# CHAPTER 9

# POWER GENERATION AND PROPULSION

### 9.1 Introduction

One of the most significant engineering applications of thermodynamics is the design and optimization of energy conversion devices which take in heat and produce useful work ("heat engines"). In Chapter 7, we discussed in very general terms the principles governing the performance of heat engines. In this chapter, we look at some specific processes which are actually used for power generation and propulsion.

# 9.2 Cycles

Any thermodynamic process which takes a substance through a sequence of states eventually returning it to the original state is known as a *cycle*. A familiar example is the Carnot cycle, discussed in Chapter 7.

Virtually all heat engines are implemented as cycles. Usually, the substance the cycle operates on is a fluid, although in principle any substance could be used, as long as it has some reversible work mode. In this chapter, we'll only consider cycles which operate on fluids, since all important engineering cycles are of this type.

The fluid used in the cycle is known as the *working fluid*. During the cycle, the working fluid may or may not change phase. In *vapor power cycles*, the working fluid changes phase from liquid to vapor and back to liquid over the course of the cycle; in *gas power cycles*, the working fluid remains gaseous during the entire cycle. As we will see, they each have certain advantages and disadvantages.

In many types of cycles, the working fluid circulates continuously in a loop. For example, the major cycles used for electric power generation are of this type. In these cycles, the thermodynamic state of the working fluid at any one *location* in the loop is constant in time. But if we focus our attention on a single "packet" of fluid (say, a unit mass) moving around the loop, its state changes as it passes through various components such as pumps, turbines, heat exchangers, etc. To analyze such cycles, we follow a fluid packet around the loop, applying the steady-flow equations expressing energy and entropy accounting to each device the fluid packet encounters to determine how its state changes.

In another type of cycle, a fixed mass of fluid is made to undergo a sequential set of operations. The Carnot cycle is an idealized example of such a cycle. Practical examples include spark-ignition and Diesel internal combustion engines. To analyze these, we apply the control-mass energy and entropy accounting equations to each step in the cycle.

To analyze real cycles, we need to account for the fact that they are irreversible (they produce entropy). As we saw in Chapter 7, some irreversibility (for example due to finite- $\Delta T$  heat transfer) is inevitable if we want to produce *power*, since reversible processes operate infinitely slowly. In addition to finite- $\Delta T$  heat transfer, some other common irreversibilities are those due to friction, fluid viscosity, shock waves, and combustion.

Nevertheless, we can learn a lot about real cycles by first considering ones which are fully reversible, and then considering how the results for reversible cycles need to be modified to account for irreversibilities. We'll do this in the next two sections, and then apply these ideas to look at the characteristics of several different cycles.

#### 9.2.1 Process Representations

To carry out thermodynamic analysis of cycles, we need to keep track of how the thermodynamic state of the fluid changes as the cycle proceeds. Since for a fluid in equilibrium only two properties are required to specify its state, a fluid in thermodynamic equilibrium may be represented as a point on a plot of one thermodynamic property vs. another one (e.g. T vs. s, P vs. v, etc.) Any such plot of the working fluid state at various points in the cycle is known as a *process representation*.

If the fluid remains in equilibrium at every point during the cycle, then the fluid state always corresponds to some point on the process representation and as the cycle proceeds, a closed curve is traced out. Since the fluid is always in equilibrium, no internal irreversible processes (which act only when a system is perturbed from equilibrium) can be occurring. Therefore, such a cycle is reversible. We call reversible cycles *ideal cycles*. Of course, real cycles can approach ideal cycles, but there will always be some entropy production in reality and the fluid will always be at least a little out of equilibrium.

To analyze real, irreversible cycles, we usually assume the fluid is approximately in equilibrium at the inlet and outlet of devices like valves, turbines, pumps, compressors, etc., but not necessarily within them. Points in the process representation plane corresponding to the inlet and outlet states are simply connected with a dashed line to represent the action of the device on the fluid.

#### 9.2.2 Analysis of Ideal Cycles

The T-s process representation is particularly useful to represent ideal cycles. Since for any reversible process dQ = Tds, the area under a curve traced out by a reversible process which takes the working fluid from some state 1 to some state 2 is the total heat added:



If we consider a complete cycle, then the *net* heat taken in by the cycle is the area enclosed by the curve in the T - s plane:

$$Q_{net} = \oint T ds. \tag{9.2}$$

Of course, on the portion of the cycle where ds < 0, dQ < 0 so heat is actually being transferred out of the working fluid to the environment.

It is convenient to break up the path of a closed cycle into the portion where heat is being added, and the portion where heat is being rejected:



Then the heat absorbed per unit mass by one complete cycle is the area under the upper portion of the curve (left), and the heat rejected to the environment per unit mass is the area under the lower portion (center). The net heat taken in is the difference, which is the enclosed area (right). From the first law, the net heat taken in must equal the net work output, since for a complete cycle  $\Delta u = 0$ .

Therefore, the area enclosed by an ideal cycle in the T-s plane is the net work produced per unit mass of working fluid for one complete cycle.

The net work per unit mass is an important quantity, since the total power produced by the cycle is the product of  $W_{net}$  and the mass flow rate  $\dot{m}$ :

$$\dot{W} = \dot{m}W_{net}.\tag{9.3}$$

The mass flow rate needed to produce a given desired power scales inversely with  $W_{net}$ . Since the overall size of a powerplant scales roughly with the working fluid mass flow rate, higher  $W_{net}$  results in smaller, more-compact (and usually less expensive) power plants. This is especially important in applications such as aircraft propulsion, where the weight of the engine must be minimized.

The thermodynamic efficiency  $\eta$  of any power cycle is defined as the net work output divided by the heat *input*:

$$\eta = \frac{W_{net}}{Q_{in}}.\tag{9.4}$$

Thus, the efficiency of an *ideal* cycle is the ratio of the area enclosed on a T-s plot to the total area under the curve.

Two important quantities for ideal cycles are the *average* temperature at which heat is added  $(\overline{T}_h)$ , and the average temperature at which heat is rejected  $(\overline{T}_c)$ . These are defined by

$$\overline{T}_h = \frac{Q_{in}}{\Delta S_{max}},\tag{9.5}$$

and

$$\overline{T}_c = \frac{Q_{out}}{\Delta S_{max}},\tag{9.6}$$

where  $\Delta S_{max} = s_B - s_A$  is the difference between the maximum entropy and minimum entropy points in the cycle. Then the rectangles shown below have area  $Q_{in}$  and  $Q_{out}$ , respectively.



The thermodynamic efficiency may be expressed in terms of  $\overline{T}_h$  and  $\overline{T}_c$ :

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$
(9.7)

=

$$= 1 - \frac{Q_{out}}{Q_{in}} \tag{9.8}$$

$$= 1 - \frac{T_c \Delta s_{max}}{\overline{T_h \Delta s_{max}}}.$$
(9.9)

Therefore,

$$\eta = 1 - \frac{\overline{T}_c}{\overline{T}_h}.$$
(9.10)

This expression holds for any ideal cycle. For the special case when all heat is taken in at a single temperature  $T_h$  and all heat is rejected at a single temperature  $T_c$  (as in the Carnot cycle),  $\overline{T}_h = T_h$  and  $\overline{T}_c = T_c$  and this expression reduces to the Carnot efficiency. We may regard the cycle as being equivalent to a Carnot cycle with temperatures  $\overline{T}_h$  and  $\overline{T}_c$ .

Equation (9.10) shows that the thermodynamic efficiency of an ideal cycle can be increased either by increasing the average temperature at which heat is added or by lowering the average temperature at which heat is rejected. This insight will prove to be vital for improving the efficiency of real cycles, although of course with real cycles we also have to be concerned with minimizing internal irreversibilities.

#### 9.2.3 Component Models

To analyze real cycles, we need models for the action of various components the working fluid encounters (pumps, compressors, heaters, turbines, nozzles, etc.) which include the irreversible character of these components. Several such models of common steady-flow devices were discussed in Section 7.8. We will use these in the analysis below, and introduce some new component models also.

#### 9.3 Vapor Power Cycles

The first power cycle to be developed was the *Rankine cycle*, and in modified form the Rankine cycle is still the most common vapor power cycle. It is in widespread use for electric power generation in large central power plants, and will be for the forseeable future. Whatever the type of fuel burned in the power plant (coal, natural gas, oil, or nuclear fuel), a modified Rankine cycle using water as the working fluid is invariably used to convert the heat produced into useful power (sometimes in conjunction with other cycles we'll discuss later).

In the sections below, we work through some specific examples of the Rankine cycle, progressively adding modifications which improve performance and lead



Figure 9.1: The Open Rankine Cycle

to the types of cycles actually used in modern power plants.

#### 9.3.1 The Open Rankine Cycle

The simplest version of the Rankine cycle is the open Rankine cycle, shown in Figure 9.1. In the open Rankine cycle, a fluid (usually water) is pumped to high pressure, and then heat from a flame is added in a boiler to boil the water at constant pressure. Saturated steam is extracted from the boiler, and expanded back to atmospheric pressure, producing work. In Figure 9.1, the expansion is shown occurring in a turbine, which is how modern implementations of the Rankine cycle operate.

The open Rankine cycle was originally developed in the 19th century to power steam locomotives and other steam-operated heavy equipment. The development of steam engines made possible the industrial revolution and everything which followed from it. In a steam locomotive, the high pressure steam entered a cylinder and pushed a piston, which propelled the train forward. At the end of the stroke, an exhaust valve opened and on the reverse stroke the low-pressure steam was expelled to the atmosphere.

Let's analyze this cycle using some typical numbers. We'll take state 1 to be water at 30 °C and 1 atm,  $P_2 = 0.8$  MPa, and assume that state 3 is saturated vapor. The pump and turbine are both adiabatic, with isentropic efficiencies of 0.6 and 0.8 respectively. To analyze the cycle, we simply work our way around it, putting a control volume around each component and writing down expressions for energy and entropy accounting.

## The Initial State

We start the analysis in state 1, since enough information is given to fully determine the state here. With T = 303.15 K and P = 1 atm, we using TPX

that

$$v_1 = 0.00100 \text{ m}^3/\text{kg}$$
  
 $h_1 = 125.2 \text{ kJ/kg}$   
 $s_1 = 0.4348 \text{ kJ/kg-K}$ 

The Pump

To analyze the pump, it is simplest to assume that the liquid water is incompressible. In this case, the work of an ideal (isentropic) pump is

$$W_{p,s} = v_1(P_2 - P_1). (9.11)$$

Since these quantities are known, we find

$$W_{p,s} = 0.7 \text{ kJ/kg.}$$

By definition of the isentropic efficiency for a pump, the actual work  $W_p$  is

$$W_p = \frac{W_{p,s}}{\eta_s} = 1.2 \text{ kJ/kg}.$$

Knowing the pump work, we can calculate the enthalpy of state 2:

$$h_2 = h_1 + W_p = 126.4 \text{ kJ/kg.}$$

Knowing  $h_2$  and  $P_2$  is sufficient to fix all other properties in state 2, and so we find

$$T_2 = 303.26 \text{ K}$$
  
 $s_2 = 0.4366 \text{ kJ/kg-K}.$ 

As required by the second law,  $s_2 > s_1$ . We see that pumping the liquid produces only a very small temperature rise (0.1 K).

The Boiler

Let us assume the heat addition in the boiler occurs at constant pressure. Then

$$P_3 = P_2 = 0.8$$
 MPa.

We are not given how much heat is added in the boiler, but we know that state 3 is saturated vapor. Calculating the properties for saturated vapor at 0.8 MPa, we find

$$T_3 = 443.58 \text{ K}$$
  
 $h_3 = 2768.7 \text{ kJ/kg}$ 

$$s_3 = 6.662 \text{ kJ/kg-K}.$$

From an energy balance on the boiler,  $h_2 + Q_b = h_3$ , so the heat added in the boiler is

$$Q_b = 2642.3 \text{ kJ/kg}.$$

The Turbine

To analyze the turbine, we first must find the outlet state which would result from an isentropic turbine. Calling this state 4s, we know

$$P_{4s} = 1 \text{ atm}$$

$$s_{4s} = s_3 = 6.662 \text{ kJ/kg-K}.$$

This is enough to fully specify state 4s, so we find

$$T_{4s} = 373.144 \text{ K}$$
  
 $h_{4s} = 2417.3 \text{ kJ/kg}$   
 $s_{4s} = 6.662 \text{ kJ/kg-K}$   
 $x_{4s} = 0.885$ 

Note that state 4s lies within the vapor dome, with nearly 12% liquid content by mass.

Having found 4s, we now can get state 4 from the given isentropic efficiency of the turbine:

$$\eta_s = \frac{h_3 - h_4}{h_3 - h_{4s}}.$$

Putting in  $\eta_s = 0.8$  and solving for  $h_4$ ,

$$h_4 = 2487.6 \text{ kJ/kg}.$$

With  $h_4$  and  $P_4$  known, we can find the remaining properties:

$$s_4 = 6.850 \text{ kJ/kg-K}$$
  
 $x_4 = 0.917$ 

We see  $s_4 > s_3$ , consistent with the second law, and the actual turbine outlet state also contains some liquid.

The vapor mass fraction x at the turbine outlet is an important parameter, since if it is too low the water droplets which form within the turbine impact on the turbine blades and cause serious erosion. Typically, x of about 0.9 is the



Figure 9.2: An open Rankin cycle (shown as closed by the environment)

lowest tolerable value. Thus, the conditions at state 4 are probably acceptable, but just barely. Since the vapor mass fraction is so important, it is often called the *steam quality*.

Process representations for this cycle in the T-s and h-s planes are shown in Figure 9.2. Note that states 1 and 2 are different, but overlap on this scale. Also, the cycle is shown in these process representations as being closed by a constant-pressure line connecting state 4 to state 1. This allows us to regard an open Rankine cycle as being closed by the environment, and the step from 4 to 1 is the heat rejection to the environment required to condense the steam and cool the liquid down to  $T_1$ .

Having now found all of the states of the cycle, we can calculate its performance. The work produced by the turbine per unit mass of steam flowing through it is

$$W_t = h_3 - h_4 = 281.1 \text{ kJ/kg}.$$

Some of this work (1.2 kJ/kg) must be used to run the pump, so the net work output is

$$W_{net} = W_t - W_p = 279.9 \text{ kJ/kg}$$

To produce this work,  $Q_b = 2642.3 \text{ kJ/kg}$  of heat had to be added to the steam

in the boiler. Therefore, the efficiency of this process at converting heat into work is

$$\eta = \frac{W_{net}}{Q_b} = 10.6\%$$

We see in this example that the work output from the turbine is much greater than the work required to run the pump. Their ratio is known as the *back work ratio* BWR:

$$BWR = \frac{W_p}{W_t}.$$
(9.12)

For this example, BWR = 0.004. This is one of the major advantages of vapor power cycles: the work required to compress the liquid is negligible compared to the work produced by the vapor expanding through the same pressure difference.

The Carnot efficiency for an ideal heat engine operating between the same maximum and minimum temperatures encountered in this cycle is

$$\eta_{Carnot} = 1 - \frac{303.15}{443.58} = 31.6\% \tag{9.13}$$

Therefore, the cycle in this example has an efficiency only 1/3 of the theoretical maximum for these temperatures. One reason the efficiency is low is the fact that the turbine and the pump are not ideal. But if we recalculated the performance of an ideal cycle with  $\eta_{s,p} = \eta_{s,t} = 1$ , we would find an efficiency of 13.2%. So even the ideal cycle has an efficiency far below the Carnot efficiency.

By examining the T-s plot in Fig. 9.2, we can see some of the reasons for the low efficiency. The turbine exhaust is steam at 373.15 K. Much of the total heat rejection to the atmosphere occurs as this steam condenses to liquid at this temperature. Therefore,  $\overline{T}_c$  is much greater than 30 °C. Thus, this cycle needlessly throws away hot steam, from which more work could still be extracted.

Another factor limiting the efficiency is the heat addition from state 2 up to the saturation point. This heat addition in the boiler occurs at less than  $T_{sat}$ , so  $\overline{T}_h < 443.58$  K.

In the next example, we consider a modification to allow rejecting heat at lower temperature. We'll look at how to raise  $\overline{T}_h$  in the following example.

#### 9.3.2 The Closed Rankine Cycle

In the last example, state 4 had a temperature of 373.15 K, which is the result of exhausting at 1 atm. But suppose we didn't simply exhaust to the atmosphere, but instead provided a low-pressure region for the turbine to exhaust into. For  $P_4 < 1$  atm,  $T_4 = T_{sat}(P_4)$  will be less than 373.15 K, and we will be able to lower the temperature at which heat is rejected.



Figure 9.3: The Closed Rankine Cycle.

To do this, we must close the cycle by adding a condenser, in which heat will be removed from the turbine exhaust, leaving saturated liquid water at state 1. This is shown in Fig. 9.3.

We can choose the temperature at which we will condense the steam based on the temperature of the reservoir into which we will be dumping the heat. In many cases, the heat will be removed by cooling water supplied from a nearby lake, river, or ocean. If a source of cooling water is available at, say, 20 °C (293 K), then we could choose a condensing temperature of, say, 300 K. The 7 degree temperature difference should be enough to drive heat transfer in the condenser. The condenser tube area is determined by the total amount of heat which must be rejected, and by the temperature difference between the steam and the cooling water. There is always a trade-off between  $\Delta T$  and total condenser area. For given  $Q_c$ , smaller  $\Delta T$  means larger A. The optimum values would be determined in reality by balancing the efficiency gain by lowering the condensing temperature with the cost required to build a bigger condenser. A  $\Delta T$  value of a few degrees is would be typical.

Let us specify  $T_4 = 300$  K. Then since state 4 will be in the vapor dome,

$$P_4 = P_{sat}(300) = 0.003536$$
 MPa.

This is about 1/29 atm. To start the cycle running (for example, after shutting down for maintainance), a vacuum pump is needed to pump out the air. Also, since the pressure is less than 1 atm, any small leaks will result in air entering the condenser. If enough air enters, the pressure will begin to rise, causing  $T_{sat}$  to rise, and lowering the cycle efficiency. So the vacuum pump must be used during operation also to remove any air entering through leaks. Thus, the system will be somewhat more complex than the open Rankine cycle. But the gain in efficiency will more than offset the increased complexity.



Figure 9.4: A closed Rankin cycle

With  $T_4$  set to 300 K, and state 1 set to saturated liquid at 300 K, we can re-do the analysis of Example 1. The numbers are summarized in the table below. (You should verify these.)

State	P (MPa)	$T(\mathbf{K})$	$h~(\rm kJ/kg)$	s ~(kJ/kg-K)
1	0.003536	300	111.7	0.3900
2	0.8	300.08	113.6	0.3924
3	0.8	443.58	2768.7	6.662
4s	.003536	300	1993.3	6.662
4	.003536	300	2148.4	7.179

These states are shown in T - s and h - s plots in Fig. 9.4.

The pump work is now

$$W_p = h_2 - h_1 = 1.33 \text{ kJ/kg}.$$

This is slightly greater than in Example 1, which is not surprising since now state 1 has a lower pressure, so  $P_2 - P_1$  is greater than before.

The turbine work output is

$$W_t = h_3 - h_4 = 620.3 \text{ kJ/kg}$$



Figure 9.5: A Rankine Cycle with Superheat.

This is a factor of 2.2 greater than the turbine work in Example 1. Lowering the outlet pressure allowed the steam to expand much more, producing more work. This is clear in comparing the h - s plots in Figures 9.2 and 9.4.

The heat added in the boiler  $Q_b$  is unchanged, so the cycle efficiency is

$$\eta = 23.3\%$$

This is quite an improvement over the open Rankine cycle efficiency.

### 9.3.3 Addition of Superheat

Having added the condenser, we've lowered the temperature at which heat is rejected about as much as possible. We now need to consider how to raise  $\overline{T}_h$  if we wish to improve the efficiency more.

The temperature of the steam in the boiler is set by the pressure, since  $T = T_{sat}(P)$ . So the next improvement to make is to increase the boiler pressure. The limits on boiler pressure are determined by how much pressure the material the tubes carrying the high-pressure steam can withstand at high temperature for long durations. Originally, boilers were constructed from cast iron, which limited the safe operating pressures. Now with high-strength steel boilers, much higher pressures can be used. Let's take a typical value of 10 MPa.

If this is all we do, we have a problem. If we expand saturated vapor at 10 MPa to 0.0035 MPa in a turbine, a very large liquid fraction would result. But there is no reason to restrict ourselves to saturated vapor. We can extract saturated steam from the boiler, but add more heat to it before sending it through the turbine. Doing this is known as *superheating* the steam, and all modern power plants employ superheat.

The upper limit on how hot the steam may be heated is determined by the materials constraints of the turbine. Hot steam is corrosive, and if it is too hot



Figure 9.6: A Rankin cycle with superheat.

when it enters the turbine it will rapidly corrode the turbine blades. Advances in high temperature materials for turbine blades and elaborate cooling methods have resulted in allowable turbine inlet temperatures of more than 800 K. We'll take this value for this example.

Finally, we can work on improving the isentropic efficiencies of the pump and turbine, to more closely approach the ideal cycle. Large-scale turbines can now be manufactured with isentropic efficiency above 90%; let's take a value of  $\eta_{s,t} = 0.9$ . The isentropic efficiency of the pump hardly matters, due to the low BWR, but these have gotten better too. We'll take  $\eta_{s,p} = 0.8$ .

The process representation is shown in Fig. 9.6.

The analysis is very similar to what we've done in the last two examples, except that state 3 is now fixed by the specified  $T_3$  and  $P_3$ . The numbers work out as shown below.

State	P (MPa)	$T(\mathbf{K})$	$h~({ m kJ/kg})$	$s \; (kJ/kg-K)$	х
1	0.003536	300	111.7	0.3900	0
2	10	300.72	124.3	0.3993	0
3	10	800	3442.0	6.683	1
4s	.003536	300	1999.7	6.683	0.774
4	.003536	300	2143.9	7.164	0.833



Figure 9.7: A Rankine cycle with superheat and one stage of reheat.

Heat added:	3317.7  kJ/kg
Turbine work:	1298.1  kJ/kg
Pump work:	12.5  kJ/kg
Efficiency:	38.7%
BWR:	0.0096

These modifications have produced a marked increase in efficiency, and this value is in the ballpark of real steam power plant efficiencies. The work output per unit mass of steam has also significantly increased. The steam quality leaving the turbine is too low, however, so we're not done yet.

#### 9.3.4 Addition of Reheat

To improve the steam quality at the turbine exit, we need to move to the right on the T-s plot. We can do this and also increase the average temperature of heat addition by employing *reheat*, as shown in Fig. 9.7.

Due to the large pressure range we're expanding the steam through, the turbine must be constructed of multiple stages or else multiple separate turbines must be used (small ones for high pressure and progressively larger ones as the pressure becomes lower). In the reheat cycle, we expand the steam first through a high pressure turbine to some intermediate pressure, extract it and add more heat, and then expand it through a second low-pressure turbine. The effect of this is seen in the process representations (Fig. 9.8). The work output (the sum of both turbines) per unit mass of steam is increased, as is the efficiency.

For this example, we will take the intermediate reheat pressure to be 1 MPa. Then carrying out an analysis similar to what we've done before, we find



Figure 9.8: A Rankin cycle with superheat and one stage of reheat.

State	P (MPa)	$T(\mathbf{K})$	$h~(\rm kJ/kg)$	s~(kJ/kg-K)	x
1	0.003536	300	111.7	0.3900	0
2	10	300.72	124.3	0.3993	0
3	10	800	3442.0	6.683	1
4s	1	471.2	2822.7	6.683	1
4	1	497.6	2884.7	6.811	1
5	1	800	3536.4	7.835	1
6s	.003536	300	2345.4	7.835	0.916
6	.003536	300	2464.5	8.232	0.965

Heat added in boiler / superheater:	3317.7  kJ/kg
Heat added in reheater:	$651.7 \mathrm{~kJ/kg}$
HP Turbine work:	$557.3 \mathrm{~kJ/kg}$
LP Turbine work:	1071.9  kJ/kg
Pump work:	12.5  kJ/kg
Efficiency:	40.7%
BWR:	0.0076

### 9.3.5 Regeneration

One remaining source of inefficiency is the heating of the water when it first enters the boiler. Since the water is initially cold (300 K), the heat addition required to bring the water to the boiling point constitutes low-temperature



Figure 9.9: A Rankine cycle with superheat, one stage of reheat, and one open feedwater heater.

heat addition. If we can eliminate some of the low-temperature heat addition, we can raise the *average* temperature at which heat is added, and thus the efficiency of the cycle.

A technique known as *regeneration* uses heat from the hot steam to preheat the water going into the boiler (the "feedwater"). There are several ways to implement regeneration. A particularly simple way (which however only partially eliminates low-temperature external heat addition) is shown in Fig. 9.9. A portion f of the steam leaving the high-pressure turbine is extracted and simply mixed with the cold water before it enters the boiler. The remaining fraction 1 - f goes through the reheater, low-pressure turbine, and condenser as usual.

The chamber where the mixing occurs is known as an *open feedwater heater*. Since the mixing must occur at the pressure at the outlet of the high-pressure turbine, this scheme requires two pumps, as shown in Fig. 9.9. The water which leaves the feedwater heater is pumped up to the boiler pressure in the second pump. Since it is now hotter than it would have been without the feedwater heater, less heat must be added in the boiler to reach the boiling point.

Let's consider the effect of adding an open feedwater heater to the previous example. We will specify that just enough steam is extracted at state 4 to produce saturated liquid at the outlet of the feedwater heater (state 7). Most of the states remain the same as in example 4. The only changes are state 2  $(P_2 = 1 \text{ MPa now, not 10 MPa})$  and the two new states 7 and 8.

State 7 is determined by our specification that it is saturated liquid at 1 MPa, and state 8 is determined by analyzing pump 2, in the same way we found state 2 earlier. The properties in each state are listed below, and the T-s plot



Figure 9.10: A Rankin cycle with superheat, one stage of reheat, and one open feedwater heater. Note that only a fraction f of the total mass flow rate flows from 4 to 7, and a fraction 1 - f flows in the loop 4-5-6-1.

State	P (MPa)	$T(\mathbf{K})$	$h~({ m kJ/kg})$	$s \; (kJ/kg-K)$	х
1	0.003536	300	111.7	0.3900	0
2	1	300.01	113.0	0.3915	0
3	10	800	3442.0	6.683	1
4s	1	471.2	2822.7	6.683	1
4	1	497.6	2884.7	6.811	1
5	1	800	3536.4	7.835	1
6s	.003536	300	2345.4	7.835	0.916
6	.003536	300	2464.5	8.232	0.965
7	1	453.06	762.5	2.138	0
8	10	454.92	775.2	2.144	0

for this cycle is shown in Fig. 9.10.

Knowing all the states, we now need to calculate the extraction fraction f which is required to produce the desired state 7 (saturated liquid at 1 MPa). To find the extraction fraction, consider an energy balance on the feedwater heater:

$$\dot{m}_4 h_4 + \dot{m}_2 h_2 = \dot{m}_7 h_7. \tag{9.14}$$

Note that we have to keep track of the mass flow rate in each stream, since they are not equal. Dividing by  $\dot{m}_7$ ,

$$fh_4 + (1 - f)h_2 = h_7. (9.15)$$

Solving for f,

$$f = \frac{h_7 - h_2}{h_4 - h_2} \tag{9.16}$$

which works out to f = 0.234.

Now we can calculate the energy transfer for each component. For the boiler, superheater, and pump 2, the full  $\dot{m}$  flows through them, and we calculate the work per unit mass in the usual way. But note that only  $(1 - f)\dot{m}$  flows through pump 1, the reheater, and turbine 2. It is most convenient to express all energy transfers as the power divided by the total  $\dot{m}$  (not the actual mass flow rate through a given component). Therefore, we must multiply the enthalpy difference by the fraction of the mass flowing through a given component:

$$Q_b = h_3 - h_8 \tag{9.17}$$

$$W_{p,2} = h_8 - h_7 \tag{9.18}$$

$$W_{t,1} = h_3 - h_4$$
(9.19)  

$$Q_r = (1 - f)(h_5 - h_4)$$
(9.20)

$$Q_r = (1-f)(h_5 - h_4) \tag{9.20}$$

$$W_{p,1} = (1-f)(h_2 - h_1)$$
 (9.21)

$$W_{t,2} = (1-f)(h_5 - h_6)$$
(9.22)

Evaluating these expressions, we find

Heat added in boiler / superheater:	2666.8  kJ/kg
Heat added in reheater:	499.0  kJ/kg
Turbine work 1:	$557.3~\mathrm{kJ/kg}$
Turbine work 2:	820.7  kJ/kg
Pump work 1:	.96  kJ/kg
Pump work 2:	12.68  kJ/kg
Extraction fraction:	23.4%
Efficiency:	43.1%
BWR:	0.0099

Using one open feedwater heater is seen to increase the efficiency of the cycle from 40.7% to 43.1%. While this may seem not much considering the added complexity, in fact a gain of 2.4 percentage points in efficiency is quite significant, and well-worth any added costs due to complexity in even a moderately-sized power plant.

A typical real power plant would use multiple feedwater heaters, with steam extracted from the turbine at various pressures, in order to achieve even more regeneration. For example, the largest unit at the Pasadena power plant makes use of 5 feedwater heaters.

## 9.4 Gas Power Cycles

A simple gas power cycle is shown below. A steady flow of air is compressed and enters a combustor, where fuel is injected into the high-pressure air stream and the fuel/air mixture burns. The hot products of combustion<sup>1</sup> expand through a turbine, producing power  $\dot{W}_t$ . Some fraction of the turbine power is used to drive the compressor, and the remaining power  $\dot{W}_{net} = \dot{W}_t - \dot{W}_c$  may be delivered to an external load.



Variations on this basic *gas turbine cycle* are widely used both for power generation and propulsion. Aircraft jet engines use a modified version of this cycle, which we will discuss in more detail shortly. Gas turbines have become increasingly popular for ground-based power generation also, as the efficiency of these cycles has improved. They are particularly attractive as topping cycles in conjunction with vapor power cycles, or when relatively small, modular power generation units are needed.

The two most important performance parameters for a gas turbine cycle are the thermal efficiency  $\eta$  and the net work developed per unit mass of air flow,  $W_{net} = \dot{W}_{net}/\dot{m}_{air}$ . The efficiency determines the fuel economy; a more efficient gas turbine cycle for electric power generation consumes less fuel per kW-hr of electricity generated. This lowers both the cost of electricity and the pollutant emissions. For an aircraft engine, higher efficiency translates into lower cost, longer range, and lower pollution.

 $W_{net}$  is especially important for transportation applications, since the size

<sup>&</sup>lt;sup>1</sup>for lean combustion, a mixture composed primarily of nitrogen, oxygen, water vapor, carbon dioxide, and smaller amounts of other species, including pollutants such as oxides of nitrogen  $(NO_x)$ .

and weight of the engine scale approximately with the air flow rate. For a fixed power output requirement, a cycle with higher  $W_{net}$  will be lighter and smaller.

Gas turbine engines are almost always run with significantly more air than required to burn all of the fuel injected (so-called "lean combustion"). A major reason lean combustion is used is to limit pollutant emissions. By burning lean, combustion is complete, so there is no unburned fuel vapor in the exhaust, nor is there much CO. Also, lean combustion limits the temperature of the gas leaving the combustor. This is important both to minimize the production of smogproducing oxides of nitrogen (NO<sub>x</sub>) and to protect the turbine blades, which cannot survive excessive temperatures.

A complete analysis of a gas turbine cycle would require considering the combustion process, including calculating the amount of heat released, and the composition of the combustion products. A design of a real gas turbine cycle would consider this in some detail, with particular focus on pollutant formation, since the emissions from both ground-based gas turbines and aircraft engines are strictly regulated by government agencies.

#### 9.4.1 The air-standard Brayton cycle

We can learn a lot about gas turbine cycles by considering a simple model which captures their essential features, without requiring full consideration of the combustion process. Since the gas mixture is mostly air, even downstream of the combustor, as a first approximation we may take the gas to be air throughout the cycle. In this approximation, the combustor is modeled as an air heater, with the heat of combustion now an external heat input. Since in this *airstandard cycle* the fluid is the same at the turbine exhaust as at the compressor inlet, we may close the cycle by adding a cooler, which removes heat from the air to the environment. The cycle which results from these approximations is called the air-standard *Brayton cycle*, shown in Fig. 9.11. It is also called the *simple* Brayton cycle, to distinguish it from modified versions we will discuss below.

To analyze the air-standard Brayton cycle, we will assume the compressor and turbine are both adiabatic, with isentropic efficiencies  $\eta_{s,c}$  and  $\eta_{s,t}$ , respectively. We will neglect any pressure drop in the heater and cooler, so these are modeled as constant-pressure heat addition or removal processes. With these assumptions, the *T*-s process representation is as shown in Fig. 9.11.

Let us put in some "typical" numbers. Suppose the inlet to the compressor is air at 1 atm and 300 K, the pressure ratio  $P^* = P_2/P_1$  across the compressor is 4, and the temperature at the inlet to the turbine is fixed at 800 K by materials



Figure 9.11: The air-standard Brayton cycle and T-s process representation.

constraints. The compressor has isentropic efficiency  $\eta_{s,c} = 0.7$ , and the turbine has isentropic efficiency  $\eta_{s,t} = 0.8$ .

To determine the performance of this cycle, we first must find all states, and their properties. The procedure is very much like what we did to analyze vapor power cycles: start with a known state (state 1), and simply work around the loop. Since this procedure is now familiar to you, we will simply summarize the set of relations needed in Table 9.1.

The equations in Table 9.1 may be easily implemented in Excel, using TPX to calculate the properties. Since TPX does not calculate properties for air, we will calculate properties for nitrogen, which of course is the major constituent (78%) of air, with most of the remainder being oxygen. For the parameter values specified above, the states are as follows:

state	T (K)	P (atm)	$h~(\rm kJ/kg)$	$s~(\rm kJ/kg\text{-}K)$
1	300	1	461.5	4.41
2s	445.5	4	613.0	4.41
2	507.1	4	677.9	4.55
3	800	4	996.4	5.05
4s	548.2	1	721.6	5.05
4	599.6	1	776.6	5.14

<u></u>				
state	T	Р	h	s
1	given	given	h(T, P)	s(T, P)
2s	T(s, P)	$P_2$	h(s, P)	$s_1$
2	T(h, P)	given	$h_1 + (h_{2s} - h_1) / \eta_{s,c}$	s(h, P)
3	given	$P_2$	h(T, P)	s(T, P)
4s	T(s, P)	$P_1$	h(s, P)	$s_3$
4	T(h, P	$P_1$	$h_3 - \eta_{s,t}(h_3 - h_{4s})$	s(h, P)

Table 9.1: Relations needed to solve for the states in a simple air-standard Brayton cycle.

The turbine work output per kg of air is given by

$$W_t = h_3 - h_4 = 219.8 \text{ kJ/kg}$$

The compressor work input is

$$W_c = h_2 - h_1 = 216.4 \text{ kJ/kg}.$$

We see that almost *all* of the turbine work output must be used simply to run the compressor! Only 3.4 kJ/kg is left over to deliver to an external load. In terms of the back work ratio, BWR = 0.98. This is in sharp contrast to the results we obtained for vapor power cycles, which had BWR < 0.01. This is a major difference between these two classes of cycles.

The heat input for this example is

$$Q_h = h_3 - h_2 = 318.5 \text{ kJ/kg},$$

so the overall thermal efficiency is

$$\eta = (W_t - W_c)/Q_h = 0.011.$$

Therefore, this cycle only converts 1.1 % of the heat input into useful work — terrible performance, considering that the Carnot efficiency evaluated between the minimum and maximum temperatures in this cycle is 1 - 300/800 = 0.625, or 62.5%.

Actually, the numbers used in this example are characteristic of the state of the art several decades ago, when gas turbine engines were first being developed. Due to intensive research, it is now possible to build compressors with isentropic efficiency in the range 0.8–0.9, and turbines with isentropic efficiency as high as 0.95. If we re-work the above example with, say,  $\eta_{s,c} = 0.85$  and  $\eta_{s,t} = 0.9$ , we find  $W_{net} = 69.1$  kJ/kg and  $\eta = 0.19$ — quite a substantial improvement.



Figure 9.12: Net work per kg of air and efficiency vs.  $P^*$ .  $T_1 = 300$  K,  $P_1 = 1$  atm,  $T_3 = 800$  K. The dashed curves are for an ideal cycle, and the solid curves for  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ .

#### Effect of Pressure Ratio on Performance

The effect of pressure ratio  $P^* = P_2/P_1$  on  $W_{net}$  and on  $\eta$  for these conditions is shown in Fig. 9.12. For an ideal cycle with  $\eta_{s,c} = \eta_{s,t} = 1$  (dashed curves), the net work per kg of air is maximized for  $P^*$  near 6, while the efficiency continues to increase with  $P^*$ . Taking the more realistic values  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ , the solid curves are obtained. In this case, the net work is maximized near  $P^* = 4$ . The efficiency reaches a maximum of about 20% near  $P^* = 5$ , and falls off at higher  $P^*$ , unlike the ideal case. A *T*-*s* plot for this cycle for  $P^* = 5$  is shown in Fig. 9.13.

The dependence of  $W_{net}$  and  $\eta$  on  $P^*$  shown in Fig. 9.12 for the *ideal* case can be explained as in Fig. 9.14. For the ideal cycle, the enclosed area is  $W_{net}$ . Since  $T_{max} = T_3$  is fixed, the area is maximized for an intermediate pressure ratio. The *efficiency* of an ideal cycle, on the other hand, is given by  $\eta = 1 - \overline{T_c}/\overline{T_h}$ . This is an increasing function of  $P^*$ , and approaches the Carnot efficiency  $1 - T_1/T_{max}$ as  $P^*$  approaches  $P^*_{max}$ , which is the pressure ratio at which the air emerges from the compressor at  $T_{max}$ .

### The Effect of Turbine Inlet Temperature on Performance

Another important development in the last couple of decades has been in advanced high-temperature materials for turbine blades, and development of blade cooling techniques. In the most advanced current gas turbines, turbine inlet temperatures up to 1700 K can be tolerated. If we recompute the above cycle performance using, say,  $T_3 = 1400$  K, we find BWR = 0.41,  $W_{net} = 261.4$  kJ/kg,



Figure 9.13: Process representation for simple Brayton cycle with  $T_1 = 300$  K,  $P_1 = 1$  atm,  $P^* = 5$ ,  $T_3 = 800$  K,  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ .



Figure 9.14: An intermediate  $P^*$  maximizes the enclosed area  $W_{net}$  for fixed  $T_{max}$ .



Figure 9.15: Net work per kg of air and efficiency vs.  $P^*$ .  $T_1 = 300$  K,  $P_1 = 1$  atm,  $T_3 = 1400$  K. The dashed curves are for the ideal cycle, and the solid curves for  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ .

and  $\eta = 0.25$ . For this higher value of  $T_3$ , the effect of pressure ratio is as shown in Fig. 9.15. The optimal pressure ratio is higher — a value of  $P^* = 10$  would be a reasonable choice. This choice would result in an efficiency of 34.9%, and  $W_{net} = 315.3 \text{ kJ/kg}$ . The *T-s* plot for this case is shown in Fig. 9.16.

#### Analysis assuming ideal gas with constant $c_p$

Since the peak pressure in a gas-turbine cycle is typically not too high (say 10 atm), air-standard cycles are often analyzed by treating air as an ideal gas. For even greater simplicity, the specific heat is often assumed constant. The ideal gas approximation should be fairly good at these pressures (cf. Fig. 4.11), but the assumption of constant  $c_p$  is more questionable, due to the large temperature variation around the cycle. Let's examine how much error is made for the conditions of Fig. 9.15.

If we assume an ideal gas with constant  $c_p$ , then

$$h(T) - h(T_0) = c_p(T - T_0)$$
(9.23)

and

$$s(T, P) - s(T_0, P_0) = c_p \ln\left(\frac{T}{T_0}\right) - R \ln\left(\frac{P}{P_0}\right).$$
 (9.24)

Therefore, for the isentropic steps such as  $1 \rightarrow 2s$ ,

$$c_p \ln\left(\frac{T_{2s}}{T_1}\right) = R \ln\left(\frac{P_{2s}}{P_1}\right). \tag{9.25}$$

Since  $R/c_p = (c_p - c_v)/c_p = 1 - c_v/c_p$  and  $P_{2s}/P_1 = P^*$ , this equation may be



Figure 9.16: Process representation for a simple Brayton cycle with  $T_1 = 300$  K,  $P_1 = 1$  atm,  $P^* = 10$ ,  $T_3 = 1400$  K,  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ . Performance:  $W_{net} = 315.3$  kJ/kg,  $\eta = 0.349$ .

written

$$T_{2s} = T_1(P^*)^{1-1/k}, (9.26)$$

where the specific heat ratio k is defined as

$$k = \frac{c_p}{c_v}.\tag{9.27}$$

Similarly,

$$T_{4s} = T_3 (1/P^*)^{1-1/k}.$$
(9.28)

These expressions may be used instead of TPX to solve for the temperatures and enthalpies in Table 9.1. For nitrogen at room temperature,  $c_p = 1.04 \text{ kJ/kg}$ , and k = (7/2)/(5/2) = 1.4 (for air,  $c_p = 1.004 \text{ kJ/kg}$ , and k is also 1.4). If we re-do the cycle analysis as a function of  $P^*$  with the ideal gas, constant  $c_p$ approximation, we come up with the dashed curves in Fig. 9.17, which are compared to the results using TPX (solid curves). The approximation is seen to be not too bad; certainly the trends with  $P^*$  are well-predicted. The major effect is an underprediction of  $W_{net}$  by roughly 10%, and a slight over-prediction of  $\eta$ .

## 9.4.2 The Regenerative Brayton Cycle

Although the cycle of Fig. 9.16 is far more efficient than the one we first considered (34.9% vs. 1.1%), more improvements are still possible. Note that due to the high turbine inlet temperature of 1400 K, the turbine exhaust (state 4) is still quite hot — 847 K. Since this temperature is higher than the air temperature at the exit of the compressor (state 2; 624 K), it should be possible to



Figure 9.17: Comparison of results calculated assuming ideal gas with constant  $c_p$  to results calculated using TPX, for cycle conditions of Fig. 9.15.



Figure 9.18: The regenerative air-standard Brayton cycle.

5				
state	T	P	h	s
1	given	given	h(T, P)	s(T, P)
2s	T(s, P)	$P_2$	h(s, P)	$s_1$
2	T(h, P)	given	$h_1 + (h_{2s} - h_1)/\eta_{s,c}$	s(h,P)
3	T(h, P)	given	$h_2+\epsilon_r(h_5-h_2)$	s(h,P)
4	given	$P_2$	h(T, P)	s(T, P)
5s	T(s, P)	$P_1$	h(s, P)	$s_4$
5	T(h, P)	$P_1$	$h_4 - \eta_{s,t}(h_4 - h_{5s})$	s(h,P)
6	T(h, P)	$P_1$	$h_5 - (h_3 - h_2)$	s(h,P)

Table 9.2: Relations needed to solve for the states in a regenerative air-standard Brayton cycle.

use the turbine exhaust to pre-heat the air entering the heater, thus requiring less external heat input  $Q_h$  to reach 1400 K. This can be done using a heat exchanger, as shown in Fig. 9.18. Since the work output is unchanged by this modification, the efficiency  $W_{net}/Q_h$  will increase.

The heat exchanger, called the *regenerator*, transfers heat from the hot turbine exhaust to the air emerging from the compressor. The heat transfer in the regenerator per kg of air is

$$Q_r = h_3 - h_2 (9.29)$$

$$= h_5 - h_6. (9.30)$$

If the regenerator were infinitely long, the air leaving it at state 3 would have the same temperature as the hot air entering at state 5. Thus, the *most* heat transfer which could occur in the regenerator is given by

$$Q_{r,max} = h_5 - h_2. (9.31)$$

It is impractical to build a regenerator so large that  $T_3$  closely approaches  $T_5$ — in practice,  $T_3$  will always be less than  $T_5$ , and therefore  $Q_r < Q_{r,max}$ .

The regenerator effectiveness is defined by

$$\epsilon_r = \frac{Q_r}{Q_{r,max}} = \frac{h_3 - h_2}{h_5 - h_2}.$$
(9.32)

A typical realistic value for  $\epsilon_r$  would be about 0.8. With  $\epsilon_r$  specified, this equation may be used to solve for  $h_3$ . The complete set of relations needed to solve the regenerative air-standard Brayton cycle is given in Table 9.2.



Figure 9.19: Net work per kg of air and efficiency vs.  $P^*$ .  $T_1 = 300$  K,  $P_1 = 1$  atm,  $T_3 = 1400$  K,  $\epsilon_r = 0.8$ . The dashed curves are for the ideal cycle, and the solid curves for  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ .

If we add a regenerator with  $\epsilon_r = 0.8$  to the simple Brayton cycle of Fig. 9.15, and use TPX to calculate the state properties, we find the performance characteristics shown in Fig. 9.19. The net work is unchanged from Fig. 9.15, but the efficiency is much different. For the ideal case ( $\epsilon_r = \eta_{s,c} = \eta_{s,t} = 1$ ), the efficiency is maximized at  $P^* = 1$  (although of course  $W_{net}$  is zero there).

For the non-ideal case, the efficiency is maximized near  $P^* = 5$ , at a value of 46.9%. This is much better than the simple Brayton cycle, which from Fig. 9.15 has an efficiency of 27.5% at  $P^* = 5$ . Adding a regenerator is seen to produce a big increase in efficiency, particularly at lower  $P^*$  values, where the difference  $(T_5 - T_2)$  is largest.

Based upon the performance results shown in Fig. 9.19, the best choice for  $P^*$  would likely be between 5 and 10, depending on whether maximizing  $W_{net}$  or  $\eta$  was more important. The *T*-s plot for  $P^* = 10$  is shown in Fig. 9.20. Note that this is the same as Fig. 9.16, except for the location of the state points.

#### 9.4.3 Intercooling and Reheat

Two more modifications are often used. In order to reduce the compressor work requirement, the compression process may be broken up into two or more stages, with heat removal between stages. This process is known as *intercooling*.



Figure 9.20: Process representation for a regenerative Brayton cycle with  $T_1 = 300$  K,  $P_1 = 1$  atm,  $P^* = 10$ ,  $T_3 = 1400$  K,  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ ,  $\epsilon_r = 0.8$ . Performance:  $W_{net} = 315.3$  kJ/kg,  $\eta = 0.447$ .



That this lowers the work requirement is easiest to see by considering an and h-s process representation of an ideal, 2-stage compressor (below). With no heat removal between stages, the total compressor work to compress from  $P_1$  to  $P_2$  is  $W_A + W_B$ . With intercooling the work is  $W_A + W_C$ . Since constant-pressure lines on an h-s plot diverge (slope = T),  $W_C < W_B$  and thus the compressor work is lower with intercooling.



Reheat is the same process as for vapor power cycles — heat is added between turbine stages. Both intercooling and reheat increase  $W_{net}$ , but may or may not



Figure 9.21: Net work per kg of air and efficiency vs.  $P^*$  for regenerative Brayton cycle with intercooling and reheat.

improve the efficiency. If they are used together with regeneration, significant gains in efficiency are possible, since they increase the potential for regeneration.

In Fig. 9.21, the performance of a regenerative cycle with one stage of intercooling and one stage of reheat is presented. Most of the parameters are the same as in Fig. 9.19. The intercooler effectiveness is 0.8, and the reheat temperature is 1400 K. The intercooling and reheat pressures are both taken to be  $(P^*)^{1/2}$ . (For an ideal cycle and assuming constant  $c_p$ , it can be shown this is the optimal value.)

It can be seen that the performance is quite good. The net work is above 400 kJ/kg, and above  $P^* = 5$  the efficiency is slightly greater than 50%.



Figure 9.22: Process representation for a regenerative Brayton cycle with intercooling and reheat.  $T_1 = 300$  K,  $P_1 = 1$  atm,  $P^* = 10$ ,  $P_i = P_r = 3.16$  atm,  $T_3 = 1400$  K,  $\eta_{s,c} = 0.85$ ,  $\eta_{s,t} = 0.9$ ,  $\epsilon_r = \epsilon_i = 0.8$ .