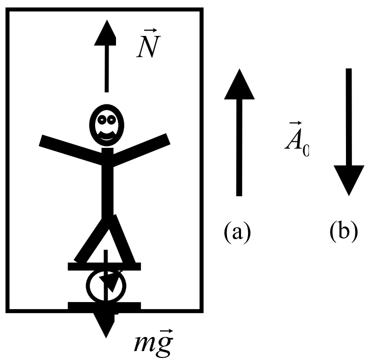
CHAPTER 5
NONINERTIAL REFERENCE SYSTEMS

5.1 (a) The non-inertial observer believes that he is in equilibrium and that the net force acting on him is zero. The scale exerts an upward force, $\vec{N}$, whose value is equal to the scale reading --- the "weight," W', of the observer in the accelerated frame. Thus



$$\vec{N} + m\vec{g} - m\vec{A}_0 = 0$$

$$N - mg - mA_0 = N - mg - m\frac{g}{4} = N - \frac{5}{4}mg = 0$$

$$W' = N = \frac{5}{4}mg = \frac{5}{4}W$$

$$W' = 150lb.$$

(b) The acceleration is downward, in the same direction as $\vec{g}$

$$N - mg + m\left(\frac{g}{4}\right) = 0 \qquad W' = W - \frac{W}{4} = \frac{3}{4}W$$

$$W' = 90lb.$$

5.2 (a) $\vec{F}_{cent} = -m\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)$

For $\vec{\omega} \perp \vec{r}'$, $\vec{F}_{cent} = m\omega^2 r'\,\hat{e}_{r'}$

$$\omega = 500\,s^{-1} = 1000\pi\,s^{-1}$$

$$\vec{F}_{cent} = 10^{-6}\times\left(1000\pi\right)^2\times 5\,\hat{e}_r = 5\pi^2 \text{ dynes outward}$$

(b) $$\frac{F_{cent}}{F_g} = \frac{m\omega^2 r'}{mg} = \frac{\left(1000\pi\right)^2 5}{980} = 5.04\times10^4$$

5.3 $m\vec{g} + \vec{T} - m\vec{A}_\circ = 0$ (See Figure 5.1.2)

$$-mg\,\hat{j} + T\cos\theta\,\hat{j} + T\sin\theta\,\hat{i} - m\left(\frac{g}{10}\right)\hat{i} = 0$$

$T\cos\theta = mg$, and $T\sin\theta = \dfrac{mg}{10}$

$$\tan\theta = \frac{1}{10}, \qquad \theta = 5.71^\circ$$

$$T = \frac{mg}{\cos\theta} = 1.005mg$$

5.4 The non-inertial observer thinks that $\vec{g}'$ points downward in the direction of the hanging plumb bob… Thus

$$\vec{g}' = \vec{g} - \vec{A}_\circ = g\,\hat{j} - \frac{g}{10}\hat{i}$$

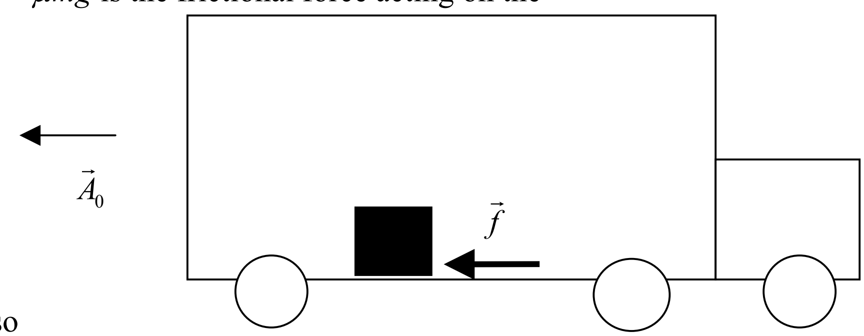
For small oscillations of a simple pendulum:

$$T = 2\pi\sqrt{\frac{l}{g'}}$$

$$g' = \sqrt{g^2 + \left(\frac{g}{10}\right)^2} = 1.005g$$

$$T = 2\pi\sqrt{\frac{l}{1.005g}} = 1.995\pi\sqrt{\frac{l}{g}}$$

5.5 (a) $f = -\mu mg$ is the frictional force acting on the



box, so

$\vec{f} - m\vec{A}_0 = m\vec{a}'$ ($\vec{a}'$ is the acceleration of the box relative to the truck. See Equation 5.1.4b.) Now, $\vec{f}$ the only real force acting horizontally, so the acceration relative to the road is

(b) $$a = \frac{f}{m} = -\frac{\mu mg}{m} = -\mu g = -\frac{g}{3}$$

(For + in the direction of the moving truck, the – indicates that friction opposes the forward sliding of the box.)

$A_\circ = -\frac{g}{2}$ (The truck is decelerating.)

from above, $ma - mA_0 = ma'$ so

(a) $$a' = a - A_\circ = -\frac{g}{3} + \frac{g}{2} = \frac{g}{6}$$

5.6 (a) $\vec{r} = \hat{i}\left(x_\circ + R\cos\Omega t\right) + \hat{j}R\sin\Omega t$

$\dot{\vec{r}} = -\hat{i}\Omega R\sin\Omega t + \hat{j}\Omega R\cos\Omega t$

$\dot{\vec{r}}\cdot\dot{\vec{r}} = v^2 = \Omega^2 R^2$ $\therefore \underline{v = \Omega R}$ circular motion of radius R

(b) $\dot{\vec{r}}' = \dot{\vec{r}} - \vec{\omega}\times\vec{r}'$ where $\vec{r}' = \hat{i}x' + \hat{j}y'$

$$= -\hat{i}\Omega R\sin\Omega t + \hat{j}\Omega R\cos\Omega t - \omega\hat{k}\times\left(\hat{i}x' + \hat{j}y'\right)$$

$$= -\hat{i}\Omega R\sin\Omega t + \hat{j}\Omega R\cos\Omega t - \hat{j}\omega x' + \hat{i}\omega y'$$
$$\dot{x}' = \omega y' - \Omega R\sin\Omega t$$
$$\dot{y}' = -\omega x' + \Omega R\cos\Omega t$$

(c) Let $u' = x' + iy'$ here $i = \sqrt{-1}$!

$$\dot{u}' = \dot{x}' + i\dot{y}' = \omega y' - \Omega R\sin\Omega t - i\omega x' + i\Omega R\cos\Omega t$$
$$i\omega u' = \quad = -\omega y' \quad + i\omega x'$$
$$\therefore \dot{u}' + i\omega u' = -\Omega R\sin\Omega t + i\Omega R\cos\Omega t = i\Omega\,\mathrm{R}e^{i\Omega t}$$

Try a solution of the form

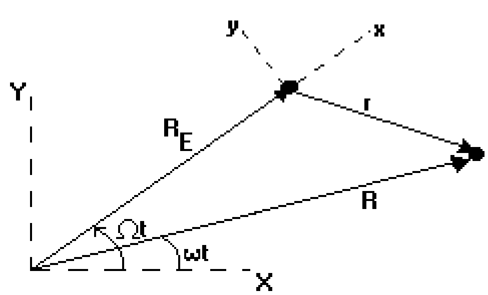
$$u' = Ae^{-i\omega t} + Be^{i\Omega t}$$
$$\dot{u}' = -i\omega Ae^{-i\omega t} + i\Omega Be^{i\Omega t}$$
$$i\omega u' = i\omega Ae^{-i\omega t} + i\omega Be^{i\Omega t}$$
$$\therefore \dot{u}' + i\omega u' = i(\omega + \Omega)Be^{i\omega t} \qquad \text{so} \quad B = \frac{\Omega R}{\omega + \Omega}$$

Also at $t = 0$ the coordinate systems coincide so

$$u' = A + B = x'(0) + iy'(0) = x_\circ + R$$
$$\therefore A = x_\circ + R - B = x_\circ + R - \frac{\Omega R}{\omega + \Omega} \qquad \text{so,} \quad A = x_\circ + \frac{\omega R}{\omega + \Omega}$$

Thus, $u' = \left[x_\circ + \dfrac{\omega R}{\omega + \Omega}\right]e^{-i\omega t} + \dfrac{\Omega R}{\omega + \Omega}e^{i\Omega t}$

5.7 The x, y frame of reference is attached to the Earth, but the x-axis always points away from the Sun. Thus, it rotates once every year relative to the fixed stars. The X,Y frame of reference is fixed inertial frame attached to the Sun.



(a) In the x, y rotating frame of reference

$$x(t) = R\cos(\Omega - \omega)t - R_\varepsilon$$
$$y(t) = -R\sin(\Omega - \omega)t$$

where R is the radius of the asteroid's orbit and R_E is the radius of the Earth's orbit. Ω is the angular frequency of the Earth's revolution about the Sun and ω is the angular frequency of the asteroid's orbit.

(b) $\dot{x}(t) = -(\Omega - \omega)R\sin(\Omega - \omega)t \to 0$ at $t = 0$

$\dot{y}(t) = -(\Omega - \omega)R\cos(\Omega - \omega)t \to -(\Omega - \omega)R$ at $t = 0$

(c) $\vec{a} = \vec{A} - \vec{A}_\varepsilon - \dot{\vec{\Omega}} \times \vec{r} - 2\vec{\Omega} \times \dot{\vec{r}} - \vec{\Omega} \times \vec{\Omega} \times \vec{r}$

Where $\vec{a}$ is the acceleration of the asteroid in the x, y frame of reference,

$\vec{A}, \vec{A}_\varepsilon$ are the accelerations of the asteroid and the Earth in the fixed, inertial frame of reference.

1^{st}: examine: $\vec{A} - \vec{A}_\varepsilon - \vec{\Omega}\times\vec{\Omega}\times\vec{r}$

$$= \vec{\omega}\times\vec{\omega}\times\vec{R} - \vec{\Omega}\times\vec{\Omega}\times\vec{R}_\varepsilon - \vec{\Omega}\times\vec{\Omega}\times\left(\vec{R}-\vec{R}_\varepsilon\right)$$

$$= \left(\vec{\omega}\times\vec{\omega} - \vec{\Omega}\times\vec{\Omega}\right)\times\vec{R} = -\left(\omega^2 - \Omega^2\right)\vec{R} \qquad \text{note: } \vec{\omega} = \omega\hat{k}\,,\ \ \vec{\Omega} = \Omega\hat{k}$$

Thus:

$$\vec{a} = \left(\Omega^2 - \omega^2\right)\vec{R} - 2\vec{\Omega}\times\vec{v}$$

Therefore:

$$\left(\hat{i}\ddot{x} + \hat{j}\ddot{y}\right) = \left(\Omega^2 - \omega^2\right)\left[\hat{i}R\cos\left(\Omega-\omega\right)t - \hat{j}R\sin\left(\Omega-\omega\right)t\right] - 2\hat{j}\Omega\dot{x} + 2\hat{i}\Omega\dot{y}$$

Thus:

$$\ddot{x} = \left(\Omega^2 - \omega^2\right)R\cos\left(\Omega-\omega\right)t + 2\Omega\dot{y}$$

$$\ddot{y} = -\left(\Omega^2 - \omega^2\right)R\sin\left(\Omega-\omega\right)t - 2\Omega\dot{x}$$

Let $\ddot{x} = \left(\Omega-\omega\right)\dot{y}$ and $\ddot{y} = \left(\Omega-\omega\right)\dot{x}$

Then, we have

$$\dot{y} = \left(\Omega-\omega\right)R\cos\left(\Omega-\omega\right)t + \frac{2\Omega}{\left(\Omega-\omega\right)}\dot{y} \qquad \text{which reduces to}$$

$$\dot{y} = -\left(\Omega-\omega\right)R\cos\left(\Omega-\omega\right)t$$

Integrating … $\boxed{y = -R\sin\left(\Omega-\omega\right)t} \to 0$ at $t = 0$

Also,

$$-\dot{x}\left(\Omega-\omega\right) = -\left(\Omega^2 - \omega^2\right)R\sin\left(\Omega-\omega\right)t - 2\Omega\dot{x}$$

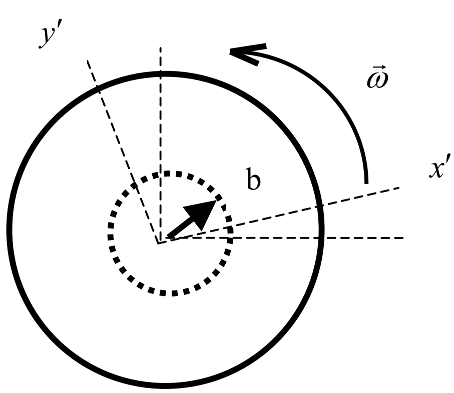
or $\dot{x} = \left(\Omega+\omega\right)R\sin\left(\Omega-\omega\right)t + 2\Omega\dot{x}$

$$\dot{x} = -\left(\Omega-\omega\right)R\sin\left(\Omega-\omega\right)t$$

Integrating … $x = R\cos\left(\Omega-\omega\right)t + const$

$$\boxed{x = R\cos\left(\Omega-\omega\right)t - R_\varepsilon} \to R - R_\varepsilon \quad \text{at} \quad t = 0$$

5.8 Relative to a reference frame fixed to the turntable the cockroach travels at a constant speed v' in a circle. Thus



$$\vec{a}' = -\frac{v'^2}{b}\hat{e}_{r'}\,.$$

Since the center of the turntable is fixed.

$$\vec{A}_\circ = 0$$

The angular velocity, ω, of the turntable is constant, so

$\vec{\omega} = \omega\hat{k}'$, with $\dot{\vec{\omega}} = 0$

$\vec{r}' = b\hat{e}_{r'}$, so $\vec{\omega}\times(\vec{\omega}\times\vec{r}') = -b\omega^2\hat{e}_{r'}$

$\vec{v}' = v'\hat{e}_{\theta'}$, so $\vec{\omega}\times\vec{v}' = -\omega v'\hat{e}_{r'}$

From eqn 5.2.14, $\vec{a} = \vec{a}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$ and putting in terms from above

$$a_{r'} = -\frac{v'^2}{b} - 2\omega v' - b\omega^2$$

For no slipping $\left|\vec{F}\right| \le \mu_s mg$, so $\left|\vec{a}\right| \le \mu_s g$

$$\frac{v'^2}{b} + 2\omega v' + b\omega^2 \le \mu_s g$$

$$v_m'^{\,2} + 2\omega b v_m' + b^2\omega^2 - b\mu_s g = 0$$

$$v_m' = -\omega b \pm \sqrt{\omega^2 b^2 - b^2\omega^2 + b\mu_s g}$$

Since v' was defined positive, the +square root is used.

$$v_m' = -\omega b + \sqrt{b\mu_s g}$$

(b) $\vec{v}' = -v'\hat{e}_{\theta'}$

$$\vec{\omega}\times\vec{v}' = +\omega v'\hat{e}_{r'}$$

$$a_{r'} = -\frac{v'^2}{b} + 2\omega v' - b\omega^2$$

$$\frac{v'^2}{b} - 2\omega v' + b\omega^2 \le \mu_s g$$

$$v_m' = \omega b + \sqrt{b\mu_s g}$$

5.9 As in Example 5.2.2, $\vec{\omega} = \frac{V_\circ}{\rho}\hat{j}'$ and $\vec{A}_\circ = \frac{V_\circ^2}{\rho}\hat{i}'$

For the point at the front of the wheel:

$$\ddot{\vec{r}}' = \frac{V_\circ^2}{b}\hat{j}' \text{ and } \vec{v}' = -V_\circ\hat{k}'$$

$$\dot{\vec{\omega}} = 0$$

$$\vec{\omega}\times\vec{r}' = \frac{V_\circ}{\rho}\hat{k}'\times\left(-b\hat{j}'\right) = \frac{V_\circ b}{\rho}\hat{i}'$$

$$\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right) = \frac{V_\circ}{\rho}\hat{k}'\times\frac{V_\circ b}{\rho}\hat{i}' = \frac{V_\circ^2 b}{\rho}\hat{j}'$$

$$\vec{\omega}\times\vec{v}' = \frac{V_\circ}{\rho}\hat{k}'\times\left(-V_\circ\hat{k}'\right) = 0$$

$$\vec{a} = \ddot{\vec{r}}' + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right) + \vec{A}_\circ = \frac{V_\circ^2}{\rho}\hat{i}' + \left(\frac{V_\circ^2}{b} + \frac{V_\circ^2 b}{\rho^2}\right)\hat{j}'$$

5.10 (See Example 5.3.3)

$$m\omega^2 x' = m\ddot{x}'$$
$$x'(t) = Ae^{\omega t} + Be^{-\omega t}$$
$$\dot{x}'(t) = \omega Ae^{\omega t} - \omega Be^{-\omega t}$$

Boundary Conditions:

$$x^2(0) = \frac{l}{2} = A + B$$
$$\dot{x}'(0) = 0 = \omega(A - B)$$
$$\therefore A = \frac{l}{4} \qquad B = \frac{l}{4}$$

(a) $x'(t) = \frac{l}{2}\cosh\omega t \qquad \dot{x}'(t) = \omega\frac{l}{2}\sinh\omega t$

(b) $x'(T) = \frac{l}{2} + \frac{l}{2} = \frac{l}{2}\cosh\omega T$ when the bead reaches the end of the rod

$\therefore \cosh\omega T = 2$ or $T = \frac{1}{\omega}\cosh^{-1} 2 = \frac{1.317}{\omega}$

(c) $\dot{x}'(T) = \omega\frac{l}{2}\sinh\omega T$

$$= \omega\frac{l}{2}\sinh\left[\cosh^{-1} 2\right] = \omega\frac{l}{2}(1.732) = 0.866\omega l$$

or $\omega\frac{l}{2}\left[\cosh^2\omega T - 1\right]^{\frac{1}{2}} = \sqrt{3}\frac{\omega l}{2} = 0.866\omega l$

5.11 $\vec{v}' = 400\hat{j}'\text{ mph} = 586.67\hat{j}'\ \text{ft}\cdot\text{s}^{-1}$

$$\vec{\omega} = 7.27\times10^{-5}\left(\cos 41^\circ\hat{j}' + \sin 41^\circ\hat{k}'\right)\ \text{s}^{-1}$$
$$\vec{\omega}\times\vec{v}' = -\left(7.27\times10^{-5}\right)(586.67)\left(\sin 41^\circ\right)\hat{i}'\ \text{ft}\cdot\text{s}^{-2}$$
$$\frac{F_{cor}}{F_{grav}} = \frac{\left|-2m\vec{\omega}\times\vec{v}'\right|}{mg}$$
$$= \frac{2\left(7.27\times10^{-5}\right)(586.67)\left(\sin 41^\circ\right)}{32}$$
$$\frac{F_{cor}}{F_{grav}} = 0.0017$$

The Coriolis force is in the $-\vec{\omega}\times\vec{v}'$ direction, i.e., $+\hat{i}'$ or east.

5.12 (See Figure 5.4.3)

$$\vec{\omega} = \omega_{y'}\hat{j}' + \omega_{z'}\hat{k}'$$
$$\vec{v}' = v_{x'}\hat{i}' + v_{y'}\hat{j}'$$

$$\vec{\omega}\times\vec{v}' = -\omega_{z'}v_{y'}\hat{i}' + \omega_{z'}v_{x'}\hat{j}' - \omega_{y'}v_{x'}\hat{k}'$$

$$\left(\vec{\omega}\times\vec{v}'\right)_{horiz} = -\omega_{z'}v'_{y'}\hat{i}' + \omega_{z'}v_{x'}\hat{j}'$$

$$\left|\left(\vec{\omega}\times\vec{v}'\right)_{horiz}\right| = \left(\omega_{z'}^2 v_{y'}^2 + \omega_{z'}^2 v_{x'}^2\right)^{\frac{1}{2}} = \omega_{z'}\left(v_{x'}^2 + v_{y'}^2\right)^{\frac{1}{2}} = \omega_{z'}v'$$

$$\vec{F}_{cor} = -2m\vec{\omega}\times\vec{v}'$$

$$\left|\left(\vec{F}_{cor}\right)_{horiz}\right| = 2m\left|\left(\vec{\omega}\times\vec{v}'\right)_{horiz}\right| = 2m\omega'_z v'$$, independent of the direction of $\vec{v}'$.

5.13 From Example 5.4.1 …

$$x'_h = \frac{1}{3}\omega\left(\frac{8h^3}{g}\right)^{\frac{1}{2}}\cos\lambda \qquad \text{and } y'_h = 0.$$

$$x'_h = \frac{1}{3}\left(7.27\times10^{-5}s^{-1}\right)\left(\frac{8\times1250^3 ft^3}{32 ft\cdot s^{-2}}\right)^{\frac{1}{2}}\cos 41^\circ$$

$x'_h = 0.404\ ft$ to the east.

5.14 From Example 5.4.2:

$\Delta \approx \dfrac{\omega H^2}{v_\circ}\left|\sin\lambda\right|$ is the deflection of the baseball towards the south since it was struck due East at Yankee Stadium at latitude $\lambda = 41^\circ N$ (problem 5.13). v_0 is the initial speed of the baseball whose range is H. From eqn 4.3.18b, without air resistance in an inertial reference frame, the horizontal range is …

$$H = \frac{v_\circ^2\sin 2\alpha}{g}$$

Solving for v_0 … $\quad v_\circ = \left(\dfrac{gH}{\sin 2\alpha}\right)^{\frac{1}{2}}$

$$v_\circ = \left(\frac{32 ft\cdot s^{-2}\times 200 ft}{\sin 30^\circ}\right)^{\frac{1}{2}} = 113 ft\cdot s^{-1}$$

$$\therefore \quad \Delta \approx \frac{\left(7.27\times10^{-5}s^{-1}\right)\left(200^2 ft^2\right)}{113 ft\cdot s^{-1}}\sin 41^\circ = 0.0169\ ft = 0.2\ in$$

A deflection of 0.2 inches should not cause the outfielder any difficulty.

5.15 Equation 5.2.10 gives the relationship between the time derivative of any vector in a fixed and rotating frame of reference. Thus …

$$\dddot{\vec{r}} = \left(\frac{d\vec{a}}{dt}\right)_{fixed} = \left(\frac{d\vec{a}}{dt}\right)_{rot} + \vec{\omega}\times\vec{a}$$

$$\vec{a} = \ddot{\vec{r}}' + \dot{\vec{\omega}}\times\vec{r}' + 2\vec{\omega}\times\dot{\vec{r}}' + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)$$

$$\left(\frac{d\vec{a}}{dt}\right)_{rot} = \dddot{\vec{r}}' + \ddot{\vec{\omega}}\times\vec{r}' + \dot{\vec{\omega}}\times\dot{\vec{r}}' + 2\dot{\vec{\omega}}\times\dot{\vec{r}}' + 2\vec{\omega}\times\ddot{\vec{r}}'$$

$$+\dot{\vec{\omega}}\times\left(\vec{\omega}\times\vec{r}'\right)+\vec{\omega}\times\left(\dot{\vec{\omega}}\times\vec{r}'\right)+\vec{\omega}\times\left(\vec{\omega}\times\dot{\vec{r}}'\right)$$

$$\vec{\omega}\times\vec{a} = \vec{\omega}\times\ddot{\vec{r}}' + \vec{\omega}\times\left(\dot{\vec{\omega}}\times\vec{r}'\right) + 2\vec{\omega}\times\left(\vec{\omega}\times\dot{\vec{r}}'\right) + \vec{\omega}\times\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)\right]$$

Now $\left(\vec{\omega}\times\vec{r}'\right)$ is $\perp$ to $\vec{\omega}$ and $\vec{r}'$. Let this define a direction $\hat{n}$:

$$\vec{\omega}\times\vec{r}' = \left|\vec{\omega}\times\vec{r}'\right|\hat{n}$$

Since $\vec{\omega}\perp\hat{n}$, $\omega\times\left(\vec{\omega}\times\vec{r}'\right)$ is in the plane defined by $\vec{\omega}$ and $\vec{r}'$ and

$$\left|\vec{\omega}\times(\vec{\omega}\times\vec{r}')\right| = \left|\vec{\omega}\times\hat{n}\right|\left|\vec{\omega}\times\vec{r}'\right| = \omega\left|\vec{\omega}\times\vec{r}'\right|.$$

Since $\vec{\omega}\perp\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)$ …

$$\left|\vec{\omega}\times\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)\right]\right| = \omega^2\left|\vec{\omega}\times\vec{r}'\right|$$

And $\vec{\omega}\times\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)\right]$ is in the direction $-\hat{n}$

Thus $\vec{\omega}\times\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)\right] = -\omega^2\left(\vec{\omega}\times\vec{r}'\right)$

$$\vec{\omega}\times\vec{a} = \vec{\omega}\times\ddot{\vec{r}}' + \vec{\omega}\times\left(\dot{\vec{\omega}}\times\vec{r}'\right) + 2\omega\times\left(\vec{\omega}\times\dot{\vec{r}}'\right) - \omega^2\left(\vec{\omega}\times\vec{r}'\right)$$

$$\dddot{\vec{r}} = \dddot{\vec{r}}' + \ddot{\vec{\omega}}\times\vec{r}' + 3\dot{\vec{\omega}}\times\dot{\vec{r}}' + 3\vec{\omega}\times\ddot{\vec{r}}' + \dot{\vec{\omega}}\times\left(\vec{\omega}\times\vec{r}'\right)$$

$$+2\vec{\omega}\times\left(\dot{\vec{\omega}}\times\vec{r}'\right) + 3\vec{\omega}\times\left(\vec{\omega}\times\dot{\vec{r}}'\right) - \omega^2\left(\vec{\omega}\times\vec{r}'\right)$$

5.16 With $x'_\circ = y'_\circ = z'_\circ = \dot{x}'_\circ = \dot{y}'_\circ = 0$, and $\dot{z}'_\circ = v'_\circ$ Equations 5.4.15a – 5.4.15c become:

$$x'(t) = \frac{1}{3}\omega g t^3\cos\lambda - \omega t^2 v'_\circ\cos\lambda$$

$$y'(t) = 0$$

$$z'(t) = -\frac{1}{2}gt^2 + v'_\circ t$$

When the bullet hits the ground $z'(t) = 0$, so

$$t = \frac{2v'_\circ}{g}$$

$$x' = \frac{1}{3}\omega g\left(\frac{8v'^3_\circ}{g^3}\right)\cos\lambda - \omega\left(\frac{4v'^2_\circ}{g^2}\right)v'_\circ\cos\lambda$$

$$x' = -\frac{4\omega v'^3_\circ}{3g^2}\cos\lambda$$

x' is negative and therefore is the distance the bullet lands to the *west*.

5.17 With $x'_\circ = y'_\circ = z'_\circ = 0$ and $\dot{x}'_\circ = v_\circ\cos\alpha$ $\quad\dot{y}'_\circ = 0$ $\quad\dot{z}'_\circ = v_\circ\sin\alpha$

we can solve equation 5.4.15c to find the time it takes the projectile to strike the ground …

$$z'(t) = -\frac{1}{2}gt^2 + v'_\circ t\sin\alpha + \omega v_\circ t^2\cos\alpha\cos\lambda = 0$$

or $$t = \frac{2v'_\circ \sin\alpha}{g - 2\omega v'_\circ \cos\alpha\cos\lambda} \approx \frac{2v'_\circ \sin\alpha}{g}$$

We have ignored the second term in the denominator—since $v'_\circ$ would have to be impossibly large for the value of that term to approach the magnitude g

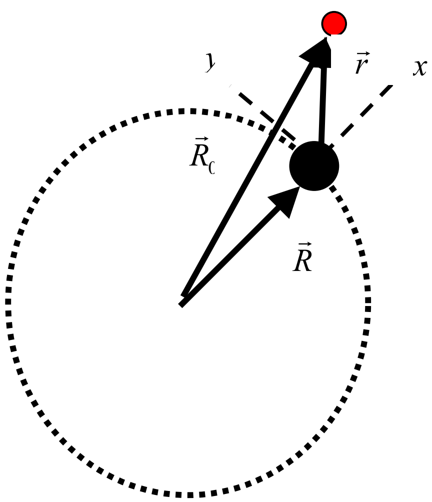
For example, for $\lambda = 41^o$ *and* $\alpha = 45^o$ $\quad g - 2\omega v'_\circ\cos\alpha\cos\lambda \approx g - \omega v'_\circ$

or $$v'_\circ \approx \frac{g}{\omega} \approx 144\frac{km}{s}\,!$$

Substituting t into equation 5.4.15b to find the lateral deflection gives

$$\dot{y}(t) = -[\omega v_\circ \cos\alpha\sin\lambda]t^2 = -\frac{4\omega v'^3_\circ}{g^2}\sin\lambda\sin^2\alpha\cos\alpha$$

5.18 Let …



$\vec{a}_\circ$ = acceleration of object relative to Earth

$\vec{\omega}_\circ = \omega_\circ\hat{k}$ = its angular speed

$\vec{A}_\circ$ = acceleration of satellite

$\vec{\omega} = \omega\hat{k}$ = its angular speed

$$\vec{a}_\circ = \vec{a} + 2\vec{\omega}\times\vec{v} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \vec{A}_\circ \quad \text{(Equation 5.2.14)}$$

$$\vec{a} = \vec{a}_\circ - \vec{A}_\circ - 2\vec{\omega}\times\vec{v} - \vec{\omega}\times\vec{\omega}\times\vec{r}$$

As in problem 5.7 Evaluate the term …

$$\vec{\Delta}_a = \vec{a}_\circ - \vec{A}_\circ - \vec{\omega}\times\vec{\omega}\times\vec{r} = \vec{\omega}_\circ\times\vec{\omega}_\circ\times\vec{R}_\circ - \vec{\omega}\times\vec{\omega}\times\vec{R} - \vec{\omega}\times\vec{\omega}\times(\vec{R}_\circ - \vec{R})$$

$$\vec{\Delta}_a = (\omega_\circ^2 - \omega^2)\vec{R}_\circ$$

… given the condition that $\quad \omega_\circ^2 R_\circ^3 = \omega^2 R^3$

$$\vec{\Delta}_a = -\omega^2\vec{R}_\circ\left(1 - \frac{R^3}{R_\circ^3}\right)$$

but $$\vec{R}_\circ\cdot\vec{R}_\circ = (\vec{R}+\vec{r})\cdot(\vec{R}+\vec{r}) = R^2 + r^2 + 2rR\cos\theta$$

Letting $x = \cos\theta$ $\quad \vec{R}_\circ\cdot\vec{R}_\circ = R^2 + r^2 + 2Rx \approx R^2\left(1+\frac{2x}{R}\right)$ for small r

and $$R_\circ^3 \approx R^3\left(1+\frac{2x}{R}\right)^{\frac{3}{2}} \quad \text{or} \quad \frac{R^3}{R_\circ^3} = \left(1+\frac{2x}{R}\right)^{-\frac{3}{2}}$$

$$\vec{\Delta}_a = -\omega^2\vec{R}_\circ\left[1 - \left(1+\frac{2x}{R}\right)^{-\frac{3}{2}}\right] \approx -3\omega^2 x\frac{\vec{R}_\circ}{R} \approx -3\omega^2 x\hat{i} \text{ for small } r$$

Hence: $\vec{a} = \vec{\Delta}_a - 2\vec{\omega}\times\vec{v} = -3\omega^2 x\hat{i} - 2\omega\hat{k}\times\left(\hat{i}\dot{x} + \hat{j}\dot{y}\right)$

$$\vec{a} = \hat{i}\ddot{x} + \hat{j}\ddot{y} = -3\omega^2 x\hat{i} + 2\omega\dot{y}\hat{i} - 2\omega\dot{x}\hat{j}$$

So $\quad \ddot{x} - 2\omega\dot{y} - 3\omega^2 x = 0$

$$\ddot{y} + 2\omega\dot{x} = 0$$

5.19 $\quad m\ddot{\vec{r}} = q\vec{E} + q\left(\vec{v}\times\vec{B}\right)$

Equation 5.2.14 $\qquad \ddot{\vec{r}} = \ddot{\vec{r}}' + \dot{\vec{\omega}}\times\vec{r}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)$

Equation 5.2.13 $\qquad \vec{v} = \vec{v}' + \vec{\omega}\times\vec{r}'$

$\vec{\omega} = -\dfrac{q}{2m}\vec{B}$ so $\dot{\vec{\omega}} = 0$

$$m\ddot{\vec{r}}' - q\left(\vec{B}\times\vec{v}'\right) - \frac{q}{2}\vec{B}\times\left(\vec{\omega}\times\vec{r}'\right) = q\vec{E} + q\left[\left(\vec{v}' + \vec{\omega}\times\vec{r}'\right)\times\vec{B}\right]$$

$$m\ddot{\vec{r}}' + q\left(\vec{v}'\times\vec{B}\right) + \frac{q}{2}\left(\vec{\omega}\times\vec{r}'\right)\times\vec{B} = q\vec{E} + q\left(\vec{v}'\times\vec{B}\right) + q\left(\vec{\omega}\times\vec{r}'\right)\times\vec{B}$$

$$m\ddot{\vec{r}}' = q\vec{E} + \frac{q}{2}\left(\vec{\omega}\times\vec{r}'\right)\times\vec{B}$$

$$\left|\frac{q}{2}\left(\vec{\omega}\times\vec{r}'\right)\times\vec{B}\right| = \frac{q}{2}\left(\frac{qB}{2m}\right)\left(r'\right)\left(\sin\theta\right)\left(B\right) \propto B^2$$

Neglecting terms in B^2, $m\ddot{\vec{r}}' = q\vec{E}$

5.20 $\quad$ For $x' = x\cos\omega' t + y\sin\omega' t$

$$y' = -x\sin\omega' t + y\cos\omega' t$$

$$\dot{x}' = \dot{x}\cos\omega' t - x\omega'\sin\omega' t + \dot{y}\sin\omega' t + y\omega'\cos\omega' t$$

$$\dot{y}' = -\dot{x}\sin\omega' t - x\omega'\cos\omega' t + \dot{y}\cos\omega' t - y\omega'\sin\omega' t$$

$$\dot{x}' = \dot{x}\cos\omega' t + \dot{y}\sin\omega' t + \omega' y'$$

$$\dot{y}' = -\dot{x}\sin\omega' t + \dot{y}\cos\omega' t - \omega' x'$$

$$\ddot{x}' = \ddot{x}\cos\omega' t - \dot{x}\omega'\sin\omega' t + \ddot{y}\sin\omega' t + \dot{y}\omega'\cos\omega' t + \omega'\dot{y}'$$

$$\ddot{y}' = -\ddot{x}\sin\omega' t - \dot{x}\omega'\cos\omega' t + \ddot{y}\cos\omega' t - \dot{y}\omega'\sin\omega' t - \omega'\dot{x}'$$

$$\ddot{x}' = \ddot{x}\cos\omega' t + \ddot{y}\sin\omega' t + 2\omega'\dot{y}' + \omega'^2 x'$$

$$\ddot{y}' = -\ddot{x}\sin\omega' t + \ddot{y}\cos\omega' t - 2\omega'\dot{x}' + \omega'^2 y'$$

Substituting into Eqns 5.6.3:

$$\ddot{x}\cos\omega' t + \ddot{y}\sin\omega' t + 2\omega'\dot{y}' + \omega'^2 x'$$

$$= -\frac{g}{l}x\cos\omega' t - \frac{g}{l}y\sin\omega' t + 2\omega'\dot{y}'$$

$$\ddot{x}\sin\omega' t + \ddot{y}\cos\omega' t - 2\omega'\dot{x}' + \omega'^2 y'$$

$$= +\frac{g}{l}x\sin\omega' t - \frac{g}{l}y\cos\omega' t - 2\omega'\dot{x}'$$

Collecting terms and neglecting terms in ω'^2:

$$\left(\ddot{x}+\frac{g}{l}x\right)\cos\omega' t+\left(\ddot{y}+\frac{g}{l}y\right)\sin\omega' t=0$$

$$\left(\ddot{x}+\frac{g}{l}x\right)\sin\omega' t-\left(\ddot{y}+\frac{g}{l}y\right)\cos\omega' t=0$$

5.21 $T=\dfrac{24}{\sin\lambda}$ hours

$$T=\frac{24}{\sin 19^\circ}=73.7\ \text{hours}$$

5.22 Choose a coordinate system with the origin at the center of the wheel, the x' and y' axes pointing toward fixed points on the rim of the wheel, and the z' axis pointing toward the center of curvature of the track. Take the initial position of the x' axis to be horizontal in the $-\vec{V}_\circ$ direction, so the initial position of the y' axis is vertical.

The bicycle wheel is rotating with angular velocity $\frac{V_\circ}{b}$ about its axis, so …

$$\vec{\omega}_1=\hat{k}'\frac{V_\circ}{b}$$

A unit vector in the vertical direction is:

$$\hat{n}=\hat{i}'\sin\frac{V_\circ t}{b}+\hat{j}'\cos\frac{V_\circ t}{b}$$

At the instant a point on the rim of the wheel reaches its highest point:

$$\vec{r}'=b\hat{n}=b\left(\hat{i}'\sin\frac{V_\circ t}{b}+\hat{j}'\cos\frac{V_\circ t}{b}\right)$$

Since the coordinate system is moving with the wheel, every point on the rim is fixed in that coordinate system.

$$\dot{\vec{r}}'=0 \quad\text{and}\quad \ddot{\vec{r}}'=0$$

The $x'y'z'$ coordinate system also rotates as the bicycle wheel completes a circle around the track:

$$\vec{\omega}_2=\hat{n}\frac{V_\circ}{\rho}=\frac{V_\circ}{\rho}\left(\hat{i}'\sin\frac{V_\circ t}{b}+\hat{j}'\cos\frac{V_\circ t}{b}\right)$$

The total rotation of the coordinate axes is represented by:

$$\vec{\omega}=\vec{\omega}_1+\vec{\omega}_2=\frac{V_\circ}{\rho}\left(\hat{i}'\sin\frac{V_\circ t}{b}+\hat{j}'\cos\frac{V_\circ t}{b}\right)+\hat{k}'\frac{V_\circ}{b}$$

$$\dot{\vec{\omega}}=\frac{V_\circ^2}{\rho b}\left(\hat{i}'\cos\frac{V_\circ t}{b}-\hat{j}'\sin\frac{V_\circ t}{b}\right)$$

$$\dot{\vec{\omega}}\times\vec{r}'=\frac{V_\circ^2}{\rho}\left(\hat{k}'\cos^2\frac{V_\circ t}{b}+\hat{k}'\sin^2\frac{V_\circ t}{b}\right)=\frac{V_\circ^2}{\rho}\hat{k}'$$

$$\vec{\omega}\times\dot{\vec{r}}'=0$$

$$\vec{\omega}\times\vec{r}'=\frac{V_\circ b}{\rho}\left(\hat{k}'\sin\frac{V_\circ t}{b}\cos\frac{V_\circ t}{b}-\hat{k}'\sin\frac{V_\circ t}{b}\cos\frac{V_\circ t}{b}\right)+V_\circ\left(\hat{j}'\sin\frac{V_\circ t}{b}-\hat{i}'\cos\frac{V_\circ t}{b}\right)$$

$$\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)=\frac{V_\circ^2}{\rho}\left(\hat{k}'\sin^2\frac{V_\circ t}{b}+\hat{k}'\cos^2\frac{V_\circ t}{b}\right)+\frac{V_\circ^2}{b}\left(-\hat{i}'\sin\frac{V_\circ t}{b}-\hat{j}'\cos\frac{V_\circ t}{b}\right)$$

$$\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)=\hat{k}'\frac{V_\circ^2}{\rho}-\hat{n}\frac{V_\circ^2}{b}$$

Since the origin of the coordinate system is traveling in a circle of radius ρ:

$$\vec{A}_\circ=\hat{k}'\frac{V_\circ^2}{\rho}$$

$$\ddot{\vec{r}}=\ddot{\vec{r}}'+\dot{\vec{\omega}}\times\vec{r}'+2\vec{\omega}\times\dot{\vec{r}}'++\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)+\vec{A}_\circ$$

$$\ddot{\vec{r}}=\hat{k}'\frac{V_\circ^2}{\rho}+\hat{k}'\frac{V_\circ^2}{\rho}-\hat{n}\frac{V_\circ^2}{b}+\hat{k}'\frac{V_\circ^2}{\rho}$$

$$\ddot{\vec{r}}=3\frac{V_\circ^2}{\rho}\hat{k}'-\frac{V_\circ^2}{b}\hat{n}$$

With appropriate change in coordinate notation, this is the same result as obtained in Example 5.2.2.
