

CHAPTER 1

FUNDAMENTAL CONCEPTS: VECTORS

1.1 (a) $\vec{A} + \vec{B} = (\hat{i} + \hat{j}) + (\hat{j} + \hat{k}) = \hat{i} + 2\hat{j} + \hat{k}$

$$|\vec{A} + \vec{B}| = (1+4+1)^{\frac{1}{2}} = \sqrt{6}$$

(b) $3\vec{A} - 2\vec{B} = 3(\hat{i} + \hat{j}) - 2(\hat{j} + \hat{k}) = 3\hat{i} + \hat{j} - 2\hat{k}$

(c) $\vec{A} \cdot \vec{B} = (1)(0) + (1)(1) + (0)(1) = 1$

(d) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i}(1-0) + \hat{j}(0-1) + \hat{k}(1-0) = \hat{i} - \hat{j} + \hat{k}$

$$|\vec{A} \times \vec{B}| = (1+1+1)^{\frac{1}{2}} = \sqrt{3}$$

1.2 (a) $\vec{A} \cdot (\vec{B} + \vec{C}) = (2\hat{i} + \hat{j}) \cdot (\hat{i} + 4\hat{j} + \hat{k}) = (2)(1) + (1)(4) + (0)(1) = 6$

$$(\vec{A} + \vec{B}) \cdot \vec{C} = (3\hat{i} + \hat{j} + \hat{k}) \cdot 4\hat{j} = (3)(0) + (1)(4) + (1)(0) = 4$$

(b) $\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{vmatrix} = -8$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = \vec{A} \cdot (\vec{B} \times \vec{C}) = -8$$

(c) $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} = 4(\hat{i} + \hat{k}) - 2(4\hat{j}) = 4\hat{i} - 8\hat{j} + 4\hat{k}$

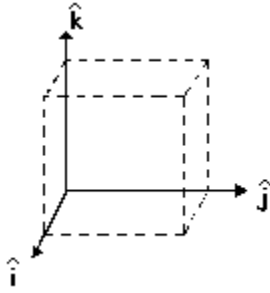
$$(\vec{A} \times \vec{B}) \times \vec{C} = -\vec{C} \times (\vec{A} \times \vec{B}) = -[(\vec{C} \cdot \vec{B})\vec{A} - (\vec{C} \cdot \vec{A})\vec{B}]$$

$$= -[0(2\hat{i} + \hat{j}) - 4(\hat{i} + \hat{k})] = 4\hat{i} + 4\hat{k}$$

$$1.3 \quad \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{(a)(a) + (2a)(2a) + (0)(3a)}{\sqrt{5a^2} \sqrt{14a^2}} = \frac{5a^2}{a^2 \sqrt{5} \sqrt{14}}$$

$$\theta = \cos^{-1} \sqrt{\frac{5}{14}} \approx 53^\circ$$

1.4



(a) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$: body diagonal

$$A = |\vec{A} \cdot \vec{A}| = \sqrt{\hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j} + \hat{k} \cdot \hat{k}} = \sqrt{3}$$

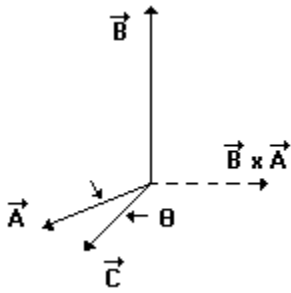
(b) $\vec{B} = \hat{i} + \hat{j}$: face diagonal

$$B = |\vec{B} \cdot \vec{B}| = \sqrt{2}$$

$$(c) \quad \vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$(d) \quad \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{1-1}{\sqrt{3}\sqrt{2}} = 0 \quad \therefore \theta = 90^\circ$$

1.5



$$B = |\vec{B}| = |\vec{A} \times \vec{C}| = AC \sin \theta \quad \therefore C_y = C \sin \theta = \frac{B}{A}$$

$$\vec{A} \cdot \vec{C} = AC \cos \theta = u \quad \therefore C_x = C \cos \theta = \frac{u}{A}$$

$$\begin{aligned} \vec{C} &= \frac{\vec{A}}{A} C_x + \frac{\vec{B} \times \vec{A}}{|\vec{B} \times \vec{A}|} C_y = \frac{u}{A^2} \vec{A} + \frac{\vec{B} \times \vec{A}}{AB} \left(\frac{B}{A} \right) \\ &= \frac{u}{A^2} \vec{A} + \frac{1}{A^2} \vec{B} \times \vec{A} \end{aligned}$$

$$1.6 \quad \frac{d\vec{A}}{dt} = \hat{i} \frac{d}{dt}(\alpha t) + \hat{j} \frac{d}{dt}(\beta t^2) + \hat{k} \frac{d}{dt}(\gamma t^3) = \hat{i} \alpha + \hat{j} 2\beta t + \hat{k} 3\gamma t^2$$

$$\frac{d^2 \vec{A}}{dt^2} = \hat{j} 2\beta + \hat{k} 6\gamma t$$

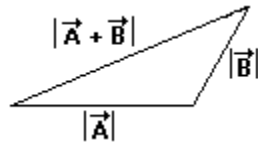
$$1.7 \quad 0 = \vec{A} \cdot \vec{B} = (q)(q) + (3)(-q) + (1)(2) = q^2 - 3q + 2$$

$$(q-2)(q-1) = 0, \quad q = 1 \text{ or } 2$$

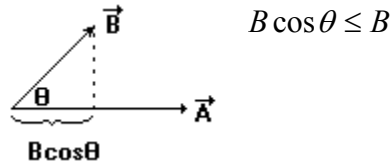
$$1.8 \quad |\vec{A} + \vec{B}|^2 = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = A^2 + B^2 + 2\vec{A} \cdot \vec{B}$$

$$\left[|\vec{A}| + |\vec{B}| \right]^2 = A^2 + B^2 + 2AB$$

$$\text{Since } \vec{A} \cdot \vec{B} = AB \cos \theta \leq AB, \quad |\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}|$$



$$|\vec{A} \cdot \vec{B}| = |AB \cos \theta| = |\vec{A}| |\vec{B}| |\cos \theta| \leq |\vec{A}| |\vec{B}|$$



$$1.9 \quad \text{Show } \vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$$

$$\text{or } \vec{A} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = (A_x C_x + A_y C_y + A_z C_z) \vec{B} - (A_x B_x + A_y B_y + A_z B_z) \vec{C}$$

$$= (A_x B_x C_x + A_y B_x C_y + A_z B_x C_z - A_x B_x C_x - A_y B_y C_x - A_z B_z C_x) \hat{i}$$

$$+ (A_x B_y C_x + A_y B_y C_y + A_z B_y C_z - A_x B_x C_y - A_y B_y C_y - A_z B_z C_y) \hat{j}$$

$$+ (A_x B_z C_x + A_y B_z C_y + A_z B_z C_z - A_x B_x C_z - A_y B_y C_z - A_z B_z C_z) \hat{k}$$

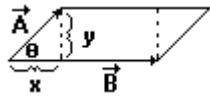
$$= (A_y B_x C_y + A_z B_x C_z - A_y B_y C_x - A_z B_z C_x) \hat{i}$$

$$+ (A_x B_y C_x + A_z B_y C_z - A_x B_x C_y - A_z B_z C_y) \hat{j}$$

$$+ (A_x B_z C_x + A_y B_z C_y - A_x B_x C_z - A_y B_y C_z) \hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_y C_z - B_z C_y & B_z C_x - B_x C_z & B_x C_y - B_y C_x \end{vmatrix} = \hat{i}(A_y B_x C_y - A_y B_y C_x - A_z B_z C_x + A_z B_x C_z) \\ + \hat{j}(A_z B_y C_z - A_z B_z C_y - A_x B_x C_y + A_x B_y C_x) \\ + \hat{k}(A_x B_z C_x - A_x B_x C_z - A_y B_y C_z + A_y B_z C_y)$$

1.10



$$y = A \sin \theta$$

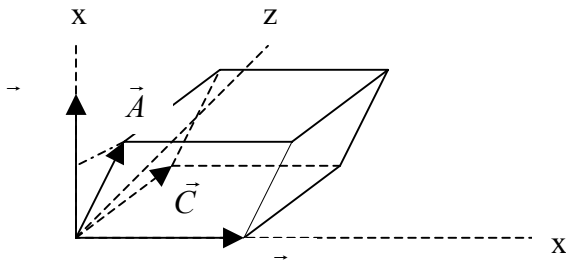
$$A = 2 \left(\frac{1}{2} xy \right) + y(B - x) = xy + yB - xy = AB \sin \theta$$

$$A = |\vec{A} \times \vec{B}|$$

1.11

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{A} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = - \begin{vmatrix} B_x & B_y & B_z \\ A_x & A_y & A_z \\ C_x & C_y & C_z \end{vmatrix} = -\vec{B} \cdot (\vec{A} \times \vec{C})$$

1.12



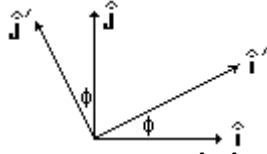
Let $\vec{A} = (A_x, A_y, A_z)$, $\vec{B} = (0, B_y, 0)$ and $\vec{C} = (0, C_y, C_z)$

C_z is the perpendicular distance between the plane \vec{A}, \vec{B} and its opposite. $\vec{u} = \vec{B} \times \vec{C}$ is directed along the x-axis since the vectors \vec{B}, \vec{C} are in the y,z plane. $u_x = |\vec{B} \times \vec{C}| = B_y C_z$

is the area of the parallelogram formed by the vectors \vec{B}, \vec{C} . Multiply that area times the height of plane $\vec{A}, \vec{B} = A_x$ to get the volume of the parallelepiped

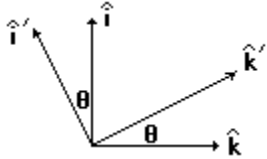
$$V = A_x u_x = A_x B_y C_z = \vec{A} \cdot (\vec{B} \times \vec{C})$$

1.13 For rotation about the z axis:



$$\begin{aligned}\hat{i} \cdot \hat{i}' &= \cos \phi = \hat{j} \cdot \hat{j}', \quad \hat{k} \cdot \hat{k}' = 1 \\ \hat{i} \cdot \hat{j}' &= -\sin \phi \\ \hat{j} \cdot \hat{i}' &= \sin \phi\end{aligned}$$

For rotation about the y' axis:



$$\begin{aligned}\hat{i} \cdot \hat{i}' &= \cos \theta = \hat{k} \cdot \hat{k}', \quad \hat{j} \cdot \hat{j}' = 1 \\ \hat{i} \cdot \hat{k}' &= \sin \theta \\ \hat{k} \cdot \hat{i}' &= -\sin \theta\end{aligned}$$

$$\vec{T} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{pmatrix}$$

1.14

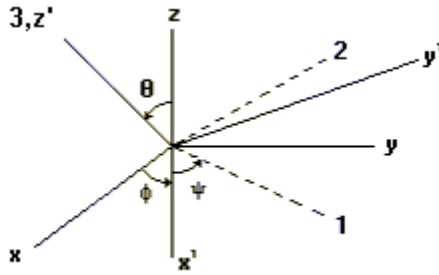
$$\begin{aligned}\hat{i} \cdot \hat{i}' &= \cos 30^\circ = \frac{\sqrt{3}}{2} & \hat{j} \cdot \hat{i}' &= \sin 30^\circ = \frac{1}{2} & \hat{k} \cdot \hat{i}' &= 0 \\ \hat{i} \cdot \hat{j}' &= -\sin 30^\circ = -\frac{1}{2} & \hat{j} \cdot \hat{j}' &= \cos 30^\circ = \frac{\sqrt{3}}{2} & \hat{k} \cdot \hat{j}' &= 0\end{aligned}$$

$$\begin{aligned}\hat{i} \cdot \hat{k}' &= 0 & \hat{j} \cdot \hat{k}' &= 0 & \hat{k} \cdot \hat{k}' &= 1\end{aligned}$$

$$\begin{bmatrix} A_{x'} \\ A_{y'} \\ A_{z'} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} + \frac{3}{2} \\ \frac{3}{2}\sqrt{3} - 1 \\ -1 \end{bmatrix}$$

$$\vec{A} = 3.232\hat{i}' + 1.598\hat{j}' - \hat{k}'$$

- 1.15**
- | | | | |
|----|---------------------------------------|---------------------|------------|
| 1. | Rotate thru ϕ about z-axis | $\phi = 45^\circ$ | R_ϕ |
| 2. | Rotate thru θ about x' -axis | $\theta = 45^\circ$ | R_θ |
| 3. | Rotate thru ψ about z' -axis | $\psi = 45^\circ$ | R_ψ |



$$R_\phi = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad R_\psi = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(\psi, \theta, \phi) = R_\psi R_\theta R_\phi = \begin{pmatrix} \frac{1}{2} - \frac{1}{2\sqrt{2}} & \frac{1}{2} + \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} - \frac{1}{2\sqrt{2}} & -\frac{1}{2} + \frac{1}{2\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = R(\psi, \theta, \phi) \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

Condition is: $\bar{x}' = R\bar{x}$ where $\bar{x}' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\bar{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

Since $\bar{x} \cdot \bar{x} = 1$ we have: $\psi^2 + \beta^2 + \alpha^2 = 1$

After a lot of algebra: $\alpha = \frac{1}{2} - \frac{\sqrt{2}}{4}$, $\beta = \frac{1}{2} + \frac{\sqrt{2}}{4}$, $\gamma = \frac{1}{2}$

1.16 $\bar{v} = v\hat{t} = ct\hat{t}$

$$\bar{a} = v\hat{t} + \frac{v^2}{\rho}\hat{n} = c\hat{t} + \frac{c^2 t^2}{b}\hat{n}$$

$$\text{at } t = \sqrt{\frac{b}{c}}, \quad \bar{v} = \hat{t}\sqrt{bc} \quad \text{and} \quad \bar{a} = c\hat{t} + c\hat{n}$$

$$\cos\theta = \frac{\bar{v} \cdot \bar{a}}{va} = \frac{c\sqrt{bc}}{\sqrt{bc}\sqrt{2c^2}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$\mathbf{1.17} \quad \bar{v}(t) = -\hat{i}b\omega \sin(\omega t) + \hat{j}2b\omega \cos(\omega t)$$

$$|\bar{v}| = (b^2\omega^2 \sin^2 \omega t + 4b^2\omega^2 \cos^2 \omega t)^{\frac{1}{2}} = b\omega(1 + 3\cos^2 \omega t)^{\frac{1}{2}}$$

$$\bar{a}(t) = -\hat{i}b\omega^2 \cos \omega t - \hat{j}2b\omega^2 \sin \omega t$$

$$|\bar{a}| = b\omega^2(1 + 3\sin^2 \omega t)^{\frac{1}{2}}$$

$$\text{at } t = 0, \quad |\bar{v}| = 2b\omega; \quad \text{at } t = \frac{\pi}{2\omega}, \quad |\bar{v}| = b\omega$$

$$\mathbf{1.18} \quad \bar{v}(t) = \hat{i}b\omega \cos \omega t - \hat{j}b\omega \sin \omega t + \hat{k}2ct$$

$$\bar{a}(t) = -\hat{i}b\omega^2 \sin \omega t - \hat{j}b\omega^2 \cos \omega t + \hat{k}2c$$

$$|\bar{a}| = (b^2\omega^4 \sin^2 \omega t + b^2\omega^4 \cos^2 \omega t + 4c^2)^{\frac{1}{2}} = (b^2\omega^4 + 4c^2)^{\frac{1}{2}}$$

$$\mathbf{1.19} \quad \bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = bke^{kt}\hat{e}_r + bce^{kt}\hat{e}_\theta$$

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = b(k^2 - c^2)e^{kt}\hat{e}_r + 2bcke^{kt}\hat{e}_\theta$$

$$\cos\phi = \frac{\bar{v} \cdot \bar{a}}{va} = \frac{b^2k(k^2 - c^2)e^{2kt} + 2b^2c^2ke^{2kt}}{be^{kt}(k^2 + c^2)^{\frac{1}{2}}be^{kt}\left[(k^2 - c^2)^2 + 4c^2k^2\right]^{\frac{1}{2}}}$$

$$\cos\phi = \frac{k(k^2 + c^2)}{(k^2 + c^2)^{\frac{1}{2}}(k^2 + c^2)} = \frac{k}{(k^2 + c^2)^{\frac{1}{2}}}, \quad \text{a constant}$$

$$\mathbf{1.20} \quad \bar{a} = (\ddot{R} - R\dot{\phi})\hat{e}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi})\hat{e}_\phi + \ddot{z}\hat{e}_z$$

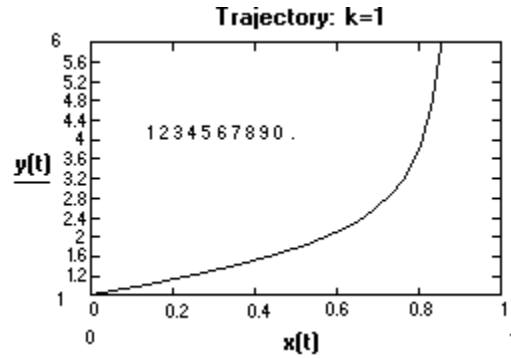
$$\bar{a} = -b\omega^2\hat{e}_R + 2c\hat{e}_z$$

$$|\bar{a}| = (b^2\omega^4 + 4c^2)^{\frac{1}{2}}$$

$$1.21 \quad \vec{r}(t) = \hat{i}(1 - e^{-kt}) + \hat{j}e^{kt}$$

$$\vec{r}(t) = \hat{i}ke^{-kt} + \hat{j}ke^{kt}$$

$$\vec{r}(t) = -\hat{i}k^2e^{-kt} + \hat{j}k^2e^{kt}$$



$$1.22 \quad \vec{v} = \hat{e}_r \dot{r} + \hat{e}_\phi r \dot{\phi} \sin \theta + \hat{e}_\theta r \dot{\theta}$$

$$\vec{v} = \hat{e}_\phi b \omega \sin \left\{ \frac{\pi}{2} \left[1 + \frac{1}{4} \cos(4\omega t) \right] \right\} - \hat{e}_\theta b \frac{\pi}{2} \omega \sin(4\omega t)$$

$$\vec{v} = \hat{e}_\phi b \omega \cos \left[\frac{\pi}{8} \cos(4\omega t) \right] - \hat{e}_\theta b \omega \frac{\pi}{2} \sin(4\omega t)$$

$$|\vec{v}| = b\omega \left[\cos^2 \left(\frac{\pi}{8} \cos 4\omega t \right) + \frac{\pi^2}{4} \sin^2 4\omega t \right]^{\frac{1}{2}}$$

Path is sinusoidal oscillation about the equator.

$$1.23 \quad \vec{v} \cdot \vec{v} = v^2$$

$$\frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt} = 2v\dot{v}$$

$$2\vec{v} \cdot \vec{a} = 2v\dot{v}$$

$$\vec{v} \cdot \vec{a} = v\dot{v}$$

$$\begin{aligned}
1.24 \quad \frac{d}{dt} [\bar{r} \cdot (\bar{v} \times \bar{a})] &= \frac{d\bar{r}}{dt} \cdot (\bar{v} \times \bar{a}) + \bar{r} \cdot \frac{d}{dt} (\bar{v} \times \bar{a}) \\
&= \bar{v} \cdot (\bar{v} \times \bar{a}) + \bar{r} \cdot \left[\left(\frac{d\bar{v}}{dt} \times \bar{a} \right) + \left(\bar{v} \times \frac{d\bar{a}}{dt} \right) \right] \\
&= 0 + \bar{r} \cdot [0 + (\bar{v} \times \dot{\bar{a}})] \\
\frac{d}{dt} [\bar{r} \cdot (\bar{v} \times \bar{a})] &= \bar{r} \cdot (\bar{v} \times \dot{\bar{a}})
\end{aligned}$$

$$1.25 \quad \bar{v} = v\hat{\tau} \quad \text{and} \quad \bar{a} = a_\tau\hat{\tau} + a_n\hat{n}$$

$$\bar{v} \cdot \bar{a} = va_\tau, \quad \text{so} \quad a_\tau = \frac{\bar{v} \cdot \bar{a}}{v}$$

$$a^2 = a_\tau^2 + a_n^2, \quad \text{so} \quad a_n = (a^2 - a_\tau^2)^{\frac{1}{2}}$$

$$1.26 \quad \text{For 1.14, } a_\tau = \frac{-b^2\omega^3 \cos \omega t \cdot \sin \omega t + b^2\omega^3 \sin \omega t \cdot \cos \omega t + 4c^2t}{(b^2\omega^2 \cos^2 \omega t + b^2\omega^2 \sin^2 \omega t + 4c^2t^2)^{\frac{1}{2}}}$$

$$a_\tau = \frac{4c^2t}{(b^2\omega^2 + 4c^2t^2)^{\frac{1}{2}}}$$

$$a_n = \left(b^2\omega^2 + 4c^2 - \frac{16c^4t^2}{b^2\omega^2 + 4c^2t^2} \right)^{\frac{1}{2}}$$

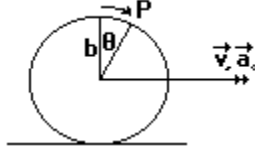
$$\text{For 1.15, } a_\tau = \frac{b^2k(k^2 - c^2)e^{2kt} + 2b^2c^2ke^{2kt}}{be^{kt}(k^2 + c^2)^{\frac{1}{2}}} = bke^{kt}(k^2 + c^2)^{\frac{1}{2}}$$

$$a_n = \left[b^2e^{2kt}(k^2 + c^2)^2 - b^2k^2e^{2kt}(k^2 + c^2) \right]^{\frac{1}{2}} = bce^{kt}(k^2 + c^2)^{\frac{1}{2}}$$

$$1.27 \quad \bar{v} = v\hat{\tau}, \quad \bar{a} = v\dot{\hat{\tau}} + \frac{v^2}{\rho}\hat{n}$$

$$|\bar{v} \times \bar{a}| = v \cdot a_n = v \frac{v^2}{\rho} = \frac{v^3}{\rho}$$

1.28



$$\vec{r}_{oP} = \hat{i}b \sin \theta + \hat{j}b \cos \theta$$

$$\vec{v}_{rel} = \hat{i}b\dot{\theta} \cos \theta - \hat{j}b\dot{\theta} \sin \theta$$

$$\vec{a}_{rel} = \hat{i}b(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) - \hat{j}b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)$$

at the point $\theta = \frac{\pi}{2}$, $\vec{v}_{rel} = -\vec{v}$

So, $|\vec{v}_{rel}| = b\dot{\theta} = v$

$$\dot{\theta} = \frac{v}{b} \quad \ddot{\theta} = \frac{\dot{v}}{b} = \frac{a_o}{b}$$

Now, $\vec{a}_{rel} = \dot{v}_{rel}\hat{t} + \frac{v_{rel}^2}{\rho}\hat{n} = a_o\hat{t} + \frac{v^2}{b}\hat{n}$

$$|\vec{a}_{rel}| = \left(a_o^2 + \frac{v^4}{b^2} \right)^{\frac{1}{2}}$$

$\vec{v}_P = \vec{v} + \vec{v}_{rel}$ and $\vec{a}_P = \vec{a}_o + \vec{a}_{rel}$

$$\vec{a}_P = \hat{i} \left[a_o + b \left(\frac{a_o}{b} \cos \theta - \frac{v^2}{b^2} \sin \theta \right) \right] - \hat{j}b \left(\frac{a_o}{b} \sin \theta + \frac{v^2}{b^2} \cos \theta \right)$$

$$|\vec{a}_P| = a_o \left(2 + 2 \cos \theta + \frac{v^4}{a_o^2 b^2} - \frac{2v^2}{a_o b} \sin \theta \right)^{\frac{1}{2}}$$

\vec{a}_P is a maximum at $\theta = 0$, i.e., at the top of the wheel.

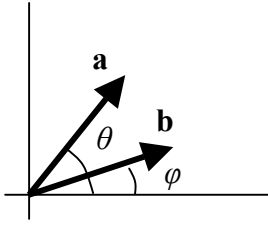
$$-2 \sin \theta - \frac{2v^2}{a_o b} \cos \theta = 0$$

$$\theta = \tan^{-1} \left(-\frac{v^2}{a_o b} \right)$$

1.29 $\tilde{R}R = \begin{pmatrix} x & -x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x & x & 0 \\ -x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 2x^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Therefore, $x = \frac{1}{\sqrt{2}}$

The transformation represents a rotation of 45° about the z-axis (see Example 1.8.2)

1.30



$$\begin{aligned}
 \text{(a)} \quad a &= \hat{i} \cos \theta + \hat{j} \sin \theta \\
 b &= \hat{i} \cos \varphi + \hat{j} \sin \varphi \\
 a \cdot b &= \cos(\theta - \varphi) = (\hat{i} \cos \theta + \hat{j} \sin \theta) \cdot (\hat{i} \cos \varphi + \hat{j} \sin \varphi) \\
 \cos(\theta - \varphi) &= \cos \theta \cos \varphi + \sin \theta \sin \varphi
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad b \times a &= |\hat{k}| \sin(\theta - \varphi) = \left| (\hat{i} \cos \theta + \hat{j} \sin \theta) \times (\hat{i} \cos \varphi + \hat{j} \sin \varphi) \right| \\
 \sin(\theta - \varphi) &= \sin \theta \cos \varphi - \cos \theta \sin \varphi
 \end{aligned}$$
