Quick Study® ACADEMIC

TRIGONOMETRY

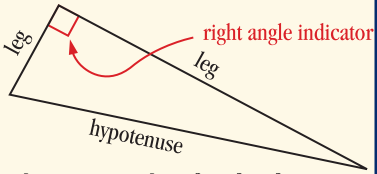
triangles.circles.trigonomic functions.sines

Trigonometry (trig) means measurement of triangles; it is usually studied as measurements of sides and angles of triangles and as points on a unit circle; this study guide is basically separated into these two main sections: trig with triangles and trig with a unit circle; in trigonometry, the measures of angles are usually represented by letters from the Greek alphabet; the Greek letters θ, α, ν, and β will be used throughout this study guide to represent angle measures

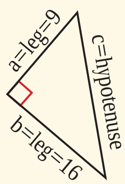
TRIG WITH TRIANGLES

A. Right Triangle

1. A **right triangle** is a triangle with exactly one 90º (right) angle
2. The **hypotenuse** is the longest side of a right triangle, and is always located opposite the right (90º) angle
3. The two shorter sides of a right triangle are both called **legs**

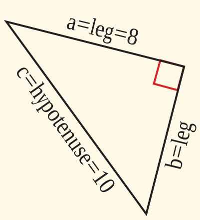


4. The **Pythagorean Theorem** ($leg^2 + leg^2 = hypotenuse^2$ or $\mathbf{a^2 + b^2 = c^2}$ where **a** and **b** are leg lengths and **c** is the hypotenuse length) may be used to find the length of any third side of a right triangle when any two side lengths are known
   a. When the two leg lengths are known, square the length of each leg, add these two squares together and square root the resulting sum; for example:



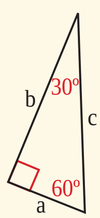
$$a^2 + b^2 = c^2$$
$$9^2 + 16^2 = c^2$$
$$81 + 256 = c^2$$
$$337 = c^2$$
$$\sqrt{337} = c$$
$$18.36 \doteq c = \text{hypotenuse}$$

   b. When the length of the hypotenuse and either leg are known, square the length of the hypotenuse, square the length of the leg, subtract these two squares, and square root the resulting difference; for example:



$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$
$$b^2 = 100 - 64$$
$$b^2 = 36$$
$$b = \sqrt{36}$$
$$\text{leg} = b = 6$$

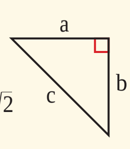
5. **Special right triangles** exist that are used so often that the relationships of the side lengths should be memorized
   a. 30º-60º-90º triangles have side lengths with ratios of $1:\sqrt{3}:2$; that is, the longest leg is always $\sqrt{3}$ times the length of the shortest leg, and the hypotenuse is always 2 times the shortest leg. This relationship can be used to find all side lengths when given only one side length; for example:



c = hypotenuse = 2x
b = longest leg = $x\sqrt{3}$
a = shortest leg = x
If a = 5 then c = 2 • 5 = 10
and b = 5 • $\sqrt{3}$ = 8.7
If b = 8 then a = 8 $\surd$ $\sqrt{3}$ = 4.6
and c = 2 • 8 $\surd$ $\sqrt{3}$ = 9.2

   b. 45º-45º-90º triangles have side lengths with ratios of $1:1:\sqrt{2}$; that is, the two legs have the same length (if two angles of a right triangle are equal, then the two legs are equal), and the hypotenuse is $\sqrt{2}$ times the length of either leg; for example:

a = leg = x
b = leg = x
c = hypotenuse = $x\sqrt{2}$
If a = 7 then b = 7
and c = 7 • $\sqrt{2}$ = 9.9
If c = 12 then a = 12 $\surd$ $\sqrt{2}$ = 8.5
and b = 8.5



B. Right Triangle Trigonometry

1. The **trigonometric (trig) functions** of an angle are related to the ratios of the sides of a right triangle
2. The trig functions are defined in the following manner where θ stands for either of the acute (less than 90º) angles in the right triangle; these definitions should be memorized:

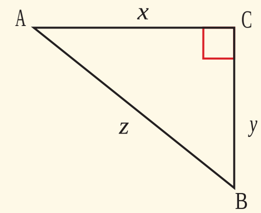
$$\text{sine } \theta = \sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad \text{cosecant } \theta = \csc\theta = \frac{\text{hypotenuse}}{\text{opposite leg}}$$

$$\text{cosine } \theta = \cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} \qquad \text{secant } \theta = \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent leg}}$$

$$\text{tangent } \theta = \tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} \qquad \text{cotangent } \theta = \cot\theta = \frac{\text{adjacent leg}}{\text{opposite leg}}$$

(***NOTE:*** *The leg of the right triangle which is considered either the opposite leg or the adjacent leg* ***changes depending on which of the acute angles is being evaluated*** *in the trig function)*

3. The **opposite leg** of a right triangle is the leg which does not touch the vertex of the angle that is named in the trig function
4. The **adjacent leg** of a right triangle is the leg which does touch the vertex of the angle that is named in the trig function; for example:
   When evaluating the trig functions for angle A in this right triangle, leg *y* is the opposite leg for angle A because it does not touch point A; however, leg *x* is the adjacent leg for angle A because it does touch point A; the hypotenuse is side *z*
   In the same right triangle, leg *x* is the opposite leg for angle B because it does not touch point B; however, leg *y* is the adjacent leg for angle B because it does touch point B



(NOTICE: The leg that is the opposite leg for angle A is the same leg that is the adjacent leg for angle B, and the leg that is the adjacent leg for angle A is the same leg that is the opposite leg for angle B; the hypotenuse is never considered as the opposite side nor as the adjacent side because it is not a leg)

5. Since trig functions are ratios, and ratios can be written as decimal numbers, trig functions are either converted to decimal numbers or left as radical expressions in lowest terms (for example, $\frac{\sqrt{3}}{2}$ or .866); for example, in the right triangles above, if:

$$z = 9,\ y = 7, \text{ and } x = \sqrt{32} = 4\sqrt{2} \text{ then}$$
$$\sin A = \cos B = \frac{y}{z} = \frac{7}{9} \doteq .7778$$
$$\cos A = \sin B = \frac{x}{z} = \frac{4\sqrt{2}}{9} \doteq .6285$$
$$\tan A = \frac{y}{x} = \frac{7}{4\sqrt{2}} \doteq 1.2374$$
$$\tan B = \frac{x}{y} = \frac{4\sqrt{2}}{7} \doteq .8081$$

6. Using the trig function decimal number values to find or use angle measures requires either a trig function chart or a calculator with trig function options; for example, if you have found that sin α = .7778 then, by using either a trig chart or calculator, the measure of angle α is about 51º

t degrees	*sin t*	*cos t*	*tan t*
•	•	•	•
•	•	•	•
•	•	•	•
51º 00'	.7771	.6293	1.235
10	.7790	.6271	1.242
20	.7808	.6248	1.250
30	.7826	.6225	1.257
40	.7844	.6202	1.265
50	.7862	.6180	1.272
52º 00'	.7880	.6157	1.280
10	.7898	.6134	1.288
•	•	•	•
•	•	•	•
•	•	•	•

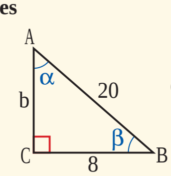
If using a calculator, follow the calculator directions

C. Triangle Trig Applications

There are two basic ways in which trig functions are used with triangles: to find angle measures and to find side lengths

1. Right Triangles
   a. **Finding Acute Angle Measures**

To find the two acute angle measures when given two sides of a right triangle, it is easiest to find the length of the third side first; for example, in the following right triangle, if you know the length of any two sides, then you may use the Pythagorean Theorem ($leg^2 + leg^2 = hypotenuse^2$) to find the length of the third side



$$8^2 + b^2 = 20^2$$
$$64 + b^2 = 400$$
$$b^2 = 336$$
$$b = \sqrt{336} = 18.33$$

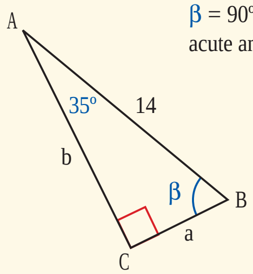
Once the three side lengths are found (it is not necessary to find the three side lengths in order to find the angle measures, but it is easier), then use the trig functions to find the degree measure of one acute angle; using the same right triangle above, the measure of α can be found using any of the trig functions, so just pick one of them, for example:

The measure of the second acute angle may be found by simply subtracting the measure of the acute angle just found from 90º because the sum of the three angles of any triangle is 180º

$$\sin\alpha = \frac{8}{20} = .4000 \text{ so } \alpha = 23^\circ\, 30'$$
$$\beta = 90^\circ - 23^\circ\, 30' = 66^\circ\, 30'$$

b. **Finding Side Lengths**

To find the side lengths of a right triangle when given only one side length and one acute angle, first, subtract the given acute angle measure from 90º because the sum of the three angles of any triangle is 180º; second, use the trig functions to find the length of another side of the triangle; for example:



β = 90º - 35º = 55º so now we use either acute angle with a trig function

$\sin 35º = \frac{a}{14}$

Use a trig chart or calculator to get this decimal value.

$\frac{.5736}{1} = \frac{a}{14}$

a = 14(.5736)

a = 8.0304

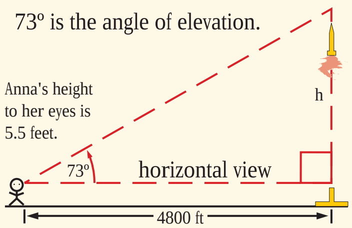
or Use the *cos* 55º, *sin* 55º, or *cos* 35º, but not the tangent function because neither leg length is given

Once two sides of the right triangle are known, the Pythagorean Theorem can be used to find the length of the third side

c. **Applying Sample Situations**

i. Definition: The **angle of elevation** is the angle formed by a horizontal line (either real or imagined) and the line of sight looking up from the horizontal; for example:

Problem: Anna stood 4,800 feet from a rocket launching pad; she measured the angle of elevation as 73º when the rocket was at its highest point; if Anna measured the angle of elevation from a height of 5.5 feet, find the greatest height that the rocket reached



$\tan 73º = \frac{h}{4800}$

$\frac{3.2709}{1} = \frac{h}{4800}$

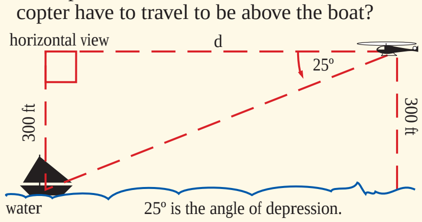
15700.32 ft = h

+ 5.50 ft = Anna

15705.82 feet above ground

ii. Definition: The **angle of depression** is the angle formed by a horizontal line (either real or imagined) and the line of sight looking down from the horizontal; for example:

Problem: A Coast Guard crew was flying a rescue mission in a helicopter; a member of the crew spotted a boat in trouble; this crewmember was looking down at about a 25º angle of depression; if the helicopter was about 300 feet above water level, how far did the helicopter have to travel to be above the boat?



$\tan 25º = \frac{300}{d}$

$\frac{.4663}{1} = \frac{300}{d}$

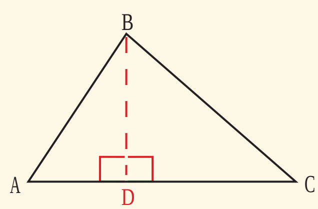
d = 643.36 ft

2. **Oblique Triangles**

Oblique triangles do not contain a right angle; therefore, any triangle that is not a right triangle is an oblique triangle

a. **Acute Triangles**

Any **acute triangle** (triangle with all acute angles) can be separated into two right triangles by constructing a line segment from one of the vertices and perpendicular to the side opposite the vertex; for example, ΔABC can be formed into right triangles ABD and BCD by drawing $\overline{BD}$ perpendicular to side $\overline{AC}$

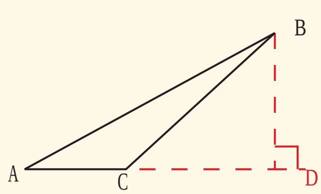


Then the trig function definitions for right triangles can be applied as discussed above

(***NOTE:*** *Another option for solving acute triangles is to leave the triangles as they are (acute) and to apply the law of cosines or the law of sines, both of which are discussed at the top of the next column)*

b. **Obtuse Triangles**

Any **obtuse triangle** (triangle with exactly one obtuse angle) can be converted into a right triangle by constructing a line segment from one of the vertices and perpendicular to the line containing the side opposite the vertex; for example, in ΔABC, ∠C is obtuse; extend side $\overline{AC}$, then draw $\overline{BD}$ perpendicular to the extension; the result is right ΔABD.



Then the trig function definitions for right triangles can be applied as discussed above

(***NOTE:*** *Another option for solving obtuse triangles is to leave the triangles as they are (obtuse) and to apply either the law of cosines or the law of sines, both of which are discussed at the top of the next column)*

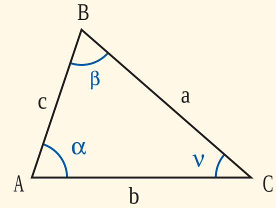
c. **Law of Cosines**

i. The **law of cosines** states that in a triangle ABC:

$a^2 = b^2 + c^2 - 2bc \cos \alpha$

$b^2 = a^2 + c^2 - 2ac \cos \beta$

$c^2 = a^2 + b^2 - 2ab \cos \nu$



ii. When to apply the law of cosines

The law of cosines may be used either when all three side lengths of the triangle are known (SSS), or when only two side lengths and the measure of the angle formed by these two sides are known (SAS, that is, two sides and the included angle)

d. **Law of Sines**

i. The **law of sines** states that in: ΔABC (as indicated in ΔABC above in the law of cosines):

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \nu}$$

ii. When to apply the law of sines:

The law of sines may be used either when one side length and two angle measures are known (SAA, that is, one of the angles must be opposite the side) or when two side lengths and one angle measure are known (SSA, that is, the angle must be opposite one of the two sides)

iii. Caution

When using the law of sines, occasionally there will be no solution; this is because not all combinations of angle measures and side lengths actually form triangles; remember that the third side of any triangle must have a length longer than the difference of the other two sides and shorter than the sum of these other two sides

TRIG WITH A UNIT CIRCLE

A. **Circles**

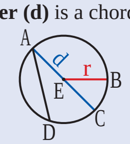
1. Definitions

a. A **circle** is the set of points in a plane that are equidistant (the same distance) from one point, the center of the circle (which is not actually a point on the circle, but only the center)

b. A **radius (r)** is a line segment whose endpoints are a point on the circle and the center of the circle

c. A **chord** is a line segment whose endpoints are both points on the circle; all other points on the chord are points in the interior of the circle

d. A **diameter (d)** is a chord that contains the center of the circle



$\overline{AD}$ is a chord.

$\overline{AC}$ is a chord and a diameter.

$\overline{EB}$ is a radius.

e. The **circumference (C) of a circle** is the distance around the circle, and may be found by using the formula C = πd where π is approximately equal to 3.14

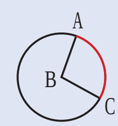
f. The **area (A) of a circle** is the number of square units that are needed to cover the interior of the circle, and may be found by using the formula $A = \pi r^2$

g. The **arc of a circle** is the set of all points on the circle between any two points on the circle; a minor arc measures less than 180º; a semicircle is an arc that measures exactly 180º; a major arc measures more than 180º

B. **Central Angles**

1. A **central angle** is an angle whose vertex is the center of a circle and whose sides contain points on the circle

2. A central angle has the same degree measure as the circular arc it intercepts (the arc located in the angle interior); additionally, an arc has the same degree measure as the central angle that intercepts it; for example, ∠ABC intercepts arc AC and their degree measure is equal



3. Degrees

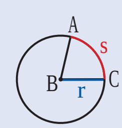
a. One **degree** is $^1/_{360º}$ of the 360º contained in a complete circle; a degree may be subdivided into 60 minutes (written 60'); a minute may be subdivided into 60 seconds (written 60")

b. The **degree measure of an angle** is the degree measure of the intercepted circular arc of the circle for which it is a central angle

4. Radians

a. One **radian** is the measure of a central angle that intercepts an arc equal in length to the radius of the circle

b. The **radian measure of a central angle** is the ratio of the circular arc length to the radius of the circle. Remember that the distance around a circle is πd; for example:



$\measuredangle ABC = \frac{s}{r}$ radians

5. Degree and Radian Conversions

a. A **semicircle** has a degree measure of 180º and a length equal to half the circle, .5πd or πr; the **radian measure** is the ratio between the circular arc length and the radius; therefore, the radian measure of a semicircle is $^{\pi r}/_{r} = \pi$; so:

i. 180º = π radians

ii. 1 radian = $^{180}/_{\pi}$

iii. 1º = $^{\pi}/_{180}$ radians

b. Degree and radian conversions can be accomplished using these proportions or equations:

i. $\frac{\text{radian measure of the angle}}{\pi \text{ radians}} = \frac{\text{degree measure of the angle}}{180º}$

ii. the radian measure of an angle $= \frac{\pi \text{ (degree measure of the angle)}}{180º}$

iii. the degree measure of an angle $= \frac{180º\text{(radian measure of the angle)}}{\pi}$

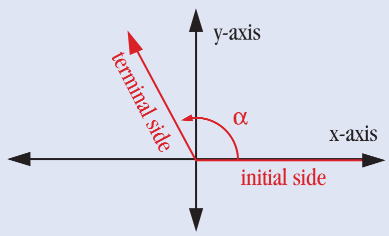
iv. For example: If $\angle A = 40^\circ$ then

$$\angle A = \frac{\pi(40)}{180} = \frac{\pi(2)}{9} = \frac{2\pi}{9} \text{ radians}$$

c. See the radians and degrees chart under the topic of *Unit Circle for the Measurements of Special Angles*

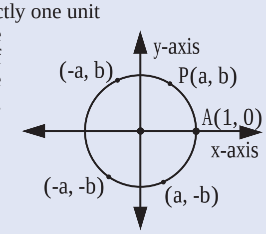
C. **Generated Angles**

1. A **generated angle** (another type of angle often used in trigonometry) is a central angle with the vertex placed at the origin of the coordinate plane, and one of the two sides placed and kept on the positive x-axis, while the second side is rotated in either a clockwise or counterclockwise direction
   a. The side that does not rotate is called the **initial side**
   b. The side that does rotate is called the **terminal side**
   c. **Negative angles** are formed when the terminal side rotates clockwise
   d. **Positive angles** are formed when the terminal side rotates counterclockwise

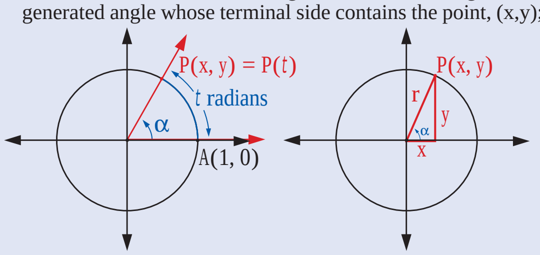


D. **Unit Circle**

1. The **unit circle** is a circle whose center is the origin (0,0) of the rectangular coordinate plane and whose radius is equal to exactly one unit (radius = 1 and diameter = 2)
2. The **equation of the unit circle** is $x^2 + y^2 = 1$
3. A point, P, is on the unit circle if and only if the distance from the center of the circle to the point is equal to the radius of exactly one unit
4. The unit circle is symmetric with respect to the x-axis, the y-axis, and the origin; therefore, if point P = (a,b) is on the unit circle, then these points are also on the unit circle (-a,b), (-a,-b), and (a,-b); for example:



5. The distance between any two points on the rectangular coordinate plane may be found by using the formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ where the points are (x_1, y_1) and (x_2, y_2).
6. The length of an arc of the unit circle is based on the circumference, $\pi d = \pi 2 = 2\pi$; because, d=2
7. Points on the Unit Circle
   a. Points can be labeled using the appropriate order pair, (x,y)
   b. Points can also be labeled using the circular arc length determined by the generated angle whose terminal side contains the point, (x,y); for example:



Remember the radius of a unit circle is one; r = 1

   c. Constructing a right triangle by drawing a perpendicular to the x-axis, and determining the side lengths of the triangle results in the following unit circle trig function definitions
8. Unit circle trig function definitions (see the diagram above):

Where t = radians
α = degrees

sin t = *sin* $\alpha = y$
cos t = *cos* $\alpha = x$
tan t = *tan* $\alpha = \frac{y}{x}$; $x \neq 0$
csc t = *csc* $\alpha = \frac{1}{y}$
sec t = *sec* $\alpha = \frac{1}{x}$
cot t = *cot* $\alpha = \frac{x}{y}$; $y \neq 0$

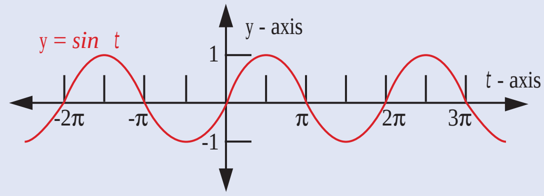
NOTE: *These functions are reciprocals:*
sin and csc; csc $\alpha = 1/\sin\alpha$
cos and sec; sec $\alpha = 1/\cos\alpha$
tan and cot; cot $\alpha = 1/\tan\alpha$

9. Frequently used **angles** and **trig functions** are indicated in the following chart

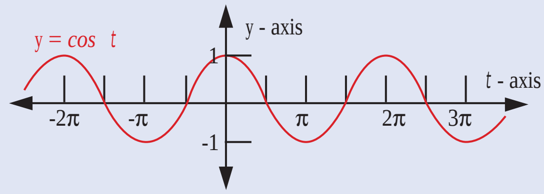
α = degree *t* = radians Ø = undefined

α	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Ø	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Ø	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0

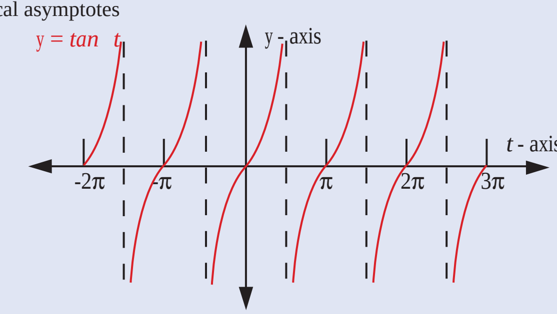
10. Trig function graphs
   a. Graphing the values of the trig functions (indicated in the chart above) on the t (radians) -axis and the y-axis yields the following results
      i. The domain of the sine function is the set of real numbers; the range is the set of real numbers between -1 and 1, inclusively; i.e., $-1 \leq y \leq 1$



      ii. Both the domain and the range of the cosine function are the same as the domain and the range of the sine function



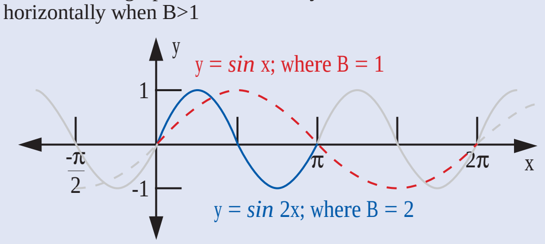
      iii. The domain of the tangent function is the set of all real numbers except those values where the function is undefined and goes off asymptotically, such as $\pm \pi/2$, $\pm 3\pi/2$, . . .
      The range is the set of all real numbers; the dashed lines are the vertical asymptotes



11. Periods of the functions
   a. A function, f, is **periodic** if there is a positive number c, such that $f(t + c) = f(t)$ for all t in the domain of the function; this may also be stated using x in place of the t value. The smallest value of c is called the period of the function; that is, the smallest value at which a function begins to repeat its range values, and thus repeat its graphing pattern, is the period of the function
   b. The period of the sine function, $f(t) = \sin t$, is 2π because $\sin(t + 2\pi) = \sin t$
   c. The period of the cosine function, $f(t) = \cos t$, is also 2π because $\cos(t + 2\pi) = \cos t$
   d. The period of the tangent function, $f(t) = \tan t$, is π because $\tan(t + \pi) = \tan t$

(NOTE: *These periods can be observed in the graphs of the functions as indicated above)*

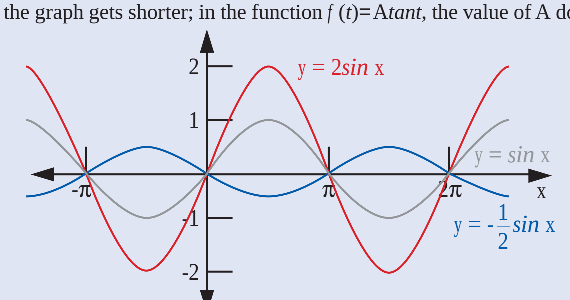
   e. The period of a function, $f(t) = \sin Bt$, is $2\pi/B$; the effect of the value of B is that it stretches the graph out horizontally when $0<B<1$ and shrinks the graph horizontally when $B>1$



(NOTE: *The red section of the graph indicates one period of y=sinx; and the blue section is one period of y=sin 2x)*

12. Amplitude
   a. The amplitude of a trig function, $y = f(t) = A\sin t$ or $y = f(t) = A\cos t$ can be defined as $|A|$

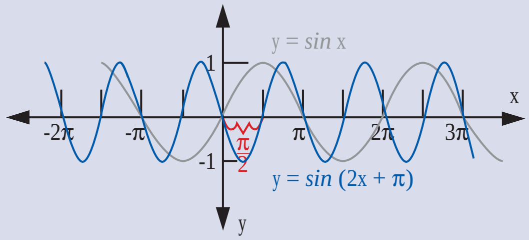
Notice that when $|A| > 1$, the maximum and the minimum values of y equal A, so the graph gets taller; likewise, when $|A| < 1$, the maximum and the minimum values of y equal A, so the graph gets shorter; in the function $f(t) = A\tan t$, the value of A does



affect the curve of the tangent graph, but not the maximum and the minimum values

   b. The amplitude may be considered to be half the difference between the minimum and the maximum values

13. Phase shifts
  a. The phase shift of $A(t)=sin(Bx+C)$ or $A(t)=cos(Bx+C)$ is $-C/B$
  b. The phase shift indicates that the graph of the function is shifted to the



right $-C/B$ units if $-C/B>0$ and to the left $-C/B$ units if $-C/B<0$

*(**NOTE:** The graph of $y = \sin(2x+\pi)$ has a $-C/B$ shift to the left because $C=\pi$, and $B=2$, so $-C/B = -\pi/2$)*

14. Inverse trig functions
  a. Inverse trig functions are denoted by ***arc*** or by -1 exponents; without restrictions on the range, the inverses of the trig functions would not be functions themselves; notice that the range of each inverse function is a subset of the domain of the corresponding trig function

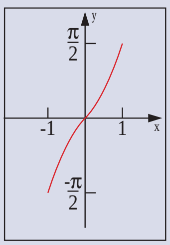
*(**NOTE:** $\sin^{-1}x$ indicates the inverse function of the sine function while $(\sin x)^{-1} = 1/\sin x$,*

$$\sin^{-1}x \neq (\sin x)^{-1} \qquad \cos^{-1}x \neq (\cos x)^{-1} \qquad \tan^{-1}x \neq (\tan x)^{-1}$$

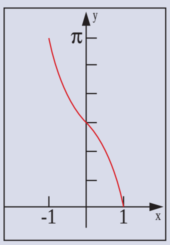
INVERSE FUNCTION	RANGE
$\arcsin x = \sin^{-1}x = y$ if and only if $\sin y = x$	$\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$
$\arccos x = \cos^{-1}x = y$ if and only if $\cos y = x$	$0 \leq y \leq \pi$
$\arctan x = \tan^{-1}x = y$ if and only if $\tan y = x$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

*which is the reciprocal of the sine function; **DO NOT** confuse these because):*

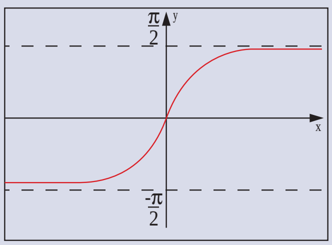
The inverse function is actually the angle measurement, y, in the range whose trig function is equal to the real number value, x; for example, arcsin .54 = the angle whose



$y = \arcsin x = \sin^{-1}x$



$y = \arccos x = \cos^{-1}x$



$y = \arctan x = \tan^{-1}x$

ANALYTIC TRIG

A. Trig **expressions** contain trig functions and relationships, but no = , < or >; they may only be simplified;
   for example: $(\cos x + \sin x) / (\sec x)$

B. Trig **equations** contain trig functions and an equals sign; they may be solved to find the values that make them true; algebraic techniques, such as factoring, may

$$\begin{aligned} \cos^2 t - \cos t - 2 &= 0 \text{ can be factored as} \\ (\cos t - 2)(\cos t + 1) &= 0 \text{ then} \\ \cos t - 2 = 0 &\text{ or } \cos t + 1 = 0 \\ \cos t = 2 &\text{ or } \cos t = -1 \text{ so in the interval } 0 \leq t \leq 2\pi \\ t \text{ has no value} &\text{ or } t = \pi \end{aligned}$$

be used to solve trig equations; for example:

C. Trig **identities** are true for all real numbers in the domain; they may be proven or varified; methods of proving or verifying identities include working the left side of the equation only until it is identical to the right side; working the right side until it is identical to the left side; or, working both sides until they are identical

D. **Fundamental Trig Identities and Formulas**

RECIPROCALS

$\csc t = \frac{1}{\sin t}$
$\sec t = \frac{1}{\cos t}$
$\cot t = \frac{1}{\tan t}$

COFUNCTIONS

$\cos\left(\frac{\pi}{2} - t\right) = \sin t$
$\sin\left(\frac{\pi}{2} - t\right) = \cos t$
$\tan\left(\frac{\pi}{2} - t\right) = \cot t$

BASIC IDENTITIES

$\tan t = \frac{\sin t}{\cos t}$
$\sin^2 t + \cos^2 t = 1$ or $\sin^2 t = 1 - \cos^2 t$
$\tan^2 t + 1 = \sec^2 t$ or $\tan^2 t = \sec^2 t - 1$
$\cot^2 t + 1 = \csc^2 t$ or $\cot^2 t = \csc^2 t - 1$

ADDITION / SUBTRACTION FORMULAS

$\cos(s \pm t) = \cos s \cos t \mp \sin s \sin t$
$\sin(s \pm t) = \sin s \cos t \pm \cos s \sin t$
$\tan(s \pm t) = \frac{\tan s \pm \tan t}{1 \mp \tan s \tan t}$

NEGATIVES

$\sin(-t) = -\sin t$
$\cos(-t) = \cos t$
$\tan(-t) = -\tan t$

DOUBLE - ANGLE FORMULAS

$\sin 2t = 2\sin t \cos t$
$\cos 2t = \cos^2 t - \sin^2 t$
$\cos 2t = 1 - 2\sin^2 t$
$\cos 2t = 2\cos^2 t - 1$
$\tan 2t = \frac{2\tan t}{1 - \tan^2 t}$
$\sin^2 t = \frac{1 - \cos 2t}{2}$
$\cos^2 t = \frac{1 + \cos 2t}{2}$

HALF - ANGLE FORMULAS

$\sin\frac{t}{2} = \pm\sqrt{\frac{1 - \cos t}{2}}$
$\cos\frac{t}{2} = \pm\sqrt{\frac{1 + \cos t}{2}}$
$\tan\frac{t}{2} = \pm\sqrt{\frac{1 - \cos t}{1 + \cos t}}$

PRODUCT - SUM FORMULAS

$\sin s \cos t = \frac{\sin(s + t) + \sin(s - t)}{2}$
$\cos s \sin t = \frac{\sin(s + t) - \sin(s - t)}{2}$
$\cos s \cos t = \frac{\cos(s + t) + \cos(s - t)}{2}$
$\sin s \sin t = \frac{\cos(s - t) - \cos(s + t)}{2}$
$\sin s + \sin t = 2\sin\left(\frac{s + t}{2}\right)\cos\left(\frac{s - t}{2}\right)$
$\sin s - \sin t = 2\cos\left(\frac{s + t}{2}\right)\sin\left(\frac{s - t}{2}\right)$
$\cos s + \cos t = 2\cos\left(\frac{s + t}{2}\right)\cos\left(\frac{s - t}{2}\right)$
$\cos s - \cos t = -2\sin\left(\frac{s + t}{2}\right)\sin\left(\frac{s - t}{2}\right)$

CREDITS

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NOTE TO STUDENT

DISCLAIMER This guide is intended only for informational purposes, and is not meant to be a substitute for professional sports instruction. Due to its condensed format, this guide cannot cover every aspect of this sport. Neither BarCharts®, its writers, designers nor editing staff, are in any way responsible or liable for the use or misuse of the information contained in this guide.

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