

Chapter 17

Coastal Processes and Tides

In the last chapter I described waves on the sea surface. Now we can consider several special and important cases: the transformation of waves as they come ashore and break; the currents and edge waves generated by the interaction of waves with coasts; tsunamis; storm surges; and tides, especially tides along coasts.

17.1 Shoaling Waves and Coastal Processes

Waves propagating into shallow water are refracted by features on the sea floor, they eventually break on the beach where the wave breaking drives near-shore currents including long-shore and rip currents.

Shoaling Waves Wave phase and group velocities are a function of depth when the depth is less than about one-quarter wavelength in deep water. Wave period and frequency are invariant (don't change as the wave comes ashore); and this is used to compute the properties of shoaling waves. Wave height increases as wave group velocity slows. Wave length decreases. Waves change direction due to refraction. Finally, waves break if the water is sufficiently shallow; and broken waves pour water into the surf zone, creating long-shore currents.

The dispersion relation (16.3) is used to calculate wave properties as the waves propagate from deep-water offshore to shallow just outside the surf zone. Because ω is constant, (16.3) leads to:

$$\frac{L}{L_0} = \frac{c}{c_0} = \frac{\sin \alpha}{\sin \alpha_0} = \tanh \frac{2\pi d}{L} \quad (17.1)$$

where

$$L_0 = \frac{gT^2}{2\pi}, \quad c_0 = \frac{gT}{2\pi} \quad (17.2)$$

and L is wave length, c is phase velocity, α is the angle of the crest relative to contours of constant depth, and d is water depth. The subscript 0 indicates values in deep water.

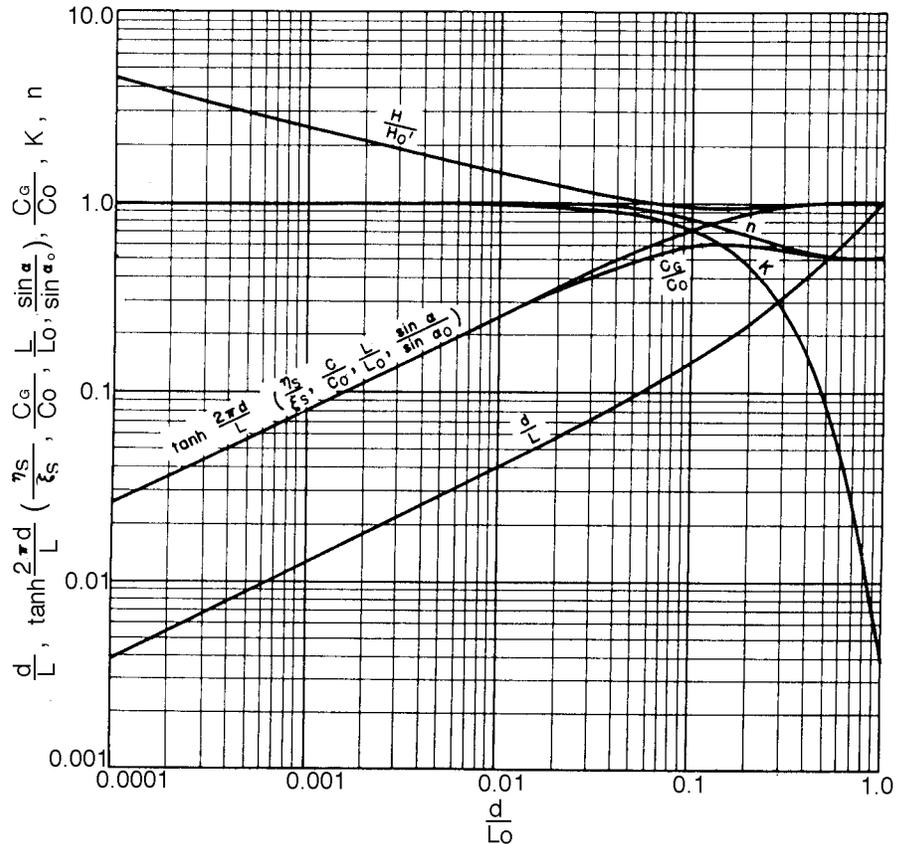


Figure 17.1 Change in wave properties as waves shoal (From Wiegel, 1964).

The quantity d/L is calculated from the solution of

$$\frac{d}{L_0} = \frac{d}{L} \tanh \frac{2\pi d}{L} \quad (17.3)$$

using an iterative technique, or from Figure 17.1 or from Table A1 of Wiegel (1964).

Because wave velocity is a function of depth in shallow water, variations in offshore water depth can focus or defocus wave energy reaching the shore. Consider the simple case of waves with deep-water crests almost parallel to a straight beach with two ridges each extending seaward from a headland (Figure 17.2). Wave group velocity is faster in the deeper water between the ridges, and the wave crests become progressively deformed as the wave propagates toward the beach. Wave energy, which propagates perpendicular to wave crests, is refracted out of the region between the headland. As a result, wave energy is focused into the headlands, and breakers there are much larger than breakers in the bay. The difference in wave height can be surprisingly large. On a calm day, breakers can be knee high shoreward of a submarine canyon at La Jolla Shores,

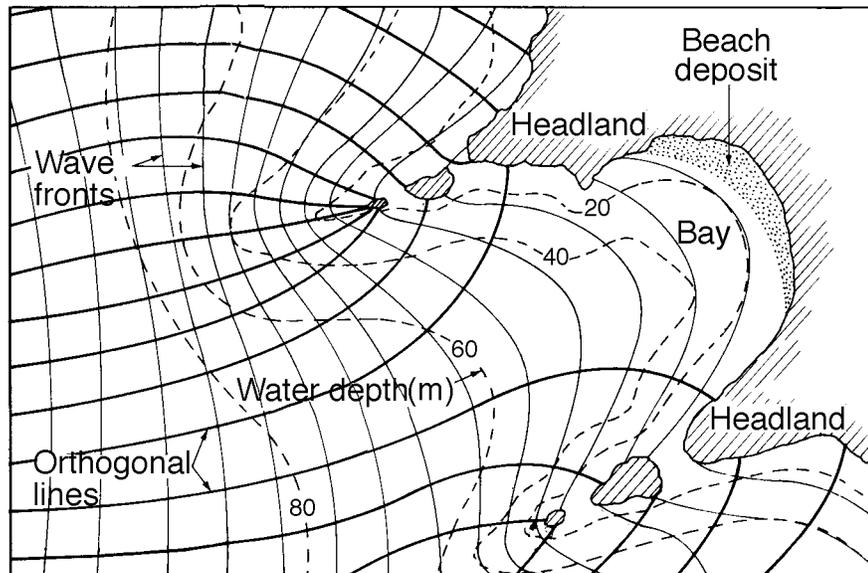


Figure 17.2 Subsea features, such as submarine canyons and ridges, offshore of coasts can greatly influence the height of breakers inshore of the features (From Thurman, 1985).

California, just south of the Scripps Institution of Oceanography. At the same time, wave just north of the canyon can be high enough to attract surfers.

Breaking Waves As waves move into shallow water, the group velocity becomes small, wave energy per square meter of sea surface increases, and non-linear terms in the wave equations become important. These processes cause waves to steepen, with short steep crests and broad shallow troughs. When wave slope at the crest becomes sufficiently steep, the wave breaks (Figure 17.3). The shape of the breaking wave depends on the slope of the bottom, and the steepness of waves offshore (Fig. 17.4).

1. Steep waves tend to lose energy slowly as the waves moves into shallower water through water spilling down the front of the wave. These are spilling breakers.
2. Less steep waves on steep beaches tend to steepen so quickly that the crest of the wave moves much faster than the trough, and the crest, racing ahead of the trough, plunges into the trough (Figure 17.5).
3. If the beach is sufficiently steep, the wave can surge up the face of the beach without breaking in the sense that white water is formed. Or if it is formed, it is at the leading edge of the water as it surges up the beach. An extreme example would be a wave incident on a vertical breakwater.

Wave-Driven Currents Waves break in a narrow band of shallow water along the beach, the *surf zone*. After breaking, waves continues as a near-vertical wall of turbulent water called a *bore* which carries water to the beach. At first, the

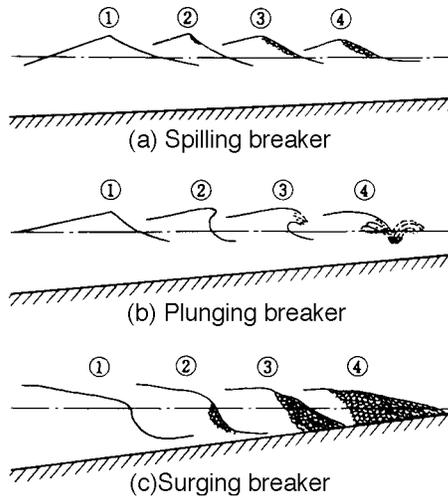


Figure 17.3 Sketch of types of breaking waves (From Horikawa, 1988).

bore surges up the beach, then retreats. The water carried by the bore is left in the shallow waters inside the surf zone.

Rip currents are produced when the water carried onshore by breaking waves returns offshore in the form of narrow swift currents. These narrow jets of fast-moving water are usually oriented perpendicular to the shore and spaced hundreds of meters apart along the shore (Figure 17.6). Usually there is a band of deeper water between the breaker zone and the beach, and the long-shore current runs in this channel. The strength of a rip current depends on the height and frequency of breaking waves and the strength of the onshore wind. The rip is a danger to unwary swimmers, especially poor swimmers bobbing along in the waves inside the breaker zone. They are carried along by the along-shore current until they are suddenly carried out to sea by the rip. Swimming

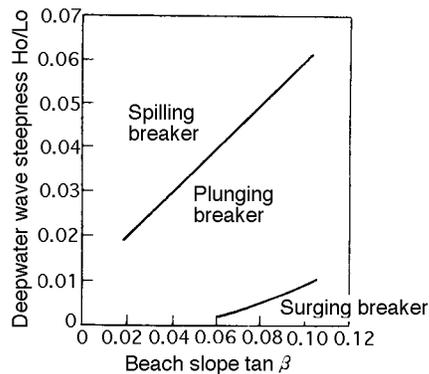


Figure 17.4 Classification of breaking waves as a function of beach steepness and wave steepness offshore (From Horikawa, 1988).

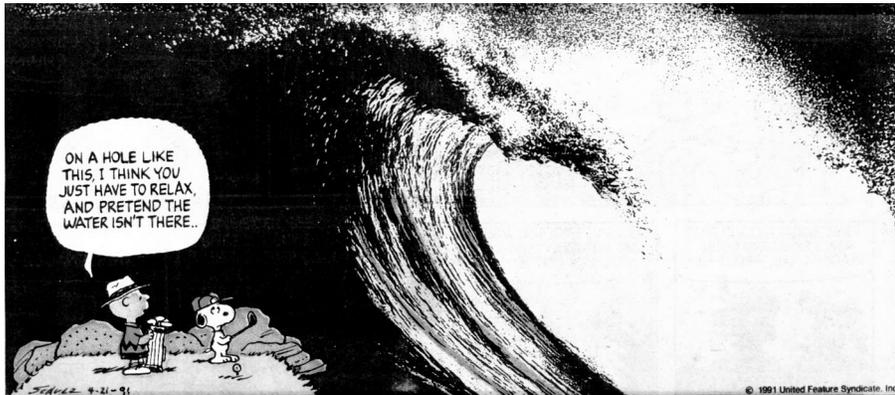


Figure 17.5 Steep, plunging breakers are the archtypical breaker. The edge of such breakers are ideal for surfing. Photo by Jeff Devine, cartoon by Schultz 4-21-91.

against the rip is futile, but swimmers can escape by swimming parallel to the beach.

Edge waves are produced by the variability of wave energy reaching shore. Waves tend to come in groups, especially when waves come from distant storms. For several minutes breakers may be smaller than average, then a few very large waves will break. The minute-to-minute variation in the height of breakers produces low-frequency variability in the along-shore current. This, in turn, drives a low-frequency wave attached to the beach, an edge wave. The waves have periods of a few minutes, a long-shore wave length of around a kilometer, and an amplitude that decays exponentially offshore (Figure 17.7).

17.2 Tsunamis

Tsunamis are low-frequency ocean waves generated by submarine earthquakes. The sudden motion of sea floor over distances of a hundred or more kilometers

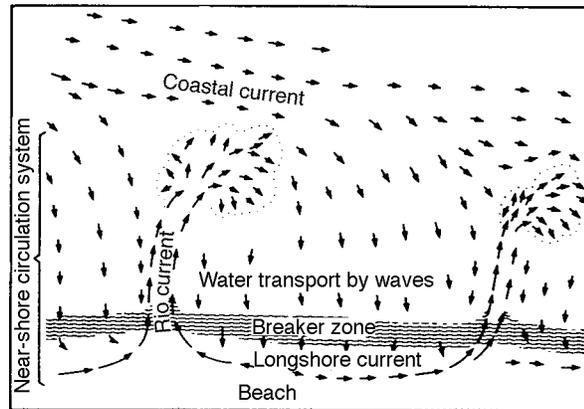


Figure 17.6 Sketch of rip currents generated by water carried to the beach by breaking waves (From Dietrich, Kalle, Kraus, & Siedler, 1980).

generates waves with periods of around 12 minutes (Figure 17.8). A quick calculation shows that such waves must be shallow-water waves, propagating at a speed of 180 m/s and having a wavelength of 130 km in water 3.6 km deep (Figure 17.9). The waves are not noticeable at sea, but after slowing on approach to the coast, and after refraction by subsea features, they can come ashore and surge to heights ten or more meters above sea level. In an extreme example, the Alaskan tsunami on 1 April 1946 destroyed the Scotch Cap lighthouse 31 m above sea level.

Shepard (1963, Chapter 4) summarized the influence of tsunamis based on his studies in the Pacific.

1. Tsunamis appear to be produced by movement (an earthquake) along a linear fault.

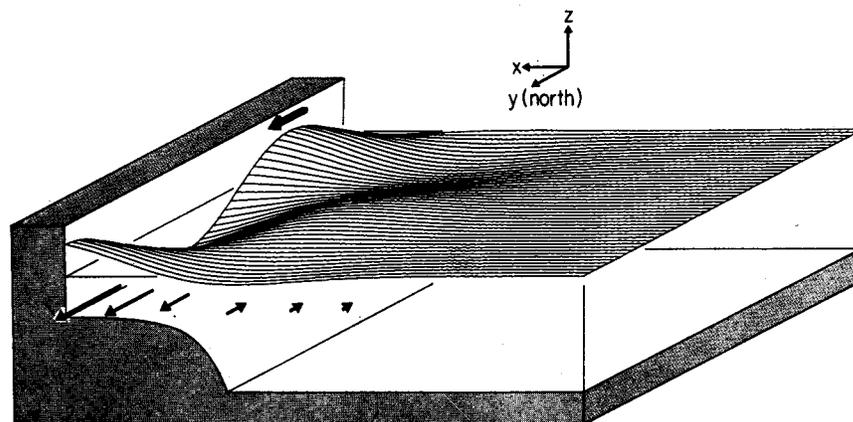


Figure 17.7 Computer-assisted sketch of an edge wave. Such waves exist in the breaker zone near the beach and on the continental shelf. (From Cutchin and Smith, 1973).

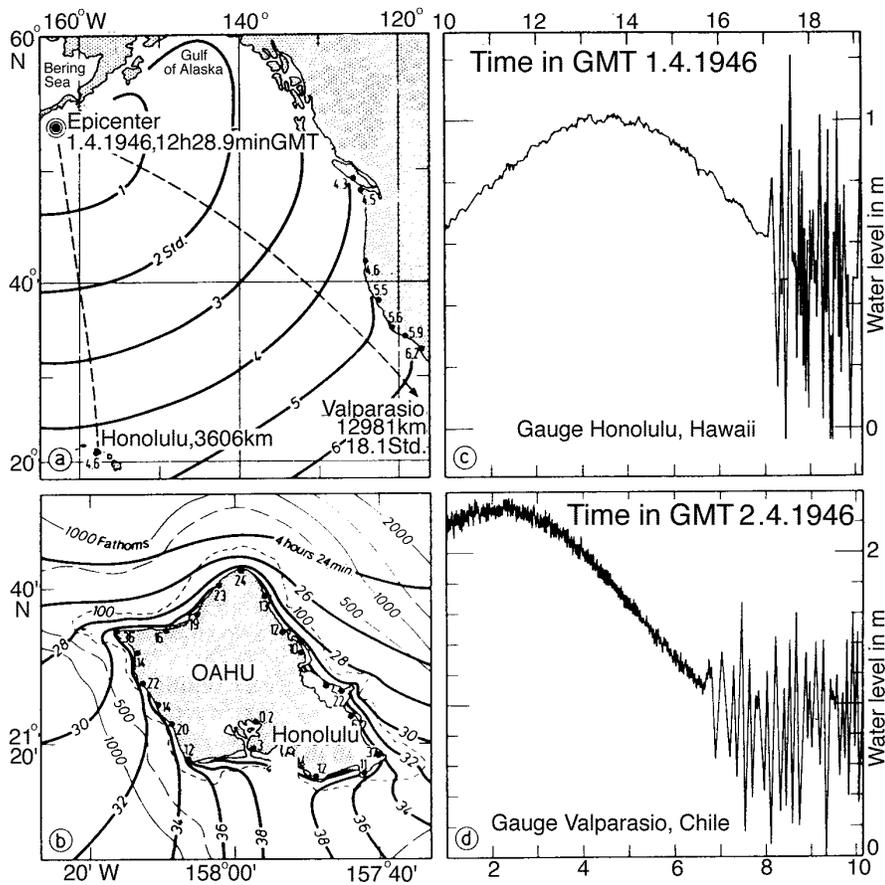


Figure 17.8 (a) Hourly positions of leading edge of tsunami generated by a large earthquake in the Aleutian Trench on April 1, 1946 at 12 h 58.9m GMT. (b) Maximum vertical extent of tsunami on Oahu Island in Hawaii and the calculated travel time in hours and minutes from the earthquake epicenter. (c) & (d) Tide gauge records of the tsunami at Honolulu and Valparaiso (From Dietrich, *et al.* 1980).

2. Tsunamis can travel thousands of kilometers and still do serious damage to coasts.
3. The first wave of a tsunami is not likely to be the biggest.
4. Wave amplitudes are relatively large shoreward of submarine ridges. They are relatively low shoreward of submarine valleys, provided the features extend into deep water.
5. Wave amplitudes are decreased by the presence of coral reefs bordering the coast.
6. Some bays have a funneling effect, but long estuaries attenuate waves.
7. Waves can bend around circular islands without great loss of energy, but they are considerably smaller on the backsides of elongated, angular islands.

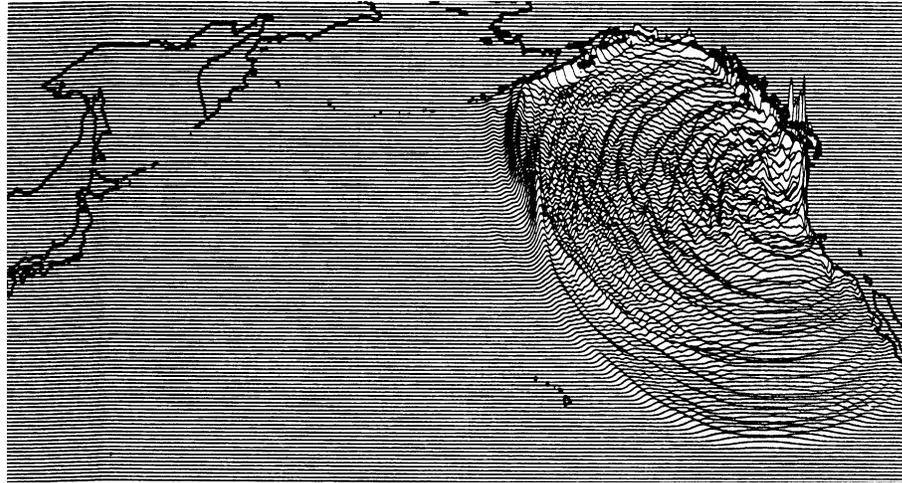


Figure 17.9 Tsunami wave height four hours after the great M9 Cascadia earthquake off the coast of Washington on 26 January 1700 calculated by a finite-element, numerical model. Maximum open-ocean wave height, about one meter, is north of Hawaii (From Satake et al 1996).

17.3 Storm Surges

Storm winds blowing over shallow, continental shelves pile water against the coast. The increase in sea level is known as a storm surge. Several processes are important:

1. Ekman transport by winds parallel to the coast transports water toward the coast causing a rise in sea level.
2. Winds blowing toward the coast push water directly toward the coast.
3. Wave run-up and other wave interactions transport water toward the coast adding to the first two processes.
4. Edge waves generated by the wind travel along the coast.
5. The low pressure inside the storm raises sea level by one centimeter for each millibar decrease in pressure through the inverted-barometer effect.
6. Finally, the storm surge adds to the tides, and high tides can change a relative weak surge into a much more dangerous one.

See Jelesnianski (1967, 1970) for a description of storm-surge models SPLASH and Sea, Lake, and Overland Surges from Hurricanes SLOSH used by the National Hurricane Center.

To a crude first approximation, wind blowing over shallow water causes a slope in the sea surface proportional to wind stress.

$$\frac{\partial \zeta}{\partial x} = \frac{\tau_0}{\rho g H} \quad (17.4)$$

where ζ is sea level, x is horizontal distance, H is water depth, T_0 is wind stress at the sea surface, ρ is water density; and g is gravitational acceleration.

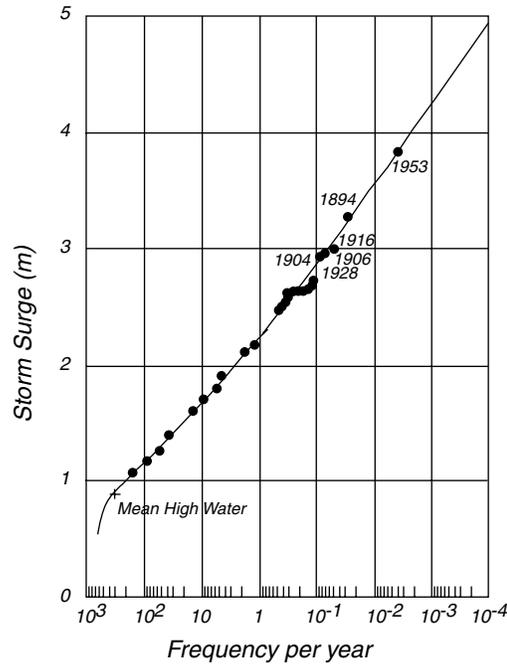


Figure 17.10 Probability (per year) density distribution of vertical height of storm surges in the Hook of Holland in the Netherlands. The distribution function is Rayleigh, and the probability of large surges is estimated by extrapolating the observed frequency of smaller, more common surges (From Wiegel, 1964).

If $x = 100$ km, $U = 40$ m/s, and $H = 20$ m, values typical of a hurricane offshore of the Texas Gulf Coast, then $T = 2.7$ Pa, and $\zeta = 1.3$ m at the shore. Figure 17.10 shows the frequency of surges at the Netherlands and a graphical method for estimating the probability of extreme events using the probability of weaker events.

17.4 Theory of Ocean Tides

Tides have been so important for commerce and science for so many thousands of years that tides have entered our everyday language: *time and tide wait for no one, the ebb and flow of events, a high-water mark, and turn the tide of battle.*

1. Tides produce strong currents in many parts of the ocean. Tidal currents can have speeds of up to 5 m/s in coastal waters, impeding navigation and mixing coastal waters.
2. Tidal currents generate internal waves over seamounts, the continental slope, and mid-ocean ridges. This contributes to mixing in the ocean.
3. Tidal currents can suspend bottom sediments, even in the deep ocean.
4. The weight of oceanic tides deforms the earth, producing signals over most continental areas. This deformation is the ocean-loading tide.
5. The deformation of the solid earth influence almost all precise measurements of earth.

6. Oceanic tides lag behind the tide-generating potential, producing forces that transfer angular momentum between the earth and the tide-producing body, especially the moon.
7. As a result of tidal forces, earth's rotation about its axis slows, increasing the length of day; the rotation of the moon about earth slows, causing the moon to move slowly away from earth; and the moon's rotation about its axis slows, causing the moon to keep the same side facing earth as the moon rotates about earth.
8. Tides influence the orbits of satellites. Hence accurate tides are needed for computing the orbit of altimetric satellites. Tides are also needed for correcting the altimetric satellite's measurements of oceanic topography.
9. Tidal forces on other planets and stars are important for understanding many aspects of solar-system dynamics and even galactic dynamics. For example, the rotation rate of Mercury, Venus, and Io result from tidal forces.

Mariners have known for at least four thousand years that tides are related to the phase of the moon. The exact relationship, however, is hidden behind many complicating factors, and some of the greatest scientific minds of the last four centuries have contributed to the understanding, calculation, and prediction of tides. Galileo, Descartes, Kepler, Newton, Euler, Bernoulli, Kant, Laplace, Airy, Lord Kelvin, Jeffreys, Munk and many others contributed. Some of the first computers were developed and used for computing and predicting tides: Ferrel built a tide-predicting machine in 1880 that was used by the U.S. Coast and Geodetic Survey to predict nineteen tidal constituents. In 1901, Harris extended the capacity to 37 constituents.

Long standing questions have remained: What is the amplitude and phase of the tides at any place on the ocean or along the coast? What is the speed and direction of tidal currents? What is the shape of the tides on the ocean? Where is tidal energy dissipated? Answers to these simple questions are difficult, and the first, accurate, global solutions were published by LeProvost et al. (1994). The problem is hard because the tides are a self-gravitating, near-resonant, sloshing of water in a rotating, elastic, ocean basin with ridges, mountains, and submarine basins.

At coastal stations and ports the problem is much simpler. Data from a tide gauge plus the theory of tidal forcing gives an accurate description of tides at that point.

Tidal Potential Tides are calculated from the hydrodynamic equations for a self-gravitating ocean on a rotating, elastic earth. The driving force is the small change in gravity due to motion of the moon and sun relative to earth.

The small variations in gravity arise from two separate mechanisms. To see how they work, consider the rotation of the moon about earth.

1. The moon and earth rotate about the center of mass of the earth-moon system. This gives rise to a centripetal acceleration at earth's surface that

drives water away from the center of mass and toward the side of earth away from the moon.

2. At the same time, mutual gravitational attraction of mass on earth and the moon causes water to be attracted toward the moon.

If earth were an ocean planet with no land, and if the ocean were very deep, the two processes would produce a pair of bulges of water on earth, one on the side facing the moon, one on the side away from the moon. A clear derivation of the forces is given by Pugh (1987) and by Dietrich, Kalle, Krauss, and Siedler (1980). Here I follow the discussion in Pugh §3.2.

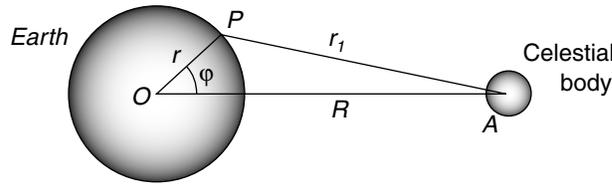


Figure 17.11 Sketch of coordinates for determining the tide-generating potential.

To calculate the amplitude and phase of the tide in the ocean, we begin by calculating the tide-generating potential. This is much simpler than calculating the forces. The tide-generating potential at earth's surface is due to the earth-moon system rotating about a common center of mass. Ignoring for now earth's rotation, the rotation of moon about earth produces a potential V_M at any point on earth's surface

$$V_M = -\frac{\gamma M}{r_1} \quad (17.5)$$

where the geometry is sketched in figure 17.11, γ is the gravitational constant, and M is the moon's mass. From the triangle OPA in the figure,

$$r_1^2 = r^2 + R^2 - 2rR \cos \varphi \quad (17.6)$$

Using this in (17.5) gives

$$V_M = -\frac{\gamma M}{R} \left\{ 1 - 2 \left(\frac{r}{R} \right) \cos \varphi + \left(\frac{r}{R} \right)^{1/2} \right\}^{-1/2} \quad (17.7)$$

$r/R \approx 1/60$, and (17.7) may be expanded in powers of r/R using Legendre polynomials (Whittaker and Watson, 1963: §15.1):

$$V_M = -\frac{\gamma M}{R} \left\{ 1 + \left(\frac{r}{R} \right) \cos \varphi + \left(\frac{r}{R} \right)^2 \left(\frac{1}{2} \right) (3 \cos^2 \varphi - 1) + \dots \right\} \quad (17.8)$$

The tidal forces are calculated from the gradient of the potential, so the first term in (17.8) produces no force. The second term produces a constant force parallel to OA . This force keeps earth in orbit about the center of mass of the

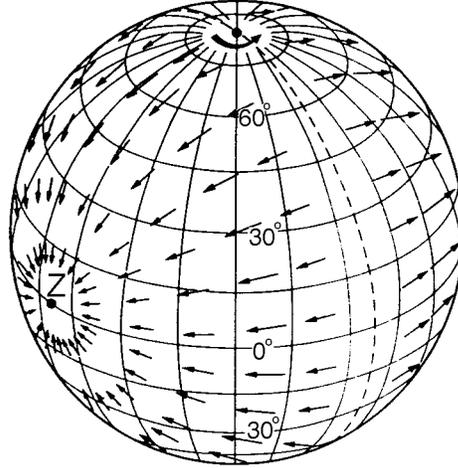


Figure 17.12 The horizontal component of the tidal force on earth when the tide-generating body is above the Equator at Z . (From Dietrich, et.al., 1980).

earth-moon system. The third term produces the tides, assuming the higher-order terms can be ignored. The tide-generating potential is therefore:

$$V = -\frac{\gamma M r^2}{2R^3} (3 \cos^2 \varphi - 1) \quad (17.9)$$

The tide-generating force can be decomposed into a force perpendicular to the sea surface and a horizontal force. The vertical force produces very small changes in the weight of the oceans. It is very small compared to gravity, and it can be ignored.

The horizontal component H of the force (Figure 17.12) is:

$$H = -\frac{1}{r} \frac{\partial V}{\partial \varphi} = \frac{2G}{r} \sin 2\varphi \quad (17.10)$$

where

$$G = \frac{3}{4} \gamma M \left(\frac{r^2}{R^3} \right) \quad (17.11)$$

If we now allow our ocean-covered earth to rotate, we see that the moon produces two tidal bulges that appear to rotate around earth. (To an observer on earth, the moon seems to rotate around the heavens at nearly one cycle per day). The bulges are symmetric about the earth-moon line, and moon produces high tides every 12 hours and 25.23 minutes on the equator if the moon is above the equator. There are not exactly two high tides per day because moon is rotating about earth. Of course, the moon is above the equator only twice per lunar month, and this complicates our simple picture of the tides on an ideal ocean-covered earth. Furthermore, moon's distance from earth varies because

moon's orbit is elliptical and because the elliptical orbit is not fixed. Thus R varies at with a period of once per month, once per 8.85 years, and once per 17.61 years. Because tides are larger when moon is closer in it's orbit around earth, the lunar tides vary with these periods.

Clearly, the calculation of tides is getting more complicated than we might have thought. Before continuing on, we note that the solar tidal forces are derived in a similar way. The relative importance of the sun and the moon are nearly the same. Although the sun is much more massive than the moon, it is much further away.

$$G_{sun} = G_S = \frac{3}{4}\gamma S \left(\frac{r^2}{R_{sun}^3} \right) \quad (17.12)$$

$$G_{moon} = G_M = \frac{3}{4}\gamma M \left(\frac{r^2}{R_{moon}^3} \right) \quad (17.13)$$

$$\frac{G_S}{G_M} = 0.46051 \quad (17.14)$$

where R_{sun} is the distance to the sun, S is the mass of the sun; R_{moon} is the distance to the moon, and M is the mass of the moon.

Coordinates of Sun and Moon Before we can proceed further we need to know the position of the moon and the sun relative to the earth. An accurate description of the positions in three dimensions is very difficult, and it involves learning arcane terms and concepts from celestial mechanics. Here, I paraphrase a simplified description from Pugh. See also figure 4.1.

A natural reference system for an observer on earth is the equatorial system described at the start of Chapter 3. In this system, *declinations* of a celestial body are measured north and south of a plane which cuts the earth's equator.

Angular distances around the plane are measured relative to a point on this celestial equator which is fixed with respect to the stars. The point chosen for this system is the *vernal equinox*, also called the 'First Point of Aries' ... The angle measured eastward, between Aries and the equatorial intersection of the meridian through a celestial object is called the *right ascension* of the object. The declination and the right ascension together define the position of the object on a celestial background ...

[Another natural reference] system uses the plane of the earth's revolution around the sun as a reference. The celestial extension of this plane, which is traced by the sun's annual apparent movement, is called the *ecliptic*. Conveniently, the point on this plane which is chosen for a zero reference is also the vernal equinox, at which the sun crosses the equatorial plane from south to north near 21 March each year. Celestial objects are located by their ecliptic latitude and ecliptic longitude. The angle between the two planes, of 23.45° , is called the obliquity of the ecliptic ... —Pugh (1987).

Tidal Frequencies Now, let's allow earth to spin about its polar axis. The changing potential at a fixed geographic coordinate on earth is:

$$\cos \varphi = \sin \varphi_p \sin \delta + \cos \varphi_p \cos \delta \cos(\tau_1 - 180^\circ) \quad (17.15)$$

where φ_p is latitude at which the tidal potential is calculated, δ is declination of the moon or sun north of the equator, and τ_1 is the hour angle of the moon or sun. The *hour angle* is the longitude where the imaginary plane containing the sun or moon and earth's rotation axis crosses the Equator.

The period of the solar hour angle is a solar day of 24 hr 0 m. The period of the lunar hour angle is a lunar day of 24 hr 50.47 m.

Earth's axis of rotation is inclined 23.45° with respect to the plane of earth's orbit about the sun. This defines the ecliptic, and the sun's declination varies between $\delta = \pm 23.45^\circ$ with a period of one solar year. The orientation of earth's rotation axis precesses with respect to the stars with a period of 26 000 years. The rotation of the ecliptic plane causes δ and the vernal equinox to change slowly, and the movement called the *precession of the equinoxes*.

Earth's orbit about the sun is elliptical, with the earth in one focus. That point in the orbit where the distance between the sun and earth is a minimum is called *perigee*. The orientation of the ellipse in the ecliptic plane changes slowly with time, causing perigee to rotate with a period of 20 900 years. Therefore R_{sun} varies with this period.

The moon's orbit is also elliptical, but a description of moon's orbit is much more complicated than a description of earth's orbit. Here are the basics. The moon's orbit lies in a plane inclined at a mean angle of 5.15° relative to the plane of the ecliptic; and lunar declination varies between $\delta = 23.45 \pm 5.15^\circ$ with a period of one tropical month of 27.32 solar days. The inclination varies between 4.97° , and 5.32° . The shape of the moon's orbit also varies. The eccentricity of the moon's orbit has a mean value of 0.0549, and it varies between 0.044 and 0.067. And, perigee rotates with a period of 8.85 years. Both processes cause variations in R_{moon} .

Note that I am a little imprecise in defining the position of the sun and the moon. Lang (1980: § 5.1.2) gives much more precise definitions.

Substituting (17.15) into (17.9) gives:

$$\begin{aligned} V = \frac{\gamma M r^2}{R^3} \frac{1}{4} & \left[(3 \sin^2 \varphi_p - 1) (3 \sin^2 \delta - 1) \right. \\ & + 3 \sin 2\varphi_p \sin 2\delta \cos \tau_1 \\ & \left. + 3 \cos^2 \varphi_p \cos^2 \delta \cos 2\tau_1 \right] \quad (17.16) \end{aligned}$$

Equation (17.16) separates the period of the lunar tidal potential into three terms with periods near 14 days, 24 hours, and 12 hours. Similarly the solar potential has periods near 180 days, 24 hours, and 12 hours. Thus there are three distinct groups of tidal frequencies: twice-daily, daily, and long period, having different latitudinal factors $\sin^2 \theta$, $\sin 2\theta$, and $1/2(1 - 3 \cos^2 \theta)$, where θ is the co-latitude ($90^\circ - \varphi$).

Table 17.1 Fundamental Tidal Frequencies

	Frequency °/hour	Period	Source
f_1	14.49205211	1 lunar day	Local mean lunar time
f_2	0.54901653	1 month	Moon's mean longitude
f_3	0.04106864	1 year	Sun's mean longitude
f_4	0.00464184	8.847 years	Longitude of Moon's perigee
f_5	-0.00220641	17.613 years	Longitude of Moon's ascending node
f_6	0.00000196	20,940 years	Longitude of sun's perigee

Doodson (1922) expanded (17.16) in a Fourier series using the cleverly chosen frequencies in Table 17.1. Other choices of fundamental frequencies are possible, for example the local, mean, solar time can be used instead of the local, mean, lunar time. Doodson's expansion, however, leads to an elegant decomposition of tidal constituents into groups with similar frequencies and spatial variability.

Using Doodson's expansion, each component of the tide has a frequency

$$f = n_1 f_1 + n_2 f_2 + n_3 f_3 + n_4 f_4 + n_5 f_5 + n_6 f_6 \quad (17.17)$$

where the integers n_i are the *Doodson numbers*. $n_1 = 1, 2, 3$ and n_2 – n_6 are between -5 and $+5$. To avoid negative numbers, Doodson added five to n_2 – n_6 . Each tidal component, sometimes called a *partial tides*, has a Doodson number. For example, the principal, twice-per-day, lunar tide has the number 255.555. Because the very long-term modulation of the tides by the change in sun's perigee is so small, the last Doodson number n_6 is usually ignored.

If the tidal potential is expanded in Doodson's Fourier series, and if the ocean surface is in equilibrium with the tidal potential, the largest tidal constituents would have frequencies and amplitudes given in Table 17.2. The expansion shows that tides with frequencies near one or two cycles per day are split into closely spaced lines with spacing separated by a cycle per month. Each of these lines is further split into lines with spacing separated by a cycle per year (Figure 17.13). Furthermore, each of these lines is split into lines with a spacing separated by a cycle per 8.8 yr, and so on. Clearly, there are very many possible tidal components.

Doodson's expansion included 399 components, of which 100 are long period, 160 are daily, 115 are twice per day, and 14 are thrice per day. Most have very small amplitudes, and only the largest are included in Table 17.2. The largest tides were named by Sir George Darwin (1911) and the names are included in the table. Thus, for example, the principal, twice-per-day, lunar tide, which has Doodson number 255.555, is the M_2 tide, called the *M-two* tide.

17.5 Tidal Prediction

If tides in the ocean were in equilibrium with the tidal potential, tidal prediction would be easy. Unfortunately, tides are far from equilibrium and tides are not easily calculated. First, the shallow-water wave which is the tide cannot move fast enough to keep up with the sun and the moon. On the equator, the tide

Table 17.2 Principal Tidal Constituents

Tidal Species	Name	n_1	n_2	n_3	n_4	n_5	Equilibrium Amplitude† (m)	Period (hr)
Semidiurnal	$n_1 = 2$							
Principal lunar	M_2	2	0	0	0	0	0.242334	12.4206
Principal solar	S_2	2	2	-2	0	0	0.112841	12.0000
Lunar elliptic	N_2	2	-1	0	1	0	0.046398	12.6584
Lunisolar	K_2	2	2	0	0	0	0.030704	11.9673
Diurnal	$n_1 = 1$							
Lunisolar	K_1	1	1	0	0	0	0.141565	23.9344
Principal lunar	O_1	1	-1	0	0	0	0.100514	25.8194
Principal solar	P_1	1	1	-2	0	0	0.046843	24.0659
Elliptic lunar	Q_1	1	-2	0	1	0	0.019256	26.8684
Long Period	$n_1 = 0$							
Fortnightly	Mf	0	2	0	0	0	0.041742	327.85
Monthly	Mm	0	1	0	-1	0	0.022026	661.31
Semiannual	Ssa	0	0	2	0	0	0.019446	4383.05

†Amplitudes from Apel (1987)

would need to propagate around the world in one day. This requires a wave speed of around 460 m/s, which is only possible in an ocean 22 km deep. Second, the continents interrupt the propagation of the wave.

We can separate the problem of tidal prediction into two parts. The first deals with the prediction of tides in ports and shallow water where tides can be measured by tide gauges. The second deals with the prediction of tides in the deep ocean where tides cannot be easily measured.

Tidal Prediction for Ports and Shallow Water Two techniques are used to predict future tides at a tide-gauge station using past observations of sea level measured at the gauge.

The Harmonic Method This is the traditional method, and it is still widely used. The method uses decades of tidal observations from a coastal tide gauge from which the amplitude and phase of each tidal constituent (the tidal harmonics) in the tide-gage record are calculated. The frequencies used in the analysis are specified in advance from the basic frequencies given in Table 17.1.

Dispite its simplicity, the technique had disadvantages compared with the response method described below.

1. More than 17.6 years of data are needed to resolve the modulation of the lunar tides.
2. Amplitude accuracy of 10^{-3} of the largest term require that at least 39 frequencies be determined. Doodson found 400 frequencies were needed for an amplitude accuracy of 10^{-4} of the largest term.
3. Non-tidal variability introduces large errors into the calculated amplitudes

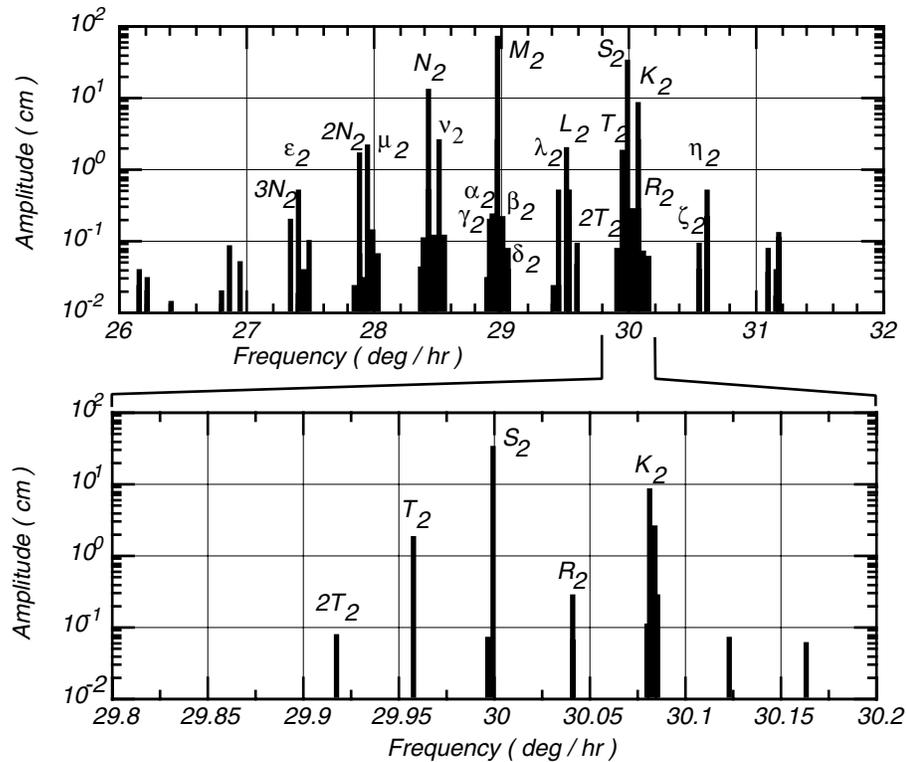


Figure 17.13 **Upper:** Spectrum of equilibrium tides with frequencies near twice per day. The spectra is split into groups separated by a cycle per month (1 deg/hr). (**Lower:** Expanded spectra of the S_2 group, showing splitting at a cycle per year (0.04 deg/hr). The finest splitting is at a cycle per 8.847 years (0.0046 deg/hr).

and phases of the weaker tidal constituents. The weaker tides have amplitudes smaller than variability at the same frequency due to other processes such as wind set up and currents near the tide gauge.

4. At many ports, the tide is non-linear, and many more tidal constituents are important. For some ports, the number of frequencies is unmanageable. When tides propagate into very shallow water, especially river estuaries, they steepen and become non-linear. This generates harmonics of the original frequencies. In extreme cases, the incoming waves steepens so much the leading edge is nearly vertical, and the wave propagates as a wall of water. This is a *tidal bore*.

The Response Method This method, developed by Munk and Cartwright (1966), calculates the relationship between the observed tide at some point and the tidal potential. The relationship is the spectral admittance between the major tidal constituents and the tidal potential at each station. The admittance is assumed to be a slowly varying function of frequency so that the admittance of the major constituents can be used for determining the response at nearby frequencies. Future tides are calculated by multiplying the tidal potential by

the admittance function.

1. The technique requires only a few months of data.
2. The tidal potential is easily calculated, and a knowledge of the tidal frequencies is not needed.
3. The admittance is $Z(f) = G(f)/H(f)$. $G(f)$ and $H(f)$ are the Fourier transforms of the potential and the tide gage data, and f is frequency.
4. The admittance is inverse transformed to obtain the admittance as a function of time.
5. The technique works only if the waves propagate as linear waves.

Tidal Prediction for Deep-Water Prediction of deep-ocean tides has been much more difficult than prediction of shallow-water tides because tide gauges were seldom deployed in deep water. All this changed with the launch of Topex/Poseidon. The satellite was placed into an orbit especially designed for observing ocean tides (Parke et al. 1987); and the altimetric system was sufficiently accurate to measure many components of the tide. Data from the satellite have now been used to determine deep-ocean tides with an accuracy of ± 2 cm. For most practical purposes, the tides are now known accurately for most of the ocean.

Several approaches have led to the new knowledge of deep-water tides using altimetry.

Prediction Using Hydrodynamic Theory Purely theoretical calculations of tides are not very accurate, especially because the dissipation of tidal energy is not well known. Nevertheless, theoretical calculations provide insight into processes influencing ocean tides. Several processes must be considered:

1. The tides in one ocean basin perturb earth's gravitational field, and the mass in the tidal bulge attracts water in other ocean basins. The self-gravitational attraction of the tides must be included.
2. The weight of the water in the tidal bulge is sufficiently great that it deforms the sea floor. The earth deforms as an elastic solid, and the deformation extends thousands of kilometers.
3. The ocean basins have a natural resonance close to the tidal frequencies. The tidal bulge is a shallow-water wave on a rotating ocean, and it propagates as a high tide rotating around the edge of the basin. Thus the tides are a nearly resonant sloshing of water in the ocean basin. The actual tide heights in deep water can be higher than the equilibrium values noted in Table 17.2.
4. Tides are dissipated by bottom friction especially in shallow seas, by the flow over seamounts and mid-ocean ridges, and by the generation of internal waves over seamounts and at the edges of continental shelves. If the tidal forcing stopped, the tides would continue sloshing in the ocean basins for several days.

5. Because the tide is a shallow-water wave everywhere, its velocity depends on depth. Tides propagate more slowly over mid-ocean ridges and shallow seas. Hence, the distance between grid points in numerical models must be proportional to depth with very close spacing on continental shelves (LeProvost et al. 1994).
6. Internal waves generated by the tides produce a small signal at the sea surface near the tidal frequencies, but not phase-locked to the potential. The noise near the frequency of the tides causes the spectral cusps in the spectrum of sea-surface elevation first seen by Munk and Cartwright (1966). The noise is due to deep-water, tidally generated, internal waves.

To reduce the computational difficulties, the theory has been supplemented at times with measurements made by tide gauges at a few sites in the deep ocean, at islands, and at well exposed coastal sites. In these cases the theory is used to interpolate between the observations. Schwiderski (1980) used the method to calculate global maps of eleven tidal constituents on a 1° by 1° grid, globally with ± 10 cm accuracy.

Altimetry Plus Response Method Several years of altimeter data from Topex/Poseidon have been used with the response method to calculate deep-sea tides almost everywhere equatorward of 66° (Ma et al. 1994). The altimeter measured sea-surface heights in geocentric coordinates at each point along the subsatellite track every 9.97 days. The temporal sampling aliased the tides into long frequencies, but the aliased periods are precisely known and the tides can be recovered (Parke et al. 1987). Because the tidal record is shorter than 8 years, the altimeter data are used with the response method to obtain predictions for a much longer time.

Recent solutions by ten different groups, have accuracy of ± 2.8 cm in deep water (Andersen, Woodworth, and Flather, 1995). Work has begun to improve knowledge of tides in shallow water.

Maps produced by this method show the essential features of the deep-ocean tides (Figure 17.14). The tide consists of a crest that rotates counterclockwise around the ocean basins in the northern hemisphere, and in the opposite direction in the southern hemisphere. Points of minimum amplitude are called *amphidromes*. Highest tides tend to be along the coast.

Altimetry Plus Numerical Models Altimeter data can be used directly with numerical models of the tides to calculate tides in all areas of the ocean from deep water all the way to the coast. Thus the technique is especially useful for determining tides near coasts and over sea-floor features where the altimeter ground track is too widely spaced to sample the tides well in space. Tide models use finite-element grids similar to the one shown in figure 15.4. Recent numerical calculations by (LeProvost et al. 1994; LeProvost, Bennett, and Cartwright, 1995) give global tides with ± 2 – 3 cm accuracy and full spatial resolution.

Further improvements will lead to solutions at the ultimate limits of practical accuracy, which is about ± 1 – 2 cm. The limit is set by noise from internal waves with tidal frequency, and the small, long-term variations of depth of the ocean. Changing heat content of the ocean produces changes in oceanic topography of

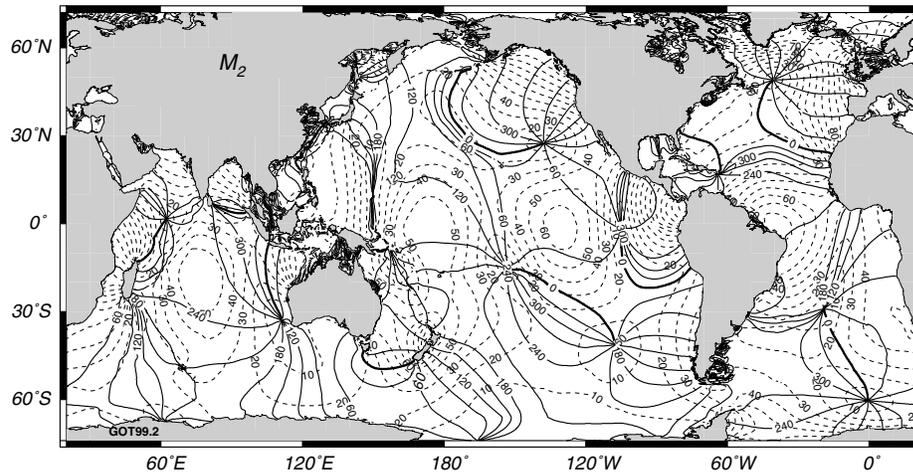


Figure 17.14 Global map of M_2 tide calculated from Topex/Poseidon observations of the height of the sea surface combined with the response method for extracting tidal information. Full lines are contours of constant tidal phase, contour interval is 30° . Dashed lines are lines of constant amplitude, contour interval is 10 cm. (From Richard Ray, NASA Goddard Space Flight Center).

a few centimeters, and this changes ever so slightly the velocity of shallow-water waves.

Tidal Dissipation Tides dissipate 3.75 ± 0.08 TW of power (Kantha, 1998), of which 3.5 TW are dissipated in the ocean, and much smaller amounts in the atmosphere and solid earth. The dissipation increases the length of day by about 2.07 milliseconds per century, it causes the semimajor axis of the moon's orbit to increase by 3.86 cm/yr, and it mixes water masses in the ocean.

The calculations of dissipation from Topex/Poseidon observations of tides are remarkably close to estimates from lunar-laser ranging, astronomical observations, and ancient eclipse records. Our knowledge of the tides is now sufficiently good that we can begin to use the information to study mixing in the ocean, which has important implications for understanding the abyssal circulation in the ocean (Munk and Wunsch, 1998).

17.6 Important Concepts

1. Waves propagating into shallow water are refracted by features of the sea floor, they eventually break on the beach where the wave breaking drives near-shore currents including long-shore currents, rip currents, and edge waves.
2. Storm surges are driven by strong winds in storms close to shore. The amplitude of the surge is a function of wind speed, the slope of the seafloor, and the propagation of the storm
3. Tides are important for navigation; they influence accurate geodetic measurements; they change the orbits and rotation of planets, moons, and stars in galaxies.

4. Tides are produced by a combination of time-varying gravitational potential of the moon and sun and the centrifugal forces generated as earth rotates about the common center of mass of the earth-moon-sun system.
5. Tides have six fundamental frequencies. The tide is the superposition of hundreds of tidal constituents, each having a frequency that is the sum and difference of five fundamental frequencies.
6. Shallow water tides are predicted using tide measurements made in ports and other locations along the coast. Tidal records of just a few months duration can be used to predict tides many years into the future.
7. Tides in deep water are calculated from altimetric measurements, especially Topex/Poseidon measurements. As a result, deep water tides are known almost everywhere with an accuracy approaching ± 2 cm.
8. The dissipation of tidal energy in the ocean transfers angular momentum from the moon to the earth, causing the day to become longer, and it mixes water masses.