

CHAPTER 7

ENTROPY ACCOUNTING AND APPLICATIONS

7.1 Introduction

In the last chapter, we found two important properties of the entropy:

1. Entropy can be *produced* by irreversible processes, but can never be destroyed: $\mathcal{P}_s \geq 0$ for every possible process.
2. When heat Q enters or leaves a system at temperature T , it carries entropy in the amount Q/T with it.

With these two results, it is possible to write down entropy accounting expressions for closed or open systems. In this chapter, we will develop these expressions, and look at several types of problems which can be solved by combining energy and entropy accounting.

7.2 Some General Accounting Principles

We must take some care in developing entropy accounting expressions, since, unlike energy, entropy is not conserved. First let's consider in general how to do accounting for things which can be produced. Consider the monthly statement you might receive from a bank if you have an interest-bearing bank account. The account balance at the end of a month is related to the previous month's balance by

$$(\text{new balance}) = (\text{old balance}) + (\text{deposits}) - (\text{withdrawals}) + (\text{interest}). \quad (7.1)$$

The account balance is the amount of money “stored” in the account; the deposits represent an “inflow” of money, the withdrawals an “outflow” of money, and we can think of the interest as money “produced” within the account during the month, since it appears in the account even though you didn't deposit it. Therefore, the accounting principle for a bank account is

$$\Delta(\text{stored money}) = (\text{money inflow}) - (\text{money outflow}) + (\text{money production}). \quad (7.2)$$

Of course, this basic accounting principle applies to *anything* which can be stored, transported, and produced. Suppose a factory manufactures a product. The change in the inventory stored at the factory over some period of time equals the number of products produced in that period, plus the number returned from stores (inflow to the factory), minus the number shipped to stores (outflow from the factory). So Eq. (7.2) can be written more generally:

$$\Delta(\text{storage}) = (\text{inflow}) - (\text{outflow}) + (\text{production}). \quad (7.3)$$

This general accounting principle can be applied to the storage, transportation, and production of physical quantities like mass, momentum, energy, and entropy. For mass and energy, the production term is always zero, since these are conserved. Therefore, the energy balances we developed in Chapters 2 and 4 were always of the form

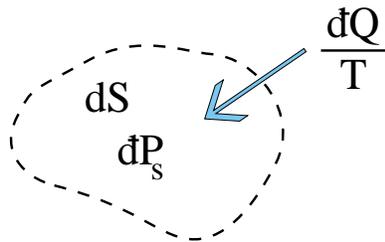
$$\Delta(\text{energy stored in system}) = (\text{energy inflow}) - (\text{energy outflow}). \quad (7.4)$$

For entropy, the production term is non-zero, so we have

$$\begin{aligned} \Delta(\text{entropy stored in system}) &= (\text{entropy inflow}) - (\text{entropy outflow}) \\ &+ (\text{entropy production}). \end{aligned} \quad (7.5)$$

7.3 Entropy Accounting for a Closed System

For a closed system, the only way entropy can enter or leave the system is with heat which enters or leaves. The simplest case occurs when heat enters at a single temperature, as shown below.



During elapsed time dt , heat dQ crosses the system boundary where the temperature is T . We saw in the last chapter that when heat dQ enters a system at temperature T , it brings with it entropy in the amount dQ/T . Therefore, the entropy inflow is dQ/T , and there is no entropy outflow.

During the elapsed time dt , entropy in the amount $d\mathcal{P}_s$ may have been also *produced* within the system due to internal irreversible processes (friction, viscous fluid flow, electrical current flow, unrestrained expansion, etc.). Therefore, according to Eq. (7.5), the increase in the amount of entropy stored within the system is

$$dS = \frac{dQ}{T} + d\mathcal{P}_s. \tag{7.6}$$

Note that work dW might have also been done on or by the system during dt , but since energy transfer as work does not carry entropy, it does not enter into the entropy accounting expression.

Since the second law requires $d\mathcal{P}_s \geq 0$,

$$dS \geq \frac{dQ}{T}. \tag{7.7}$$

Thus, the system entropy increases by *more* than the amount which enters with heat; the difference is what is produced inside. Equation (7.7) also holds if $dQ < 0$, in which case it says the the entropy decreases by less than the amount of entropy which leaves with heat. For a reversible process, Eq. (7.7) reduces to

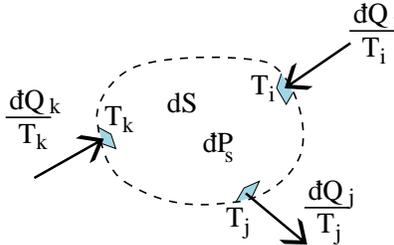
$$dQ_{rev} = TdS_{rev}. \tag{7.8}$$

From Eq. (7.6), a process which is both adiabatic ($dQ = 0$) and reversible ($d\mathcal{P}_s = 0$) would not change the entropy ($dS = 0$). Therefore,

the entropy is constant during any reversible, adiabatic process in a closed system.

We call processes which occur at constant entropy *isentropic*.

Equation (7.6) can be easily generalized to the case where heat flows across the boundary at various spots, each of which may have a different temperature (which might even be time-dependent).



In time dt , the total entropy S of the system changes due to the entropy entering or leaving with heat, and the entropy produced by irreversible processes

inside. If heat enters through I places, and leaves through O places, then

$$dS = \sum_{\text{in}} \left(\frac{dQ_{\text{in}}}{T_{\text{in}}} \right) - \sum_{\text{out}} \left(\frac{dQ_{\text{out}}}{T_{\text{out}}} \right) + d\mathcal{P}_s. \quad (7.9)$$

Note that the dQ/T terms are evaluated *at the temperature on the system boundary where the heat flow is occurring*.

Equation (7.9) can also be written on a per-unit-time basis (a rate basis). Dividing by dt ,

$$\frac{dS}{dt} = \sum_{\text{in}} \left(\frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} \right) - \sum_{\text{out}} \left(\frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} \right) + \dot{\mathcal{P}}_s. \quad (7.10)$$

If the temperatures on the boundaries where heat is entering or leaving are constant in time, this equation can be integrated for a finite time interval $(0, t)$:

$$\Delta S = S(t) - S(0) = \sum_{\text{in}} \left(\frac{Q_{\text{in}}}{T_{\text{in}}} \right) - \sum_{\text{out}} \left(\frac{Q_{\text{out}}}{T_{\text{out}}} \right) + \mathcal{P}_s. \quad (7.11)$$

Here each Q is just the total heat which enters: $Q = \int_0^t dQ$.

Equations (7.9) – (7.10) all are simply statements of entropy accounting. They equate the net change in the total entropy of a system to the *net* entropy inflow (inflow - outflow) plus the amount of entropy produced inside the system. They apply to any closed (control mass) system.

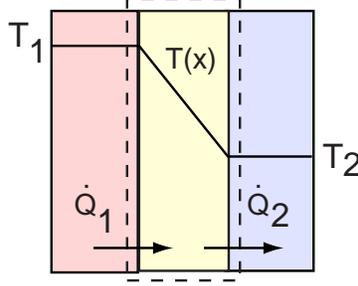
In many (but not all) processes we will consider, the system is in steady state. In this case, the entropy contained within the system is not changing, so on a rate basis $dS/dt = 0$, or on a total time basis $\Delta S = 0$. For a steady-state problem, Eq. (7.10) reduces to

$$\sum_{\text{in}} \left(\frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} \right) - \sum_{\text{out}} \left(\frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} \right) + \dot{\mathcal{P}}_s = 0. \quad (7.12)$$

A great deal can be learned about real processes by carrying out energy and entropy accounting. Let's consider a few examples of processes occurring in closed systems.

7.4 Heat Transfer Through Finite ΔT

Consider the situation shown below. Heat is flowing from a high-temperature region at T_1 to low-temperature region at T_2 through a thin slab of insulation material, which has thickness L and area A . The temperature is everywhere constant in time, and the insulation is in steady state.



From the theory of heat conduction in solids, the heat flow through the insulation is related to the temperature difference across it by

$$\dot{Q} = \frac{\kappa A}{L} (T_1 - T_2), \quad (7.13)$$

where κ is a material property called the *thermal conductivity* (SI units: W/m-K). This equation is analogous to $I = V/R$ for current flow through a resistor, with \dot{Q} playing the role of current, ΔT the potential difference, and $L/\kappa A$ the resistance. For this reason, $L/\kappa A$ is called the *thermal resistance* R_{th} of the insulation.

We will define a system to consist of the insulation, with the left system boundary at T_1 and the right one at T_2 . We will assume the heat flow is one dimensional, so heat only crosses the system boundaries perpendicular to x . Since the insulation is in steady state, no energy or entropy are building up in the system: $dU/dt = 0$ and $dS/dt = 0$. Therefore, the energy balance reduces to

$$\dot{Q}_h = \dot{Q}_c, \quad (7.14)$$

and so we may drop the subscripts and call the heat transfer rate \dot{Q} .

Applying Eq. (7.10) with $dS/dt = 0$ and one heat inflow and one heat outflow, the entropy accounting expression becomes

$$0 = \frac{\dot{Q}}{T_1} - \frac{\dot{Q}}{T_2} + \dot{\mathcal{P}}_s. \quad (7.15)$$

Solving for $\dot{\mathcal{P}}_s$,

$$\dot{\mathcal{P}}_s = \dot{Q} \left(\frac{1}{T_2} - \frac{1}{T_1} \right). \quad (7.16)$$

Since $T_1 > T_2$, $\dot{\mathcal{P}}_s > 0$, in accord with the Second Law. Note that as $T_2 \rightarrow T_1$, the ratio $\dot{\mathcal{P}}_s/\dot{Q} \rightarrow 0$. Therefore, the amount of entropy produced per unit of heat transferred can be made arbitrarily small by making $\Delta T = T_1 - T_2$ sufficiently small. We conclude that heat transfer through a finite ΔT is *irreversible* (produces entropy), but in the limit as T_2 approaches T_1 , the process approaches reversibility.

Of course, from Eq. (7.13), decreasing ΔT decreases the heat transfer rate, and as $T_2 \rightarrow T_1$ $\dot{Q} \rightarrow 0$. Therefore, in the reversible limit transferring a finite amount of heat would take infinite time.

7.5 Heat Engines

The original engineering problem which led to the development of thermodynamics was how to efficiently use the thermal energy of a fire (produced by burning wood or coal) to do useful work, for example to turn the wheels of a locomotive.

It's easy to design a process which can take in heat from a high-temperature source and produce work for a little while, but much harder to design one which can go on doing it indefinitely, which is what is really desired. For example, gas in a cylinder fitted with a piston will expand when heated, pushing the piston, doing work against an external load. This is fine until the piston has reached the top of the cylinder, at which time the process must stop. Figuring out how to get the piston back to its initial position to start the "cycle" again without having to put more work in than you get out is the challenge.

A *heat engine* is a device which operates either steadily or in a cycle, which takes in energy as heat and converts at least part of it into useful work. Heat engines are a vital part of modern technology. Most current and proposed electric power plants are heat engines, which take in heat at high temperature given off by a chemical or nuclear reaction, and convert part of this heat into electrical work delivered to the grid. Even very sophisticated concepts such as nuclear fusion are simply different sources of heat — a heat engine of some sort is still needed to convert this to useful work.¹ Automobile, aircraft, and rocket engines are other examples of practical heat engines.

Since a heat engine operates either continuously or in a cycle, in which it returns to its initial state after some time period T , we can use a steady-

¹One promising power plant concept which is not a heat engine is a fuel cell, which converts the chemical energy in fuel directly into electrical energy, without first converting it into thermal energy in a flame. Thermodynamics still governs fuel cell performance, however, as we'll discuss later.

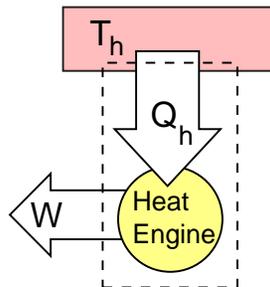


Figure 7.1: A hypothetical one-temperature heat engine.

state analysis to study heat engines. For continuous operation, $dE/dt = 0$ and $dS/dt = 0$, while for cyclic operation we will choose the time period T for the analysis, in which case $\Delta E = 0$ and $\Delta S = 0$. In the following, we will assume continuous operation, but the results apply equally for cyclic operation.

To analyze heat engines, it is useful to introduce the conceptual device of a *thermal reservoir*. A thermal reservoir is defined as a substance with a very large heat capacity, so that heat can be taken from it or added to it without significantly perturbing its temperature. Furthermore, it has a very large thermal conductivity, so that transferring heat to or from it causes only negligible temperature differences within it [Eq. (7.13)]. A large block of copper or a large lake would *approximate* a thermal reservoir.

In the limit where the heat capacity and thermal conductivity become infinite, the temperature of a thermal reservoir remains perfectly uniform, and constant in time, no matter how much heat is added or removed. In this limit, heat flow *within* the reservoir is perfectly reversible. (The electrical analog of a thermal reservoir would be an infinitely-large perfect conductor, which would have constant electrical potential even if current were being drawn from it.)

7.5.1 A (Hypothetical) One-Temperature Heat Engine

Consider the heat engine shown in Fig. 7.1. The engine takes in heat \dot{Q}_h from a thermal reservoir at temperature T_h , and outputs power \dot{W} . Since the engine is in steady state, energy accounting yields

$$\dot{Q}_h = \dot{W} \quad (7.17)$$

and entropy accounting [Eq. (7.12)] yields

$$\frac{\dot{Q}_h}{T_h} + \dot{\mathcal{P}}_s = 0. \quad (7.18)$$

Since the Second Law requires $\dot{\mathcal{P}}_s \geq 0$ and also $T_h > 0$, Eq. (7.18) requires $\dot{Q}_h < 0$, and therefore $\dot{W} < 0$.

Thus, it is impossible to produce positive power *by any process* which only takes in heat at a single temperature T_h .

It is not hard to see why this doesn't work. Entropy is entering the engine at a rate \dot{Q}_h/T_h , but no entropy is leaving. (Remember work does not carry entropy with it.) Thus, operation of this engine would require entropy to be destroyed inside the engine, but that violates the Second Law. Therefore, we conclude

a one-temperature heat engine cannot be built.

This statement is sometimes called the *Kelvin-Planck statement of the Second Law*. In some classical treatments of thermodynamics which make no reference to the existence of atoms or microstates, this is introduced as a *postulate*. With this postulate, it is possible to work “backwards” (from our point of view) using some very clever thought experiments to prove that there must exist a property, the entropy, which has all of the macroscopic characteristics we've discussed already.

7.5.2 A Two-Temperature Heat Engine

To allow production of positive power, some means of continually removing entropy from the engine must be provided. The only way to do this in a closed system is to remove some heat from the engine, which will carry entropy with it. Since we want to convert at least part of \dot{Q}_h to \dot{W} , we can only afford to remove a portion of \dot{Q}_h . But we need to remove *at least as much* entropy as entered (since some might have also been produced inside), so we need to remove heat in such a way that the ratio (entropy flow)/(heat flow) is larger going out than coming in. Since the amount of entropy carried with a unit amount of heat flow is $1/T$, this suggests we should try rejecting some heat to a reservoir at a cold temperature T_c (large $1/T_c$).

Doing energy and entropy accounting for the two-temperature heat engine in Fig. 7.2, we have

$$\dot{Q}_h = \dot{W} + \dot{Q}_c, \quad (7.19)$$

and

$$\frac{\dot{Q}_h}{T_h} + \dot{\mathcal{P}}_s = \frac{\dot{Q}_c}{T_c}. \quad (7.20)$$

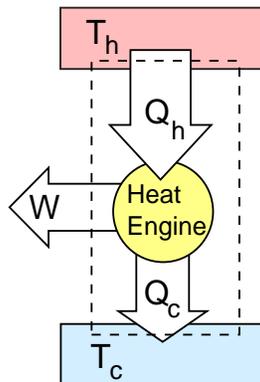


Figure 7.2: A two-temperature heat engine.

Eliminating \dot{Q}_c between these equations,

$$\dot{W} = \dot{Q}_h \left(1 - \frac{T_c}{T_h} \right) - T_c \dot{\mathcal{P}}_s. \quad (7.21)$$

We see that positive power *can* be produced by a two-temperature heat engine. Since the second law requires $\dot{\mathcal{P}}_s \geq 0$, the *maximum* power for a given \dot{Q}_h is produced for $\dot{\mathcal{P}}_s = 0$. Therefore a reversible heat engine produces the most power possible for a given heat transfer rate \dot{Q}_h , and given T_h and T_c . For a real engine, the term $-T_c \dot{\mathcal{P}}_s$ represents power “lost” due to irreversibilities.

The *thermal efficiency* η of a heat engine is defined as

$$\eta = \frac{\dot{W}}{\dot{Q}_h}. \quad (7.22)$$

This definition makes sense, since the desired output is \dot{W} , and typically it is \dot{Q}_h which you must “pay for” in the cost of fuel burned to produce it.

A reversible heat engine has the highest thermal efficiency of any heat engine operating between T_h and T_c . From Eq. (7.21), this limiting value of the efficiency is

$$\eta^{max} = \left(1 - \frac{T_c}{T_h} \right), \quad (7.23)$$

This upper-limit efficiency is often called the *Carnot efficiency*, after Sadi Carnot, a French engineer who was the first person to systematically analyze heat engines in the 19th century. Note that it depends only on T_h and T_c , and is the same for all possible heat engines operating between these temperatures.

Most practical engines and generators, including automotive engines, aircraft engines, and electric power generation facilities, can be approximated as

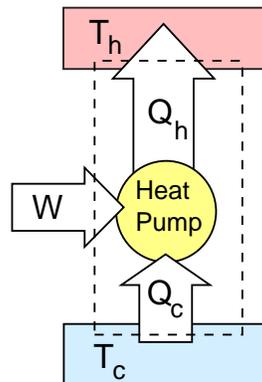


Figure 7.3: A heat pump or refrigerator “pumps” heat from low temperature to high temperature.

two-temperature heat engines. In order to produce a fuel-efficient engine, η should be as large as practically possible. Of course, the first thing to do is to try to eliminate as much unnecessary entropy production within the engine as possible, to minimize the term $-T_c \dot{\mathcal{P}}_s$. But once this has been done to the extent practical, further increases in η require either increasing T_h , or decreasing T_c .

For most engines, T_c is the temperature of the ambient surroundings and can't be easily varied. This leaves increasing T_h . A flame can easily be used to produce a temperature of, say, 2000 K. If we use $T_h = 2000$ K and $T_c = 300$ K, we find $\eta^{max} = 1 - 300/2000 = 0.85$. This is a very high efficiency, but unfortunately not many materials can withstand this temperature for long. Metals (which turbines are usually made of) fail at much lower temperatures, so the peak temperature in a practical engine is usually limited by materials constraints. Some of the most significant developments in recent years have been ceramic components in engines which can take much higher temperatures and allow designing engines with higher T_h .

7.6 Heat Pumps and Refrigerators

Although heat will not spontaneously flow from low temperature to high, a process can be designed which takes in heat at low temperature and expels heat at high temperature. However, such a process will not run by itself — it requires work input to “pump” the heat from low to high temperature.

A *heat pump* is a device which operates either continuously or in a cycle,

which takes in heat at low temperature and expels it at higher temperature (Fig. 7.3). A refrigerator is a common example of a heat pump. Heat continually leaks into the refrigerated compartment from the surroundings through the insulation, or when the door is opened. This heat must be removed and expelled to the surroundings at higher temperature to keep the refrigerated space cold.

We may analyze heat pumps assuming steady-state, as we did to analyze heat engines, since they operate continuously ($dS/dt = 0$) or in a cycle ($S(T) - S(0) = 0$). For the heat pump shown in Fig. 7.3, energy accounting yields

$$\dot{W} + \dot{Q}_c = \dot{Q}_h, \tag{7.24}$$

and entropy accounting yields

$$\frac{\dot{Q}_c}{T_c} - \frac{\dot{Q}_h}{T_h} + \dot{\mathcal{P}}_s = 0. \tag{7.25}$$

If the heat pump is used as a refrigerator, what we really care about is how much power \dot{W} we have to supply to run the heat pump for a given rate of heat removal \dot{Q}_c from the cold space. In this case, eliminating \dot{Q}_h between equations (7.24) and (7.25),

$$\dot{W} = \dot{Q}_c \left(\frac{T_h}{T_c} - 1 \right) + T_h \dot{\mathcal{P}}_s. \tag{7.26}$$

Since the Second Law requires $\dot{\mathcal{P}}_s \geq 0$, clearly a reversible heat pump ($\dot{\mathcal{P}}_s = 0$) requires the least power input for given \dot{Q}_c , T_c , and T_h . Note that if \dot{W} were set to zero in Eq. (7.26), there would be no solution with $\dot{Q}_c > 0$, even if the heat pump were reversible. Therefore,

a zero-work heat pump is impossible to build.

This statement is sometimes called the *Clausius statement of the Second Law*. Like the Kelvin-Planck statement, it is sometimes introduced as a postulate in purely macroscopic treatments of thermodynamics, from which the existence of entropy is inferred.

The second law requires $\dot{\mathcal{P}}_s \geq 0$, so for a given \dot{Q}_c , the *minimum* power required is obtained when $\dot{\mathcal{P}}_s = 0$. Therefore, a hypothetical heat pump which produces no entropy (a reversible heat pump) would require the least work:

$$\dot{W}_{min} = \dot{Q}_c \left(\frac{T_h}{T_c} - 1 \right). \tag{7.27}$$

Since no real refrigerator is truly reversible, the actual power required to run it will be greater than this value. The term $T_h \dot{\mathcal{P}}_s$ in Eq. (7.26) represents the

“extra” work a real refrigerator does, due to irreversibilities such as friction and finite-temperature-difference heat transfer. Design of an energy-efficient refrigerator would require a careful thermodynamic analysis to find where entropy is being produced, and how entropy production could be minimized.

The performance of a refrigerator is characterized by the *coefficient of performance*, defined as the watts of heat removed from the cold space per watt of power input: $\text{COP}_{ref} = \dot{Q}_c/\dot{W}$. From Eq. (7.27), the maximum value is

$$\text{COP}_{ref}^{(max)} = \frac{1}{T_h/T_c - 1}. \quad (7.28)$$

Note that this can be greater than one: more energy can be removed as heat than is required as input to run the process.

An interesting thing about this equation is that it only depends on T_h and T_c , not on the details of how we implement the heat pump – *every* possible refrigerator design has the same limiting COP as it approaches reversibility, no matter whether the working substance is a fluid, or a magnetic crystal, or anything else.

If a heat pump is used to heat a house by pumping heat \dot{Q}_h into the house at T_h from the outside surroundings at T_c , then eqs. (7.24) and (7.25) still apply, but we are most concerned with the ratio \dot{Q}_h/\dot{W} . If we eliminate \dot{Q}_c between equations (7.24) and (7.25), we find

$$\dot{W} = \dot{Q}_h \left(1 - \frac{T_c}{T_h} \right) + T_c \dot{\mathcal{P}}_s. \quad (7.29)$$

The performance parameter which is used to characterize heat pumps used for heating is also called the coefficient of performance, but it is defined slightly differently:

$$(\text{COP})_{hp} = \frac{\dot{Q}_h}{\dot{W}}. \quad (7.30)$$

For a reversible heat pump the coefficient of performance is maximized:

$$(\text{COP})_{hp}^{(max)} = \frac{1}{1 - T_c/T_h}. \quad (7.31)$$

7.6.1 The Carnot Cycle

We’ve shown based on very general considerations that no heat engine can have an efficiency greater than $1 - T_c/T_h$. In deriving this result, we didn’t have to say anything about *how* the engine operates – it is a general result, which applies to all conceivable heat engines.

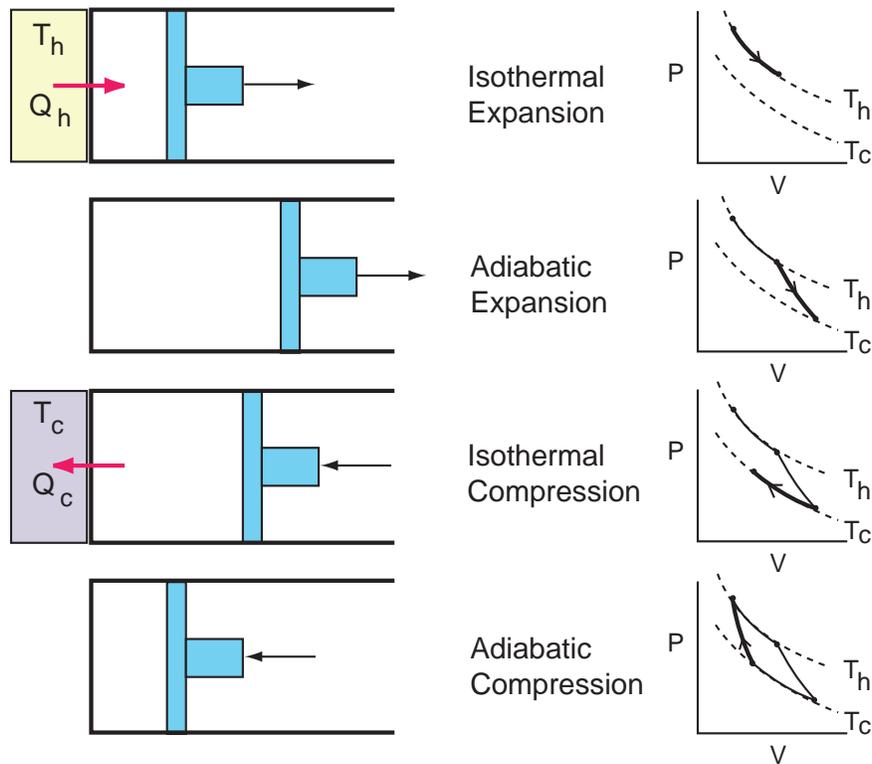


Figure 7.4: The Carnot Cycle two-temperature heat engine. Each step is performed quasi-statically, and therefore the process is reversible. By reversing the cycle, it also can function as a heat pump.

It's instructive to consider a simple idealized, reversible cycle which achieves this limiting efficiency. While not a very practical design for an engine, this cycle, known as the *Carnot cycle*, is conceptually the simplest possible reversible two-temperature heat engine. By running it in reverse, it functions as a heat pump.

In a Carnot-cycle engine, a fluid (not necessarily an ideal gas) is placed in a piston/cylinder system. The following 4 steps are executed in a reversible manner in sequence:

1. The cylinder is placed in contact with a thermal reservoir at T_h , and the fluid is expanded isothermally, drawing in heat Q_h .
2. Now the thermal reservoir is removed, and the expansion continues adiabatically. Since no heat is being added in this step, but the fluid is doing

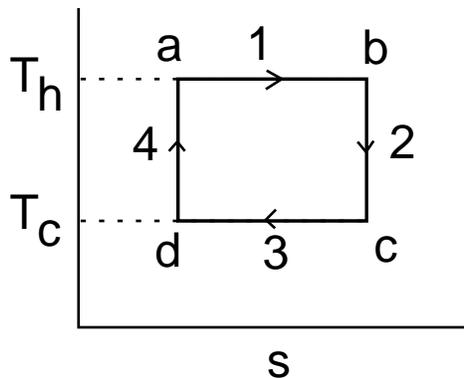


Figure 7.5: The Carnot cycle as represented on a $T - S$ plot.

work, its temperature drops. This step continues until the temperature drops to T_c .

3. Now a thermal reservoir at T_c is placed in contact with the cylinder, and the the fluid is isothermally compressed, expelling heat Q_c .
4. Finally, the thermal reservoir is removed, and the compression continues adiabatically, ending with the piston reaching its initial position and the fluid temperature back at T_h .

In Fig. 7.4, this process is represented on a $P - V$ plot, assuming the fluid is a gas. Since the process is carried out quasi-statically, the fluid remains in equilibrium at all times during the cycle. Therefore, its state may be represented at every time by a point in the $P - V$ plane, and the entire cycle is a closed curve in the $P - V$ plane.

A simpler representation of this cycle is a temperature-entropy ($T - S$) plot, as shown in Fig. 7.5.

The Carnot cycle traces out a rectangle on a $T - S$ plot, and this is true no matter what the fluid is. Consider first steps 2 and 4, which are both reversible and adiabatic. We have shown that reversible, adiabatic processes are isentropic, so the entropy of the fluid does not change during steps 2 and 4. They must appear as vertical lines on a $T - S$ plot.

Steps 1 and 3 are isothermal, so they must appear as horizontal lines on a $T - S$ plot. Step 1 draws in heat Q_h , which brings in entropy Q_h/T_h with it. Since the process is reversible, $\mathcal{P}_s = 0$ and therefore

$$\Delta S_1 = S_b - S_a = \frac{Q_h}{T_h}. \quad (7.32)$$

For step 3, heat Q_c is rejected and the entropy of the system decreases by Q_c/T_c :

$$\Delta S_3 = S_a - S_b = -\frac{Q_c}{T_c}. \tag{7.33}$$

Since step 3 brings the system entropy back to its initial value (it is unchanged by step 4), we must have

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h} = \Delta S_1. \tag{7.34}$$

The net heat taken in by one complete cycle is then

$$Q_h - Q_c = T_h \Delta S - T_c \Delta S = (T_h - T_c) \Delta S, \tag{7.35}$$

which is simply the area enclosed by the cycle on the $T - S$ plot.

An energy balance for one complete cycle is

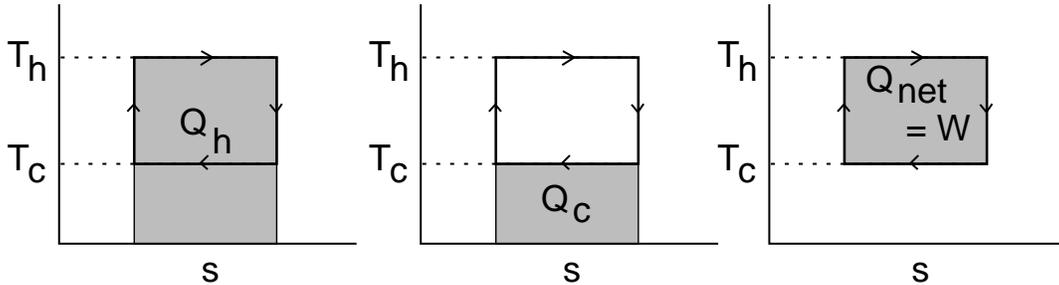
$$Q_h = W_{net} + Q_c, \tag{7.36}$$

where W_{net} is the *net* work output for the entire cycle. (If we denote the work output done in step n by W_n , then $W_{net} = W_1 + W_2 + W_3 + W_4$, where W_1 and W_2 are positive, and W_3 and W_4 are negative.) Therefore,

$$W_{net} = Q_h - Q_c = (T_h - T_c) \Delta S \tag{7.37}$$

is also given by the area enclosed on a $T - S$ plot.

We can also relate Q_h , Q_c , and W to areas on the $T - S$ plot:



Then the efficiency $\eta = W/Q_h$ is simply the ratio of the areas of two rectangles, and is easily seen to be $(1 - T_c/T_h)$.

Temperature-entropy plots are very useful for analyzing any reversible cycle. For a reversible process, $dQ = TdS$. Integrating this around a complete cycle, returning to the initial state,

$$Q_{net} = \oint dQ = \oint TdS. \tag{7.38}$$

Thus, the net heat taken in during one complete cycle equals the area enclosed by the cycle on a $T - S$ plot, no matter what the shape of the curve is (not only for a rectangle). From the First Law for a complete cycle,

$$\oint dU = 0 = \oint \bar{d}Q - \oint \bar{d}W \tag{7.39}$$

where we are taking work to be positive which is done *by* the system. Therefore, the net work done during one cycle is also given by the enclosed area:

$$W_{net} = Q_{net} = \oint TdS. \tag{7.40}$$

7.6.2 Maximum Power Output of a Heat Engine

Anyone who has compared the performance and fuel efficiency of a Geo Metro and a Corvette will not be surprised to learn that there is a *trade-off* between the efficiency of an engine and its power output. Let’s look at a simple example which shows this clearly.

Suppose we have a two-temperature heat engine operating between specified T_h and T_c , and we want to maximize the power output

$$\dot{W} = \eta \dot{Q}_h. \tag{7.41}$$

The highest η will be achieved by minimizing $\dot{\mathcal{P}}_s$, in which case η will approach the Carnot efficiency.

To minimize $\dot{\mathcal{P}}_s$, the engine must be run very slowly — in the limit, quasi-statically. This minimizes the work lost to friction, and also insures that heat transfer from the hot reservoir to the engine and from the engine to the cold reservoir is slow enough that large temperature differences (needed to “drive” the heat transfer) don’t develop. But if we run the engine that slowly, \dot{Q}_h will be very small!

The engine will be very *efficient* at converting Q_h into W , but the *rate* of heat extraction \dot{Q}_h will be very low, as will $\dot{W} = \eta \dot{Q}_h$, and in the limit of a reversible process (infinitely slow) both will go to zero. So it seems that building a reversible engine is not what we want to do if we want to maximize power output. The conditions which maximize η don’t maximize the product $\eta \dot{Q}_h$.

To maximize *power* instead of *efficiency*, we must allow some irreversibility. Let’s consider a simple case which only has one type of irreversibility – finite ΔT heat transfer. In reality, this source of irreversibility is inescapable. For example, suppose heat is taken from a reservoir at T_h by passing a fluid through tubes immersed in it. In order for heat to flow from T_h into the fluid in the tubes,

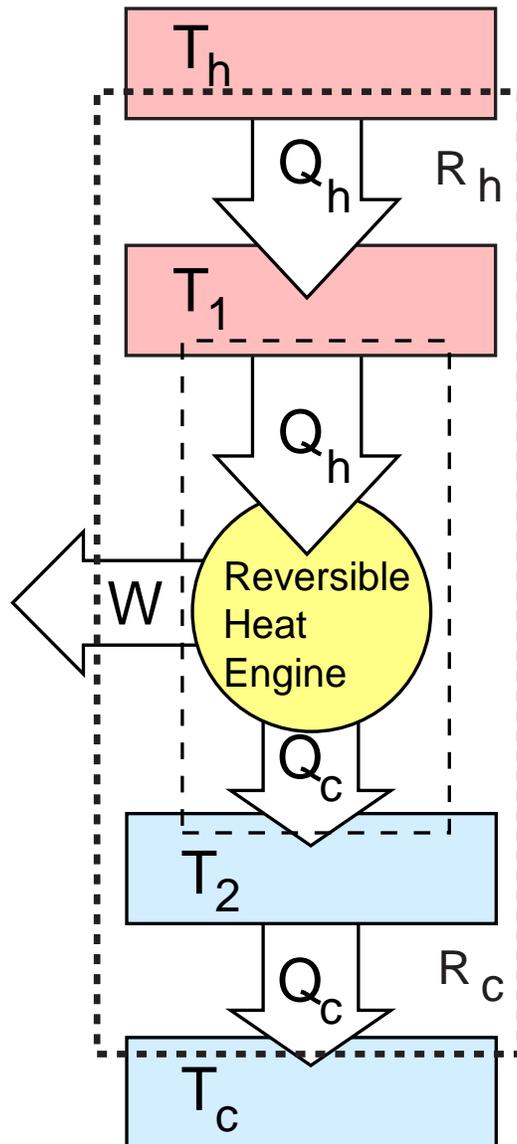


Figure 7.6: An “endoreversible” engine, in which the only irreversibility is due to the thermal resistances for heat transfer.

the fluid temperature must be less than T_h ; call it T_1 . The heat transfer rate is generally proportional to the temperature difference:

$$\dot{Q}_h = \frac{T_h - T_1}{R_h} \quad (7.42)$$

where R_h is the overall thermal resistance, which depends on factors such as the total tube area, the fluid flow rate, and the thermal conductivities of the materials involved. But if these are constant, then $Q_h \propto (T_h - T_1)$.

Also, to reject heat to a reservoir at T_c , the fluid (again flowing through tubes) must be hotter than T_c ; let's say it has temperature $T_2 > T_c$. Then the heat rejection rate will be

$$\dot{Q}_c = \frac{T_2 - T_c}{R_c}, \quad (7.43)$$

where R_c is the thermal resistance for heat transfer to the cold reservoir.

Assume for simplicity that there are no other irreversibilities – the engine is a reversible one, operating between its own maximum and minimum temperatures T_1 and T_2 . An hypothetical engine which is internally reversible, but must transfer heat in or out through thermal resistances is called *endoreversible*, meaning that the interior (*endo*) is reversible.

Since the engine is internally reversible, its efficiency is

$$\eta = \frac{\dot{W}}{\dot{Q}_h} = \left(1 - \frac{T_2}{T_1}\right). \quad (7.44)$$

But since $T_1 < T_h$ for $Q_h > 0$, and $T_2 > T_c$ for $Q_c > 0$, this efficiency is less than the value it would have if it were operating reversibly between T_h and T_c .

Temperatures T_1 and T_2 are not fixed, but change depending on the heat extraction rate. We can use Eq. (7.42) and Eq. (7.43) to substitute for T_1 and T_2 in terms of the constant reservoir temperatures T_h and T_c . If we also invoke the first law ($\dot{Q}_h = \dot{W} + \dot{Q}_c$), we find (after a bit of algebra):

$$\eta = 1 - \frac{T_c}{T_h - \dot{Q}_h(R_h + R_c)}. \quad (7.45)$$

As we increase the rate at which we take heat from the hot reservoir, the engine efficiency goes down, since T_1 decreases and T_2 increases. If we keep increasing \dot{Q}_h , eventually T_1 and T_2 approach each other, and $\eta \rightarrow 0$. From Eq. (7.45), this occurs when

$$\dot{Q}_h^{(max)} = \frac{T_h - T_c}{R_h + R_c}. \quad (7.46)$$

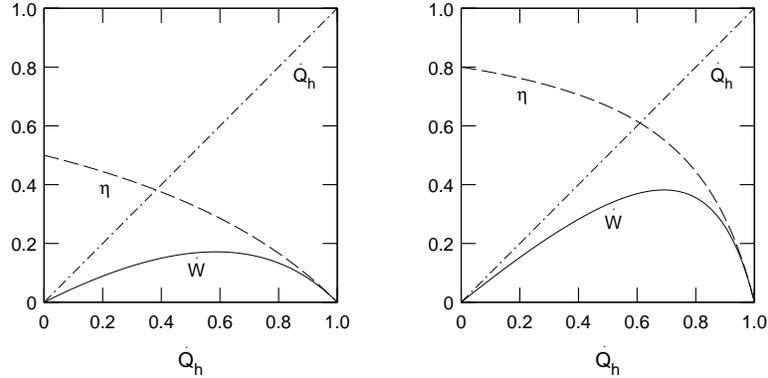


Figure 7.7: Efficiency, heat input, and power output for an endoreversible engine. Both \dot{Q}_h and \dot{W} are shown normalized by $\dot{Q}_h^{(max)}$ [Eq. (7.46)]. Left: $T_c/T_h = 0.5$; right: $T_c/T_h = 0.2$.

Equation (7.45) may be written as

$$\eta = 1 - \frac{(T_c/T_h)}{1 - (\dot{Q}_h/\dot{Q}_h^{(max)})(1 - T_c/T_h)}. \quad (7.47)$$

The power output is given by

$$\dot{W} = \eta \dot{Q}_h. \quad (7.48)$$

The quantities η , \dot{Q}_h , and \dot{W} shown below in Fig. 7.7 as a function of \dot{Q}_h for two values of T_c/T_h (0.2 and 0.5). The maximum *efficiency* is obtained for $\dot{Q}_h \rightarrow 0$, but the maximum power output occurs at a value of $\dot{Q}_h > 0$, where the engine efficiency is less than its maximum value.

To find the maximum power point, we can differentiate \dot{W} with respect to \dot{Q}_h and set the resulting expression to zero. Doing this, the maximum power condition is found to occur for

$$\dot{Q}_h^{(maxpower)} = \dot{Q}_h^{(max)} \frac{1}{1 + \sqrt{T_c/T_h}}. \quad (7.49)$$

Substituting this expression into Eq. (7.47), the engine efficiency at the maximum power condition is

$$\eta^{(maxpower)} = 1 - \sqrt{\frac{T_c}{T_h}}. \quad (7.50)$$

Note that this (unlike the Carnot efficiency) is not an upper limit on the efficiency of an engine. It's simply the efficiency an internally reversible engine will

have in the presence of thermal resistance for heat transfer, when it is operating at the maximum-power-output condition.

The surprising thing about this result is that $\eta^{(maxpower)}$ depends only on T_c and T_h , not on R_c and R_h . This analysis was first done by Curzon and Ahlborn at the University of British Columbia in 1975 and for this reason $1 - \sqrt{T_c/T_h}$ is sometimes called the Curzon and Ahlborn efficiency.

Curzon and Ahlborn pointed out that this efficiency is in practice quite close to that of many large power plants. This suggests that finite- ΔT heat transfer is one of the largest irreversibilities encountered in real power plants, and that they are in effect designed for maximum power output achievable for given T_h , T_c , R_h , and R_c . This in turn suggests that capital cost, not fuel cost, is the dominant factor in determining how many power plants are designed. (If fuel cost were dominant, it would make sense to operate below maximum power, where the efficiency is higher. To produce the required power, a larger (and more expensive) power plants could be built.)

7.7 Entropy Accounting for Open Systems

To extend our analysis to open systems, we have to modify the entropy accounting expressions of Section 7.3 to account for entropy carried by matter entering or leaving the system. We've already seen that when matter enters a system, it carries with it *energy* of e units per kg. Recall e is the total specific energy: $e = u + |\vec{V}|^2/2 + gz$.

It should come as no surprise that matter also carries entropy into a system when it enters. After all, entropy is an extensive property like energy or mass. The amount brought in is s units of entropy per kg, where s is the specific entropy (J/K-kg) of the matter entering. So if a fluid is flowing into a control volume at rate \dot{m} , and at the inlet the fluid has specific entropy s_{in} , the rate at which entropy is carried into the system by the fluid will be $\dot{m}s_{in}$.

We can easily add terms to our entropy accounting equations to account for this *convected* entropy. For example, Eq. (7.10) can be generalized as:

$$\frac{dS}{dt} = \sum_{\text{heat in}} \left(\frac{\dot{Q}_{in}}{T_{in}} \right) - \sum_{\text{heat out}} \left(\frac{\dot{Q}_{out}}{T_{out}} \right) \quad (7.51)$$

$$+ \sum_{\text{inlets}} (\dot{m}s)_{in} - \sum_{\text{outlets}} (\dot{m}s)_{out} + \dot{P}_s. \quad (7.52)$$

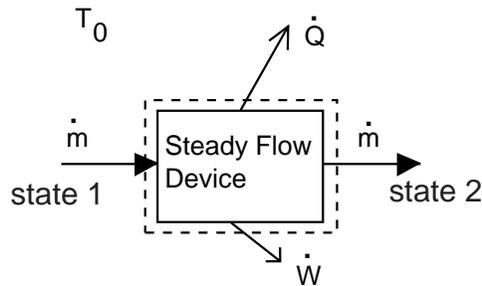


Figure 7.8: A steady-flow device.

7.8 Steady-Flow Devices

In Chapter 5, a set of useful steady-flow devices was introduced: turbines, compressors, pumps, nozzles, diffusers, and heat exchangers. These devices change the state of a fluid flowing steadily through them. In Chapter 5, we did an energy accounting on each of them. Let's now combine this with an entropy accounting.

We'll make the assumptions of steady state (no change in the amount of energy or entropy contained within the device), steady flow (constant \dot{m} in and out), and equilibrium properties at the inlet and outlet, where we need to evaluate them. We will assume that heat transfer occurs at a steady rate \dot{Q} to the surroundings at T_0 , and power \dot{W} is being produced at a steady rate. With these assumptions, the system is as shown in Fig. 7.8.

With the system as shown in the sketch, the entropy accounting is

$$\dot{m}s_{in} + \dot{\mathcal{P}}_s = \dot{m}s_{out} + \frac{\dot{Q}}{T_0} \quad (7.53)$$

Note that we are taking the system boundary just *outside* the device, where the temperature is T_0 . In that way, the irreversibility due to any finite- ΔT heat transfer occurs within the system, and is contained in the $\dot{\mathcal{P}}_s$ term. If we put the system boundary inside the device, then entropy would be being produced in the environment – we could do the analysis that way, too, but it would be more complicated.

Let's apply this equation together with an energy balance to some specific devices.

7.8.1 Adiabatic steady-flow devices

In Chapter 5, we introduced five steady-flow devices which are often modeled as adiabatic. These were (1) compressors, (2) turbines, (3) nozzles, (4) diffusers, and (5) valves.

For any *adiabatic* steady-flow device, Eq. (7.53) reduces to

$$\dot{m}s_{in} + \dot{\mathcal{P}}_s = \dot{m}s_{out}. \quad (7.54)$$

Since the second law requires $\dot{\mathcal{P}}_s \geq 0$, $s_{out} \geq s_{in}$:

the specific entropy of the fluid leaving an adiabatic steady-flow device is greater than or equal to the specific entropy of the fluid entering.

Equation (7.54), together with the steady-flow energy equation introduced in Chapter 4, is what we need to analyze any of these devices. Recall the forms of the steady-flow energy equation appropriate for adiabatic compressors, turbines, nozzles, and diffusers:

$$W_c = \dot{W}_c/\dot{m} = h_2 - h_1 \quad (\text{compressor}) \quad (7.55)$$

$$W_t = \dot{W}_t/\dot{m} = h_1 - h_2 \quad (\text{turbine}) \quad (7.56)$$

$$\Delta [V^2/2] = h_1 - h_2 \quad (\text{nozzle or diffuser}) \quad (7.57)$$

$$(7.58)$$

Since h and s appear in the energy and entropy accounting expressions, an $h - s$ plot is a convenient way to represent the change in the state of the fluid as it passes through any of these devices, as shown in Fig. 7.9.

In Fig. 7.9, the inlet state 1 and the outlet state 2 are shown connected by a dashed line. This is because the process may be irreversible, and so the fluid *inside* the device may not be in an equilibrium state, and therefore would not correspond to any point on the plot. Since the second law requires $s_2 \geq s_1$, state 2 can be *anywhere to the right of point 1, but not to the left*.

In these plots, another state “2s” is also shown. This is the outlet state which *would* result for an “ideal” reversible adiabatic device operating between P_1 and P_2 . The fluid coming out of the idealized device ends up in a different state than the outlet state of the real, irreversible device. State 2s has the pressure of state 2, but the entropy of state 1.

The turbine work output, compressor work input, or nozzle/diffuser change in kinetic energy is given by $|h_2 - h_1|$. This is shown on the plot. Since constant-

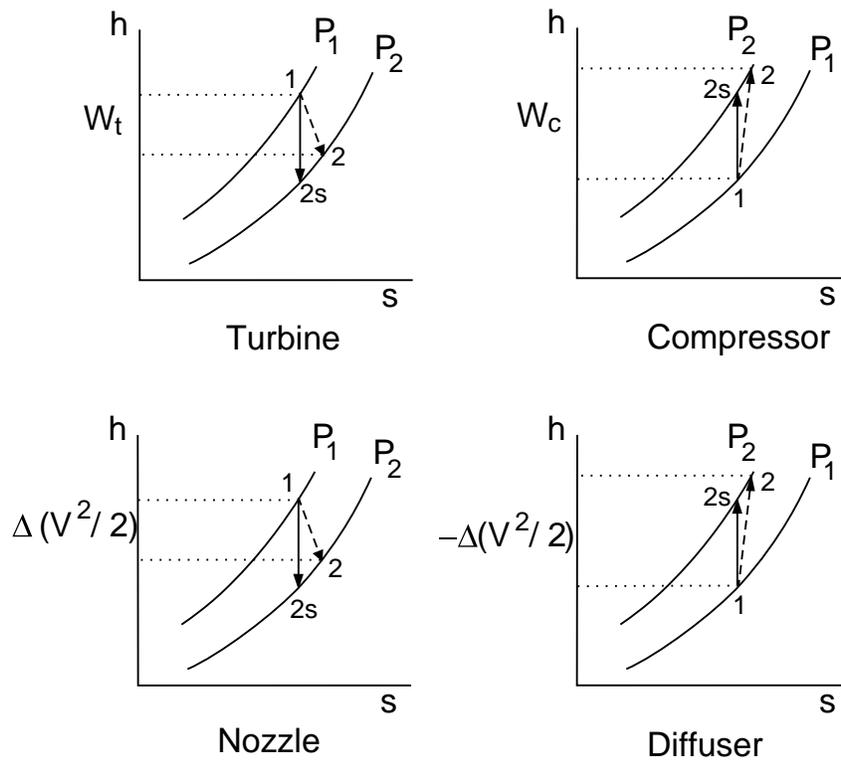


Figure 7.9: Enthalpy-entropy process representations for some adiabatic steady-flow devices. Isobars at P_1 and P_2 are also shown.

pressure lines have positive slope on an $h - s$ plot [i.e., $(\partial h/\partial s)_P > 0$],² we see that the reversible device always has the best performance. Specifically,

1. A reversible turbine gives the *maximum* work output W_t
2. A reversible compressor requires the *least* work input W_c
3. A reversible nozzle gives the *maximum* change in kinetic energy.

Example 7.1 Steam enters an adiabatic turbine at 900 K and 10 MPa and leaves at 1 MPa. What is the maximum work output per unit mass of steam, and what is the exit temperature under maximum work conditions?

Solution: at the inlet,

$$\begin{aligned} h_1 &= \text{h}(\text{"h2o"}, \text{"tp"}, 900, 10) = 3691.5 \text{ kJ/kg} \\ s_1 &= \text{s}(\text{"h2o"}, \text{"tp"}, 900, 10) = 6.977 \text{ kJ/kg-K}. \end{aligned}$$

Since the Second Law requires $s_2 \geq s_1$, $s_2 \geq 6.977 \text{ kJ/kg-K}$, and we're given $P_2 = 1 \text{ MPa}$. From TPX, the state with $s_2 = s_1$ at 1 MPa has temperature 535.87 K and enthalpy 2970.6 kJ/kg. Any state at this pressure with *greater* entropy has a higher temperature, and a higher enthalpy (try it and see).

The work output per kg of steam is $W_t = h_1 - h_2$. Thus, the most work is produced when h_2 is as small as possible; the smallest h_2 is achieved when $s_2 = s_1$. Therefore, the greatest work output possible is $W_{max} = 3691.5 - 2970.6 \text{ kJ/kg} = 720.9 \text{ kJ/kg}$, and the exit temperature is 535.87 K.

The performance of an adiabatic steady-flow device is often characterized by its *isentropic efficiency* η_s . The definition of η_s is different for each type of device, but in each case is a comparison of the actual performance (work input or output, or change in kinetic energy) with the performance of an ideal isentropic device operating between the same pressures. The isentropic efficiency is always defined so that it is ≤ 1 .

For a turbine,

$$\eta_{s,t} = \frac{W_t}{W_{t,s}} = \frac{h_1 - h_2}{h_1 - h_{2s}}. \quad (7.59)$$

Here W_t is the actual work output, and $W_{t,s}$ is the work output of the ideal isentropic turbine. Values of η_s for real turbines depend on size, varying from perhaps 60% for a very small turbine to 95% for a very large, well-designed turbine. A typical value would be around 90%.

²Since $dh = d(u + Pv)$, $dh = (Tds - Pdv) + vdP + Pdv = Tds + vdP$. Therefore, $(\partial h/\partial s)_P = T$, which is positive.

For a compressor,

$$\eta_{s,c} = \frac{W_{c,s}}{W_c} = \frac{h_{2s} - h_1}{h_2 - h_1}. \quad (7.60)$$

Compressor efficiencies are typically lower than turbine efficiencies, with a typical value of about 85%. A very large compressor might approach 90% efficiency.

7.9 Steady-Flow Availability

A question which is often of interest is how much power can be produced by *any conceivable* steady-flow process which takes a steadily-flowing fluid from a specified inlet state to a specified outlet state. To answer this question, consider again Fig. 7.8. From Eq. (7.53), the entropy accounting is

$$\dot{m}s_1 + \dot{\mathcal{P}}_s = \dot{m}s_2 + \frac{\dot{Q}}{T_0}. \quad (7.61)$$

The energy accounting for this steady flow process is

$$\dot{m}(e + Pv)_1 = \dot{W} + \dot{Q} + \dot{m}(e + Pv)_2. \quad (7.62)$$

(We are allowing the fluid to have kinetic and/or potential energy in addition to internal energy in states 1 and 2.)

Eliminating \dot{Q} between these equations and solving for \dot{W} ,

$$\dot{W} = \dot{m} [(e + Pv)_1 - (e + Pv)_2] - [T_0\dot{m}(s_1 - s_2) + T_0\dot{\mathcal{P}}_s], \quad (7.63)$$

or

$$\dot{W} = \dot{m}(b_1 - b_2) - T_0\dot{\mathcal{P}}_s, \quad (7.64)$$

where

$$b = e + Pv - T_0s \quad (7.65)$$

is the *steady flow availability function*. (Note that it is the environment temperature T_0 , not the fluid temperature T , which appears in this expression.)

From Eq. (7.64), we see that a real, irreversible process always produces less power than a reversible process acting between the same inlet and outlet states, for the same mass flow rate. The term $T_0\dot{\mathcal{P}}_s$ represents power which is “lost” due to the irreversibilities in the real process. For this reason, it is called the *irreversibility rate* \dot{I} :

$$\dot{I} = T_0\dot{\mathcal{P}}_s. \quad (7.66)$$

The maximum power produced occurs for $\dot{I} = 0$, and is

$$\dot{W}^{(max)} = \dot{m}(b_1 - b_2). \quad (7.67)$$

In some cases, the outlet state 2 is not fixed, and therefore we may ask what state 2 conditions maximize $\dot{W}^{(max)}$. Clearly, this will be the state which minimizes b_2 .

To minimize

$$b_2 = u_2 + e_{k,2} + e_{p,2} + P_2 v_2 - T_0 s_2,$$

the first thing to do is pick state 2 to have zero kinetic energy, and the lowest achievable potential energy. For example, if the potential energy is due to gravitation, pick state 2 at the lowest achievable elevation. (Elevations below the surface of the earth result in lower potential energy, but are not usually achievable.)

Once this is done, we are left with the task of minimizing $h_2 - T_0 s_2$. It can be shown (using methods to be discussed in the next chapter) that this is minimized for $T_2 = T_0$ and P_2 as small as possible. The lowest possible P_2 is P_0 , since eventually the fluid must flow to the environment. Thus, the optimal state 2 is the one which is in thermal and mechanical equilibrium with the environment.

Example 7.2

A certain industrial process exhausts 10 kg/s of steam at 2 atm and 400 K to the atmosphere through an exhaust valve. It has been suggested that the valve should be replaced with some process which can generate electrical power. Before deciding whether to pursue this, it is desired to know how much power could be produced in principle. The environment conditions are $P_0 = 1$ atm and $T_0 = 300$ K.

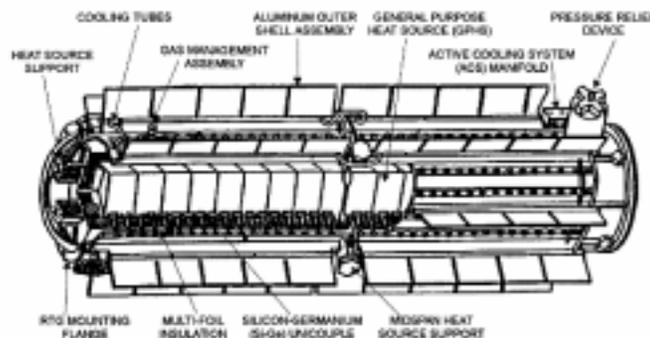
Solution: The maximum power output is $\dot{m}(b_1 - b_2)$. We will take state 2 to be liquid water at 1 atm and 300 K. Using TPX, the properties are

State	T (K)	P (atm)	h (kJ/kg)	s (kJ/kg-K)	b (kJ/kg)
1	400	2	2720.4	7.156	573.6
2	300	1	112.7	0.393	-5.3

Therefore, $\dot{W}^{(max)} = (10 \text{ kg/s})(578.9 \text{ kJ/kg}) = 5.8 \text{ MW}$. Of course, any real process will produce less power, but a well-designed process should be able to produce at least a couple of megawatts. Whether it makes sense to pursue this would depend on how much the process costs to build and run, and on the cost of electricity saved.

Problems

- 7.1** Gaseous water at 5 MPa in a piston/cylinder system is adiabatically expanded until its temperature is 400 K. If it is desired that there be no liquid droplets in the final state, what is the minimum possible initial temperature?
- 7.2** An insulated container contains saturated liquid nitrogen at 1 atm. You wish to design a process to produce work from this liquid nitrogen.
1. If the surroundings are at 300 K and 1 atm, what is the maximum work which could be produced in principle?
 2. Suppose you use a simple process in which you transfer the liquid to a piston-cylinder system, and let it warm up to room temperature at constant pressure. Sketch this process on an appropriate process diagram, and calculate the work produced.
 3. If the work produced in (b) is less than the maximum calculated in (a), discuss why.
 4. Can you suggest ways to improve this process?
- 7.3** The Cassini mission to Saturn was recently launched by NASA, and should arrive at Saturn in 2004. Like all NASA deep space missions, on-board electrical power on Cassini is generated by RTGs (Radioisotope Thermoelectric Generators). These consist of pellets of radioactive plutonium fuel which produce heat, bundles of silicon-germanium thermocouples, and radiator fins which radiate waste heat into space. The thermocouples produce electrical power directly when connected between the high-temperature “General Purpose Heat Source” containing the plutonium and the cold radiator fins.



The heat rejected from the radiator fins follows the Stefan-Boltzmann law:

$$\dot{Q}_c = \epsilon A \sigma T_c^4.$$

Here T_c is the temperature of the radiator fins, A is the total fin area, and ϵ is a material property known as the *emissivity* ($0 < \epsilon < 1$).

The array of thermocouples constitutes a heat engine operating between the plutonium fuel temperature (T_h) and the radiator temperature T_c .

To minimize the weight and spacecraft size, it is desired to keep the radiator fins as small as possible. For a given power generation requirement \dot{W} , ϵ , and T_h , determine

1. the *minimum radiator fin area* required, if the thermocouples act as “ideal” heat engines.
2. the thermodynamic efficiency $\eta = \dot{W}/\dot{Q}_h$ and fin temperature T_c under the conditions of part (a).

Assume that there is no heat transfer limitation on the rate at which heat may be taken from the radioactive source.

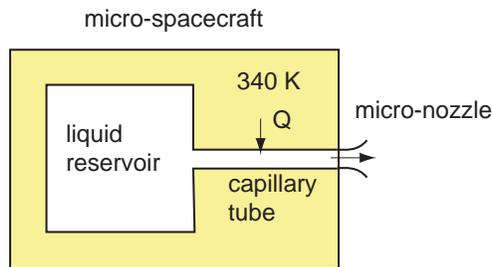
7.4 You have 2 identical copper blocks of mass M . One is initially at temperature $T_1(0)$, and the other is initially at temperature $T_2(0) < T_1(0)$. Both may be assumed to be incompressible substances with constant c . You now let them interact through some process which extracts heat from block 1, generates work, and dumps some heat into block 2. Since the blocks have finite mass, $T_1(t)$ and $T_2(t)$ change with time. Eventually, they come to the same temperature $T_1(\infty) = T_2(\infty) = T_f$ and no more work can be done.

1. Taking both blocks together as the system and assuming no heat is transferred to or from the environment, write down expressions for $\Delta U = U(\infty) - U(0)$ and $\Delta S = S(\infty) - S(0)$. Assume that the total entropy produced during the process is \mathcal{P}_s .
2. Combine these equations to obtain expressions for W and T_f . Your expression should contain only M , c , $T_1(0)$, $T_2(0)$, and \mathcal{P}_s .
3. Show that the *maximum possible* work is

$$W_{max} = Mc \left(\sqrt{T_1} - \sqrt{T_2} \right)^2$$

and determine T_f in this case.

- 7.5** Nitrogen is held in a container at its critical pressure and temperature. If the environment is at 300 K and 1 atm, what is the maximum work which could be done per kg of N_2 ?
- 7.6** A concept for a positioning thruster for a very small (1 kg) spacecraft is shown below.



Liquid HFC-134a at $T = 250$ K and 0.8 MPa flows from a small on-board reservoir into a capillary tube 0.5 mm in diameter and 1 cm long, where it absorbs waste heat given off by the on-board electronic equipment, causing it to evaporate. It emerges from the tube as vapor at 320 K at a pressure of 0.65 MPa. (Due to wall friction, there is a substantial pressure drop down the tube.) The vapor emerging from the tube enters an adiabatic micro-nozzle and is accelerated to produce thrust. The micro-nozzle has an isentropic efficiency of 0.9, and the pressure at its exit is 0.05 MPa.

Determine:

1. The irreversibility rate \dot{I} in the capillary tube, if the environment surrounding the tube is at 340 K.
 2. The velocity and temperature at the exit of the micro-nozzle
 3. The mass flow rate in mg/s required to produce a thrust of 1 mN. (Thrust is mass flow rate multiplied by exit velocity.)
- 7.7** What is the minimum power required by an adiabatic compressor which compresses 10 kg/min of saturated water vapor at 0.2 MPa to 0.4 MPa?
- 7.8** What is the minimum power required by *any* process (not necessarily adiabatic) which compresses 10 kg/min of saturated water vapor at 0.2 MPa to 0.4 MPa? The surroundings are at $P_0 = 1$ atm and $T_0 = 300$ K. Compare this process to that in problem 1. Are the final states the same? Does this process involve heat transfer? If so, in which direction?

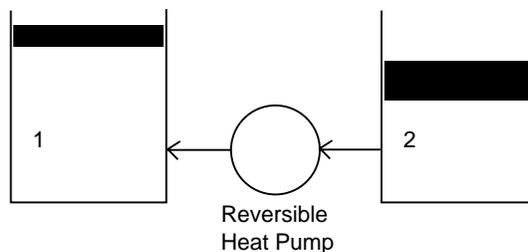
7.9 An industrial process to produce liquid nitrogen starts with gaseous nitrogen at 300 K and 1 atm, which enters at a steady rate of 1 kg/s. The product is saturated liquid nitrogen at 1 atm.

1. If the environment is at 300 K and 1 atm, does this process require work input, or can it produce work?
2. Determine the minimum work input, or maximum work output (depending on your answer to (a)).
3. Determine the heat transfer rate for the conditions of part (b).

7.10 (NOTE: Requires Helium data; can't be done with current version of TPX.)

Two cylinders with pistons each contain 10 kg of helium. The weight of piston 1 keeps P_1 constant at 0.05 MPa, and the weight of piston 2 keeps P_2 constant at 0.2 MPa. Initially, both cylinders are at a temperature of 5.2 K. They are now thermally connected to a reversible heat pump, which heats cylinder 1 and cools cylinder 2 with no heat transfer to the surroundings. When cylinder 1 reaches 8 K, the process stops.

1. Write equations expressing energy and entropy accounting for this process.
2. Find the final state of the helium in cylinder 2.
3. Find the work input to the heat pump.



7.11 A heat-powered portable air compressor consists of three components: (a) an adiabatic compressor; (b) a constant pressure heater (heat supplied from an outside source); and (c) an adiabatic turbine. The compressor and turbine both may be idealized as reversible, and air may be assumed to be an ideal gas with $k = c_p/c_v = 1.4$.

Ambient air enters the compressor at 0.1 MPa and 300 K, and is compressed to 0.6 MPa. All of the power from the turbine goes to run the

compressor, and the turbine exhaust is the supply of compressed air. If this pressure is required to be 0.2 MPa, what must the temperature be at the exit of the heater?

- 7.12** For a particular industrial process, a stream of gaseous oxygen at 2 MPa and 1000 K is required. You have available a stream of saturated liquid oxygen at 100 K, and you wish to design a process to continuously produce the desired gaseous oxygen from this liquid oxygen. The surroundings are at 300 K. Determine whether such a process requires work input or can produce work output, and the minimum work input or maximum work output per kg of O₂.
- 7.13** An adiabatic pump pumps 200 kg/hr of saturated liquid HFC-134a at 320 K to 2.5 MPa. The temperature of the emerging liquid is 321.26 K. Determine:
1. The power required to run the pump
 2. The isentropic efficiency of the pump
 3. The entropy production rate
 4. The irreversibility rate if the environment is at 300 K.