Chapter 9

Response of the Upper Ocean to Winds

If you have had a chance to travel around the United States, you may have noticed that the climate of the east coast differs considerably from that on the west coast. Why? Why is the climate of Charleston, South Carolina so different from that of San Diego, although both are near 32°N, and both are on or near the ocean? Charleston has 125–150 cm of rain a year, San Diego has 25–50 cm, Charleston has hot summers, San Diego has cool summers. Or why is the climate of San Francisco so different from that of Norfolk, Virginia?

If we look closely at the characteristics of the atmosphere along the two coasts near 32°N, we find more differences that may explain the climate. For example, when the wind blows onshore toward San Diego, it brings a cool, moist, marine, boundary layer a few hundred meters thick capped by much warmer, dry air. On the east coast, when the wind blows onshore, it brings a warm, moist, marine, boundary layer that is much thicker. Convection, which produces rain, is much easier on the east coast than on the west coast. Why then is the atmospheric boundary layer over the water so different on the two coasts? The answer can be found in part by studying the ocean's response to local winds, the subject of this chapter.

9.1 Inertial Motion

To begin our study of currents near the sea surface, let's consider first a very simple solution to the equations of motion, the response of the ocean to an impulse that sets the water in motion. For example, the impulse can be a strong wind blowing for a few hours. The water then moves under the influence of coriolis force and gravity. No other forces act on the water.

Such motion is said to be inertial. The mass of water continues to move due to its inertia. If the water were in space, it would move in a straignt line according to Newton's second law. But on a rotating earth, the motion is much different. From (7.18) the equations of motion for a frictionless ocean are:

$$\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \varphi \tag{9.1a}$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \sin \varphi$$
(9.1b)

$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + 2\Omega u \cos \varphi - g \tag{9.1c}$$

where p is pressure, $\Omega = 2\pi/(\text{sidereal day}) = 7.292 \times 10^{-5} \text{ rad/s}$ is the rotation of the Earth in fixed coordinates, and φ is latitude. We have also used $F_i = 0$ because the fluid is frictionless.

Let's now look for simple solutions to these equations. To do this we must simplify the momentum equations. First, if only gravity and coriolis force act on the water, there must be no pressure gradient:

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

Furthermore, we can assume that the vertical velocity is small, $w \ll u, v$, so $2\Omega \cos \varphi \ll g$, and (9.1) becomes:

$$\frac{du}{dt} = 2\Omega v \sin \varphi = f v \tag{9.2a}$$

$$\frac{dv}{dt} = -2\Omega \, u \sin \varphi = -f u \tag{9.2b}$$

where:

$$f = 2\Omega \sin \varphi \tag{9.3}$$

is the Coriolis Parameter.

Equations (9.2) are two coupled, first-order, linear, differential equations which can be solved with standard techniques. If we solve the second equation for u, and insert it into the first equation we obtain:

$$\frac{du}{dt} = -\frac{1}{f}\frac{d^2v}{dt^2} = fv$$

Rearranging the equation puts it into a standard form we should recognize, the equation for the harmonic oscillator:

$$\frac{d^2v}{dt} + f^2v = 0 (9.4)$$

which has the solution (9.5). This current is called an *inertial corrent* or *inertial oscillation*:

$$u = V \sin ft$$

$$v = V \cos ft$$

$$V^{2} = u^{2} + v^{2}$$
(9.5)



Figure 9.1 Trajectory of a water parcel calculated from current measured from August 17 to August 24, 1933 at $57^{\circ}49$ 'N and $17^{\circ}49$ 'E west of Gotland (From Sverdrup, Johnson, and Fleming, 1942).

Notice that (9.5) are the parametric equations for a circle with diameter $D_i = 2V/f$ and period $T_i = (2\pi)/f = T_{sd}/(2\sin\varphi)$ where T_{sd} is a siderial day (Figure 9.1).

 T_i is the *inertial period*, and it is one half the time required for the rotation of a local plane on Earth's surface (Table 9.1). The direction of rotation is *anti-cyclonic*: clockwise in the northern hemisphere, counterclockwise in the southern. Notice that at latitudes near 30°, inertial oscillations have periods very close to once-per-day tidal periods, and it is difficult to separate inertial oscillations from tidal currents at these latitudes.

Table 9.1 Inertial Oscillations				
Latitude (φ)	T_i (hr)	D (km)		
	for $V =$	20 cm/s		
90°	11.97	2.7		
35°	20.87	4.8		
10°	68.93	15.8		

Inertial currents are the free motion of parcels of water on a rotating plane. They are very common, and they occur everywhere in the ocean. Webster (1968) reviewed many published reports of inertial currents and found that currents have been observed at all depths in the ocean and at all latitudes. The motions are transient and decay in a few days. Oscillations at different depths or at different nearby sites are usually incoherent.

Inertial currents are usually caused by wind, with rapid changes of strong winds producing the largest oscillations. The forcing can be directly through the wind stress, or it can be indirect through non-linear interactions among ocean waves at the sea surface (Hasselmann, 1970). Although we have derived the equations for the oscillation assuming frictionless flow, friction cannot be completely neglected. With time, the oscillations decay into other surface currents. (See, for example, Apel, 1987: §6.3 for more information.)

Fridtjof Nansen	(1898)	Qualitative theory, currents transport water at an angle to the wind.
Vagn Walfrid Ekman	(1902)	Quantitative theory for wind-driven transport at the sea surface.
Harald Sverdrup	(1947)	Theory for wind-driven circulation in the eastern Pacific.
Henry Stommel	(1948)	Theory for westward intensification of wind-driven circulation (western boundary currents).
Walter Munk	(1950)	Quantitative theory for main features of the wind- driven circulation.
Kirk Bryan	(1963)	Numerical models of the oceanic circulation.
Bert Semtner and Robert Chervin	(1988)	Global, eddy-resolving, realistic model of the ocean's circulation.

Table 9.2 Contributions to the Theory of the Wind-Driven Circulation

9.2 Ekman Layer at the Sea Surface

Steady winds blowing on the sea surface produce a thin, horizontal boundary layer, the *Ekman layer*. By thin, I mean a layer that is at most a few-hundred meters thick, which is thin compared with the depth of the water in the deep ocean. A similar boundary layer exists at the bottom of the ocean, the *bottom Ekman layer*, and at the bottom of the atmosphere just above the sea surface, the planetary boundary layer or frictional layer described in §4.3. The Ekman layer is named after Professor Walfrid Ekman, who worked out its dynamics for his doctoral thesis.

Ekman's work was the first of a remarkable series of studies conducted during the first half of the twentieth century that led to an understanding of how winds drive the ocean's circulation (Table 9.1). In this chapter we consider Nansen and Ekman's work. The rest of the story is given in the next chapter and in Chapter 16.

Nansen's Qualitative Arguments Fridtjof Nansen, while drifting on the *Fram*, noticed that wind tended to drive ice at an angle of $20^{\circ}-40^{\circ}$ to the right of the wind in the Arctic, by which he meant that the track of the iceberg was to the right of the wind looking downwind (See figure 9.2). He subsequently worked out the basic balance of forces that must exist when wind tried to push icebergs downwind on a rotating earth.

Nansen argued that three forces must be important:

- 1. Wind Stress, W;
- 2. Friction **F** (otherwise the iceberg would move as fast as the wind);
- 3. Coriolis Force, **C**.

Nansen argued further that the forces must have the following attributes:

- 1. Drag must be opposite the direction of the ice's velocity;
- 2. Coriolis force must be perpendicular to the velocity;
- 3. The forces must balance for steady flow.

$$\mathbf{W} + \mathbf{F} + \mathbf{C} = 0$$



Figure 9.2 The balance of forces acting on an iceberg in a wind on a rotating Earth.

Ekman's Solution When Nansen returned to Stockholm, he asked Vilhelm Bjerknes to let one of Bjerknes' students make a theoretical study of the influence of Earth's rotation on wind-driven currents. Walfrid Ekman was chosen, and he presented the results in his thesis at Uppsala. Ekman later expanded the study to include the influence of continents and differences of density of water (Ekman, 1905). The following follows Ekman's line of reasoning in that paper.

Ekman assumed a steady, homogeneous, horizontal flow with friction on a rotating Earth. Thus horizontal and temporal derivatives are zero:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \tag{9.6}$$

Ekman further assumed a constant vertical eddy viscosity of the form (8:13):

$$T_x = \rho_w A_z \frac{\partial u}{\partial z}, \qquad T_y = \rho_w A_z \frac{\partial v}{\partial z}$$

$$(9.7)$$

where T_x , T_y are the components of the wind stress in the x, y directions, and ρ_w is the density of sea water.

With these assumptions, and using (9.7) in (8.15), the x and y components of the momentum equation have the simple form:

$$fv + A_z \frac{\partial^2 u}{\partial z^2} = 0 \tag{9.8a}$$

$$-fu + A_z \frac{\partial^2 v}{\partial z^2} = 0 \tag{9.8b}$$

where f is the Coriolis parameter.

It is easy to verify that the equations (9.9) have solutions:

$$u = V_0 \exp(az) \sin(\pi/4 - az) \tag{9.9a}$$

$$v = V_0 \exp(az) \cos(\pi/4 - az) \tag{9.9b}$$

when the wind is blowing to the north $(T = T_y)$. The constants are

$$V_0 = \frac{T}{\sqrt{\rho_w^2 f A_z}} \qquad \text{and} \qquad a = \sqrt{\frac{f}{2A_z}} \tag{9.10}$$

and V_0 is the velocity of the current at the sea surface.

Now let's look at the form of the solutions. At the sea surface z = 0, $\exp(z = 0) = 1$, and

$$u(0) = V_0 \cos(\pi/4) \tag{9.11a}$$

$$v(0) = V_0 \sin(\pi/4)$$
 (9.11b)

The current has a speed of V_0 to the northeast. In general, the surface current is 45° to the right of the wind when looking downwind in the northern hemisphere. The current is 45° to the left of the wind in the southern hemisphere. Below the surface, the velocity decays exponentially with depth (Figure 9.3):

$$\left[u^{2}(z) + v^{2}(z)\right]^{1/2} = V_{0} \exp(az)$$
(9.12)



Figure 9.3. Vertical distribution of current due to wind blowing on the sea surface (From Dietrich, $et \ al.$, 1980).

Evaluation of Ekman's Solution To proceed further, we need values for any two of the free parameters: the velocity at the surface, V_0 ; the coefficient of eddy viscosity, A_z ; or the wind stress T.

The wind stress is well known, and Ekman used the bulk formula (4.2):

$$T = \rho_{air} C_D U_{10}^2 \tag{9.13}$$

where ρ_{air} is the density of air, C_D is the drag coefficient, and U_{10} is the wind speed at 10 m above the sea. Ekman turned to the literature to obtain values for V_0 as a function of wind speed. He found:

$$V_0 = \frac{0.0127}{\sqrt{\sin|\varphi|}} U_{10}, \qquad |\varphi| \ge 10$$
(9.14)

With this information, he could then calculate the velocity as a function of depth knowing the wind speed U_{10} and wind direction.

Ekman Layer Depth The thickness of the Ekman layer is arbitrary because the Ekman currents decrease exponentially with depth. Ekman proposed that the thickness be the depth D_E at which the current velocity is opposite the velocity at the surface, which occurs at a depth $D_E = \pi/a$, and the *Ekman layer depth* is:

$$D_E = \sqrt{\frac{2\pi^2 A_z}{f}} \tag{9.15}$$

Using (9.13) in (9.10), dividing by U_{10} , and using (9.14) and (9.15) gives:

$$D_E = \frac{7.6}{\sqrt{\sin|\varphi|}} U_{10} \tag{9.16}$$

in SI units; wind in meters per second gives depth in meters. The constant in (9.16) is based on $\rho_w = 1027 \text{ kg/m}^3$, $\rho_{air} = 1.25 \text{ kg/m}^3$, and Ekman's value of $C_D = 2.6 \times 10^{-3}$ for the drag coefficient.

Using (9.16) with typical winds, the depth of the Ekman layer varies from about 10 to 25 meters (Table 9.3), and the velocity of the surface current varies from 2.5% to 1.1% of the wind speed depending on latitude.

Table 9.3 Typical Ekman Depths

	Latitude		
$U_{10} [m/s]$	15°	45°	
5	10 m	6 m	
10	$20 \mathrm{m}$	$12 \mathrm{m}$	
20	$39 \mathrm{m}$	24 m	

The Ekman Number: Coriolis and Frictional Forces The depth of the Ekman layer is closely related to the depth at which frictional force is equal to the Coriolis force in the momentum equation (9.9). The Coriolis force is fu, and the frictional force is $A_z \partial^2 U/\partial z^2$. The ratio of the forces, which is non dimensional, is called the *Ekman Number* E_z :

$$E_{z} = \frac{\text{Friction Force}}{\text{Coriolis Force}} = \frac{A_{z} \frac{\partial^{2} u}{\partial z^{2}}}{f u} = \frac{A_{z} \frac{u}{d^{2}}}{f u}$$

$$\boxed{E_{z} = \frac{A_{z}}{f d^{2}}}$$
(9.17)

where we have approximated the terms using typical velocities u, and typical depths d. The subscript z is needed because the ocean is stratified and mixing in the vertical is much less than mixing in the horizontal. Note that as depth increases, friction becomes small, and eventually, only the Coriolis force remains.

Solving (9.17) for d gives

$$d = \sqrt{\frac{A_z}{fE_z}} \tag{9.18}$$

which agrees with the functional form (9.15) proposed by Ekman. Equating (9.18) and (9.15) requires $E_z = 1/(2\pi^2) = 0.05$ at the Ekman depth. Thus Ekman chose a depth at which frictional forces are much smaller than the Coriolis force.

Bottom Ekman Layer The Ekman layer at the bottom of the ocean and the atmosphere differs from the layer at the ocean surface. The solution for a bottom layer below a fluid with velocity U in the x-direction is:

$$u = U[1 - \exp(-az)\cos az] \tag{9.19a}$$

$$v = U \exp(-az) \sin az \tag{9.19b}$$

The velocity goes to zero at the boundary, u = v = 0 at z = 0. The direction of the flow close to the boundary is 45° to the left of the flow U outside the boundary layer in the northern hemisphere; and the direction of the flow rotates with distance above the boundary (Figure 9.4). The direction of rotation is anticyclonic with distance above the bottom.

Winds above the planetary boundary layer are perpendicular to the pressure gradient in the atmosphere and parallel to lines of constant surface pressure. Winds at the surface are 45° to the left of the winds aloft, and surface currents are 45° to the right of the wind at the surface. Therefore we expect currents at the sea surface to be nearly in the direction of winds above the planetary boundary layer and parallel to lines of constant pressure. Observations of surface drifters in the Pacific tend to confirm the hypothesis (Figure 9.5).



Figure 9.4 Ekman layer for the lowest kilometer in the atmosphere (solid line), together with wind velocity measured by Dobson (1914) - - - . The numbers give height above the surface in meters. The boundary layer at the bottom of the ocean has a similar shape. (From Houghton, 1977).

Examining Ekman's Assumptions Before considering the validity of Ekman's theory for describing flow in the surface boundary layer of the ocean, let's first examine the validity of Ekman's assumptions. He assumed:

- 1. No boundaries. This is valid away from coasts.
- 2. Deep water. This is valid if depth $\gg 200$ m.
- 3. *f*-plane. This is valid.
- 4. Steady state. This is valid if wind blows for longer than a pendulum day. Note however that Ekman also calculated a time-dependent solution, as



Figure 9.5 Trajectories of surface drifters in April 1978 together with surface pressure in the atmosphere averaged for the month. Note that drifters tend to follow lines of constant pressure except in the Kuroshio where ocean currents are fast compared with velocities in the Ekman layer in the ocean. (From McNally, *et al.* 1983).

did Hasselmann (1970).

- 5. A_z is a function of U_{10}^2 only. It is assumed to be independent of depth. This is not a good assumption. The mixed layer may be thinner than the Ekman depth, and A_z will change rapidly at the bottom of the mixed layer because mixing is a function of stability. Mixing across a stable layer is much less than mixing through a layer of a neutral stability. More realistic profiles for the coefficient of eddy viscosity as a function of depth change the shape of the calculated velocity profile. We reconsider this problem below.
- 6. Homogeneous density. This is probably good, except as it effects stability.

Observations of Flow Near the Sea Surface Does the flow close to the sea surface agree with Ekman's theory? Measurements of currents made during several, very careful experiments indicate that Ekman's theory is remarkably good. The theory accurately describes the flow averaged over many days. The measurements also point out the limitations of the theory.

Weller and Plueddmann (1996) measured currents from 2 m to 132 m using 14 vector-measuring current meters deployed from the Floating Instrument Platform FLIP in February and March 1990 500 km west of point Conception, California. This was the last of a remarkable series of experiments coordinated by Weller using instruments on FLIP.

Davis, DeSzoeke, and Niiler (1981) measured currents from 2 m to 175 m using 19 vector-measuring current meters deployed from a mooring for 19 days in August and September 1977 at 50°N, 145°W in the northeast Pacific.

Ralph and Niiler (1999) tracked 1503 drifters drogued to 15 m depth in the Pacific from March 1987 to December 1994. Wind velocity was obtained every 6 hours from the European Centre for Medium-Range Weather Forcasts ECMWF.

The results of the experiments indicate that:

- 1. Inertial currents are the largest component of the flow.
- 2. The flow is nearly independent of depth within the mixed layer for periods near the inertial period. Thus the mixed layer moves like a slab at the inertial period. Current shear is concentrated at the top of the thermocline.
- 3. The flow averaged over many inertial periods is almost exactly that calculated from Ekman's theory. The shear of the Ekman currents extends through the averaged mixed layer and into the thermocline. Ralph and Niiler found:

$$D_E = \frac{7.12}{\sqrt{\sin|\varphi|}} U_{10} \tag{9.20}$$

$$V_0 = \frac{0.0068}{\sqrt{\sin|\varphi|}} U_{10} \tag{9.21}$$

The Ekman-layer depth D_E is almost exactly that proposed by Ekman (9.16), but the surface current V_0 is half his value (9.14).

9.2. EKMAN LAYER AT THE SEA SURFACE

4. The transport is 90° to the right of the wind in the northern hemisphere. The transport direction agrees well with Ekman's theory.

Note that few experiments are able to produce useful measurements of Ekman currents. This is because because it is hard to make accurate, direct measurements of currents within the Ekman layer at the sea surface. Two important difficulties must be overcome:

- 1. Direct measurements of currents are difficult to make. Ekman currents are fastest within a few meters of the sea surface after winds have been blowing strongly for at least a day. But strong winds blowing for a day produce large waves that have large oscillating currents within a few meters of the surface. Only vector-measuring moored current meters and holey-sock drifters work accurately during these conditions. (see §10.8).
- 2. Ekman currents are difficult to separate from other near-surface currents, including those in geostrophic balance (see Chapter 10), and currents due to waves, inertial oscillations, Langmuir circulation, and tides. Currents must be mesaures once a second for many days to separate the different contributions to the signal.

Langmuir Circulation The measurements of surface currents show that the near-surface flow in the ocean with variable stratification and changing wind and wave conditions is much more complicated than the simple Ekman layer we have been describing. Other processes complicate the picture. One important process is the Langmuir circulation. According to Langmuir, surface currents spiral around an axis parallel to the wind direction. Weller, *et al.* (1985) observed such a flow during an experiment to measure the wind-driven circulation in the upper 50 meters of the sea. They found that during a period when the wind speed was 14 m/s, surface currents were organized into Langmuir cells spaced 20 m apart, the cells were aligned at an angle of 15° to the right of the wind; and vertical velocity at 23 m depth was concentrated in narrow jets under the areas of surface convergence (Figure 9.6). Maximum vertical velocity was -0.18 m/s. The seasonal thermocline was at 50 m, and no downward velocity was observed in or below the thermocline.

Influence of Stability in the Ekman Layer Ralph and Niiler (1999) point out that Ekman's choice of an equation for surface currents (9.14), which leads to (9.16), is consistent with theories that include the influence of stability in the upper ocean. Currents with periods near the inertial period produce shear in the thermocline. The shear mixes the surface layers when the Richardson number falls below the critical value (Pollard et al. 1973). This idea, when included in mixed-layer theories, leads to a surface current V_0 that is proportional to \sqrt{Nf}

$$V_0 \sim U_{10}\sqrt{N/f} \tag{9.22}$$

furthermore

$$A_z \sim U_{10}^2/N$$
 and $D_E \sim U_{10}/\sqrt{Nf}$ (9.23)

Notice that (9.22) and (9.23) are now dimensionally correct. The equations used earlier, (9.14), (9.16), (9.20), and (9.21) all required a dimensional coefficient.



Figure 9.6 A three-dimensional view of the Langmuir circulation at the surface of the Pacific observed from the Floating Instrument Platform FLIP. The heavy dashed line on the sea surface indicate lines of convergence marked by cards on the surface. Vertical arrows are individual values of vertical velocity measured every 14 seconds at 23 m depth as the platform drifted through the Langmuir currents. Horizontal arrows, which are drawn on the surface for clarity, are values of horizontal velocity at 23 m. The broad arrow gives the direction of the wind (From Weller *et al.* 1985).

9.3 Ekman Mass Transports

Flow in the Ekman layer at the sea surface carries mass. For many reasons we may want to know the total mass transported in the layer. The *Ekman mass transport* M_E is defined as the integral of the Ekman velocity U_E, V_E from the surface to a depth d below the Ekman layer. The two components of the transport are M_{Ex} , M_{Ey} :

$$M_{Ex} = \int_{-d}^{0} \rho U_E \, dz, \qquad M_{Ey} = \int_{-d}^{0} \rho V_E \, dz \tag{9.24}$$

The transport has units kg/(m·s); and it is the mass of water passing through a vertical plane one meter wide that is perpendicular to the transport and extending from the surface to depth -d (Figure 9.7).

Volume transport Q is the mass transport divided by the density of water and multiplied by the width perpendicular to the transport.

$$Q_x = \frac{YM_x}{\rho}, \qquad Q_y = \frac{XM_y}{\rho} \tag{9.25}$$

where Y is the north-south distance across which the eastward transport Q_x is calculated, and X in the east-west distance across which the northward transport Q_y is calculated. Volume transport has dimensions of cubic meters per



Figure 9.7 Sketch for defining Left: mass transports, and Right: volume transports.

second. A convenient unit for volume transport in the ocean is a million cubic meters per second. This unit is called a *Sverdrup*, and it is abbreviated Sv.

We calculate the Ekman mass transports by integrating (8.15) in (9.24):

$$f \int_{-d}^{0} \rho V_E \, dz = f \, M_{Ey} = -\int_{-d}^{0} dT_x$$
$$f \, M_{Ey} = -T_x \big|_{z=0} + T_x \big|_{z=-d}$$
(9.26)

A few hundred meters below the surface the Ekman velocities approach zero, and the last term of (9.26) is zero. Thus mass transport is due only to wind stress at the sea surface (z = 0). In a similar way, we can calculate the transport in the x direction to obtain the two components of the *Ekman mass transport*:

$$f M_{Ey} = -T_x(0)$$
 (9.27a)

$$f M_{Ex} = T_y(0)$$
 (9.27b)

where T(0) is the stress at the sea surface.

Notice that the transport is perpendicular to the wind stress, and to the right of the wind in the northern hemisphere. If the wind is to the north in the positive y direction (a south wind), then $T_x(0) = 0$, $M_{Ey} = 0$; and $M_{Ex} = T_y(0)/f$. In the northern hemisphere, f is positive, and the mass transport is in the x direction, to the east.

It may seem strange that the drag of the wind on the water leads to a current at right angles to the drag. The result follows from the assumption that friction is confined to a thin surface boundary layer, that it is zero in the interior of the ocean, and that the current is in equilibrium with the wind so that it is no longer accelerating.

Recent observations of Ekman transport in the ocean agree with the theoretical values (9.27). Chereskin and Roemmich (1991) measured the Ekman volume transport across 11°N in the Atlantic using an acoustic Doppler current profiler described in Chapter 10. They calculated a southward transport $Q_y = 12.0 \pm 5.5$ Sv from direct measurements of current, $Q_y = 8.8 \pm 1.9$ Sv from measured winds using (9.27) and (9.25), and $Q_y = 13.5 \pm 0.3$ Sv from mean winds averaged over many years at 11°N. Advantages of Use of Transports The calculation of mass transports has two important advantages. First, the calculation is much more robust than calculations of velocities in the Ekman layer. By robust, I mean that the calculation is based on fewer assumptions, and that the results are more likely to be correct. Thus the calculated mass transport does not depend on knowing the distribution of velocity in the Ekman layer or the eddy viscosity.

Second, the variability of transport in space has important consequences. Let's look at a few applications.

9.4 Application of Ekman Theory

Because winds blowing on the sea surface produce an Ekman layer that transports water at right angles to the wind direction, any spatial variability of the wind, or winds blowing along some coasts, can lead to upwelling. And upwelling is important:

- 1. Enhanced biological productivity in upwelling regions leads to important fisheries.
- 2. Cold upwelled water alters local weather. Weather onshore of regions of upwelling tend to have fog, low stratus clouds, a stable stratified atmosphere, little convection, and little rain.
- 3. Spatial variability of transports leads to upwelling and downwelling, which leads to redistribution of mass in the ocean, which leads to wind-driven geostrophic currents via Ekman pumping, a process we will consider in Chapter 11.

Coastal Upwelling To see how winds lead to upwelling, consider north winds blowing parallel to the California Coast (Figure 9.8 left). The wind produces a mass transport away from the shore everywhere along the shore. The water pushed offshore can be replaced only by water from below the Ekman layer. This is *upwelling* (Figure 9.8 right). Because the upwelled water is cold, the upwelling leads to a region of cold water at the surface along the coast. Figure 10.17 shows the distribution of cold water off the coast of California.

Upwelled water is colder than water normally found on the surface, and it is richer in nutrients. The nutrients fertilize phytoplankton in the mixed layer, which are eaten by zooplankton, which are eaten by small fish, which are eaten by larger fish and so on to infinity. As a result, upwelling regions are productive waters supporting the world's major fisheries. The important regions are offshore of Peru, California, Somalia, Morocco, and Namibia.

Now we can answer the question we asked at the beginning of the chapter: Why is the climate of San Francisco so different from that of Norfolk, Virginia? Figures 4.2 or 9.8 show that wind along the California and Oregon coasts has a strong southward component. The wind causes upwelling along the coast; which leads to cold water close to shore. The shoreward component of the wind brings warmer air from far offshore over the colder water, which cools the incoming air close to the sea, leading to a thin, cool atmospheric boundary layer. As the air cools, fog forms along the coast. Finally, the cool layer of air is blown over



Figure 9.8 Sketch of Ekman transport along a coast leading to upwelling of cold water along the coast. **Left:** Cross section. The water transported offshore must be replaced by water upwelling from below the mixed layer. **Right:** Plan view. North winds along a west coast in the northern hemisphere cause Ekman transports away from the shore.

San Francisco, cooling the city. The warmer air above the boundary layer, due to downward velocity of the Hadley circulation in the atmosphere (see Figure 4.3), inhibits vertical convection, and rain is rare. Rain forms only when winter storms coming ashore bring strong convection higher up in the atmosphere.

In addition to upwelling, there are other processes influencing weather in California and Virginia.

- 1. The mixed layer at the sea surface tends to be thin on the eastern side of oceans, and upwelling can easily bring up cold water.
- 2. Currents along the eastern side of oceans at mid-latitudes tend to bring colder water from higher latitudes.
- 3. The marine boundary layer in the atmosphere, that layer of moist air above the sea, is only a few hundred meters thick in the eastern Pacific near California. It is over a kilometer thick near Asia.

All these processes are reversed offshore of east coasts, leading to warm water close to shore, thick atmospheric boundary layers, and frequent convective rain. Thus Norfolk is much different that San Francisco due to upwelling and the direction of the coastal currents.

Ekman Pumping The spatial variability of the wind at the sea surface leads to spatial variability of the Ekman transports. Because mass must be conserved, the spatial variability of the transports must lead to vertical velocities at the top of the Ekman layer (see §7.8). To calculate this velocity, we first integrate the continuity equation (7.19) in the vertical:

$$\rho \int_{-d}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dz = 0$$

$$\frac{\partial}{\partial x} \int_{-d}^{0} \rho \, u \, dz + \frac{\partial}{\partial y} \int_{-d}^{0} \rho \, v \, dz = -\frac{\partial}{\partial z} \int_{-d}^{0} \rho \, w \, dz$$

$$\frac{\partial M_{Ex}}{\partial x} + \frac{\partial M_{Ey}}{\partial y} = -\rho \left[w(0) - w(-d) \right]$$
(9.28)

By definition, the Ekman velocities approach zero at the base of the Ekman layer, and the vertical velocity at the base of the layer $w_E(-d)$ due to divergence of the Ekman flow must be zero. Therefore:

$$\frac{\partial M_{Ex}}{\partial x} + \frac{\partial M_{Ey}}{\partial y} = -\rho \, w_E(0) \tag{9.29a}$$

$$\nabla_H \cdot \mathbf{M}_E = -\rho \, w_E(0) \tag{9.29b}$$

Where \mathbf{M}_E is the vector mass transport due to Ekman flow in the upper boundary layer of the ocean, and ∇_H is the horizontal divergence operator. (9.29) states that the horizontal divergence of the Ekman transports leads to a vertical velocity in the upper boundary layer of the ocean, a process called *Ekman Pumping*.

If we use the Ekman mass transports (9.27) in (9.29) we can relate Ekman pumping to the wind stress.

$$w_E(0) = -\frac{1}{\rho} \left[\frac{\partial}{\partial x} \left(\frac{T_x(0)}{f} \right) - \frac{\partial}{\partial y} \left(\frac{T_y(0)}{f} \right) \right]$$
(9.30a)

$$w_E(0) = -\operatorname{curl}\left(\frac{\mathbf{T}}{\rho f}\right) \tag{9.30b}$$

where \mathbf{T} is the vector wind stress.

The vertical velocity at the sea surface w(0) must be zero because the surface cannot rise into the air, so $w_E(0)$ must be balanced by another vertical velocity. We will see in Chapter 12 that it is balanced by a geostrophic velocity $w_G(0)$ at the top of the interior flow in the ocean.

Note that the derivation above follows Pedlosky (1996), and it differs from the traditional approach that leads to a vertical velocity at the base of the Ekman layer. Pedlosky points out that if the Ekman layer is very thin compared with the depth of the ocean, it makes no difference whether the velocity is calculated at the top or bottom of the Ekman layer, but this is usually not true for the ocean. Hence, we must compute vertical velocity at the top of the layer. Note also that in deriving (9.29) we have implicitly assumed that the sea surface is at z = 0. If it is not, the limits of the integration are not constant. For this case, see Fofonoff (1962b).

9.5 Important Concepts

- 1. Changes in wind stress produce transient oscillations in the ocean called inertial currents
 - (a) Inertial currents are very common in the ocean.
 - (b) The period of the current is $(2\pi)/f$.
- 2. Steady winds produce a thin boundary layer, the Ekman layer, at the top of the ocean. Ekman boundary layers also exist at the bottom of the ocean and the atmosphere. The Ekman layer in the atmosphere above the sea surface is called the planetary boundary layer.

- 3. The Ekman layer at the sea surface has the following characteristics:
 - (a) *Direction*: 45° to the right of the wind looking downwind in the Northern Hemisphere.
 - (b) Surface Speed: 1–2.5% of wind speed depending on latitude.
 - (c) *Depth*: approximately 5–30 m depending on latitude and wind velocity.
- 4. Careful measurements of currents near the sea surface show that:
 - (a) Inertial oscillations are the largest component of the current in the mixed layer.
 - (b) The flow is nearly independent of depth within the mixed layer for periods near the inertial period. Thus the mixed layer moves like a slab at the inertial period.
 - (c) An Ekman layer exists in the atmosphere just above the sea (and land) surface.
 - (d) Surface drifters tend to drift parallel to lines of constant atmospheric pressure at the sea surface. This is consistent with Ekman's theory.
 - (e) The flow averaged over many inertial periods is almost exactly that calculated from Ekman's theory.
- 5. Transport is 90° to the right of the wind in the northern hemisphere.
- 6. Spatial variability of Ekman transport, due to spatial variability of winds over distances of hundreds of kilometers and days, leads to convergence and divergence of the transport.
 - (a) Winds blowing toward the equator along west coasts of continents produces upwelling along the coast. This leads to cold, productive waters within about 100 km of the shore.
 - (b) Upwelled water along west coasts of continents modifies the weather along the west coasts.