

# List of mathematical symbols

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This is a list of symbols found within all branches of **mathematics** to express a formula or to represent a **constant**.

When reading the list, it is important to recognize that a mathematical concept is independent of the symbol chosen to represent it. For many of the symbols below, the symbol is usually synonymous with the corresponding concept (ultimately an arbitrary choice made as a result of the cumulative history of mathematics), but in some situations a different convention may be used. For example, depending on context, "**≡**" may represent congruence or a definition. Further, in mathematical logic, numerical equality is sometimes represented by "**≐**" instead of "**=**", with the latter representing equality of **well-formed formulas**. In short, convention dictates the meaning.

Each symbol is shown both in **HTML**, whose display depends on the browser's access to an appropriate font installed on the particular device, and in **T<sub>E</sub>X**, as an image.

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## Symbols [edit]

Symbol in HTML	Symbol in T <sub>E</sub> X	Name Read as Category	Explanation	E
<b>=</b>	<b>=</b>	equality is equal to; equals everywhere	$x = y$ means $x$ and $y$ represent the same thing or value.	$2 = 2$ $1 + 1 = 2$
<b>≠</b>	<b>≠</b>	inequality is not equal to; does not equal everywhere	$x \neq y$ means that $x$ and $y$ do not represent the same thing or value. <i>(The forms !=, /= or &lt;&gt; are generally used in programming languages where ease of typing and use of ASCII text is preferred.)</i>	$2 + 2 \neq 5$
<b>&lt;</b>	<b>&lt;</b>	strict inequality is less than, is greater than order theory	$x < y$ means $x$ is less than $y$ . $x > y$ means $x$ is greater than $y$ .	$3 \leq 4$ $5 > 4$
<b>&gt;</b>	<b>&gt;</b>	proper subgroup is a proper subgroup of group theory	$H < G$ means $H$ is a proper subgroup of $G$ .	$5\mathbb{Z} \leq \mathbb{Z}$ $A_3 < S_3$
<b>≪</b>	<b>≪</b>	significant (strict) inequality is much less than, is much greater than order theory	$x \ll y$ means $x$ is much less than $y$ . $x \gg y$ means $x$ is much greater than $y$ .	$0.003 \ll 1000000$
<b>≫</b>	<b>≫</b>	asymptotic comparison is of smaller order than, is of greater order than analytic number theory	$f \ll g$ means the growth of $f$ is asymptotically bounded by $g$ . <i>(This is I. M. Vinogradov's notation. Another notation is the Big O notation, which looks like <math>f = O(g)</math>.)</i>	$x \ll e^x$
<b>≤</b>	<b>≤</b>	inequality is less than or equal to, is greater than or equal to order theory	$x \leq y$ means $x$ is less than or equal to $y$ . $x \geq y$ means $x$ is greater than or equal to $y$ . <i>(The forms &lt;= and &gt;= are generally used in programming languages, where ease of typing and use of ASCII text is preferred.)</i>	$3 \leq 4$ and $5 \leq 5$ $5 \geq 4$ and $5 \geq 5$
<b>≦</b>	<b>≦</b>	subgroup is a subgroup of group theory	$H \leq G$ means $H$ is a subgroup of $G$ .	$\mathbb{Z} \leq \mathbb{Z}$ $A_3 \leq S_3$
<b>≧</b>	<b>≧</b>	reduction is reducible to computational complexity theory	$A \leq B$ means the <b>problem</b> $A$ can be reduced to the <b>problem</b> $B$ . Subscripts can be added to the $\leq$ to indicate what kind of reduction.	If $\exists f \in F . \forall x \in \mathbb{N} . x \in A \Leftrightarrow$ then $A \leq_F B$
<b>≡</b>	<b>≡</b>	congruence relation ...is less than ... is greater than... modular arithmetic	$7k \equiv 28 \pmod{2}$ is only true if $k$ is an even integer. Assume that the problem requires $k$ to be non-negative; the domain is defined as $0 \leq k \leq \infty$ .	$10a \equiv 5 \pmod{5}$ for $1 \leq a \leq 10$
<b>≦</b>	<b>≦</b>	vector inequality	$x \leq y$ means that each component of vector $x$ is less than or equal to each corresponding component of vector $y$ .	
<b>≧</b>	<b>≧</b>	... is less than or equal... is greater than or equal...	$x \geq y$ means that each component of vector $x$ is greater than or equal to each corresponding component of vector $y$ .	

		order theory	<i>It is important to note that <math>x \leq y</math> remains true if every element is equal. However, if the operator is changed, <math>x \leq y</math> is true if and only if <math>x \neq y</math> is also true.</i>	
$\wedge$	$\gamma$	Karp reduction is Karp reducible to; is polynomial-time many-one reducible to computational complexity theory	$L_1 < L_2$ means that the problem $L_1$ is Karp reducible to $L_2$ . <sup>[1]</sup>	If $L_1 < L_2$ and $L_2 \in \mathbf{P}$ , then $L_1 \in \mathbf{P}$ .
$\propto$	$\propto$	proportionality is proportional to; varies as everywhere Karp reduction <sup>[2]</sup> is Karp reducible to; is polynomial-time many-one reducible to computational complexity theory	$y \propto x$ means that $y = kx$ for some constant $k$ .  $A \propto B$ means the problem $A$ can be polynomially reduced to the problem $B$ .	if $y = 2x$ , then $y \propto x$ .  If $L_1 \propto L_2$ and $L_2 \in \mathbf{P}$ , then $L_1 \in \mathbf{P}$ .
$+$	$+$	addition plus; add arithmetic disjoint union the disjoint union of ... and ... set theory	$4 + 6$ means the sum of 4 and 6.  $A_1 + A_2$ means the disjoint union of sets $A_1$ and $A_2$ .	$2 + 7 = 9$  $A_1 = \{3, 4, 5, 6\} \wedge A_2 = \{7, 8, 9, 10\} \Rightarrow A_1 + A_2 = \{(3,1), (4,1), (5,1), (6,1), (7,2)\}$
$-$	$-$	subtraction minus; take; subtract arithmetic negative sign negative; minus; the opposite of arithmetic set-theoretic complement minus; without set theory	$9 - 4$ means the subtraction of 4 from 9.  $-3$ means the <b>negative</b> of the number 3.  $A - B$ means the set that contains all the elements of $A$ that are not in $B$ . ( $\setminus$ can also be used for set-theoretic complement as described below.)	$8 - 3 = 5$  $\{1,2,4\} - \{1,3,4\} = \{2\}$
$\pm$	$\pm$	plus-minus plus or minus arithmetic plus-minus plus or minus measurement	$6 \pm 3$ means both $6 + 3$ and $6 - 3$ .  $10 \pm 2$ or equivalently $10 \pm 20\%$ means the range from $10 - 2$ to $10 + 2$ .	The equation $x = 5 \pm \sqrt{4}$ , has two solutions  If $a = 100 \pm 1$ mm, then $a \geq 99$ mm and $a \leq 101$ mm.
$\mp$	$\mp$	minus-plus minus or plus arithmetic	$6 \pm (3 \mp 5)$ means $6 + (3 - 5)$ and $6 - (3 + 5)$ .	$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$ .
$\times$	$\times$	multiplication times; multiplied by arithmetic Cartesian product the Cartesian product of ... and ...; the direct product of ... and ... set theory cross product cross linear algebra group of units the group of units of ring theory	$3 \times 4$ means the multiplication of 3 by 4.  (The symbol $*$ is generally used in programming languages, where ease of typing and use of ASCII text is preferred.)  $X \times Y$ means the set of all <b>ordered pairs</b> with the first element of each pair selected from $X$ and the second element selected from $Y$ .  $\mathbf{u} \times \mathbf{v}$ means the cross product of <b>vectors</b> $\mathbf{u}$ and $\mathbf{v}$  $R^\times$ consists of the set of units of the ring $R$ , along with the operation of multiplication.  <i>This may also be written <math>R^*</math> as described below, or <math>U(R)</math>.</i>	$7 \times 8 = 56$  $\{1,2\} \times \{3,4\} = \{(1,3),(1,4),(2,3),(2,4)\}$  $(1,2,5) \times (3,4,-1) = (-22, 16, -2)$  $(\mathbb{Z}/5\mathbb{Z})^\times = \{[1], [2], [3], [4]\} \cong C_4$
$*$	$*$	multiplication times; multiplied by arithmetic convolution convolution; convolved with functional analysis complex conjugate conjugate complex numbers	$a * b$ means the product of $a$ and $b$ .  (Multiplication can also be denoted with $\times$ or $\cdot$ , or even simple juxtaposition. $*$ is generally used where ease of typing and use of ASCII text is preferred, such as programming languages.)  $f * g$ means the convolution of $f$ and $g$ .  $z^*$ means the complex conjugate of $z$ . ( $\bar{z}$ can also be used for the conjugate of $z$ , as described below.)	$4 * 3$ means the product of 4 and 3, or 12  $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau$  $(3 + 4i)^* = 3 - 4i$

		group of units the group of units of ring theory	$R^*$ consists of the set of units of the ring $R$ , along with the operation of multiplication.  <i>This may also be written <math>R^*</math> as described above, or <math>U(R)</math>.</i>	$(\mathbb{Z}/5\mathbb{Z})^* = \{[1], [2], [3], [4]\}$ $\cong C_4$
		hyperreal numbers the (set of) hyperreals non-standard analysis	${}^*\mathbf{R}$ means the set of hyperreal numbers. Other sets can be used in place of $\mathbf{R}$ .	${}^*\mathbf{N}$ is the hypernatural numbers.
		Hodge dual Hodge dual; Hodge star linear algebra	${}^*v$ means the Hodge dual of a vector $v$ . If $v$ is a $k$ -vector within an $n$ -dimensional oriented inner product space, then ${}^*v$ is an $(n-k)$ -vector.	If $\{e_i\}$ are the standard basis vectors of
·	·	multiplication times; multiplied by arithmetic	$3 \cdot 4$ means the multiplication of 3 by 4.	$7 \cdot 8 = 56$
		dot product dot linear algebra	$\mathbf{u} \cdot \mathbf{v}$ means the dot product of vectors $\mathbf{u}$ and $\mathbf{v}$	$(1,2,5) \cdot (3,4,-1) = 6$
		placeholder (silent) functional analysis	$A \cdot$ means a placeholder for an argument of a function. Indicates the functional nature of an expression without assigning a specific symbol for an argument.	$\  \cdot \ $
		tensor product, tensor product of modules tensor product of linear algebra	$V \otimes U$ means the tensor product of $V$ and $U$ . <sup>[3]</sup> $V \otimes_R U$ means the tensor product of modules $V$ and $U$ over the ring $R$ .	$\{1, 2, 3, 4\} \otimes \{1, 1, 2\} = \{\{1, 2, 3, 4\}, \{1, 2, 3, 4\}, \{2, 4, 6, 8\}\}$
		Kulkarni–Nomizu product Kulkarni–Nomizu product tensor algebra	Derived from the tensor product of two symmetric type (0,2) tensors; it has the algebraic symmetries of the Riemann tensor. $f = g \otimes h$ has components $f_{\alpha\beta\gamma\delta} = g_{\alpha\gamma}h_{\beta\delta} + g_{\beta\delta}h_{\alpha\gamma} - g_{\alpha\delta}h_{\beta\gamma} - g_{\beta\gamma}h_{\alpha\delta}$ .	
÷  /	÷  /	division (Obelus) divided by; over arithmetic	$6 \div 3$ or $6/3$ means the division of 6 by 3.	$2 \div 4 = 0.5$ $12/4 = 3$
		quotient group mod group theory	$G/H$ means the quotient of group $G$ modulo its subgroup $H$ .	$\{0, a, 2a, b, b+a, b+2a\} / \{0, b\} = \{\{0, b\}, \dots\}$
		quotient set mod set theory	$A/\sim$ means the set of all $\sim$ equivalence classes in $A$ .	If we define $\sim$ by $x \sim y \Leftrightarrow x - y \in \mathbb{Z}$ , then $\mathbb{R}/\sim = \{x + n : n \in \mathbb{Z}\} : x \in [0,1)$
		square root the (principal) square root of real numbers	$\sqrt{x}$ means the nonnegative number whose square is $x$ .	$\sqrt{4} = 2$
		complex square root the (complex) square root of complex numbers	if $z = r \exp(i\phi)$ is represented in polar coordinates with $-\pi < \phi \leq \pi$ , then $\sqrt{z} = \sqrt{r} \exp(i\phi/2)$ .	$\sqrt{-1} = i$
$\bar{x}$	$\bar{x}$	mean overbar; ... bar statistics	$\bar{x}$ (often read as "x bar") is the mean (average value of $x_i$ ).	$x = \{1, 2, 3, 4, 5\}; \bar{x} = 3$
		complex conjugate conjugate complex numbers	$\bar{z}$ means the complex conjugate of $z$ .  <i>(<math>z^*</math> can also be used for the conjugate of <math>z</math>, as described above.)</i>	$\overline{3 + 4i} = 3 - 4i$
		finite sequence, tuple finite sequence, tuple model theory	$\bar{a}$ means the finite sequence/tuple $(a_1, a_2, \dots, a_n)$ .	$\bar{a} := (a_1, a_2, \dots, a_n)$
		algebraic closure algebraic closure of field theory	$\bar{F}$ is the algebraic closure of the field $F$ .	The field of algebraic numbers is some closure of the rational numbers $\mathbb{Q}$ .
		topological closure (topological) closure of topology	$\bar{S}$ is the topological closure of the set $S$ .  <i>This may also be denoted as <math>\text{cl}(S)</math> or <math>\text{Cl}(S)</math>.</i>	In the space of the real numbers, $\overline{\mathbb{Q}} = \mathbb{R}$ .
		unit vector hat geometry	$\hat{\mathbf{a}}$ (pronounced "a hat") is the normalized version of vector $\mathbf{a}$ , having length 1.	
		estimator estimator for statistics	$\hat{\theta}$ is the estimator or the estimate for the parameter $\theta$ .	The estimator $\hat{\mu} = \frac{\sum_i x_i}{n}$ produces a
				absolute value; modulus absolute value of; modulus of numbers  Euclidean norm or Euclidean length or

...	...	magnitude	x  means the (Euclidean) length of <b>vector</b> <b>x</b> .	$ x  = \sqrt{3^2 + (-4)^2} = 5$
		Euclidean norm of geometry		
		determinant	A  means the determinant of the matrix <b>A</b>	$\begin{vmatrix} 1 & 2 \\ 2 & 9 \end{vmatrix} = 5$
		determinant of matrix theory		
X	X	cardinality	X  means the cardinality of the set <b>X</b> .  (# may be used instead as described below.)	{3, 5, 7, 9}  = 4.
		cardinality of; size of;		
		order of set theory		
...	...	norm	x   means the <b>norm</b> of the element <b>x</b> of a <b>normed vector space</b> . <sup>[4]</sup>	x + y   ≤   x   +   y
		norm of; length of linear algebra		
		nearest integer function	x   means the nearest integer to <b>x</b> .  (This may also be written [x], ⌊x⌋, nint(x) or Round(x).)	1   = 1,   1.6   = 2,   -2.4   = -2,   3.49   =
		nearest integer to numbers		
		conditional event	P(A B) means the probability of the event <b>a</b> occurring given that <b>b</b> occurs.	If <b>x</b> is a uniformly random day of the year
		given probability		
		restriction	f <sub>A</sub> means the function <b>f</b> restricted to the set <b>A</b> , that is, it is the function with domain <b>A</b> ∩ dom( <b>f</b> ) that agrees with <b>f</b> .	The function <b>f</b> : <b>R</b> → <b>R</b> defined by <b>f(x) =</b>
		restriction of ... to ...; restricted to set theory		
		such that		
such that; so that	means "such that", see ":" (described below).	S = {(x,y)   0 < y < f(x)} The set of (x,y) such that y is greater than		
everywhere				
		divisor, divides	<b>a</b>   <b>b</b> means <b>a</b> divides <b>b</b> . <b>a</b> † <b>b</b> means <b>a</b> does not divide <b>b</b> .	Since 15 = 3×5, it is true that 3   15 and
		divides number theory		
†	†		(The symbol   can be difficult to type, and its negation is rare, so a regular but slightly shorter vertical bar † character is often used instead.)	
		exact divisibility	<b>p</b> <sup><b>a</b></sup>    <b>n</b> means <b>p</b> <sup><b>a</b></sup> exactly divides <b>n</b> (i.e. <b>p</b> <sup><b>a</b></sup> divides <b>n</b> but <b>p</b> <sup><b>a+1</b></sup> does not).	2 <sup>3</sup>    360.
		exactly divides number theory		
		parallel	<b>x</b>    <b>y</b> means <b>x</b> is parallel to <b>y</b> . <b>x</b> † <b>y</b> means <b>x</b> is not parallel to <b>y</b> . <b>x</b> # <b>y</b> means <b>x</b> is equal and parallel to <b>y</b> .  (The symbol    can be difficult to type, and its negation is rare, so two regular but slightly longer vertical bar    characters are often used instead.)	If <b>l</b>    <b>m</b> and <b>m</b> ⊥ <b>n</b> then <b>l</b> ⊥ <b>n</b> .
		is parallel to geometry		
		incomparability		
#	#	is incomparable to order theory	<b>x</b>    <b>y</b> means <b>x</b> is incomparable to <b>y</b> .	{1,2}    {2,3} under set containment.
#	#	cardinality	# <b>X</b> means the cardinality of the set <b>X</b> .  ( ...  may be used instead as described above.)	#{4, 6, 8} = 3
		cardinality of; size of; order of set theory		
		connected sum	<b>A</b> # <b>B</b> is the connected sum of the manifolds <b>A</b> and <b>B</b> . If <b>A</b> and <b>B</b> are knots, then this denotes the knot sum, which has a slightly stronger condition.	<b>A</b> # <b>S</b> <sup><b>m</b></sup> is homeomorphic to <b>A</b> , for any manifold <b>S</b>
		connected sum of; knot sum of; knot composition of topology, knot theory		
primorial	<b>n</b> # is product of all prime numbers less than or equal to <b>n</b> .	12# = 2 × 3 × 5 × 7 × 11 = 2310		
ℵ	ℵ	aleph number	ℵ <sub>α</sub> represents an infinite cardinality (specifically, the α-th one, where α is an ordinal).	ℕ  = ℵ <sub>0</sub> , which is called aleph-null.
		aleph set theory		
⊃	⊃	beth number	⊃ <sub>α</sub> represents an infinite cardinality (similar to ℵ, but ⊃ does not necessarily index all of the numbers indexed by ℵ. )	⊃ <sub>1</sub> =  P(ℕ)  = 2 <sup>ℵ<sub>0</sub></sup> .
		beth set theory		
c	c	cardinality of the continuum	The cardinality of <b>R</b> is denoted by   <b>R</b>   or by the symbol <b>c</b> (a lowercase Fraktur letter C).	<b>c</b> = ⊃ <sub>1</sub>
		cardinality of the continuum; <b>c</b> ; cardinality of the real numbers set theory		
:	:	such that	: means "such that", and is used in proofs and the set-builder notation (described below).	∃ n ∈ ℕ : n is even.
		such that; so that		
		everywhere		

:	:	field extension	$K : F$ means the field $K$ extends the field $F$ .	$\mathbb{R} : \mathbb{Q}$
		extends; over	<i>This may also be written as <math>K \geq F</math>.</i>	
		field theory		
		inner product of matrices	$A : B$ means the Frobenius inner product of the matrices $A$ and $B$ .	$A : B = \sum_{i,j} A_{ij}B_{ij}$
		inner product of	<i>The general inner product is denoted by <math>(u, v)</math>, <math>(u   v)</math> or <math>(u   v)</math>, as described below. For spatial vectors, the dot product notation, <math>x \cdot y</math> is common. See also bra-ket notation.</i>	
		linear algebra		
		index of a subgroup	The index of a subgroup $H$ in a group $G$ is the "relative size" of $H$ in $G$ : equivalently, the number of "copies" (cosets) of $H$ that fill up $G$	$ G : H  = \frac{ G }{ H }$
		index of subgroup		
		group theory		
!	!	logical negation	The statement $\neg A$ is true if and only if $A$ is false.	$\neg(A) \Leftrightarrow A$
		not	A slash placed through another operator is the same as "!" placed in front.	$x \neq y \Leftrightarrow \neg(x = y)$
		propositional logic	<i>(The symbol ! is primarily from computer science. It is avoided in mathematical texts, where the notation <math>\neg A</math> is preferred.)</i>	
		factorial	$n!$ means the product $1 \times 2 \times \dots \times n$ .	$4! = 1 \times 2 \times 3 \times 4 = 24$
		factorial		
		combinatorics		
()	()	combination; binomial coefficient	$\binom{n}{k} = \frac{n!/(n-k)!}{k!} = \frac{(n-k+1) \cdots (n-2) \cdot (n-1) \cdot n}{k!}$	$\binom{73}{5} = \frac{73!/(73-5)!}{5!} = \frac{69 \cdot 70 \cdot 71 \cdot 72 \cdot 73}{5!} = \frac{69 \cdot 7 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = \frac{69 \cdot 7}{120} = \frac{483}{120} = \frac{161}{40}$
		$n$ choose $k$	means (in the case of $n =$ positive integer) the number of combinations of $k$ elements drawn from a set of $n$ elements.	$\binom{5}{7} = \frac{-5 \cdot -4 \cdot -3 \cdot -2 \cdot -1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = -\frac{1}{120}$
		combinatorics	<i>(This may also be written as <math>C(n, k)</math>, <math>C(n; k)</math>, <math>{}_n C_k</math>, <math>{}^n C_k</math>, or <math>\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle</math>.)</i>	
(( ))	(( ))	multiset coefficient		
		$u$ multichoose $k$	$\left(\!\!\binom{u}{k}\!\!\right) = \binom{u+k-1}{k} = \frac{(u+k-1)!}{k!(u-1)!}$	$\left(\!\!\binom{-5.5}{7}\!\!\right) = \frac{-5.5 \cdot -4.5 \cdot -3.5 \cdot -2.5 \cdot -1.5 \cdot -0.5 \cdot 0.5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} = \frac{1}{1008}$
		combinatorics	(when $u$ is positive integer)	
			means reverse or rising binomial coefficient.	
~	~	probability distribution		
		has distribution	$X \sim D$ , means the random variable $X$ has the probability distribution $D$ .	$X \sim N(0,1)$ , the standard normal distribution
		statistics		
		row equivalence	$A \sim B$ means that $B$ can be generated by using a series of elementary row operations on $A$	$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$
		is row equivalent to		
		matrix theory		
		same order of magnitude	$m \sim n$ means the quantities $m$ and $n$ have the same order of magnitude, or general size.	$2 \sim 5$
		roughly similar; poorly approximates		$8 \times 9 \sim 100$
		approximation theory	<i>(Note that <math>\sim</math> is used for an approximation that is poor, otherwise use <math>\approx</math>.)</i>	but $\pi^2 \approx 10$
		similarity		
is similar to <sup>[5]</sup>	$\triangle ABC \sim \triangle DEF$ means triangle $ABC$ is similar to (has the same shape) triangle $DEF$ .			
geometry				
asymptotically equivalent				
is asymptotically equivalent to	$f \sim g$ means $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 1$ .	$x \sim x+1$		
asymptotic analysis				
equivalence relation				
are in the same equivalence class	$a \sim b$ means $b \in [a]$ (and equivalently $a \in [b]$ ).	$1 \sim 5 \pmod{4}$		
everywhere				
≈	≈	approximately equal	$x \approx y$ means $x$ is approximately equal to $y$ .	$\pi \approx 3.14159$
		is approximately equal to everywhere	<i>This may also be written <math>\cong</math>, <math>\simeq</math>, <math>\sim</math>, <math>\doteq</math> (Libra Symbol), or <math>\approx</math>.</i>	
		isomorphism	$G \approx H$ means that group $G$ is isomorphic (structurally identical) to group $H$ .	$Q / \{1, -1\} \approx V$ , where $Q$ is the quaternion group and $V$ is
		is isomorphic to	<i>(<math>\cong</math> can also be used for isomorphic, as described below.)</i>	
group theory				
{}	{}	wreath product	$A \wr H$ means the wreath product of the group $A$ by the group $H$ .	$S_n \wr Z_2$ is isomorphic to the automorphisms of a set of $2n$ vertices.
		wreath product of ... by ...	<i>This may also be written <math>A_{\text{wr}} H</math>.</i>	
Δ	Δ	normal subgroup		
		is a normal subgroup of	$N \triangleleft G$ means that $N$ is a normal subgroup of group $G$ .	$Z(G) \triangleleft G$
		group theory		
		ideal		
∇	∇	is an ideal of	$I \triangleleft R$ means that $I$ is an ideal of ring $R$ .	$(2) \triangleleft \mathbb{Z}$
		ring theory		
▷	▷	antijoin		
		the antijoin of	$R \triangleright S$ means the antijoin of the relations $R$ and $S$ , the tuples in $R$ for which there is not a tuple in $S$ that is equal on their common attribute names.	$R \triangleright S = R - R \times S$
relational algebra				
⋈		semidirect product	$N \rtimes_{\varphi} H$ is the semidirect product of $N$ (a normal subgroup) and $H$ (a subgroup), with respect to $\varphi$ . Also, if $G = N \rtimes_{\varphi} H$ , then $G$ is said to split over $N$ .	$D_{2n} \cong C_n \rtimes C_2$
		the semidirect product of		

$\bowtie$	$\bowtie$	group theory	( $\bowtie$ may also be written the other way round, as $\bowtie$ , or as $\times$ .)	
		semijoin	$R \bowtie S$ is the semijoin of the relations $R$ and $S$ , the set of all tuples in $R$ for which there is a tuple in $S$ that is equal on their common attribute names.	$R \bowtie S = \prod_{a_1, \dots, a_n} (R \bowtie S)$
		the semijoin of relational algebra		
$\bowtie$	$\bowtie$	natural join	$R \bowtie S$ is the natural join of the relations $R$ and $S$ , the set of all combinations of tuples in $R$ and $S$ that are equal on their common attribute names.	
		the natural join of relational algebra		
$\therefore$	$\therefore$	therefore; so; hence	Sometimes used in proofs before logical consequences.	All humans are mortal. Socrates is a hun
		everywhere		
$\because$	$\because$	because	Sometimes used in proofs before reasoning.	11 is prime $\because$ it has no positive integer fa
		because; since		
		everywhere		
$\blacksquare$	$\blacksquare$	end of proof	Used to mark the end of a proof.	
		QED; tombstone; Halmos symbol	(May also be written Q.E.D.)	
		everywhere		
$\square$	$\square$	D'Alembertian	It is the generalisation of the Laplace operator in the sense that it is the differential operator which is invariant under the isometry group of the underlying space and it reduces to the Laplace operator if restricted to time independent functions.	$\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$
		non-Euclidean Laplacian		
		vector calculus		
$\curvearrowright$	$\curvearrowright$	downwards zigzag arrow	Denotes that contradictory statements have been inferred. For clarity, the exact point of contradiction can be appended.	Statement: Every finite, non-empty, orde assume that $X$ is a finite, non-empty, or some $x_1 \in X$ , there exists an $x_2 \in X$ an $x_3 \in X$ with $x_2 < x_3$ , and so on. $\curvearrowright X$ is finite.
		contradiction; this contradicts that		
		everywhere		
$\Rightarrow$	$\Rightarrow$	material implication	$A \Rightarrow B$ means if $A$ is true then $B$ is also true; if $A$ is false then nothing is said about $B$ .	
		implies; if ... then	( $\rightarrow$ may mean the same as $\Rightarrow$ , or it may have the meaning for functions given below.)	$x = 2 \Rightarrow x^2 = 4$ is true, but $x^2 = 4 \Rightarrow x =$
		propositional logic, Heyting algebra	( $\supset$ may mean the same as $\Rightarrow$ , <sup>[6]</sup> or it may have the meaning for superset given below.)	
$\Leftrightarrow$	$\Leftrightarrow$	material equivalence	$A \Leftrightarrow B$ means $A$ is true if $B$ is true and $A$ is false if $B$ is false.	$x + 5 = y + 2 \Leftrightarrow x + 3 = y$
		if and only if; iff		
		propositional logic		
$\neg$	$\neg$	logical negation	The statement $\neg A$ is true if and only if $A$ is false.	
		not	A slash placed through another operator is the same as "-" placed in front.	$\neg(\neg A) \Leftrightarrow A$ $x \neq y \Leftrightarrow \neg(x = y)$
		propositional logic	(The symbol $\sim$ has many other uses, so $\neg$ or the slash notation is preferred. Computer scientists will often use ! but this is avoided in mathematical texts.)	
$\wedge$	$\wedge$	logical conjunction or meet in a lattice	The statement $A \wedge B$ is true if $A$ and $B$ are both true; else it is false.	$n < 4 \wedge n > 2 \Leftrightarrow n = 3$ when $n$ is a natu
		and; min; meet	For functions $A(x)$ and $B(x)$ , $A(x) \wedge B(x)$ is used to mean $\min(A(x), B(x))$ .	
		propositional logic, lattice theory		
		wedge product	$u \wedge v$ means the wedge product of any multivectors $u$ and $v$ . In three-dimensional Euclidean space the wedge product and the cross product of two vectors are each other's Hodge dual.	$u \wedge v = *(u \times v)$ if $u, v \in \mathbb{R}^3$
		wedge product; exterior product		
		exterior algebra		
$\wedge$	$\wedge$	exponentiation	$a \wedge b$ means $a$ raised to the power of $b$	
		... (raised) to the power of ...	( $a \wedge b$ is more commonly written $a^b$ . The symbol $\wedge$ is generally used in programming languages where ease of typing and use of plain ASCII text is preferred.)	$2 \wedge 3 = 2^3 = 8$
		everywhere		
$\vee$	$\vee$	logical disjunction or join in a lattice	The statement $A \vee B$ is true if $A$ or $B$ (or both) are true; if both are false, the statement is false.	$n \geq 4 \vee n \leq 2 \Leftrightarrow n \neq 3$ when $n$ is a natu
		or; max;		

		<p>join</p> <p>propositional logic, lattice theory</p>	For functions $A(x)$ and $B(x)$ , $A(x) \vee B(x)$ is used to mean $\max(A(x), B(x))$ .	
$\oplus$	$\oplus$	<p>exclusive or</p> <p>xor</p> <p>propositional logic, Boolean algebra</p>	The statement $A \oplus B$ is true when either $A$ or $B$ , but not both, are true. $A \vee B$ means the same.	$(\neg A) \oplus A$ is always true, $A \oplus A$ is always false.
$\underline{\vee}$	$\underline{\vee}$	<p>direct sum</p> <p>direct sum of</p> <p>abstract algebra</p>	The direct sum is a special way of combining several objects into one general object. <i>(The bun symbol <math>\oplus</math>, or the coproduct symbol <math>\amalg</math>, is used; <math>\underline{\vee}</math> is only for logic.)</i>	Most commonly, for vector spaces $U, V, W$ : $U = V \oplus W \Leftrightarrow (U = V + W) \wedge (V \cap W = \{0\})$
$\forall$	$\forall$	<p>universal quantification</p> <p>for all; for any; for each</p> <p>predicate logic</p>	$\forall x: P(x)$ means $P(x)$ is true for all $x$ .	$\forall n \in \mathbb{N}: n^2 \geq n$ .
$\exists$	$\exists$	<p>existential quantification</p> <p>there exists; there is; there are</p> <p>predicate logic</p>	$\exists x: P(x)$ means there is at least one $x$ such that $P(x)$ is true.	$\exists n \in \mathbb{N}: n$ is even.
$\exists!$	$\exists!$	<p>uniqueness quantification</p> <p>there exists exactly one</p> <p>predicate logic</p>	$\exists! x: P(x)$ means there is exactly one $x$ such that $P(x)$ is true.	$\exists! n \in \mathbb{N}: n + 5 = 2n$ .
$=:$				
$:=$				
$\equiv$	$:=$ $:=$ $\equiv$	<p>definition</p> <p>is defined as; is equal by definition to</p> <p>everywhere</p>	<p><math>x := y, y =: x</math> or <math>x \equiv y</math> means <math>x</math> is defined to be another name for <math>y</math>, under certain assumptions taken in context.</p> <p><i>(Some writers use <math>\equiv</math> to mean congruence).</i></p> <p><math>P \Leftrightarrow Q</math> means <math>P</math> is defined to be logically equivalent to <math>Q</math>.</p>	$\cosh x := \frac{e^x + e^{-x}}{2}$
$\triangleq$	$\triangleq$ $\stackrel{\text{def}}{=}$ $\stackrel{\text{def}}{=}$			
$\stackrel{\text{def}}{=}$				
$\stackrel{\text{def}}{=}$				
$\cong$	$\cong$	<p>congruence</p> <p>is congruent to</p> <p>geometry</p> <p>isomorphic</p> <p>is isomorphic to</p> <p>abstract algebra</p>	<p><math>\triangle ABC \cong \triangle DEF</math> means triangle <math>ABC</math> is congruent to (has the same measurements as) triangle <math>DEF</math>.</p> <p><math>G \cong H</math> means that group <math>G</math> is isomorphic (structurally identical) to group <math>H</math>. <i>(<math>\simeq</math> can also be used for isomorphic, as described above.)</i></p>	$\mathbb{R}^2 \cong \mathbb{C}$
$\equiv$	$\equiv$	<p>congruence relation</p> <p>... is congruent to ... modulo ...</p> <p>modular arithmetic</p>	$a \equiv b \pmod{n}$ means $a - b$ is divisible by $n$	$5 \equiv 2 \pmod{3}$
$\{, \}$	$\{, \}$	<p>set brackets</p> <p>the set of ...</p> <p>set theory</p>	$\{a, b, c\}$ means the set consisting of $a, b$ , and $c$ . <sup>[7]</sup>	$\mathbb{N} = \{1, 2, 3, \dots\}$
$\{:\}$	$\{:\}$	<p>set builder notation</p> <p>the set of ... such that</p> <p>set theory</p>	$\{x : P(x)\}$ means the set of all $x$ for which $P(x)$ is true. <sup>[7]</sup> $\{x \mid P(x)\}$ is the same as $\{x : P(x)\}$ .	$\{n \in \mathbb{N} : n^2 < 20\} = \{1, 2, 3, 4\}$
$\{ \}$	$\{ \}$			
$\{;\}$	$\{;\}$			
$\emptyset$	$\emptyset$	<p>empty set</p> <p>the empty set</p> <p>set theory</p>	$\emptyset$ means the set with no elements. <sup>[7]</sup> $\{\}$ means the same.	$\{n \in \mathbb{N} : 1 < n^2 < 4\} = \emptyset$
$\{\}$	$\{\}$			
$\in$	$\in$	<p>set membership</p> <p>is an element of:</p>	$a \in S$ means $a$ is an element of the set $S$ . <sup>[7]</sup> $a \notin S$ means $a$ is not an element of $S$ .	$(1/2)^{-1} \in \mathbb{N}$

$\notin$	$\notin$	is not an element of everywhere, set theory	element of $S$ . <sup>[7]</sup>	$2^{-1} \notin \mathbb{N}$
$\ni$	$\ni$	such that symbol such that mathematical logic set membership contains an element set theory	often abbreviated as "s.t."; $:$ and $ $ are also used to abbreviate "such that". The use of $\ni$ goes back to early mathematical logic and its usage in this sense is declining. $S \ni e$ means the same thing as $e \in S$ , where $S$ is a set and $e$ is an element of $S$ .	Choose $x \ni 2 x$ and $3 x$ . (Here $ $ is use
$\not\ni$	$\not\ni$	set membership does not contain as an element set theory	$S \not\ni e$ means the same thing as $e \notin S$ , where $S$ is a set and $e$ is not an element of $S$ .	
$\subseteq$	$\subset$	subset is a subset of set theory	(subset) $A \subseteq B$ means every element of $A$ is also an element of $B$ . <sup>[8]</sup> (proper subset) $A \subset B$ means $A \subseteq B$ but $A \neq B$ . (Some writers use the symbol $\subset$ as if it were the same as $\subseteq$ .)	$(A \cap B) \subseteq A$ $\mathbb{N} \subset \mathbb{Q}$ $\mathbb{Q} \subset \mathbb{R}$
$\supseteq$	$\supseteq$	superset is a superset of set theory	$A \supseteq B$ means every element of $B$ is also an element of $A$ . $A \supset B$ means $A \supseteq B$ but $A \neq B$ . (Some writers use the symbol $\supset$ as if it were the same as $\supseteq$ .)	$(A \cup B) \supseteq B$ $\mathbb{R} \supset \mathbb{Q}$
$\cup$	$\cup$	set-theoretic union the union of ... or ...; union set theory	$A \cup B$ means the set of those elements which are either in $A$ , or in $B$ , or in both. <sup>[8]</sup>	$A \subseteq B \Leftrightarrow (A \cup B) = B$
$\cap$	$\cap$	set-theoretic intersection intersected with; intersect set theory	$A \cap B$ means the set that contains all those elements that $A$ and $B$ have in common. <sup>[8]</sup>	$\{x \in \mathbb{R} : x^2 = 1\} \cap \mathbb{N} = \{1\}$
$\Delta$	$\Delta$	symmetric difference symmetric difference set theory	$A \Delta B$ (or $A \ominus B$ ) means the set of elements in exactly one of $A$ or $B$ . (Not to be confused with delta, $\Delta$ , described below.)	$\{1, 5, 6, 8\} \Delta \{2, 5, 8\} = \{1, 2, 6\}$ $\{3, 4, 5, 6\} \ominus \{1, 2, 5, 6\} = \{1, 2, 3, 4\}$
$\setminus$	$\setminus$	set-theoretic complement minus; without set theory	$A \setminus B$ means the set that contains all those elements of $A$ that are not in $B$ . <sup>[8]</sup> ( $-$ can also be used for set-theoretic complement as described above.)	$\{1, 2, 3, 4\} \setminus \{3, 4, 5, 6\} = \{1, 2\}$
$\rightarrow$	$\rightarrow$	function arrow from ... to set theory, type theory	$f: X \rightarrow Y$ means the function $f$ maps the set $X$ into the set $Y$ .	Let $f: \mathbb{Z} \rightarrow \mathbb{N} \cup \{0\}$ be defined by $f(x) := x^2$
$\mapsto$	$\mapsto$	function arrow maps to set theory	$f: a \mapsto b$ means the function $f$ maps the element $a$ to the element $b$ .	Let $f: x \mapsto x+1$ (the successor function).
$\circ$	$\circ$	function composition composed with set theory	$f \circ g$ is the function, such that $(f \circ g)(x) = f(g(x))$ . <sup>[9]</sup>	if $f(x) := 2x$ , and $g(x) := x + 3$ , then $(f \circ g)($
$\circ$	$\circ$	Hadamard product entrywise product linear algebra	For two matrices (or vectors) of the same dimensions $A, B \in \mathbb{R}^{m \times n}$ the Hadamard product is a matrix of the same dimensions $A \circ B \in \mathbb{R}^{m \times n}$ with elements given by $(A \circ B)_{i,j} = (A)_{i,j} \cdot (B)_{i,j}$ . This is often used in matrix based programming such as MATLAB where the operation is done by $A.*B$	$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \circ \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$
$\mathbb{N}$	$\mathbb{N}$	natural numbers $\mathbb{N}$ ; the (set of) natural numbers numbers	$\mathbb{N}$ means either $\{0, 1, 2, 3, \dots\}$ or $\{1, 2, 3, \dots\}$ . <i>The choice depends on the area of mathematics being studied; e.g. number theorists prefer the latter; analysts, set theorists and computer scientists prefer the former. To avoid confusion, always check an author's definition of <math>\mathbb{N}</math>.</i> <i>Set theorists often use the notation <math>\omega</math> (for least infinite ordinal) to denote the set of natural numbers (including zero), along with the standard ordering relation <math>\leq</math>.</i>	$\mathbb{N} = \{ a  : a \in \mathbb{Z}\}$ or $\mathbb{N} = \{a > 0 : a \in \mathbb{Z}\}$
$\mathbb{Z}$	$\mathbb{Z}$	integers $\mathbb{Z}$ ; the (set of) integers numbers	$\mathbb{Z}$ means $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ . $\mathbb{Z}^+$ or $\mathbb{Z}^>$ means $\{1, 2, 3, \dots\}$ . $\mathbb{Z}^*$ or $\mathbb{Z}^z$ means $\{0, 1, 2, 3, \dots\}$ .	$\mathbb{Z} = \{p, -p : p \in \mathbb{N} \cup \{0\}\}$
$\mathbb{Z}_n$	$\mathbb{Z}_n$	integers mod $n$ $\mathbb{Z}_n$ ; the (set of) integers mod $n$ numbers	$\mathbb{Z}_n$ means $\{[0], [1], [2], \dots, [n-1]\}$ with addition and multiplication modulo $n$ . <i>Note that any letter may be used instead of <math>n</math>, such as <math>p</math>. To avoid confusion with <math>p</math>-adic numbers, use <math>\mathbb{Z}/p\mathbb{Z}</math> or <math>\mathbb{Z}(p)</math> instead.</i>	$\mathbb{Z}_3 = \{[0], [1], [2]\}$
$\mathbb{Z}_p$	$\mathbb{Z}_p$			
$\mathbb{Z}_n$	$\mathbb{Z}_n$			



$Z_n$	$Z_p$	$p$ -adic integers the (set of) $p$ -adic integers numbers	Note that any letter may be used instead of $p$ , such as $n$ or $l$ .	
$P$	$P$	projective space $P$ ; the projective space; the projective line; the projective plane topology	$P$ means a space with a point at infinity.	$P^1, P^2$
$P$	$P$	probability the probability of probability theory	$P(X)$ means the probability of the event $X$ occurring. <i>This may also be written as <math>P(X)</math>, <math>Pr(X)</math>, <math>P[X]</math> or <math>Pr[X]</math>.</i>	If a fair coin is flipped, $P(\text{Heads}) = P(\text{Tail})$
		Power set the Power set of Powerset	Given a set $S$ , the power set of $S$ is the set of all subsets of the set $S$ . The power set of $S$ is denoted by $P(S)$ .	The power set $P(\{0, 1, 2\})$ is the set of all $P(\{0, 1, 2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$
$Q$	$Q$	rational numbers $Q$ ; the (set of) rational numbers; the rationals numbers	$Q$ means $\{p/q : p \in \mathbb{Z}, q \in \mathbb{N}\}$ .	$3.14000... \in Q$ $\pi \notin Q$
$R$	$R$	real numbers $R$ ; the (set of) real numbers; the reals numbers	$R$ means the set of real numbers.	$\pi \in R$ $\sqrt{-1} \notin R$
$C$	$C$	complex numbers $C$ ; the (set of) complex numbers numbers	$C$ means $\{a + b i : a, b \in R\}$ .	$i = \sqrt{-1} \in C$
$H$	$H$	quaternions or Hamiltonian quaternions $H$ ; the (set of) quaternions numbers	$H$ means $\{a + b i + c j + d k : a, b, c, d \in R\}$ .	
$O$	$O$	Big O notation big-oh of Computational complexity theory	The Big O notation describes the limiting behavior of a function, when the argument tends towards a particular value or infinity.	If $f(x) = 6x^4 - 2x^3 + 5$ and $g(x) = x^4$ , then
$\infty$	$\infty$	infinity infinity numbers	$\infty$ is an element of the extended number line that is greater than all real numbers; it often occurs in limits.	$\lim_{x \rightarrow 0} \frac{1}{ x } = \infty$
$[...]$	$[...]$	floor floor; greatest integer; entier numbers	$[x]$ means the floor of $x$ , i.e. the largest integer less than or equal to $x$ . <i>(This may also be written <math>[x]</math>, <math>\text{floor}(x)</math> or <math>\text{int}(x)</math>.)</i>	$[4] = 4$ , $[2.1] = 2$ , $[2.9] = 2$ , $[-2.6] = -3$
$[...]$	$[...]$	ceiling ceiling numbers	$[x]$ means the ceiling of $x$ , i.e. the smallest integer greater than or equal to $x$ . <i>(This may also be written <math>\text{ceil}(x)</math> or <math>\text{ceiling}(x)</math>.)</i>	$[4] = 4$ , $[2.1] = 3$ , $[2.9] = 3$ , $[-2.6] = -2$
$[...]$	$[...]$	nearest integer function nearest integer to numbers	$[x]$ means the nearest integer to $x$ . <i>(This may also be written <math>[x]</math>, <math>  x  </math>, <math>\text{nint}(x)</math> or <math>\text{Round}(x)</math>.)</i>	$[2] = 2$ , $[2.6] = 3$ , $[-3.4] = -3$ , $[4.49] = 4$
$[ : ]$	$[ : ]$	degree of a field extension the degree of field theory	$[K : F]$ means the degree of the extension $K : F$ .	$[Q(\sqrt{2}) : Q] = 2$ $[C : R] = 2$ $[R : Q] = \infty$
		equivalence class the equivalence class of abstract algebra	$[a]$ means the equivalence class of $a$ , i.e. $\{x : x \sim a\}$ , where $\sim$ is an equivalence relation. $[a]_R$ means the same, but with $R$ as the equivalence relation.	Let $a \sim b$ be true iff $a \equiv b \pmod{5}$ . Then $[2] = \{\dots, -8, -3, 2, 7, \dots\}$ .
		floor floor; greatest integer; entier numbers	$[x]$ means the floor of $x$ , i.e. the largest integer less than or equal to $x$ . <i>(This may also be written <math>[x]</math>, <math>\text{floor}(x)</math> or <math>\text{int}(x)</math>. Not to be confused with the nearest integer function, as described below.)</i>	$[3] = 3$ , $[3.5] = 3$ , $[3.99] = 3$ , $[-3.7] = -4$
$[ ]$		nearest integer function nearest integer to numbers	$[x]$ means the nearest integer to $x$ . <i>(This may also be written <math>[x]</math>, <math>  x  </math>, <math>\text{nint}(x)</math> or <math>\text{Round}(x)</math>. Not to be confused with the floor function, as described above.)</i>	$[2] = 2$ , $[2.6] = 3$ , $[-3.4] = -3$ , $[4.49] = 4$

[ , ]	[ , ]	lverson bracket	1 if true, 0 otherwise	[S] maps a true statement S to 1 and a false statement S to 0.	[0=5]=0, [7>0]=1, [2 ∈ {2,3,4}]=1, [5 ∈ {2	
		propositional logic				
		image	image of ... under ... everywhere	f[X] means { f(x) : x ∈ X }, the image of the function f under the set X ⊆ dom(f). <i>(This may also be written as f(X) if there is no risk of confusing the image of funder X with the function application f of X. Another notation is Im f, the image of funder its domain.)</i>	sin[ℝ] = [-1, 1]	
		closed interval	closed interval	[a, b] = { x ∈ ℝ : a ≤ x ≤ b }.	0 and 1/2 are in the interval [0, 1].	
		order theory				
		commutator	the commutator of	[g, h] = g <sup>-1</sup> h <sup>-1</sup> gh (or ghg <sup>-1</sup> h <sup>-1</sup> ), if g, h ∈ G (a group).	x <sup>y</sup> = x[x, y] (group theory).	
		group theory, ring theory		[a, b] = ab - ba, if a, b ∈ R (a ring or commutative algebra).	[AB, C] = A[B, C] + [A, C]B (ring theory).	
		triple scalar product	the triple scalar product of	[a, b, c] = a × b · c, the scalar product of a × b with c.	[a, b, c] = [b, c, a] = [c, a, b].	
		vector calculus				
		( )	( )	function application	of	f(x) means the value of the function f at the element x.
set theory						
image	image of ... under ... everywhere			f(X) means { f(x) : x ∈ X }, the image of the function f under the set X ⊆ dom(f). <i>(This may also be written as f(X) if there is a risk of confusing the image of funder X with the function application f of X. Another notation is Im f, the image of funder its domain.)</i>	sin(ℝ) = [-1, 1]	
precedence grouping	parentheses			Perform the operations inside the parentheses first.	(8/4)/2 = 2/2 = 1, but 8/(4/2) = 8/2 = 4.	
tuple	tuple; n-tuple; ordered pair/triple/etc; row vector; sequence			An ordered list (or sequence, or horizontal vector, or row vector) of values. <i>(Note that the notation (a,b) is ambiguous: it could be an ordered pair or an open interval. Set theorists and computer scientists often use angle brackets ⟨ ⟩ instead of parentheses.)</i>	(a, b) is an ordered pair (or 2-tuple). (a, b, c) is an ordered triple (or 3-tuple). ( ) is the empty tuple (or 0-tuple).	
highest common factor	highest common factor; greatest common divisor; hcf; gcd			(a, b) means the highest common factor of a and b. <i>(This may also be written hcf(a, b) or gcd(a, b).)</i>	(3, 7) = 1 (they are coprime); (15, 25) = 5	
number theory						
( , )	( , )			open interval	(a, b) = { x ∈ ℝ : a < x < b }.	4 is not in the interval (4, 18). (0, +∞) equals the set of positive real num
open interval	order theory			<i>(Note that the notation (a,b) is ambiguous: it could be an ordered pair or an open interval. The notation ]a,b[ can be used instead.)</i>		
( , ]	( , ]			left-open interval	half-open interval; left-open interval	(a, b] = { x ∈ ℝ : a < x ≤ b }.
		order theory				
[ , )	[ , )	right-open interval	half-open interval; right-open interval	[a, b) = { x ∈ ℝ : a ≤ x < b }.	[4, 18) and [1, +∞)	
		order theory				
⟨ ⟩	⟨ ⟩	inner product	inner product of	(u,v) means the inner product of u and v, where u and v are members of an inner product space. <i>Note that the notation (u, v) may be ambiguous: it could mean the inner product or the linear span.</i>	The standard inner product between two (x, y) = 2 × -1 + 3 × 5 = 13	
		linear algebra		<i>There are many variants of the notation, such as ⟨u   v⟩ and ⟨u   v⟩, which are described below. For spatial vectors, the dot product notation, x·y is common. For matrices, the colon notation A : B may be used. As ( and ) can be hard to type, the more "keyboard friendly" forms &lt; and &gt; are sometimes seen. These are avoided in mathematical texts.</i>		
		average	average of	let S be a subset of N for example, ⟨S⟩ represents the average of all the element in S.	for a time series :g(t) (t = 1, 2,...) we can define the structure functions S <sub>q</sub> ( S <sub>q</sub> = ⟨ g(t + τ) - g(t)  <sup>q</sup> ⟩ <sub>t</sub>	
		statistics				
		linear span	(linear) span of; linear hull of	(S) means the span of S ⊆ V. That is, it is the intersection of all subspaces of V which contain S. (u <sub>1</sub> , u <sub>2</sub> , ...) is shorthand for ⟨{u <sub>1</sub> , u <sub>2</sub> , ...}⟩.	⟨(1/0), (0/1), (0/1)⟩ = ℝ <sup>3</sup> .	
		linear algebra		<i>Note that the notation (u, v) may be ambiguous: it could mean the inner product or the linear span.</i>		
				<i>The span of S may also be written as Sp(S).</i>		
		subaroud generated by a				

		set	$\langle S \rangle$ means the smallest subgroup of $G$ (where $S \subseteq G$ , a group) containing every element of $S$ .	In $S_3$ , $\langle (1\ 2) \rangle = \{id, (1\ 2)\}$ and $\langle \langle$
		the subgroup generated by		
		group theory	$\langle g_1, g_2, \dots \rangle$ is shorthand for $\langle g_1, g_2, \dots \rangle$ .	
		tuple		
		tuple; $n$ -tuple; ordered pair/triple/etc; row vector; sequence	An ordered list (or sequence, or horizontal vector, or row vector) of values. (The notation $(a,b)$ is often used as well.)	$\langle a, b \rangle$ is an ordered pair (or 2-tuple). $\langle a, b, c \rangle$ is an ordered triple (or 3-tuple). $\langle \rangle$ is the <b>empty tuple</b> (or 0-tuple).
		everywhere		
$\langle   \rangle$	$\langle   \rangle$	inner product	$(u   v)$ means the inner product of $u$ and $v$ , where $u$ and $v$ are members of an inner product space. <sup>[10]</sup> $(u   v)$ means the same.	
$(   )$	$(   )$	inner product of		
		linear algebra	Another variant of the notation is $(u, v)$ which is described above. For spatial vectors, the dot product notation, $x \cdot y$ is common. For matrices, the colon notation $A : B$ may be used. As $\langle$ and $\rangle$ can be hard to type, the more "keyboard friendly" forms $<$ and $>$ are sometimes seen. These are avoided in mathematical texts.	
$  \rangle$	$  \rangle$	ket vector		
		the ket ...; the vector ...	$ \varphi\rangle$ means the vector with label $\varphi$ , which is in a Hilbert space.	A qubit's state can be represented as $\alpha 0\rangle + \beta 1\rangle$ with $ \alpha ^2 +  \beta ^2 = 1$ .
		Dirac notation		
$\langle  $	$\langle  $	bra vector		
		the bra ...; the dual of ...	$\langle \varphi $ means the dual of the vector $ \varphi\rangle$ , a linear functional which maps a ket $ \psi\rangle$ onto the inner product $\langle \varphi \psi\rangle$ .	
		Dirac notation		
$\Sigma$	$\Sigma$	summation		
		sum over ... from ... to ... of	$\sum_{k=1}^n a_k$ means $a_1 + a_2 + \dots + a_n$ .	$\sum_{k=1}^4 k^2 = 1^2 + 2^2 + 3^2 + 4^2 ::=$
		arithmetic		
$\Pi$	$\Pi$	product		
		product over ... from ... to ... of	$\prod_{k=1}^n a_k$ means $a_1 a_2 \dots a_n$ .	$\prod_{k=1}^4 (k+2) = (1+2)(2+2)(3+2)$
		arithmetic		
		Cartesian product		
		the Cartesian product of; the direct product of	$\prod_{i=0}^n Y_i$ means the set of all $(n+1)$ -tuples $(Y_0, \dots, Y_n)$ .	$\prod_{n=1}^3 \mathbb{R} = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3$
		set theory		
$\sqcup$	$\sqcup$	coproduct		
		coproduct over ... from ... to ... of	A general construction which subsumes the disjoint union of sets and of topological spaces, the free product of groups, and the direct sum of modules and vector spaces. The coproduct of a family of objects is essentially the "least specific" object to which each object in the family admits a morphism.	
		category theory		
$\Delta$	$\Delta$	delta	$\Delta x$ means a (non-infinitesimal) change in $x$ .	
		delta; change in		
		calculus	(If the change becomes infinitesimal, $\delta$ and even $d$ are used instead. Not to be confused with the symmetric difference, written $\Delta$ , above.)	$\frac{\Delta y}{\Delta x}$ is the gradient of a straight line
		Laplacian		
		Laplace operator	The Laplace operator is a second order differential operator in $n$ -dimensional Euclidean space	If $f$ is a twice-differentiable real-valued $f$ by $\Delta f = \nabla^2 f = \nabla \cdot \nabla f$
		vector calculus		
$\delta$	$\delta$	Dirac delta function		
		Dirac delta of		
		hyperfunction	$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$	$\delta(x)$
		Kronecker delta		
		Kronecker delta of	$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$	$\delta_{ij}$
		hyperfunction		
		Functional derivative	$\left\langle \frac{\delta F[\varphi(x)]}{\delta \varphi(x)}, f(x) \right\rangle = \int \frac{\delta F[\varphi(x)]}{\delta \varphi(x')} f(x') dx'$	
		Functional derivative of	$= \lim_{\epsilon \rightarrow 0} \frac{F[\varphi(x) + \epsilon f(x)] - F[\varphi(x)]}{\epsilon}$	$\frac{\delta V(r)}{\delta \rho(r')} = \frac{1}{4\pi\epsilon_0  r - r' }$
		Differential operators	$= \frac{d}{d\epsilon} F[\varphi + \epsilon f] \Big _{\epsilon=0}$	
$\partial$	$\partial$	partial derivative		
		partial; d	$\partial f / \partial x_i$ means the partial derivative of $f$ with respect to $x_i$ , where $f$ is a function on $(x_1, \dots, x_n)$ .	If $f(x,y) := x^2y$ , then $\partial f / \partial x = 2xy$
		calculus		
		boundary		
		boundary of	$\partial M$ means the boundary of $M$	$\partial \{x :   x   \leq 2\} = \{x :   x   = 2\}$
		topology		
		degree of a polynomial		
		degree of	$\partial f$ means the degree of the polynomial $f$ .	$\partial(x^2 - 1) = 2$
		algebra	(This may also be written $\deg f$ .)	
		gradient		
		del; nabla; gradient of	$\nabla f(x_1, \dots, x_n)$ is the vector of partial derivatives $(\partial f / \partial x_1, \dots, \partial f / \partial x_n)$ .	If $f(x,y,z) := 3xy + z^2$ , then $\nabla f = (3y, 3x, 2z)$

∇	∇	vector calculus			
		divergence	del dot; divergence of	$\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	If $\vec{v} := 3xy\mathbf{i} + y^2z\mathbf{j} + 5\mathbf{k}$ , then $\nabla$
		curl	curl of	$\nabla \times \vec{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \mathbf{i}$ $+ \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \mathbf{j} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \mathbf{k}$	If $\vec{v} := 3xy\mathbf{i} + y^2z\mathbf{j} + 5\mathbf{k}$ , then $\nabla$
		derivative	... prime; derivative of	$f'(x)$ means the derivative of the function $f$ at the point $x$ , i.e., the slope of the tangent to $f$ at $x$ . <i>(The single-quote character ' is sometimes used instead, especially in ASCII text.)</i>	If $f(x) := x^2$ , then $f'(x) = 2x$
		derivative	... dot; time derivative of	$\dot{x}$ means the derivative of $x$ with respect to time. That is $\dot{x}(t) = \frac{\partial}{\partial t}x(t)$ .	If $x(t) := t^2$ , then $\dot{x}(t) = 2t$ .
∫	∫	indefinite integral or antiderivative	indefinite integral of the antiderivative of	$\int f(x) dx$ means a function whose derivative is $f$ .	$\int x^2 dx = x^3/3 + C$
		definite integral	integral from ... to ... of ... with respect to	$\int_a^b f(x) dx$ means the signed area between the $x$ -axis and the graph of the function $f$ between $x = a$ and $x = b$ .	$\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$
		line integral	line/ path/ curve/ integral of... along...	$\int_C f ds$ means the integral of $f$ along the curve $C$ , $\int_a^b f(\mathbf{r}(t)) \mathbf{r}'(t)  dt$ where $\mathbf{r}$ is a parametrization of $C$ . <i>(If the curve is closed, the symbol <math>\oint</math> may be used instead, as described below.)</i>	
∮	∮	Contour integral; closed line integral	contour integral of	Similar to the integral, but used to denote a single integration over a closed curve or loop. It is sometimes used in physics texts involving equations regarding Gauss's Law, and while these formulas involve a closed surface integral, the representations describe only the first integration of the volume over the enclosing surface. Instances where the latter requires simultaneous double integration, the symbol $\oint$ would be more appropriate. A third related symbol is the closed volume integral, denoted by the symbol $\iiint$ . The contour integral can also frequently be found with a subscript capital letter $C$ , $\oint_C$ , denoting that a closed loop integral is, in fact, around a contour $C$ , or sometimes dually appropriately, a circle $C$ . In representations of Gauss's Law, a subscript capital $S$ , $\oint_S$ , is used to denote that the integration is over a closed surface.	If $C$ is a Jordan curve about 0, then $\oint_C$ .
π	π	projection	Projection of	$\pi_{a_1, \dots, a_n}(R)$ restricts $R$ to the $\{a_1, \dots, a_n\}$ attribute set.	$\pi_{Age, Weight}(Person)$
		mathematical constant	pi; 3.1415926...; ≈22÷7	Used in various formulas involving circles; $\pi$ is equivalent to the amount of area a circle would take up in a square of equal width with an area of 4 square units, roughly 3.14/4. It is also the ratio of the circumference to the diameter of a circle.	$A = \pi R^2 = 314.16 \rightarrow R = 10$
σ	σ	selection	Selection of	The selection $\sigma_{a\theta b}(R)$ selects all those tuples in $R$ for which $\theta$ holds between the $a$ and the $b$ attribute. The selection $\sigma_{a\theta v}(R)$ selects all those tuples in $R$ for which $\theta$ holds between the $a$ attribute and the value $v$ .	$\sigma_{Age \geq 34}(Person)$ $\sigma_{Age = Weight}(Person)$
<	<	cover	is covered by	$x < y$ means that $x$ is covered by $y$ .	$\{1, 8\} < \{1, 3, 8\}$ among the subsets of $\{$
		subtype	is a subtype of	$T_1 < T_2$ means that $T_1$ is a subtype of $T_2$ .	If $S < T$ and $T < U$ then $S < U$ (transitive)
†	†	conjugate transpose	conjugate transpose; adjoint; Hermitian adjoint/conjugate/transpose	$A^\dagger$ means the transpose of the complex conjugate of $A$ . <sup>[11]</sup> <i>This may also be written <math>A^{*T}</math>, <math>A^{T*}</math>, <math>A^*</math>, <math>\overline{A^T}</math> or <math>\overline{A^*}</math>.</i>	If $A = (a_{ij})$ then $A^\dagger = (\overline{a_{ji}})$ .
T	T	transpose	transpose	$A^T$ means $A$ , but with its rows swapped for columns. <i>This may also be written <math>A^*</math>, <math>A^t</math> or <math>A^{tr}</math>.</i>	If $A = (a_{ij})$ then $A^T = (a_{ji})$ .
T	T	top element	the top element	$T$ means the largest element of a lattice.	$\forall x : x \vee T = T$
		top type	the top type; top	$T$ means the top or universal type; every type in the type system of interest is a subtype of top.	$\forall \text{ types } T, T < T$

⊥	⊥	perpendicular	x ⊥ y means x is perpendicular to y; or more generally x is <b>orthogonal</b> to y.	If l ⊥ m and m ⊥ n in the plane, then l    n
		is perpendicular to geometry		
		orthogonal complement	W <sup>⊥</sup> means the orthogonal complement of W (where W is a subspace of the <b>inner product space</b> V), the set of all vectors in V orthogonal to every vector in W.	Within $\mathbb{R}^3, (\mathbb{R}^2)^\perp \cong \mathbb{R}$ .
		orthogonal/ perpendicular complement of; perp		
		linear algebra		
		coprime	x ⊥ y means x has no factor greater than 1 in common with y.	34 ⊥ 55.
		is coprime to number theory		
		independent	A ⊥ B means A is an event whose probability is independent of event B.	If A ⊥ B, then P(A B) = P(A).
		is independent of probability		
		bottom element	⊥ means the smallest element of a lattice.	$\forall x : x \wedge \perp = \perp$
the bottom element lattice theory				
bottom type				
the bottom type; bot	⊥ means the bottom type (a.k.a. the zero type or empty type); bottom is the subtype of every type in the <b>type system</b> .	$\forall \text{ types } T, \perp <: T$		
type theory				
comparability	x ⊥ y means that x is comparable to y.	{e, π} ⊥ {1, 2, e, 3, π} under set contain		
is comparable to order theory				
⊨	⊨	entailment	A ⊨ B means the sentence A entails the sentence B, that is in every model in which A is true, B is also true.	A ⊨ A ∨ ¬A
		entails model theory		
⊢	⊢	inference	x ⊢ y means y is derivable from x.	A → B ⊢ ¬B → ¬A.
		infers; is derived from propositional logic, predicate logic		
		partition	p ⊢ n means that p is a partition of n.	$(4,3,1,1) \vdash 9, \sum_{\lambda \vdash n} (f_\lambda)^2 = n!$
⋮	⋮	vertical ellipsis	Denotes that certain constants and terms are missing out (e.g. for clarity) and that only the important terms are being listed.	P(r, t) = χ: E(r, t <sub>1</sub> )E(r, t <sub>2</sub> )E(r
		vertical ellipsis everywhere		
E	E	expected value	the value of a random variable one would "expect" to find if one could repeat the random variable process an infinite number of times and take the average of the values obtained	$\mathbb{E}[X] = \frac{x_1 p_1 + x_2 p_2 + \dots + x_k}{p_1 + p_2 + \dots + p_k}$
		expected value probability theory		