# Chapter 10

# **Geostrophic Currents**

Within the ocean's interior away from the top and bottom Ekman layers, for horizontal distances exceeding a few tens of kilometers, and for times exceeding a few days, horizontal pressure gradients in the ocean almost exactly balance the Coriolis force resulting from horizontal currents. This balance is known as the *geostrophic approximation*.

The dominant forces acting in the vertical are the vertical pressure gradient and the weight of the water. The two balance within a few parts per million. Thus pressure at any point in the water column is due almost entirely to the weight of the water in the column above the point. The dominant forces in the horizontal are the pressure gradient and the Coriolis force. They balance within a few parts per thousand over large distances and times (See Box).

Both balances require that viscosity and nonlinear terms in the equations of motion be negligible. Is this reasonable? Consider viscosity. We know that a rowboat weighing a hundred kilograms will coast for maybe ten meters after the rower stops. A super tanker moving at the speed of a rowboat may coast for kilometers. It seems reasonable, therefore that a cubic kilometer of water weighing  $10^{15}$  kg would coast for perhaps a day before slowing to a stop. And oceanic mesoscale eddies contain perhaps 1000 cubic kilometers of water. Hence, our intuition may lead us to conclude that neglect of viscosity is reasonable. Of course, intuition can be wrong, and we need to refer back to scaling arguments.

#### 10.1 Hydrostatic Equilibrium

Before describing the geostrophic balance, let's first consider the simplest solution of the momentum equation, the solution for an ocean at rest. It gives the hydrostatic pressure within the ocean. To obtain the solution, we assume the fluid is stationary:

$$u = v = w = 0;$$
 (10.1)

the fluid remains stationary:

$$\frac{du}{dt} = \frac{dv}{dt} = \frac{dw}{dt} = 0; \tag{10.2}$$

#### Scaling the Equations: The Geostrophic Approximation

We wish to simplify the equations of motion to obtain solutions that describe the deep-sea conditions well away from coasts and below the Ekman boundary layer at the surface. To begin, let's examine the typical size of each term in the equations in the expectation that some will be so small that they can be dropped without changing the dominant characteristics of the solutions. For interior, deep-sea conditions, typical values for distance L, horizontal velocity U, depth H, Coriolis parameter f, gravity g, and density  $\rho$  are:

$L \approx 10^6 \text{ m}$	$f \approx 10^{-4} \ {\rm s}^{-1}$
$U \approx 10^{-1} \mathrm{m/s}$	$g \approx 10 \text{ m/s}^2$
$H \approx 10^3 \text{ m}$	$ hopprox10^3~{ m kg/m}^3$

From these variables we can calculate typical values for vertical velocity W, pressure P, and time T:

$$\frac{\partial W}{\partial z} = -\left(\frac{\partial U}{\partial x} + \frac{\partial v}{\partial y}\right)$$
$$\frac{W}{H} = \frac{U}{L}; \quad W = \frac{UH}{L} = \frac{10^{-1} \, 10^3}{10^6} \text{ m/s} = 10^{-4} \text{m/s}$$
$$P = \rho g z = 10^3 \, 10^1 \, 10^3 = 10^7 \text{ Pa}$$
$$T = L/U = 10^7 \text{ s}$$

The momentum equation for vertical velocity is therefore:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + 2\Omega u \cos \varphi - g$$
$$\frac{W}{T} + \frac{UW}{L} + \frac{UW}{L} + \frac{W^2}{L} = \frac{P}{\rho H} + f U - g$$
$$10^{-11} + 10^{-11} + 10^{-11} + 10^{-14} = 10 + 10^{-5} - 10$$

and the only important balance in the vertical is hydrostatic:

$$\frac{\partial p}{\partial z} = -\rho g$$
 Correct to  $1:10^6$ 

The momentum equation for horizontal velocity in the x direction is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + fv$$
$$10^{-8} + 10^{-8} + 10^{-8} + 10^{-8} = 10^{-5} + 10^{-5}$$

Thus the Coriolis force balances the pressure gradient within one part per thousand. This is called the *geostrophic balance*, and the *geostrophic equations* are:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv; \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = -fu; \quad \frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

This balance applies to oceanic flows with horizontal dimensions larger than roughly 50 km and times greater than a few days.

and, there is no friction:

$$f_x = f_y = f_z = 0. (10.3)$$

With these assumptions, (7.18) becomes:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = 0; \qquad \qquad \frac{1}{\rho}\frac{\partial p}{\partial y} = 0; \qquad \qquad \frac{1}{\rho}\frac{\partial p}{\partial z} = -g(\varphi, z) \qquad (10.4)$$

where we have explicitly noted that gravity g is a function of latitude  $\varphi$  and height z. We will show later why we have kept this explicit.

Equations (10.4a) require surfaces of constant pressure to be level surface. A surface of constant pressure is an *isobaric surface*. The last equation can be integrated to obtain the pressure at any depth h. Recalling that  $\rho$  is a function of depth for an ocean at rest.

$$p = \int_{-h}^{0} g(\varphi, z) \rho(z) dz \qquad (10.5)$$

Later, we will show that (10.5) applies with an accuracy of about one part per million even if the ocean is not at rest.

#### **10.2** Geostrophic Equations

The geostrophic balance requires that the Coriolis force balance the horizontal pressure gradient. The equations for geostrophic balance are derived from the equations of motion assuming the flow has no acceleration, du/dt = dv/dt = dw/dt = 0; that horizontal velocities are much larger than vertical,  $w \ll u, v$ ; that the only external force is gravity; and that friction is small. With these assumptions (7.15) become

$$\frac{\partial p}{\partial x} = \rho f v; \quad \frac{\partial p}{\partial y} = -\rho f u; \quad \frac{\partial p}{\partial z} = -\rho g$$
(10.6)

where  $f = 2\Omega \sin \varphi$  is the Coriolis parameter. These are the *geostrophic equations*.

The equations can be written:

$$u = -\frac{1}{f\rho}\frac{\partial p}{\partial y}; \qquad v = \frac{1}{f\rho}\frac{\partial p}{\partial x}$$
 (10.7a)

$$p = p_0 + \int_{-h}^{\zeta} g(\varphi, z)\rho(z)dz \qquad (10.7b)$$

where  $p_0$  is atmospheric pressure at z = 0, and  $\zeta$  is the height of the sea surface. Note that we have allowed for the sea surface to be above or below the surface z = 0; and the pressure gradient at the sea surface is balanced by a surface current  $u_s$ .



Figure 10.1 Sketch defining  $\zeta$  and r, used for calculating pressure just below the sea surface.

Substituting (10.7b) into (10.7a) gives:

$$u = -\frac{1}{f\rho} \frac{\partial}{\partial y} \int_{-h}^{0} g(\varphi, z) \rho(z) dz - \frac{g}{f} \frac{\partial \zeta}{\partial y}$$
$$u = -\frac{1}{f\rho} \frac{\partial}{\partial y} \int_{-h}^{0} g(\varphi, z) \rho(z) dz - u_s$$
(10.8a)

where we have used the Boussinesque approximation, retaining full accuracy for  $\rho$  only when calculating pressure.

In a similar way, we can derive the equation for v.

$$v = \frac{1}{f\rho} \frac{\partial}{\partial x} \int_{-h}^{0} g(\varphi, z) \rho(z) dz + \frac{g}{f} \frac{\partial \zeta}{\partial x}$$
$$v = \frac{1}{f\rho} \frac{\partial}{\partial x} \int_{-h}^{0} g(\varphi, z) \rho(z) dz + v_s$$
(10.8b)

If the occean is homogeneous and density and gravity are constant, the first term on the right-hand side of (10.8) is equal to zero; and the horizontal pressure gradients within the ocean are the same as the gradient at the surface.

If the ocean is stratified, the horizontal pressure gradient has two components, one due to the slope at the sea surface, and an additional term due to horizontal density differences. The first term on the right-hand side of (10.8) is called the relative velocity. Thus calculation of geostrophic currents from the density distribution requires the velocity  $(u_0, v_0)$  at the sea surface or at some other depth.

# 10.3 Surface Geostrophic Currents From Altimetry

The geostrophic approximation applied at the sea surface leads to a very simple relation between surface slope and surface current. Consider a level surface slightly below the sea surface, say two meters below the sea surface, at z = -r. A *level surface* is a surface of constant gravitational potential, and no work is required to move along a frictionless, level surface (Figure 10.1).

The pressure on the level surface is:

$$p = \rho g \ (\zeta + r) \tag{10.9}$$



Figure 10.2 The slope of the sea surface relative to the geoid  $(\partial \zeta / \partial x)$  is directly related to surface geostrophic currents  $v_s$ . The slope of 1 meter per 100 kilometers (10  $\mu$ rad) is typical of strong currents.

assuming  $\rho$  and g are essentially constant in the upper few meters of the ocean.

Substituting this into (10.8a, b), gives the two components  $(u_s, v_s)$  of the surface geostrophic current:

$$u_s = -\frac{g}{f}\frac{\partial\zeta}{\partial y};$$
  $v_s = \frac{g}{f}\frac{\partial\zeta}{\partial x}$  (10.10)

where g is gravity, f is the Coriolis parameter, and  $\zeta$  is the height of the sea surface above a level surface.

The Oceanic Topography In §3.4 we define the topography of the sea surface  $\zeta$  to be the height of the sea surface relative to a particular level surface, the geoid; and we defined the geoid to be the level surface that coincided with the surface of the ocean at rest. Thus, according to (10.10) the surface geostrophic currents are proportional to the slope of the topography (Figure 10.2), a quantity that can be measured by satellite altimeters if the geoid is known.

Because the geoid is a level surface, it is a surface of constant geopotential. To see this, consider the work done in moving a mass m by a distance h perpendicular to a level surface. The work is W = mgh, and the change of potential energy per unit mass is gh. Thus level surfaces are surfaces of constant geopotential, where the geopotential  $\Phi = gh$ .

Topography is due to process that cause the ocean to move: tides, ocean currents, and the changes in barometric pressure that produce the inverted barometer effect. Because the ocean's topography is due to dynamical processes, it is usually called *dynamic topography*. The topography is approximately one hundredth of the geoid undulations. This means that the shape of the sea surface is dominated by local variations of gravity. The influence of currents is much smaller. Typically, sea-surface topography has amplitude of  $\pm 1$  m. Typical slopes are  $\partial \zeta / \partial x \approx 1$ –10 microradians for v = 0.1–1.0 m/s at mid latitude.

Errors in knowing the height of the geoid are larger than the topographic signal for wavelengths shorter than roughly 1600 km (Nerem, *et al.* 1994). The height of the geoid, smoothed over horizontal distances greater than roughly 1,600 km, is known with an accuracy of  $\pm 15$  cm (Tapley, *et al.* 1994a). The unsmoothed geoid is less well known. The height of the unsmoothed, local geoid is known with an accuracy of only around  $\pm 50$  cm (Figure 10.3).



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Figure 10.3 Topex/Poseidon altimeter observations of the Gulf Stream. When the altimeter observations are subtracted from the local geoid, they yield the oceanic topography, which is due primarily to ocean currents in this example. The gravimetric geoid was determined by the Ohio State University from ship and other surveys of gravity in the region. From Center for Space Research, University of Texas.

Satellite Altimetry Very accurate, satellite-altimeter systems are needed for measuring the oceanic topography. The first systems, carried on Seasat, Geosat, ERS-1, and ERS-2 were designed to measure the variability of currents with horizontal dimensions of less than a thousand kilometers. Only Topex/Poseidon, launched in 1992, was designed to make the much more accurate measurements necessary for observing the permanent (time-averaged) surface circulation of the oceans, tides, and the variability of gyre-scale currents.

Because the geoid is not well known locally, altimeters are usually flown in orbits that have an exactly repeating ground track. Thus Topex/Poseidon flies over the same ground track every 9.9156 days. By subtracting sea-surface height from one traverse of the ground track from height measured on a later traverse, changes in topography can be observed without knowing the geoid. The geoid is constant in time, and the subtraction removes the geoid, revealing changes due to changing currents, such as mesoscale variability, assuming tides have been removed from the data (Figure 10.4). Mesoscale variability includes eddies with diameters between roughly 20 and 500 km.

The great accuracy and precision of Topex/Poseidon's altimetric system allow the measurements of the oceanic topography over ocean basins with an accuracy of  $\pm 5$  cm. Such an accurate satellite-altimeter system can measure:

 Changes in the global mean volume of the ocean (Born et al. 1986, Nerem, 1995);



Figure 10.4 Global distribution of variance of topography from Topex/Poseidon altimeter data from 10/3/92 to 10/6/94. The height variance is an indicator of variability of currents. (From Center for Space Research, University of Texas).

- 2. Seasonal heating and cooling of the ocean (Chambers, Tapley, and Stewart, 1998);
- 3. Tides (Andersen, Woodworth, and Flather, 1995);
- 4. The permanent surface geostrophic current system (Figure 10.5);
- 5. Changes in surface geostrophic currents on all scales (Figure 10.4); and
- 6. Variations in topography of equatorial current systems such as those associated with El Niño (Figure 10.6).

Altimeter Errors (Topex/Poseidon) The most accurate observations of the sea-surface topography are from Topex/Poseidon. Errors for this satellite altimeter system are due to:

- 1. Instrument noise, ocean waves, water vapor, free electrons in the ionosphere, and mass of the atmosphere. Topex/Poseidon carries a precise altimeter system able to observed the height of the satellite above the sea surface between  $\pm 66^{\circ}$  latitude with a precision of  $\pm 2$  cm and an accuracy of  $\pm 3.2$  cm (Fu, *et al.* 1994). The system consists of a two-frequency radar altimeter to measure height above the sea, the influence of the ionosphere, and wave height. The system also included a three-frequency microwave radiometer able to measure water vapor in the troposphere.
- 2. Tracking errors. The satellite carries three tracking systems that enable its position in space, its ephemeris, to be determined with an accuracy of  $\pm 3.5$  cm (Tapley *et al.* 1994a).
- 3. Sampling error. The satellite measures height along a ground track that repeats within  $\pm 1$  km every 9.9156 days. Each repeat is a cycle. Because



Four-Year Mean Sea-Surface Topography (cm)

Figure 10.5 Global distribution of time-averaged topography of the ocean from Topex/Poseidon altimeter data from 10/3/92 to 10/6/99 relative to the JGM-3 geoid. Geostrophic currents at the ocean surface are parallel to the contours. Compare with Figure 2.8 calculated from hydrographic data. (From Center for Space Research, University of Texas).

currents are measured only along the subsatellite track, there is a sampling error. The satellite cannot map the topography between ground tracks, nor can it observe changes with periods less than  $2 \times 9.9156$  d (see §17.3).

4. Geoid error. The permanent topography is not well known over distances shorter than 1,600 km because geoid errors dominate for shorter distances. Maps of the topography smoothed over 1,600 km are used to study the dominant features of the permanent geostophic currents at the sea surface (Figure 10.5).

Taken together, the measurements of height above the sea and the satellite position give sea-surface height in geocentric coordinates with an accuracy of  $\pm 4.7$  cm. The geoid error adds further errors that depend on the size of the area being measured.

#### 10.4 Geostrophic Currents From Hydrography

The geostrophic equations are widely used in oceanography to calculate currents at depth. The basic idea is to use hydrographic measurements of temperature, salinity or conductivity, and pressure to calculate the density field of the ocean using the equation of state of sea water. Density is used in (10.7b) to calculate the internal pressure field, from which the geostrophic currents are calculated using (10.8a, b). Usually, however, the constant of integration in (10.8) is not known, and only the relative velocity field can be calculated.

At this point, you may ask, why not just measure pressure directly as is done in meteorology, where direct measurements of pressure are used to calculate winds. And, aren't pressure measurements needed to calculate density from the equation of state? The answer is that very small changes in depth make



Figure 10.6 Time-longitude plot of sea-level anomalies in the Equatorial Pacific observed by Topex/Poseidon. Warm anomalies are light gray, cold anomalies are dark gray. The anomalies are computed from 10-day deviations from a mean surface computed from 10/3/1992 to 10/8/1995. The data are smoothed with a Gaussian weighted filter with a longitudinal span of 5° and a latitudinal span of 2°. The annotations on the left are cycles of satellite data. The black stripe indicates data missing at time plot wqs made.

large changes in pressure because water is so heavy. Errors in pressure caused by errors in determining the depth of a pressure gauge are much larger than the pressure signal due to currents. For example, using (10.7a), we calculate that the pressure gradient due to a 10 cm/s current at 30°latitude is  $7.5 \times 10^{-3}$ Pa/m, which is 750 Pa in 100 km. From the hydrostatic equation (10.5), 750 Pa is equivalent to a change of depth of 7.4 cm. Therefore, for this example, we must know the depth of a pressure gauge with an accuracy of much better than 7.4 cm. This is not possible.

While simple in concept, the calculation of geostrophic currents from hydrographic data is difficult, and the difficulties lie in the details. The first detail is to understand how variations in gravity influence pressure.

**Geopotential Surfaces Within the Ocean** Calculation of pressure gradients within the ocean must be done along surfaces of constant geopotential just as we calculated pressure gradients on the geoid at the sea surface to calculate surface geostrophic currents. As long ago as 1910, Vilhelm Bjerknes (1910) realized that such surfaces are not at fixed heights in the atmosphere because g is not constant; and (10.4c) must include the variability of gravity in both the horizontal and vertical directions.

The geopotential  $\Phi$  is:

$$\Phi = \int_0^z g dz \tag{10.11}$$

and in SI units,  $\Phi/9.8$  has almost the same numerical value as height in meters. The meteorological community accepted Bjerknes' proposal that height be replaced by *dynamic meters*  $D = \Phi/10$  to obtain a natural vertical coordinate. Later, this was replaced by the *geopotential meter* (gpm)  $Z = \Phi/9.80$ . The geopotential meter is a measure of the work required to lift a unit mass from sea level to a height z against the force of gravity. Harald Sverdrup, Bjerknes' student, carried the concept to oceanography; and depths in the ocean are often quoted in geopotential meters. Hence, geopotential surfaces in the ocean are defined by different values of  $\Phi$ , and the geometric distance between two geopotential surfaces cannot be constant over thousands of kilometers.

Gravity can be written as the product of a term that varies with latitude times a term that varies with height (List, 1966: 217 & 488):

$$g = g(\varphi, z) = g_{\varphi} \left(\frac{a}{a+z}\right)^2 \tag{10.12a}$$

$$g_{\varphi} = 9.806160 \left[ 1 - 2.64 \times 10^{-3} \cos 2\varphi + 5.9 \times 10^{-6} \cos^2 \varphi \right]$$
(10.12b)

$$a = 6,378,134.9 \text{ m}$$
 (10.12c)

where a is the Earth's equatorial radius, and  $\varphi$  is latitude. Here z is measured from the geoid, and it is negative downward. The difference between depths of constant vertical distance and constant potential can be relatively large. For example, the geometrical depth of the 1000 dynamic meter surface is 1017.40 m at the north pole and 1022.78 m at the equator, a difference of 5.38 m. Depth in geopotential meters, depth in meters, and pressure in decibars are almost the same numerically, where a decibar is  $10^4$  Pa (Table 10.1), and a pascal (Pa) is the SI unit for pressure. For this reason, oceanographers prefer to state pressure in decibars. At a depth of 1 meter the pressure is approximately 1.007 decibars and the depth is 1.00 geopotential meters.

Fable 10.1	Unit	ts of Pressure
1 Pa	=	$1 \text{ N/m}^2 = 1 \text{ kg.s}^{-2}.\text{m}^{-1}$
1 Bar	=	$10^5$ Pa
1 decibar	=	$10^4$ Pa
1 millihar	_	100 Pa

Dutton (1995: §4.2) shows that by writing  $Z = \Phi/g_{38}$ , where  $g_{38} = 9.80$  m/s<sup>2</sup>, and Z = geopotential height, then the hydrostatic equation is  $\partial P/\partial Z = g_{38}\rho$ . Writing z for Z, and g for 9.8 m/s<sup>2</sup>, we obtain the hydrostatic equation in familiar form:  $\partial p/\partial z = -g\rho$ .

Equations for Geostrophic Currents Within the Ocean To calculate geostrophic currents, we need to calculate the horizontal pressure gradient within the ocean. This can be done using either of two approaches:

- 1. Calculate the slope of an isobaric surface. We used this approache when we used sea-surface slope from altimetry to calculate surface geostrophic currents. The sea surface is an isobaric surface.
- 2. Calculate the change in pressure on a surface of constant geopotential. Such a surface is called a *geopotential surface*.

Oceanographers usually calculate the slope of isobaric surfaces. The important steps are:

- 1. Calculate differences in heights  $(\Phi_A \Phi_B)$  between two isobaric surfaces  $(P_1, P_2)$  at hydrographic stations A and B (Figure 10.7). This is similar to the calculation of  $\zeta$  of the surface layer.
- 2. Calculate the slope of the upper isobaric surface relative to the lower.



Figure 10.7. Sketch of geometry used for calculating geostrophic current from hydrography.

- 3. Calculate the geostrophic current at the upper surface relative to the current at the lower. This is the current shear.
- 4. Integrate the current shear from some depth where currents are known to obtain currents as a function of depth. For example, from the surface downward, using surface geostrophic currents observed by satellite altimetry, or upward from an assumed level of no motion.

To calculate geostrophic currents oceanographers use a modified form of the hydrostatic equation. The vertical pressure gradient (10.3c) is written

$$\frac{\delta p}{\rho} = \alpha \, \delta p = -g \, \delta z \tag{10.13a}$$

$$\alpha \, \delta p = \delta \Phi \tag{10.13b}$$

where  $\alpha = \alpha(S, t, p)$  is the *specific volume*; and (10.13b) follows from (10.11). Differentiating (10.13b) with respect to horizontal distance x allows the geostrophic balance to be written in terms of the slope of the isobaric surface:

$$\alpha \,\frac{\partial p}{\partial x} = \frac{1}{\rho} \,\frac{\partial p}{\partial x} = -2\,\Omega \,v \sin\varphi \tag{10.14a}$$

$$\frac{\partial \Phi \left( p = p_0 \right)}{\partial x} = -2 \,\Omega \, v \, \sin \varphi \tag{10.14b}$$

where  $\Phi$  is the geopotential height of an isobaric surface. Note that the terminology is a little confusing;  $\Phi$  is not necessarily a geopotential surface.

Now let's see how hydrographic data are used for evaluating  $\partial \Phi / \partial x$ . Consider two isobaric surfaces  $(P_1, P_2)$  in the ocean as shown in Figure 10.7. The geopotential difference between two isobaric surfaces at station A is:

$$\Phi(P_{1A}) - \Phi(P_{2A}) = \int_{P_{1A}}^{P_{2A}} \alpha(S, t, p) \, dp \tag{10.15}$$

The specific volume anomaly is written as the sum of two parts:

$$\alpha(S, t, p) = \alpha(35, 0, p) + \delta \tag{10.16}$$

where  $\alpha(35, 0, p)$  is the specific volume of sea water with salinity of 35 psu, temperature of 0°C, and pressure p. The second term  $\delta$  is the *specific volume anomaly*. Using (10.11) in (10.10) gives:

$$\Phi(P_{1A}) - \Phi(P_{2A}) = \int_{P_{1A}}^{P_{2A}} \alpha(35, 0, p) + \int_{P_{1A}}^{P_{2A}} \delta \, dp$$
$$\Phi(P_{1A}) - \Phi(P_{2A}) = (\Phi_1 - \Phi_2)_{std} + \Delta \Phi_A$$

where  $(\Phi_1 - \Phi_2)_{std}$  is the standard geopotential distance between two isobaric surfaces  $P_1$  and  $P_2$ ; and

$$\Delta \Phi_A = \int_{P_{1A}}^{P_{2A}} \delta \, dp \tag{10.17}$$

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is the anomaly of the geopotential distance between the surfaces. It is called the *geopotential anomaly*. If pressure is in decibars, the standard geopotential distance is numerically approximately z, where z is the geometric distance between the two surfaces. The geopotential anomaly is much smaller, being approximately 0.1% of the standard geopotential distance.

Consider now the geopotential anomaly between two pressure surfaces  $P_1$ and  $P_2$  calculated at two hydrographic stations A and B a distance L meters apart (Figure 10.7). For simplicity we assume the lower isobaric surface is a level surface. Hence the isobaric and geopotential surfaces coincide, and there is no geostrophic velocity at this depth. The slope of the upper surface is

$$\frac{\Delta \Phi_B - \Delta \Phi_A}{L} = \text{slope of isobaric surface } P_2$$

because the standard geopotential distance is the same at stations A and B. The geostrophic velocity at the upper surface calculated from (10.14b) is:

$$V = \frac{\left(\Delta \Phi_B - \Delta \Phi_A\right)}{2\Omega L \sin \varphi} \tag{10.18}$$

where V is the velocity at the upper geopotential surface. The velocity V is perpendicular to the plane of the two hydrographic stations and directed into the plane of Figure 10.7 if the flow is in the northern hemisphere. A useful rule of thumb is that the flow is such that warmer, lighter water is to the right looking downstream in the northern hemisphere.

If the pressure is measured in decibars, and L in meters, (10.18) becomes:

$$V = \frac{10 \left(\Delta \Phi_B - \Delta \Phi_A\right)}{2\Omega L \sin \varphi} \tag{10.19}$$

Note that we could have calculated the slope of the isobaric surfaces using density  $\rho$  instead of specific volume  $\alpha$ . We have chosen to use  $\alpha$  because it is the common practice in oceanography, and tables of specific volume anomalies and computer code to calculate the anomalies are widely available. The common practice follows from numerical methods developed before calculators and computers were available, when all calculations were done by hand or by mechanical calculators with the help of tables and nomograms. Because the computations must be done with an accuracy of a few parts per million, and because all scientific fields tend to be conservative, the common practice has continued to use specific volume anomalies rather than density anomalies.

**Barotropic and Baroclinic Flow:** If the ocean were homogeneous with constant density, then isobaric surfaces would always be parallel to the sea surface, and the geostrophic velocity would be independent of depth. In this case the relative velocity is zero, and hydrographic data cannot be used to measure the geostrophic current. If density varies with depth, but not with horizontal distance, the isobaric surfaces are always parallel to the sea surface and the levels of constant density, the *isopycnal surfaces*. In this case, the relative flow is also zero. Both cases are examples of *barotrophic flow*.

*Barotropic flow* occurs when levels of constant pressure in the ocean, the *isobaric surfaces*, are always parallel to the surfaces of constant density, the *isopycnal surfaces*. Note, some authors call the vertically averaged flow the baroclinic component of the flow. Wunsch (1996: 74) points out that baroclinic is used in so many different ways that the term is meaningless and should not be used.

*Baroclinic flow* occurs when levels of constant pressure are inclined to surfaces of constant density. In this case, density varies with depth and horizontal position. A good example is seen in Figure 10.12 which shows levels of constant density changing depth by more than 1 km over horizontal distances of 100 km at the Gulf Stream. Baroclinic flow varies with depth, and the relative current can be calculated from hydrographic data. Note, constant-density surfaces cannot be inclined to constant-pressure surfaces for a fluid at rest.

In general, the variation of flow in the vertical can be decomposed into a barotropic component which is independent of depth, and a baroclinic component which varies with depth.

#### 10.5 An Example Using Hydrographic Data

Let's now consider a specific numerical calculation of geostrophic velocity using generally accepted proceedures from *Processing of Oceanographic Station Data* (JPOTS Editorial Panel, 1991). The book has worked examples using hydrographic data collected by the R/V *Endeavor* in the North Atlantic. Data were collected on Cruise 88 along 71°W across the Gulf Stream south of Cape Cod, Massachusetts at stations 61 and 64. Station 61 is on the Sargasso Sea side of the Gulf Stream in water 4260 m deep. Station 64 is north of the Gulf Stream in water 3892 m deep. The measurements were made by a Conductivity-Temperature-Depth-Oxygen Profiler, Mark III CTD/02, made by Neil Brown Instruments Systems. It had a rosette of 24 1.2-liter Niskin water bottles. Salinity and oxygen samples from the bottles were used to calibrate the CTD.

The CTD sampled temperature, salinity, and pressure 22 times per second, and the digital data were averaged over 2 dbar intervals as the CTD was lowered in the water. Data were tabulated at 2 dbar pressure intervals centered on odd values of pressure because the first observation is at the surface, and the first averaging interval extends to 2 dbar, and the center of the first interval is at 1 dbar. Data were further smoothed with a binomial filter and linearly interpolated to standard levels reported in the first three columns of Tables 10.2 and 10.3. All processing was done electronically.

 $\delta(S, t, p)$  in the fourth column of Tables 10.2 and 10.3 is calculated from the values of t, S, p in the layer.  $\langle \delta \rangle$  is the average value of specific volume anomaly for the layer between standard pressure levels. It is the average of the values of  $\delta(S, t, p)$  at the top and bottom of the layer. The last column  $(10^{-5}\Delta\Phi)$  is the product of the average specific volume anomaly of the layer times the thickness of the layer in decibars. Therefore, the last column is the geopotential anomaly  $\Delta\Phi$  calculated by integrating (10.17) between  $p_1$  at the bottom of each layer and  $p_2$  at the top of each layer.

		,			,	,
Pressure	$\mathbf{t}$	$\mathbf{S}$	$\sigma(\theta)$	$\delta(S, t, p)$	$<\delta>$	$10^{-5}\Delta\Phi$
decibar	$^{\circ}\mathrm{C}$	$\mathbf{psu}$	$\mathrm{kg/m^{3}}$	$10^{-8} \mathrm{m}^3/\mathrm{kg}$	$10^{-8}\mathrm{m}^3/\mathrm{kg}$	$\mathrm{m}^2/\mathrm{s}^2$
0	25.698	35.221	23.296	457.24	457.00	0.0040
1	25.698	35.221	23.296	457.28	457.20	0.0046
10	26.763	36.106	23.658	423.15	440.22	0.0396
20	26.678	36.106	23.658	423.66	423.41	0.0423
30	26.676	36.107	23.659	423.98	423.82	0.0424
50	24.528	36.561	24.670	328.48	376.23	0.0752
75	22.753	36.614	25.236	275.66	302.07	0.0755
100	21.427	36.637	25.630	239.15	257.41	0.0644
125	20.633	36.627	25.841	220.06	229.61	0.0574
150	19.522	36.558	26.086	197.62	208.84	0.0522
200	18.798	36.555	26.273	181.67	189.65	0.0948
250	18.431	36.537	26.354	175.77	178.72	0.0894
300	18.189	36.526	26.408	172.46	174.12	0.0871
400	17.726	36.477	26.489	168.30	170.38	0.1704
500	17.165	36.381	26.557	165.22	166.76	0.1668
600	15.952	36.105	26.714	152.33	158.78	0.1588
700	13.458	35.776	26.914	134.03	143.18	0.1432
800	11.109	35.437	27.115	114.36	124.20	0.1242
900	8.798	35.178	27.306	94.60	104.48	0.1045
1000	6.292	35.044	27.562	67.07	80.84	0.0808
1100	5.249	35.004	27.660	56.70	61.89	0.0619
1200	4.813	34.995	27.705	52.58	54.64	0.0546
1300	4.554	34.986	27.727	50.90	51.74	0.0517
1400	4.357	34.977	27.743	49.89	50.40	0.0504
1500	4.245	34.975	27.753	49.56	49.73	0.0497
1750	4.028	34.973	27.777	49.03	49.30	0.1232
2000	3.852	34.975	27.799	48.62	48.83	0.1221
2500	3.424	34.968	27.839	46.92	47.77	0.2389
3000	2.963	34.946	27.868	44.96	45.94	0.2297
3500	2.462	34.920	27.894	41.84	43.40	0.2170
4000	2.259	34.904	27.901	42.02	41.93	0.2097
	-			-		

Table 10.2 Computation of Relative Geostrophic Currents.Data from Endeavor Cruise 88, Station 61(36°40.03'N, 70°59.59'W; 23 August 1982; 1102Z)

The distance between the stations is L = 110,935 m; the average Coriolis parameter is  $f = 0.88104 \times 10^{-4}$ ; and the factor 10/fL in (10.19) is 1.0231 s/m. This was used to calculate the relative geostrophic currents reported in Table 10.4 and plotted in Figure 10.8. The currents are calculated relative to the current at 2000 decibars. Notice that there is no indication of Ekman currents in the current profile. Ekman currents are not geostrophic, and they do not contribute to the topography.

	<b>`</b>	,	,	0	, ,	
Pressure	t	S	$\sigma(\theta)$	$\delta(S, t, p)$	$<\delta>$	$10^{-5}\Delta\Phi$
decibar	$^{\circ}\mathrm{C}$	psu	$kg/m^3$	$10^{-8} { m m}^3 / { m kg}$	$10^{-8} \mathrm{m}^3/\mathrm{kg}$	$\mathrm{m}^2/\mathrm{s}^2$
0	26.148	34.646	22.722	512.09	×10.1×	0.00×1
1	26.148	34.646	22.722	512.21	512.15	0.0051
10	26.163	34.645	22.717	513.01	512.61	0.0461
20	26.167	34.655	22.724	512.76	512.89	0.0513
30	25.640	35.733	23.703	419.82	466.29	0.0466
50	18.967	35.944	25.755	224.93	322.38	0.0645
75	15.371	35.904	26.590	146.19	185.56	0.0464
100	14.356	35.897	26.809	126.16	136.18	0.0340
125	13.059	35.696	26.925	115.66	120.91	0.0302
150	12.134	35.567	27.008	108.20	111.93	0.0280
200	10.307	35.360	27.185	92.17	100.19	0.0501
250	8.783	35.168	27.290	82.64	87.41	0.0437
300	8.046	35.117	27.364	76.16	79.40	0.0397
400	6.235	35.052	27.568	57.19	66.68	0.0667
500	5.230	35.002	27.667	48 23	52.71	0.0527
600	5.005	35 044	27.001 27.710	45 29	46.76	0.0468
700	4756	35.027	27.710 27.731	44 04	44.67	0.0447
800	4 399	34 992	27.701 27.744	43.33	43.69	0.0437
900	4 291	34 991	27.711 27.756	43 11	43.22	0.0432
1000	1.201	3/ 986	27.100 27.764	43.12	43.12	0.0431
1100	4 077	3/ 082	27.101 27.773	43.07	43.10	0.0431
1200	3 060	34.075	27.110 27.770	43.17	43.12	0.0431
1200	3 000	34.975	27.113 27.786	43.17	43.28	0.0433
1400	3 831	34.073	27.703	43.36	43.38	0.0434
1500	3.767	34.975	27.135	43.26	43.31	0.0433
1750	3.600	34.975	27.802	43.20	43.20	0.1080
2000	3.000 2.401	24.975	21.021	40.10	43.00	0.1075
2000	0.401 0.040	34.900 24.049	21.031	42.00	42.13	0.2106
2000 2000	2.942 2.475	34.948 34.099	27.007	41.39 20.26	40.33	0.2016
2500	2.470	04.920 24.004	27.091	39.20 20.17	39.22	0.1961
3000	2.219	34.904	27.900	39.17	40.08	0.2004
4000	2.177	34.896	27.901	40.98		

Table 10.3 Computation of Relative Geostrophic Currents.Data from Endeavor Cruise 88, Station 64(37°39.93'N, 71°0.00'W; 24 August 1982; 0203Z)

# 10.6 Comments on Geostrophic Currents

Knowing the theory of geostrophic currents and how the theory can be applied to measurements to calculate currents, let's now consider some of the limitations of the theory and techniques.

**Converting Relative Velocity to Velocity** Hydrographic data give geostrophic currents relative to geostrophic currents at some reference level. How can we convert the relative geostrophic velocities to velocities relative to the earth?

1. Assume a Level of no Motion: Traditionally, oceanographers assume there is a level of no motion, sometimes called a reference surface, roughly 2,000

Pressure	$10^{-5}\Delta\Phi_{61}$	$\Sigma \Delta \Phi dz$	$10^{-5}\Delta\Phi_{64}$	$\Sigma \Delta \Phi dz$	V
decibar	$\mathrm{m}^2/\mathrm{s}^2$	at $61^*$	$\mathrm{m}^2/\mathrm{s}^2$	at $64^*$	(m/s)
0	0.0046	2.1872	0.0051	1.2583	0.95
1	0.0040	2.1826	0.0051	1.2532	0.95
10	0.0390	2.1430	0.0401	1.2070	0.96
20	0.0425	2.1006	0.0315	1.1557	0.97
30	0.0424 0.0752	2.0583	0.0400	1.1091	0.97
50	0.0752	1.9830	0.0645	1.0446	0.96
75	0.0755	1.9075	0.0464	0.9982	0.93
100	0.0644 0.0574	1.8431	0.0340	0.9642	0.90
125	0.0574	1.7857	0.0302	0.9340	0.87
150	0.0322	1.7335	0.0280	0.9060	0.85
200	0.0948	1.6387	0.0301	0.8559	0.80
250	0.0894	1.5493	0.0437	0.8122	0.75
300	0.0871	1.4623	0.0397	0.7725	0.71
400	0.1704	1.2919	0.0007	0.7058	0.60
500	0.1008	1.1252	0.0327	0.6531	0.48
600	0.1388	0.9664	0.0408	0.6063	0.37
700	0.1452 0.1249	0.8232	0.0447 0.0427	0.5617	0.27
800	0.1242 0.1045	0.6990	0.0437	0.5180	0.19
900	0.1045	0.5945	0.0432	0.4748	0.12
1000	0.0608	0.5137	0.0431	0.4317	0.08
1100	0.0019	0.4518	0.0431	0.3886	0.06
1200	0.0540	0.3972	0.0431	0.3454	0.05
1300	0.0517	0.3454	0.0435	0.3022	0.04
1400	0.0304	0.2950	0.0434 0.0422	0.2588	0.04
1500	0.0497	0.2453	0.0433	0.2155	0.03
1750	0.1232 0.1221	0.1221	0.1080	0.1075	0.01
2000	0.1221	0.0000	0.1075	0.0000	0.00
2500	0.2569	-0.2389	0.2100	-0.2106	-0.03
3000	0.2297	-0.4686	0.2010 0.1061	-0.4123	-0.06
3500	0.2170	-0.6856	0.1901	-0.6083	-0.08
4000	0.2097	-0.8952	0.2004	-0.8087	-0.09

Table 10.4 Computation of Relative Geostrophic Currents. Data from Endeavor Cruise 88, Station 61 and 64

\* Geopotential anomaly integrated from 2000 dbar level. Velocity is calculated from (10.18)

m below the surface. This is the assumption used to derive the currents in Table 10.4. Currents are assumed to be zero at this depth, and relative currents are integrated up to the surface and down to the bottom to obtain current velocity as a function of depth. There is some experimental evidence that such a level exists on average for mean currents (see for example, Defant, 1961: 492), although current meters tend to measure strong, variable currents at all levels.

Defant recommends choosing the reference level at the depth where the current shear in the vertical is smallest, which is usually near 2 km. The as-



Figure 10.8 Relative current as a function of depth calculated from hydrographic data collected by the R/V Endeavor cruise south of Cape Cod in August 1982. The Gulf Stream is the fast current shallower than 1000 decibars. The assumed depth of no motion is at 2000 decibars.

sumption leads to useful maps of surface currents because surface currents tend to be faster than deeper currents. Figure 10.9 shows the geopotential anomaly and surface currents in the Pacific relative to the 1,000 dbar pressure level. When the specific volume anomaly is integrated from the level of no motion to the surface, the height of the surface is often called the *dynamic topography*.

Note that even a small error in the assumed velocity at the level of no motion leads to a large error in the calculation of transport, even though the error in the calculation of near surface currents is small.

2. Use known currents: The known currents could be measured by current meters or by satellite altimetry. Problems arise if the currents are not measured at the same time as the hydrographic data. For example, the hydrographic data may have been collected over a period of months to decades, while the currents may have been measured over a period of only a few months. Hence, the hydrography may not be consistent with



Figure 10.9. Mean geopotential anomaly of the Pacific Ocean relative to the 1,000 dbar surface based on 36,356 observations. Height is in geopotential centimeters. If the velocity at 1,000 dbar were zero, the map would be the surface topography of the Pacific. (From Wyrtki, 1974).

the current measurements. Sometimes currents and hydrographic data are measured at nearly the same time (Figure 10.10). In this example, currents were measured continuously by moored current meters (points) in a deep western boundary current and from CTD data taken just after the current meters were deployed and just before they were recovered (smooth curves). The solid line is the current assuming a level of no motion at 2,000 m, the dotted line is the current adjusted using the current meter observations smoothed for various intervals before or after the CTD casts.

3. Use Conservation Equations: Lines of hydrographic stations across a strait or an ocean basin may be used with conservation of mass and salt to calculate currents. This is an example of an inverse problem (see Wunsch, 1996 on how inverse methods are used in oceanography). The solution may not be unique, but bounds on the error can be calculated.



Figure 10.10 Current meter measurements can be used with CTD measurements to determine current as a function of depth avoiding the need for assuming a depth of no motion. Solid line: profile assuming a depth of no motion at 2000 decibars. Dashed line: profile adjusted to agree with currents measured by current meters 1–7 days before the CTD measurements. (Plots from Tom Whitworth, Texas A&M University)

**Disadvantage of Calculating Currents from Hydrographic Data** Currents calculated from hydrographic data have provided important insights into the circulation of the ocean over the decades from the turn of the 20th century to the present. Nevertheless, it is important to review the limitations of the technique.

- 1. Hydrographic data can be used to calculate only the current relative a current at another level.
- 2. The assumption of a level of no motion may be suitable in the deep ocean, but it is usually not a useful assumption when the water is shallow such as over the continental shelf.
- 3. Geostrophic currents cannot be calculated from hydrographic stations that are close together. Stations must be tens of kilometers apart.
- 4. Hydrographic stations must be repeated to obtain the mean and variable components of the current. This is impractical, and geostrophic currents calculated from hydrographic data have usually been used to map only the time-averaged circulation of the oceans or the change in circulation from decate to decade.

Limitations of the Geostrophic Equations We began this section by showing that the geostrophic balance applies with good accuracy to flows that exceed a few tens of kilometers in extent and with periods greater than a few days. The balance cannot, however, be perfect. If it were, the flow in the ocean would never change because the balance ignores any acceleration of the flow. The important limitations of the geostrophic assumption are:

- 1. Geostrophic currents cannot evolve with time.
- 2. The balance ignores acceleration of the flow, therefore it does not apply to oceanic flows with horizontal dimensions less than roughly 50 km and times less than a few days.
- 3. The geostrophic balance does not apply near the equator where the Coriolis force goes to zero because  $\sin \varphi \to 0$ .
- 4. The geostrophic balance ignores the influence of friction.

Despite these problems, currents in the ocean are almost always very close to being in geostrophic balance even within a few degrees of the Equator. Strub et al. (1997) showed that currents calculated from satellite altimeter measurements of sea-surface slope have an accuracy of  $\pm$  3–5 cm/s. Later, Uchida, Imawaki, and Hu (1998) compared currents measured by drifters in the Kuroshio with currents calculated from satellite altimeter measurements of sea-surface slope assuming geostrophic balance. Using slopes over distances of 12.5 km, they found the difference between the two measurements was  $\pm 16$  cm/s for currents up to 150 cm/s, or about 10%. Johns, Watts, and Rossby (1989) measured the velocity of the Gulf Stream northeast of Cape Hatteras and compared the measurements with velocity calculated from hydrographic data assuming geostrophic balance. They found that the measured velocity in the core of the stream, at depths less than 500 m, was 10-25 cm/s faster than the velocity calculated from the geostrophic equations using measured velocities at a depth of 2000 m. The maximum velocity in the core was greater than 150 cm/s, so the error was  $\approx 10\%$ . When they added the influence of the curvature of the Gulf Stream, which adds an acceleration term to the geostrophic equations, the difference in the calculated and observed velocity dropped to less than 5–10 cm/s ( $\approx 5\%$ ).

## 10.7 Currents From Hydrographic Sections

Lines of hydrographic data along ship tracks are often used to produce contour plots of density in a vertical section along the track. Cross-sections of currents sometimes show sharply dipping density surfaces with a large contrast in density on either side of the current. The baroclinic currents in the section can be estimated using a technique first proposed by Margules (1906) and described by Defant (1961: Chapter 14). The technique allows oceanographers to estimate the speed and direction of currents perpendicular to the section by a quick look at the section.

To derive Margules' equation, consider the slope  $\partial z/\partial y$  of the boundary between two water masses with densities  $\rho_1$  and  $\rho_2$  (see Figure 10.11). To calculate the change in velocity across the interface we assume homogeneous layers of density  $\rho_1 < \rho_2$  both of which are in geostrophic equilibrium. Although the ocean does not have an idealized interface that we assumed, and the water masses do not have uniform density, and the interface between the water masses is not sharp, the concept is still useful in practice. The change in pressure on the interface is:

$$\delta P \frac{\partial P}{\partial x} \,\delta x + \frac{\partial P}{\partial z} \,\delta z,\tag{10.20}$$

and the vertical and horizontal pressure gradients are obtained from (10.6):

$$\frac{\partial P}{\partial z} = \rho_1 g \frac{\partial P}{\partial z} = \rho_1 f v_1 \tag{10.21}$$

Therefore:

$$\delta P_1 = -\rho_1 f v_1 \delta x + \rho_1 g dz \tag{10.22a}$$

$$\delta P_2 = -\rho_2 f v_2 \delta x + \rho_2 g dz \tag{10.22b}$$

The boundary conditions require  $\delta P_1 = \delta P_2$  on the boundary. Equating (10.22a) with (10.22b), dividing by  $\delta x$ , and solving for  $\delta z / \delta x$  gives:

$$\frac{\delta z}{\delta x} \equiv \tan \gamma = \frac{f}{g} \left( \frac{\rho_2 \, v_2 - \rho_1 \, v_1}{\rho_1 - \rho_2} \right)$$

$$\tan \gamma \approx \frac{f}{g} \left(\frac{\rho_1}{\rho_1 - \rho_2}\right) (v_2 - v_1) \tag{10.23a}$$

$$\tan \beta_1 = -\frac{f}{g} v_1 \tag{10.23b}$$

$$\tan\beta_2 = -\frac{f}{g}v_2 \tag{10.23c}$$

Because the internal differences in density are small,  $\gamma \approx 1000 \tan \beta$ . Thus the slope of the interface between the two water masses is 1000 times larger than the slope at the sea surface.

Consider the application of the technique to the Gulf Stream (Figure 10.12). From the figure:  $\varphi = 36^{\circ}$ ,  $\rho_1 = 1026.6 \text{ kg/m}^3$ ; and  $\rho_2 = 1027.6 \text{ kg/m}^3$ . If we



Figure 10.11 Inclination of the isobaric surfaces and interface between two homogeneous, moving water layers in the Northern Hemisphere.

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Figure 10.12 Cross section of potential density  $\sigma_{\theta}$  across the Gulf Stream along 63.66°W calculated from CTD data collected from *Endeavor* on 25–28 April 1986. The Gulf Stream is centered on the steeply sloping contours shallower than 1000m between 40° and 41°. Notice that the vertical scale is 425 times the horizontal scale. (Data contoured by Lynn Talley, Scripps Institution of Oceanography).

use the  $\sigma_t = 27.2$  surface to estimate the slope between the two water masses, we see that the surface changes from a depth of 450 m to a depth of 810 m over a distance of 40.7 km. Therefore,  $\tan \gamma = -8840 \times 10^{-6} = 0.00884$ ; and  $\Delta v = v_2 - v_1 = -0.98$  m/s. Assuming  $v_2 = 0$ , then  $v_1 = 0.98$  m/s. This rough estimate of the velocity of the Gulf Stream compares well with velocity at a depth of 600m calculated from hydrographic data. Assuming a level of no motion at 1200 m, the hydrographic calculation gives a velocity of 0.80 m/s.

The slope of the isopycnal surfaces are clearly seen in the figure. And plots of isopycnal surfaces can be used to quickly estimate current directions and a rough value for the speed. In contrast, the slope of the sea surface is  $8.6 \times 10^{-6}$  or 0.86 m in 100 km. This is easily observed by an altimeter, but impossible to see by eye.

Note that isopycnal surfaces in the Gulf Stream slope downward to the

east, and that sea-surface topography slopes upward to the east. Isobaric and isopycnal surfaces have opposite slope, current decreases as depth increases, and currents are baroclinic.

If the sharp interface between two water masses reaches the surface, it is an oceanic front. Such fronts have properties that are very similar to atmospheric fronts.

Eddies in the vicinity of the Gulf Stream can have warm or cold cores (Figure 10.13). Application of Margules' method these mesoscale eddies gives the direction of the flow. Anticyclonic eddies (clockwise rotation in the northern hemisphere) have warm cores ( $\rho_1$  is deeper in the center of the eddy than elsewhere) and the isobaric surfaces bow upward. In particular, the sea surface is higher at the center of the ring. Cyclonic eddies are the reverse.



Figure 10.13 Shape of isobaric surfaces  $p_i$  and the interface between two water masses of density  $\rho_1, \rho_2$  if the upper is rotating faster than the lower. **Left:** Anticylconic motion, warm-core eddy. **Right:** Cyclonic, cold-core eddy. Note that the sea surface  $p_0$  slopes up toward the center of the warm-core ring, and the isopycnal surfaces slope down toward the center (From Defant, 1929).

#### **10.8** Lagrangean Measurements of Currents

Oceanography and fluid mechanics distinguishes between two types of velocity: Lagrangian and Eulerian velocities. Lagrangian velocity is the velocity of a water particle. Eulerian velocity is the velocity of water at a fixed position. Because of the importance of measurements of currents, many techniques have been developed, although no one technique dominates.

**Basic Technique** Lagrangean techniques track the position of a drifter that follows a water parcel either on the surface or deeper within the water column. The mean velocity over some period is calculated from the distance between positions at the beginning and end of the period divided by the period. Errors are due to:

- 1. Errors in determining the position of the drifter.
- 2. The failure of the drifter to follow a parcel of water. We assume the drifter stays in a parcel of water, but external forces acting on the drifter can cause it to drift relative to the water.
- 3. Sampling errors. Drifters go only where drifters want to go. And drifters want to go to convergent zones. Hence drifters tend to avoid areas of

#### divergent flow.

**Satellite Tracked Surface Drifters** Surface drifters consist of a drogue plus a float that is usually tracked by the Argos system on meteorological satellites. The buoy carries a simple radio transmitter with a very stable frequency  $F_0$ . A receiver on the satellite receives the signal and determines the Doppler shift F as a function of time t (Figure 10.14). The Doppler frequency is

$$F = \frac{dR}{dt} \frac{F_0}{c} + F_0$$

where R is the distance to the buoy, c is the velocity of light. The closer the buoy to the satellite the more rapidly the frequency changes. When  $F = F_0$  the range is a minimum. This is the time of closest approach, and the satellite's velocity vector is perpendicular to the line from the satellite to the buoy. The time of closest approach and the time rate of change of Doppler frequency at that time gives the bouy's position relative to the orbit with a 180° ambiguity. Because the orbit is accurately known, and because the buoy can be observed many times, its position can be determined without ambiguity.

The accuracy of the position depends on the stability of the frequency transmitted by the buoy. The Argos system tracks buoys with an accuracy of  $\pm 1-2$ 



Figure 10.14 Satellite systems, especially System Argos, use radio signals transmitted from surface buoys to determine the position of the buoy. The satellite S receives a radio signal from the buoy B. The time rate of change of the signal, the Doppler shift, is a function of buoy position and distance from the satellite's track. The recorded Doppler signal is transmitted to ground stations E, which relays the information to processing centers A via control stations K. (From Dietrich *et al.*, 1980)

km, collecting 1–8 positions per day depending on latitude. Because 1 cm/s  $\approx$  1 km/day, and because typical values of currents in the ocean range from one to two hundred centimeters per second, this is an acceptable accuracy.

Holey-Sock Drifters Many types of surface drifters have been developed, culminating with the holey-sock drifter now widely used to track surface currents. The drifter consists of a circular, cylindrical drogue of cloth 1 m in diameter by 15 m long with 14 large holes cut in the sides. The weight of the drogue is supported by a submerged float set 3 m below the surface. The submerged float is tethered to a partially submerged surface float carrying the Argos transmitter.

Niiler et al. (1995) carefully measured the rate at which wind blowing on the surface float pulls the drogue through the water, and they found that the buoy moves  $12 \pm 9^{\circ}$  to the right of the wind at a speed

$$U_s = (4.32 \pm 0.67 \times) 10^{-2} \frac{U_{10}}{DAR} + (11.04 \pm 1.63) \frac{D}{DAR}$$
(10.24)

where DAR is the drag area ratio defined as the drogue's drag area divided by the sum of the tether's drag area and the surface float's drag area, and D is the difference in velocity of the water between the top of the cylindrical drogue and the bottom. If DAR > 40, then the drift  $U_s < 1$  cm/s for  $U_{10} < 10$  m/s.

Subsurface Drifters (Swallow and Richardson Floats) Subsurface drifters are widely used for measuring currents below the mixed layer. Subsurface drifters are neutrally buoyant chamber tracked by sonar using the SOFAR—Sound Fixing and Ranging—system for listening to sounds in the sound channel. The chamber can be a section of aluminum tubing containing electronics and carefully weighed to have the same density as water at a predetermined depth. Aluminum is chosen because it has a compressibility less than water.

The drifter has errors due to the failure of the drifter to stay within the same water mass. Often the errors are sufficiently small that the only important error is due to tracking accuracy.

The primary disadvantage of the neutrally buoyant drifter is that tracking systems are not available throughout the ocean.

**Subsurface Drifters (ALACE Drifters)** : Autonomous Lagrangian Circulation Explorer (ALACE) drifters (Figure 10.15) are designed to cycle between the surface and some predetermined depth. The drifter spends roughly 30 days at depth, and periodically returns to the surface to report it's position and other information using the Argos system (Davis et al., 1992). The drifter thus combines the best aspects of surface and neutrally-buoyant drifters. It is able to track deep currents, it is autonomous of acoustic tracking systems, and it can be tracked anywhere in the ocean by satellite. The maximum depth is near 2 km, and the drifter carries sufficient power to complete 70 30-day cycles to 1,000 m or 50 30-day cycles to 2,000 m.

The drifters are widely used in the World Ocean Circulation Experiment to determine mid-level currents in remote regions, especially the Antarctic Circumpolar Current.

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Figure 10.15 The Autonomous Lagrangian Circulation Explorer (ALACE) drifters are widely used by the World Ocean Circulation Experiment to measure deeper currents within the ocean. Left: Schematic of the drifter. To ascend, the hydraulic pump moves oil from an internal reservoir to an external bladder, reducing the drifter's density. To descend, the latching valve is opened to allow oil to flow back into the internal reservoir. The antenna is mounted to the end cap. **Right:** Expanded schematic of the hydraulic system. The motor rotates the wobble plate actuating the piston which pumps hydraulic oil. (From Davis et al., 1992).

Lagrangean Current Measurements Using Tracers Perhaps the best way for following water parcels is to tag the parcel with molecules not normally found in the ocean. Thanks to atomic bomb tests in the 1950s and the recent exponential production of chlorofluorocarbons, such tracers have been introduced into the ocean in large quantities. See §13.5 for a list of tracers used in oceanography. Surveys of the trace molecules are used for inferring the movement of the water. The technique is especially useful for calculating velocity of deep water masses averaged over decades and for calculating eddy diffusivities.

The distribution of trace molecules is calculated from the concentration of the molecules in water samples collected on hydrographic sections and surveys. Because the collection of data is expensive and slow, the number of repeated sections is not large. Figure 10.16 shows two maps of the distribution of tritium in the North Atlantic collected in 1972–1973 by the Geosecs Program and in 1981, a decade later. The sections show that tritium, introduced into the atmosphere during the atomic bomb tests in the atmosphere in the 1950s to 1972, penetrated to depths below 4 km only north of 40°N by 1971 and to 35°N by 1981. This shows that deep currents are very slow, about 1.6 mm/s in this example.

Because the deep currents are so small, we can question what process are responsible for the observed distribution of tracers. Both turbulent diffusion



Figure 10.16 Distribution of tritium along a section through the western basins in the North Atlantic, measured in 1972 (**Top**) and remeasured in 1981 (**Bottom**). Units are tritium units, where one tritium unit is  $10^{18}$  (tritium atoms)/(hydrogen atoms) corrected to the activity levels that would have been observed on 1 January 1981. Compare this figure to the density in the ocean shown in Figure 13.9. From Toggweiler (1994)

and advection by currents can fit the observations. Hence, does Figure 10.16 give mean currents in the deep Atlantic, or the turbulent diffusion of tritium?

Another useful tracer is the temperature and salinity of the water. We will consider these observations in Chapter 13 when we describe the core method for studying the deep circulation. Here, we note that AVHRR observations of surface temperature of the ocean are an additional source of information about currents.

Sequential infrared images of surface temperature are used to calculate the



Figure 10.17 Ocean temperature and current patterns are combined in this AVHRR analysis. Surface currents were computed by tracking the displacement of small thermal or sediment features between a pair of images. A directional edge-enhancement filter was applied here to define better the different water masses. (From Ocean Imaging, Solana Beach, Caliornia, with permission).

displacement of features in the images (Figure 10.17). The technique is especially useful for surveying the variability of currents near shore. Land provides reference points from which displacement can be calculated accurately, and large temperature contrasts can be found in many regions in some seasons.

There are two important difficulties.

- 1. Many regions have extensive cloud cover, and the ocean cannot be seen.
- 2. Flow is primarily parallel to temperature fronts, and strong currents can exist along fronts even though the front may not move. It is therefore



Figure 10.18 Trajectories that spilled rubber duckies would have followed had they been spilled on January 10 of different years. Five trajectories were selected from a set of 48 simulations of the spill each year between 1946 and 1993. The trajectories begin on January 10 (T) and end 2 years later (double symbols). Large symbols enclosing dates are the positions on November 16 of the year of the spill. Hence the circle with 92 inside is the location where rubber ducks first came ashore near Sitka. The code at lower left gives the dates of the trajectories: 1959, when the toys would have traveled in a loop around the Gulf of Alaska gyre; 1961, the most southerly trajectory; 1984, when the toys would have looped back to the northeast in an area of slow drift; 1990, when the toys after passing Sitka would go westward then northward through Unimak Pass into the Bering Sea. Note that the drifters tended to follow only one track. They could not be used for mapping currents away from the track. (From Ebbesmeyer and Ingraham, 1994).

essential to track the motion of small eddies embedded in the flow near the front and not the position of the front.

An example of Langrangean Current Measurements: The Rubber Duckie Spill On January 10, 1992 a 12.2-m container with 29,000 bathtub toys (including rubber ducks) washed overboard from a container ship at 44.7°N, 178.1°E. Ten months later the toys began washing ashore near Sitka, Alaska. A similar accident on May 27, 1990 released 80,000 Nike-brand shoes at 48°N, 161°W when waves washed containers from the *Hansa Carrier* (Figure 10.18). The spill and the eventual recovery of the toys proved to be a good test of a numerical model for calculating the trajectories of oil spills developed by Ebbesmeyer and Ingraham (1992, 1994). They calculated the possible trajectories of the spilled rubber ducks using the Ocean Surface Current Simulations OSCURS numerical model driven by winds calculated from the Fleet Numerical



Figure 10.19 **Left:** An example of a surface mooring of the type deployed by the Woods Hole Oceanographic Institution's Buoy Group. **Right:** An example of a subsurface mooring deployed by the same group. (From Baker, 1981).

Oceanography Center's daily sea-level pressure data. The calculated trajectories agreed well with observed locations of drifters found on the shore. Using a 50% increase in windage coefficient and a 5° decrease in the angle of deflection function, the toys arrived near Sitka, Alaska at the time of the first recoveries on November 16, 1992.

#### **10.9** Eulerian Measurements of Currents

Eulerian currents are measured using many types of current meters attached to many types of moorings or ships. The instruments can be mechanical or acoustic, and many different configurations and techniques have been used at one time or another.

Moorings are deployed by ships, and they may last for months to longer than a year (Figure 10.19). Because the mooring must be deployed and recovered by deep-sea research ships, the technique is expensive. Yet, it is one of the most widely used method for directly measuring currents. Submerged moorings are preferred for several reasons: the surface float is not forced by high frequency, strong, surface currents; the mooring is out of sight and it does not attract the attention of fishermen; and the floatation is usually deep enough to avoid being caught by fishing nets.

Errors in measurements of Eulerian currents arise from:

- 1. Mooring motion. Subsurface moorings move least. Surface moorings in strong currents move most, and are seldom used.
- 2. Inadequate Sampling. Moorings tend not to last long enough to give accurate estimates of mean velocity or interannual variability of the velocity.
- 3. Fouling of the sensors by marine organisms, especially instruments deployed for more than a few weeks close to the surface.

Moored Current Meters Moored current meters are perhaps the most common type of Eulerian current-measuring device. Many different types of mechanical current meters have been used. Examples include:

- 1. Aanderaa current meters which uses a vane and a Savonius rotor (Figure 10.20).
- 2. Vector Averaging Current Meters which uses a vane and propellers.
- 3. Vector Measuring Current Meters, which uses a vane and specially designed pairs of propellers oriented at right angles to each other. The propellers are designed to respond to the cosine of the vector velocity (weller and Davis, 1980).

Errors are due to the failure to accurately measure the flow past the instrument: i) The response may be nonlinear; ii) the instrument may not respond to rapid changes in the current; and iii) it may not respond accurately to flow that is not horizontal. Special care must be taken in selecting meters to be used near the surface where wave-produced currents are large.

Acoustic-Doppler Current Profiler: For many applications, mechanical current meters are being replaced by acoustic current meters that measure the Doppler shift of acoustic signals reflected from bubbles, phytoplankton and zooplankton in the water in several directions and distances from the acoustic transducer. One type of acoustic device is particularly useful, the Acoustic-Doppler Current Profiler, commonly called the ADCP. Ship-board instruments are widely used for profiling currents within 200 to 300 m of the sea surface while the ship steams between hydrographic stations. Instruments mounted on CTDs are used to profile currents from the surface to the bottom at hydrographic stations.

The instrument measures Doppler shift in several directions using three to four acoustic beams. Each beam gives the velocity in the direction of the beam, and the combination of several beams gives two or three components of the velocity.

The accuracy of the ship-borne instrument depends on the accuracy with which the ship's velocity and orientation are known. Note that the ship can be headed in one direction, yet drift in a slightly different direction. Because the ship's velocity is much faster than the current, small errors in determining the ship's velocity can produce large errors in the measurement of current. The error can be reduced by steaming along a closed, rectangular track. The net flow into a rectangular box a kilometer on a side traversed in a few minutes must be zero, and this can be used to infer the accuracy of the measurements.



Figure 10.20 An example of a moored current meter with a Savonius roter to measure current speed, a vane to measure current direction, and a pressure-resistant housing for power and circuits to record the signal. The turns of the rotor are measured by the acoustic transducer. (From Dietrich, et al. 1980)

Acoustic Tomography Another acoustic technique uses acoustic signals transmitted through the sound channel to and from a few moorings spread out across oceanic regions. The technique is expensive because it requires many deep moorings and loud sound sources. It promises, however, to obtain information difficult to obtain by other means. The number of acoustic paths across a region rises as the square of the number of moorings. And, the signal propagating along the sound channel has many modes, some that stay near the axis of the channel, others that propagate close to the sea surface and bottom (See Figure 3.16). The various modes give information about the vertical temperature structure in the ocean, and the many paths in the horizontal give the spatial distribution of temperature. If one mooring retransmits the signal it receives from another mooring, the time for the signal to propagate in one direction minus the time for the signal to propagate in the reverse direction, the reciprocal travel time, is proportional to current component parallel to the acoustic path.

**Other Methods (Mostly of Historical Interest)** A variety of techniques widely used in the past, are now seldom used. One of the most popular for a few years was the Geomagnetic ElectroKinetograph GEK current meter. It measured currents by measuring the electrical potential induced in sea water when a conductor (sea water) moving in a magnetic field (Earth's field). It consisted of a pair of electrodes towed behind a ship. The electrodes were at the beginning and end of line several hundred meters long. Or, the electrodes were at ends of submarine telephone cables. The accuracy of the technique was

difficult to quantify and the technique fell from favor. The primary error was due to unknown shorting of current by conduction through the sea floor and in still water below moving surface currents.

## 10.10 Important Concepts

- 1. Pressure distribution is almost precisely the hydrostatic pressure obtained by assuming the ocean is at rest. Pressure is therefore calculated very accurately from measurements of temperature and conductivity as a function of pressure using the equation of state of seawater. Hydrographic data give the relative, internal pressure field of the ocean.
- 2. Flow in the ocean is in almost exact geostropic balance except for flow in the upper and lower boundary layers. Corolis force almost exactly balances the horizontal pressure gradient.
- 3. Satellite altimetric observations of the oceanic topography give the surface geostrophic current. The calculation of topography requires an accurate geoid, which is known with sufficient accuracy only over distances exceeding a few thousand kilometers. If the geoid is not known, altimeters can measure the change in topography as a function of time, which gives the change in surface geostrophic currents.
- 4. Topex/Poseidon is the most accurate altimeter system, and it can measure the topography or changes in topography with an accuracy of  $\pm 4.7$  cm.
- 5. Hydrographic data are used to calculate the internal geostrophic currents in the ocean relative to known currents at some level. The level can be surface currents measured by altimetery or an assumed level of no motion at depths below 1-2 m.
- 6. Flow in the ocean that is independent of depth is called barotropic flow, flow that depends on depth is called baroclinic flow. Hydrographic data give only the baroclinic flow.
- 7. Geostrophic flow cannot change with time, so the flow in the ocean is not exactly geostrophic. The geostrophic method does not apply to flows at the equator where the Coriolis force vanishes.
- 8. Slopes of constant density or temperature surfaces seen in a cross-section of the ocean can be used to estimate the speed of flow through the section.
- 9. Measurements of the position of a parcel of water give the Lagrangean flow in the ocean. The position can be determined using surface or subsurface drifters, or chemical tracers such as tritium.
- 10. Measurements of the velocity of flow past a point gives the Eulearian flow in the ocean. The velocity of the flow can be measured using moored current meters or acoustic velocity profilers on ships, CTDs or moorings.