.46)$$

is the Integral Quantum Hall Effect (IQHE)

FRACTIONAL QUANTUM HALL EFFECT

For two superconducting wells separated by the magnetic length,

ℓ_0 ,

$$\ell_0 = \frac{\hbar c}{eB}^{1/2} = \frac{c}{B} \phi_0^{1/2} \quad (17.47)$$

where ϕ_0 given by Eq. (17.23) is the magnetic flux quantum, the wells are linked. Electrons can propagate from one well to the other with activation energy

$$\left| \frac{E_{mag}}{E_{mag}} \right| \frac{e^2}{\ell_0} \quad (17.48)$$

In the case that a magnetic field is applied to both well one and well two, and that an exact balance between the Lorentzian force (Eq. (17.4)) and the electric force corresponding to the Hall voltage (Eq. (17.5)) exists, each superconducting electron propagates along the z-axis where

$$\frac{E_1}{B_1} = v_1 \quad (17.49)$$

$$\frac{E_2}{B_2} = v_2 \quad (17.50)$$

Because the two wells are linked

$$v_1 = jv_2 \quad (17.51)$$

where j is an integer. Eq. (17.51) provides that the electrons are in phase with

$$\frac{2}{\lambda_1} = k_1 = j k_2 = j \frac{2}{\lambda_2} \quad (17.52)$$

where the de Broglie wavelength is given by Eq. (17.19). Otherwise, $E_z = 0$, and the state is not superconducting. It follows from the derivation of Eq. (17.40) of the Integral Quantum Hall Effect Section that

$$\frac{E_1}{n_1 B_{01}} = v_1 \quad (17.53)$$

And,

$$\frac{E_2}{n_2 B_{01}} = v_2 \quad (17.54)$$

where n_1 and n_2 are integers. From Eqs. (17.52), (17.53), and (17.54)

$$E_1 = j \frac{n_1}{n_2} \frac{E_2}{B_{01}} \quad (17.55)$$

The resistance of each well is proportional to the transverse velocity as shown previously, and the resistance across both linked wells which are in series is the sum of the individual resistances. Thus, the total resistance is proportional to the sum of the individual velocities.

$$R = \frac{E_1}{n_1 B_{01}} + \frac{E_2}{n_2 B_{01}} \quad (17.56)$$

Substitution of Eq. (17.55) into Eq. (17.56) gives

$$R = \frac{E_2}{B_{01}} \frac{1}{n_2} (j + 1) \quad (17.57)$$

It follows from the derivation of Eq. (17.40) of the Integral Quantum Hall Effect Section that Hall resistance, R_H , is

$$R_H = \frac{V_H}{i_z} = \frac{(j+1) \frac{\mu_0 e \hbar^2 k^3}{m_e^2}}{\frac{n_2 \mu_0 e^3 \hbar k^3}{2m_e^2}} = \frac{h}{ne^2} \quad (17.58)$$

where n is a fraction. This superconducting phenomenon whereby the Hall resistance occurs as inverse fractional multiples of

$$\frac{h}{e^2} \quad (17.59)$$

is the Fractional Quantum Hall Effect (FQHE)

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AHARONOV-BOHM EFFECT

The resistance of a circuit corresponds to the decrease in the energy of the current carrying electrons as they propagate through the circuit. Scattering of the electrons is a principal mechanism. In the case where a magnetic field is applied such that the field lines are perpendicular to the plane of a current carrying ring, the current carrying electrons lose energy through the effect of the field on the current.

The application of the magnetic field to the current carrying ring initially gives rise to a changing flux through the ring. The changing flux gives rise to an electric field which reduces the current in the ring; thus, the magnetic field contributes a term called magnetoresistance to the resistance of the ring. This term can be derived from the change in velocity (assuming no scattering) of a current carrying electron of charge e and mass m_e by the application of a magnetic field of strength B which is given as Eq. (29) of Purcell [1]

$$\frac{v}{r} = \frac{eB}{2m_e} \quad (18.1)$$

where r is the radius of the ring. The changes in the force on the electron due to the electric field is

$$F = e E \quad (18.2)$$

The change in kinetic energy of the electron over length s is

$$\frac{1}{2} m_e v^2 = F s = e E s = e V \quad (18.3)$$

where V is the change in voltage over the distance s . From Eq. (18.3), the voltage change is

$$V = \frac{m_e v^2}{2e} \quad (18.4)$$

The change in current, i , per electron due to the change in velocity, v , is given by Eq. (20) of Purcell [1].

$$i = \frac{e v}{2\pi r} \quad (18.5)$$

And, the total change in current, Δi , is

$$\Delta i = NWt \frac{e v}{2\pi r} \quad (18.6)$$

where N is the density of current carrying electrons in the current ring cross section, W is the width of the current ring, and t is the thickness of the ring.

The resistance change, R , follows from Eqs. (18.4) and (18.6)

$$R = \frac{V}{i} = \frac{2\pi r m_e v^2}{NWt 2e v} = \frac{\pi r m_e v}{NWt e^2} \quad (18.7)$$

Substitution of v given by Eq. (18.1) into Eq. (18.7) gives the change in

resistance corresponding to the magnetoresistance.

$$R = \frac{\pi r^2 B}{NWt2e} \quad (18.8)$$

An additional critically damped, over damped, or underdamped oscillatory resistive term may arise due to both the magnetoresistance and the vector potential of the electron. The electron possesses an angular momentum of \hbar . As shown in the Electron g Factor Section, the electron angular momentum comprises kinetic and vector potential components. Angular momentum is conserved in the presence of an applied magnetic field when the electron links flux in units of the magnetic flux quantum,

$$\phi_0 = \frac{h}{2e} \quad (18.9)$$

This occurs when the electron rotates by $\frac{\pi}{2}$ radians about an axis perpendicular to the axis parallel to the magnetic flux lines. This electron rotation corresponds to an $\frac{\hbar}{2}$ magnitude, 180° , rotation of the electron's angular momentum vector. In the case that the electrons carry current, this change in momentum of a given current carrying electron increases or decreases the current depending on the vector projection of the momentum change onto the direction of the current. Recently, it has been demonstrated that 50-nm-diameter rings of *InAs* on a *GaAs* surface have can host a single circulating electron in a pure quantum state, that is easily controlled by magnetic fields and voltages on nearby plates. The electrons were observed to link flux in the unit of the magnetic flux quantum with a gain in a unit of angular momentum in a specific direction with the linkage [2].

At low temperature, the de Broglie wavelength of an electron

$$\lambda = \frac{h}{m_e v} \quad (18.10)$$

has macroscopic dimensions, and the electron scattering length for a given electron in a current carrying ring may be comparable to the dimensions of the ring. A current carrying ring having a magnetic field applied perpendicularly to the plane of the ring may be constructed and operated at a temperature, current, and applied magnetic field strength such that resonance occurs between the vector potential of a current carrying electron and the flux of the applied magnetic field. This coupling can give rise to a contribution to the resistance which behaves as an underdamped harmonic oscillator in response to the applied magnetic flux. The general form of the equation for this component of the resistance is the product of an exponential dampening function and a harmonic function as given by Fowles [3]. Each electron links flux only

in units of the magnetic flux quantum, ϕ_0 , given by Eq. (18.9). Thus, the natural frequency in terms of the applied flux, ϕ , is the magnetic flux quantum, ϕ_0 . According to Eq. (18.8), the magnetoresistance is proportional to the applied flux where

$$R = \pi r^2 B \quad (18.11)$$

Thus, the argument of the dampening function is proportional to $\frac{\phi}{\phi_0}$.

Furthermore, the magnetoresistance gives rise to a distribution of electron velocity changes centered about the average velocity change given by Eq. (18.1) where each electron's current contributing drift velocity along the ring contributes a component to the kinetic term of the electron's angular momentum. The distribution of velocity changes, dampens the coupling between each electron vector potential and the applied magnetic flux at the natural frequency corresponding to the average electron velocity. And, each electron de Broglie wavelength change corresponding to its velocity change alters the electron-lattice scattering cross section which also contributes to the dampening of the oscillatory resistance behavior. The argument of the dampening

function is the product of $\frac{\phi}{\phi_0}$ and the corresponding dimensionless

damping factor, α_D , which incorporates both dampening effects. The underdamped oscillatory resistance change due to the applied magnetic field is

$$R = \frac{\pi r^2 B}{NWt2e} e^{-\alpha_D \frac{\phi}{\phi_0}} \cos 2\pi \frac{\phi}{\phi_0} \quad (18.12)$$

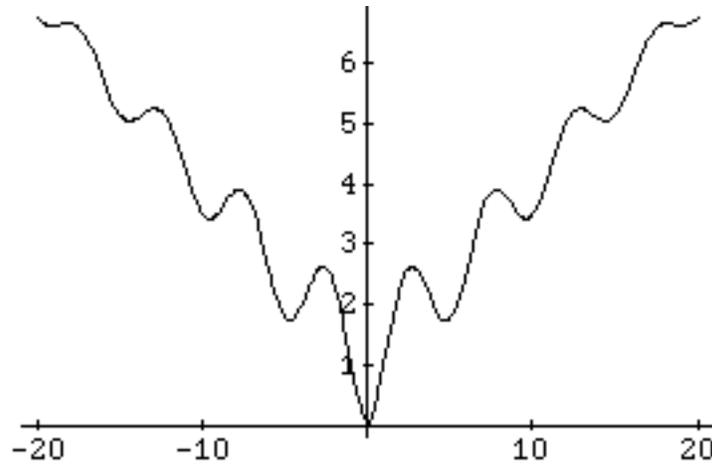
The total resistance change due to the applied field is the sum of the magnetoresistance and the underdamped oscillatory resistance

$$R = \frac{\pi r^2 B}{NWt2e} \left[1 + e^{-\alpha_D \frac{\phi}{\phi_0}} \cos 2\pi \frac{\phi}{\phi_0} \right] \quad (18.13)$$

This type of contribution to the resistance that is an oscillatory function of the applied flux with a period of $\phi_0 = \frac{h}{2e}$ is known as the

Aharonov-Bohm Effect. The resistance contribution given by Eq. (18.13) is consistent with the observed behavior [4] as shown in Figure 18.1.

Figure 18.1. The change in the resistance divided by the resistance as a function of the applied flux which demonstrates the Aharonov-Bohm effect.



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CREATION OF MATTER FROM ENERGY

[The general result of equations and relationships derived in the Pair Production and Gravity Sections are given herein.]

Matter and energy are interconvertible and are in essence different *states* of the same entity. The state, matter or energy, is determined by the laws of nature and the properties of spacetime. A photon propagates according to Maxwell's Equations at the speed of light in spacetime having intrinsic impedance η . Matter as a fundamental particle is created in spacetime from a photon. Matter obeys the laws of Special Relativity, the relationship of motion to spacetime, and spacetime is curved by matter according to the laws of General Relativity. Relationships must exist between these laws and the implicit fundamental constants. The fundamental elements which determine the evolution of the universe are the fundamental constants of spacetime, ϵ_0 and μ_0 with the property of charge; the capacity of spacetime to be curved by mass/energy; and the photon's angular momentum of \hbar . The conversion of energy into matter requires a transition state for which the identification of the entity as matter or energy is impossible. From the properties of the entity, as matter or energy, and from the physical laws and the properties of spacetime, the transition state hereafter called a transition state orbitsphere are derived. Concomitantly, the equations for the interconversion of matter and energy are determined, and the fundamental constant relationships are determined exactly. The results are: matter and energy possess mass; matter possesses charge, and energy is stored in the electric and magnetic fields of matter as a consequence of its charge and the motion of its charge. Matter can trap photons as an absorption event. The mass of the matter possessing a "trapped photon" increases by the mass/energy of the photon, and the photon acts as if it possesses charge. (The electric fields of "trapped photons" is given in the Excited States of the One Electron Atom (Quantization) Section). Photons obey Maxwell's Equations. At the two dimensional surface of the orbitsphere containing a "trapped photon", the relationship between the photon's electric field and its charge at the orbitsphere (See Eq. 2.10) is

$$\mathbf{n} \cdot (\mathbf{E}_1 - \mathbf{E}_2) = \frac{\sigma}{\epsilon_0} \quad (19.1)$$

Thus, the photon's electric field acts as surface charge. This property of a photon is essential because *charge arises from electromagnetic radiation in the creation of matter*. Furthermore, energy is proportional to the mass of matter as given by

$$E = mc^2 \quad (19.2)$$

And, energy is proportional to frequency as given by Planck's equation,

$$E = \hbar\omega \quad (19.3)$$

It is shown in the Gravity Section (Eq. (23.29)) that the de Broglie relationship can be derived from Planck's equation.

$$\lambda = \frac{h}{mv} \quad (19.4)$$

Matter and light obey the wave equation relationship

$$v = \lambda \frac{\omega}{2\pi} \quad (19.5)$$

and Eqs. (19.2) through (19.4). Light and matter exist as orbitspheres, as given in the Photon Equation Section and the One Electron Atom Section, respectively.

The boundary condition for nonradiation by a *transition state orbitsphere* is

$$2\pi(r_n^*) = 2\pi(nr_1^*) = n\lambda_1^* = \lambda_n^* \quad (19.6)$$

where r^* and λ^* are allowed radii and allowed wavelengths for the transition state matter in question, and n is a positive real number. A general relationship derived for the electron in the Pair Production Section is that when $r = \alpha a_o$, v of Eq. (19.5) of a transition state orbitsphere equals the velocity of light in the inertial reference frame of the photon of angular frequency ω^* and energy $\hbar\omega^* = m_e c^2$ which forms the transition state orbitsphere of rest mass m_e . Substitution of Eq. (19.4) into Eq. (19.6) with $v = c$ and $r^* = \alpha a_o$ gives the result that the radius of the transition state orbitsphere is the Compton wavelength bar, $\tilde{\lambda}_c$, which gives the general condition for particle production.

$$r_{\alpha}^* = \alpha a_o = \tilde{\lambda}_c = \frac{\hbar}{m_0 c} \quad (19.7)$$

With the substitution of Eq. (19.7) and the appropriate special relativistic corrections into the orbitsphere energy equations, the following energies, written in general form, are equal

$$E = \hbar\omega^* = m_0 c^2 = V \quad (19.8)$$

where V is the potential energy. In the case of an electron orbitsphere, the rest mass $m_0 = m_e$, the radius $r_{\alpha}^* = \alpha a_o$, and the electron and positron each experience an effective charge of $\alpha^{-1}e$.

$$V = \frac{\alpha^{-1}e^2}{4\pi\epsilon_o\alpha a_o} \quad (19.9)$$

This energy and mass are that of the transition state orbitsphere which can be considered to be created from the photon of frequency ω^* . Furthermore, the relativistic factor, γ ,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (19.10)$$

for the lab frame relative to the photon frame of the transition state orbitsphere of radius αa_o is 2π where Eq. (19.10) is transformed from Cartesian coordinates to spherical coordinates¹. (For example, the relativistic mass of the electron transition state orbitsphere of radius αa_o is $2\pi m_e$.) Using the relativistic mass, the Lorentzian invariance of charge, and the radius of the transition state orbitsphere as αa_o , it is demonstrated in the Pair Production Section that the electrical potential energy is equal to the energy stored in the magnetic field which gives the following equalities of energies written in general form

$$E = V = E_{mag} = \hbar\omega^* = m_0c^2 \quad (19.11)$$

The energy stored in the electric and magnetic fields of any photon are equal, and equivalence of these energies occurs for an LC circuit excited at its resonance frequency

$$\omega^* = \frac{1}{\sqrt{LC}} \quad (19.12)$$

where L is the inductance and C is the capacitance of the circuit. Spacetime is an LC circuit with resonance frequency

$$\omega^* = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\epsilon_o\mu_o d^2}} = \frac{1}{\sqrt{\epsilon_o\mu_o \lambda_c^2}} \quad (19.13)$$

where d is the circuit dimensions. (This equation is derived in the Pair Production Section.) For $d = \alpha a_o$, this frequency is equivalent to that of a photon of energy $m_e c^2$. When the resonance frequency of an LC circuit is excited, the impedance becomes infinite. Thus, spacetime is excited at its resonance frequency when a photon of angular frequency ω^* forms a transition state orbitsphere of mass/energy $m_e c^2$. At this event, the

¹ For time harmonic motion, with angular velocity, ω , the relationship between the radius and the wavelength given Eq. (1.43) by is

$$2\pi r_n = \lambda_n$$

The de Broglie wave length is given by Eq. (1.46)

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n}$$

In the relativistically corrected case given by Eq. (1.45),

$$r_n = \lambda_n$$

Then from Eq. (1.46),

$$r_n = \lambda_n = \frac{h}{p_n} = \frac{h}{(2\pi m_e) v_n}$$

Thus, the relativistically corrected electron mass is $2\pi m_e$.

equivalence of all energies given previously provides that matter and energy are indistinguishable. (For the transition state orbitsphere, the potential energy corresponds to the stored electrical energy of an LC circuit which, in turn, corresponds to the energy stored in the electric field of a photon.) The impedance for the propagation of electromagnetic radiation becomes infinite and a photon of energy $m_e c^2$ becomes a fundamental particle as the transition state orbitsphere becomes real. The energy of the photon is equal to the rest mass of the particle at zero potential energy. Therefore, in the case of charged particle production, a particle and an antiparticle each of mass $\frac{\hbar\omega^*}{c^2}$ is produced at infinity relative to the mutual central field of

$$\mathbf{E} = \frac{+e}{4\pi\epsilon_0 r^2} \quad (19.14)$$

And, momentum is conserved by a third body such as an atomic nucleus.

The boundary condition, Eq. (1.43) and Eq. (19.6), precludes the existence of the Fourier components of the current-density function of the orbitsphere that are synchronous with waves traveling at the speed of light. The nonradiative condition is Lorentzian invariant because the velocity is perpendicular to the radius. However, the constancy of the speed of light must also hold which requires relativistic corrections to spacetime. The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity. Thus, the creation of matter causes local spacetime to become curved. The geometry of spacetime is transformed from flat (Euclidean) to curved (Riemannian). Time and distances are distorted. At particle production, the proper time of the particle must equal the coordinate time given by Special Relativity for Riemannian geometry affected by the creation of matter of mass m_0 where the metric of spacetime is given by the Schwarzschild metric. This boundary condition determines the masses of the fundamental particles. The gravitational radius, α_G or r_G , which arises from the solution of the Schwarzschild metric is defined as

$$\alpha_G = \frac{Gm_0}{c^2} = r_G \quad (19.15)$$

where G is the gravitational constant. The radius of the transition state orbitsphere is

$$r_\alpha^* = \tilde{\lambda}_c = \frac{\hbar}{m_0 c} \quad (19.16)$$

These radii are equal when the gravitational potential, E_{grav} , is

$$E_{grav} = \frac{Gm_0^2}{r_\alpha^*} = \frac{Gm_0^2}{\tilde{\lambda}_c} = \hbar\omega = V = E_{mag} \quad (19.17)$$

These relationships represent the unification of the fundamental laws of the universe, Maxwell's Equations, Newtonian Mechanics, Special and General Relativity, and the Planck equation and the de Broglie relationship where the latter two can be derived from Maxwell's Equations as demonstrated in the Gravity Section.

PAIR PRODUCTION

Matter and energy are interconvertible and are in essence different states of the same entity. The state, matter or energy, is determined by the laws of nature and the properties of spacetime. A photon propagates according to Maxwell's Equations at the speed of light in spacetime having intrinsic impedance η . Matter as a fundamental particle is created in spacetime from a photon. Matter obeys the laws of Special Relativity, the relationship of motion to spacetime, and spacetime is curved by matter according to the laws of General Relativity. Relationships must exist between these laws and the implicit fundamental constants. The conversion of energy into matter requires a transition state for which the identification of the entity as matter or energy is impossible. From the properties of the entity, as matter or energy, and from the physical laws and the properties of spacetime, the transition state hereafter called a transition state orbitsphere is derived. For example, a photon of energy 1.02 MeV in the presence of a third particle becomes a positron and an electron. This phenomenon, called pair production, involves the conservation of mass/energy, charge, and angular and linear momentum. Pair production occurs as an event in spacetime where all boundary conditions are met according to the physical laws: Maxwell's Equations, Newton's Laws, and Special and General Relativity, where matter and energy are indistinguishable by any physical property. Matter and photons exist as orbitspheres; thus, the conversion of energy to matter must involve the orbitsphere equations derived in the previous sections. It must also depend on the equations of electromagnetic radiation and the properties of spacetime because matter is created from electromagnetic radiation as an event in spacetime.

Matter and light obey the wave equation relationship

$$v = \lambda \frac{\omega}{2\pi} \quad (20.1)$$

The boundary condition for nonradiation by a transition state orbitsphere is

$$2\pi(r_n^*) = 2\pi(nr_1^*) = n\lambda_1^* = \lambda_n^* \quad (20.2)$$

where r^* and λ^* are allowed radii and allowed wavelengths for the transition state matter in question, and n is a positive real number.

Consider the production of an electron and a positron providing a mutual central field. The relationship between the potential energy of an electron orbitsphere and the angular velocity of the orbitsphere is

$$V = \hbar\omega = \frac{1}{n} \frac{e^2}{4\pi\epsilon_0 na_0} \quad (20.3)$$

It can be demonstrated that the velocity of the electron orbitsphere satisfies the relationship for the velocity of a wave by substitution of Eqs. (1.43) and (1.55) into Eq. (20.1), which gives Eq. (1.56). Similarly, the relationship between c , the velocity of light in free space, and frequency ω and wavelength λ is

$$c = \lambda \frac{\omega}{2\pi} \quad (20.4)$$

And, the energy of a photon of frequency ω is

$$E = \hbar\omega \quad (20.5)$$

Recall from the Excited States of the One Electron (Quantization) Section that a photon of discrete frequency, ω , can be trapped in the orbitsphere of an electron which serves as a resonator cavity of radius r_n where the resonance excitation energy of the cavity is given by Eq. (20.3).

As demonstrated in the Excited States of the One Electron Atom (Quantization) Section, with the inclusion of the contribution of the electron kinetic energy change, the change in the orbitsphere angular velocity is equal to the angular velocity of the resonant photon of the corresponding electron transition. The ratio of their linear velocities is given by Eq. (20.4).

$$\frac{v_n}{c_{photon}} = \frac{\lambda_n \frac{\omega_n}{2\pi}}{\lambda_{photon} \frac{\omega_{photon}}{2\pi}} = \frac{\lambda_n}{\lambda_{photon}} \quad (20.6)$$

where the n subscripts refer to orbitsphere quantities.

Consider a transition state electron orbitsphere which is defined as the transition state between light and matter where light and matter are indistinguishable, and where $v_n = v_n$ and $\lambda_n = \lambda_n$. For this case, the velocity of the electron transition state orbitsphere is the speed of light in the inertial reference frame of the photon which formed the transition state orbitsphere. The result of the substitution into Eq. (20.1) of c for v , of λ_n given by Eq. (2.2) where r_1 is given by Eq. (1.169) for λ , and of ω_n given by Eq. (1.55) for ω is

$$c = 2\pi n a_o \frac{\hbar}{m_e (n a_o)^2 2\pi} \quad (20.7)$$

Maxwell's Equations provide that

$$c = \sqrt{\frac{1}{\epsilon_o \mu_o}} \quad (20.8)$$

The result of substitution of Eqs. (1.168) and (20.8) into Eq. (20.7) is

$$n^{-1} = \frac{m_e c a_o}{\hbar} = \frac{m_e}{\hbar \sqrt{\epsilon_o \mu_o}} \frac{4\pi \epsilon_o \hbar^2}{e^2 m_e} = 4\pi \sqrt{\frac{\epsilon_o}{\mu_o}} \frac{\hbar}{e^2} = \alpha^{-1} \quad (20.9)$$

In fact, α is the fine structure constant (a dimensionless constant for pair production) [1]. The experimental value is 0.0072973506. Recently, alterations to the most up-to-date, self consistent set of the recommended values of the MKS basic constants and conversion factors of physics and chemistry resulting from the 1986 least-squares adjustment have been proposed [2]. Eq. (20.9), the equations of pair production given below, and the equations in the Unification of Spacetime, the Forces, Matter, and Energy Section and Gravity Section permit the derivation of a more accurate self consistent set.

Continuing with the present MKS units, the radius of the transition state electron orbitsphere is αa_0 , and the potential energy, V , is given by Eq. (20.3) where n is α where α arises from Gauss's law surface integral and the relativistic invariance of charge.

$$V = \frac{-\alpha^{-2}e^2}{4\pi\epsilon_0 a_0} \quad (20.10)$$

$$V = m_e c^2 \quad (20.11)$$

Furthermore, the result of the multiplication of both sides of Eq. (1.55) by \hbar , $r_n = n a_0$, and the substitution of $n = \alpha$ yields

$$\hbar\omega_\alpha^* = m_e c^2 \quad (20.12)$$

The relativistic factor,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (20.13)$$

for an orbitsphere at radius r_α^* (αa_0 in the case of the electron) is 2π where Eq. (20.13) is transformed from Cartesian coordinates to spherical coordinates. The energy stored in the magnetic field of the electron orbitsphere is

$$E_{mag} = \frac{\pi\mu_0 e^2 \hbar^2}{(m_e)^2 r_n^3} \quad (20.14)$$

Eq. (20.15) is the result of the substitution of αa_0 for r_n , the relativistic mass, $2\pi m_e$, for m_e , and multiplication by the relativistic correction, α^{-1} , which arises from Gauss's law surface integral and the relativistic invariance of charge.

$$E_{mag} = m_e c^2 \quad (20.15)$$

Thus, the energy stored in the magnetic field of the transition state electron orbitsphere equals the electrical potential energy of the transition state orbitsphere. The magnetic field is a relativistic effect of the electrical field; thus, equivalence of the potential and magnetic energies when $v = c$ is given by Special Relativity where these energies are calculated using Maxwell's Equations. The energy stored in the electric and magnetic fields of a photon are equivalent. The corresponding

equivalent energies of the transition state orbitsphere are the electrical potential energy and the energy stored in the magnetic field of the orbitsphere.

Spacetime is an electrical LC circuit with an intrinsic impedance of exactly

$$\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} \quad (20.16)$$

The lab frame circumference of the transition state electron orbitsphere is $2\pi\alpha a_o$; whereas, the circumference for the $v = c$ inertial frame is αa_o .

The relativistic factor for the radius of αa_o is 2π ; thus, due to relativistic length contraction, the total capacitance of free space of the transition state orbitsphere of radius αa_o is

$$C = \frac{2\pi\alpha a_o \epsilon_o}{2\pi} = \epsilon_o \alpha a_o \quad (20.17)$$

where ϵ_o is the capacitance of spacetime per unit length (F/m).

Similarly, the inductance is

$$L = \frac{2\pi\alpha a_o \mu_o}{2\pi} = \mu_o \alpha a_o \quad (20.18)$$

where μ_o is the inductance per unit length (H/m).

Thus, the resonance frequency of a transition state electron orbitsphere is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\epsilon_o \alpha a_o \mu_o \alpha a_o}} \quad (20.19)$$

Thus,

$$\omega = \frac{m_e c^2}{\hbar} \quad (20.20)$$

Thus, the LC resonance frequency of free space for a transition state electron orbitsphere equals the angular frequency of the photon which forms the transition state orbitsphere.

The impedance of any LC circuit goes to infinity when it is excited at the resonance frequency. Thus, the electron transition state orbitsphere is an LC circuit excited at the corresponding resonance frequency of free space. The impedance of free space becomes infinite, and electromagnetic radiation cannot propagate. At this event, the frequency, wavelength, velocity, and energy of the transition state orbitsphere equals that of the photon. The energy of the photon is equal to the rest mass of the particle at zero potential energy, and charge is conserved. Therefore, a free electron and a free positron each of mass $\frac{\hbar\omega}{c^2}$ is produced at infinity relative to the mutual central field of

$$\mathbf{E} = \frac{+e}{4\pi\epsilon_0 r^2} \quad (20.21)$$

where all of the electron transition state orbitsphere equations developed herein apply to this central field. The equation of the free electron is given in the Electron in Free Space Section. The transition state is equivalent to the equation of the photon given in the Photon Equation Section. Photons superimpose; thus, pair production occurs with a single photon of energy equal to twice the rest mass of an electron. Linear momentum is conserved by a third body such as a nucleus which recoils in the opposite direction as the particle pair; thus, permitting pair production to occur.

For pair production, angular momentum is conserved. All photons carry $\pm\hbar$ of angular momentum, and the angular momentum of all matter as orbitspheres is $\pm\hbar$; see Eq. (1.57). The radius of particle creation is αr_1^* . This radius is equal to $\hat{\lambda}_c$, the Compton wavelength bar, where $\hat{\lambda}_c = \frac{\hbar}{m_e c}$. It arises naturally from the boundary condition of no radiation, Eq. (1.43) and Eq. (20.2) where $n = \alpha$, the de Broglie relationship, Eq. (1.46), and that the velocity of the transition state orbitsphere equals c .

$$r_\alpha^* = \frac{\hbar}{m_e c} = \hat{\lambda}_c \quad (20.22)$$

The equations derived for the electron in the present section are generally applicable to all fundamental particles, and it is shown in the Gravity Section that the masses of the fundamental particles are determined by these equations and the curvature of spacetime by matter. During the creation of matter, the constancy of the speed of light must hold which requires relativistic corrections to spacetime. The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity. Thus, the creation of matter causes local spacetime to become curved. The geometry of spacetime is transformed from flat (Euclidean) to curved (Riemannian). Time and distances are distorted. At particle production, the proper time of the particle must equal the coordinate time given by Special Relativity for Riemannian geometry affected by the creation of matter of mass m_0 (in the case of pair production, $m_0 = m_e$) where the metric of spacetime is given by the Schwarzschild metric. This boundary condition determines the masses of the fundamental particles.

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2. Cohen, B., Taylor, B., "The fundamental physical constants", Physics Today, August, (1991), BG 9-BG13.

POSITRONIUM

Pair production, the creation of a positron/electron pair occurs such that the radius of one orbitsphere has a radius infinitesimally greater than the radius of the antiparticle orbitsphere. The inner orbitsphere is held in force balance by a photon such as the case of the proton and neutron (See Proton and Neutron Section). The forces are central, and the radius of the outer orbitsphere (electron or positron) is calculated as follows. The centrifugal force is given by Eq. (1.152). The centripetal electric force of the inner orbitsphere on the outer orbitsphere is given by Eq. (1.153). A second centripetal force is the relativistic corrected magnetic force, \mathbf{F}_{mag} , between each point of the particle and the inner antiparticle given by Eq. (1.164) with m_e substituted for m . The force balance equation is given by Eq. (1.165) with m_e substituted for m . Thus,

$$r_1 = \frac{4\pi\epsilon_0\hbar^2}{e^2\mu} \quad (21.1)$$

where $r_1 = r_2$ is the radius of the positron and the electron and where the reduced mass, μ , is

$$\mu = \frac{m_e}{2} \quad (21.2)$$

The Bohr radius given by Eq. (21.3) and Eq. (21.2) is substituted into Eq. (21.1),

$$a_o = \frac{4\pi\epsilon_0\hbar^2}{e^2 m_e} \quad , \text{ and} \quad (21.3)$$

$$r_1 = 2a_o \quad (21.4)$$

Energy Calculations

The potential energy V between the particle and the antiparticle having the radius r_1 is,

$$V = \frac{-e^2}{4\pi\epsilon_o r_1} = \frac{-Z^2 e^2}{8\pi\epsilon_o a_o} = -2.18375 \times 10^{-18} \text{ J} = 13.59 \text{ eV}. \quad (21.5)$$

The calculated ionization energy is $\frac{1}{2}V$ which is

$$E_{ele} = 6.795 \text{ eV}. \quad (21.6)$$

The experimental ionization energy is 6.795 eV.

Parapositronium, a singlet state hydrogen-like atom comprising an electron and a positron, can absorb a photon which excites the atom to the first triplet state, orthopositronium. In parapositronium, the electron and positron angular momentum vectors are parallel; whereas, the magnetic moment vectors are antiparallel. The opposite relationships exist for orthopositronium. The principal energy levels for

the singlet excited states are given by Eq. (2.22) with the electron reduced mass substituted for the mass of the electron.

$$E_n = \frac{e^2 \mu}{8\pi \epsilon_o a_o n^2} = \frac{6.8}{n^2} \text{ eV}; \quad n \text{ is an integer} \quad (21.7)$$

which are the experimental energy levels. The energy of the transition from parapositronium to orthopositronium is the hyperfine structure interval. Parapositronium possesses orbital angular momentum states corresponding to the quantum number $m_l = 0$; whereas, orthopositronium possesses orbital angular momentum states corresponding to $m_l = 0, \pm 1$. The orbital angular momentum states of orthopositronium are degenerate in the absence of an applied magnetic field. The magnitude of the central field of the first excited triplet state of positronium is $\frac{1}{2}e$ as given in the Excited States of the One Electron Atom (Quantization) Section and F_{mag} of Eq. (1.164) with m_e substituted for m is zero for the parallel spins having antiparallel angular momentum vectors. Thus, the radii of the two orbitspheres of the first triplet excited state are given by the force balance equating the centrifugal and centripetal forces:

$$\frac{m_e v_1^2}{4\pi r_1^2 r_1} = \frac{e}{4\pi r_1^2} \frac{1}{2} \frac{Ze}{4\pi \epsilon_o r_1^2} \quad (21.8)$$

$$r_1 = r_2 = 2a_o \quad (21.9)$$

Thus, E_{ele} , Eq. (21.6) is unchanged from the "ground" state energy. The hyperfine structure interval of positronium can be calculated from the spin/spin coupling energy and the magnetic energy stored in the surface currents produced by the "trapped resonant photon". The spin/spin coupling energy between the inner orbitsphere and the outer orbitsphere is given by Eq. (1.136) where μ_B , the magnetic moment of the outer orbitsphere is given by Eq. (1.137). The magnetic flux, \mathbf{B} , of the inner orbitsphere at the position of the outer is

$$\mathbf{B} = \frac{\mu_o e \hbar}{2m_e r_2^3} \quad (21.10)$$

Substitution of Eq. (21.10) and (1.137) into Eq. (1.136) gives

$$E = \frac{2\mu_o e^2 \hbar^2}{4m_e^2 r_2^3} \quad (21.11)$$

Photons obey Maxwell's Equations. At the two dimensional surface of the orbitsphere containing a "trapped photon", the relationship between the photon's electric field and its charge at the orbitsphere is

$$\mathbf{n} \cdot (\mathbf{E}_1 - \mathbf{E}_2) = \frac{\sigma}{\epsilon_o} \quad (21.12)$$

Thus, the photon's electric field acts as surface charge. According to Eq.

(21.12), the photon standing wave in the positronium orbitsphere resonator cavity gives rise to a two dimensional surface charge at the orbitsphere two dimensional surface at r_1^+ , infinitesimally greater than the radius of the inner orbitsphere, and r_2^- , infinitesimally less than the radius of the outer orbitsphere. For an electron in a central field, the magnitude of the field strength of the first excited state corresponding to a transition from the state with $n = 1$ and radius $2a_0$ to the state with $n = 2$ is $\frac{1}{4}e$ as given in the Excited States of the One Electron Atom

(Quantization) Section. The energy corresponding to the surface charge which arises from the "trapped photon standing wave" is given by the energy stored in the magnetic fields of the corresponding currents. The surface charge is given by Eq. (21.12) for a central field strength equal in magnitude to $\frac{1}{4}e$. This surface charge possesses the same angular velocity as each orbitsphere; thus, it is a current with a corresponding stored magnetic energy. The energy corresponding to the surface currents, E_{sc} , is the difference of $E_{mag \text{ internal}}$ and $E_{mag \text{ external}}$ for a single charge of $\frac{1}{4}e$ substituted into Eqs. (1.125) and (1.127).

$$E_{sc} = \frac{2}{3} \frac{\pi \mu_o e^2 \hbar^2}{4^2 m_e^2 r_1^3} - \frac{1}{3} \frac{\pi \mu_o e^2 \hbar^2}{4^2 m_e^2 r_1^3} \quad (21.13)$$

The hyperfine structure interval of positronium, E_{HF} , is given by the sum of Eq. (21.11) and Eq. (21.13) where $r_1 = 2a_0$.

$$E_{HF} = \frac{\mu_o e^2 \hbar^2}{4m_e^2 8a_0^3} - \frac{\pi \mu_o e^2 \hbar^2}{(4^2)(3)m_e^2 8a_0^3} \quad (21.14)$$

Eq. (21.14) is the hyperfine structure interval calculated for an electron or a positron magnetic moment of one Bohr magneton; however, the exact magnetic moment in the case of an orbitsphere includes the electron (fluxon) g factor which is given by Eq. (1.151). Thus, Eq. (21.14) becomes

$$E_{HF} = 2g \frac{\mu_o e^2 \hbar^2}{32m_e^2 a_0^3} \left(1 - \frac{\pi}{12} \right) \quad (21.15)$$

$$E_{HF, \text{calculated}} = 8.4111 \times 10^{-4} \text{ eV}$$

$$E_{HF, \text{experimental}} = 8.4111 \times 10^{-4} \text{ eV}$$