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WORLD'S #1 ACADEMIC OUTLINE

Quick Study® ACADEMIC

LOGIC

THE BASIC PRINCIPLES OF PROPOSITIONAL & SYLLOGISTIC LOGIC - PLUS QUANTIFICATION THEORY

PROPOSITIONAL LOGIC

BASIC ELEMENTS

EQUIVALENT NAMES

Propositional Calculus - Sentential Logic
Truth-Functional Logic - Algebra of Statements

SYMBOLS

- **Sentence Variables**: p, q, r, s,...
- **Sentence Abbreviations**: A, B, C,...
- **Punctuation Marks**: (parentheses), [brackets],{braces}
- **Symbols**: "~" not, "→" if, "**V**" or, "≡" if and only (IFF)
- **Operators**:
  1. <u>Monadic</u>:
     Negation: ~p, ~q, ~A, ~B,...
     Read as: "not-p"; "p is false"; "p is not true"
  2. <u>Dyadic</u>:
     a. **Conjunction**: p & q, A & B,...
        Read as "p and q"; "p and q are both true"; "p while q"; "p nevertheless q"; "p however q"; "p but q."
        Component parts = ***conjuncts***.
     b. **Disjunction**: p **V** q, A **V** B.... Read as: "p or q"; "Either p or q is true"; "p or q or both are true"; "p and/or q"; "p unless q"; "Unless p, q."
        Component parts = ***disjuncts***.
        NOTE: *The foregoing is a <u>weak</u> (**inclusive**) disjunction. If a <u>strong</u> (or **exclusive**) disjunction ("p or q but not both") requires symbolization, the weak disjunction need merely be conjoined to the denial of the truth of both disjuncts*: **[(p V q) & ~ (p & q)]**.
     c. **Conditional**: p → q, A → B,...
        Read: "If p, then q"; "p only if q"; "Only if q, p"; "Provided that p, q"; "On the condition that p, q."
        Sentence before "→" is the ***antecedent***.
        Sentence following "→" is the ***consequent***.
     d. **Biconditional**: p ≡ q, A ≡ B,...
        Read: "p if and only if q" (sometimes abbreviated as "p if f q") or "p triple bar q" (do *not* read as "p is identical to q"); "If and only if p, q"; "p when and only when q"; "p whenever q"; "p exactly when q."
     e. **Unconventional Translations**: "Neither p nor q" or "not either p or q" :: ~ (p V q) or this may also be symbolized as (~p & ~q). "If p then q" or "p only if q" :: p → q. Note, however, that "p if only q" :: q → p. "p is a sufficient condition for q" :: p → q. "p is a necessary condition for q" :: q → p.

Sufficient *conditions* = ***antecedent*** *of the conditional.*
Necessary *conditions* = ***consequent*** *of the conditional.*
'SUN' - ***S*** *= the sufficient condition*
U *= the conditional operator (→)*
N *= the necessary condition*

TRUTH-TABLE DEFINITIONS OF OPERATORS

COLUMNS

Truth Functions:	p	q	~p	p& q	p V q	p → q	p ≡ q
Lines:	T	T	F	T	T	T	T
	T	F	F	F	T	F	F
	F	T	T	F	T	T	F
	F	F	T	F	F	T	T

Note: ***Two truth values ("T" and "F") yield 16 truth functions. The above matrice details only five of these as they correspond to ordinary language expressions (i.e. not, and, or, etc.).***

WELL-FORMED FORMULA (wff)

- **p (q, r, s, t, ...) is a wff.**
- **If S is a wff, then ~S is a wff.**
- **If S1 and S2 are wffs, then "S1 & S2," "S1 VS2," "S1 → S2," "S1 ≡ S2" are wffs.**

TRUTH-TABLE METHOD

OUTLINE OF METHOD

- An **inference** (or sentence) is valid only if its symbolized wff yields a tautology on the final (defining) column of a truth-table. A sentence is consistent if its symbolized wff yields a tautology or a contingency on the final column of a truth-table. If the purpose for generating a truth-table is to determine validity, then there is no need to complete the table as soon as a single "F" is detected in its final (defining) column — since this automatically makes the symbolized inference invalid.
- **Tautology**: A wff yielding a "<u>T</u>" <u>on every line</u> of its final (defining) column of a truth-table.
- **Contingency**: A wff yielding <u>both Ts and Fs</u> on the final column of a truth-table.
- **Contradiction**: A wff yielding an "<u>F</u>" <u>on every line</u> of its final column of a truth-table.

INFERENCE: PREMISES & CONCLUSION:

*An **Inference** (or argument) is usually indicated by the presence of a premise-word or conclusion-word, or both:*

- **Premise-words**: since, because, for, for the reason that, etc.
- **Conclusion-words**: therefore, hence, thus, so, consequently, it follows that, ergo, etc.

Note: The conclusion of an inference, while always appearing as the final consequent of a conditional wff, is usually not the last sentence of an inference in ordinary language.

SHORTER TRUTH-TABLE METHOD

OUTLINE OF METHOD

- **Indirect Proof** (*reductio ad absurdum*): Assumes a wff is false. If presupposition leads to a contradiction, then the wff is necessarily true; inference is valid. If a truth-value assignment can be found that is consistent with the initial presupposition of falsity, then the wff is not necessarily true; inference is invalid.
- **Limitations to Method**: Not strictly effective (cannot be used in a unique mechanical procedure on all wffs). Conjunctions, biconditionals, negations of conditionals or disjunctions can be falsified in more than one way. Method is applicable to inference. The wff that symbolizes an inference is *always* a *conditional*, which is falsifiable in only one way: **The antecedent must be true while the consequent is false**. Only one consistent falsifying assignment of truth-values is required for determining invalidity.
- **Valid Inference**: (***Symbolized inference***)

```
[(p Vq) & ~p] → q          a. inference is falsified.
                F
[(p Vq) & ~p] → q          b. antecedent must be true and
       T       F F            the consequent false.
[(p V q) & ~p] → q         c. a true conjunction requires
   T    T   T  F F            all true conjuncts.
[(p V q) & ~p] → q         d. carry over falsity of "p" from
F T F  T  TF   F F            2nd premise and falsity of
                              "q" from the conclusion.
[(p V q) & ~p] → q         e. but "p" must also be true since
F T F  T TF    F F            a true disjunction requires
T                             at least one true disjunct.
X
```

NOTE: *As indicated by the "X," a contradiction has been derived. Thus, consistent falsification is impossible. Consequently, the inference may be pronounced **valid**.*

- Invalid Inference:

```
[(p V q) & p]→ ~q          No contradiction.
 T T T  T T   T  FT        Wff can be falsified.
 6 5 4  2 5   1  2 3       (numbered in order assigned).
```

TRUTH-TREE METHOD

OUTLINE OF METHOD

- **General**: Truth-trees use the **reductio ad absurdum** (or indirect) method of proof (see "Indirect Proof" under Shorter Truth Tables). The formula being tested is first denied (negated with the tilde). Each molecular formula is decomposed until the result is either an atomic proposition or its negation. If and only if the tree reveals a contradiction on every branch is the formula a tautology.
- **Setting up the Test**:
  1. **Formula for an Inference**:
     a. List each premise.
     b. Add the negation of the conclusion.
     c. Decompose every molecular formula, checking off each formula sequentially.
  2. **Any Well-Formed Formula**:
     a. Negate formula.
     b. Decompose every molecular formula. It will be noted that the procedure 2.a, above, simply speeds up the procedure for 2.b, since any inference form, when negated, will decompose into a list of premises with the negated conclusion added.
- **Rules of Decomposition**:
  1. **Rules:**

```
√ p & q        √ p V q        √ p → q        √ p ≡ q
    p            /  \           /  \           /  \
    q           p    q        ~p    q         p    ~p
                                              q    ~q

√(p & q)     √~ (p V q)    √ ~ ( p → q)   √ ~ (p ≡ q)
  /  \           ~p             p             /  \
~p    ~q         ~q            ~q            p    ~p
                                            ~q     q
```

  2. **Interpretation**:
     a. <u>The two basic operations</u> are **conjunction** and **disjunction**, represented on the tree respectively by a listing or a branching of sentences. Decomposition of all other truth-functions should be interpreted as either a listing or a branching operation.
     b. <u>The decomposition</u> of a "→" formula should be facilitated by reference to the rule of Implication: (p → q) ≡ ~ p **V** q. That is, the decomposition of an "→" sentence should branch or split into a **denial of the antecedent, and an affirmation of the consequent**.
     c. <u>DeMorgans Laws</u>: *Denial of a disjunction or conjunction*...Distribute the tilde through parentheses so as to append to atomic formulae eventually, listing and splitting accordingly.
     d. <u>Biconditional</u>: A biconditional statement is true if and only if both sides are true or (disjunction) if both sides are false. But a biconditional is false if and only if both sides have different truth values.
- **Growing The Tree**
  1. The order in which molecular formulae are decomposed is of no consequence to the **effectiveness** of the test; however, it is of consequence to the **complexity of the tree** generated. *Perform the listing operations first so as to minimize the number of branches generated.*
  2. The decomposition of a molecular sentence must be added to <u>every open branch of the tree</u>. <u>Closed branches</u> (branches blocked off by a contradiction) require no additional work. If and only if every branch is closed is the formula in question a tautology (or the inference symbolized valid).

EXAMPLES

- ***VALID*** **INFERENCE**:
  1. **Symbolized Inference**:
     [(R → E) &~E] → ~ R
     R → E
     ~E   /: . ~R
  2. **The Proof**:

```
√ R → E       All branches
   ~E         are closed. The
    R         conjunction of
   / \        premises with
 ~R   E       the negation of
  X   X       the conclusion
              is contradictory.
```

- ***INVALID*** **INFERENCE**:
  1. **Symbolized Inference**:
     [(E**V**~R) & ~R] →~E
     E **V** ~R
     ~R   /: . ~ E
  2. **The Proof**:

```
√E V ~R       At least one open
   ~R         branch remains.
    E         Conjunction of
   / \        premises with the
  E   R       negation of the con-
              clusion is not con-
              tradictory. The infer-
              ence is invalid.
```

PROOFS BY NATURAL DEDUCTION

THE BASIC METHOD

A proof by natural deduction of an inference requires that we list and number the premises and append the conclusion to be deduced. Next, in a series of consecutive numbered steps, each justified by reference to a previous step or steps and to a rule of derivation or a rule of replacement, consequences of the premises are deduced until the conclusion is established.

RULES OF INFERENCE

- **Rules of Derivation**:
  1. **Modus Ponens (MP)**: From p → q and p, derive q.
  2. **Modus Tollens (MT)**: From p → q and ~q, derive ~p.
  3. **Disjunctive Syllogism (DS)**: From pVq and ~p, derive q.
  4. **Hypothetical Syllogism (HS)**: From p → q and q → r, derive p → r.
  5. **Conjunction (Conj.)**: From p and q, derive p & q.
  6. **Simplification (Simp.)**: From p & q, derive p.
  7. **Dilemma (Dil.)**: From p → q, r → s and p V r, derive q V s.
  8. **Addition* (Add.)**: From p, derive p V q.

     ***NOTE ON ADDITION**: The addition rule may seem somewhat counter-intuitive insofar as the above shows, it seems that via disjunction we can add any arbitrary wff signified by q. The rationale for the justification of this rule may be found by consideration of the truth-table definition for disjunction (see "Truth-Table Definitions of Operators"). As this definition reveals, a disjunction is only false if both disjuncts are false; hence, if p is true, then p V q must be true — no matter what q signifies. Since even if q is false, the entire disjunction will remain true, given the previous assertion of the truth of p.

     NOTE: Rules of derivation may only be applied to entire lines (or steps) of a proof. (E.g., when using MP (Rule 1 above), the p → q must be the form of the entire numbered step, and the p must be another separate step. In other words, if a wff of the form "p → q" is only part of the wff at some step in a proof, it cannot be used as one of the premises of an MP derivation.)

RULES OF REPLACEMENT (SUBSTITUTION)

- **Commutation (Comm.):** Replace q & p with p & q, or vice versa. Replace p V q with q V p, or vice versa.*
- **Association (Assoc.)**: **Replace p & (q & r) with (p & q) & r, or vice versa. Replace p V (q V r) with (p V q) V r, *or vice versa. Let the "vice versa" clause be understood to apply to all remaining rules of replacement.
- **Tautology (Taut.)**: Replace p & p with p. Replace p V p with p.
- **Distribution (Dist.)**: Replace p & (q V r) with (p & q) V (p & r). Replace p V (q & r) with (p V q) & (p V r).
- **Double Negation (DN)**: Replace p with ~~p.*
- **Transposition (Trans.)**: Replace p → q with~q → ~p.
- **Implication (Imp.)**: Replace p → q with ~p V q.
- **DeMorgan's Rules (DM)**: Replace ~(p & q) with ~p V ~q. Replace ~(p V q) with ~p & ~q.
- **Equivalence (Equ.)**: Replace p≡q with (p → q) & (q → p).
- **Exportation (Exp.)**: Replace (p & q) → r with p → (q → r).

NOTE: Rules of replacement are logically stronger than rules of derivation. Since rules of replacement all involve equivalences, the "whole-step" restriction for rules of derivation no longer applies. (E.g., if "p → q" appears any place in a proof, either as an entire step or as part of a step, we may replace it with "~q → ~p" by the rule of transposition.)

FORMAL PROOFS

- **Direct Proof:**
  1. P1
  2. P2 / ∴ C
  3. deduced WFF (line number) (deduction rule)

     .
     .

  n. QED (line number) (deduction rule)
- **Conditional Proof:**
  1. P!
  2. P2 / ∴ F → G
  3. F (provisionally assumed) / ∴ G

     .
     .

  4. deduced WFF (line number) (deduction rule)

     .

  n. G (line number) (deduction rule)

  n + 1. F → G (3 - n), CP
- **Indirect Proof:**
  1. P1
  2. P2 / ∴ C
  3. - C
  4. deduced WFF (line number) (deduction rule)

     .
     .

  n. F & -F (derive a contradiction)

  n + 1. QED (3-n), IP

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PREDICATE LOGIC

QUANTIFICATION THEORY

SYMBOL NOTATION

- **Predicate letters**: A, B, C,...
- **Predicate variables**: F, G, H,...
- **Individual variables**: Z, Y, Z,... (Use super- or subscripts to indicate further individual variables).
- **Individual (Singular) terms**: d thru s.
- **Dummy ("John Doe") terms**: a,b,c,...
- **Either a dummy or a singular term**: t
- **Sentence variables**: p,q,r,...
- **Truth-functional operators**: ~, &, V, →, ≡
- **Universal quantifier**: (x), (y),...(Read: "For all x,...")
- **Existential quantifier**: (∃x),(∃y),..(Read: "There is an x, such that...")
- **Open sentences**: Fx,Gx,Gy,...(Read: "x has the property F," etc.)
- **Closed sentences**: (x)Fx, (∃y)(Gy), (x)(Fx → Gx),... (Read: "For all x, x has the property F," "Something has the property G," "All F are G," etc.)
- **Singular sentences**: Fd, Ge,...(Read(for example): "Dorothy is famous," "Edgar is great," etc.)

RULES OF INFERENCE

- **Truth-Functional**: (as per previous description, see Truth-Tree Method for Propositional Logic on page 1)
  1. **Rules of Derivation**
  2. **Rules of Replacement**
- **Quantifier**:
  1. **Universal Instantiation (UI)**: (x)Fx instatiate Ft
  2. **Existential Instantiation (EI)**: (∃x)Fx instatiate Fa (provided that 'a' has **not** been used in a previous step of the proof).
  3. **Existential Generalization (EG)**: Ft generalize to (∃x)Fx
  4. **Universal Generalization (UG)**: Fa generalize to (x)Fx (provided that 'a' appears in **no** step introduced by EI).
  5. **Change of Quantifier (CQ)**: -(x)Fx change to (∃x)-Fx-(∃x)Fx change to (x)-Fx

SYMBOLIZING STANDARD FORM SENTENCES

(Note: See "Introduction to Syllogistic Logic" [sections II & III] for translational equivalencies for nonstandard form sentences.)

- **Universal Affirmative Sentences: ("A" Sentences)**

  Standard Form: "All F are G": (x)(Fx → Gx)
- **Universal Negative Sentences: ("E" Sentences)**

  Standard Form: "No F are G": (x)(Fx → ~ Gx) equivalently: ~ (∃x)(Fx & Gx)
- **Particular Affirmative Sentences: ("I" Sentences)**

  Standard Form: "Some F are G": (∃x)(Fx & Gx)
- **Particular Negative Sentences: ("O" Sentences)**

  Standard Form: "Some F are not G": (∃x)(Fx & ~Gx)
- **Singular Sentences**:

  Standard Form: Ge (for: "Edgar" is great; i.e., a specific individual is named).
- **Conjunctive "A" Sentences: ("Class-identity" Sentences)**

  Standard Form: "All F are G and all G are F": (x)(Fx ≡ Gx) equivalently: (x)[(Fx → Gx) & (Gx → Fx)]
- **Conjunctive "I-O" Sentences**:

  Standard Form: "Some F are G and some F are not G": (∃x)[(Fx & Gx) & (Fx & ~ Gx)] equivalently: (∃x)(∃y)[(Fx & Gx) & (Fy & ~ Gy)]
- **Polyadic Predicate Sentences: (Relational Sentences)**

  "All F are like all G": (x)[Fx → (y)(Gy → Lxy)] (wherein "Lxy" designates "x is like y")

  "All F are like some G": (x)[Fx → (∃y)(Gy & Lxy)]

"Kelly can fool all the people some of the time, and some of the people all of the time, but she cannot fool all the people all of the time."

Let: k = Kelly; Ty = y is a time; Px = x is a person; Fkxy = Kelly fools x at y. (x)[Px → (∃y)(Ty & Fkxy)] & (∃x)[Px & (y)(Ty → Fkxy)] & ~(x)[Px → (y)(Ty → Fkxy)]

TRUTH-TREE METHOD FOR QUANTIFICATION THEORY

RULES OF DECOMPOSITION

It is herewith presupposed that the "truth-tree" method for propositional logic has already been mastered. The rules to follow supplement the rules of propositional logic and are all that are required to augment the method to predicate logic.

1. *(x) Fx

   Ft

   "t" = every dummy variable or singular term, and only the variables and singular terms appearing on the tree. If there are no such terms, introduce one. Sentence is starred at left, showing that it can be decomposed any number of times.
2. √(∃x) Fx

   Fa

   "a" = a new variable, not previously used on the branch leading back to "(∃x)(Fx)." Sentence is checked, indicating that it cannot be decomposed again.
3. √ ~ (x) Fx

   (∃x) ~ Fx
4. √ ~ (∃x) Fx

   (x) ~ Fx

   Rules 3 and 4 are the familiar CQ rules, and only serve to reduce the task of decomposition to rules 1 and 2.

DEFINITIONS

- **Grown Branch**: A pathway (branch) on which every compound sentence is completely decomposed, leaving only literals. Note that a branch/pathway must always be traced straight back to its previous assumptions. No turning away at a split in the branch is allowed.
- **Literals**: Simple atomic sentences or negations thereof.
- **Grown Tree**: A tree with all grown branches.
- **Dead Branch**: A grown branch displaying a sentence and its negation. Such a branch (unlike its purely truth-functional relative) must be killed at once with an x-mark; else, spuriously perpetual trees may result.
- **Living Tree**: A tree on which at least one branch is not dead.
- **Dead Tree**: An inconsistent tree on which all branches are dead.
- **Mortal Tree**: A tree such that, after application of the decomposition rules, either dies or at least is grown.
- **Perpetual Tree**: A tree containing at least one branch that never stops growing. This complication, which did not threaten a purely truth-functional tree, is introduced by decomposition rule number 1 above.
- **Valid Sentence**: A sentence which, when negated and decomposed, results in a dead tree.
- **Invalid Sentence**: A sentence which, when negated and decomposed, reveals a living tree.

INTERPRETATIONS

- **All branches are dead**: No problem. Sentence being tested is invalid; hence, if originally negated, then, by *reductio ad absurdum*, is valid.
- **At least one grown branch remains alive**: Again, no problem. Sentences being tested are consistent; hence, if sentences were originally negated, then, by *reductio ad absurdum*, they are consistent and invalid. The respective invalidating truth-value assignments can be read off the living branch.
- **No branch fully grows**: Due to endless decomposition by Rule 1 ∴
  1. **Case 1** ∴ But such endless decomposition is obviously pointless and at least one non-grown branch will yield a truth-value assignment for a mortal tree. Sentence is therefore consistent, and, if originally negated, is invalid.
  2. **Case 2** ∴ Decomposition is endless because it produces variously sized, looped, or circular, branches. Sentence, nevertheless, is consistent although this cannot be demonstrated. Nor, of course, can a negated sentence in this case be demonstrated invalid. (For further details concerning this problem of "undecidability," see Church's 1936 Theorem.)

***Note**: In summation, we may deem the polyadic (i.e. relational) predicate logic to be not (fully) decidable, in that there is no effective procedure for determining the validity or invalidity of every putative valid argument. Notwithstanding, the monadic predicate logic is decidable.*

QuickStudy

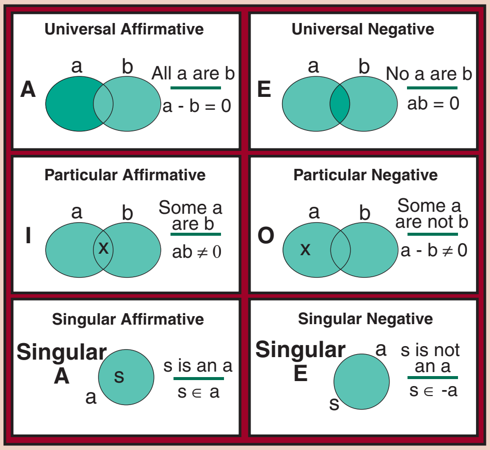
SYLLOGISTIC LOGIC

FORMS OF STATEMENTS

SYMBOLS

- **Class-variables**: {a,b,c,...}
- **Class-abbreviations**: {A,B,C,...}
- **Class product (or "intersect")**: AB, ab, a(b)...
- **Class sum (or "union")**: a + b, A + B...
- **Complementary class**: -a, -b, -A, -B,...
- **Universe class**: U (U = df. a + -a)
- **Null class**: 0 (o = df. a-a)
- **Class identity**: a = b, A = B,...
- **Class non-identity**: a ≠ b, a ≠ 0,...
- **Singular membership (epsilon "∈")**: s ∈ a, s ∈ B,...

STANDARD FORM SENTENCES
With Boolean Symbolism & Venn Diagrams



NON-STANDARD FORM STATEMENTS

Exemplar: "All logicians are shy." LS= 0

- **"A" Statements**: LS= 0
  1. Every (the, a, any, each) logician is shy.
  2. Logicians are shy. 3. If he/she is a logician, then he/she is shy.
- **Exclusive Statements**: ("A" statements with terms reversed) SL=0
  1. Only logicians are shy (i.e. all shy persons are logicians).
  2. Only if he/she is a logician is he/she shy.
  3. None but logicians are shy.
  4. Logicians alone are shy.
- **Exceptive Statements**: ("A" statements with negative subject terms and usually conjoined with an "E" statement.)
  1. All except logicians are shy (i.e. all non-logicians are shy and (usually) no logicians are shy: LS= 0 or: LS=0 & LS=0).
  2. Only logicians are not shy.
  3. Logicians alone are not shy.
- **"E" Statements**: No logicians are shy. LS = 0
  1. Logicians are not shy.
  2. Not a single logician is shy.
- **"I" Statements**: Some logicians are shy. LS ≠ 0
  1. At least one logician is shy.
  2. Several logicians are shy.
  3. Many logicians are shy.
  4. Most logicians are shy.
- **"O" Statements**: Some logicians are not shy. LS ≠ 0
  1. Not all logicians are shy.
  2. Logicians are not all shy.
  3. Not every logician is shy.
  4. At least one (many, most, several) logician is not shy.
- **Conjunctive "A" Statements**: (Class identity) LS= 0 & LS= 0
  1. All logicians are shy and all shy persons are logicians.
  2. All logicians and only logicians are shy.
  3. Logicians and logicians alone are shy.
  4. He/she is a logician if and only if he/she is shy.
  5. "Shy" is synonymous with "logician."
- **Conjunctive "I & O" Statements**: LS ≠ 0 & LS ≠ 0
  1. Some logicians are, but some are not, shy.
  2. Some logicians are shy, but some are not.

  3. Almost all logicians are shy.
  4. Not quite all logicians are shy.
  5. All but a few logicians are shy.
  6. The majority of logicians are shy.
  7. About half (one-quarter, one-third, etc.) of logicians are shy.
- **Singular Statements**:
  1. Albert is a logician. s ∈ L
  2. Lester is not a logician. s ∈ L
  3. Tom is a shy logician. s ∈ SL
- **Existential Universal Statements**:
  1. **Existential "A" Statements**: All logicians are shy (and there are logicians; i.e. logicians do exist). LS= 0 & L ≠ 0 * applies to all variants of "A" statements.
  2. **Existential "E" Statements**: No logicians are shy (and logicians and/or shy people exist). LS = 0 & L ≠ 0 S ≠ 0

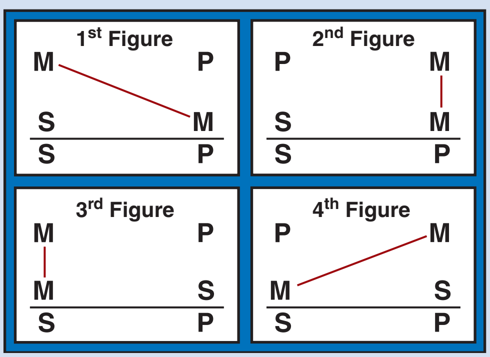
SYLLOGISTIC REASONING

REQUIREMENTS OF A SYLLOGISM

- **Syllogism**: Must be composed of exactly three standard form sentences, two of which are premises and the third a conclusion.
- **Major (first) premise** = predicate term of the conclusion.
  Minor (second) premise = subject term of the conclusion...*minor-term of the syllogism.*
  Conclusion + Major premise = *major term* of the syllogism.
- A syllogism may contain only one term, the middle term, in addition to the major or minor terms. The middle term is the term common to the two premises.
- Summary: It follows as a consequence of the above that an argument is a syllogism if and only if, it contains:
  1. **3 standard form sentences**: 2 premises and 1 conclusion;
  2. **3 terms**: The major, minor, and middle terms, each appearing twice in the argument, and never twice in the same sentence.

IDENTIFICATION OF SYLLOGISMS

- If singular and existential universal sentences are treated like hypothetical universals, then there are only 256 possible syllogisms (of which only 24 are valid in classical syllogistic logic), each of which is identified according to its **mood** and **figure**.
- The **mood** of a syllogism is signified by listing in sequential order the standard form abbreviations of the major premise, minor premise, and conclusion — respectively. For instance, the mood of the argument: "No logicians are shy and some auto mechanics are logicians, therefore some auto mechanics are not shy" is "EIO."
- The **figure** of the syllogism refers to the location in the premises of the middle term. Since the middle term can be the subject or predicate of either the major or minor premise, there are four possible figures, easily learned by studying the following diagrams. In each case, the horizontal line stands for the relationship "therefore" between premises and conclusion.



Thus, the complete identification of any syllogism is given by specifying the mood and figure. For instace, the syllogism spelled out in "2" above is "EIO-1" (i.e. the mood is EIO in the first figure).

DISTRIBUTION OF TERMS

- A term in a standard form sentence is said to be distributed when it refers to every member of the class denoted. Otherwise, it is undistributed.
  A: All a (dist.) are b (undist.)
  E: No a (dist.) are b (dist.)
  I: Some a (undist.) are b (undist.)
  O: Some a (undist.) are not b (dist.)
  As: s (dist.) is an a (undist.)
  Es: s (dist.) is not an a (dist.)

TRADITIONAL RULES OF THE SYLLOGISM

- **Mood Rules**:
  1. If the conclusion is negative, one premise must be negative. Otherwise, "**the fallacy of negative conclusion from affirmative premises**" is committed.
  2. At least one premise must be affirmative. Otherwise, "**the fallacy of two negative premises**" is committed.
- **Distribution Rules**:
  1. The middle term must be distributed in at least one of the premises. Otherwise, "**the fallacy of the undistributed middle term**" is committed.
  2. If the minor or major term is distributed in the conclusion, it must also be distributed in the respective minor or major premise, and vice versa. Otherwise, the "**fallacy of the illicit process of the minor (or major) term**" (or more succinctly: "illicit minor" or "illicit major") is committed.

VENN DIAGRAM TECHNIQUE FOR TESTING SYLLOGISMS

- Count the number of class terms in the syllogism and draw the corresponding number of circles on a Venn diagram. Enter the information from the premises (and only the premises) on the diagram. If and only if the diagram then displays the conclusion is the argument valid.

***NOTE:** If both premises of the syllogism are universal, while the conclusion is particular, then the premises must be (ordinarily) assumed to be existential, otherwise every such argument would be invalid.*

DISTINGUISHING PREMISES & CONCLUSION

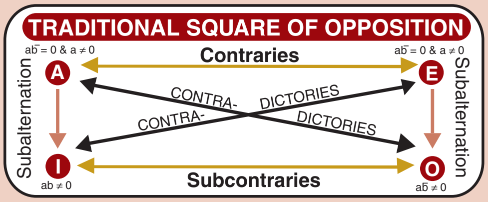
- Syllogistic arguments in ordinary language **do not come in "logical" order**, that is, first the major premise, then the minor premise, and, finally, the conclusion. More often, the conclusion is asserted first, followed by the premises. Usually, this is the case when the arguer asserts a sentence, and then feels impelled to furnish reasons (premises) for that assertion. Or, the conclusion may be asserted after one premise and before another. In the absence of an arguer to whom we might direct the question: "What is it that you are trying to prove?", we must observe that:
  1. **Conclusion words**: Prefix the sentence being proved; (e.g., ***therefore, consequently, hence, thus, so, it follows that, ergo, etc.***)
  2. **Premise words**: Prefix the reason or reasons given for the sentence being proved: (e.g., ***since, because, for, for the reason that, etc.***)
  3. **Conjunction words**: Signify a parallel status as compound premises or conclusions: (e.g., ***and, while, but, also, in addition, at the same time, moreover, furthermore, likewise, etc.***) For example, if a known premise is jointly asserted by one of the above conjunction words with another sentence, we know that this last is also a premise.
- **Summary**:

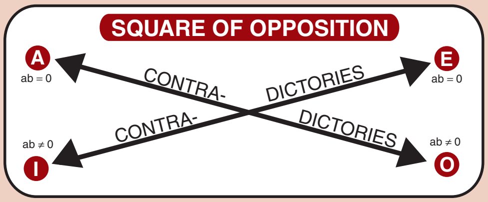
Conclusion words - conclusion follows,
- preceding sentences are premises.

Premise words - Premise follows
- preceding sentence, if any, is a conclusion.

QuickStudy

IMMEDIATE INFERENCES BASED ON CLASSICAL SQUARE OF OPPOSITION





- **If the universal sentences (A & E) are existential, then:**
  1. **Contradictories**: (A & O, E & I) always have opposite truth values.
  2. **Contraries**: (A & E) both may be false, but both cannot be true.
  3. **Subcontraries**: (I & O) both may be true, but both cannot be false.
  4. **Subalternation**: Is valid, i.e. if the A is true, so is the I, and if the E is true, so is the O. Superalternation, i.e. reasoning from the I to the A, or from the O to the E, is invalid. (Note: Subalternation and superalternation are sometimes referred to as *subimplication* and *superimplication*, respectively.)
- **If the universal sentences (A & E)** are hypothetical in that (unlike in classical syllogistic logic) we do not presuppose existential class membership, then:
  1. Contradiction still holds, but
  2. Contriety fails,
  3. Subcontriety fails, and
  4. Subalternation fails.

CONVERSION

- Conversion is an immediate (reciprocal) inference in which the subject and predicate terms are transposed. The quantity and quality of the standard form sentences remains the same.
  1. **"E" sentences**: "No *a* are *b*" converts to "No *b* are *a*."
  2. **"I" sentences**: "Some *a* are *b*" converts to "Some *b* are *a*."
- Conversion is invalid for "A," "O," and singular sentences. (Note: The converse of a singular sentence is not even meaningful, and its symbolization is said to be not well-formed or ill-formed formula.)

OBVERSION

- Obversion is an immediate (reciprocal) inference that is valid for all standard form sentences. The obverse of any standard form sentence can be obtained by changing the quality of the sentence and negating the predicate term.
  1. **"A" sentences**:
     "All *a* are *b*" obverts to "No *a* are non-*b*."
  2. **"E" sentences**:
     "No *a* are *b*" obverts to "All *a* are non-*b*."
  3. **"I" sentences**:
     "Some *a* are *b*" obverts to "Some *a* are not non-*b*" (i.e. the sentence form is preserved).
  4. **"O" sentences**:
     "Some *a* are not *b*" obverts to "Some *a* are non-*b*" (Note: "Not-*a*" is class exclusion, whereas "non-*a*" is membership exclusion.)
  5. **"As" sentences**:
     "*s* is an *a*" obverts to "*s* is not a non-*a*."
  6. **"Es" sentences**:
     "*s* is not an *a*" obverts to "*s* is a non-*a*."

CONTRAPOSITION

- Contraposition is an immediate (reciprocal) inference that permits transposing the subject and predicate terms of "A" and "O" sentences (cf., "conversion" above). However, in transposing the terms of "A" and "O" sentences, the two terms must be negated.
  1. **"A" sentences**:
     "All *a* are *b*" contraposes to "All non-*b* are non-*a*."
  2. **"O" sentences**:
     "Some *a* are not *b*" contraposes to "Some non-*b* are not-*a*."
- Just as the "A" and "O" sentences do not validly convert, so the "E" and "I" sentences do not validly contrapose.

ENTHYMEMES

TYPES OF ENTHYMEMES

- **1st Order**: Major premise suppressed
- **2nd Order**: Minor premise suppressed
- **3rd Order**: Conclusion suppressed

SUPPLYING A SUPRESSED PREMISE

- **Basic Requirements**:
  Always make the argument valid. A suppressed premise is an implicit presupposition of the argument. What is presupposed is determined and formulated by the listener. And the principle of charity in logic would clearly demand that we make the argument valid, if at all possible. But, as it turns out, is it always possible to make the argument valid, given sufficient ingenuity (and charity?)
- **Satisfying Mood Requirements**:
  Refer to syllogistic mood rules. If the conclusion is negative, for example, and the given premise is affirmative, clearly the missing premise must be negative. And if the conclusion is affirmative, the supplied premise must also be affirmative. Knowledge of Venn diagrams will reveal that a universal conclusion demands a universal premise, while a particular conclusion demands a particular premise.
- **Satisfying Distribution Requirements**:
  Refer to syllogistic distribution rules. If, for example, a universal affirmative ("A") premise must be supplied, choose the form ("All a are b" or "All b are a") which will avoid introducing an undistributed middle term.
- **Desperation Tactics**:
  If none of the preceding steps prove successful in making the argument valid, **draw a Venn diagram** in which the given premise or premises are entered. What other standard form sentence will establish the conclusion? If no single standard form sentence will do the job, will two supplied standard form sentences work? If not, will one non-standard form sentence, in conjunction with the given premise or premises, yield the required conclusion? Make the argument valid, even at the expense of sacrificing the form of a syllogism. The logical principle of charity is often times more important than adhering strictly to syllogistic form.

SORITES

DEFINITION

A sorites is an argument composed of three or more premises, each of which is in standard form, and a single conclusion in standard form. In general, a sorites is a concatenated argument which can be decomposed into a series of component syllogisms.

NUMBER OF COMPONENT SYLLOGISMS & ENTHYMEMES

- **Component Syllogisms**: An argument with three premises will decompose into two syllogisms, a sorites with four premises will decompose into three component syllogisms, etc.; in general, the number of component syllogisms contained in a sorites will be one fewer than the number of premises in the sorites.
- **Suppressed Premises**: Since each component syllogism in a sorites, except the last, will require that a suppressed premise be supplied, the number of suppressed premises in a sorites will equal the number of component syllogisms minus one (i.e. in view of II.A above, the number of premises minus two).

TESTING SORITES FOR VALIDITY

In order to test a sorites for validity (i.e. completeness), the argument must be decomposed into component syllogisms. An easy method for this is the following:

- Find the conclusion for the entire sorites.
- Find the minor premise as determined by the subject term of the conclusion.
- Construct and supply the major premise, using the same techniques used in completing enthymemes.
- Using the premise derived in step 3 as a new ancillary conclusion, find its required minor premise and supply a validating major premise (via step 3).
- Continue constructing component syllogisms until only two premises remain.
- The remaining two premises should establish as a conclusion the last supplied ancillary conclusion.
- Test each syllogism by mood and distribution rules, and demonstrate validity by a Venn diagram for each component syllogism.

IMPORTANT NOTE*: In those cases wherein a component syllogism is invalid, though it may be validated by supplying additional validating premises, nonetheless, it is the supplied enthymatic premise that is utilized for the subsequent ancillary conclusion. The reason for the preceding is that the enthymatic premise that is derived from the sorites is implicitly contained in it, whereas an extraneous validating premise is not; hence, it is the information implicitly contained in the sorites that must be utilized to produce its ancillary conclusions.*

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NOTE

This ***QUICKSTUDY***® guide is an **outline of the major topics** taught in introductory Logic courses. Due to its condensed format, use it as a study guide, but not as a replacement for assigned class work.

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