Chapter 15

Numerical Models

We saw earlier that analytic solutions to the equations of motion are difficult or impossible to obtain for typical oceanic flows. The problem is due to the non-linear terms, friction, and the need for realistic shapes for the sea floor and coastlines. We have also seen how difficult it is to describe the ocean from measurements. Satellites can observe some processes almost everywhere every few days. But they observe only some processes, and only near or at the surface. Ships can measure more variables, and deeper into the water, but the measurements are sparse. Hence, numerical models provide the only useful, global view of ocean currents. Let's look at the accuracy and validity of the models, keeping in mind that although they are only models, they provide a remarkably detailed and realistic view of the ocean.

15.1 Introduction–Some Words of Caution

Numerical models of ocean currents have many advantages. They simulate flows in realistic ocean basins with a realistic sea floor. They include the influence of viscosity and non-linear dynamics. And they can calculate possible future flows in the ocean. Perhaps, most important, they interpolate between sparse observations of the ocean produced by ships, drifters, and satellites.

Numerical models are not without problems. "There is a world of difference between the character of the fundamental laws, on the one hand, and the nature of the computations required to breathe life into them, on the other"—Berlinski (1996). The models can never give complete descriptions of the oceanic flows even if the equations are integrated accurately. The problems arise from several sources.

Discrete equations are not the same as continuous equations. In Chapter 7 we wrote down the differential equations describing the motion of a continuous fluid. Numerical models use algebraic approximations to the differential equations. We assume that the ocean basins are filled with a grid of points, and time moves forward in tiny steps. The value of the current, pressure, temperature, and salinity are calculated from their values at nearby points and previous times. Ian Stewart (1992), a noted mathematician, points out that

Discretization is essential for computer implementation and cannot be dispensed with. The essence of the difficulty is that the dynamics of discrete systems is only loosely related to that of continuous systems—indeed the dynamics of discrete systems is far richer than that of their continuous counterparts—and the approximations involved can create spurious solutions.

Calculations of turbulence are difficult. Numerical models provide information only at grid points of the model. They provide no information about the flow between the points. Yet, the ocean is turbulent, and any oceanic model capable of resolving the turbulence needs grid points spaced millimeters apart, with time steps of milliseconds. Clearly, such a model can be used only for flow in a small box.

Practical ocean models have grid points spaced tens to hundreds of kilometers apart in the horizontal, and tens to hundreds of meters apart in the vertical. This means that turbulence cannot be calculated directly, and the influence of turbulence must be parameterized. Holloway (1994) states the problem succinctly:

Ocean models retain fewer degrees of freedom than the actual ocean (by about 20 orders of magnitude). We compensate by applying 'eddyviscous goo' to squash motion at all but the smallest retained scales. (We also use non-conservative numerics.) This is analogous to placing a partition in a box to prevent gas molecules from invading another region of the box. Our oceanic models cannot invade most of the real oceanic degrees of freedom simply because the models do not include them.

Given that we cannot do things 'right', is it better to do nothing? That is not an option. 'Nothing' means applying viscous goo and wishing for the ever bigger computer. Can we do better? For example, can we guess a higher entropy configuration toward which the eddies tend to drive the ocean (that tendency to compete with the imposed forcing and dissipation)?

By "degrees of freedom" Holloway means all possible motions from the smallest waves and turbulence to the largest currents. We will return to turbulence later in this chapter.

Practical models must be simpler than the real ocean. Models of the ocean must run on available computers. This means oceanographers further simplify their models. We use the hydrostatic and Boussinesq approximations, and we often use equations integrated in the vertical, the shallow-water equations (Haid-vogel and Beckmann, 1999: 37). We do this because we cannot yet run the most detailed models of oceanic circulation for thousands of years to understand the role of the ocean in climate.

Initial conditions are not well known. How to initialize the model? We do not know accurately the present velocity and density in the ocean, the starting point for running any model. The best we can do is to start at rest using the best estimates of the ocean's density field, such as that contained in the digital atlas produced by Levitus (1982, 1994), or we can use the output from an earlier run of the model or a similar model. Still, there are difficulties. The Levitus atlas is based on slightly inaccurate observations made over many decades; and the oceans take hundreds of years to come to equilibrium with the atmosphere, so models must run for hundreds of years to get the right deep circulation.

Numerical code has errors. Do you know of any software without bugs? Numerical models use many subroutines each with many lines of code which are converted into instructions understood by processors using other software called a compiler. Eliminating all software errors is impossible. With careful testing, the output may be correct, but the accuracy cannot be guaranteed. Plus, numerical calculations cannot be more accurate than the accuracy of the floating-point numbers and integers used by the computer. Round-off errors cannot be ignored. Lawrence et al (1999), examining the output of an atmospheric numerical model found an error in the code produced by the FORTRAN-90 compiler used on the CRAY Research supercomputer used to run the code. They also found round-off errors in the concentration of tracers calculated from the model. Both errors produced important errors in the output of the model.

Summary Despite these many sources of error, most are small in practice. Numerical models of the ocean are giving the most detailed and complete views of the circulation available to oceanographers. Some of the simulations contain unprecedented details of the flow. Langer (1999), writing about the use of computers in physics wrote:

All of who are involved in the sciences know that the computer has become an essential tool for research ... Scientific computation has reached the point where it is on a par with laboratory experiment and mathematical theory as a tool fro research in science and engineering.

I included the words of warning not to lead you to believe the models are wrong, but to lead you to accept the output with a grain of salt.

15.2 Numerical Models in Oceanography

Numerical models are used for many purposes in oceanography. For our purpose we can divide models into two classes:

Mechanistic models are simplified models used for studying processes. Because the models are simplified, the output is easier to interpret than output from more complex models. Many different types of simplified models have been developed, including models for describing planetary waves, the interaction of the flow with sea-floor features, or the response of the upper ocean to the wind. These are perhaps the most useful of all models because they provide insight into the physical mechanisms influencing the ocean. The development and use of mechanistic models is, unfortunately, beyond the scope of this book.

Simulation models are used for calculating realistic circulation of oceanic regions. The models are often very complex because all important processes are included, and the output is difficult to interpret. Let's look at a few of the more widely used models.

15.3 Simulation Models

The first simulation models were developed by Kirk Bryan and Michael Cox at the Geophysical Fluid Dynamics laboratory in Princeton. Their model (Bryan, 1969) calculated the 3-dimensional flow in the ocean using the continuity and momentum equation with the hydrostatic and Boussinesq approximations and a simplified equation of state. Such models are called *primitive equation* models because they use the most basic, or primitive form of the equations of motion. The equation of state allows the model to calculate changes in density due to fluxes of heat and water through the sea surface, so the model includes thermodynamic processes.

The Bryan-Cox model used large horizontal and vertical viscosity and diffusion to eliminate turbulent eddies having diameters smaller about 500 km, which is a few grid points in the model. It also had complex coastlines, smoothed seafloor features, and a rigid lid.

The rigid lid was necessary for eliminating ocean-surface waves, such as tides and tsunamis which move far too fast for the coarse time steps used by all simulation models. The rigid lid has, however, disadvantages. Islands substantially slow the computation, and the sea-floor features must be smoothed to eliminate steep gradients.

The first simulation model was regional. It was quickly followed by a global model (Cox, 1975) with a horizontal resolution of 2° and with 12 levels in the vertical. The model ran far too slowly even on the fastest computers of the day, but it laid the foundation for more recent models. The coarse spatial resolution required that the model have large values for viscosity, and even regional models were too viscous to have realistic western boundary currents or mesoscale eddies.

Since those times, the goal has been to produce models with ever finer resolution. The hope is that if the resolution is sufficiently fine, the output will be sufficiently realistic. Computer technology has changed rapidly, and models have evolved rapidly. The output from the most recent models of the North Atlantic, which have resolution of 0.1° look very much like the real ocean (Smith et al, 2000). Let's look at a few typical and widely used models.

15.4 Primitive-Equation Models

The Bryan-Cox models evolved into many, widely used models which are providing impressive views of the global ocean circulation. The models include the influence of heat and water fluxes, eddy dynamics, and the meridionaloverturning circulation. The models range in complexity from those that can run on desktop workstations to those that require the world's fastest computers. Semtner (1995) gives a good summary of resent results from computers with multiple, parallel processors.

Eddy-admitting, primitive-equation models have sufficient horizontal resolution that they produce mesoscale eddies. The resolution of these models is a few tenths of a degree of latitude and longitude, which is sufficient to resolve the largest eddies, those with diameter larger than two to three times the distance between grid points, such as those seen in Figures 11.11, 11.12, and 15.2. Verti-

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cal resolution is typically around 30 vertical levels. The models include realistic coasts and bottom features. The models are possible thanks to the development of fast, parallel processors with large memory. To obtain high vertical and horizontal resolution, the models require more than a million grid points. Typically they can simulate the global oceanic circulation for several decades.

Geophysical Fluid Dynamics Laboratory Modular Ocean Model MOM is perhaps the most widely used model growing out of the original Bryan-Cox code. It consists of a large set of modules that can be configured to run on many different computers to model many different aspects of the circulation. The source code is open and free, and it is in the public domain. The model is widely use for climate studies and and for studying the ocean's circulation over a wide range of space and time scales (Pacanowski and Griffies, 1999).

Because MOM is used to investigate processes which cover a wide range of time and space scales, the code and manual are lengthy. However, it is far from necessary for the typical ocean modeler to become acquainted with all of its aspects. Indeed, MOM can be likened to a growing city with many different neighborhoods. Some of the neighborhoods communicate with one another, some are mutually incompatible, and others are basically independent. This diversity is quite a challenge to coordinate and suport. Indeed, over the years certain "neighborhoods" have been jettisoned or greatly renovated for various reasons.—Pacanowski and Griffies.

The model uses the momentum equations, equation of state, and the hydrostatic and Boussinesq approximations. Subgrid-scale motions are reduced by use of eddy viscosity. Version 3 of the model has a free surface, realistic bottom features, and it can be coupled to atmospheric models.

Semtner and Chervin's Global Model was perhaps the first, global, eddyadmitting model based on the Bryan-Cox models (Semtner and Chervin, 1988). It has much in common with the Modular Ocean Model (Semtner helped write the MOM code), and it provided the first high resolution view of ocean dynamics. It has a resolution of $0.5^{\circ} \times 0.5^{\circ}$ with 20 levels in the vertical. It has simple eddy viscosity, which varies with scale; and it does not allow static instability. In contrast with earlier models, it is global, excluding only the Arctic Sea, it includes the largest turbulent eddies, and it has realistic bottom features and coastlines. Originally, it had a rigid lid to eliminate fast-moving waves such as tides, so the bottom features were smoothed and it had few islands. More recent versions of the model have a free surface, eliminating the restrictions of the rigid lid.

The model was started from rest with observed vertical density distribution. Then it was was spun up for 22.5 yr with mean-annual wind stress, heat fluxes, and water fluxe. Then, it was integrated for ten more years with monthly wind stress, heat fluxes, and water fluxes. The integration was extended for an additional 12.5 years and the results were reported in Semtner and Chervin (1992). The results give a realistic picture of the global ocean circulation, its eddies, the transport of heat and mass, and the statistics of the variability.

Parallel Ocean Climate Model POCM is the latest version of the Semtner-Chervin model, being optomized to run on parallel computers. The model uses equally spaced grid points on a Mercator projection extending between $\pm 75^{\circ}$ with a resolution of $0.4^{\circ} \times 0.4^{\circ} \cos \theta \times 20$ so the resolution varyies from 0.4° at the equator to 0.1° near the poles. The average resolution is about 0.25° . It has a free surface, and realistic coasts, islands, and bottom features. It is forced by ECMWF wind stress and surface heat and water fluxes (Barnier et al, 1995).

The model was initialized with the fields calculated from the 0.5° Semtner-Chervin 1992 model which had been spun up for 33 years starting with the distribution of temperature and salinity from the Levitus atlas. The $1/2^{\circ}$ fields were interpolated to $1/4^{\circ}$, and the model was integrated starting from 1985 using ECMWF fluxes.

Output from the model has been compared with altimeter data from Topex/ Poseidon (Stammer et al, 1996). The comparisons indicate that numerical circulation models forced with the best known fluxes, and with resolution sufficient to resolve the larger eddies in the ocean, give realistic results.

Parallel Ocean Program Model produced by Smith and colleagues at Los Alamos National Laboratory (Maltrud et al, 1998) is a variation of the Semtner-Chervin model modified to run on a fast, parallel-processor computer, the CM-5 Connection Machine at Los Alamos. The modifications included improved numerical algorithms, realistic coasts, islands, and unsmoothed bottom features.

The model has 1280×896 equally spaced grid points on a Mercator projection extending from 77°S to 77°N, and 20 levels in the vertical. Thus it has a resolution of $0.28^{\circ} \times 0.28^{\circ} \cos \theta$, which varies from 0.28° (31.25 km) at the equator to 0.06° (6.5 km) at the highest latitudes. The average resolution is about 0.2° . The model was is forced by ECMWF wind stress and surface heat and water fluxes (Barnier et al, 1995).

The model was initialized using temperature and salinity interpolated from the integration of Semtner and Chervin's (1992) 0.25° degree model, which was initialized from the integration of Semtner and Chervin's (1988) model. The model was then integrated for a 10-year period beginning in 1985 using various surface-forcing functions. Figure 15.1 was calculated using surface wind stress from the ECMWF averaged over 3-day periods and surface heat and fresh-water fluxes obtained by restoring the surface temperature and salinity to the monthly mean values of Levitus (1982).

Miami Isopycnal Coordinate Ocean Model MICOM All the models just described use x, y, z coordinates. Such a coordinate system has disadvantages. For example, mixing in the ocean is easy along surfaces of constant density, and difficult across such surfaces. A more natural coordinate system uses x, y, ρ , where ρ is density. A model with such coordinates is called an *isopycnal model*. Essentially, $\rho(z)$ is replaced with $z(\rho)$. Furthermore, because isopycnal surfaces are surfaces of constant density, horizontal mixing is always on constant-density surfaces in this model.

The Miami model is a prominant example of this class of models. It is a primitive-equation model driven by wind stress and heat fluxes. It has been integrated from 65°N to 69°S using 20 million grid points with horizontal spacing of $0.225^{\circ} \times 0.225^{\circ} \cos \theta$, where θ is latitude. The model was forced with COADS data supplemented with ECMWF data south of 30°S and with rain calculated



Figure 15.1 Instantaneous, near-surface geostrophic currents in the Atlantic for October 1, 1995 calculated from the Parallel Ocean Program numerical model developed at the Los Alamos National Laboratory. The length of the vector is the mean speed in the upper 50 m of the ocean; the direction is the mean direction of the current. From Richard Smith.

from the spaceborne microwave sounding unit. The model was initialized using temperature and salinity distributions from the Levitus (1994) atlas. The model produces realistic transports and currents (Figure 15.2).

Primitive-equation climate models are used for studies of large-scale hydrographic structure, climate dynamics, and water-mass formation. These mod-



Figure 15.2 Output of Bleck's Miami Isopycnal Coordinate Ocean Model MICOM. It is a high-resolution model of the Atlantic showing the Gulf Stream, its variability, and the circulation of the North Atlantic (From Bleck).

els are the same as the eddy-admitting, primitive equation models I have just described except the horizontal resolution is much coarser. Because the models must simulate ocean flows for centuries, they must have coarse horizontal resolution and they cannot simulate msoscale eddies. Hence, they must have high dissipation for numerical stability. Typical horizontal resolutions are 2° to 4°. The models tend, however, to have high vertical resolution necessary for describing the meridional-overturning circulation important for climate. Often, the models are coupled with atmospheric and land models to simulate earth's climate.

15.5 Coastal Models

The great economic importance of the coastal zone has led to the development of many different numerical models for describing coastal currents, tides, and storm surges. The models extend from the beach to the continental slope, and they can include a free surface, realistic coasts and bottom features, river runoff, and atmospheric forcing. Because the models don't extend very far into deep water, they need additional information about deep-water currents or conditions at the shelf break.

The many different coastal models have many different goals, and many different implementations. Several of the models described above, including MOM and MICOM, have been used to model coastal processes. But many other specialized models have also been developed. Heaps (1987), Lynch et al (1996), and Haidvogel (1998) provide good overviews of the subject. Rather than look at a menu of models, let's look at two typical models.

Princeton Ocean Model developed by Blumberg and Mellor (1987) is widely

used for describing coastal currents. It is a direct descendant of the Bryan-Cox model. It includes thermodynamic processes, turbulent mixing, and the Boussinesq and hydrostatic approximations. The Coriolis parameter is allowed to vary using a beta-plane approximation. Because the model must include a wide reage of depths, Blumberg and Mellor used a vertical coordinate σ scaled by the depth of the water:

$$\sigma = \frac{z - \eta}{H + \eta} \tag{15.1}$$

where $z = \eta(x, y, t)$ is the sea surface, and z = -H(x, y) is the bottom.

Sub-grid turbulence is parameterized using a closure scheme proposed by Mellor and Yamada (1982) whereby eddy diffusion coefficients vary with the size of the eddies producing the mixing and the shear of the flow.

The model is driven by wind stress and heat and water fluxes from meteorological models. The model uses known geostrophic, tidal, and Ekman currents at the outer boundary.

The model has been used to calculate the three-dimensional distribution of velocity, salinity, sea level, temperature, and turbulence for up to 30 days over a region roughly 100-1000 km on a side with grid spacing of 1-50 km.

Dartmouth Gulf of Maine Model developed by Lynch et al (1996) is a 3dimensional model of the circulation using a triangular, finite-element grid. The size of the triangles is proportional to both depth and the rate of change of depth. The triangles are small in regions where the bottom slopes are large and the depth is shallow, and they are large in deep water. The variable mesh is especially useful in coastal regions where the depth of water varies greatly. Thus the variable grid gives highest resolution where it is most needed.

The model uses roughly 13,000 triangles to cover the Gulf of Maine and nearby waters of the North Atlantic (Figures 15.3 and 15.4). Minimum size of the elements is roughly one kilometer. The model has 10 to 40 horizontal layers. The vertical spacing of the layers is not uniform. Layers are closer together near the top and bottom and they are more widely spaced in the interior. Minimum spacing is roughly one meter in the bottom boundary layer.

The model integrates the three-dimensional, primitive equations, in shallowwater form. The model has a simplified equation of state and a depth-averaged continuity equation, and it uses the hydrostatic and Boussinesq assumptions. Sub-grid mixing of momentum, heat and mass is parameterized using the Mellor and Yamada (1982) turbulence-closure scheme which gives vertical mixing coefficients that vary with stratification and velocity shear. Horizontal mixing coefficients were calculated from Smagorinski (1963). A carefully chosen, turbulent, eddy viscosity is used in the bottom boundary layer. The model is forced by wind, heating, and tidal forcing from the deep ocean.

The model is spun up from rest for a few days using a specified density field at all grid points, usually from a combination of CTD data plus historical data. This gives a velocity field consistent with the density field. The model is then forced with local winds and heat fluxes to calculate the evolution of the density and velocity fields.



Figure 15.3 Topographic map of the Gulf of Maine showing important features of the Gulf. (From Lynch et al, 1996).

Comments on Coastal Models Roed et al. (1995) examined the accuracy of coastal models by comparing the ability of five models, including Blumberg and Mellor's to describe the flow in typical cases. They found that the models produced very different results, but that after the models were adjusted, the differences were reduced. The differences were due to differences in vertical and horizontal mixing and spatial and temporal resolution.

Hackett et al. (1995) compared the ability of two of the five models to describe observed flow on the Norwegian shelf. They conclude that

... both models are able to qualitatively generate many of the observed features of the flow, but neither is able to quantitatively reproduce detailed currents ... [Differences] are primarily attributable to inadequate parameterizations of subgrid scale turbulent mixing, to lack of horizontal resolution and to imperfect initial and boundary conditions.

Storm-Surge Models Storms coming ashore across wide, shallow, continental shelves drive large changes of sea level at the coast called storm surges (see §17.3 for a description of surges and processes influencing surges). The surges can cause great damage to coasts and coastal structures. Intense storms in the Bay of Bengal have killed hundrend of thousands in a few days in Bangladesh. Because surges are so important, government agencies in many countries have developed models to predict the changes of sea level and the extent of coastal flooding.



Figure 15.4 Triangular, finite-element grid used to compute flow in the Gulf of Maine shown in the previous figure. Note that the size of the triangles varies with depth and rate of change of depth. (From Lynch et al, 1996).

Calculating storm suges is not easy. Here are some reasons, in a rough order of importance.

- 1. The distribution of wind over the ocean is not well known. Numerical weather models calculate wind speed at a constant pressure surface, storm-surge models need wind at a constant height of 10 m. Winds in bays and lagoons tend to be weaker than winds just offshore because nearby land distorts the airflow, and this is not included in the weather models.
- 2. The shoreward extent of the model's domain changes with time. For example, if sea level rises, water will flood inland, and the boundary between water and sea moves inland with the water.
- 3. The drag coefficient of wind on water is not well known for hurricane force winds.
- 4. The drag coefficient of water on the seafloor is also not well known.
- 5. The models must include waves and tides which influence sea level in shallow waters.
- 6. Storm surge models must include the currents generated in a stratified, shallow sea by wind.

To reduce errors, models are tuned to give results that match conditions seen in past storms. Unfortunately, those past conditions are not well known. Changes in sea level and wind speed are rarely recorded accurately in storms except at a few, widely paced locations. Yet storm-surge heights can change by more than a meter over distances of tens of kilometers.

Despite these problems, models give very useful results. Let's look at one, commonly-used model.

Sea, Lake, and Overland Surges Model SLOSH is used by NOAA for forecasting storm surges prduced by hurricanes coming ashore along the Atlantic and Gulf coasts of the United States (Jelesnianski, Chen, and Shaffer, 1992).

The model is the result of a lifetime of work by Chester Jelesnianski. In developing the model, Jelesnianski paid careful attention to the relative importance of errors in the model. He worked to reduce the largest errors, and ignored the smaller ones. For example, the distribution of winds in a hurricane is not well known, so it makes little sense to use a spatially varying drag coefficient for the wind. Thus, Jelesnianski used a constant drag coefficient in the air, and a constant eddy stress coefficient in the water.

SLOSH calculates water level from depth-integrated, quasi-linear, shallowwater equations. Thus it ignores stratification. It also ignores river inflow, rain, and tides. The latter may seem strange, but the model is designed for forecasting. The time of landfall cannot be foreast accurately, and hence the height of the tides is mostly unknown. Tides can be added to the calculated surge, but the nonlinear interaction of tides and surge is ignored.

The model is forced by idealized hurricane winds. It needs only atmospheric pressure at the center of the storm, the distance from the center to the area of maximum winds, the forecast storm track and speed along the track.

In preparation for hurricanes coming ashore near populated areas, the model has been adapted for 27 basins from Boston Harbor Massachusetts to Laguna Madre Texas. The model uses a fixed polar mesh. Mesh spacing begins with a fine mesh near the pole, which is located near the coastal city for which the model is adapted. The grid stretches continuously to a coarse mesh at distant boundaries of a large basin. Such a mesh gives high resolution in bays and near the coast where resolution is most needed. Using measured depths at sea and elevations on land, the model allows flooding of land, overtopping of levees and dunes, and sub-grid flow through channels between offshore islands.

Sea level calculated from the model has been compared with heights measured by tide gauges for 13 storms, including Betsy (1965), Camile (1969), Donna (1960), and Carla (1961). The overall accuracy is $\pm 20\%$.

15.6 Assimilation Models

None of the models we have described so far have output, such as current velocity or surface topography, constrained by oceanic observations. Thus we may ask: Can we model currents more accurately if we include observations of the variables we are trying to calculate? For example, can we use satellite altimetric measurements of the sea-surface topography and WOCE measurements of currents and internal density in the ocean to make a better model of the present ocean currents? Models which accept data that they are also trying to calculate are called *assimilation models*.

Here is a simple example. Suppose we are running a primitive-equation, eddy admitting numerical model to calculate the position of the Gulf Stream. Let's assume that the model is driven with real-time surface winds from the ECMWF weather model. Using the model, we can calculate the position of the current and also the sea-surface topography associated with the current. We find that the position of the Gulf Stream wiggles offshore of Cape Hattaras due to instabilities, and the position calculated by the model is just one of many possible positions for the same wind forcing. Which position is correct, that is, what is the position of the current today? We know, from satellite altimetry, the position of the current at a few points a few days ago. Can we use this information to calculate the current's position today? How do we assimilate this information into the model?

Many different approaches are being explored (Malanotte-Rizzoli, 1996). Roger Daley (1991) gives a complete description of how data are used with atmospheric models. Andrew Bennet (1992) and Carl Wunsch (1996) describe oceanic applications.

Assimilation of data into models is not easy.

1. Data assimilation is an *inverse problem*: A finite number of observations are used to estimate a continuous field—a function, which has an infinite number of points. The calculated fields, the solution to the inverse problem, are completely under-determined. There are many fields that fit the observations and the model precisely; and the solutions are not unique. In our example, the position of the Gulf Stream is a function. We may not need an infinite number of values to specify the position of the stream if we assume the position is somewhat smooth in space. But we certainly need hundreds of values along the stream's axis. Yet, we have only a few satellite points to constrain the position of the Stream.

To learn more about inverse problems and their solution, read Parker (1994) who gives a very good introduction based on geophysical examples.

- 2. Ocean dynamics are non-linear, while most methods for calculating solutions to inverse problems depend on linear approximations. For example the position of the Gulf Stream is a very nonlinear function of the forcing by wind and heat fluxes over the North Atlantic.
- 3. Both the model and the data are incomplete and both have errors. We have few altimeter data points, and altimeter measurements of topography have errors, although the errors are small.
- 4. Most data available for assimilation into data comes from the surface, such as AVHRR and altimeter data. Surface data obviously constrain the surface geostrophic velocity, and surface velocity is related to deeper velocities. The trick is to couple the surface observations to deeper currents.

While various techniques are used to constrain numerical models in oceanography, perhaps the most practical are techniques borrowed from meteorology.

Most major ocean currents have dynamics which are significantly nonlinear. This precludes the ready development of inverse methods ... Accordingly, most attempts to combine ocean models and measurements have followed the practice in operational meteorology: measurements are used to prepare initial conditions for the model, which is then integrated forward in time until further measurements are available. The model is thereupon re-initialized. Such a strategy may be described as sequential.—Bennet (1992).

Let's see how Professor Allan Robinson and colleagues at Harvard University used sequential estimation techniques to forecast the position of the Gulf Stream.

The Harvard Open-Ocean Model is an eddy-admitting, quasi-geostropic model of the Gulf Stream downstraem of Cape Hatteras (Robinson et al. 1989). It has six levels in the vertical, 15 km resolution, and one-hour time steps. It uses a simple filter to smooth high-frequency variability and to damp grid-scale variability.

By quasi-geostrophic we mean that the flow field is close to geostrophic balance. The equations of motion include the acceleration terms D/Dt, where D/Dt is the substantial derivative and t is time. The flow can be stratified, but there is no change in density due to heat fluxes or vertical mixing. Thus the quasi-geostrophic equations are simpler than the primitive equations, and they can be integrated much faster. Cushman-Roisin (1994: 204) gives a good description of the development of quasi-geostrophic equations of motion.

The model reproduces the important features of the Gulf Stream and it's extension, including meanders, cold- and warm-core rings, the interaction of rings with the stream, and baroclinic instability. Because the model was designed to forecast the dynamics of the Gulf Stream, it must be constrained by oceanic measurements:

- 1. Data provide the initial conditions for the model. Satellite measurements of sea-surface temperature from the AVHRR and topography from an altimeter are used to determine the location of features in the region. Expendable bathythermograph AXBT measurements of subsurface temperature, and historical measurements of internal density are also used. The features are represented by simple analytic functions in the model.
- 2. The data are introduced into the numerical model, which interpolates and smoothes the data to produce the best estimate of the initial fields of density and velocity. The resulting fields are called *nowcasts*.
- 3. The model is integrated forward for one week, when new data are available, to produce a forecast.
- 4. Finally, the new data are introduced into the model as in the first step above, and the processes is repeated.

The model has been used for making successful, one-week forecasts of the Gulf Stream and region (Figure 15.4). Similar models have been developed to study the Azores current.



Figure 15.5 Output from the Harvard Open-Ocean Model: (Top) Data used for initializing the model on 2 March 1988 and the initial state of the model, the nowcast. (Center) The forecast for 9 March 1988. (Lower) The data used for determining currents on 9 March 1988 and the state of the ocean on that date calculated by the model using the new data. Although the Gulf Stream changed substantially in one week, the model forecasts the changes well. (From Robinson et al. 1989)

15.7 Coupled Ocean and Atmosphere Models

Coupled numerical models of the atmosphere and the ocean are used to study the climate system, its natural variability, and its response to external forcing. The most important use of the models has been to study how Earth's climate might respond to a doubling of CO_2 in the atmosphere. Much of the literature on climate change is based on studies using such models. Other important uses of coupled models include studies of El Niño and the meridional overturning circulation. The former varies over periods of a few years, the latter varies over a period of a few centuries.

Development of the work tends to be coordinated through the World Climate Research Program of the World Meteorological Organization WCRP/WMO, and recent progress is summarized in Chapter 5 of the *Climate Change 1995* report by the Intergovernmental Panel on Climate Change (Gates, et al, 1996).

Comments on Accuracy of Coupled Models Models of the coupled, landair-ice-ocean climate system must simulate hundreds to thousands of years. Yet,

It will be very hard to establish an integration framework, particularly on a global scale, as present capabilities for modelling the Earth system are rather limited. A dual approach is planned. On the one hand, the relatively conventional approach of improving coupled atmosphere-oceanland-ice modles will be pursued. Ingenuity aside, the computational demands are extreme, as is borne out by the Earth System Simulator — 640 linked supercomputers providing 40 teraflops $[10^{12}$ floating-point operations per second] and a cooling system from hell under one roof — to be built in Japan by 2003.— Newton, 1999.

Because models must be simplified to run on existing computers, the simplification introduces important errors. First, the models must be simplier than models that simulate flow for a few years (WCRP, 1995).

Second, the coupled model must be integrated for many years for the ocean and atmosphere to approach equilibrium. This causes a new type of error. The coupled system tends to drift away from reality due to errors in calculating fluxes of heat and momentum between the ocean and atmosphere. For example, very small errors in precipitation over the Antarctic Circumpolar Current leads to small changes the salinity of the current, which leads to large changes in deep convection in the Weddel Sea, which influences the meridional overturning circulation.

Some modelers allow the system to drift, others adjust sea-surface temperature and the calculated fluxes between the ocean and atmosphere. Returning to the example, the flux of fresh water in the circumpolar current could be adjusted to keep salinity close to the observed value in the current. There is no good scientific basis for the adjustments except the desire to produce a "good" coupled model. Hence, the adjustments are ad hoc and controversial. Such adjustments are called *flux adjustments* or *flux corrections*.

Fortunately, as models have improved, the need for adjustment or the magnitude of the adjustment has been reduced. For example, using the Gent-McWilliams scheme for mixing along constant-density surfaces in a coupled ocean-atmosphere model greatly reduced climate drift in a coupled ocean-atmosphere model because the mixing scheme reduced deep convection in the Antarctic Circumpolar Current and elsewhere (Hirst, O'Farrell, and Gordon, 2000).

Grassl (2000) lists four capabilities of a credible coupled general circulation model:

- 1. "Adequate representation of the present climate.
- 2. "Reproduction (within typical interannual and decadal time-scale climate variability) of the changes since the start of the instrumental record for a given history of external forcing;

- 3. "Reproduction of a different climate ephisode in the past as derived from paleo climate records for given estimates of the history of external forcing; and
- 4. "Successful simulation of the gross features of an abrupt climate change event from the past."

Gates et al (1996) compared the output from sixteen coupled models, including models with and without flux adjustments. They found substantial differences among the models. For example, only three models calculated a meridional overturning circulation within the observed range of 13–18 Sv. Some had values as low as 2 Sv, others had values as large as 26 Sv. Furthermore, the observed root-mean-square difference (standard deviation) of the difference between the observed and calculated heat fluxes was 17–30 W/m² depending on season and hemisphere.

Grassl (2000) found four years later that many models, including models with and without flux adjustment, meet the first criterion. Some models meet the second criterion, but external solar forcing is still not well known and more work is needed. And a few models are starting to reproduce some aspects of the warm event of 6,000 years ago.

But how useful are these models in making projections of future climate? Opinion is polarized. At one extreme are those who take the model results as gospel; at the other are those who denigrate results simply because they distrust models, or on the grounds that the model preformance is obviously wrong in some respects or that a process is not adequately included. The truth lies in between. All models are of course wrong because, by design, they depict a simplified view of the system being modelled. Nevertheless, many—but not all—models are very useful.—Trenberth, 1997.

Coupled models Many coupled ocean and atmosphere models have been developed. Some include only physical processes in the ocean, atmosphere, and the ice-covered polar seas. Others add the influence of land and biological activity in the ocean. Let's look at the oceanic components of a few models.

Climate System Model The Climate System Model developed by the National Center for Atmospheric Research NCAR includes physical and biogeochemical influence on the climate system (Boville and Gent, 1998). It has atmosphere, ocean, land-surface, and sea-ice components coupled by fluxes between components. The atmospheric component is the NCAR Community Climate Model, the oceanic component is a modified version of the Princeton Modular Ocean Model, using the Gent and McWilliams (1990) scheme for parameterizing mesoscale eddies. Resolution is approximately $2^{\circ} \times 2^{\circ}$ with 45 vertical levels in the ocean.

The model has been spun up and integrated for 300 years, the results are realistic, and there is no need for a flux adjustment. (See the special issue of *Journal of Climate*, June 1998).

Princeton Coupled Model The model consists of an atmospheric model with a horizonatal resolution of 7.5° longitude by 4.5° latitude and 9 levels in the

vertical, an ocean model with a horizontal resolution of 4° and 12 levels in the vertical, and a land-surface model. The ocean and atmosphere are coupled through heat, water, and momentum fluxes; land and ocean are coupled through river runoff; and land and atmosphere are coupled through water and heat fluxes.

Hadley Center Model This is an atmosphere-ocean-ice model that minimizes the need for flux adjustments (Johns et al, 1997). The ocean component is based on the Bryan-Cox primitive equation model, with realistic bottom features, vertical mixing coefficients from Pacanowski and Philander (1981). Both the ocean and the atmospheric component have a horizontal resolution of 96×73 grid points, the ocean has 20 levels in the vertical.

In contrast to most coupled models, this one is spun up as a coupled system with flux adjustments during spin up to keep sea surface temperature and salinity close to observed mean values. The coupled model was integrated from rest using Levitus values for temperature and salinity for September. The initial integration was from 1850 to 1940. The model was then integrated for another 1000 years. No flux adjustment was necessary after the initial 140-year integration because drift of global-averaged air temperature was ≤ 0.016 K/century.

15.8 Important Concepts

- 1. Numerical models are used to simulate oceanic flows with realistic and useful results. The most recent models include heat fluxes through the surface, wind forcing, mesoscale eddies, realistic coasts and sea-floor features, and more than 20 levels in the vertical.
- 2. Numerical models are not perfect. They solve discrete equations, which are not the same as the equations of motion described in earlier chapters. And,
- 3. Numerical models cannot reproduce all turbulence of the coean because the grid points are tens to hundreds of kilometers apart. The influence of turbulent motion over smaller distances must be calculated from theory; and this introduces errors.
- 4. Numerical models can be forced by real-time oceanographic data from ships and satellites to produce forecasts of oceanic conditions, including El Niño in the Pacific, and the position of the Gulf Stream in the Atlantic.
- 5. Coupled ocean-atmosphere models have much coarser spatial resolution so that that they can be integrated for hundreds of years to simulate the natural variability of the climate system and its response to increased CO_2 in the atmosphere.