2. The axioms of set theory

Here, for ease of reference, is a list of the axioms of set theory, expressed informally. In Handout 3, I indicate how they can be expressed in the language of set theory.

The *Zermelo-Fraenkel axioms*, or ZF, are all the axioms except the Axiom of Choice. The entire list is known as ZFC.

Axiom of extensionality *Two sets are equal if and only if they have the same elements.*

Empty set axiom *The empty set* $\varnothing$ *exists.*

Axiom of Pairs *If* a *and* b *are sets, then so is* $\{a, b\}$.

Axiom of Unions *Suppose* A *is a set. Then so is the union* $\bigcup A$ *of its elements.*

Subset axiom scheme *Suppose* A *is a set and* $\phi(x)$ *is a statement in the language of set theory. Then*

$$\{x \in A : \phi(x)\}$$

is a set.

Foundation axiom *Suppose* A *is a non-empty set. Then* A *has an* $\in$*-minimal element; that is, there exists* $m \in A$ *such that* $m \cap A = \varnothing$.

Powerset axiom *Let* X *be a set. Then* $\wp X$ *is a set.*

Axiom of Infinity *There is a successor set.*

Replacement Axiom Scheme *Given a set* X*, and a rule which associates, with each element* x *of* X*, a unique set* $\Phi(x)$*,*

$$\{y : \exists x \in X \, y = \Phi(x)\}$$

is a set.

Axiom of Choice (AC) *Let* $\mathscr{A}$ *be a non-empty set of disjoint non-empty sets. Then there exists a set* B *such that for all* $A \in \mathscr{A}$, $|A \cap B| = 1$.