Subject Index

A

Abstract determinant, 74
Abstract matrix, 71, 74
Abstract pivot, 80
Addition
  binary matroid, 62
  dual of binary matroid, 62
  graph, 10
  graphic matroid, 36
  matroid, 80
AG(2,3) affine geometry, 66
Agrees with α, 275
Algorithm, *see* Polynomial algorithm
Almost α-balanced, with respect to a path, 277
Almost regular matroid, 106, 296, *see also* Construction, Existence condition, Parity condition, Restricted
Almost representative matrix, 286, 290, *see also* Construction
Applications, 109, 110, 241–243, 260–269, 312, 313, 348, 349
Arc, 6
Articulation point, 15
Axioms
  abstract matrix, 75
  matroid, 72, 73
α-balanced graph, 274, 275, *see also* Characterization

B

B^8 matrix, 318, *see also* N_8 matroid
$B^{8.1}$, $B^{8.2}$ matrix, 66
B^{11} matrix, 107, 109, 299, 300
B^9 matrix, 320, *see also* N_9 matroid
B^7 matrix, 65, 107–109, 196, 298–300, *see also* F_7 matroid
  with labels, 107–109, 298–300, *see also* F_7 matroid with labels
B^{10} matrix, 193, *see also* R_{10} matroid
$B^{10.1}$, $B^{10.2}$ matrix, 261, *see also* R_{10} matroid
B^{12} matrix, 105, 194, 255, *see also* R_{12} matroid
  component matrices, 194

totally unimodular version, 194
Balanced
graph, 282
matrix, 282, 312
Balancedness-inducing matrix, 281
Base
binary matroid, 53, 54
matroid, 72, 73
minor of binary matroid, 60
Binary, *see also* Characterization, Construction
field (= GF(2)), *see* Field
matroid, 52, 53
space, 56
Bipartite graph, 17
Black box, 84
Bridge, 9
Building block, 234, 237, 336
β-condition (graph), 277

C

C_3^2 graph, 93, 94
Capacity
cutset of graph, 314
edge, 314
Cardinality
intersection problem, 119
set, 5
Census, binary matroids, 65, 66
Characteristic
column vector, 19
row vector, 19
vector, 19
Characterization
α-balanced graph, 277
binary matroid, 86
cographic matroid, 225
conjecture for ΔY graphs, 100, 101
cycle polytope, 267
graph 2-isomorphic to a suspended tree, 94
graphic matroid, 225
graphic matroid of suspended tree, 95
matroid representable over
every field (= regular matroid), 196
GF(4), 204
GF(3) (= ternary matroid), 199
GF(2) (= binary matroid), 86
max-flow min-cut matroid, 326
outerplanar graph, 93
planar graph, 163
planar matroid, 211, 225
regular matroid, 196
SP graph, 91
SP matroid, 95, 109
ternary matroid, 199
Chord, 8, *see also* G-chord
Chromatic number, 242
Circuit
binary matroid, 53, 54
matroid, 72, 73
minor of binary matroid, 60
Circuit/cocircuit condition, 59
Clique, 203
Clique/node incidence matrix, 203, 312
Cobase
binary matroid, 53, 55
matroid, 72
minor of binary matroid, 60
Cocircuit
binary matroid, 53, 56
matroid, 72
minor of binary matroid, 60
Cocycle, 9
graphic matroid, 33
minor of graph, 13
minor of graphic matroid, 36
Cographic, *see also* Characteri-

zation, Planar, Regular
matrix, 34
matroid, 34, 192
Cohyperplane, 297
Co-independence
binary matroid, 53
graphic matroid, 34
Collection of weighted circuits, 332
Coloop
binary matroid, 56
graph, 9
matroid, 79
Colorable with n colors, 241
Column
complement of matrix, 284
Eulerian matrix, 24, 54
submatrix, 18
vector, terminology, 19
Common vertex, 39
Complement totally unimodular matrix, 285, *see also* Construction
Complete
bipartite graph $K_{m,n}$, 17, *see also* $K_{4,3}$, $K_{3,3}$, $K_{2,3}$ graph
graph K_n, 17, *see also* K_5, K_4, K_6 graph
Complexity of algorithms, 24, *see also* Polynomial algorithm
Component matrices of B^{12}, *see* B^{12} matrix
Components of sum, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
Composition, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
Conjecture, nonisomorphic minimal matroids for inherited properties, 244
Connected
binary matroid, 63, 64
block, 23, 178
components of graph, 9
graph, 9
graphic matroid, 43
in cycle fashion, 39
in tree fashion, 39
matrix, 23
matroid, 80, 82
Connecting
graph, 180
matroid, 176
minor, 176, 180
nodes, 15
triad, 184
triangle, 183
vertex, 39
Connectivity, *see also* k-connectivity
function, 87, 88
types, graph, 15
Construction
almost regular matroid, 106–109, 298–300
almost representative matrix (over GF(2) or GF(3)), 309
binary representation matrix, 55, 56
complement totally unimodular matrix, 305, 306
complement totally unimodular matrix, example, 310
graph without G_{12} minors, 237
graph without K_5 minors, 234
graph without $K_5\backslash y$ minors, 237
graph without odd-K_4 minors, 347, 348
max-flow min-cut matroid, 336
minimal violation matrix of regularity (= minimal nonregular matrix), 310
minimal violation matrix of total unimodularity (= minimal non-totally unimodular matrix), 309

regular matroid, 257
sequence of nested 3-connected minors, 157–163
SP graph, 90
SP matroid, 94, 109
totally unimodular $\{0, 1\}$ matrix, 264, 265
totally unimodular $\{0, \pm 1\}$ matrix, 260–264
ΔY graph, 96
ΔY matroid, 101–104
Contraction
binary matroid, 58
dual of binary matroid, 62, 63
graph, 10
graph with T nodes, 209, 210
graphic matroid, 36
matroid, 80
minor, binary matroid, 61
minor, graph, 61
minor, matroid, 81, 82
Coparallel
class, graph, 10
class, graphic matroid, 34
edges, graph, 10
edges, graphic matroid, 34
elements, binary matroid, 56
elements, matroid, 79
Corank gap, 157
Corner node, 216
Cosimple graph, 10
Cotree, 9
graphic matroid, 33
Cubic node, 221
Cycle, 8
agrees with α, 275
connectivity, 15
graphic matroid, 32
k-connected, 15
k-separable, 15
k-separation, 15
length, 8
minor of graph, 12, 13
minor of graphic matroid, 36
polytope, binary matroid, 267, 268
Cycle/cocycle condition, 12, 35

D

Decomposition, *see also* Δ-, k-, 1-, 3-, 2-, Y-sum
circle graph, 243
combinatorial structure, 243
graph into 3-connected components, 243
minimally 3-connected graph, 243
optimization problem, 243
perfect graph, 243
Degree (vertex), 7
Deletion
binary matroid, 58
dual of binary matroid, 62, 63
graph, 10
graph with T nodes, 209
graphic matroid, 36
matroid, 80
minor, binary matroid, 62
minor, graph, 62
minor, matroid, 82
Delta-sum, *see* Δ-sum
Delta-wye, *see* ΔY
Dependent set, matroid, 72
det (abstract matrix), 74
$\det_{\mathcal{F}}$ ($\mathcal{F}$-determinant), 19
$\det_3$ (GF(3)-determinant), 19
$\det_2$ (GF(2)-determinant), 19
Determinant, 19
Difference of two sets, 5
Display
convention, matrix, 24
minor, 60
Double triangle, *see* C_3^2 graph
Dual
binary matroid, 53
graph, 213
graphic matroid, 34

linear program, 126
matroid, 72, 80
plane graph, 14
Duality theorem, *see* Linear program
Δ-sum
binary matroid, 183
components, 183
composition, binary matroid, 183
composition, graph, 183
decomposition, binary matroid, 183
decomposition, graph, 183
graph, 183, 234–238, 346, 347
max-flow min-cut matroid, 318, 333–335
regular matroid, 251, 257
signed graph, 346, 347
Δ-to-Y graph, 110
ΔY, *see also* Characterization, Construction
exchange, almost regular matroid, 108, 299
exchange, binary matroid, 103, 104, 183, 184
exchange, graph, 96, 183, 184
exchange, regular matroid, 249
exchange, restricted, 108, 299
extension sequence, almost regular matroid, 108, 299
extension sequence, binary matroid, 104
extension sequence, restricted, 108, 299
graph, 96
matroid, 104, 250
reducibility, binary matroid, 104
reducibility, graph, 96
reducibility, planar graph, 97
reduction sequence, binary matroid, 104
reduction sequence, graph, 96
extension sequence, binary matroid, 104
extension sequence, graph, 96,

E

Ear decomposition, 243
Edge, 6
return edge (SP graph), *see* SP
subdivision, *see* Subdivision
Element, 5
Empty
graph, 9
matrix, 18, 37
matroid, 37
set, 5
Endpoints
edge, 6
path, 8
Equality, matrices, 18, 19
Equivalence (signed graph), *see* Signed graph
Eulerian matrix, 24, 291
Even edge, *see* Signed graph
Exact k-separation
binary matrix, 63
binary matroid, 63
inducing another k-separation, 129–146
in terms of matroid rank function, 68, 83
Existence condition, almost regular matroid, 296
Expansion
binary matroid, 62
dual of binary matroid, 62, 63
graph, 11
graphic matroid, 36
matroid, 80
Extension
binary matroid, 62
graph, 11
matroid 80

F

F_7 matroid, 65, 107, 196, 199, 259, 268, 298, 316, 318, 366, *see also* B^7 matrix
  graph with T nodes, 208
  with labels, 107, 298, 366, *see also* B^7 matrix with labels
  with special element l, 316, *see also* Odd double triangle
  splitter, 259
F_7^* matroid, 65, 196, 199, 259, 268, 316, 326
  graph with T nodes, 208
  with special element l, 316, 326, *see also* Odd K_4 graph
  splitter, 259
Face, plane graph, 14
Fano matroid, *see* F_7, F_7^* matroid
$\mathcal{F}$-determinant (= $\det_{\mathcal{F}}$), 19
Field
  binary (= GF(2)), 18
  $\mathcal{F}$ (arbitrary field), 5, 18
  GF(4), 204
  GF(3), 18
  GF(2), 18
  $\mathbb{R}$ (real numbers), 18
  ternary (= GF(3)), 18
Finding 1-, 2-, 3-sums, *see* Polynomial algorithm
Flow value
  collection of weighted circuits, 332
  max flow problem, 314
Forest, 9
Four-color
  conjecture, 241
  theorem, 241
$\mathcal{F}$-pivot, 20, 21
  properties, 21
Fractional
  matching polyhedron, 341
  max flow problem, 327
  min cut problem, 327
$\mathcal{F}$-rank, 19
Fundamental
  circuit, binary matroid, 55
  circuit, matroid, 78, 79
  cocircuit, binary matroid, 56
  cocircuit, matroid, 79
  cocycle, graphic matroid, 33
  cycle, graphic matroid, 32

G

G_8 graph, 232–237
  splitter, 233
G_{15}^i graph, 236, 237
G_{14}^i graph, 236, 237
G_9 graph, 236
G_{13} graph, 236, 237
G_{12} graph, 236, 237
Gap, 157
G-chord, 275
Generalized network, recognition problem, 243
Geometric viewpoint, 67
GF(4), 204, *see also* Field
GF(3), *see also* Field
  -determinant (= $\det_3$), 19
  -pivot, *see* $\mathcal{F}$-pivot
  -rank, *see* $\mathcal{F}$-rank
GF(2), *see also* Field
  -dependence, 28
  -determinant (= $\det_2$), 19
  -mindependence, 28
  -pivot, 20, 34, 35, 80
  -rank, *see* $\mathcal{F}$-rank
Graph, 6–17, *see also* Characterization, Construction
  addition, *see* Addition
  coloop, *see* Coloop
  contraction, *see* Contraction
  deletion, *see* Deletion
  expansion, *see* Expansion
  extension, *see* Extension
  without G_{12} minors, 236

without K_5 minors, 228–234
without $K_5\backslash y$ minors, 236
k-connectivity, *see* k-connectivity
k-separation, *see* k-separation
loop, *see* Loop
max cut problem, *see* Max cut problem
max flow problem, *see* Max flow problem
min cut problem, *see* Min cut problem
without odd-K_4 minors, 340–348
reduction, *see* Reduction
subdivision, *see* Subdivision
sum, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
sum (special), 39
with T nodes, 206–211
Graphic, *see also* Characterization, Planar, Regular
matrix, 31
matroid, 29, 34, 192
Graphicness test
binary matroid, 238–240
matroid, 243
subroutine, 47, 48
Grid graph, 98
Groundset
binary matroid, 53
graphic matroid, 29
matroid, 72

H

Hadwiger's conjecture (graph coloring), 241
Hamiltonian cycle problem, 127
Has
a $\overline{G}$ minor, terminology, 13
an $\overline{M}$ minor, terminology, 63
Homeomorphism, graph, 13
Hyperplane, 297

I

Identification, nodes, 15
Incidence, edge, 7
Independent set
binary matroid, 53
graphic matroid, 29
matroid, 72
Independent vertex set problem, 203
Induced k-separation, 129, *see also* Separation algorithm
graph, 145
sufficient conditions, 143–145
under L-isomorphism, 146
Induced subgraph, by node subset, 7, 8
Integer
hull of polyhedron, 340
program, 195
Internal vertex, 39
Internally node-disjoint paths, 8
Intersection
algorithm, 119–123, 127, 181
circuits and cocircuits, 57
problem, applications, 124, 125
problem, generalization, 126
problem of at least three matroids, 127
problem of two matroids, 119–125
sets, 5
theorem, 123, 124
Isolated node, 7
Isomorphism
graph, 13
matrix, 19
matroid, 63
Isthmus, 9

K

K_5 graph, 17, 49, 96, 157, 163,

230–237
splitter, 157
K_4 graph, 17, 91, 93, *see also* Signed graph, W_3 graph
$K_{4,3}$ graph, 185
$K_{m,n}$ graph, 17
K_n graph, 17
K_6 graph, 100
$K_{3,3}$ graph, 17, 49, 157, 163, 217, 218, 233–237
$K^i_{3,3}$ graph, 229, 231, 233
$K_{2,3}$ graph, 17, 93
k-connected, *see* k-connectivity
k-connectivity
abstract matrix, 80
binary matrix, 63
binary matroid, 63
graph, 16
graphic matrix, 38
graphic matroid, 38
matroid, 80
k-separable, *see* k-separation
k-separation, *see also* Polynomial algorithm
abstract matrix, 80
binary matrix, 63
binary matroid, 63
with given subsets, 124, 125
graph, 15
graphic matrix, 38
graphic matroid, 38
link between graph and matroid separations, 39, 40
matroid, 80
minimal k, 39
rank function, 67, 68, 83
k-star, 7
k-sum, *see also* Δ-, 1-, 3-, 2-, Y-sum
binary matroid, 173–176
components, 174
composition, binary matroid, 174, 175
connecting matroid, 176
connecting minor, 176
decomposition, binary matroid, 174
regular matroid ($k \geq 4$), 270

L

Label sum, graph, 275
Lattice viewpoint, 67
Leaf
edge, 9
node, 9
Length
circuit, 337
cycle, 8
matrix, 18
path, 8
Line of graph, 216
Linear program
duality theorem version, 328
intersection problem, 126
max flow problem, 327
min cut problem, 327
partitioning problem, 126
polynomial algorithm, 339
with totally unimodular coefficient matrix, 195, 330
Link between graph and matroid separations, 39, 40
l-isomorphism, 316
L-isomorphism, 146
Loop
binary matroid, 54
graph, 7
matroid, 79, 80

M

$M(C^2_3)$ matroid, 95
$M(G_8)^*$ matroid, 268
$M(K_5)$ matroid, 49, 211, 214, 225
splitter, 214
$M(K_5)^*$ matroid, 51, 208, 211,

225, 268
graph with T nodes, 208
$M(K_4)$ matroid, 95
$M(K_{3,3})$ matroid, 49, 211, 214, 216, 219, 225, 268
$M(K_{3,3})^*$ matroid, 51, 208, 211, 225, 268
graph with T nodes, 208
$M(W_4)$ matroid, 66, 116
$M(W_n)$ matroid, 116
$M(W_3)$ matroid, 65, 116–118, 176
$M(W_2)$ matroid, 116
Matching
graph, 124, 126
matroid, 4
Matrix, 18–24, *see also* Construction, Polynomial algorithm
decomposition, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
determinant, *see* Determinant
display convention, *see* Display
over GF(3), *see* Field
over GF(2), *see* Field
isomorphism, *see* Isomorphism
length, *see* Length
pivot, *see* Pivot
over $\mathbb{R}$, *see* Field
rank, *see* Rank
over the real numbers, *see* Field
representation, *see* Representation
support, *see* Support
terminology, 18
viewpoint, 66
Matroid, *see also* Characterization, Construction, Polynomial algorithm
addition, *see* Addition
axioms, *see* Axioms
composition, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
connectivity, *see* Connectivity
contraction, *see* Contraction
decomposition, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
definition, *see* Axioms
deletion, *see* Deletion
delta-sum, *see* Δ-sum
dual, *see* Dual
Δ-sum, *see* Δ-sum
expansion, *see* Expansion
extension, *see* Extension
intersection algorithm, *see* Intersection
intersection theorem, *see* Intersection
isomorphism, *see* Isomorphism
k-connectivity, *see* k-connectivity
k-separation, *see* k-separation
k-sum, *see* k-sum
max flow problem, *see* Max flow problem
min cut problem, *see* Min cut problem
minor, *see* Minor
1-sum *see* 1-sum
oracle, *see* Oracle
rank function, *see* Rank
reduction, *see* Reduction
regularity, *see* Regular, Regularity test
representability, *see* Representability, Representation
representation, *see* Representability, Representation
separation algorithm, *see* Separation algorithm
split, *see* Split
splitter, *see* Splitter
standard representation matrix, *see* Standard representation matrix
submodularity, *see* Submodularity

sum, *see* Δ-, k-, 1-, 3-, 2-, Y-sum
3-sum, *see* 3-sum
2-sum, *see* 2-sum
wye-sum, *see* Y-sum
Y-sum, *see* Y-sum
Max cut problem, 242, 267
Max flow problem
graph, 314
matroid, 315, 327, 339
Max-flow min-cut, *see also* Characterization, Construction, Polynomial algorithm
matroid, 315
property, 315
property test, 337
theorem, graph, 314
Maximal
subgraph, 9
submatrix, 9
subset, 5
Maximum matching, *see* Matching
Menger's Theorem, 8, 16, 127
Min cut problem
graph, 314
matroid, 315, 327, 339
Mindependence, 28
Minimal
in context of induced separations, 135, 146
in context of $K_{3,3}$ minor, 218
cutset, 9
GF(2)-dependence, 28
under isomorphism, without induced k-separation, 135, 142, 143
under L-isomorphism, without induced k-separation, 146
matroid without induced k-separation, 135
nonregular matrix, 283, 310
non-totally unimodular matrix, 107, 246, 290, 309, 311
subgraph, 9
submatrix, 9
subset, 5
support vector, binary space, 56
violation matrix of a property, 273, 294–296
violation matrix of regularity, 283, 310
violation matrix of total unimodularity, 107, 246, 290, 309, 311, *see also* Characterization, Construction, Polynomial algorithm
Minimum node cover, 124
Minor
binary matroid, 59
with given elements, 126, 166
graph, 12
graph, terminology, 13
graphic matroid, 36
matroid, 80
matroid, terminology, 63

N

N_8 matroid, 318–321, *see also* B^8 matrix
N_9 matroid, 320, 321, *see also* B^9 matrix
n-cycle, 275
Nested 3-connected minors
algorithm, 162
binary matroid, 158–162
graph, 159–162
Network flow
problem, 195
property, 239–241
property test, 240, 241
Network matrix, 261
Node, 6
cover, 124
Node/arc incidence matrix, 189

Node/edge incidence matrix, 27, 28, 189
Nonsingular matrix (abstract), 74
Nonstandard representation matrix, 31, 54
$\mathcal{NP}$, 24
  -complete, 24
  -hard, 25
n-path, 275
n-subgraph, 275
Nullspace, 56

O

Odd, *see also* Signed graph
  cycle, 269
  cycle (signed graph), 341
  double triangle, 347
  edge, 341
  K_4 graph, 342, 343
Odd-K_4 minor, 315, 342, 343, *see also* Signed graph
1-sum
  binary matroid, 169, 170
  components, 169
  composition, binary matroid, 169
  composition, graph, 170
  decomposition, binary matroid, 169, 180
  decomposition, graph, 170
  graph, 233
  regular matroid, 247, 256
Optimum branching, 126
Oracle, 84
Order
  algorithm, 24
  matrix, 18
Orthogonal complement, binary space, 56
Outerplanar graph, 93, *see also* Characterization, Planar, SP

P

Parallel, *see also* SP
  class, graph, 9
  class, graphic matroid, 32
  edges, graph, 9
  edges, graphic matroid, 32
  elements, binary matroid, 55
  elements, matroid, 79
  vectors, 23
Parity condition, almost regular matroid, 296
Partially odd vertex, *see* Signed graph
Partitioning problem, 119, 123, 124, *see also* Intersection algorithm, Intersection problem
  generalization, 126
  graph into forests, 126
Path, 8
  length, 8
Path shortening technique, 112–114
  applications, 114–119
Perfect
  graph, 243, 312
  matrix, 312
Petersen graph, 100, 237
PG(2,2) projective geometry, 65, 196
Pivot, *see* Abstract pivot, GF(2)-pivot, $\mathcal{F}$-pivot
Pivot element, 20, 21
Planar, *see also* Characterization
  graph, 13
  graph, ΔY reducibility, 97
  matrix, 34
  matroid, 34
  matroid, 1-element extension, 226
Plane graph, 13, 14
Point (= vertex), 6
Polyhedral combinatorics, 126,

127, 340
Polymatroid, 126
Polynomial algorithm, 24, *see also* Characterization, Construction
graphicness testing subroutine, *see* Graphicness test
induced separation, *see* Separation algorithm
induced separation or extension, 145, 149
intersection problem, *see* Intersection algorithm
k-separation with given subsets, *see* k-separation
linear program, 339
max flow problem, max-flow min-cut matroid, 339, 340
min cut problem, max-flow min-cut matroid, 339, 340
1-, 2-, or 3-sum decomposition, 180–182
partitioning problem, *see* Intersection algorithm
shortest circuit problem, max-flow min-cut matroid, 337–339
signing of regular matrix, *see* Signing
test of graphicness, *see* Graphicness test
test of matrix total unimodularity, *see* Total unimodularity test
test of matroid regularity, *see* Regularity test
test of max-flow min-cut property, *see* Max-flow min-cut property test
test of network flow property, *see* Network flow property test
test of representability over every field, 204
2-separation or sequence of nested 3-connected minors, 162
Polynomial reduction, 24
Power set, 5
Problem instance, 24
Projective geometry PG(2,2), *see* PG(2,2)
Proper
subgraph, 7
submatrix, 18
subset, 5

R

$\mathbb{R}$ (real numbers), *see* Field
R_{10} matroid, 193, 203, 209, 219, 224–226, 252, 256, 257, 268, 301, 311, 336, *see also* B^{10}, $B^{10.1}$, $B^{10.2}$ matrix
alternate representation matrix, 226
graph with T nodes, 209
splitter, 252
R_{12} matroid, 105, 194, 203, 209, 219, 224, 225, 251, 255, 256, 311, *see also* B^{12} matrix
complement totally unimodular representation matrix, 310
graph with T nodes, 209
Rank
abstract matrix, 74
binary matroid, 53
field $\mathcal{F}$, *see* $\mathcal{F}$-rank
field GF(3), *see* $\mathcal{F}$-rank
field GF(2), *see* $\mathcal{F}$-rank
function, binary matroid, 53, 54, 67
function, matroid, 73, 78
function viewpoint, 67
gap, 157
graph, 9
matrix, 19

matroid, 72
Real numbers (IR), *see* Field
Recognition problem, generalized network, 243
Reduction
binary matroid, 58
graph, 11
graphic matroid, 35, 36
matroid, 80
Regular, *see also* Characterization, Cographic, Construction, Graphic, Planar, Polynomial algorithm, Regularity test, Representability over every field, Total unimodularity test, Totally unimodular matrix
matrix, 64, 106, 190
matroid, 64, 106, 191
matroid decomposition theorem, 256
Regularity test, 259–260, 270, *see also* Regular
Removal, edge, 7
Representability, *see also* Characterization, Construction, Polynomial algorithm, Regular, Regularity test
over every field (= regularity), 84, 196–199, 204
general discussion, 83, 84, 204
over GF(4), 204
over GF(3), 196–202
over GF(3) and GF(q), 204
over GF(2), 85–87
Representation, *see also* Representability
abstract matrix, 75–77, 83, 84
matrix, binary matroid, 53
matrix, graphic matroid, 31
of matroid by abstract matrix, 78, 79
Restricted, *see also* Almost regular matroid
ΔY exchange, 108, 299
ΔY extension sequence, 108, 299
SP step, 108, 299
Return edge (SP graph), *see* SP
Rim, wheel graph, 17
Row
complement of matrix, 284
Eulerian matrix, 24
submatrix, 18
vector, terminology, 19

S

Scaling
graph, 275
signed graph, *see* Signed graph
Separable, *see* k-separation
Separation, *see* k-separation
Separation algorithm, 132, 133, 137, 138, 181
Sequence of nested 3-connected minors, 157–163
Series, *see also* SP
class, graph, 10
class, graphic matroid, 33, 34
edges, graph, 10
edges, graphic matroid, 33, 34
elements, binary matroid, 56
elements, matroid, 79
Series-parallel, *see* SP
Set, 5
Shortest circuit problem, 337, *see also* Polynomial algorithm
Signed graph (with odd and even edges), 341, *see also* Max-flow min-cut
Δ-sum, *see* Δ-sum
equivalence, 341
even edge, 341
odd cycle, 341
odd double triangle, 347
odd edge, 341
odd-K_4 minor, 315, 342, 343

without odd-K_4 minors, 340–348
partially odd vertex, 343
scaling, 341
two odd faces, 345
2-sum of type 1, *see* 2-sum
2-sum of type 2, *see* 2-sum
Signing
graph (for α-balancedness), 275–277
regular matrix (for total unimodularity), 191
Simple
graph, 10
rounding of vector, 341
Singular matrix (abstract), 74
Solid
staircase matrix, 303
triangular matrix, 303
SP, *see also* Characterization, Construction, Parallel, Series
extension, graph, 90
extension, matroid, 94
graph, 90
graph, return edge, 109
matroid, 94
reduction, graph, 90
reduction, matroid, 94
step, almost regular matroid, 108, 299
step, restricted, 108, 299
Special edge, max flow problem, 314
Split (= 2-separation), 151
Splitter, 151, 152, 214, *see also* F_7, F_7^* matroid, G_8, K_5 graph, $M(K_5)$, R_{10} matroid, W_3 graph
theorem, binary matroid, 152, 252
theorem, graph, 156, 228, 229
Spoke, wheel graph, 17
Standard representation matrix, 31, 54
Star, 7
Steinitz's Theorem, 110
Straddling element, 3-separation, 322, 344
Strong perfect graph conjecture, 312
Subdivision
edge, 13,
graph, 217
Subgraph, 7
Submatrix, 18
Submodular, *see* Submodularity
Submodularity
abstract matrix rank function, 75
binary matroid rank function, 68–71, 136
$\mathcal{F}$-rank function, 22
function on sets, 68
matrix rank function, 22, 70–71
matroid rank function, 73
Subset, 5
Subvector, 18
Sum, *see also* Δ-, k-, 1-, 3-, 2-, Y-sum
of circuits property, 270
of several graphs (special), 39
Support, matrix, 19
Suspended tree, *see* Tree
Switching, 45, 46
graph with T nodes, 210, 211
Symmetric difference
circuits, binary matroid, 57
disjoint unions of circuits, binary matroid, 57
sets, 5

T

T node, 207
Ternary, *see also* Characterization

field (= GF(3)), *see* Field
matroid, 199
matroid without $M(K_4)$ minors, 204
Test, *see* Polynomial algorithm
3-connected extension
binary matroid, 133, 147
graph, 133
3-connectedness
binary matroid, 64
matroid, 82
3-sum
binary matroid, 176–179
components, 177
composition, binary matroid, 176, 177
composition, graph, 179, 180
decomposition, binary matroid, 176–179, 180–182
decomposition, graph, 179, 180
graph, 179, 180, 233
regular matroid, 248–250, 256
Tip node, 9
Total dual integrality, 329
Total unimodularity test, 259, 260, *see also* Signing, Totally unimodular matrix
Totally unimodular matrix, 64, 106, 189, *see also* $B^{10.1}$, $B^{10.2}$ matrix, B^{12} matrix, totally unimodular version, Complement totally unimodular matrix, Construction, Linear program, Minimal violation matrix of total unimodularity, Polynomial algorithm, Regular, Regularity test, Signing, U^i matrix
Transpose, abstract matrix, 75
Tree, 9, 28
decomposition, 243
graphic matroid, 31
suspended, 93
Triad, binary matroid, 56, 65
Triangle, binary matroid, 55, 65
Triples in circuits, 269
trivial
matrix, 18
matroid, 37
Tutte
connectivity, graphs, 15
k-connected, *see* k-connectivity
k-connectivity, *see* k-connectivity
k-separable, *see* k-separation
k-separation, *see* k-separation
Two odd faces, *see* Signed graph
2-connected components, graph, 16, 17
2-isomorphism, 44–46
2-sum
binary matroid, 171
components, 171
composition, binary matroid, 171
composition, graph, 172
decomposition, binary matroid, 171, 180–182
decomposition, graph, 172
graph, 172, 233–237
max-flow min-cut matroid, 316, 317, 332, 333
regular matroid, 247, 256, 257
of type 1 (signed graph), 344
of type 2 (signed graph), 344

U

U_i matrix, 305
U_5^3 matroid, 199
U_5^2 matroid, 199
U_4^2 matroid, 85, 86, 126, 326
minor with given element, 126
U_n^m matroid, 85
Union, sets, 5
Uniqueness of reductions

binary matroid, 59
graph, 11

V

Valid inequality, 267
Vector, terminology, 19
Vertex, 6
connectivity, 15
k-connected, 15
k-separable, 15
k-separation, 15
2-isomorphism, 87
Visible minor, binary matroid, 60, 61

W

W_4 graph, 17, 66, 157
W_n graph, 17, 236
W_3 graph, 17, 65, 157, 179, *see also* K_4 graph
splitter, 157
$\mathcal{W}_i$ matroid, 125, 126, *see also* U_4^2 matroid
Wheel
graph, 17, 115, 116, 156, 159, *see also* K_4, W_4, W_n, W_3 graph
matroid, 116, 152, 158, *see also* $M(W_4)$, $M(W_n)$, $M(W_3)$, $M(W_2)$ matroid
theorem, 159, 211, 252
Wheels and whirls theorem, 166
Whirl, 125, 126, 165
Wye-sum, *see* Y-sum, 182

Y

Y-sum
binary matroid, 184
components, 184
composition, binary matroid, 184
composition, graph, 184
decomposition, binary matroid, 184
decomposition, graph, 184
graph, 184
regular matroid, 251, 257
Y-to-Δ graph, 110